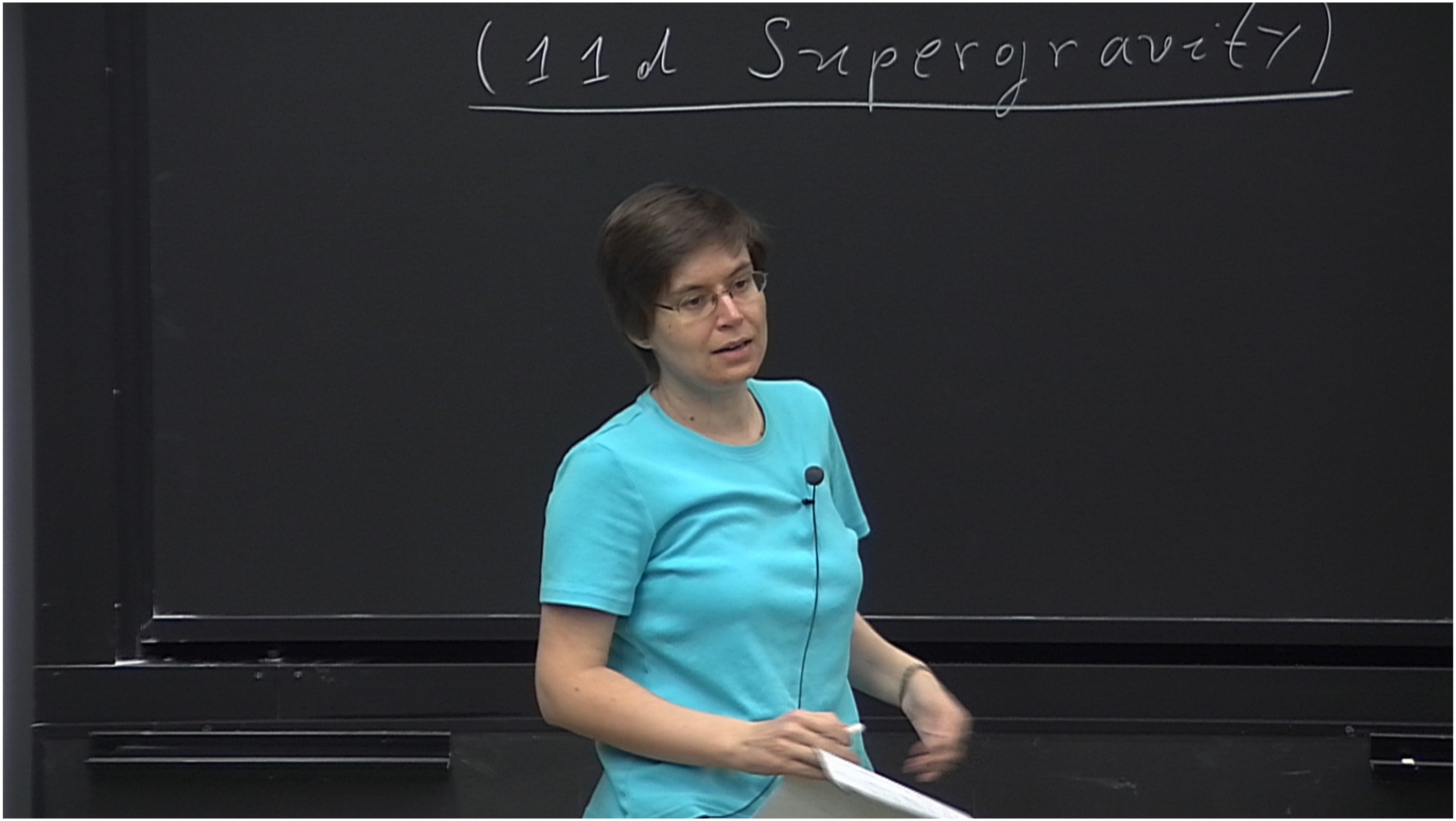
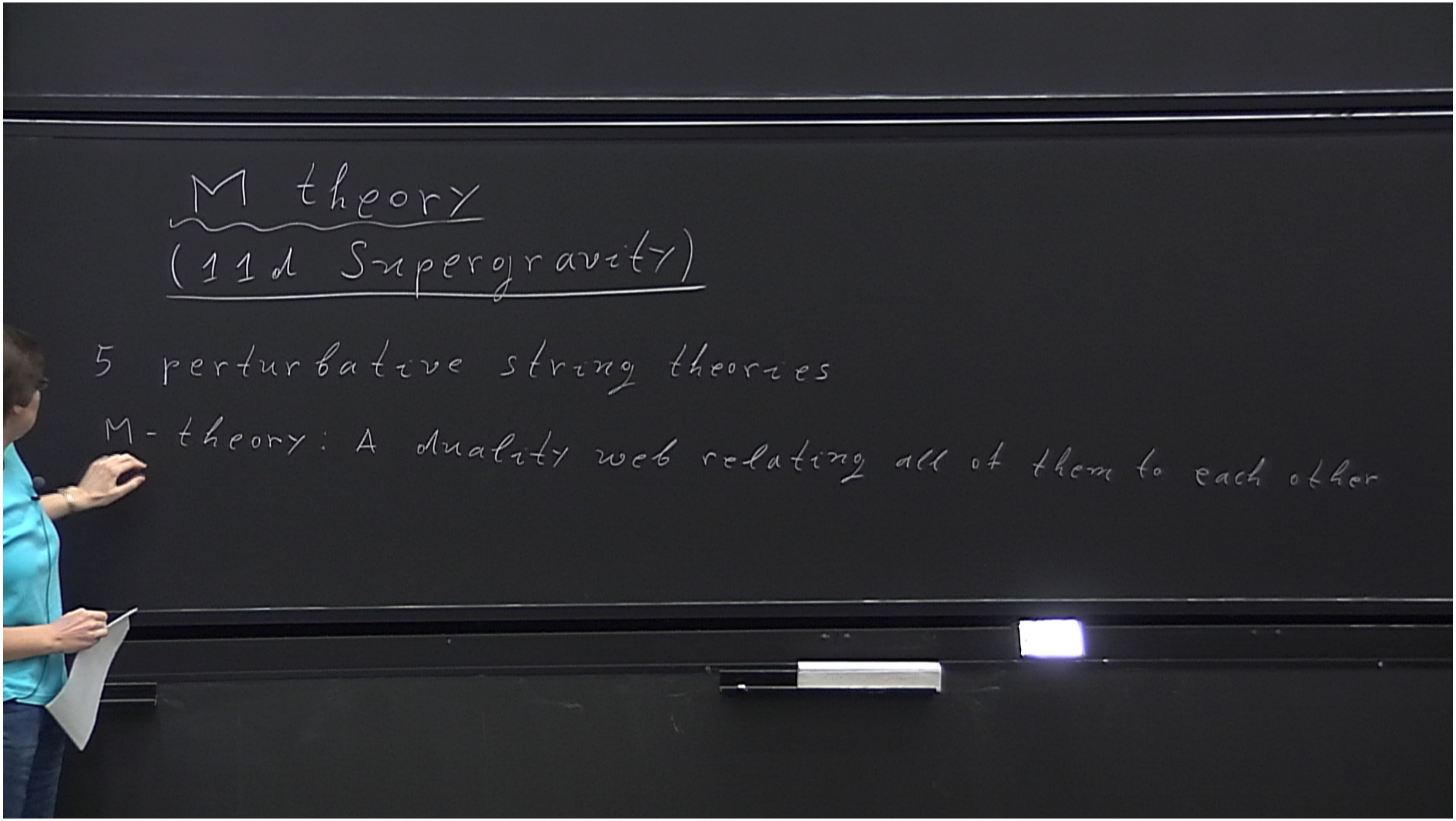
Title: String Theory (Review) - Lecture 8

Date: Feb 23, 2012 11:30 AM

URL: http://pirsa.org/12020057

Abstract:


(11d Supergravity)


M theory
(11d Supergravity)
5 perturbative string theories
M-theory: A duality web relating all of them to each other

M theory
(11d Supergravity)

11d SUGRA : Low-energy
(supergravity)

5 perturbative string theories

M-theory: A duality web relating all of them to each other

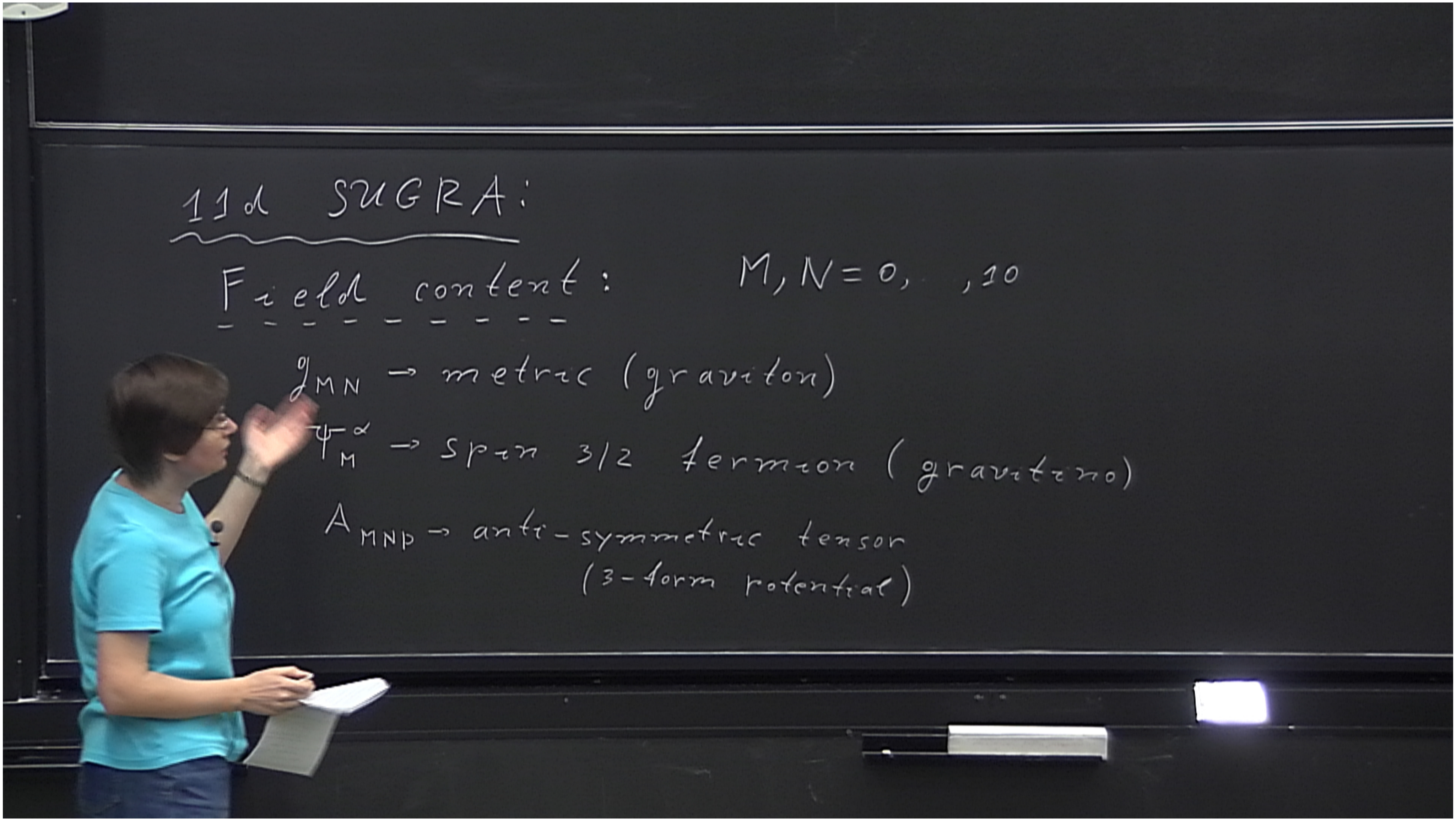
2 string theories (IIA, Het. $E_8 \times E_8$) exhibit an 11^{th} dim. at strong coupl.

11d SUGRA : Low-energy effective description of
(supergravity) M theory

[Fundam. objects : membranes]

... of them to each other

... but an 11^{th} dim. at strong coupl.


11d SUGRA:
Field content:
M, N = 0, ,10
g_{MN} → metric (graviton)
ψ_M^α → spin 3/2 fermion (gravitino)
A_{MNP} → anti-symmetric tensor
(3-form potential)

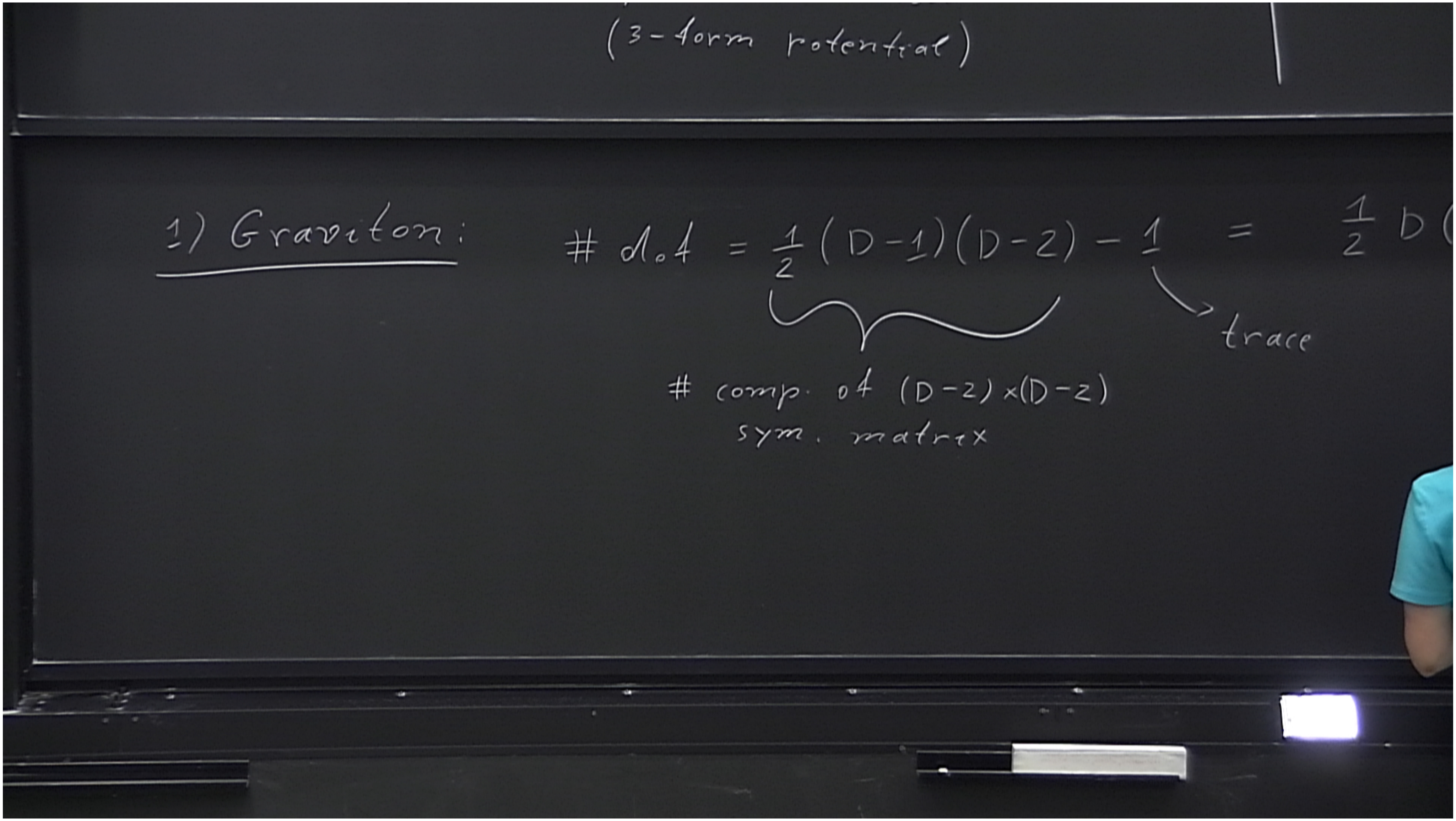
...it an 11th dim. at strong coupl.

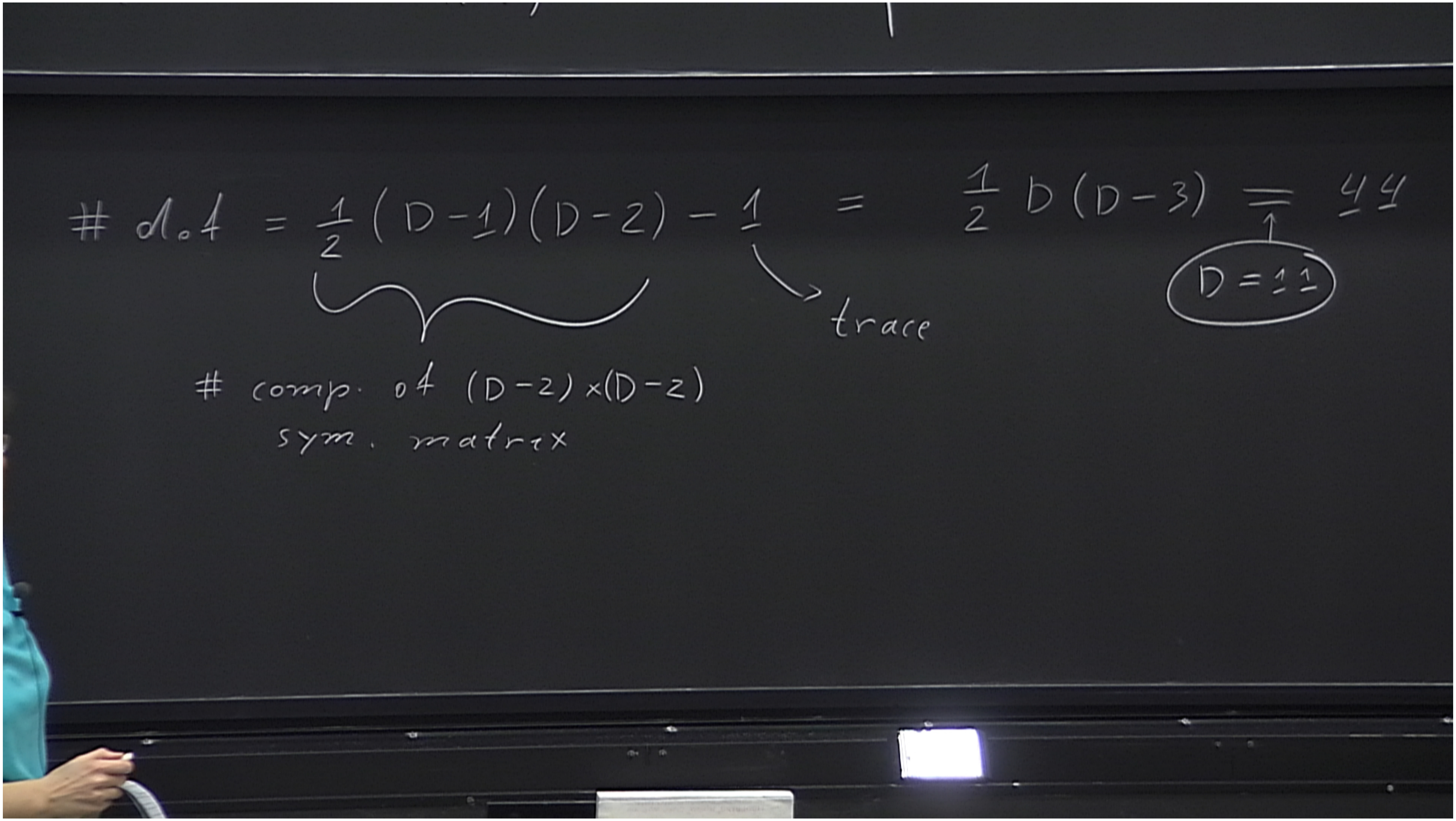
Let's count the phys. degrees of freedom
[polarizations]

In D dimensions the little group: SO(D-2)

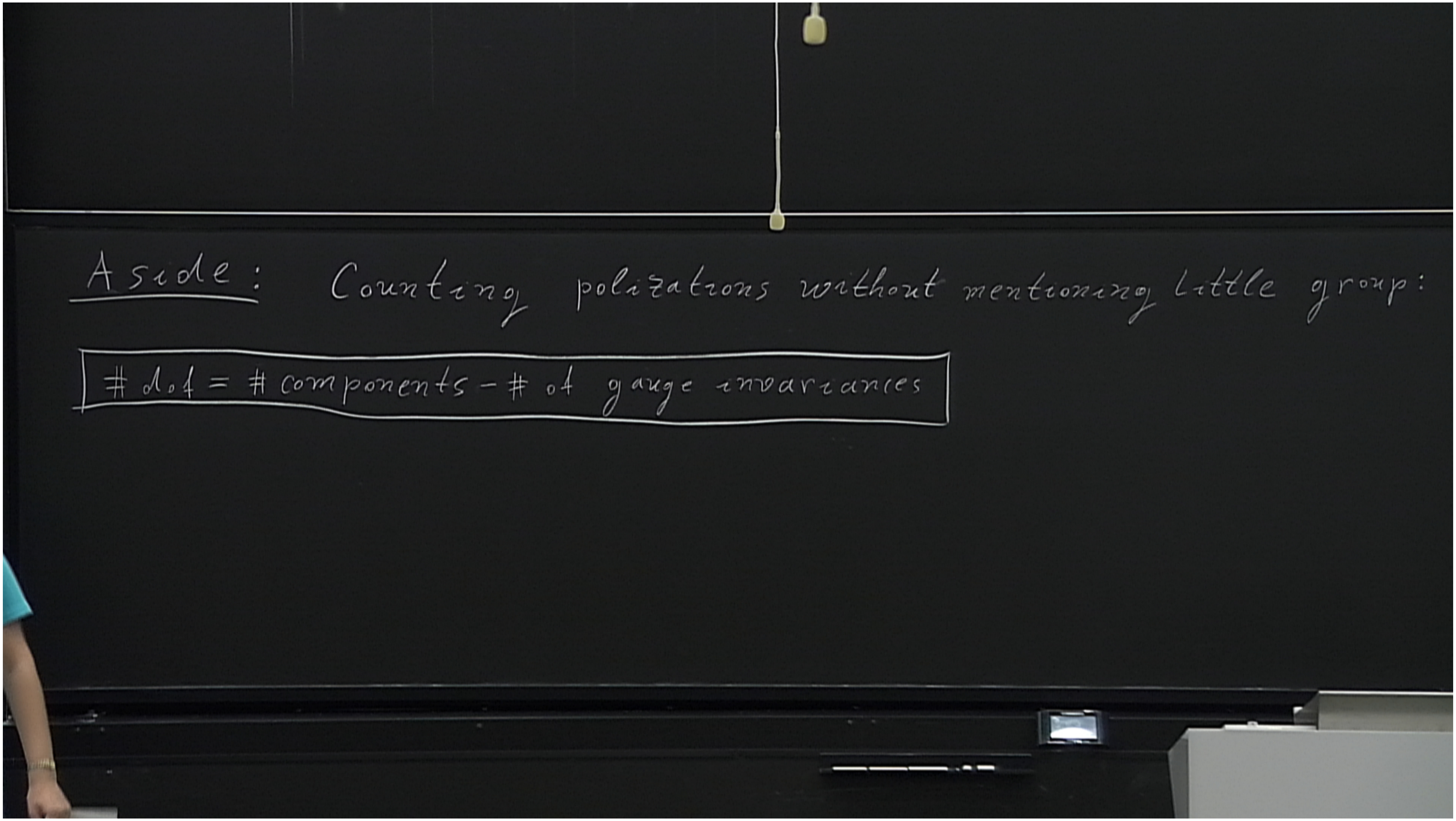
1) Graviton:

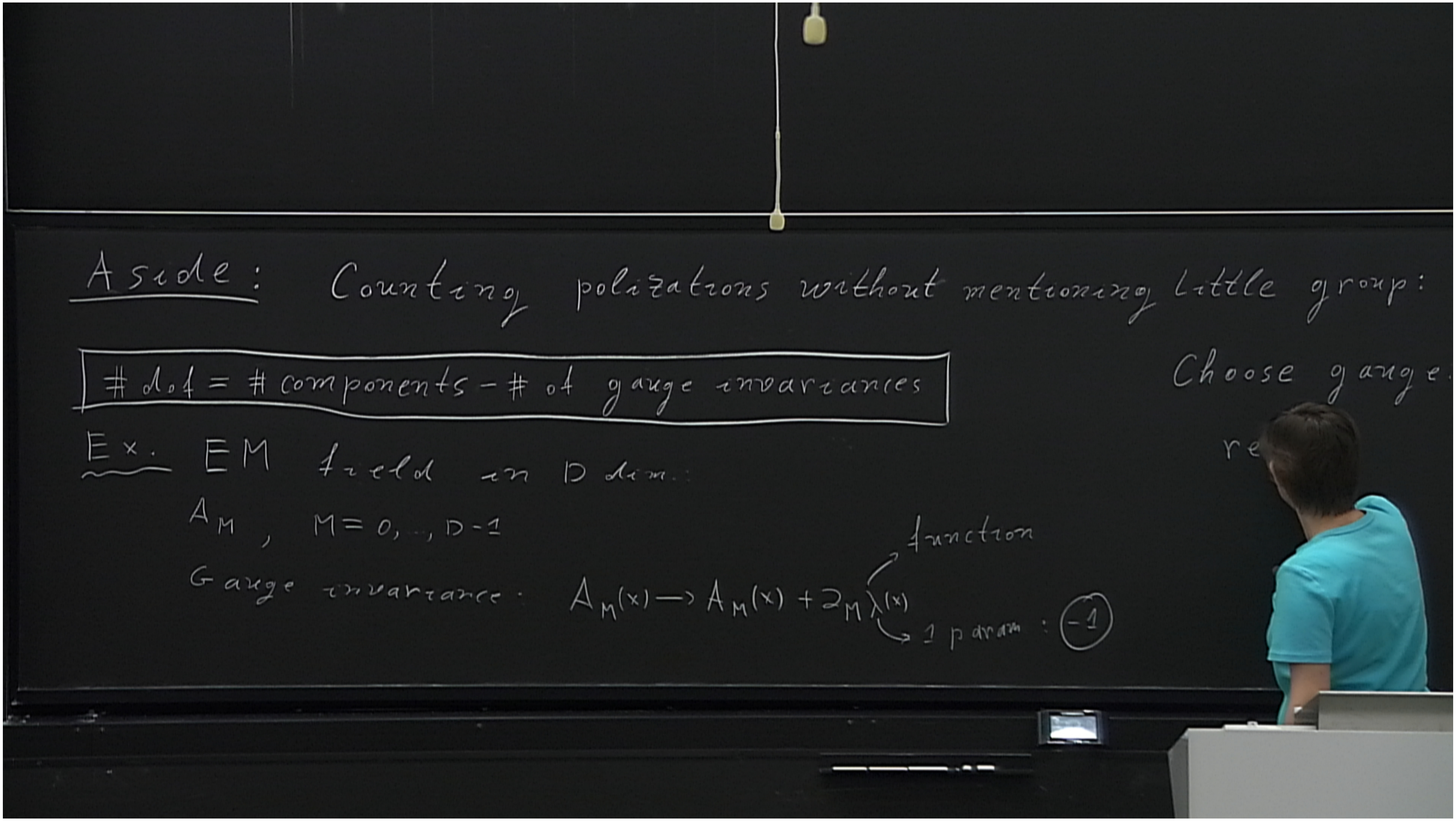
indep. degrees of freedom = # of components of a symmetric traceless (D-2)×(D-2) matrix

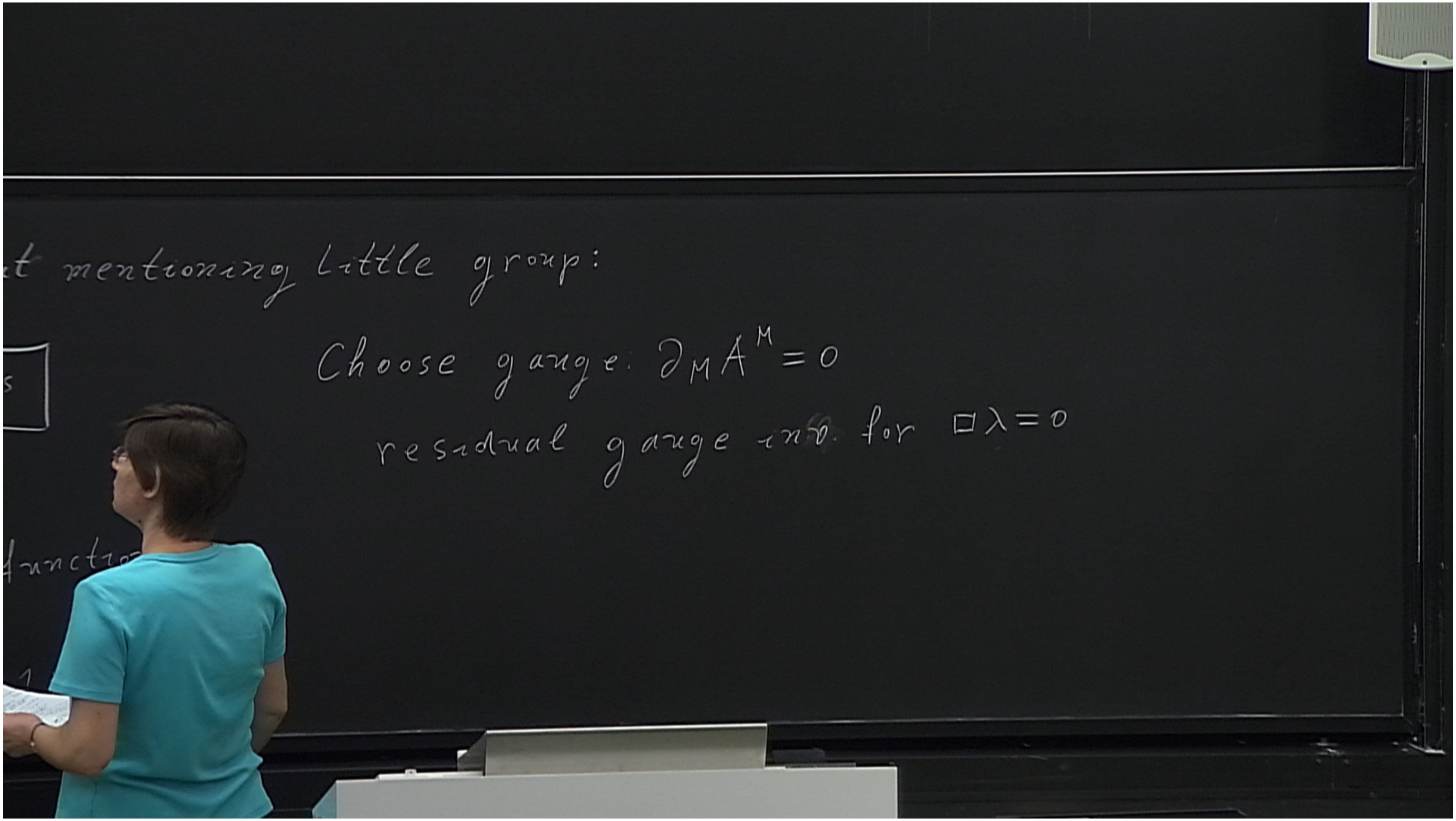

(3-form potential)
1) Graviton:
d.o.f = 1/2 (D-1)(D-2) - 1 = 1/2 D(
comp. of (D-2)×(D-2)
sym. matrix
trace

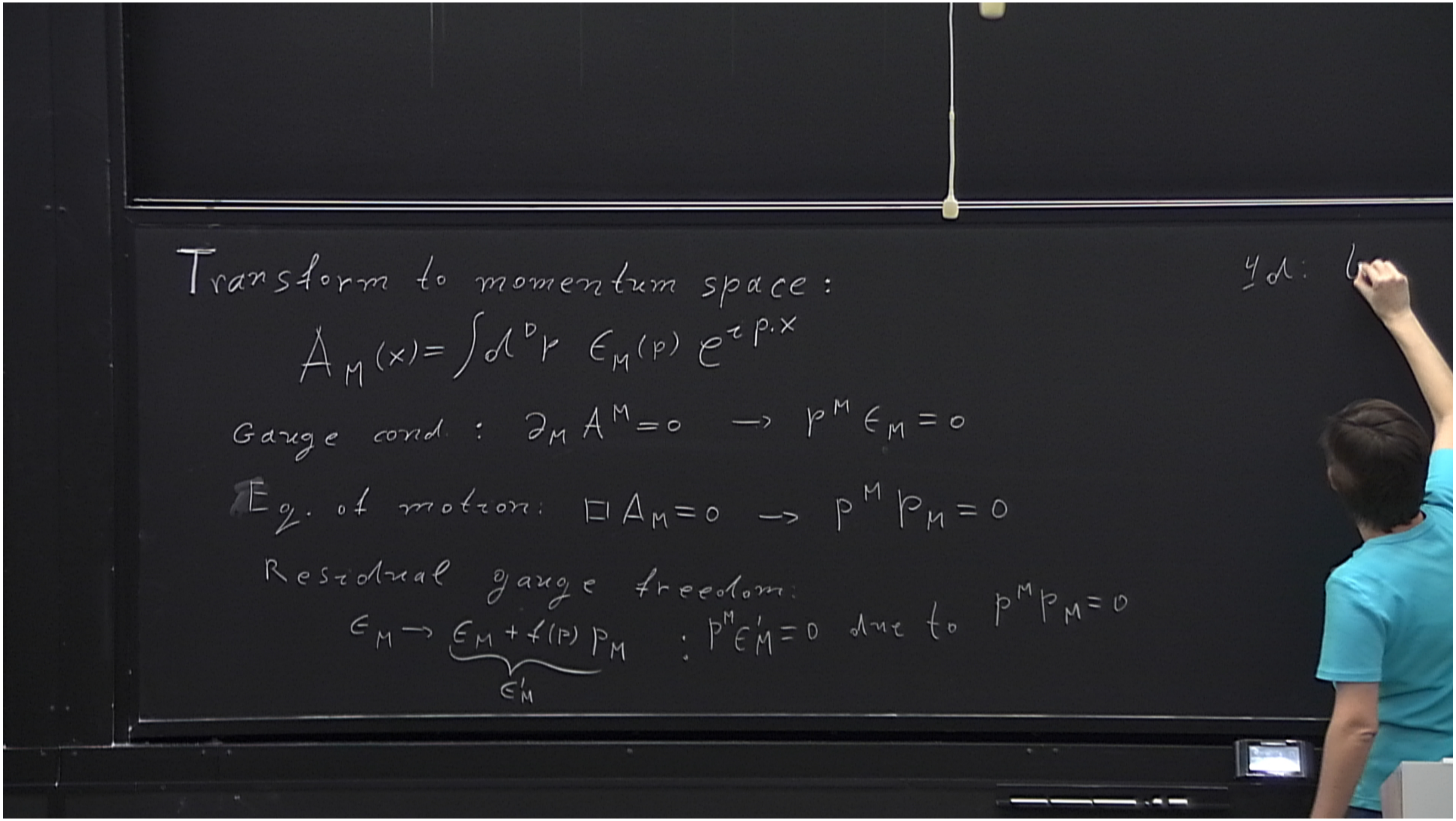


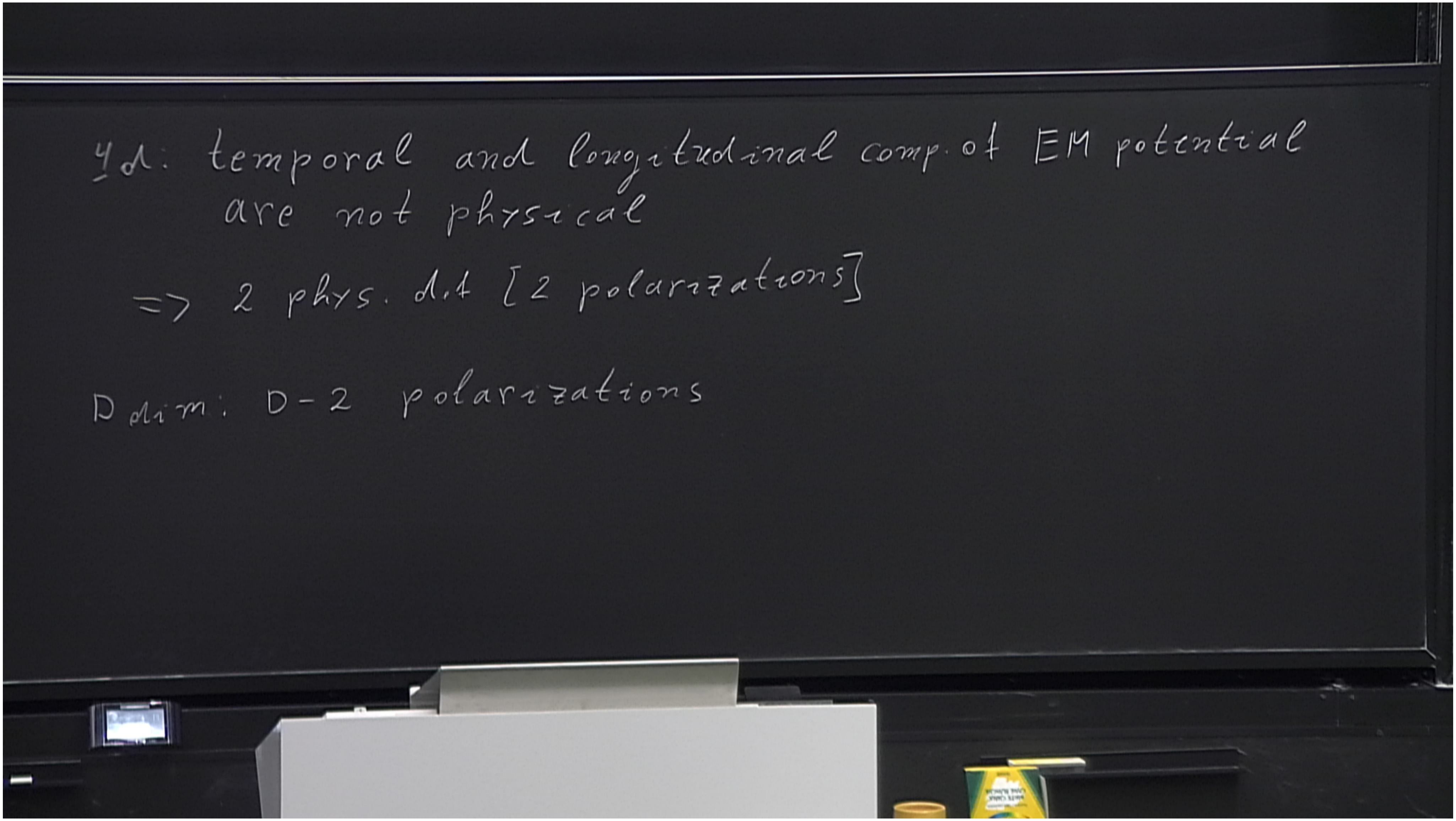
d.o.f = 1/2 (D−1)(D−2) − 1 = 1/2 D(D−3) = 44
trace
D = 11
# comp. of (D−2)×(D−2)
sym. matrix


Aside: Counting polizations without mentioning little group:
d.o.f = # components - # of gauge invariances


Aside: Counting polizations without mentioning little group:
d.o.f = # components - # of gauge invariances
Choose gauge.
Ex. EM field in D dim.:
A_M , M = 0, ..., D-1
function
Gauge invariance: A_M(x) → A_M(x) + ∂_M λ(x)
1 param : -1


mentioning little group:
Choose gauge: $\partial_M A^M = 0$
residual gauge inv. for $\Box\lambda = 0$


Transform to momentum space:
$A_M(x) = \int d^D p \; \epsilon_M(p)\, e^{ip\cdot x}$
Gauge cond. : $\partial_M A^M = 0 \longrightarrow p^M \epsilon_M = 0$
Eq. of motion: $\Box A_M = 0 \longrightarrow p^M p_M = 0$
Residual gauge freedom:
$\epsilon_M \to \underbrace{\epsilon_M + f(p)\, p_M}_{\epsilon'_M}$: $p^M \epsilon'_M = 0$ due to $p^M p_M = 0$
4d:

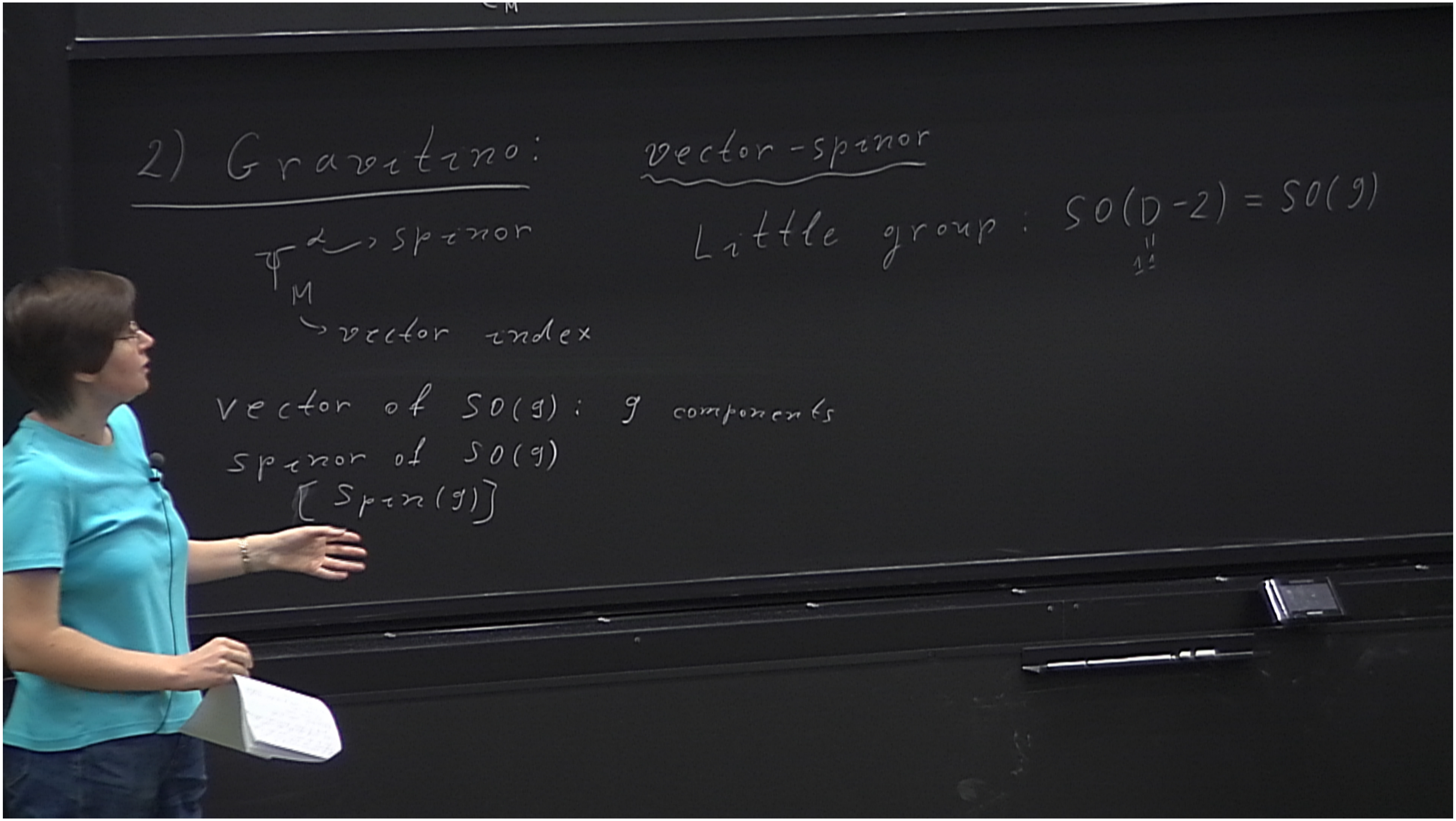

4d: temporal and longitudinal comp. of EM potential
are not physical
=> 2 phys. d.o.f [2 polarizations]
D dim: D-2 polarizations

$$g_{MN} \to g_{MN} + \partial_M \epsilon_N + \partial_N \epsilon_M$$

Choosing a gauge:
removes D dof

Residual gauge inv.:
removes D more dof

$$\frac{D(D+1)}{2} - D - D = \frac{1}{2} D(D-3)$$


2) Gravitino:
vector-spinor
Little group: SO(D-2) = SO(9)
11
Ψ^α_M → spinor
→ vector index
vector of SO(9): 9 components
spinor of SO(9)
[Spin(9)]

2) Gravitino: vector-spinor

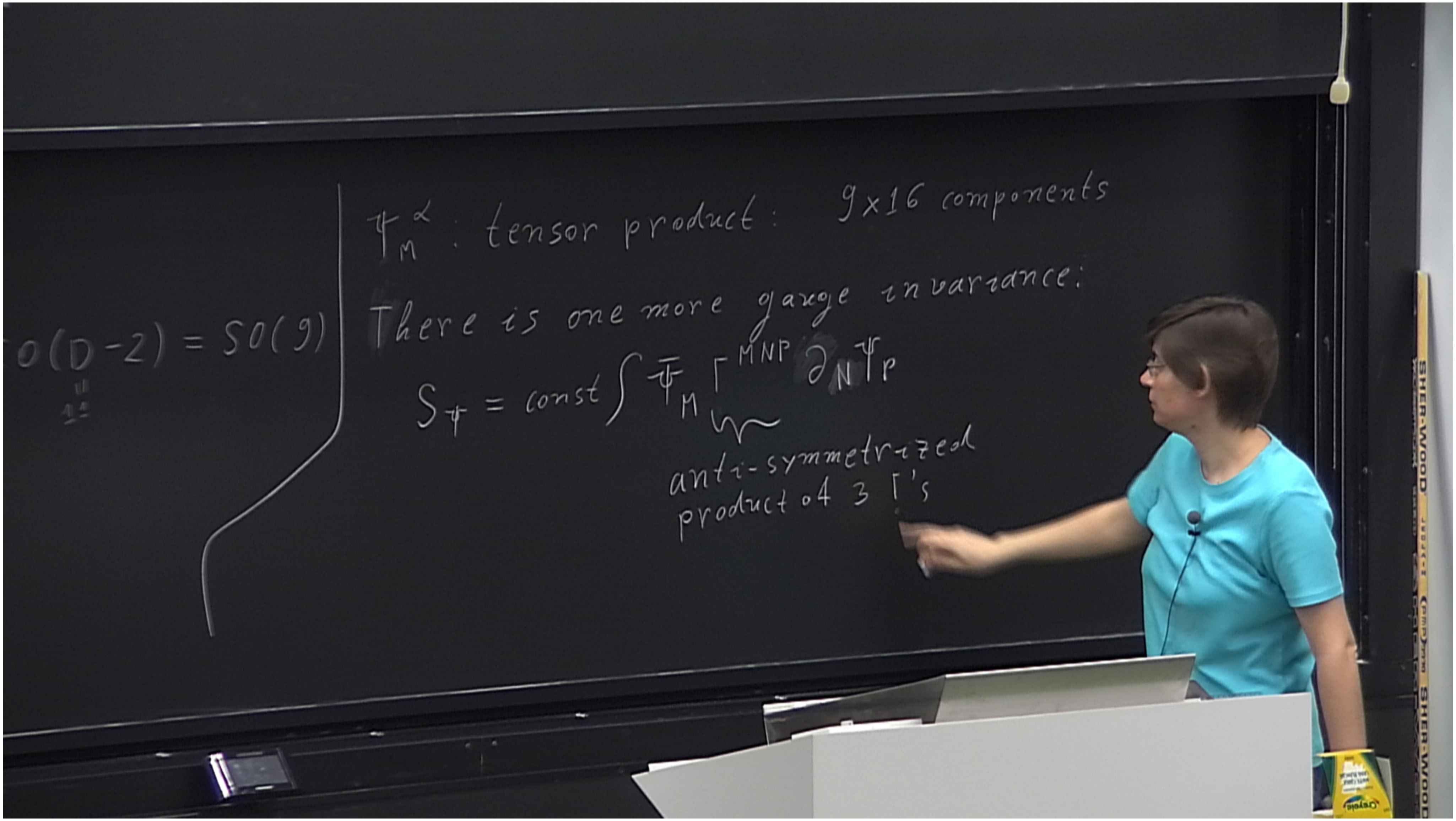
ψ_M^{α} — α → spinor; M → vector index

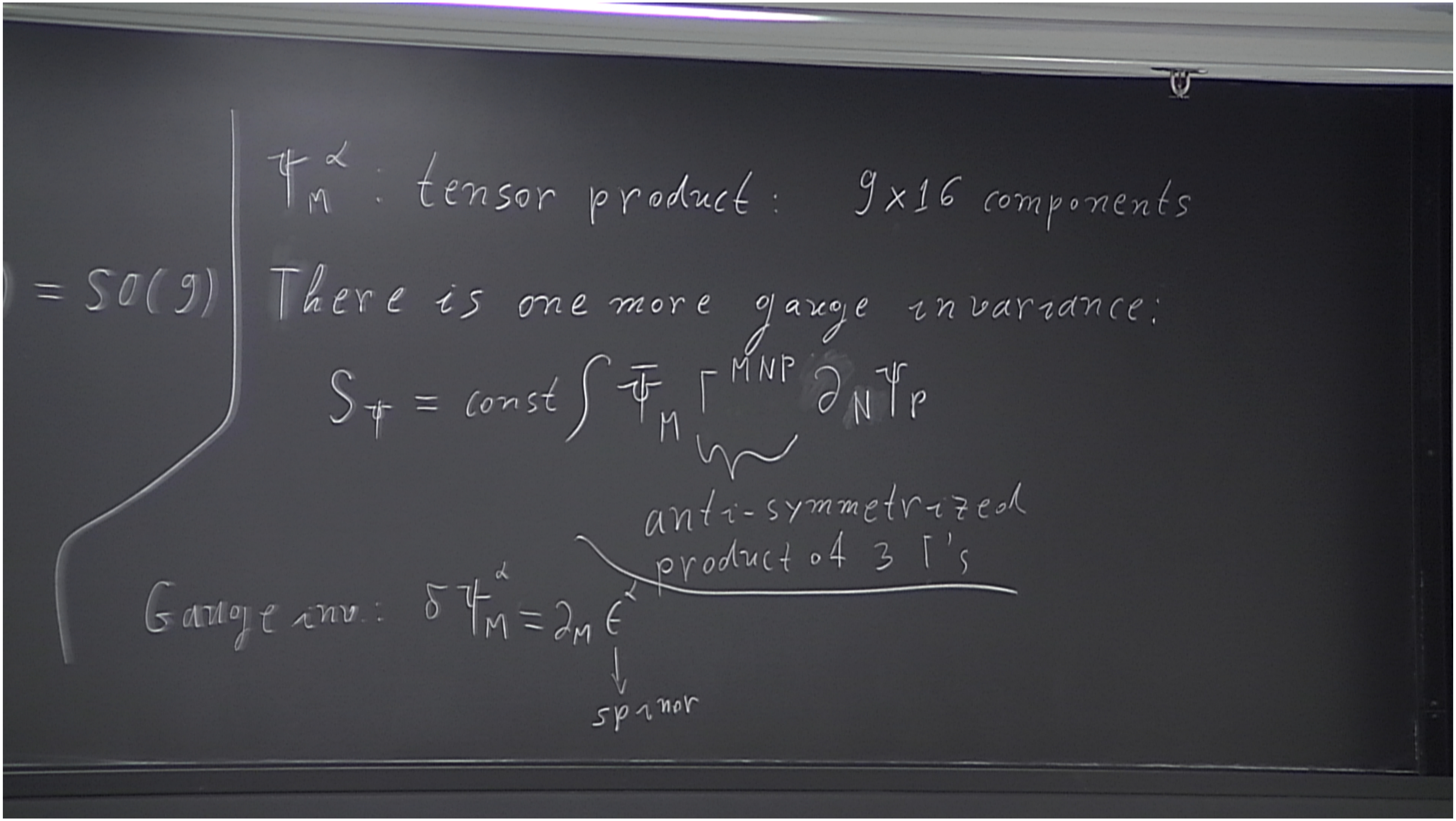
Little group: $SO(D-2) = SO(9)$, with $D = 11$

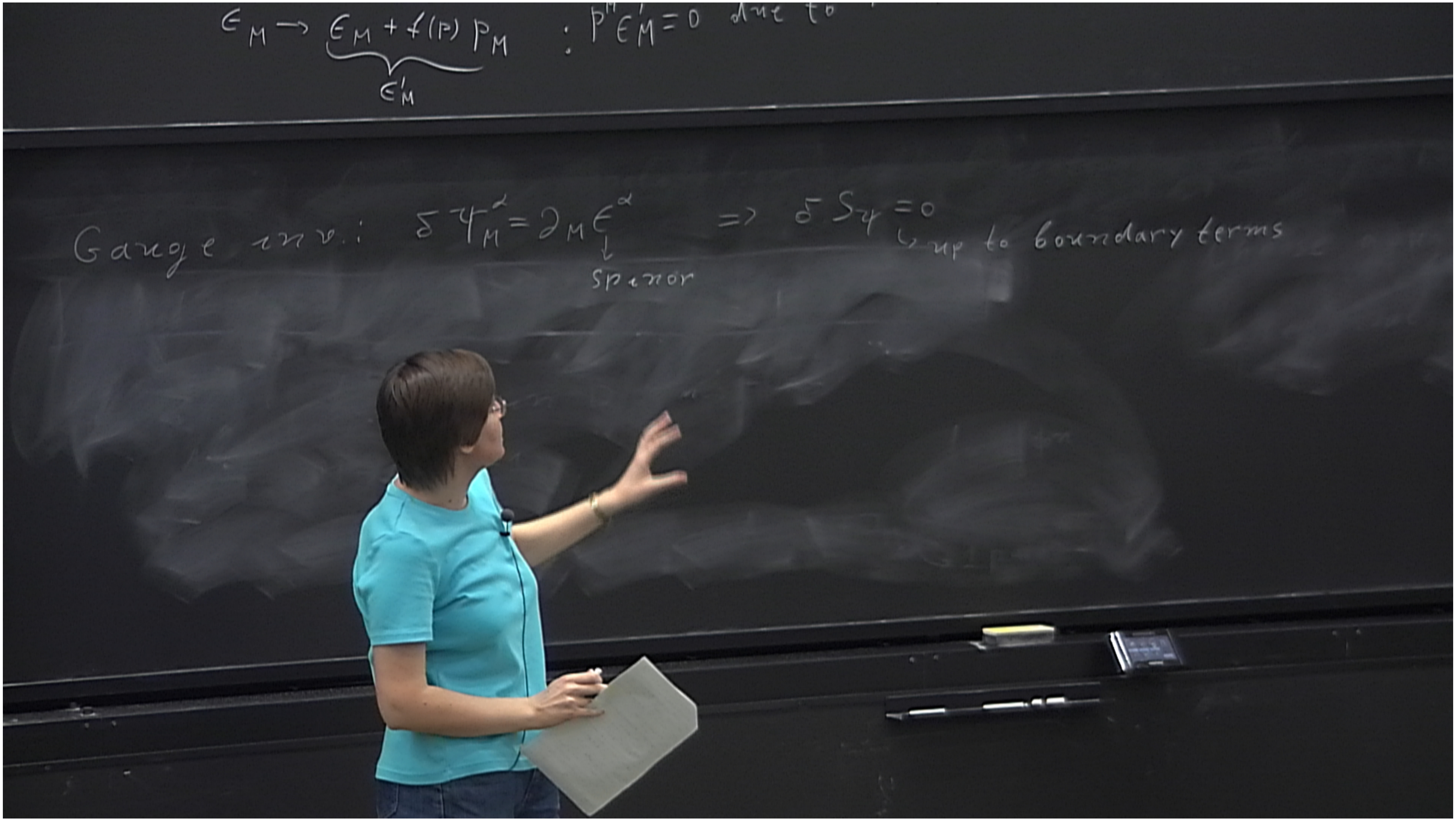
$SO(n) \to Spin(n)$

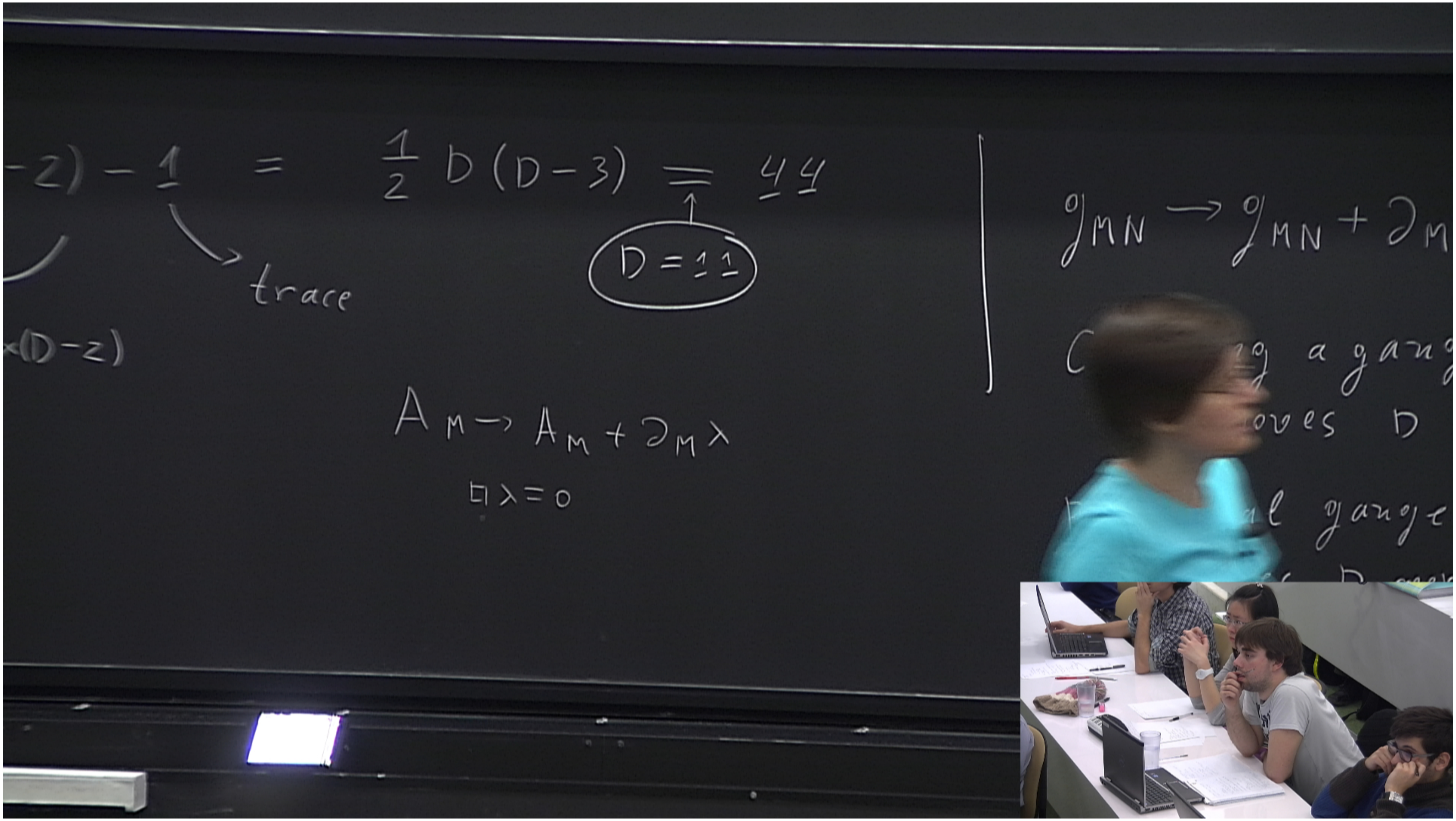
vector of $SO(9)$: 9 components

spinor of $SO(9)$ [$Spin(9)$]: $2^{\frac{8}{2}} = 16$ components


SO(D-2) = SO(9)
Ψ_M^α : tensor product: 9×16 components
There is one more gauge invariance:
S_ψ = const ∫ Ψ̄_M Γ^MNP ∂_N Ψ_P
anti-symmetrized product of 3 Γ's
SHER-WOOD


Ψ_M^α : tensor product : 9×16 components
= SO(9)
There is one more gauge invariance:
$S_\psi = \text{const} \int \bar{\Psi}_M \Gamma^{MNP} \partial_N \Psi_P$
anti-symmetrized product of 3 Γ's
Gauge inv.: $\delta \Psi_M^\alpha = \partial_M \epsilon^\alpha$
spinor


$\epsilon_M \to \epsilon_M + f(p)\, p_M \quad : p^M \epsilon'_M = 0$ due to
ϵ'_M
Gauge inv.: $\delta \psi^\alpha_M = \partial_M \epsilon^\alpha \Rightarrow \delta S_\psi = 0$
spinor
up to boundary terms


-2) - 1 = 1/2 D (D-3) = 44
trace
D = 11
(D-2)
A_M → A_M + ∂_M λ
□λ = 0
g_MN → g_MN + ∂_M
a gaug
oves D
l gauge

g_{MN} → metric (graviton) $\quad \boxed{\mathcal{N} = 1}$

Ψ_M^α → spin 3/2 fermion (gravitino)

A_{MNP} → anti-symmetric tensor (3-form potential)

In D dimensions the little group: …

1) Graviton:

indep. degrees of freedom = # of componen… symmetric traceless $(D-2)\times(D-2)$ matrix

3) 3-form field: A_{MNP} potential → field strength F_{MNPQ} $\quad F_{(4)} = dA_{(3)}$

# of polarizations = # of components of rank 3 anti-sym matrix in $(D-2)$ dim = $\dfrac{9\times 8\times 7}{3!} = 84$ $\quad (D-2 = 9)$

11 d SUGRA: bosonic d.o.f. $= 84 + 44 = 128$

fermionic d.o.f. $= 128$

Action (only bosonic part):

$$S = \frac{1}{2\kappa_{11}^2}\int d^{11}x\,\sqrt{-\det g}\left(R - \frac{1}{2}F_4^2\right) - \frac{1}{6}\int A_3 \ldots$$

$2\kappa_{11}^2 = 16\pi G_{11}$ (Newton's const.)

$F_4 = dA_3 \quad \Big| \quad F_4^2 = \frac{1}{4!}F_{M_1\ldots M_4}\ldots$

(3-form potential)

3) 3-form field : A_{MNP} potential → field str

\# of polarizations = # of components of rank

$$= \frac{9 \times 8 \times 7}{3!} = 84$$

11d SUGRA : bosonic d.o.f $= 84 + 44 = 128$

fermionic d.o.f $= 128$

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rank 3 anti-sym. matrix in $(D-2)$ dim =
 9

128

Action (only bosonic part):

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-\det g} \left(R - \frac{1}{2} F_4^2 \right) - \frac{1}{6} \int A_3 \wedge F_4 \wedge F_4$$

$2\kappa_{11}^2 = 16\pi G_{11}$

Newton's const.

$$F_4 = dA_3 \quad \Big| \quad F_4^2 = \frac{1}{4!} F_{M_1 \ldots M_4} F_{N_1 \ldots N_4} g^{M_1 N_1} \ldots g$$

...matrix

...trength F_{MNPQ} : $F_{(4)} = dA_{(3)}$

...k 3 anti-sym. matrix in $(D-2)$ dim =
$\qquad\qquad\qquad\qquad\qquad\qquad 9$

$A_3 \to A_3 + d\Lambda_2$

Action (only bosonic part):

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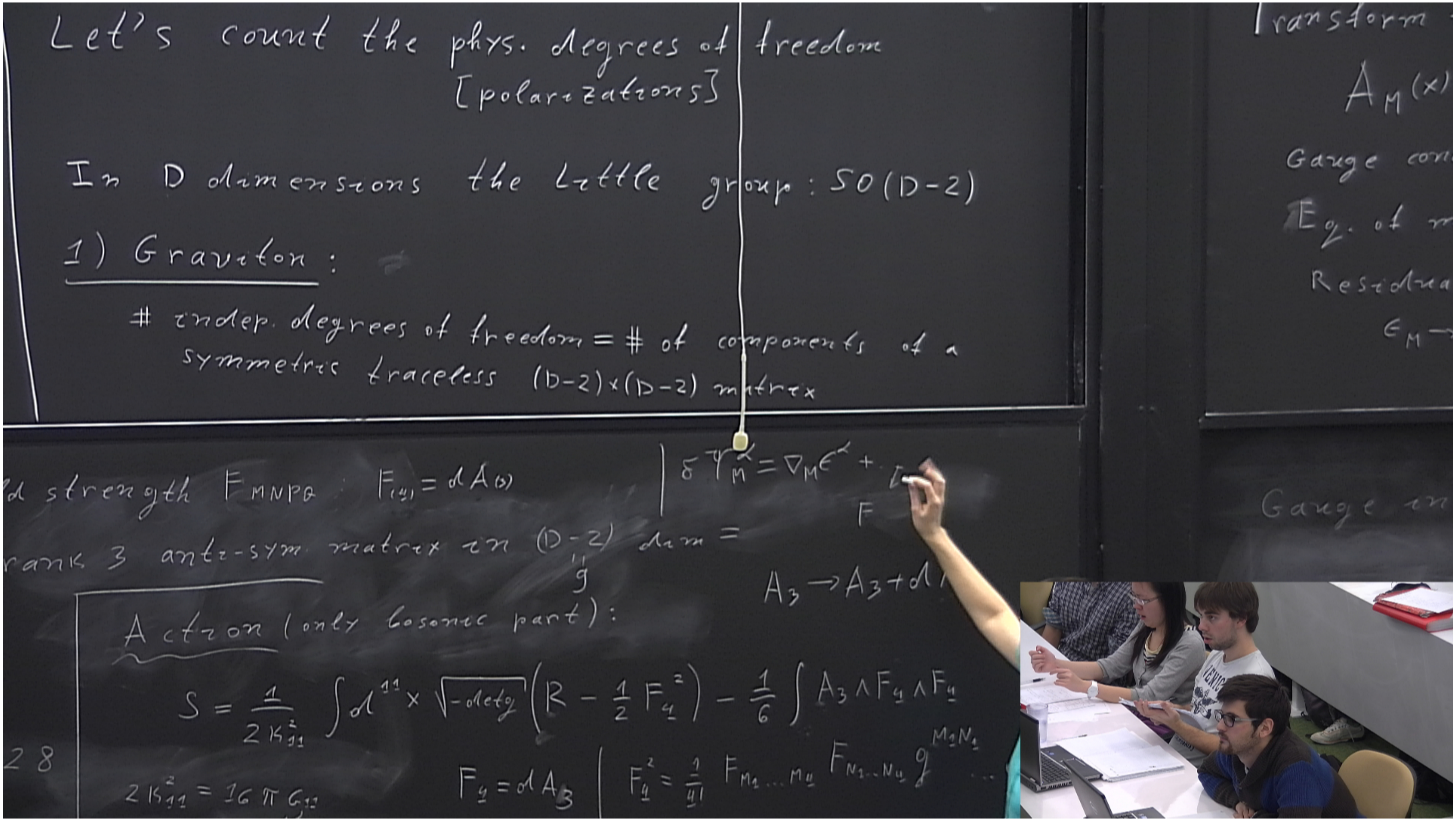
Newton's const.

$F_4 = dA_3 \quad \Big| \quad F_4^2 = \frac{1}{4!} F_{M_1\ldots M_4} F_{N_1\ldots N_4} g^{M_1N_1}\cdots g^{M_4N_4}$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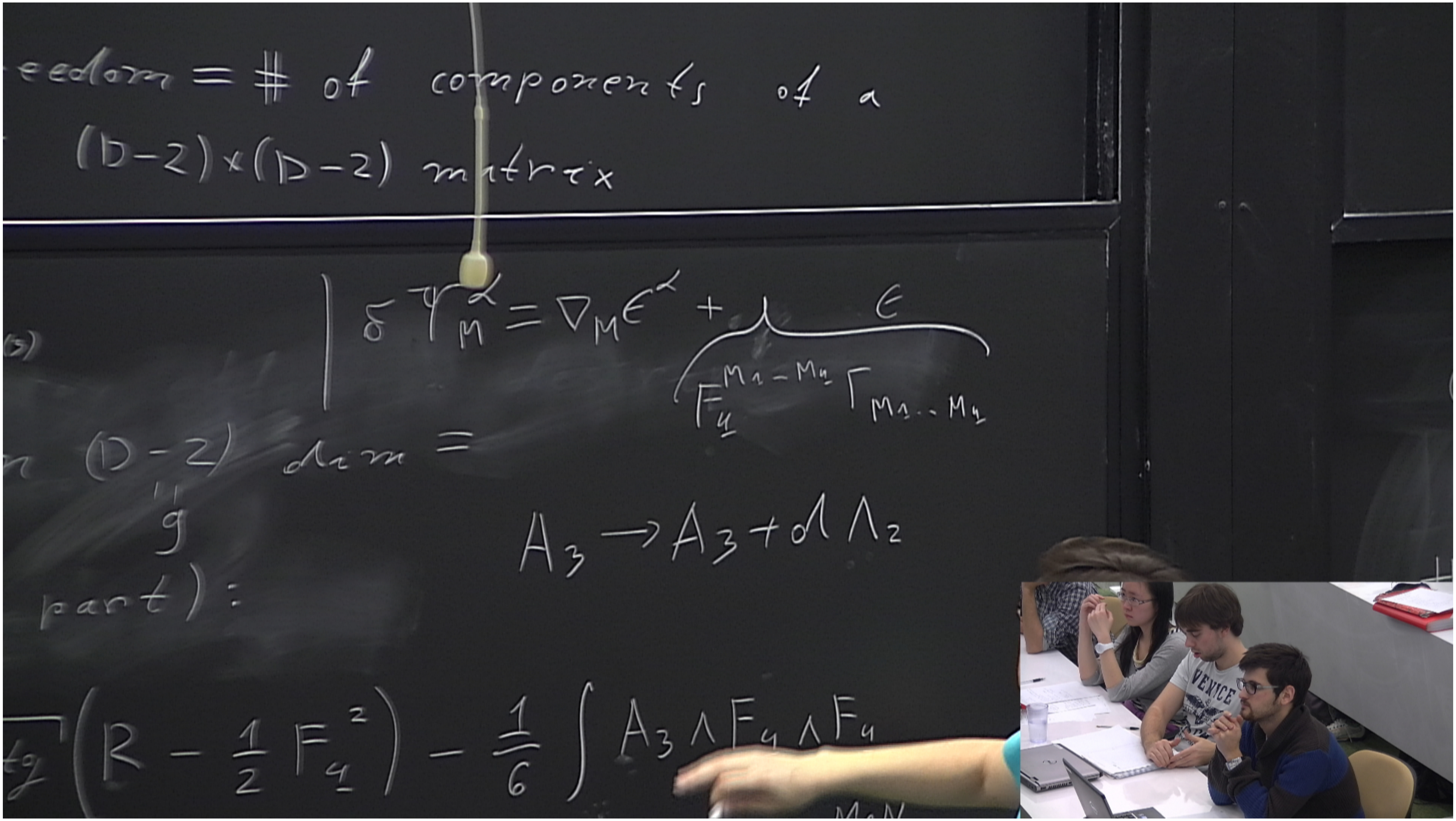
[polarizations]
In D dimensions the little group : SO(D-2)
1) Graviton :
indep. degrees of freedom = # of components of a symmetric traceless (D-2)×(D-2) matrix
Transform
Gauge
Eq. of
Residua
strength
rank 3 anti-sym. matrix in (D-2) dim =
Action (only bosonic part):
S = 1/2κ²₁₁ ∫d¹¹x √-det g (R − 1/2 F₄²) − 1/6 ∫ A₃ ∧ F₄ ∧ F₄
2κ²₁₁ = 16 π G₁₁
F₄ = d A₃
A₃ → A₃ + d
Gauge


eedom = # of components of a
(D-2)×(D-2) matrix
δΨ^α_M = ∇_M ε^α + F_4^{M1...M4} Γ_{M1...M4} ε
(D-2)
dim =
A_3 → A_3 + dΛ_2
part):
(R - 1/2 F_4^2) - 1/6 ∫ A_3 ∧ F_4 ∧ F_4
VENICE