Title: Scale-invariant Alternatives to General Relativity

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Abstract: We study the general class of gravitational field theories constructed on the basis of scale invariance (and therefore absence of any mass parameters) and invariance under transverse diffeomorphisms (TDiff), which are the 4-volume conserving coordinate transformations. We show that these theories are equivalent to a specific type of scalar-tensor theories of gravity (invariant under all diffeomorphisms) with a number of properties, making them phenomenologically interesting. In particular, they lead to the evolution of the universe supported by present observations: inflation in the past, followed by the radiation and matter dominated stages and accelerated expansion at present. All mass scales in this type of theories come from one and the same source. The massless particle spectrum of these theories contains the graviton and a new particle -- dilaton, which has only derivative couplings and thus escapes the fifth force constraints.


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE
Scale-invariant alternatives to general relativity
Mikhail Shaposhnikov
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Based on:

- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 162
- M.S., Daniel Zenhäusern, Phys. Lett. B **671** (2009) 187
- Diego Blas, M.S., Daniel Zenhäusern, Phys. Rev. D84 (2011) 044001
- Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern, Phys. Rev. D84 (2011) 123504

GR

Einstein gravity is a theory which is invariant under all diffeomorphisms, Diff: $x^\mu \to f^\mu(x^\nu)$.

Pros - consistence with all tests of GR. One of the main predictions - existence of massless graviton.

Problems of GR

- Large dimensionfull coupling constant $G_N^{-1} = M_P^2$, leading to hierarchy problem $m_H \ll M_P$.
- Extra arbitrary fundamental parameter - cosmological constant - which is known to be very small.
- Quantum gravity?

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Our proposal

$$Diff \rightarrow TDiff \times Dilatations$$

TDiff - volume conserving coordinate transformations,

$$det\left[\frac{\partial f^{\mu}}{\partial x^{\nu}}\right] = 1 \ .$$

Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \ \rightarrow \ \sigma^{n}\Psi(\sigma x) \ ,$$

$n = 0$ for the metric, $n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

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Similarity with GR: consistency with all tests

Differences with GR:

- Dynamical origin of all mass scales
- Hierarchy problem gets a different meaning - an alternative (to SUSY, techicolor, little Higgs or large extra dimensions) solution of it may be possible.
- Cosmological constant problem acquires another formulation.
- Natural chaotic cosmological inflation
- Low energy sector contains a massless dilaton
- There is Dark Energy even without cosmological constant
- Quantum gravity?

Outline

- Field theory: classical scale invariance and its spontaneous breakdown
- Unimodular gravity
- Scale invariance, unimodular gravity, cosmological constant, inflation and dark energy
- Quantum scale invariance
- Dilaton as a part of the metric in TDiff gravity
- Conclusions

Scale invariance

Trivial statement: multiply all mass parameters in the theory

$$M_W, \;\; \Lambda_{QCD}, \;\; M_H, M_{Pl}, ...$$

by one and the same number : $M \to \sigma M$. Physics is not changed!

Indeed, this change, supplemented by a dilatation of space-time coordinates $x^\mu \to \sigma x^\mu$ and an appropriate redefinition of the fields does not change the complete quantum effective action of the theory.

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Dilatation symmetry

Dilatations:

$$\phi(x) \rightarrow \sigma^n \phi(\sigma x)$$

n-canonical dimension of the field: $n = 1$ for scalars and vectors, $n = 3/2$ for fermions, while the metric transforms as $g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x)$.

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Ruled out by observations?

No, if it is spontaneously broken!

First step: consider classical physics only (no parameters like Λ_{QCD}), just tree explicit mass parameters such as M_H, M_W, M_{Pl}.

Classical scale invariant theory

Unique regular scale-invariant Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM[M\to 0]} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi, \chi)$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda \left(\varphi^\dagger \varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2}\,,$$

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Spontaneous breaking of scale invariance

Forget first about gravity. Consider scalar potential

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Requirements: vacuum state exists if $\lambda \geq 0, \quad \beta \geq 0$

For $\lambda > 0, \ \ \beta > 0$ the vacuum state is unique: $\chi = 0, \ \ \varphi = 0$ and scale invariance is exact.

Field propagators: scalar $1/p^2$, fermion $\not{p}/p^2$. Greenberg, 1961:

free quantum field theory!!

If not - theory does not describe particles !!

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For $\lambda > 0, \;\; \beta = 0$ the scale invariance can be spontaneously broken.

The vacuum manifold:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \;\; M_{Pl} \sim \chi_0$$

Phenomenological requirement:

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \lll 1$$

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Ordinary gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates.

Invariant under general coordinate transformations

Unimodular gravity:

the metric $g_{\mu\nu}$ is an arbitrary function of space-time coordinates with set$[g] = -1$. Invariant under general coordinate transformations which conserve the 4-volume.

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Origin of UG: Field theory describing spin 2 massless particles is either GR or UG

Number of physical degrees of freedom is the same.

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Unimodular gravity and cosmological constant

Theories are equivalent everywhere except the way the cosmological constant appears

GR. Λ is the fundamental constant:

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\,[R+\Lambda]$$

UG. Λ does not appear in the action:

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UG is equivalent to

$$S = -\frac{1}{M_P^2}\int d^4x\sqrt{-g}\left[R + \Lambda(x)\left(1 - \frac{1}{\sqrt{-g}}\right)\right]$$

Equations of motion ($G_{\mu\nu}$ - Einstein tensor):

$$G_{\mu\nu} = -\Lambda(x)\, g_{\mu\nu}\ ,\ \sqrt{-g} = 1$$

Bianchi identity: $\Lambda(x)_; = 0 \rightarrow \Lambda(x) = const.$

Solutions of UG are the same as solutions of GR with an arbitrary cosmological constant.

Conclusion: in UG cosmological constant reappears, but as an integral of motion, related to initial conditions

However: quantum matter fluctuations do not contribute to Λ - no need for fine-tuning of quartic divergences! Weinberg, Smolin

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Equations of motion for Unimodular Gravity:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 8\pi G_N (T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T)$$

Perfect example of "degravitation" - the "$g_{\mu\nu}$" part of energy-momentum tensor does not gravitate. Solution of the "technical part" of cosmological constant problem - quartically divergent matter loops do not change the geometry. But - no solution of the "main" cosmological constant problem - why $\Lambda \ll M_P^4$? Scale invariance can help!

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Scale invariance + unimodular gravity

Solutions of scale-invariant UG are the same as the solutions of scale-invariant GR with the action

$$S = -\int d^4x \sqrt{-g} \left[\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi \right) \frac{R}{2} + \Lambda + ... \right] ,$$

Physical interpretation: Einstein frame

$$g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu} \; , \;\; (\xi_\chi \chi^2 + \xi_h h^2) \Omega^2 = M_P^2$$

Λ is not a cosmological constant, it is the strength of a peculiar potential!

Relevant part of the Lagrangian (scalars + gravity) in Einstein frame:

$$\mathcal{L}_E = \sqrt{-\tilde{g}} \left(-M_P^2 \frac{\tilde{R}}{2} + K - U_E(h,\chi) \right) ,$$

K - complicated non-linear kinetic term for the scalar fields,

$$K = \Omega^2 \left(\frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu h)^2) \right) - 3M_P^2 (\partial_\mu \Omega)^2 .$$

The Einstein-frame potential $U_E(h,\chi)$:

$$U_E(h,\chi) = M_P^4 \left[\frac{\lambda \left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2}{4(\xi_\chi \chi^2 + \xi_h h^2)^2} + \frac{\Lambda}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \right] ,$$

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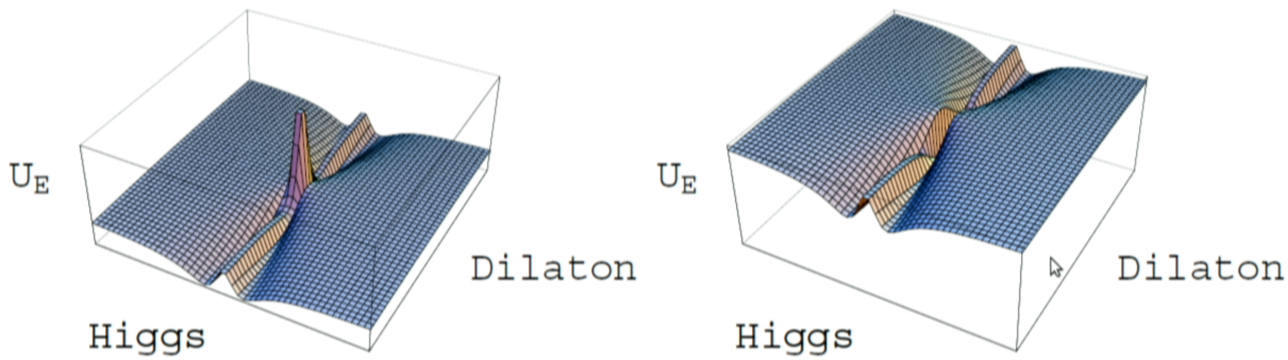

Potential for the Higgs field and dilaton in the Einstein frame.
Left: $\mathbf{\Lambda > 0}$, right $\mathbf{\Lambda < 0}$.

$\mathbf{50\%}$ chance ($\mathbf{\Lambda < 0}$): inflation + late collapse

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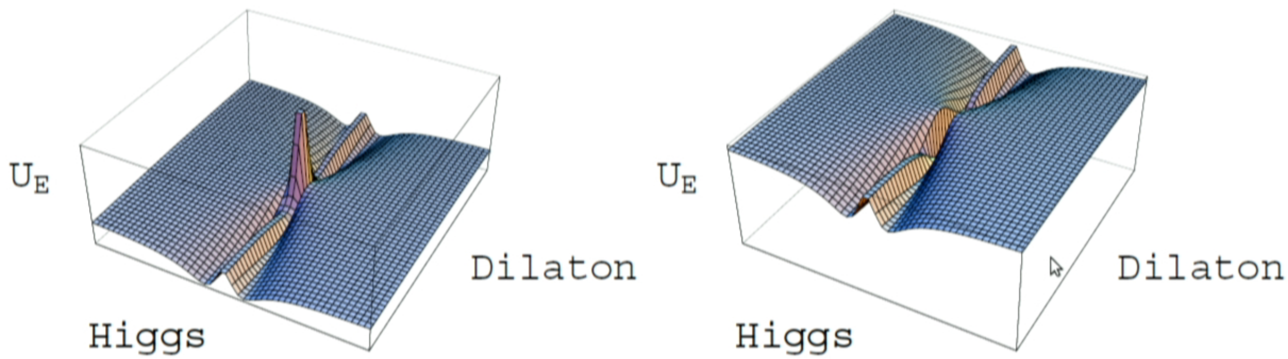

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Inflation

Chaotic initial condition: fields χ and h are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \quad \xi_\chi \ll 1$ (will be justified later)

Then - dynamics of the Higgs field is more essential, $\chi \simeq const$ and is frozen. Denote $\xi_\chi \chi^2 = M_P^2$.

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Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\tilde{h}}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \quad \begin{cases} h \simeq \tilde{h} & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\tilde{h}}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

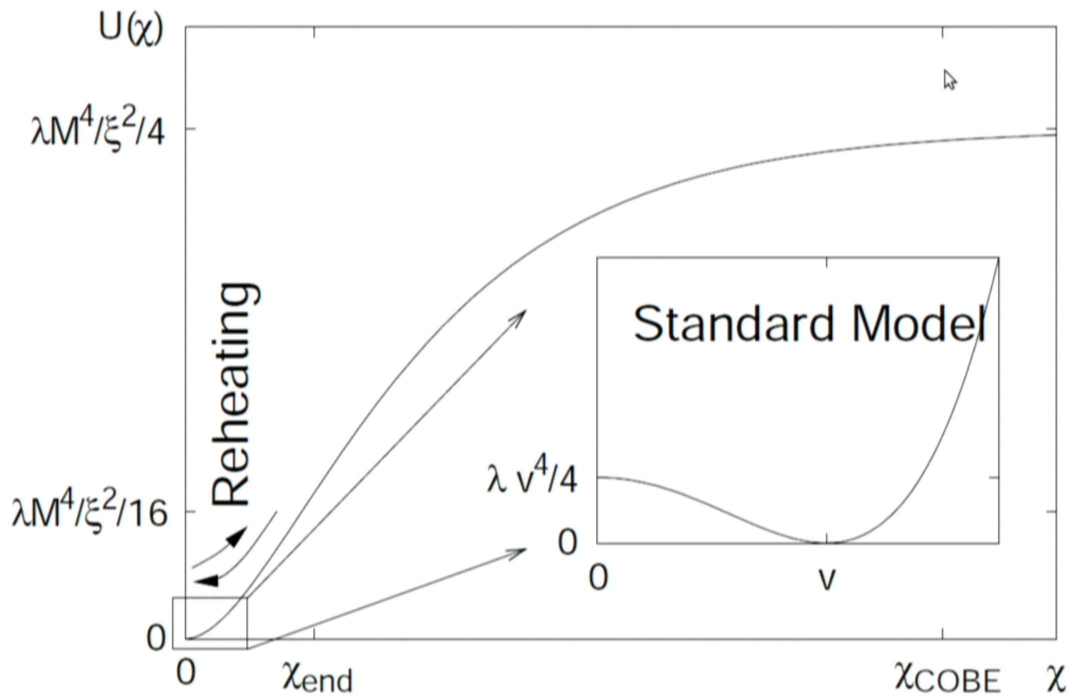
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \tilde{h} \partial^\mu \tilde{h}}{2} - \frac{1}{\Omega(\tilde{h})^4}\frac{\lambda}{4} h(\tilde{h})^4 \right\}$$

Potential:

$$U(\tilde{h}) = \begin{cases} \frac{\lambda}{4}\tilde{h}^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\tilde{h}}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



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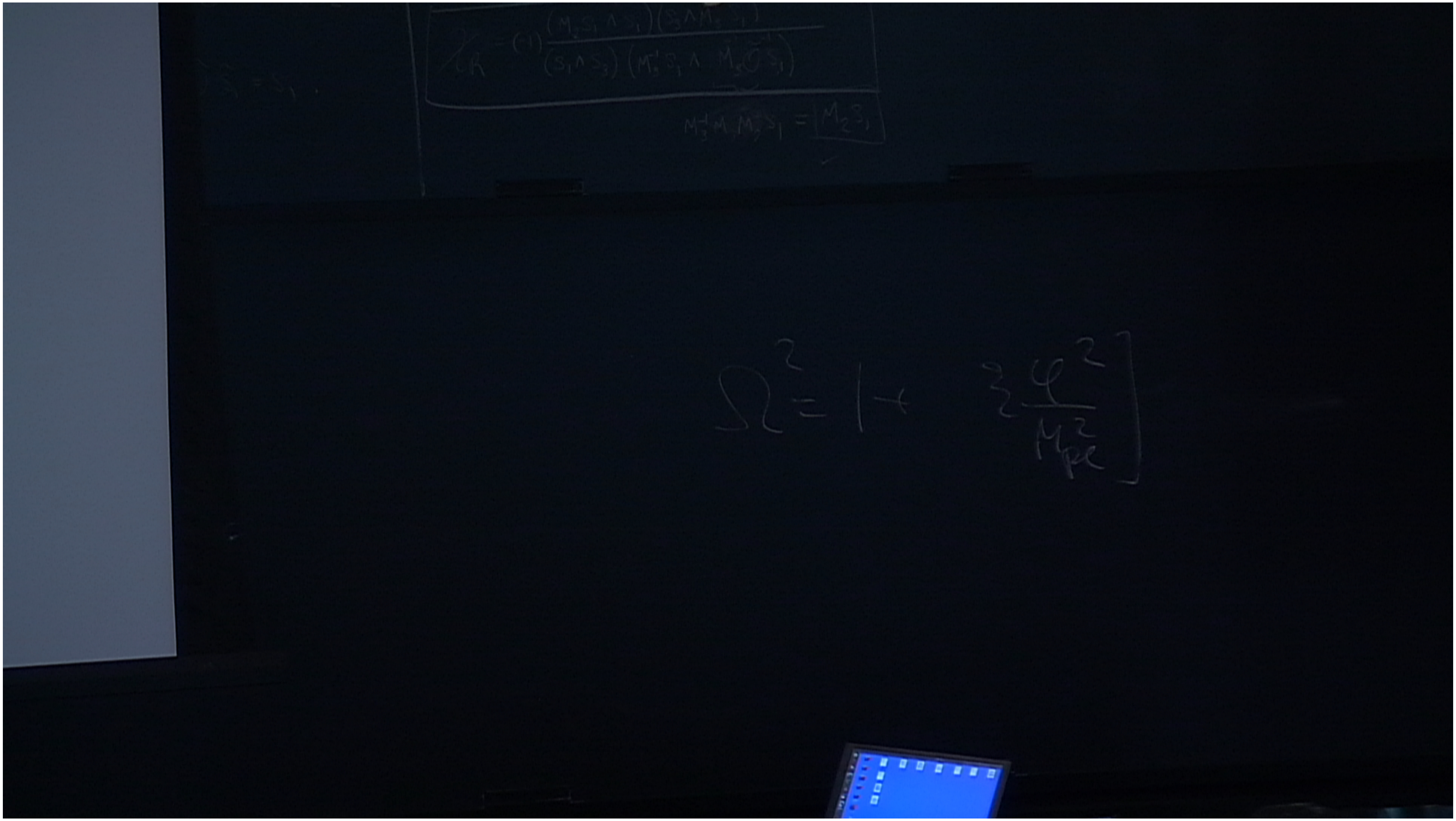
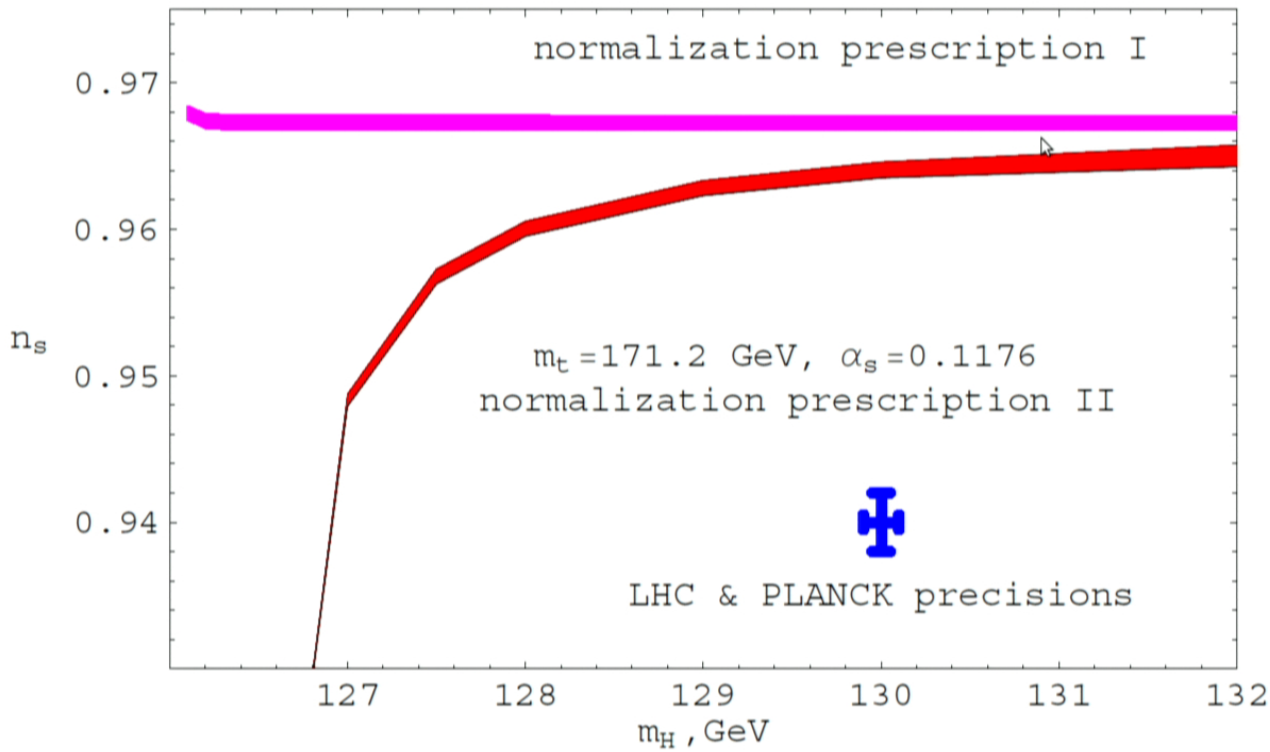


Experimental precision



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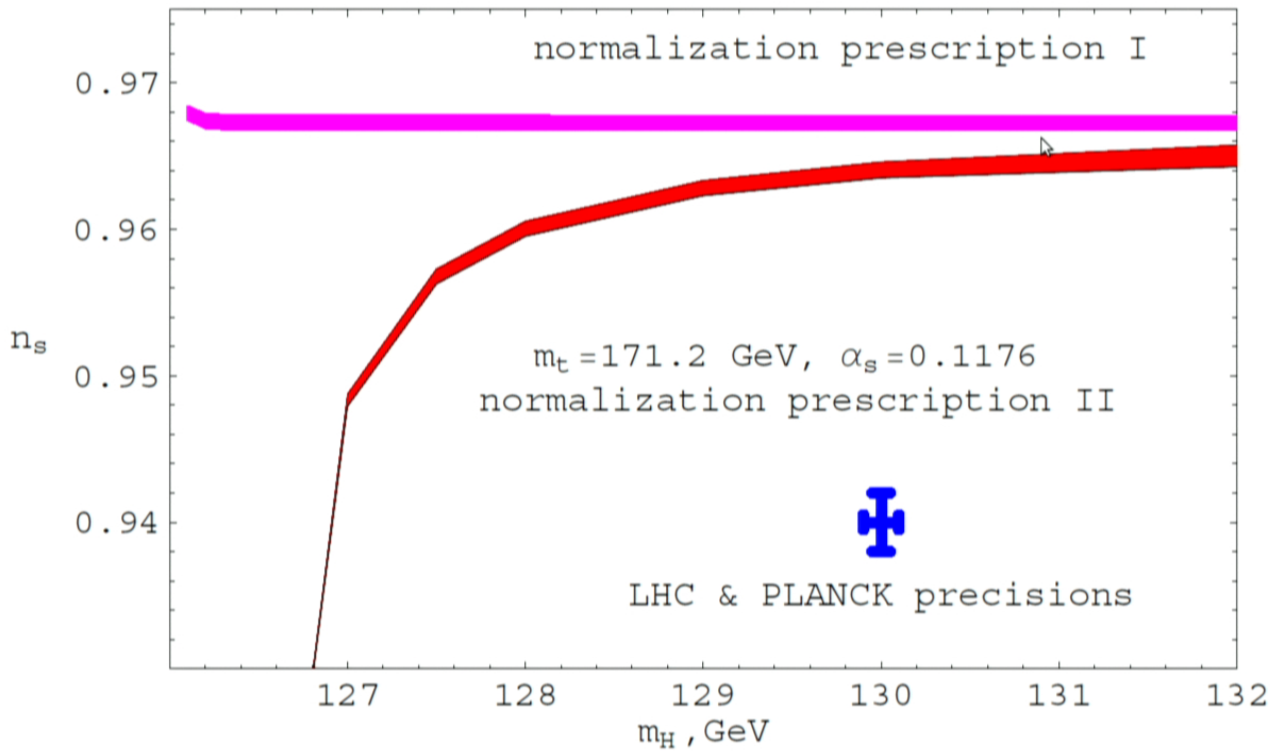

Higgs mass from inflation

Previous consideration tells nothing about the Higgs mass: change λ as $\propto \xi^2$ - no modifications!

However: λ is not a constant, it depends on the energy. Typical scale at inflation $\sim M_P/\sqrt{\xi}$.

Therefore, SM must be a valid quantum field theory up to the Inflation (or, to be on safe side, up to the Planck scale).

$$m_{\min} < m_H < m_{\max}$$

$$m_{\min} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 0.6]\ \mathrm{GeV}$$

$$m_{\max} = [173.5 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.118}{0.002} \times 0.1]\ \mathrm{GeV}$$

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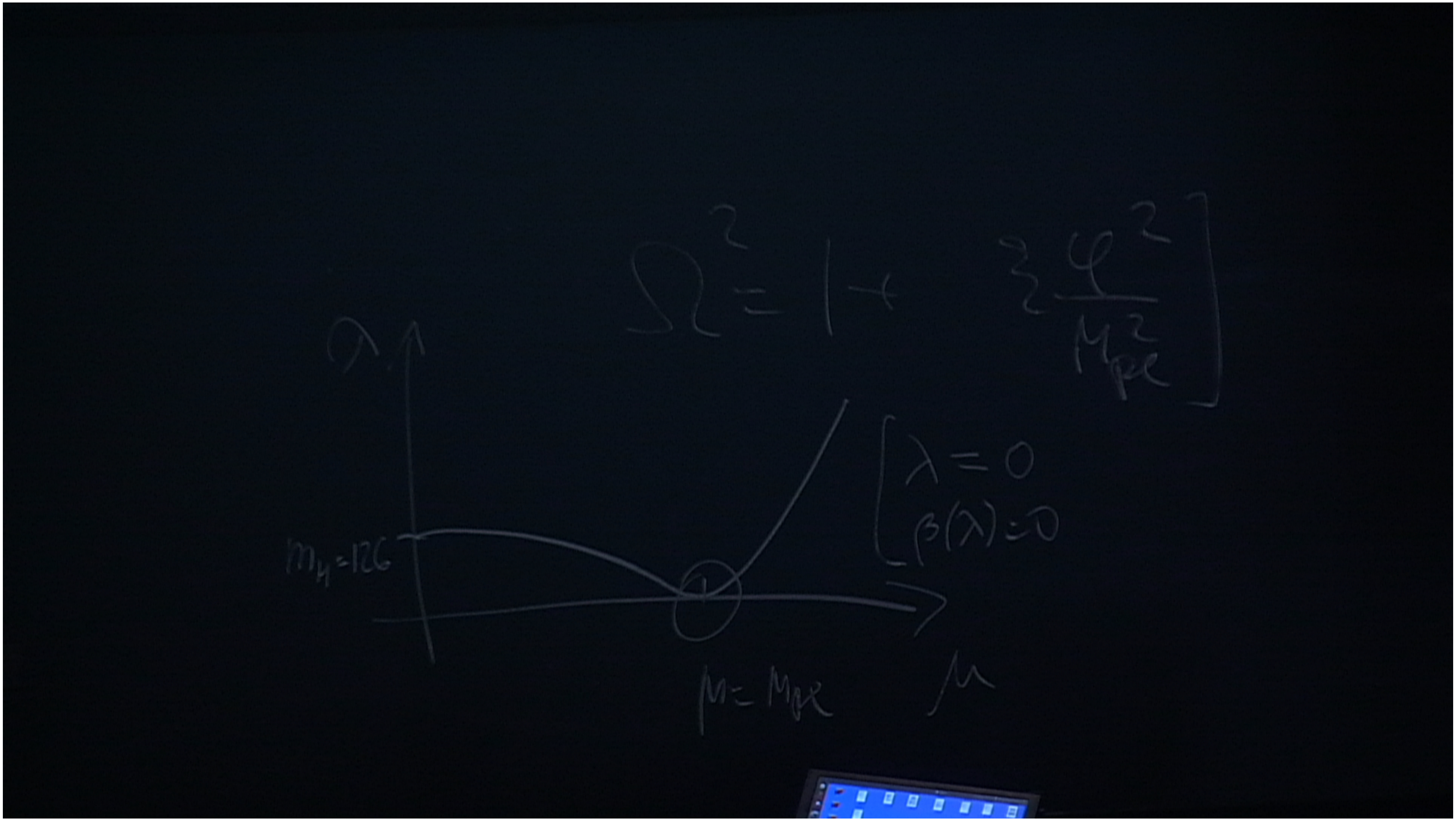
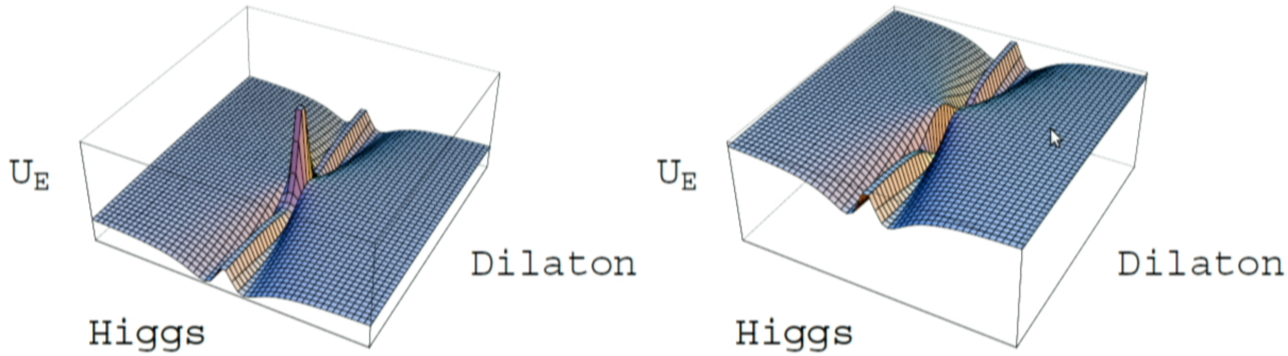


$\Omega^2 = 1 + \xi \frac{\varphi^2}{M_{pl}^2}$
λ
$m_H = 126$
$\lambda = 0$
$\beta(\lambda) = 0$
$\mu = M_{pl}$
μ

Dark energy



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Dark energy

Late time evolution of dilaton ρ along the valley, related to χ as

$$\chi = M_P \exp\left(\frac{\gamma\rho}{4M_P}\right), \quad \gamma = \frac{4}{\sqrt{6+\frac{1}{\xi_\chi}}} .$$

Potential: Wetterich; Ratra, Peebles

$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) .$$

From observed equation of state: $0 < \xi_\chi < 0.09$

Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.

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$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) .$$

From observed equation of state: $0 < \xi_\chi < 0.09$

Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.

Higgs-dilaton cosmology: Strategy

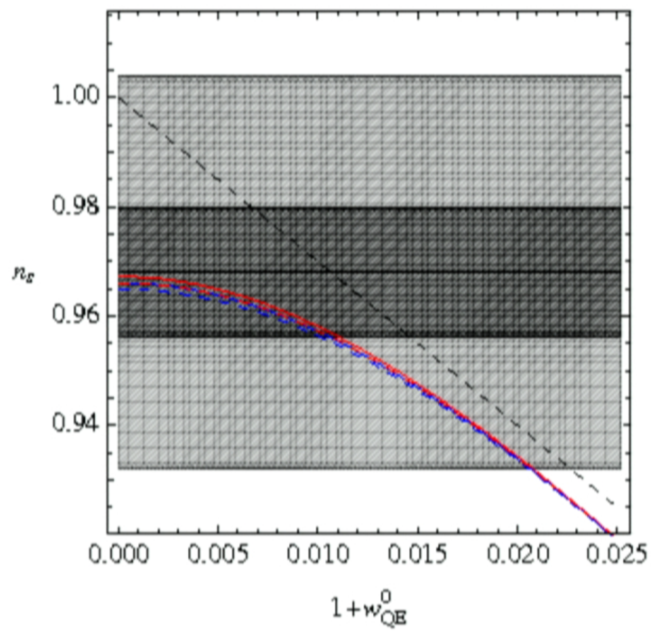
Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern

Both fields together:

- Take arbitrary initial conditions for the Higgs and the dilaton
- Find the region on the $\{\chi, h\}$ plane that lead to inflation
- Find the region on the $\{\chi, h\}$ plane that lead to exit from inflation
- Find the region on the $\{\chi, h\}$ plane that lead to observed abundance of Dark Energy

Inflation-dark energy relation

Value of n_s is determined by ξ_h and ξ_χ, and equation of state of DE ω by $\xi_\chi \Longrightarrow n_s - \omega$ relation:



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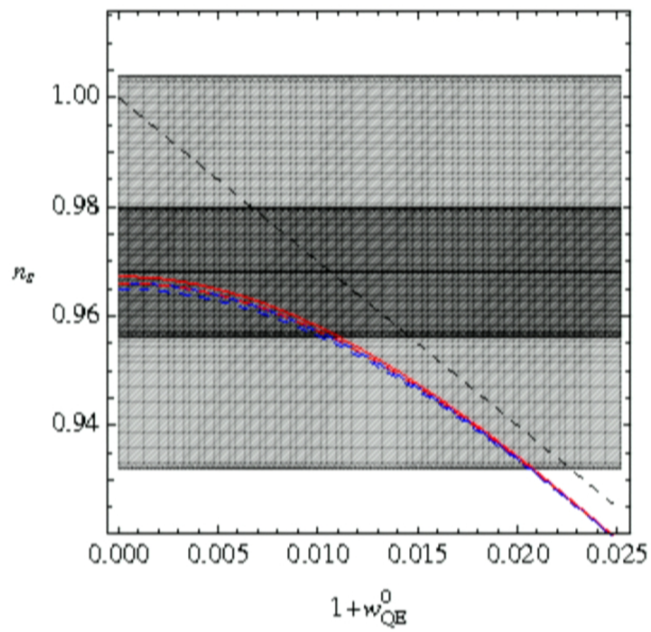

Intermediate summary

- Spontaneously broken scale invariance :
  - All mass scales originate from one and the same source - vev of the massless dilaton
  - Zero cosmological constant – $\beta = 0$ – existence of particles
  - Scale invariance naturally leads to flat potentials and thus to cosmological inflation
- TDiff or Unimodular gravity:
  - New parameter – strength of a particular potential for the dilaton
  - Dynamical Dark Energy

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Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

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Quantum scale invariance

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$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\ a} \ ,$$

Sidney Coleman: "For scale invariance,..., the situation is hopeless; any cutoff procedure necessarily involves a large mass, and a large mass necessarily breaks scale invariance in a large way."

Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

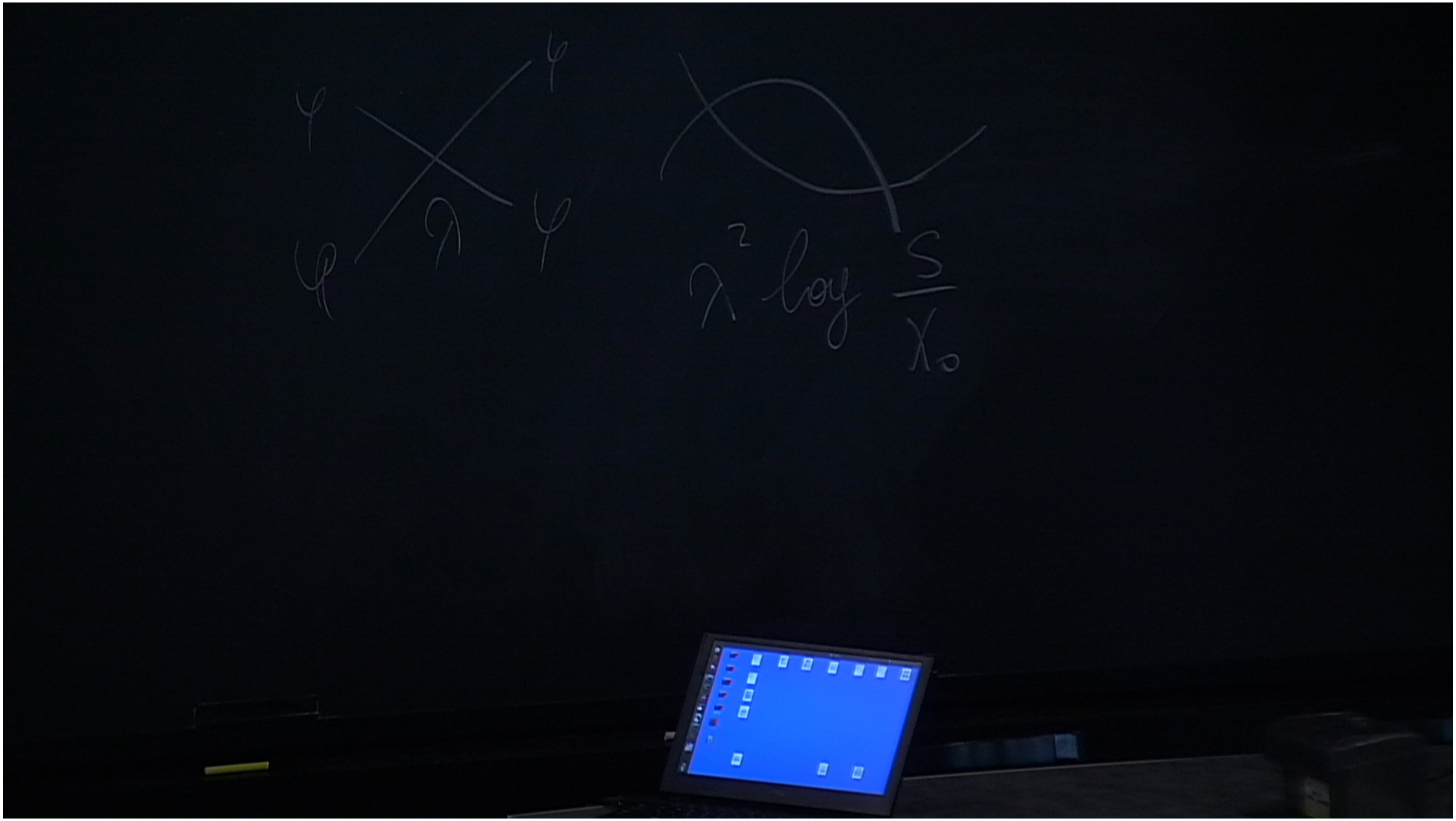
Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

μ is a dimensionfull parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$


φ
φ
φ
φ
λ
λ² log S/χ₀

Origin of Λ_{QCD}

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \text{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \ .$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

Quantum effective action is scale invariant in all orders of perturbation theory!!!

Problems

- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is "no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory

Consequences

- The dilaton is massless in all orders of perturbation theory

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- The dilaton is massless in all orders of perturbation theory
- Since it is a Goldstone boson of spontaneously broken symmetry it has only derivative couplings to matter (inclusion of gravity is essential!)

Dilaton as a part of the metric

Previous discussion - ad hoc introduction of scalar field χ. It is massless, as is the graviton. Can it come from gravity?

Yes - it automatically appears in scale-invariant TDiff gravity as a part of the metric!

Consider arbitrary metric $g_{\mu\nu}$ (no constraints). Determinant g of $g_{\mu\nu}$ is TDiff invariant. Generic scale-invariant action for scalar field and gravity:

$$\mathcal{S} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}\phi^2 f(-g) R - \frac{1}{2}\phi^2 \mathcal{G}_{gg}(-g)(\partial g)^2$$

$$-\frac{1}{2}\mathcal{G}_{\phi\phi}(-g)(\partial\phi)^2 + \mathcal{G}_{g\phi}(-g)\phi\,\partial g \cdot \partial\phi - \phi^4 v(-g)\Big] .$$

Conclusions

- Scale-invariance and TDiff gravity lead to:
  - Unique source for all mass scales.
  - Natural inflation
  - Higgs mass is stable against radiative corrections (at least if dimensional regularization is used) - no SUSY, or technicolor, or little Higgs, or large extra dimensions are needed
  - Cosmological constant may be zero due to quantum scale-invariance and the requirement of existence of particles
  - Even if $\Lambda = 0$, Dark Energy is present
  - The massless sector of the theory contains dilaton, which has only derivative couplings to matter and can be a part of the metric.