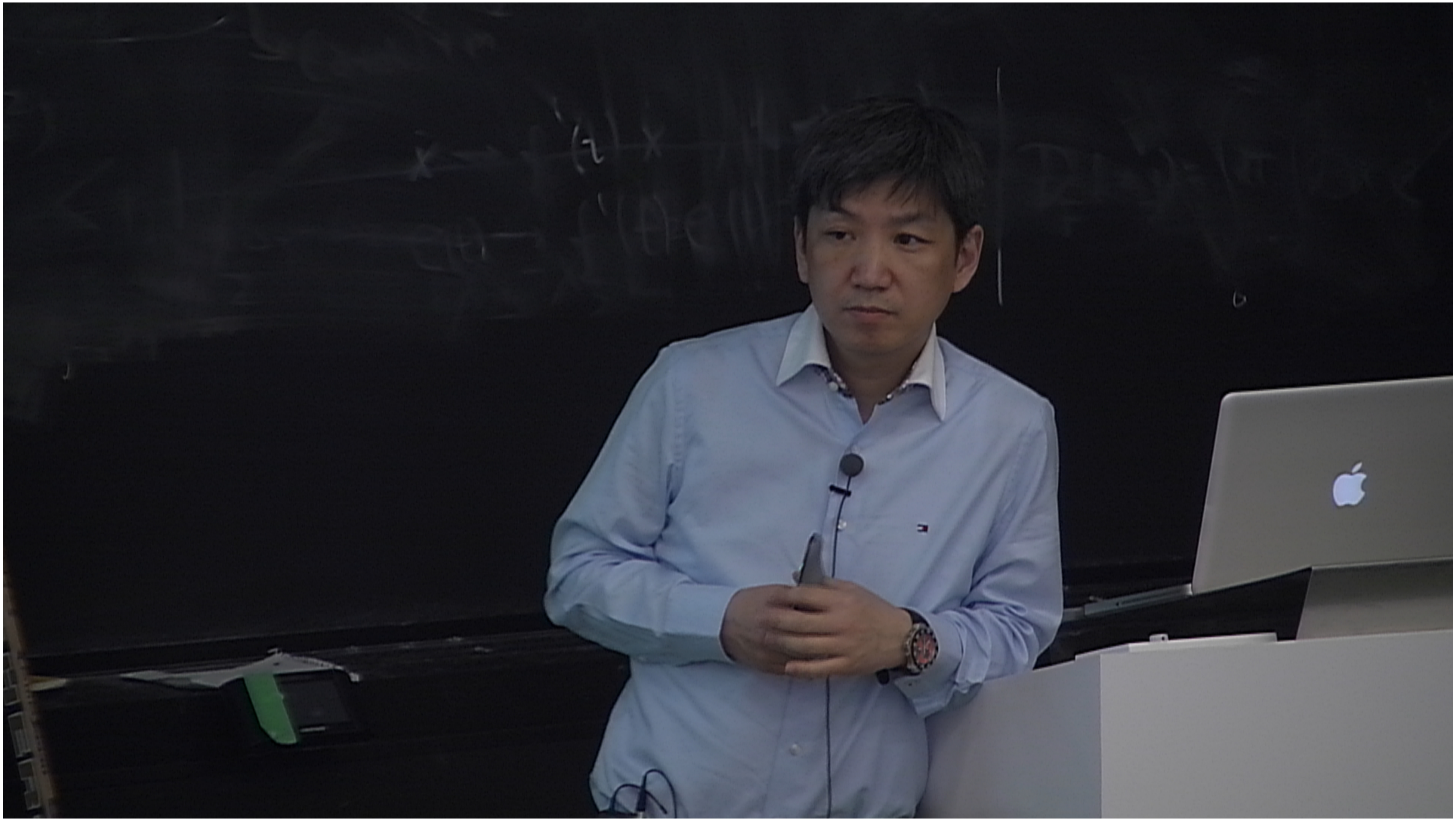
Title: Where Are We in Spin Liquid Research?

Date: Mar 16, 2012 02:30 PM

URL: http://pirsa.org/12020095

Abstract: <span>We provide a brief introduction to quantum spin liquid and review current status of theoretical and experimental progresses on this subject. Spin liquid phases that arise in different situations are examined in the light of both theoretical models and experimental systems.</span>



# Where are we in spin liquid research ?

Yong Baek Kim
University of Toronto

Perimeter Institute, March 16, 2012

Canada Research Chairs
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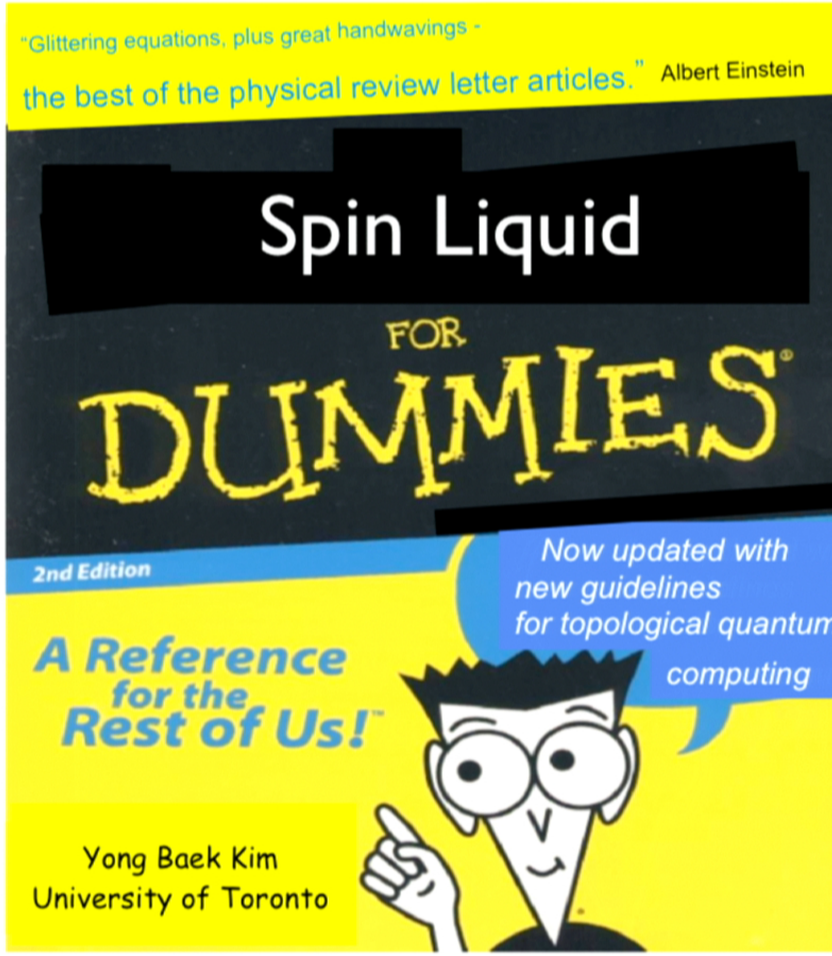
"Glittering equations, plus great handwavings -
the best of the physical review letter articles." Albert Einstein
Spin Liquid
FOR
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Now updated with
new guidelines
for topological quantum
computing
A Reference for the Rest of Us!™
Yong Baek Kim
University of Toronto

# Outline

What is "Quantum" Spin Liquid ?

Overview of Experiments

Overview of Theories

Interpretation of Experiments

Topological Mott Insulator

# What is Quantum Spin Liquid ?

1) Different from Cooperative Paramagnet

Large number of equally unhappy (classical) ground states ~ $e^{\alpha N}$


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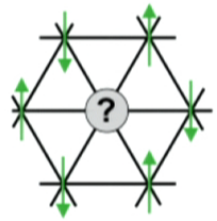


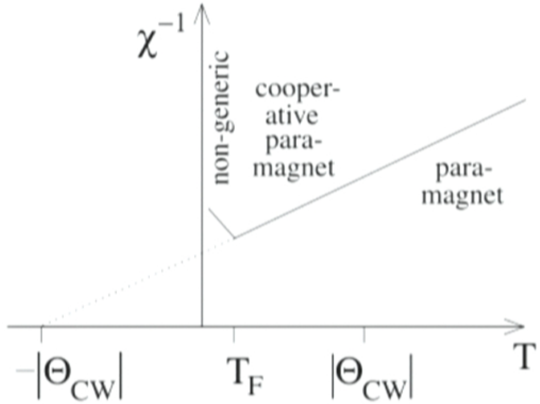


$\Theta_{CW}$ : Curie-Weiss temperature

$$\chi \sim \frac{1}{T - \Theta_{CW}} \quad T \gg |\Theta_{CW}|$$

$$f = \frac{|\Theta_{CW}|}{T_F}$$ useful diagnostic of frustration

$f \gg 1$ strong frustration

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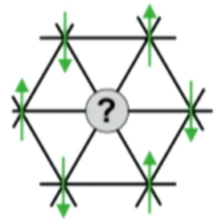

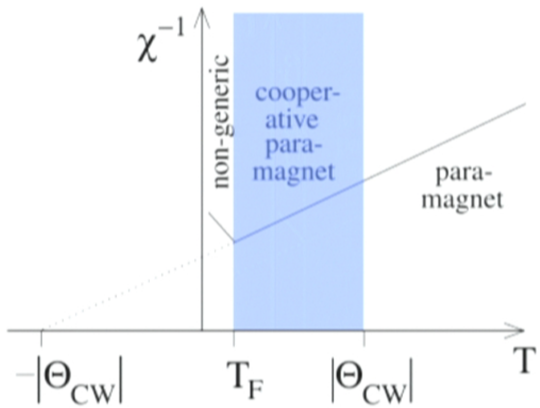


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highly degenerate classical ground state manifold

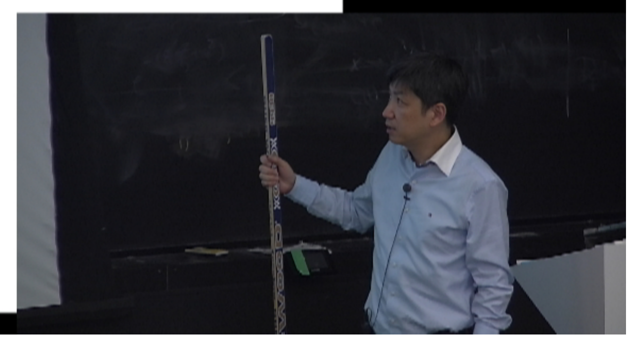
"classical spin liquid"

# What is Quantum Spin Liquid ?

2) Different from boring incoherent (high temp) Paramagnet

Spins fluctuate independently; don't talk to each other

No coherent excitations



# What is Quantum Spin Liquid ?

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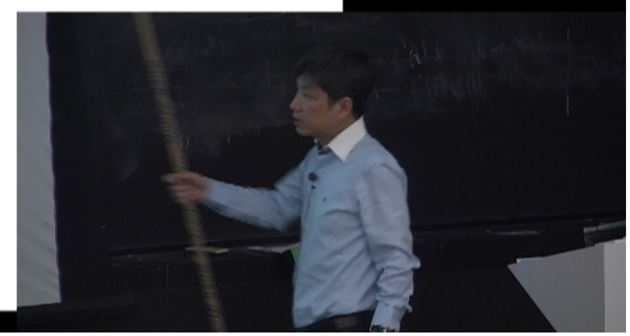
3) Spin Liquid in 1D

Spin-1/2 Heisenberg Model in 1D : Spin Liquid Ground State

Quantum paramagnet, No translational symmetry breaking

Coherent Excitations; S=1/2 spinons



# What is Quantum Spin Liquid ?

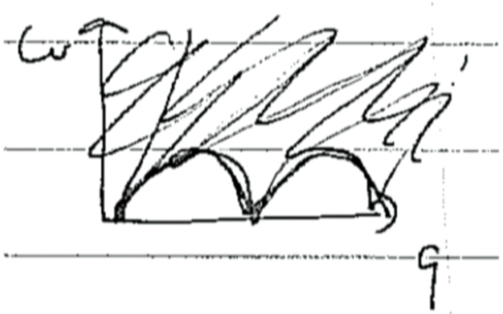
Neutron Scattering -- Spin-1 excitations

Spinon-Antispinon pair excitations

Well-defined dispersion --> Threshold energy for pair excitations

$$\omega_{\mathbf{q}} \sim \min \left[ \varepsilon_{\frac{\mathbf{q}}{2}+\frac{\mathbf{p}}{2}} + \varepsilon_{\frac{\mathbf{q}}{2}-\frac{\mathbf{p}}{2}} \right]$$

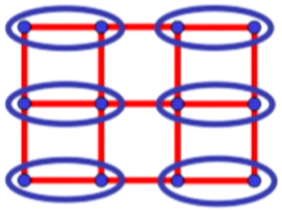
for all possible $\mathbf{p}$



# Higher dimensions ?

Quantum Paramagnet $\langle \mathbf{S} \rangle = 0$

i) Translational symmetry breaking; Valence Bond Solid (VBS)



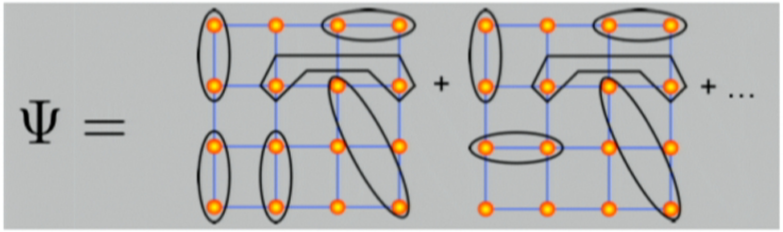
$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Valence Bond

ii) No broken translational symmetry in Mott insulator

Spin Liquid: charge neutral spin-1/2 excitations with/without spin gap

Resonating Valence Bond state (RVB); Superposition of Valence Bond coverings

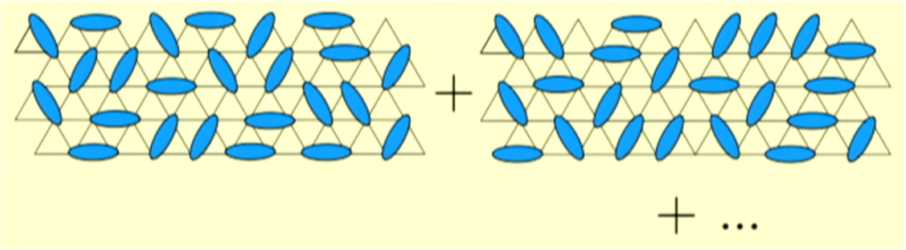



P.W.Anderson

Rokhsar-Kivelson

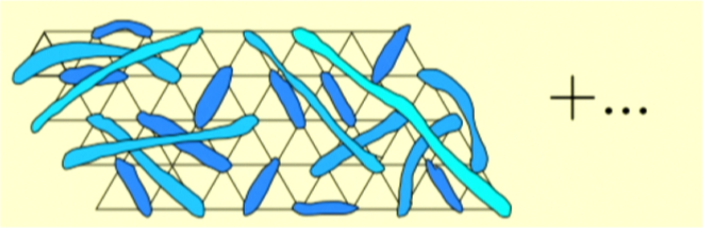
# Many Flavors of Spin Liquid States

Short-range RVB



Gapped spinons, Finite spin gap $\langle \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{0}) \rangle \sim e^{-r/\xi}$

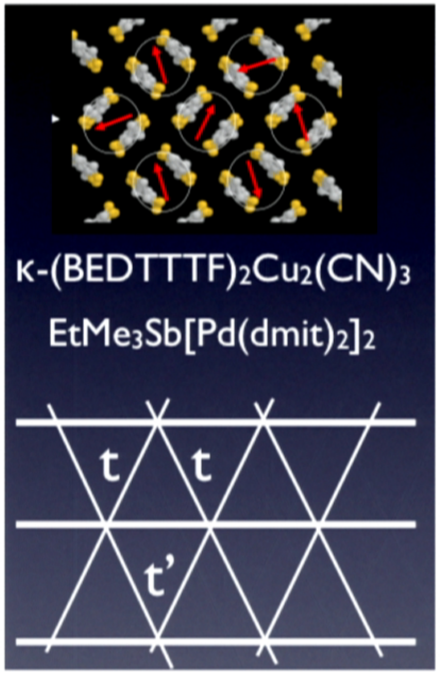
Long-range RVB



Gapless spinons; "Critical" Spin Liquid
Zero spin gap

$$\langle \mathbf{S}(\mathbf{r}) \cdot \mathbf{S}(\mathbf{0}) \rangle \sim \frac{1}{r^{\alpha}}$$

# S=1/2 Candidate Materials for Spin Liquid

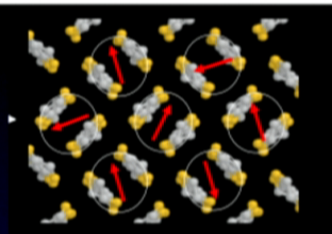



K. Kanoda
R. Kato

# S=1/2 Candidate Materials for Spin Liquid

D. G. Nocera, Y. S. Lee

Z. Hiori



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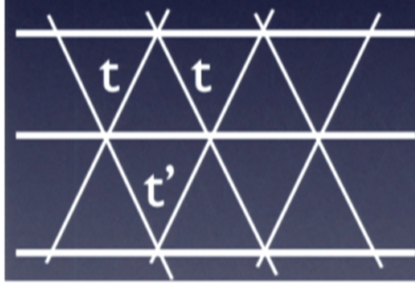
EtMe$_3$Sb[Pd(dmit)$_2$]$_2$

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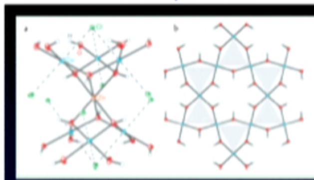
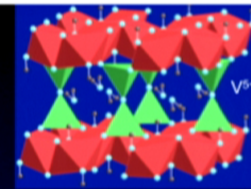
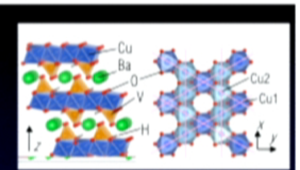


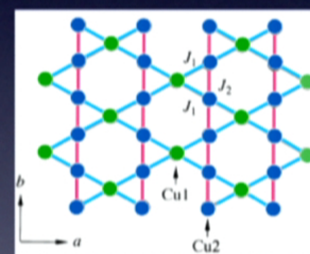
herbertsmithite




volborthite




vesignieite



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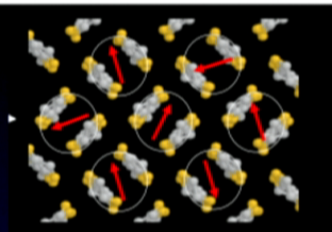

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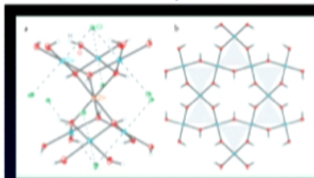
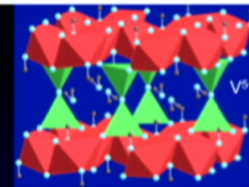


herbertsmithite

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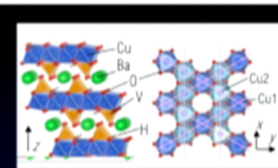


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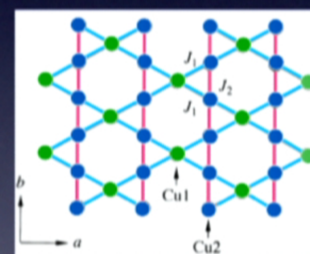


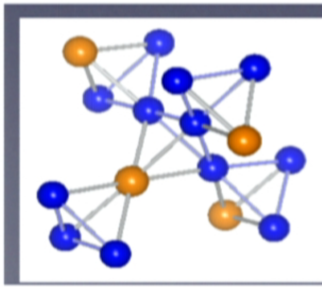

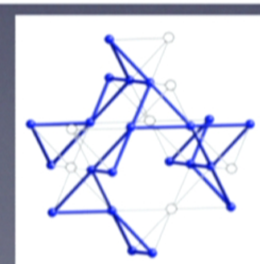

$Na_4Ir_3O_8$

H. Takagi

# Typical Behaviors


Charge Insulator

Constant susceptibility at low temperatures

Power-law specific heat

A lot of low energy (charge-neutral) excitations that carry spin quantum number

--> Spin Liquid with gapless spinon excitations ?

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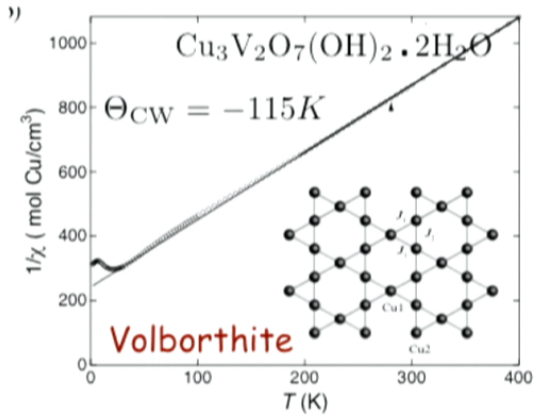
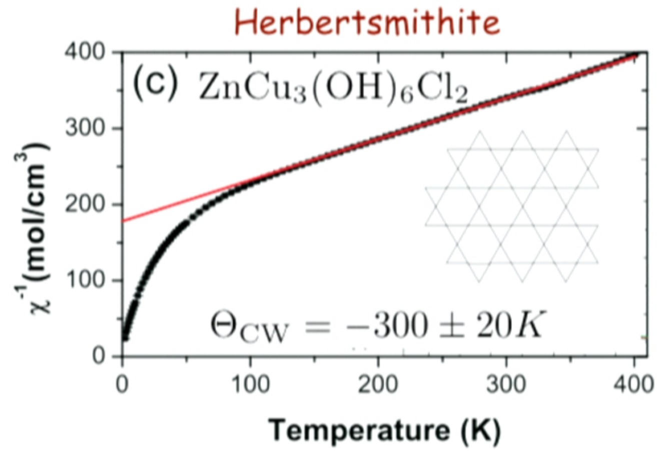
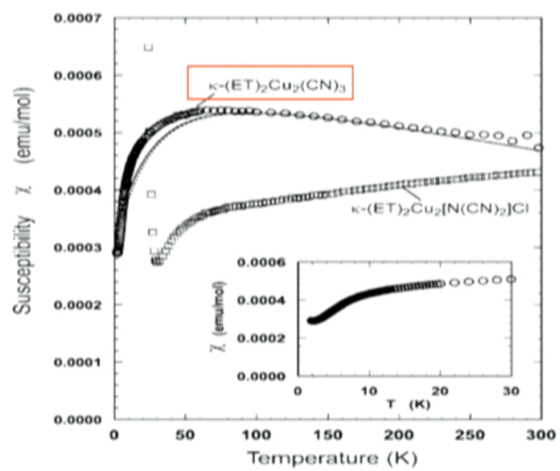
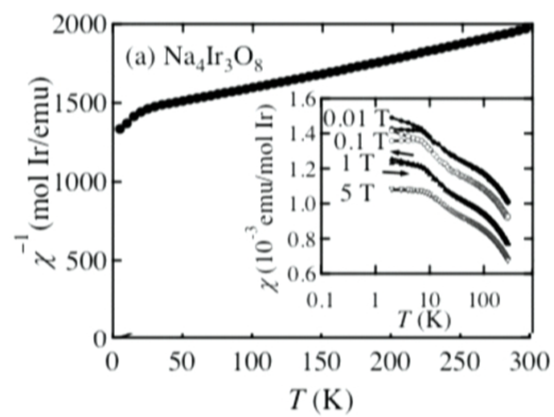
# Spin Susceptibility

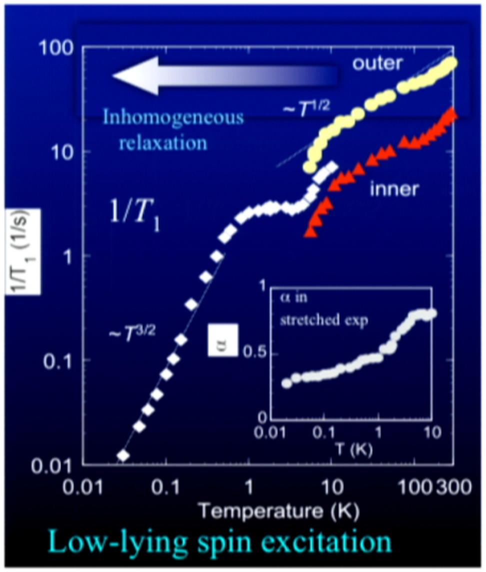


# Abundance of low energy spin excitations

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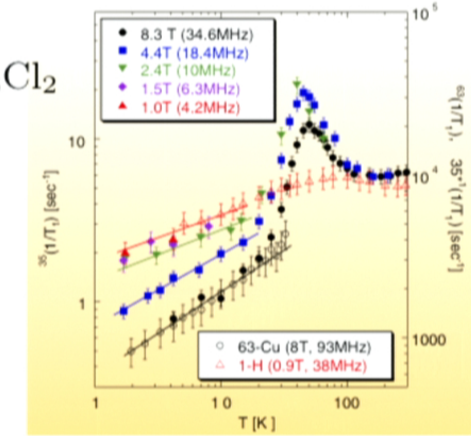
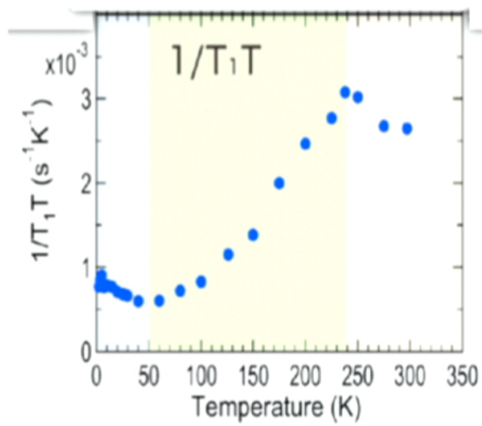
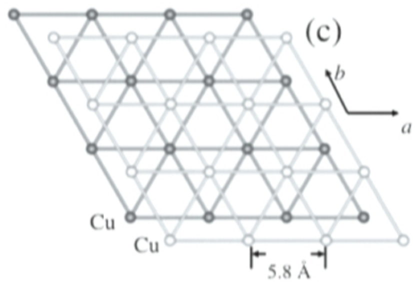



$Na_4Ir_3O_8$

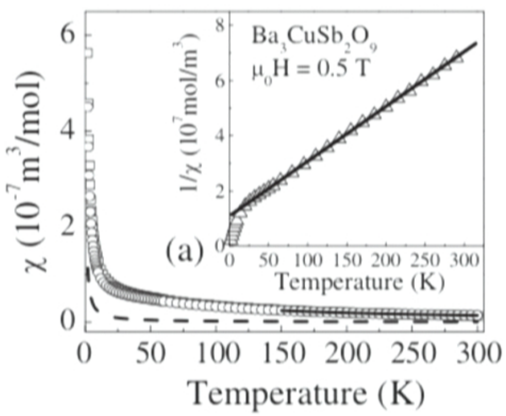
# Spin Liquid State in the $S = 1/2$ Triangular Lattice $Ba_3CuSb_2O_9$

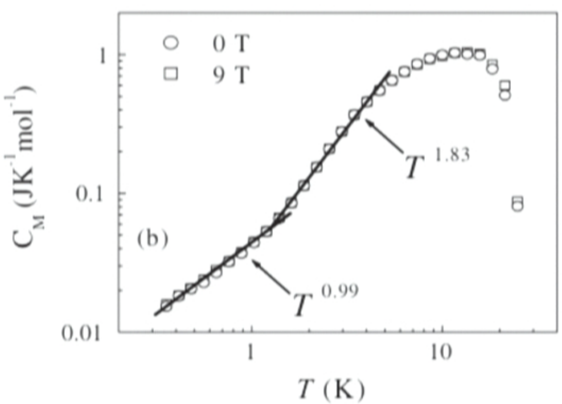
H. D. Zhou,[1,*] E. S. Choi,[1] G. Li,[1] L. Balicas,[1] C. R. Wiebe,[1,2,3] Y. Qiu,[4,5] J. R. D. Copley,[4] and J. S. Gardner[4,6]




$$\theta_{CW} = -55 \text{ K}.$$

$$\gamma = 43.4 \text{ mJ K}^{-2} \text{ mol}^{-1}$$



PRL **106,** 147204 (2011) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2011

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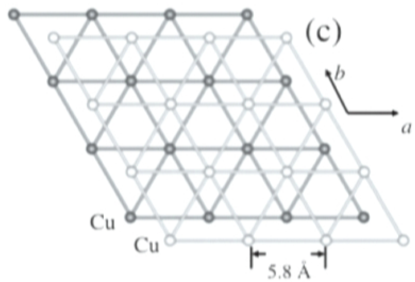


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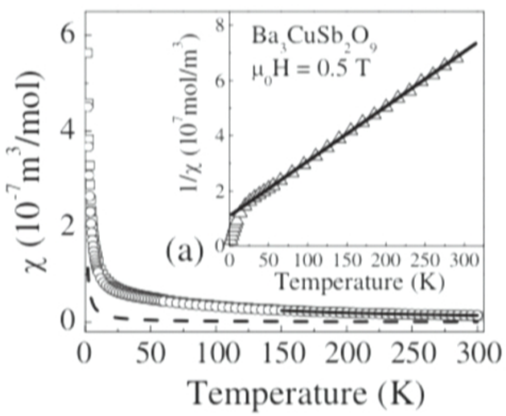
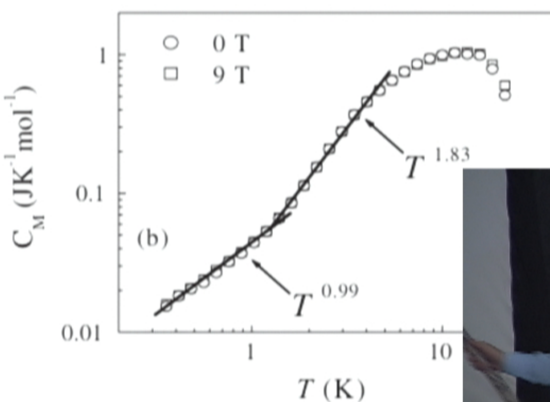


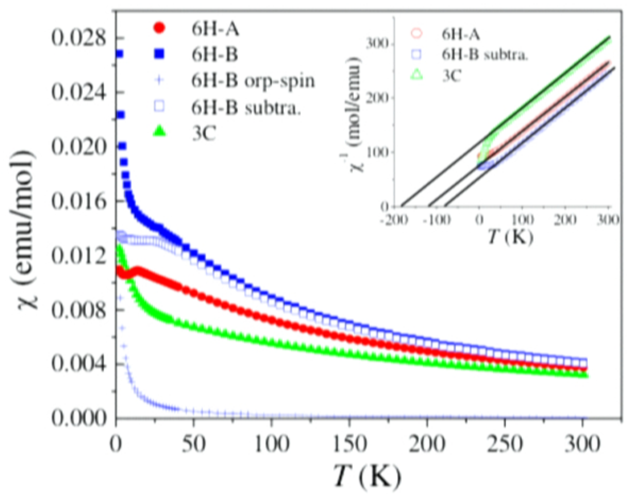

PRL **107,** 197204 (2011) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2011

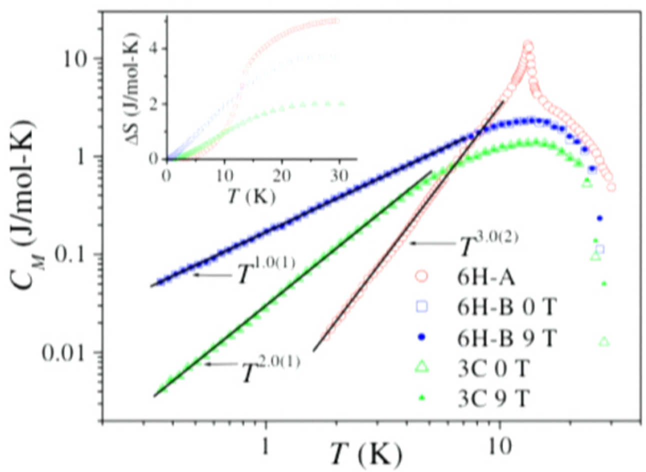
## High-Pressure Sequence of $Ba_3NiSb_2O_9$ Structural Phases: New $S = 1$ Quantum Spin Liquids Based on $Ni^{2+}$

J. G. Cheng,[1] G. Li,[2] L. Balicas,[2] J. S. Zhou,[1] J. B. Goodenough,[1] Cenke Xu,[3] and H. D. Zhou[2,*]

$\theta_{CW}$ of $-75.5$ (6$H$-B) K

$\gamma = 168$ mJ/mol K$^2$



# S=1/2 Candidate Materials for Spin Liquid

| Material | Lattice | Ground State | $f=\|\Theta\|_{CW}/T_F$ |
|---|---|---|---|
| $\kappa$-$(BEDTTTF)_2Cu_2(CN)_3$ | triangular | QSL? | $> 10^3$ |
| $ETMe_3Sb[Pd(dmit)_2]_2$ | triangular | QSL? | $> 10^3$ |
| $ZnCu_3(OH)_6Cl_2$ (Herbertsmithite) | kagome | QSL? | $> 10^3$ |
| $Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ (Volborthite) | anisotropic-kagome | AF? Glassy? | $\approx 100$ |
| $BaCu_3V_2O_8(OH)_2$ (vesigniete) | anisotropic-kagome | QSL? | $> 100$ |
| $Na_4Ir_3O_8$ | hyperkagome | QSL? | $> 10^3$ |

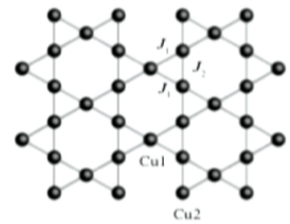
# Search for Quantum Spin Liquid ("S=1/2")

Herbertsmithite "Ideal" Kagome lattice

$ZnCu_3(OH)_6Cl_2$ D. G. Nocera, Y. S. Lee

Volborthite Distorted Kagome lattice

$Cu_3V_2O_7(OH)_2 \cdot 2H_2O$ Z. Hiori



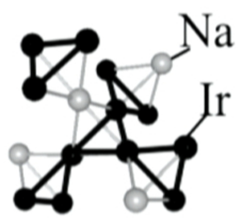

$\kappa$-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ K. Kanoda

$EtMe_3Sb[Pd(dmit)_2]_2$ R. Kato

Organic Material
Triangular Lattice

Hyperkagome $Na_4Ir_3O_8$

H. Takagi



## Weak Mott Insulator ?

$\kappa$-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$

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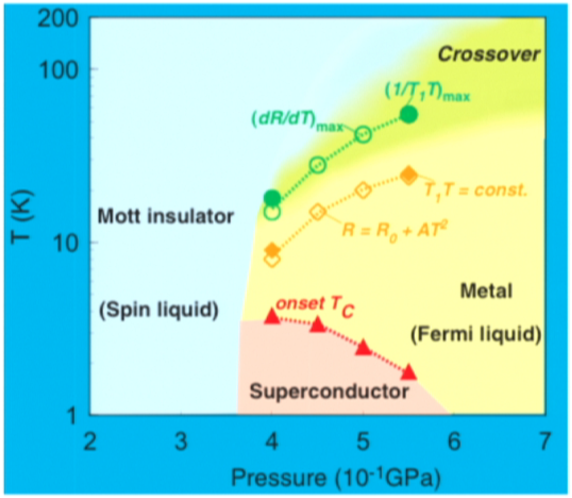
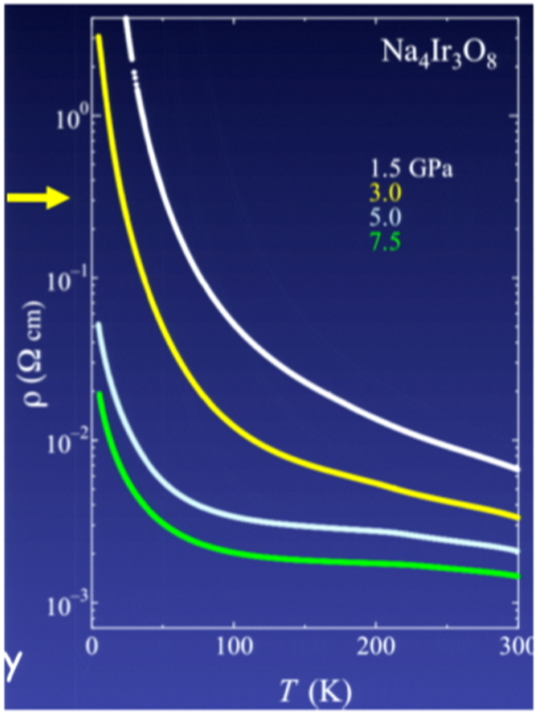
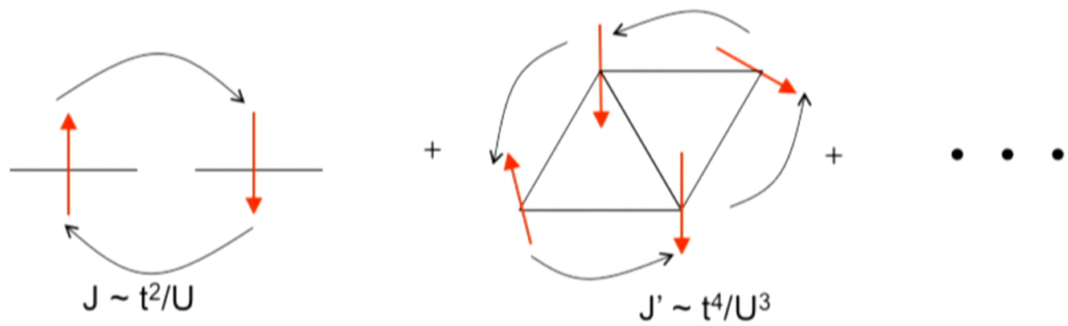


# Importance of Charge Fluctuations

$$H \sim H_{\text{heisenberg}} + H_{\text{ring}} + \cdots$$




Charge fluctuations are important near
the Mott transition even in the insulating phase

Imada (2003)

Motrunich (2005)

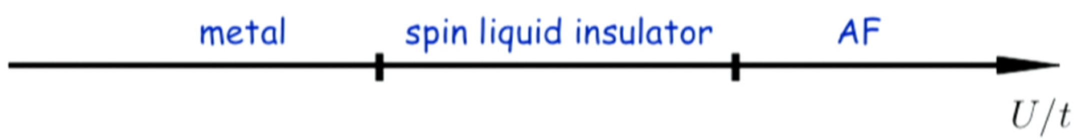
S. S. Lee and P. A. Lee (2005)

## Weak Mott Insulator ?

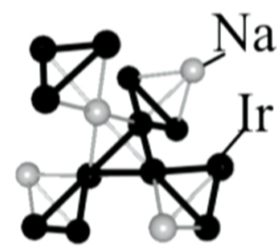
$\kappa$-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ K. Kanoda

$EtMe_3Sb[Pd(dmit)_2]_2$ R. Kato, Y. Matsuda

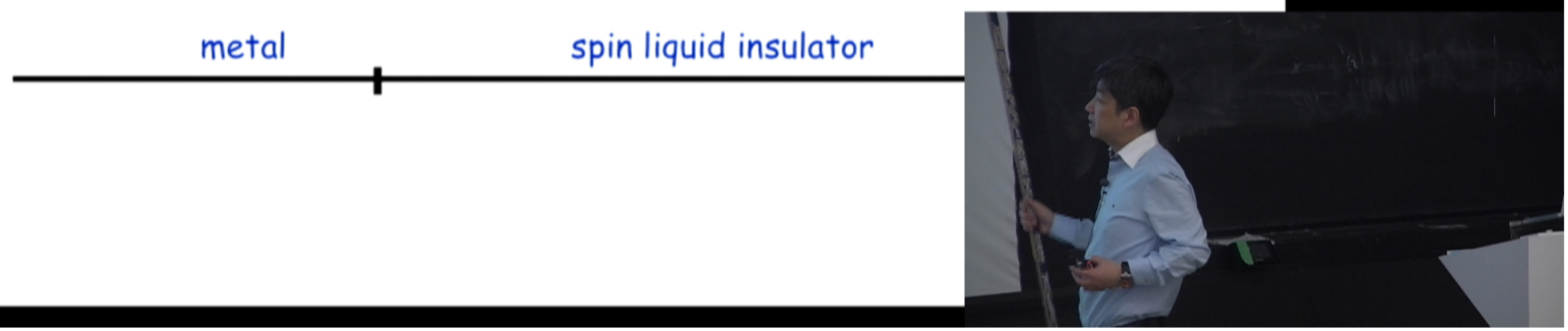
Triangular Lattice; near Mott transition




Hyper-Kagome $Na_4Ir_3O_8$ H. Takagi




5d transition metal oxides; intermediate coupling



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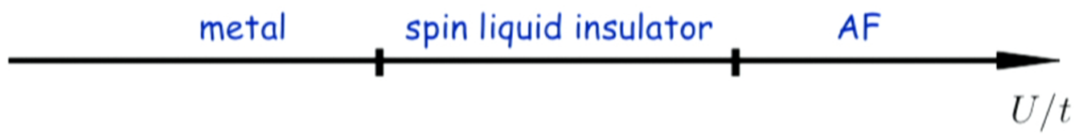
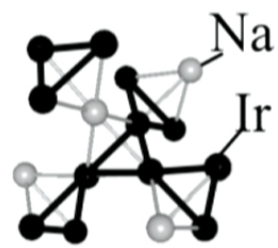
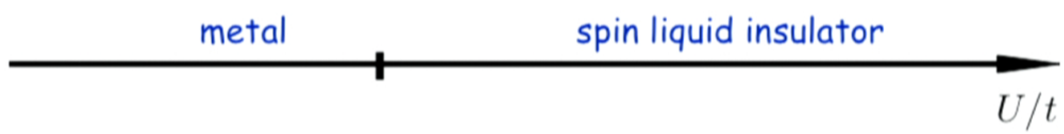



Hyper-Kagome $Na_4Ir_3O_8$ H. Takagi

5d transition metal oxides; intermediate coupling




Spinons inherit the Fermi Surface of Electrons

# Variational Wavefunctions for Spin Liquids

## Slave-Particle Approach

$\vec{S}_i = \frac{1}{2} f^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} f_{i\beta}$ with the constraint $\sum_\alpha f^\dagger_{i\alpha} f_{i\alpha} = 1 \quad \alpha, \beta = \{\uparrow, \downarrow\}$

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$$H = \sum_{ij} J_{ij}\ \vec{S}_i \cdot \vec{S}_j \quad \longrightarrow \quad H = \sum_{ij} J_{ij}\ f^\dagger_{i\alpha} f^\dagger_{i\beta} f_{j\beta} f_{j\alpha} + \cdots$$

fermion-fermion interaction

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## Mean-Field Theory

$$\chi_{ij} \sim \langle f^\dagger_{i\alpha} f_{i\alpha} \rangle$$ fermion "kinetic" energy dynamically generated

$$\Delta_{ij} \sim \langle \epsilon_{\alpha\beta} f_{i\alpha} f_{j\beta} \rangle$$ possible pairing correlation

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$$H_{MF} = \sum_{ij} [\chi_{ij} f^\dagger_{j\alpha} f_{i\alpha} + \Delta_{ij} \epsilon_{\alpha\beta} f^\dagger_{j\beta} f^\dagger_{i\alpha} + h.c.]$$

$\sum_\alpha \langle f^\dagger_{i\alpha} f_{i\alpha} \rangle = 1$ Constraint imposed only on average

# Variational Wavefunctions for Spin Liquids

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Not every choice of $\chi_{ij}$ and $\Delta_{ij}$ is independent

The mean-field Hamiltonian is invariant under certain gauge and lattice symmetry transformation

Classification requires "equivalence" study --
PSG (projected symmetry Group) analysis by X. G. Wen

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## Topological Order and Quantum Order

(gapped) (gapless)

## Variational Wavefunctions for Spin Liquids

Projected Wavefunction (project out double-occupancy)

$$\Psi_{proj} = P_G \Psi_{MF}$$

$\chi_{ij}$ and $\Delta_{ij}$ can be taken as variational parameters

Find $\chi_{ij}$ and $\Delta_{ij}$ that give the lowest energy solution

Ranking of ground state energy for $\Psi_{MF}$ and $\Psi_{proj}$ changes quite often

# Possible Spin Liquid Phases; Why Gauge Theory ?

1) U(1) Spin Liquid $\Delta_{ij} = 0 \quad \chi_{ij} \neq 0$

$$H \sim \sum_{ij} \chi_{ij} f^\dagger_{i\alpha} f_{j\alpha} \sim \sum_{ij} |\chi| e^{ia_{ij}} f^\dagger_{i\alpha} f_{j\alpha}$$

$$f_{i\alpha} \to f_{i\alpha} e^{i\theta_i}$$
$$a_{ij} \to a_{ij} + \theta_i - \theta_j$$

U(1) gauge invariance

$f_{i\alpha}$ spinons interact with a U(1) gauge field

2) $Z_2$ Spin Liquid $\Delta_{ij} \neq 0 \quad \chi_{ij} \neq 0$ (a BCS state of the spinons)

Only $f_{i\alpha} \to \pm f_{i\alpha}$ is the gauge symmetry Only $a_{ij} = 0, \pi$ are allowed

Ising gauge field $e^{ia_{ij}} = \sigma_{ij} = \pm 1$

$$f_{i\alpha} \to \epsilon_i f_{i\alpha}$$
$$\sigma_{ij} \to \epsilon_i \sigma_{ij} \epsilon_j$$

$\epsilon_i = \pm 1$

$Z_2$ gauge invariance

# Possible Spin Liquid Phases; Wavefunctions

1) U(1) Spin Liquid

$$\Psi_{proj} \sim P_G \Psi_{\text{"free" fermions}}$$

2) $Z_2$ Spin Liquid

$$\Psi_{proj} \sim P_G \Psi_{BCS}$$

# Properties of Spin Liquid Phases

1) U(1) Spin Liquid

| | Gapped | Gapless | Specific Heat |
|---|---|---|---|
| 2D | Not stable | Stable<br>Fermi Surface<br>Dirac point | $C_{MF}(T) \sim T$<br>$C_{MF}(T) \sim T^2$ |
| 3D | Stable | Stable<br>Fermi Surface<br>Line Node | $C_{MF}(T) \sim T$<br>$C_{MF}(T) \sim T^2$ |

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Gauge field fluctuation effect

**Gappless Cases**

2D Fermi surface case: $C(T) \sim T^{2/3}$

3D Fermi surface case: $C(T) \sim T \ln(1/T)$

**Gapped Case ("photons")**

3D $C(T) \sim T^3$

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Stable in 2D and 3D

Can support Dirac point (d-wave gap) and line nodes etc.
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Can support Dirac point (d-wave gap) and line nodes etc.
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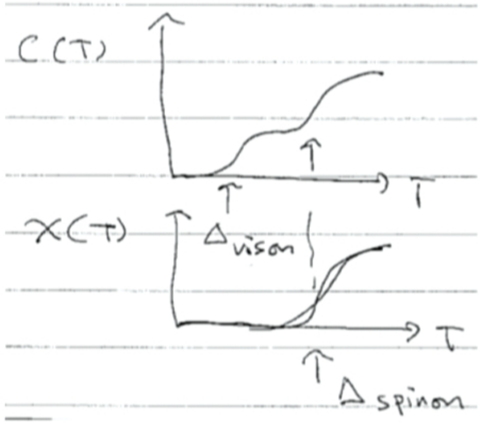
i) Fully gapped case: C(T) ~ $e^{-\Delta_{\rm vison}/T}$

$\Delta_{\rm vison}$ ~ energy gap for visons
- topological defect in $\sigma_{ij}$

charge-neutral spinless;
only carries entropy



# Properties of Spin Liquid Phases

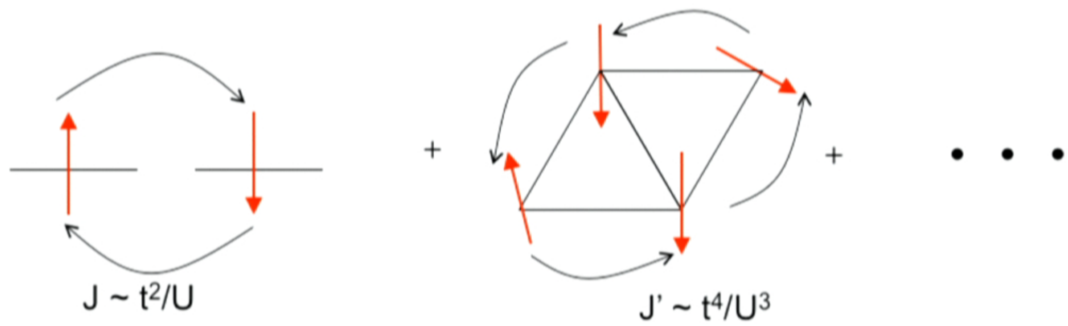
Notice C(T) ~ $T^{\alpha}$ often in candidate spin liquid materials

indicates the presence of low energy spin-carrying excitations ?

Spinons ?

# Organics: Triangular Lattice

Motrunich $H \sim H_{\text{heisenberg}} + H_{\text{ring}} + \cdots$




Variational wavefunction - Uniform RVB: $\chi_{ij} = \chi \quad \Delta_{ij} = 0$

Fermi surface of spinons $C_{\text{MF}}(T) \sim \gamma T$ but ... $C_{\text{fluc}}(T) \sim T^{2/3}$

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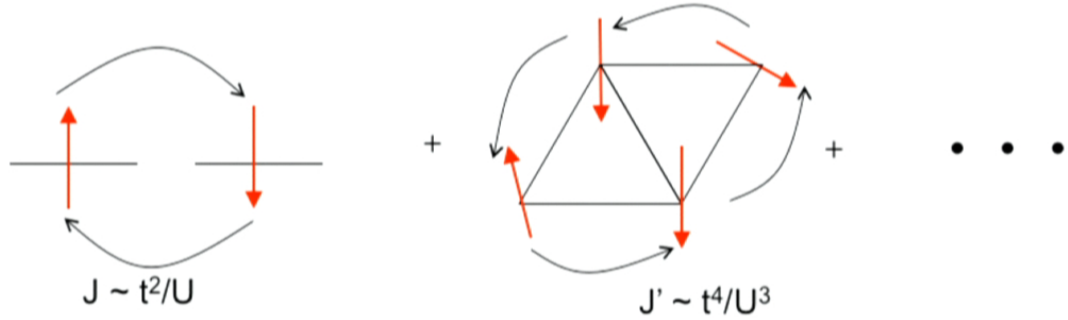


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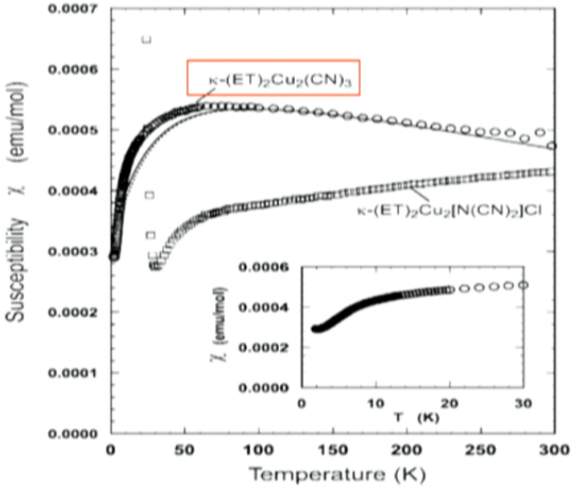
S. S. Lee + P. A. Lee, M. Imada, A.-M. Tremblay,

T. Senthil: Charge fluctuations near Metal-Insulator transition

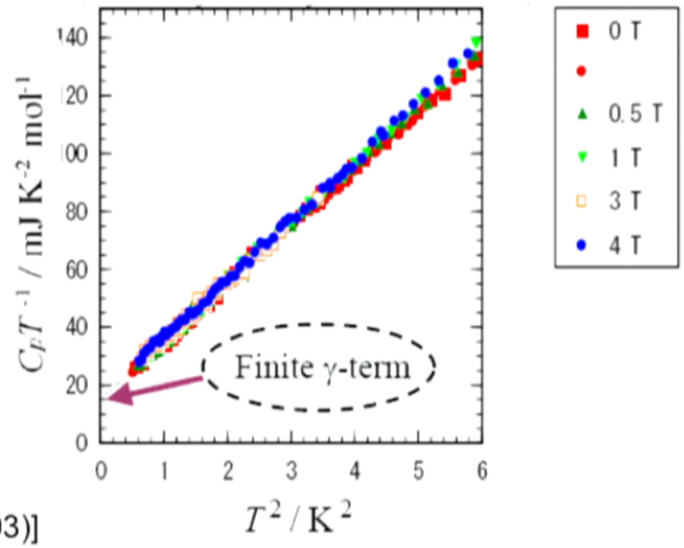
$$C_{\text{fluc}}(T) \sim T^{2/3} \rightarrow C_{\text{fluc}} \sim T \ln(1/T)$$

# Organics: Triangular Lattice

$\kappa\text{-(BEDT-TTF)}_2\text{Cu}_2(\text{CN})_3$

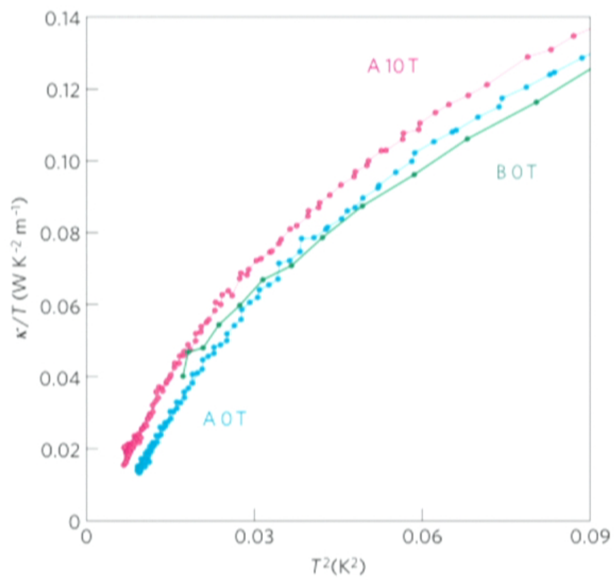



[Y. Shimizu et al., PRL 91, 107001 (03)]




No magnetic ordering;
Constant T=0 susceptibility;
C(T) ~ T;
$1/T_1$ Power-law in T;
Wilson Ratio: Order O(1)

# Organics: Triangular Lattice; Further Challenges




$\kappa/T$ goes to zero as T -> 0 ?

(Y. Matsuda, Nature Physics)

inconsistent with
"Fermi surface" state
of spinons

C(T) shows a kink around 5K ?
(K. Kanoda)

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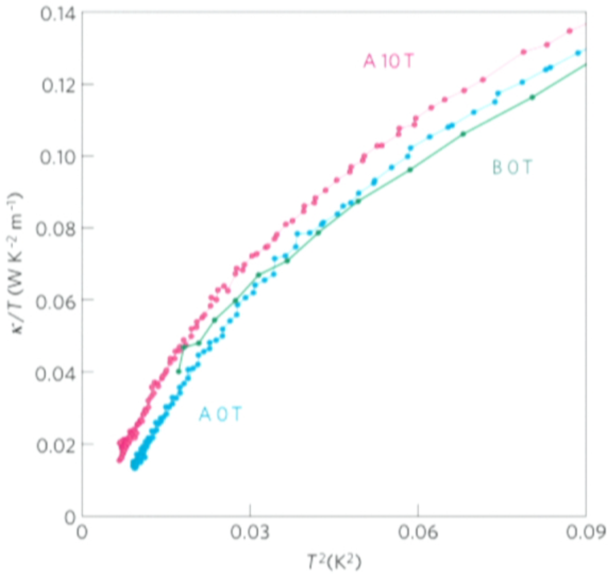


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$Z_2$ Spin Liquid ?

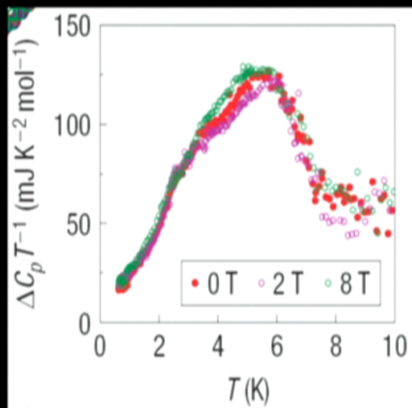
T. Senthil + P. A. Lee -- d-wave nematic spin liquid

S. S. Lee + P. A. Lee -- Finite momentum Pairing via amperian pairing

V. Galitski + Y. B. Kim -- spin-triplet pairing (no change in Knight shift)

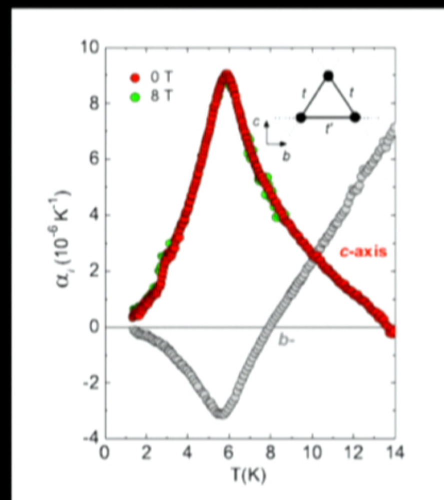
# *Thermodynamic anomaly at 5-6K in* κ-$(ET)_2Cu_2(CN)_3$

***Specific heat*** S. Yamashita *et al*., *Nature Phys*. 4 (2008) 459



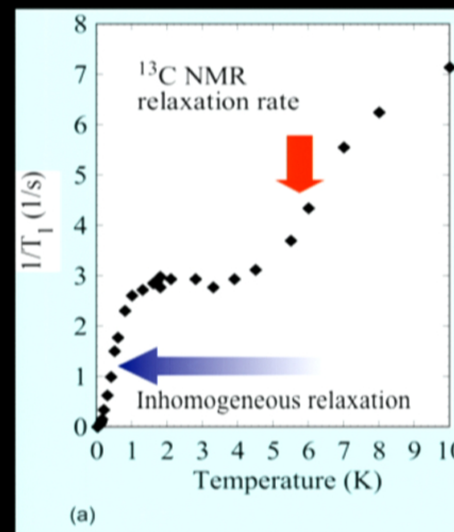

***Thermal expansion coefficient***

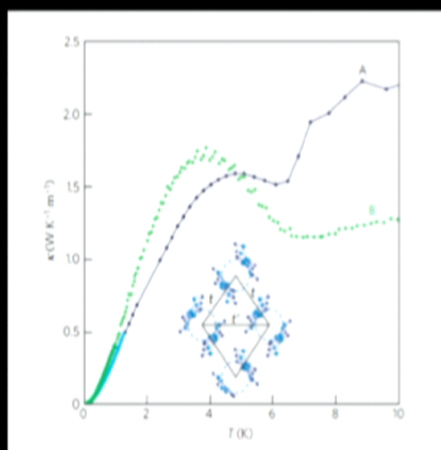
Manna et al., *PRL* **104** (2010) 016403




***NMR Relaxation rate***

Shimizu et al., *PRB* **70** (2006) 060510



***Thermal conductivity***

M. Yamashita *et al*., *Nature Phys*. **5** (2009) 44

# Organics: Triangular Lattice

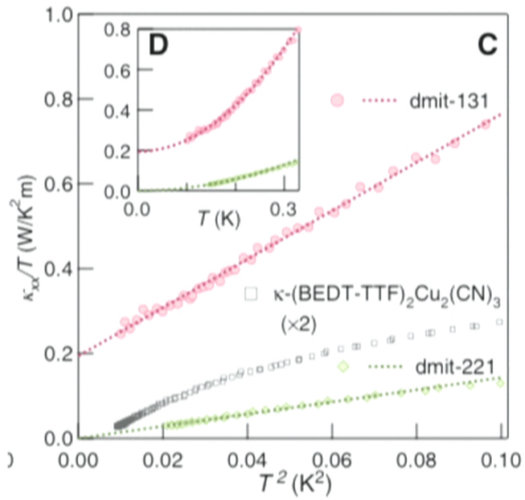
$\mathrm{EtMe_3Sb[Pd(dmit)_2]_2}$

C(T) ~ T (but C(T) at very low temperatures may be complicated...)

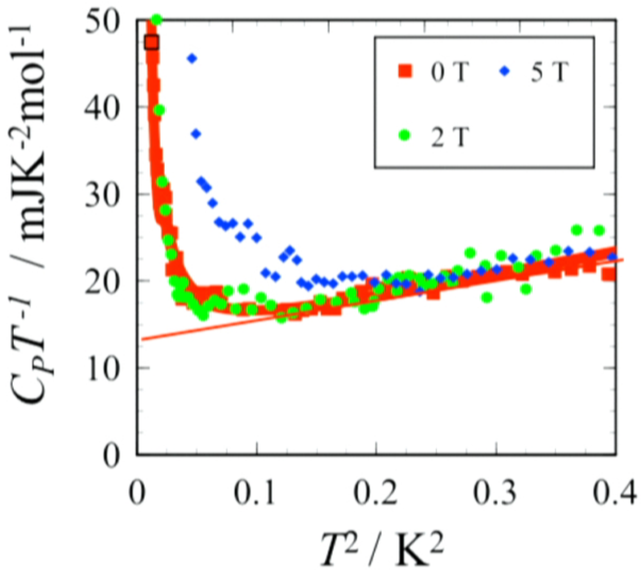
$\kappa/T$ is finite as T -> 0 (Y. Matsuda, Science)

$1/T_1 \sim T^3$ in another experiment; d-wave spinon gap ?

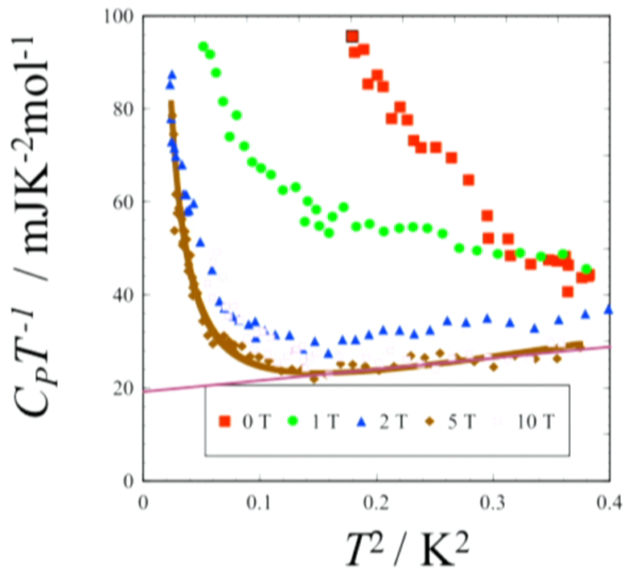
but it was obtained from $e^{-c(t/T_1)^\alpha}$ with $\alpha \neq 1$



$\kappa$-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$




EtMe$_3$Sb[Pd(dmit)$_2$]$_2$




$\gamma$ = 12.6 mJK$^{-2}$mol$^{-1}$ ($\kappa$-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$)

$\gamma$ = 19.9 mJK$^{-2}$mol$^{-1}$ (EtMe$_3$Sb[Pd(dmit)$_2$]$_2$)

Existence of *T*-linear contribution suggests the realization of gap-less quantum spin liquid state.

# Kagome Lattice; Herbertsmithite

$$\mathrm{ZnCu_3(OH)_6Cl_2}$$

$C(T) \sim T^{\alpha} \quad \alpha = ?$ constant susceptibility (extrinsic?)

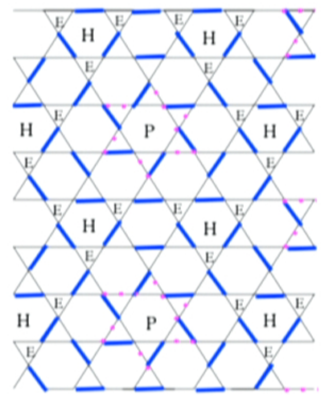
Ying Ran + P. A. Lee + X. G. Wen; U(1) Dirac Spin Liquid

spinons have Dirac spectrum $C(T) \sim T^2 \quad \chi \sim T$

need "disorder" to be consistent with the experiments

VBS state with 36-site unit cell; small spin gap

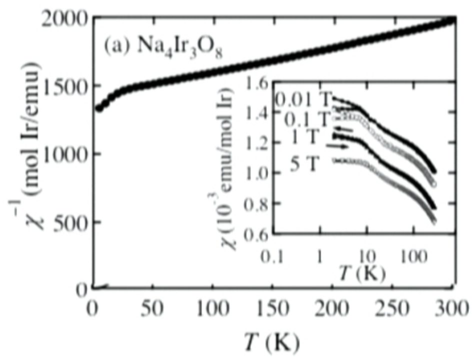
Marston+Zeng, Huse+Singh ... $C(T) \sim e^{-\Delta/T}$
G. Vidal

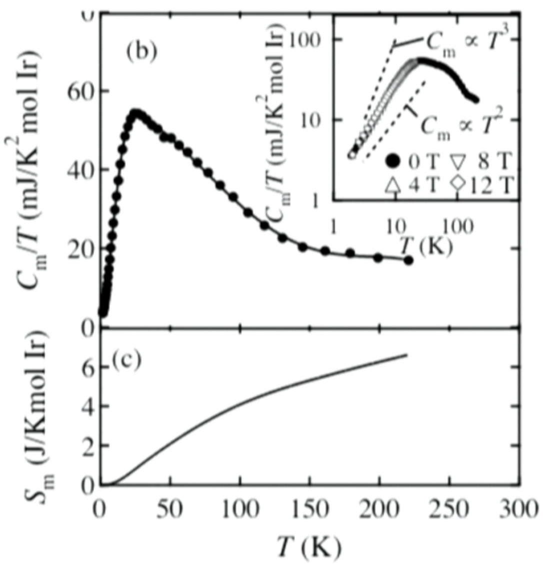


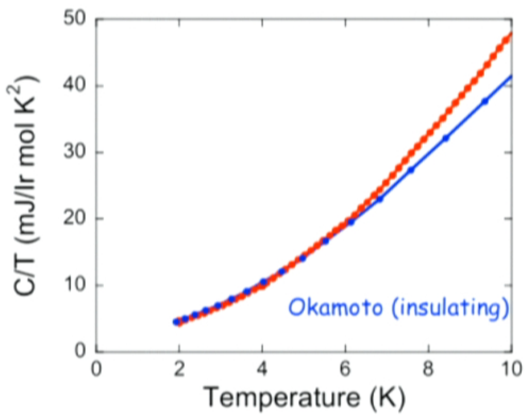

Recent DMRG study by White + Huse;
Spin Liquid with finite spin gap

$Z_2$ Spin Liquid ?

# Hyper-Kagome Lattice; $\mathrm{Na_4Ir_3O_8}$




$C(T) \sim \gamma T$

$\chi$ ~ constant

$1/T_1 \sim T$

Wilson ratio ~ 40-50

Large spin-orbit coupling ?

# Hyper-Kagome Lattice; $Na_4Ir_3O_8$

Spin Liquid with small particle-like and hole-like pockets $C_{\mathrm{MF}}(T) \sim \gamma T$

M. Lawler, A. Paramekanti, Y. B. Kim, L. Balents

Y. Zou, T.-K. Ng, F.-C. Zhang, P. A. Lee

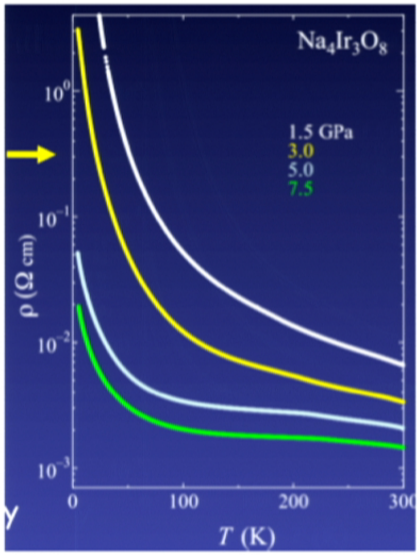
Charge-fluctuations due to the proximity to Metal-Insulator transition

D. Podolsky, A. Paramekanti, Y. B. Kim, T. Senthil

$$C_{\mathrm{fluc}}(T) \sim T \ln\ln(1/T)$$

instead of

$$C_{\mathrm{fluc}}(T) \sim T \ln(1/T)$$



# Topological Mott Insulator

# Type I Topological Phases (Gapped Phases)

Topological Phases

No "Path"
(Local unitary transformations)
without closing the bulk gap

"simple phases" (fully characterized by local order parameter/information)

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Quantum Hall States

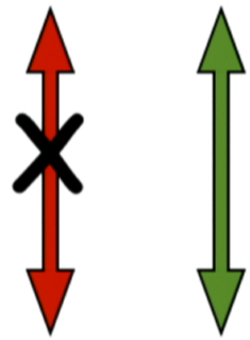
Spin Liquids

(correlated quantum paramagnetic state)

# Type II "Symmetry-Protected" Topological Phases (Gapped Phases)

Topological Phases

No Symmetry-Preserving "Path" without closing the bulk gap



Symmetry-Breaking "Path" without closing the bulk gap

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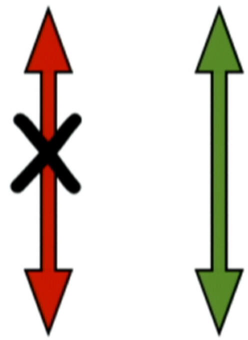

Symmetry-Breaking "Path" without closing the bulk gap

"simple phases"

Topological Band Insulator
(e.g. time-reversal symmetry)

## Can we have topological phases with both Type I and Type II characters ?

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Three-dimensional spin liquid with spinons having topological band structure

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spinon topological band insulator;
spinons are gapped, but interacting with a U(1) gauge field

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Fractionalized topological insulator;
time-reversal symmetry

Such phases in 2D is unstable

# Can we have topological phases with both Type I and Type II characters ?

$J_1$-$J_2$ model on the pyrochlore lattice    $J_1 > 0$ and $J_2 < 0$

S. Bhattacharjee, S.-S.Lee, Y.B.Kim, D.H.Lee, arXiv:1202.0291

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Emergent Spin-Orbit Coupling for Spinons

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$$H_{MF} = -\frac{3J_1\chi}{8} \sum_{\langle i\alpha, j\beta \rangle} \sum_\tau \left[ f^\dagger_{i,\alpha,\tau} f_{j,\beta,\tau} + h.c. \right] - \frac{i3J_2}{8} \sum_{\langle\langle i\alpha, j\beta \rangle\rangle} \sum_{\tau\tau'} \left[ \left( E^{\alpha\beta}_a f^\dagger_{i,\alpha,\tau} \sigma^a_{\tau\tau'} f_{j,\beta,\tau'} - h.c \right) \right] \tag{7}$$

S. Bhattacharjee, S.-S.Lee, Y.B.Kim, D.H.Lee, arXiv:1202.0291

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Mean-field theory ($J_1 \gg |J_2|$);
spin liquid with
"strong topological insulator" for spinons

S. Bhattacharjee, S.-S.Lee, Y.B.Kim, D.H.Lee, arXiv:1202.0291

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$$\vec{S} = \frac{1}{2} f^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} f_{j\beta} \qquad \sum_\alpha f^\dagger_{i\alpha} f_{i\alpha} = 1$$

$$\begin{aligned} H_{MF} = &- \frac{3J_1\chi}{8} \sum_{\langle i\alpha, j\beta \rangle} \sum_\tau \left[ f^\dagger_{i,\alpha,\tau} f_{j,\beta,\tau} + h.c. \right] \\ &- \frac{i3J_2}{8} \sum_{\langle\langle i\alpha, j\beta \rangle\rangle} \sum_{\tau\tau'} \left[ \left( E^{\alpha\beta}_a f^\dagger_{i,\alpha,\tau} \sigma^a_{\tau\tau'} f_{j,\beta,\tau'} - h.c \right) \right] \end{aligned} \qquad (7)$$

S. Bhattacharjee, S.-S.Lee, Y.B.Kim, D.H.Lee, arXiv:1202.0291

## Can we have topological phases with both Type I and Type II characters ?

Mean-field theory ($J_1 \gg |J_2|$):
spin liquid with
"strong topological insulator" for spinons

Such phases also have bond-spin-nematic order

$$\langle Q_{ij}^{ab} \rangle = -\frac{1}{2}\left[ E_{ij,a}E_{ij,b}^{*} - \frac{\delta_{ab}}{3}|\mathbf{E}_{ij}|^2 \right] \neq 0$$

R. Shindou and T. Momoi (2009)

S. Bhattacharjee, S.-S.Lee, Y.B.Kim, D.H.Lee, arXiv:1202.0291

# Contemplation

Our theory of spin liquid is incomplete; Standard Model ?

Shall we call it S-theory ??

Landau Fermi Liquid theory v.s. Spinon Fermi Liquid theory

density-density interaction <----> current-current interaction

Instability of electron FS --> Broken Symmetry States

Instability of spinon FS --> A variety of Mott insulators ?

But the spinons do not have to be well-defined;
gauge-invariant response functions still know about them

Is gauge theory necessary ?

long-range current-current interaction of spinons

non-locality; consequence of requiring fermions/bosons

# Contemplation

We need more spectroscopic tools: Theory and Experiments

Neutron ? Magnetic X-ray ? Direct probe of spinon FS ?

We need better single crystals, thermal transport, and specific heat data

Control and understanding of disorder effect