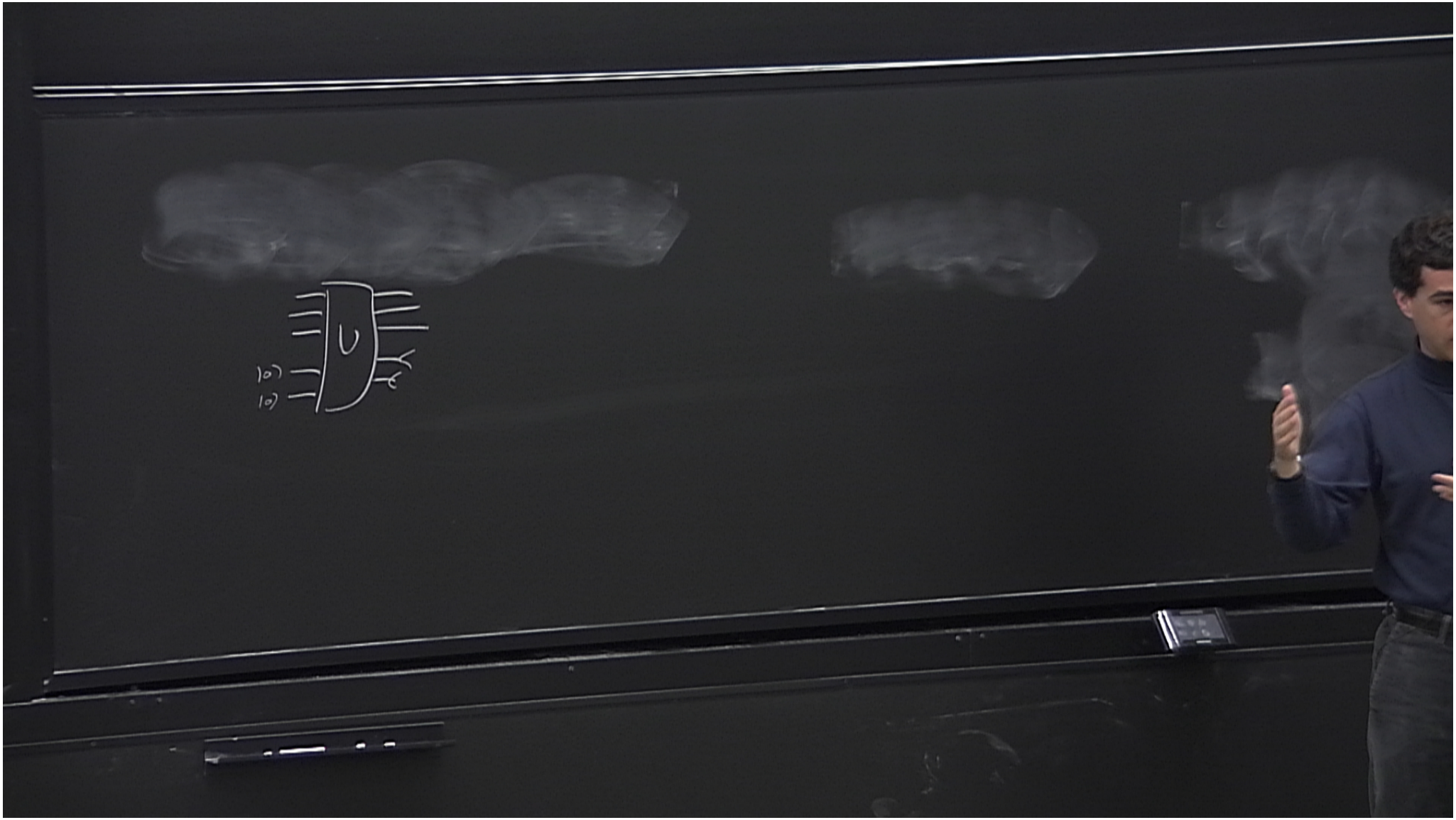
Title: Quantum Information (Review) - Lecture 11

Date: Feb 28, 2012 10:15 AM

URL: http://pirsa.org/12020045

Abstract: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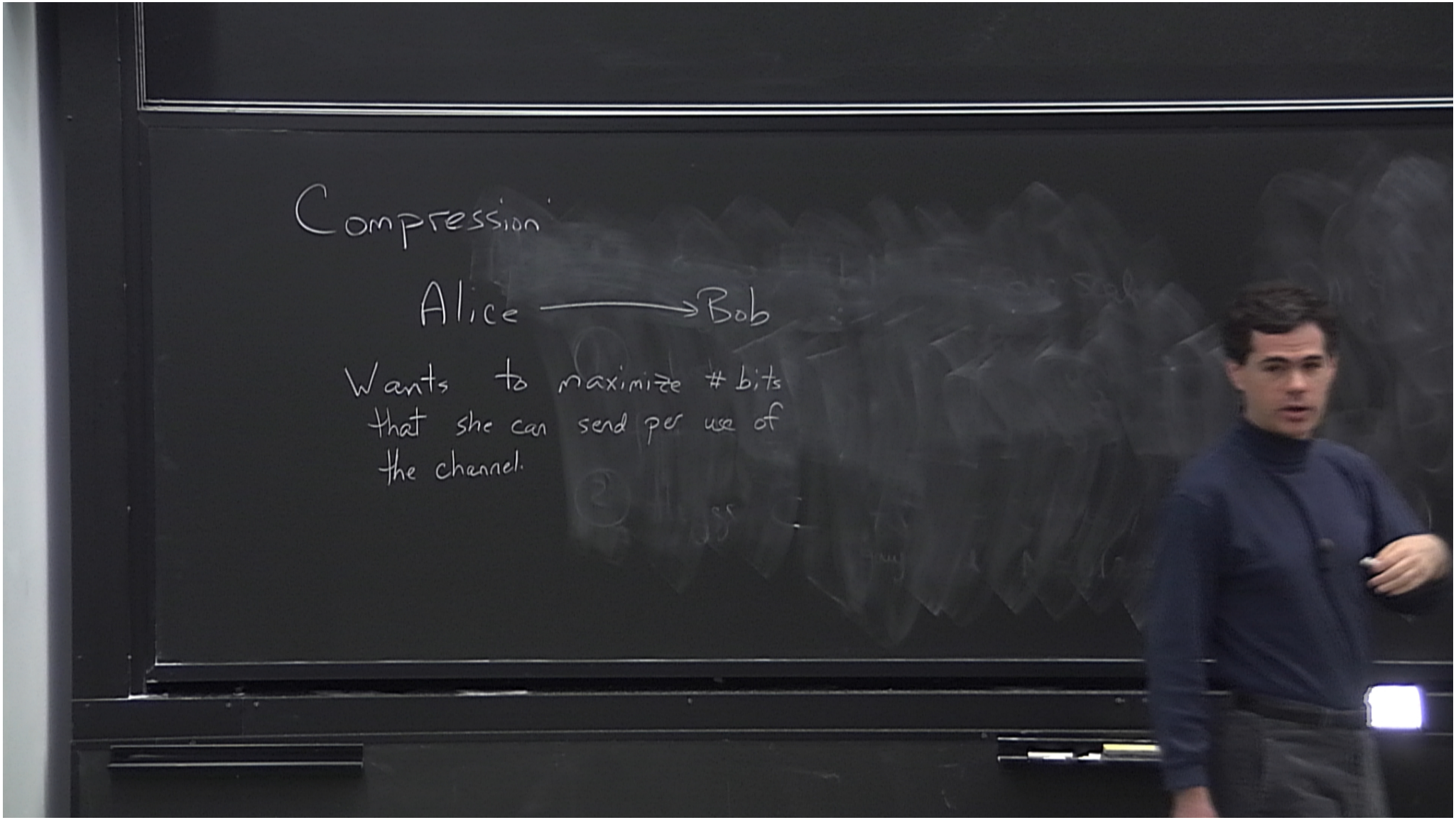
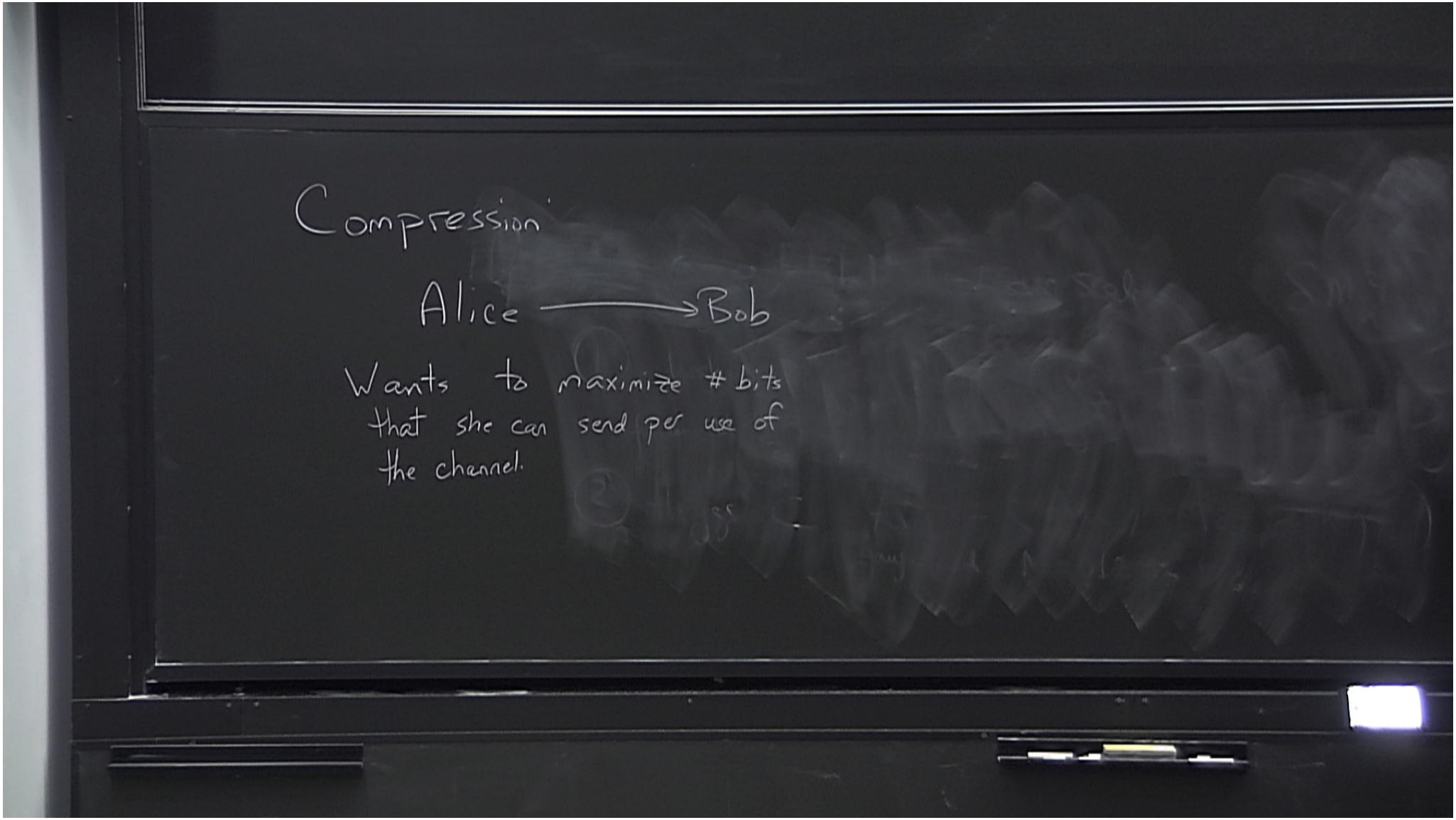

Compression
Alice
Bob
Wants to maximize # bits that she can send per use of the channel.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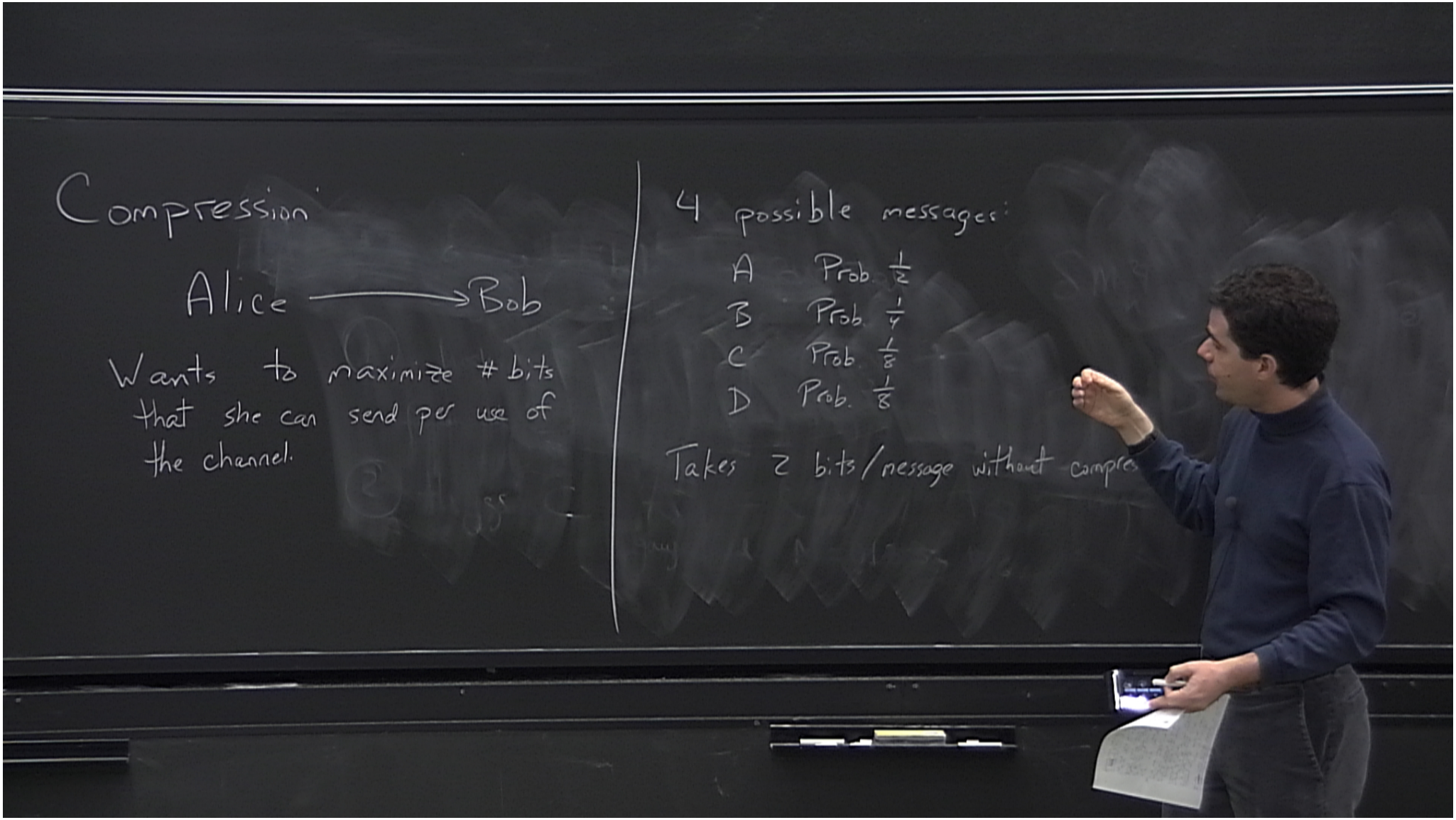
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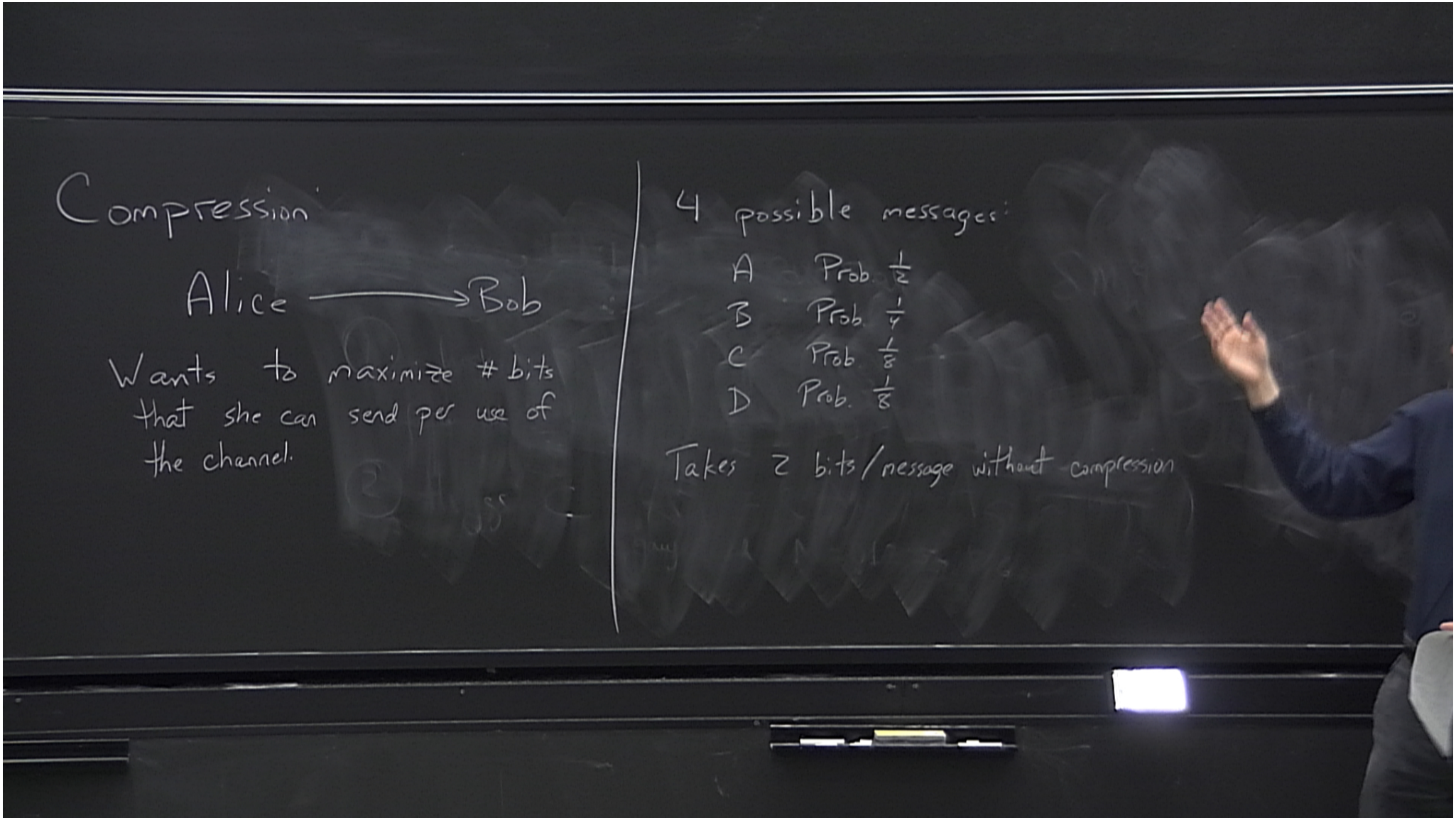
Alice → Bob
Wants to maximize # bits that she can send per use of the channel.
4 possible messages:
A Prob. 1/2
B Prob. 1/4
C Prob. 1/8
D Prob. 1/8
Takes 2 bits/message without compres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Compression

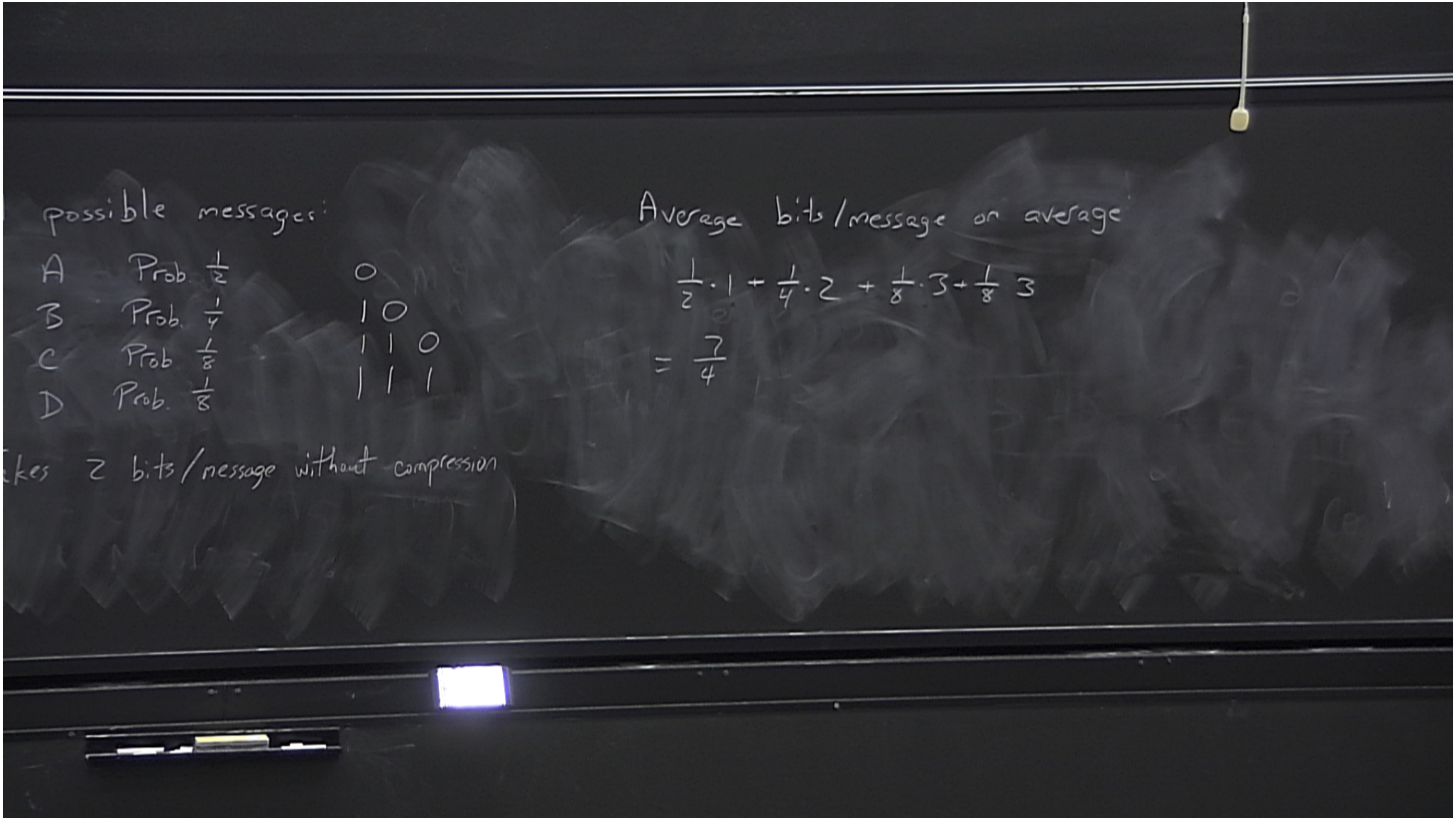
Alice ⟶ Bob

Wants to maximize # bits that she can send per use of the channel.

4 possible messages:

A	Prob $\frac{1}{2}$	0
B	Prob $\frac{1}{4}$	1 0
C	Prob $\frac{1}{8}$	1 1 0
D	Prob $\frac{1}{8}$	1 1 1

Takes 2 bits/message without compression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A Prob. 1/2 0
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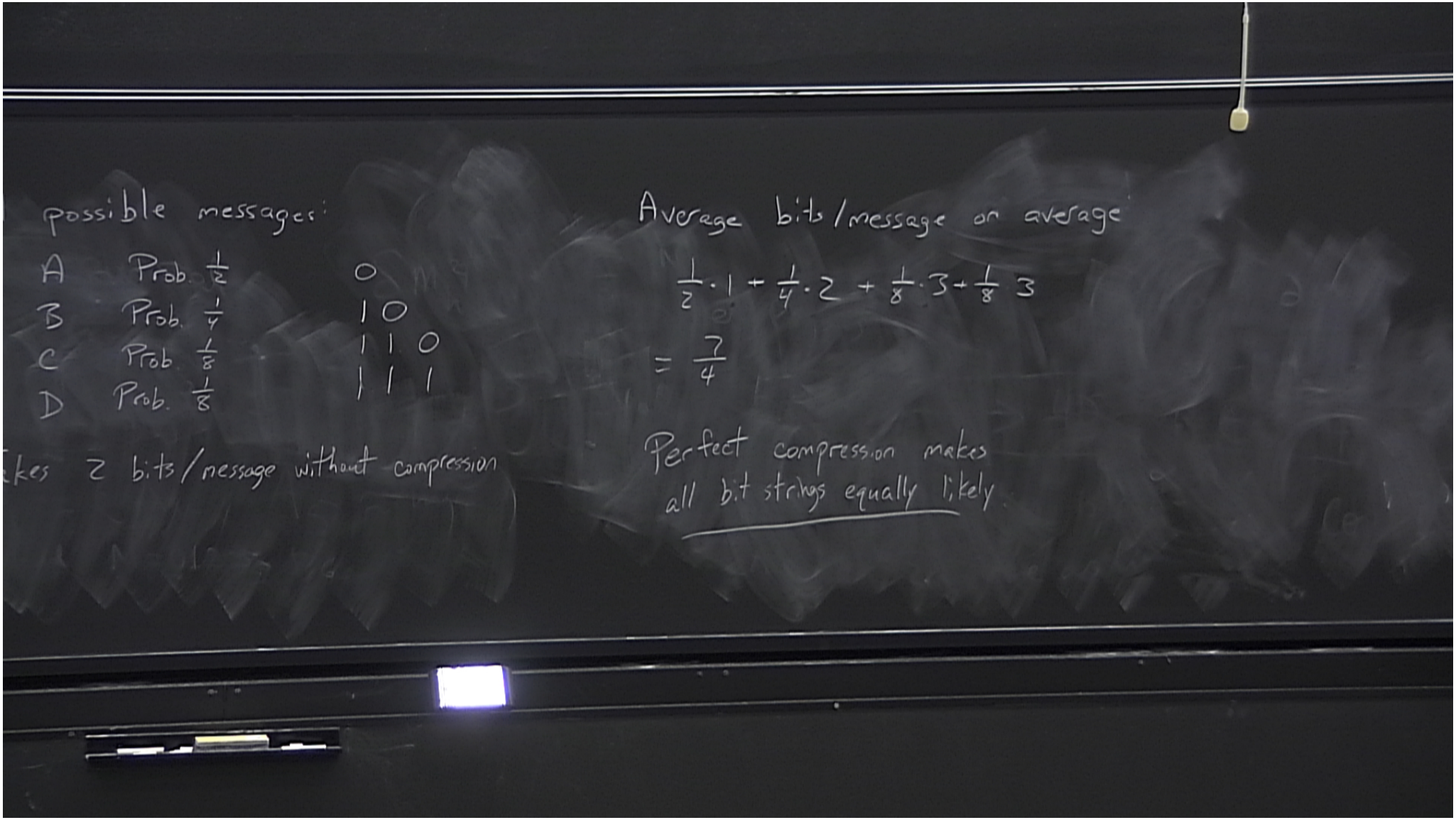
Average bits/message or average
1/2 · 1 + 1/4 · 2 + 1/8 · 3 + 1/8 3
= 7/4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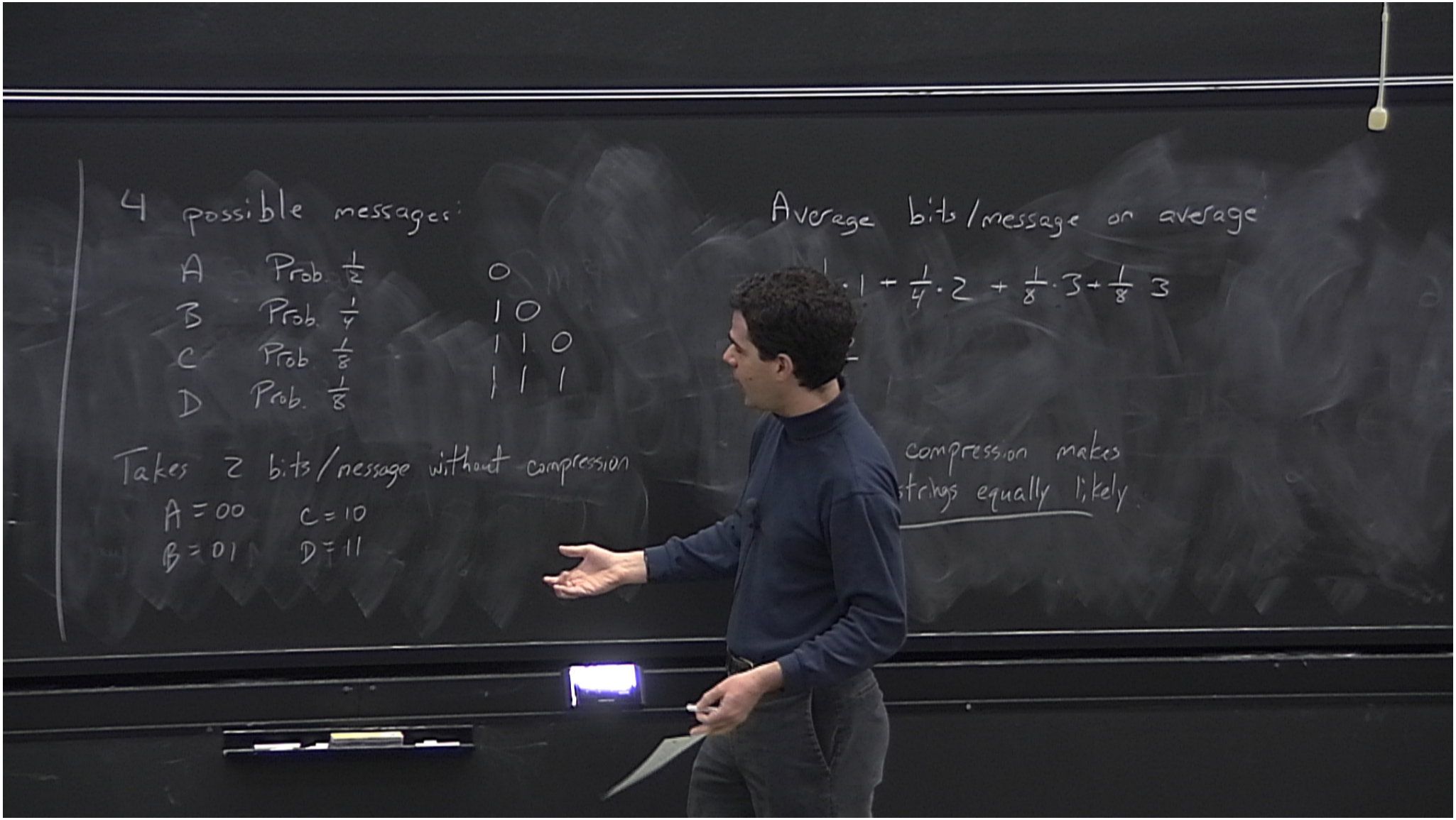
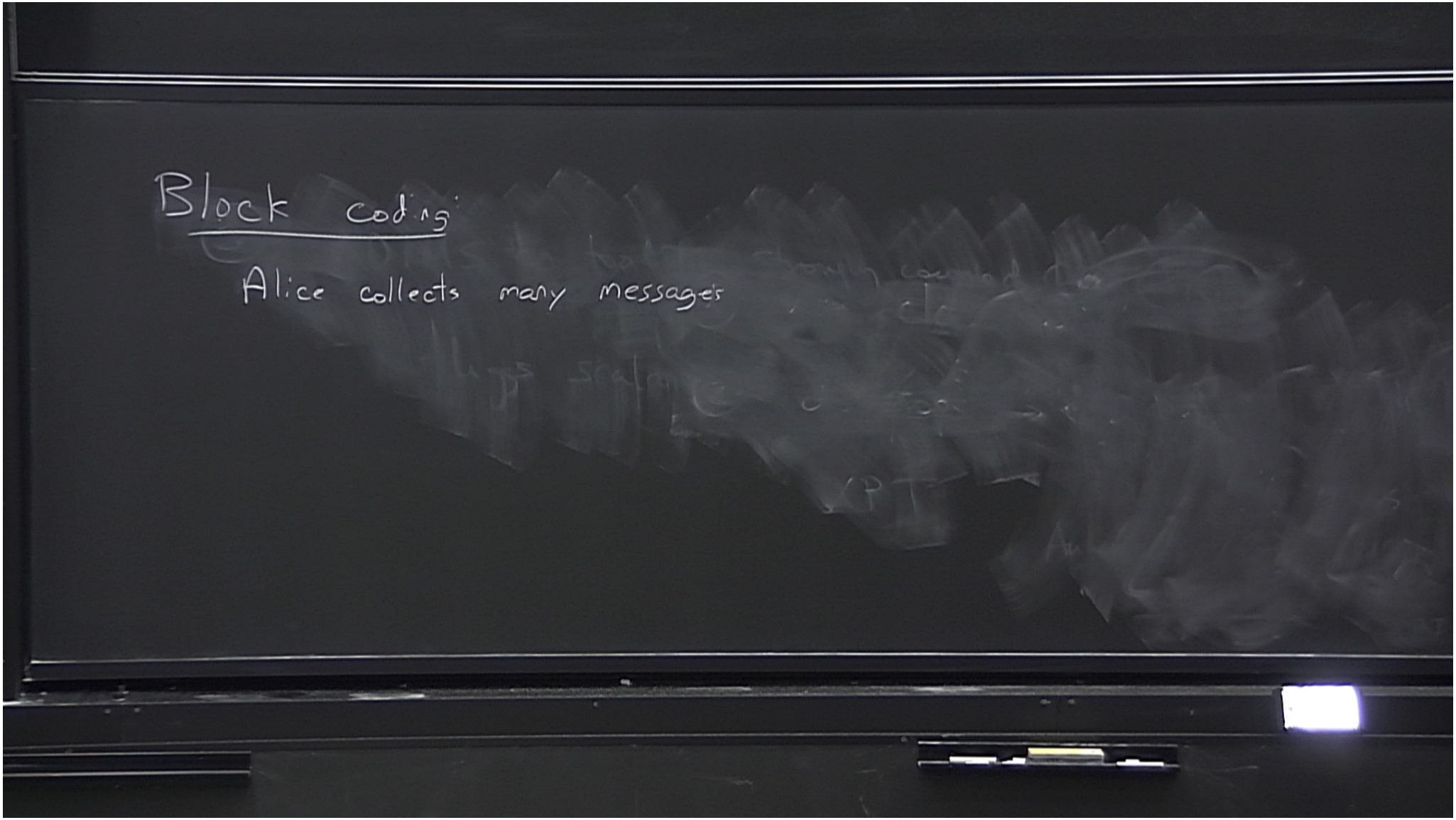
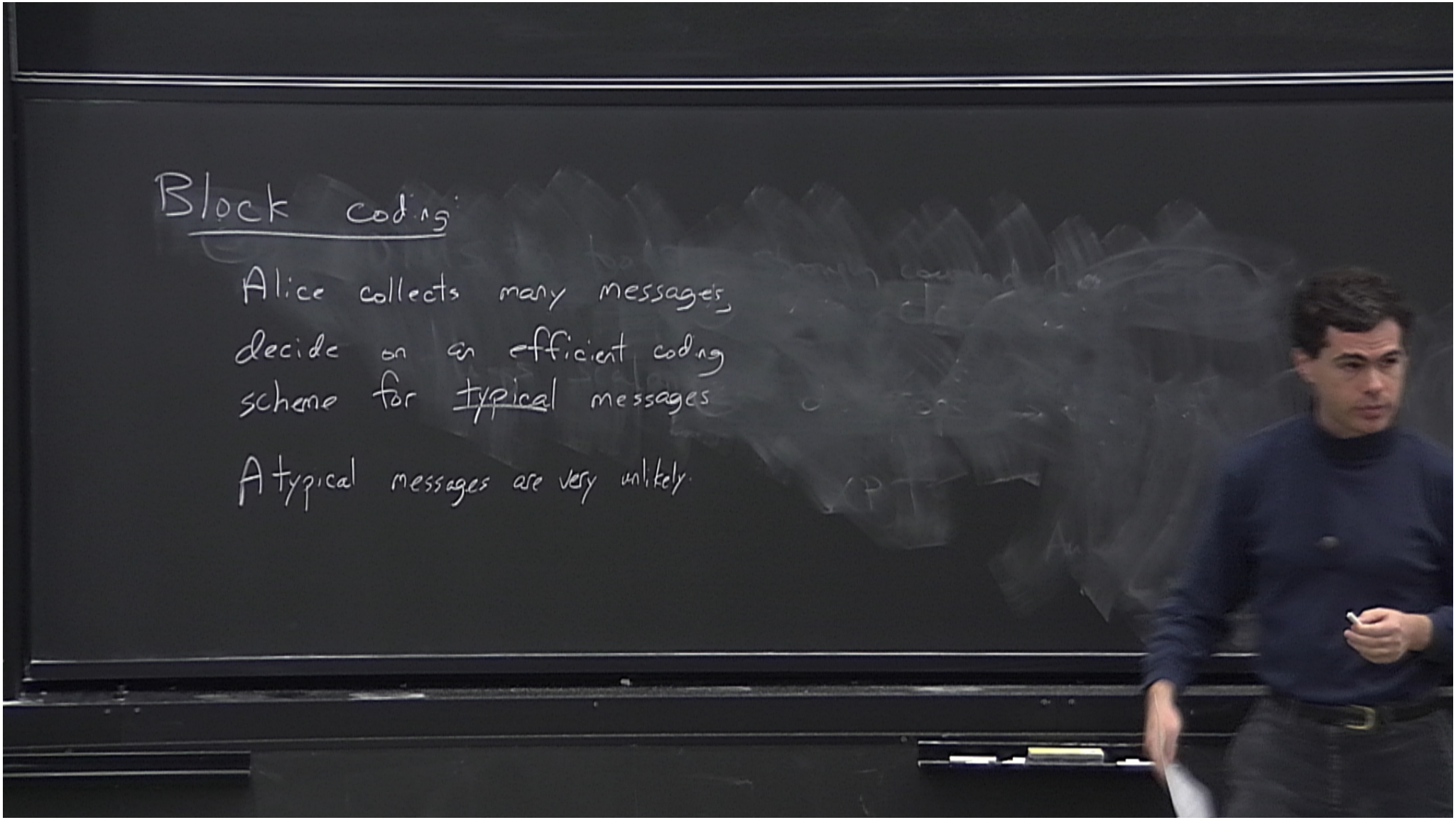
Perfect compression makes
all bit strings equally likely


4 possible messages:
A Prob. 1/2 0
B Prob. 1/4 1 0
C Prob 1/8 1 1 0
D Prob. 1/8 1 1 1
Takes 2 bits/message without compression
A = 00 C = 10
B = 01 D = 11
Average bits/message on average
1/2 · 1 + 1/4 · 2 + 1/8 · 3 + 1/8 3
compression makes
strings equally likely


Block coding
Alice collects many messages


Block coding
Alice collects many messages,
decide on an efficient coding
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Atypical messages are very unlikely.

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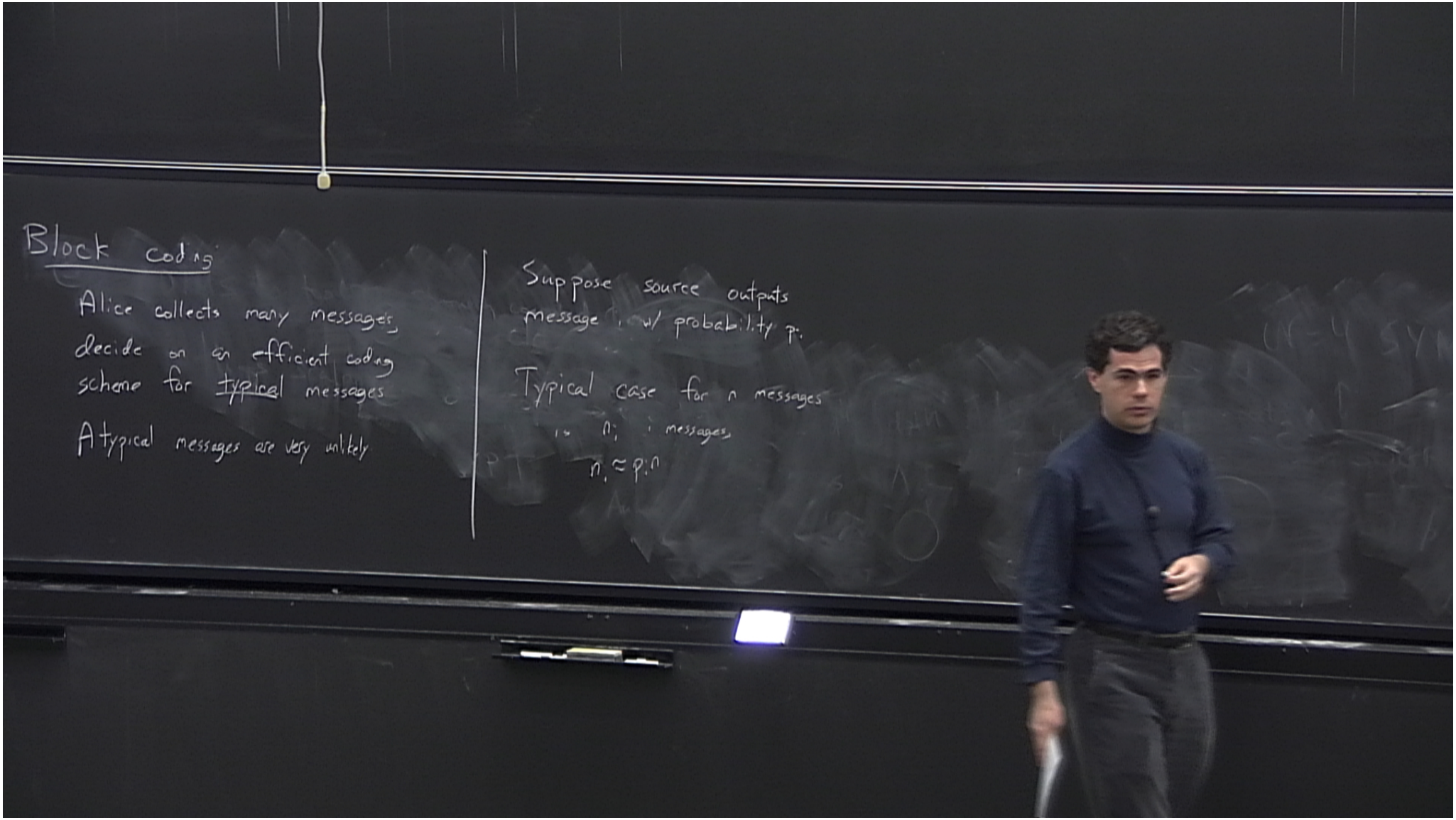
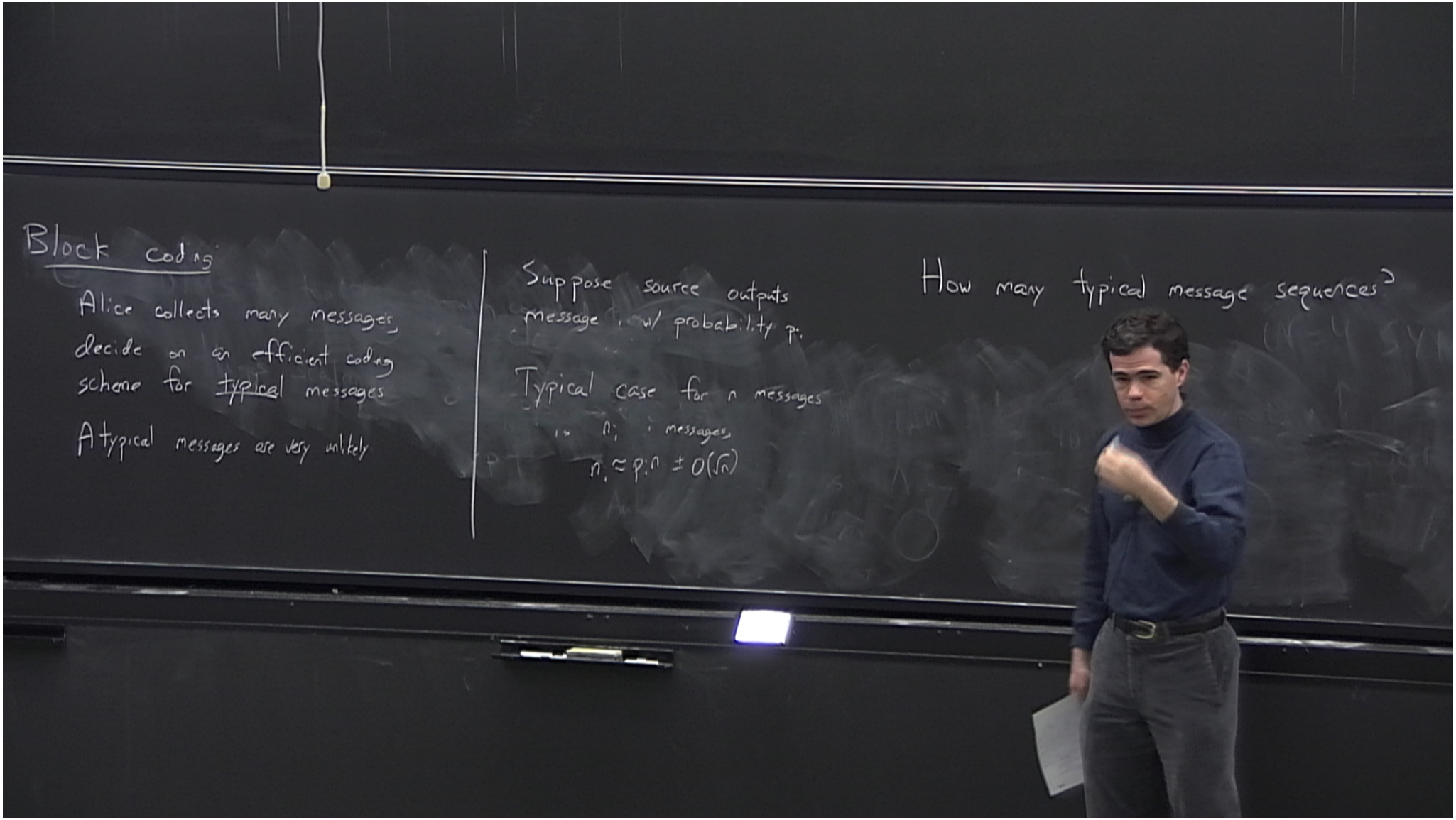
Suppose source outputs message i w/ probability p_i.


Block coding
Alice collects many messages, decide on an efficient coding scheme for typical messages
Atypical messages are very unlikely
Suppose source outputs message i w/ probability p_i
Typical case for n messages
$\approx n_i$: messages i
$n_i \approx p_i n$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How many typical message sequences?

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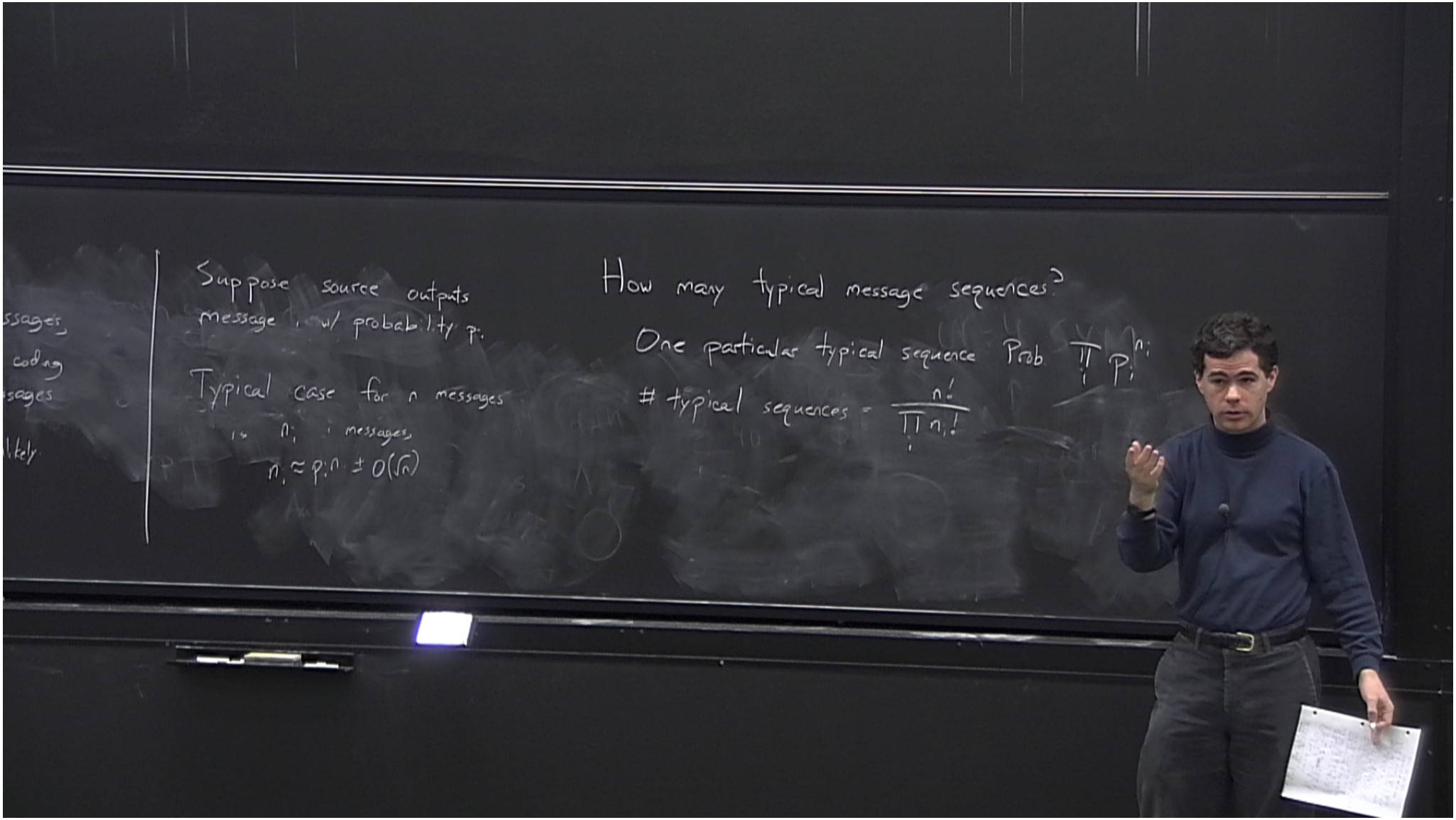
One particular typical sequence Prob $\prod_i p_i^{n_i}$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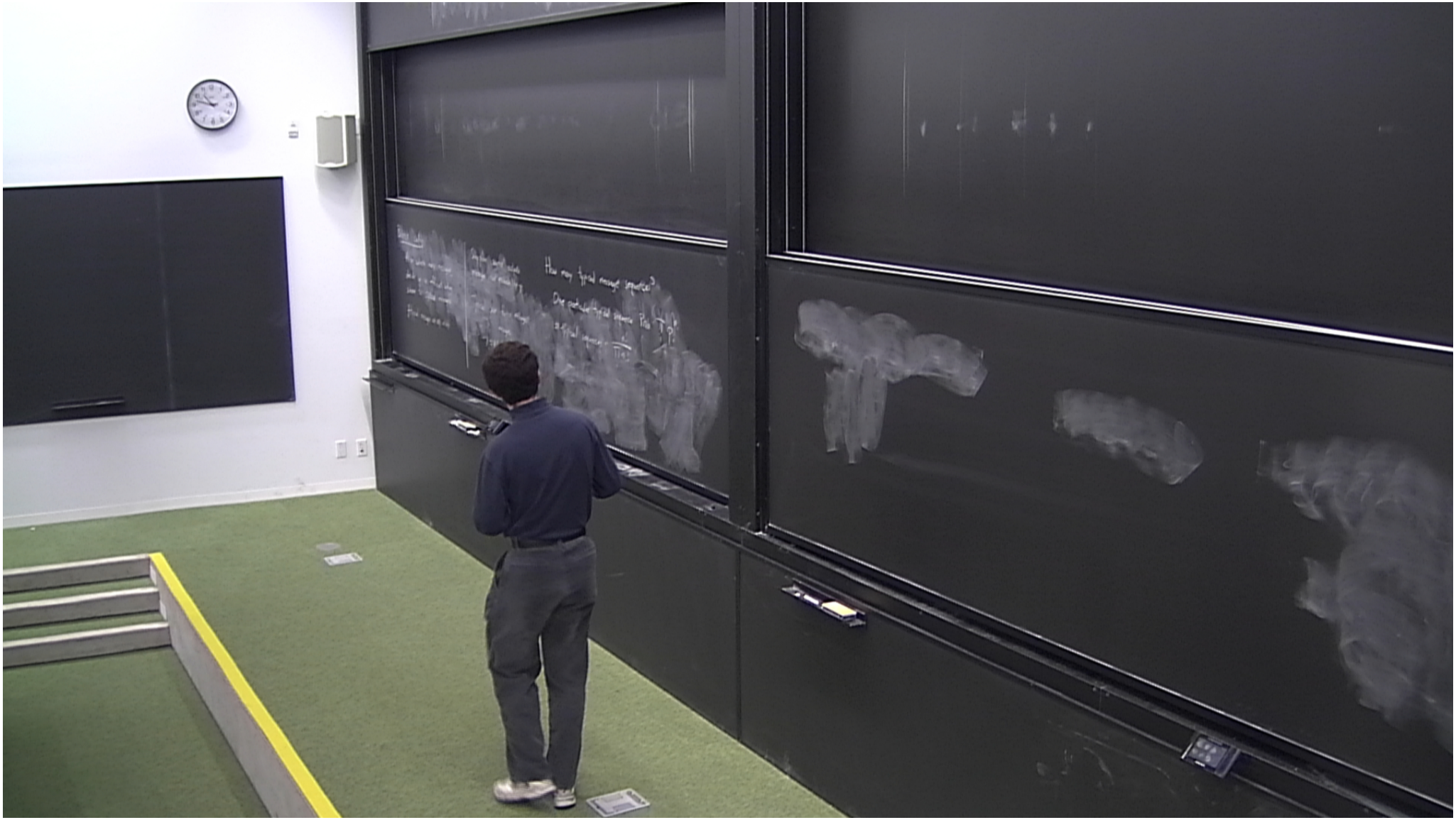
typical sequences = $\frac{n!}{\prod_i n_i!}$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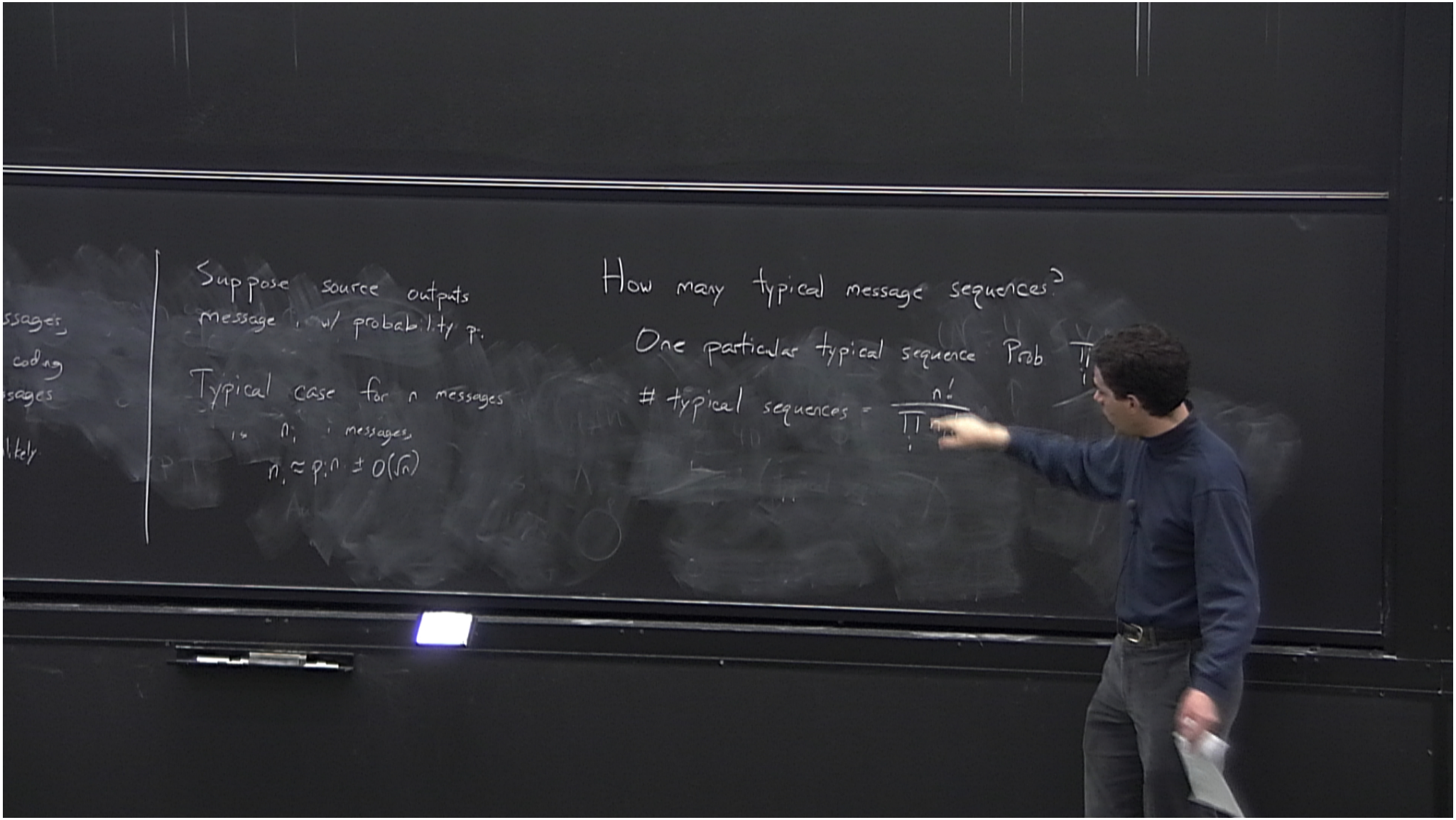
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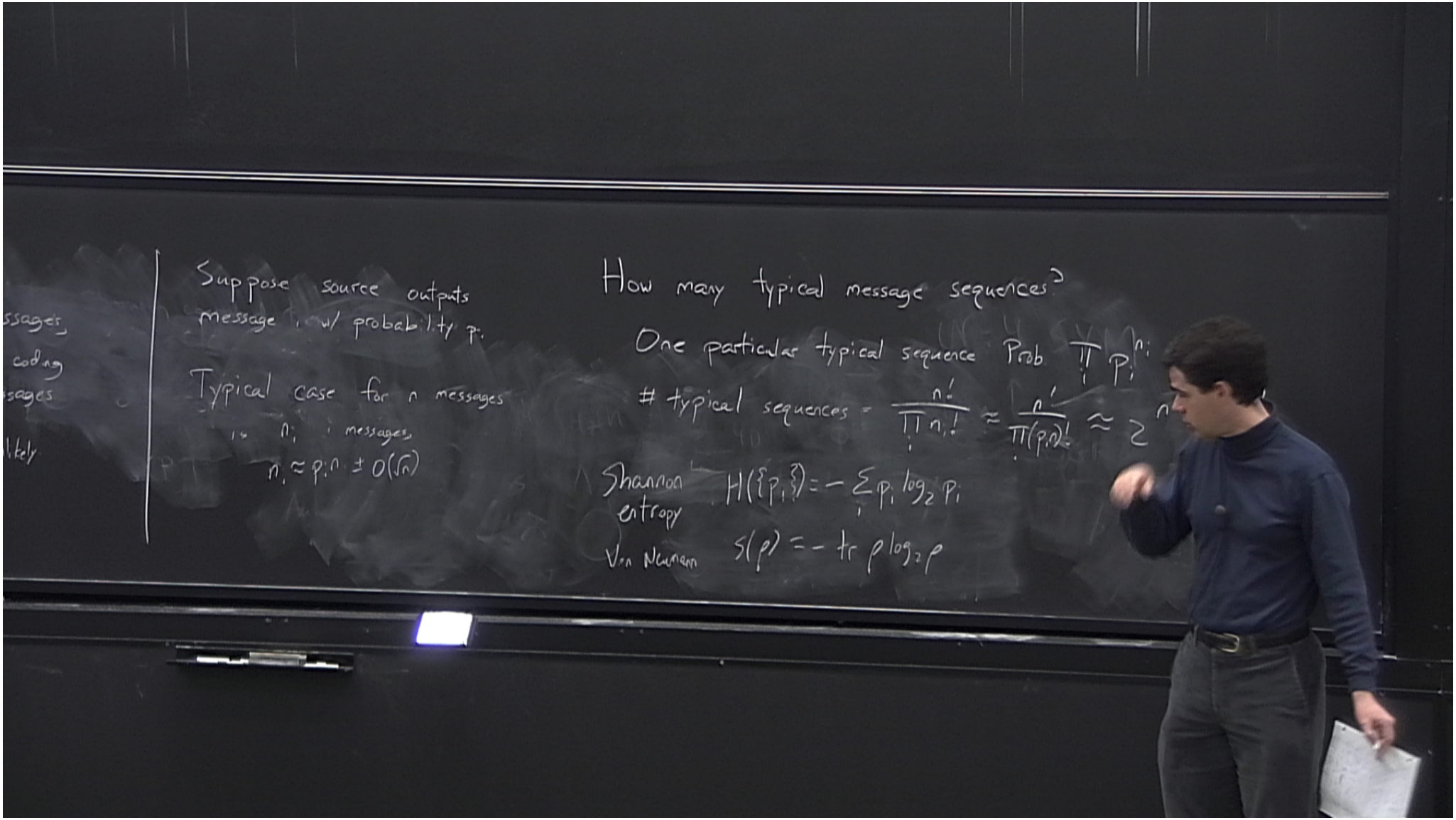
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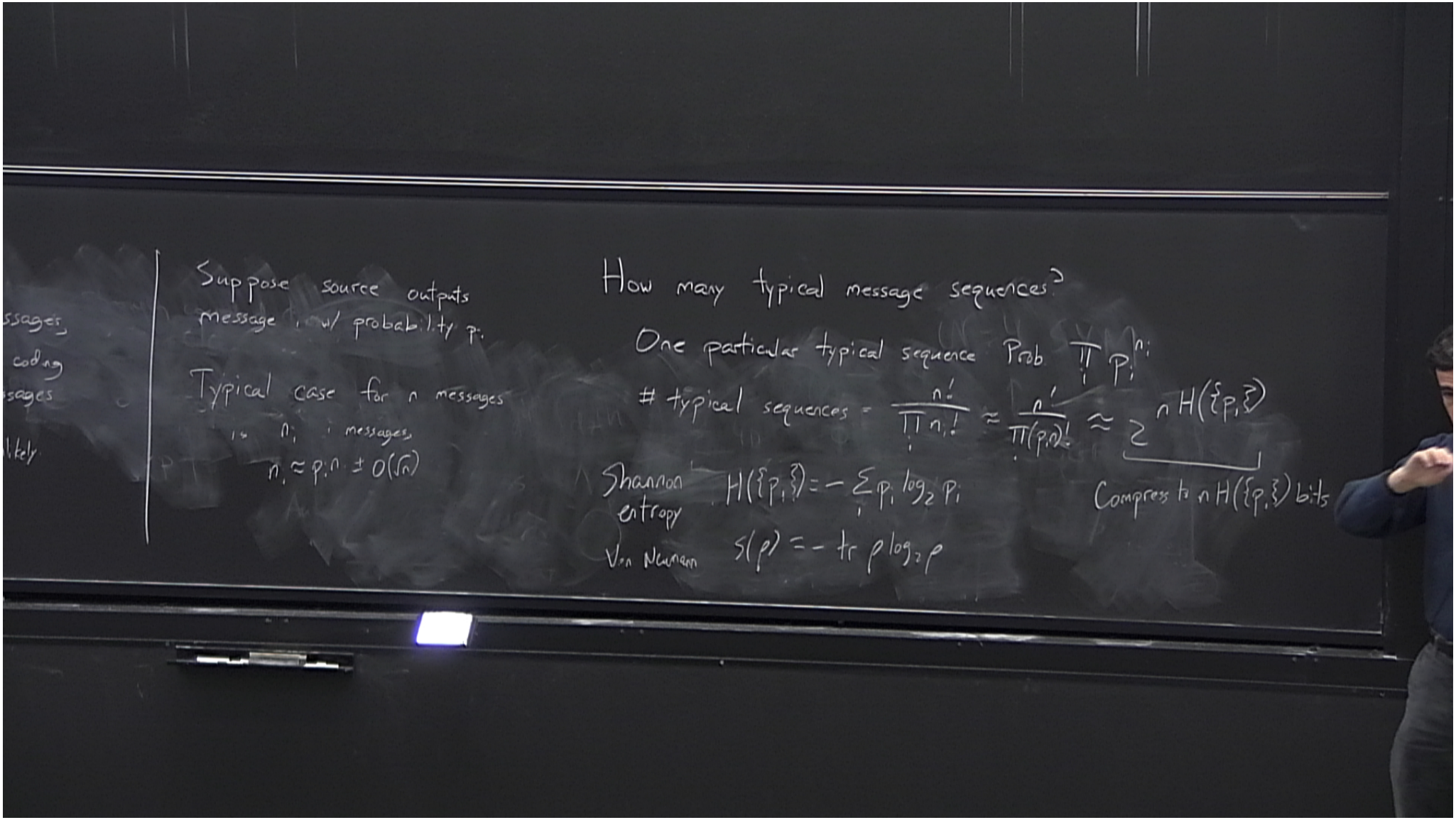
typical sequences = $\frac{n!}{\prod_i n_i!} \approx \frac{n^n}{\prod_i (p_i n)^{p_i n}} \approx 2^n$
Shannon entropy $H(\{p_i\}) = -\sum_i p_i \log_2 p_i$
Von Neumann $S(\rho) = -\mathrm{tr}\, \rho \log_2 \rho$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Compress to $n H(\{p_i\})$ bits
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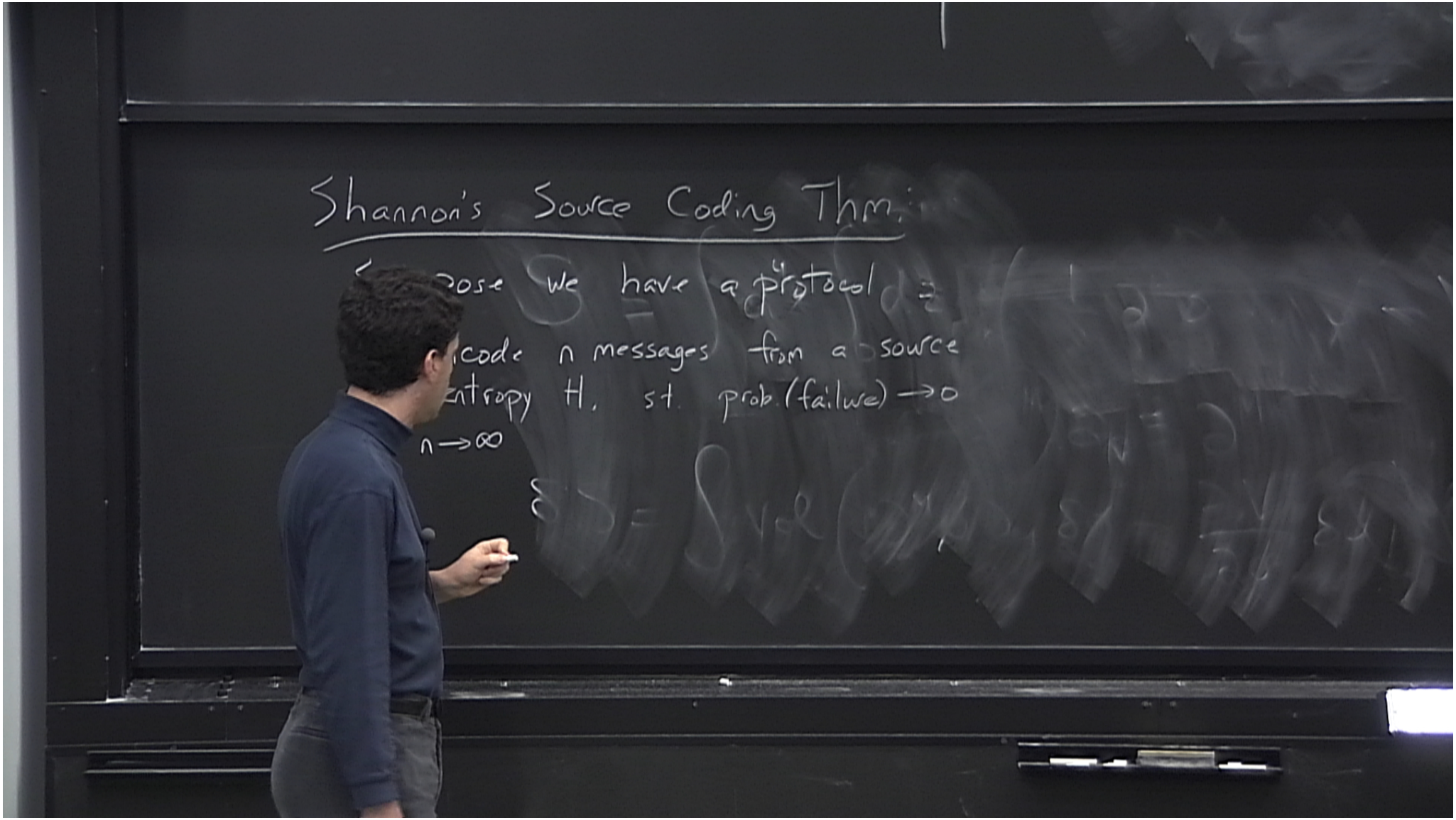
Shannon's Source Coding Thm:
code n messages from a source
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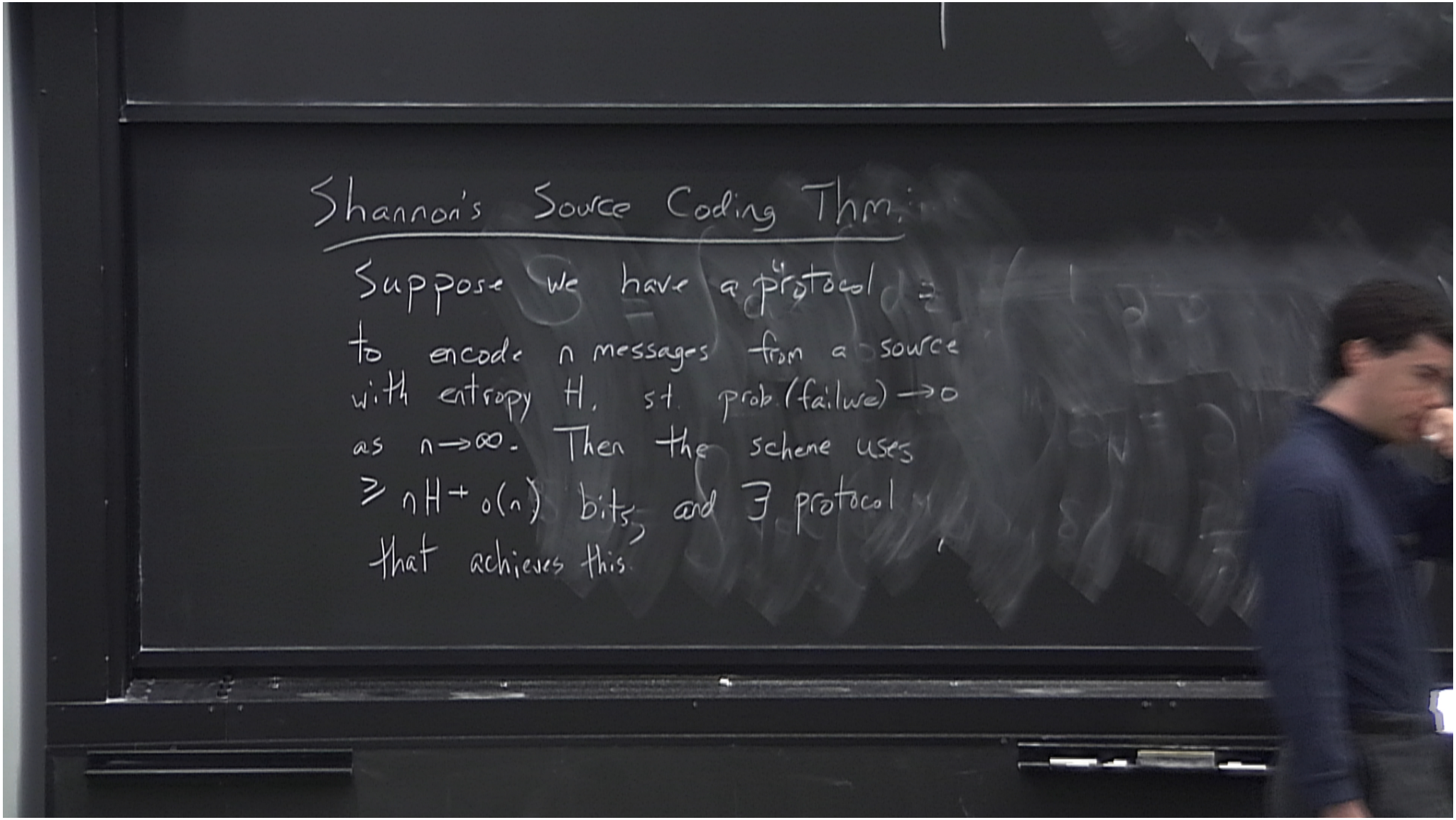
n → ∞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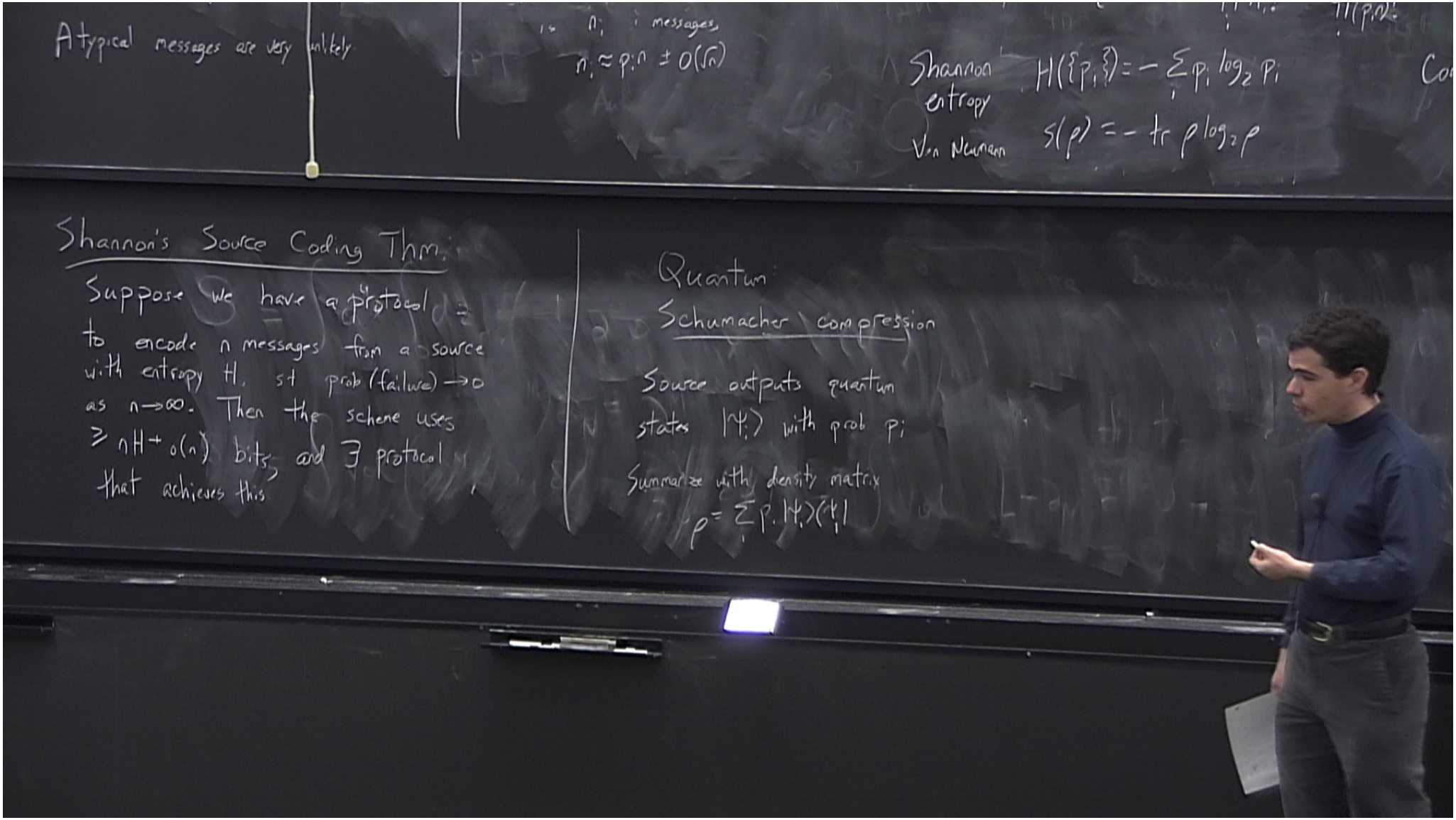
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Quantum Schumacher compression
Source outputs quantum states $|\psi_i\rangle$ with prob p_i
Summarize with density matrix
$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

messages are very unlikely

n_i : messages

$n_i \simeq p_i n \pm O(\sqrt{n})$

Shannon entropy $H(\{p_i\}) = -\sum_i p_i \log_2 p_i$

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Compression

Diagonalize ρ

In that basis, does classical block coding

⇒ Uses $S(\rho)\, n$ qubits

Entropy

Von Neumann $S(\rho) = -\mathrm{tr}\,\rho \log_2 \rho$

compression

quantum

prob. p_i

matrix

$|\psi_i\rangle\langle\psi_i|$

Compression

Diagonalize ρ

In that basis, does classical block coding

$\Rightarrow$ Uses $S(\rho)n$ qubits

- Alice sets entangled states with R. (reference system R.)

Bob's output BR should have fidelity $\to 1$ with AR as $n \to \infty$

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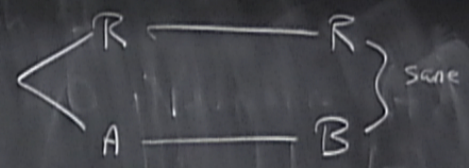

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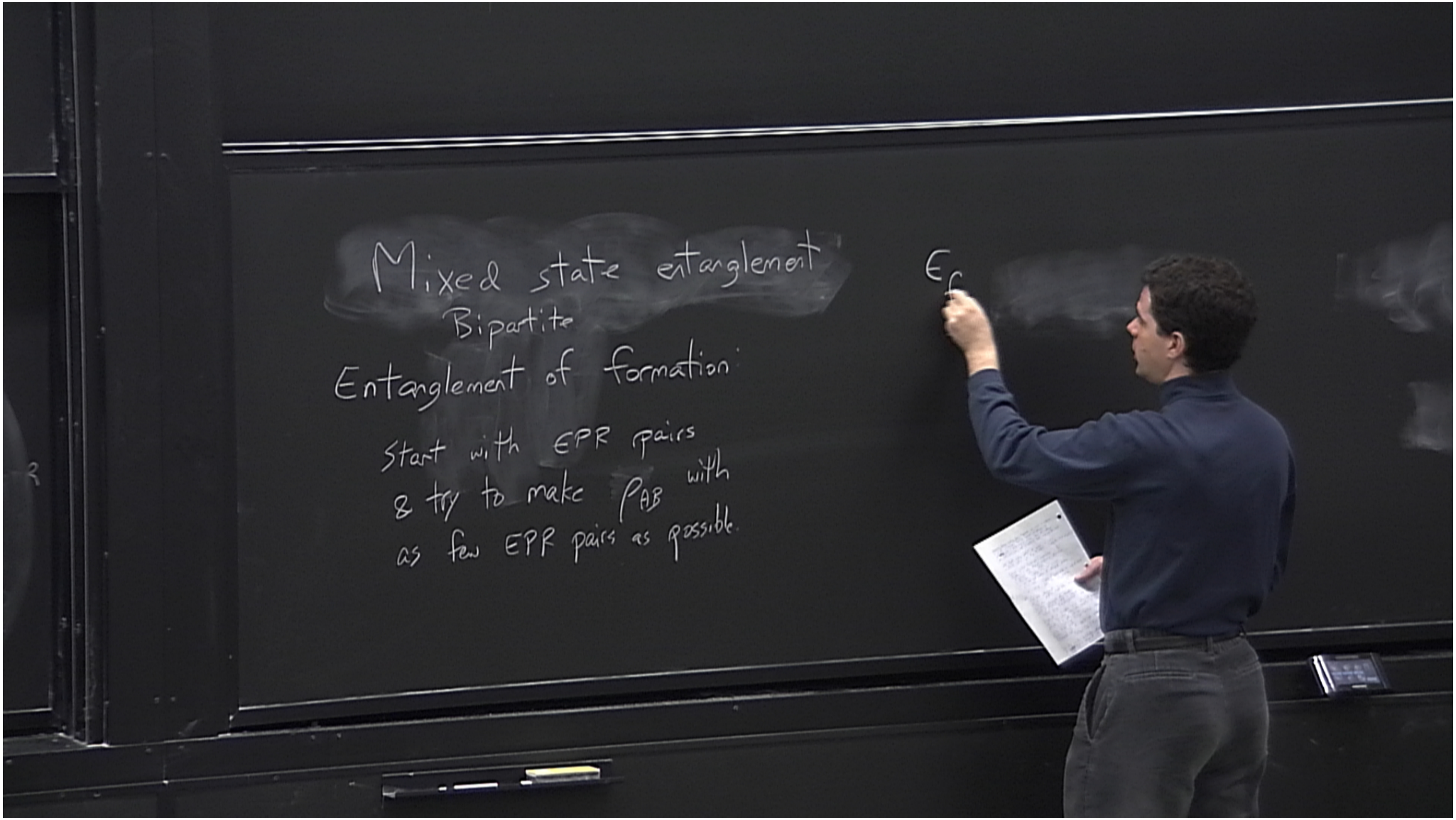
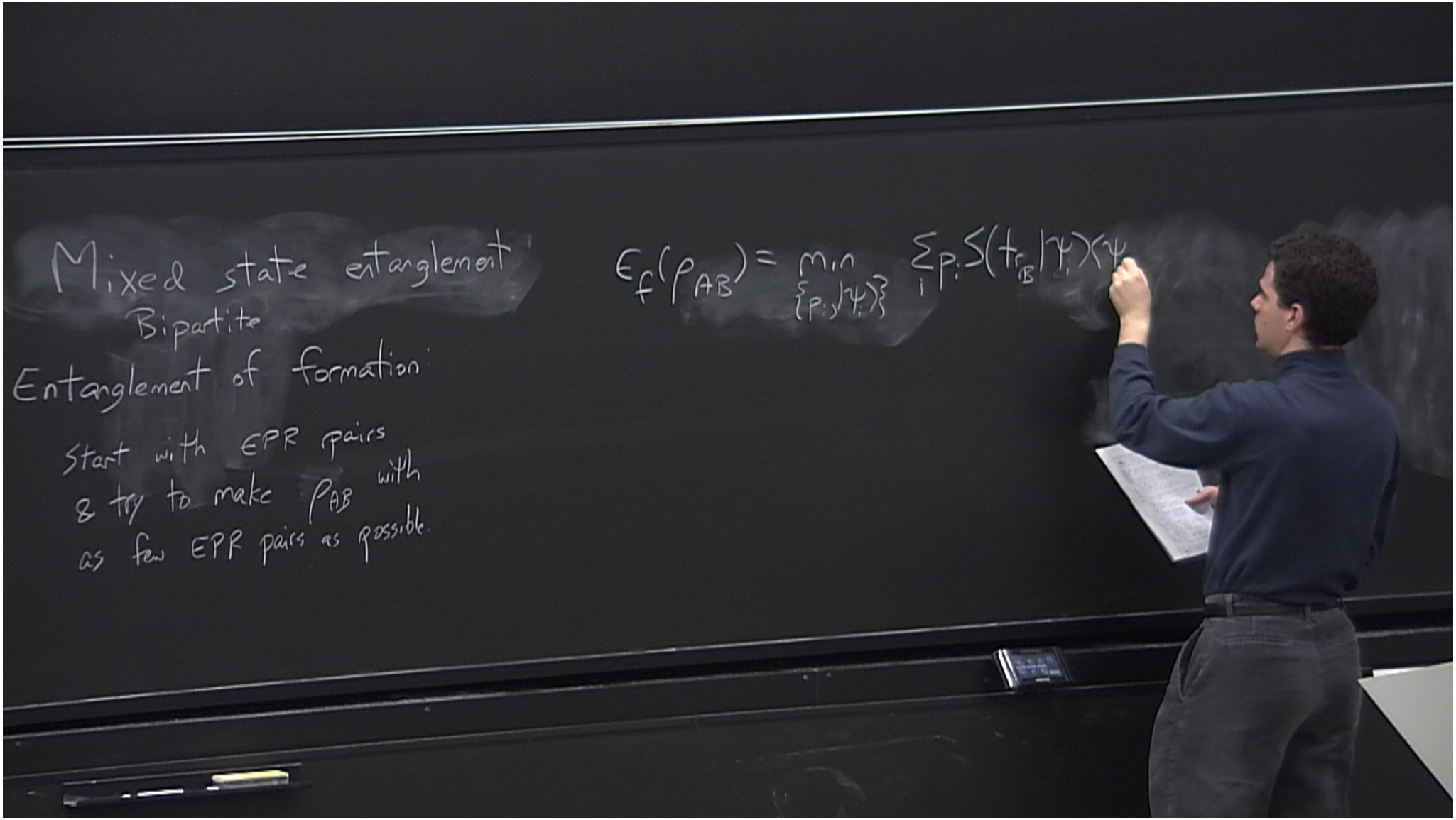

Mixed state entanglement
Bipartite
Entanglement of formation
Start with EPR pairs
& try to make ρ_{AB} with
as few EPR pairs as possible.
E_f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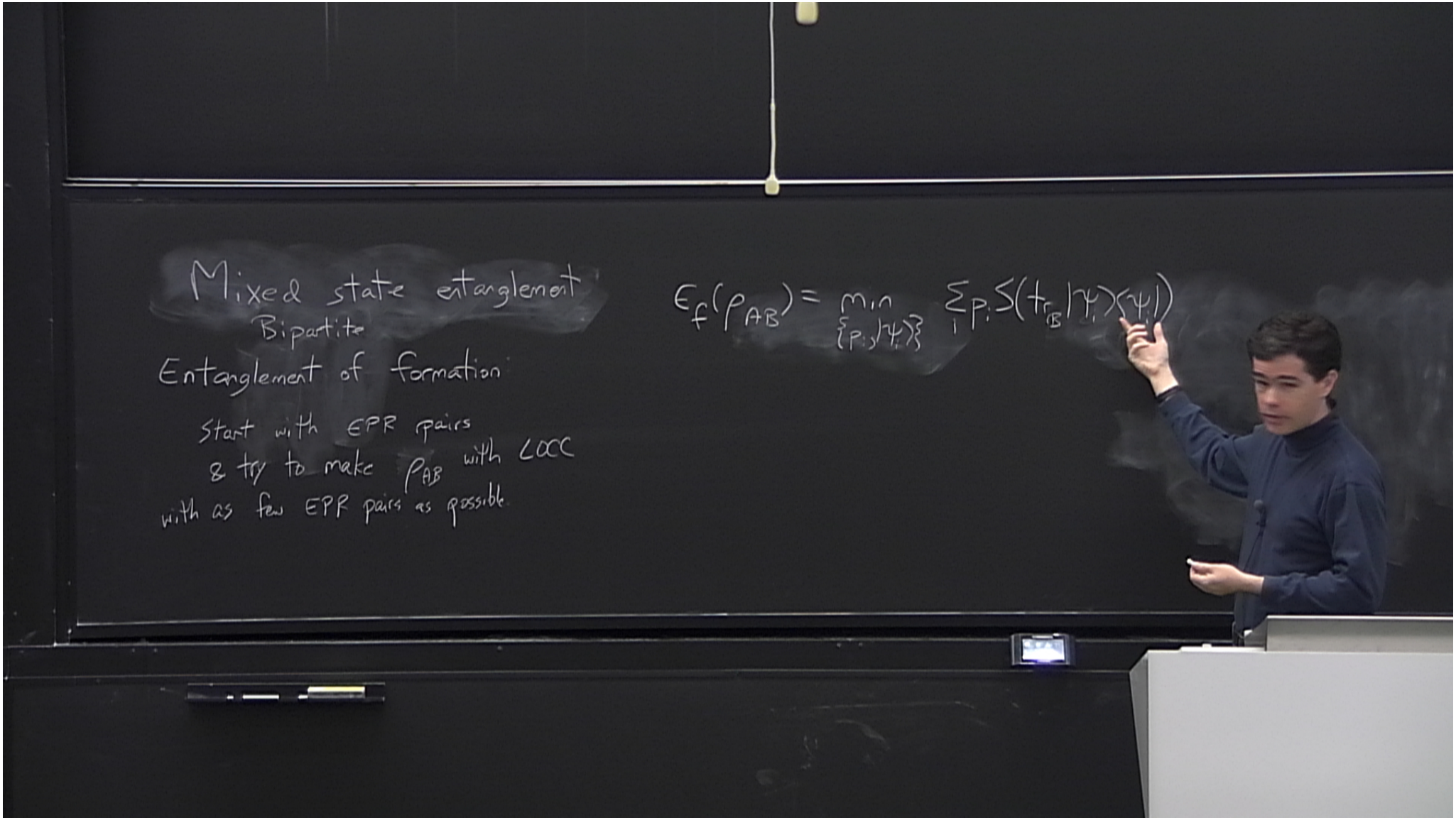
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Start with EPR pairs
& try to make ρ_{AB} with LOCC
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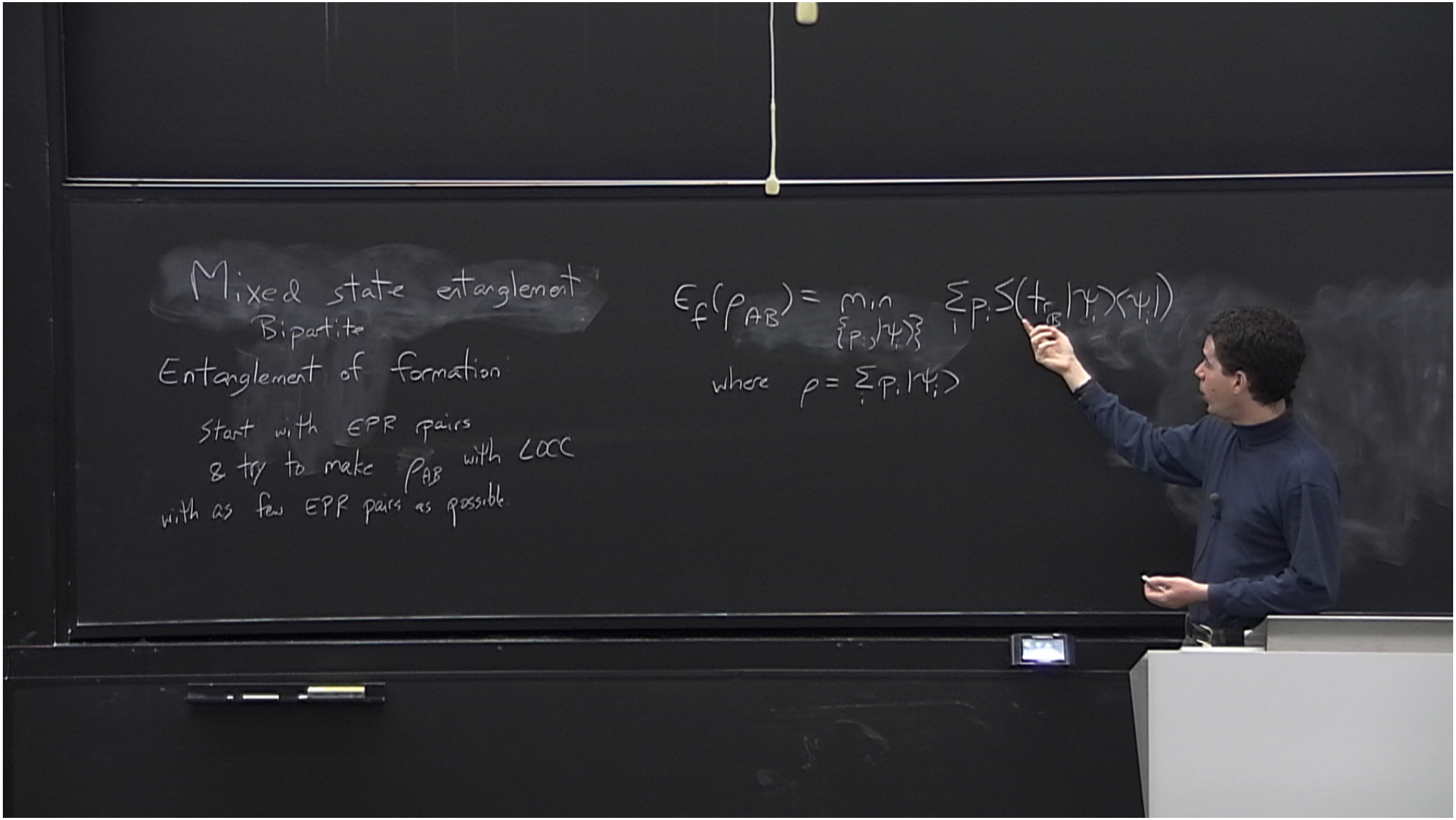
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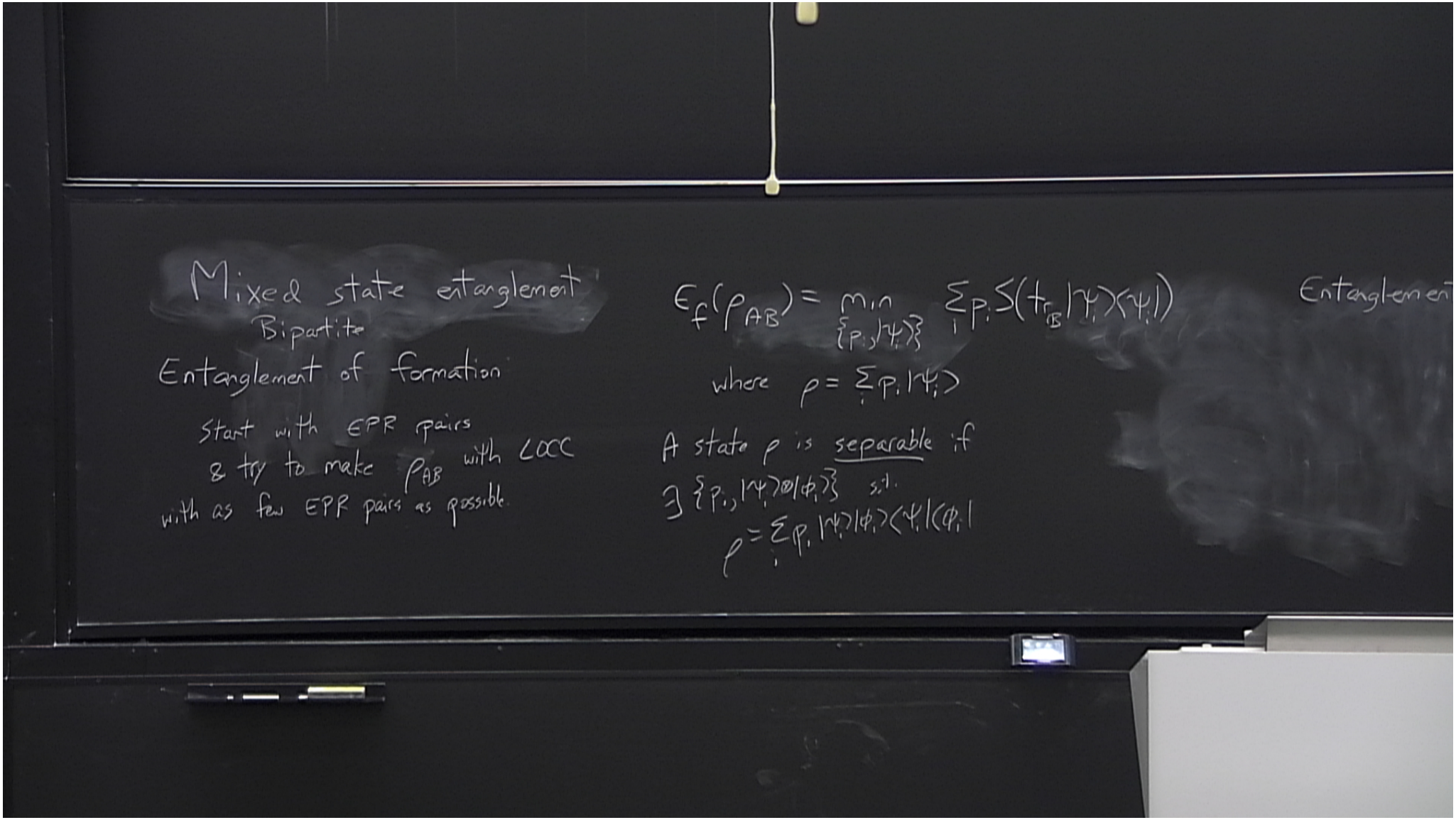
where $\rho = \sum_i p_i |\psi_i\rangle$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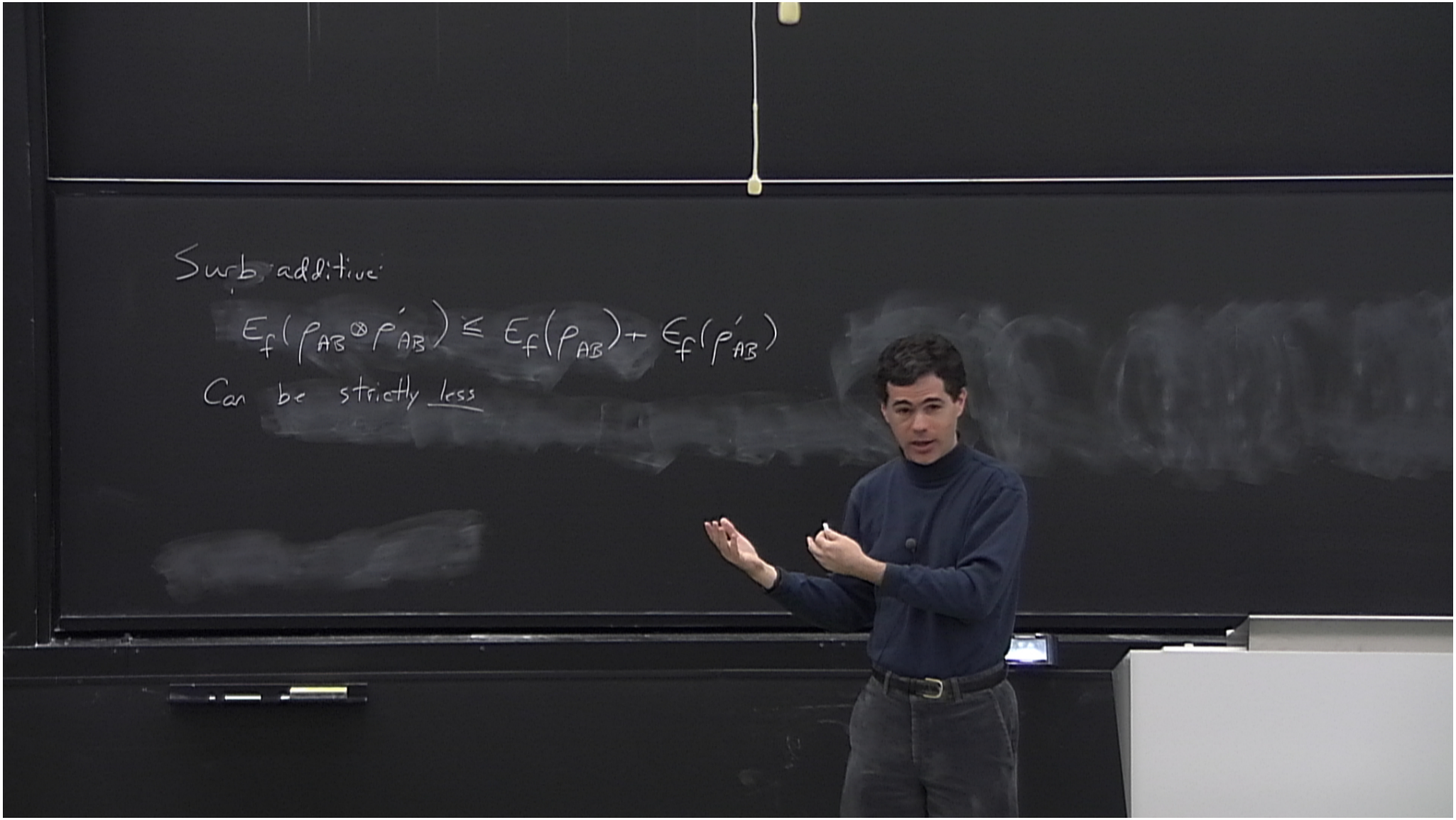
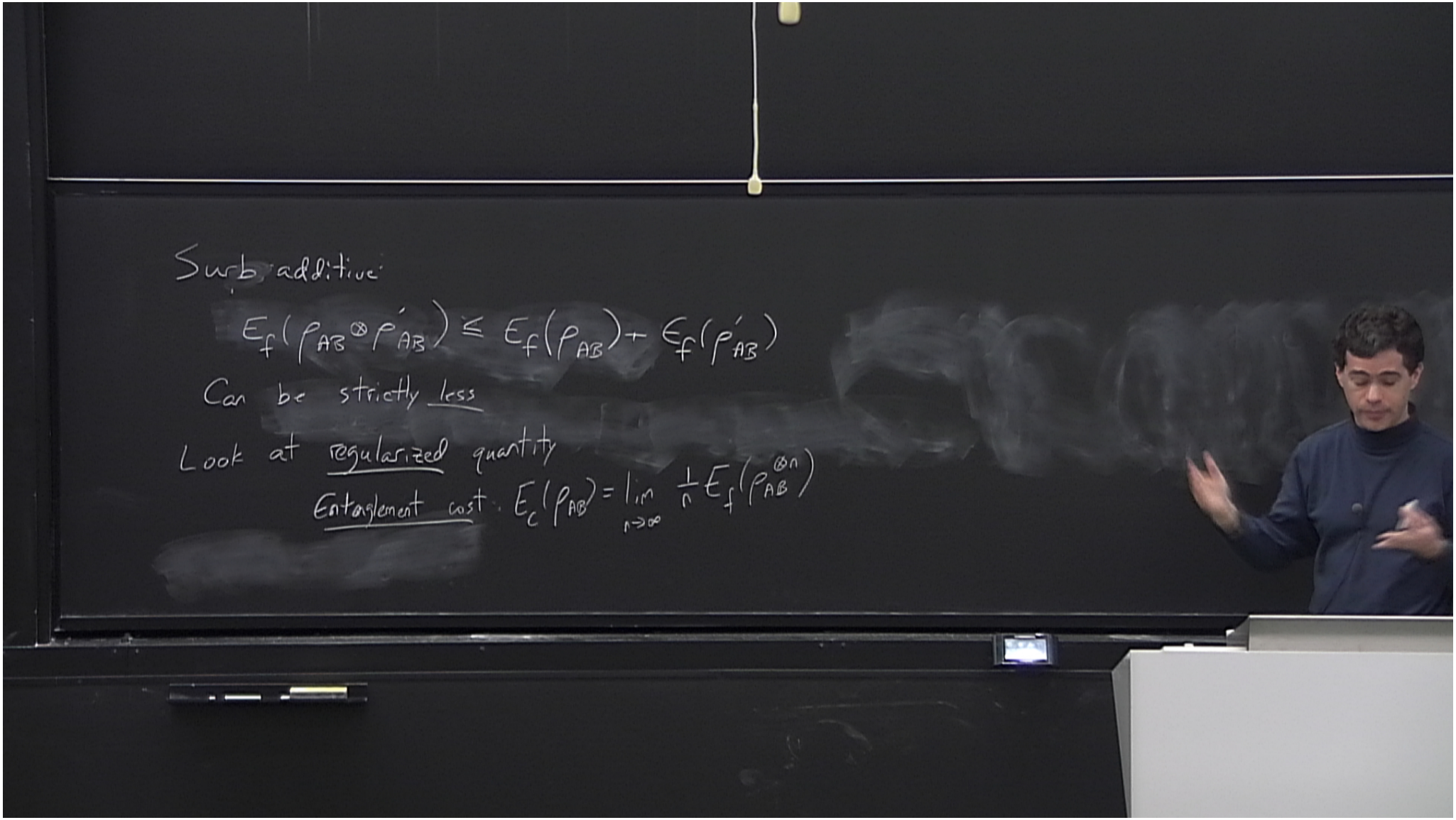
where $\rho = \sum_i p_i |\psi_i\rangle$
A state ρ is separable if
$\exists \{p_i, |\psi_i\rangle \otimes |\phi_i\rangle\}$ s.t.
$\rho = \sum_i p_i |\psi_i\rangle|\phi_i\rangle\langle\psi_i|\langle\phi_i|$
Entanglement


Subadditive
$E_f(\rho_{AB} \otimes \rho'_{AB}) \leq E_f(\rho_{AB}) + E_f(\rho'_{AB})$
Can be strictly less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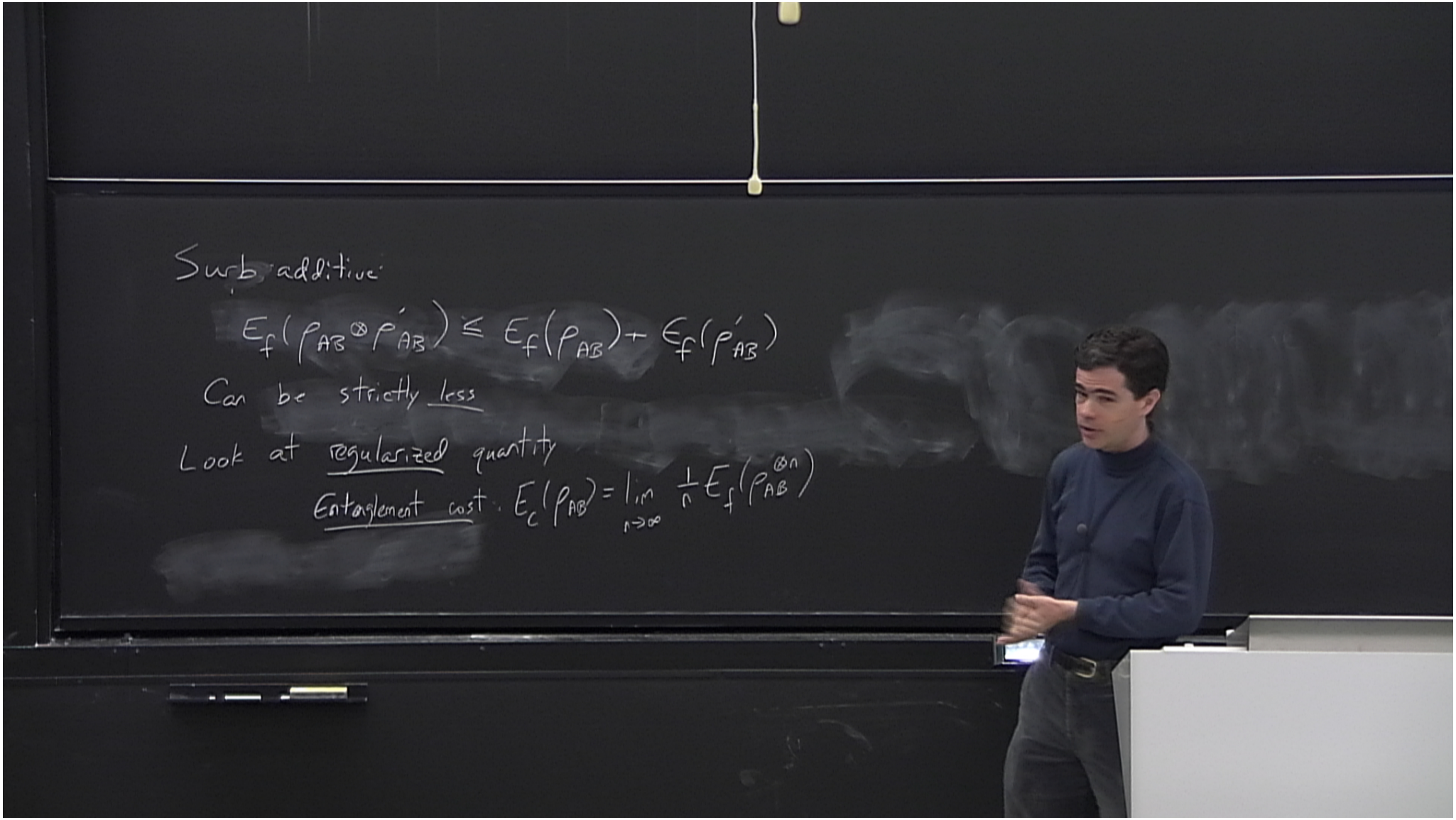
Look at regularized quantity
Entanglement cost $E_c(\rho_{AB}) = \lim_{n\to\infty} \frac{1}{n} E_f(\rho_{AB}^{\otimes n})$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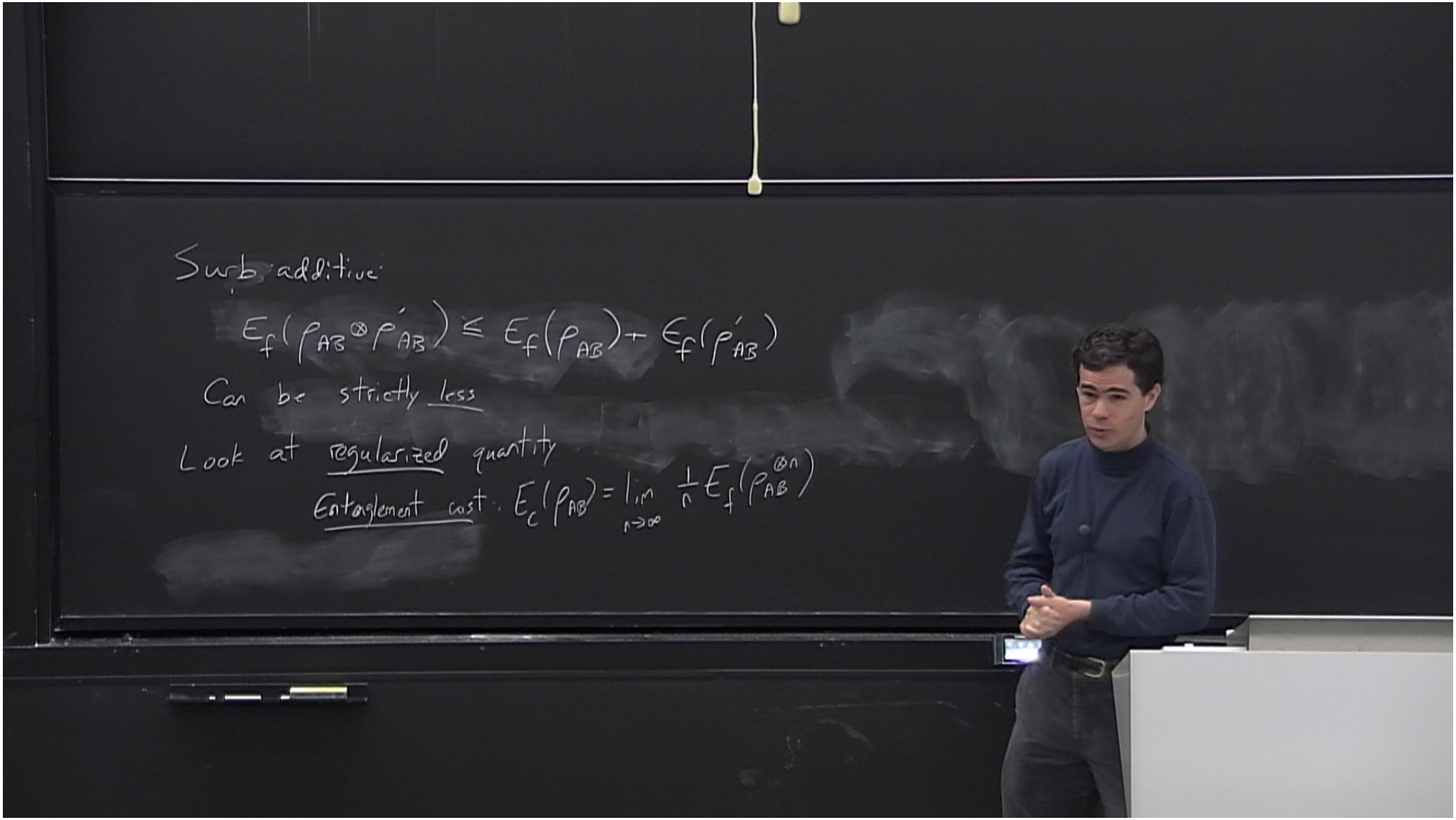
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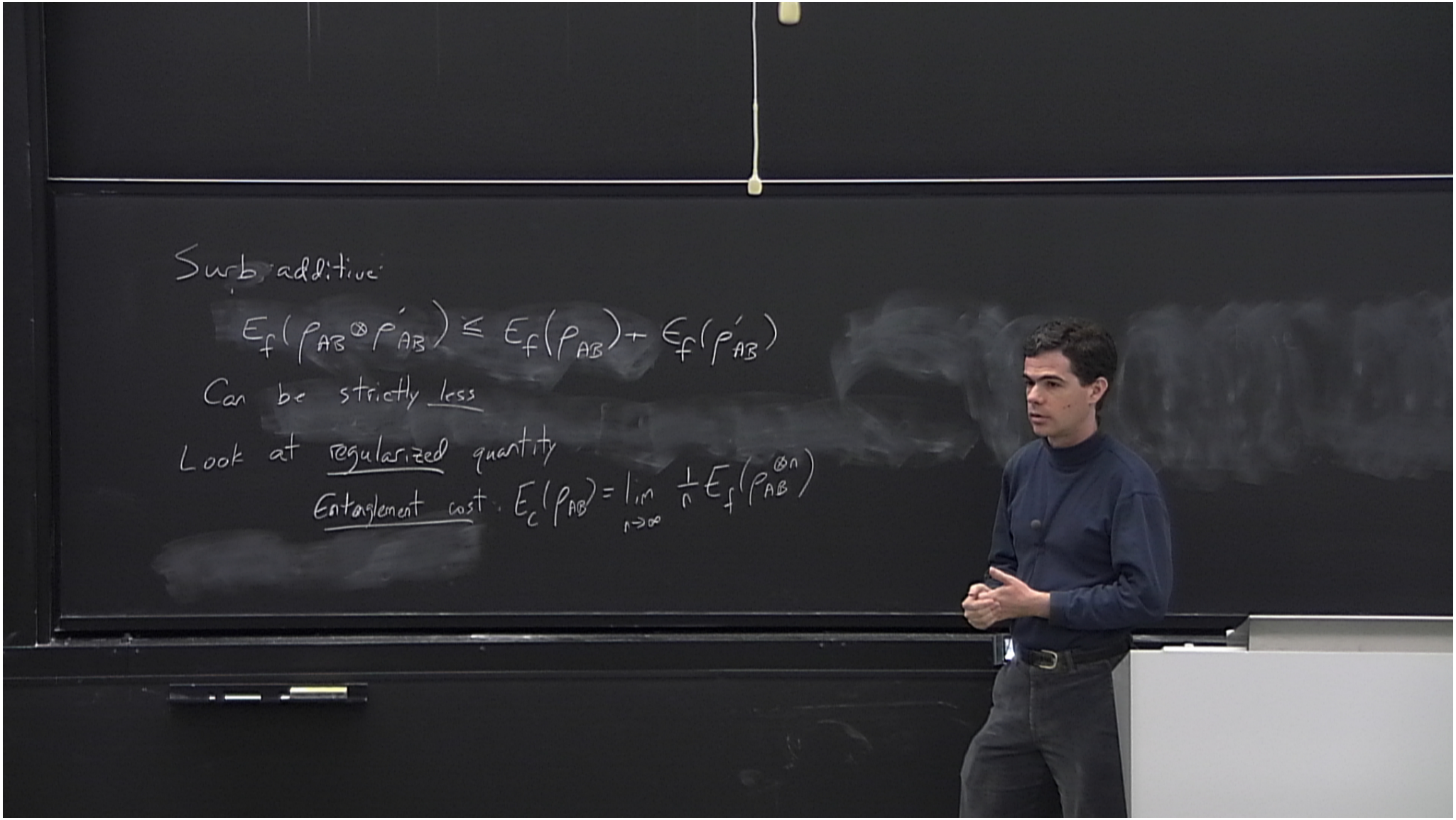
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