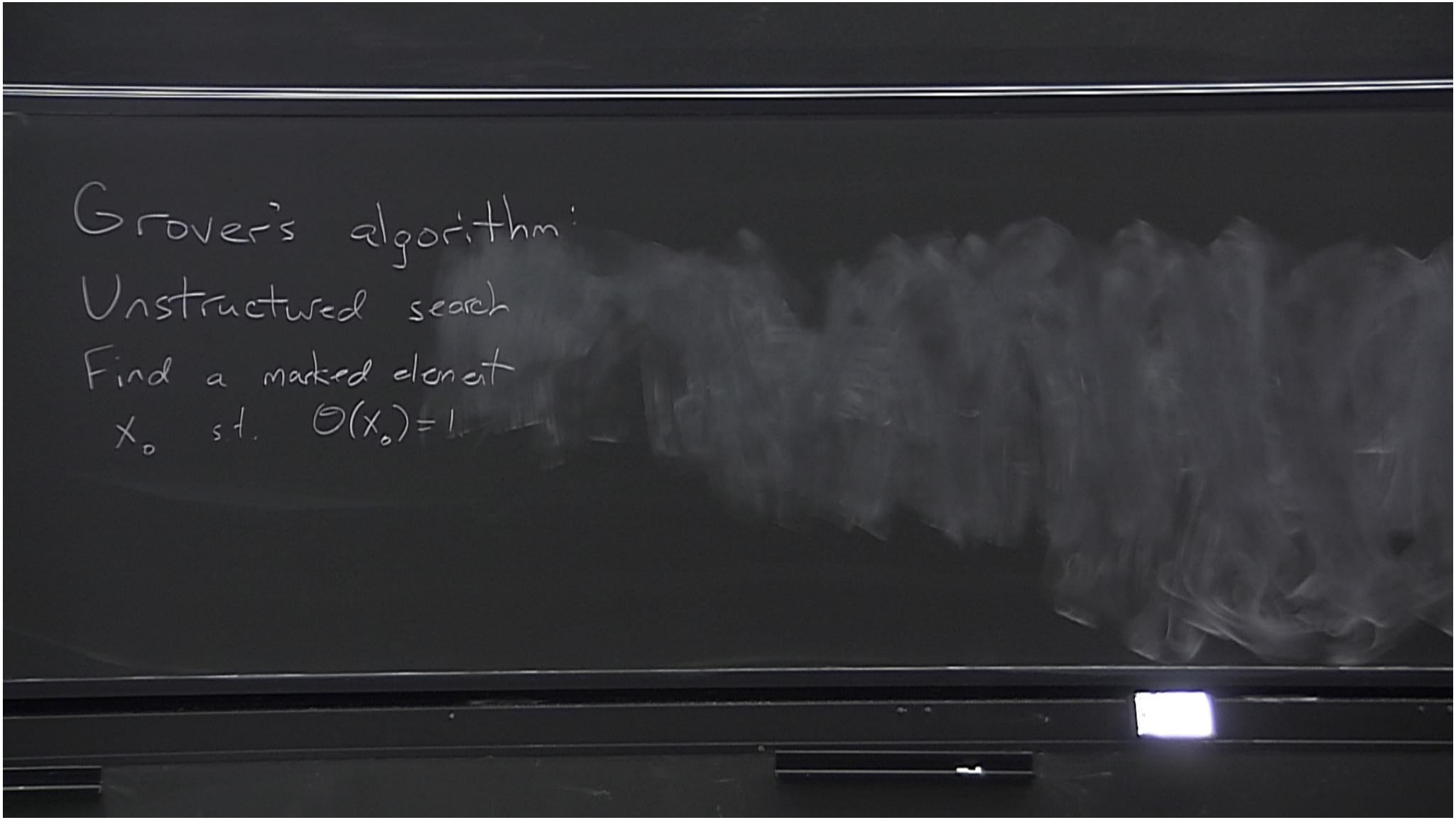
Title: Quantum Information (Review) - Lecture 9

Date: Feb 24, 2012 10:15 AM

URL: http://www.pirsa.org/12020041

Abstract:


Grover's algorithm:
Unstructured search
Find a marked element
x_0 s.t. $\Theta(x_0) = 1$

Grover's algorithm:

Unstructured search

Find a marked element

x_0 s.t. $\Theta(x_0) = 1$

Use $O(\sqrt{N})$ queries

$\Theta: \mathbb{Z}_N \to \mathbb{Z}_2$


Grover's algorithm
Unstructured search

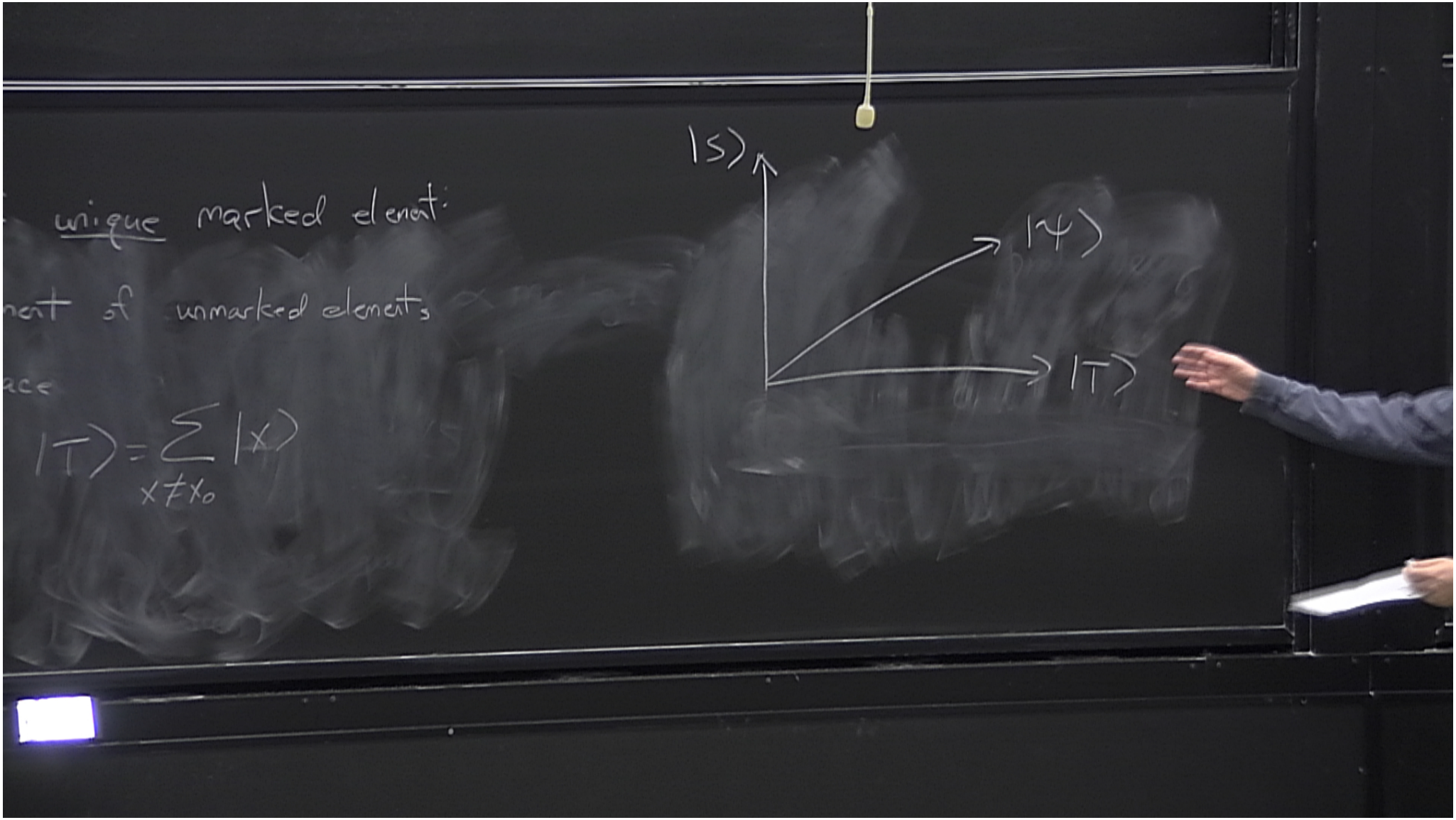

Grover's algorithm
Unstructured search
Consider case of unique marked element

Consider case of unique marked element:

Symmetric treatment of unmarked elements

2-D Hilbert space

$|S\rangle = |x_0\rangle \qquad |T\rangle = \sum_{x \neq x_0} |x\rangle$


unique marked element:
ment of unmarked elements
ace
$|T\rangle = \sum_{x \neq x_0} |x\rangle$
$|s\rangle$
$|\psi\rangle$
$|T\rangle$

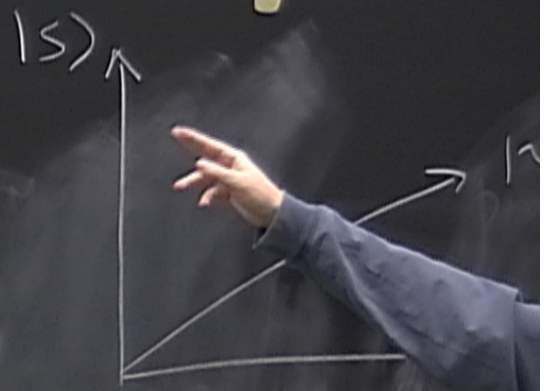
Consider case of unique marked element:

Symmetric treatment of unmarked elements

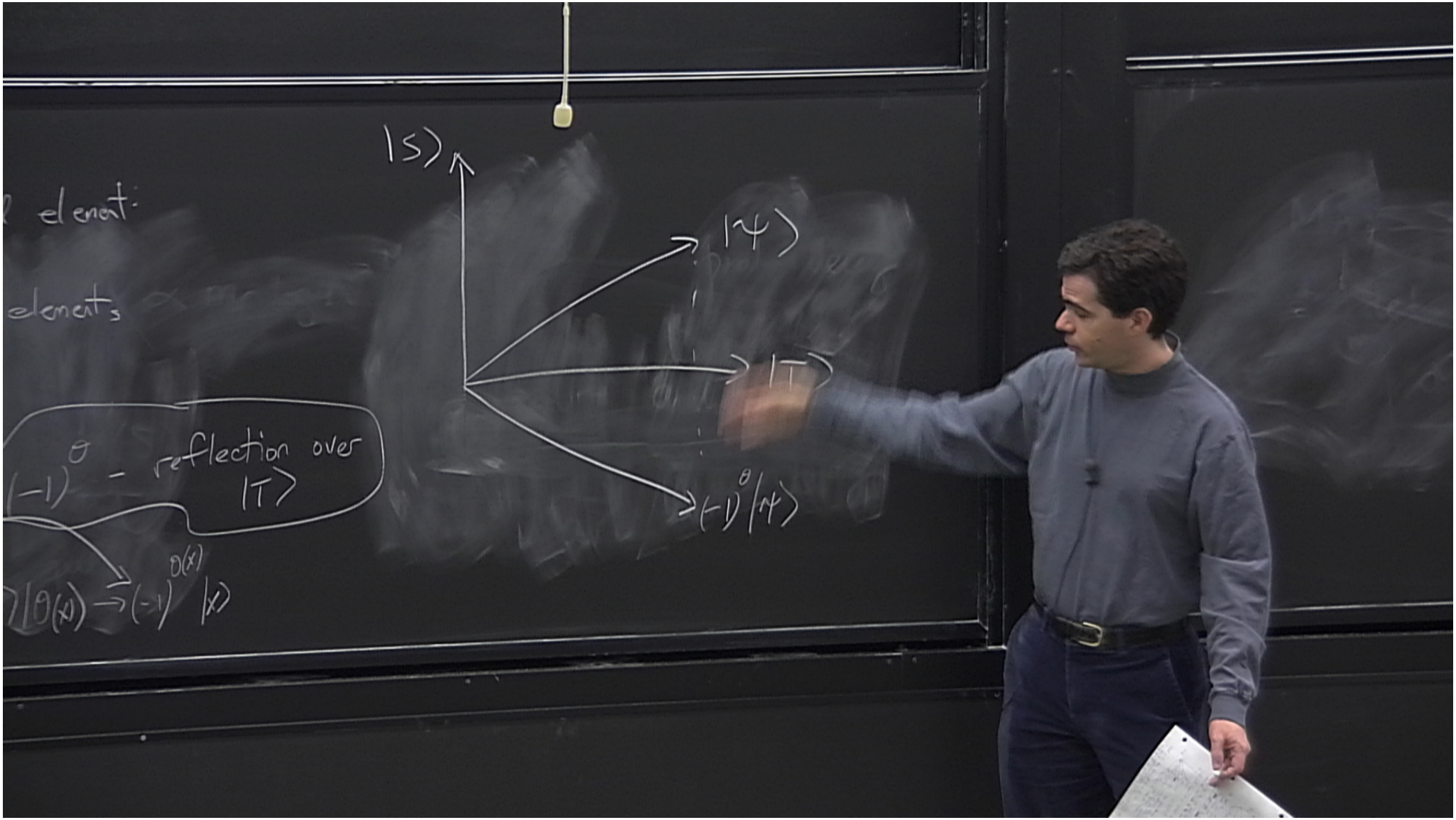
2-D Hilbert space

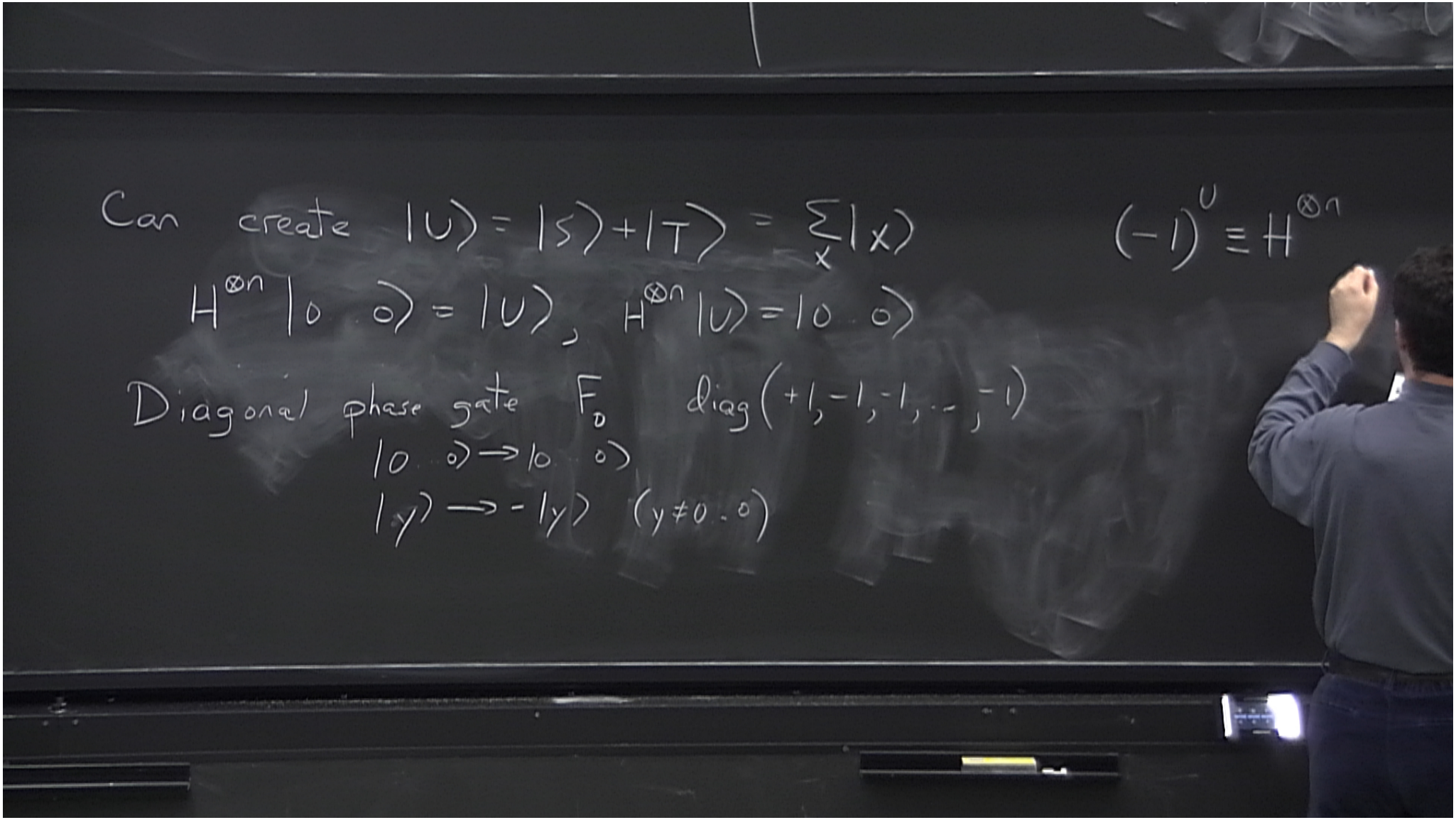
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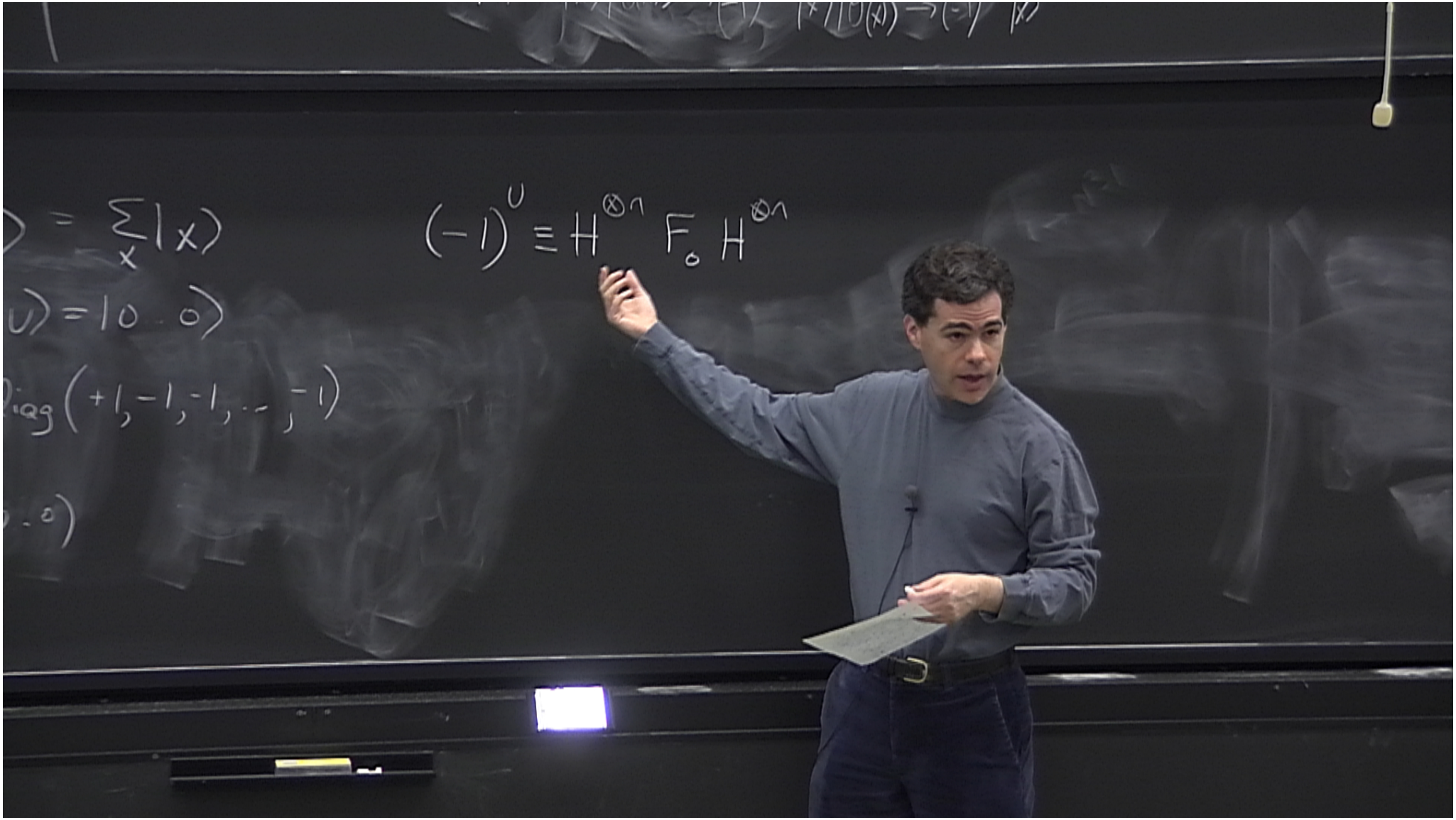
Use oracle $|x\rangle \rightarrow |x\rangle|0(x)\rangle \rightarrow (-1)^{0(x)}|x\rangle|0(x)\rangle \rightarrow (-1)^{0(x)}|x\rangle$

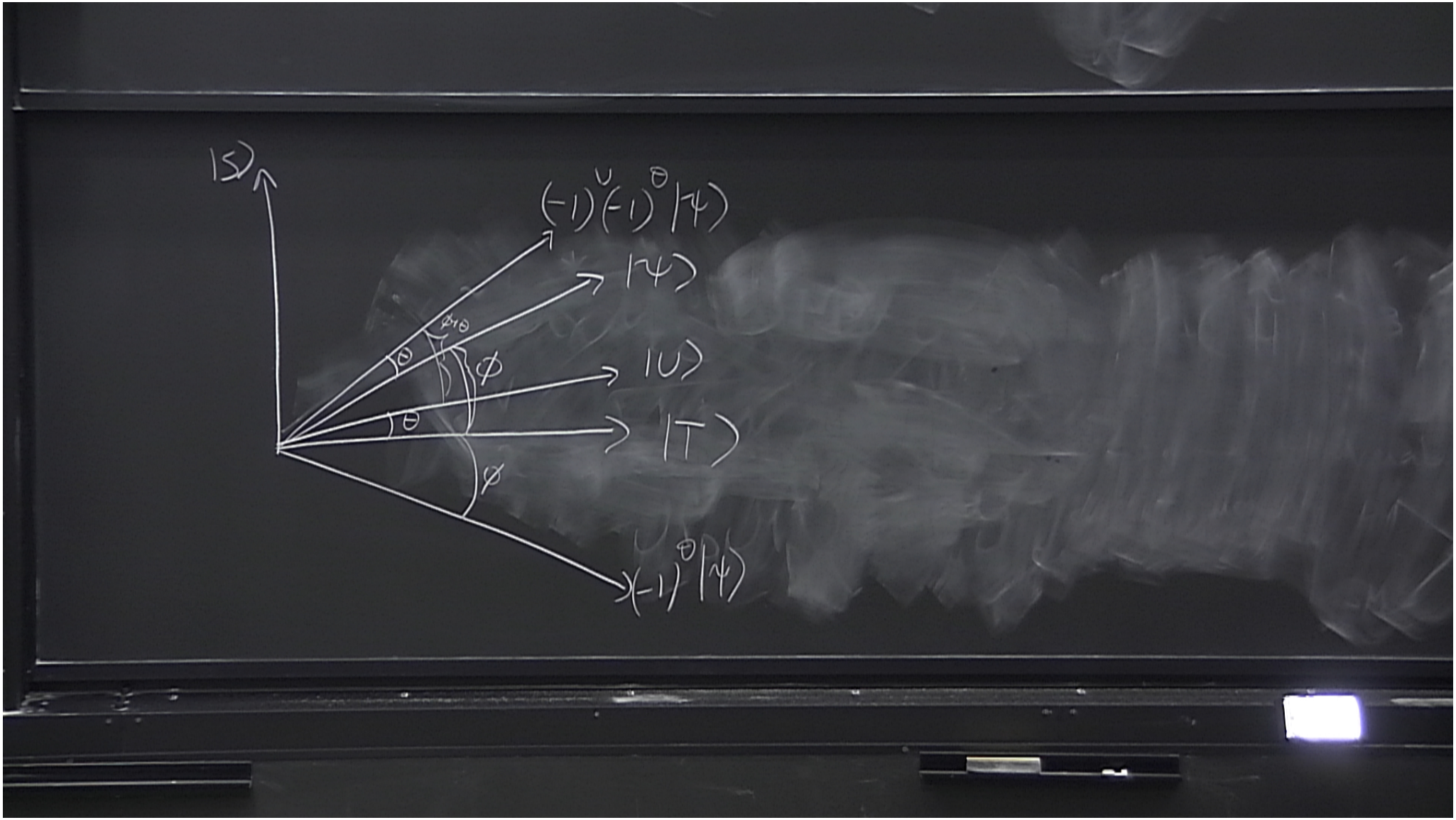


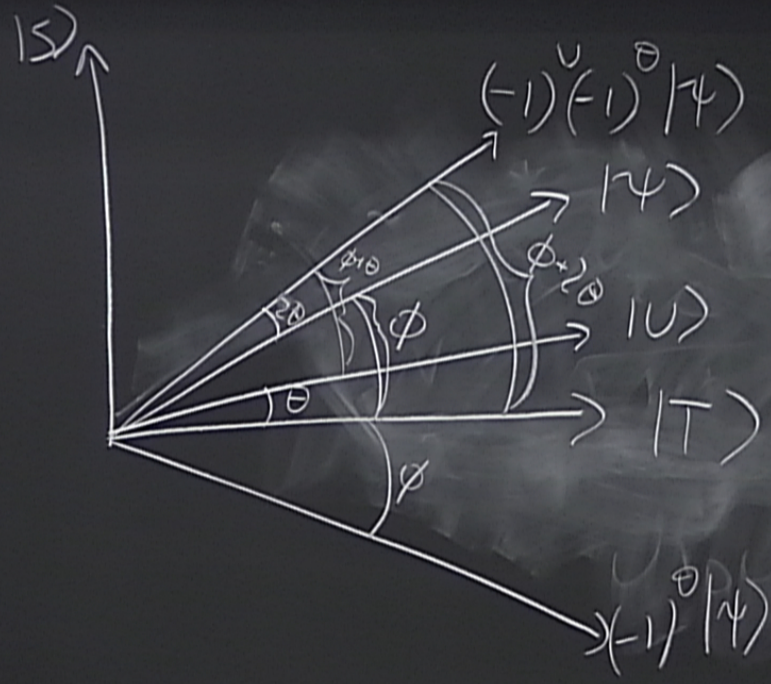

Grover's algorithm
Unstructured search
Find a marked element
Use O(√N) queries
Consider case of unique marked element
Symmetric treatment of unmarked elements
Hilbert space
Use circuit


element:
elements
$(-1)^{\theta}$ - reflection over $|T\rangle$
$|\theta(x)\rangle \to (-1)^{\theta(x)}|x\rangle$
$|S\rangle$
$|\psi\rangle$
$|T\rangle$
$(-1)^{\theta}|\psi\rangle$


Can create $|U\rangle = |S\rangle + |T\rangle = \sum_x |x\rangle$
$(-1)^U \equiv H^{\otimes n}$
$H^{\otimes n}|0\ 0\rangle = |U\rangle$, $H^{\otimes n}|U\rangle = |0\ 0\rangle$
Diagonal phase gate F_0 $\mathrm{diag}(+1,-1,-1,\ldots,-1)$
$|0\ 0\rangle \to |0\ 0\rangle$
$|y\rangle \to -|y\rangle$ $(y \neq 0.0)$

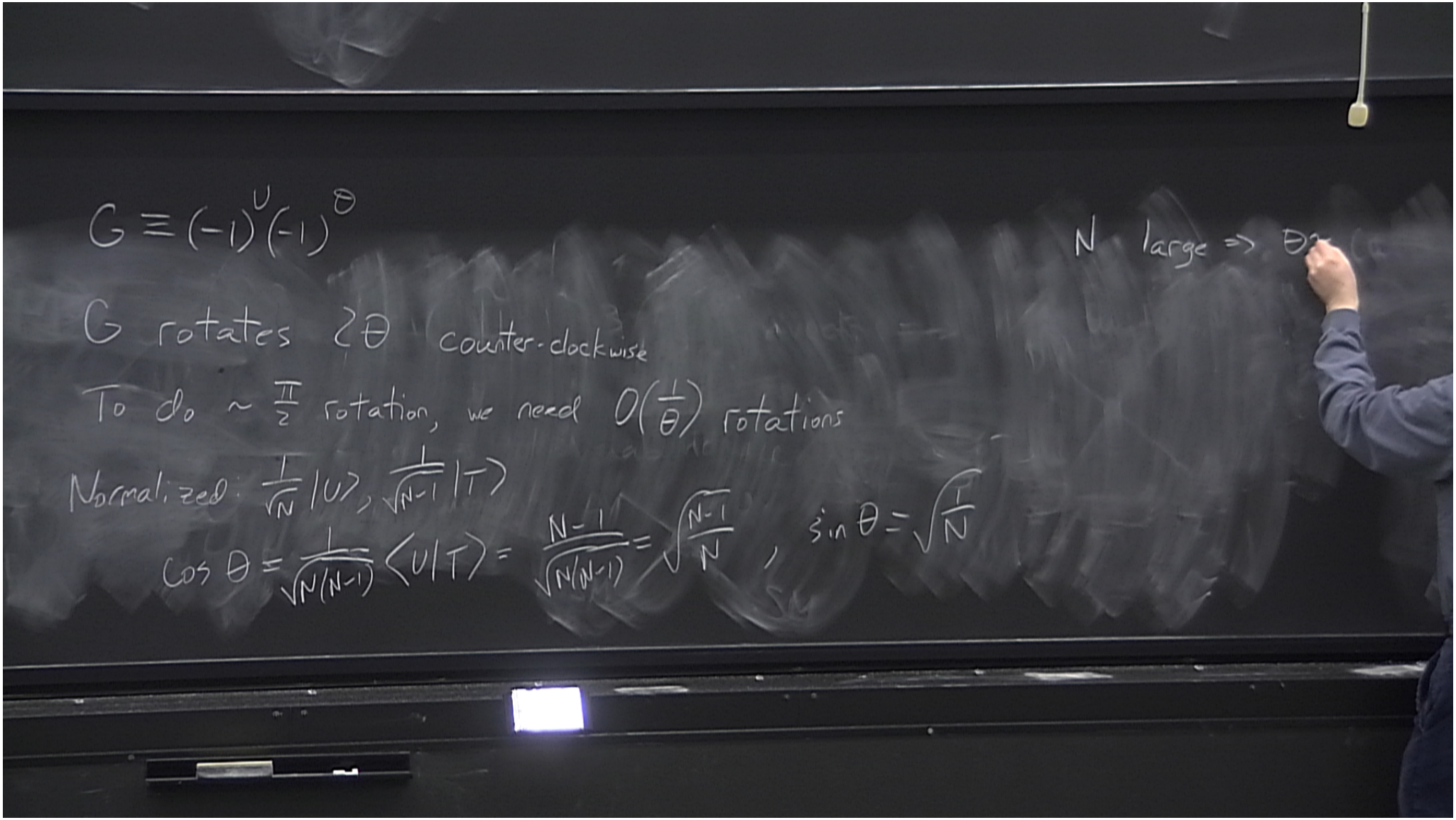

$(-1)^{U} \equiv H^{\otimes n} F_0 H^{\otimes n}$

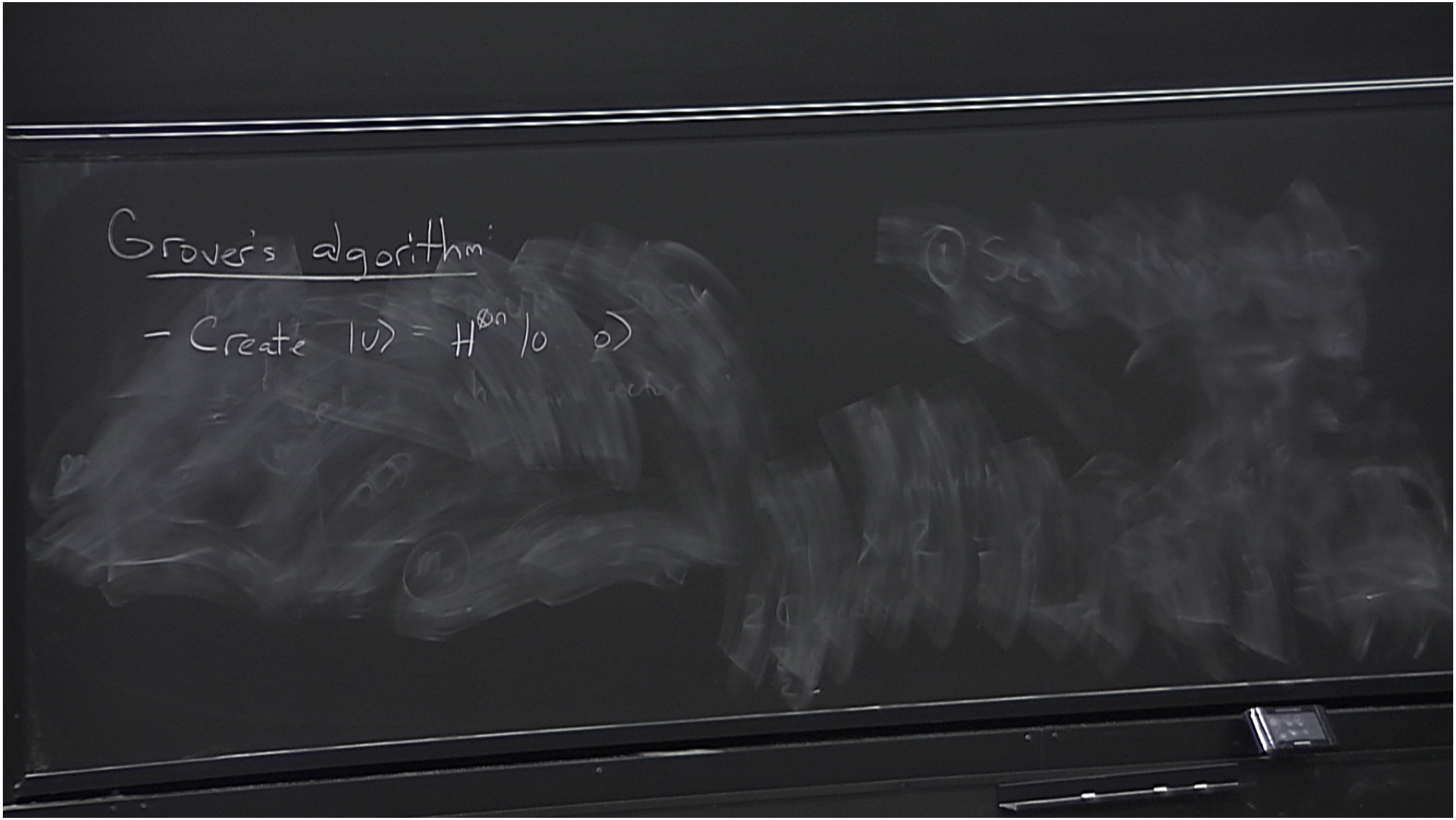

|S⟩
(-1)^U (-1)^θ |↑⟩
|↑⟩
|U⟩
|T⟩
(-1)^θ |↑⟩
θ
φ+θ
φ
θ
φ

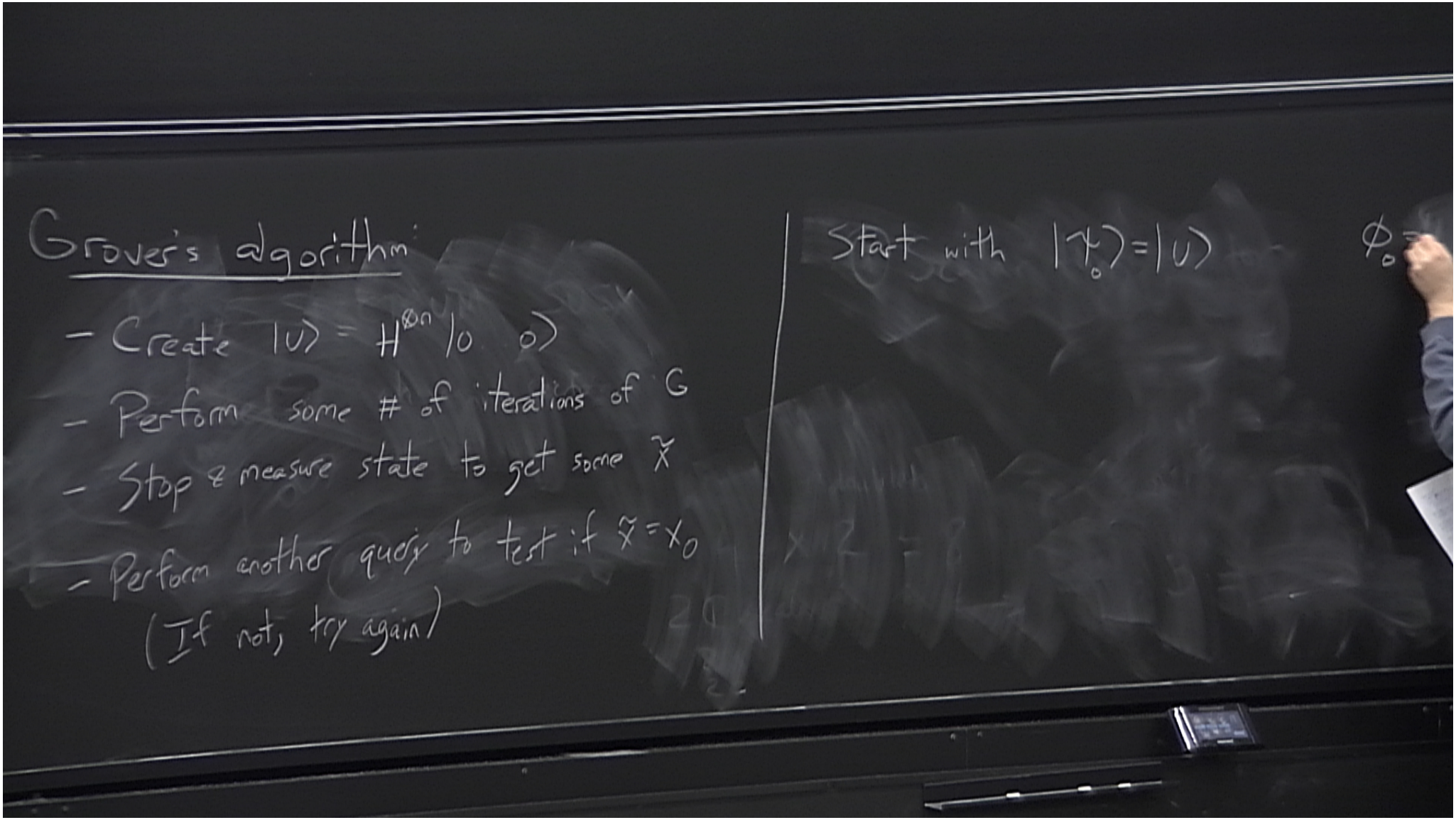


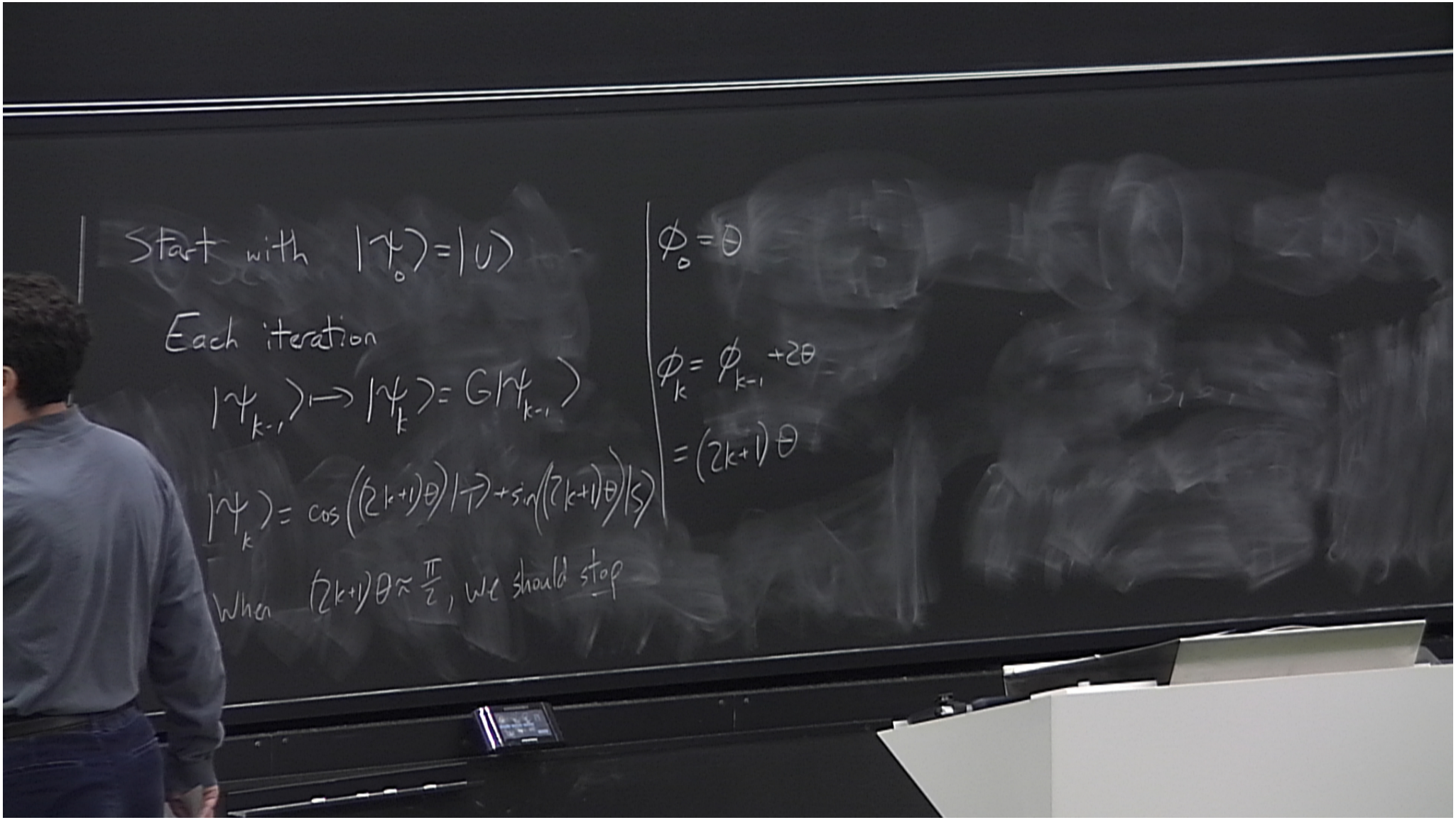
$G \equiv (-1)^U (-1)^\theta$

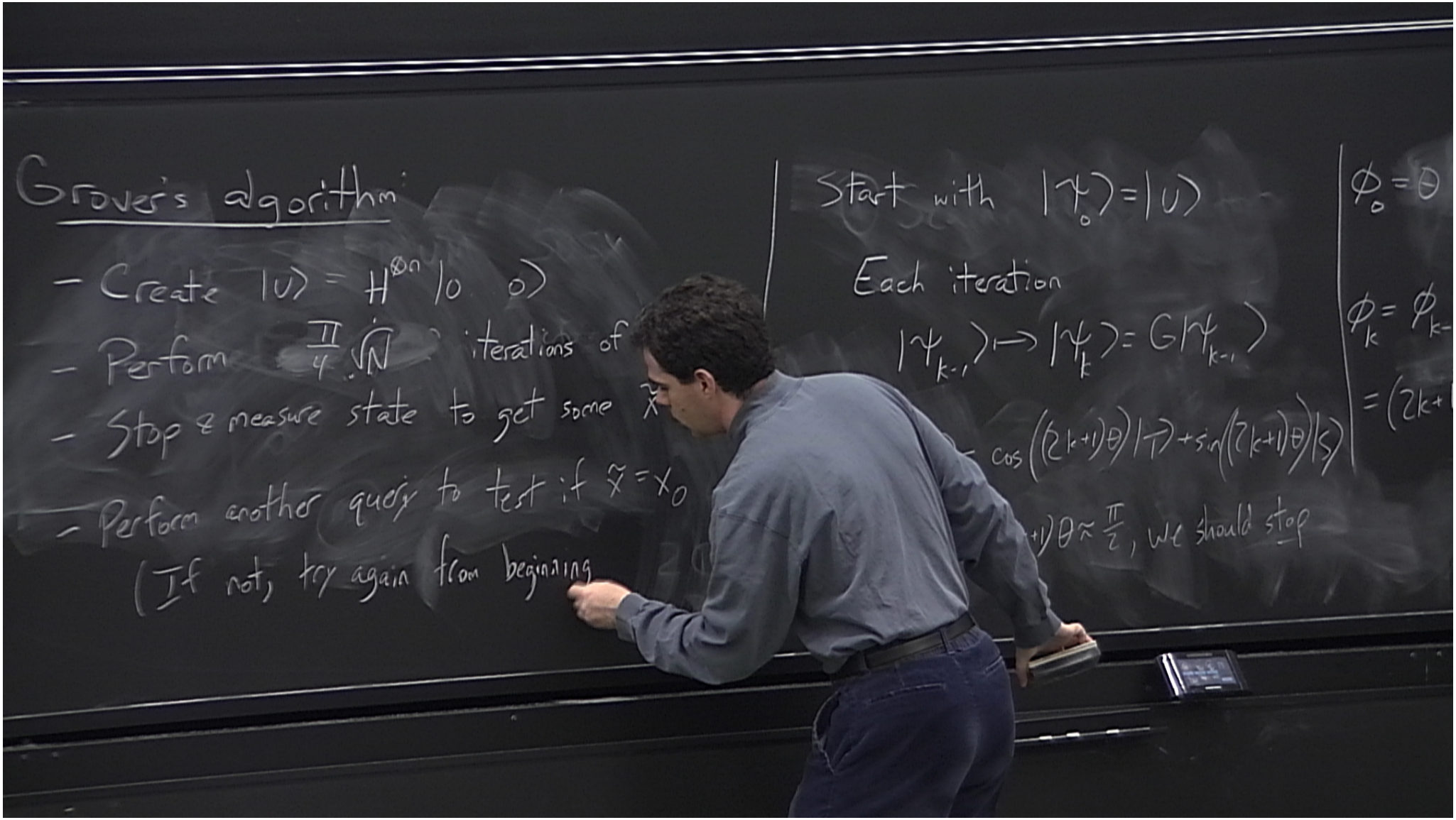
G rotates 2θ counter-clockwise


$G \equiv (-1)^U (-1)^\theta$
N large => θ
G rotates 2θ counter-clockwise
To do ~ π/2 rotation, we need O(1/θ) rotations
Normalized: $\frac{1}{\sqrt{N}}|U\rangle, \frac{1}{\sqrt{N-1}}|T\rangle$
$\cos\theta = \frac{1}{\sqrt{N(N-1)}}\langle U|T\rangle = \frac{N-1}{\sqrt{N(N-1)}} = \sqrt{\frac{N-1}{N}}$, $\sin\theta = \sqrt{\frac{1}{N}}$

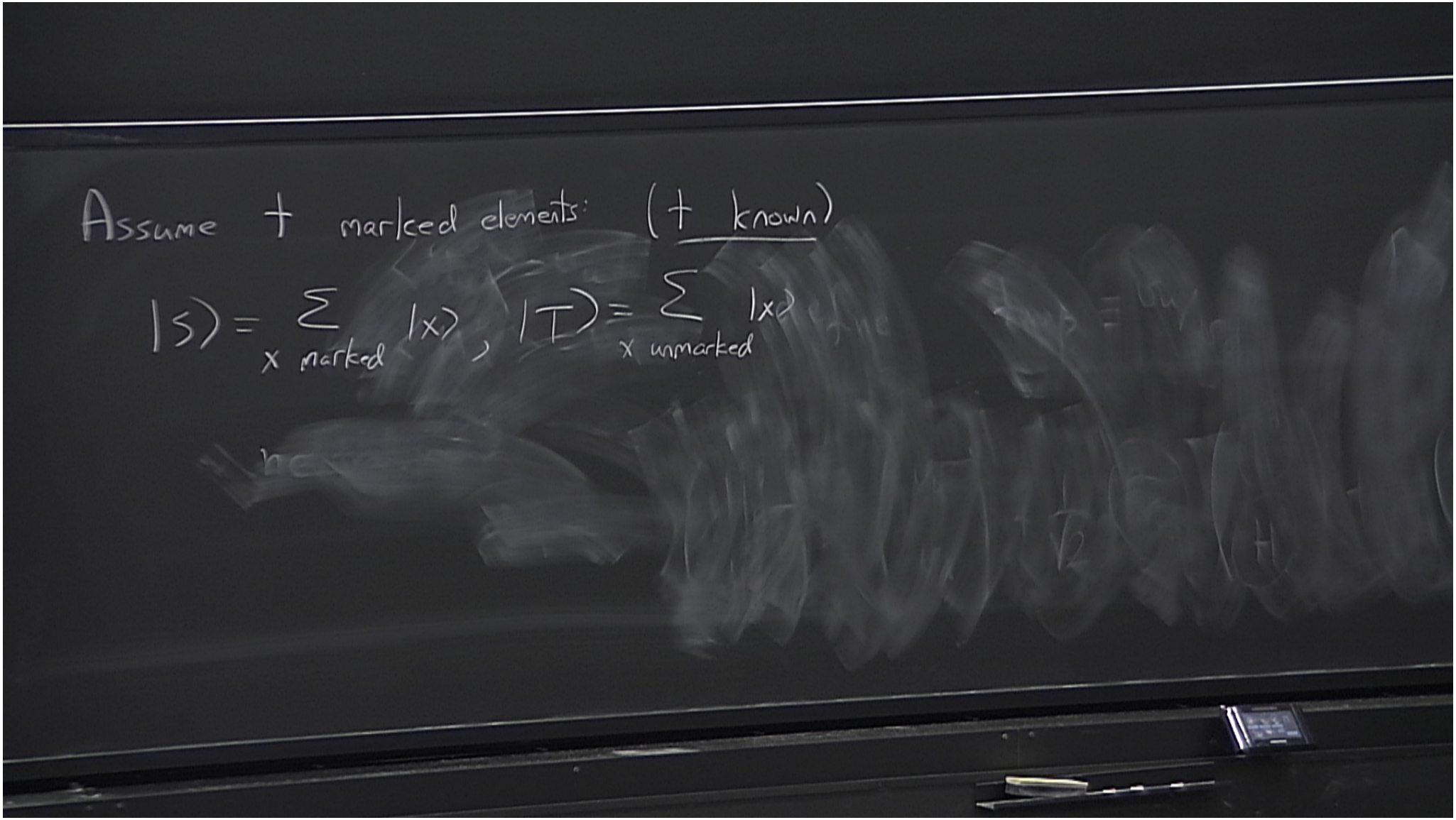

Grover's algorithm
- Create $|u\rangle = H^{\otimes n}|0\ 0\rangle$

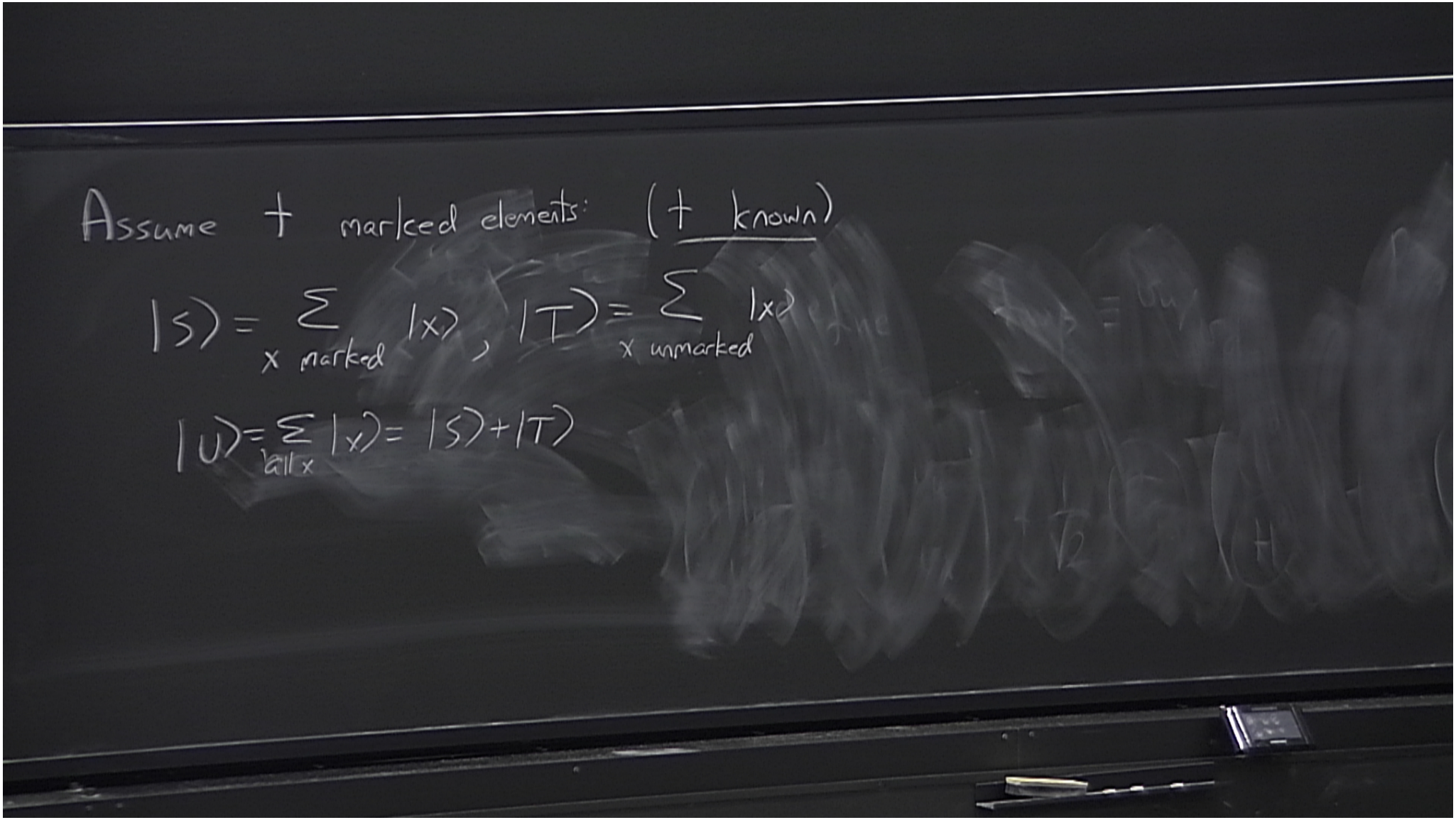

Grover's algorithm
- Create $|U\rangle = H^{\otimes n}|0\ \ 0\rangle$
- Perform some # of iterations of G
- Stop & measure state to get some $\tilde{x}$
- Perform another query to test if $\tilde{x} = x_0$
(If not, try again)
Start with $|\psi_0\rangle = |U\rangle$
ϕ_0

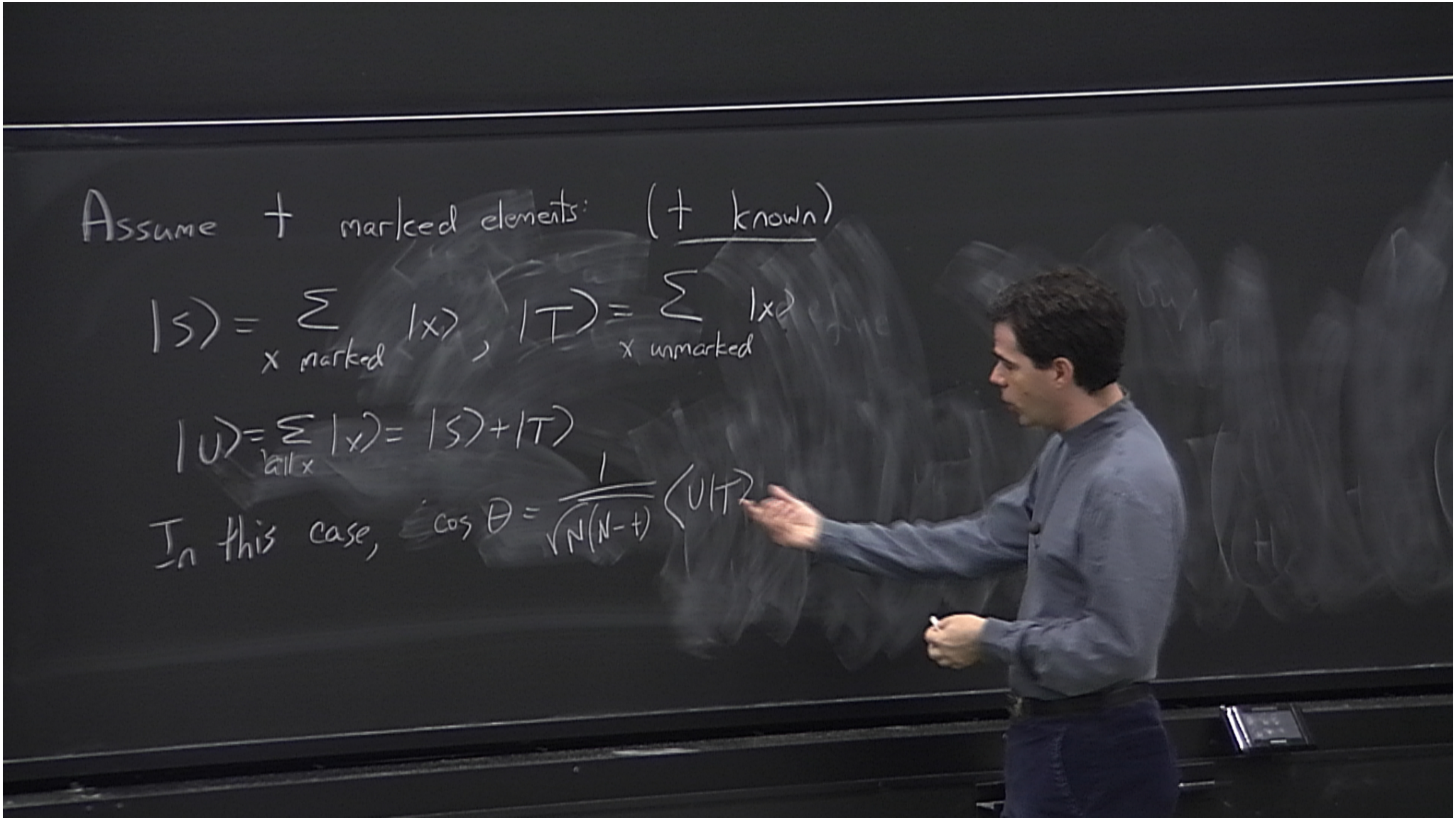

Start with $|\psi_0\rangle = |U\rangle$
Each iteration
$|\psi_{k-1}\rangle \mapsto |\psi_k\rangle = G|\psi_{k-1}\rangle$
$|\psi_k\rangle = \cos((2k+1)\theta)|T\rangle + \sin((2k+1)\theta)|S\rangle$
When $(2k+1)\theta \approx \frac{\pi}{2}$, we should stop
$\phi_0 = \theta$
$\phi_k = \phi_{k-1} + 2\theta$
$= (2k+1)\theta$

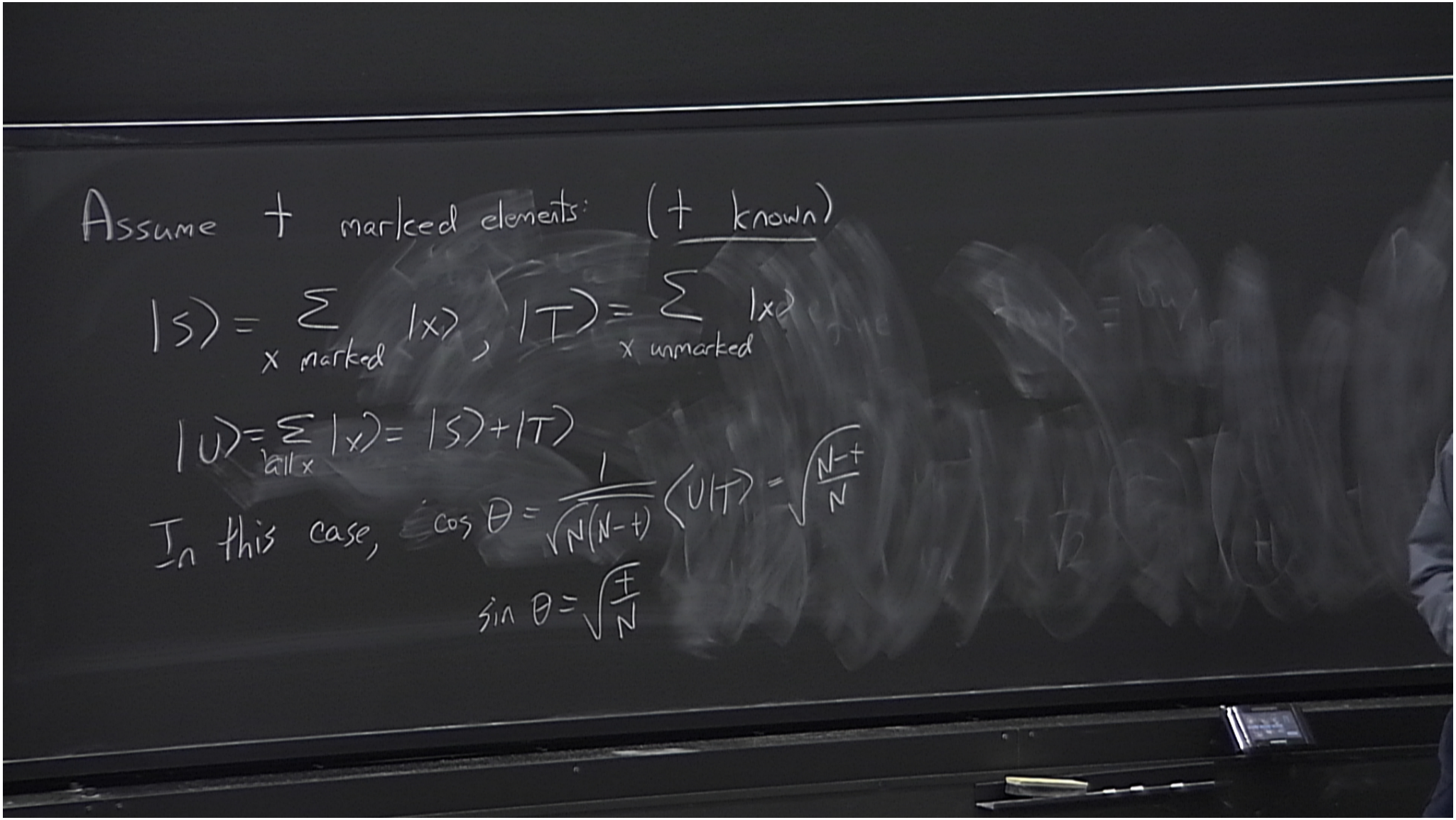

Grover's algorithm
- Create |U⟩ = H^⊗n |0 ... 0⟩
- Perform (π/4)√N iterations of
- Stop & measure state to get some x̃
- Perform another query to test if x̃ = x_0
(If not, try again from beginning
Start with |ψ_0⟩ = |U⟩
Each iteration
|ψ_{k-1}⟩ ↦ |ψ_k⟩ = G|ψ_{k-1}⟩
cos((2k+1)θ)|T⟩ + sin((2k+1)θ)|S⟩
+1)θ ≈ π/2, we should stop
φ_0 = θ
φ_k = φ_{k-
= (2k+

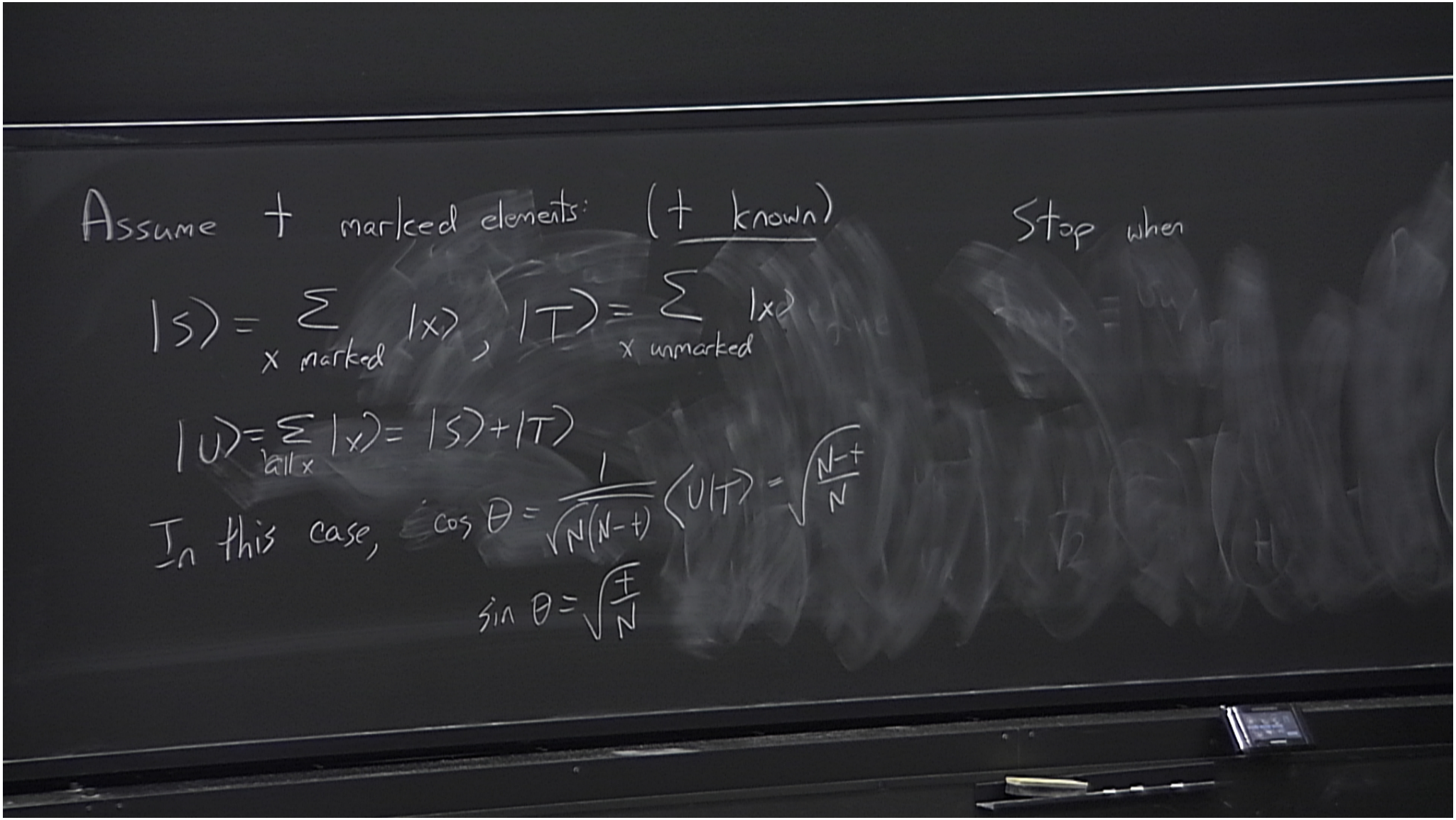

Assume + marked elements:

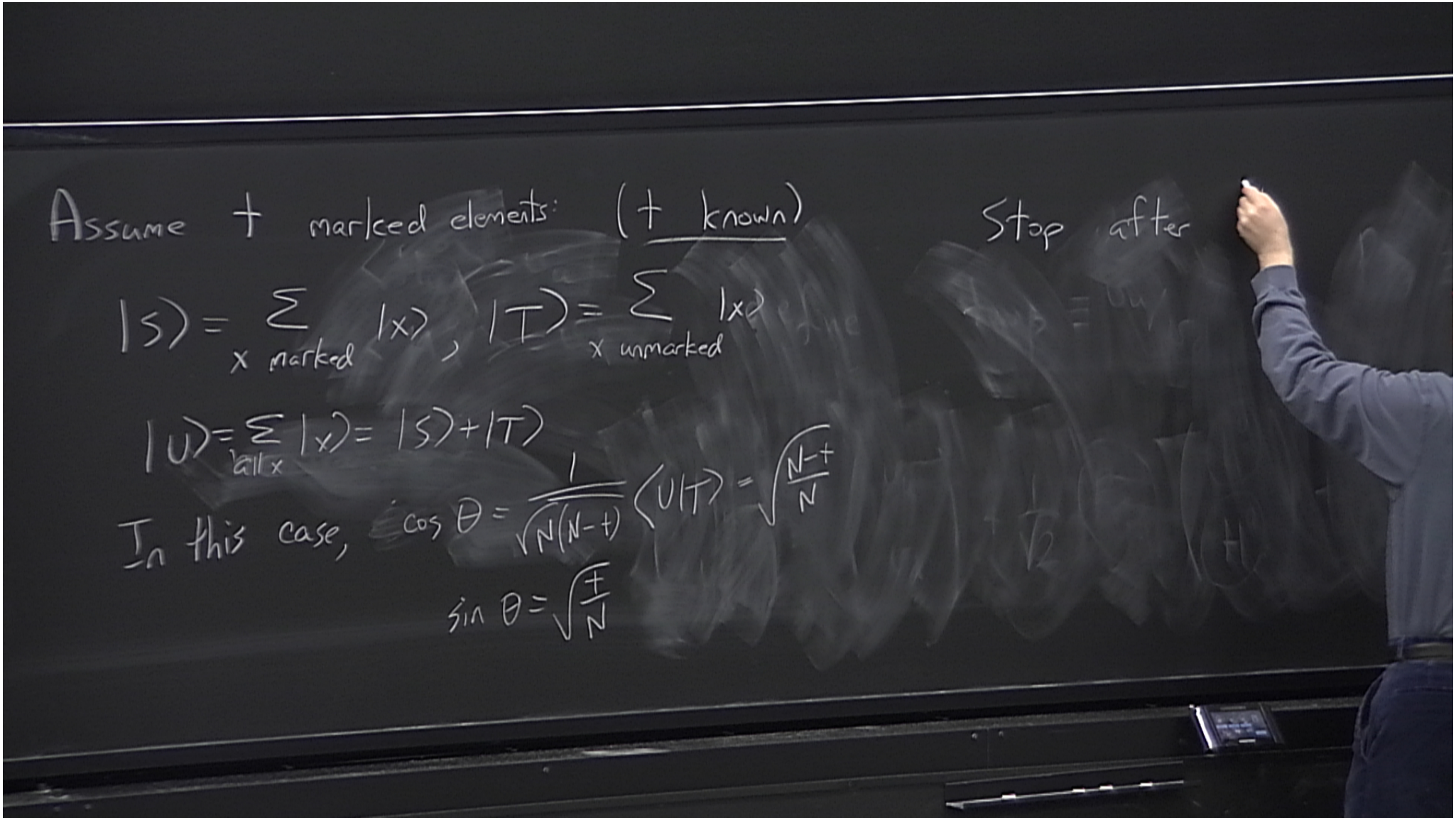

Assume t marked elements: (t known)
$|S\rangle = \sum_{x \text{ marked}} |x\rangle$, $|T\rangle = \sum_{x \text{ unmarked}} |x\rangle$

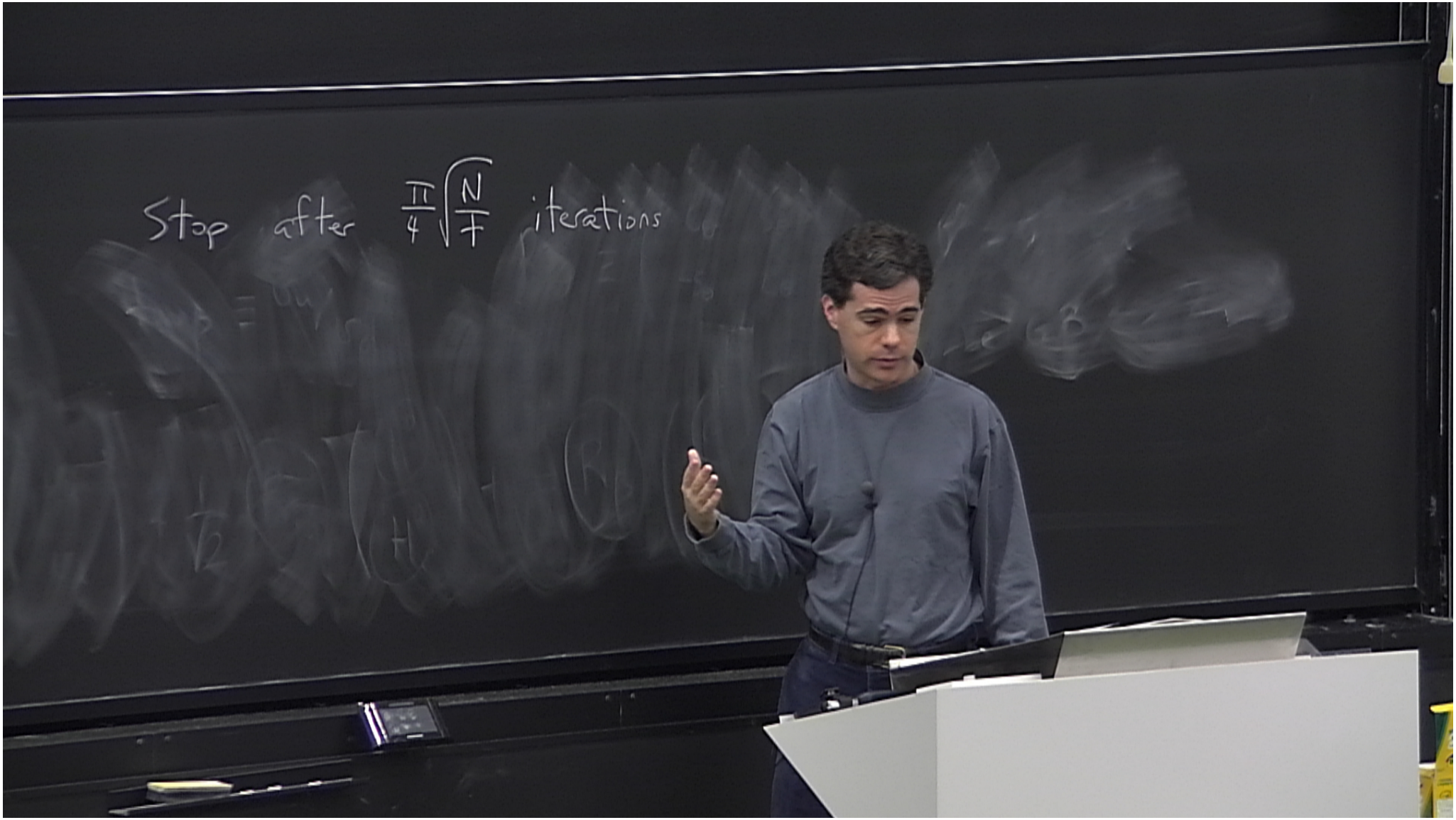

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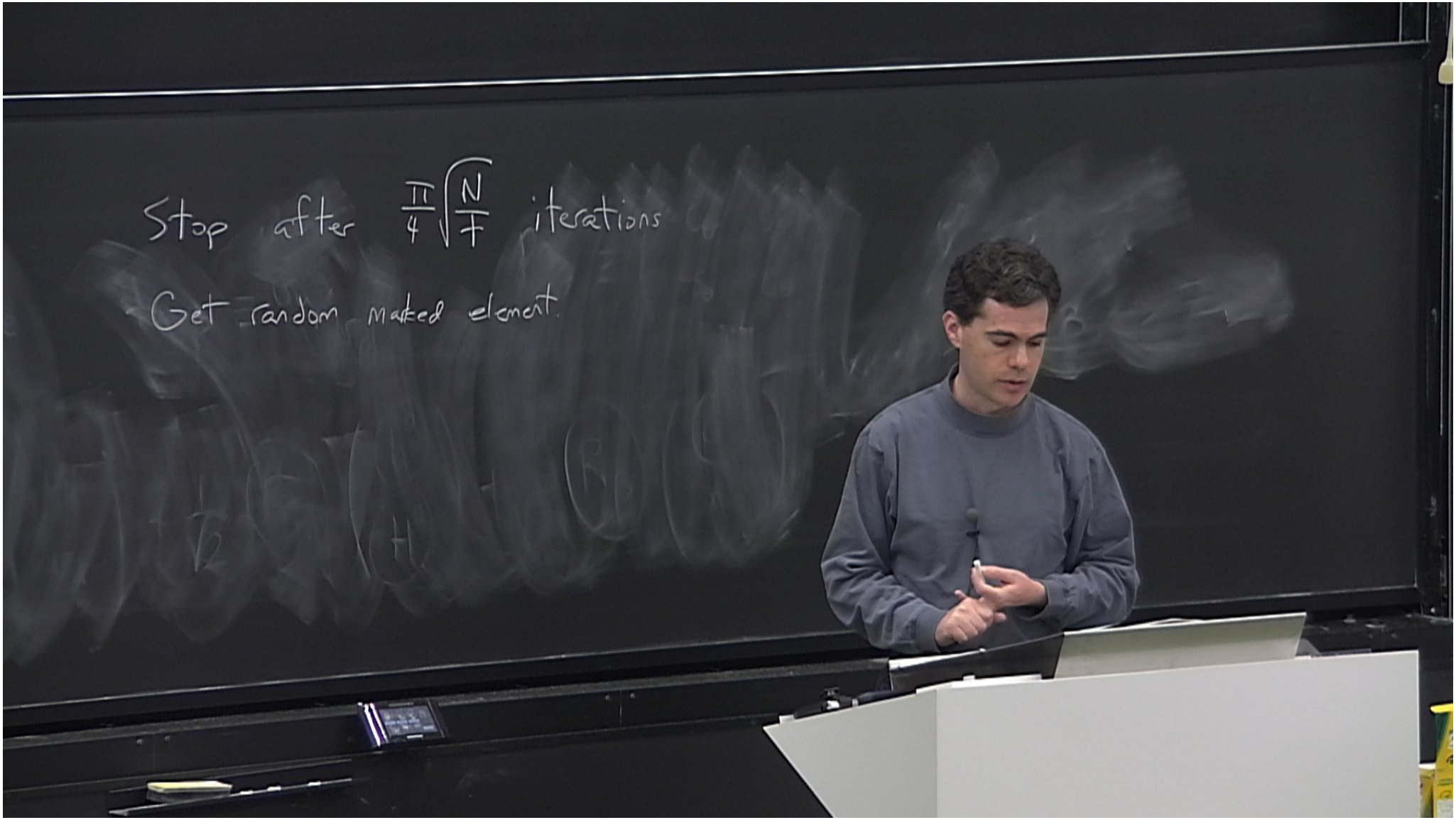

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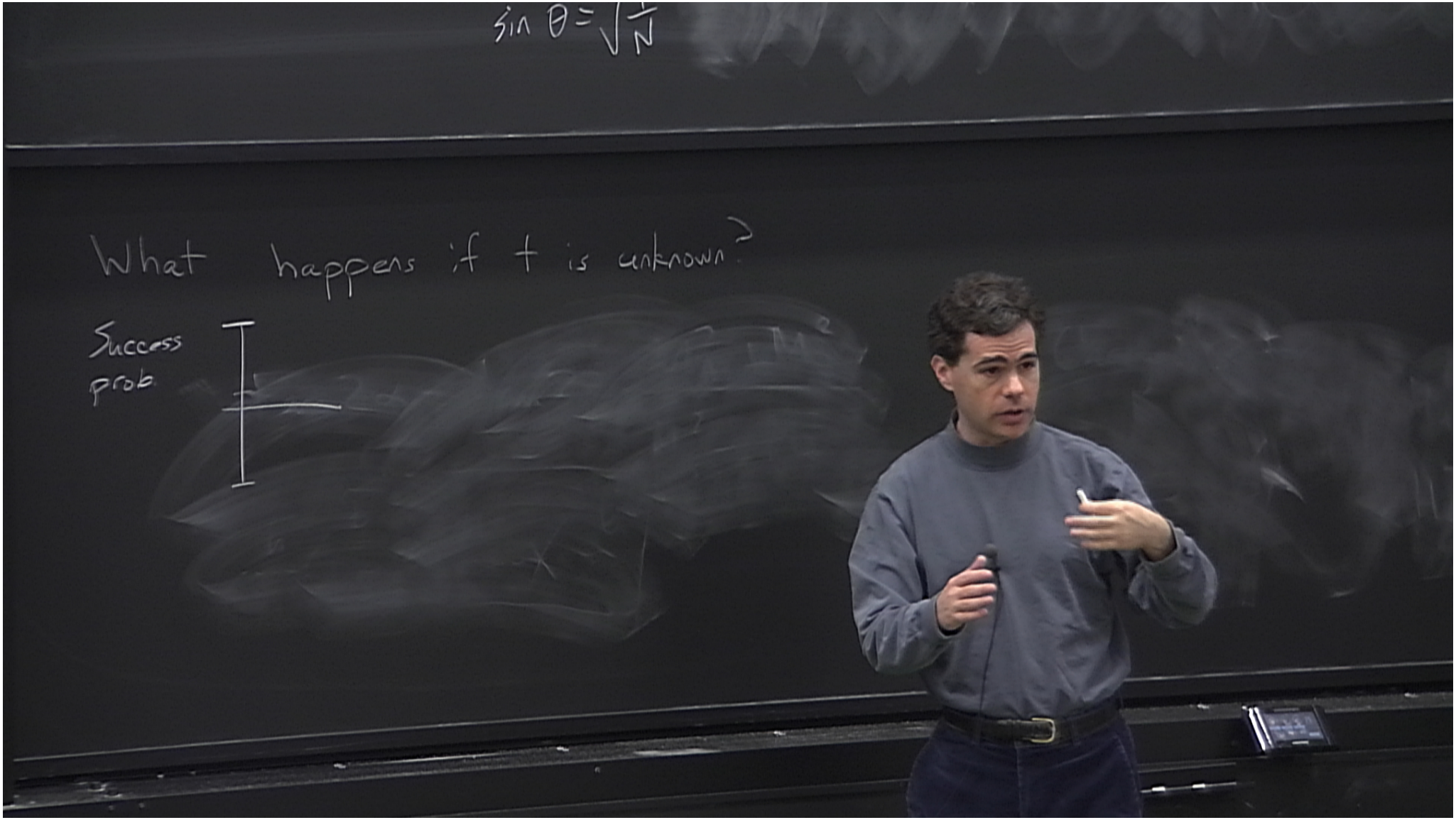

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$\sin\theta = \sqrt{\frac{t}{N}}$


Assume t marked elements: (t known)
Stop when
$|S\rangle = \sum_{x \text{ marked}} |x\rangle$, $|T\rangle = \sum_{x \text{ unmarked}} |x\rangle$
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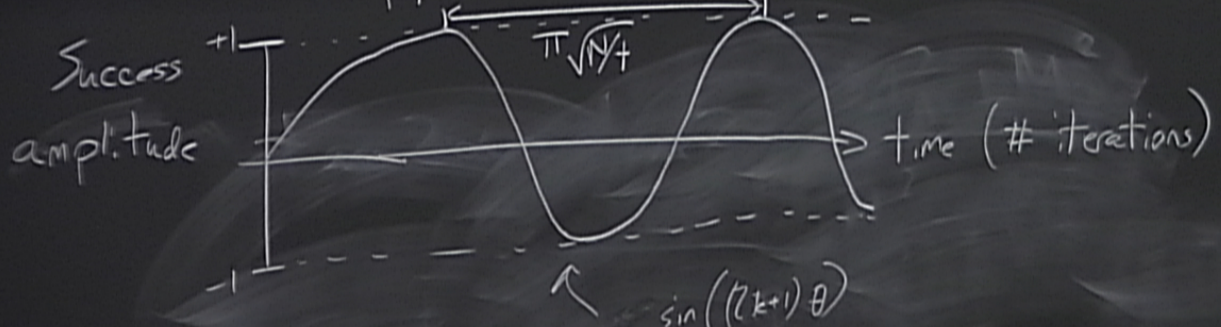

Stop after $\frac{\pi}{4}\sqrt{\frac{N}{f}}$ iterations


Stop after $\frac{\pi}{4}\sqrt{\frac{N}{f}}$ iterations
Get random marked element


$\sin\theta = \sqrt{\frac{1}{N}}$
What happens if t is unknown?
Success prob

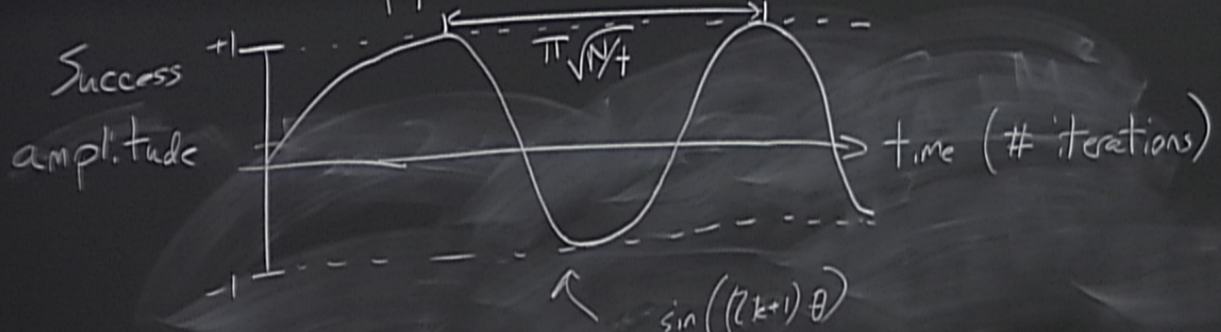
$\sin\theta = \sqrt{\frac{t}{N}}$

What happens if t is unknown?



$\sin\theta = \sqrt{\frac{t}{N}}$

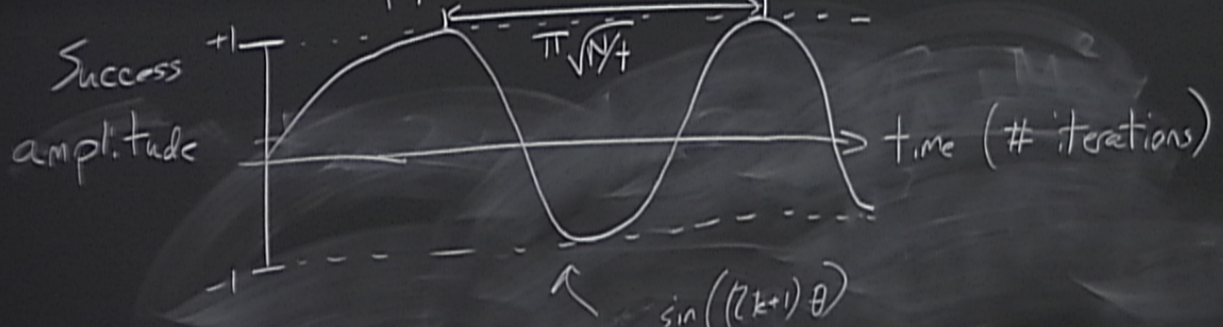
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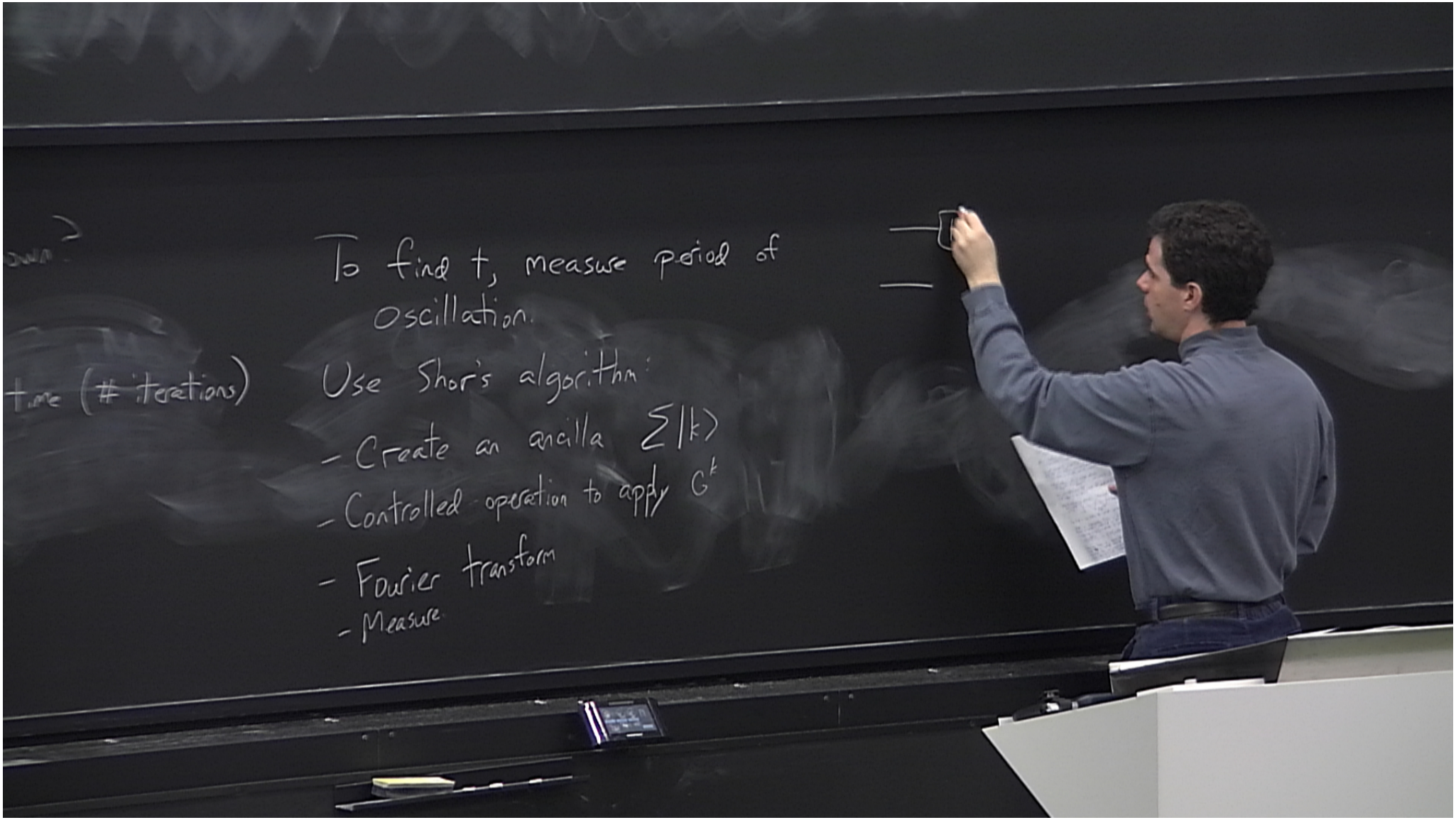
To find t,

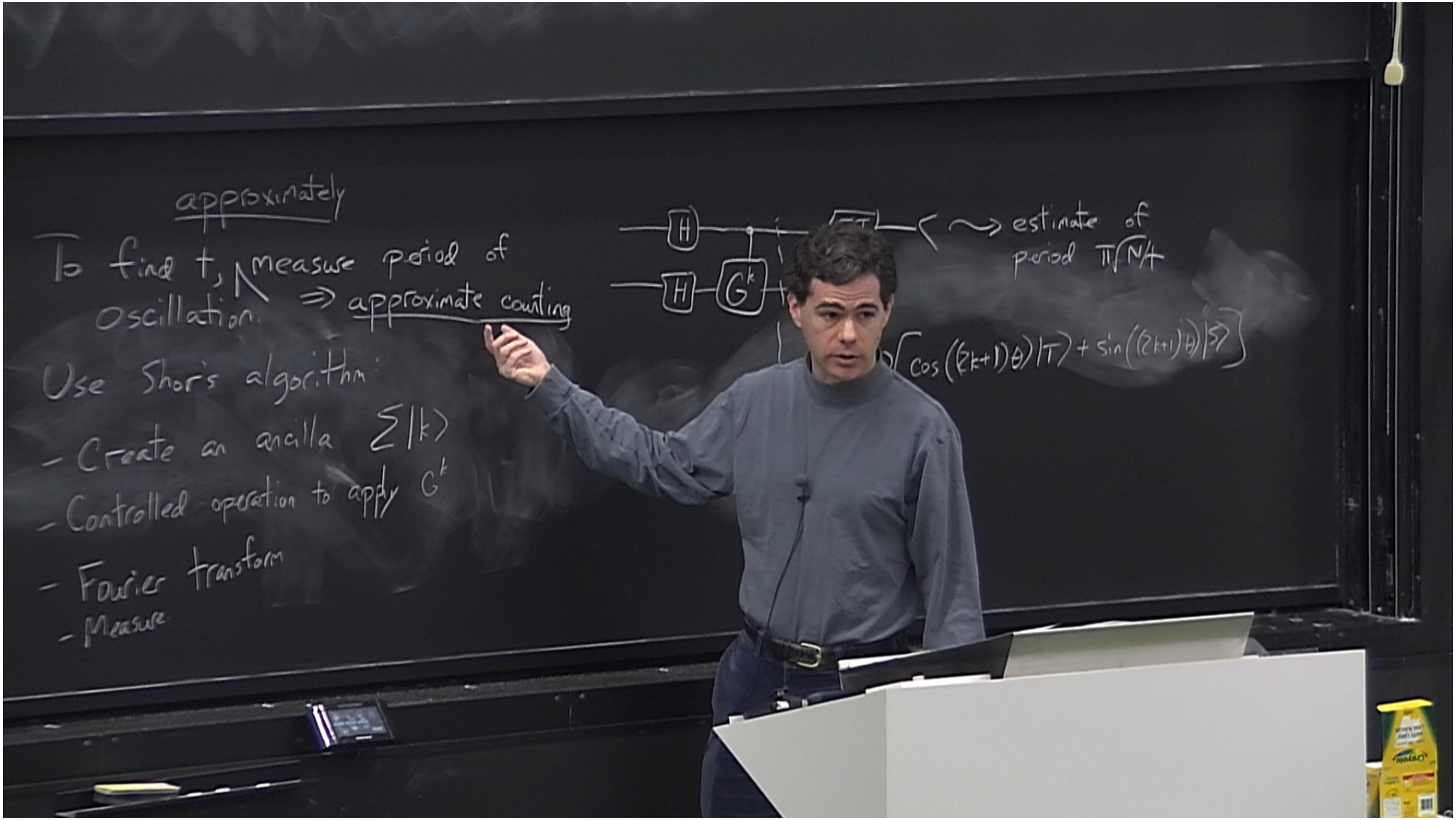
$\sin\theta = \sqrt{\frac{t}{N}}$

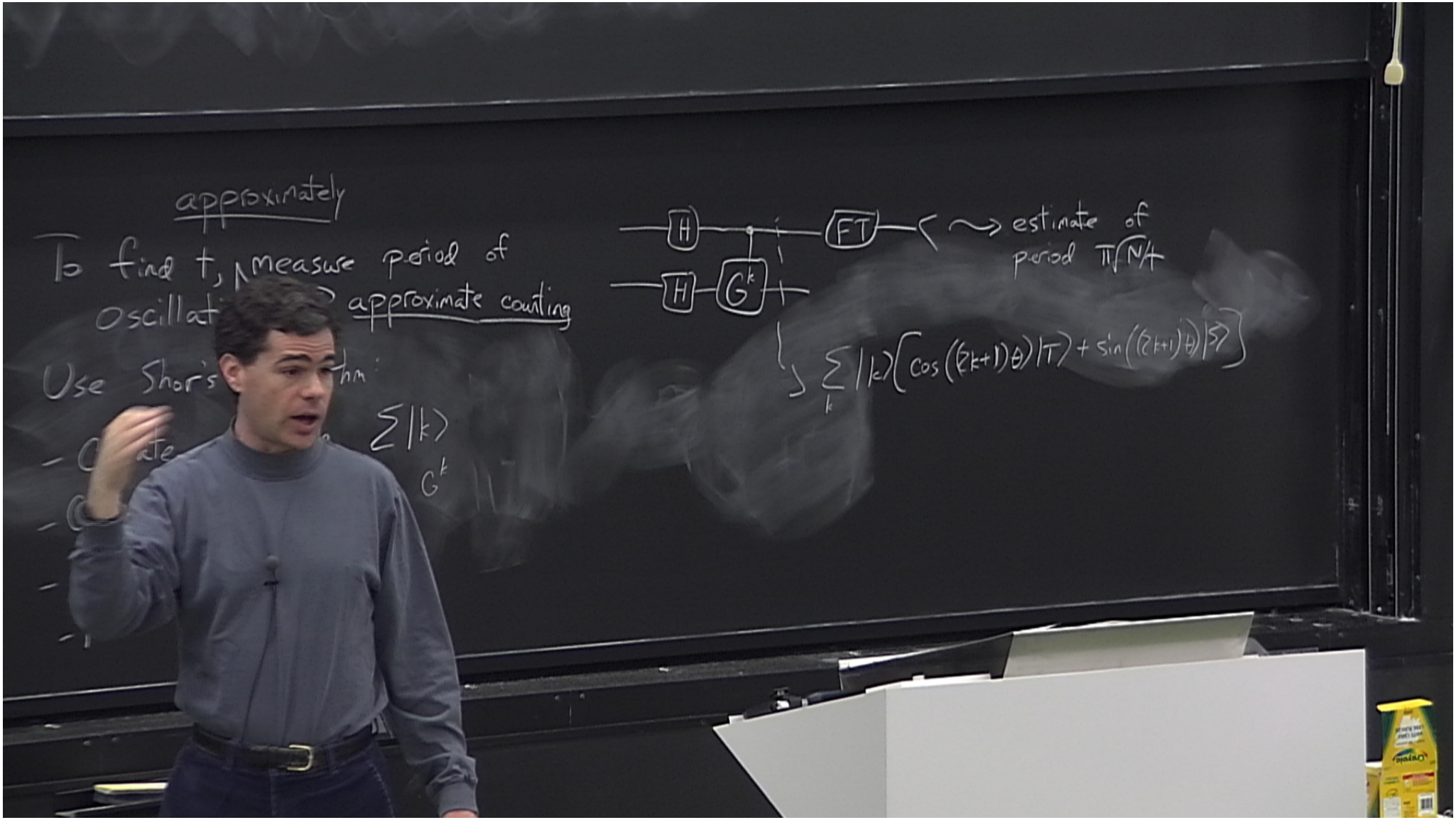
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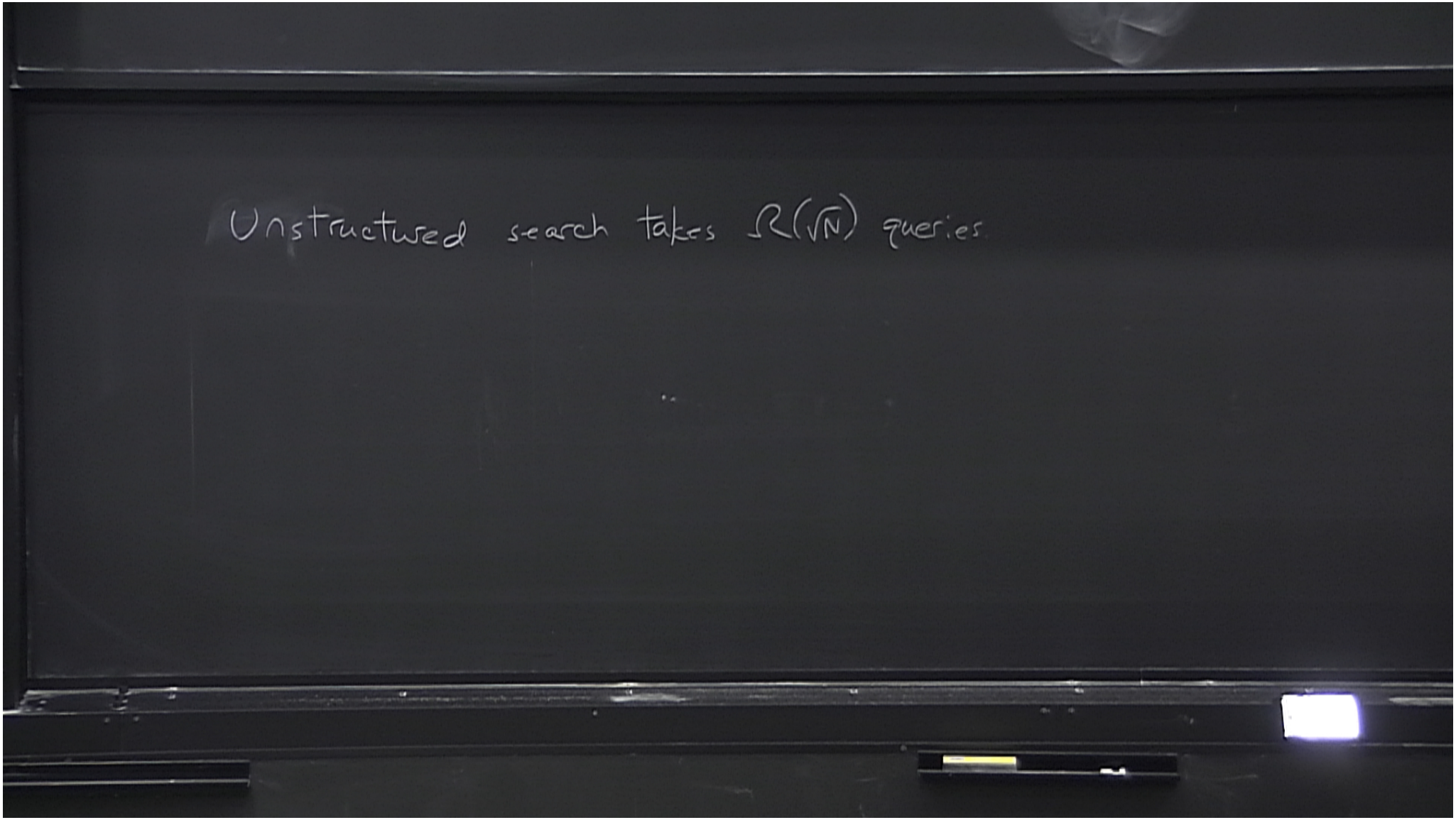


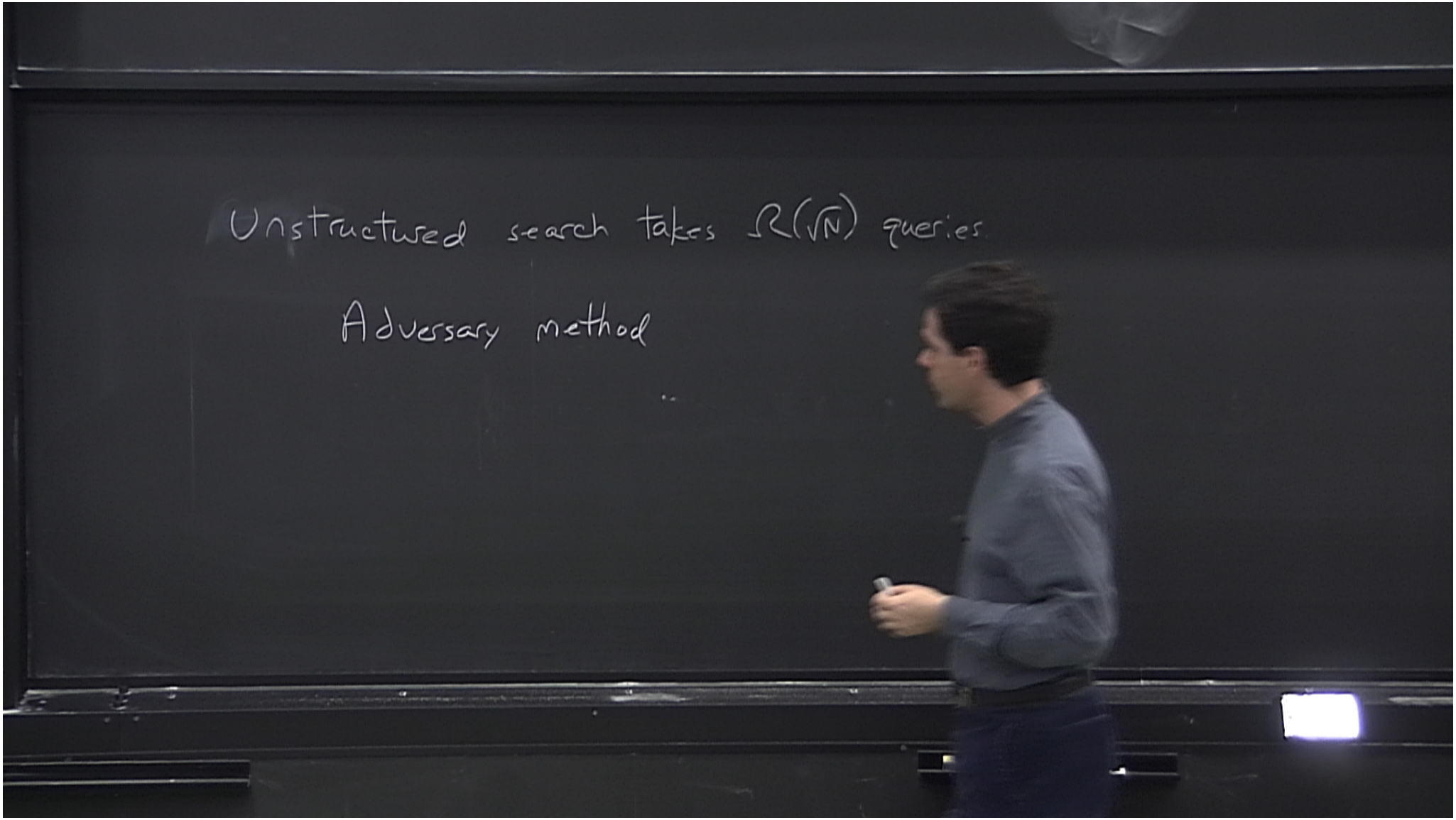
To find t, measure [illegible] of oscillation.

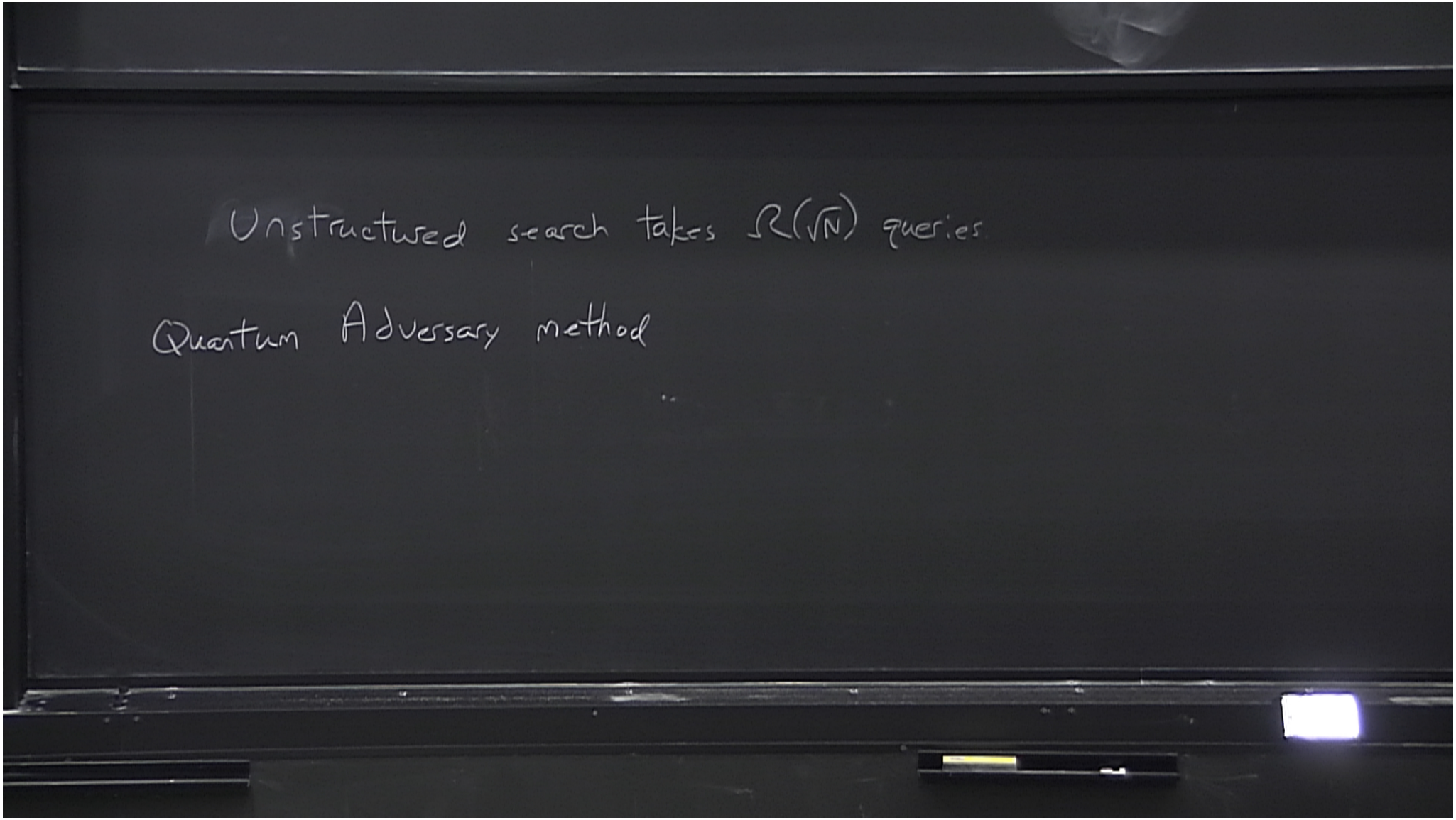

To find t, measure period of oscillation.
Use Shor's algorithm:
- Create an ancilla $\Sigma|k\rangle$
- Controlled operation to apply G^k
- Fourier transform
- Measure


approximately
To find t, measure period of oscillation. ⇒ approximate counting
Use Shor's algorithm:
- Create an ancilla Σ|k⟩
- Controlled operation to apply G^k
- Fourier transform
- Measure
H
H
G^k
estimate of period π√N/t
cos((2k+1)θ)|T⟩ + sin((2k+1)θ)|S⟩


approximately
To find t, measure period of
oscillat
approximate counting
Use Shor's
H
H
FT
estimate of
period $\pi\sqrt{N/t}$
$\sum_k |k\rangle \left[\cos((2k+1)\theta)|T\rangle + \sin((2k+1)\theta)|S\rangle \right]$


Unstructured search takes $\Omega(\sqrt{N})$ queries.


Unstructured search takes $\Omega(\sqrt{N})$ queries.
Adversary method


Unstructured search takes $\Omega(\sqrt{N})$ queries
Quantum Adversary method

Unstructured search takes $\Omega(\sqrt{N})$ queries.

Quantum Adversary method

Superposition over all oracles w/ 1 marked element.

Unstructured search takes $\Omega(\sqrt{N})$ queries

Quantum Adversary method

Superposition over all oracles w/ 1 marked element

Start with $|0\ldots0\rangle \sum_\theta |\theta\rangle$ — tensor product

End with $\sum_\theta |\theta\rangle|\theta\rangle$ — entangled