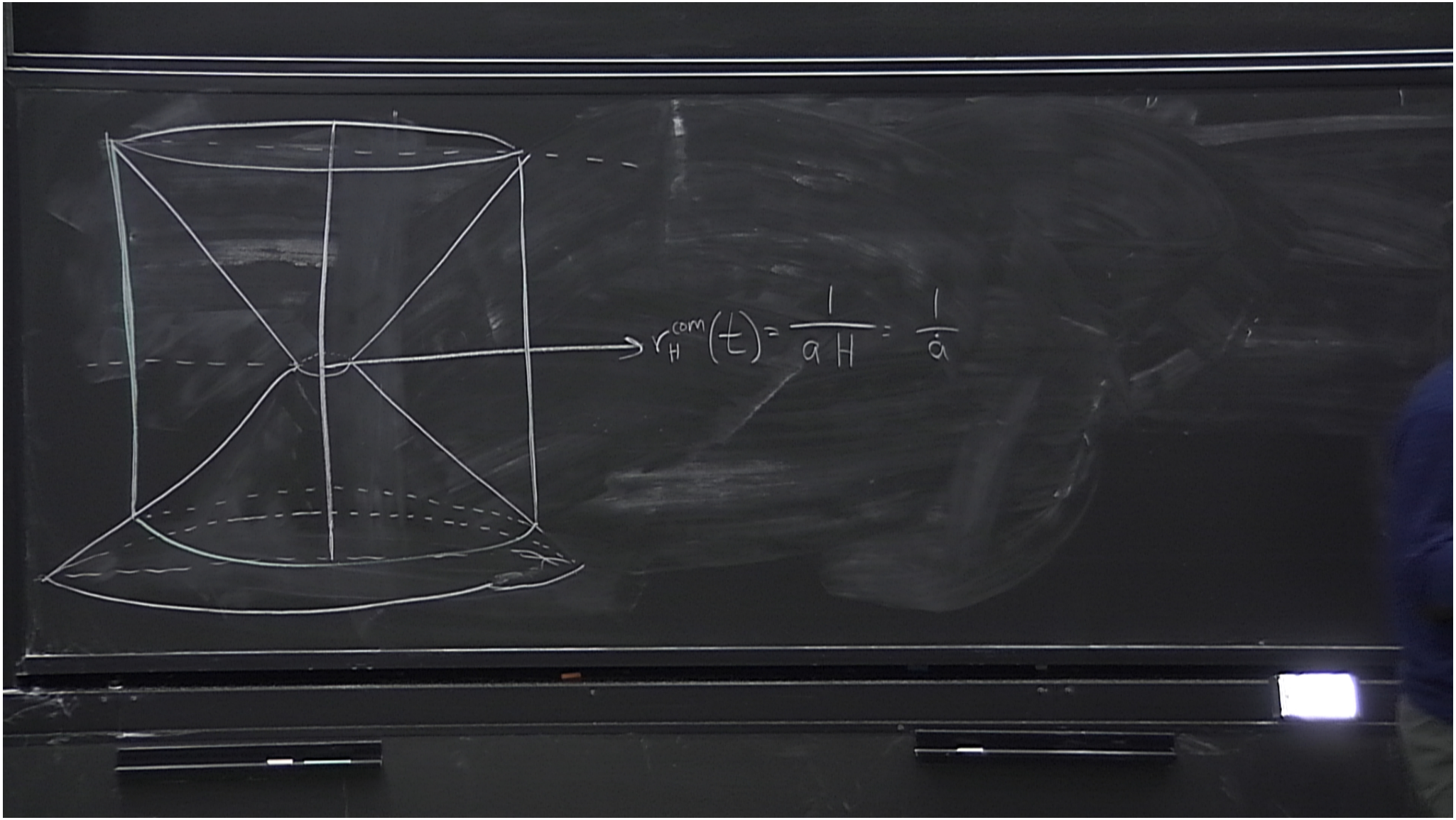
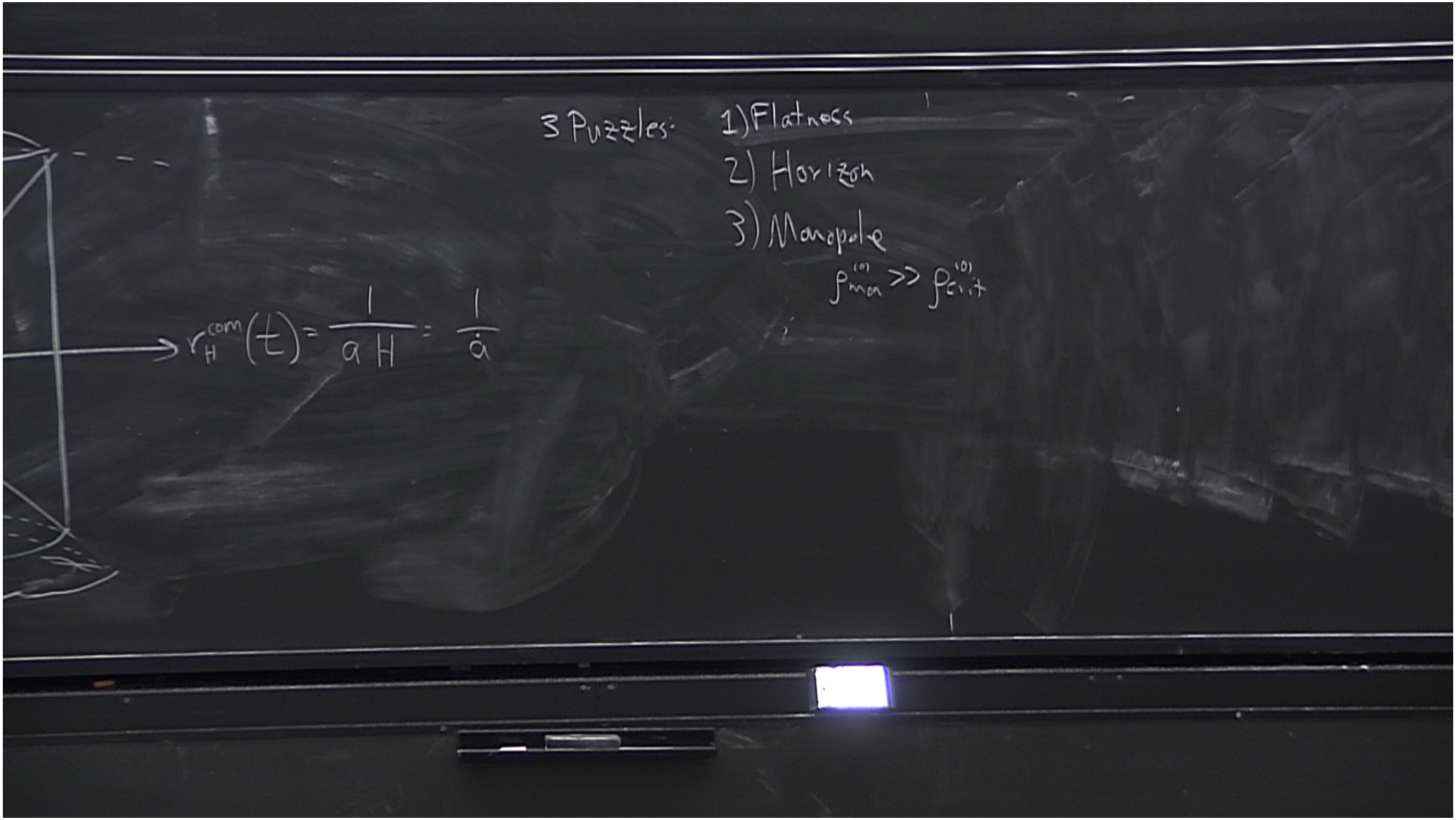
Title: Cosmology (Review) - Lecture 15

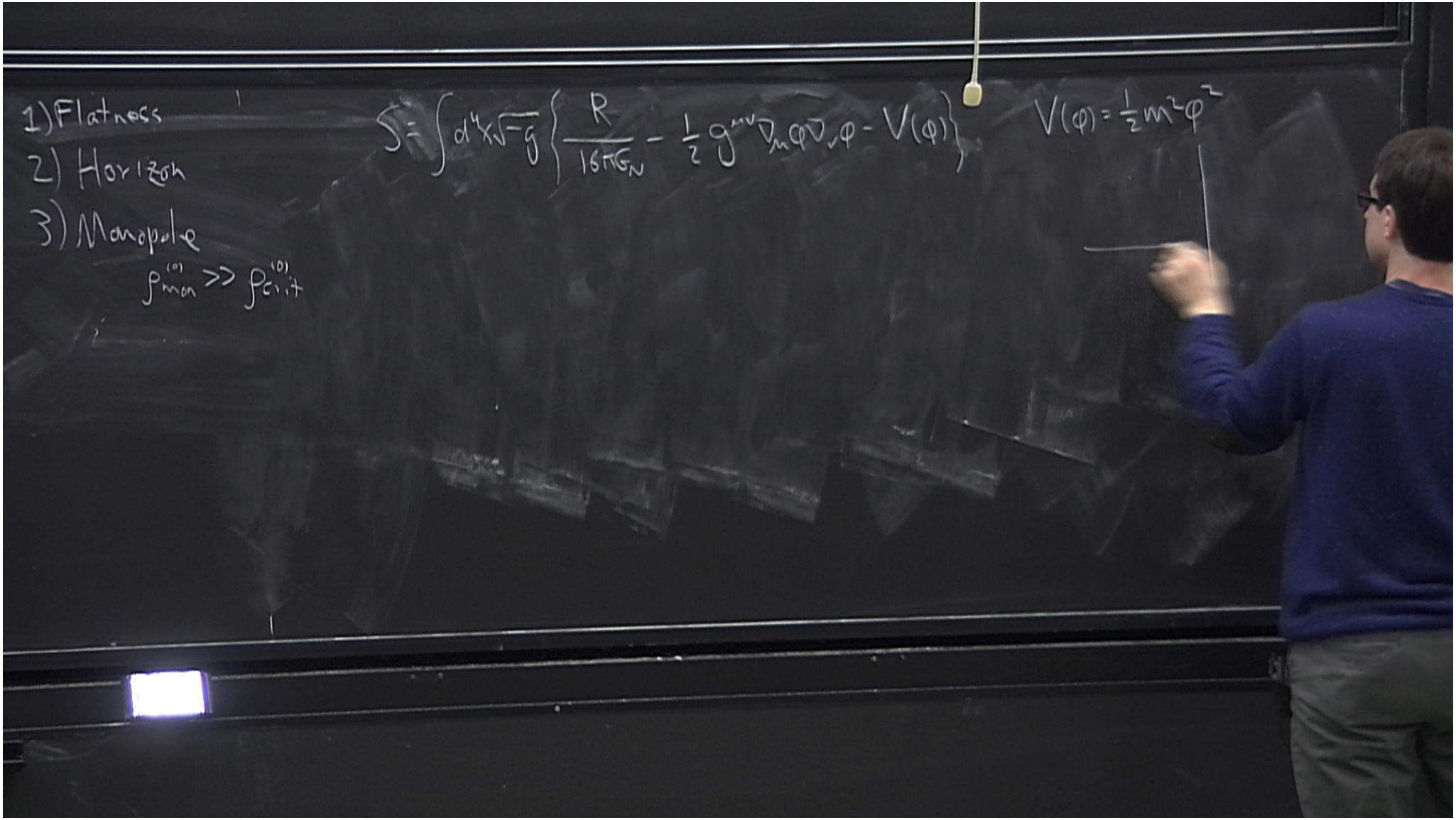
Date: Feb 10, 2012 11:30 AM

URL: http://pirsa.org/12020029

Abstract:


$r_H^{com}(t) = \frac{1}{aH} = \frac{1}{\dot{a}}$


3 Puzzles:
1) Flatness
2) Horizon
3) Monopole
$\rho_{mon}^{(a)} >> \rho_{crit}^{(t)}$
$r_H^{com}(t) = \frac{1}{aH} = \frac{1}{\dot{a}}$


1) Flatness
2) Horizon
3) Monopole
$\rho^{(0)}_{mon} >> \rho^{(0)}_{crit}$
$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\}$
$V(\varphi) = \frac{1}{2} m^2 \varphi^2$

1) Flatness

2) Horizon

3) Monopole

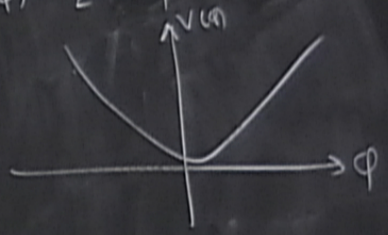
$\rho^{(0)}_{mon} >> \rho^{(0)}_{curv}$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\}$$

$$\epsilon = \frac{1}{2} M_{Pl}^2 \left[\frac{V'(\varphi)}{V(\varphi)} \right]^2$$

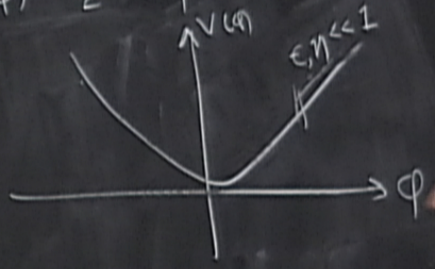
$$\eta = M_{Pl}^2 \frac{V''(\varphi)}{V(\varphi)}$$

$V(\varphi) = \frac{1}{2} m^2 \varphi^2$



$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\}$$

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

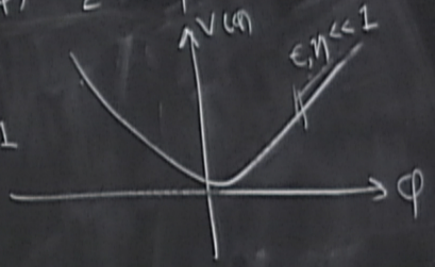


$$\epsilon = \frac{1}{2} M_{Pl}^2 \left[\frac{V'(\varphi)}{V(\varphi)} \right]^2 = \frac{1}{2} M_{Pl}^2 \frac{4}{\varphi^2}$$

$$\eta = M_{Pl}^2 \frac{V''(\varphi)}{V(\varphi)}$$

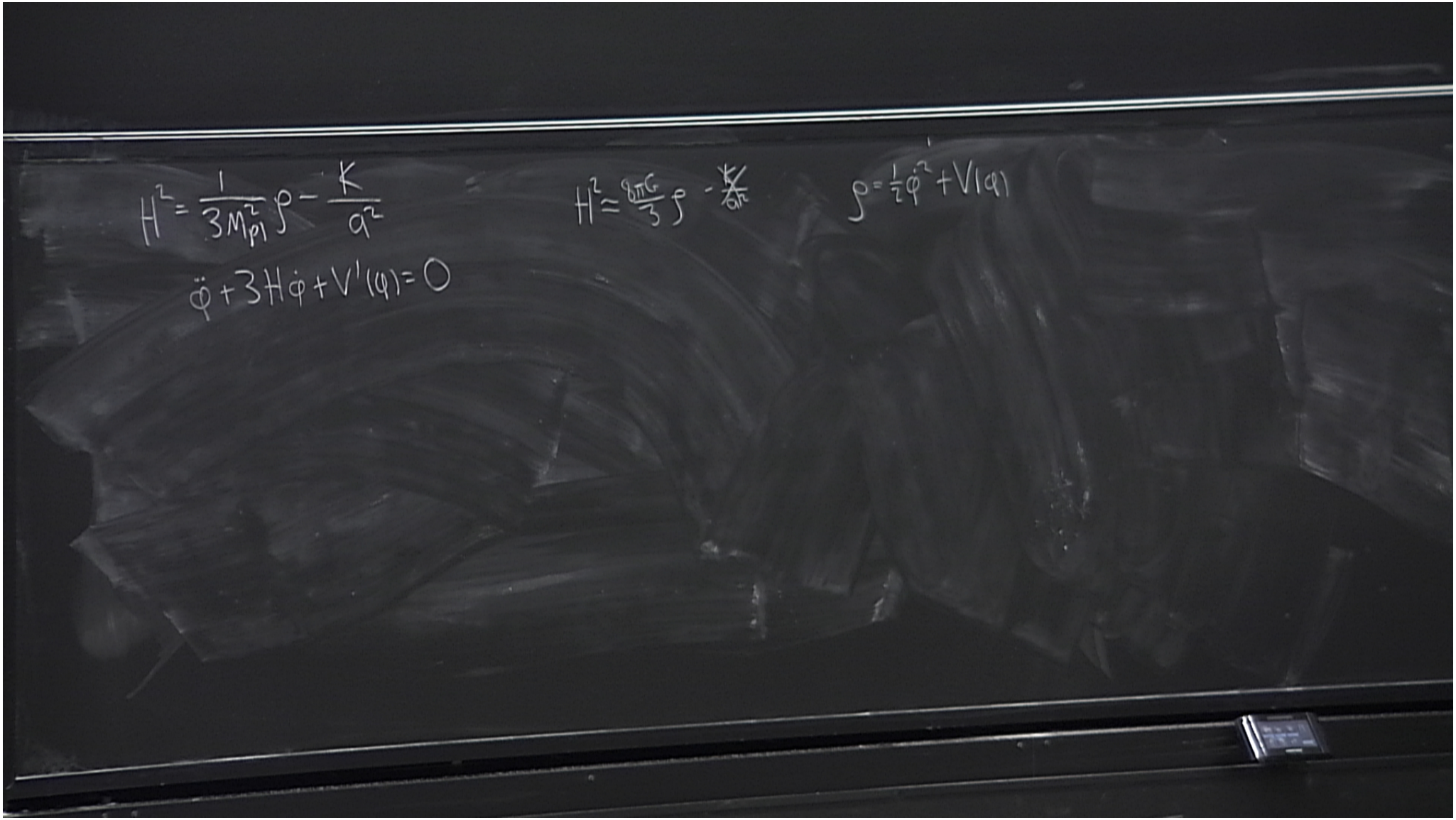
$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right\}$$

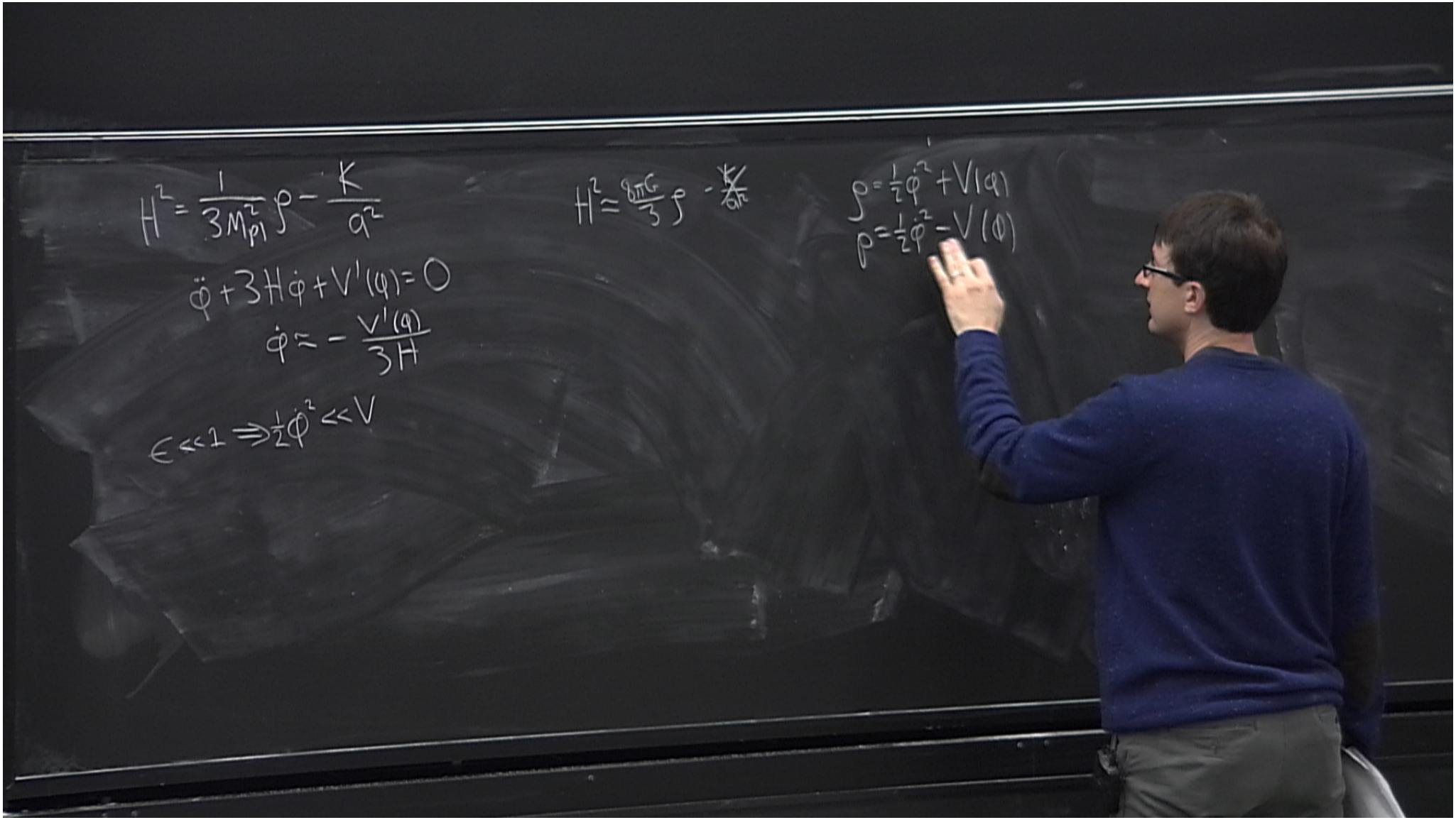
$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

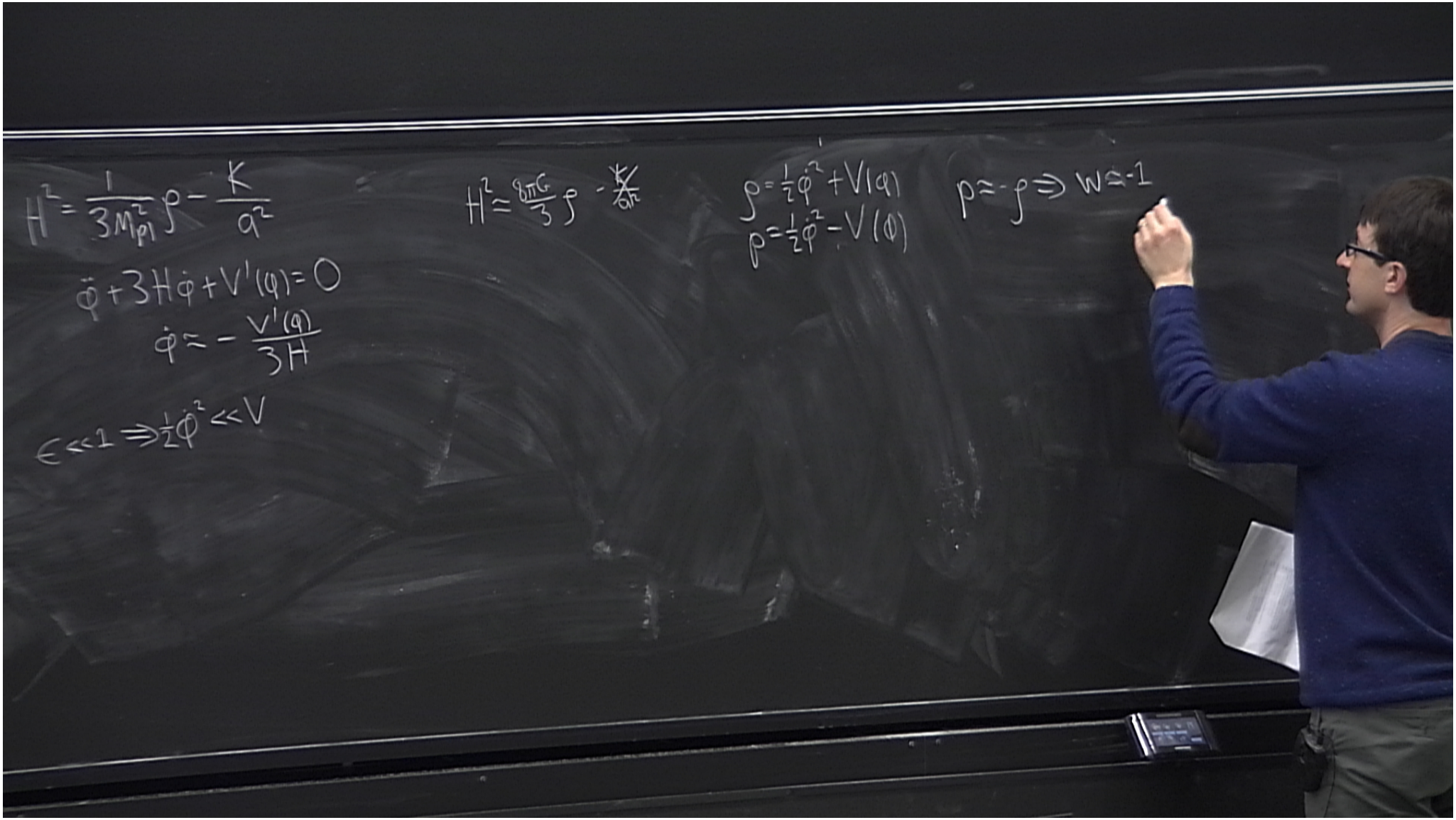


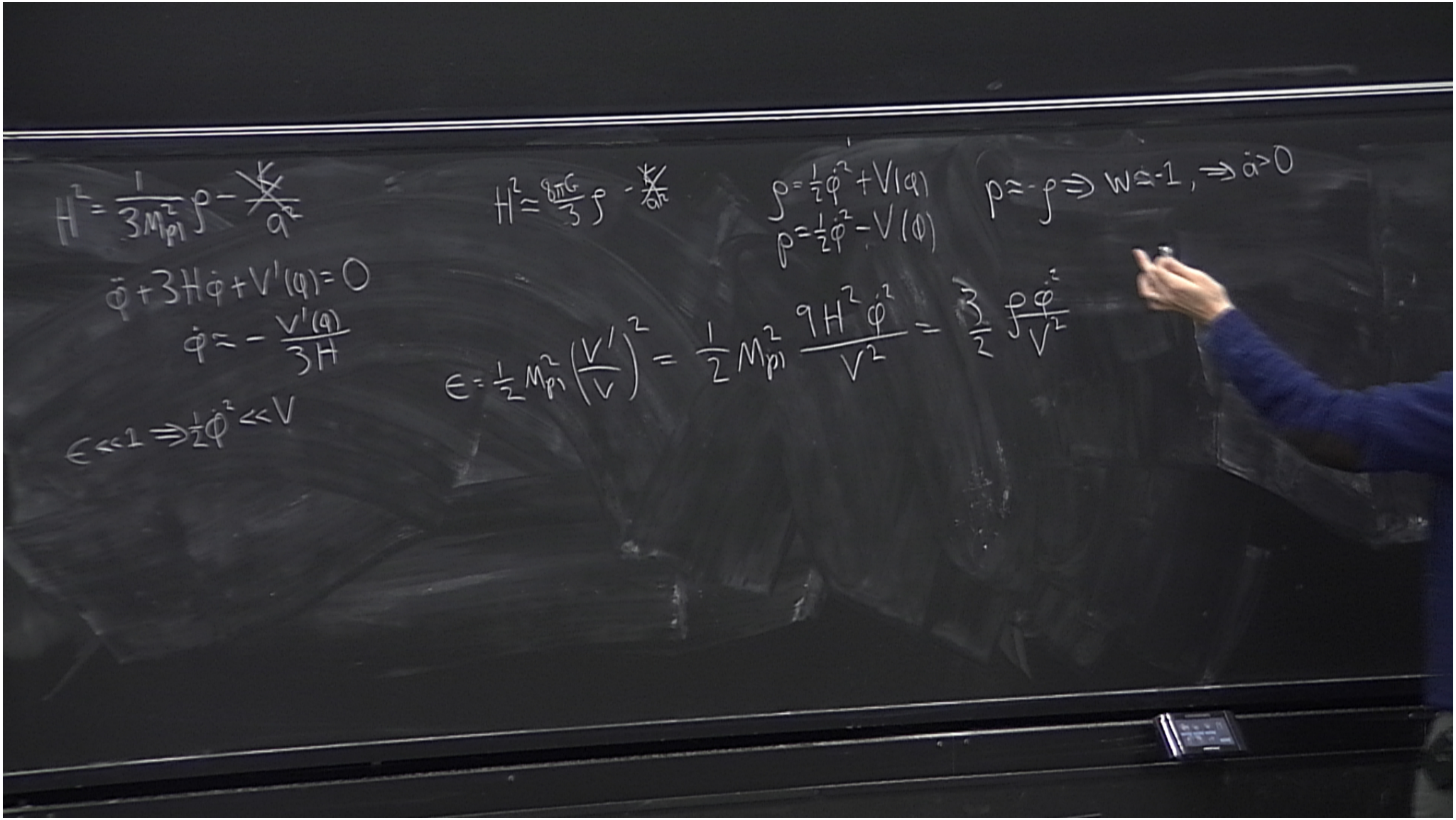
$$\epsilon = \frac{1}{2} M_{Pl}^2 \left[\frac{V'(\varphi)}{V(\varphi)} \right]^2 = \frac{1}{2} M_{Pl}^2 \frac{4}{\varphi^2} \to \ll 1$$

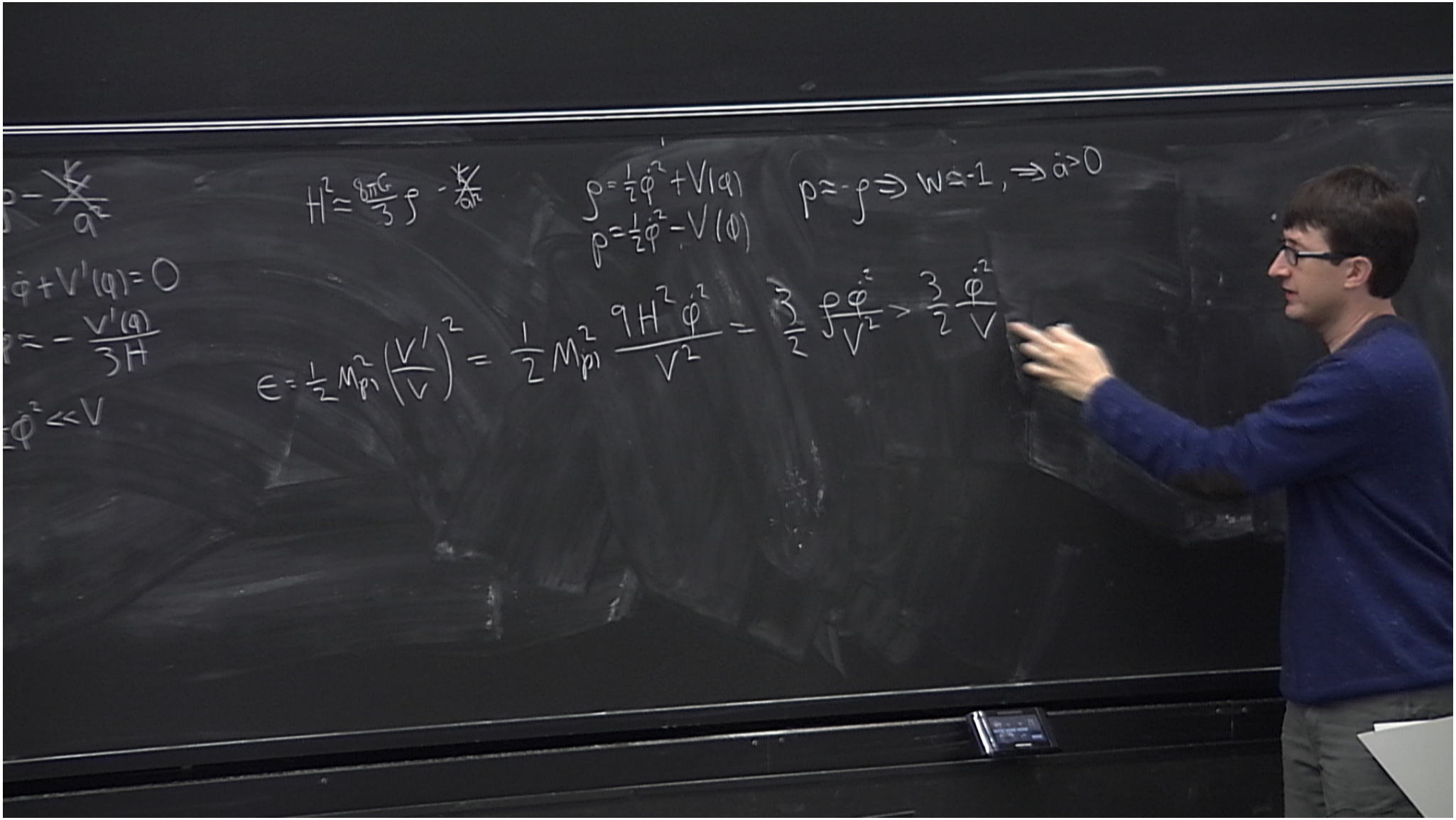
$$\eta = M_{Pl}^2 \frac{V''(\varphi)}{V(\varphi)} = 2 M_{Pl}^2 \frac{1}{\varphi^2}$$

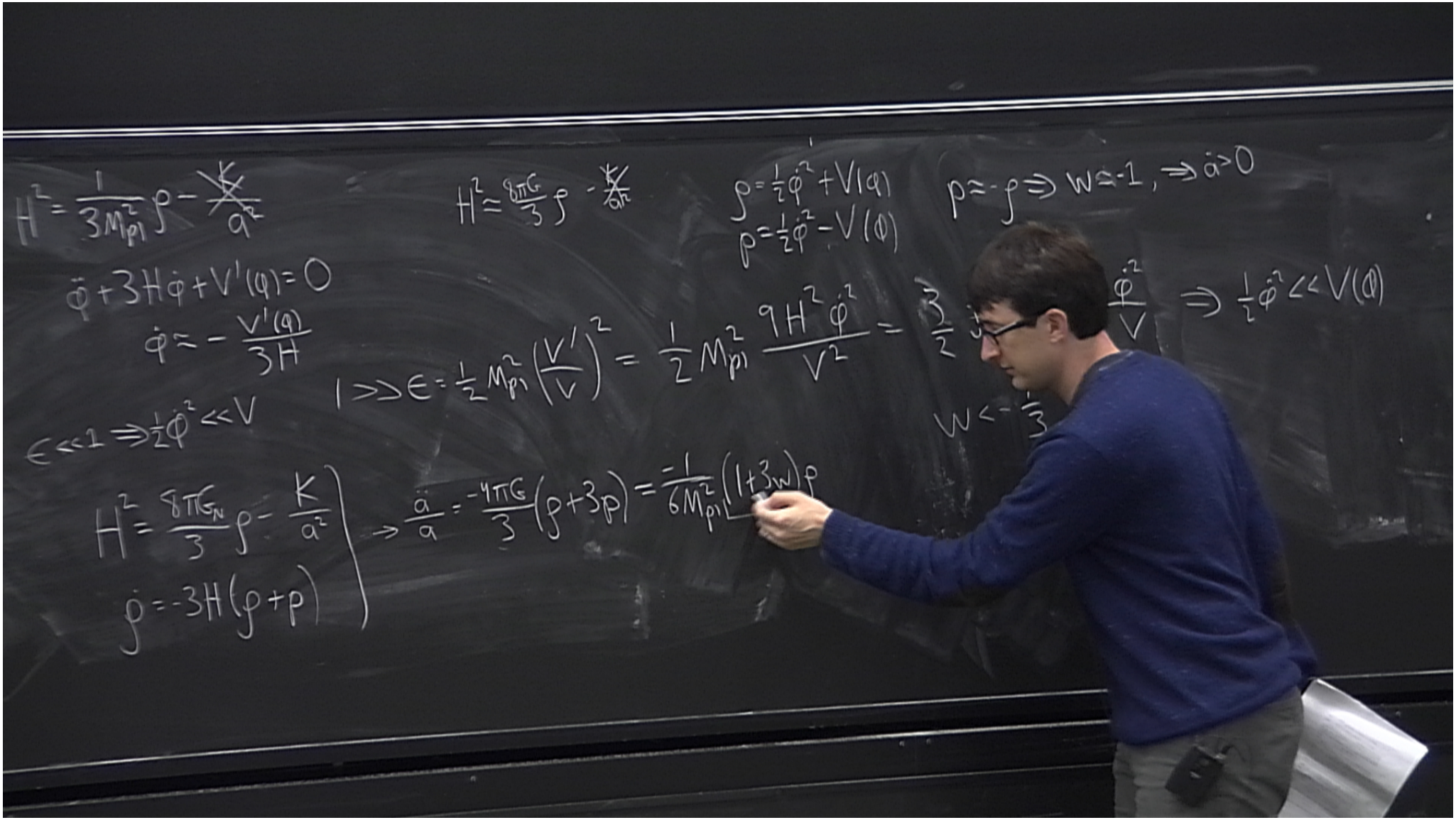

$H^2 = \frac{1}{3M_{Pl}^2}\rho - \frac{K}{a^2}$
$H^2 \simeq \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$

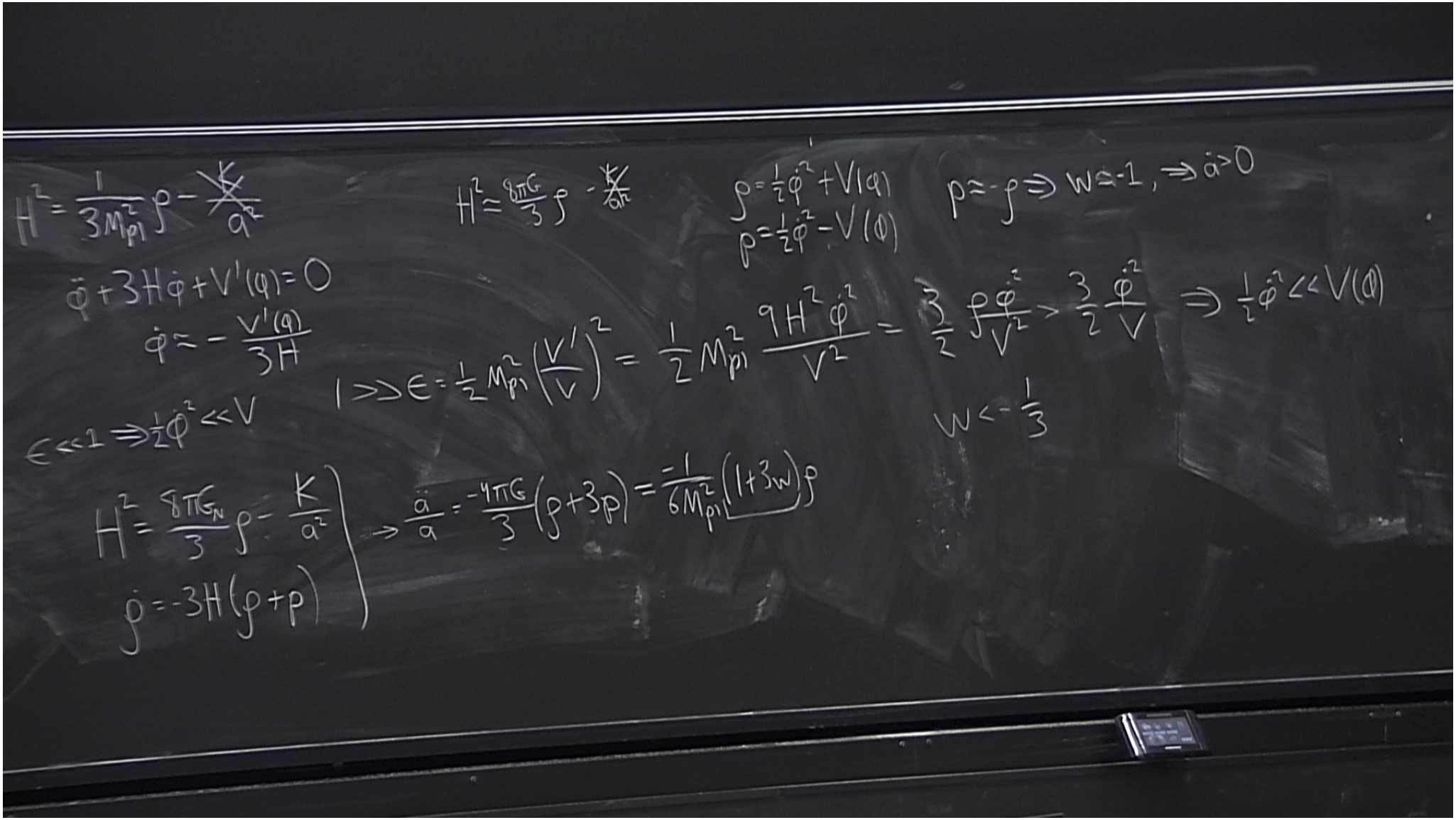

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$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$
$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$
$\dot{\varphi} \simeq -\frac{V'(\varphi)}{3H}$
$\epsilon \ll 1 \Rightarrow \frac{1}{2}\dot{\varphi}^2 \ll V$

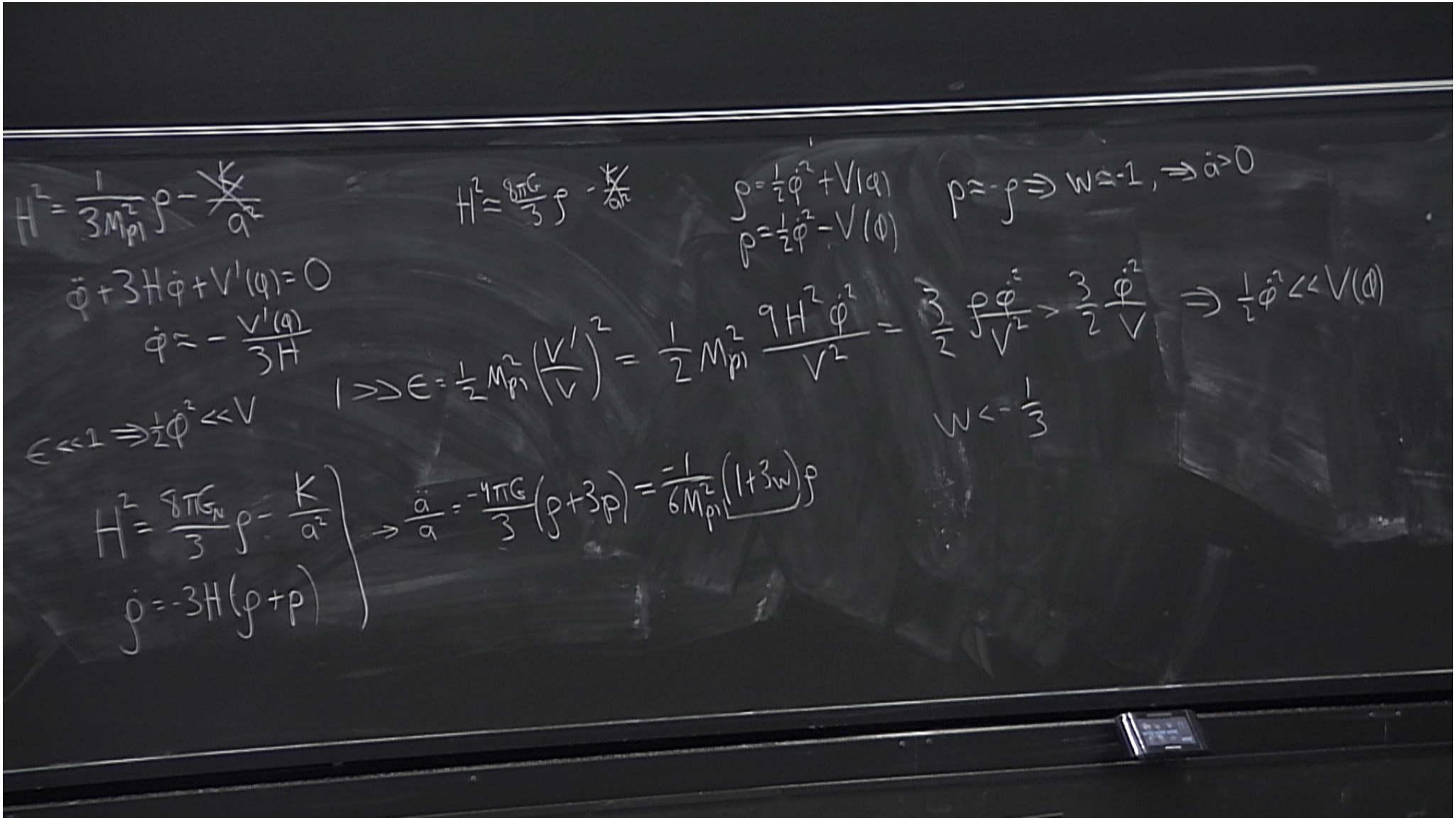

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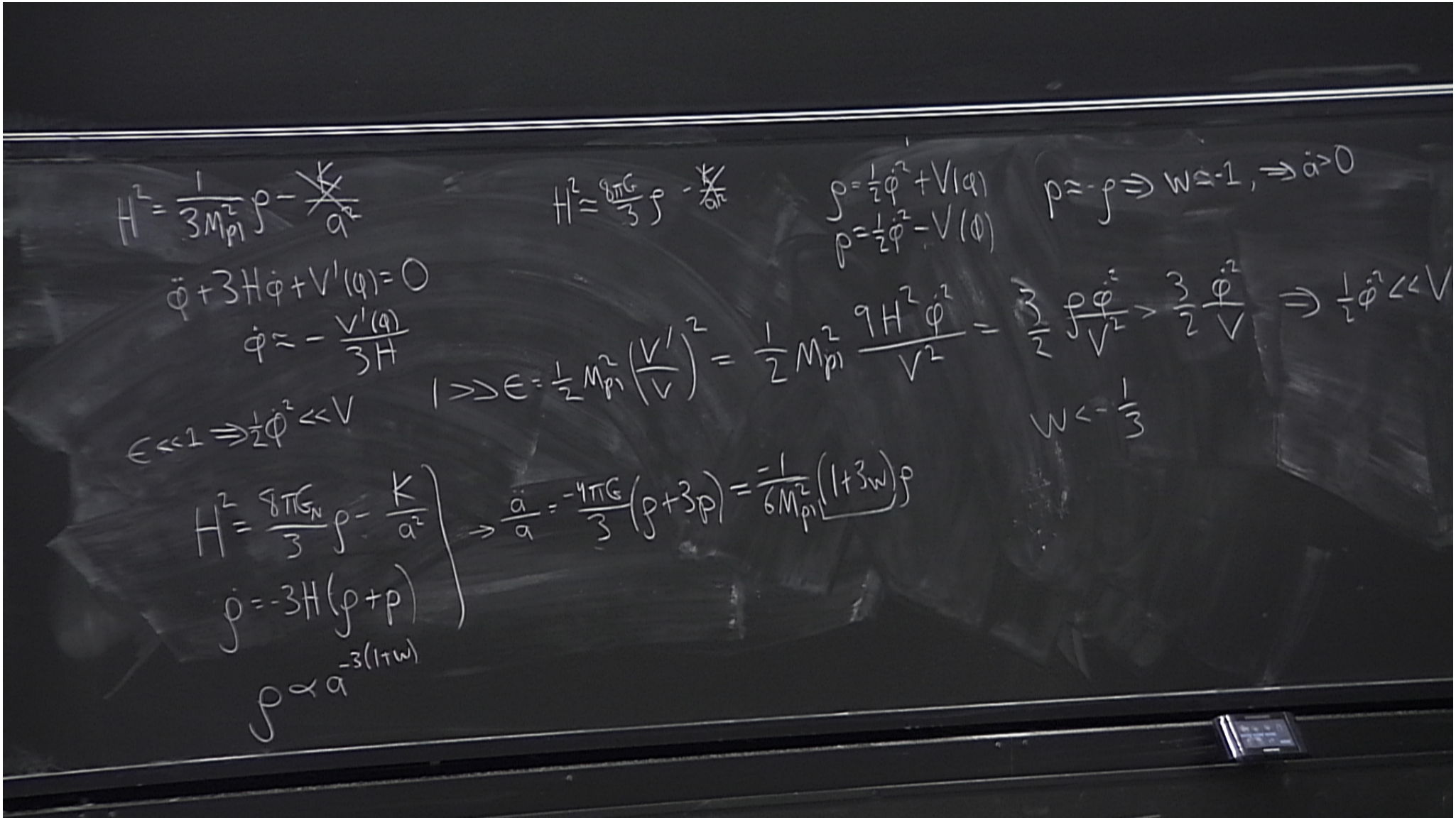

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$\epsilon \ll 1 \Rightarrow \frac{1}{2}\dot{\phi}^2 \ll V$
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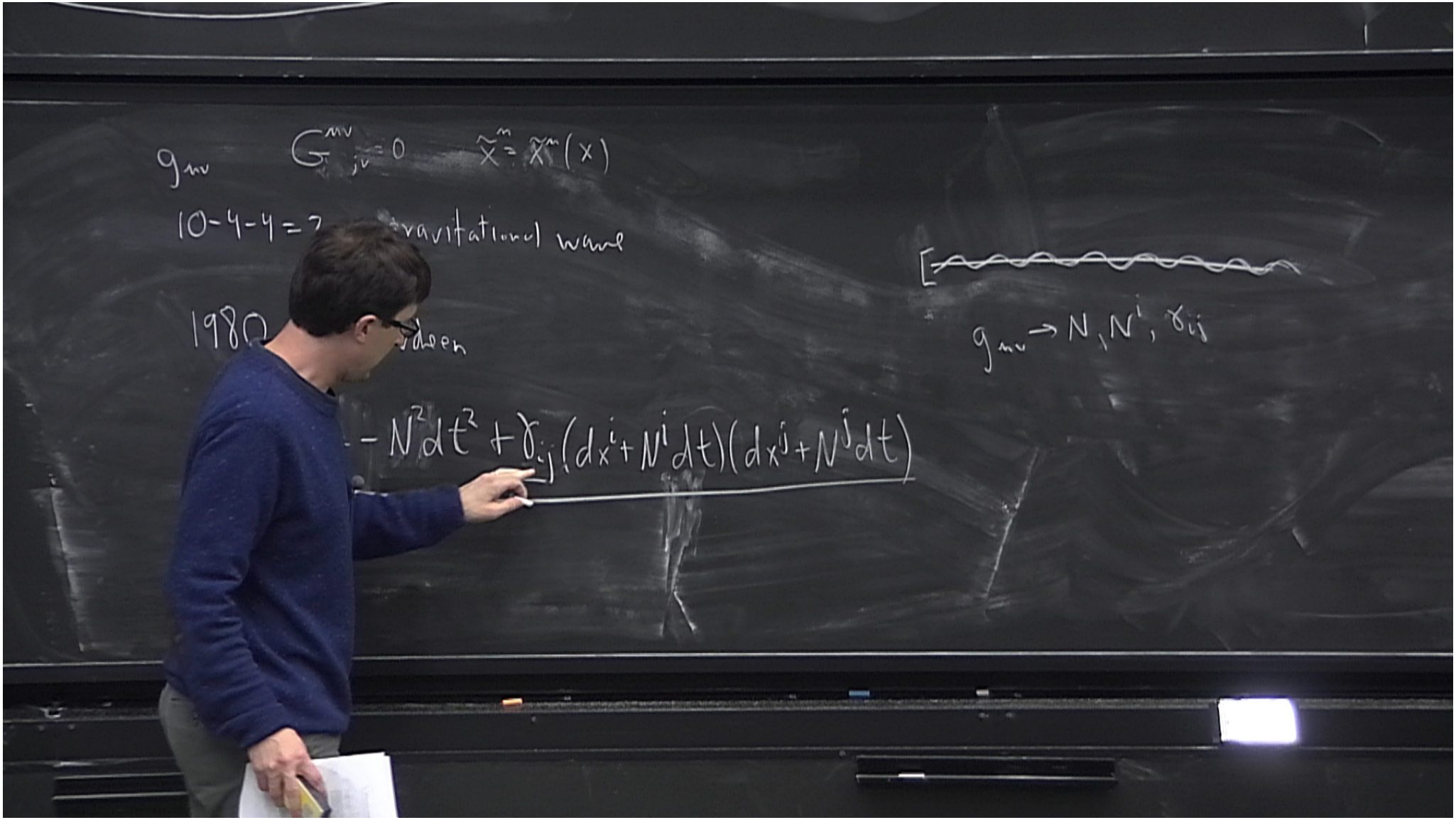

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$\frac{\dot{\varphi}^2}{V}$
$\Rightarrow \frac{1}{2}\dot{\varphi}^2 << V(\varphi)$
$\epsilon << 1 \Rightarrow \frac{1}{2}\dot{\varphi}^2 << V$
$w < -\frac{1}{3}$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p)$
$\Rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = \frac{-1}{6M_{Pl}^2}(1+3w)\rho$


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$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p)$
$\rho \propto a^{-3(1+w)}$
$\rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = \frac{-1}{6M_{Pl}^2}(1+3w)\rho$


$g_{\mu\nu}$
$G^{\mu\nu}{}_{;\nu} = 0$
$\tilde{x}^m = \tilde{x}^m(x)$
10-4-4=
gravitational wave
1980
$-N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$g_{\mu\nu} \to N, N^i, \gamma_{ij}$

$g_{\mu\nu}$ $\quad G^{\mu\nu}{}_{;\nu} = 0 \quad \tilde{x}^{\mu} = \tilde{x}^{\mu}(x)$

$10 - 4 - 4 = 2$ ← gravitational wave

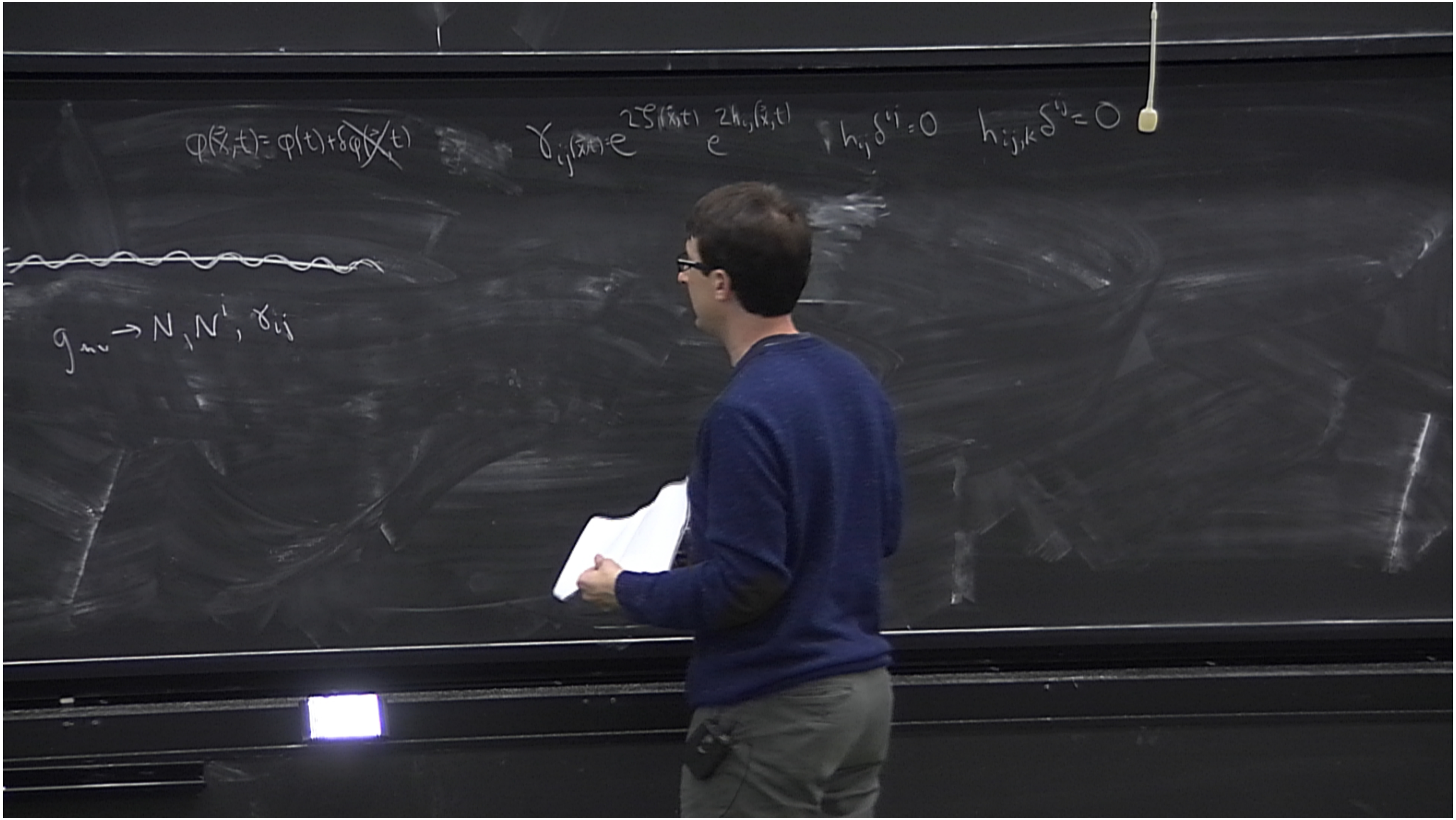
$\downarrow$ 3

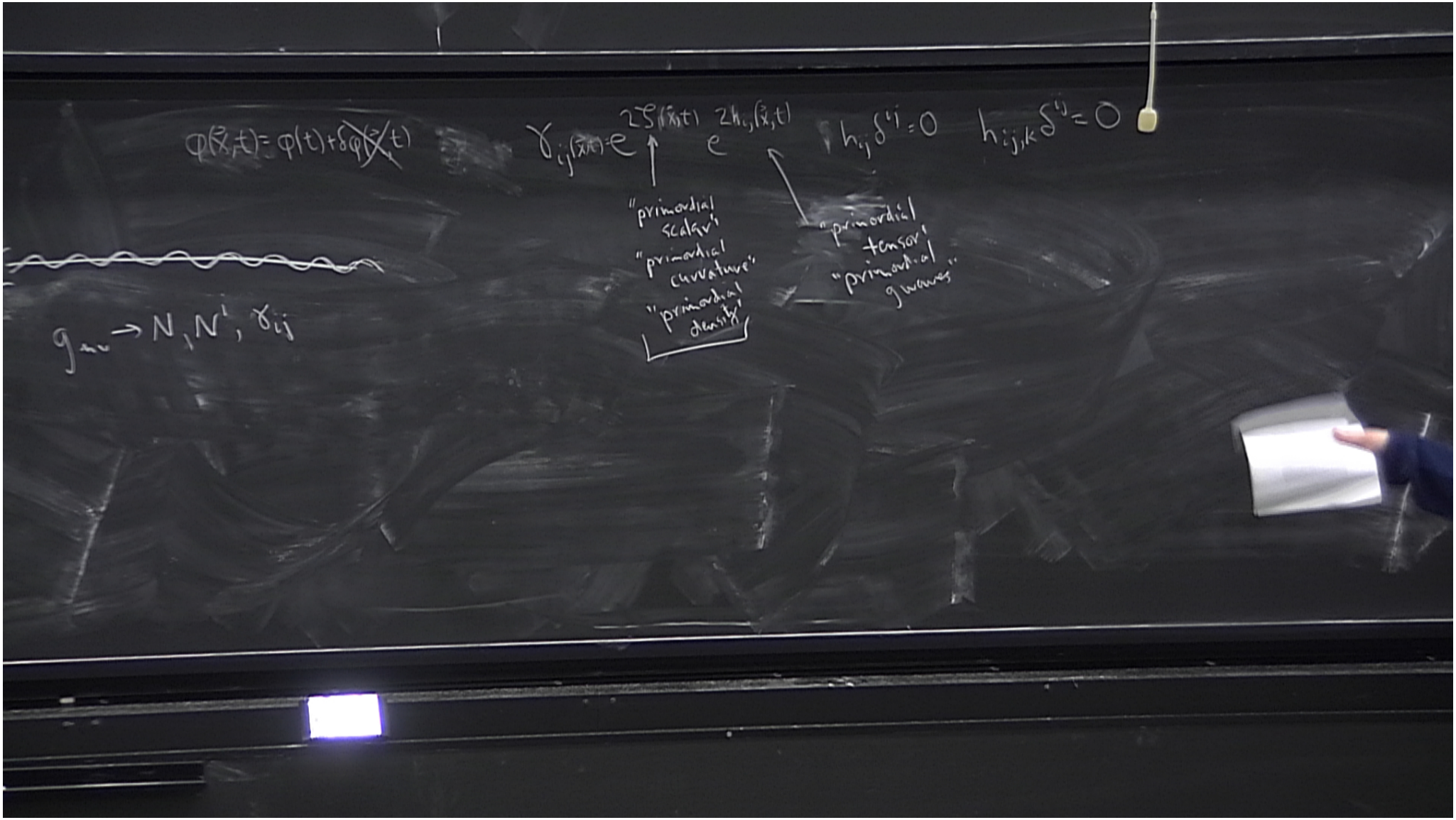
1980 J. Bardeen

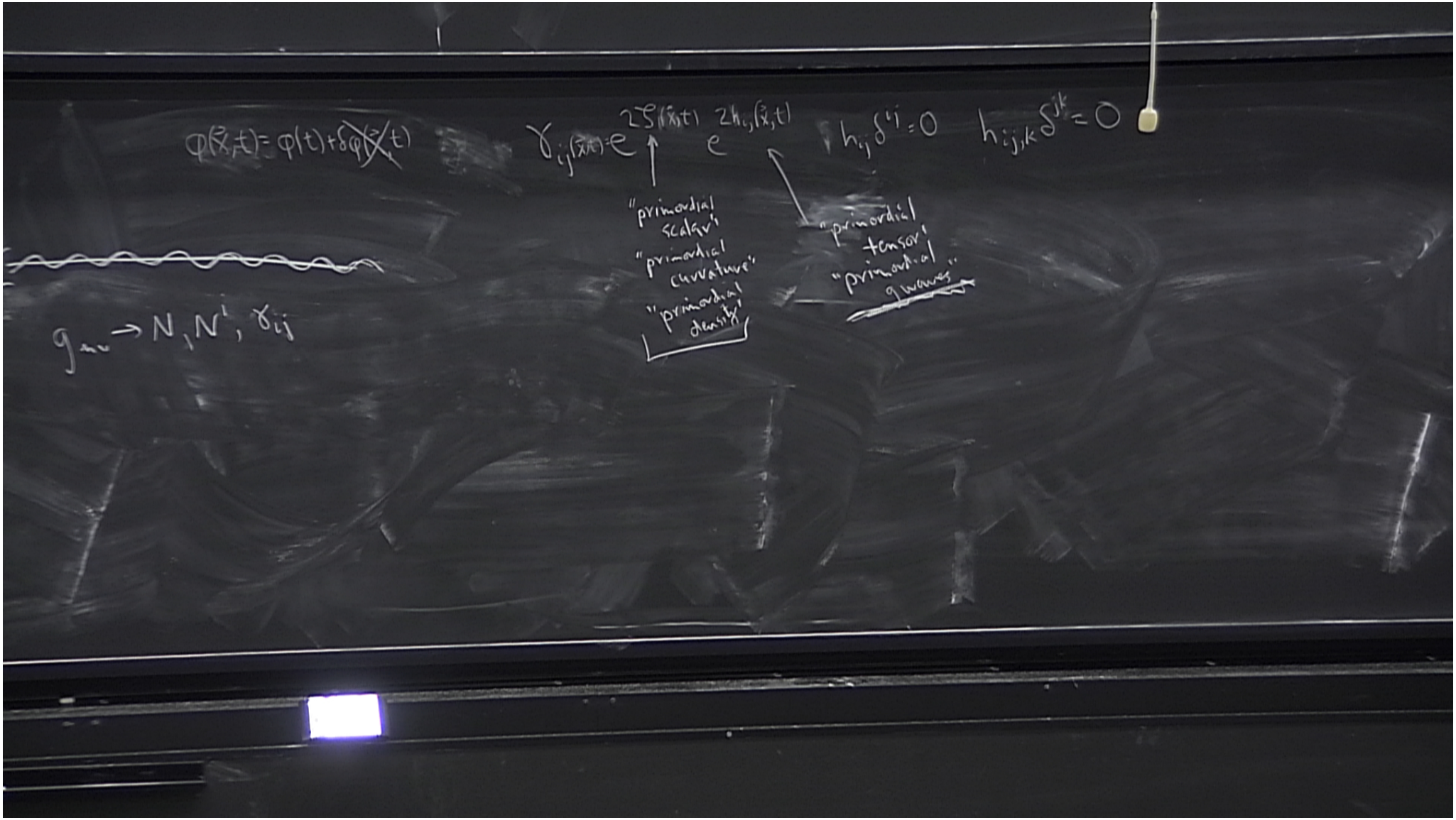


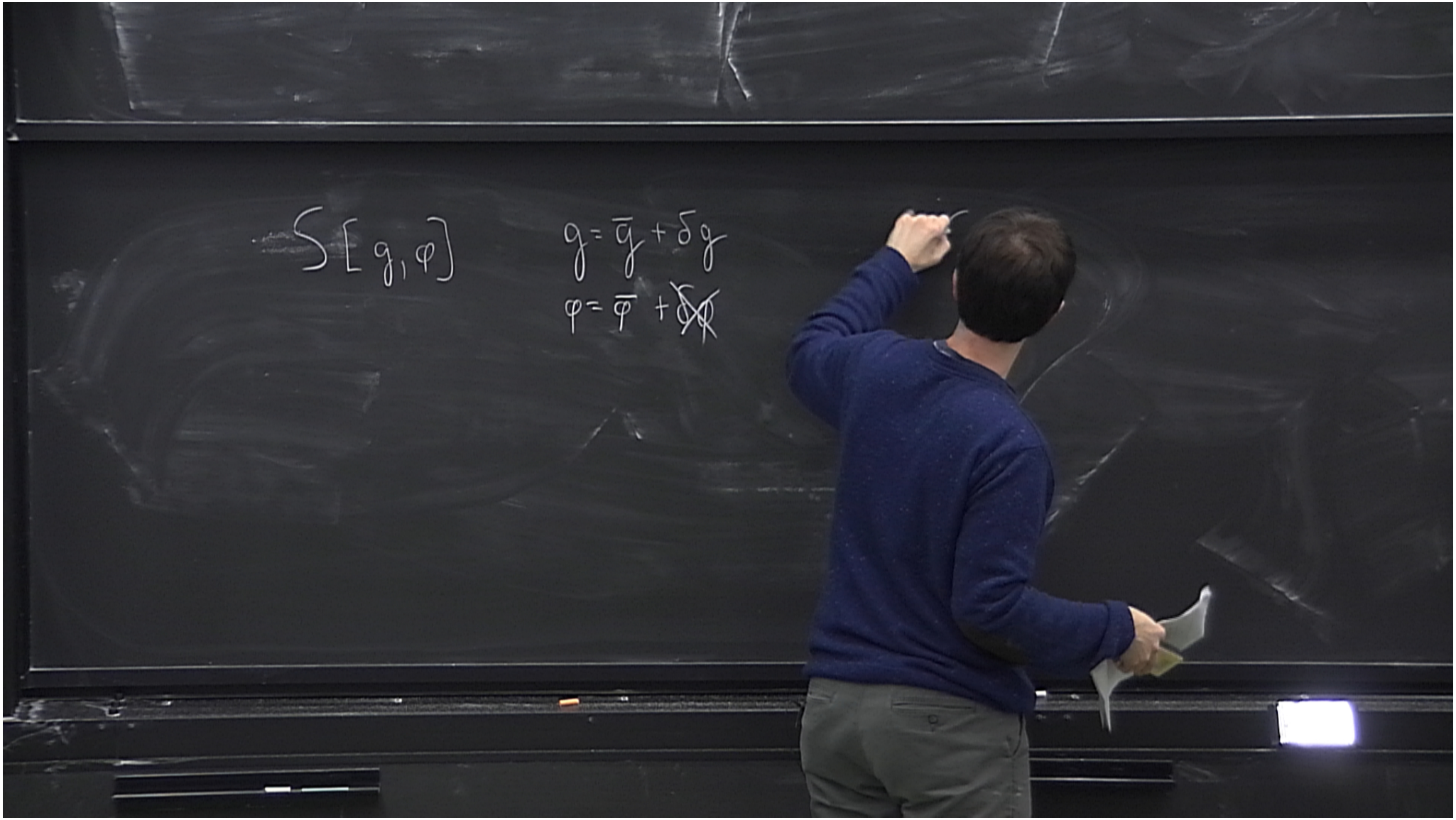
$g_{\mu\nu} \rightarrow N, N^i, \gamma_{ij}$

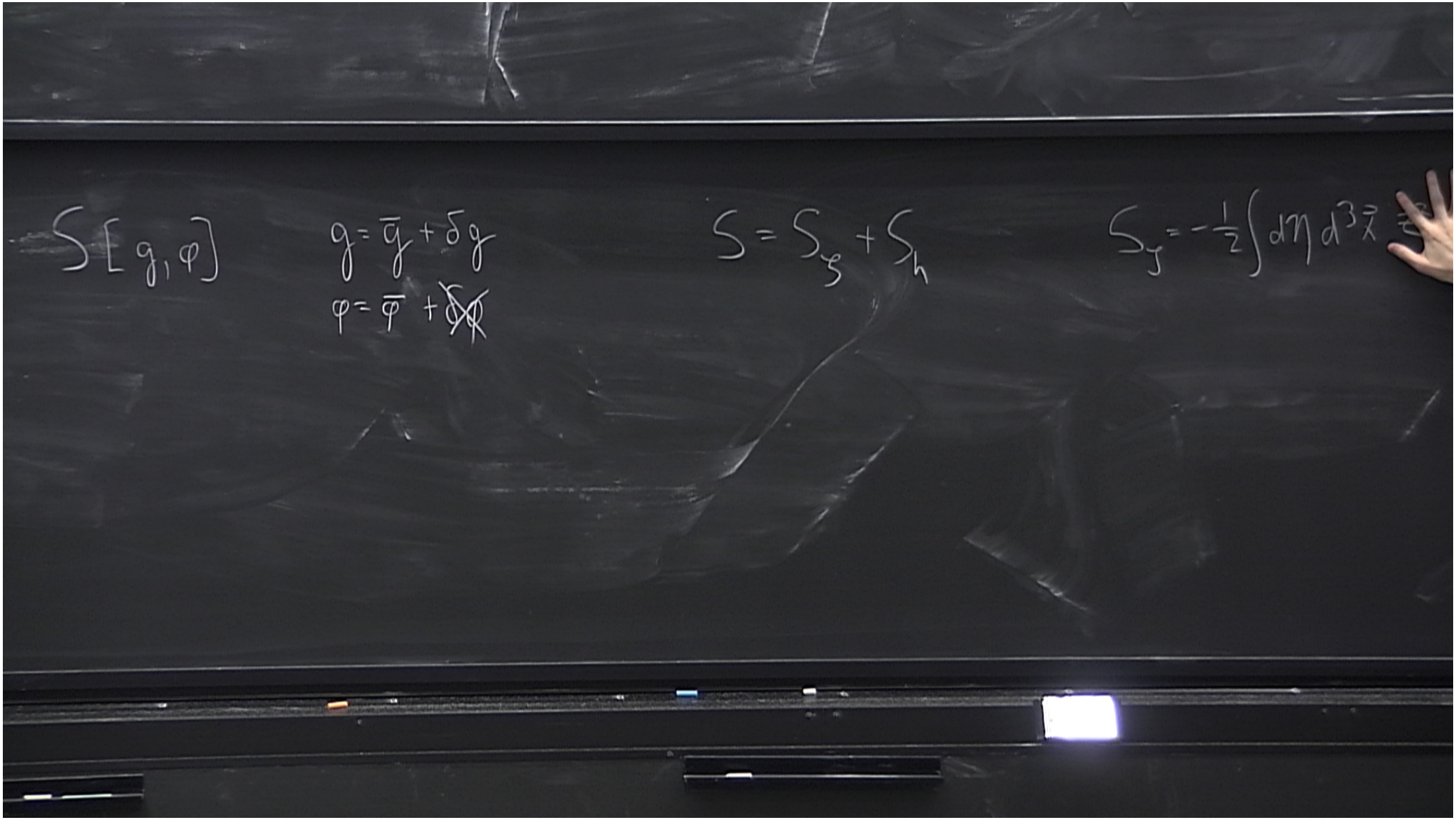
$\delta\phi$ $\quad ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

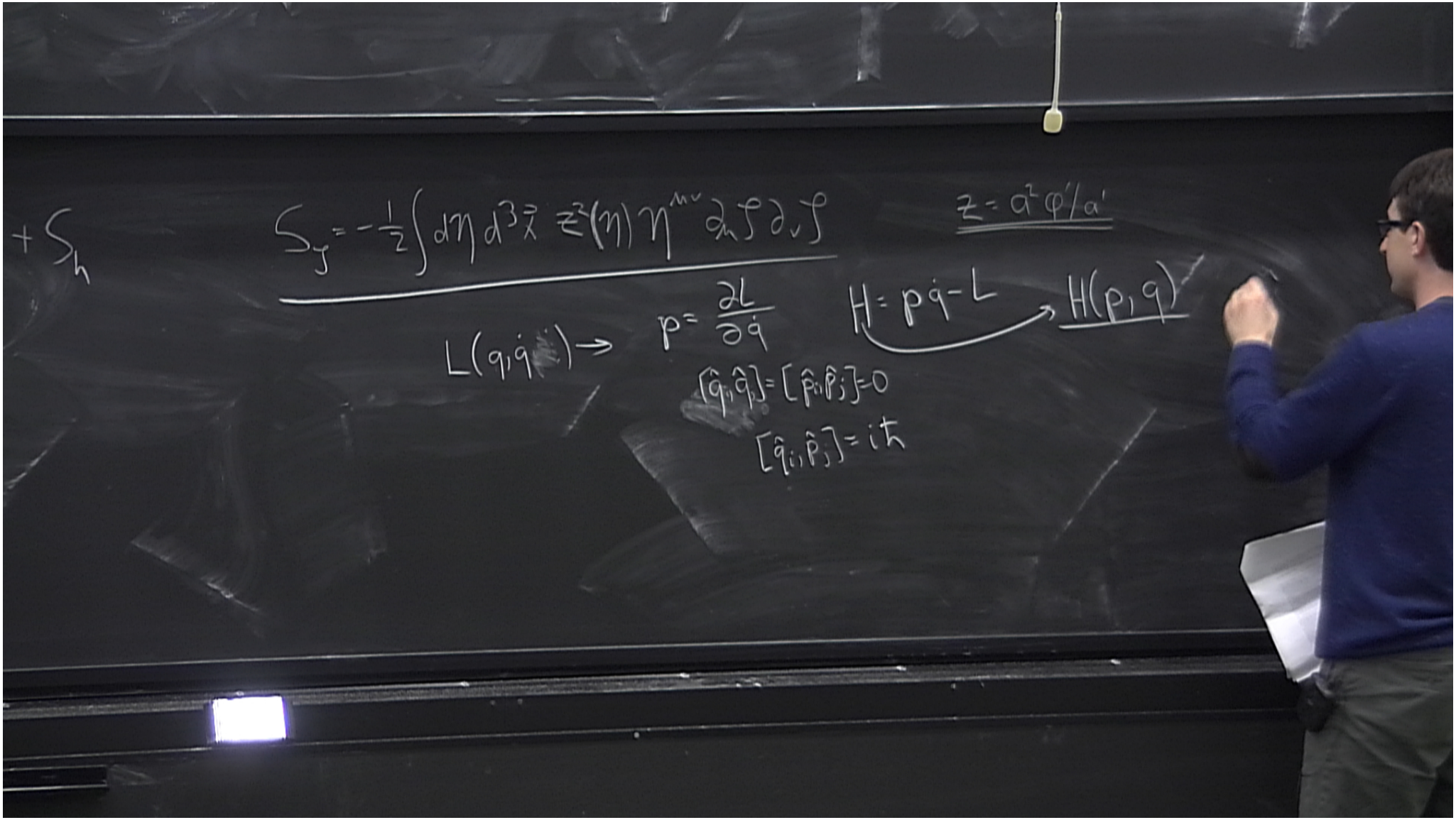

$\varphi(\vec{x},t) = \varphi(t) + \delta\varphi(\vec{x},t)$
$\gamma_{ij}(\vec{x},t) = e^{2\zeta(\vec{x},t)} e^{2h_{ij}(\vec{x},t)}$
$h_{ij}\delta^{ij} = 0$
$h_{ij,k}\delta^{ij} = 0$
$g_{\mu\nu} \to N, N^i, \gamma_{ij}$

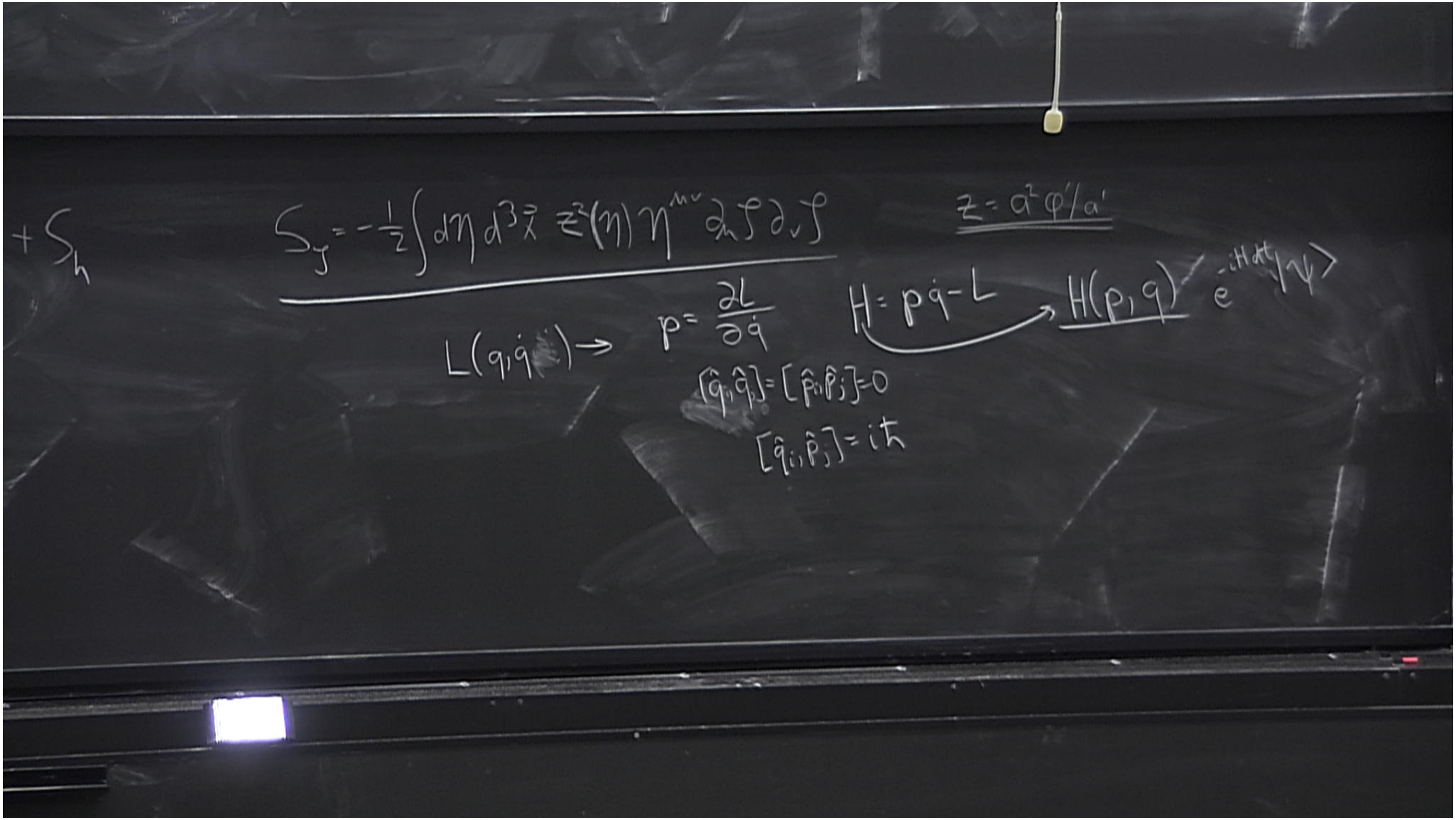

$\varphi(\vec{x},t) = \varphi(t) + \delta\varphi(\vec{x},t)$
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$h_{ij}\delta^{ij} = 0$
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"primordial scalar"
"primordial curvature"
"primordial density"
"primordial tensor"
"primordial gravity"
$g_{\mu\nu} \to N, N^i, \gamma_{ij}$

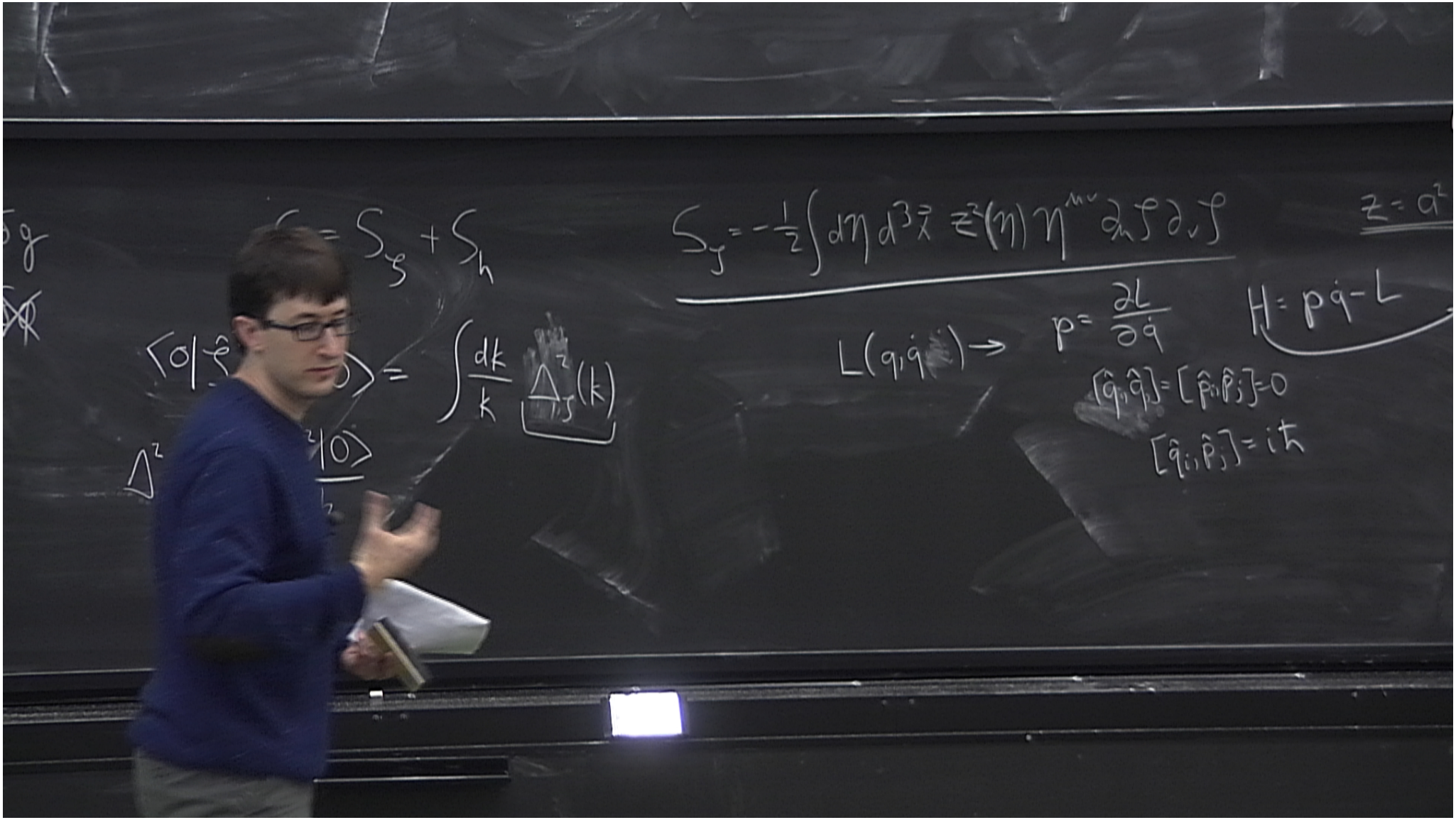

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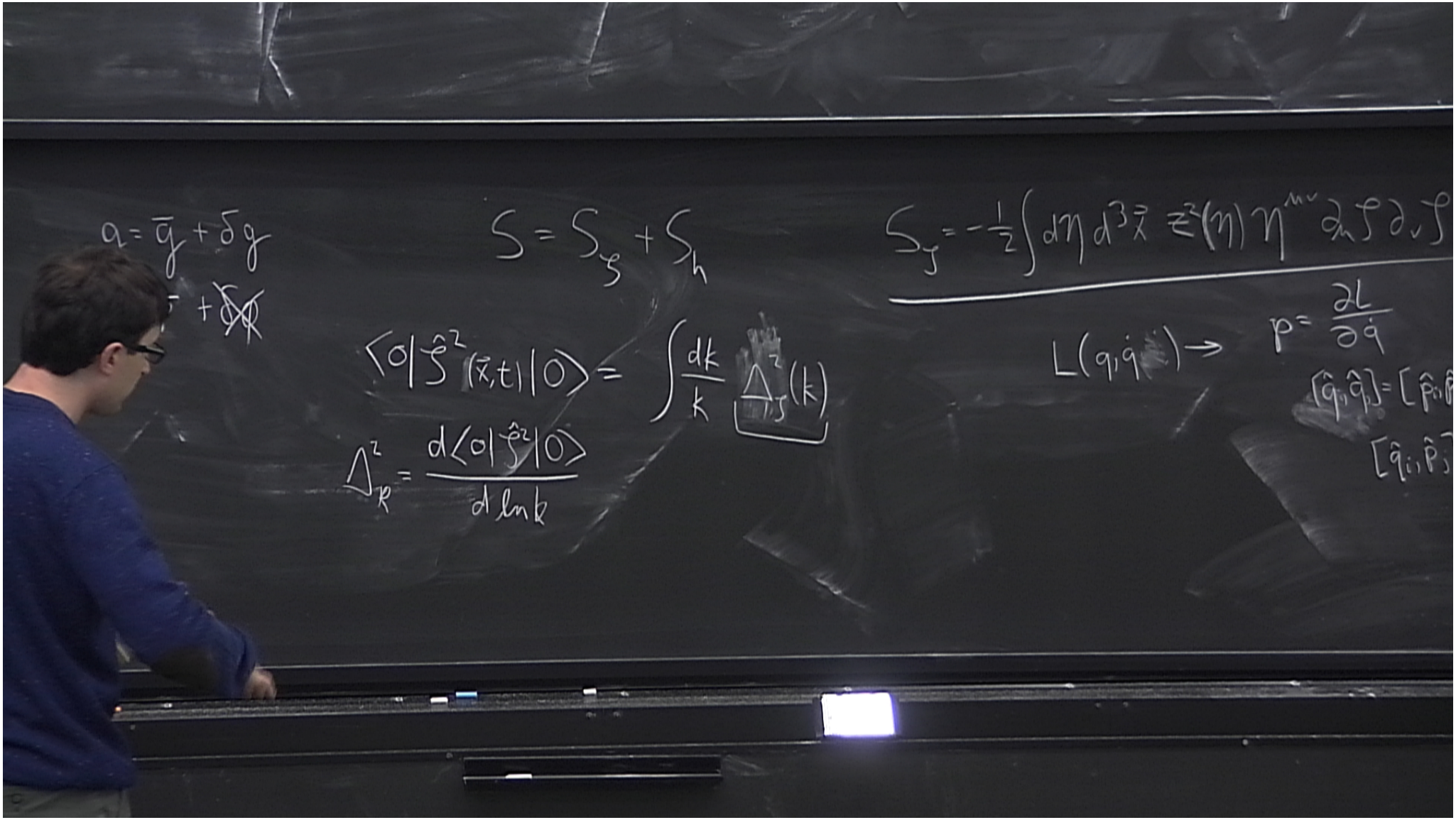

$S[g,\varphi]$
$g = \bar{g} + \delta g$
$\varphi = \bar{\varphi} + \delta\varphi$

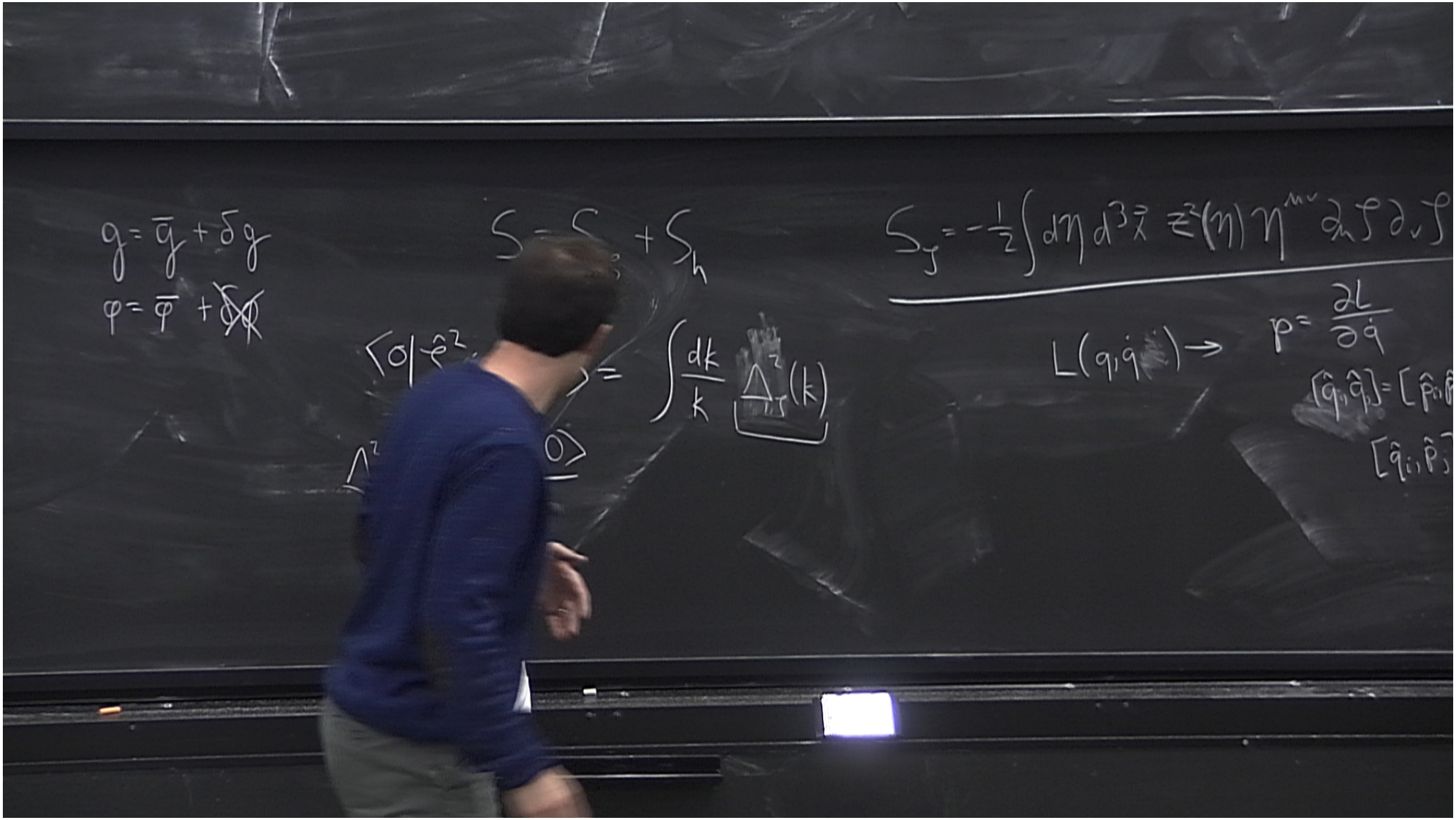

$S[g,\varphi]$
$g = \bar{g} + \delta g$
$\varphi = \bar{\varphi} + \delta\varphi$
$S = S_g + S_h$
$S_g = -\frac{1}{2}\int d\eta\, d^3\vec{x}$

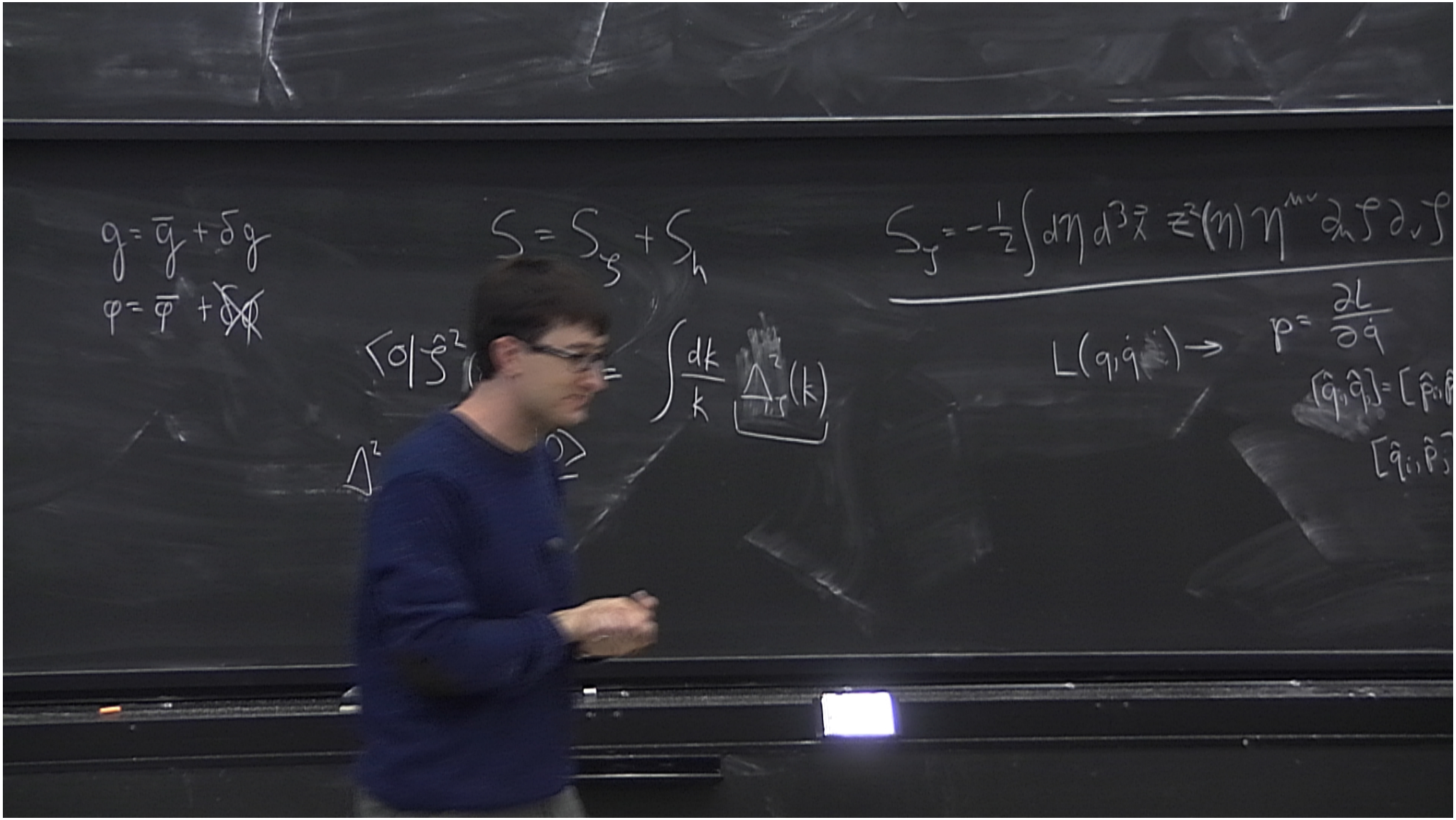

$+S_h$
$S_f = -\frac{1}{2}\int d\eta\, d^3x\, z^2(\eta)\,\eta^{\mu\nu}\partial_\mu f\,\partial_\nu f$
$z = a^2\varphi'/a'$
$L(q,\dot{q}) \rightarrow$
$p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$
$H(p,q)$
$[\hat{q}_i, \hat{q}_j] = [\hat{p}_i, \hat{p}_j] = 0$
$[\hat{q}_i, \hat{p}_j] = i\hbar$


$+S_h$
$S_\chi = -\frac{1}{2}\int d\eta\, d^3x\, z^2(\eta)\,\eta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$
$z = a^2\varphi'/a'$
$L(q,\dot{q}) \rightarrow$
$p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$
$H(p,q)$
$e^{-iHdt}|\psi\rangle$
$[\hat{q}_i,\hat{q}_j] = [\hat{p}_i,\hat{p}_j] = 0$
$[\hat{q}_i,\hat{p}_j] = i\hbar$


$S = S_s + S_h$
$S_s = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, a^2(\eta)\,\eta^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi$
$z = a^2$
$\int \frac{dk}{k}\,\Delta^2_{h,s}(k)$
$L(q,\dot{q}) \rightarrow p = \frac{\partial L}{\partial \dot{q}}$
$H = p\dot{q} - L$
$[\hat{q}_i, \hat{q}_j] = [\hat{p}_i, \hat{p}_j] = 0$
$[\hat{q}_i, \hat{p}_j] = i\hbar$


$g = \bar{g} + \delta g$
$S = S_s + S_h$
$S_s = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, z^2(\eta)\, \eta^{\mu\nu}\, \partial_\mu s\, \partial_\nu s$
$\langle 0|\hat{s}^2(\vec{x},t)|0\rangle = \int \frac{dk}{k}\, \Delta^2_s(k)$
$\Delta^2_R = \frac{d\langle 0|\hat{s}^2|0\rangle}{d\ln k}$
$L(q,\dot{q}) \rightarrow$
$p = \frac{\partial L}{\partial \dot{q}}$
$[\hat{q}_i, \hat{q}_j] = [\hat{p}_i$
$[\hat{q}_i, \hat{p}_j$


$g = \bar{g} + \delta g$
$\varphi = \bar{\varphi} + \delta\varphi$
$S = S_0 + S_h$
$\langle 0|\hat{\varphi}^2 \ldots = \int \frac{dk}{k} \Delta^2_{\zeta T}(k)$
$S_J = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, a^2(\eta)\, \eta^{\mu\nu}\, \partial_\mu J\, \partial_\nu J$
$L(q, \dot{q}) \rightarrow$
$p = \frac{\partial L}{\partial \dot{q}}$
$[\hat{q}_i, \hat{q}_j] = [\hat{p}_i, \hat{p}$
$[\hat{q}_i, \hat{p}_j]$


$g = \bar{g} + \delta g$
$\varphi = \bar{\varphi} + \delta\varphi$
$S = S_g + S_h$
$S_\zeta = -\frac{1}{2}\int d\eta\, d^3\vec{x}\, z^2(\eta)\, \eta^{\mu\nu}\, \partial_\mu \zeta\, \partial_\nu \zeta$
$\int \frac{dk}{k}\, \Delta^2_{\zeta T}(k)$
$L(q, \dot{q}) \rightarrow$
$p = \frac{\partial L}{\partial \dot{q}}$
$[\hat{q}_i, \hat{p}_j]$