

Title: Quantum Information (Review) - Lecture 3

Date: Feb 15, 2012 10:15 AM

URL: <http://pirsa.org/12020032>

Abstract:



DiVincenzo criteria:

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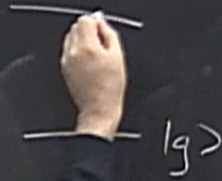
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Ion trap:

Group II atoms
Ionize once $\Rightarrow$ one
electron in outer shell
Energy state of that
electron is qubit.



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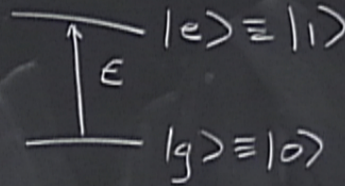
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$$\text{---} |e\rangle \equiv |1\rangle$$

$$\text{---} |g\rangle \equiv |0\rangle$$

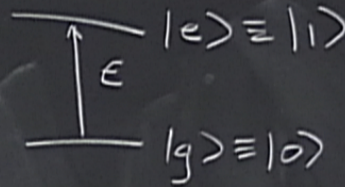
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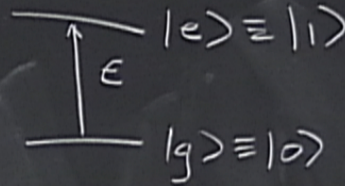
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Atom is charged.

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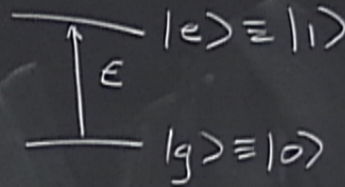
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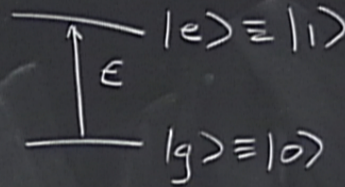


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Single ion:
Hamiltonian $H = H_{\text{atom}}$

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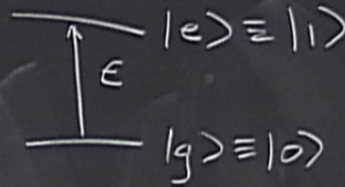
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$$\text{Hamiltonian } H = H_{\text{atom}} + H_{\text{laser}}$$

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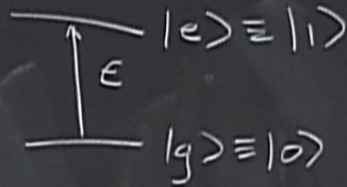
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$$H_{\text{atom}} = \epsilon |e\rangle\langle e| +$$

$$H_{\text{laser}} = J \left(e^{-i\omega t} |g\rangle\langle e| + e^{i\omega t} |e\rangle\langle g| \right)$$

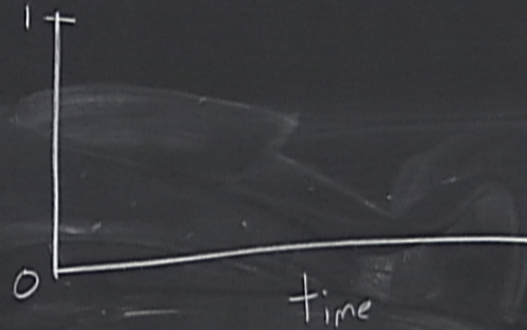
laser frequency $\omega = \epsilon/\hbar$

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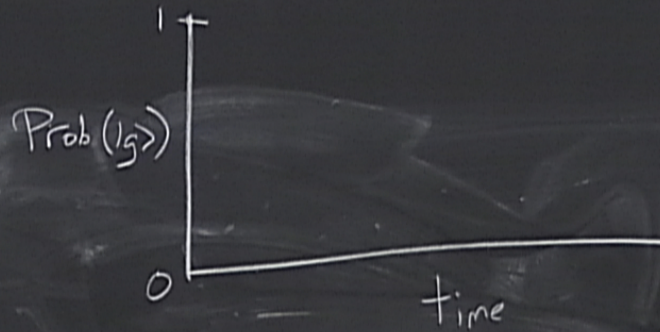


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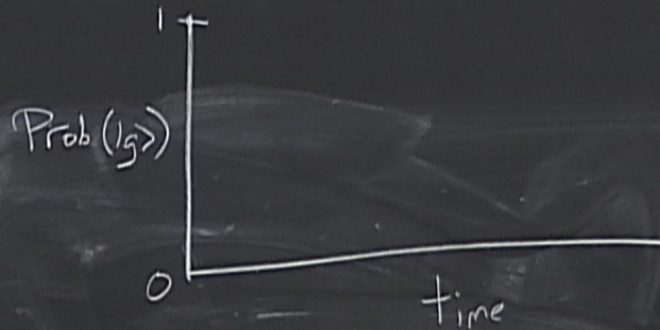


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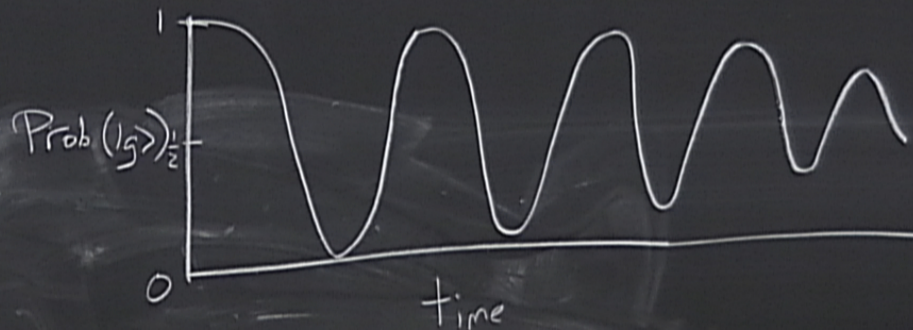
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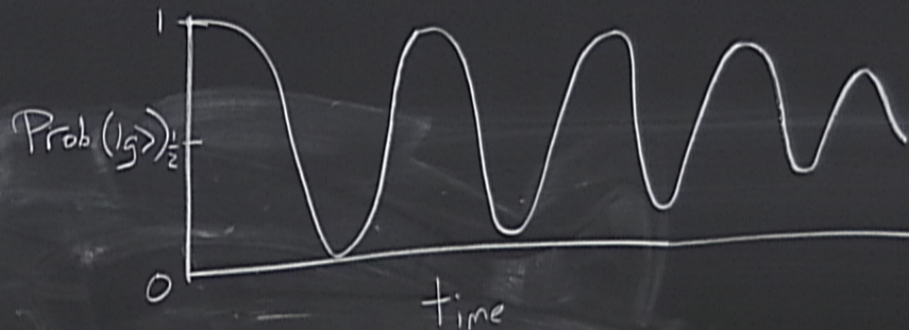
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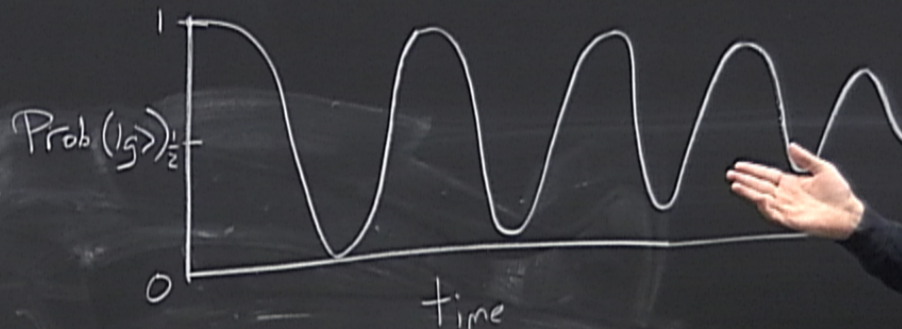
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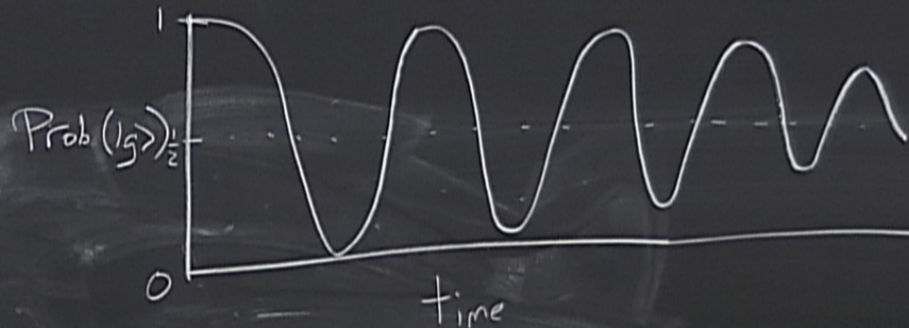
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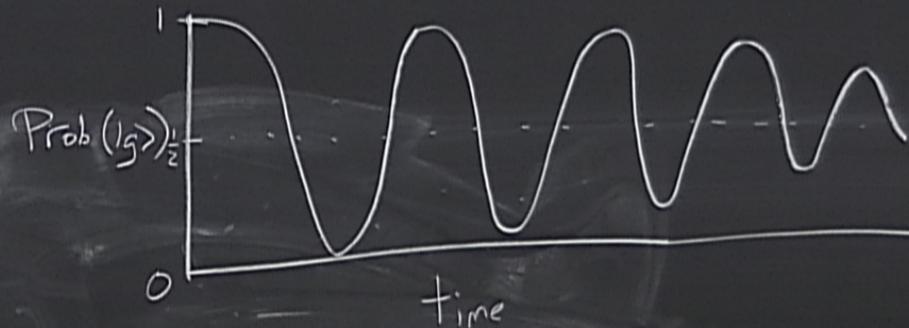
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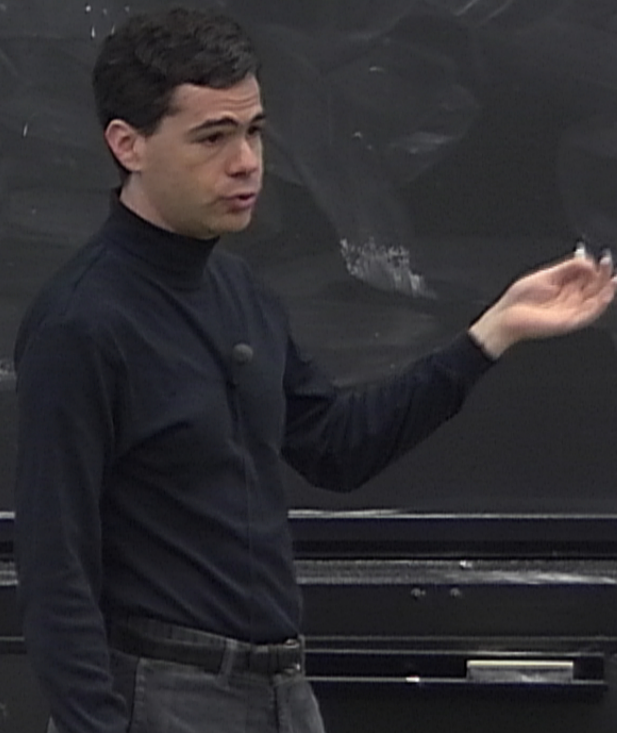
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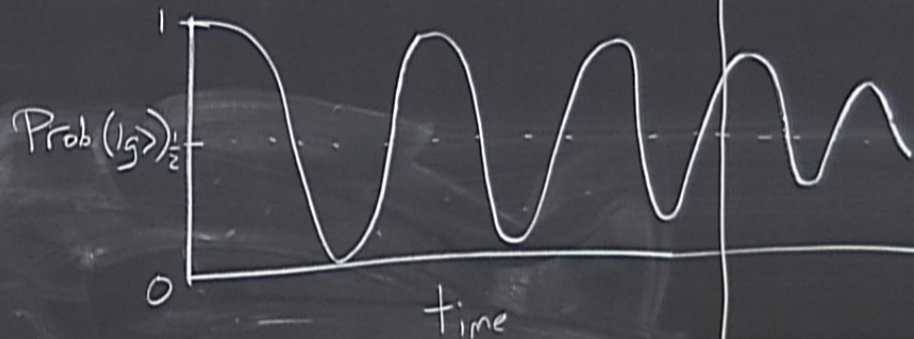
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Diagonal phase gates

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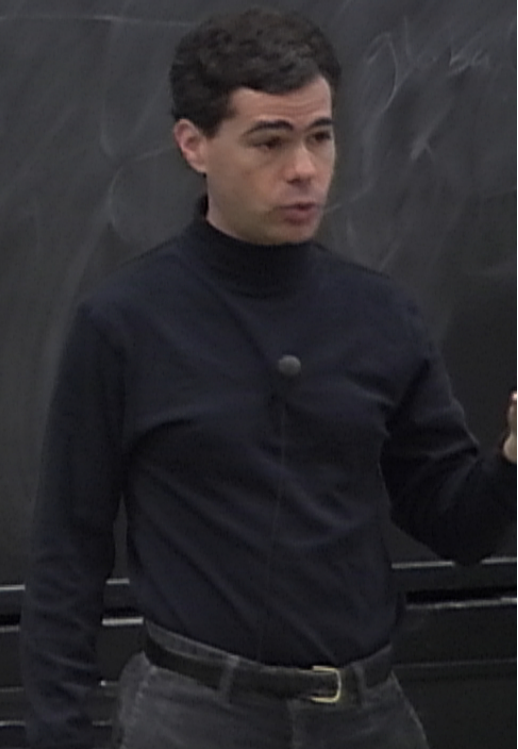
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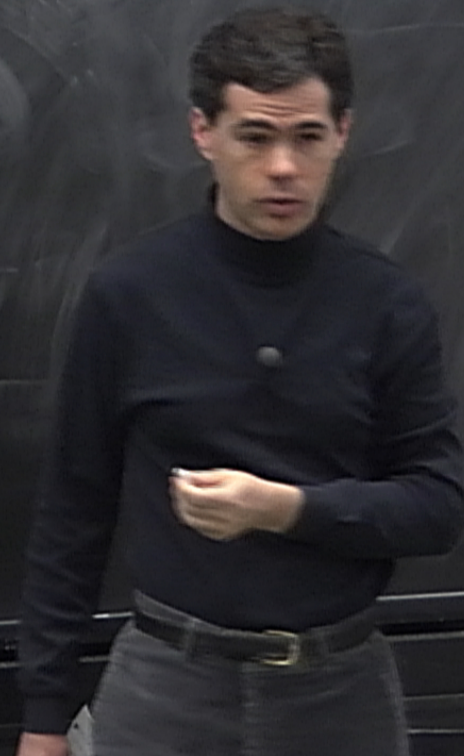
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From these, we can
get any single-qubit gate

Can put multiple ions in trap



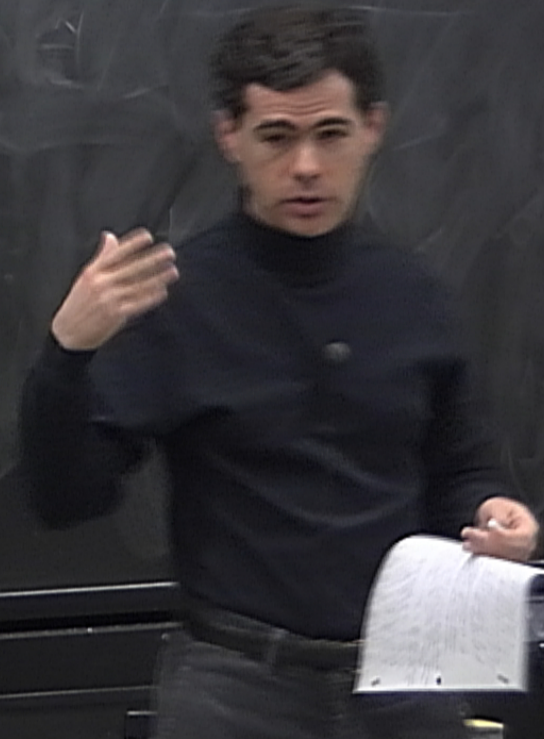
Can put multiple ions in trap
Repel each other, so they try
to spread out
(.....)



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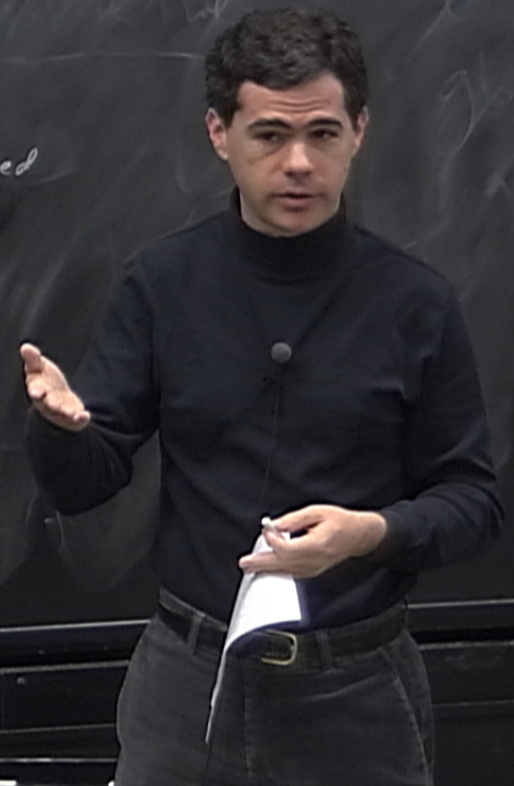
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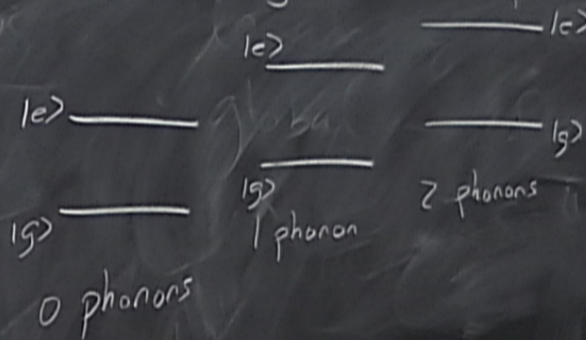
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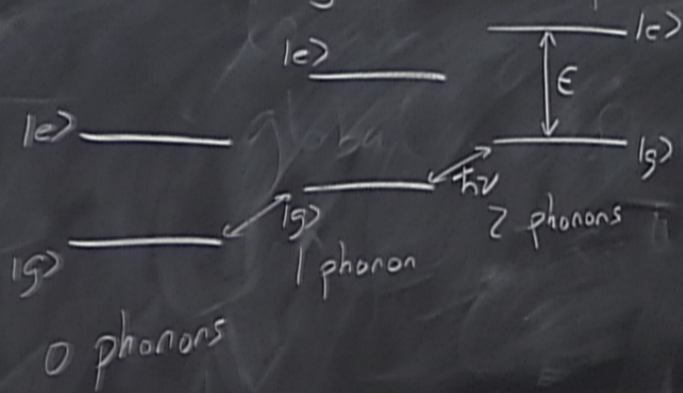
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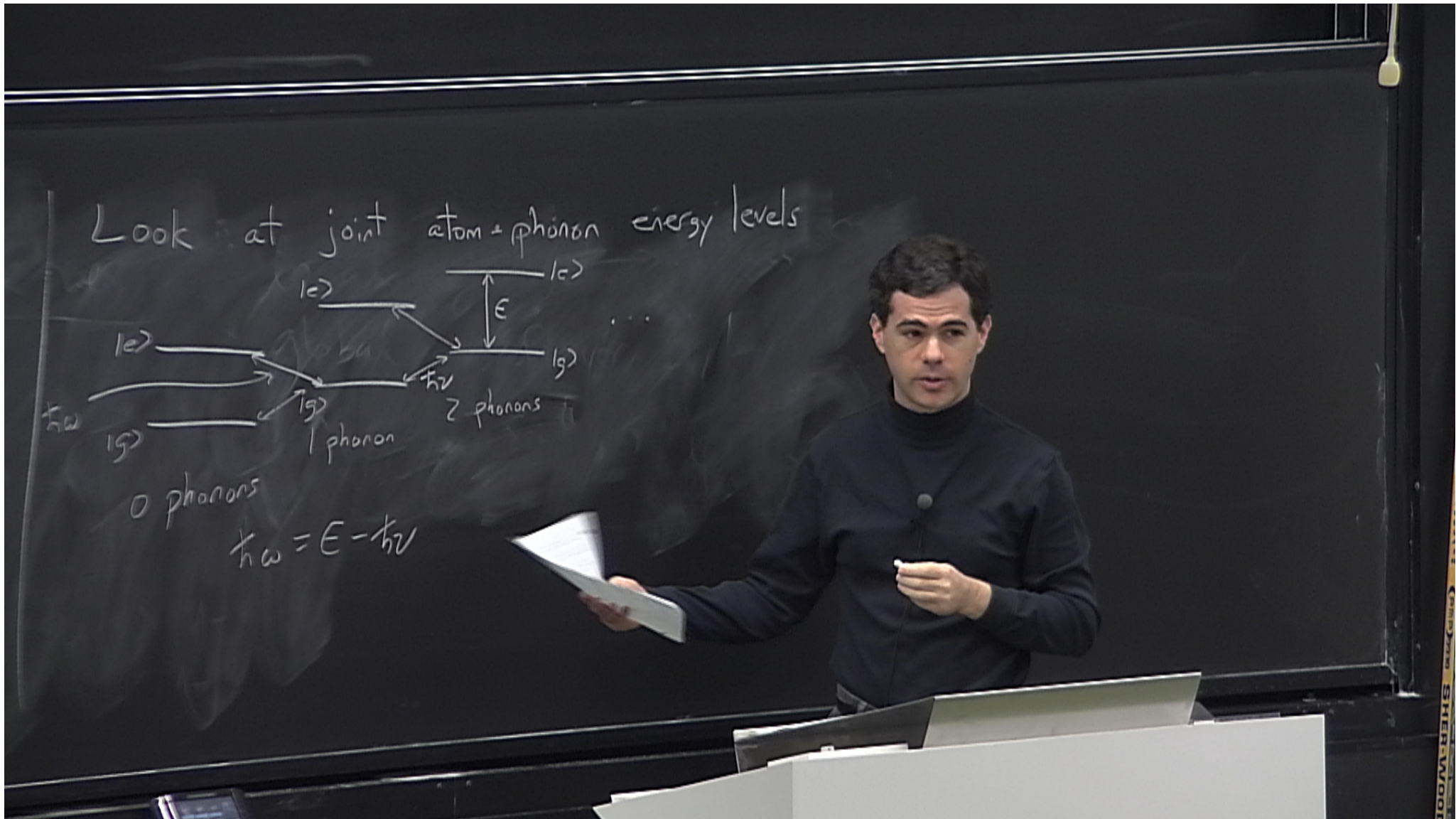
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Look at joint atom + phonon energy levels

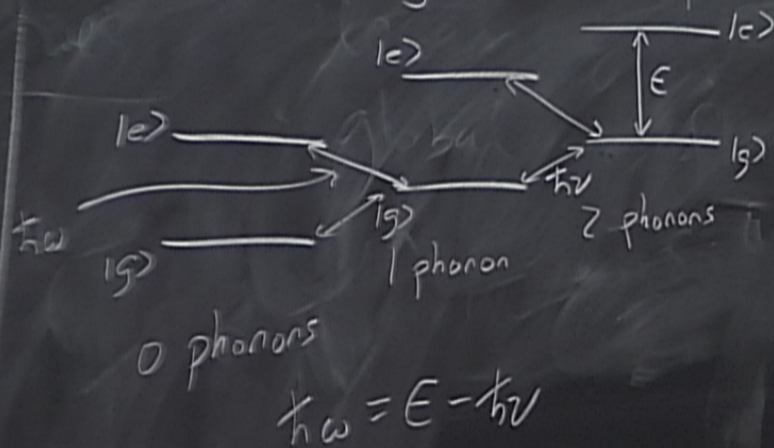


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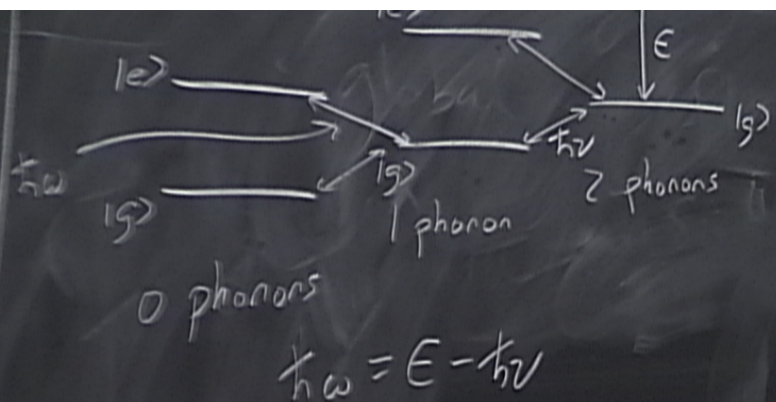




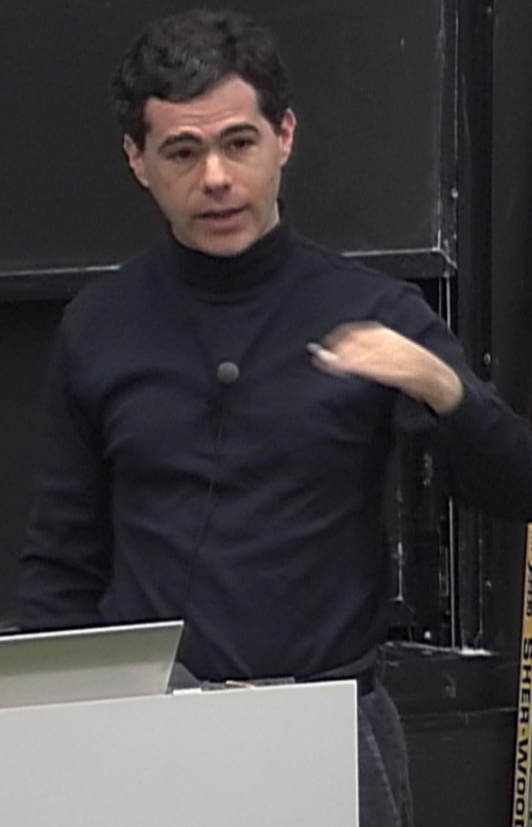
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 couple $|e, n \text{ phonons}\rangle \leftrightarrow |g, n+1 \text{ phonons}\rangle$



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 $|g, 0 \text{ phonons}\rangle$ does not couple.



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Additional electronic level $|e'\rangle$

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2) π pulse on atom 2, w/
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$|e\rangle_2 |1 \text{ phonon}\rangle$ couples to $|e'\rangle_2 |0 \text{ phonons}\rangle$
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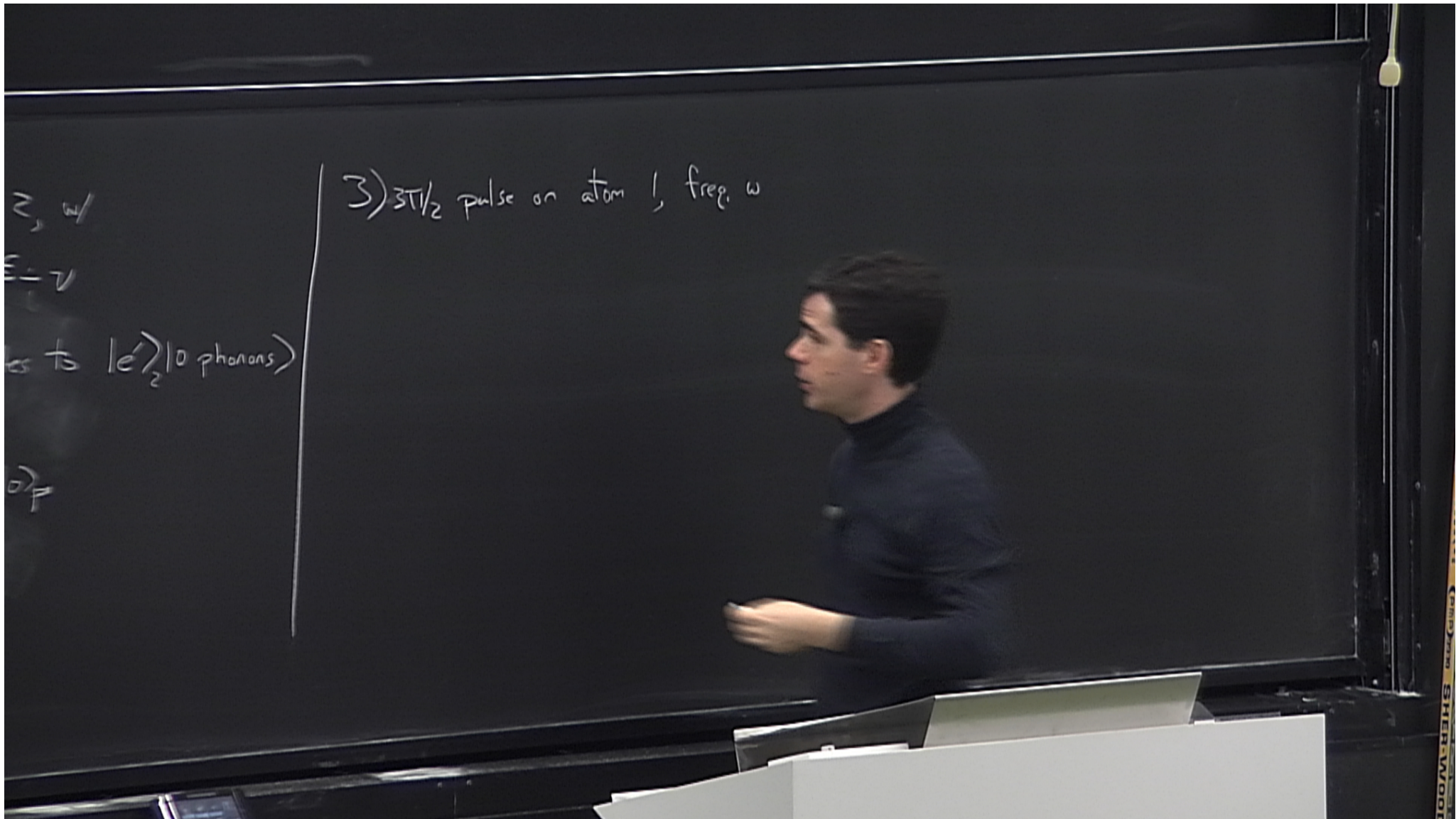
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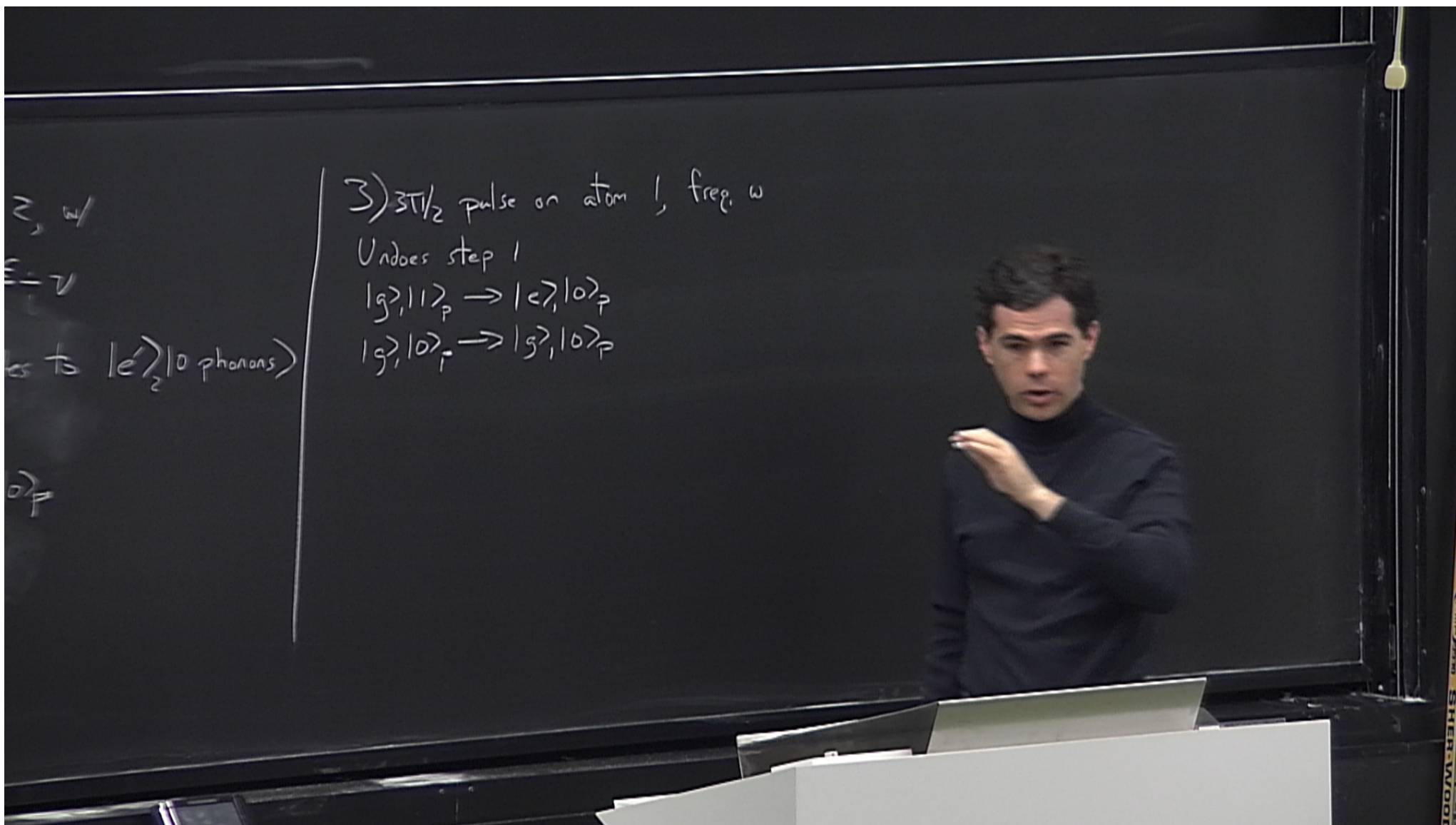
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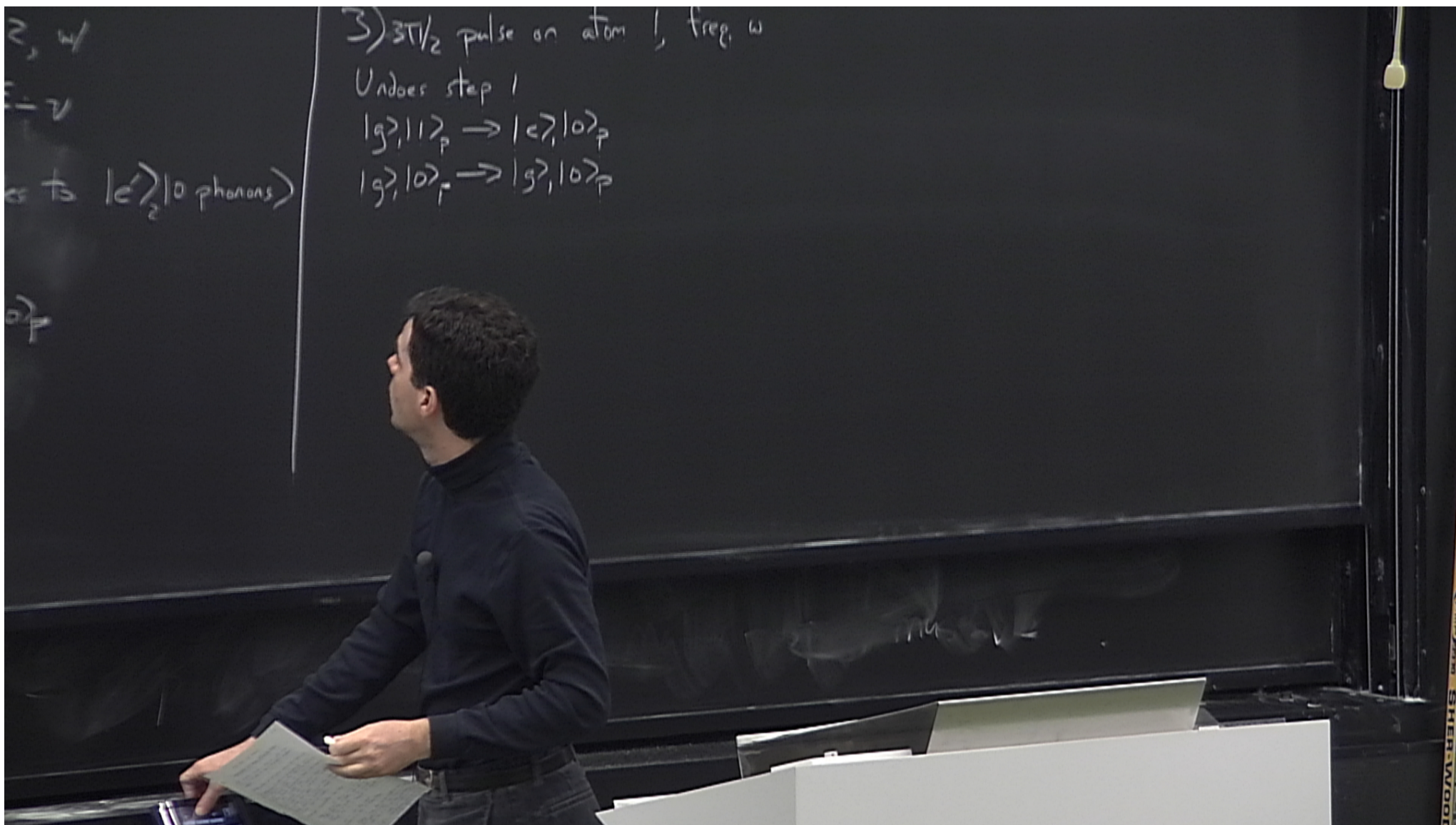
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$\hat{z}, \hat{w}$
3) $3\pi/2$ pulse on atom 1, freq. ω

Undoes step 1

$$|g\rangle_1 |1\rangle_p \rightarrow |e\rangle_1 |0\rangle_p$$

as to $|e\rangle_2 |0 \text{ phonons}\rangle$

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Overall gate:

$$|g\rangle_1 |g\rangle_2 \rightarrow$$

$$|g\rangle_1 |e\rangle_2 \rightarrow$$

$$|e\rangle_1 |g\rangle_2$$

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$\hat{z}, \hat{w}$

$\hat{F} = \hat{V}$

to $|e\rangle_2 |0 \text{ phonons}\rangle$

$|g\rangle_P$

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$\hat{z}, \hat{w}$

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to $|e'\rangle_2 |0 \text{ phonons}\rangle$

$\hat{\sigma}_P$

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Undoes step 1

$$|g\rangle_1 |1\rangle_P \rightarrow |e\rangle_1 |0\rangle_P$$

$$|g\rangle_1 |0\rangle_P \rightarrow |g\rangle_1 |0\rangle_P$$

Overall gate:

$$|g\rangle_1 |g\rangle_2 \rightarrow |g\rangle_1 |g\rangle_2$$

$$|g\rangle_1 |e\rangle_2 \rightarrow |g\rangle_1 |e\rangle_2$$

$$|e\rangle_1 |g\rangle_2 \rightarrow |e\rangle_1 |g\rangle_2$$

$$|e\rangle_1 |e\rangle_2 \rightarrow -|e\rangle_1 |e\rangle_2$$

$\hat{z}, \omega$

$\hat{p} = \hat{p}$

as to $|e\rangle_2 |0 \text{ phonons}\rangle$

$\hat{p}$

3) $3\pi/2$ pulse on atom 1, freq. ω

Undoes step 1

$$|g\rangle_1 |1\rangle_p \rightarrow |e\rangle_1 |0\rangle_p$$

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$$|e\rangle_1 |e\rangle_2 \rightarrow -|e\rangle_1 |e\rangle_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



$\hat{z}, \omega$

$\hat{F} = \nu$

to $|e\rangle_2 |0 \text{ phonons}\rangle$

$|g\rangle_P$

3) $3\pi/2$ pulse on atom 1, freq. ω

Undoes step 1

$$|g\rangle_1 |g\rangle_P \rightarrow |e\rangle_1 |g\rangle_P$$

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$$|e\rangle_2 \rightarrow -|e\rangle_1 |e\rangle_2$$

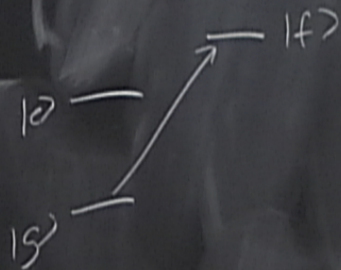
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Universal set of gates

(4)

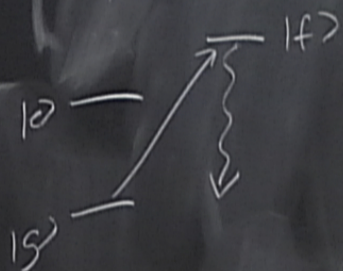
Measurement:

Short-lived $|f\rangle$ that
spontaneously decay to $|g\rangle$



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Short-lived $|f\rangle$ that
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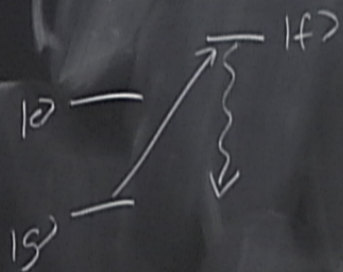
Strong laser coupling
 $|g\rangle$ to $|f\rangle$

"cycling transition"

Many photons emitted - can measure.

Measurement:

Short-lived $|f\rangle$ that
spontaneously decay to $|g\rangle$



Strong laser coupling
 $|g\rangle$ to $|f\rangle$

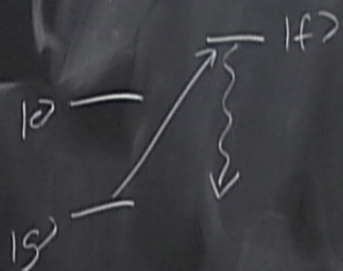
"cycling transition"

(5)

Many photons emitted - can measure.

Measurement:

Short-lived $|f\rangle$ that
spontaneously decay to $|g\rangle$



Strong laser coupling
 $|g\rangle$ to $|f\rangle$

"cycling transition"

(5) 4 (2)

Many photons emitted - can measure.