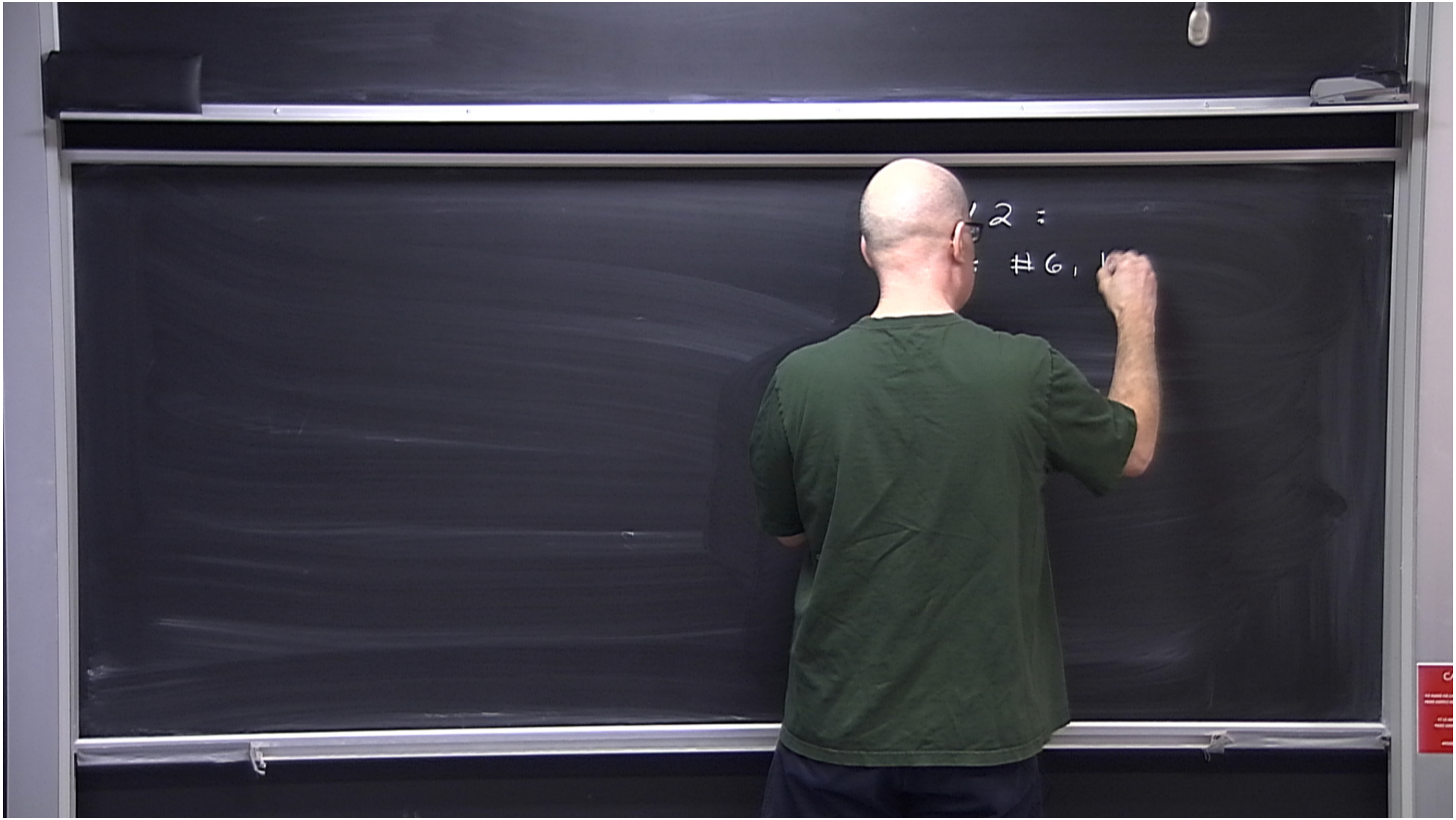
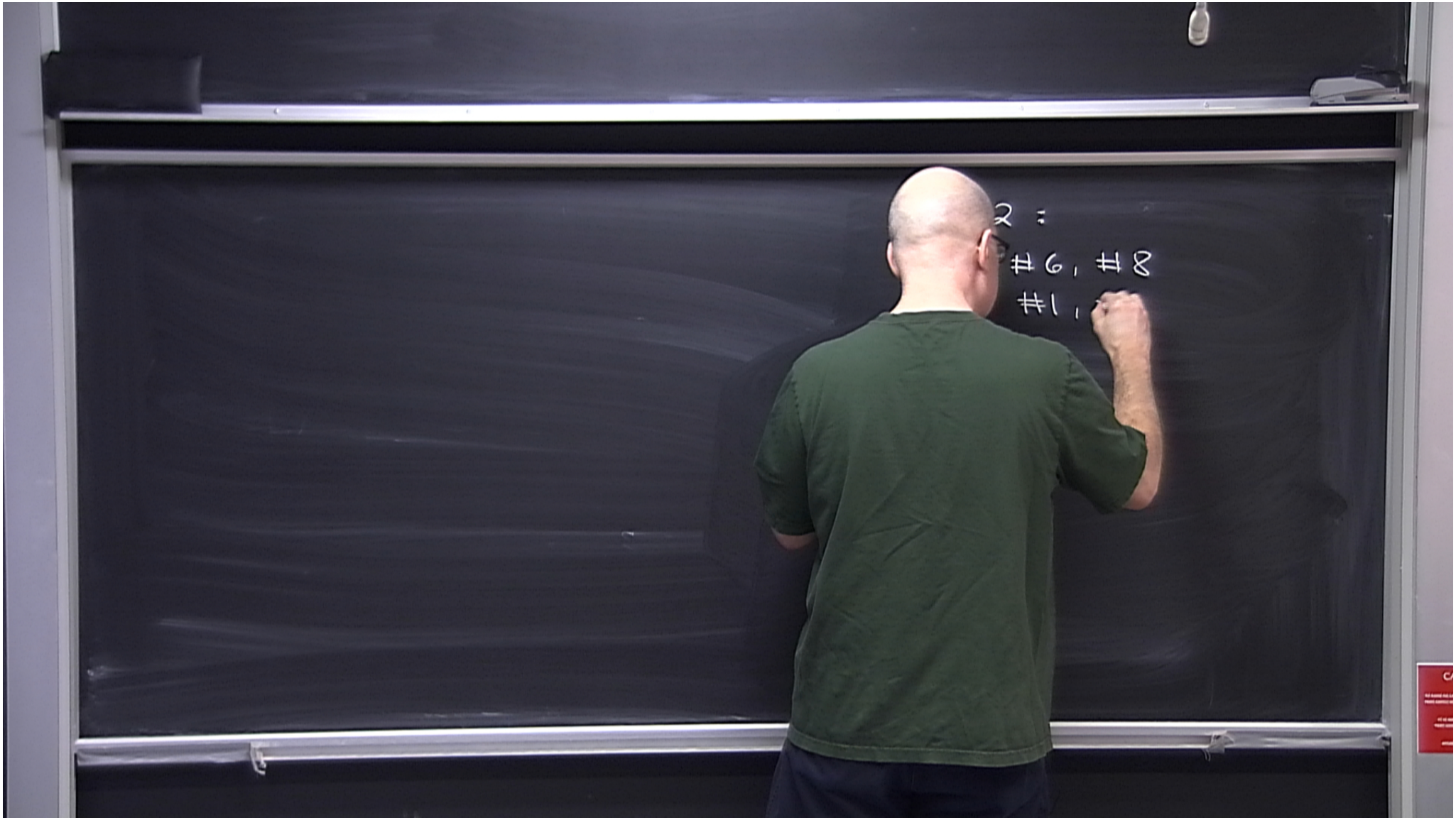
Title: Advanced General Relativity - Lecture 9

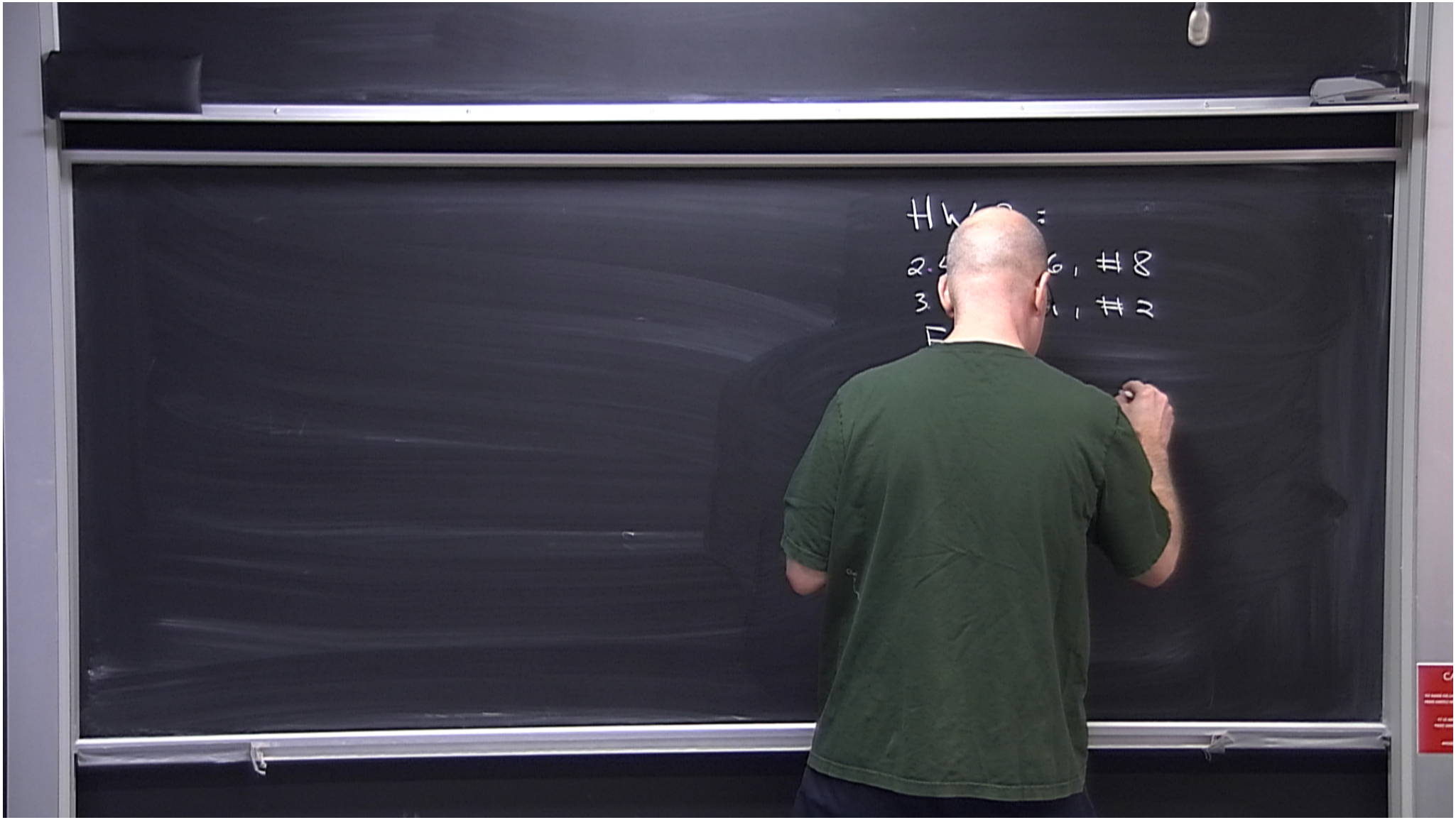
Date: Feb 08, 2012 10:00 AM

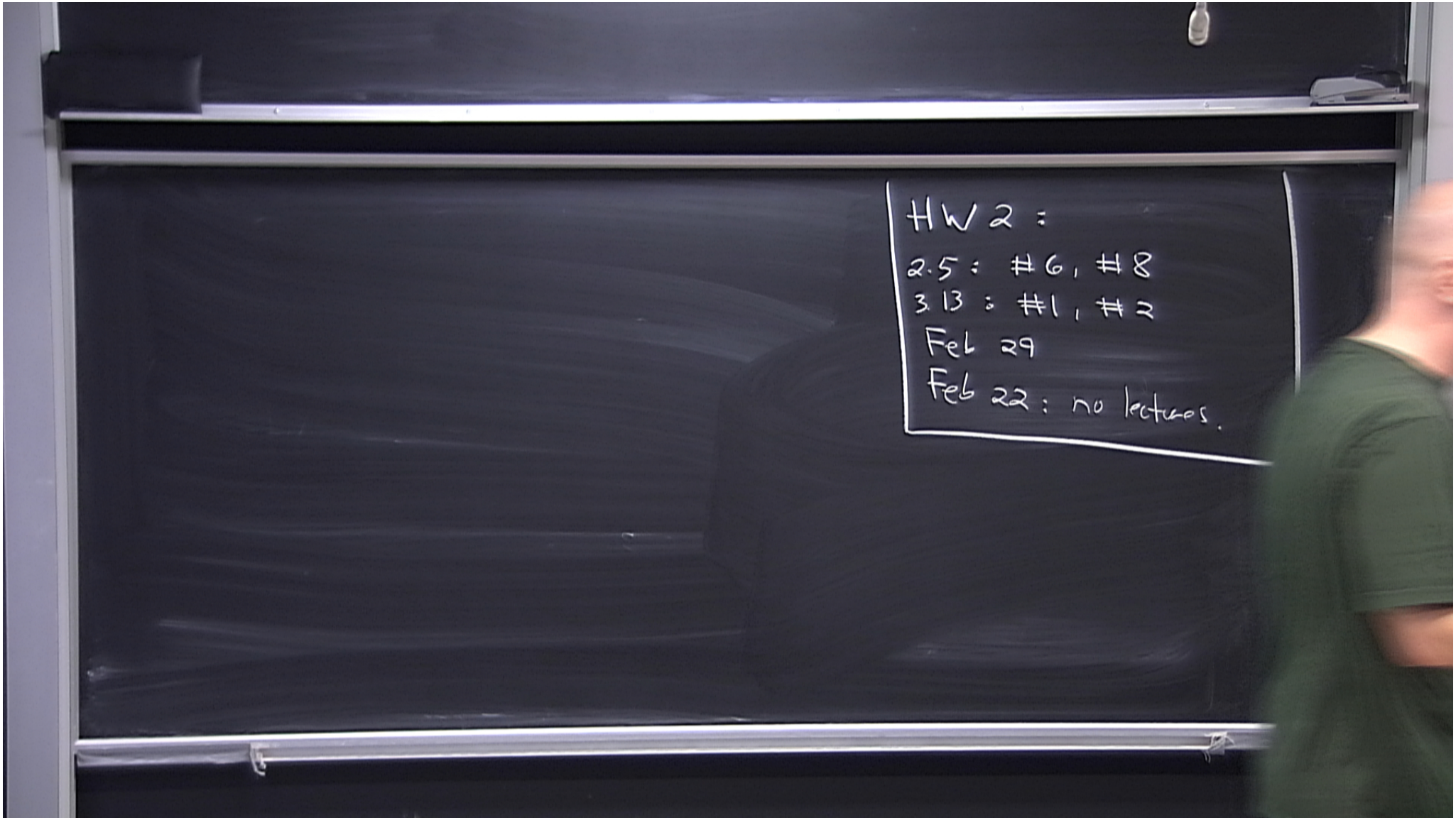
URL: http://pirsa.org/12020107

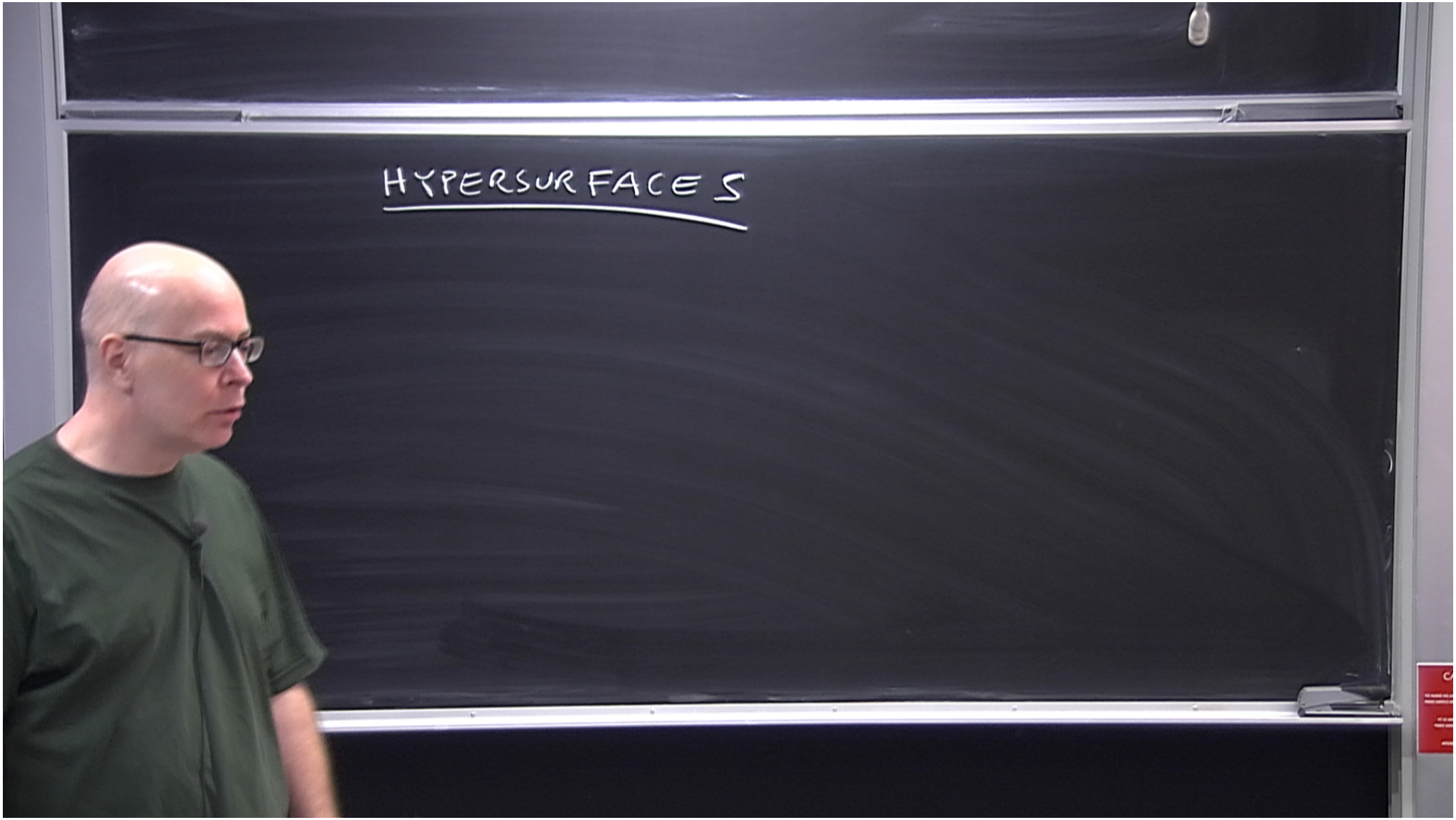
Abstract:


12 =
#6,


2 :
#6, #8
#1,


#8
#2


HW 2 :
2.5 : #6, #8
3.13 : #1, #2
Feb 29
Feb 22 : no lectures.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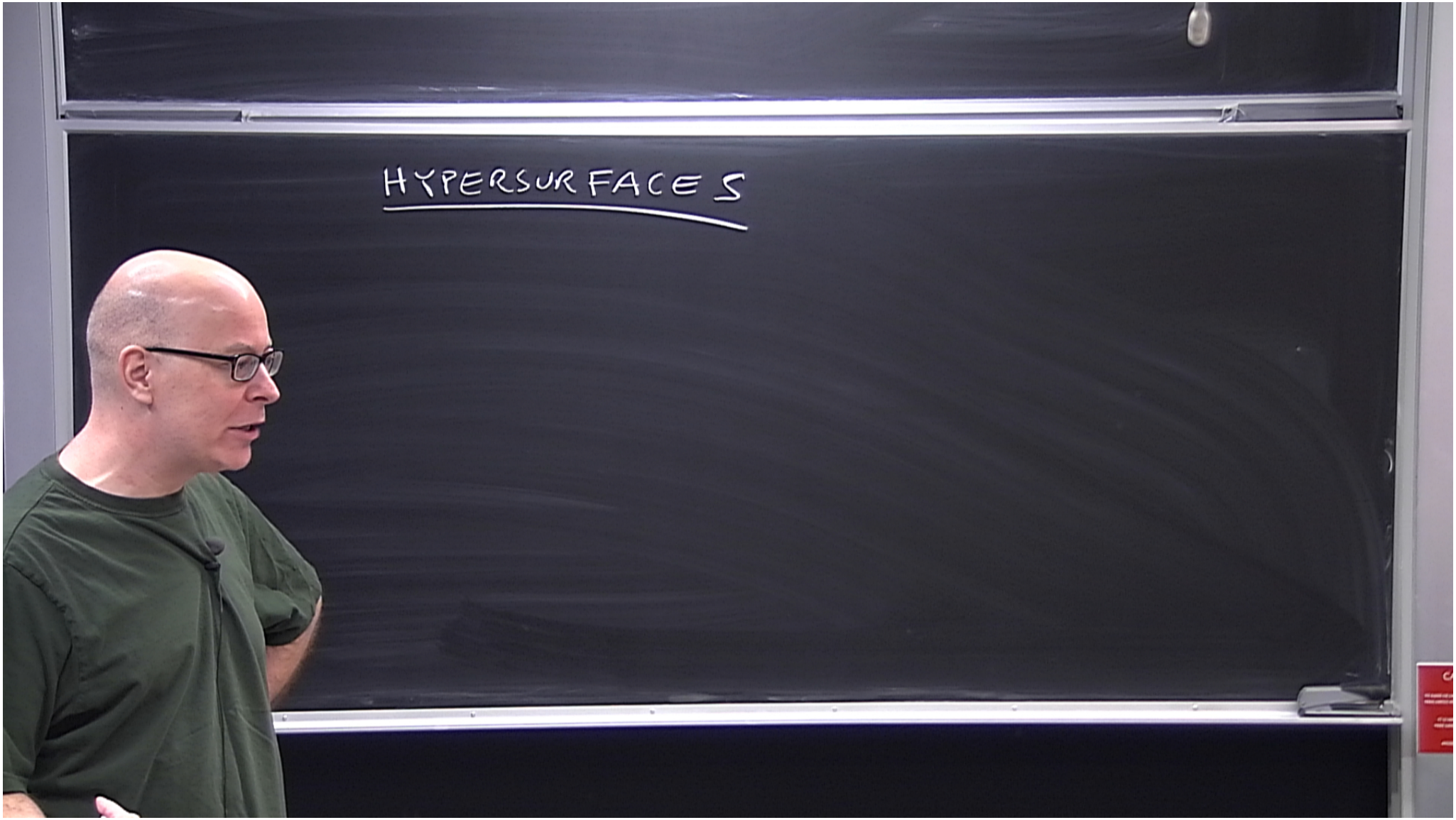
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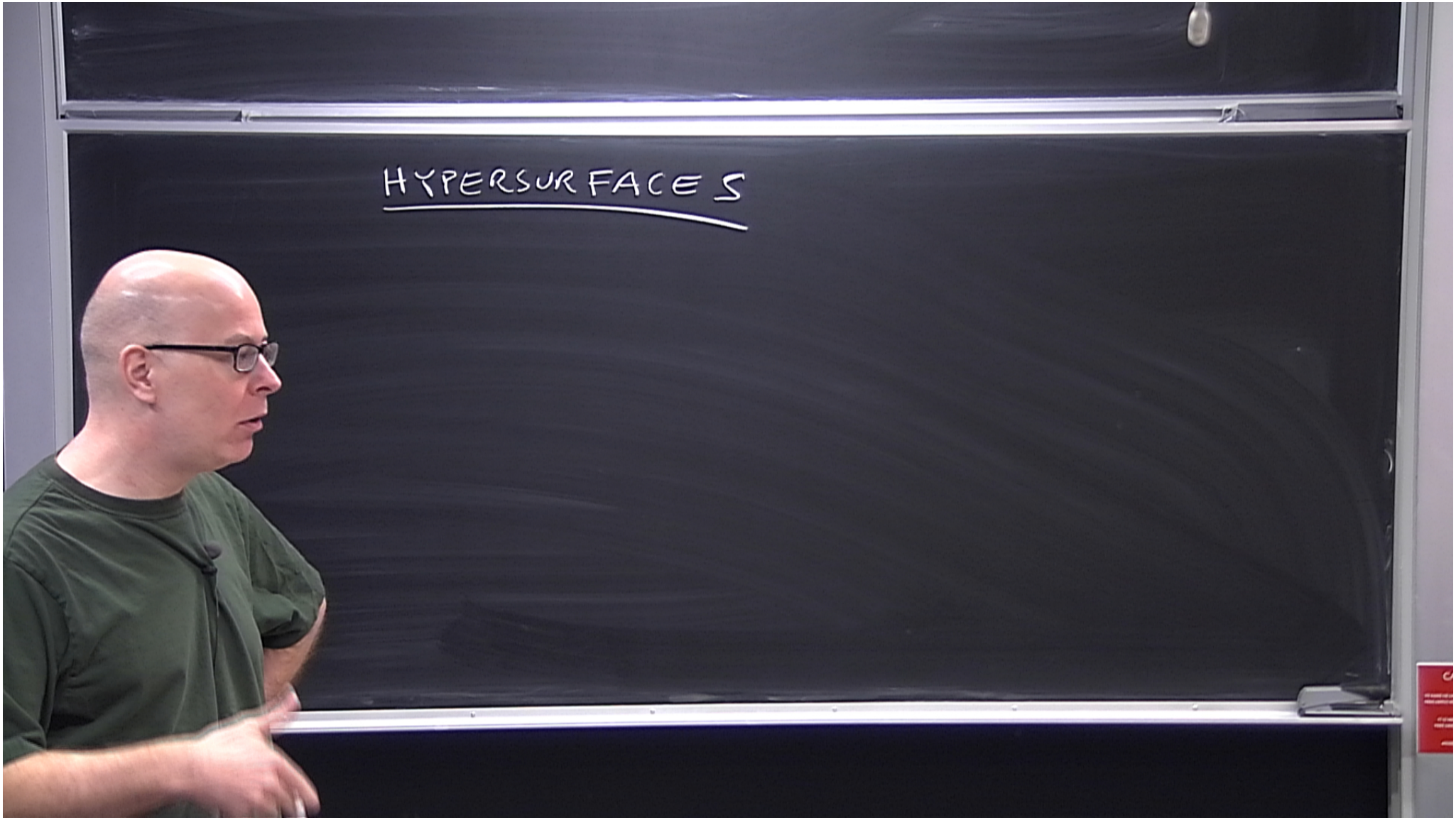
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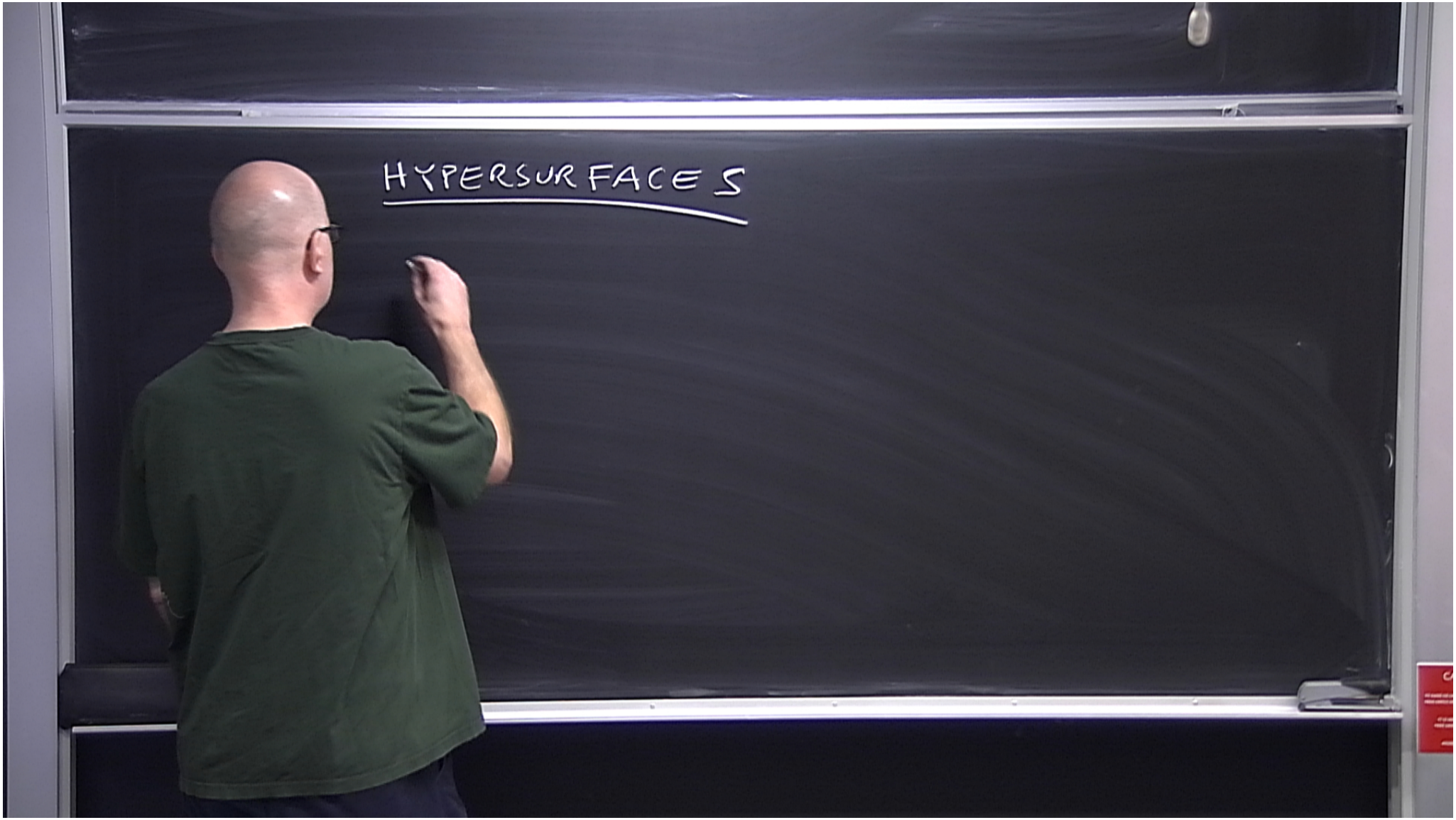
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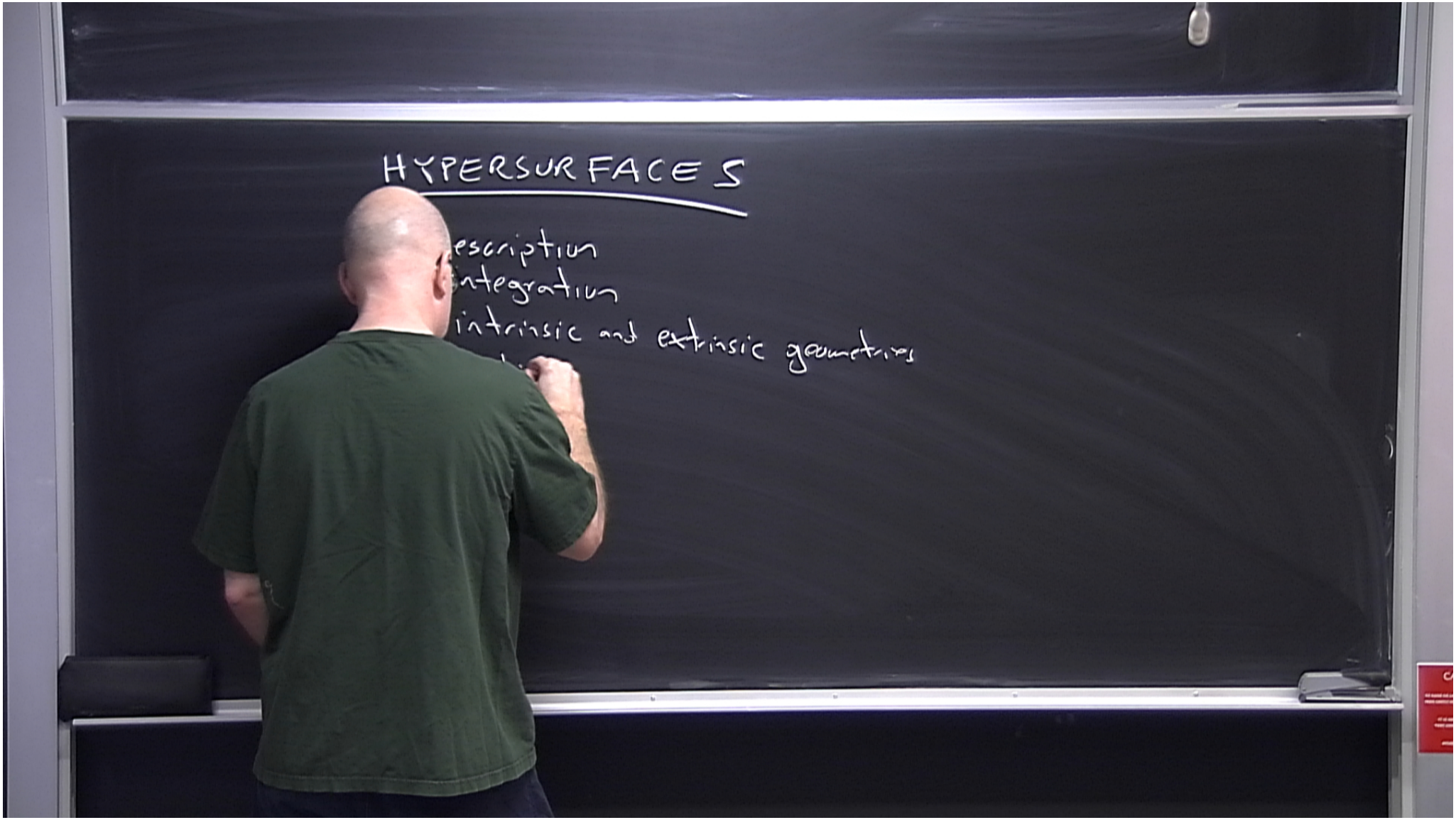
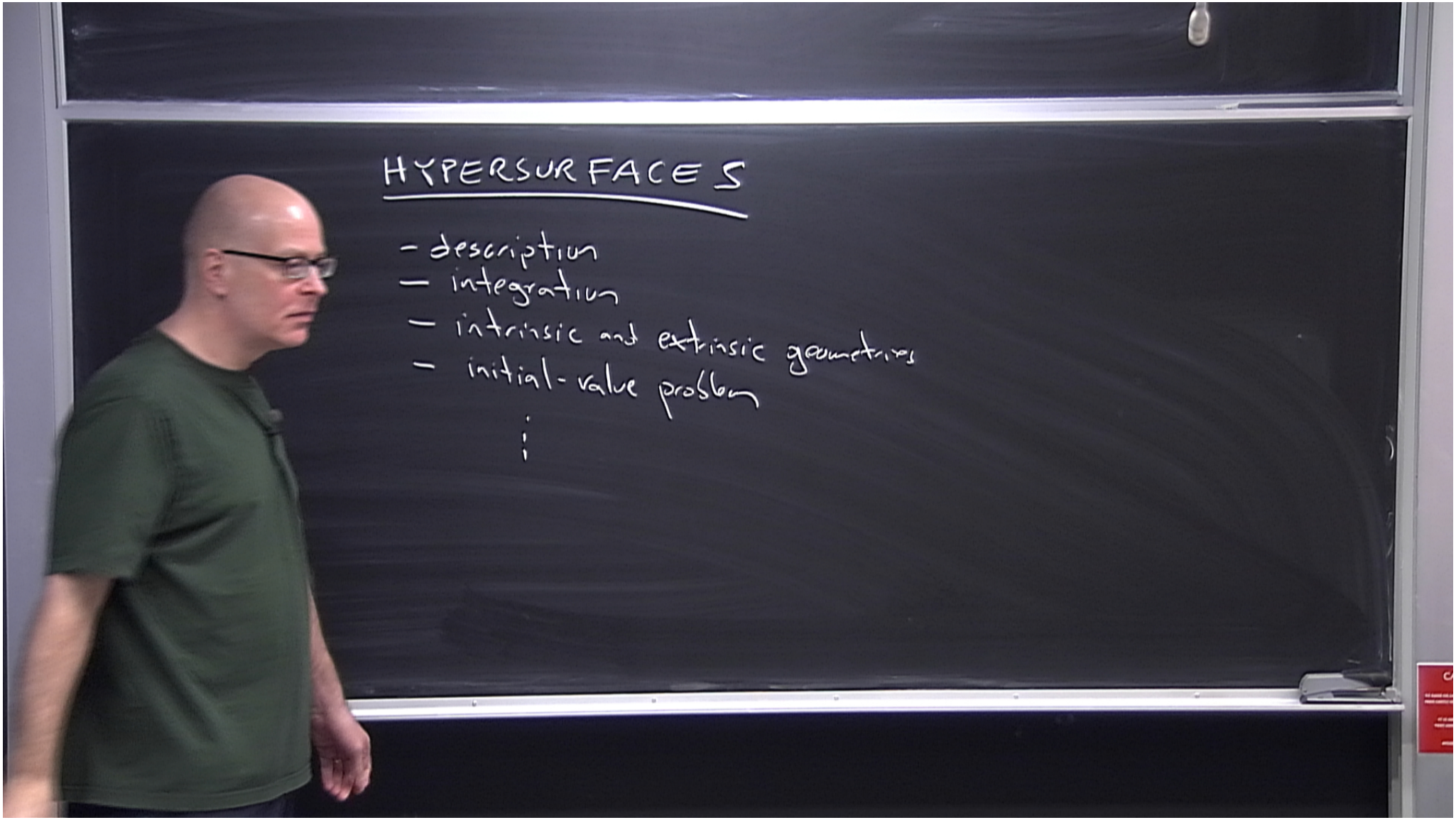
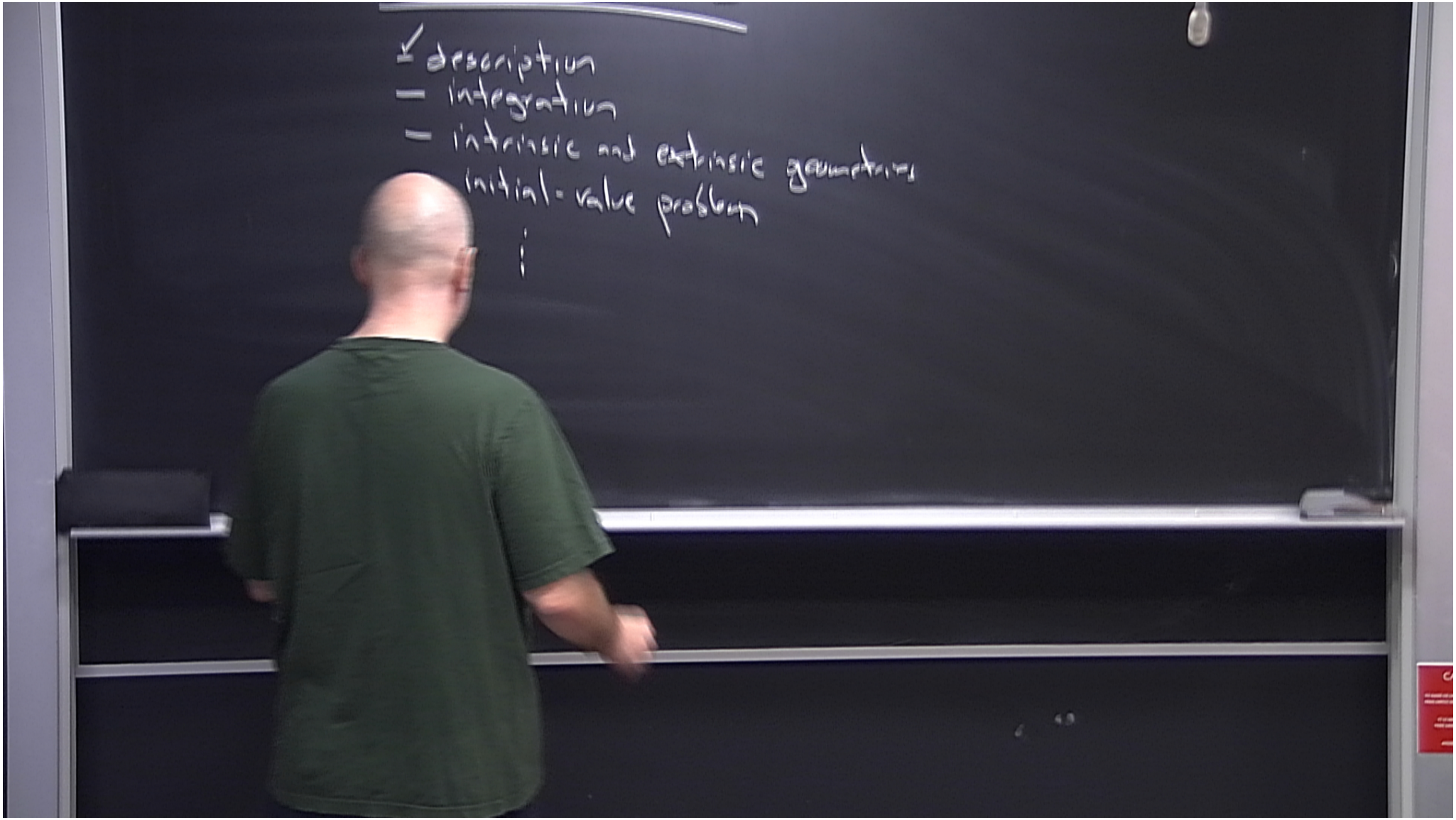
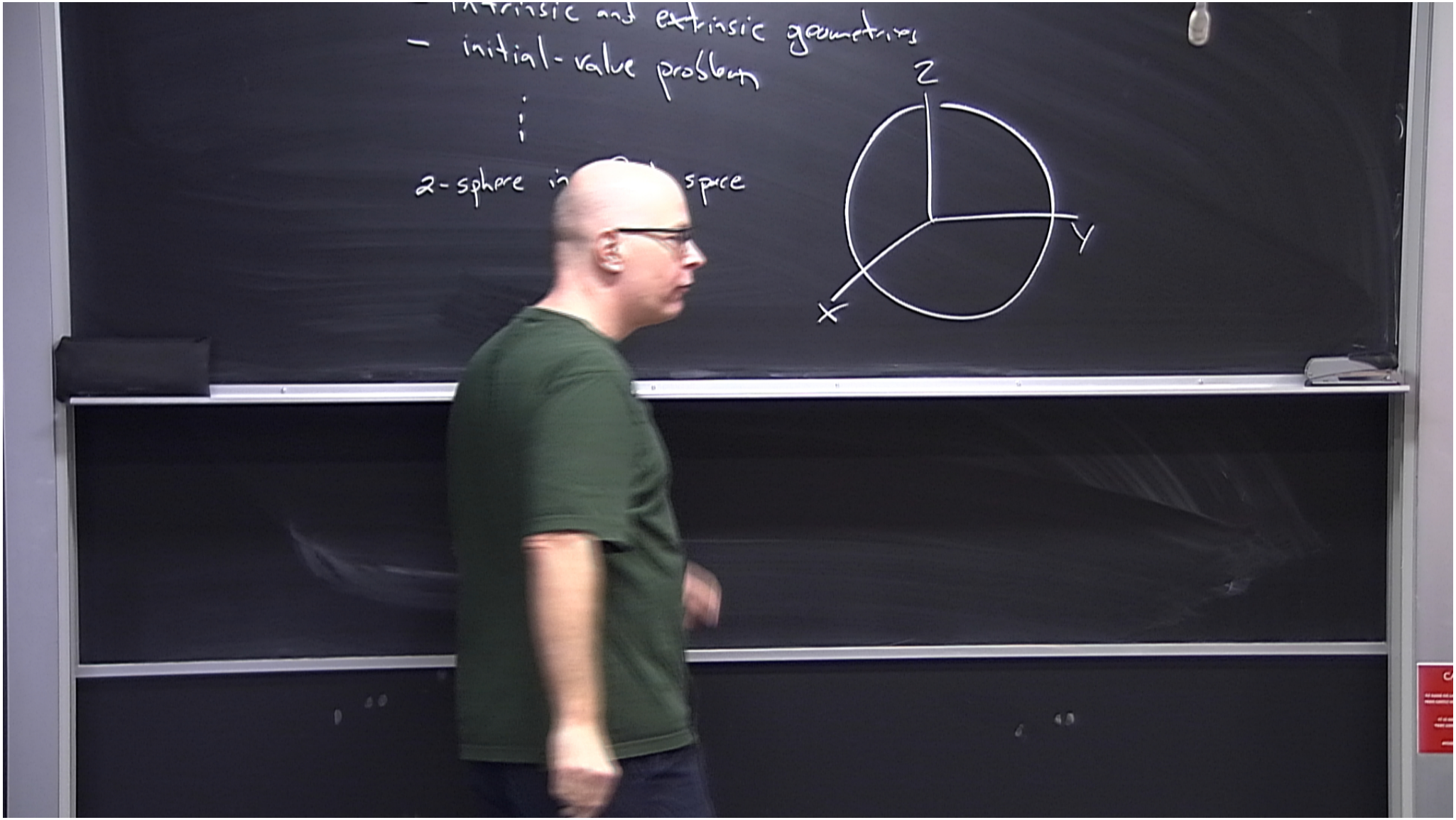
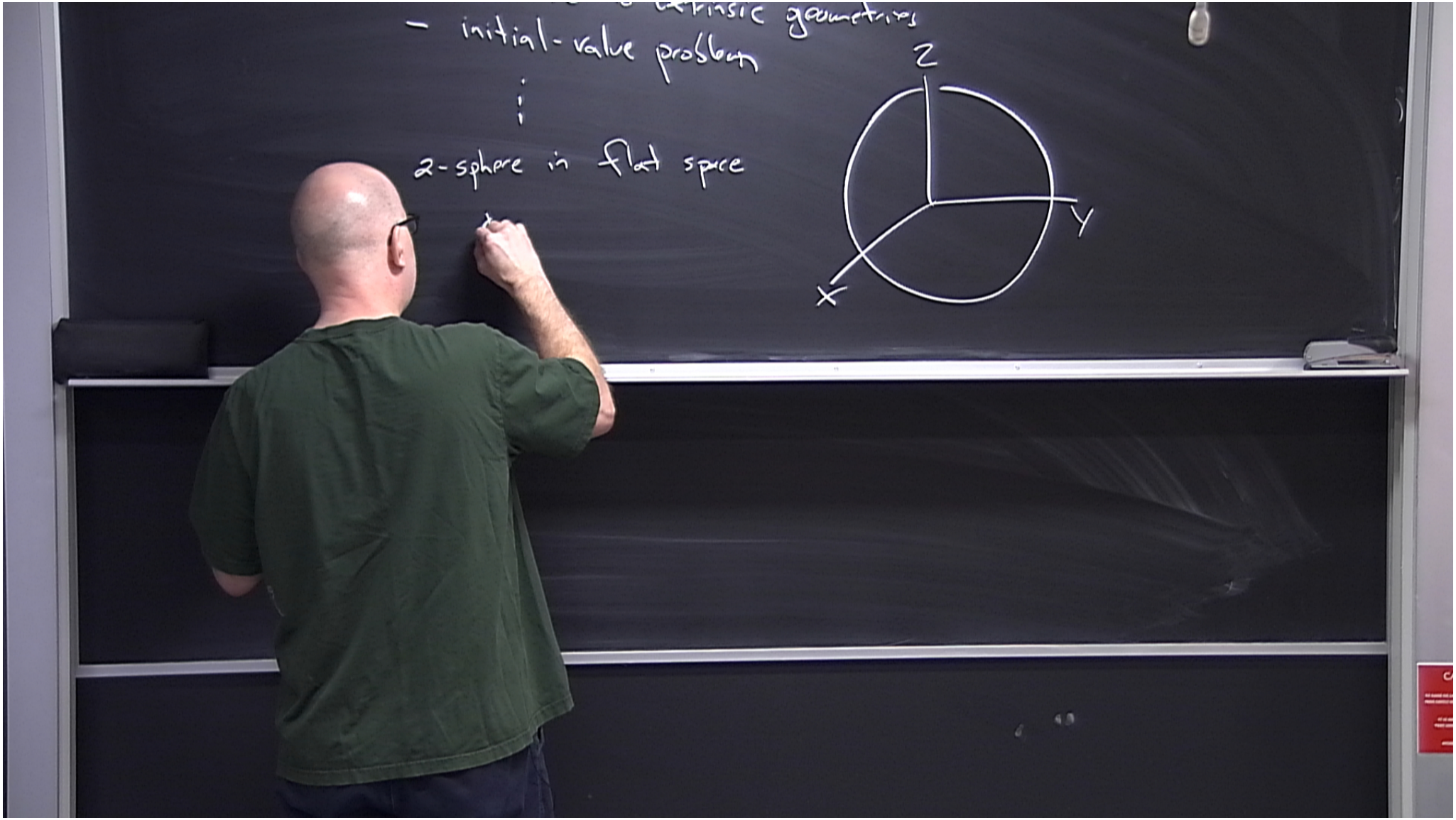
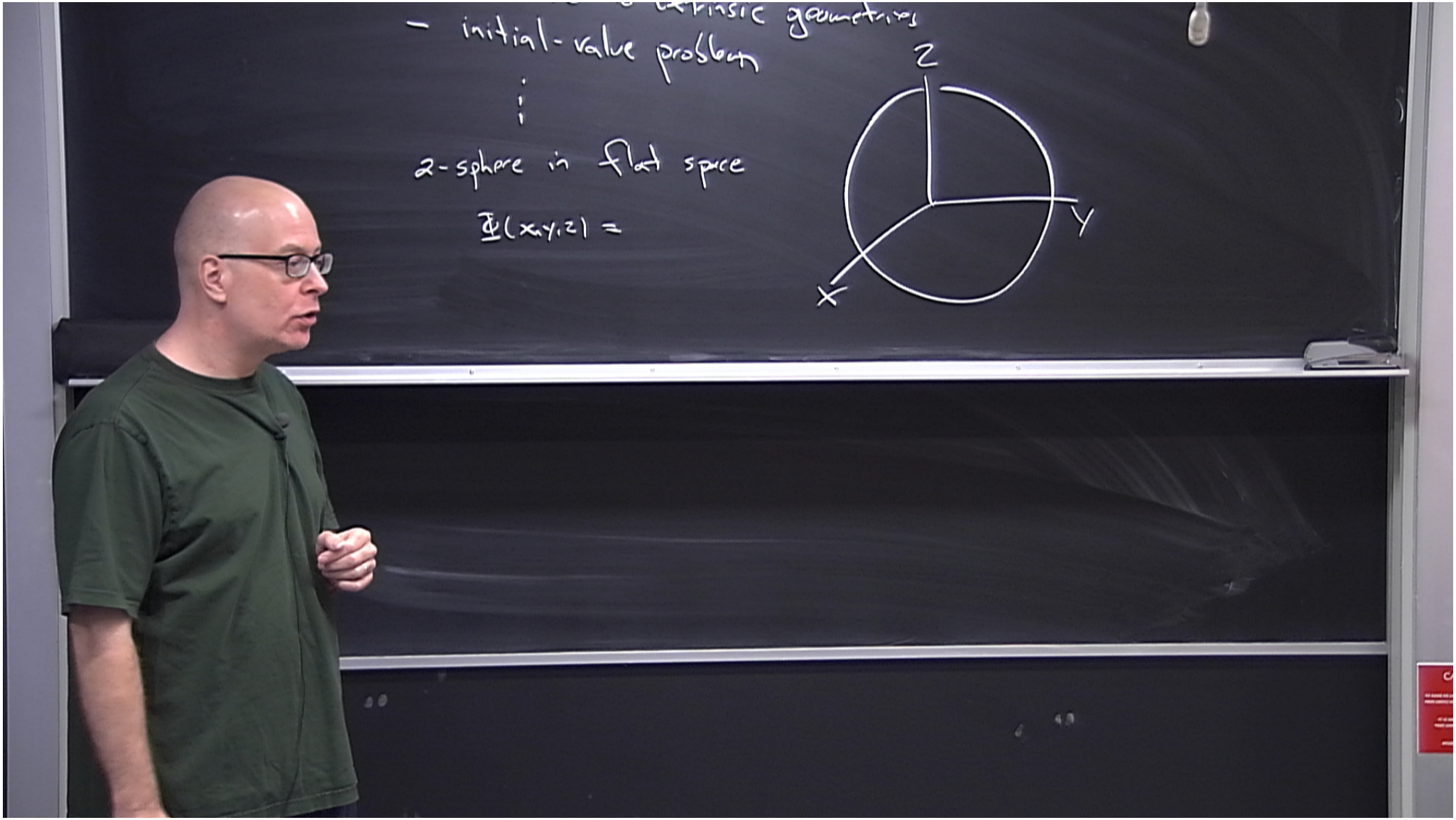

HYPERSURFACES
escription
ntegration
intrinsic and extrinsic geometries

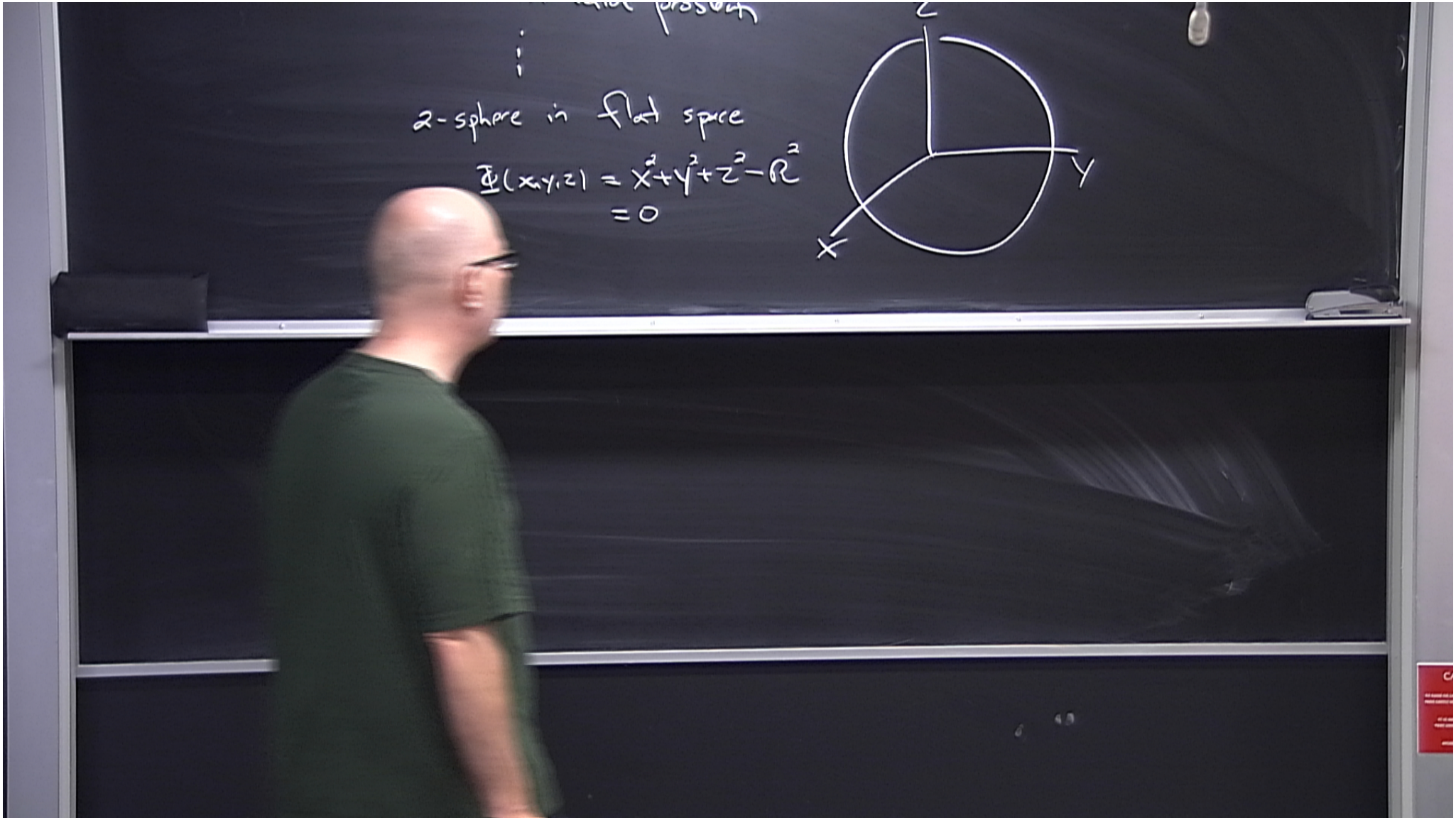

HYPERSURFACES
- description
- integration
- intrinsic and extrinsic geometries
- initial-value problem
⋮

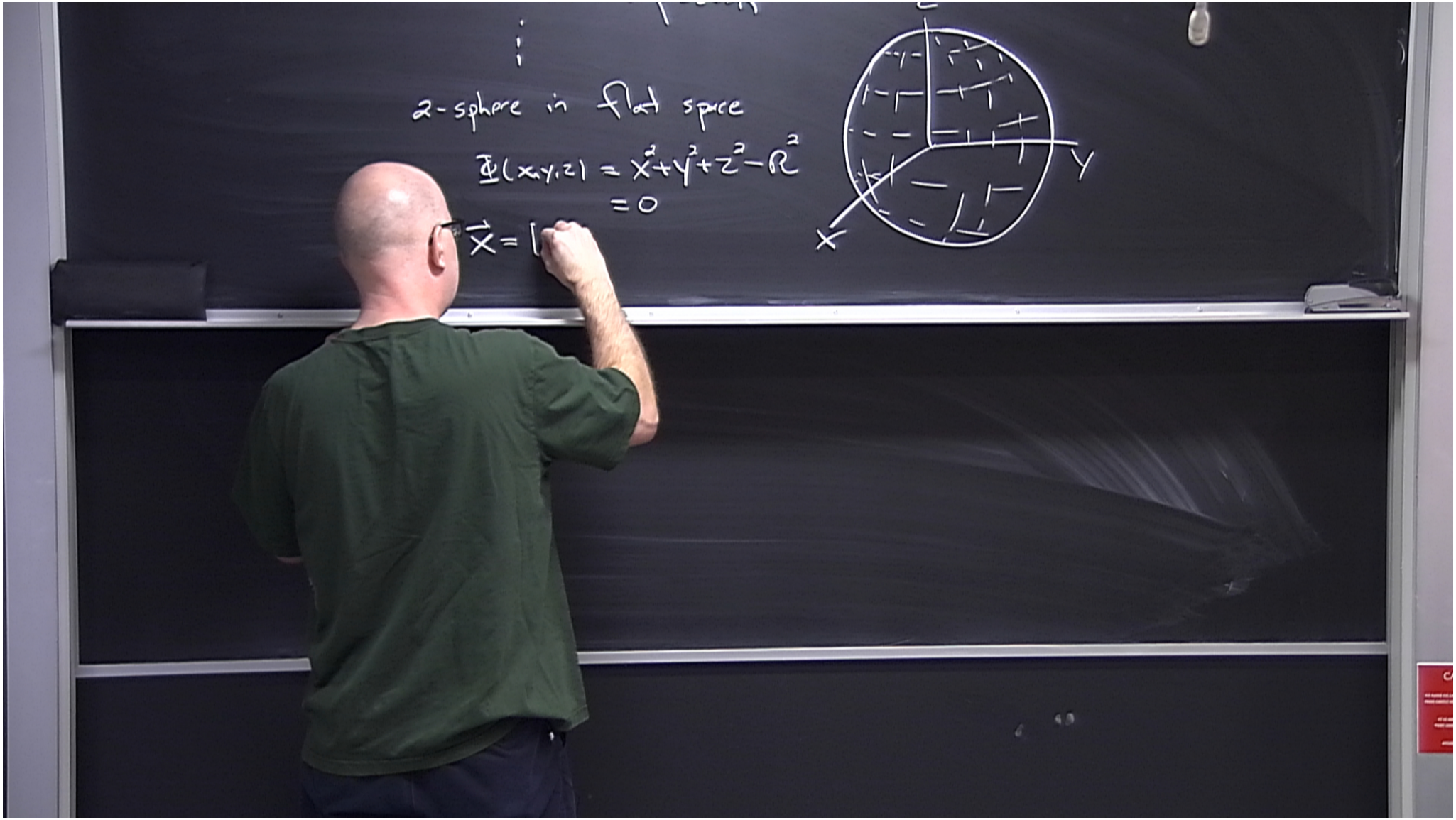

description
integration
intrinsic and extrinsic geometry
initial-value problem

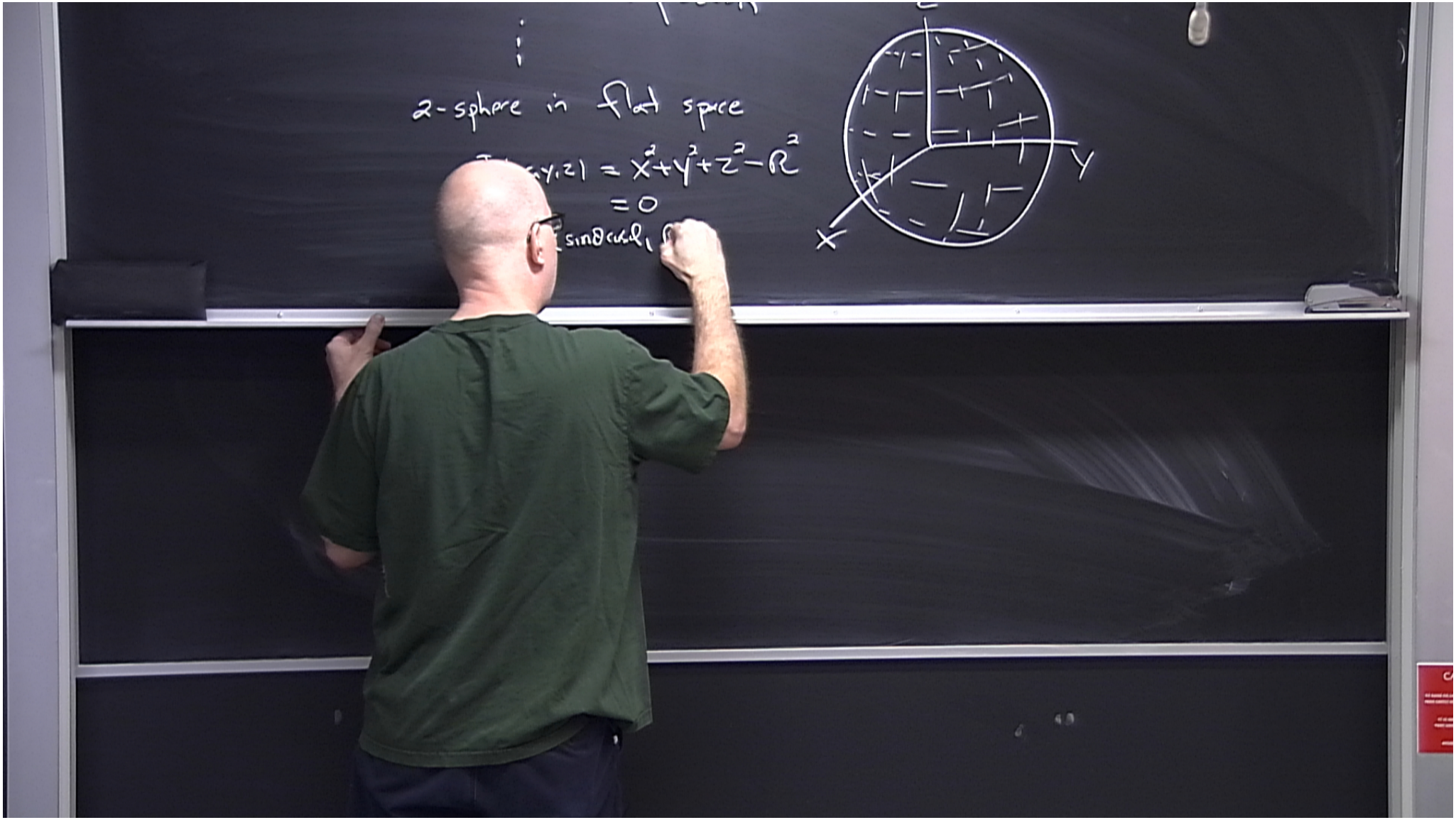

intrinsic and extrinsic geometries
- initial-value problem
2-sphere
space
Z
Y
X

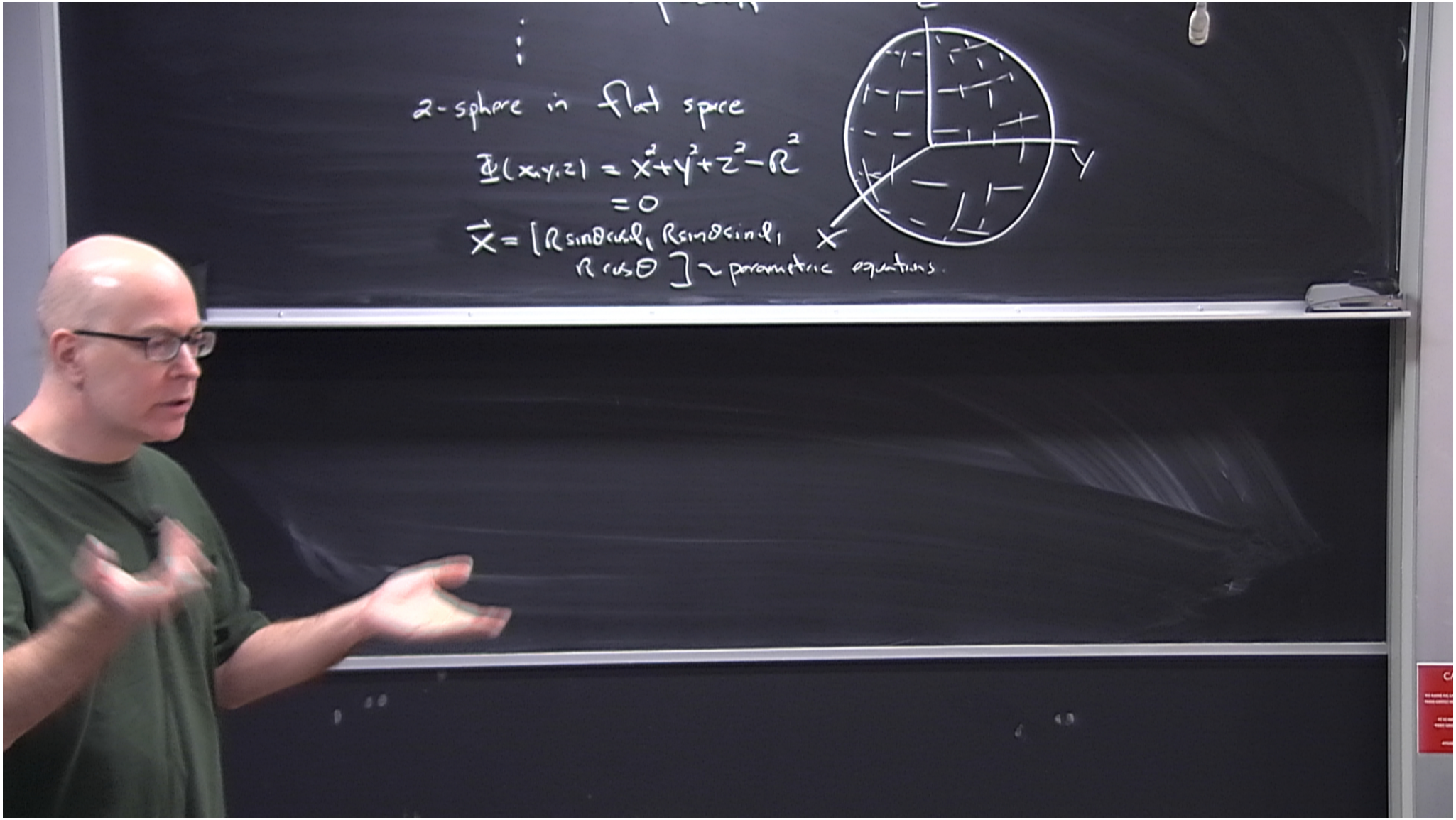

- initial-value problem
2-sphere in flat space
Z
Y
X

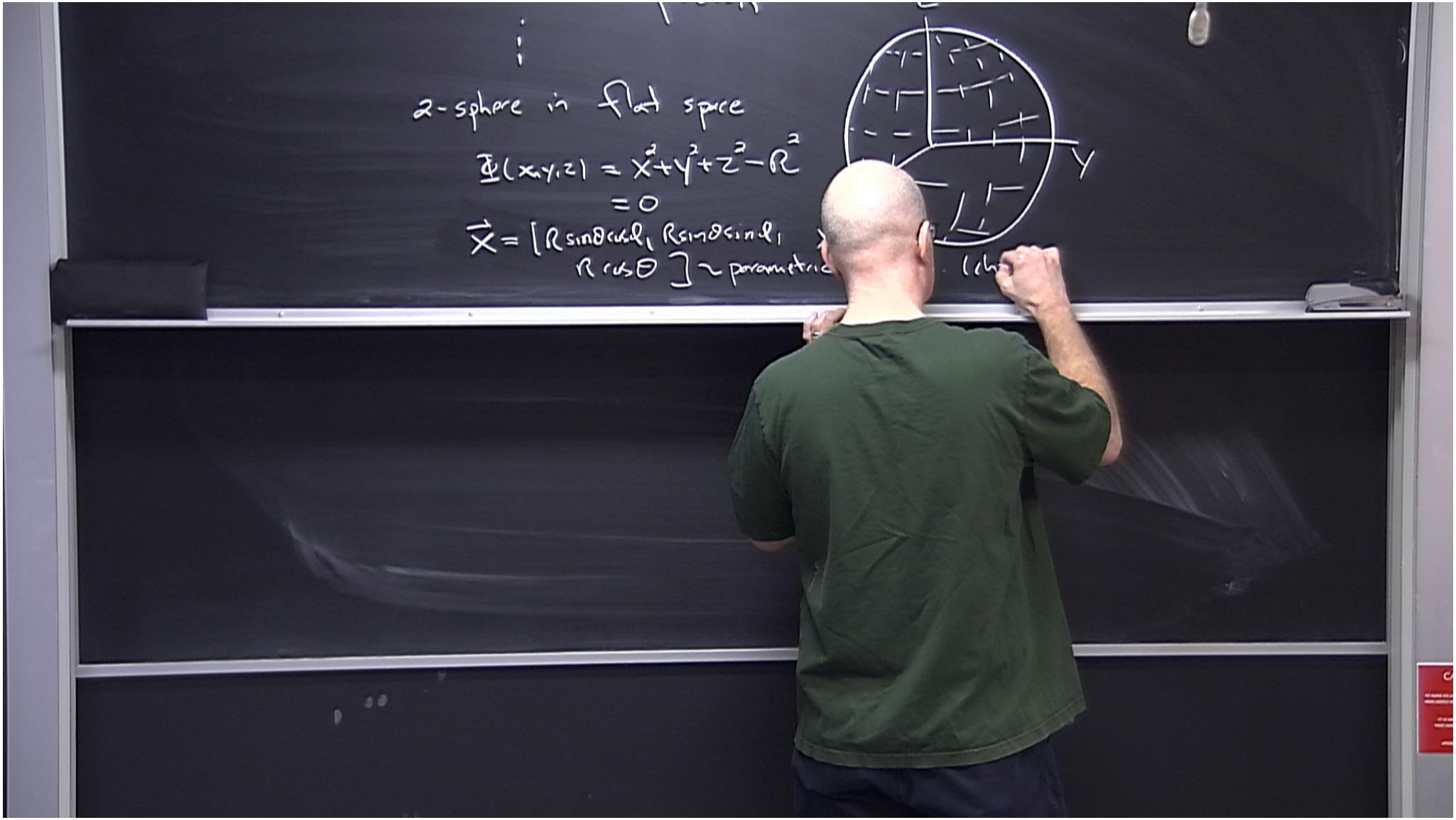

- initial-value problem
2-sphere in flat space
Φ(x,y,z) =
Z
Y
X

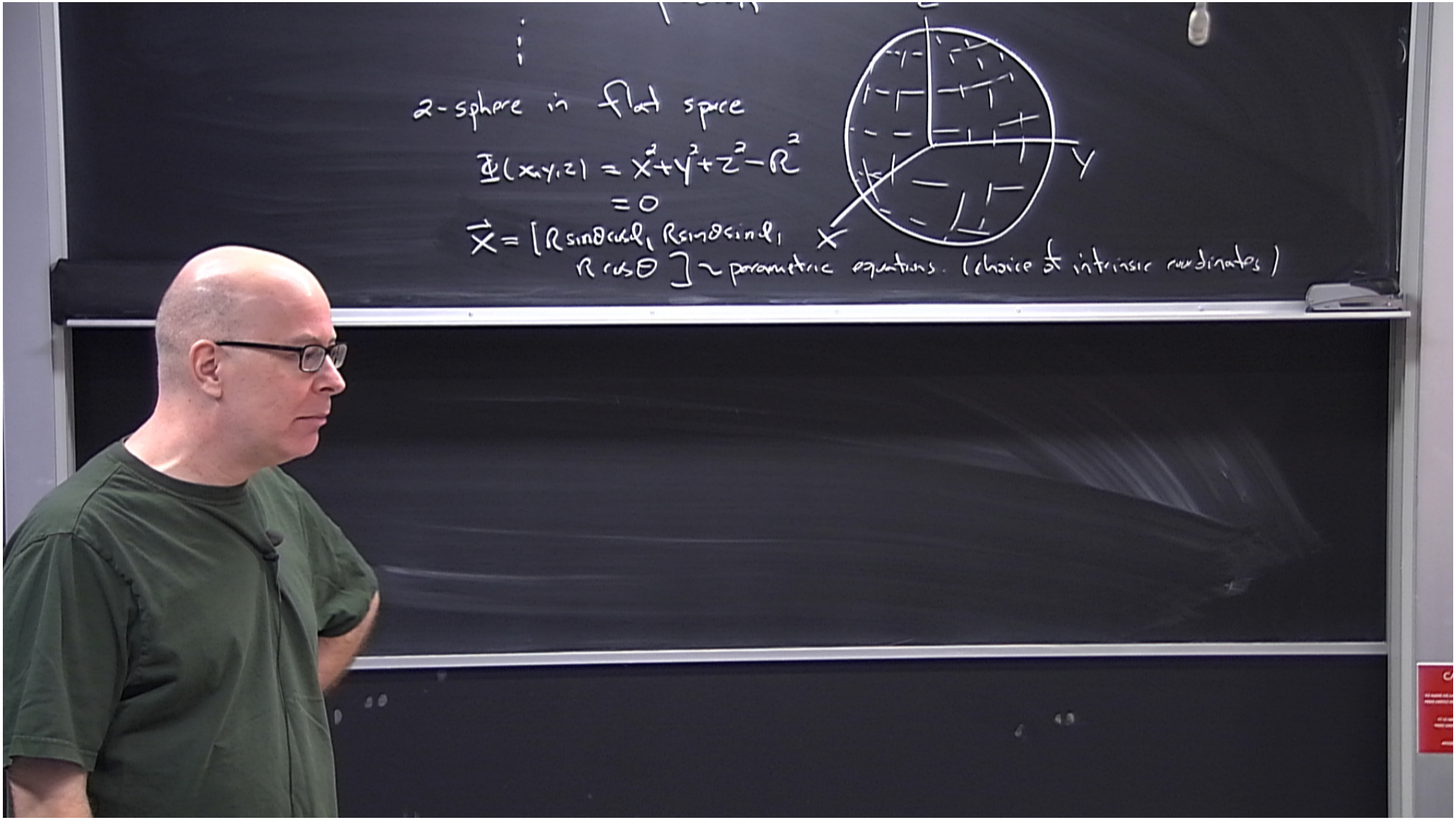

2-sphere in flat space
$\Phi(x,y,z) = x^2+y^2+z^2-R^2$
$= 0$
Z
Y
X

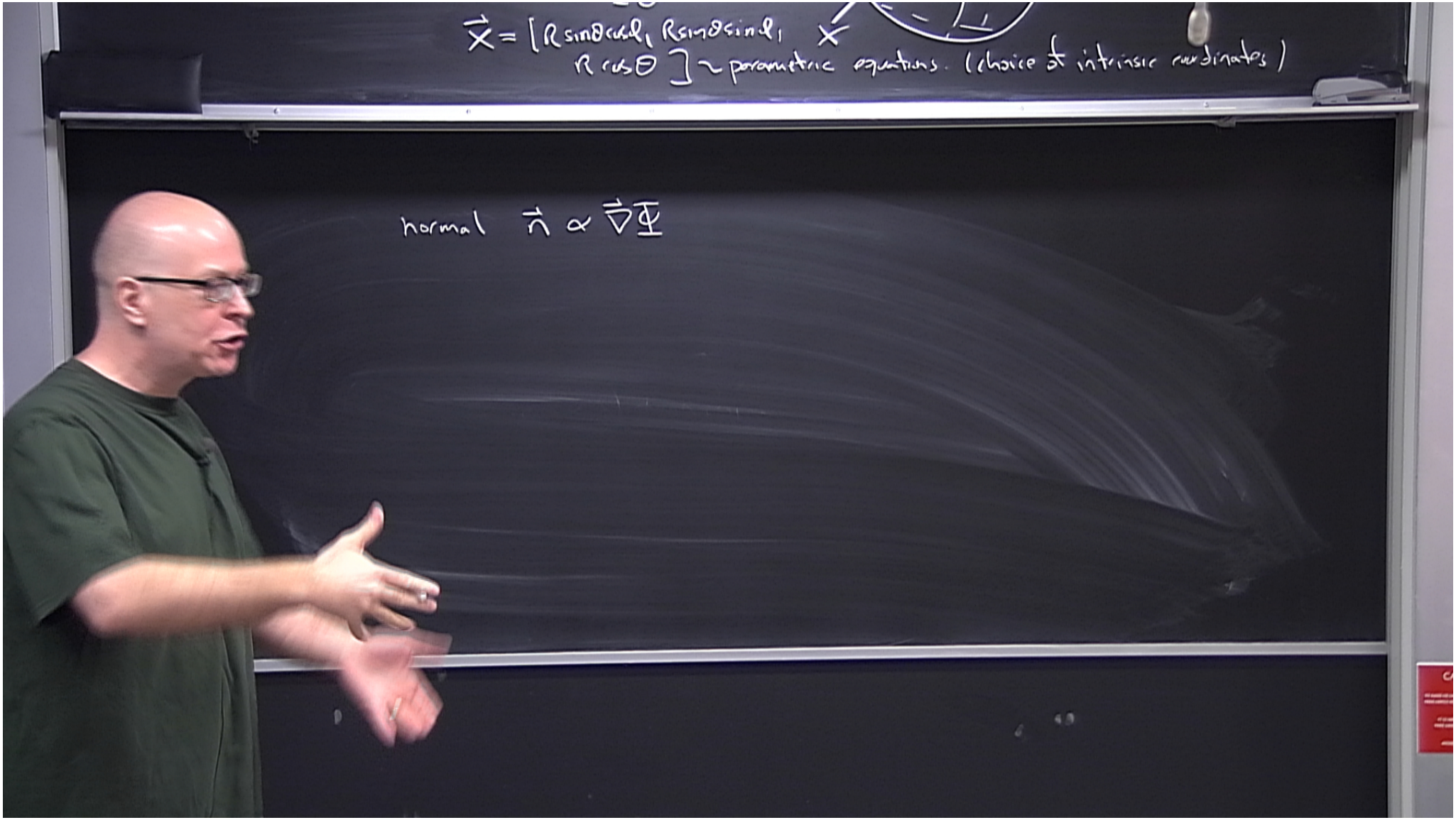

2-sphere in flat space
$\Phi(x,y,z) = x^2+y^2+z^2-R^2$
$= 0$
$\vec{x} =$
Y
X

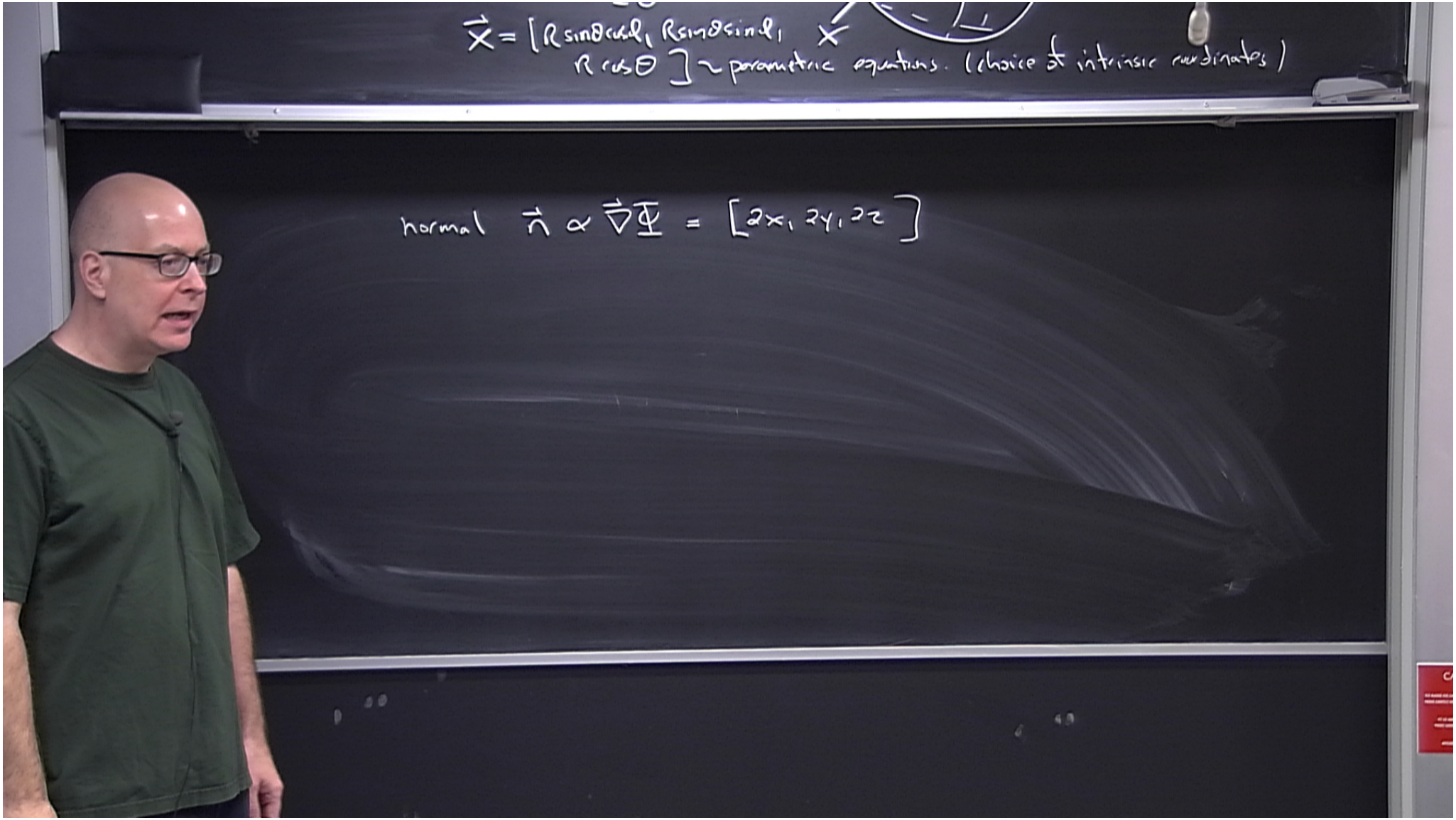

2-sphere in flat space
$= X^2+Y^2+Z^2-R^2$
$= 0$
Y
X

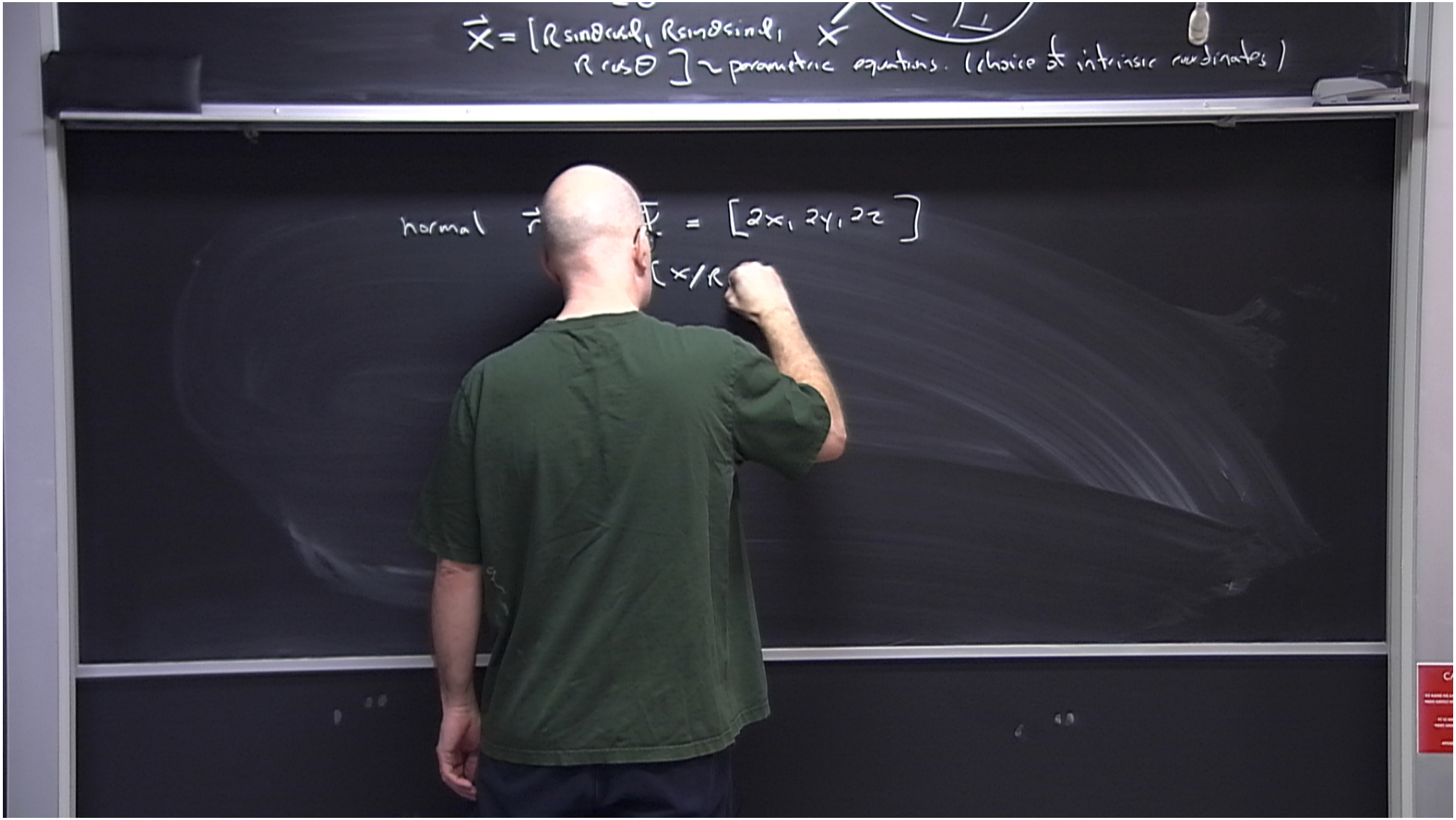

2-sphere in flat space
$\Phi(x,y,z) = x^2+y^2+z^2-R^2$
$= 0$
$\vec{x} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,$
$R\cos\theta]$ ~ parametric equations.
Y
X

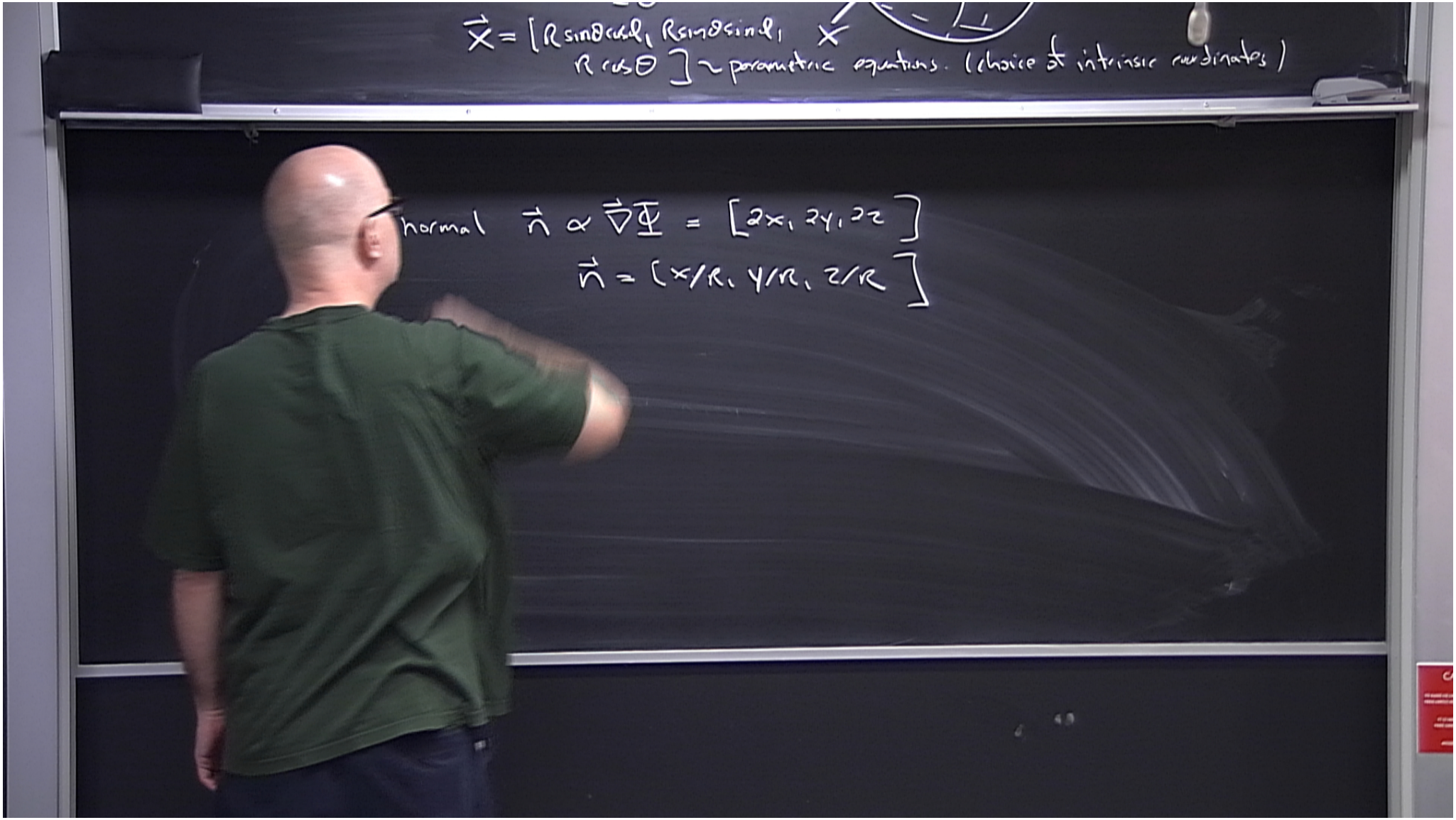

2-sphere in flat space
$\Phi(x,y,z) = x^2+y^2+z^2-R^2$
$= 0$
$\vec{X} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,$
$R\cos\theta\ ]$ ~ parametric
Y

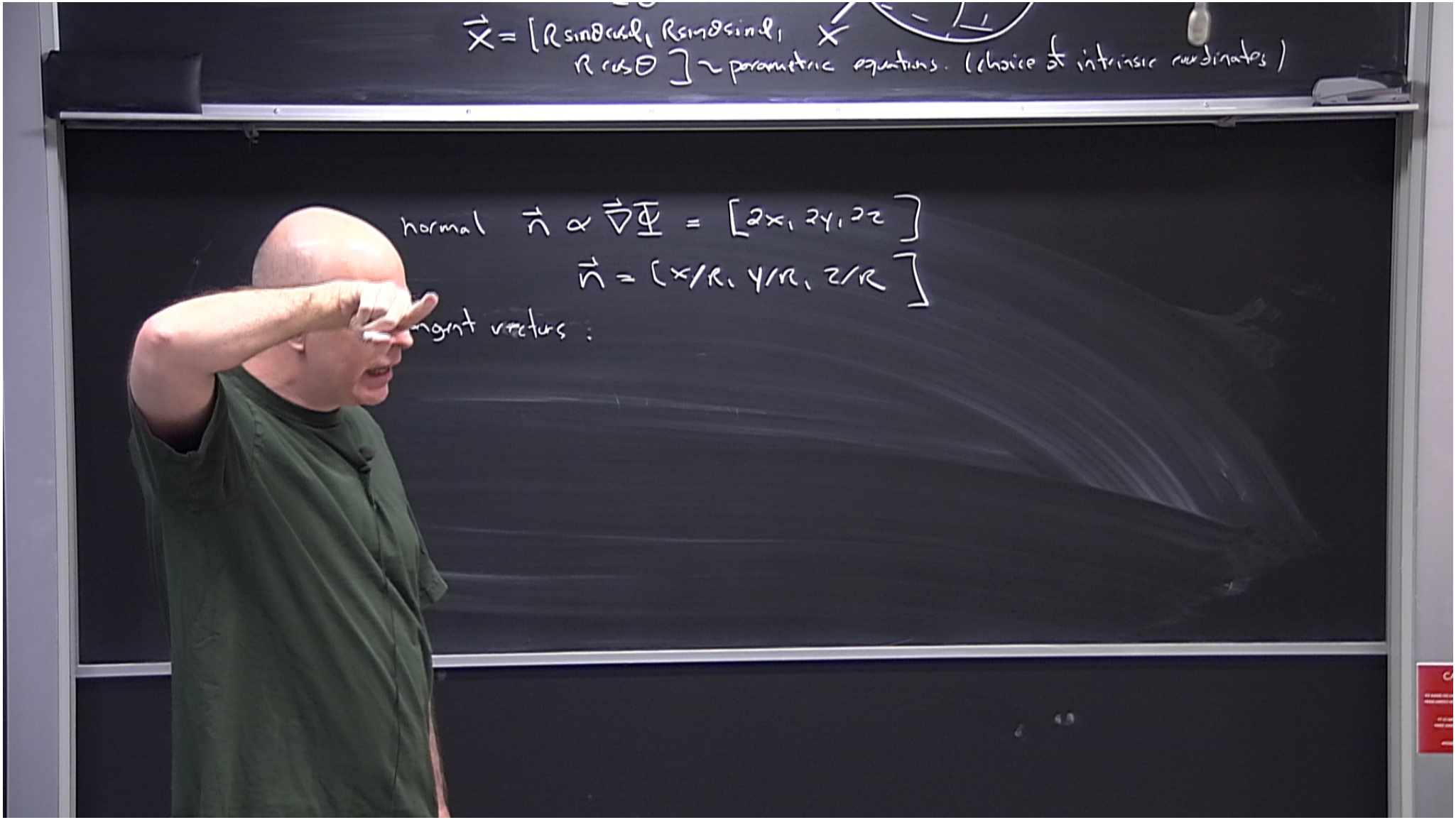

2-sphere in flat space
$\Phi(x,y,z) = x^2+y^2+z^2-R^2$
$= 0$
$\vec{X} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,$
$R\cos\theta\,]$ ~ parametric equations. (choice of intrinsic coordinates)
Y
X

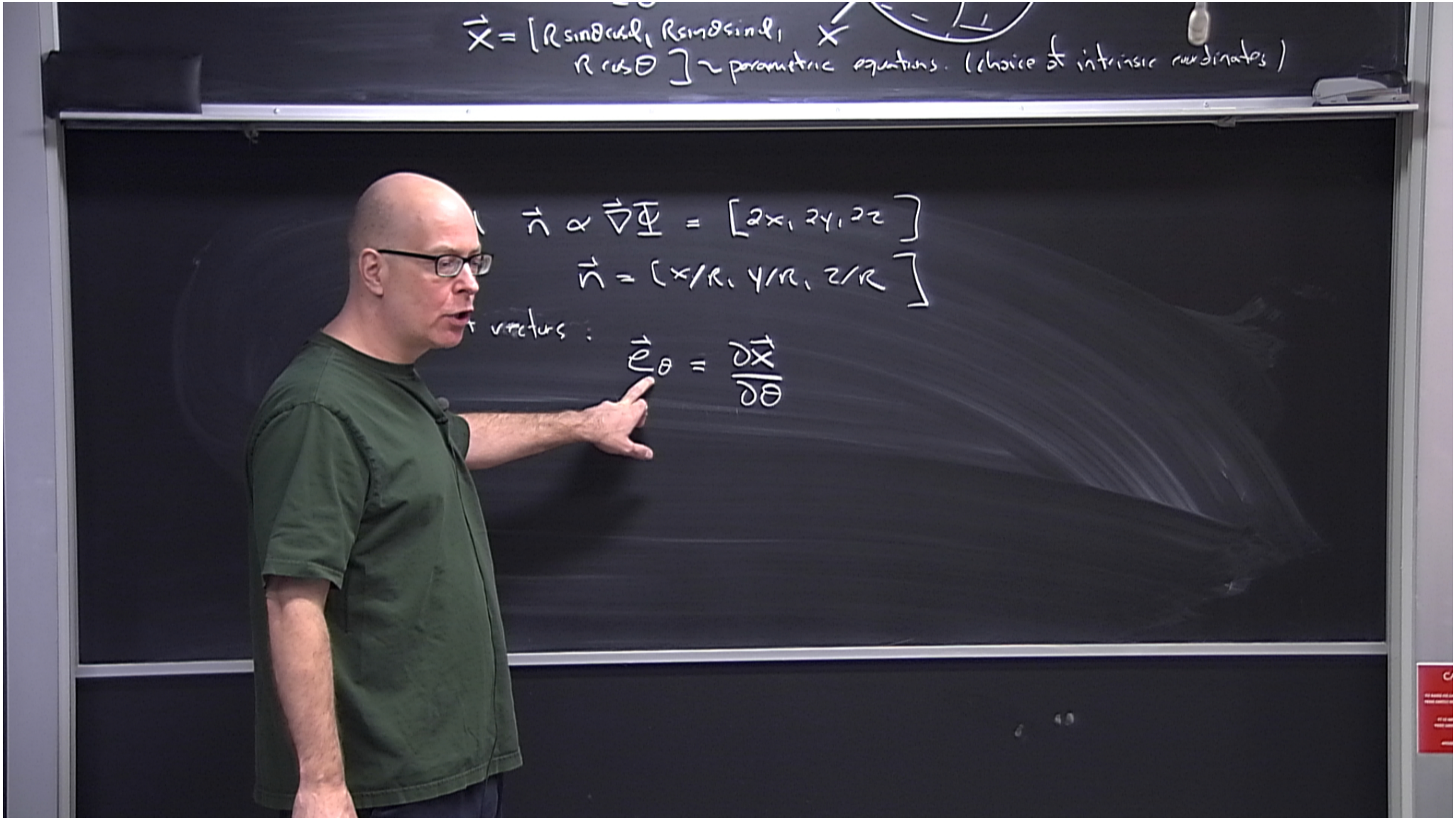

$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi,$
$R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi$

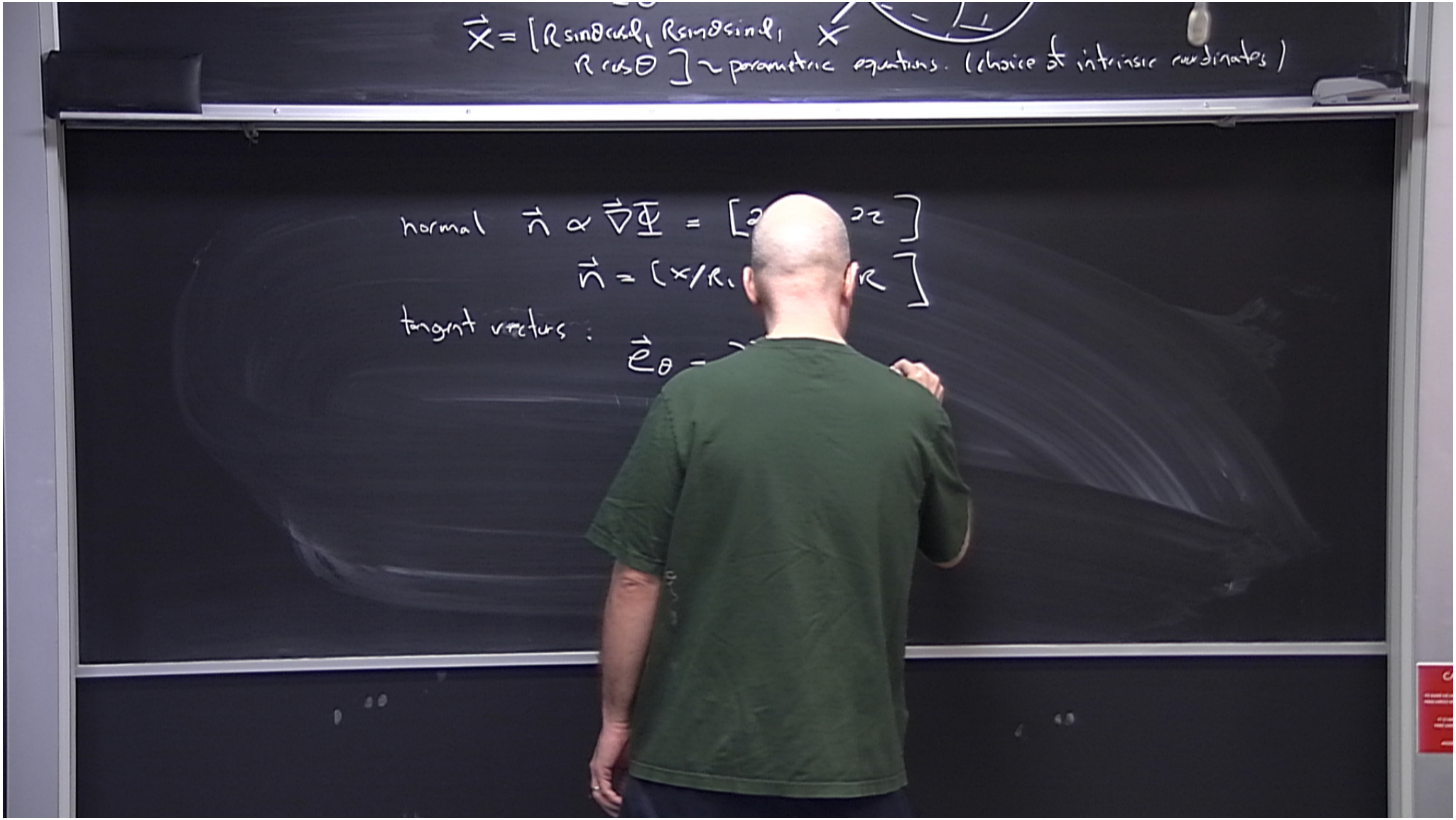

$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi, R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$

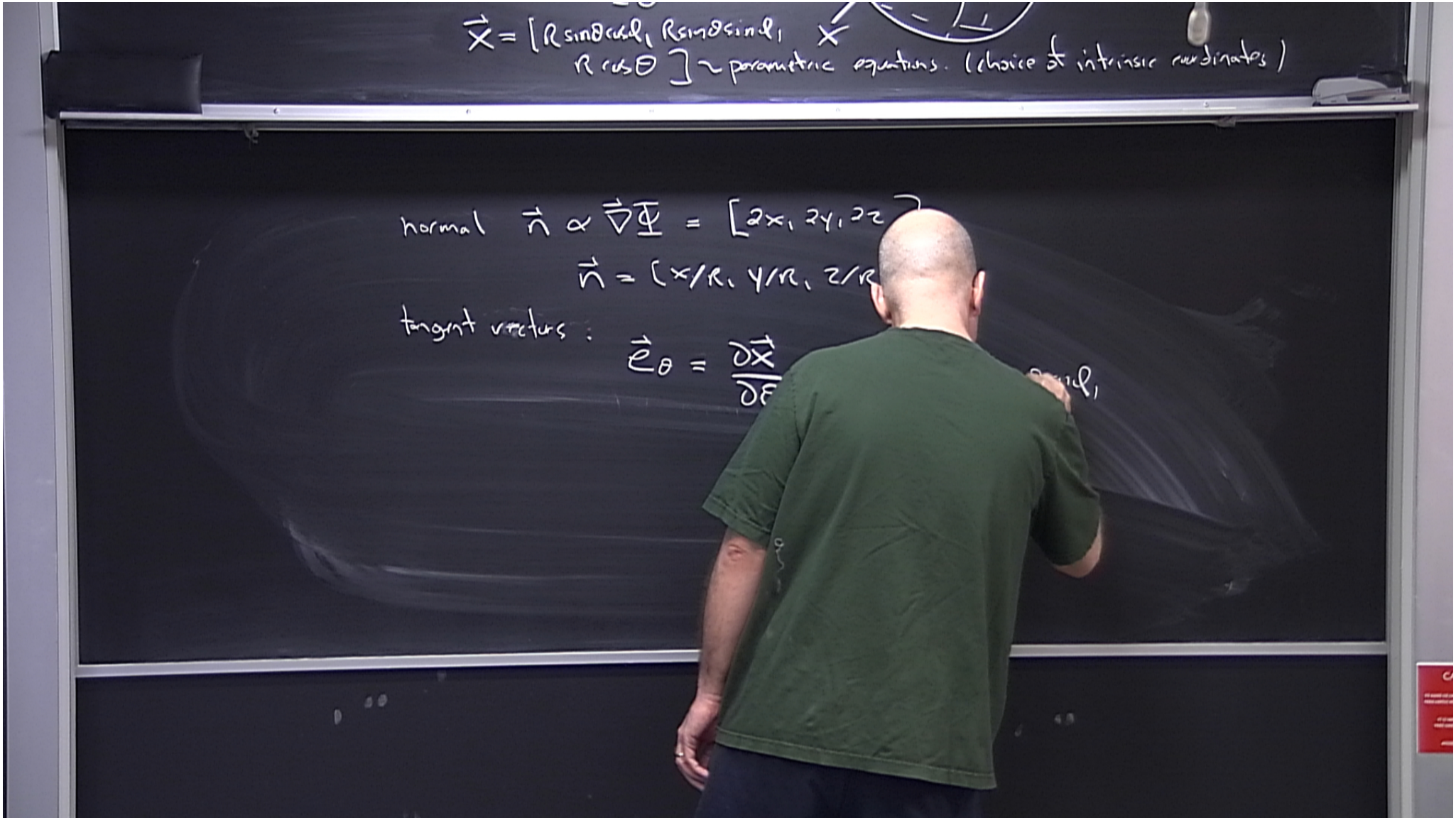

$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi, R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal
$= [2x, 2y, 2z]$
$[x/R$

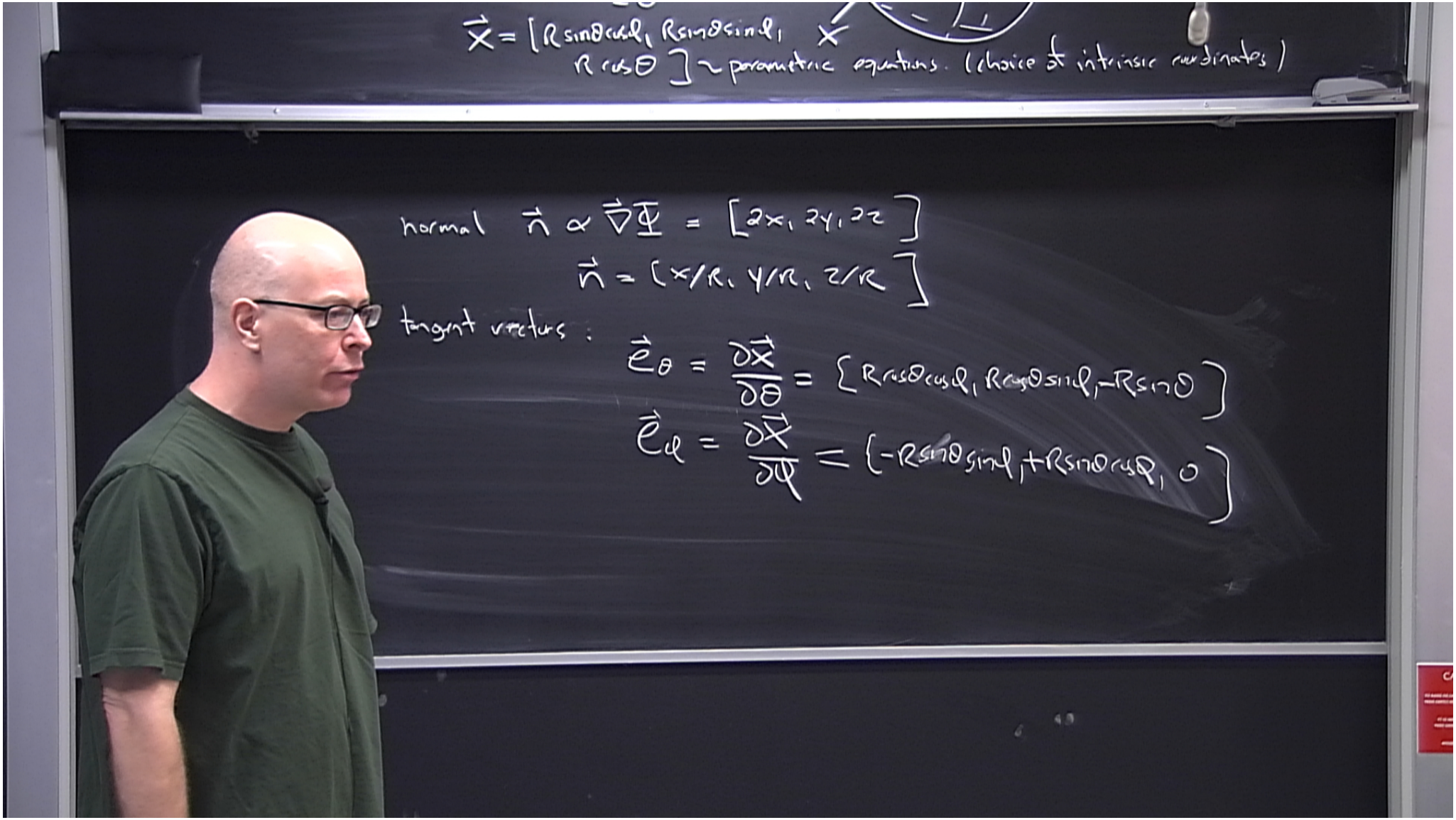

$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi, R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$
$\vec{n} = [x/R, y/R, z/R]$

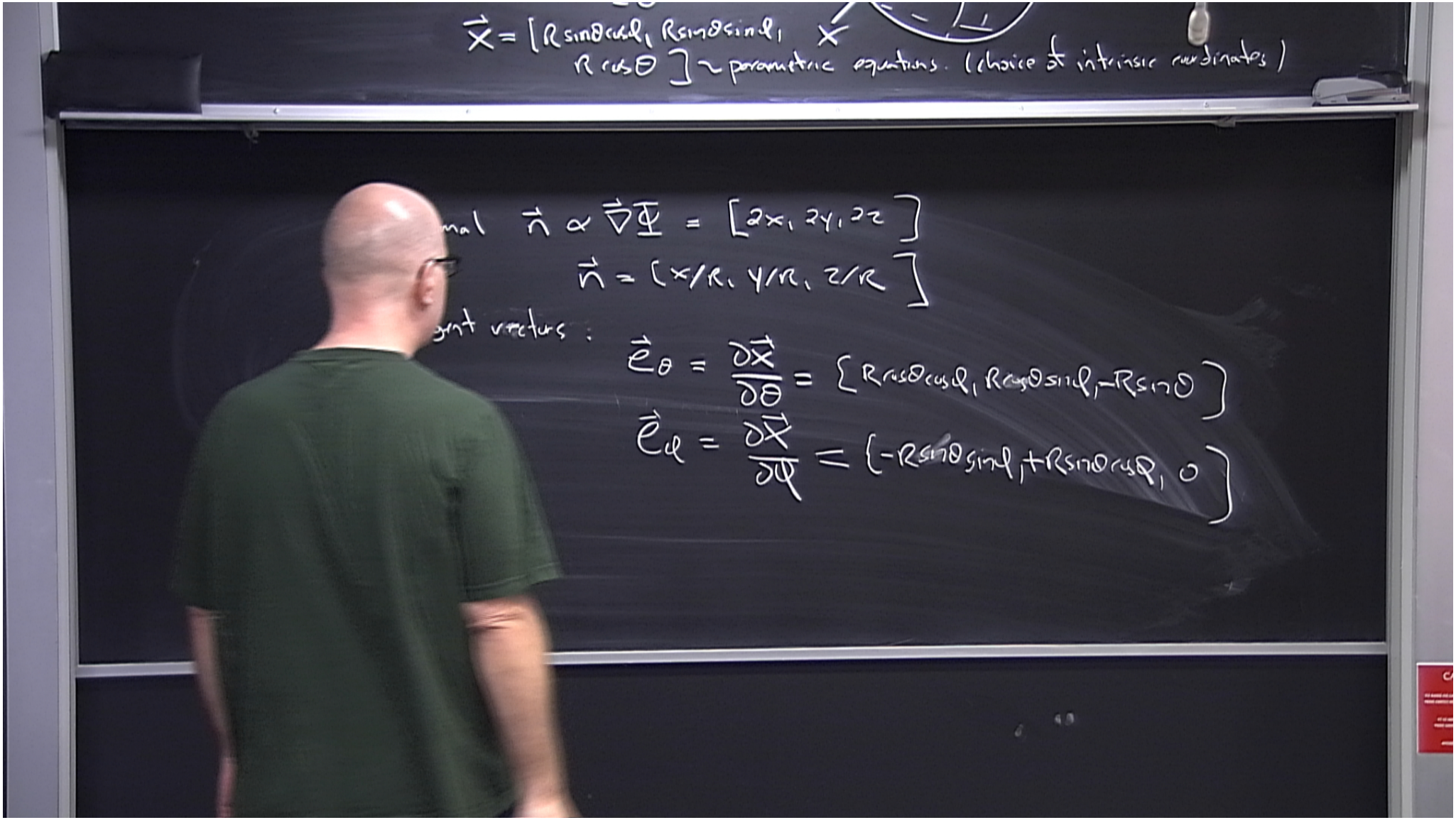

$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi, R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$
$\vec{n} = [x/R, y/R, z/R]$


$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi, R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
$\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$
$\vec{n} = [x/R, y/R, z/R]$
vectors:
$\vec{e}_\theta = \frac{\partial \vec{x}}{\partial \theta}$


$\vec{x} = [R\sin\theta\cos\phi, R\sin\theta\sin\phi,$
$R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [$
$\vec{n} = [x/R,$
tangent vectors :
$\vec{e}_\theta$


$\vec{x} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,$
$R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z$
$\vec{n} = [x/R, y/R, z/R$
tangent vectors:
$\vec{e}_\theta = \frac{\partial \vec{x}}{\partial$


$\vec{x} = [R\sin\theta\cos\varphi,\ R\sin\theta\sin\varphi,\ R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$
$\vec{n} = [x/R, y/R, z/R]$
tangent vectors:
$\vec{e}_\theta = \frac{\partial \vec{x}}{\partial \theta} = \{R\cos\theta\cos\varphi, R\cos\theta\sin\varphi, -R\sin\theta\}$
$\vec{e}_\varphi = \frac{\partial \vec{x}}{\partial \varphi} = \{-R\sin\theta\sin\varphi, +R\sin\theta\cos\varphi, 0\}$


$\vec{x} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,\ R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)
$\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$
$\vec{n} = [x/R, y/R, z/R]$
$\vec{e}_\theta = \frac{\partial \vec{x}}{\partial \theta} = [R\cos\theta\cos\phi,\ R\cos\theta\sin\phi,\ -R\sin\theta]$
$\vec{e}_\phi = \frac{\partial \vec{x}}{\partial \phi} = [-R\sin\theta\sin\phi,\ +R\sin\theta\cos\phi,\ 0]$

$\vec{x} = [R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,\ R\cos\theta]$ ~ parametric equations. (choice of intrinsic coordinates)

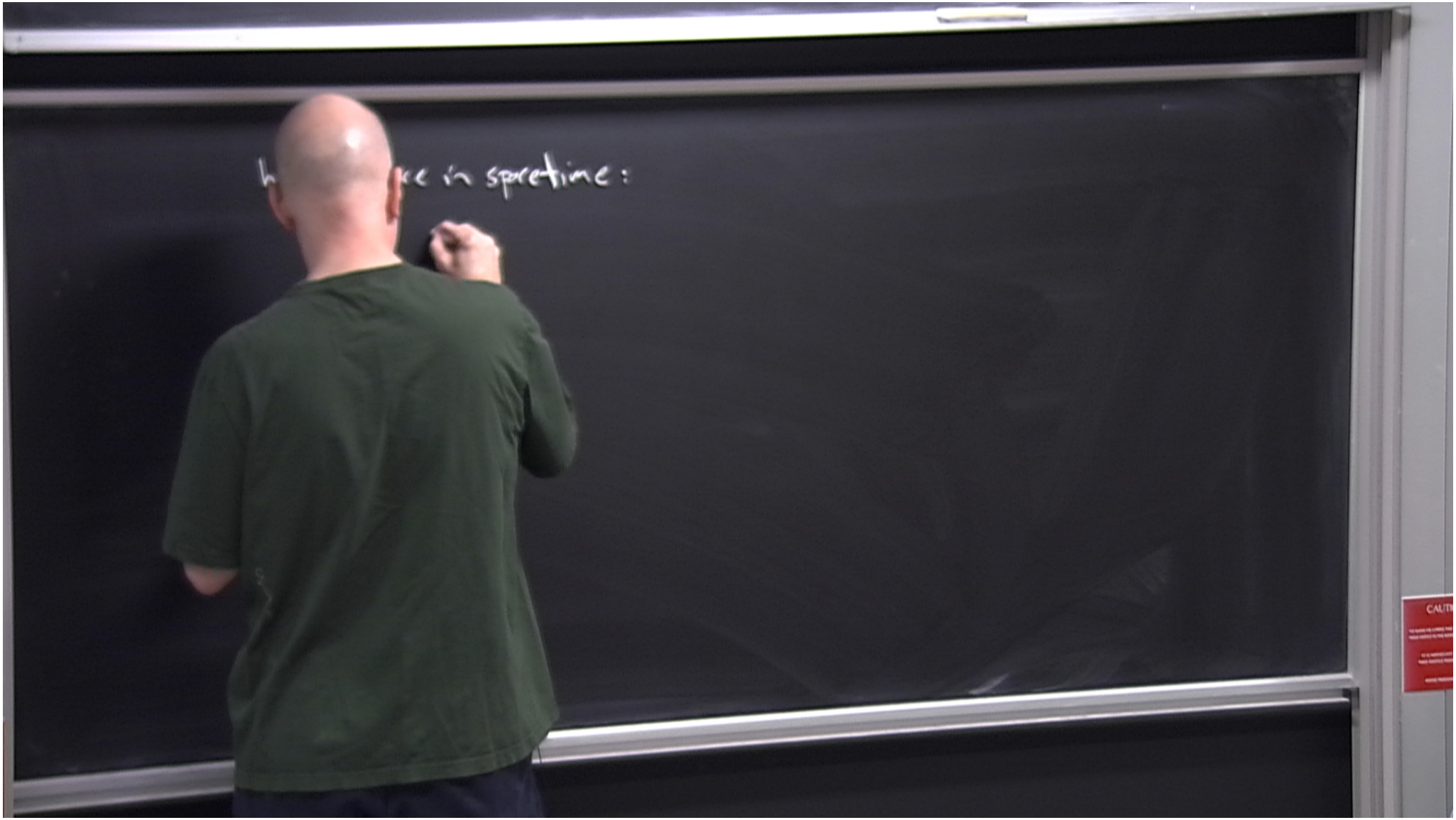
normal $\vec{n} \propto \vec{\nabla}\Phi = [2x, 2y, 2z]$

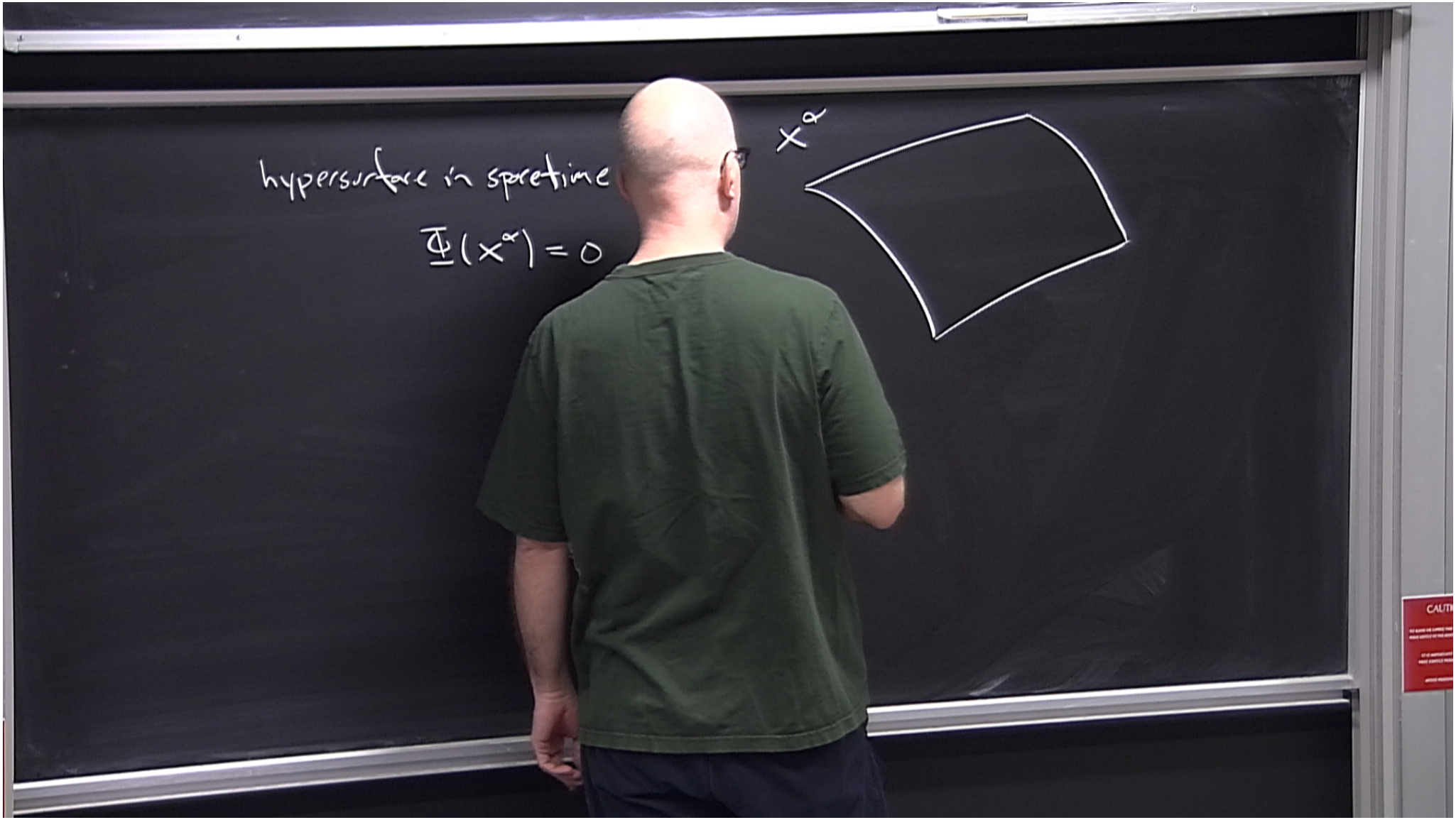
$\vec{n} = [x/R, y/R, z/R]$

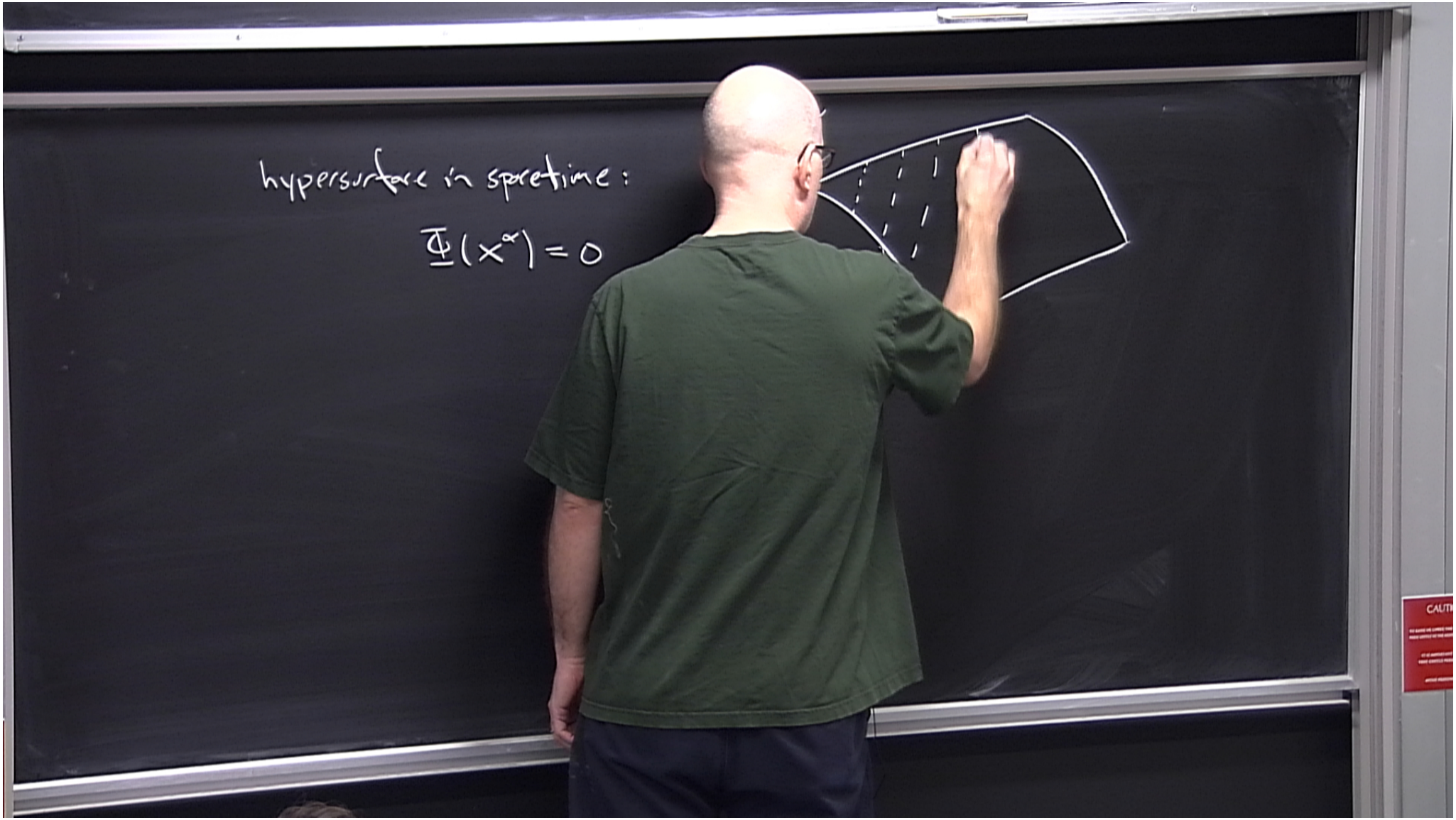
tangent vectors:

$$\vec{e}_\theta = \frac{\partial \vec{x}}{\partial \theta} = \{R\cos\theta\cos\phi,\ R\cos\theta\sin\phi, -R\sin\theta\}$$

$$\vec{e}_\phi = \frac{\partial \vec{x}}{\partial \phi} = \{-R\sin\theta\sin\phi, +R\sin\theta\cos\phi,\ 0\}$$

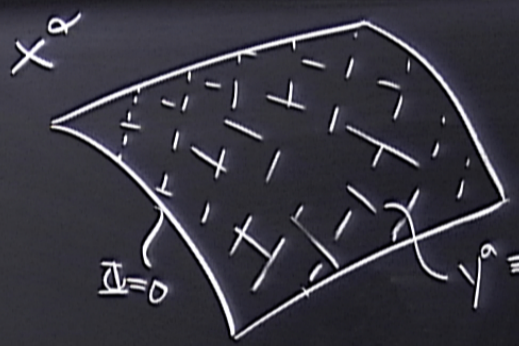

ce in spacetime:

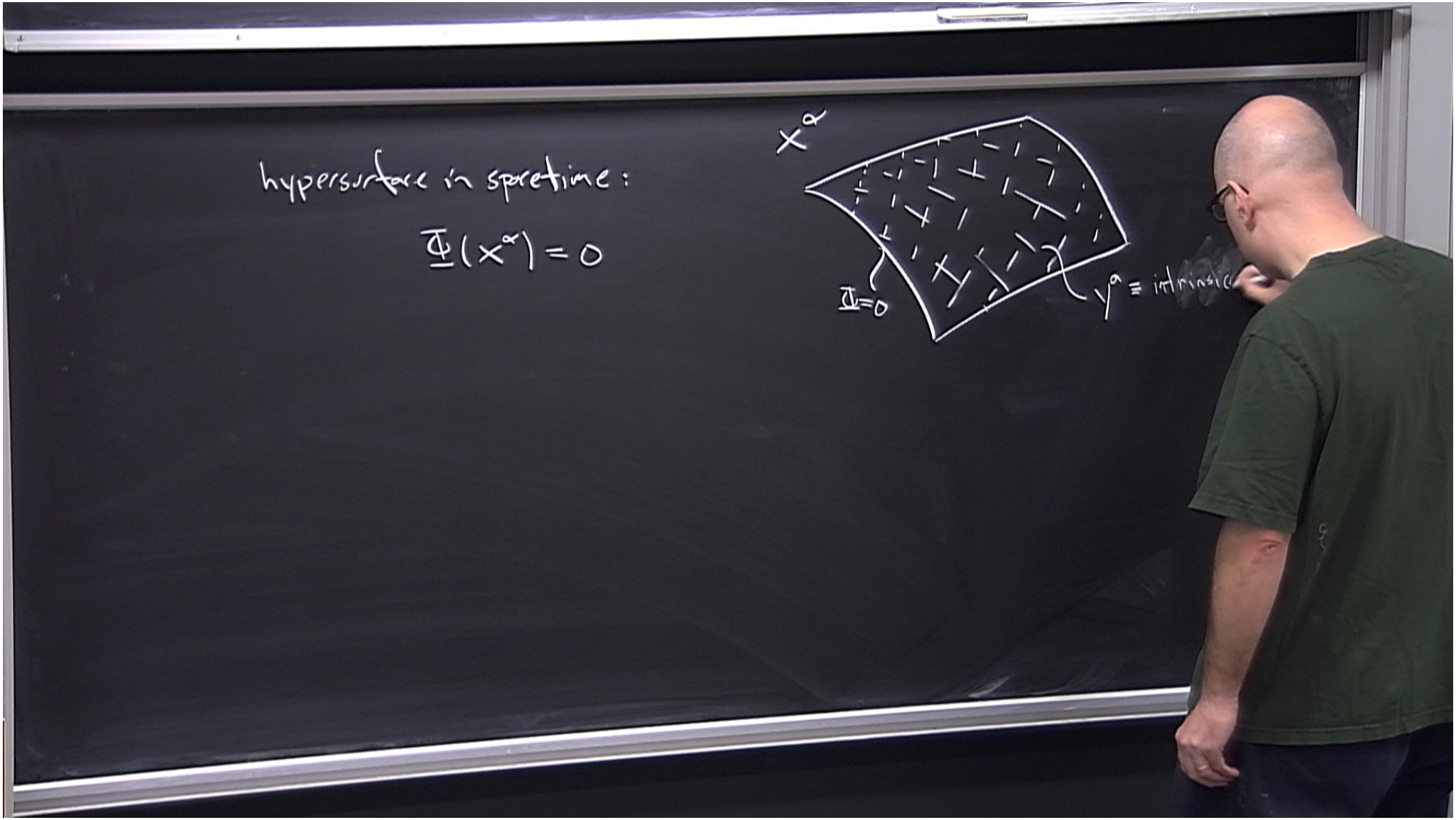

hypersurface in spacetime
$\Phi(x^\alpha) = 0$
x^α

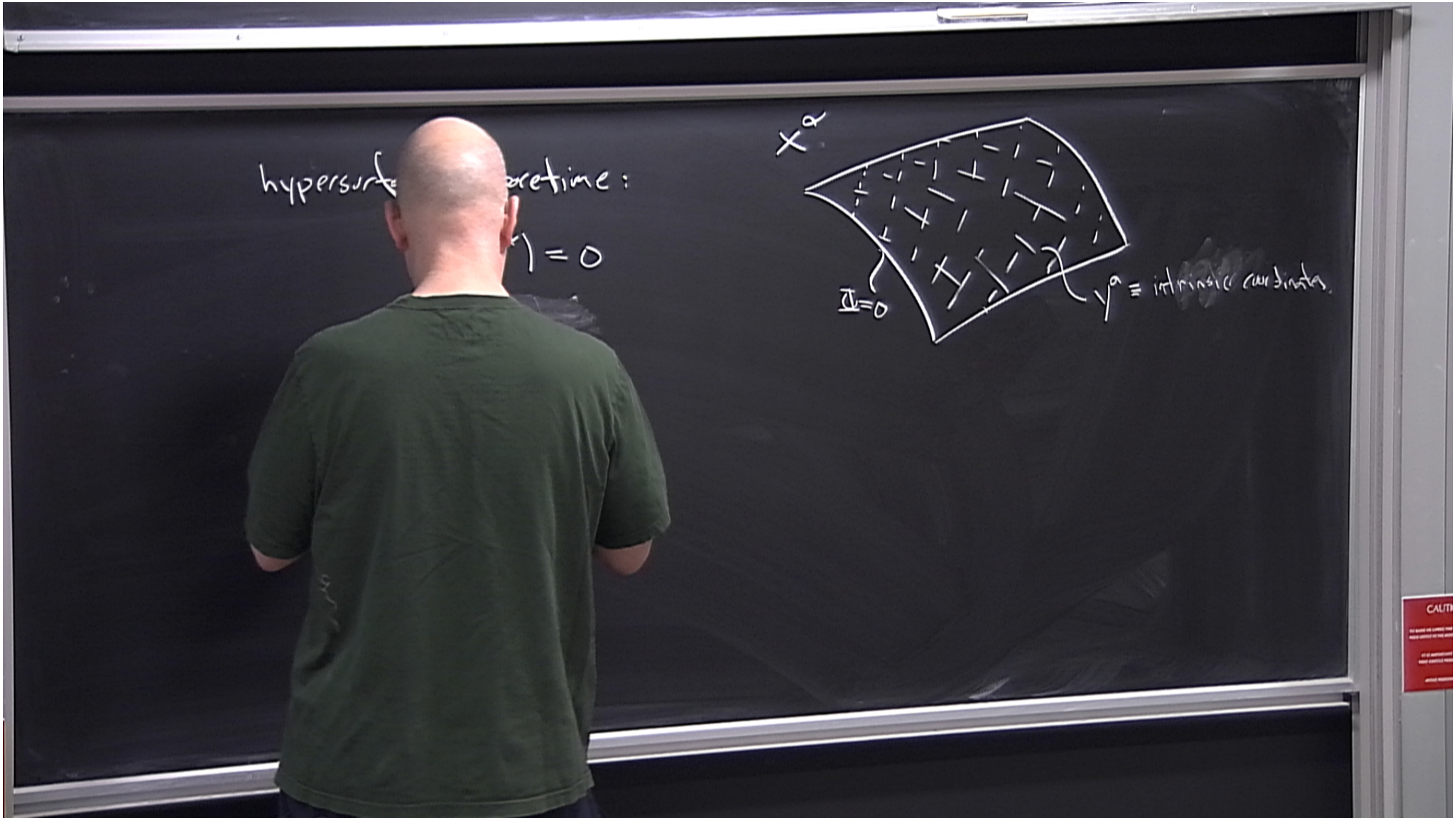

hypersurface in spacetime:
$\Phi(x^\alpha) = 0$

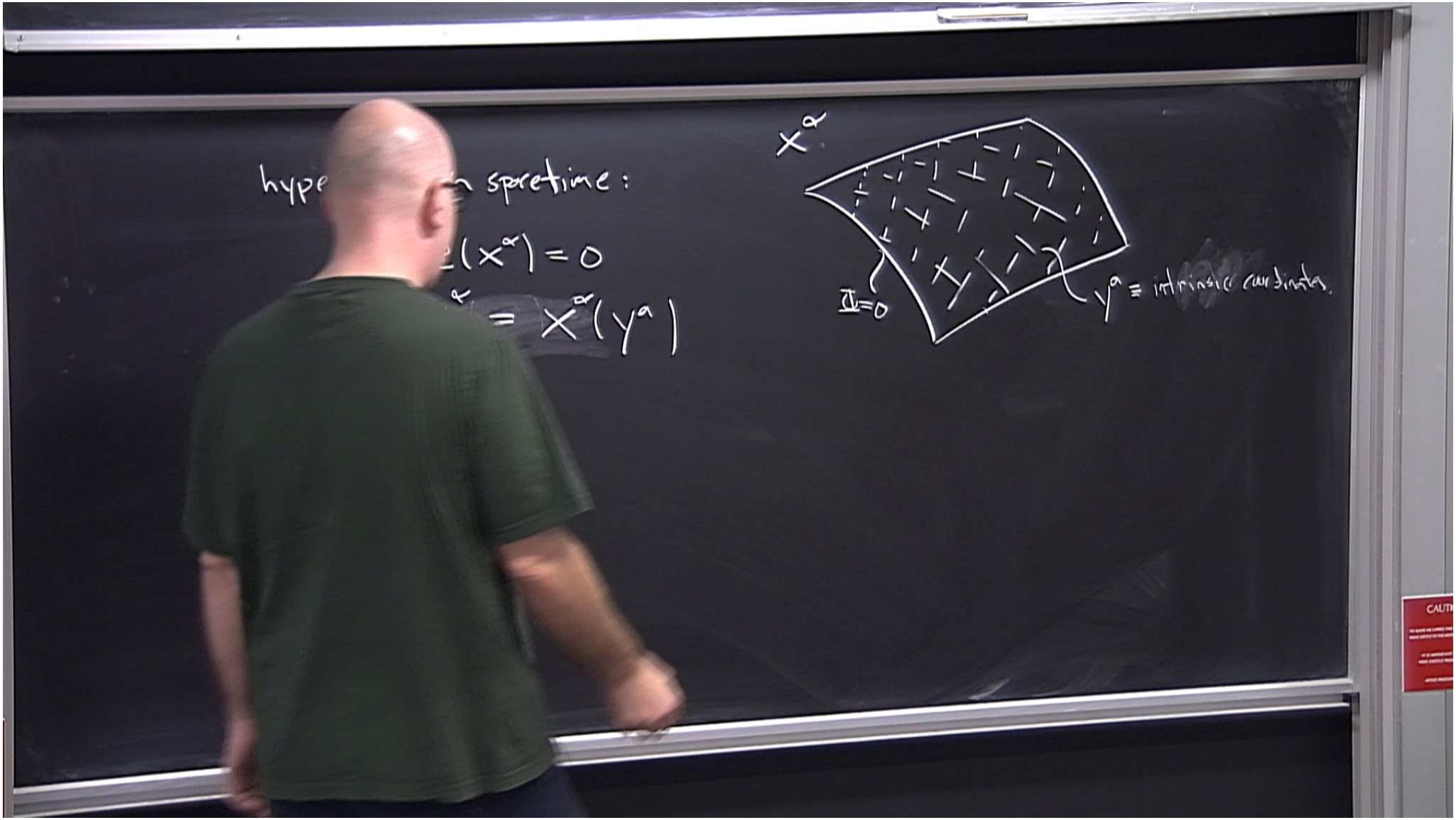
hypersurface in spacetime:

$\Phi(x^\alpha) = 0$




hypersurface in spacetime:
$\Phi(x^\alpha) = 0$
x^α
$\Phi = 0$
$y^a \equiv$ intrinsic

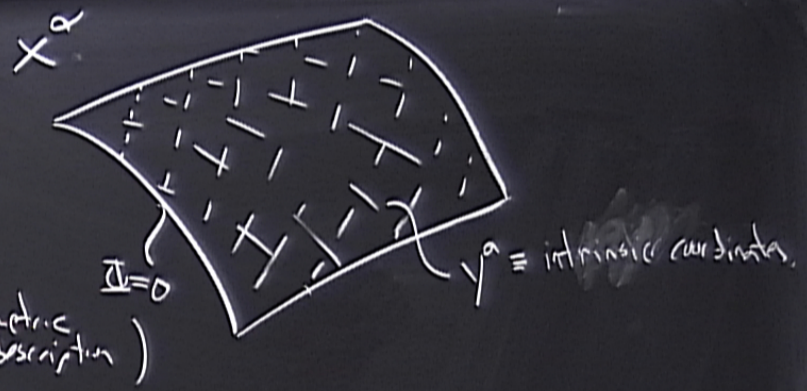

hypersurf
ocetime :
) = 0
x^α
$\Phi = 0$
$y^a \equiv$ intrinsic coordinates


hype
spacetime:
$(x^a) = 0$
$= X^a(y^a)$
x^a
$\Phi = 0$
y^a = intrinsic coordinates

hypersurface in spacetime:

$\Phi(x^\alpha) = 0$

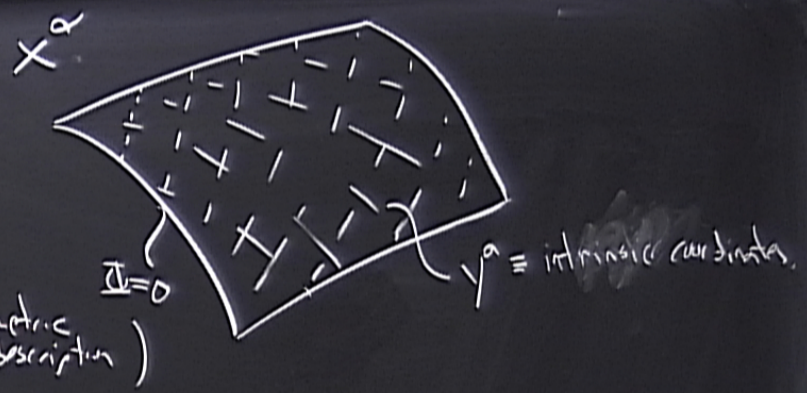
$x^\alpha = X^\alpha(y^a)$ (parametric description)

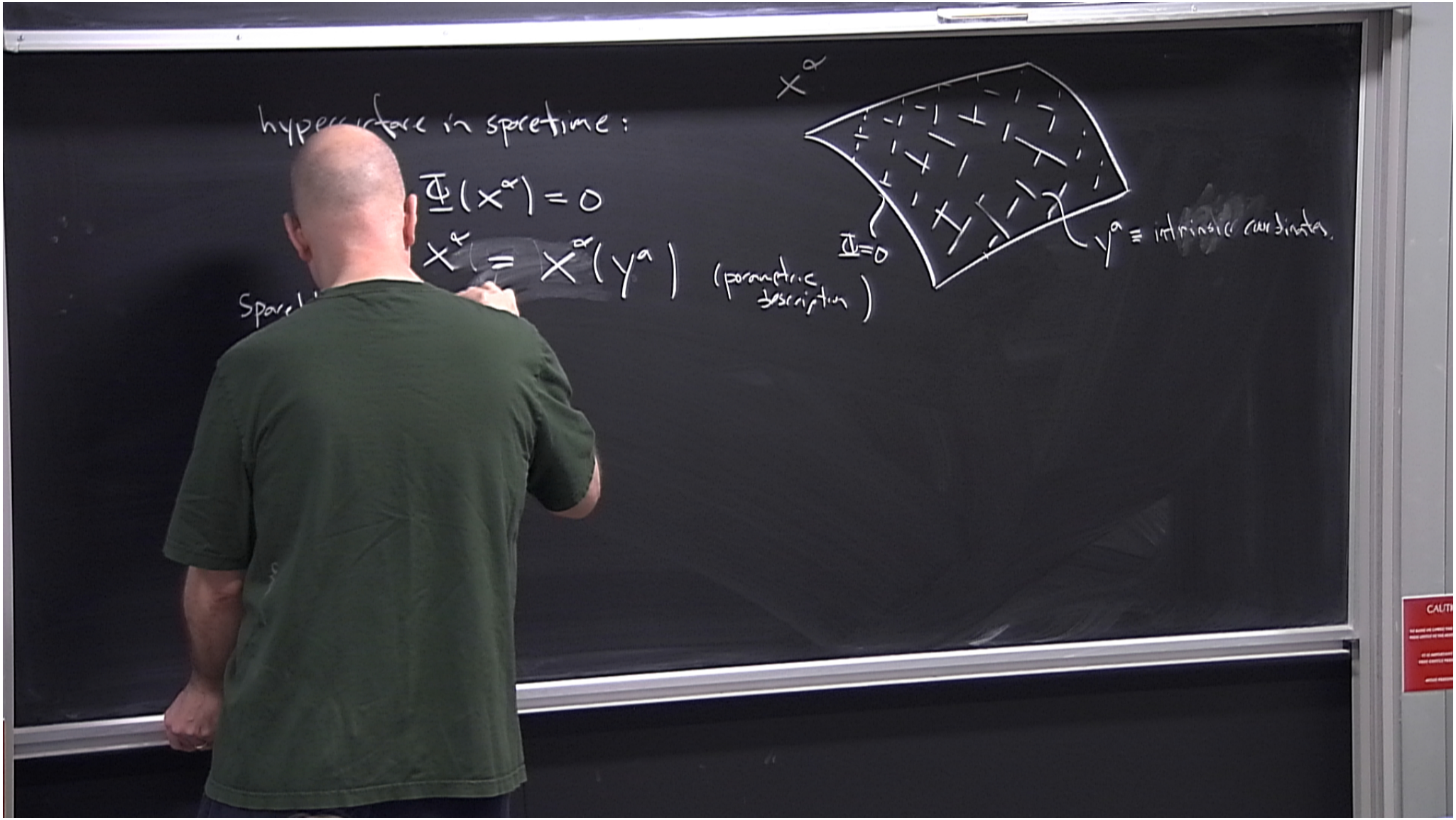


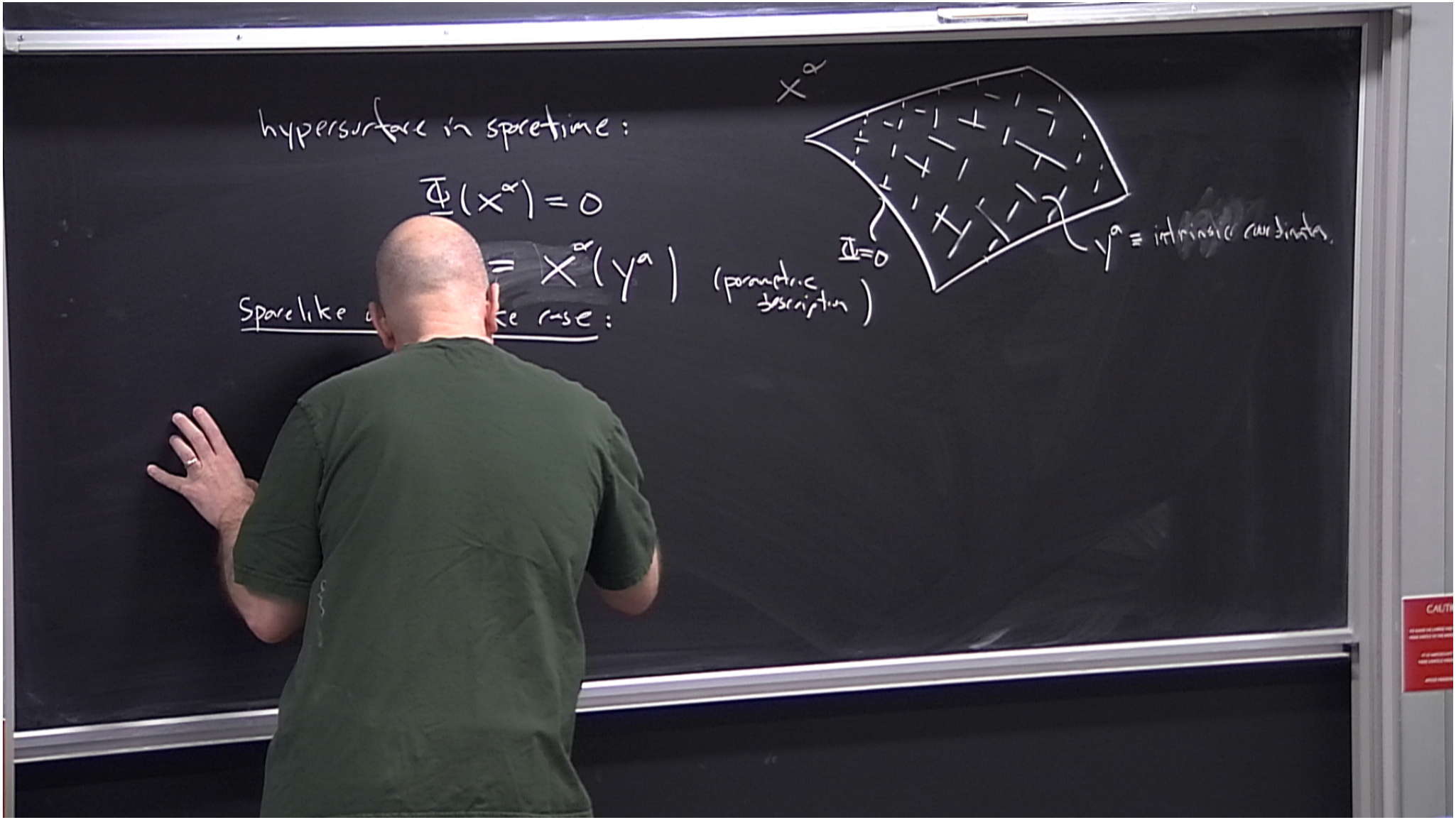
hypersurface in spacetime:

$\Phi(x^\alpha) = 0$

$x^\alpha = X^\alpha(y^a)$ (parametric description)




hypersurface in spacetime:
$\Phi(x^\alpha) = 0$
$x^\alpha = X^\alpha(y^a)$ (parametric description)
x^α
$\Phi = 0$
y^a = intrinsic coordinates


hypersurface in spacetime:
$\Phi(x^\alpha) = 0$
$= X^\alpha(y^a)$ (parametric description)
Spacelike
case:
x^α
$\Phi=0$
$y^a \equiv$ intrinsic coordinates

hypersurface in spacetime:

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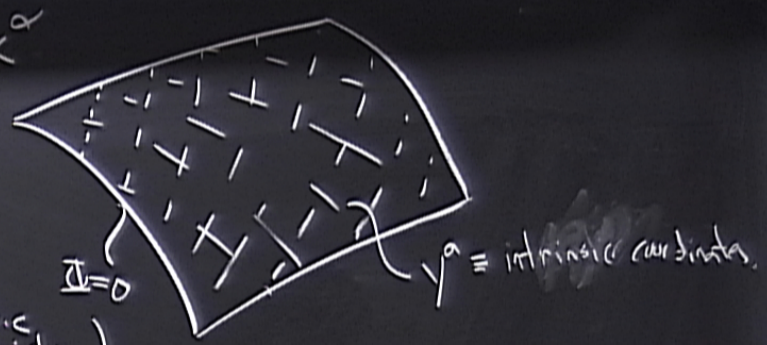


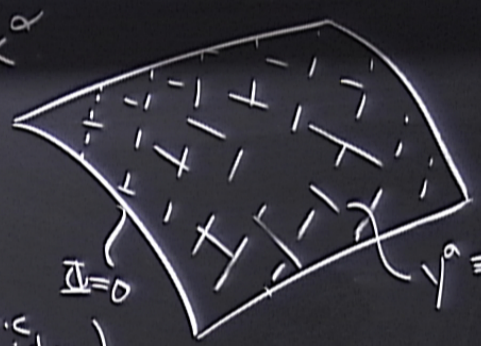
Spacelike or timelike case:

$$n_\alpha \propto \partial_\alpha \Phi$$

hypersurface in spacetime:

$$\Phi(x^\alpha) = 0$$

$$x^\alpha = X^\alpha(y^a) \quad \text{(parametric description)}$$



Spacelike or timelike case:

$$n_\alpha \propto \partial_\alpha \Phi \Rightarrow \text{normalize.}$$

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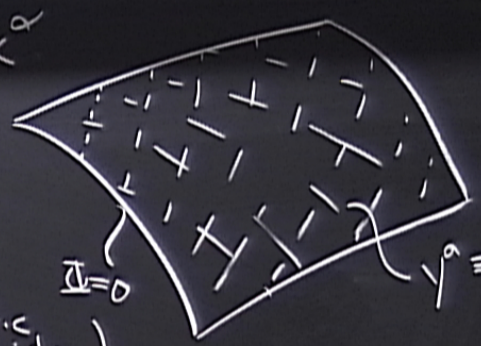

Spacelike or timelike case:

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by convention, n^α point

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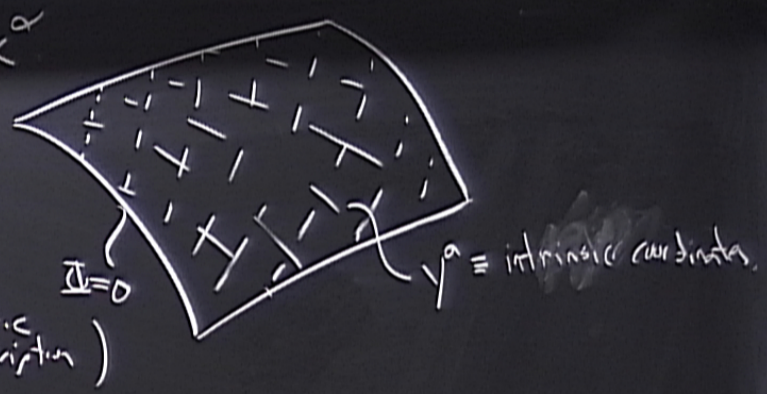

Spacelike or timelike case:

$$n_\alpha \propto \partial_\alpha \Phi \Rightarrow \text{normalize.}$$

by convention, n^α points in direction of increasing Φ

$$n^\alpha \partial_\alpha \Phi > 0$$

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Spacelike or timelike case:

$n_\alpha \propto \partial_\alpha \Phi \Rightarrow$ normalize.

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$$n^\alpha \partial_\alpha \Phi > 0$$

$$\Rightarrow \quad n_\alpha = \frac{\varepsilon \partial_\alpha \Phi}{|g^{\mu\nu}}$$

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$n^\alpha \partial_\alpha \Phi > 0$ $\qquad \varepsilon = n_\alpha n^\alpha = \begin{cases} -1 \\ \end{cases}$

$$\Rightarrow \quad n_\alpha = \frac{\varepsilon \partial_\alpha \Phi}{|g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi|}$$

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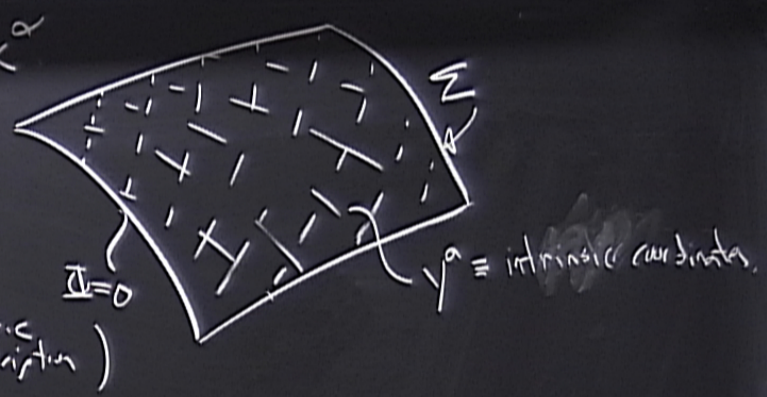

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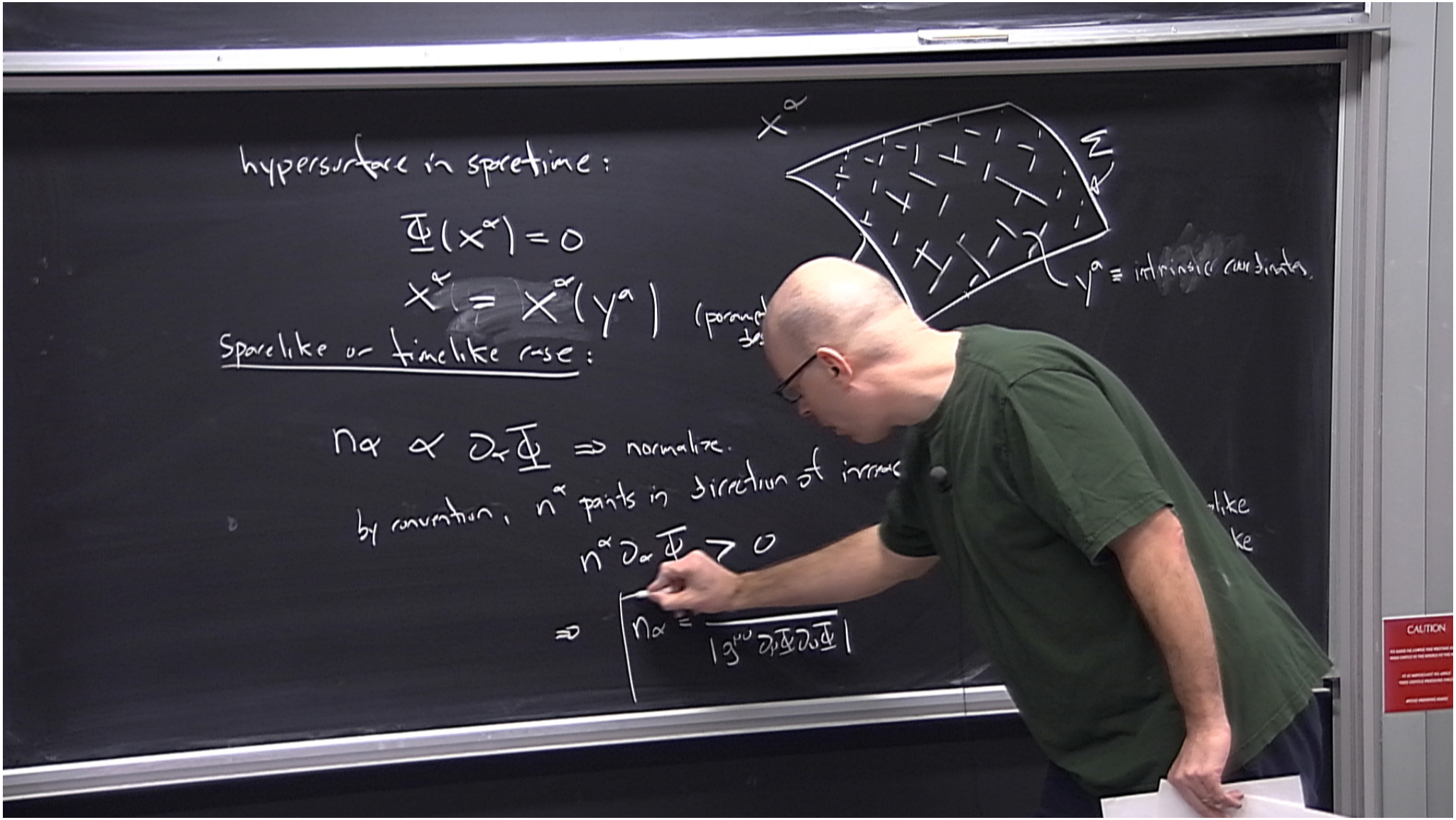
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$\Rightarrow$
$n_\alpha = \frac{\partial_\alpha \Phi}{\sqrt{|g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi|}}$
x^α
Σ
y^a = intrinsic coordinates
CAUTION

hypersurface in spacetime:

$$\Phi(x^\alpha) = 0$$

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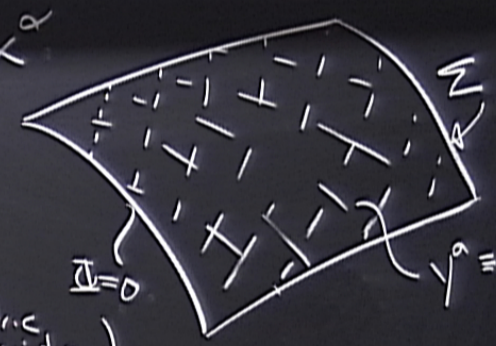

Spacelike or timelike case:

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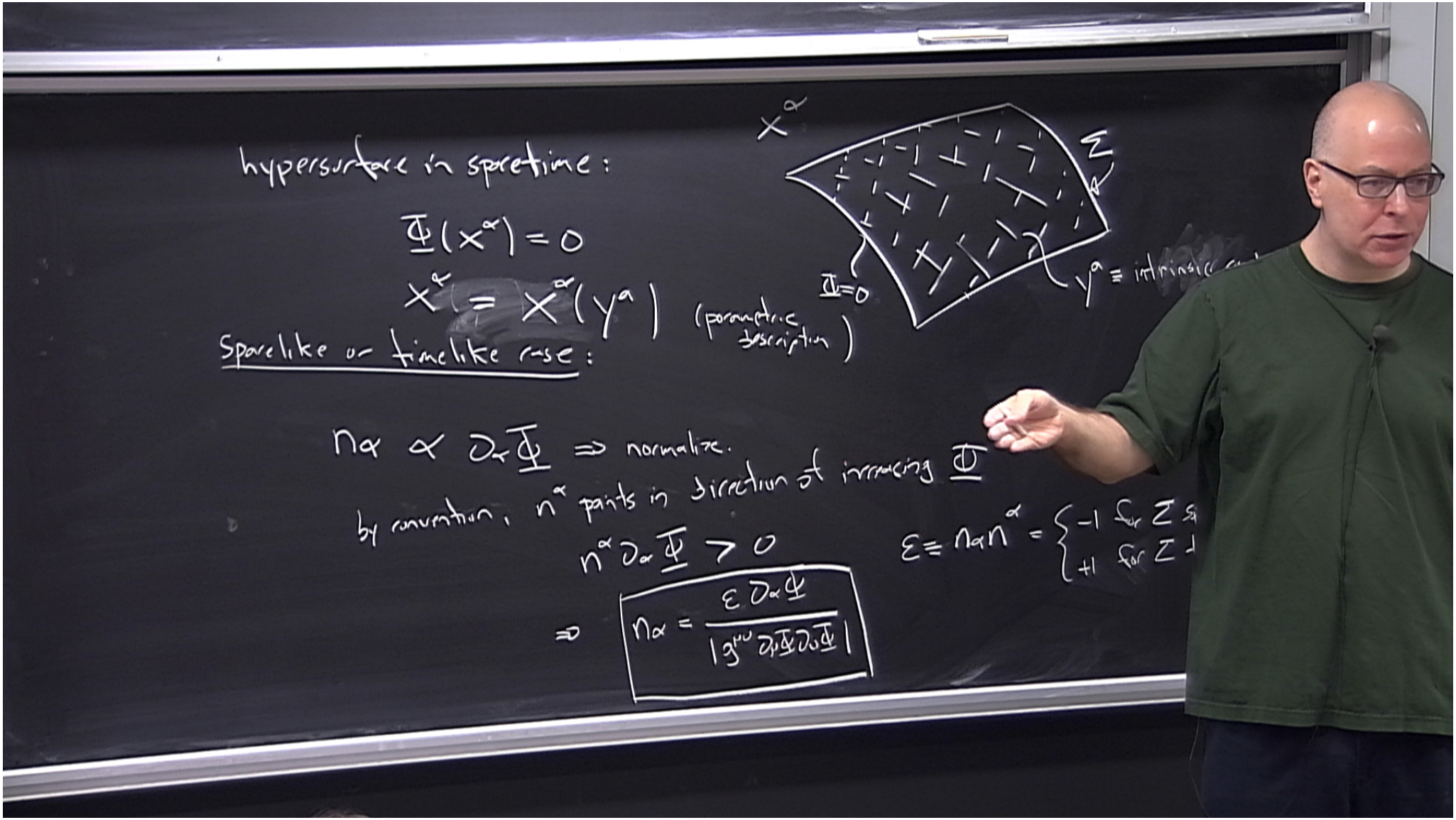
$$\Rightarrow \boxed{n_\alpha = \frac{\varepsilon \partial_\alpha \Phi}{|g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi|}}$$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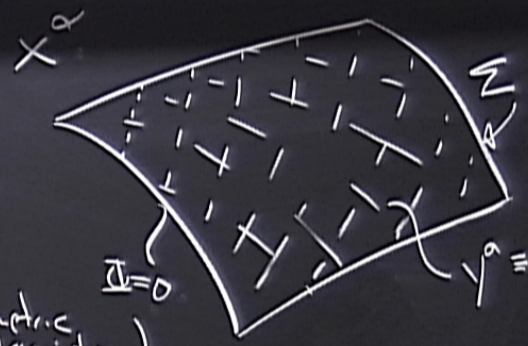

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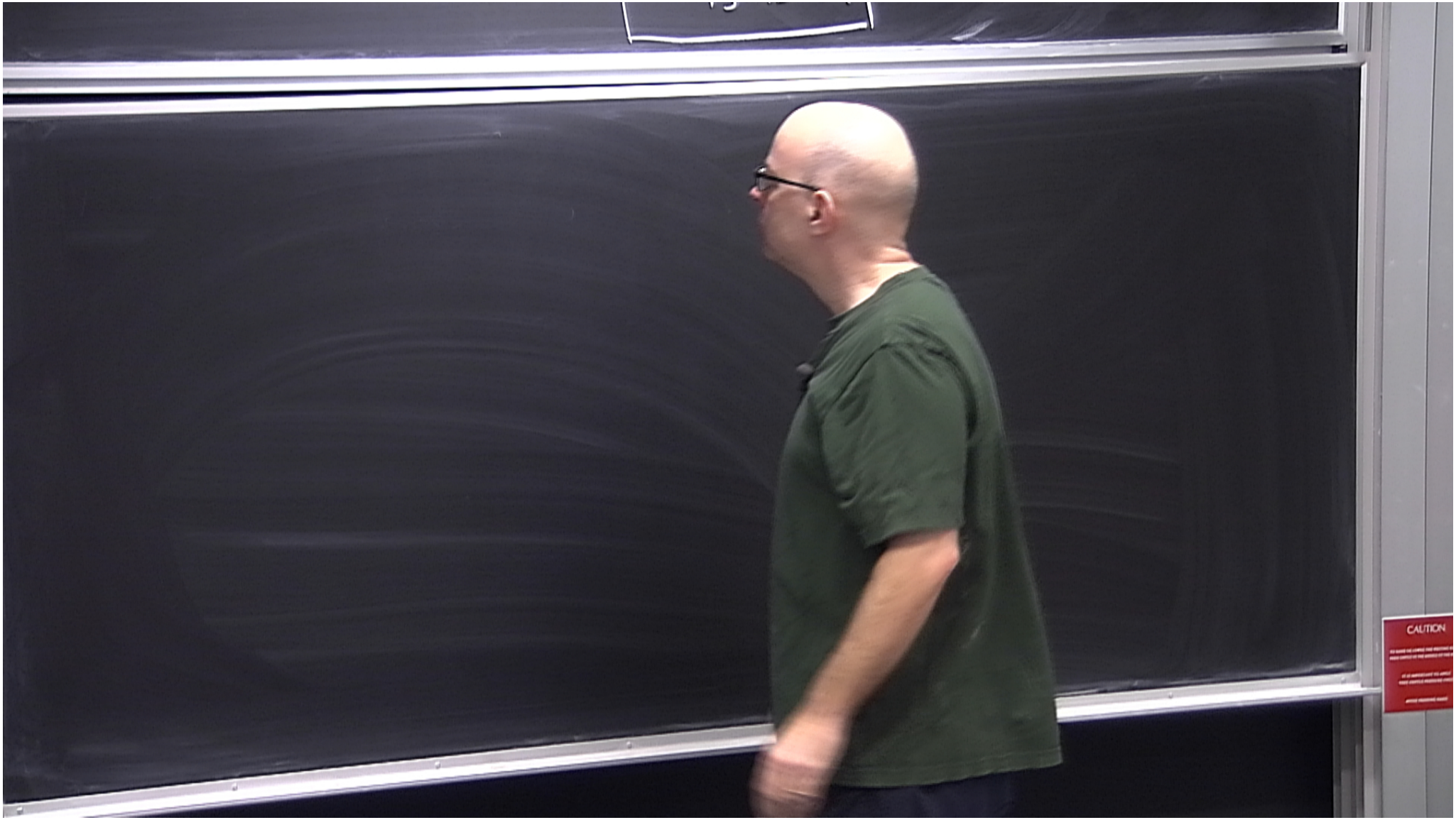
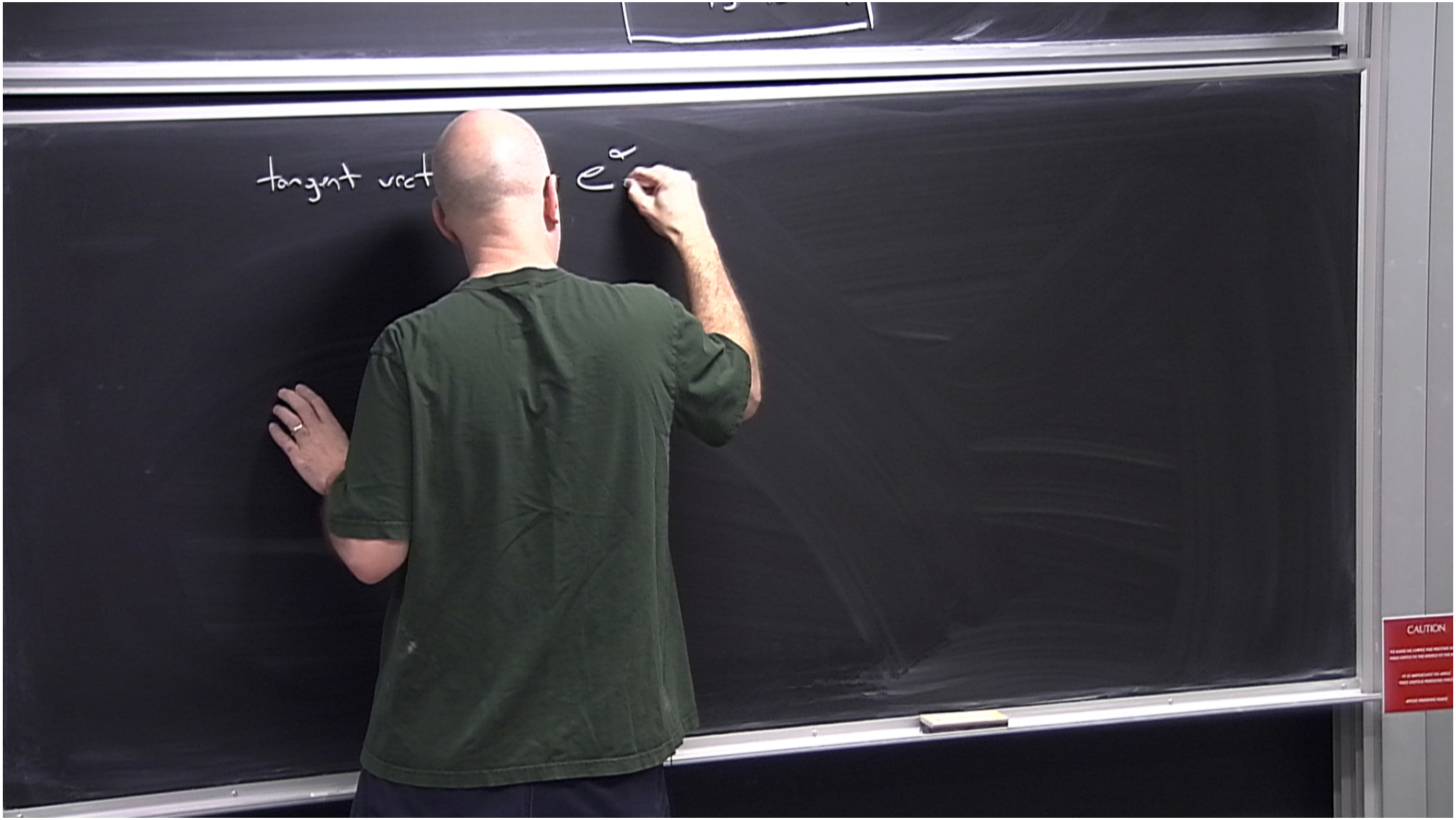
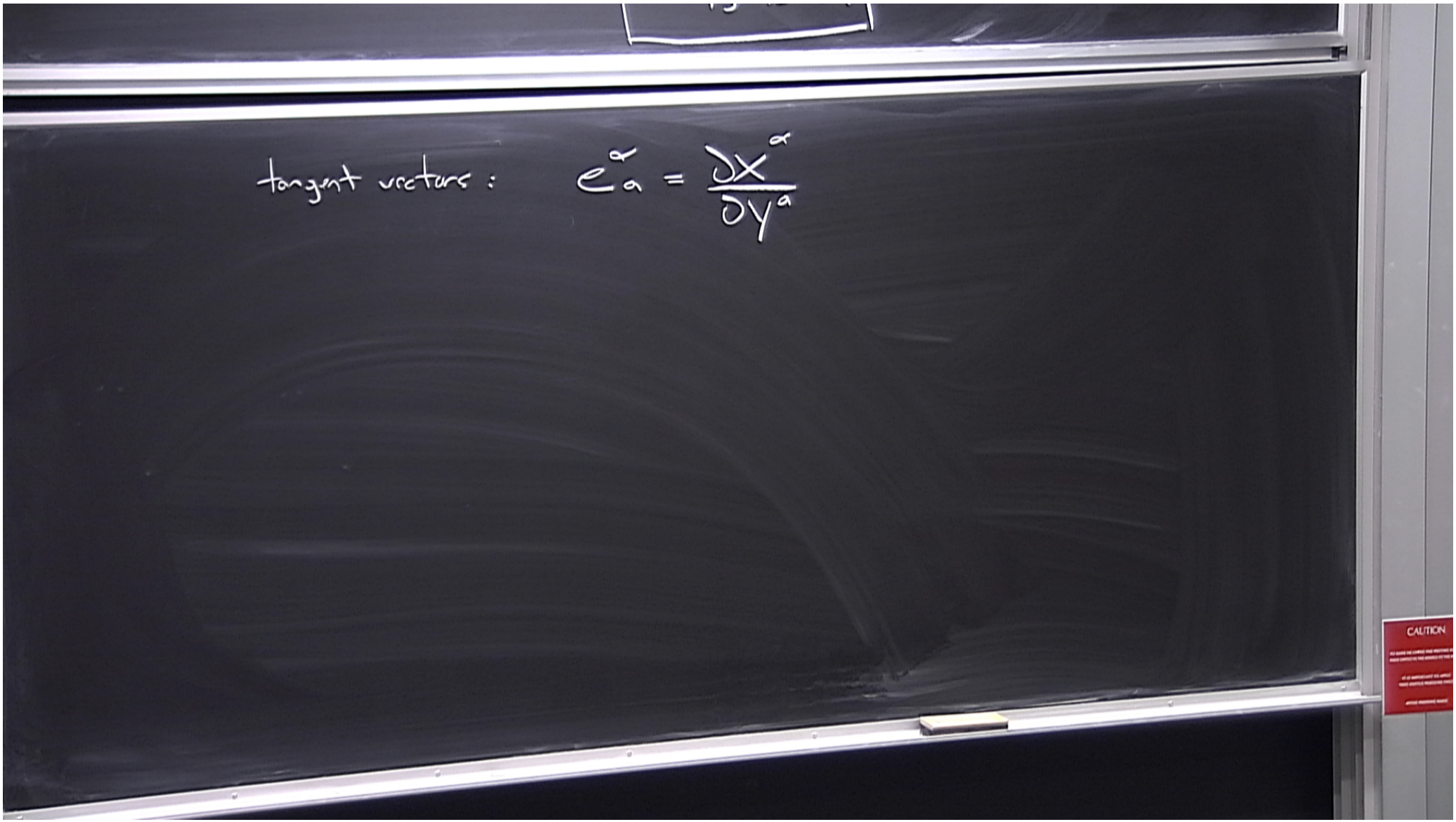
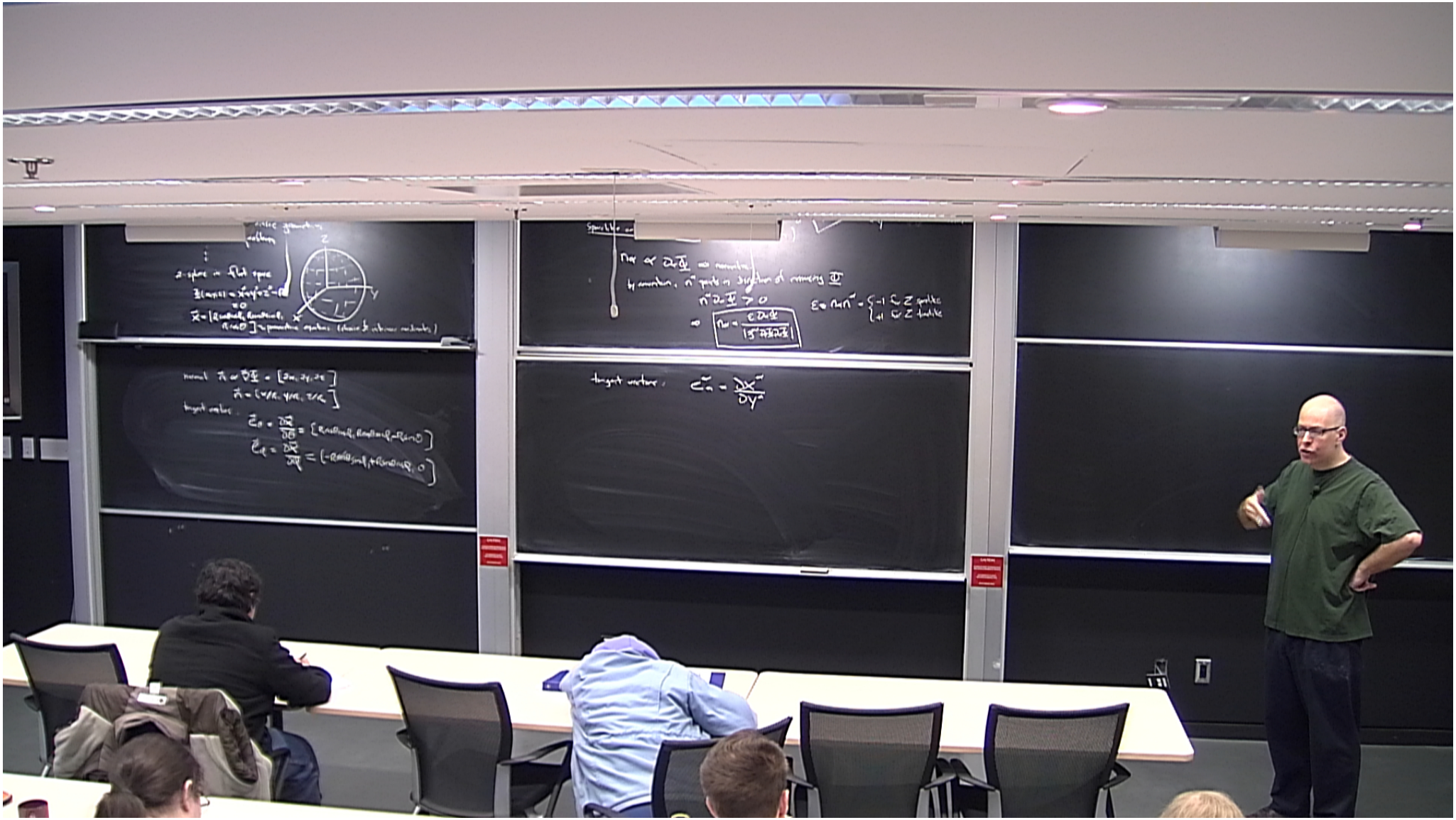
$$\Rightarrow \boxed{n_\alpha = \frac{\varepsilon \partial_\alpha \Phi}{|g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi|}}$$

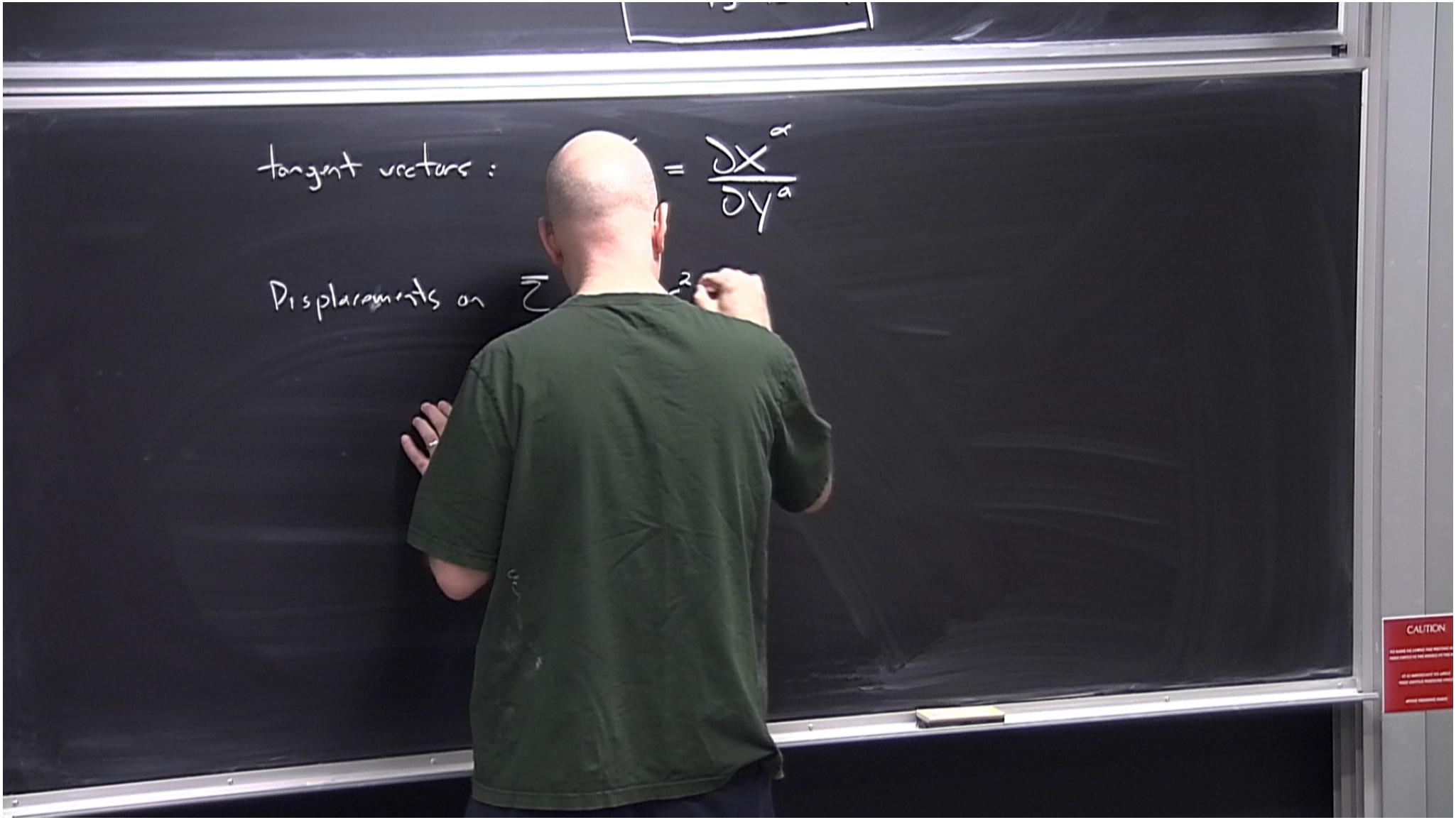
CAUTION


CAUTION


tangent vect
e^{α}
CAUTION

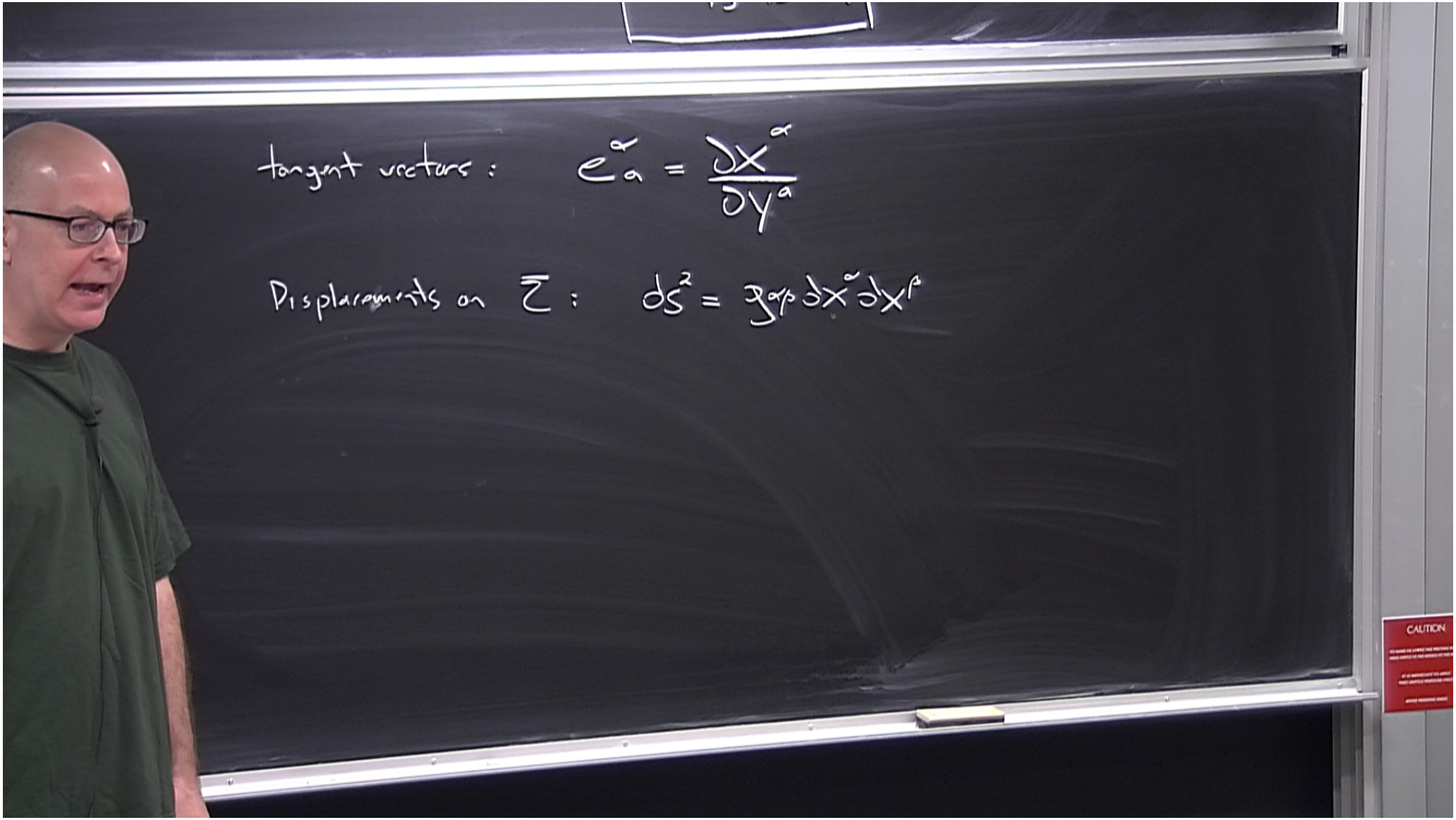

tangent vectors : $e^{\alpha}_{a} = \frac{\partial x^{\alpha}}{\partial y^{a}}$
CAUTION




tangent vectors :
$= \frac{\partial x^a}{\partial y^a}$
Displacements on

tangent vectors : $e_a^\alpha = \frac{\partial x^\alpha}{\partial y^a}$

Displacements on Σ : $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$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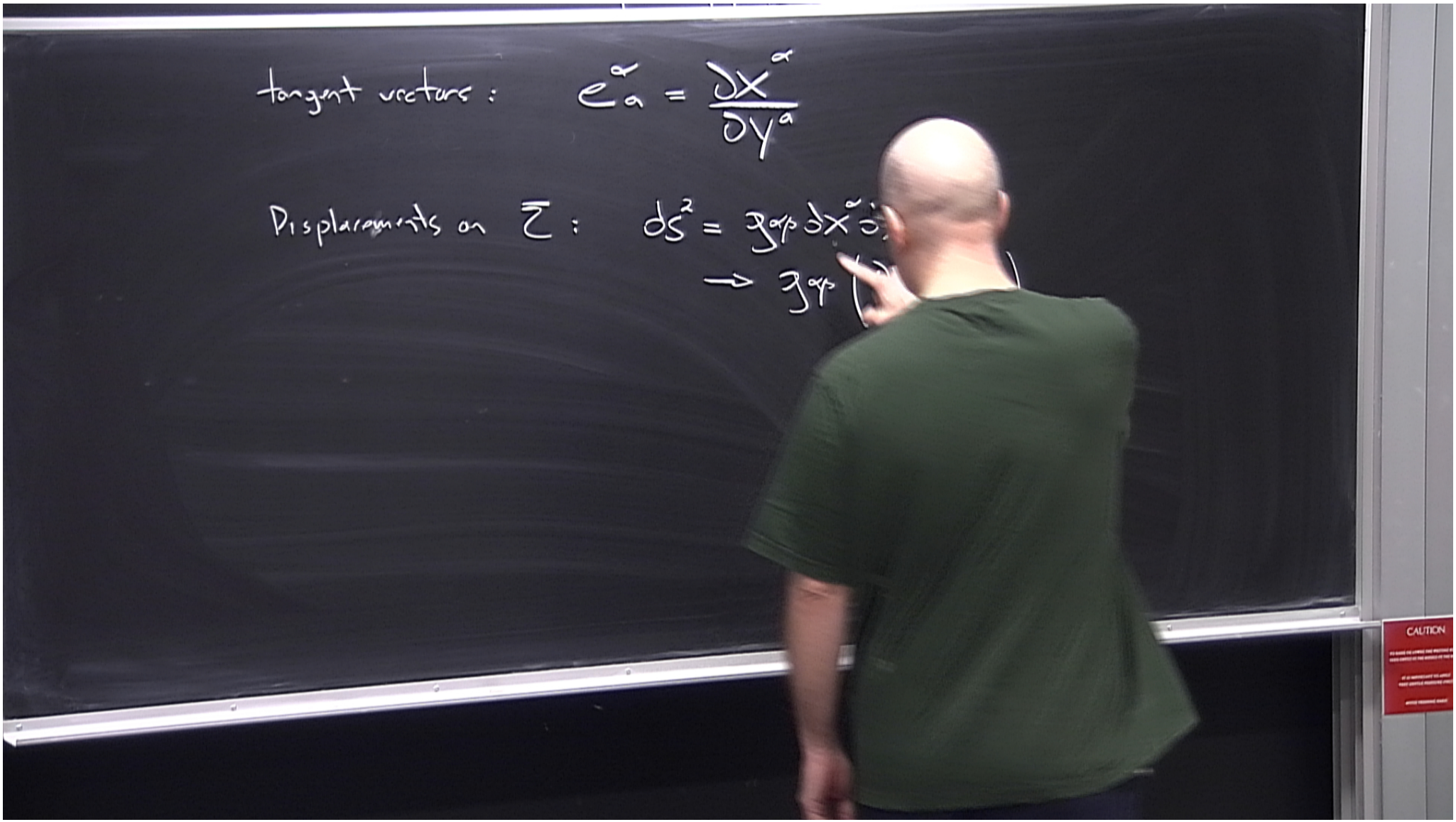
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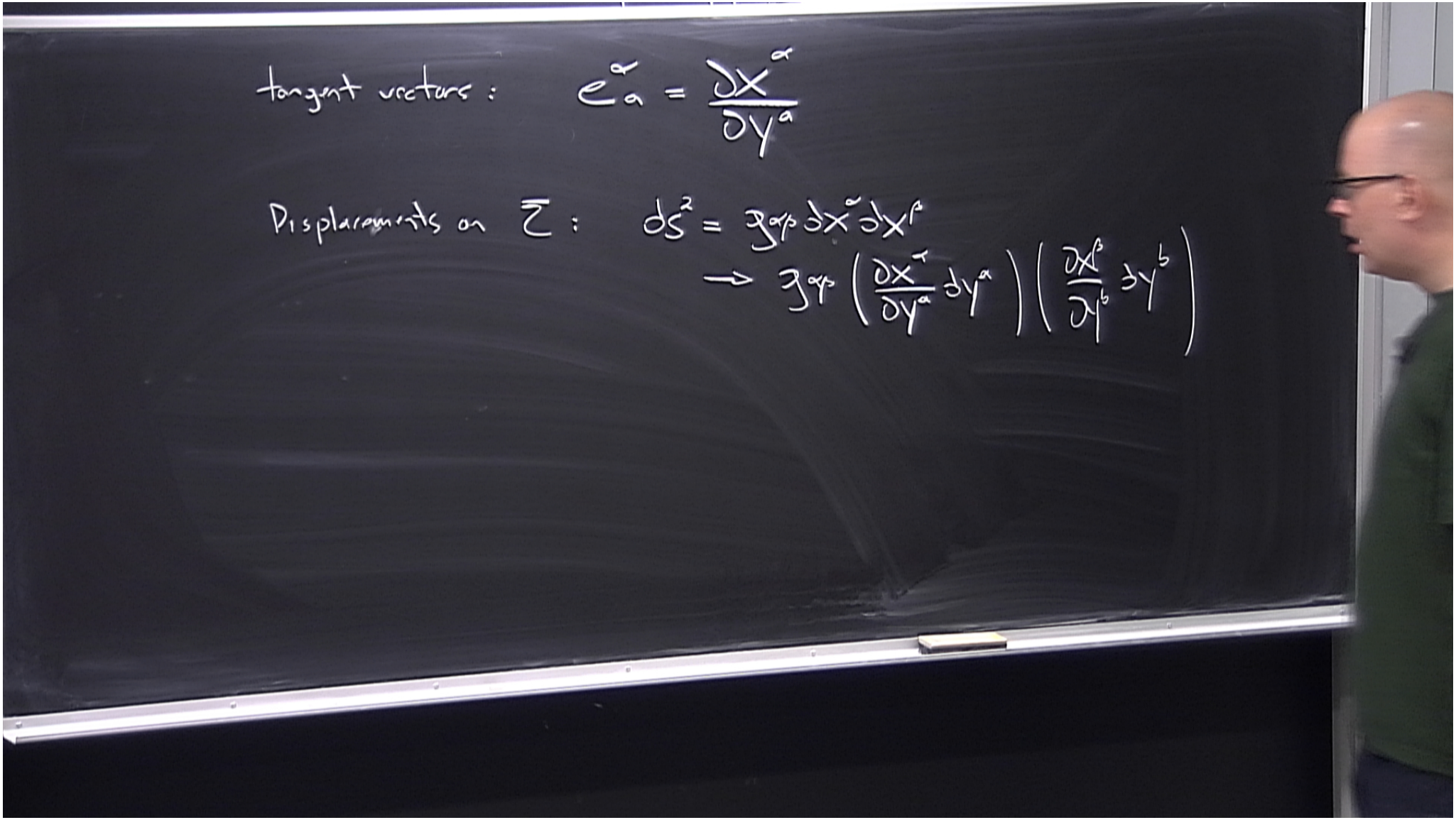
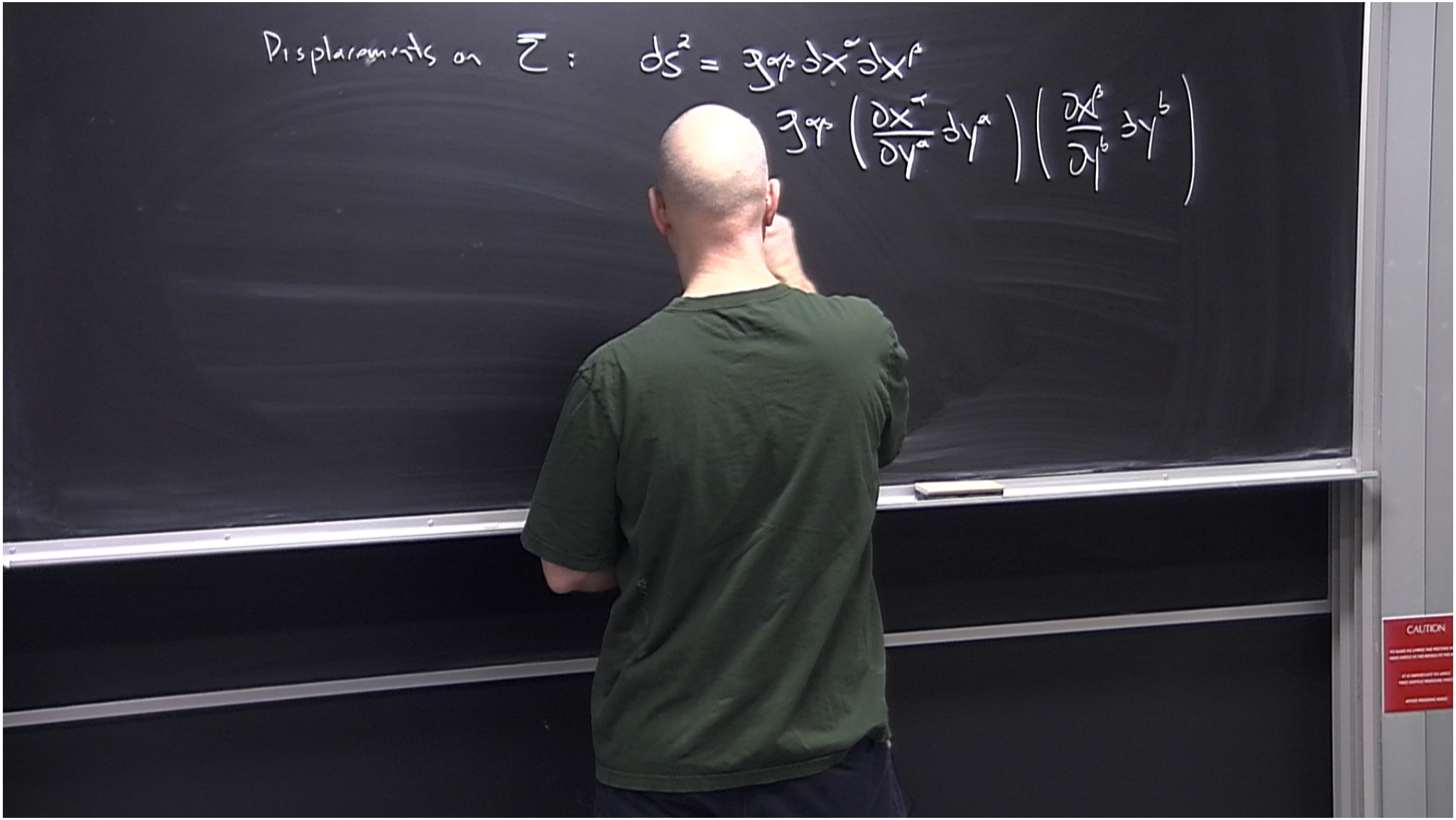
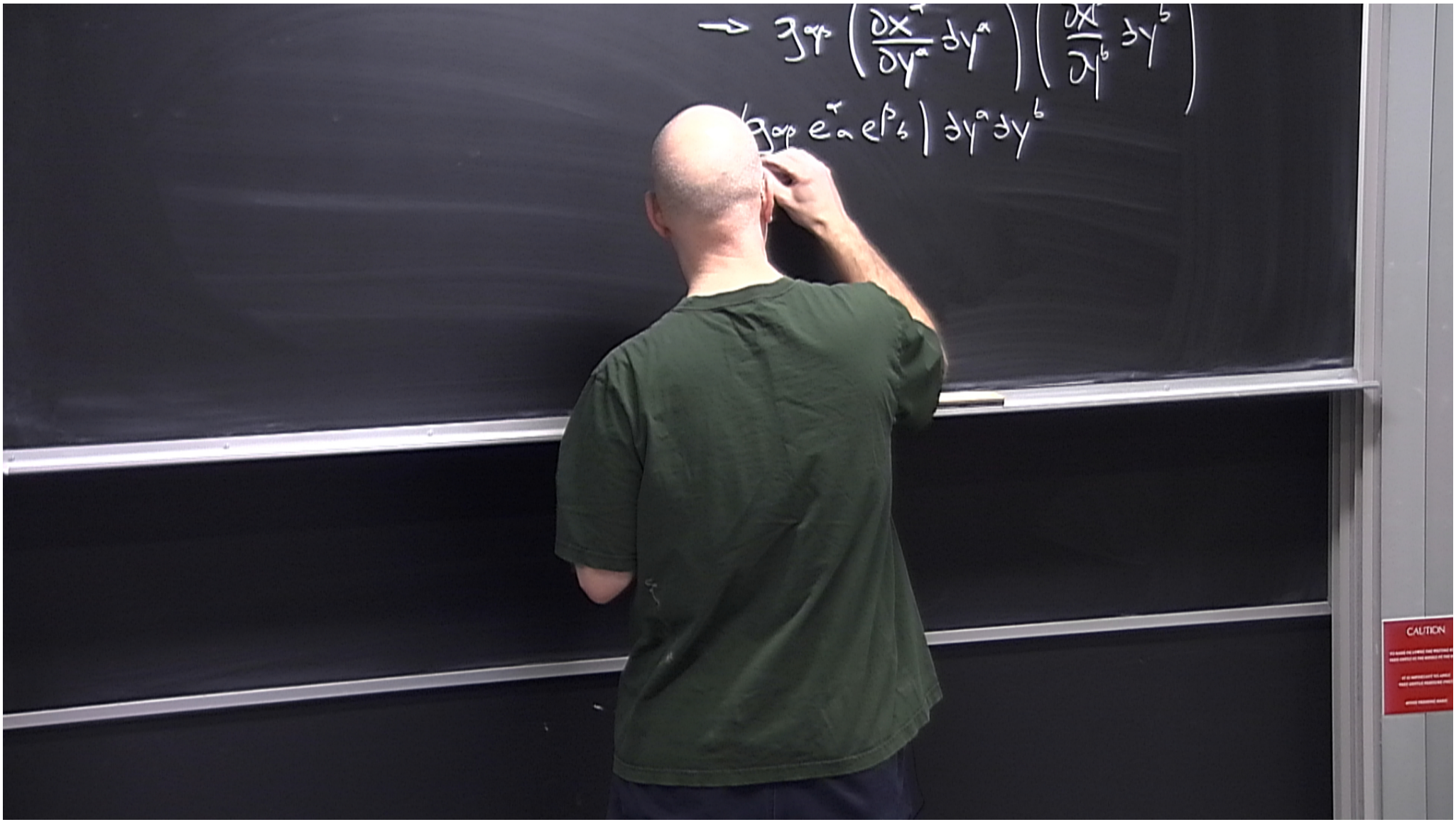
$\rightarrow g_{ab}$
CAUTION


tangent vectors : $e^{\alpha}_{a} = \frac{\partial x^{\alpha}}{\partial y^{a}}$
Displacements on Σ : $ds^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta}$
$\rightarrow g_{\alpha\beta} \left(\frac{\partial x^{\alpha}}{\partial y^{a}} dy^{a} \right) \left(\frac{\partial x^{\beta}}{\partial y^{b}} dy^{b} \right)$


Displacements on Σ : $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$g_{\mu\nu} \left(\frac{\partial x^\mu}{\partial y^a} dy^a \right) \left(\frac{\partial x^\nu}{\partial y^b} dy^b \right)$
CAUTION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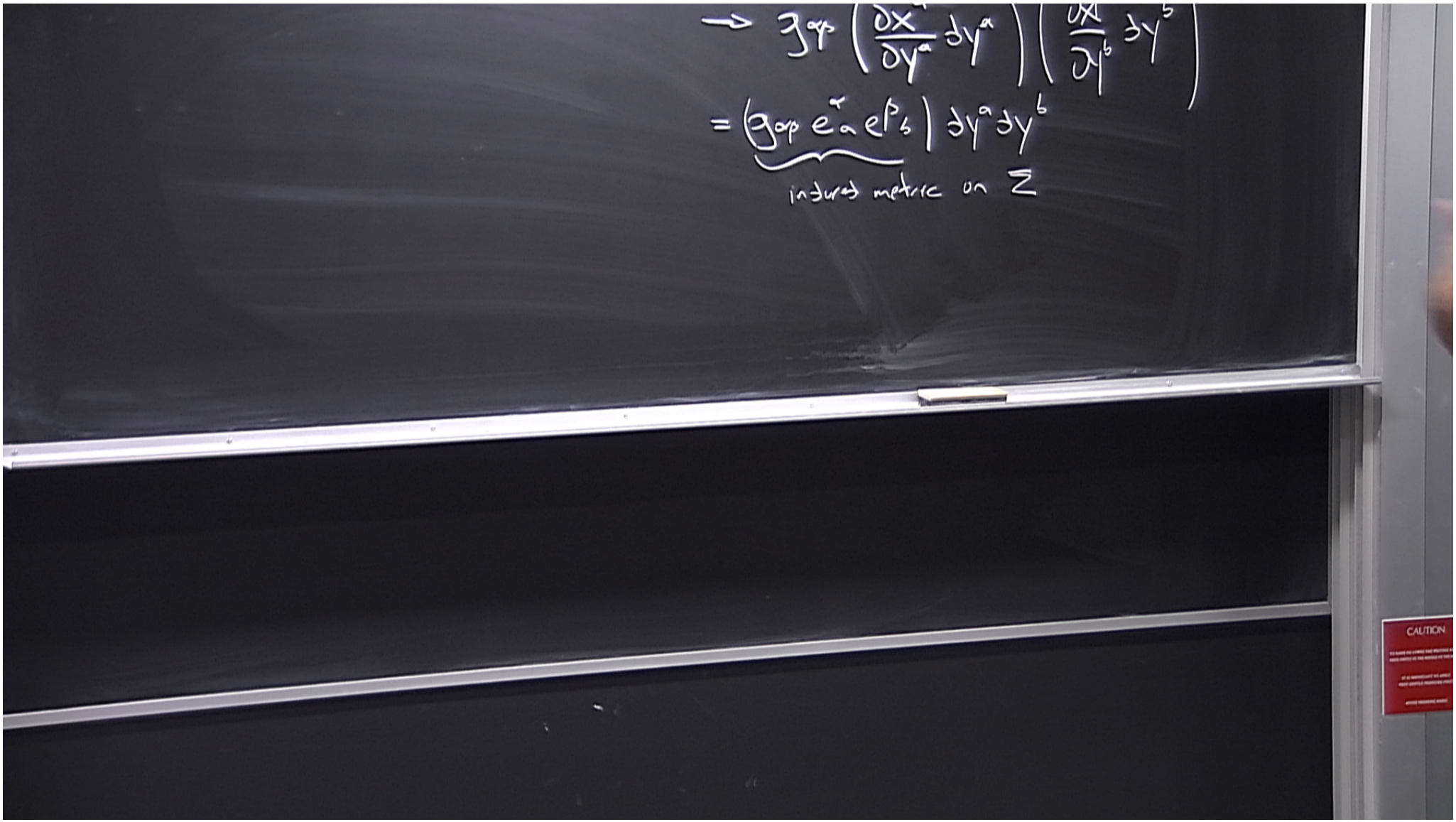
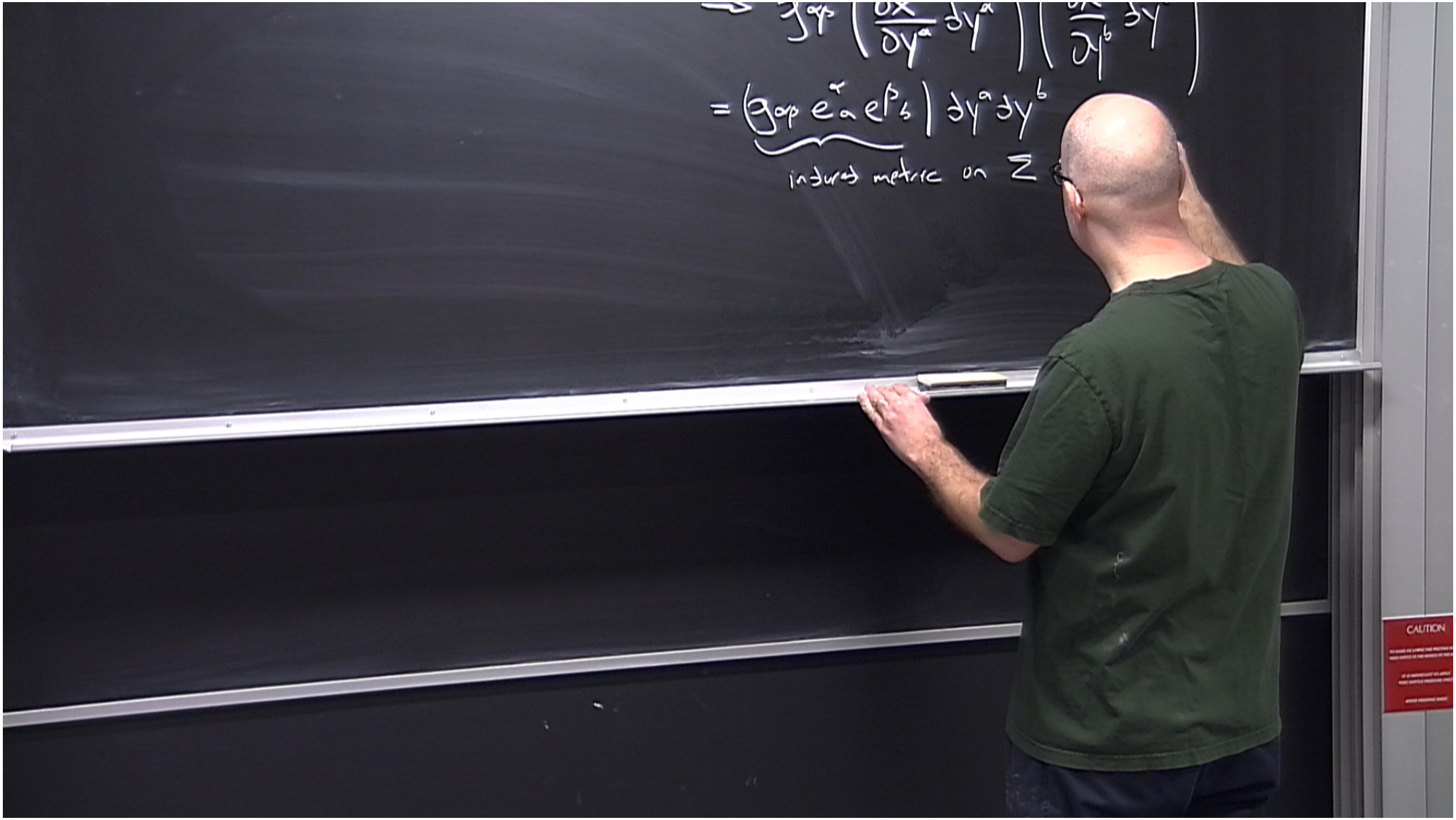
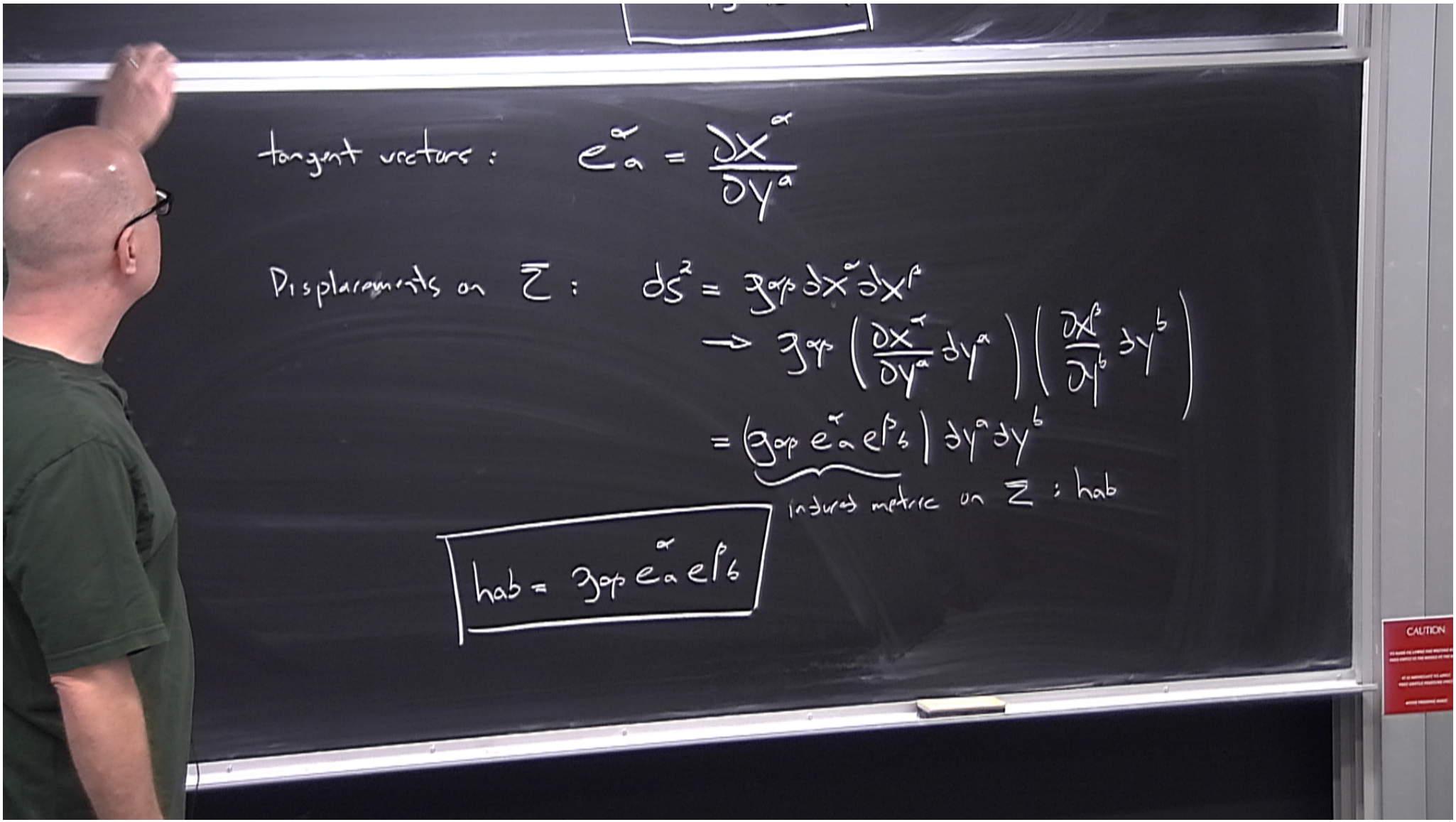
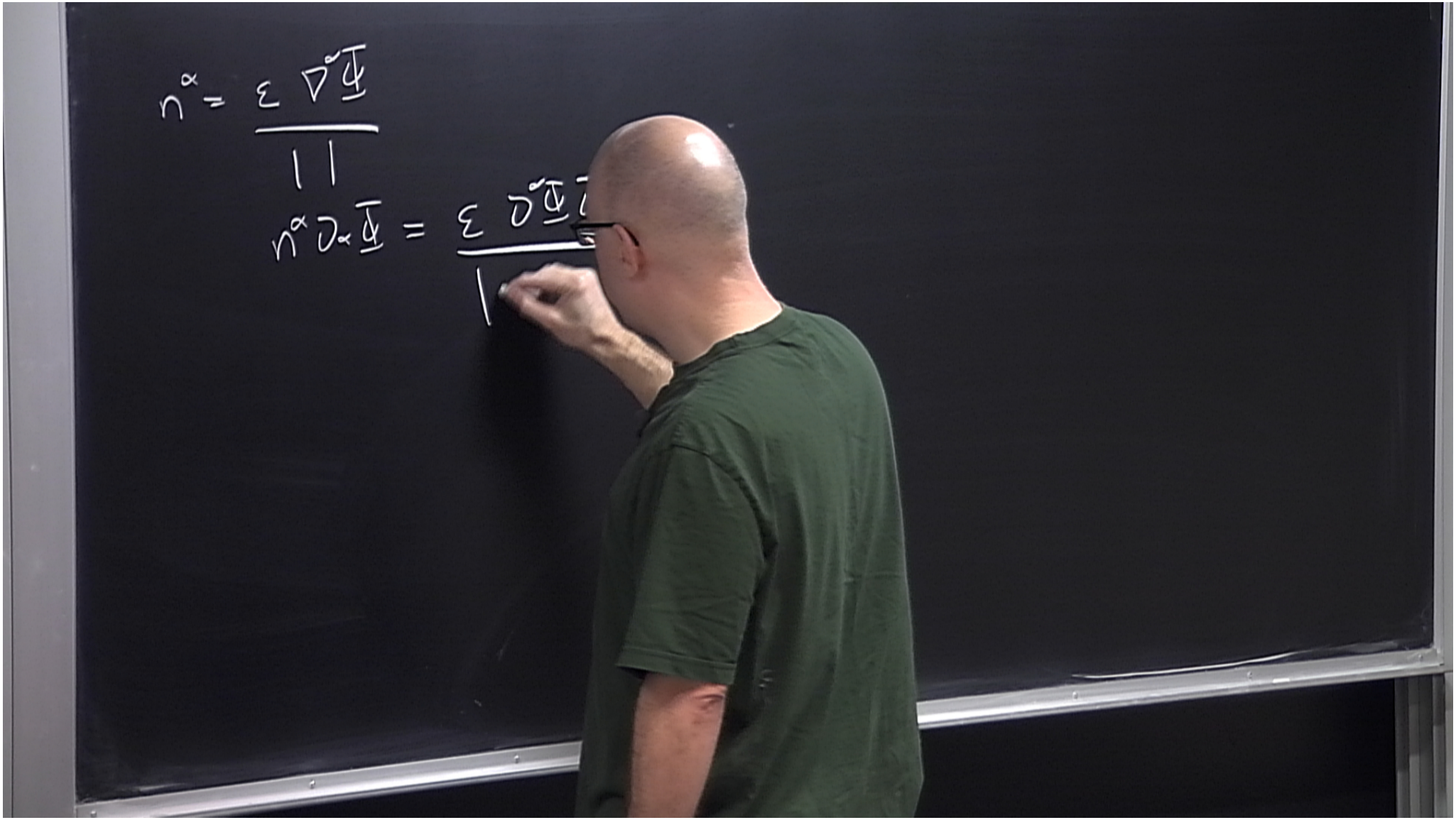
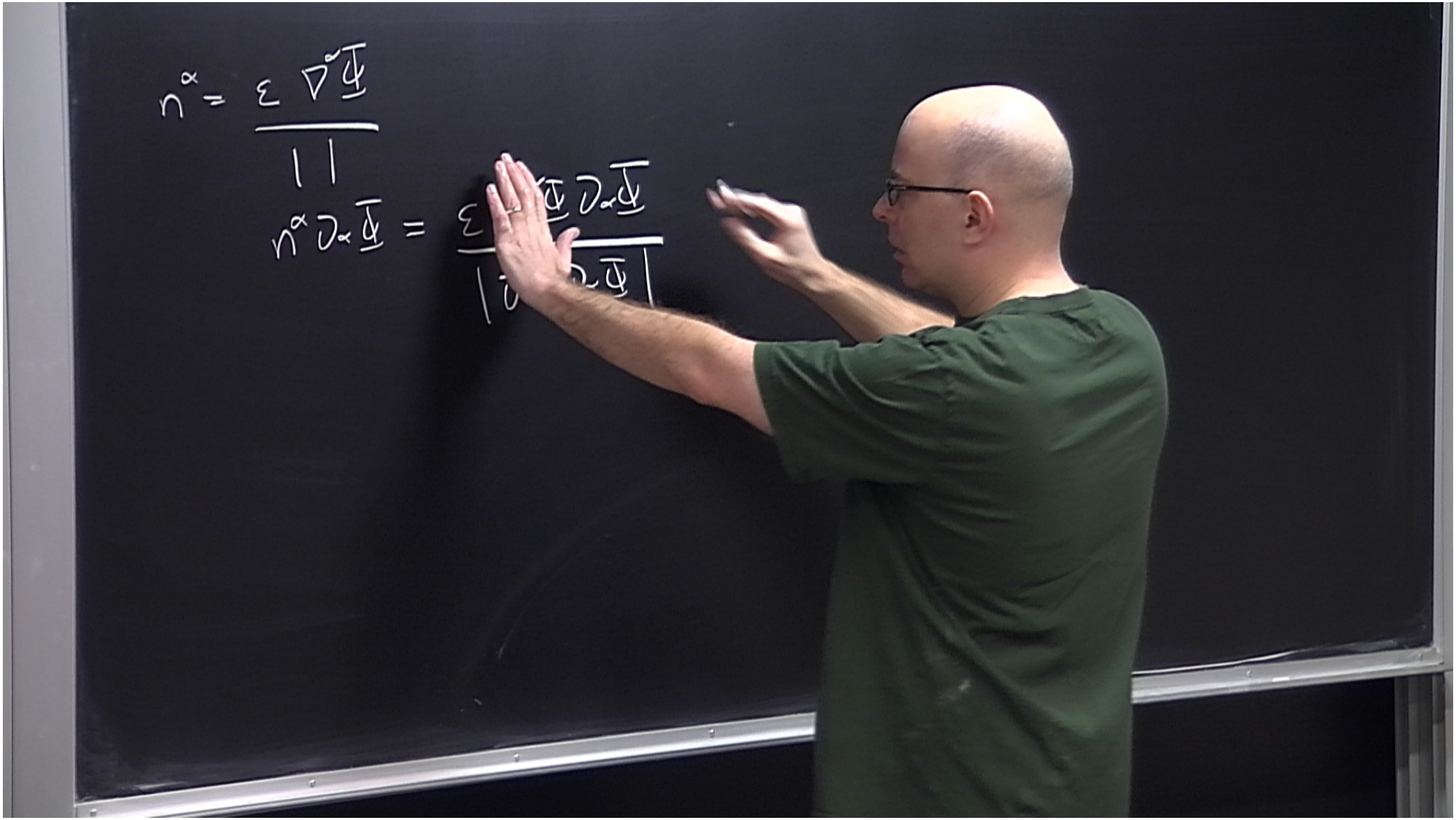
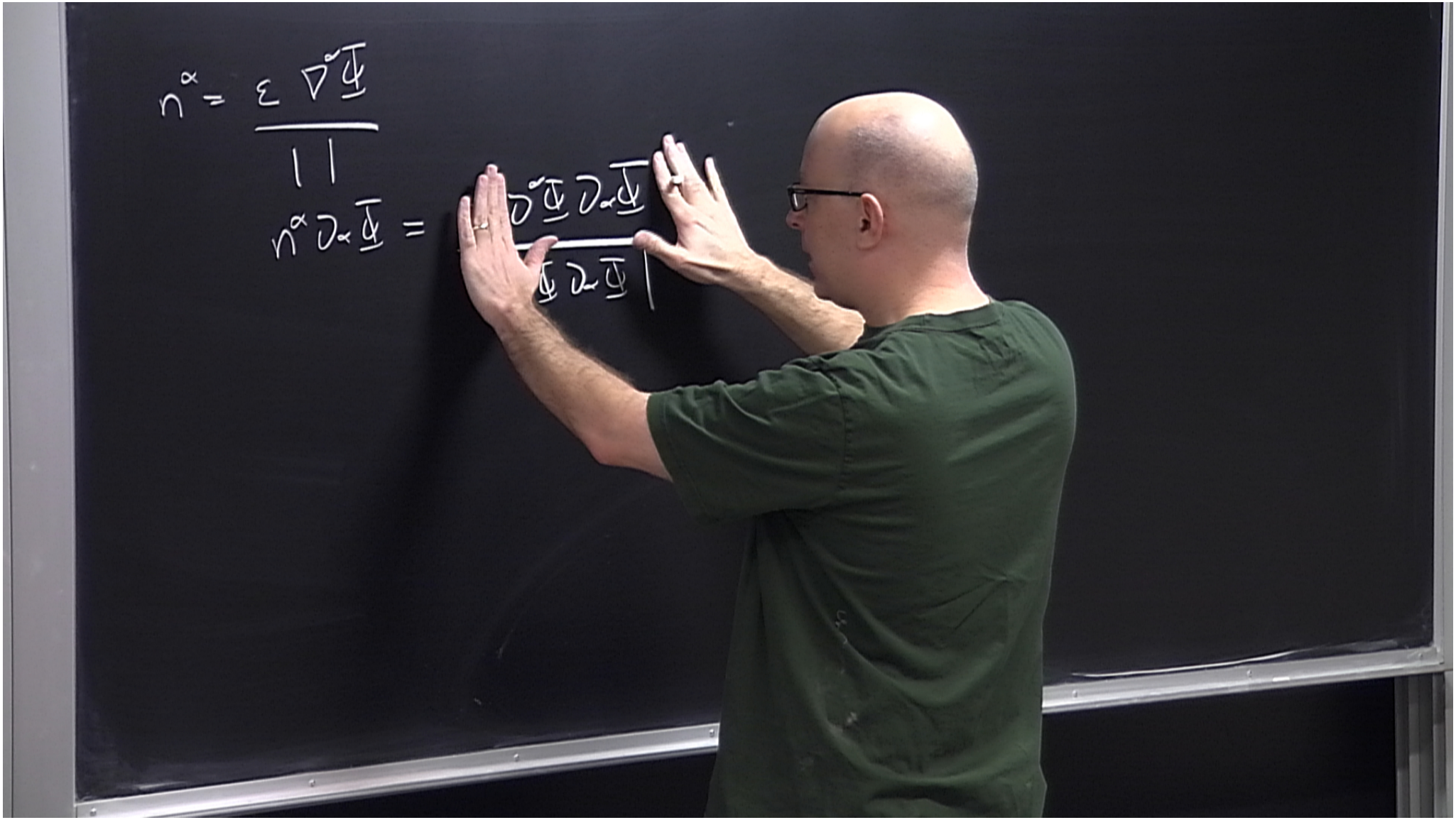

$\rightarrow g_{\alpha\beta}\left(\frac{\partial x^\alpha}{\partial y^a}dy^a\right)\left(\frac{\partial x^\beta}{\partial y^b}dy^b\right)$
$= g_{\alpha\beta}\, e^\alpha_a e^\beta_b \, dy^a dy^b$
induced metric on Σ

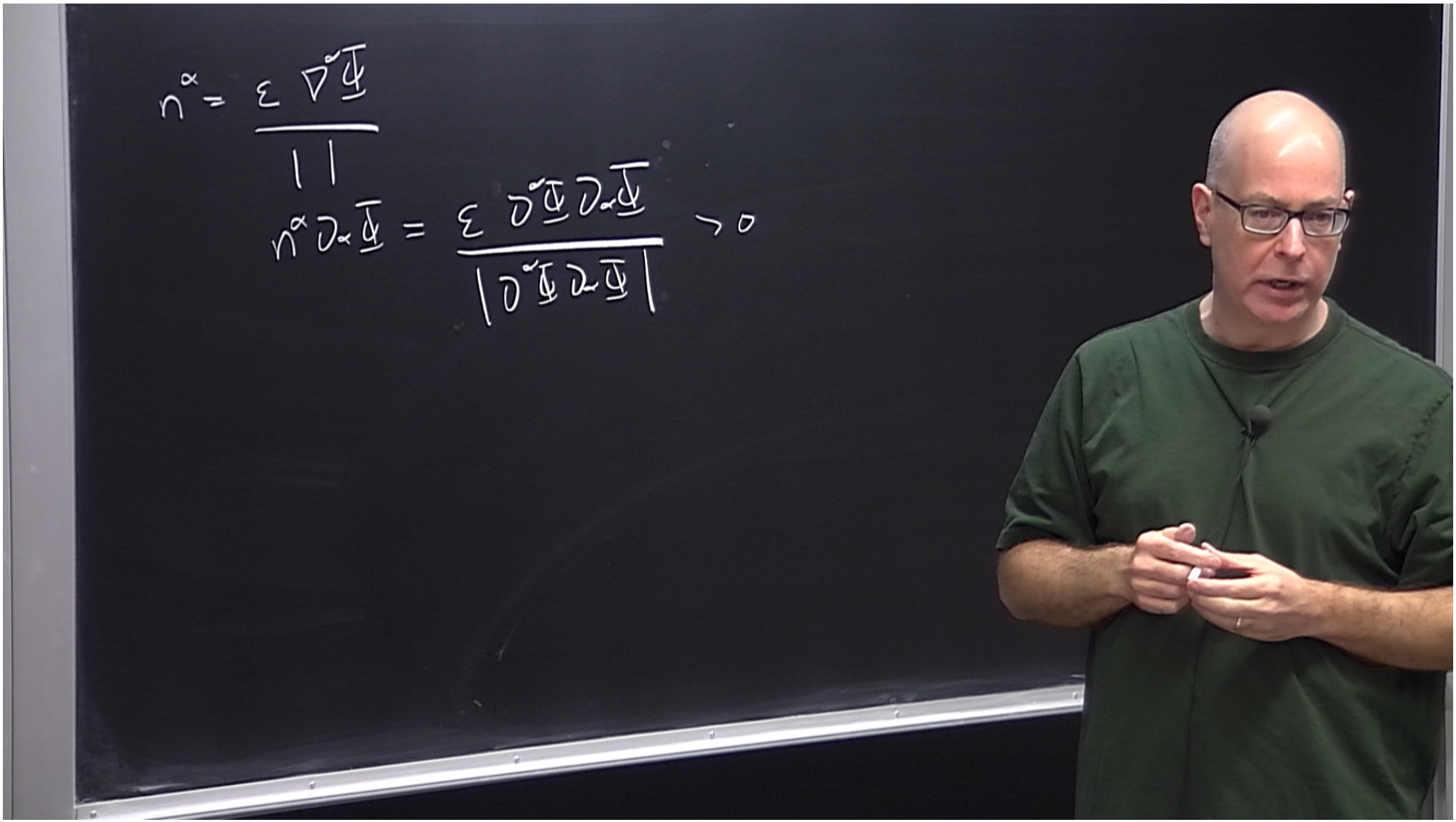

$= (g_{\alpha\beta}\, e^{\alpha}_{a}\, e^{\beta}_{b})\, dy^a dy^b$
induced metric on


tangent vectors : $e^{\tilde{\alpha}}_a = \frac{\partial x^{\tilde{\alpha}}}{\partial y^a}$
Displacements on Σ : $ds^2 = g_{\tilde{\alpha}\tilde{\beta}} dx^{\tilde{\alpha}} dx^{\tilde{\beta}}$
$\rightarrow g_{\tilde{\alpha}\tilde{\beta}} \left(\frac{\partial x^{\tilde{\alpha}}}{\partial y^a} dy^a \right) \left(\frac{\partial x^{\tilde{\beta}}}{\partial y^b} dy^b \right)$
$= \left(g_{\tilde{\alpha}\tilde{\beta}} e^{\tilde{\alpha}}_a e^{\tilde{\beta}}_b \right) dy^a dy^b$
induced metric on Σ : h_{ab}
$h_{ab} = g_{\tilde{\alpha}\tilde{\beta}} e^{\tilde{\alpha}}_a e^{\tilde{\beta}}_b$
CAUTION



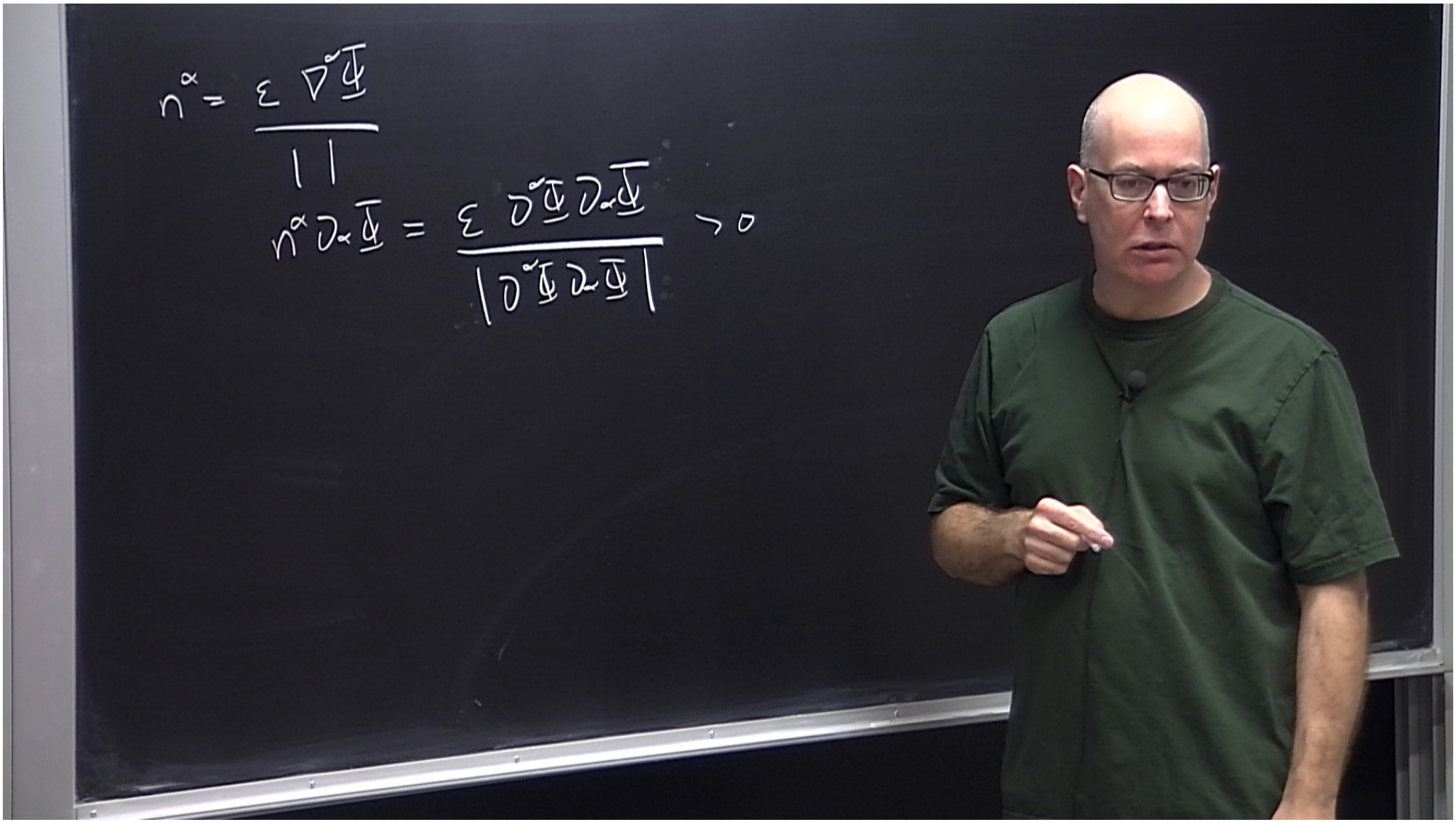





$n^\alpha = \frac{\varepsilon \nabla^\alpha \Phi}{|\ |}$
$n^\alpha \partial_\alpha \Phi = \frac{\varepsilon \partial^\alpha \Phi \partial_\alpha \Phi}{|\partial^\alpha \Phi \partial_\alpha \Phi|} > 0$

$$n^{\alpha} = \frac{\varepsilon^{\alpha\beta\gamma}}{11}$$

$$n^{\alpha} = \frac{\varepsilon^{\alpha\beta\gamma}}{|\varepsilon^{\alpha\beta\gamma}|} \quad 0 <$$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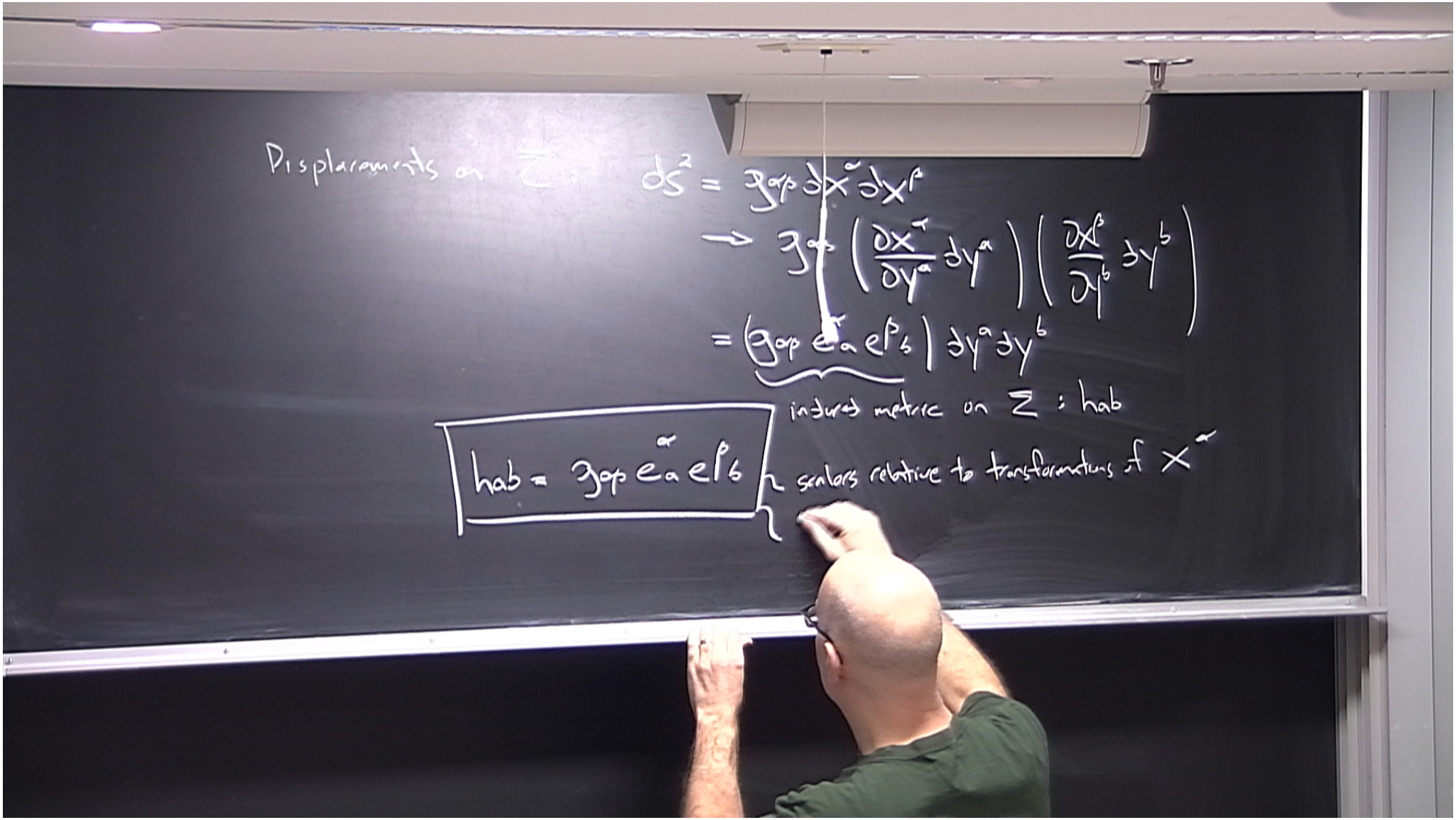
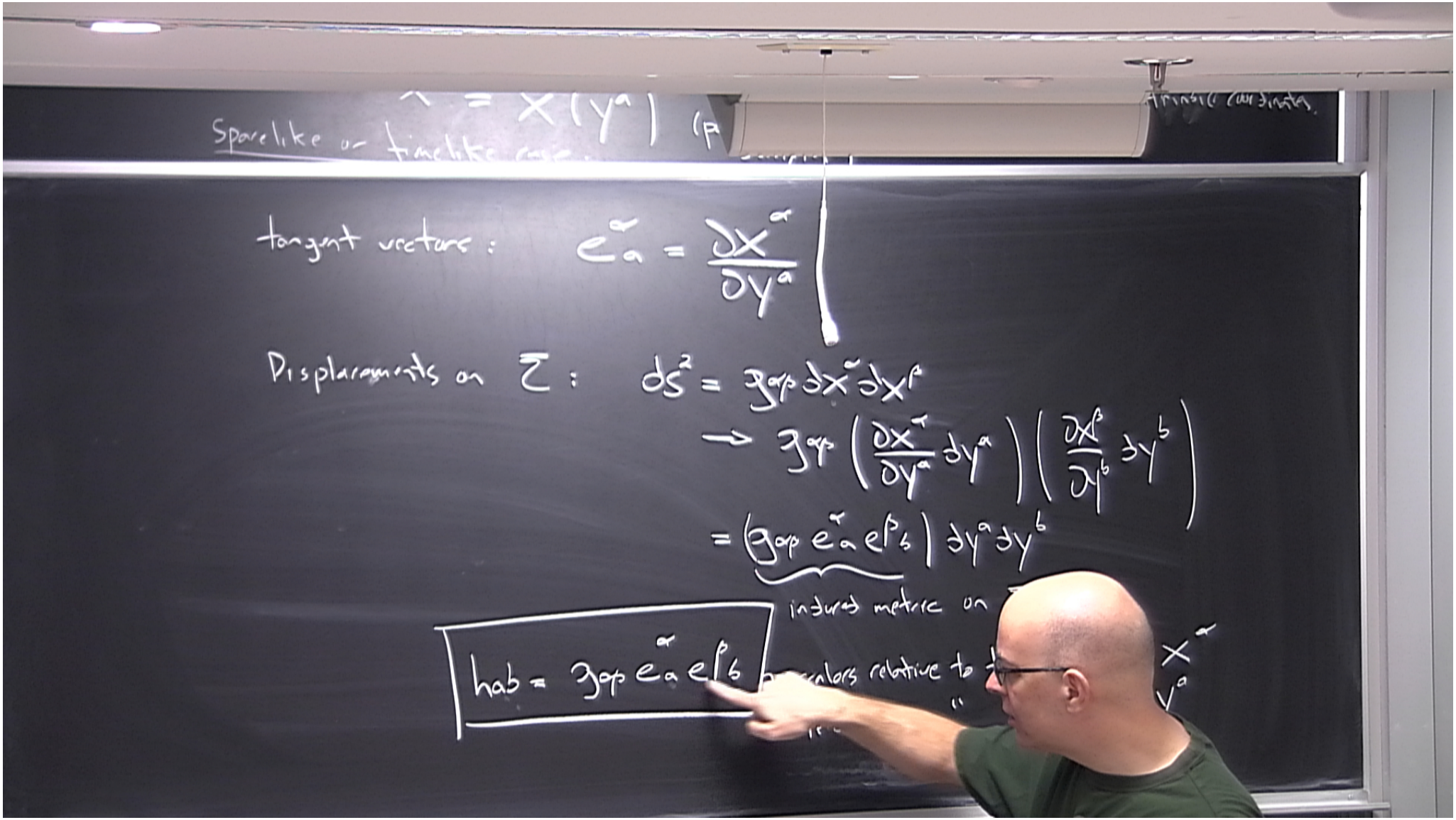

Displacements on Σ: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$\rightarrow g_{\mu\nu} \left(\frac{\partial x^\mu}{\partial y^a} dy^a\right)\left(\frac{\partial x^\nu}{\partial y^b} dy^b\right)$
$= \underbrace{g_{\mu\nu} e^\mu_a e^\nu_b}\, dy^a dy^b$
induced metric on Σ : h_{ab}
$h_{ab} = g_{\mu\nu} e^\mu_a e^\nu_b$
scalars relative to transformations of x^μ


Spacelike or timelike case.
tangent vectors: $e^{\tilde{\alpha}}_a = \frac{\partial x^{\tilde{\alpha}}}{\partial y^a}$
Displacements on Σ: $ds^2 = g_{\tilde{\alpha}\tilde{\beta}} dx^{\tilde{\alpha}} dx^{\tilde{\beta}}$
$\rightarrow g_{\tilde{\alpha}\tilde{\beta}} \left(\frac{\partial x^{\tilde{\alpha}}}{\partial y^a} dy^a\right)\left(\frac{\partial x^{\tilde{\beta}}}{\partial y^b} dy^b\right)$
$= (g_{\tilde{\alpha}\tilde{\beta}} e^{\tilde{\alpha}}_a e^{\tilde{\beta}}_b)\, dy^a dy^b$
induced metric on
$h_{ab} = g_{\tilde{\alpha}\tilde{\beta}} e^{\tilde{\alpha}}_a e^{\tilde{\beta}}_b$
components relative to

$\hat{} = X(y^a)$ (P

Spacelike or timelike

tangent vectors: $e^\alpha_a = \frac{\partial x^\alpha}{\partial y^a}$

Displacements on Σ: $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$

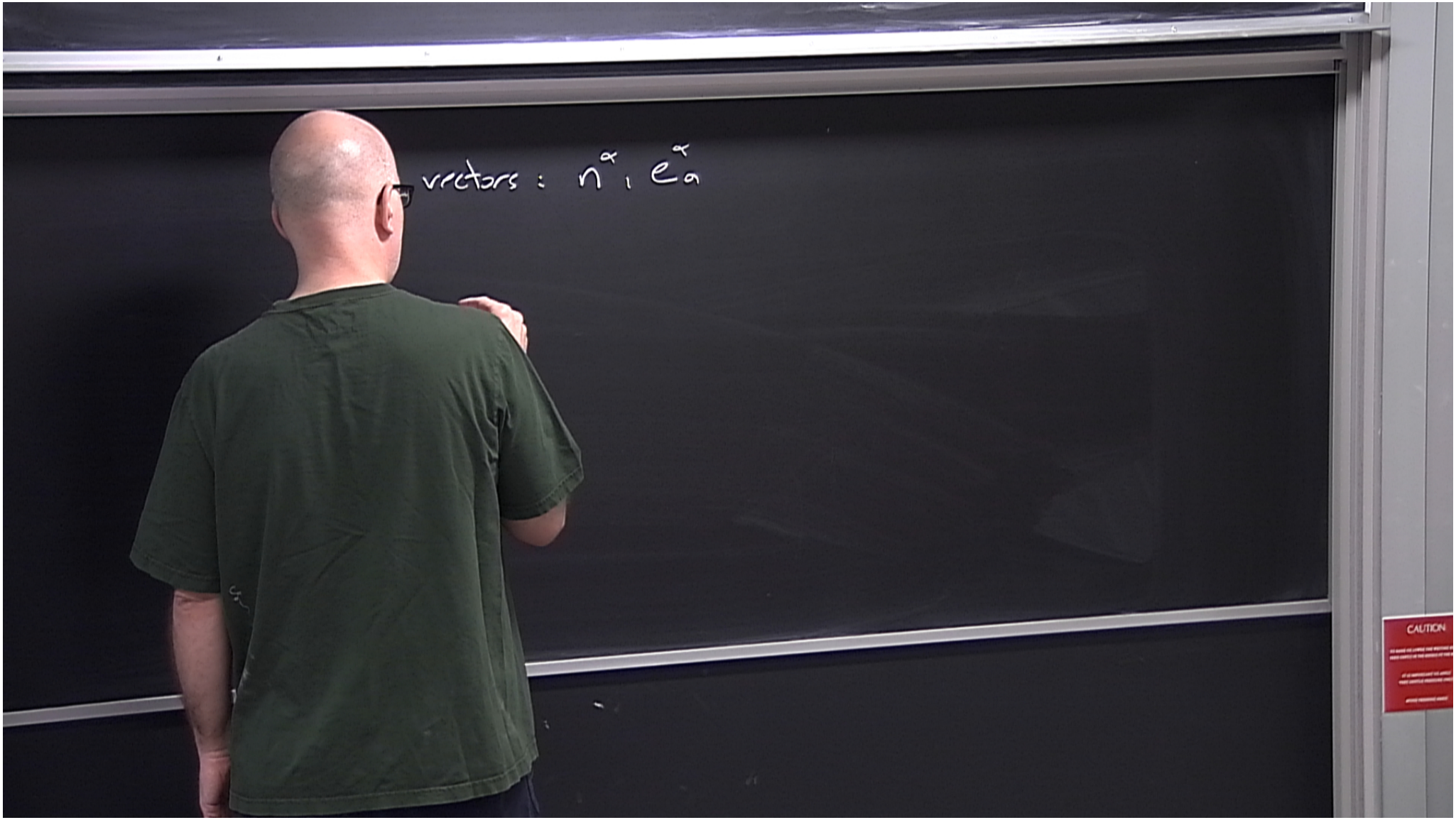
$\rightarrow g_{\alpha\beta}\left(\frac{\partial x^\alpha}{\partial y^a} dy^a\right)\left(\frac{\partial x^\beta}{\partial y^b} dy^b\right)$

$= \underbrace{g_{\alpha\beta}\, e^\alpha_a e^\beta_b}\, dy^a dy^b$

induced metric on Σ: h_{ab}

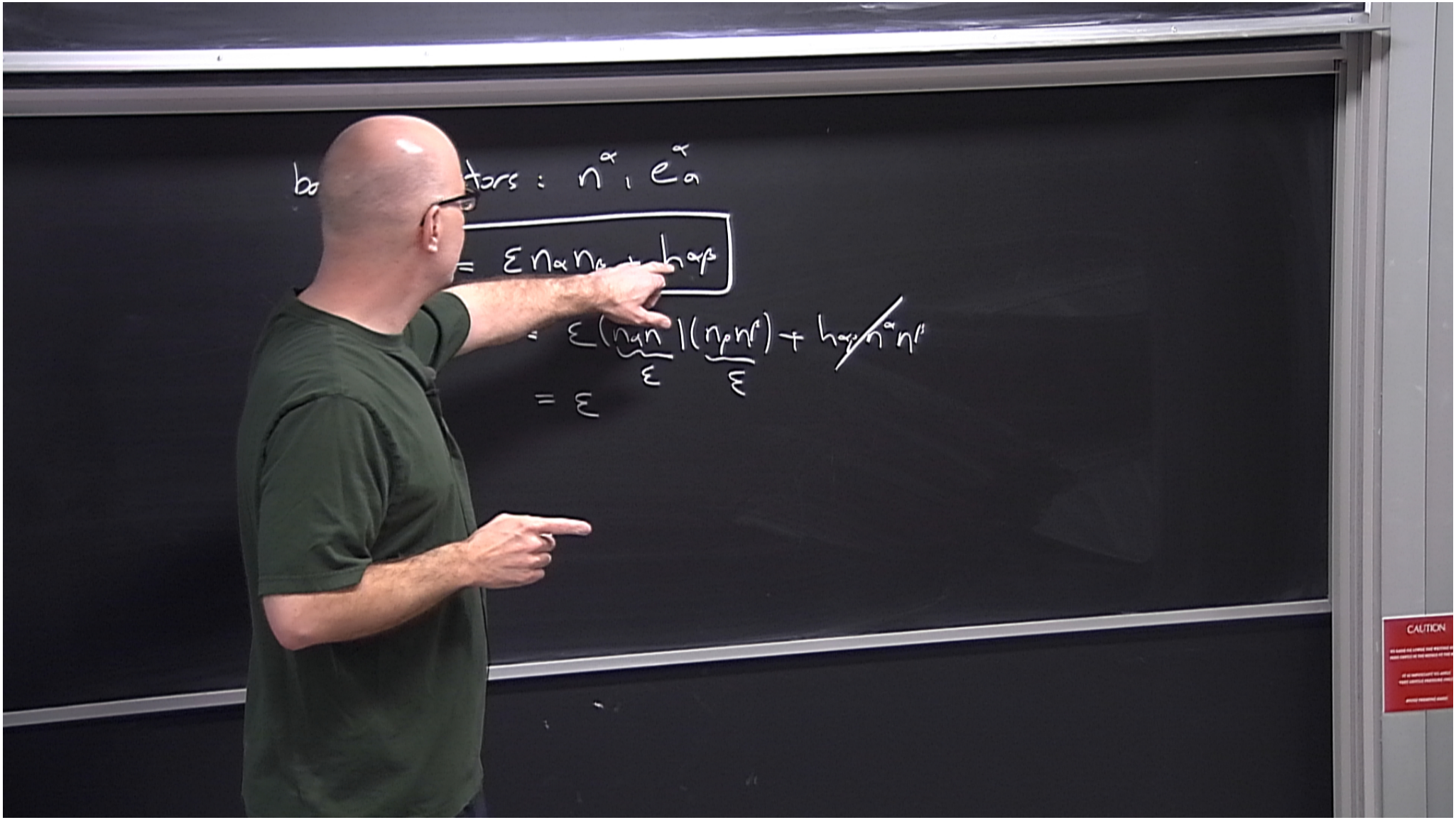
$h_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b$

- scalars relative to transformations of X^α
- tensor " " " of y^a


vectors : n^a, e^a_a
CAUTION

basis of vectors : n^a ; e^a_a

$g_{ab} =$

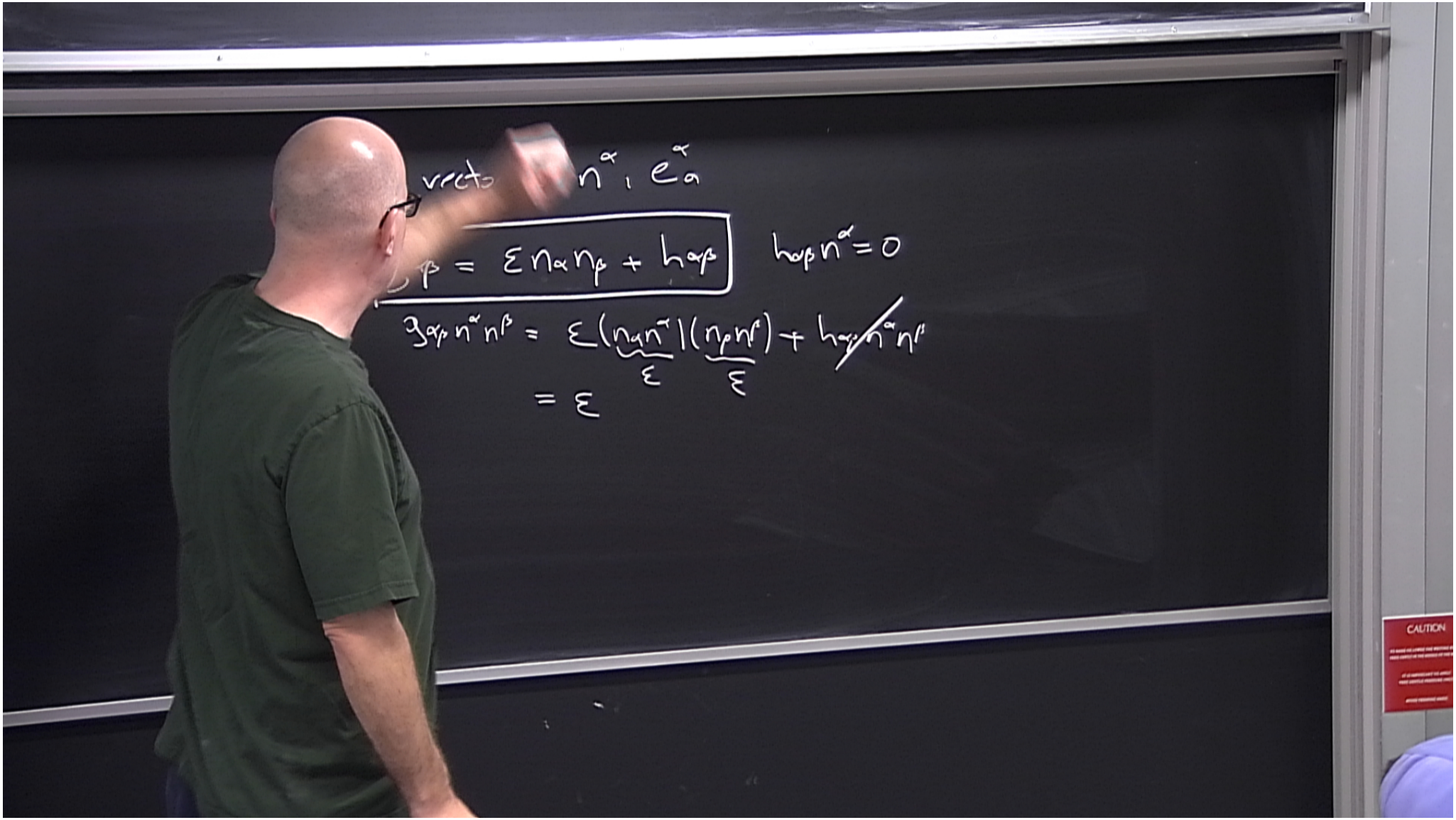

n^α, e^α_a
$= \varepsilon n_\alpha n_\beta + h_{\alpha\beta}$
$= \varepsilon(\underbrace{n_\alpha n^\alpha}_{\varepsilon})(\underbrace{n_\beta n'^\beta}_{\varepsilon}) + h_{\alpha\beta} n^\alpha n'^\beta$
$= \varepsilon$
CAUTION

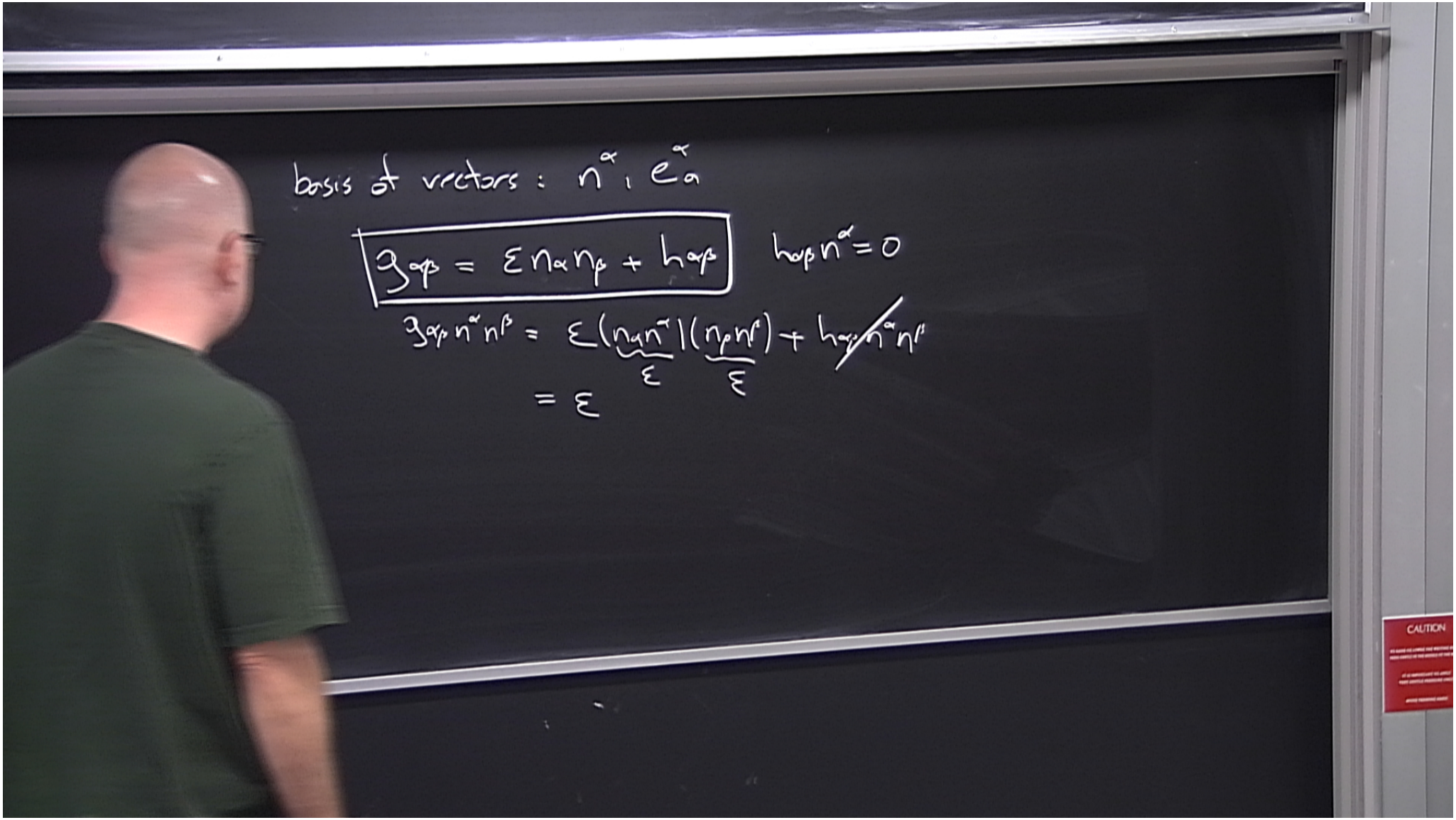
basis of vectors : n^α ; e^α_a

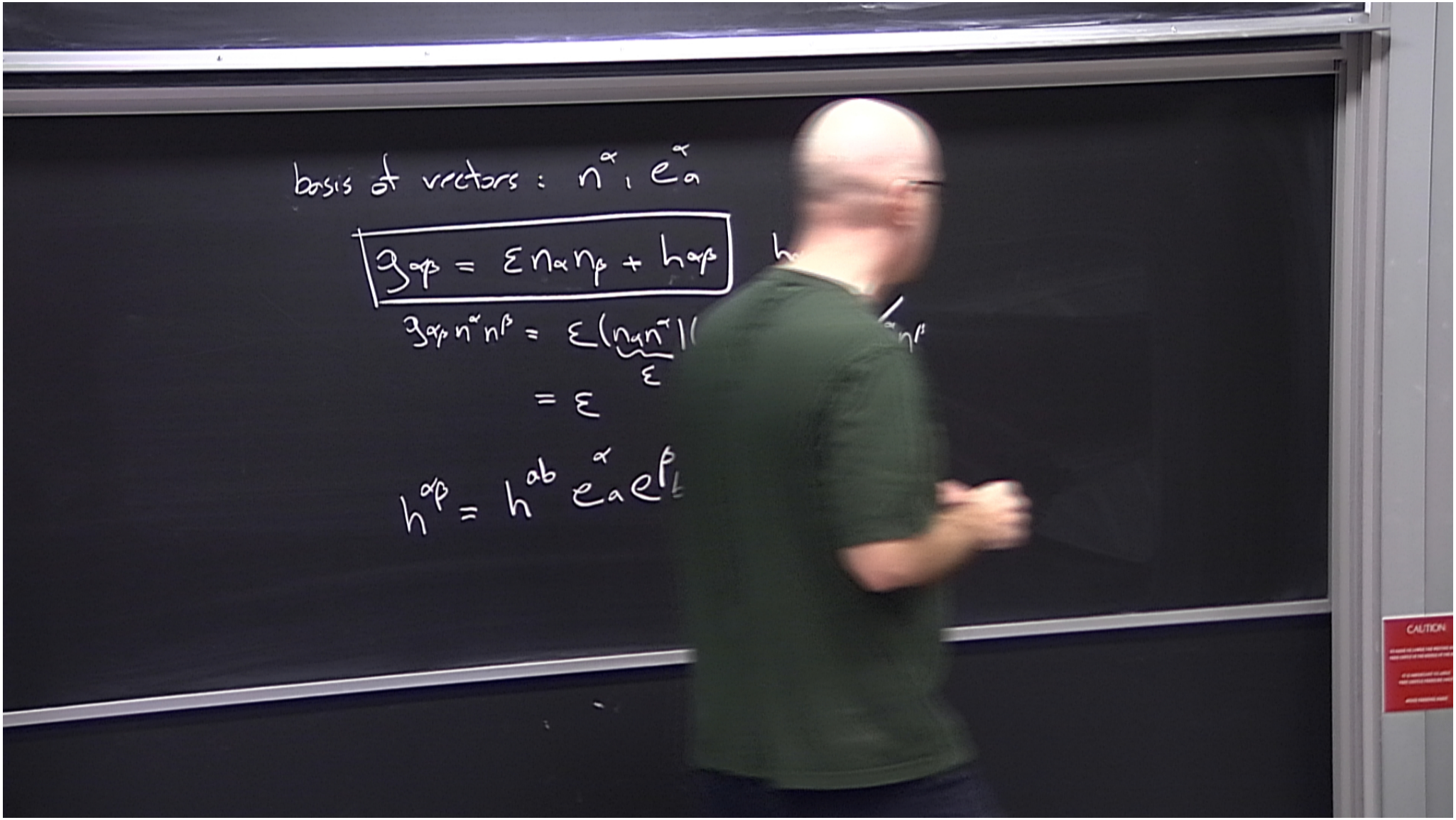
$$\boxed{g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}} \qquad h_{\alpha\beta} n^\alpha = 0$$

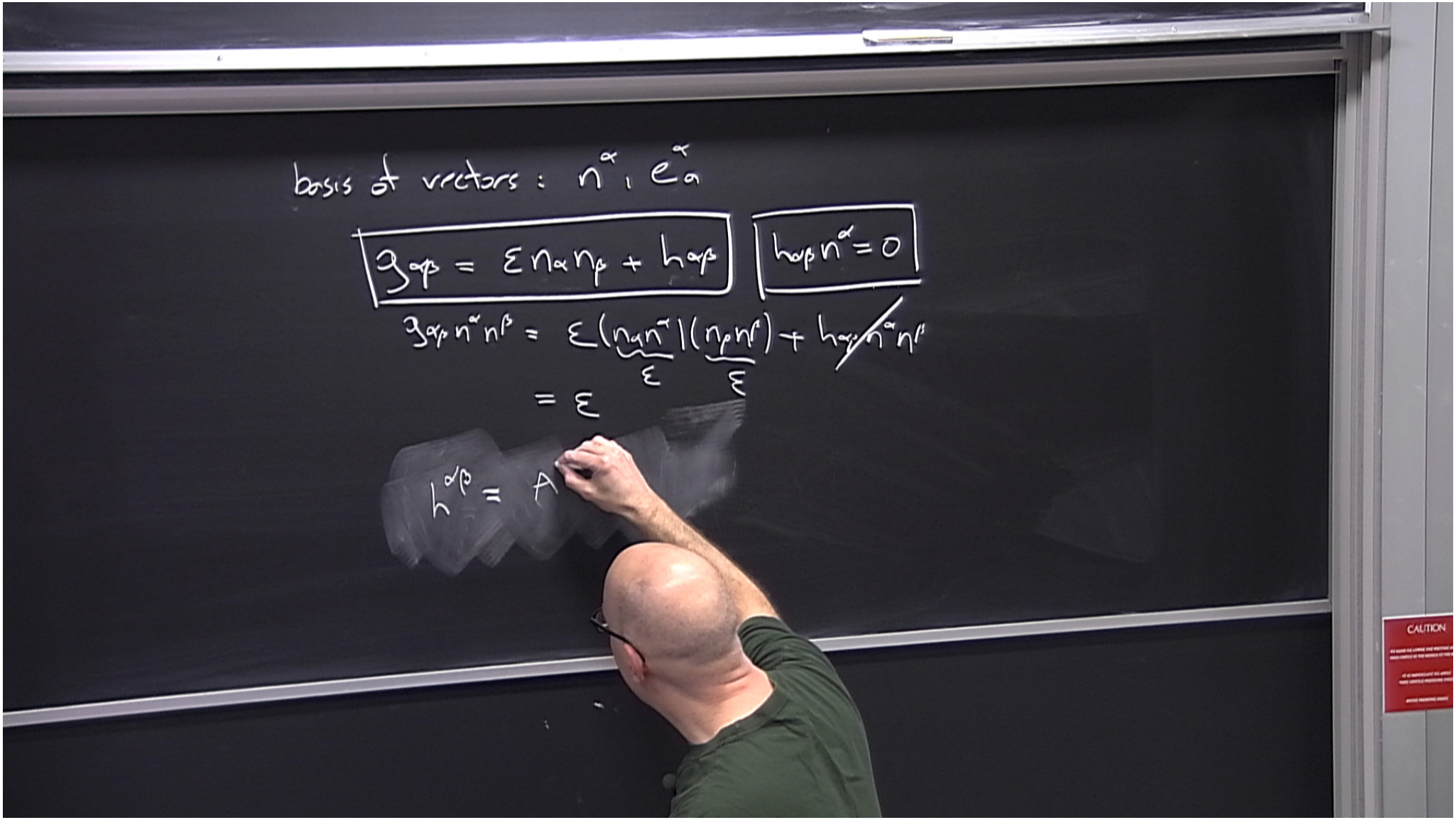
$$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (\underbrace{n_\alpha n^\alpha}_{\varepsilon})(\underbrace{n_\beta n^\beta}_{\varepsilon}) + h_{\alpha\beta} n^\alpha n^\beta$$

$$= \varepsilon$$


vects n^α, e^α_a
$g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}$
$h_{\alpha\beta} n^\alpha = 0$
$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (n_\alpha n^\alpha)(n_\beta n^\beta) + h_{\alpha\beta} n^\alpha n^\beta$
$= \varepsilon$
CAUTION


basis of vectors : n^α ; e^α_a
$g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}$
$h_{\alpha\beta} n^\alpha = 0$
$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (n_\alpha n^\alpha)(n_\beta n^\beta) + h_{\alpha\beta} n^\alpha n^\beta$
$= \varepsilon$
CAUTION


basis of vectors : n^α, e^α_a
$g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}$
$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (n_\alpha n^\alpha)$
ε
$= \varepsilon$
$h^{\alpha\beta} = h^{ab} e^\alpha_a e^\beta$
CAUTION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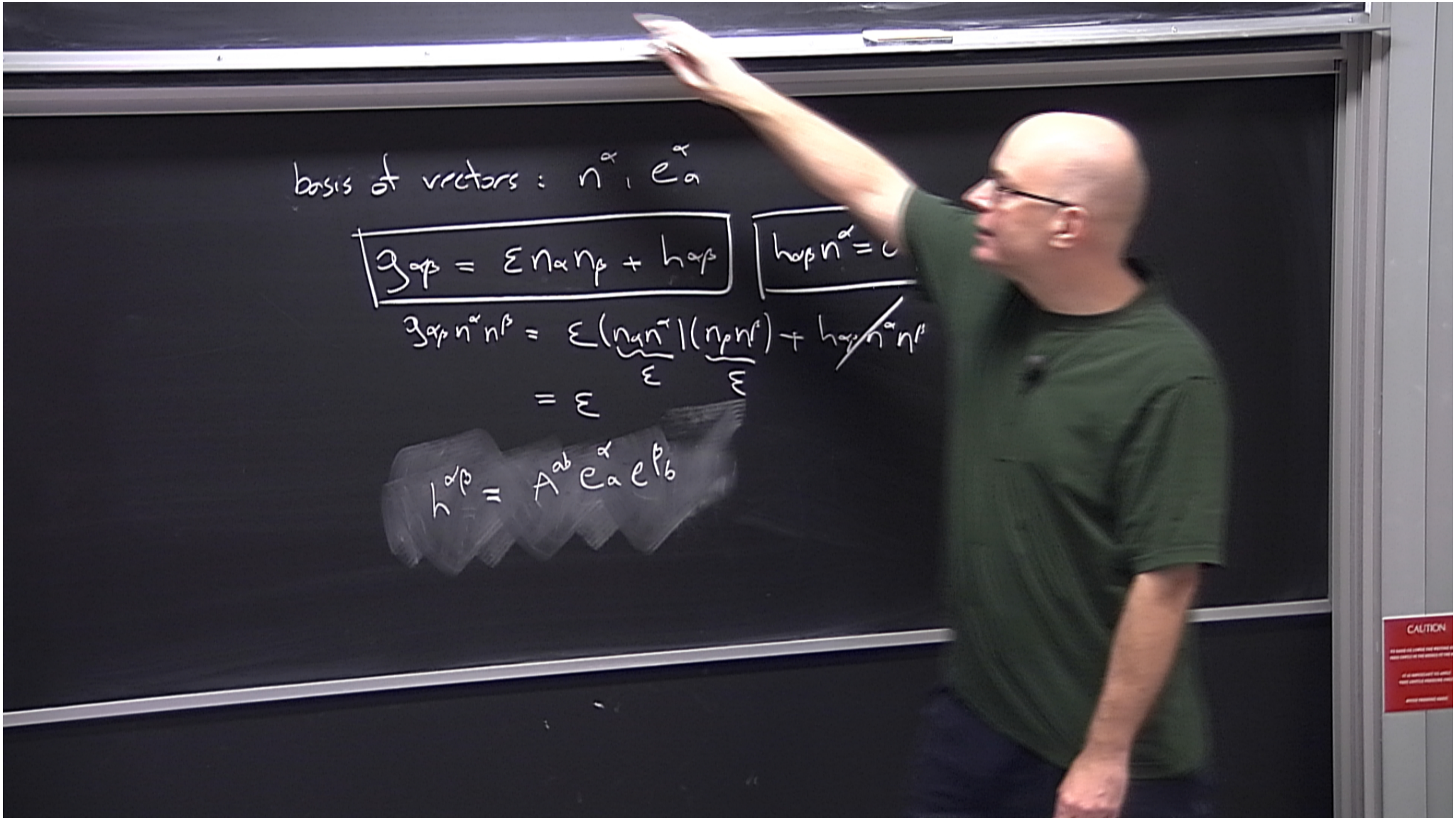
$h_{\alpha\beta} n^\alpha = 0$
$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (n_\alpha n^\alpha)(n_\beta n^\beta) + h_{\alpha\beta} n^\alpha n^\beta$
$\varepsilon \quad \varepsilon$
$= \varepsilon$
$h^{\alpha\beta} = A$
CAUTION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$g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}$
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$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (n_\alpha n^\alpha)(n_\beta n^\beta) + h_{\alpha\beta} n^\alpha n^\beta$
$= \varepsilon$
$h^{\alpha\beta} = A^{ab} e^\alpha_a e^\beta_b$
CAUTION

basis of vectors: n^α, e^α_a

$$\boxed{g_{\alpha\beta} = \varepsilon n_\alpha n_\beta + h_{\alpha\beta}} \qquad \boxed{h_{\alpha\beta} n^\alpha = 0}$$

$$g_{\alpha\beta} n^\alpha n^\beta = \varepsilon (\underbrace{n_\alpha n^\alpha}_{\varepsilon})(\underbrace{n_\beta n^\beta}_{\varepsilon}) + h_{\alpha\beta} n^\alpha n^\beta$$

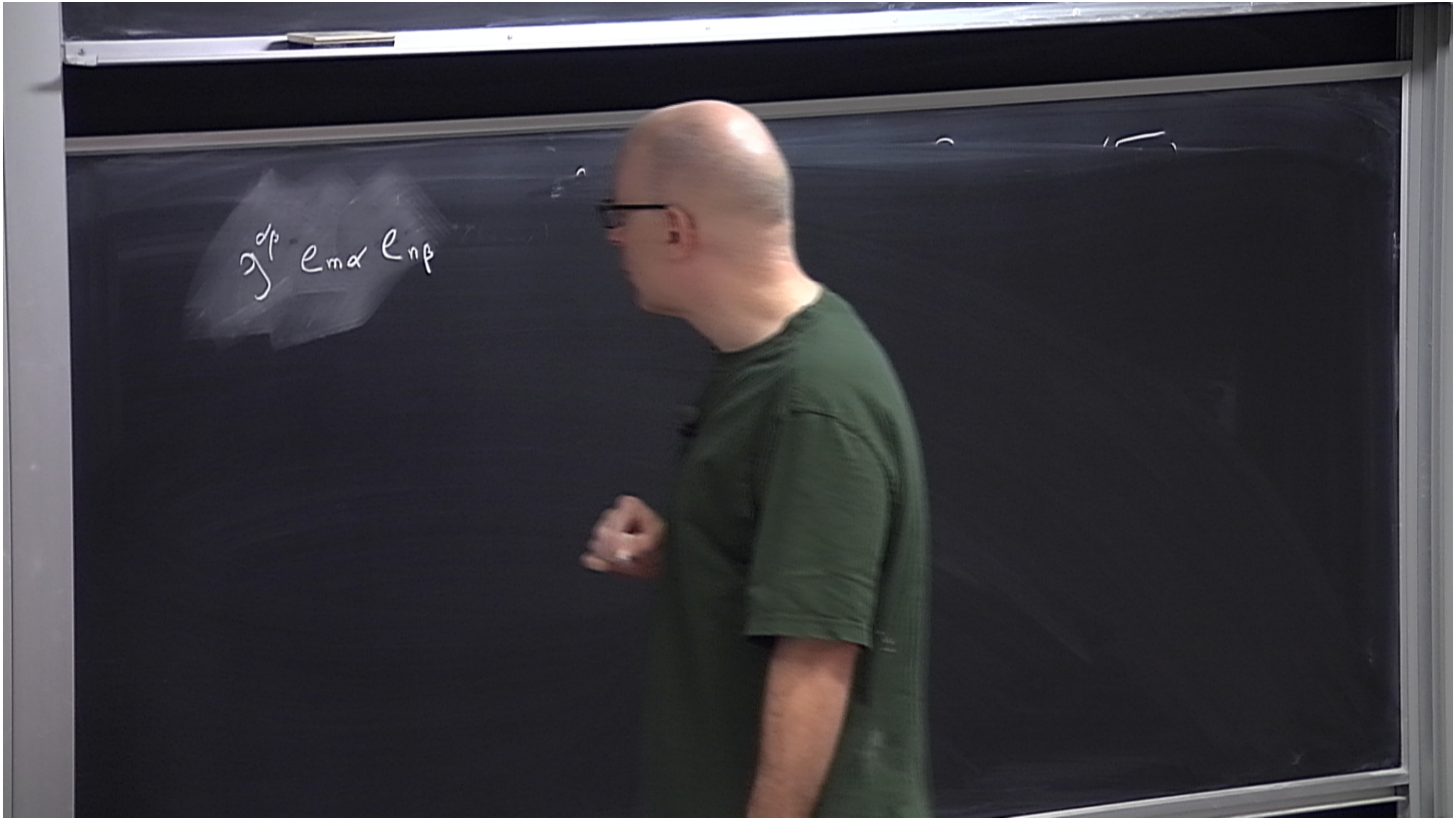
$$= \varepsilon$$

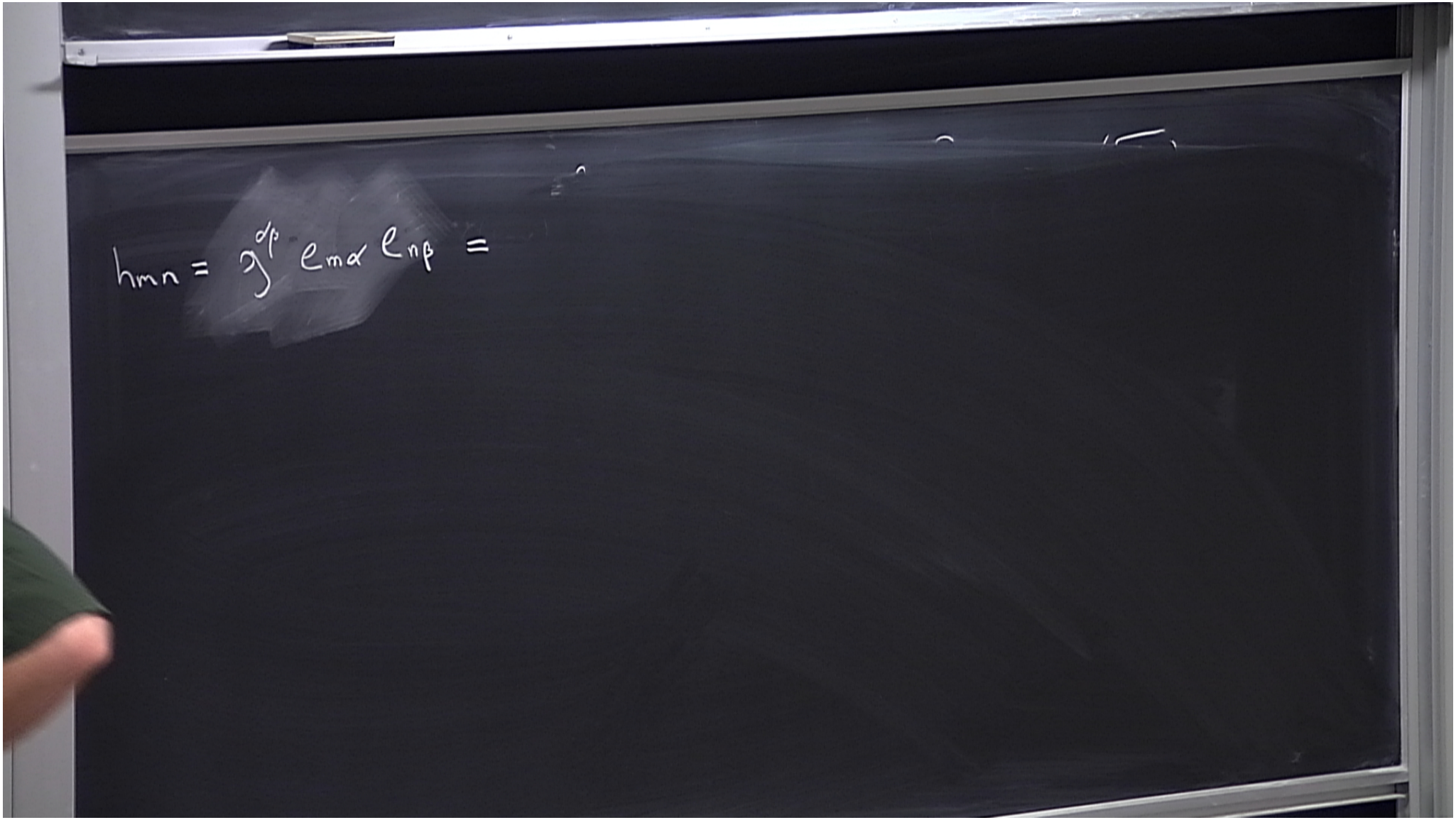
$$h^{\alpha\beta} = A^{ab} e^\alpha_a e^\beta_b$$

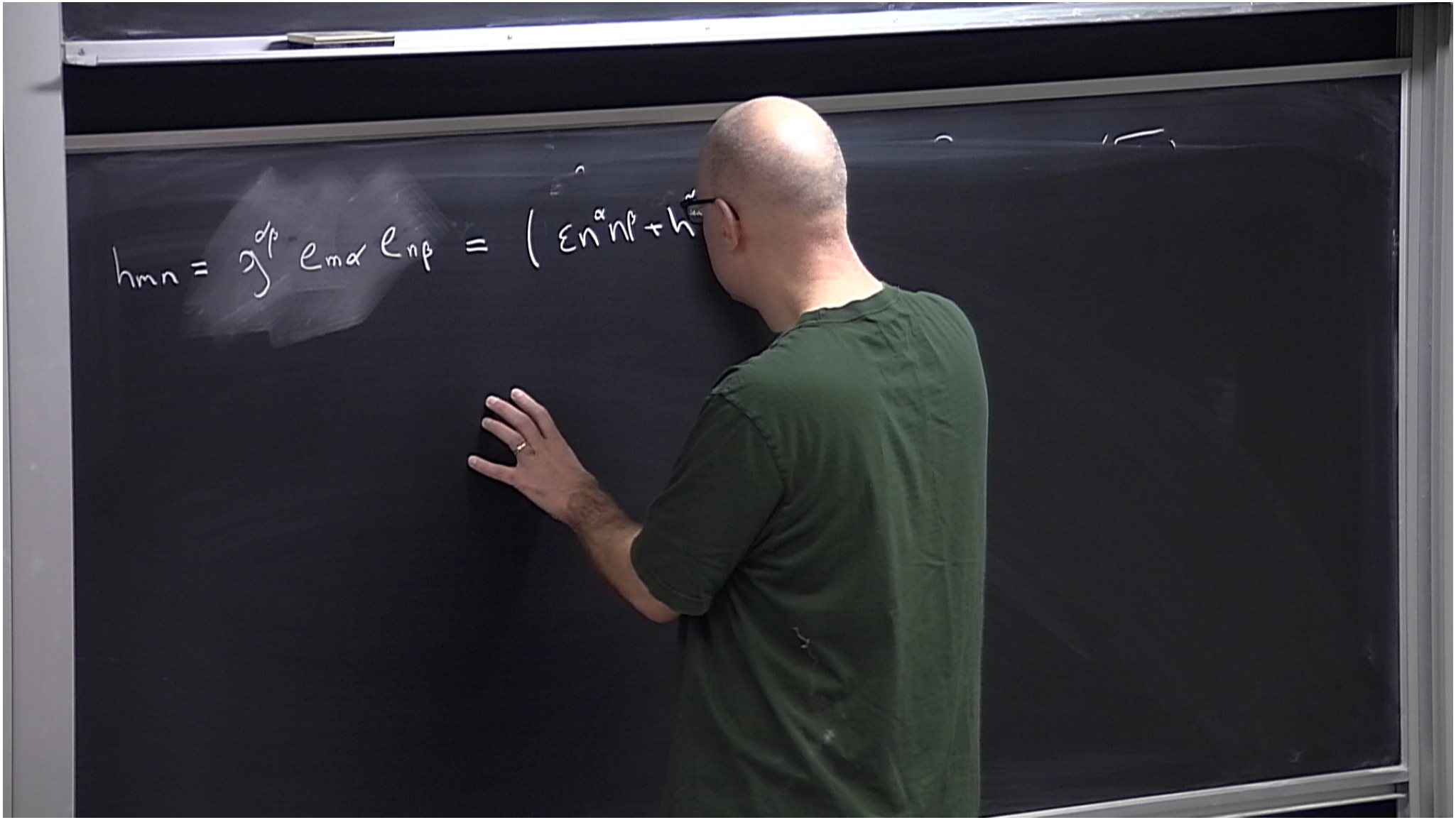
claim $A^{ab} = h^{ab}$; $h^{ab} h_{bc} = \delta^a_c$

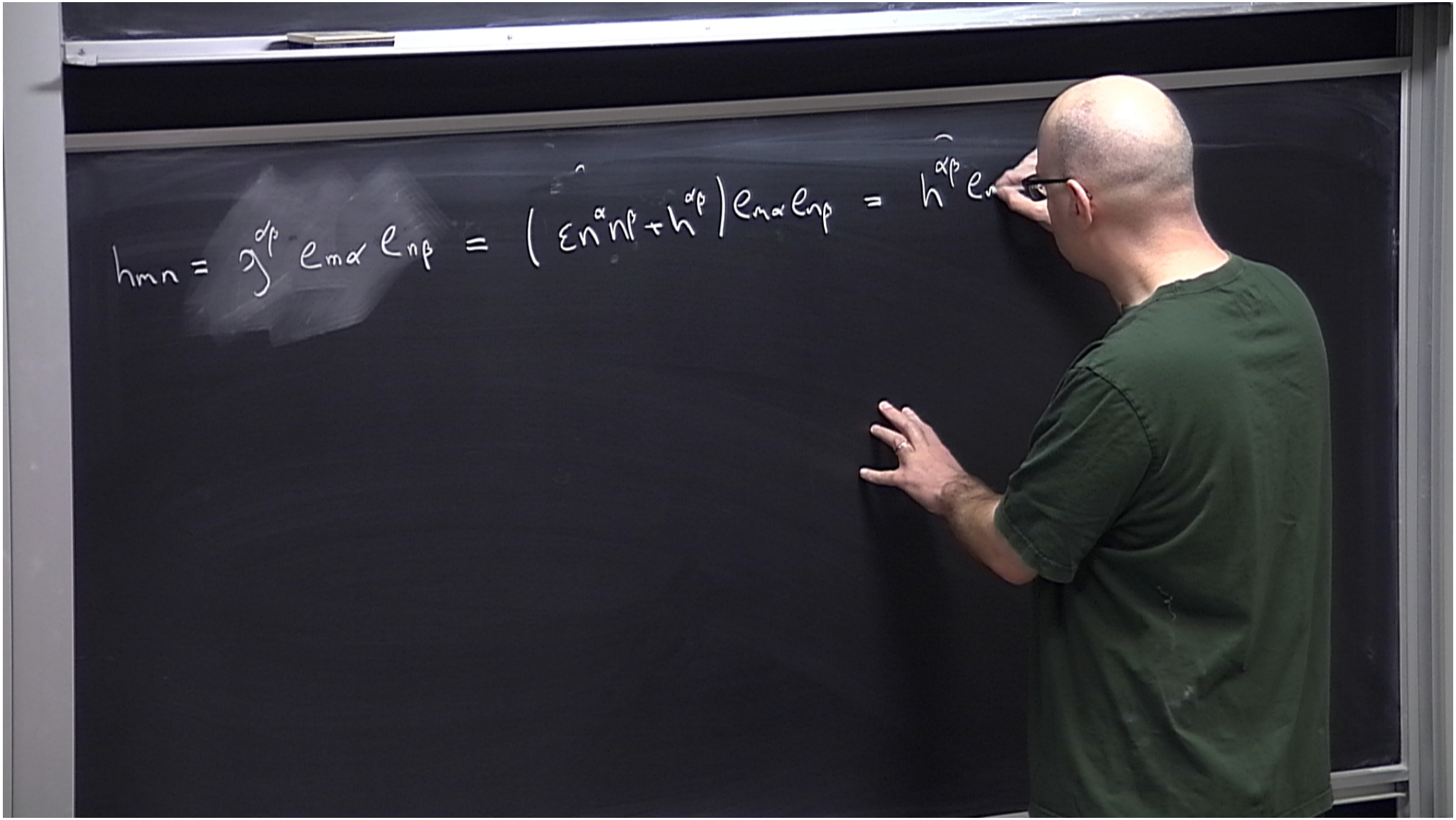
↳ inverse intrinsic metric.

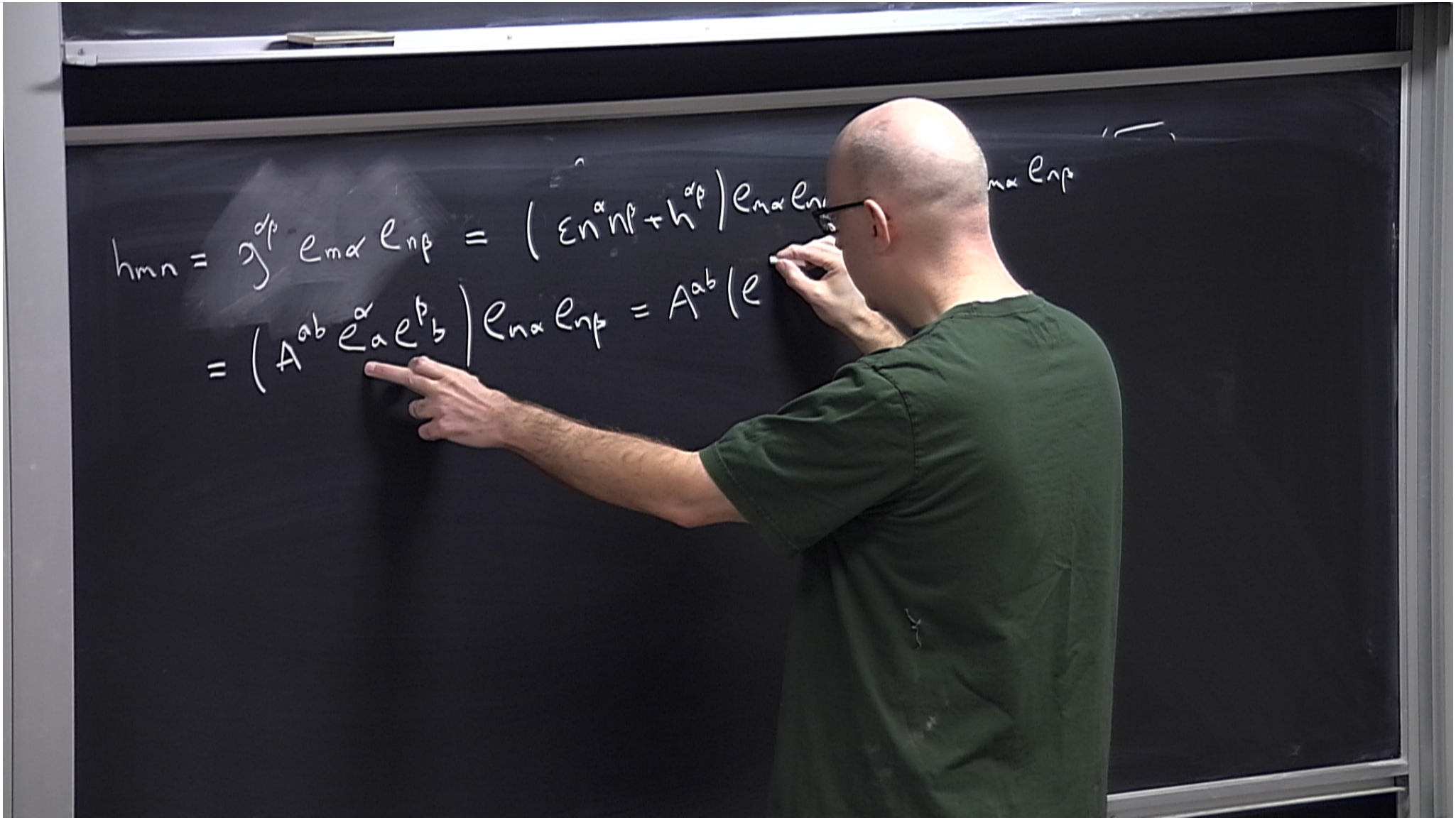
CAUTION

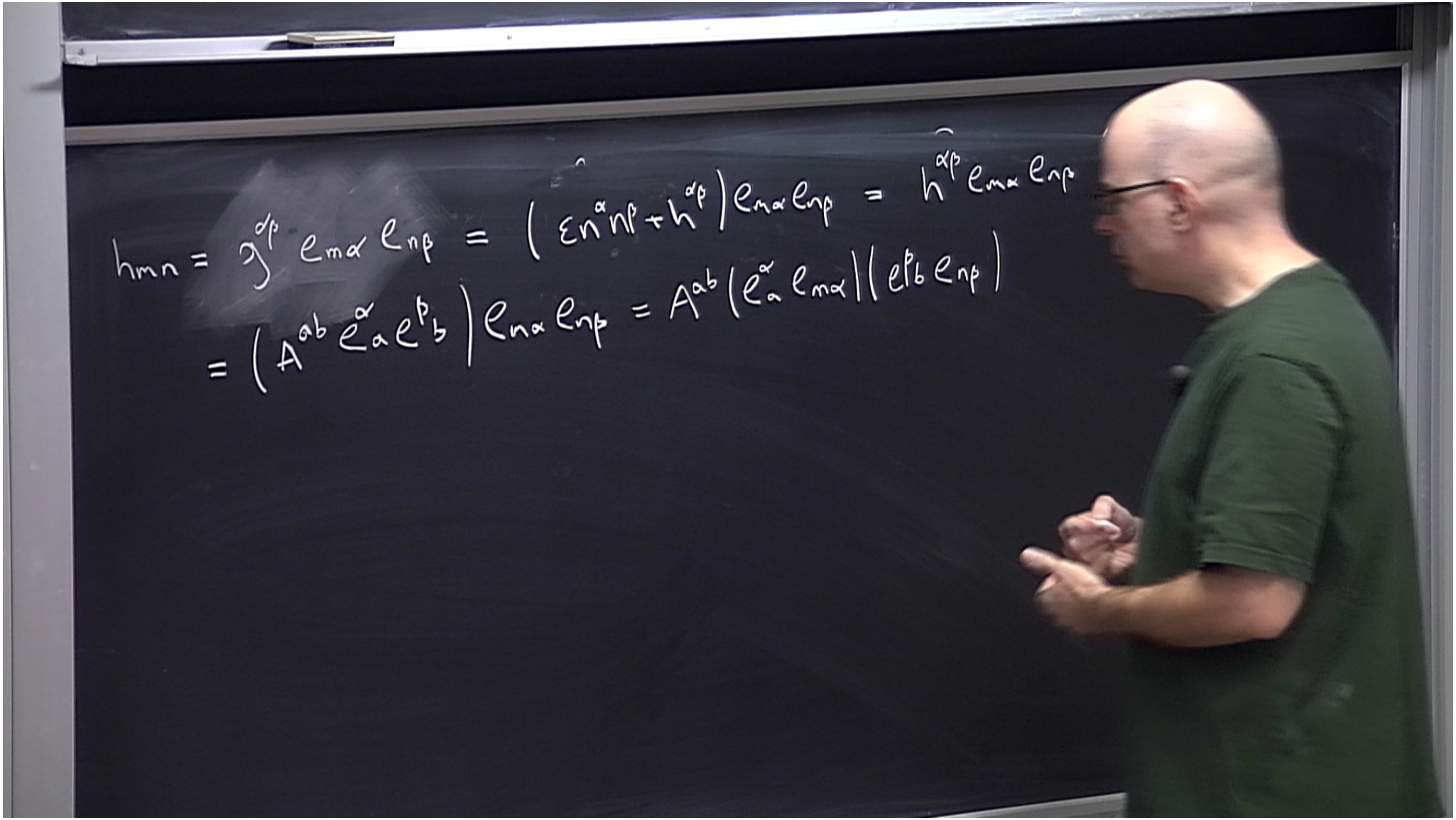

$\int^{\alpha\beta} e_{m\alpha} e_{n\beta}$

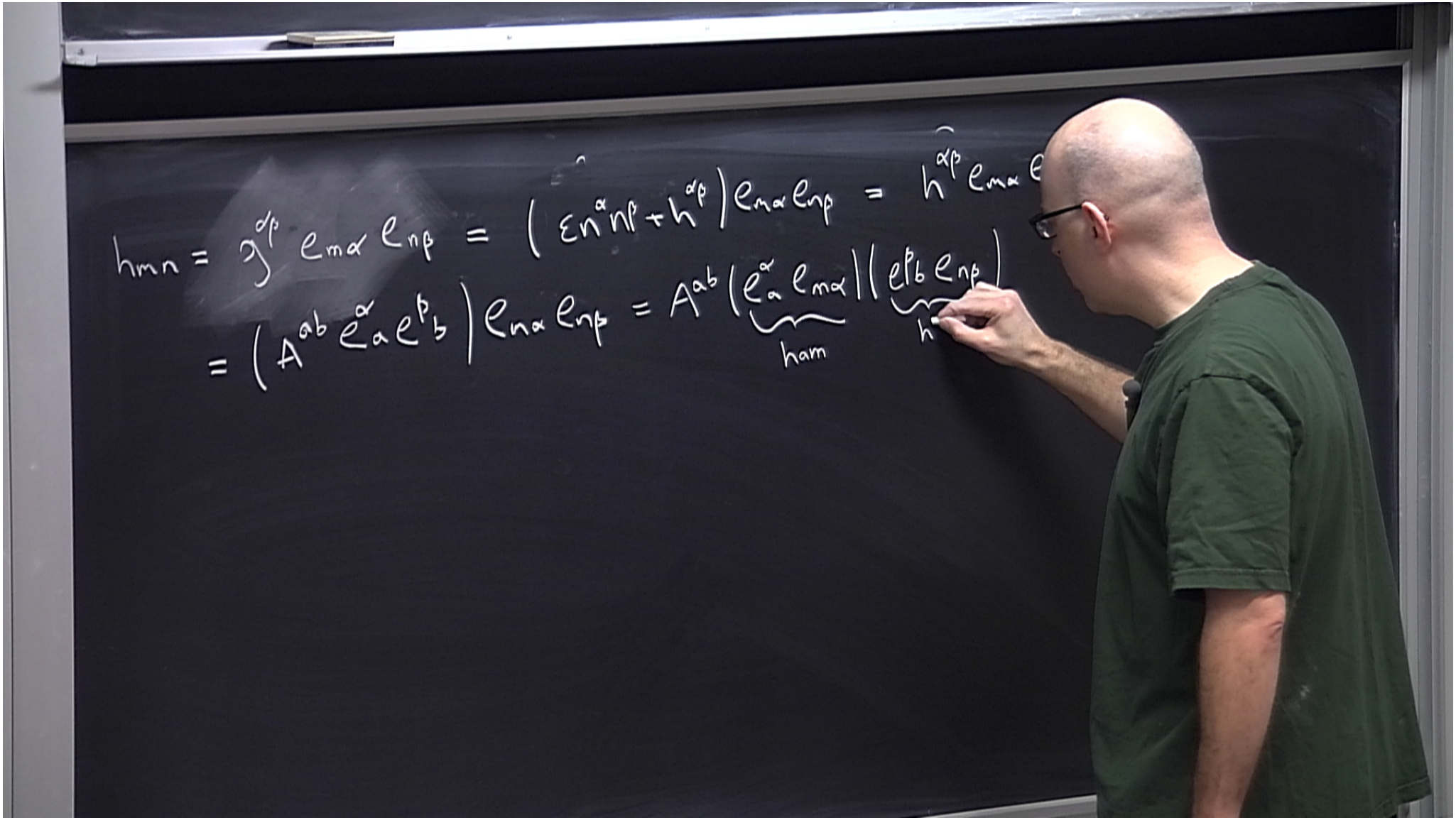

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} =$

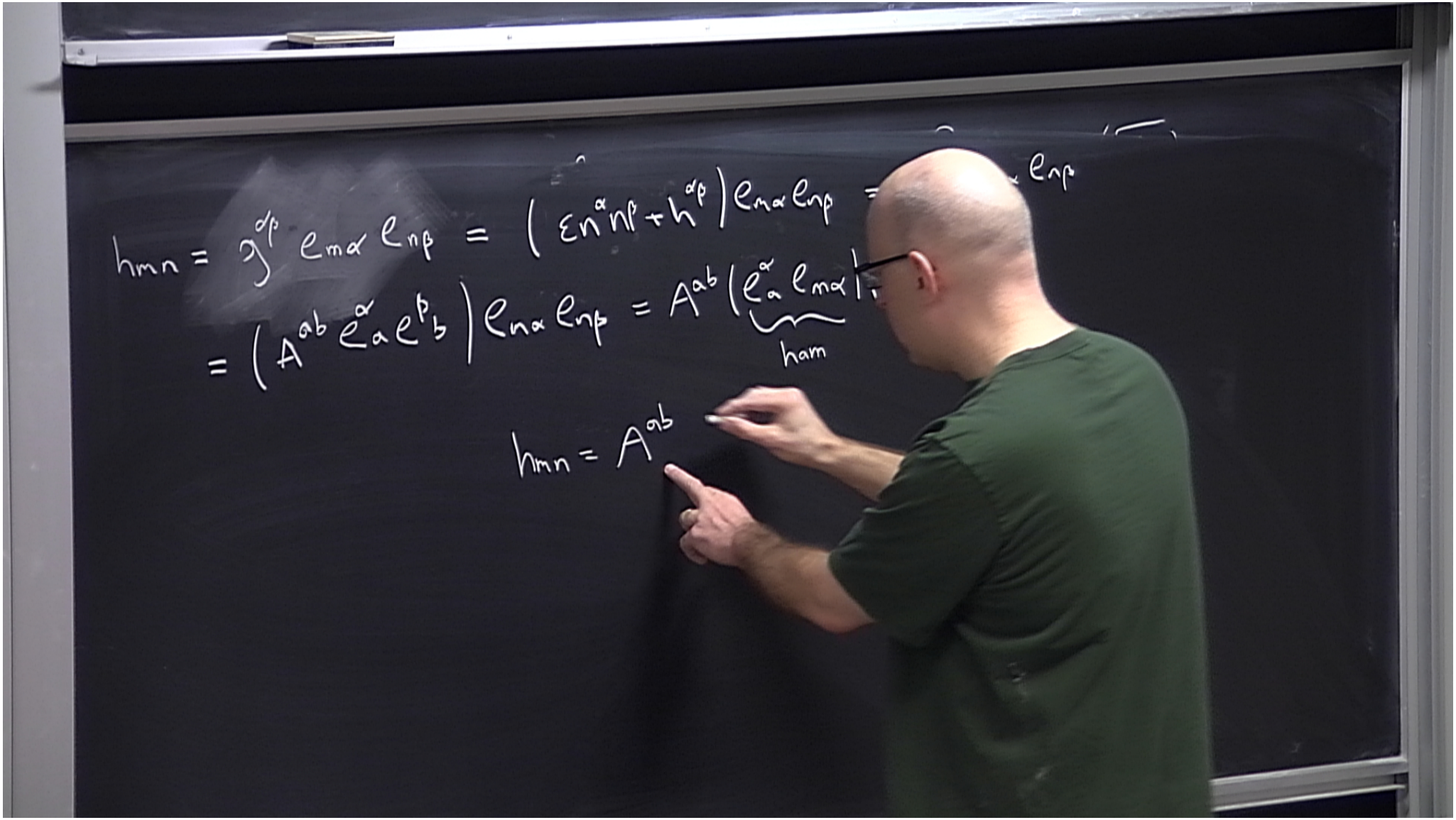

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = ($


$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^{\alpha} n^{\beta} + h^{\alpha\beta}) e_{m\alpha} e_{n\beta} = h^{\alpha\beta} e_{m}$


$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^\alpha n^\beta + h^{\alpha\beta}) e_{m\alpha} e_{n\beta}$
$= (A^{ab} e^\alpha_a e^\beta_b) e_{m\alpha} e_{n\beta} = A^{ab} (e$


$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^\alpha n^\beta + h^{\alpha\beta}) e_{m\alpha} e_{n\beta} = h^{\alpha\beta} e_{m\alpha} e_{n\beta}$
$= (A^{ab} e^\alpha_a e^\beta_b) e_{m\alpha} e_{n\beta} = A^{ab} (e^\alpha_a e_{m\alpha})(e^\beta_b e_{n\beta})$

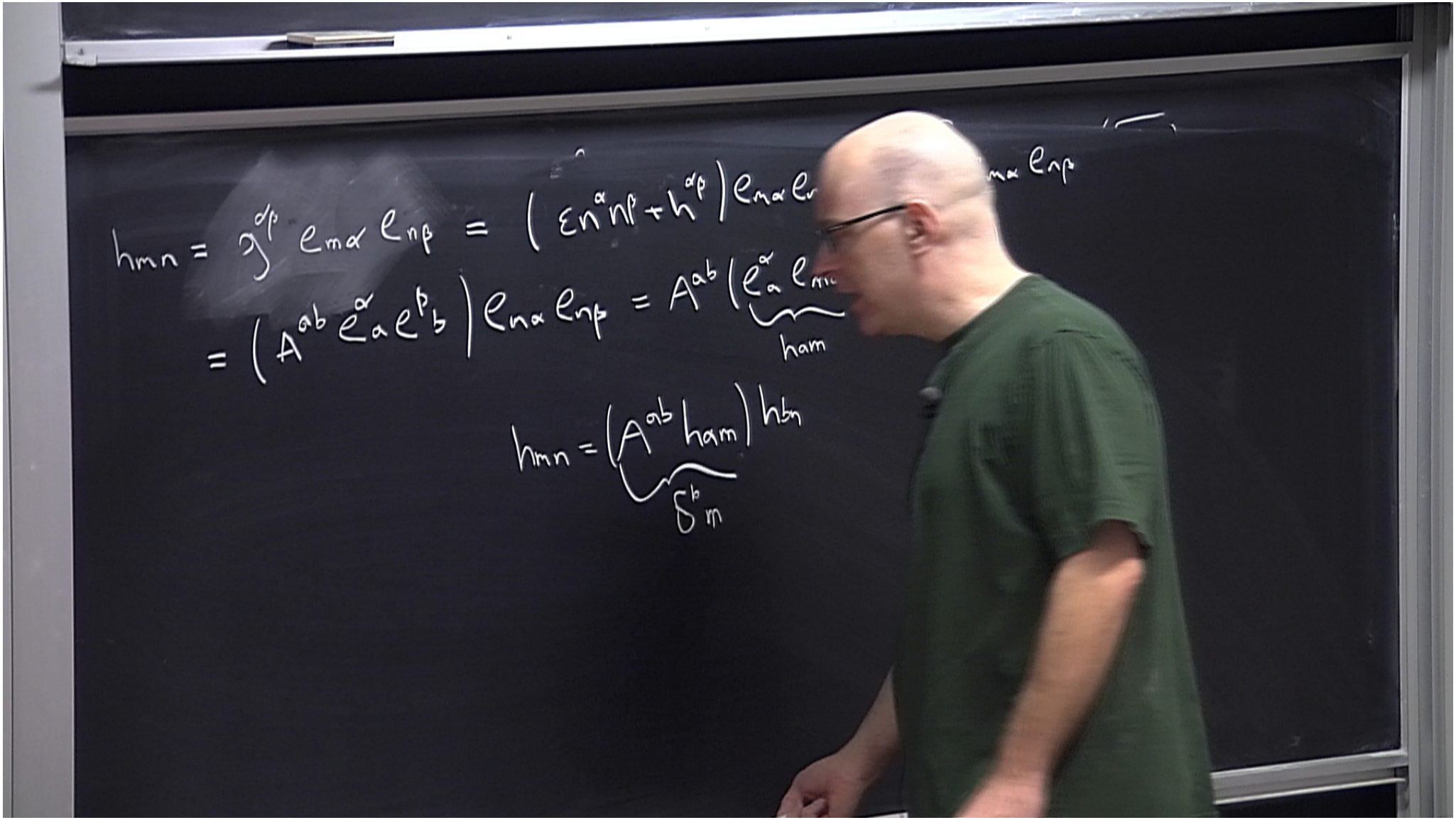

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^\alpha n^\beta + h^{\alpha\beta}) e_{m\alpha} e_{n\beta} = h^{\alpha\beta} e_{m\alpha} e$
$= (A^{ab} e^\alpha_a e^\beta_b) e_{m\alpha} e_{n\beta} = A^{ab} (e^\alpha_a e_{m\alpha})(e^\beta_b e_{n\beta})$
ham

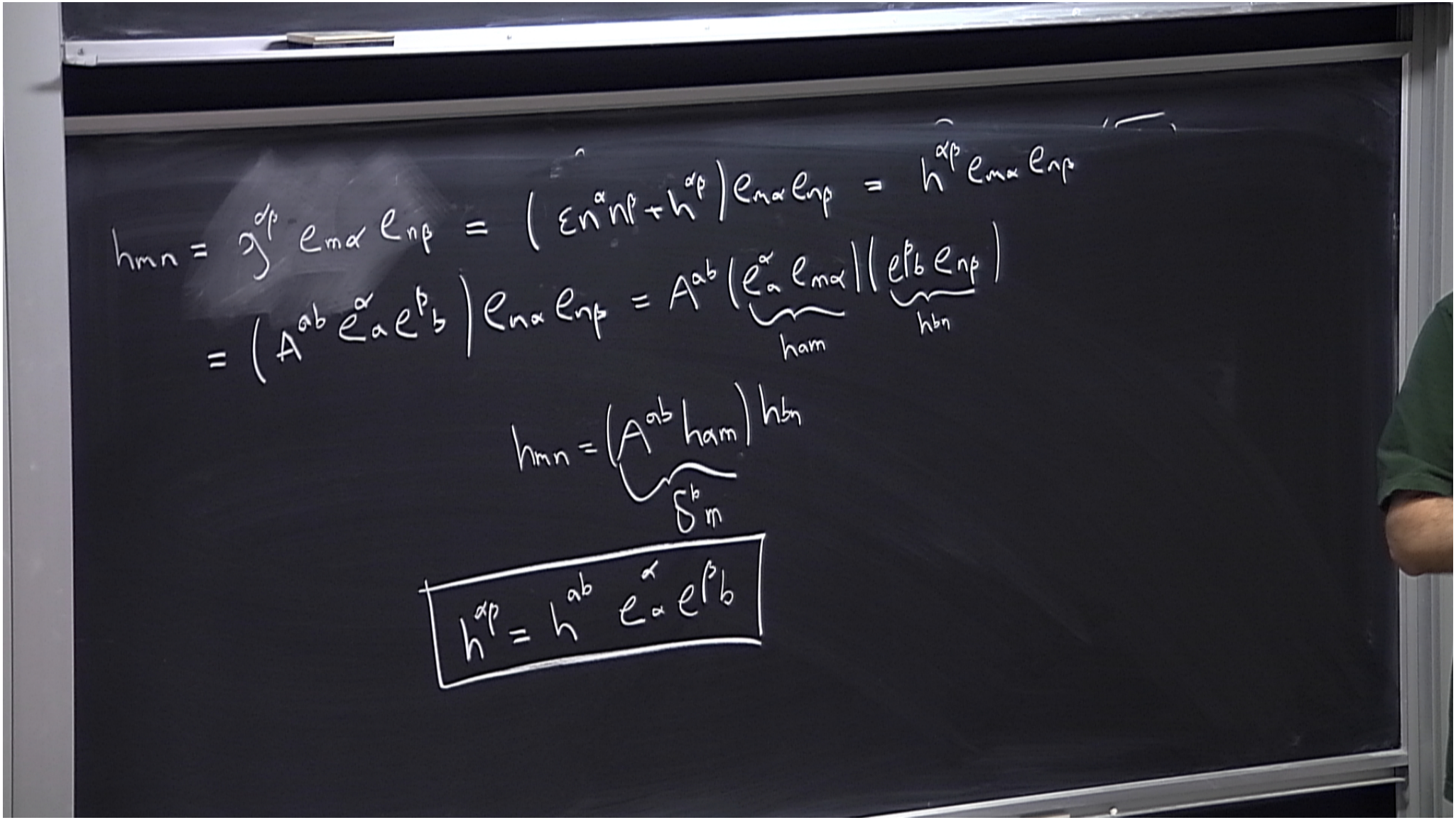

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^\alpha n^\beta + h^{\alpha\beta}) e_{m\alpha} e_{n\beta}$
$= (A^{ab} e^\alpha_a e^\beta_b) e_{m\alpha} e_{n\beta} = A^{ab} (e^\alpha_a e_{m\alpha})$
ham
$h_{mn} = A^{ab}$

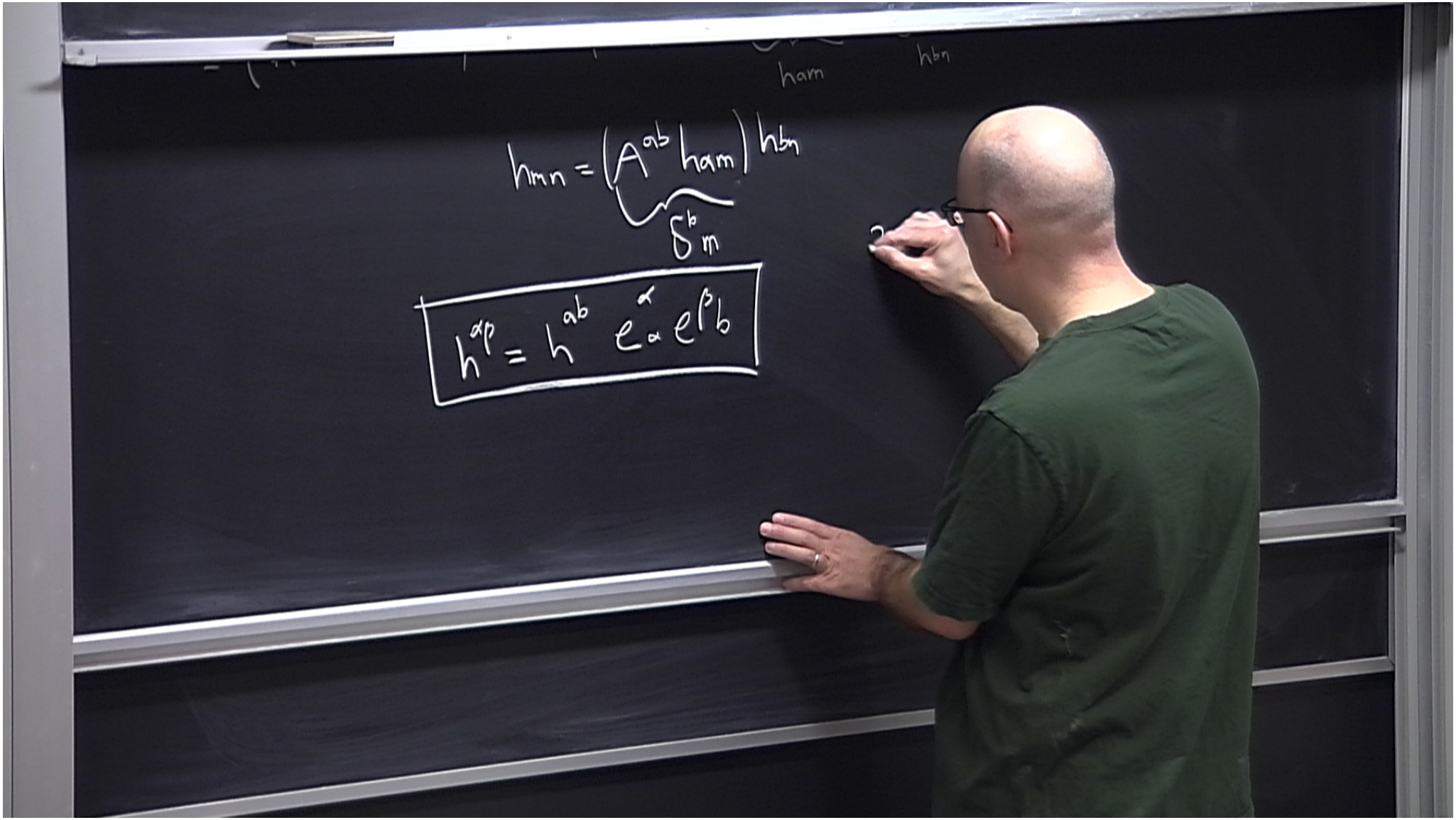
$$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = \left(\varepsilon n^{\alpha} n^{\beta} + h^{\alpha\beta} \right) e_{m\alpha} e_{n\beta} = h^{\alpha\beta} e_{m\alpha} e_{n\beta}$$

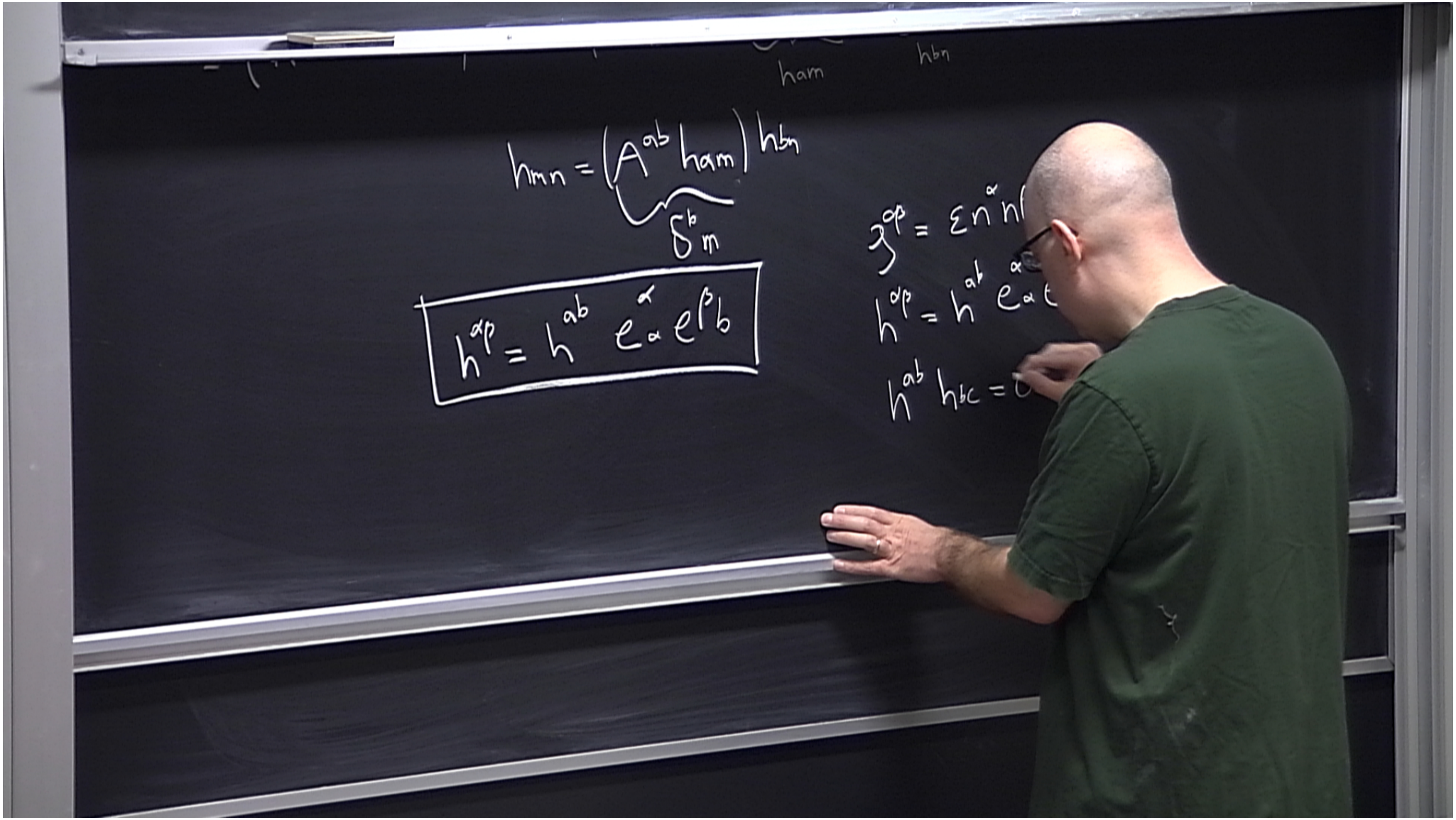
$$= \left(A^{ab} e^{\alpha}_{a} e^{\beta}_{b} \right) e_{m\alpha} e_{n\beta} = A^{ab} \underbrace{\left(e^{\alpha}_{a} e_{m\alpha} \right)}_{h_{am}} \underbrace{\left(e^{\beta}_{b} e_{n\beta} \right)}_{h_{bn}}$$

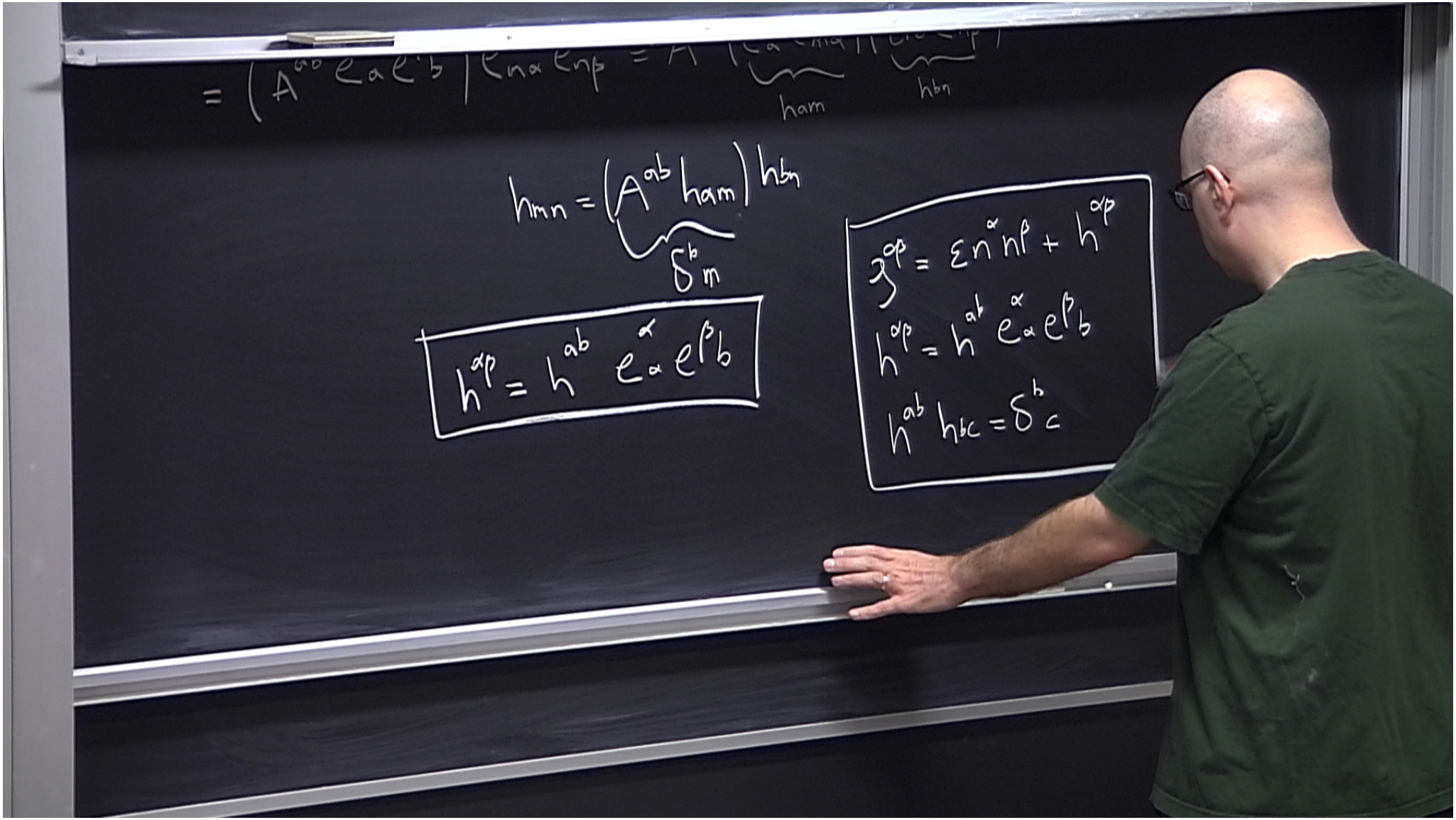
$$h_{mn} = \left(A^{ab} h_{am} \right) h_{bn}$$

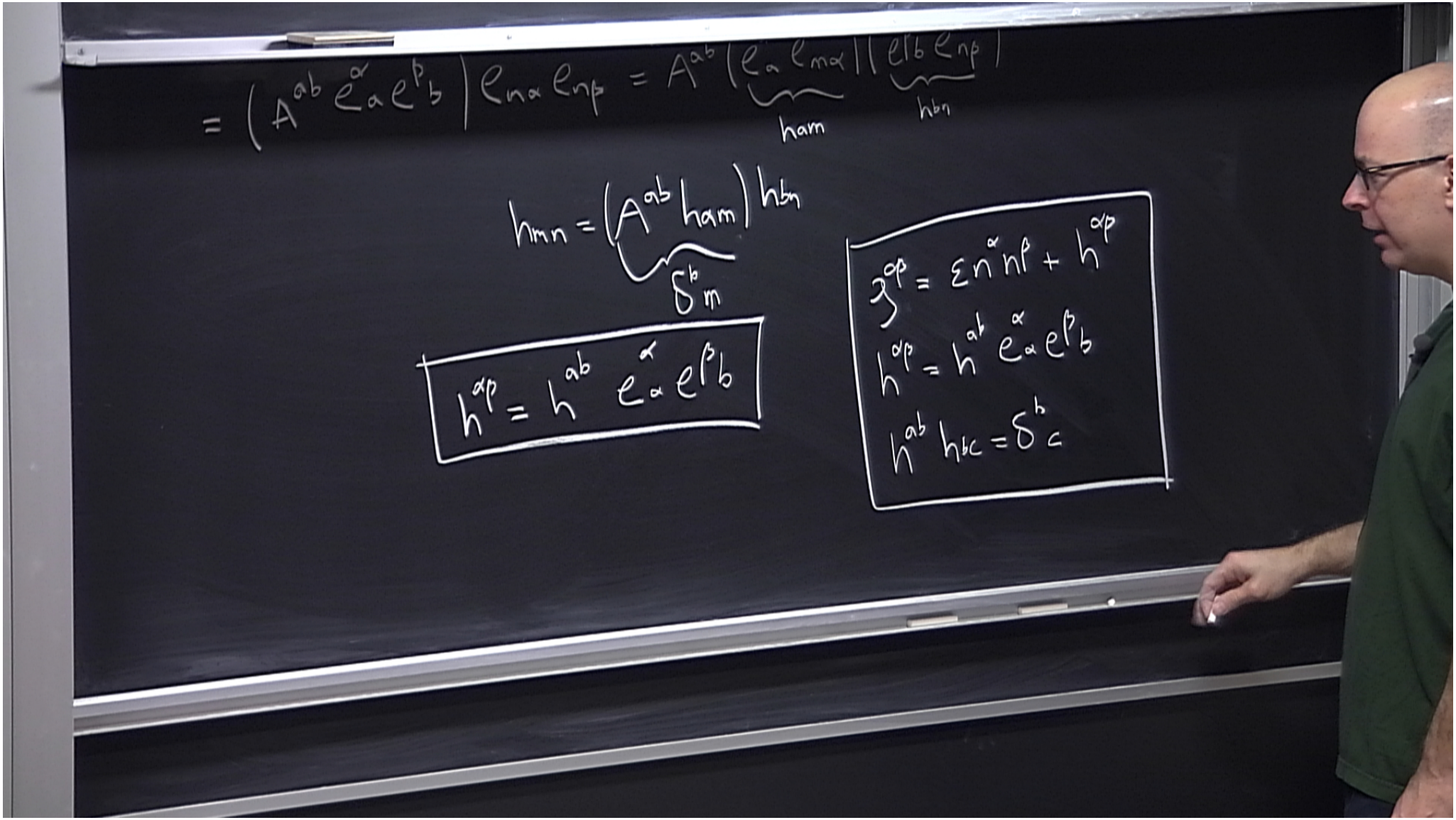

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^{\alpha} n^{\beta} + h^{\alpha\beta}) e_{m\alpha} e_{n\beta}$
$= (A^{ab} e^{\alpha}_{a} e^{\beta}_{b}) e_{m\alpha} e_{n\beta} = A^{ab} (e^{\alpha}_{a} e_{m\alpha})$
h_{am}
$h_{mn} = (A^{ab} h_{am}) h_{bn}$
δ^{b}_{m}

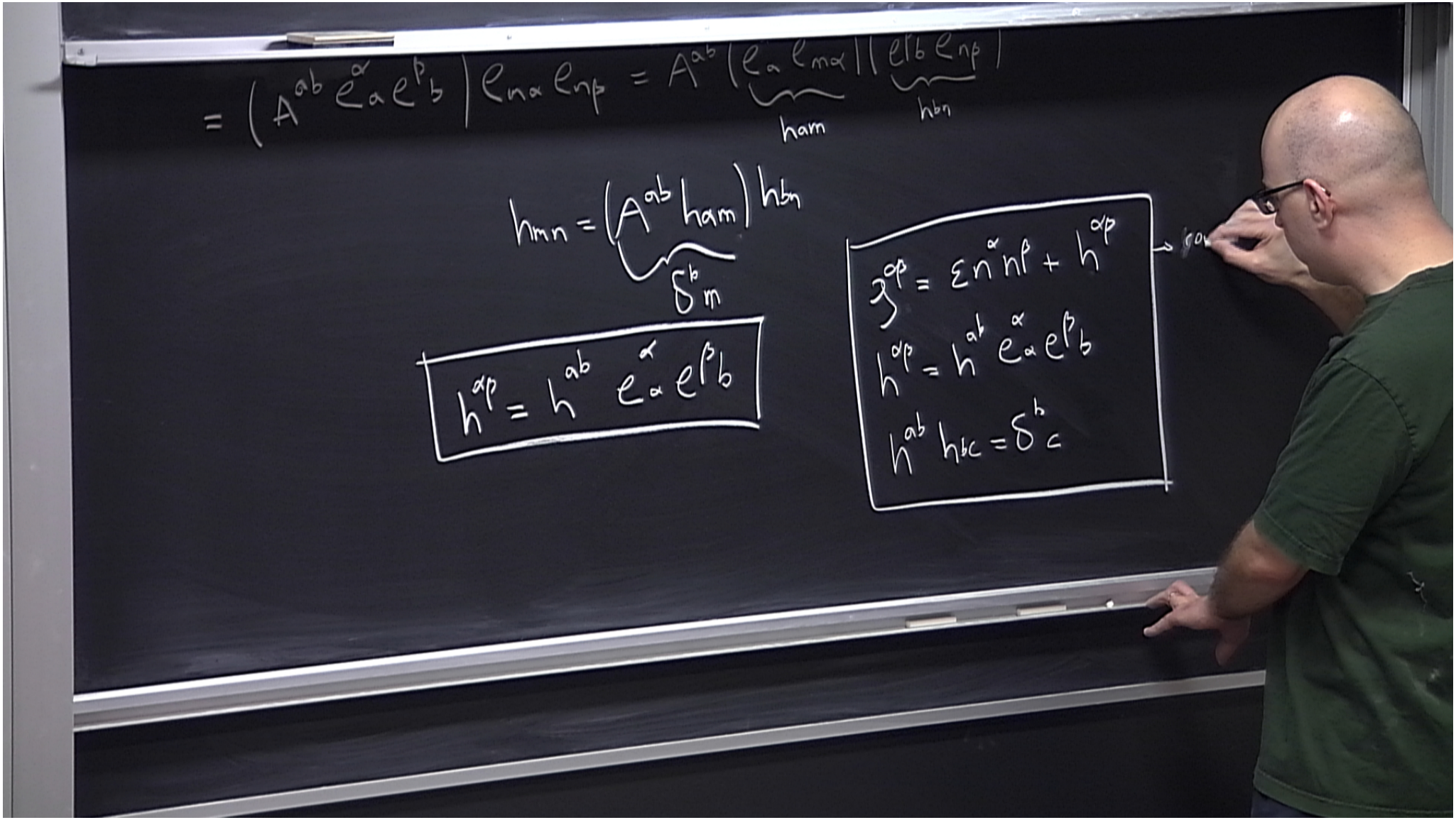

$h_{mn} = g^{\alpha\beta} e_{m\alpha} e_{n\beta} = (\varepsilon n^\alpha n^\beta + h^{\alpha\beta}) e_{m\alpha} e_{n\beta} = h^{\alpha\beta} e_{m\alpha} e_{n\beta}$
$= (A^{ab} e^\alpha_a e^\beta_b) e_{m\alpha} e_{n\beta} = A^{ab} (\underbrace{e^\alpha_a e_{m\alpha}}_{h_{am}} | \underbrace{e^\beta_b e_{n\beta}}_{h_{bn}})$
$h_{mn} = (\underbrace{A^{ab} h_{am}}_{\delta^b_m}) h_{bn}$
$h^{\alpha\beta} = h^{ab} e^\alpha_a e^\beta_b$

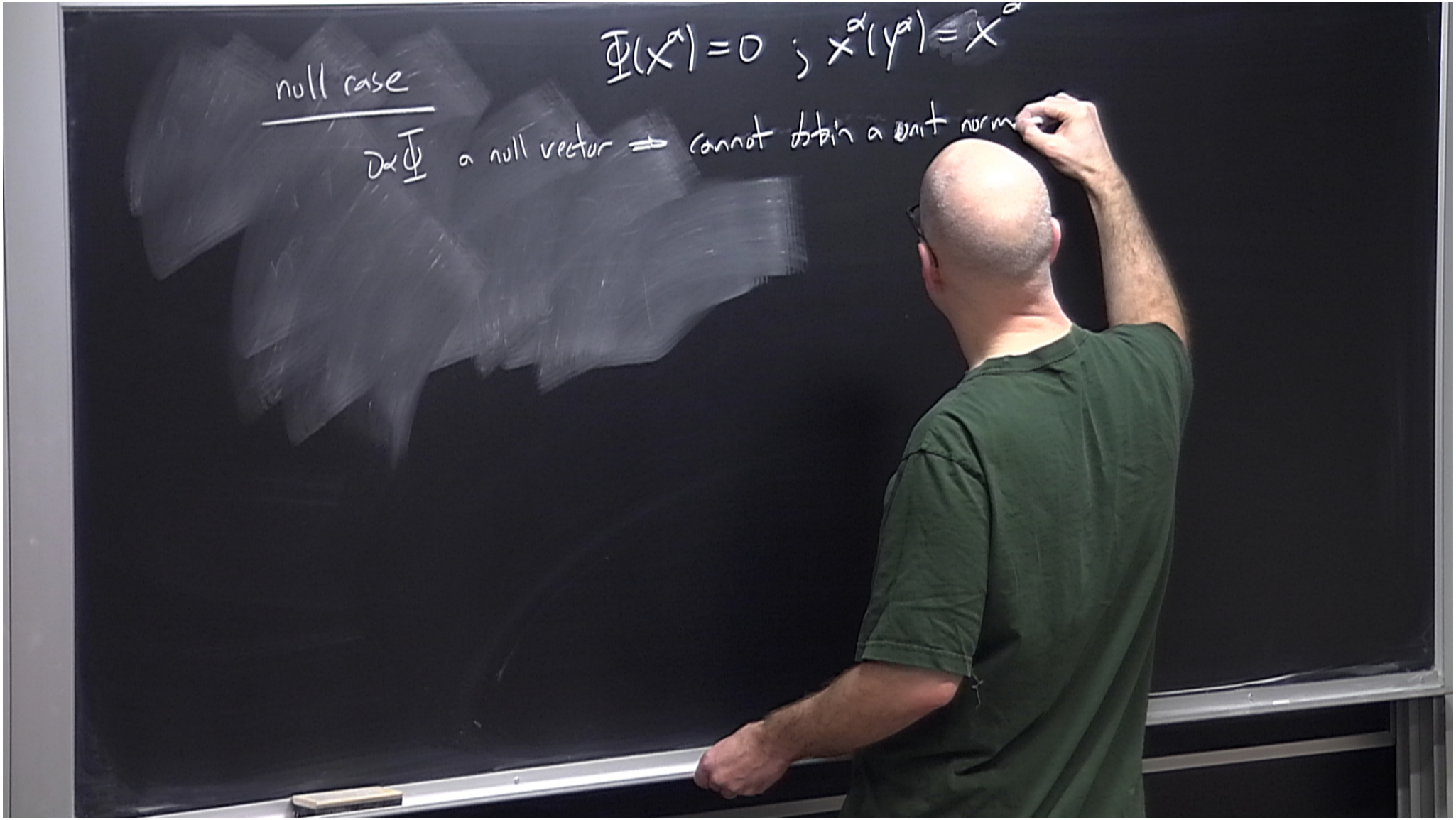

$h_{mn} = (A^{ab} h_{am}) h_{bn}$
δ^b_m
$h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$

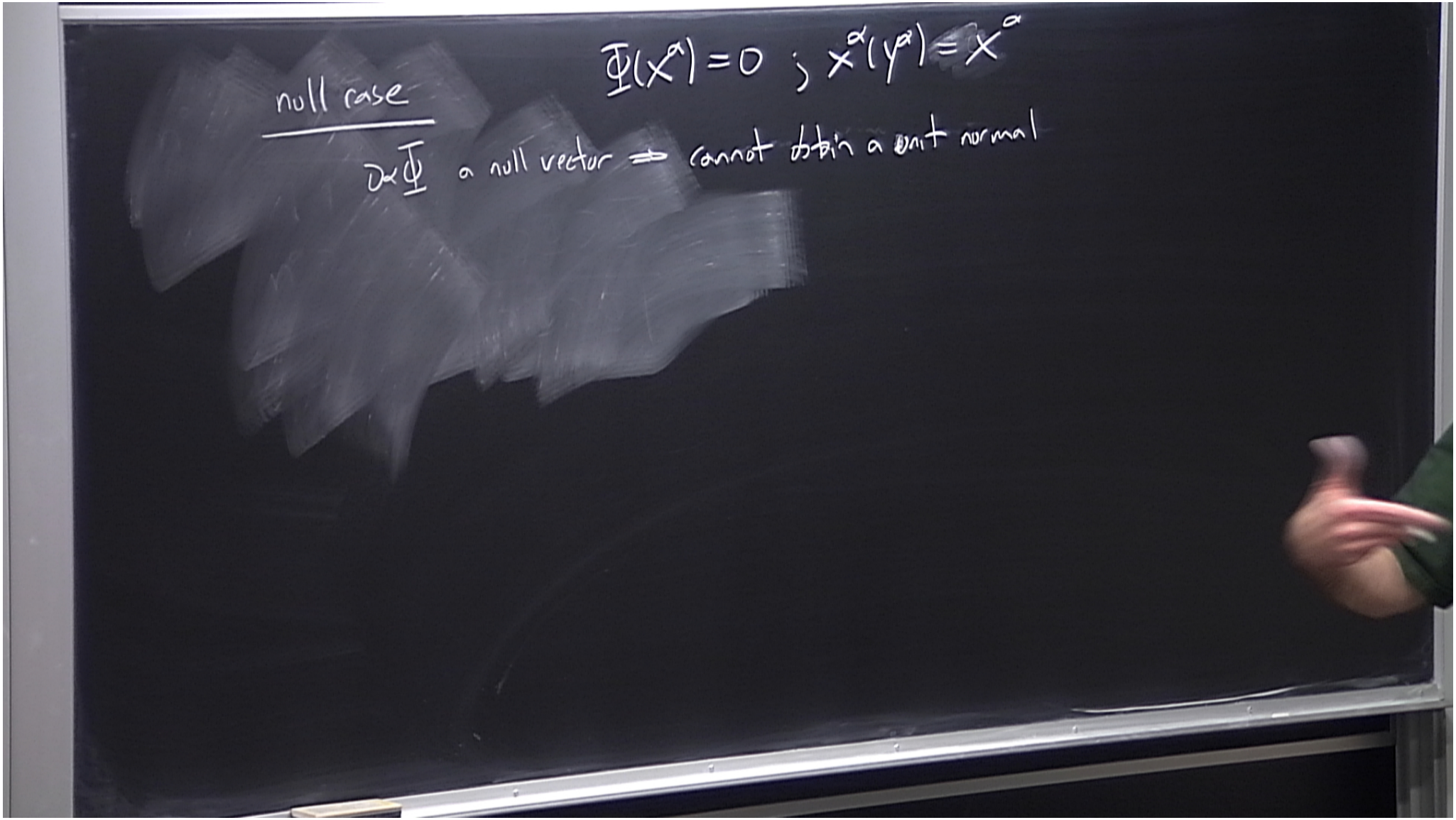

$h_{mn} = (A^{ab} h_{am}) h_{bn}$
δ^b_m
$h^{\alpha\beta} = h^{ab} e^\alpha_a e^\beta_b$
$h^{ab} h_{bc} =$

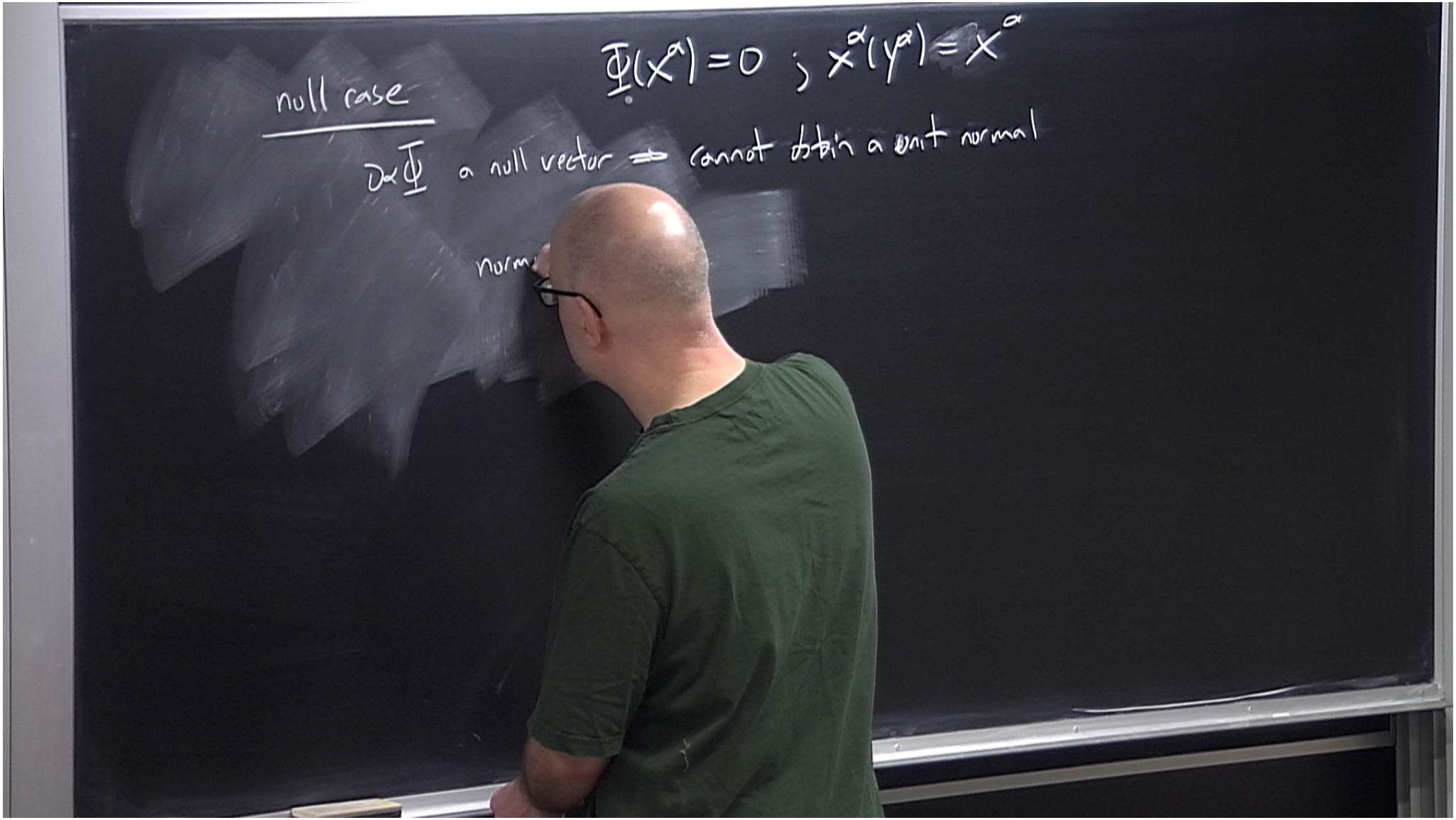

$h_{mn} = (A^{ab} h_{am}) h_{bn}$
δ^b_m
$h^{\alpha\beta} = h^{ab} e^\alpha_a e^\beta_b$
$g^{\alpha\beta} = \varepsilon n^\alpha n^\beta + h^{\alpha\beta}$
$h^{\alpha\beta} = h^{ab} e^\alpha_a e^\beta_b$
$h^{ab} h_{bc} = \delta^b_c$


$= (A^{ab} e^{\alpha}_{a} e^{\beta}_{b} \mid e_{m\alpha} e_{n\beta} = A^{ab} (\underbrace{e_{a} e_{m\alpha}}_{h_{am}} \mid \underbrace{e_{b} e_{n\beta}}_{h_{bn}})$
$h_{mn} = (\underbrace{A^{ab} h_{am}}_{\delta^{b}_{m}}) h_{bn}$
$h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$
$g^{\alpha\beta} = \varepsilon n^{\alpha} n^{\beta} + h^{\alpha\beta}$
$h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$
$h^{ab} h_{bc} = \delta^{b}_{c}$


$= (A^{ab} e^{\alpha}_{a} e^{\beta}_{b} \mid e_{m\alpha} e_{n\beta} = A^{ab} (e_{a} e_{m\alpha} \mid e_{b} e_{n\beta})$
h_{am}
h_{bn}
$h_{mn} = (A^{ab} h_{am}) h_{bn}$
δ^{b}_{m}
$h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$
$g^{\alpha\beta} = \varepsilon n^{\alpha} n^{\beta} + h^{\alpha\beta}$
$h^{\alpha\beta} = h^{ab} e^{\alpha}_{a} e^{\beta}_{b}$
$h^{ab} h_{bc} = \delta^{b}_{c}$


null case
$\Phi(x^\alpha) = 0$; $x^\alpha(\gamma^\alpha) = x^\alpha$
$\partial_\alpha \Phi$ a null vector ⇒ cannot obtain a unit norm


null case
$\Phi(x^\alpha) = 0$; $x^\alpha(y^a) = x^\alpha$
$\partial_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal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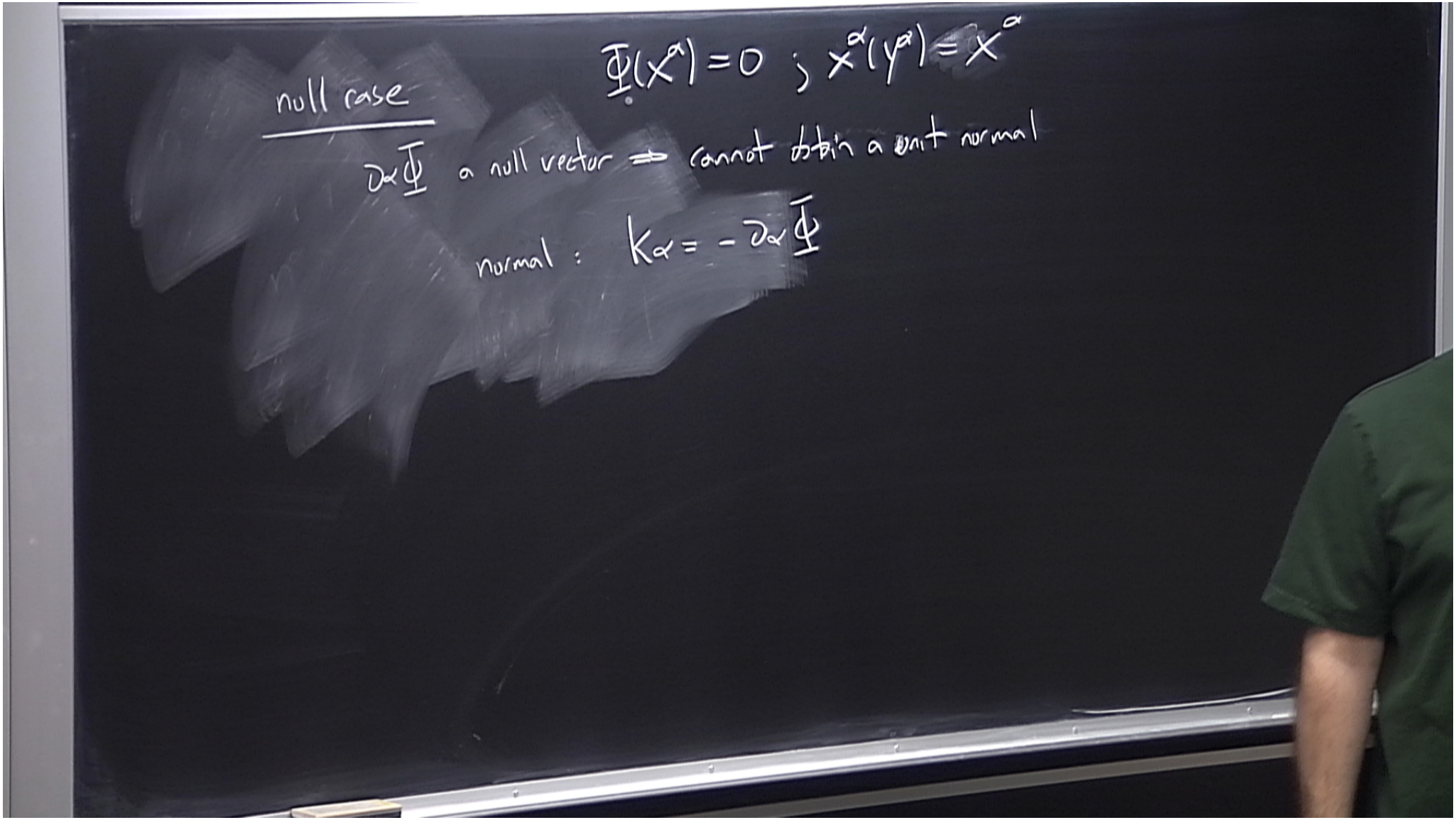
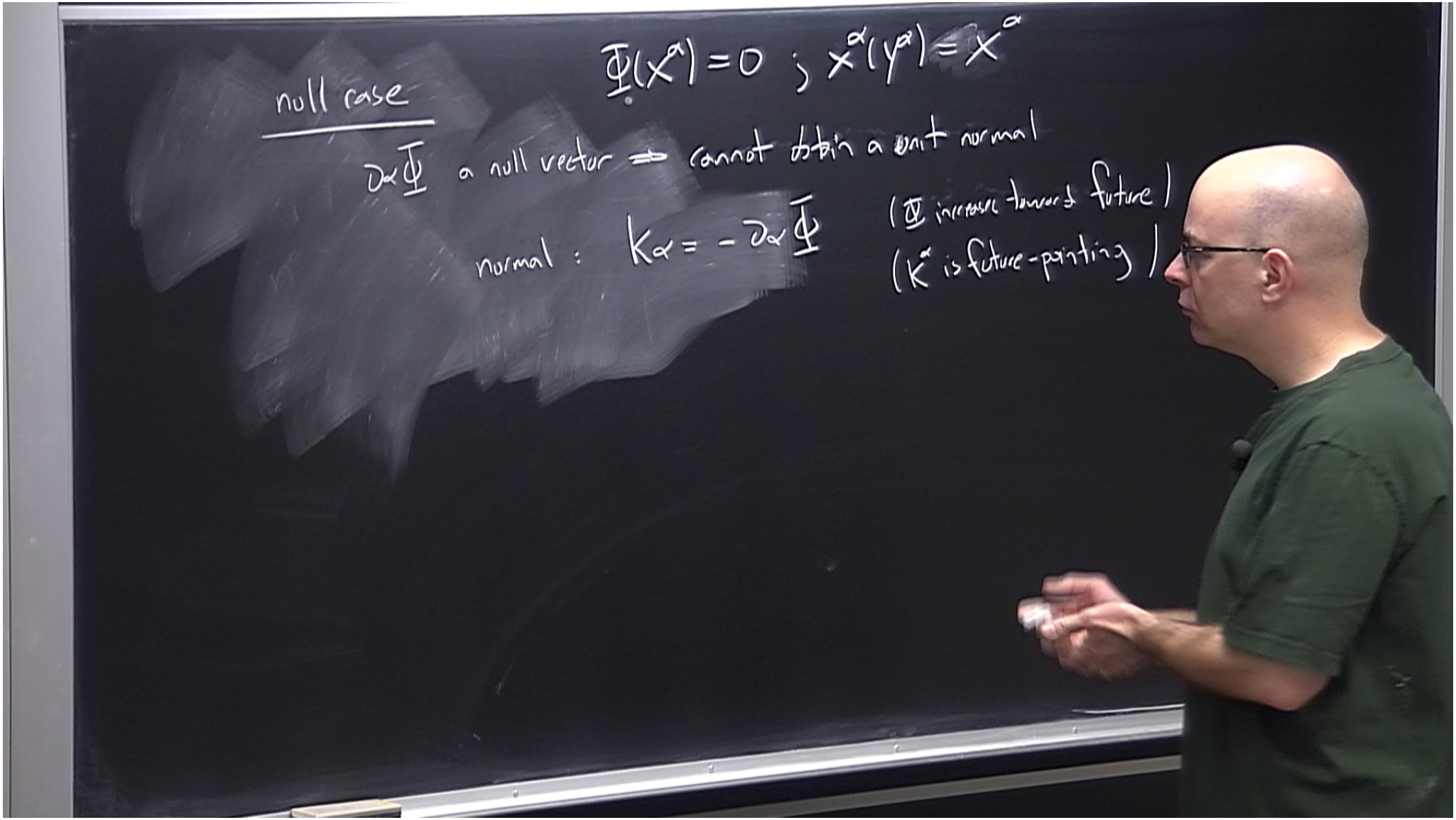
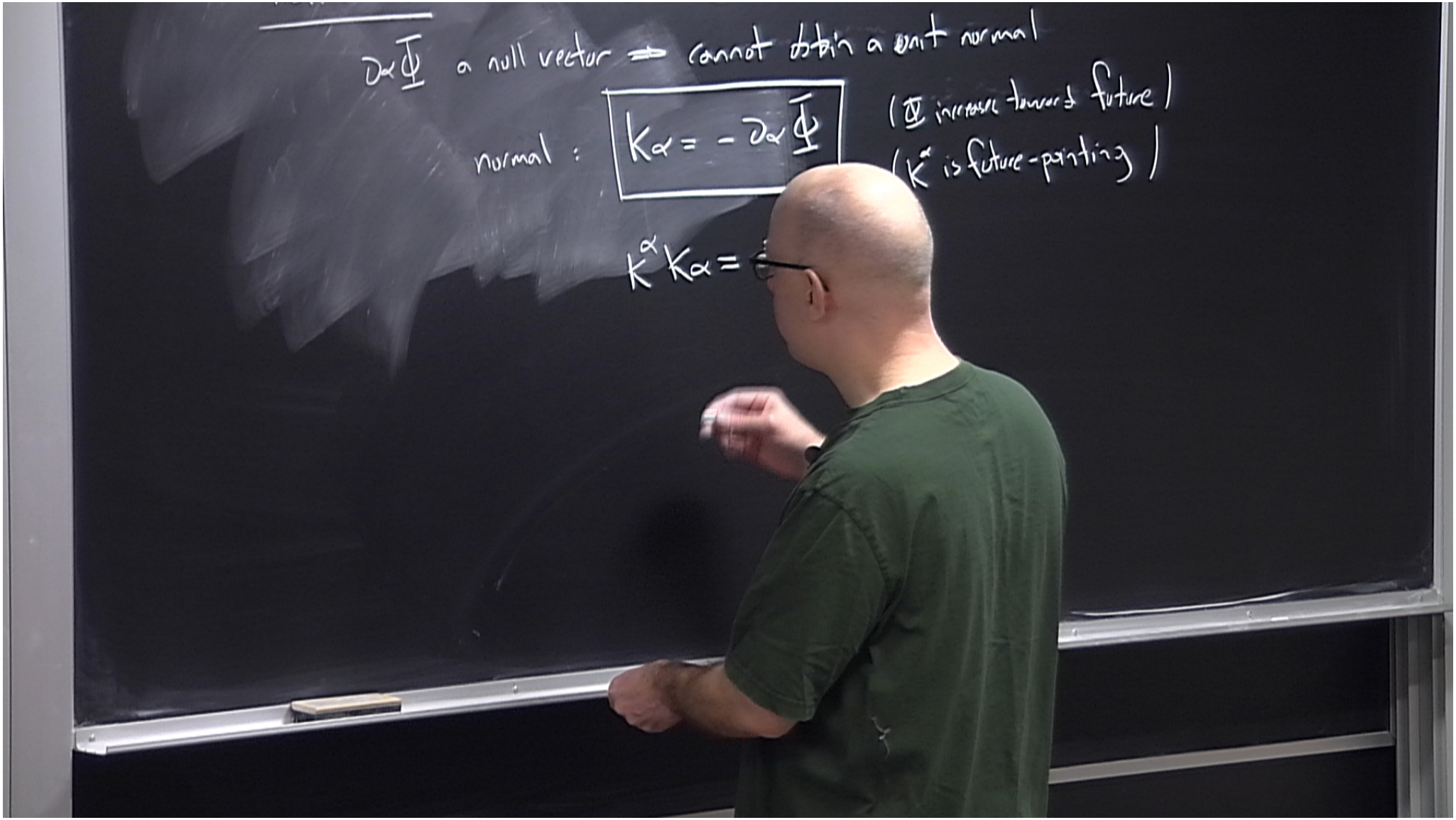

null case
$\Phi(x^\alpha) = 0$; $x^\alpha(y^a) = x^a$
$\partial_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal
normal : $k_\alpha = -\partial_\alpha \Phi$


null case
$\Phi(x^\alpha) = 0$; $x^\alpha(\gamma^\beta) = x^\alpha$
$\partial_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal
normal : $k_\alpha = -\partial_\alpha \Phi$
(Φ increases toward future)
(k^α is future-pointing)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normal : $k_\alpha = -\partial_\alpha \Phi$
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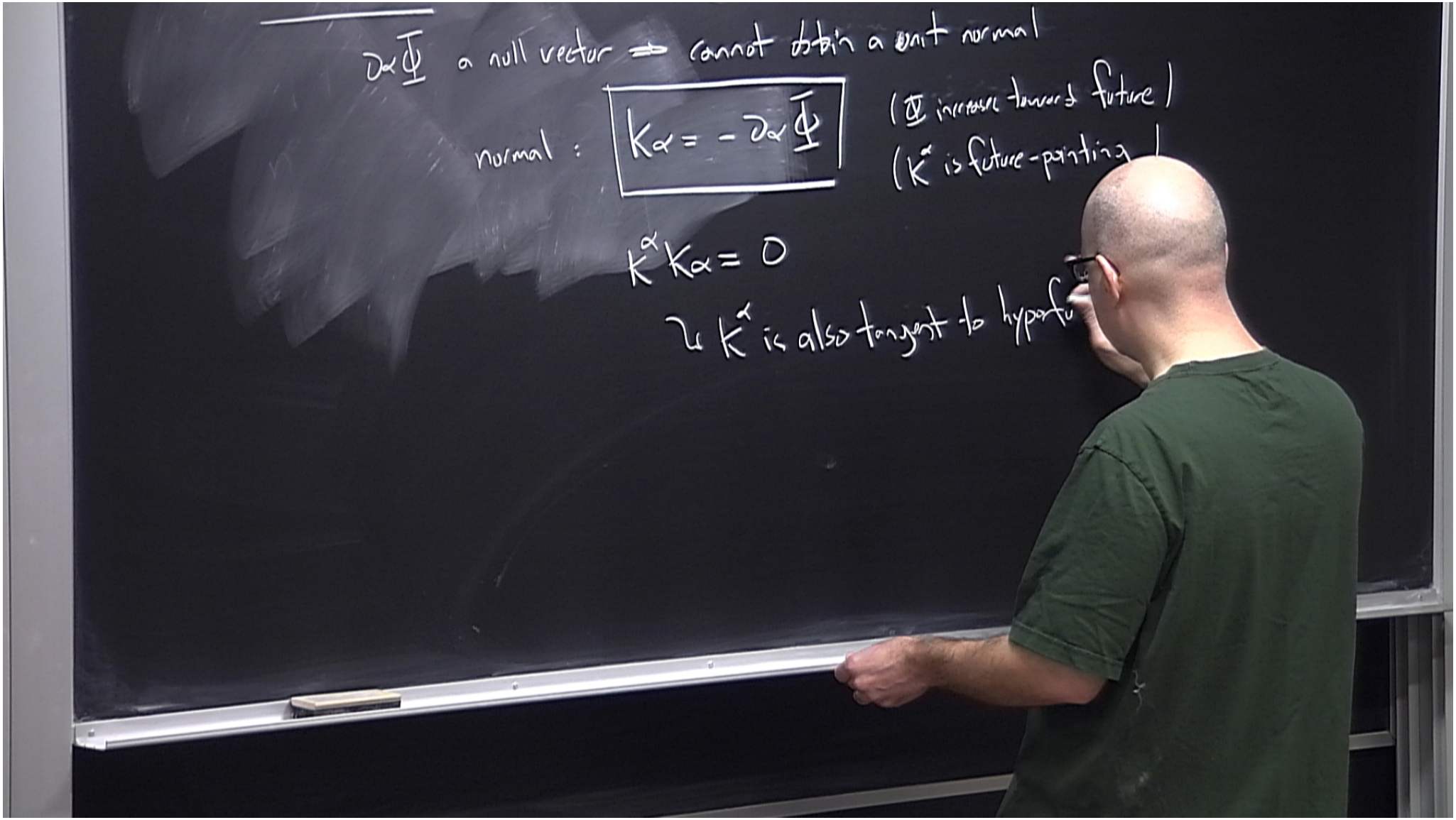
$k^\alpha k_\alpha =$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$k^\alpha k_\alpha = 0$
$\Rightarrow k^\alpha$ is also tangent to hypersu

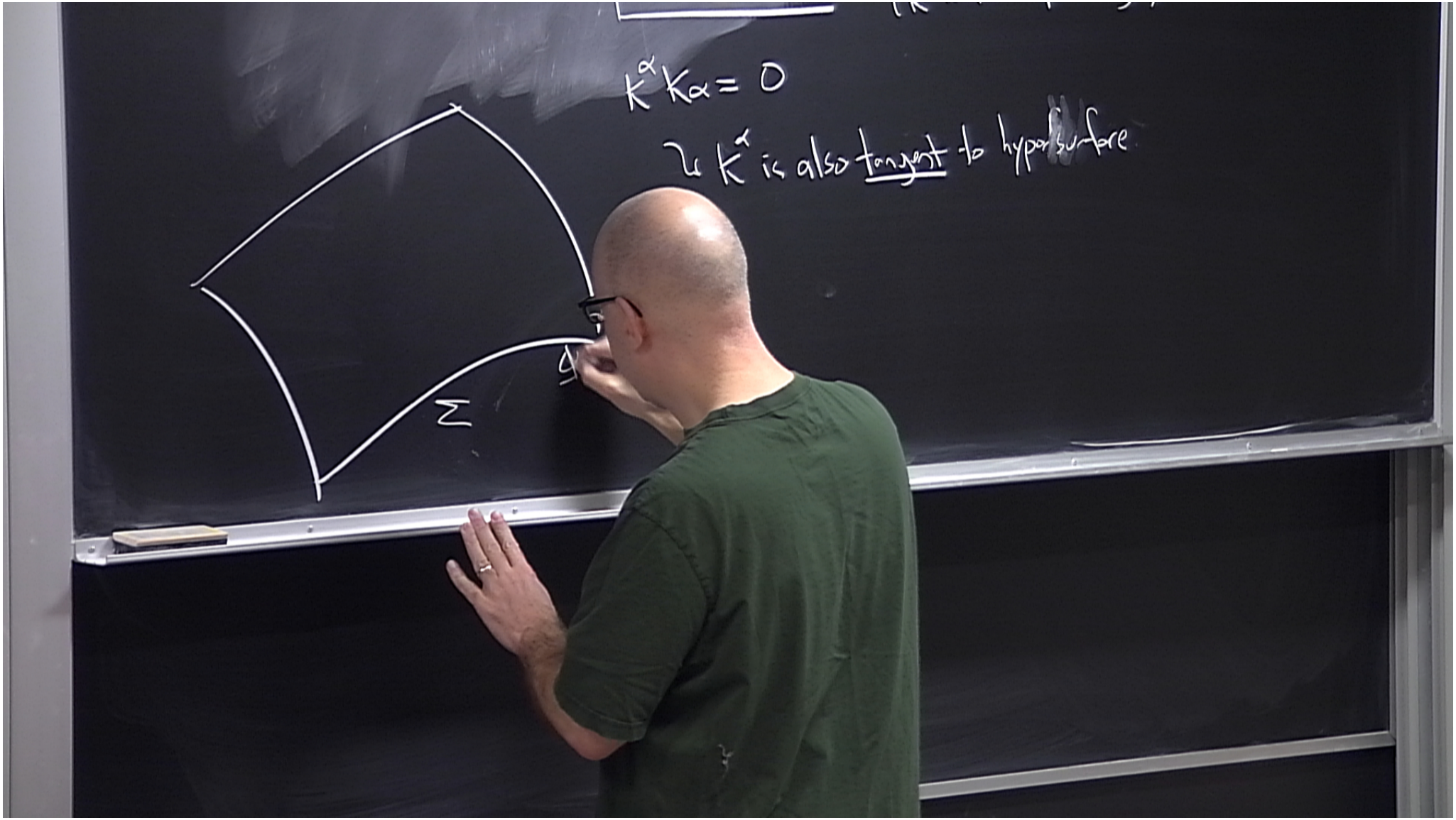
$\partial_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal

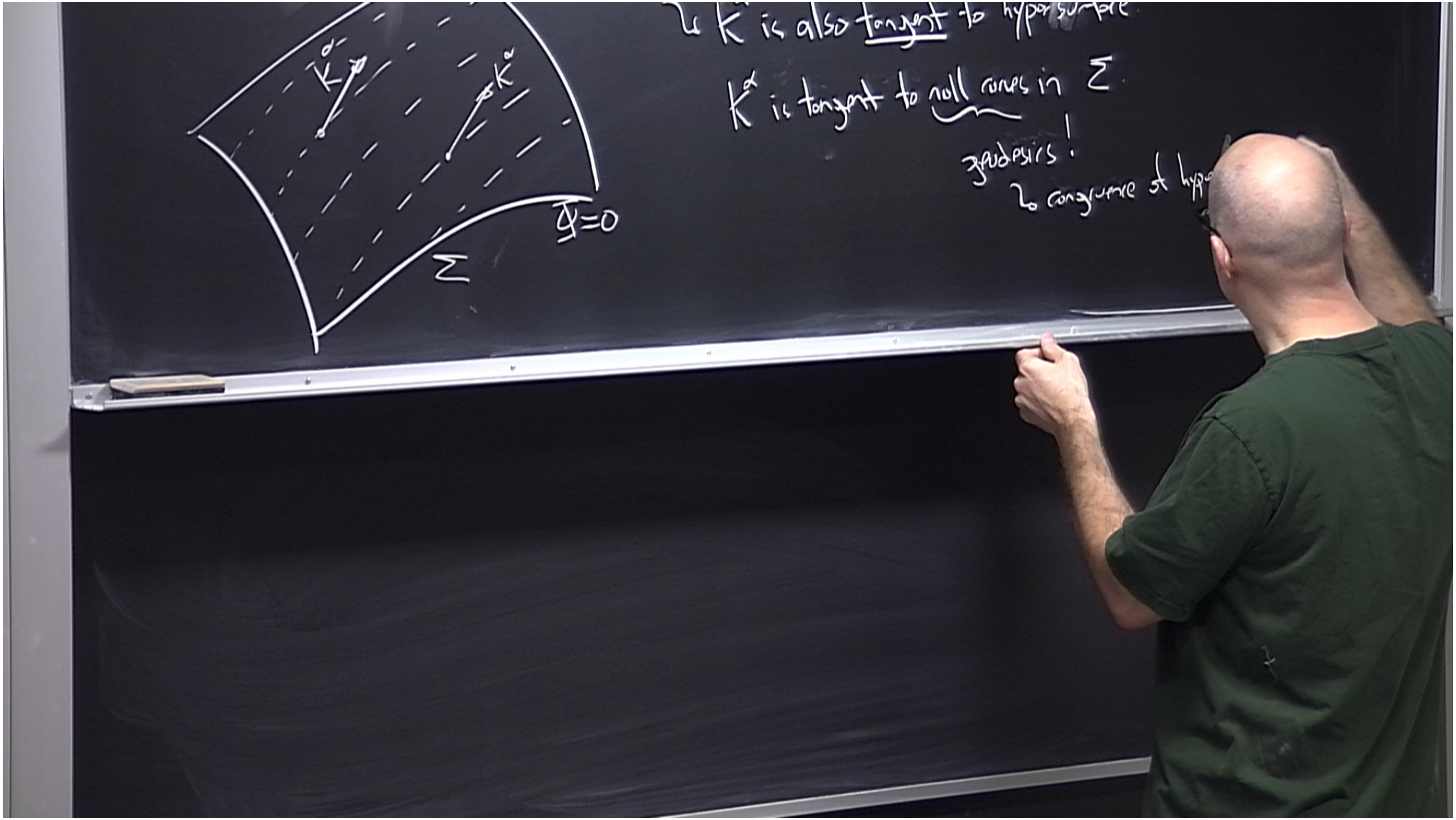
normal : $\boxed{k_\alpha = -\partial_\alpha \Phi}$ (Φ increases toward future)

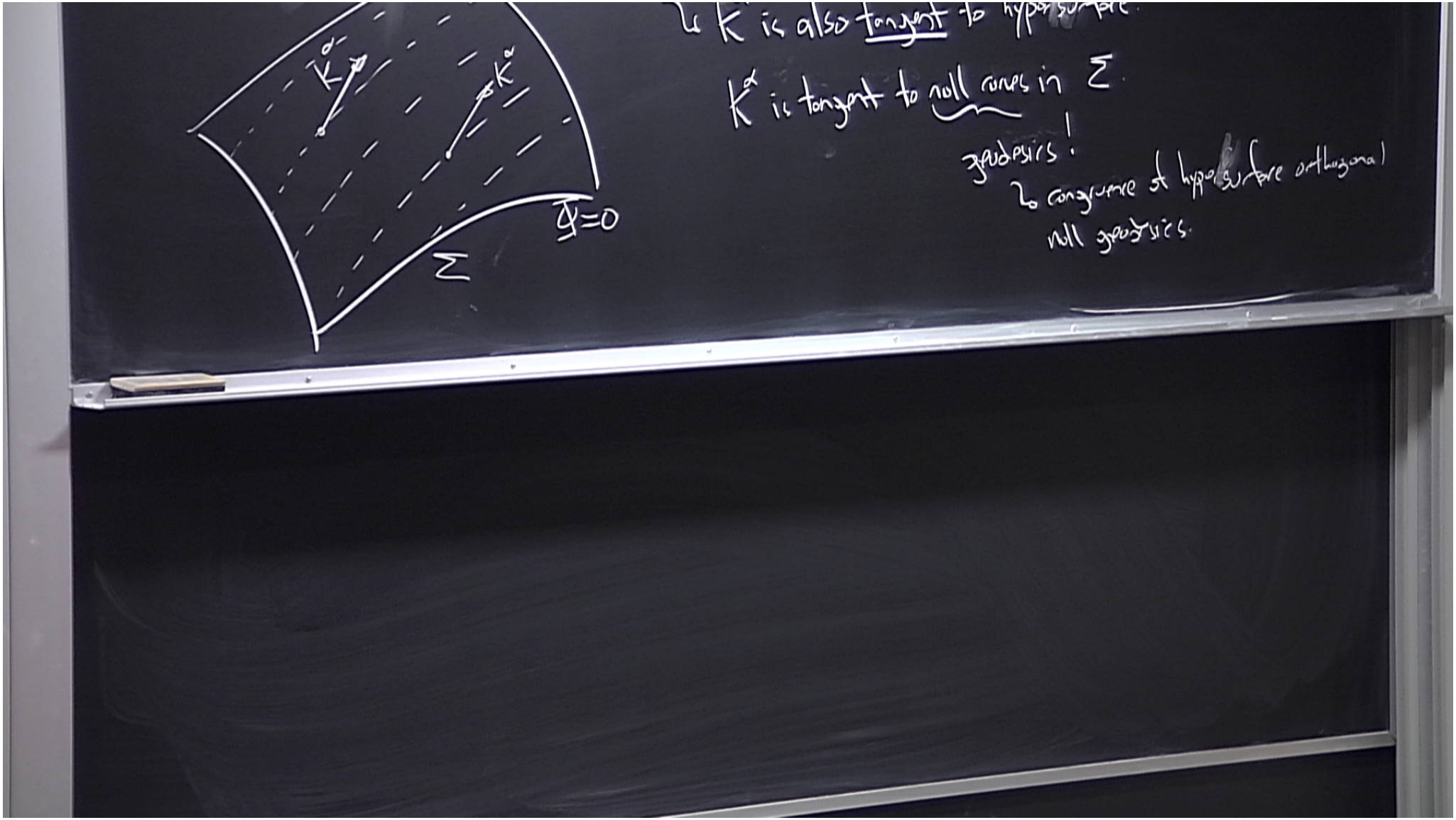
(k^α is future-pointing)

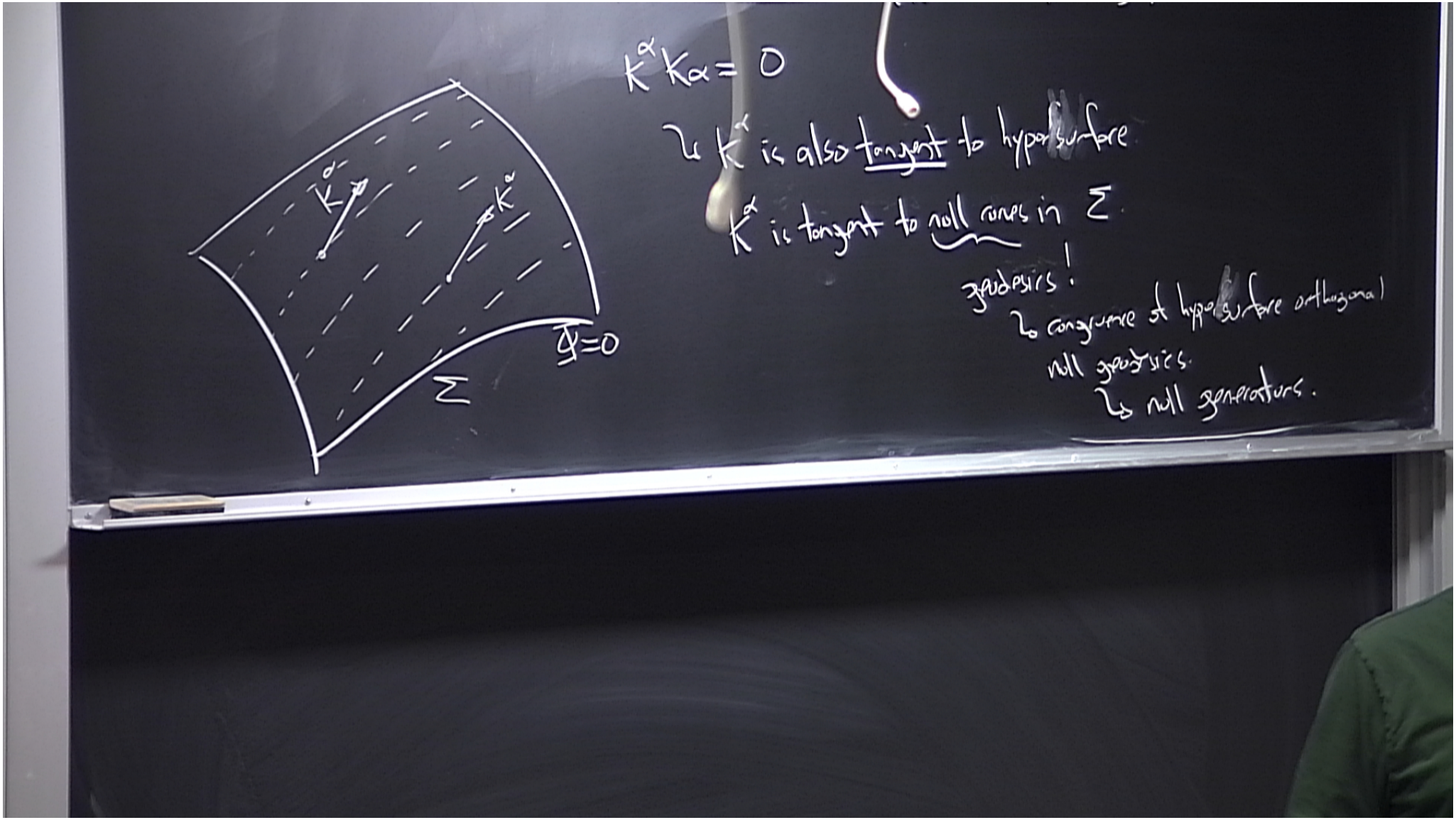
$k^\alpha k_\alpha = 0$

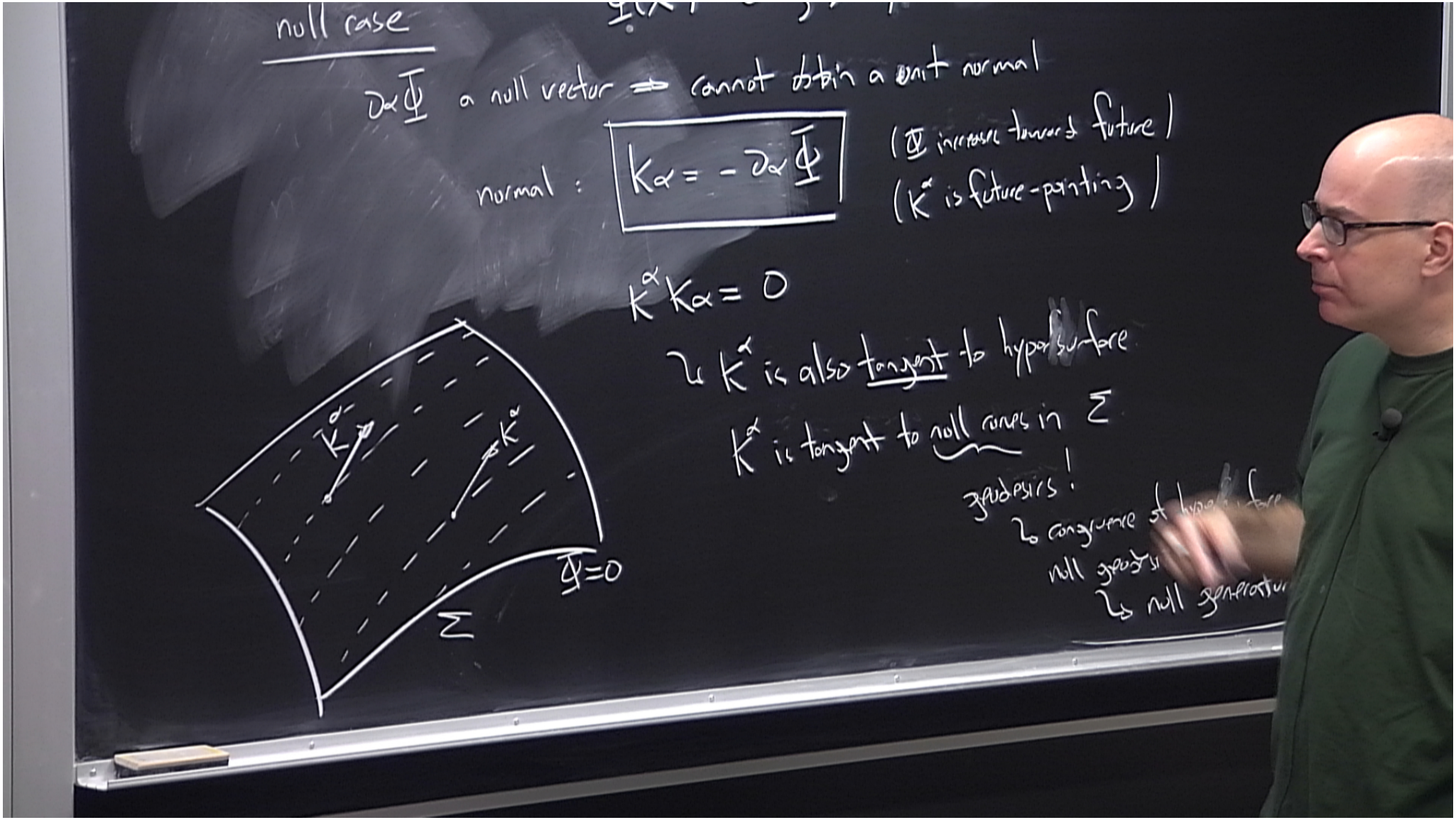
$\Rightarrow k^\alpha$ is also tangent to hypersurface


$k^\alpha k_\alpha = 0$
k^α is also tangent to hypersurface
Σ


K^α
K^β
Σ
$\Psi=0$
K^α is also tangent to hypersurface
K^α is tangent to null curves in Σ
geodesics!
congruence of hyp


k^α
k^α
$\Psi=0$
Σ
k^α is also tangent to hypersurface
k^α is tangent to null curves in Σ.
geodesics!
2 congruence of hypersurface orthogonal
null geodesics.


$k^\alpha k_\alpha = 0$
k^α is also tangent to hypersurface
k^α is tangent to null curves in Σ.
geodesics!
congruence of hypersurface orthogonal null geodesics.
null generators.
k^α
k^α
$\Phi = 0$
Σ


null case
$\nabla_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal
normal : $k_\alpha = -\nabla_\alpha \Phi$
(Φ increases toward future)
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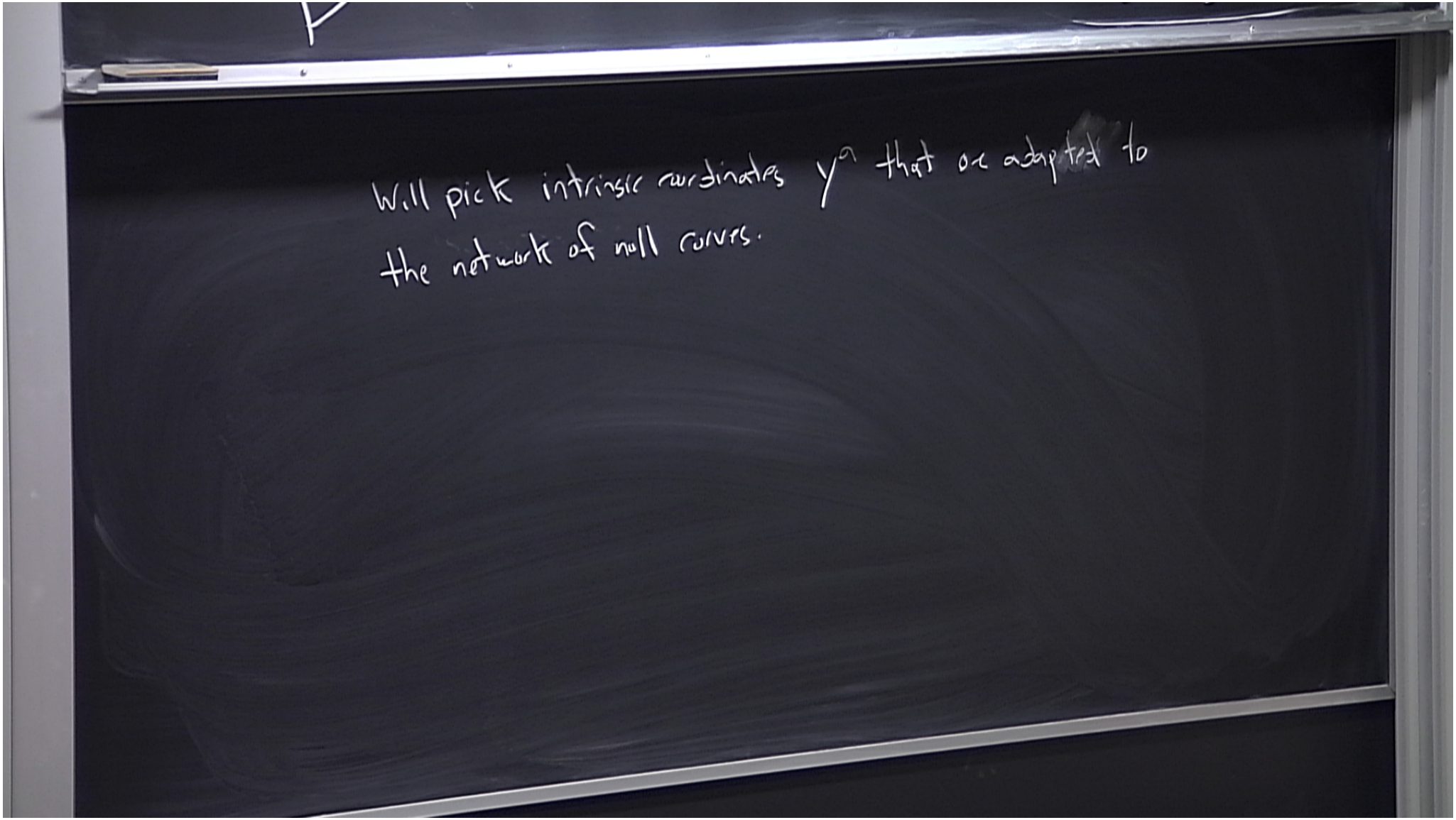
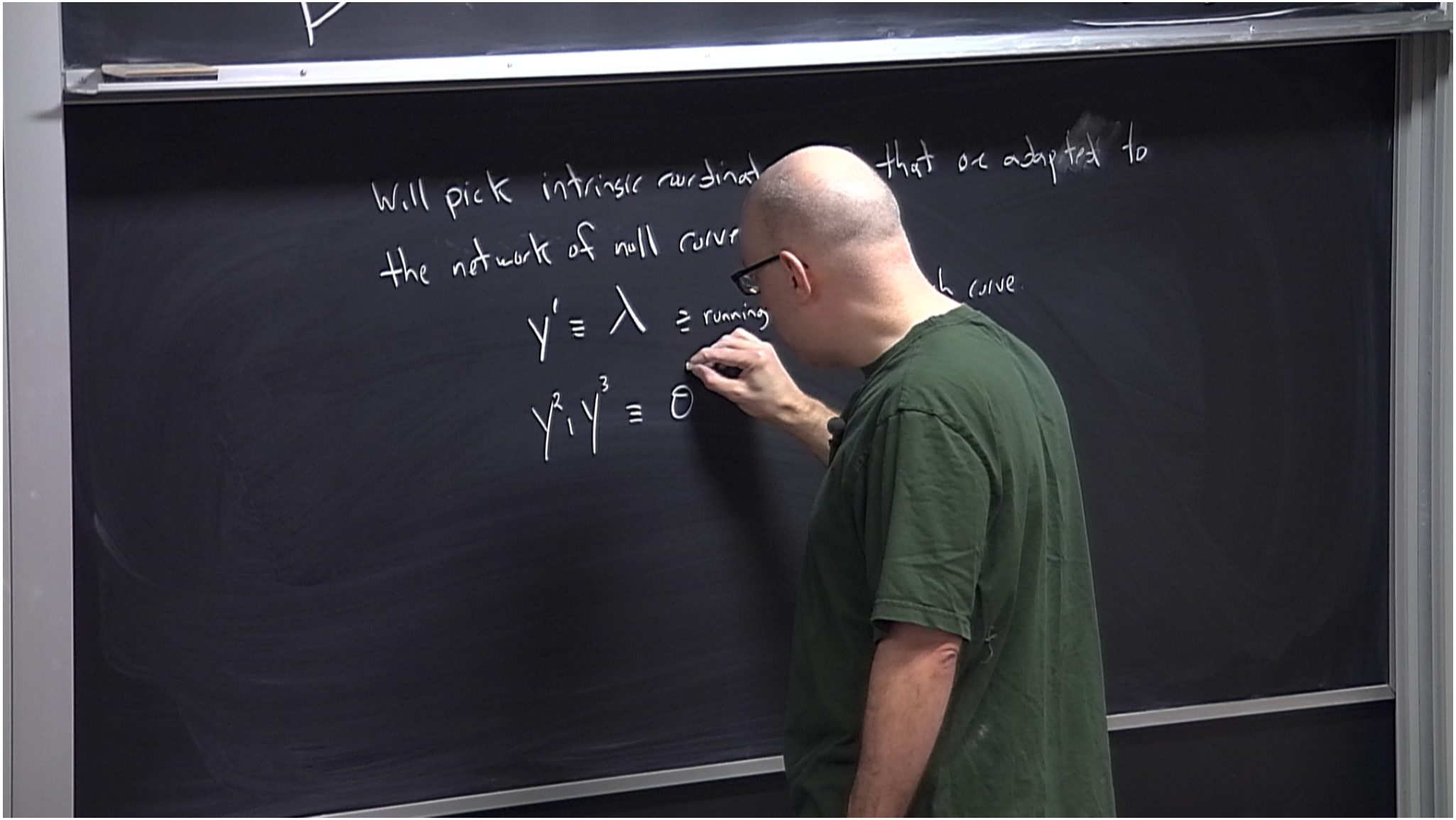
$k^\alpha k_\alpha = 0$
k^α is also tangent to hypersurface
k^α is tangent to null curves in Σ
geodesics!
k^α
k^α
$\Phi = 0$
Σ

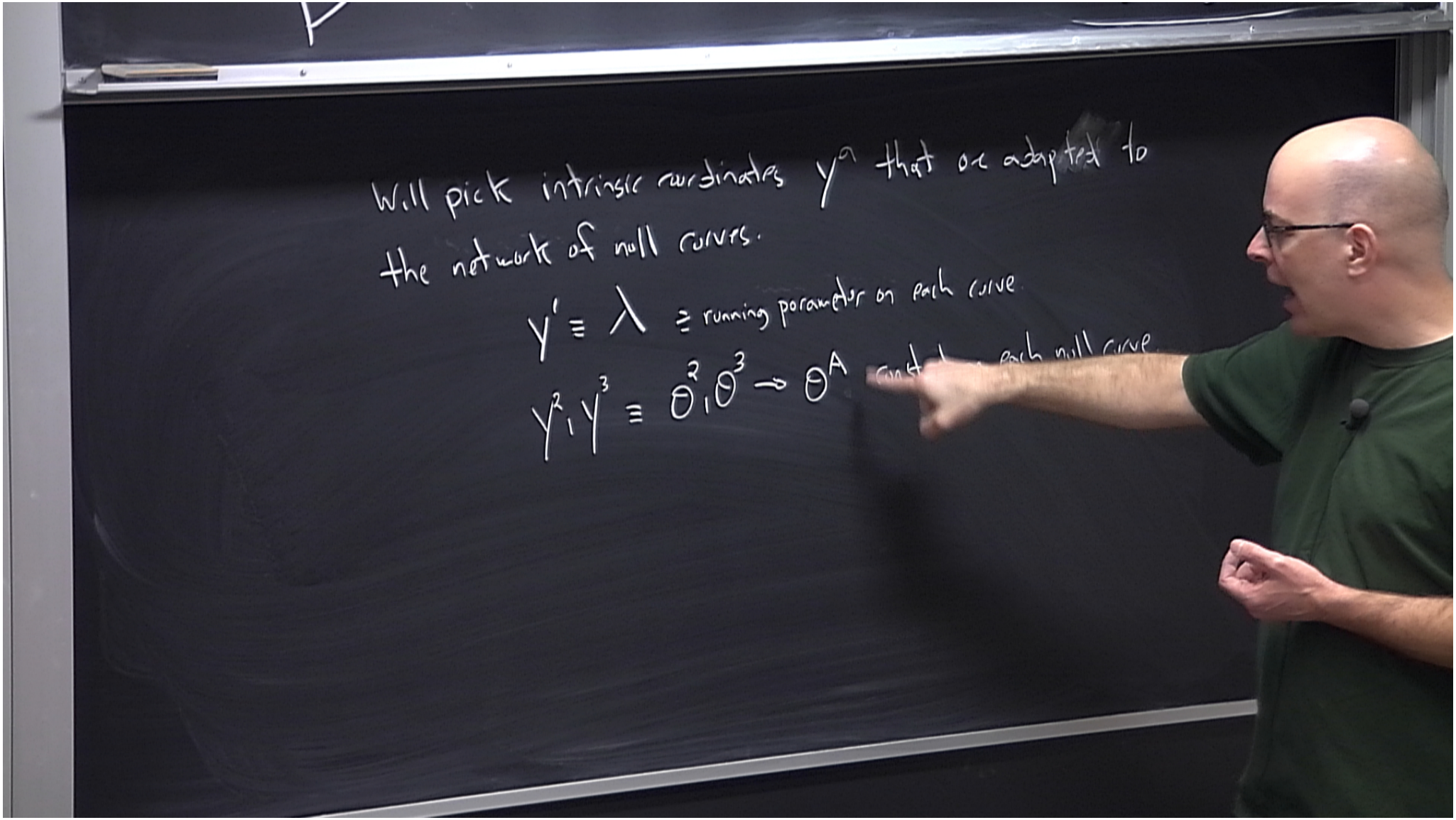

Will pick intrinsic coordinates y^a that are adapted to
the network of null curves.


Will pick intrinsic coordinat
that are adapted to
the network of null curve
h curve.
y' ≡ λ ≡ running
y², y³ ≡ θ

Will pick intrinsic coordinates y^a that are adapted to the network of null curves.

$y^1 \equiv \lambda$ ≡ running parameter on each curve

$y^2, y^3 \equiv \theta^2, \theta^3 \to \theta^A$ constant on each null curve.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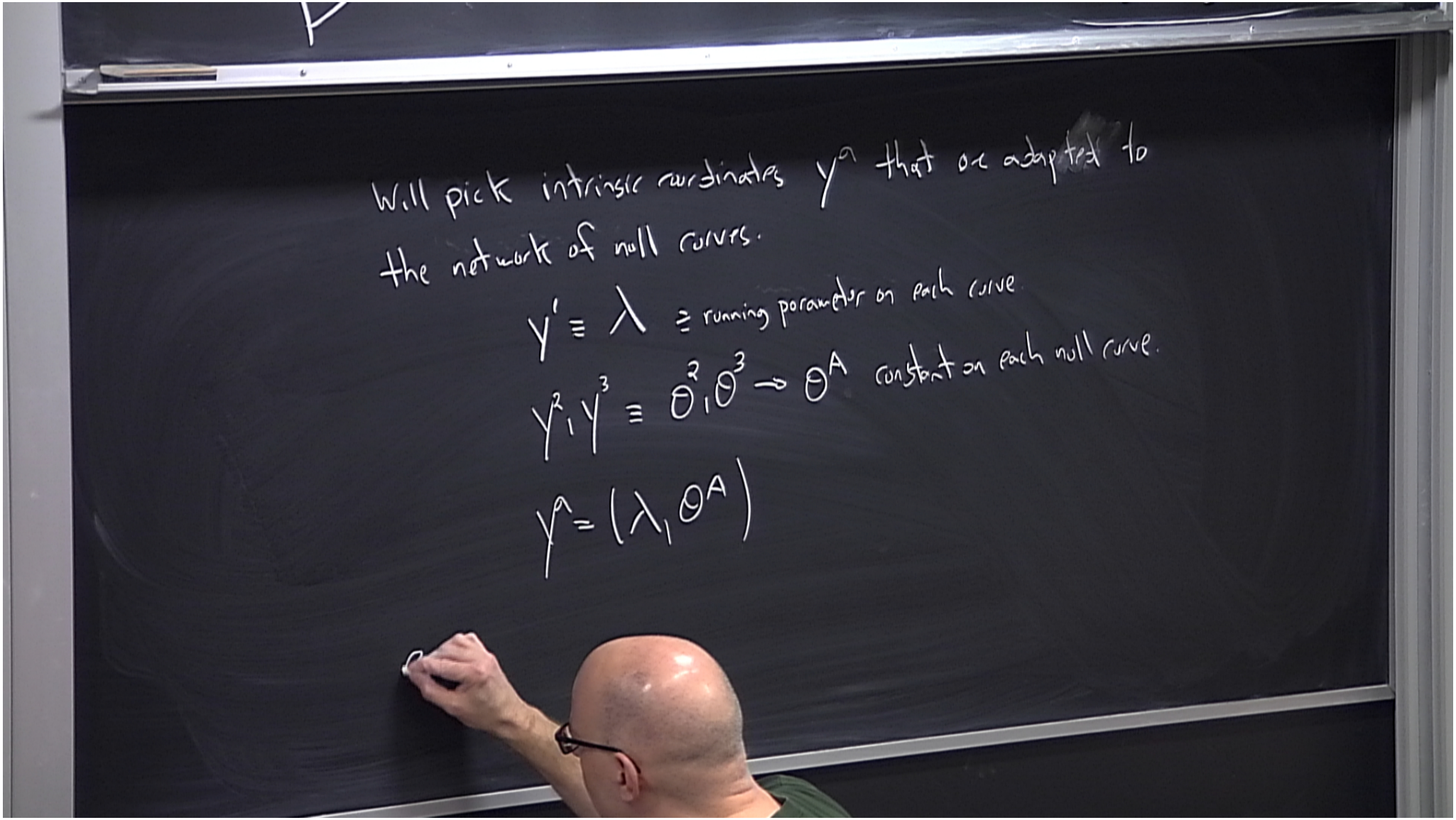
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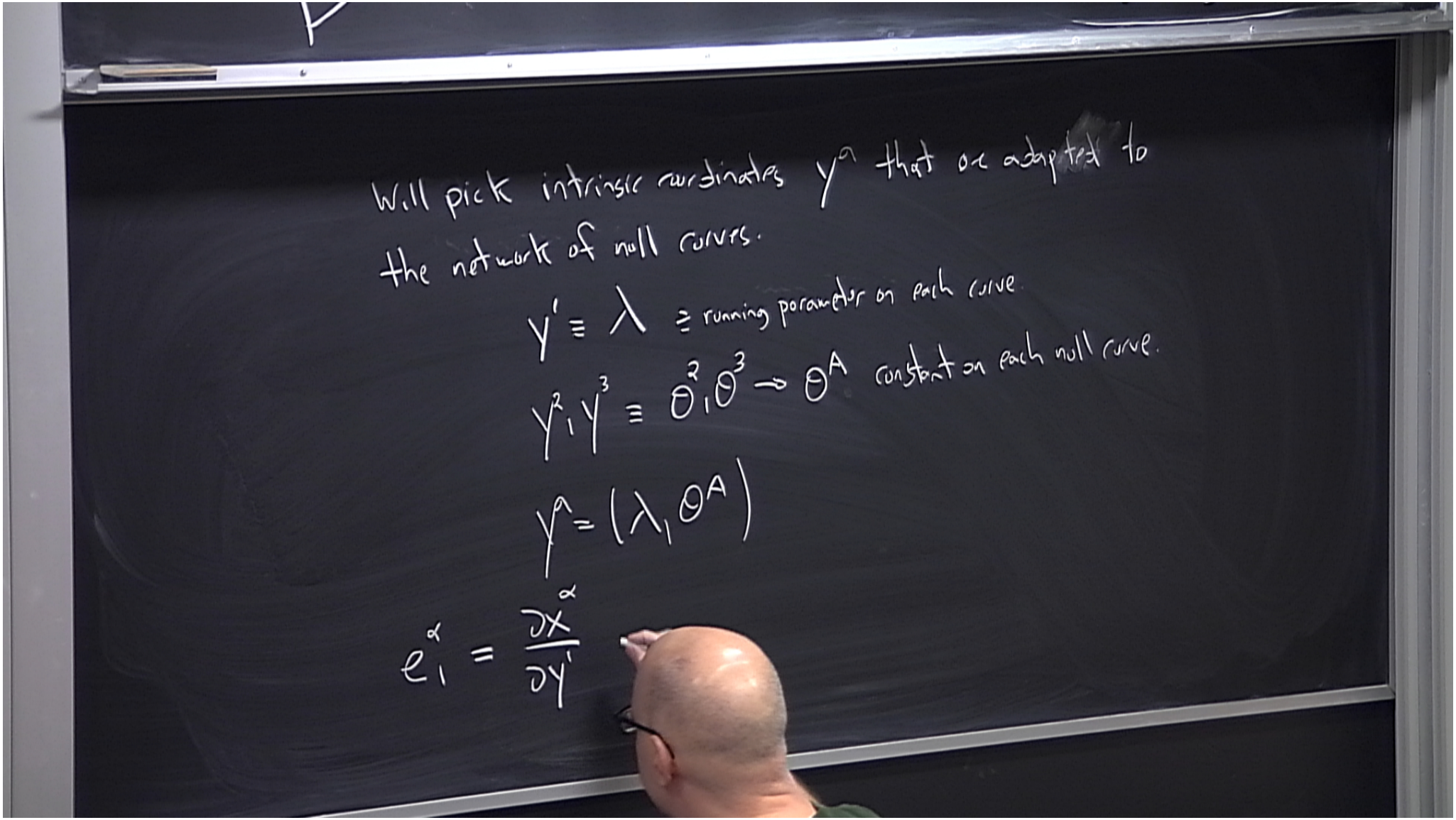
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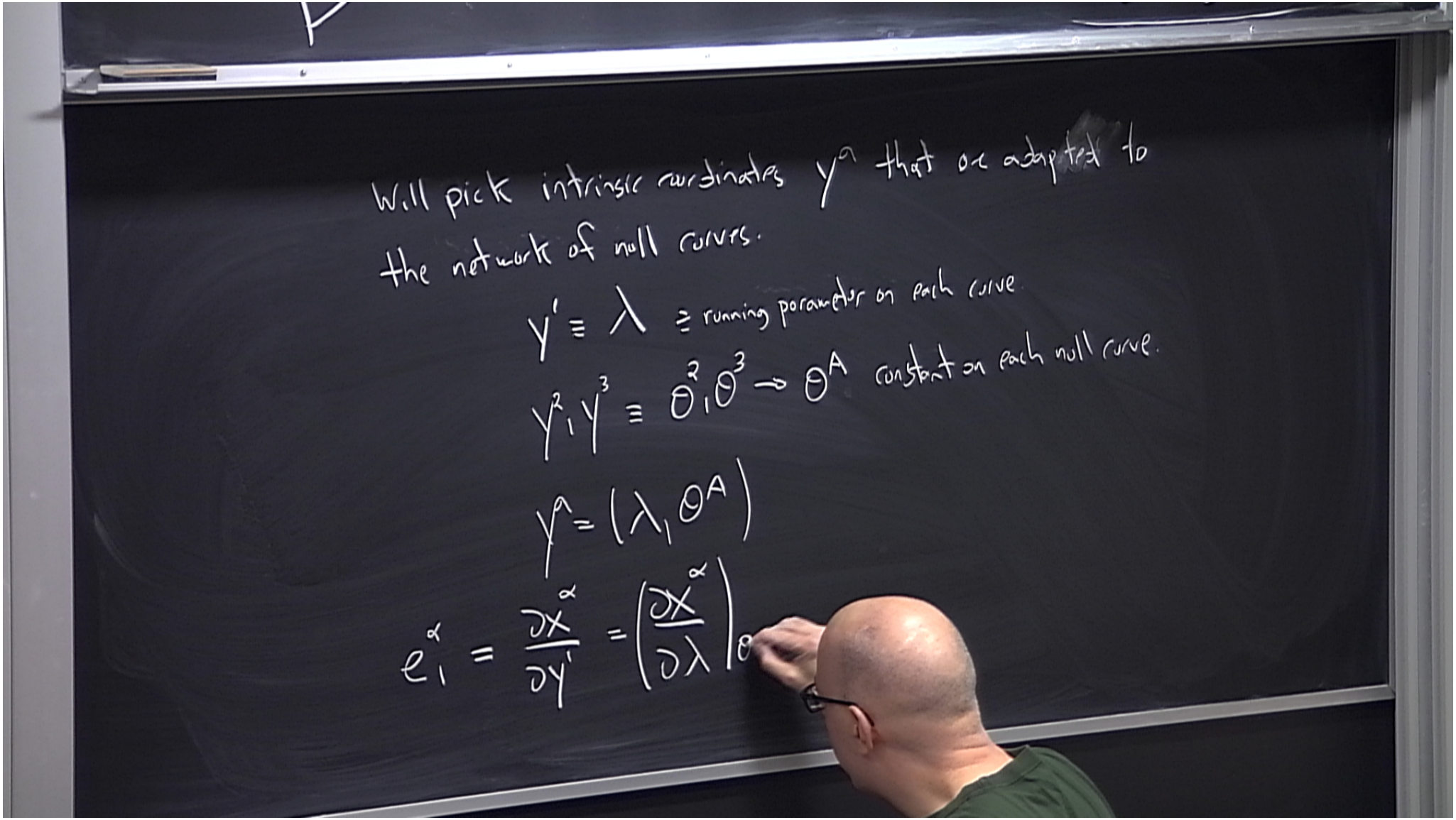
$e_1^\alpha = \frac{\partial x^\alpha}{\partial y^1}$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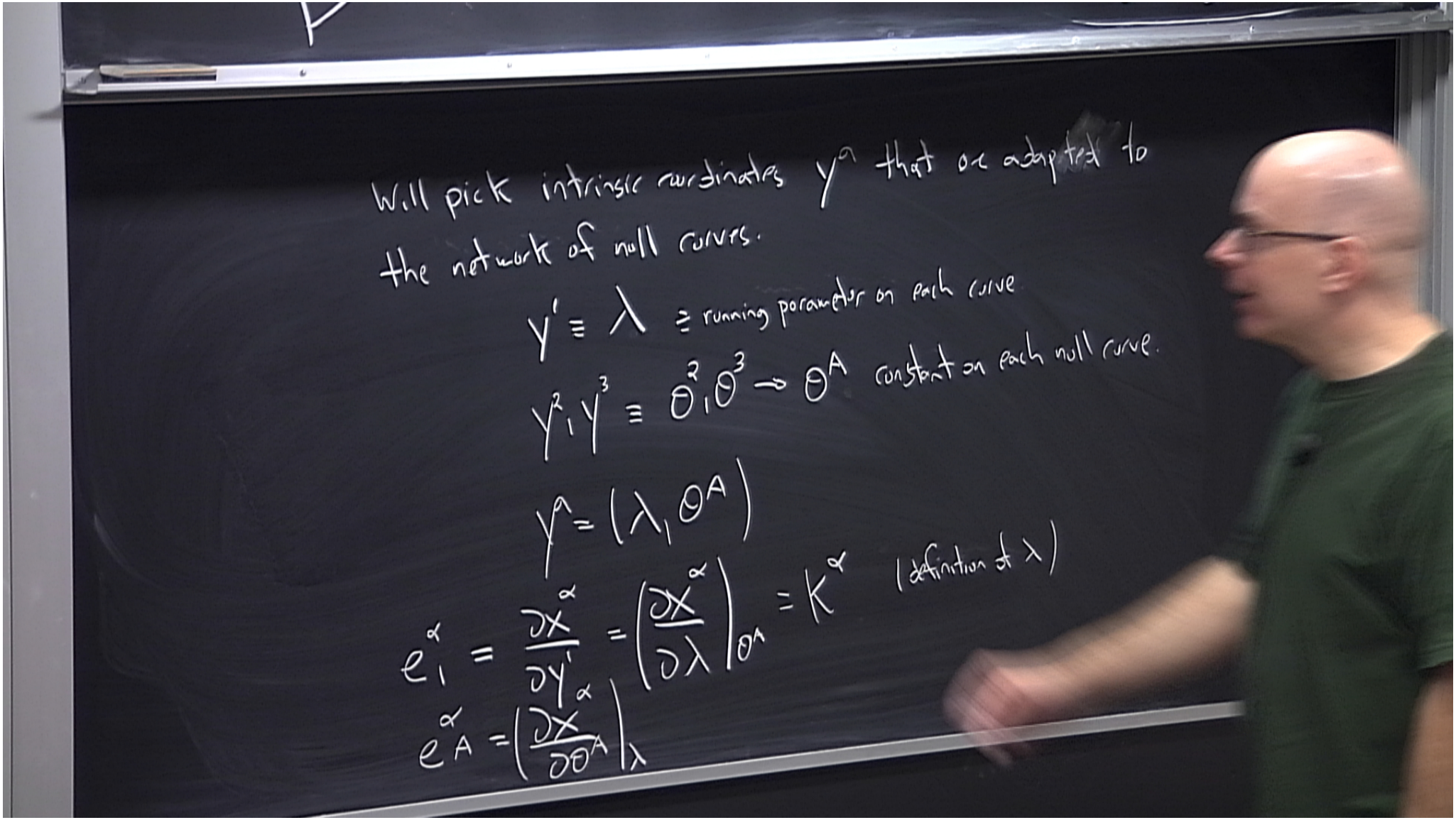
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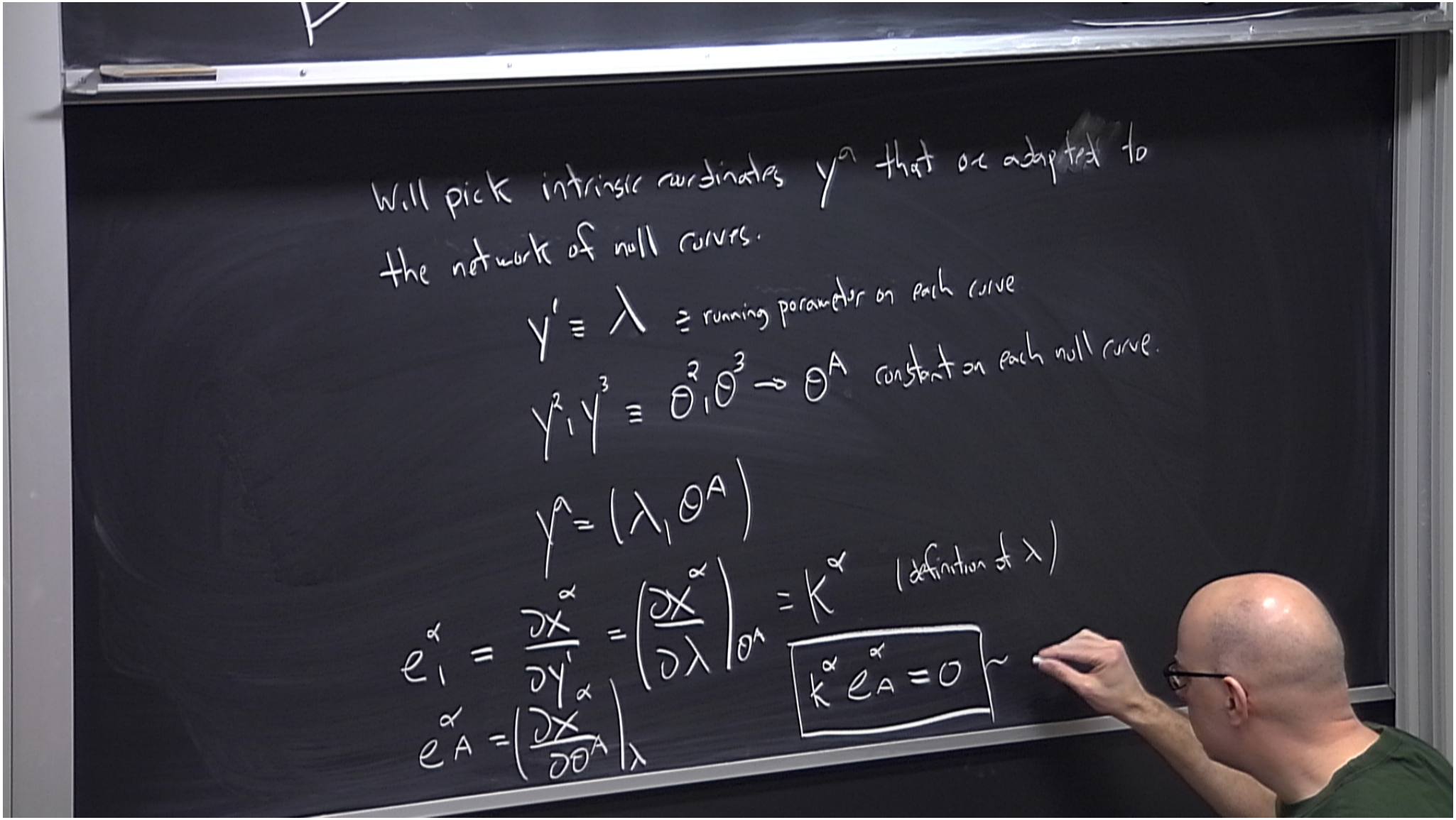
$k^\alpha e_A^\alpha = 0$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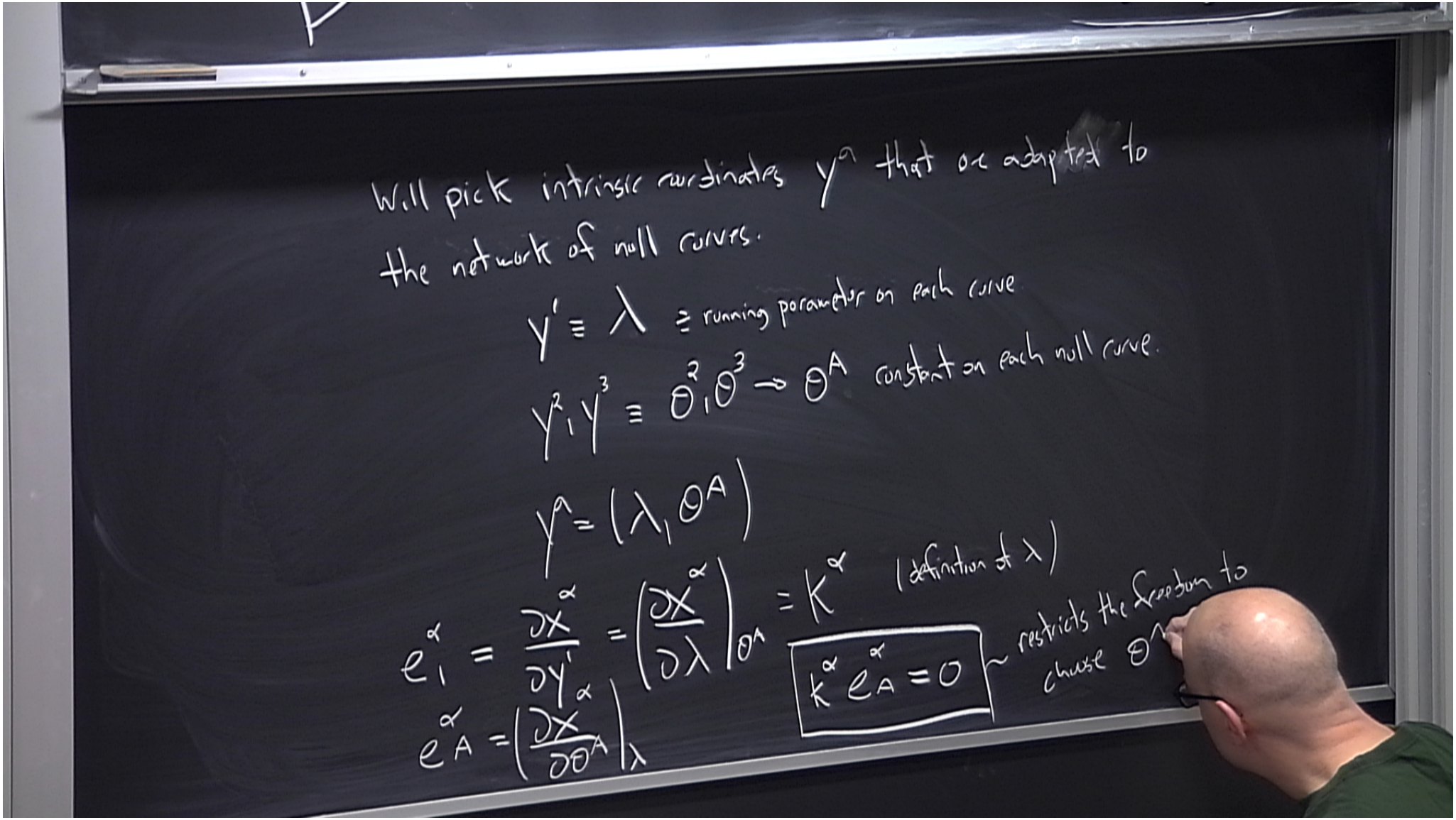
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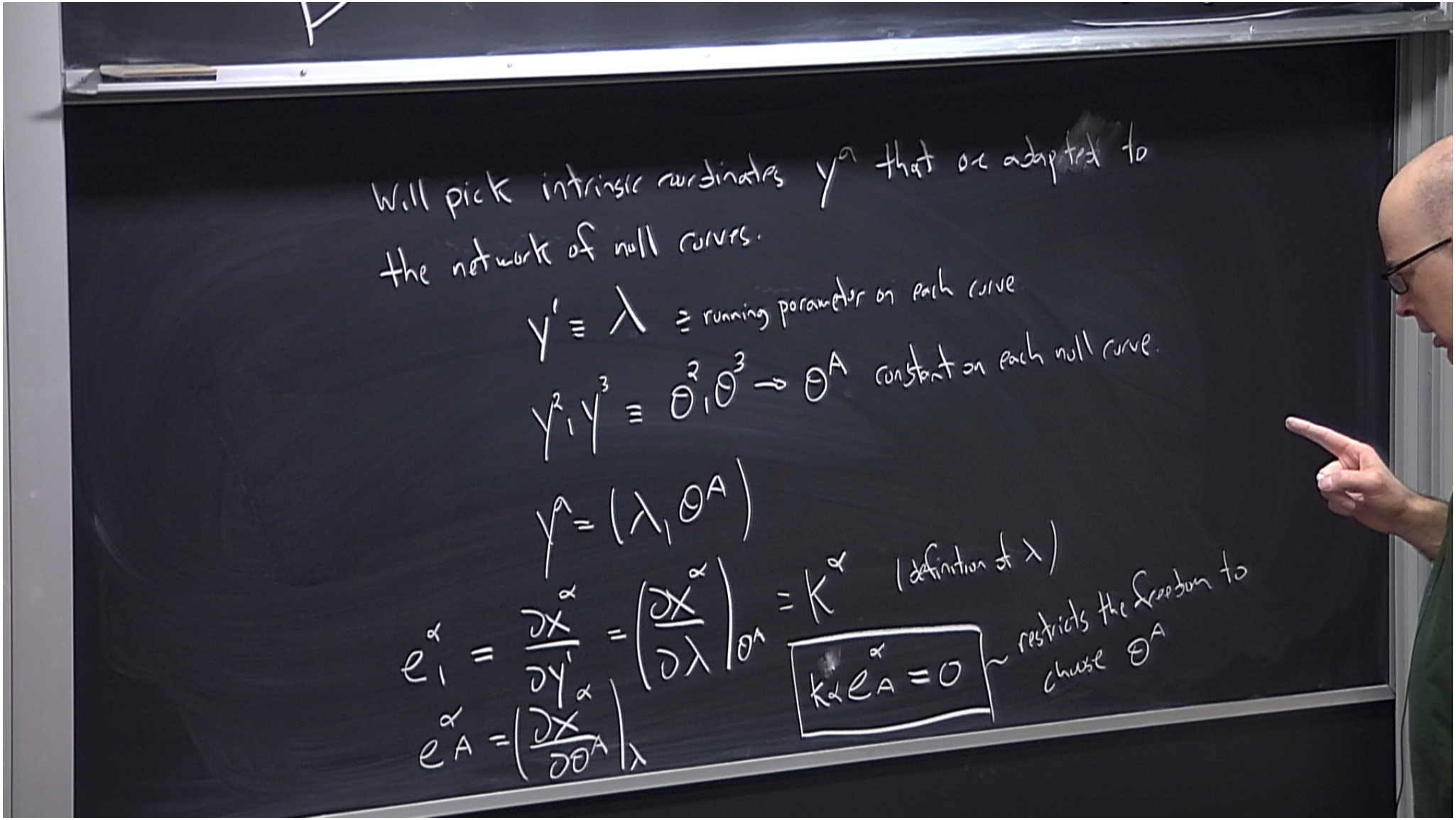
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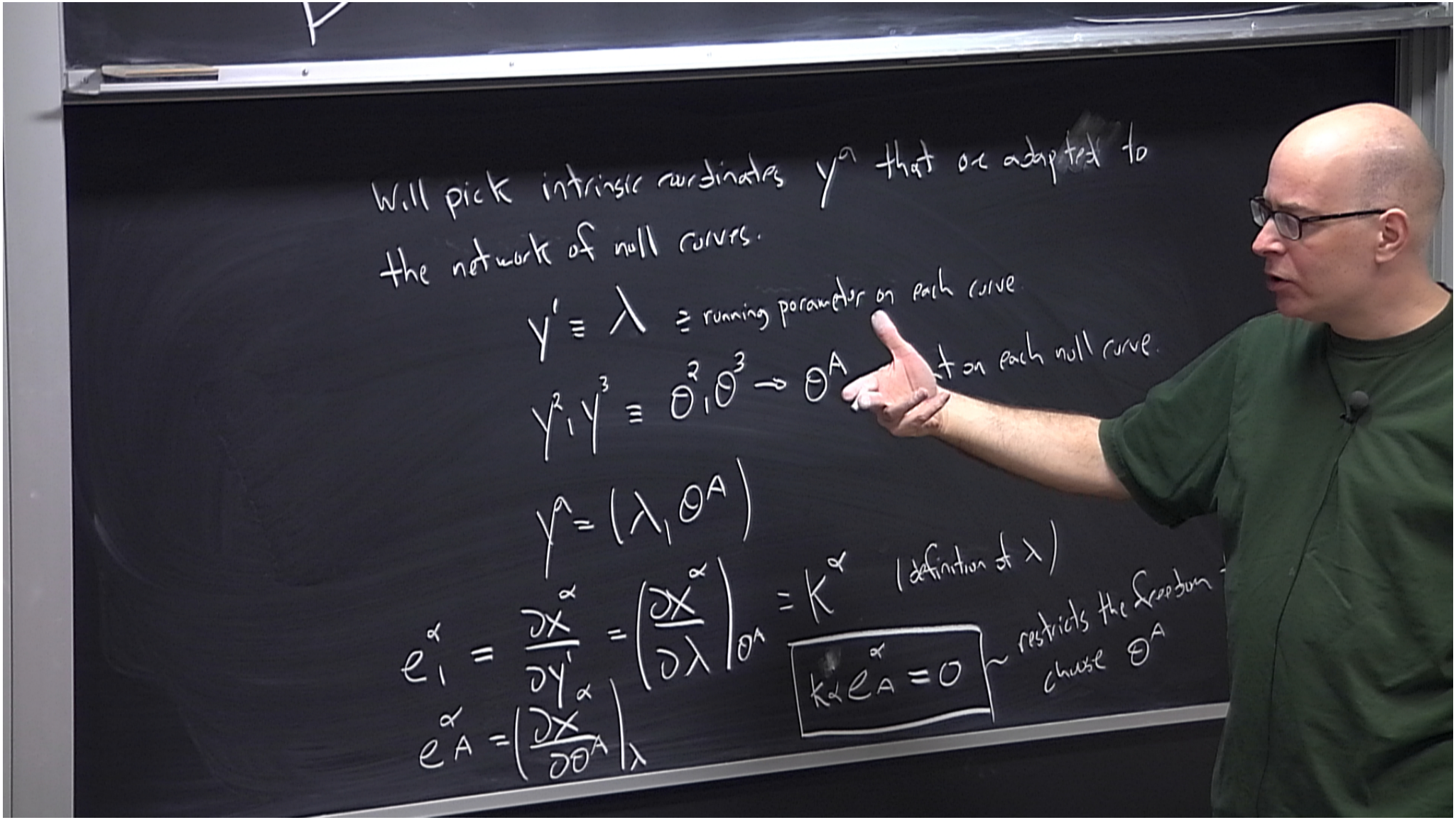
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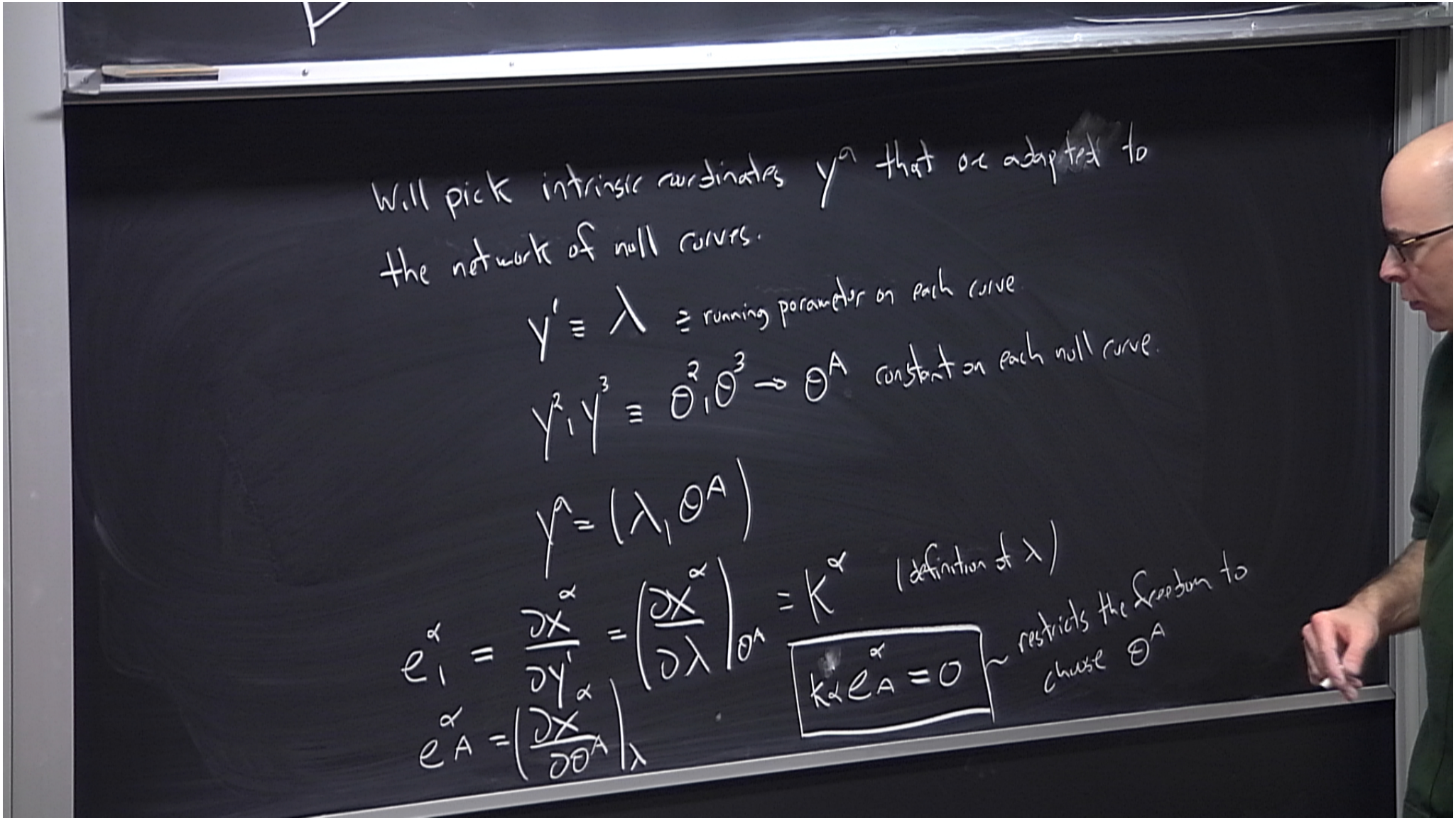
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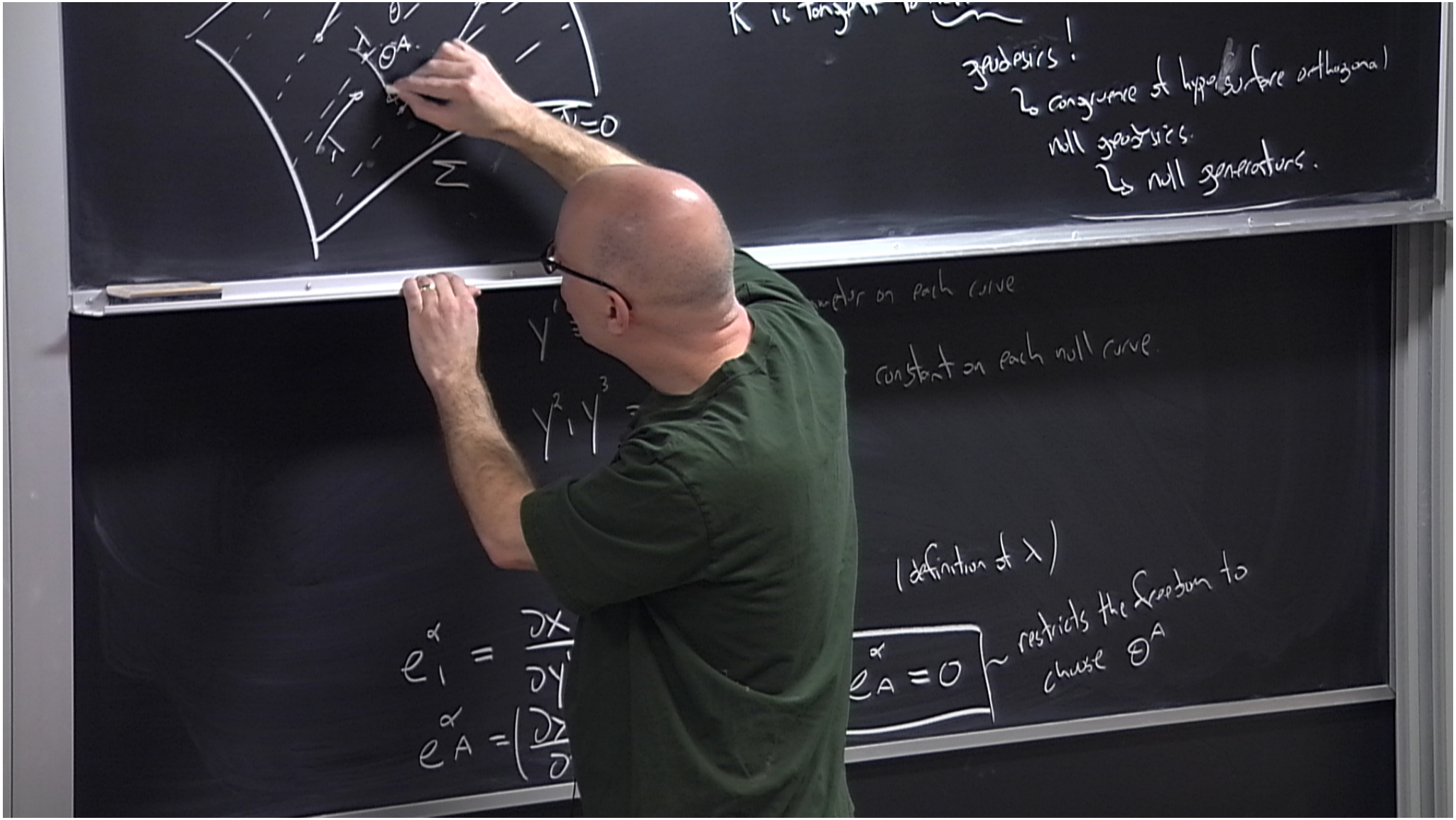
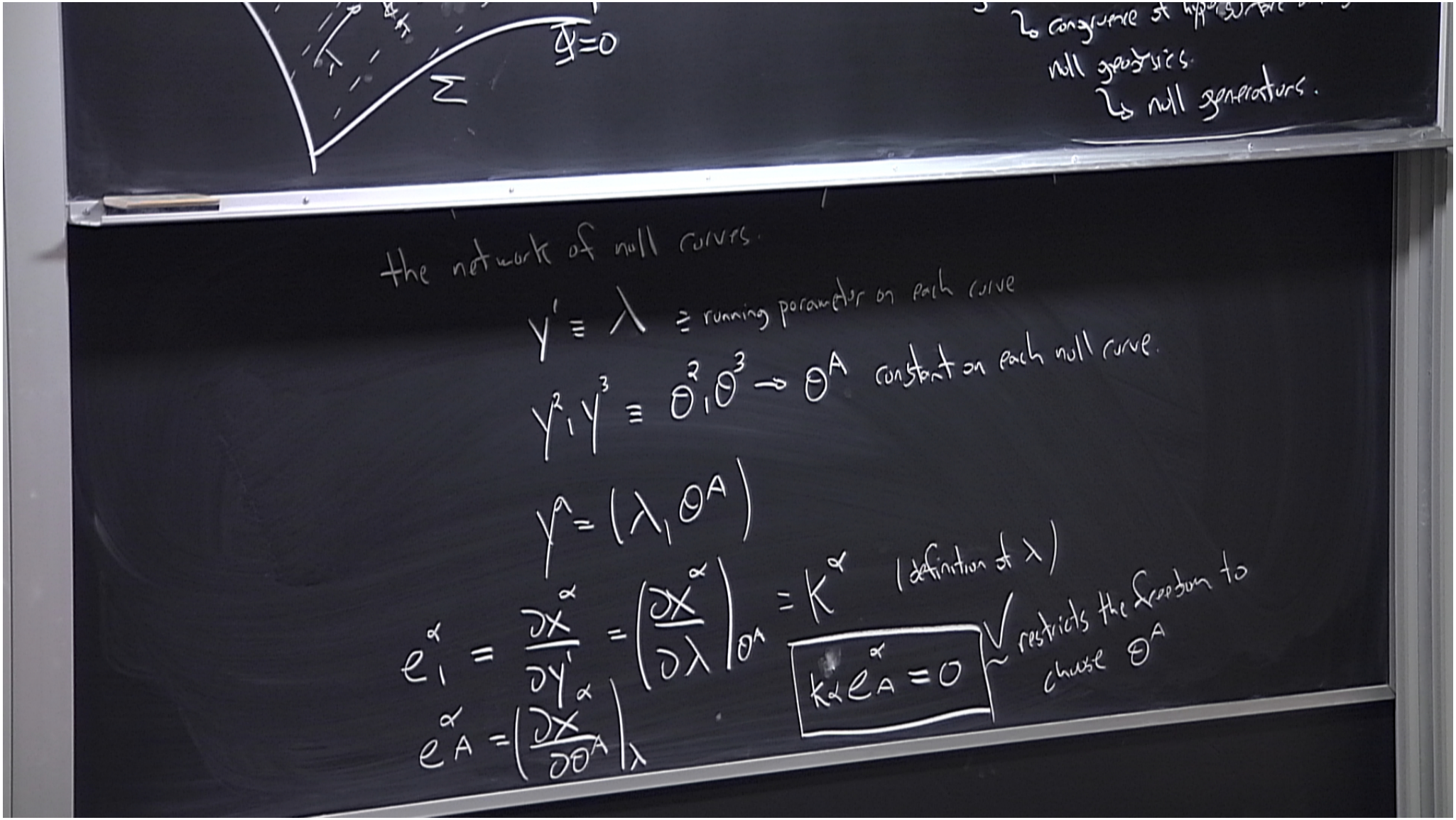

geodesics!
null geodesics.
null generators.
constant on each null curve.
(definition of λ)
restricts the freedom to
choose Θ^A



$\Phi = 0$
Σ
↳ congruence of hypersurface
null geodesics.
↳ null generators.
the network of null curves.
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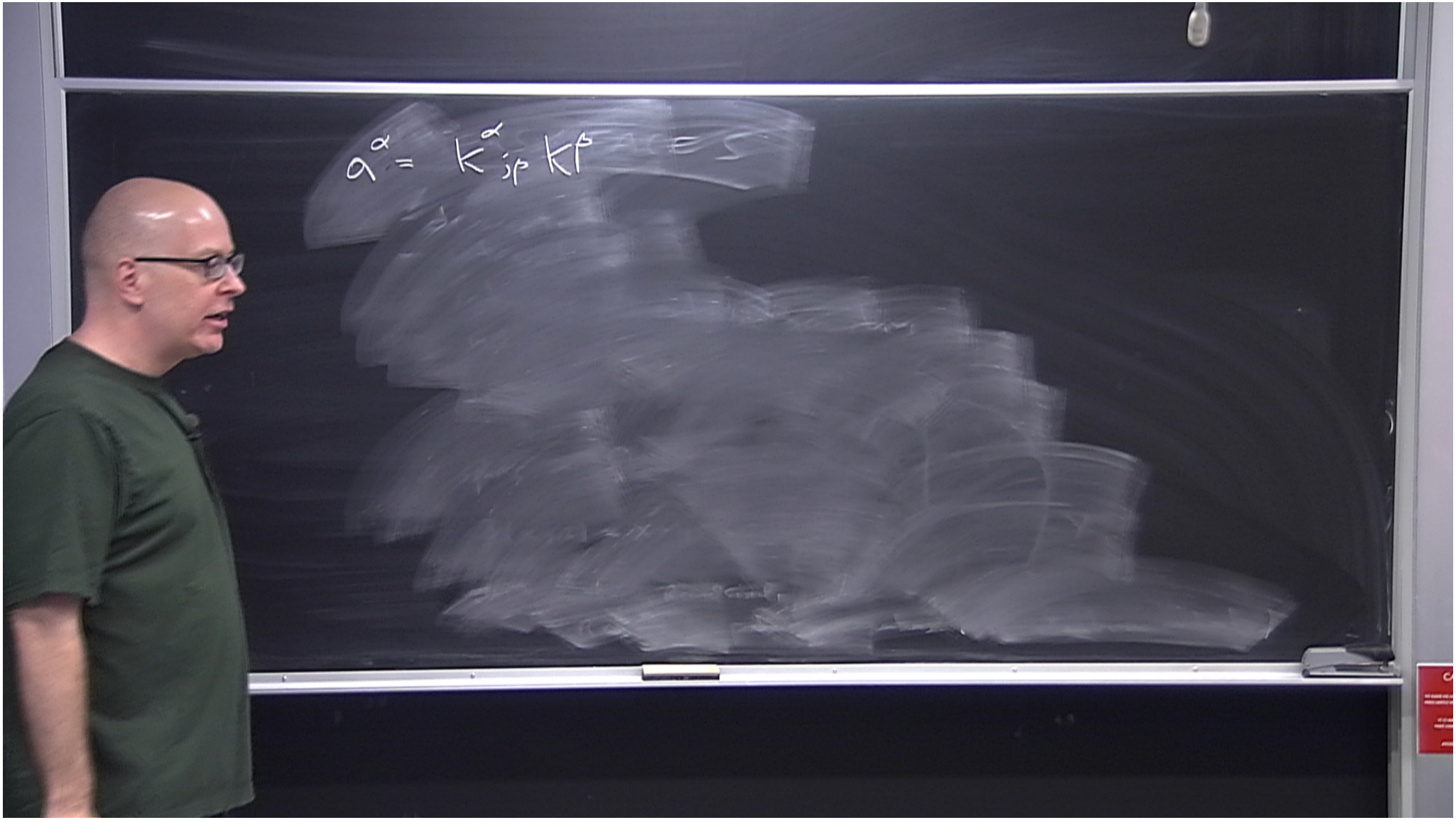
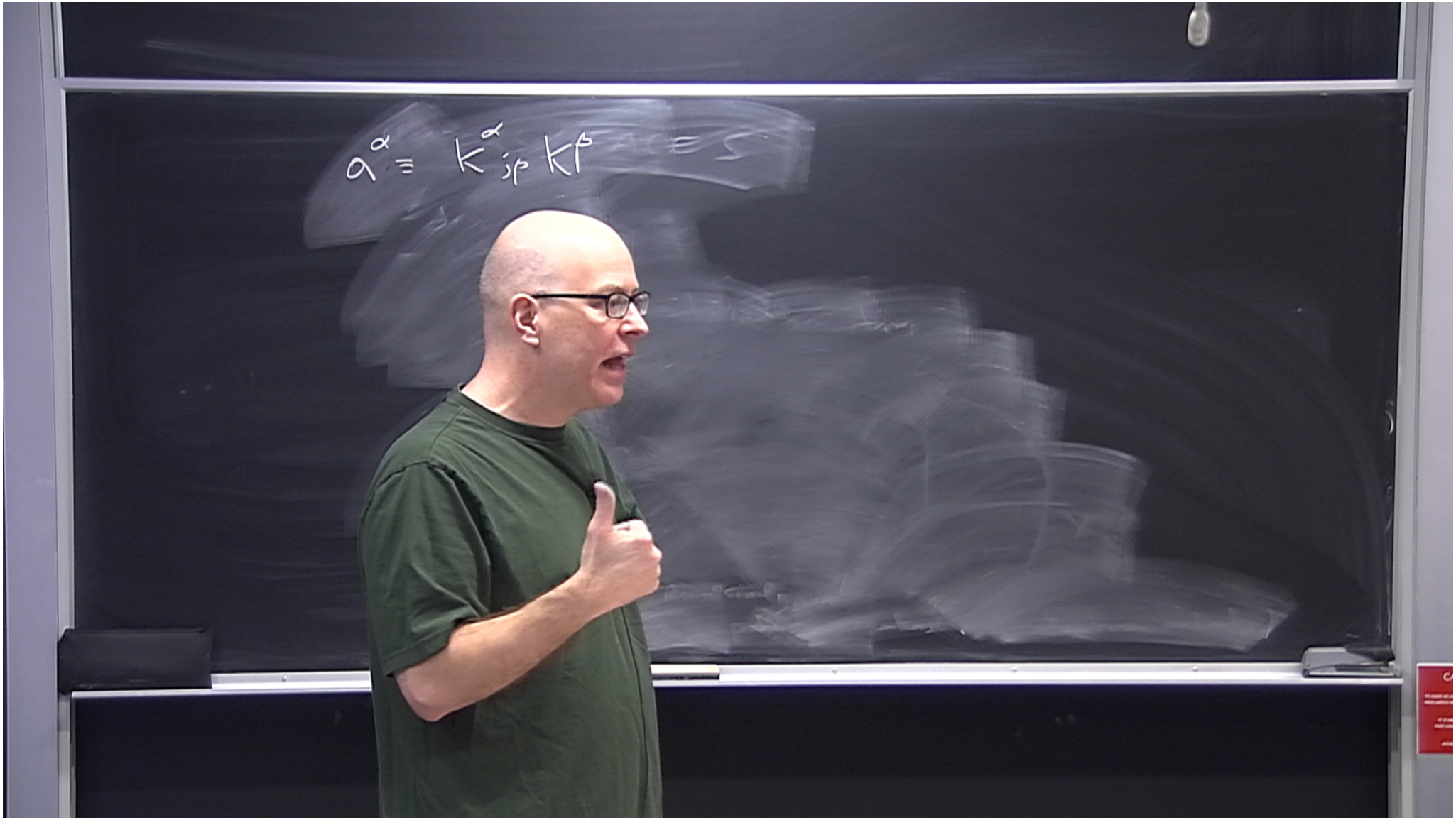
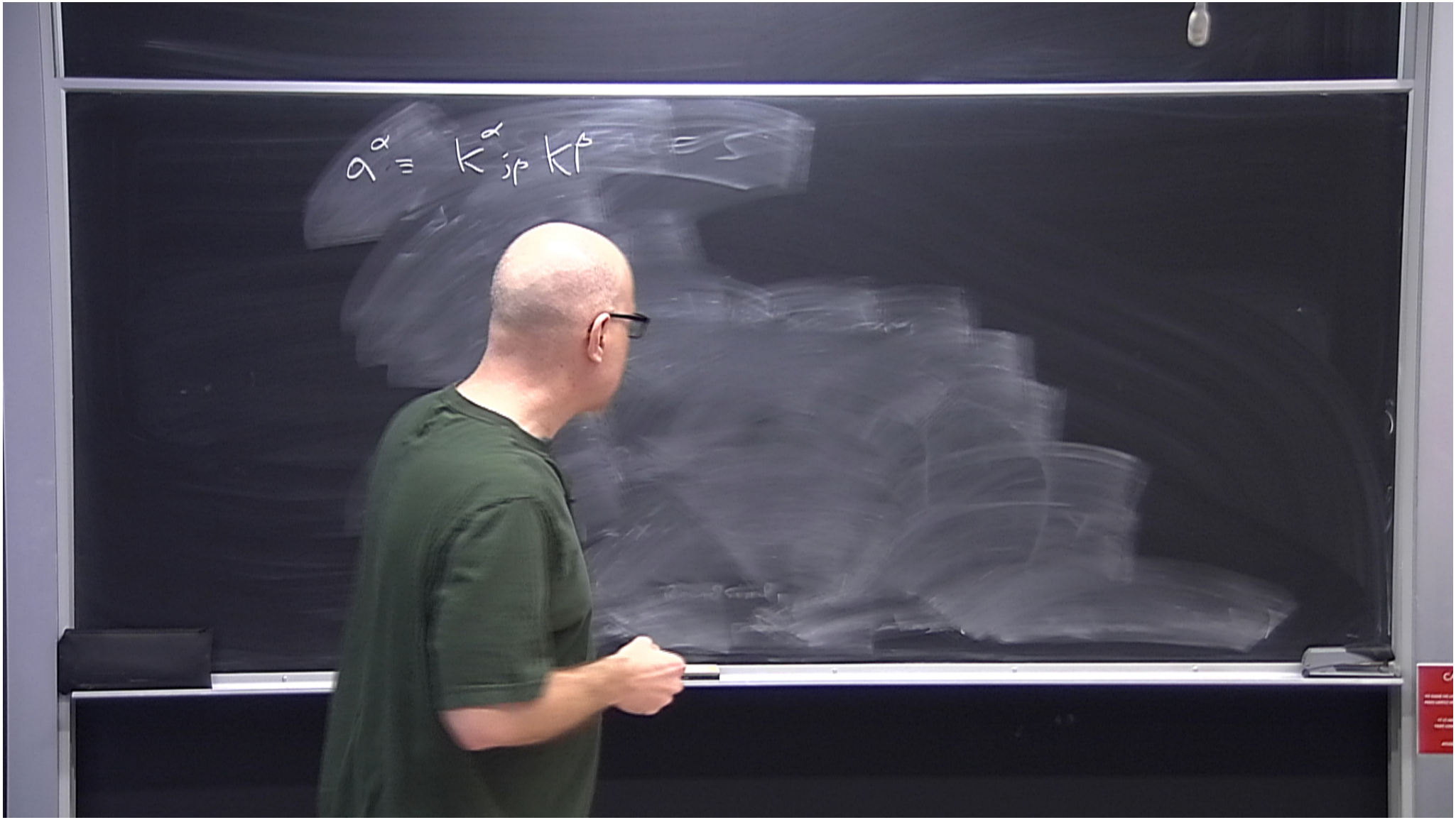
$\boxed{k_\alpha e^\alpha_A = 0}$ ✓ restricts the freedom to choose θ^A


$q^\alpha = k^\alpha_{\beta\rho} k^\rho$


$q^\alpha \equiv k^\alpha_{jp} k^\beta$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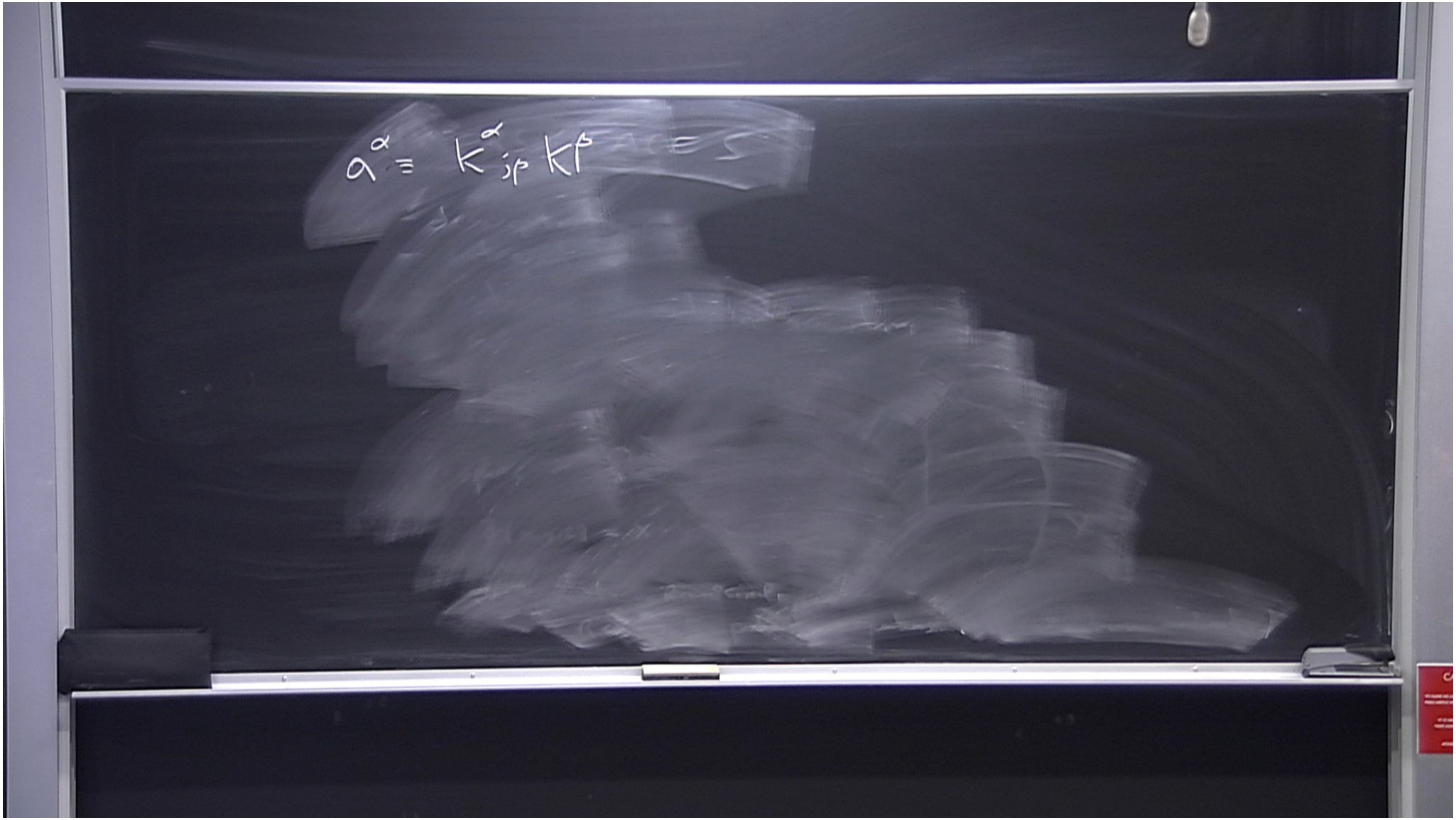
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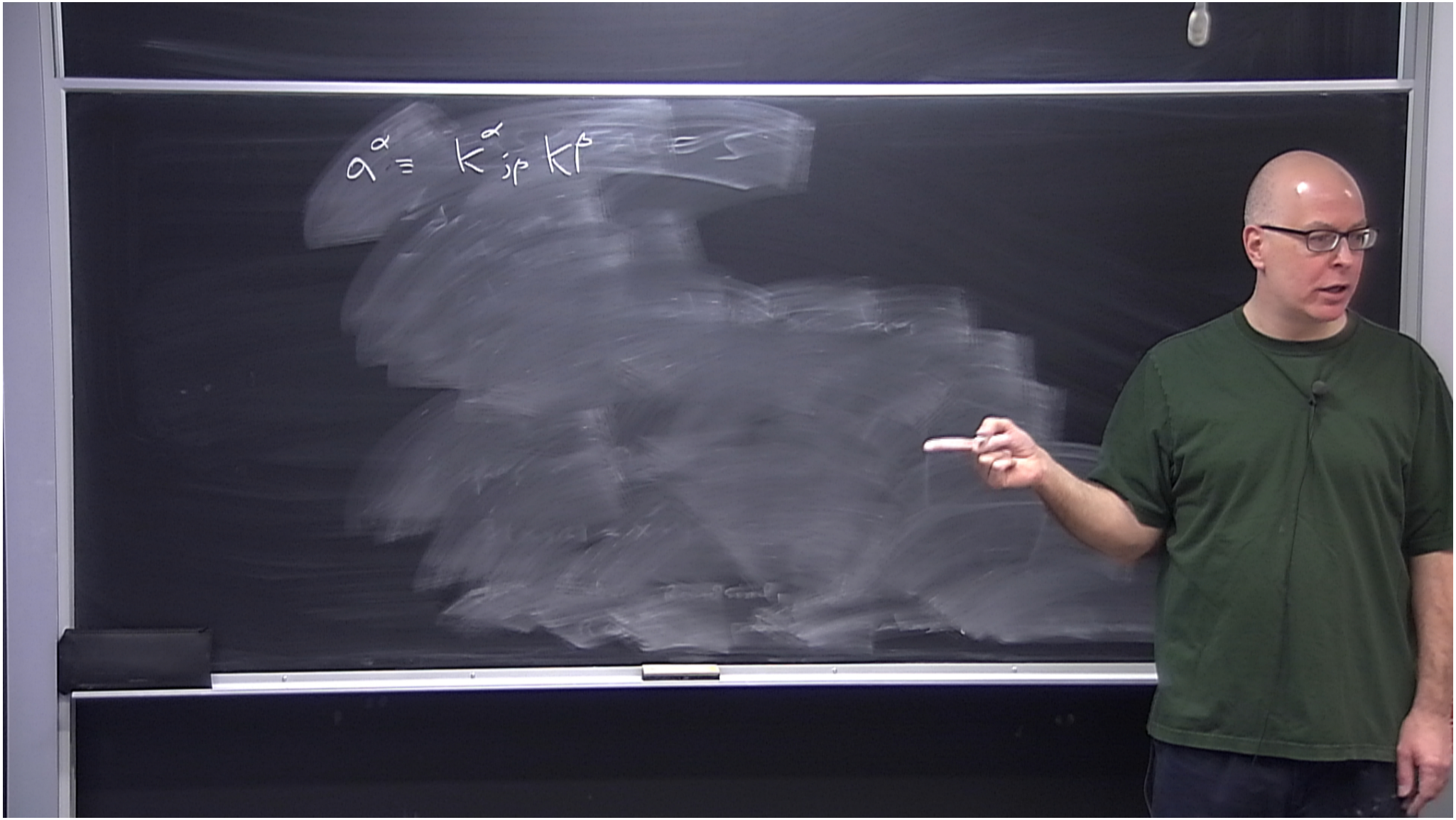
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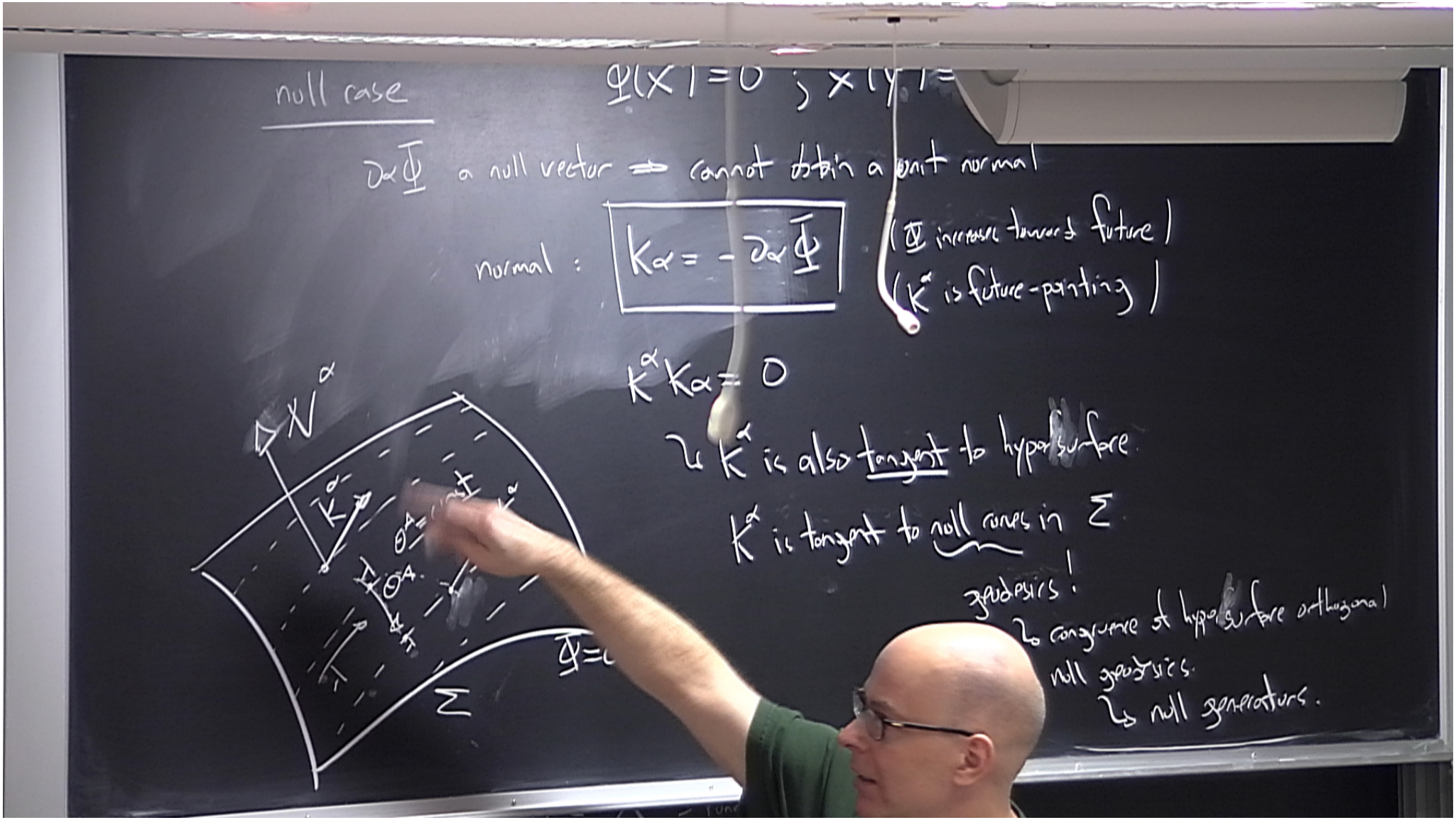
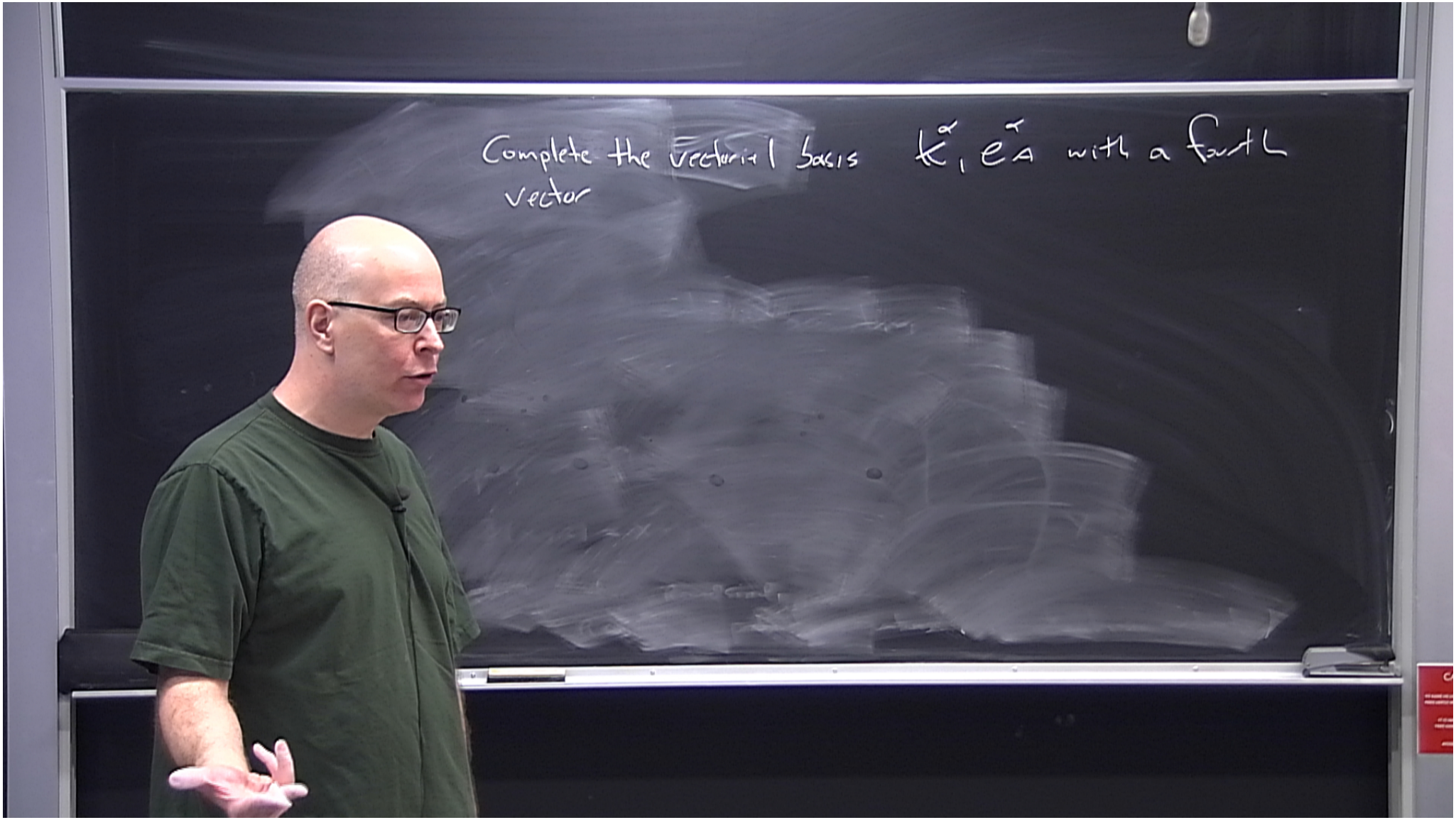
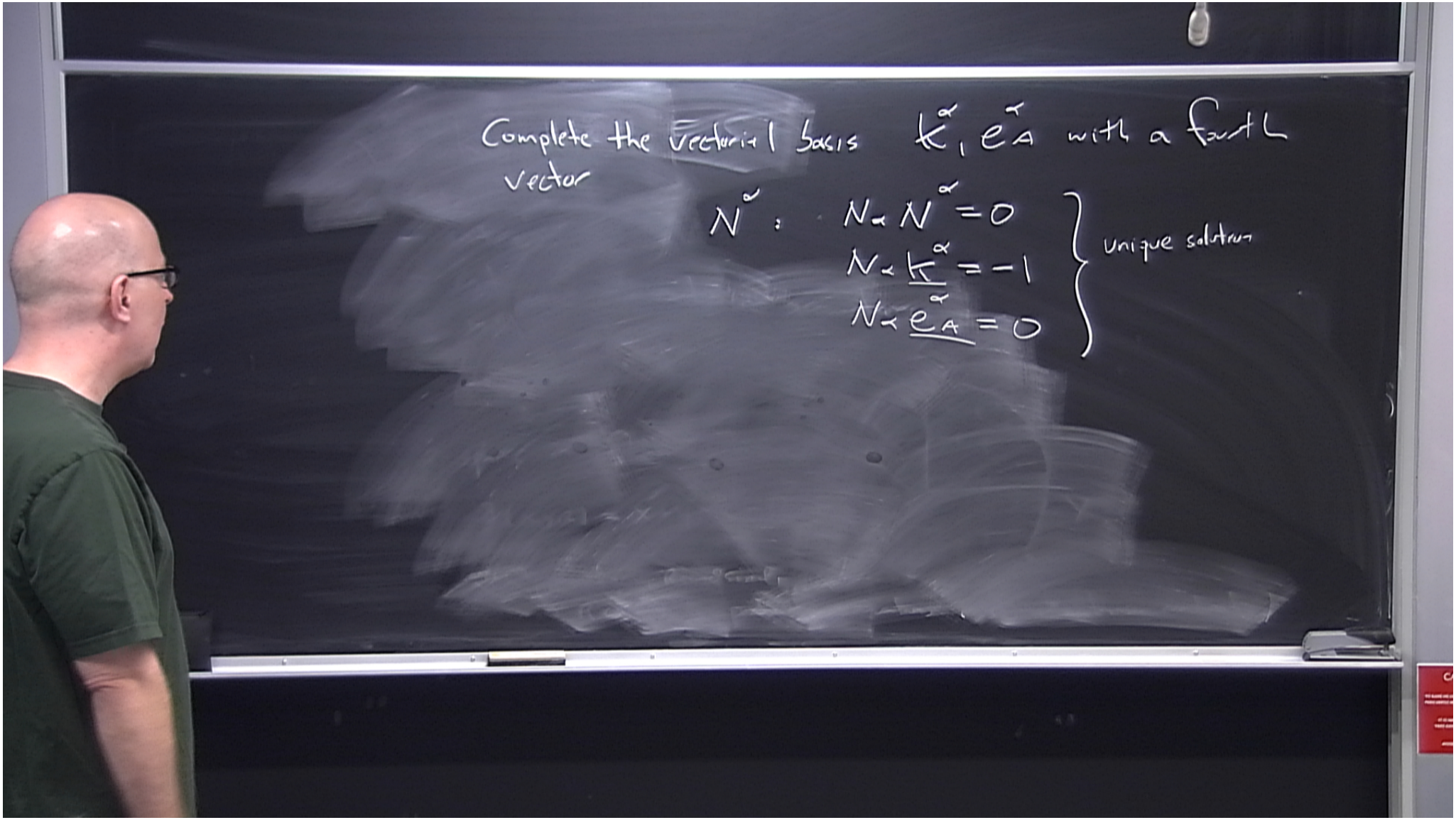
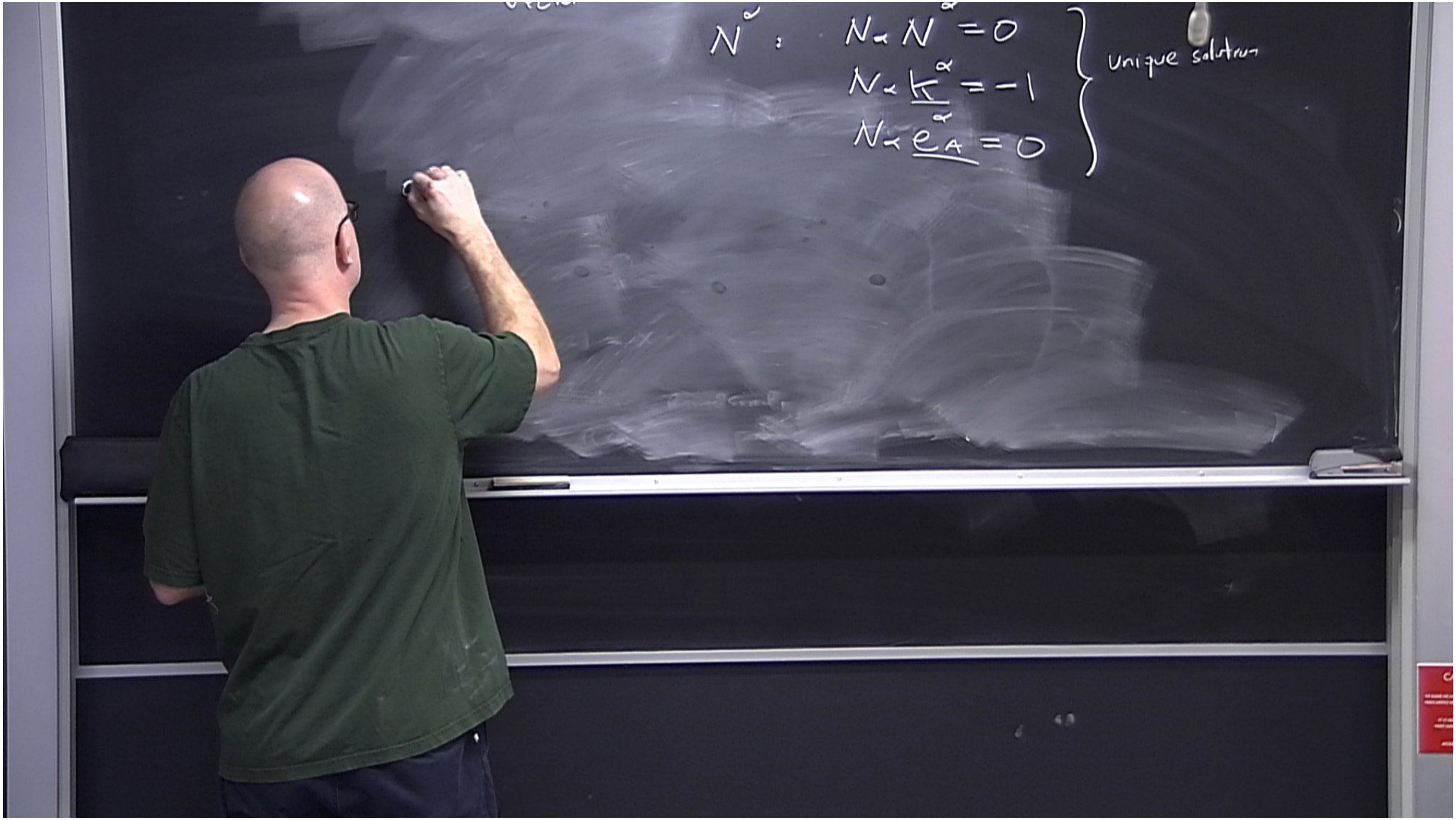
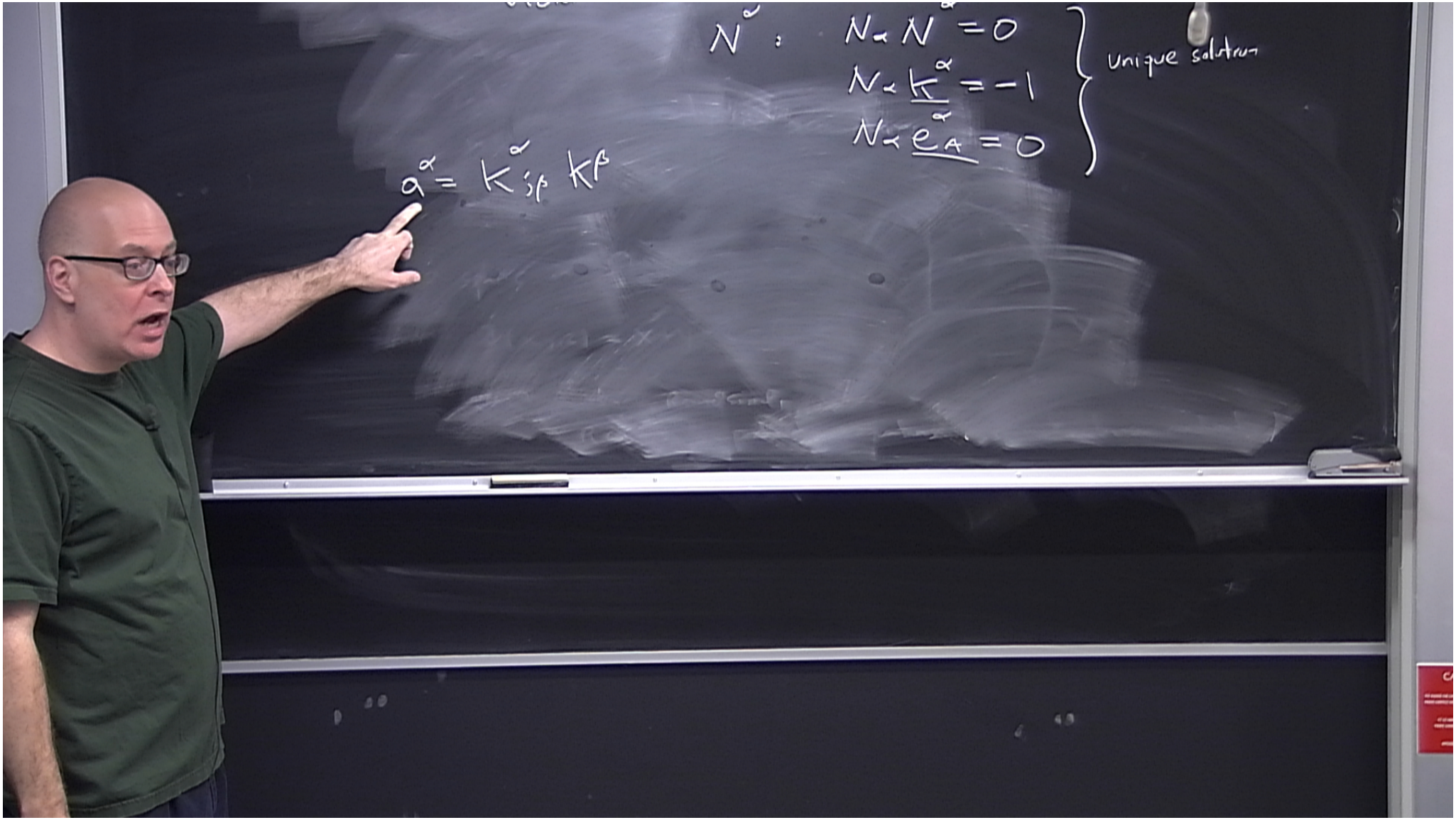
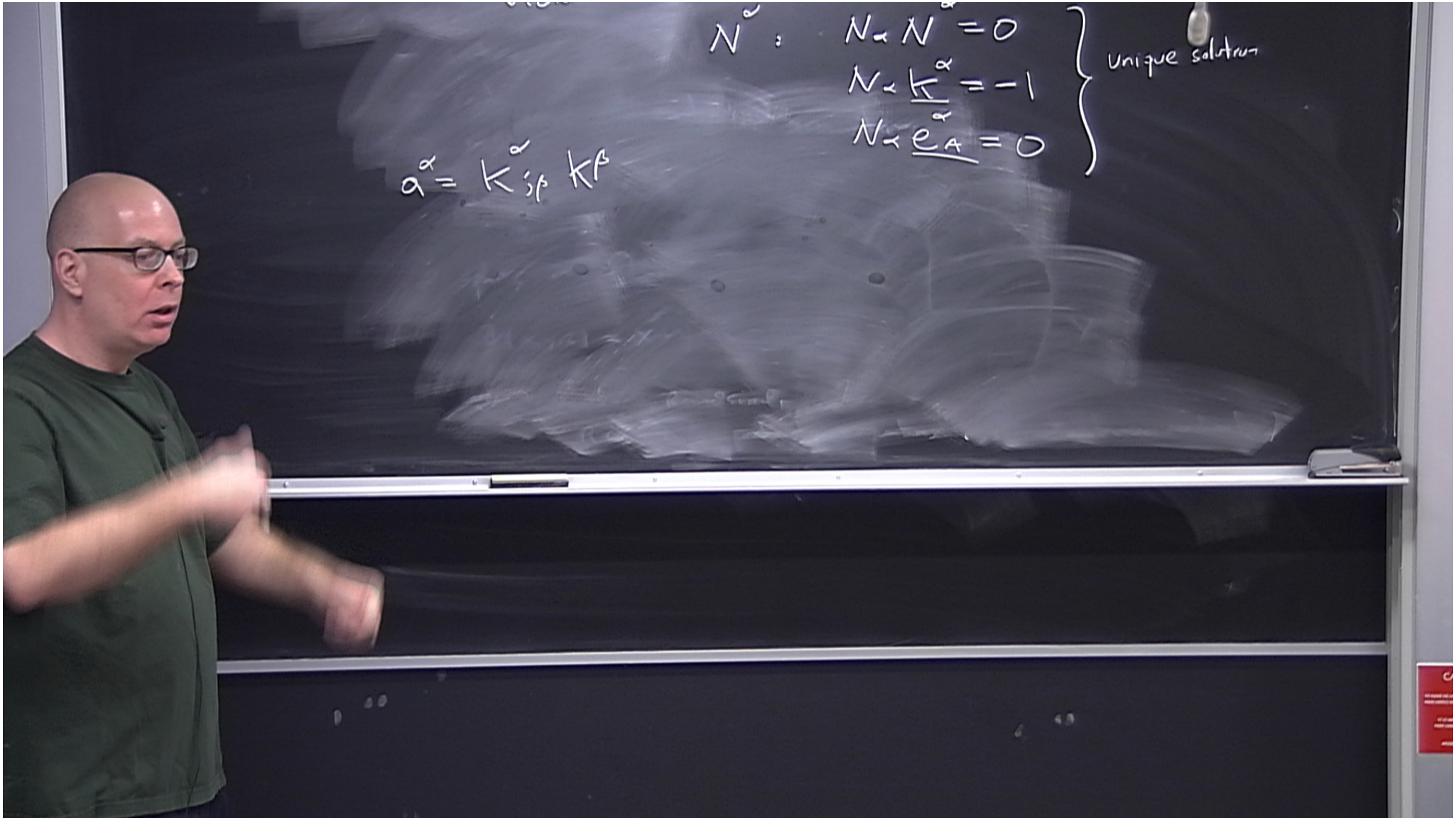

null case
$\Phi(x) = 0$; $x(y) =$
$\partial_\alpha \Phi$ a null vector $\Rightarrow$ cannot obtain a unit normal
normal : $k_\alpha = -\partial_\alpha \Phi$
(Φ increases toward future)
(k^α is future-pointing)
$k^\alpha k_\alpha = 0$
ΔN^α
k^α
$\theta^A =$ const
$\Phi =$
Σ
k^α is also tangent to hypersurface
k^α is tangent to null curves in Σ
geodesics!
congruence of hypersurface orthogonal null geodesics.
null generators.


Complete the vectorial basis k^a, e^A_a with a fourth vector


Complete the vectorial basis k^α, e^α_A with a fourth vector
N^α : $N_\alpha N^\alpha = 0$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
unique solution


$N^{\tilde{\alpha}}$:
$N_\alpha N^\alpha = 0$
$N_\alpha \underline{k}^\alpha = -1$
$N_\alpha \underline{e}_A^\alpha = 0$
Unique solution


N^α :
$N_\alpha N^\alpha = 0$
$N_\alpha \underline{k}^\alpha = -1$
$N_\alpha \underline{e}_A^\alpha = 0$
Unique solution
$a^\alpha = k^\alpha{}_{;\beta} k^\beta$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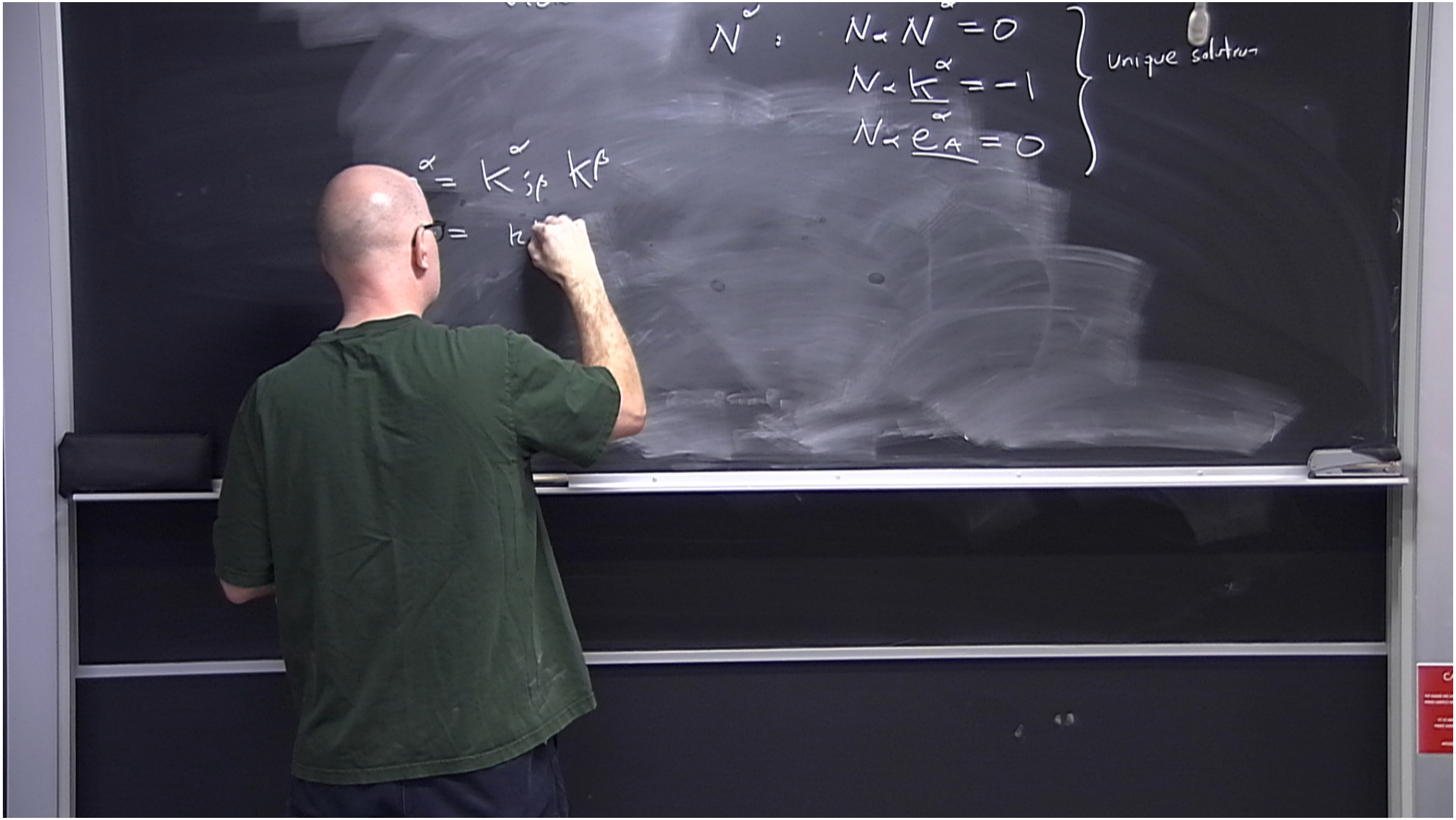
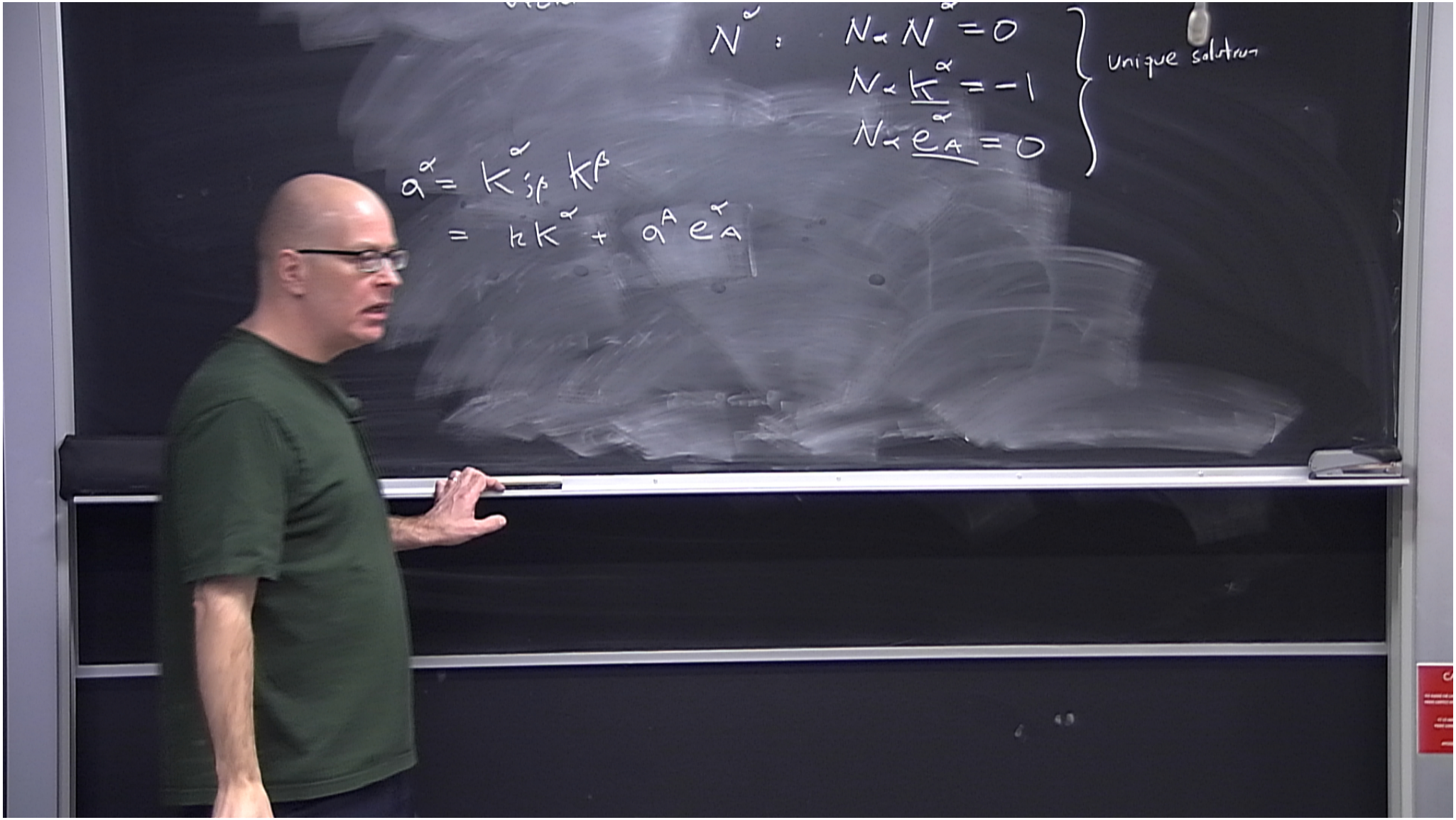
Unique solution
$q^\alpha = k^\alpha_{;\beta} k^\beta$


N^α :
$N_\alpha N^\alpha = 0$
$N_\alpha \underline{k}^\alpha = -1$
$N_\alpha \underline{e}_A^\alpha = 0$
Unique solution
$= k^\alpha{}_{;\beta} k^\beta$


N^α :
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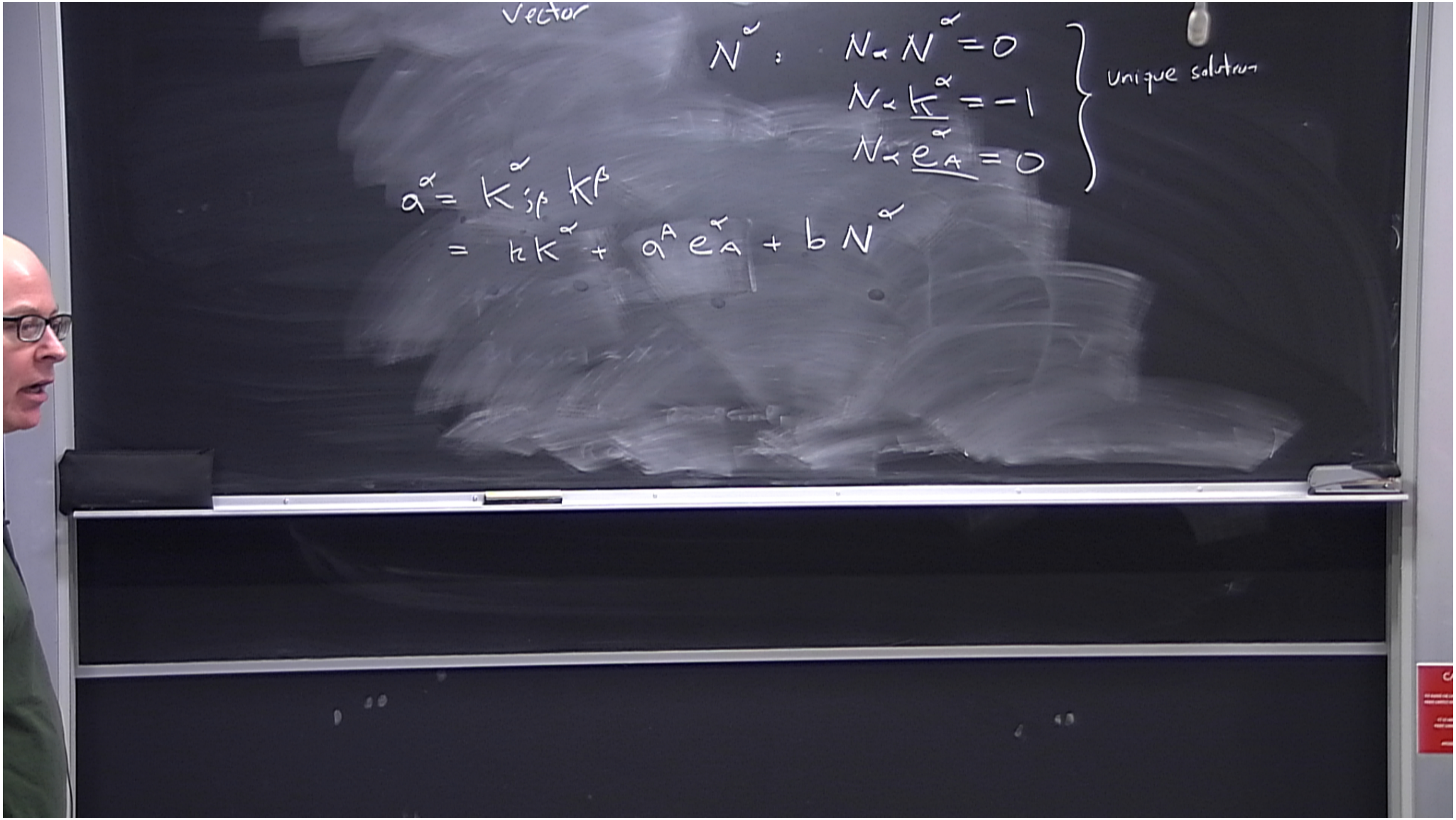
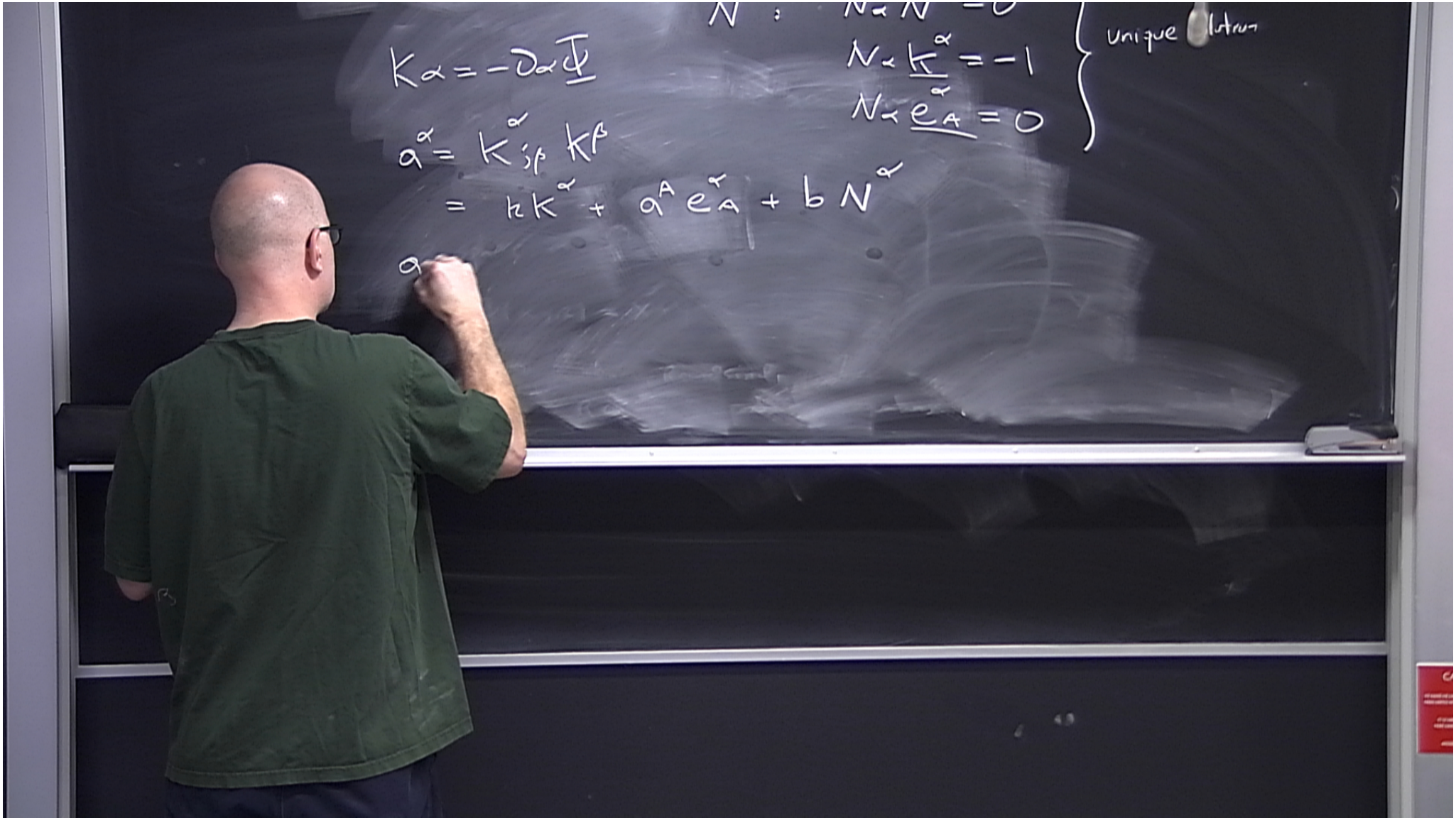
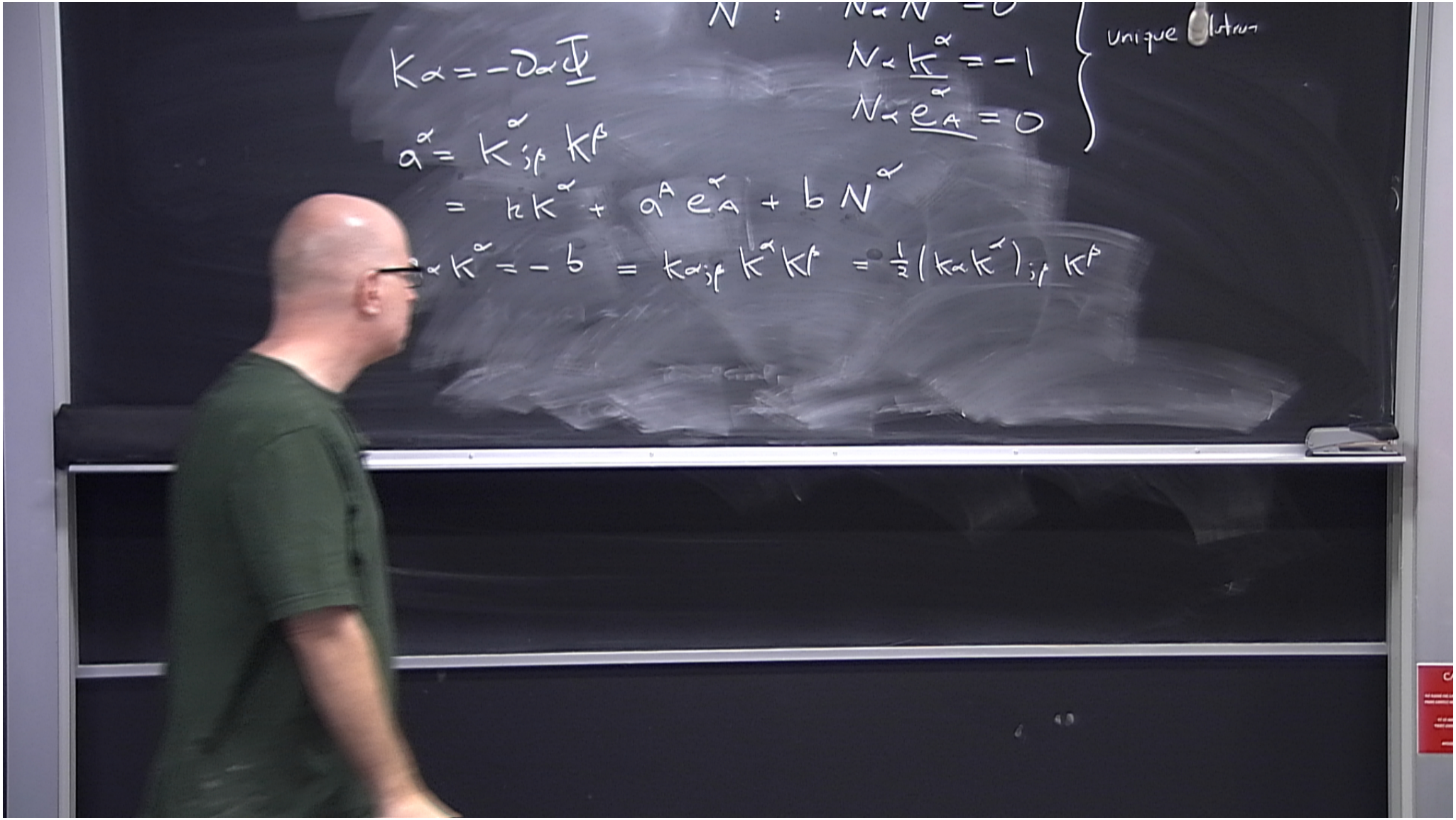
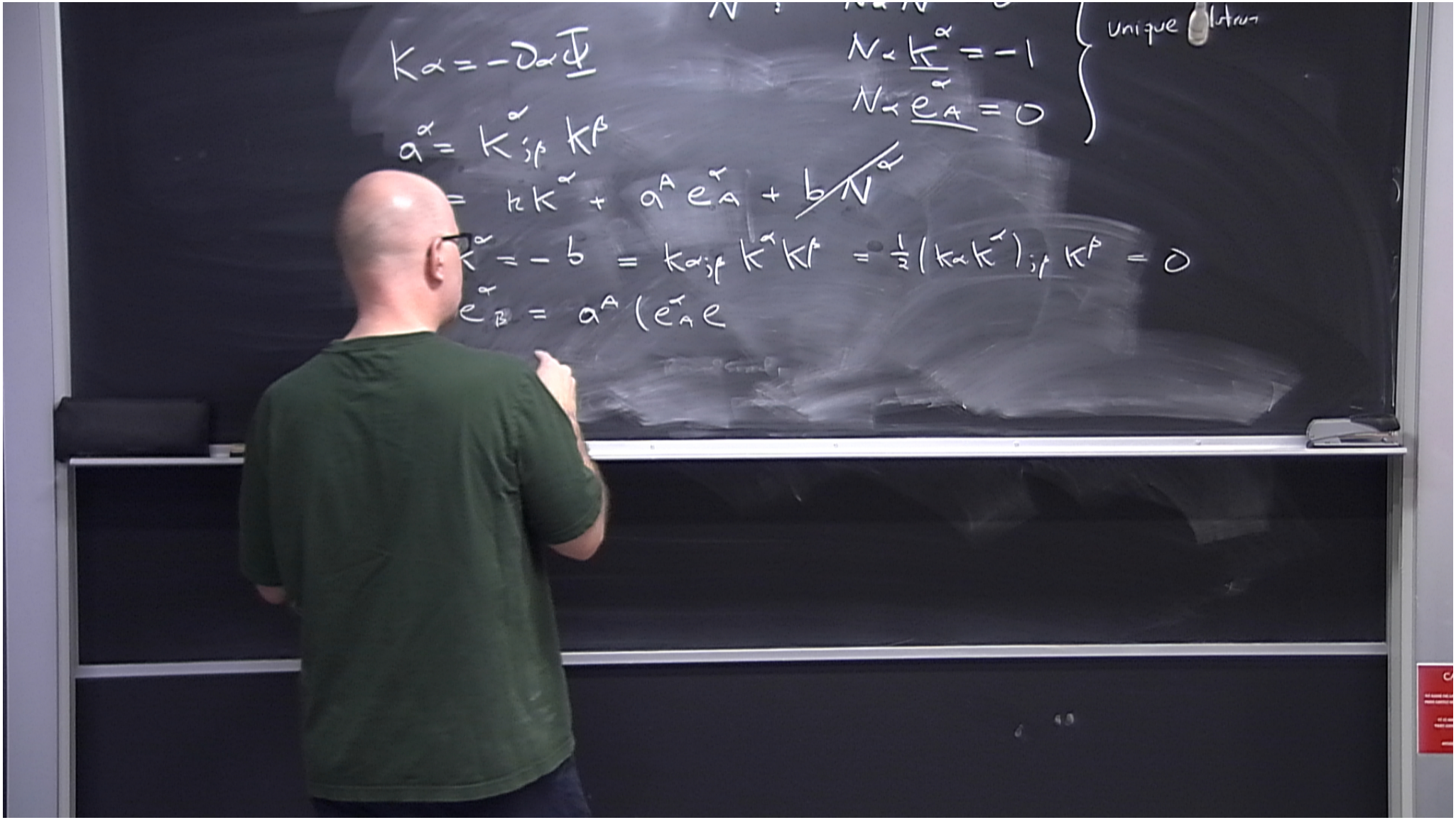
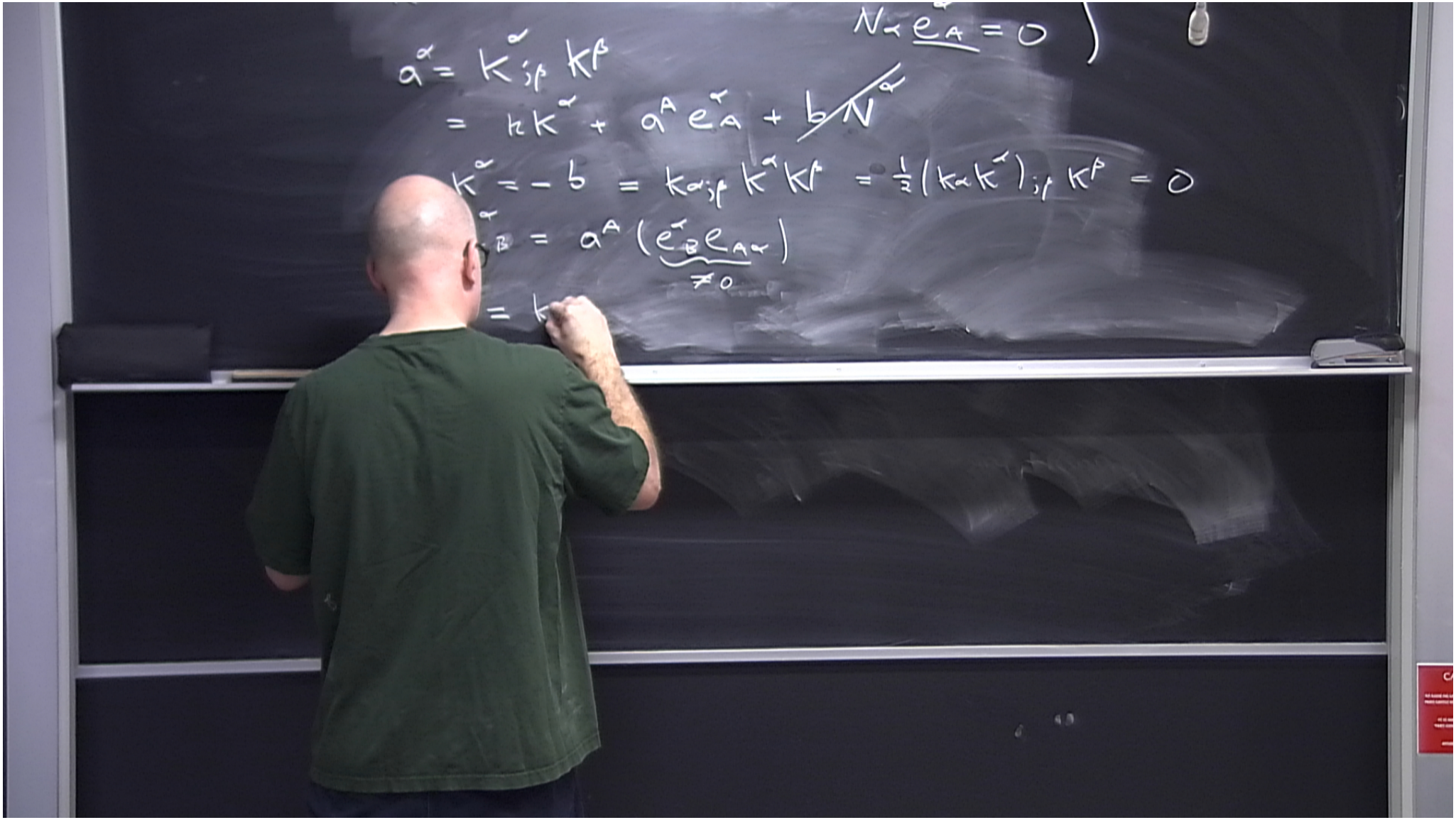
Unique solution
$q^\alpha = k^\alpha{}_{;\beta} k^\beta$
$= \kappa k^\alpha + q^A e^\alpha_A$

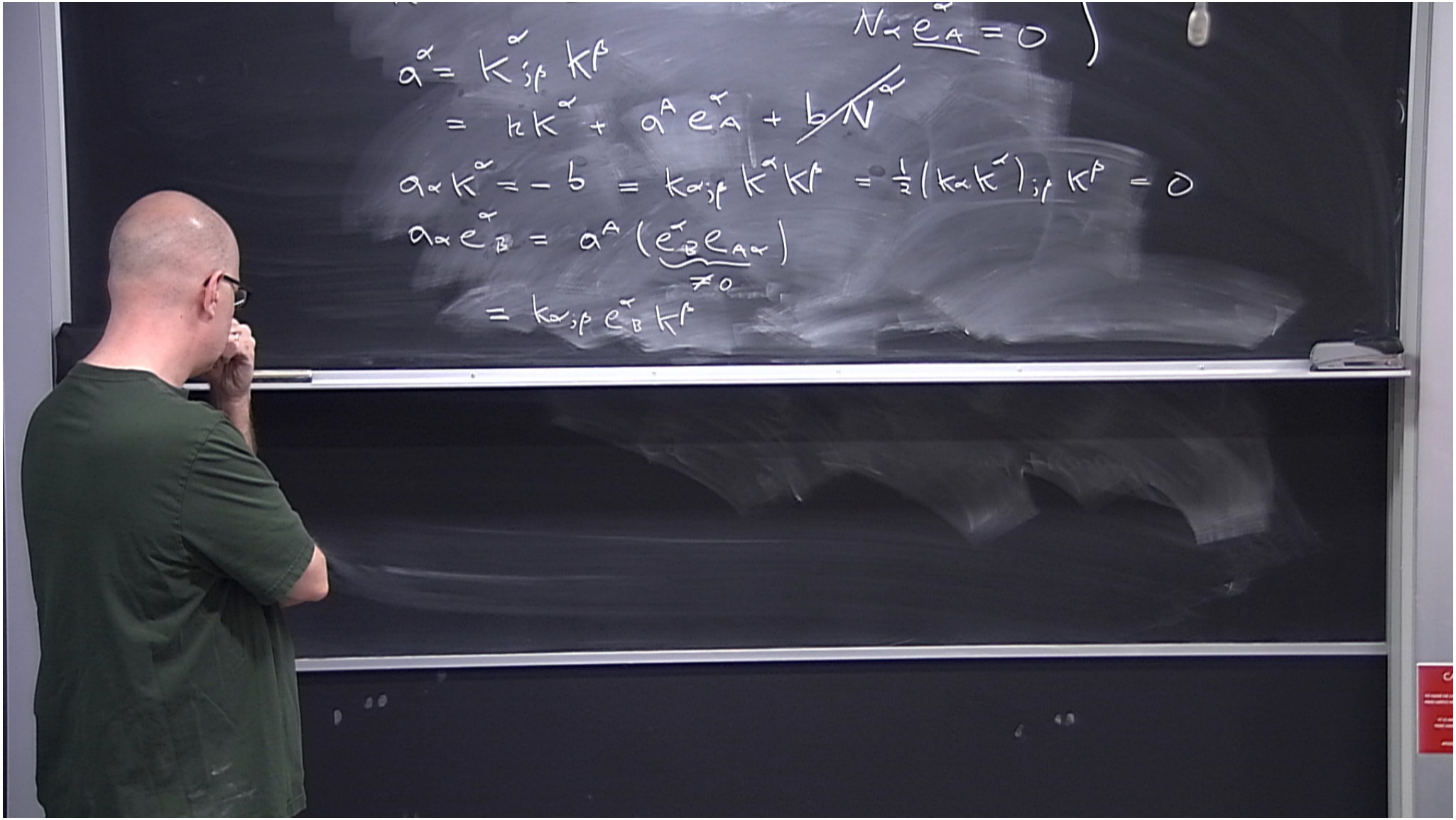

vector
N^α : $N_\alpha N^\alpha = 0$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
Unique solution
$q^\alpha = k^\alpha_{;\beta} k^\beta$
$= \kappa k^\alpha + q^A e^\alpha_A + b N^\alpha$

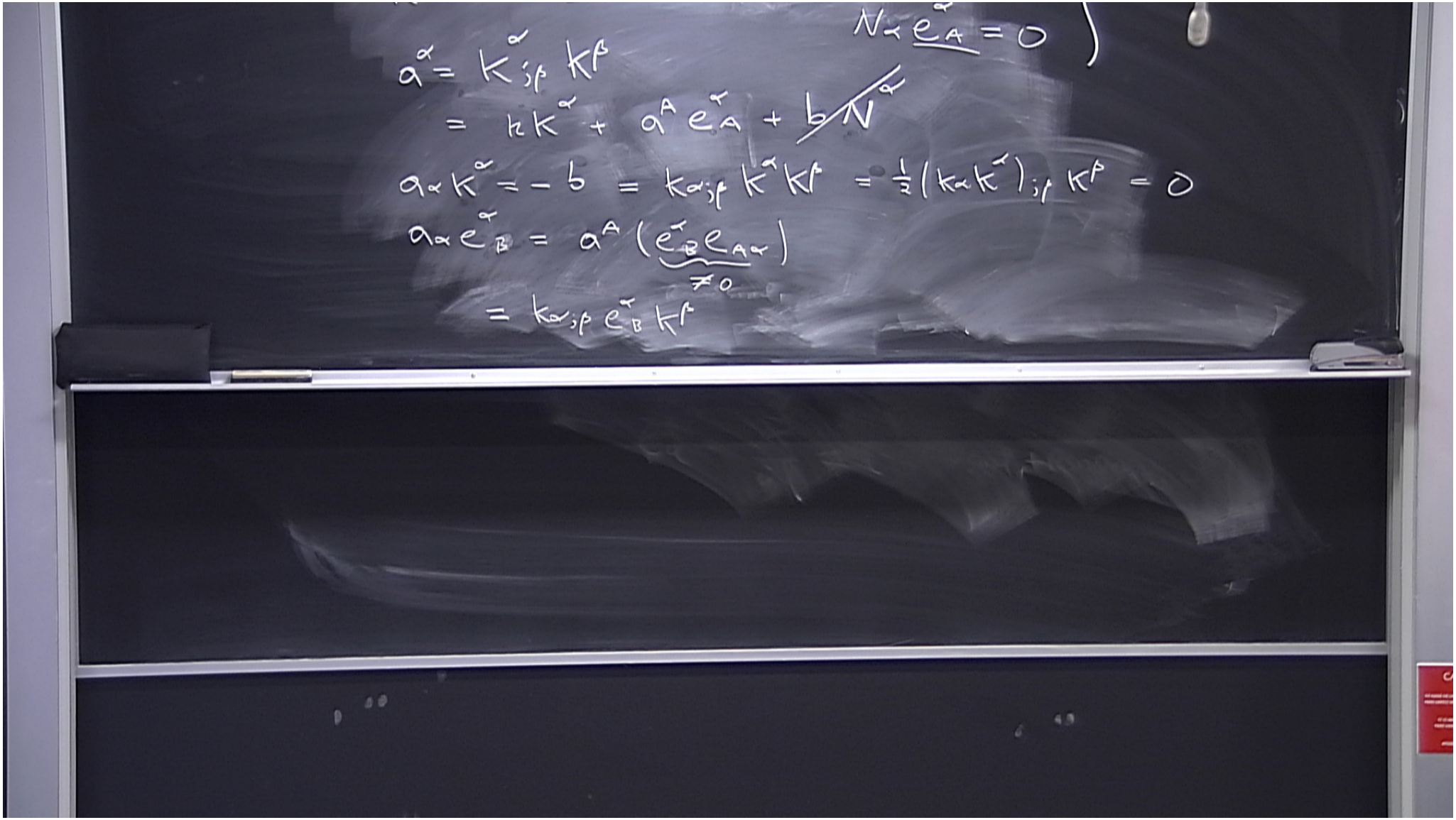

$K_\alpha = -\partial_\alpha \Psi$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
unique
$a^\alpha = k^\alpha_{;\beta} k^\beta$
$= \kappa k^\alpha + a^A e^\alpha_A + b N^\alpha$

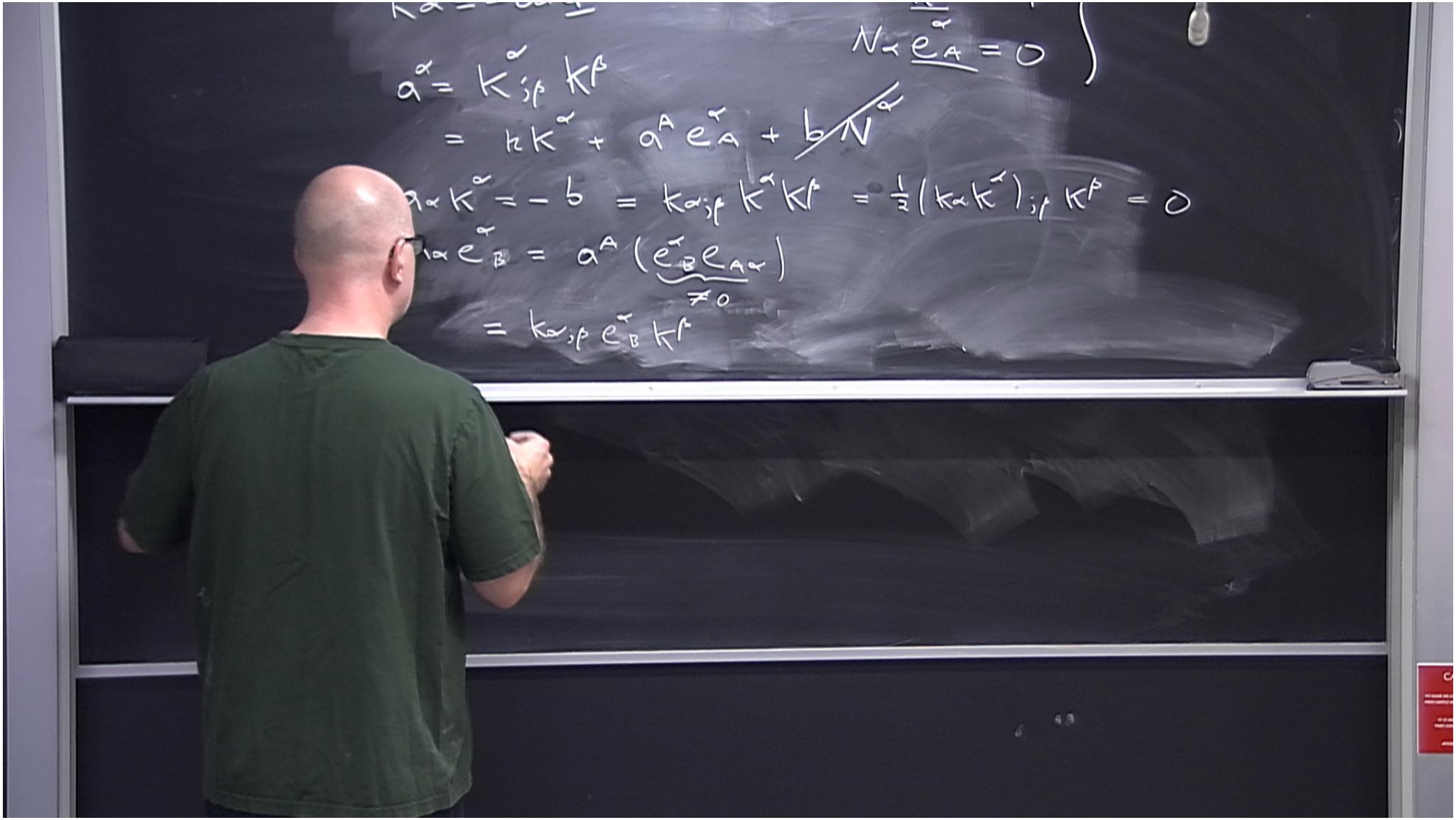

$K_\alpha = -\partial_\alpha \Psi$
$a^\alpha = K^\alpha{}_{;\beta} K^\beta$
$= \kappa K^\alpha + a^A e^\alpha_A + b N^\alpha$
$N_\alpha K^\alpha = -b = K_{\alpha;\beta} K^\alpha K^\beta = \frac{1}{2}(K_\alpha K^\alpha)_{;\beta} K^\beta$
$N_\alpha K^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
unique

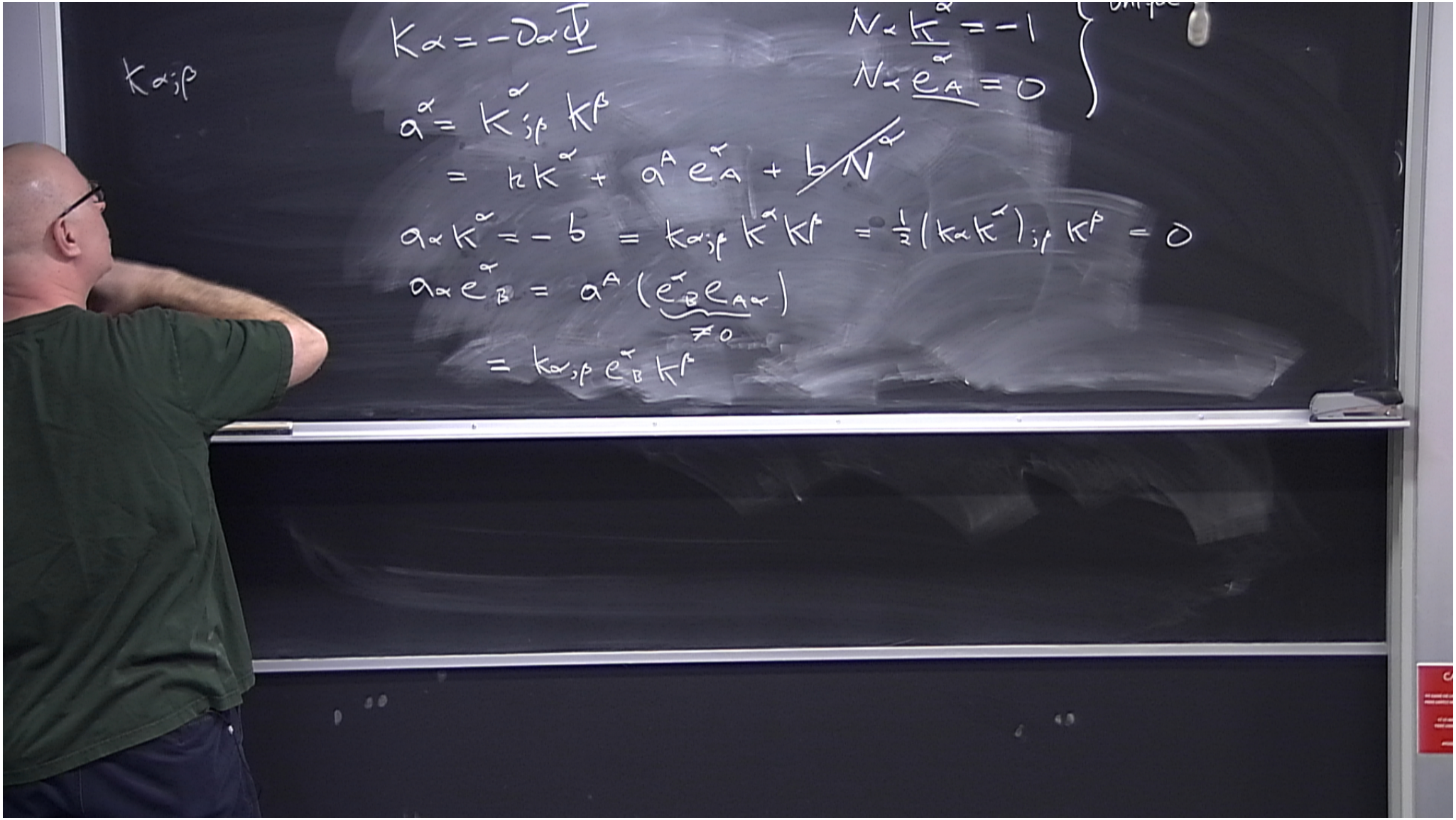

$K_\alpha = -\partial_\alpha \Psi$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
Unique
$a^\alpha = k^\alpha{}_{;\beta} k^\beta$
$= k k^\alpha + a^A e^\alpha_A + b N^\alpha$
$k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$e^\alpha_B = a^A (e^\alpha_A e$

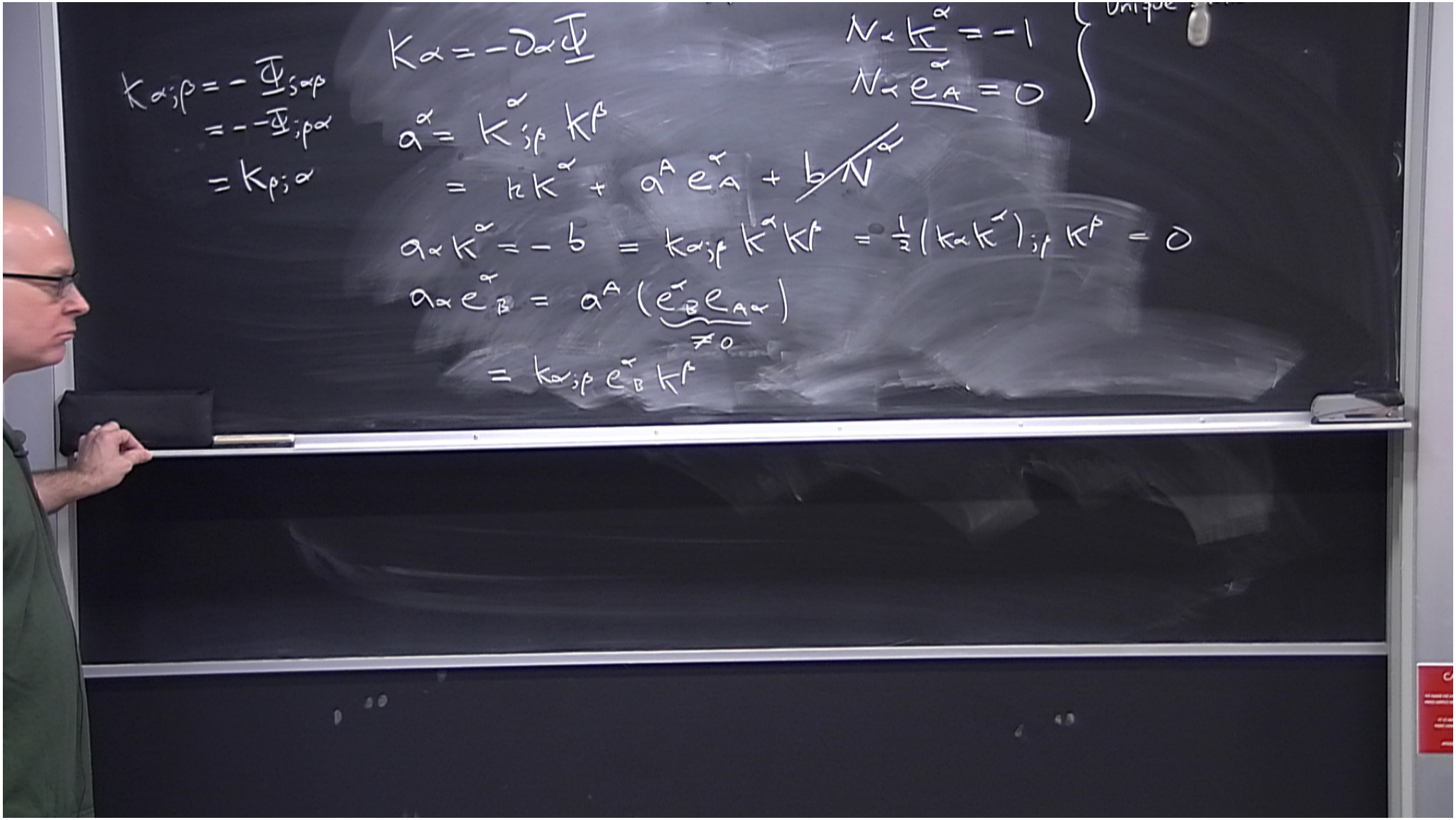

$N_\alpha e^\alpha_A = 0$
$a^\alpha = k^\alpha{}_{;\beta} k^\beta$
$= k k^\alpha + a^A e^\alpha_A + b N^\alpha$
$k^\alpha = -b = k^\alpha{}_{;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$

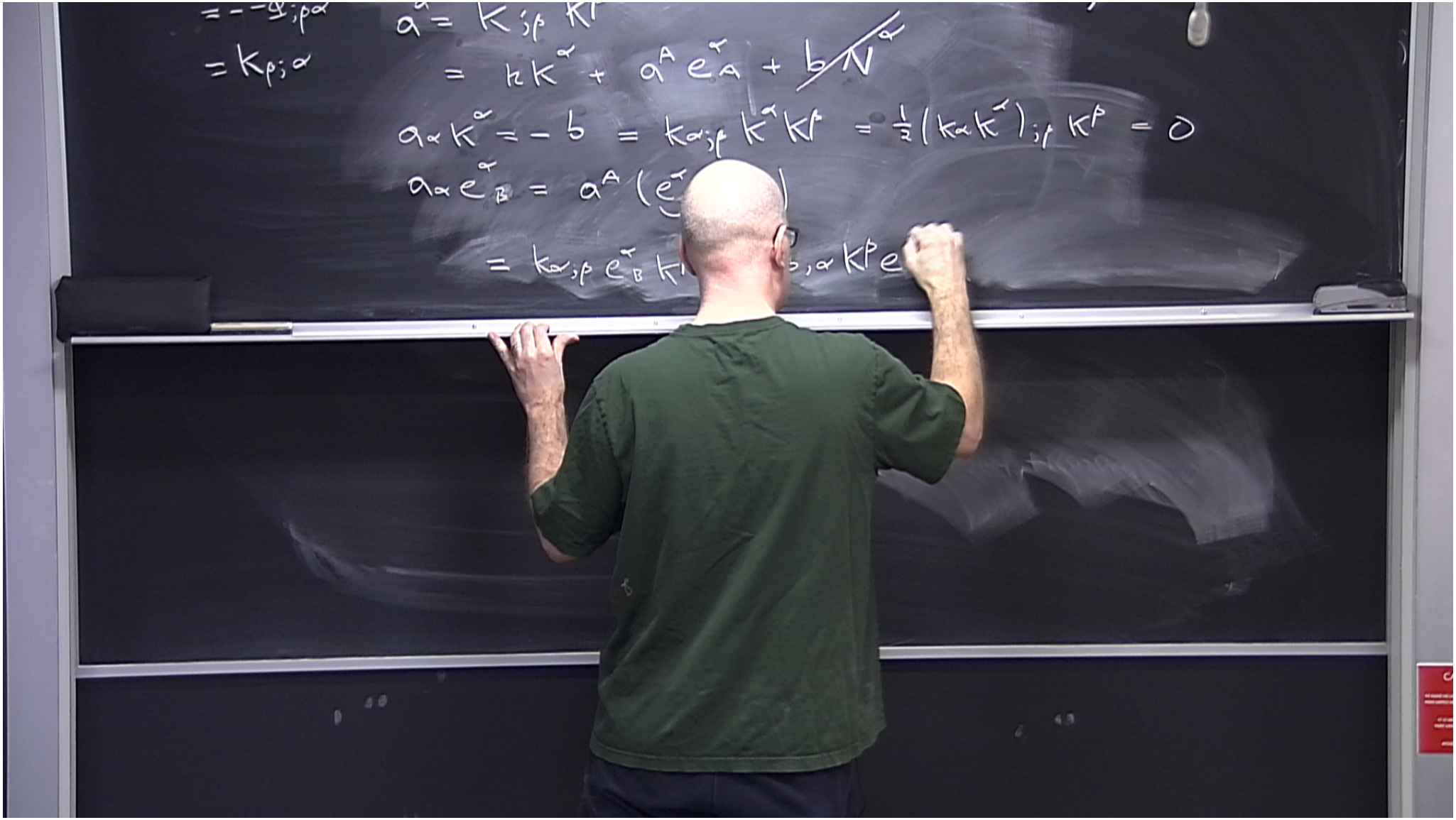

$N_\alpha e^\alpha_A = 0$
$a^\alpha = k^\alpha{}_{;\beta} k^\beta$
$= \kappa k^\alpha + a^A e^\alpha_A + b N^\alpha$
$a_\alpha k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a_\alpha e^\alpha_B = a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$
$= k_{\alpha;\beta} e^\alpha_B k^\beta$

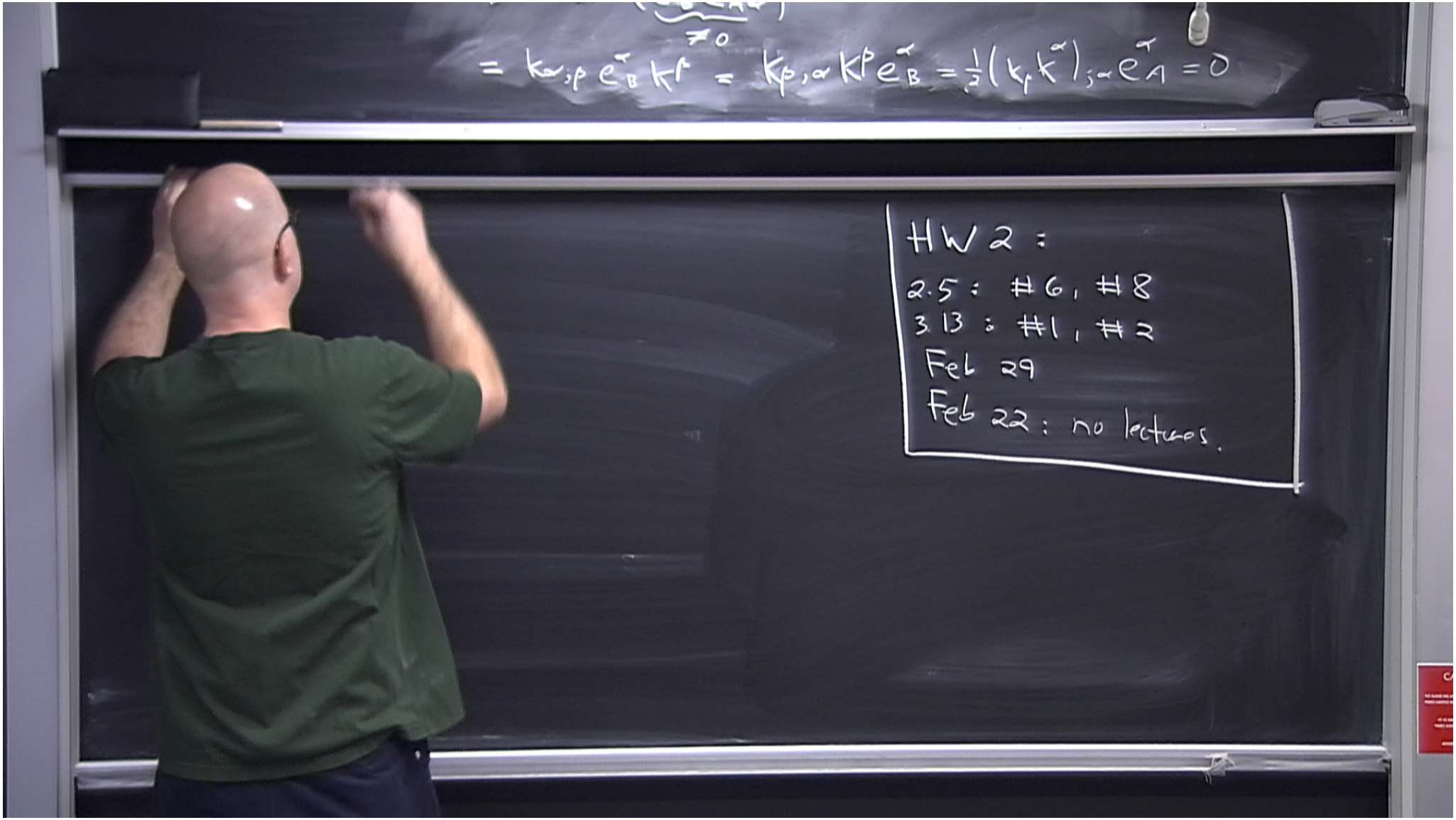

$N_\alpha e^\alpha_A = 0$
$a^\alpha = K^\alpha{}_{;\beta} k^\beta$
$= k k^\alpha + a^A e^\alpha_A + b N^\alpha$
$a_\alpha k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a_\alpha e^\alpha_B = a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$
$= k_{\alpha;\beta} e^\alpha_B k^\beta$

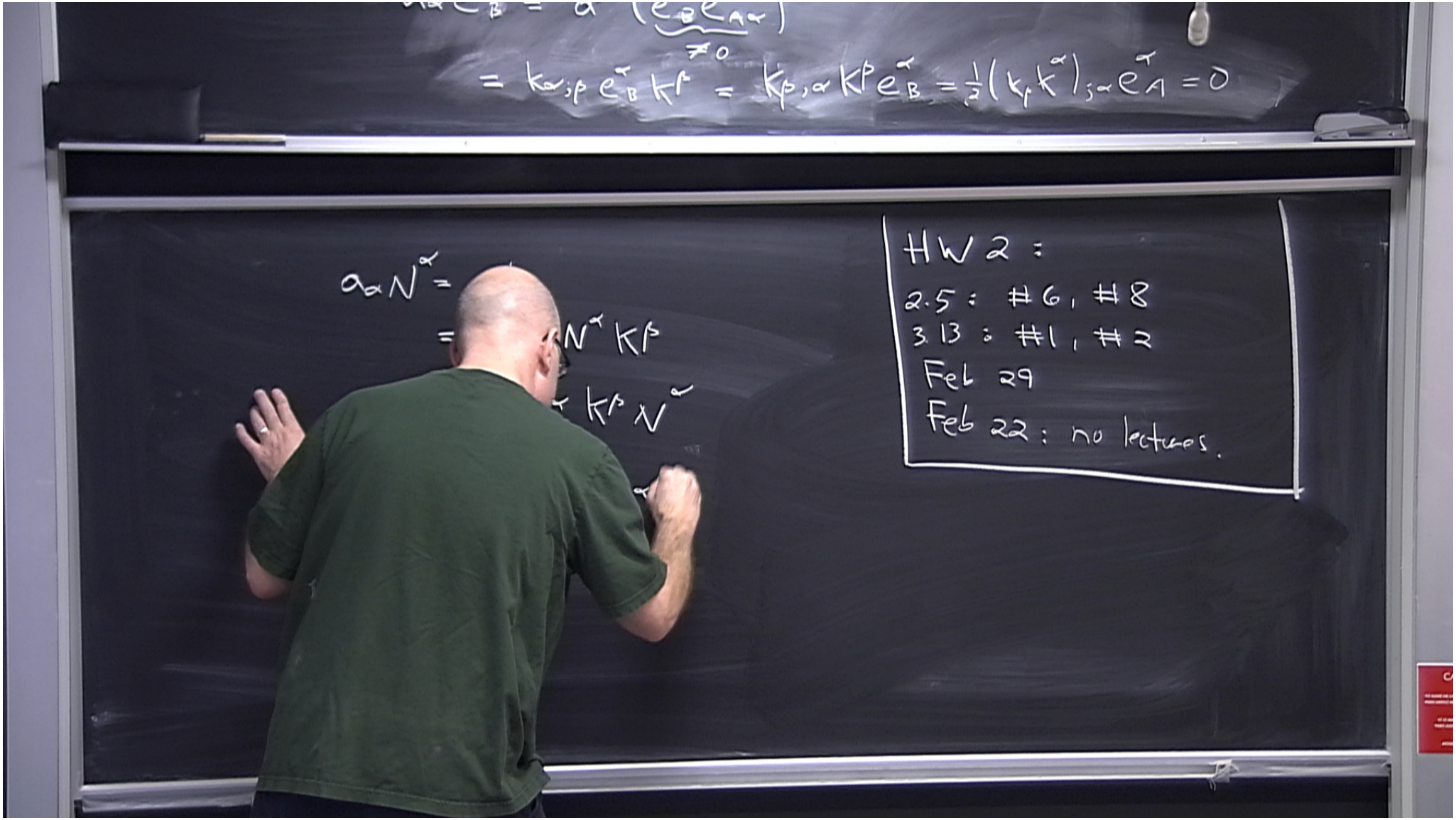

$N_\alpha e^\alpha_A = 0$
$a^\alpha = K^\alpha_{;\beta} k^\beta$
$= k k^\alpha + a^A e^\alpha_A + b N^\alpha$
$a_\alpha k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a_\alpha e^\alpha_B = a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$
$= k_{\alpha;\beta} e^\alpha_B k^\beta$


$k_\alpha = -\partial_\alpha \Psi$
$k_{\alpha;\beta}$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$
$a^\alpha = k^\alpha_{;\beta} k^\beta$
$= \kappa k^\alpha + a^A e^\alpha_A + b N^\alpha$
$a_\alpha k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a_\alpha e^\alpha_B = a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$
$= k_{\alpha;\beta} e^\alpha_B k^\beta$


$k_{\alpha;\beta} = -\Phi_{;\alpha\beta}$
$= -(-\Phi_{;\beta\alpha})$
$= k_{\beta;\alpha}$
$k_\alpha = -\partial_\alpha \Phi$
$a^\alpha = k^\alpha{}_{;\beta} k^\beta$
$= \kappa k^\alpha + a^A e^\alpha_A + b N^\alpha$
$a_\alpha k^\alpha = -b = k_{\alpha;\beta} k^\alpha k^\beta = \frac{1}{2}(k_\alpha k^\alpha)_{;\beta} k^\beta = 0$
$a_\alpha e^\alpha_B = a^A (e^\alpha_B e_{A\alpha})$
$\neq 0$
$= k_{\alpha;\beta} e^\alpha_B k^\beta$
$N_\alpha k^\alpha = -1$
$N_\alpha e^\alpha_A = 0$




$= k_{\alpha;\rho} e^{\alpha}_{B} k^{\rho} = k_{\rho;\alpha} k^{\rho} e^{\alpha}_{B} = \frac{1}{2}(k_\rho k^\alpha)_{;\alpha} e^{\alpha}_{A} = 0$
HW 2 :
2.5 : #6, #8
3.13 : #1, #2
Feb 29
Feb 22 : no lectures.


HW 2 :
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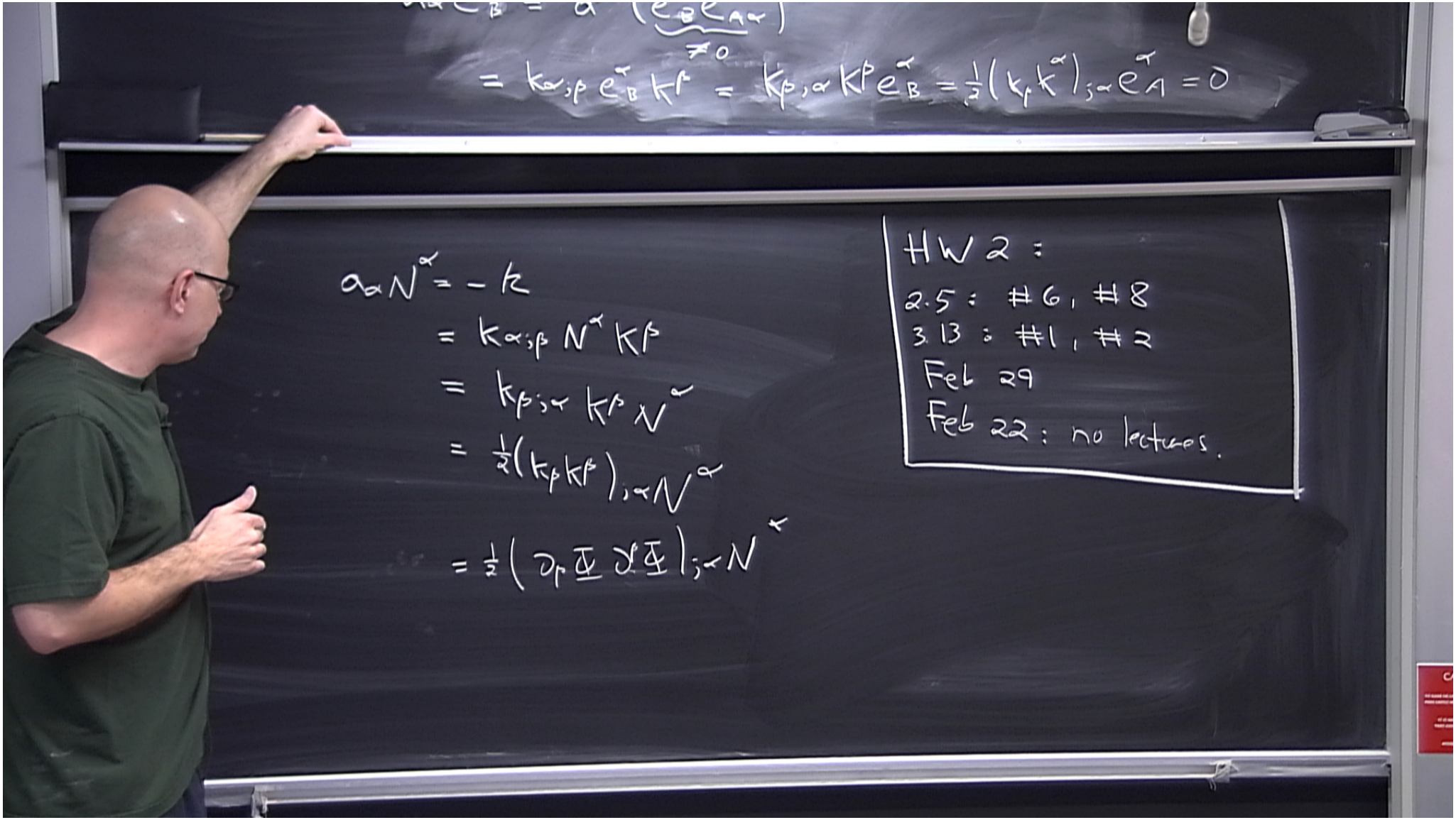
$= k_{\alpha;\beta} e_B^\alpha k^\beta = k_{\beta,\alpha} k^\beta e_B^\alpha = \frac{1}{2}(k_\beta k^\beta)_{;\alpha} e_A^\alpha = 0$
$a_\alpha N^\alpha = -k$
$= k_{\alpha;\beta} N^\alpha k^\beta$
$= k_{\beta;\alpha} k^\beta N^\alpha$
$= \frac{1}{2}(k_\beta k^\beta)_{;\alpha} N^\alpha$
$= \frac{1}{2}(\partial_\rho \Xi \partial^\rho \Xi)_{;\alpha} N^\alpha$
HW 2 :
2.5 : #6, #8
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Feb 29
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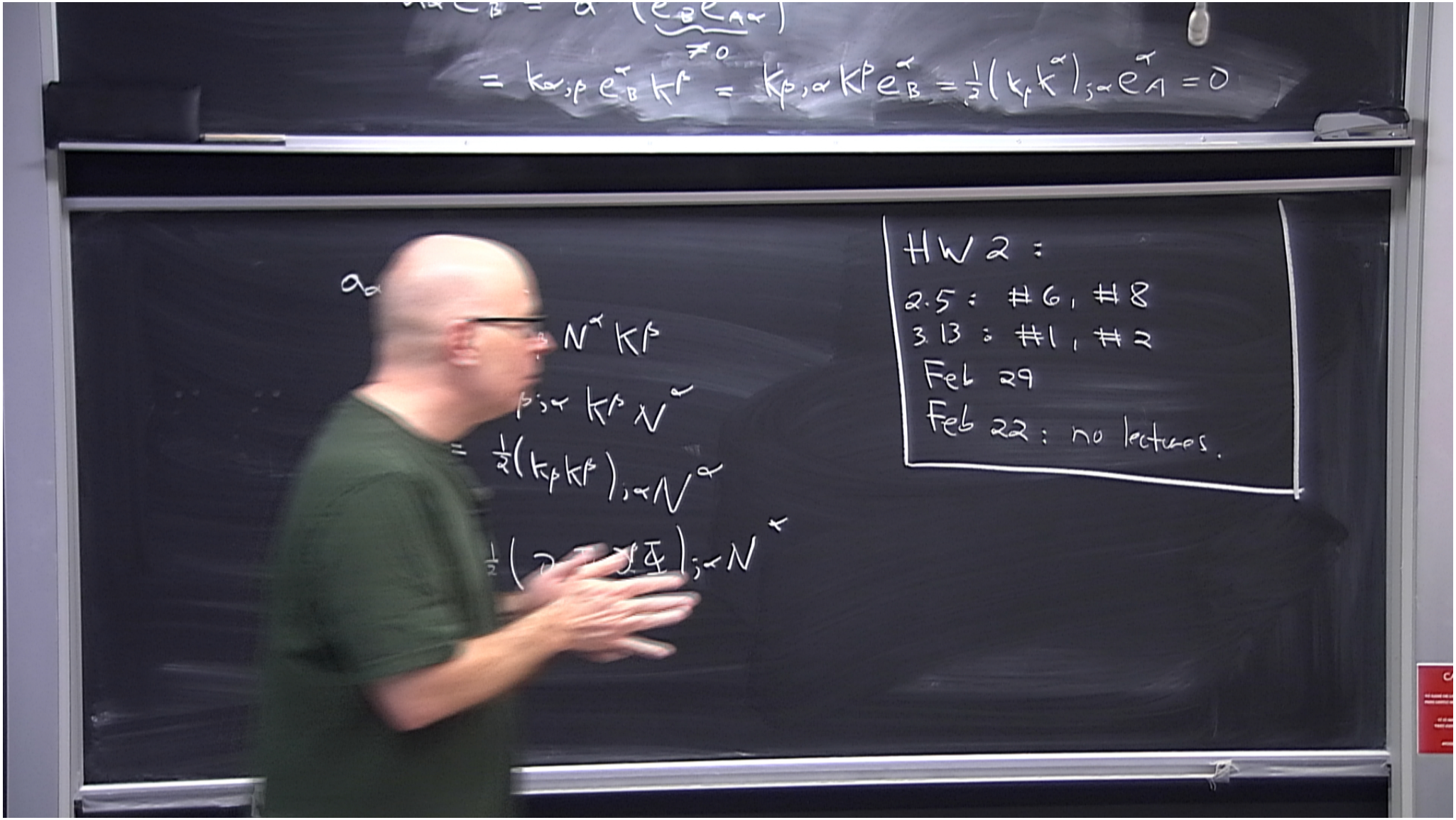
HW 2 :
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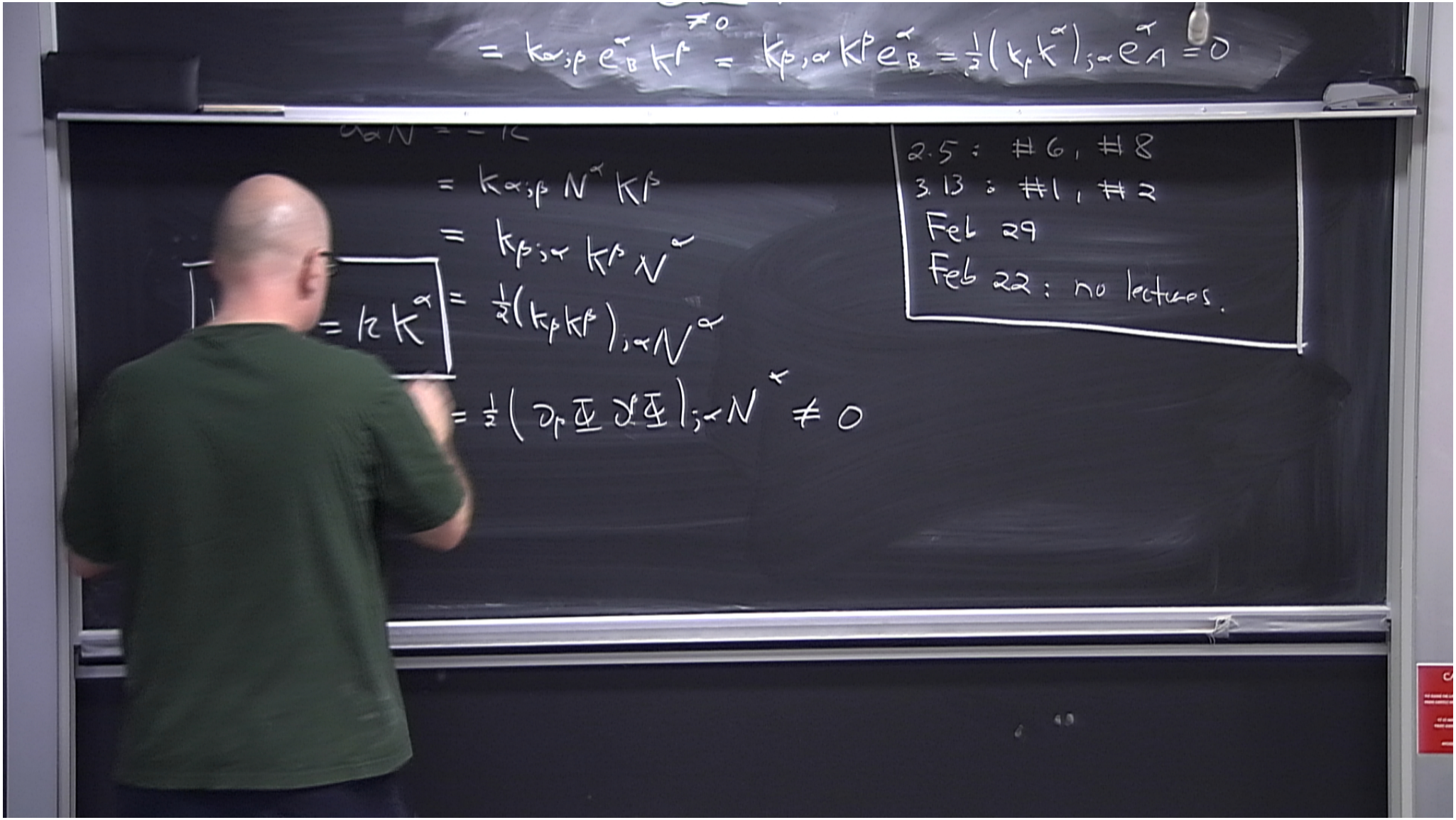
2.5 : #6, #8
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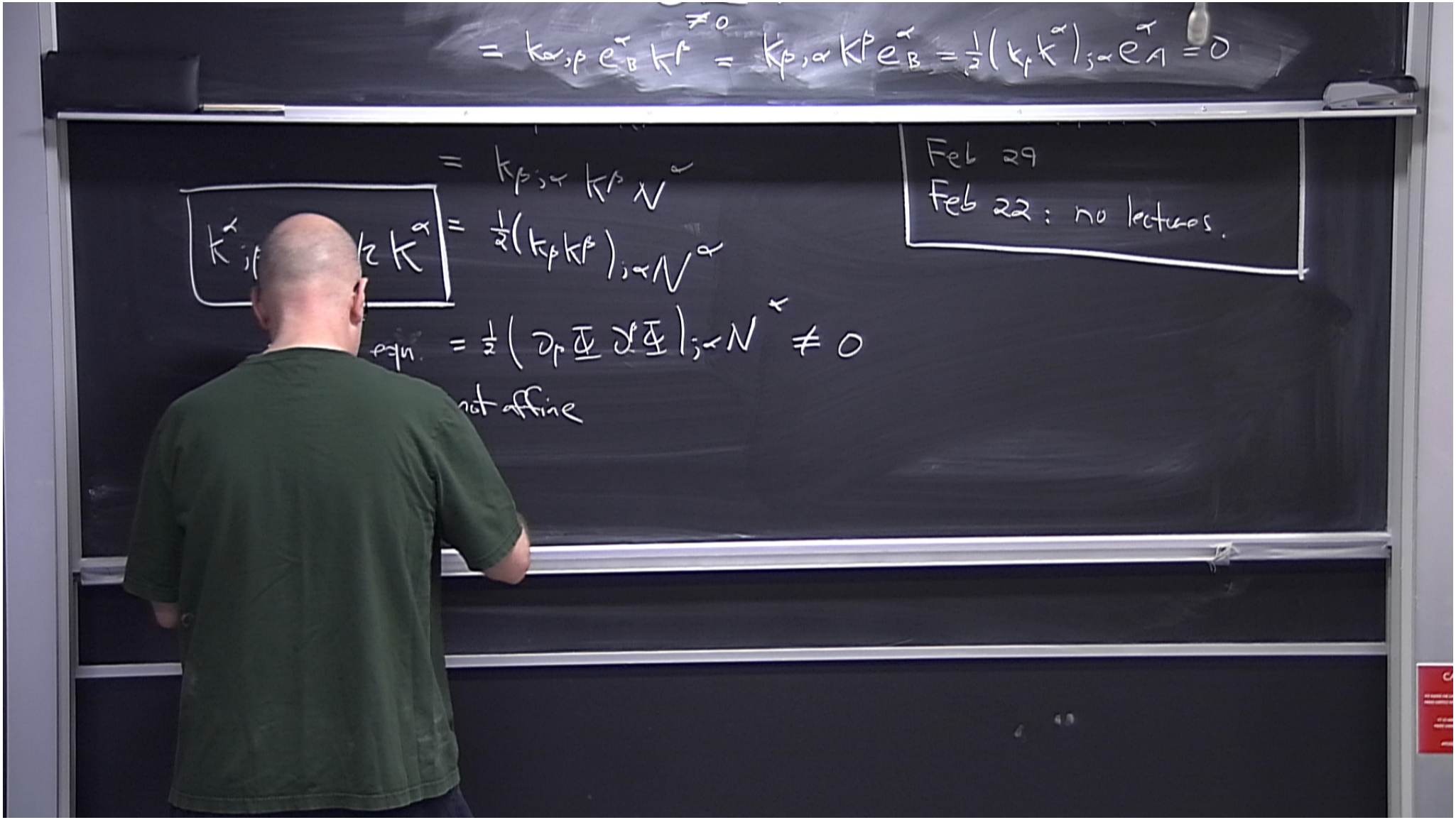
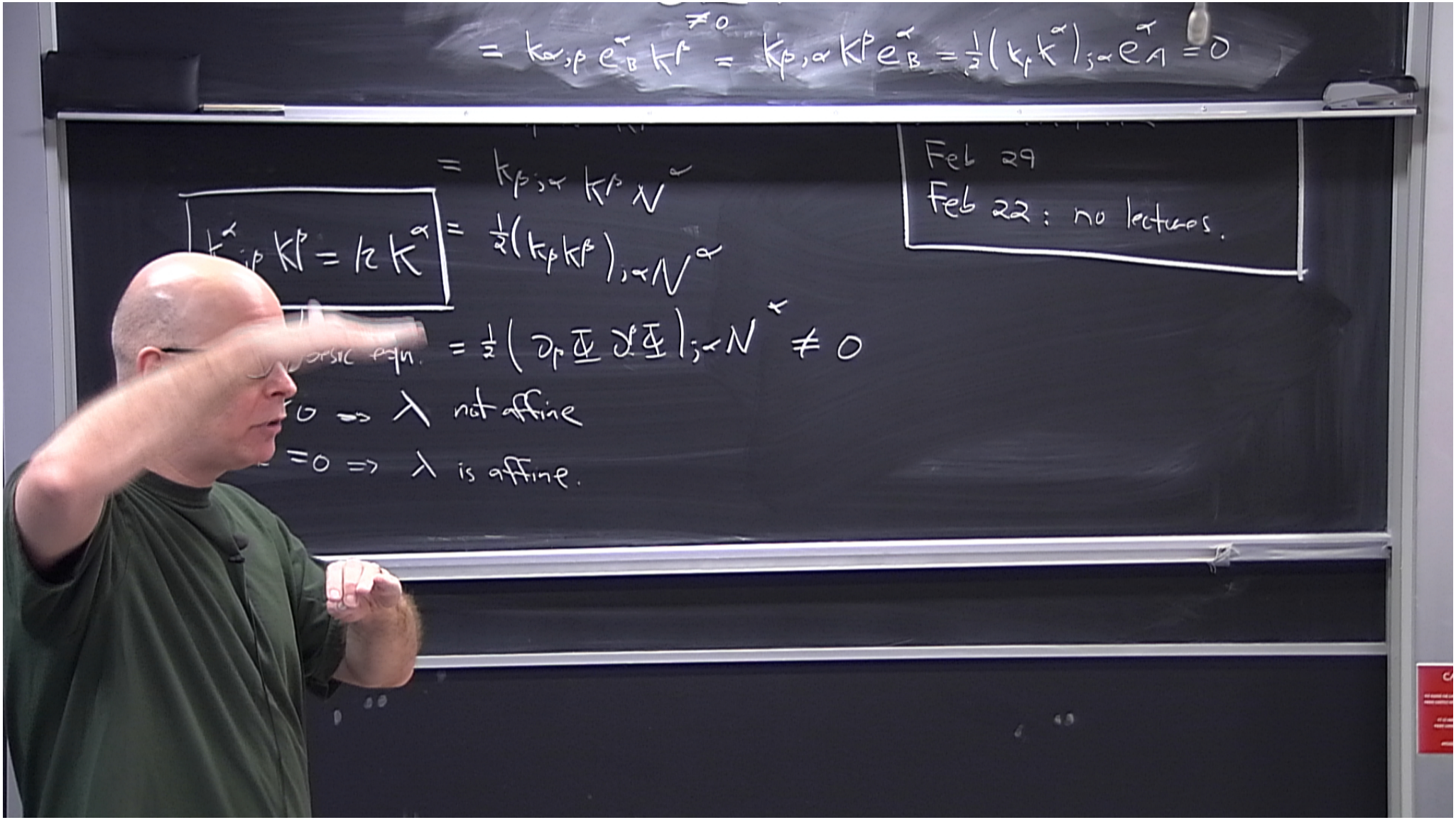
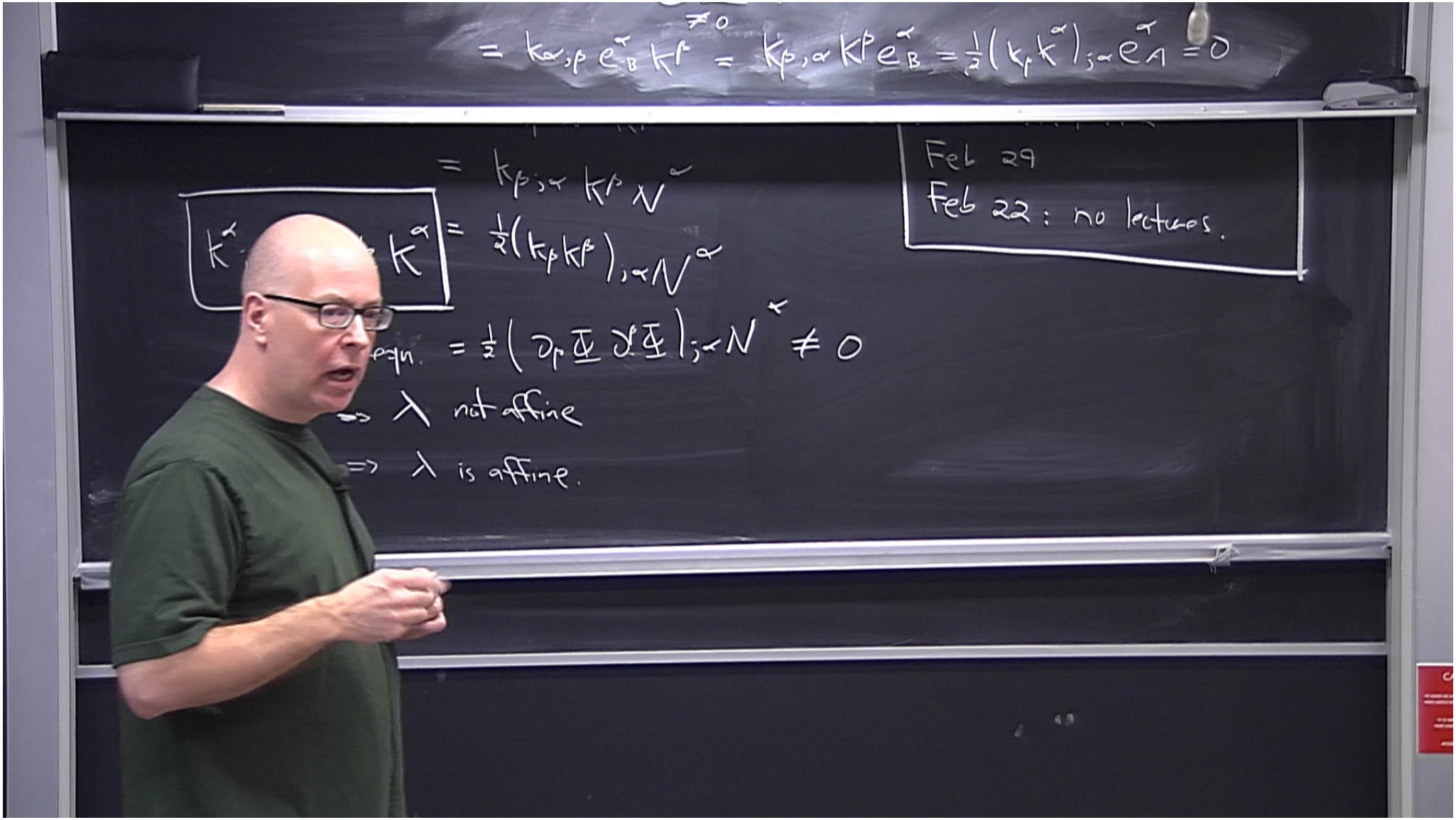
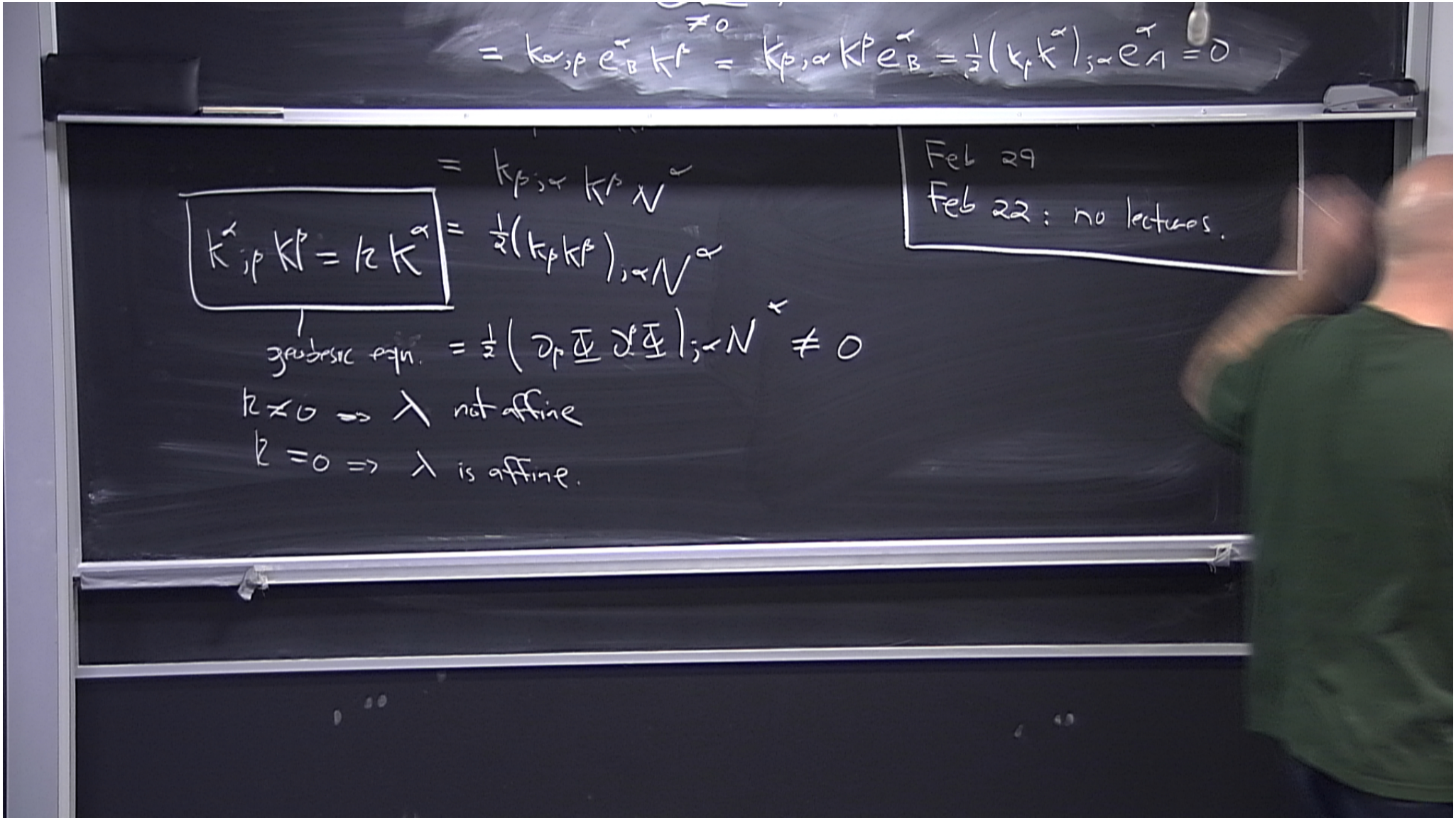

Feb 29
Feb 22: no lectures.


$= k_{\alpha;\beta} e^{\alpha}_{B} k^{\beta} = k_{\beta;\alpha} k^{\beta} e^{\alpha}_{B} = \frac{1}{2}(k_\beta k^\alpha)_{;\alpha} e^{\alpha}_{A} = 0$
$\neq 0$
$= k_{\beta;\alpha} k^{\beta} N^{\alpha}$
$k^{\alpha}{}_{;\beta} k^{\beta} = \kappa k^{\alpha}$
$= \frac{1}{2}(k_\beta k^\beta)_{;\alpha} N^{\alpha}$
$= \frac{1}{2}(\partial_\beta \Phi \partial^\beta \Phi)_{;\alpha} N^{\alpha} \neq 0$
$\neq 0 \Rightarrow \lambda$ not affine
$= 0 \Rightarrow \lambda$ is affine.
Feb 29
Feb 22: no lectures.


$= k_{\alpha;\rho}\, e^{\tau}_{B} k^{\rho} \neq 0 = k_{\rho;\alpha} k^{\rho} e^{\tau}_{B} = \frac{1}{2}(k_\rho k^\rho)_{;\alpha} e^{\tau}_{A} = 0$
$= k_{\rho;\alpha} k^{\rho} N^{\alpha}$
$= \frac{1}{2}(k_\rho k^\rho)_{;\alpha} N^{\alpha}$
eqn. $= \frac{1}{2}(\partial_\rho \underline{x} \partial \underline{x})_{;\alpha} N^{\alpha} \neq 0$
$\Rightarrow \lambda$ not affine
$\Rightarrow \lambda$ is affine.
Feb 29
Feb 22: no lectures.


$= k_{\alpha;\rho} e^{\alpha}_{B} k^{\rho} \neq 0 = k_{\rho;\alpha} k^{\rho} e^{\alpha}_{B} = \frac{1}{2}(k_\rho k^\rho)_{;\alpha} e^{\alpha}_{A} = 0$
$= k_{\rho;\alpha} k^{\rho} N^{\alpha}$
$k^{\alpha}_{;\rho} k^{\rho} = \kappa k^{\alpha}$
$= \frac{1}{2}(k_\rho k^\rho)_{;\alpha} N^{\alpha}$
geodesic eqn.
$= \frac{1}{2}(\partial_\rho \underline{x} \partial^\rho \underline{x})_{;\alpha} N^{\alpha} \neq 0$
$\kappa \neq 0 \Rightarrow \lambda$ not affine
$\kappa = 0 \Rightarrow \lambda$ is affine.
Feb 29
Feb 22: no lectures.