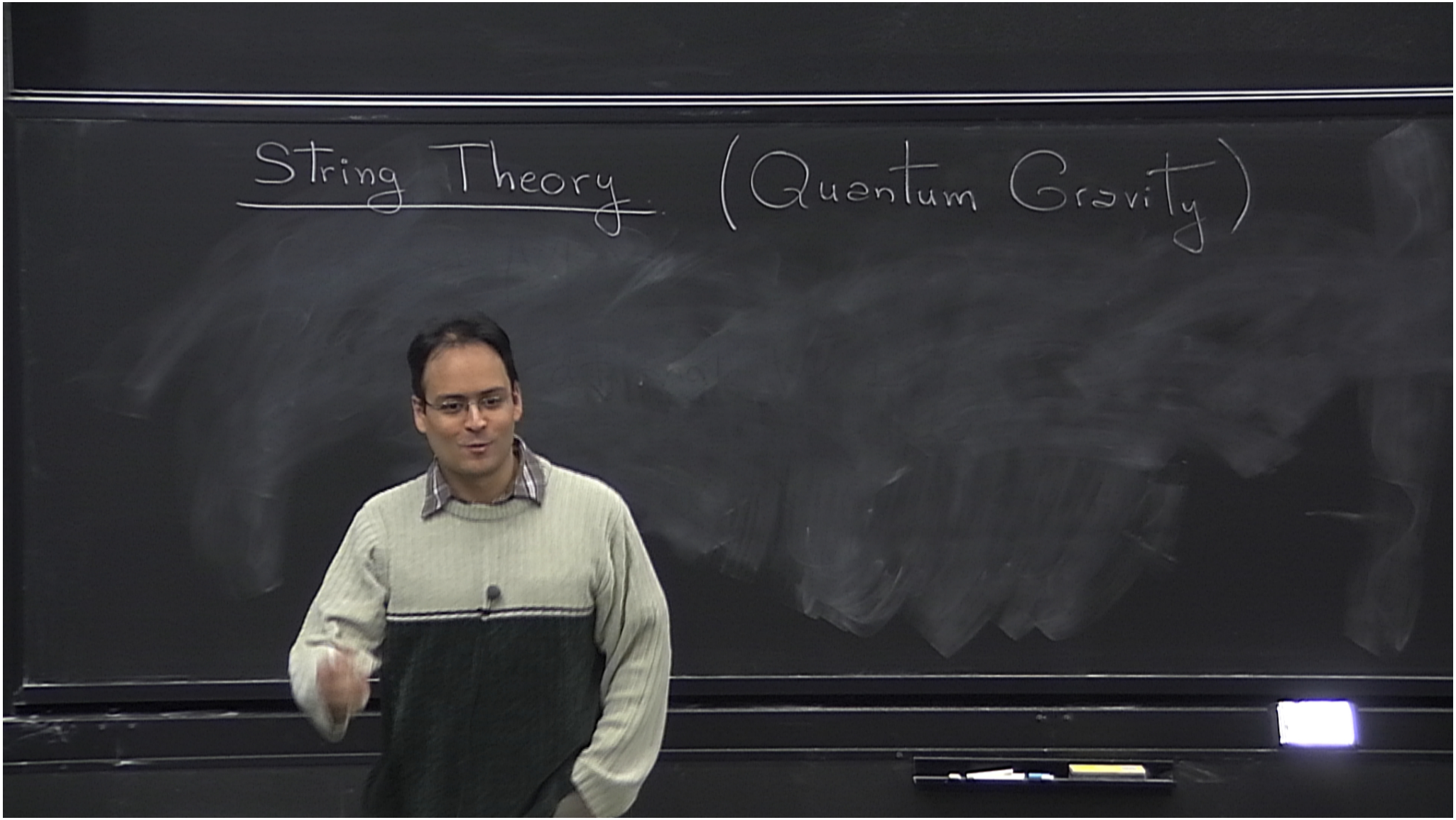
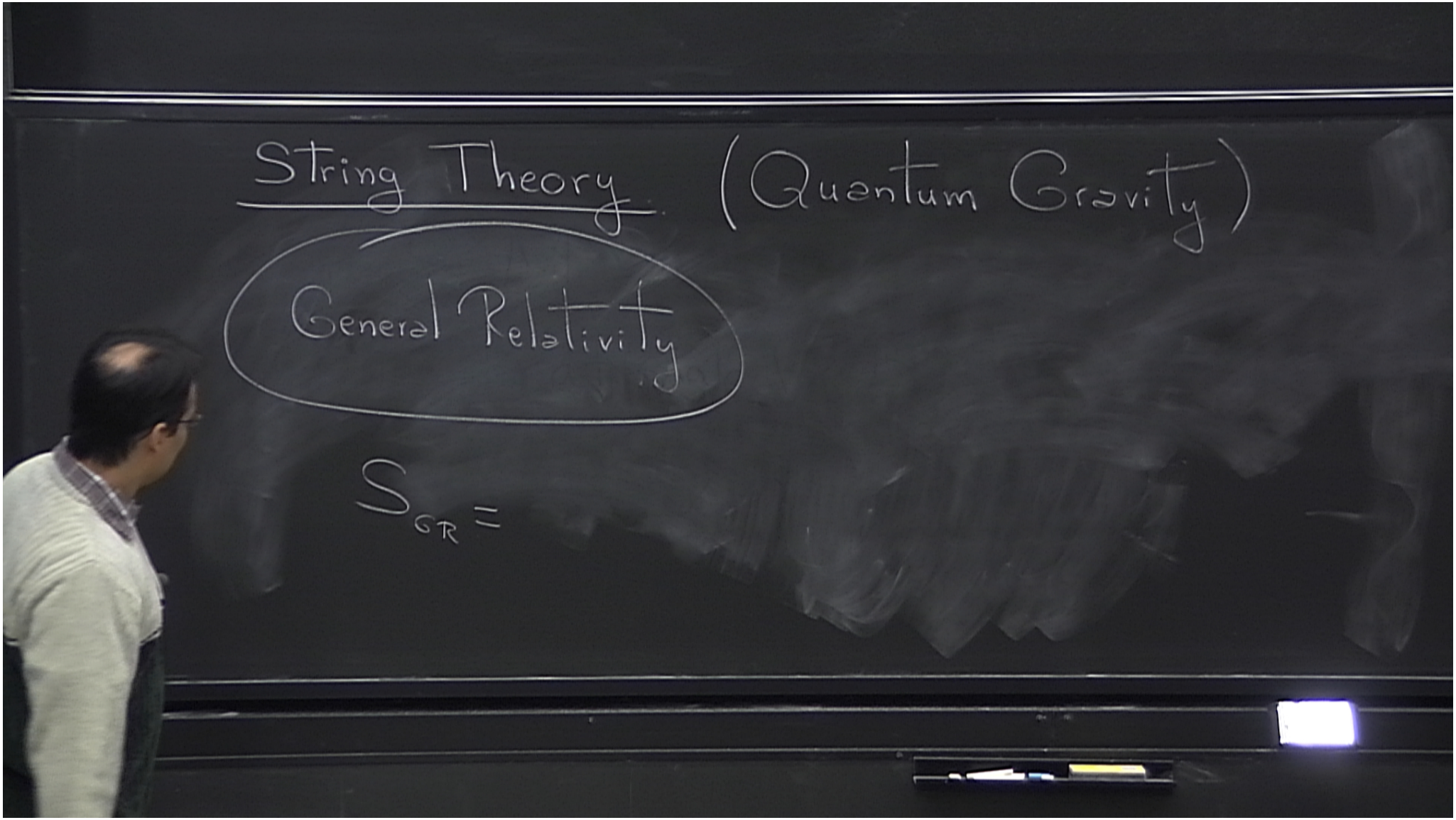
Title: String Theory (Review) - Lecture 1

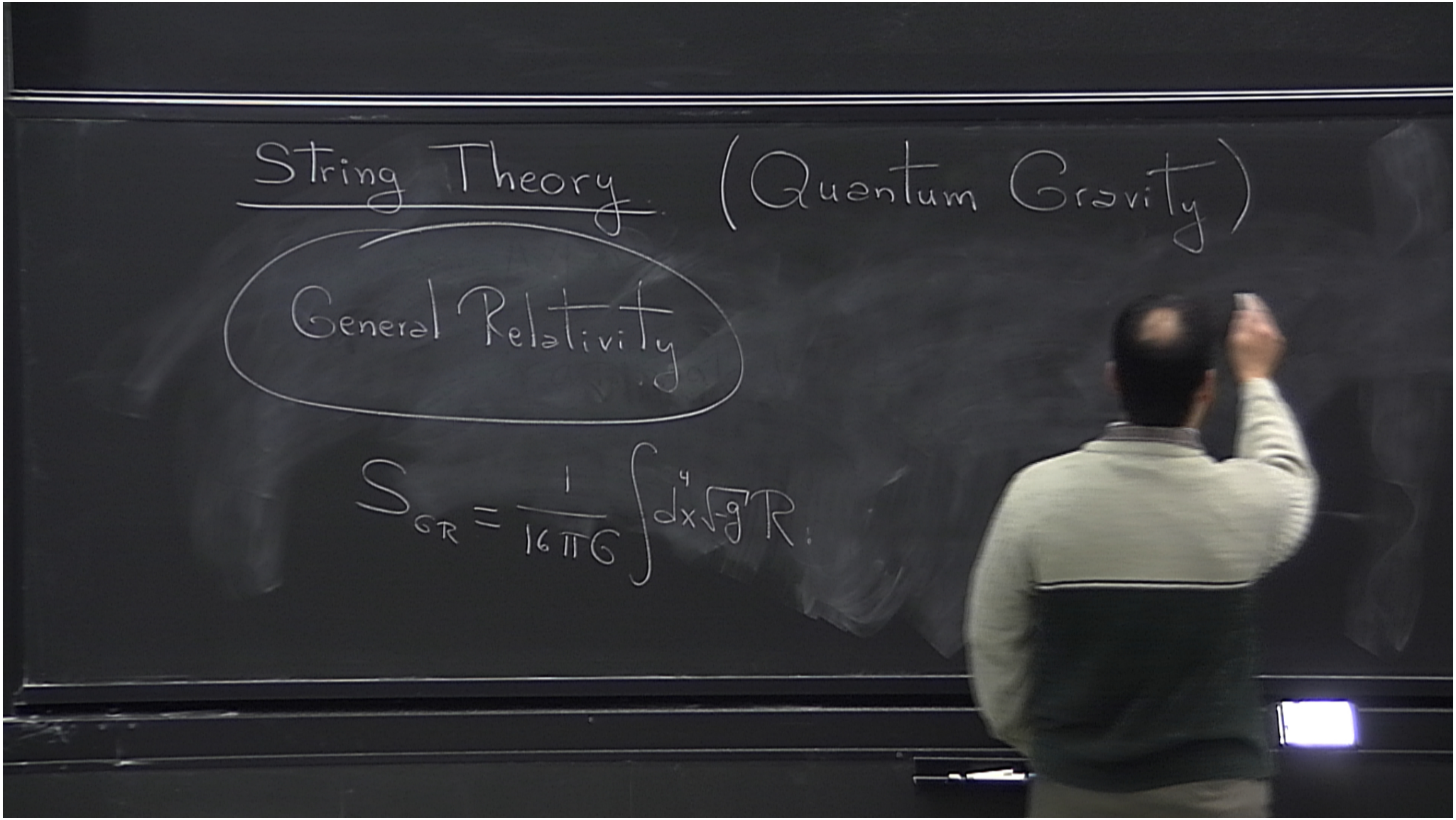
Date: Feb 13, 2012 11:30 AM

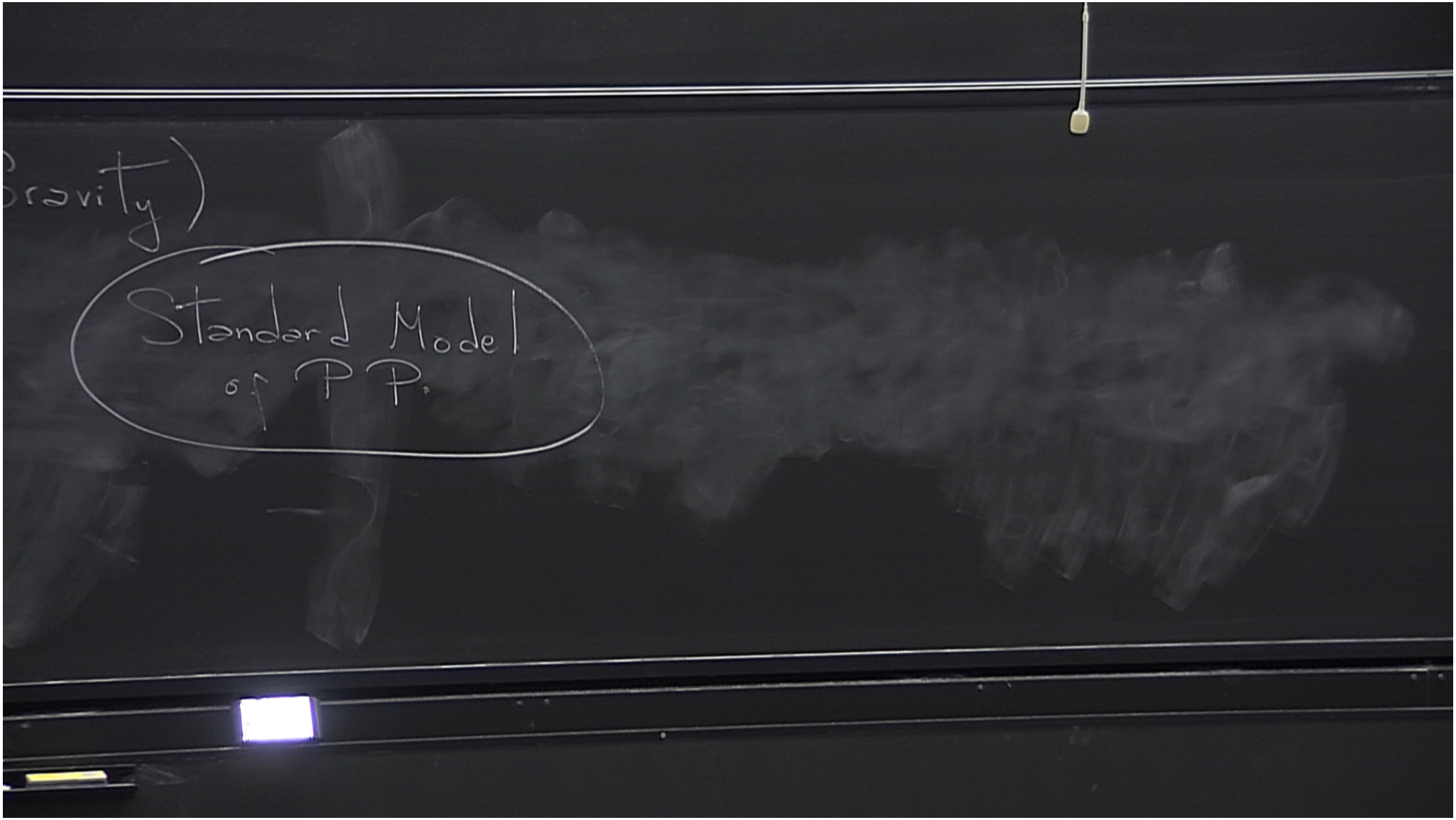
URL: http://pirsa.org/12020047

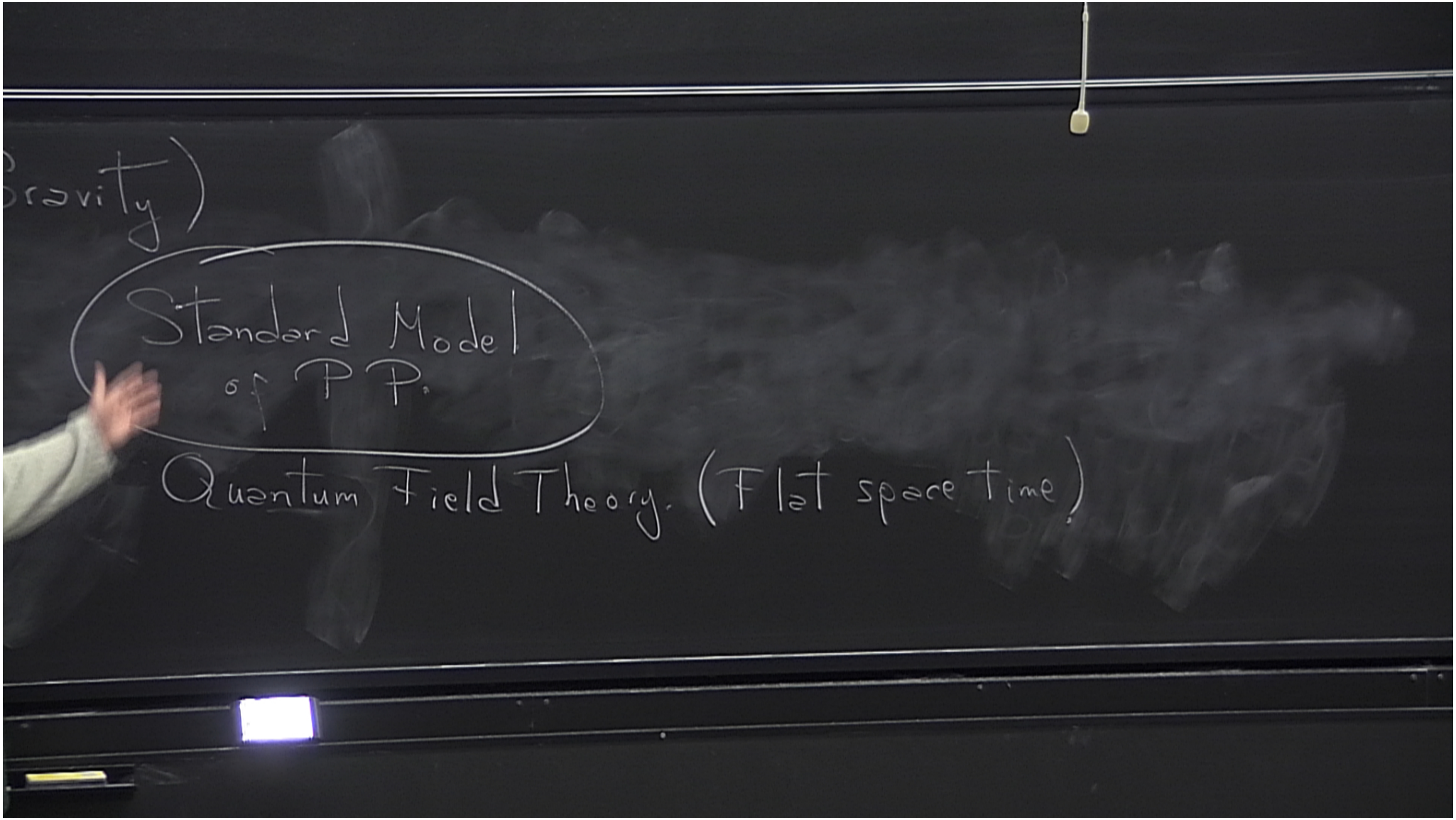
Abstract:


String Theory (Quantum Gravity)


String Theory (Quantum Gravity)
General Relativity
$S_{GR} =$


String Theory (Quantum Gravity)
General Relativity
$S_{GR} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R$


Standard Model
of P P.


Gravity)
Standard Model
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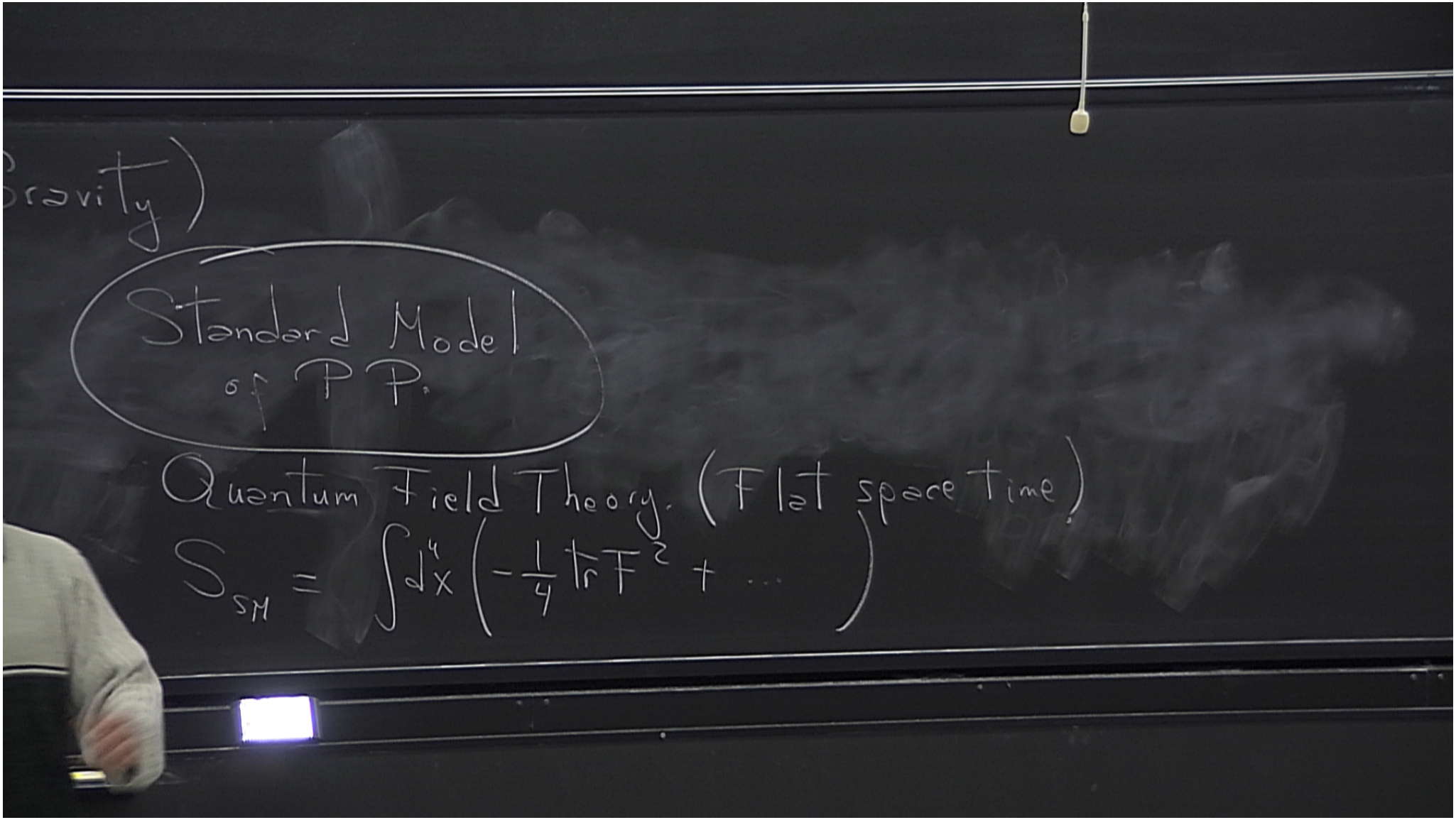
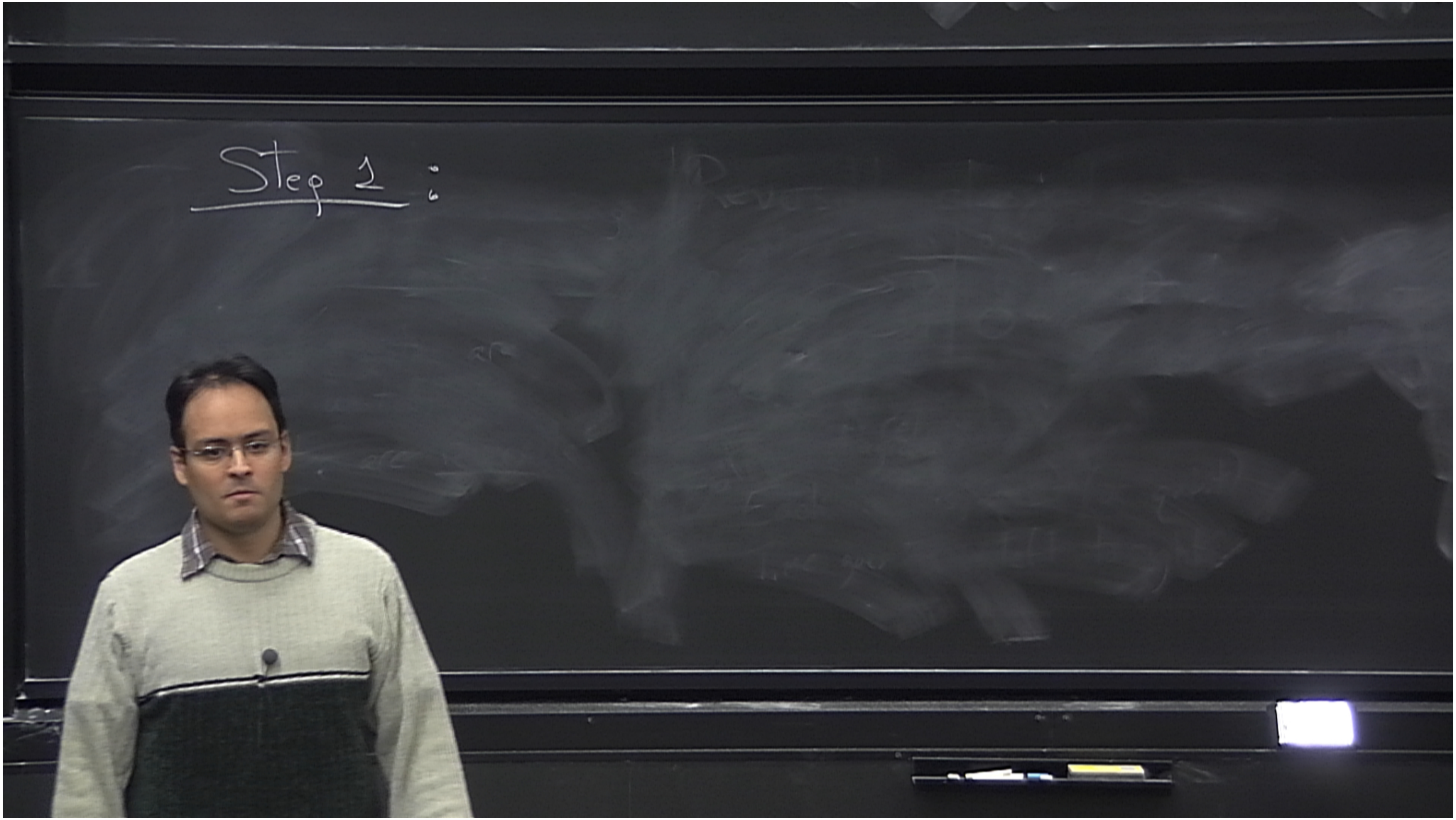
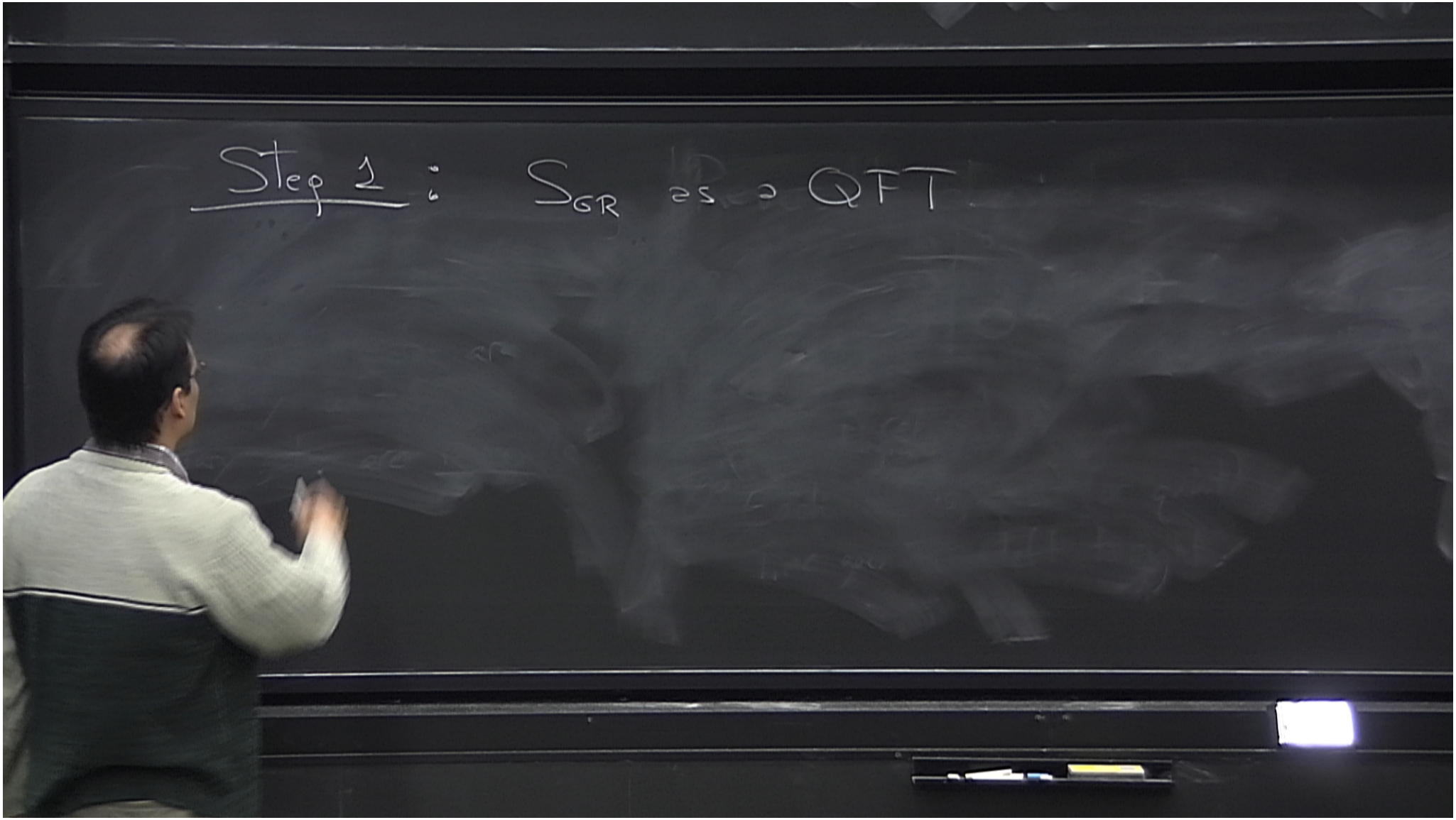
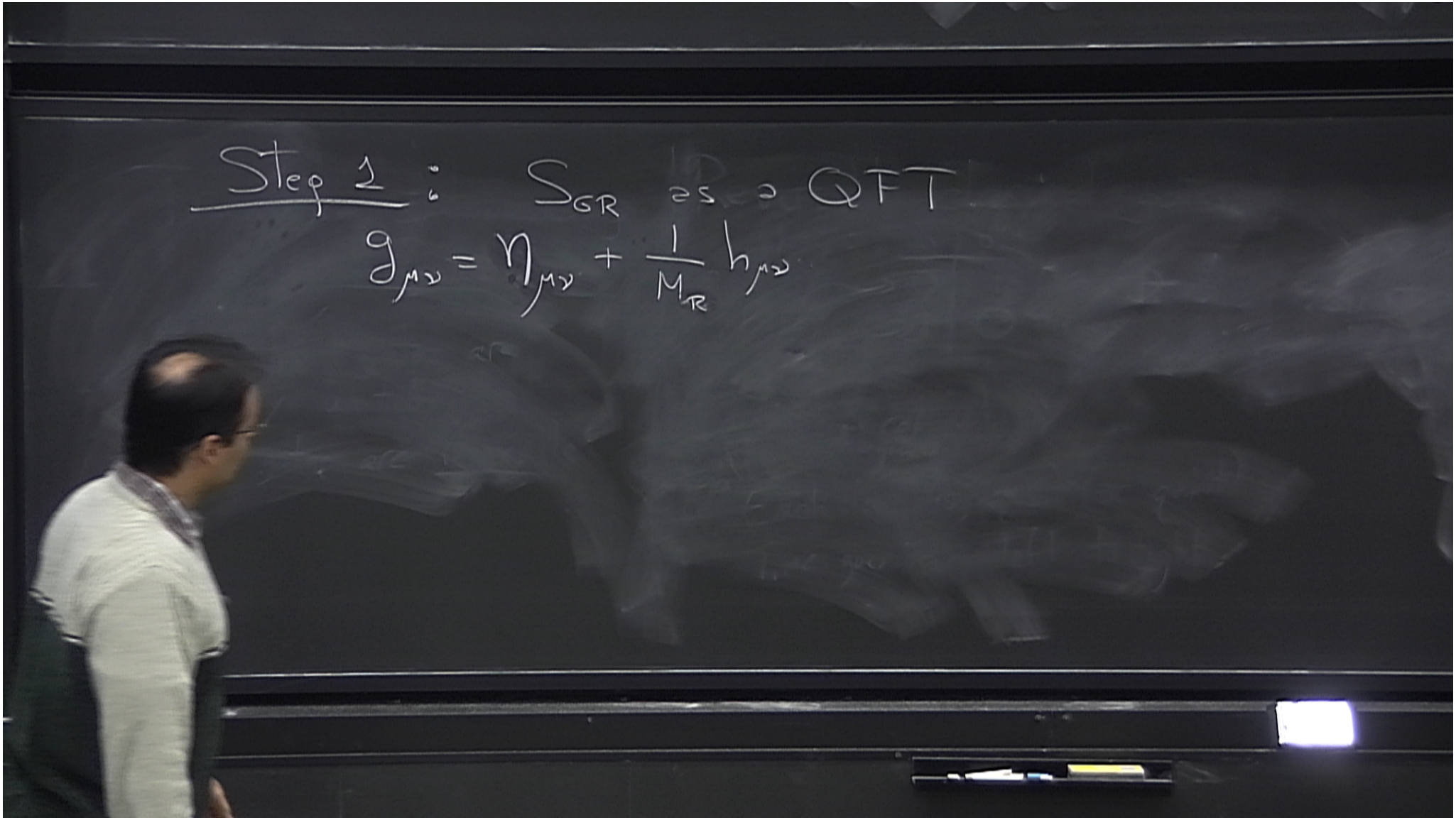
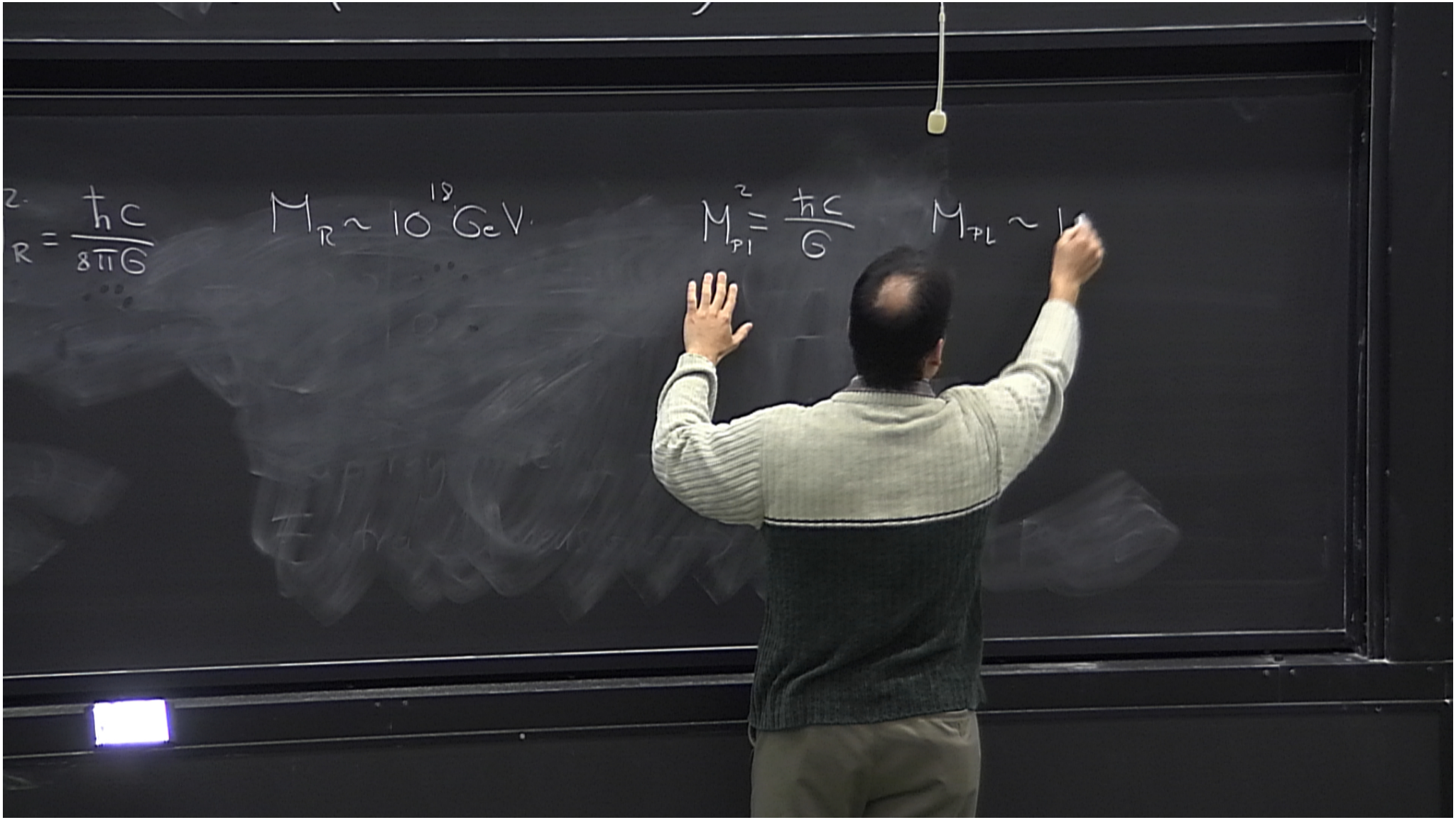
Quantum Field Theory. (Flat space time)

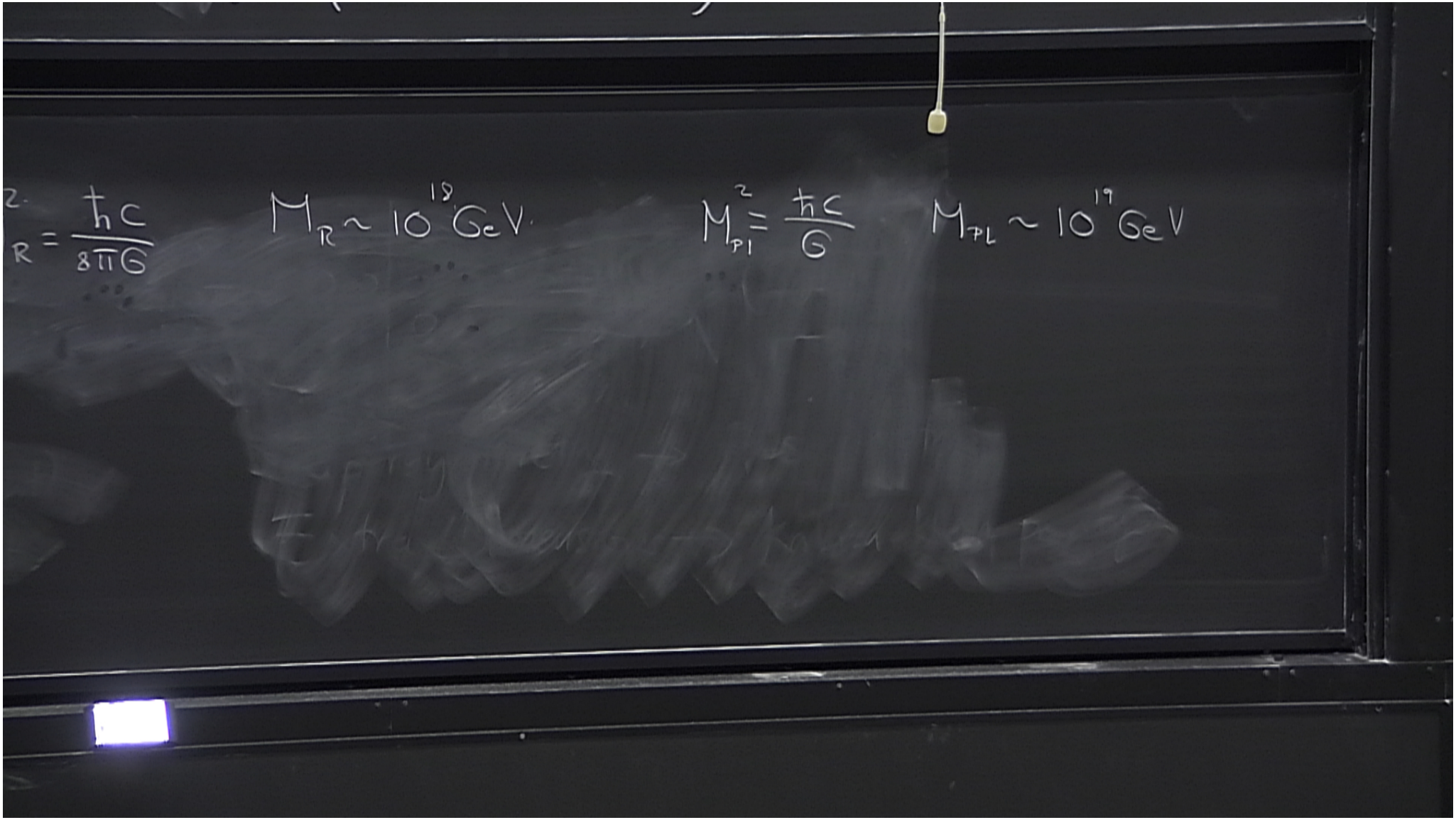

Gravity)
Standard Model of P P.
Quantum Field Theory. (Flat space time)
$S_{SM} = \int d^4x \left(-\frac{1}{4} \mathrm{Tr} F^2 + \cdots \right)$


Step 1 :


Step 1 : S_{GR} as a QFT


Step 1 : S_{GR} as a QFT
$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_R} h_{\mu\nu}$


$R = \frac{\hbar c}{8\pi G}$
$M_R \sim 10^{19}\,\mathrm{GeV}$
$M_{Pl}^2 = \frac{\hbar c}{G}$
$M_{Pl} \sim$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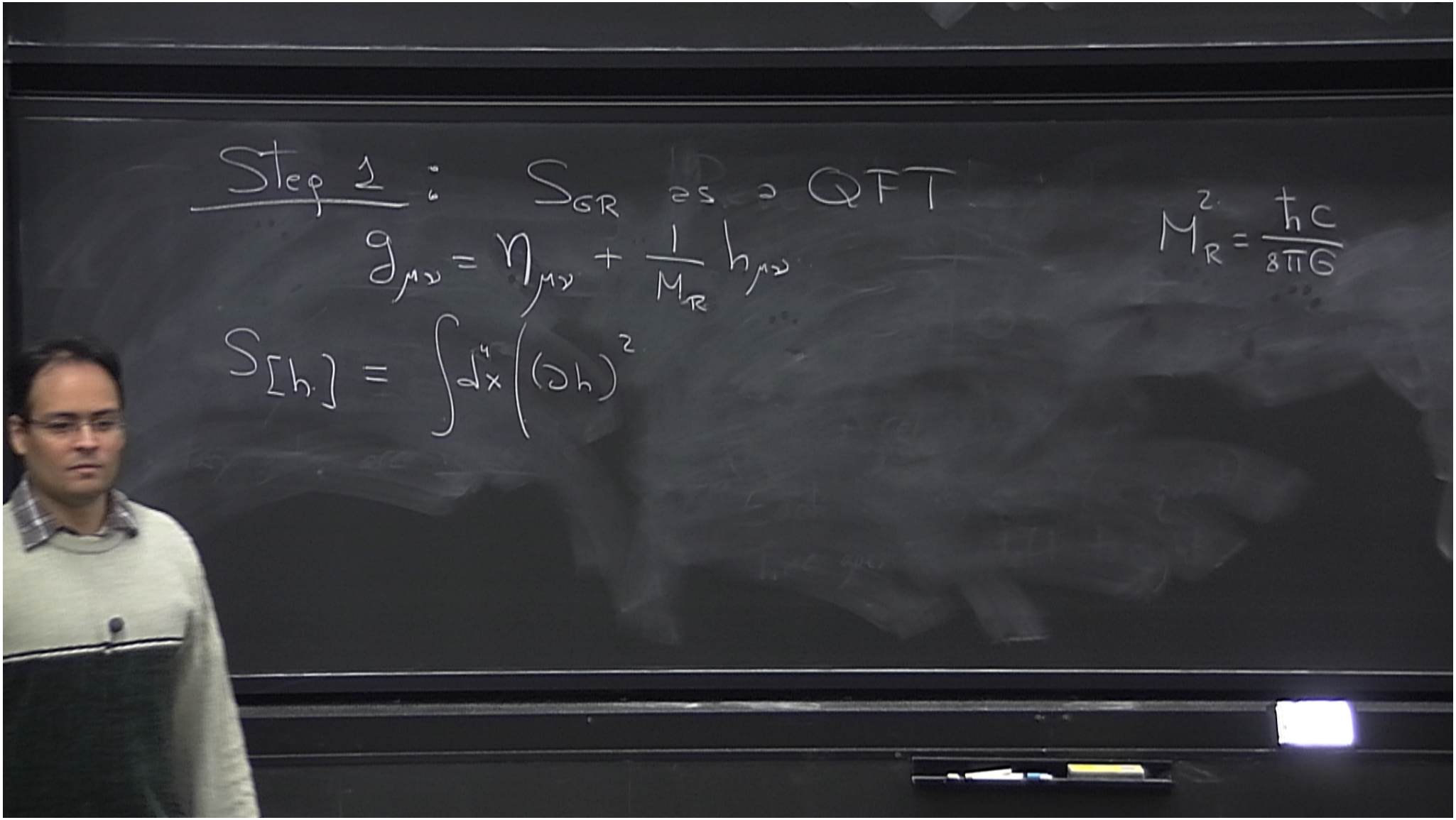
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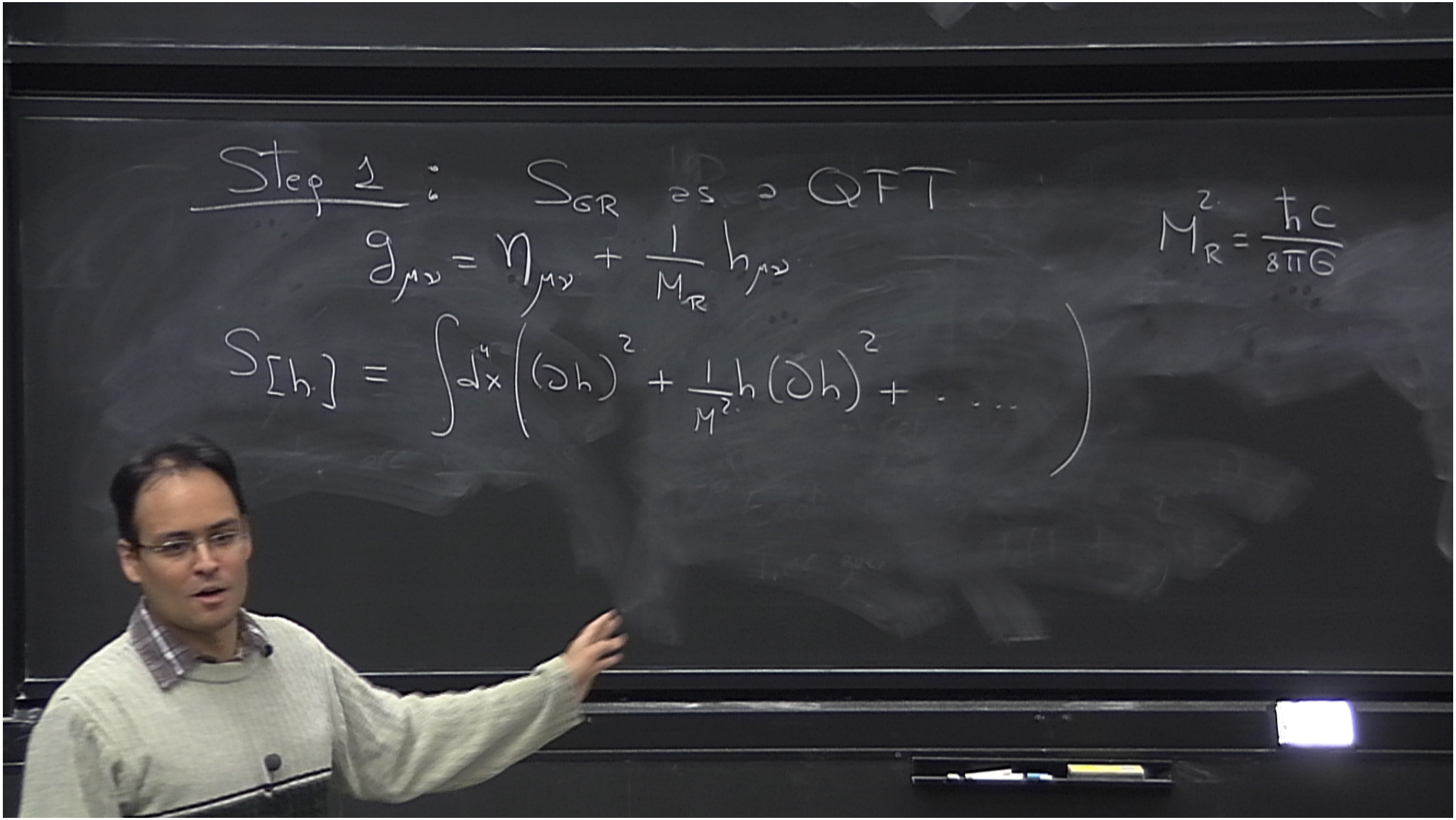
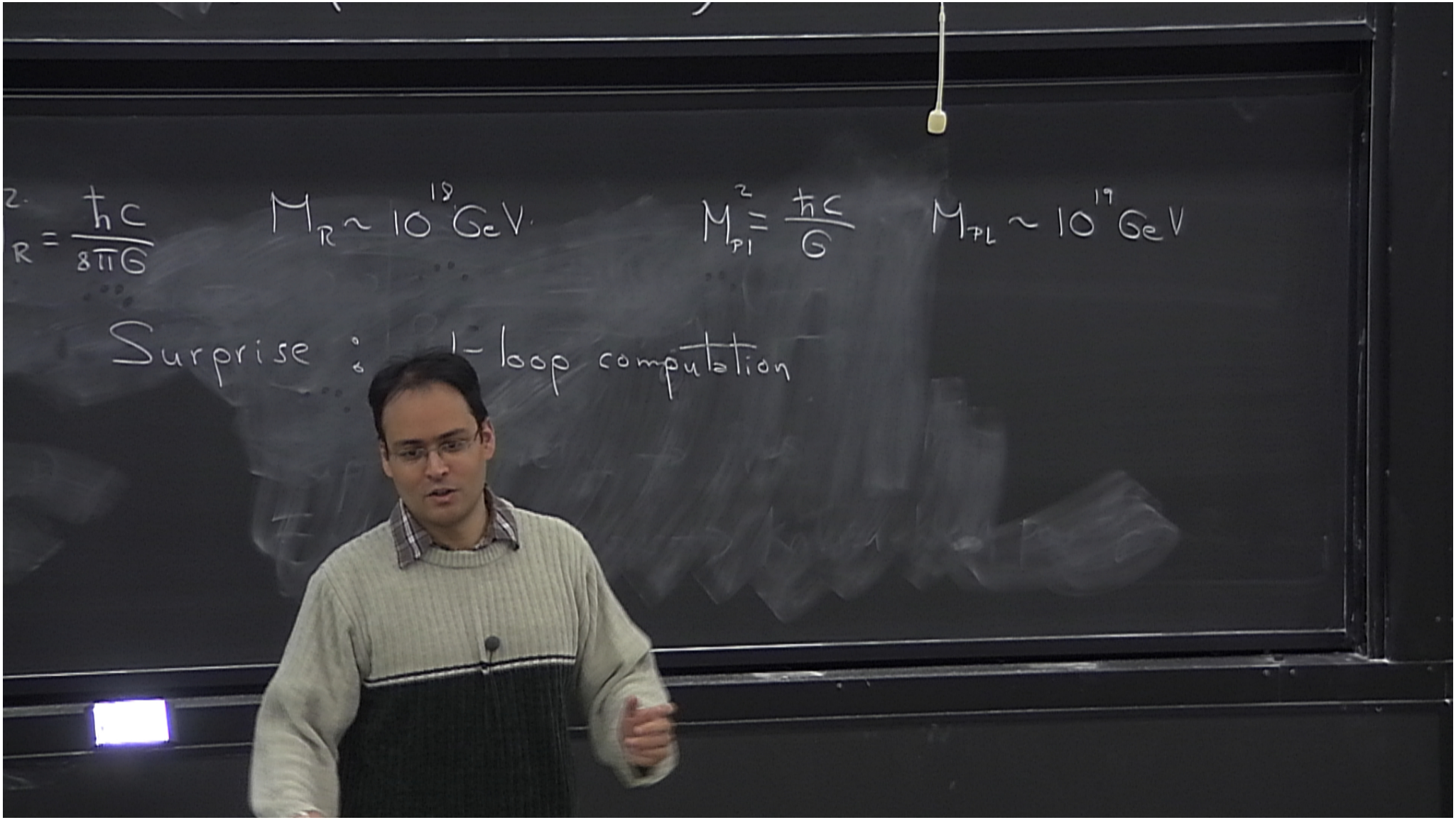
$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_R} h_{\mu\nu}$
$M_R^2 = \frac{\hbar c}{8\pi G}$
$S_{[h]} = \int d^4x \left((\partial h)^2 \right.$


Step 1 : S_{GR} as a QFT
$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_R} h_{\mu\nu}$
$M_R^2 = \frac{\hbar c}{8\pi G}$
$S_{[h]} = \int d^4x \left((\partial h)^2 + \frac{1}{M^2} h (\partial h)^2 + \dots \right)$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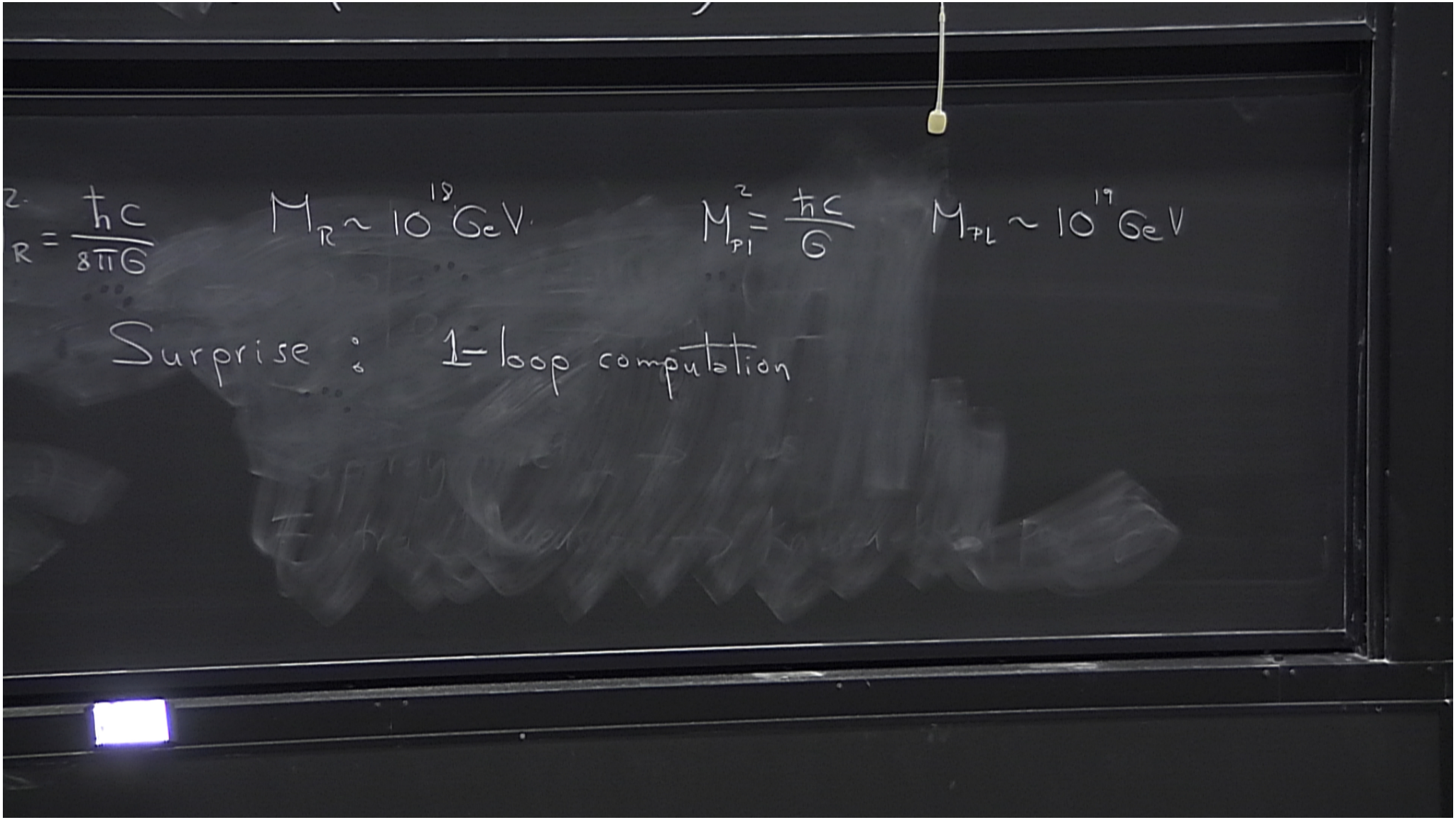
Surprise : 1-loop computation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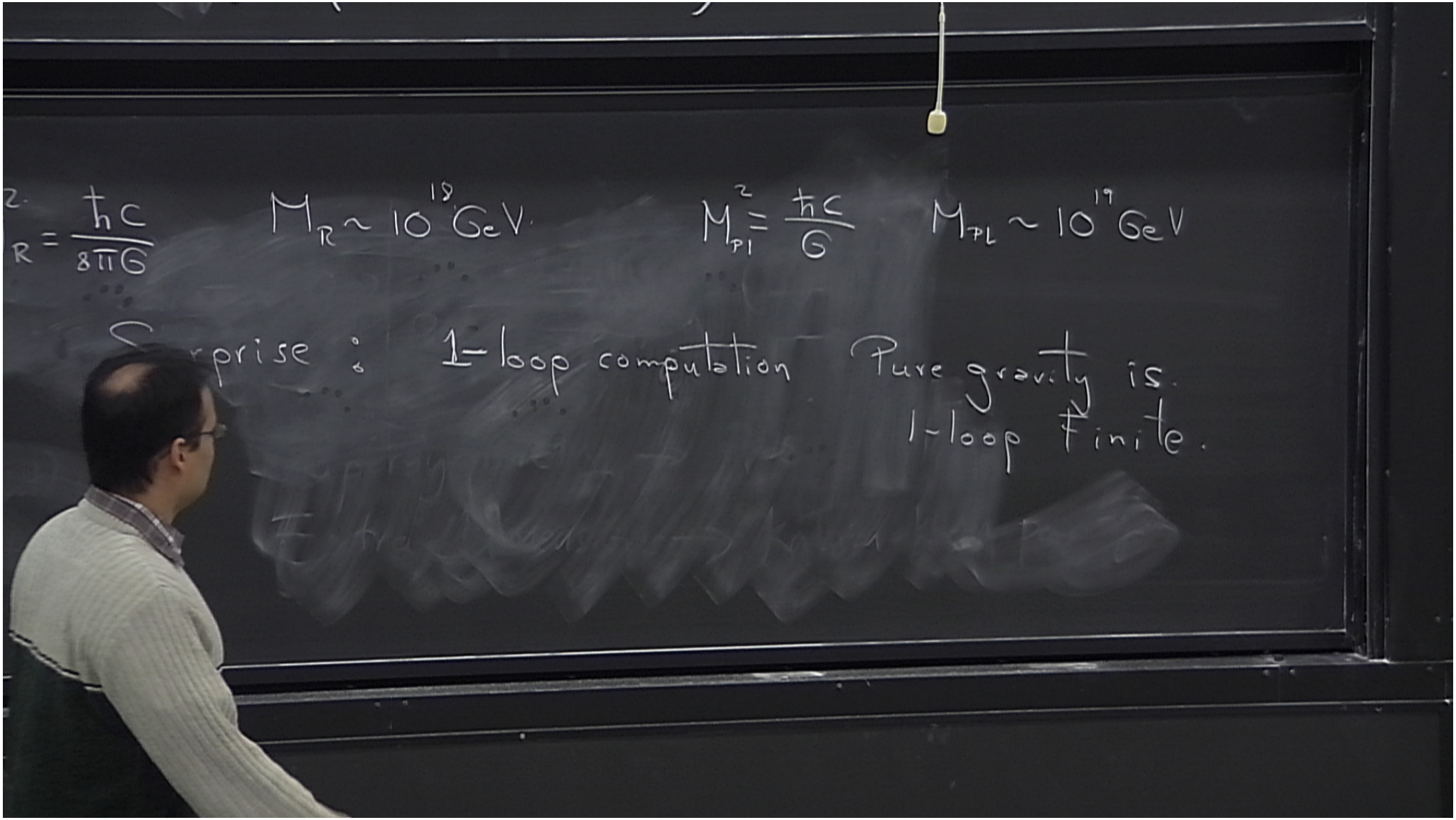
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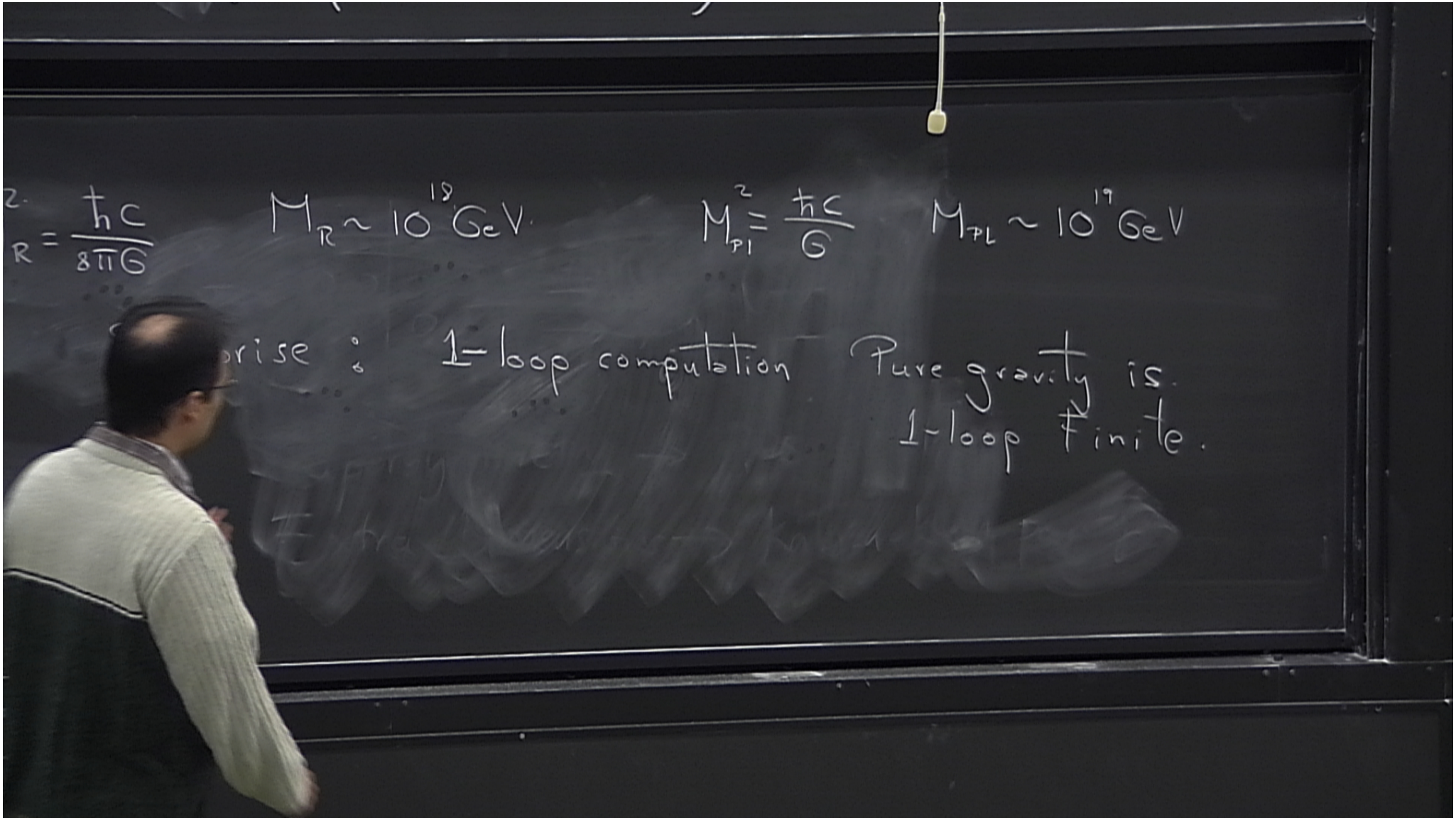
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2-loop

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2-loop Divergence

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Surprise : 1-loop computation Pure gravity is 1-loop finite.

2-loop Divergence

$$\frac{1}{\epsilon}\frac{1}{M_P^4}\int d^4x\sqrt{-g}\, R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\gamma\delta} R^{\gamma\delta}{}_{\mu\nu}$$

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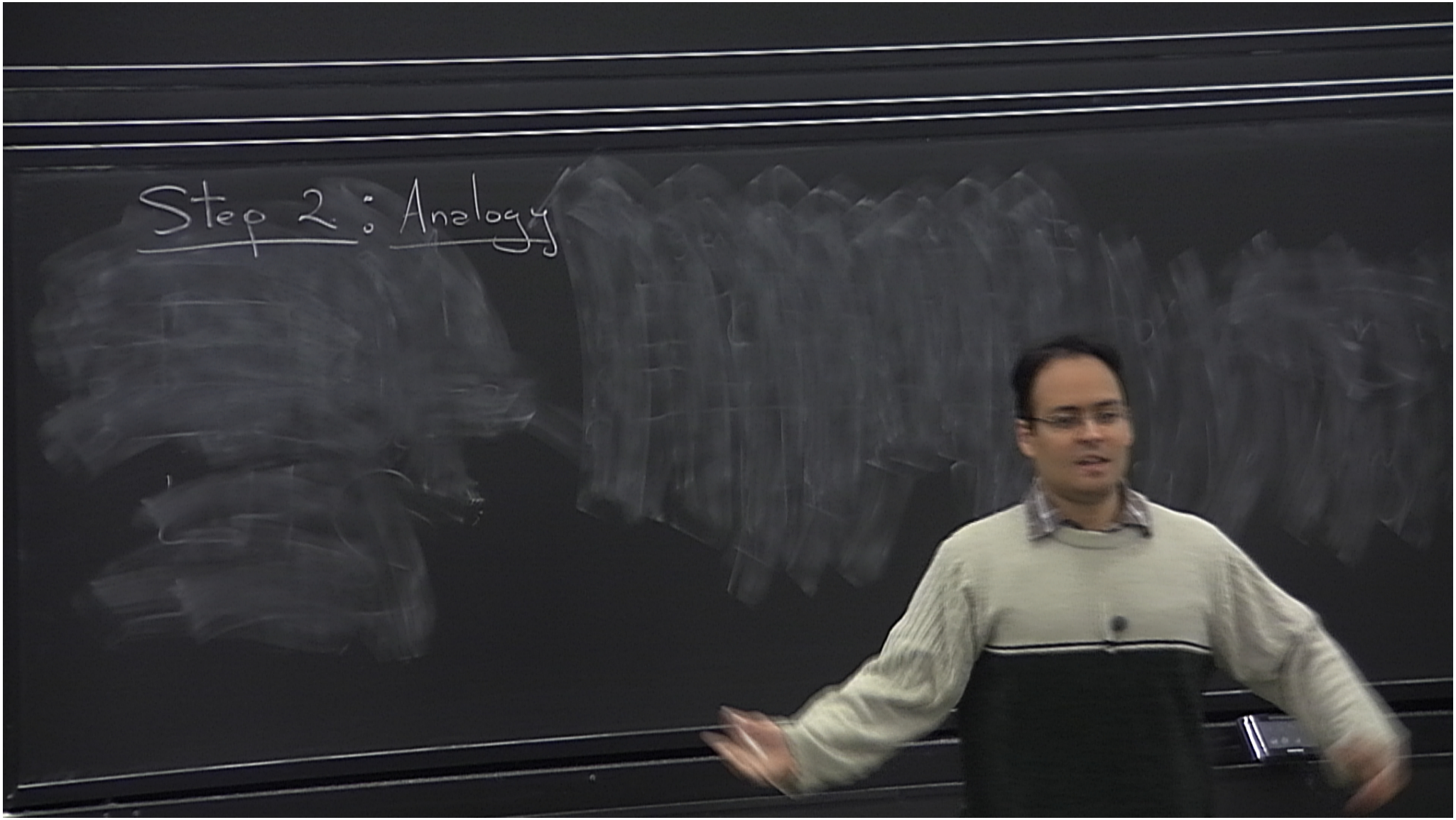
Surprise: 1-loop computation Pure gravity is 1-loop finite.

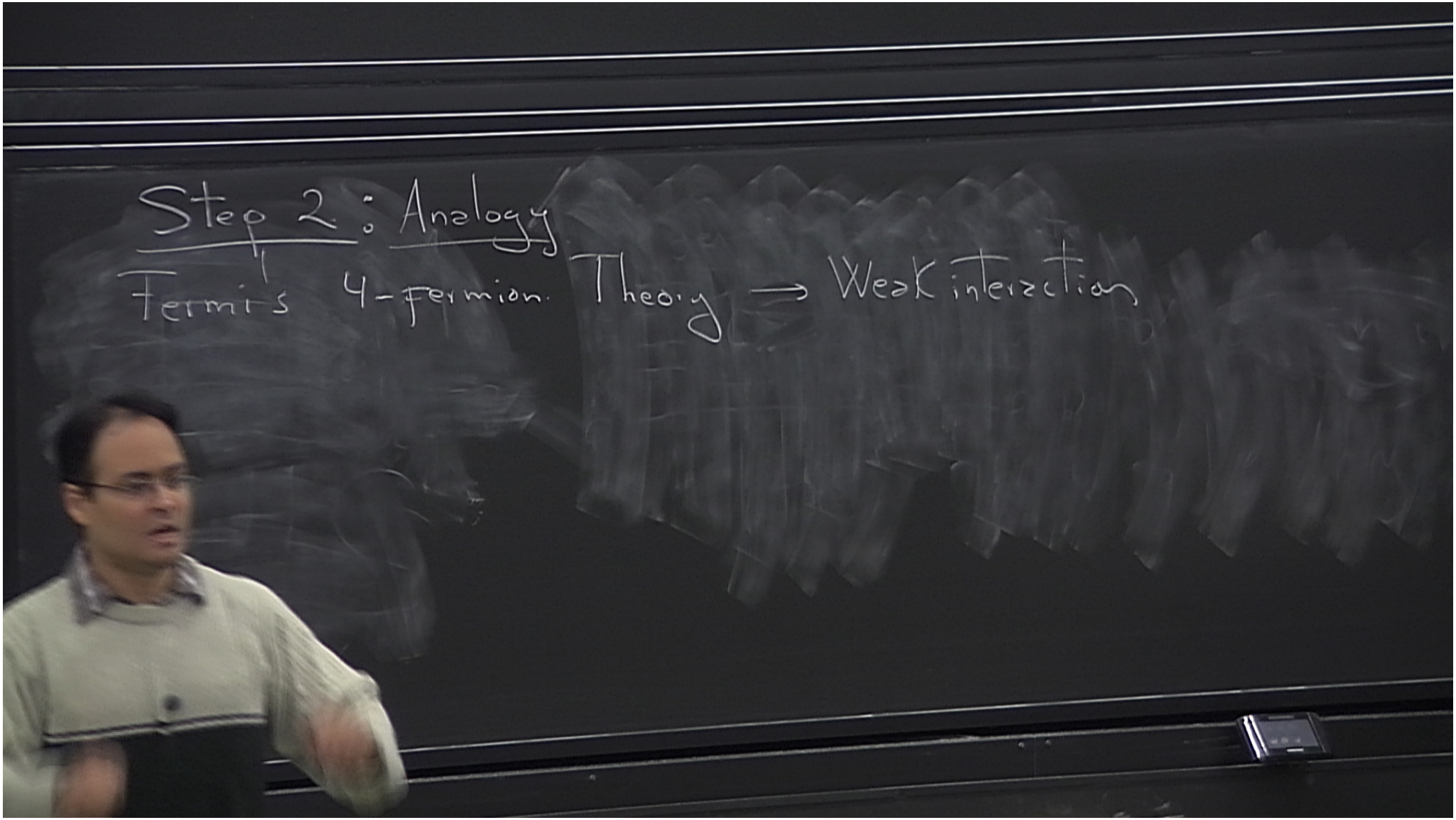
$+SM$

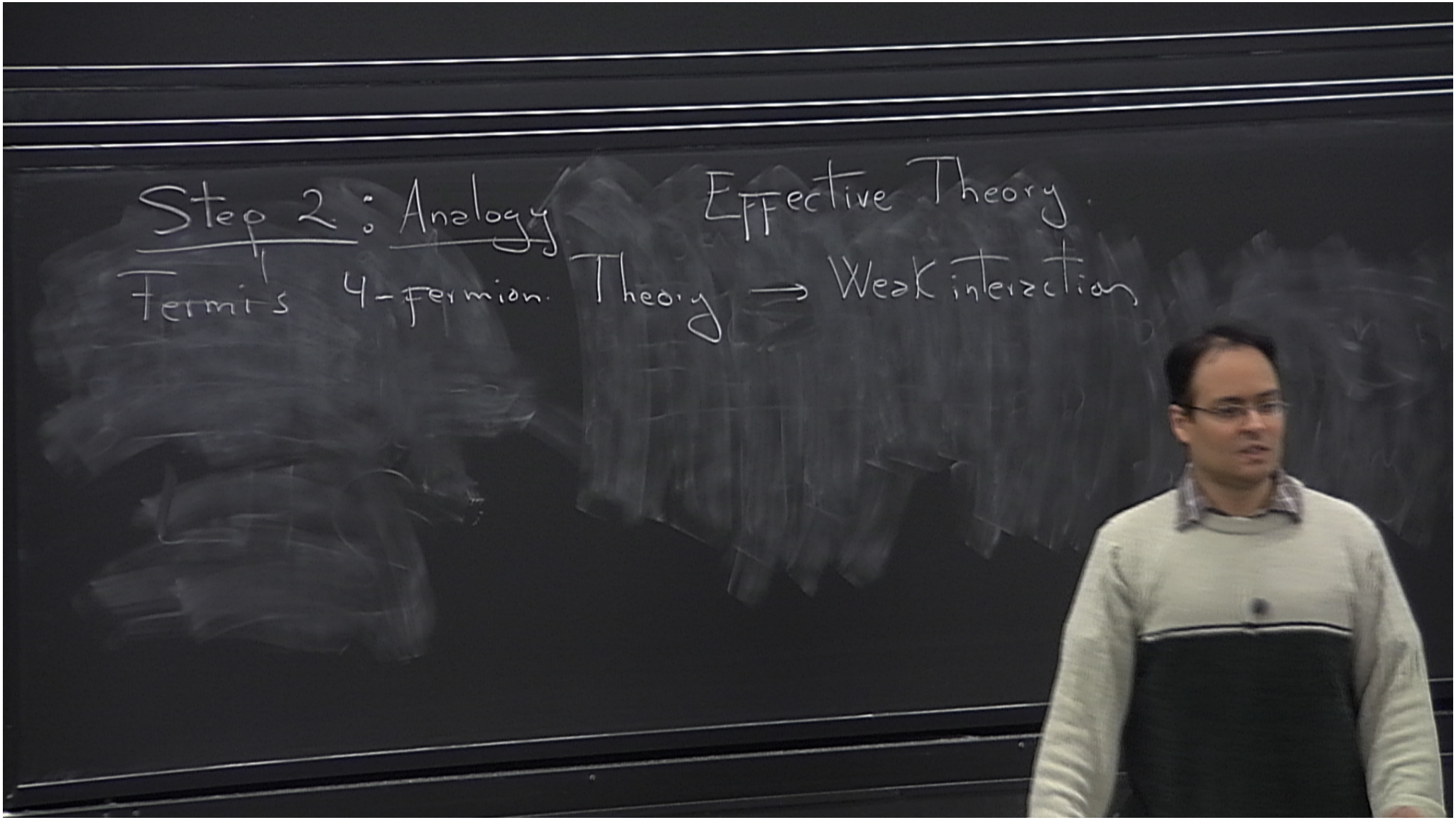
Diverges.

2-loop Divergence

$$\frac{1}{\epsilon}\frac{1}{M_P^4}\int d^4x\sqrt{-g}\, R^{\mu\nu}{}_{\rho\sigma} R^{\rho\sigma}{}_{\gamma\delta} R^{\gamma\delta}{}_{\mu\nu} \qquad \epsilon = 4$$


Step 2: Analogy


Step 2: Analogy
Fermi's 4-fermion Theory → Weak interaction


Step 2: Analogy
Effective Theory
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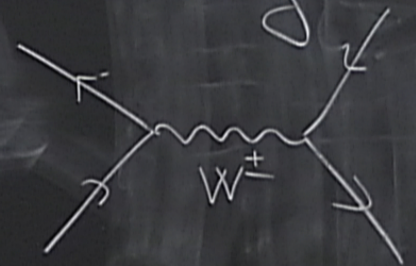
Step 2: Analogy Effective Theory

- Fermi's 4-fermion Theory → Weak interaction

- $W^{\pm}$, Z



→ Renormalizable.

Step 2: Analogy Effective Theory

- Fermi's 4-fermion Theory → Weak interaction
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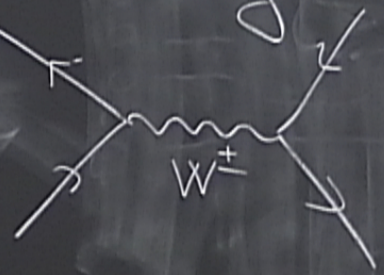
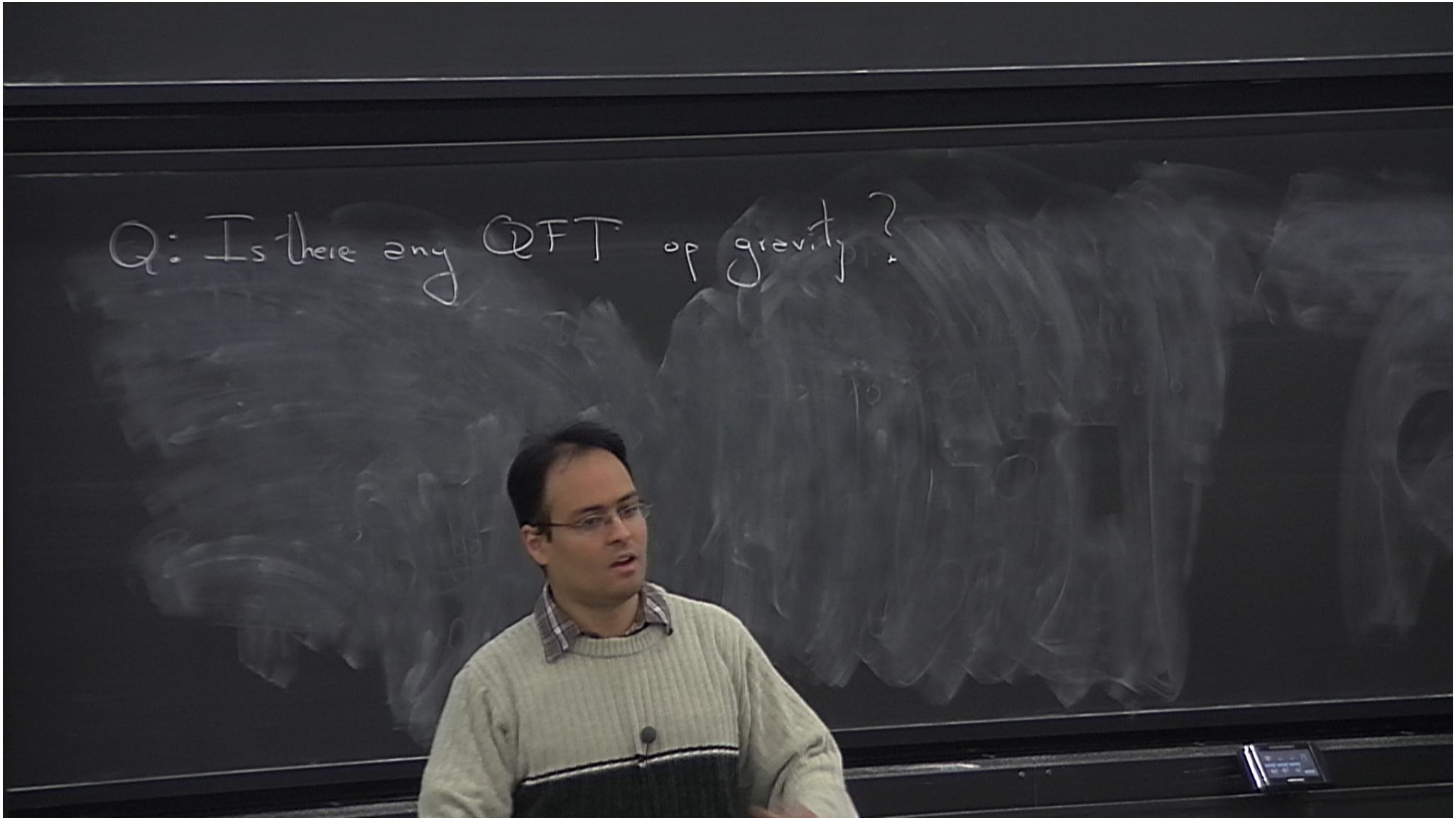


→ Renormalizable.

Q: Could GR be an effective theory of another QFT?


Q: Is there any QFT of gravity?

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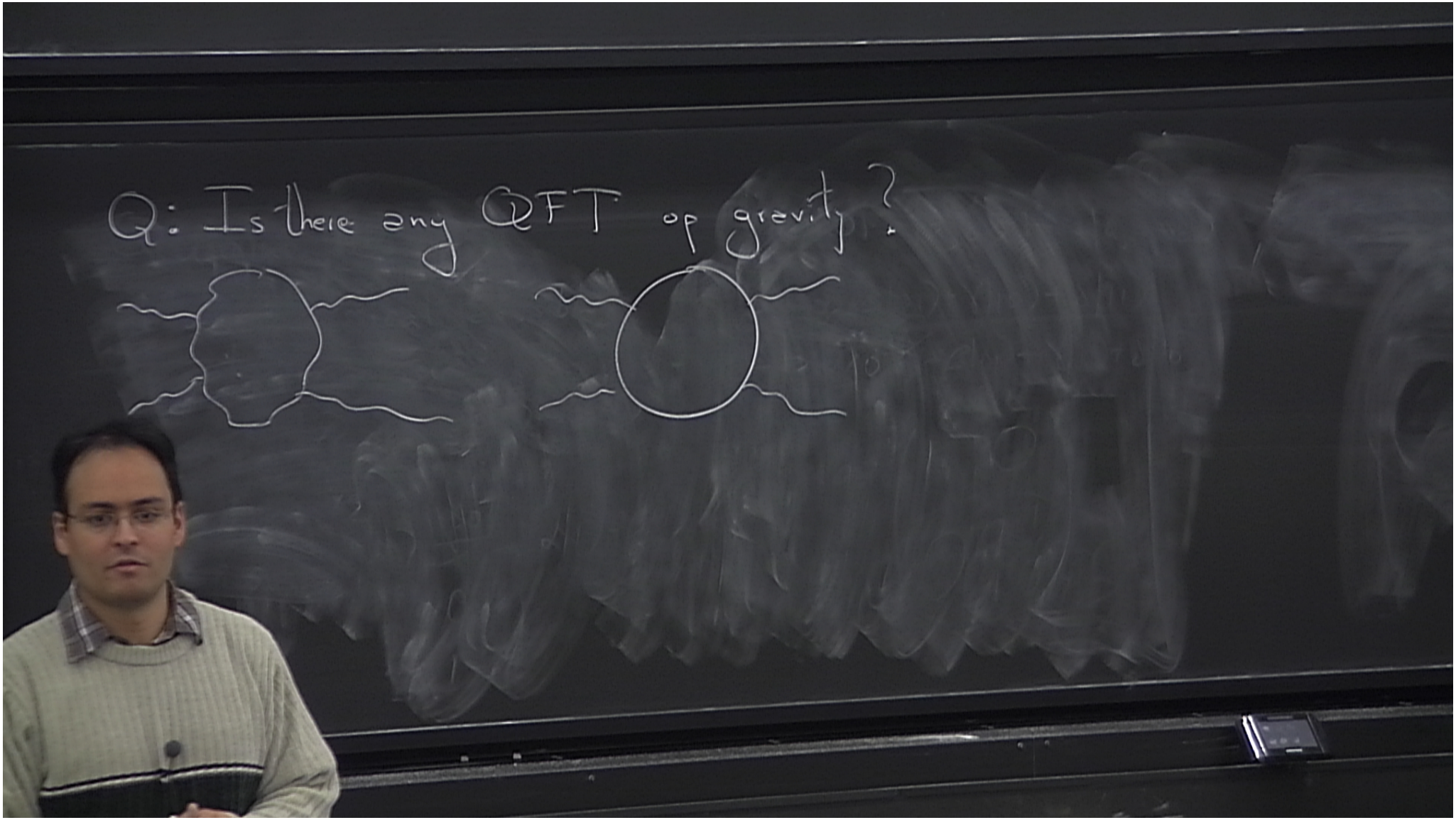
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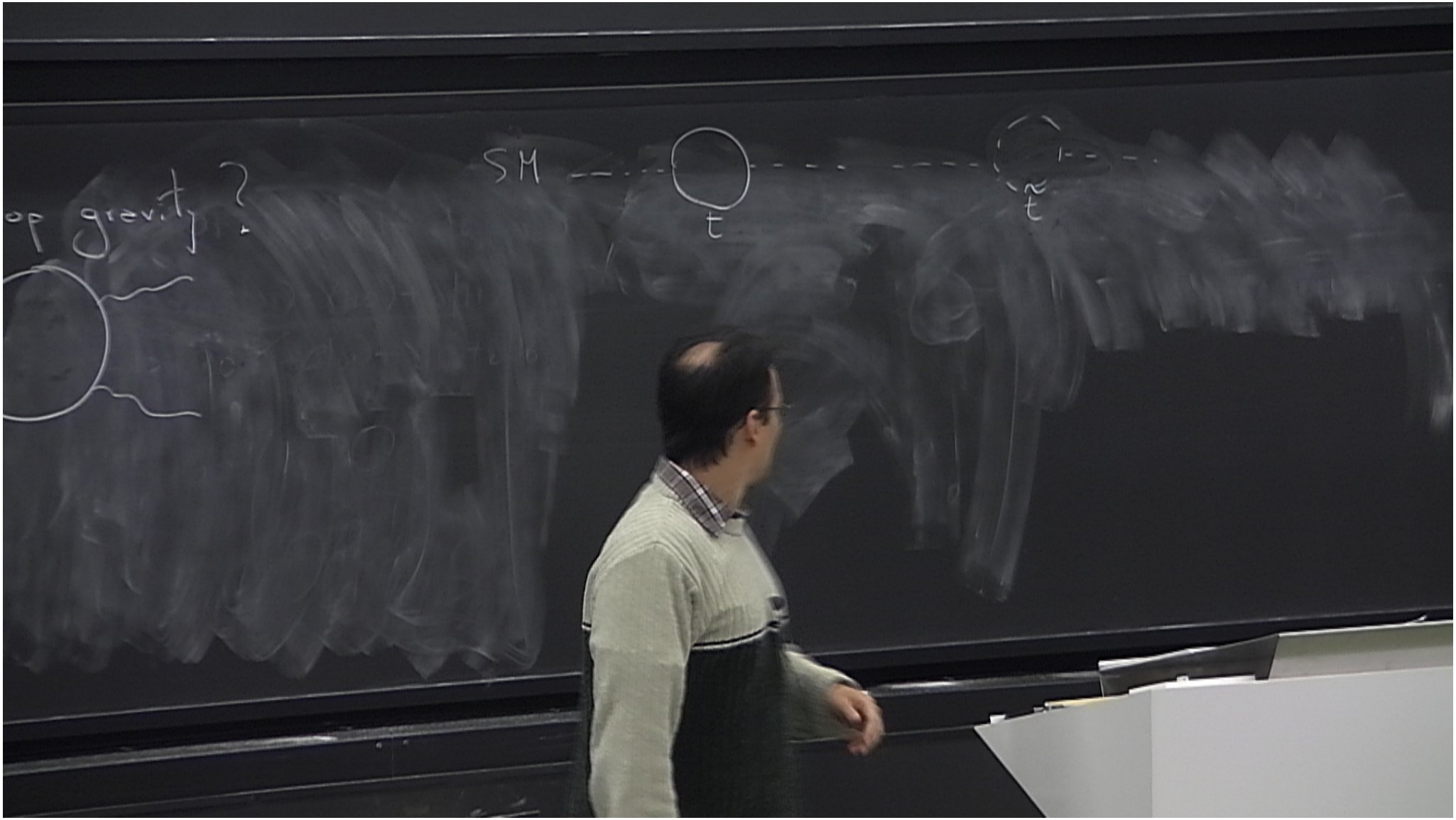

SM
t
t̃

Q: Is there any QFT of gravity?

Symmetry Bosons — Fermions.

SM

t

Q: Is there any QFT of gravity?

SM ---- t ----

Symmetry Bosons — Fermions. } Supersymmetry.

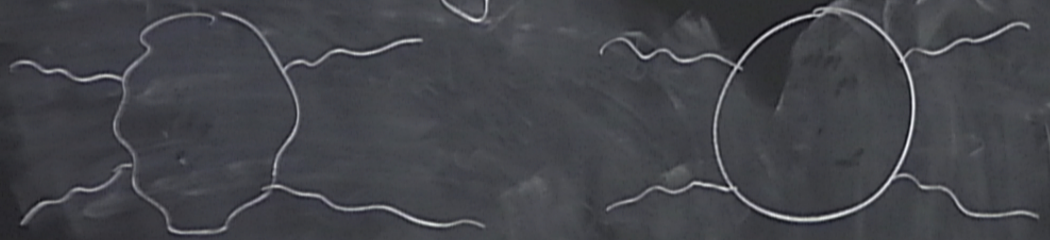
Q: Is there any QFT of gravity?

SM ---- t ----

Symmetry Bosons — Fermions. } Supersymmetry.

Massless Particles.

Q: Is there any QFT of gravity?



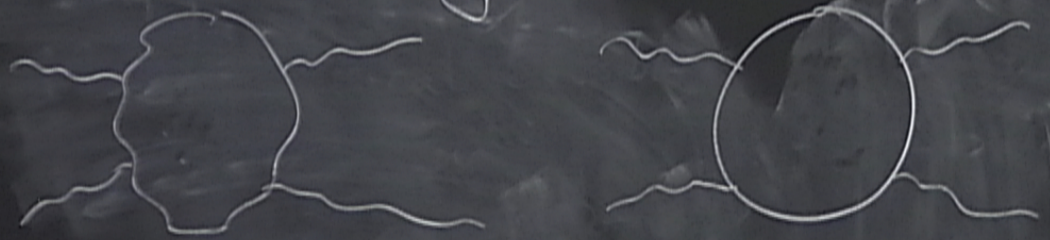
SM ---- t ----

Symmetry Bosons — Fermions. } Supersymmetry.

Massless Particles : helicity

Q: Is there any QFT of gravity?

SM ---- t ----



Symmetry Bosons — Fermions. } Supersymmetry.

Massless Particles : helicity Graviton $h = \pm 2$.

Q: Is there any QFT of gravity?

SM ---- t

Symmetry Bosons — Fermions. } Supersymmetry.

Massless Particles : helicity Graviton $h = \pm 2$.

$|-2\rangle$ $|+2\rangle$

Q: Is there any QFT of gravity?

SM ---- t ----

Symmetry Bosons — Fermions. } Supersymmetry. Q, $\bar{Q}$

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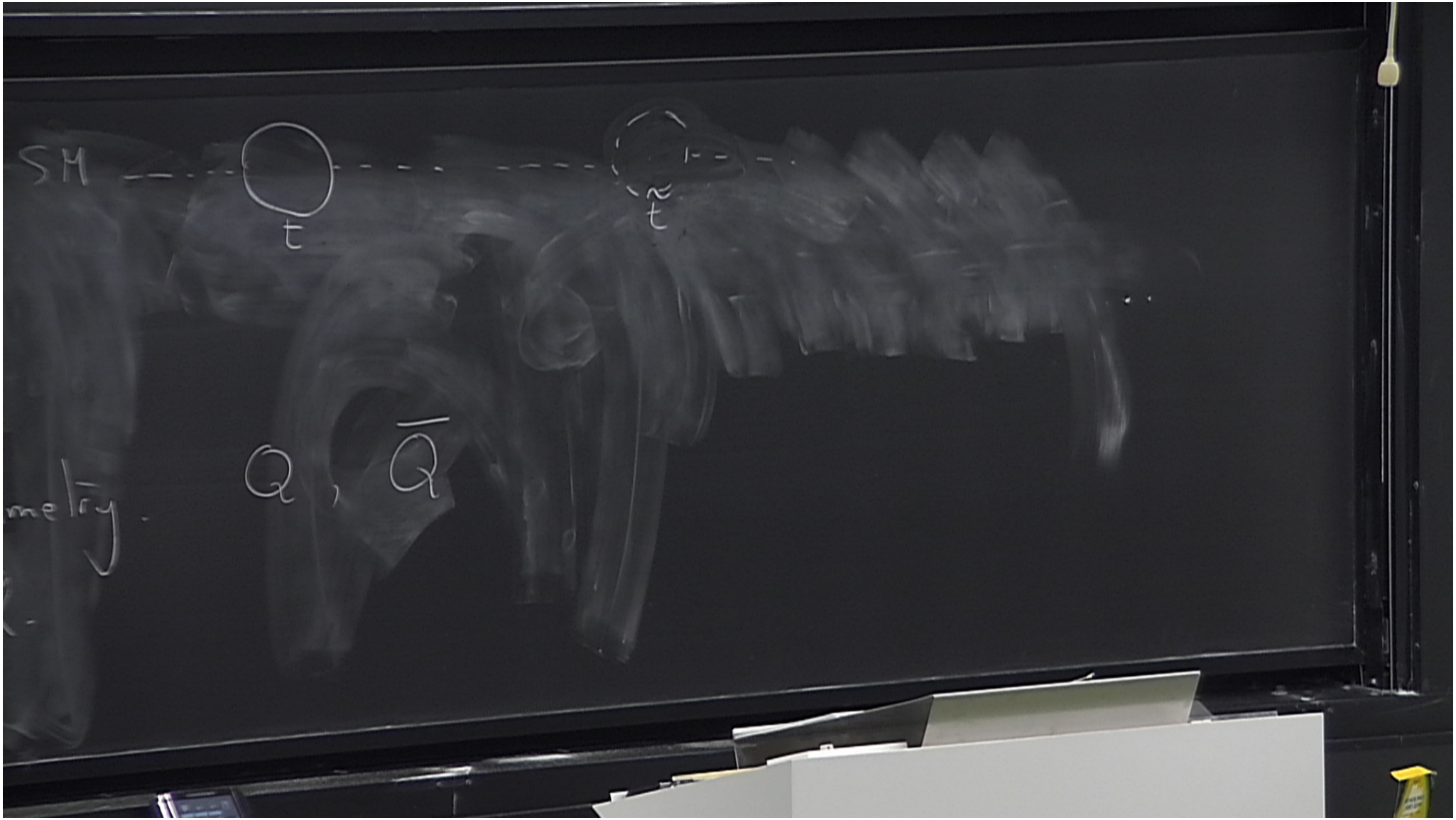
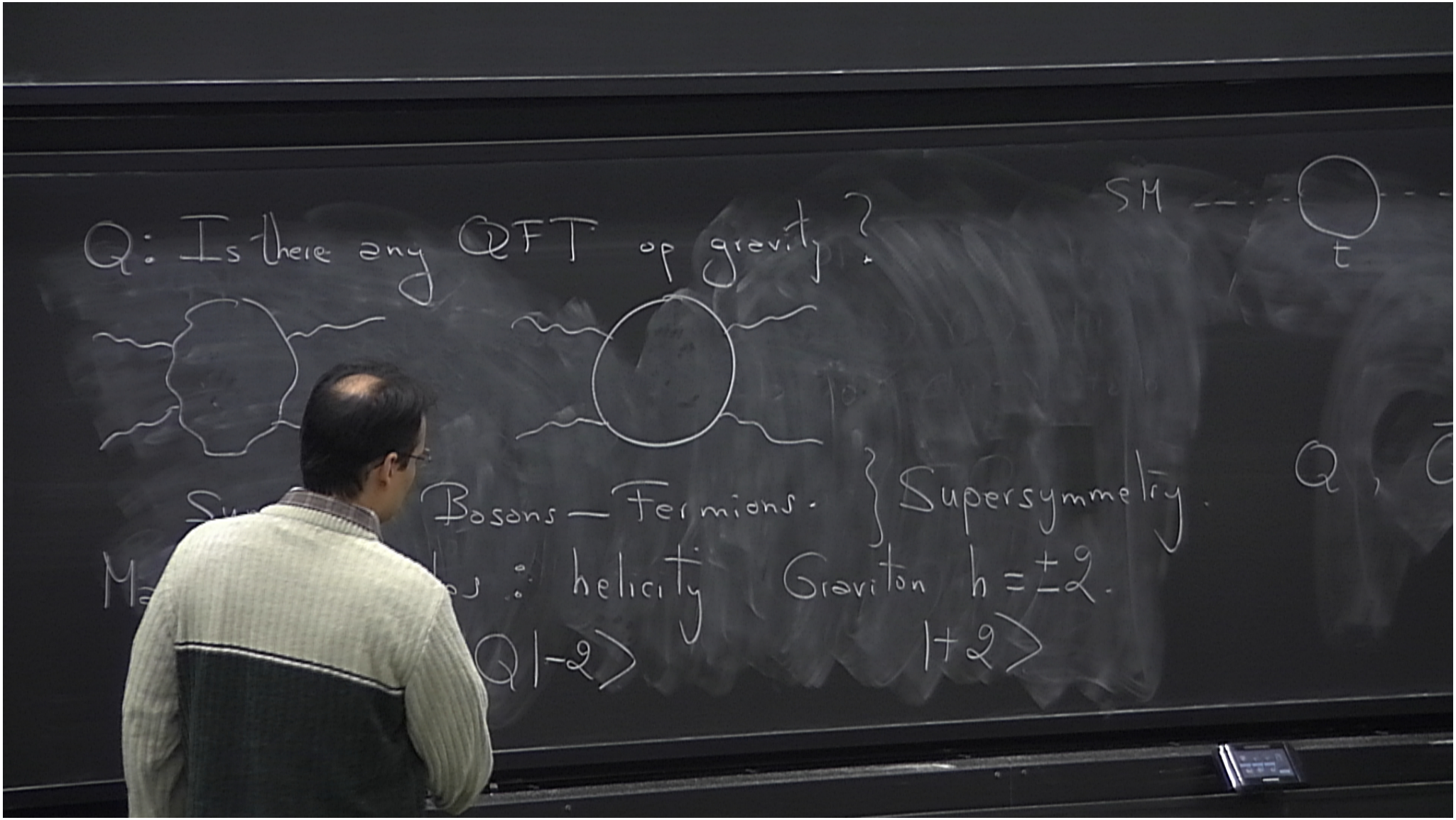

SM
t
t̃
Q, Q̄


Q: Is there any QFT of gravity?
SM
t
Bosons — Fermions.
Supersymmetry.
helicity
Graviton h = ±2.
Q|-2>
|+2>

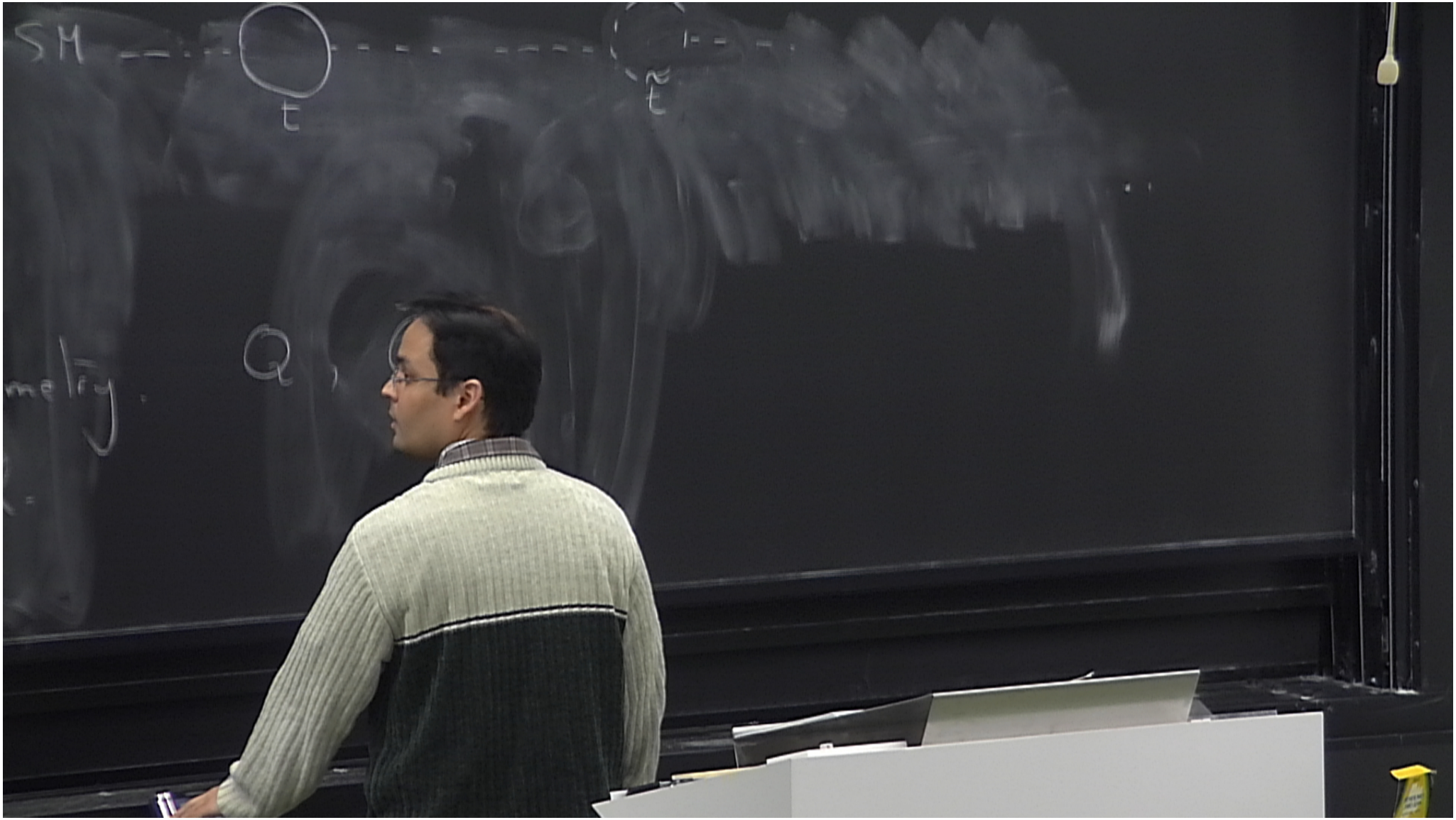
Q: Is there any QFT of gravity?

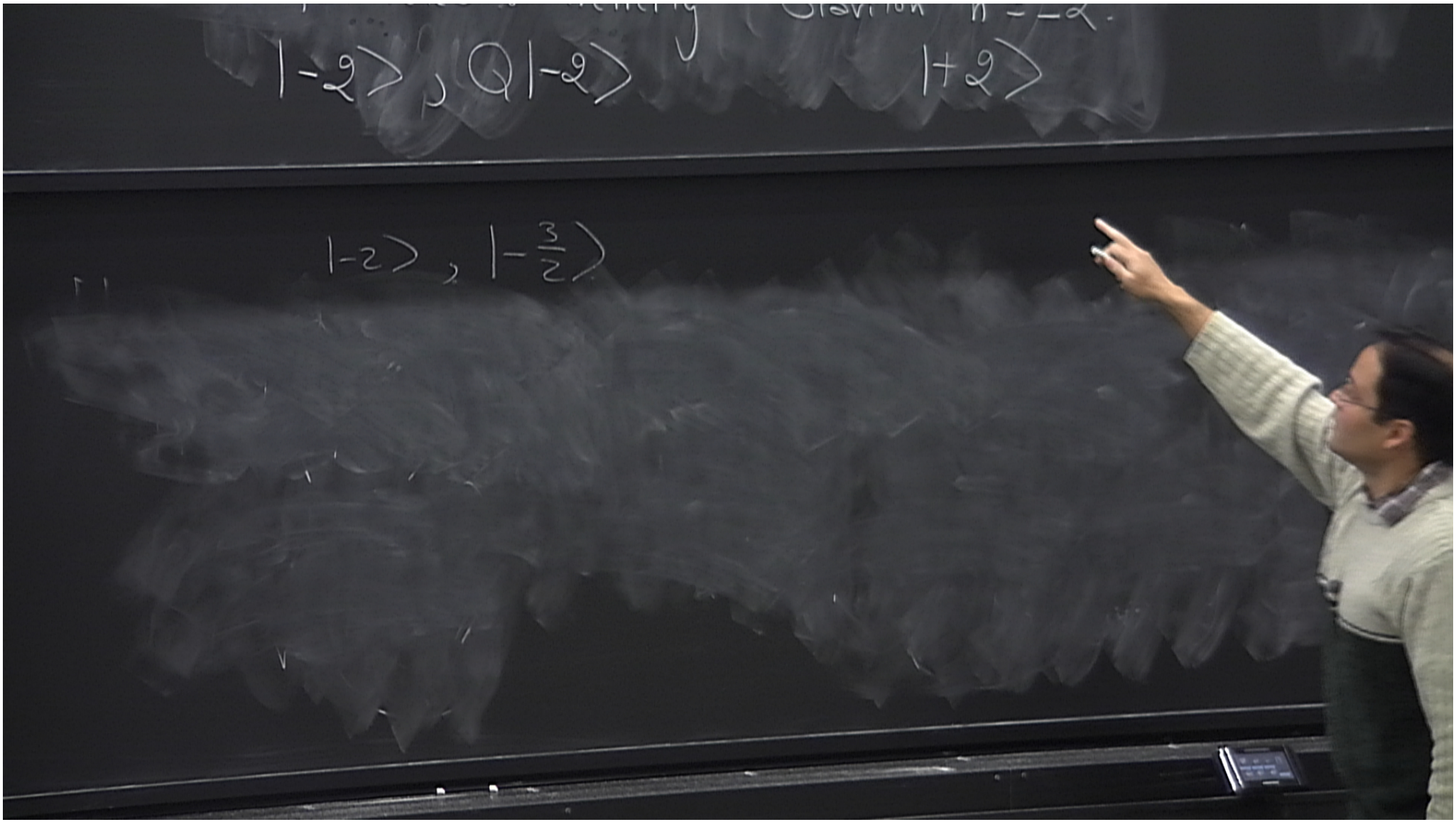
SM ---- t ----

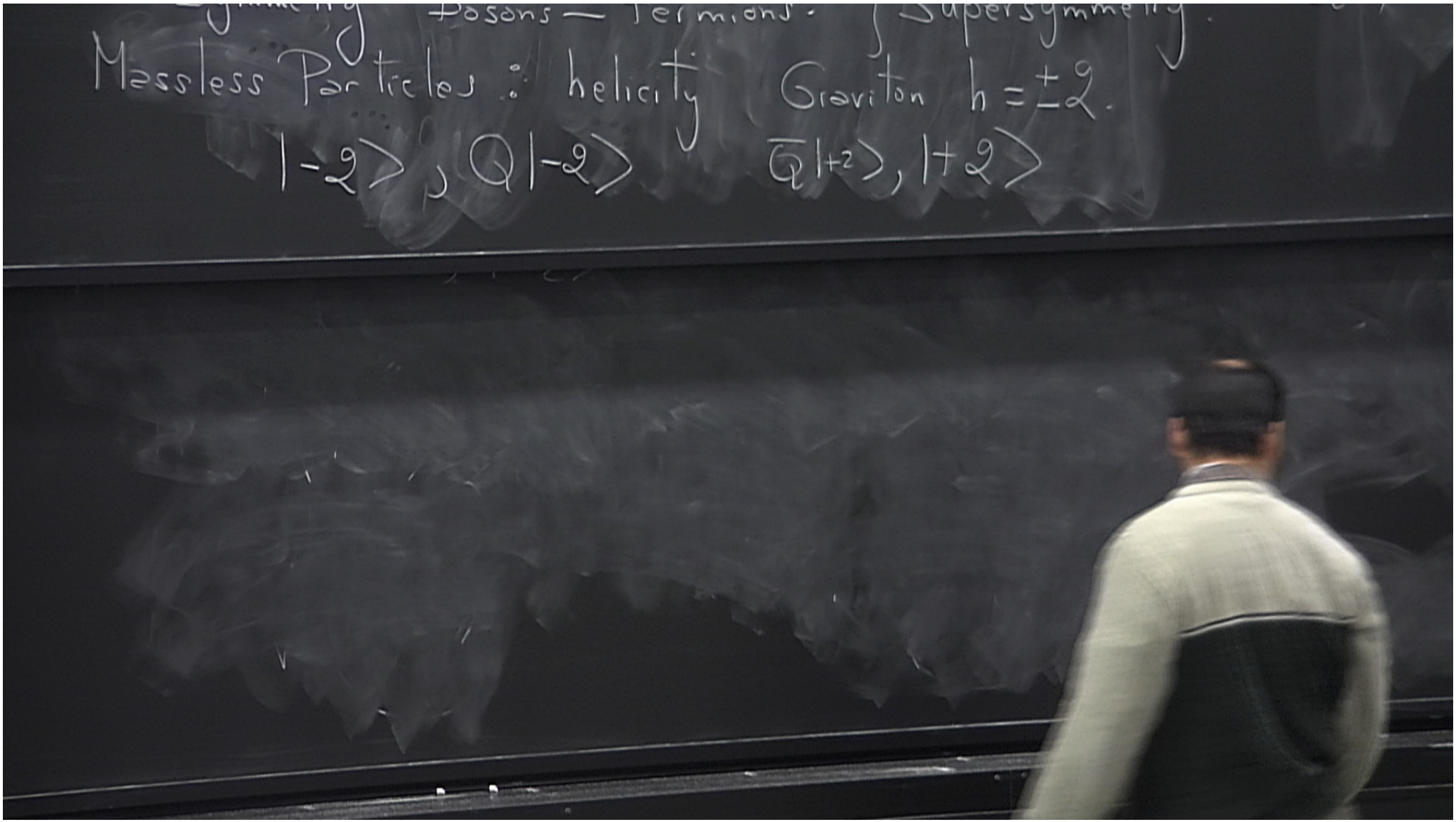
Symmetry Bosons — Fermions. } Supersymmetry. Q

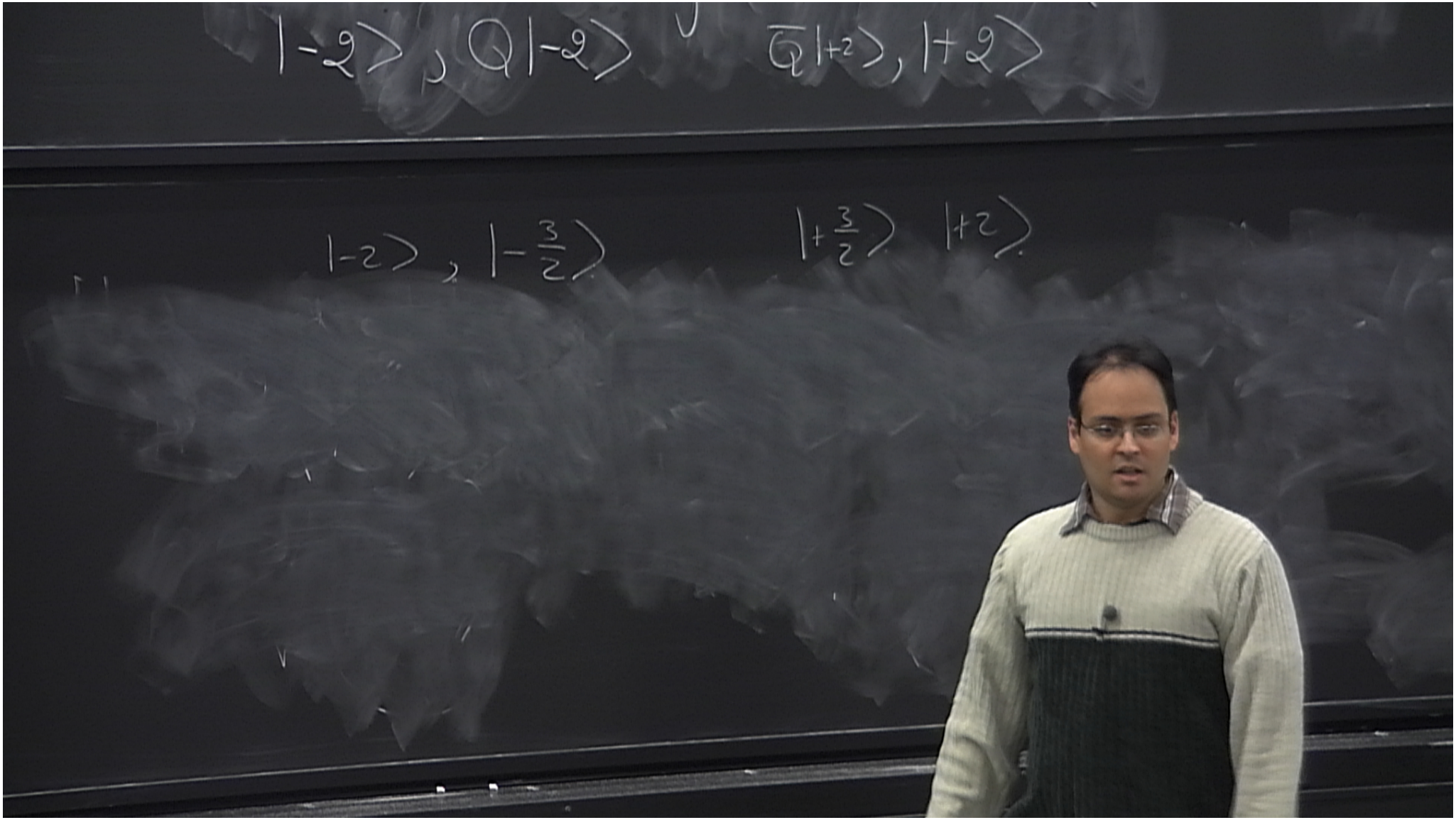
Massless Particles : helicity Graviton $h = \pm 2$.

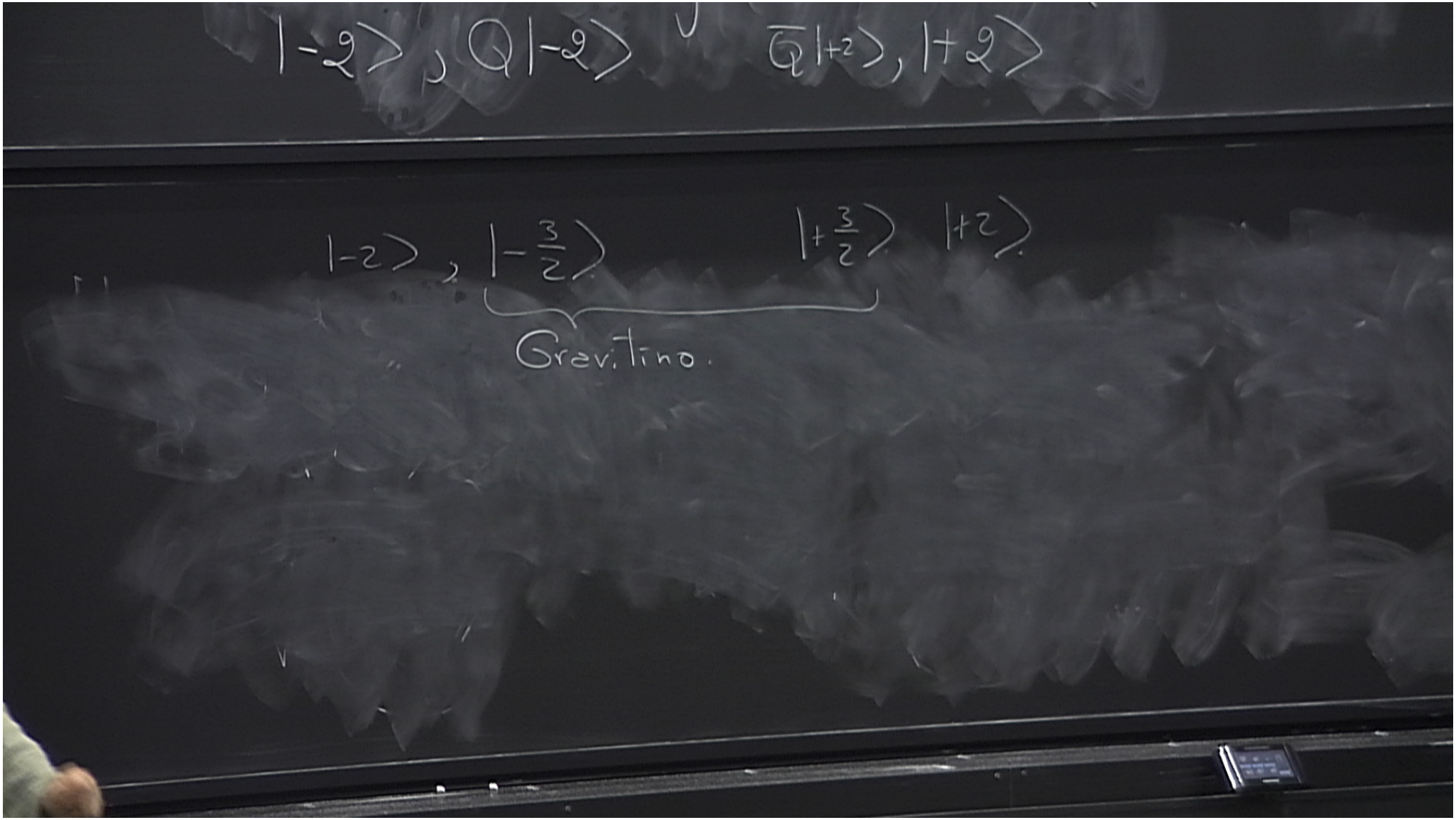
$|-2\rangle$, $Q|-2\rangle$ $|+2\rangle$

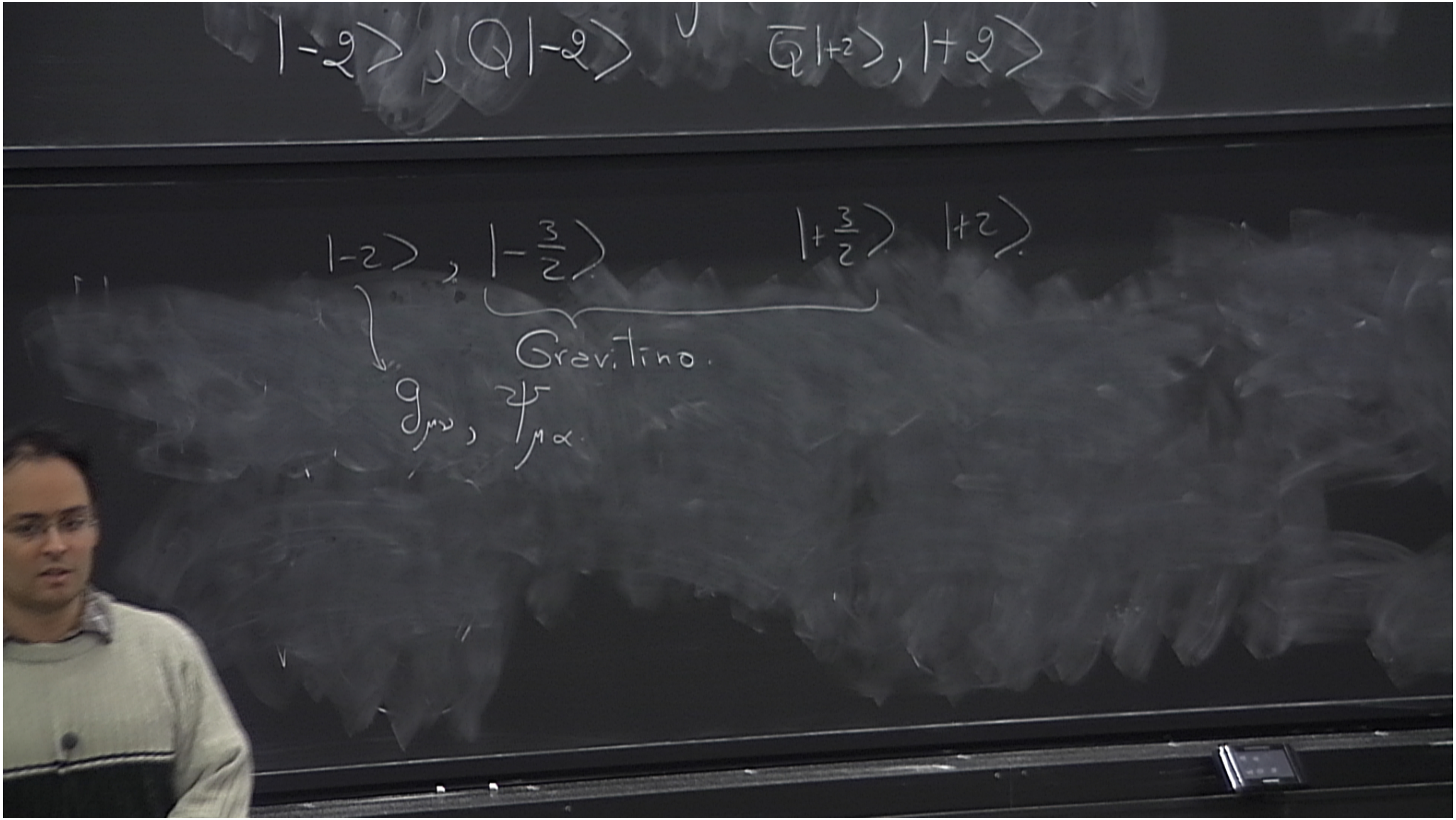

SM
t
t̃

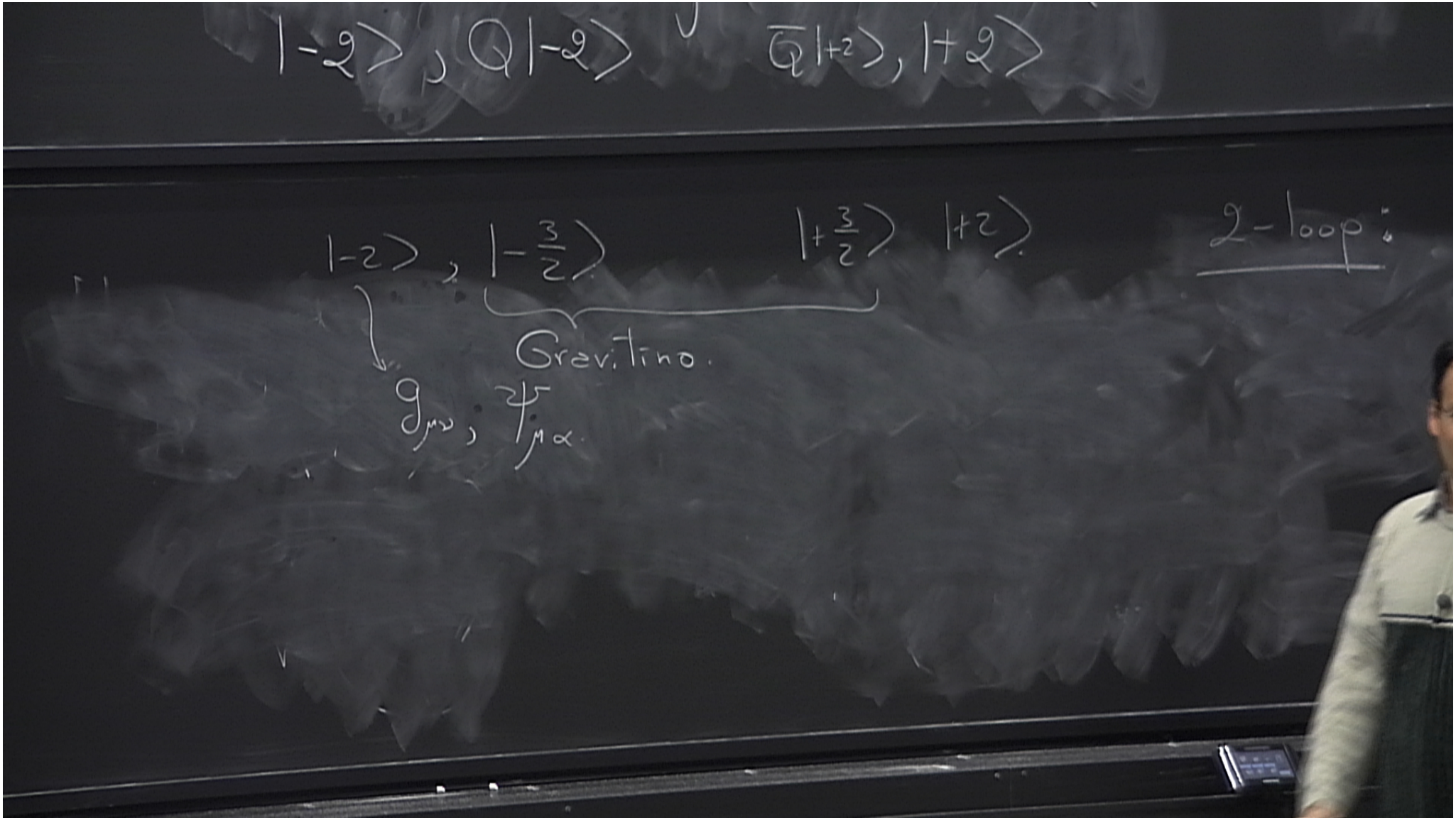

$|-2\rangle$, $Q|-2\rangle$
$|+2\rangle$
$|-2\rangle$, $|-\frac{3}{2}\rangle$

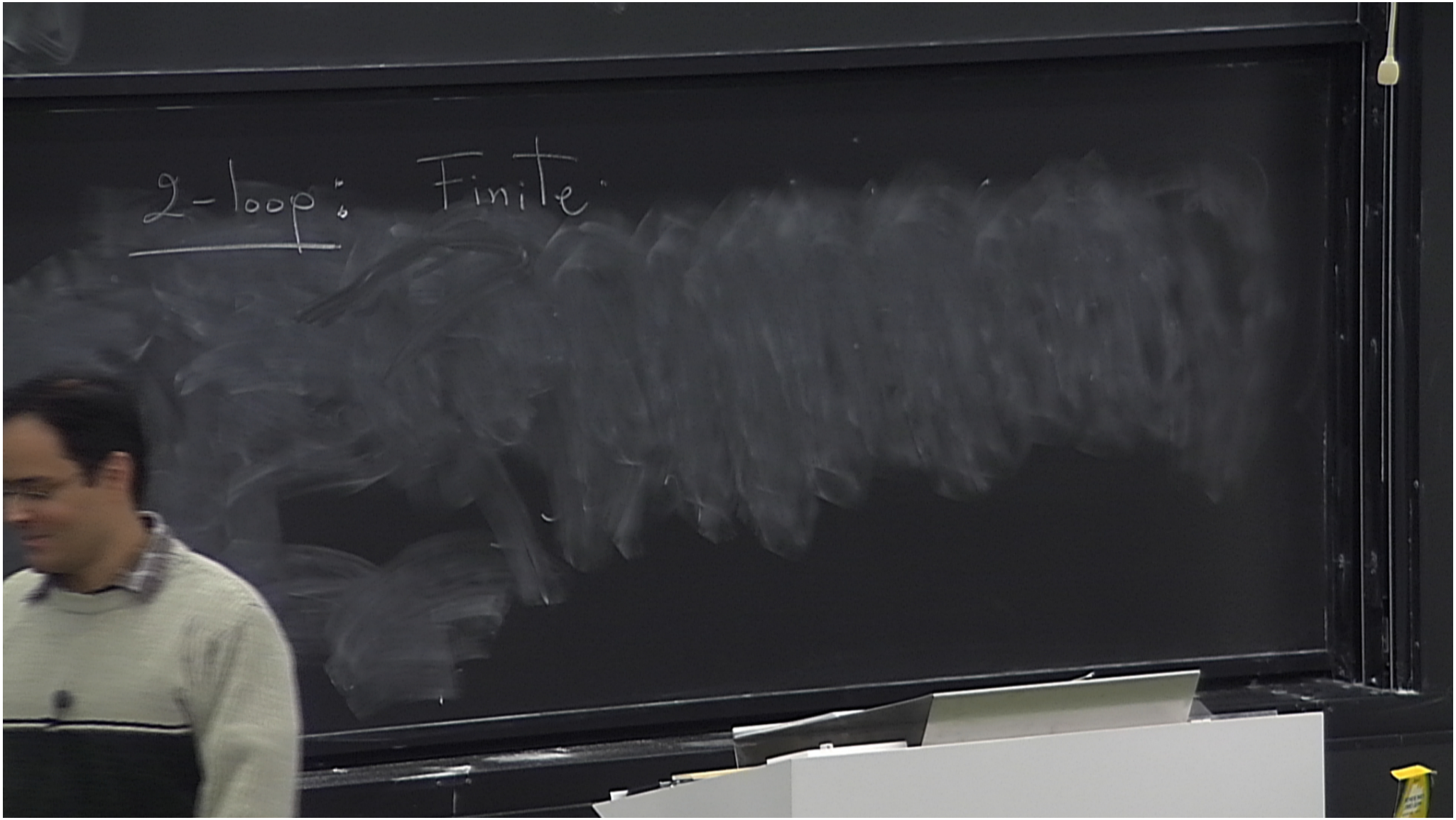

Bosons — Fermions.
Massless Particles : helicity
Graviton $h = \pm 2$.
$|-2\rangle$, $Q|-2\rangle$
$\bar{Q}|+2\rangle$, $|+2\rangle$

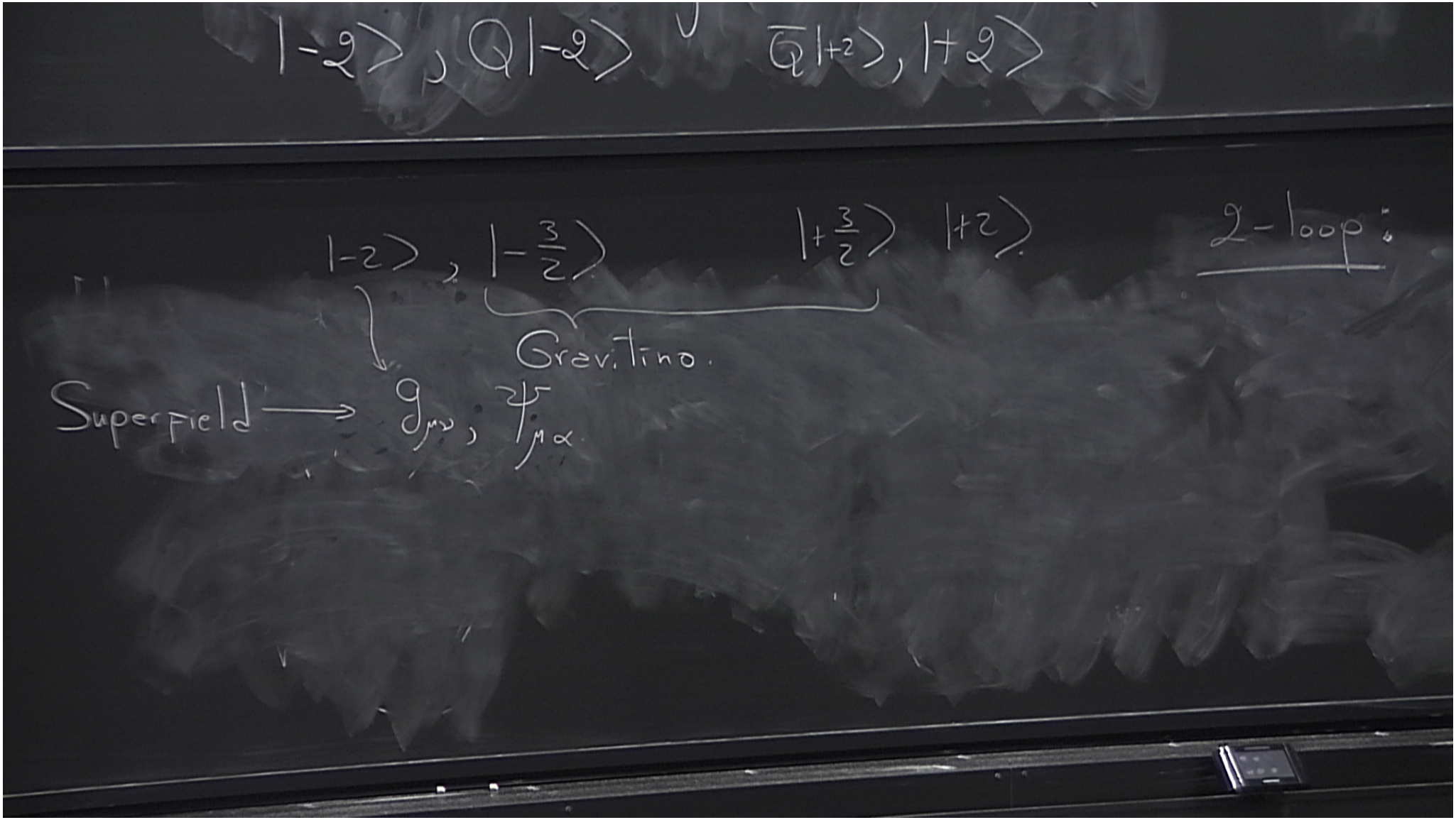

$|-2\rangle, Q|-2\rangle$
$Q|+2\rangle, |+2\rangle$
$|-2\rangle, |-\frac{3}{2}\rangle$
$|+\frac{3}{2}\rangle$ $|+2\rangle$


$|-2\rangle$, $Q|-2\rangle$ $\bar{Q}|+2\rangle$, $|+2\rangle$
$|-2\rangle$, $|-\frac{3}{2}\rangle$ $|+\frac{3}{2}\rangle$ $|+2\rangle$
Gravitino.


$|-2\rangle$, $Q|-2\rangle$ ... $\bar{Q}|+2\rangle$, $|+2\rangle$
$|-2\rangle$, $|-\frac{3}{2}\rangle$ ... $|+\frac{3}{2}\rangle$ $|+2\rangle$
Gravitino.
$g_{\mu\nu}$, $\psi_{\mu\alpha}$


$|-2\rangle$, $Q|-2\rangle$ $\overline{Q}|+2\rangle$, $|+2\rangle$
$|-2\rangle$, $|-\frac{3}{2}\rangle$ $|+\frac{3}{2}\rangle$ $|+2\rangle$
2-loop:
Gravitino.
$g_{\mu\nu}$, $\psi_{\mu\alpha}$.


2-loop: Finite

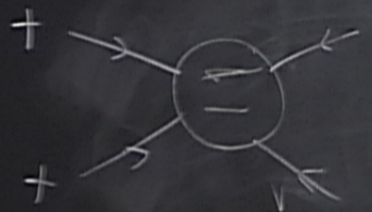

|-2>, Q|-2>
Q̄|+2>, |+2>
|-2>, |-3/2>
|+3/2>
|+2>
2-loop:
Gravitino.
Superfield ⟶ g_μν, ψ_μα

$|-2\rangle$, $Q|-2\rangle$ $\quad$ $\bar{Q}|+2\rangle$, $|+2\rangle$

$|-2\rangle$, $|-\frac{3}{2}\rangle$ $\quad$ $|+\frac{3}{2}\rangle$ $|+2\rangle$ $\quad$ 2-loop:

Gravitino.

Superfield $\longrightarrow$ $g_{\mu\nu}$, $\psi_{\mu\alpha}$

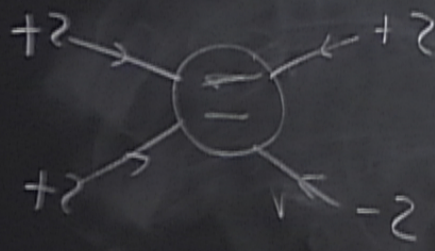


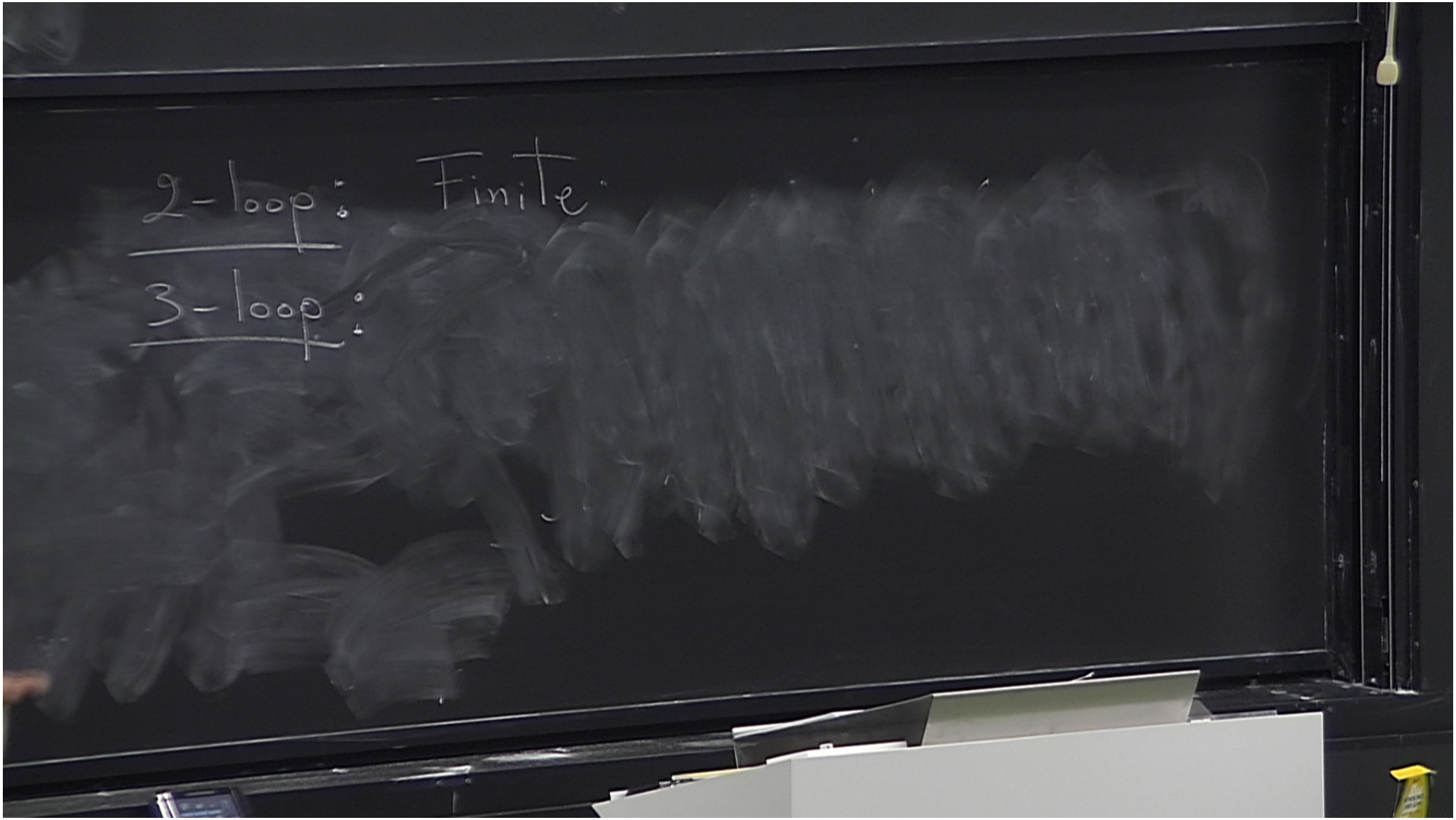
$|-2\rangle$, $Q|-2\rangle$ $\cdots$ $\bar{Q}|+2\rangle$, $|+2\rangle$

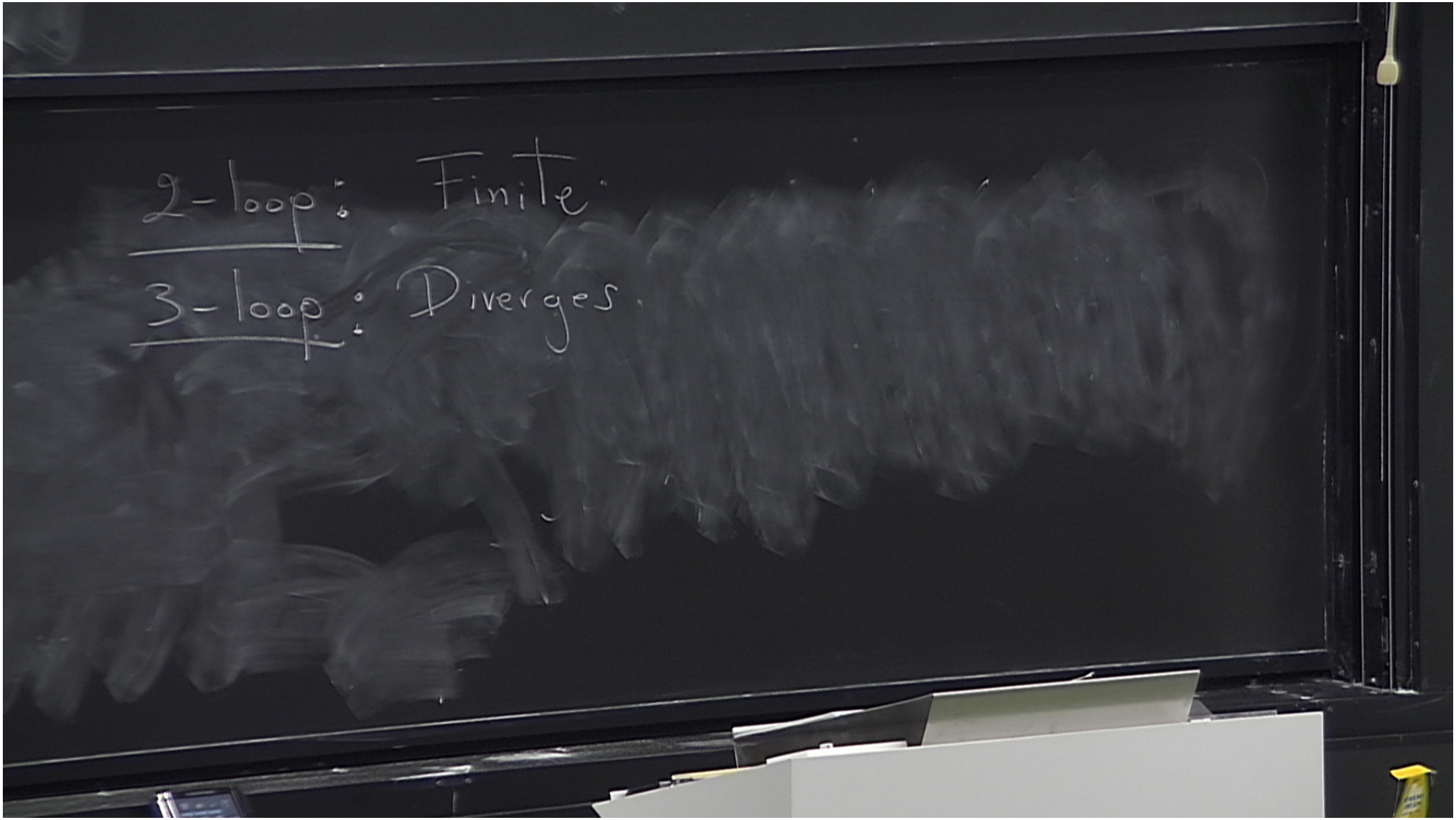
$|-2\rangle$, $|-\frac{3}{2}\rangle$ $|+\frac{3}{2}\rangle$ $|+2\rangle$ 2-loop:

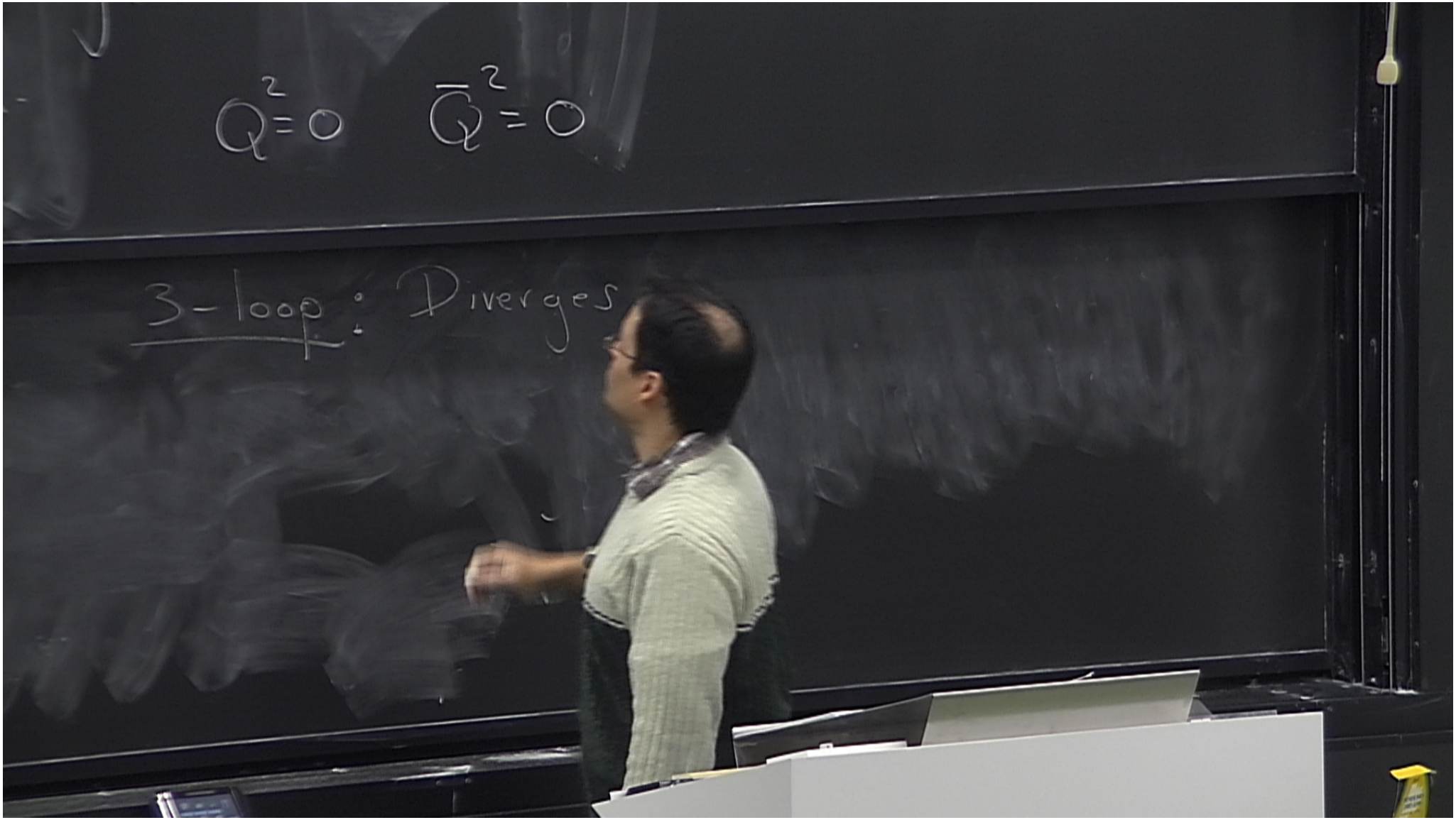
Gravitino.

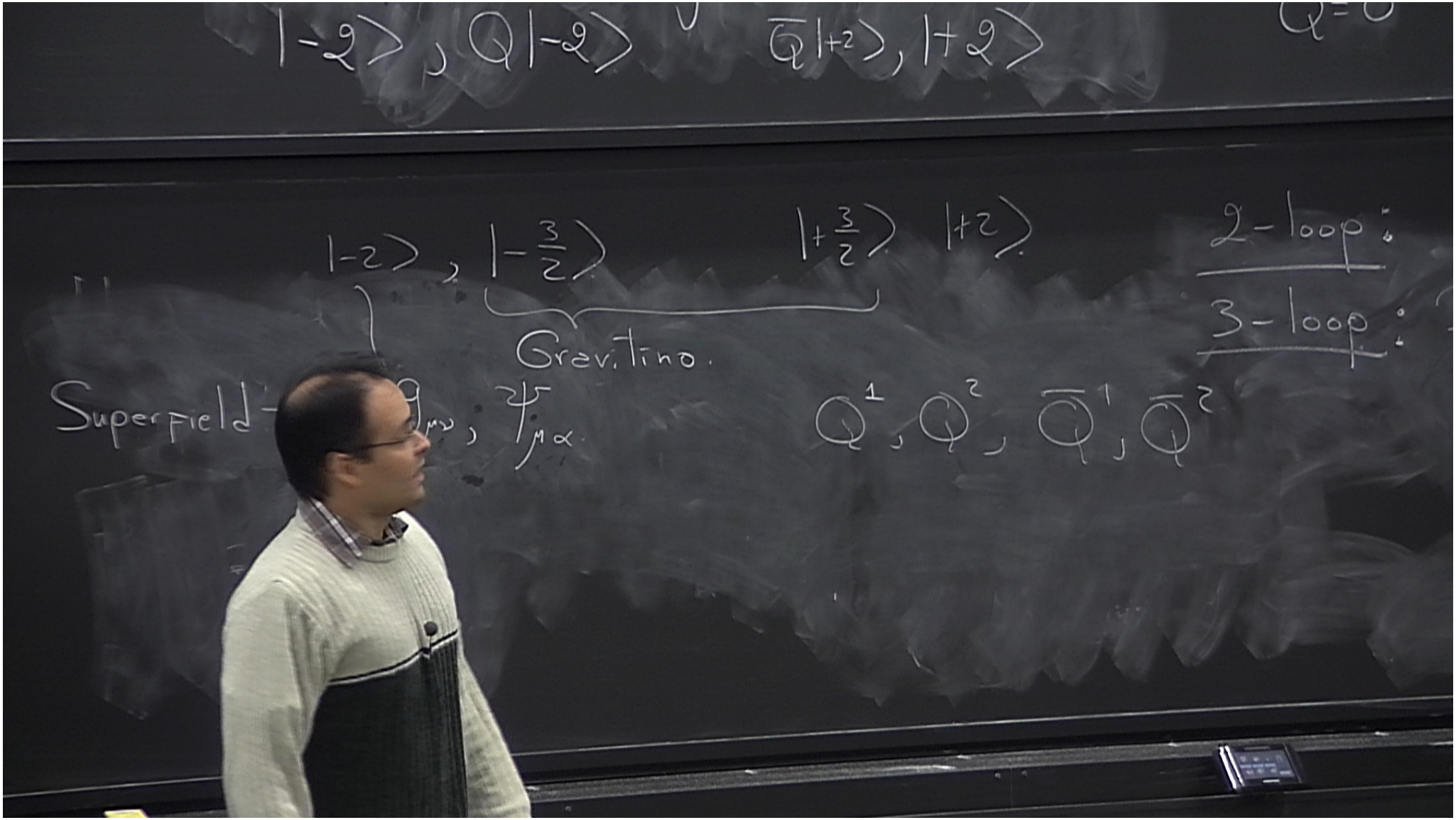
Superfield $\longrightarrow$ $g_{\mu\nu}$, $\psi_{\mu\alpha}$

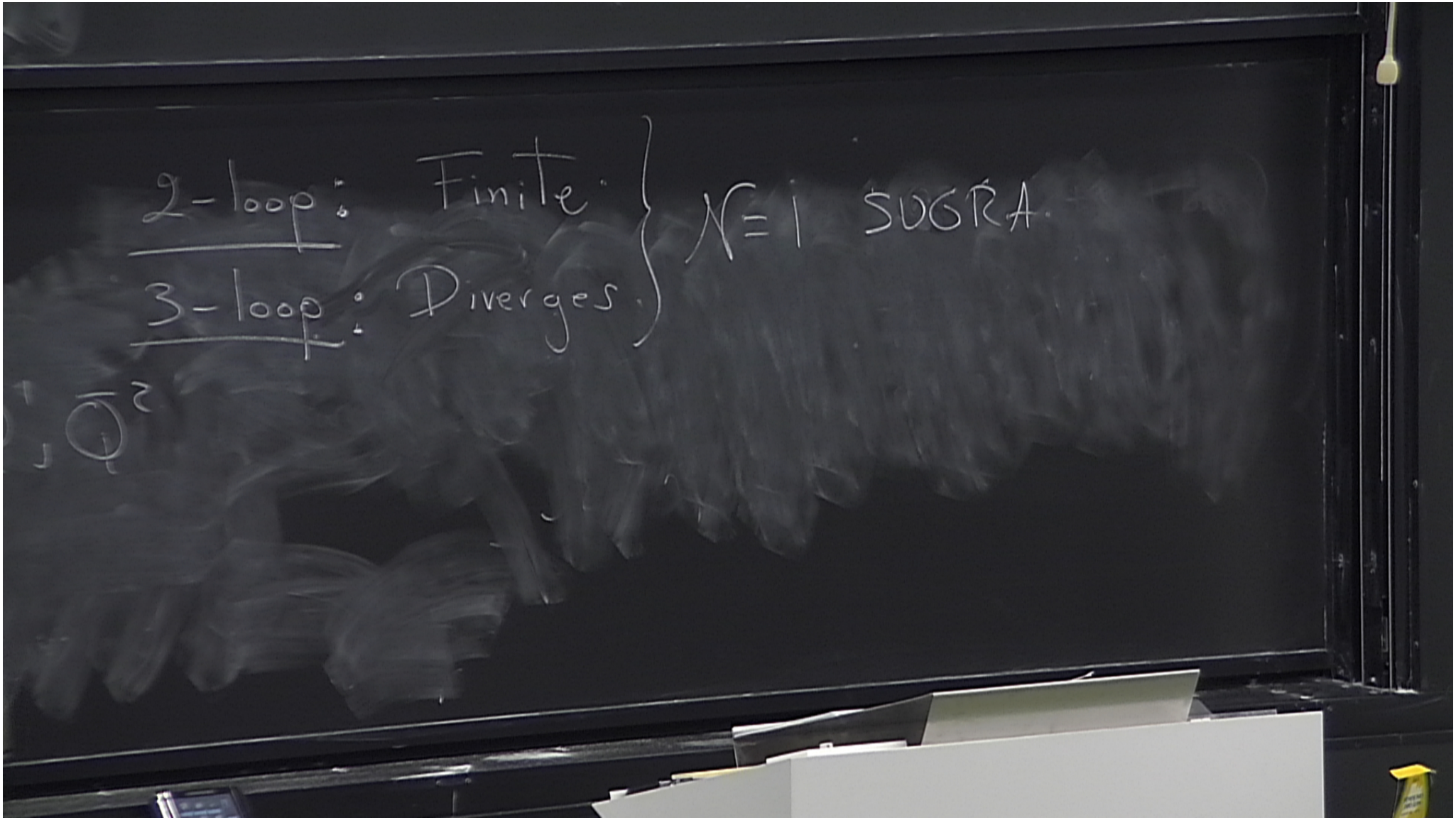


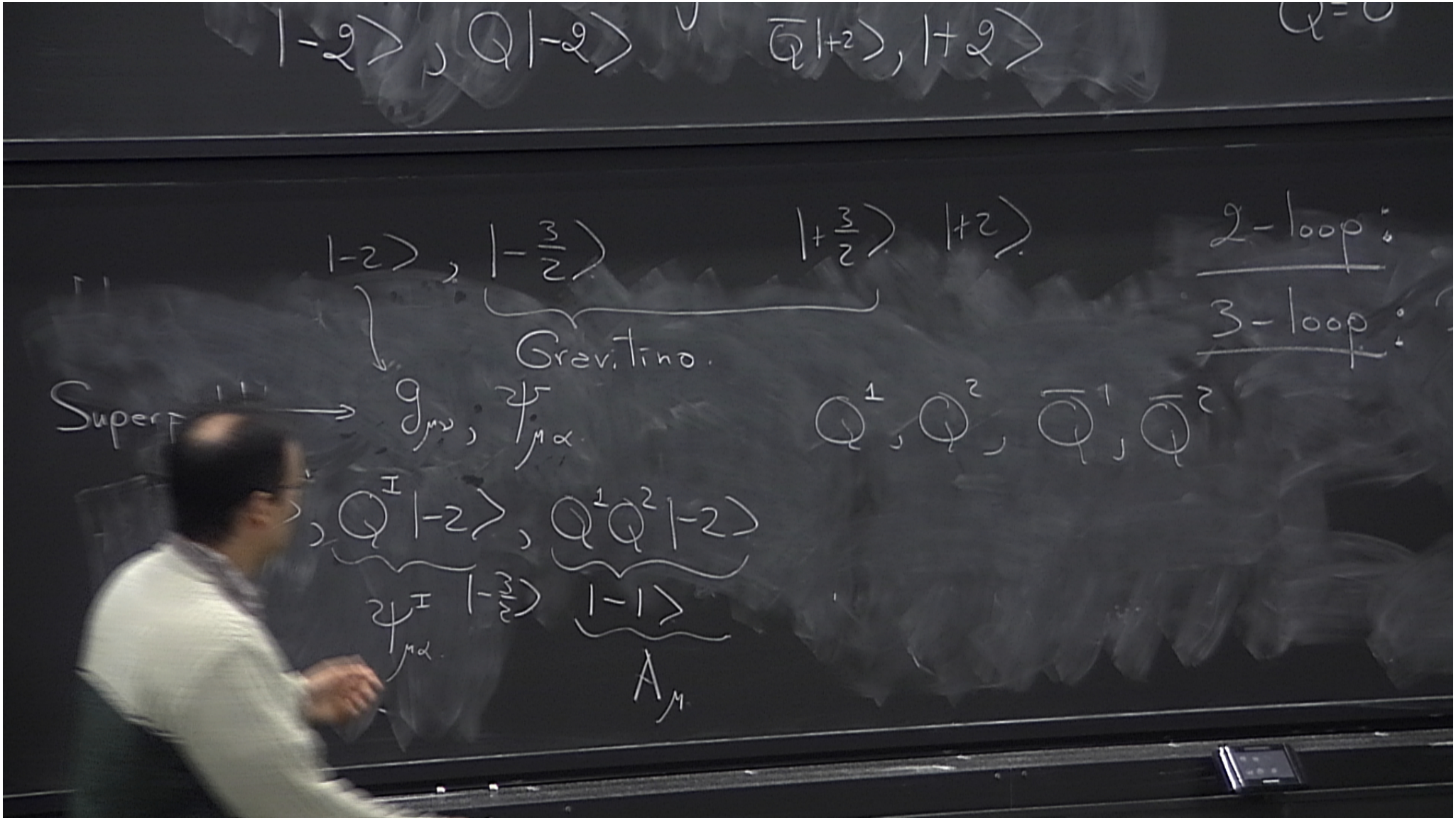

2-loop: Finite
3-loop:

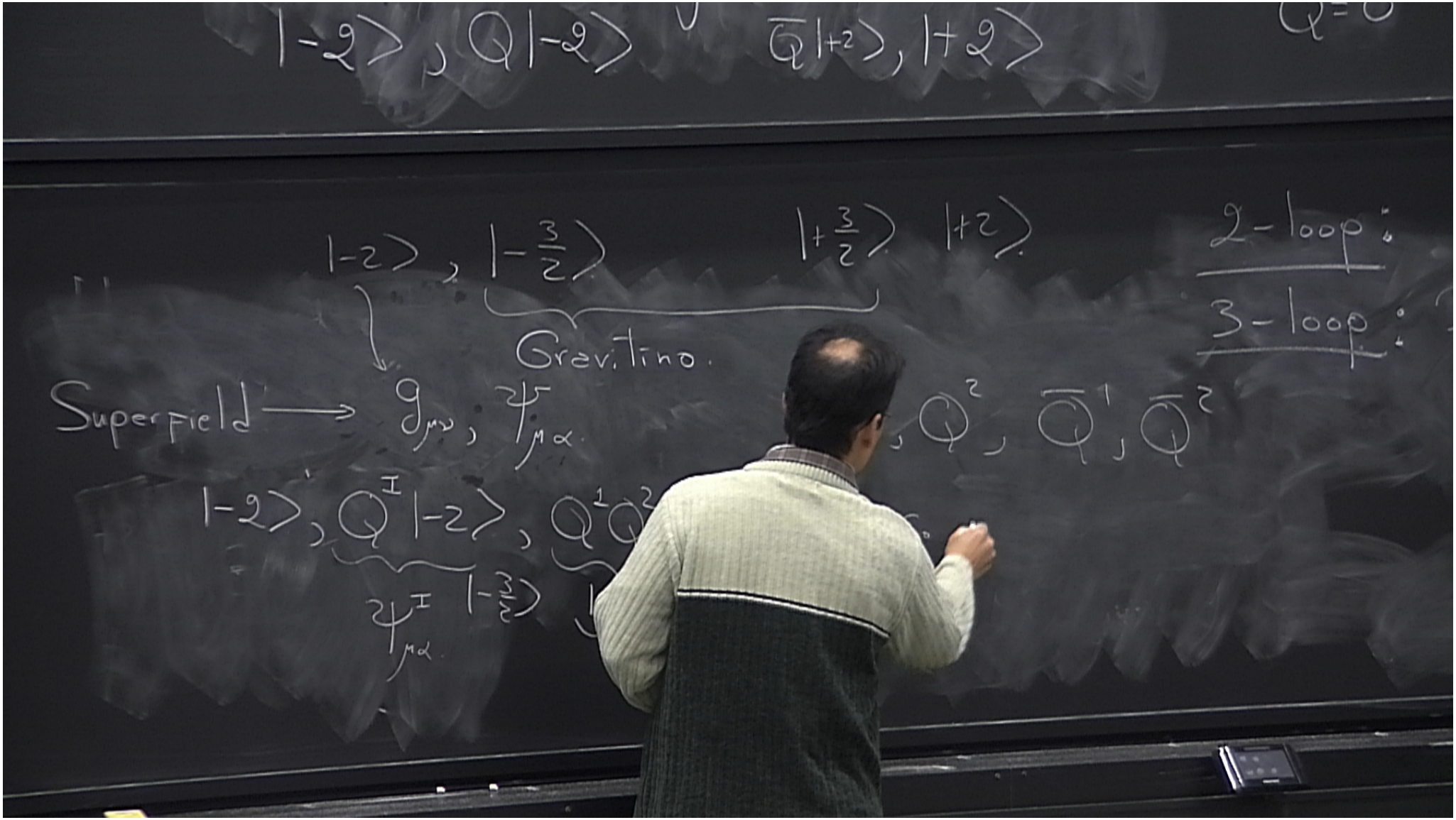

2-loop: Finite
3-loop: Diverges

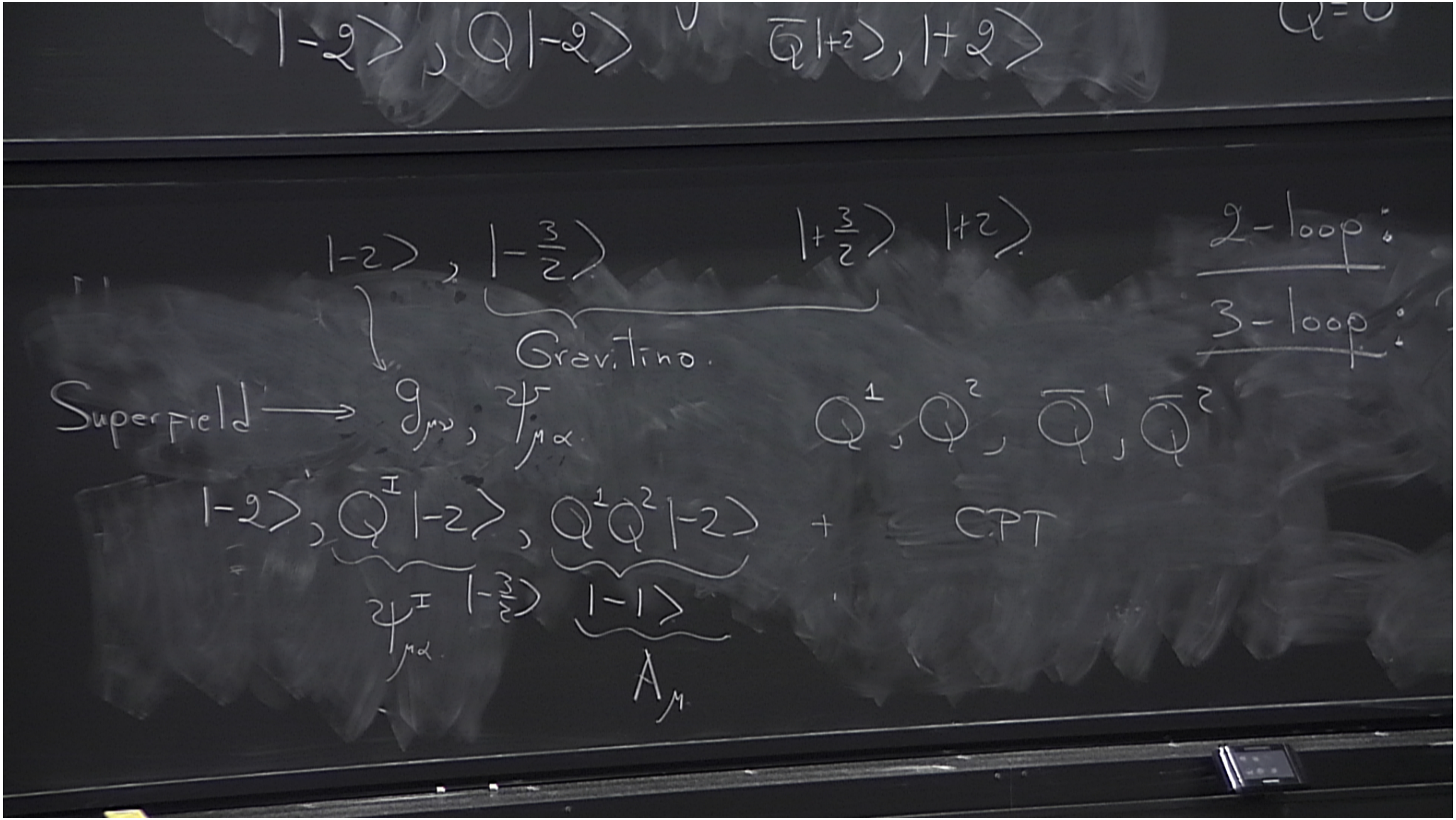

$Q^2 = 0$ $\bar{Q}^2 = 0$
3-loop: Diverges

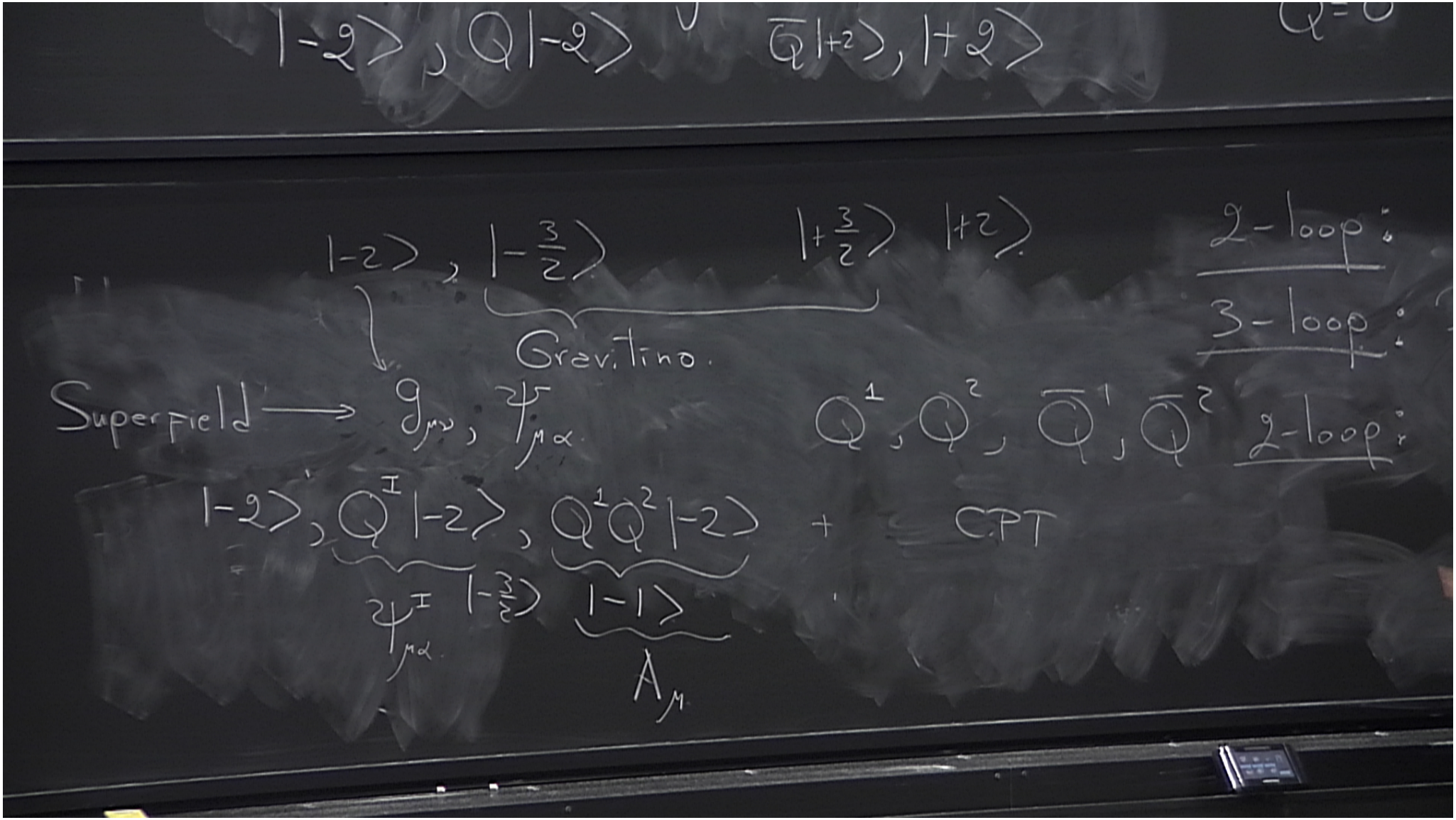

|-2>, Q|-2>
Q̄|+2>, |+2>
|-2>, |-3/2>
|+3/2> |+2>
Gravitino.
2-loop:
3-loop:
Superfield
Q¹, Q², Q̄¹, Q̄²

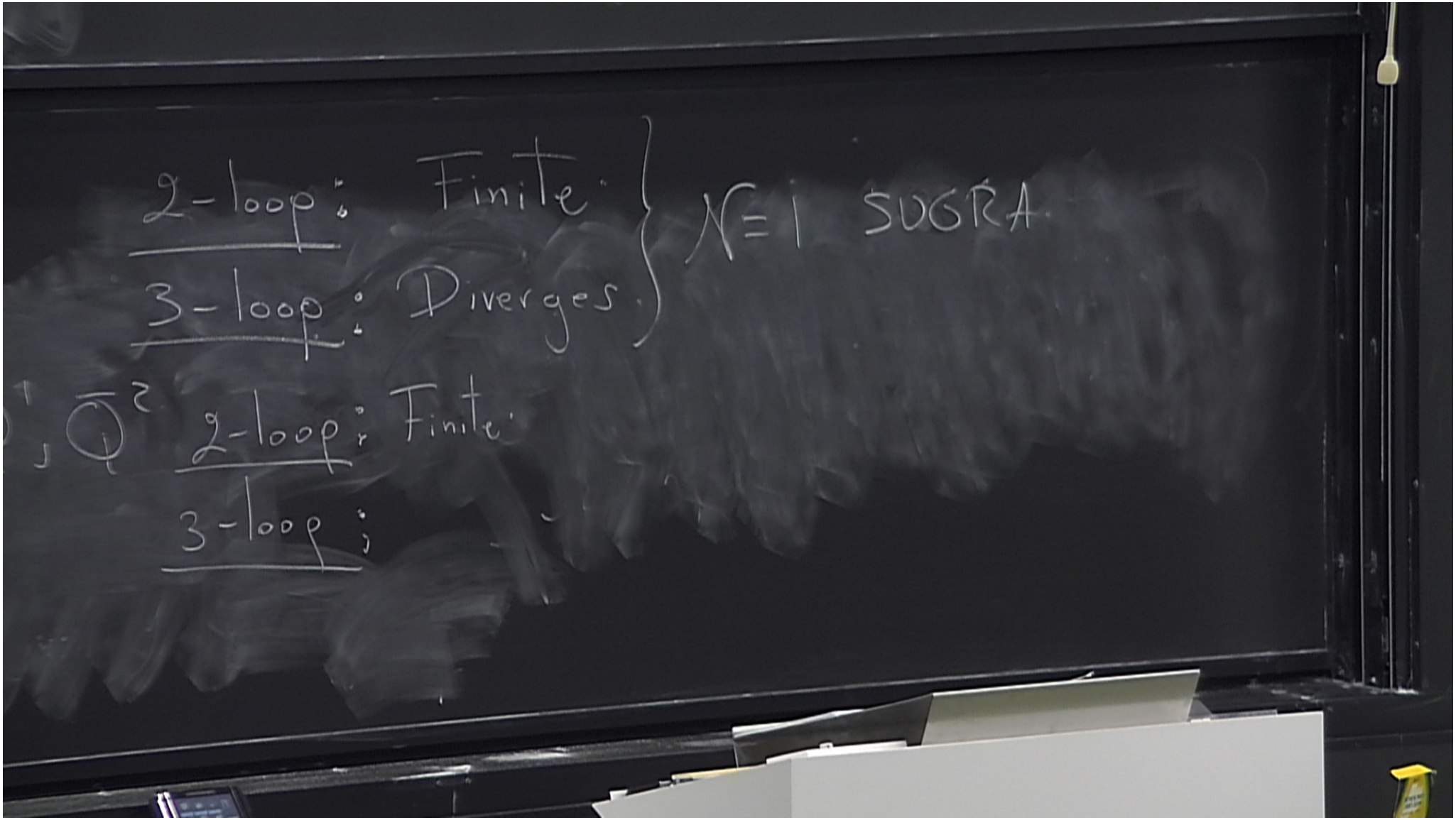

2-loop: Finite
3-loop: Diverges
$\mathcal{N}=1$ SUGRA
$\bar{Q}^2$

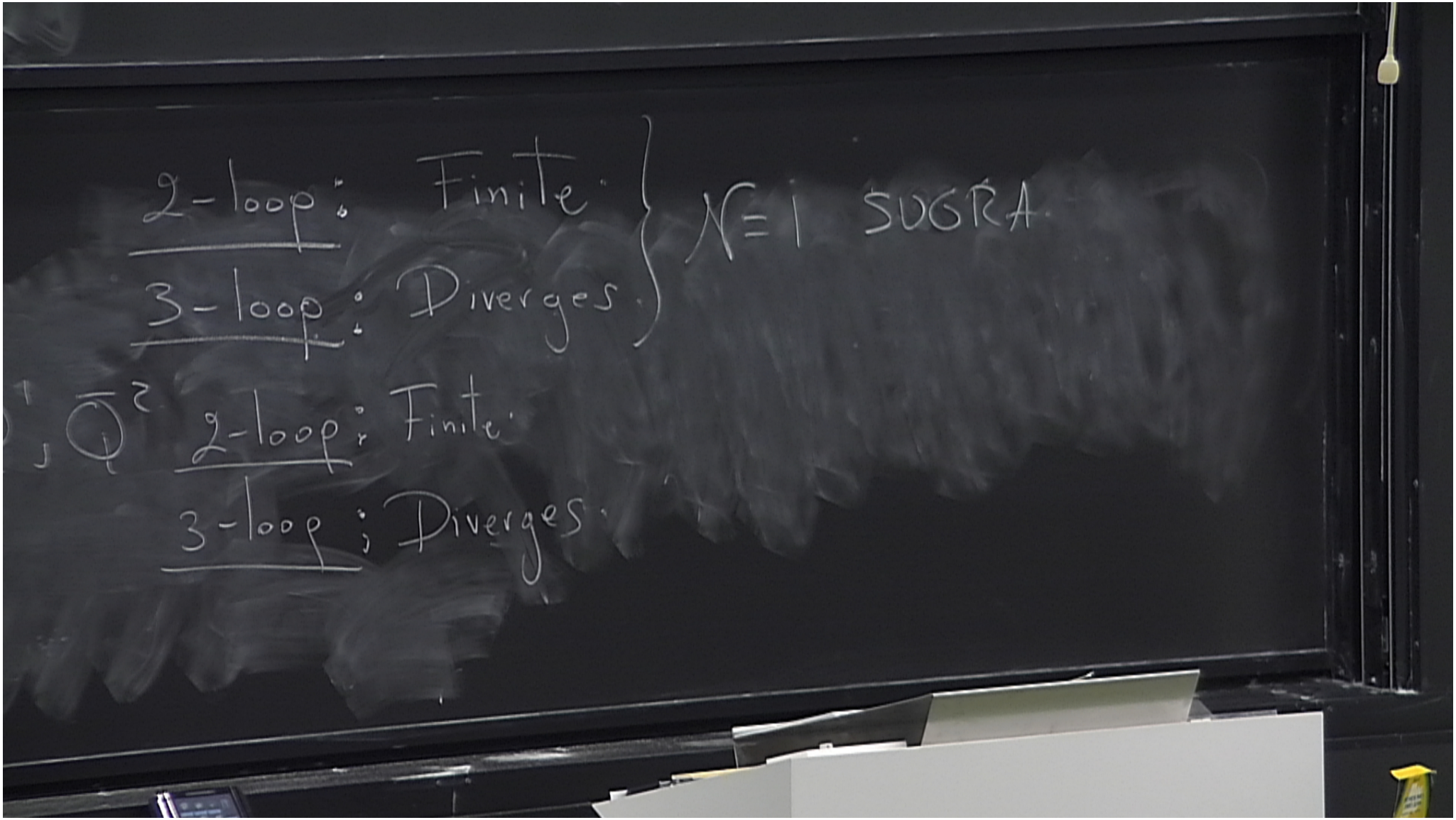

|-2>, Q|-2>
Q̄|+2>, |+2>
|-2>, |-3/2>
|+3/2> |+2>
Gravitino.
2-loop:
3-loop:
Super
g_μν, ψ_μα
Q^1, Q^2, Q̄^1, Q̄^2
Q^I|-2>, Q^1Q^2|-2>
ψ^I_μα |-3/2> |-1>
A_μ

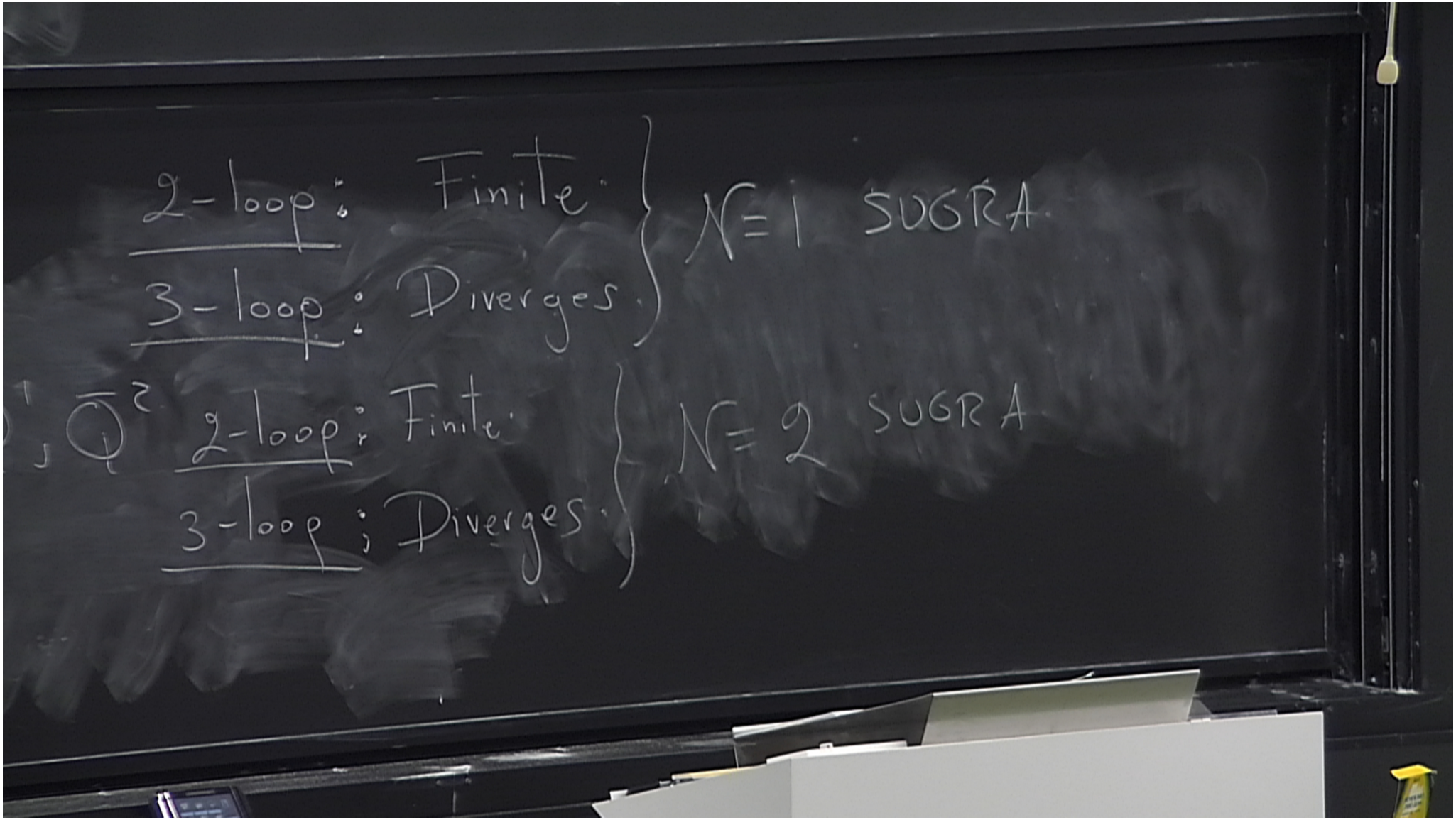

|-2>, Q|-2>
Q̄|+2>, |+2>
|-2>, |-3/2>
|+3/2>
|+2>
Gravitino.
2-loop:
3-loop:
Superfield ⟶ g_μν, ψ_μα
Q², Q̄¹, Q̄²
|-2>, Q^I|-2>, Q¹Q²
ψ^I_μα |-3/2>
Q=0

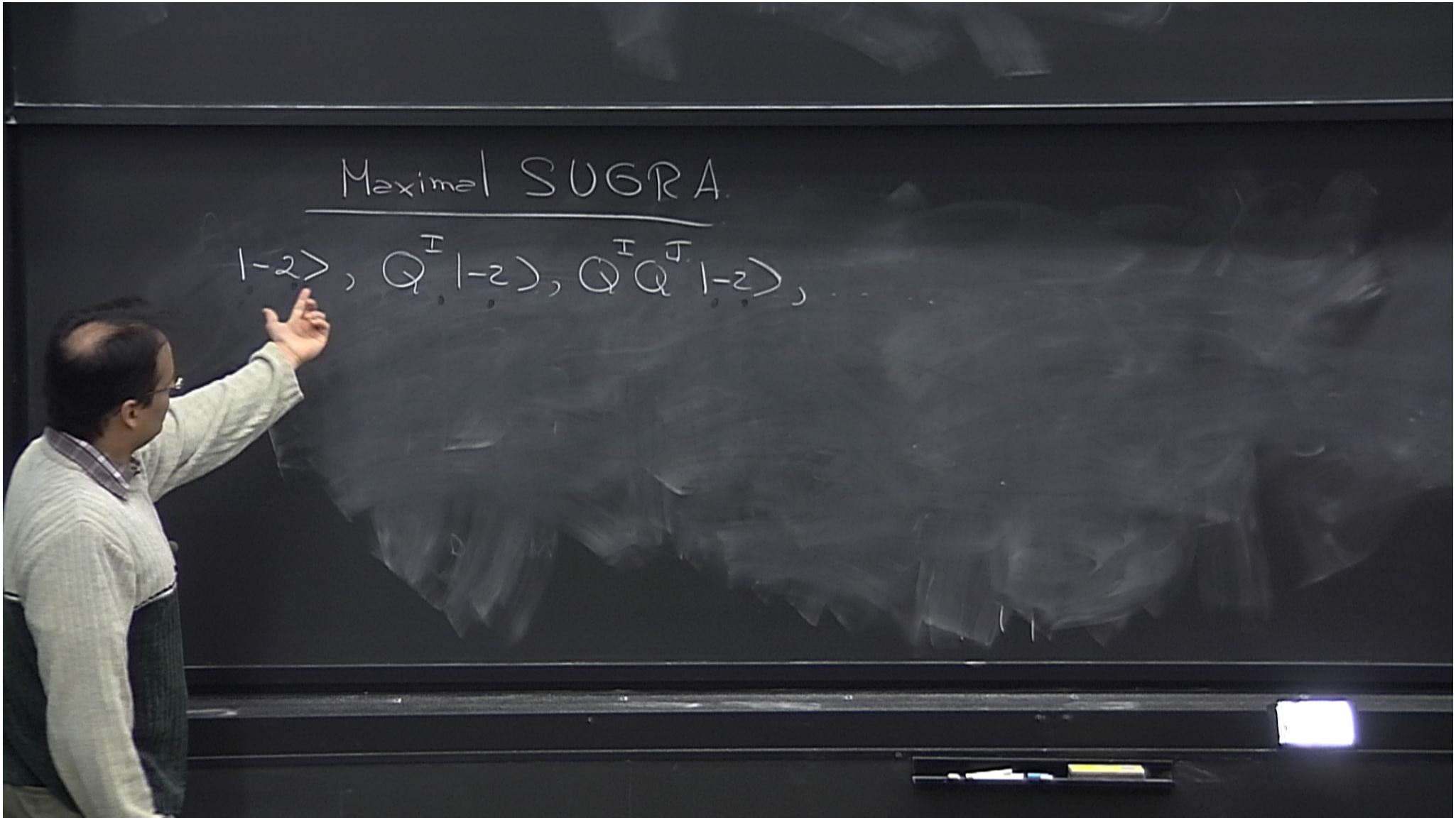

|-2>, Q|-2>
$\bar{Q}|+2\rangle$, |+2>
Q=0
|-2>, $|-\frac{3}{2}\rangle$
$|+\frac{3}{2}\rangle$ |+2>
Gravitino.
2-loop:
3-loop:
Superfield ⟶ $g_{\mu\nu}$, $\psi_{\mu\alpha}$
$Q^1, Q^2, \bar{Q}^1, \bar{Q}^2$
|-2>, $Q^I|-2\rangle$, $Q^1Q^2|-2\rangle$ + CPT
$\psi^I_{\mu\alpha}$ $|-\frac{3}{2}\rangle$
|-1>
A_μ

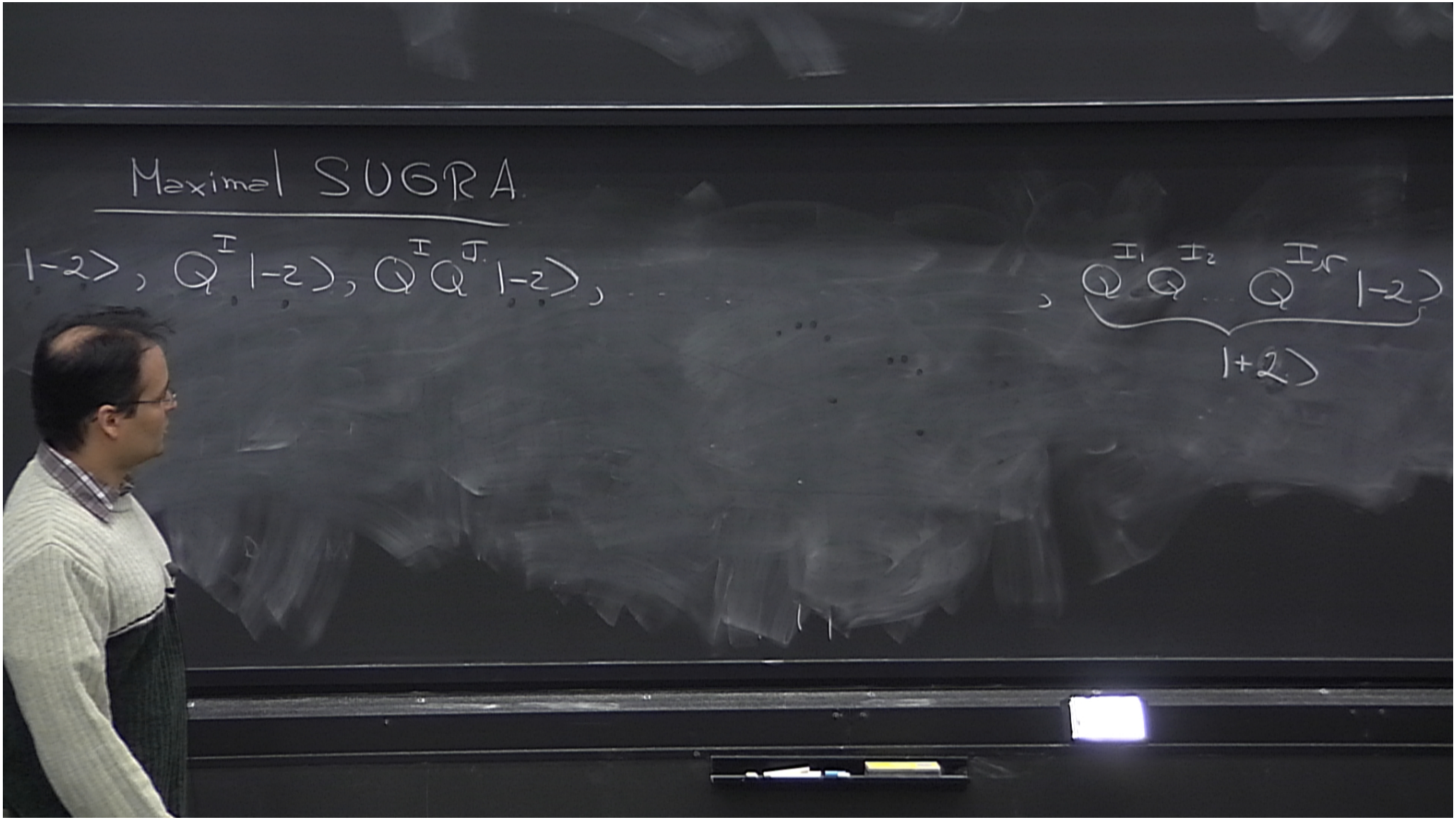

|-2>, Q|-2>
$\bar{Q}|+2\rangle$, |+2>
Q=0
|-2>, $|-\frac{3}{2}\rangle$
$|+\frac{3}{2}\rangle$ |+2>
Gravitino.
2-loop:
3-loop:
Superfield ⟶ $g_{\mu\nu}$, $\psi_{\mu\alpha}$
$Q^1, Q^2, \bar{Q}^1, \bar{Q}^2$ 2-loop:
|-2>, $Q^I|-2\rangle$, $Q^1Q^2|-2\rangle$ + C.P.T
$\psi^I_{\mu\alpha}$ $|-\frac{3}{2}\rangle$ |-1>
A_μ

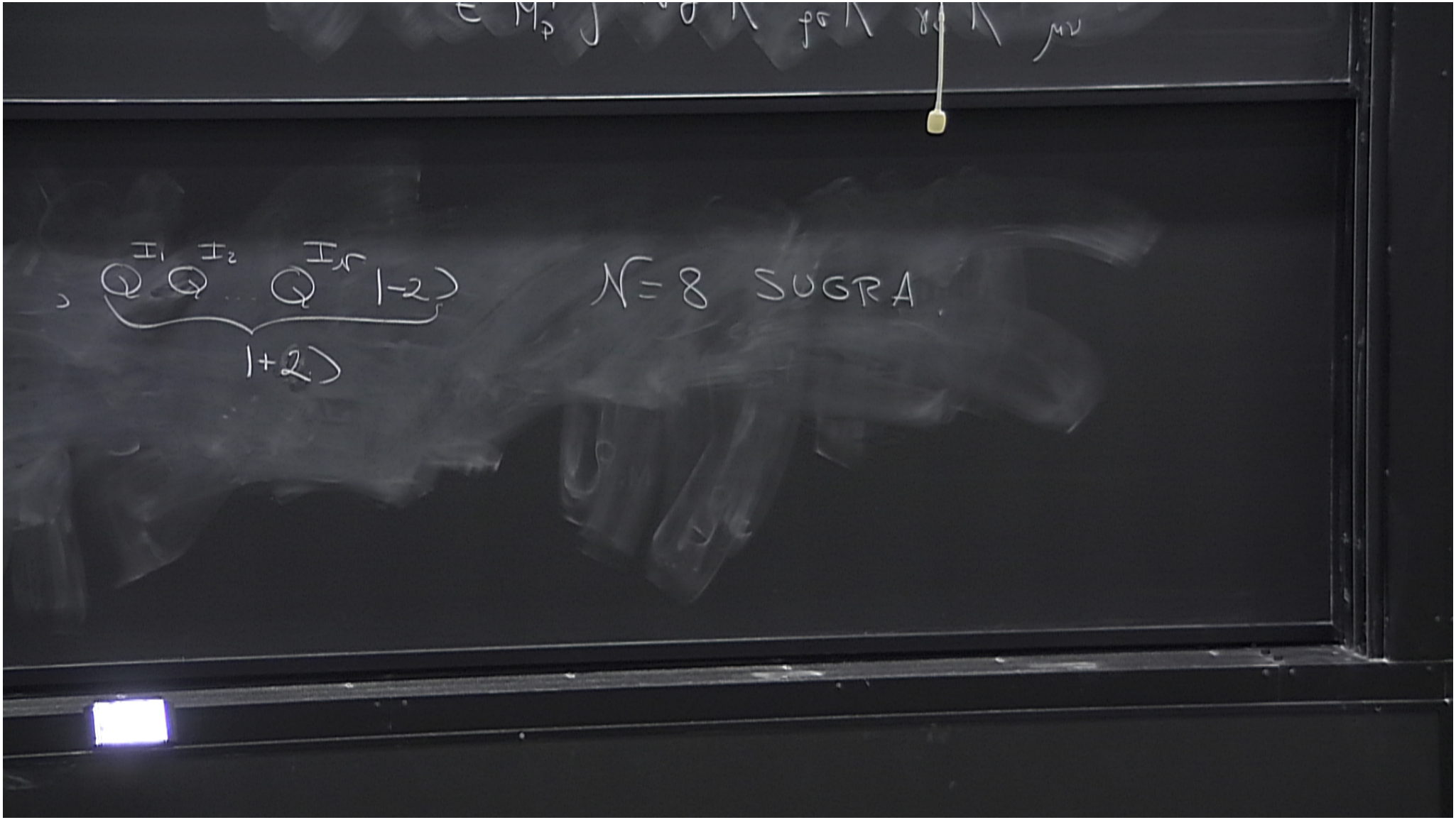

2-loop: Finite
3-loop: Diverges
$\mathcal{N}=1$ SUGRA
$\bar{Q}^2$
2-loop: Finite
3-loop:

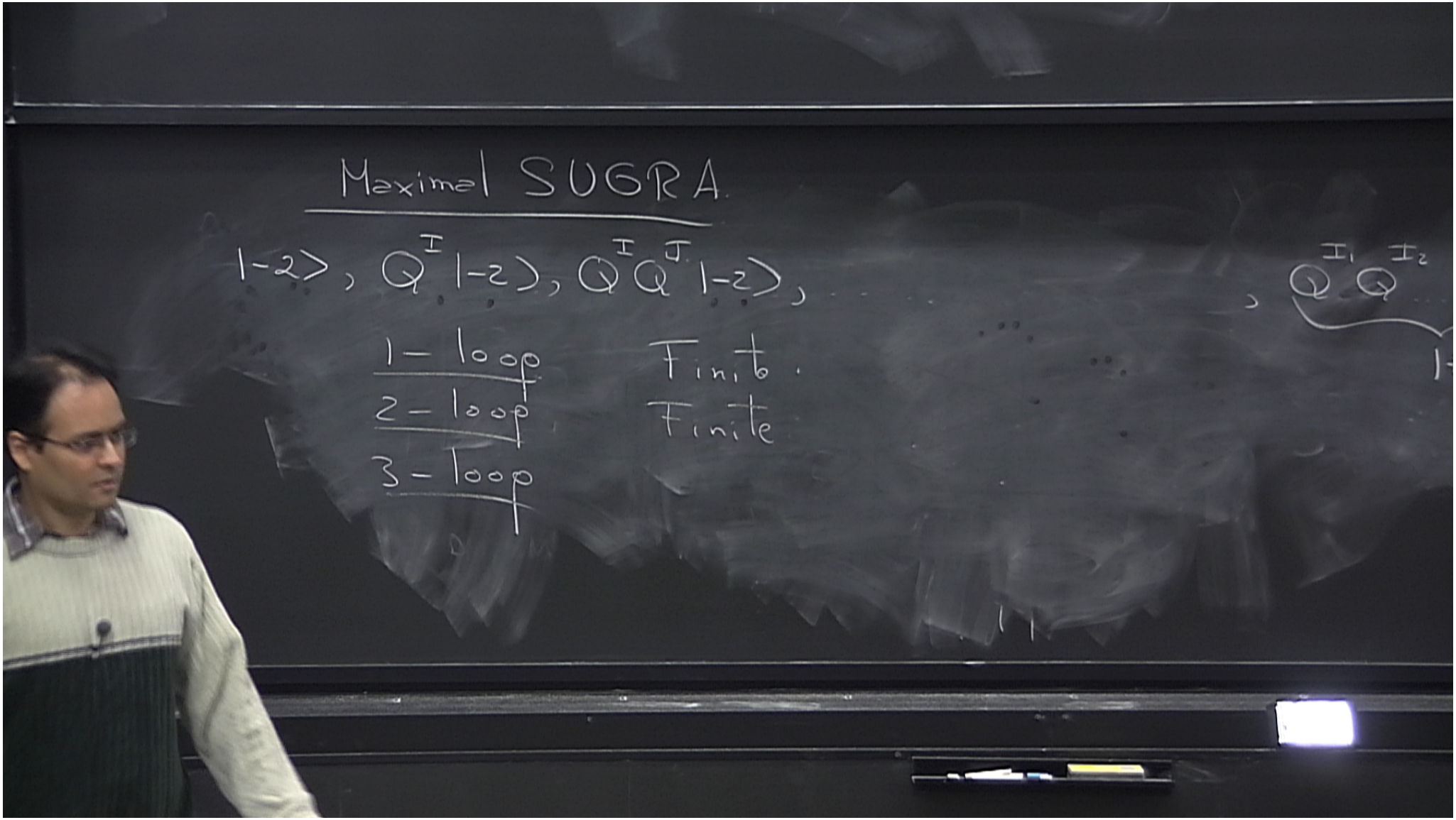

2-loop: Finite
3-loop: Diverges
} N=1 SUGRA
$\bar{Q}^2$
2-loop: Finite
3-loop; Diverges

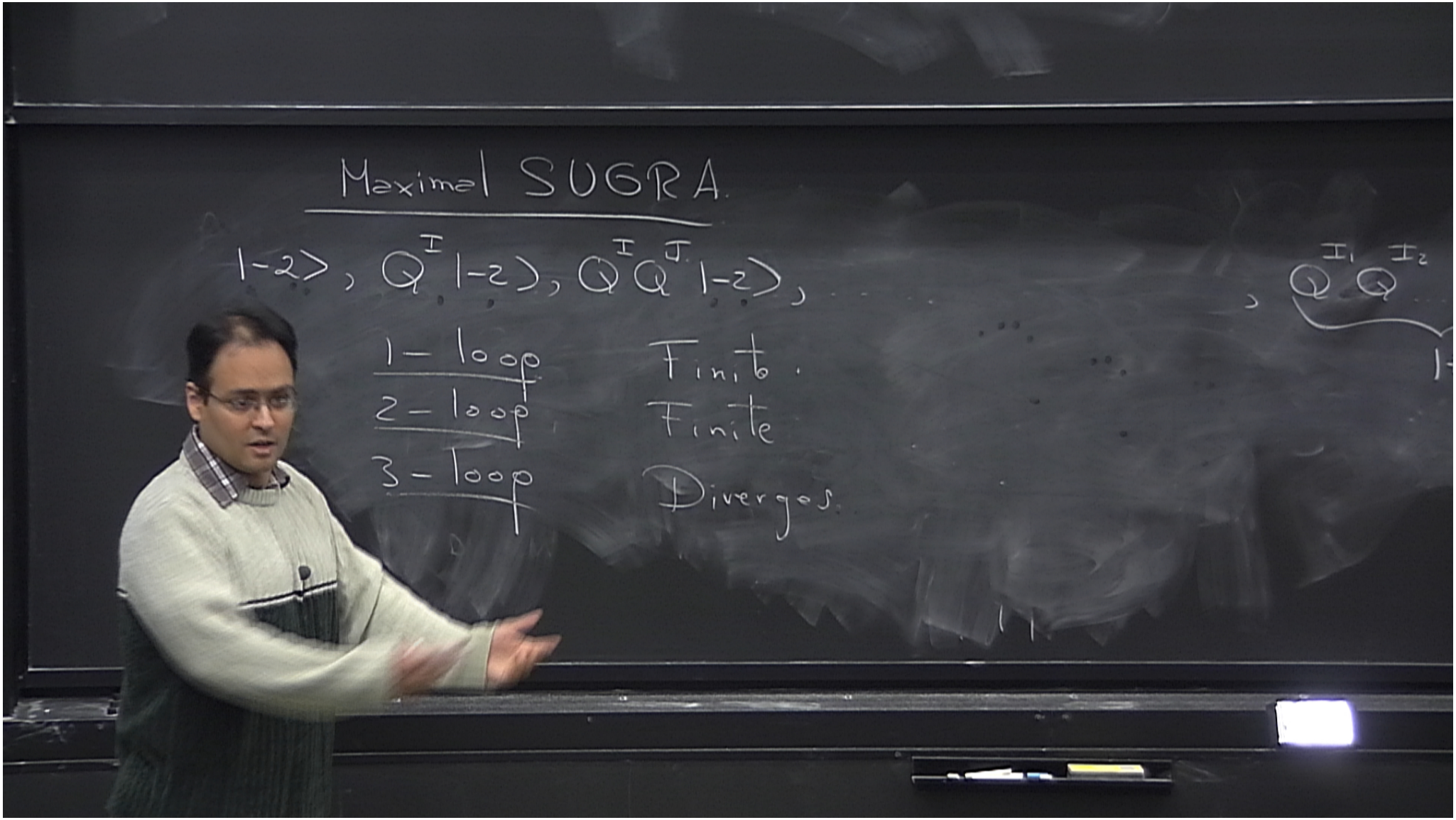

2-loop: Finite
3-loop: Diverges
$\mathcal{N}=1$ SUGRA
$\bar{Q}^2$
2-loop: Finite
3-loop: Diverges
$\mathcal{N}=2$ SUGRA

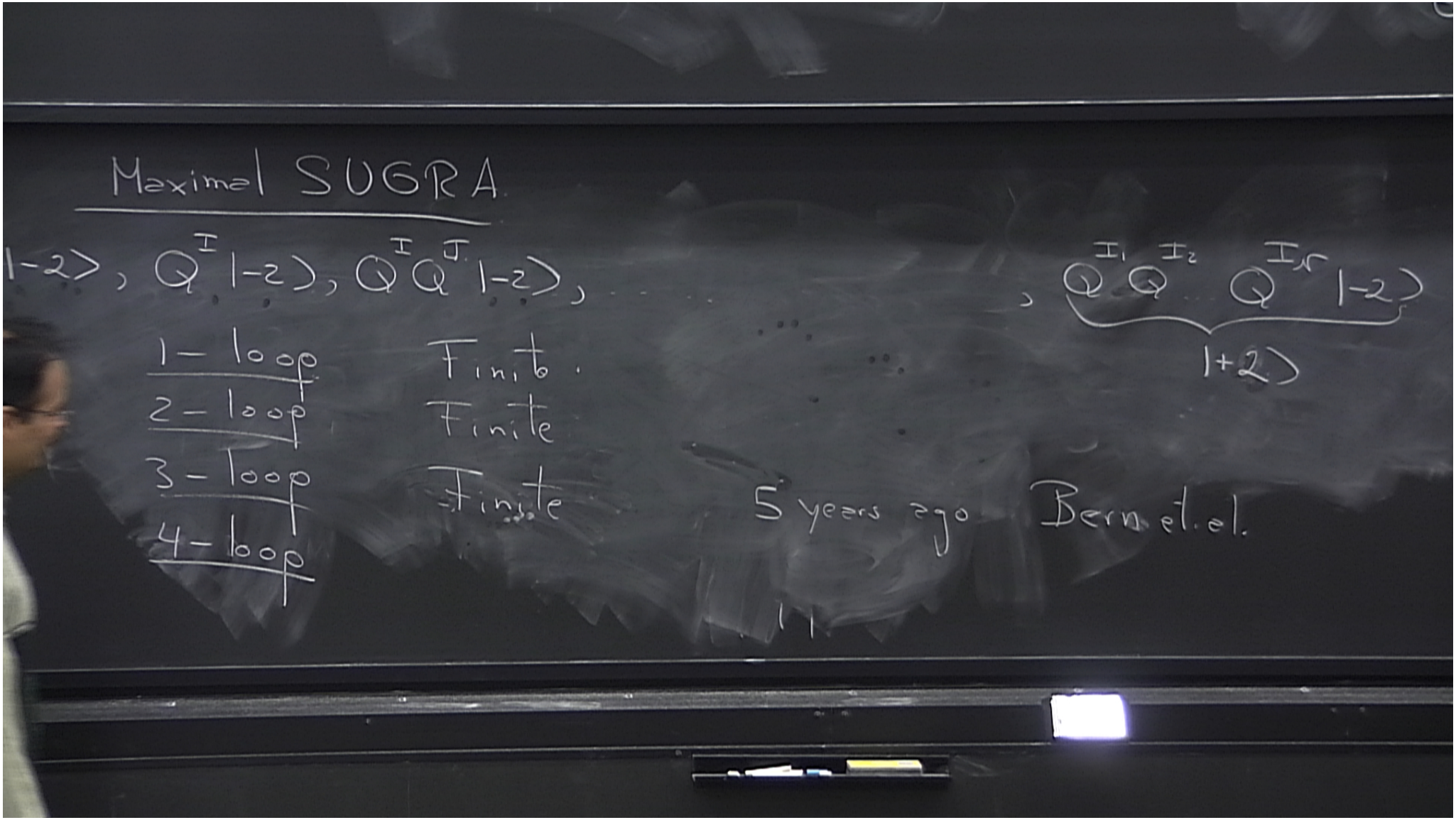

Maximal SUGRA
|-2>, Q^I |-2>, Q^I Q^J |-2>,


Maximal SUGRA
|-2>, Q^I |-2>, Q^I Q^J |-2>, ... , Q^{I1} Q^{I2} ... Q^{Ir} |-2>
|+2>


$Q^{I_1} Q^{I_2} \cdots Q^{I_r} |-2\rangle$
$|+2\rangle$
$\mathcal{N}=8$ SUGRA


Maximal SUGRA
|-2>, Q^I |-2>, Q^I Q^J |-2>,
, Q^{I1} Q^{I2}
1-loop Finite.
2-loop Finite
3-loop


Maximal SUGRA
|-2>, Q^I |-2>, Q^I Q^J |-2>,
, Q^{I1} Q^{I2}
1-loop Finite.
2-loop Finite
3-loop Diverges.


Maximal SUGRA
$|-2\rangle$, $Q^I|-2\rangle$, $Q^I Q^J|-2\rangle$, ... , $Q^{I_1} Q^{I_2} \cdots Q^{I_r}|-2\rangle$
$|+2\rangle$
1-loop Finite.
2-loop Finite
3-loop Finite
4-loop
5 years ago Bern et. el.

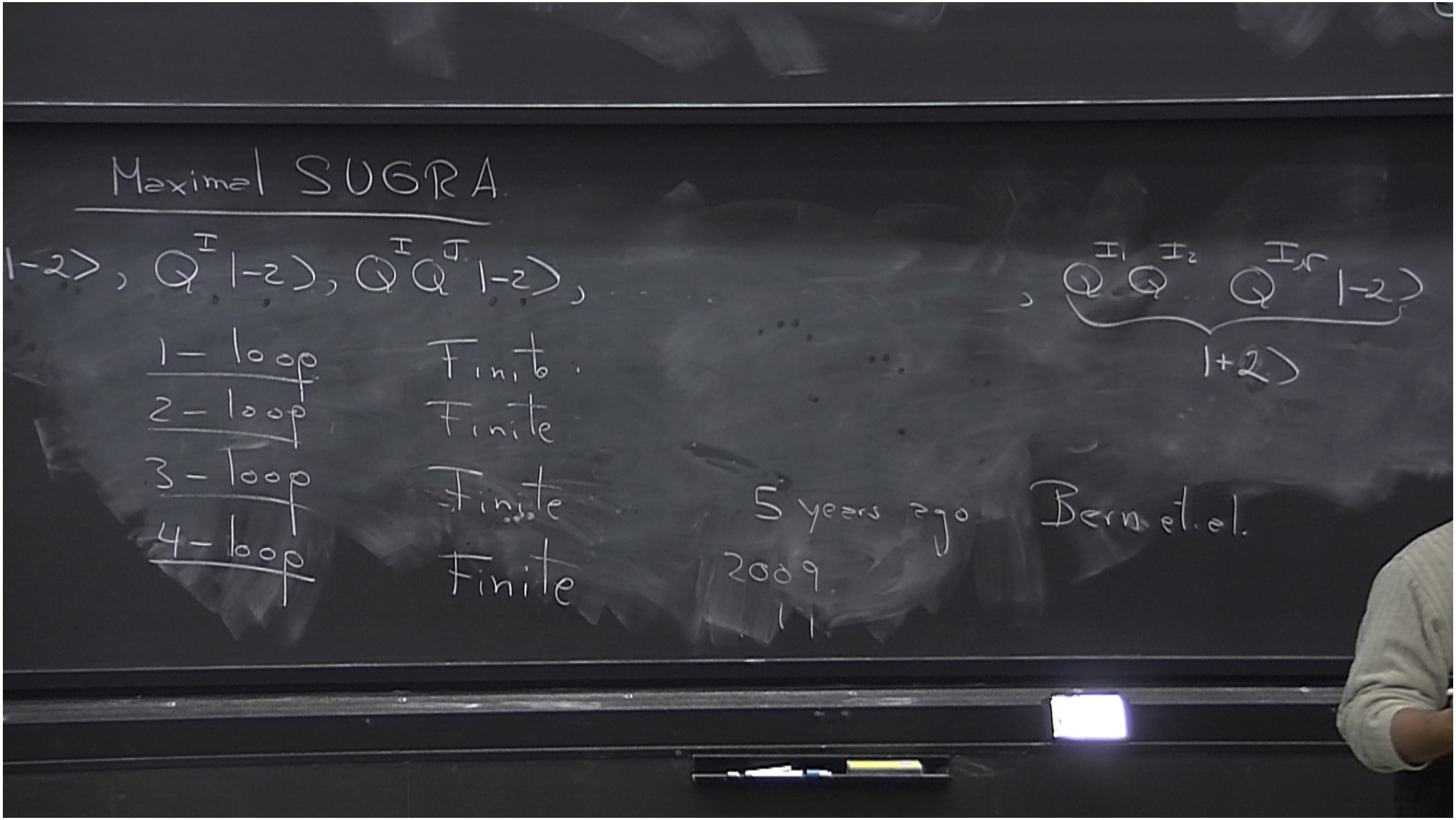
Maximal SUGRA

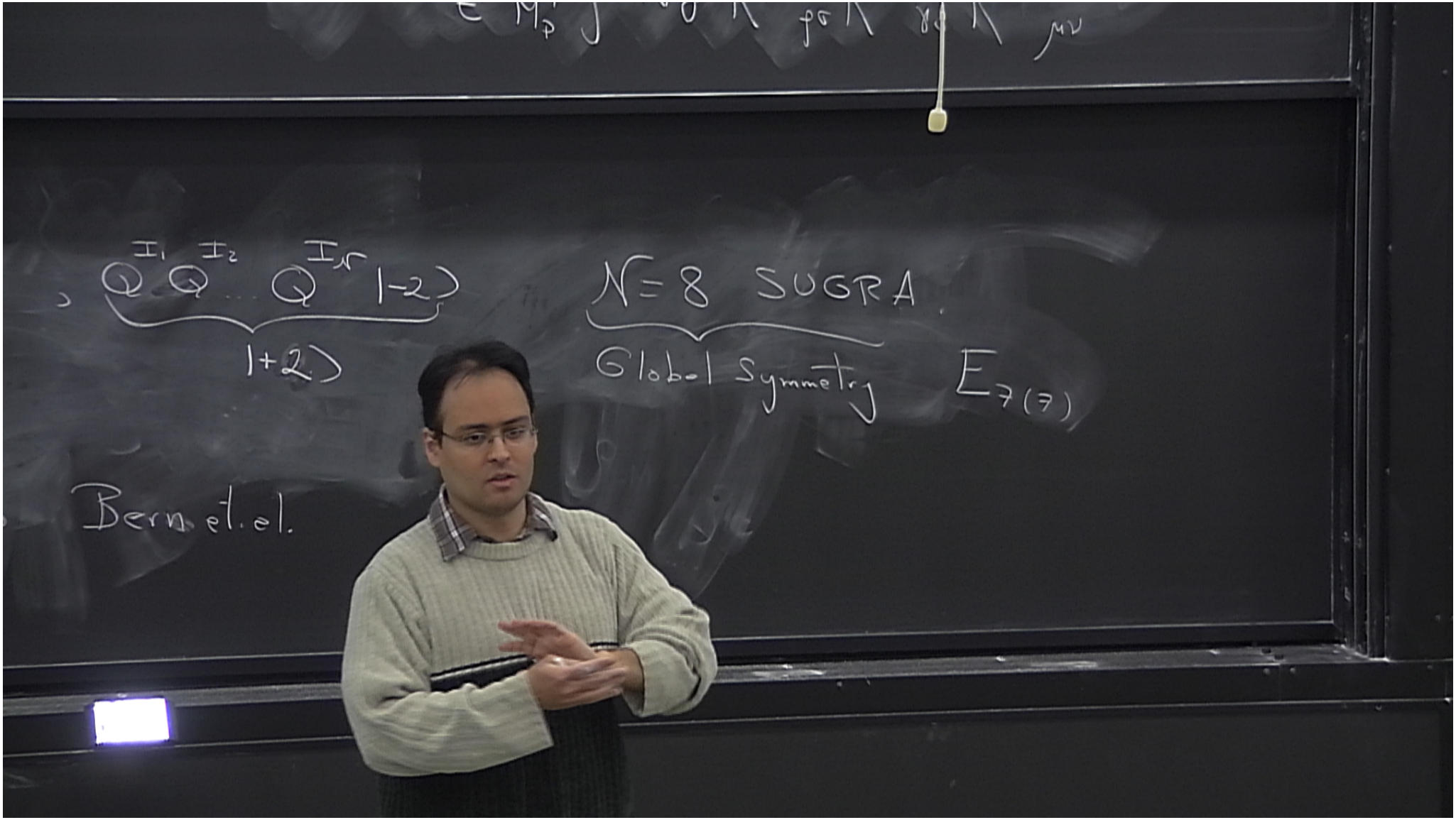
$|-2\rangle$, $Q^I|-2\rangle$, $Q^I Q^J|-2\rangle$, ... , $\underbrace{Q^{I_1} Q^{I_2} \cdots Q^{I_r}|-2\rangle}_{|+2\rangle}$

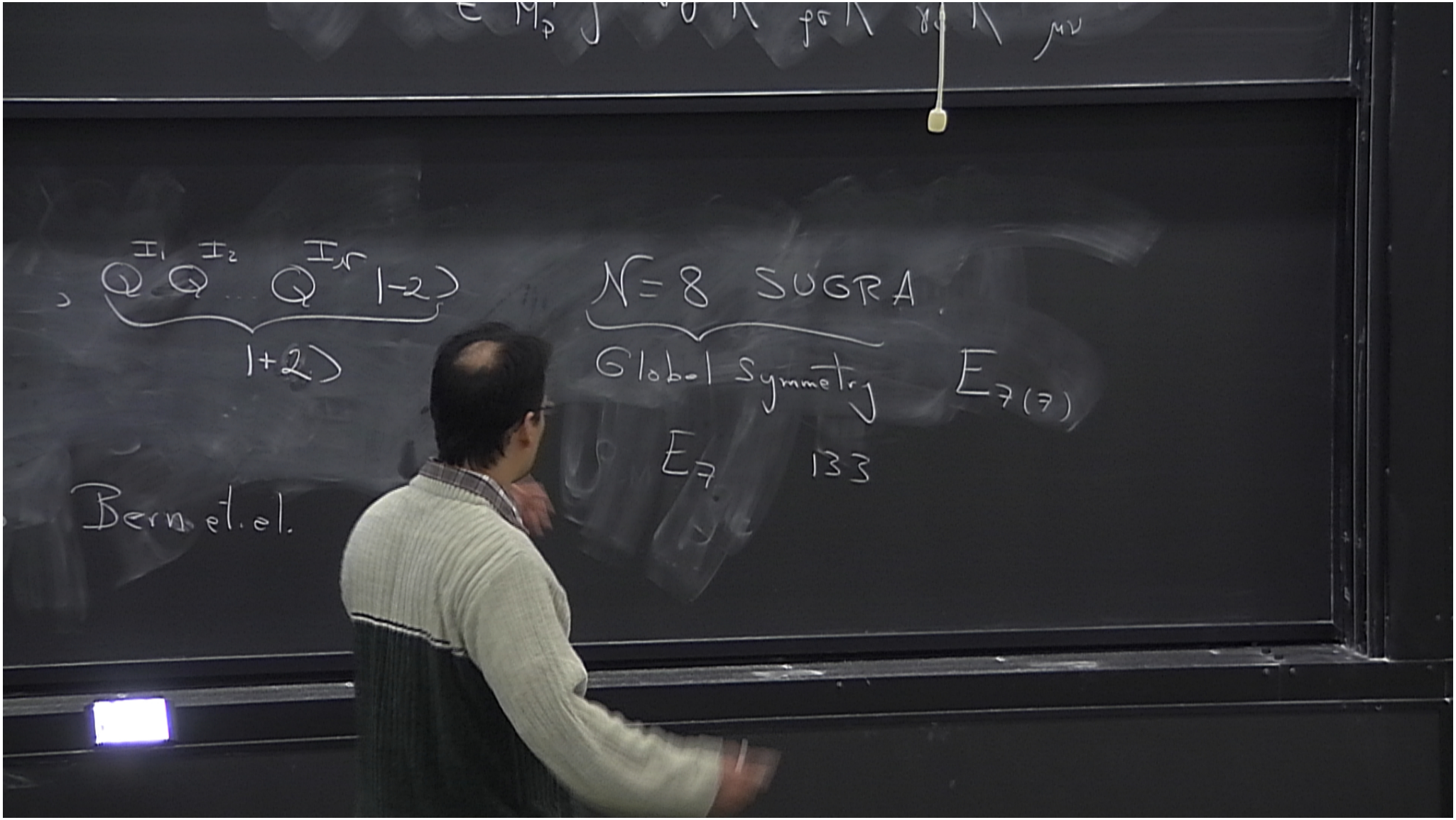
1-loop	Finite
2-loop	Finite
3-loop	Finite
4-loop	Finite

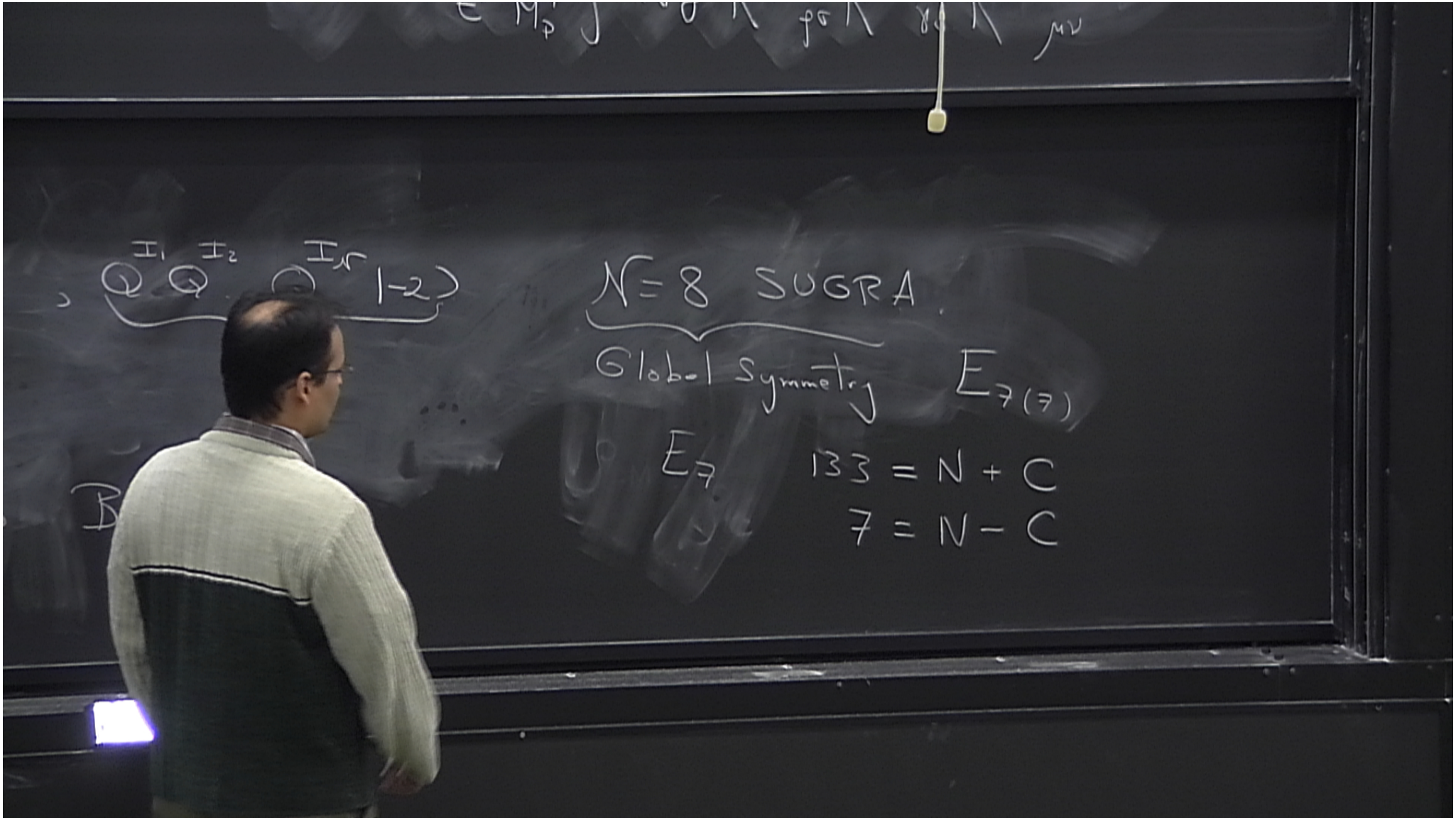
5 years ago Bern et. el.

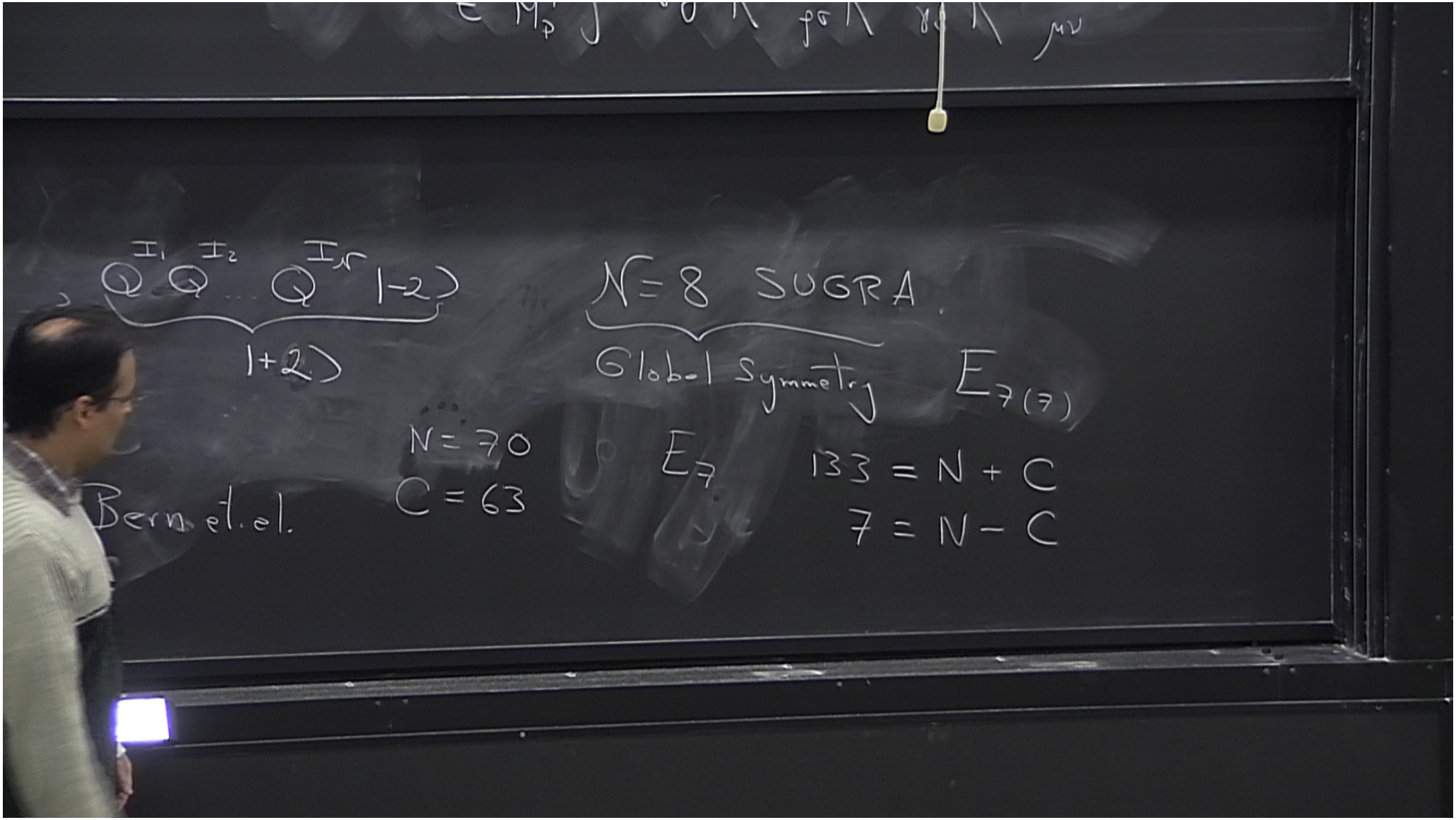
2009


Maximal SUGRA
$|-2\rangle$, $Q^I|-2\rangle$, $Q^I Q^J|-2\rangle$, ..., $Q^{I_1} Q^{I_2} \cdots Q^{I_r}|-2\rangle$
$|+2\rangle$
1-loop Finite
2-loop Finite
3-loop Finite
4-loop Finite
5 years ago Bern et.el.
2009


$Q^{I_1} Q^{I_2} \cdots Q^{I_r} |-2\rangle$
$|+2\rangle$
$\mathcal{N}=8$ SUGRA
Global Symmetry $E_{7(7)}$
Bern et. el.


$Q^{I_1} Q^{I_2} \cdots Q^{I_r} |-2\rangle$
$|+2\rangle$
$\mathcal{N}=8$ SUGRA
Global Symmetry $E_{7(7)}$
E_7 133
Bern et. el.


I_1 I_2 I_r
Q Q Q $|-2\rangle$
$\mathcal{N}=8$ SUGRA
Global Symmetry $E_{7(7)}$
E_7 $133 = N + C$
$7 = N - C$


$Q^{I_1} Q^{I_2} \cdots Q^{I_r} |-2\rangle$
$|+2\rangle$
$\mathcal{N}=8$ SUGRA
Global Symmetry $E_{7(7)}$
$N=70$
$C=63$
E_7
$133 = N + C$
$7 = N - C$
Bern et. el.

$Q^{I_1} Q^{I_2} \cdots Q^{I_r} |-2\rangle$

$|1+2\rangle$

$\mathcal{N}=8$ SUGRA

Global Symmetry $E_{7(7)}$

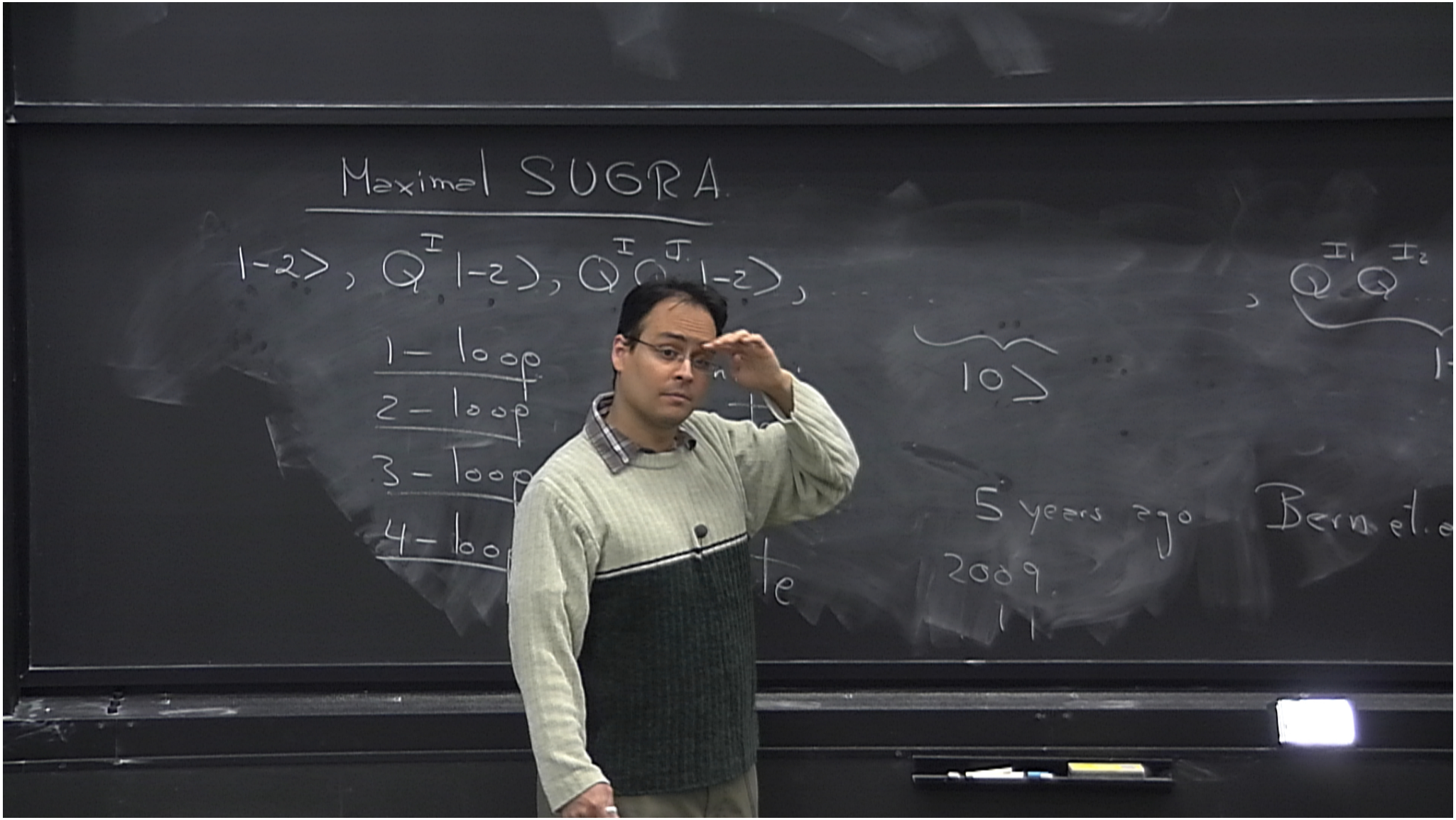
$N=70$

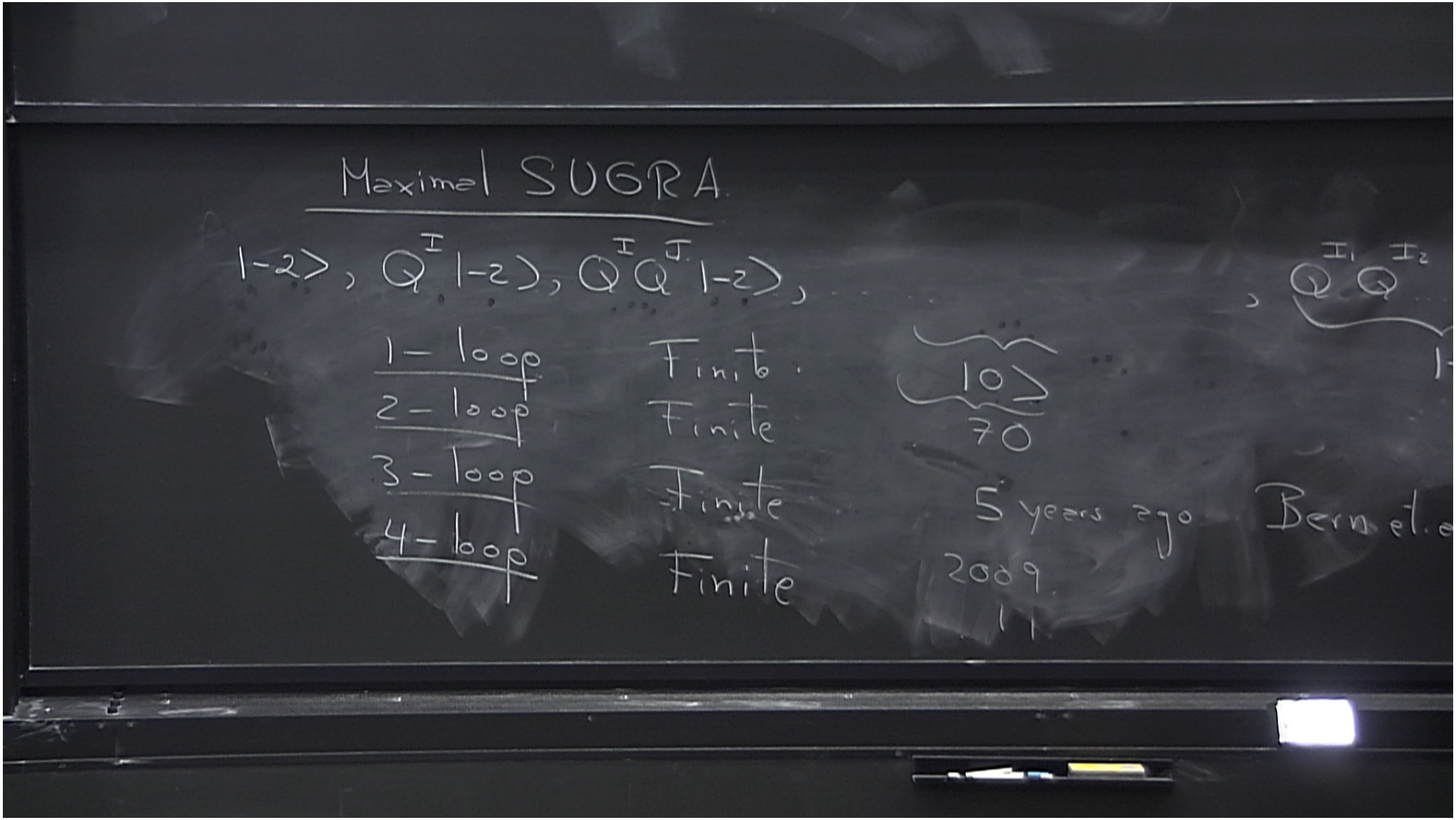
$C = 63 = SU(8)^C$ $E_{7(7)}$

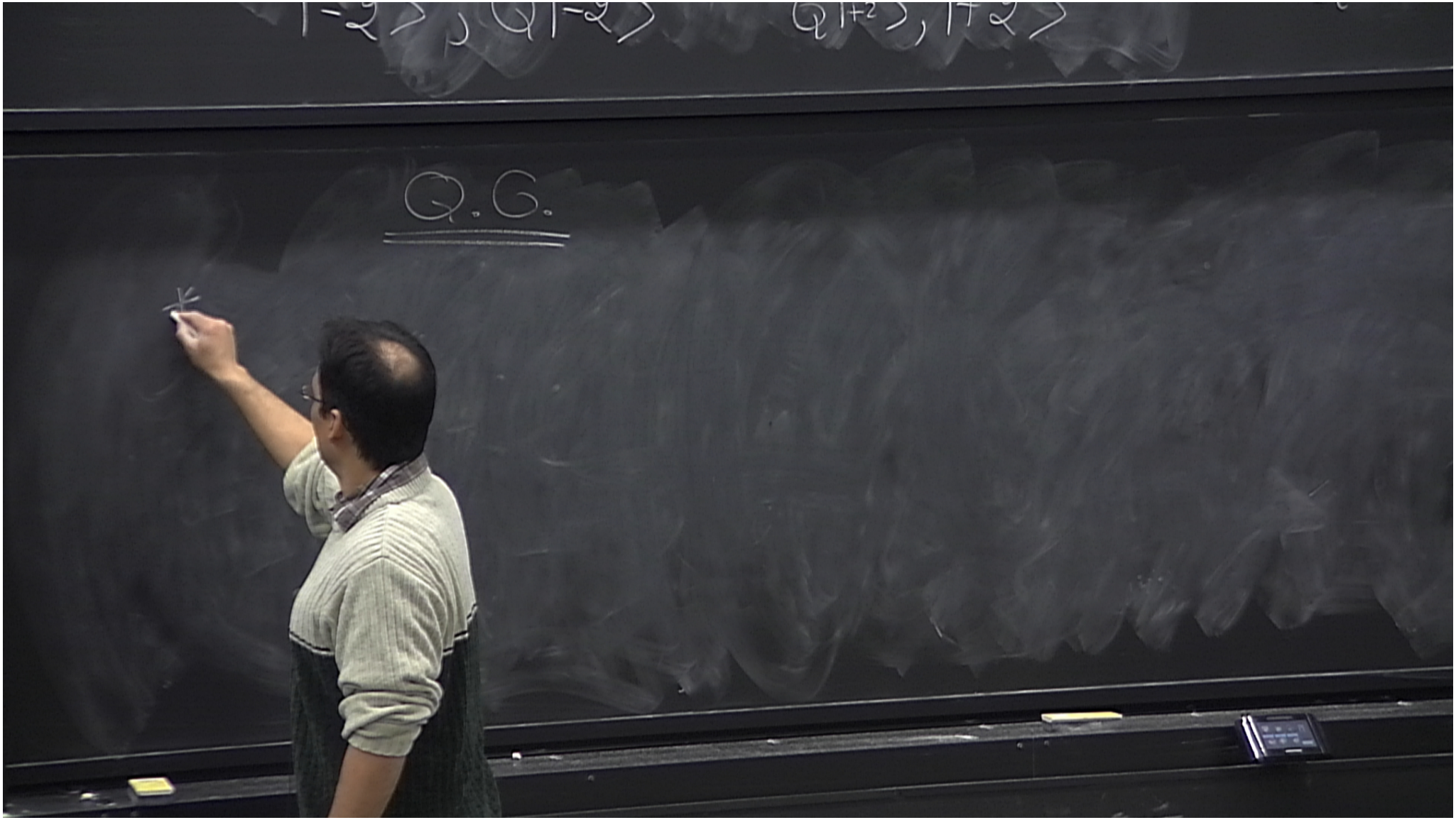
$133 = N + C$

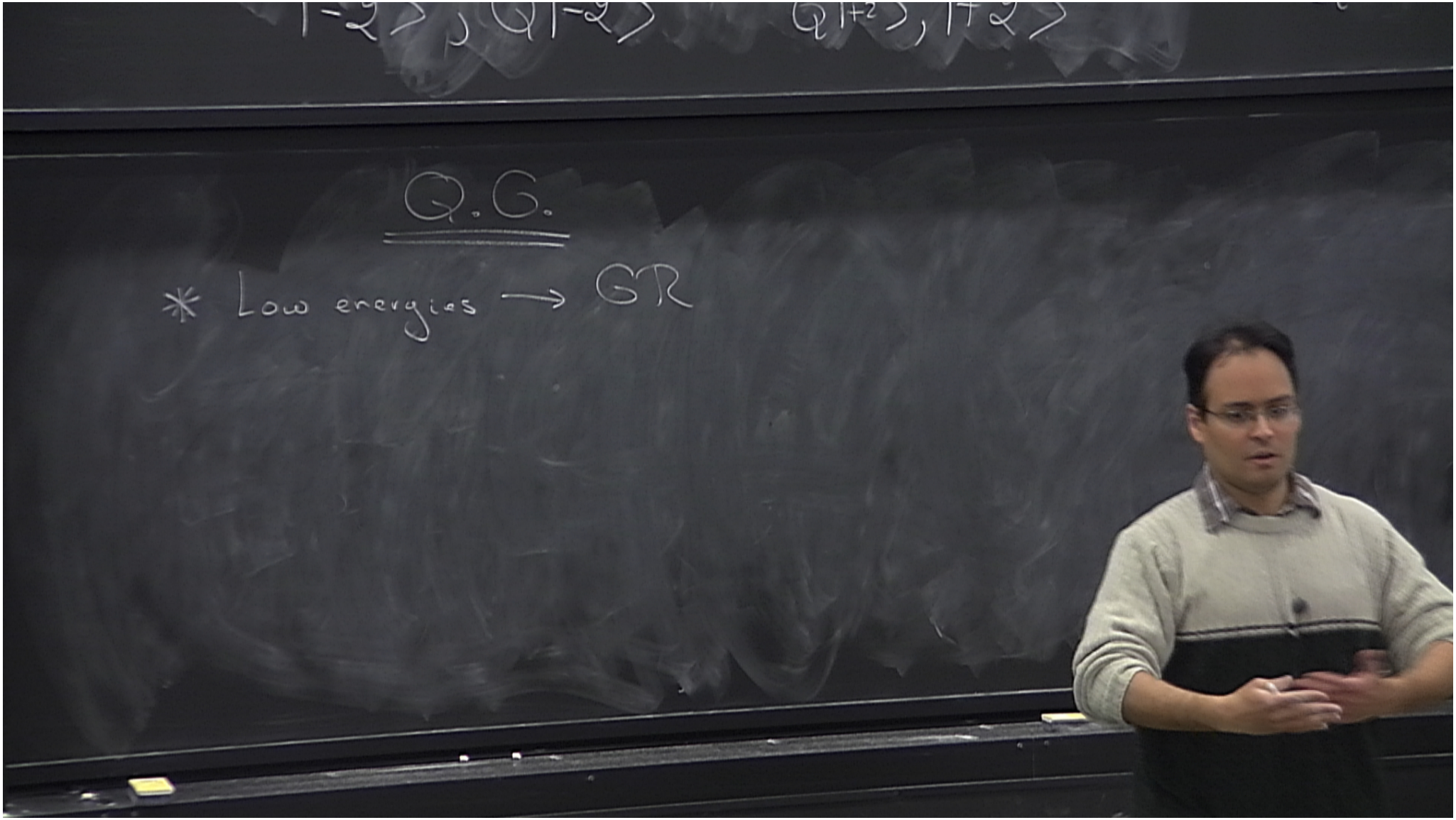
$7 = N - C$

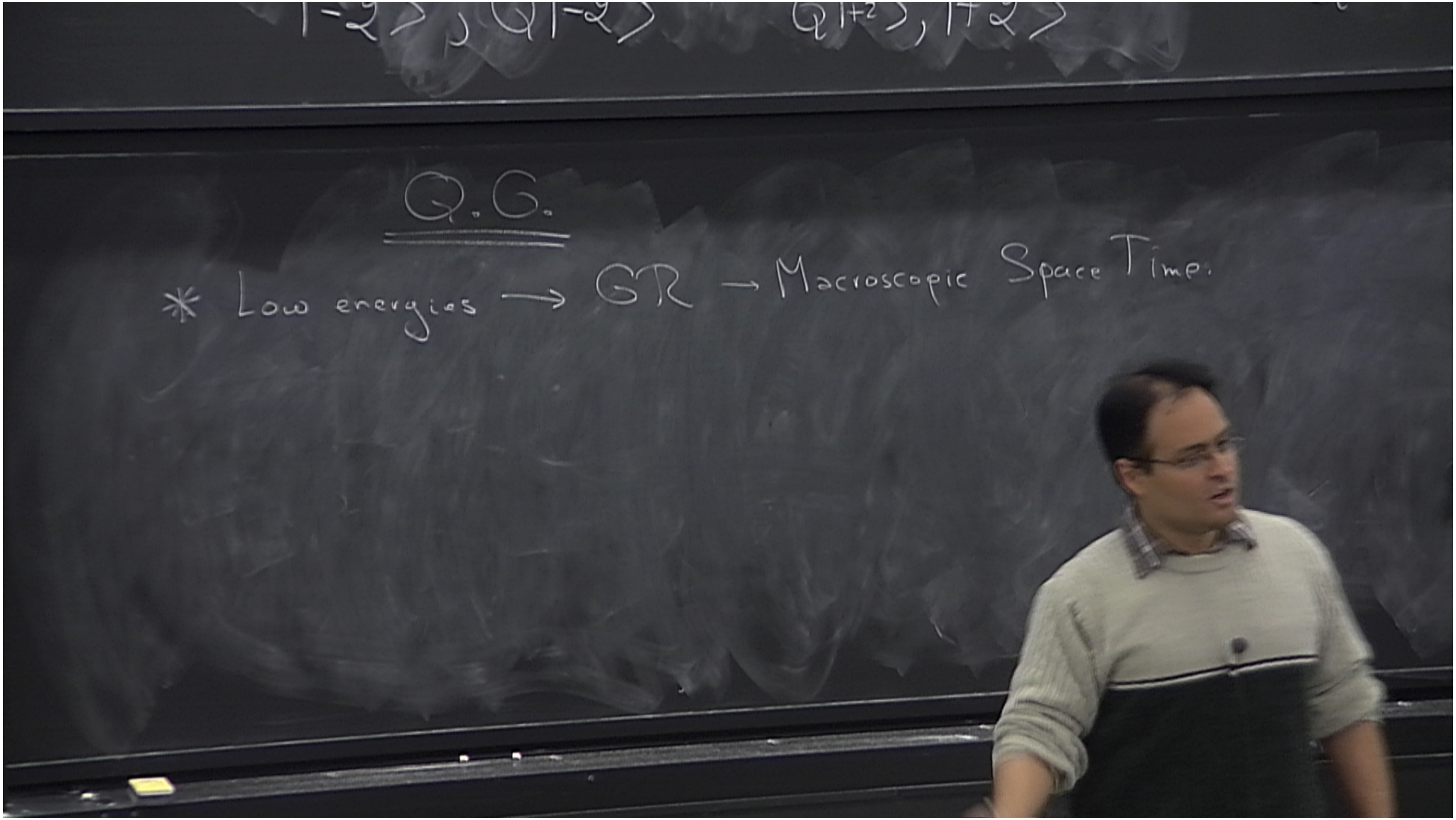
Bern et. el.

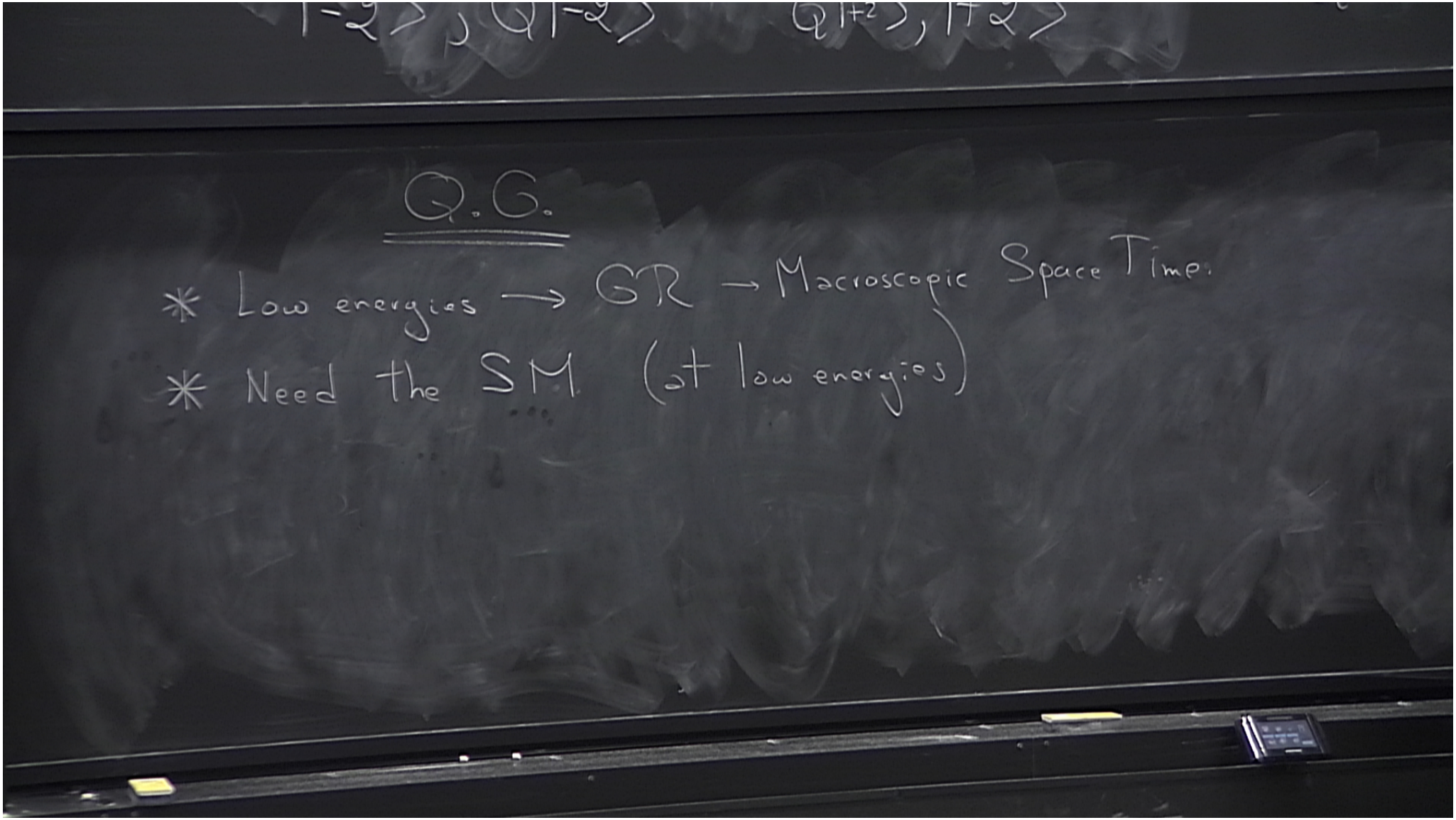

Maximal SUGRA
1-2>, Q^I 1-2), Q^I Q^J 1-2>,
1-loop
2-loop
3-loop
4-loop
10>
5 years ago
2009
Berne

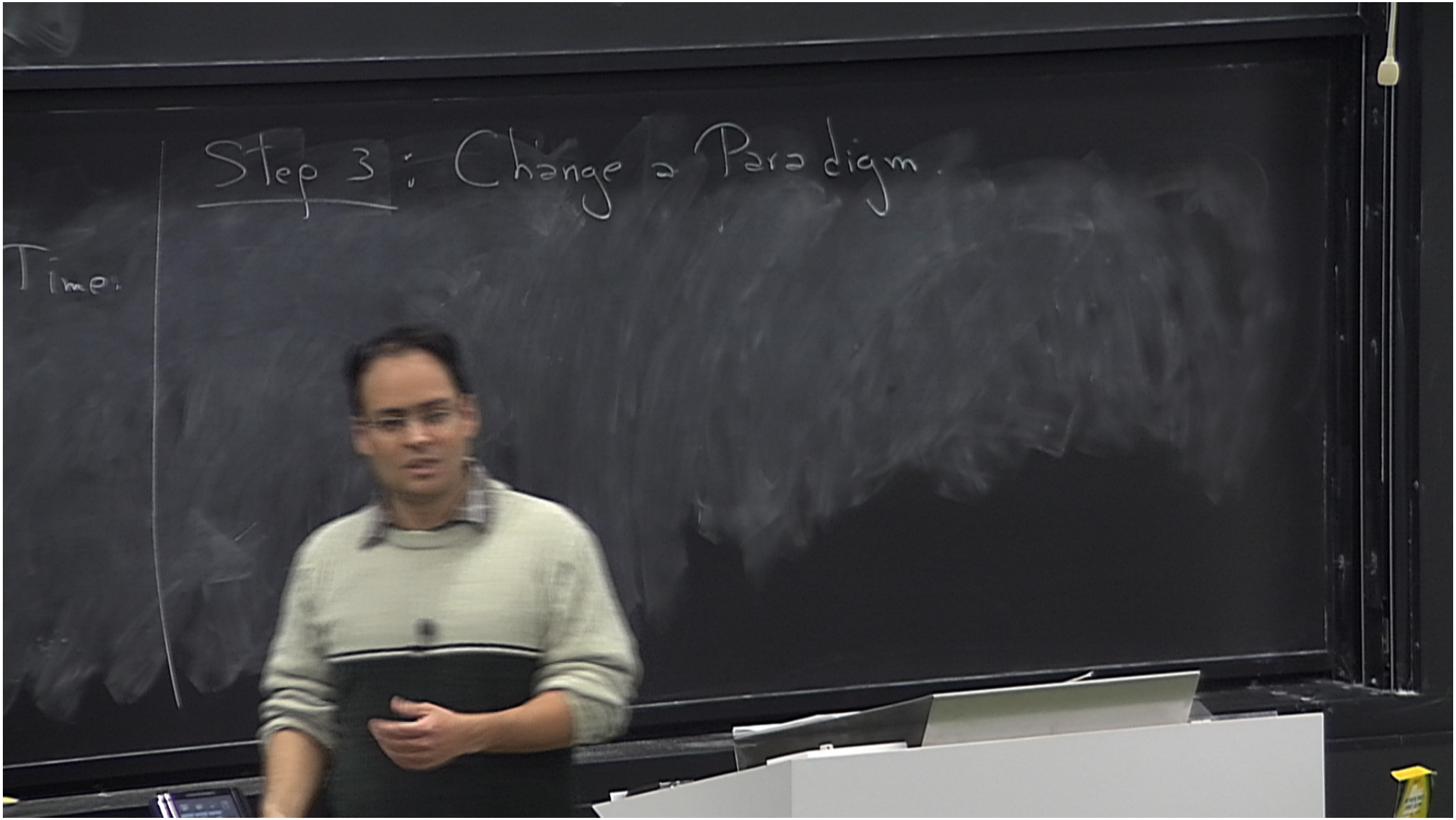

Maximal SUGRA
|-2>, Q^I |-2>, Q^I Q^J |-2>, ... , Q^{I1} Q^{I2} ...
1-loop Finite
2-loop Finite
3-loop Finite
4-loop Finite
|0>
70
5 years ago Bern et.
2009
11

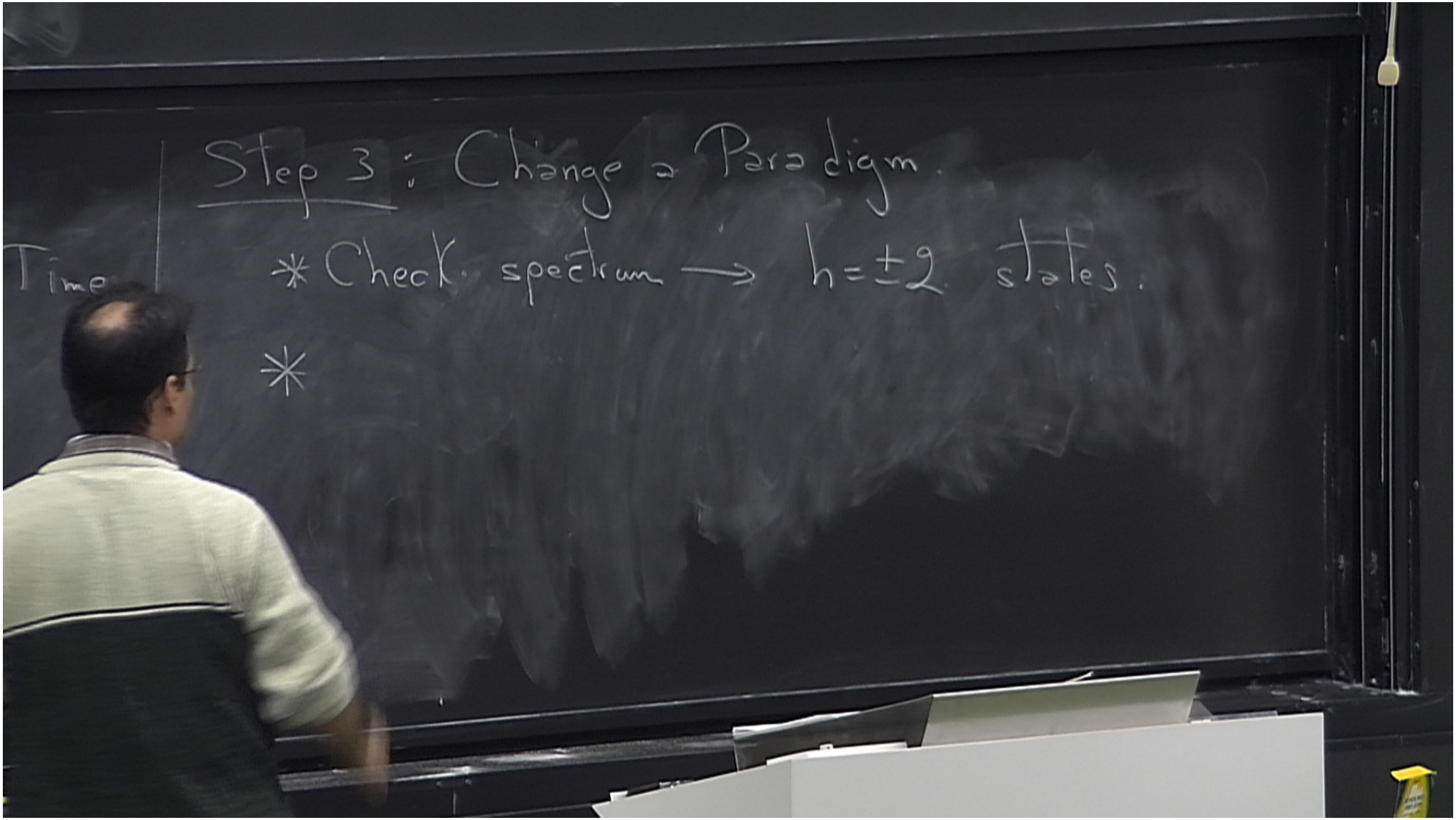

Q.G.

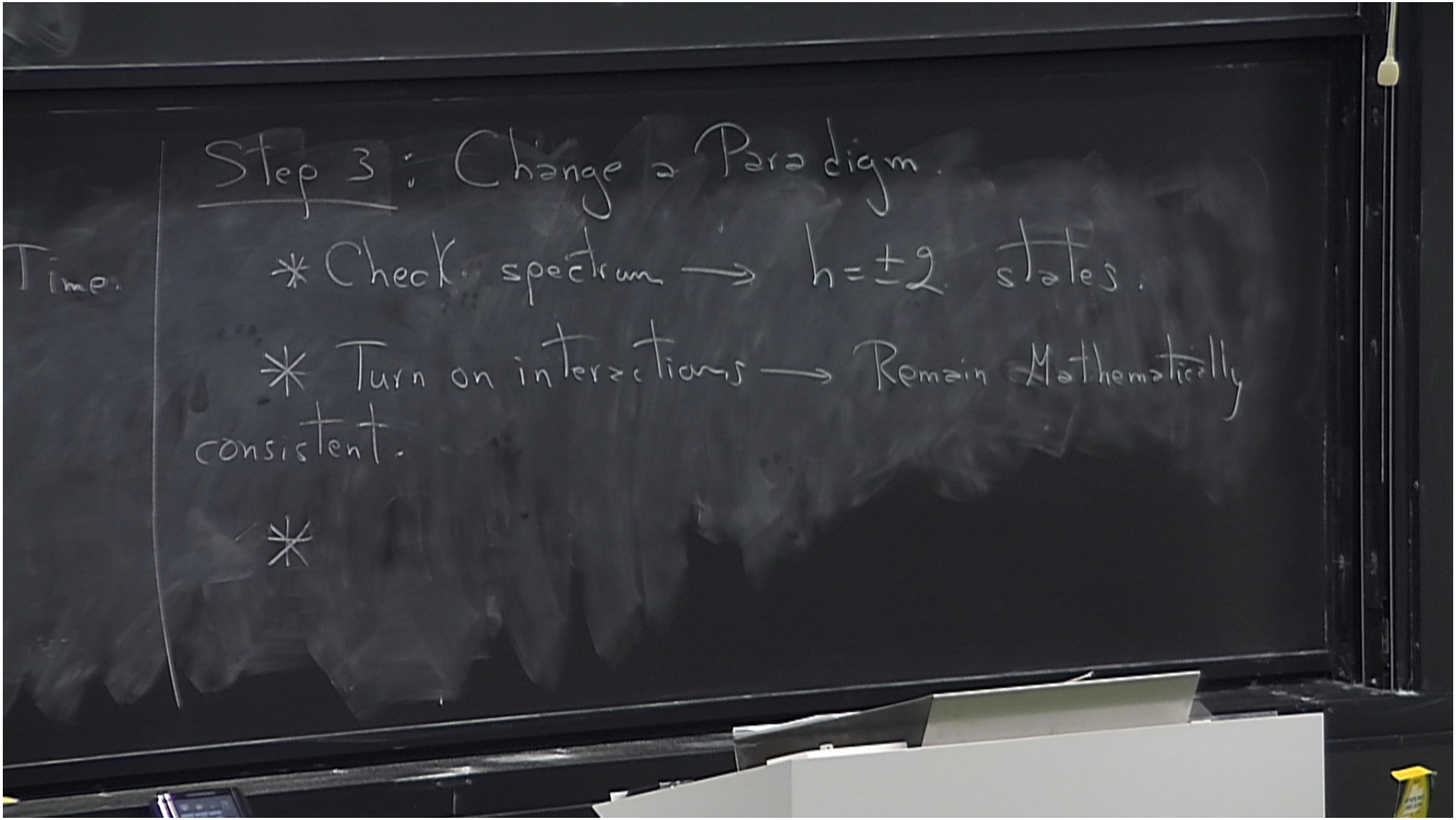

Q.G.
* Low energies → GR

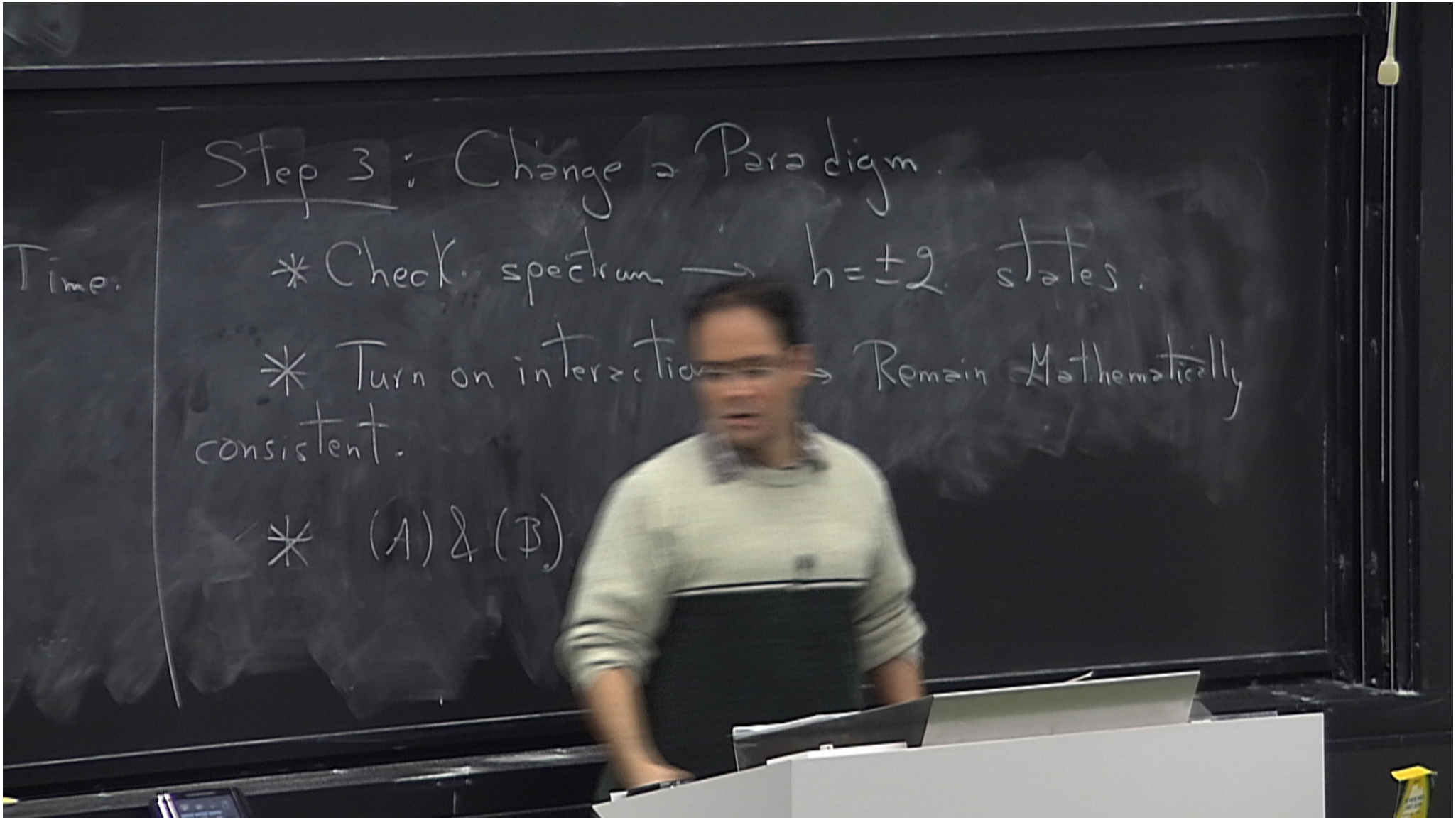

Q.G.
* Low energies → GR → Macroscopic Space Time.

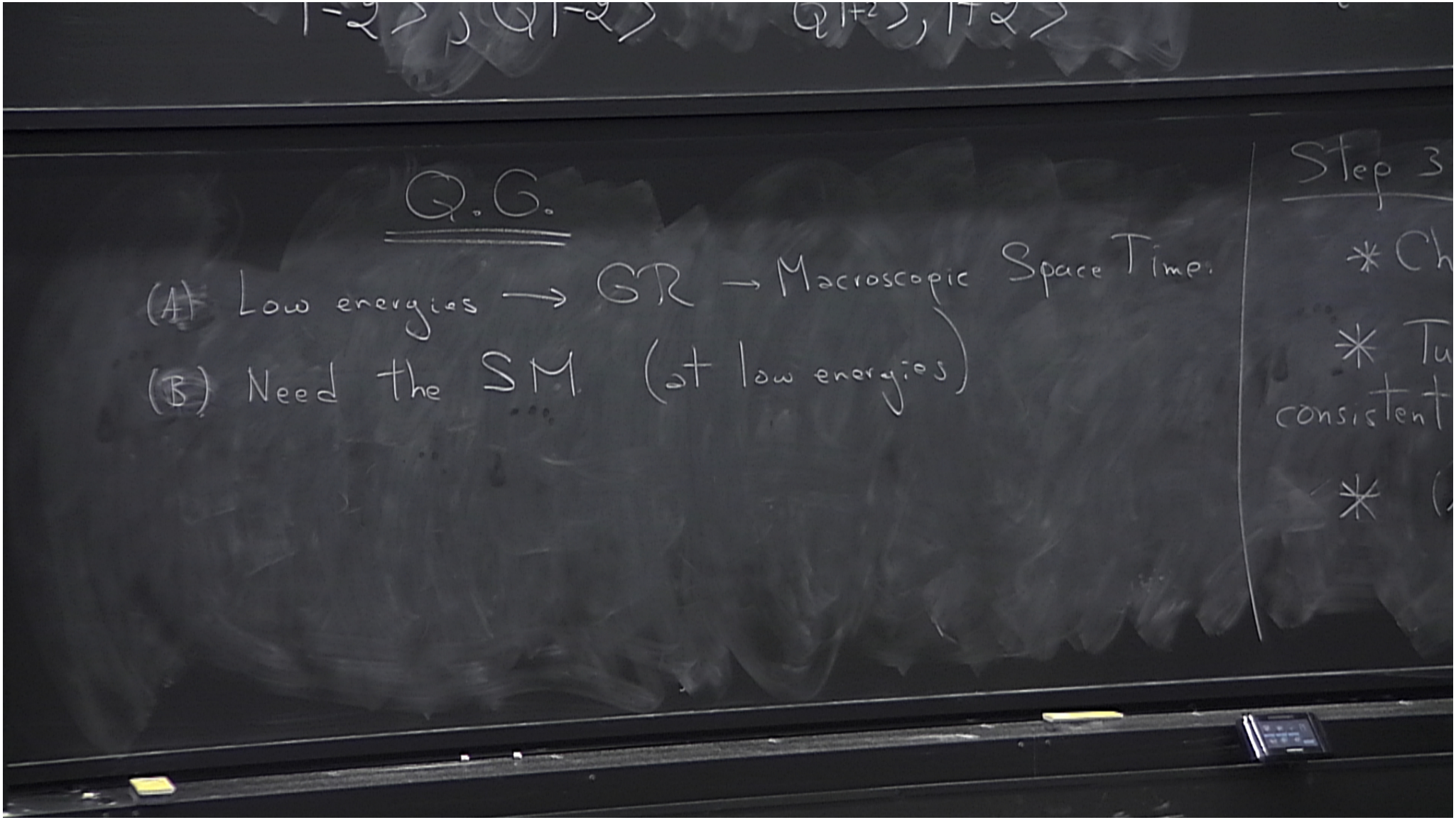

Q.G.
* Low energies → GR → Macroscopic Space Time.
* Need the SM (at low energies)

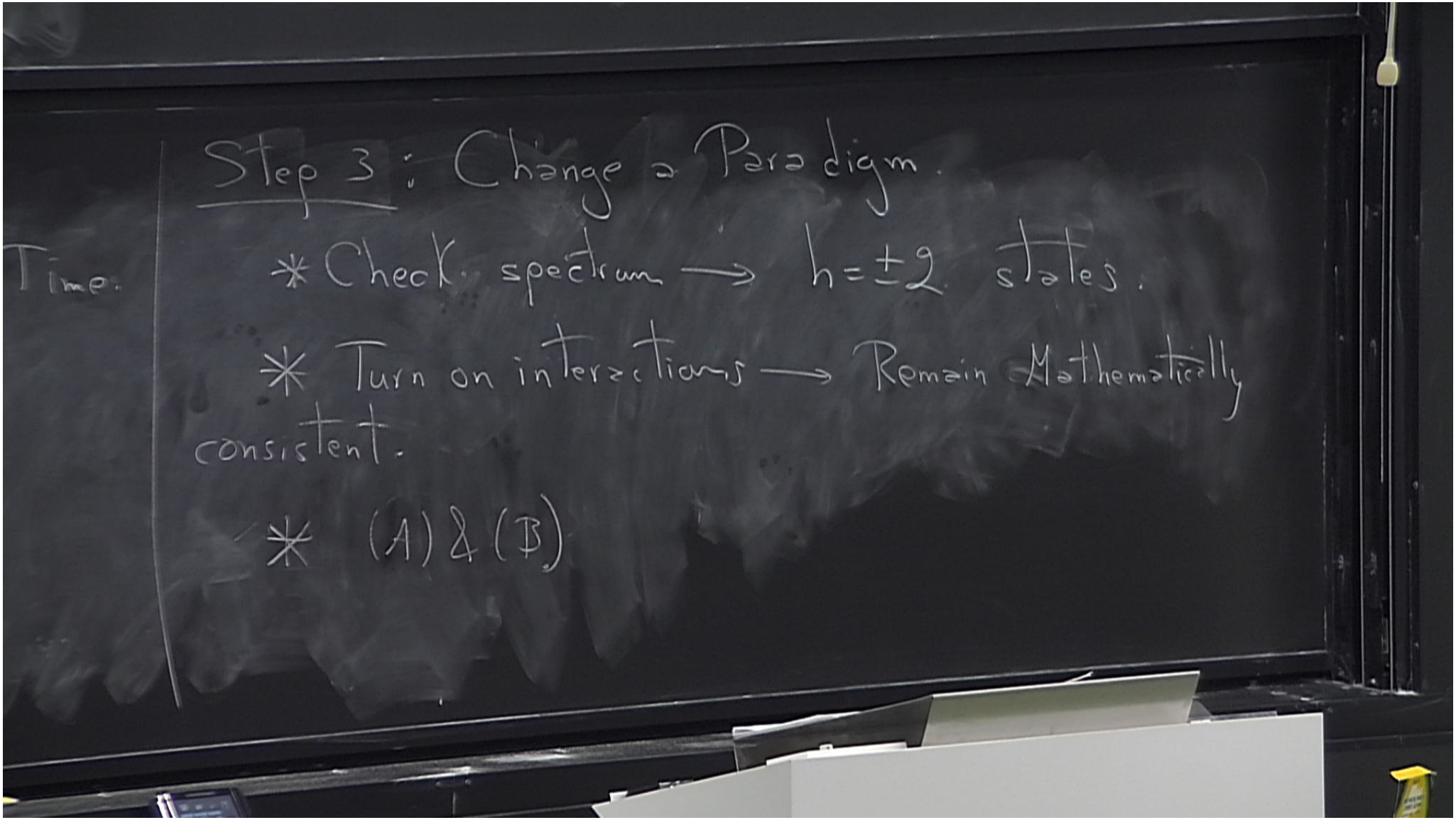

Step 3: Change a Paradigm
Time:

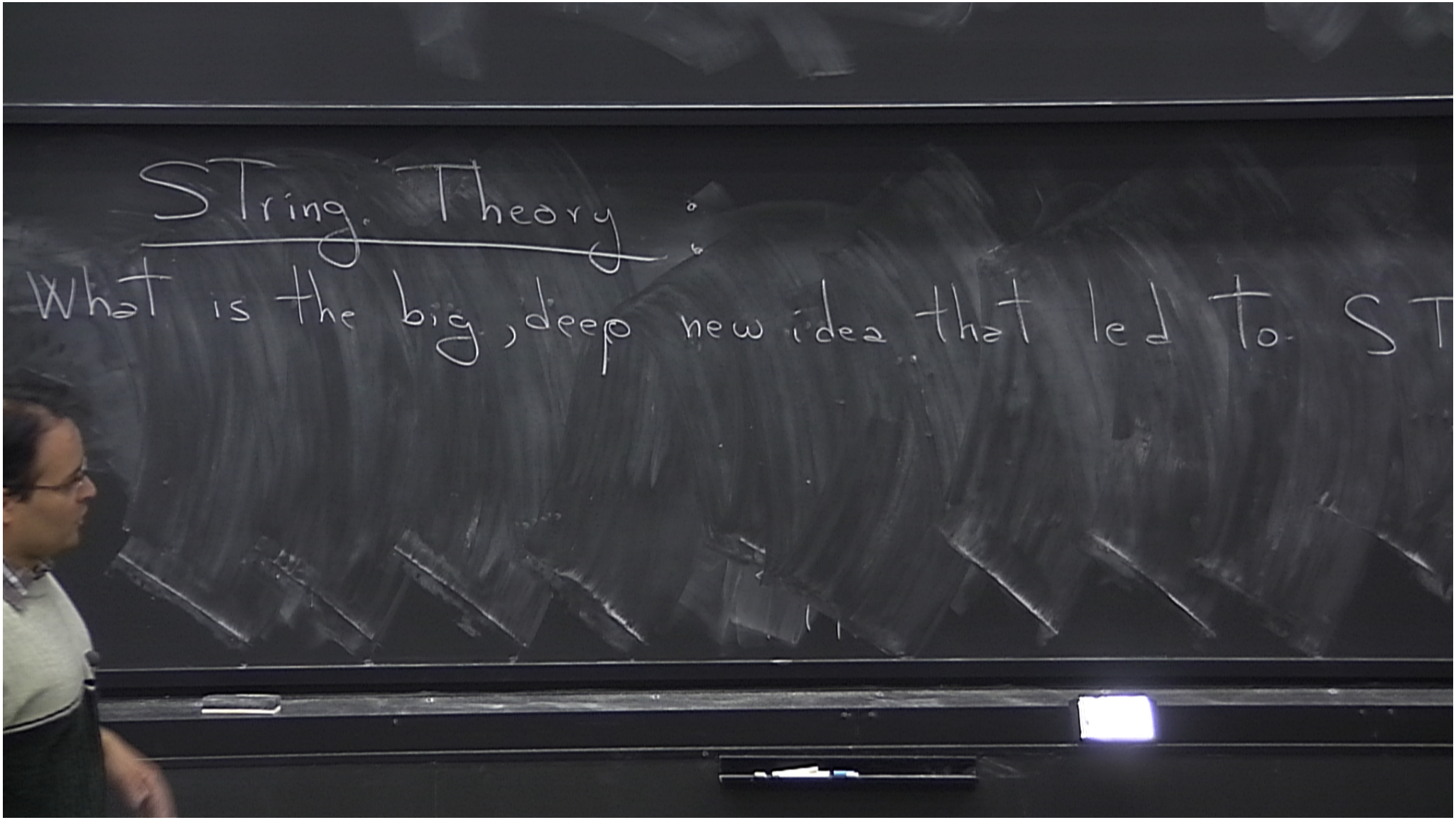

Step 3 : Change a Paradigm.
Time
* Check spectrum → h=±2 states.
*

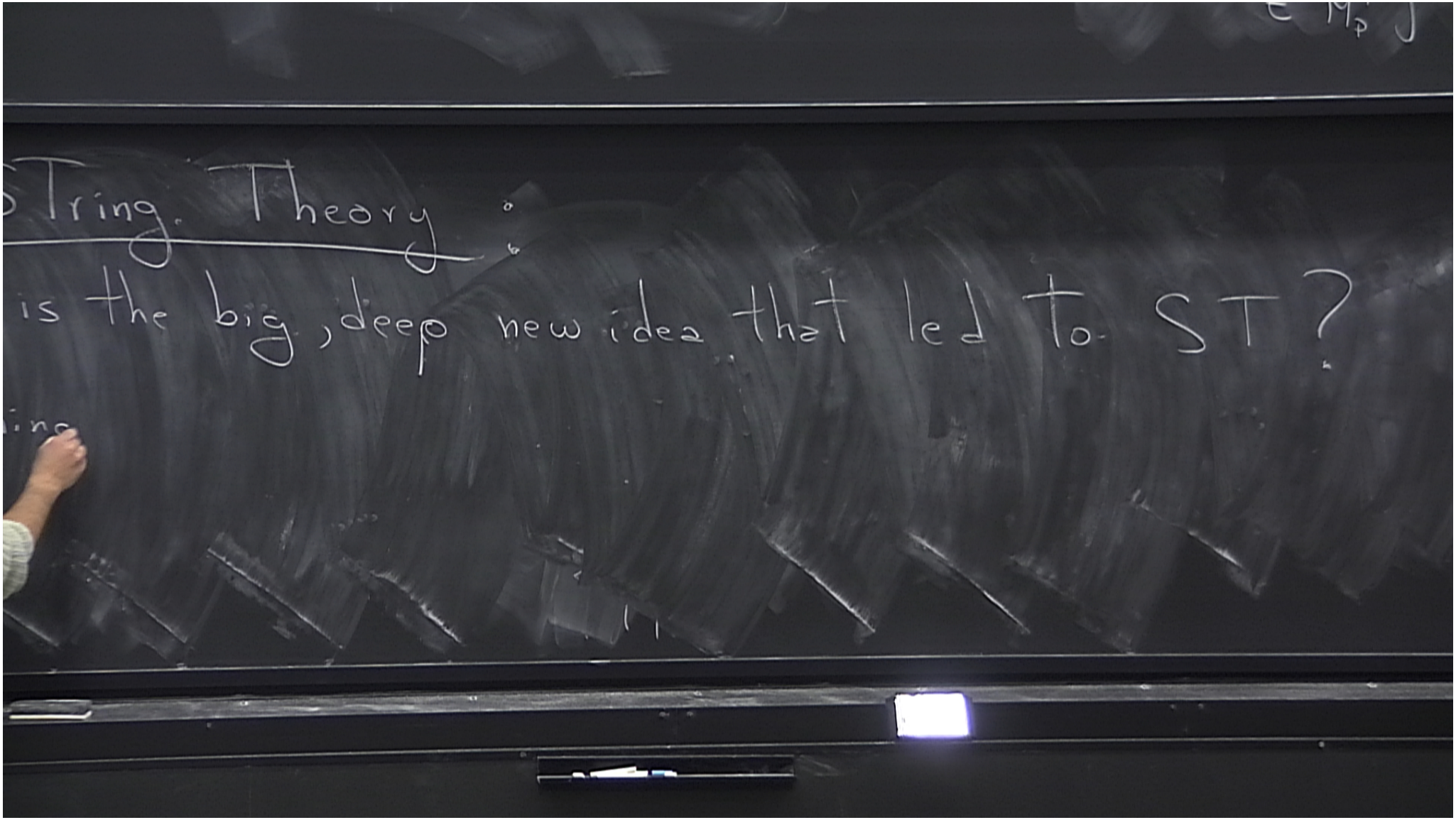

Step 3 : Change a Paradigm.
Time.
* Check spectrum → h = ±2 states.
* Turn on interactions → Remain Mathematically consistent.
*

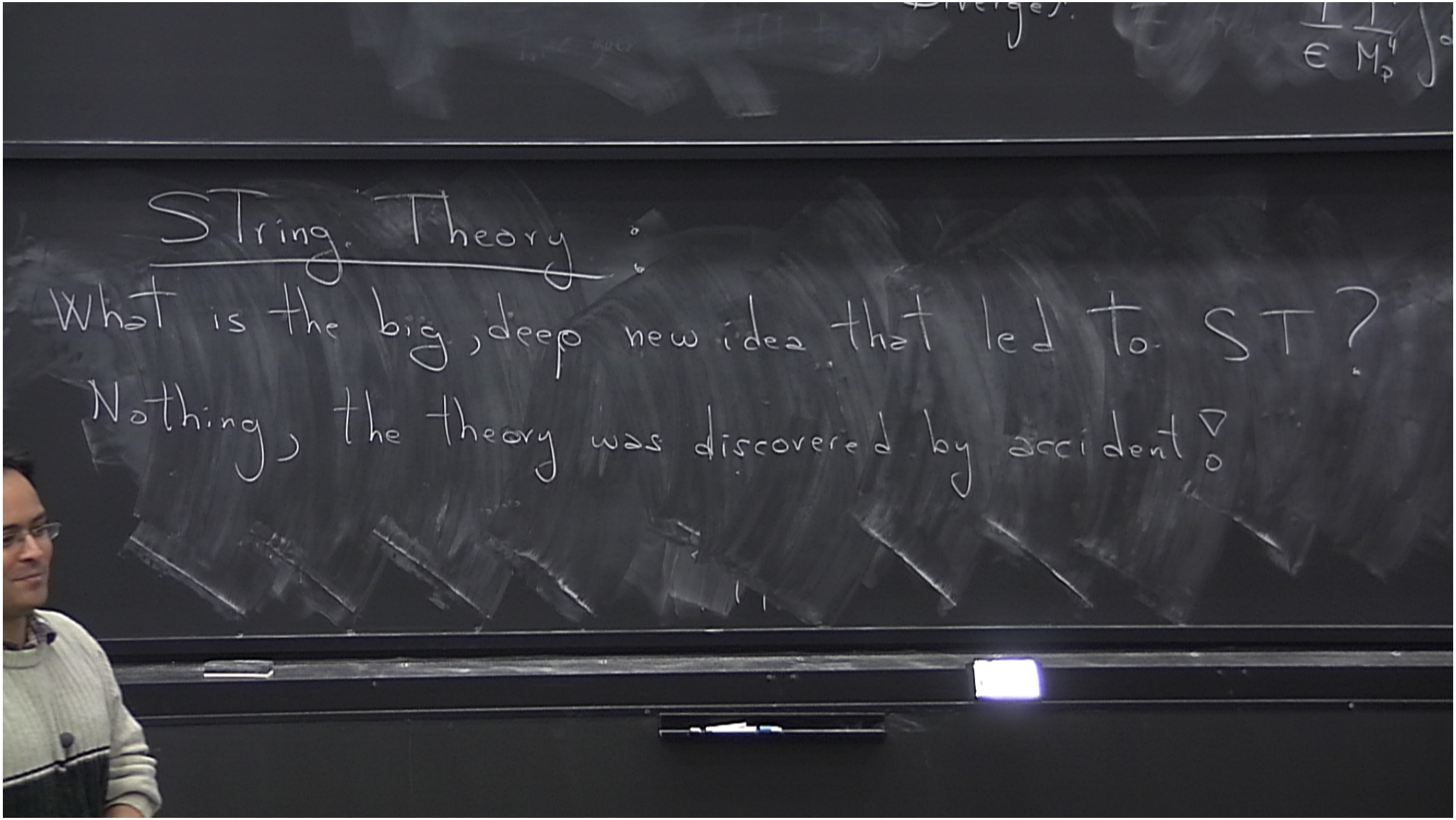

Step 3 : Change a Paradigm.
Time.
* Check spectrum → h=±2 states.
* Turn on interactions → Remain Mathematically consistent.
* (1) & (2)

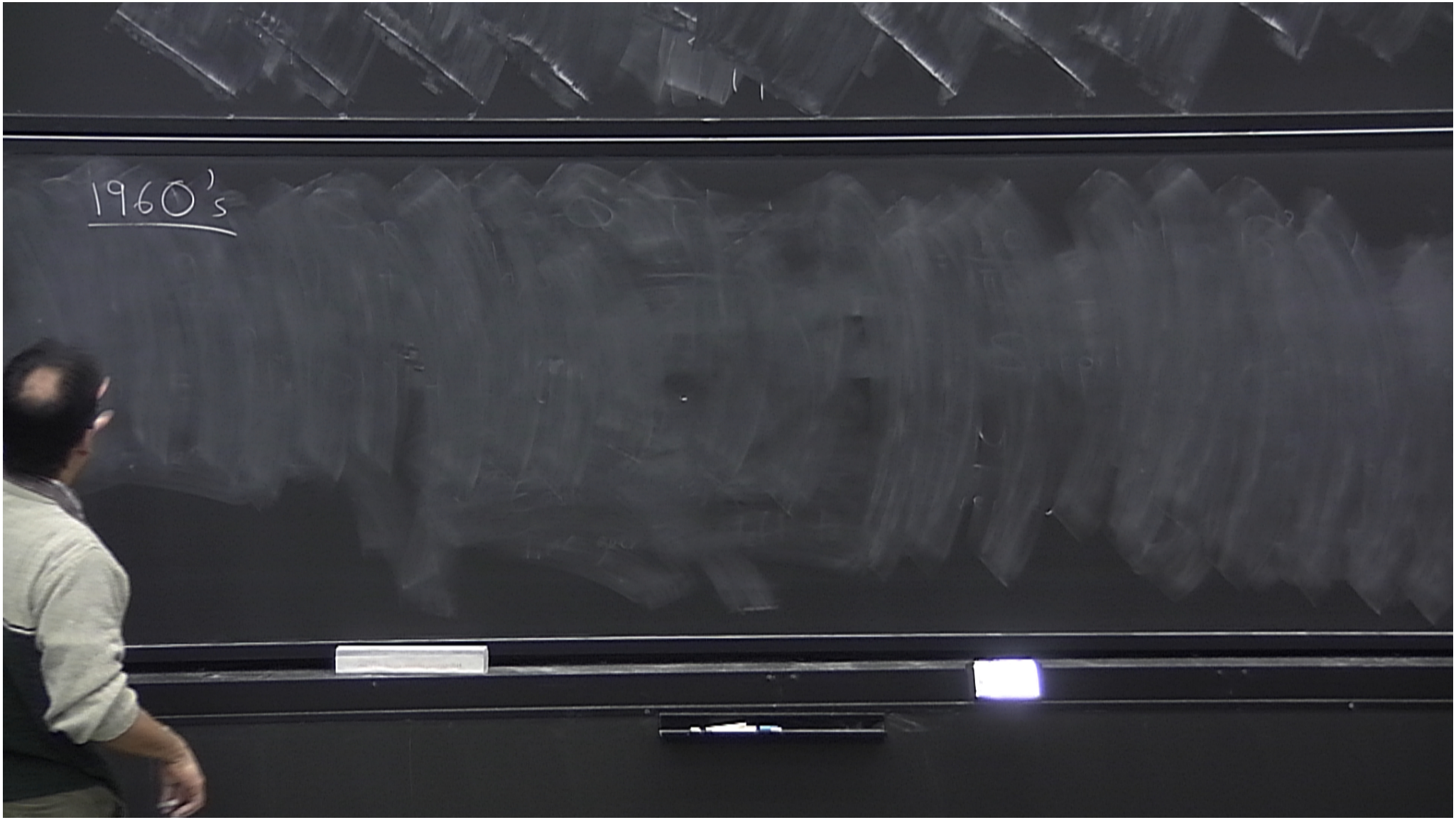

Q.G.
(A) Low energies → GR → Macroscopic Space Time.
(B) Need the SM (at low energies)
Step 3
consistent

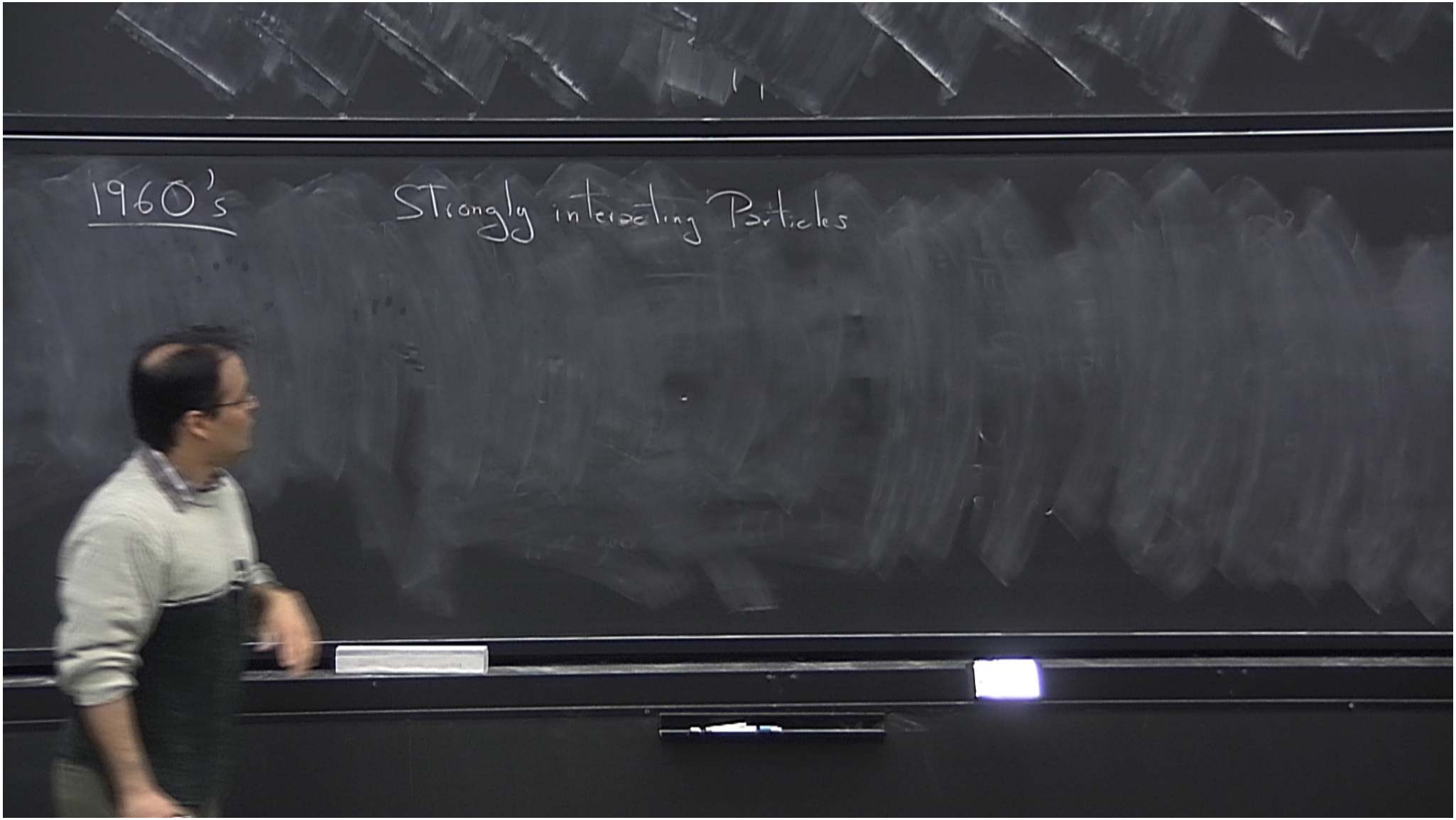

Step 3 : Change a Paradigm.
Time.
* Check spectrum → h = ±2 states.
* Turn on interactions → Remain Mathematically consistent.
* (A) & (B)

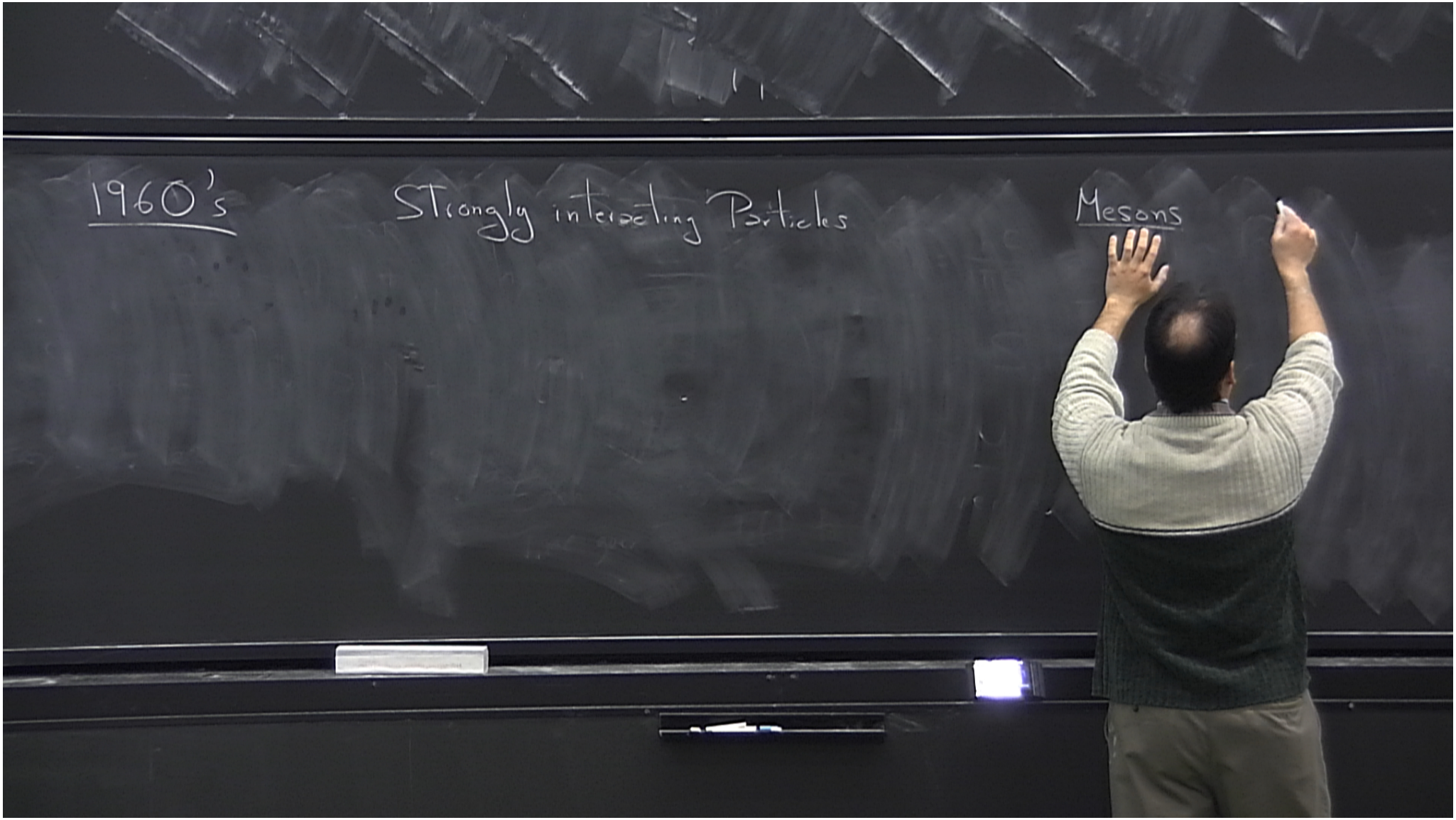

STring Theory :
What is the big, deep new idea that led to ST

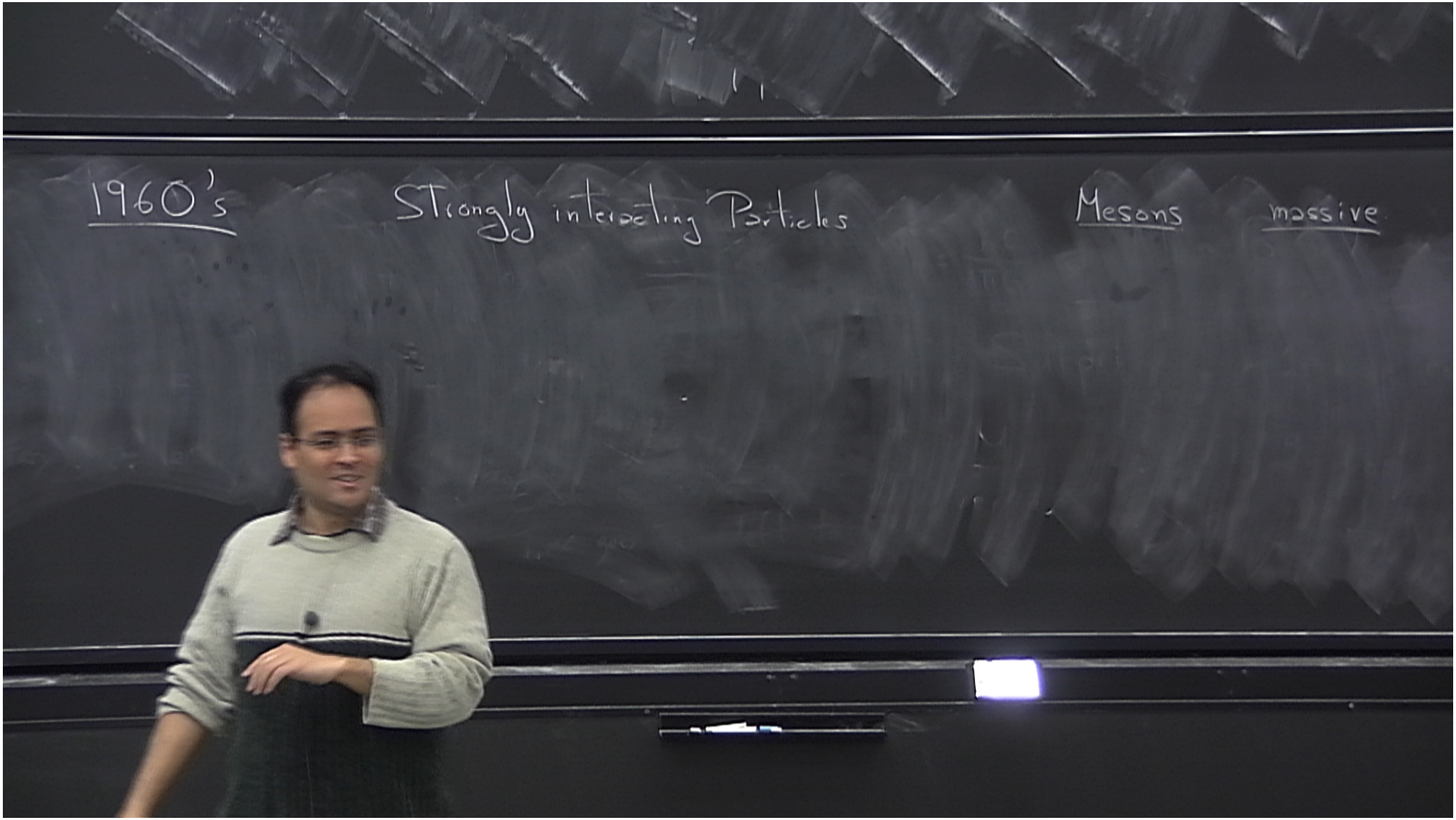

String Theory:
is the big, deep new idea that led to ST?

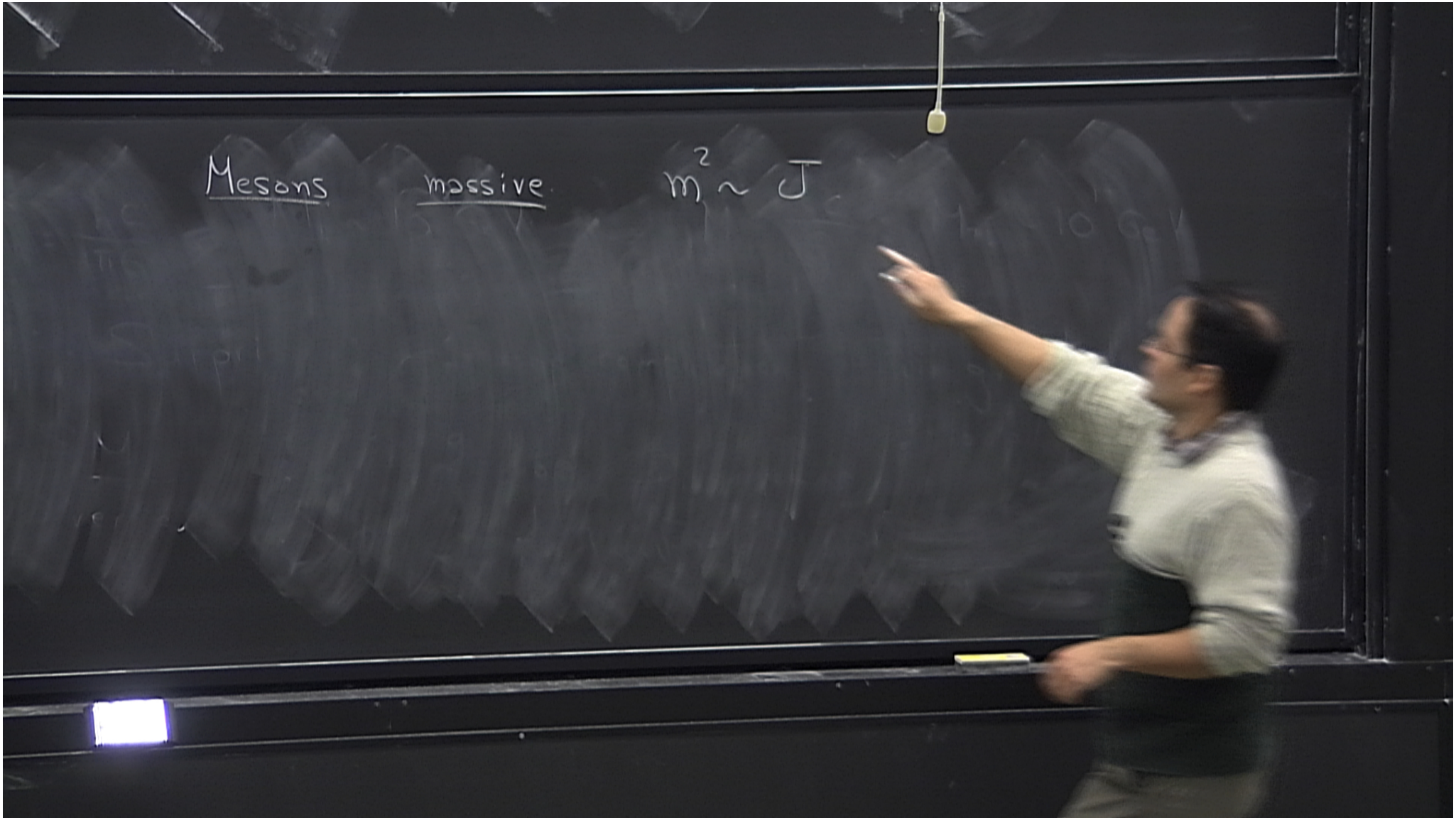

String Theory:
What is the big, deep new idea that led to ST?
Nothing, the theory was discovered by accident!

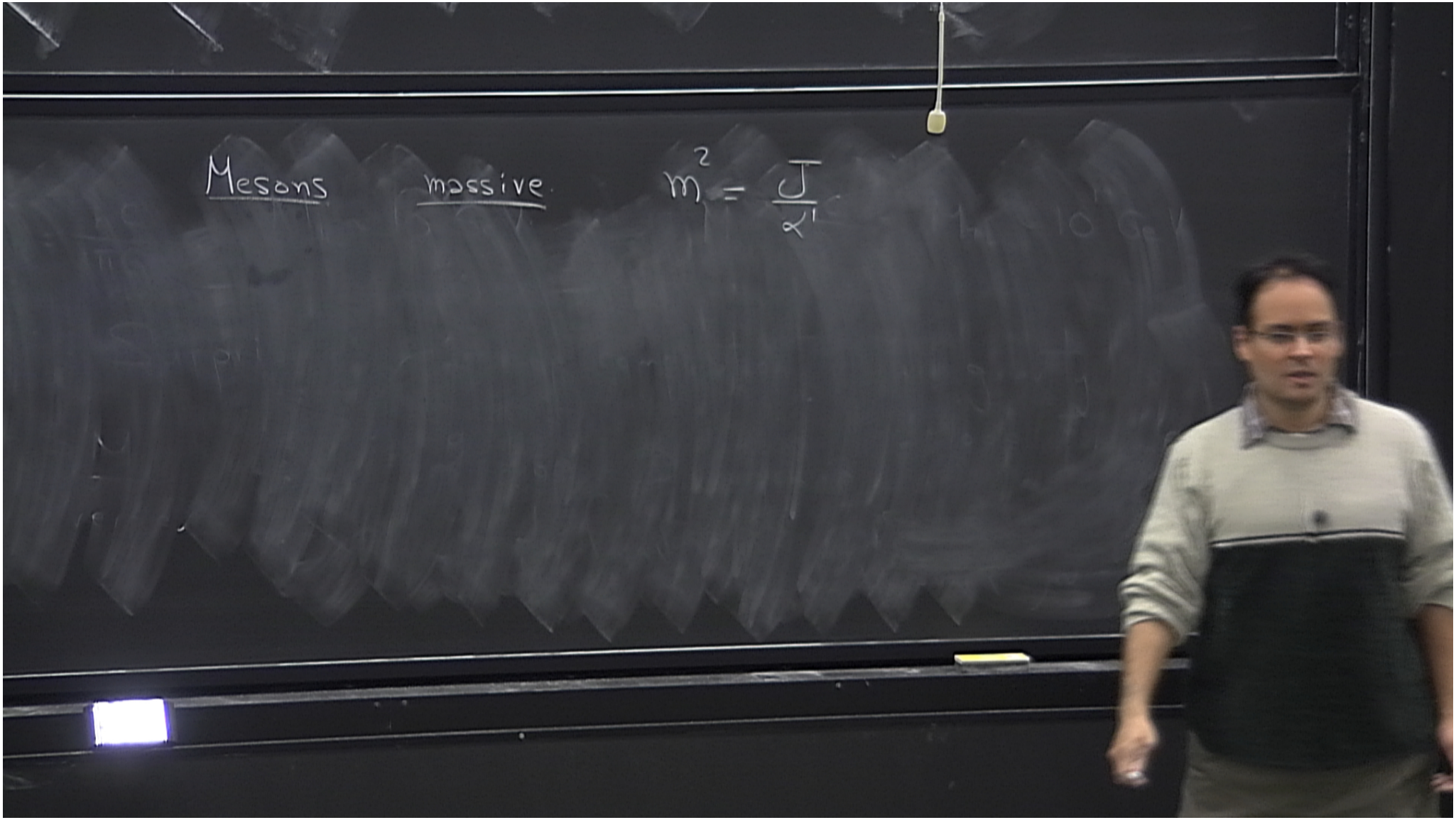

1960's

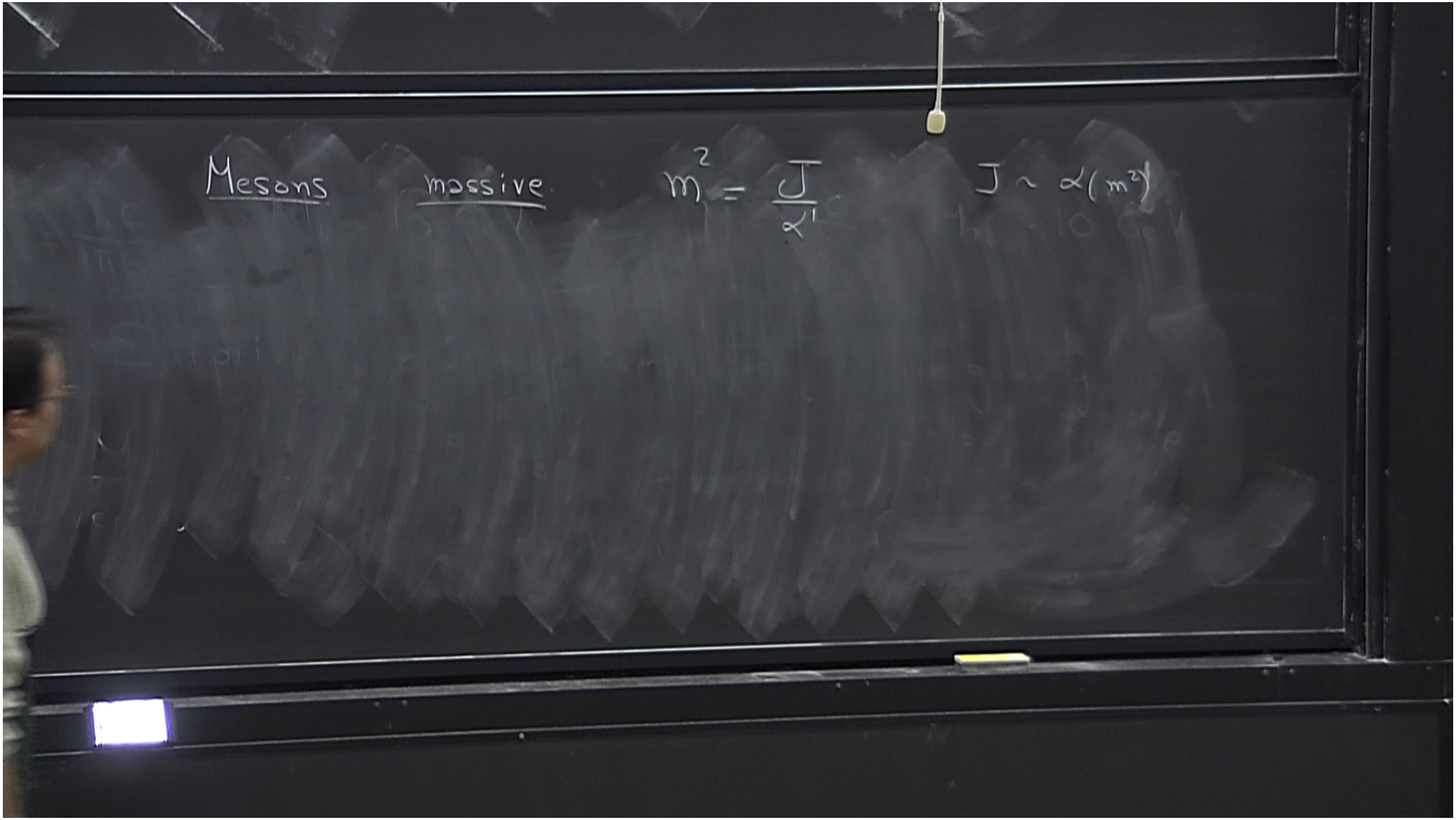

1960's
Strongly interacting Particles

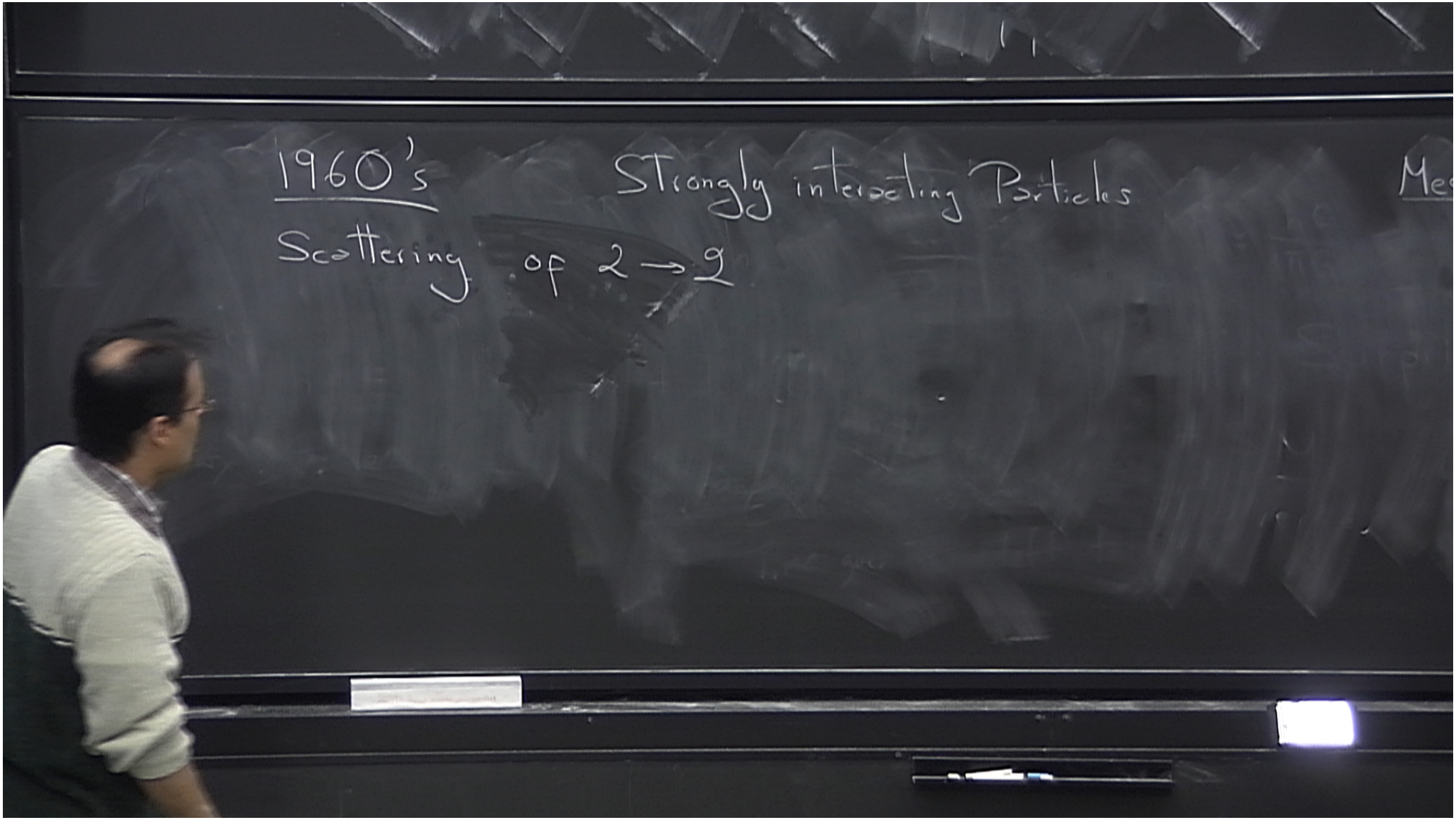

1960's
STrongly interacting Particles
Mesons

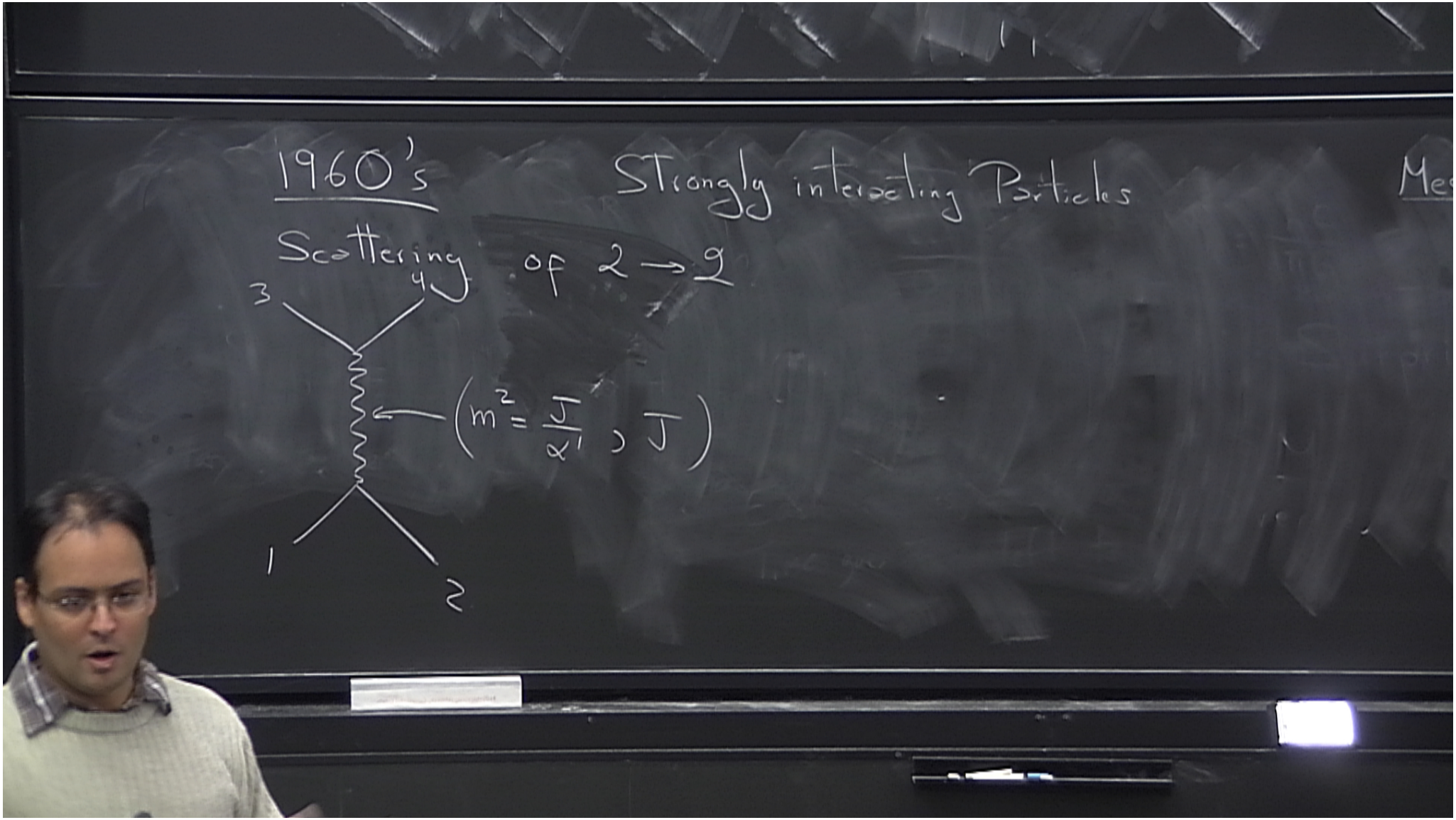

1960's
STrongly interacting Particles
Mesons
massive


Mesons massive $m^2 \sim J$


Mesons
massive
$m^2 = \frac{J}{\alpha'}$

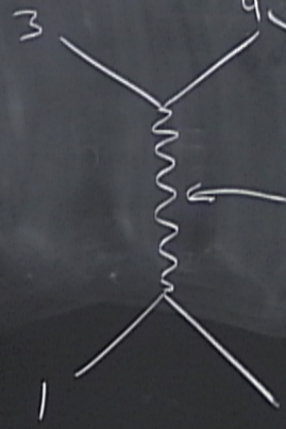

Mesons massive $m^2 = \frac{J}{\alpha'}$ $J \sim \alpha(m^2)$


1960's
Strongly interacting Particles
Scattering of 2 → 2


1960's
Strongly interacting Particles
Scattering of 2 → 2
3
4
$m^2 = \frac{J}{\alpha'}$, J
1
2

1960's Strongly interacting Particles Me

Scattering of $2 \to 2$

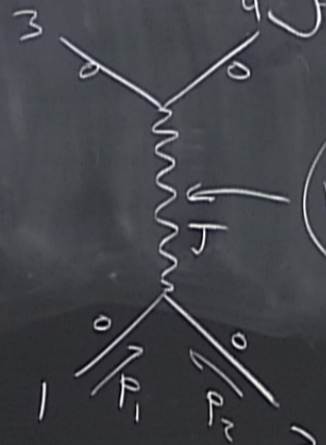


$\left(m^2 = \frac{J}{\alpha'}, J\right)$

$s = (p_1 + p_2)^2 \quad t = (p_2 + p_3)^2 \quad u = ($ $)^2$

1960's Strongly interacting Particles Me

Scattering of $2 \to 2$



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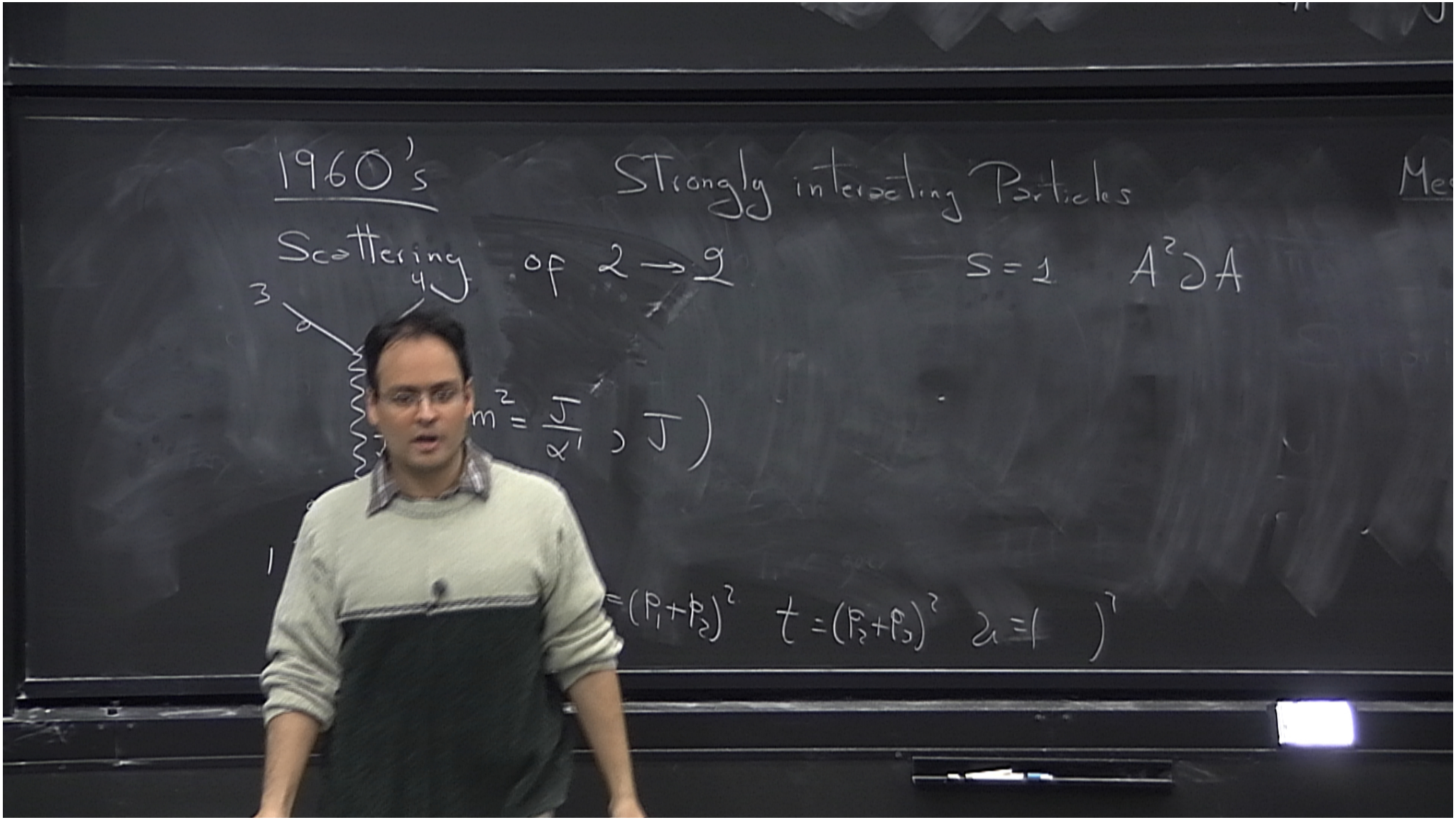
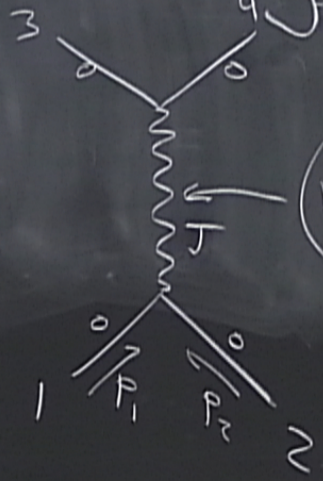

1960's
Strongly interacting Particles
Scattering of 2 → 2
s = 1
A²∂A
m² = J/α' , J)
= (p₁+p₂)²
t = (p₂+p₃)²

1960's Strongly interacting Particles

Scattering of $2 \to 2$

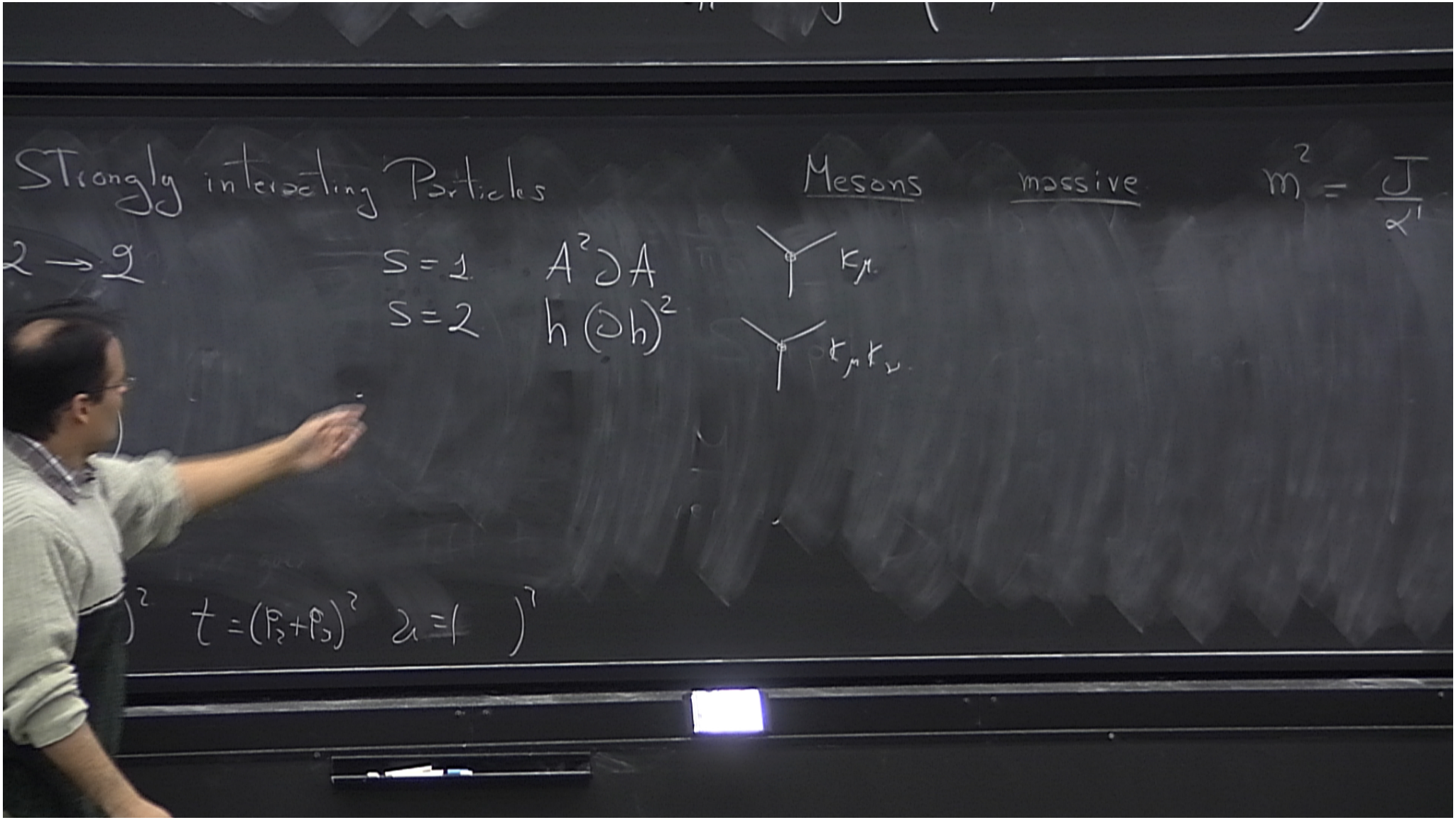


$\left(m^2 = \frac{J}{\alpha'}\right)$

$s = 1$ $A^2 \partial A$

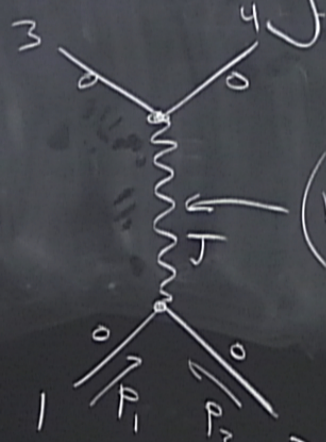
$s = 2$ $h(\partial h)^2$

$t = (p_2 + p_3)^2$ $\alpha' = 1$)


Strongly interacting Particles
Mesons
massive
$m^2 = \frac{J}{\alpha'}$
$2 \to 2$
$s = 1$ $A^2 \partial A$
$s = 2$ $h(\partial h)^2$
k_μ
$k_\mu k_\nu$
$t = (p_2 + p_3)^2$

1960's Strongly interacting Particles

Scattering of $2 \to 2$

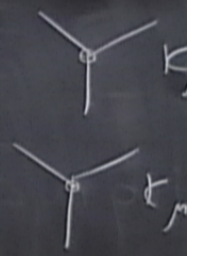


$\left(m^2 = \frac{J}{\alpha'} , J\right)$

$\frac{t^J}{s - m_J^2}$

$s = 1$ $A^2 \partial A$

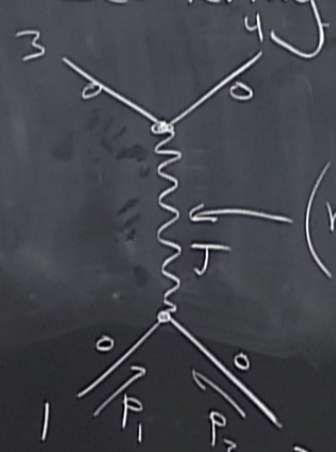
$s = 2$ $h(\partial h)^2$



$s = (p_1 + p_2)^2$ $t = (p_2 + p_3)^2$ $\alpha' = 1$)

1960's Strongly interacting Particles

Scattering of $2 \to 2$



$S = 1$ $A^2 \partial A$

$S = 2$ $h(\partial h)^2$

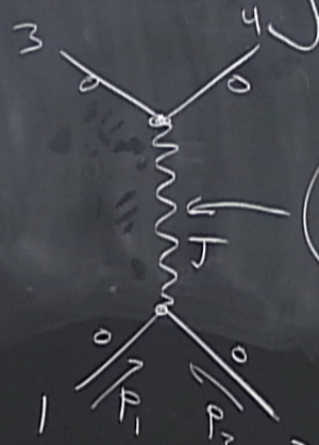
$\left(m^2 = \frac{J}{\alpha'}, J\right) \sim \frac{t^J}{s - m_J^2}$

$s = (p_1 + p_2)^2$ $t = (p_2 + p_3)^2$ $\alpha' = 1$)

Mesons massive $m_J^2 = \frac{J}{\alpha'}$ $J \sim \alpha(m^2)$

1960's Strongly interacting Particles

Scattering of $2 \to 2$



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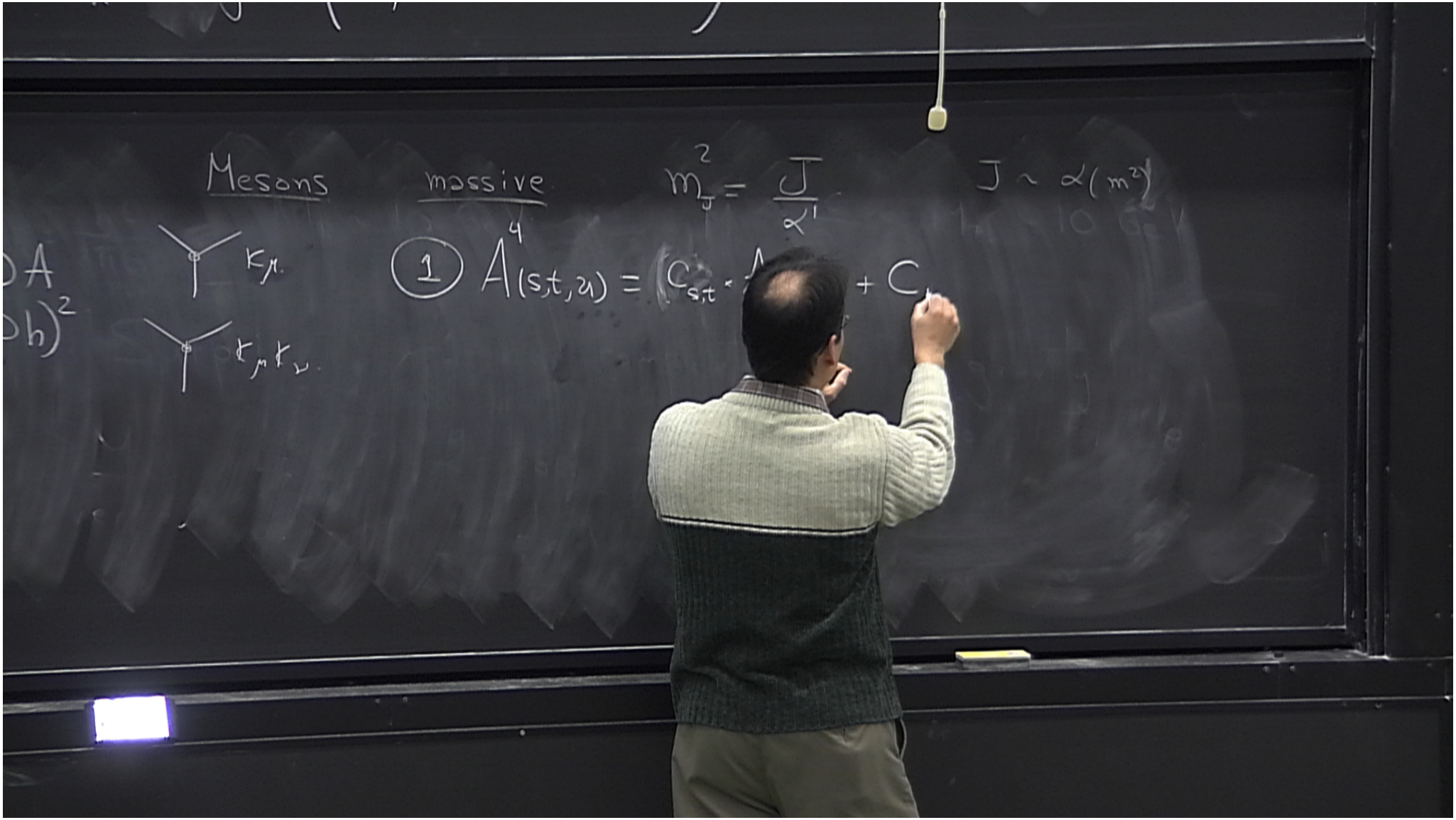
k_μ

$\epsilon_\mu \epsilon_\nu$

(1) $A^4(s,t,u) =$


Mesons
massive
$m_J^2 = \frac{J}{\alpha'}$
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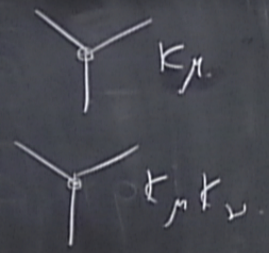
$A^4(s,t,u) =$

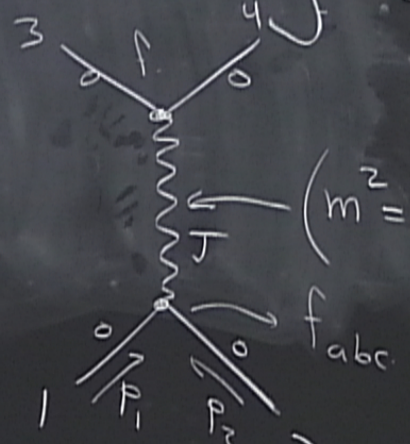
Mesons massive $m_J^2 = \frac{J}{\alpha'}$ $J \sim \alpha(m^2)$



(1) $A^4(s,t,u) = C_{s,t} \cdot A(s,t) + C_{t,u} A(t,u) + C_{u,s} A(u,s)$

1960's Strongly interacting Particles

Scattering of $2 \to 2$



$\left(m^2 = \frac{J}{\alpha'}, J\right) \sim \frac{t^J}{s - m_J^2}$

$s = (p_1+p_2)^2 \quad t = (p_2+p_3)^2 \quad \alpha = (\quad)'$

$s = 1 \quad A^2 \partial A$

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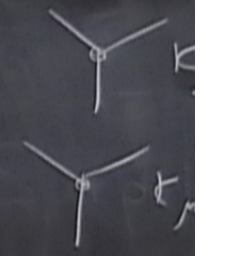

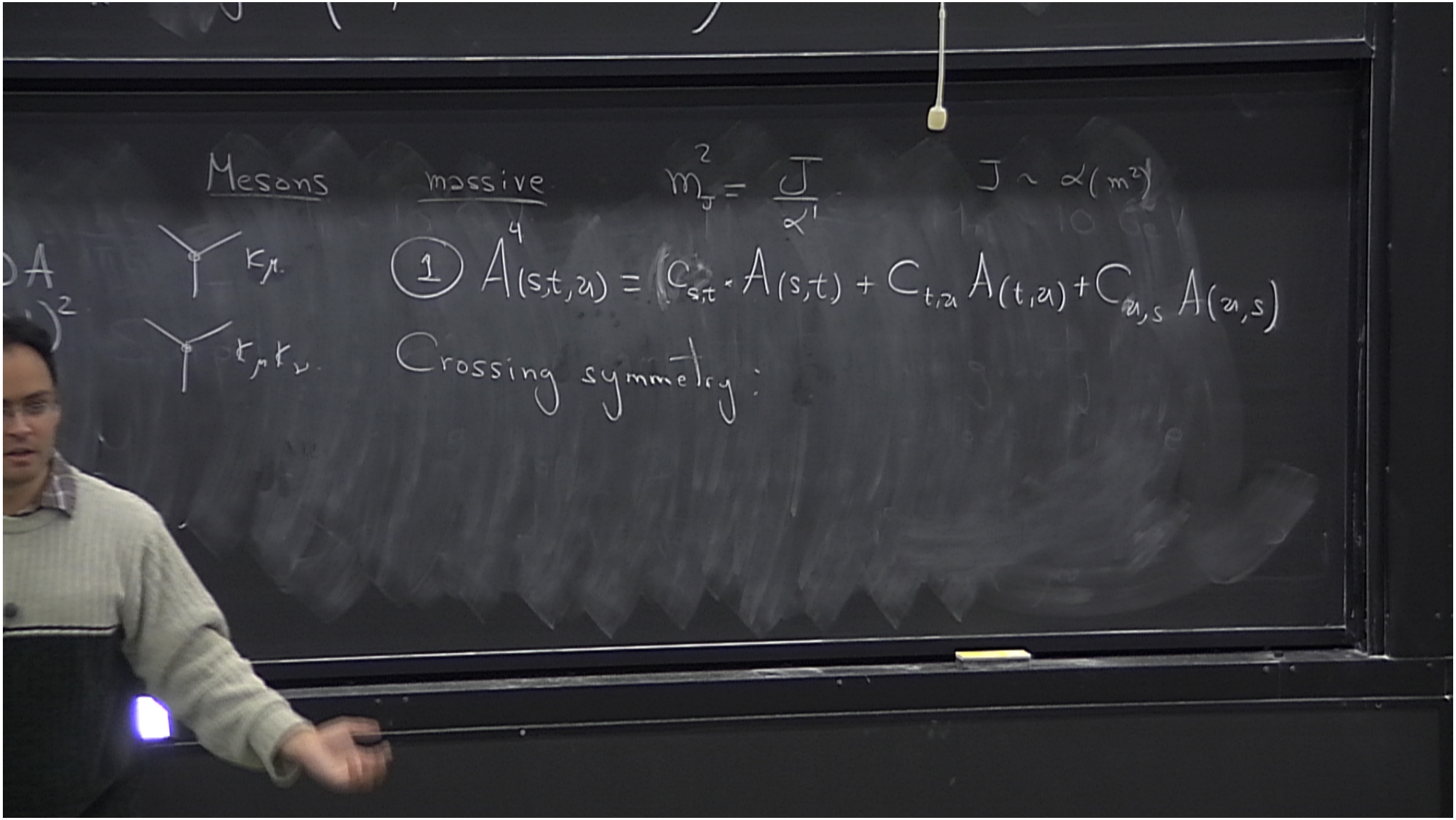
Mesons
massive
$m_J^2 = \frac{J}{\alpha'}$
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k_μ
k_μ, k_ν
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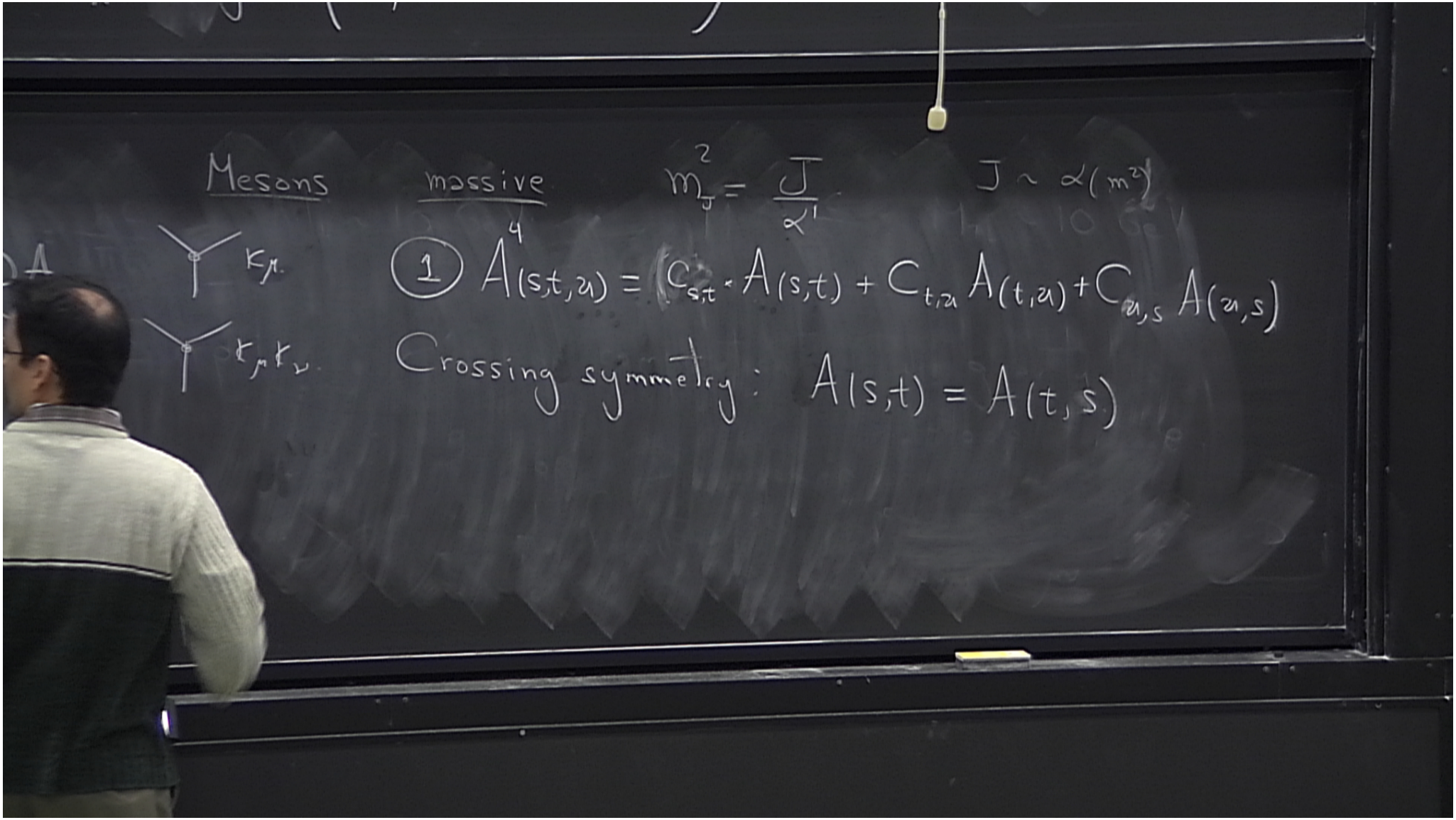
Crossing symmetry:


Mesons
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$m_J^2 = \frac{J}{\alpha'}$
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Crossing symmetry: $A(s,t) = A(t,s)$

Mesons massive $m_J^2 = \frac{J}{\alpha'}$ $J \sim \alpha(m^2)$

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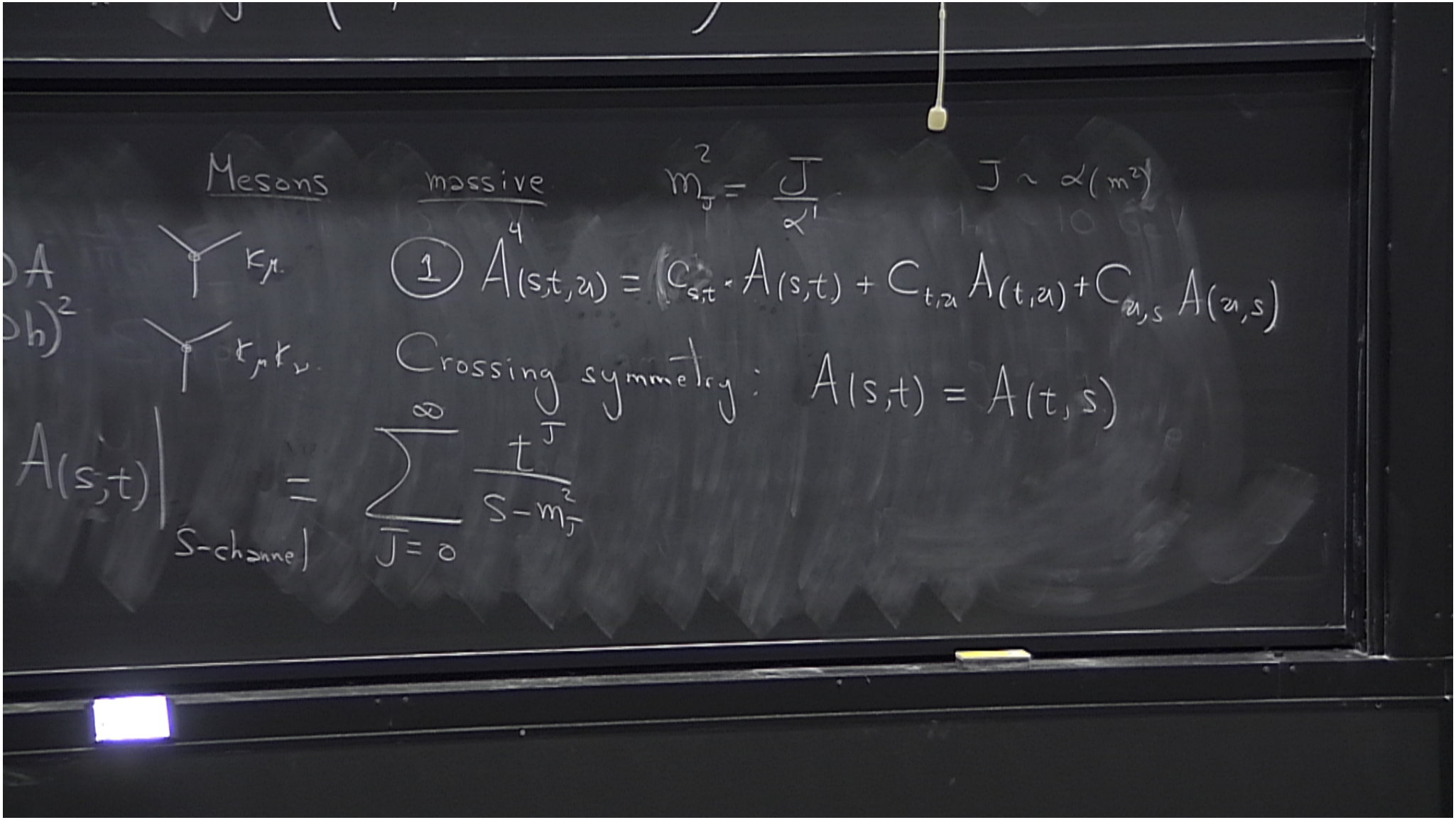
Crossing symmetry: $A(s,t) = A(t,s)$

$A(s,t)\Big|_{S\text{-channel}} =$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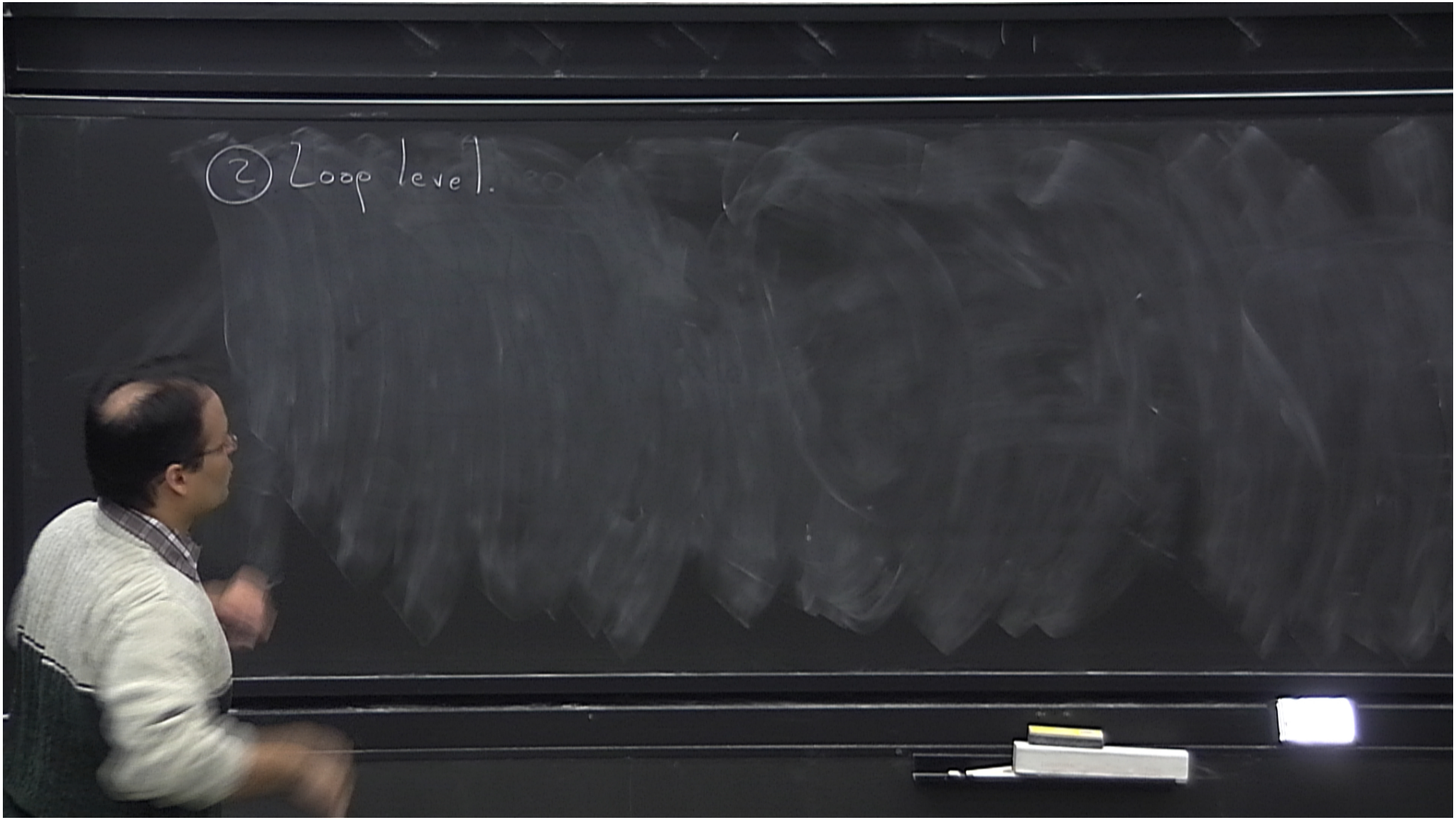
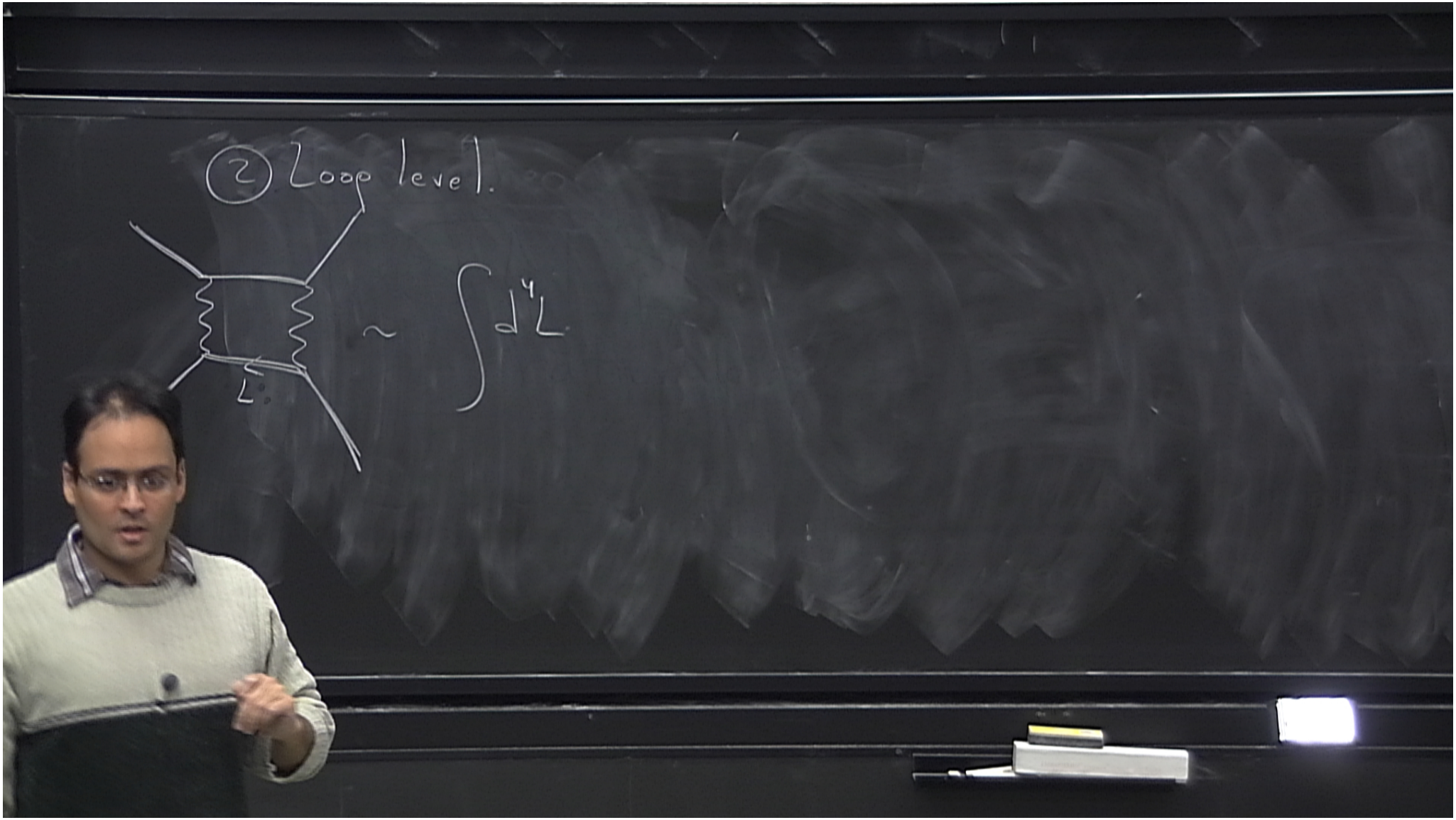
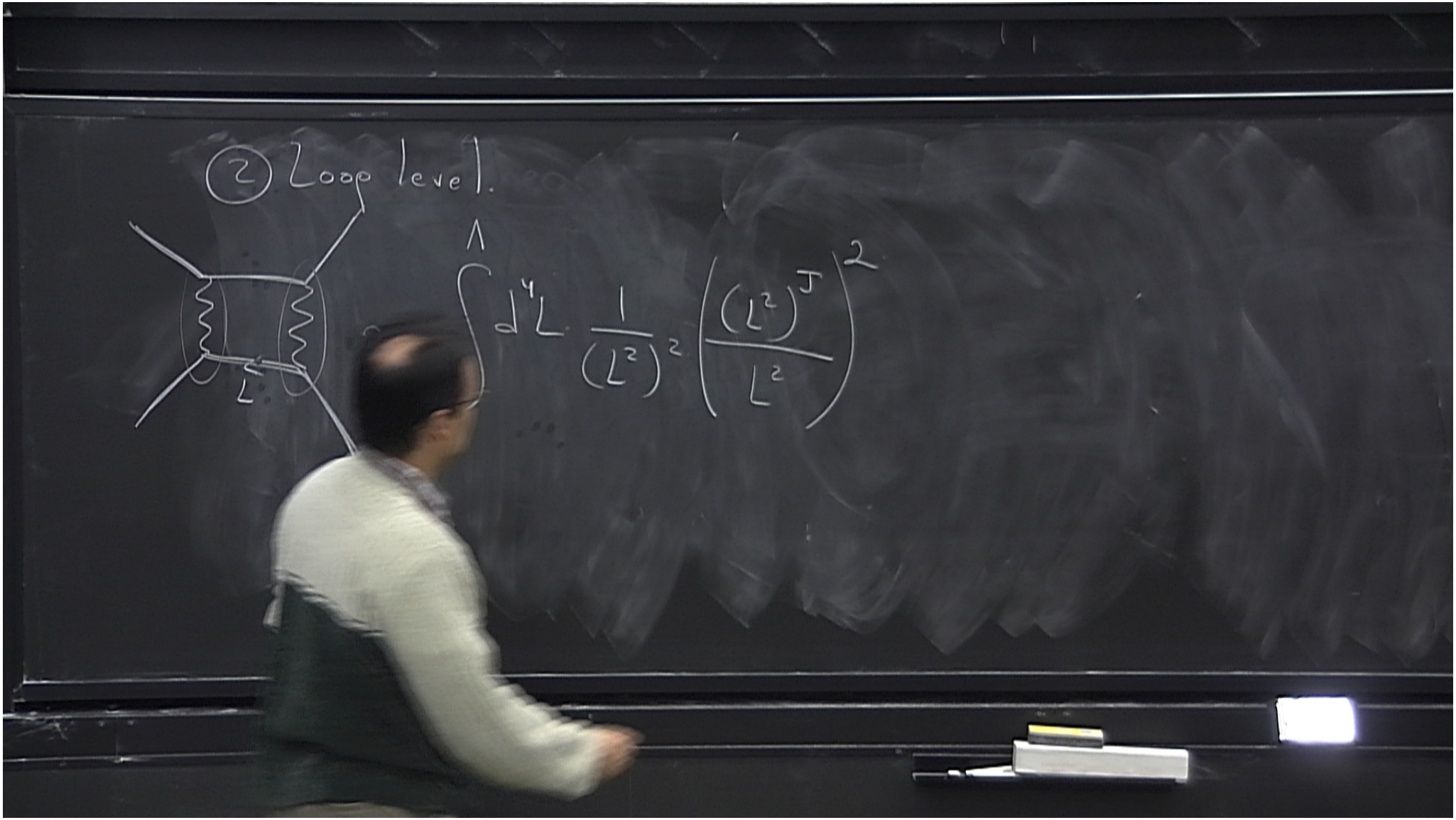
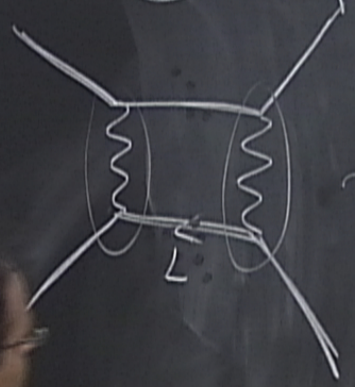
$$A(s,t)\Big|_{t\text{-channel}} = \sum_{J=0}^{\infty} \frac{s^J}{t - m_J^2}$$


② Loop level.

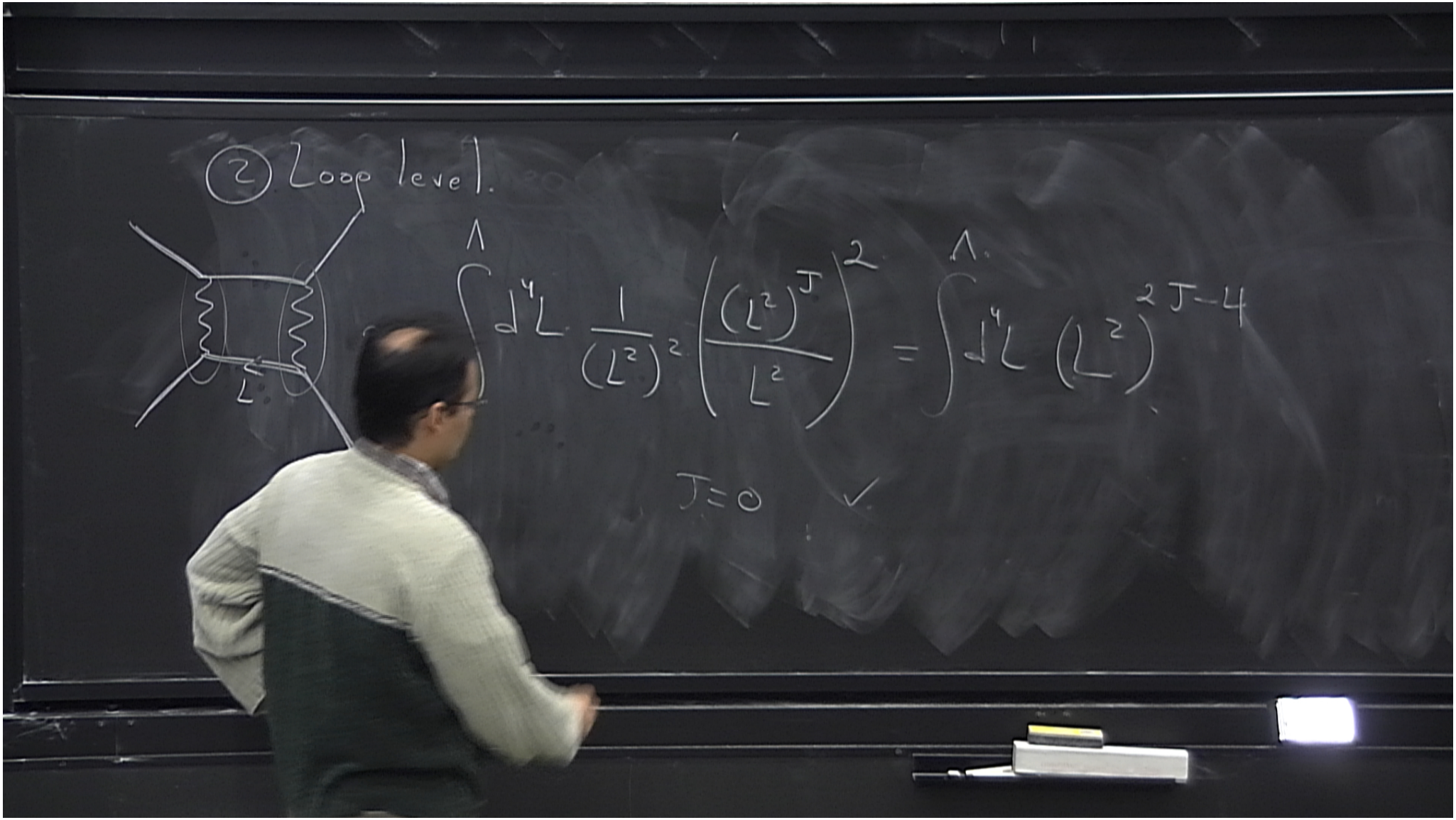

② Loop level.
L
~ ∫ d⁴L

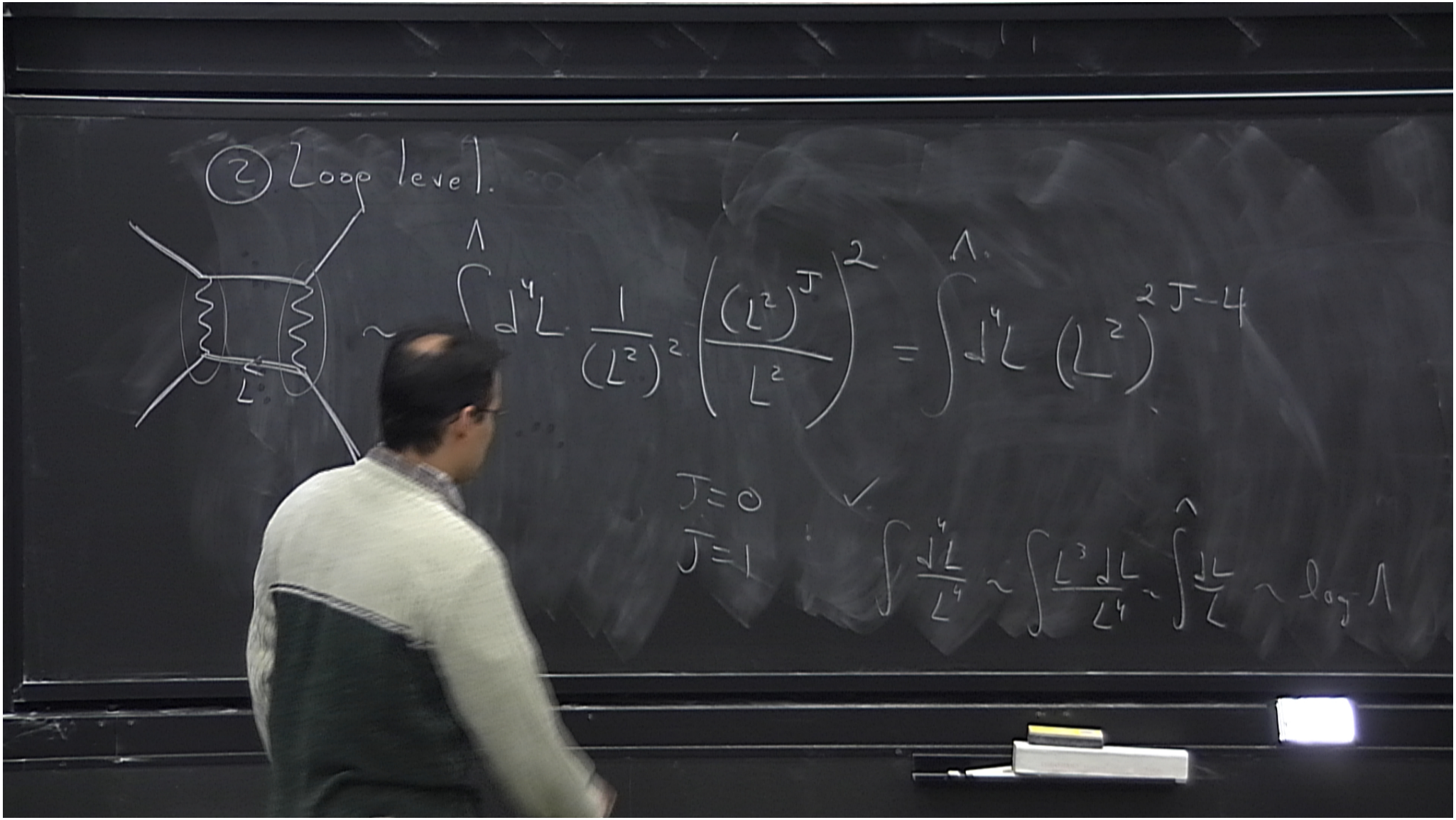

(2) Loop level.
L
$\int_{\Lambda} d^4L \frac{1}{(L^2)^2} \left(\frac{(L^2)^J}{L^2} \right)^2$

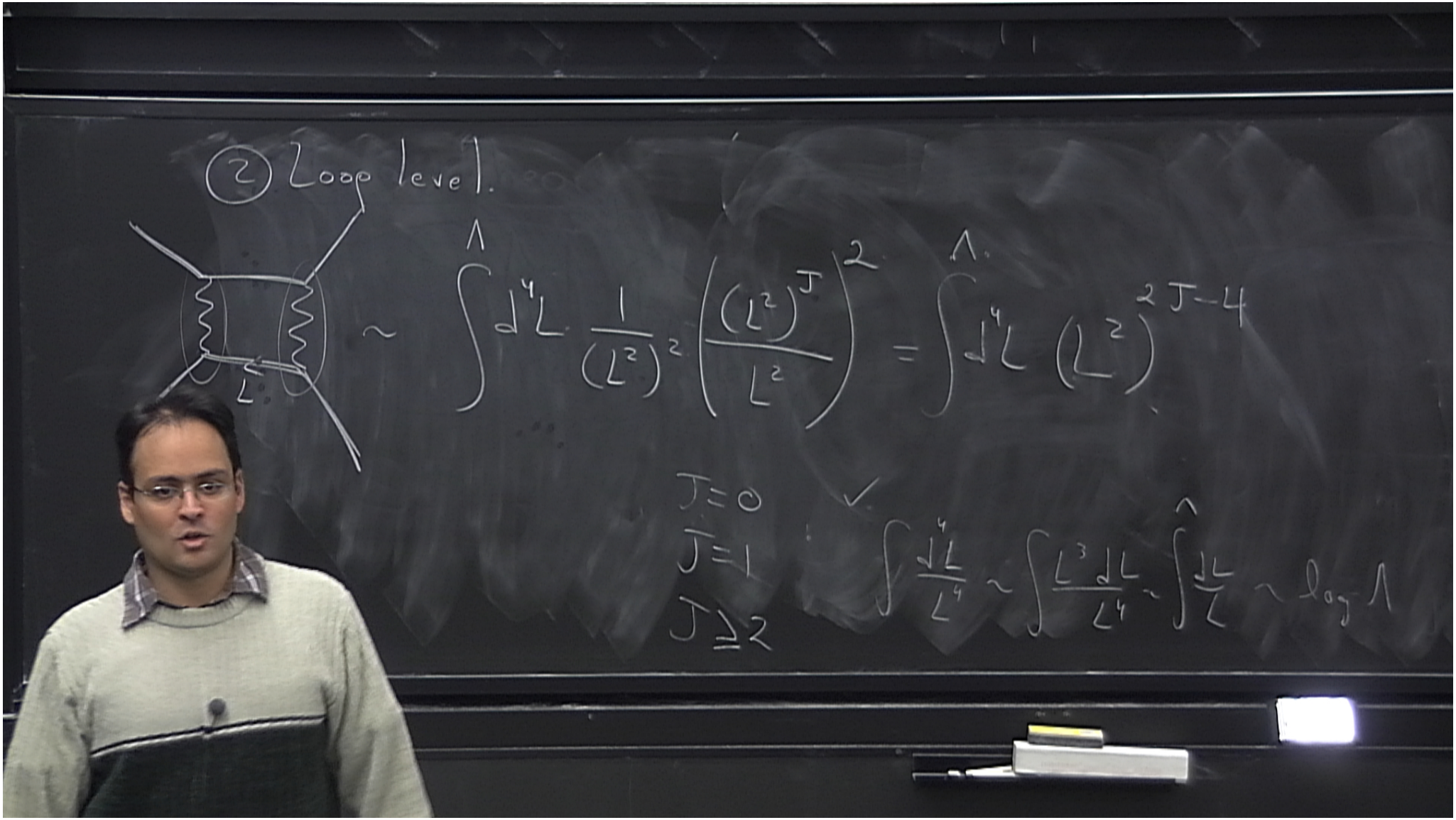
(2) Loop level.

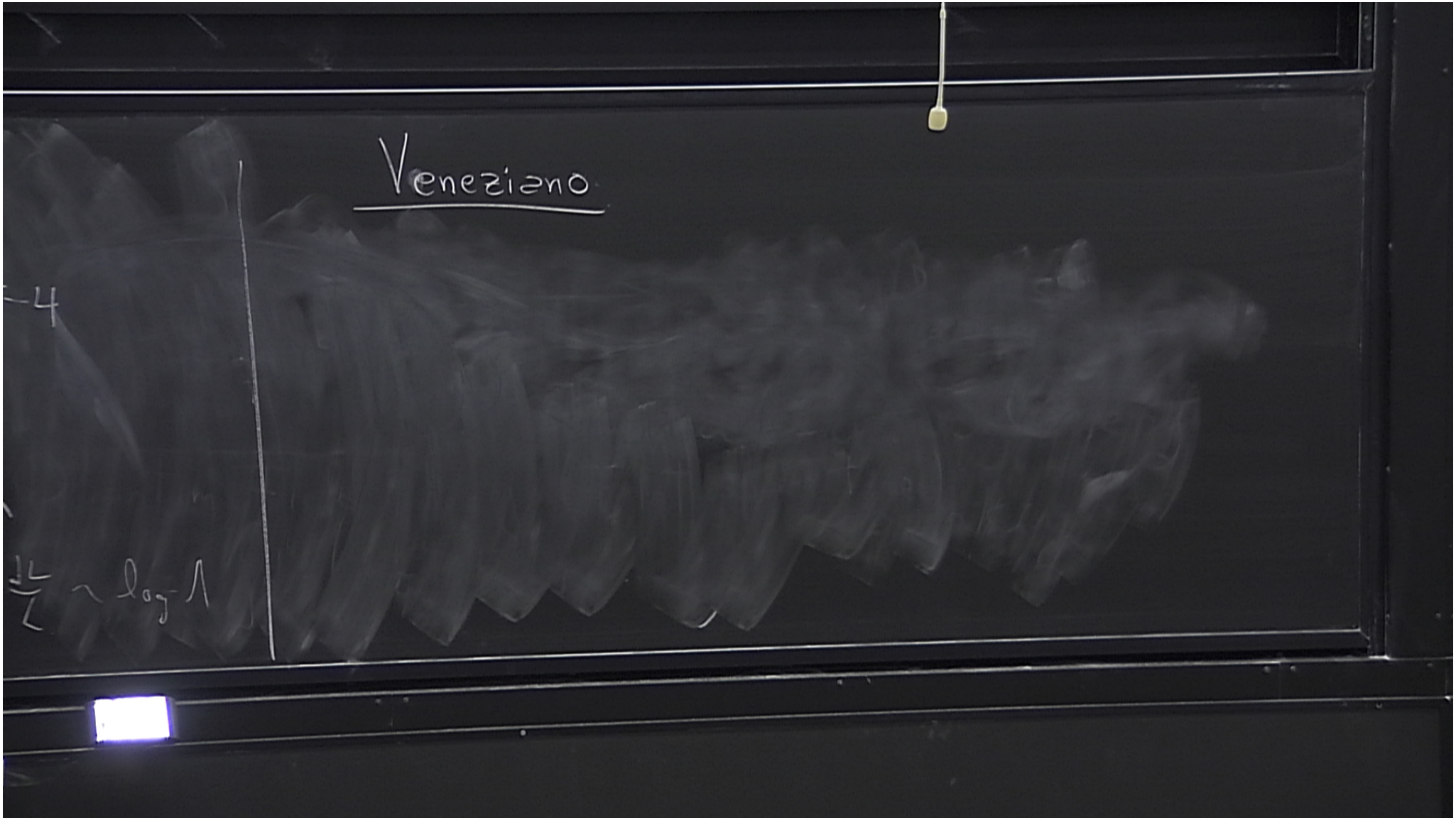


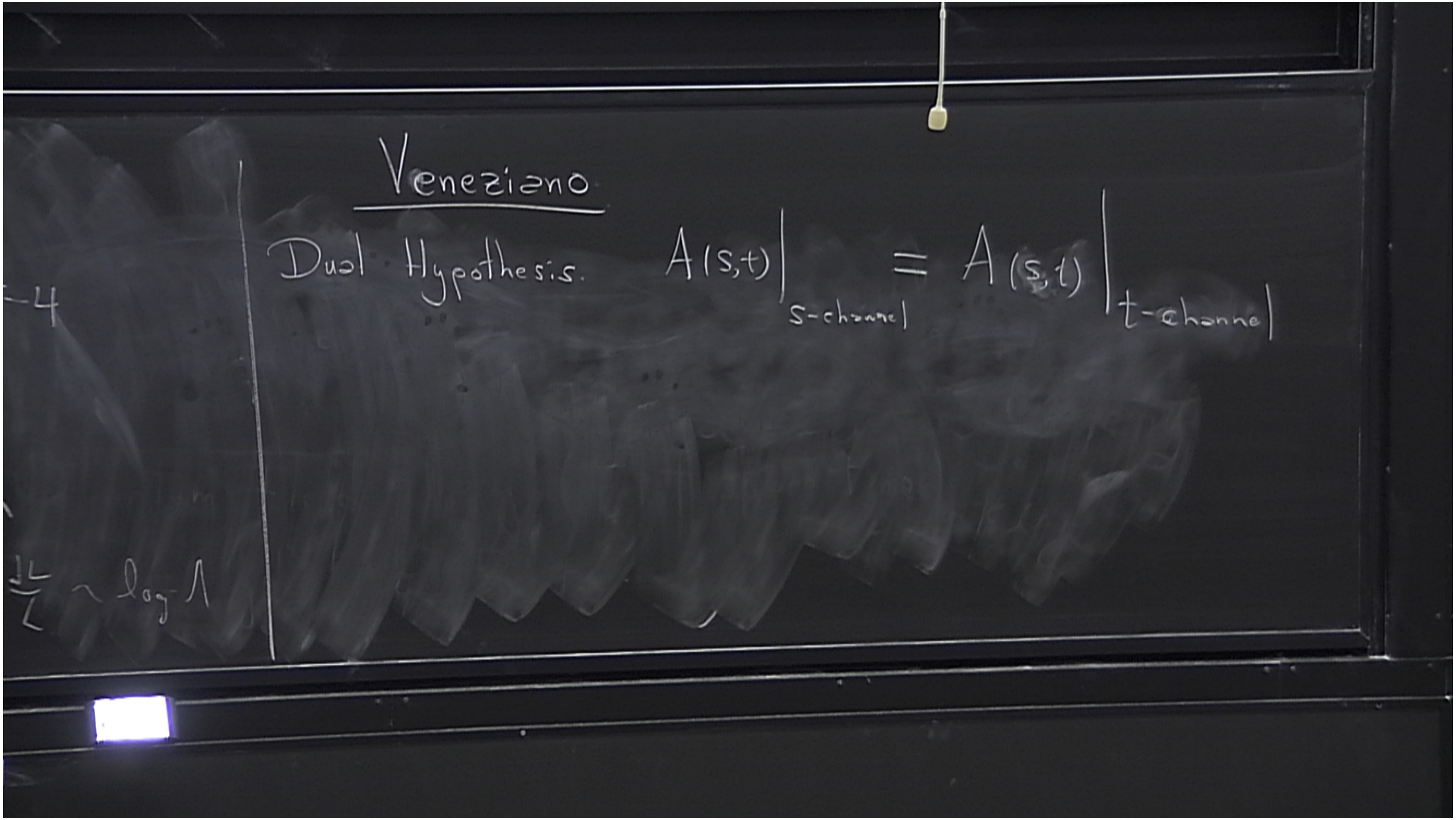
$$\sim \int^{\Lambda} d^4L \, \frac{1}{(L^2)^2} \left(\frac{(L^2)^J}{L^2} \right)^2 = \int^{\Lambda} d^4L \, (L^2)^{2J-4}$$

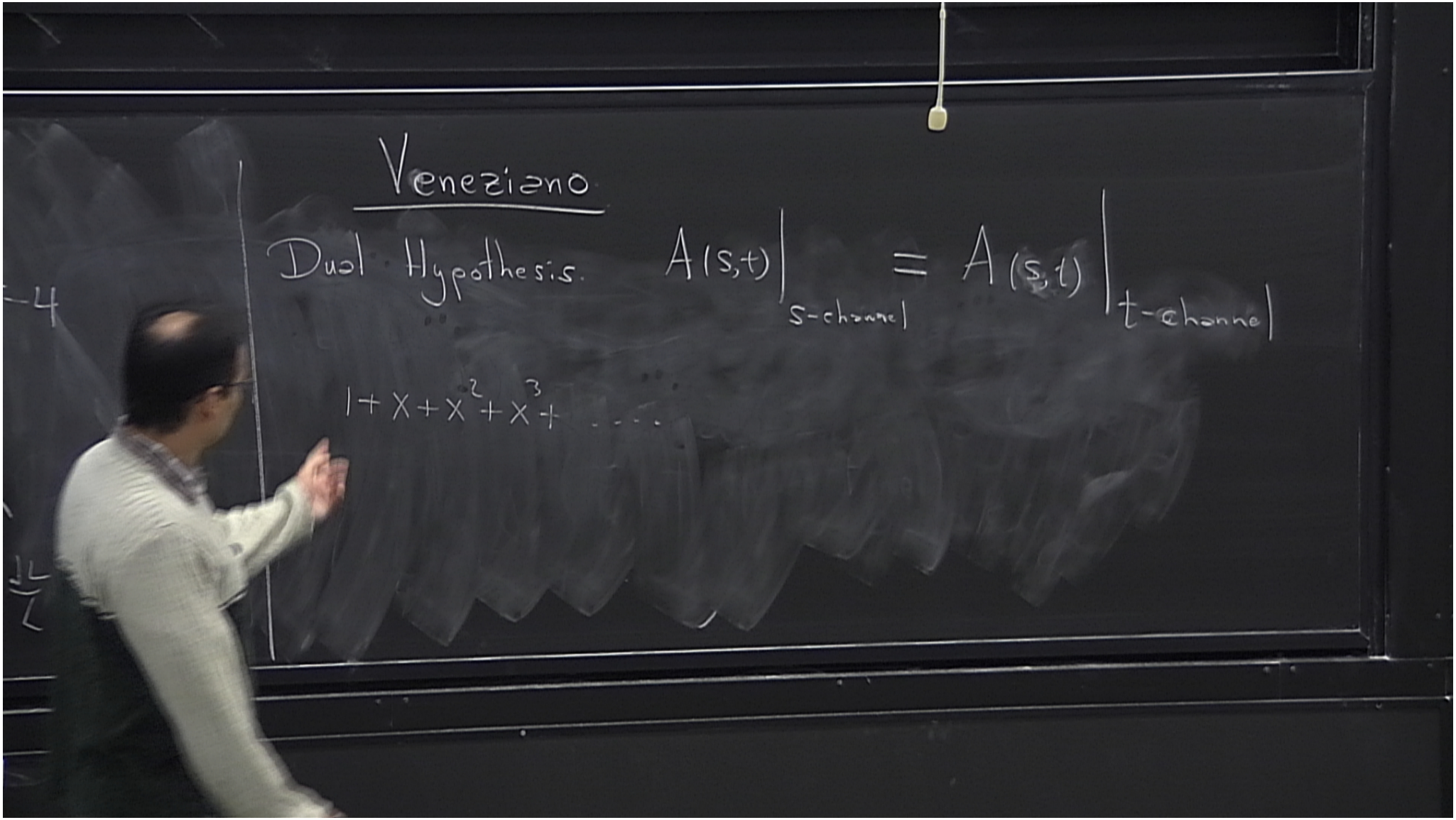

(2) Loop level.
L
$\int_{\Lambda} d^4L \frac{1}{(L^2)^2} \left(\frac{(L^2)^J}{L^2}\right)^2 = \int_{\Lambda} d^4L \, (L^2)^{2J-4}$
J=0 ✓


② Loop level.
L
$\sim \int_{\Lambda} d^4L \frac{1}{(L^2)^2} \left(\frac{(L^2)^J}{L^2}\right)^2 = \int_{\Lambda} d^4L \, (L^2)^{2J-4}$
J=0
J=1 :
$\int \frac{d^4L}{L^4} \sim \int \frac{L^3 dL}{L^4} \sim \int \frac{dL}{L} \sim \log \Lambda$


(2) Loop level.
$\sim \int^{\Lambda} d^4L \frac{1}{(L^2)^2} \left(\frac{(L^2)^J}{L^2}\right)^2 = \int^{\Lambda} d^4L \, (L^2)^{2J-4}$
$J=0$ ✓
$J=1$: $\int \frac{d^4L}{L^4} \sim \int \frac{L^3 dL}{L^4} \sim \int \frac{dL}{L} \sim \log \Lambda$
$J \geq 2$


Veneziano


Veneziano
Dual Hypothesis. $A(s,t)\big|_{s\text{-channel}} = A(s,t)\big|_{t\text{-channel}}$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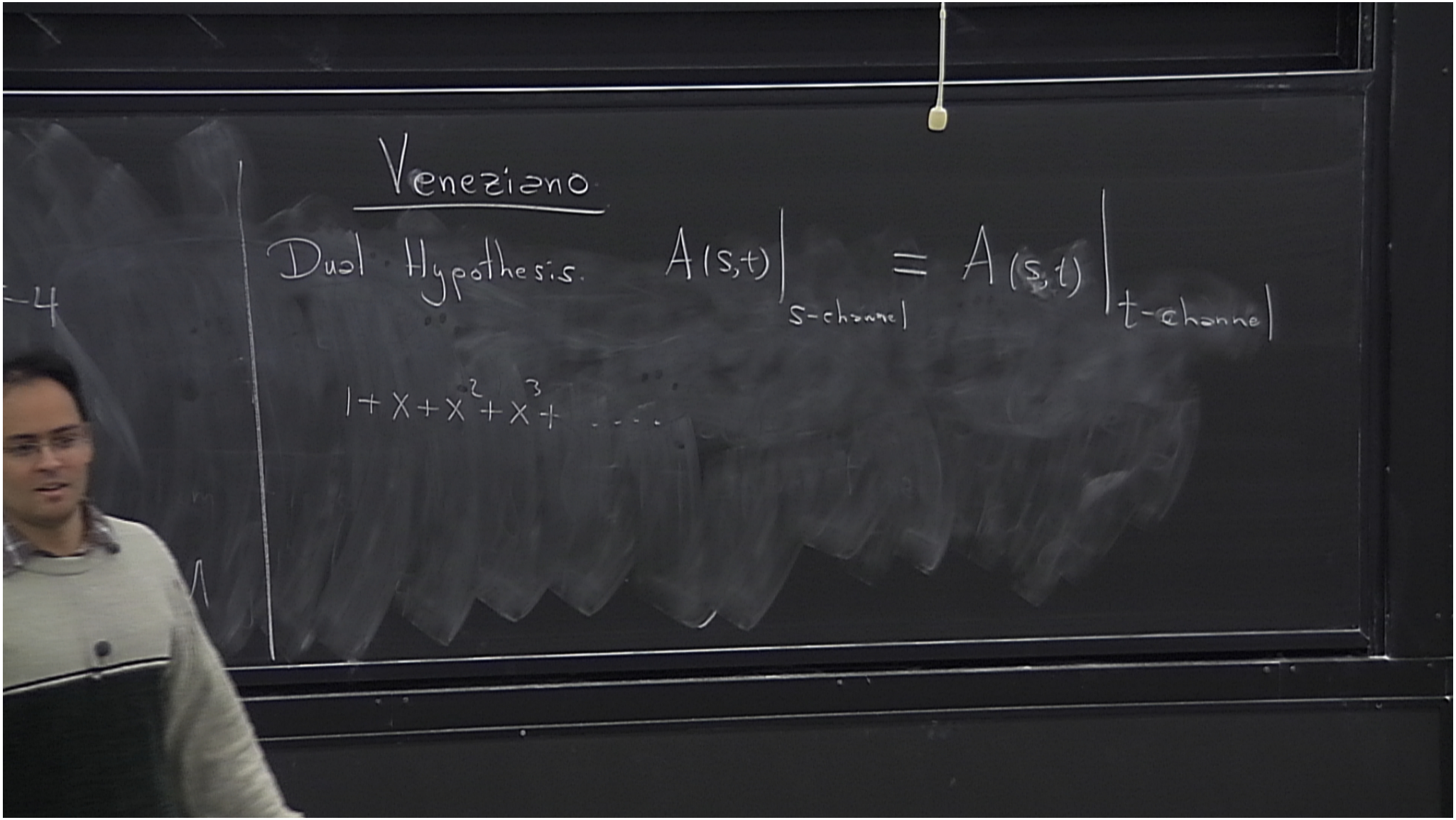
$1 + X + X^2 + X^3 + \ldots$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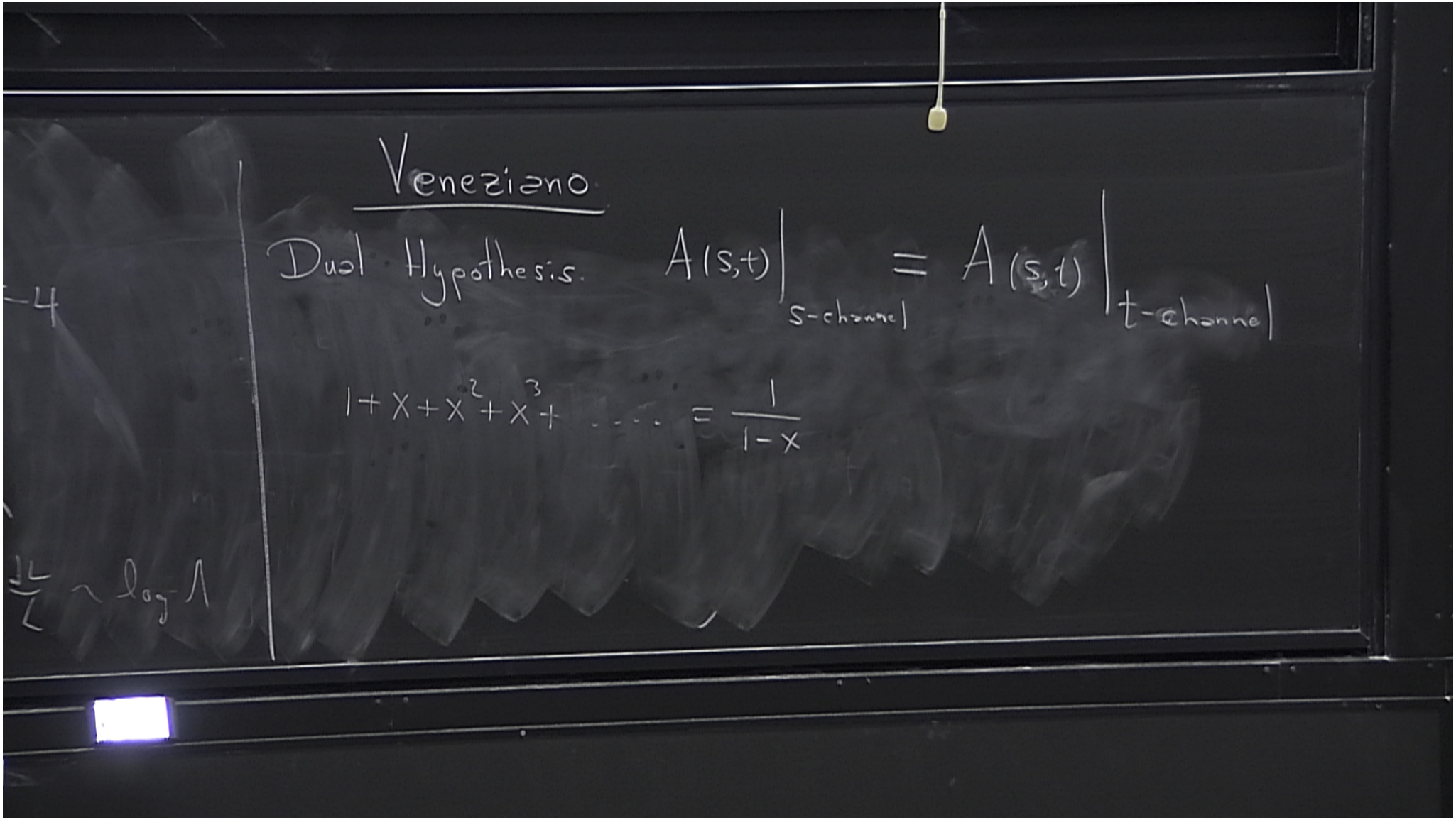
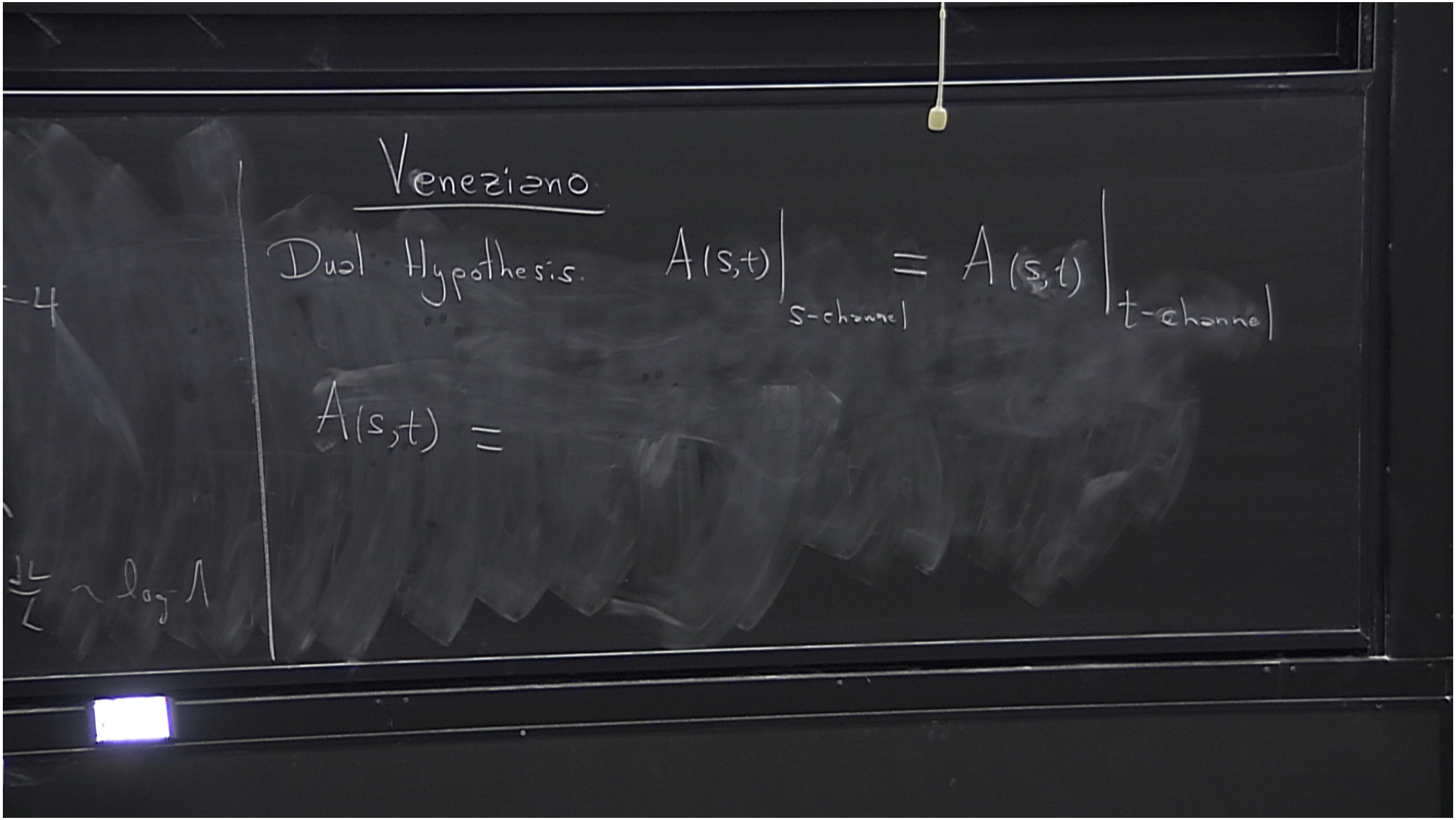
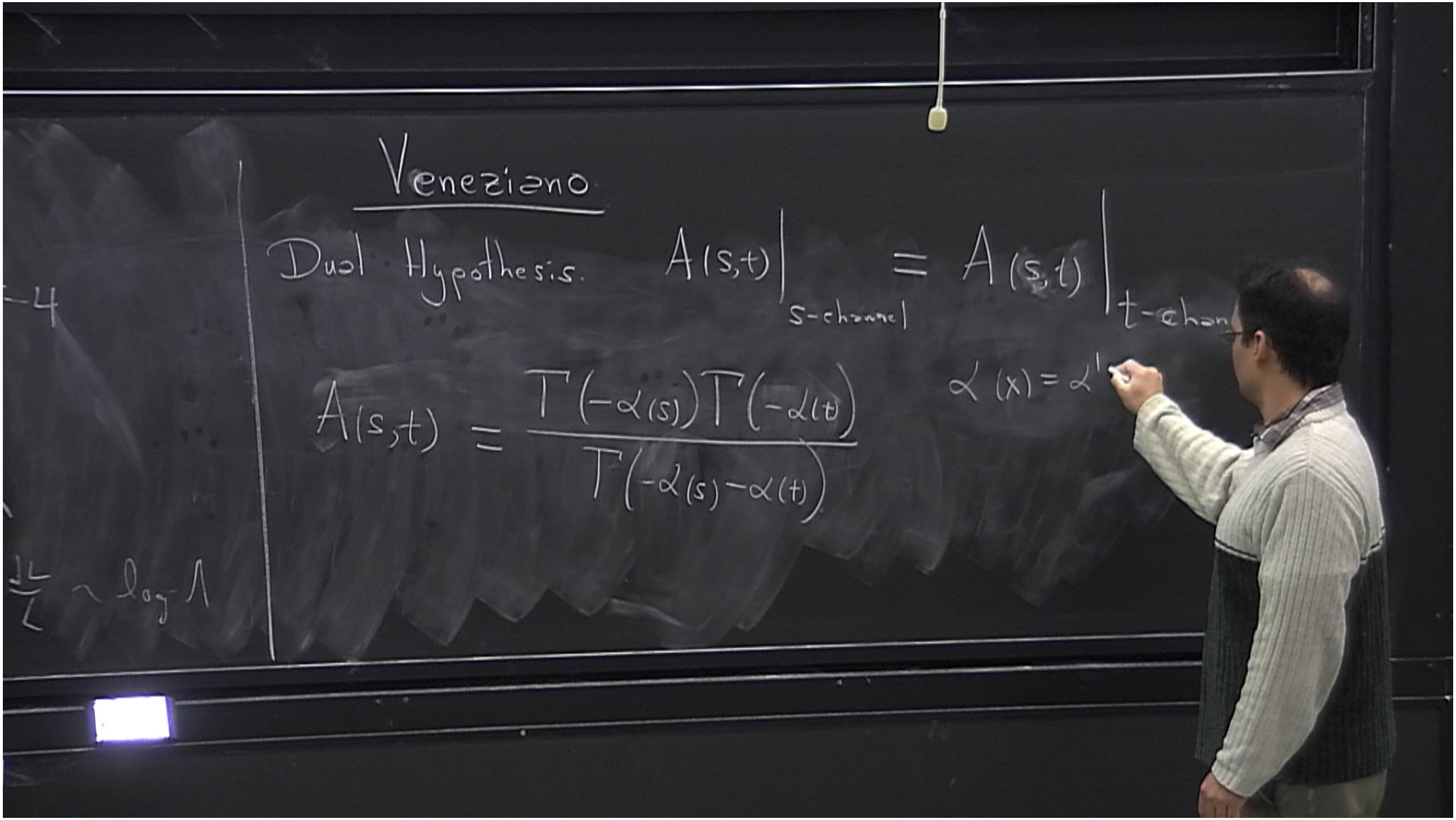
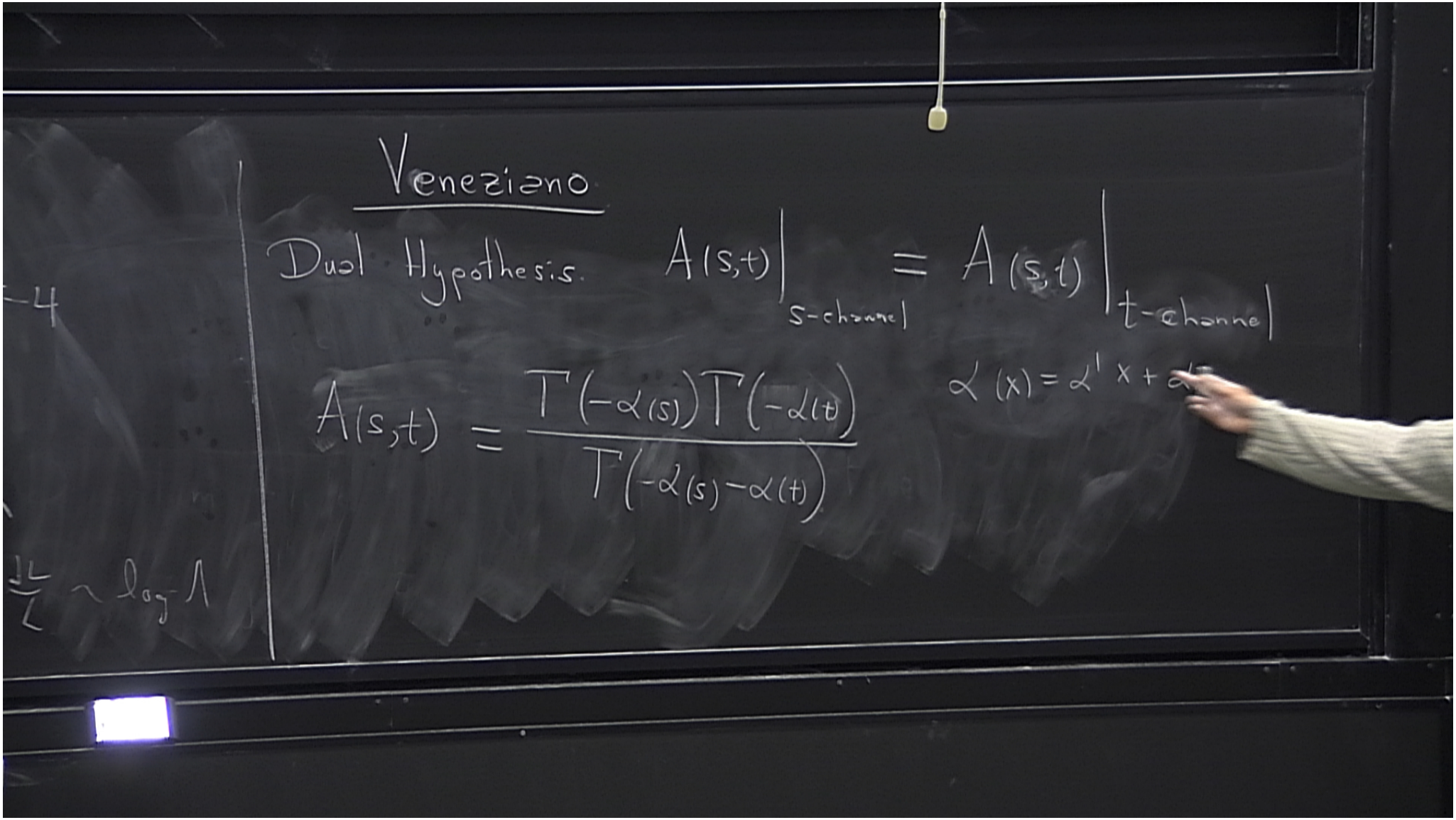

Veneziano
Dual Hypothesis. $A(s,t)\big|_{s\text{-channel}} = A(s,t)\big|_{t\text{-channel}}$
$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$


Veneziano
Dual Hypothesis. $A(s,t)\big|_{s\text{-channel}} = A(s,t)\big|_{t\text{-channel}}$
$A(s,t) =$


Veneziano
Dual Hypothesis. $A(s,t)\big|_{s\text{-channel}} = A(s,t)\big|_{t\text{-chan}}$
$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$
$\alpha(x) = \alpha'$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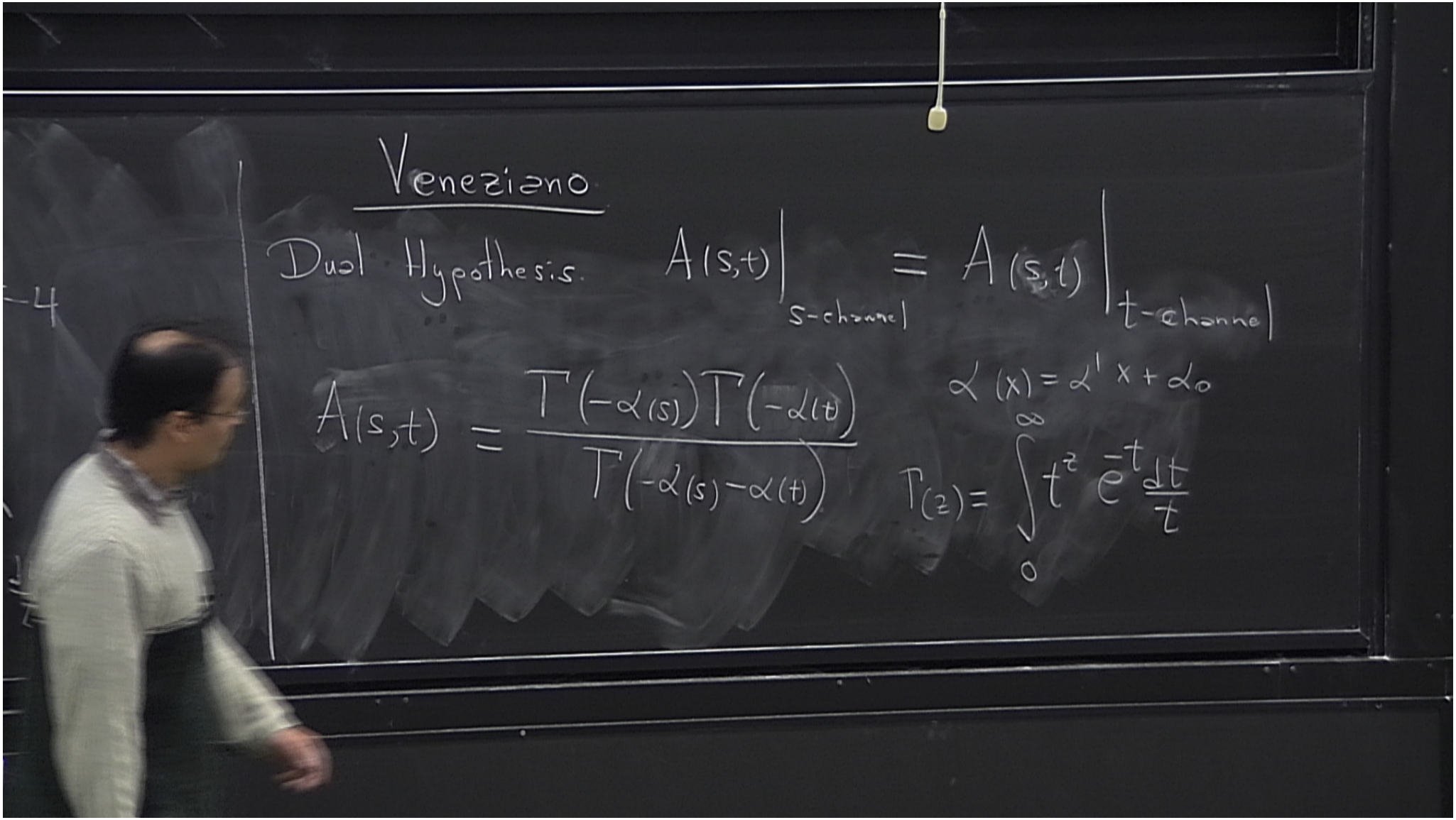
$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$
$\alpha(x) = \alpha' x + \alpha$
$\frac{1}{L} \sim \log \Lambda$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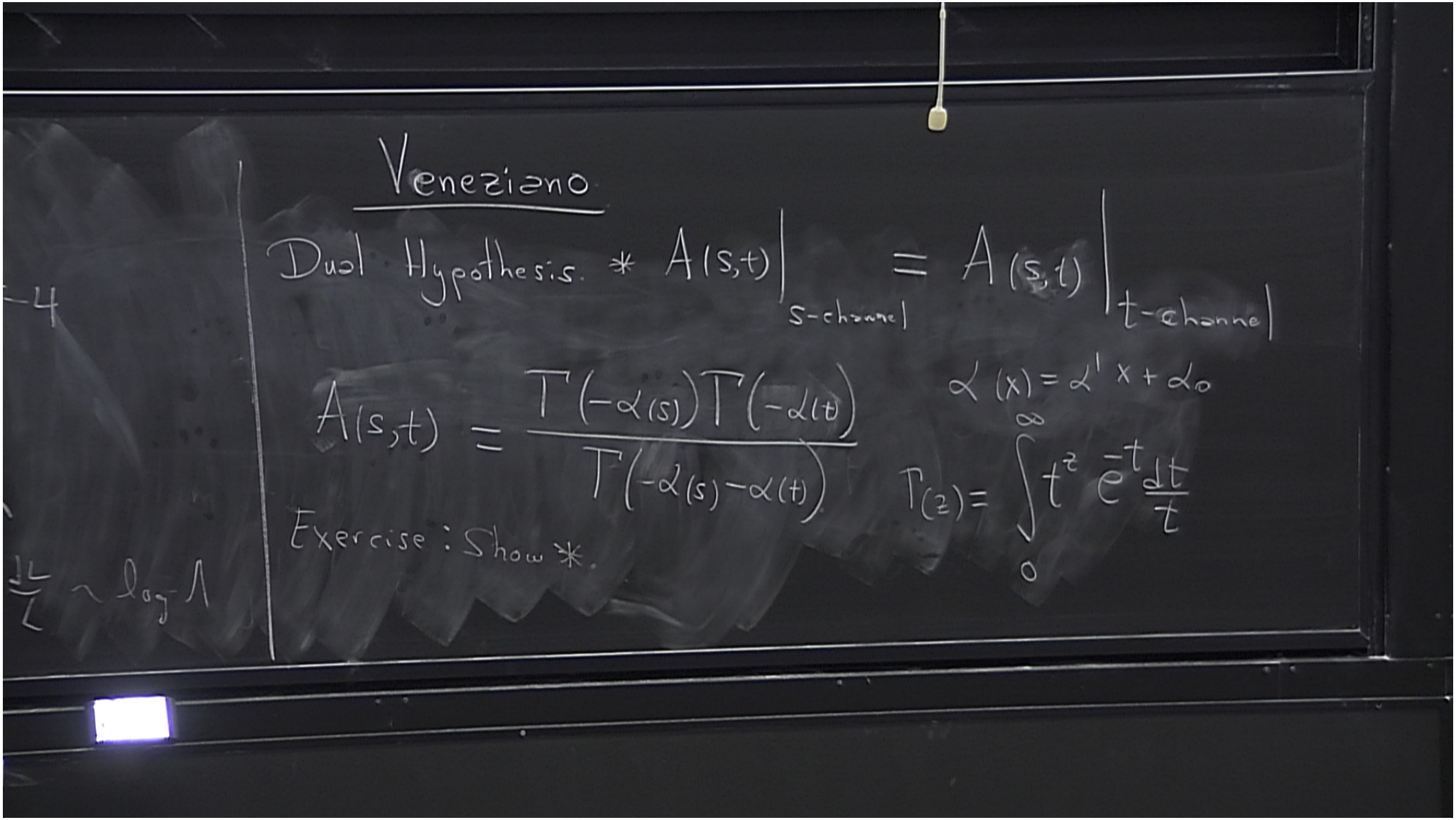
$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$
$\alpha(x) = \alpha' x + \alpha_0$
$\Gamma(z) = \int_0^\infty t^z e^{-t} \frac{dt}{t}$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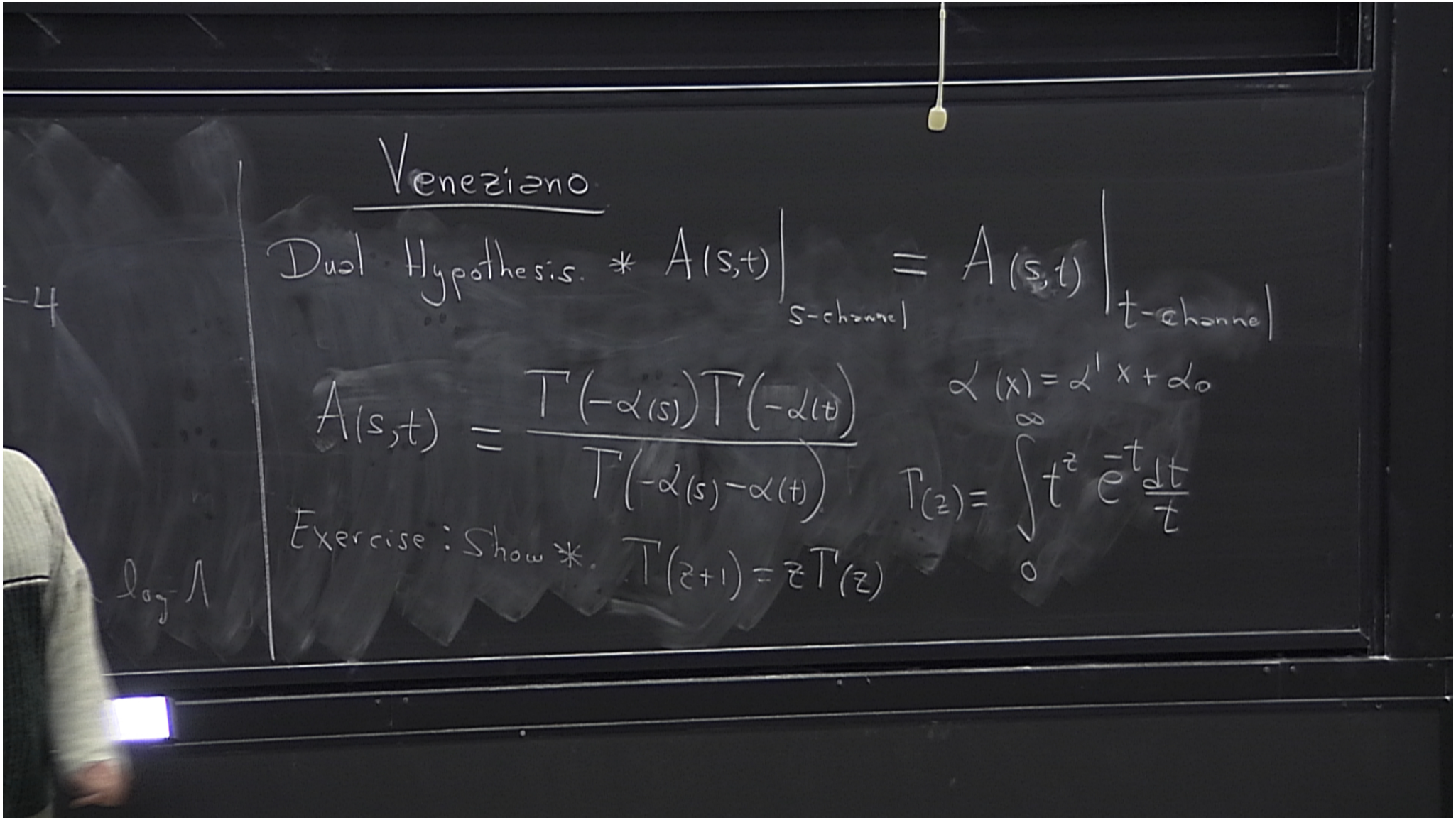
$\Gamma(z) = \int_0^\infty t^z e^{-t} \frac{dt}{t}$
Exercise: Show *.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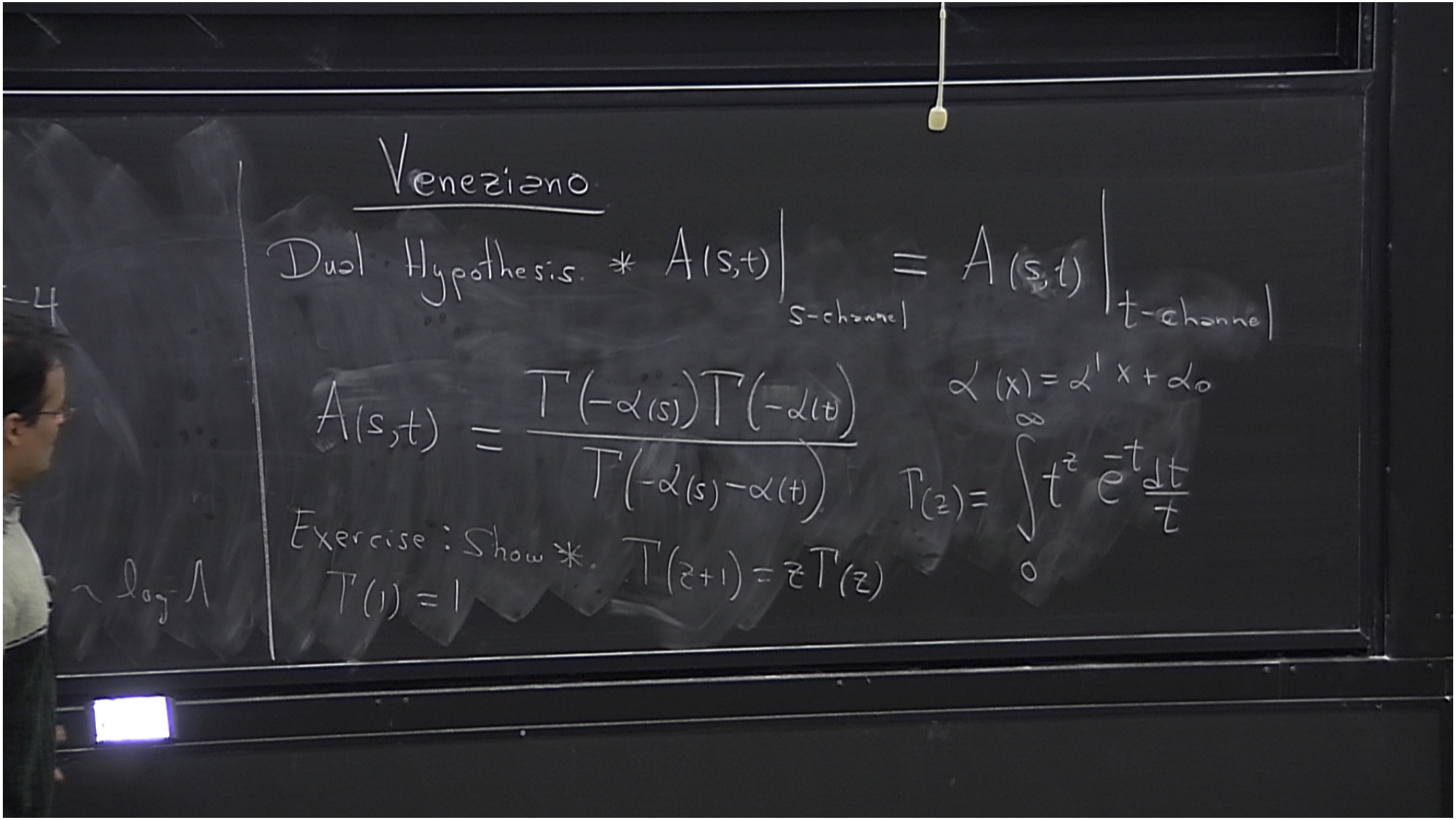
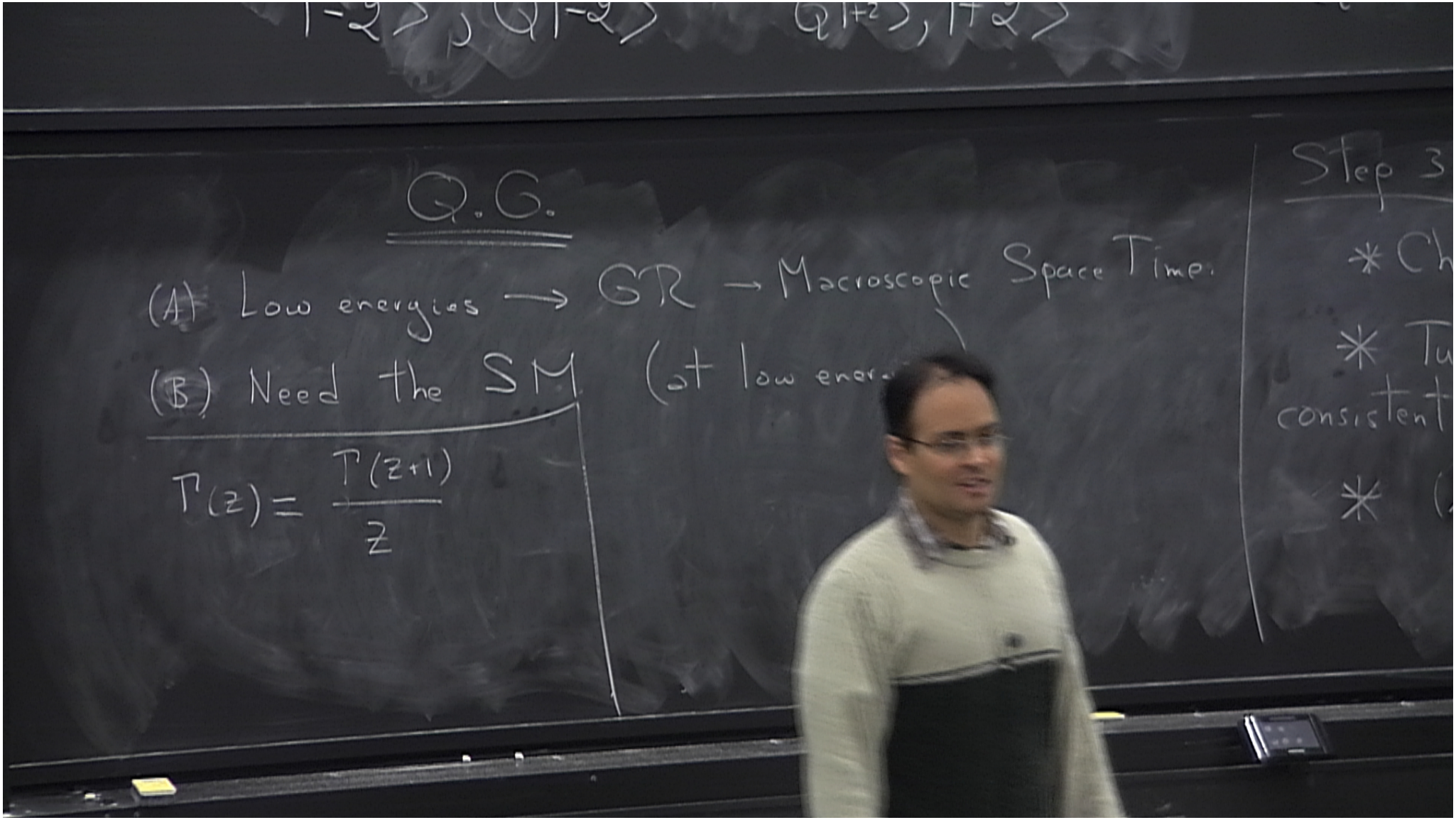
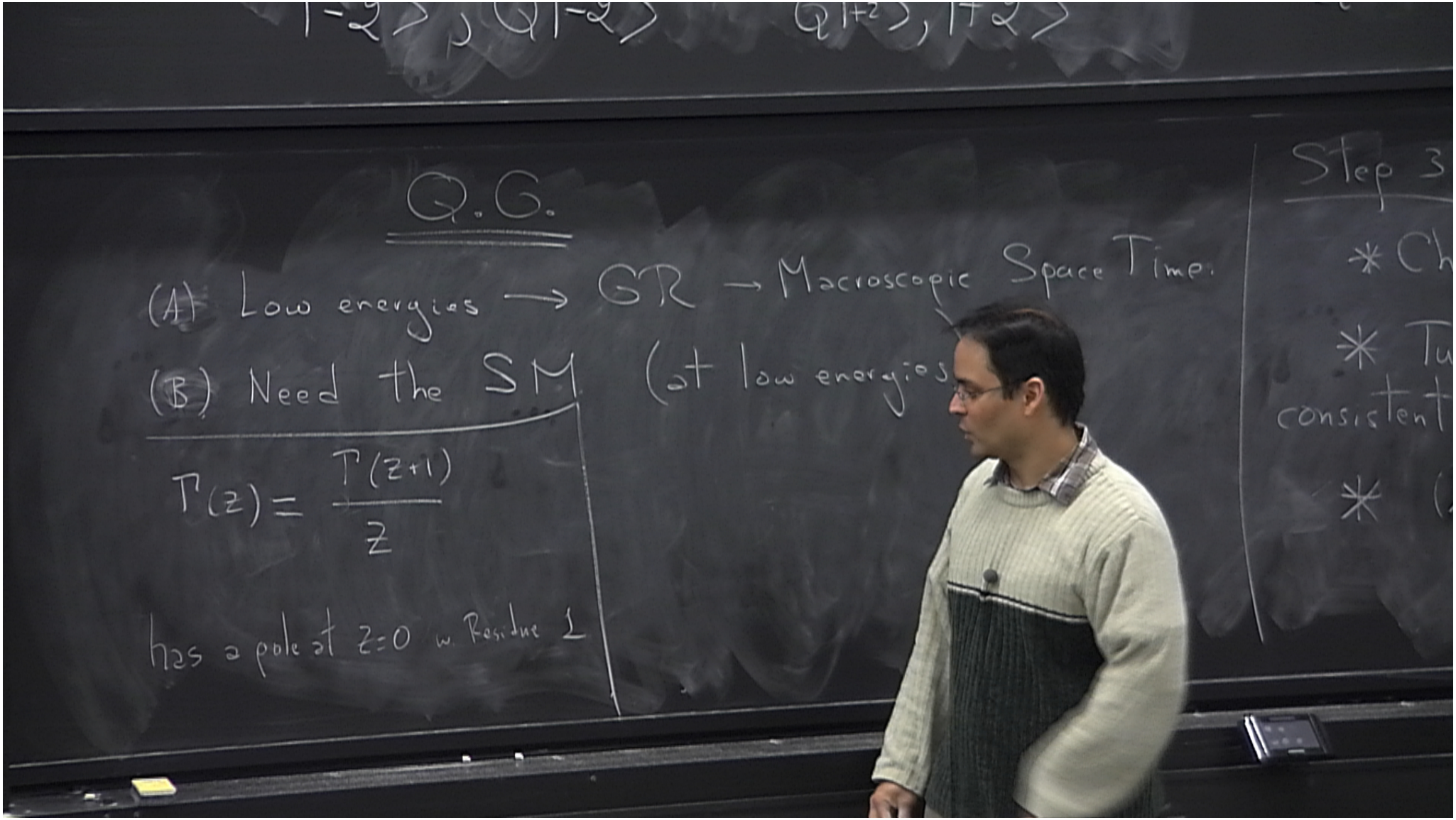
Exercise: Show *. $\Gamma(z+1) = z\Gamma(z)$


Veneziano
Dual Hypothesis. * $A(s,t)\big|_{s\text{-channel}} = A(s,t)\big|_{t\text{-channel}}$
$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$
$\alpha(x) = \alpha' x + \alpha_0$
$\Gamma(z) = \int_0^\infty t^z e^{-t} \frac{dt}{t}$
Exercise: Show *. $\Gamma(z+1) = z\Gamma(z)$
$\Gamma(1) = 1$


Q.G.
(A) Low energies → GR → Macroscopic Space Time.
(B) Need the SM
$\Gamma(z) = \frac{\Gamma(z+1)}{z}$
Step 3
consistent


Q.G.
(A) Low energies → GR → Macroscopic Space Time.
(B) Need the SM (at low energies
$\Gamma(z) = \frac{\Gamma(z+1)}{z}$
has a pole at z=0 w. Residue 1
Step 3
consistent