

Title: Quantum Information (Review) - Lecture 10

Date: Feb 27, 2012 10:15 AM

URL: <http://www.pirsa.org/12020044>

Abstract:



Distance measures

Pure states: inner product.

Fidelity = absolute value of inner product

$$F(|\phi\rangle, |\psi\rangle) = |\langle \phi | \psi \rangle|$$

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Operational meaning:

Probability of "impersonation"

Distance measures

Pure states: inner product.

Fidelity = absolute value of inner product

$$F(|\phi\rangle, |\psi\rangle) = |\langle\phi|\psi\rangle|$$

Operational meaning:

Probability of 'impersonation'

Measure $\{|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|\}$

But state is $|\phi\rangle$ -

the probability of $|\psi\rangle\langle\psi|$ is

$$|\langle\phi|\psi\rangle|^2 = F^2(|\phi\rangle, |\psi\rangle)$$

Suppose we replace $|\phi\rangle$ by σ

$$F(|\psi\rangle, \sigma)^2 = \text{tr}(|\psi\rangle\langle\psi|\sigma)$$

ng:

personation:

$$I - |\psi\rangle\langle\psi|$$

$$|\psi\rangle\langle\psi|$$

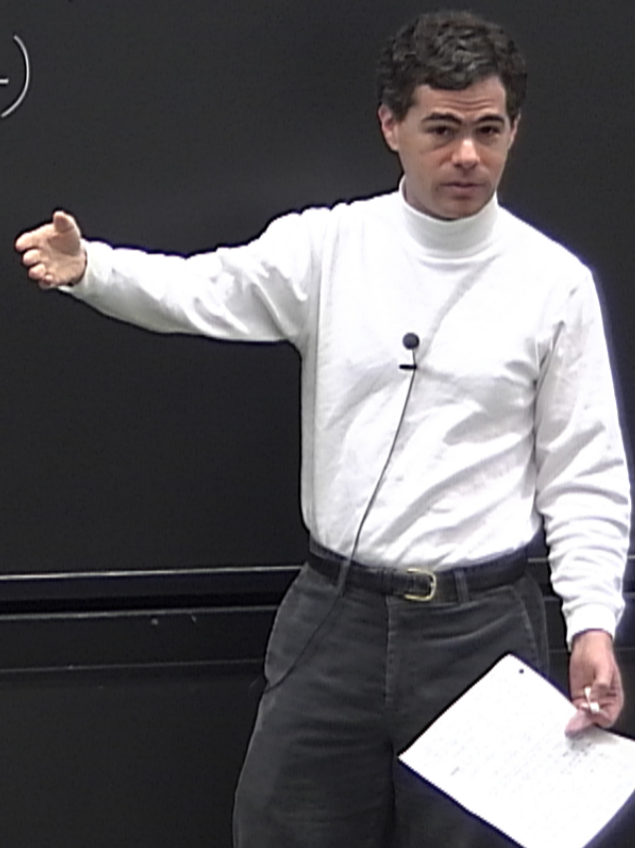
2)

Suppose we replace $|\psi\rangle$ by σ

$$F(|\psi\rangle, \sigma)^2 = \text{tr}(|\psi\rangle\langle\psi| \sigma)$$

For both mixed:

$$F(\rho, \sigma) = \text{tr} \sqrt{\rho^{1/2} \sigma \rho^{1/2}}$$



Thm. (Uhlmann's thm) $F(\rho, \sigma) = \max_{|\psi\rangle, |\phi\rangle} |\langle \psi | \phi \rangle|$

where ρ, σ on Hilbert space $\mathcal{Q}$, purify w/ Hilbert space $\mathcal{R}$,
 $\rho = \text{tr}_{\mathcal{R}} |\psi\rangle\langle\psi|$, $\sigma = \text{tr}_{\mathcal{R}} |\phi\rangle\langle\phi|$.

Properties of fidelity

$$0 \leq F \leq 1$$

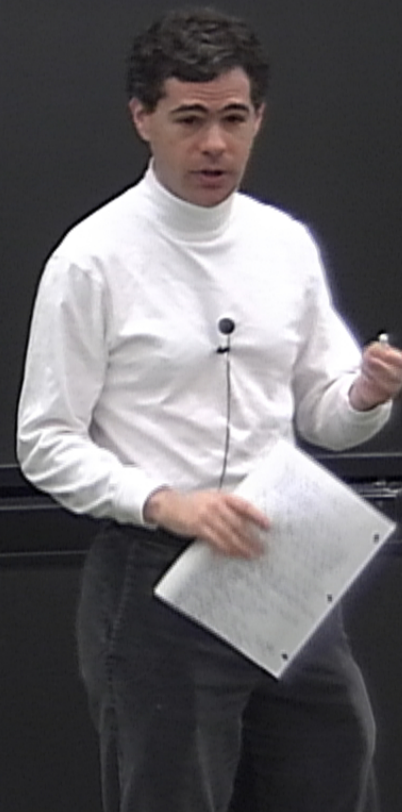
$$a) F(\rho, \sigma) = 1 \Leftrightarrow \rho = \sigma$$

$$b) F(\rho, \sigma) = 0 \Leftrightarrow \rho \text{ \& } \sigma \text{ have orthogonal support}$$

$$c) F(\rho, \sigma) = F(\sigma, \rho)$$

$$d) F(U\rho U^\dagger, U\sigma U^\dagger) = F(\rho, \sigma) \text{ when } U \text{ is unitary}$$

e)



e) F(E)

Trace distance

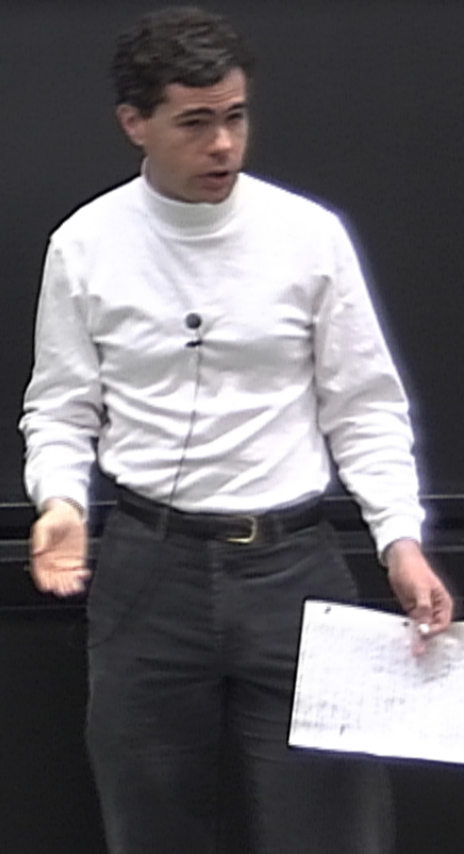
$$D(\rho, \sigma) = \frac{1}{2} \text{tr} |\rho - \sigma|$$

Diagonalize $\rho - \sigma$: sum of positive
eigenvalues + sum |negative eigenvalues|

$$e) f(\mathcal{E}(\rho), \mathcal{E}(\sigma)) \geq f(\rho, \sigma) \quad \text{for any CPTP } \mathcal{E}.$$

$$\text{Thm.: } D(\rho, \sigma) = \max_{\text{POVMs } \{E_m\}} \underbrace{D(\text{tr}(\rho E_m), \text{tr}(\sigma E_m))}_{\text{classical statistical distance}}$$

Optimal POVM: Project on the subspaces spanned by + eigenvalues of $\rho - \sigma$, - eigenvalues of $\rho - \sigma$



$L(\sigma) \geq F(\rho, \sigma)$ for any CPTP L .

$\text{tr}(\rho E_m) + \text{tr}(\sigma E_m)$
classical statistical distance
subspaces spanned
eigenvalues of $\rho - \sigma$

Suppose you are given
 ρ with prob. $\frac{1}{2}$ & σ with
Can guess correctly which you
have w/ prob

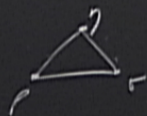
Properties of trace distance

$$0 \leq D(\rho, \sigma) \leq 1$$

$$a) D(\rho, \sigma) = 0 \Leftrightarrow \rho = \sigma$$

$$b) D(\rho, \sigma) = D(\sigma, \rho)$$

$$c) D(\rho, \sigma) \leq D(\rho, \eta) + D(\eta, \sigma)$$



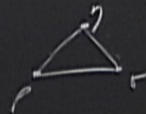
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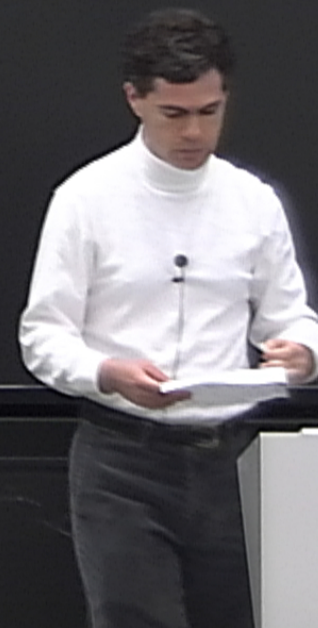
$$b) D(\rho, \sigma) = D(\sigma, \rho)$$

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$$d) D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho, \sigma) \text{ for unitary } U.$$

$$e) D(\mathcal{E}(\rho), \mathcal{E}(\sigma)) \leq D(\rho, \sigma) \text{ for all CPTP } \mathcal{E}.$$



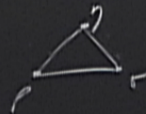
Properties of trace distance:

$$0 \leq D(\rho, \sigma) \leq 1$$

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d) $D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho, \sigma)$ for unitary U .

e) $D(\mathcal{E}(\rho), \mathcal{E}(\sigma)) \leq D(\rho, \sigma)$ for all CPTP $\mathcal{E}$.

Thm: $1 - F(\rho, \sigma) \leq D(\rho, \sigma) \leq$

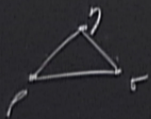
Trace distance

≤ 1

$\rho \Leftrightarrow \sigma \Leftrightarrow \rho = \sigma$

(σ, ρ)

$D(\rho, \eta) + D(\eta, \sigma)$



$D(\rho, \sigma) = D(\rho, \sigma)$ for unitary U .

$D(\rho, \sigma)$ for all CPTP E .

Thm: $1 - F(\rho, \sigma) \leq D(\rho, \sigma) \leq \sqrt{1 - F(\rho, \sigma)^2}$



Quantifying entanglement

Bipartite entanglement

$A \sim B$

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Pure state case

$|\psi\rangle_{AB} \rightarrow A \text{ has } \rho_A = \text{tr}_B |\psi\rangle\langle\psi|$
B has ρ_B

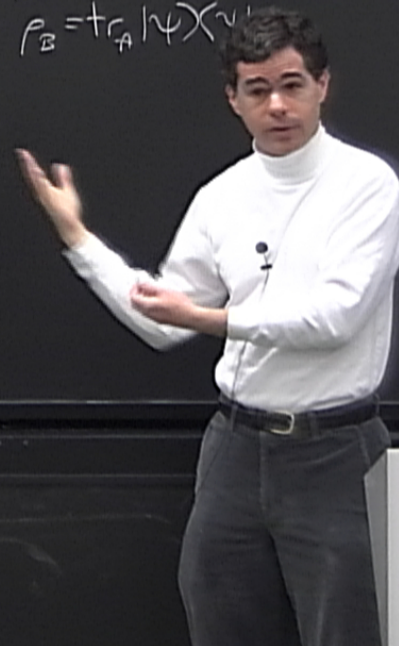
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tanglement:

tanglement

B

se

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Von Neumann entropy of ρ_A is $S(\rho) = -\text{tr} \rho \log_2 \rho$

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Properties of entropy

a) On a Hilbert space of dimension D , $S(\rho) \leq \log_2 D$

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Von Neumann entropy of ρ_A is $S(\rho) = -\text{tr} \rho \log_2 \rho$

Properties of entropy

a) On a Hilbert space of dimension D , $S(\rho) \leq \log_2 D$ (when $\rho = \frac{1}{D} I$)

b) $S(\rho) = 0 \Leftrightarrow \rho$ is pure

c) $S(A, B) \geq |S(A) - S(B)|$

d) Subadditivity $S(A, B) \leq S(A) + S(B)$ (= when $\rho_{AB} = \rho_A \otimes \rho_B$)

e) Strong subadditivity $S(A, B, C) + S(B) \leq S(A, B) + S(B, C)$



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Global pure state $S(A, B) = 0 \geq |S(A) - S(B)|$
 $\Rightarrow S(A) = S(B)$

Quantifying entanglement

Bipartite entanglement

$A \sim B$

Pure state case

$$|\psi\rangle_{AB} \rightarrow A \text{ has } \rho_A = \text{tr}_B |\psi\rangle\langle\psi|$$

$$B \text{ has } \rho_B = \text{tr}_A |\psi\rangle\langle\psi|$$

Von Neumann entropy of ρ_A is $S(\rho) = -\text{tr} \rho \log_2 \rho$

Entanglement of $|\psi\rangle_{AB}$ is $S(\rho_A)$

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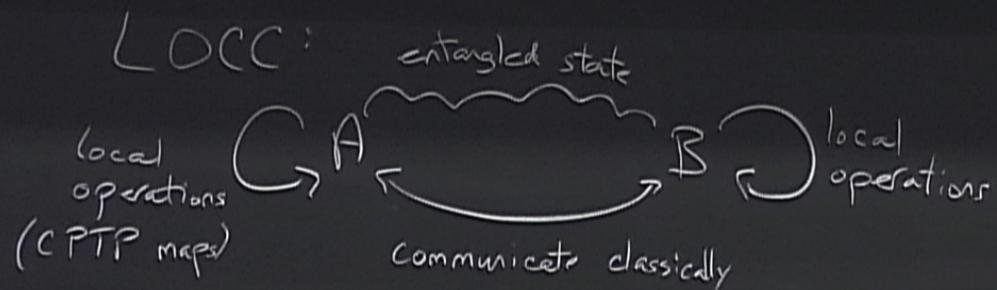
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e) Strong subadditivity $S(A, B, C) + S(B) \leq S(A, B) + S(B, C)$

e) $F(C(p), C(b)) \geq F(p, b)$ for any CPTP C .



"local operations and classical communication"

