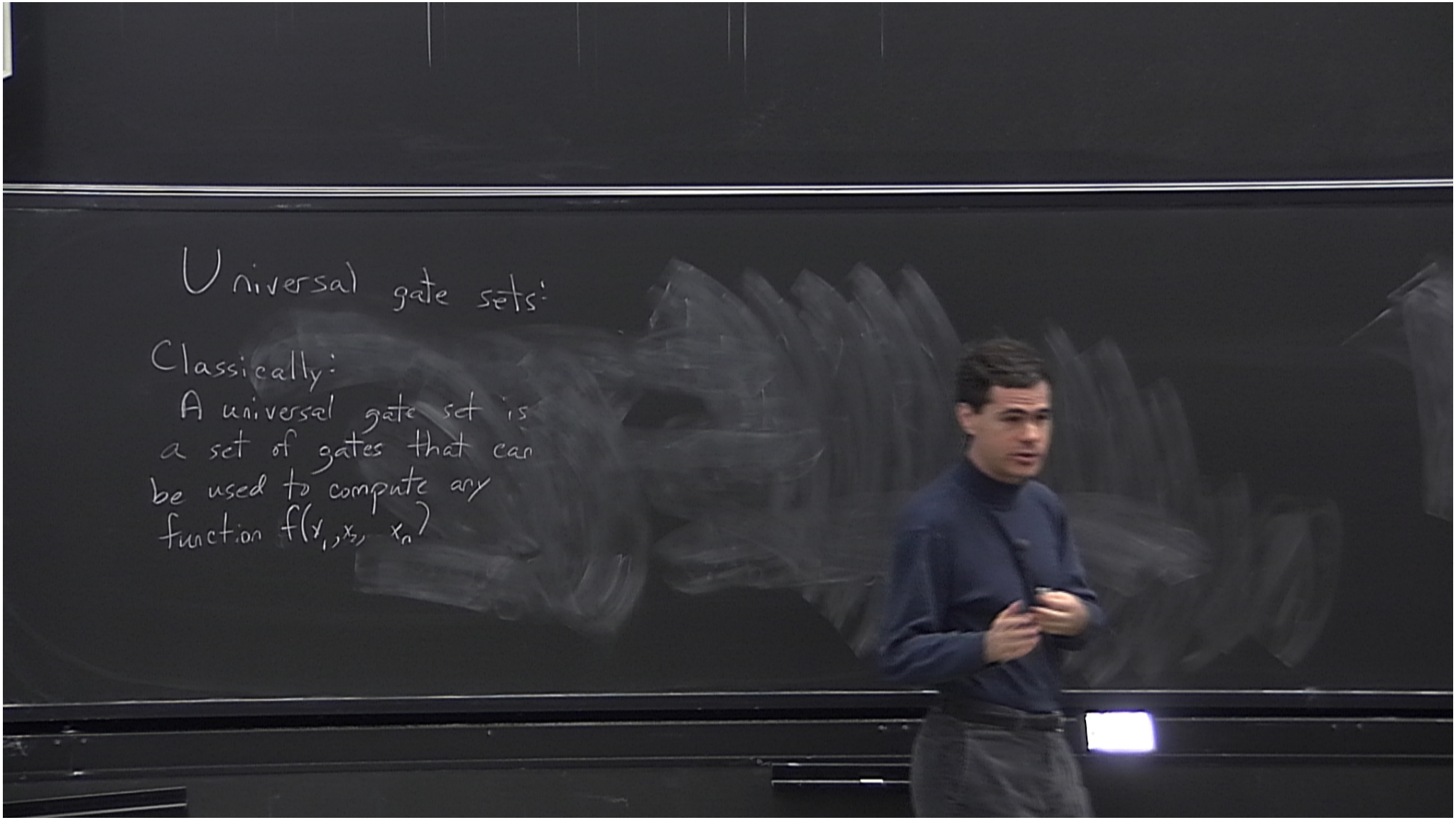
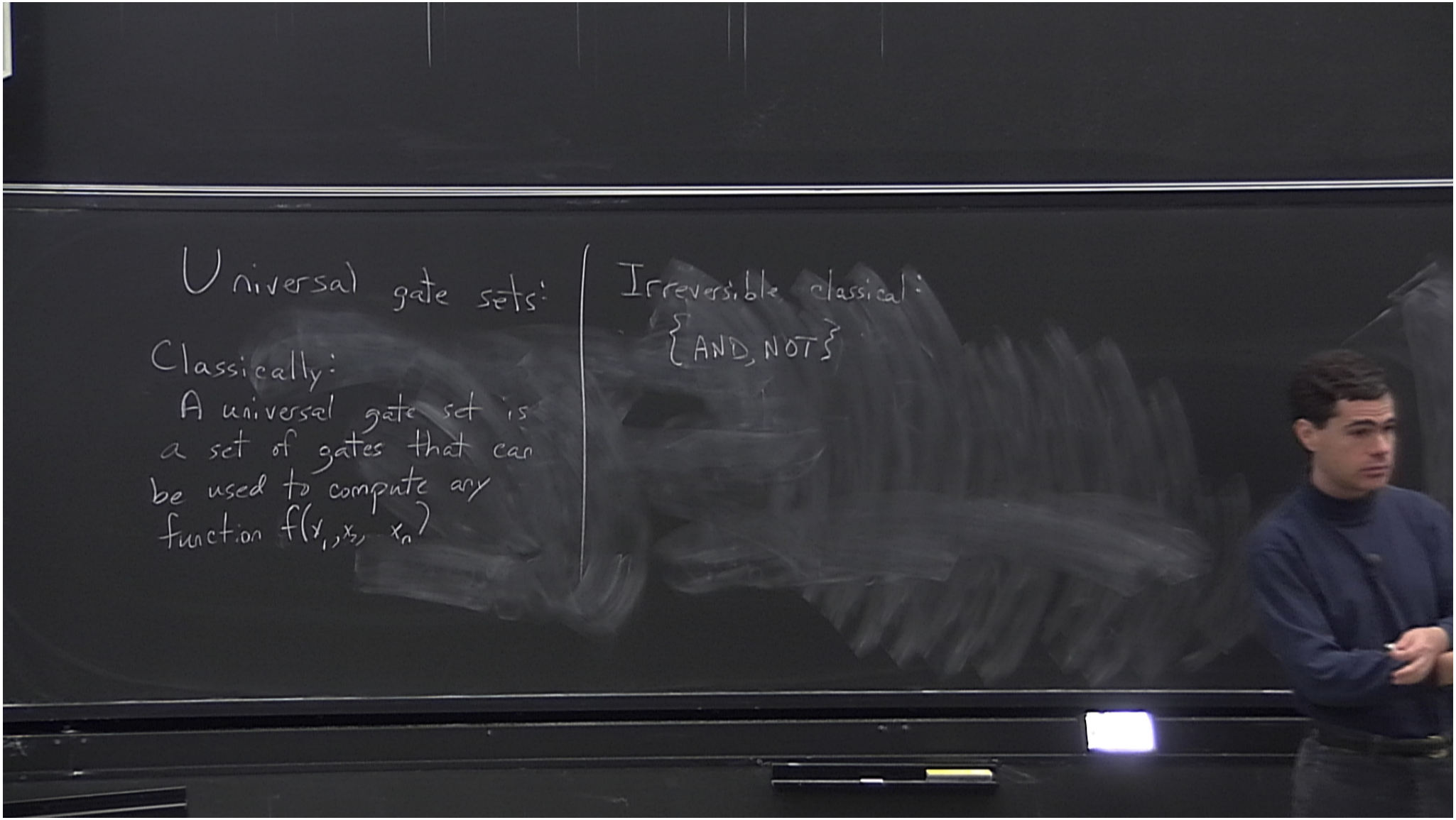
Title: Quantum Information (Review) - Lecture 2

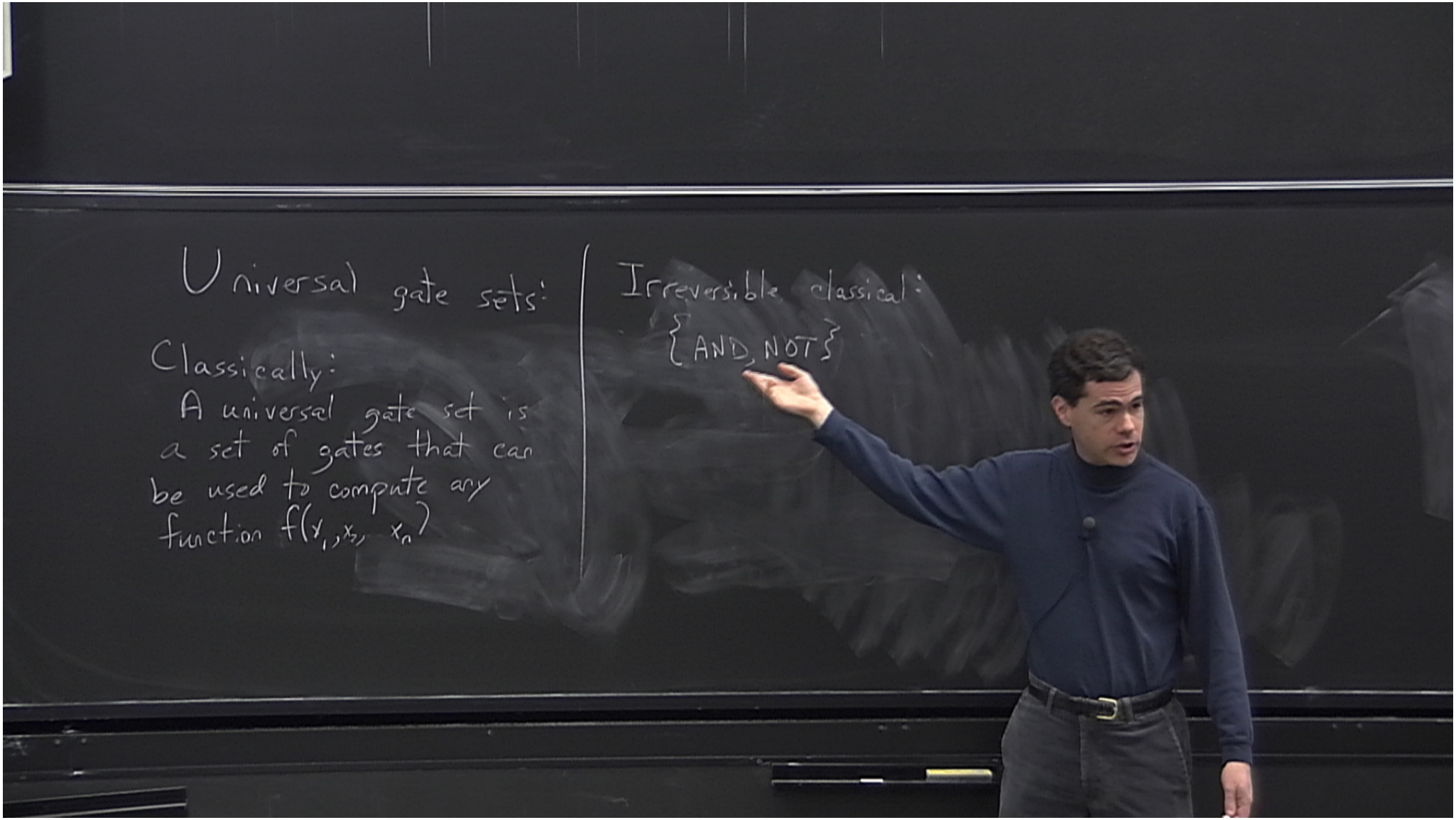
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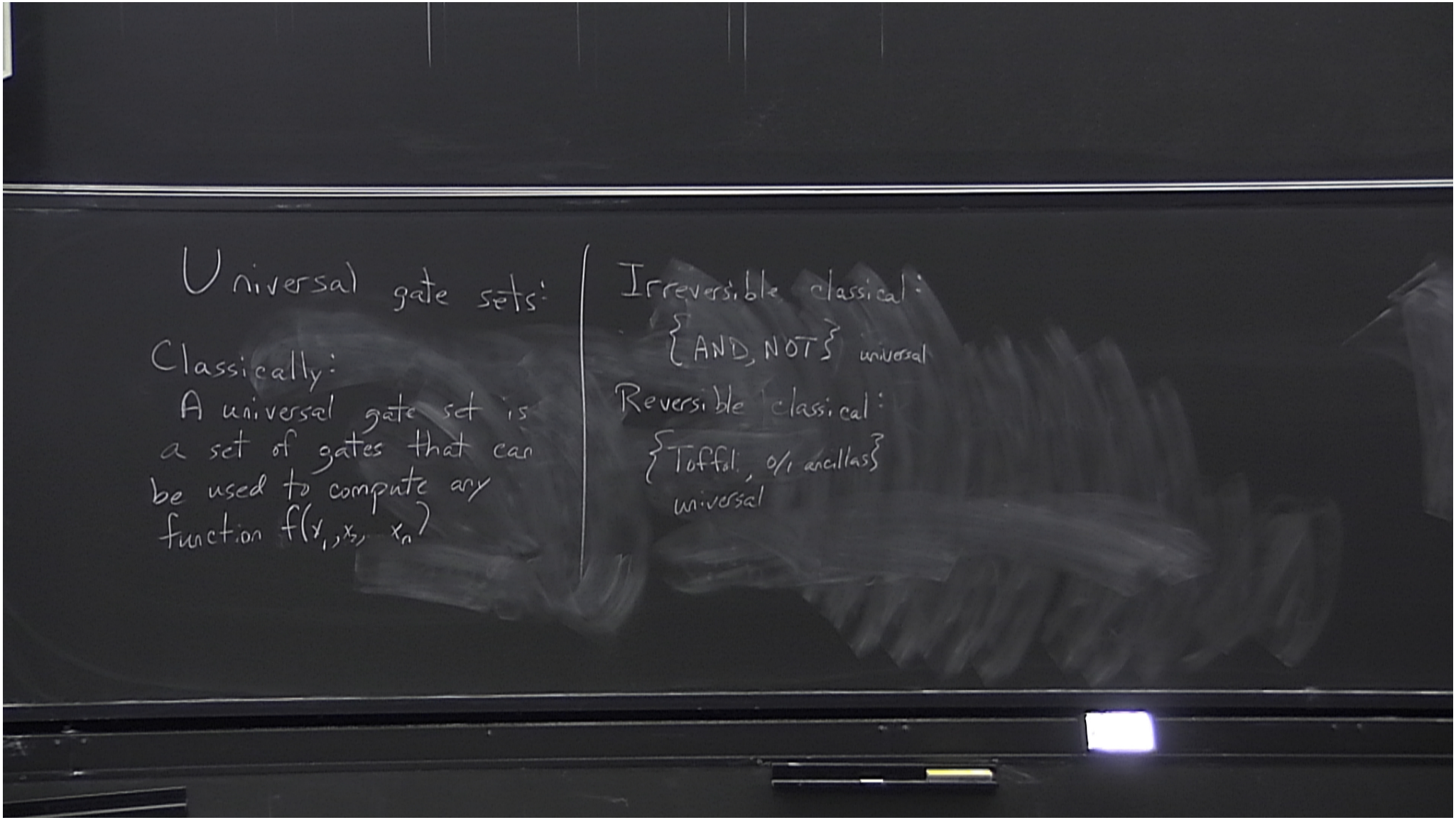
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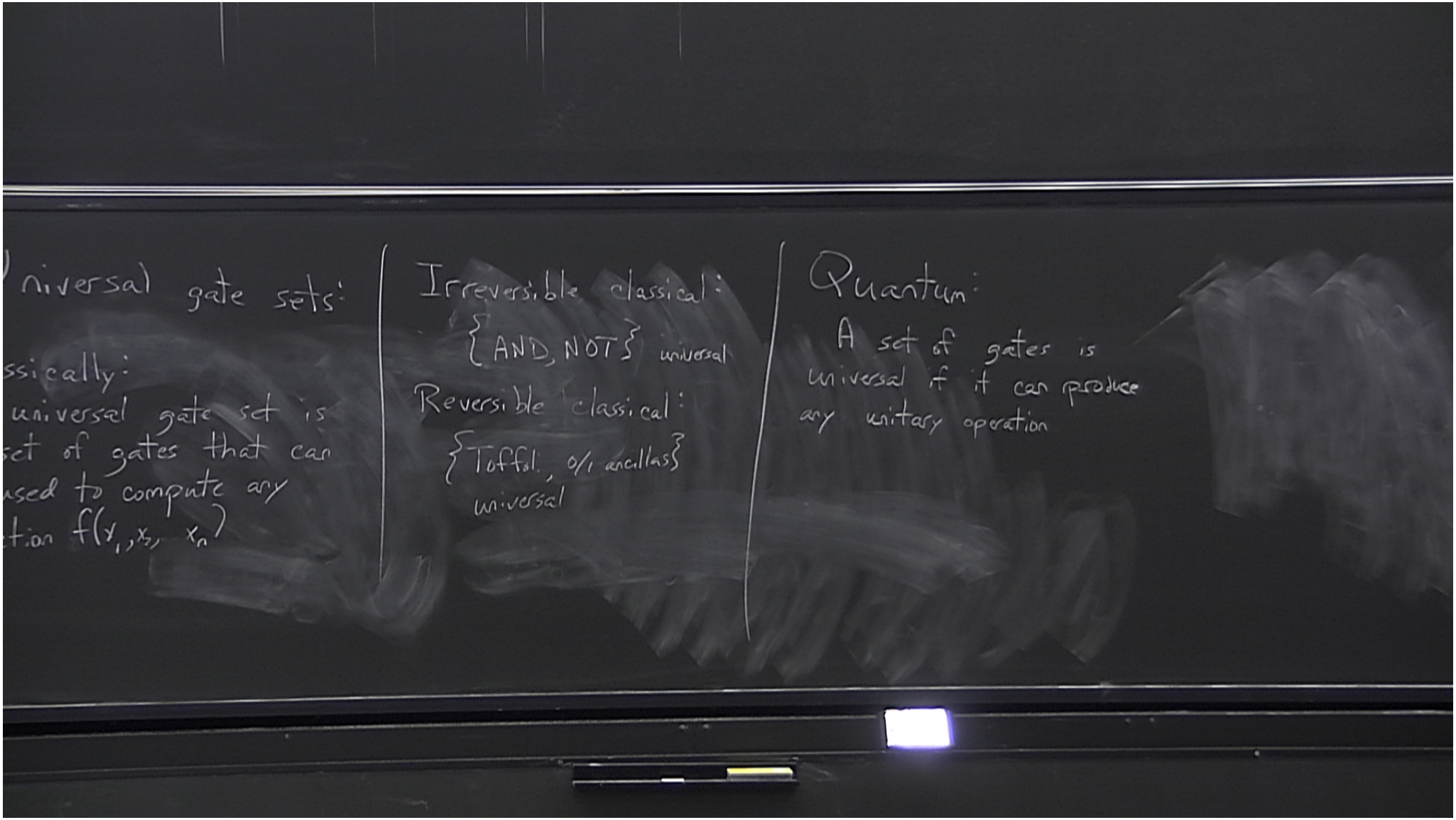
Abstract:

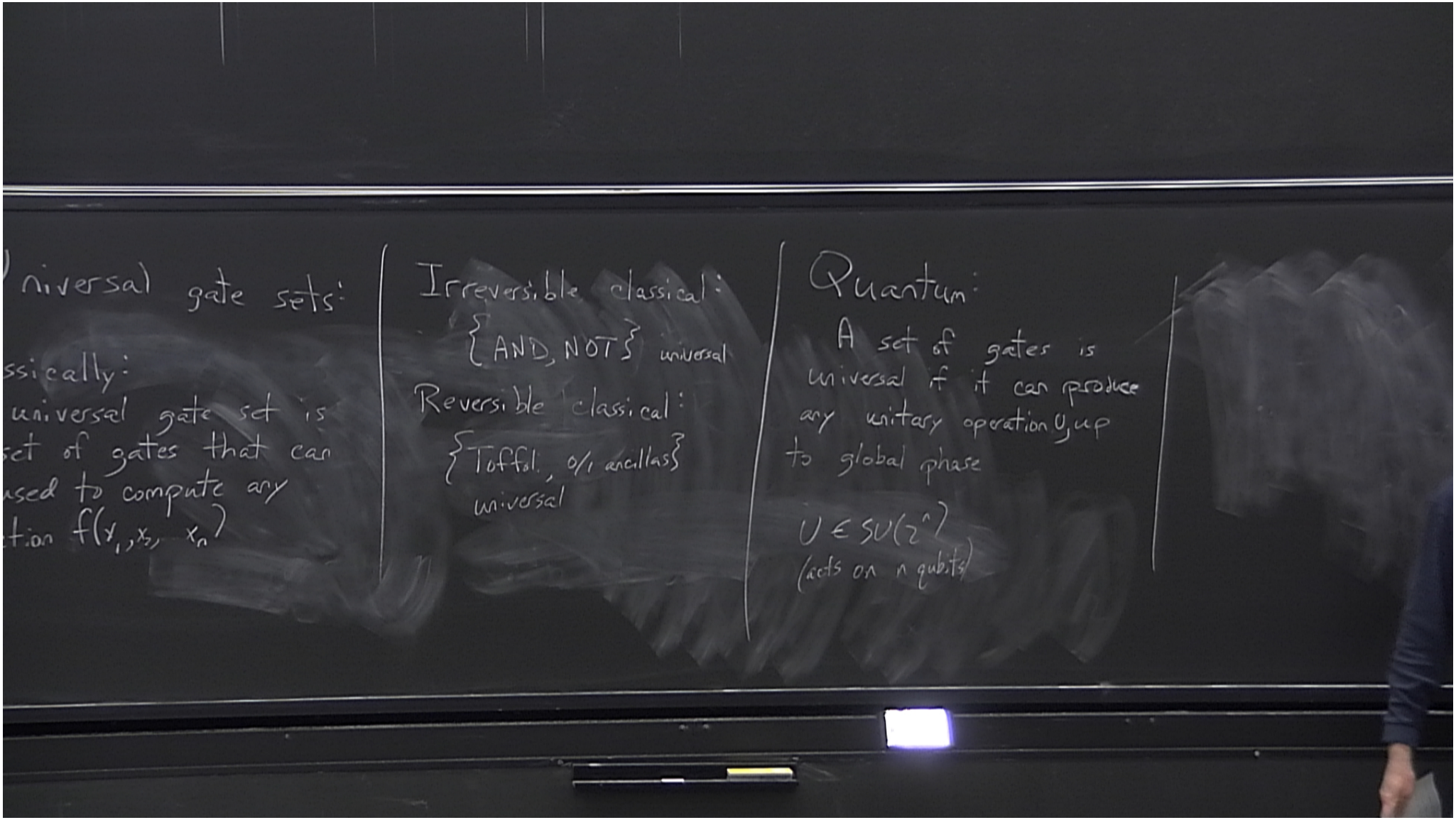

Universal gate sets:
Classically:
A universal gate set is
a set of gates that can
be used to compute any
function $f(x_1, x_2, \ldots x_n)$

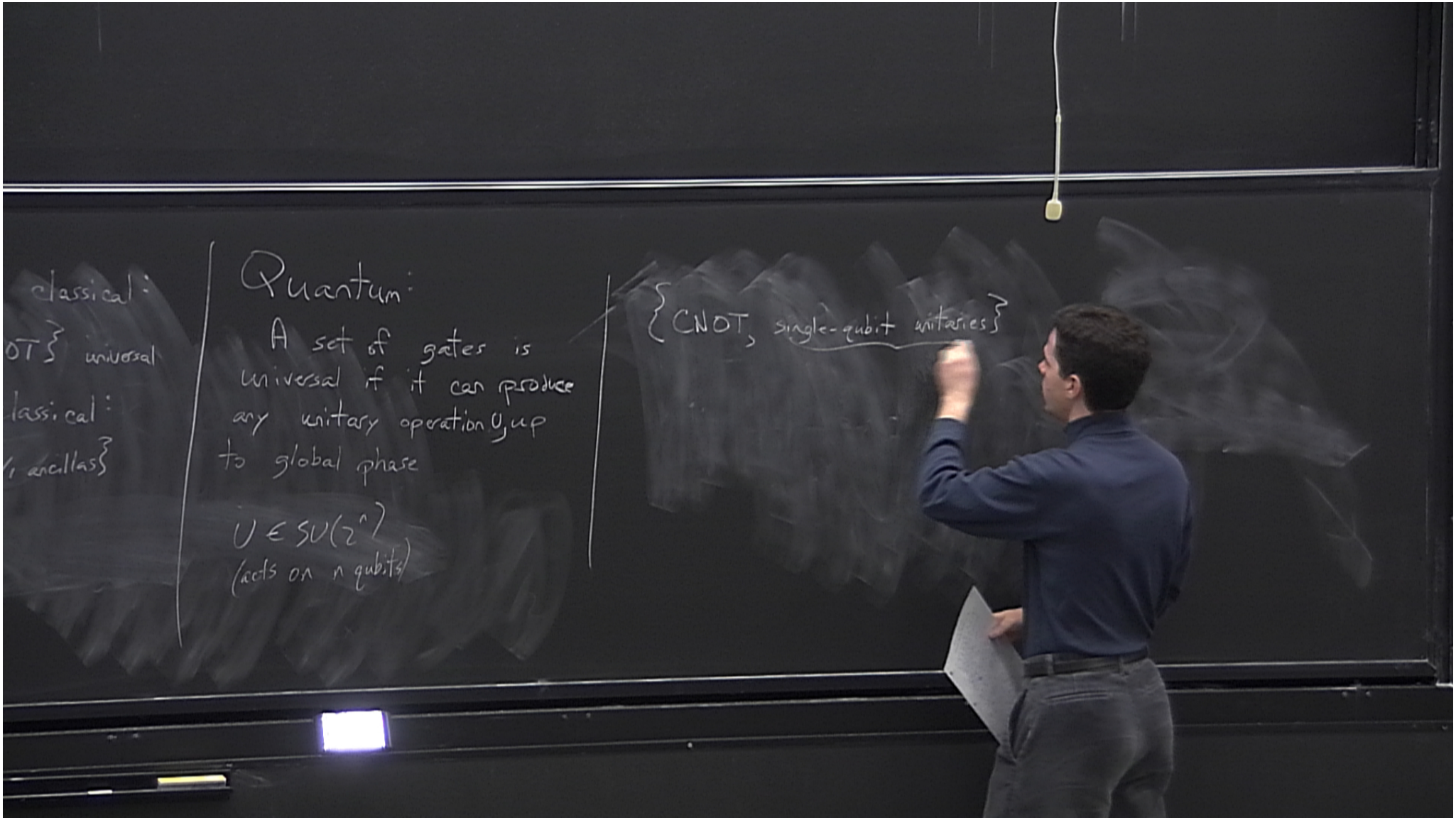

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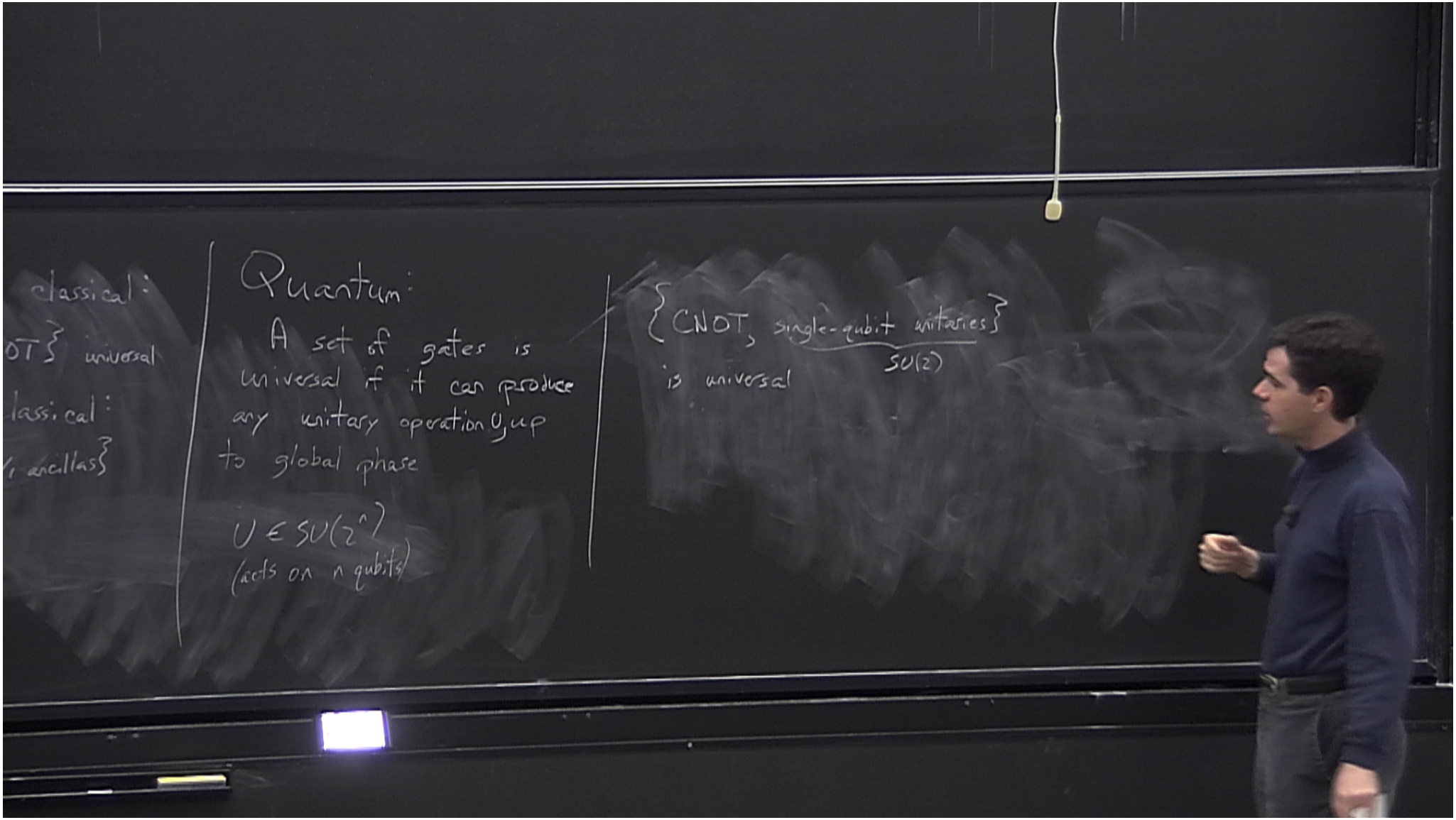

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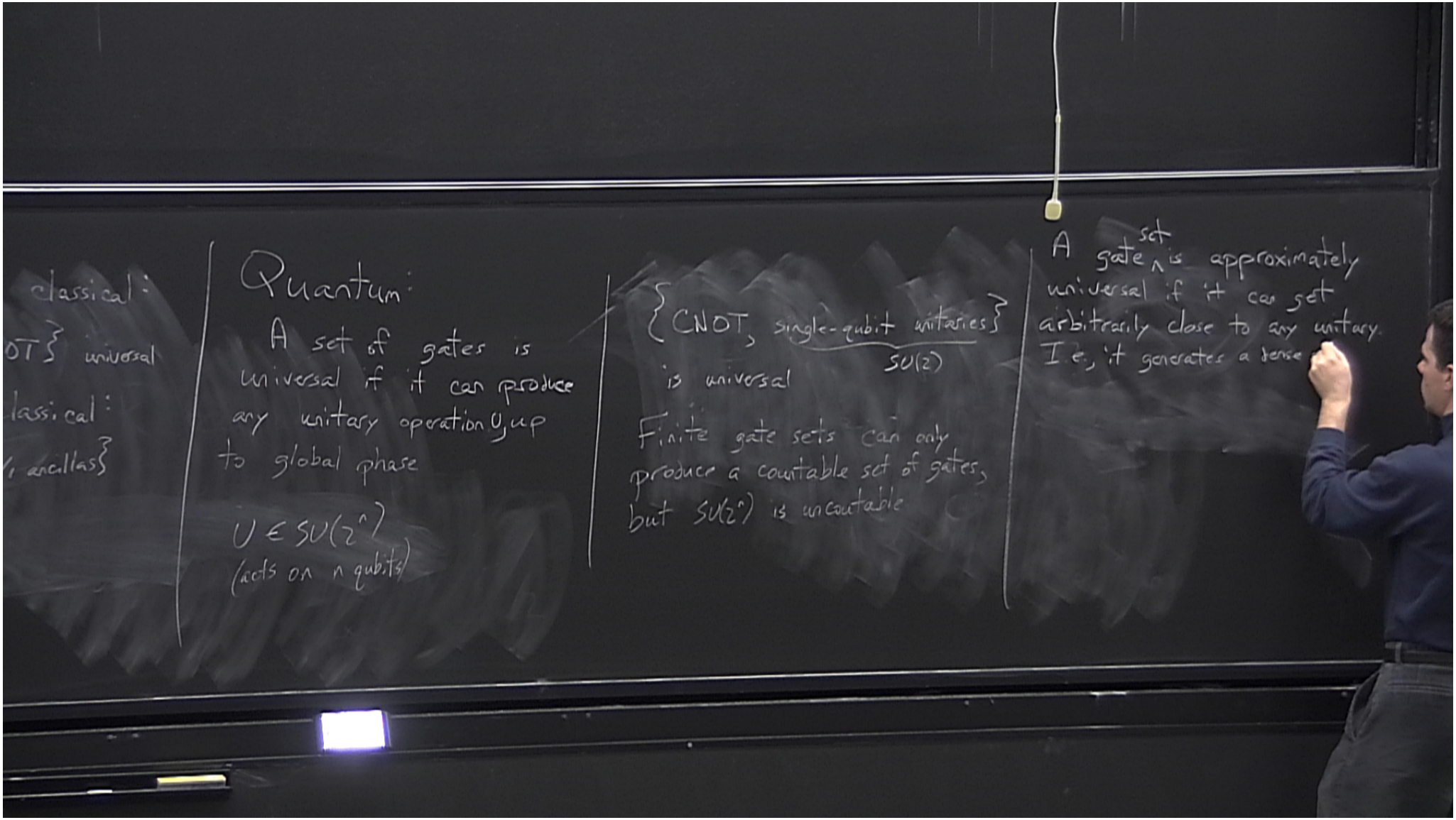

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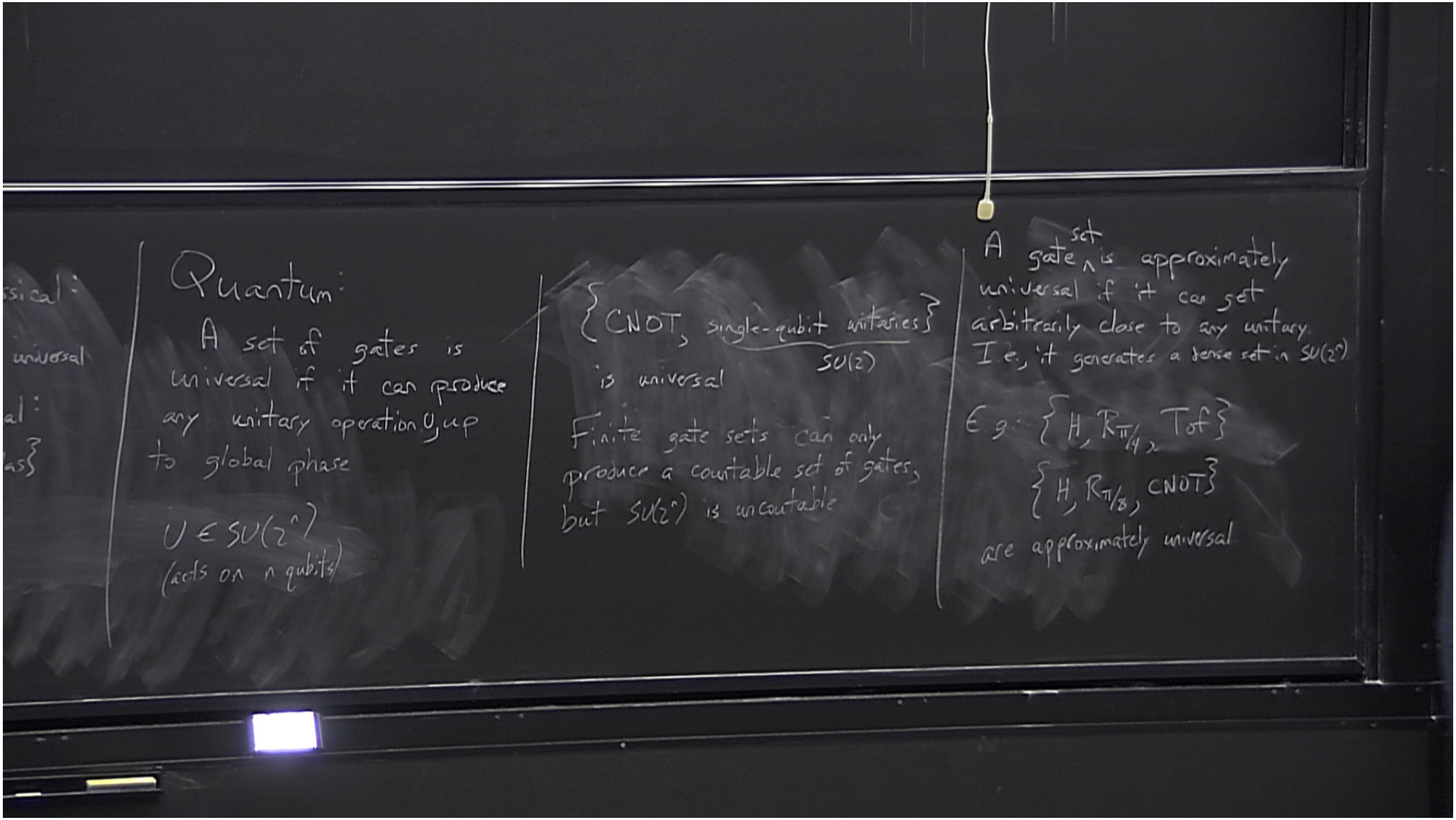

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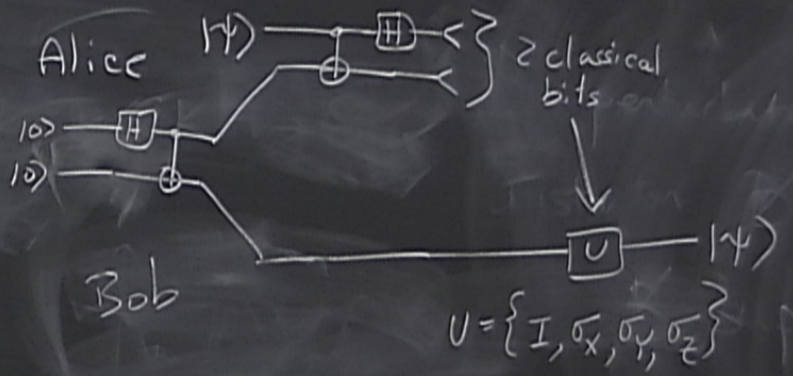

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E.g.: $\{H, R_{\pi/4}, Tof\}$
$\{H, R_{\pi/8}, CNOT\}$
are approximately universal

Quantum teleportation:

Alice ⟶ Bob
send qubit

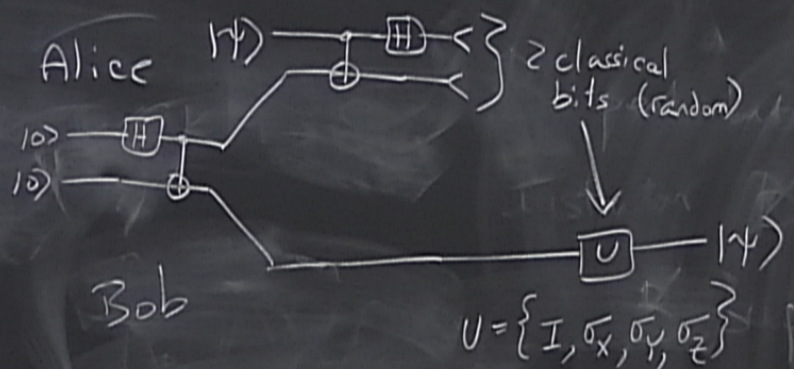
Share entangled state
$|00\rangle + |11\rangle$



portation:

→ Bob

state

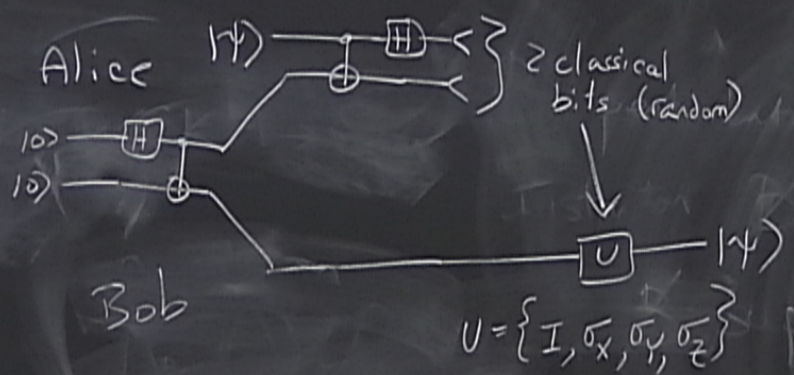


Useful for:

portation:

→ Bob

state



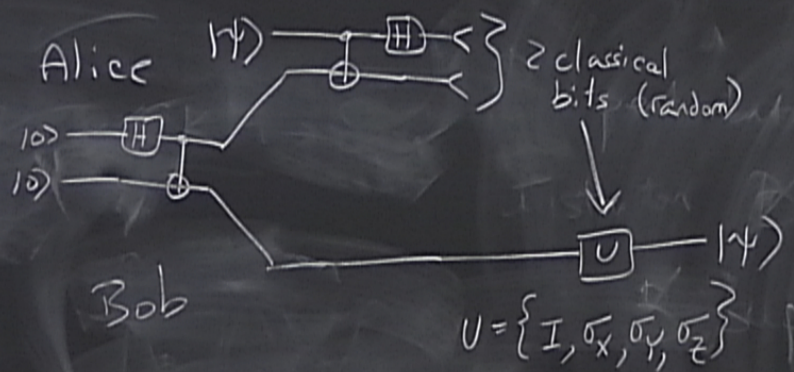
Useful for:

- Sending qubits under various non-ideal conditions.

portation:

→ Bob

state



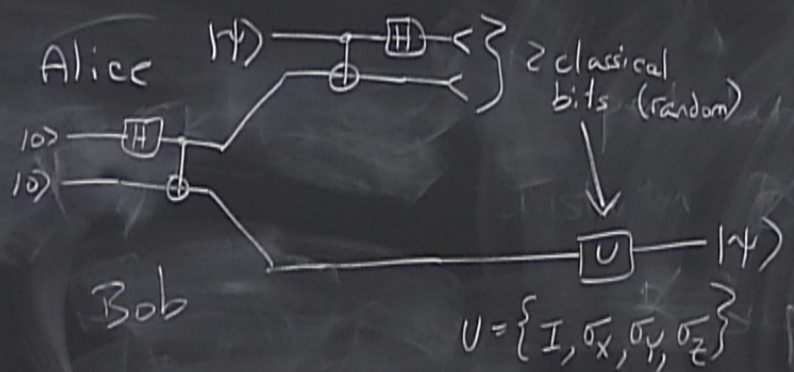
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- Sending qubits under various non-ideal conditions.
- As a building block for more complicated quantum protocols.

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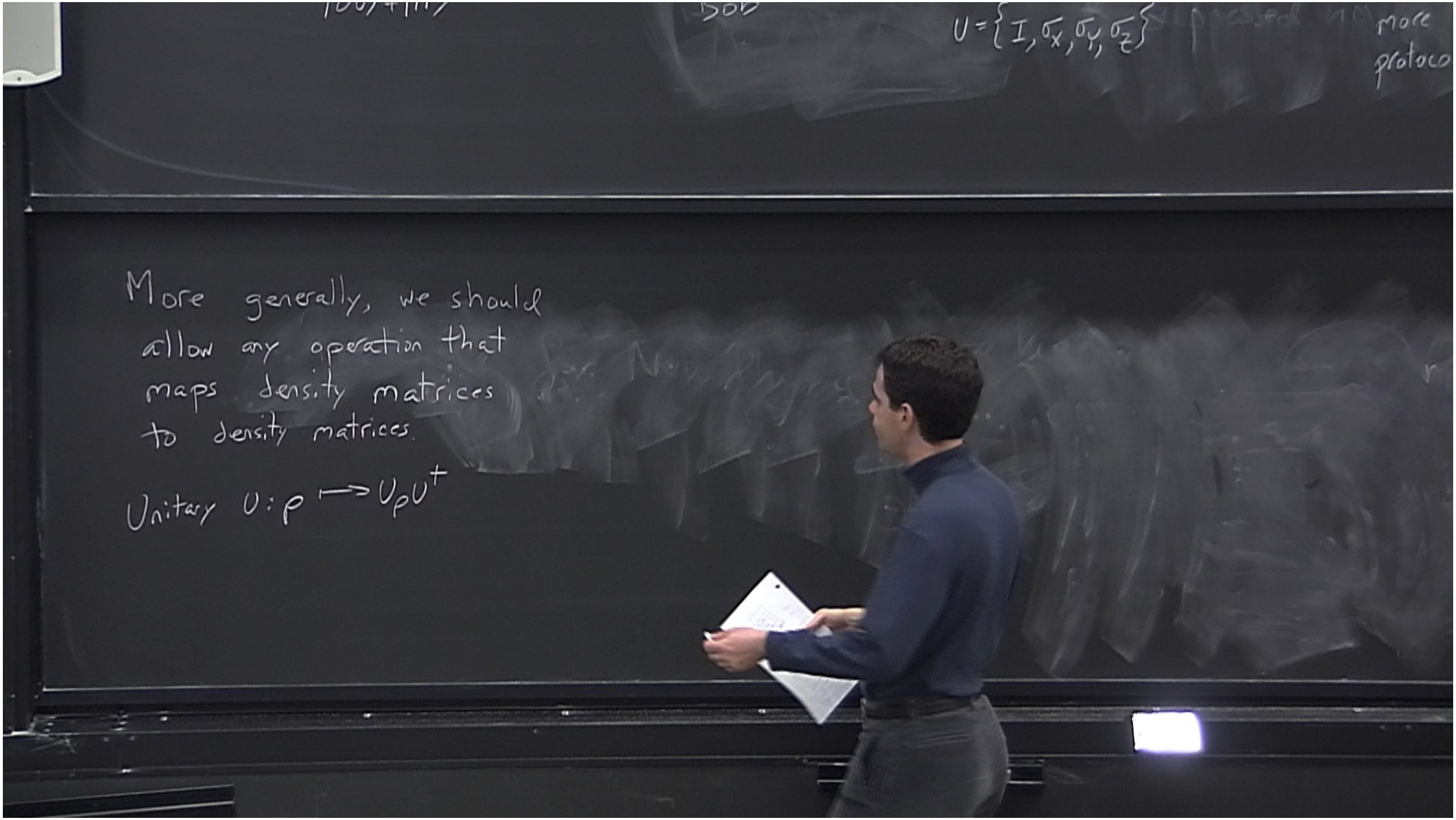
→ Bob

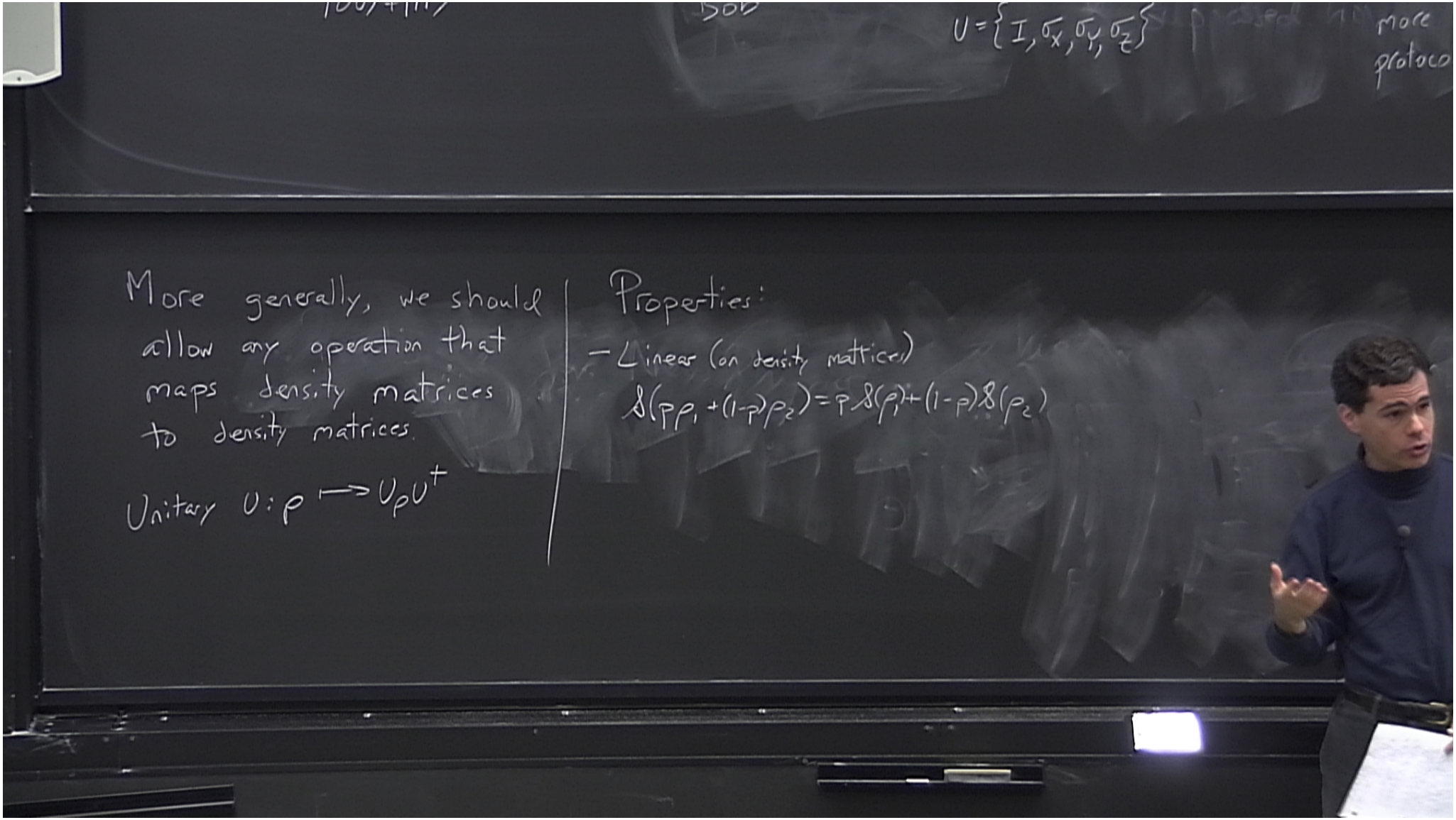
state



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$U = \{I, \sigma_x, \sigma_y, \sigma_z\}$
more
protoco
More generally, we should
allow any operation that
maps density matrices
to density matrices.
Unitary $U: \rho \longmapsto U\rho U^\dagger$


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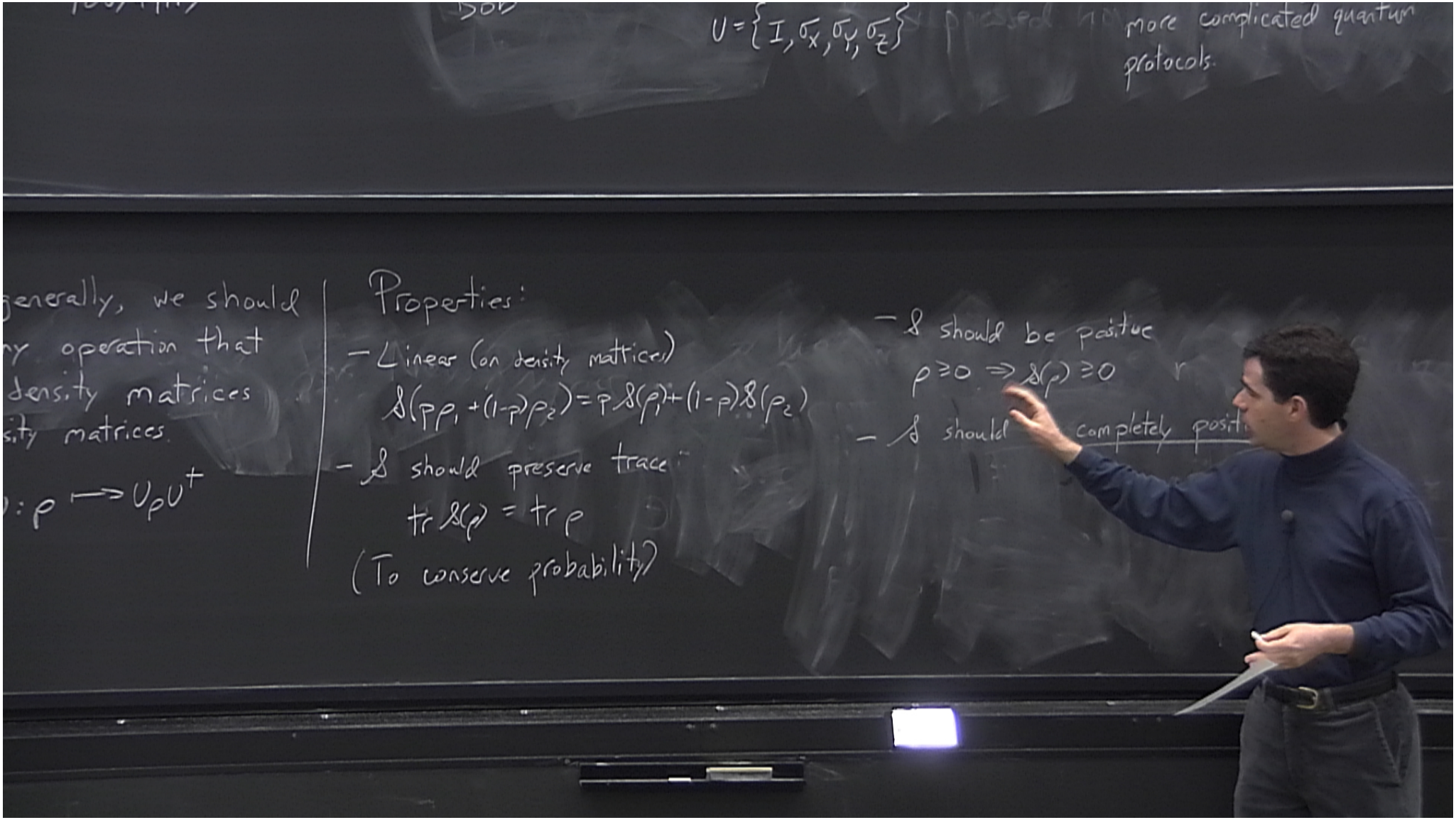
- $\$$ should preserve trace

  $\mathrm{tr}\,\$(\rho) = \mathrm{tr}\,\rho$

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- $\$$ should be positive

  $\rho \geq 0 \Rightarrow \$(\rho) \geq 0$


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more complicated quantum protocols.
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Properties:
- $\mathcal{L}$ linear (on density matrices)
$\mathcal{L}(p\rho_1 + (1-p)\rho_2) = p\mathcal{L}(\rho_1) + (1-p)\mathcal{L}(\rho_2)$
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  i.e. $\$$ should still positive applied to part of an entangled state

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more complicated quantum protocols

trace preserving maps = CPTP)

- $\mathcal{E}$ should be positive

$\rho \geq 0 \Rightarrow \mathcal{E}(\rho) \geq 0$

ρ_2)

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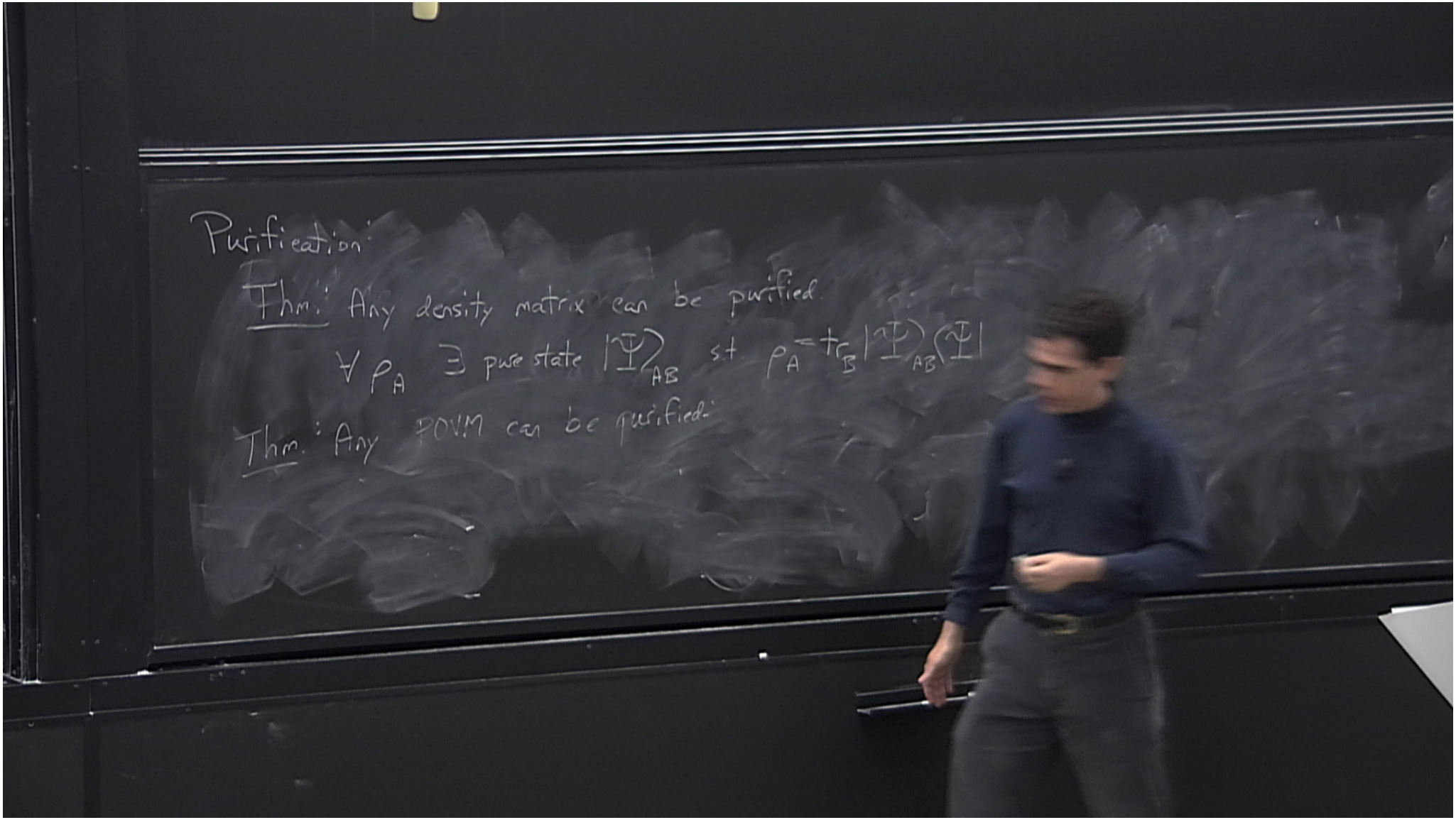
Most general measurement:

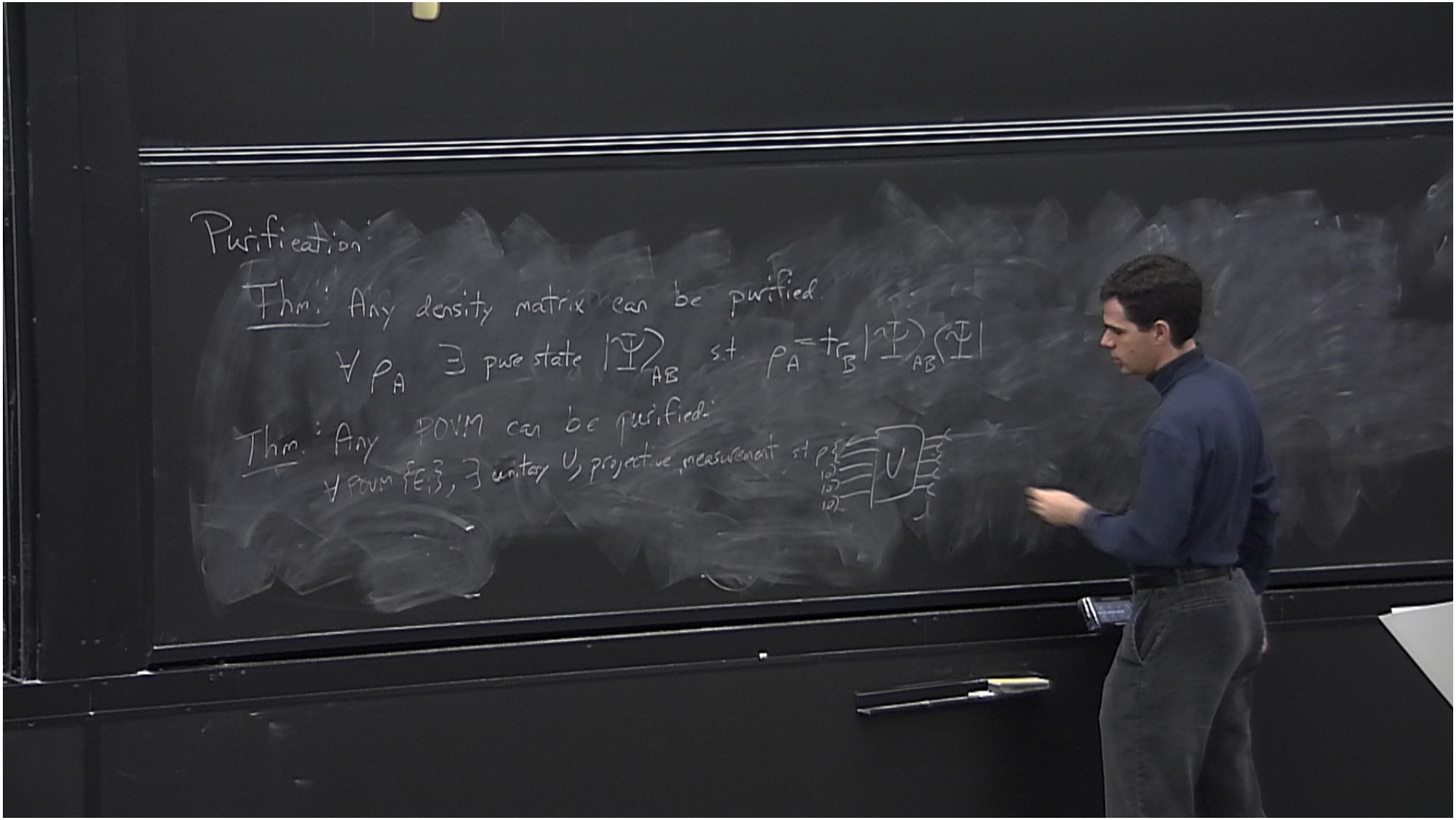
POVM $\{E_i\}$, $E_i \geq 0$, $\sum E_i = I$

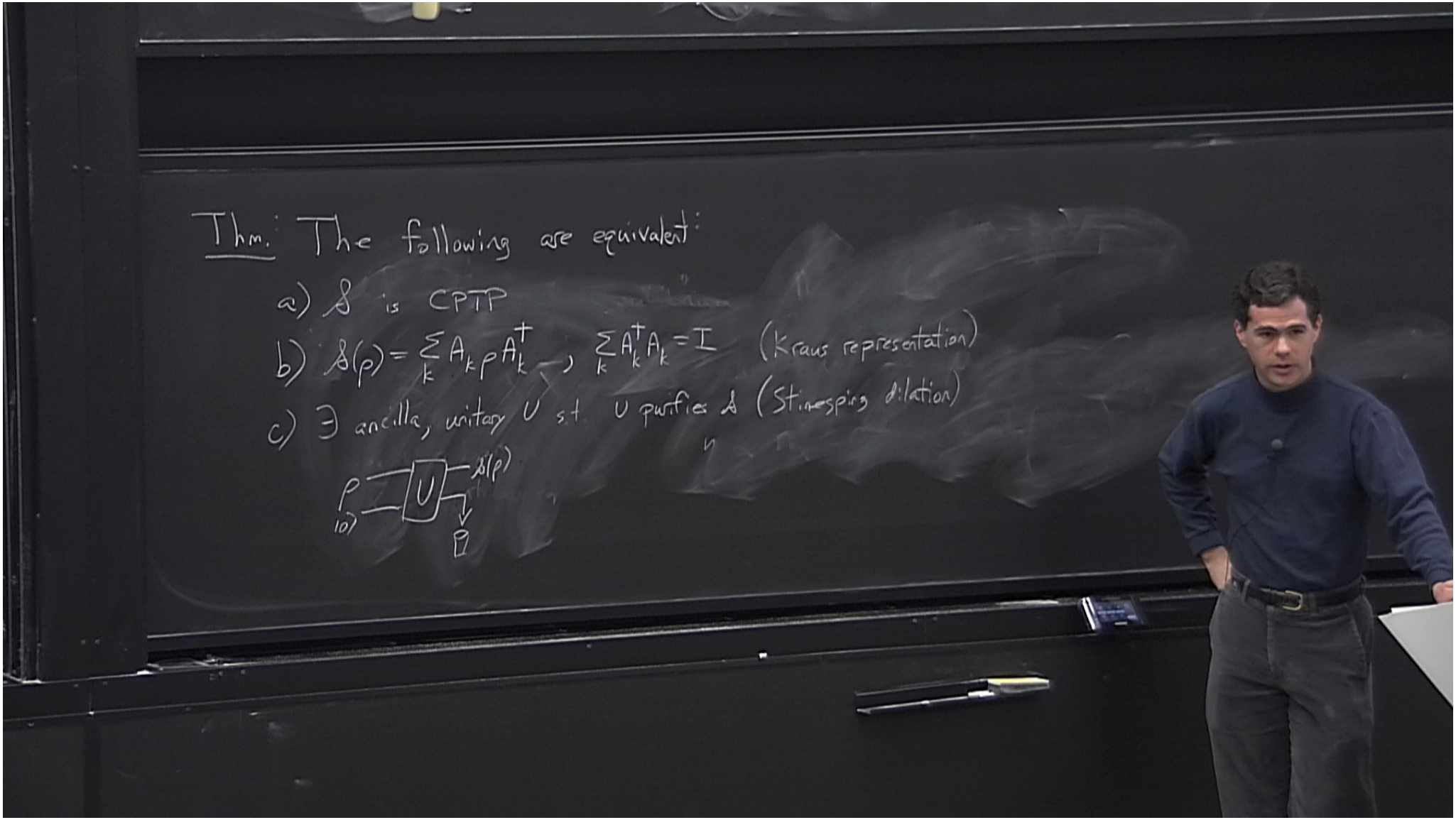
Outcome i occurs w/ probability $\mathrm{tr}(E_i \rho)$

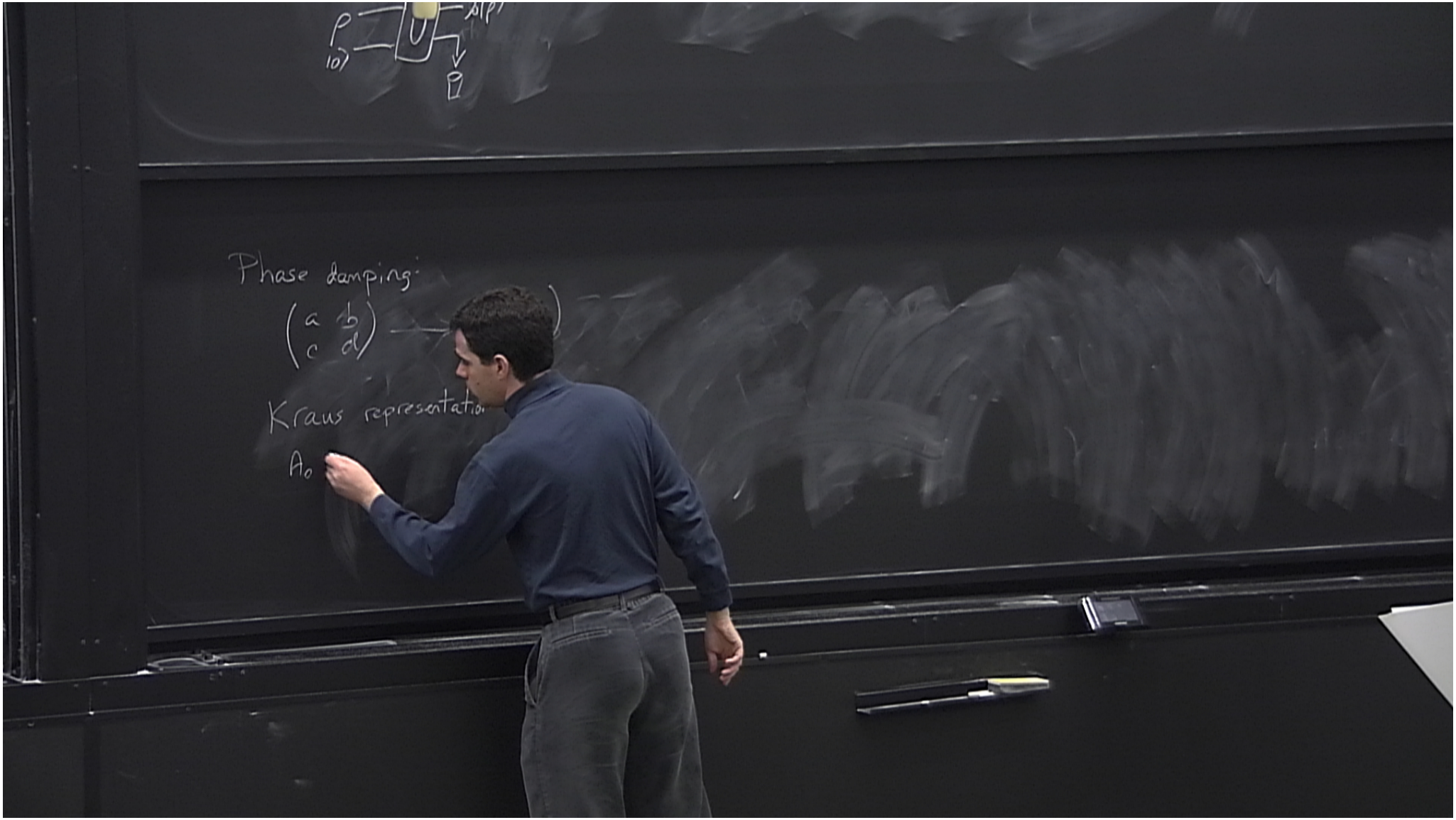
probabilities ≥ 0

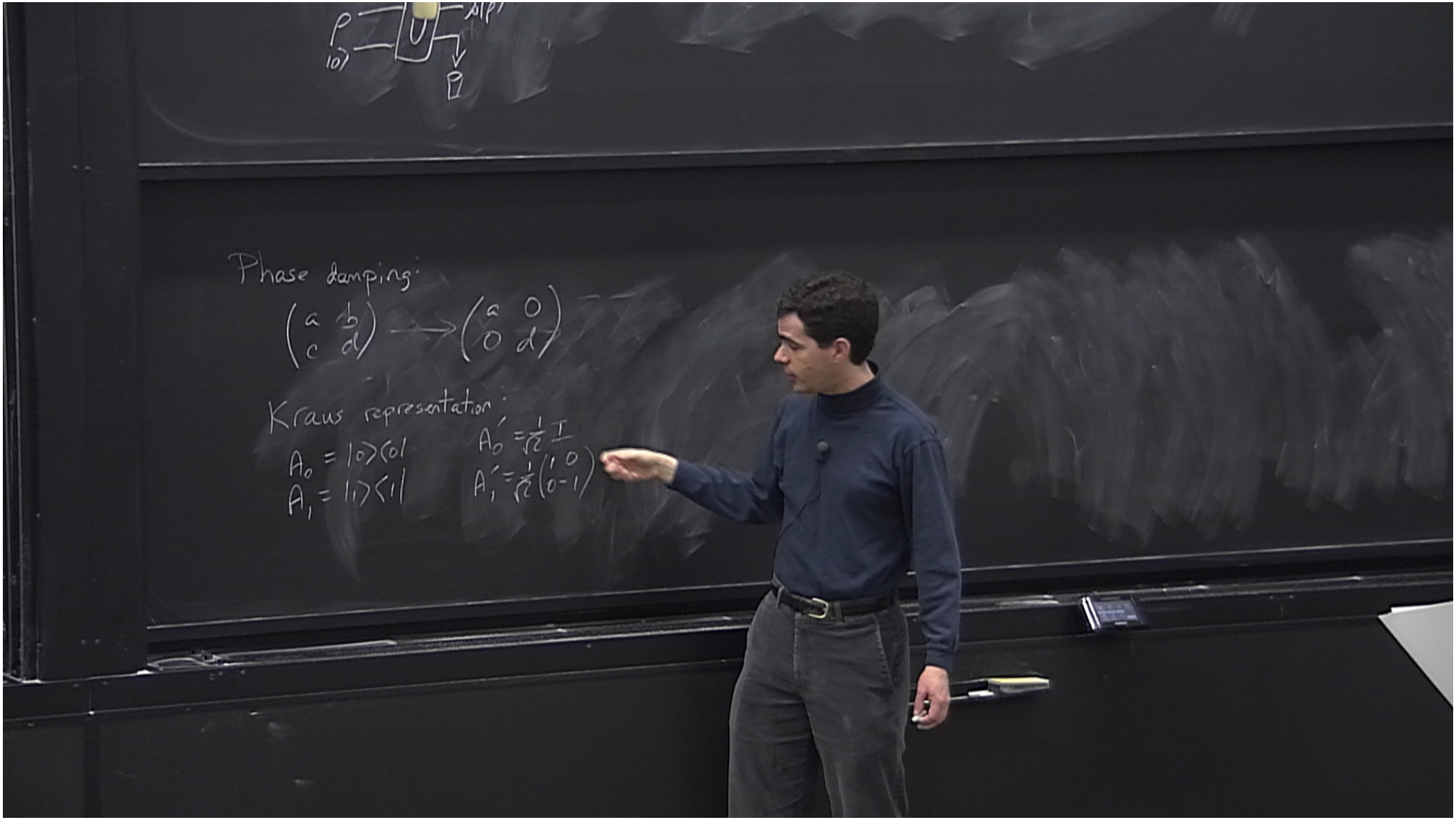
probs sum to 1


Purification:
Thm: Any density matrix can be purified.
$\forall \rho_A \ \exists$ pure state $|\Psi\rangle_{AB}$ s.t. $\rho_A = \mathrm{tr}_B |\Psi\rangle_{AB}\langle\Psi|$
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Thm: Any POVM can be purified:
$\forall$ POVM $\{E_i\}$, $\exists$ unitary U, projective measurement s.t.
$|0\rangle$
$|0\rangle$
$|0\rangle$
U


Thm: The following are equivalent:
a) $\mathcal{E}$ is CPTP
b) $\mathcal{E}(\rho) = \sum_k A_k \rho A_k^\dagger$, $\sum_k A_k^\dagger A_k = I$ (Kraus representation)
c) $\exists$ ancilla, unitary U s.t. U purifies $\mathcal{E}$ (Stinespring dilation)
ρ
$|0\rangle$
U
$\mathcal{E}(\rho)$


Phase damping:
Kraus representation


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Kraus representation: