

Title: String Theory (Review) - Lecture 10

Date: Feb 27, 2012 11:30 AM

URL: <http://pirsa.org/12020061>

Abstract:

RNS string.



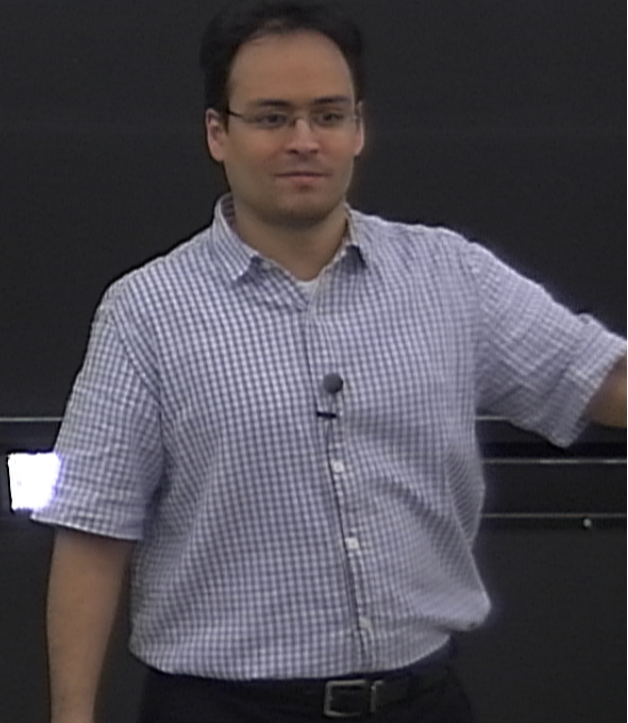
RNS string theory

$$S = \int \mathbb{R}^2 \mathcal{L}$$

RNS string theory

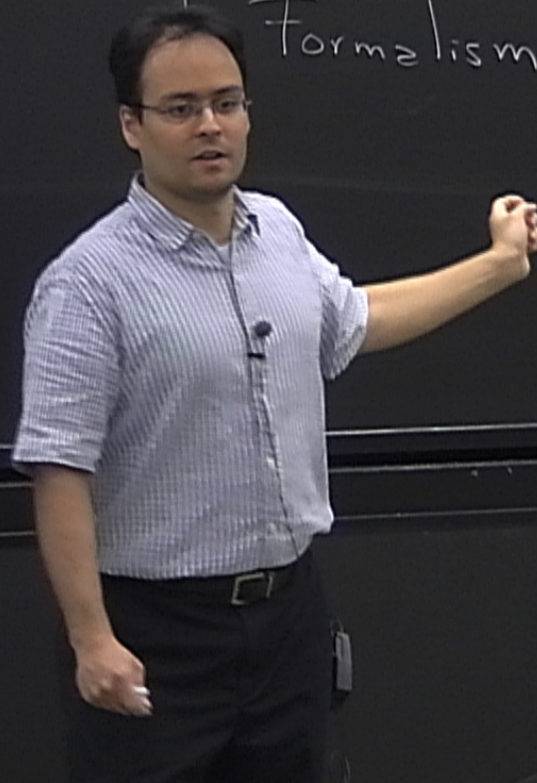
$$S^{(X)} = -\frac{1}{2\pi} \int d\sigma \partial_\alpha X^\mu \partial^\alpha X^\nu \eta_{\mu\nu}$$

$$S^{(\psi)} = -\frac{1}{2\pi} \int d\sigma \bar{\psi}^\mu \not{\partial} \psi^\nu \eta_{\mu\nu}$$



$\delta\psi^\nu \eta_{\mu\nu}$

Another approach: (X^μ, θ)
Green-Schwarz
Formalism.





RNS string theory

$$S^{(X)} = -\frac{1}{2\pi} \int d\sigma^2 \partial_\alpha X^\mu \partial^\alpha X^\nu \eta_{\mu\nu}$$

$$S^{(\psi)} = -\frac{1}{2\pi} \int d\sigma^2 \bar{\psi}^\mu \psi^\nu \eta_{\mu\nu}$$

Majorana-Weyl

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$$

$$\int d\sigma^2 d\sigma \left(\psi_-^\mu \partial_+ \psi_{-\mu} + \psi_+^\mu \partial_- \psi_{+\mu} \right)$$

Field eqs: $\partial_+ \psi_-^\mu = 0$

$$\partial_- \psi_+^\mu = 0$$

Bdr

Field eqs: $\partial_+ \psi_-^M = 0$ $\partial_- \psi_+^M = 0$

Bdry conditions

Open String.

R:

$$\psi_+^M(\tau, \pi) = \psi_-^M(\tau, \pi)$$

$$\psi_+^M(\tau, 0) = \psi_-^M(\tau, 0)$$

$$\underline{NS} : \psi_{+}^{\mu}(\tau, \pi) = -\psi_{-}^{\mu}(\tau, \pi)$$

$$\psi_{+}^{\mu}(\tau, 0) = \psi_{-}^{\mu}(\tau, 0)$$

Bdry conditions

Open String.

R:

$$\psi^M_{+}(\tau, \pi) = + \psi^M_{-}(\tau, \pi)$$

$$\psi^M_{+}(\tau, 0) = \psi^M_{-}(\tau, 0)$$

$$\psi^M_{+}(\tau, \pi) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma_+}$$

$$\psi^M_{-}(\tau, 0) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma_-}$$

$$\Rightarrow d_n^M = d_n^M$$

$$d_n^{M+} = d_{-n}^M$$

$$\underline{NS} : \psi_{+}^{\mu}(\tau, \pi) = -\psi_{-}^{\mu}(\tau, \pi)$$

$$\psi_{+}^{\mu}(\sigma_{+}) = \sum_r b_r^{\mu} e^{-in\sigma_{+}}$$

$$\psi_{-}^{\mu}(\sigma_{-}) = \sum_r b_r^{\mu} e^{-in\sigma_{-}}$$

$$\psi_{+}^{\mu}(\tau, 0) = \psi_{-}^{\mu}(\tau, 0)$$

$$\Downarrow$$
$$b_r^{\mu} = b_r^{\mu}$$

At

NS: $\psi_{+}^{\prime}(z, \pi) = -\psi_{-}^{\prime}(z, \pi)$

$\psi_{+}^{\prime}(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_r^{\prime} e^{-ir\sigma_+}$

$\psi_{-}^{\prime}(z) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_r^{\prime} e^{-ir\sigma_-}$

$\sigma_{\pm} = \tau \pm \sigma$

$\psi_{+}^{\prime}(z, 0) = \psi_{-}^{\prime}(z, 0)$

\downarrow

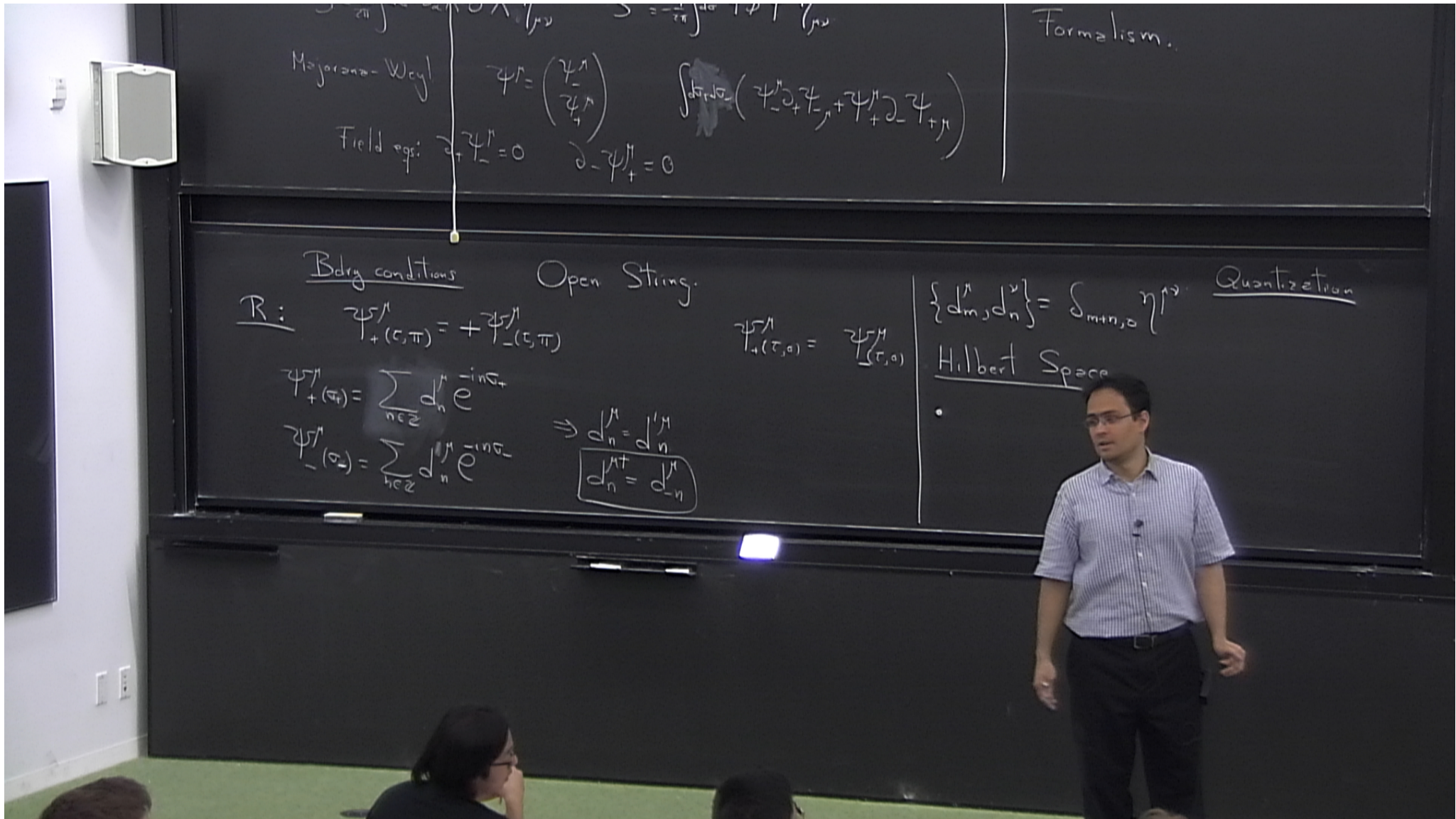
$b_r^{\prime} = b_r^{\prime\prime}$

At $\sigma = \pi$, let's add up both: $\psi_{+}^{\prime} + \psi_{-}^{\prime}$

$0 = \sum_r b_r^{\prime} e^{-ir\pi} \cos(r\pi) \Rightarrow r \in \mathbb{Z}$

Quantization





Majorana-Weyl $\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$

Field eqs: $\partial_+ \psi_- = 0$ $\partial_- \psi_+ = 0$

$\int_{\sigma_1, \sigma_2} (\psi_- \partial_+ \psi_+ + \psi_+ \partial_- \psi_-)$

Formalism.

Bdry conditions Open String.

R: $\psi_+^M(\sigma, \pi) = +\psi_-^M(\sigma, \pi)$

$\psi_+^M(\sigma) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma}$

$\psi_-^M(\sigma) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma}$

$\Rightarrow d_n^M = d_n^M$
 $d_n^{M+} = d_{-n}^M$

$\psi_+^M(\sigma, \sigma) = \psi_-^M(\sigma, \sigma)$

$\{d_m^M, d_n^N\} = \delta_{m+n,0} \eta^{MN}$ Quantization

Hilbert Space

Quantization

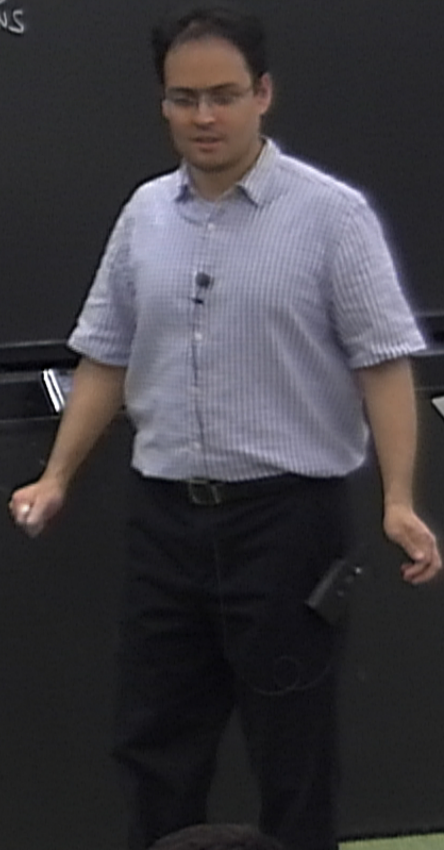
$$\left\{ \begin{matrix} \psi^{\mu} \\ \psi^{\nu} \end{matrix} \right\}_{(c,c)} = \delta_{(\mu-\nu)} \eta^{\mu\nu}$$

Hilbert Space.

$$[\alpha_m^{\mu}, \alpha_n^{\nu}] = m \delta_{m+n,0} \eta^{\mu\nu}$$

$$b_r^{\circ} |0\rangle_{NS}$$

$$\{b_r^{\mu}, b_s^{\nu}\} = \delta_{r+s,0} \eta^{\mu\nu}$$



Quantization

$$\left\{ \begin{matrix} \psi^{\mu} \\ \psi^{\nu} \end{matrix} \right\}_{(CC, \eta)} = \delta_{(\mu-\nu)} \eta^{\mu\nu}$$

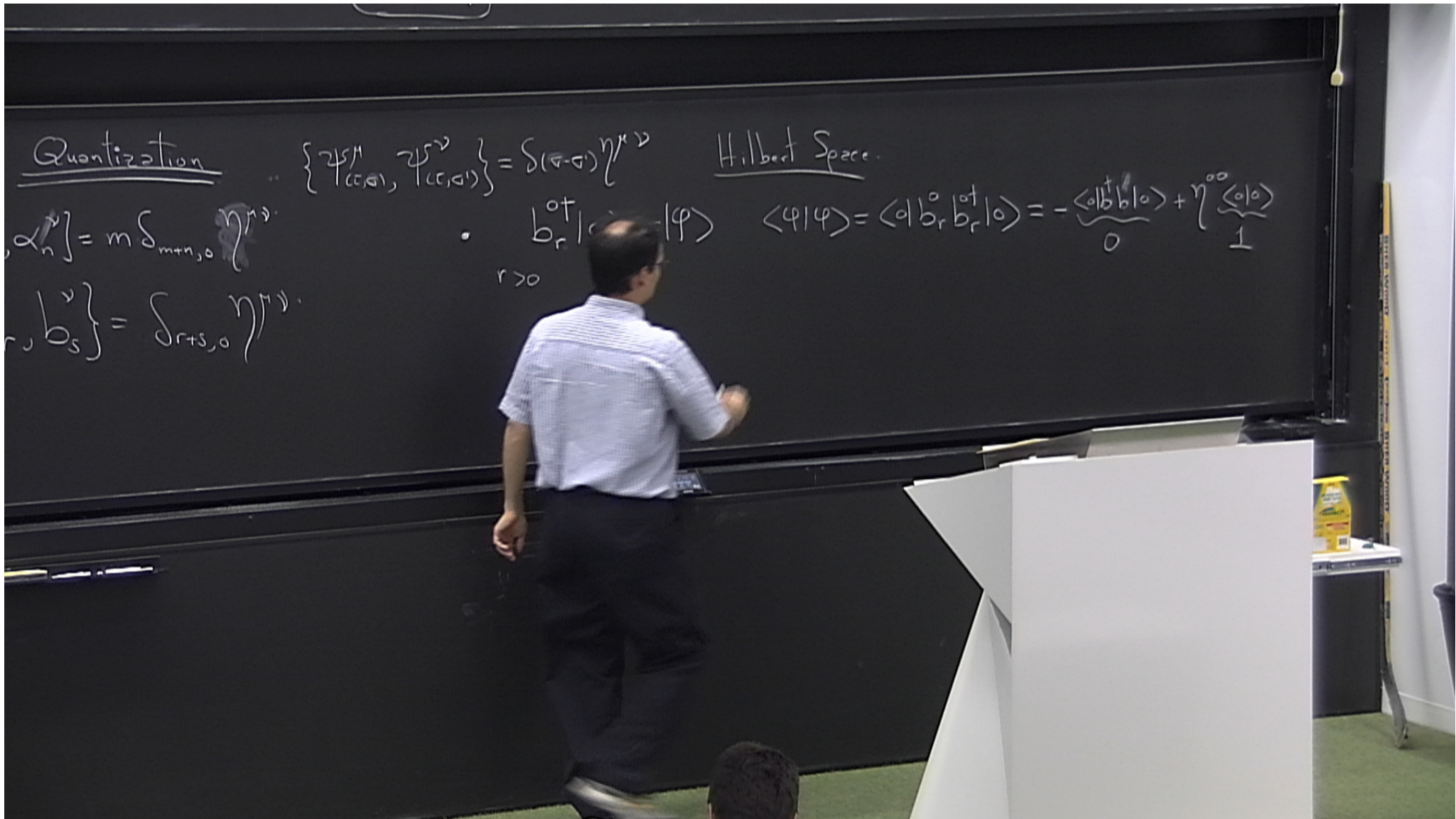
Hilbert Space

$$[\alpha_m^{\mu}, \alpha_n^{\nu}] = m \delta_{m+n,0} \eta^{\mu\nu}$$

$$\bullet \quad b_r^{\dagger} |0\rangle_{NS} = |\varphi\rangle \quad r > 0$$

$$\langle \varphi | \varphi \rangle = \langle 0 | b_r^{\dagger} b_r^{\dagger} |0\rangle =$$

$$\left\{ b_r^{\mu}, b_s^{\nu} \right\} = \delta_{r+s,0} \eta^{\mu\nu}$$



Quantization

$$\{ \alpha_n^\nu \} = m \delta_{m+n,0} \eta^{m, \nu}$$

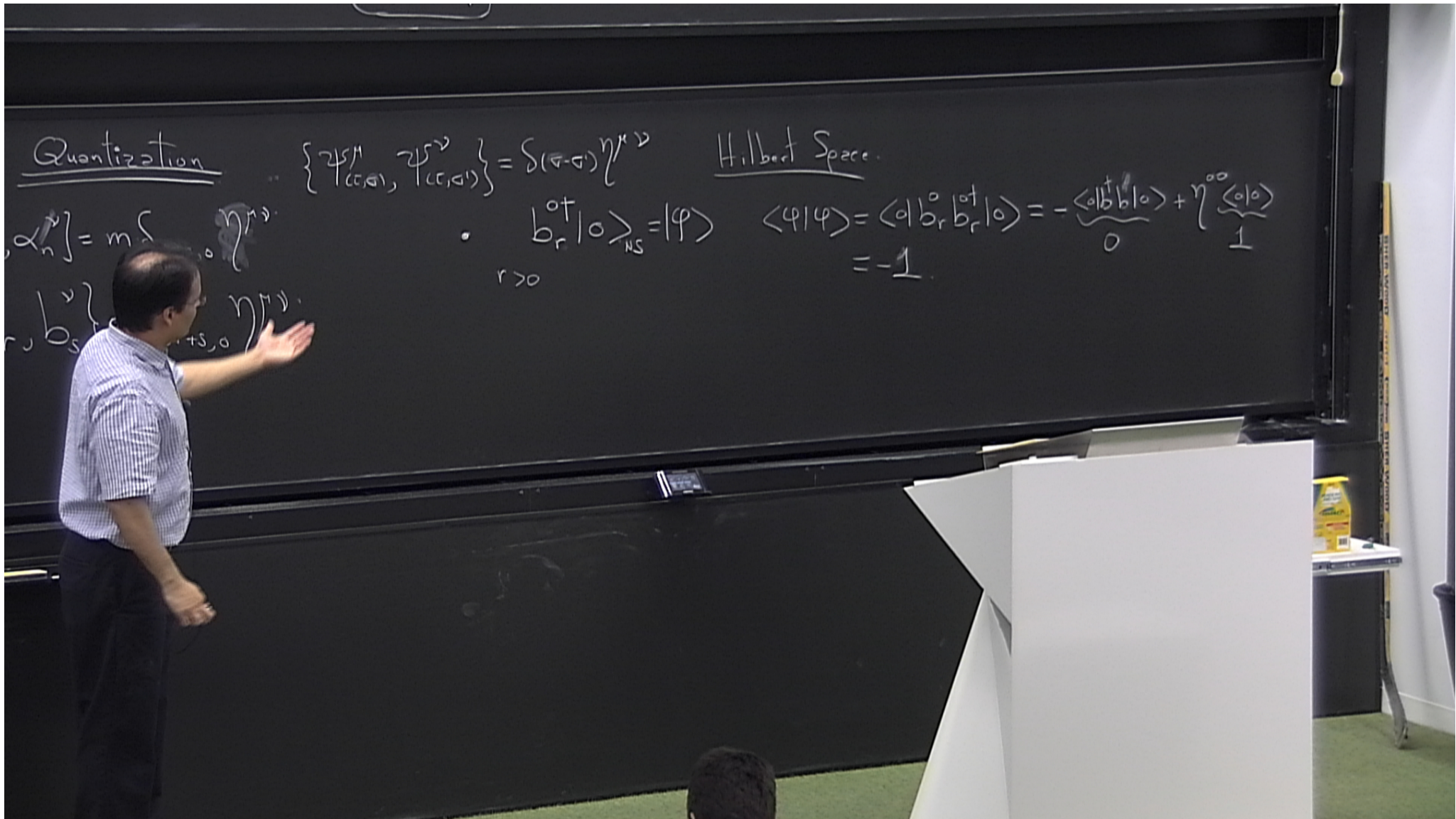
$$\{ b_r^\nu, b_s^\nu \} = \delta_{r+s,0} \eta^{r, \nu}$$

$$\{ \psi_{(cc, \sigma)}^\mu, \psi_{(cc, \sigma)}^\nu \} = \delta_{(\sigma-\sigma')} \eta^{\mu, \nu}$$

Hilbert Space

$$b_r^{\dagger \nu} |0\rangle = |r\rangle \quad r > 0$$

$$\langle 4|4\rangle = \langle 0| b_r^{\dagger 0} b_r^{\dagger 0} |0\rangle = - \underbrace{\langle 0| b_r^{\dagger 0} b_r^{\dagger 0} |0\rangle}_0 + \eta^{00} \underbrace{\langle 0|0\rangle}_1$$



Quantization

$$\{\psi^{\mu}_{(CC, \sigma)}, \psi^{\nu}_{(CC, \sigma)}\} = \delta_{(\sigma-\sigma')} \eta^{\mu\nu}$$

Hilbert Space

$$[\alpha_n^{\mu}, \alpha_m^{\nu}] = m \delta_{n+m, 0} \eta^{\mu\nu}$$

$$[b_r^{\mu}, b_s^{\nu}] = r \delta_{r+s, 0} \eta^{\mu\nu}$$

$$b_r^{\dagger} |0\rangle_{NS} = |\varphi\rangle$$

$r > 0$

$$\langle \varphi | \varphi \rangle = \langle 0 | b_r^{\dagger} b_r^{\dagger} | 0 \rangle = - \underbrace{\langle 0 | b_r b_r | 0 \rangle}_0 + \eta^{\mu\nu} \underbrace{\langle 0 | 0 \rangle}_1 = -1$$

Majorana-Weyl

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

$$\int d\tau d\sigma (\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+)$$

formalism.

Field eqs: $\partial_+ \psi_- = 0$ $\partial_- \psi_+ = 0$

Bdry conditions

Open String.

R:

$$\psi_+^{\mu}(\sigma, \pi) = +\psi_-^{\mu}(\sigma, \pi)$$

$$\psi_+^{\mu}(\tau, 0) = \psi_-^{\mu}(\tau, 0)$$

$$\psi_+^{\mu}(\sigma_+) = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma_+}$$

$$\psi_-^{\mu}(\sigma_-) = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma_-}$$

$$\Rightarrow d_n^{\mu} = d_n^{\mu} \\ \boxed{d_n^{\mu+} = d_{-n}^{\mu}}$$

$$\{d_m^{\mu}, d_n^{\nu}\} = \delta_{m+n,0} \eta^{\mu\nu}$$

Hilbert Space

Quantization

$$\psi_{-}(\sigma) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_r^{\dagger} e^{-i r \sigma}$$

$$\psi_{\pm} = \psi \pm \bar{\psi} \quad b_r = b_r^{\dagger}$$

$$b_r^{\dagger} = b_{-r}^{\dagger}$$

Quantization

$$\{\psi_{\mu}^{\alpha}, \psi_{\nu}^{\beta}\} = \delta_{\mu\nu} \eta^{\alpha\beta}$$

Hilbert Space

$$[\alpha_m^{\dagger}, \alpha_n^{\dagger}] = m \delta_{m+n,0} \eta^{\dagger\dagger}$$

$$\{b_r^{\dagger}, b_s^{\dagger}\} = \delta_{r+s,0} \eta^{\dagger\dagger}$$

$$\begin{aligned} \bullet \quad b_r^{\dagger} |0\rangle_{NS} &= |\varphi\rangle \quad \langle \varphi | \varphi \rangle = \langle 0 | b_r^{\dagger} b_r^{\dagger} | 0 \rangle = - \underbrace{\langle 0 | b^{\dagger} b | 0 \rangle}_0 + \eta^{\dagger\dagger} \underbrace{\langle 0 | 0 \rangle}_1 \\ &= -1 \end{aligned}$$

• Define Vacuum: $b_r^{\dagger} |0\rangle_{NS} = 0 \quad r > 0 \quad \alpha_m^{\dagger} |0\rangle_{NS} = 0 \quad m > 0$

R: $\psi_{+(\tau, \pi)}^{\mu} = + \psi_{-(\tau, \pi)}^{\mu}$

$\psi_{+(\tau, 0)}^{\mu} = \psi_{-(\tau, 0)}^{\mu}$

$\psi_{+(\sigma_+)}^{\mu} = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma_+}$

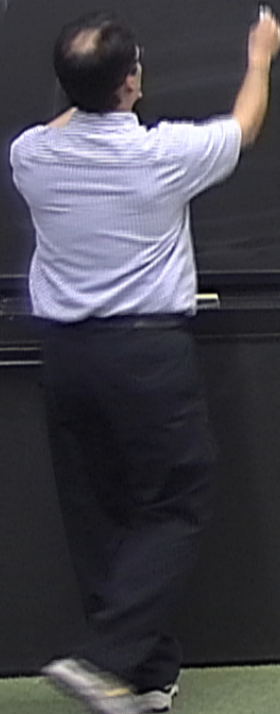
$\psi_{-(\sigma_-)}^{\mu} = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma_-}$

$\Rightarrow d_n^{\mu} = d_{-n}^{\mu}$
 $d_n^{\mu\dagger} = d_{-n}^{\mu}$

Hilbert Space

- Vacuum: $d_m^{\mu} |0\rangle_R = 0 \quad \forall m > 0$
 $m=0 \quad n=0$
 $\{d_0^{\mu}, d_0^{\nu}\} = \eta^{\mu\nu}$

$\{\sqrt{2}d_0^{\mu}, \sqrt{2}d_0^{\nu}\}$



$R:$ $\psi_+^M(\tau, \pi) = + \psi_-^M(\tau, \pi)$

$\psi_+^M(\tau, \sigma) = \psi_-^M(\tau, \sigma)$

$\psi_+^M(\sigma_+) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma_+}$

$\psi_-^M(\sigma_-) = \sum_{n \in \mathbb{Z}} d_n^M e^{-in\sigma_-}$

$\Rightarrow d_n^M = d_{-n}^M$
 $d_n^{M\dagger} = d_{-n}^M$

$\{d_m, d_n\} = \delta_{m+n, 0}$

Hilbert Space

- Vacuum: $d_m^M |0\rangle_R = 0 \quad \forall m > 0$
- $m=0 \quad n=0$
- $\{d_0^M, d_0^N\} = \eta^{MN}$

$\{\sqrt{2}d_0^M, \sqrt{2}d_0^N\} = 2\eta^{MN}$

$T^M = \sqrt{2}d_0^M$

$\{T^M, T^N\} = 2\eta^{MN}$

Clifford Algebra \mathcal{D}

$[N, d_0^M] = 0$

$N|0\rangle_R = 0$

\Rightarrow Vacuum (or any state) must furnish a representation of the Clifford Algebra
 dim $2^{D/2}$

Give a label to the different vacua.
 $|a\rangle$

$$R: \quad \psi_{+(\tau, \pi)}^{\mu} = + \psi_{-(\tau, \pi)}^{\mu}$$

$$\psi_{+(\tau, \sigma)}^{\mu} = \psi_{-(\tau, \sigma)}^{\mu}$$

$$\psi_{+}^{\mu}(\sigma) = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma}$$

$$\psi_{-}^{\mu}(\sigma) = \sum_{n \in \mathbb{Z}} d_n^{\mu} e^{-in\sigma}$$

$$\Rightarrow d_n^{\mu} = d_{-n}^{\mu}$$

Hilbert Space

- Vacuum: $d_m^{\mu} |0\rangle_{\mathbb{R}} = 0 \quad \forall m > 0$
 $m=0 \quad n=0$
 $\{d_{\alpha}^{\mu}, d_{\beta}^{\nu}\} = \eta^{\mu\nu}$

$$\{\sqrt{2}d_0^{\mu}, \sqrt{2}d_0^{\nu}\} = 2\eta^{\mu\nu} \quad T^{\mu} = \sqrt{2}d_0^{\mu} \quad \{T^{\mu}, T^{\nu}\} = 2\eta^{\mu\nu} \quad \text{Clifford Algebra } \mathbb{D}$$

$$\bullet [N, d_0^{\mu}] = 0$$

$N|0\rangle_{\mathbb{R}} = 0 \Rightarrow$ Vacuum (or any state) must furnish a representation of the Clifford Algebra $\dim 2^{D/2}$

Give a label to the different vacua.

$$d_0^{\mu} |a\rangle_{\mathbb{R}} = T_{ab}^{\mu} |b\rangle_{\mathbb{R}}$$

$$a=1, \dots, 2^{D/2}$$

$$\psi_{(-\sigma)} = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_r^\dagger e^{-ir\sigma}$$

$$b_r^\dagger = b_{-r}^\dagger$$

Quantization

$$\{\psi_{(\sigma, \alpha)}, \psi_{(\sigma, \alpha)}^\dagger\} = \delta_{(\sigma-\sigma')} \eta^{\alpha\alpha'}$$

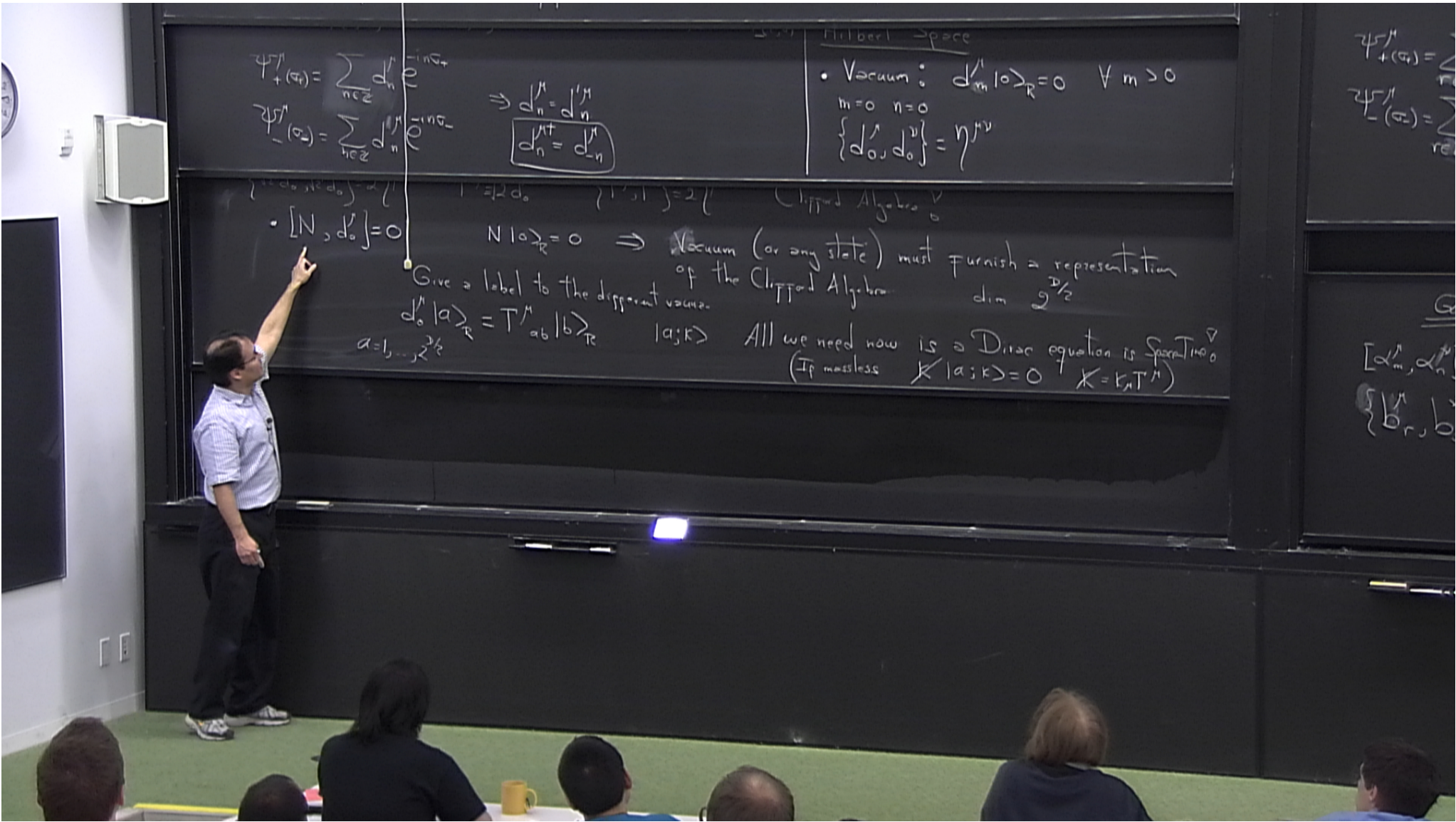
Hilbert Space

$$[\alpha_m^\dagger, \alpha_n^\dagger] = m \delta_{m+n, 0} \eta^{\alpha\alpha'}$$

$$\{b_r^\dagger, b_s^\dagger\} = \delta_{r+s, 0} \eta^{\alpha\alpha'}$$

$$\bullet \quad b_r^{\dagger 0} |0\rangle_{NS} = |1\rangle \quad \langle 1|1\rangle = \langle 0|b_r^0 b_r^{\dagger 0}|0\rangle = -\frac{\langle 0|b_r^\dagger b_r|0\rangle}{0} + \eta^{\alpha\alpha'} \frac{\langle 0|0\rangle}{1} = -1$$

• Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0 \quad \alpha_{-m}^\dagger |0\rangle_{NS} = 0 \quad m > 0$

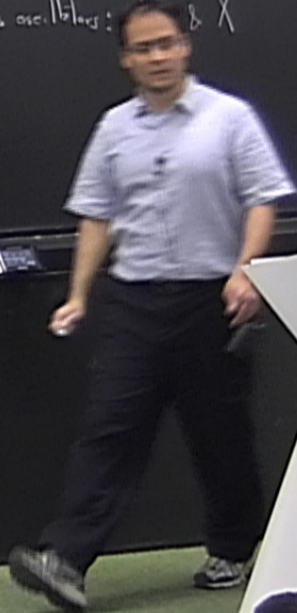


representation
 $g^{D/2}$
 equation is Spacetime
 $\nabla \cdot X = k, T^M$

$\{b_r, b_s\} = \delta_{r+s,0} \eta^{rs}$
 $r \in \mathbb{Z} + \frac{1}{2}$

- Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0 \quad \alpha_m^- |0\rangle = 0 \quad m > 0$
- $|0; k\rangle_{NS}$

Light Cone Gauge Quantization $S = \int d\sigma d\tau 2X \cdot \dot{X}$
 $(U^M) = (\underbrace{U^+, U^-, U^i}_{i=1,2,\dots,D-2}, \underbrace{U^0, U^{D-1}}_{\text{Divergent oscillators: } X^+ \& X^-})$



representation
 $g^{D/2}$
 ac equation is Spacetime
 ∇
 $= 0$ $X = k_\mu T^\mu$

$$\{b_r, b_s\} = \delta_{r+s,0} \eta^{rs}$$

$$r \in \mathbb{Z} + \frac{1}{2}$$

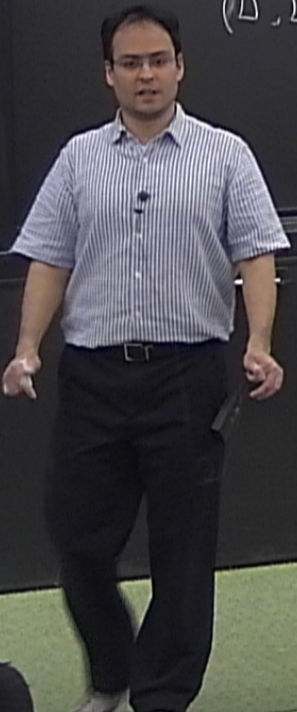
- Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0 \quad \alpha_{-m}^{\mu} |0\rangle_{NS} = 0 \quad m > 0$
- $|0; k\rangle_{NS}$

Light Cone Gauge Quantization

$$S = \int d\sigma d\tau 2X \cdot \partial_\alpha X$$

$$(\underline{U}^\mu) = (\underbrace{U^+, U^-, U^i}_{\rightarrow 1, 2, \dots, D-2}, \underbrace{U^0, U^{D-1}})$$

Dynamical oscillators: X^+ & X^-
 Residual Symmetry: Conformal invariance
 $X_{(CG)}^+ = x^+ + \theta \tau^+ c$



representation
 $g^{D/2}$
 equation is Spacetime
 ∇
 $\square = 0$
 $X = k_\mu T^\mu$

$\{b_r, b_s\} = \delta_{r+s,0} \eta^{rs}$
 $r \in \mathbb{Z} + \frac{1}{2}$

- Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0 \quad \alpha_m^- |0\rangle_{NS} = 0 \quad m > 0$
- $|0; k\rangle_{NS}$

Light Cone Gauge Quantization

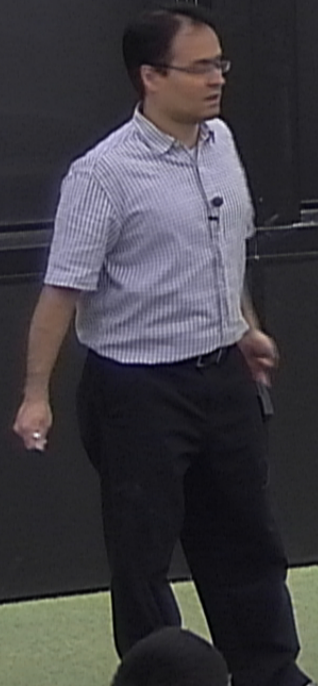
$S = \int d\sigma d\tau 2X \cdot \dot{X}$

$(L^M) = (\underbrace{L^+, L^-, L^i}_{(L^0, L^{D-1})}) \rightarrow 1, 2, \dots, D-2$

Dynamical oscillators: X^+ & X^-
 Residual Symmetry: Conformal invariance

$X_{(reg)}^+ = x^+ + \theta \tau^+ c$

$\dot{X}^+ X^+ = 0 \quad -\dot{X}^+ X^- = \dot{X}^- X^+ \quad \dot{X}^i X^i = 0$



representation
 $\frac{D}{2}$
 equation is $\text{Spin}(1, D-1)$
 $X = \tau_1 T^{\mu\nu}$

$\{b_r^\mu, b_s^\nu\} = \delta_{rs,0} \eta^{\mu\nu}$
 $r \in \mathbb{Z} + \frac{1}{2}$

$b_r^\mu = b_{-r}^\mu$

Define Vacuum: $b_r^\mu |0\rangle_{NS} = 0 \quad r > 0$
 $\alpha_m^\mu |0\rangle_{NS} = 0 \quad m > 0$
 $|0; k\rangle_{NS}$

Light Cone Gauge Quantization

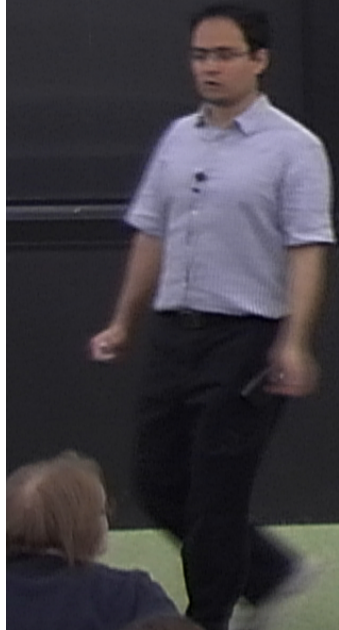
$(L^\mu) = (\underbrace{L^+, L^-}_{L^0, L^{D-1}}, L^i) \rightarrow 1, 2, \dots, D-2$

$\dot{X}^\mu \dot{X}_\mu = 0$
 $-\dot{X}^+ \dot{X}^- = \dot{X}^i \dot{X}^i = 0$

$S = \int d\sigma d\tau 2\dot{X}^+ \dot{X}^-$

Dangerous oscillators: X^+ & X^-
 Residual Symmetry: Conformal invariance
 $X^+_{(\sigma, \tau)} = x^+ + \alpha' p^+ \tau$
 $\alpha_m^- = f(\alpha_n^i)$

$S = \int d\sigma d\tau \bar{\psi} \not{\partial} \psi$
 $\psi_+^{\pm}, \psi_-^{\pm}$ chirality on the World Sheet



$$\{b_r, b_s\} = \delta_{r+s,0} \eta^{rs}$$

$$r \in \mathbb{Z} + \frac{1}{2}$$

$r > 0$
 Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0$
 $\alpha_m |0\rangle_{NS} = 0 \quad m > 0$
 $|0; k\rangle_{NS}$

Light Cone Gauge Quantization

$$(L^{\mu\nu}) = (\underbrace{L^+, L^-, L^i}_{(L^0, L^{D-1})}) \rightarrow 1, 2, \dots, D-2$$

$$\dot{X}^\mu \dot{X}^\nu = 0 \quad -\dot{X}^+ \dot{X}^- = \dot{X}^- \dot{X}^+ + \sum_i \dot{X}^i \dot{X}^i = 0$$

$$S = \int d\sigma d\tau 2 \dot{X}^- \dot{X}^+$$

Dangerous oscillators: X^+ & X^-
 Residual Symmetry: Conformal invariance
 $X^+_{(\tau\sigma)} = X^+ + D^+_{\sigma} \tau$
 $\alpha_m^- = f(\alpha_n^i)$

$S = \int d\sigma d\tau \bar{\psi} \not{\partial} \psi$
 ψ^+_{\pm}, ψ^{\pm} chirality on the World-Sheet
 Super-Conformal
 $\psi^+_{\pm} = 0$

$\cdot [N, d_0^\mu] = 0$ $N|0\rangle_R = 0 \Rightarrow$ Vacuum (or any state) must furnish a representation of the Clipped Algebra $\dim g^{D/2}$

Sector with $N=0$ \rightarrow Give a label to the different vacua. $d_0^\mu |a\rangle_R = T_{ab}^\mu |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spec } T_{ab}^\mu = 0$ (If massless $K|a; k\rangle = 0$ $K = k_\mu T^{\mu\nu}$)

Back to the Drawing Board.

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-h} (h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu)$$

$$h_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}$$

$N=0 \rightarrow d_0^m |a\rangle_R = T^m_{ab} |b\rangle_R \quad |a; k\rangle$
 $a=1, \dots, 2^{D/2}$

All we need now is a Dirac equation is for
 (If massless $K|a; k\rangle = 0$ $K=k, T^m$)

Back to the Drawing Board

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \left(h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right) + \bar{\psi}^\mu \not{D} \psi^\nu \eta_{\mu\nu}$$

$$h_{\alpha\beta} = e^a_\alpha e^b_\beta \eta_{ab} \quad a, b = 0, 1$$

$$\not{D} = \rho^\alpha \not{D}_\alpha$$

$$\{p^\alpha, p^\beta\} = 2h^{\alpha\beta}$$

$$p_\alpha = e^a_\alpha p_a$$

$$\{b_r, b_s\} = \delta_{r+s,0} \eta^{rs}$$

$$r \in \mathbb{Z} + \frac{1}{2}$$

$r > 0$
 Define Vacuum: $b_r |0\rangle_{NS} = 0$ $r > 0$ $\alpha_{-m} |0\rangle_{NS} = 0$ $m > 0$
 $|0; k\rangle_{NS}$

Light Cone Gauge Quantization

$$(L^{\mu}) = (\underbrace{L^+, L^-, L^i}_{(L^0, L^{D-1})}) \rightarrow 1, 2, \dots, D-2$$

$$\dot{X}^\mu \dot{X}_\mu = 0 \quad -\dot{X}^+ \dot{X}^- = \dot{X}^- \dot{X}^+ + \sum_i \dot{X}^i \dot{X}^i = 0$$

$$S = \int d\sigma d\tau 2 \dot{X}^- \dot{X}^+$$

Dangerous oscillators: X^+ & X^-
 Residual Symmetry: Conformal invariance.
 $X^+_{(\tau\sigma)} = \dot{X}^+ + \partial_\sigma^+ \tau^+$ $\alpha_m^- = f(\alpha_n^i)$

$S = \int d\sigma d\tau \bar{\psi} \not{\partial} \psi$
 ψ^+_{\pm}, ψ^{\pm} chirality on the World-Sheet
 Super-Conformal.
 $\psi^+_{\pm} = 0$

$N=0 \rightarrow d_0^M |a\rangle_R = T^M_{ab} |b\rangle_R \quad |a; k\rangle$
 $a=1, \dots, 2^{D/2}$

All we need now is a Dirac equation is $\not{K} |a; k\rangle = 0$
 (If massless $K = k, T^M$)

Back to the Drawing Board

$$S = -\frac{1}{2\pi} \int d^2\sigma \left(h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu \right) + \bar{\psi}^\mu \not{D} \psi^\nu \eta_{\mu\nu}$$

$h_{\alpha\beta} = e^a_\alpha e^b_\beta \eta_{ab} \quad a, b = 0, 1$
 • Diffeos + Weyl

$$\not{D} = \rho^\alpha \not{D}_\alpha$$

$$\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_\alpha X \cdot \partial_\beta X + \text{Fermions} - \text{traces} = 0$$

$$\{p^a, p^b\} = 2h^{ab}$$

$$p_\alpha = e^a_\alpha p_a$$

Review:

$$D_\alpha = \partial_\alpha + [p^a, p^b] \omega_\alpha^{ab}$$

Spin connection

$$\{b_r, b_s\} = \delta_{r+s, 0}$$

$$r \in \mathbb{Z} + \frac{1}{2}$$

Define Vacuum: $b_r |0\rangle_{NS} = 0 \quad r > 0$ $\alpha_m^- |0\rangle_{NS} = 0 \quad m > 0$

$|0; k\rangle_{NS}$

Light Cone Gauge Quantization

$$T_{op} = 0 \Rightarrow X'^- = f(X^i, \psi^i)$$

$$\Rightarrow \alpha_m^- = f(\alpha^i, b^i \text{ or } d^i)$$

Nothing seems to kill the ψ_{\pm}^+ oscillators

$$S = \int d\sigma d\tau \mathcal{L} = \int d\sigma d\tau \mathcal{L}_X + \mathcal{L}_\psi$$

Dangerous oscillators: X^+ & X^-

Residual Symmetry: Conformal invariance

$$X_{(\tau, \sigma)}^+ = x^+ + \alpha' p^+ \tau + \dots$$

$$\alpha_m^- = f(\alpha_n^i)$$

$S = \int d\sigma d\tau \bar{\psi} \not{\partial} \psi$

$\psi_{\pm}^+, \psi_{\pm}^-$ chirality on the World-Sheet

Super-Conformal!

Sector with $N=0$ \rightarrow Give a label to the different values of the Clifford Algebra must furnish a representation of dimension $2^{D/2}$
 $d_0^R |a\rangle_R = T_{ab}^M |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spin-T} \nabla$
 (If massless $K|a; k\rangle = 0$ $K = k_\mu T^{\mu\nu}$)

Back to the Drawing Board (Supergravity) $\{h_{\alpha\beta}, \chi_\alpha\}$
 $S = -\frac{1}{2\pi} \int d^2x e (h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu) - i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\chi}_\alpha \gamma^{\mu\nu} \psi) \partial_\mu X_\nu + 2 (\bar{\psi} \gamma_\mu \psi) (\bar{\chi}_\alpha \gamma^{\mu\nu} \chi_\nu)$
 $h_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}$ $a, b = 0, 1$
 • Diffeos + Weyl $\mathcal{D} = \int d^2x \mathcal{D}_\alpha$
 $\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu + \text{Fermions} - \text{Trac} + 2 \psi^\mu \psi_\mu$



Sector with $N=0$ \rightarrow Give a label to the different values of the Clifford Algebra must furnish a representation $\dim 2^{D/2}$
 $d_0^a |a\rangle_R = T_{ab}^a |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spin-T} \nabla_0$
 (If massless $K|a; k\rangle = 0$ $K = k_i T^{i1}$)

Back to the Drawing Board (Supergravity) $\{h_{\alpha\beta}, \chi_\alpha\}$
 $S = -\frac{1}{2\pi} \int d^2x e (h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu) X_{,\mu\nu} - i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\chi}_\alpha \gamma^\mu \gamma^\nu \psi) \partial_\mu X_{,\nu} + 2 (\bar{\psi} \gamma_\mu \psi) (\bar{\chi}_\alpha \gamma^\mu \gamma^\nu \chi_\beta)$
 $h_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}$ $a, b = 0, 1$ $\mathcal{D} = \int \rho^\alpha \mathcal{D}_\alpha$
 $\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_\alpha X \partial_\beta X + \text{Fermion - Trace}$
 • Diffeos + Weyl



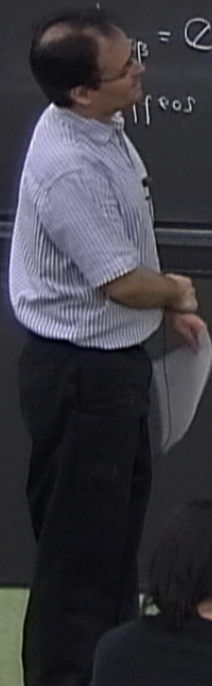
Sector with $N=0$ \rightarrow Give a label to the different vacua. of the Clifford Algebra must furnish a representation $\dim 2^{D/2}$
 $d_0^a |a\rangle_R = T_{ab}^a |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spin-T} \nabla$
 (If massless $K|a; k\rangle = 0$ $K = k_\mu T^{\mu\nu}$)

Back to the Drawing Board (Supergravity) $\{h_{\mu\nu}, \chi_\mu\}$
 $S = -\frac{1}{2\pi} \int d^2x e (h^{\mu\nu} \partial_\mu X^\nu)_p X_{,\mu} - i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\chi}_\mu \gamma^\mu \psi) \partial_\mu X_\nu + 2 (\bar{\psi} \gamma_\mu \psi) (\bar{\chi}_\nu \gamma^\mu \psi)$
 $h_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$ $a, b = 0, 1$
 • Diff + Weyl



Sector with $N=0$ \rightarrow Give a label to the different values of the Clifford Algebra $\dim \mathcal{S}^{D/2}$
 $d_0^a |a\rangle_R = T_{ab}^a |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spin}(1, D-1)$
 (If massless $K|a; k\rangle = 0$ $K = k_\mu T^{\mu\nu}$)

Back to the Drawing Board (Supergravity) $\{h_{\mu\nu}, \chi_\alpha\}$
 $S = -\frac{1}{2\pi} \int d^2x e (h^{\mu\nu} \partial_\mu X^\nu) \partial_\rho X_\rho - i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\chi}_\alpha \gamma^\mu \gamma^\nu \psi) \partial_\mu X_\nu + 2 (\bar{\psi} \gamma_\mu \psi) (\bar{\chi}_\alpha \gamma^\mu \gamma^\nu \chi_\rho)$
 $\eta_{ab} = e_\alpha^a e_\beta^b \eta_{ab}$ $a, b = 0, 1$
 Local Supersymmetry + Super Weyl $\Rightarrow h_{\mu\nu} = \eta_{\mu\nu}, \chi_\alpha = 0$



Sector with $N=0$ → Give a label to the different values of the Clifford Algebra must furnish a representation of $\dim 2^{D/2}$
 $d_0^M |a\rangle_R = T_{ab}^M |b\rangle_R$ $|a; k\rangle$ All we need now is a Dirac equation is $\text{Spin}(1, D-1)$
 (If massless $K|a; k\rangle = 0$ $K = k_\mu T^{\mu\nu}$)

Back to the Drawing Board (Supergravity) $\{h_{\alpha\beta}, \chi_\alpha\}$

$$S = -\frac{1}{2\pi} \int d^2x e (h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X_\mu) - i \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\bar{\chi}_\alpha \gamma^{\mu\nu} \psi) \partial_\mu X_\nu + 2 (\bar{\psi} \gamma_\mu \psi) (\bar{\chi}_\alpha \gamma^{\mu\nu} \chi_\nu)$$

$$h_{\alpha\beta} = e_\alpha^a e_\beta^b \eta_{ab}, \quad b=0,1$$

• Diffeos + Weyl

• Local Supersymmetry + Super Weyl $\Rightarrow h_{\alpha\beta} = \eta_{\alpha\beta}, \chi_\alpha = 0$
 $T_{\alpha\beta} = 0 \quad \leftarrow \quad \bar{\psi} \gamma_\alpha \psi = 0$