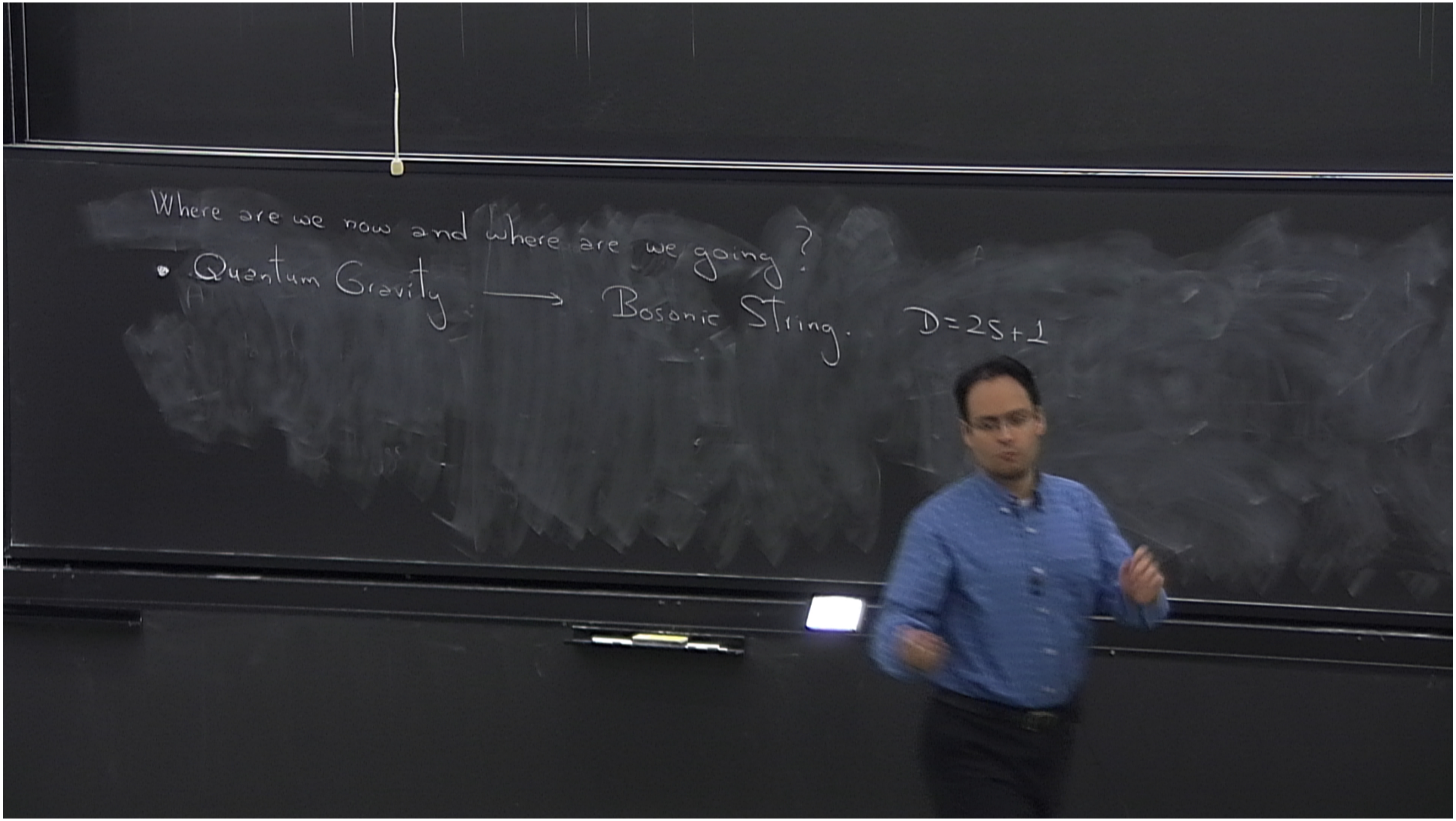
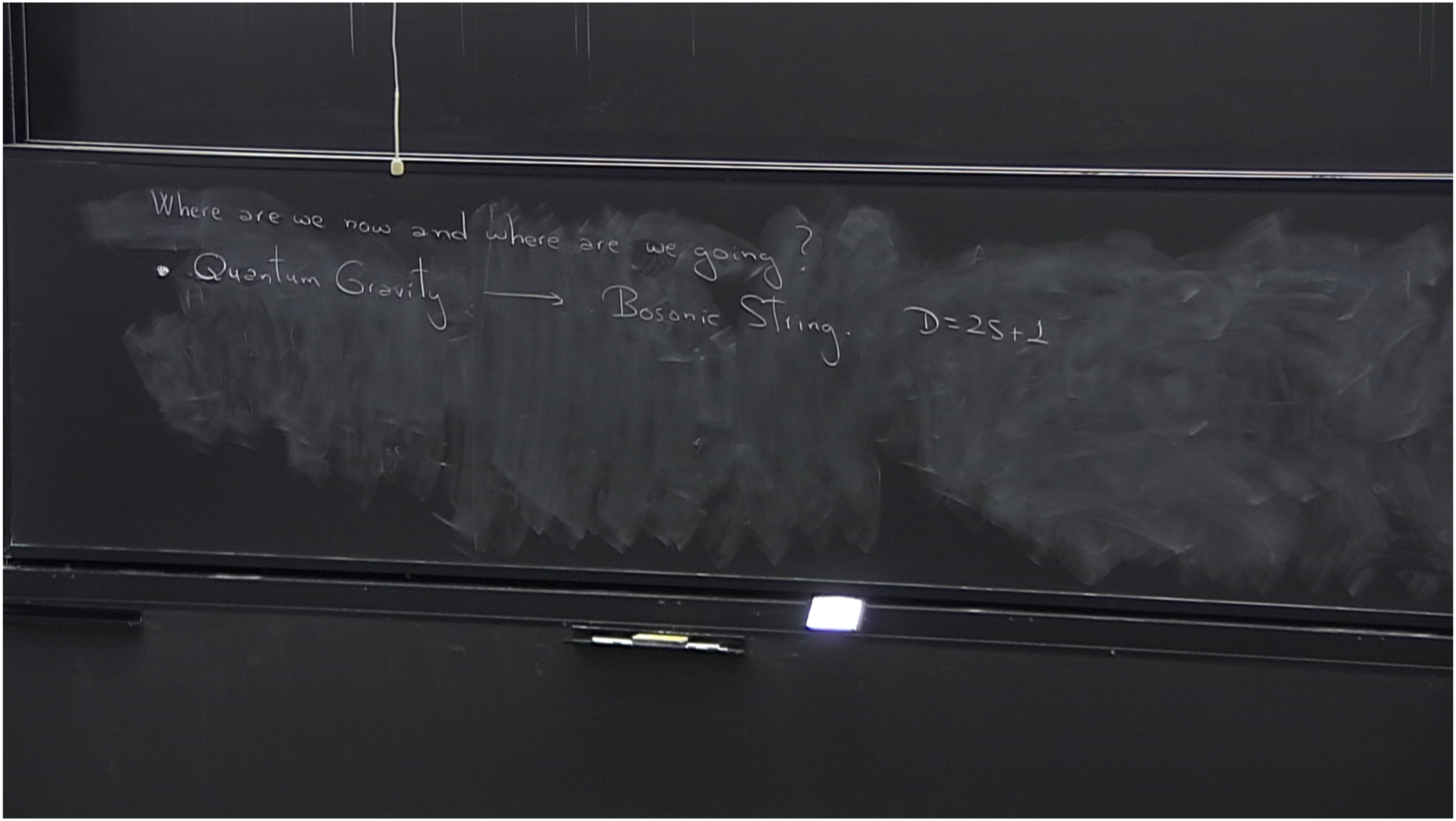
Title: String Theory (Review) - Lecture 11

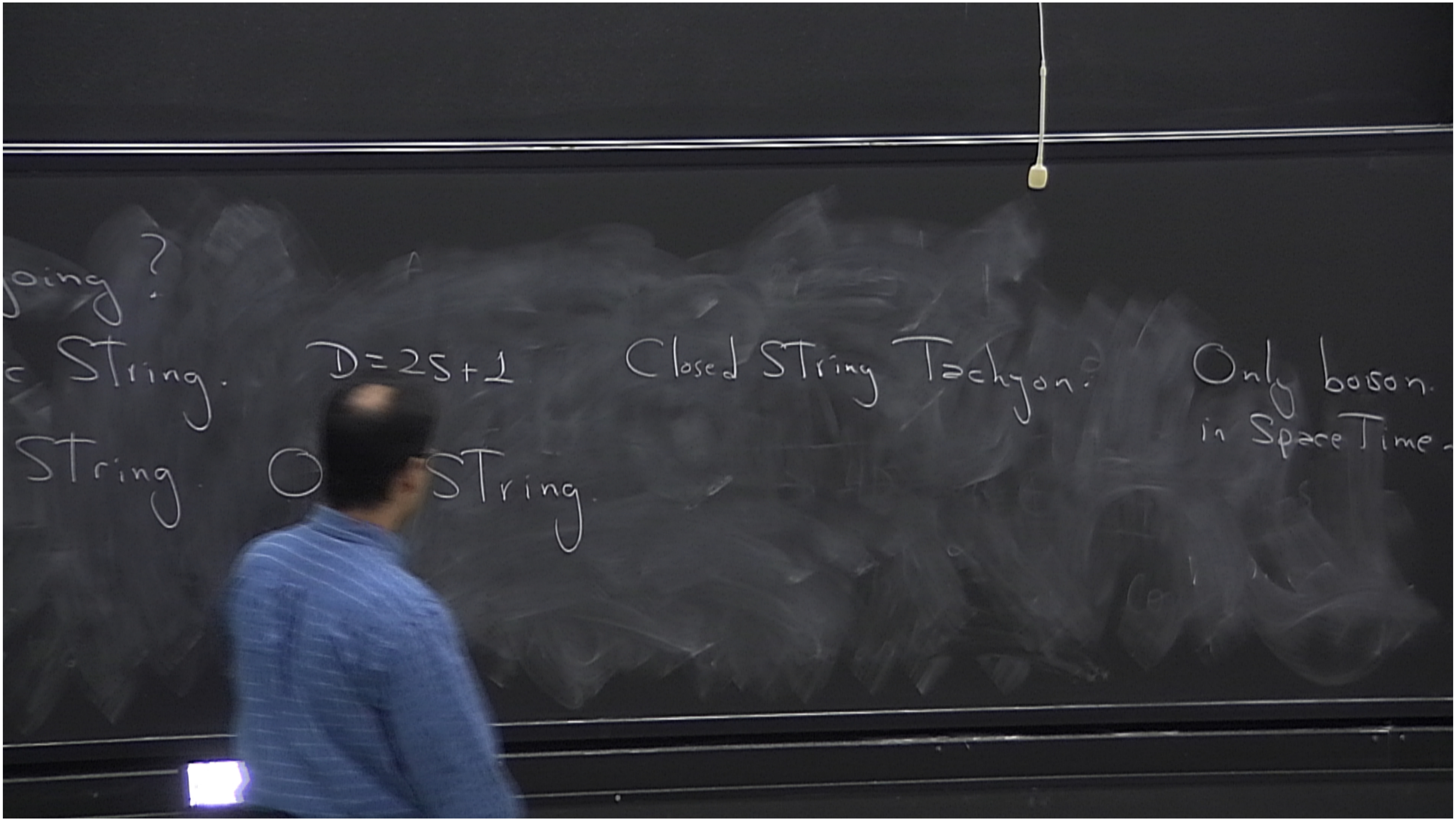
Date: Feb 28, 2012 11:30 AM

URL: http://www.pirsa.org/12020062

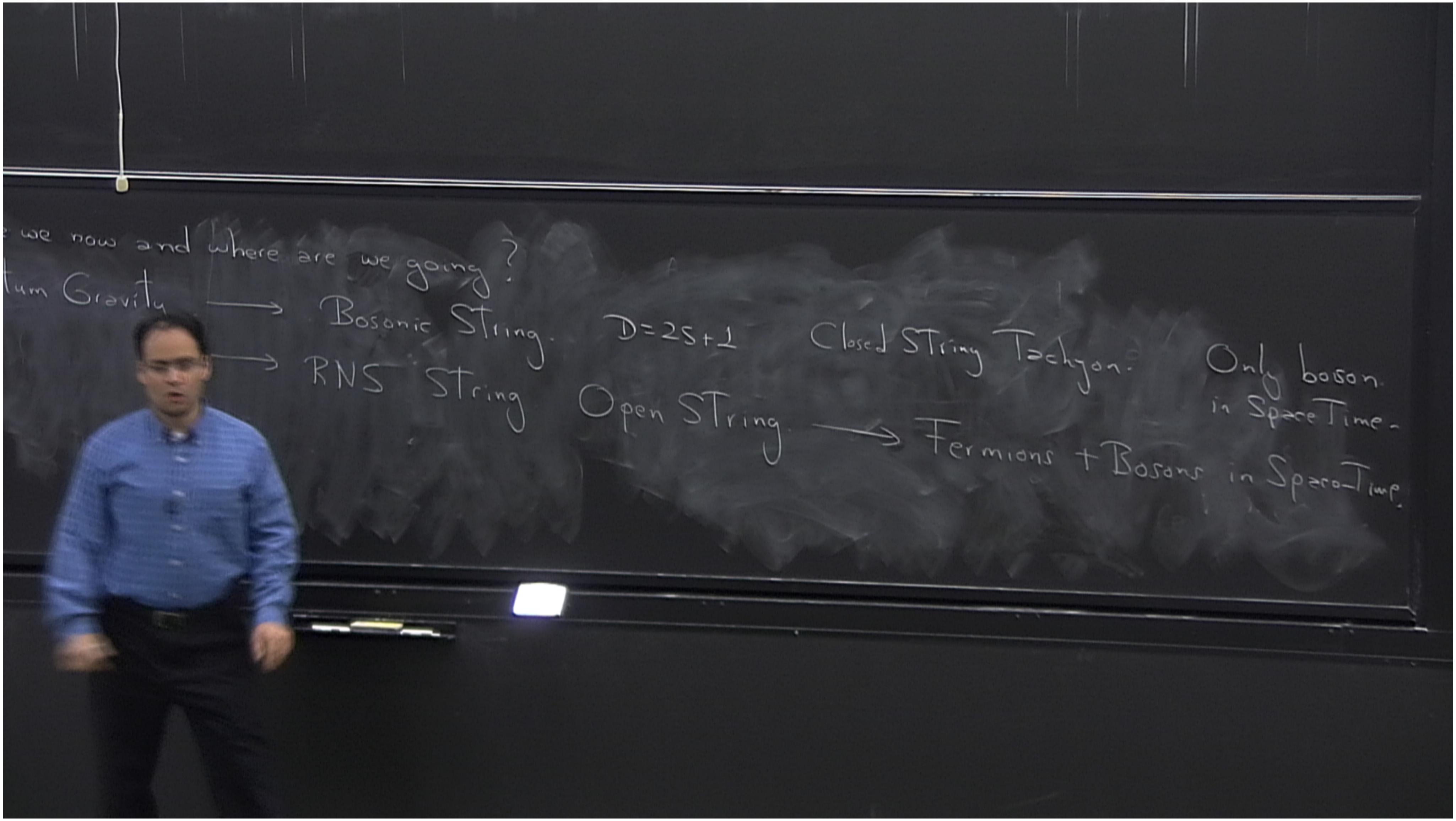
Abstract:

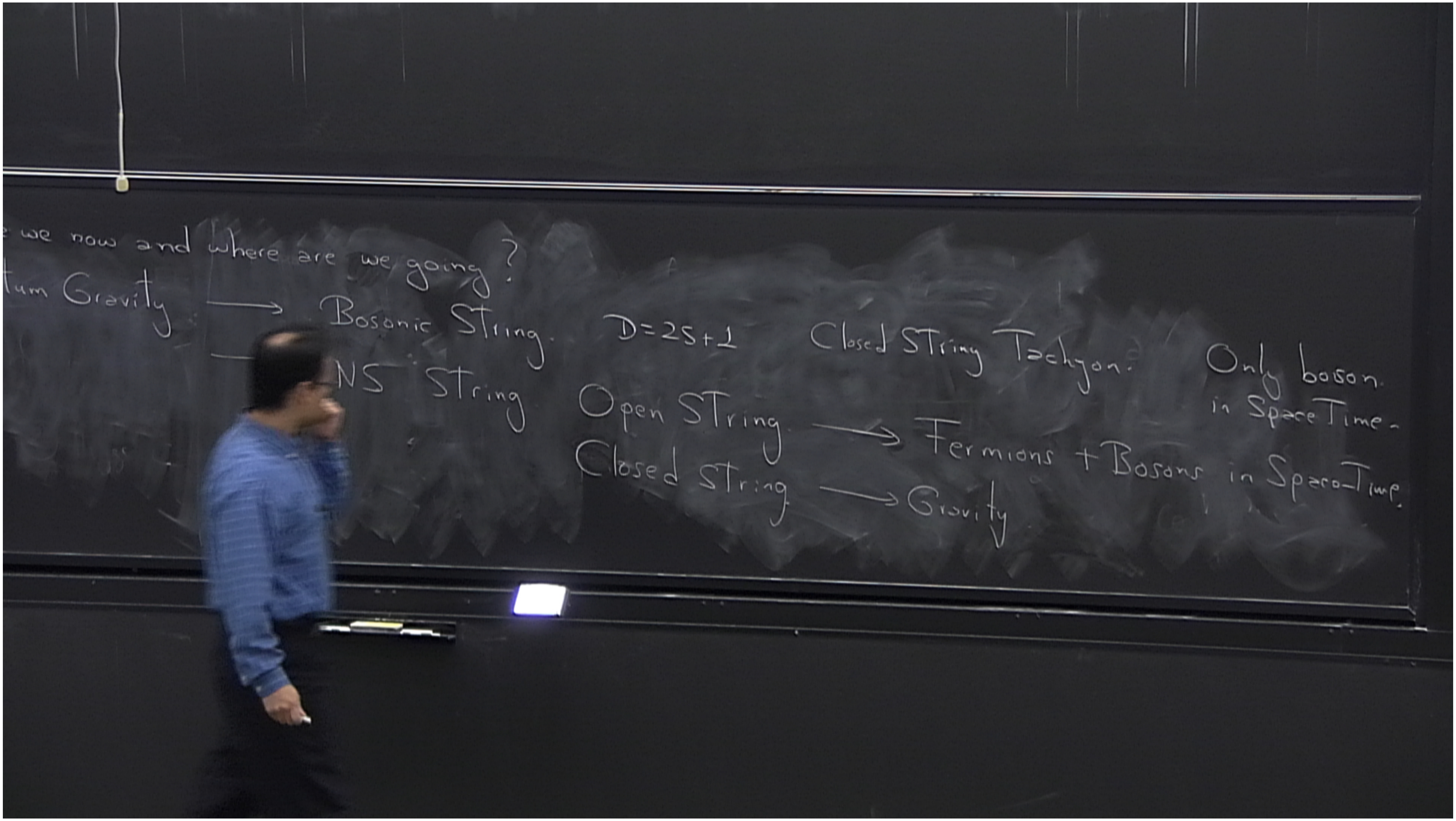

Where are we now and where are we going?
• Quantum Gravity → Bosonic String. D=25+1

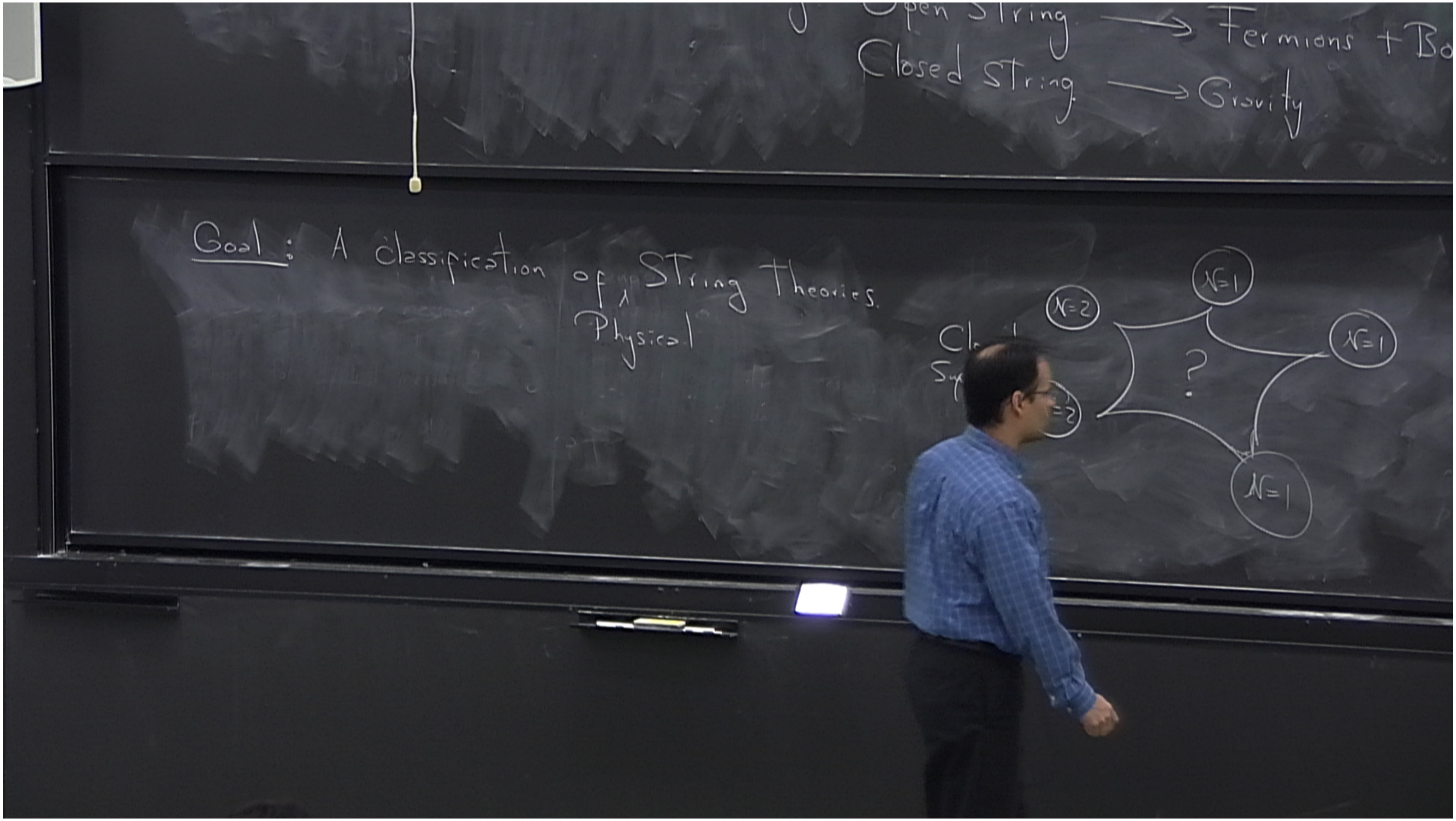

Where are we now and where are we going?
• Quantum Gravity → Bosonic String. D=25+1

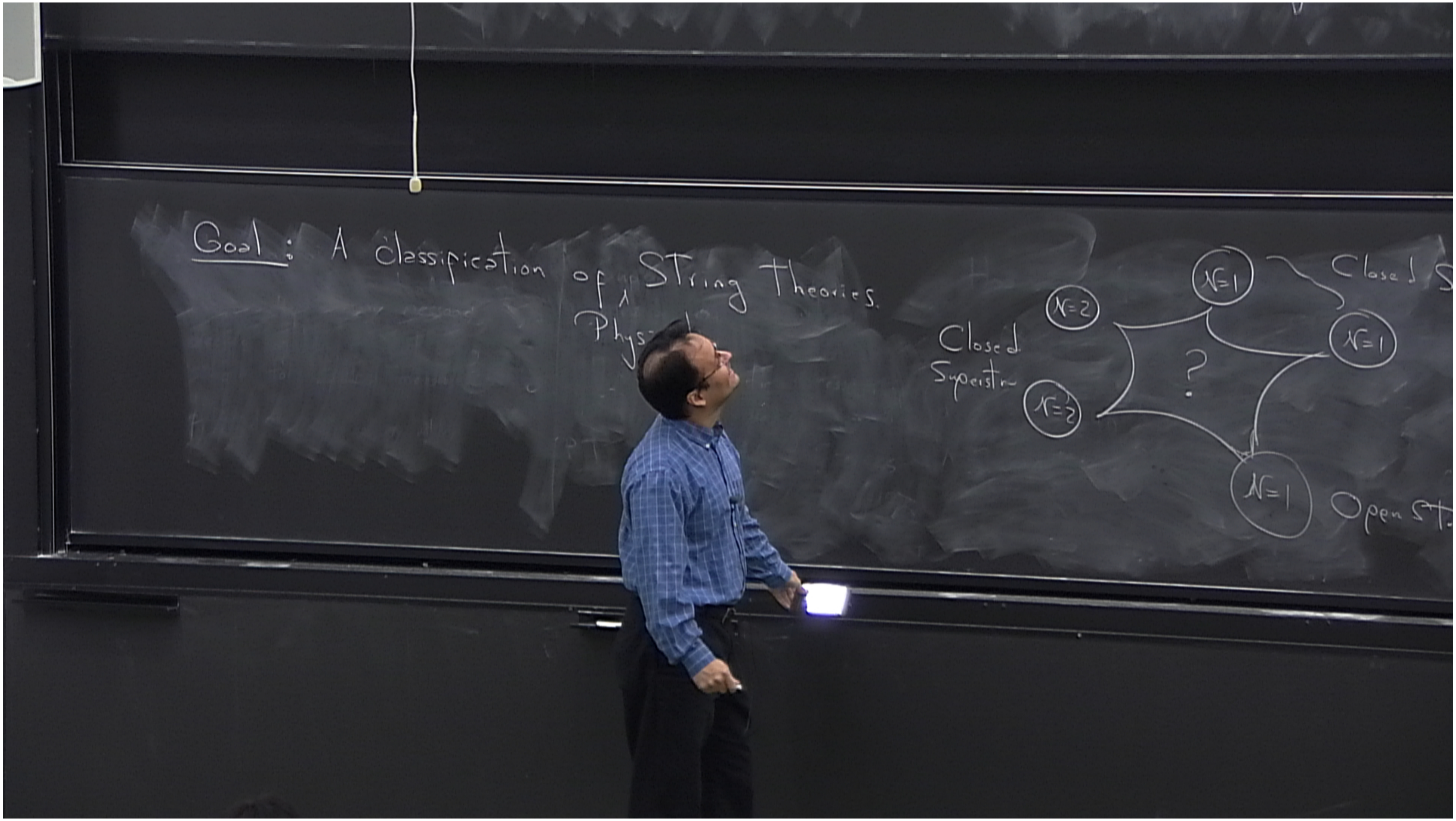


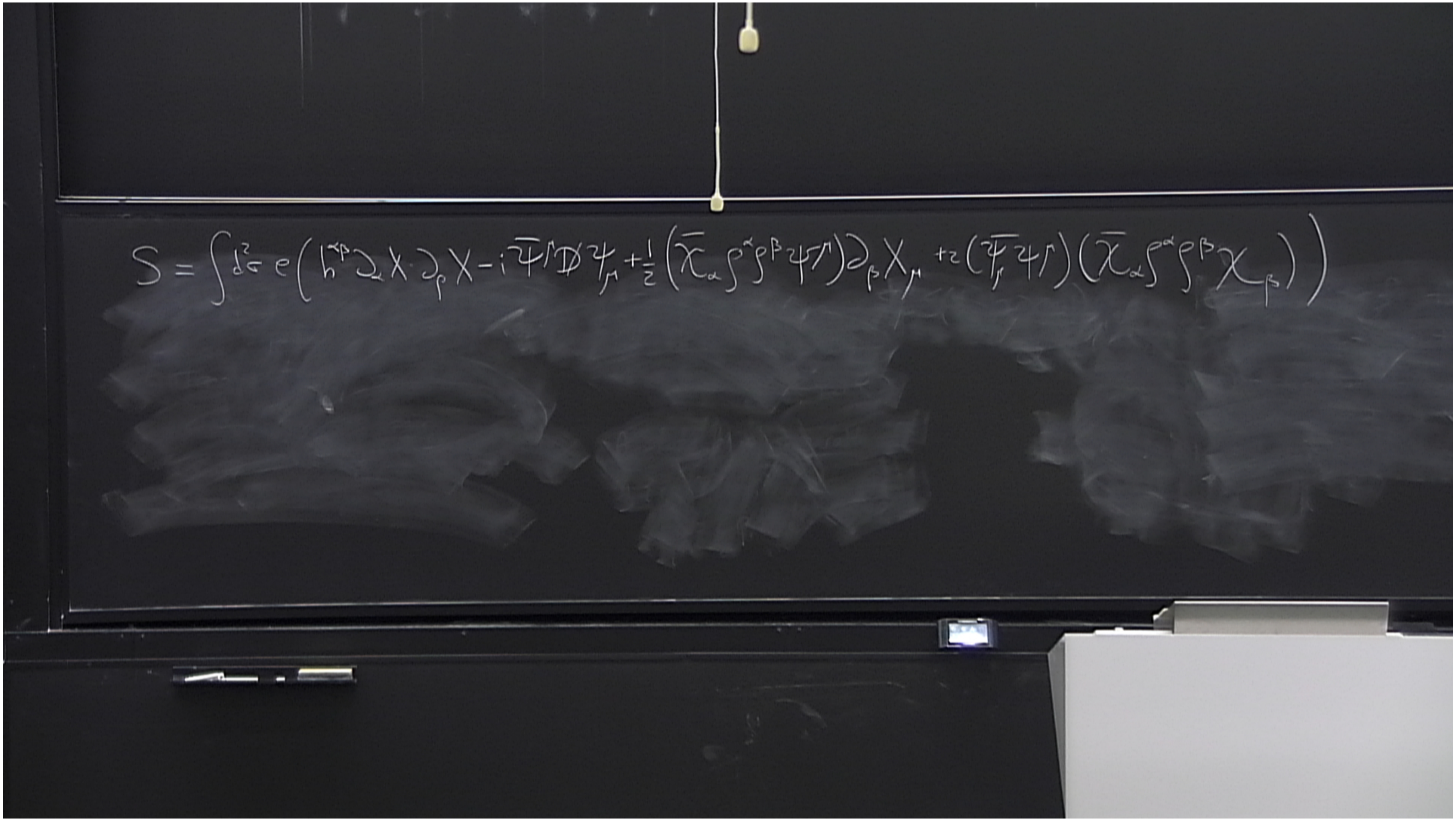
String.
D=25+1
Closed String Tachyon.
Only boson.
in SpaceTime
String.
String.

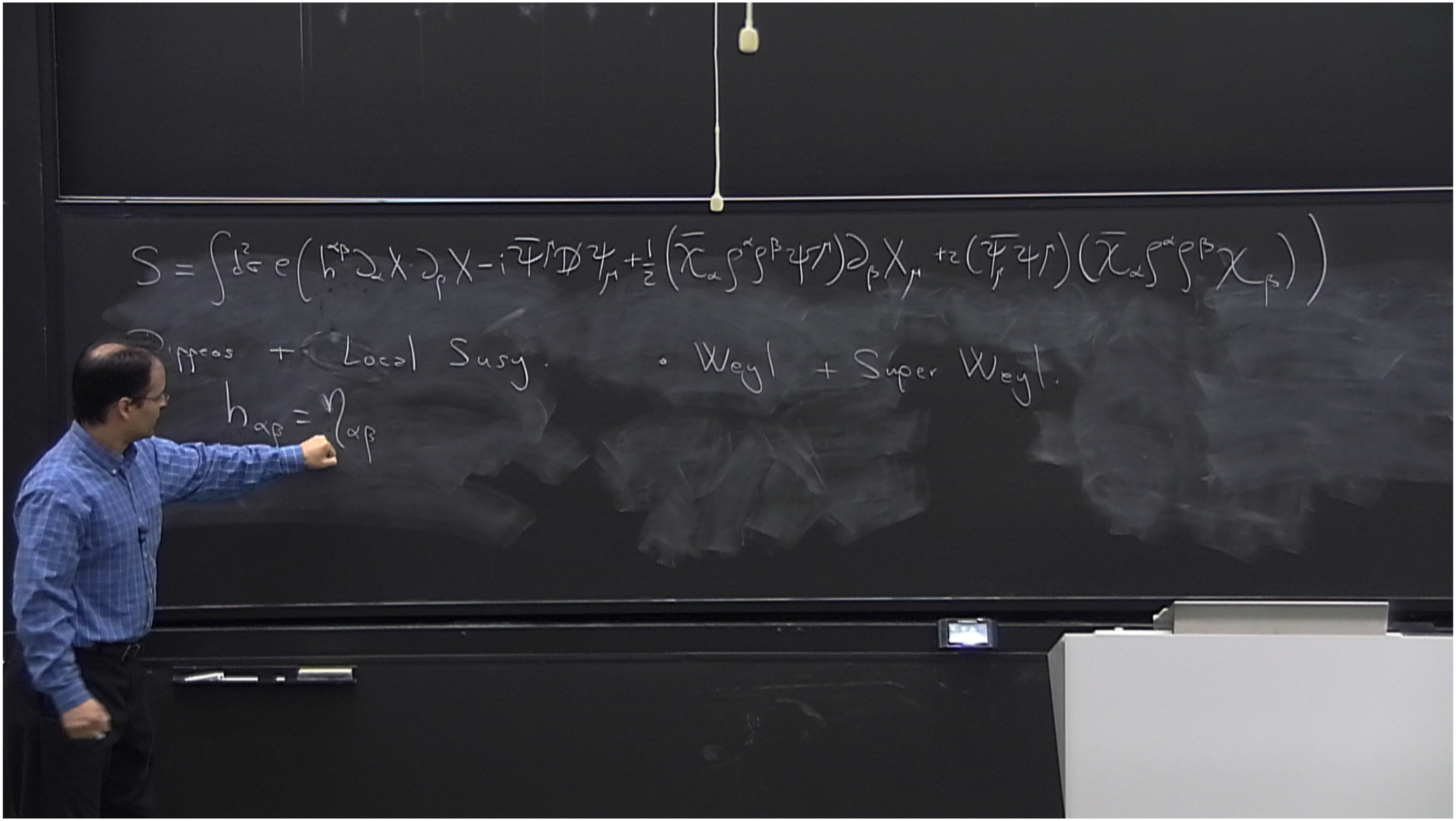

we now and where are we going?
tum Gravity → Bosonic String. D=25+1 Closed String Tachyon! Only boson in SpaceTime.
→ RNS String Open String. → Fermions + Bosons in Space-Time.

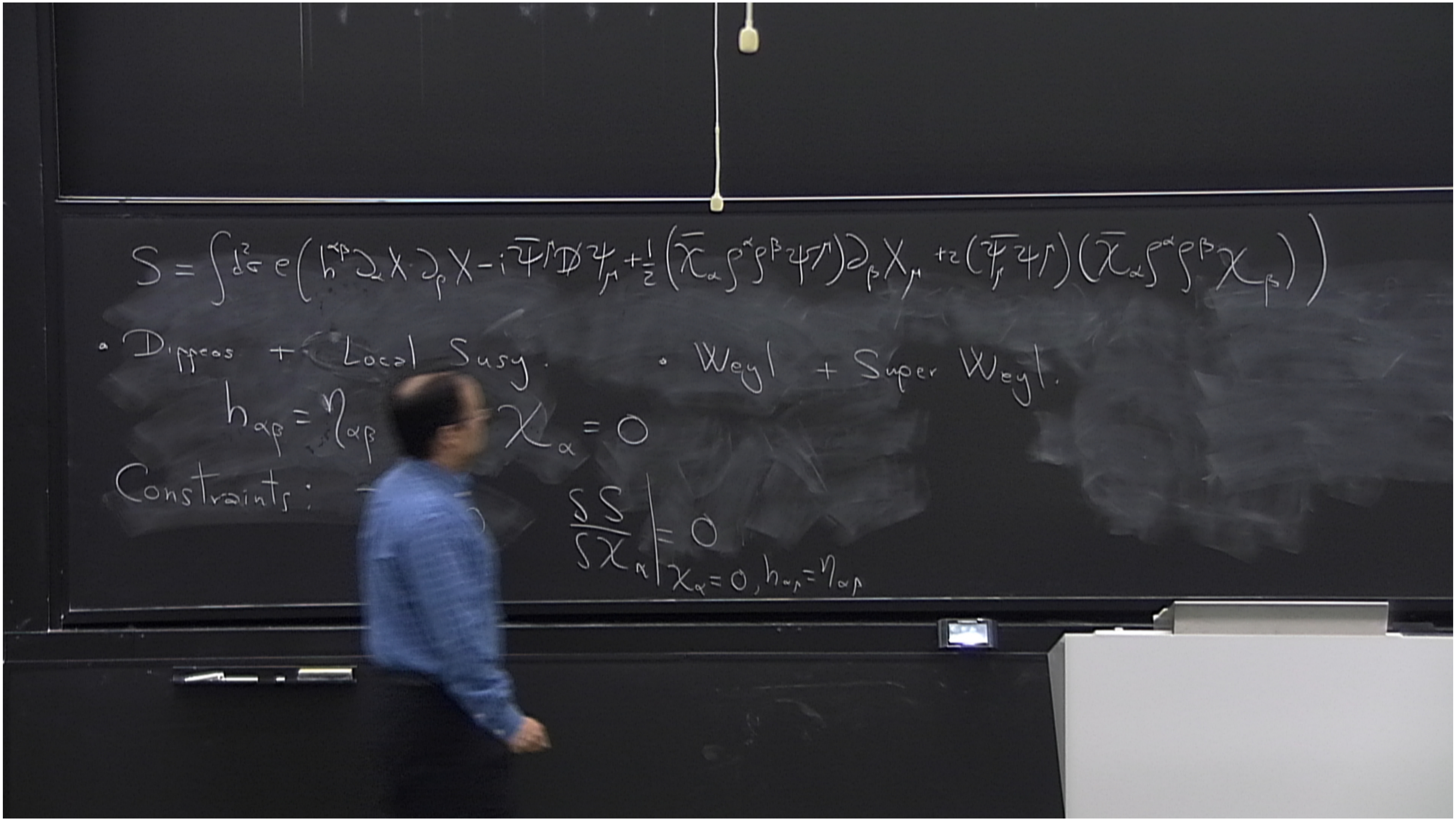

e we now and where are we going?
tum Gravity
Bosonic String.
D=25+1
Closed String Tachyon.
Only boson.
in SpaceTime.
NS String
Open String
Fermions + Bosons in Space-Time.
Closed String
Gravity


Open String → Fermions + Bo
Closed String → Gravity
Goal: A classification of Physical String Theories.
N=2
N=1
N=1
N=1
?


Goal: A classification of String theories.
Phys
Closed Superstr
N=2
N=2
N=1
N=1
N=1
?
Closed
Open


$S = \int d^2\sigma\, e\left(h^{\alpha\beta}\partial_\alpha X \cdot \partial_\beta X - i\bar{\psi}^\mu \not{D}\psi_\mu + \frac{1}{2}(\bar{\chi}_\alpha \rho^\alpha \rho^\beta \psi^\mu)\partial_\beta X_\mu + 2(\bar{\psi}_\mu \psi^\mu)(\bar{\chi}_\alpha \rho^\alpha \rho^\beta \chi_\beta)\right)$


$S = \int d^2\sigma \, e \left(h^{\alpha\beta} \partial_\alpha X \cdot \partial_\beta X - i \bar{\psi}^\mu \not{D} \psi_\mu + \frac{1}{2} (\bar{\chi}_\alpha \rho^\beta \rho^\alpha \psi^\mu) \partial_\beta X_\mu + \frac{1}{4} (\bar{\psi}_\mu \psi^\mu)(\bar{\chi}_\alpha \rho^\beta \rho^\alpha \chi_\beta) \right)$
Local Susy.
Weyl + Super Weyl.
$h_{\alpha\beta} = \eta_{\alpha\beta}$


$S = \int d^2\sigma\, e \left(h^{\alpha\beta} \partial_\alpha X \cdot \partial_\beta X - i \bar{\Psi}^\mu \not{D} \Psi_\mu + \frac{1}{2} (\bar{\chi}_\alpha \rho^\alpha \rho^\beta \Psi^\mu) \partial_\beta X_\mu + 2 (\bar{\Psi}_\mu \Psi^\mu)(\bar{\chi}_\alpha \rho^\alpha \rho^\beta \chi_\beta) \right)$
• Diffeos + Local Susy.
• Weyl + Super Weyl.
$h_{\alpha\beta} = \eta_{\alpha\beta}$
$\chi_\alpha = 0$
Constraints:
$\frac{\delta S}{\delta \chi_\alpha}\Big|_{\chi_\alpha = 0,\, h_{\alpha\beta} = \eta_{\alpha\beta}} = 0$

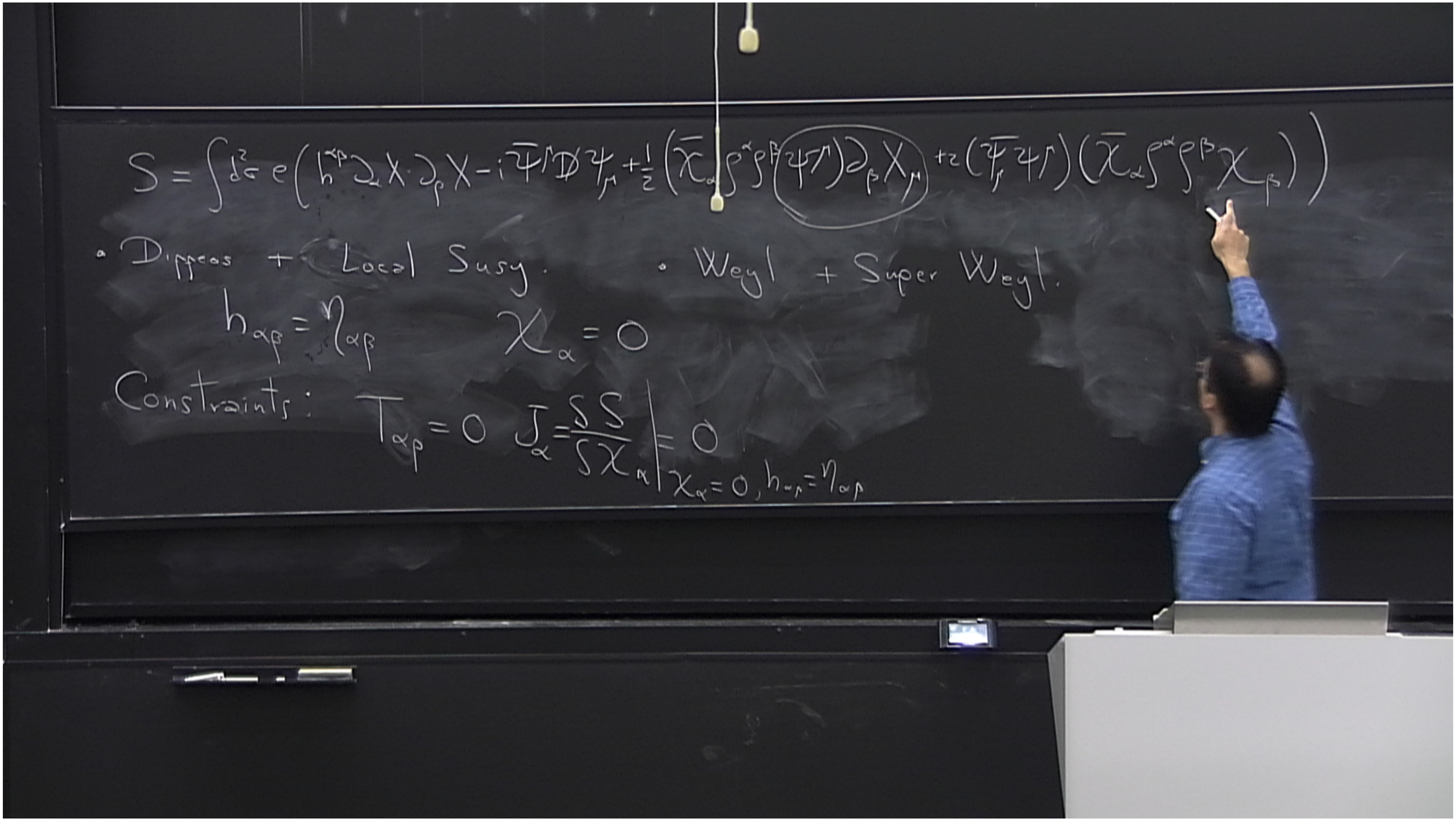
Constraints: $T_{\alpha\beta} = 0$ $\qquad \left.\frac{\delta S}{\delta \chi_\alpha}\right|_{\chi_\alpha = 0,\, h_{\alpha\beta} = \eta_{\alpha\beta}} = 0$

New coord. $\sigma^{\pm} = \tau \pm \sigma$

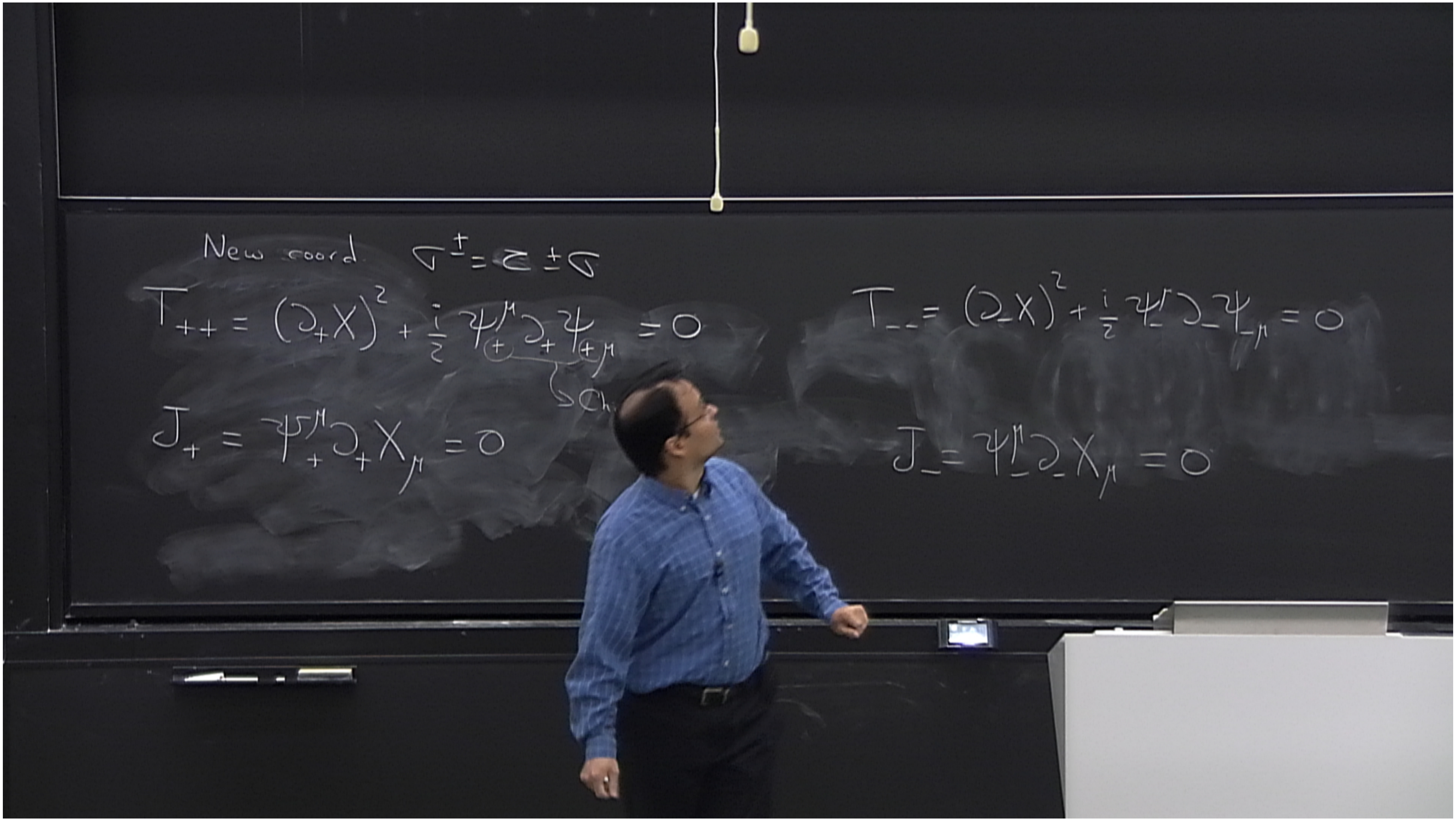
$$T_{++} = (\partial_+ X)^2 + \frac{i}{2}\psi^\mu_+ \partial_+ \psi_{+\mu} = 0$$

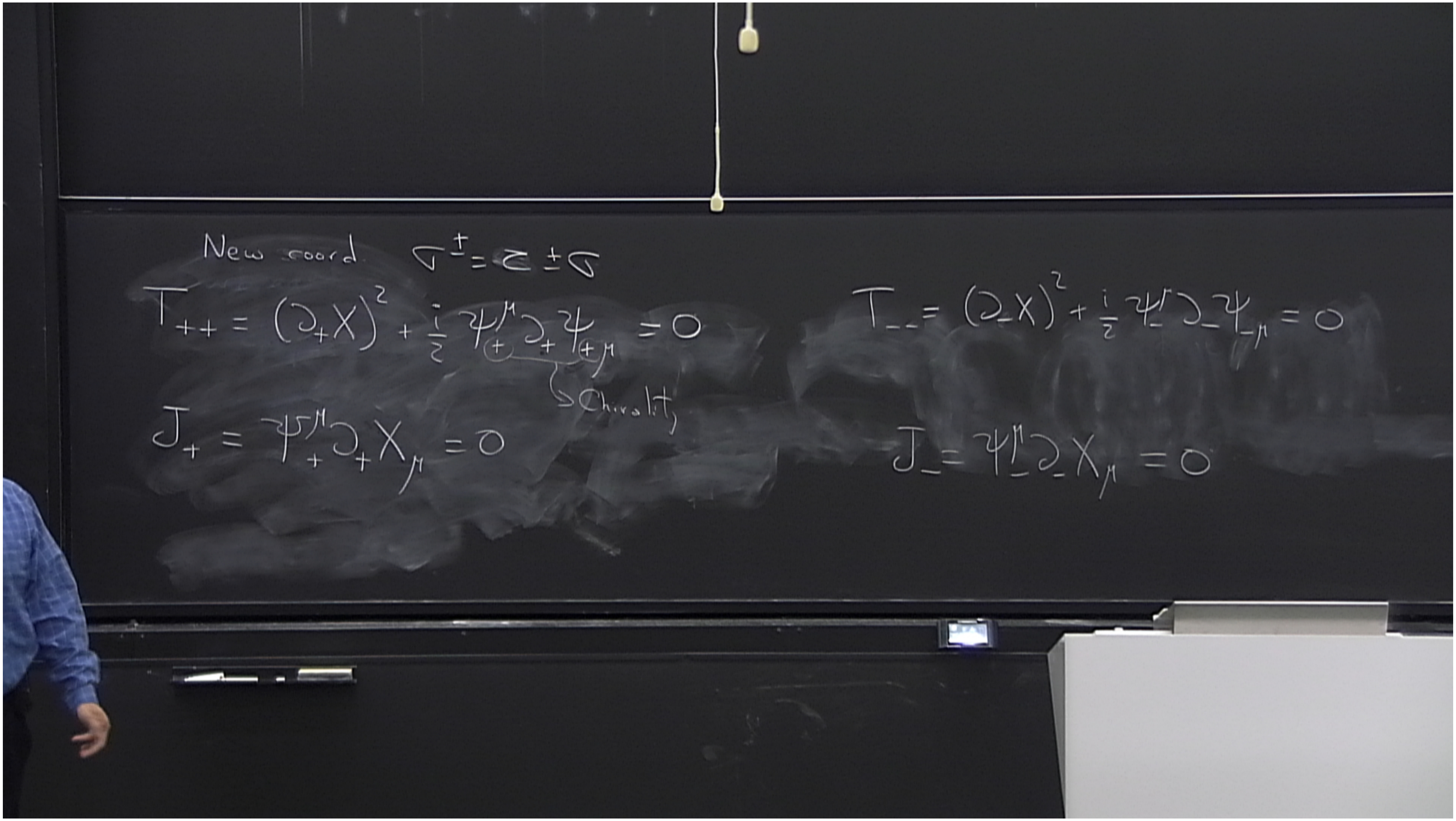
↳ Chirality

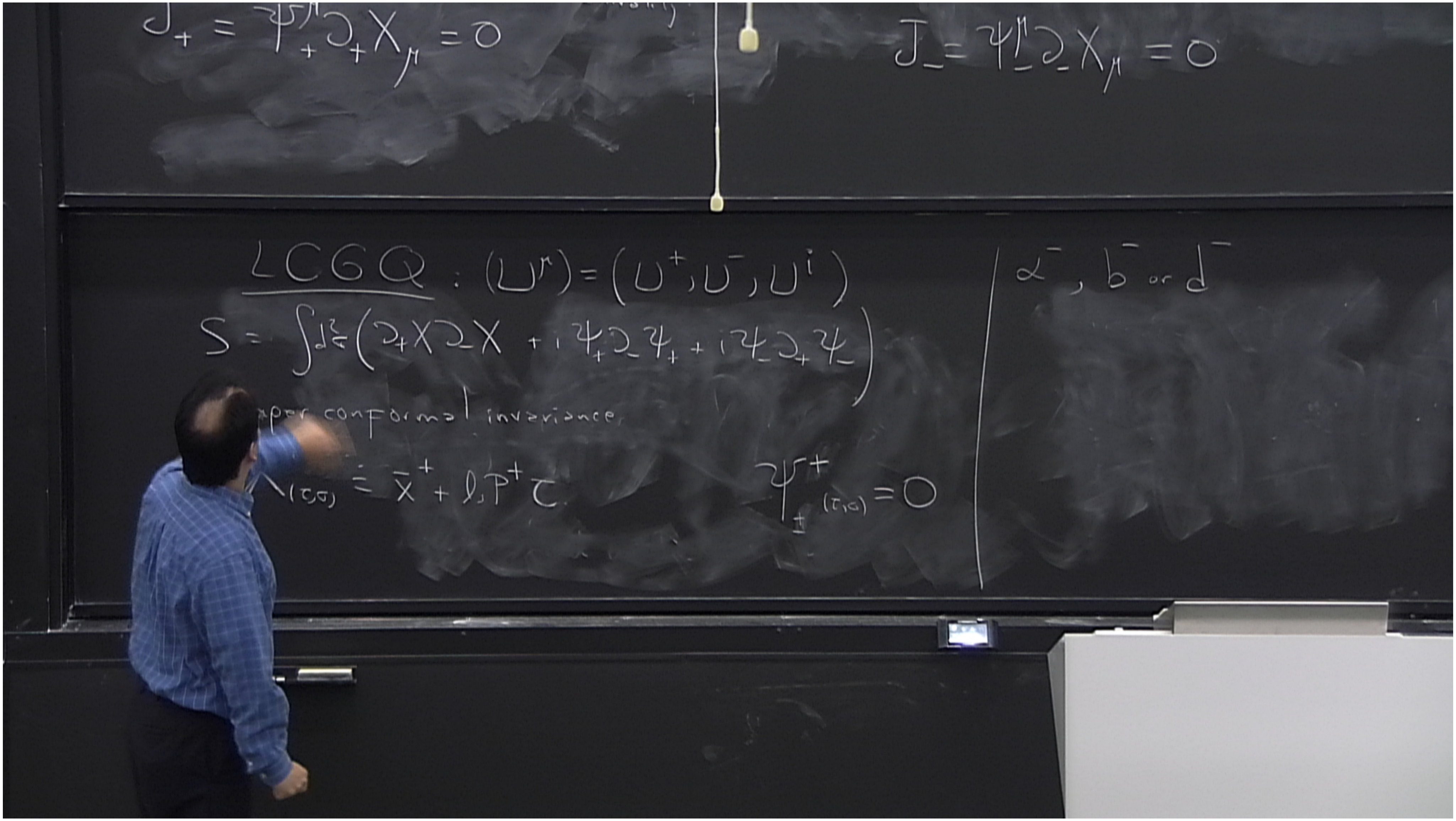
$$T_{--} = (\partial_- X)^2 + \frac{i}{2}\psi^\mu_- \partial_- \psi_{-\mu} = 0$$

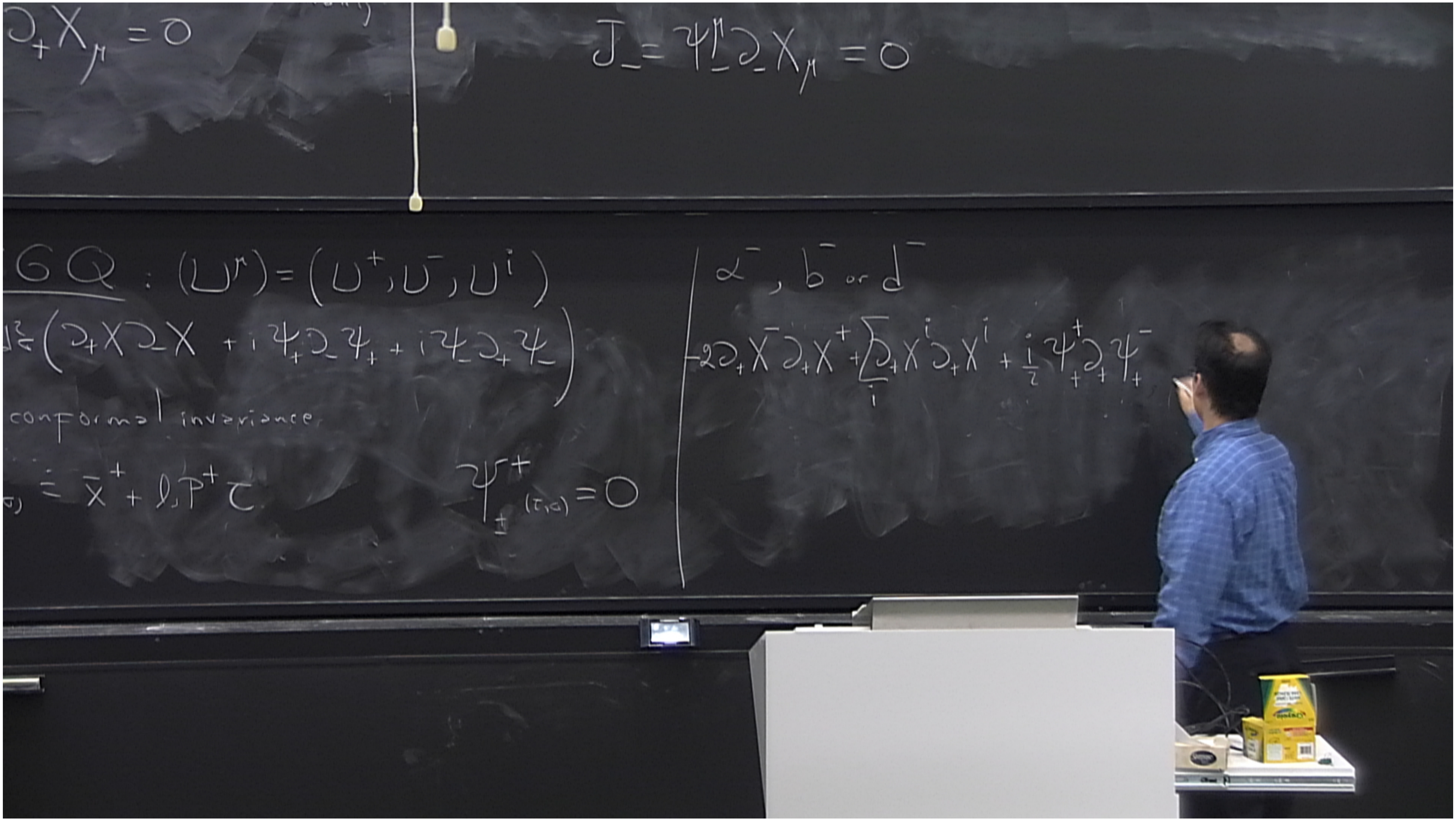


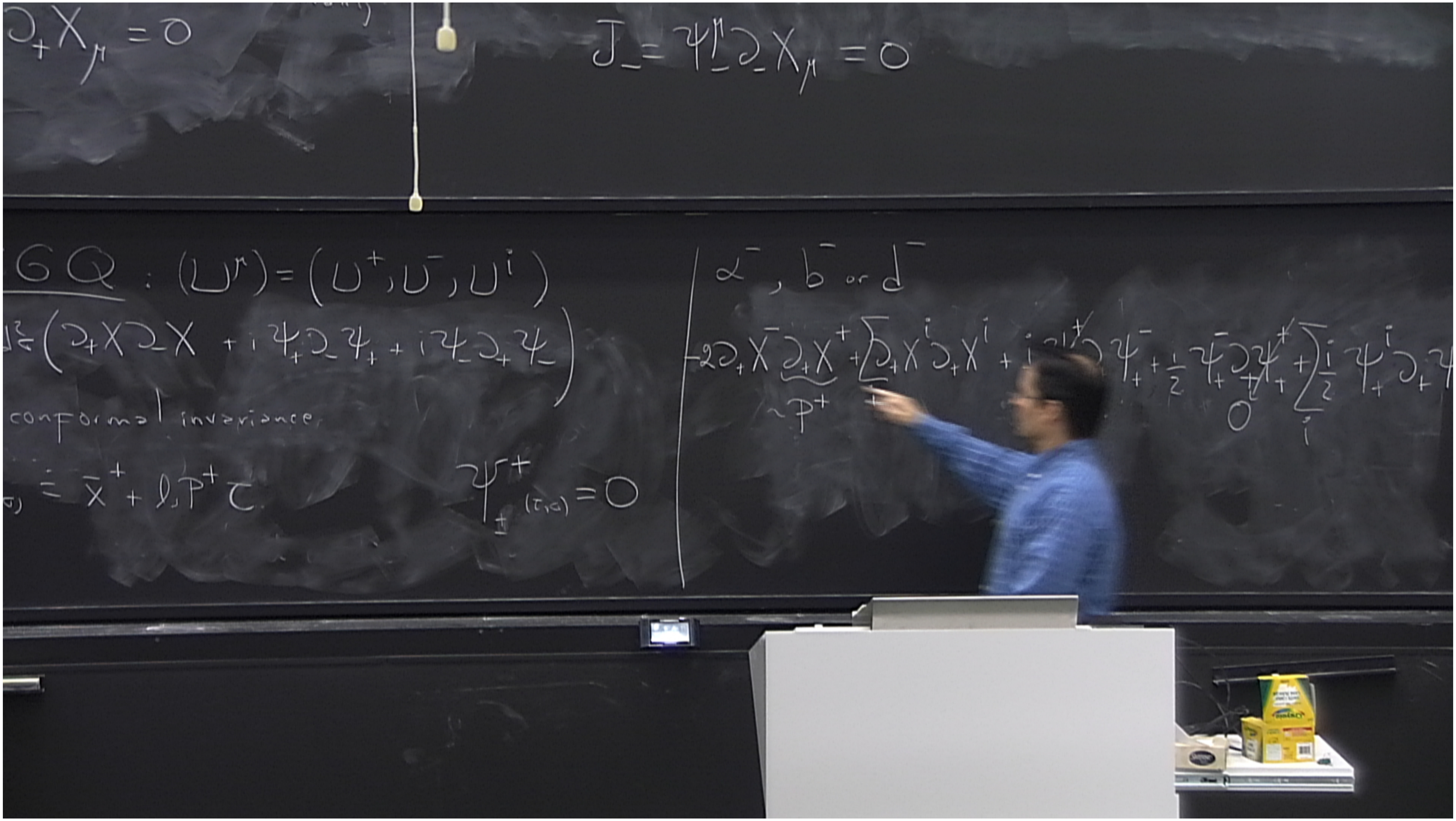
$S = \int d^2\sigma\, e\left(h^{\alpha\beta}\partial_\alpha X\cdot\partial_\beta X - i\bar{\psi}^\mu \not{D}\psi_\mu + \frac{1}{2}(\bar{\chi}_\alpha \rho^\alpha\rho^\beta \psi^\mu)\partial_\beta X_\mu + 2(\bar{\psi}_\mu\psi^\mu)(\bar{\chi}_\alpha\rho^\alpha\rho^\beta\chi_\beta)\right)$
• Diffeos + Local Susy.
• Weyl + Super Weyl.
$h_{\alpha\beta} = \eta_{\alpha\beta}$
$\chi_\alpha = 0$
Constraints: $T_{\alpha\beta} = 0$ $J_\alpha = \frac{\delta S}{\delta \chi_\alpha}\Big|_{\chi_\alpha = 0,\, h_{\alpha\beta} = \eta_{\alpha\beta}} = 0$

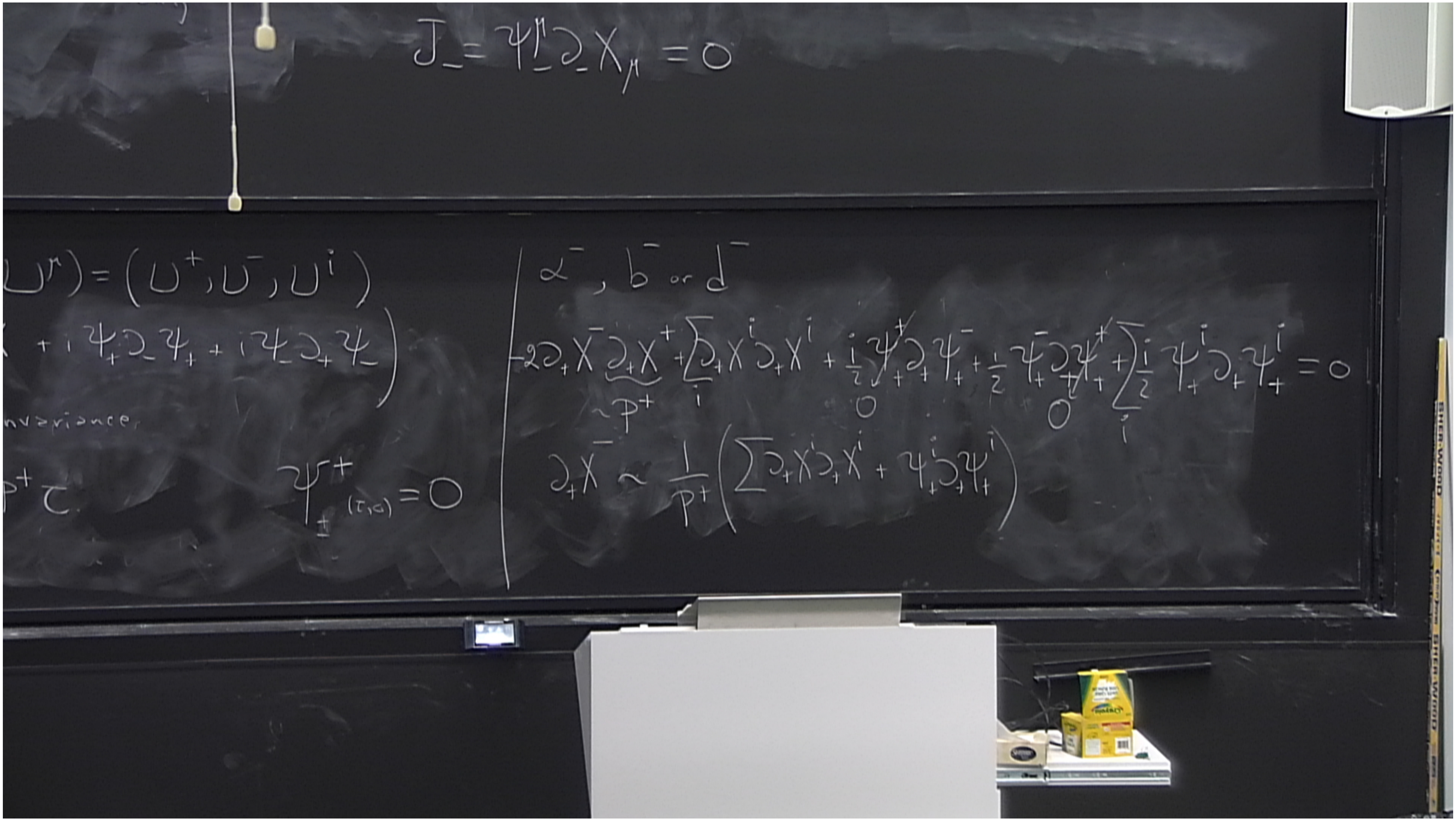

New coord $\sigma^{\pm} = \tau \pm \sigma$
$T_{++} = (\partial_+ X)^2 + \frac{i}{2} \psi^{\mu}_{+} \partial_+ \psi_{+\mu} = 0$
$J_+ = \psi^{\mu}_{+} \partial_+ X_{\mu} = 0$
$T_{--} = (\partial_- X)^2 + \frac{i}{2} \psi^{\mu}_{-} \partial_- \psi_{-\mu} = 0$
$J_- = \psi^{\mu}_{-} \partial_- X_{\mu} = 0$

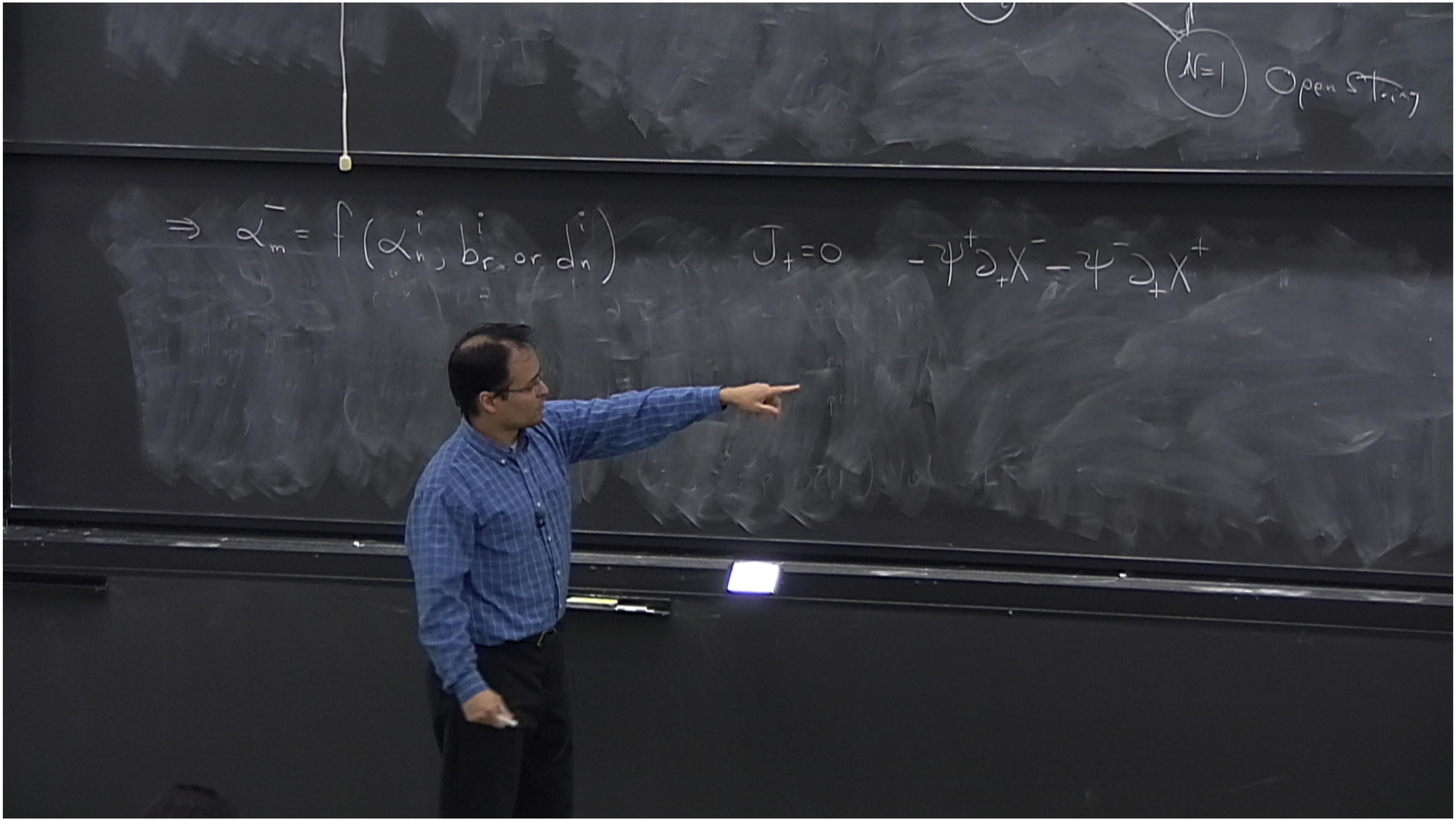

New coord. $\sigma^{\pm} = \tau \pm \sigma$
$T_{++} = (\partial_+ X)^2 + \frac{i}{2}\psi^\mu_+ \partial_+ \psi_{+\mu} = 0$
Chirality
$J_+ = \psi^\mu_+ \partial_+ X_\mu = 0$
$T_{--} = (\partial_- X)^2 + \frac{i}{2}\psi^\mu_- \partial_- \psi_{-\mu} = 0$
$J_- = \psi^\mu_- \partial_- X_\mu = 0$

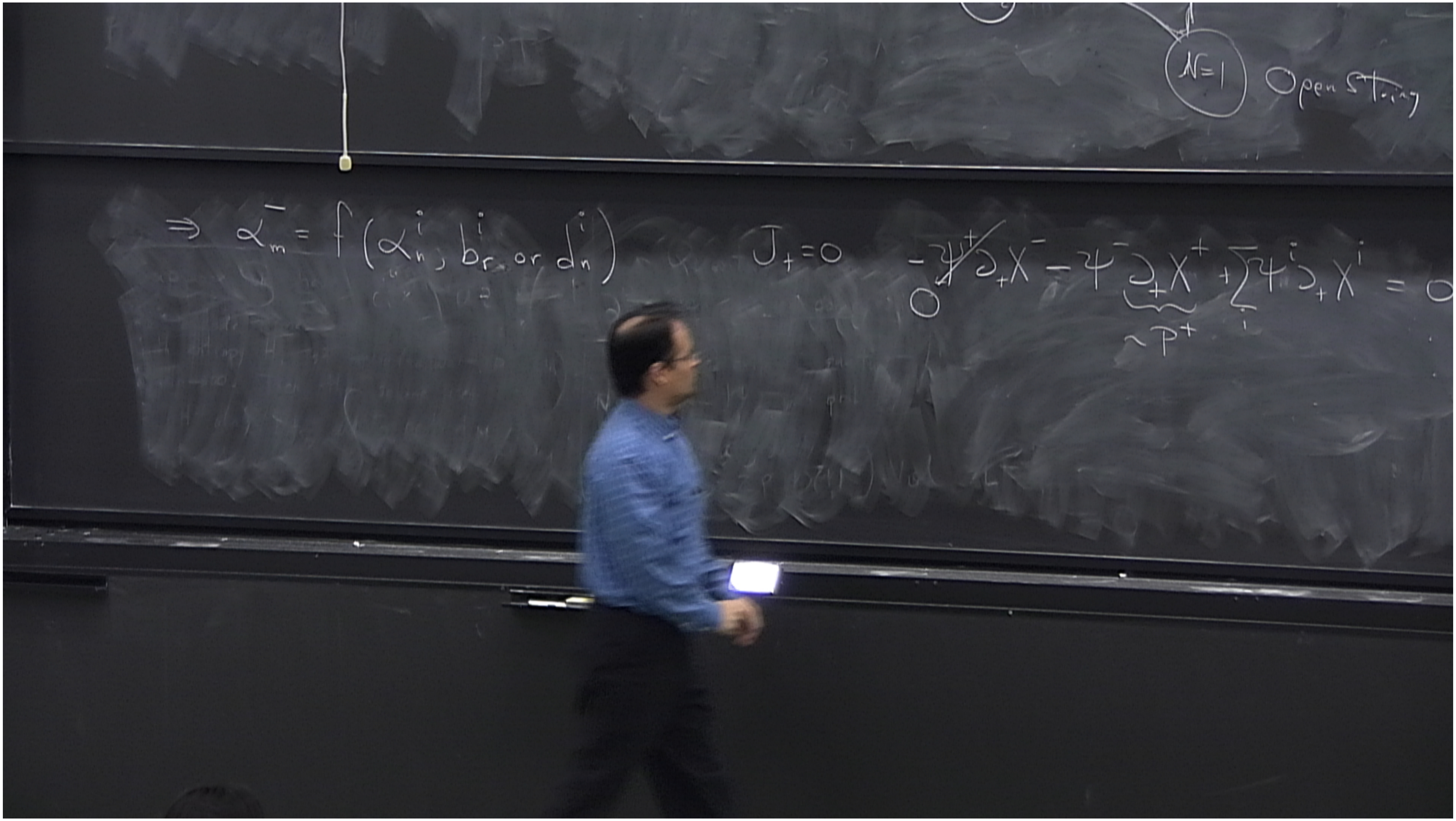

LCGQ : (U^μ) = (U^+, U^-, U^i)
α^-, b^- or d^-
conformal invariance

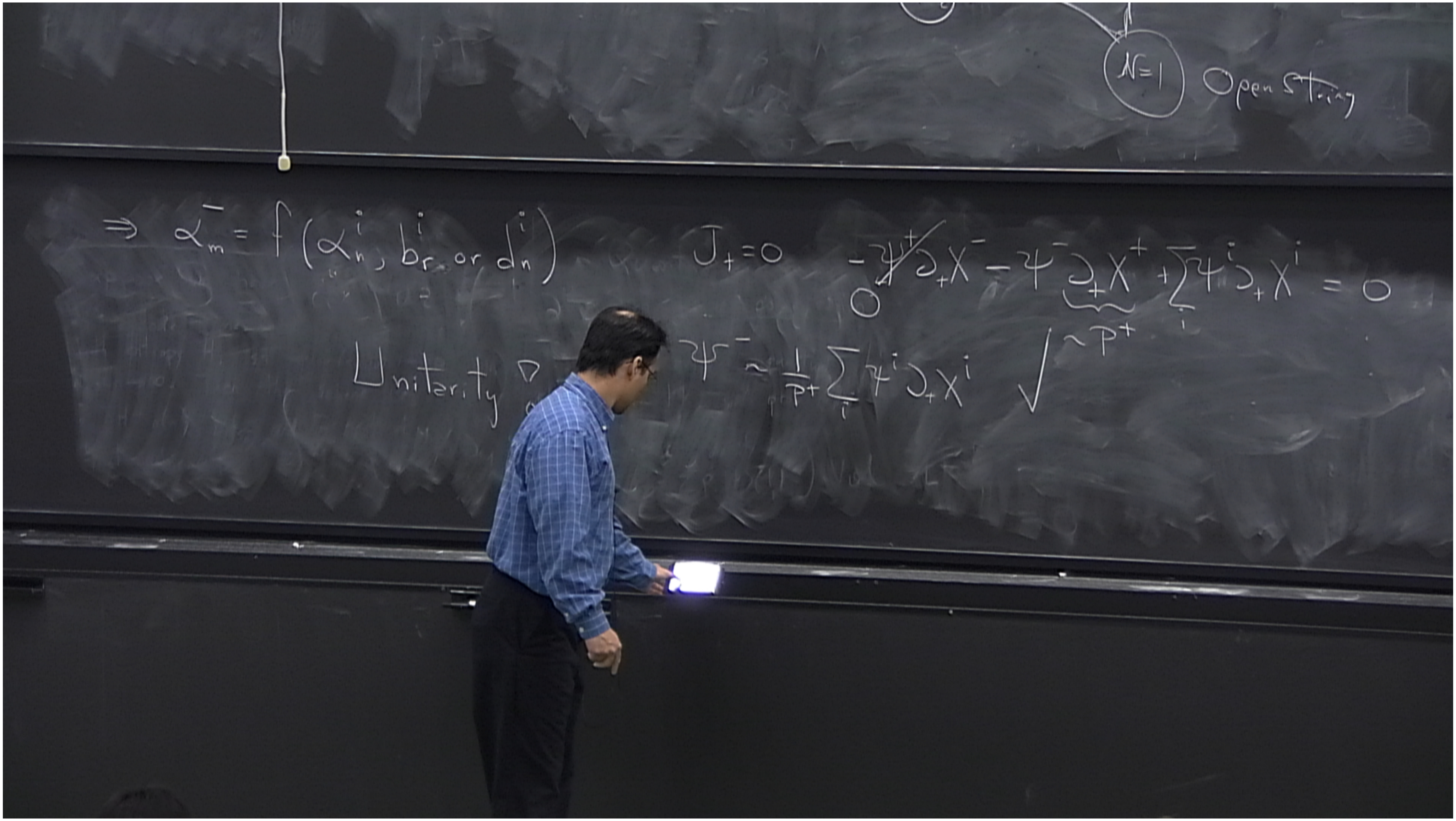

J_ = ψ^μ_ ∂_ X_μ = 0
conformal invariance
α⁻, b⁻ or d⁻

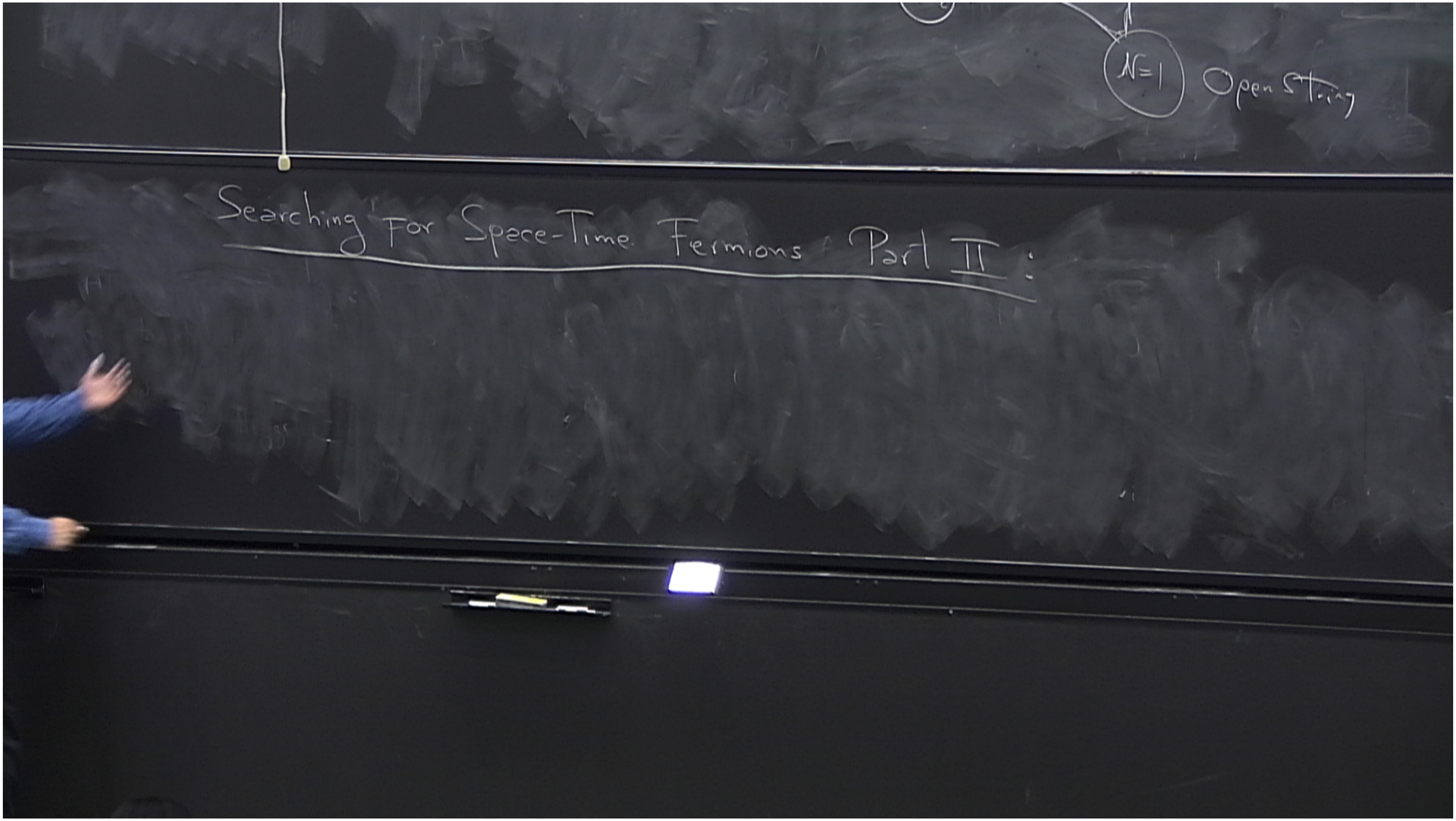

J_- = ψ^μ_- ∂_- X_μ = 0
GQ : (U^μ) = (U^+, U^-, U^i)
conformal invariance
α^-, b^- or d^-

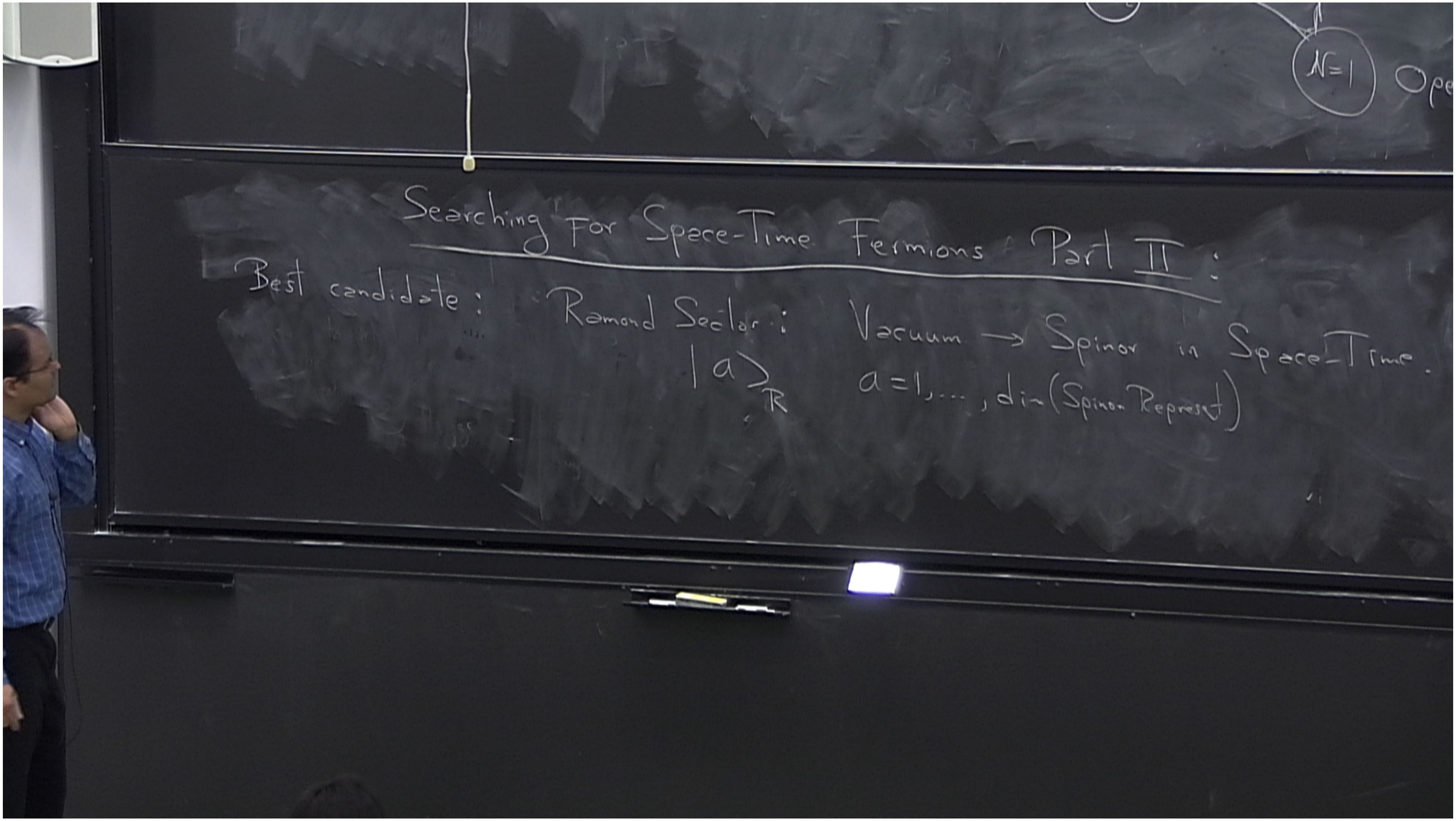

J_ = ψ^μ_ ∂_ X_μ = 0
(U^μ) = (U^+, U^-, U^i)
α^-, b^- or d^-
invariance,
ψ^+_± (τ,σ) = 0
∂_+ X^- ~ 1/p^+ (Σ ∂_+X^i ∂_+X^i + ψ^i_+ ∂_+ ψ^i_+)

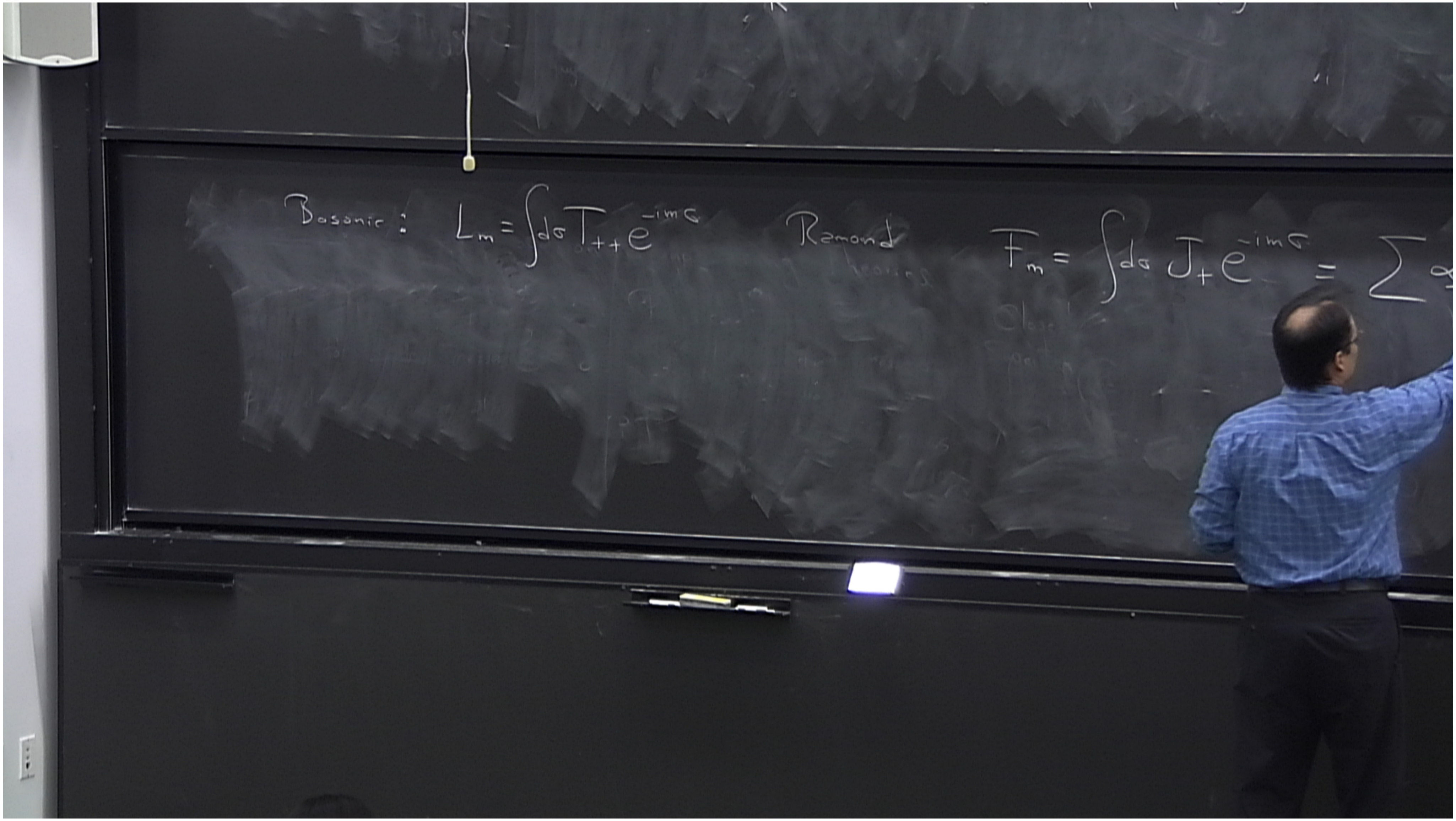

N=1 Open String
⇒ α⁻m = f(αin, bir or din)
J+ = 0
-ψ+∂+X- - ψ-∂+X+

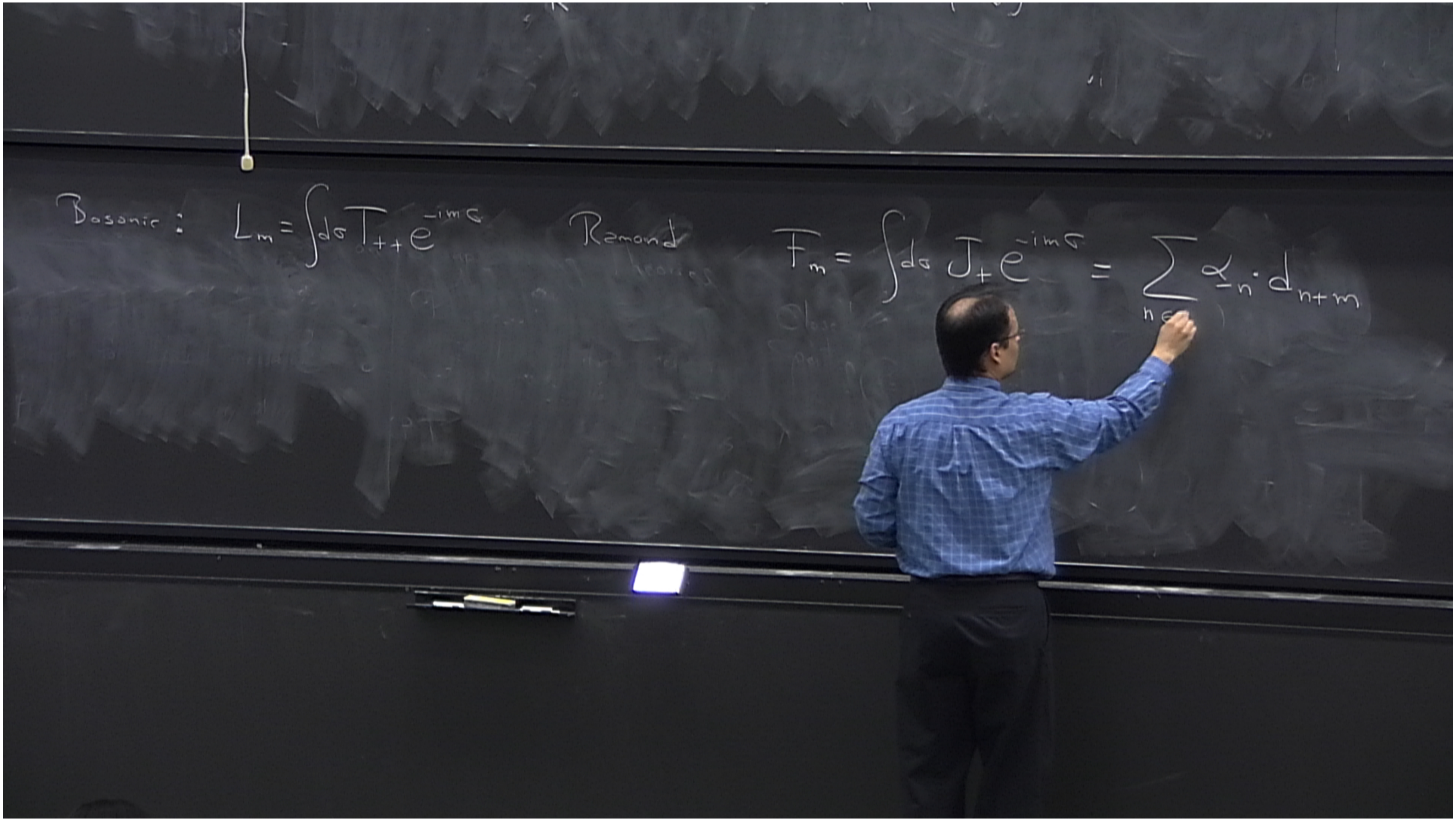

N=1
Open String
J+ = 0


Open String
$\Rightarrow \alpha^-_m = f(\alpha^i_n, b^i_r \text{ or } d^i_n)$
$J_+ = 0$
$- \psi^- \partial_+ X^+ + \sum_i \psi^i \partial_+ X^i = 0$
Unitarity
$\psi^- \sim \frac{1}{p^+} \sum_i \psi^i \partial_+ X^i$


N=1
Open String
Searching For Space-Time Fermions : Part II :


N=1
Searching For Space-Time Fermions Part II :
Best candidate :
Ramond Sector : Vacuum → Spinor in Space-Time.
|a⟩R
a=1,..., dim(Spinor Represent)


Bosonic :
$L_m = \int d\sigma \, T_{++} e^{-im\sigma}$
Ramond
$F_m = \int d\sigma \, J_+ e^{-im\sigma} = \sum$

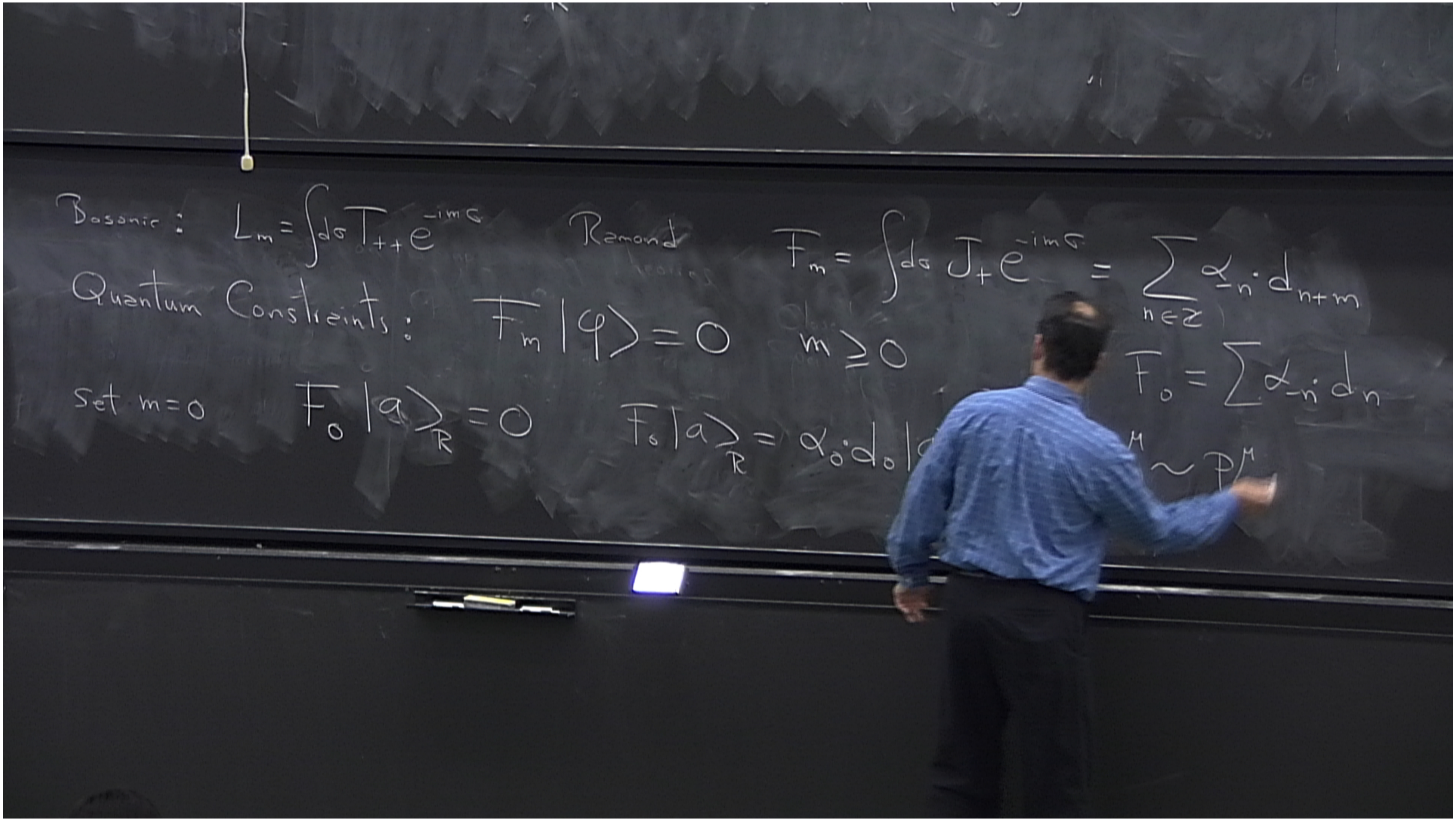

Bosonic : $L_m = \int d\sigma \, T_{++} e^{-im\sigma}$
Ramond
$F_m = \int d\sigma \, J_+ e^{-im\sigma} = \sum_{n \in} \alpha_{-n} \cdot d_{n+m}$

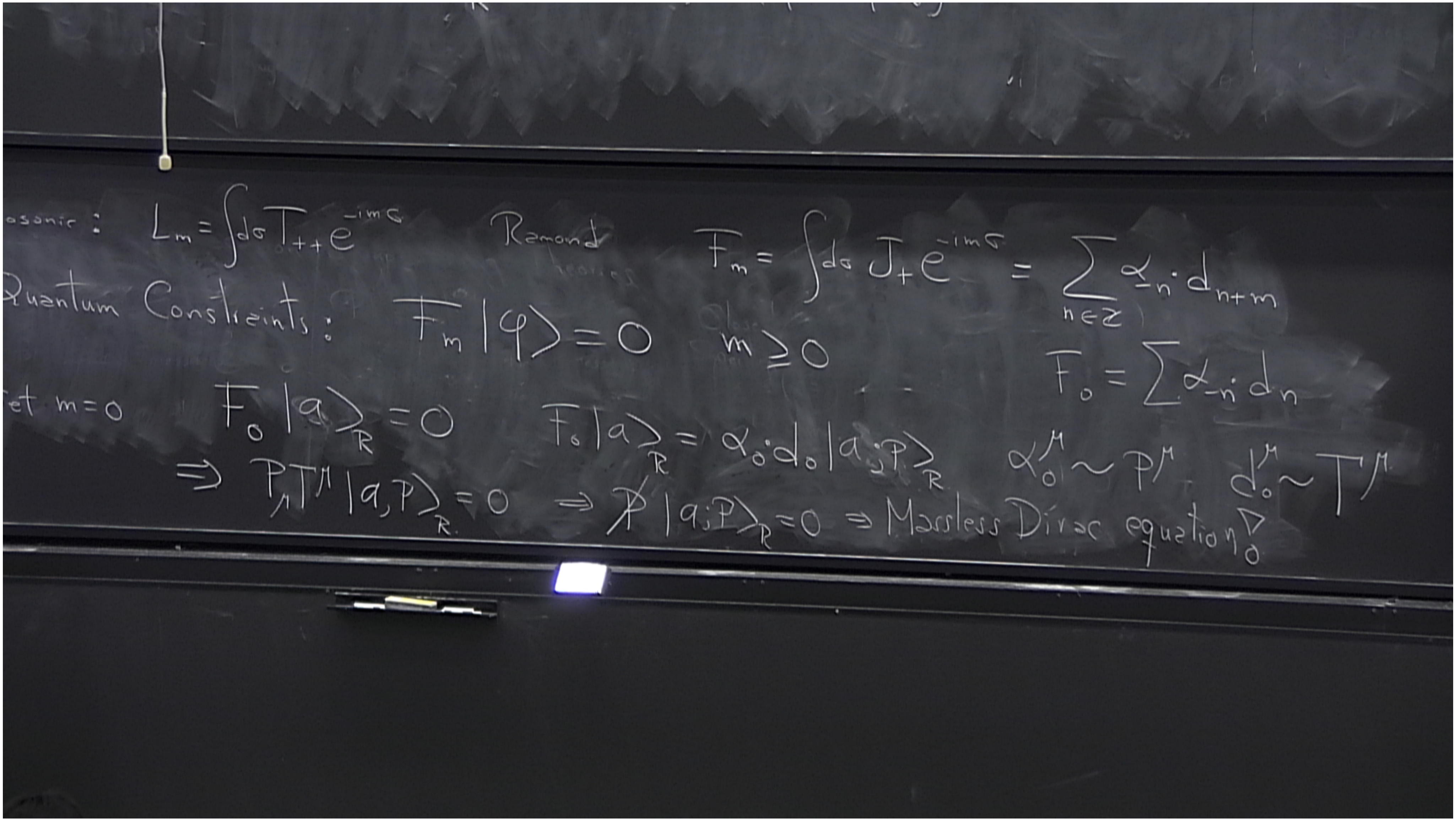
Bosonic: $L_m = \int d\sigma \, T_{++} e^{-im\sigma}$

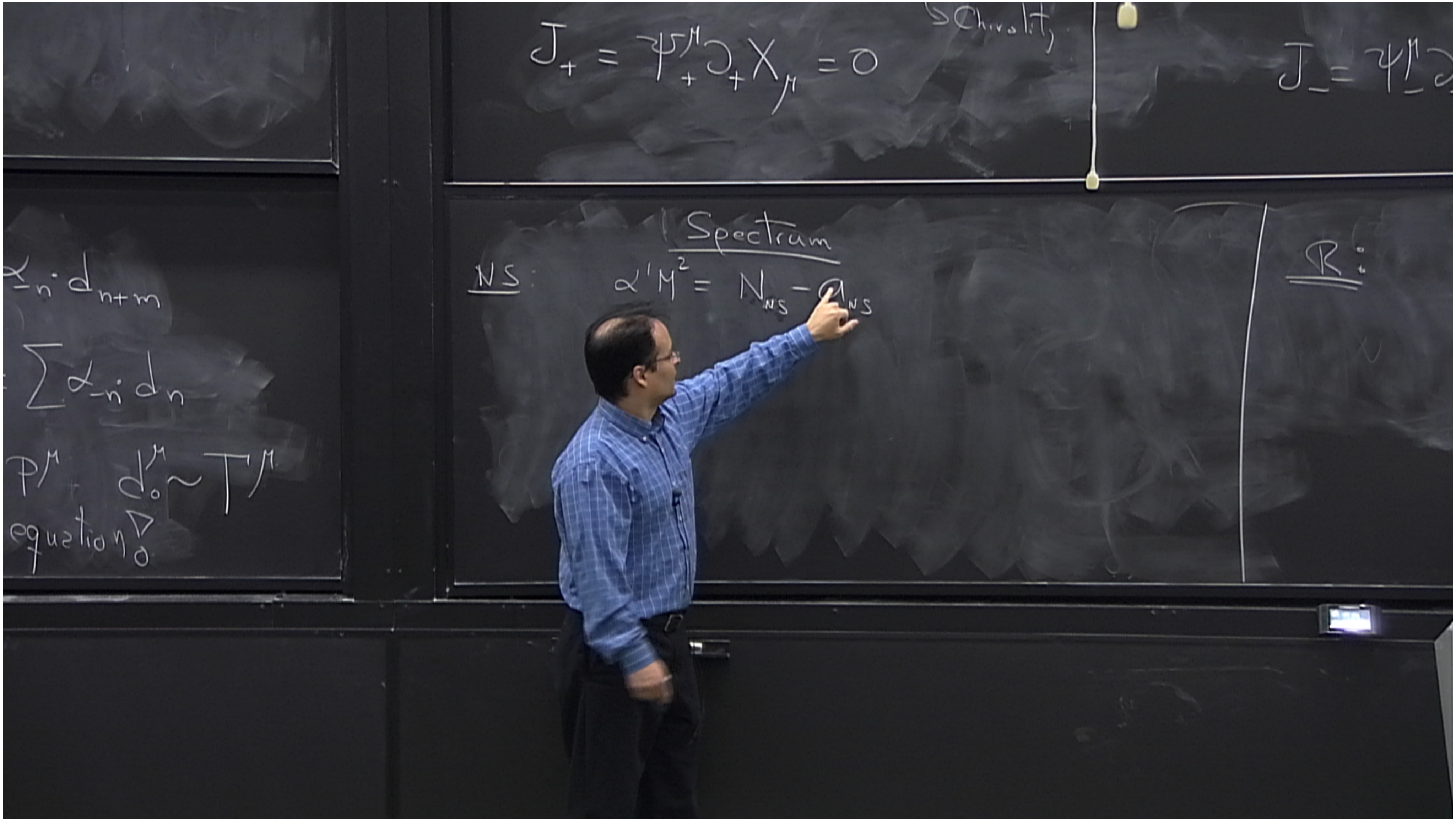
Ramond: $F_m = \int d\sigma \, J_+ e^{-im\sigma} = \sum_{n \in \mathbb{Z}} \alpha_{-n} \cdot d_{n+m}$

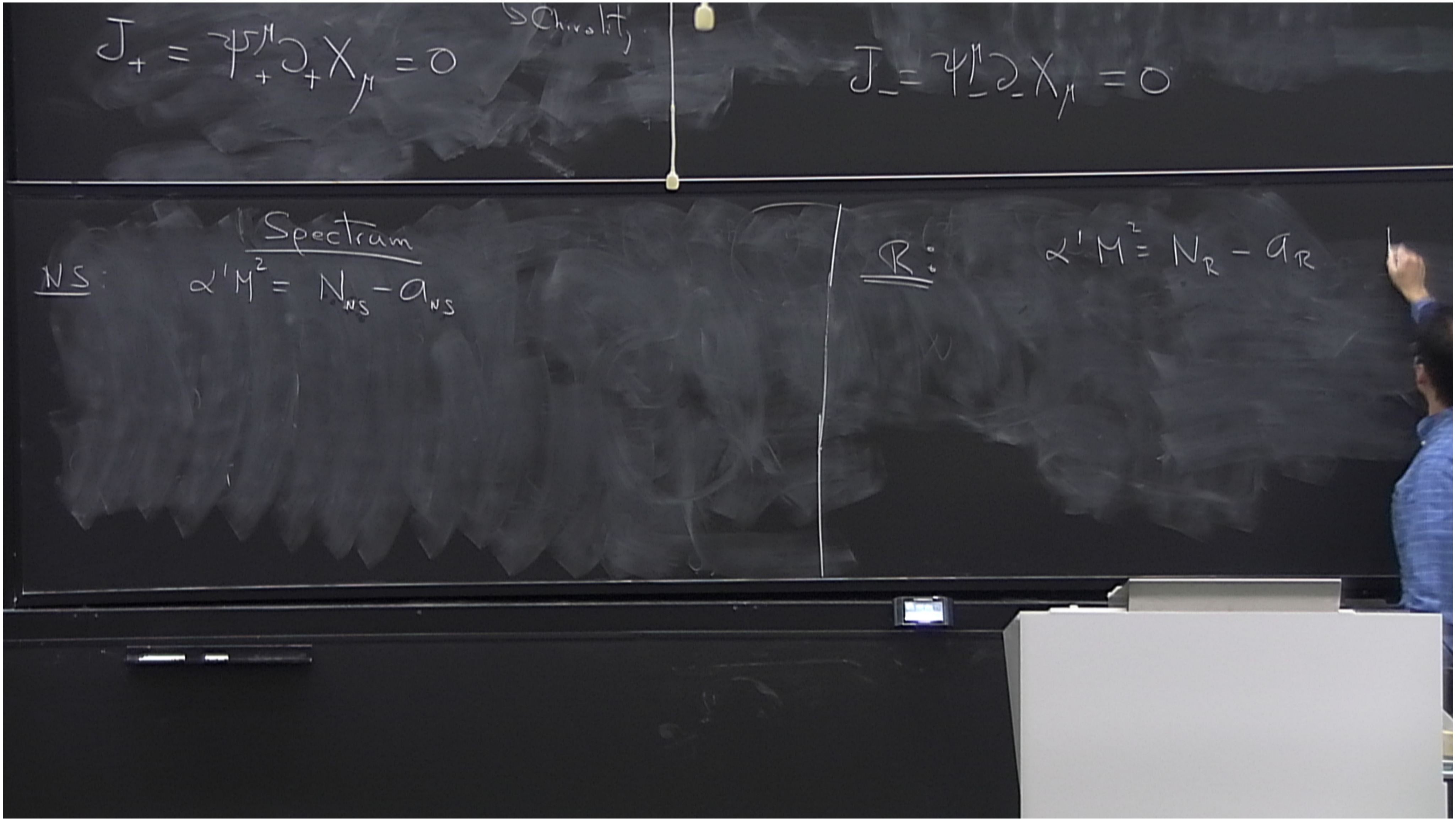
Quantum Constraints: $F_m |\varphi\rangle = 0 \quad m \geq 0$

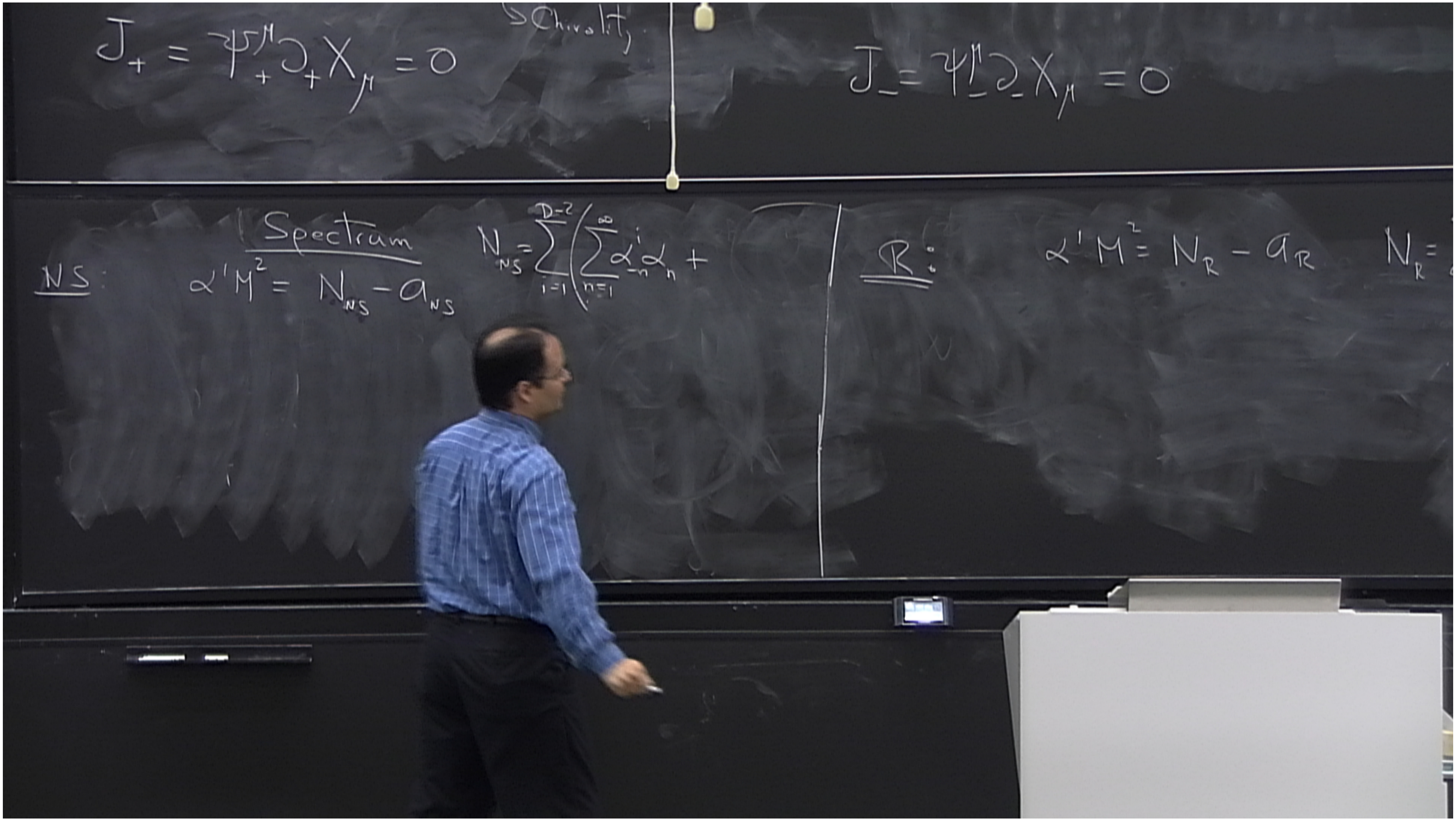
$|\varphi\rangle_R$

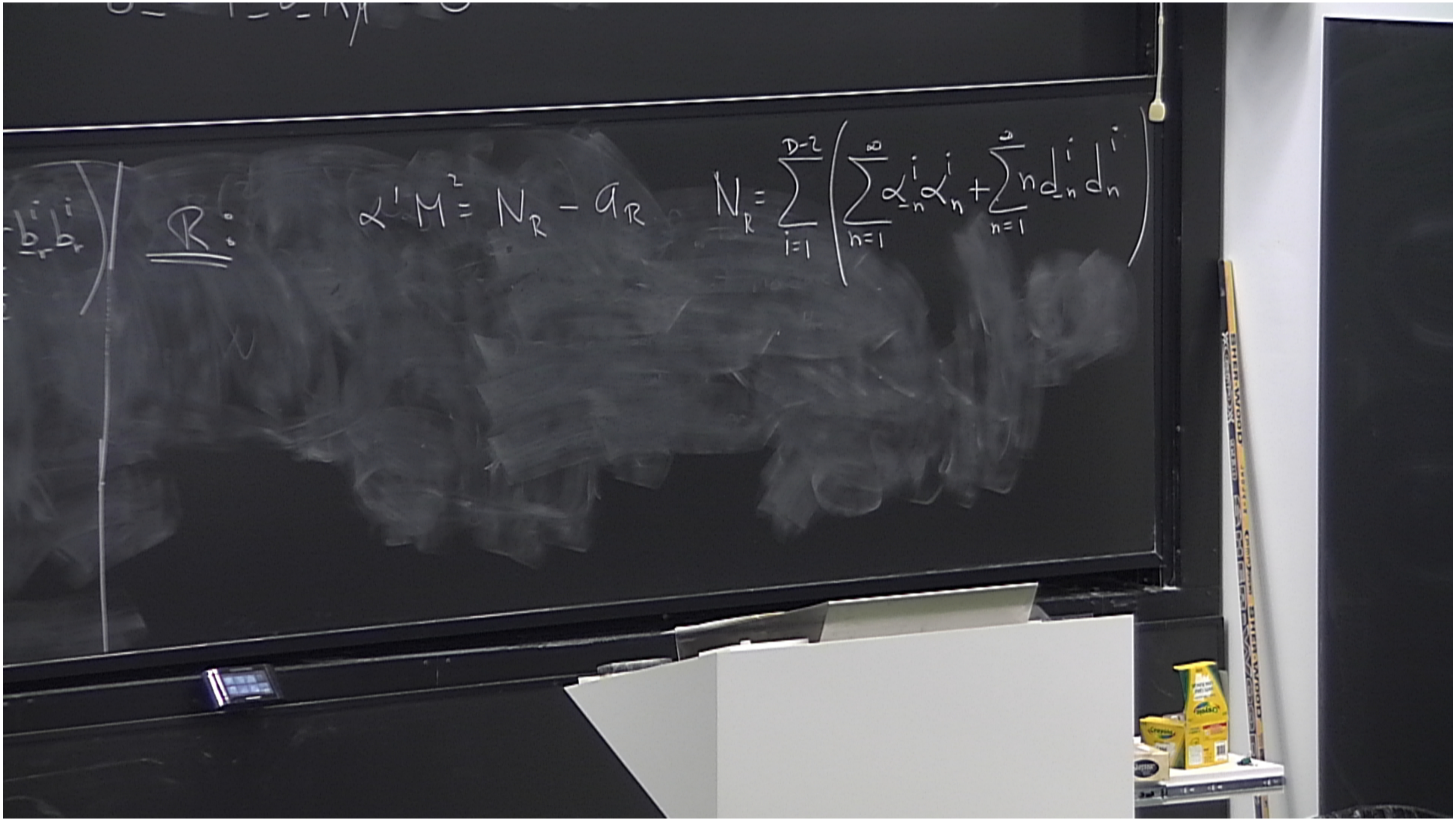

Bosonic: L_m = ∫dσ T_++ e^-imσ
Ramond
F_m = ∫dσ J_+ e^-imσ = Σ_n∈Z α_-n · d_n+m
Quantum Constraints: F_m |φ⟩ = 0 m ≥ 0
F_0 = Σ α_-n · d_n
Set m=0 F_0 |a⟩_R = 0 F_0 |a⟩_R = α_0 · d_0 |
~ p^μ

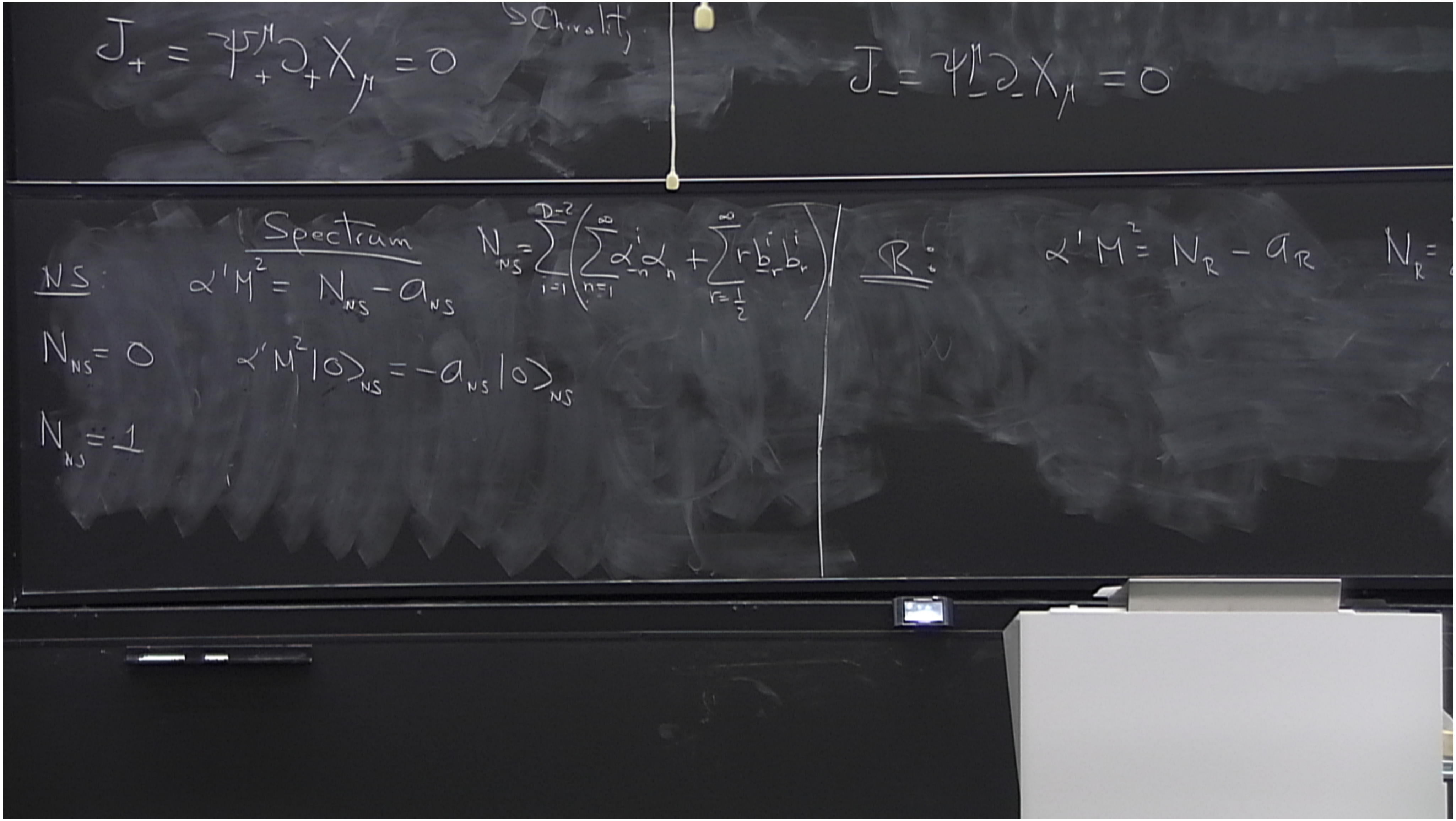

osonic : $L_m = \int d\sigma\, T_{++} e^{-im\sigma}$
Ramond
$F_m = \int d\sigma\, J_+ e^{-im\sigma} = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot d_{n+m}$
uantum Constraints : $F_m |\varphi\rangle = 0 \quad m \geq 0$
$F_0 = \sum \alpha_{-n}\cdot d_n$
et m=0 $F_0 |a\rangle_R = 0$
$F_0 |a\rangle_R = \alpha_0\cdot d_0 |a;P\rangle_R$
$\alpha_0^\mu \sim P^\mu \quad d_0^\mu \sim \Gamma^\mu$
$\Rightarrow P_\mu \Gamma^\mu |a,P\rangle_R = 0 \Rightarrow \not{P} |a;P\rangle_R = 0 \Rightarrow$ Massless Dirac equation!


$J_+ = \psi^\mu_+ \partial_+ X_\mu = 0$
Chirality
Spectrum
NS
$\alpha' M^2 = N_{NS} - a_{NS}$
R
equation?

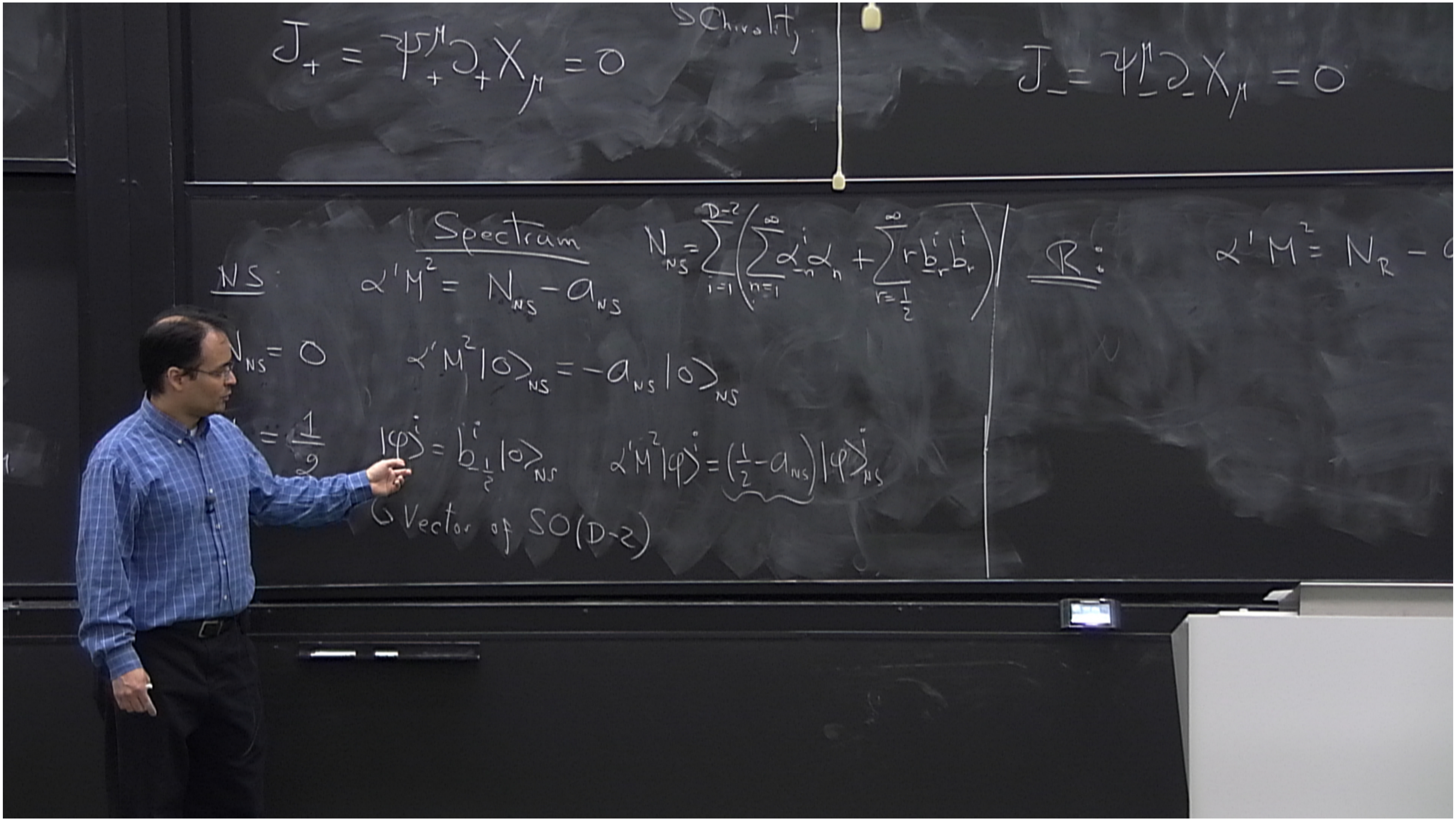

$J_+ = \psi^\mu_+ \partial_+ X_\mu = 0$
Chirality
$J_- = \psi^\mu_- \partial_- X_\mu = 0$
Spectrum
NS: $\alpha' M^2 = N_{NS} - a_{NS}$
R: $\alpha' M^2 = N_R - a_R$

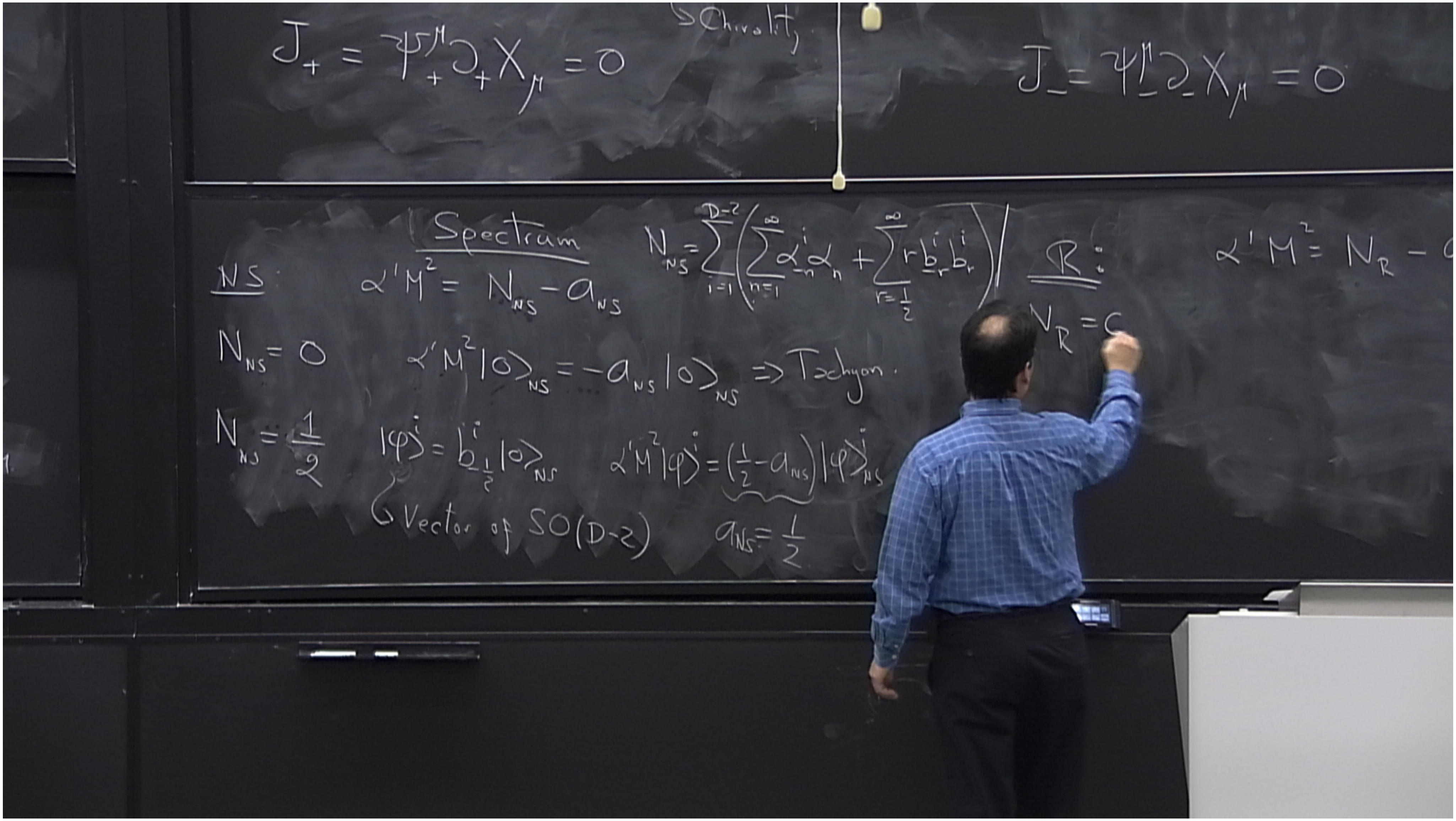

J+ = ψ+μ ∂+ Xμ = 0
Chirality
J− = ψ−μ ∂− Xμ = 0
Spectrum
NS:
α'M² = N_NS − a_NS
N_NS = Σ(i=1 to D−2) (Σ(n=1 to ∞) α^i_−n α^i_n +
R:
α'M² = N_R − a_R
N_R =

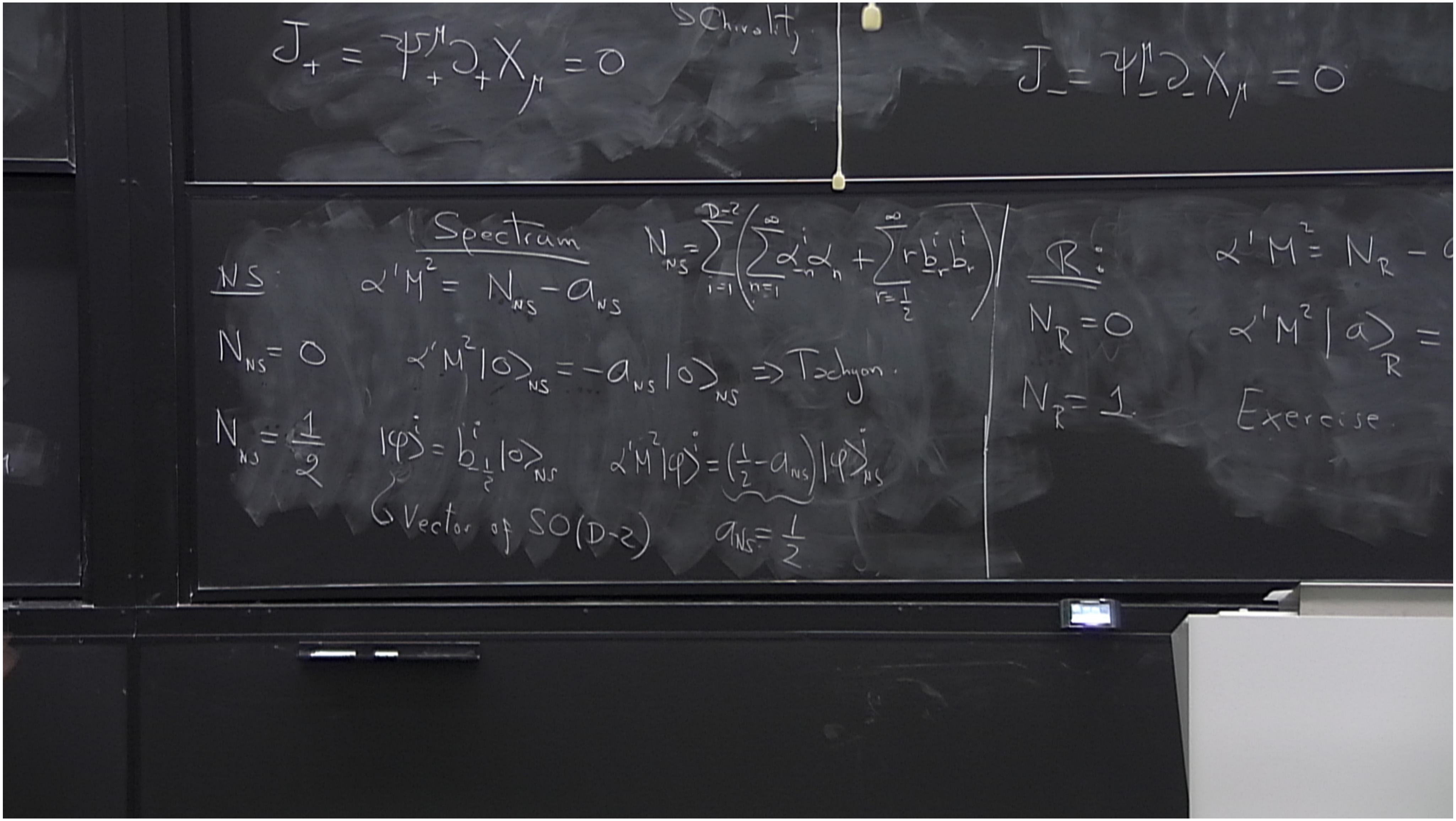

R :
$\alpha' M^2 = N_R - a_R$
$N_R = \sum_{i=1}^{D-2}\left(\sum_{n=1}^{\infty}\alpha_{-n}^i\alpha_n^i + \sum_{n=1}^{\infty} n\, d_{-n}^i d_n^i\right)$

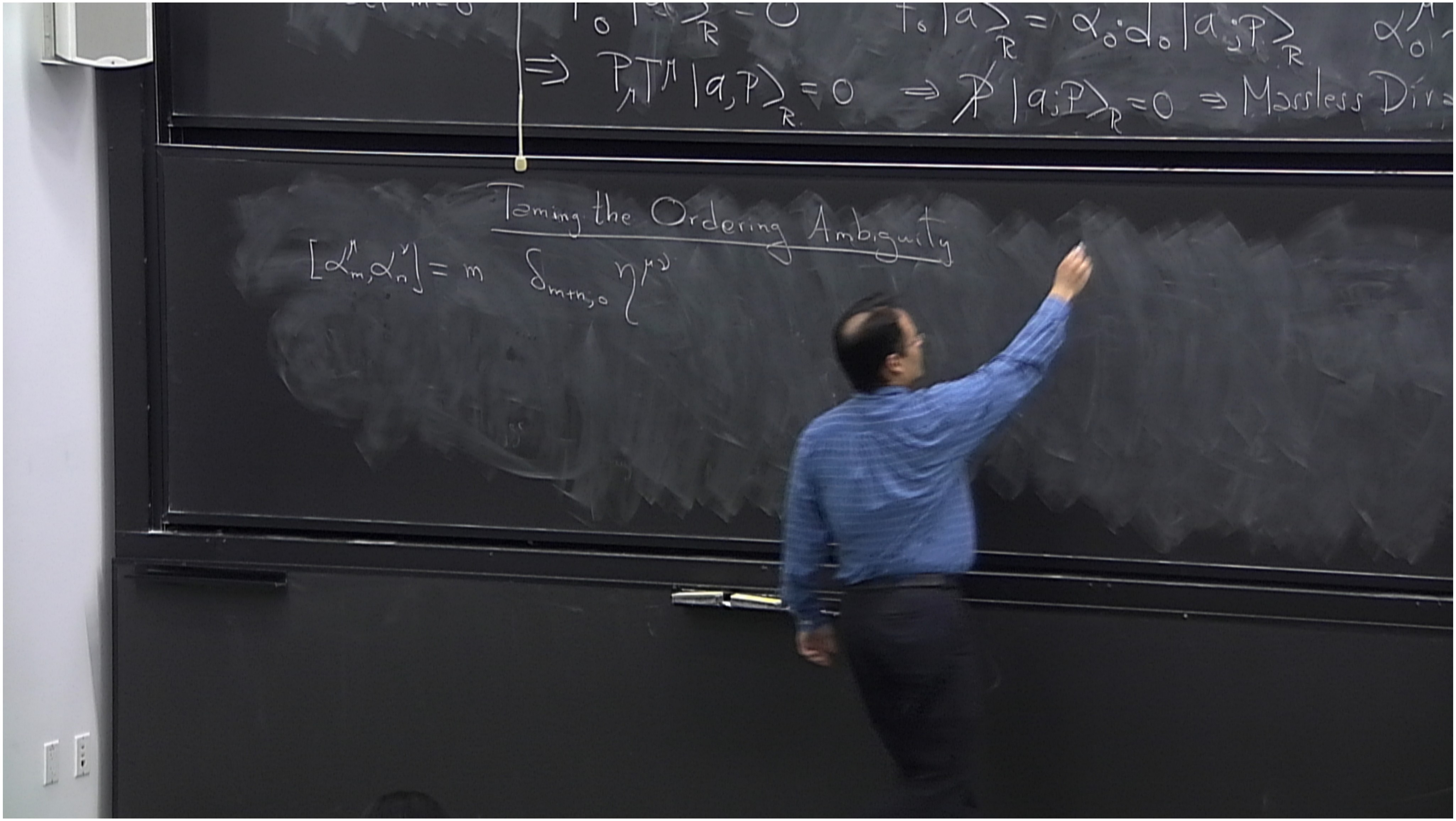


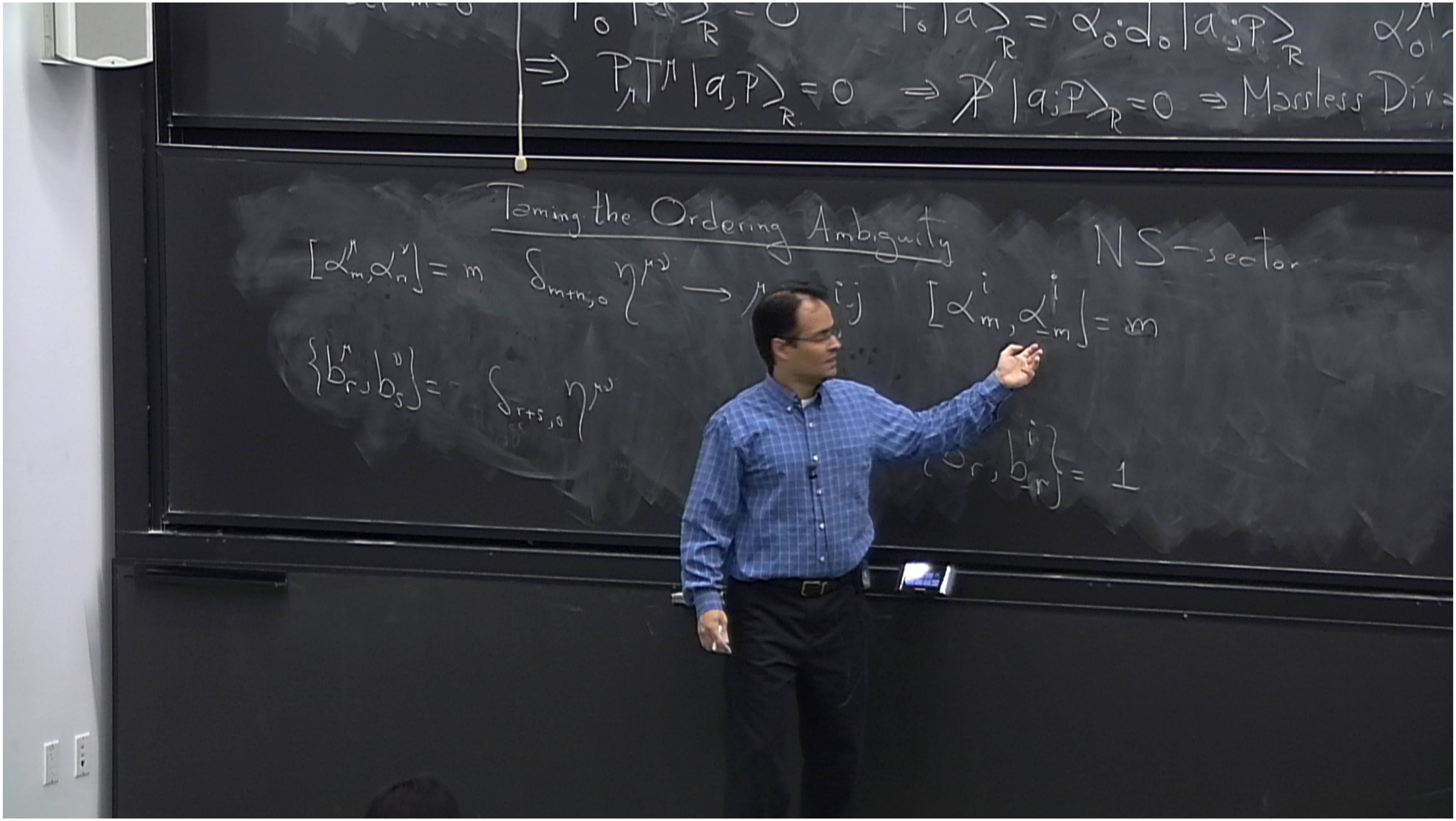
$J_+ = \psi_+^\mu \partial_+ X_\mu = 0$
Chirality
$J_- = \psi_-^\mu \partial_- X_\mu = 0$
Spectrum
NS:
$\alpha' M^2 = N_{NS} - a_{NS}$
$N_{NS} = \sum_{i=1}^{D-2} \left(\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r}^i b_r^i \right)$
R:
$\alpha' M^2 = N_R - a_R$
$N_{NS} = 0$
$\alpha' M^2 |0\rangle_{NS} = -a_{NS} |0\rangle_{NS}$
$N_{NS} = 1$

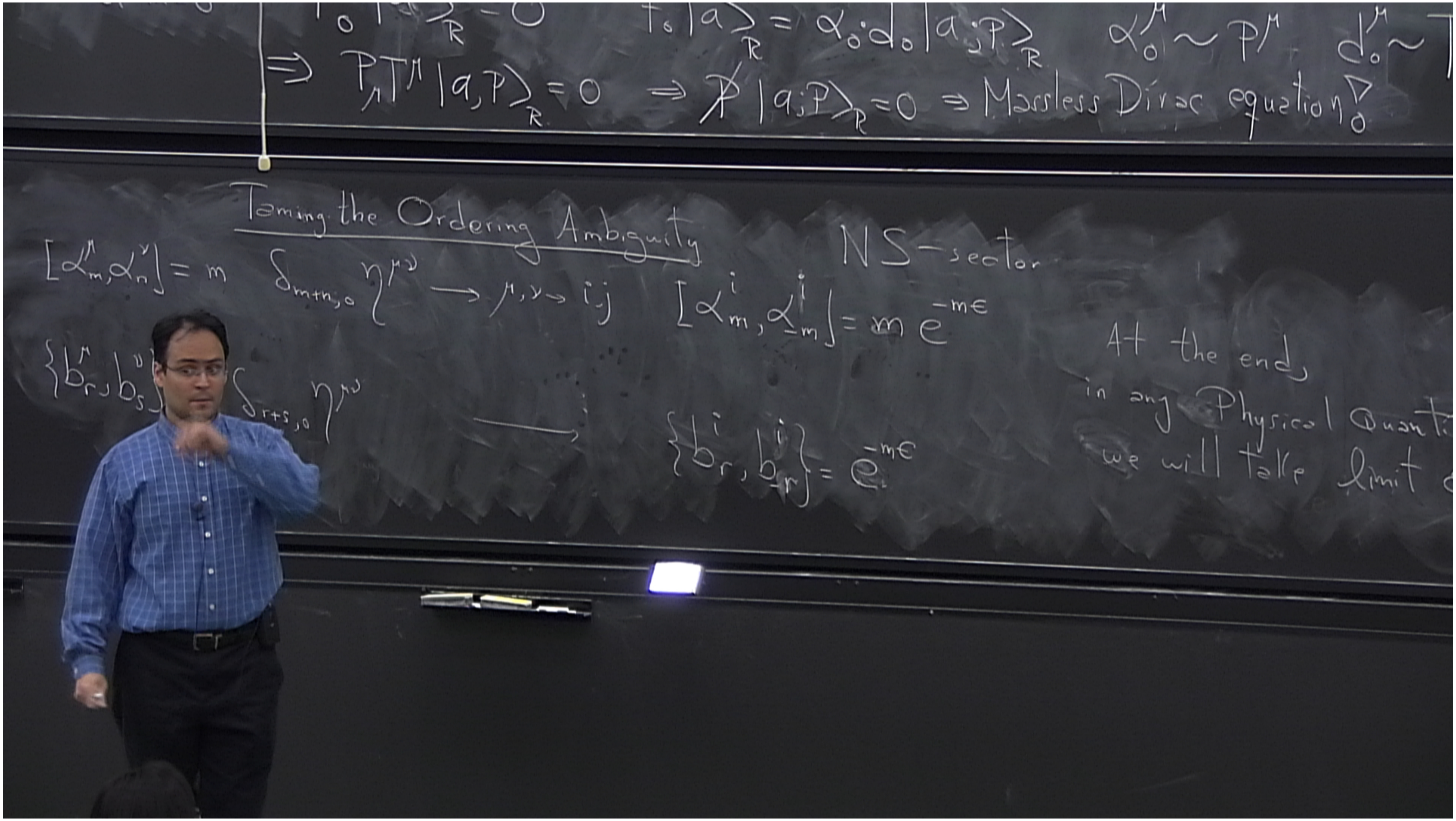

$J_+ = \psi^\mu_+ \partial_+ X_\mu = 0$
Chirality
$J_- = \psi^\mu_- \partial_- X_\mu = 0$
Spectrum
NS:
$\alpha' M^2 = N_{NS} - a_{NS}$
$N_{NS} = \sum_{i=1}^{D-2} \left(\sum_{n=1}^{\infty} \alpha^i_{-n} \alpha^i_n + \sum_{r=\frac{1}{2}}^{\infty} r b^i_{-r} b^i_r \right)$
R:
$\alpha' M^2 = N_R -$
$N_{NS} = 0$
$\alpha' M^2 |0\rangle_{NS} = -a_{NS} |0\rangle_{NS}$
$|\varphi\rangle^i = b^i_{-\frac{1}{2}} |0\rangle_{NS}$
$\alpha' M^2 |\varphi\rangle^i = (\frac{1}{2} - a_{NS}) |\varphi\rangle^i_{NS}$
Vector of SO(D-2)

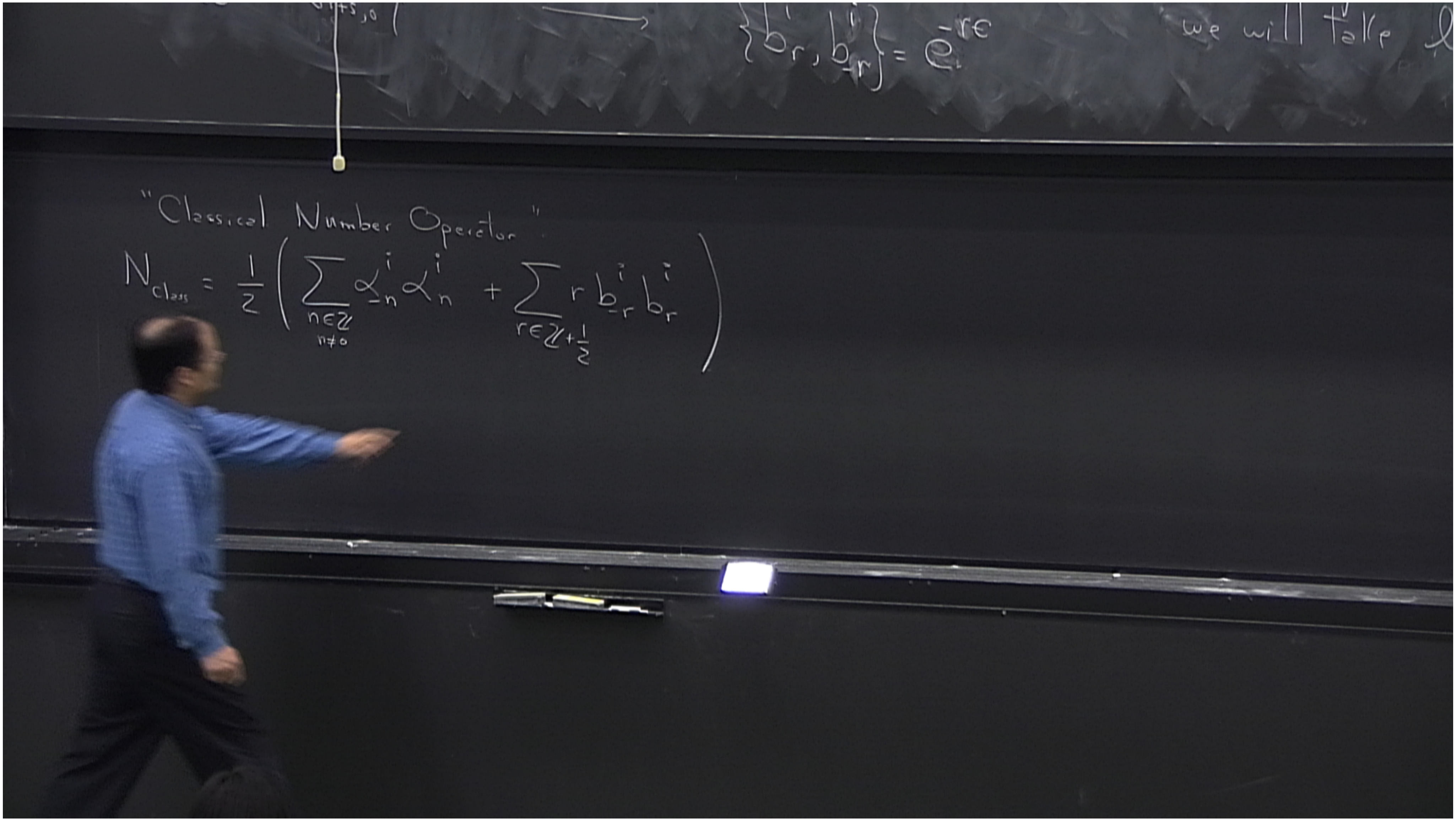

Chirality
$J_+ = \psi_+^\mu \partial_+ X_\mu = 0$
$J_- = \psi_-^\mu \partial_- X_\mu = 0$
Spectrum
NS
$\alpha' M^2 = N_{NS} - a_{NS}$
$N_{NS} = \sum_{i=1}^{D-2} \left(\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r}^i b_r^i \right)$
R:
$\alpha' M^2 = N_R -$
$N_R = 0$
$N_{NS} = 0$
$\alpha' M^2 |0\rangle_{NS} = -a_{NS} |0\rangle_{NS}$ => Tachyon.
$N_{NS} = \frac{1}{2}$
$|\varphi\rangle^i = b_{-\frac{1}{2}}^i |0\rangle_{NS}$
$\alpha' M^2 |\varphi\rangle^i = (\frac{1}{2} - a_{NS}) |\varphi\rangle^i_{NS}$
Vector of SO(D-2)
$a_{NS} = \frac{1}{2}$

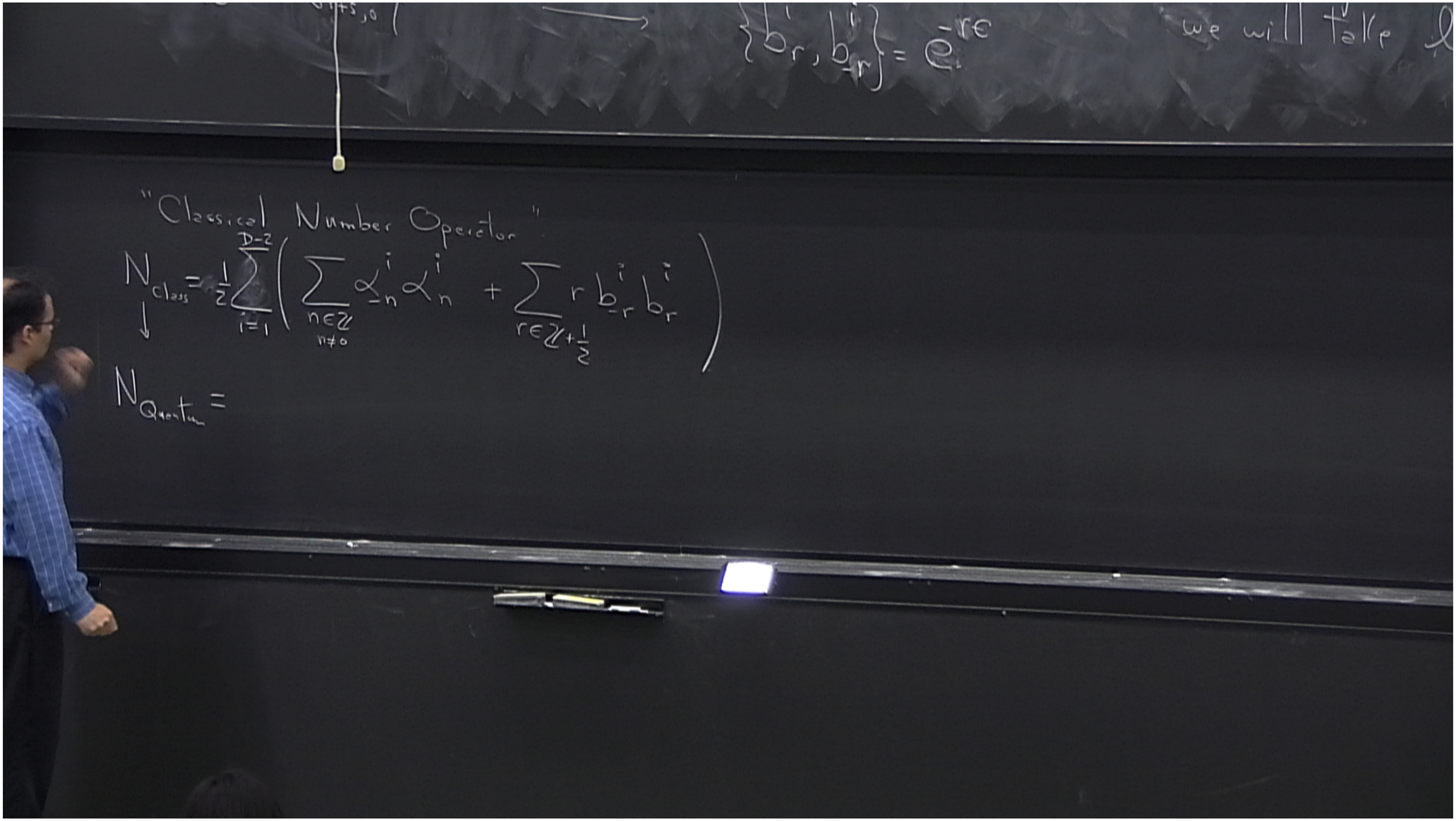

Chirality
J+ = ψ+^μ ∂+ X_μ = 0
J− = ψ−^μ ∂− X_μ = 0
Spectrum
N_NS = Σ_{i=1}^{D−2} (Σ_{n=1}^∞ α^i_{−n} α^i_n + Σ_{r=1/2}^∞ r b^i_{−r} b^i_r)
NS:
α′M² = N_NS − a_NS
N_NS = 0
α′M² |0⟩_NS = −a_NS |0⟩_NS ⇒ Tachyon.
N_NS = 1/2
|φ⟩^i = b^i_{−1/2} |0⟩_NS
α′M² |φ⟩^i = (1/2 − a_NS) |φ⟩^i_NS
Vector of SO(D−2)
a_NS = 1/2
R:
α′M² = N_R −
N_R = 0
α′M² |a⟩_R =
N_R = 1
Exercise.


⇒ P_μ Γ^μ |a, P⟩_R = 0 ⇒ P̸ |a; P⟩_R = 0 ⇒ Massless Dir
Taming the Ordering Ambiguity
[α^μ_m, α^ν_n] = m δ_{m+n,0} η^{μν}


Taming the Ordering Ambiguity
NS-sector


⇒ Massless Dirac equation
Taming the Ordering Ambiguity
NS-sector
At the end,
in any Physical Quanti
we will take limit


$\{b^i_r, b^i_{-r}\} = e^{-r\epsilon}$
we will take
"Classical Number Operator"
$N_{class} = \frac{1}{2}\left(\sum_{n\in\mathbb{Z}, n\neq 0} \alpha^i_{-n}\alpha^i_n + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b^i_{-r} b^i_r \right)$


$\{b_r, b_{-r}\} = e^{-r\epsilon}$
we will take
"Classical Number Operator"
$N_{Class} = \frac{1}{2}\sum_{i=1}^{D-2}\left(\sum_{n\in\mathbb{Z}, n\neq 0} \alpha^i_{-n}\alpha^i_n + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b^i_{-r} b^i_r\right)$
$N_{Quantum} =$

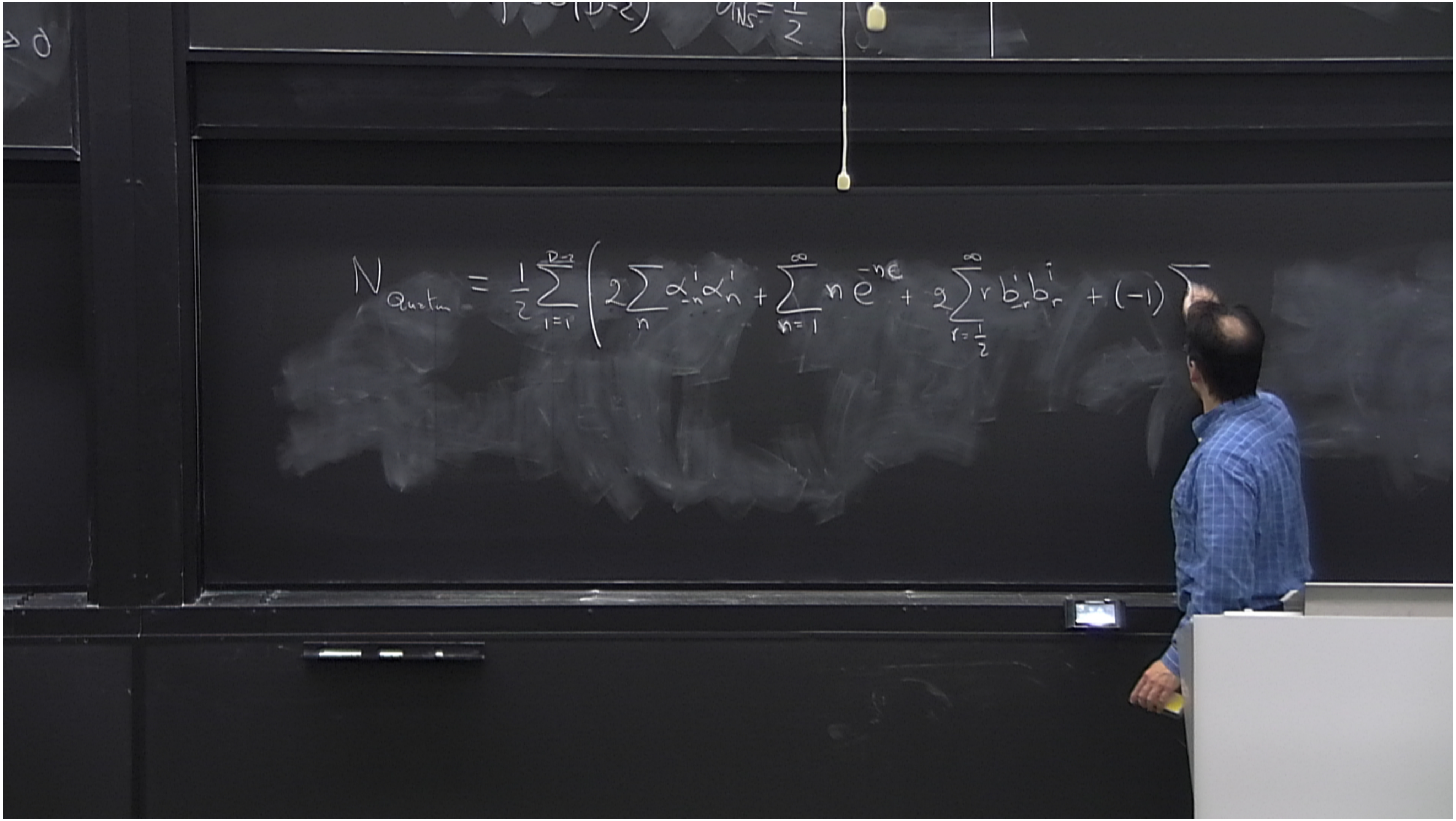
$\{b^i_r, b^i_{-r}\} = e^{-r\epsilon}$ we will take

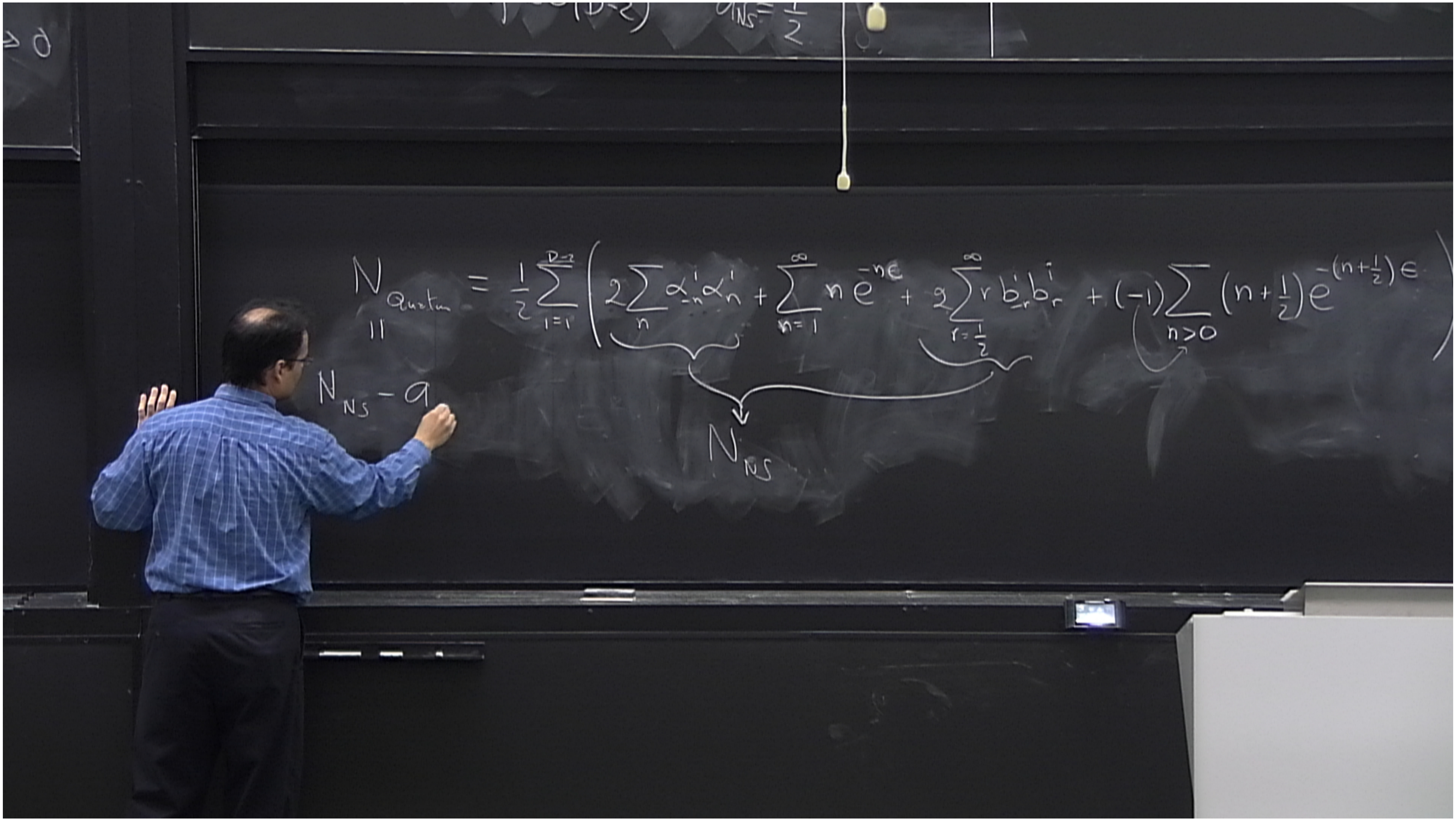
"Classical Number Operator"

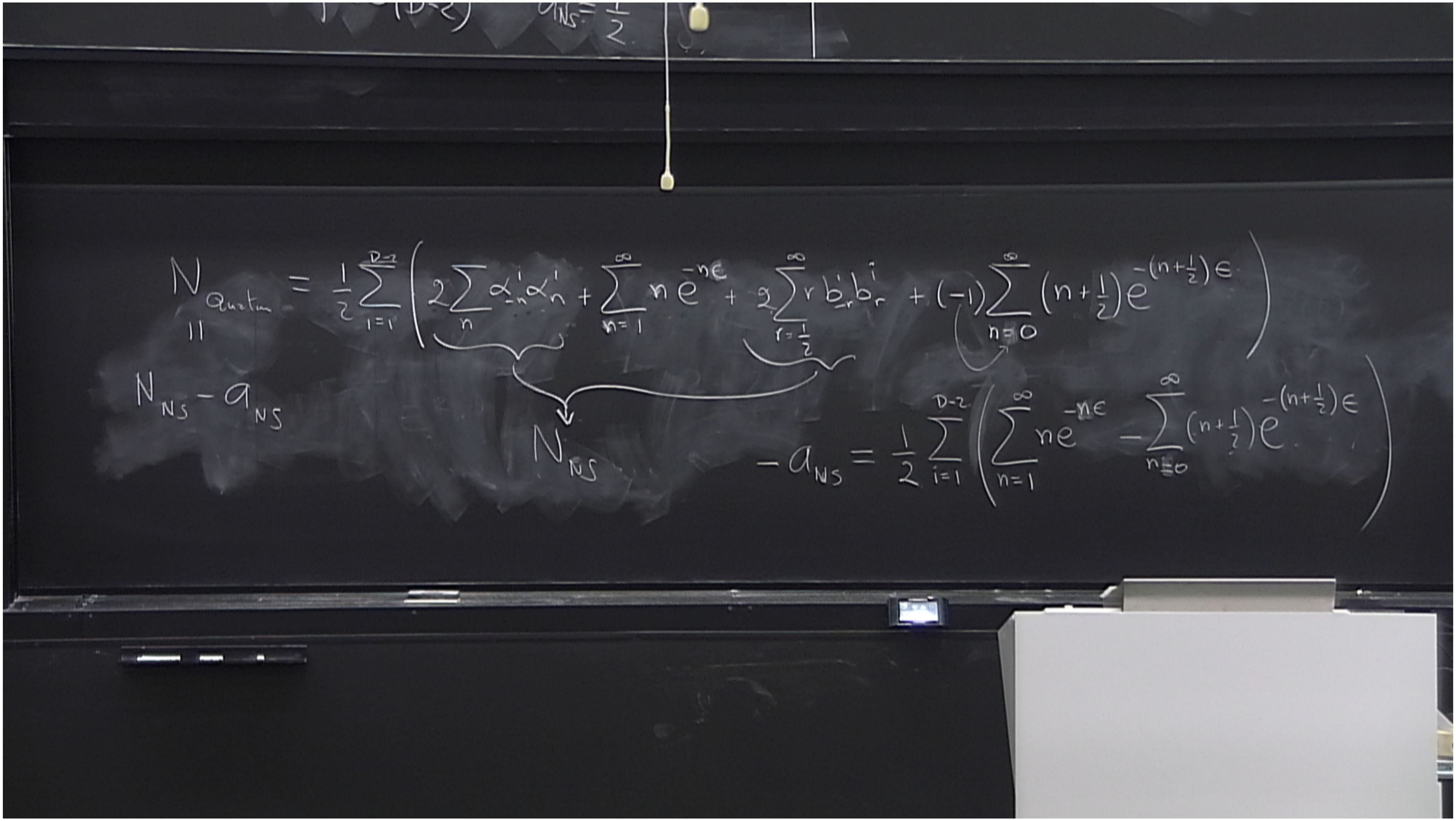
$$N_{Class} = \frac{1}{2}\sum_{i=1}^{D-2}\left(\sum_{\substack{n\in\mathbb{Z}\\ n\neq 0}}\alpha^i_{-n}\alpha^i_n + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b^i_{-r} b^i_r\right)$$

$\downarrow$

$$\text{"}N_{Quantum}\text{"} = \frac{1}{2}\sum_{i=1}^{D-2}\left(\sum_{n>0}\alpha^i_{-n}\alpha^i_n + \sum_{n<0}\alpha^i_{-n}\alpha^i_n + \sum_{r=\frac{1}{2}}^{\infty} r\, b^i_{-r} b^i_r + \sum_{n<0}(n+\tfrac{1}{2})\, b^i_{-(n+\frac{1}{2})} b^i_{(n+\frac{1}{2})}\right)$$


$N_{\text{Quantum}} = \frac{1}{2}\sum_{i=1}^{D-2}\left(2\sum_{n}\alpha^i_{-n}\alpha^i_n + \sum_{n=1}^{\infty} n e^{-n\epsilon} + 2\sum_{r=\frac{1}{2}}^{\infty} r\, b^i_{-r} b^i_r + (-1)\sum\right.$


N_Quantum = 1/2 Σ_{i=1}^{D-2} (2 Σ_n α^i_{-n} α^i_n + Σ_{n=1}^∞ n e^{-nε} + 2 Σ_{r=1/2}^∞ r b^i_{-r} b^i_r + (-1) Σ_{n>0} (n+1/2) e^{-(n+1/2)ε})
N_NS - a
N_NS


$N_{\text{Quantum}} = \frac{1}{2}\sum_{i=1}^{D-2}\left(2\sum_n \alpha_{-n}^i\alpha_n^i + \sum_{n=1}^{\infty} n e^{-n\epsilon} + 2\sum_{r=\frac{1}{2}}^{\infty} r\, b_{-r}^i b_r^i + (-1)\sum_{n=0}^{\infty}(n+\tfrac{1}{2})e^{-(n+\frac{1}{2})\epsilon}\right)$
$N_{NS} - a_{NS}$
N_{NS}
$-a_{NS} = \frac{1}{2}\sum_{i=1}^{D-2}\left(\sum_{n=1}^{\infty} n e^{-n\epsilon} - \sum_{n=0}^{\infty}(n+\tfrac{1}{2})e^{-(n+\frac{1}{2})\epsilon}\right)$

$$(D-2) \qquad a_{NS} = \frac{1}{2}$$

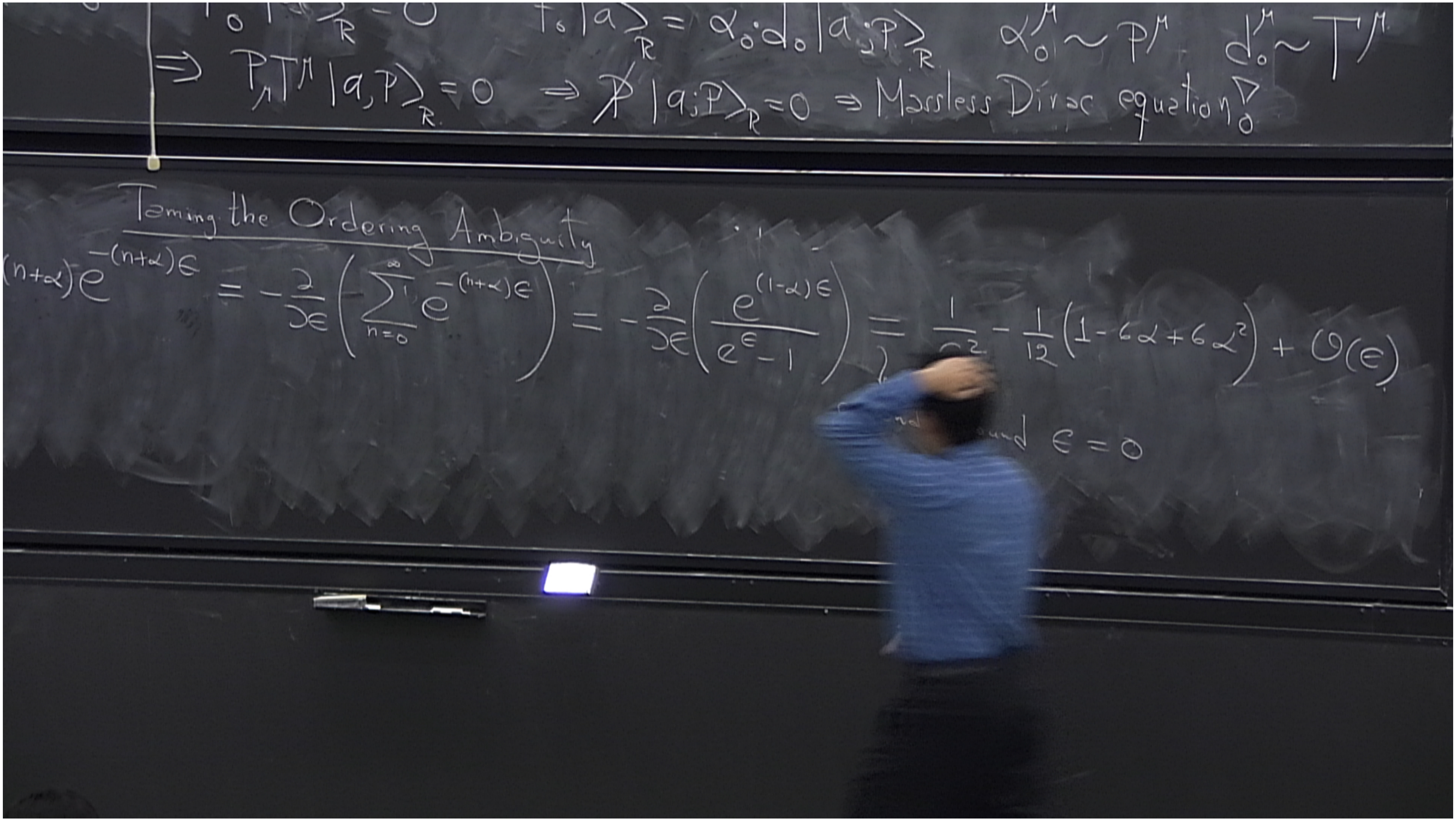
$$N_{\text{Quantum}} = \frac{1}{2}\sum_{i=1}^{D-2}\left(2\sum_{n}\alpha^i_{-n}\alpha^i_n + \sum_{n=1}^{\infty} n e^{-n\epsilon} + 2\sum_{r=\frac{1}{2}}^{\infty} r\, b^i_{-r} b^i_r + (-1)\sum_{n=0}^{\infty}\left(n+\tfrac{1}{2}\right) e^{-(n+\frac{1}{2})\epsilon}\right)$$

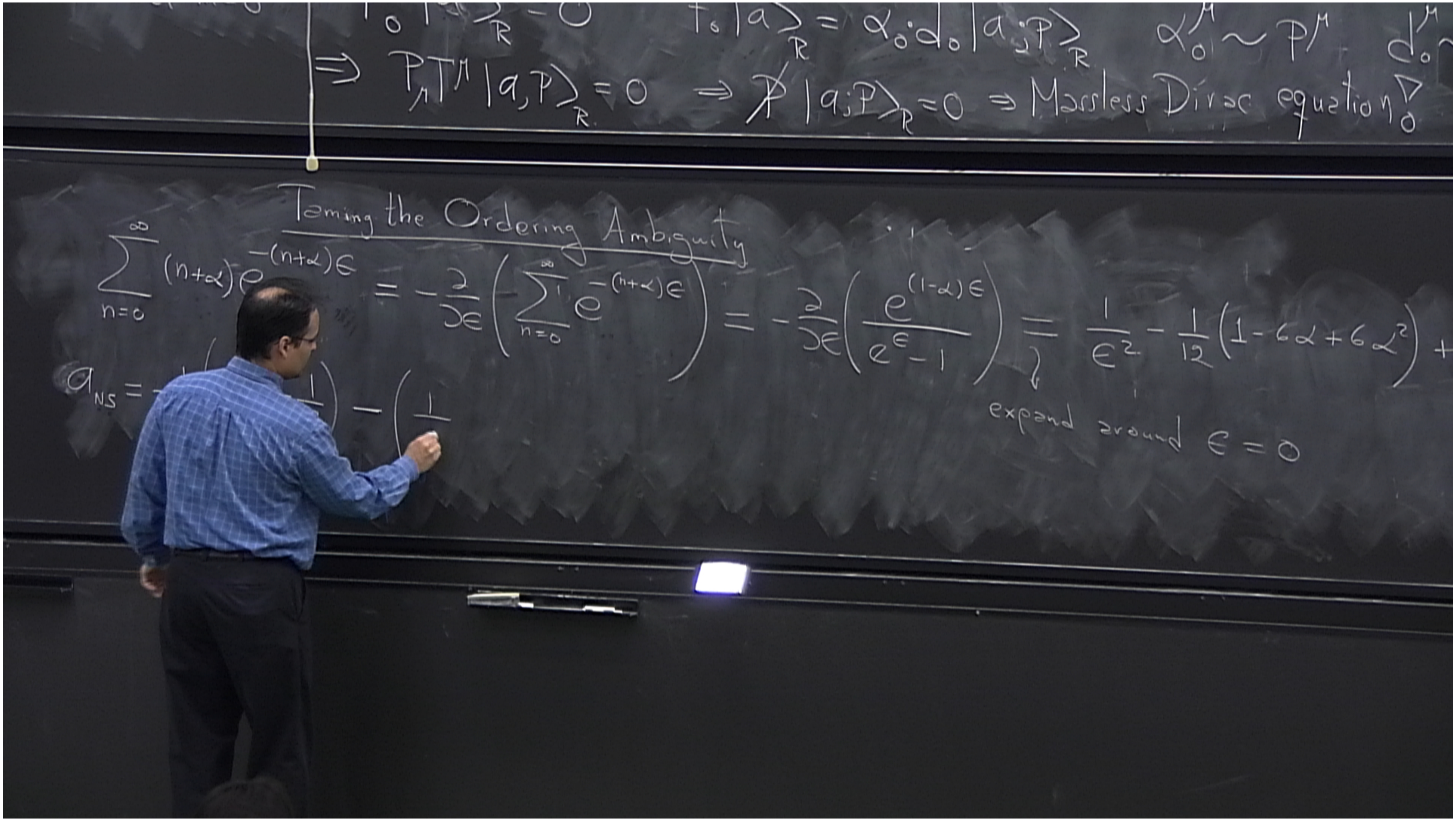
$$\parallel$$

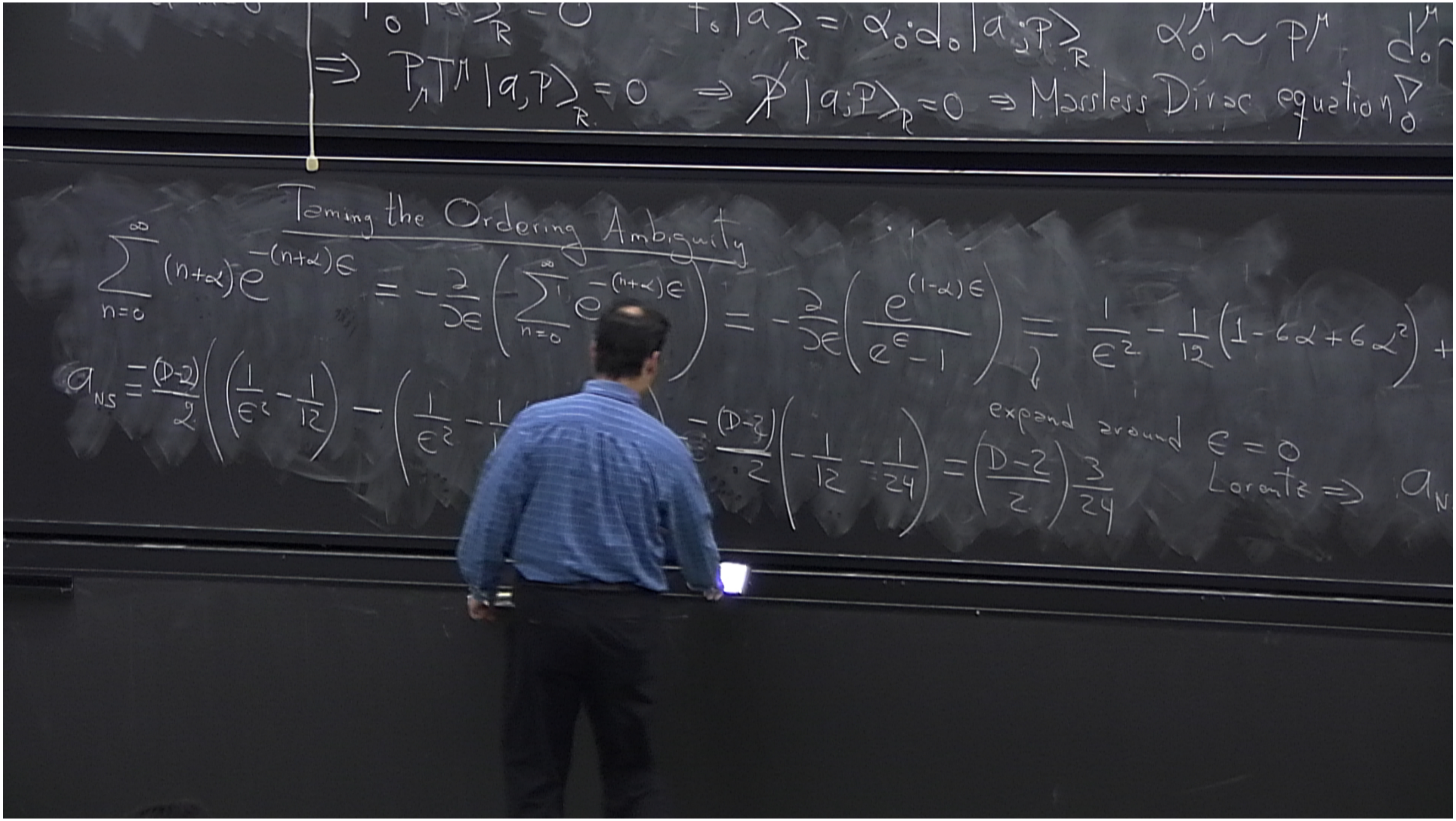
$$N_{NS} - a_{NS}$$

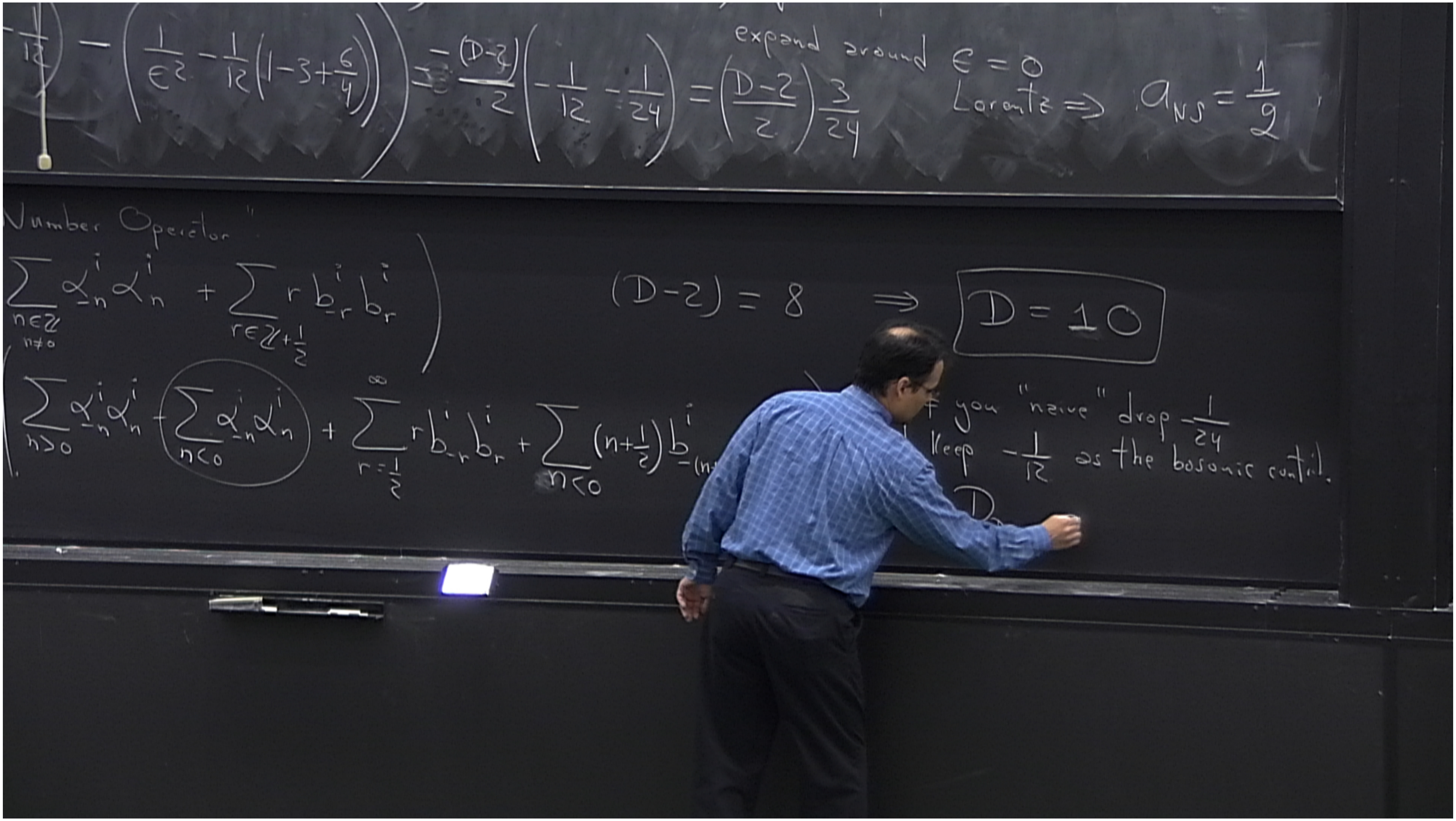
$$N_{NS}$$

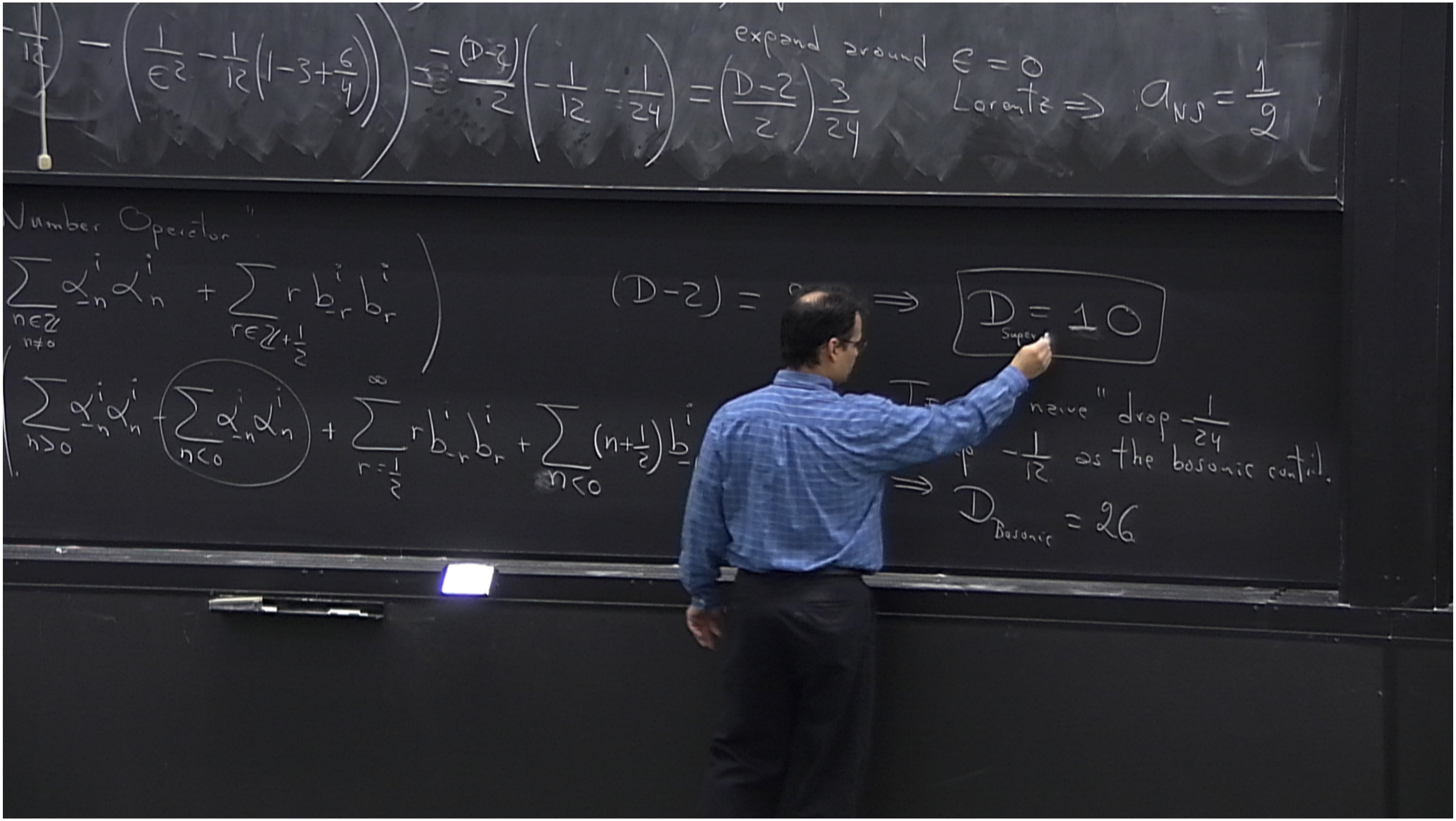
$$-a_{NS} = \frac{1}{2}\sum_{i=1}^{D-2}\left(\sum_{n=1}^{\infty} n e^{-n\epsilon} - \sum_{n=0}^{\infty}\left(n+\tfrac{1}{2}\right) e^{-(n+\frac{1}{2})\epsilon}\right)$$


⇒ Massless Dirac equation?
Taming the Ordering Ambiguity
and ε = 0


⇒ Massless Dirac equation
Taming the Ordering Ambiguity
expand around ε = 0




expand around ε = 0
Lorentz ⇒ a_NS = 1/2
Number Operator
(D-2) = 8 ⇒ D = 10
"naive" drop -1/24
keep -1/12 as the bosonic contrib.


expand around ε = 0
Lorentz ⇒ a_NS = 1/2
Number Operator
(D−2) =
D_Super = 10
"naive" drop −1/24
−1/12 as the bosonic contrib.
D_Bosonic = 26

$$\frac{1}{12}\Big) - \left(\frac{1}{\epsilon^2} - \frac{1}{12}\left(1 - 3 + \frac{6}{4}\right)\right)\Bigg) = -\frac{(D-2)}{2}\left(-\frac{1}{12} - \frac{1}{24}\right) = \left(\frac{D-2}{2}\right)\frac{3}{24}$$

expand around $\epsilon = 0$

Lorentz $\Rightarrow$ $a_{NS} = \frac{1}{2}$

"Number Operator"

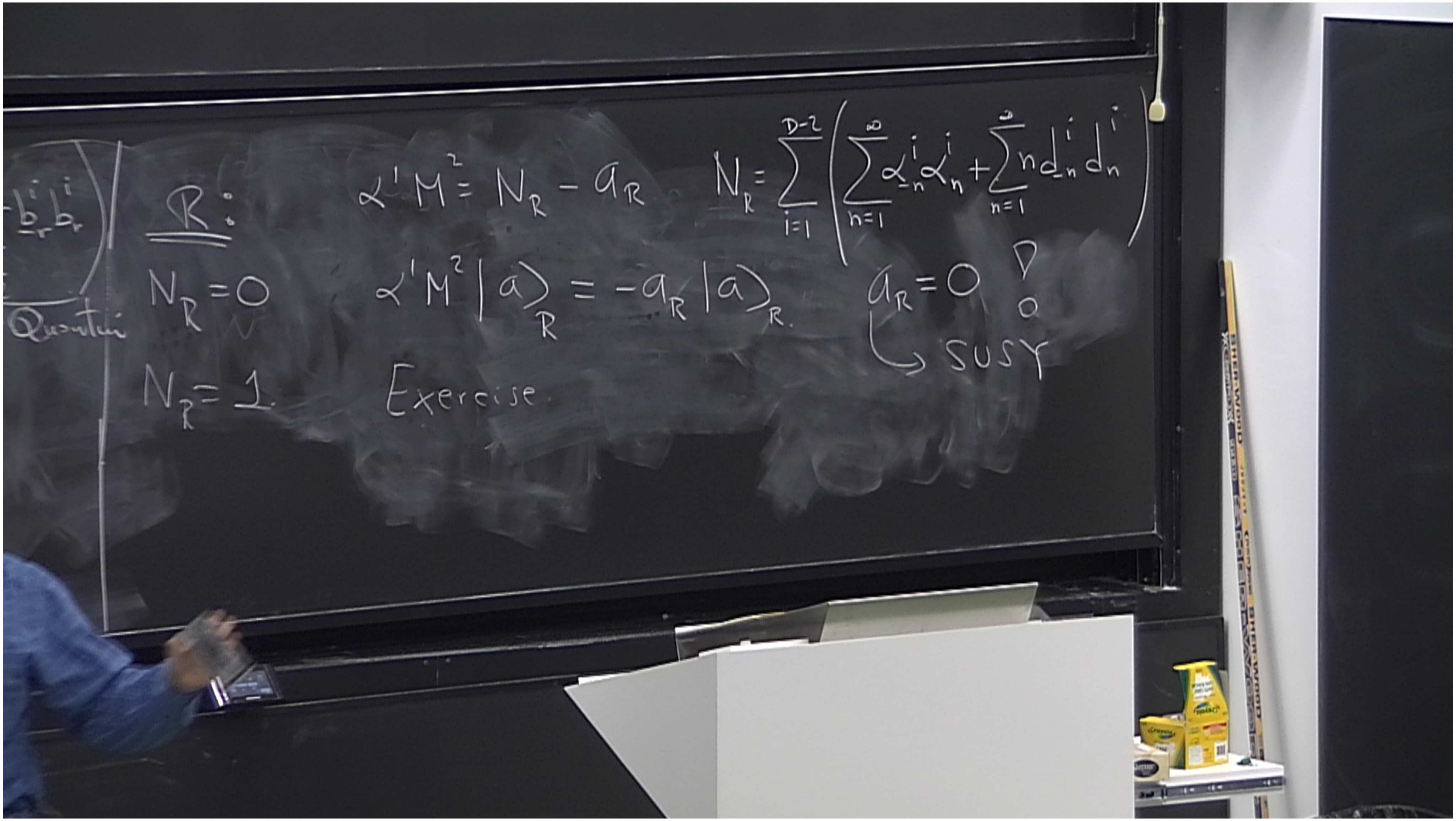
$$\sum_{\substack{n\in\mathbb{Z}\\ n\neq 0}} \alpha^i_{-n}\alpha^i_n + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r\, b^i_{-r} b^i_r \Bigg)$$

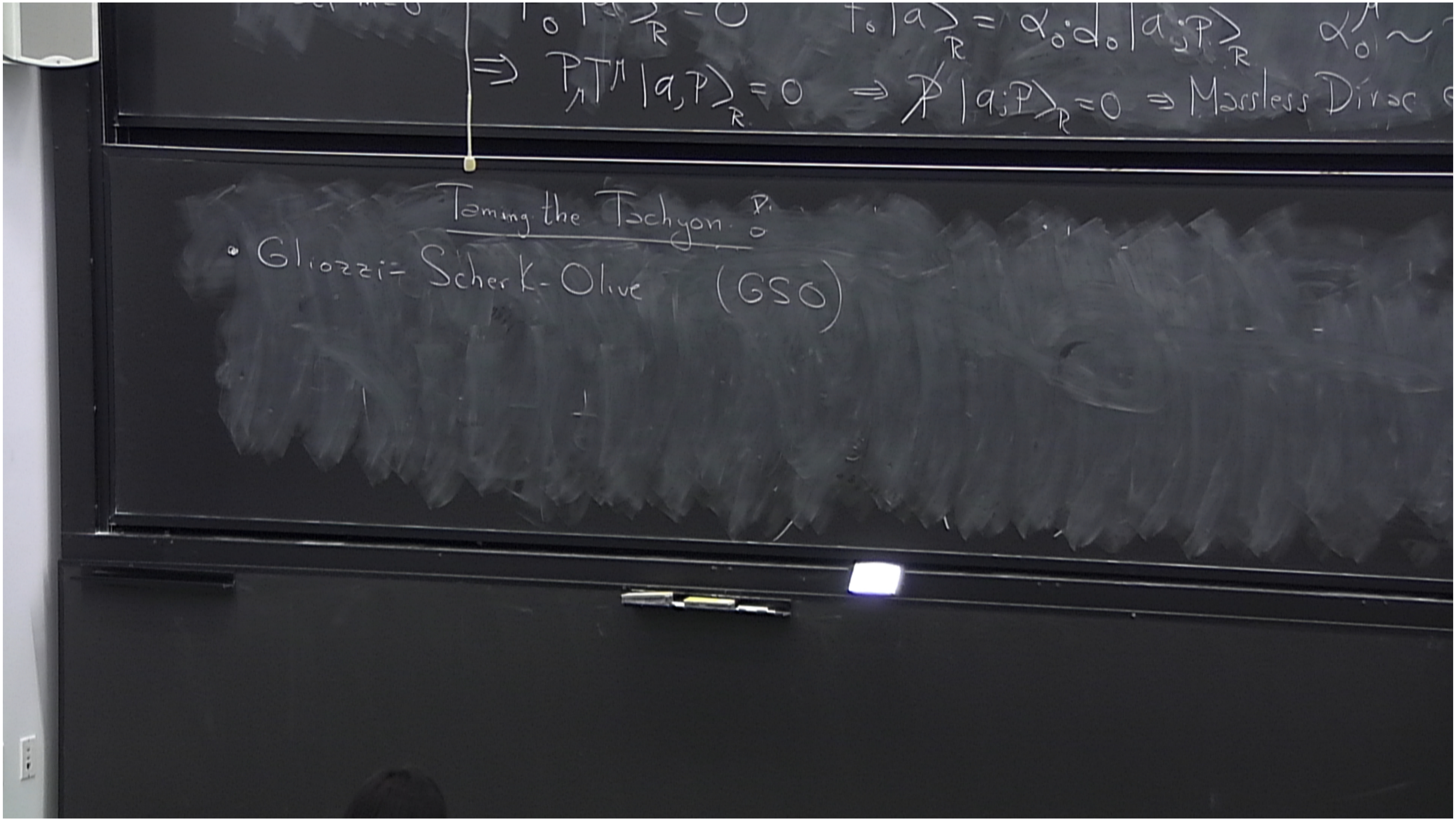
$$(D-2) = 8 \Rightarrow \boxed{D_{\text{Super}} = 10}$$

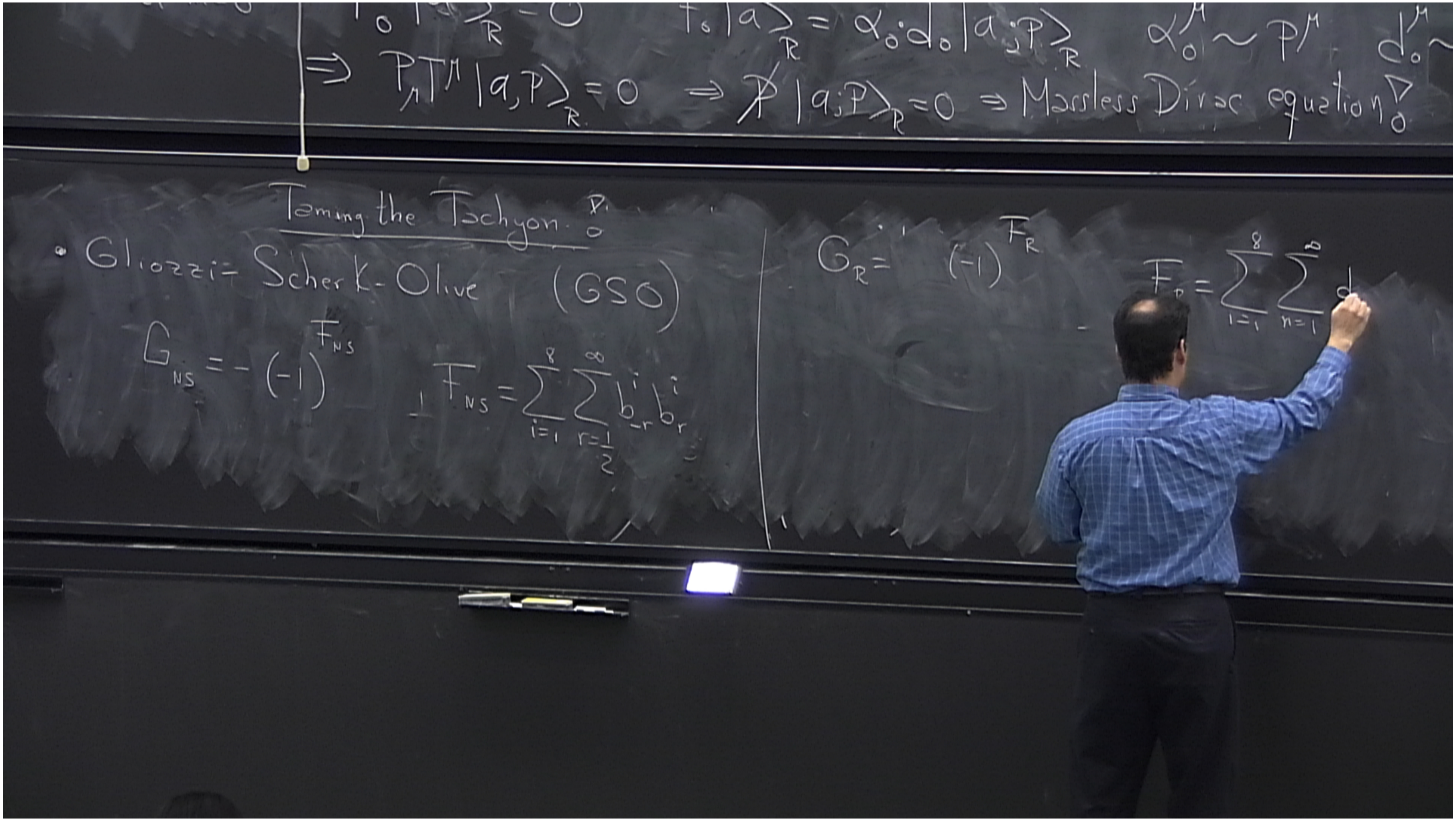
$$\Bigg(\sum_{n>0} \alpha^i_{-n}\alpha^i_n + \sum_{n<0} \alpha^i_{-n}\alpha^i_n + \sum_{r=\frac{1}{2}}^{\infty} r\, b^i_{-r} b^i_r + \sum_{n<0} \left(n+\tfrac{1}{2}\right) b^i_{-(n+\frac{1}{2})} b^i_{(n+\frac{1}{2})}\Bigg)$$

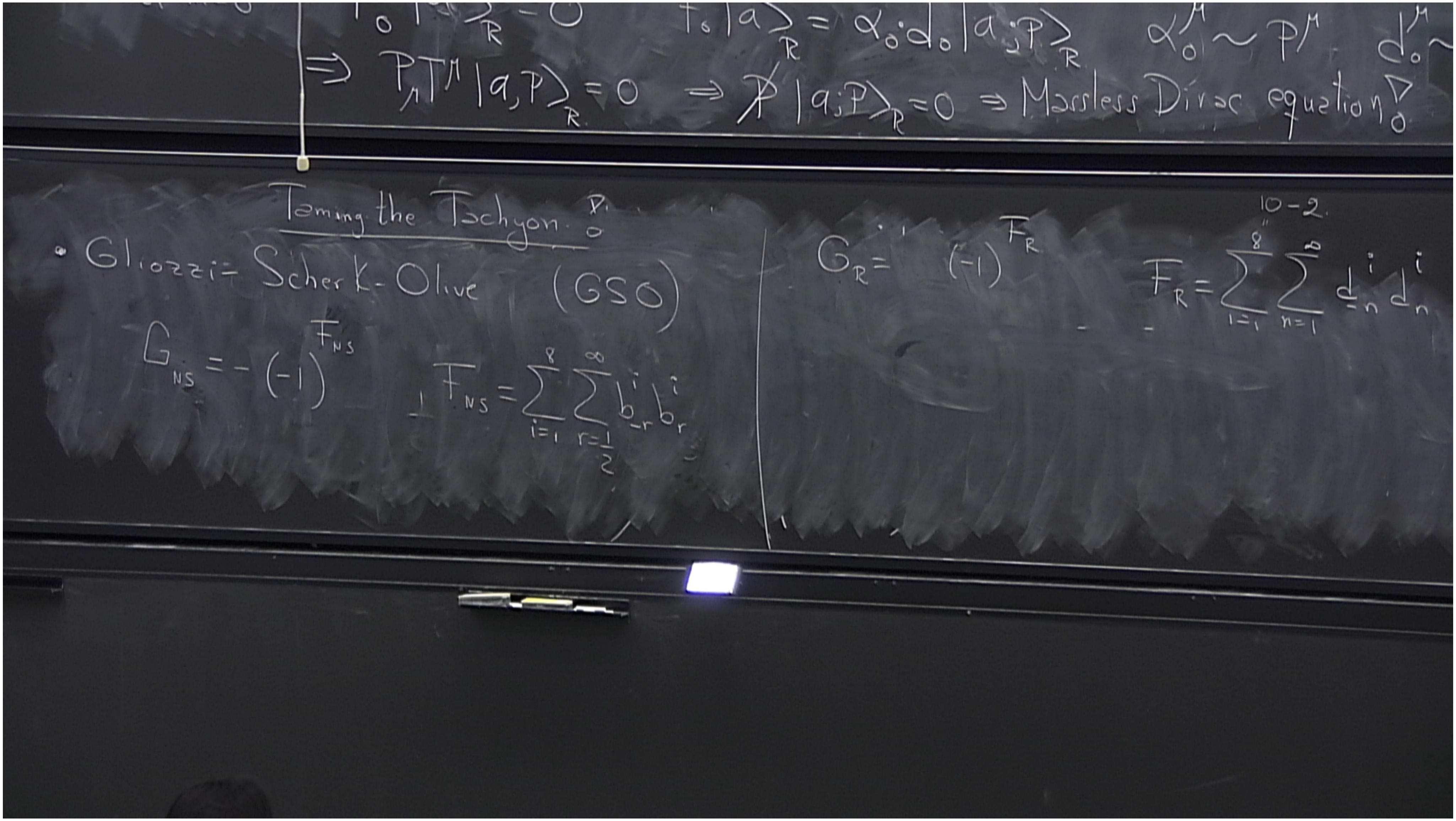
If you "naive" drop $-\frac{1}{24}$ and keep $-\frac{1}{12}$ as the bosonic contrib.

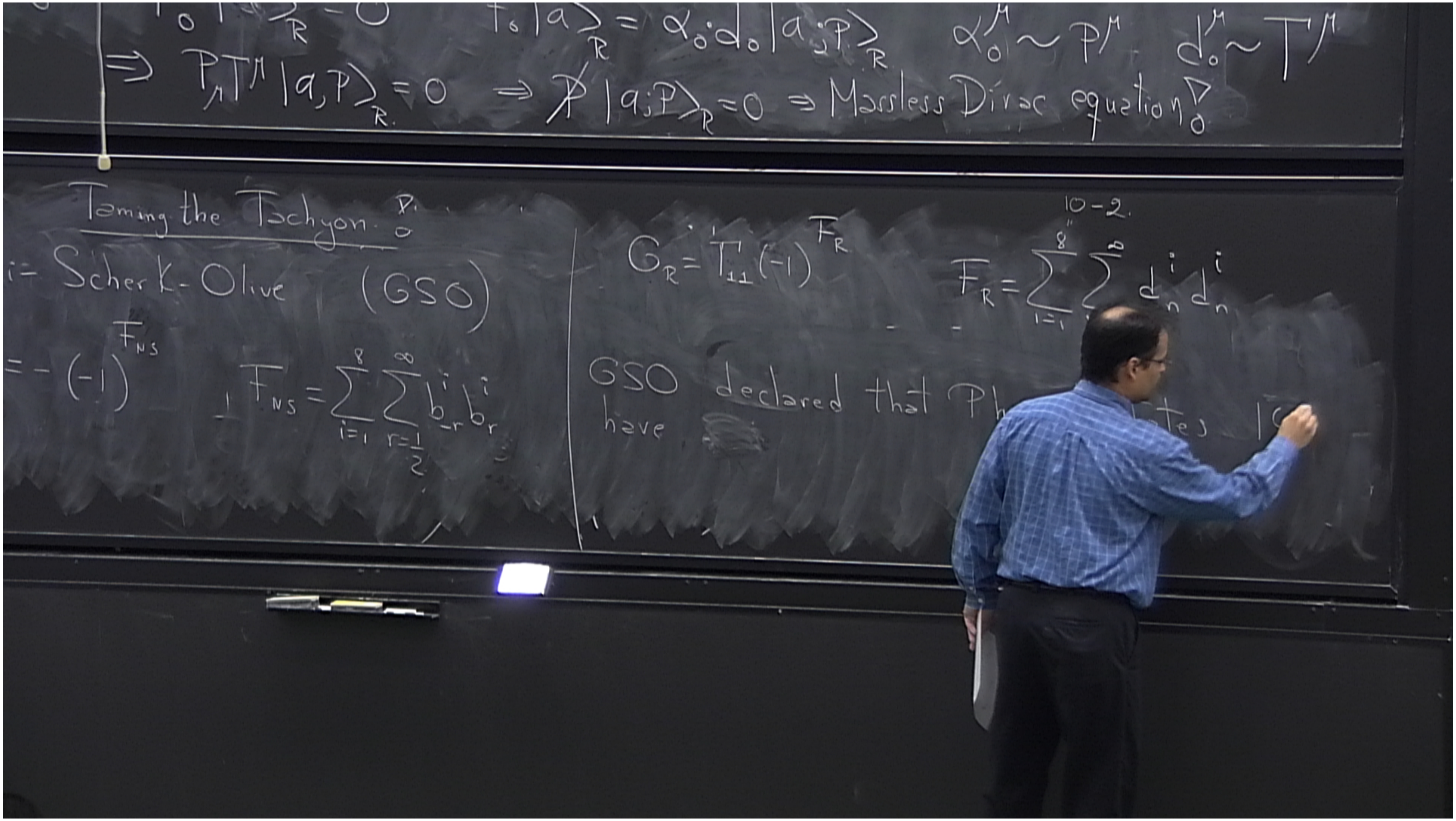
$\Rightarrow D_{\text{Bosonic}} = 26 \quad a = 1$


R :
$\alpha' M^2 = N_R - a_R$
$N_R = \sum_{i=1}^{D-2} \left(\sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{n=1}^{\infty} n d_{-n}^i d_n^i \right)$
$N_R = 0$
$\alpha' M^2 |a\rangle_R = -a_R |a\rangle_R$
$a_R = 0$
SUSY
$N_R = 1$
Exercise
Quantu

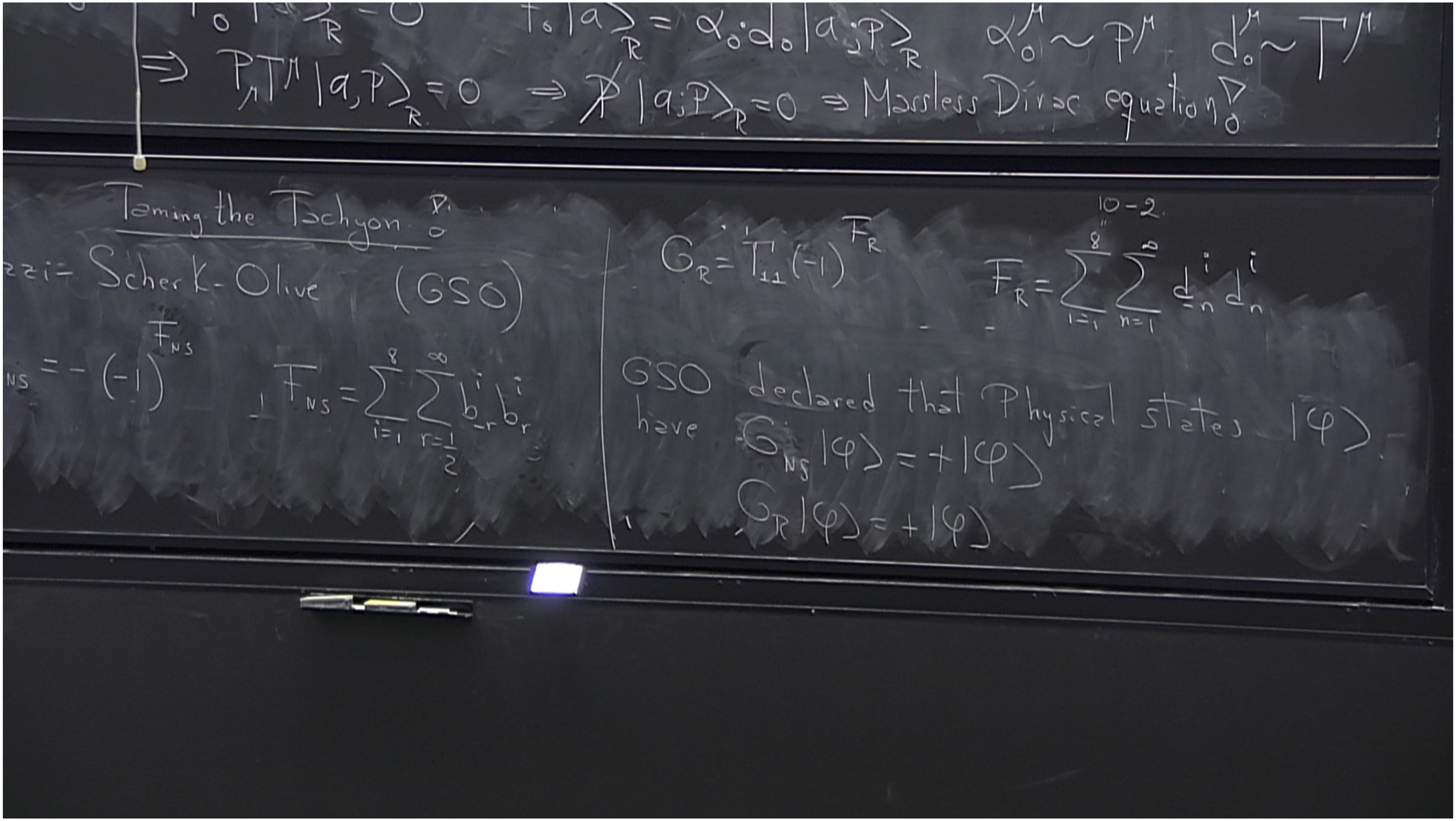

⇒ Massless Dirac
Taming the Tachyon
• Gliozzi-Scherk-Olive (GSO)

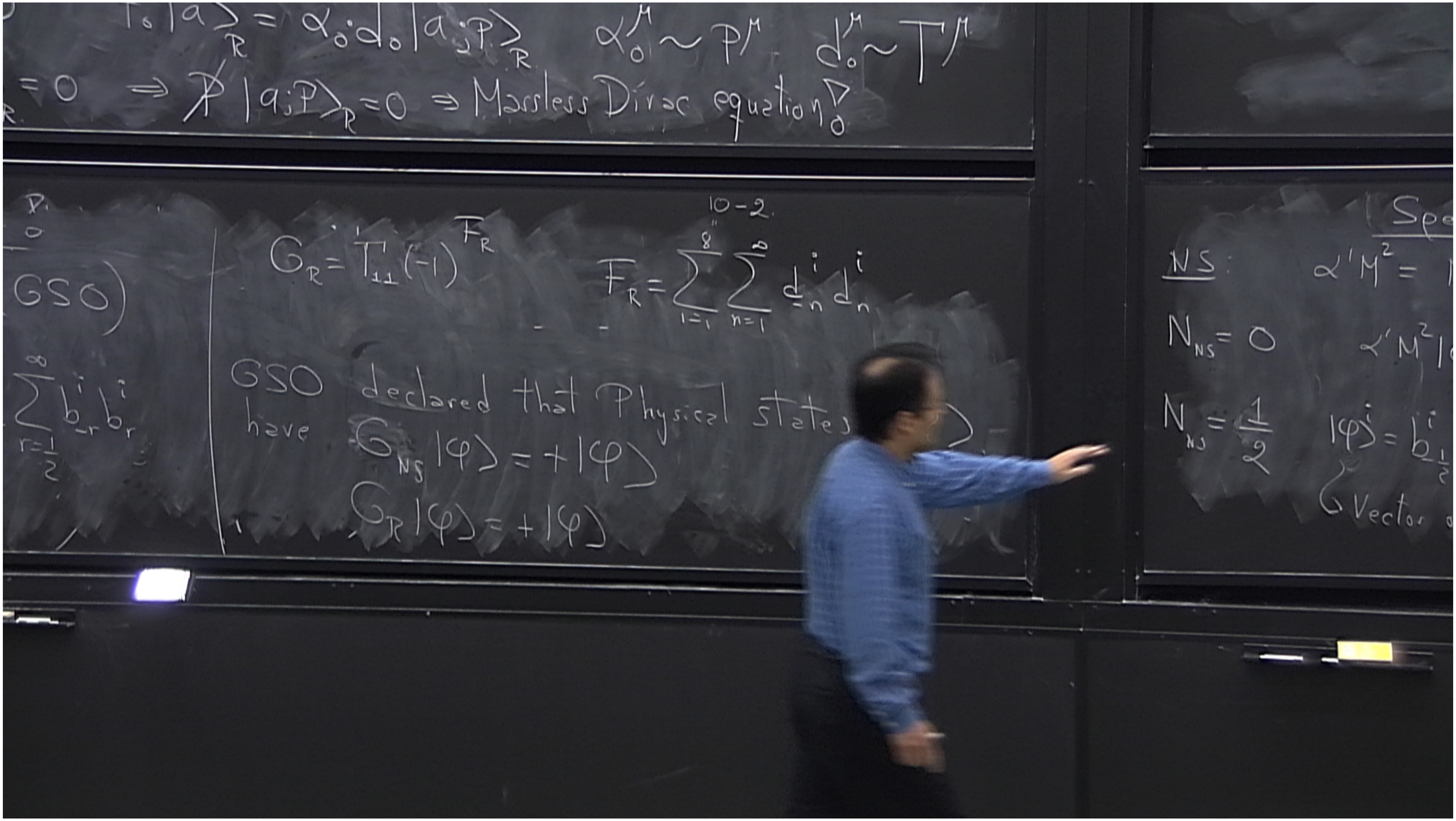

⇒ Massless Dirac equation
Taming the Tachyon
Gliozzi-Scherk-Olive (GSO)

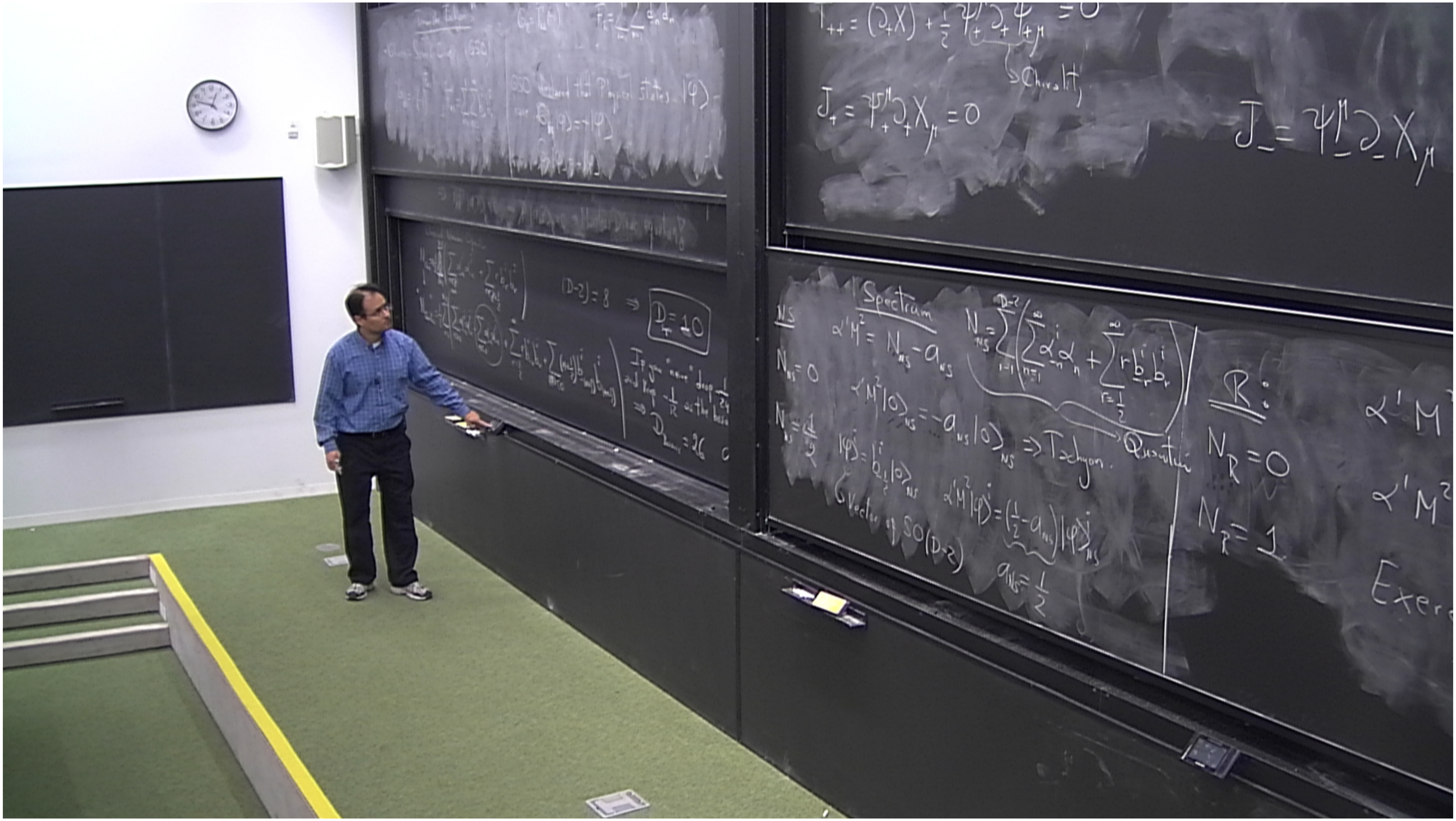

⇒ $p_\mu \Gamma^\mu |a,p\rangle_R = 0$ ⇒ $\not{p}|a;p\rangle_R = 0$ ⇒ Massless Dirac equation!
Taming the Tachyon.
• Gliozzi-Scherk-Olive (GSO)
$G_{NS} = -(-1)^{F_{NS}}$
$F_{NS} = \sum_{i=1}^{8} \sum_{r=\frac{1}{2}}^{\infty} b^i_{-r} b^i_r$
$G_R = (-1)^{F_R}$
$F_R = \sum_{i=1}^{8} \sum_{n=1}^{\infty} d^i_{-n} d^i_n$

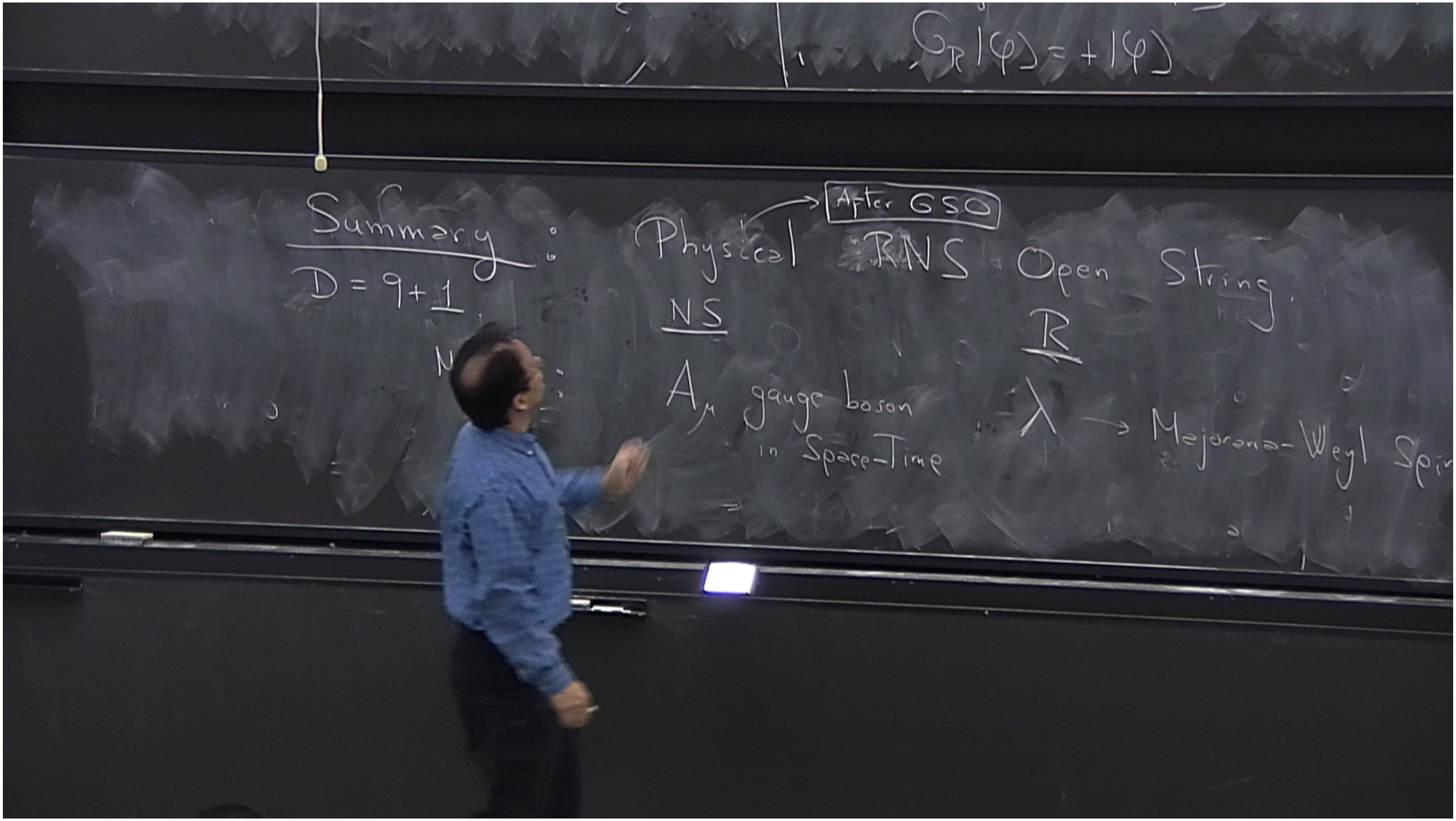


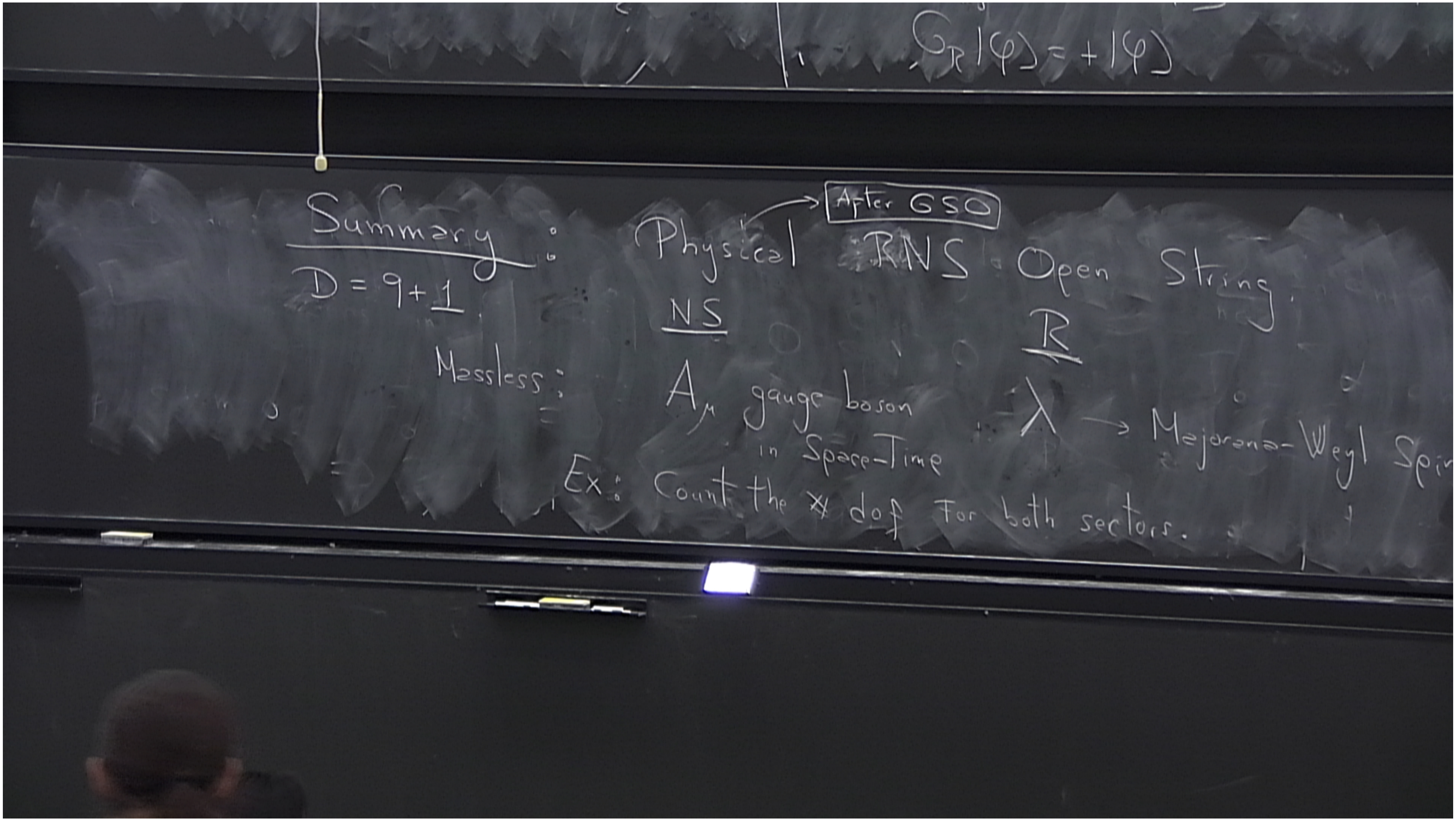
Massless Dirac equation
Taming the Tachyon
Scherk-Olive (GSO)
GSO declared that
have


$F_0|a\rangle_R = 0$ $F_0|a\rangle_R = \alpha_0 \cdot d_0|a;P\rangle_R$ $\alpha_0^\mu \sim P^\mu$ $d_0^\mu \sim \Gamma^\mu$
$\Rightarrow P_\mu \Gamma^\mu |a,P\rangle_R = 0 \Rightarrow \not{P}|a;P\rangle_R = 0 \Rightarrow$ Massless Dirac equation
Taming the Tachyon
zzi- Scherk-Olive (GSO)
$_{NS} = -(-1)^{F_{NS}}$
$F_{NS} = \sum_{i=1}^{8}\sum_{r=\frac{1}{2}}^{\infty} b^i_{-r} b^i_r$
$G_R = \Gamma_{11}(-1)^{F_R}$
$F_R = \sum_{i=1}^{8}\sum_{n=1}^{\infty} d^i_{-n} d^i_n$
10-2
GSO declared that Physical states $|\varphi\rangle$
have $G_{NS}|\varphi\rangle = +|\varphi\rangle$
$G_R|\varphi\rangle = +|\varphi\rangle$


Massless Dirac equation
GSO declared that Physical states have
NS:
Vector


Spectrum
Tachyon
Exer


$G_R|\varphi\rangle = +|\varphi\rangle$
After GSO
Summary :
Physical RNS Open String :
D = 9+1
NS
R
A_μ gauge boson
in Space-Time
λ → Majorana-Weyl


$G_R|\varphi\rangle = +|\varphi\rangle$
After GSO
Summary:
Physical RNS Open String:
D = 9+1
NS
R
Massless:
A gauge boson in Space-Time
λ → Majorana-Weyl Spin
Ex: Count the # dof for both sectors.