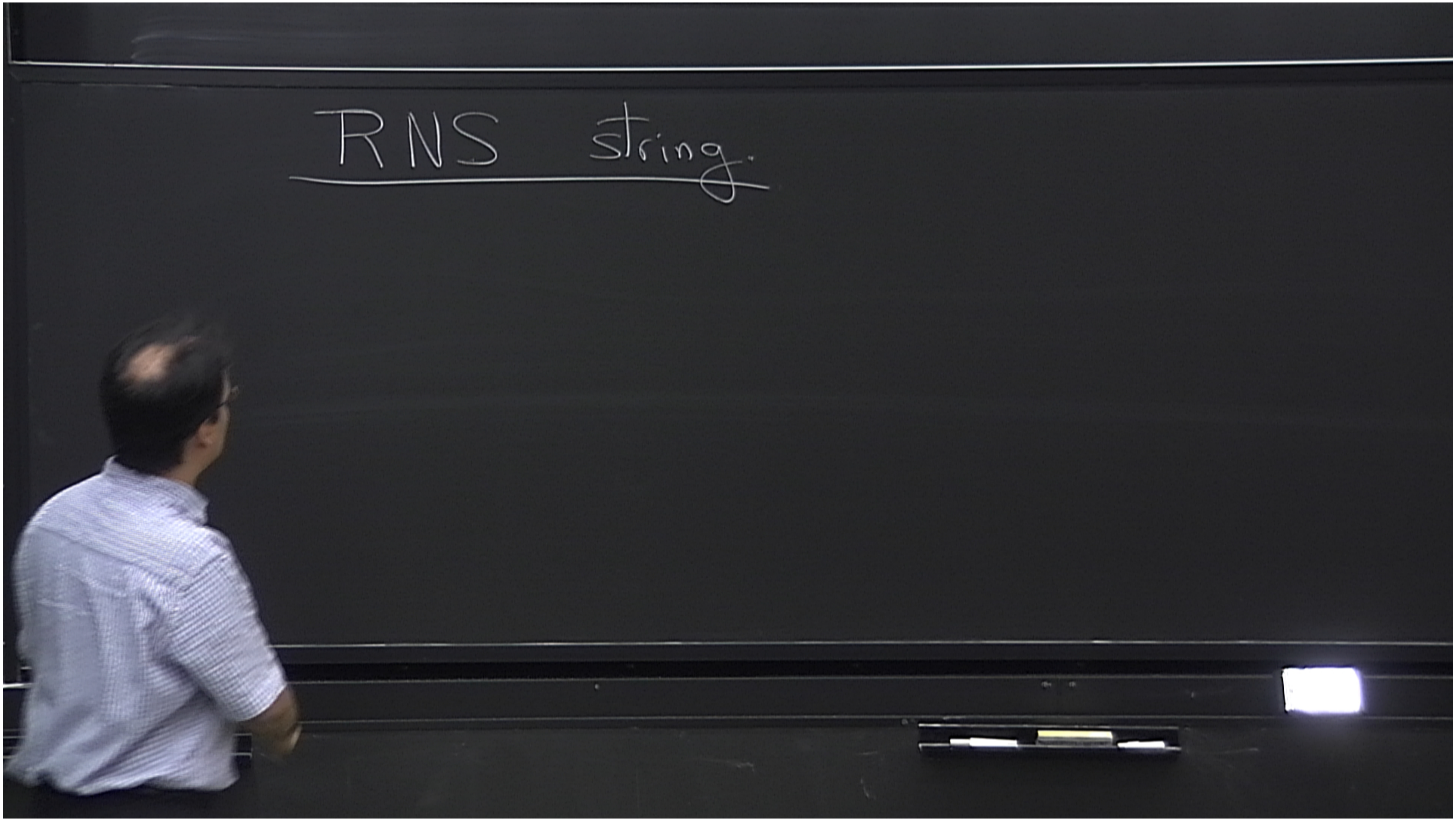
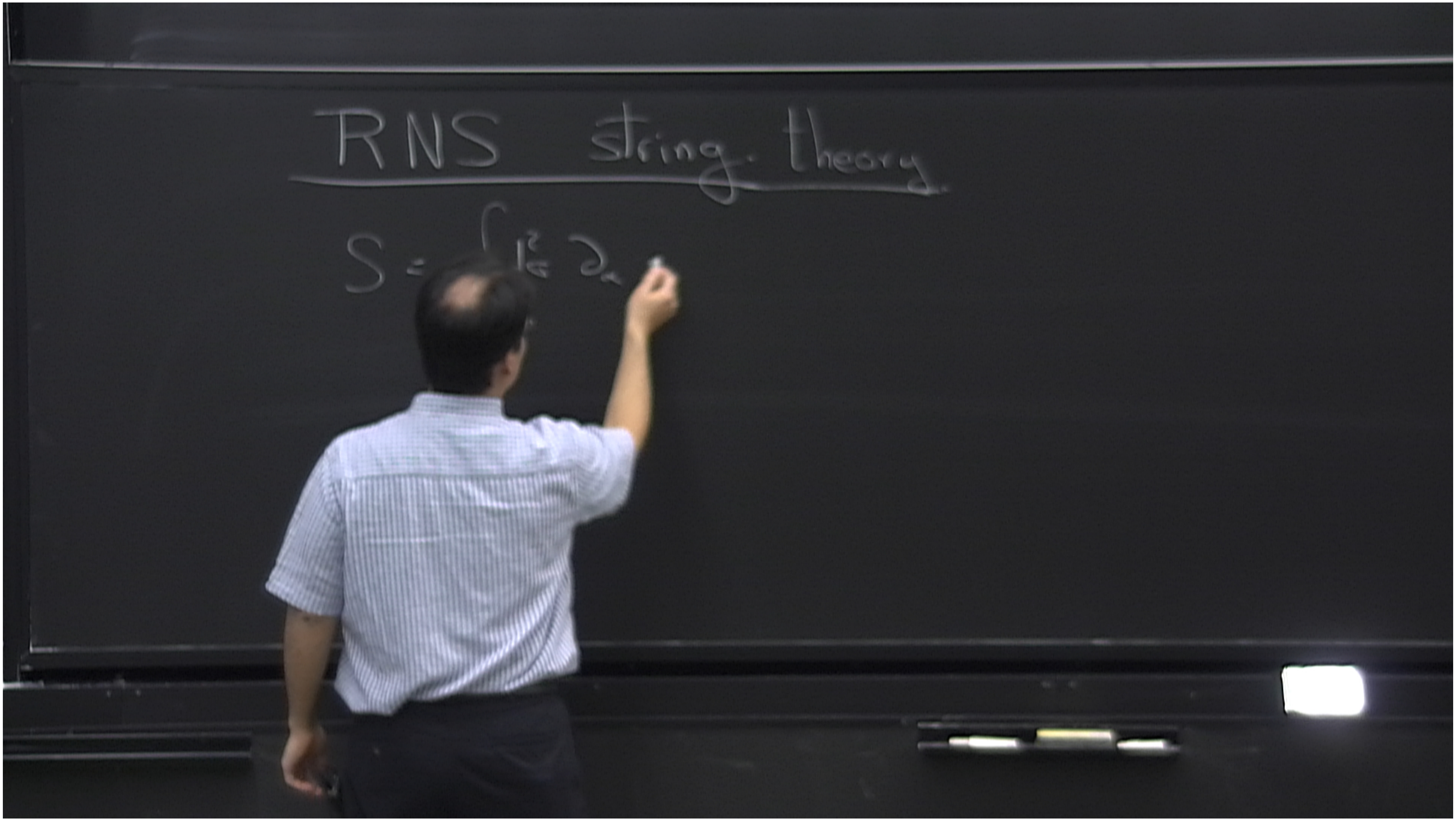
Title: String Theory (Review) - Lecture 10

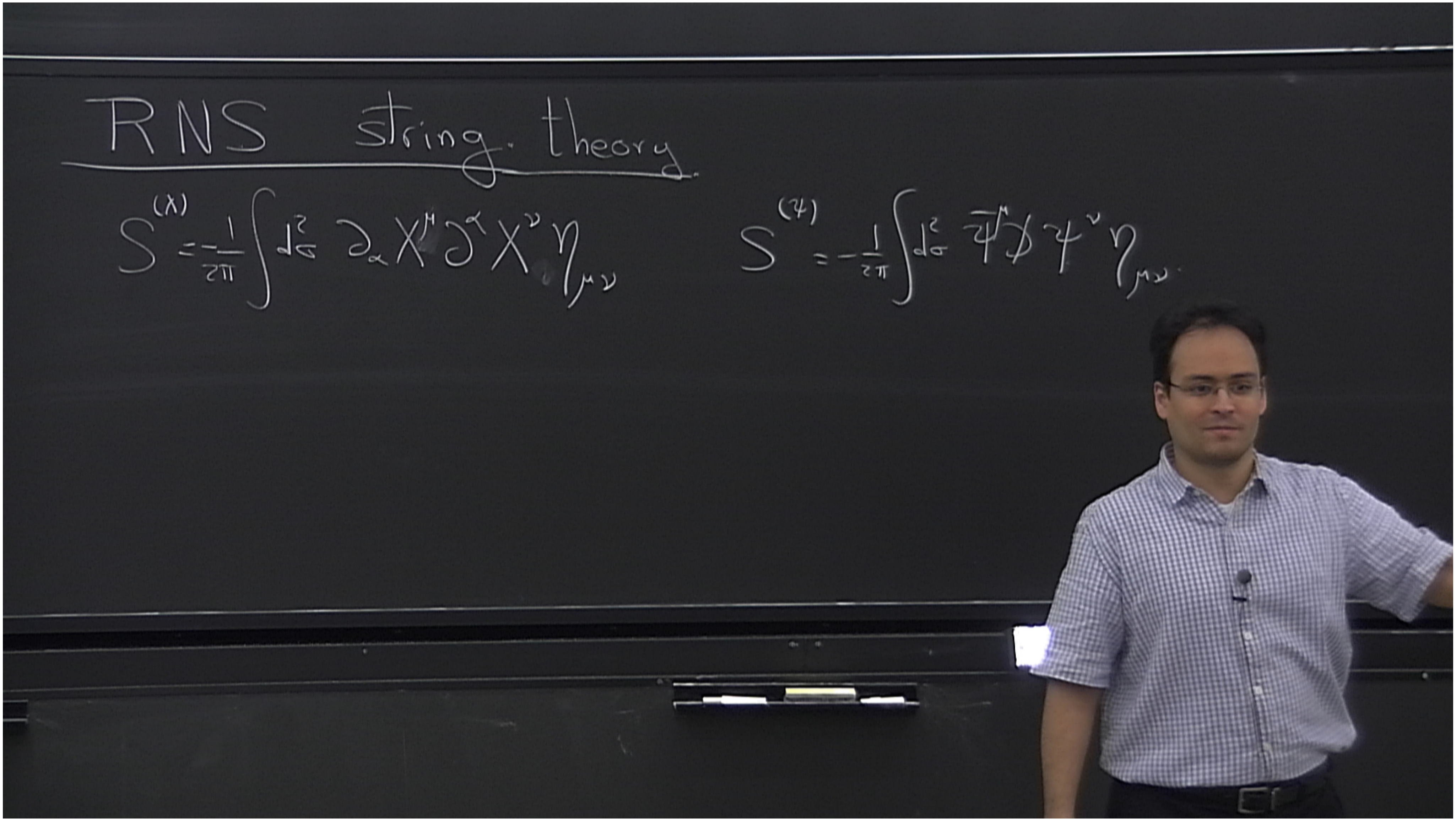
Date: Feb 27, 2012 11:30 AM

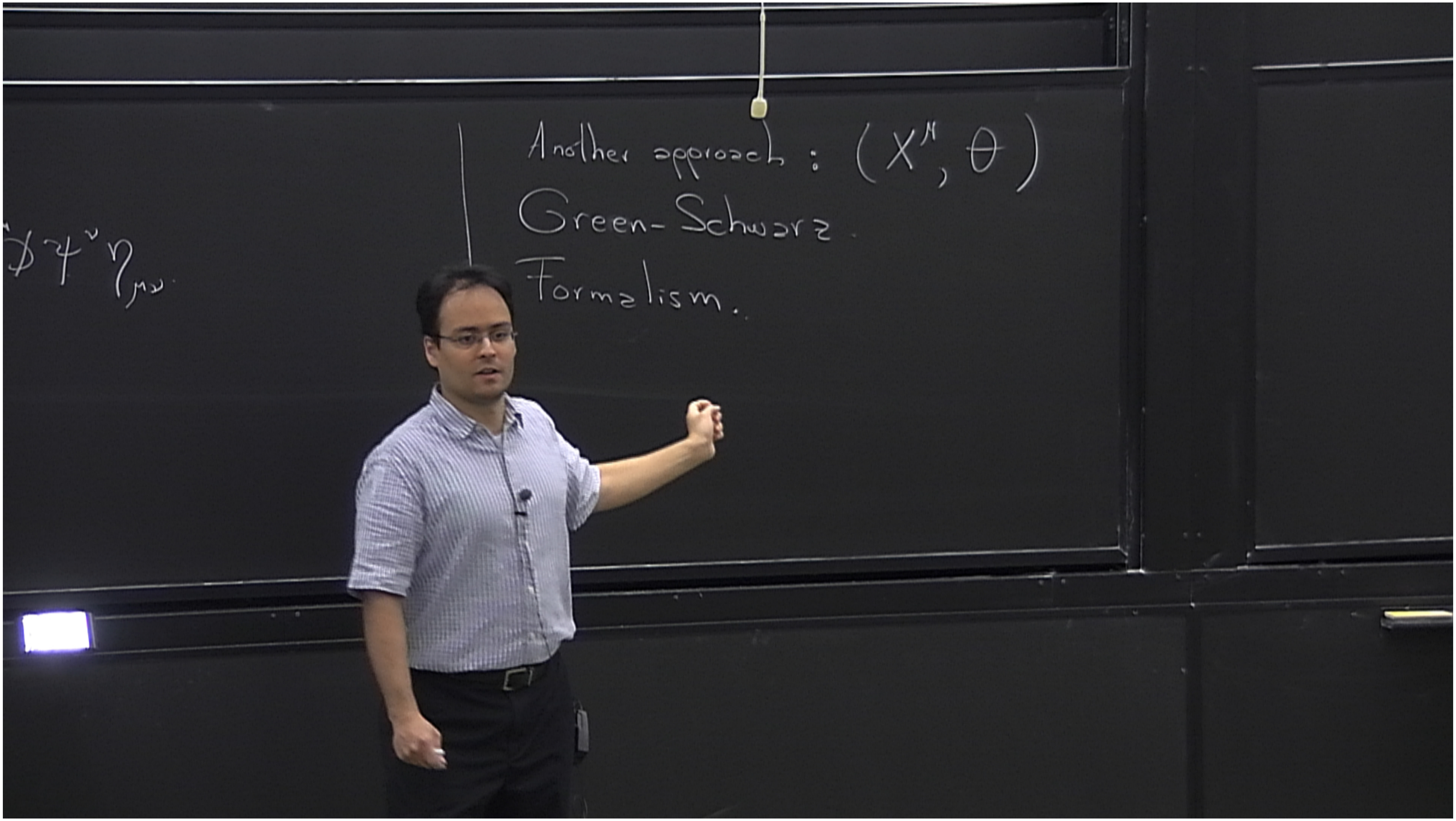
URL: http://pirsa.org/12020061

Abstract:


RNS string.


RNS string theory


RNS string theory
$S^{(X)} = -\frac{1}{2\pi}\int d^2\sigma \, \partial_\alpha X^\mu \partial^\alpha X^\nu \eta_{\mu\nu}$
$S^{(\psi)} = -\frac{1}{2\pi}\int d^2\sigma \, \bar{\psi}^\mu \not{\partial} \psi^\nu \eta_{\mu\nu}$


Another approach : (X^μ, θ)
Green-Schwarz
Formalism.

RNS string theory

$$S^{(X)} = -\frac{1}{2\pi}\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X^\nu \eta_{\mu\nu}$$

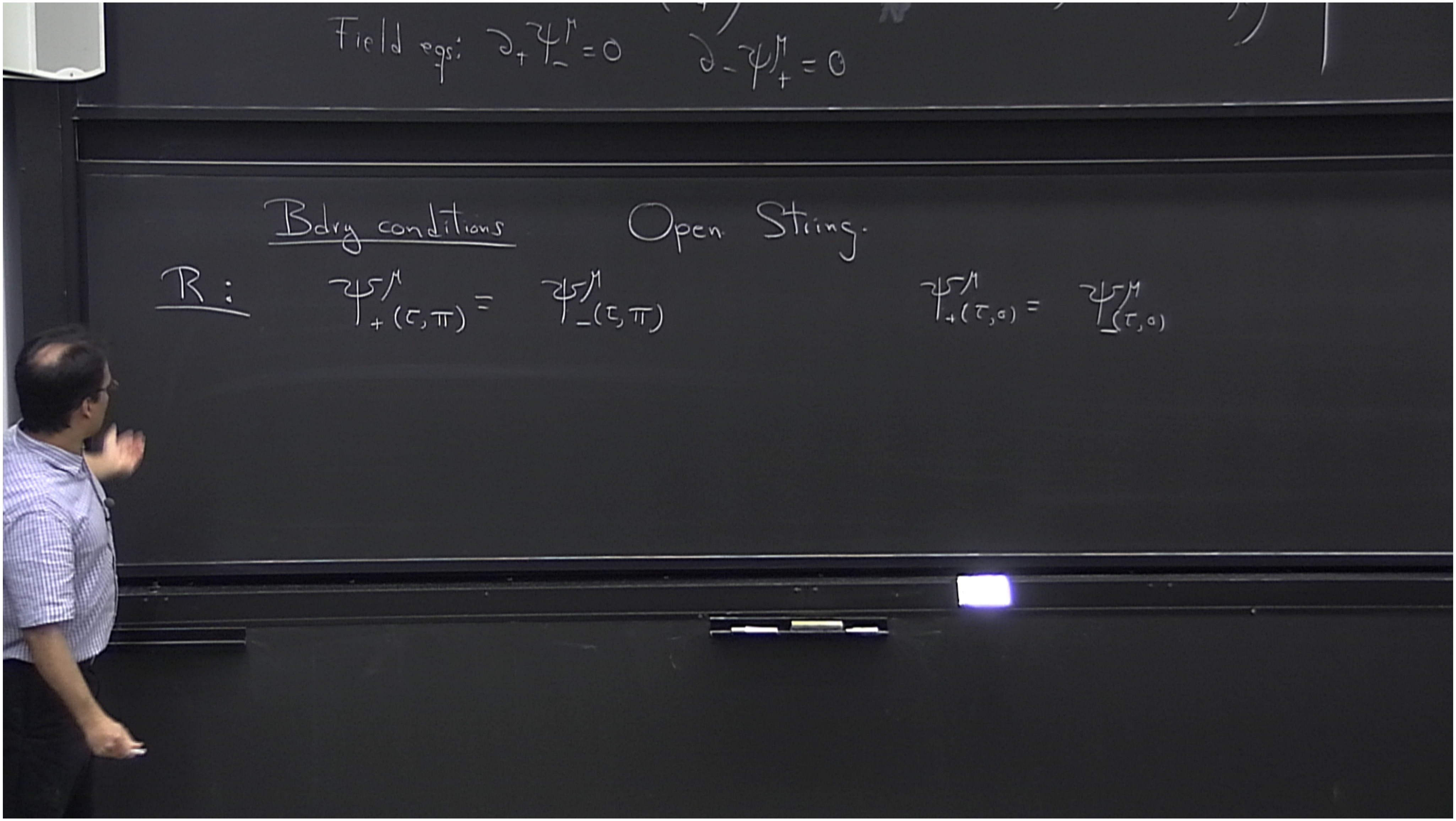
$$S^{(\psi)} = -\frac{1}{2\pi}\int d^2\sigma\, \bar{\psi}^\mu \not{\partial} \psi^\nu \eta_{\mu\nu}$$

Majorana-Weyl $\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$

$$\int d\sigma_+ d\sigma_- \left(\psi_-^\mu \partial_+ \psi_{-,\mu} + \psi_+^\mu \partial_- \psi_{+\mu} \right)$$

Field eqs: $\partial_+ \psi_-^\mu = 0$ $\quad \partial_- \psi_+^\mu = 0$

Bdr


Field eqs: $\partial_+ \psi^\mu_- = 0 \quad \partial_- \psi^\mu_+ = 0$
Bdry conditions
Open String.
R :
$\psi^\mu_+(\tau,\pi) = \psi^\mu_-(\tau,\pi)$
$\psi^\mu_+(\tau,0) = \psi^\mu_-(\tau,0)$

NS: $\psi^\mu_+(\tau,\pi) = -\psi^\mu_-(\tau,\pi)$ $\psi^\mu_+(\tau,0) = \psi^\mu_-(\tau,0)$

Bdry conditions   Open String.

R:

$$\psi^\mu_+(\tau,\pi) = +\psi^\mu_-(\tau,\pi) \qquad \psi^\mu_+(\tau,0) = \psi^\mu_-(\tau,0)$$

$$\psi^\mu_+(\sigma_+) = \sum_{n\in\mathbb{Z}} d^\mu_n e^{-in\sigma_+}$$

$$\psi^\mu_-(\sigma_-) = \sum_{n\in\mathbb{Z}} d'^\mu_n e^{-in\sigma_-}$$

$$\Rightarrow d^\mu_n = d'^\mu_n$$

$$\boxed{d^{\mu\dagger}_n = d^\mu_{-n}}$$

NS: $\psi^{\mu}_{+}(\tau,\pi) = -\psi^{\mu}_{-}(\tau,\pi)$

$$\psi^{\mu}_{+}(\sigma_+) = \sum_r b^{\mu}_r e^{-in\sigma_+}$$

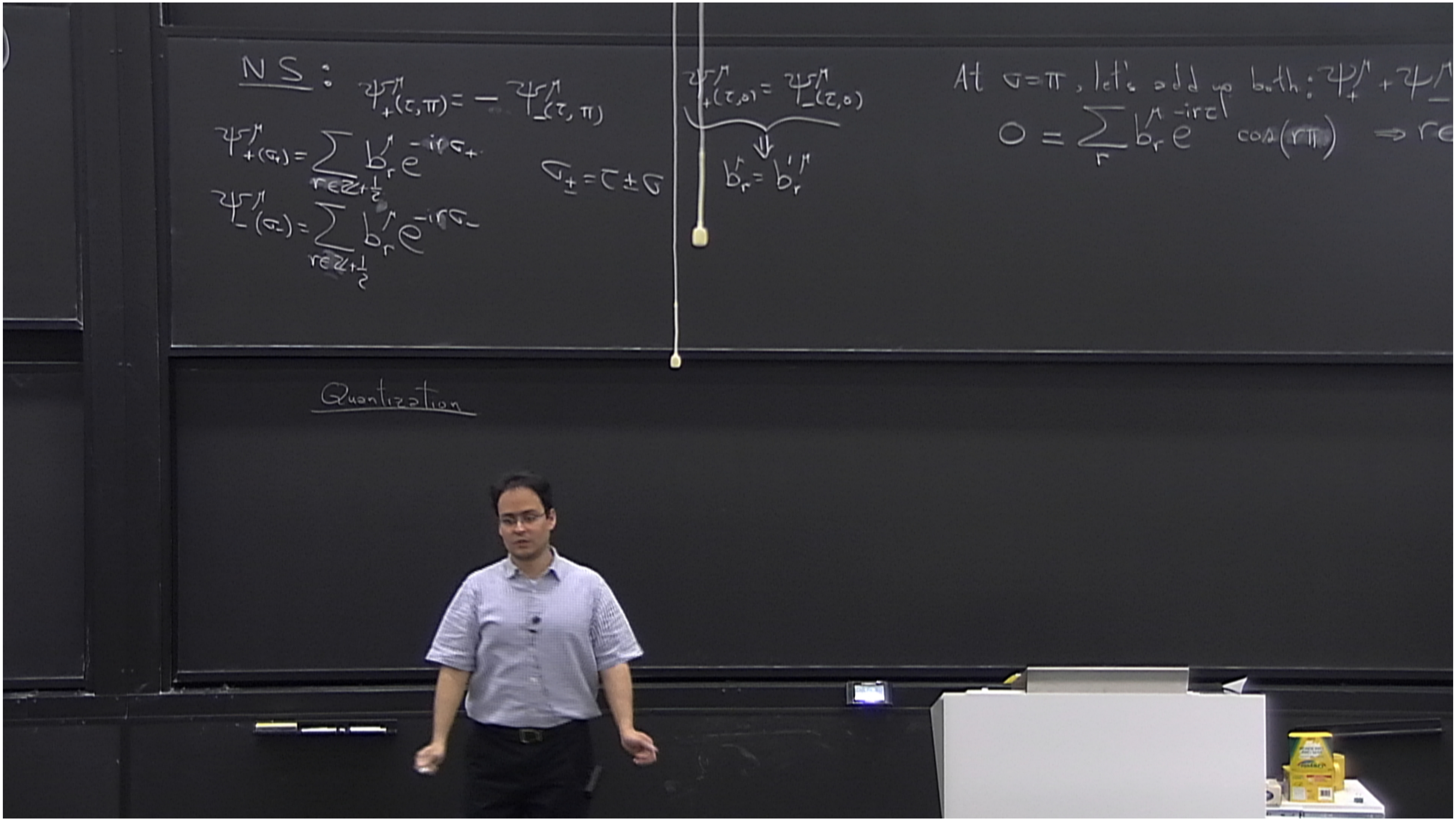
$$\psi^{\mu}_{-}(\sigma_-) = \sum_r b'^{\mu}_r e^{-in\sigma_-}$$

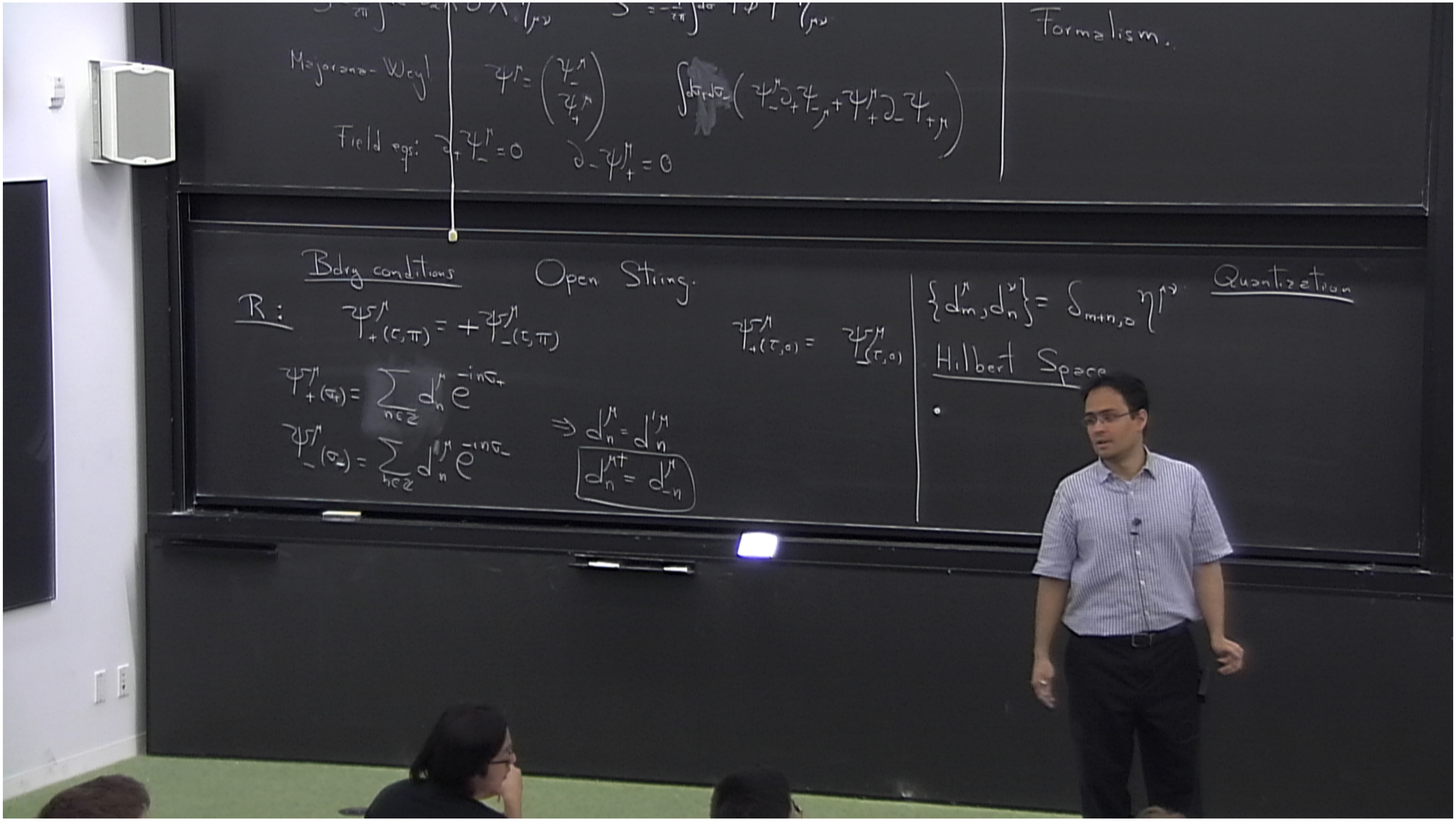
$$\underbrace{\psi^{\mu}_{+}(\tau,0) = \psi^{\mu}_{-}(\tau,0)}$$

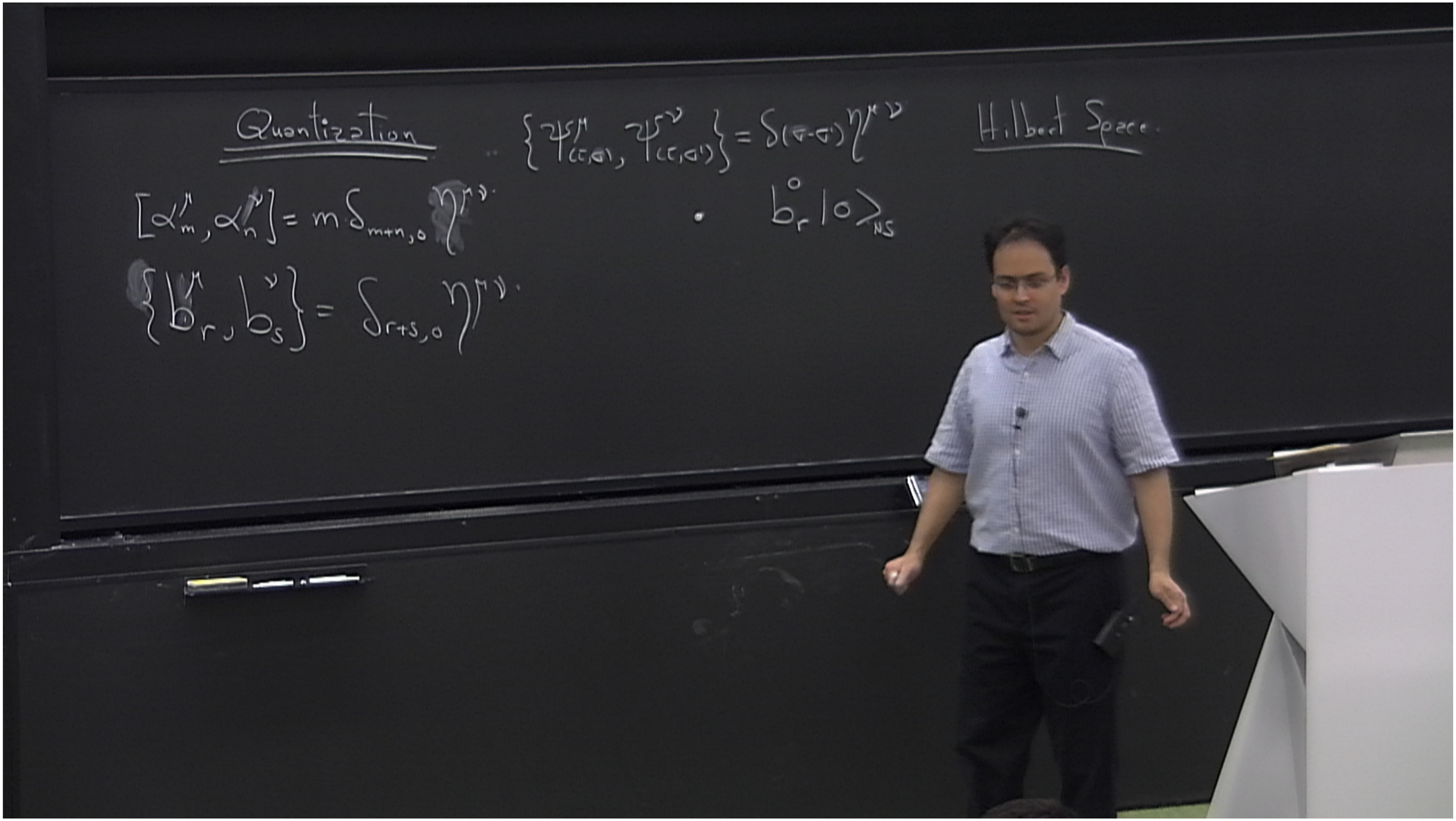
$$\Downarrow$$

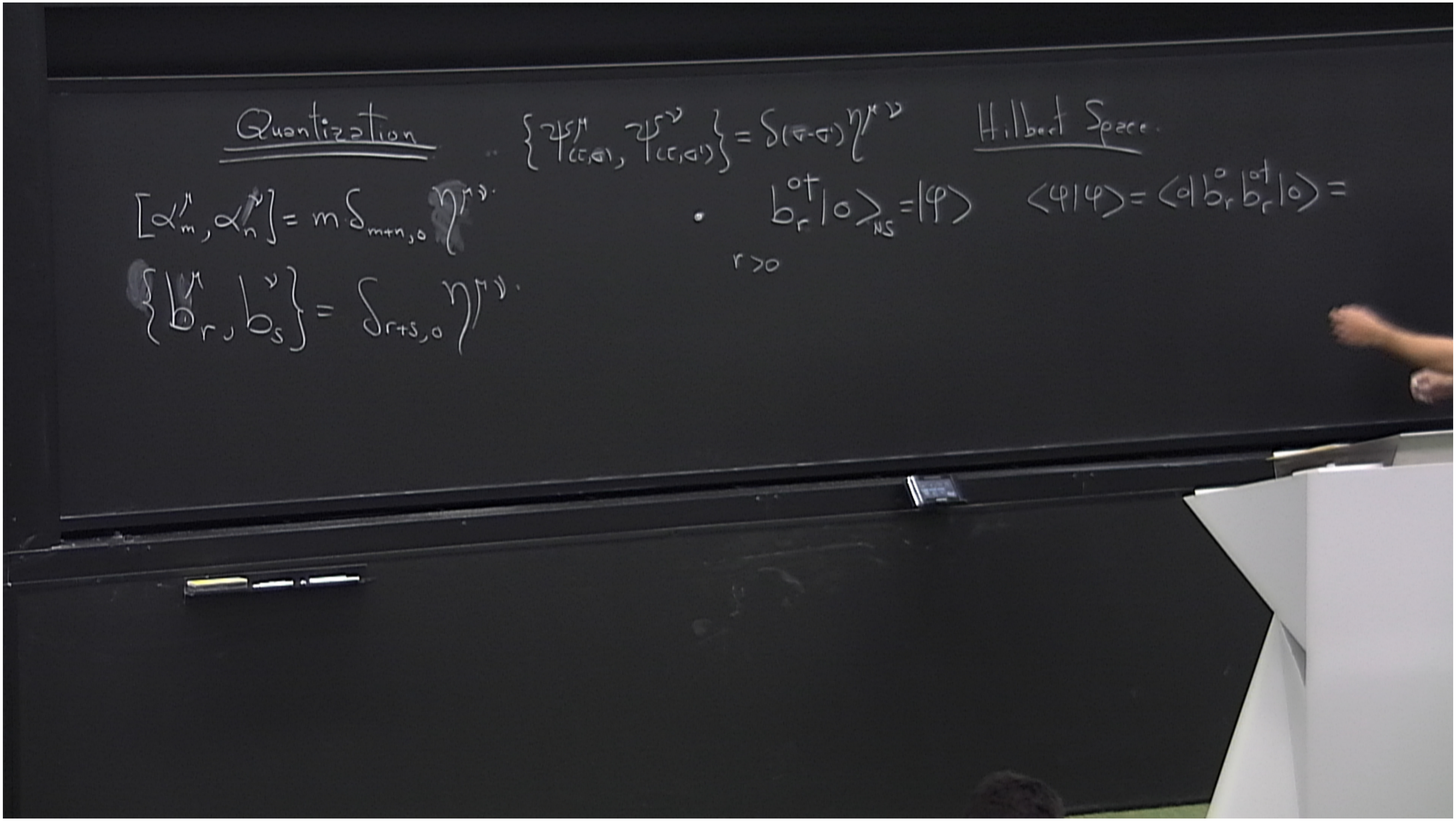
$$b^{\mu}_r = b'^{\mu}_r$$

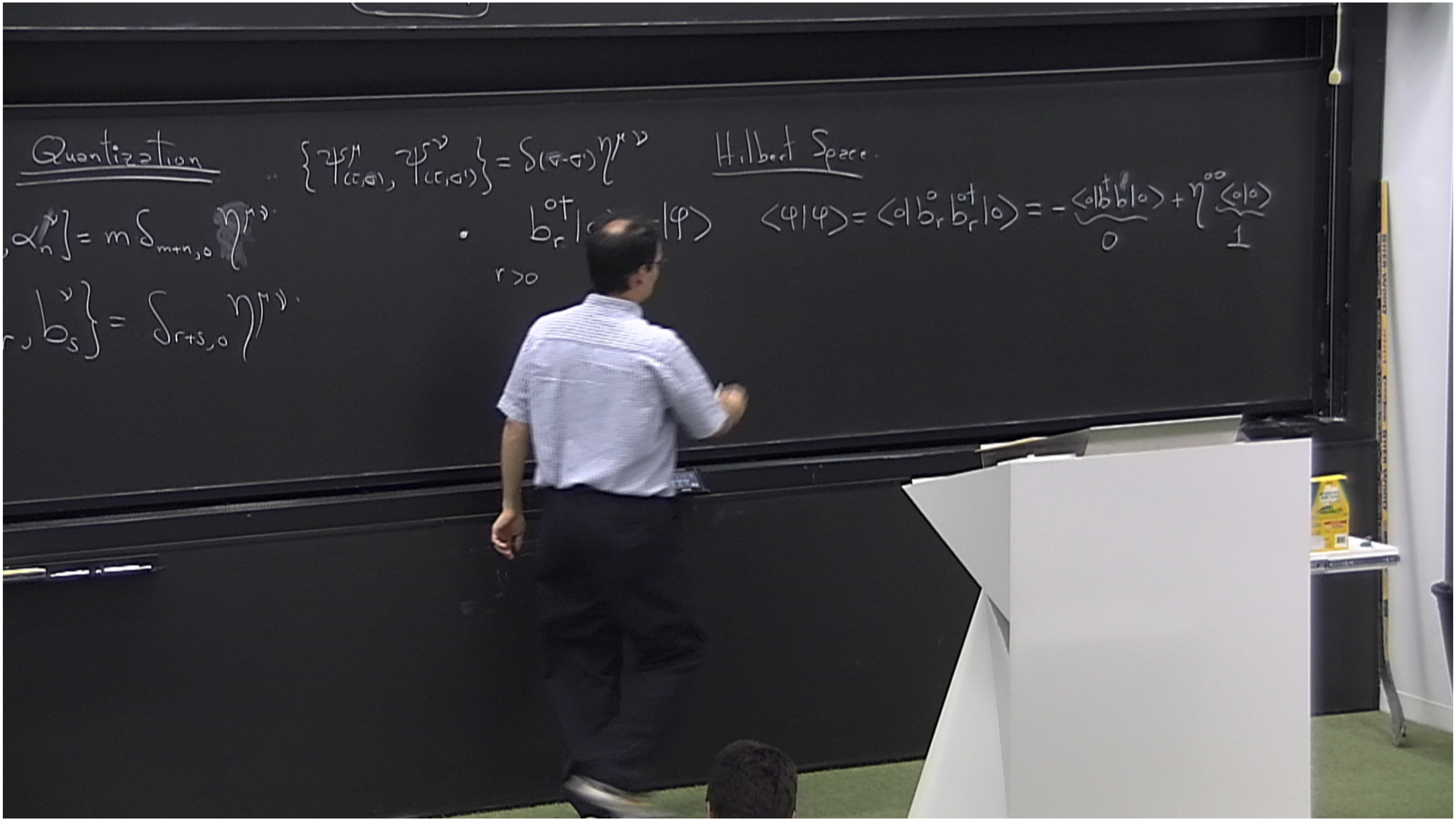
At

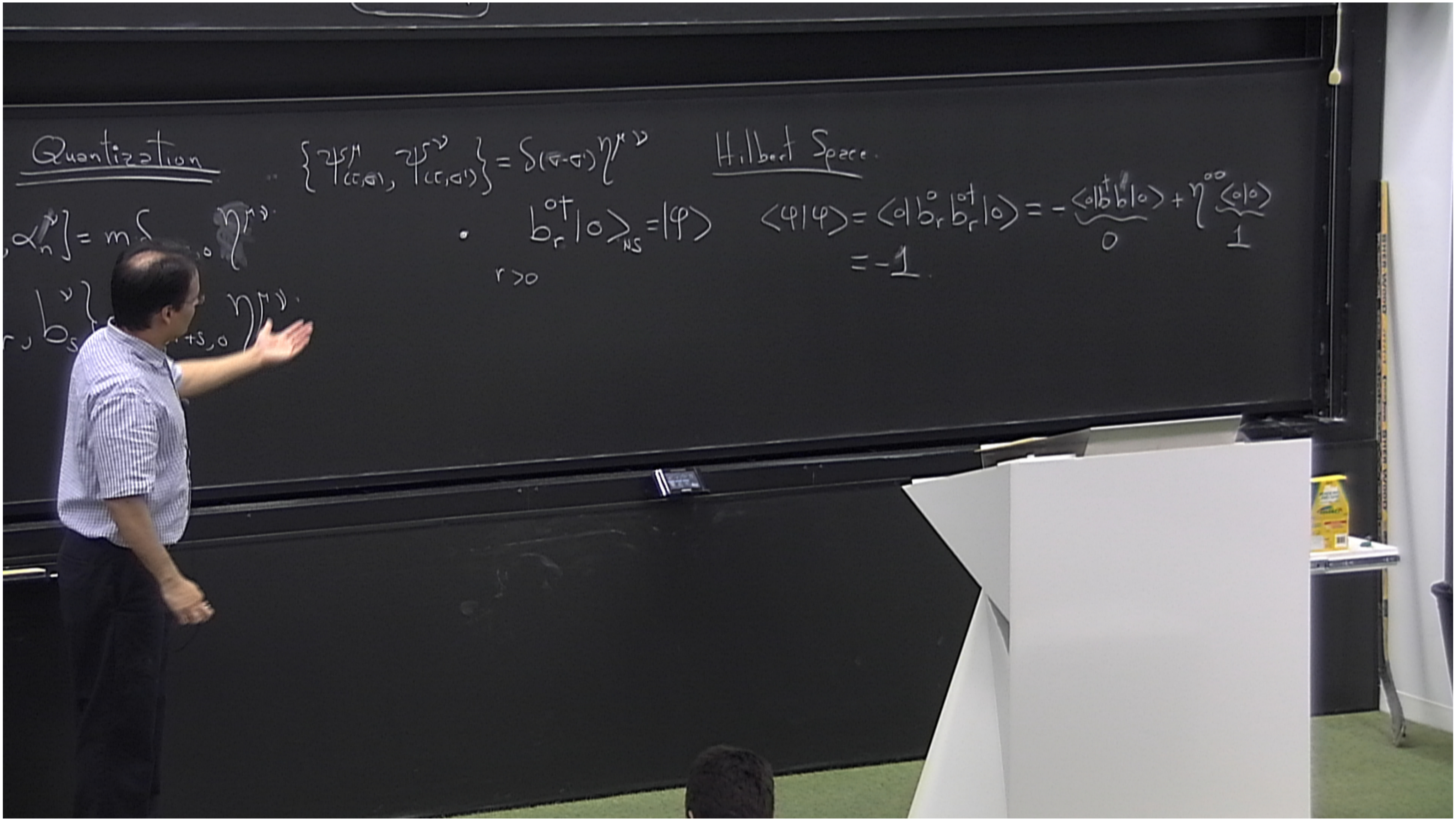

NS:
$\psi^\mu_+(\tau,\pi) = -\psi^\mu_-(\tau,\pi)$
$\psi^\mu_+(\sigma_+) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b^\mu_r e^{-ir\sigma_+}$
$\psi^\mu_-(\sigma_-) = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b'^\mu_r e^{-ir\sigma_-}$
$\sigma_\pm = \tau \pm \sigma$
$\psi^\mu_+(\tau,0) = \psi^\mu_-(\tau,0)$
$b^\mu_r = b'^\mu_r$
At $\sigma=\pi$, let's add up both: $\psi^\mu_+ + \psi^\mu_-$
$0 = \sum_r b^\mu_r e^{-ir\tau} \cos(r\pi) \Rightarrow r\in$
Quantization

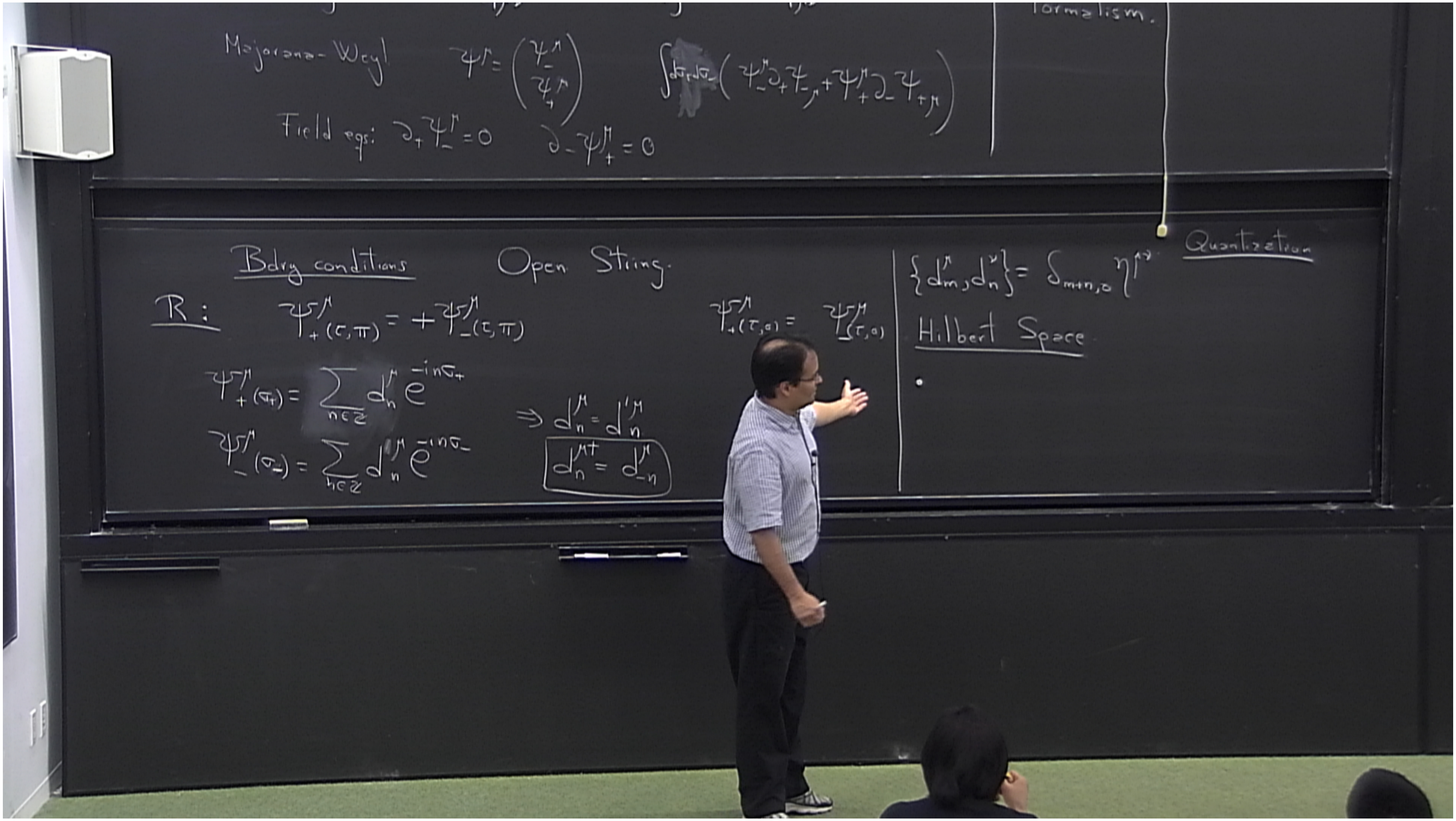

Formalism.
Majorana-Weyl
Field eqs:
Bdry conditions
Open String.
R:
Quantization
Hilbert Space

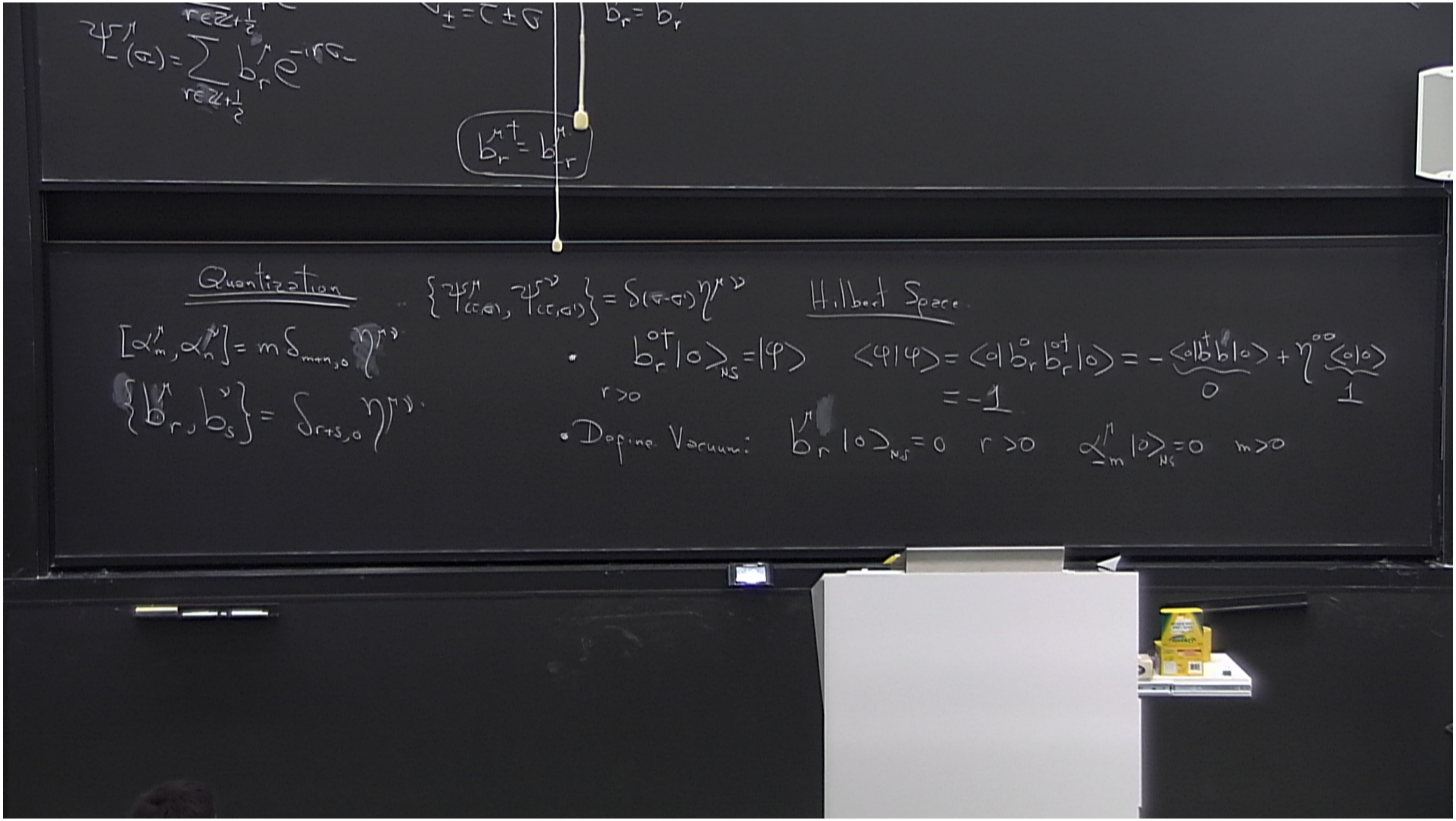

Quantization
{ψ^μ(τ,σ), ψ^ν(τ,σ')} = δ(σ-σ') η^μν
Hilbert Space.
[α^μ_m, α^ν_n] = m δ_m+n,0 η^μν.
b^0_r |0>_NS
{b^μ_r, b^ν_s} = δ_r+s,0 η^μν.


Quantization
$\{\psi^\mu_{(\tau,\sigma)}, \psi^\nu_{(\tau,\sigma')}\} = \delta(\sigma-\sigma')\eta^{\mu\nu}$
Hilbert Space
$[\alpha^\mu_m, \alpha^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$
$\{b^\mu_r, b^\nu_s\} = \delta_{r+s,0}\,\eta^{\mu\nu}$
$b^{0\dagger}_r|0\rangle_{NS} = |\varphi\rangle$
$r>0$
$\langle\varphi|\varphi\rangle = \langle 0|b^0_r b^{0\dagger}_r|0\rangle =$

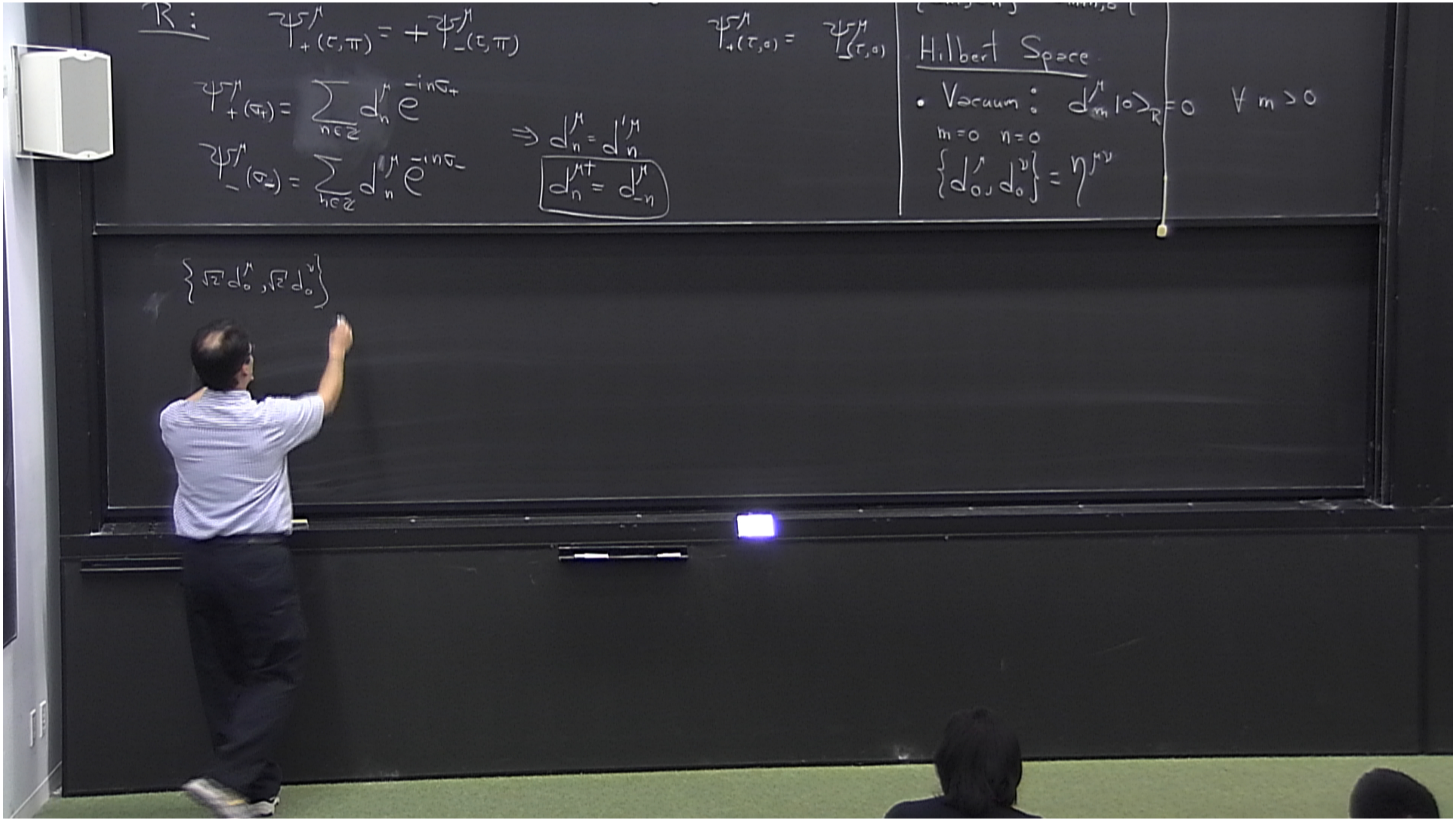

Quantization
Hilbert Space.

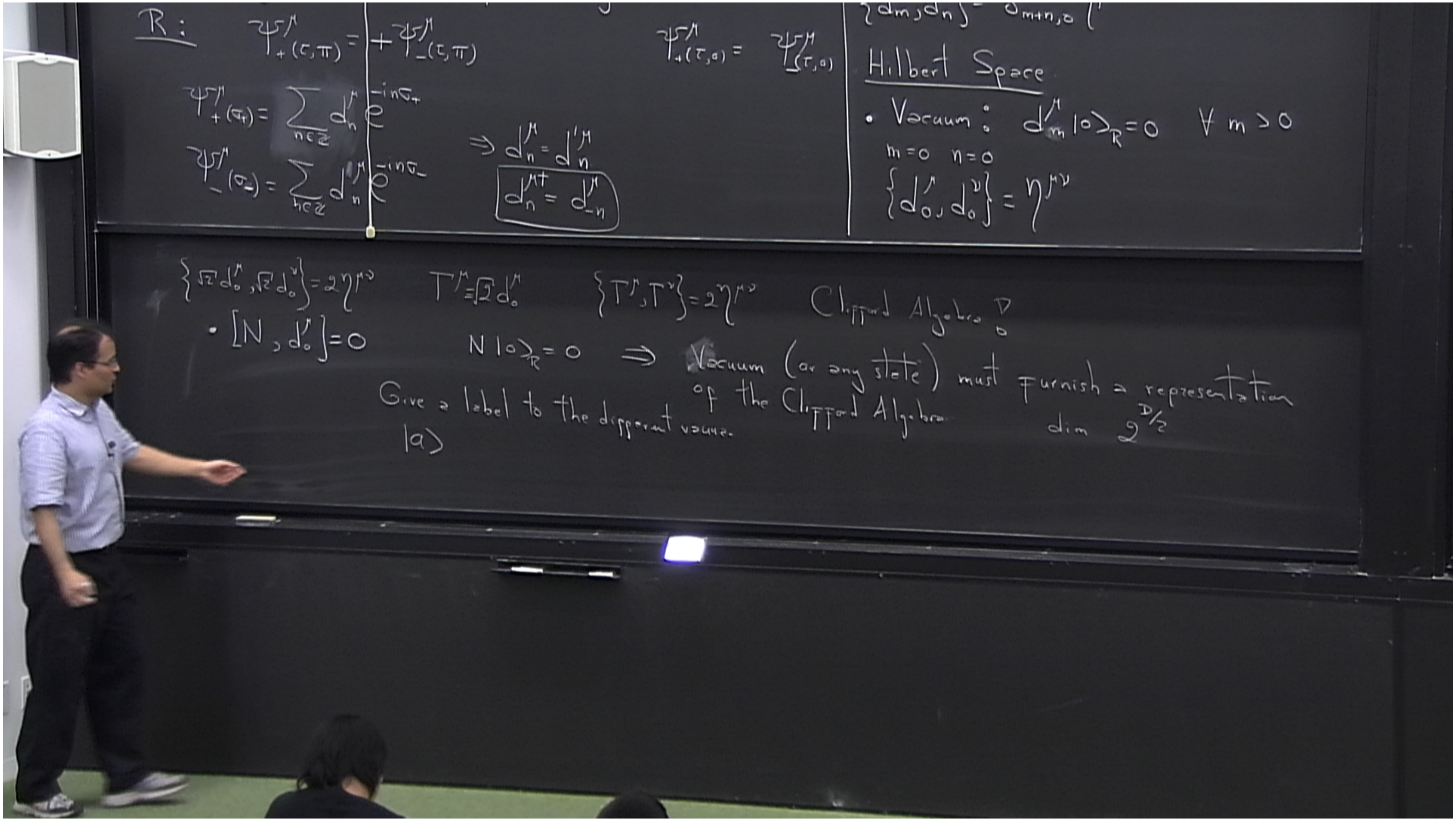

Quantization
Hilbert Space
r > 0
= -1

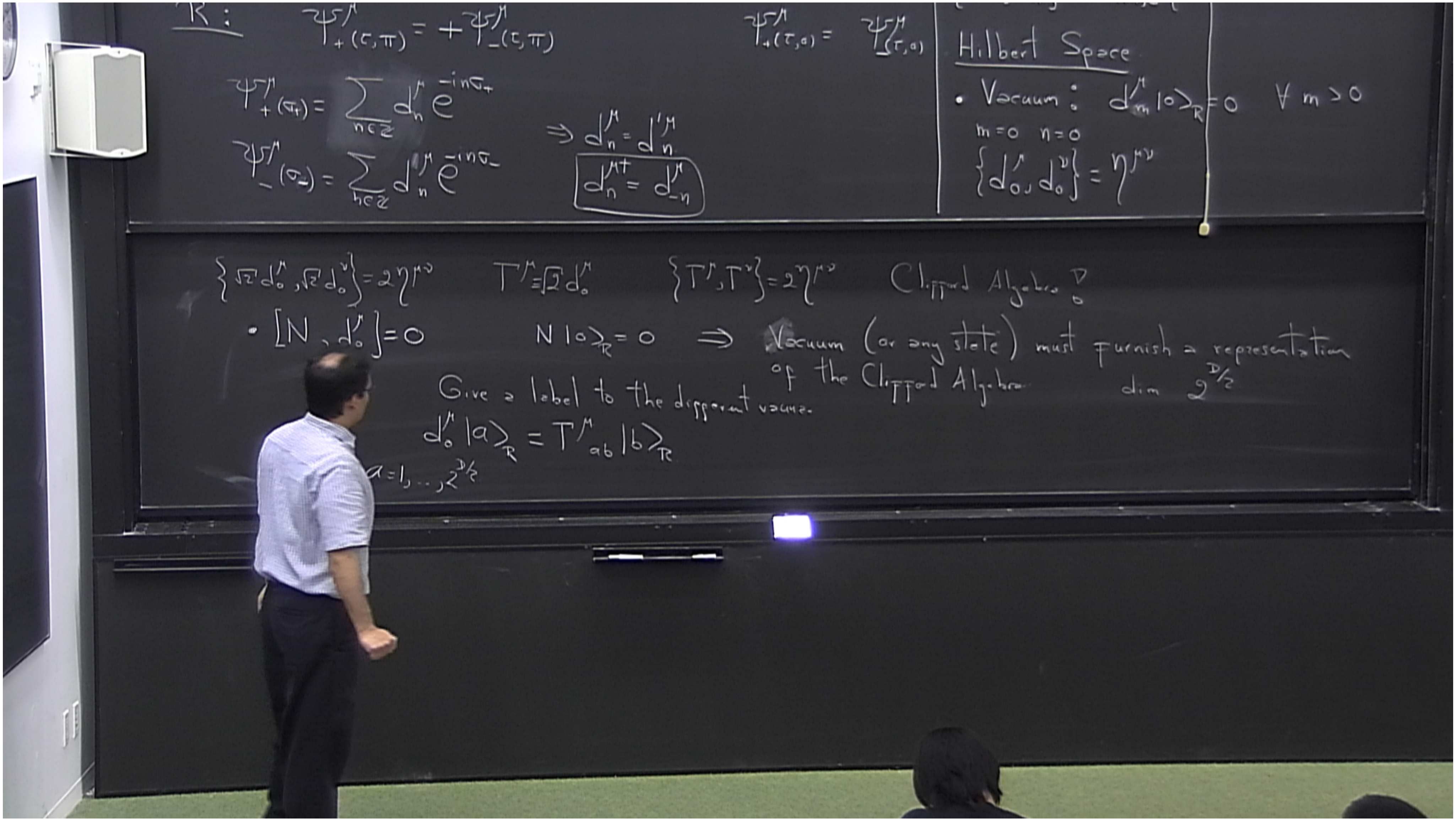

formalism.
Majorana-Weyl
Field eqs:
Bdry conditions
Open String.
R :
Hilbert Space
Quantization

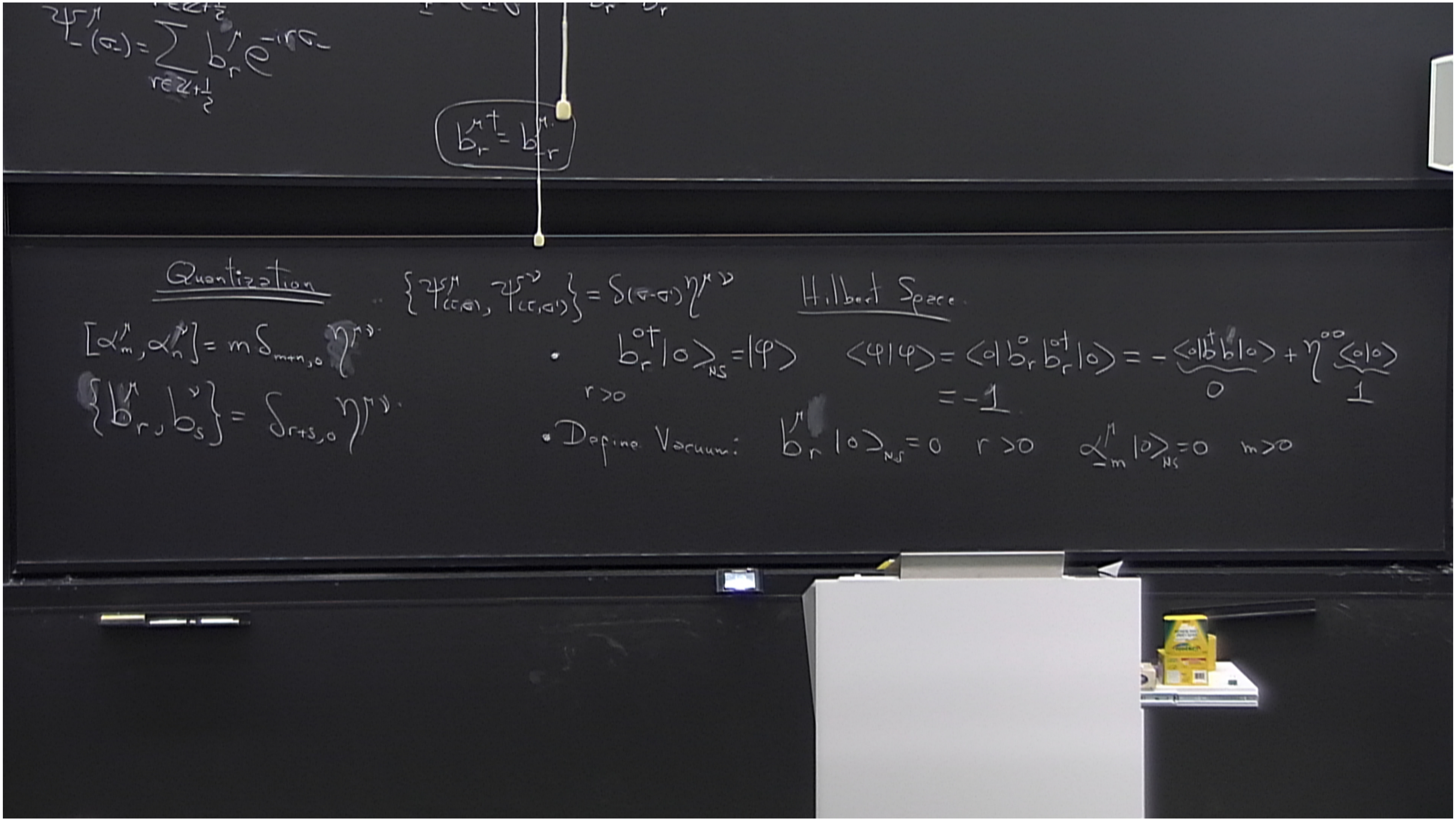


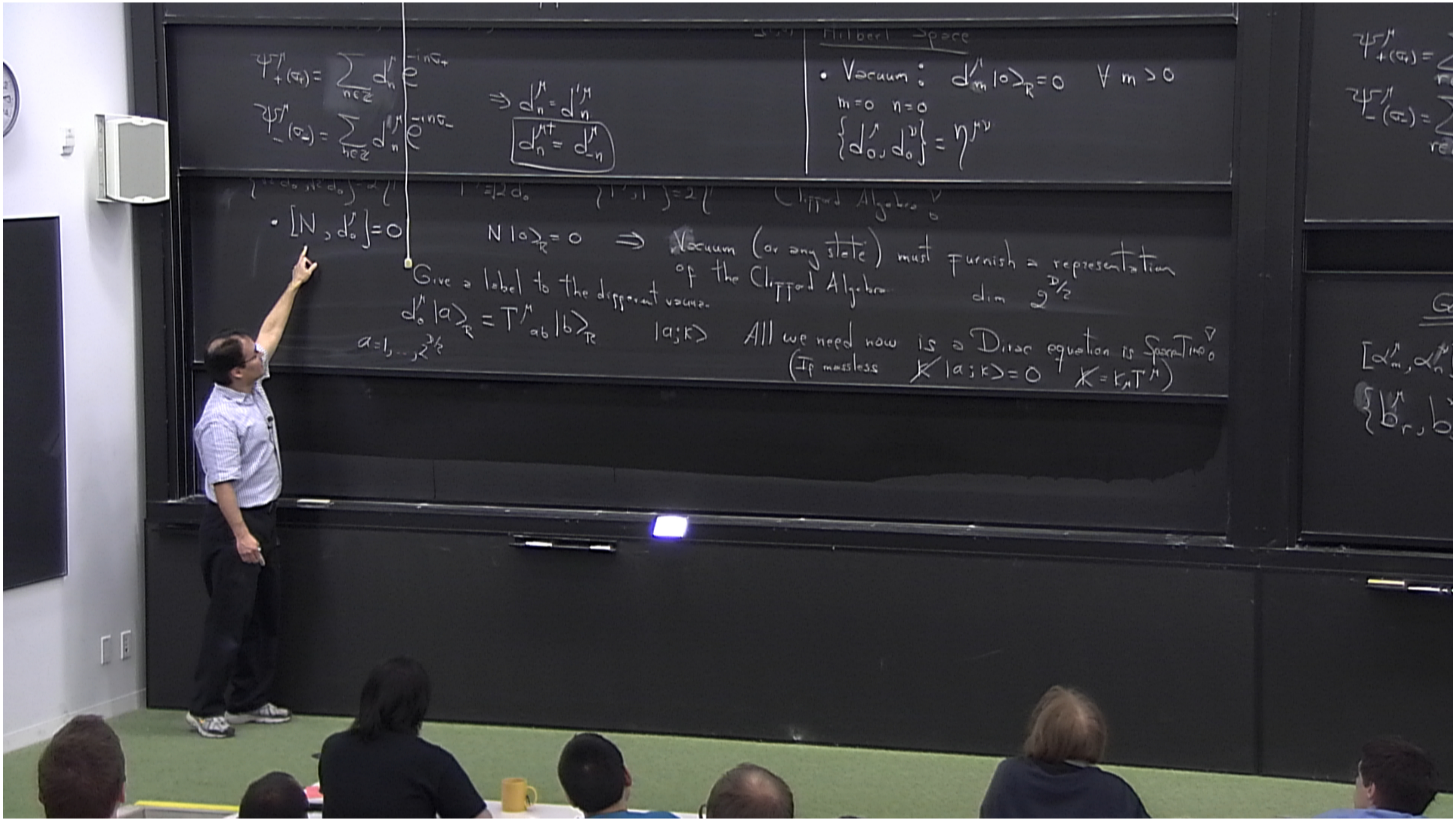
Quantization
Hilbert Space

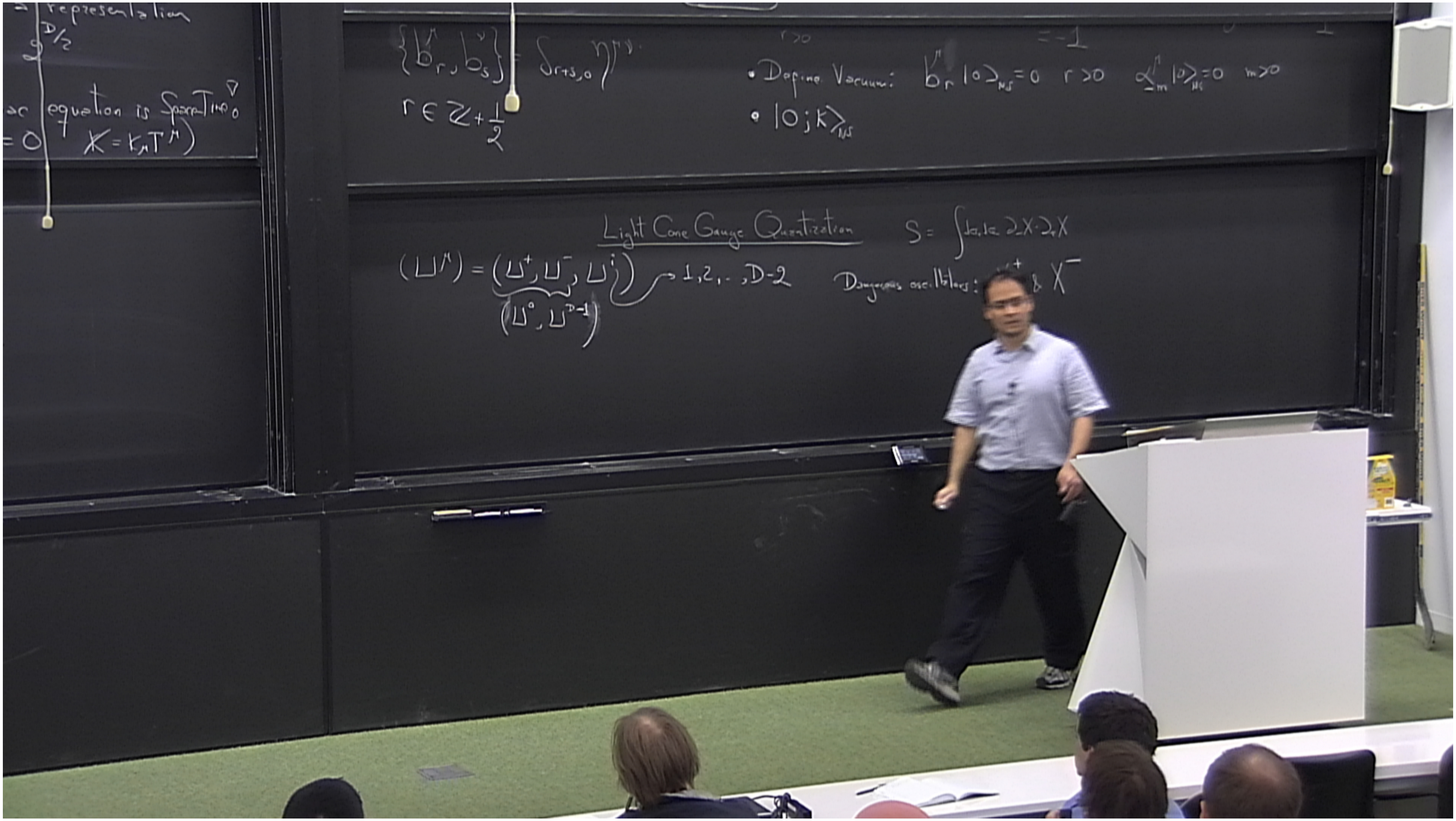

R :
$\psi^{\mu}_{+}(\tau,\pi) = +\psi^{\mu}_{-}(\tau,\pi)$
$\psi^{\mu}_{+}(\tau,0) = \psi^{\mu}_{-}(\tau,0)$
$\psi^{\mu}_{+}(\sigma_+) = \sum_{n\in\mathbb{Z}} d^{\mu}_n e^{-in\sigma_+}$
$\psi^{\mu}_{-}(\sigma_-) = \sum_{n\in\mathbb{Z}} d'^{\mu}_n e^{-in\sigma_-}$
$\Rightarrow d^{\mu}_n = d'^{\mu}_n$
$d^{\mu\dagger}_n = d^{\mu}_{-n}$
Hilbert Space
• Vacuum : $d^{\mu}_m |0\rangle_R = 0 \quad \forall\, m > 0$
m = 0 n = 0
$\{d^{\mu}_0, d^{\nu}_0\} = \eta^{\mu\nu}$
$\{\sqrt{2}\, d^{\mu}_0, \sqrt{2}\, d^{\nu}_0\}$

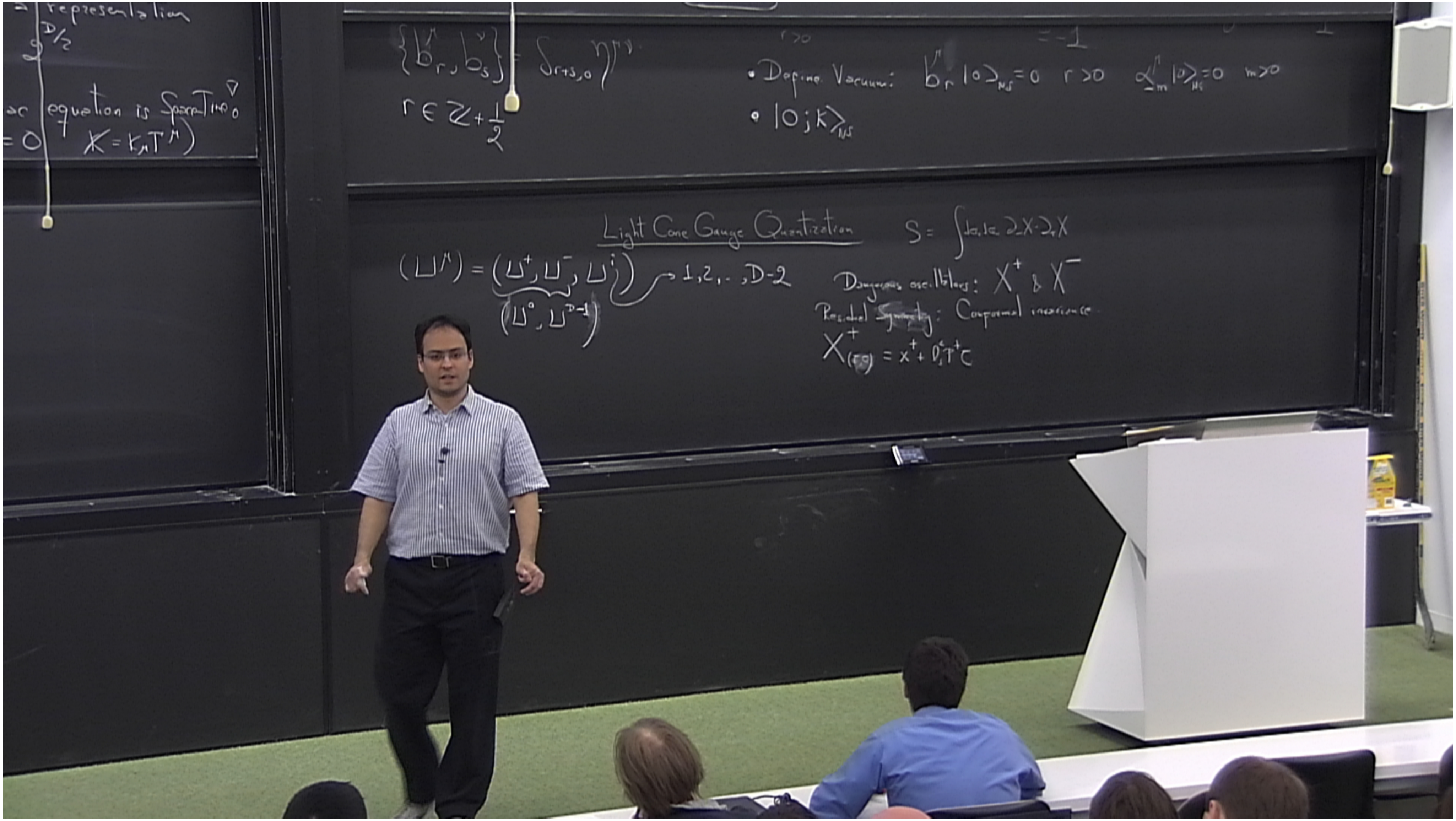

R :
Hilbert Space
• Vacuum : $d_m^\mu |0\rangle_R = 0 \quad \forall\, m > 0$
m = 0 n = 0
$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}$
$\{\sqrt{2}d_0^\mu, \sqrt{2}d_0^\nu\} = 2\eta^{\mu\nu}$
$\Gamma^\mu = \sqrt{2}d_0^\mu$
$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$
Clifford Algebra
• $[N, d_0^\mu] = 0$
$N|0\rangle_R = 0$
⇒ Vacuum (or any state) must furnish a representation of the Clifford Algebra
dim $2^{D/2}$
Give a label to the different vacua
$|a\rangle$

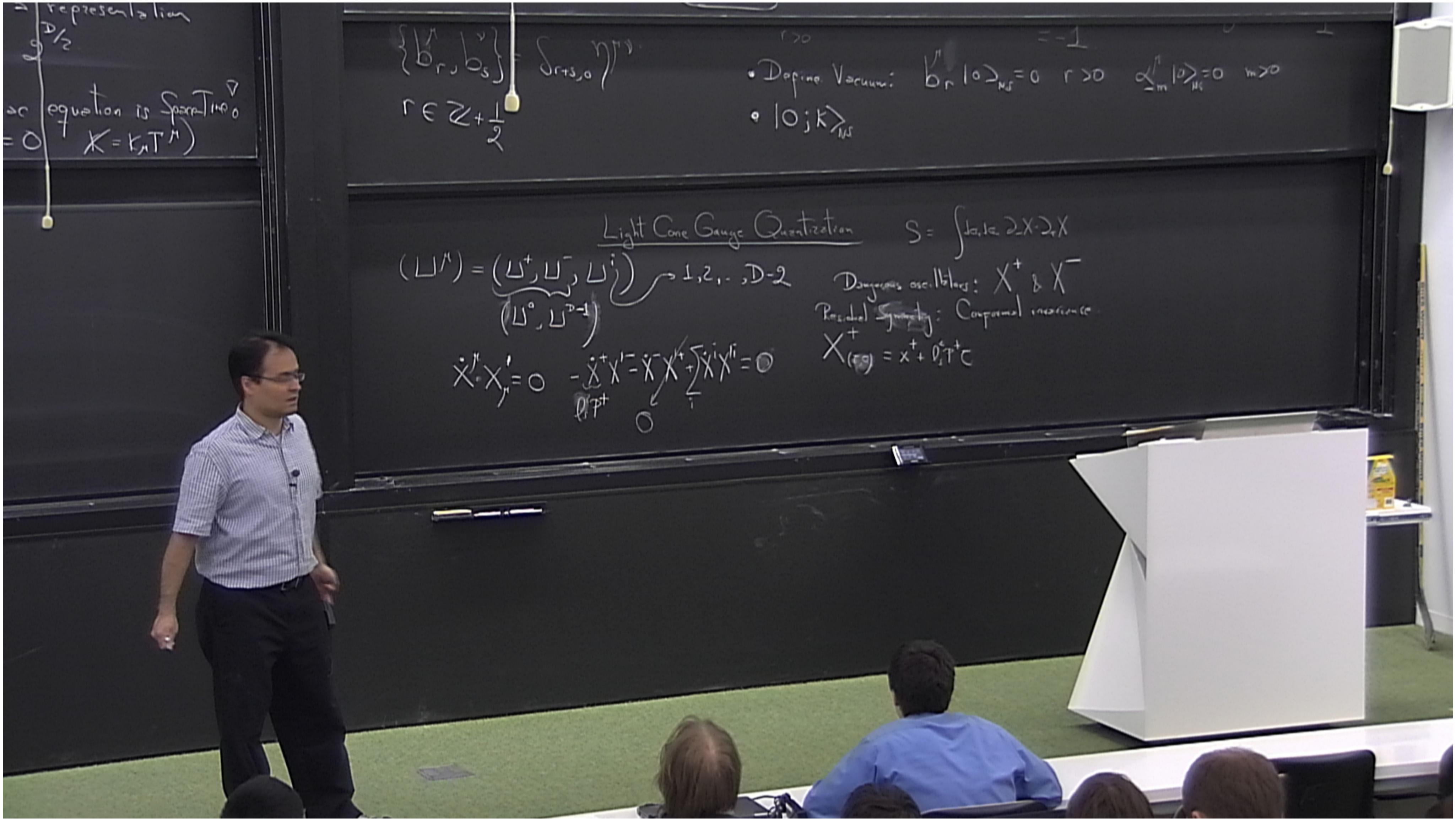

R : ψ^μ_+(τ,π) = +ψ^μ_−(τ,π)   ψ^μ_+(τ,0) = ψ^μ_−(τ,0)
ψ^μ_+(σ_+) = Σ_{n∈Z} d^μ_n e^{−inσ_+}
ψ^μ_−(σ_−) = Σ_{n∈Z} d^μ_n e^{−inσ_−}
⇒ d^μ_n = d'^μ_n
d^{μ†}_n = d^μ_{−n}
Hilbert Space
• Vacuum : d^μ_m |0⟩_R = 0 ∀ m > 0
m = 0 n = 0
{d^μ_0, d^ν_0} = η^{μν}
{√2 d^μ_0, √2 d^ν_0} = 2η^{μν}   Γ^μ ≡ √2 d^μ_0   {Γ^μ, Γ^ν} = 2η^{μν}   Clifford Algebra D
• [N, d^μ_0] = 0   N |0⟩_R = 0 ⇒ Vacuum (or any state) must furnish a representation of the Clifford Algebra   dim 2^{D/2}
Give a label to the different vacua.
d^μ_0 |a⟩_R = Γ^μ_{ab} |b⟩_R
a = 1, ..., 2^{D/2}

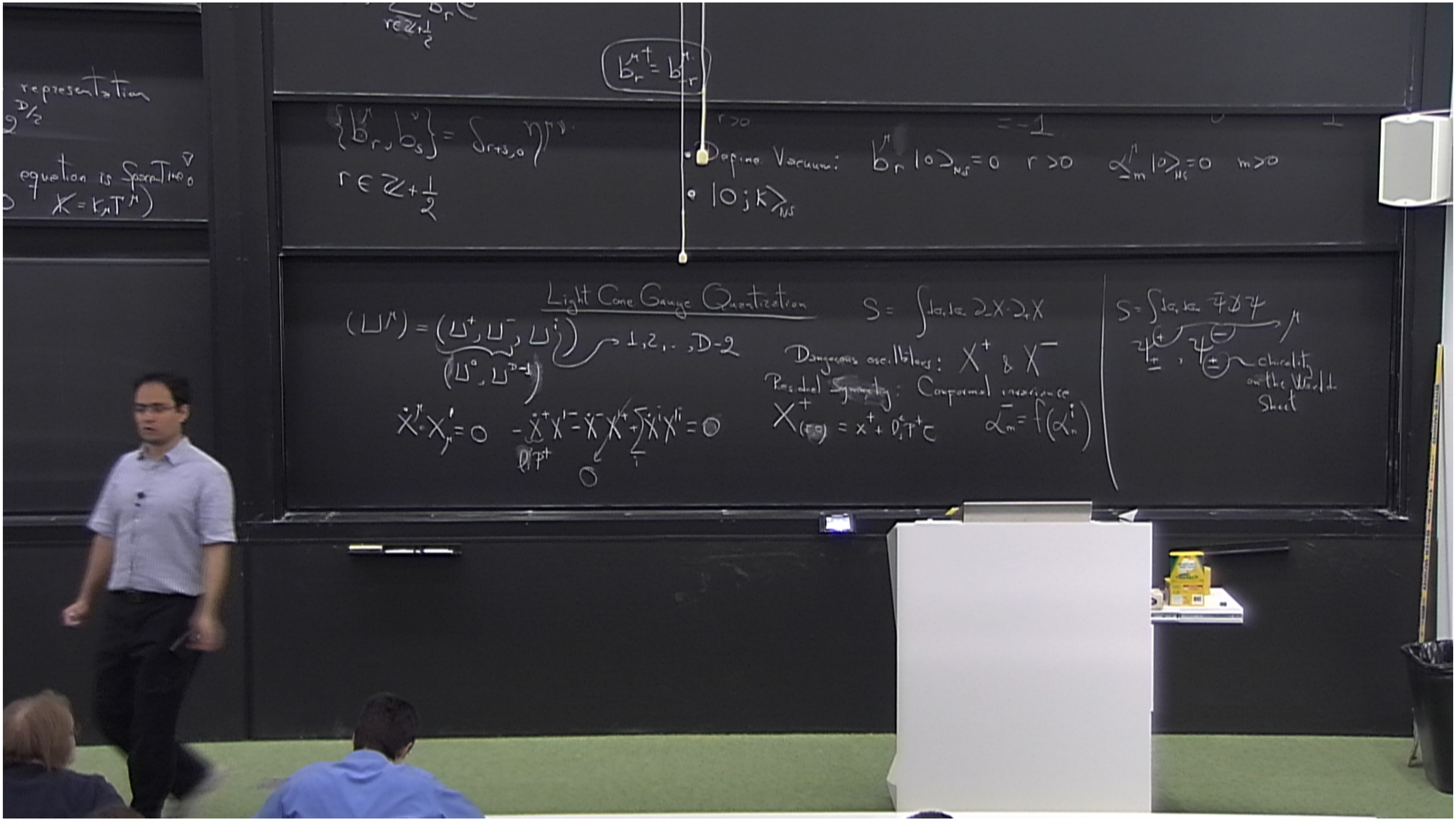

Quantization
Hilbert Space
Define Vacuum:

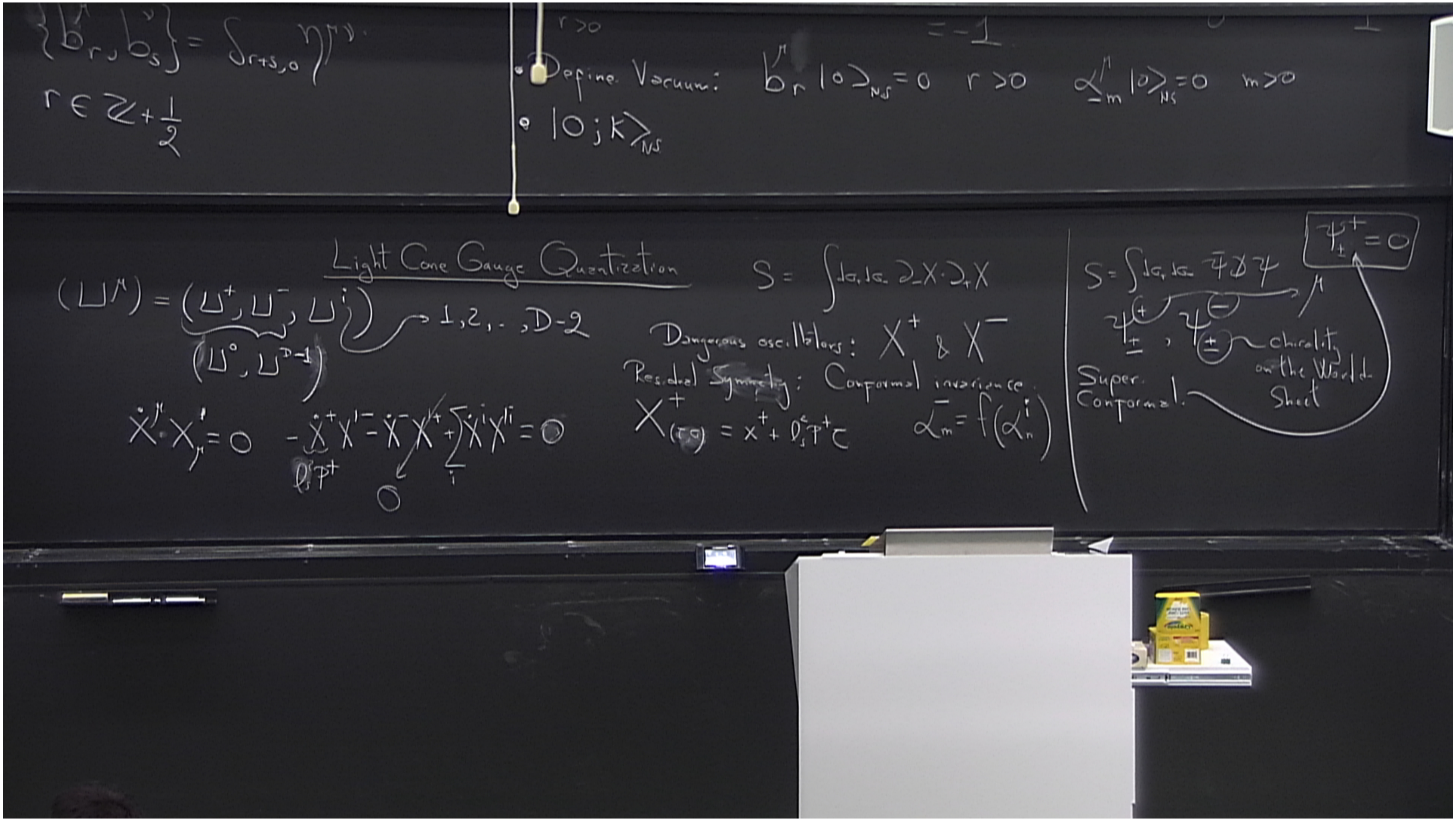

Hilbert Space
• Vacuum: d^μ_m |0>_R = 0 ∀ m > 0
m=0 n=0
{d^μ_0, d^ν_0} = η^μν
⇒ d^μ_n = d'^μ_n
d^μ†_n = d^μ_-n
• [N, d^μ_0] = 0
N |0>_R = 0 ⇒ Vacuum (or any state) must furnish a representation of the Clifford Algebra dim 2^D/2
Give a label to the different vacua.
d^μ_0 |a>_R = Γ^μ_ab |b>_R
a = 1, ..., 2^D/2
|a; k>
All we need now is a Dirac equation in SpaceTime?
(If massless k̸ |a; k> = 0 k̸ = k_μ Γ^μ)

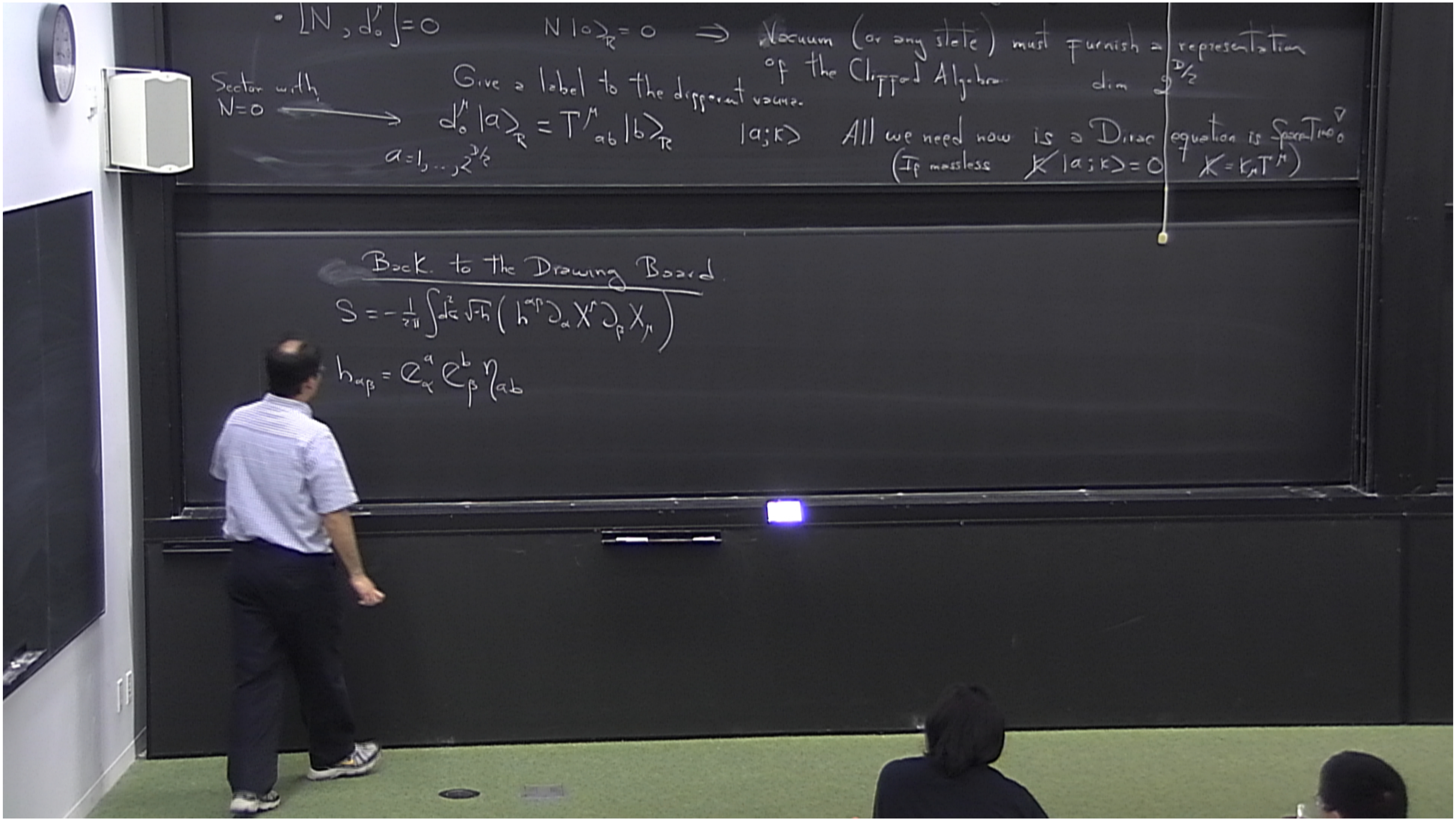

Light Cone Gauge Quantization
Define Vacuum:
Dangerous oscillators:

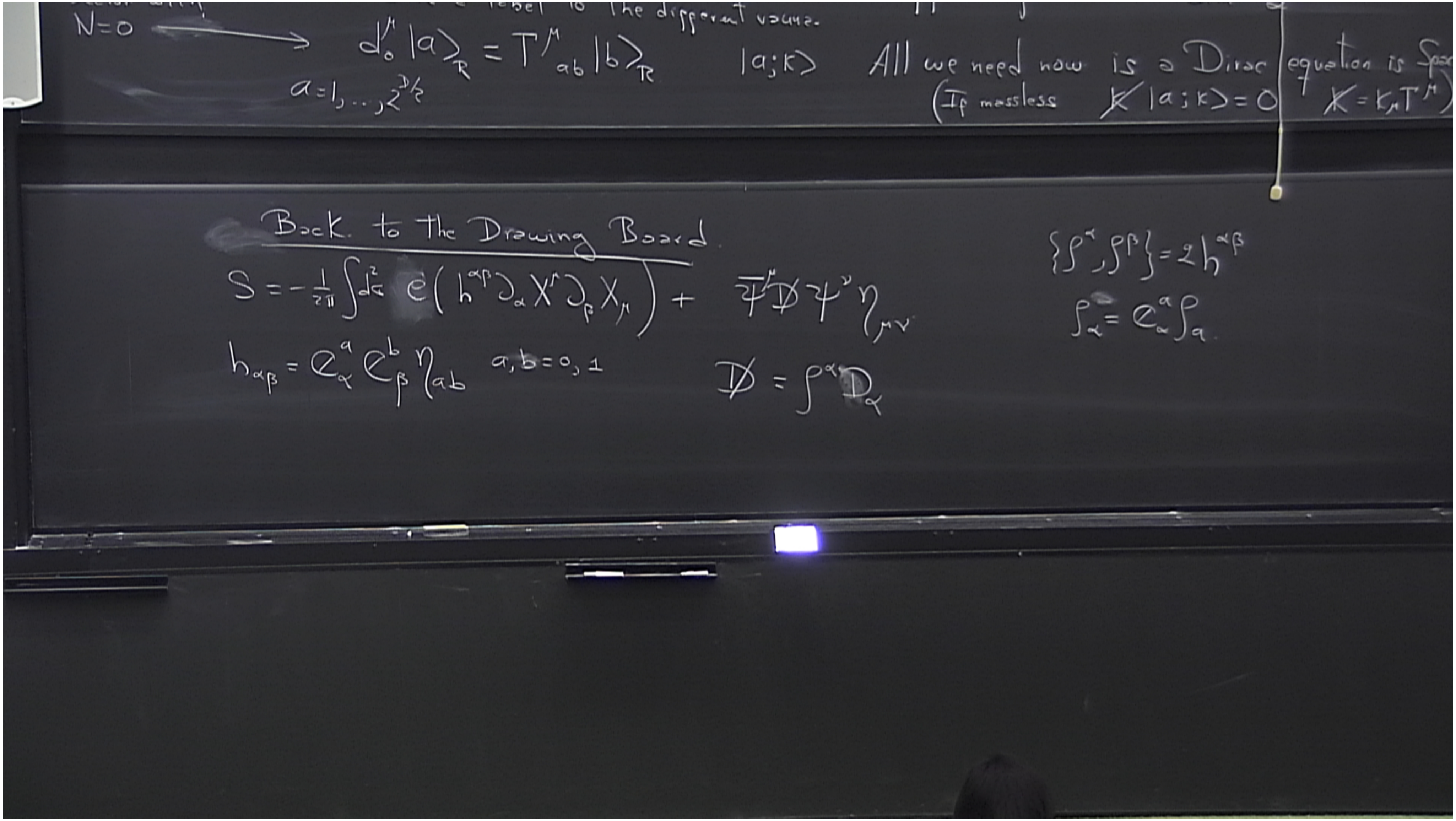



Light Cone Gauge Quantization
Define Vacuum:
Dangerous oscillators: X+ & X−
Residual Symmetry: Conformal invariance

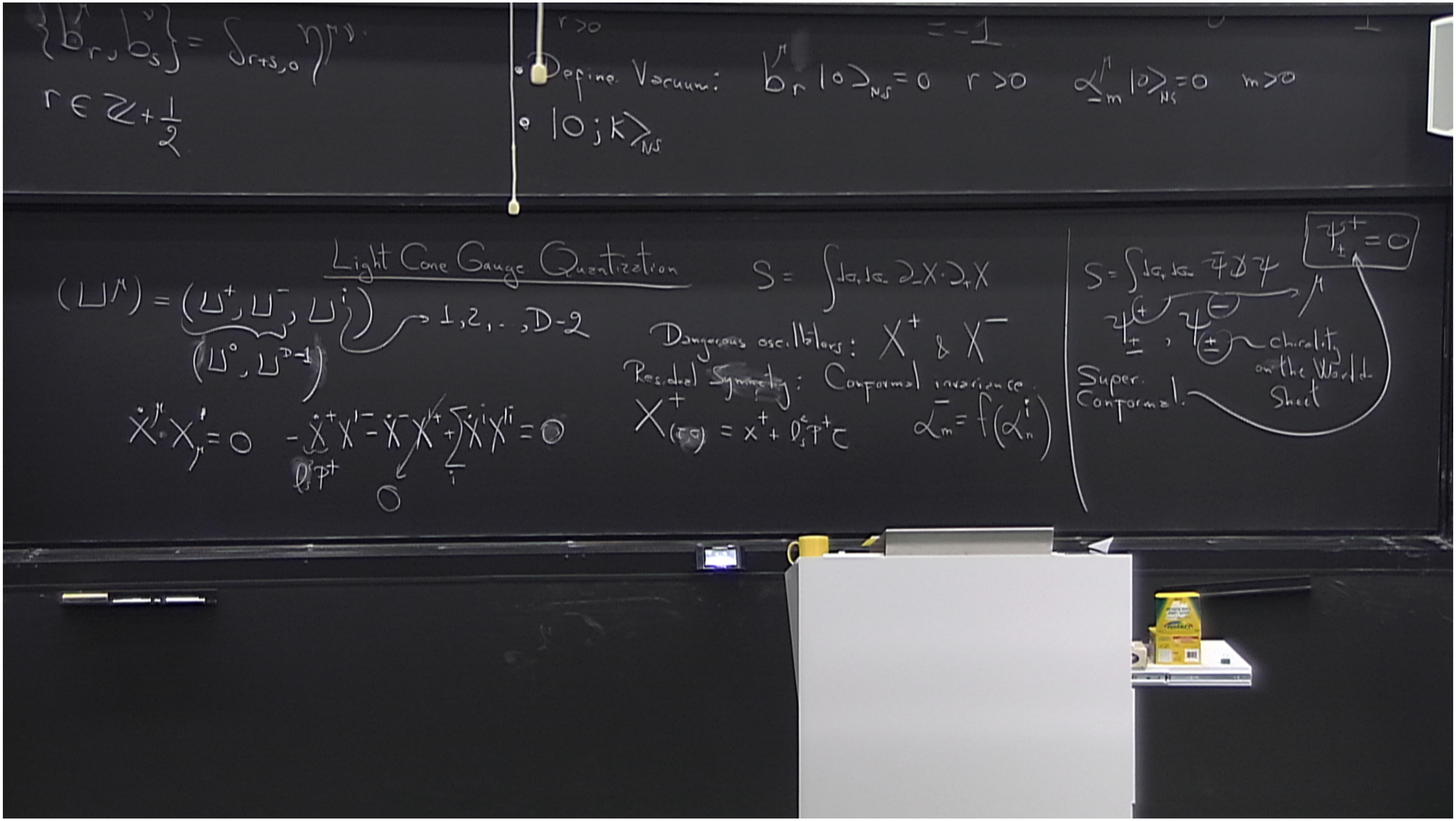

Light Cone Gauge Quantization
Define Vacuum:
Dangerous oscillators:
Residual Symmetry: Conformal invariance
chirality on the World Sheet


{b^μ_r , b^ν_s} = δ_{r+s,0} η^{μν}
r ∈ Z + 1/2
Define Vacuum: b^μ_r |0⟩_NS = 0 r > 0 α^μ_m |0⟩_NS = 0 m > 0
|0; k⟩_NS
Light Cone Gauge Quantization
(U^μ) = (U^+, U^-, U^i) → 1, 2, .., D-2
(U^0, U^{D-1})
S = ∫dσ+ dσ- ∂-X · ∂+X
Dangerous oscillators: X^+ & X^-
Residual Symmetry: Conformal invariance
X^+(τ,σ) = x^+ + l_s^2 p^+ τ
α^-_m = f(α^i_n)
S = ∫dσ+ dσ- ψ̄ ∂̸ ψ
ψ^+_± = 0
chirality on the World Sheet
Super Conformal

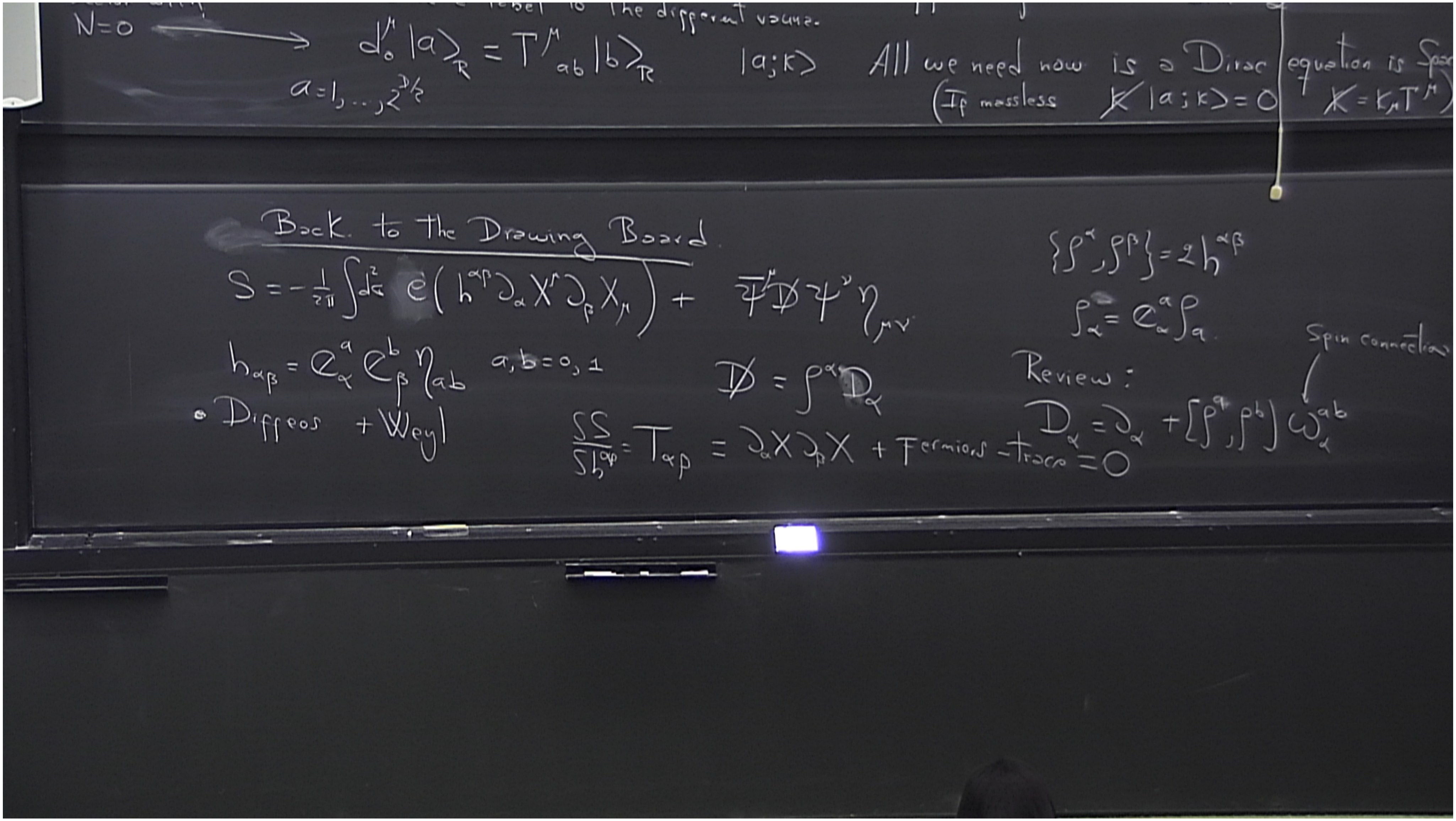

[N, d0μ] = 0
N |0>R = 0 ⇒ Vacuum (or any state) must furnish a representation
of the Clifford Algebra dim 2D/2
Sector with N=0
Give a label to the different vacua.
d0μ |a>R = Γμab |b>R
a = 1, .., 2D/2
|a;k>
All we need now is a Dirac equation in SpaceTime
(If massless k̸ |a;k> = 0 k̸ = kμΓμ)
Back to the Drawing Board
S = −1/2π ∫d²σ √−h (hαβ ∂α Xμ ∂β Xμ)
hαβ = eαa eβb ηab

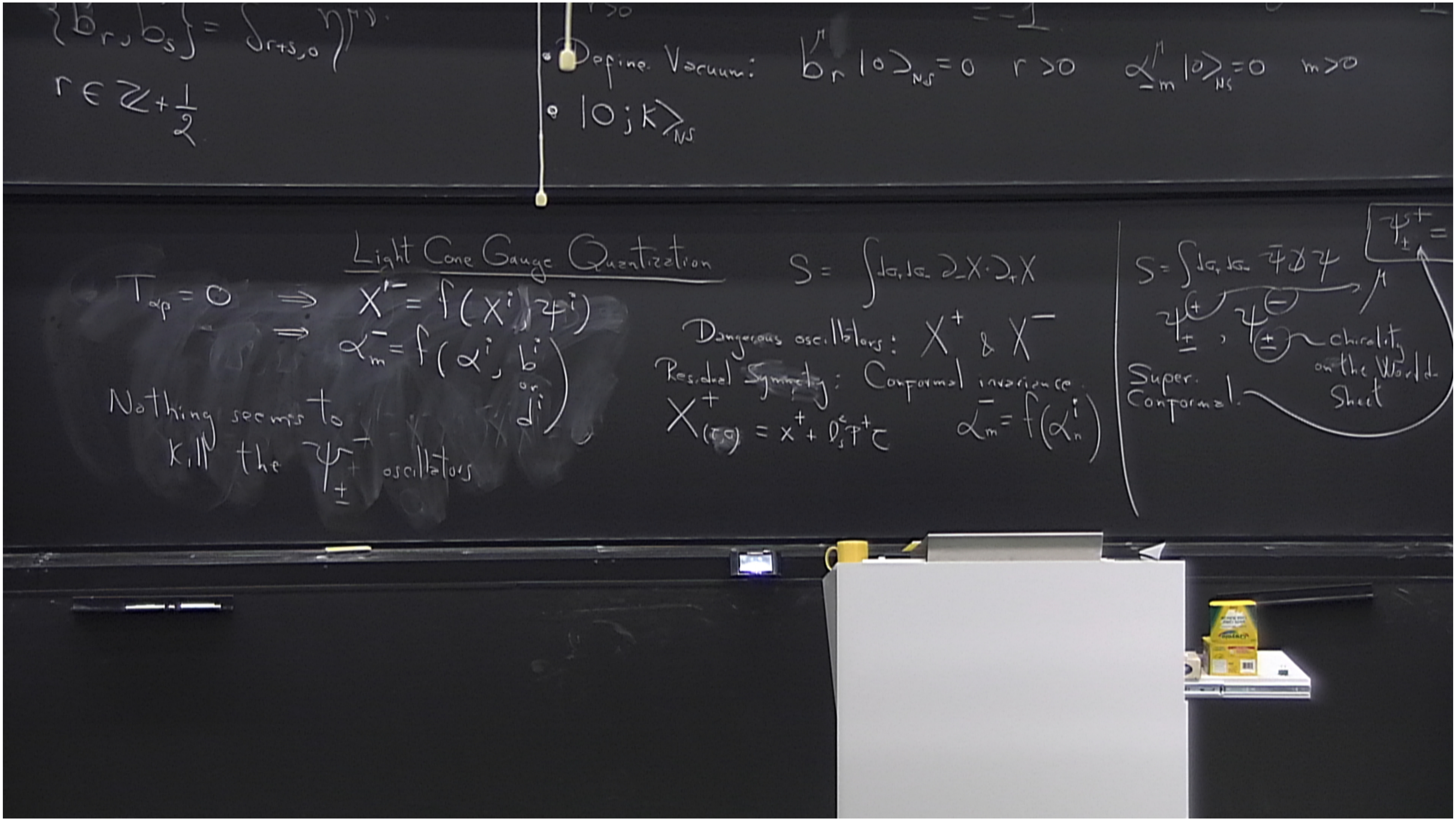


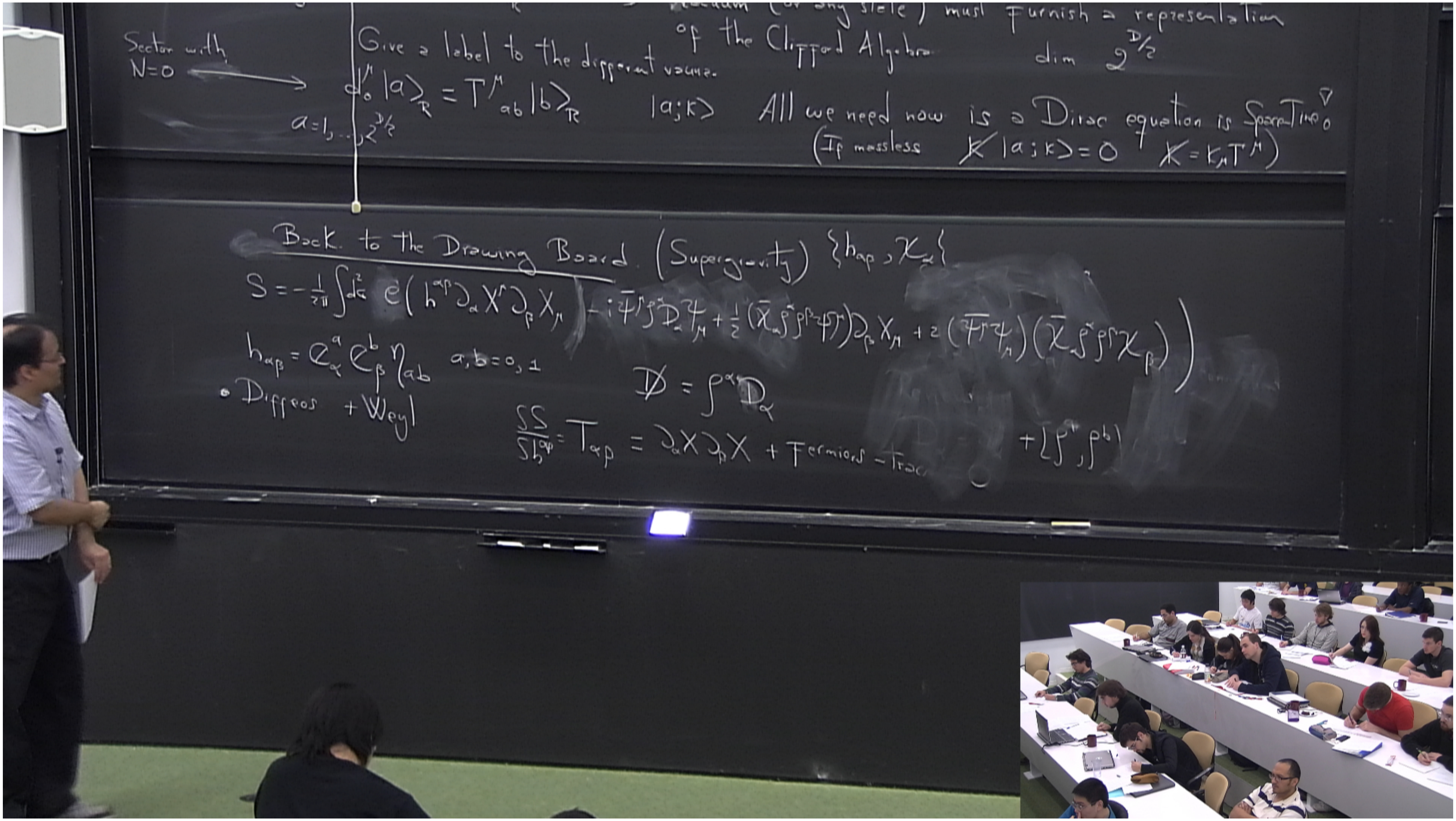
N=0
$d_0^\mu |a\rangle_R = T^\mu{}_{ab} |b\rangle_R$
$|a;k\rangle$
All we need now is a Dirac equation
(If massless $\not{k}|a;k\rangle = 0$
$\not{k} = k_\mu T^\mu$)
$a = 1, \dots, 2^{D/2}$
Back to the Drawing Board
$S = -\frac{1}{2\pi}\int d^2\sigma\, e\left(h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu\right) + \bar{\psi}^\mu \not{D} \psi^\nu \eta_{\mu\nu}$
$h_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}$
$a, b = 0, 1$
$\not{D} = \rho^\alpha D_\alpha$
$\{\rho^\alpha, \rho^\beta\} = 2h^{\alpha\beta}$
$\rho_\alpha = e_\alpha^a \rho_a$

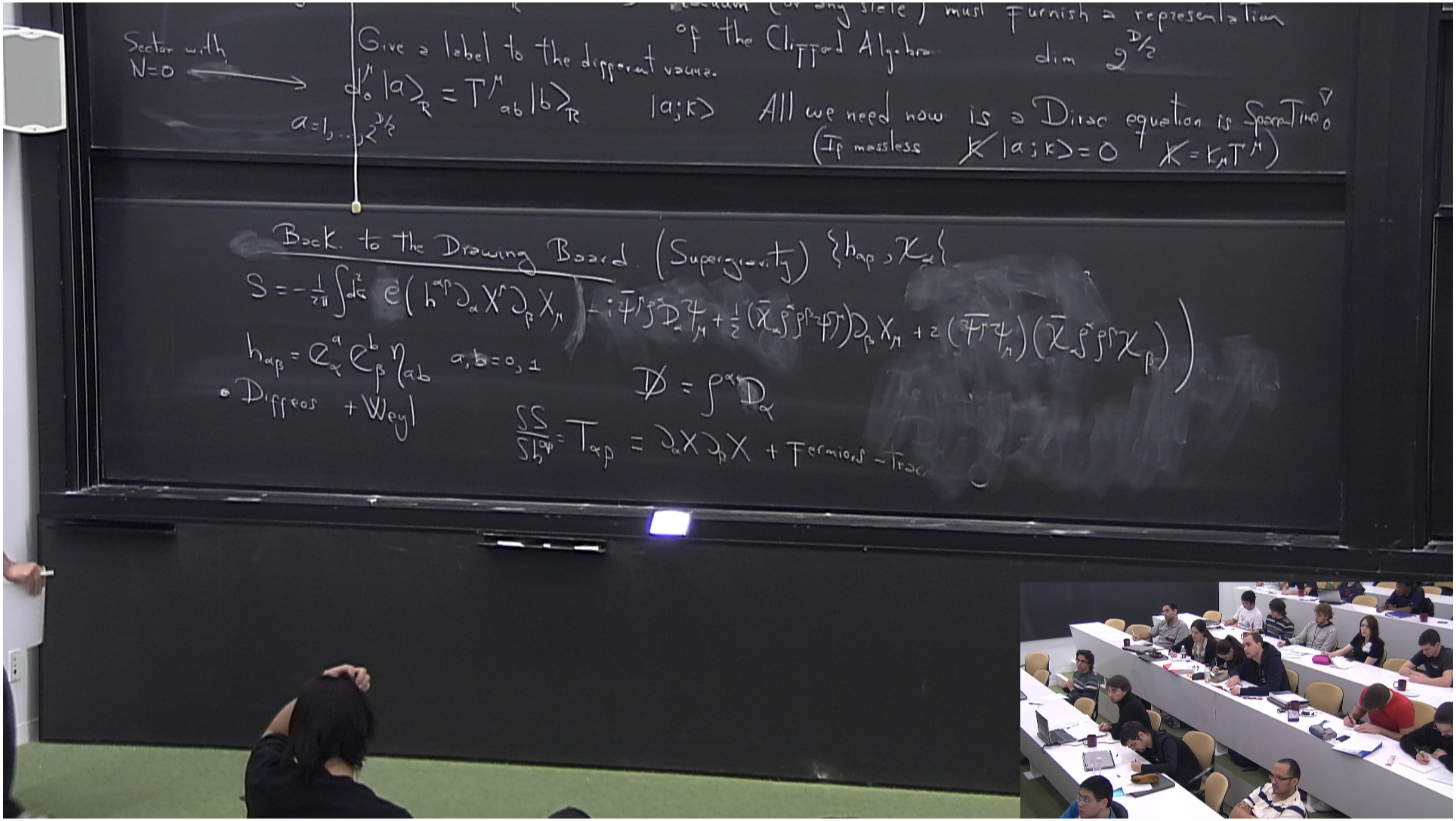


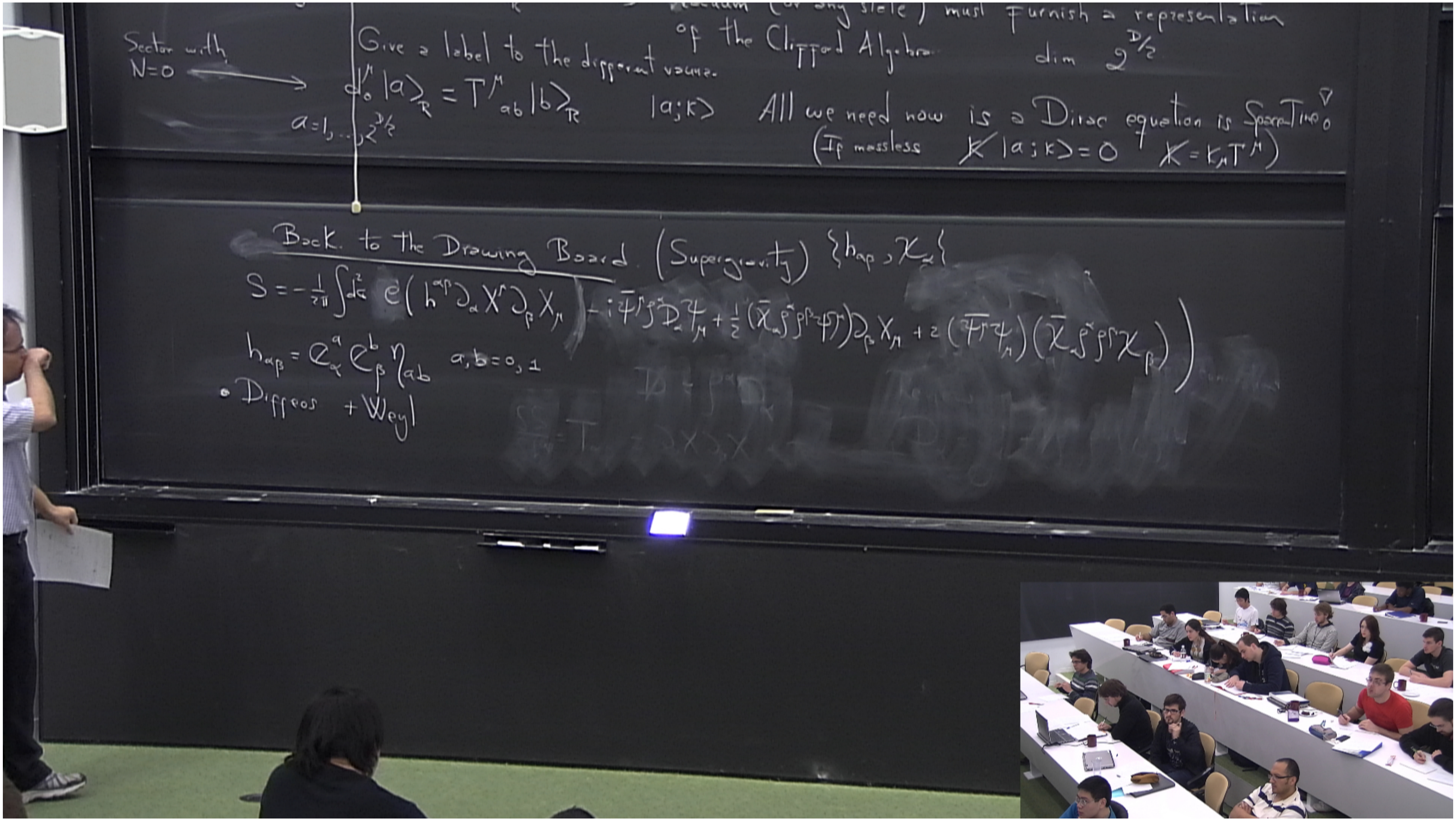
Define Vacuum:
Light Cone Gauge Quantization
Dangerous oscillators: X+ & X−
Residual Symmetry: Conformal invariance.
chirality on the World Sheet
Super Conformal.


N=0 → $d_0^{\mu} |a\rangle_R = T^{\mu}{}_{ab} |b\rangle_R$ $|a;k\rangle$
$a = 1, \dots, 2^{D/2}$
All we need now is a Dirac equation
(If massless $\not{k} |a;k\rangle = 0$ $\not{k} = k_{\mu} T^{\mu}$)
Back to the Drawing Board
$S = -\frac{1}{2\pi} \int d^2\sigma\, e \left(h^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X_{\mu} \right) + \bar{\psi}^{\mu} \not{D} \psi^{\nu} v_{\mu\nu}$
$\{\rho^{\alpha}, \rho^{\beta}\} = 2 h^{\alpha\beta}$
$\rho_{\alpha} < e_{\alpha}^{a} \rho_{a}$
$h_{\alpha\beta} = e_{\alpha}^{a} e_{\beta}^{b} \eta_{ab}$ $a, b = 0, 1$
$\not{D} = \rho^{\alpha} D_{\alpha}$
Review:
Spin connection
$D_{\alpha} = \partial_{\alpha} + [\rho^{a}, \rho^{b}] \omega_{\alpha}^{ab}$
Diffeos + Weyl
$\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_{\alpha} X \partial_{\beta} X$ + Fermions - trace = 0


{b_r, b_s} = δ_{r+s,0} η^{μν}
r ∈ Z + 1/2
• Define Vacuum: b_r |0>_NS = 0 r > 0 α_m |0>_NS = 0 m > 0
• |0; k>_NS
Light Cone Gauge Quantization
T_αβ = 0 ⇒ X⁻ = f(X^i, ψ^i)
⇒ α⁻_m = f(α^i, b^i or d^i)
Nothing seems to kill the ψ⁻_± oscillators
S = ∫dσ₊dσ₋ ∂₋X·∂₊X
Dangerous oscillators: X⁺ & X⁻
Residual Symmetry: Conformal invariance.
X⁺(τ,σ) = x⁺ + l_s² p⁺ τ α⁻_m = f(α^i_n)
S = ∫dσ₊dσ₋ ψ̄ ∂ ψ
ψ_± , ψ_±
Chirality on the World Sheet
Super. Conformal.


must furnish a representation of the Clifford Algebra
dim 2^{D/2}
Sector with N=0
Give a label to the different vacua.
a=1,...,2^{D/2}
|a;k>
All we need now is a Dirac equation in SpaceTime
(If massless
Back to the Drawing Board (Supergravity) {h_{αβ}, χ_α}
h_{αβ} = e^a_α e^b_β η_{ab}   a,b=0,1
• Diffeos + Weyl
+ Fermions - Trace


must furnish a representation of the Clifford Algebra
dim $2^{D/2}$
Sector with N=0
Give a label to the different vacua.
$|a;k\rangle$
All we need now is a Dirac equation in SpaceTime
(If massless $\not{k}|a;k\rangle = 0$, $\not{k} = k_\mu \Gamma^\mu$)
$a = 1, \ldots, 2^{D/2}$
Back to the Drawing Board (Supergravity) $\{h_{\alpha\beta}, \chi_\alpha\}$
$h_{\alpha\beta} = e^a_\alpha e^b_\beta \eta_{ab}$ $a,b = 0,1$
Diffeos + Weyl
$\not{D} = \rho^\alpha D_\alpha$
$\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_\alpha X \partial_\beta X$ + Fermions - Trace



must furnish a representation
of the Clifford Algebra
dim
Sector with
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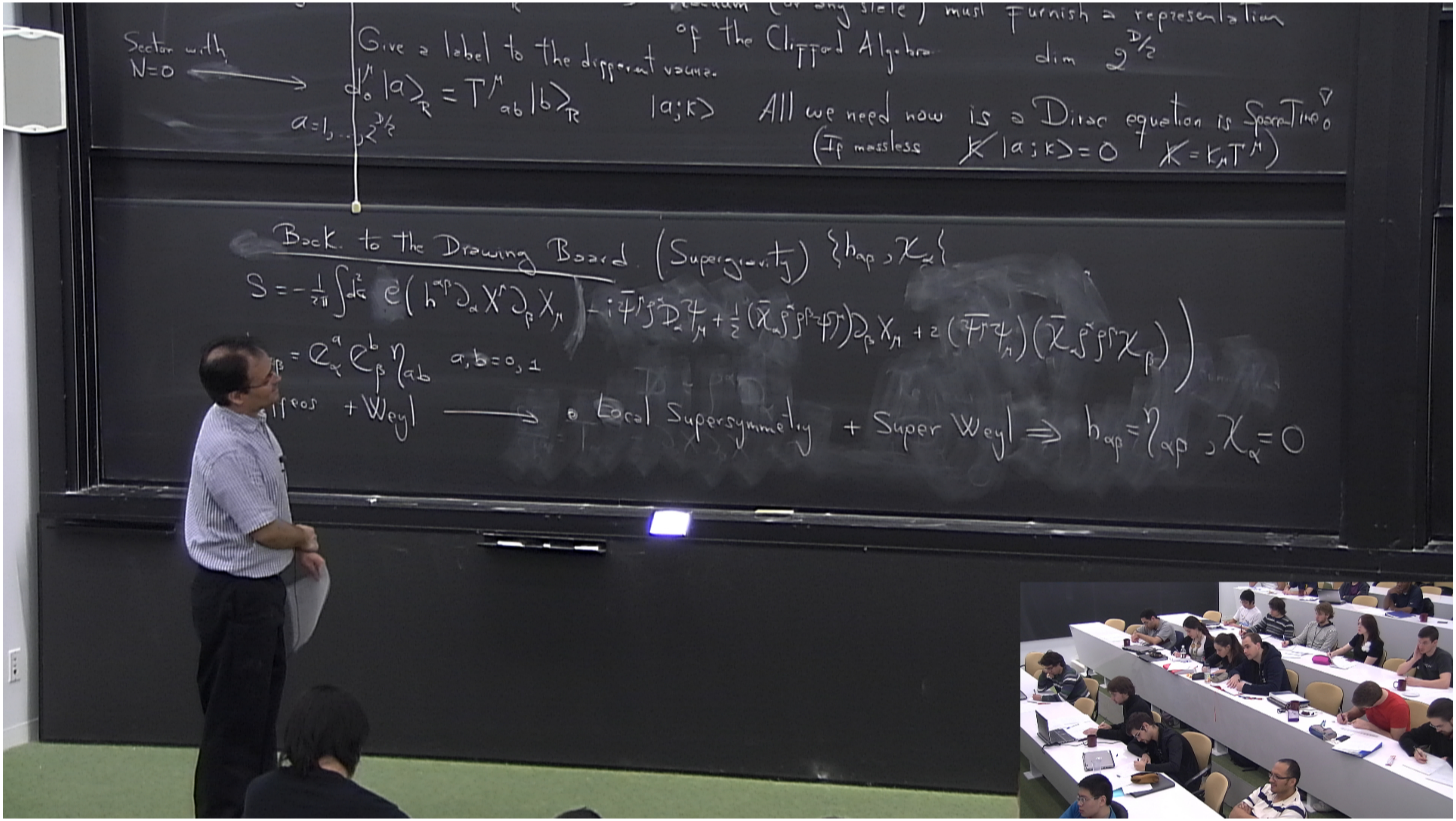
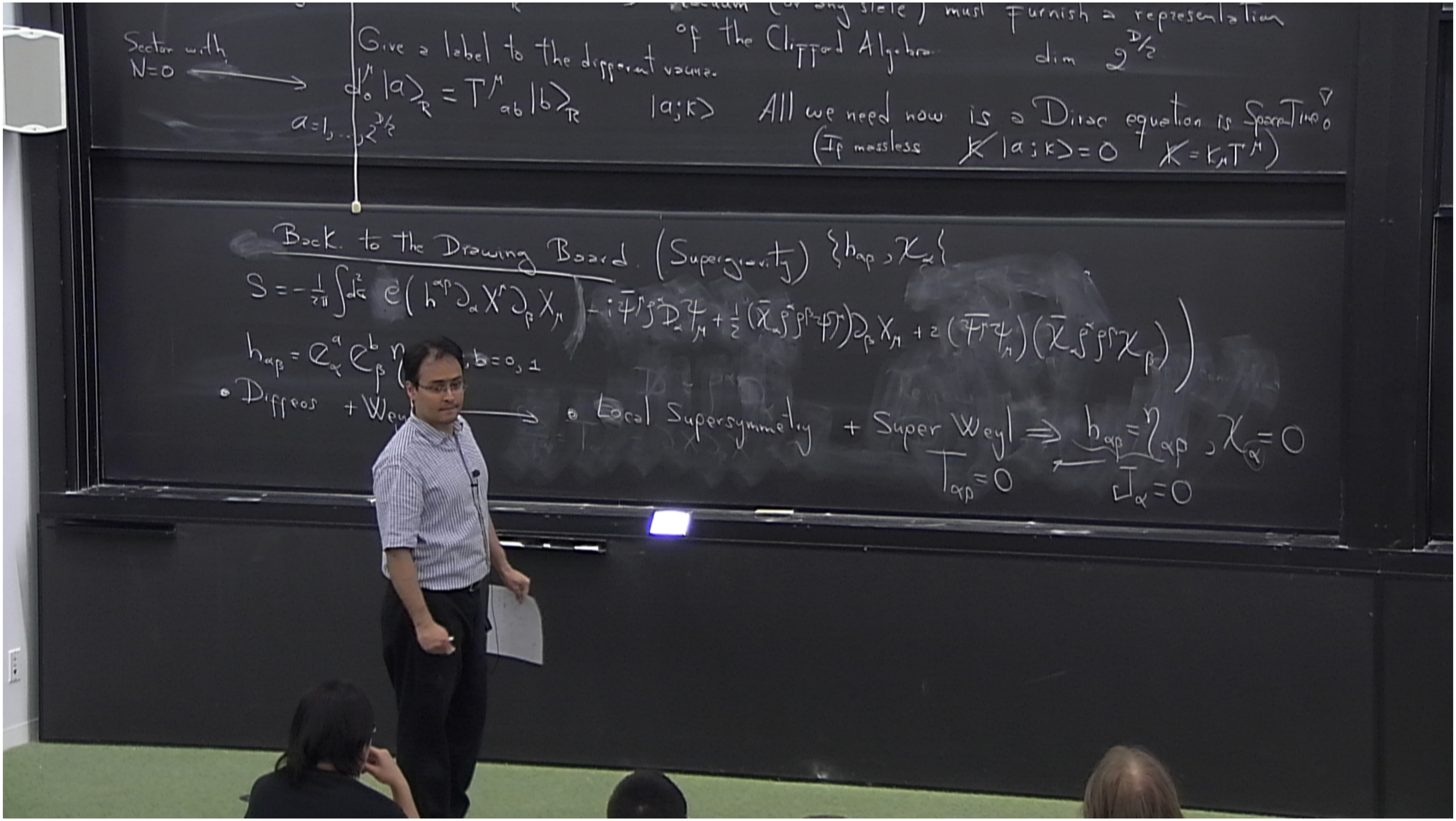
Give a label to the different vacua.
|a;k>
All we need now is a Dirac equation in SpaceTime
(If massless
Back to the Drawing Board (Supergravity)
a,b=0,1
Diffeos + Weyl



must furnish a representation
of the Clifford Algebra
Sector with
N=0
Give a label to the different vacua.
All we need now is a Dirac equation in SpaceTime
massless
Back to the Drawing Board (Supergravity)
Local Supersymmetry + Super Weyl


Sector with N=0
Give a label to the different vacua.
a=1,...,2^{D/2}
must Furnish a representation of the Clifford Algebra
dim 2^{D/2}
|a;k>
All we need now is a Dirac equation in SpaceTime?
(If massless)
Back to the Drawing Board (Supergravity) {h_{αβ}, χ_α}
h_{αβ} = e^a_α e^b_β
b = 0, 1
• Diffeos + Weyl
• Local Supersymmetry + Super Weyl ⇒ h_{αβ} = η_{αβ}, χ_α = 0
T_{αβ} = 0
J_α = 0