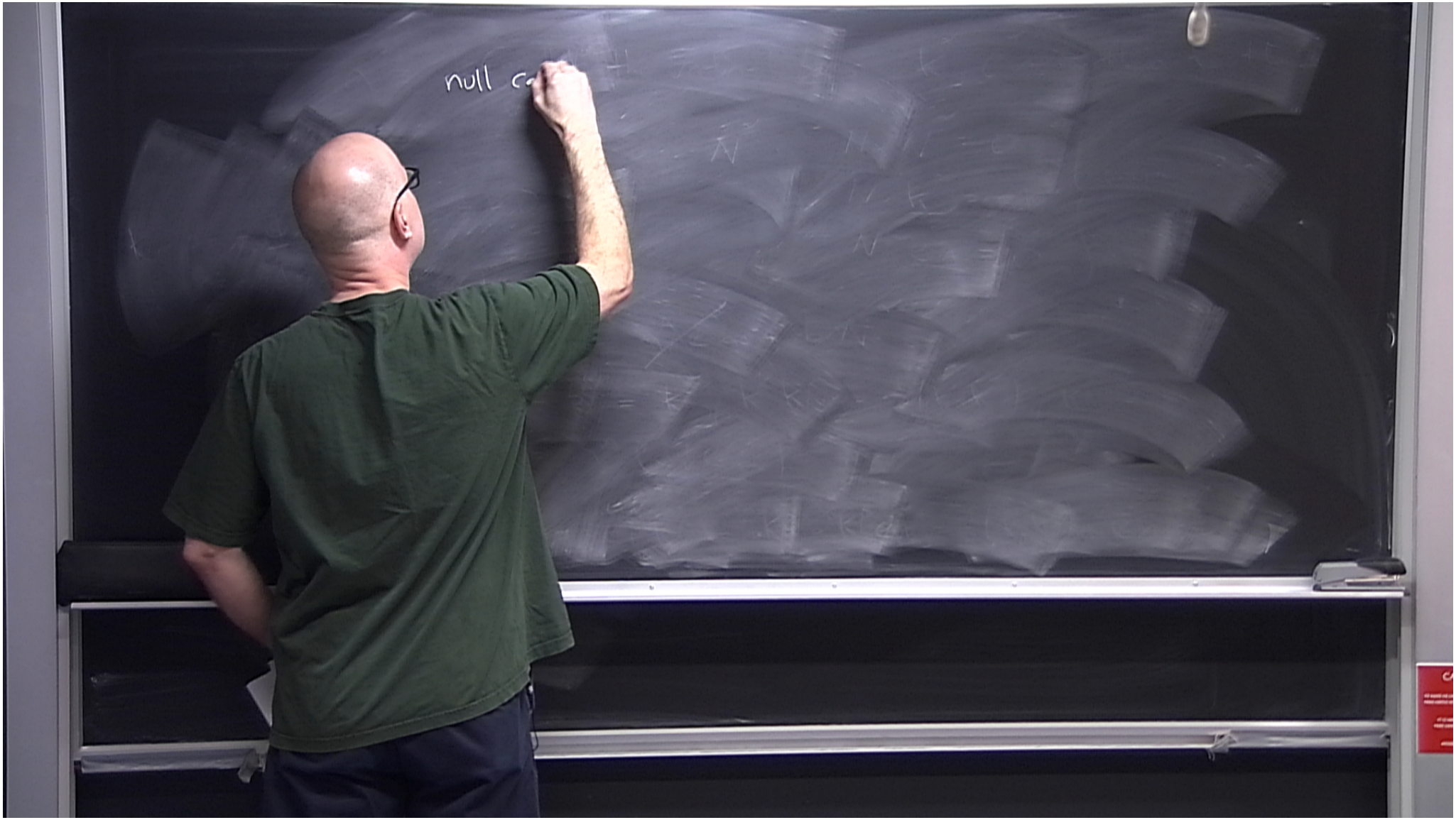
Title: Advanced General Relativity - Lecture 10

Date: Feb 08, 2012 03:30 PM

URL: http://pirsa.org/12020112

Abstract: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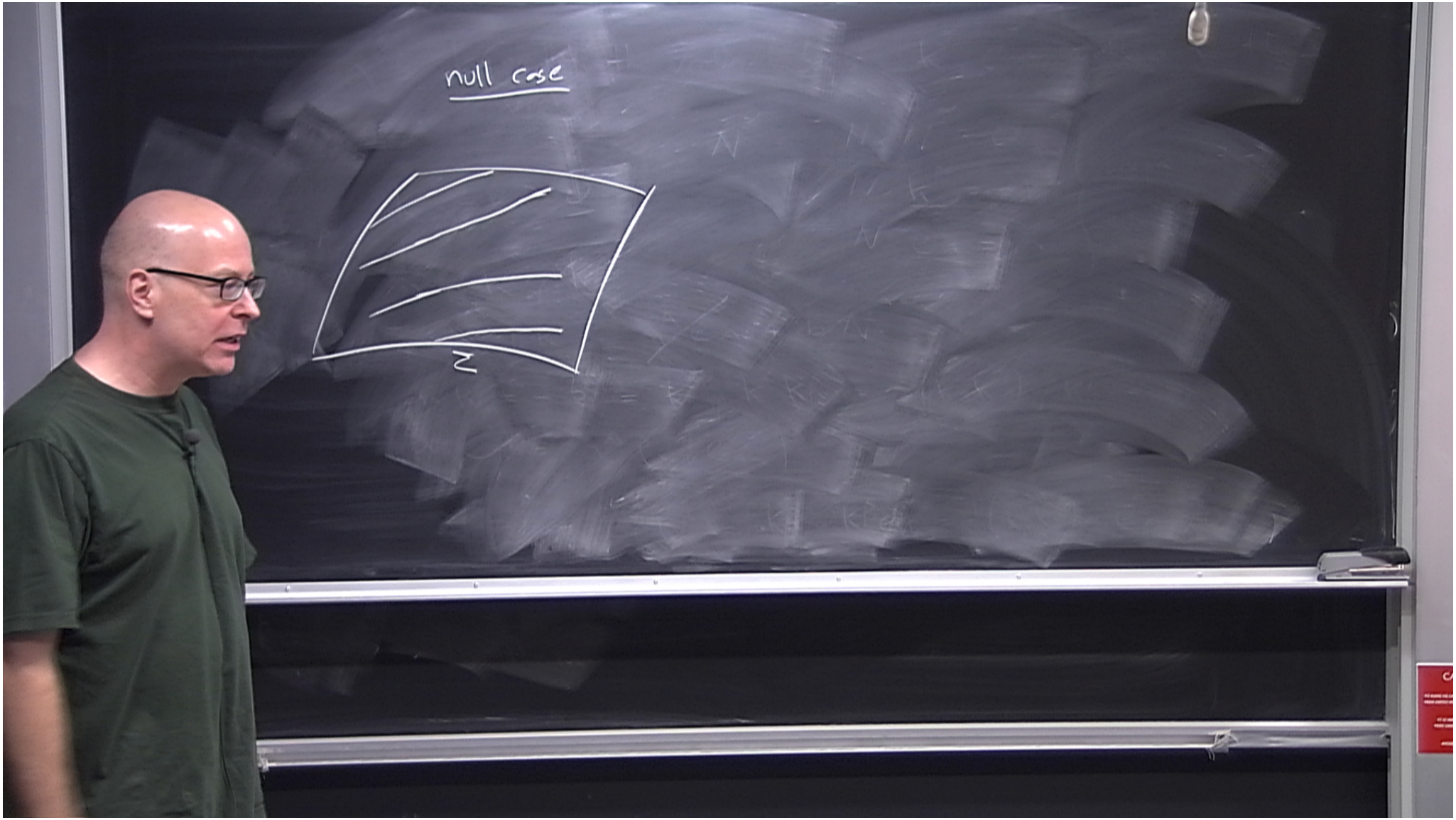
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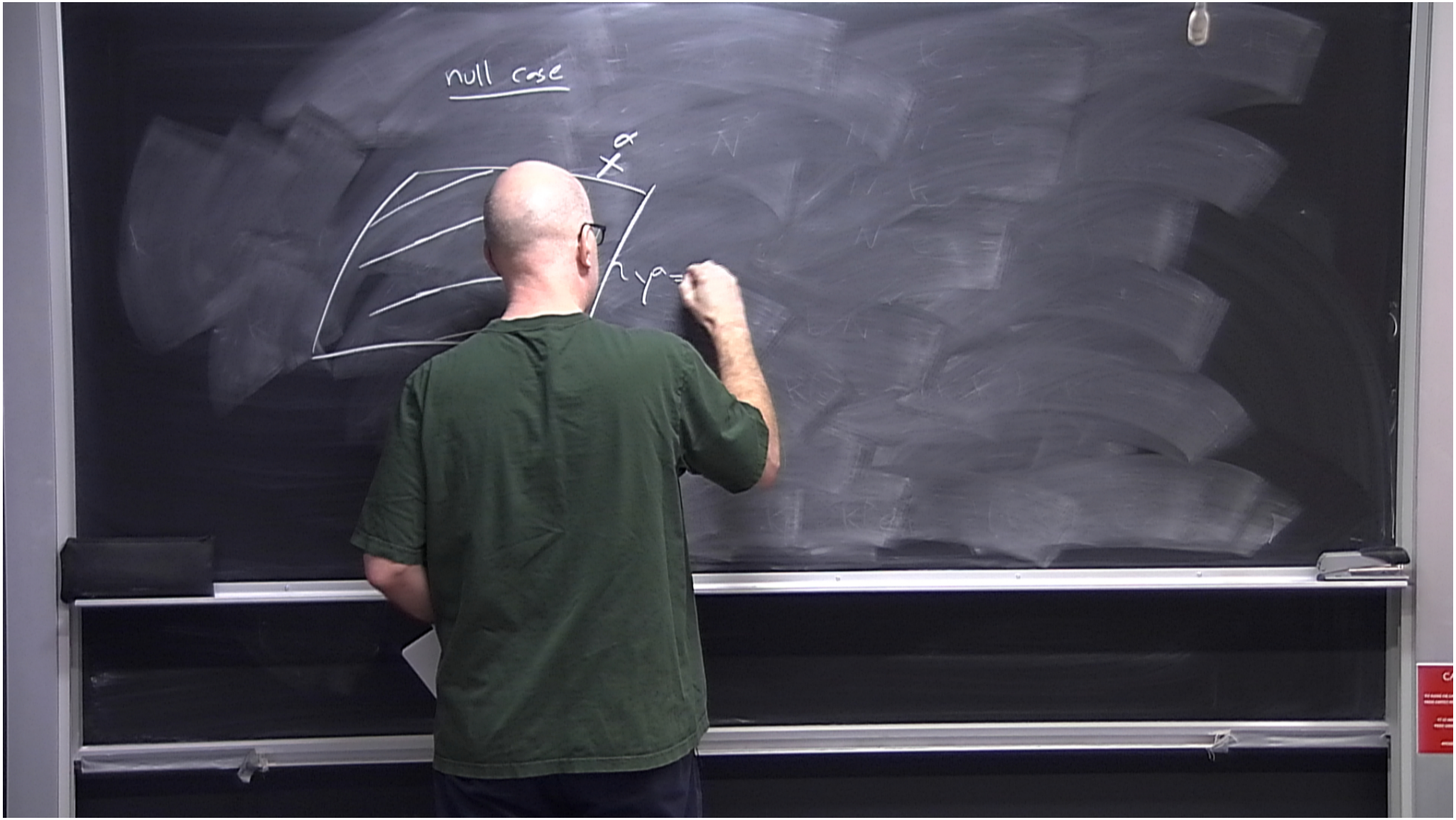
N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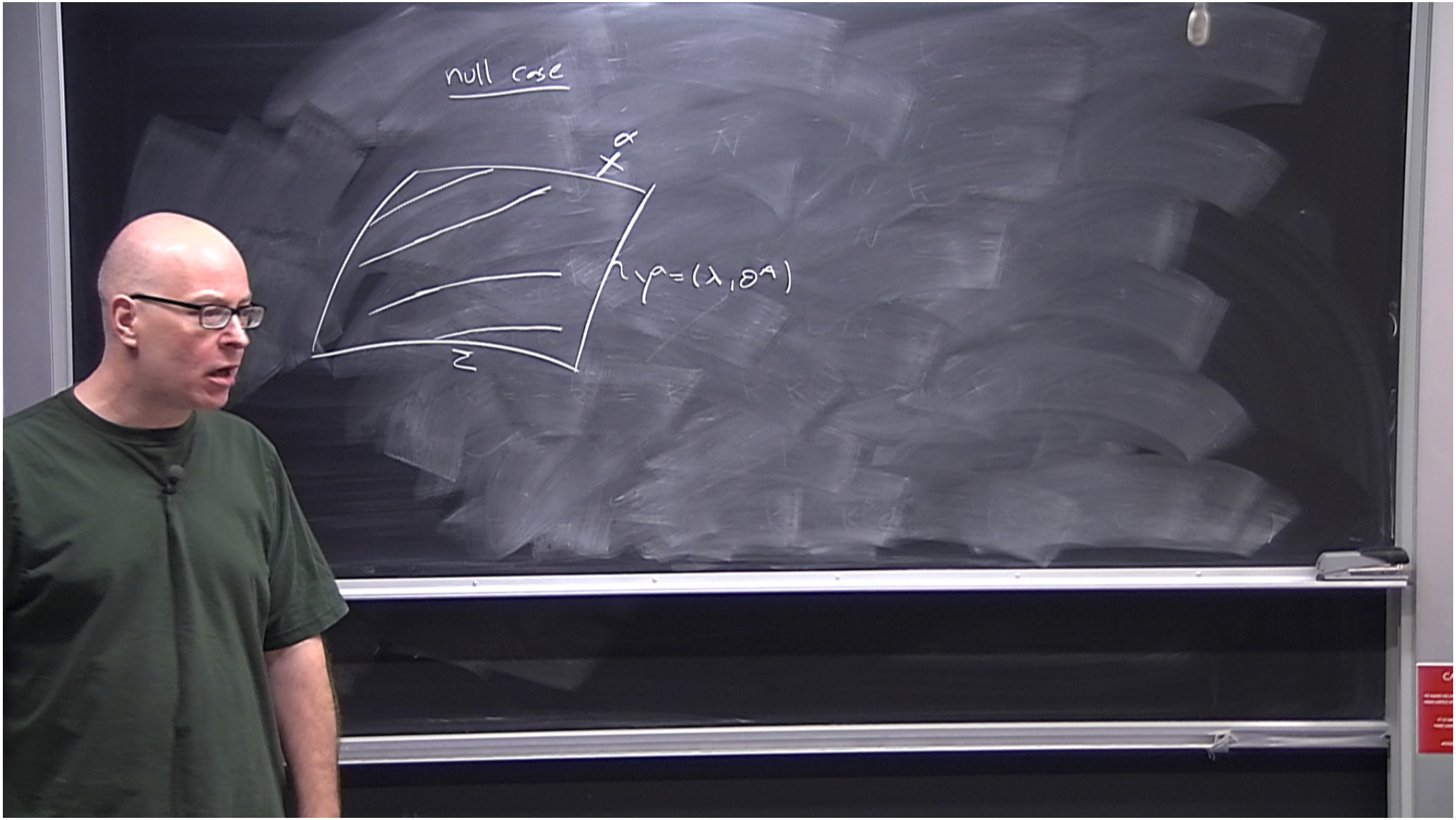
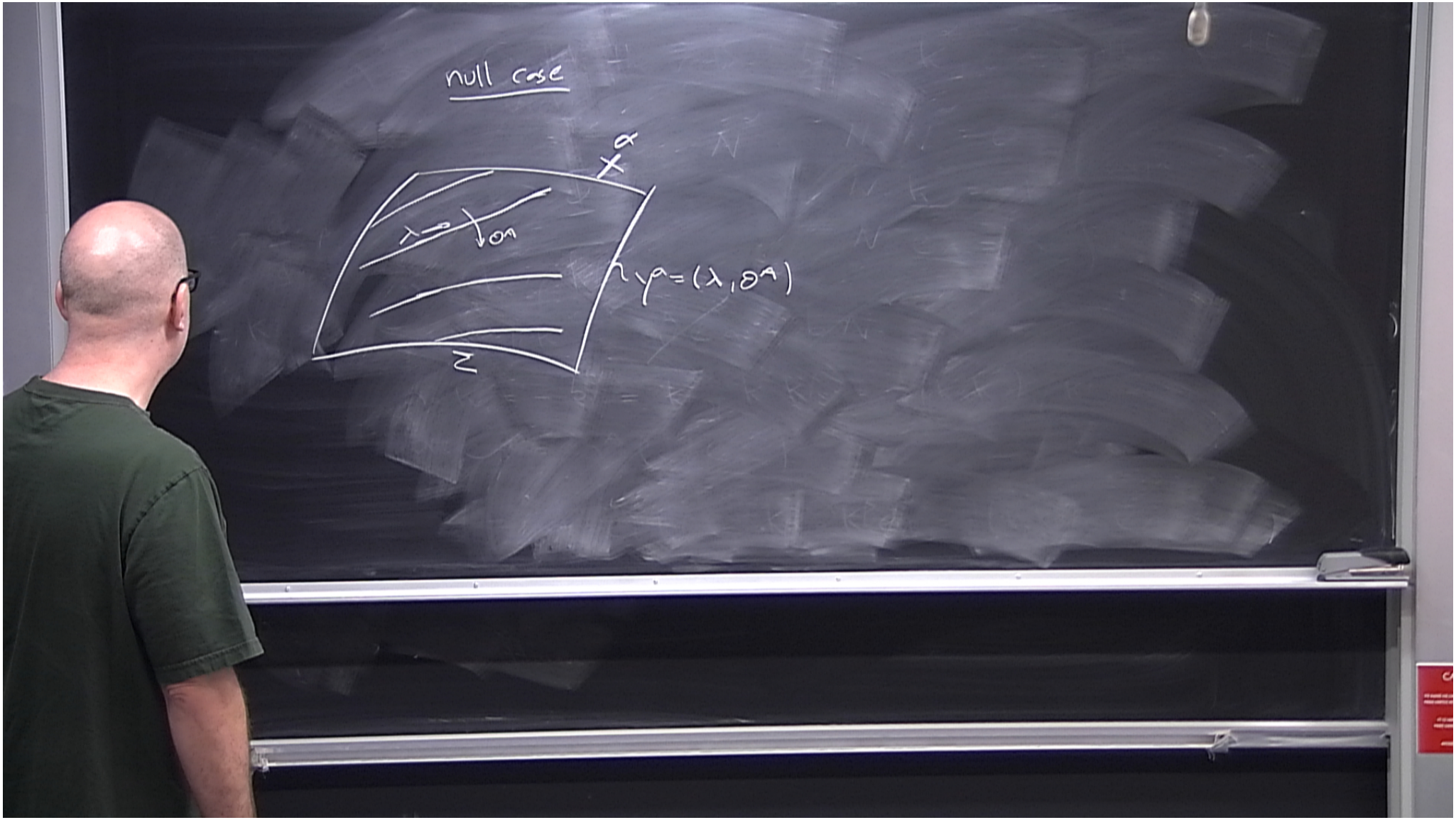
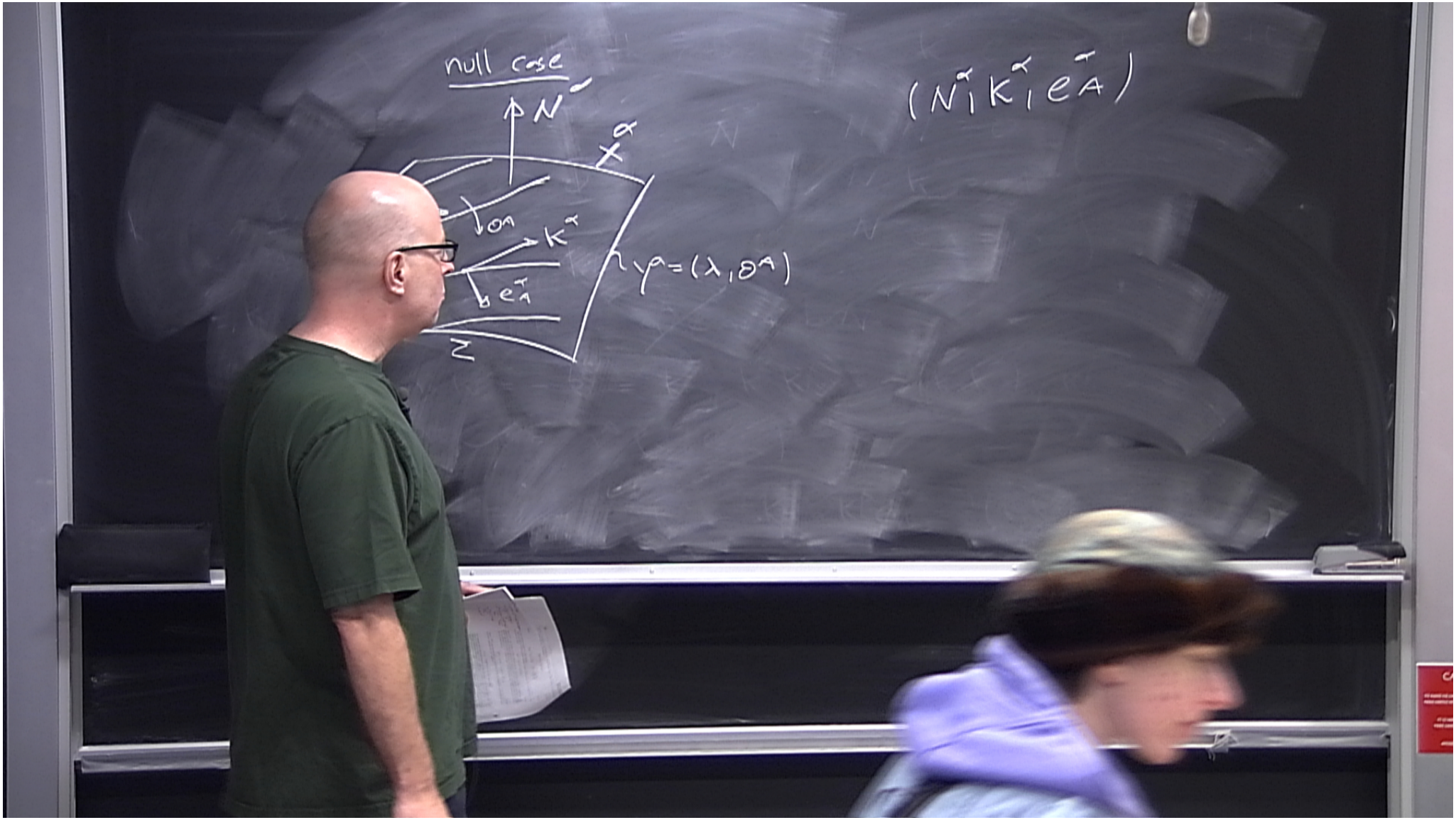
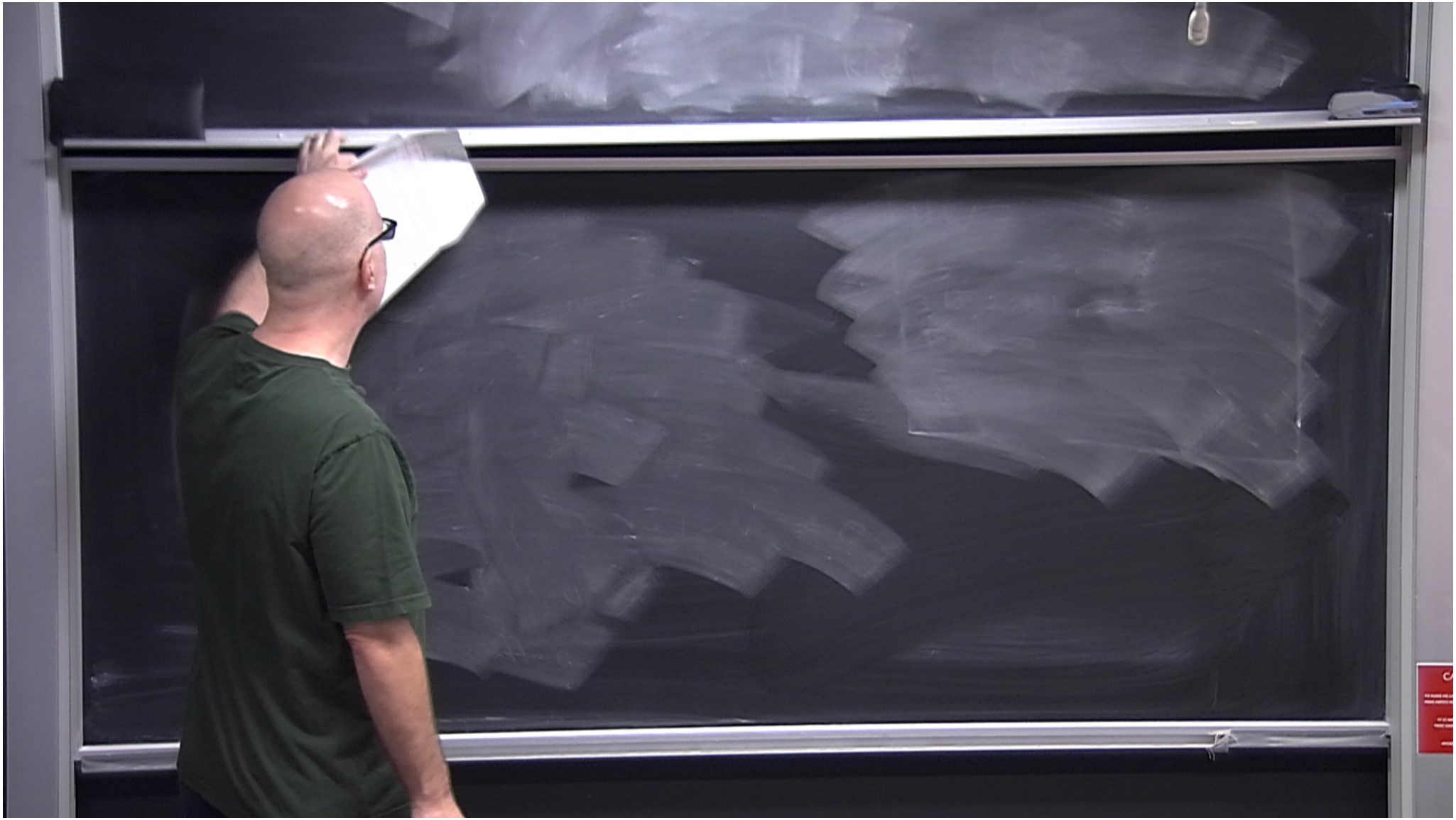
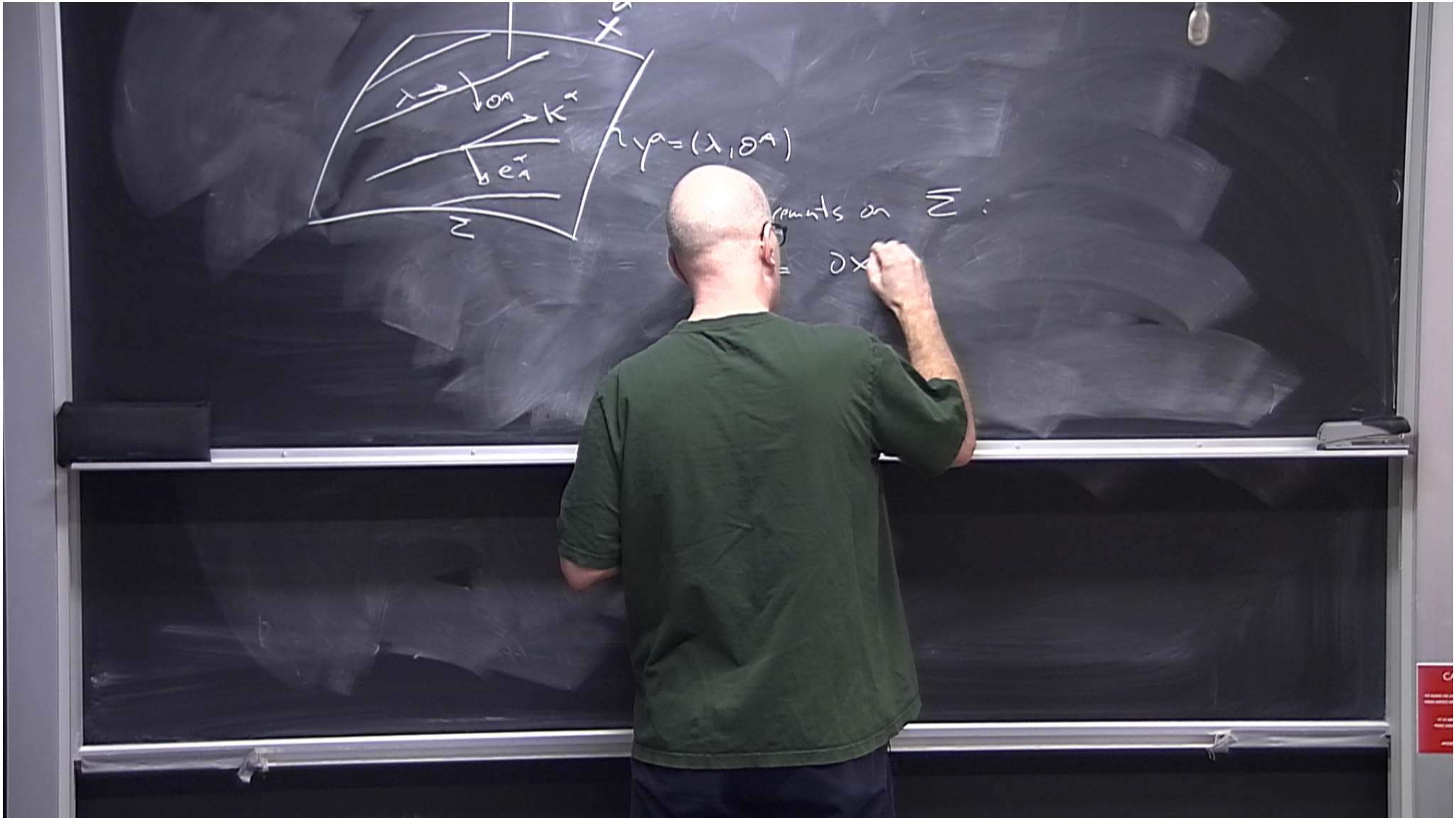

null case
x^a
z
λ, y^a = (λ, θ^A)

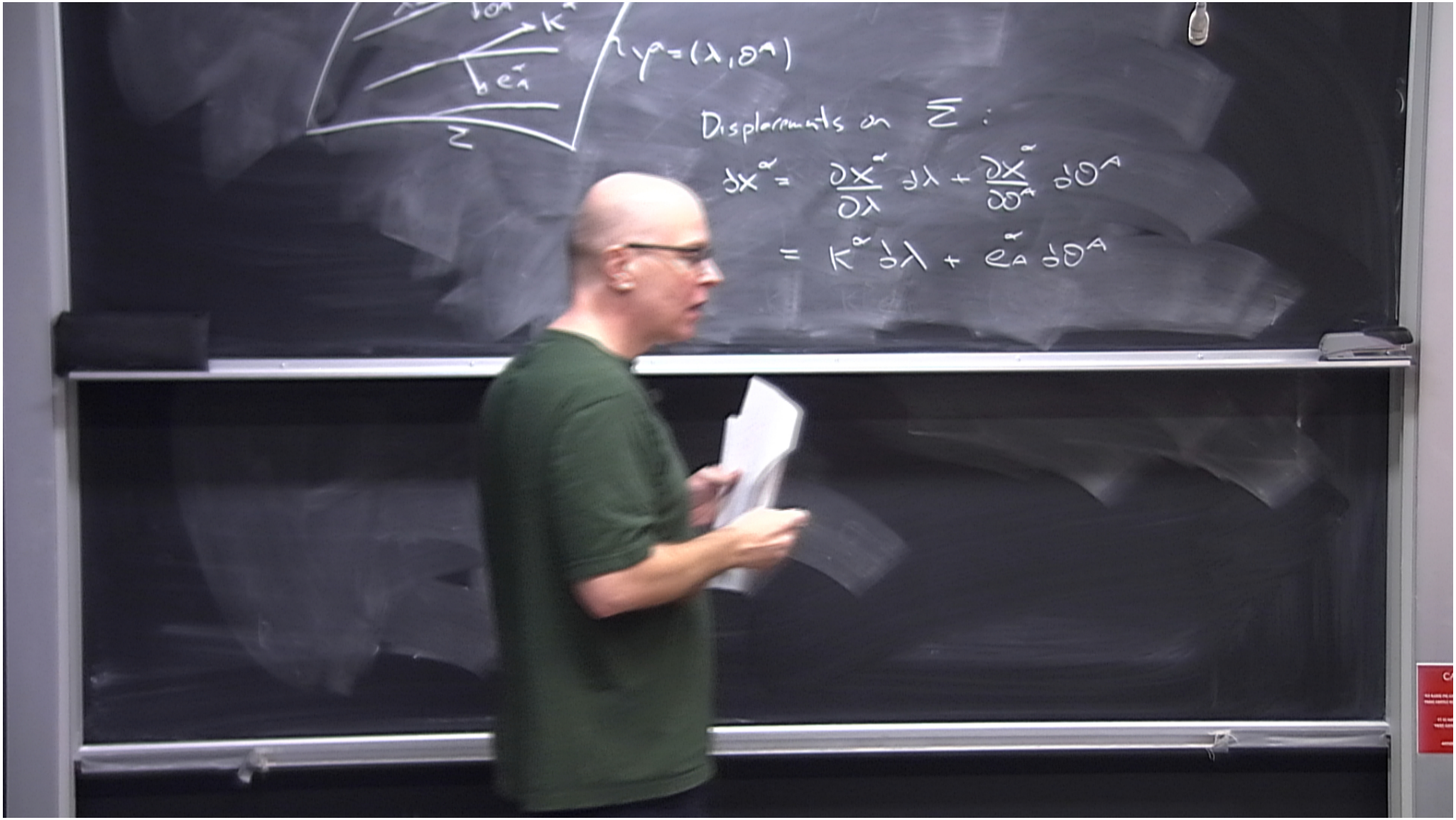

null case
x^a
$\lambda = 0$
$\gamma^a = (\lambda, \theta^A)$
Σ

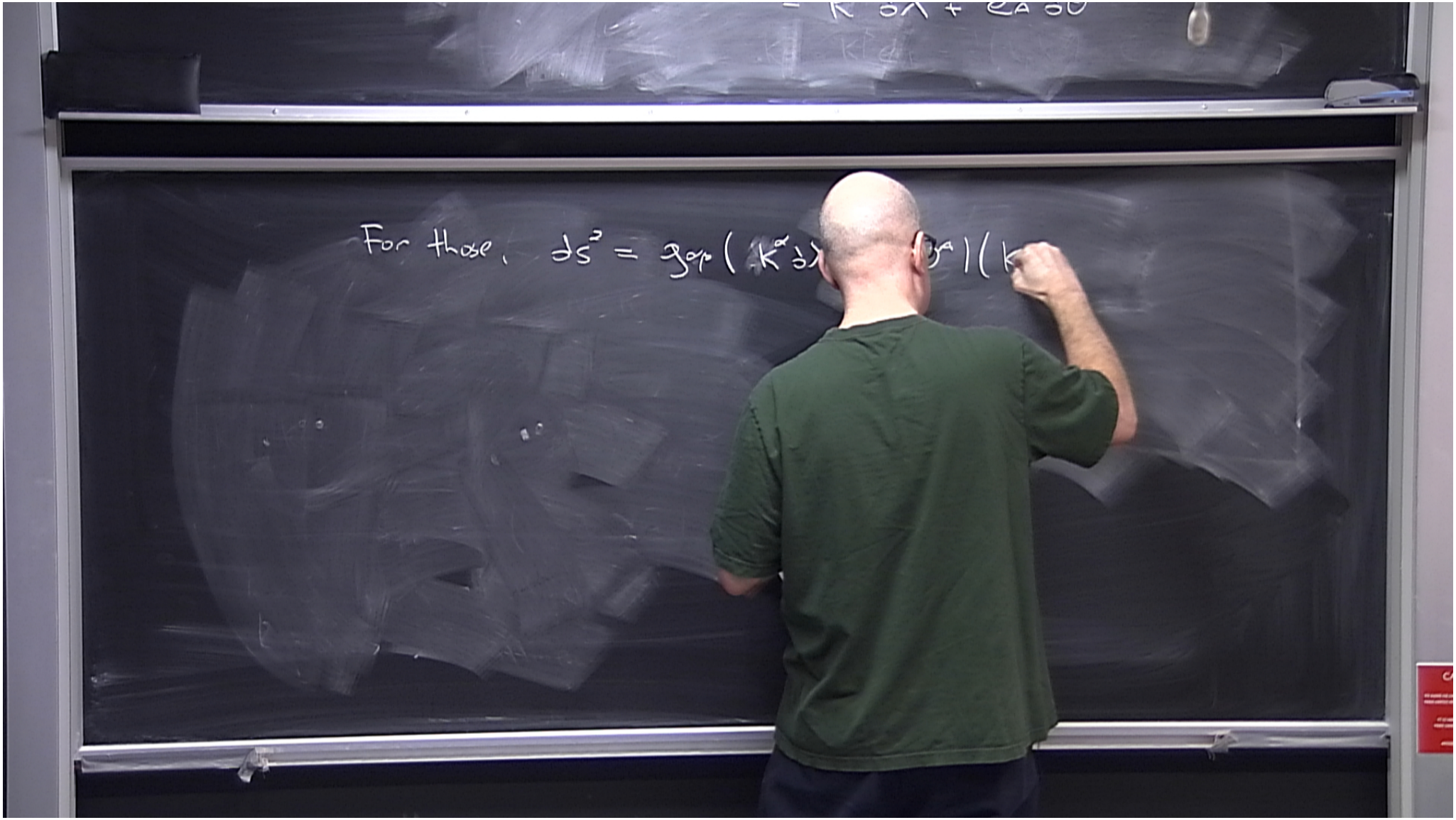

null case
N^α
x^α
θ^A
K^α
e^α_A
Σ
$y^a = (\lambda, \theta^A)$
$(N^\alpha, K^\alpha, e^\alpha_A)$

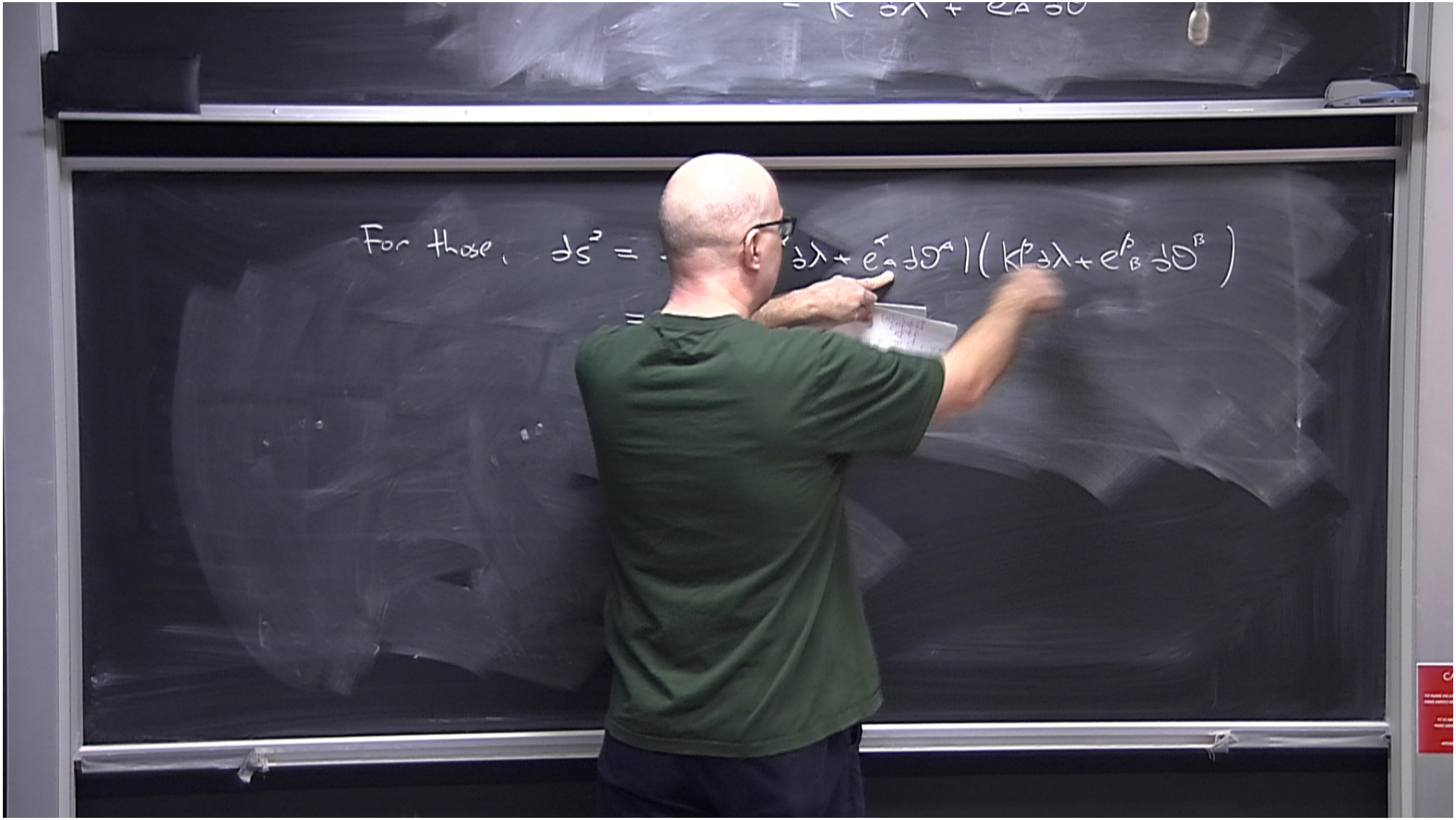


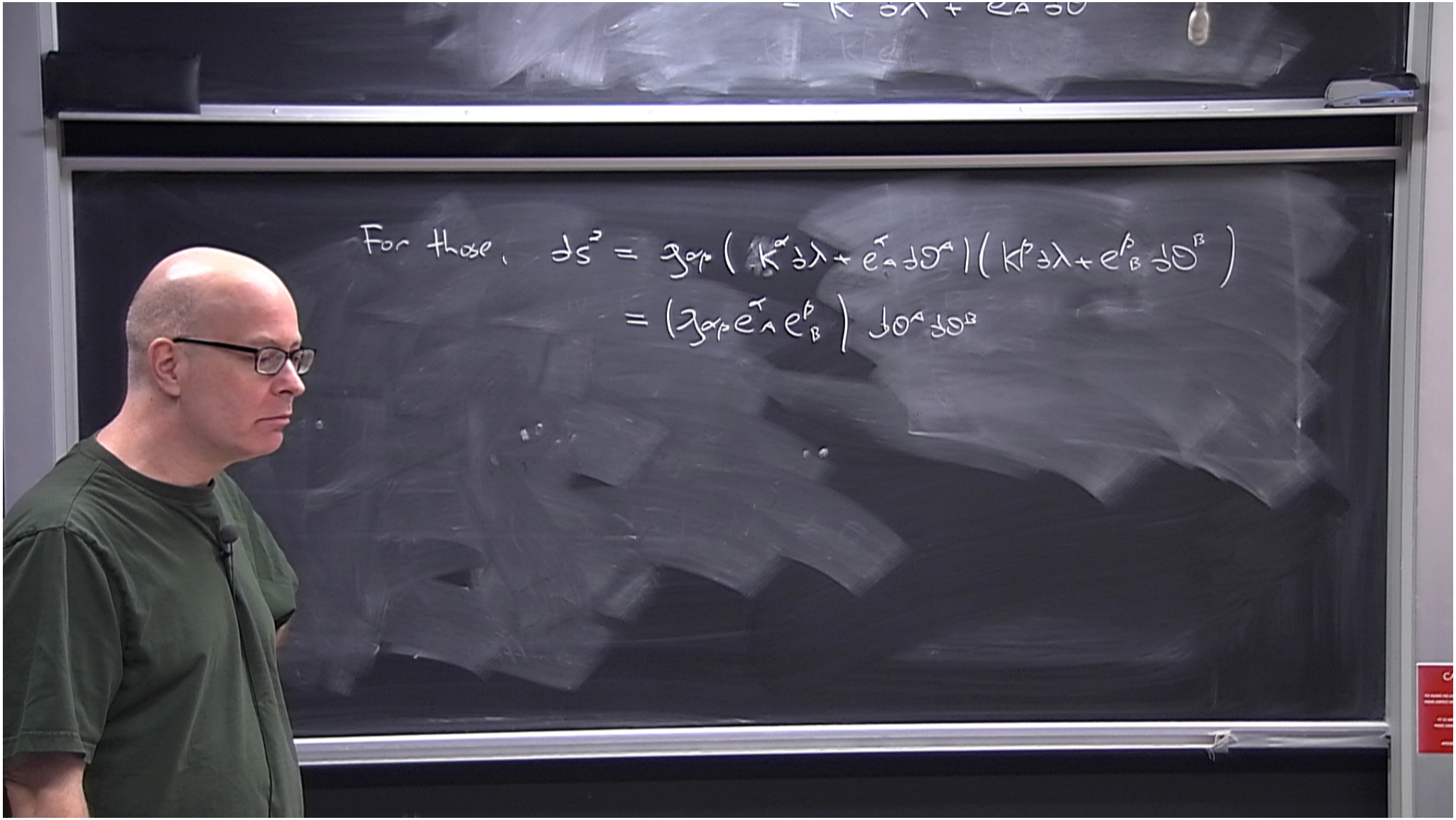


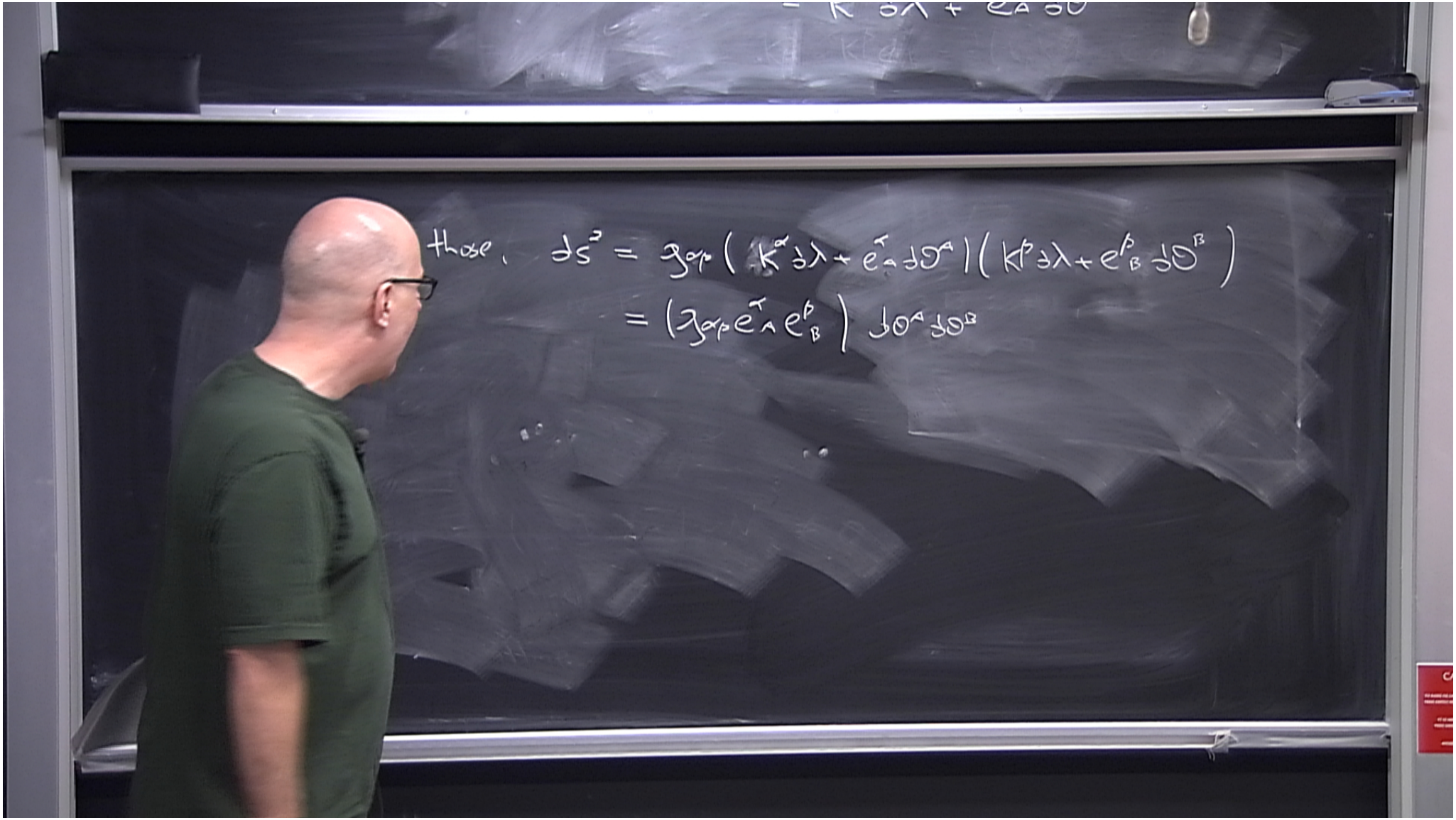
λ
θ^A
K^α
e^α_A
Σ
$y^a = (\lambda, \theta^A)$
on Σ :
∂x


k^α
e^α_A
Σ
$y^\alpha = (\lambda, \theta^A)$
Displacements on Σ:
$\delta x^\alpha = \frac{\partial x^\alpha}{\partial \lambda} \delta\lambda + \frac{\partial x^\alpha}{\partial \theta^A} \delta\theta^A$
$= k^\alpha \delta\lambda + e^\alpha_A \delta\theta^A$


For those, $ds^2 = g_{\alpha\beta}(k^\alpha d\lambda$


For those,


For those, $ds^2 = g_{\alpha\beta}(-k^\alpha d\lambda + e^\alpha_A d\theta^A)(k^\beta d\lambda + e^\beta_B d\theta^B)$
$= (g_{\alpha\beta} e^\alpha_A e^\beta_B) d\theta^A d\theta^B$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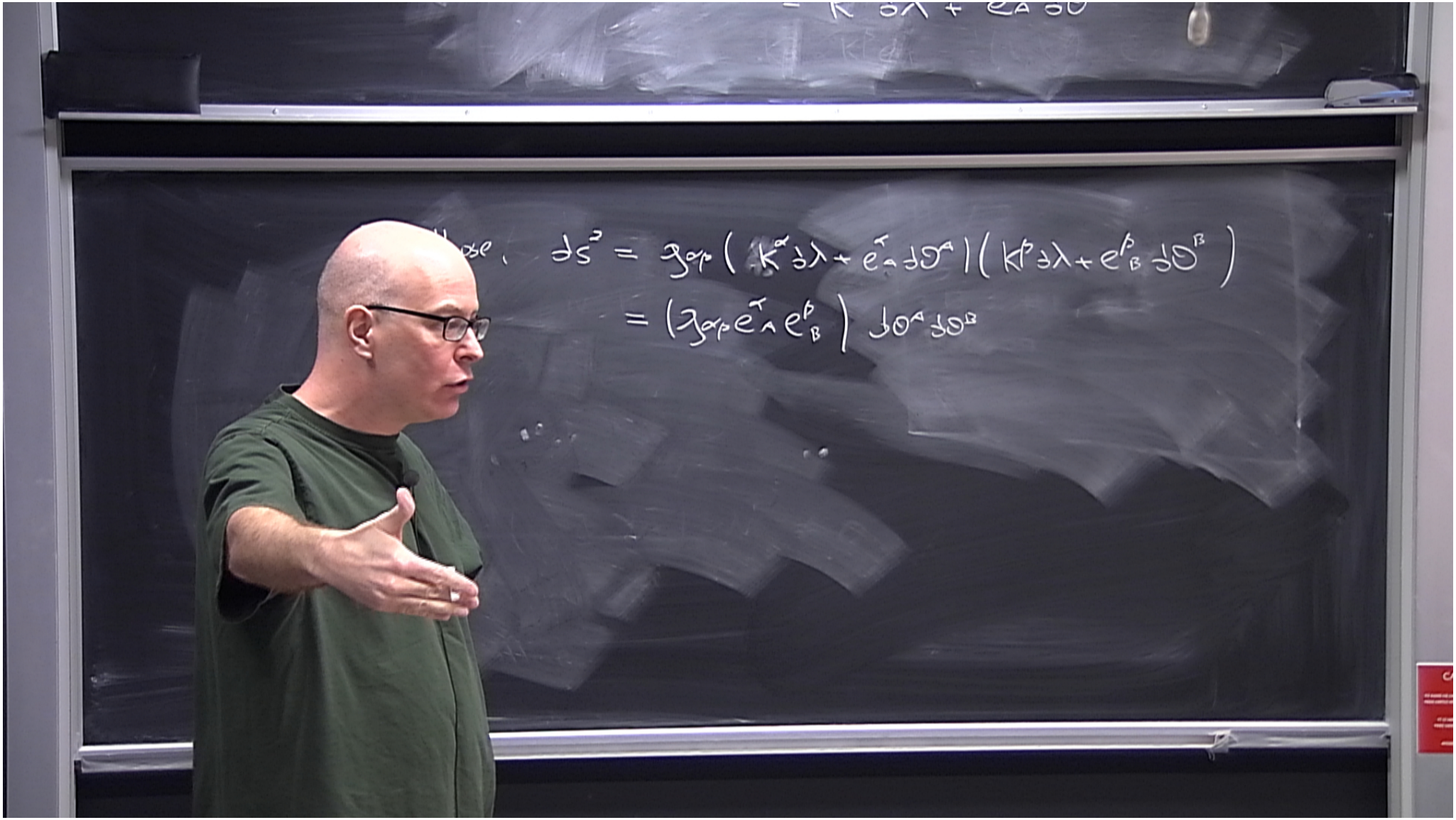
$ds^2 = g_{\alpha\beta}\left(K^\alpha d\lambda + e^\alpha_A d\theta^A\right)\left(K^\beta d\lambda + e^\beta_B d\theta^B\right)$
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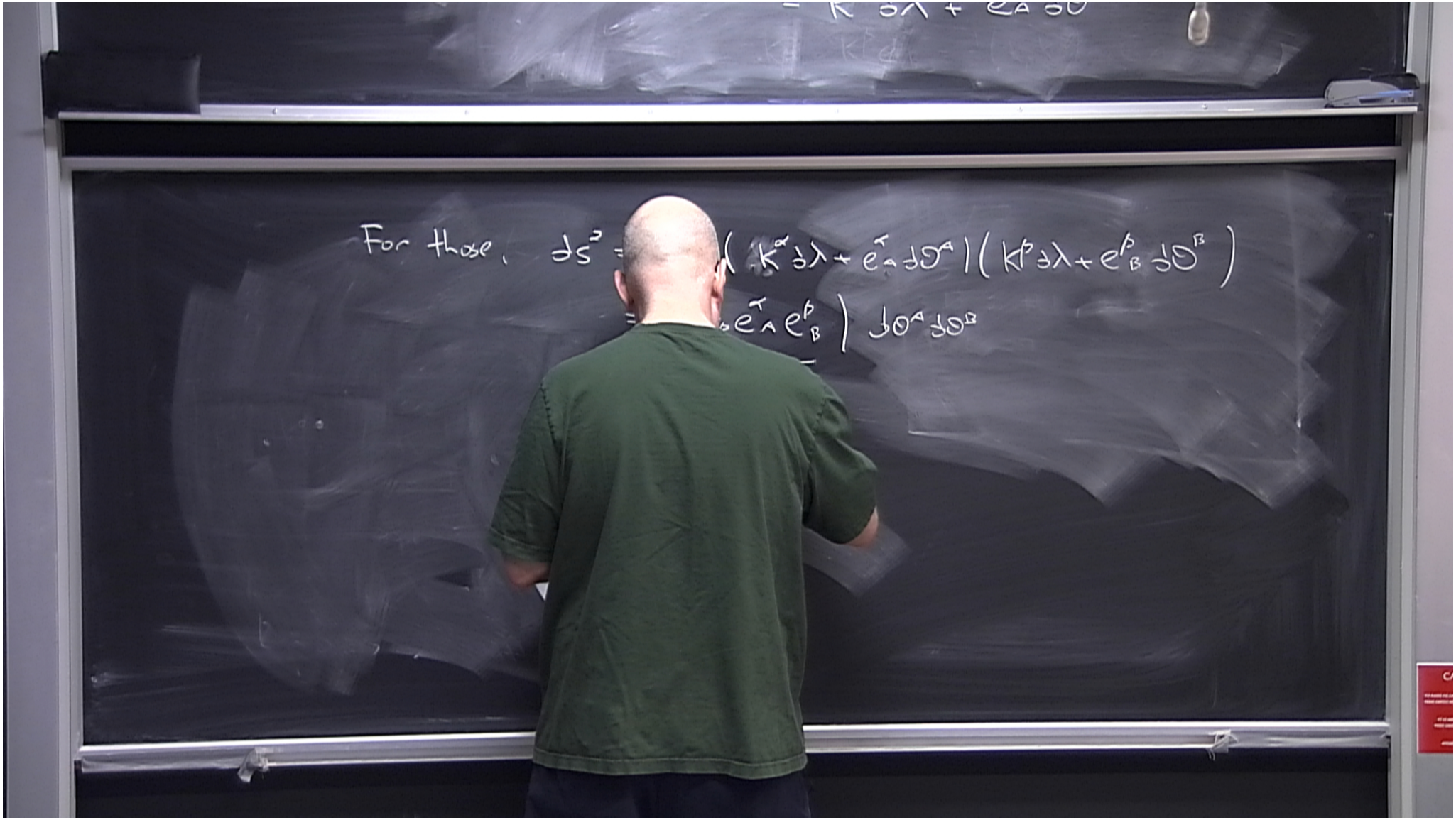
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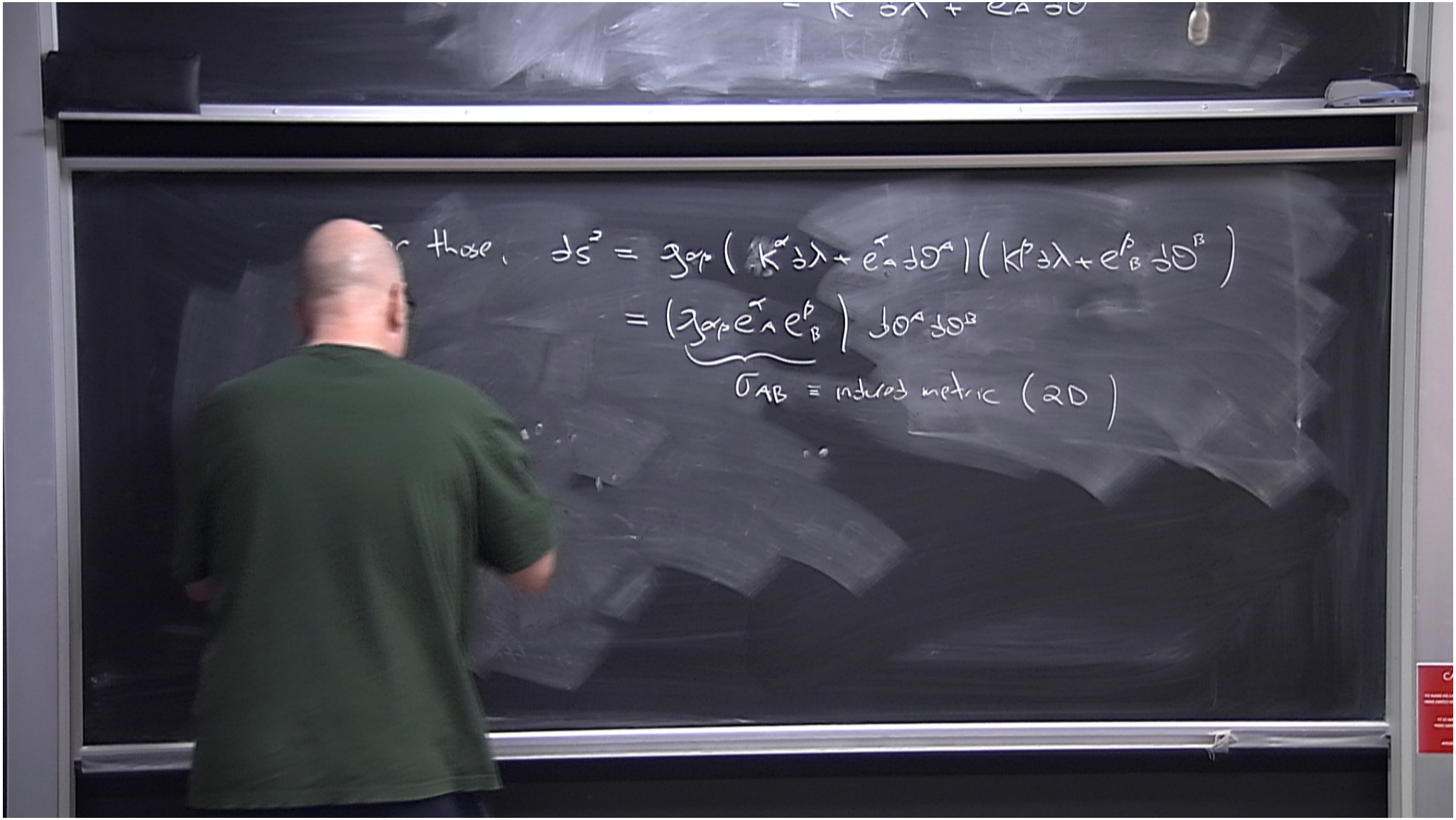
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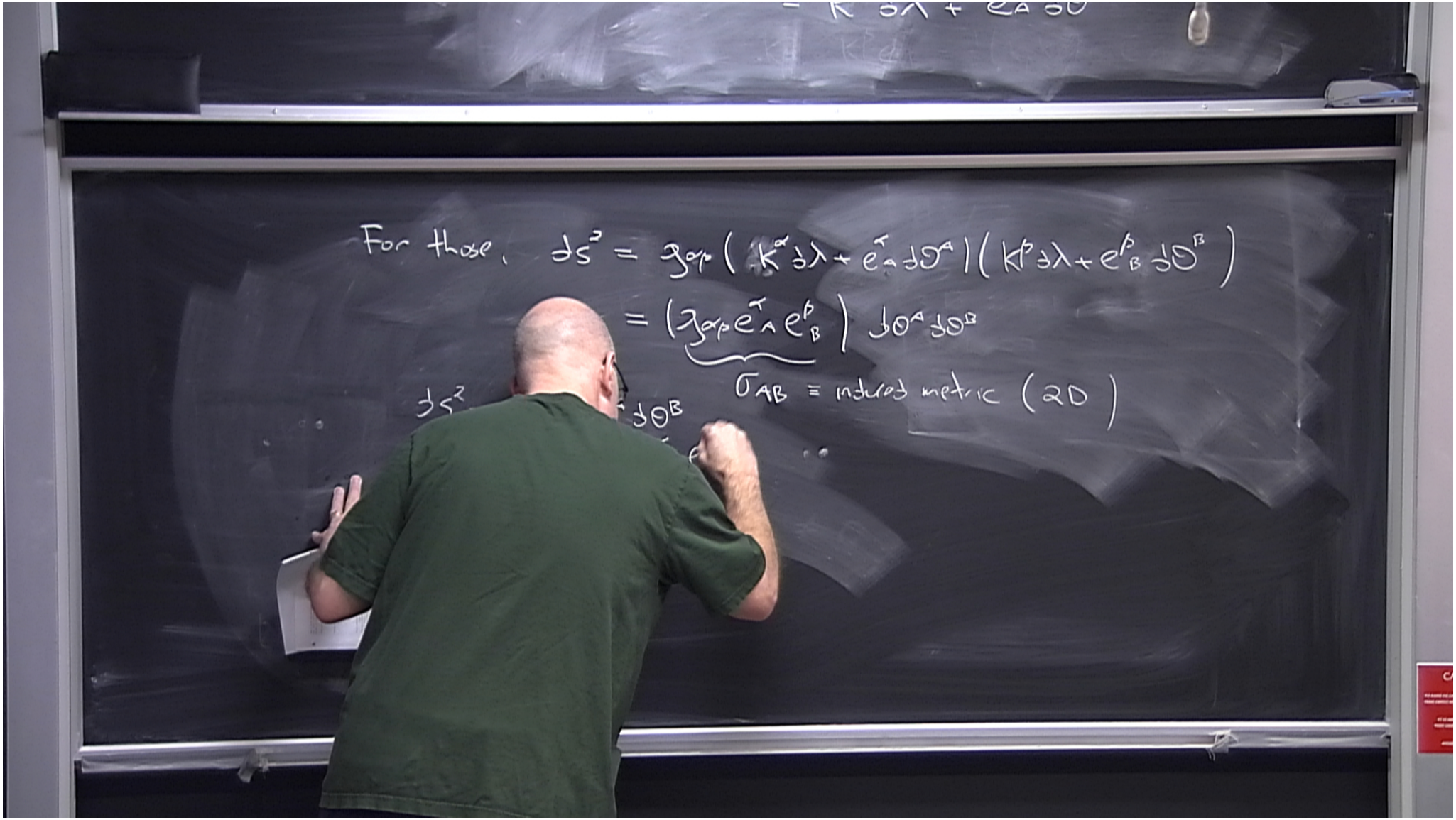
$= (g_{\alpha\beta} e^\alpha_A e^\beta_B)\, d\theta^A d\theta^B$
σ_{AB} ≡ induced metric (2D)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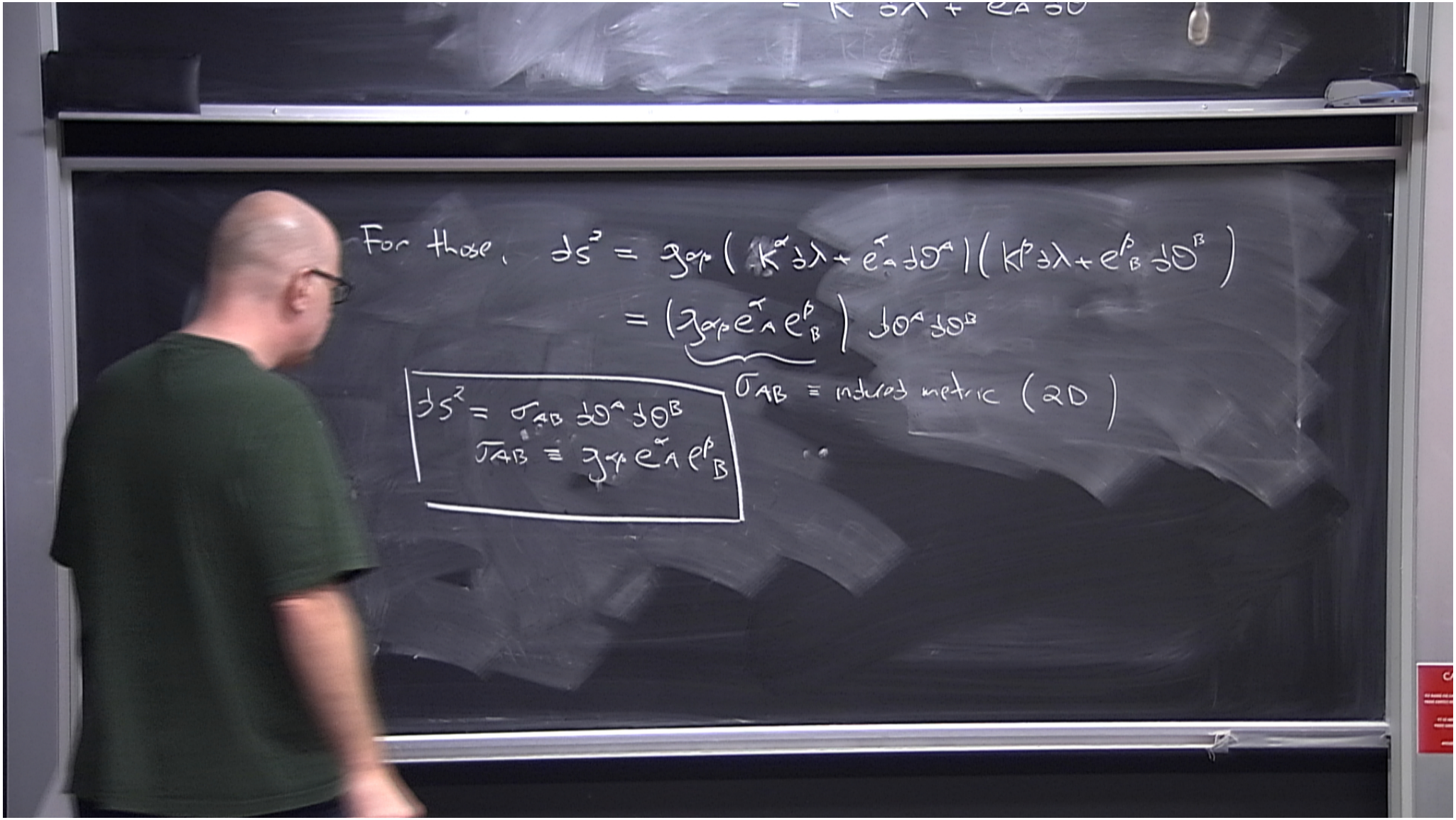
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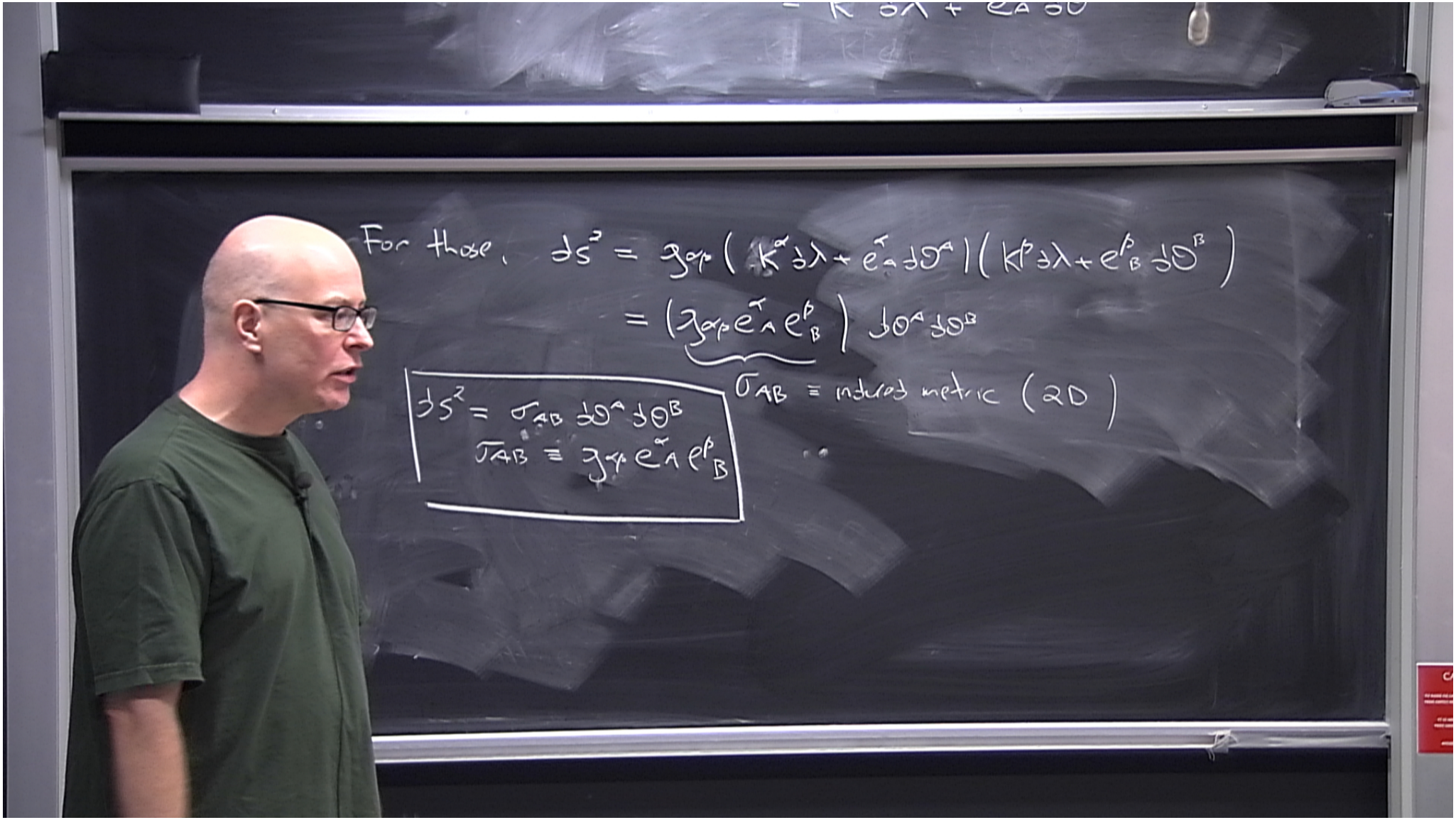
$= (g_{\alpha\beta} e^\alpha_A e^\beta_B)\, d\theta^A d\theta^B$
$ds^2 = \sigma_{AB} d\theta^A d\theta^B$
$\sigma_{AB} \equiv g_{\alpha\beta} e^\alpha_A e^\beta_B$
$\sigma_{AB} \equiv$ induced metric (2D)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$\sigma_{AB} \equiv$ induced metric (2D)
$ds^2 = \sigma_{AB} d\theta^A d\theta^B$
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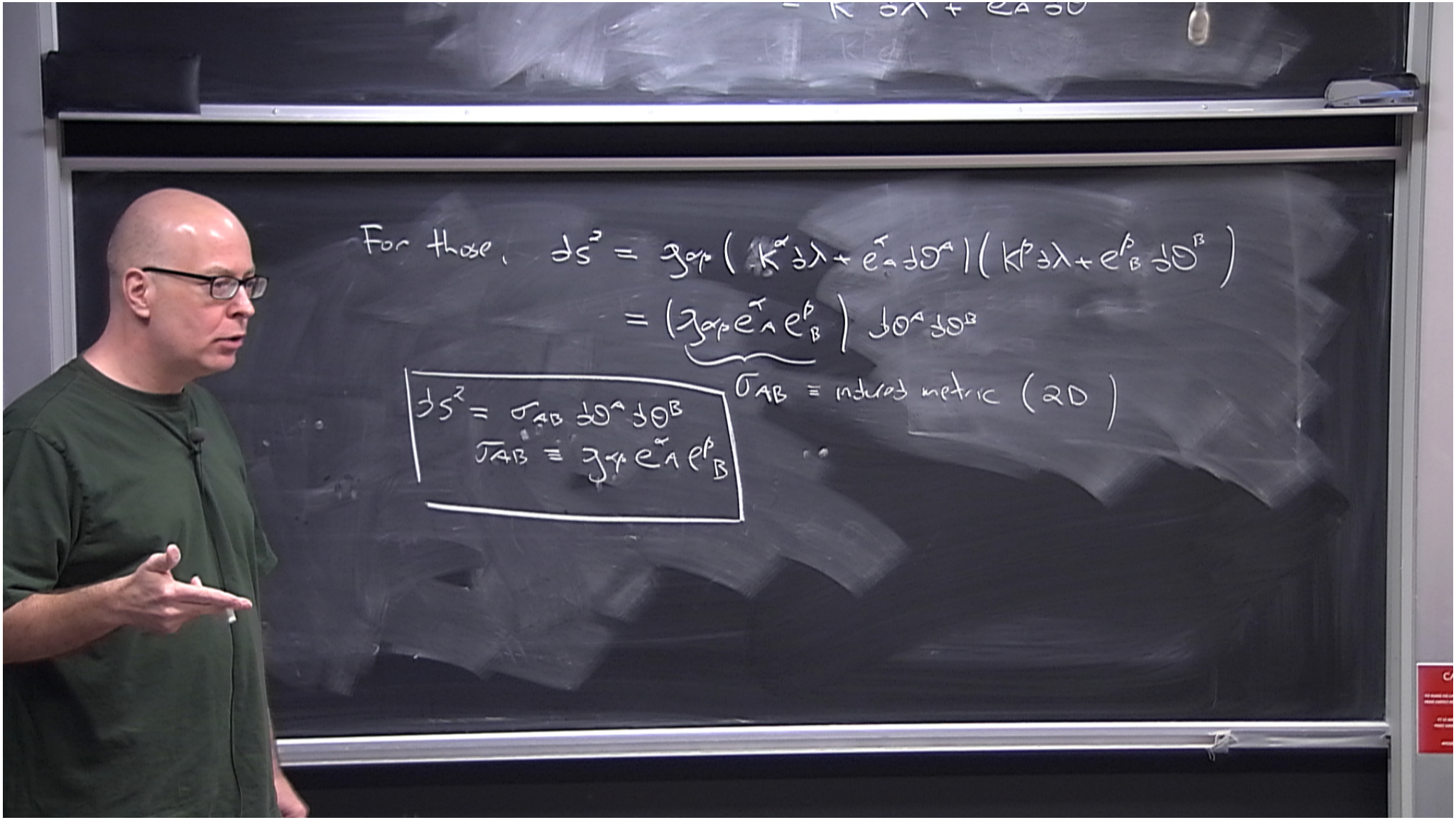
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σ_{AB} ≡ induced metric (2D)
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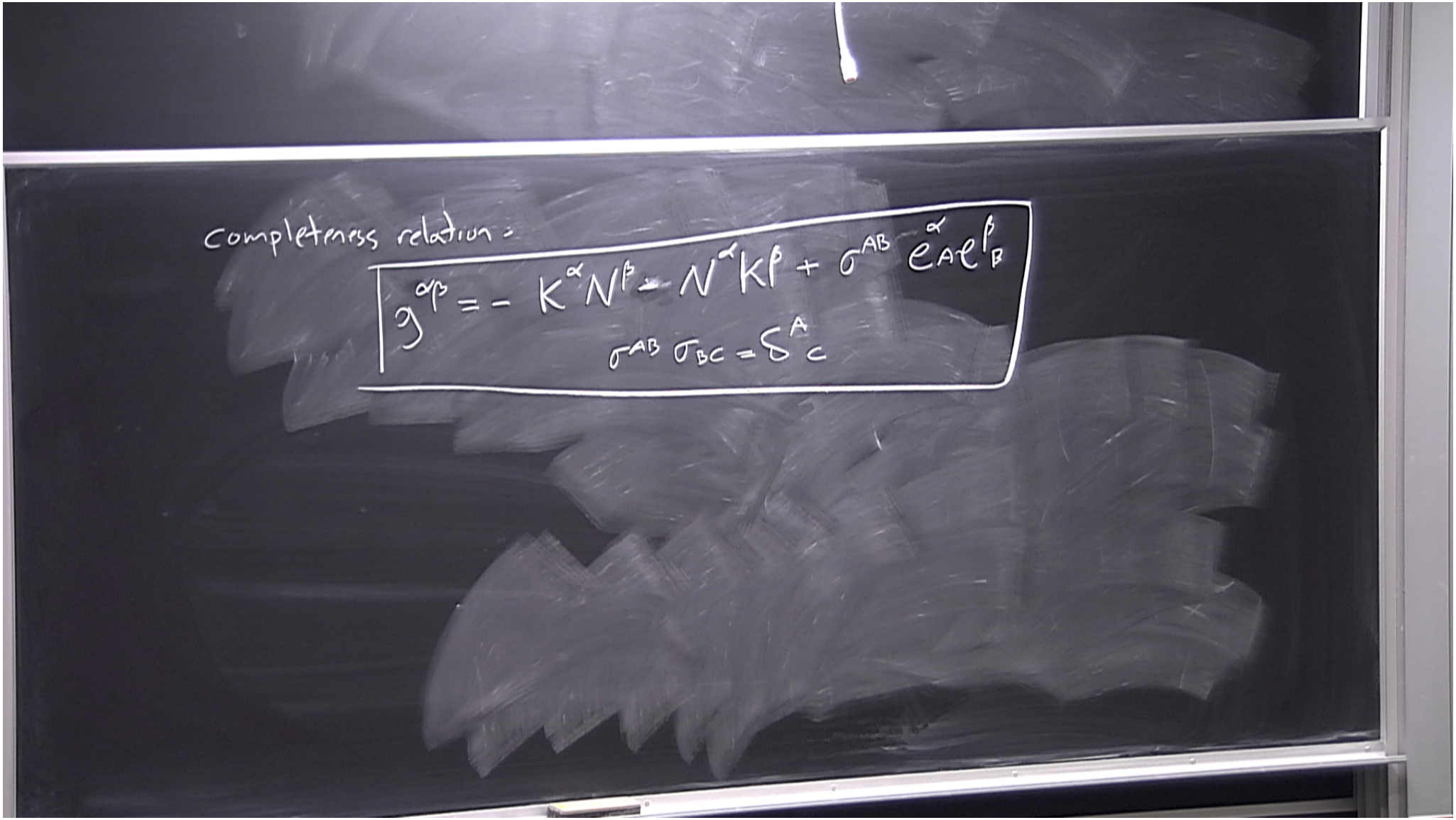
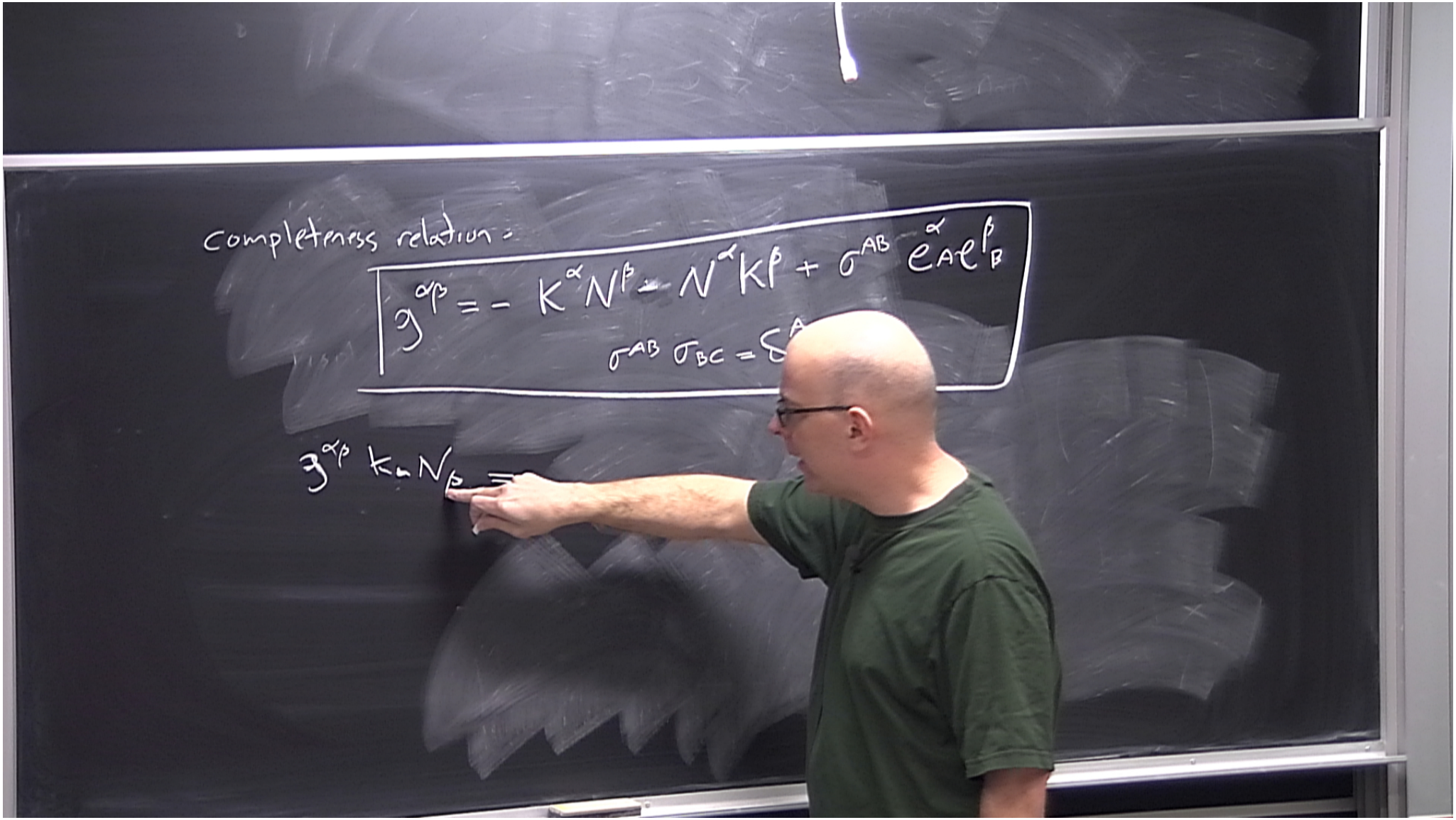
For those, $ds^2 = g_{\alpha\beta}\left(k^\alpha d\lambda + e^\alpha_A d\theta^A\right)\left(k^\beta d\lambda + e^\beta_B d\theta^B\right)$

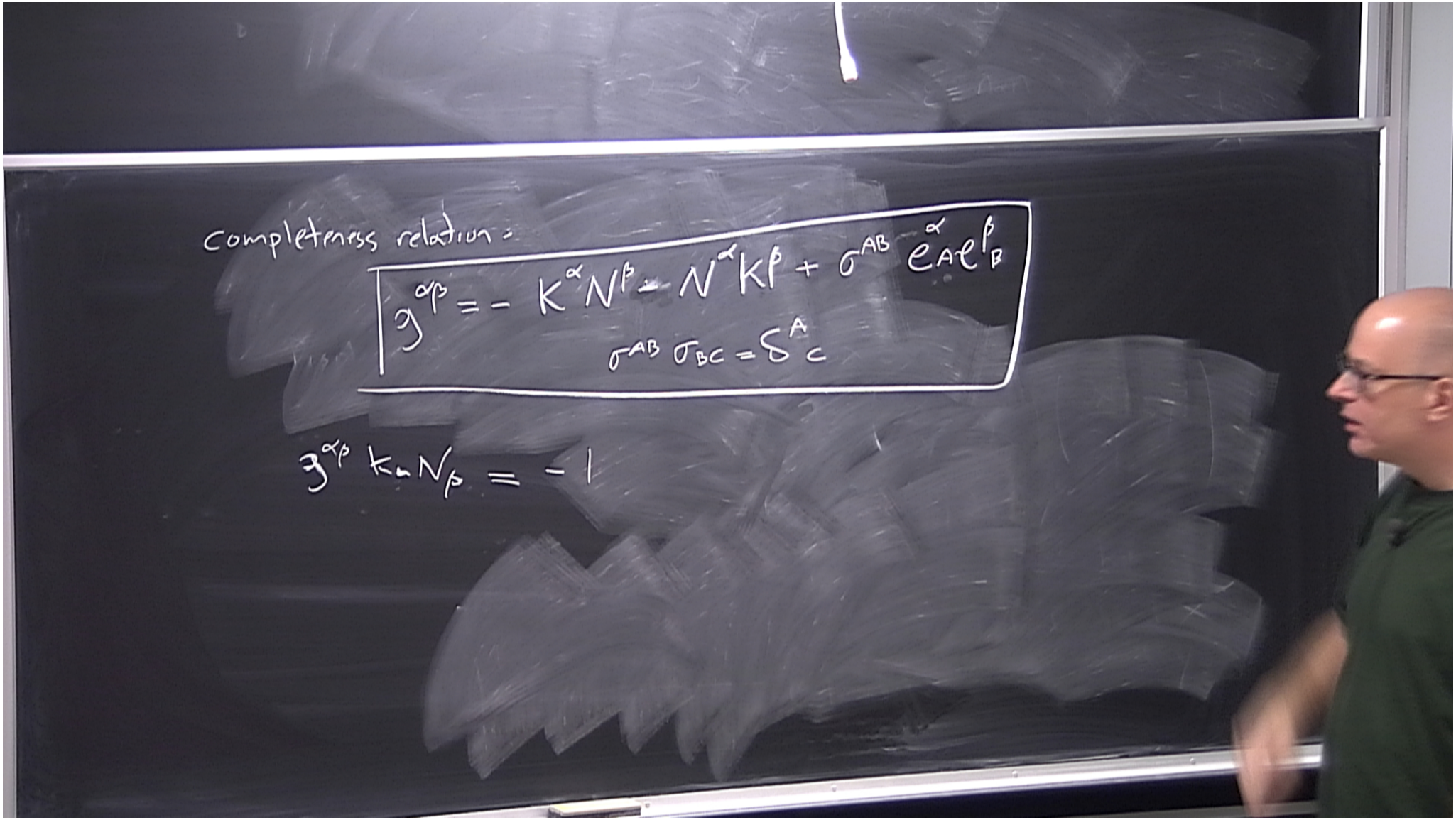
$= \left(g_{\alpha\beta} e^\alpha_A e^\beta_B\right) d\theta^A d\theta^B$

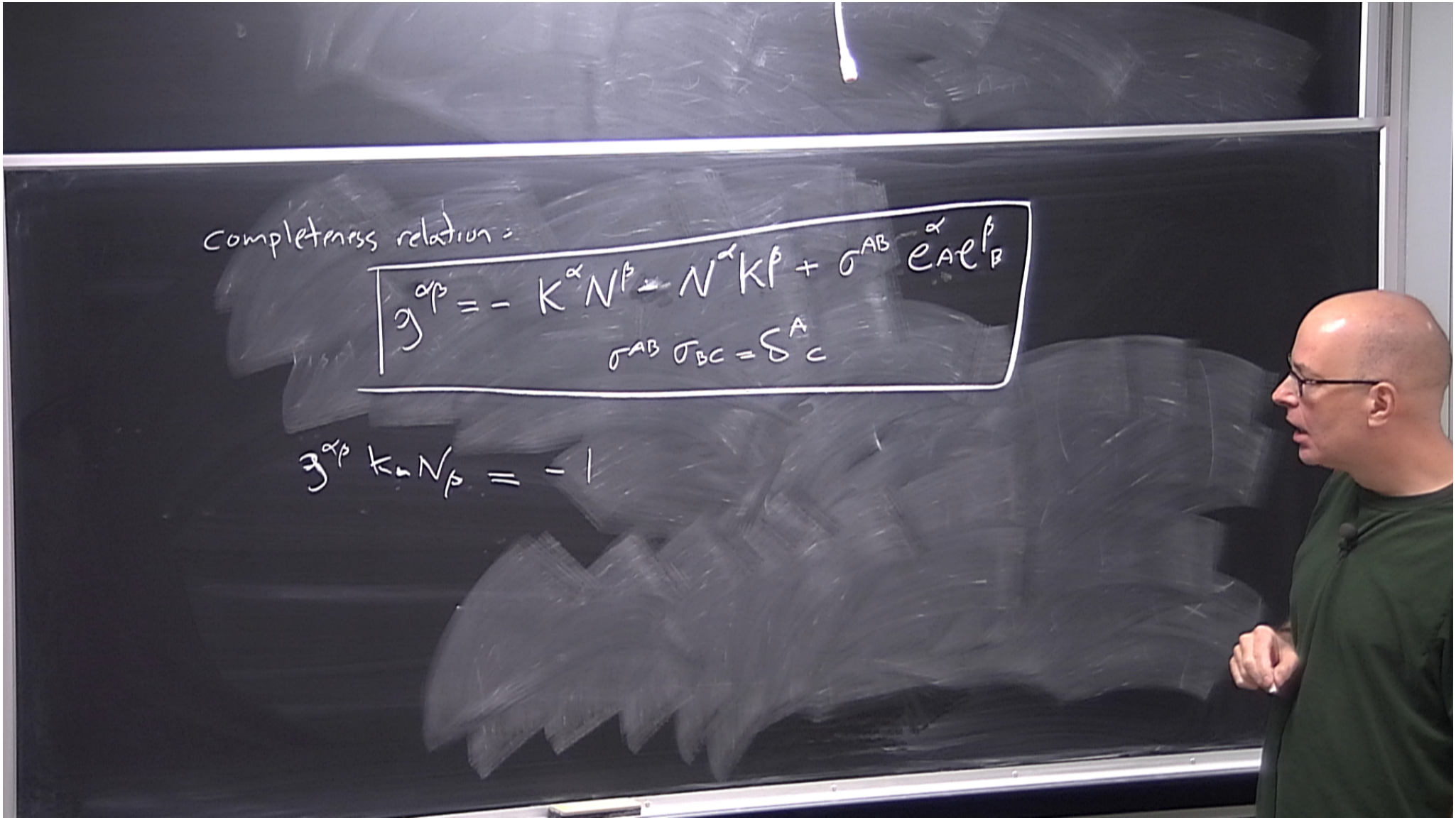
σ_{AB} = induced metric (2D)

$$ds^2 = \sigma_{AB}\, d\theta^A d\theta^B$$
$$\sigma_{AB} \equiv g_{\alpha\beta} e^\alpha_A e^\beta_B$$


completeness relation:
$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$
$\sigma^{AB}\sigma_{BC} = \delta^A_C$


Completeness relation:
$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$
$\sigma^{AB}\sigma_{BC} = \delta^A$
$g^{\alpha\beta} k_\alpha N_\beta =$


Completeness relation:
$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$
$\sigma^{AB}\sigma_{BC} = \delta^A_C$
$g^{\alpha\beta} k_\alpha N_\beta = -1$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$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$
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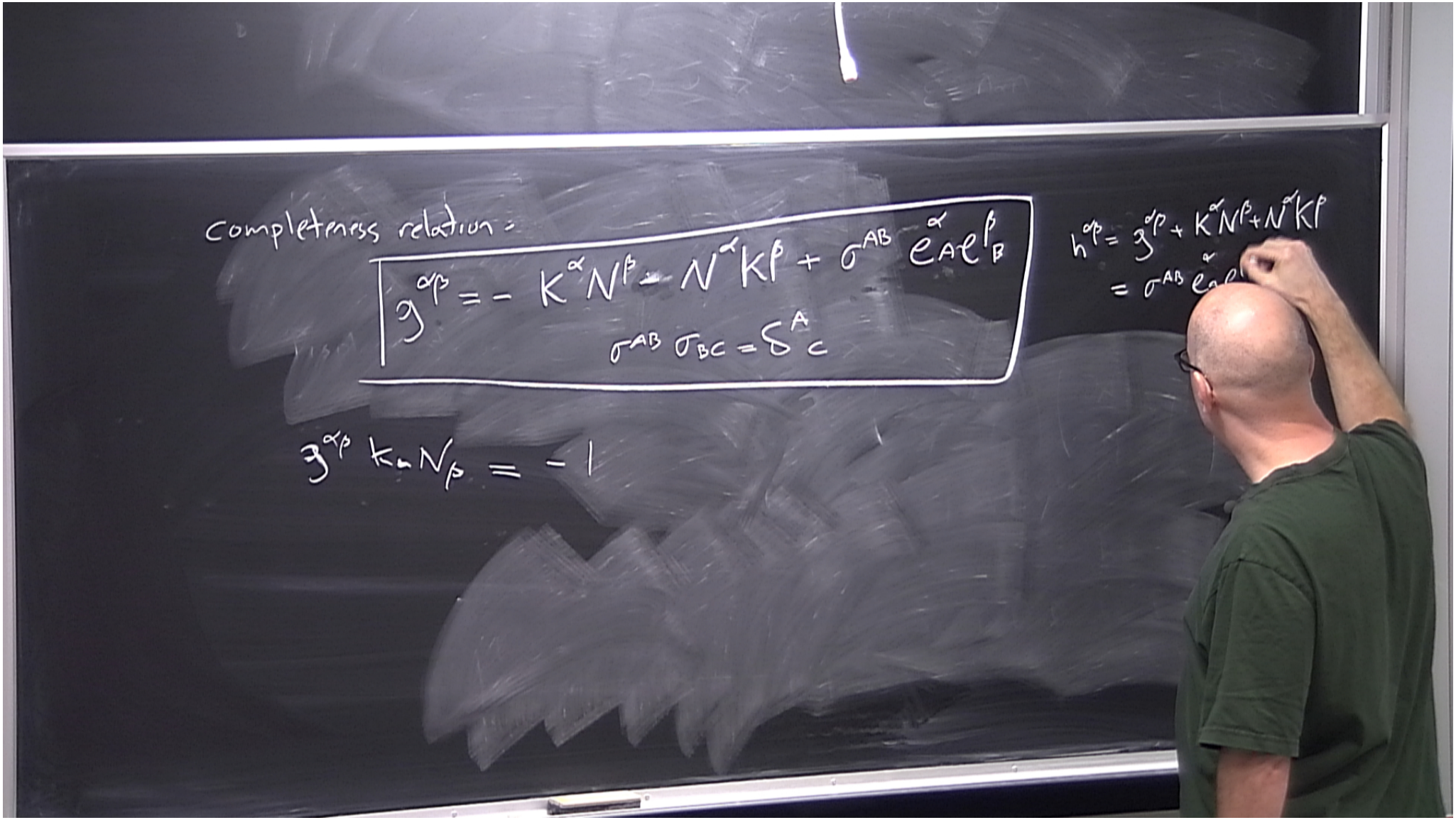
Completeness relation:
$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$
$\sigma^{AB}\sigma_{BC} = \delta^A_C$
$g^{\alpha\beta} k_\alpha N_\beta = -1$
$h^{\alpha\beta} = g^{\alpha\beta} + K^\alpha N^\beta + N^\alpha K^\beta$
$= \sigma^{AB} e^\alpha$

Completeness relation:

$$g^{\alpha\beta} = -K^\alpha N^\beta - N^\alpha K^\beta + \sigma^{AB} e^\alpha_A e^\beta_B$$

$$\sigma^{AB}\sigma_{BC} = \delta^A_C$$

$$h^{\alpha\beta} = g^{\alpha\beta} + K^\alpha N^\beta + N^\alpha K^\beta = \sigma^{AB} e^\alpha_A e^\beta_B$$

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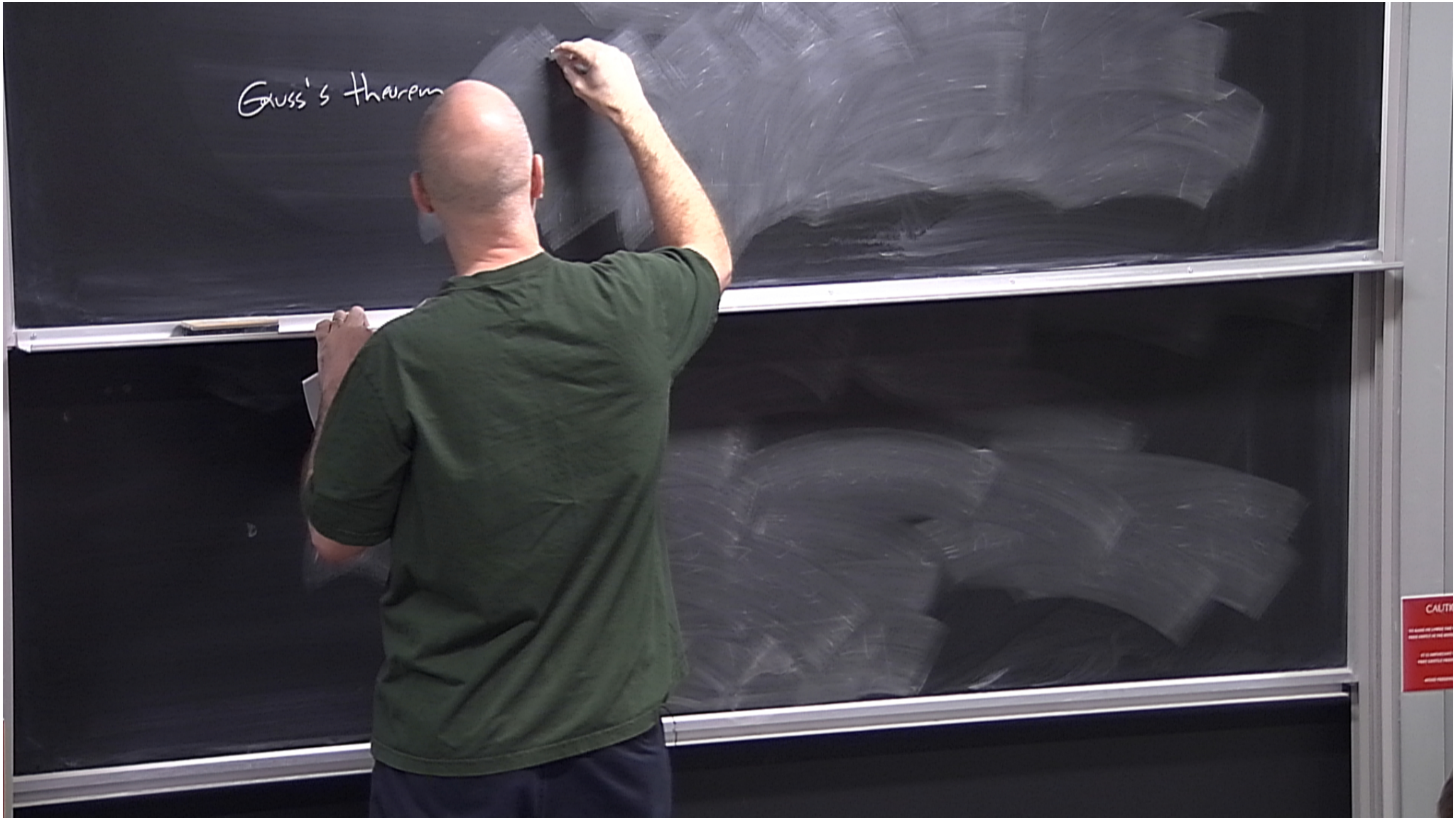
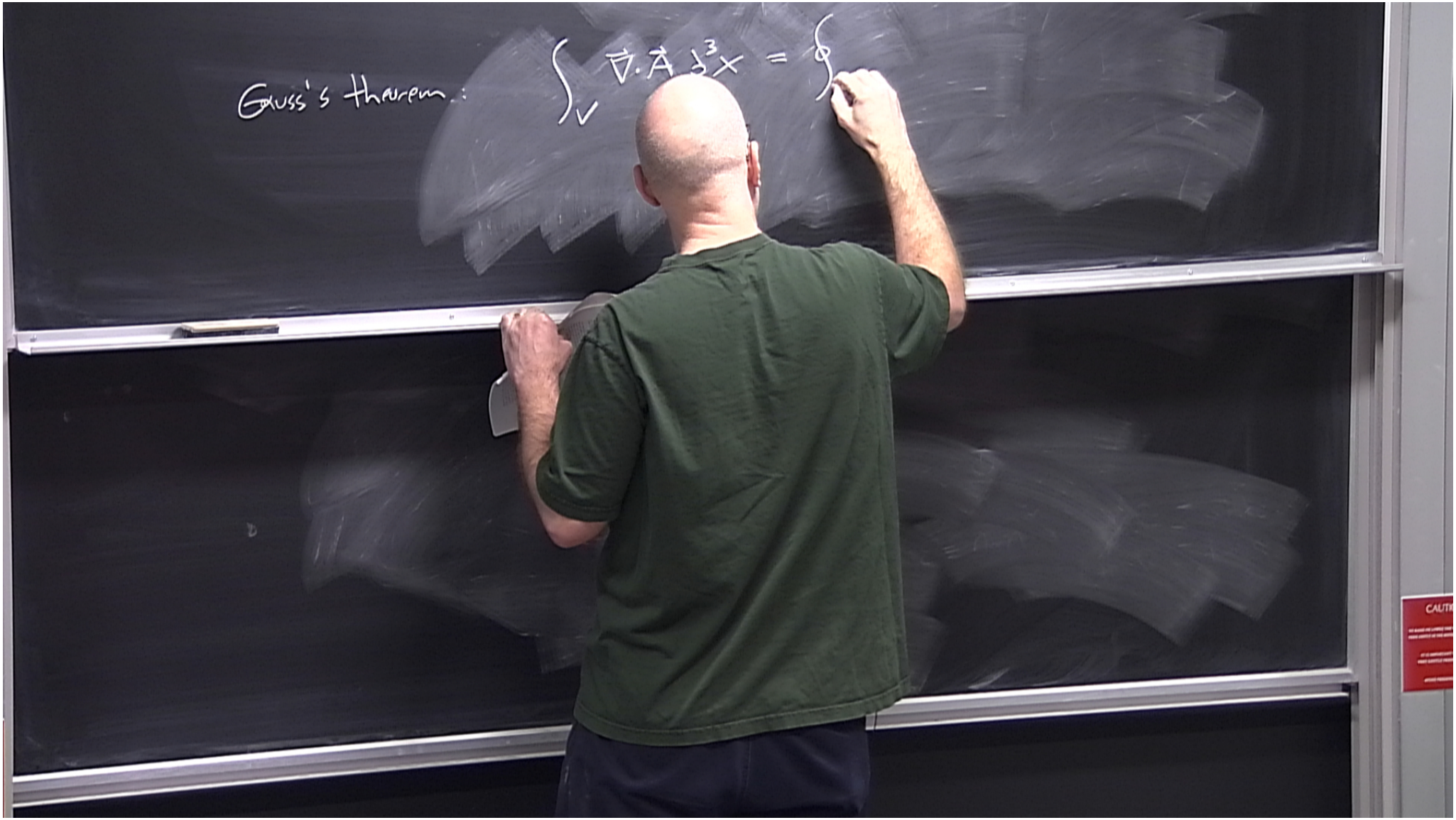
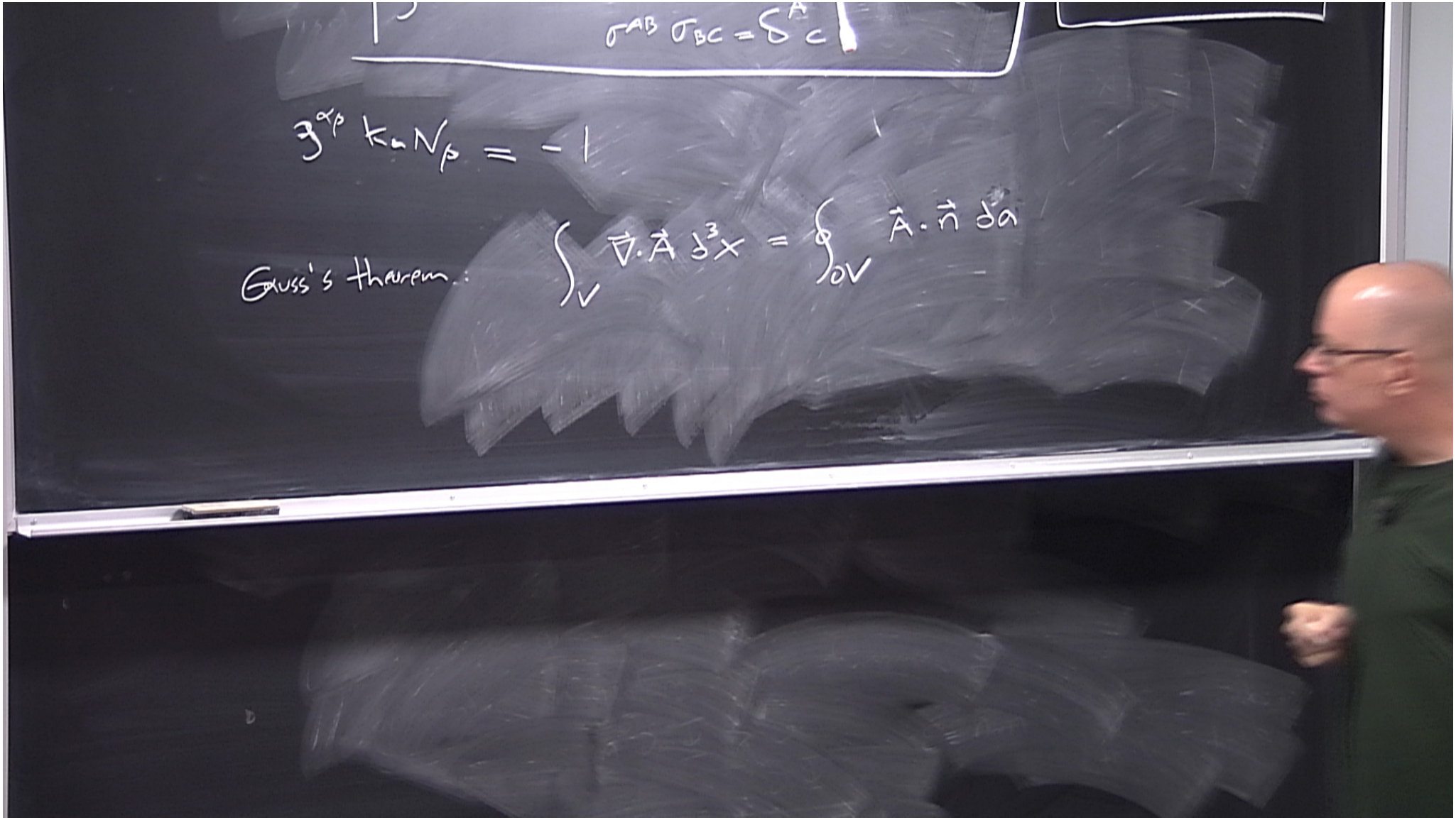
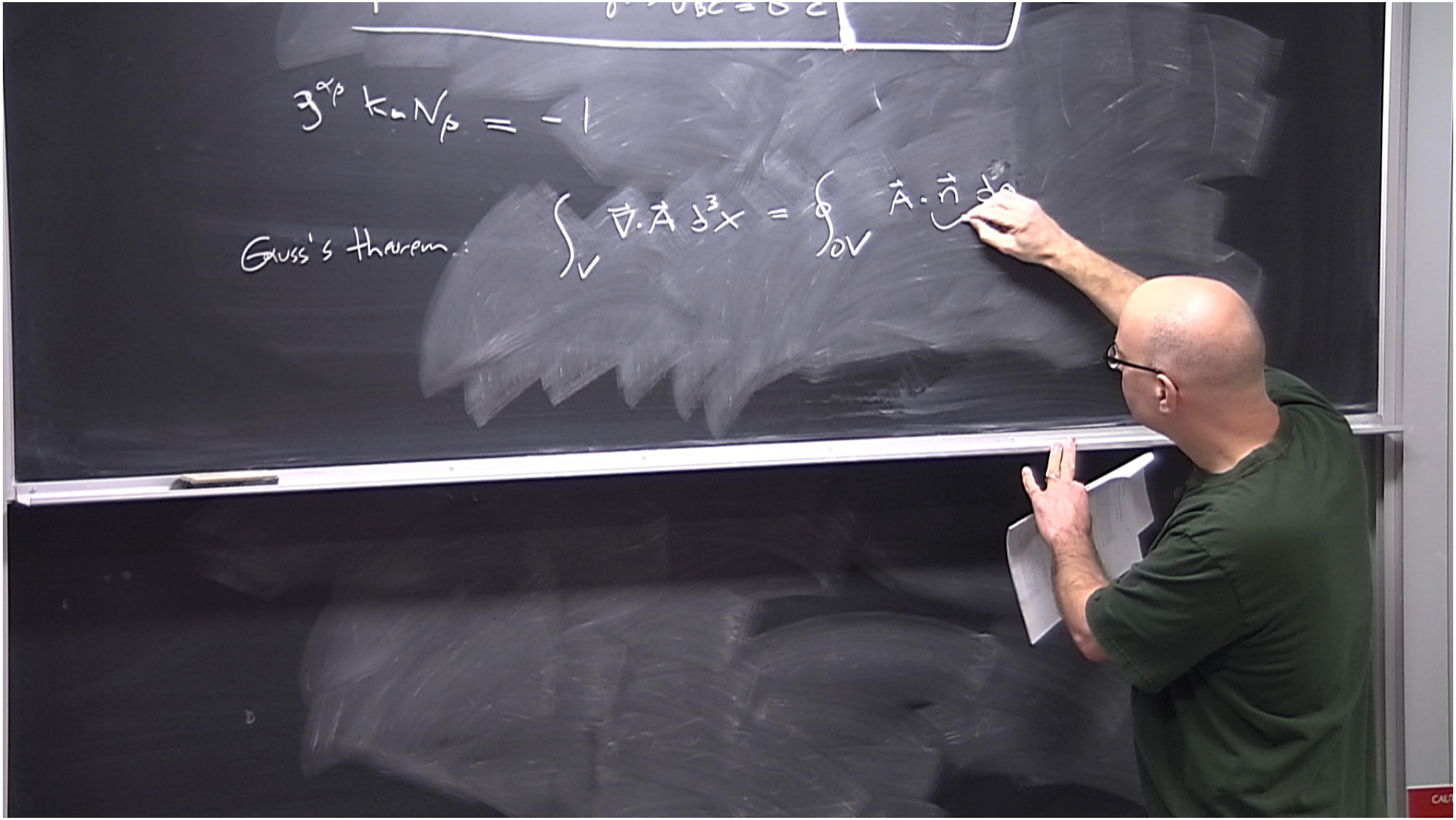

Gauss's theorem


Gauss's theorem:
$\int_V \vec{\nabla}\cdot\vec{A}\, d^3x = \oint$


$\sigma^{AB}\sigma_{BC} = \delta^{A}{}_{C}$
$g^{\alpha\beta} k_\alpha N_\beta = -1$
Gauss's theorem: $\int_V \vec{\nabla}\cdot\vec{A}\, d^3x = \oint_{\partial V} \vec{A}\cdot\vec{n}\, da$


$\mathcal{J}^{\alpha\rho}\, k_\alpha N_\rho = -1$
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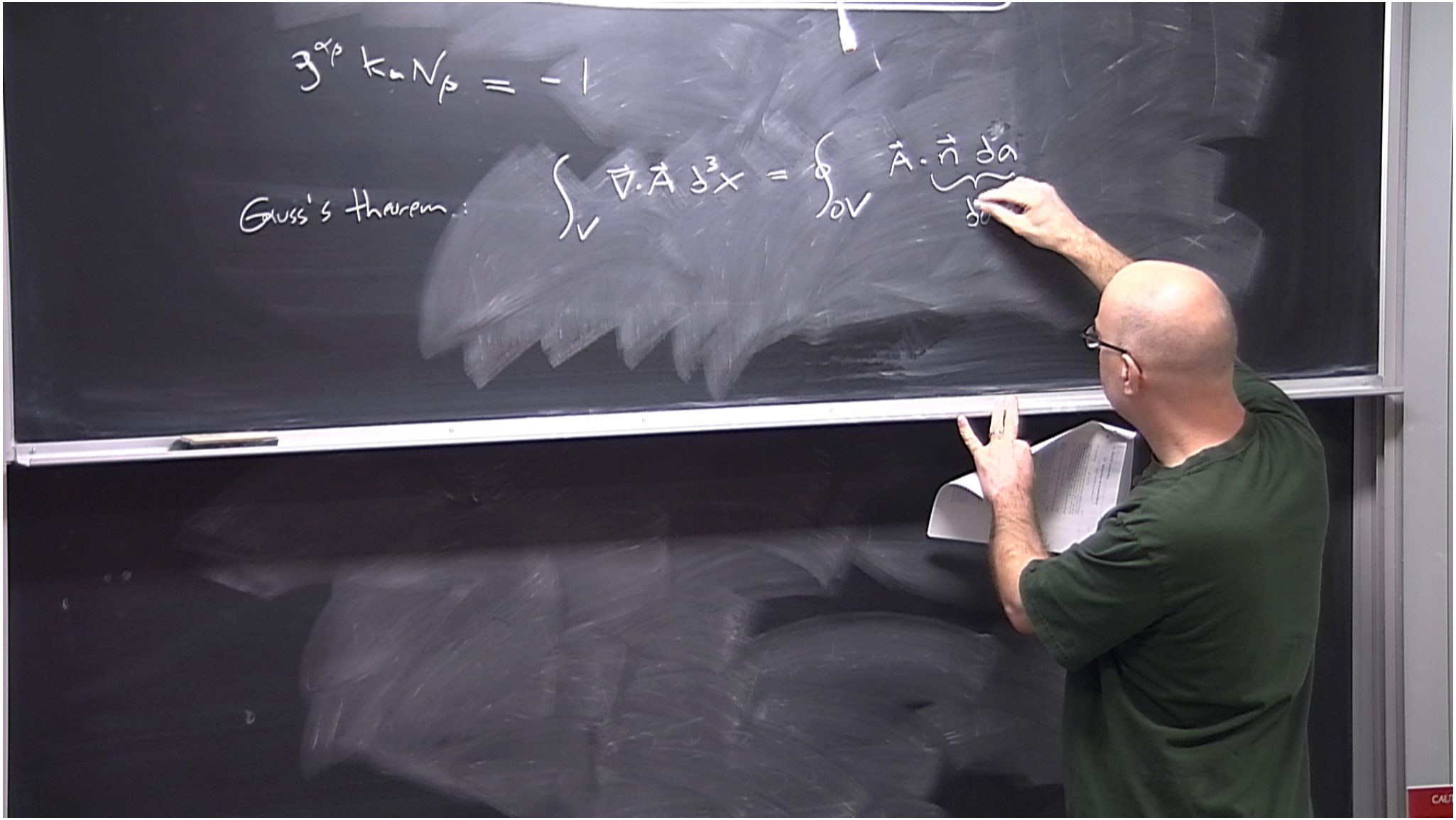
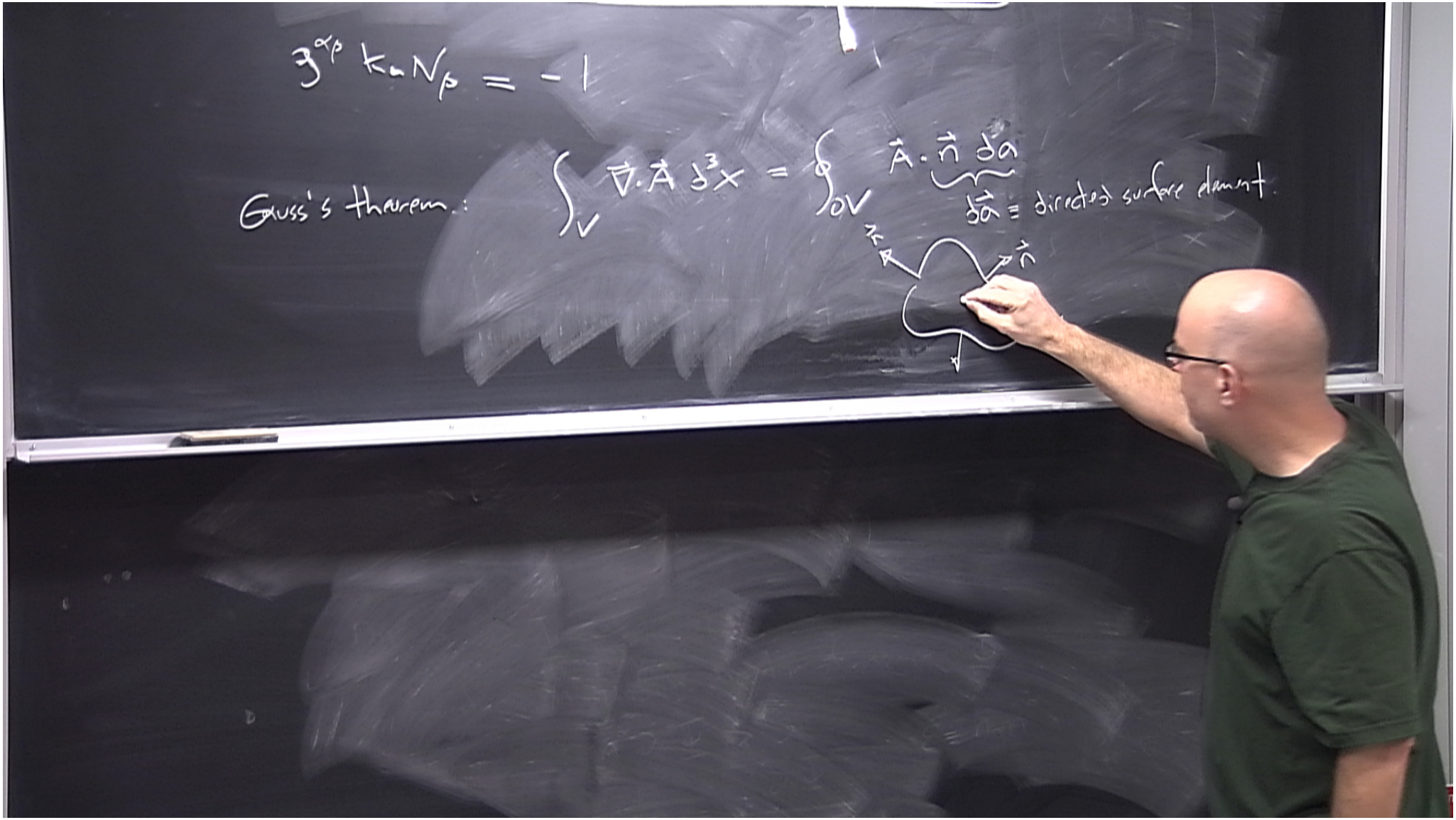
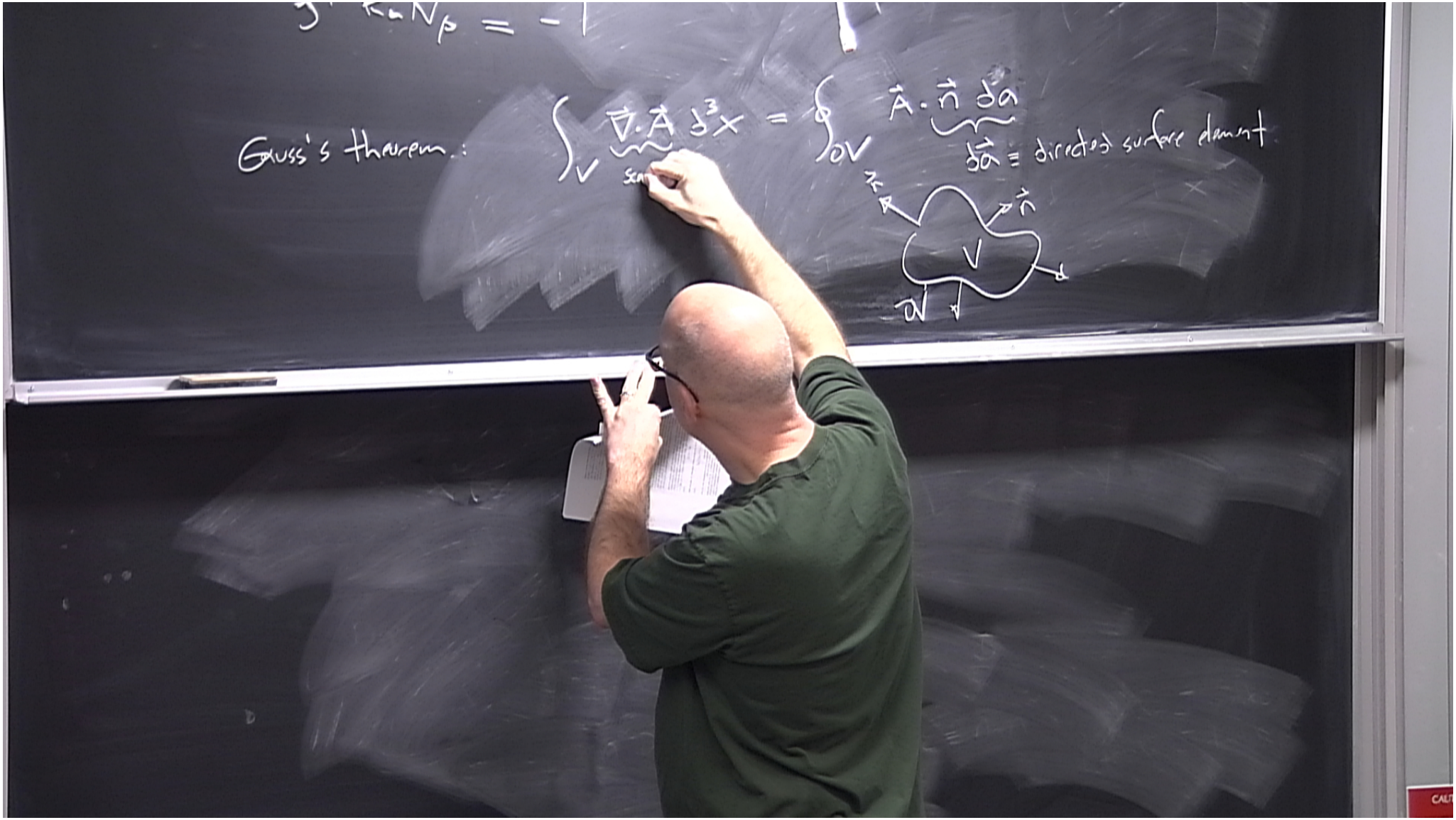
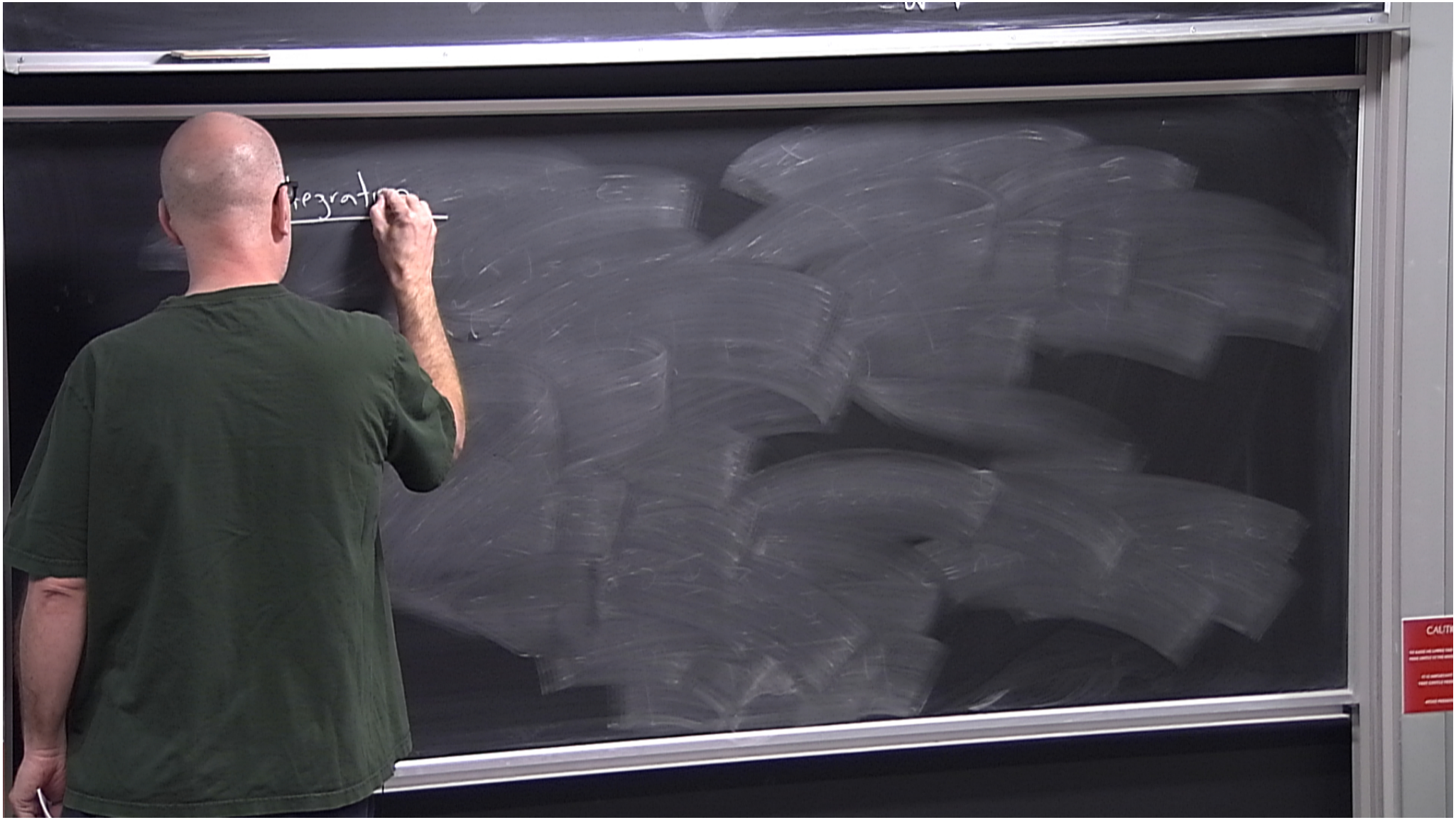
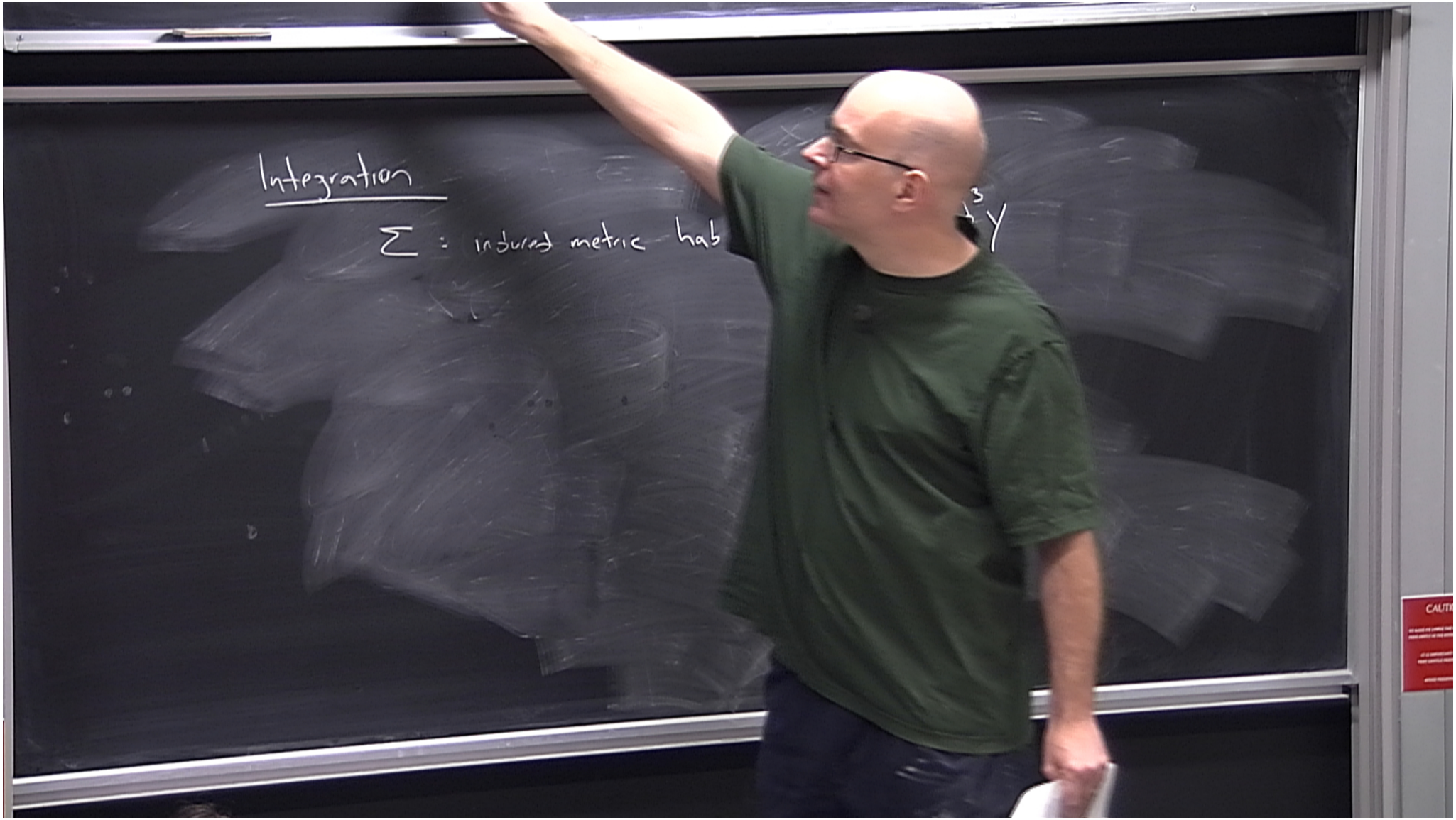
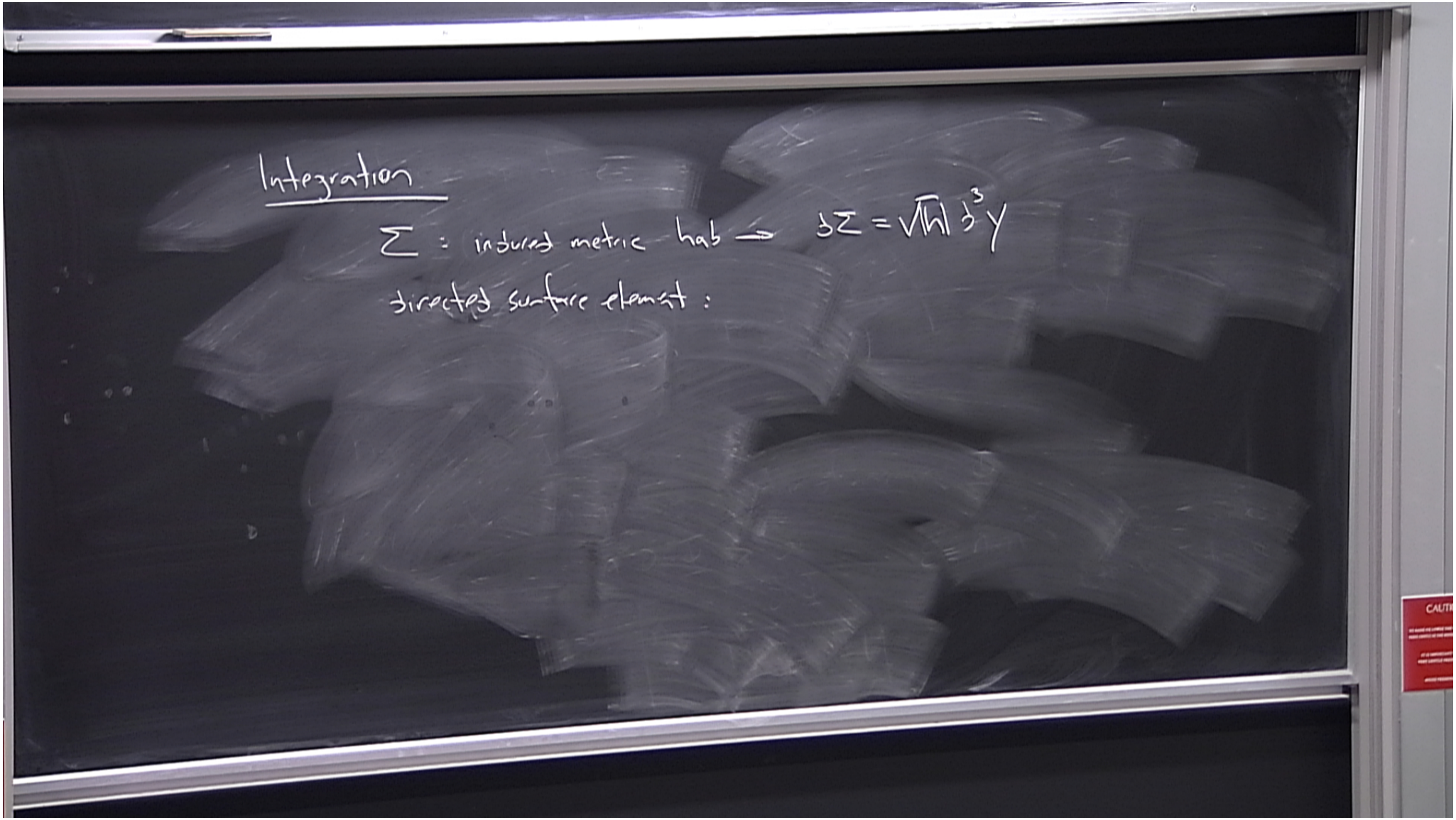

$\exists^{\alpha_p} k_\alpha N_p = -1$
Gauss's theorem: $\int_V \vec{\nabla}\cdot\vec{A}\, d^3x = \oint_{\partial V} \vec{A}\cdot\vec{n}\, da$

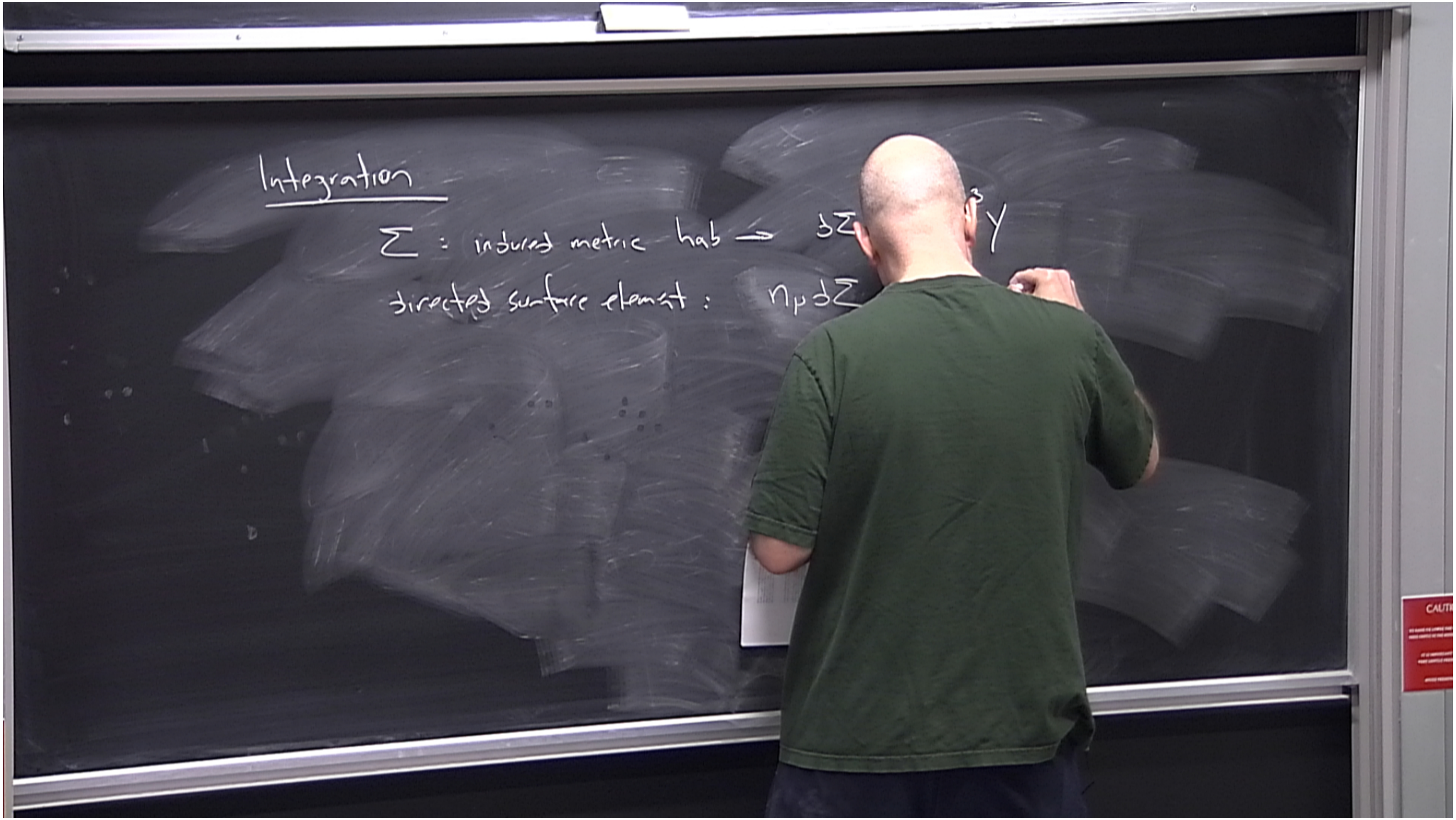

$g^{\alpha\beta} k_\alpha N_\beta = -1$
Gauss's theorem:
$\int_V \vec{\nabla}\cdot\vec{A}\, d^3x = \oint_{\partial V} \vec{A}\cdot\underbrace{\vec{n}\, da}$
$d\vec{a} \equiv$ directed surface element.
$\vec{n}$
$\vec{n}$

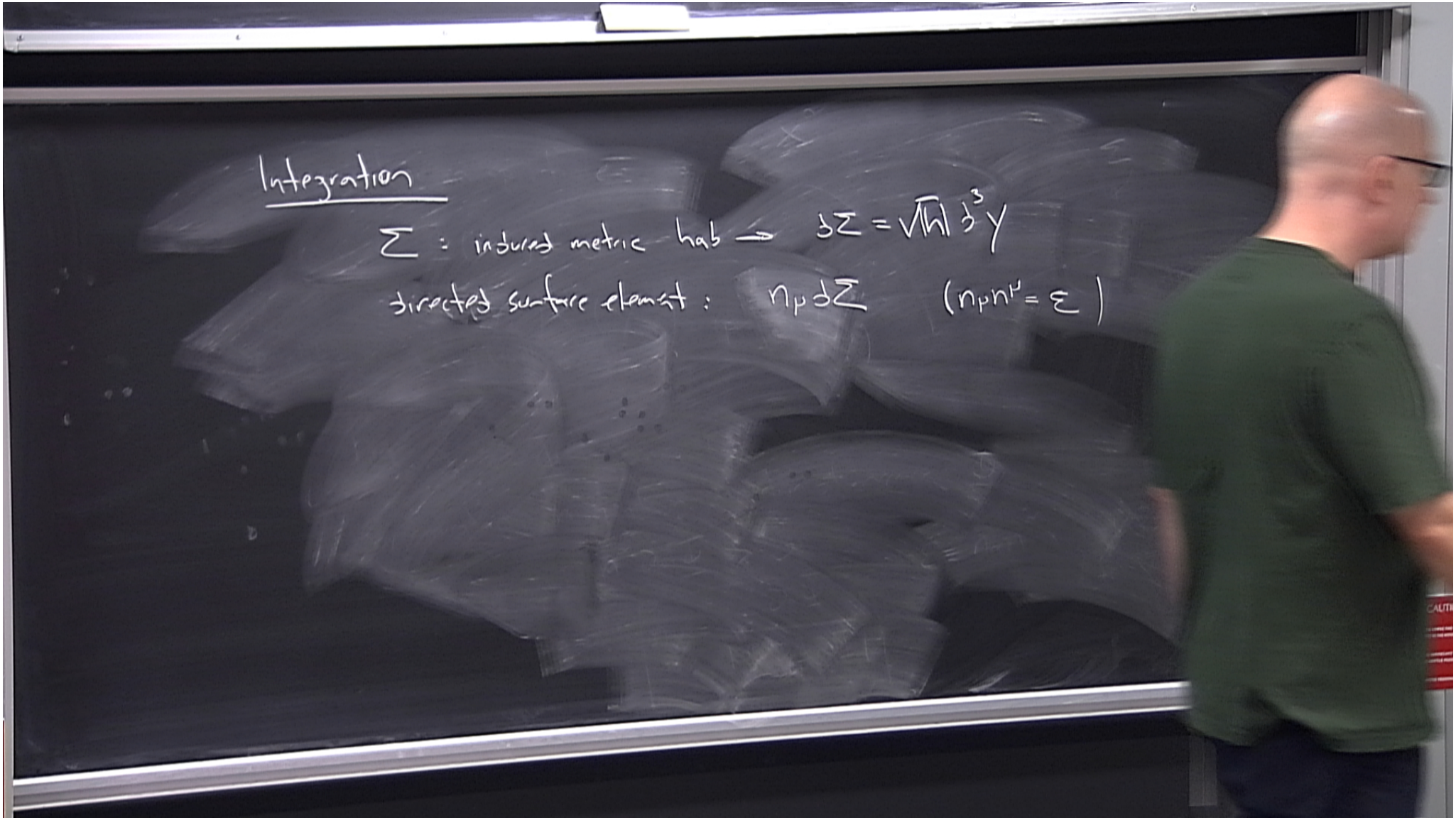

Gauss's theorem:
$\int_V \vec{\nabla}\cdot\vec{A}\, d^3x = \oint_{\partial V} \vec{A}\cdot\vec{n}\, da$
$\vec{da}$ = directed surface element
$\vec{n}$
∂V

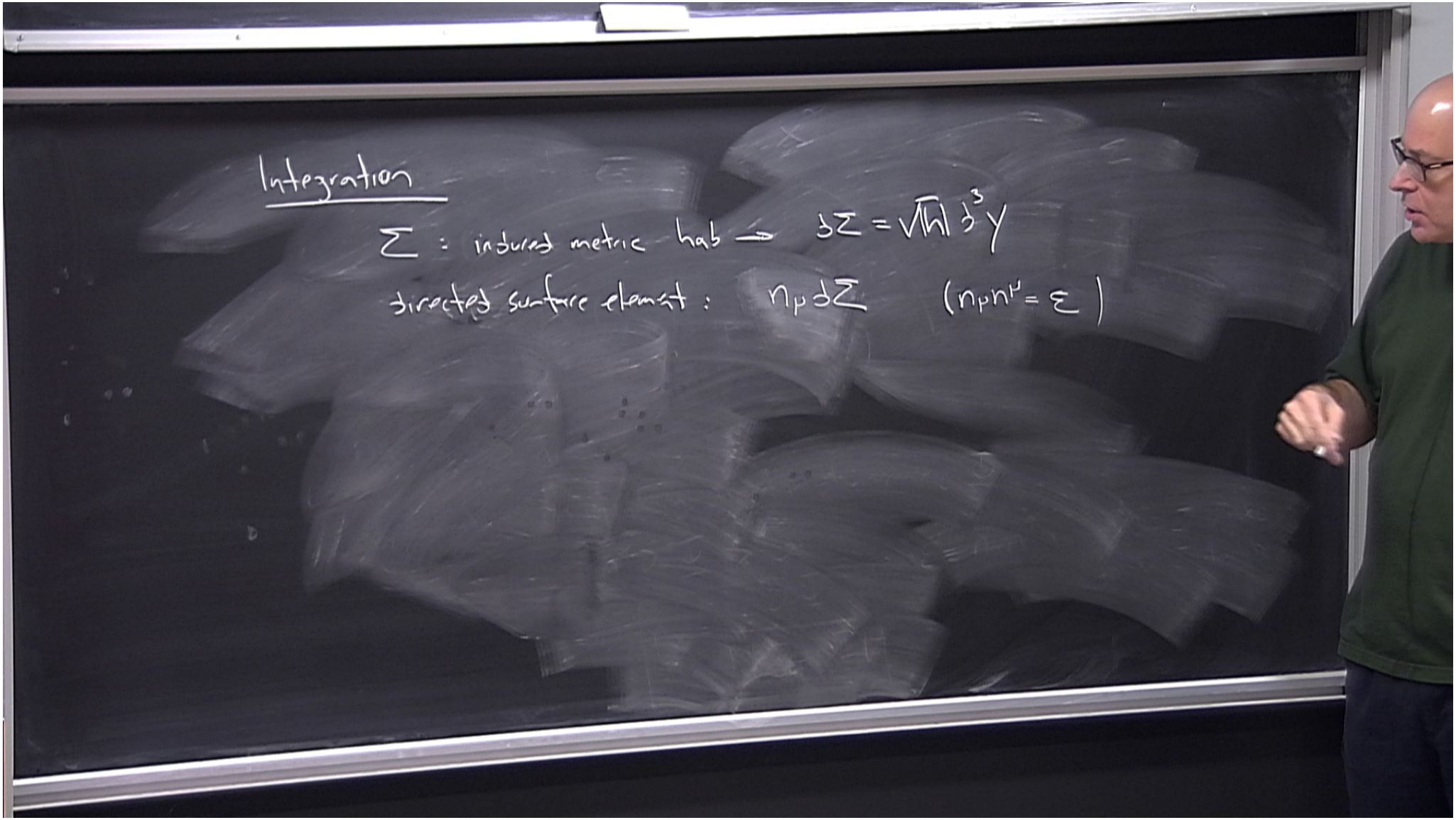


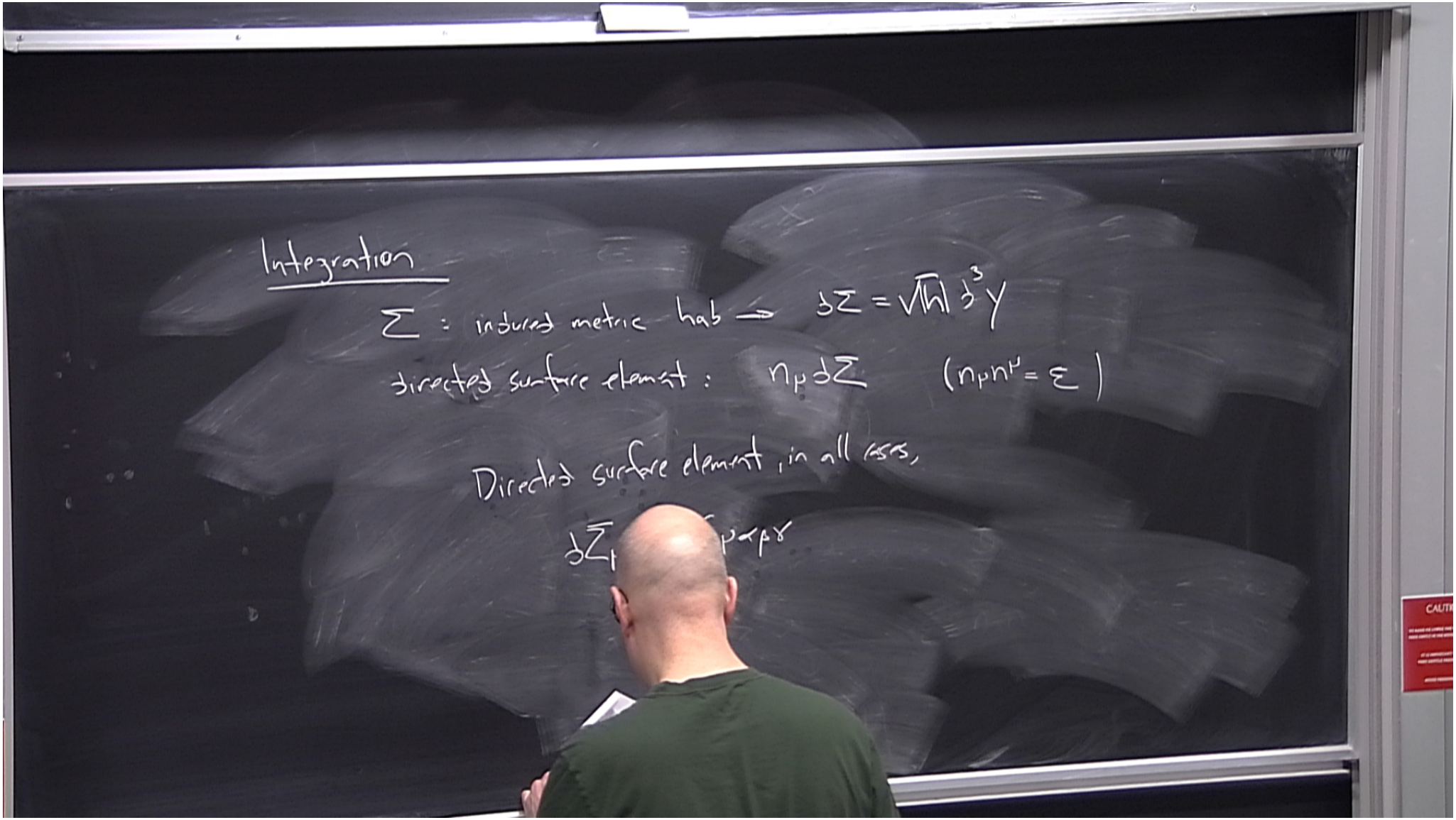

Integration
Σ : induced metric hab


Integration
Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$
directed surface element :


Integration
Σ : induced metric hab →
directed surface element : nμ dΣ


Integration
Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$
directed surface element : $n_\mu d\Sigma$ $(n_\mu n^\nu = \varepsilon)$


Integration
Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$
directed surface element : $n_\mu d\Sigma$ $(n_\mu n^\mu = \varepsilon)$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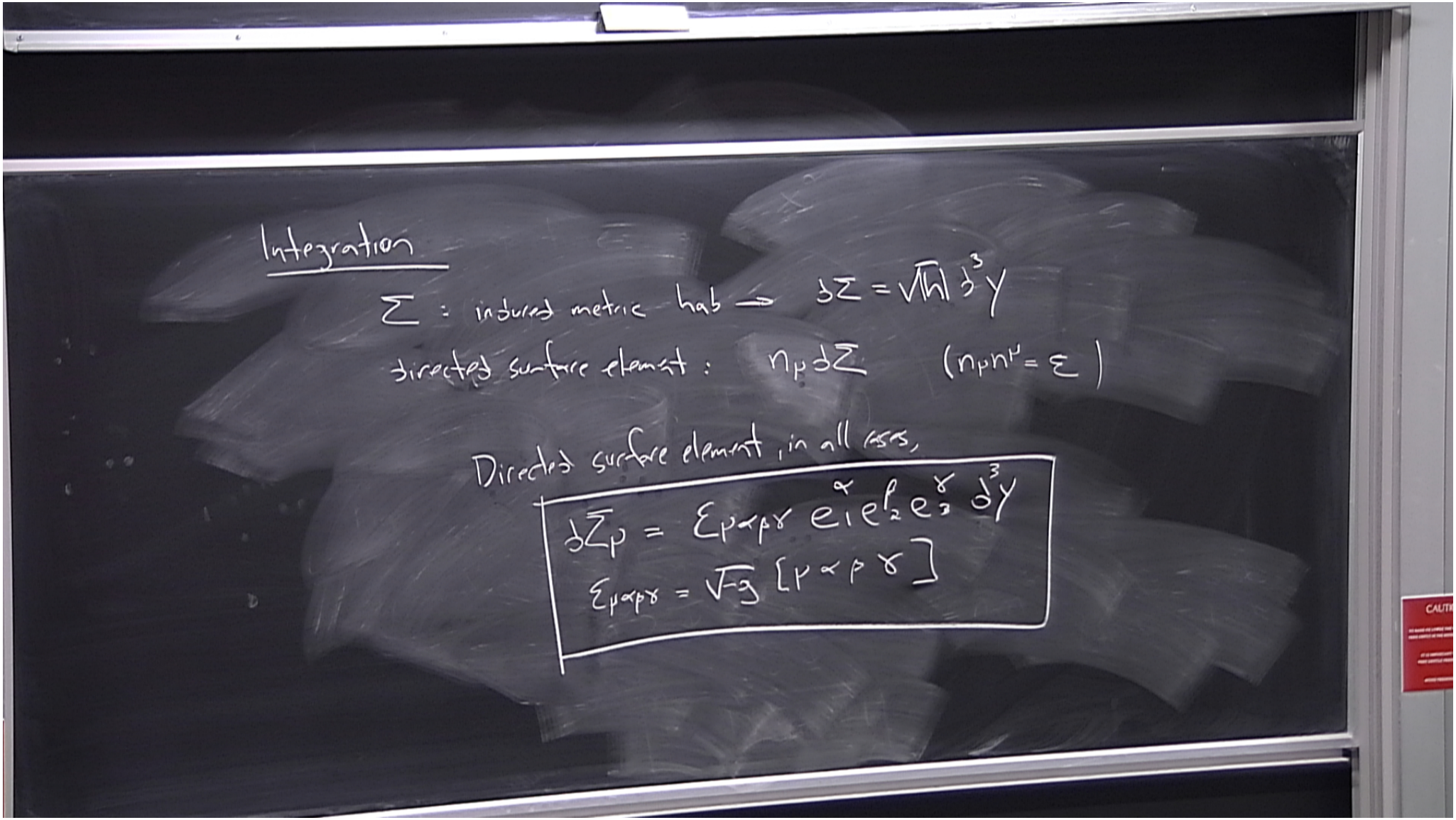
Directed surface element, in all cases,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Directed surface element, in all cases,
$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$\epsilon_{\mu\alpha\beta\gamma} = \sqrt{-g}\,[\mu\alpha\beta\gamma]$

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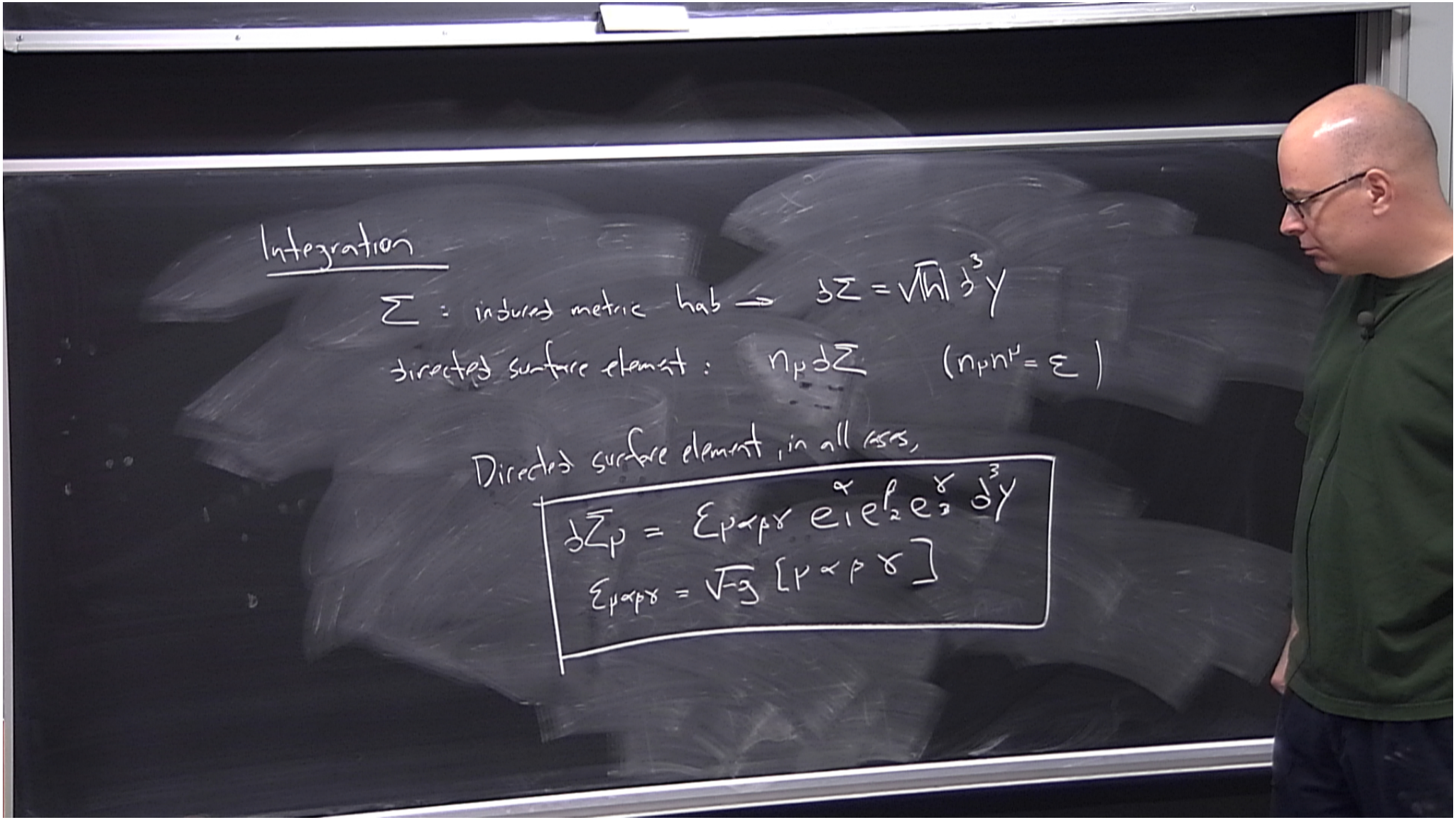
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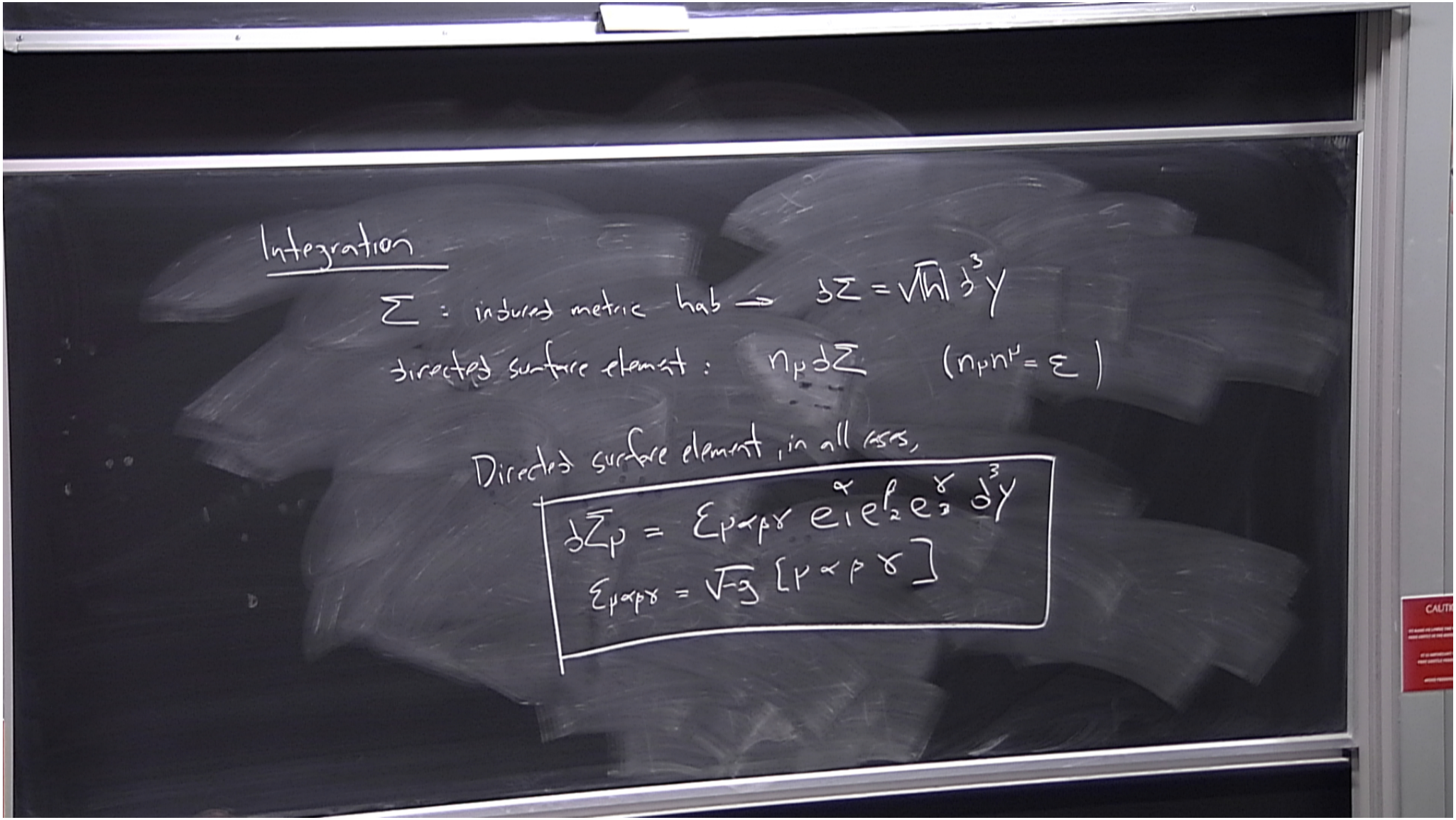
directed surface element : nμ dΣ (nμnμ = ε)
Directed surface element, in all cases,
dΣμ = εμαβγ e₁^α e₂^β e₃^γ d³y
εμαβγ = √-g [μαβγ]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Directed surface element, in all cases,
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Integration

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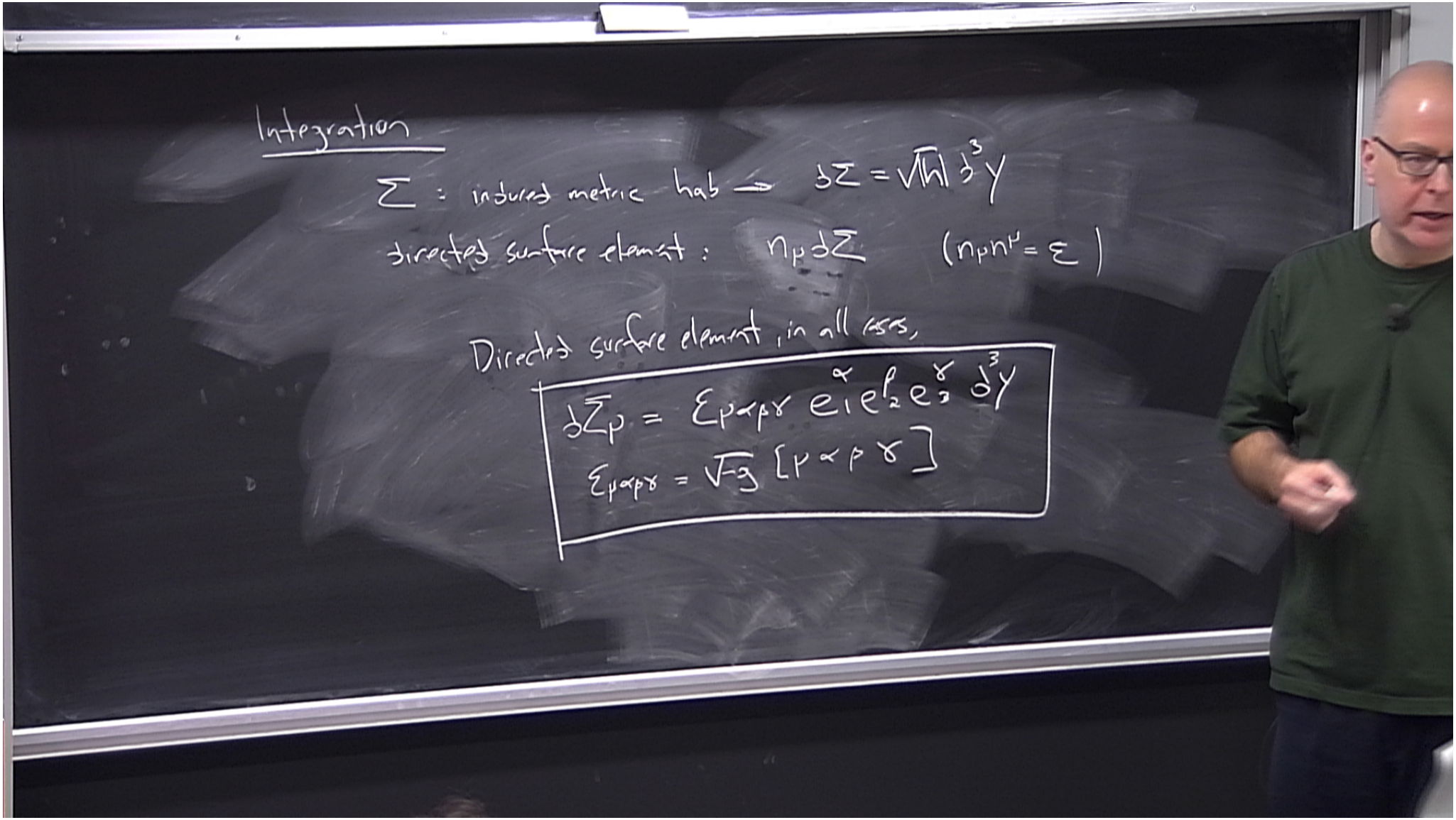
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Directed surface element, in all cases,
dΣμ = εμαβγ e¹α e²β e³γ d³y
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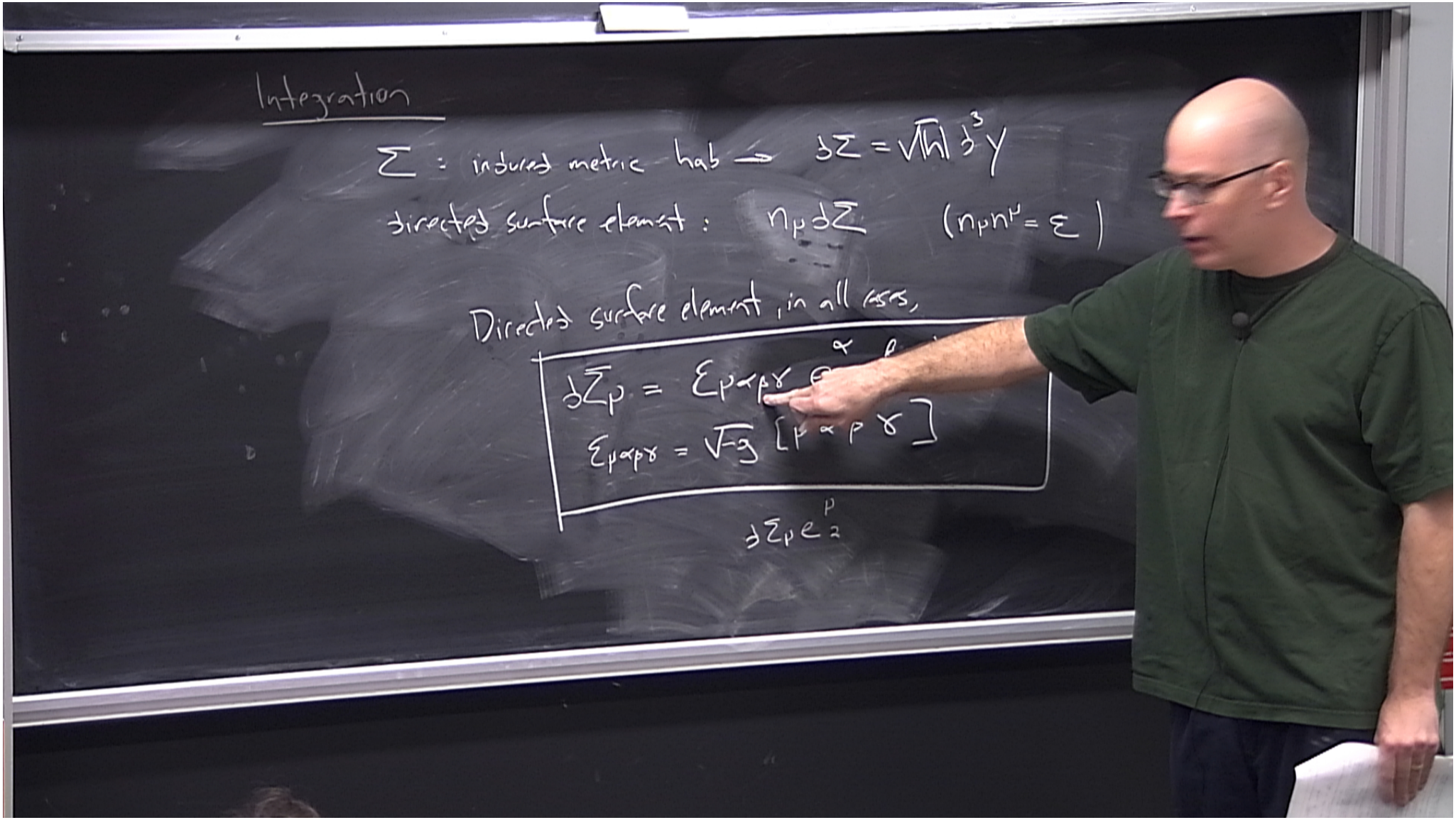
Integration
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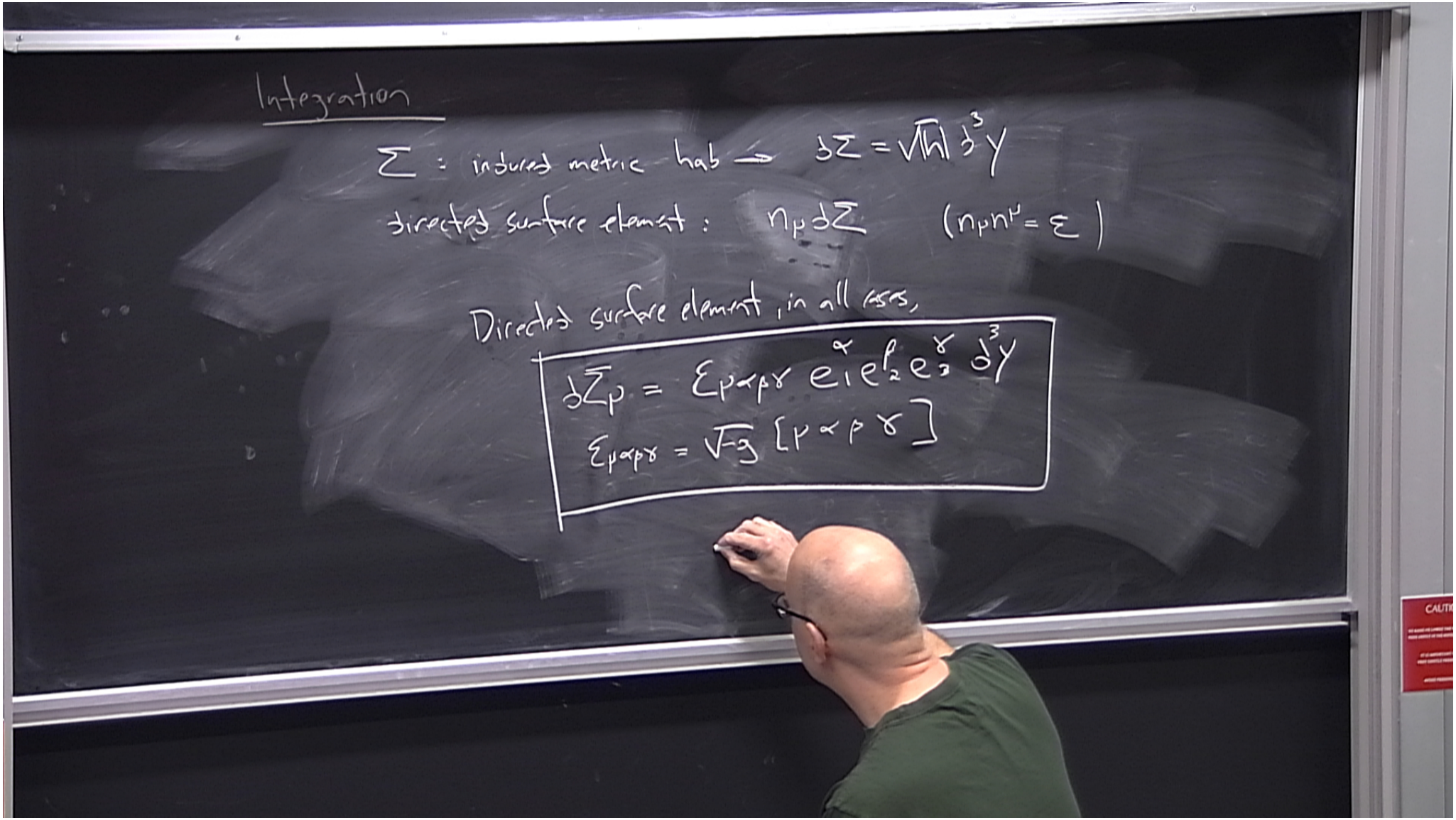
directed surface element : $n_\mu d\Sigma$ $(n_\mu n^\nu = \epsilon)$
Directed surface element, in all cases,
$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma}$
$\epsilon_{\mu\alpha\beta\gamma} = \sqrt{-g}\,[\mu\alpha\beta\gamma]$
$d\Sigma_\mu e^\mu_2$


Integration
Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$
directed surface element : $n_\mu d\Sigma$ $(n_\mu n^\nu = \epsilon)$
Directed surface element, in all cases,
$d\vec{\Sigma}_\rho = \epsilon_{\rho\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$\epsilon_{\rho\alpha\beta\gamma} = \sqrt{-g}\,[\rho\alpha\beta\gamma]$

Integration

Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$

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Directed surface element, in all cases,

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$$d\Sigma_\mu = f\, n_\mu$$

Integration

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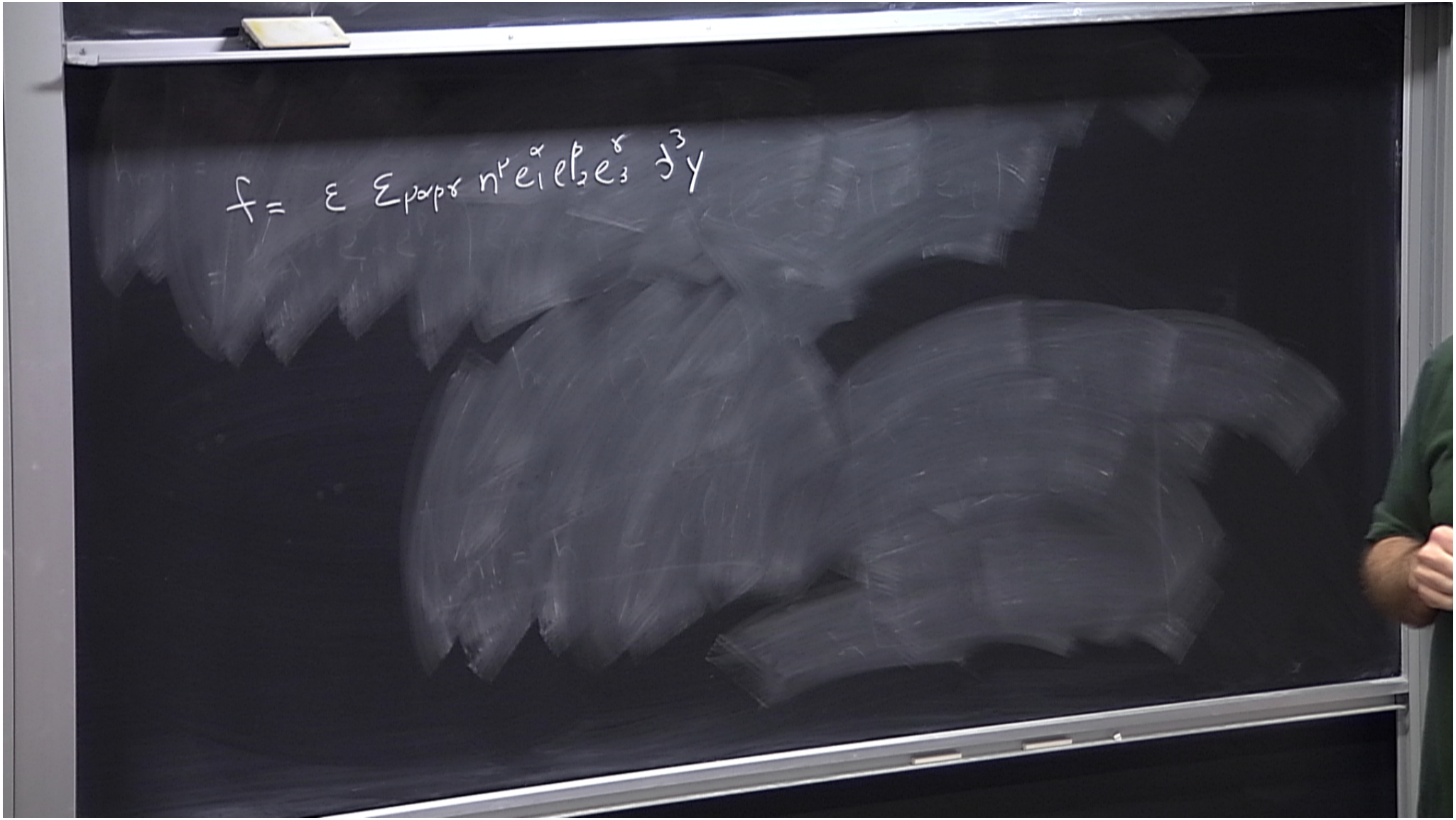
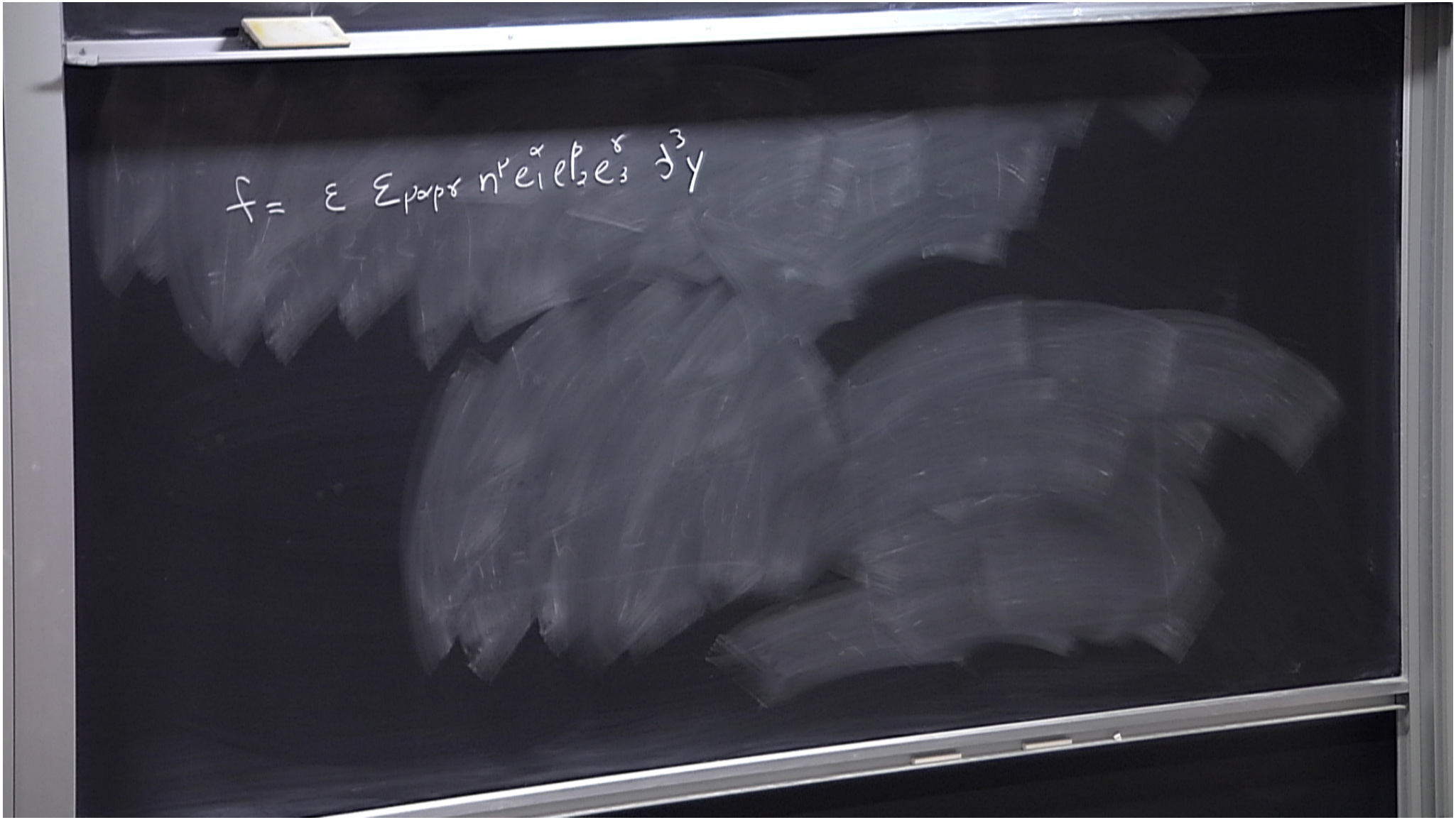
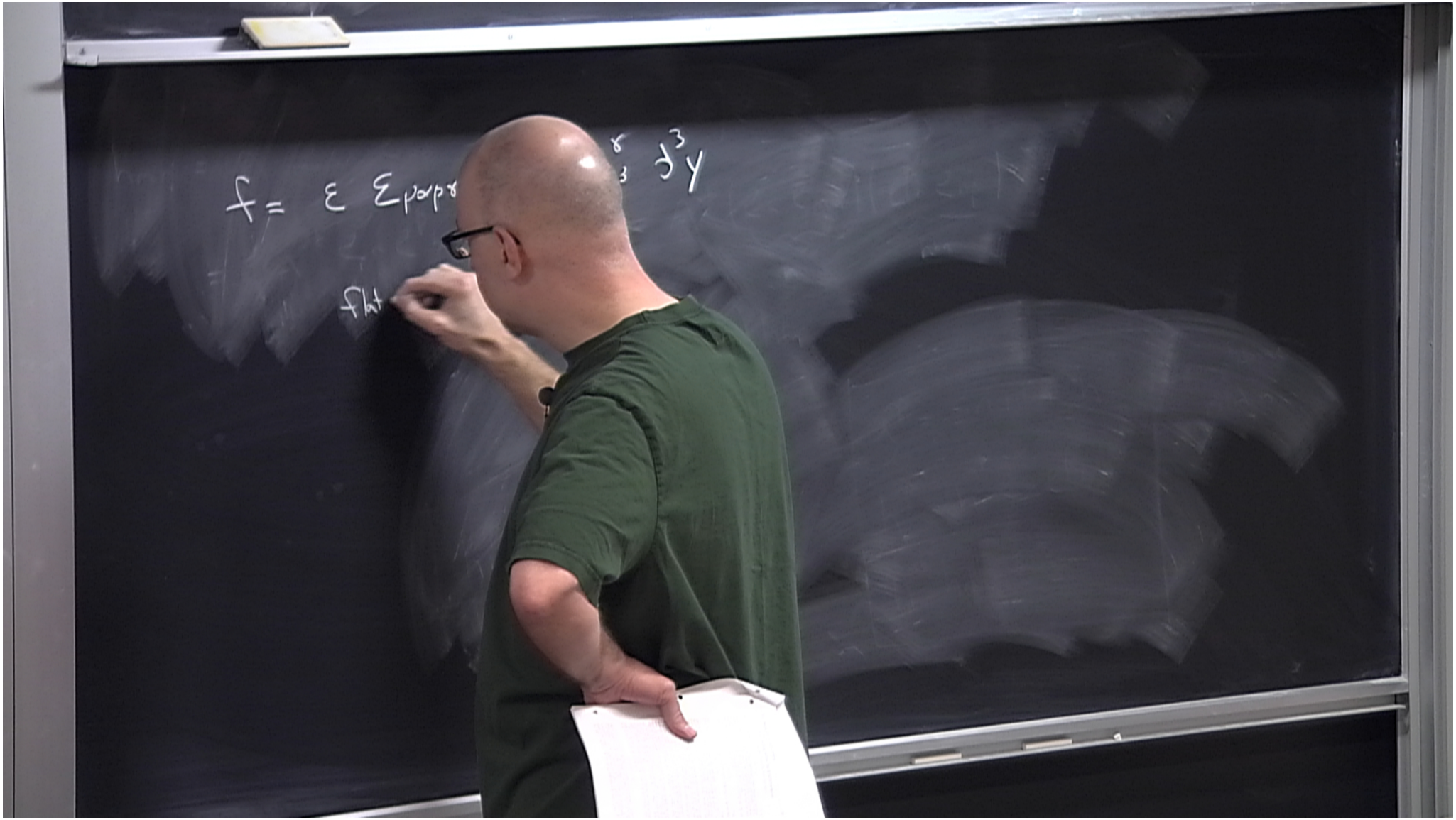
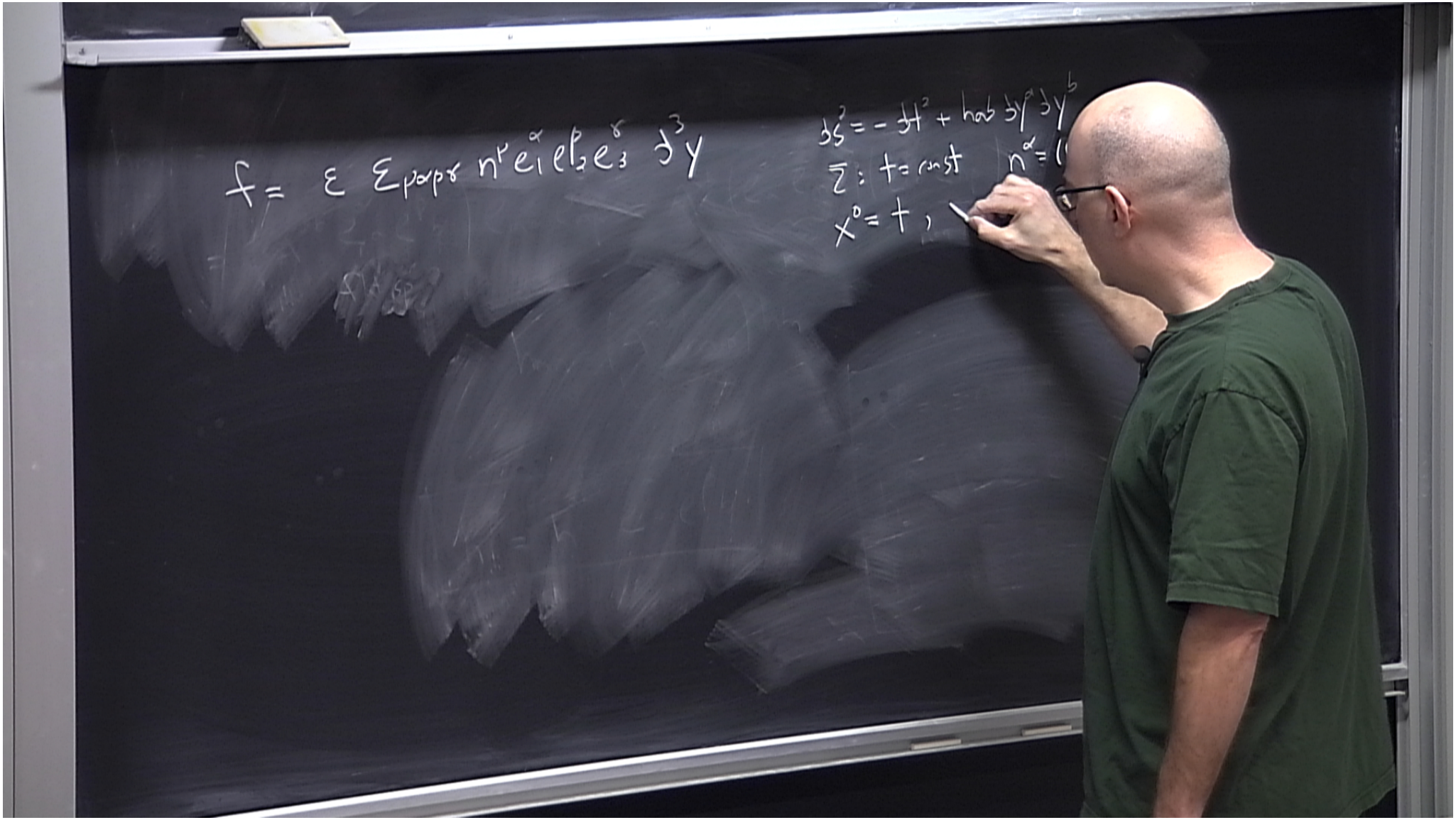
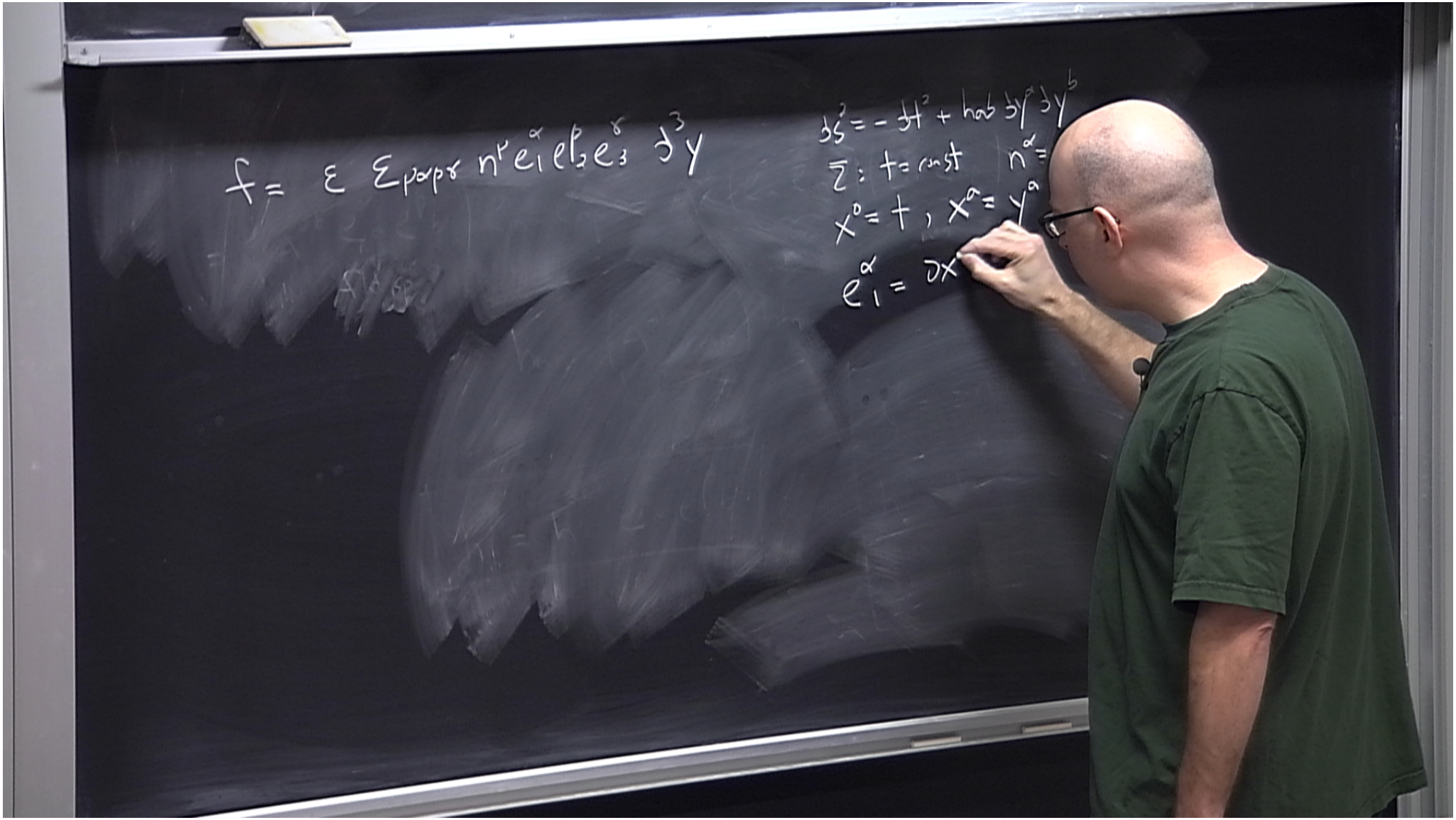
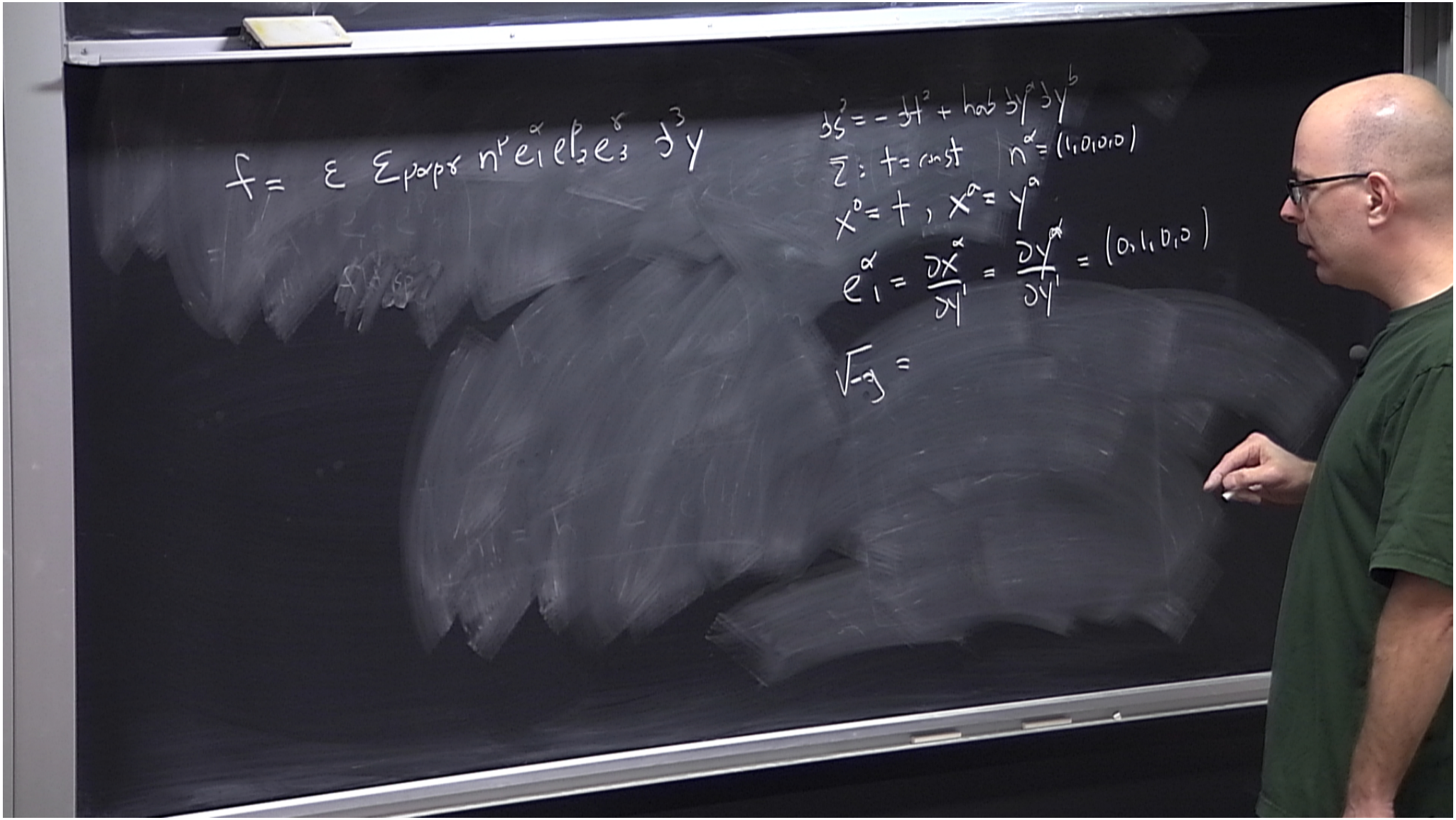

$f = \varepsilon \, \varepsilon_{\rho\alpha\beta\gamma} \, n^\rho e_1^\alpha e_2^\beta e_3^\gamma \, d^3y$

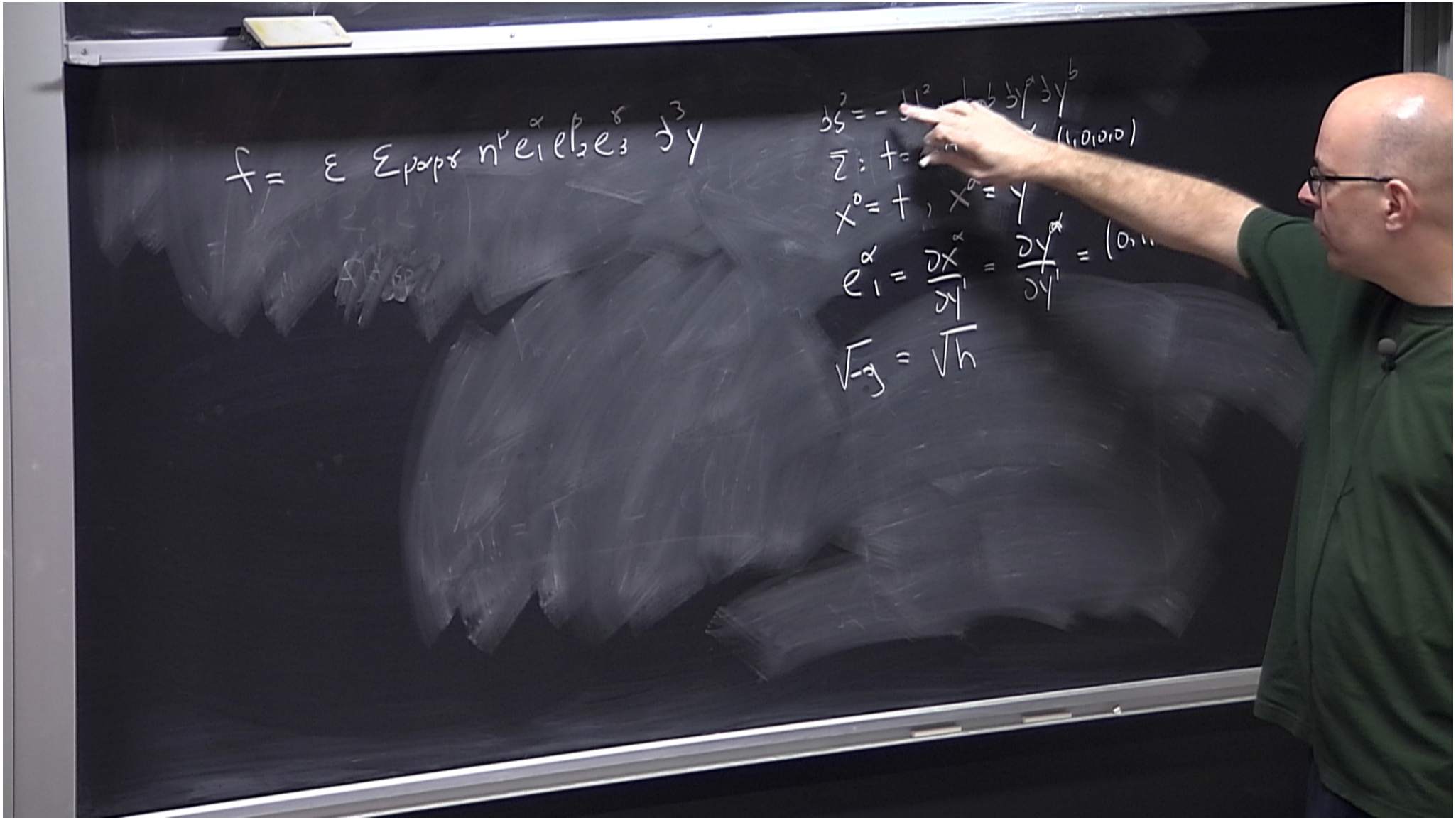

$f = \varepsilon\, \varepsilon_{\mu\alpha\beta\gamma}\, n^\mu e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$

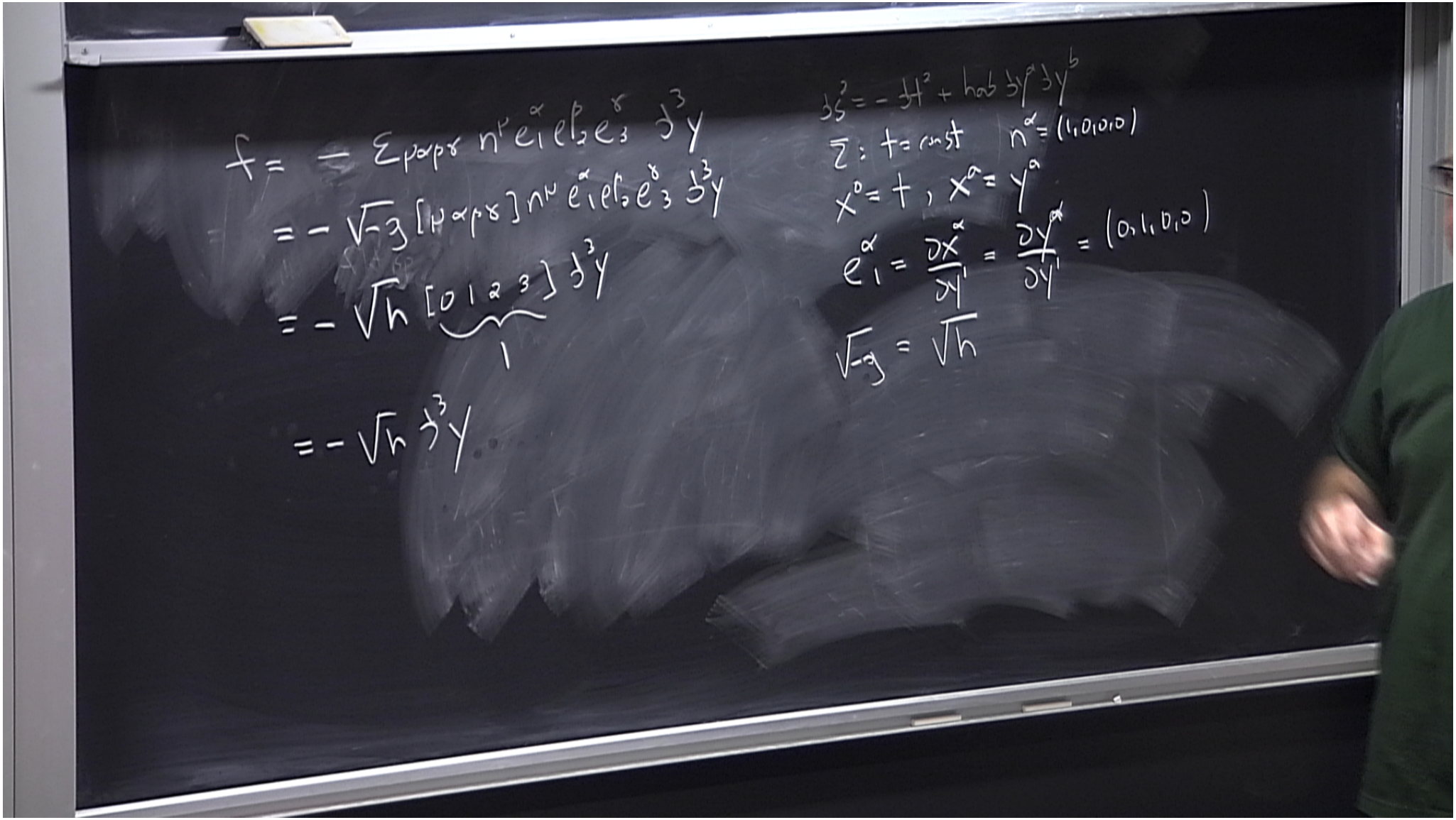



$f = \varepsilon \, \varepsilon_{\mu\alpha\beta\gamma} \, n^\mu e_1^\alpha e_2^\beta e_3^\gamma \, d^3y$
$ds^2 = -dt^2 + h_{ab} dy^a dy^b$
$\Sigma : t = \text{const}$
$x^0 = t,$


$f = \varepsilon \, \varepsilon_{\mu\rho\sigma} \, n^\nu e_1^\alpha e_2^\beta e_3^\gamma \, d^3y$
$ds^2 = -dt^2 + h_{ab} dy^a dy^b$
$\Sigma : t = \text{const}$
$x^0 = t , x^a = y^a$
$e_1^\alpha = \partial x$


$f = \varepsilon \Sigma_{\mu\rho\sigma} n^\tau e_1^\mu e_2^\rho e_3^\sigma \, d^3y$
$ds^2 = -dt^2 + h_{ab} dy^a dy^b$
Σ : t = const $n^\alpha = (1,0,0,0)$
$x^0 = t$, $x^a = y^a$
$e_1^\alpha = \frac{\partial x^\alpha}{\partial y^1} = \frac{\partial y^\alpha}{\partial y^1} = (0,1,0,0)$
$\sqrt{-g} =$


f = ε ε_μαβγ n^μ e_1^α e_2^β e_3^γ d^3y
x^0 = t , x^a = y
√-g = √h


$f = = \varepsilon_{\mu\alpha\rho\sigma} n^\mu e_1^\alpha e_2^\rho e_3^\sigma \, d^3y$
$= -\sqrt{-g}\,[\mu\alpha\rho\sigma]\, n^\mu e_1^\alpha e_2^\rho e_3^\sigma \, d^3y$
$= -\sqrt{h}\,\underbrace{[0123]}_{1}\, d^3y$
$= -\sqrt{h}\, d^3y$
$ds^2 = -dt^2 + h_{ab}\, dy^a dy^b$
$\Sigma : t = \text{const}$ $n^\alpha = (1,0,0,0)$
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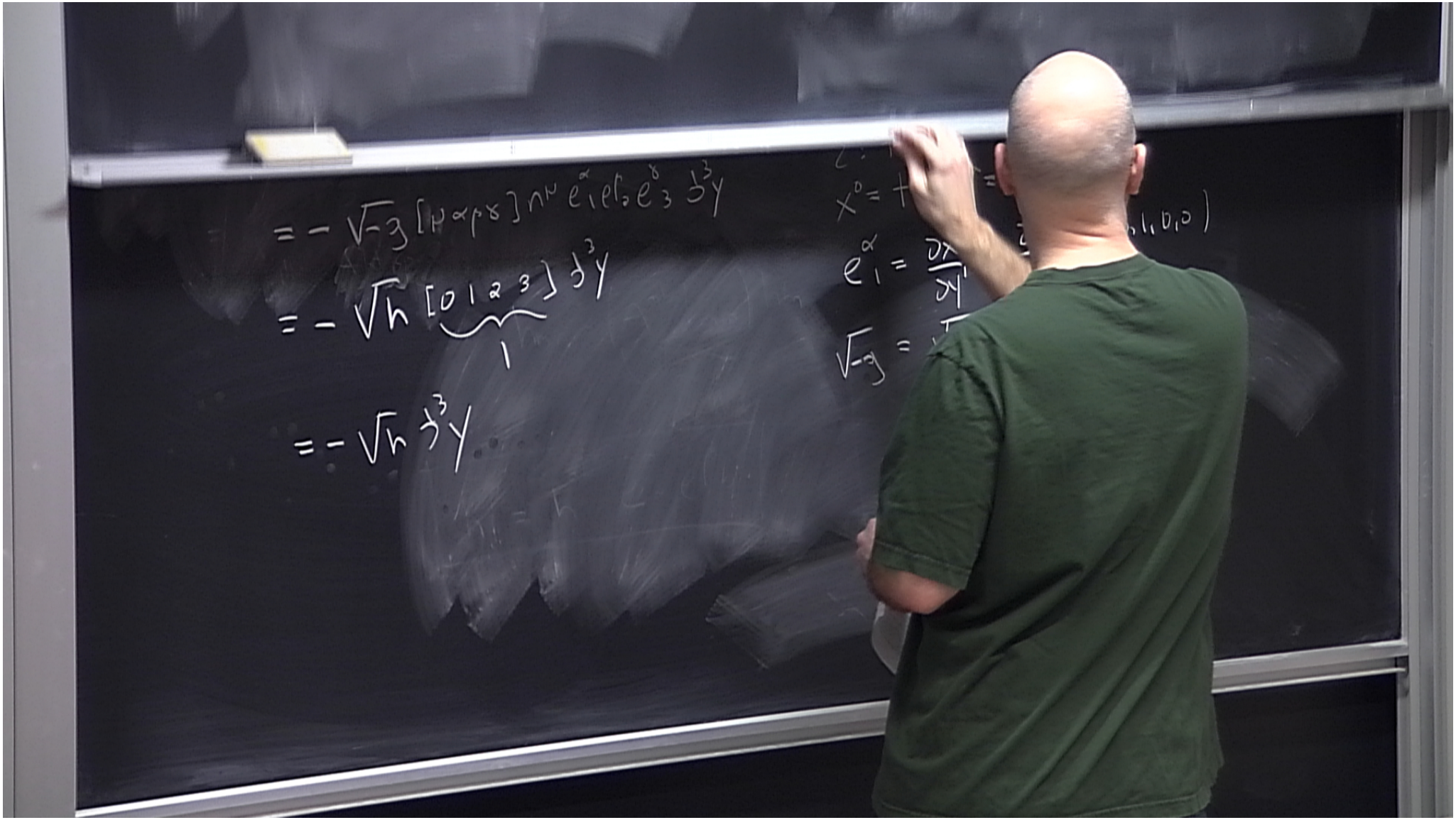
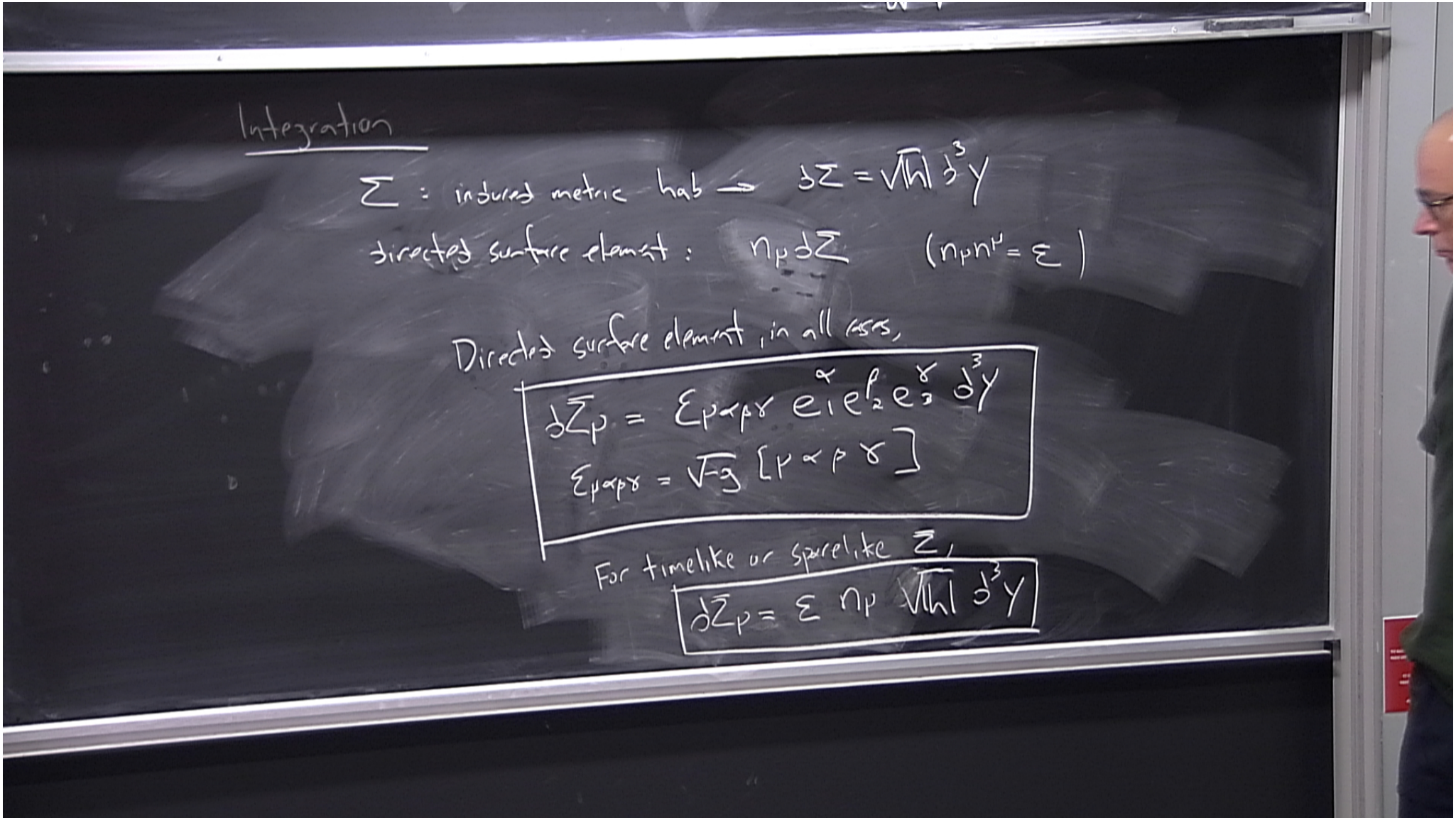
$\sqrt{-g} = \sqrt{h}$


$= -\sqrt{-g}\,[\mu\alpha\rho\sigma]\,n_\nu\, e^\nu_1 e^\alpha_2 e^\beta_3\, d^3y$
$= -\sqrt{h}\,[0123]\,d^3y$
$= -\sqrt{h}\,d^3y$
$x^0 = t$
$e^\alpha_1 = \frac{\partial x}{\partial y}$
$\sqrt{-g} =$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Σ : induced metric hab → dΣ = √|h| d³y
directed surface element : nμ dΣ (nμnμ = ε)
Directed surface element, in all cases,
dΣμ = εμαβγ e¹α e²β e³γ d³y
εμαβγ = √-g [μαβγ]
For timelike or spacelike Σ,
dΣμ = ε nμ √|h| d³y

Integration

Σ : induced metric h_{ab} $\rightarrow$ $d\Sigma = \sqrt{|h|}\, d^3y$

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For timelike or spacelike Σ,

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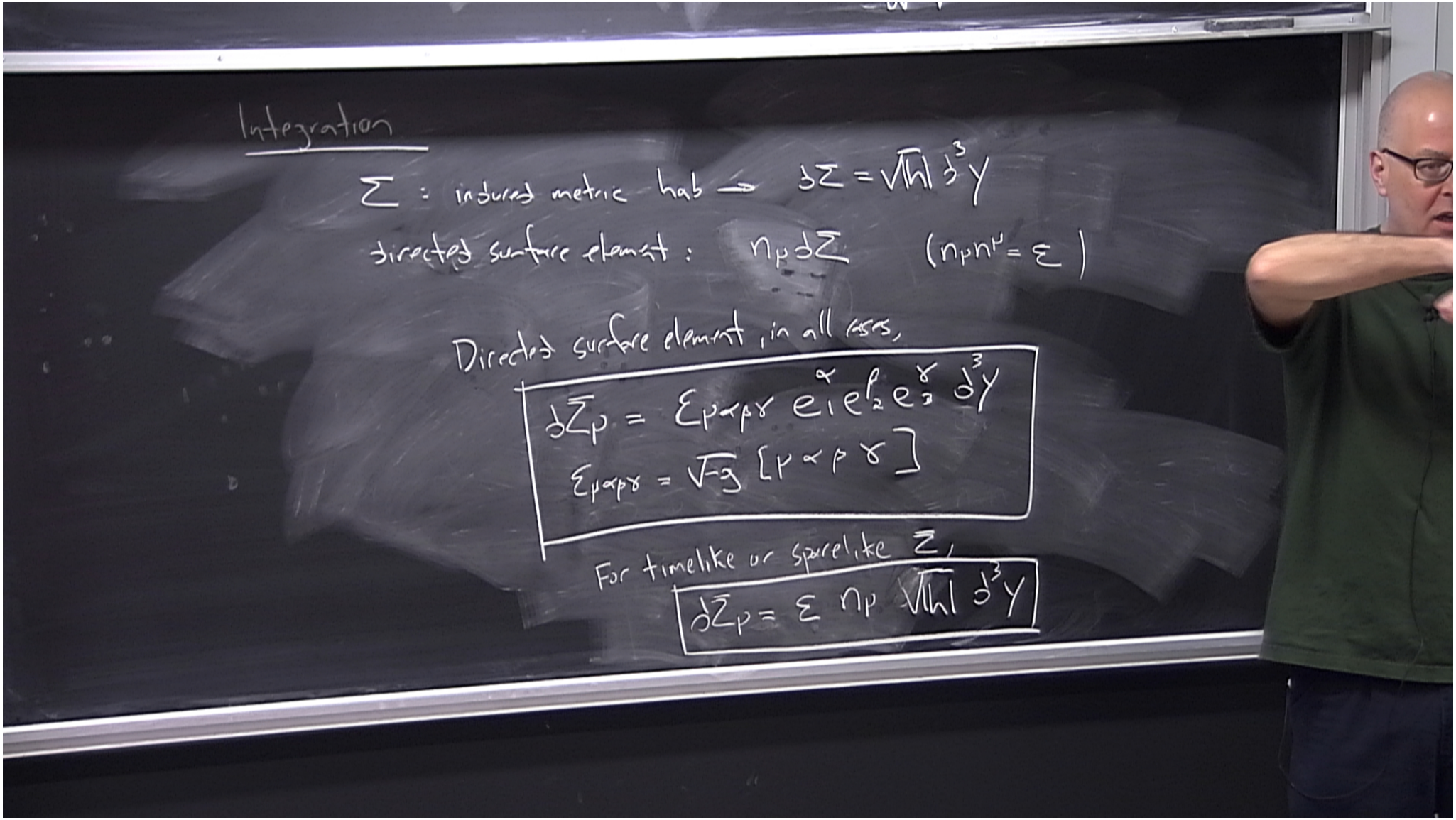
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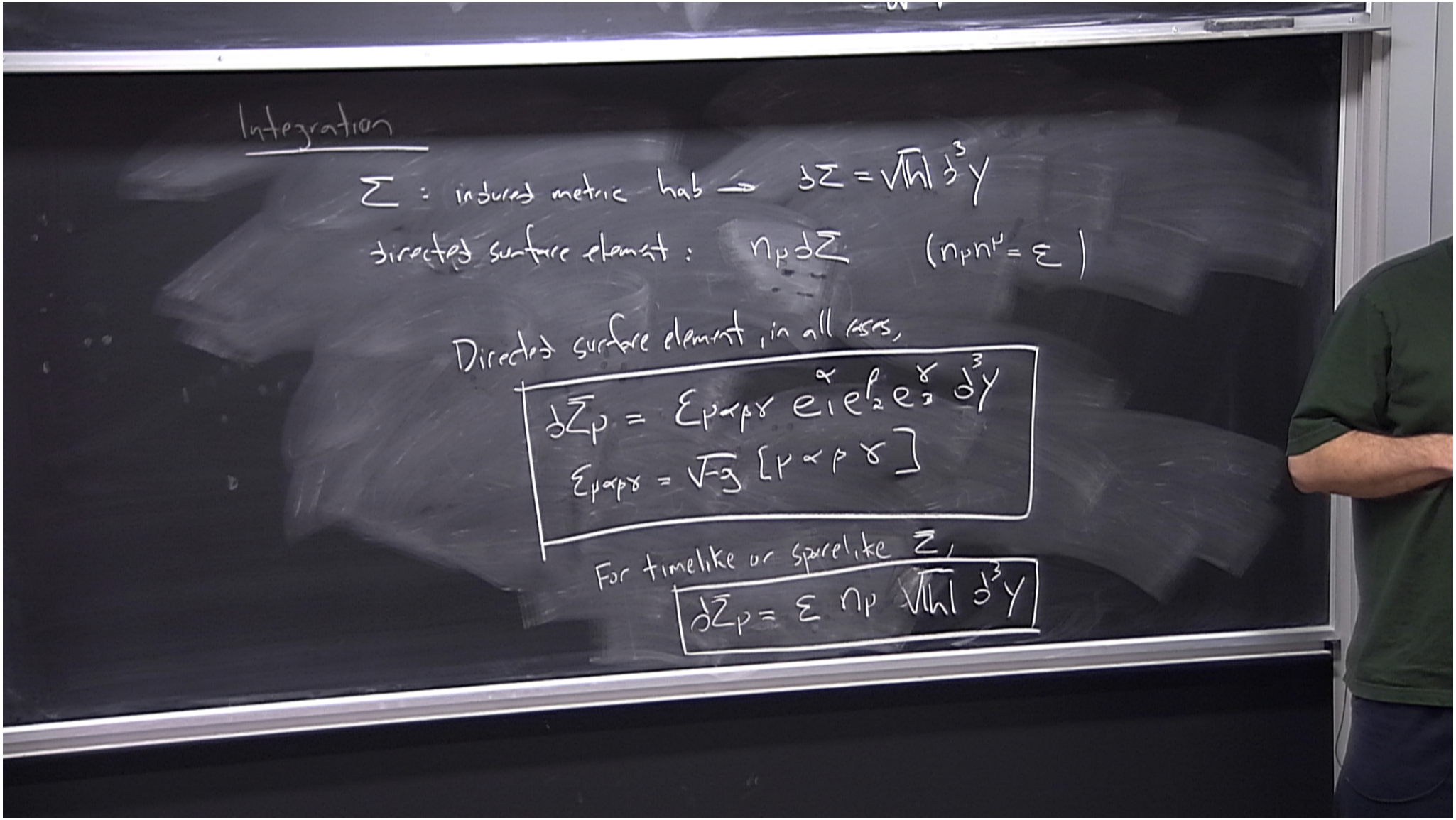
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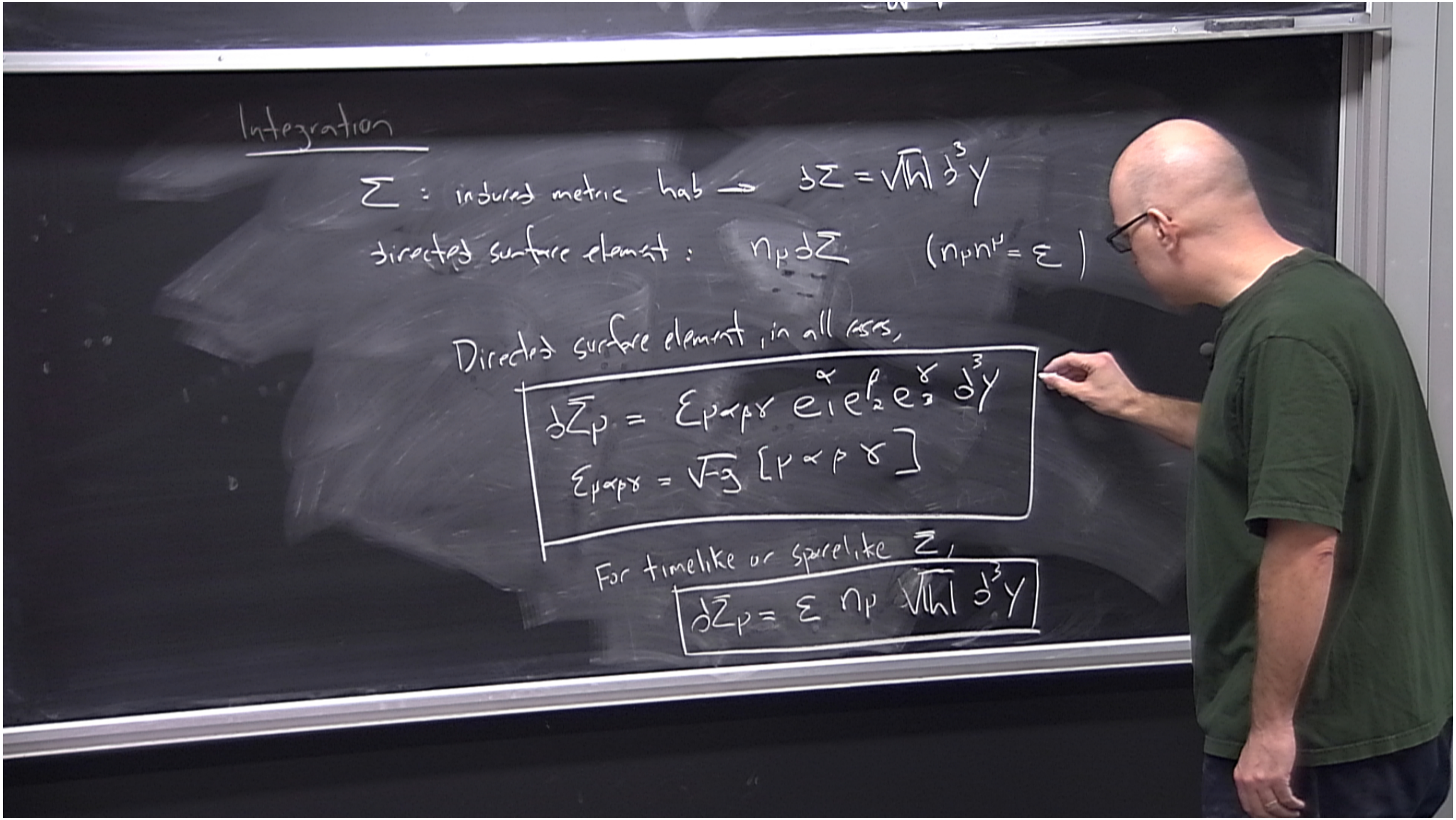
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$\varepsilon_{\mu\alpha\beta\gamma} = \sqrt{-g}\,[\mu\alpha\beta\gamma]$
For timelike or spacelike Σ,
$d\vec{\Sigma}_\mu = \varepsilon\, n_\mu \sqrt{|h|}\, d^3y$

Integration

Σ : induced metric h_{ab} → $d\Sigma = \sqrt{|h|}\, d^3y$

directed surface element : $n_\mu d\Sigma$ $\quad (n_\mu n^\mu = \varepsilon)$

Directed surface element, in all cases,

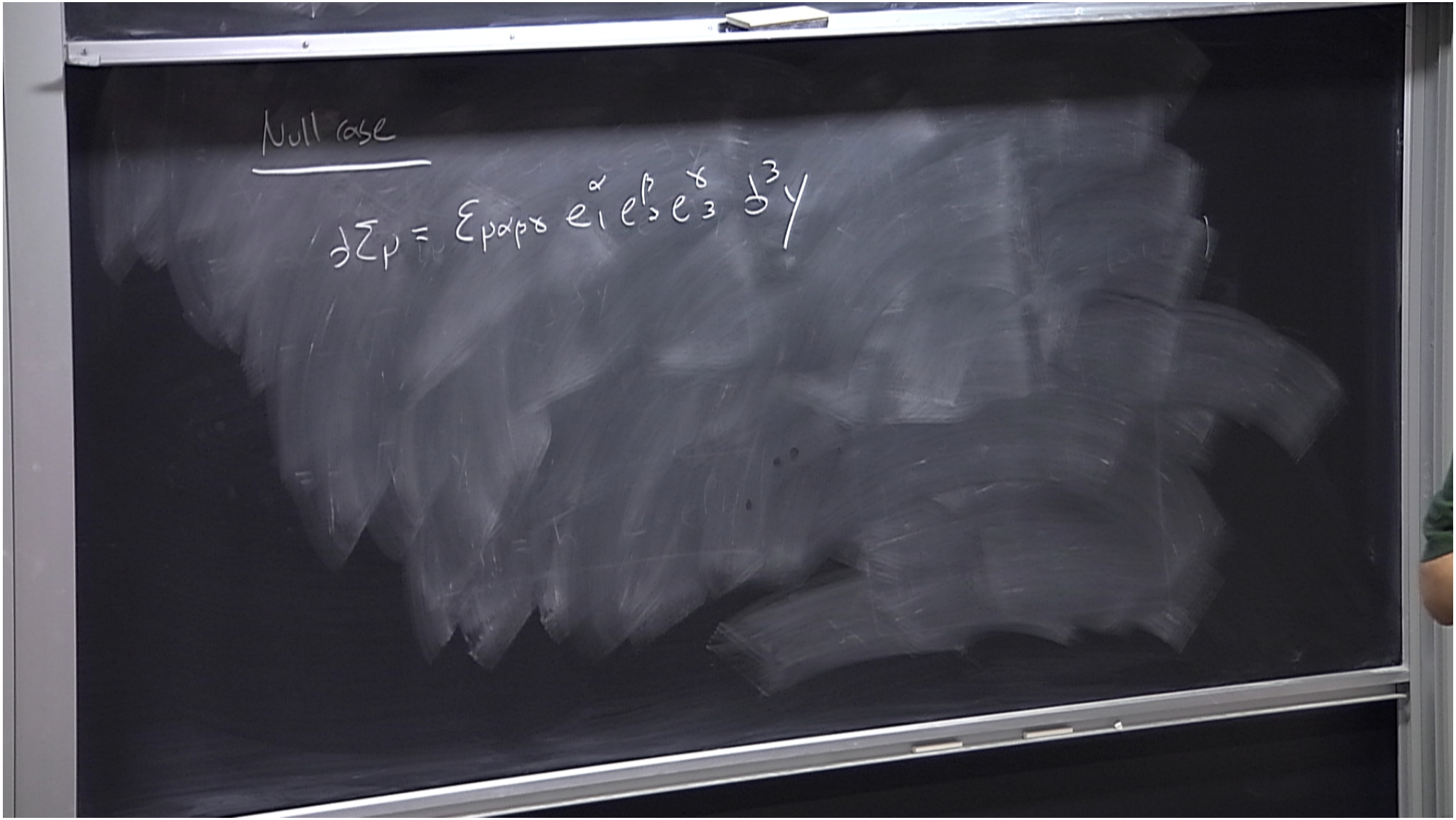
$$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$$

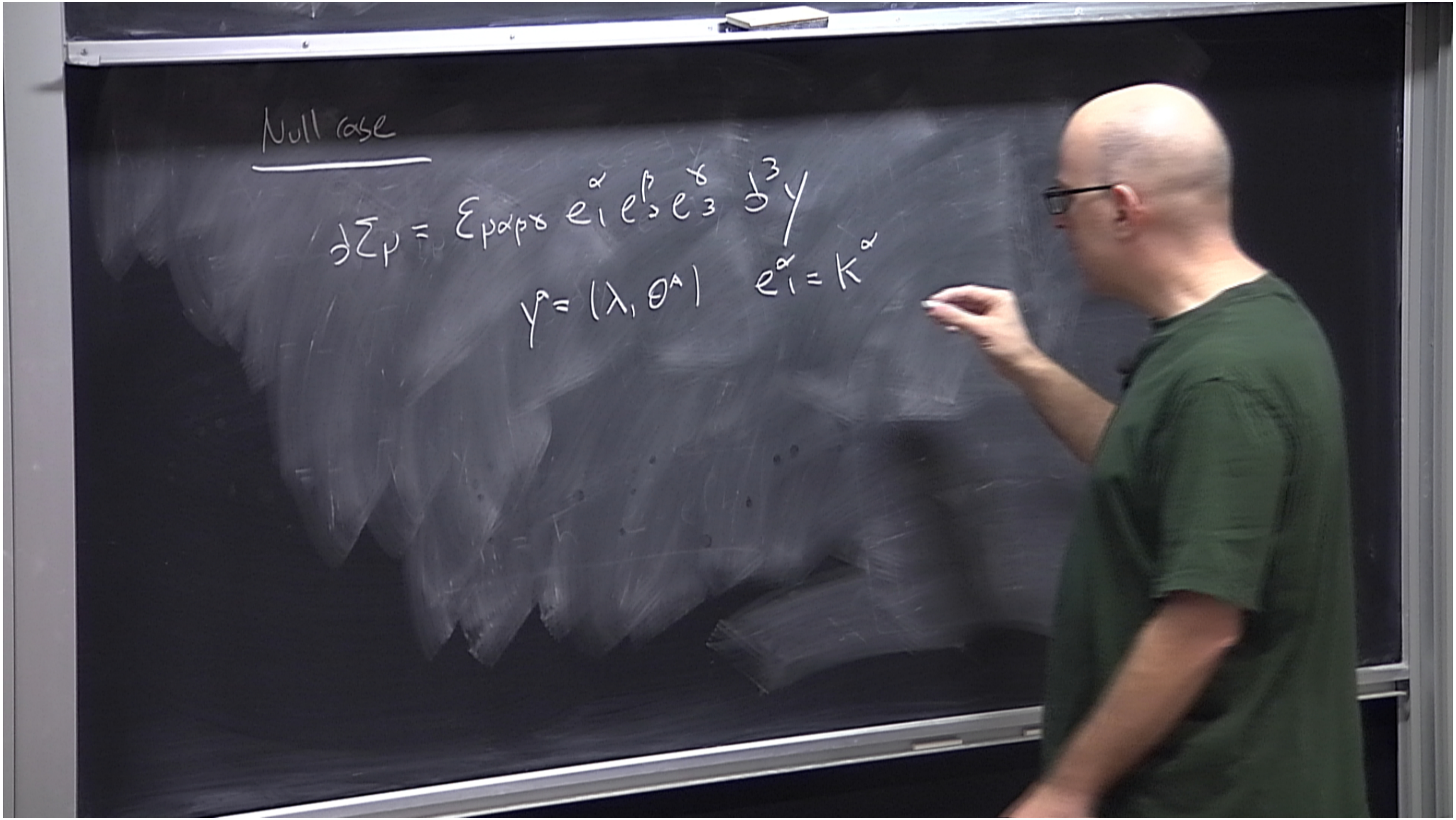
$$\varepsilon_{\mu\alpha\beta\gamma} = \sqrt{-g}\,[\mu\alpha\beta\gamma]$$

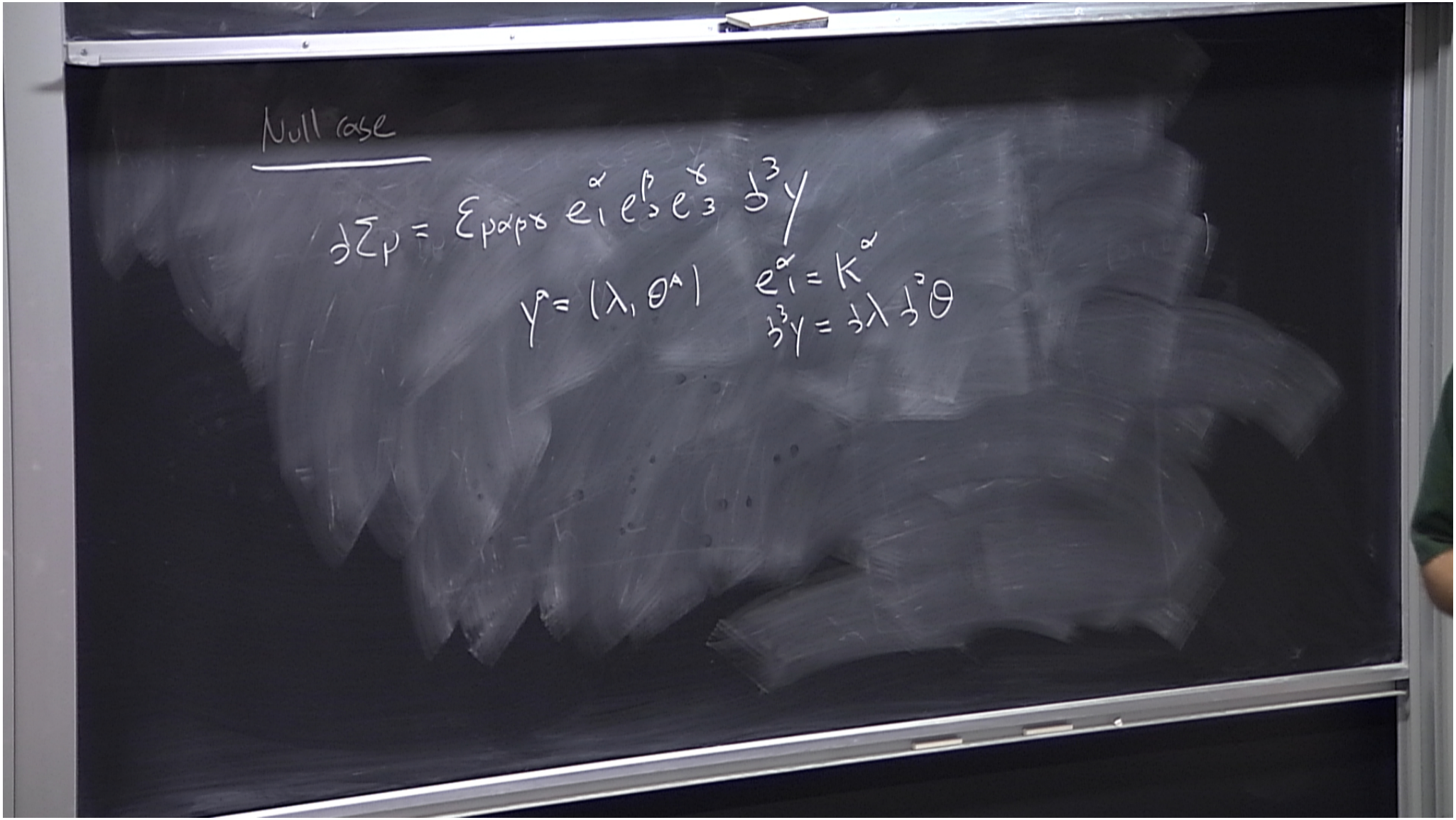
→ a good null limit

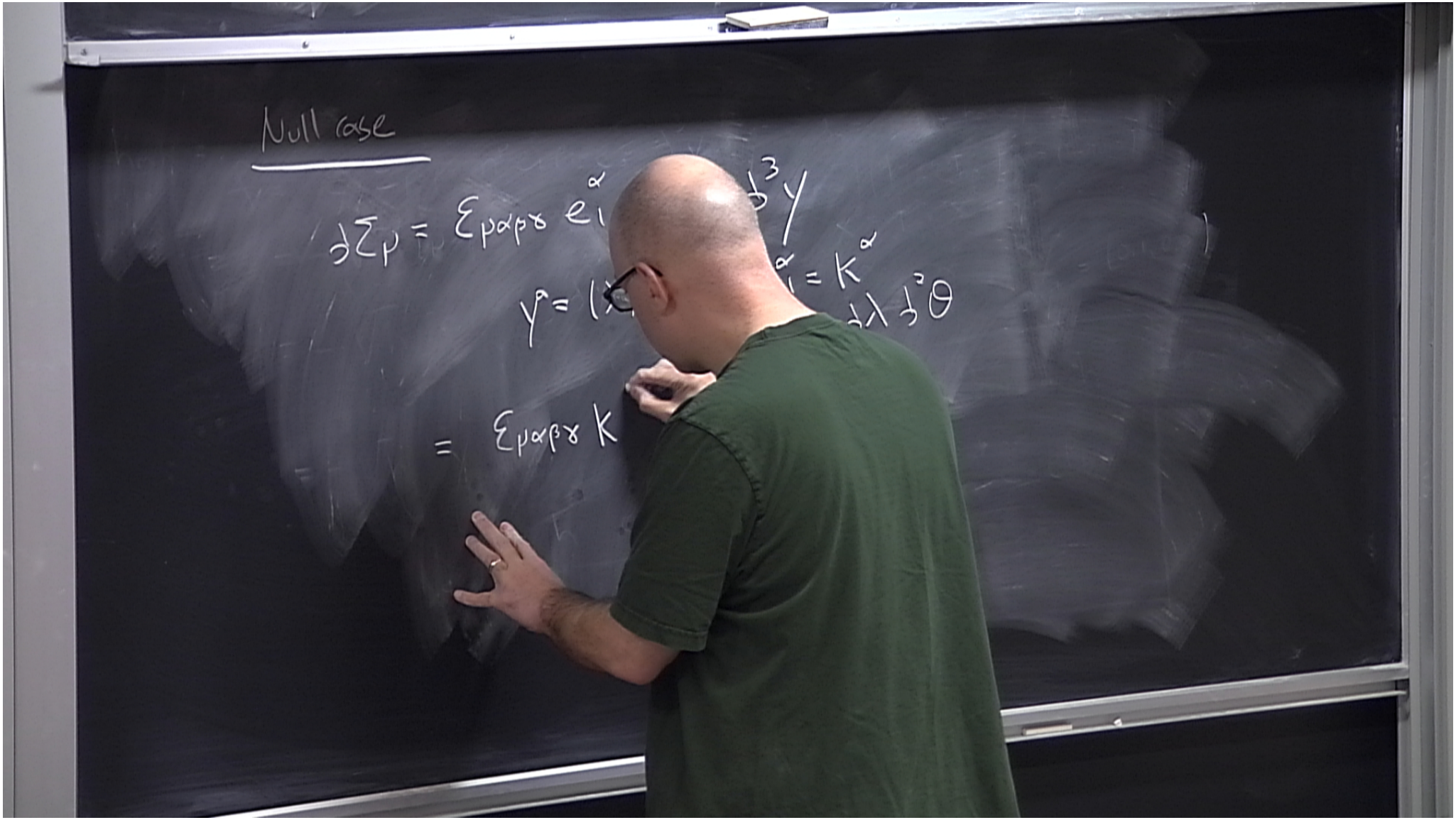
For timelike or spacelike Σ,

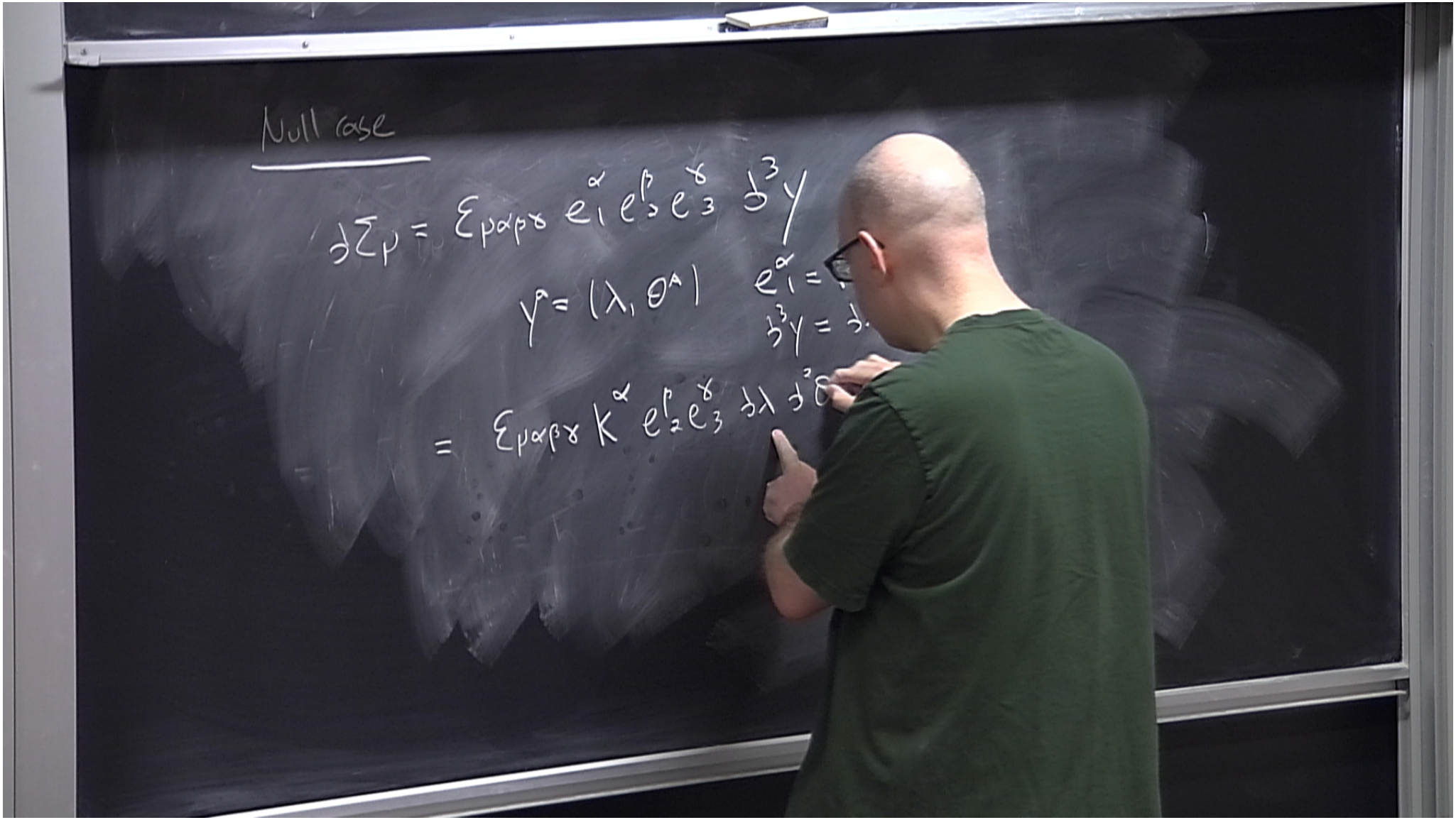
$$d\Sigma_\mu = \varepsilon\, n_\mu \sqrt{|h|}\, d^3y$$

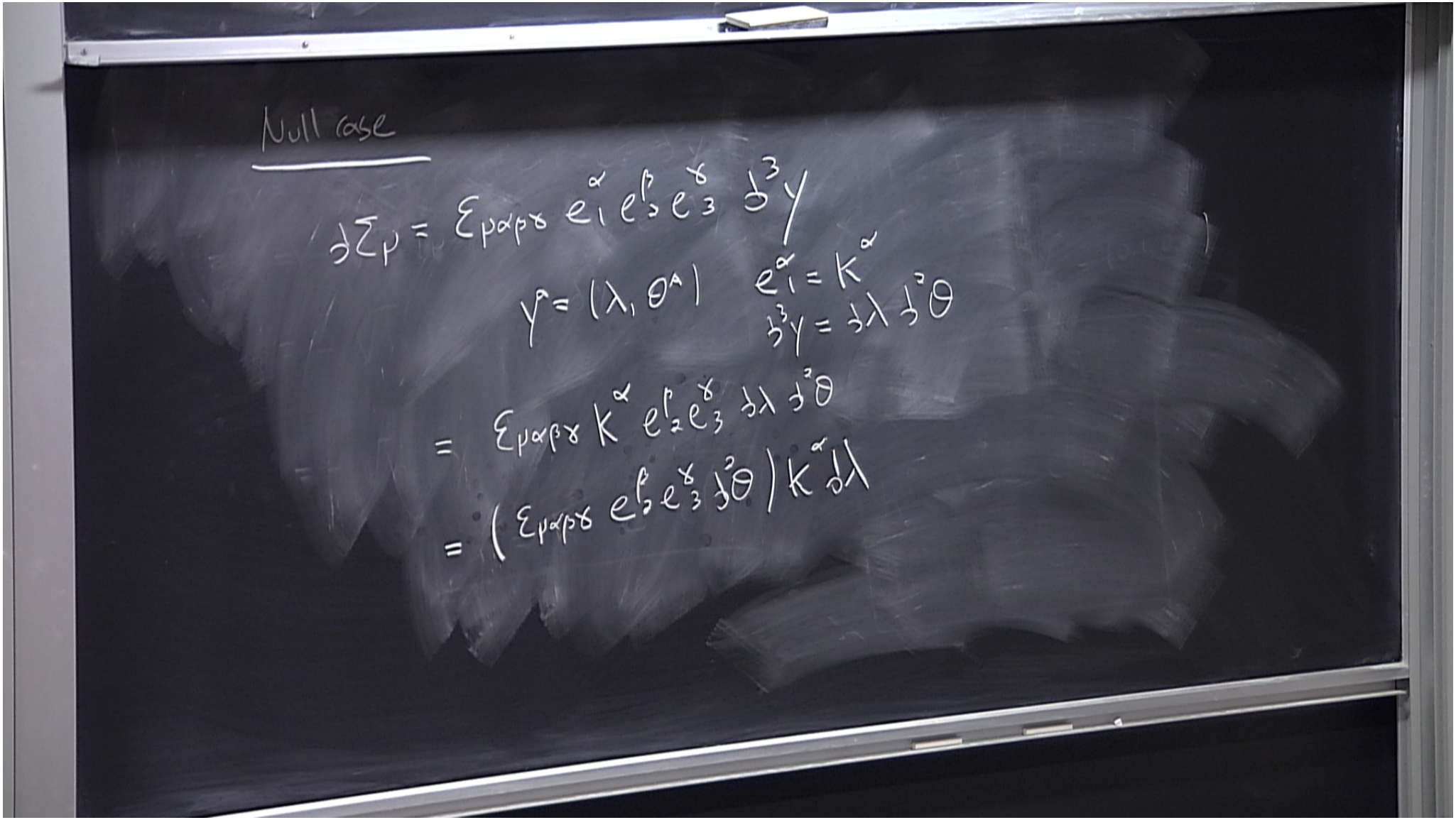

Null case
$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$


Null case
$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$y^\alpha = (\lambda, \theta^A)$ $e_1^\alpha = k^\alpha$


Null case
$d\Sigma_\mu = \epsilon_{\mu\alpha\beta\gamma} e_1^\alpha e_2^\beta e_3^\gamma d^3y$
$y^a = (\lambda, \theta^A)$
$e_1^\alpha = k^\alpha$
$d^3y = d\lambda d^2\theta$


Null case


Null case
$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$y^a = (\lambda, \theta^A)$
$= \varepsilon_{\mu\alpha\beta\gamma} k^\alpha e_2^\beta e_3^\gamma$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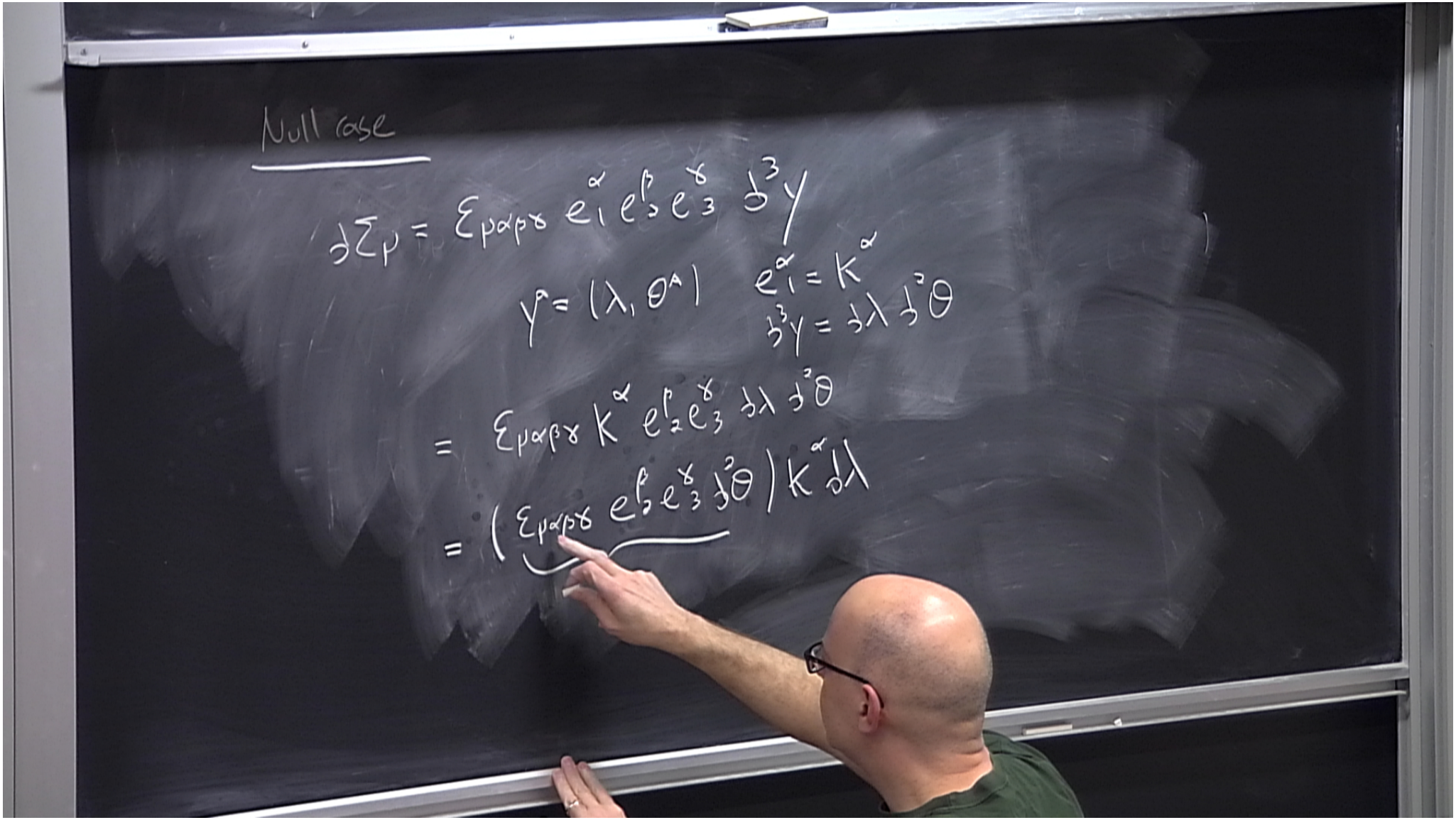
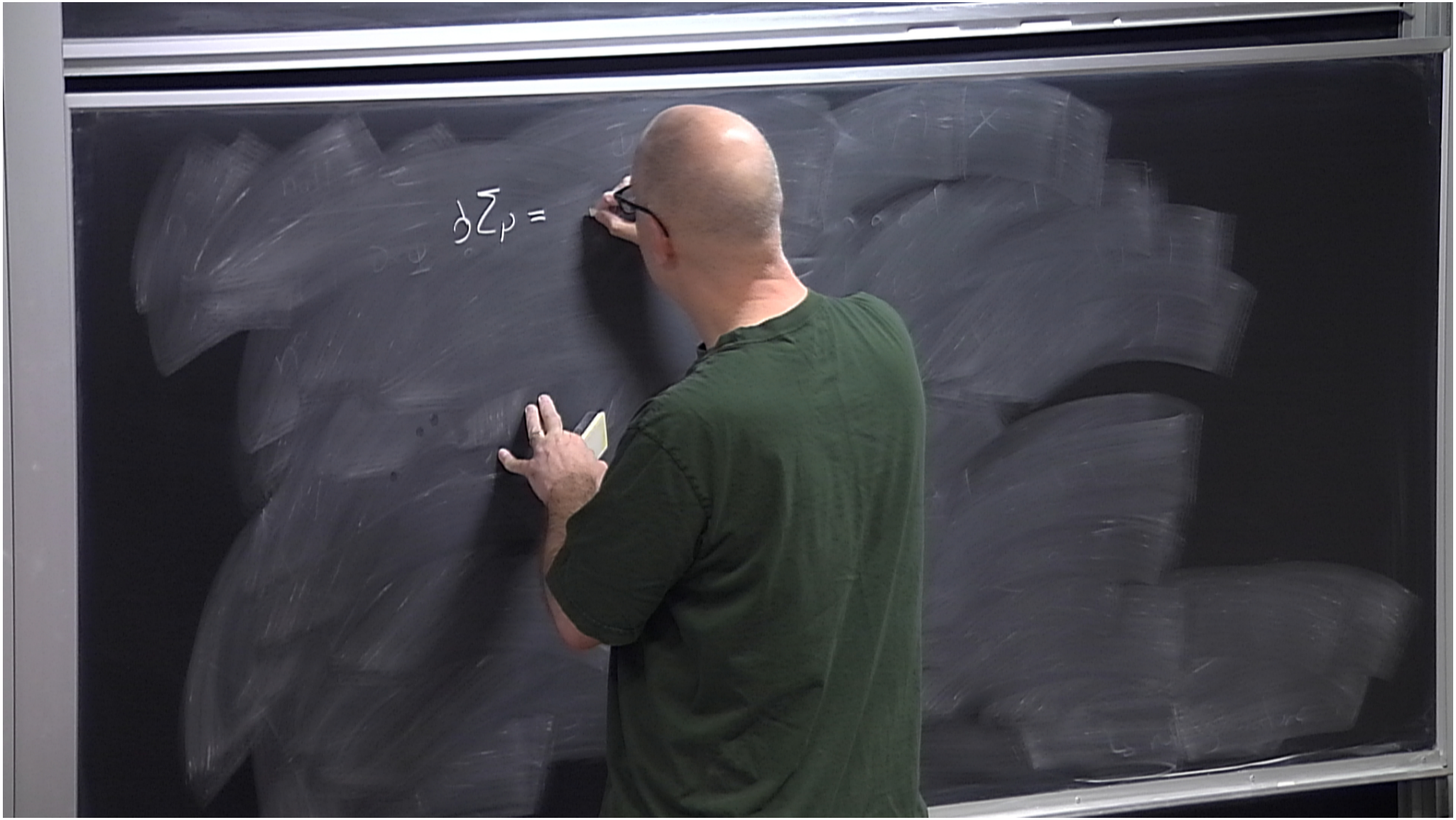
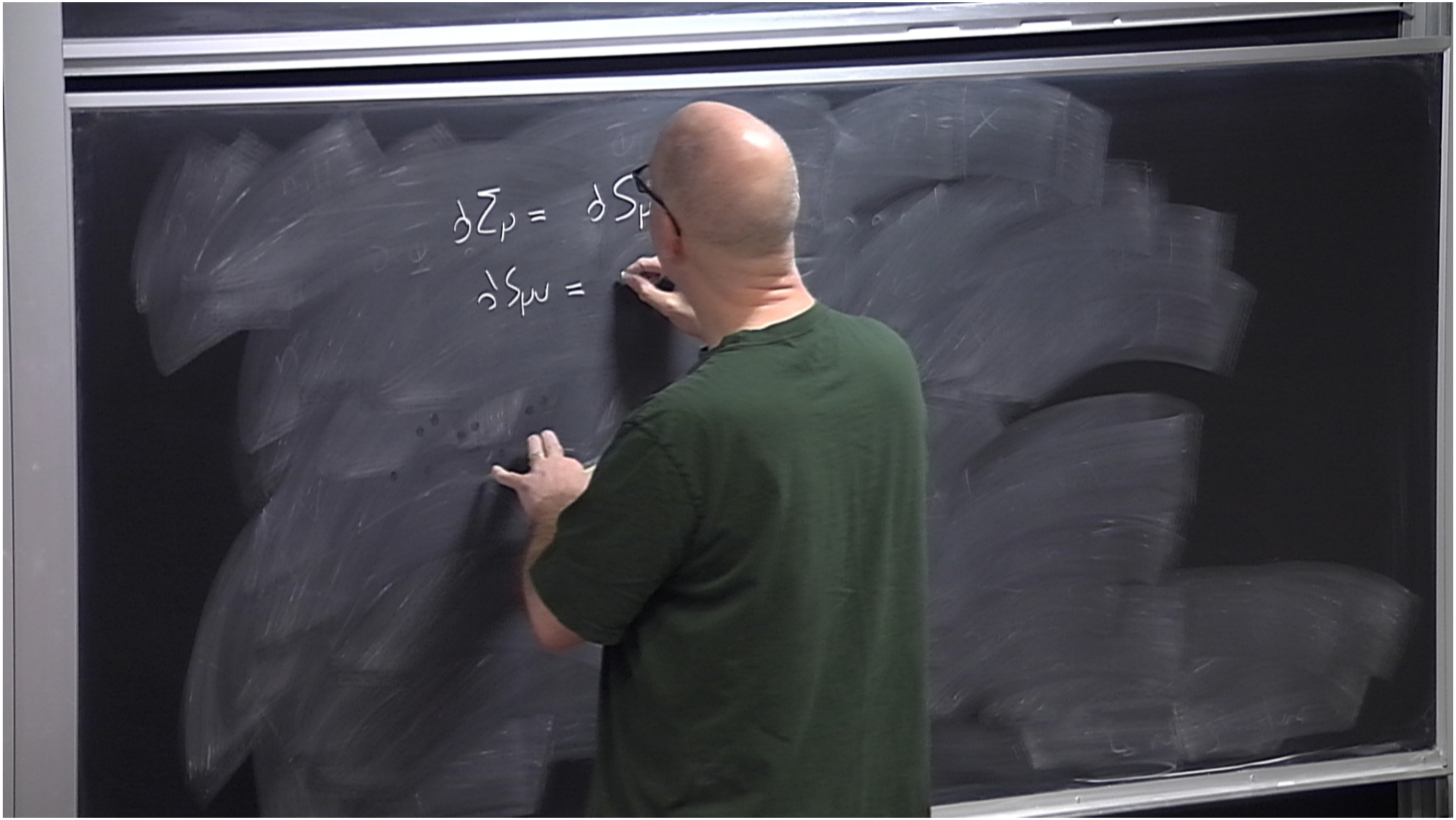
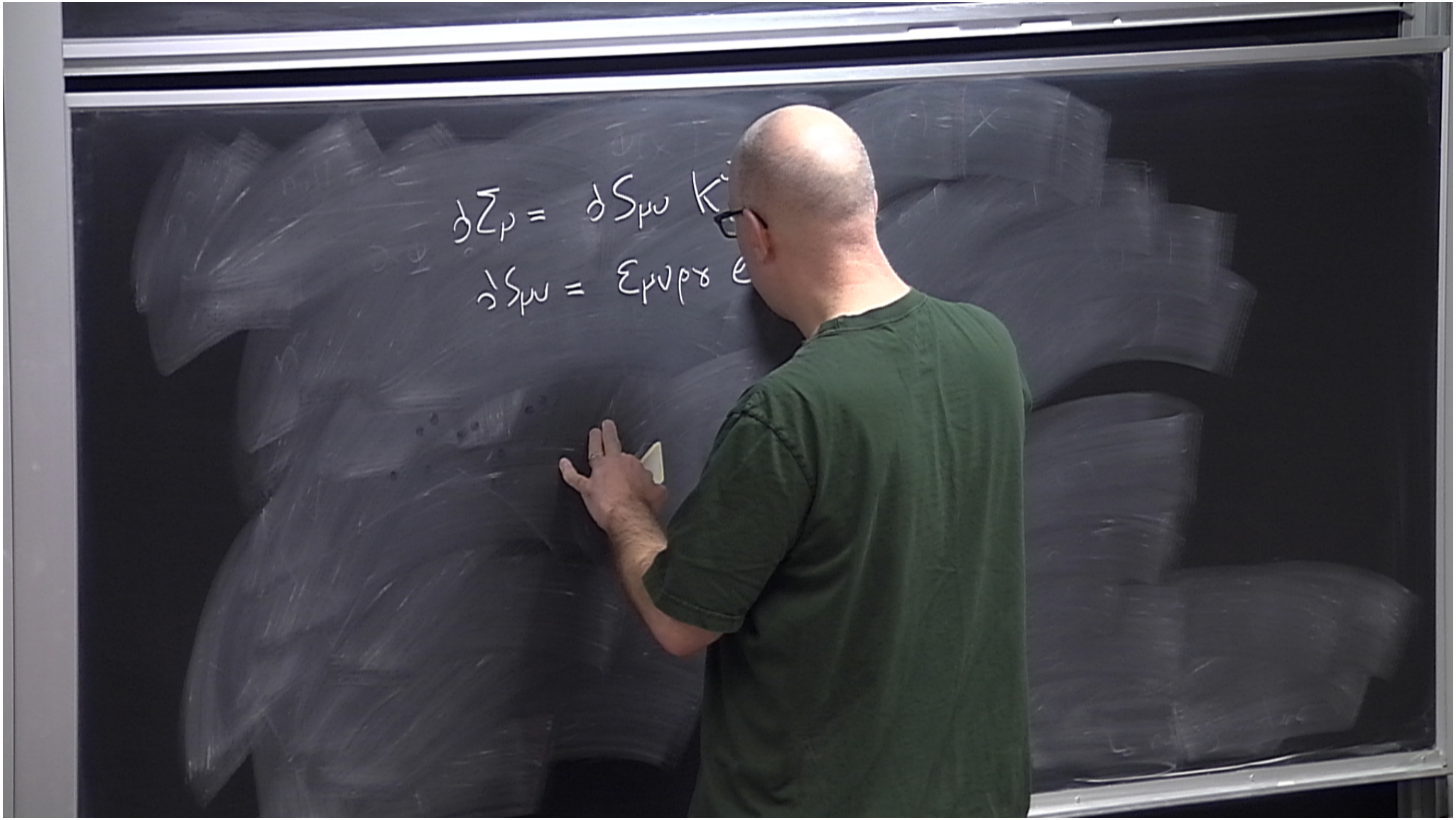
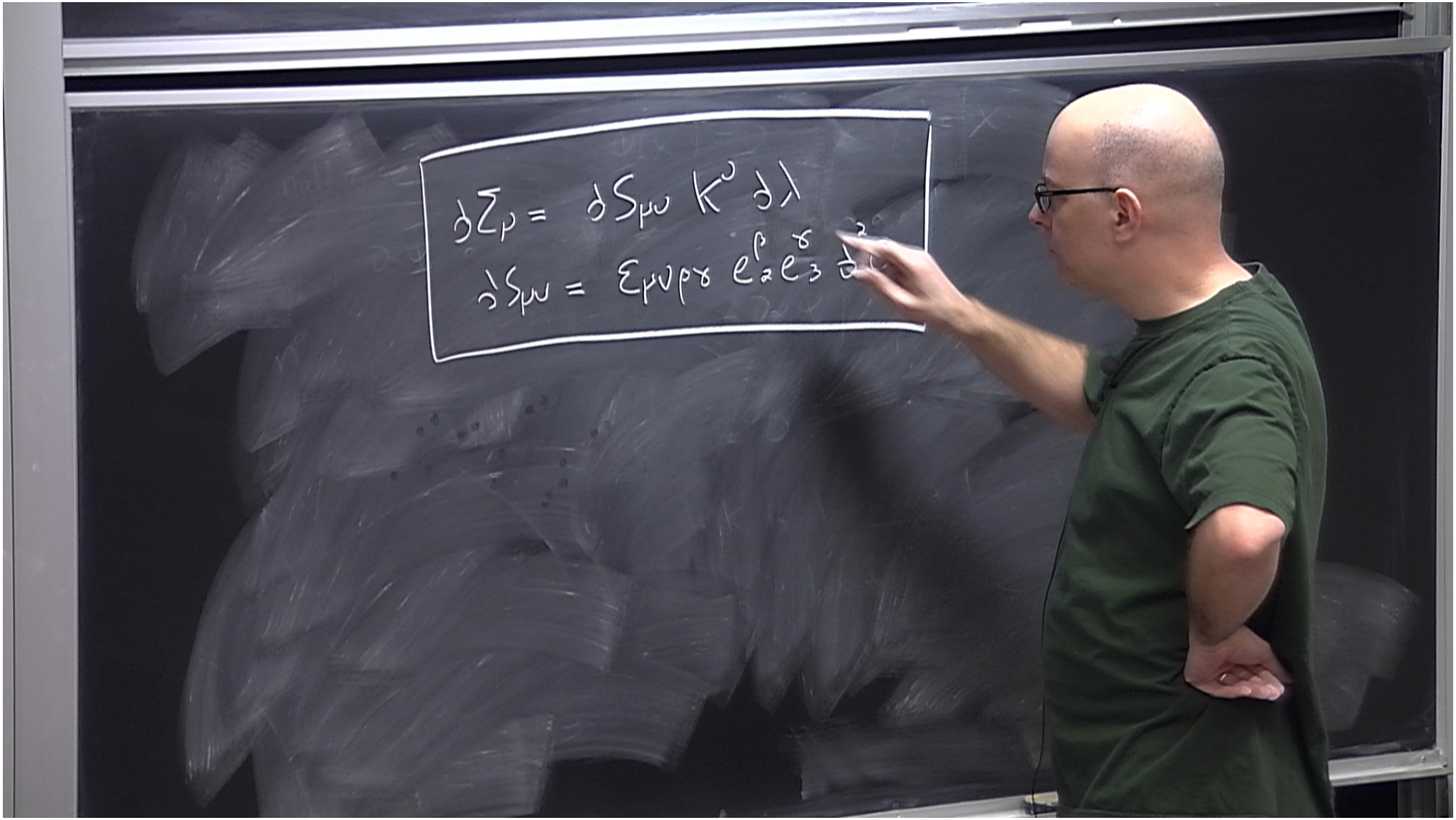
$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$y^a = (\lambda, \theta^A)$ $\quad e_1^\alpha = k^\alpha$
$d^3y = d\lambda\, d^2\theta$
$= \varepsilon_{\mu\alpha\beta\gamma} k^\alpha e_2^\beta e_3^\gamma\, d\lambda\, d^2\theta$
$= \left(\varepsilon_{\mu\alpha\beta\gamma} e_2^\beta e_3^\gamma\, d^2\theta\right) k^\alpha d\lambda$

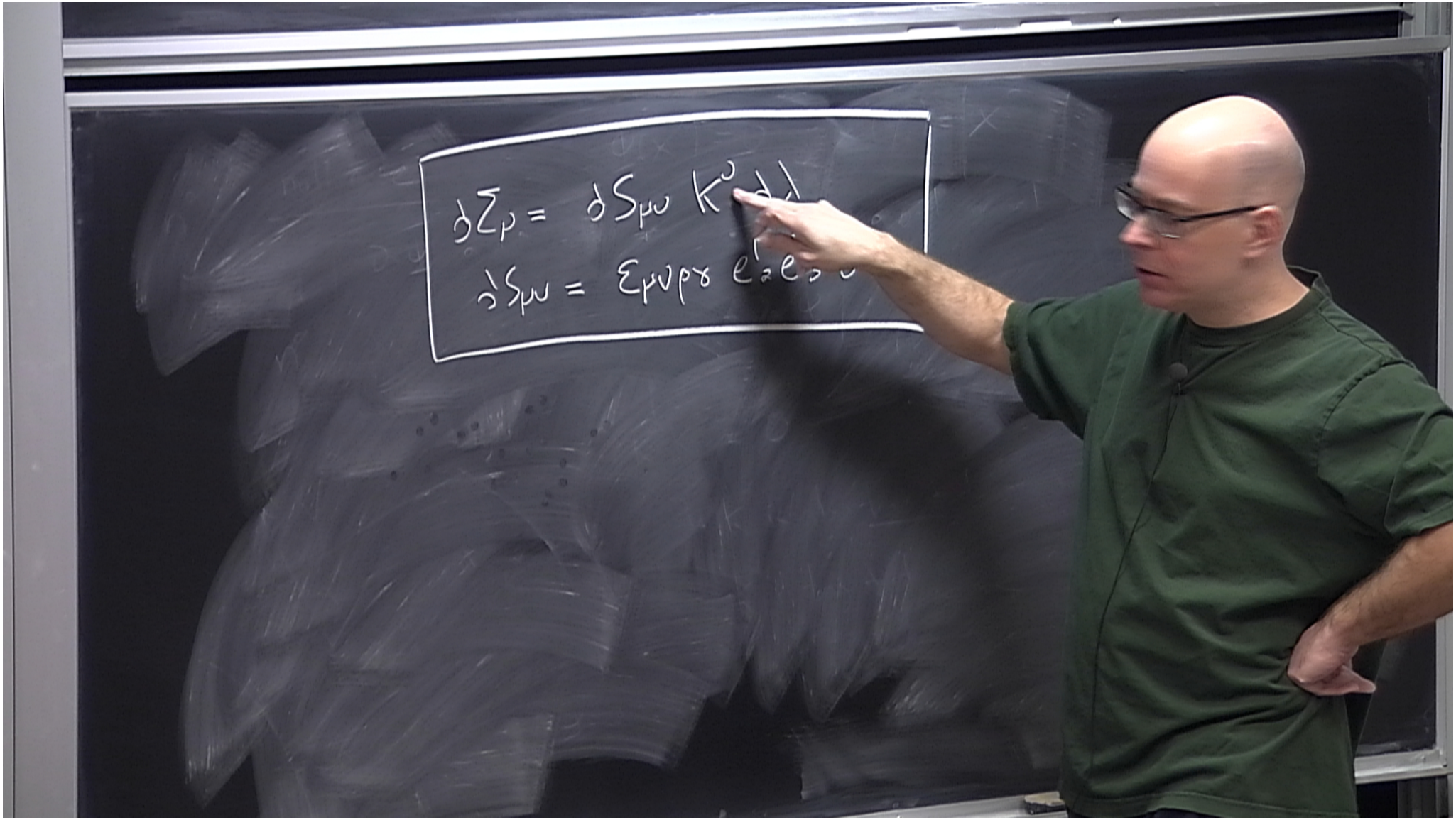

Null case
$d\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\, e_1^\alpha e_2^\beta e_3^\gamma\, d^3y$
$y^a = (\lambda, \theta^A)$ $\quad e_1^\alpha = k^\alpha$
$d^3y = d\lambda\, d^2\theta$
$= \varepsilon_{\mu\alpha\beta\gamma} k^\alpha e_2^\beta e_3^\gamma\, d\lambda\, d^2\theta$
$= \left(\varepsilon_{\mu\alpha\beta\gamma} e_2^\beta e_3^\gamma\, d^2\theta\right) k^\alpha d\lambda$

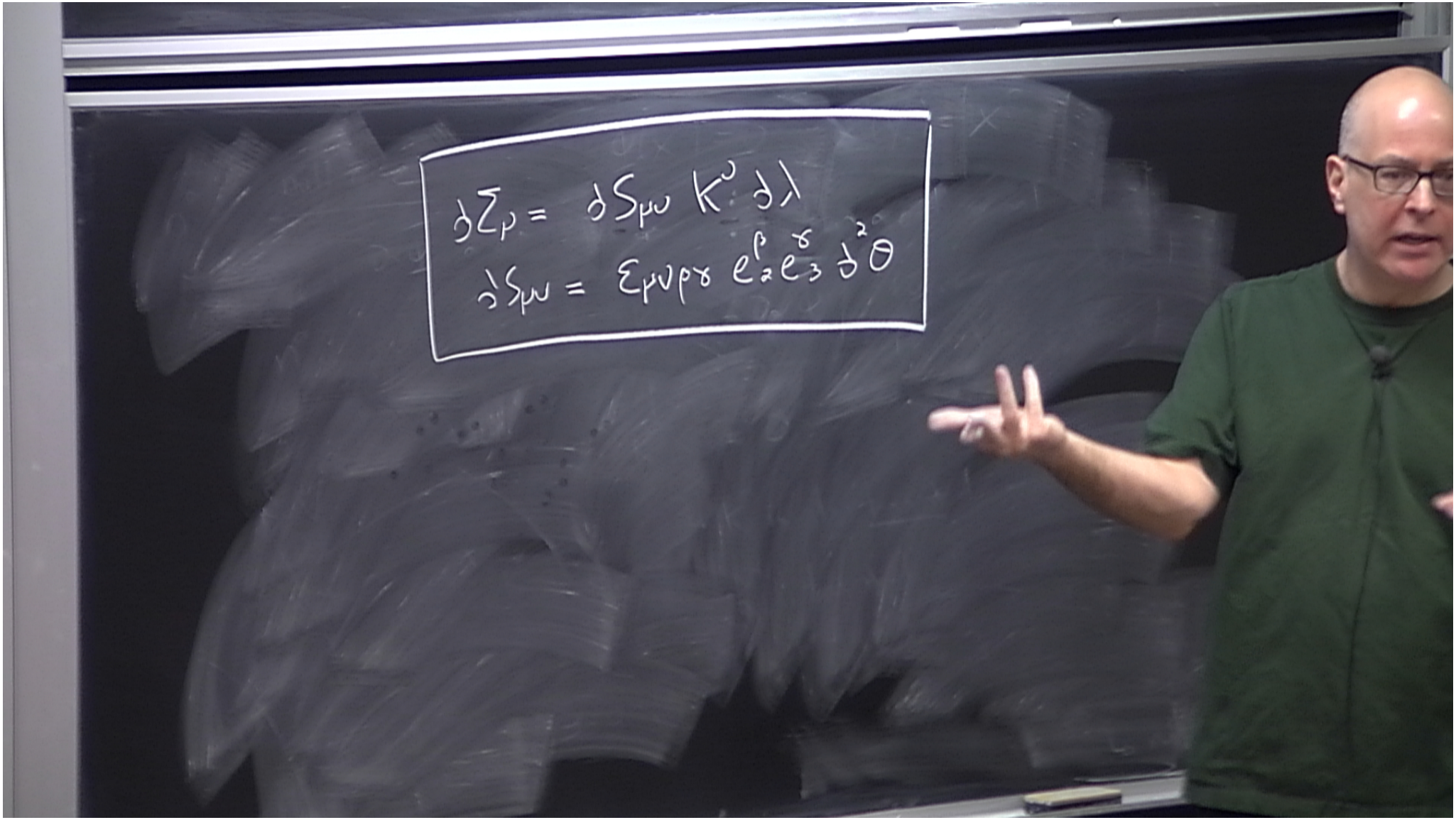

$\delta\Sigma_p =$

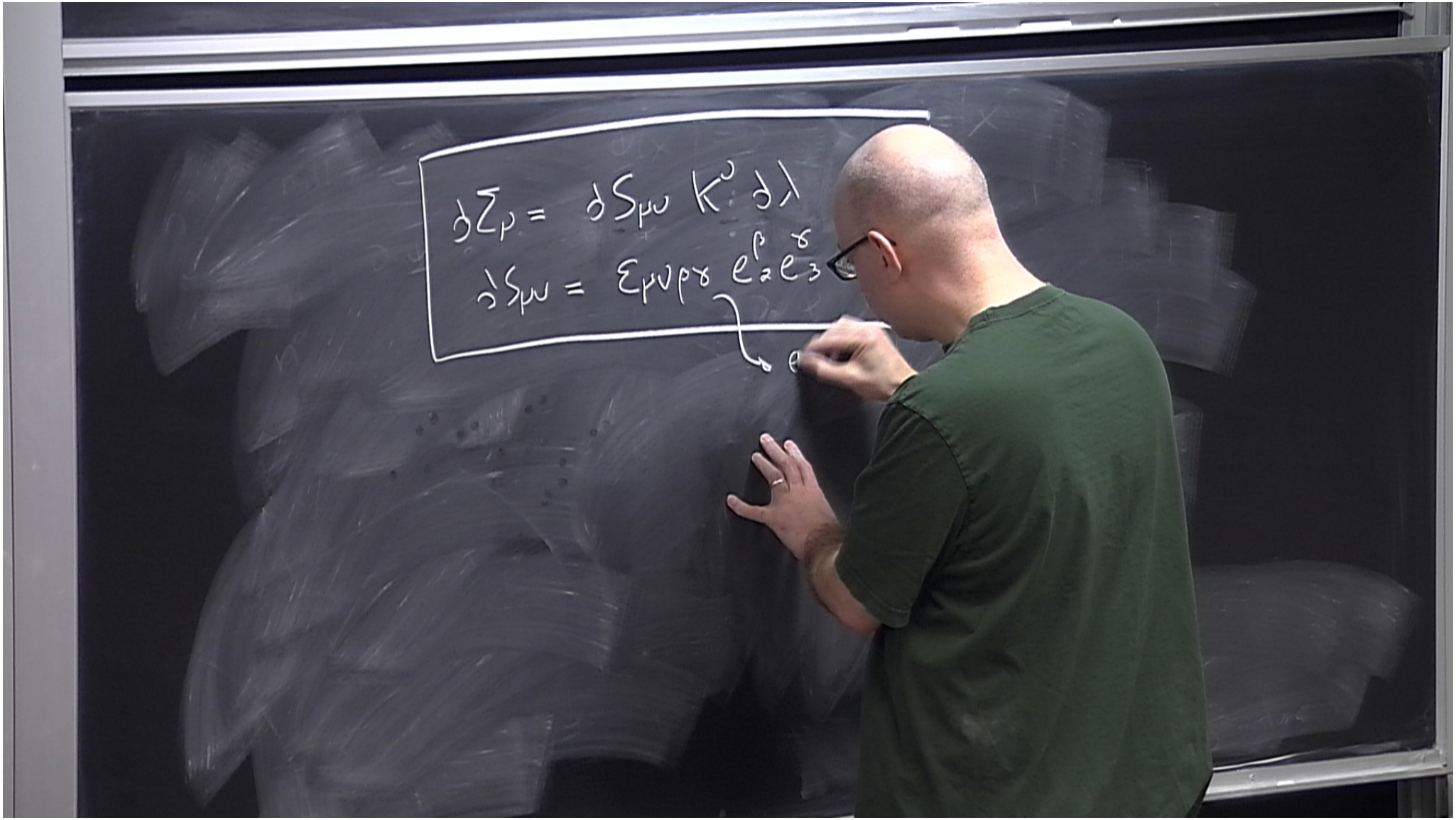

$\delta Z_\mu = \delta S_\mu$
$\delta S_{\mu\nu} =$

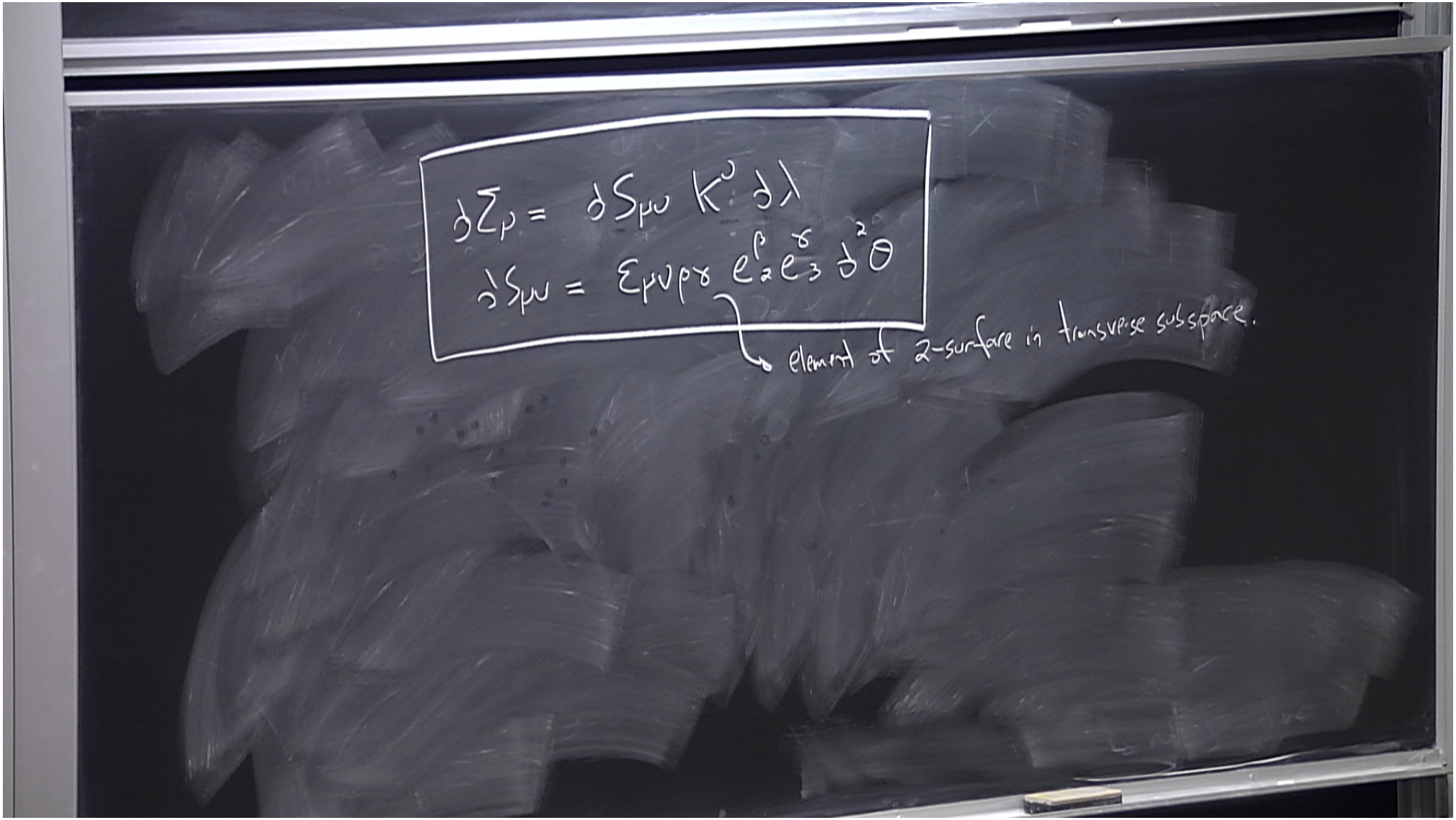

$\delta\Sigma_\rho = \delta S_\mu$
$\delta S_\mu = \varepsilon_{\mu\nu\rho\alpha}$


$\partial \Sigma_\mu = \partial S_\mu K^\nu \partial \lambda$
$\partial S_\mu = \varepsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma$


$d\Sigma_\mu = d S_\nu K^\nu$
$d S_\mu = \varepsilon_{\mu\nu\rho\sigma} e^\nu_1 e^\rho_2$


$d\Sigma_\mu = dS_{\mu\nu} K^\nu d\lambda$
$dS_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma d^2\theta$

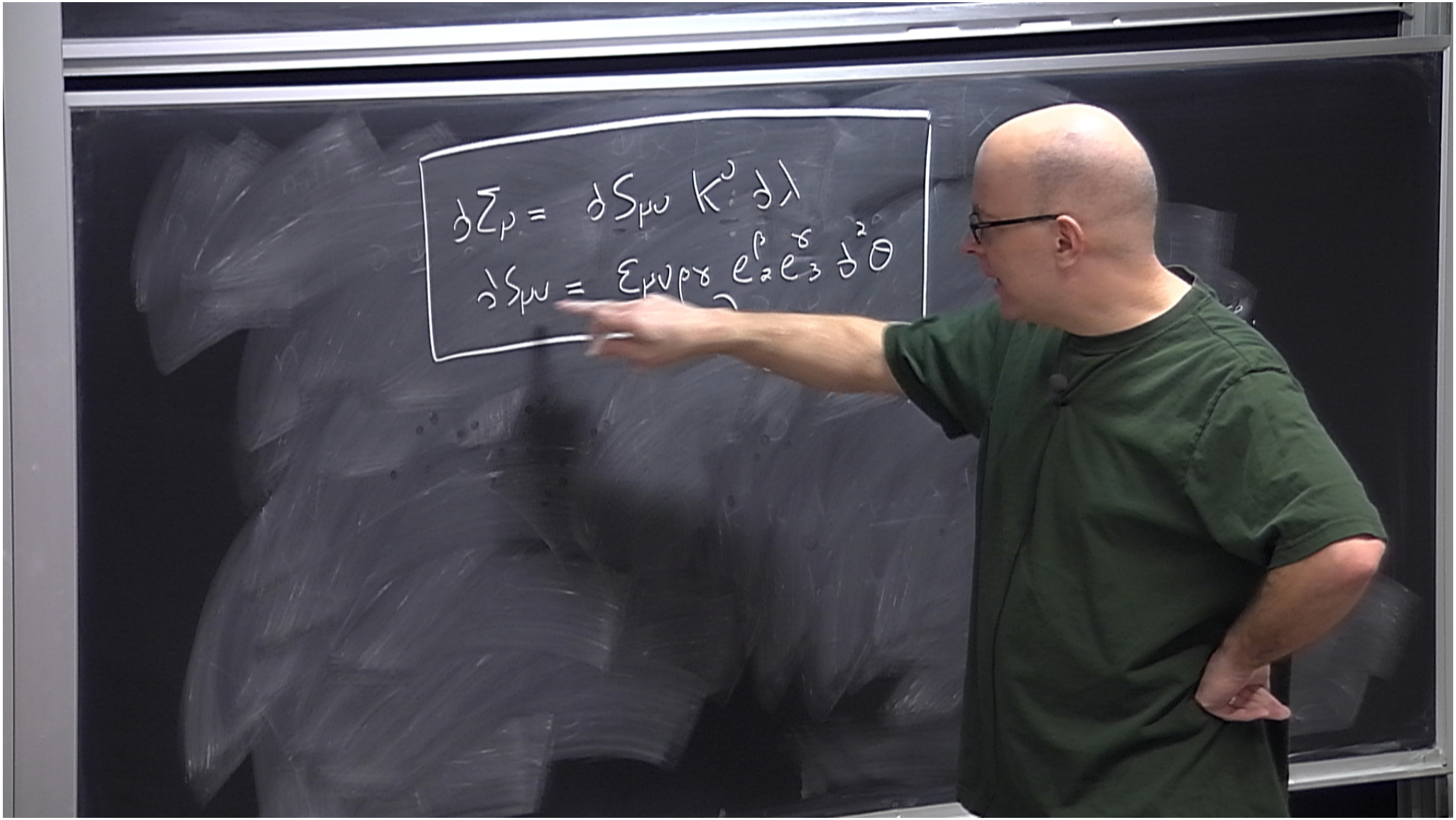

$d\Sigma_\rho = dS_\mu K^\mu d\lambda$
$dS_\mu = \varepsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma$

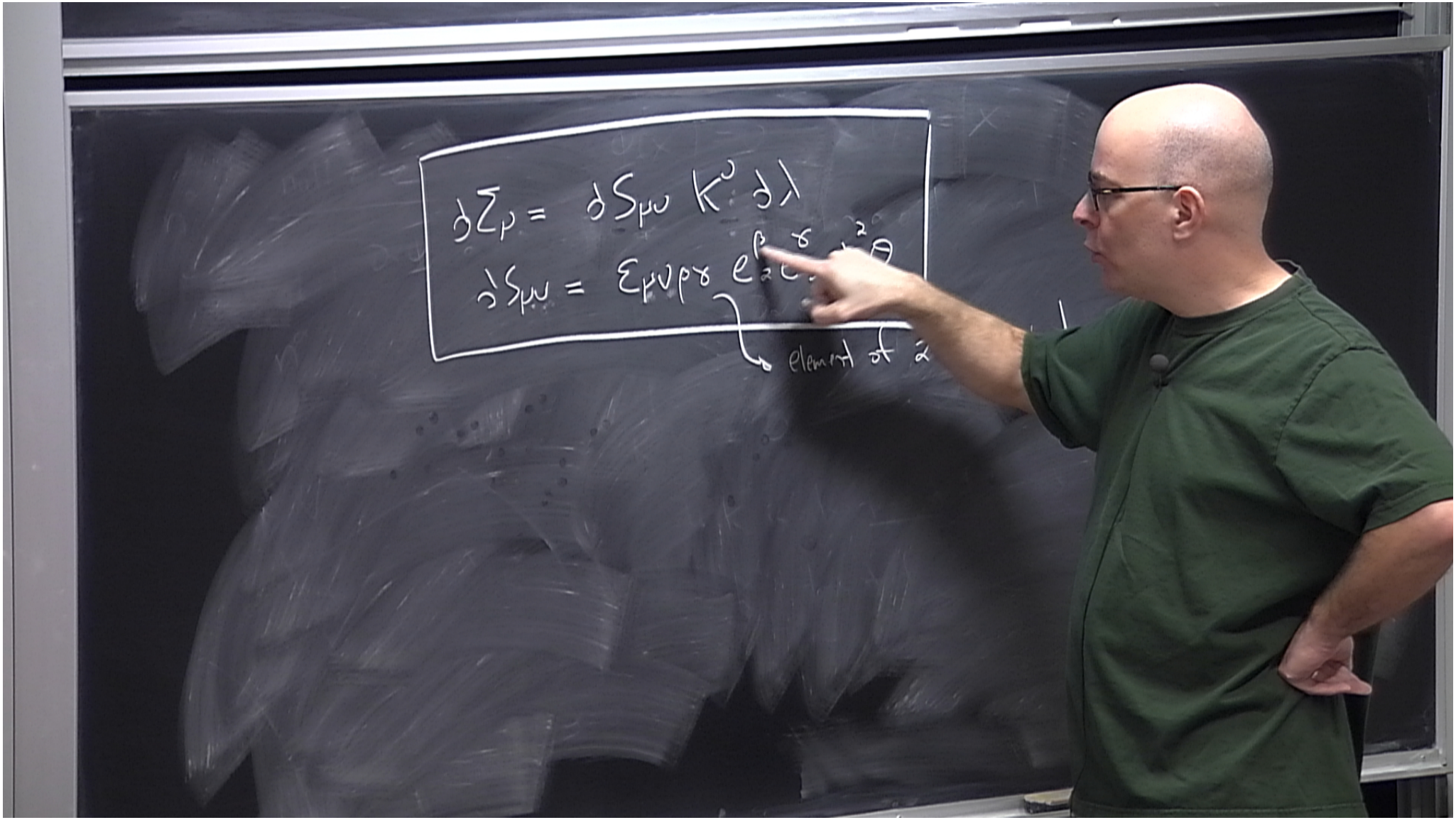

$d\Sigma_\mu = dS_{\mu\nu} K^\nu d\lambda$
$dS_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma d^2\theta$
element of 2-surface in transverse subspace.

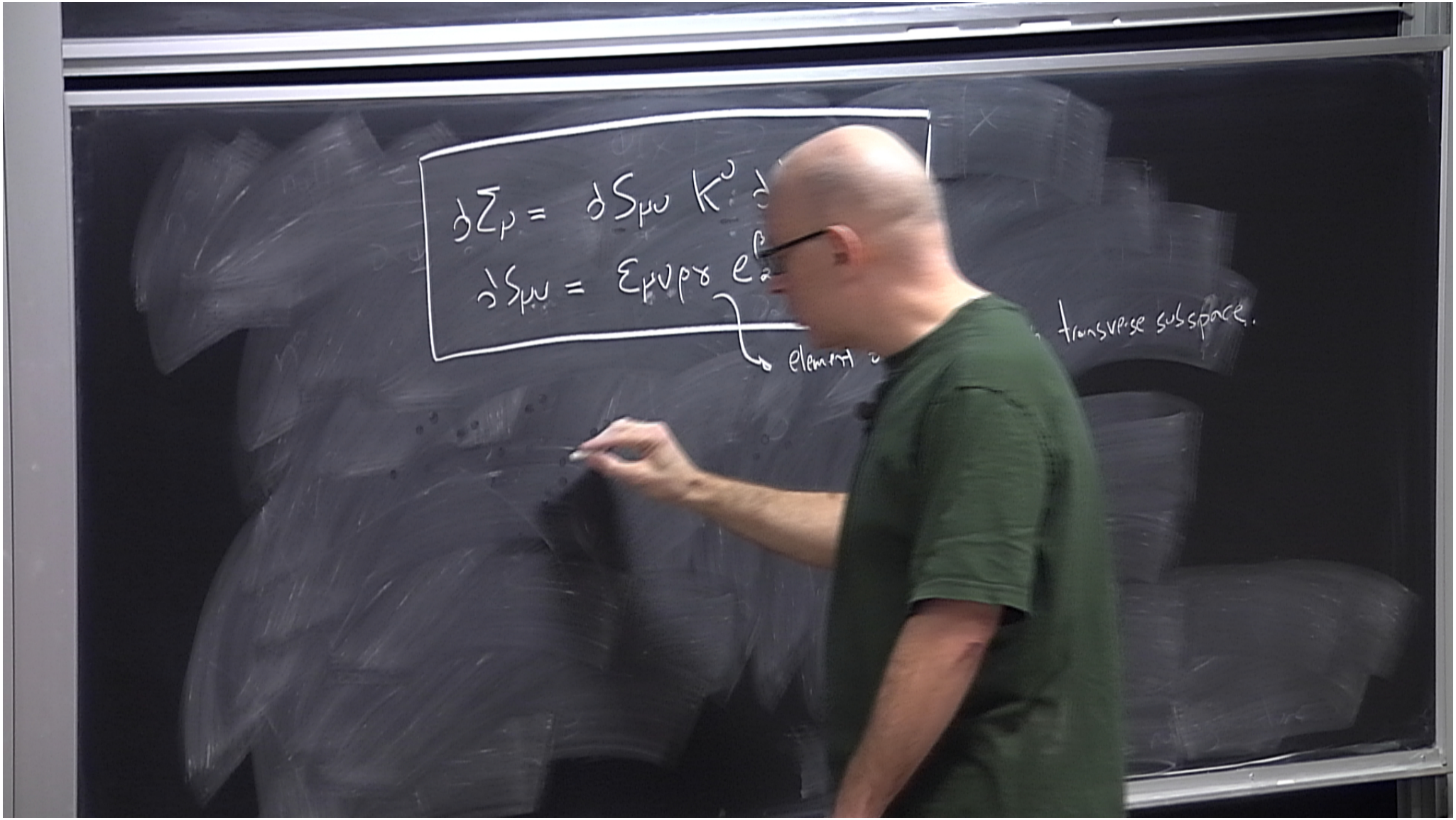
$$\partial \Sigma_\rho = \partial S_{\mu\nu} K^\nu \partial\lambda$$

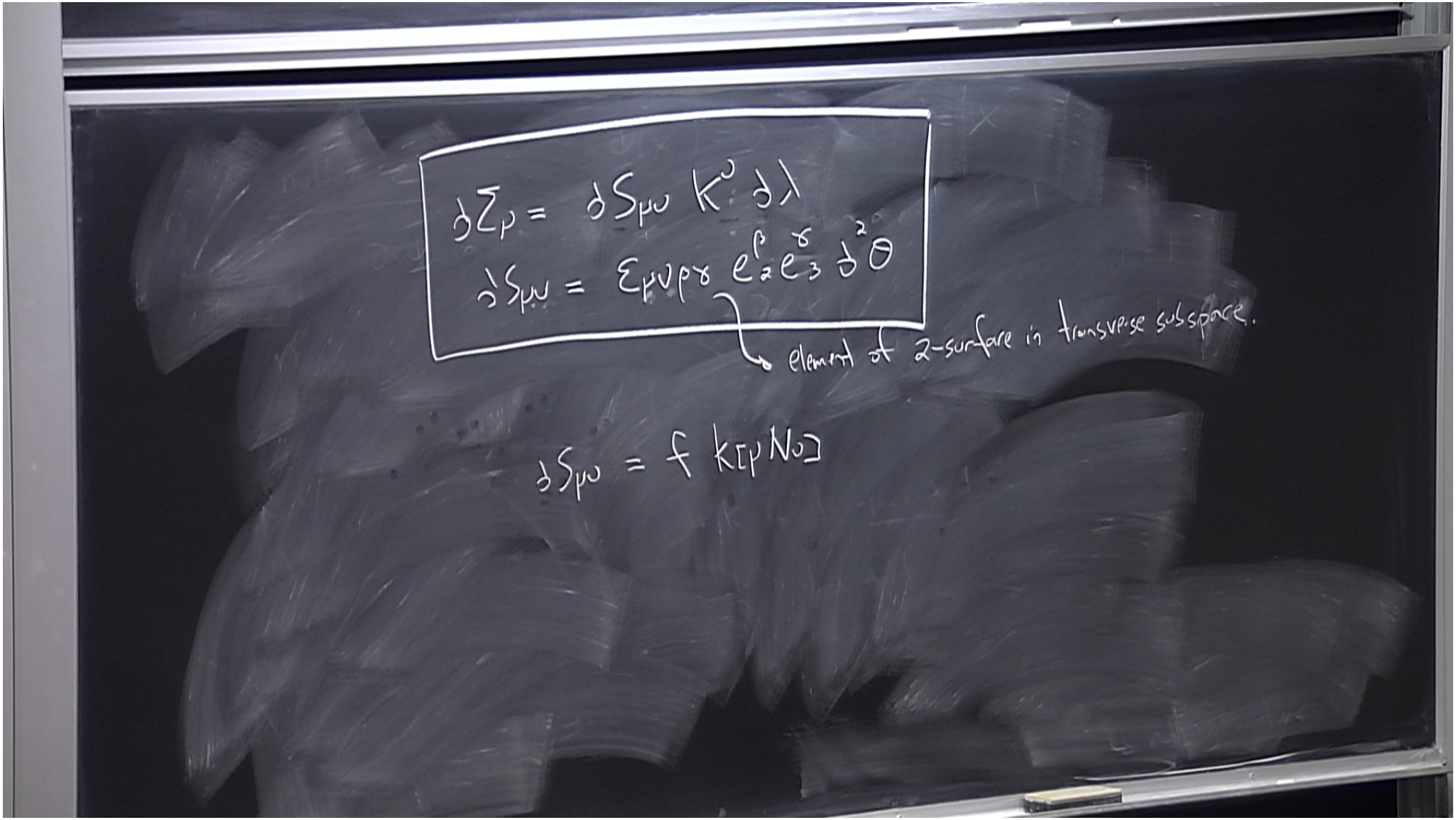
$$\partial S_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma \partial^2\theta$$

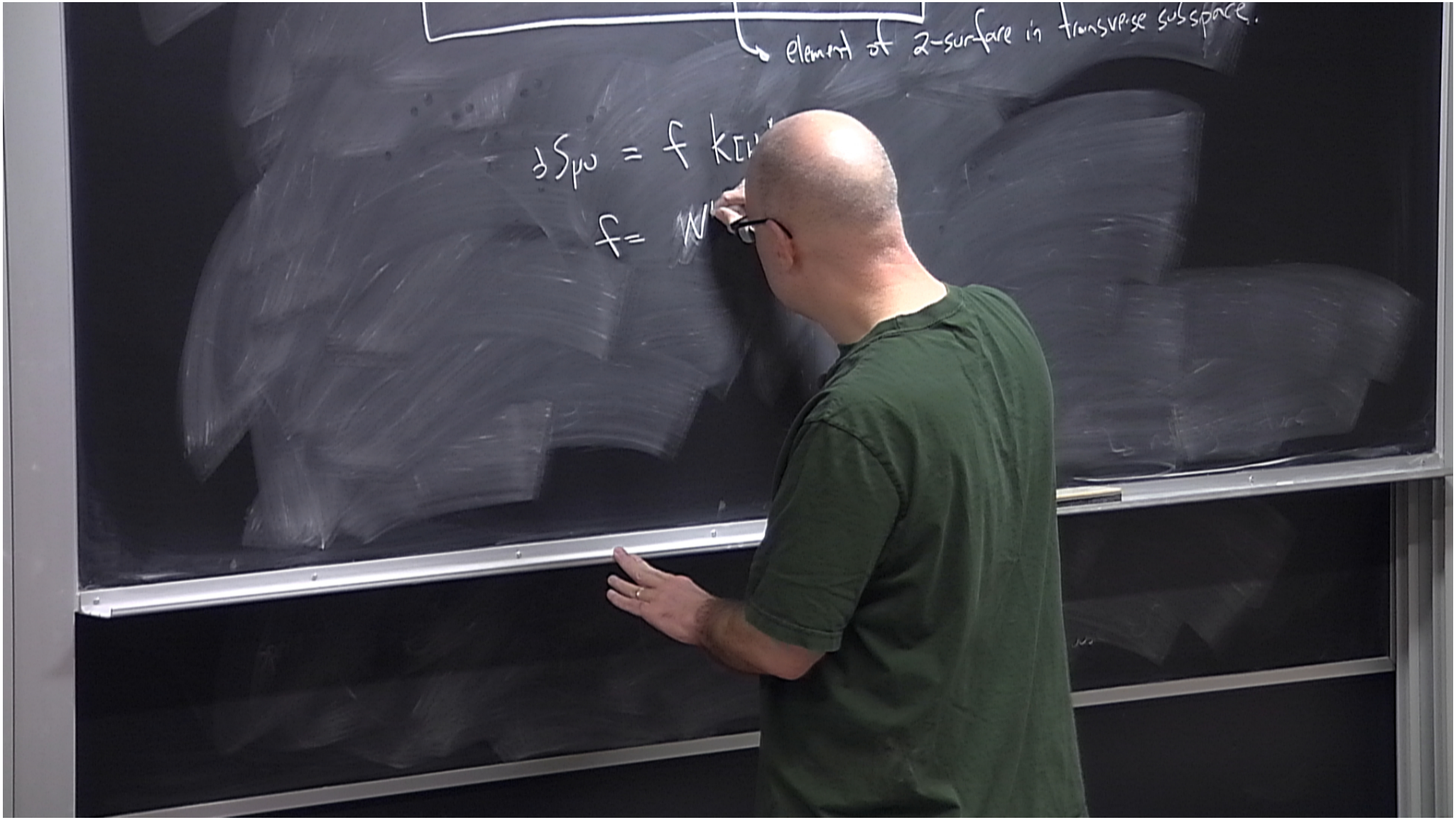
element of 2-surface in transverse subspace.

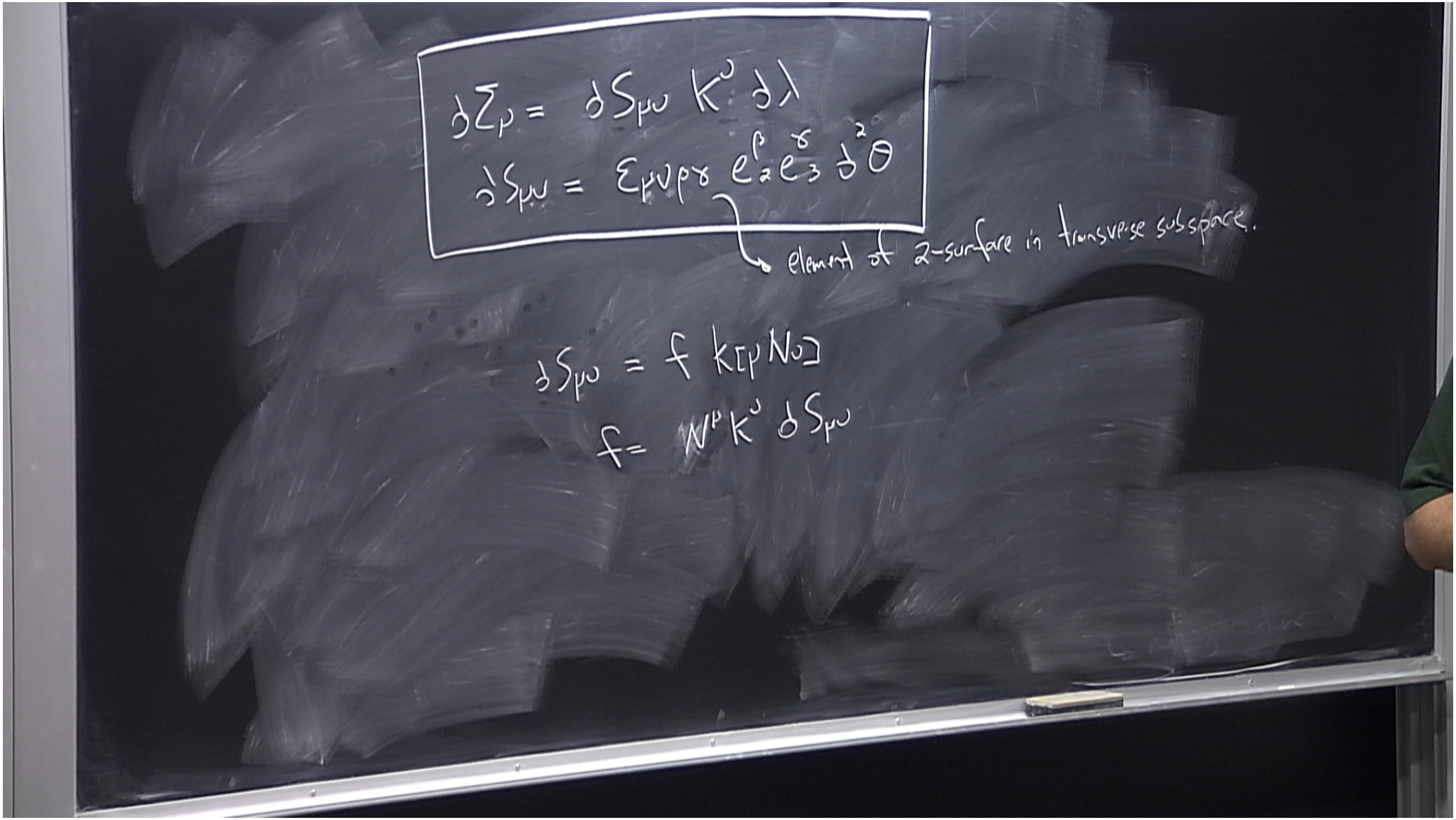

$\delta\Sigma_\rho = \delta S_\mu K^\mu \delta\lambda$
$\delta S_\mu = \varepsilon_{\mu\nu\rho\sigma} e_1^\rho e_2^\sigma \delta^2\theta$

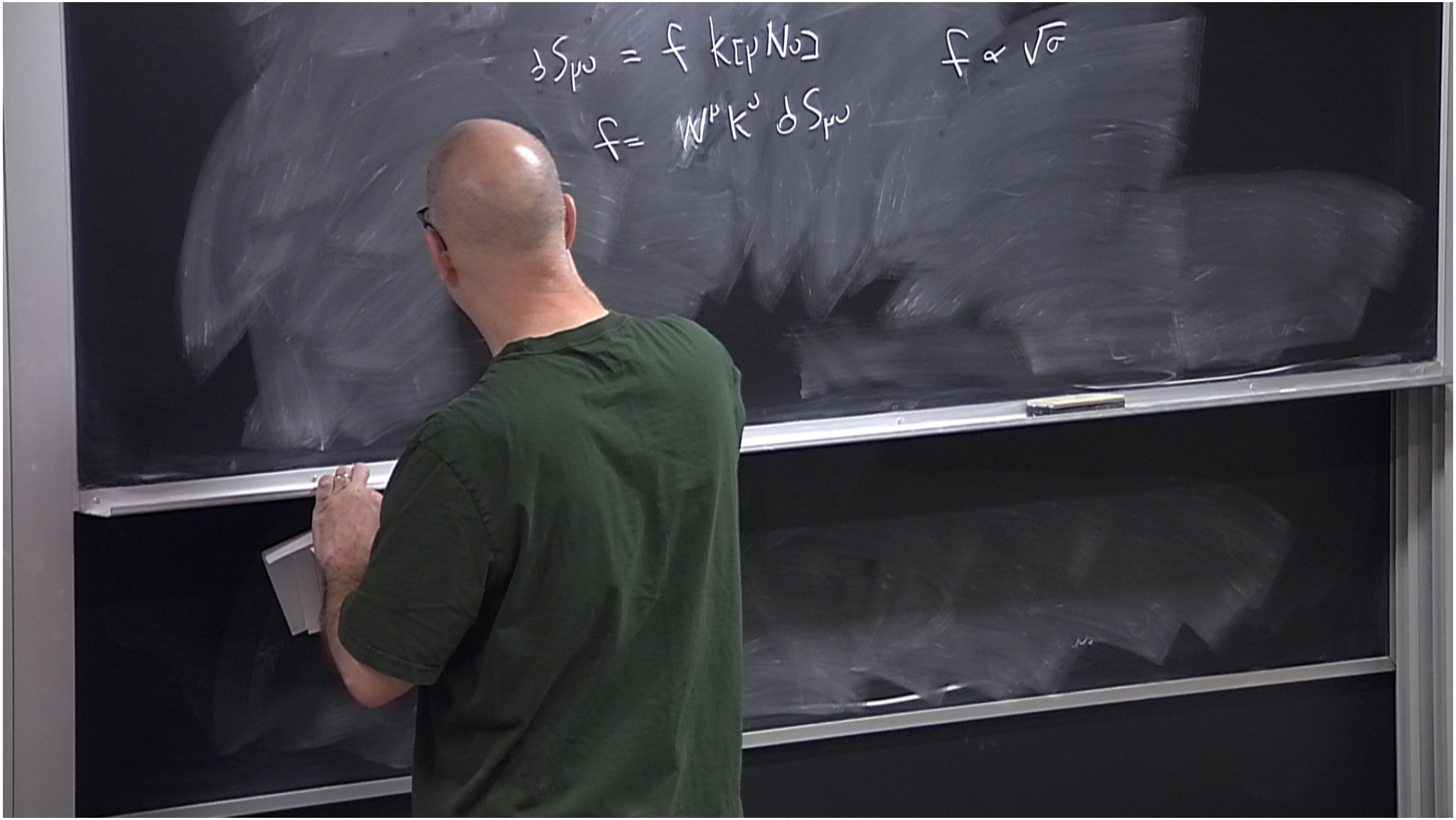

$d\Sigma_\mu = dS_\mu K^\nu d\lambda$
$dS_\mu = \varepsilon_{\mu\nu\rho\sigma} e^\rho_2 e^\sigma_3 d^2\theta$
element of

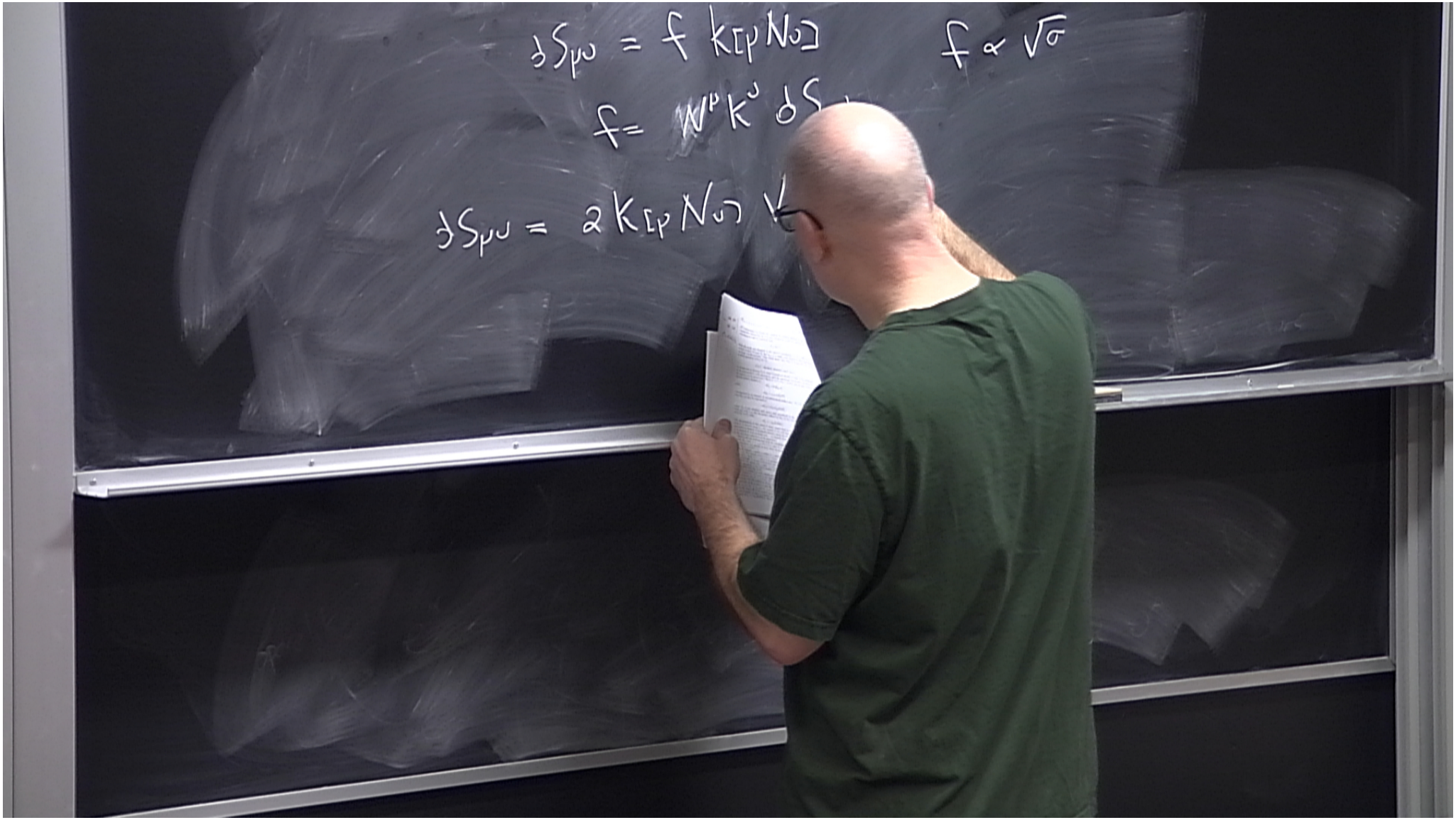

transverse subspace.

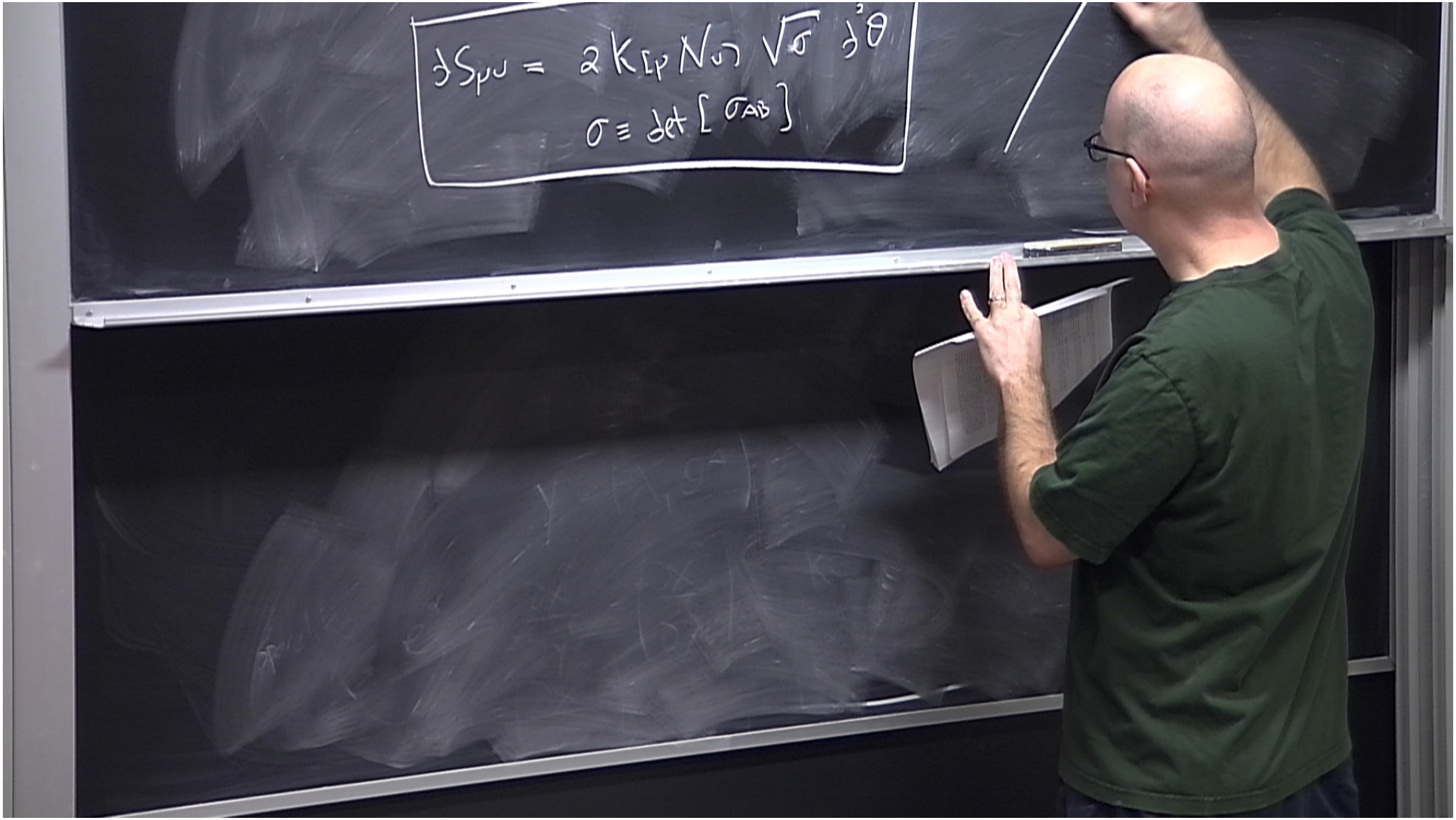

$d\Sigma_\rho = dS_{\mu\nu} k^\nu d\lambda$
$dS_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma d^2\theta$
element of 2-surface in transverse subspace.
$dS_{\mu\nu} = f k_{[\mu} N_{\nu]}$

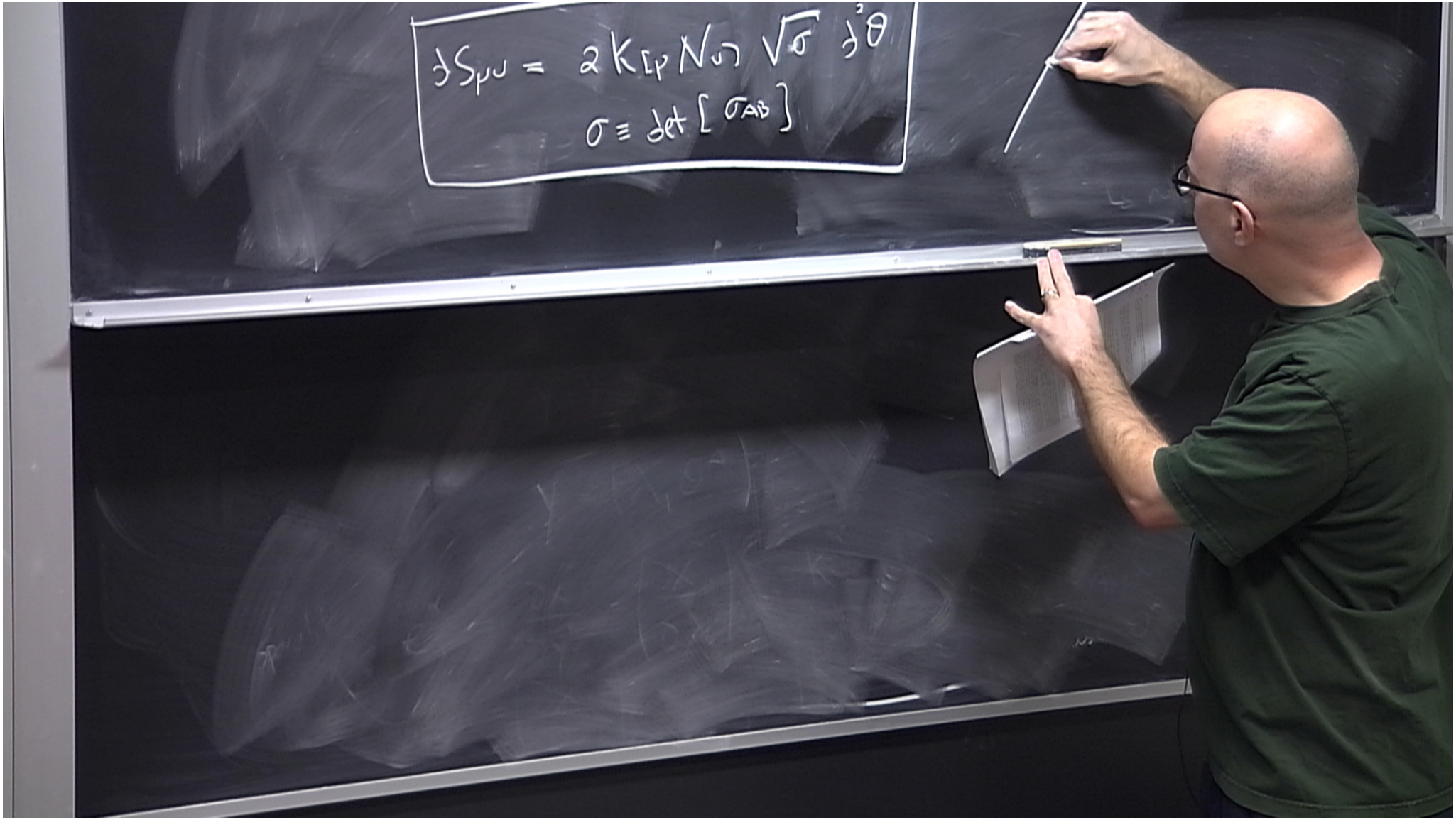

element of 2-surface in transverse subspace.

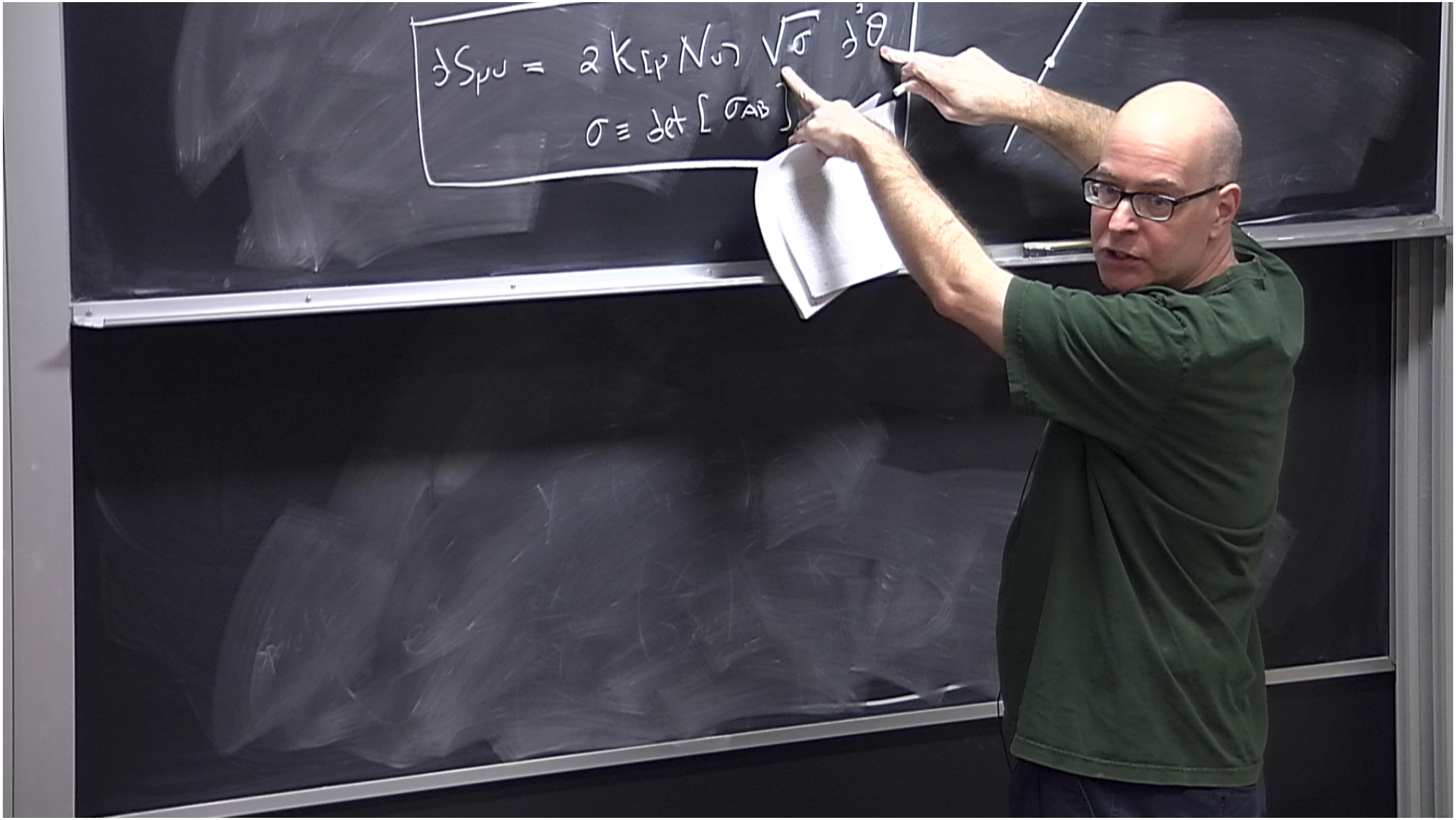

$\partial Z_\mu = \partial S_{\mu\nu} K^\nu \partial\lambda$
$\partial S_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} e_2^\rho e_3^\sigma \partial^2\theta$
element of 2-surface in transverse subspace.
$\partial S_{\mu\nu} = f\, k_{[\mu} N_{\nu]}$
$f = N^\nu K^\mu \partial S_{\mu\nu}$

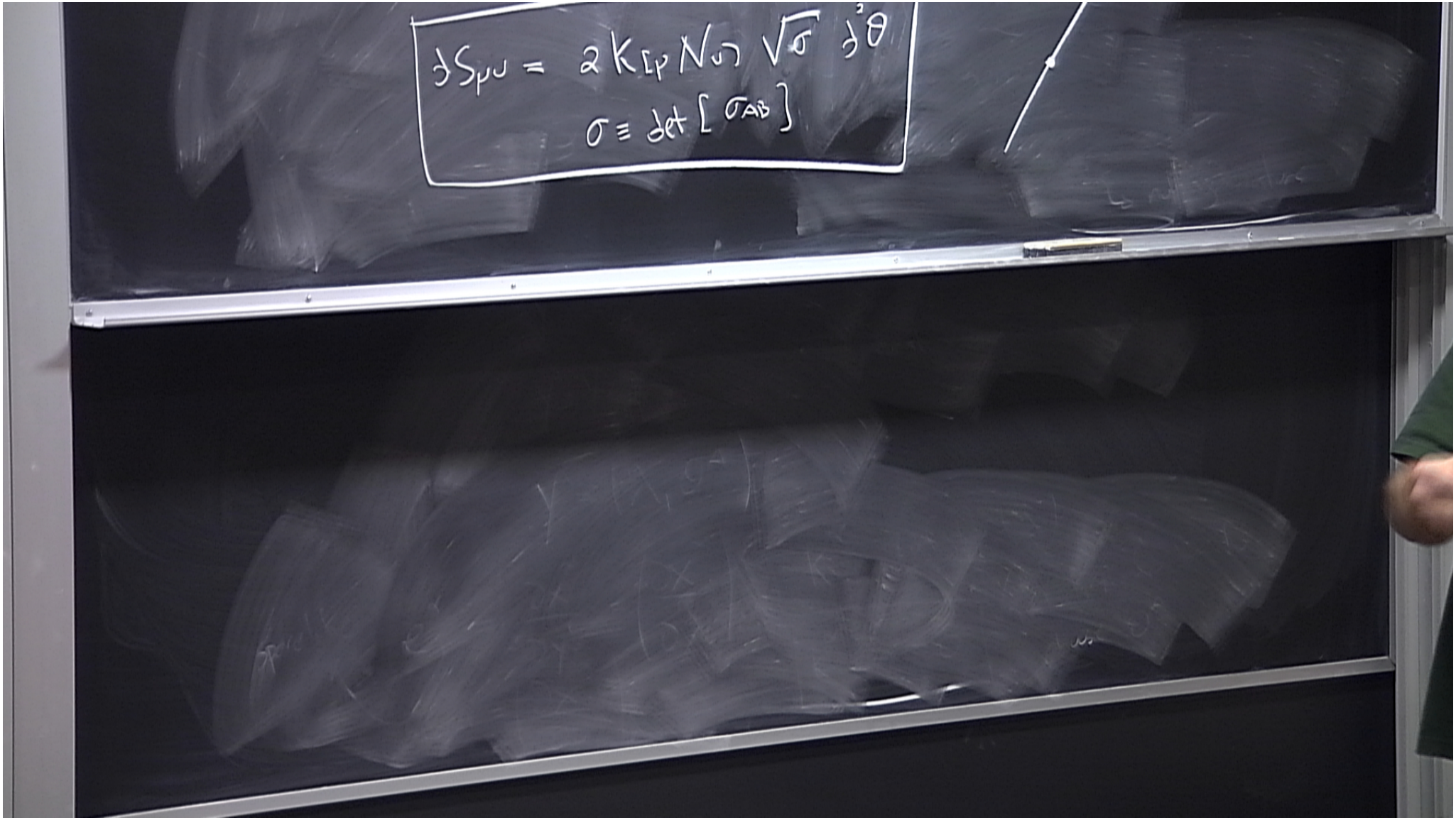

$\partial S_{\mu\nu} = f\, k_{[\mu} N_{\nu]}$
$f \propto \sqrt{\sigma}$
$f = W^{\mu} k^{\nu} \partial S_{\mu\nu}$

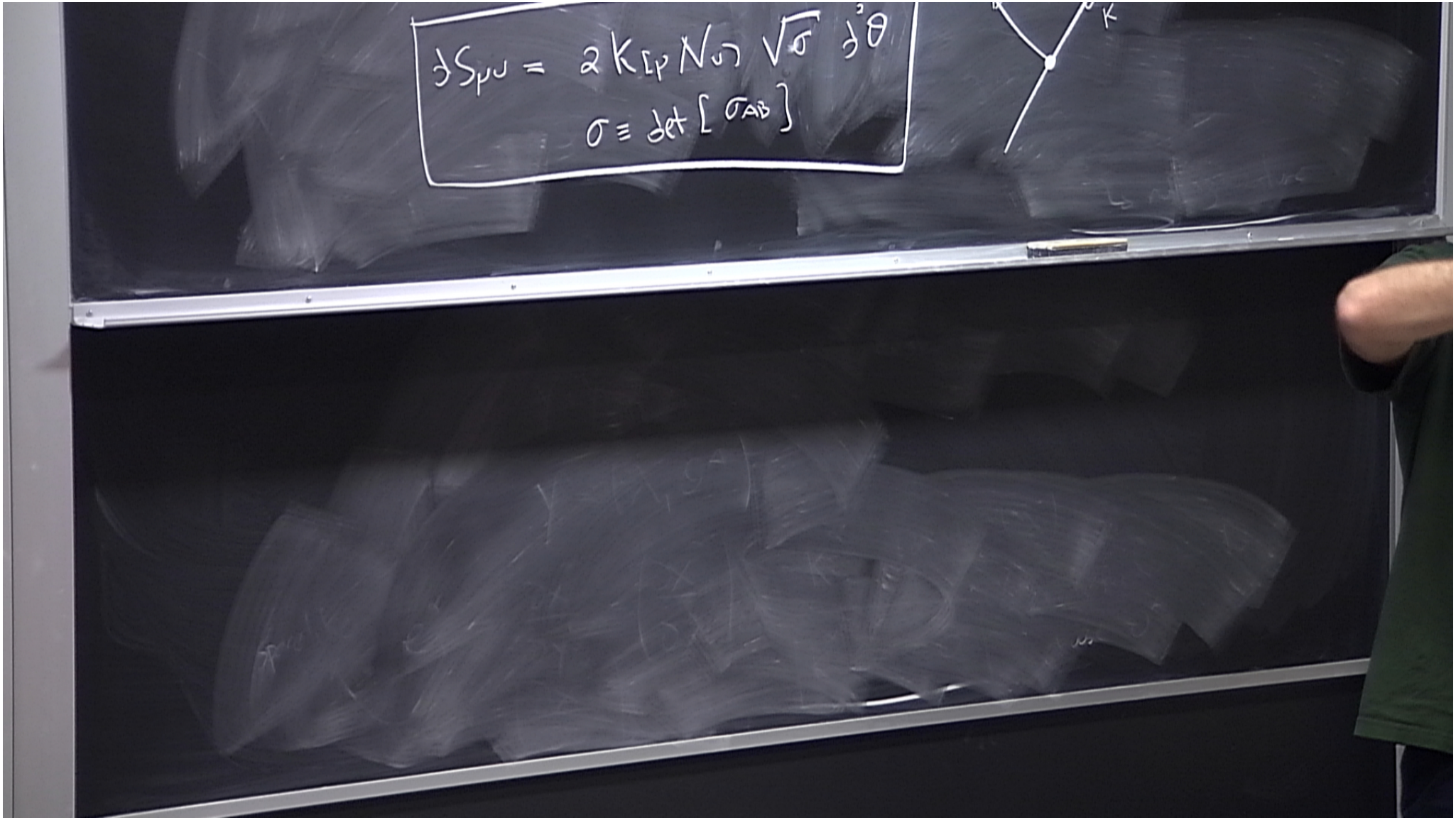

f ∝ √σ

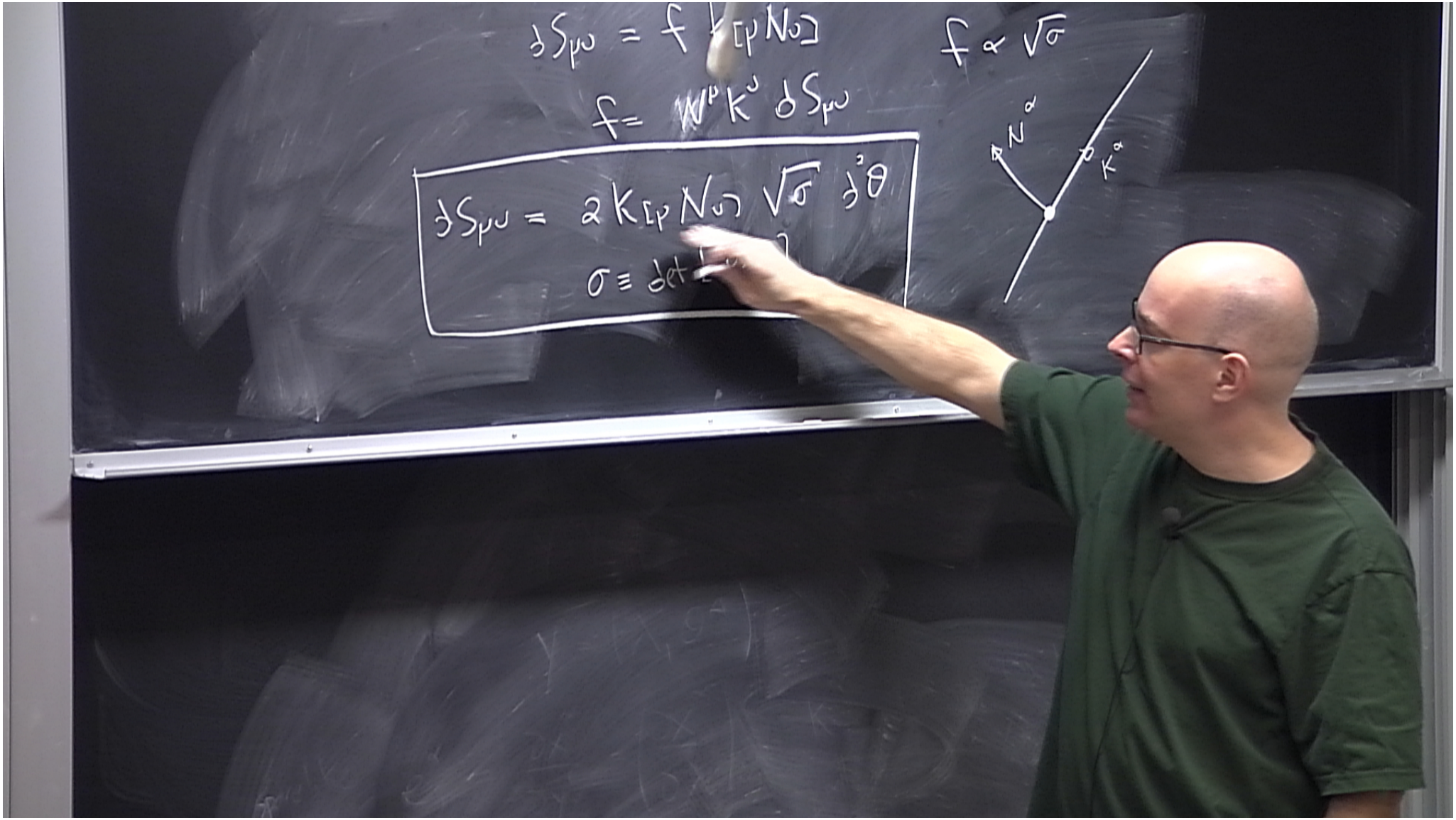

$\delta S_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, \delta^2\theta$
$\sigma \equiv \det[\sigma_{AB}]$

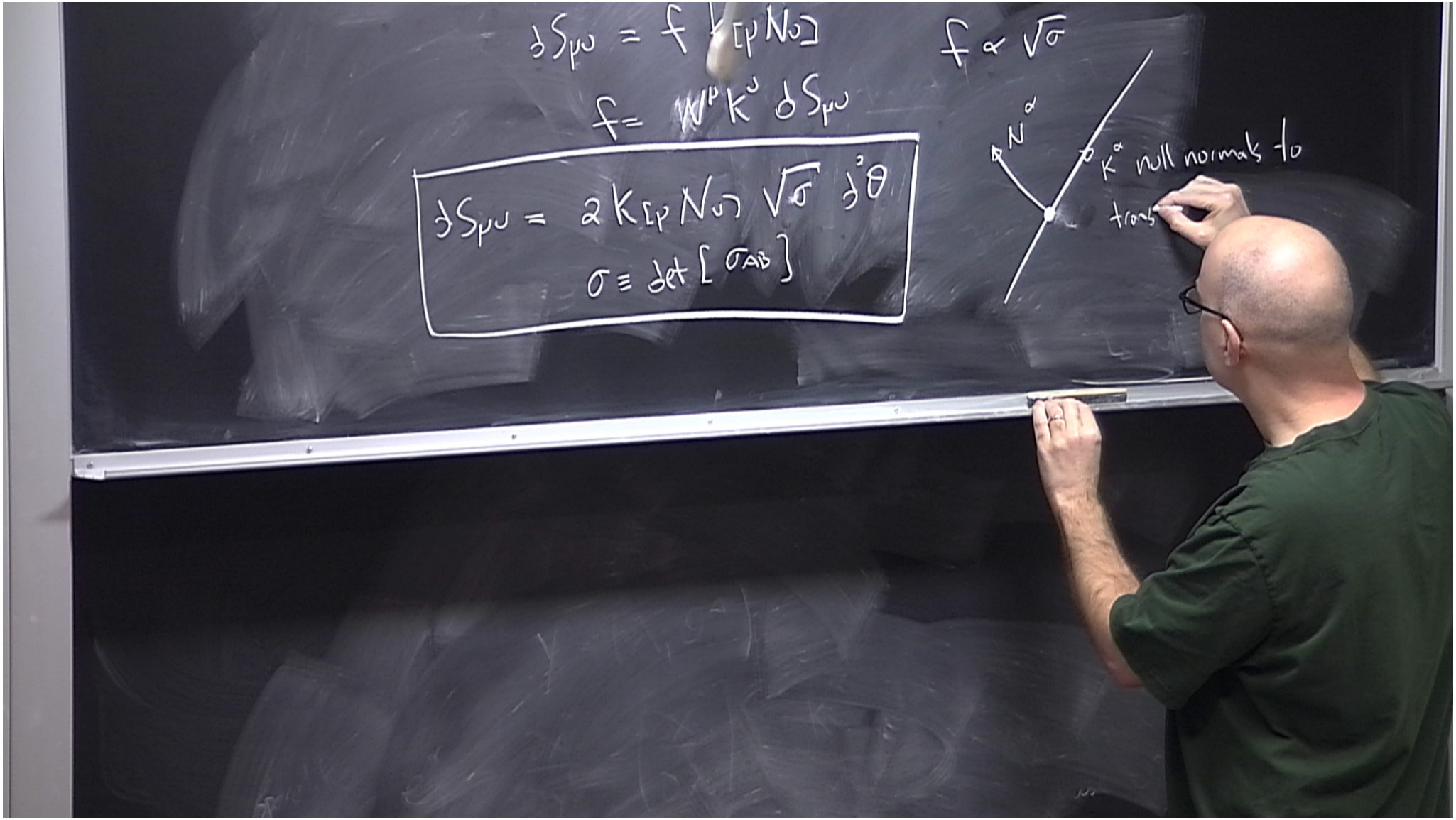

$\delta S_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, \delta\theta$
$\sigma \equiv \det[\sigma_{AB}]$

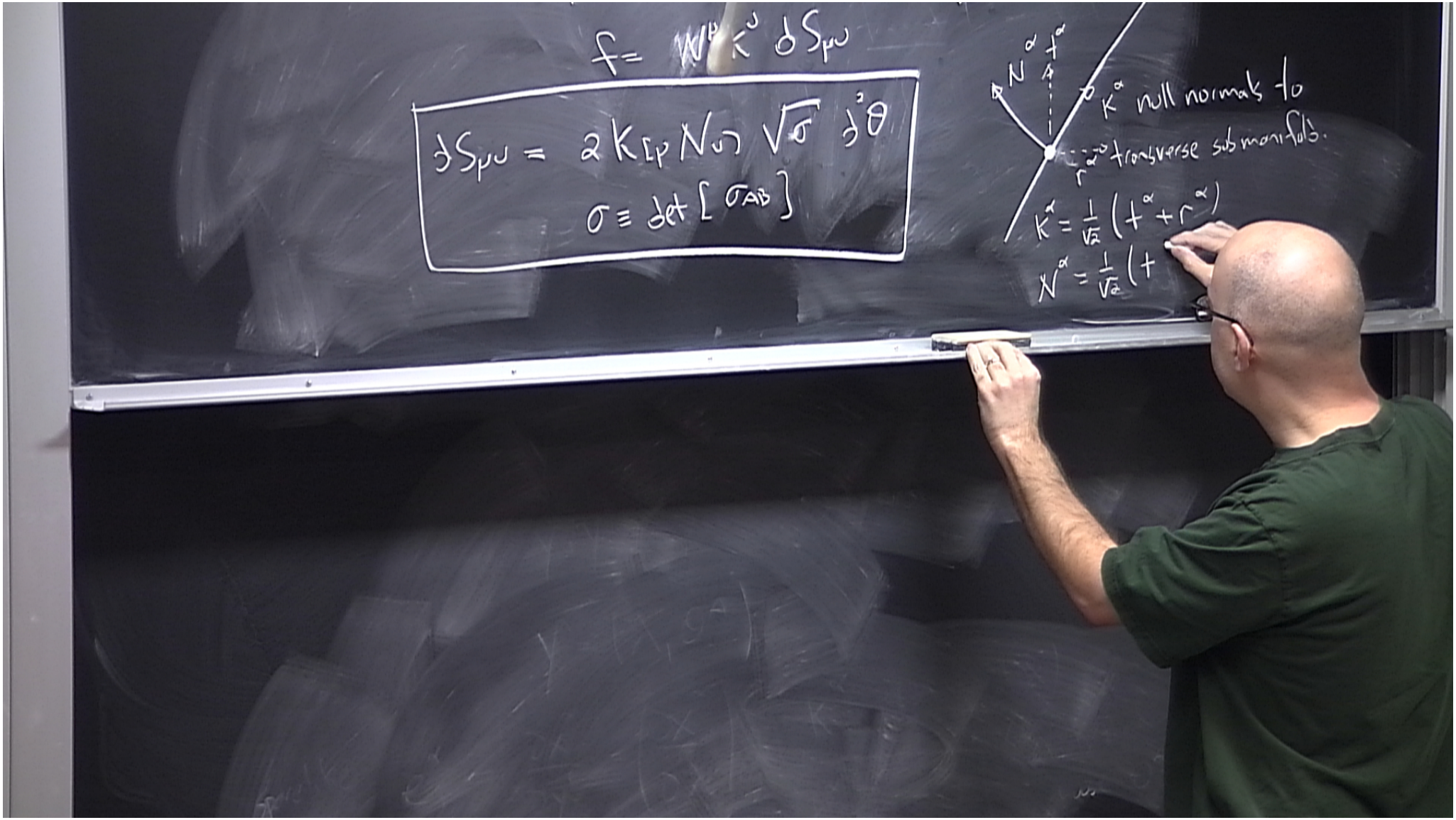

$\partial S_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, \partial\theta$
$\sigma \equiv \det[\sigma_{AB}]$

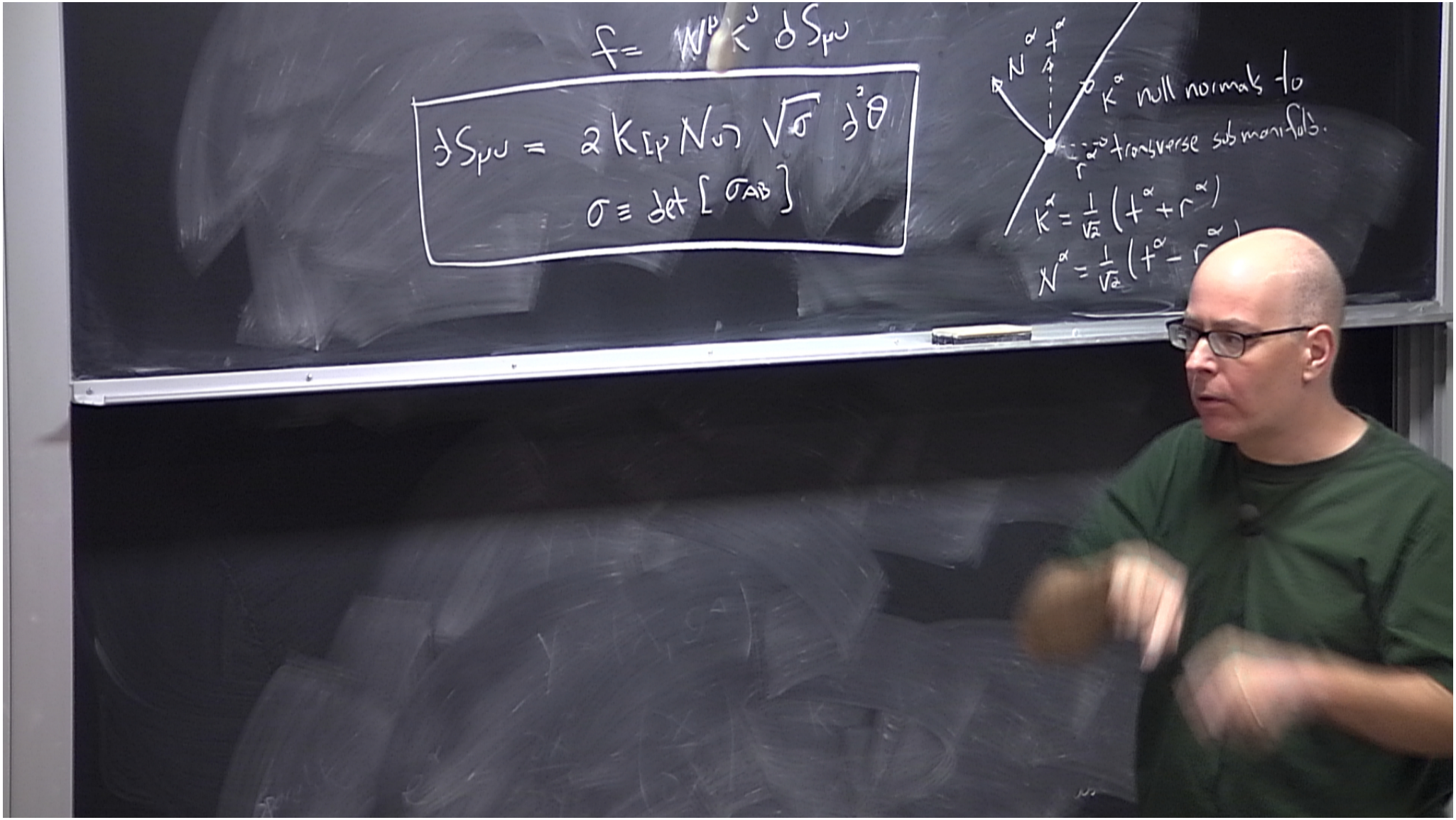

$\partial S_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, \partial^2\theta$
$\sigma \equiv \det[\sigma_{AB}]$

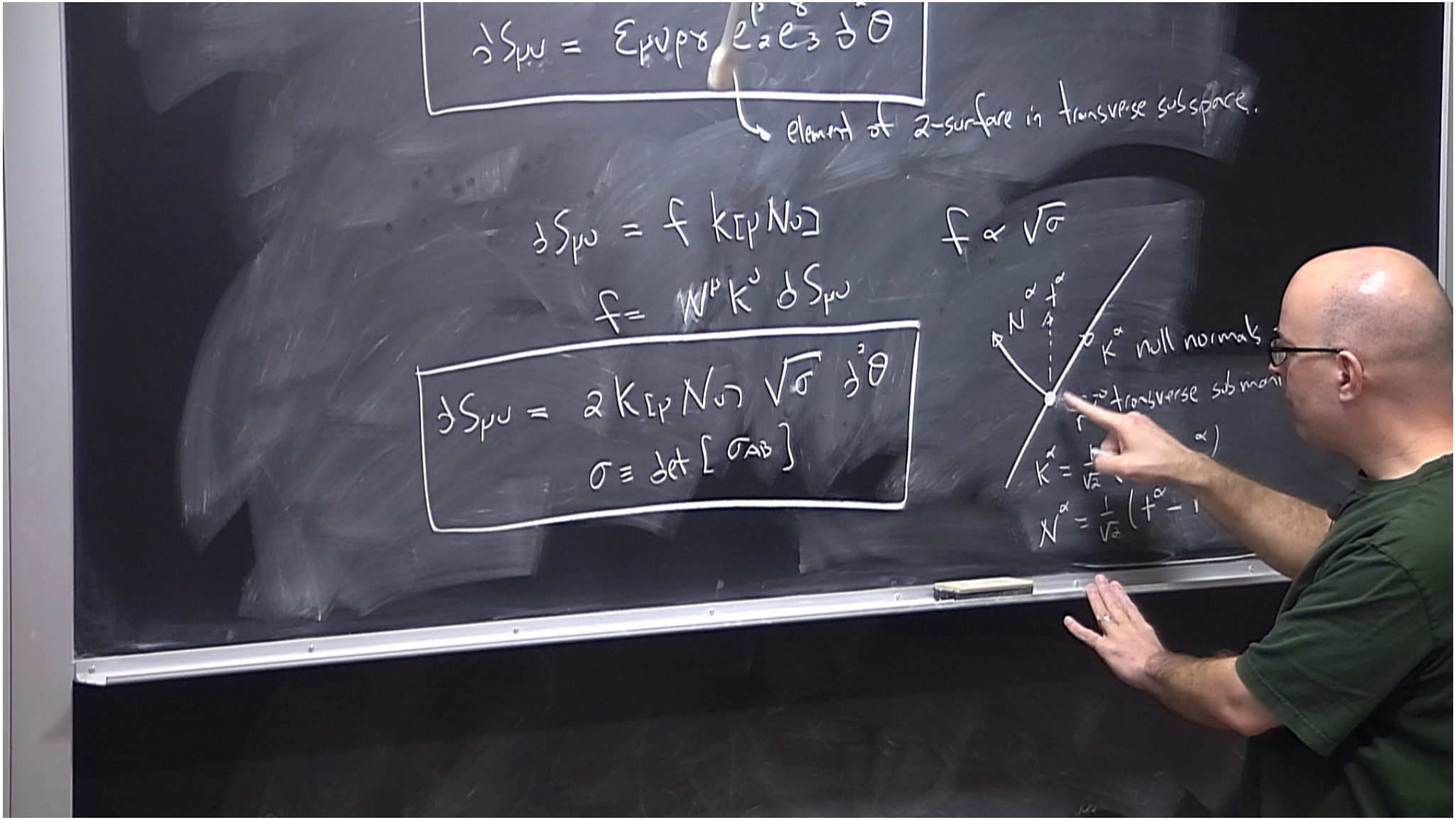

$\partial S_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, \partial\theta$
$\sigma \equiv \det[\sigma_{AB}]$


$f \propto \sqrt{\sigma}$
N^a
K^a


$dS_{\mu\nu} = f\, k_{[\mu} N_{\nu]}$
$f = W^\mu k^\nu dS_{\mu\nu}$
$f \propto \sqrt{\sigma}$
$dS_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, d^2\theta$
$\sigma \equiv \det[\sigma_{AB}]$
N^α
K^α null normals to


$f = W^{\mu\nu} \xi^{\nu} dS_{\mu\nu}$
$dS_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, d^2\theta$
$\sigma \equiv \det[\sigma_{AB}]$
N^α
t^α
k^α null normals to
transverse submanifolds.
$k^\alpha = \frac{1}{\sqrt{2}}(t^\alpha + r^\alpha)$
$N^\alpha = \frac{1}{\sqrt{2}}(t$


f = W^μ K^ν dS_μν
dS_μν = 2 K_[μ N_ν] √σ d²θ
σ ≡ det [σ_AB]
N^α t^α
K^α null normals to
r^α transverse submanifolds.
K^α = 1/√2 (t^α + r^α)
N^α = 1/√2 (t^α − r^α)


$dS_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} e^{\rho}_{2} e^{\sigma}_{3} d^2\theta$
element of 2-surface in transverse subspace.
$dS_{\mu\nu} = f\, k_{[\mu} N_{\nu]}$
$f = N^\mu k^\nu dS_{\mu\nu}$
$f \propto \sqrt{\sigma}$
$dS_{\mu\nu} = 2 k_{[\mu} N_{\nu]} \sqrt{\sigma}\, d^2\theta$
$\sigma \equiv \det[\sigma_{AB}]$
N^α
t^α
k^α null normals
transverse submani
$k^\alpha = \frac{1}{\sqrt{2}}$
$N^\alpha = \frac{1}{\sqrt{2}}(t^\alpha - 1$

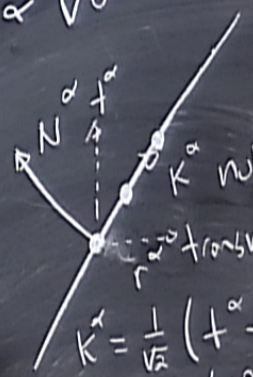
$$\partial S_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\, e_2^\rho e_3^\sigma\, d^2\theta$$

element of 2-surface in transverse subspace.

$$\partial S_{\mu\nu} = f\, k_{[\mu} N_{\nu]} \qquad f \propto \sqrt{\sigma}$$

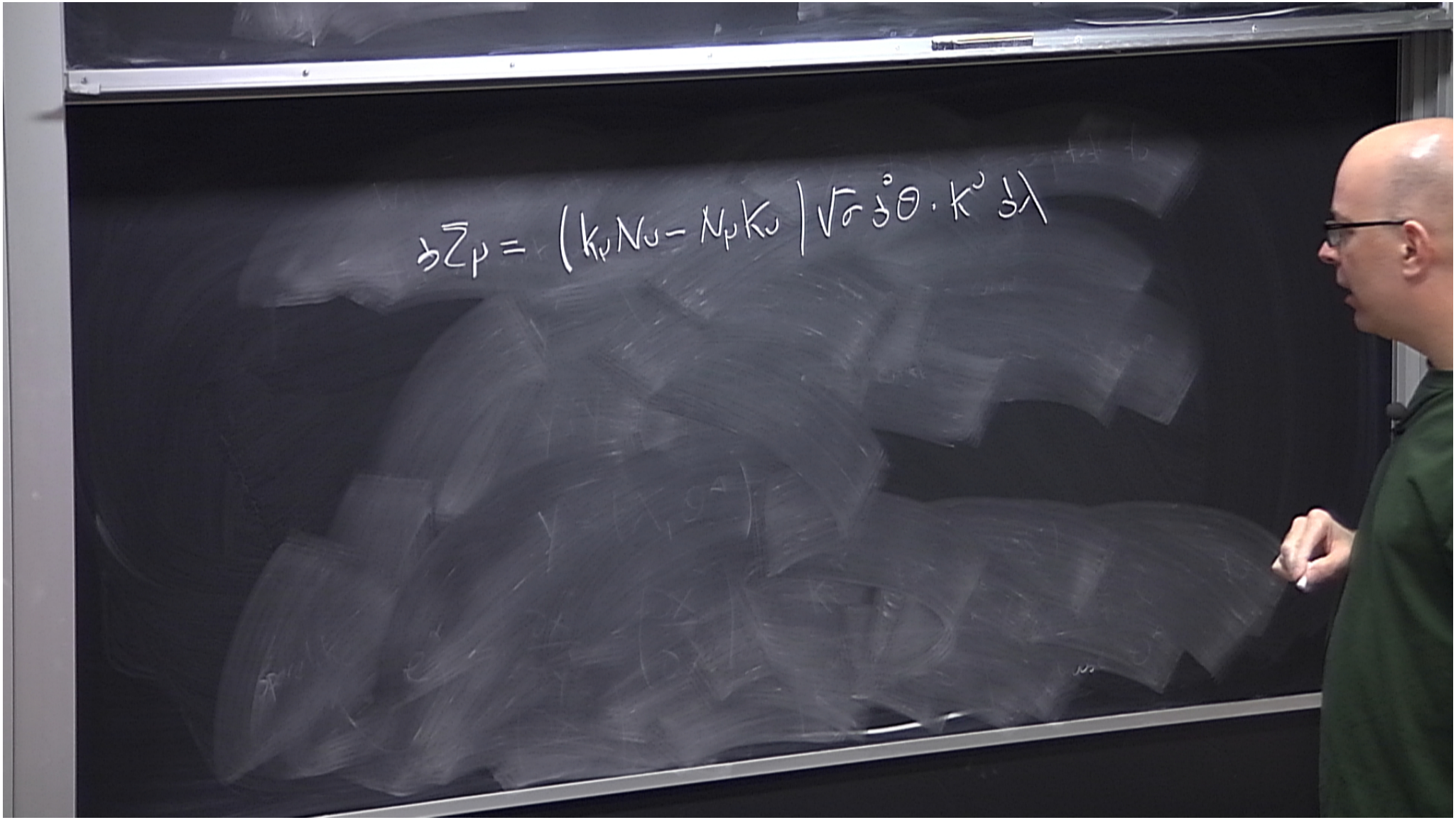
$$f = N^\mu k^\nu\, \partial S_{\mu\nu}$$

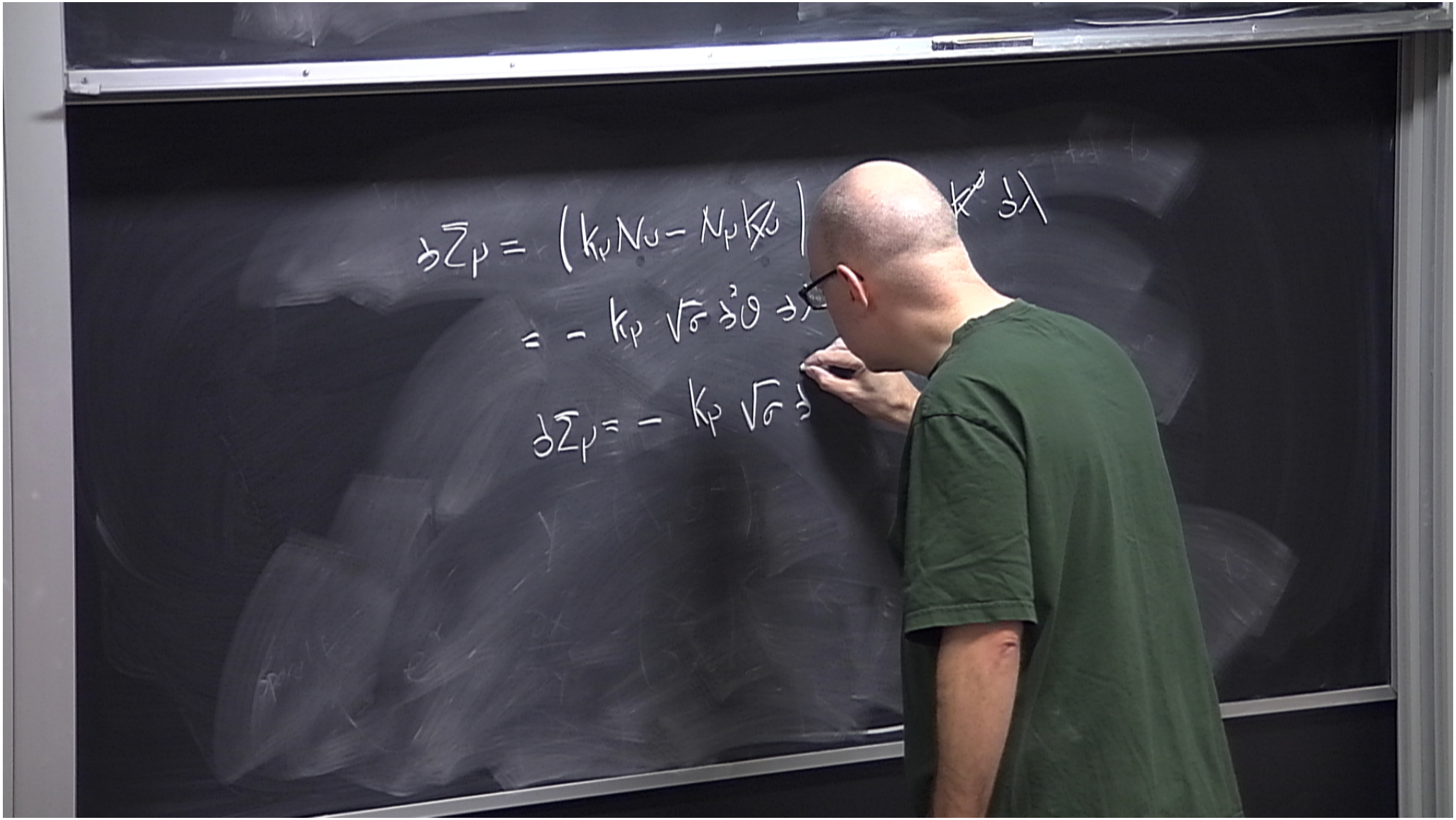
$$\boxed{\partial S_{\mu\nu} = 2\, k_{[\mu} N_{\nu]}\, \sqrt{\sigma}\, d^2\theta \qquad \sigma \equiv \det[\sigma_{AB}]}$$

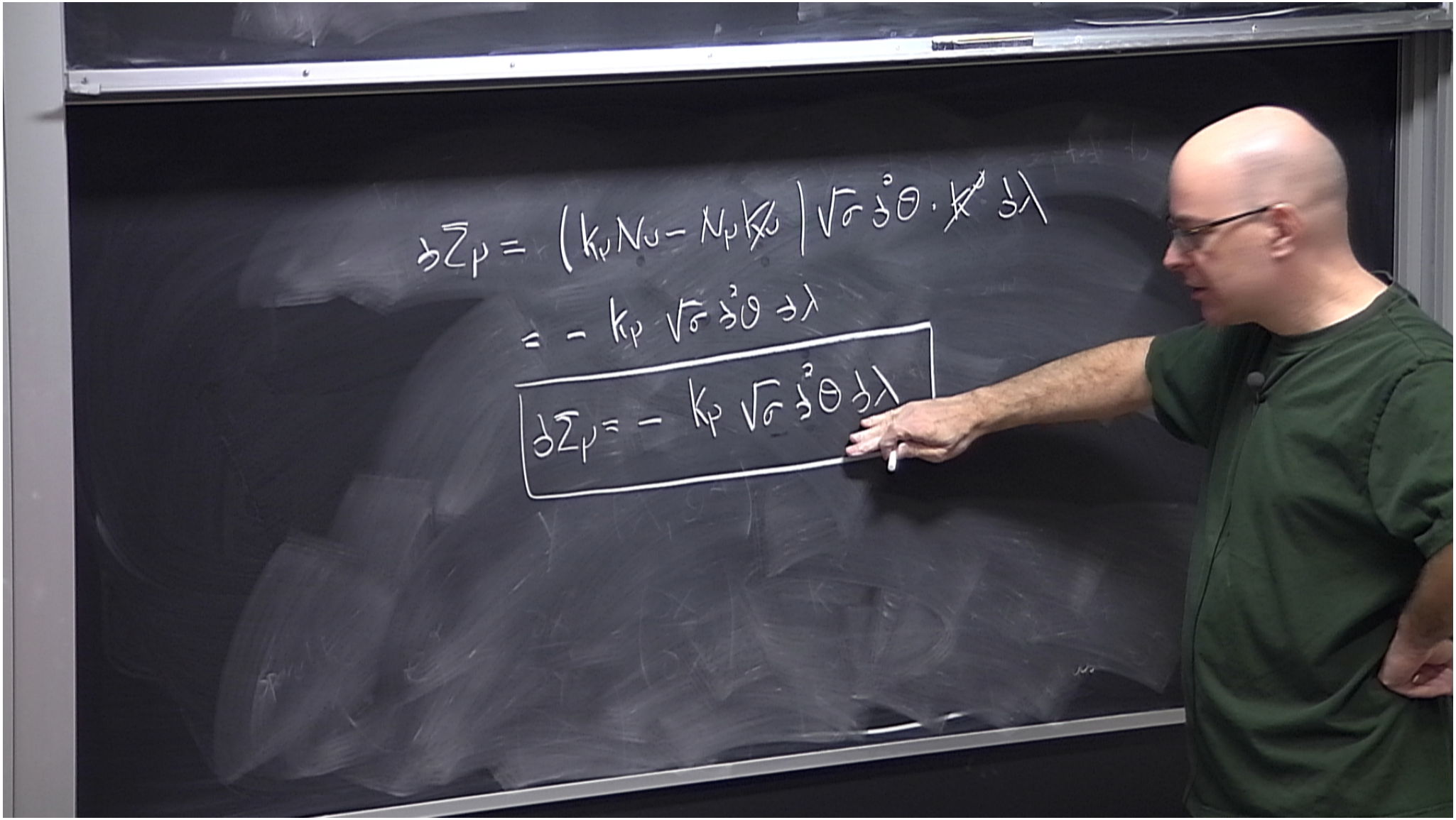


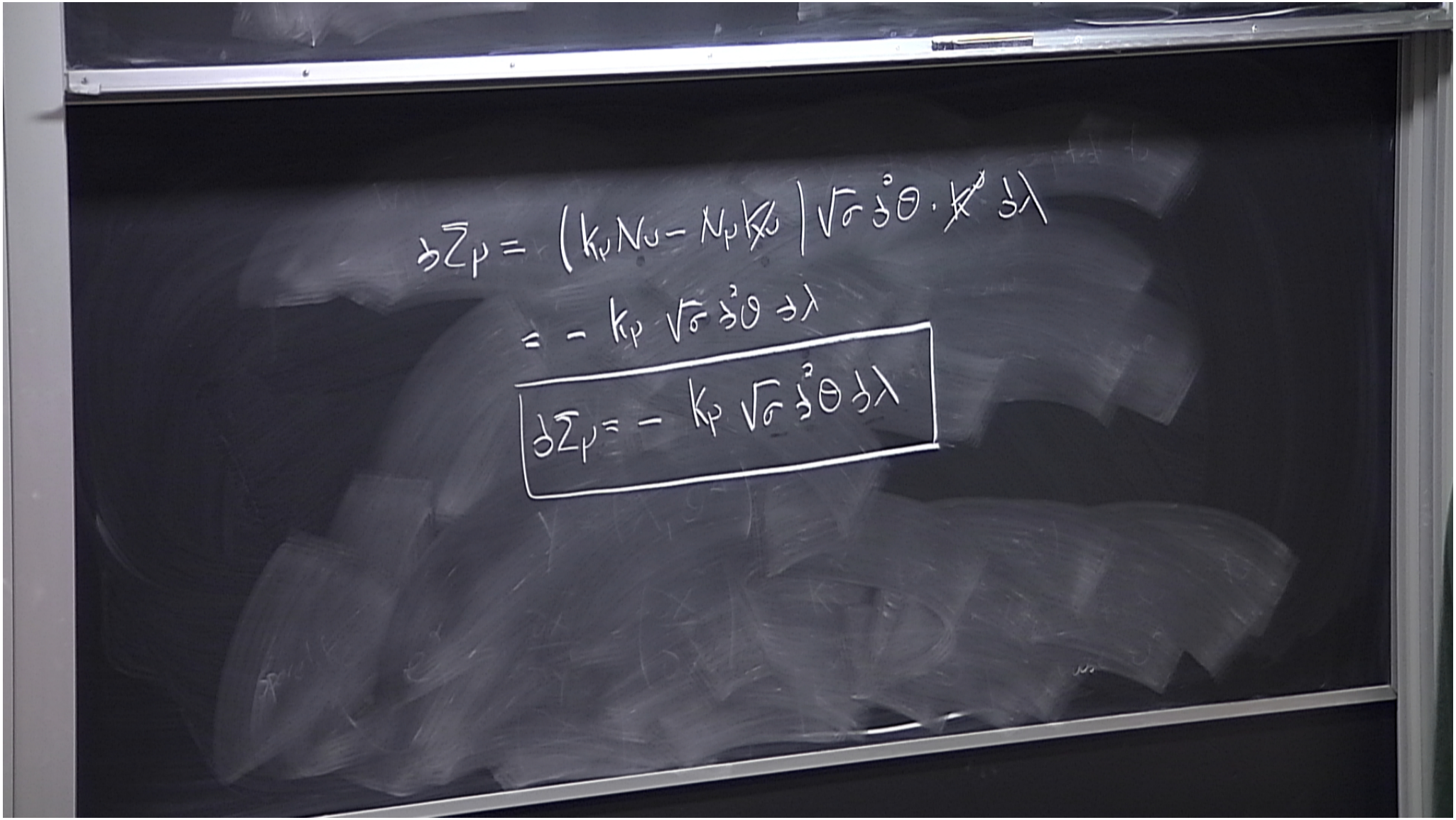
$$k^\alpha = \frac{1}{\sqrt{2}}(t^\alpha + r^\alpha)$$

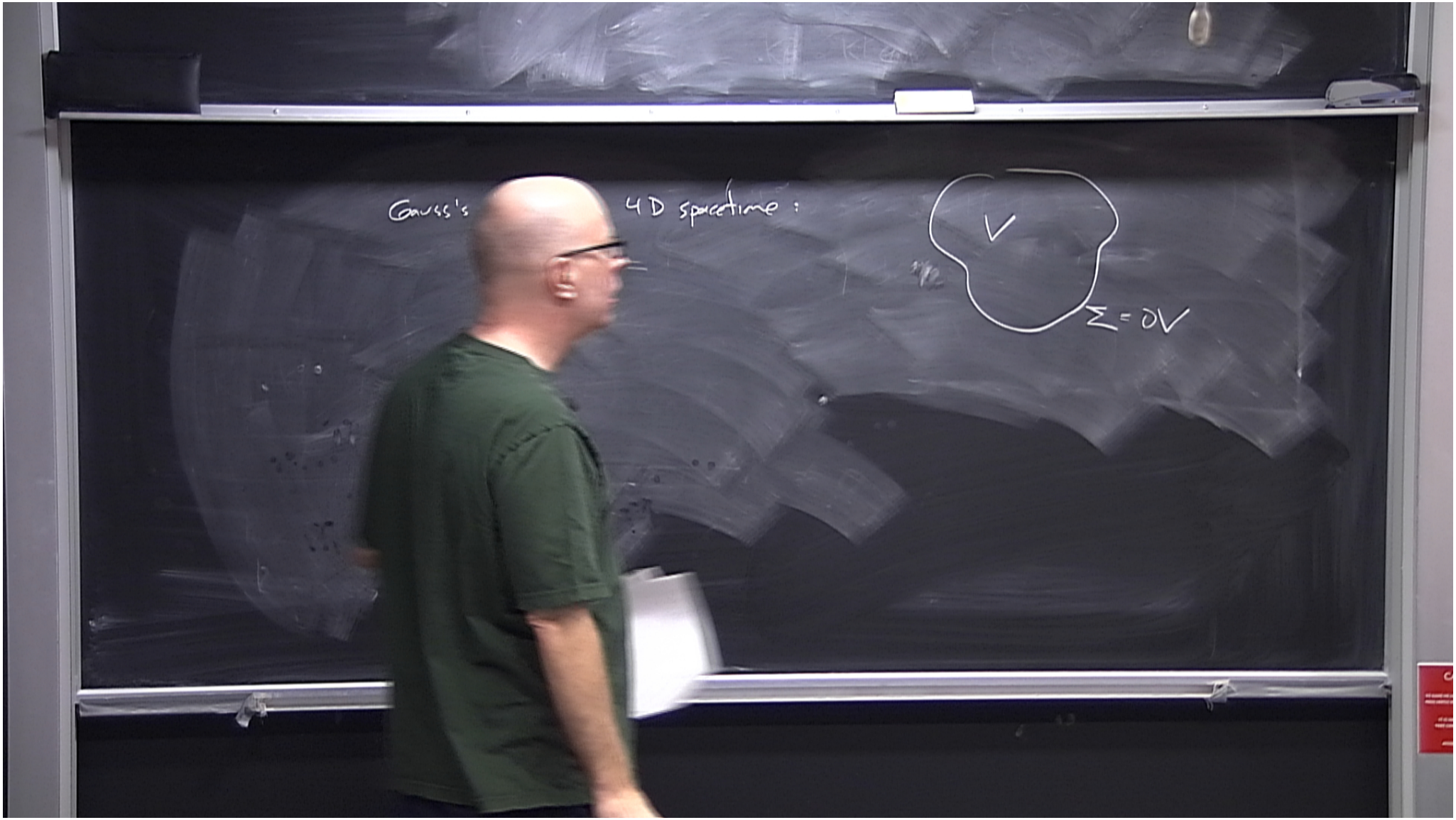
$$N^\alpha = \frac{1}{\sqrt{2}}(t^\alpha - r^\alpha)$$


$\partial \bar{Z}_\mu = (k_\mu N_\nu - N_\mu k_\nu) \sqrt{\sigma}\, \partial\theta \cdot k^\nu\, d\lambda$


$\partial \Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu$
$= - k_\mu \sqrt{\sigma}\, \partial\theta\, \partial$
$\partial \Sigma_\mu = - k_\mu \sqrt{\sigma}\, \partial$

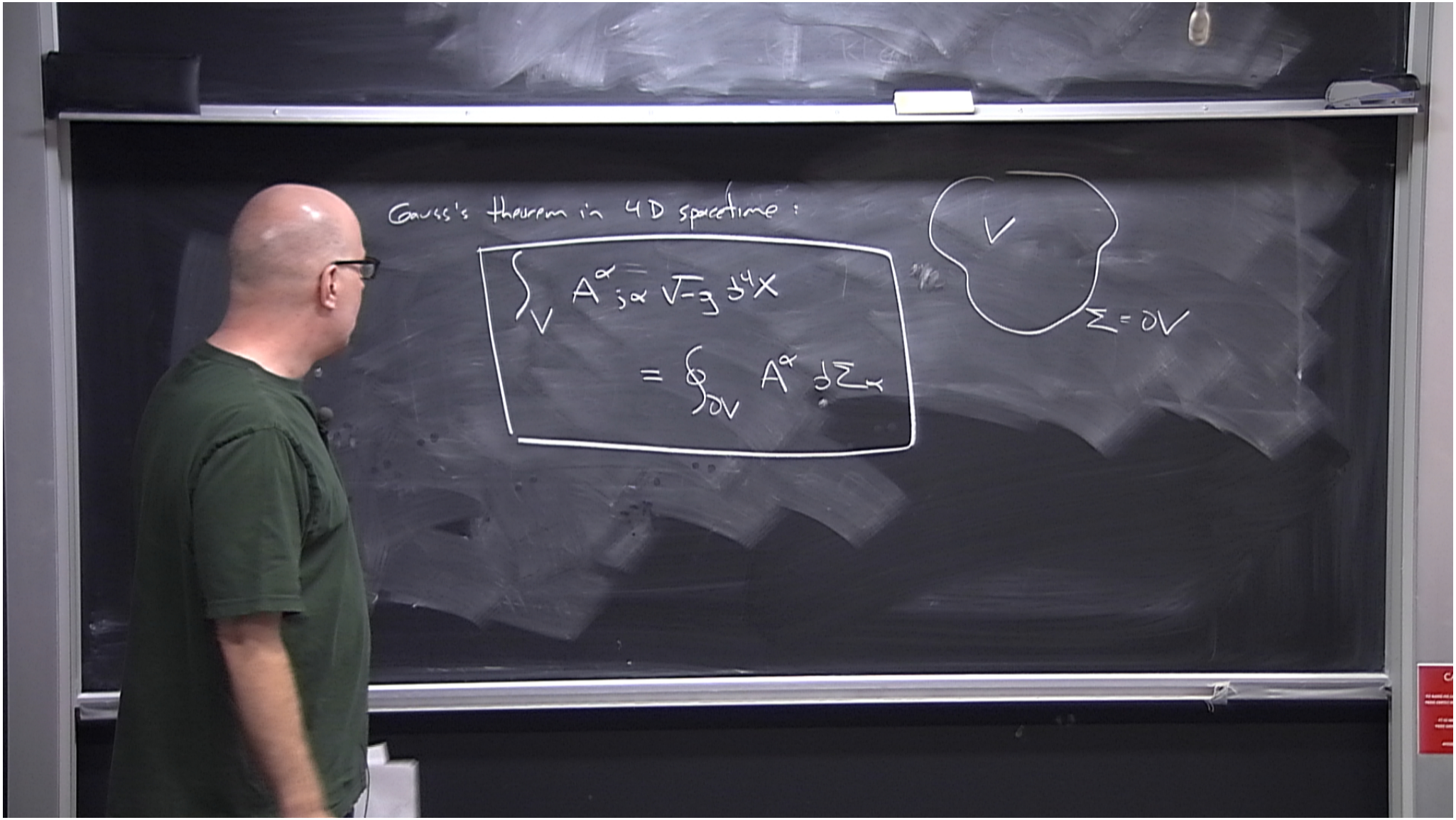

$\partial\Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu)\sqrt{\sigma}\,\partial^2\theta\cdot k\,\partial\lambda$
$= -k_\mu\sqrt{\sigma}\,\partial^2\theta\,\partial\lambda$
$\partial\Sigma_\mu = -k_\mu\sqrt{\sigma}\,\partial^2\theta\,\partial\lambda$

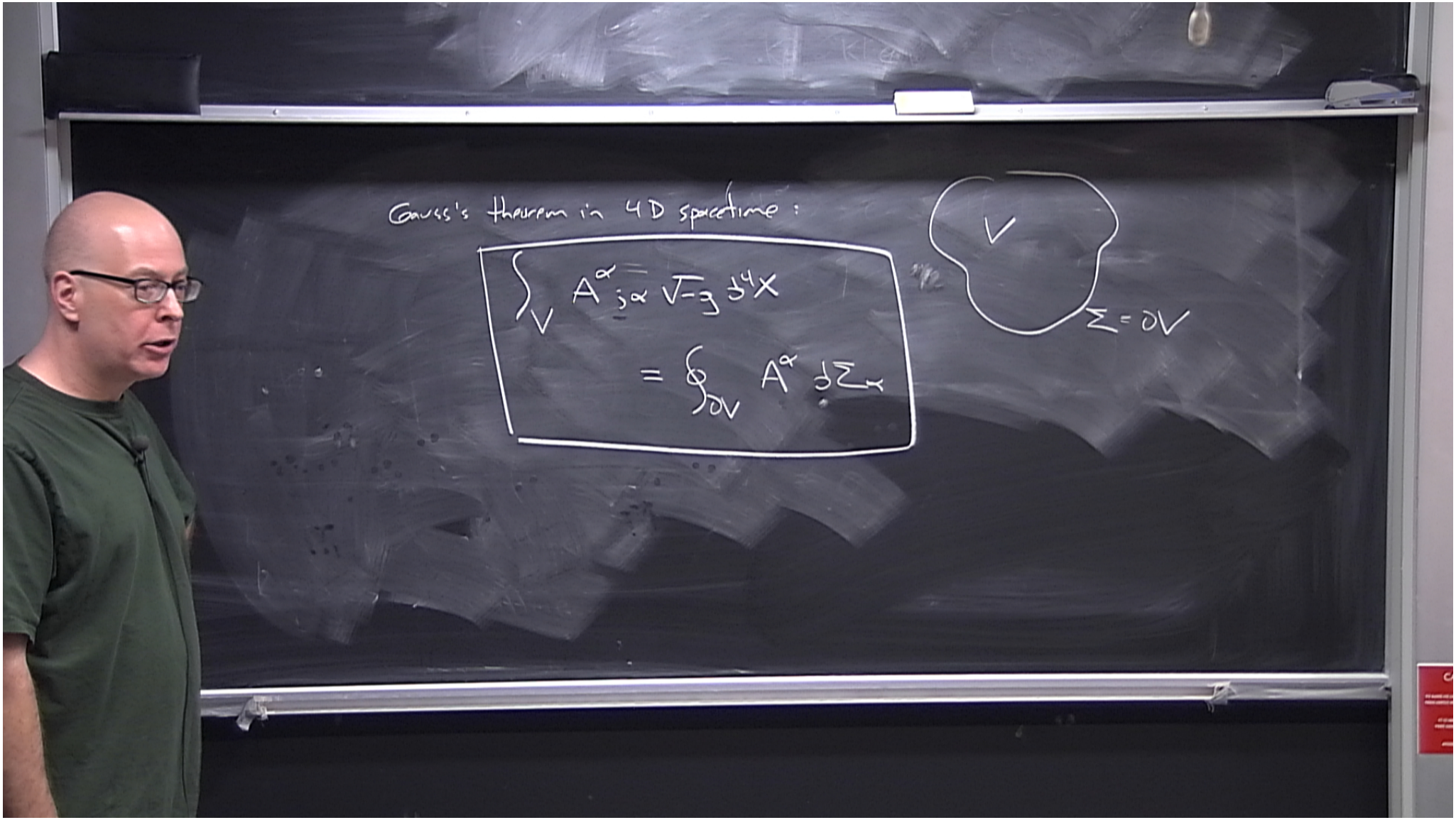

$\partial \Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu) \sqrt{\sigma}\, \partial^2\theta \cdot k\, \partial\lambda$
$= - k_\mu \sqrt{\sigma}\, \partial^2\theta\, \partial\lambda$
$\boxed{\partial \Sigma_\mu = - k_\mu \sqrt{\sigma}\, \partial^2\theta\, \partial\lambda}$

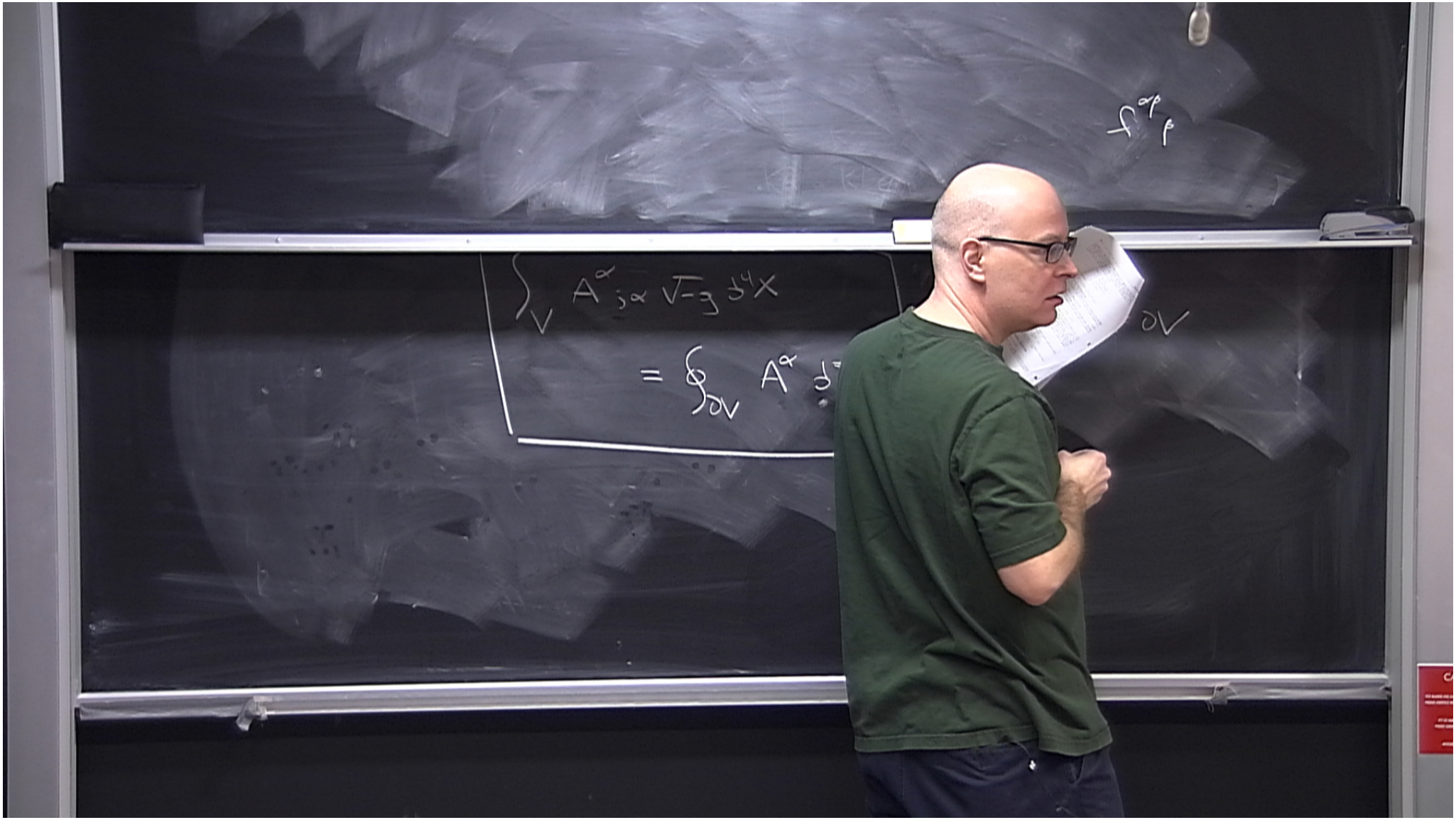

Gauss's
4D spacetime :
V
Σ = ∂V

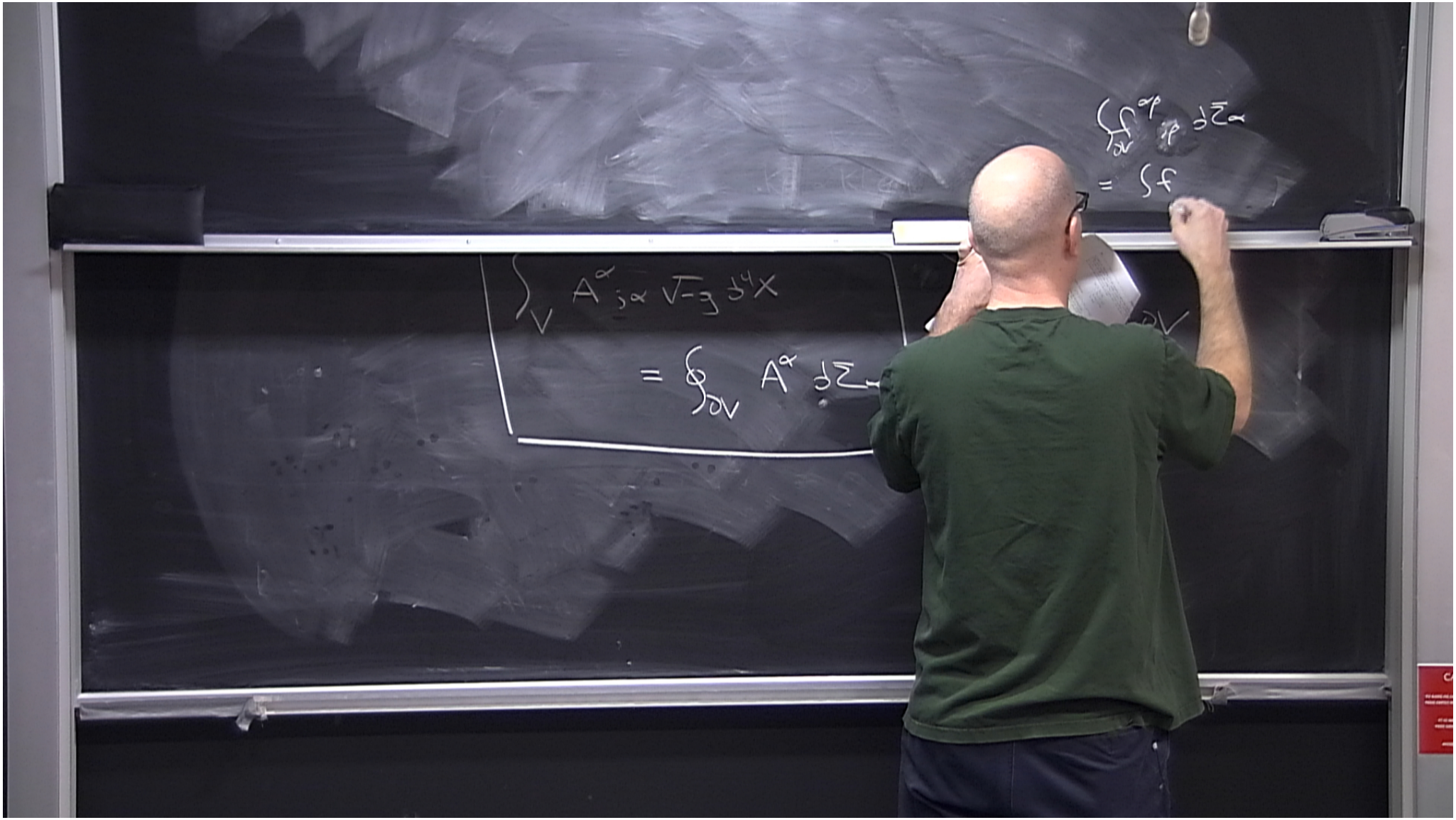
Gauss's theorem in 4D spacetime:

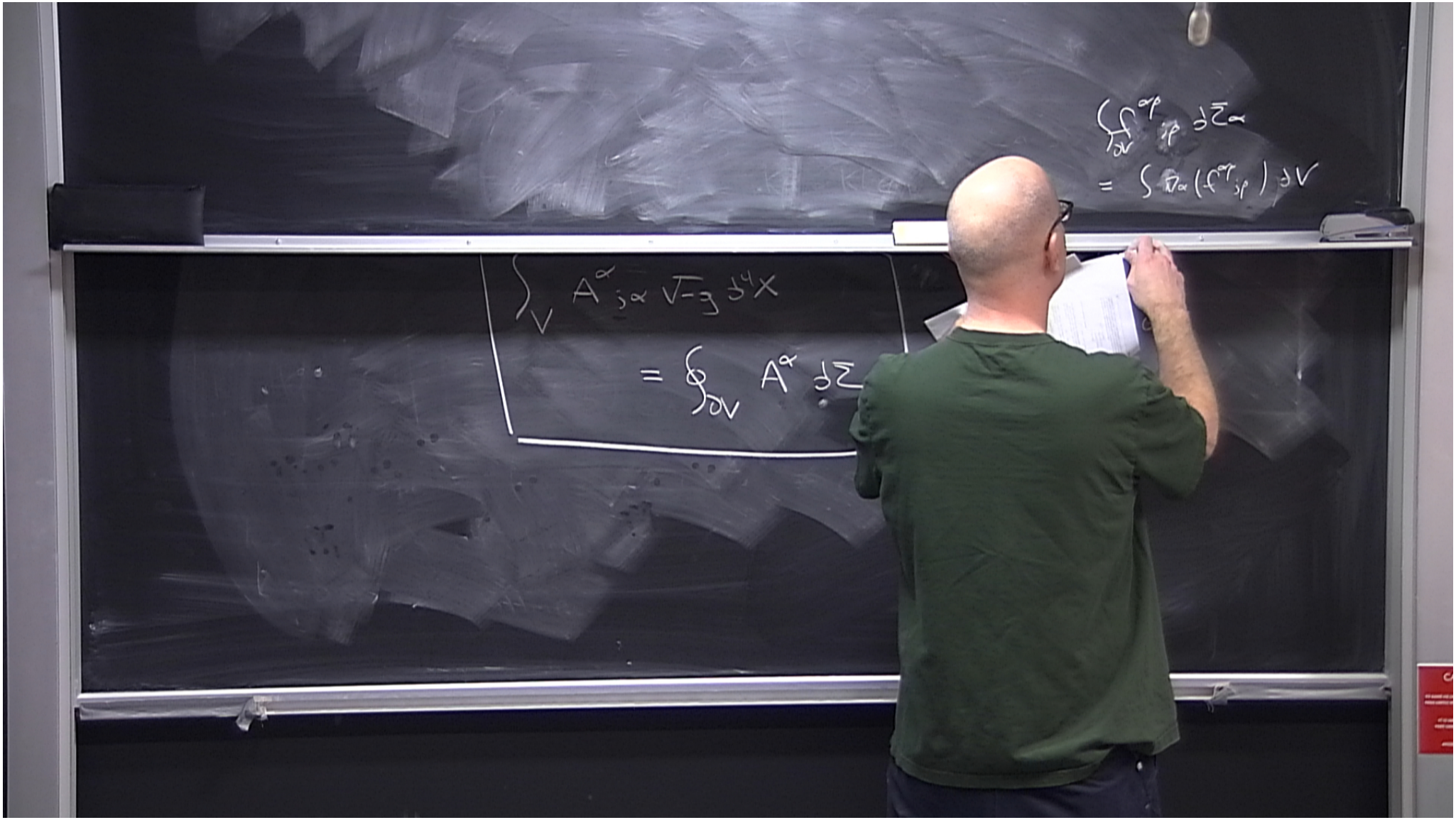
$$\int_V A^{\alpha}{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} \Sigma = \partial V$$

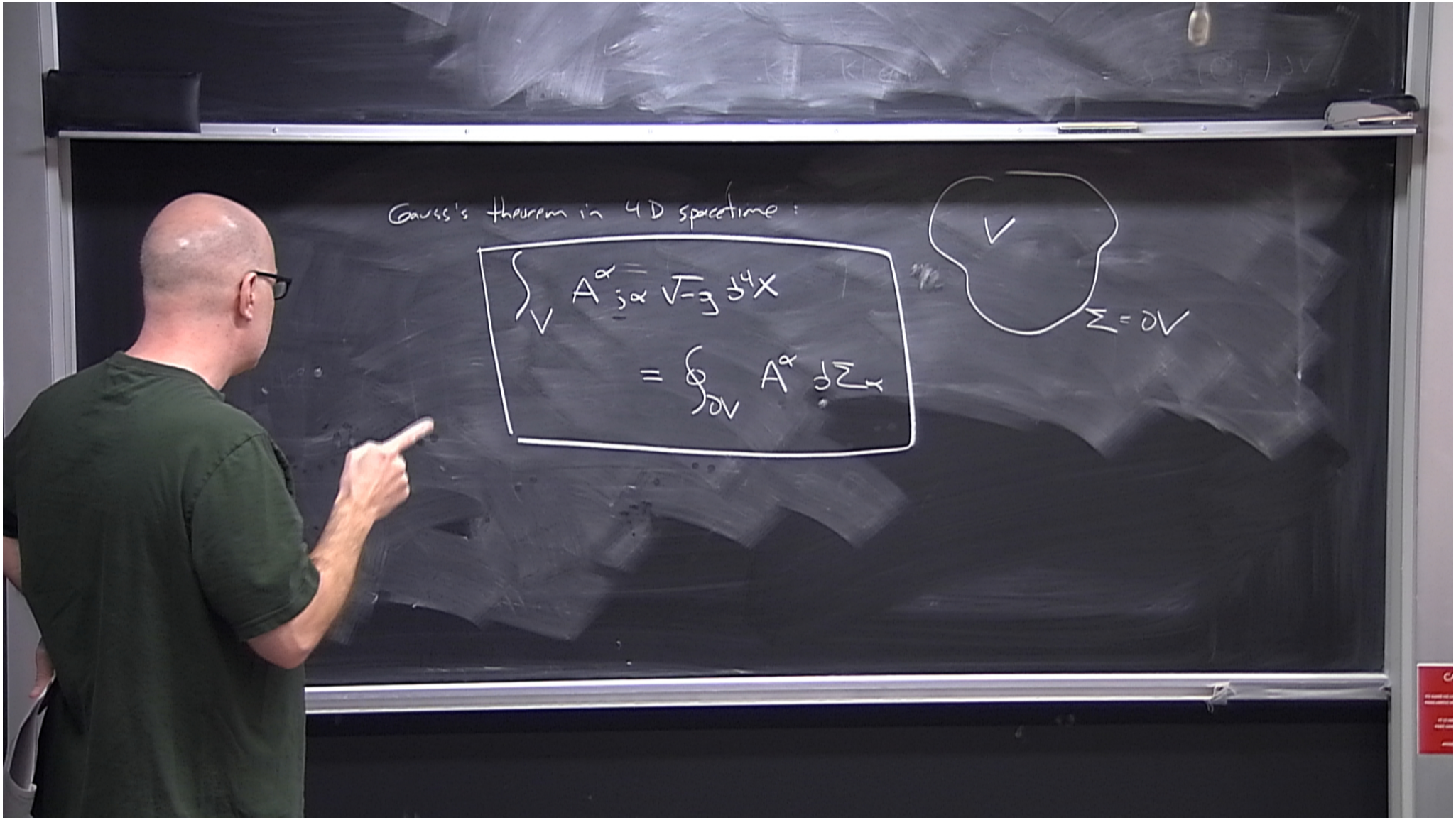

Gauss's theorem in 4D spacetime :
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$

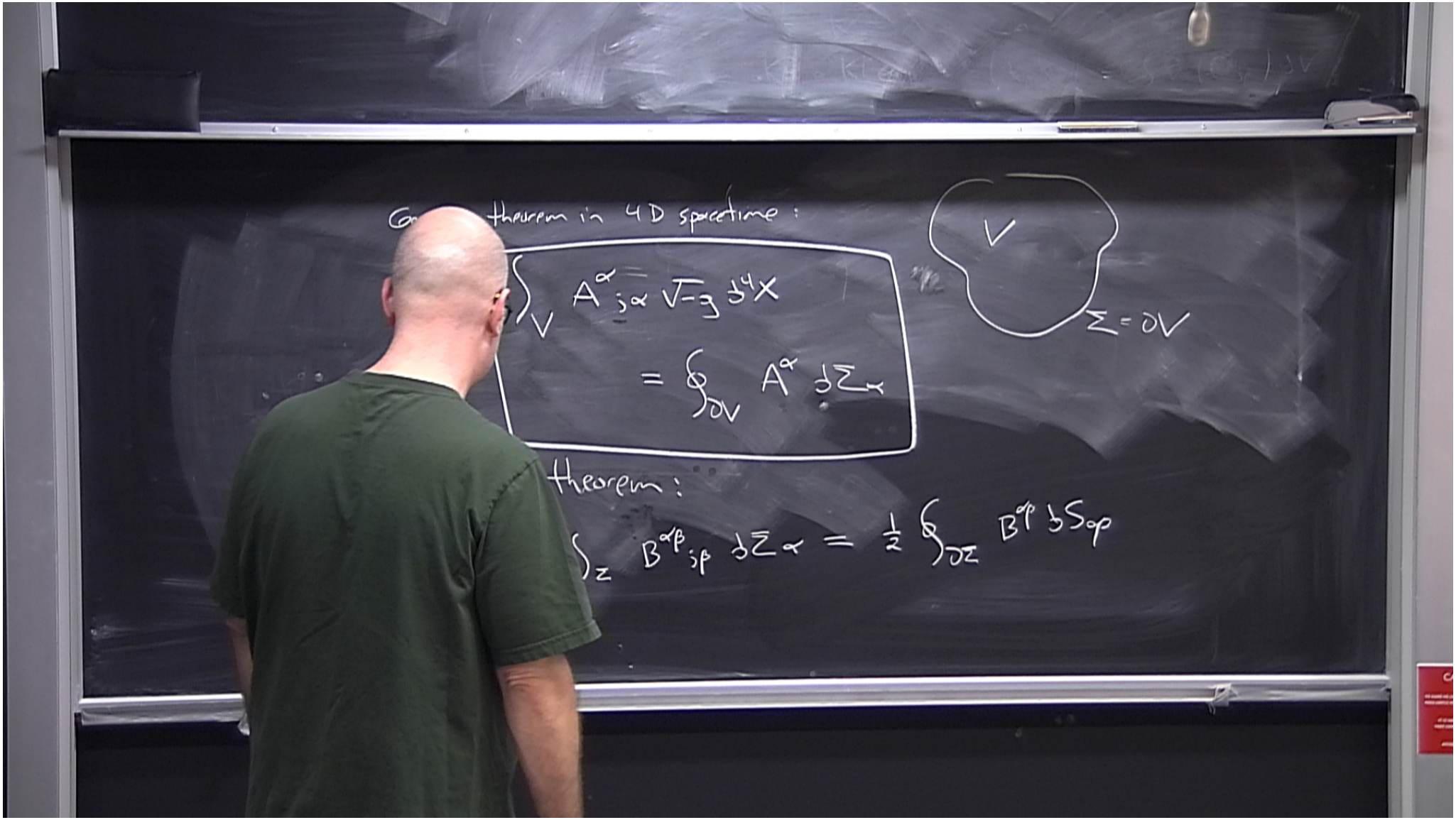

Gauss's theorem in 4D spacetime :
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$

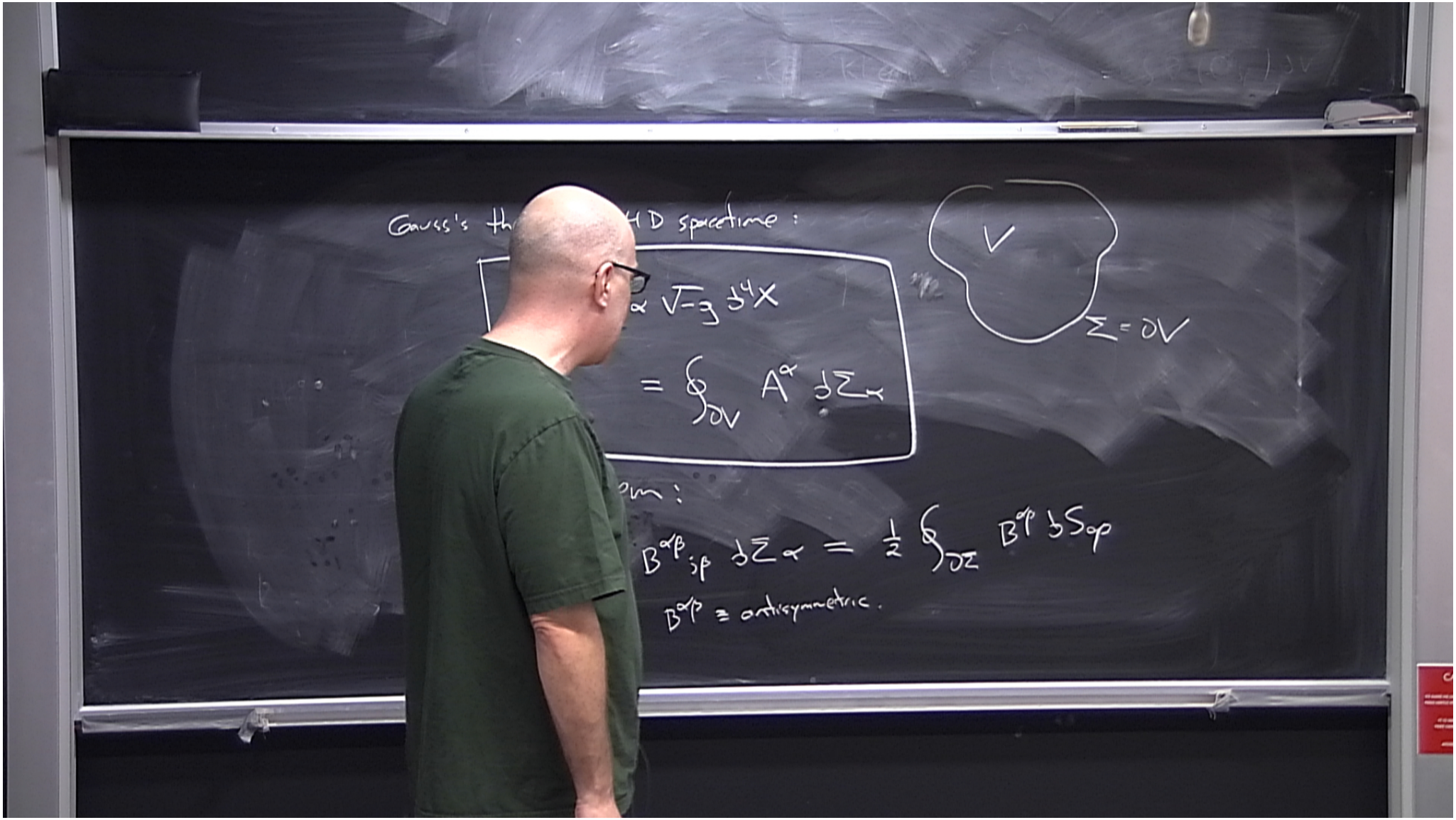

$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x$
$= \oint_{\partial V} A^\alpha$
∂V

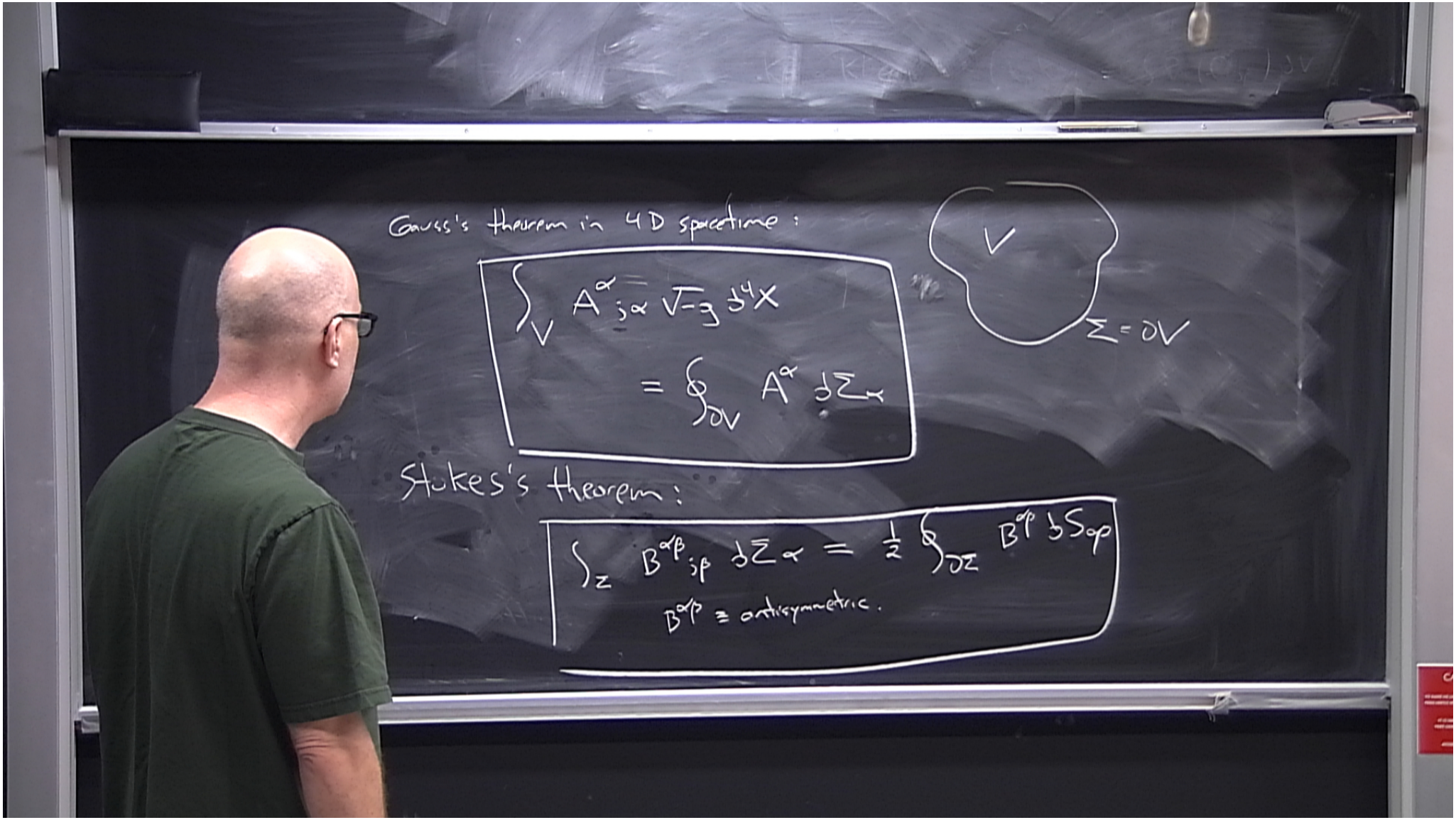

$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x$
$= \oint_{\partial V} A^\alpha$

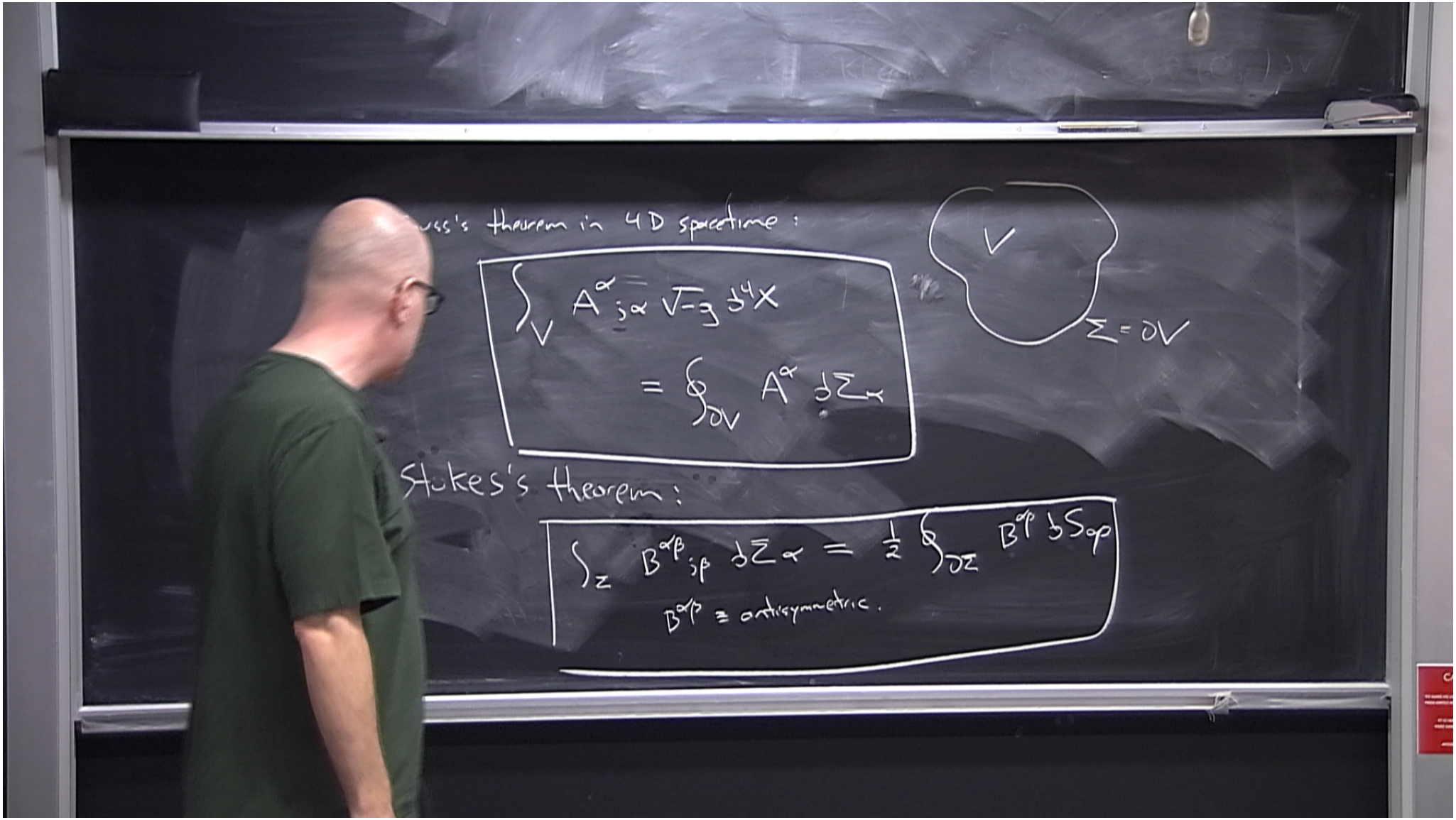

$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x$
$= \oint_{\partial V} A^\alpha \, d\Sigma$
$= \int_V \nabla_\alpha (A^\alpha{}_\beta)\, dV$


Gauss's theorem in 4D spacetime:
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x$
$= \oint_{\partial V} A^\alpha \, d\Sigma_\alpha$
V
$\Sigma = \partial V$


theorem in 4D spacetime :
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$
theorem :
$\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha = \frac{1}{2} \oint_{\partial\Sigma} B^{\alpha\beta}\, dS_{\alpha\beta}$


Gauss's th
4D spacetime :
$\int \sqrt{-g}\, d^4x$
$= \oint_{\partial V} A^\alpha \, d\Sigma_\alpha$
V
$\Sigma = \partial V$
$B^{\alpha\beta}{}_{;\beta} \, d\Sigma_\alpha = \frac{1}{2} \oint_{\partial \Sigma} B^{\alpha\beta} \, dS_{\alpha\beta}$
$B^{\alpha\beta}$ = antisymmetric.


Gauss's theorem in 4D spacetime:
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$
Stokes's theorem:
$\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha = \frac{1}{2} \oint_{\partial\Sigma} B^{\alpha\beta}\, dS_{\alpha\beta}$
$B^{\alpha\beta}$ = antisymmetric.


uss's theorem in 4D spacetime:
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$
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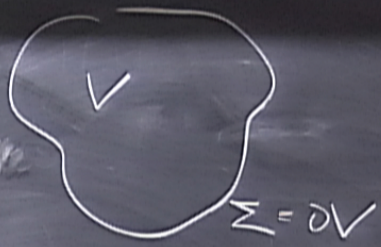
$\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha = \frac{1}{2} \oint_{\partial\Sigma} B^{\alpha\beta}\, dS_{\alpha\beta}$
$B^{\alpha\beta}$ = antisymmetric.

Gauss's theorem in 4D spacetime:

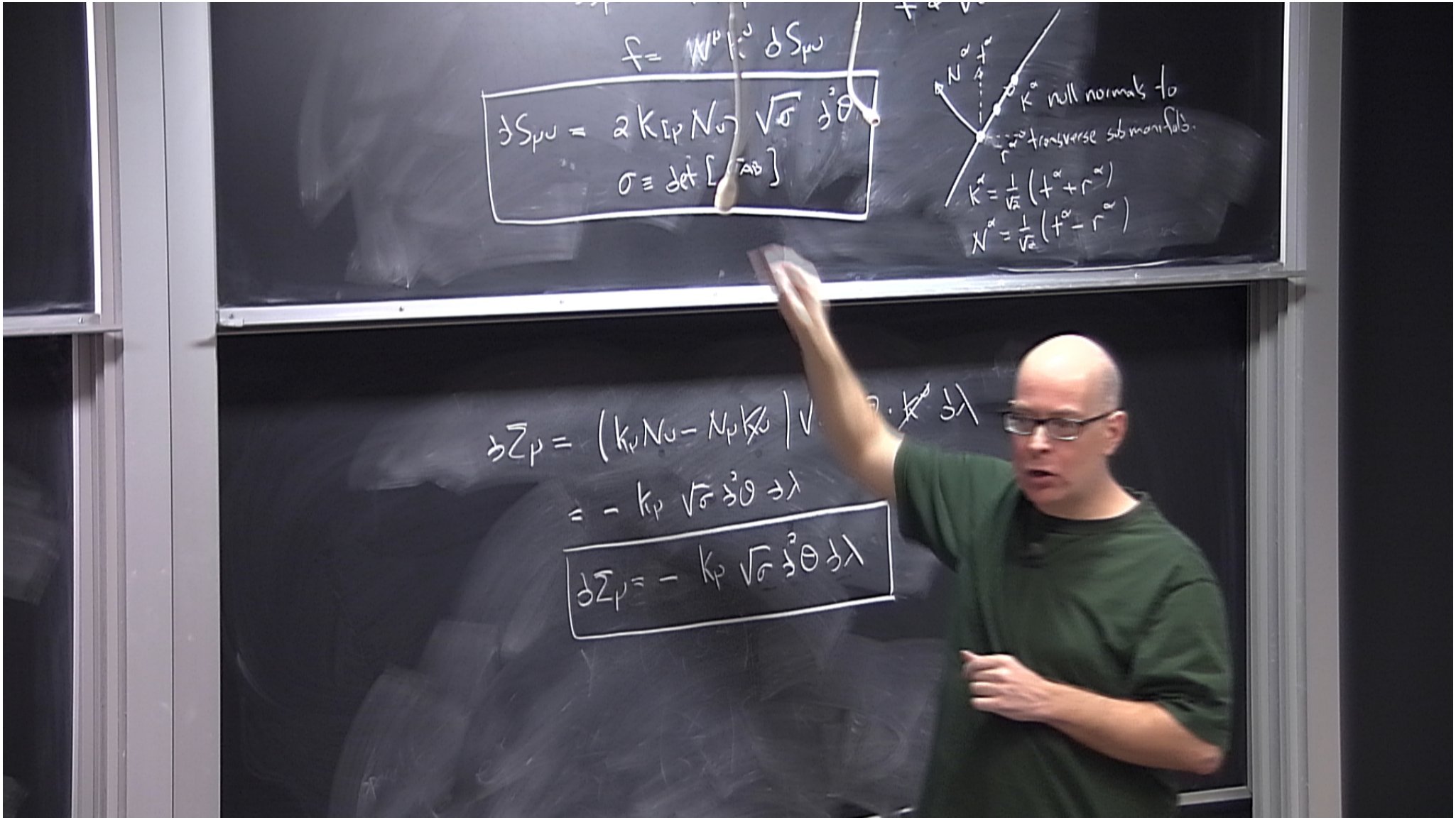
$$\int_V A^\alpha{}_{;\alpha}\sqrt{-g}\,d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$$

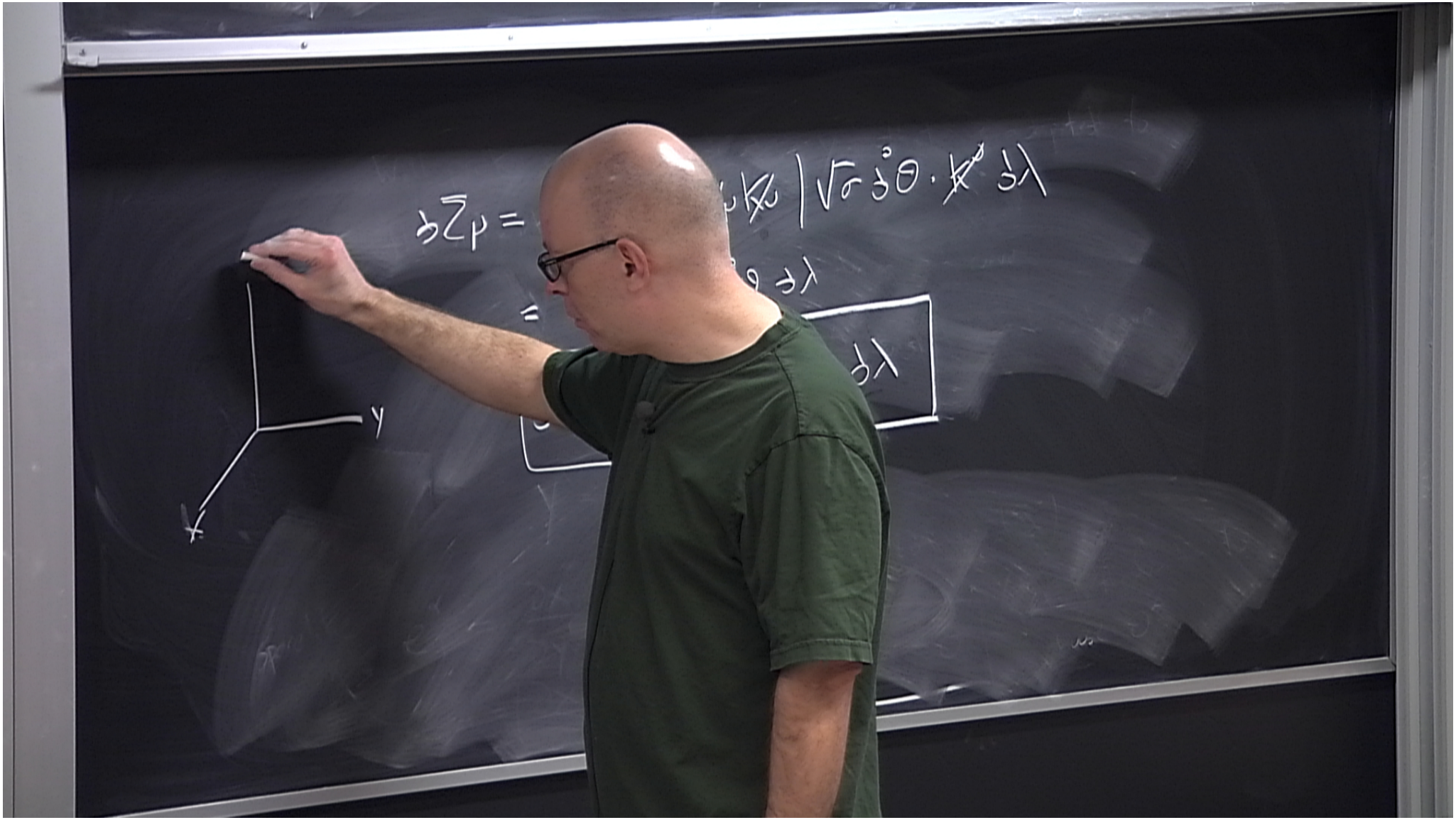


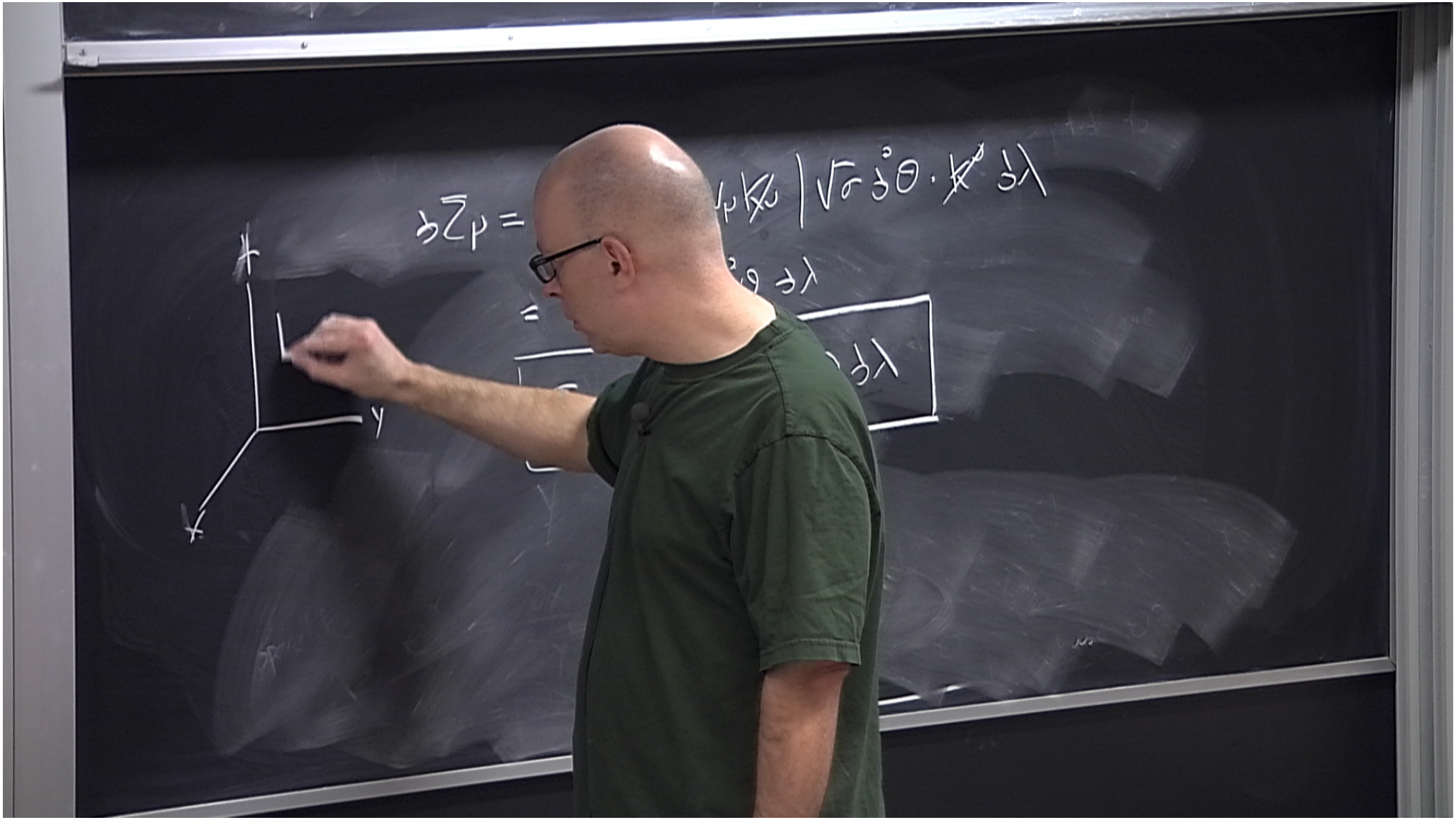
Stokes's theorem:

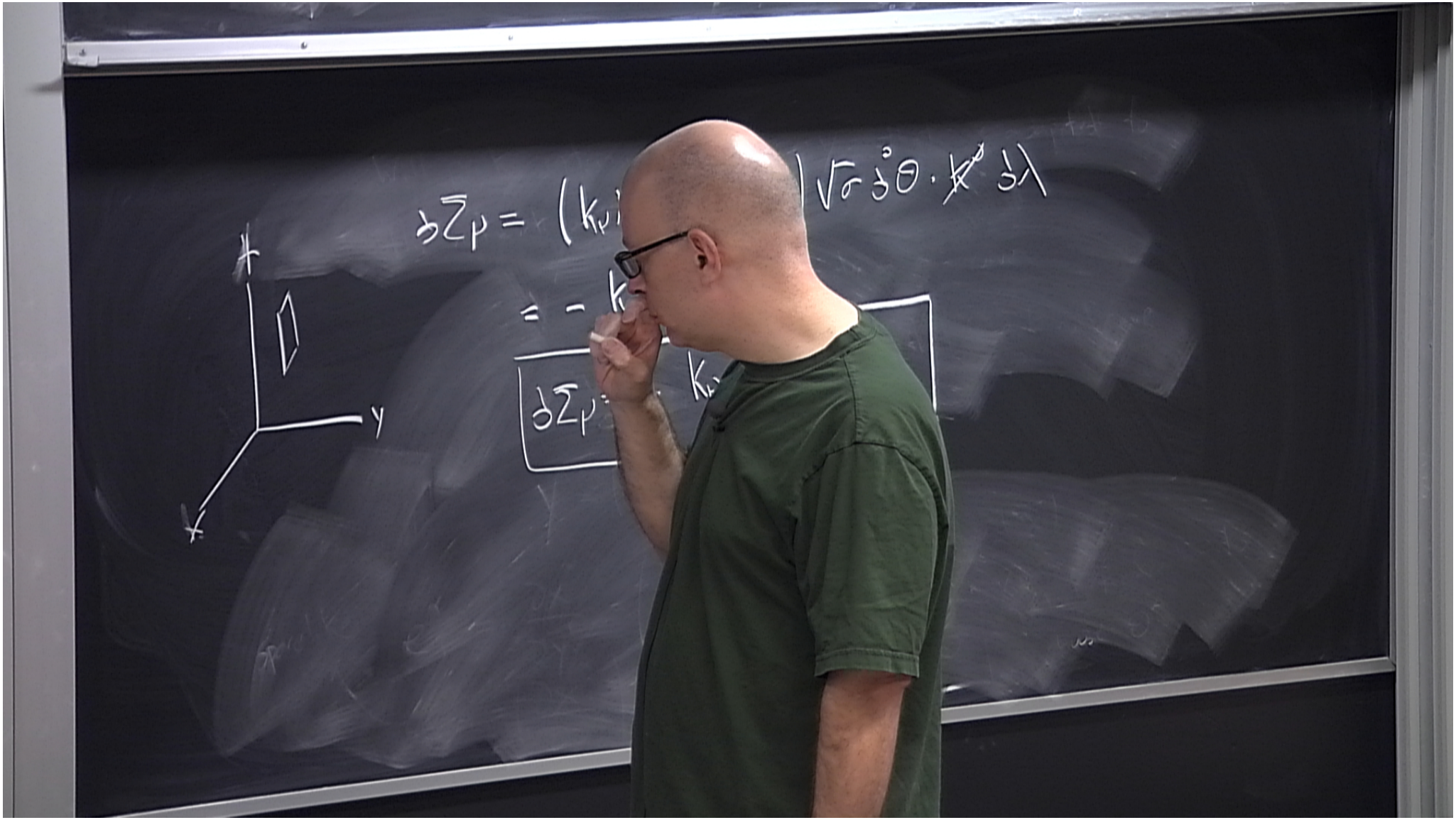
$$\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha = \frac{1}{2}\oint_{\partial\Sigma} B^{\alpha\beta}\, dS_{\alpha\beta}$$

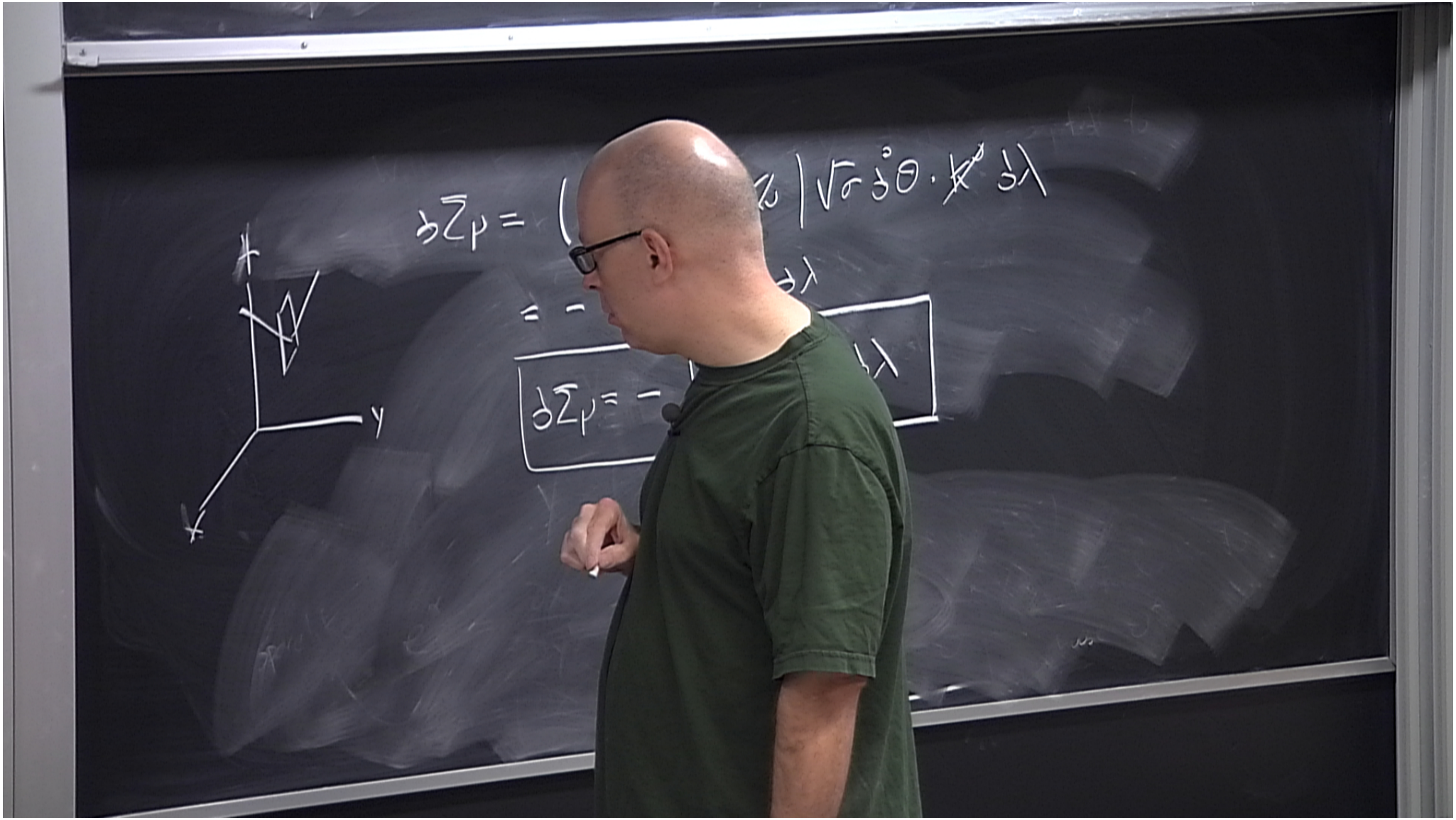
$B^{\alpha\beta}$ = antisymmetric.

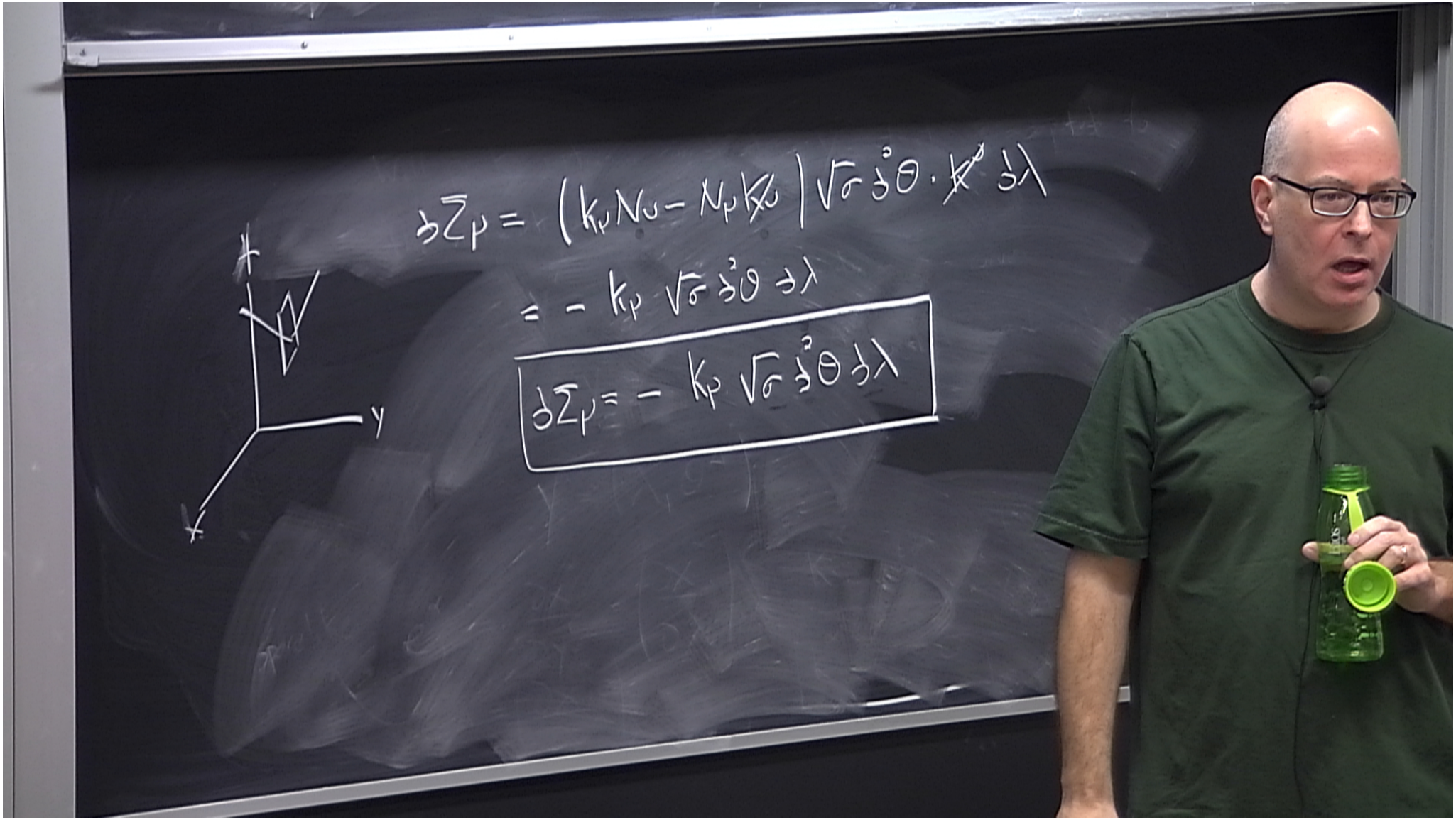

f = W^μ k^ν dS_μν
dS_μν = 2 k_[μ N_ν] √σ d²θ
σ ≡ det [ ]
N^α t^α
k^α null normals to
r^α transverse submanifolds.
k^α = 1/√2 (t^α + r^α)
N^α = 1/√2 (t^α − r^α)
dΣ_μ = (k_μ N_ν − N_μ k_ν) √ · k^ν dλ
= − k_μ √σ d²θ dλ
dΣ_μ = − k_μ √σ d²θ dλ

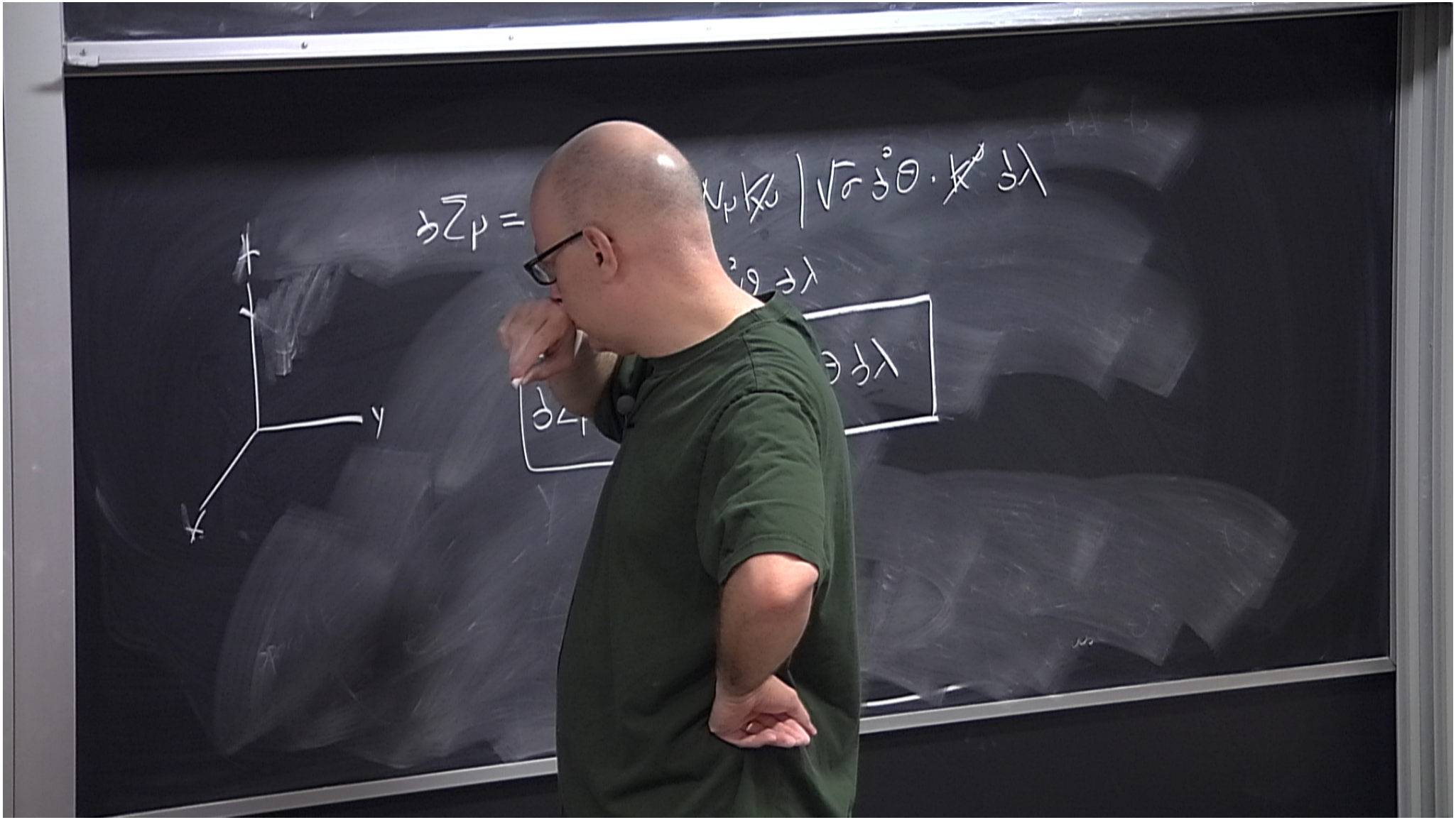


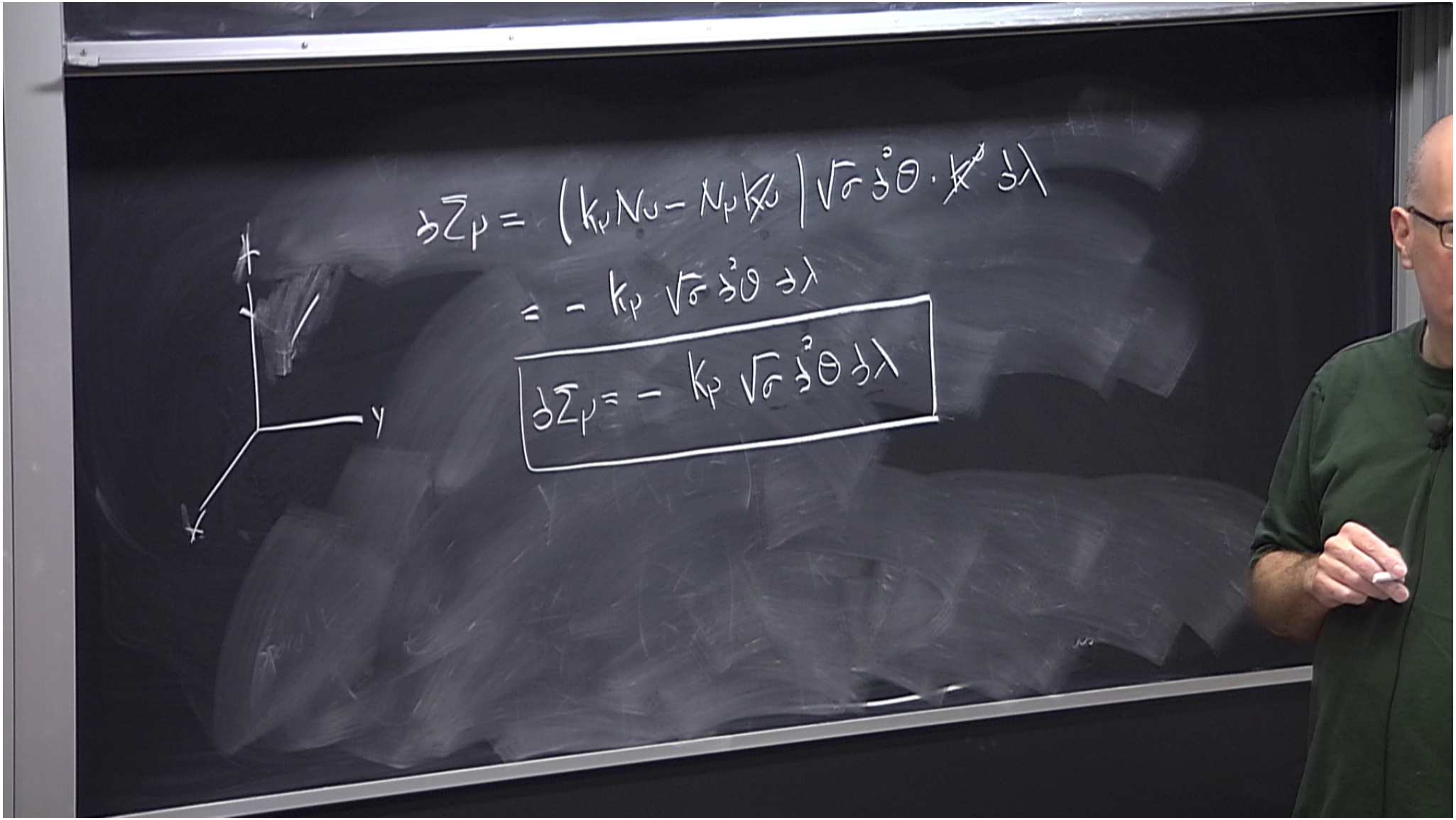



y


y


$\delta\Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu)\sqrt{\sigma}\, d^2\theta \cdot k^\nu\, d\lambda$
$= -k_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda$
$\boxed{\delta\Sigma_\mu = -k_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda}$
y




$\delta\Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu)\sqrt{\sigma}\, d^2\theta \cdot k^\nu\, d\lambda$
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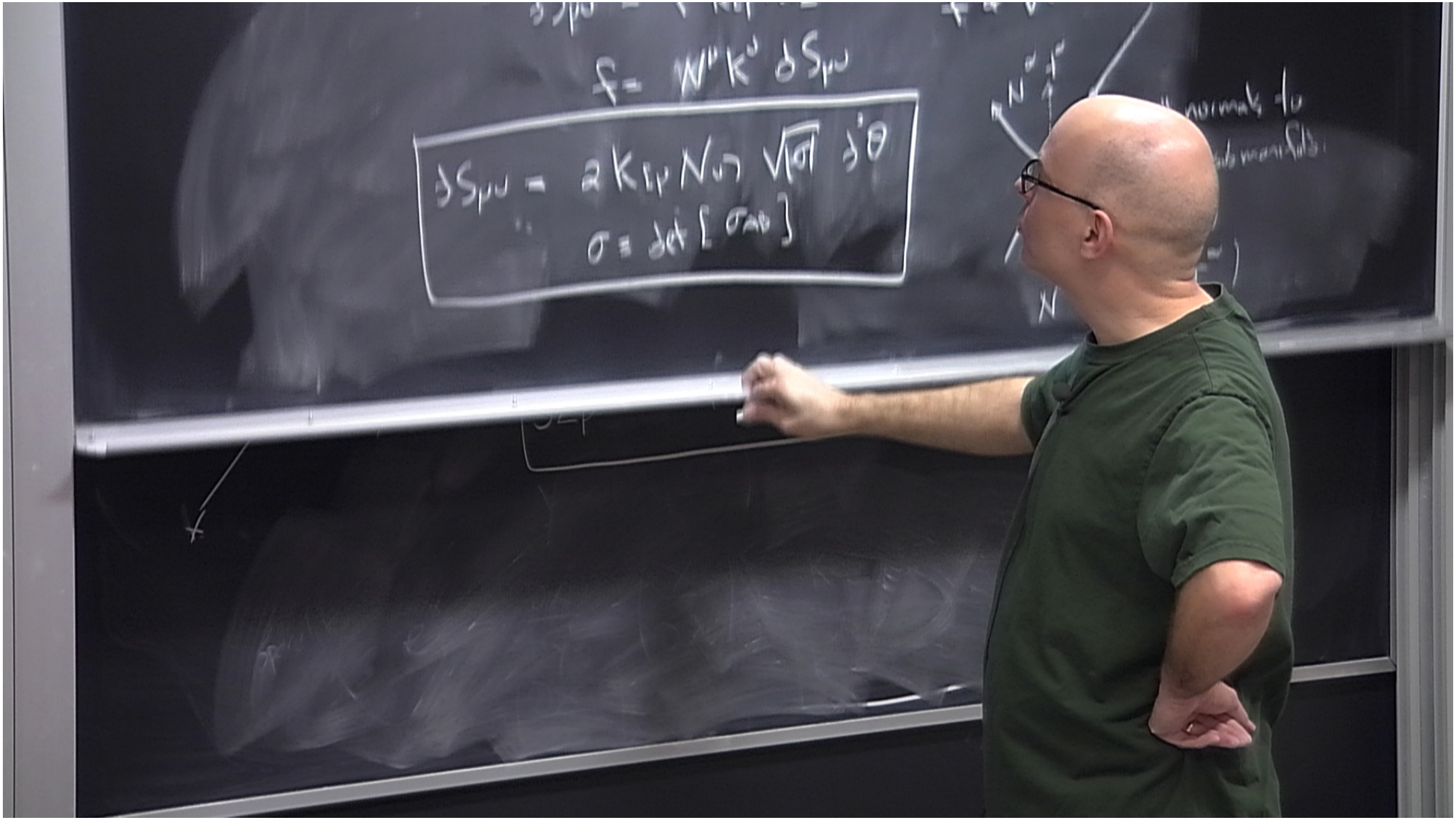
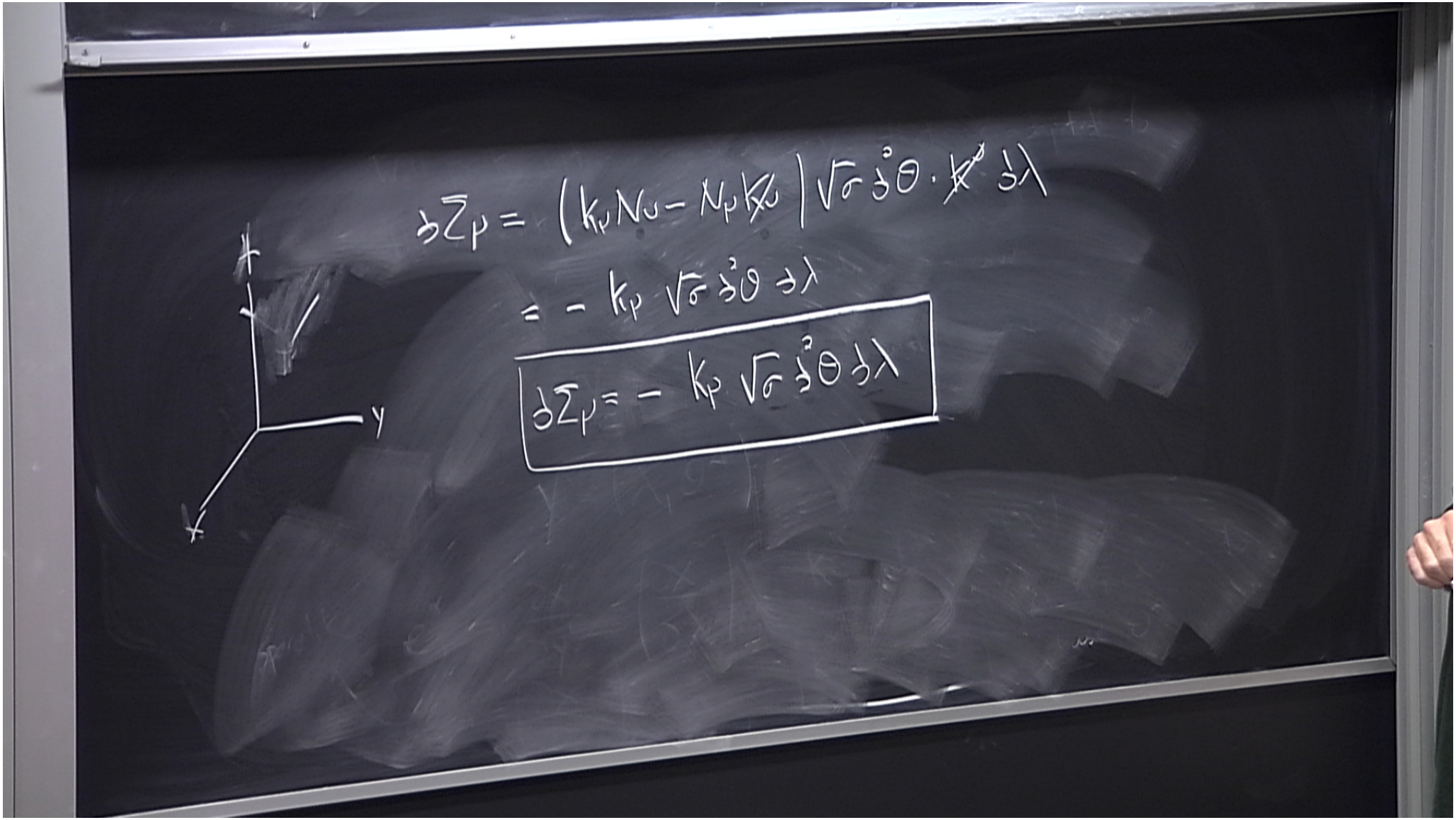
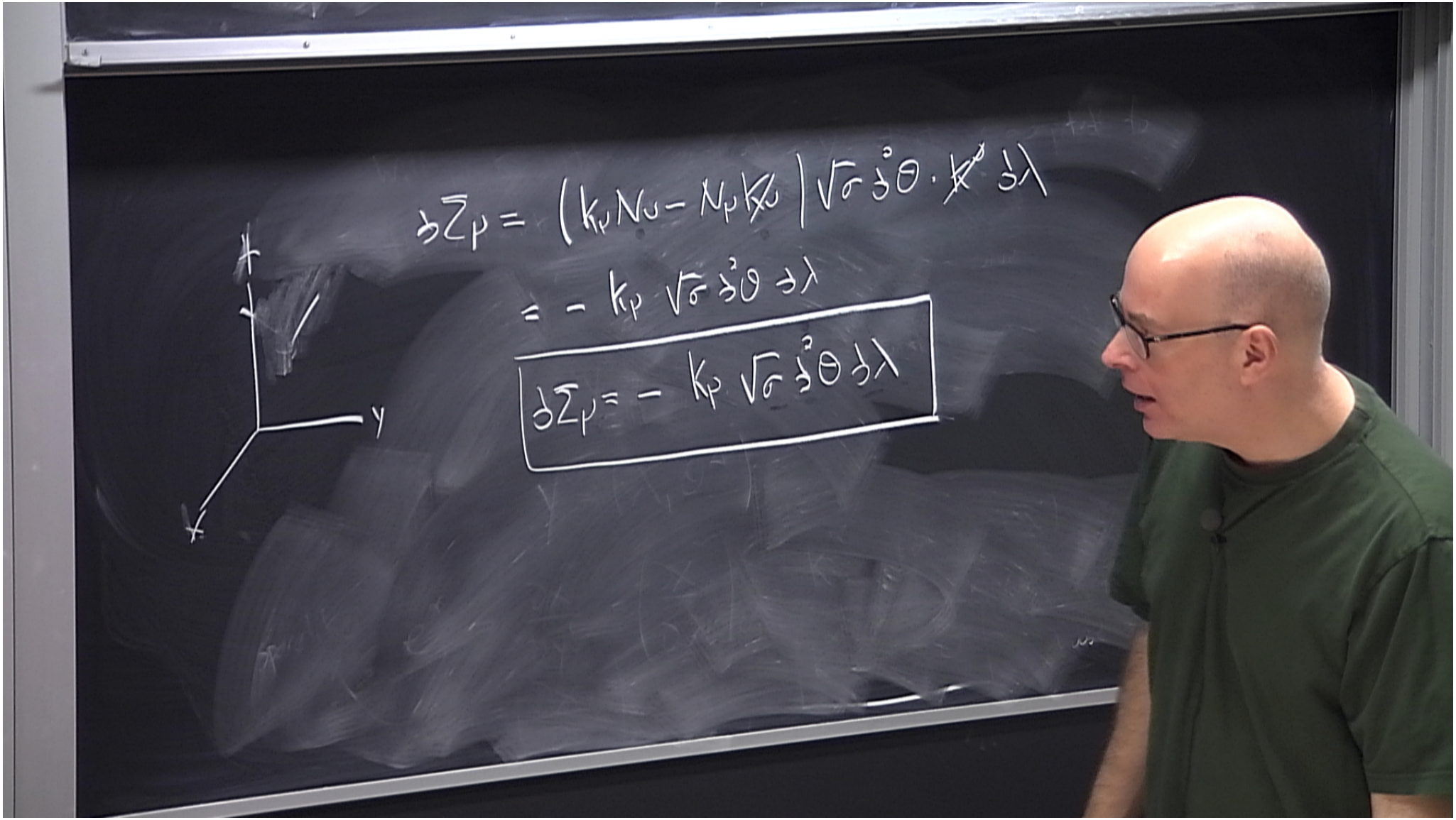
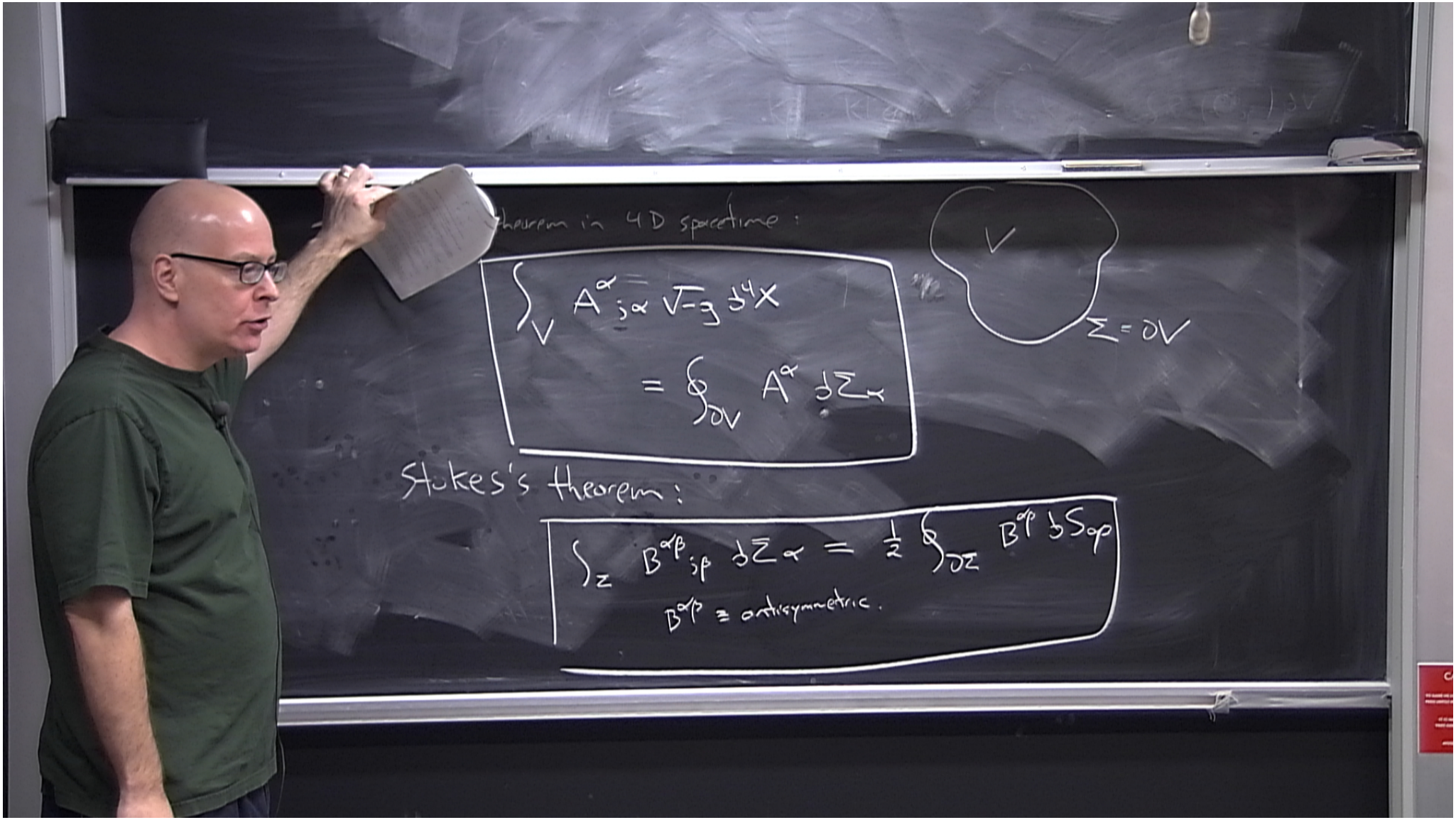
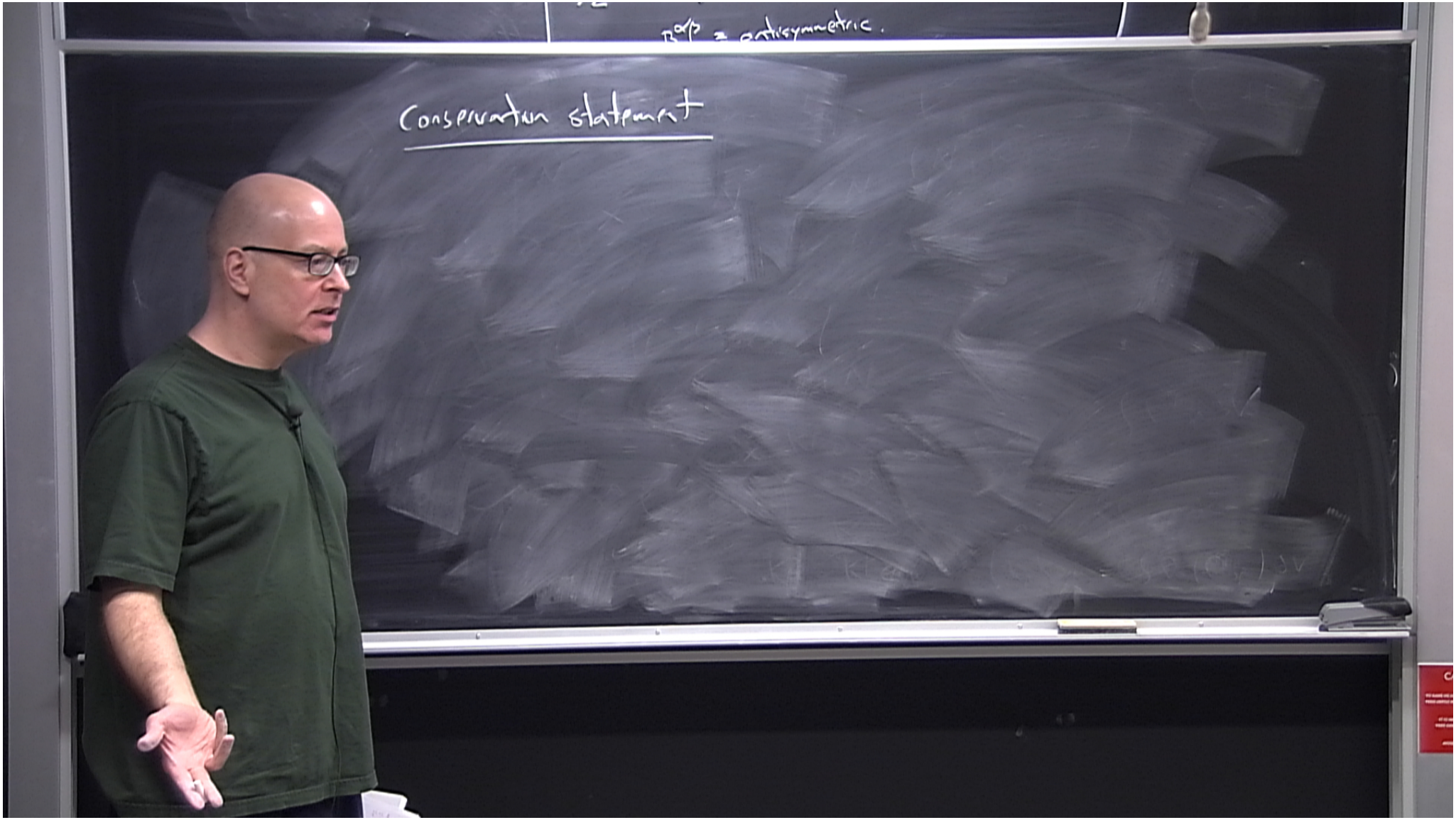
y

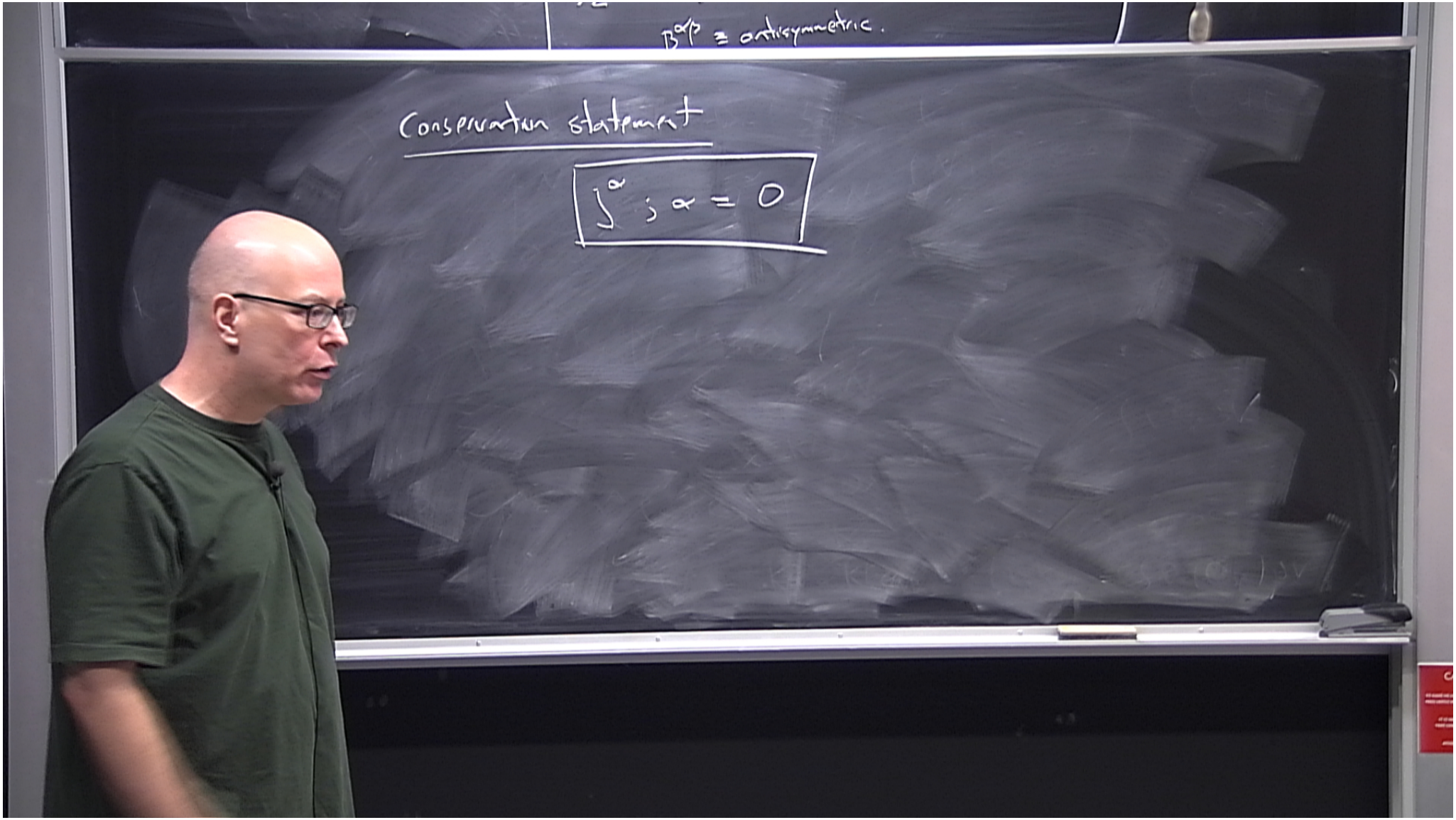

f = N^μ K^ν dS_μν
dS_μν = 2 K_[μ N_ν] √|σ| d²θ
σ = det [σ_AB]

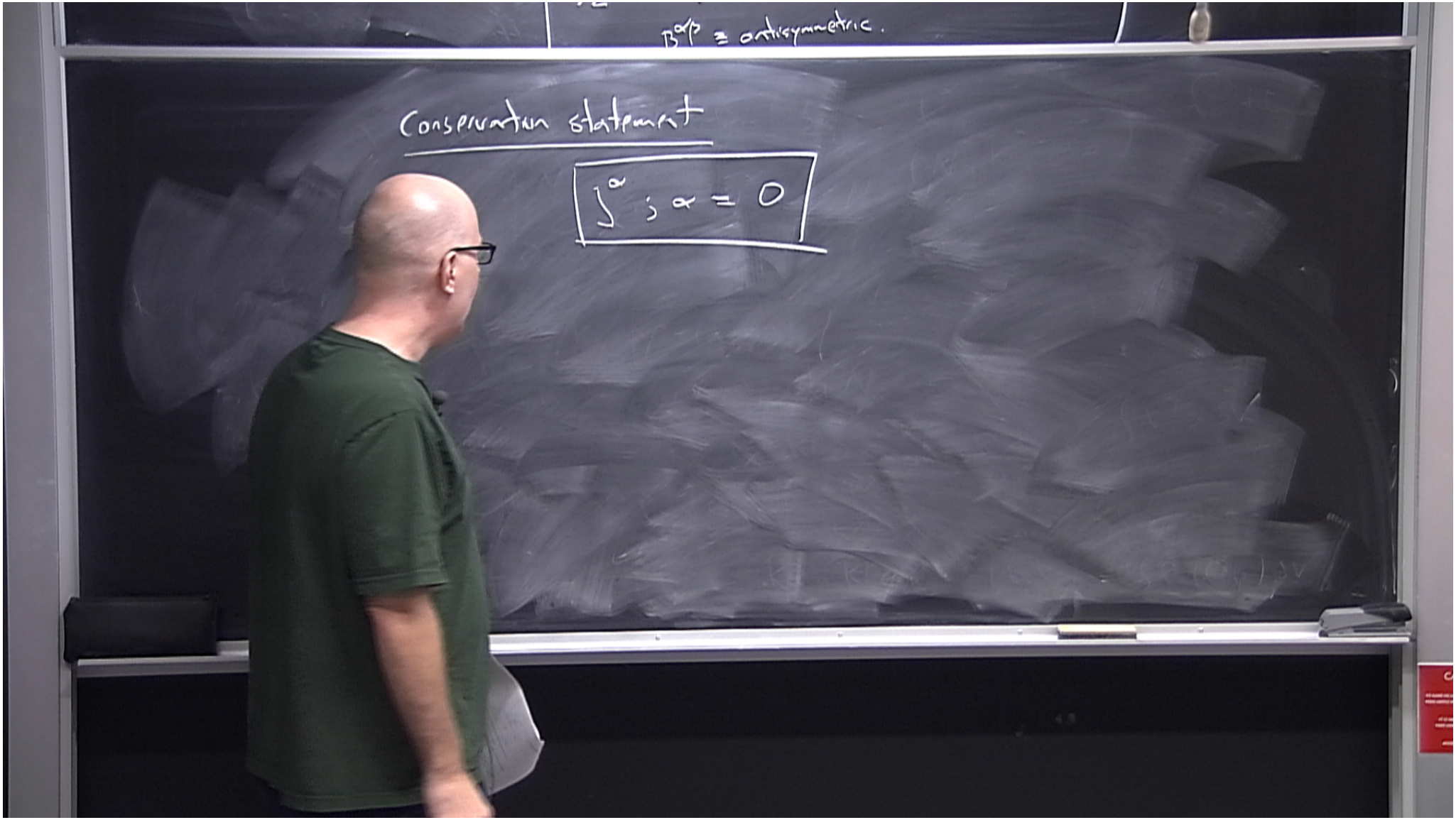

$\delta\Sigma_\mu = (k_\mu N_\nu - N_\mu k_\nu)\sqrt{\sigma}\,\delta^2\theta\cdot k\,\delta\lambda$
$= -k_\mu\sqrt{\sigma}\,\delta^2\theta\,\delta\lambda$
$\boxed{\delta\Sigma_\mu = -k_\mu\sqrt{\sigma}\,\delta^2\theta\,\delta\lambda}$
y

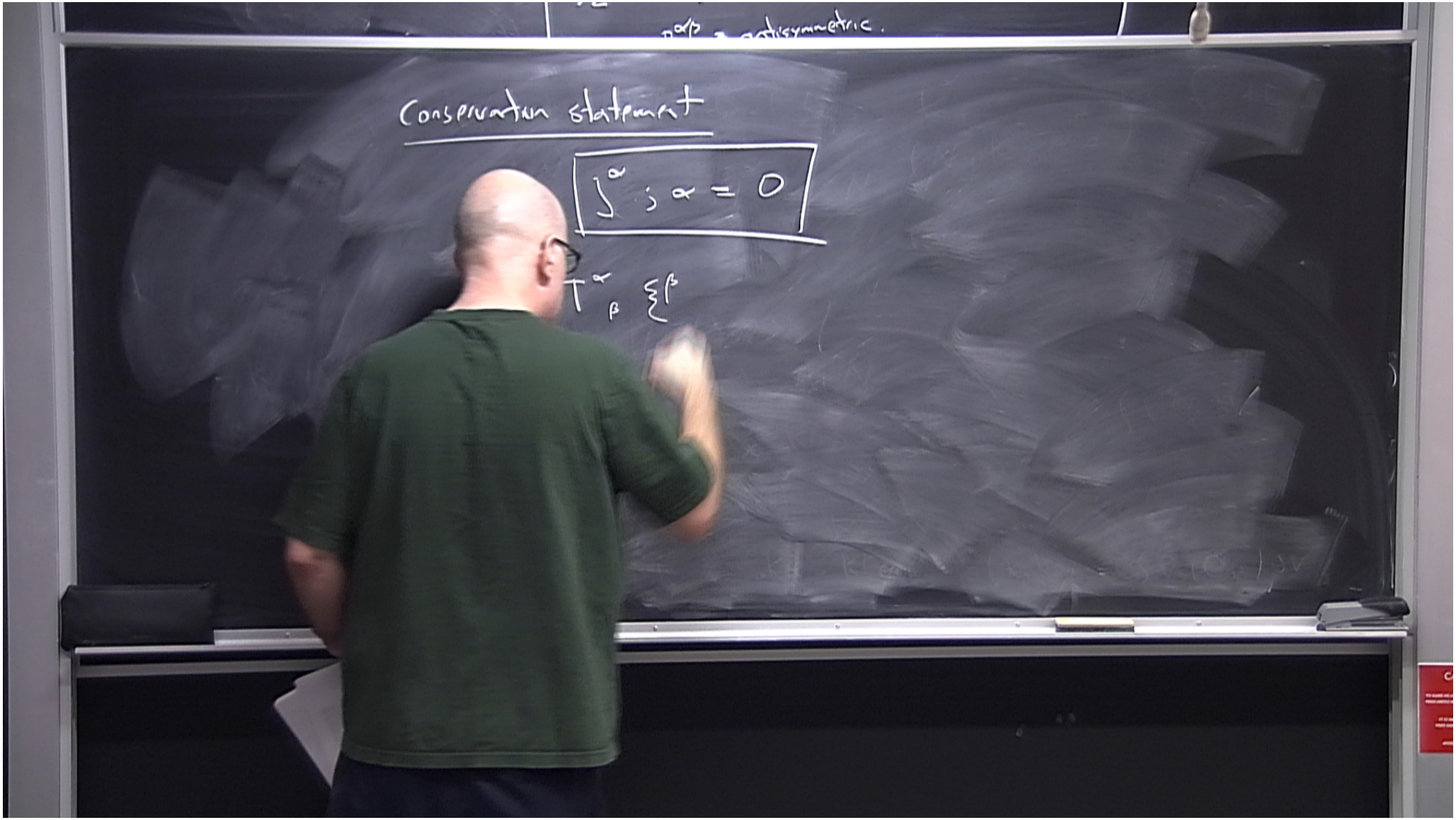

$\delta Z_\mu = (k_\mu N_\nu - N_\mu k_\nu)\sqrt{\sigma}\, d^2\theta \cdot \not{k}\, d\lambda$
$= -k_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda$
$\boxed{\delta Z_\mu = -k_\mu \sqrt{\sigma}\, d^2\theta\, d\lambda}$
y

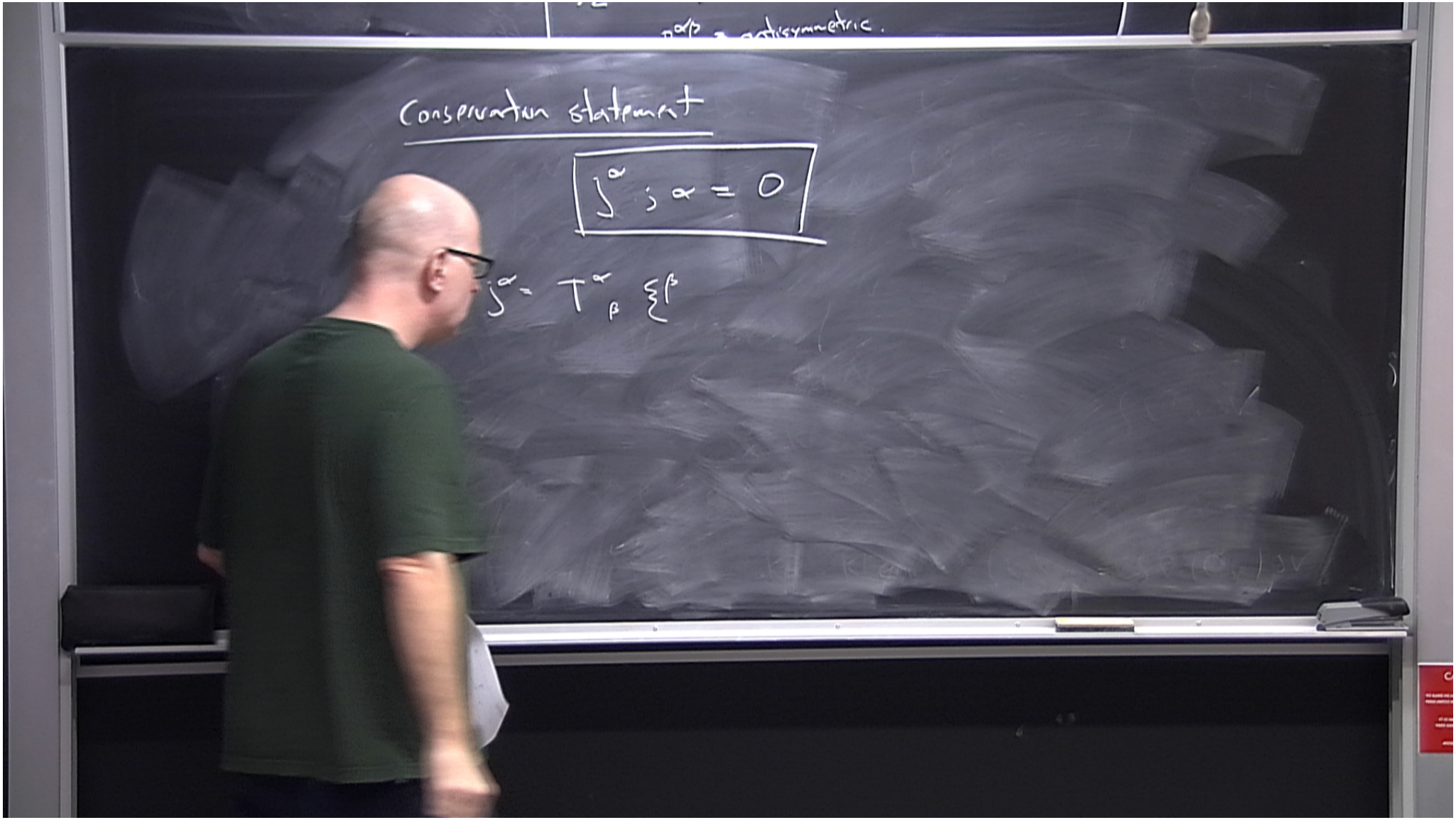

theorem in 4D spacetime:
$\int_V A^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} A^\alpha\, d\Sigma_\alpha$
V
$\Sigma = \partial V$
Stokes's theorem:
$\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha = \frac{1}{2} \oint_{\partial\Sigma} B^{\alpha\beta}\, dS_{\alpha\beta}$
$B^{\alpha\beta}$ = antisymmetric.

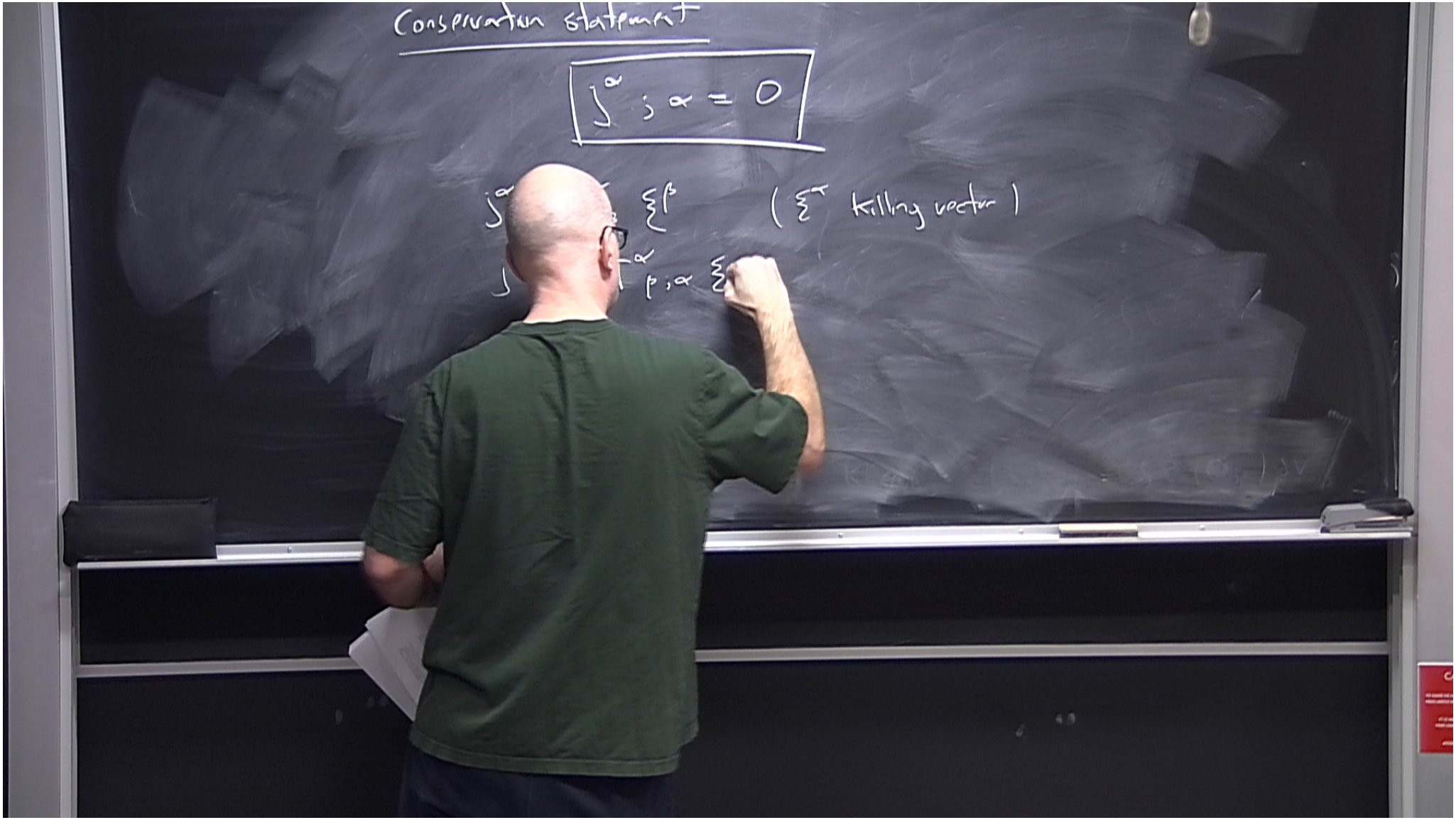

$R^{\alpha\beta}$ = antisymmetric.
Conservation statement

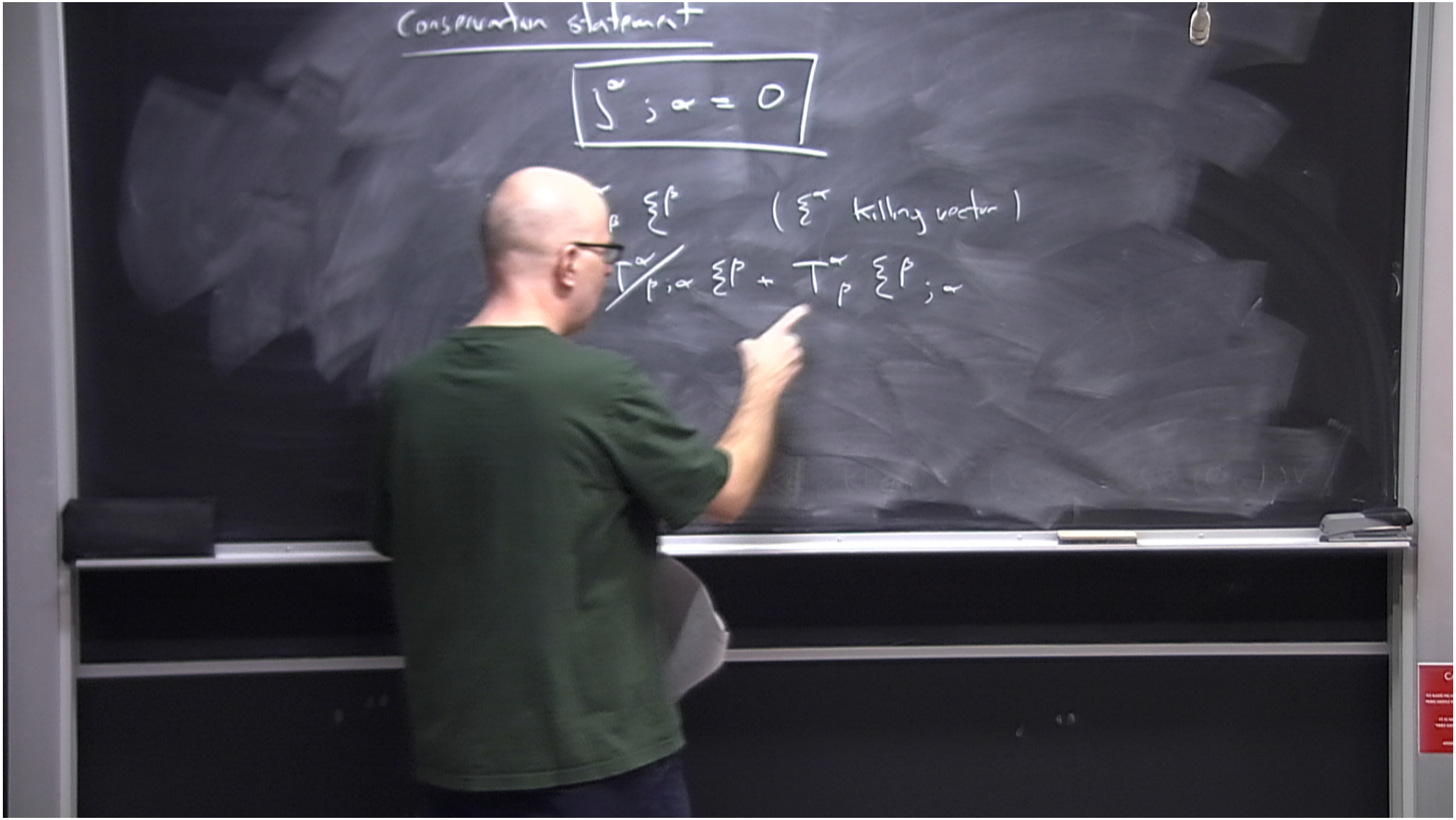

$B^{\alpha\beta}$ = antisymmetric.
Conservation statement
$J^{\alpha}{}_{;\alpha} = 0$

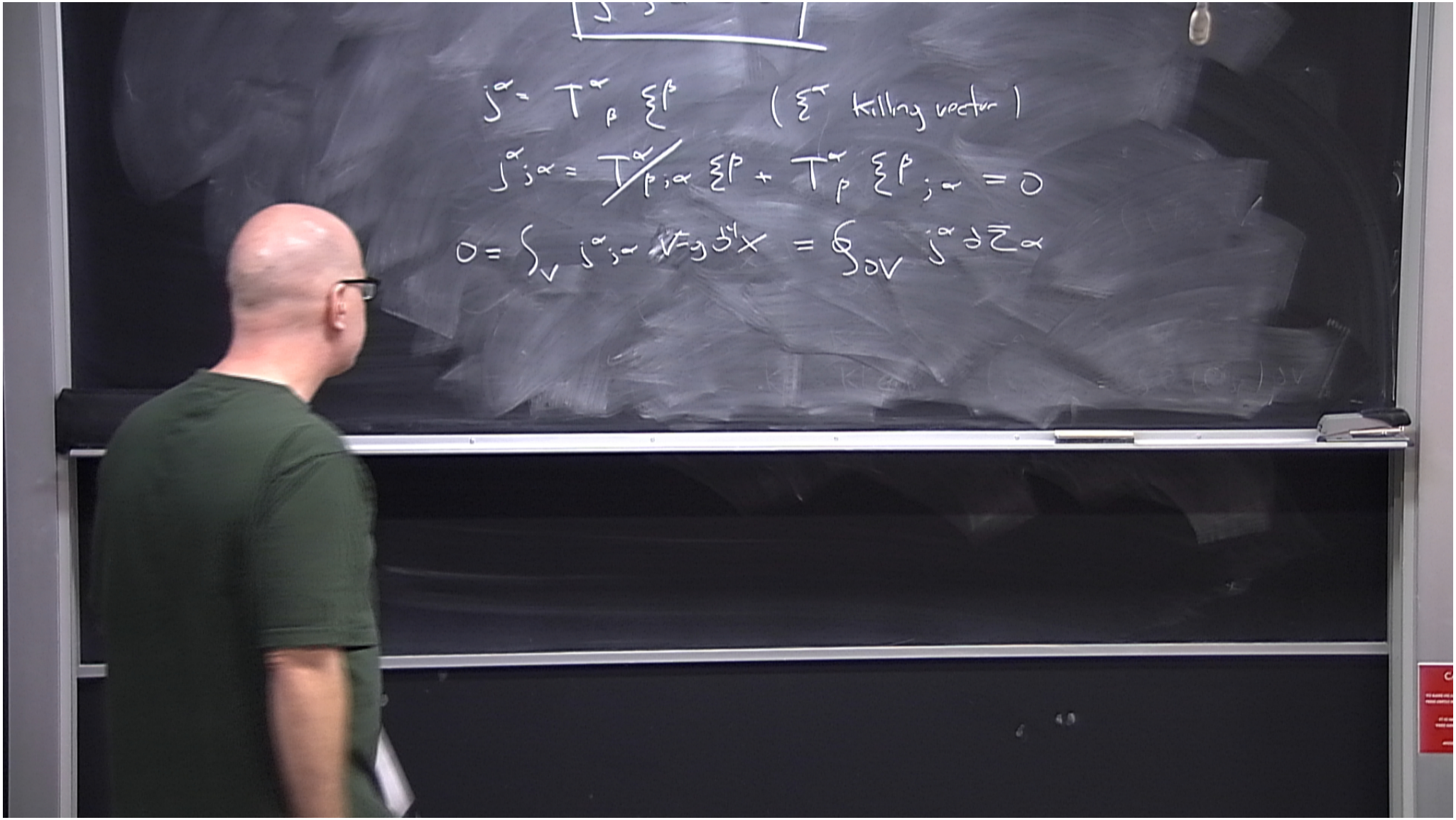

$B^{\alpha\beta}$ = antisymmetric.
Conservation statement
$\int^{\alpha} ; \alpha = 0$

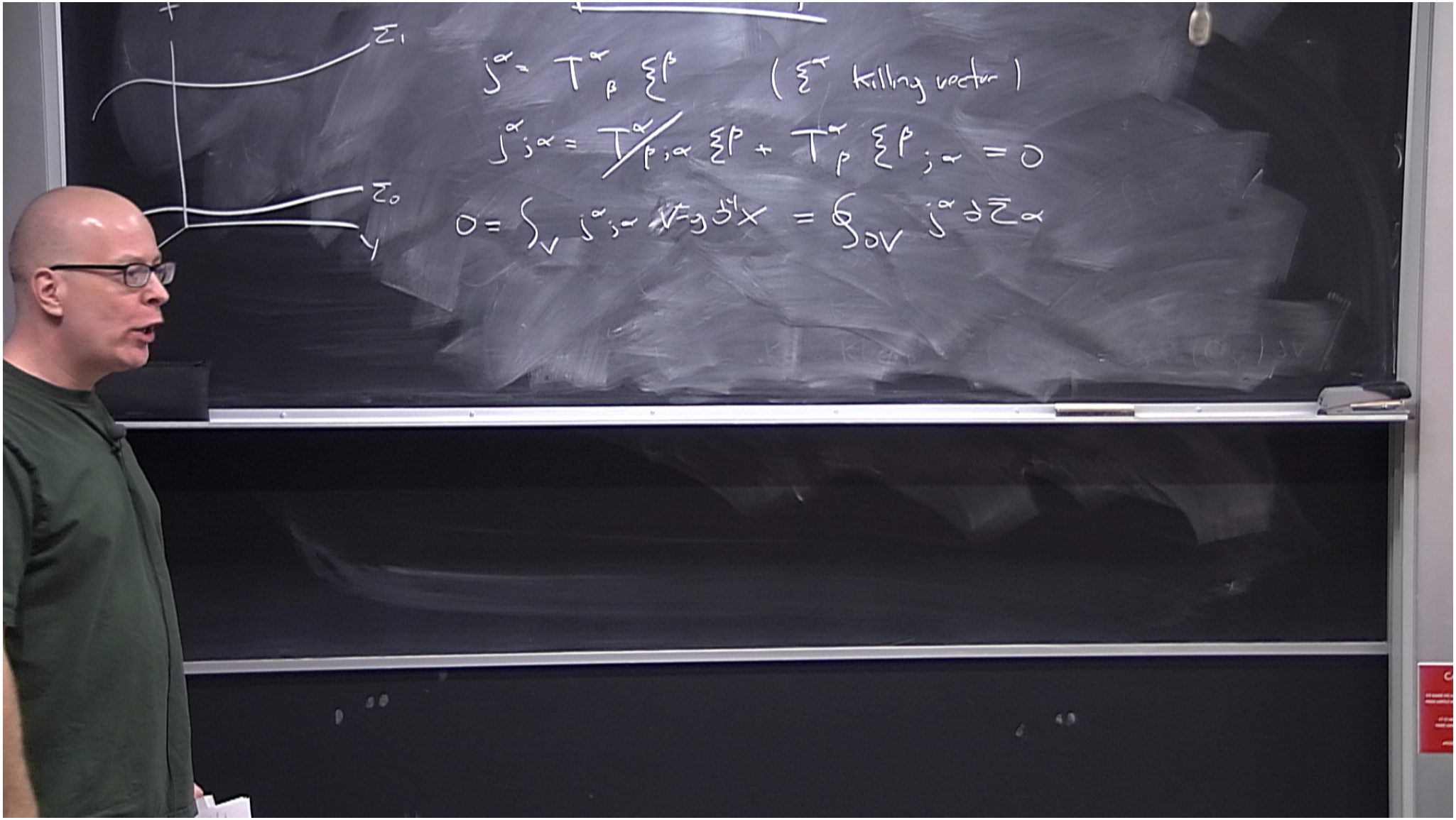

antisymmetric.
Conservation statement
$J^\alpha{}_{;\alpha} = 0$
$T^\alpha{}_\beta \xi^\beta$

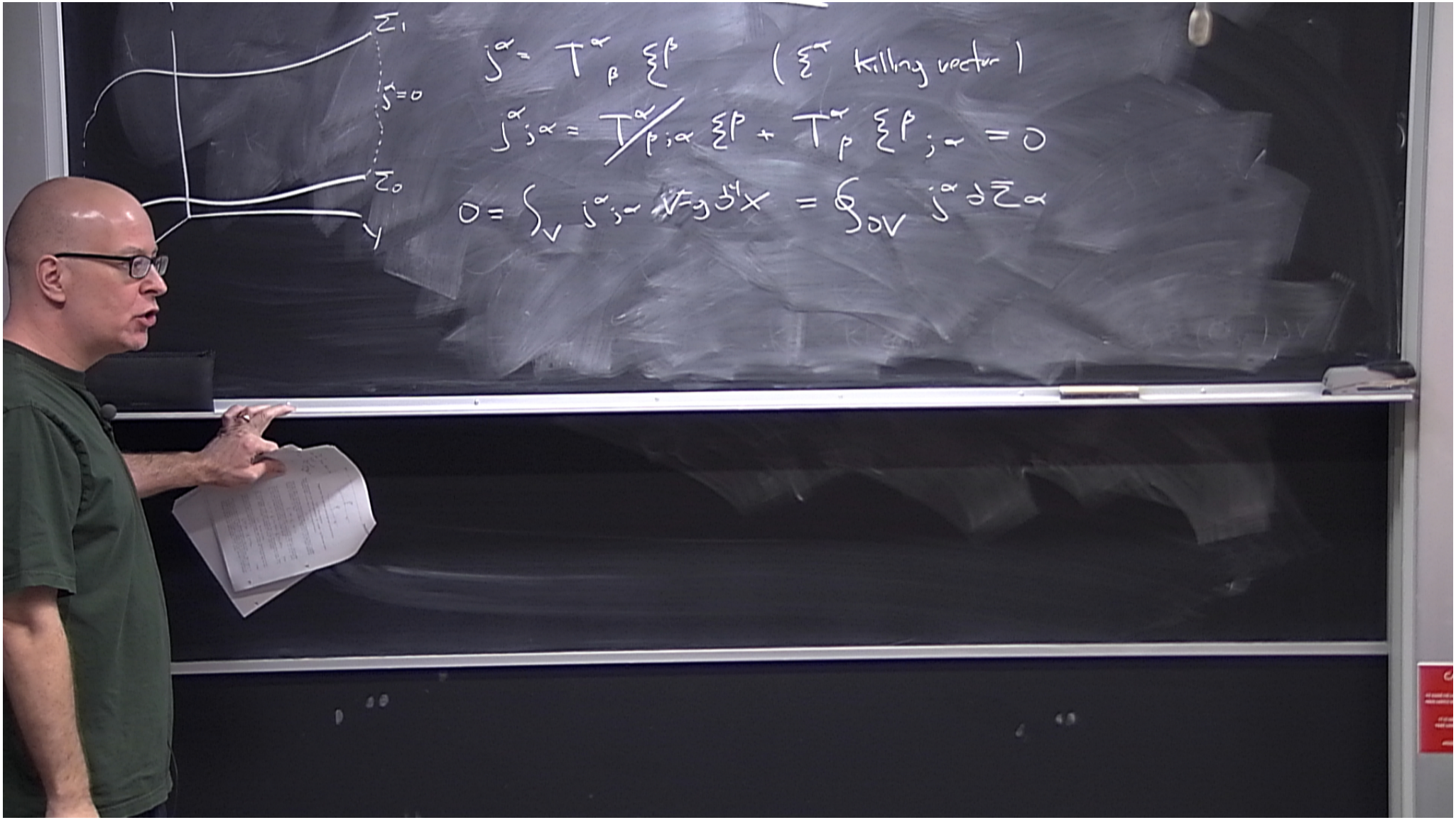

antisymmetric.
Conservation statement
$J^\alpha{}_{;\alpha} = 0$
$J^\alpha = T^\alpha{}_\beta \xi^\beta$

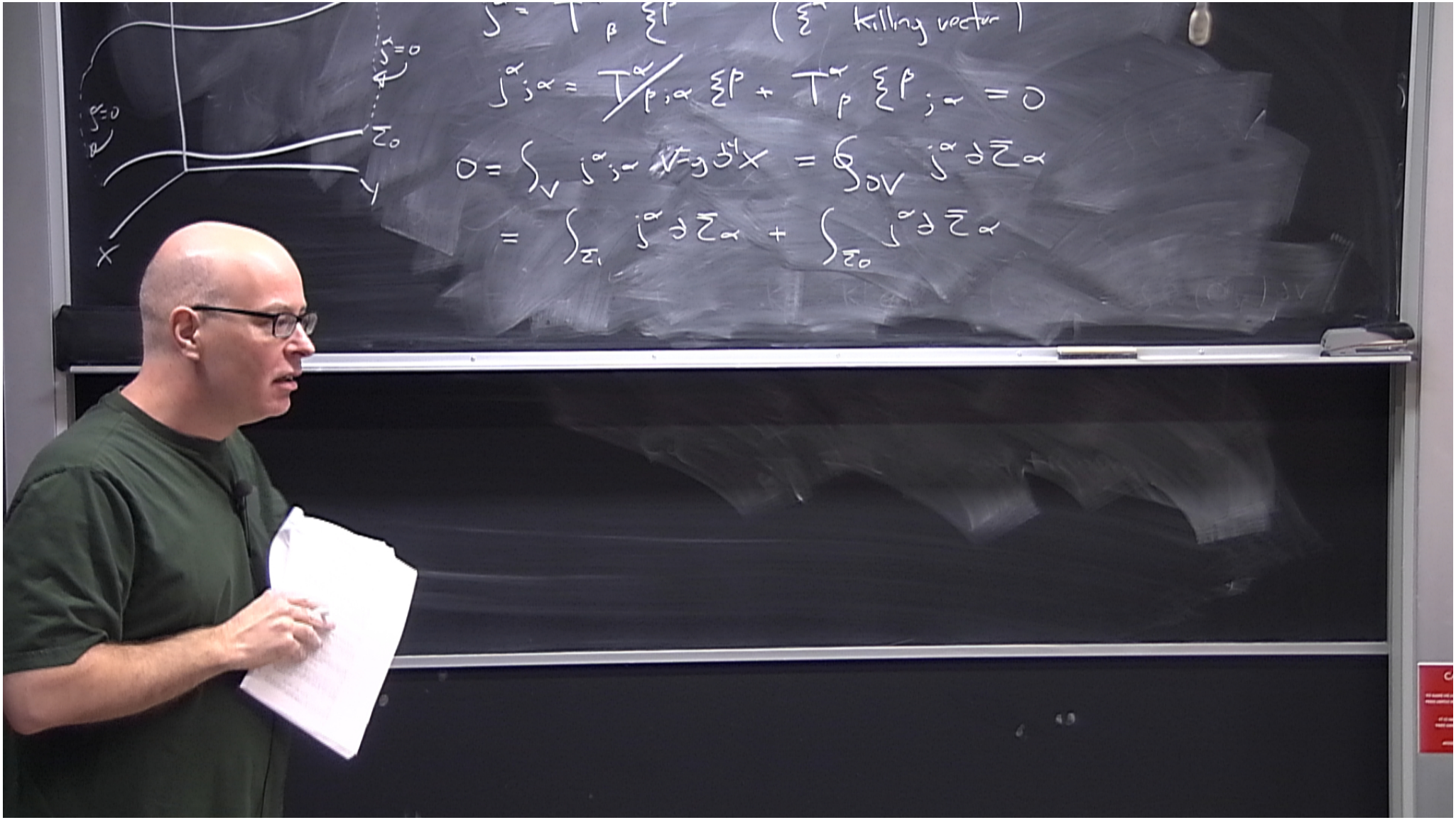

Conservation statement
$j^\alpha{}_{;\alpha} = 0$
$j^\alpha = \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = \xi^\beta{}_{;\alpha}$


Conservation statement
$J^{\alpha}{}_{;\alpha} = 0$
ξ^{ρ} (ξ^{τ} killing vector)
$\Gamma^{\alpha}_{\rho;\alpha}\,\xi^{\rho} + T^{\tau}_{\rho}\,\xi^{\rho}{}_{;\tau}$

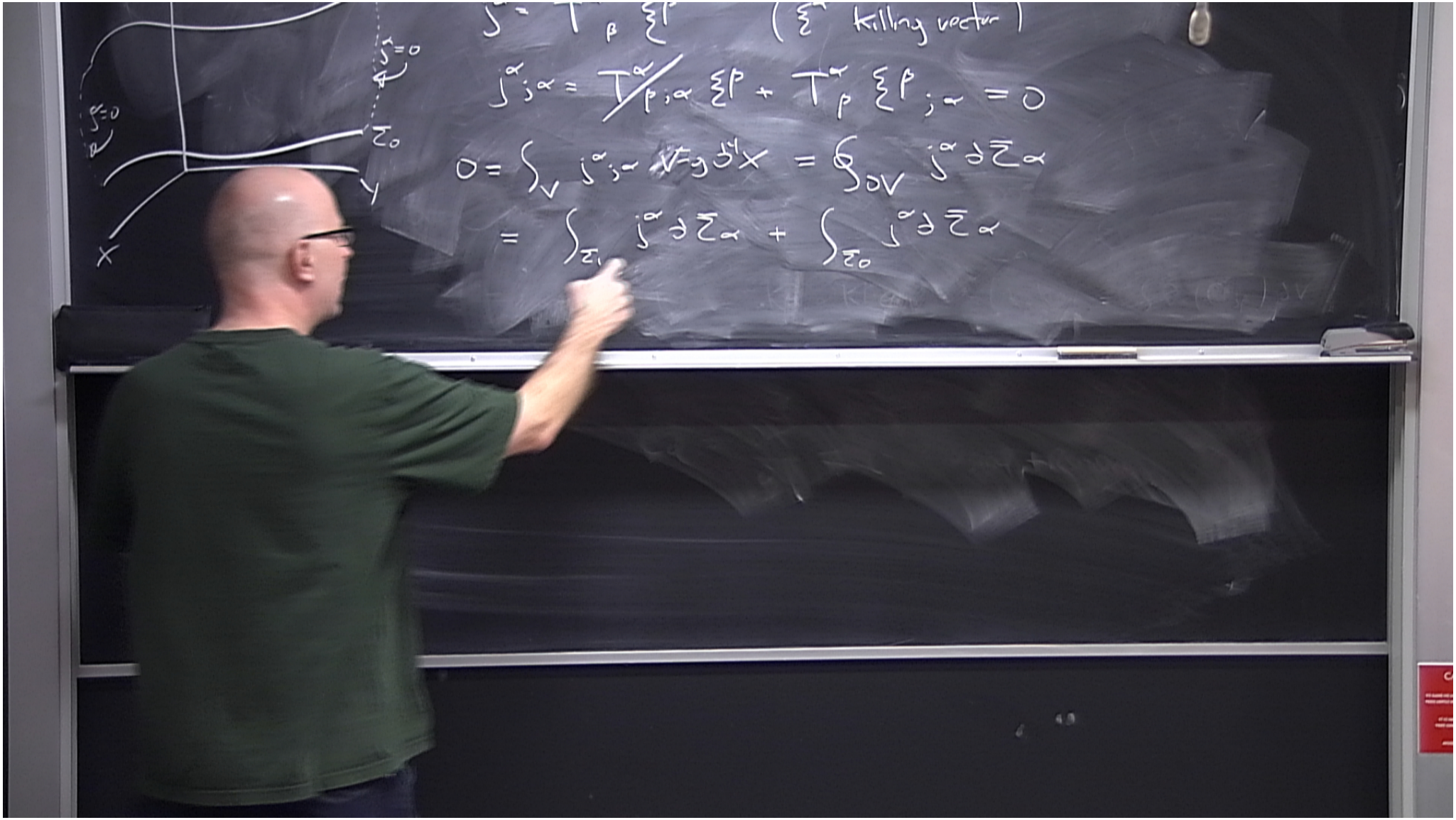

$j^\alpha = T^\alpha_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha_{\beta;\alpha} \xi^\beta + T^\alpha_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$


Σ_1
Σ_0
$j^\alpha = T^\alpha_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha_{\beta;\alpha} \xi^\beta + T^\alpha_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha \, d\Sigma_\alpha$

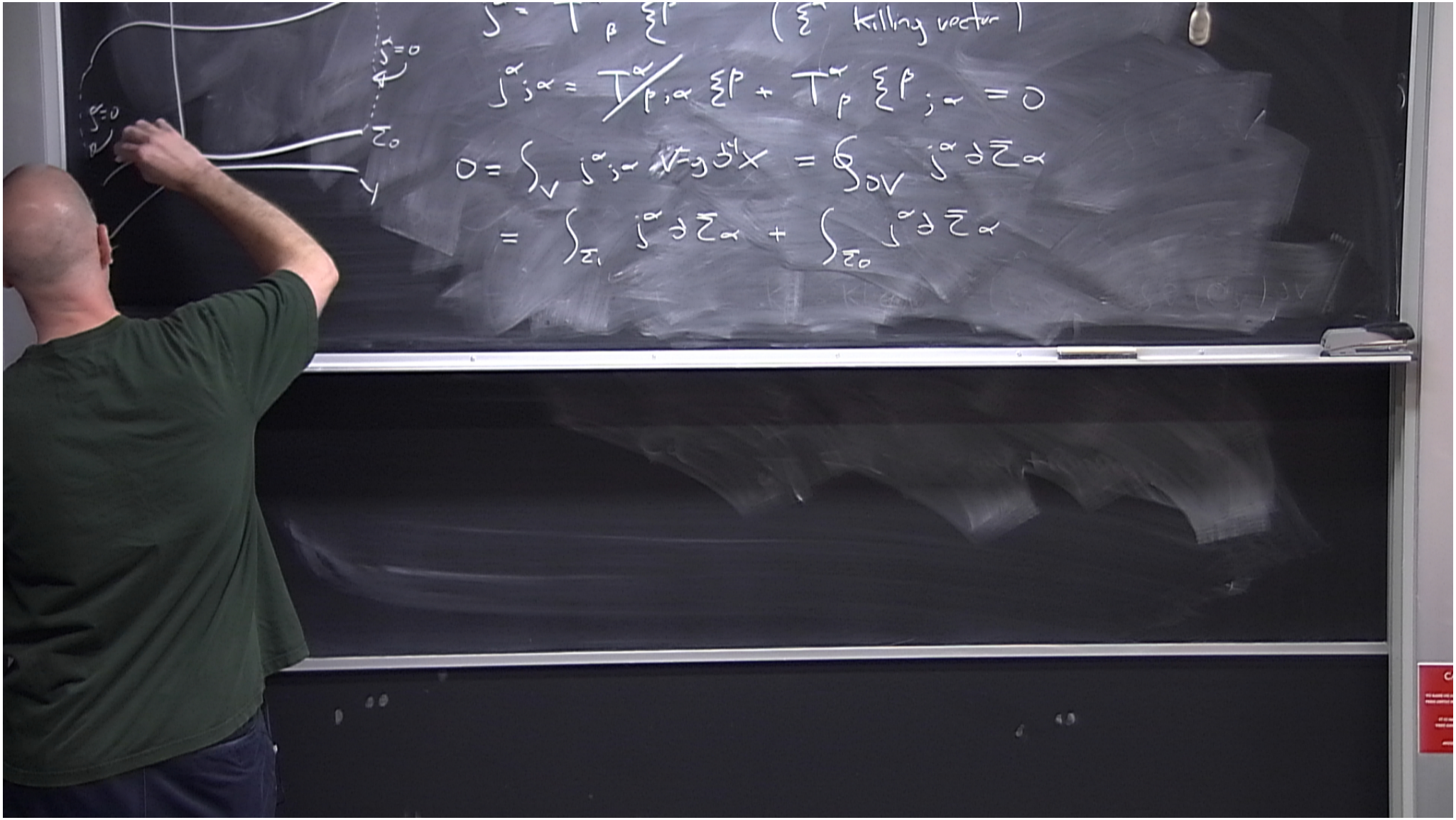

Σ_1
$j^\alpha = 0$
Σ_0
γ
$j^\alpha = T^\alpha_\beta \xi^\beta$ (ξ^τ killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha_{\beta;\alpha} \xi^\beta + T^\alpha_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha \partial\Sigma_\alpha$

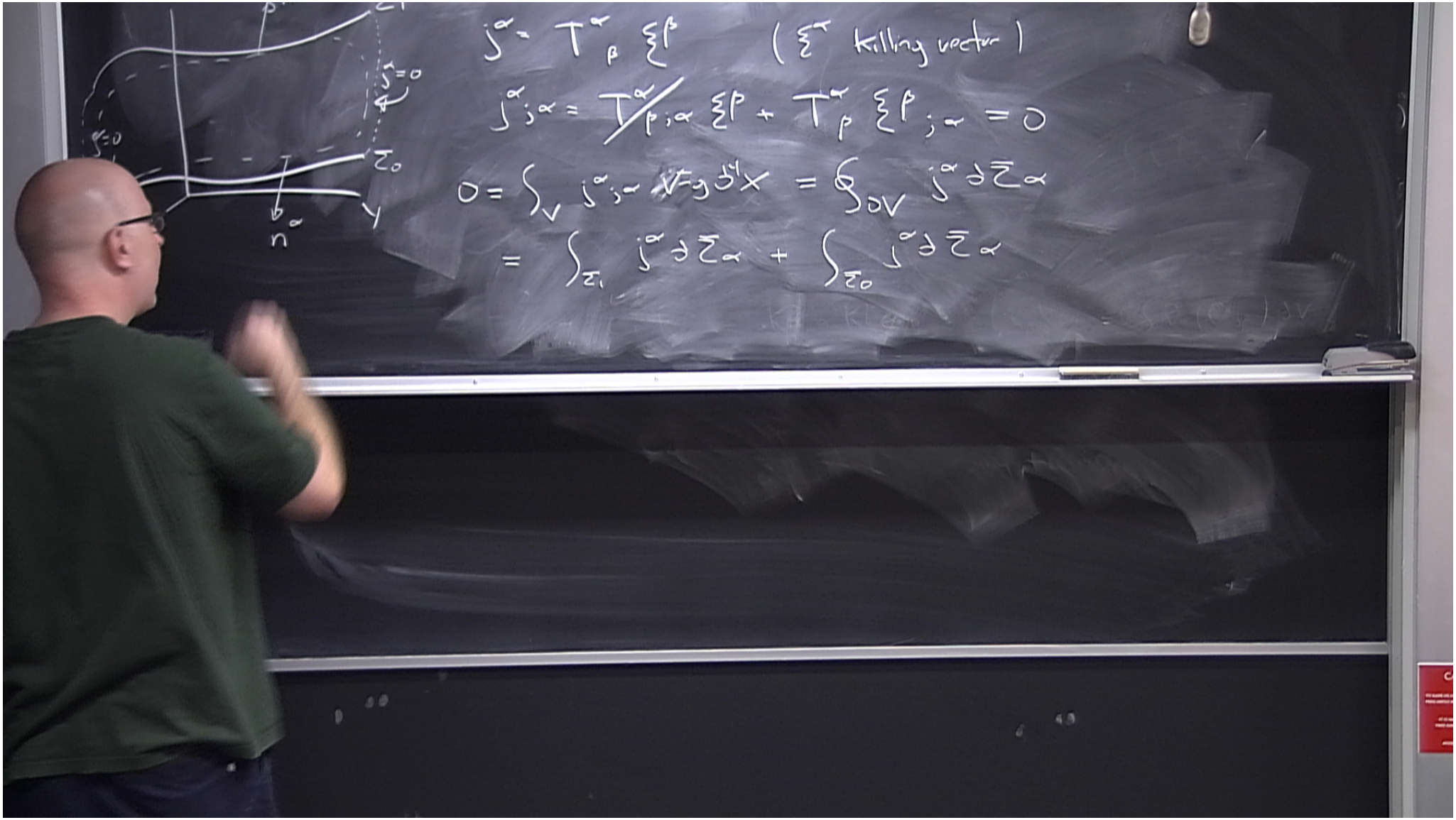


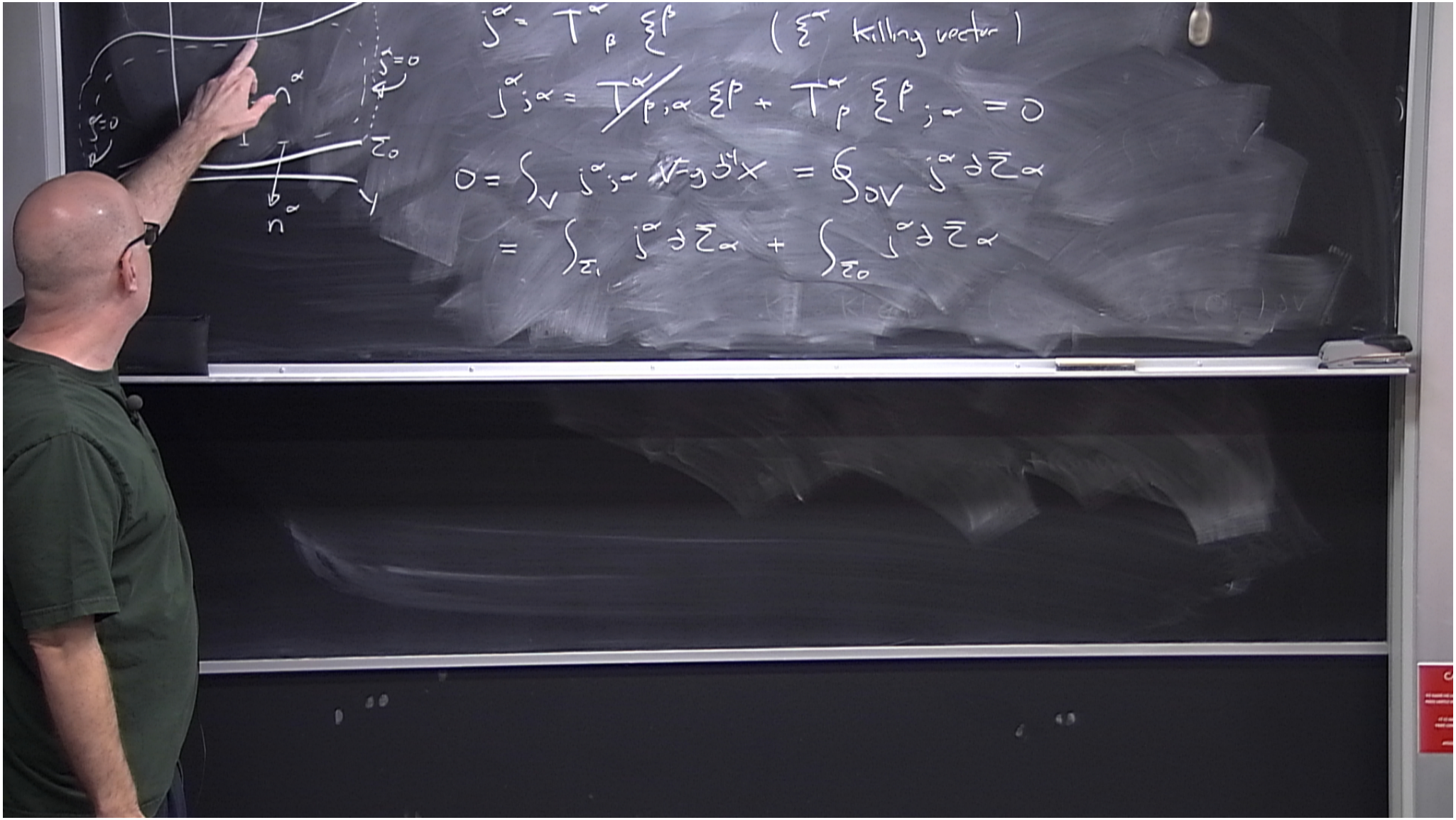
$j^\alpha = T^\alpha{}_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha{}_{\beta;\alpha} \xi^\beta + T^\alpha{}_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} j^\alpha d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha d\Sigma_\alpha$
$j^\alpha = 0$
Σ_0
y
x

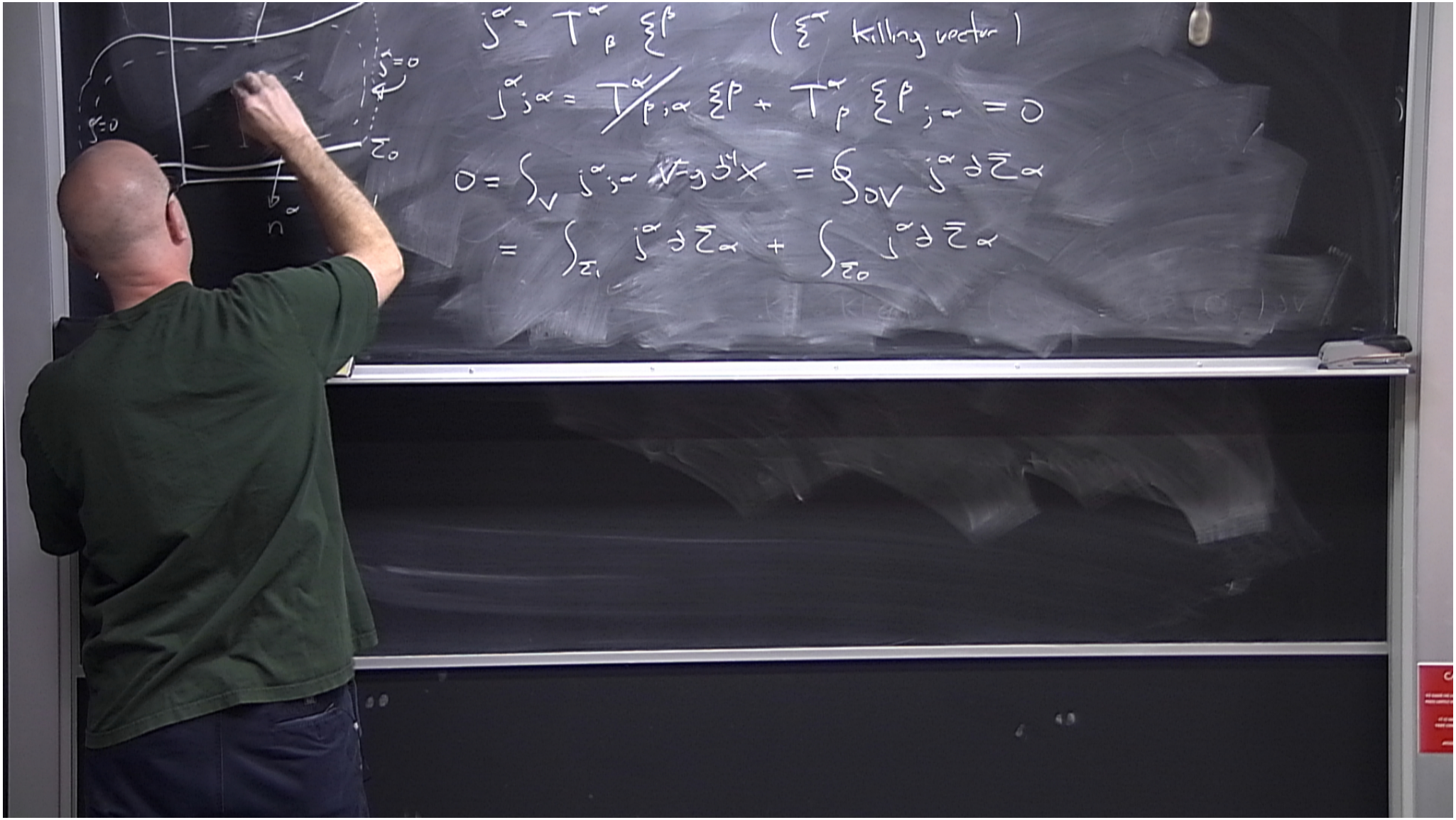


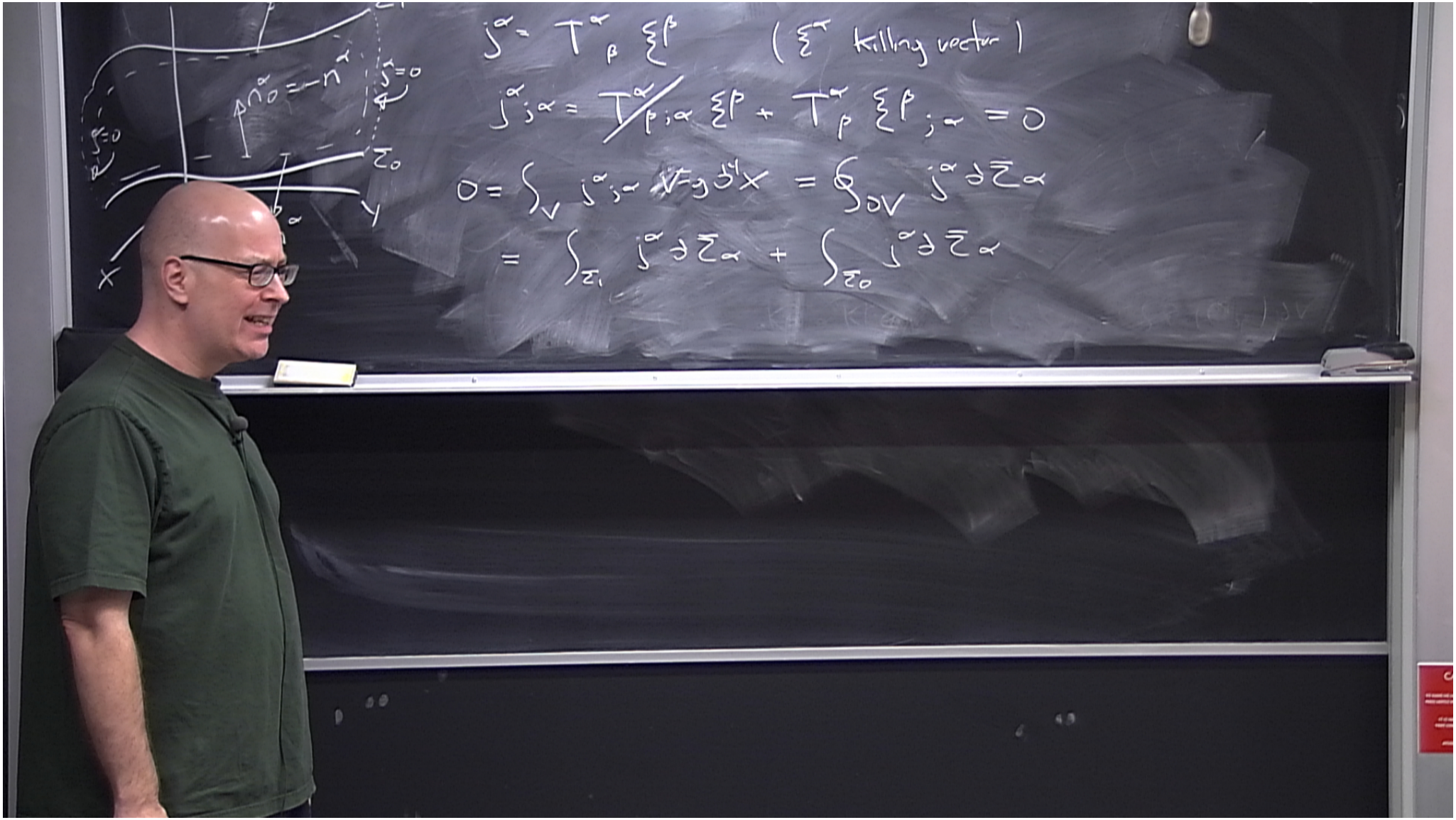
$j^\alpha = T^\alpha{}_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha{}_{\beta;\alpha} \xi^\beta + T^\alpha{}_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} j^\alpha d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha d\Sigma_\alpha$
$j^\alpha = 0$
$j^\alpha = 0$
Σ_0
Y
X


killing vector
Σ_2
Σ_1


$j^\alpha = T^\alpha_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = T^\alpha_{\beta;\alpha}\xi^\beta + T^\alpha_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha}\sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} j^\alpha d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha d\Sigma_\alpha$
$t=0$
Σ_0
n^α


$j^\alpha = T^\alpha{}_\beta \xi^\beta$ (ξ^α killing vector)
$j^\alpha{}_{;\alpha} = \cancel{T^\alpha{}_{\beta;\alpha}} \xi^\beta + T^\alpha{}_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} j^\alpha d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha d\Sigma_\alpha$
n^α
Σ_0
$j^\alpha = 0$


$J^\alpha = T^\alpha_\beta \xi^\beta$ (ξ^α killing vector)
$J^\alpha{}_{;\alpha} = T^\alpha_{\beta;\alpha} \xi^\beta + T^\alpha_\beta \xi^\beta{}_{;\alpha} = 0$
$0 = \int_V J^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} J^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} J^\alpha d\Sigma_\alpha + \int_{\Sigma_0} J^\alpha d\Sigma_\alpha$
$t=0$
Σ_0
n^α


$j^\alpha = T^\alpha{}_\beta \xi^\beta$ (ξ^α killing vector)
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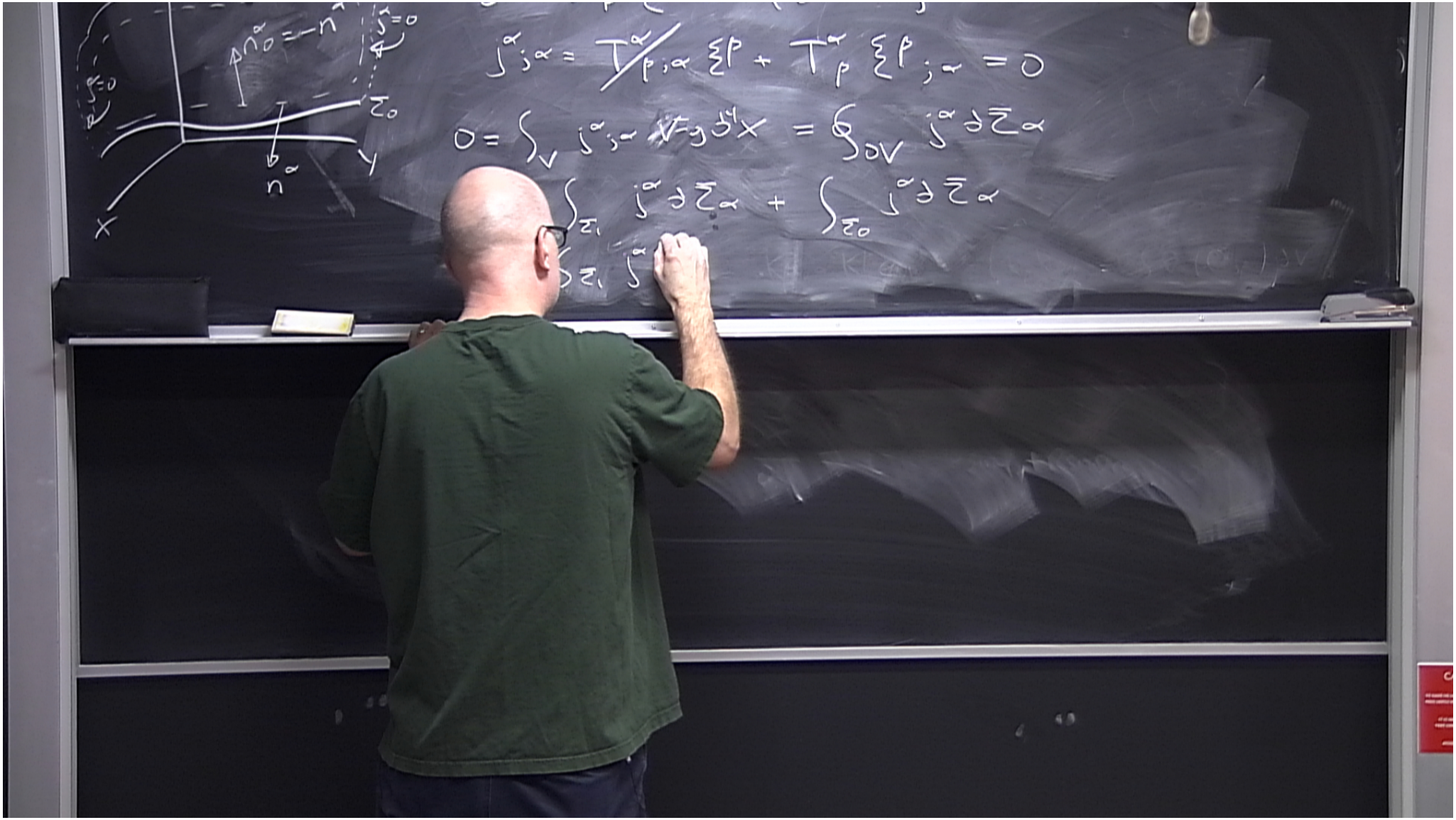
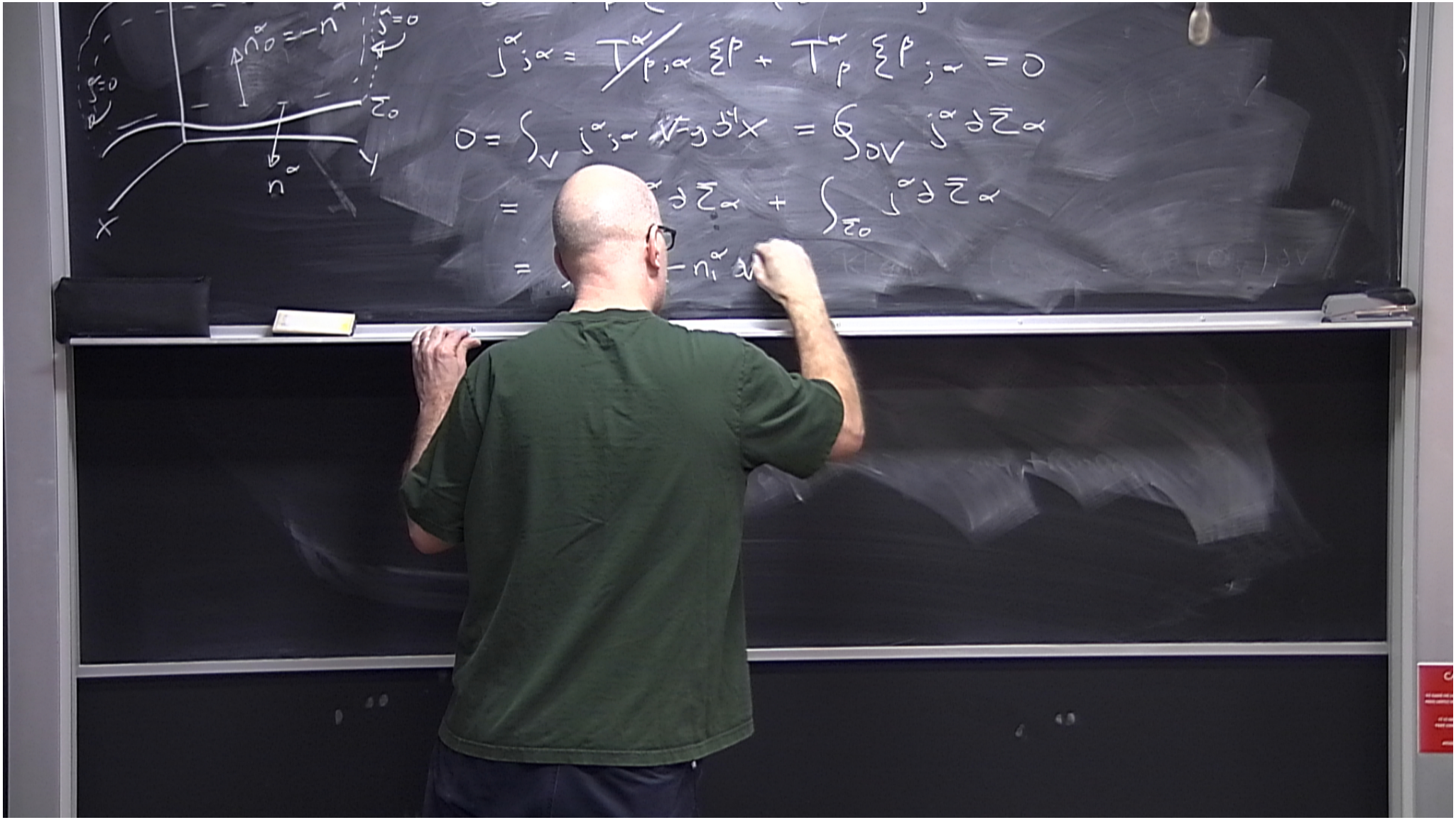
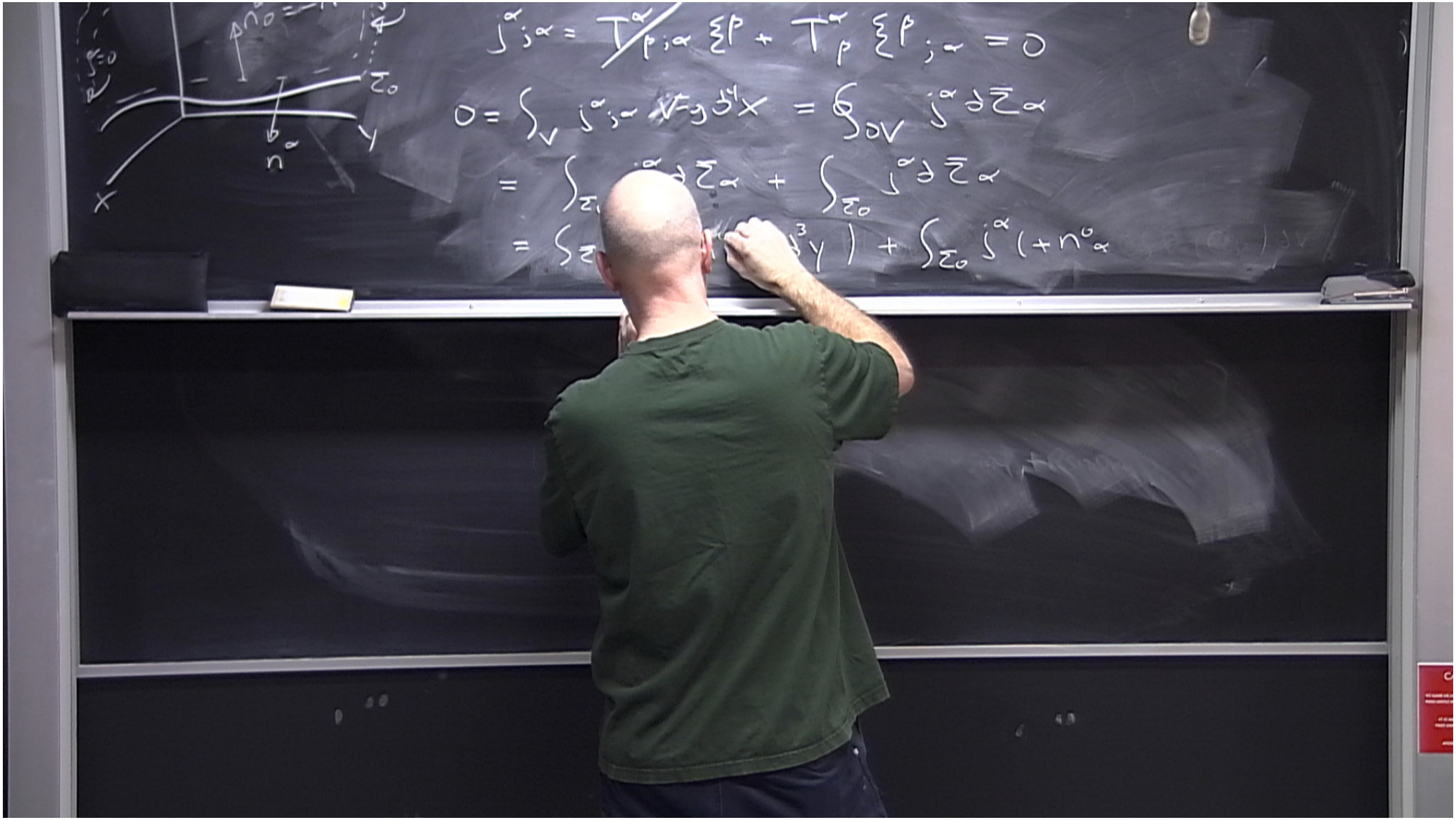
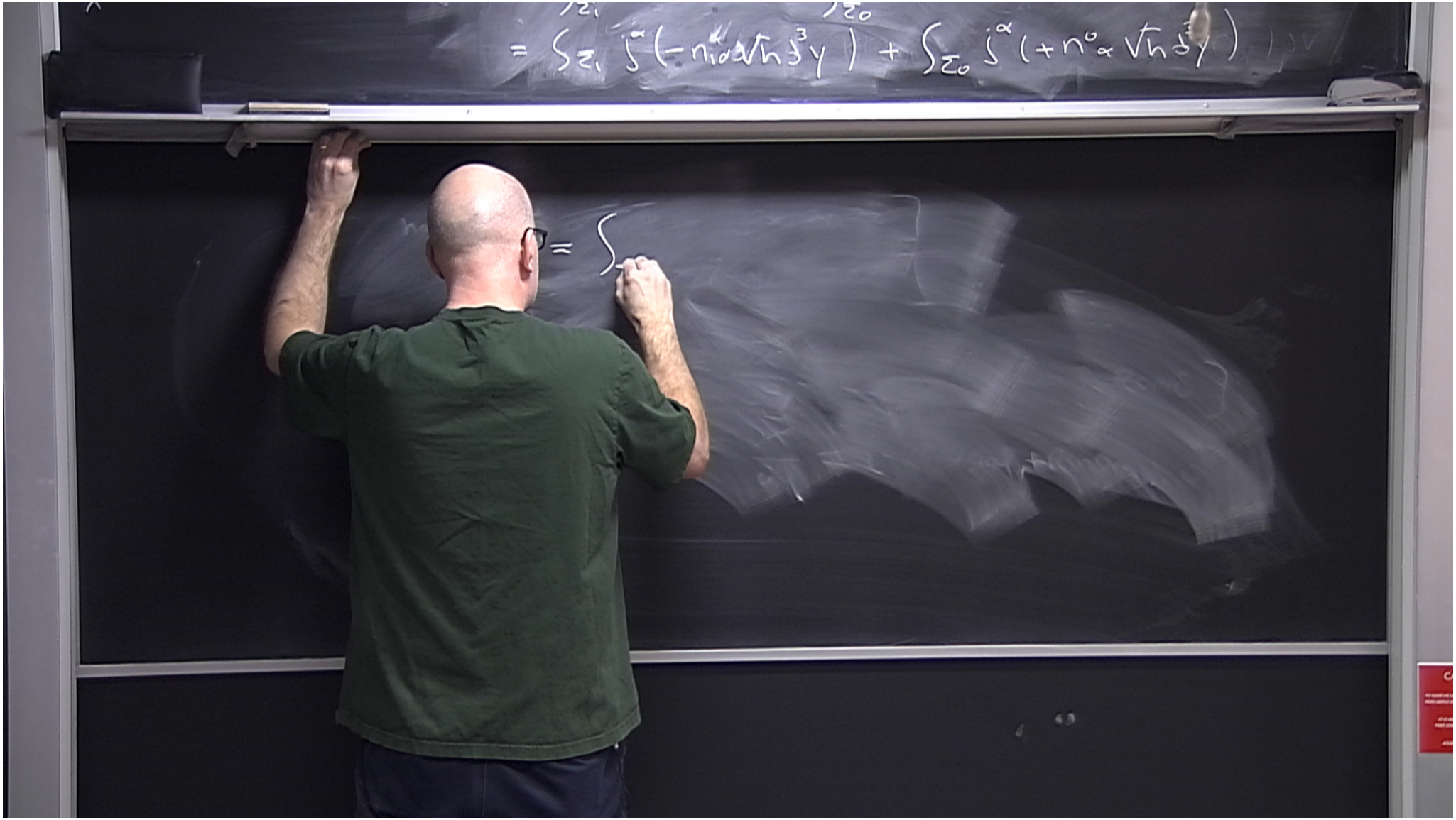
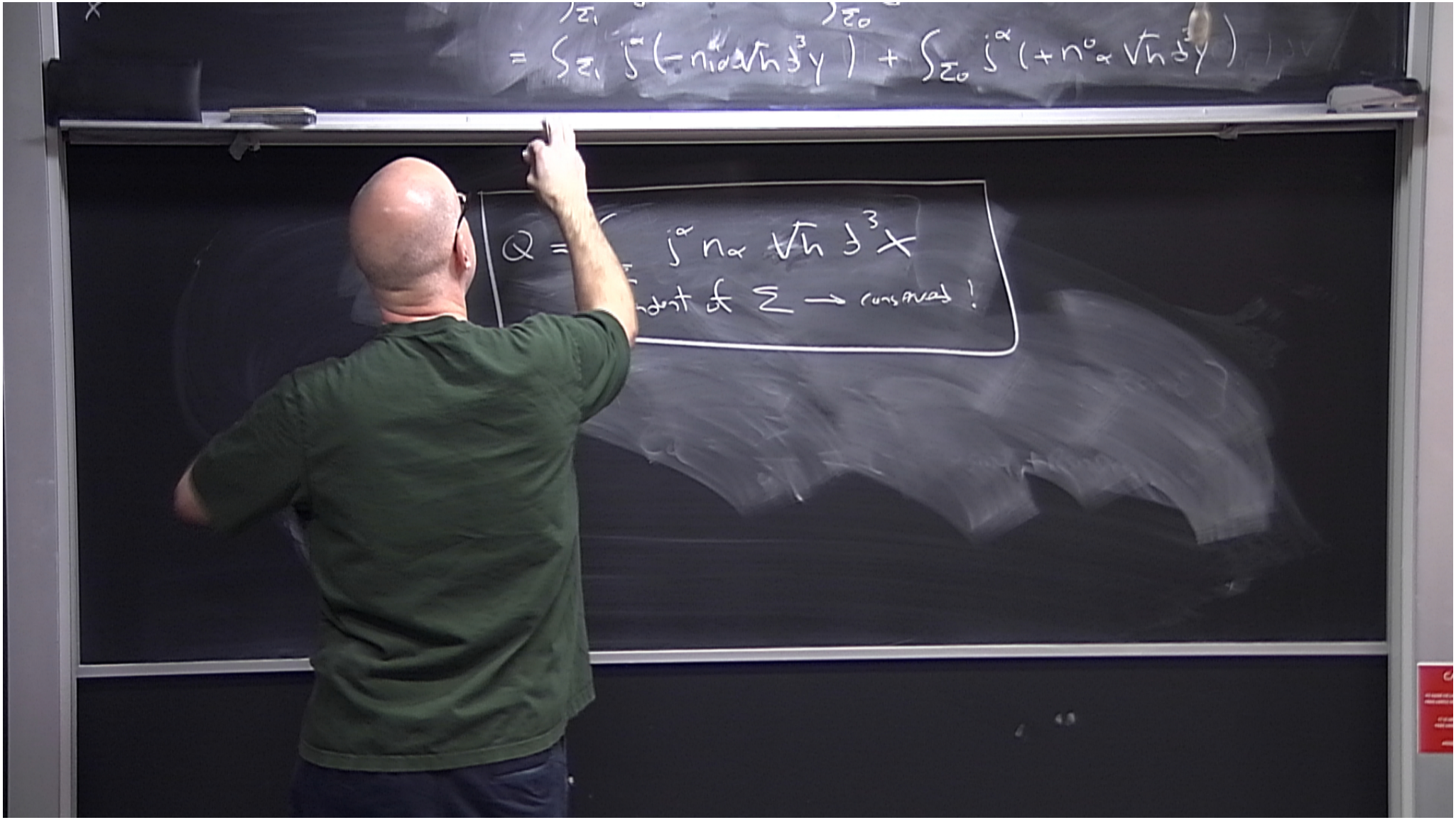
$0 = \int_V j^\alpha{}_{;\alpha} \sqrt{-g}\, d^4x = \oint_{\partial V} j^\alpha d\Sigma_\alpha$
$= \int_{\Sigma_1} j^\alpha d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha d\Sigma_\alpha$
$n^\alpha_0 = -n^\alpha$
$j^\alpha = 0$
Σ_0
x
y

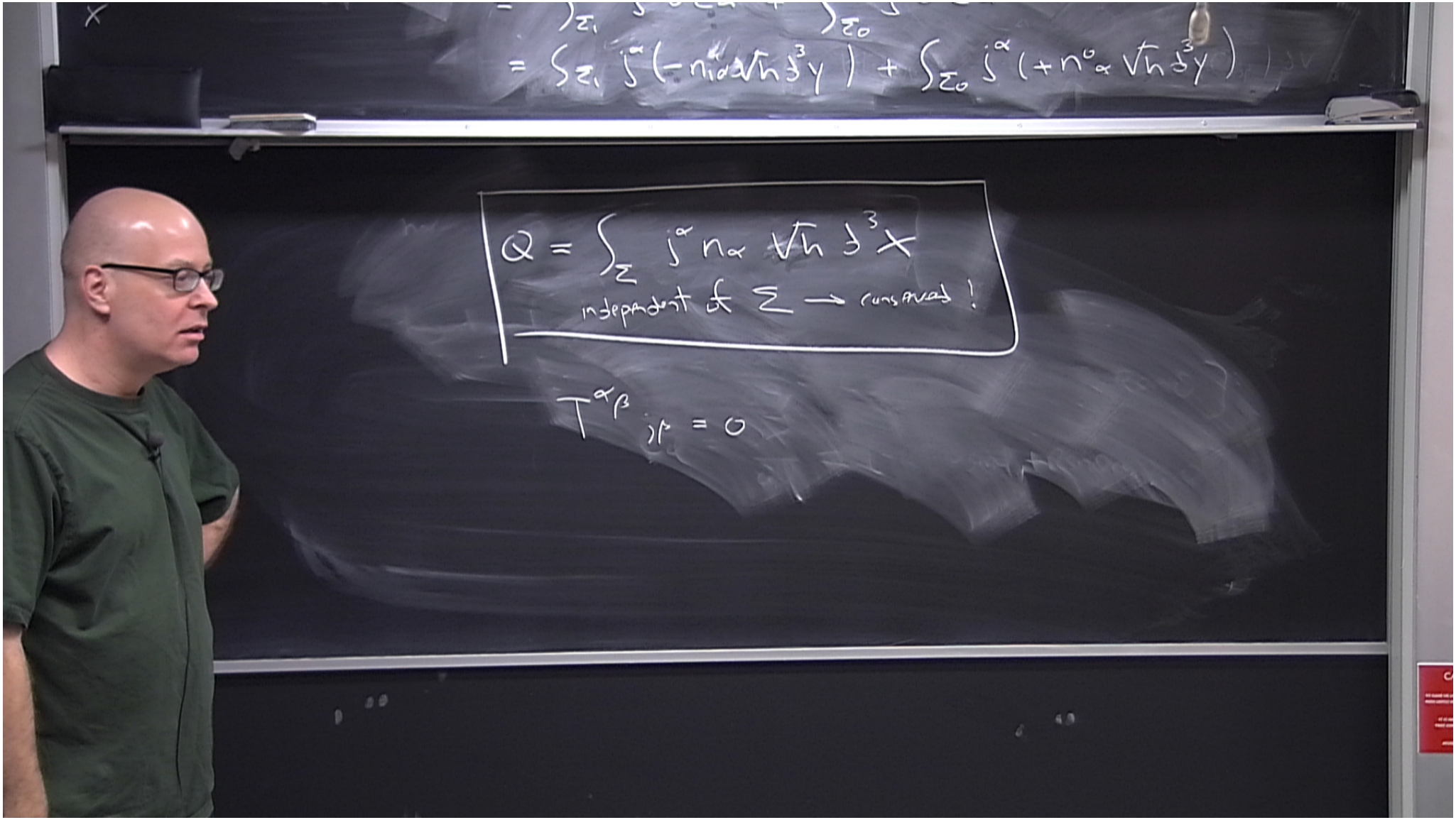

$n^\alpha_0 = -n^\alpha$
$j^\alpha = 0$
z_0
n^α
y
x
$j^\alpha{}_{;\alpha} = T^\alpha{}_{\beta;\alpha}\,\xi^\beta + T^\alpha{}_\beta\,\xi^\beta{}_{;\alpha} = 0$
$0 = \int_V j^\alpha{}_{;\alpha}\sqrt{-g}\,d^4x = \oint_{\partial V} j^\alpha\,d\Sigma_\alpha$
$\int_{\Sigma_1} j^\alpha\,d\Sigma_\alpha + \int_{\Sigma_0} j^\alpha\,d\Sigma_\alpha$
$\int_{\Sigma_1} j^\alpha$

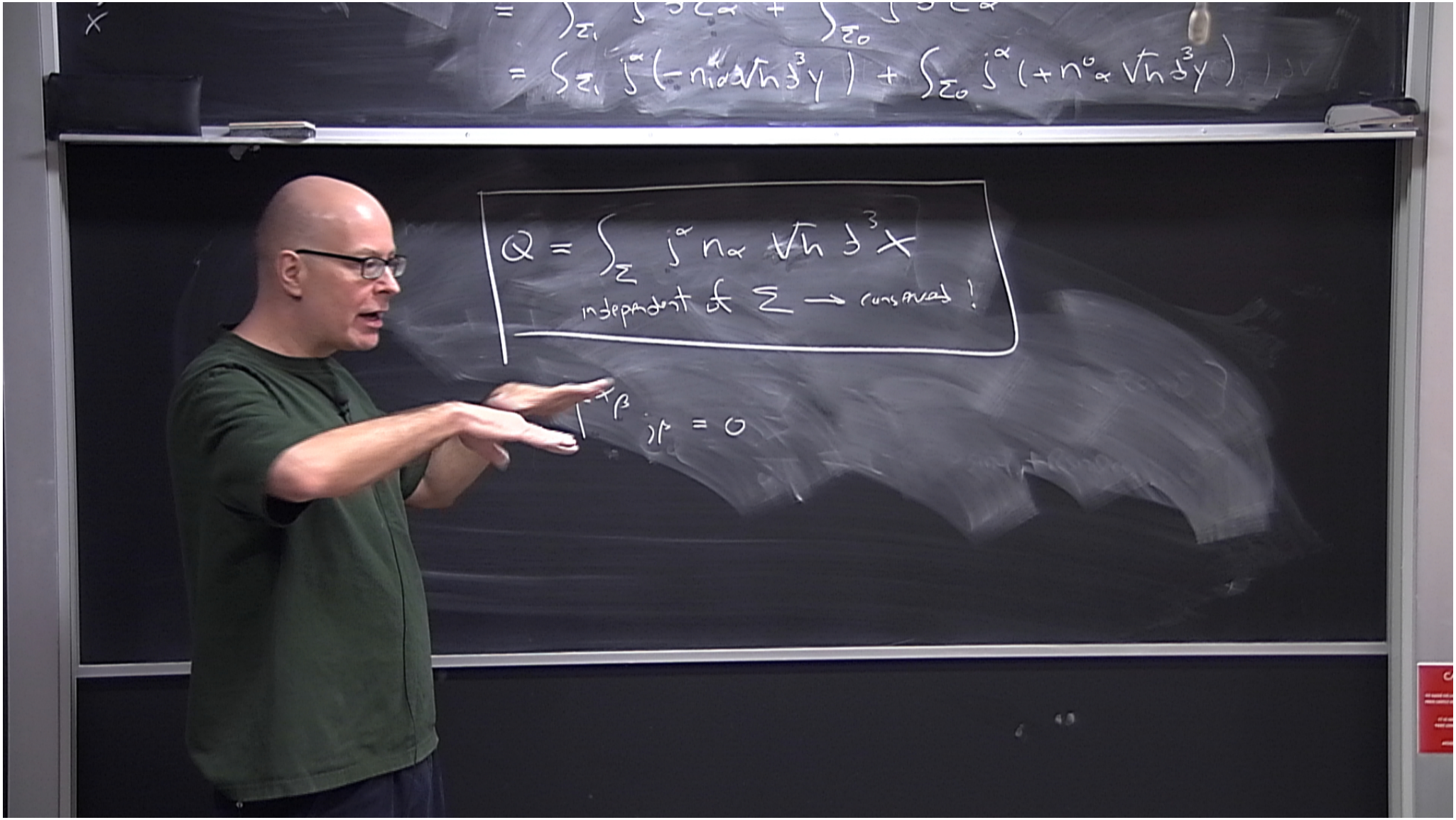


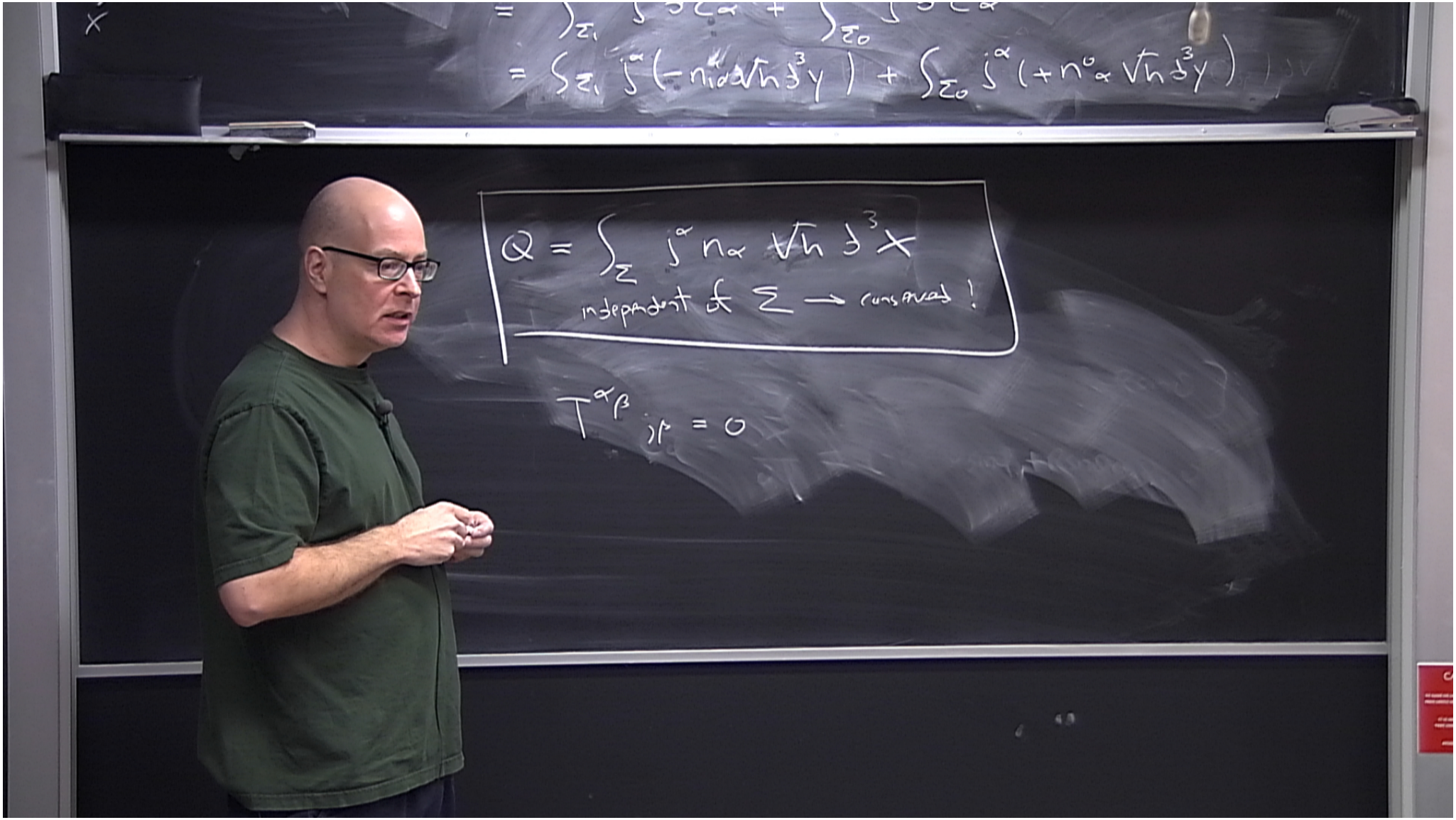


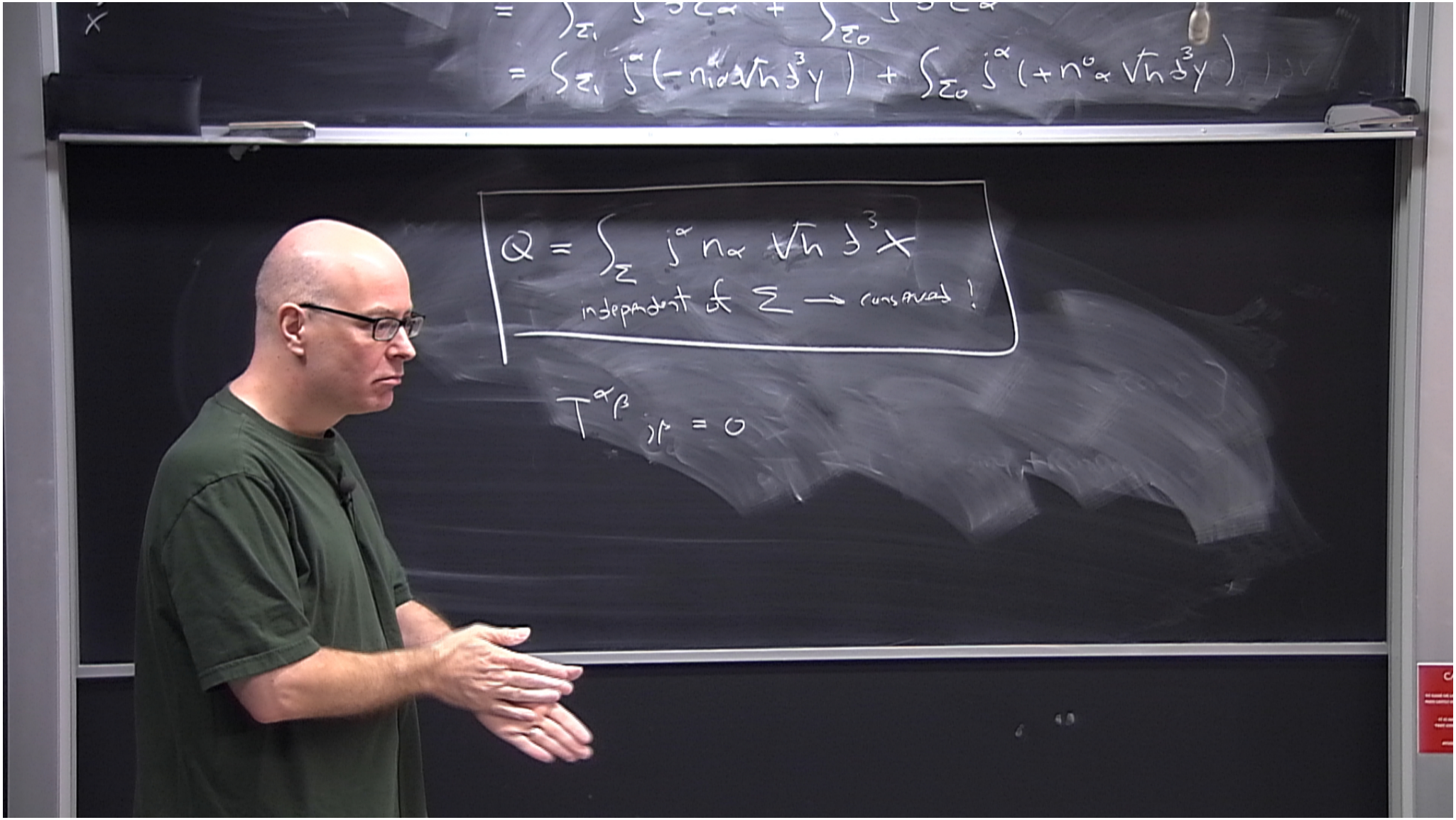



$= \int_{\Sigma_1} j^\alpha (-n_{1\alpha} \sqrt{h}\, d^3y) + \int_{\Sigma_0} j^\alpha (+n^0_\alpha \sqrt{h}\, d^3y)$
$Q = \int j^\alpha n_\alpha \sqrt{h}\, d^3x$
$\Sigma \rightarrow$ constant !


$= \int_{\Sigma_1} j^\alpha (-n_{1\alpha} \sqrt{h}\, d^3y) + \int_{\Sigma_0} j^\alpha (+n^0_\alpha \sqrt{h}\, d^3y)$
$Q = \int_\Sigma j^\alpha n_\alpha \sqrt{h}\, d^3x$
independent of Σ → conserved !
$T^{\alpha\beta}{}_{;\beta} = 0$


$= \int_{\Sigma_1} j^\alpha (-n_{1\alpha} \sqrt{h}\, d^3y) + \int_{\Sigma_0} j^\alpha (+n^0_\alpha \sqrt{h}\, d^3y)$
$Q = \int_\Sigma j^\alpha n_\alpha \sqrt{h}\, d^3x$
independent of Σ → conserved !
$\partial_\mu = 0$


$= \int_{\Sigma_1} j^\alpha (-n_{1\alpha} \sqrt{h}\, d^3y) + \int_{\Sigma_0} j^\alpha (+n^0{}_\alpha \sqrt{h}\, d^3y)$
$Q = \int_\Sigma j^\alpha n_\alpha \sqrt{h}\, d^3x$
independent of Σ → conserved !
$T^{\alpha\beta}{}_{;\beta} = 0$


$= \int_{\Sigma_1} j^\alpha (-n_{1\alpha} \sqrt{h}\, d^3y) + \int_{\Sigma_0} j^\alpha (+n^0{}_\alpha \sqrt{h}\, d^3y)$
$Q = \int_\Sigma j^\alpha n_\alpha \sqrt{h}\, d^3x$
independent of Σ → conserved !
$T^{\alpha\beta}{}_{;\beta} = 0$