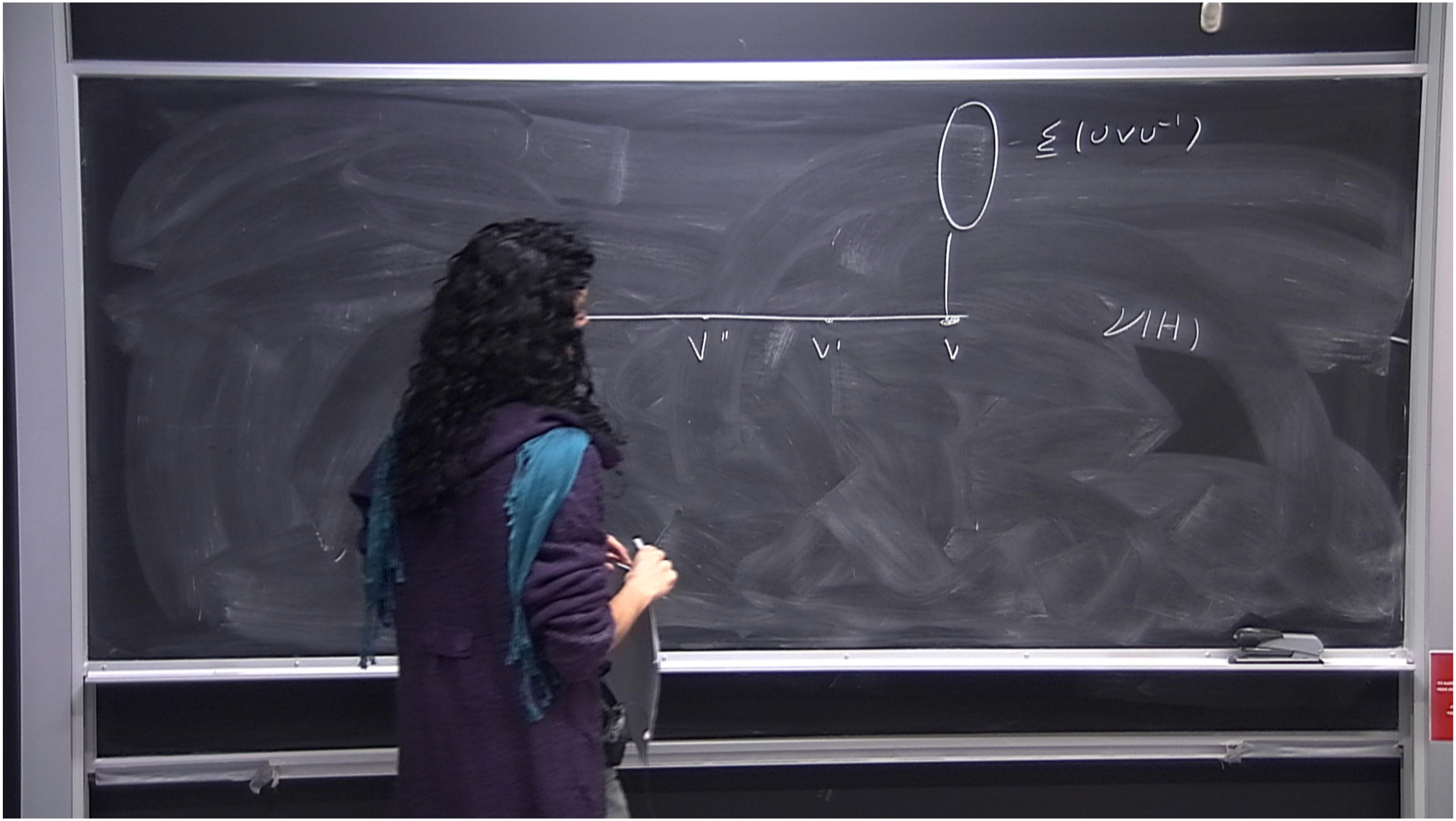
Title: Topos Quantum Physics - Lecture 16

Date: Feb 14, 2012 01:30 PM

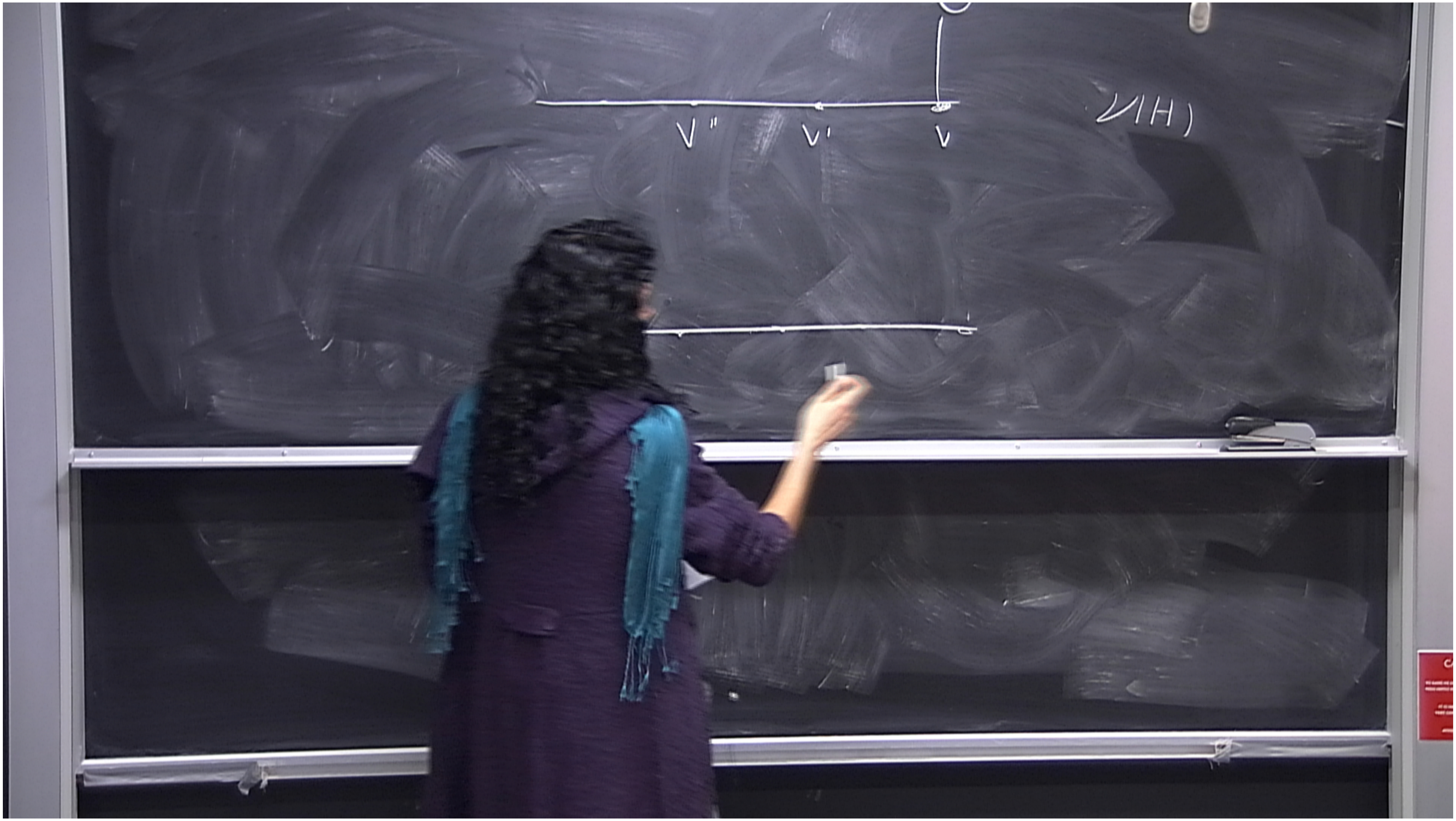
URL: http://pirsa.org/12020101

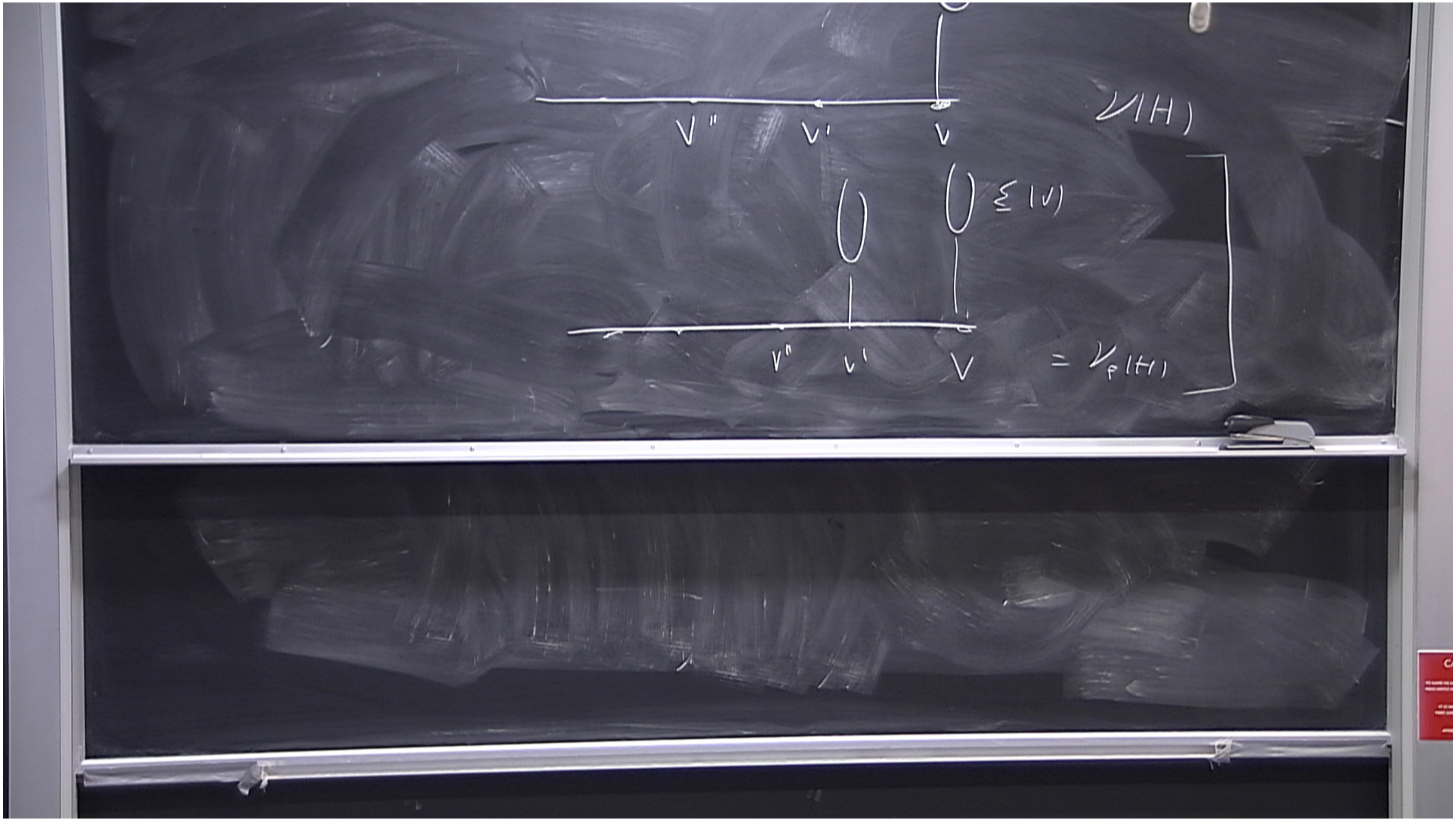
Abstract:


ξ(UVU⁻¹)
V''
V'
V
V(H)

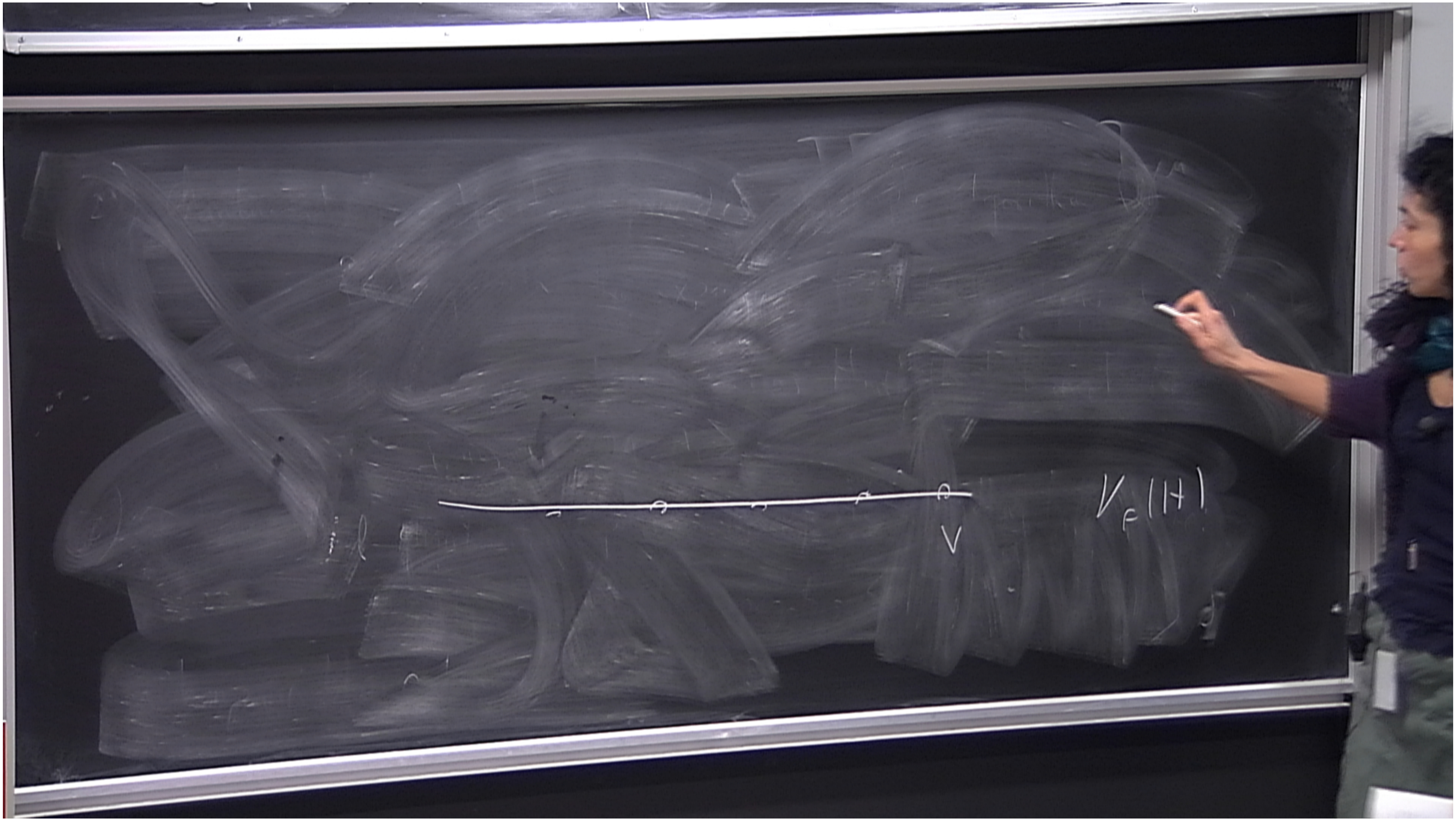
$$l_g^* : sh(V|H) \to sh(V_1|H_1)$$

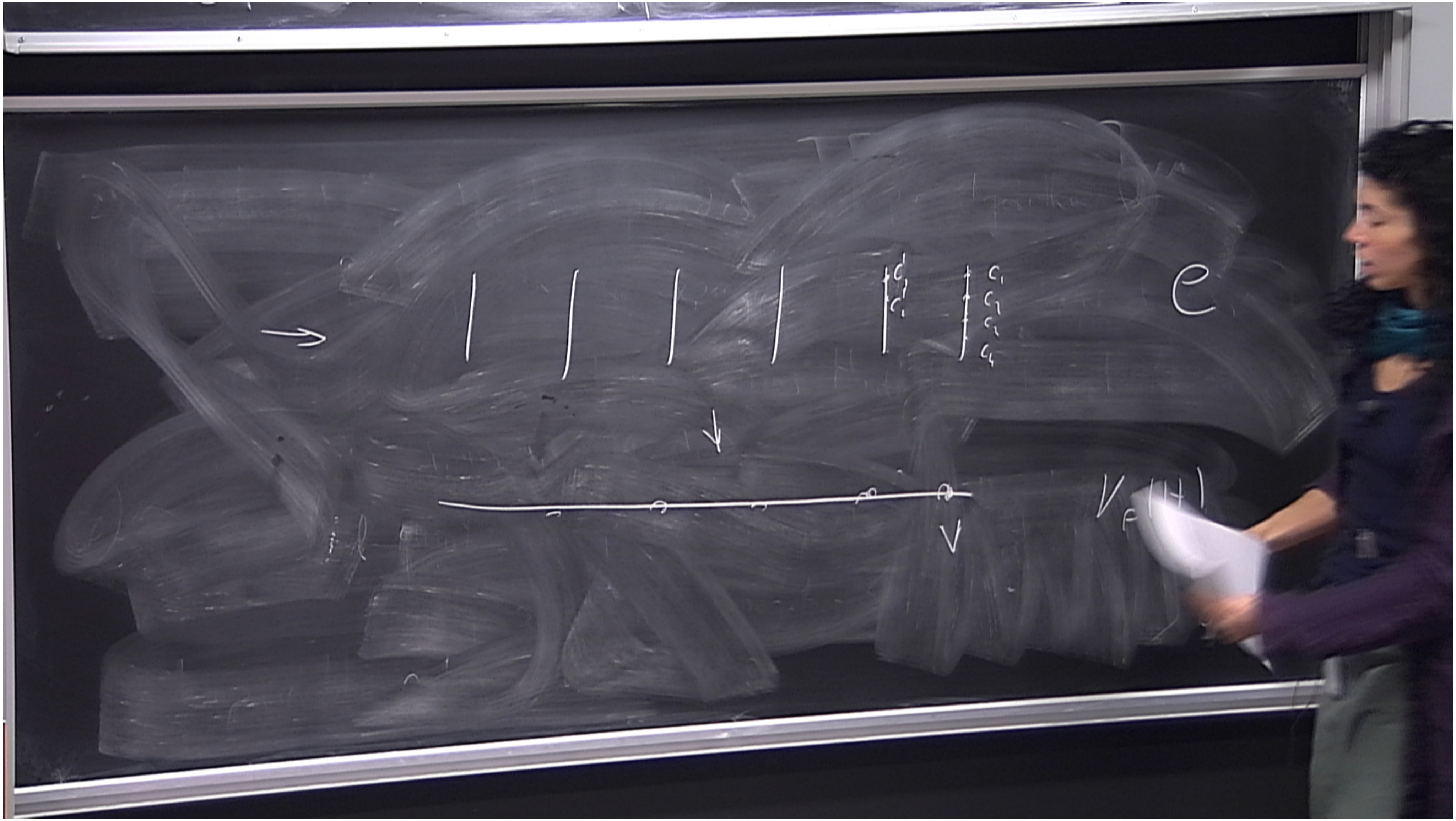
$$\underline{\xi} \to \left(\underline{\xi} \circ l_g\right)(v)$$


V''
V'
V

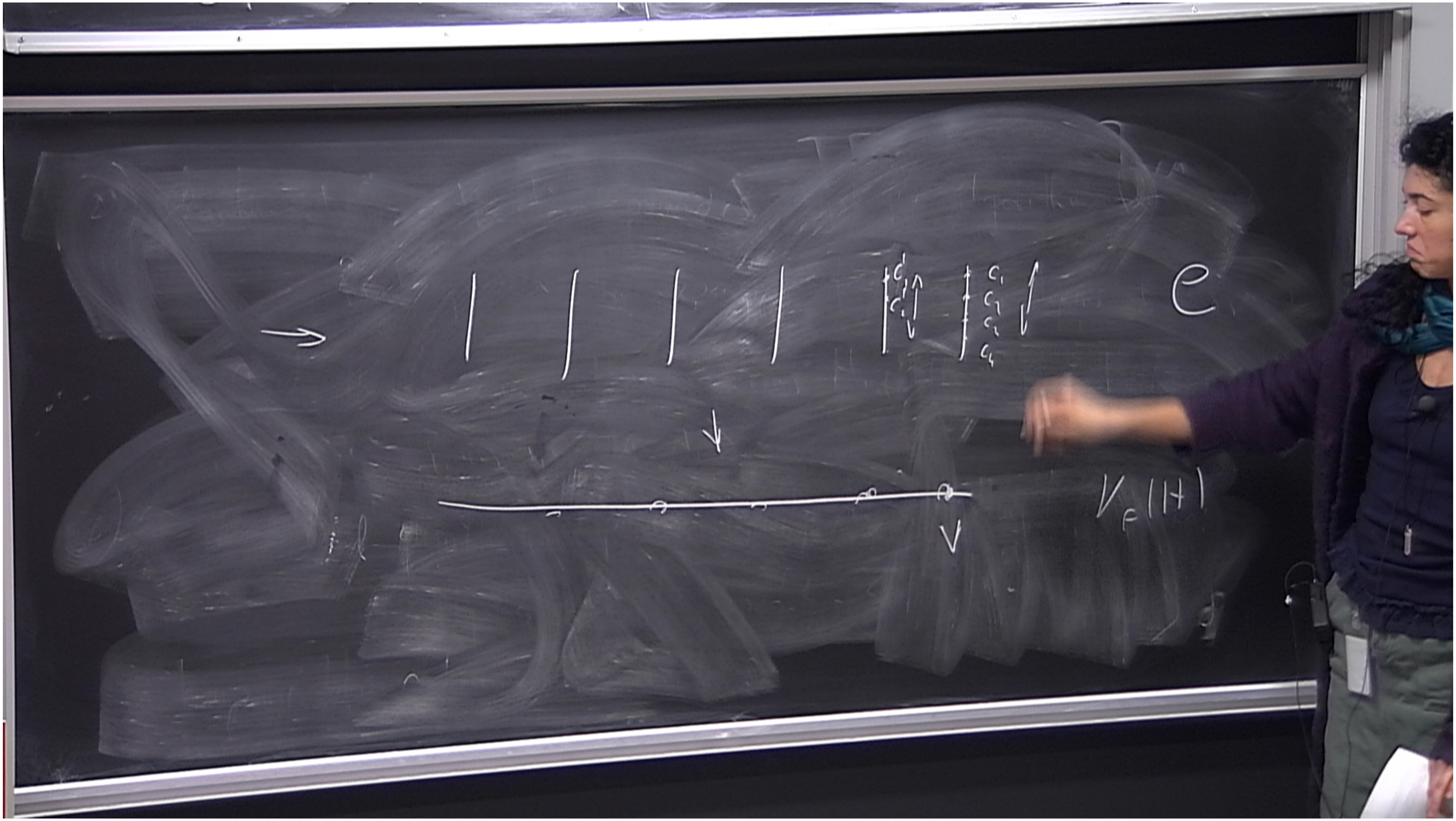


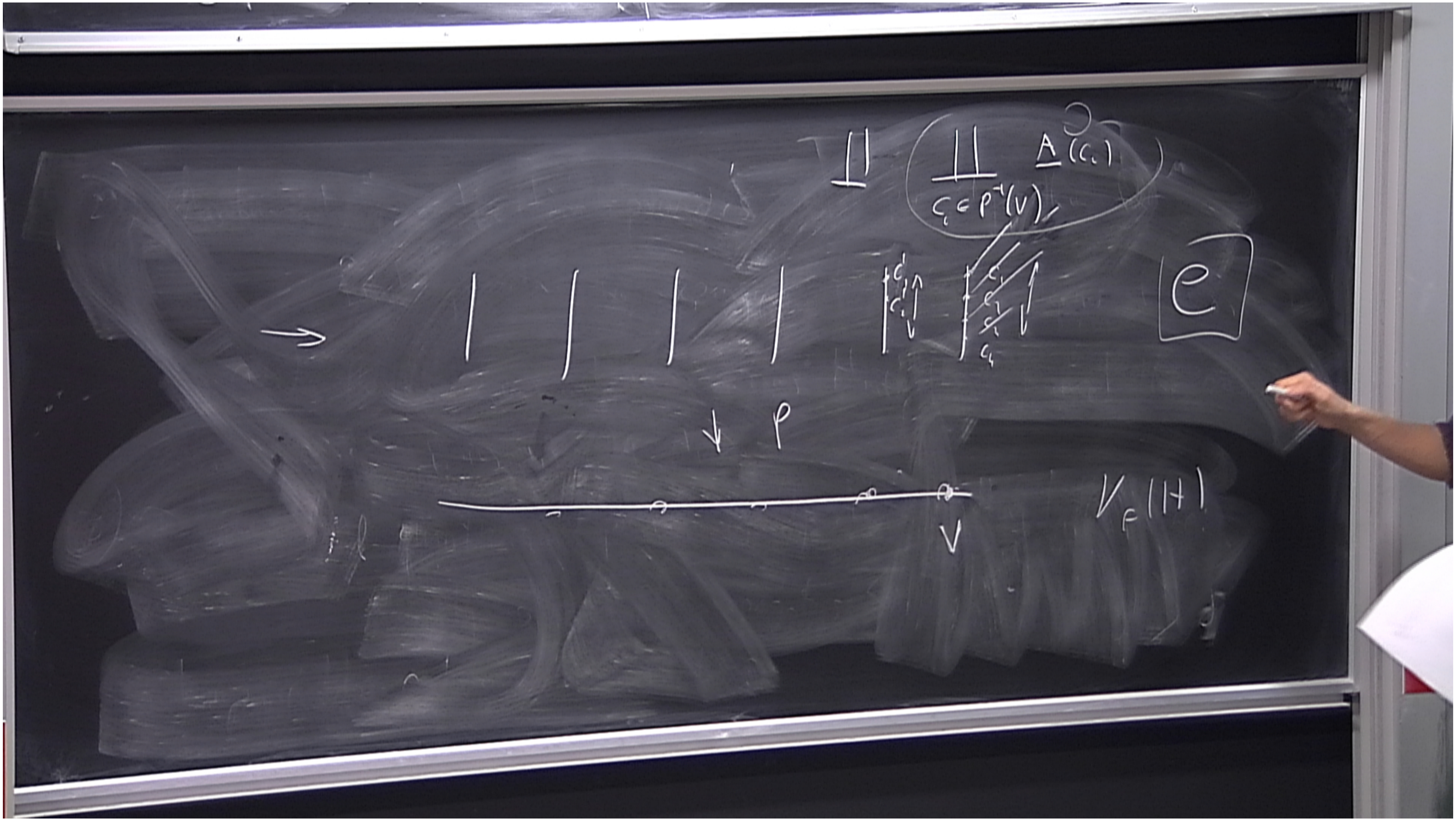
V''
V'
V
ν(H)
ε(V)
V''
V'
V
= ν_p(H)

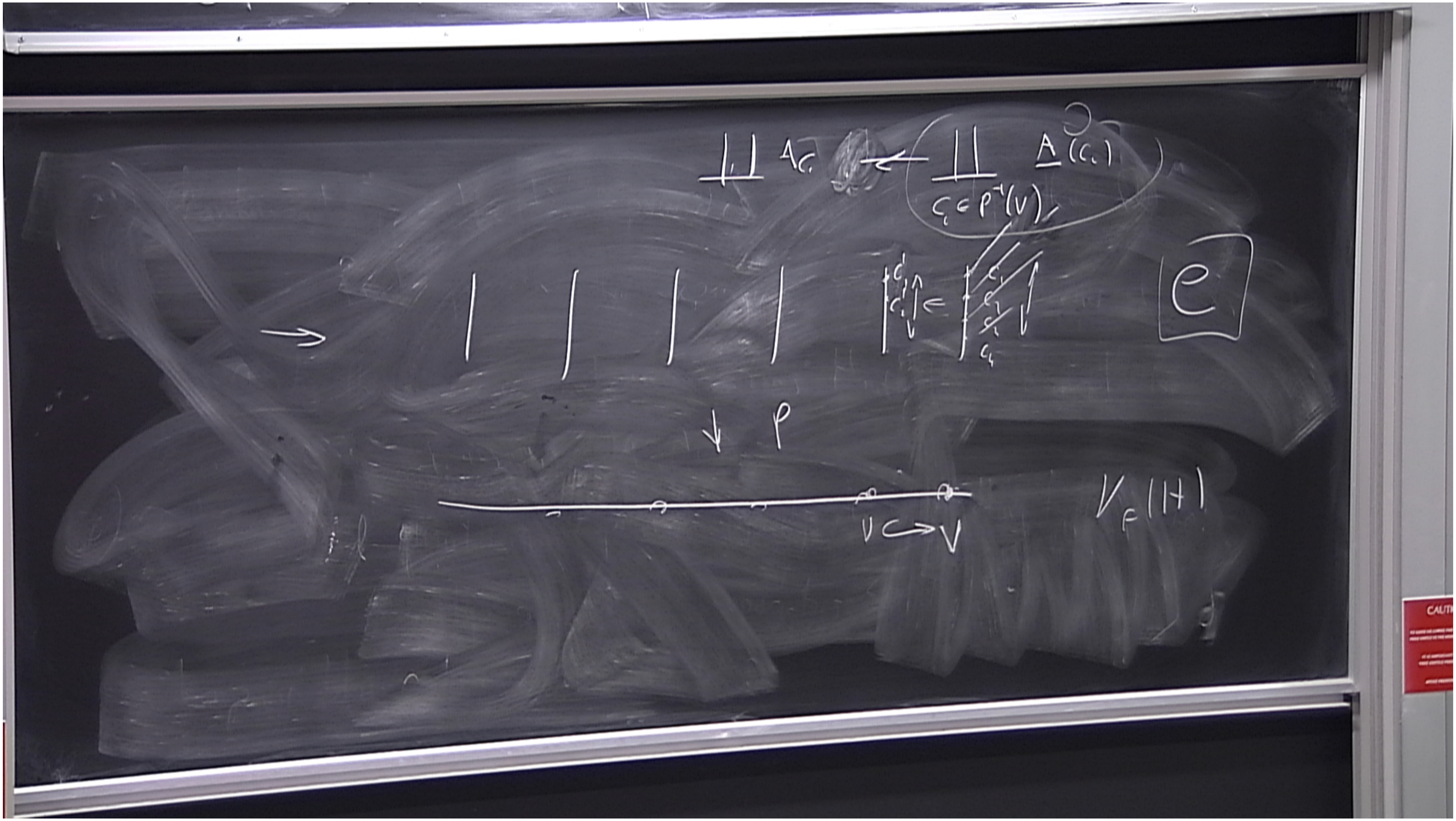

V
$V_F(H)$

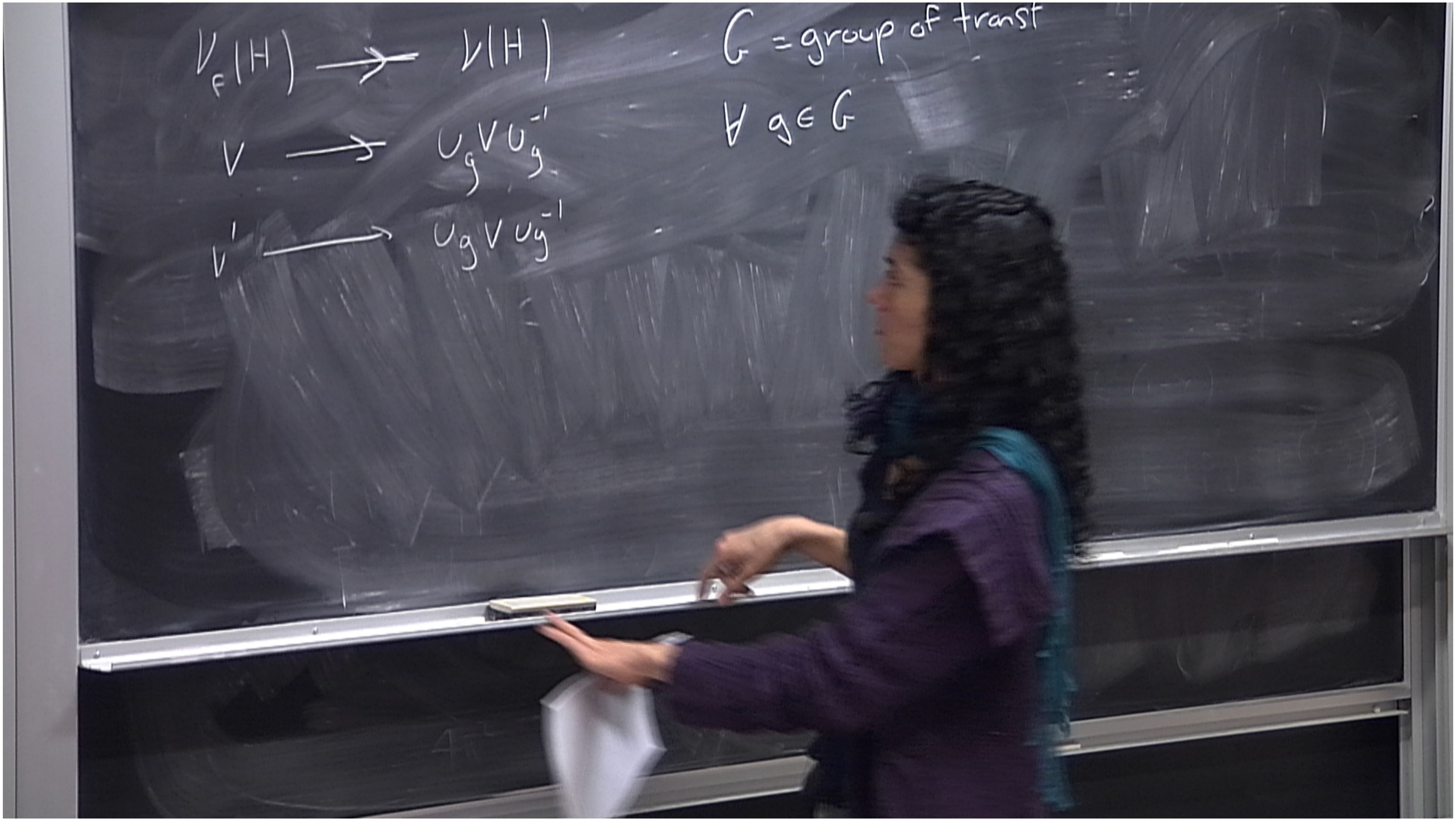


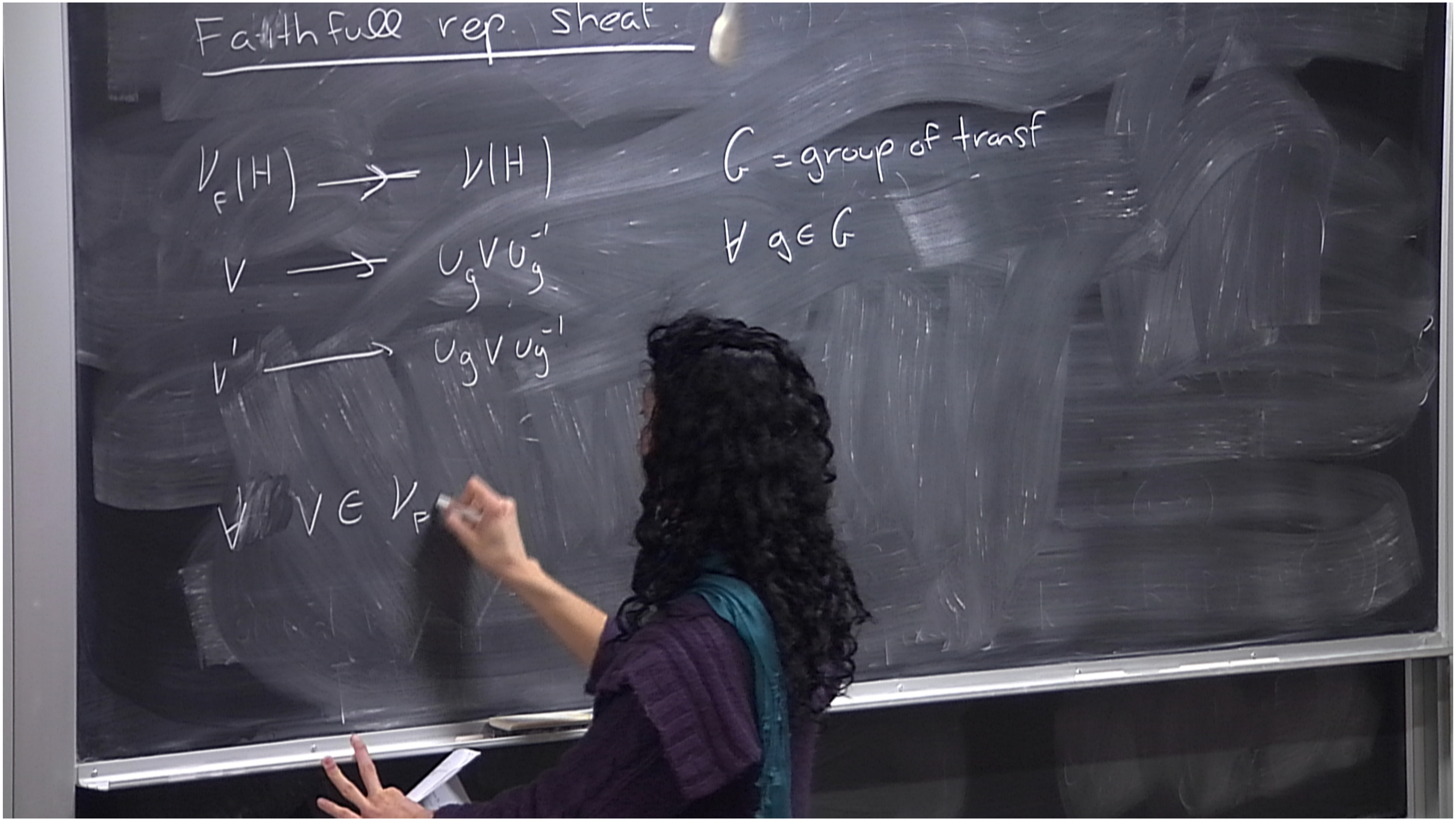
c_1
c_2
c_3
c_4
e
$V_e(t)$




A(c_i)
c_i ∈ ρ^{-1}(V)
e
ρ
V
V_ρ(H)


∐ A_{c_i} ← ∐ A(c_i)
c_i ∈ ρ^{-1}(V)
e
ρ
U ↪ V
V_ρ(H)


$V_F(H) \rightarrow V(H)$
G = group of transf
$V \rightarrow U_g V U_g^{-1}$
$\forall g \in G$
$V' \rightarrow U_g V U_g^{-1}$


Faithfull rep. sheaf
$V_F(H) \longrightarrow V(H)$
$V \longrightarrow U_g V U_g^{-1}$
$V' \longrightarrow U_g V U_g^{-1}$
$\forall V \in V_F$
G = group of transf
$\forall g \in G$

$$\underline{\mathrm{Hom}_{\mathrm{faith}}\left(V_f|H|,\ V|H|\right)} \quad \text{s.t} \quad \forall \ V \in V|H|$$

$$\mathrm{Hom}\left(\downarrow V,\ V|H|\right) \equiv \text{objects}$$

- morphisms $\quad V' \hookrightarrow V \Rightarrow \mathrm{Hom}\left(\downarrow V,\ V|H|\right) \longrightarrow \mathrm{Hom}\left(\downarrow V',\ V|H|\right)$

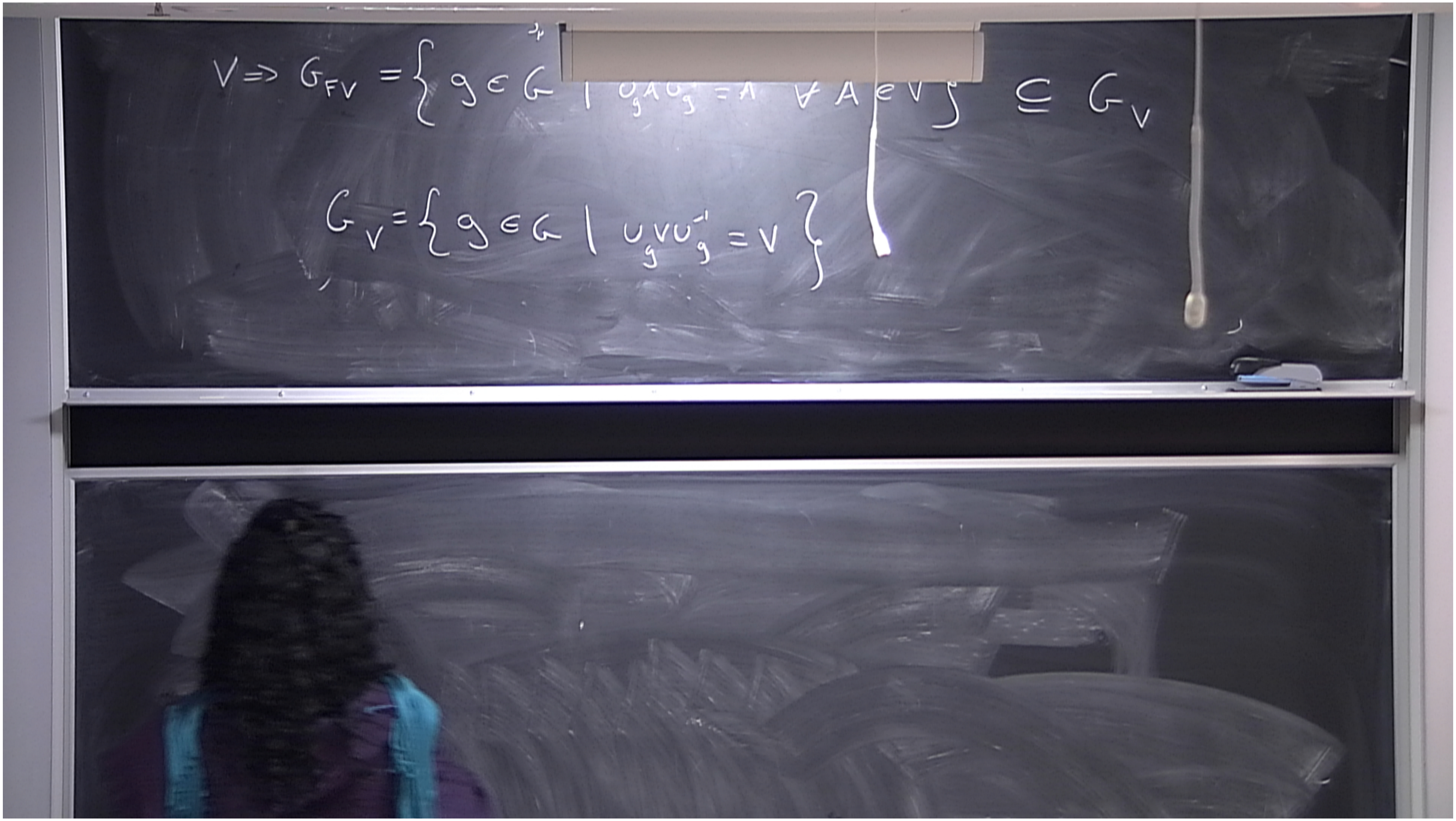
$$\phi_g \longrightarrow \phi_g|_{\downarrow V'}$$

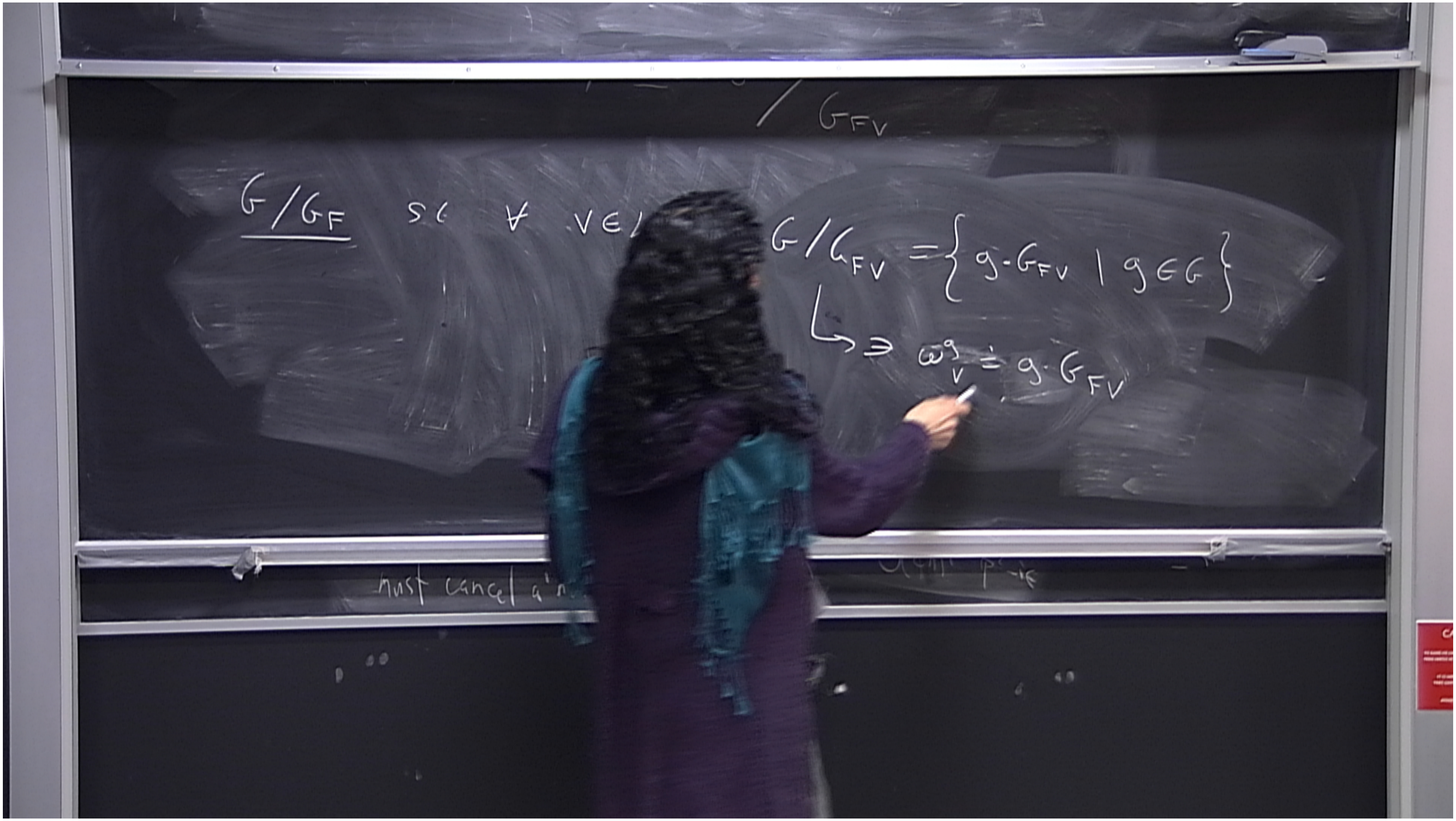
• $\underline{\mathrm{Hom}_{\text{faith}}(\mathcal{V}_f|H|, \mathcal{V}|H|)}$ s.t. $\forall\ V \in \mathcal{V}|H|$

$\mathrm{Hom}(\downarrow V, \mathcal{V}|H) \equiv$ objects

• morphisms $V' \hookrightarrow V \Rightarrow \mathrm{Hom}(\downarrow V, \mathcal{V}|H|) \longrightarrow \mathrm{Hom}(\downarrow V', \mathcal{V}|H|)$

$$\phi_g \longmapsto \phi_g|_{\downarrow V'}$$


$V \Rightarrow G_{FV} = \{ g \in G \mid U_g A U_g^{-1} = A \ \forall A \in V \} \subseteq G_V$
$G_V = \{ g \in G \mid U_g V U_g^{-1} = V \}$

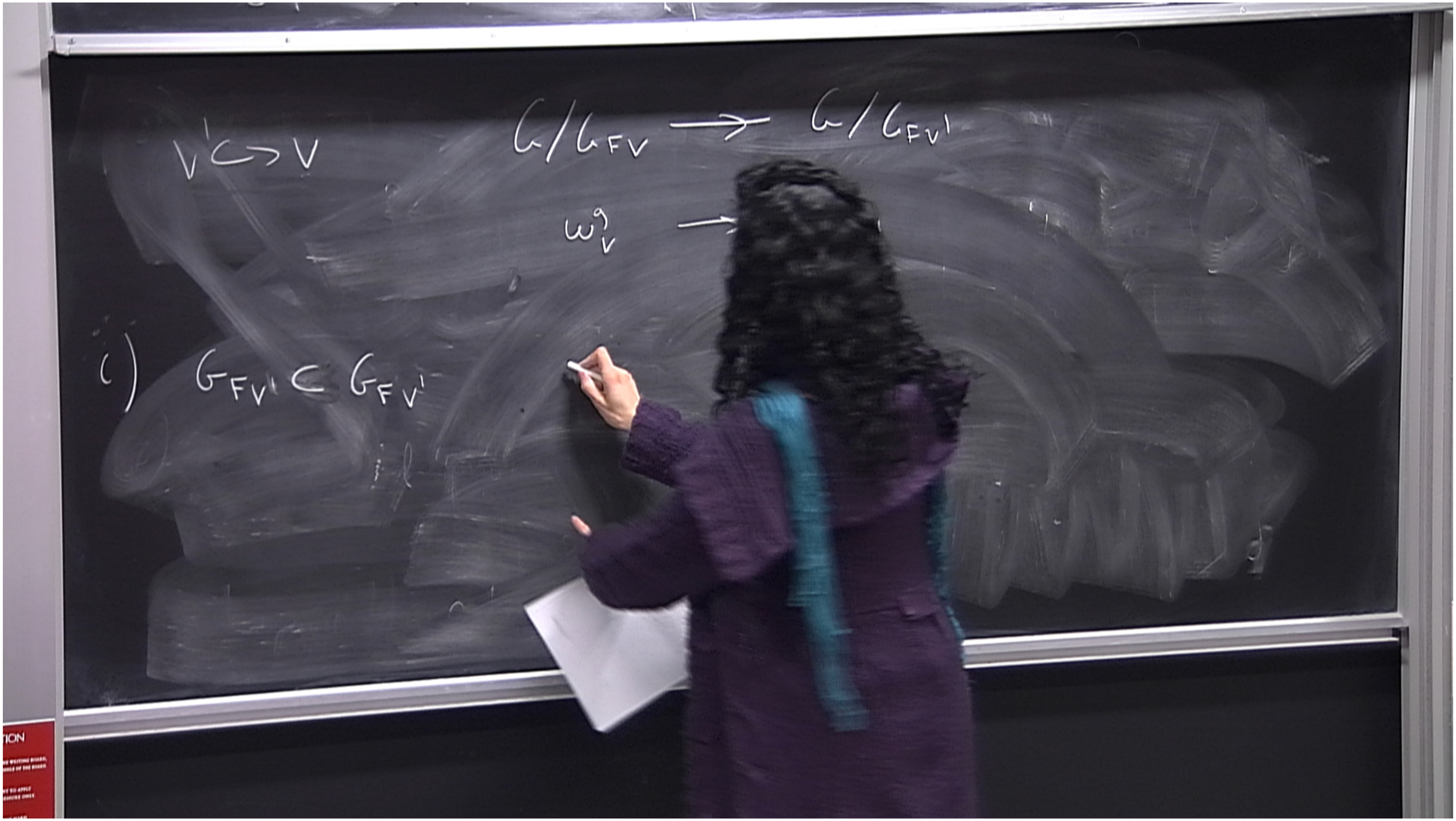

G_FV
G/G_F s.t. ∀ v∈
G/G_FV = {g·G_FV | g∈G}
∋ ω_v^g ≐ g·G_FV
must cancel a'n

$$\mathrm{Hom}_\Gamma(\downarrow V, \nu(H)) \cong G/G_{FV}$$

$\underline{G/G_F}$ s.t. $\forall\ V \in \nu_F(H)$ $\quad G/G_{FV} = \{ g \cdot G_{FV} \mid g \in G \}$

$\quad \hookrightarrow \ni \omega^g_V \doteq g \cdot G_{FV}$

$\quad\quad \hookrightarrow \phi_g$


$V' \hookrightarrow V$
$G/G_{FV} \longrightarrow G/G_{FV'}$
$\omega^g_V \longrightarrow$
i) $G_{FV'} \subset G_{FV'}$

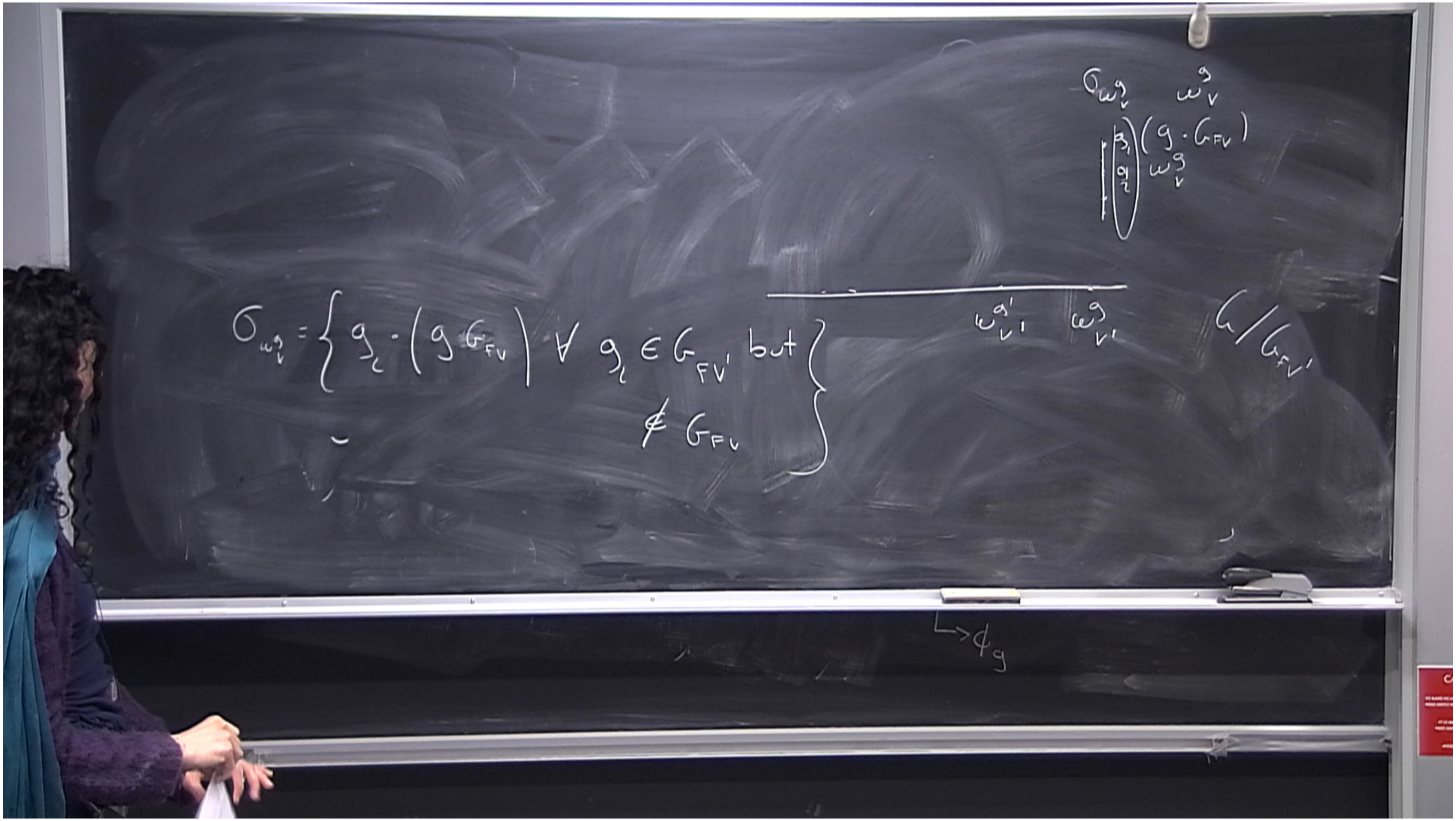
$V' \hookrightarrow V$

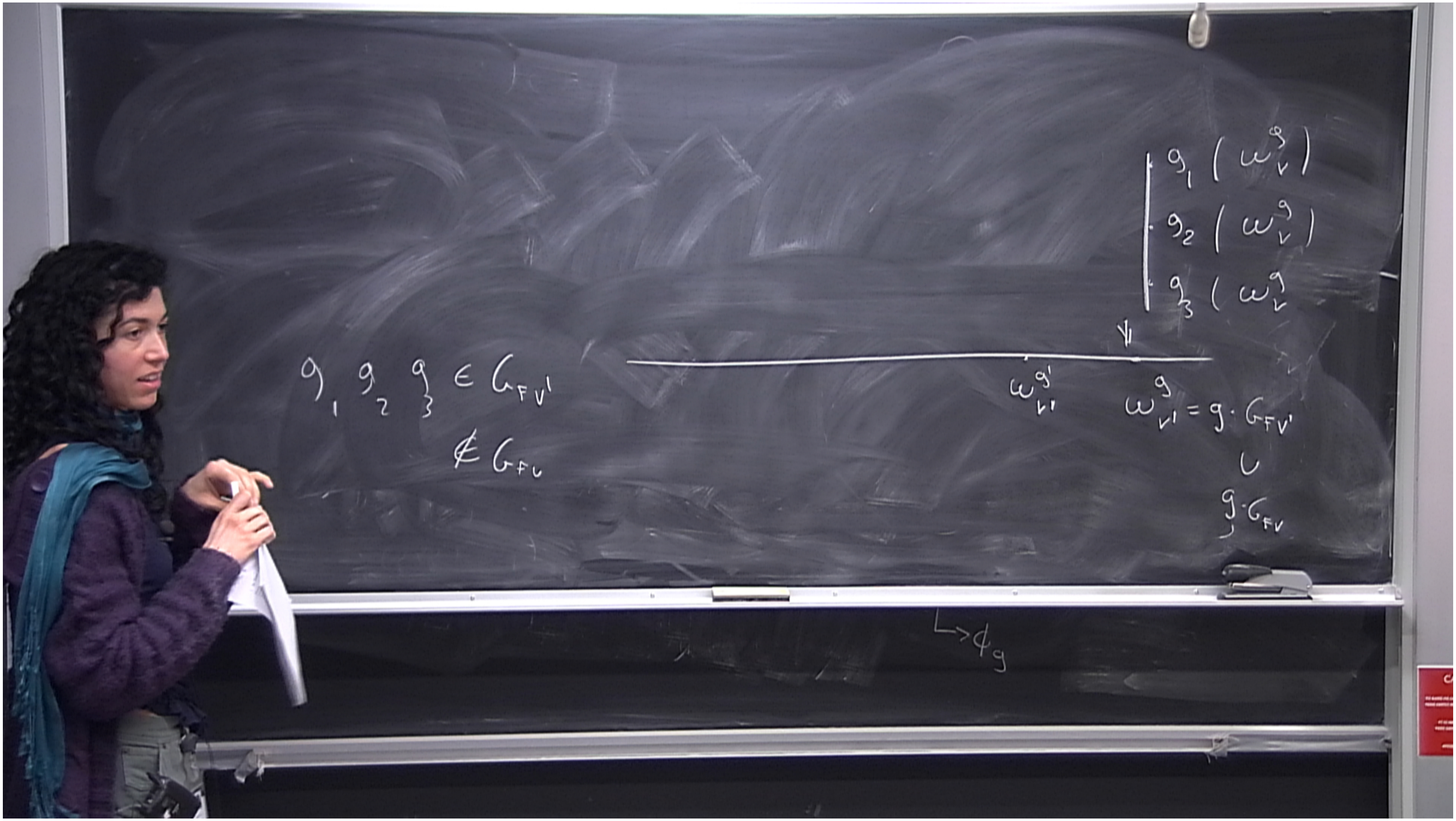
$$G/G_{FV} \longrightarrow G/G_{FV'}$$

$$\omega^g_V \longrightarrow \omega^g_{V|V'}$$

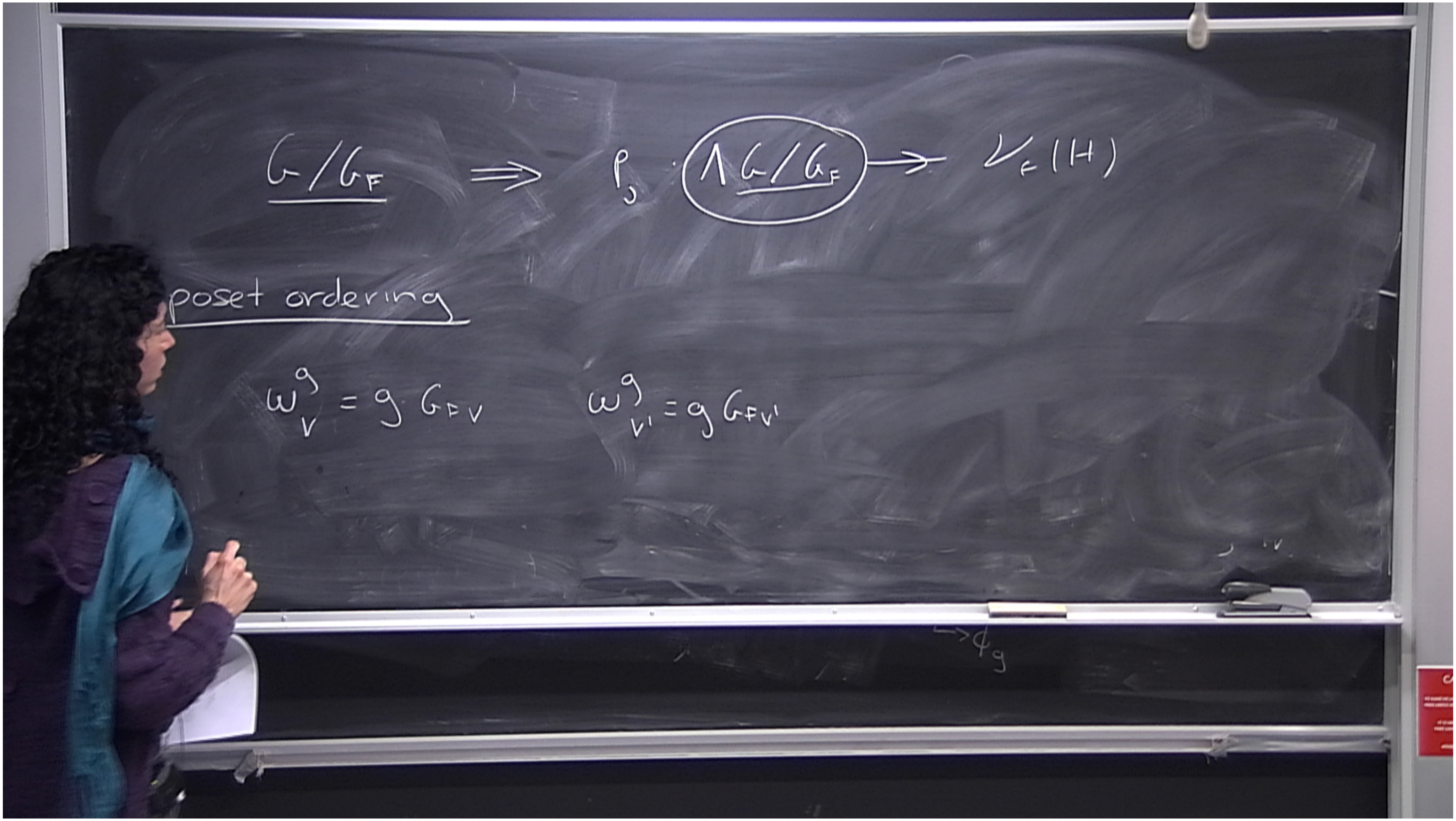
i) $G_{FV} \subset G_{FV'}$

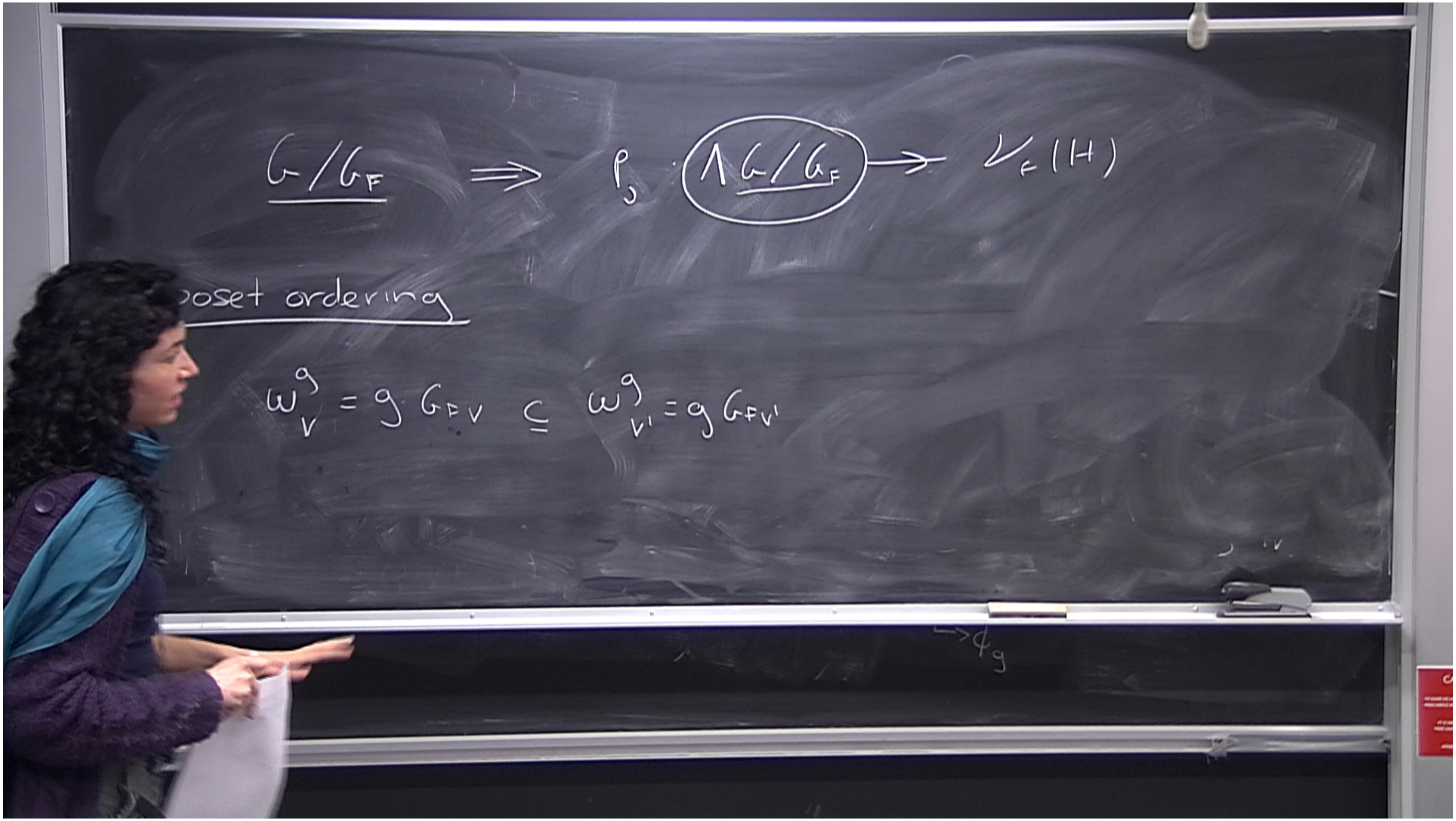
$$\begin{tikzcd} G_{FV'}/G_{FV} \arrow[r] & G/G_{FV} \arrow[d] \\ & G/G_{FV'} \end{tikzcd}$$


$\sigma_{\omega_v^g} = \{ g_l \cdot (g\, G_{FV}) \mid \forall\, g_l \in G_{FV'}$ but $\notin G_{FV} \}$
$\sigma_{\omega_v^g}$ ω_v^g
$(g \cdot G_{FV})$
ω_v^g
$\omega_{v'}^{g'}$ $\omega_{v'}^{g}$
$G/G_{FV'}$
$\hookrightarrow \phi_g$

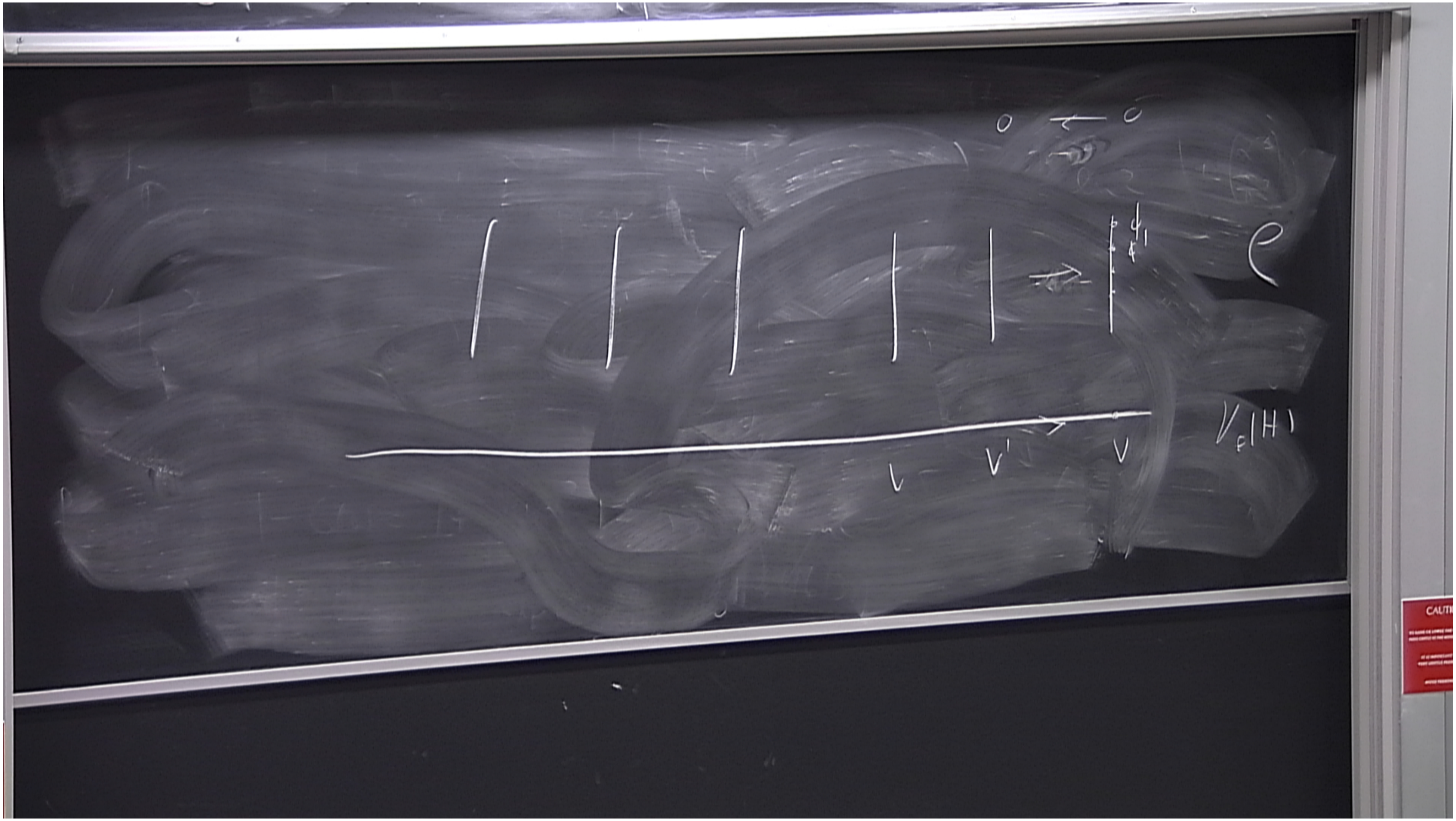


$g_1 (\omega_v^g)$
$g_2 (\omega_v^g)$
$g_3 (\omega_v^g$
$g_1\ g_2\ g_3 \in G_{FV'}$
$\notin G_{Fv}$
$\omega_{v'}^{g'}$
$\omega_{v'}^{g} = g \cdot G_{FV'}$
$\cup$
$g \cdot G_{FV}$
$\hookrightarrow \phi_g$


G/G_F ⟹ ρ_J : ΛG/G_F → L_F(H)
poset ordering
ω^g_v = g G_F v
ω^g_v' = g G_F v'
↪ ϕ_g

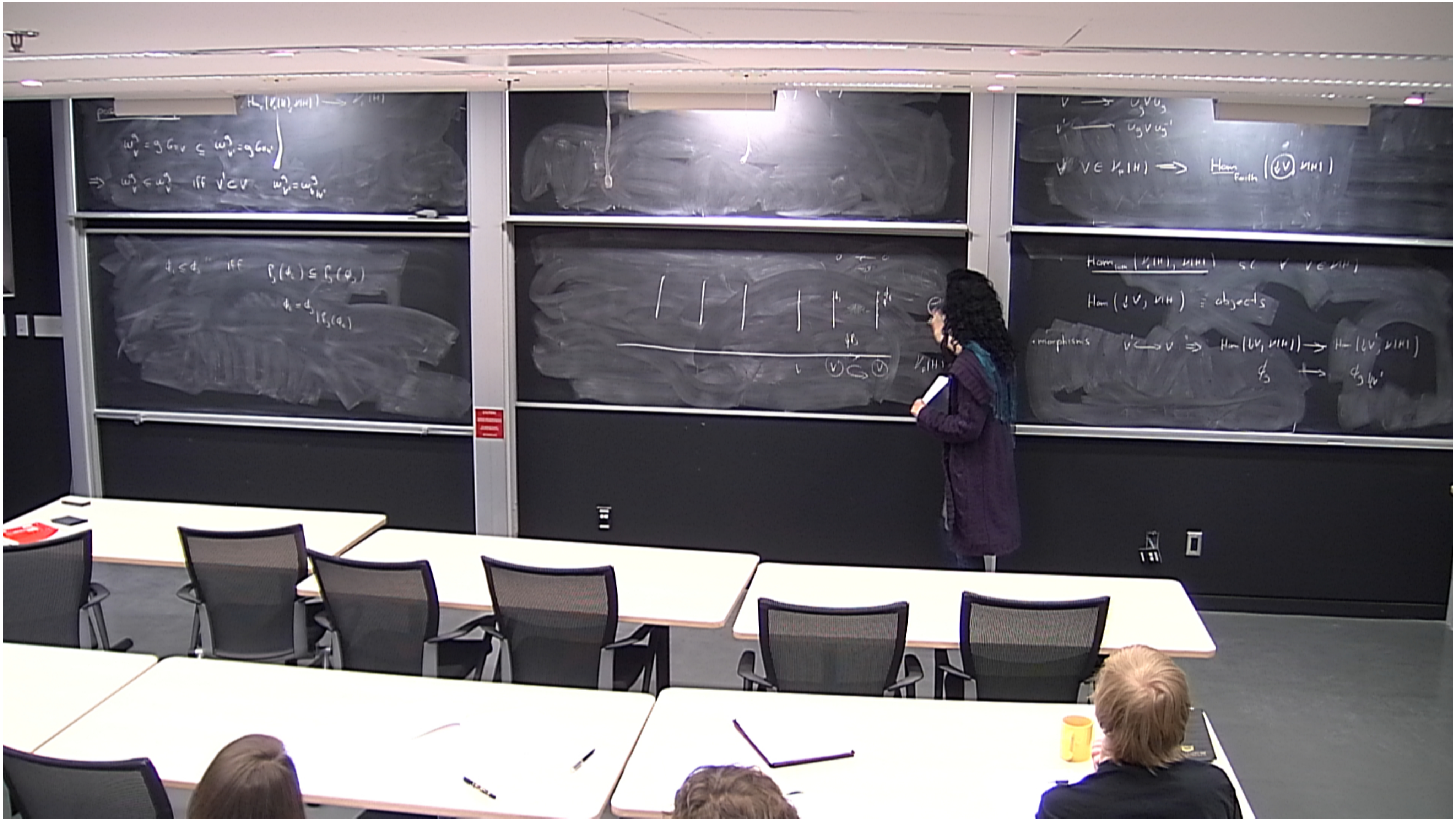


$G/G_F \Longrightarrow \rho_J : \Lambda G/G_F \longrightarrow \mathcal{L}_F(H)$
oset ordering
$\omega^g_v = g G_{Fv} \subseteq \omega^g_{v'} = g G_{Fv'}$


$V_\ell(H)$
V'
V

$\Rightarrow \omega_{V'} \leq \omega_{V}$ iff $V' \subset V$ $\quad \omega_{V'} = \omega_{V|_{V'}}$

$\phi_i \leq \phi_j$" iff $\rho_j(\phi_i) \subseteq \rho_j(\phi_j)$


objects
morphisms

$\Rightarrow \omega^{J}_{V'} \le \omega^{J}_{V}$ iff $V' \subset V$ $\quad \omega^{J}_{V'} = \omega^{J}_{V}|_{V'}$

"$\phi_i \le \phi_j$" iff $\rho_j(\phi_i) \subseteq \rho_j(\phi_j)$

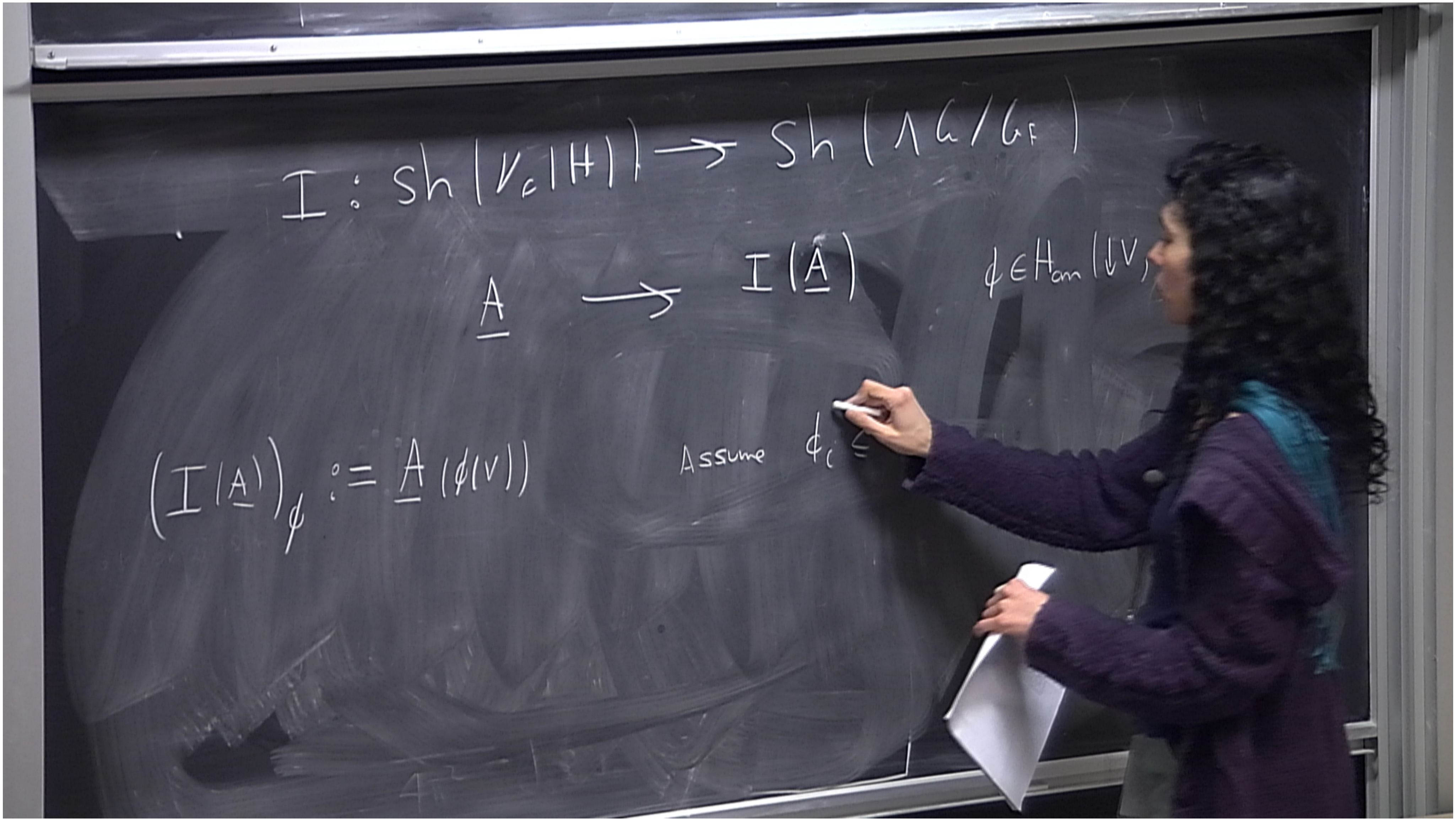
$\phi_i = \phi_j|_{\rho_j(\phi_i)}$

$Sh(\Lambda G/G_P)$

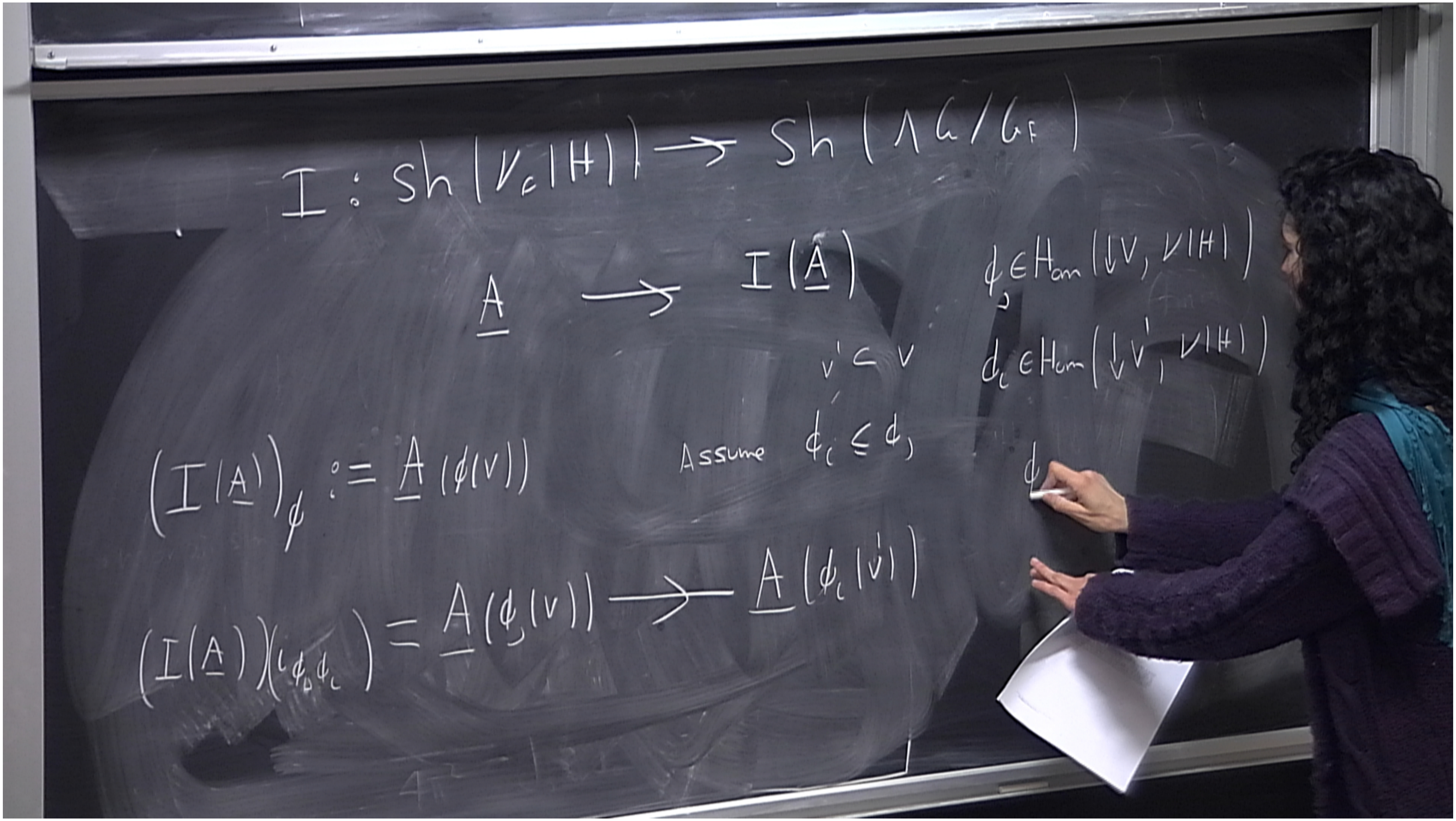
$$I : Sh(\mathcal{V}_c(H)) \to Sh(\Lambda G / G_F)$$

$$\underline{A} \longrightarrow I(\underline{A}) \qquad \phi \in Hom(V, \mathcal{V}(H))$$

$$(I(\underline{A}))_\phi := \underline{A}(\phi(V))$$



$I : Sh(V_c(H)) \to Sh(\Lambda G / G_F)$
$\underline{A} \to I(\underline{A})$
$\phi \in Hom(\downarrow V,$
$(I(\underline{A}))_\phi := \underline{A}(\phi(V))$
Assume $\phi_i \leq$


$\underline{I} : Sh(\mathcal{V}_c(H)) \to Sh(\Lambda G/G_F)$
$\underline{A} \longrightarrow \underline{I}(\underline{A})$
$\phi_j \in Hom(\downarrow V, \mathcal{V}(H))$
$V' \subset V$
$\phi_i \in Hom(\downarrow V', \mathcal{V}(H))$
Assume $\phi_i \subseteq \phi_j$
$(\underline{I}(\underline{A}))_\phi := \underline{A}(\phi(V))$
ϕ
$(\underline{I}(\underline{A}))(i_{\phi_j \phi_i}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_i(V'))$

$$I : Sh(V_c \| H) \longrightarrow Sh(\Lambda G / G_F)$$

$$\underline{A} \longrightarrow I(\underline{A})$$

$\phi_j \in Hom(\downarrow V, V \| H)$

$V' \subseteq V$ $\quad$ $\phi_i \in Hom(\downarrow V', V \| H)$

$$(I(\underline{A}))_\phi := \underline{A}(\phi(V))$$

Assume $\phi_i \leq \phi_j$ $\quad$ $\phi_j(V) \supset \phi_i(V')$

$\phi_i = \phi_j | \downarrow V'$

$$(I(\underline{A}))(\iota_{\phi_j \phi_i}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_i(V'))$$

$$\underline{A}(V_1) \longrightarrow \underline{A}(V_2)$$

$$I : Sh(V_c(H)) \longrightarrow Sh(\Lambda G/G_F)$$

$$\underline{A} \longrightarrow I(\underline{A})$$

$\phi_j \in Hom(\downarrow V, V(H))$

$V' \subseteq V$

$\phi_i \in Hom(\downarrow V', V(H))$

$$(I(\underline{A}))_\phi := \underline{A}(\phi(V))$$

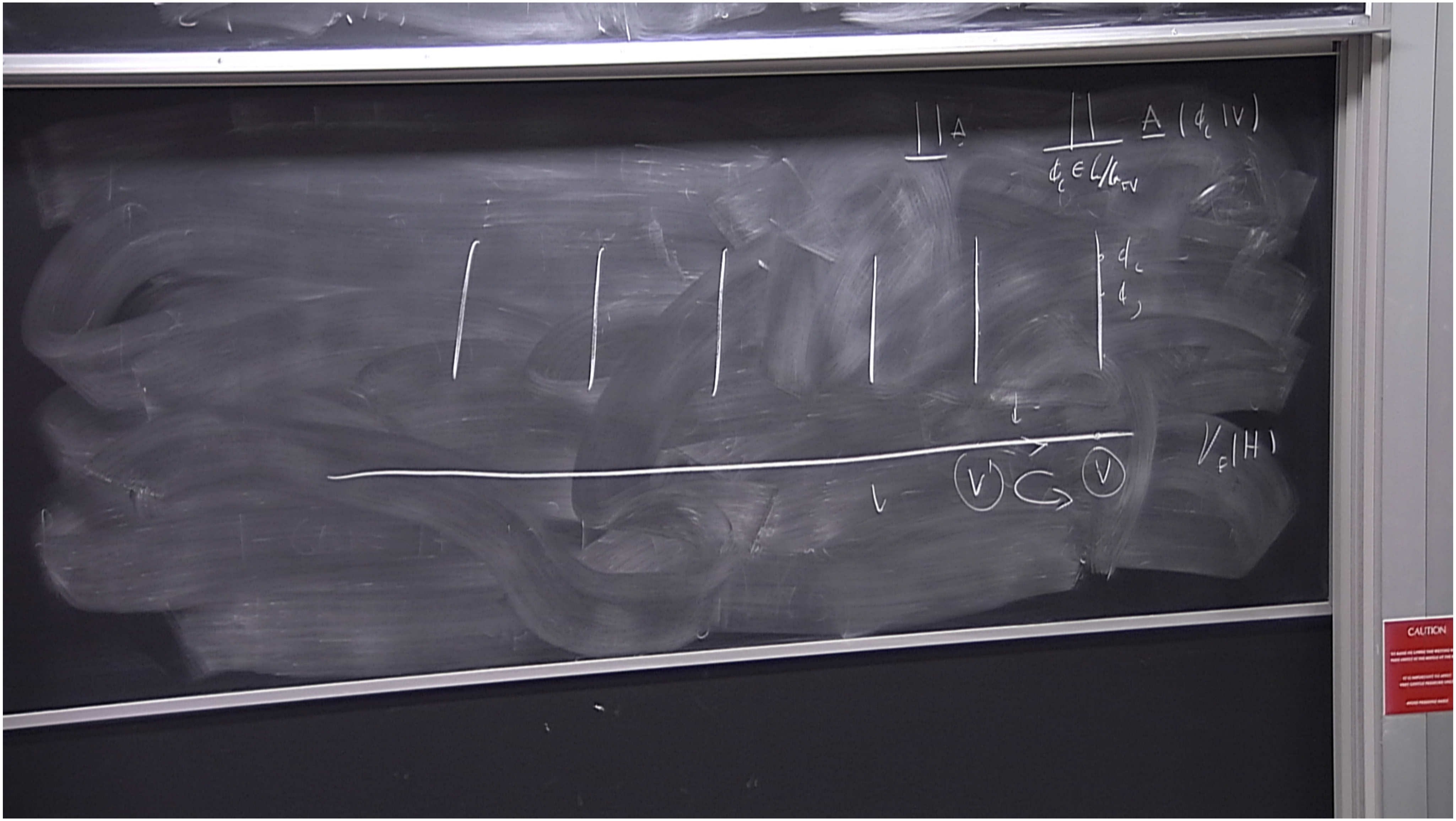
Assume $\phi_i \leq \phi_j$

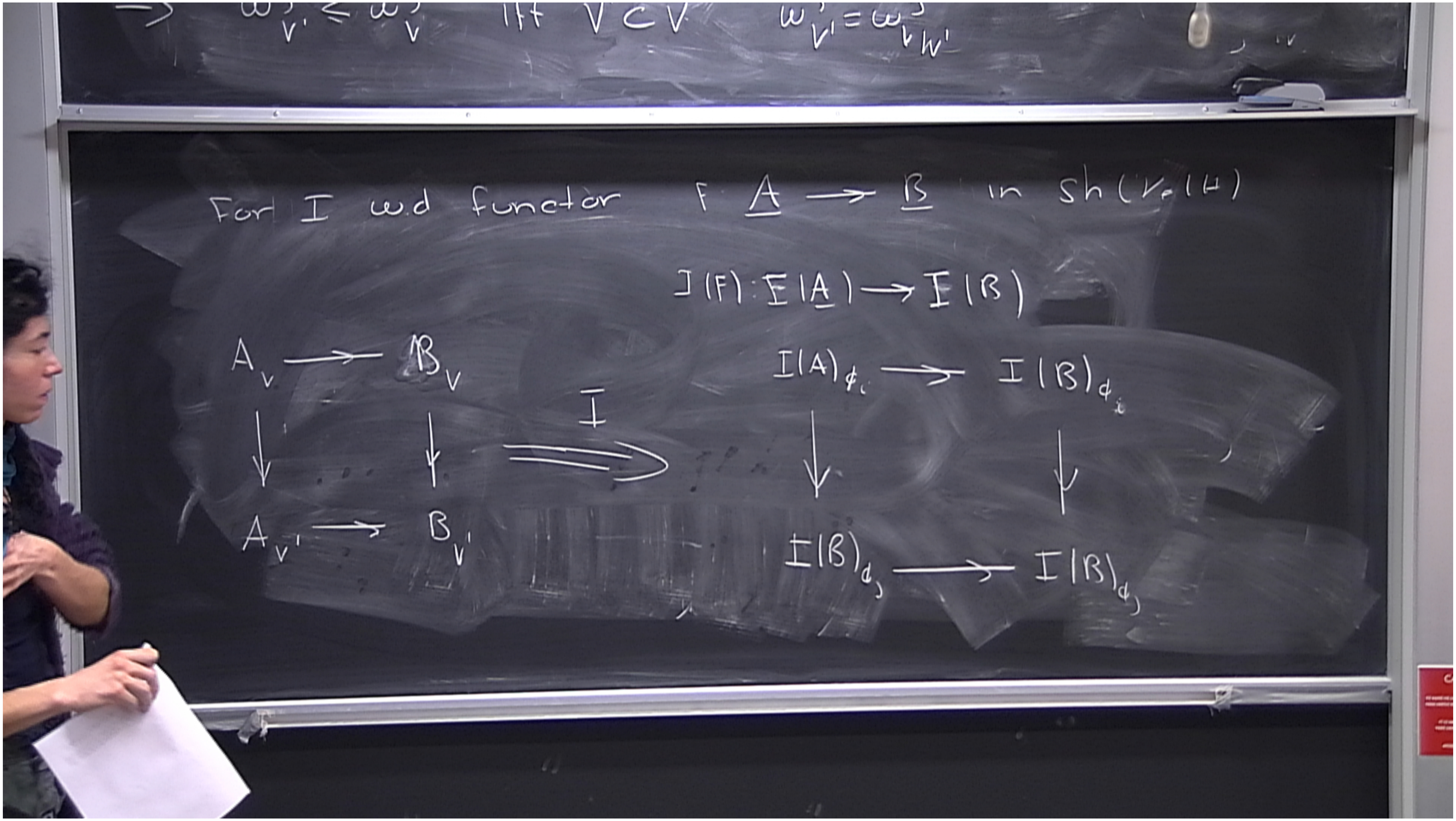
$\phi_j(V) \supset \phi_i(V')$

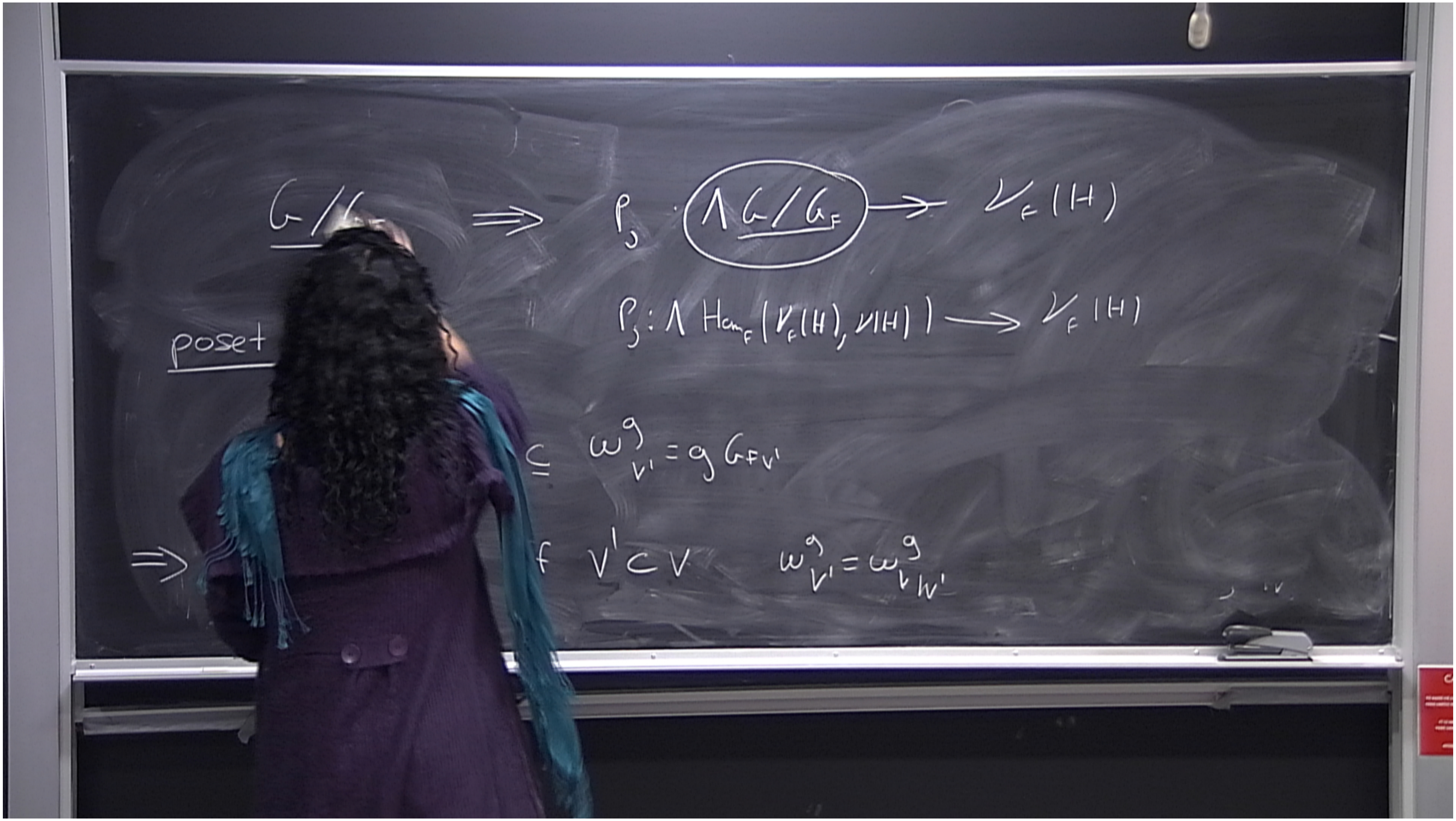
$\phi_i = \phi_j | \downarrow V'$

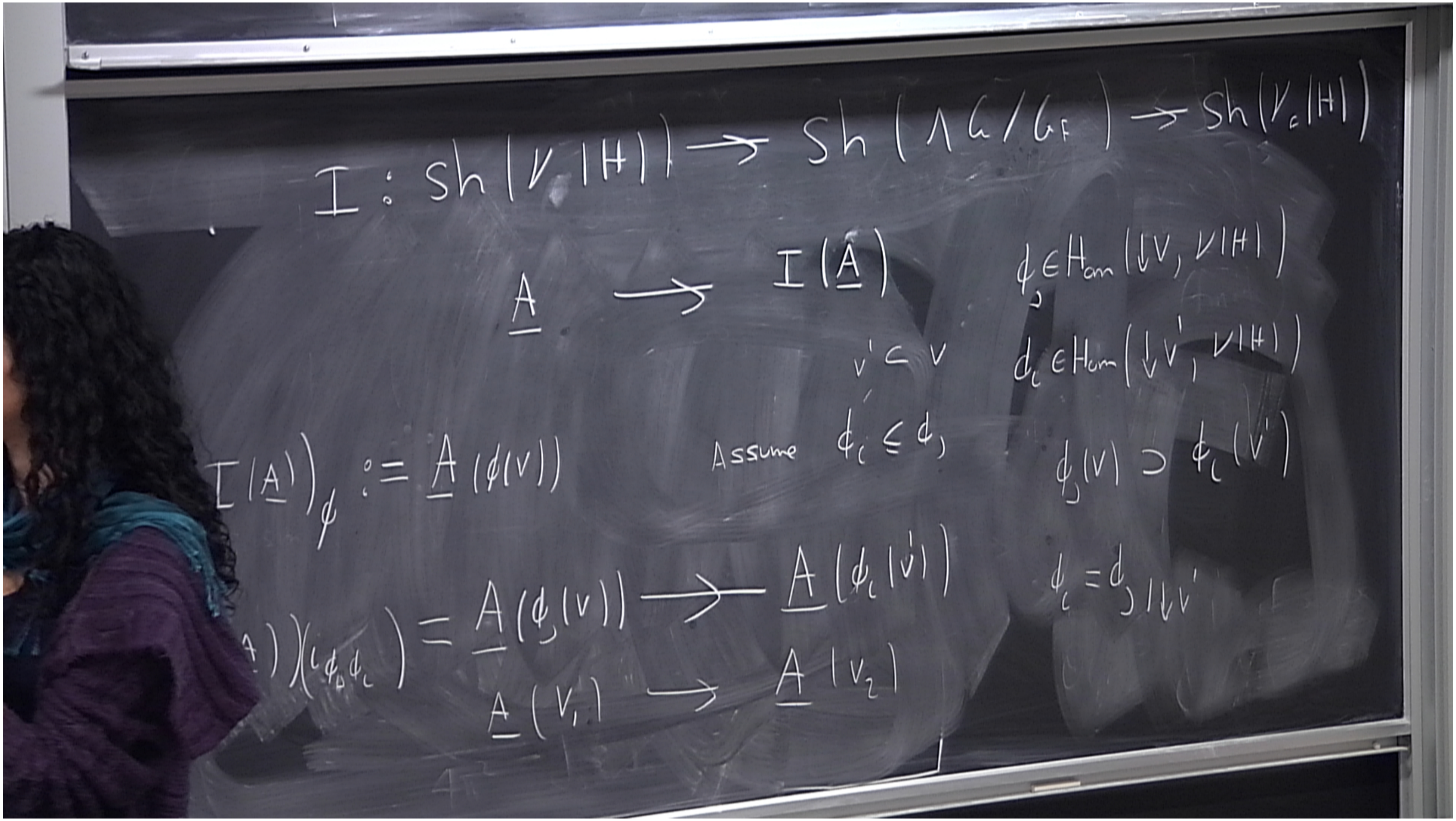
$$(I(\underline{A}))(i_{\phi_j \phi_i}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_i(V'))$$

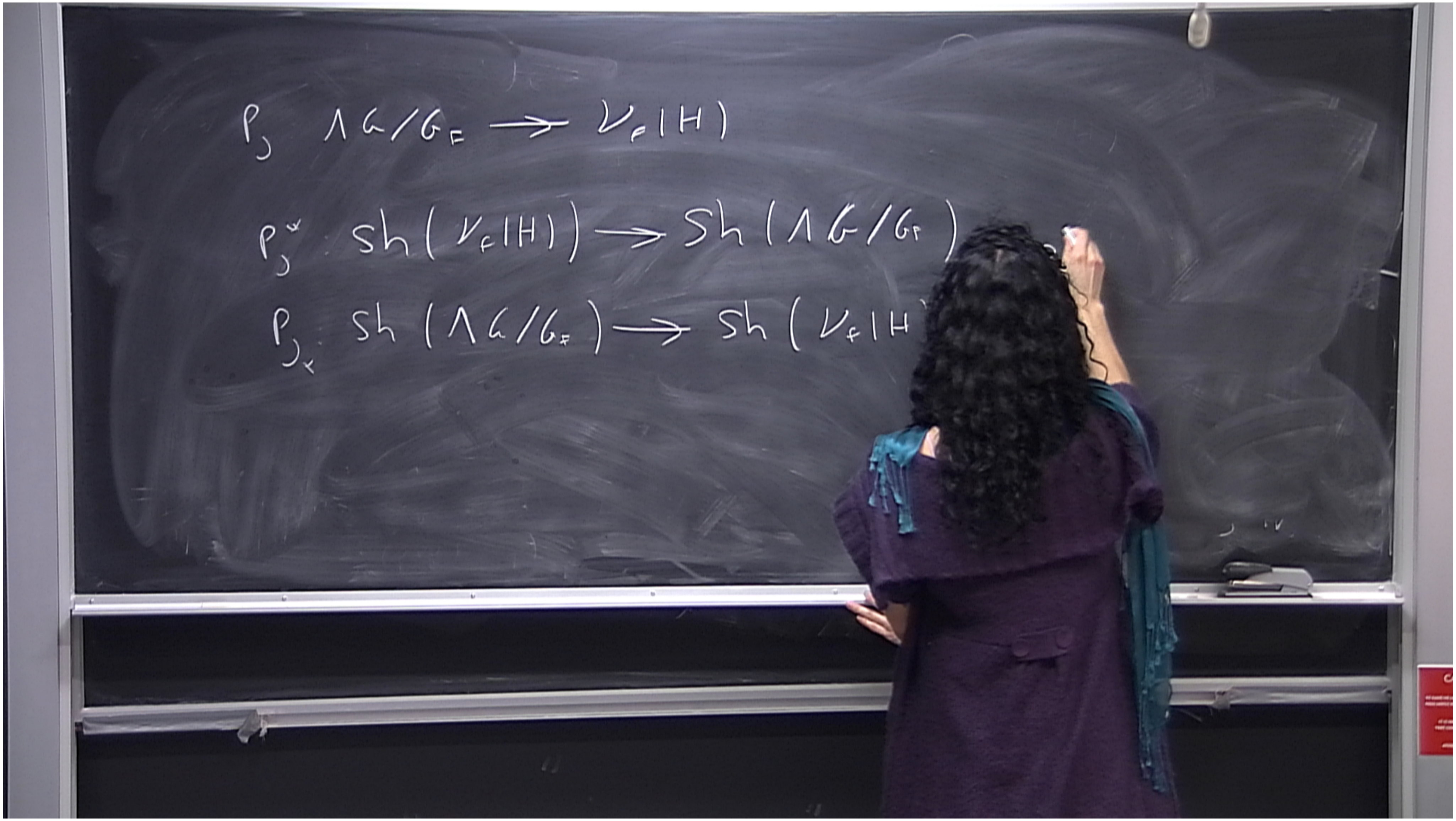
$$\underline{A}(V_1) \longrightarrow \underline{A}(V_2)$$

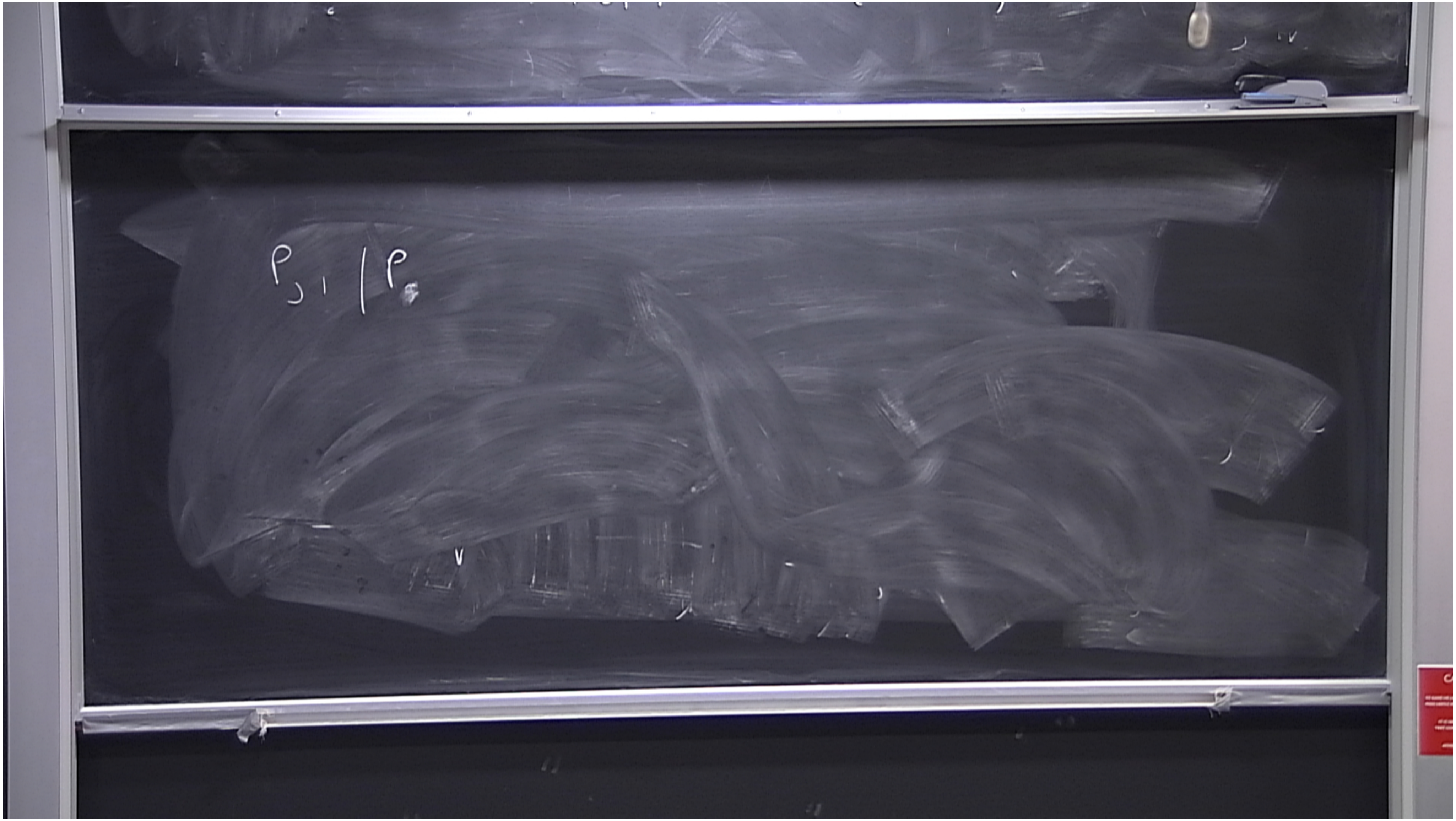

$\coprod A$
$\coprod_{\phi_c \in G/G_{TV}} A(\phi_c | V)$
ϕ_c
ϕ_j
$V_f(H)$
V'
V


For I w.d functor F: A → B in Sh(V(H))
I(F): I(A) → I(B)
A_V → B_V
A_V' → B_V'
I
I(A)_φi → I(B)_φi
I(B)_φj → I(B)_φj


poset
ρj : Λ G/GF → VF(H)
ρj : Λ HomF(VF(H), V(H)) → VF(H)
ωgv' = g GFv'
v' ⊂ V   ωgv' = ωgv|v'


I : Sh(V(H)) → Sh(ΛG/G_F) → Sh(V_c(H))
A → I(A)
φ_j ∈ Hom(↓V, V(H))
φ_i ∈ Hom(↓V', V(H))
V' ⊂ V
Assume φ_i ≤ φ_j
I(A))_φ := A(φ(V))
φ_j(V) ⊃ φ_i(V')
A(φ_j(V)) → A(φ_i(V'))
φ_i = φ_j|↓V'
A(V_1) → A(V_2)


$p_j \;\; \Lambda G/G_F \longrightarrow V_F(H)$
$p_j^* : \; Sh(V_F(H)) \longrightarrow Sh(\Lambda G/G_F)$
$p_{j*} : \; Sh(\Lambda G/G_F) \longrightarrow Sh(V_F(H$

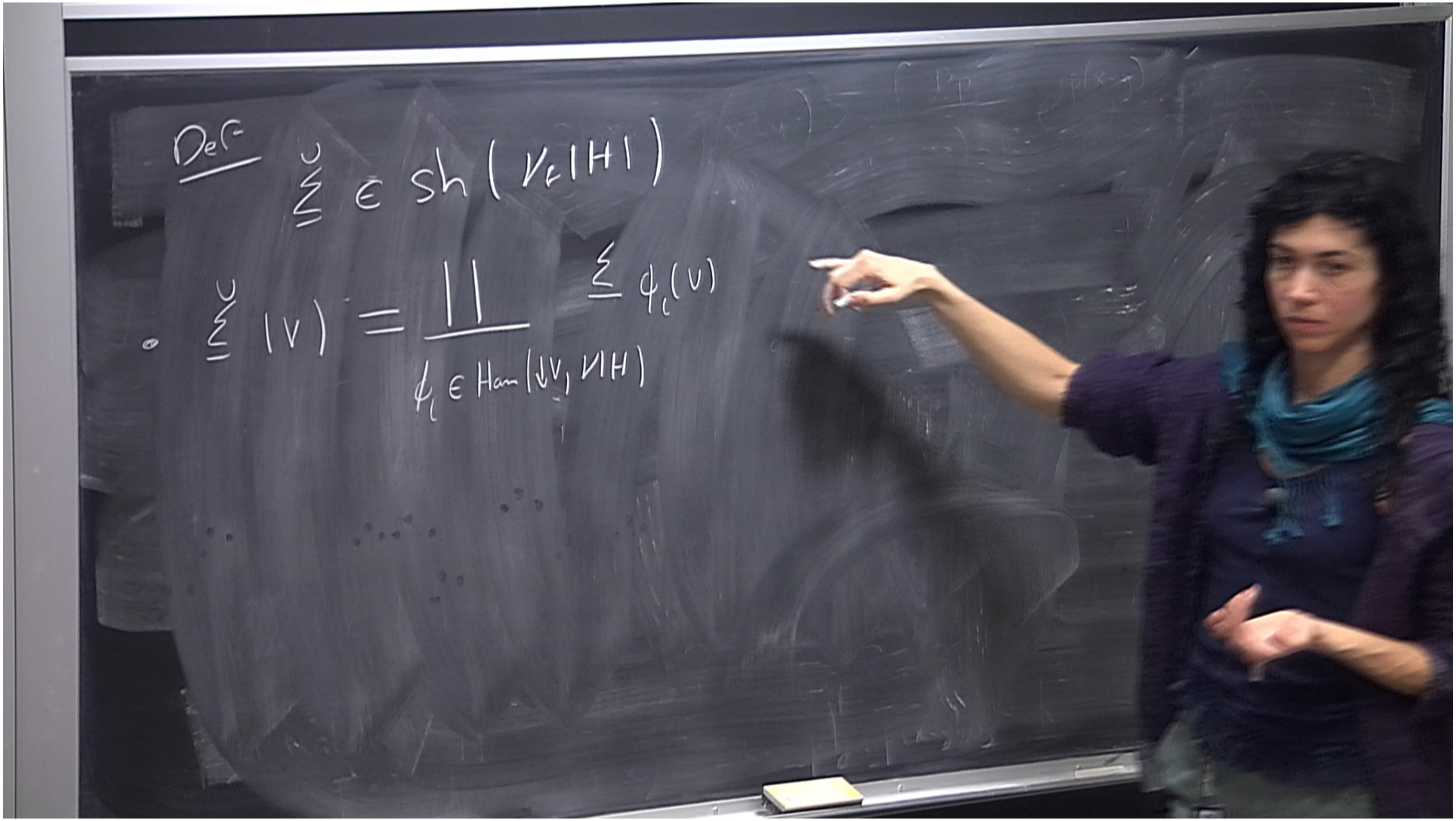


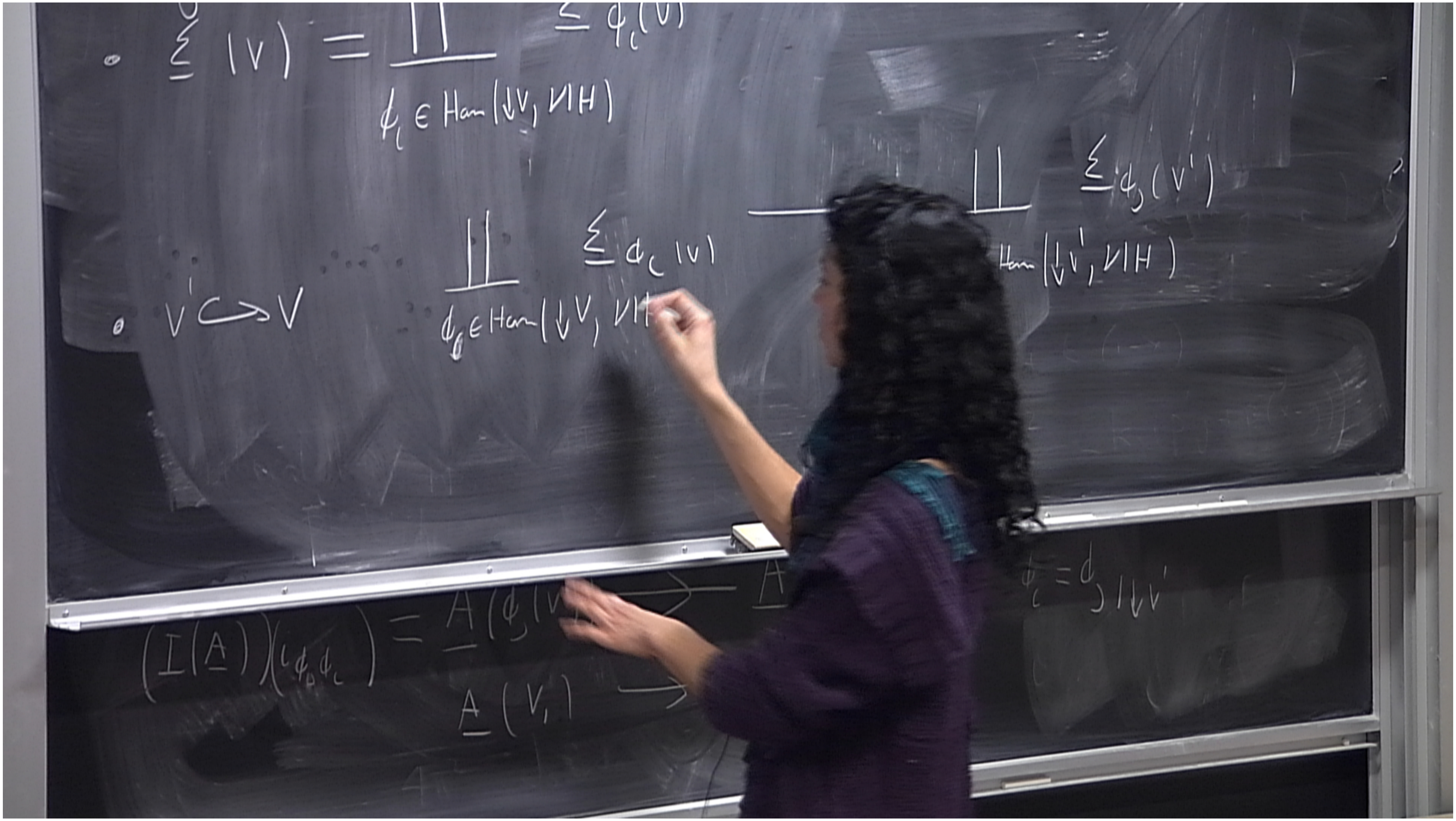
$$p_{J!}\left(p_A \quad A \longrightarrow \Lambda G/G_F\right) = \left(p_J \circ p_A\right) \quad A \longrightarrow \nu_F(H)$$

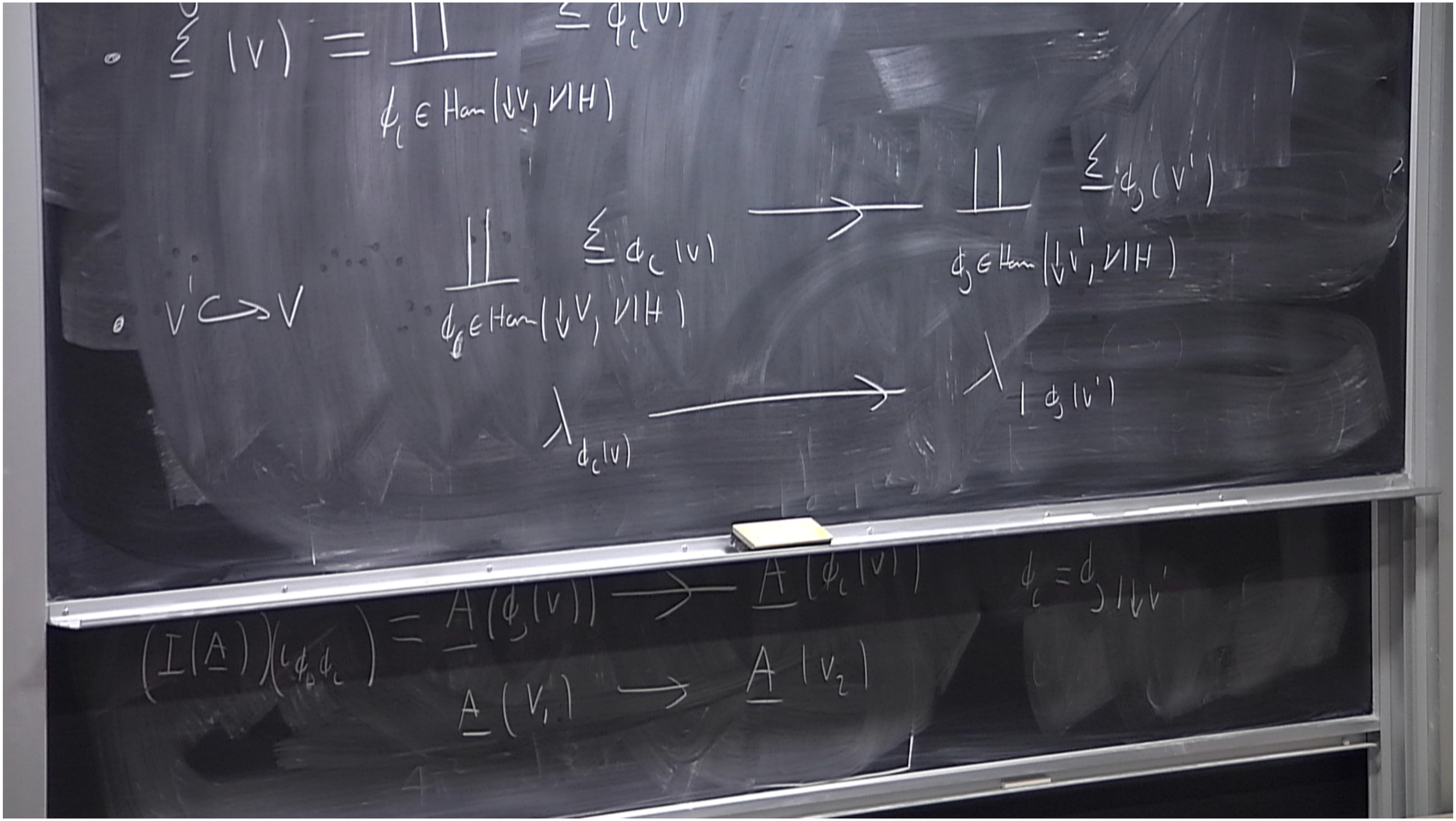
$$p_{J}!\left(\rho_A \quad A \longrightarrow \Lambda G/G_F\right)\Big] = (p_J \circ \rho_A) \quad A \longrightarrow \nu_F(H)$$

$$P_{J!}\left(\rho_A \quad A \longrightarrow \Lambda G/G_F\right) = \left(P_J \circ \rho_A\right) \quad A \longrightarrow V_F(H)$$

$$\underline{\mathcal{E}} \longrightarrow P_{J!} \circ I(\underline{\mathcal{E}}) = \underline{\breve{\mathcal{E}}}$$


Def^n
$\check{\Sigma} \in sh(V_L|H|)$
$\check{\Sigma}(V) = \coprod_{\phi_i \in Hom(\downarrow V, V|H|)} \Sigma\, \phi_i(V)$




$\underline{\Sigma}(V) = \coprod_{\phi_i \in Hom(\downarrow V, V(H))} \underline{\Sigma}\phi_i(V)$
$V' \hookrightarrow V$
$\coprod_{\phi_i \in Hom(\downarrow V, V(H))} \underline{\Sigma}\phi_i(V) \longrightarrow \coprod_{\phi_j \in Hom(\downarrow V', V(H))} \underline{\Sigma}\phi_j(V')$
$\lambda_{\phi_i(V)} \longrightarrow \lambda_{|\phi_j(V')}$
$(I(\underline{A}))(\iota_{\phi_j \phi_i}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_i(V))$
$\phi_i = \phi_j|\downarrow V'$
$\underline{A}(V_1) \longrightarrow \underline{A}(V_2)$

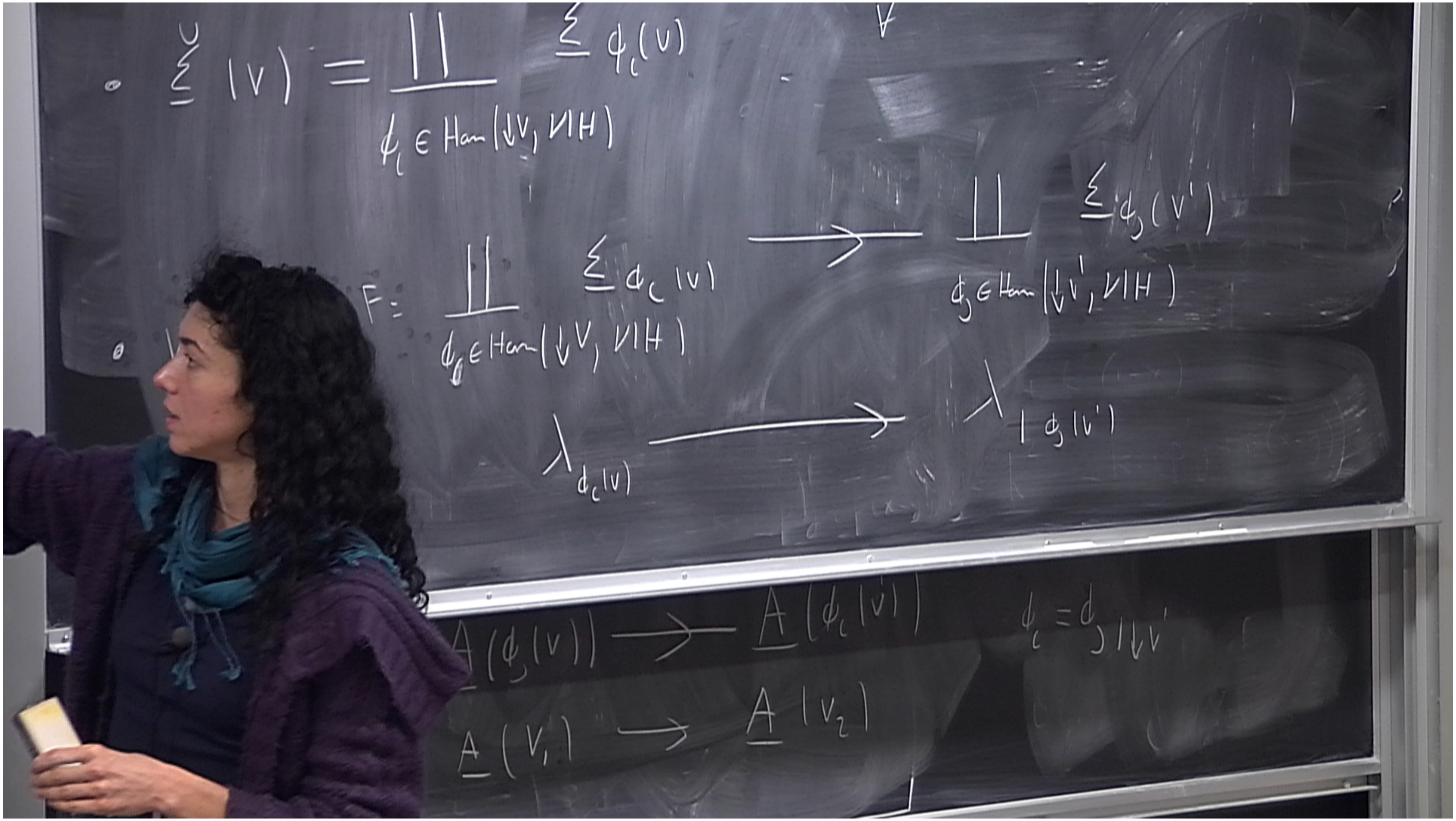
- $\underline{\Sigma}^{\vee}(V) = \coprod_{\phi_i \in \mathrm{Hom}(\downarrow V,\, V|H)} \underline{\Sigma}_{\phi_i(V)}$

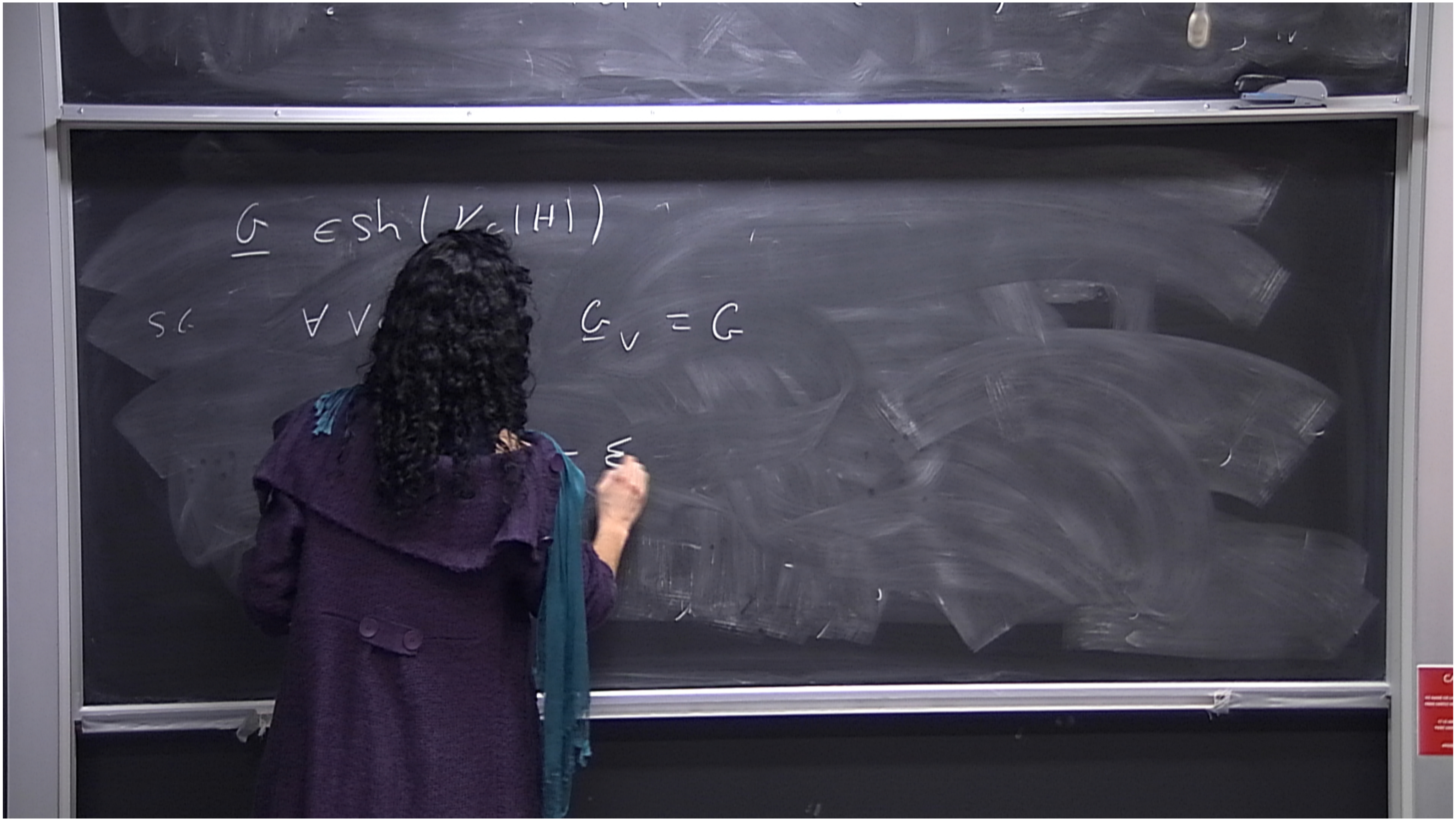
- $V' \hookrightarrow V$ $\quad F : \coprod_{\phi_i \in \mathrm{Hom}(\downarrow V,\, V|H)} \underline{\Sigma}_{\phi_i(V)} \longrightarrow \coprod_{\phi_j \in \mathrm{Hom}(\downarrow V',\, V|H)} \underline{\Sigma}_{\phi_j(V')}$

$$\lambda_{\phi_i(V)} \longrightarrow \lambda|_{\phi_j(V')}$$

$$(I(\underline{A}))(\iota_{\phi_j \phi_i}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_i(V)) \qquad \phi_i = \phi_j|_{\downarrow V'}$$

$$\underline{A}(V_1) \longrightarrow \underline{A}(V_2)$$


Ṽ(V) = ∐ Ṽ φ_i(V)
φ_i ∈ Hom(↓V, V|H)
F: ∐ Ṽ φ_c(V) ⟶ ∐ Ṽ φ_j(V')
φ_c ∈ Hom(↓V, V|H)
φ_j ∈ Hom(↓V', V|H)
λ_{φ_c(V)} ⟶ λ_{φ_j(V')}
A(φ_j(V)) ⟶ A(φ_c(V))
φ_c = φ_j|↓V
A(V_1) ⟶ A(V_2)



$\underline{G} \in sh($
$\underline{G}_V = G$

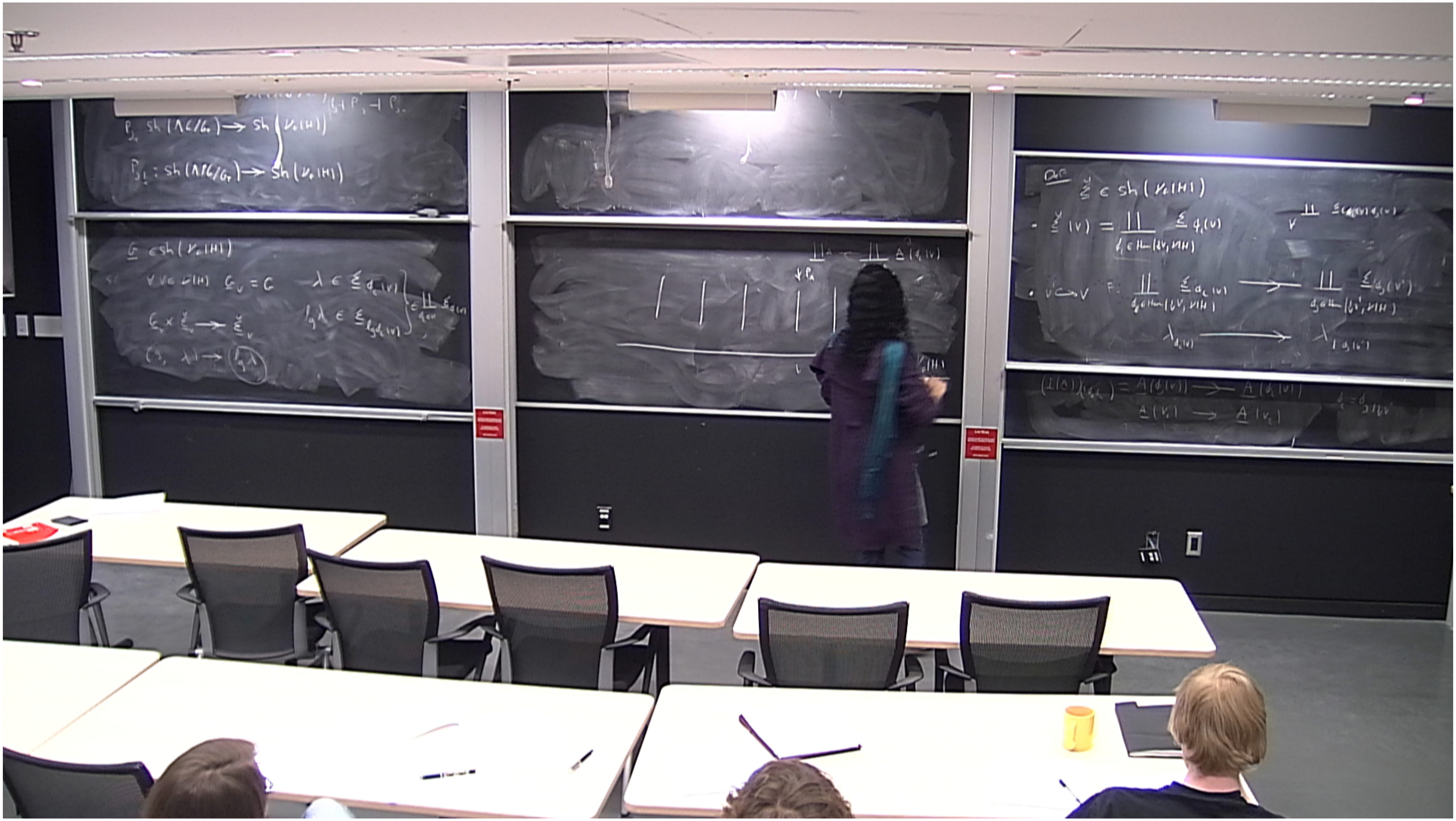
$\underline{G} \in sh(\mathcal{V}_f(H))$

s.t. $\forall V \in \mathcal{V}(H)$ $\underline{G}_V = G$ $\lambda \in \underline{\Sigma}_{\delta_c(V)}$

$l_g \lambda \in \underline{\Sigma}_{l_g \delta_c(V)}$

$\underline{G}_V \times \check{\underline{\Sigma}}_V \longrightarrow \check{\underline{\Sigma}}_V$

$(g, \lambda) \longrightarrow l_g \lambda$



• $\underline{\Sigma}(V) = \coprod_{\phi_c \in Hom(\downarrow V, \nu H)} \underline{\Sigma}_{\phi_c}(V)$

• $V' \hookrightarrow V$ $\quad F : \coprod_{\phi_c \in Hom(\downarrow V, \nu H)} \underline{\Sigma}_{\phi_c}(V) \longrightarrow \coprod_{\phi_j \in Hom(\downarrow V', \nu H)} \underline{\Sigma}_{\phi_j}(V')$

$\lambda_{\phi_c(V)} \longrightarrow \lambda_{|\phi_j(V')}$

$(\underline{I}(\underline{A}))(\iota_{\phi_j \phi_c}) = \underline{A}(\phi_j(V)) \longrightarrow \underline{A}(\phi_c(V))$ $\qquad \phi_c = \phi_j|_{\downarrow V'}$

$\underline{A}(V_1) \longrightarrow \underline{A}(V_2)$