

Title: Cosmology (Review) - Lecture 10

Date: Feb 06, 2012 11:30 AM

URL: <http://pirsa.org/12020025>

Abstract:



Lecture 1: Diff. Geom.  $\begin{cases} \text{Manifold} \\ \text{Connection} \\ \text{metric} \end{cases}$

Lecture 2:  $\mathbb{R} \xrightarrow{F} T$   
 $\int \frac{R}{16\pi G} \sqrt{-g}$

Lecture 3: Einstein Eq

Lecture 1: Diff. Geom.  $\begin{cases} \text{Manifold} \\ \text{Connection} \\ \text{metric} \end{cases}$

Lecture 2:  $R \xrightarrow{F} \int \text{Tr}[F^2]$   
 $\int \frac{R}{16\pi G} \sqrt{-g}$

Lecture 3: Einstein Eqs.  $T$

Lecture 4: Best Metrics

$\rightarrow T_{\mu\nu}$   
sym.  $\rightarrow$

Lecture 1: Diff. Geom.  $\rightarrow$  Manifold  
 $\rightarrow$  Connection  
 $\rightarrow$  metric

Lecture 2:  $R \rightarrow F$   
 $\int \frac{R}{16\pi G} \sqrt{-g}$   $\rightarrow$   $\int \text{Tr}[F^2] \sqrt{-g}$

Lecture 3: Einstein Eqs.  $T_{\mu\nu}, \lambda_{\text{scalar}} \rightarrow T_{\mu\nu}$

Lecture 4: Best Metrics  $\rightarrow$  maximally sym.  $\rightarrow R_{abcd} = (g_{ac}g_{bd} - g_{ad}g_{bc})$   
 $\rightarrow$  FRW  $\begin{cases} K=+1 \\ K=0 \\ K=-1 \end{cases}$   $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $\rho = -3H(p+p)$   
BH metrics

Lecture 5: Kinematics of FRW:  $x^1, x^2, x^3$

$$(g_{ac}g_{bd} - g_{ad}g_{bc})$$
$$\dot{\rho} = -3H(\rho + p)$$

Lecture 5: Kinematics of FRW:  $x^1, x^2, x^3 \rightarrow r, \theta, \varphi$   $x, \theta, \varphi$   $r = S_K(x)$   
comoving observers,

$$(g_{ac}g_{bd} - g_{ad}g_{bc})$$
$$\dot{\rho} = -3H(\rho + p)$$

Lecture 5: Kinematics of FRW:  $x^1, x^2, x^3 \rightarrow \underline{r}, \theta, \varphi$   $\chi, \theta, \varphi$   $r = S_K(\chi)$   
comoving observer / comoving distance / physical dist

$$(g_{ac}g_{bd} - g_{ad}g_{bc})$$
$$\dot{\rho} = -3H(\rho + p)$$

Lecture 5: Kinematics of FRW:  $x^1, x^2, x^3 \rightarrow \underline{r}, \theta, \varphi$   $\underline{x}, \theta, \varphi$   $r = S_K(\chi)$   $t, \eta$   $d\eta = \frac{dt}{a(t)}$   
 comoving observers, (comoving distance)  $a(t)$  - physical dist  
 $p = |\vec{p}| \propto \frac{1}{a} \rightarrow$  massless

$g_{\alpha\beta}(t)$



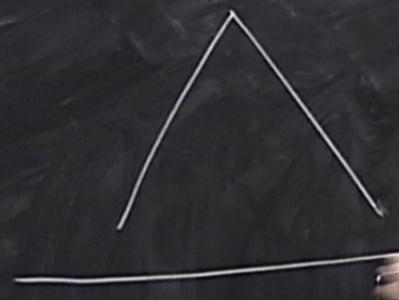






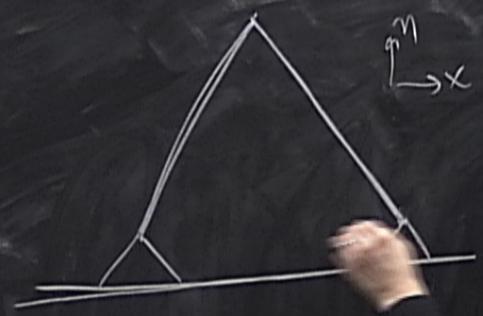
# Lecture 5: Kinematics of FRW:

$x^1, x^2, x^3 \Rightarrow \underline{r}, \theta, \varphi$      $\underline{x}, \theta, \varphi$      $r = S_K(\underline{x})$      $t, \eta$      $d\eta = \frac{dt}{a(t)}$   
 comoving     $\rightarrow$  massless     $\omega \propto \frac{1}{a}$   
 $p = |$     massive     $p \sim mv$      $v \propto \frac{1}{a}$   
 horizon     $a > 0$     peculiar velocity  
                    $a < 0$

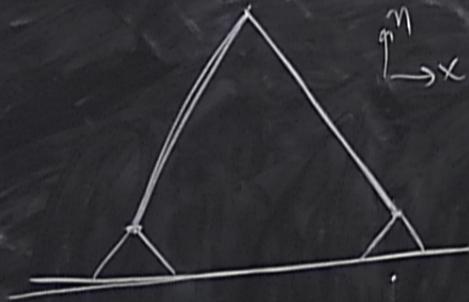


# Lecture 5: Kinematics of FRW:

$x^1, x^2, x^3 \rightarrow r, \theta, \varphi$      $\underline{x}, \theta, \varphi$      $r = S_K(\underline{x})$      $t, \eta$      $d\eta = \frac{dt}{a(t)}$   
 comoving observers, (comoving distance)  $a(t) = \text{physical dist}$   
 $\frac{1}{a} \rightarrow$  massless     $\omega \propto \frac{1}{a}$   
 $\rightarrow$  massive     $p \sim mv$      $v \propto \frac{1}{a}$   
 $\dot{a} > 0, \ddot{a} > 0$     peculiar velocity  
 $\dot{a} > 0, \ddot{a} < 0$

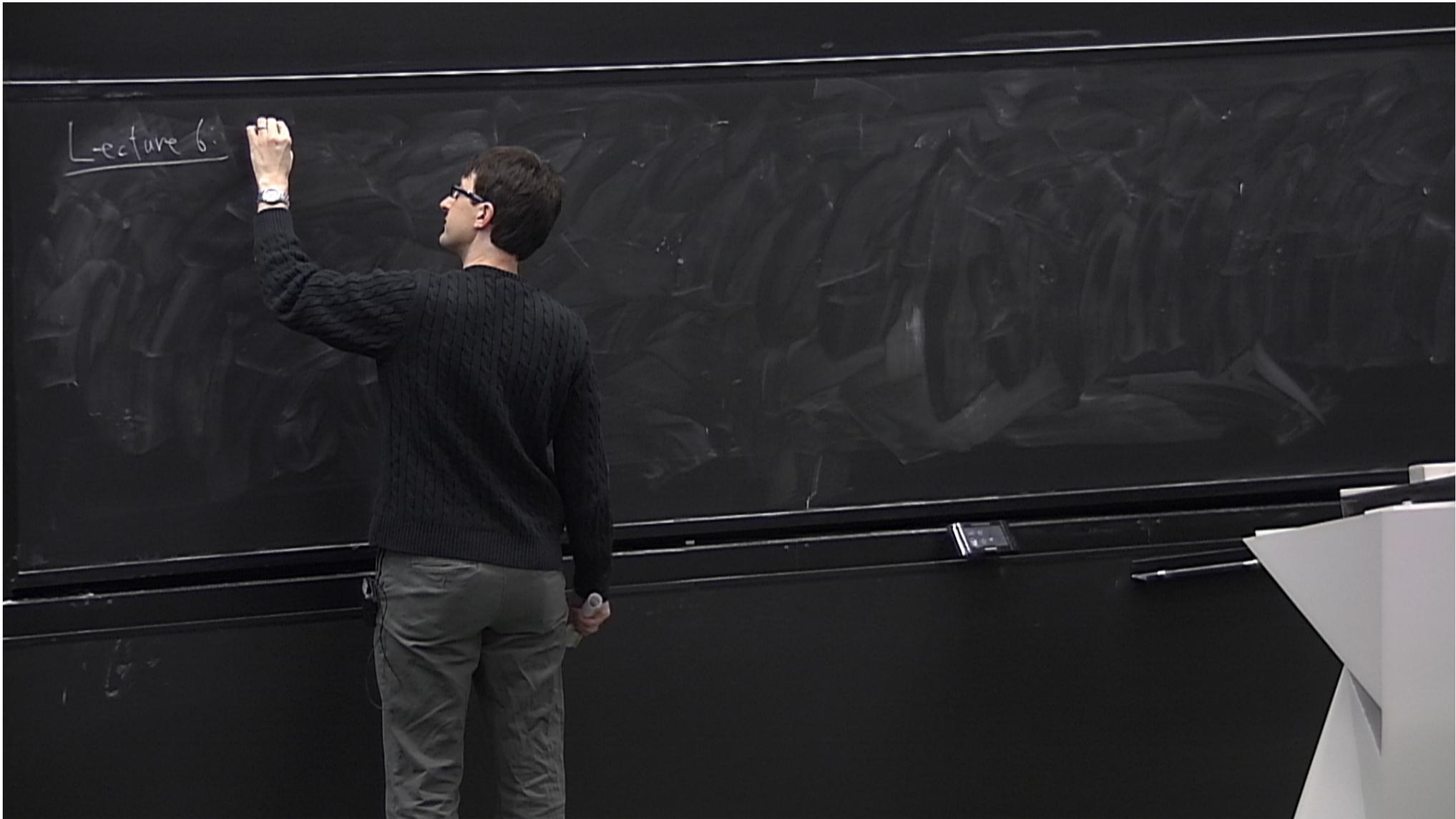


# Lecture 5: Kinematics of FRW:

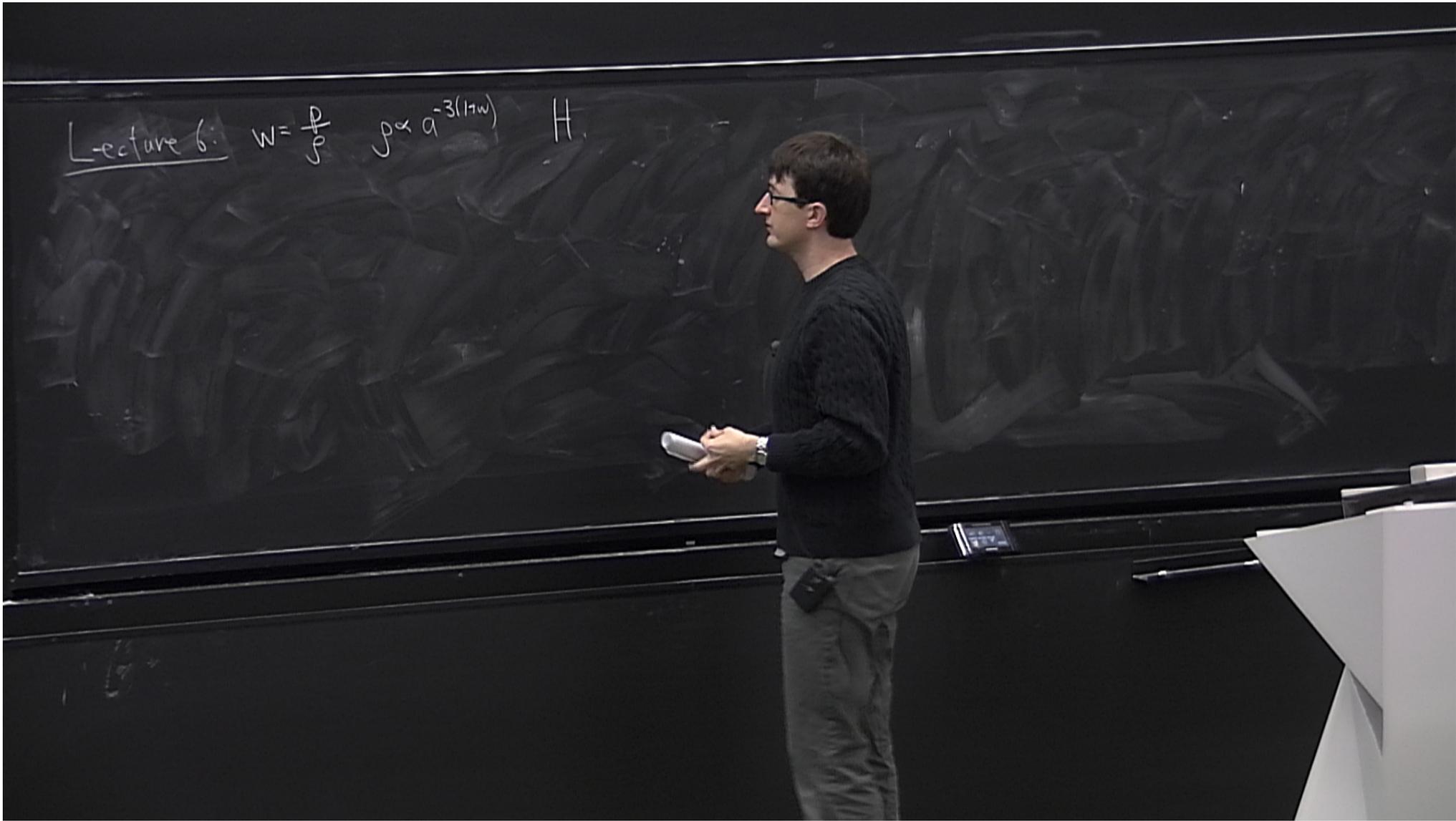


$x^1, x^2, x^3 \rightarrow r, \theta, \varphi$      $\underline{x}, \theta, \varphi$      $r = S_K(\underline{x})$      $t, \eta$      $d\eta = \frac{dt}{a(t)}$   
 comoving observers, (comoving distance)  $a(t) = \text{physical dist}$   
 $p = |\vec{p}| \propto \frac{1}{a} \begin{cases} \rightarrow \text{massless } \omega \propto \frac{1}{a} \\ \rightarrow \text{massive } p \sim mv \end{cases}$   
 horizons  $\begin{cases} \rightarrow \dot{a} > 0, \ddot{a} > 0 \\ \rightarrow \dot{a} > 0, \ddot{a} < 0 \end{cases}$





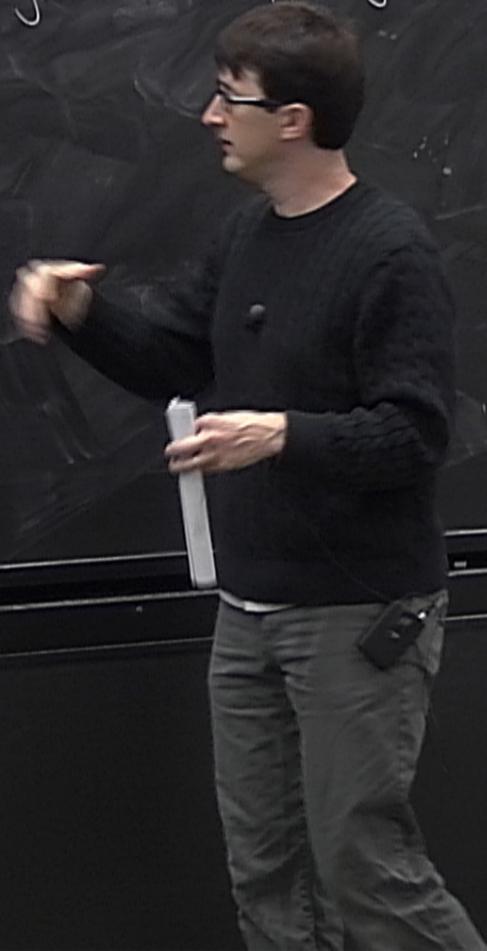
Lecture 6:  $w = \frac{p}{f}$   $\rho \propto a^{-3(1+w)}$  H.



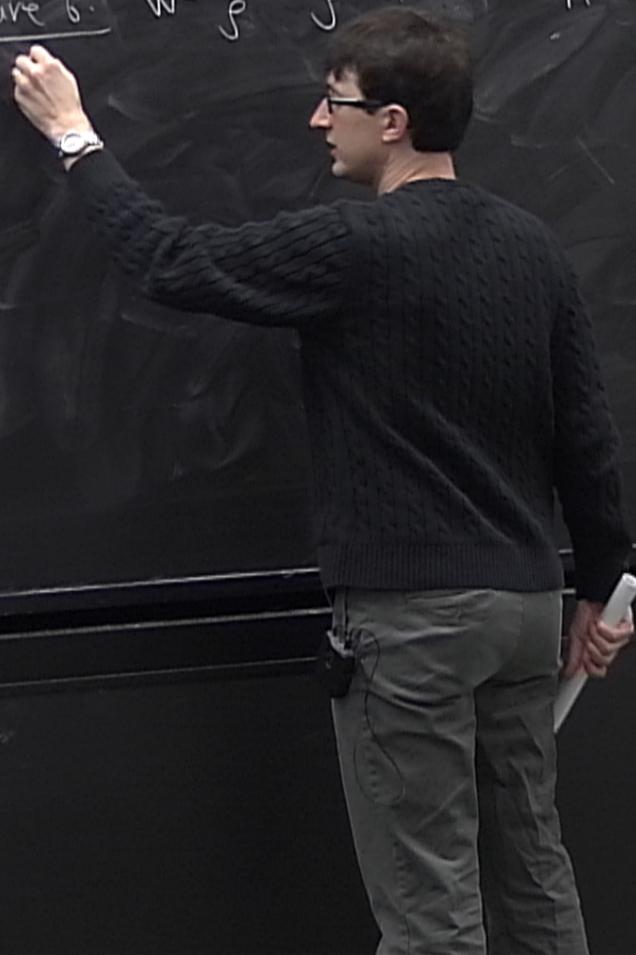
Lecture 6:  $w = \frac{p}{\rho}$   $\rho \propto a^{-3(1+w)}$   $H^2 = \frac{8\pi G}{3} \rho = \frac{K}{a^2}$   $a(\eta) \sim \int \frac{d\eta}{a}$   $\alpha = \frac{2}{1+3w}$



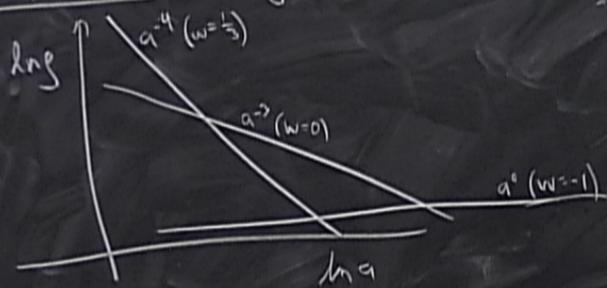
Lecture 6:  $w = \frac{p}{\rho}$   $\rho \propto a^{-3(1+w)}$   $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \sim \int_K^{1/a} \frac{1}{\sqrt{K}} d\eta$   $\alpha = \frac{2}{1+3w}$



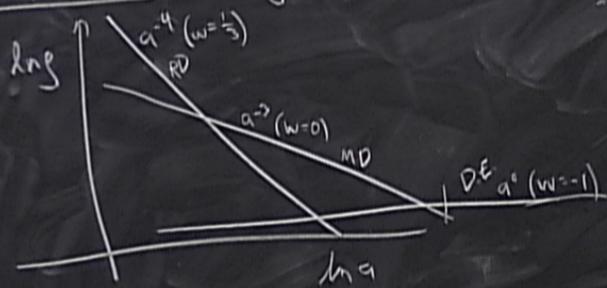
Lecture 6:  $w = \frac{p}{\rho}$   $\rho \propto a^{-3(1+w)}$   $H^2 = \frac{8\pi G}{3} \rho = \frac{k}{a^2}$   $a(\eta) \propto S_K(\eta/\alpha)$   $\alpha = \frac{2}{1+3w}$



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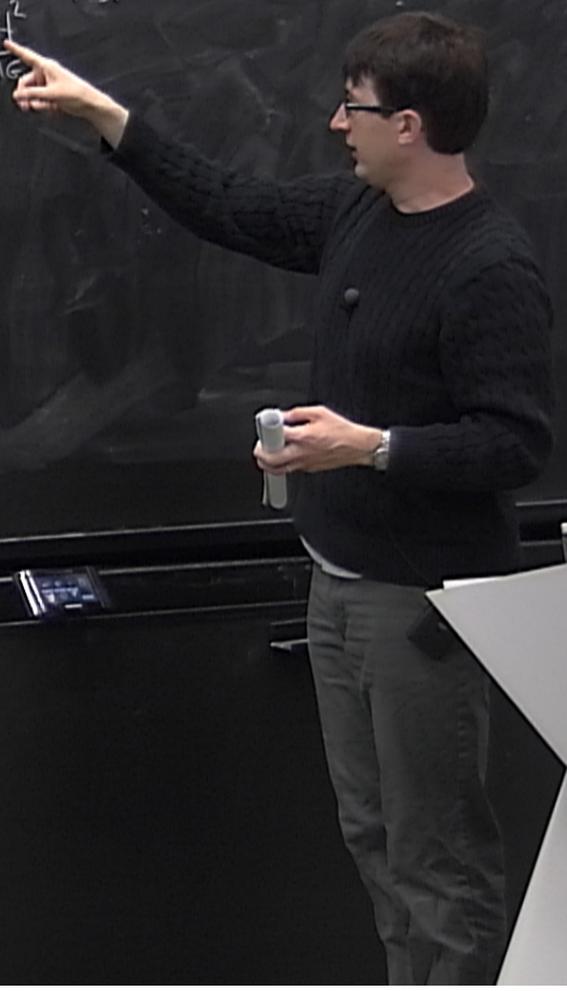
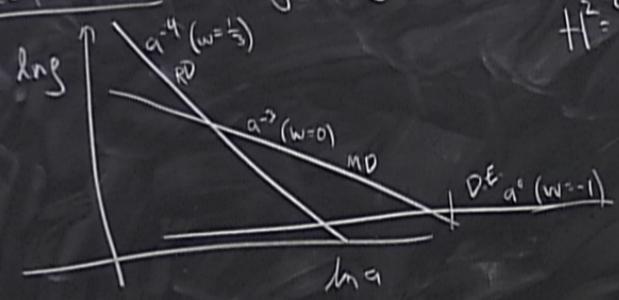


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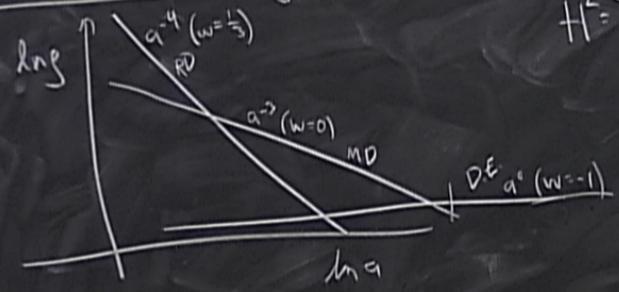
$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{k}{a^2 H^2}$   $\rho_{crit} = \frac{3H^2}{8\pi G}$



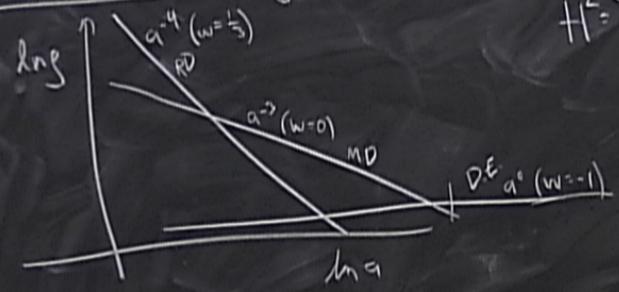
Lecture 6:  $w = \frac{p}{f}$   $p \propto a^{-3(1+w)}$   $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \sim \int_K^\eta (1/a)$   $\alpha = \frac{2}{1+3w}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{f}{f_{crit}} - \frac{K}{a^2 H^2}$   $f_{crit} = \frac{3H^2}{8\pi G}$

- K=+1
- K=-1
- K=0



Lecture 6:  $w = \frac{p}{f}$   $p \propto a^{-3(1+w)}$   $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \sim \int \frac{d\eta}{K^{1/2}}$   $\alpha = \frac{2}{1+3w}$

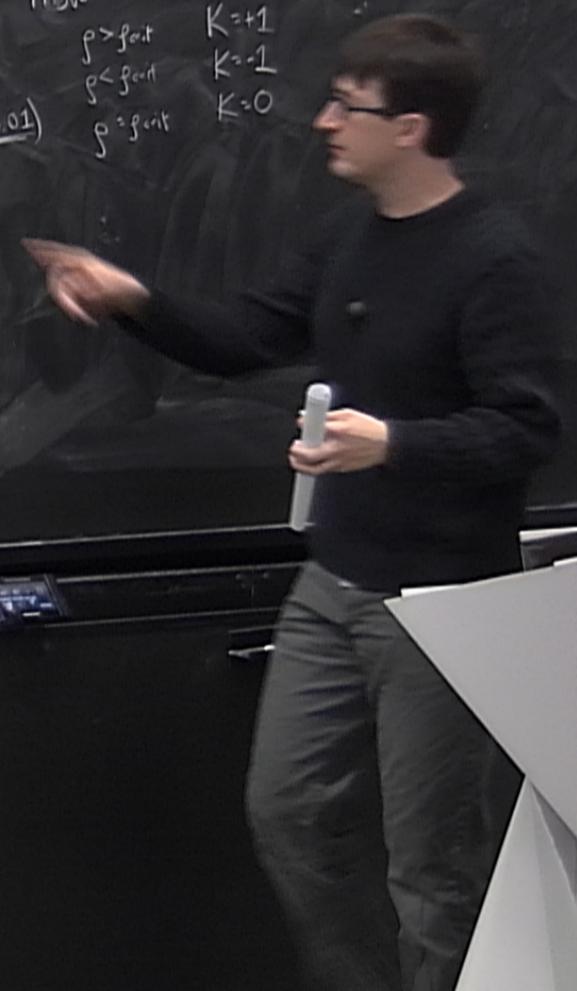


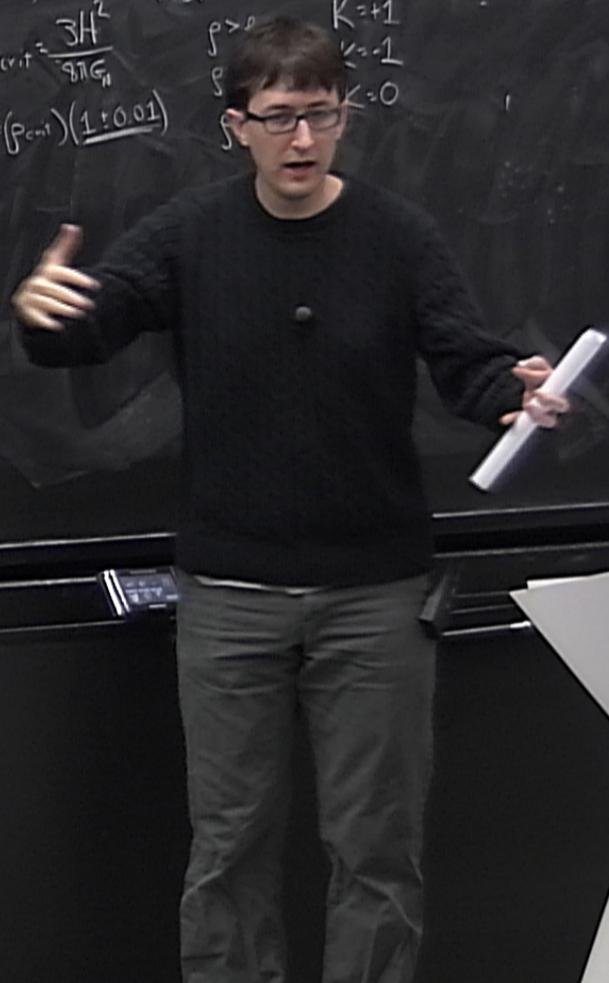
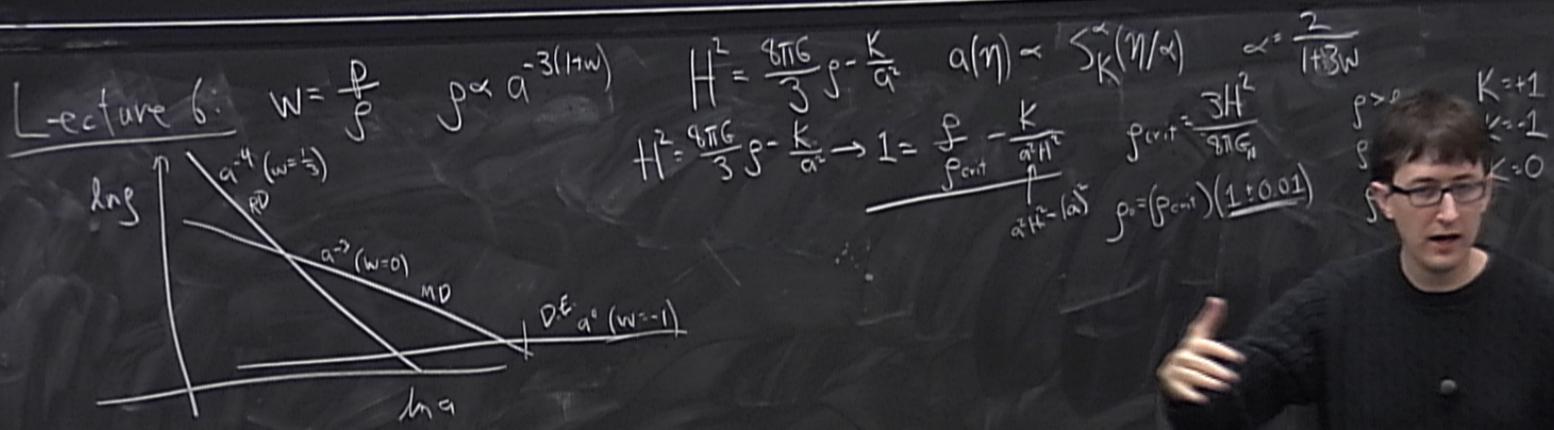
$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\rho = \rho_{crit} (1 \pm 0.01)$$

- $\rho > \rho_{crit}$   $K=+1$
- $\rho < \rho_{crit}$   $K=-1$
- $\rho = \rho_{crit}$   $K=0$





Lecture 6:

$$w = \frac{p}{f}$$

$$p \propto a^{-3(1+w)}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$$

$$a(\eta) \sim \int \frac{d\eta}{a^2}$$

$$\alpha = \frac{2}{1+3w}$$

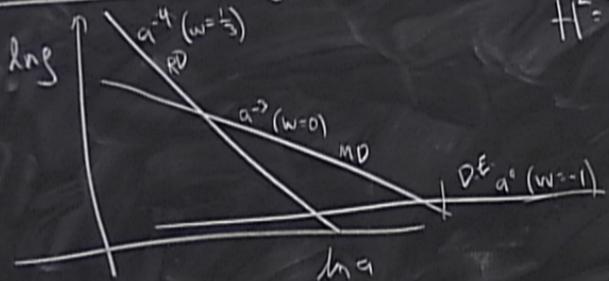
$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{f}{f_{crit}} - \frac{K}{a^2 H^2}$$

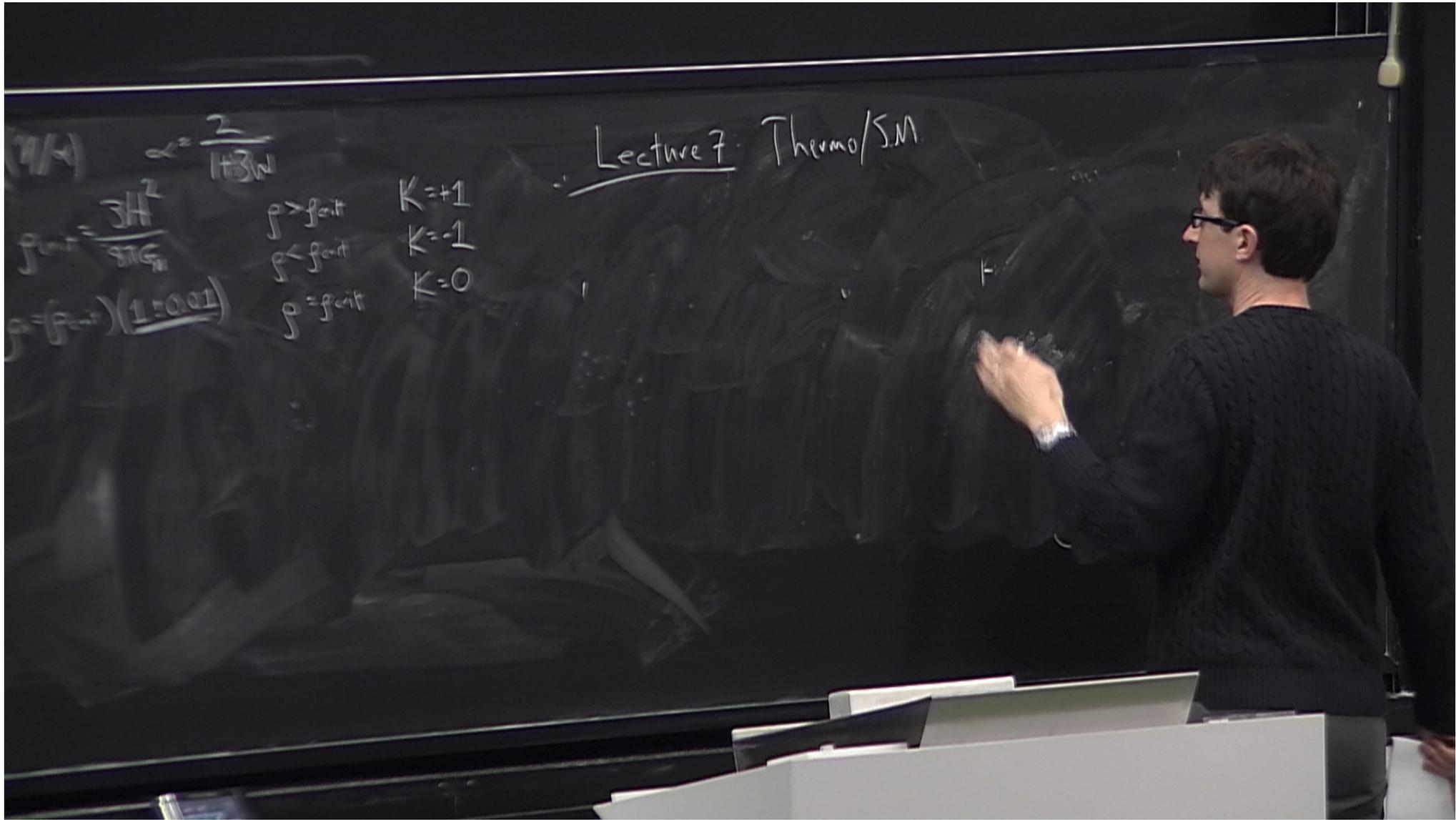
$$f_{crit} = \frac{3H^2}{8\pi G}$$

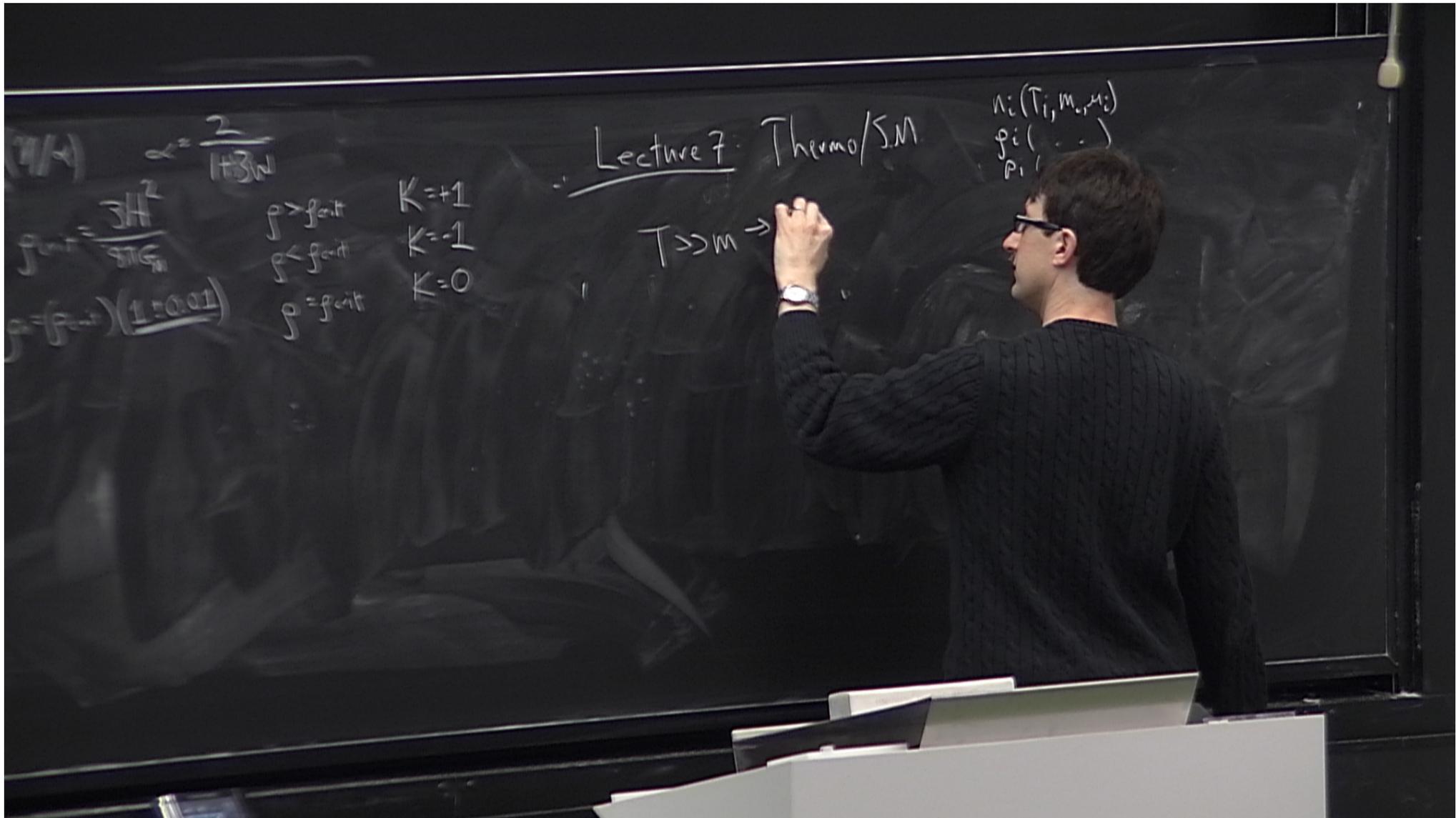
- $p > p_{crit} \quad K = +1$
- $p < p_{crit} \quad K = -1$
- $p = p_{crit} \quad K = 0$

$$f_0 = (f_{crit}) (1 \pm 0.01)$$

Lecture 7







$$\alpha = \frac{2}{1+3N}$$

$$f_{\text{rot}} = \frac{3H^2}{8\pi G}$$

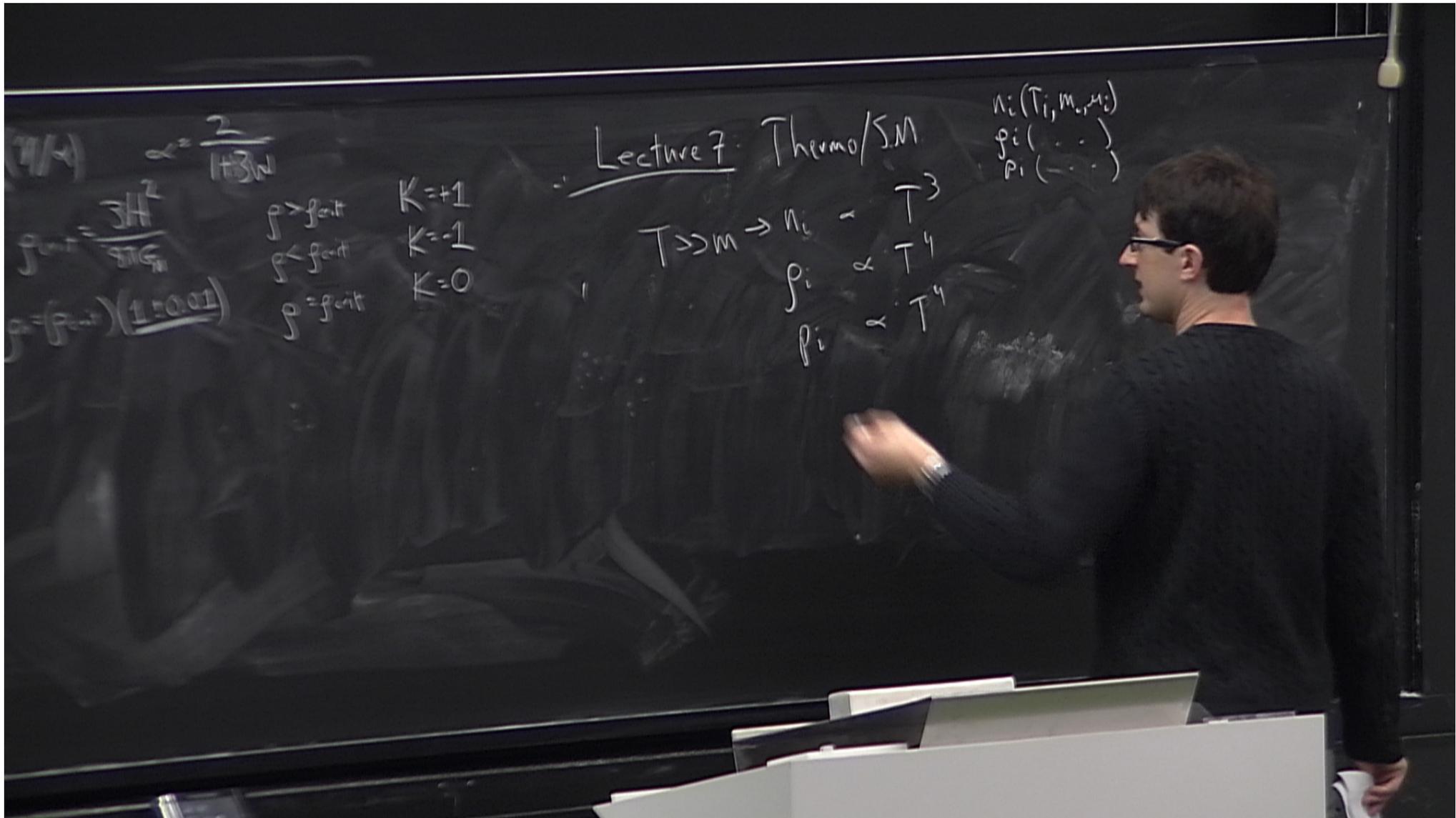
$$f = f_{\text{rot}} (1 \pm 0.01)$$

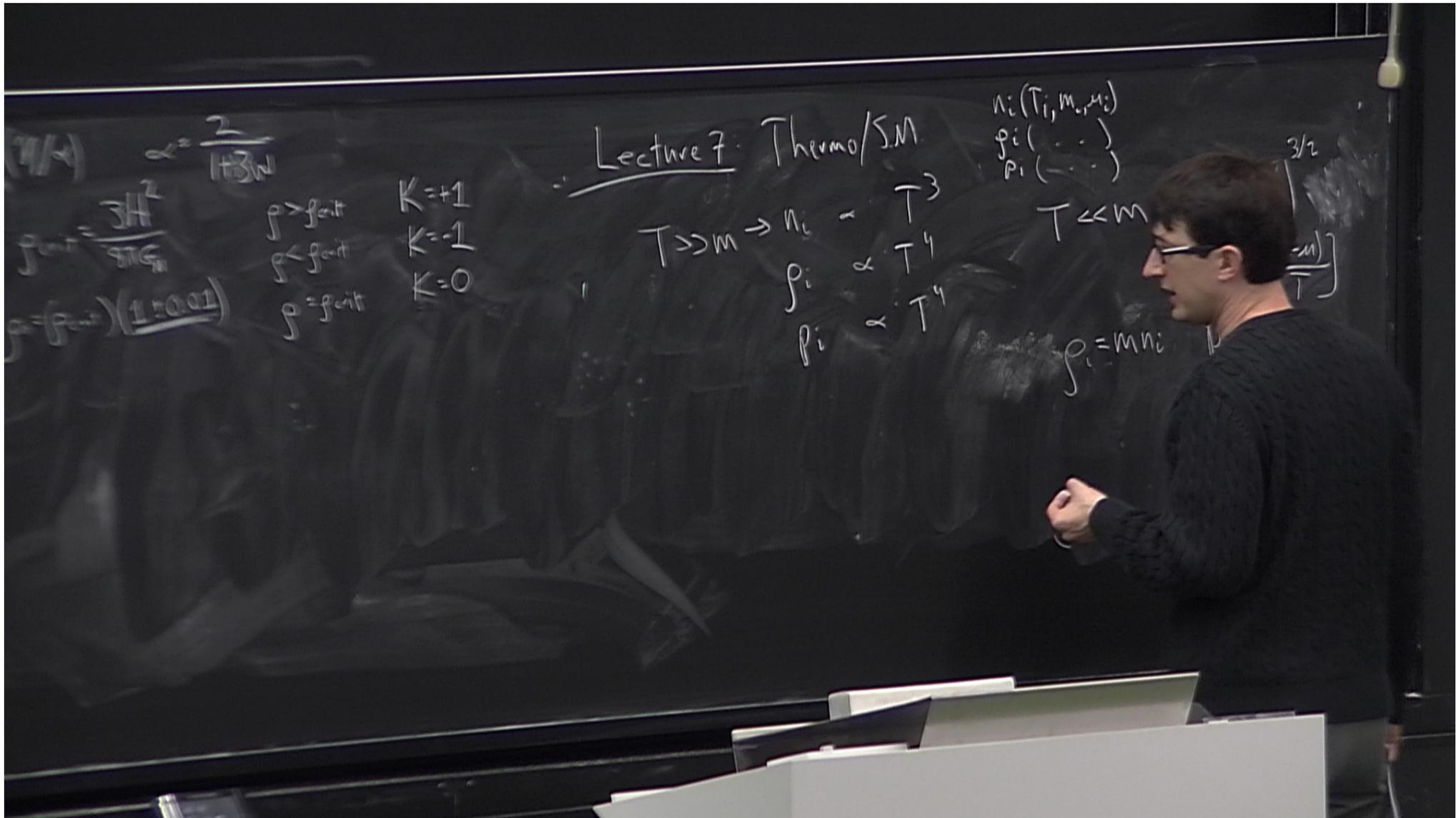
$p > f_{\text{rot}}$   $K = +1$   
 $p < f_{\text{rot}}$   $K = -1$   
 $p = f_{\text{rot}}$   $K = 0$

## Lecture 7: Thermo/S.M

$T > M \rightarrow$

$n_i(T_i, m_i, \mu_i)$   
 $p_i(\dots)$   
 $\rho_i(\dots)$





Lecture 7. Thermo/S.M

$\lambda = \frac{2}{\sqrt{3N}}$   
 $f_{\text{part}} = \frac{3H^2}{8\pi G_m}$   
 $f = f_{\text{part}} (1 \pm 0.01)$

$\rho > f_{\text{crit}} \quad K = +1$   
 $\rho < f_{\text{crit}} \quad K = -1$   
 $\rho = f_{\text{crit}} \quad K = 0$

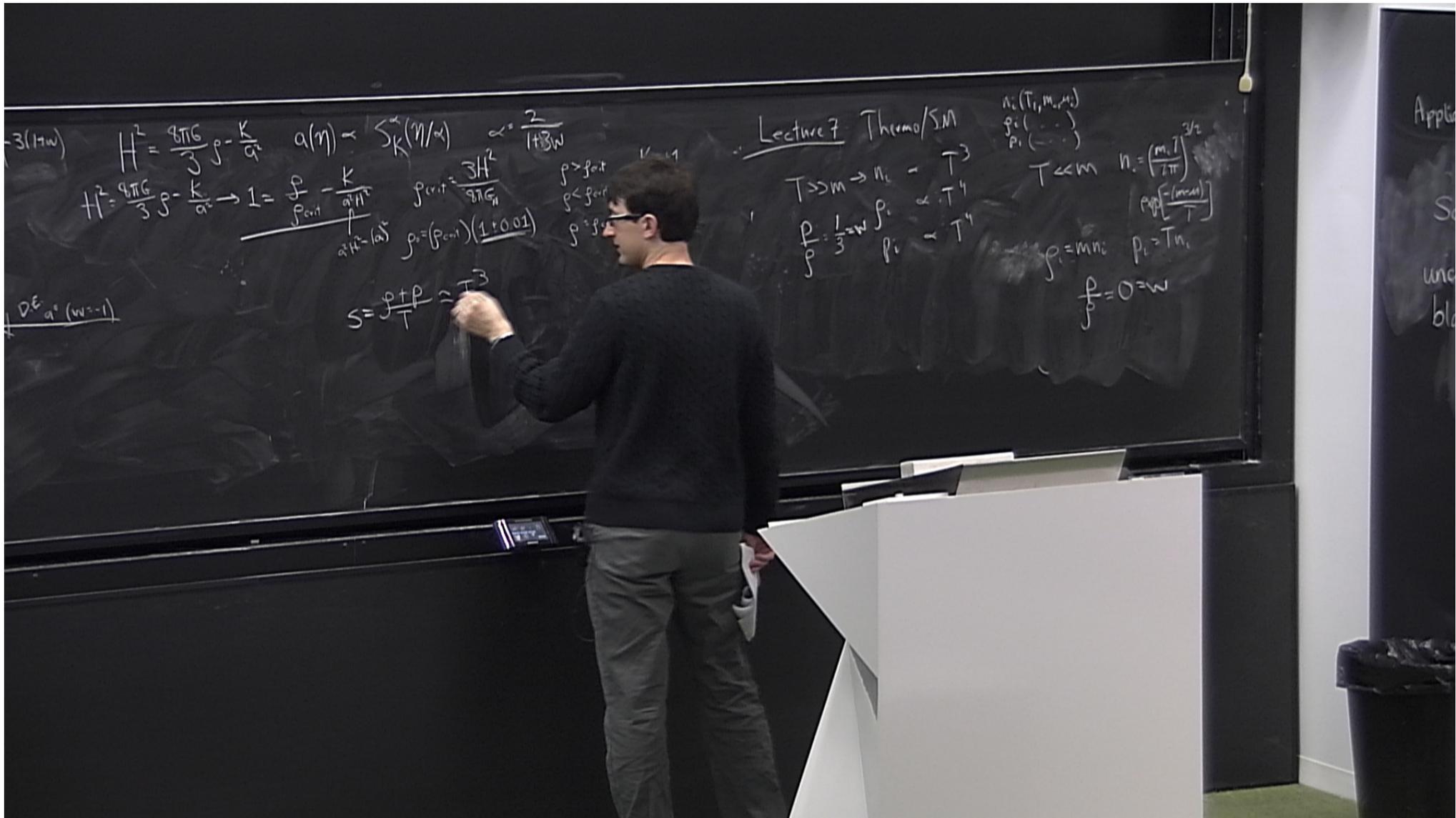
$T \gg m \rightarrow n_i \propto T^3$   
 $\rho_i \propto T^4$   
 $p_i \propto T^4$

$n_i(T_i, m_i, \mu_i)$   
 $p_i(\dots)$   
 $\rho_i(\dots)$

$T \ll m$

$\rho_i = m n_i$

$\frac{3}{2}$   
 $\frac{-m}{T}$



$$H^2 = \frac{9\pi G}{3} \rho - \frac{K}{a^2}$$

$$a(\eta) \sim S_K(\eta/\alpha) \sim \frac{2}{1+3w}$$

$$H^2 = \frac{9\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\rho = \rho_{crit} (1+0.01)$$

$$S = \frac{\rho + p}{T} \sim T^{-3}$$

Lecture 7 Thermo/SM

$$T \gg m \rightarrow n_i \sim T^3$$

$$\frac{\rho}{\rho} = \frac{1}{3} = w \quad \rho_i \propto T^4$$

$$p_i \propto T^4$$

$$n_i(T_i, m_i, g_i)$$

$$g_i(\dots)$$

$$p_i(\dots)$$

$$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i c^2)}{T}\right]$$

$$g_i = m_i g_i \quad p_i = T n_i$$

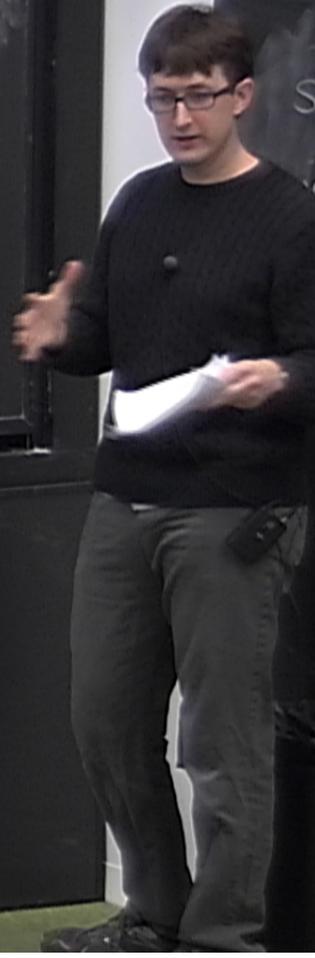
$$\frac{\rho}{\rho} = 0 = w$$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$      $a(\eta) \sim \int \frac{d\eta}{\sqrt{2K - 8\pi G \rho a^4}}$      $\propto \frac{2}{1+3w}$   
 $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$      $\rho_{crit} = \frac{3H^2}{8\pi G}$      $\rho > \rho_{crit} \quad K = +1$   
 $\rho < \rho_{crit} \quad K = -1$   
 $\rho = \rho_{crit} \quad K = 0$   
 $\rho = \rho_{crit} (1 + 0.01)$      $\rho = \rho_{crit}$   
 $S = \frac{\rho + p}{T} = T^3 \propto a^{-3} \Rightarrow T \propto \frac{1}{a}$

Lecture 7 Thermo/SM

$T \gg m \rightarrow n_i \sim T^3$   
 $\frac{\rho}{\rho} = \frac{1}{3} = w \quad \rho_i \propto T^4$   
 $\rho_i \propto T^4$

$n_i(T, m_i, g_i)$   
 $\rho_i(\dots)$   
 $p_i(\dots)$   
 $T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i T)}{T}\right]$   
 $\rho_i = m_i n_i \quad p_i = T n_i$   
 $\frac{\rho}{\rho} = 0 = w$



-3(1+w)

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \quad a(\eta) \sim S_K(\eta/\alpha) \sim \frac{2}{1+3w}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2} \quad \rho > \rho_{crit} \quad K = +1$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \rho < \rho_{crit} \quad K = -1$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \rho = \rho_{crit} \quad K = 0$$

$$\frac{dE}{dt} \sim a^w (w-1) \quad \rho = \rho_{crit} (1+0.01) \quad \rho = \rho_{crit}$$

$$S = \frac{\rho + p}{T} \approx T^3 \sim a^{-3} \Rightarrow T \sim \frac{1}{a}$$

### Lecture 7 Thermo/SM

$$T \gg m \rightarrow n_i \sim T^3$$

$$\frac{p}{\rho} = \frac{1}{3} = w \quad \rho_i \propto T^4$$

$$\quad \quad \quad \quad \quad \rho_i \propto T^4$$

$$n_i(T_i, m_i, g_i)$$

$$g_i(\dots)$$

$$p_i(\dots)$$

$$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i + p_i)}{T}\right]$$

$$g_i = m_i \quad p_i = T n_i$$

$$\frac{p}{\rho} = 0 = w$$

$$\frac{dE}{dt} \sim a^w (w-1)$$

$$\Gamma = H$$

BBN  $\rightarrow$  He 25%, H = 75%  
MeV n, p  $\rightarrow$   $^4\text{He}$

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CMB  $\rightarrow$   $e^- + p^+ \leftrightarrow H + \gamma$

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CMB  $\rightarrow$   $e^- + p^+ \leftrightarrow H + \gamma$     few  $\times 10^2 \times 10^3$  yrs

Gamow: DNA triplet code.

BBN  $\rightarrow$  He 25%, H = 75%  
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CMB  $\rightarrow$   $e^- + p^+$   $\leftarrow$   $\gamma$       few  $\times 10^2 \times 10^3$  yrs

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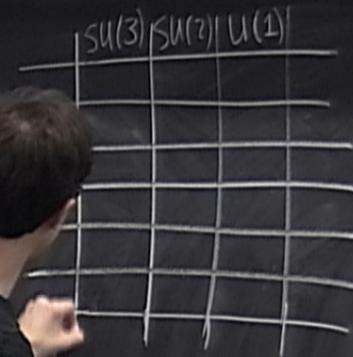
CMB  $\rightarrow e^- + p^+ \leftrightarrow H + \gamma$   $\text{few} \times 10^2 \times 10^3 \nu$

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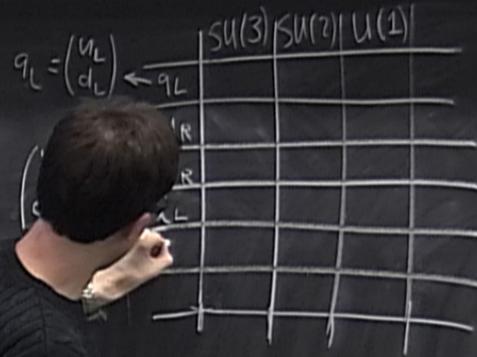
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 MeV  $n, p \rightarrow$  He

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Garrow: DNA triplet code.

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

	SU(3)	SU(2)	U(1)
$q_L$	3		
$u_R$	3		
$d_R$	3		
$l_L$	1		
$\nu_R$	1		
$e_R$	1		

BBN  $\rightarrow$  He 25%, H = 75%  
 MeV n, p  $\rightarrow$   ${}^4\text{He}$

CMB  $\rightarrow e^- + p^+ \leftrightarrow H + \gamma$      $\text{few} \times 10^2 \times 10^3$

Garrow: DNA triplet code.

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$l_L$	1	2	$-\frac{1}{2}$
$\nu_R$	1	1	0
$e_R$	1	1	-1

BBN  $\rightarrow$  He 25%, H = 75%  
 MeV n, p  $\rightarrow$   ${}^4\text{He}$

CMB  $\rightarrow e^- + p^+ \leftrightarrow H + \gamma$     few  $\times 10^2 \times 10^3$  yrs  
 Garrow: DNA triplet code.

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$   $\leftarrow q_L$

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$\bar{q}_L$	$\bar{3}$	1	$+\frac{2}{3}$
$\ell_L$	3	1	$-\frac{1}{3}$
$\bar{\ell}_L$	$\bar{1}$	2	$-\frac{1}{2}$
$\bar{q}_R$	1	1	0
$\ell_R$	1	1	-1

$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \leftarrow \ell_L$

BBN  $\rightarrow$  He 25%, H = 75%  
 MeV n, p  $\rightarrow$   ${}^4\text{He}$

CMB  $\rightarrow e^- + p^+ \leftrightarrow H + \gamma$      $\text{few} \times 10^2 \times 10^{-}$   
 Garrow: DNA triplet code.

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$   $\leftarrow$   $q_L$

	SU(3)	SU(2)	U(1)
$u_L$	3	2	$+\frac{1}{6}$
$d_L$	3	2	$-\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$\nu_L$	3	2	$-\frac{1}{2}$
$e_L$	3	1	0
$\nu_R$	3	1	-1
$h$	1	2	$+\frac{1}{2}$

$\leftarrow$   $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

BBN  $\rightarrow$  He 25%, H = 75%  
 MeV n, p  $\rightarrow$   ${}^4\text{He}$

CMB  $\rightarrow e^- + p^+ \leftrightarrow H + \gamma$     few  $\times 10^2 \times 10^3$  yrs  
 Garrow: DNA triplet code.

$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$   $\leftarrow$

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ $\leftarrow$	1	2	$-\frac{1}{2}$
$\nu_R$	1	1	0
$e_R$	1	1	
$h$	1	2	

Lorentz  
 Gauge  
 Renormalizable

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$\ell_L$	1	2	$-\frac{1}{2}$
$\nu_R$	1	1	0
$e_R$	1	1	-1
$h$	1	2	$+\frac{1}{2}$

Lorentz  
Gauge  
Renormalizable

$$\bar{q}_L^i h_{ij} \nu_R^j$$

super - iso

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$\ell_L$	1	2	$-\frac{1}{2}$
$\nu_R$	1	1	0
$e_R$	1	1	-1
$h$	1	2	$+\frac{1}{2}$

Lorentz  
Gauge  
Renormalizable

$$\bar{q}_L^i h^j U_R^k \Gamma_{ij}$$

20-30  
↑

super - 150

	SU(3)	SU(2)	U(1)
$q_L$	3	2	$+\frac{1}{6}$
$u_R$	3	1	$+\frac{2}{3}$
$d_R$	3	1	$-\frac{1}{3}$
$\nu_L$	1	2	$-\frac{1}{2}$
$\nu_R$	1	1	0
$e_R$	1	1	-1
$h$	1	2	$+\frac{1}{2}$

Lorentz  
Gauge  
Renormalizable

$$\bar{q}_L^i h U_R^j \Gamma_{ij}$$

20-30  
↑

2000

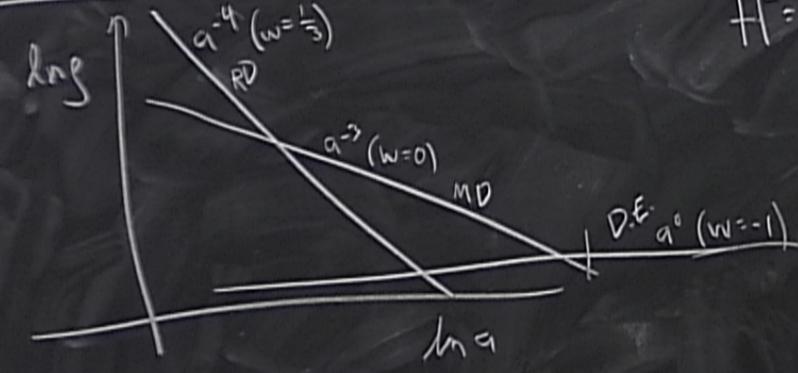
Lecture 6:

$W = \frac{p}{p}$      $p \propto a^{-3(1+w)}$

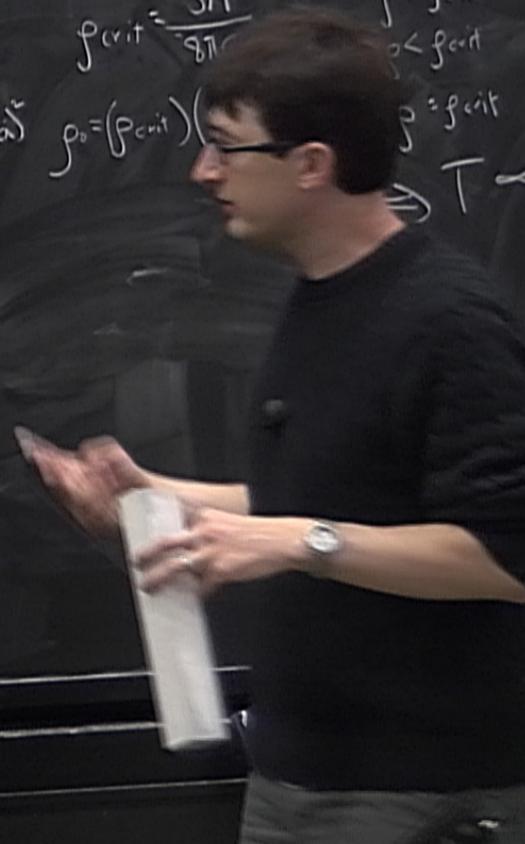
$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$      $a(\eta) \propto \int \frac{d\eta}{K(\eta/a)}$      $\alpha = \frac{2}{1+3W}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$

$\rho_{crit} = \frac{3H^2}{8\pi G}$   
 $\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit}$



$H_0 \rightarrow \rho_{crit} = \frac{3H_0}{8\pi G}$



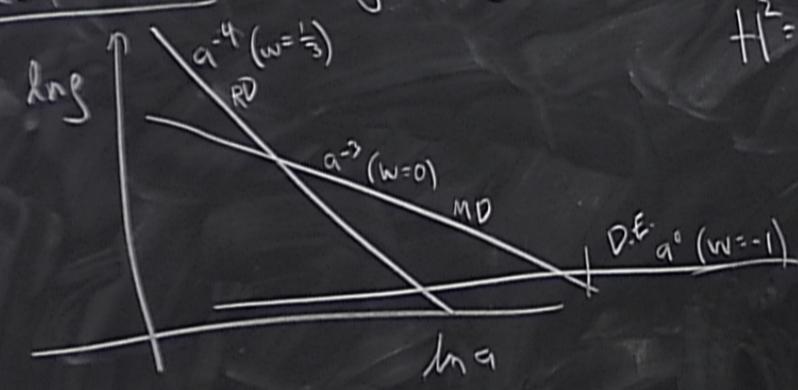
Lecture 6.

$w = \frac{p}{p}$      $p \propto a^{-3(1+w)}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$      $a(\eta) \propto \int \frac{d\eta}{K^{1/2}}$      $\alpha = \frac{2}{1+3w}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{p}{\rho} - \frac{K}{a^2 H^2}$      $p_{crit} = \frac{3H^2}{8\pi G}$      $p > p_{crit}$   
 $p < p_{crit}$   
 $p = p_{crit}$

$p_0 = (p_{crit})(1 \pm 0.01)$      $-3 \Rightarrow T \leftarrow$

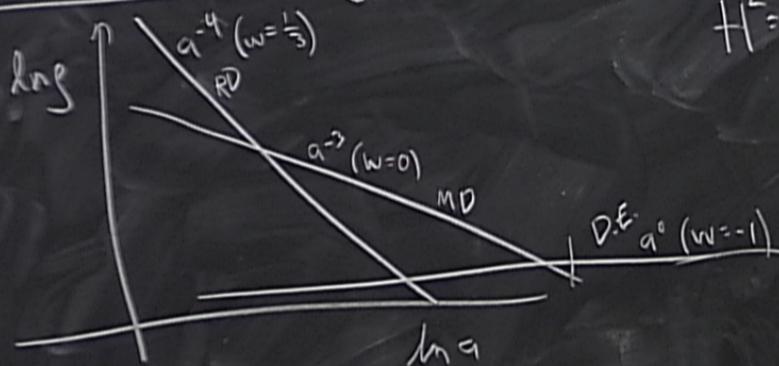


$H_0$   $\rightarrow p_{crit} = \frac{3}{8\pi G}$

Lecture 6.

$$W = \frac{p}{p}$$

$$p \propto a^{-3(1+w)}$$



w=0

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \quad a(\eta) \propto \int \frac{d\eta}{K^{1/2}}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G_N}$$

$$\rho_0 = (\rho_{crit})(1 \pm 0.01)$$

$$\alpha = \frac{2}{1+3w}$$

$p > p_{crit}$   
 $p < p_{crit}$   
 $p = p_{crit}$

$-3 \Rightarrow T$

$$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G_N}$$

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$$

$$\Omega_{DM} =$$

Lecture 6.

$$W = \frac{p}{p}$$

$$p \propto a^{-3(1+w)}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$$

$$a(\eta) \propto \int \frac{d\eta}{K(\eta/a)} \quad \alpha = \frac{2}{1+3W}$$

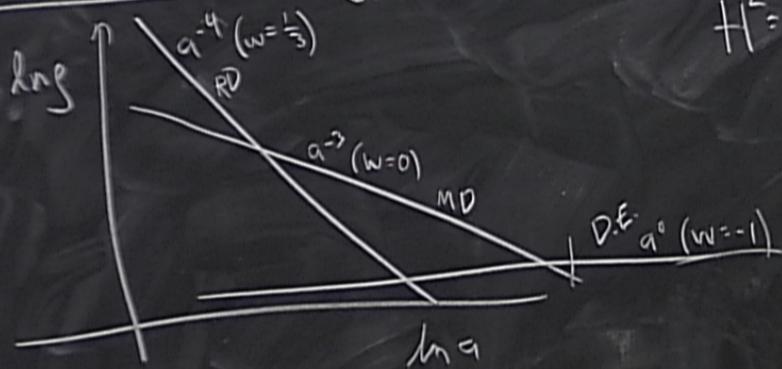
$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G_N}$$

$\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit}$

$$\rho_0 = (\rho_{crit})(1 \pm 0.01)$$

$-3 \Rightarrow T \leftarrow$



$w=0$

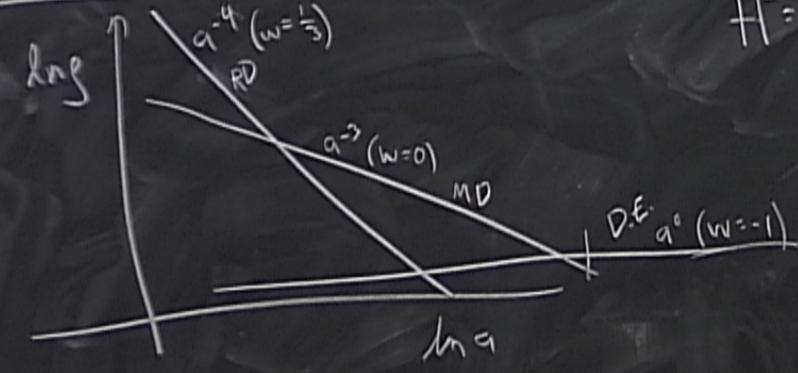
$$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G_N}$$

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$$

$$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$$

Lecture 6:

$W = \frac{p}{p}$      $p \propto a^{-3(1+w)}$



$w=0$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$      $a(\eta) \propto \int \frac{d\eta}{K(\eta/a)}$      $\alpha = \frac{2}{1+3w}$   
 $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$      $\rho_{crit} = \frac{3H^2}{8\pi G_N}$      $\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit} (1 \pm 0.01)$      $\rho = \rho_{crit}$   
 $-3 \Rightarrow T$

$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G_N}$

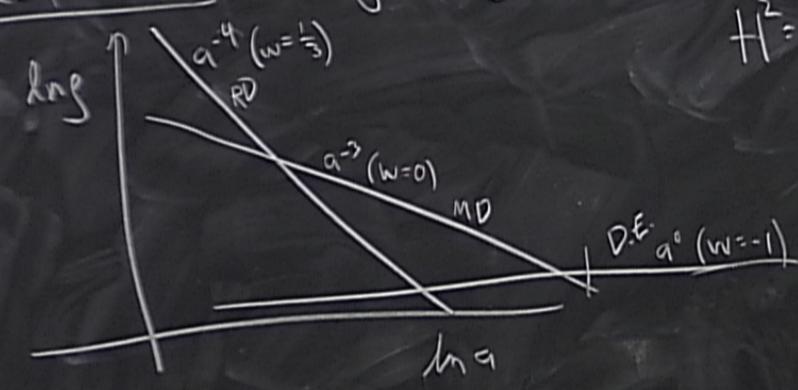
$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$

$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$

Lecture 6.

$W = \frac{p}{p}$   $p \propto a^{-3(1+w)}$



$w=0$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \propto \int \frac{d\eta}{K(\eta/a)}$   $\alpha = \frac{2}{1+3w}$   
 $H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$   $\rho_{crit} = \frac{3H^2}{8\pi G_N}$   $p > p_{crit}$   
 $\rho_0 = (\rho_{crit})(1 \pm 0.01)$   $p < p_{crit}$   
 $p = p_{crit}$   $-3 \Rightarrow T \leftarrow$

$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G_N}$

$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$

$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$

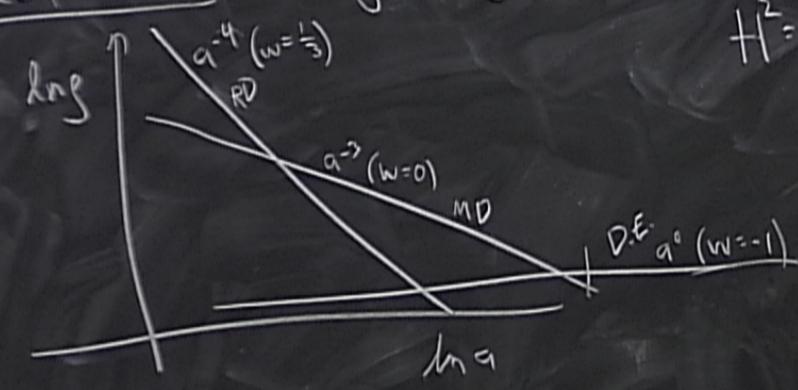
Lecture 6:

$w = \frac{p}{\rho}$      $\rho \propto a^{-3(1+w)}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$      $a(\eta) \propto \int \frac{d\eta}{\sqrt{K(\eta/a)}}$      $\alpha = \frac{2}{1+3w}$      $p > p_{crit}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$      $\rho_{crit} = \frac{3H^2}{8\pi G_N}$

$\rho = (\rho_{crit})(1 \pm 0.01)$



$H_0 \rightarrow \rho_{crit} = \frac{3H_0}{8\pi G_N}$

$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit} |_0}$

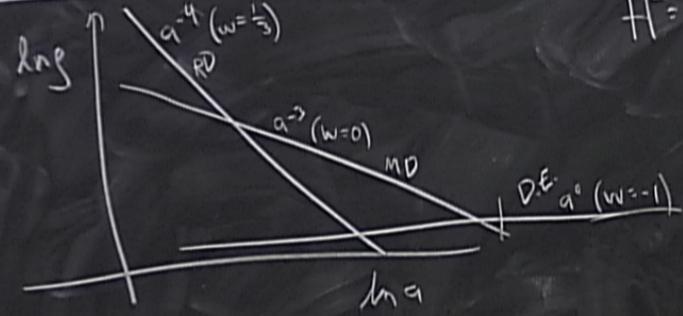
$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit} |_0}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit} |_0}$

$\Omega_{Baryons} = \frac{\rho_{baryon}}{\rho_{crit} |_0}$

Lecture 6:

$w = \frac{p}{\rho}$   $\rho \propto a^{-3(1+w)}$



$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \propto \int \frac{1}{\sqrt{K(\eta/a)}}$   $\alpha = \frac{2}{1+3w}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$   $\rho_{crit} = \frac{3H^2}{8\pi G}$

$\rho_0 = (\rho_{crit})(1 \pm 0.01)$

- $p > p_{crit}$   $K = +1$
- $p < p_{crit}$   $K = -1$
- $p = p_{crit}$   $K = 0$

$\Rightarrow T \propto \frac{1}{a}$

$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G}$

$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit}|_0}$

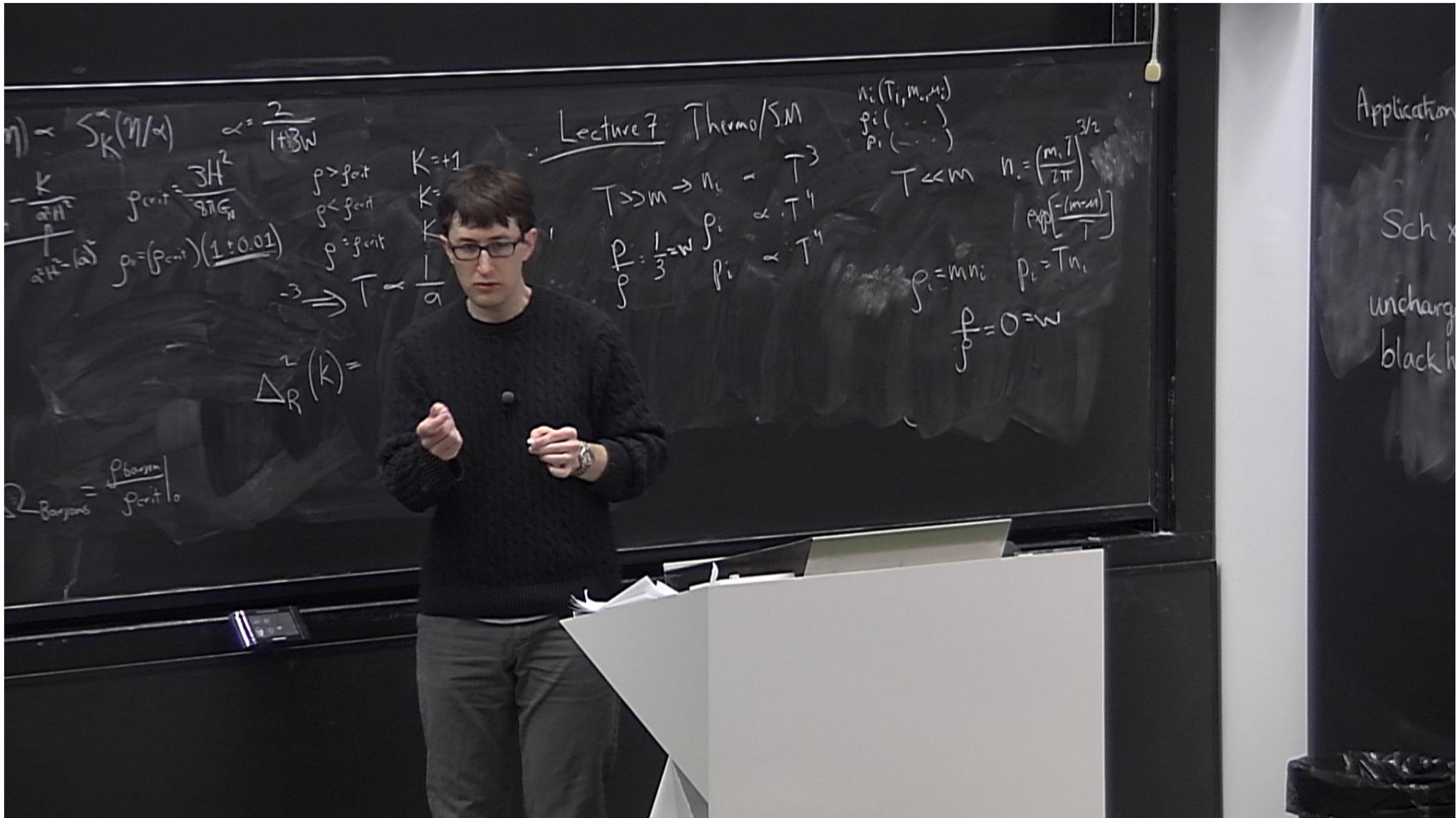
$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit}|_0}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit}|_0}$

$\Omega_{Baryons} = \frac{\rho_{baryons}}{\rho_{crit}|_0}$

$\Delta_R^2(K) =$

w=0



$$\eta \propto \sum_K (\eta/\alpha) \propto \frac{2}{1+3W}$$

$$p_{crit} = \frac{3H^2}{8\pi G_n}$$

$$p_0 = (p_{crit})(1 \pm 0.01)$$

$$\Rightarrow T \propto \frac{1}{a}$$

$$\Delta_R^2(k) =$$

$$\rho_{barions} = \frac{\rho_{barions}}{\rho_{crit} 10}$$

Lecture 7 Thermo/SM

$T \gg m \rightarrow n_i \propto T^3$

$\frac{p}{\rho} = \frac{1}{3} = w$

$p_i \propto T^4$

$\rho_i \propto T^4$

$n_i(T_i, m_i, \mu_i)$

$p_i(\dots)$

$\rho_i(\dots)$

$T \ll m$

$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{m_i}{T}}$

$p_i = T n_i$

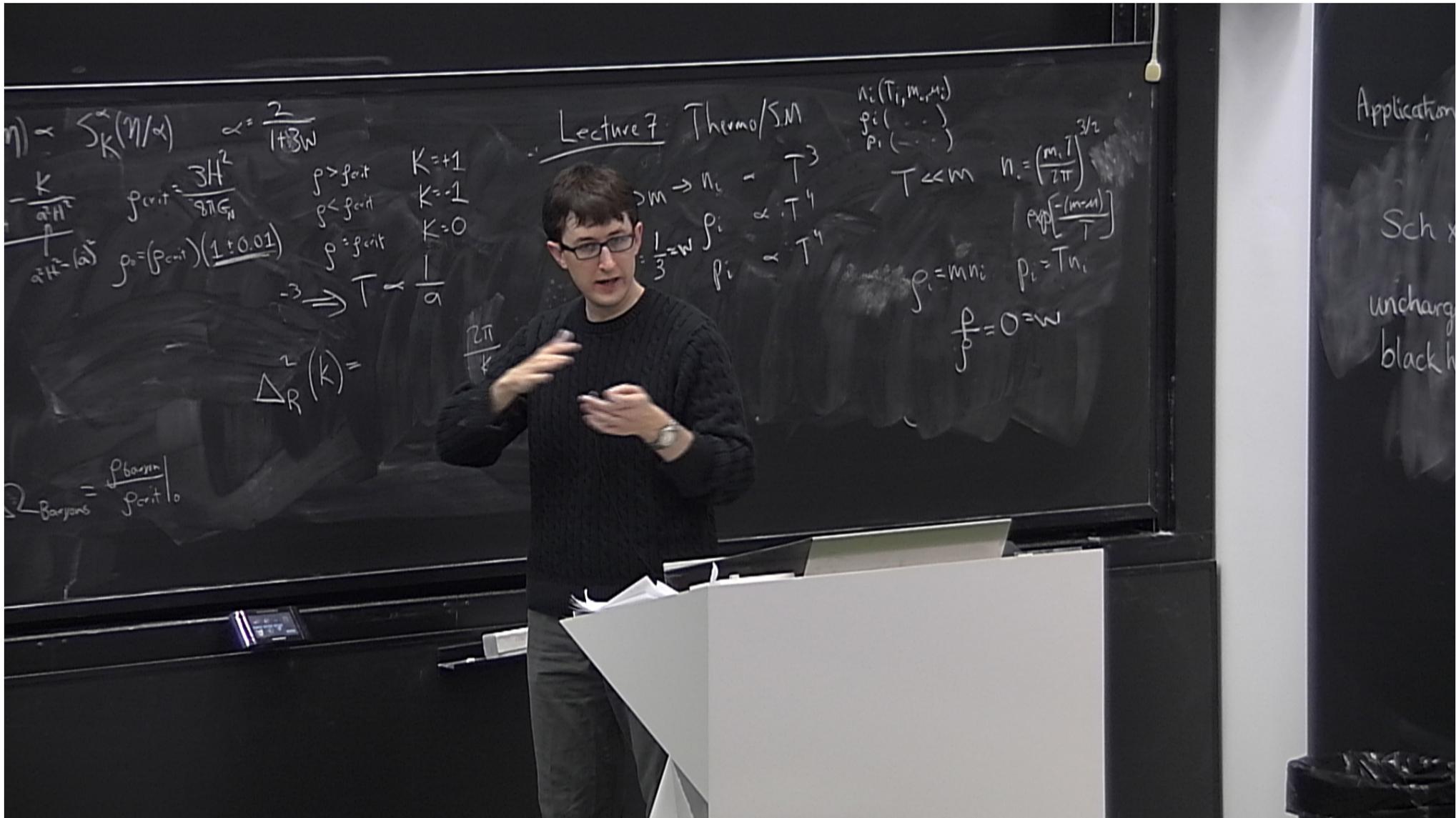
$\frac{p}{\rho} = 0 = w$

Application

Sch x

uncharg

black h



Lecture 7 Thermo/SM

$$\eta) \propto \sum_K (\eta/\alpha) \propto \frac{2}{1+3W}$$

$$\frac{k}{a^3 H^2} \quad p_{crit} = \frac{3H^2}{8\pi G_m}$$

$$\rho = (\rho_{crit})(1 \pm 0.01)$$

$$\Rightarrow T \propto \frac{1}{a}$$

$$\Delta_R^2(k) = \frac{2\pi}{k}$$

$p > p_{crit} \quad K = +1$   
 $p < p_{crit} \quad K = -1$   
 $p = p_{crit} \quad K = 0$

$> m \rightarrow n_i \propto T^3$   
 $\rho_i \propto T^4$   
 $\frac{1}{3} = w \quad \rho_i \propto T^4$

$n_i(T_i, m_i, \mu_i)$   
 $p_i(\dots)$   
 $\rho_i(\dots)$   
 $T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i - \mu_i)}{T}\right]$   
 $\rho_i = m n_i \quad p_i = T n_i$   
 $\frac{p}{\rho} = 0 = w$

$\rho_{Barions} = \frac{\rho_{barion}}{\rho_{crit} \Omega_b}$

Application

Sch

uncharg  
black h

$$\eta \propto \sum_K (\eta/\alpha) \quad \alpha = \frac{2}{1+3W}$$

$$-\frac{k}{a^2 H^2} \quad p_{\text{crit}} = \frac{3H^2}{8\pi G_m}$$

$$\frac{f}{a^2 H^2} \quad p_0 = (p_{\text{crit}})(1 \pm 0.01)$$

$$p > p_{\text{crit}} \quad K = +1$$

$$p < p_{\text{crit}} \quad K = -1$$

$$p = p_{\text{crit}} \quad K = 0$$

$$\Rightarrow T \propto \frac{1}{a}$$

$$\Delta_R^2(k) = \Delta_R^2(k)$$

$$\text{Barions} = \frac{p_{\text{barion}}}{p_{\text{crit}0}}$$

### Lecture 7 Thermo/SM

$$n_i(T_i, m_i, \mu_i)$$

$$g_i(\dots)$$

$$p_i(\dots)$$

$$n_i \propto T^3$$

$$\rho_i \propto T^4$$

$$p_i \propto T^4$$

$$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i - \mu_i)}{T}\right]$$

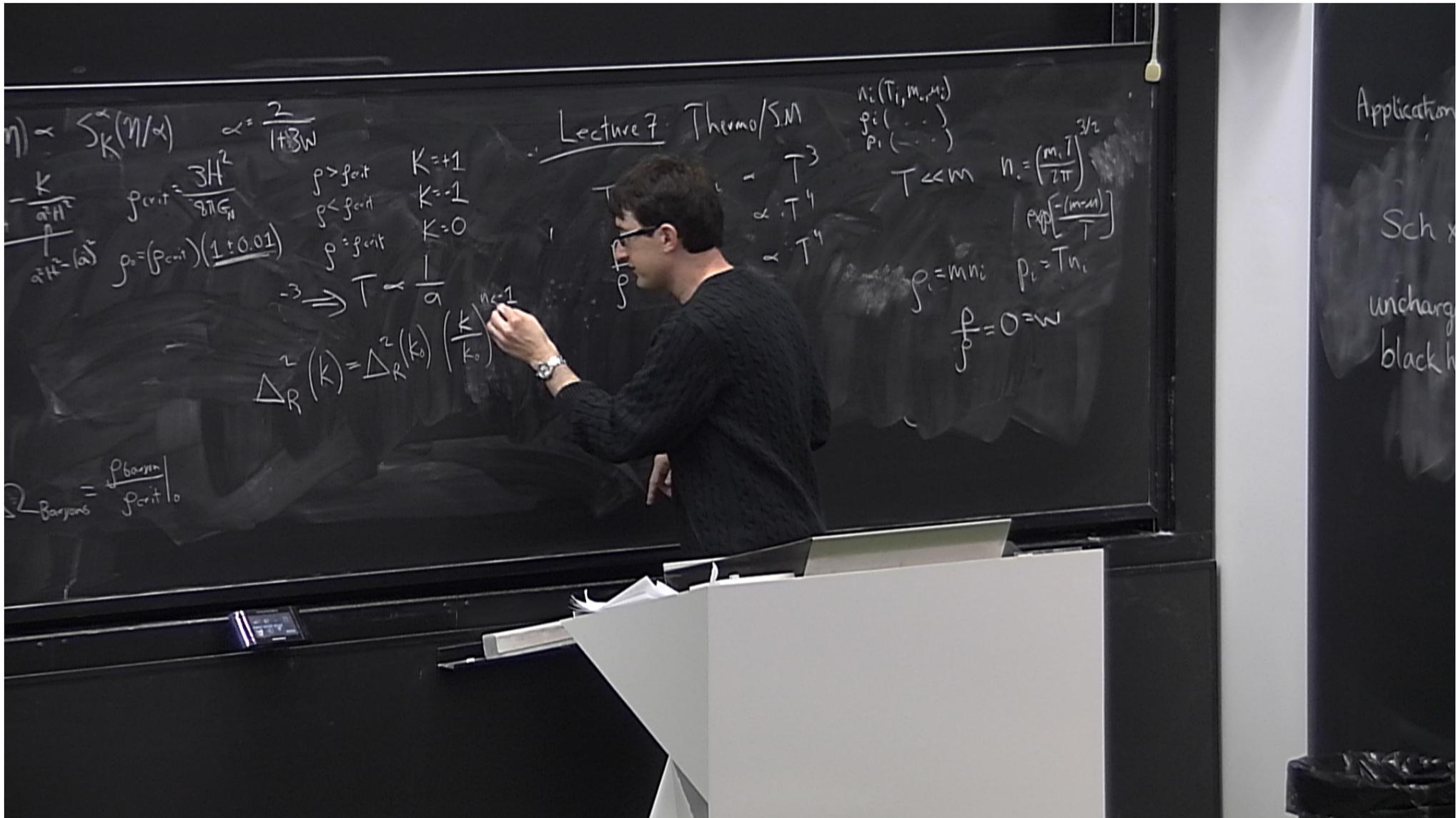
$$p_i = m n_i \quad p_i = T n_i$$

$$\frac{p}{\rho} = 0 = w$$

Application

Sch x

uncharg  
black h



$$\eta) \propto \sum_K (\eta/\alpha) \propto \frac{2}{1+3W}$$

$$\frac{k}{a^2 H^2} \frac{f}{a^2 H^2 (a)^2}$$

$$p_{crit} = \frac{3H^2}{8\pi G_n}$$

$$p_0 = (p_{crit})(1 \pm 0.01)$$

$p > p_{crit} \quad K = +1$   
 $p < p_{crit} \quad K = -1$   
 $p = p_{crit} \quad K = 0$

$$\Rightarrow T \propto \frac{1}{a^{n-1}}$$

$$\Delta_R^2(K) = \Delta_R^2(K_0) \left(\frac{K}{K_0}\right)^{n-1}$$

$$\rho_{Baryons} = \frac{p_{barion}}{p_{crit} 10}$$

Lecture 7 Thermo/SM

$T \propto T^3$   
 $\propto T^4$   
 $\propto T^4$

$n_i(T_i, m_i, \mu_i)$   
 $f_i(\dots)$   
 $p_i(\dots)$

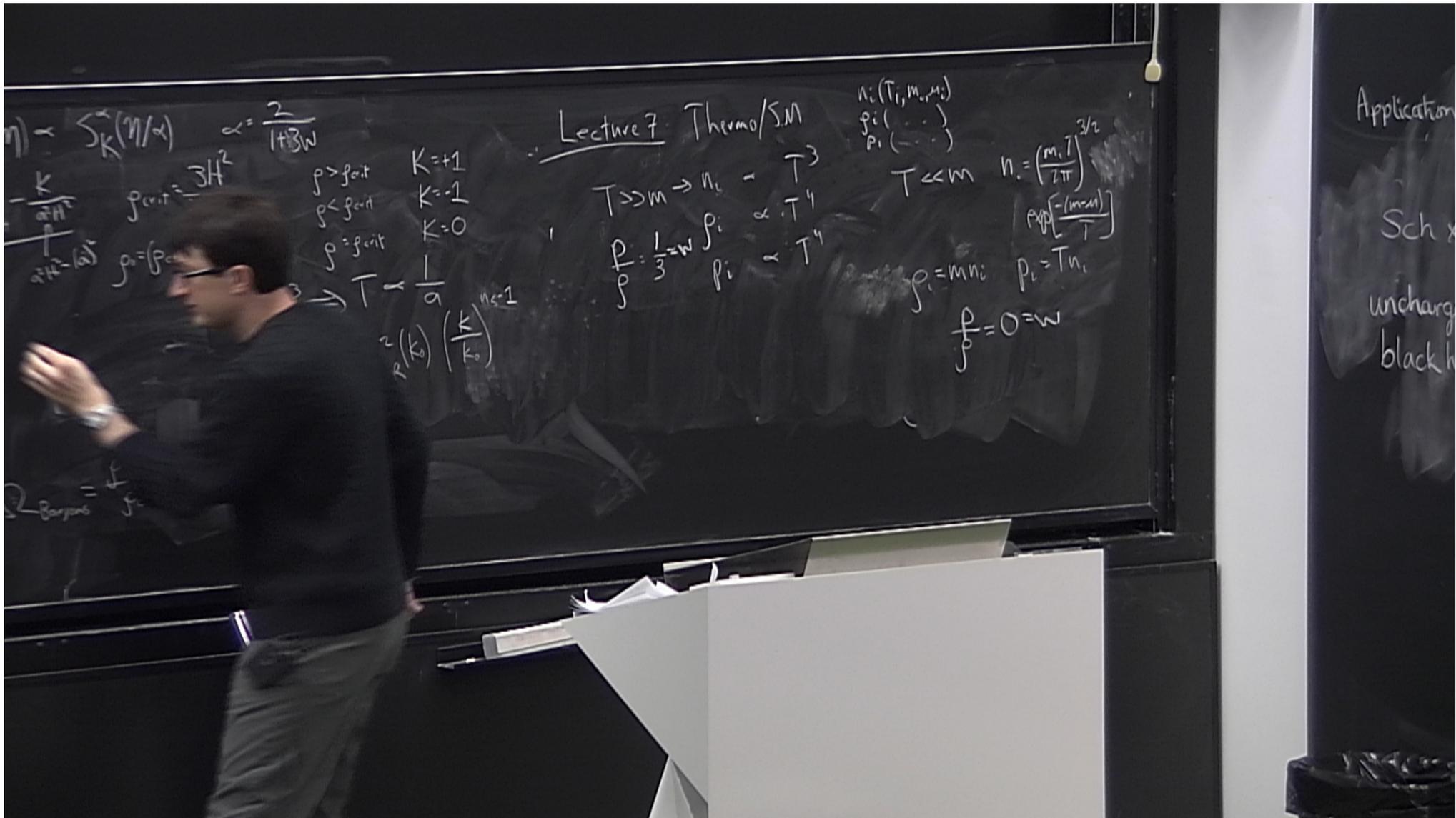
$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i - \mu_i)}{T}\right]$   
 $p_i = m n_i \quad p_i = T n_i$   
 $\frac{p}{p_0} = 0 = w$

Application

Sch x

uncharg

black h



$$\eta = \sum_K (\eta/\alpha) \quad \alpha = \frac{2}{1+3W}$$

$$p_{crit} = 3H^2$$

$$p > p_{crit} \quad K = +1$$

$$p < p_{crit} \quad K = -1$$

$$p = p_{crit} \quad K = 0$$

$$T \propto \frac{1}{a} \quad n \propto 1$$

$$R \left( \frac{K}{K_0} \right) \left( \frac{K}{K_0} \right)^{n-1}$$

Lecture 7 Thermo/SM

$$T \gg m \rightarrow n_i \propto T^3$$

$$\frac{p}{\rho} = \frac{1}{3} = w \quad p_i \propto T^4$$

$$p_i \propto T^4$$

$$n_i(T_i, m_i, \mu_i)$$

$$p_i(\dots)$$

$$p_i(\dots)$$

$$T \ll m \quad n_i = \left( \frac{m_i T}{2\pi} \right)^{3/2} e^{-\frac{m_i}{T}}$$

$$p_i = T n_i$$

$$\frac{p}{\rho} = 0 = w$$

Application

Sch x

uncharg  
black h

SU(3)	SU(2)	U(1)
3	2	$+\frac{1}{6}$
3	1	$+\frac{2}{3}$
3	1	$-\frac{1}{3}$
1	2	$-\frac{1}{2}$
1	1	0
1	1	-1
1	2	$+\frac{1}{2}$

Lorentz  
Gauge  
Renormalizable

$$\bar{q}_L^i h U_R^j \epsilon_{ij}$$

20-30  
↑

2000 ΛCDM

$T=$



Lecture 6.

$$w = \frac{\rho}{\rho_0}$$

$$\rho \propto a^{-3(1+w)}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$$

$$a(\eta) \propto \int \frac{d\eta}{K(\eta/\alpha)} \quad \alpha = \frac{2}{1+3w}$$

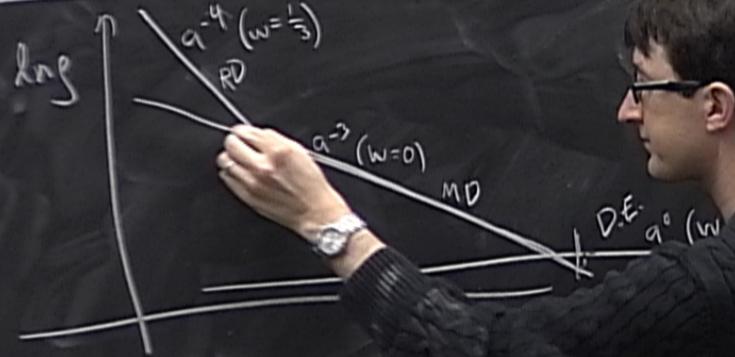
$$\frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G_N}$$

$\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit}$

$$\rho_0 = (\rho_{crit})(1 \pm 0.01)$$

$-3 \Rightarrow T \propto$



$$\rho_{crit} = \frac{3H_0}{8\pi G_N}$$

$$\frac{\rho_{DE}}{\rho_{crit,0}} = \frac{\rho_M}{\rho_{crit,0}} = \frac{\rho_{rad}}{\rho_{crit,0}}$$

$$\Omega_{Baryons} = \frac{\rho_{Baryons}}{\rho_{crit,0}}$$

$$\Delta_R^2(k) = \frac{\Delta_R^2(k_0)}{k_0^2}$$

Lecture 6.

$$w = \frac{p}{\rho}$$

$$\rho \propto a^{-3(1+w)}$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

$$a(t) \propto \int \frac{dt}{K(t/a)}$$

$$\alpha = \frac{2}{1+3w}$$

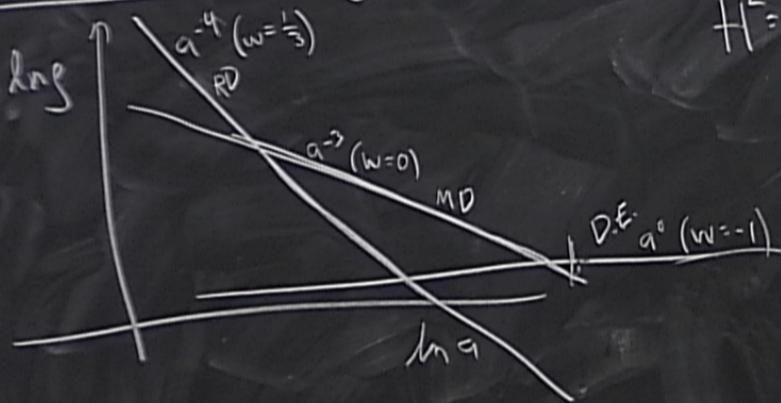
$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{k}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G_N}$$

$$\rho_0 = (\rho_{crit})(1 \pm 0.01)$$

$\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit}$

$-3 \Rightarrow T \propto$



$$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G_N}$$

$$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit} |_0}$$

$$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit} |_0}$$

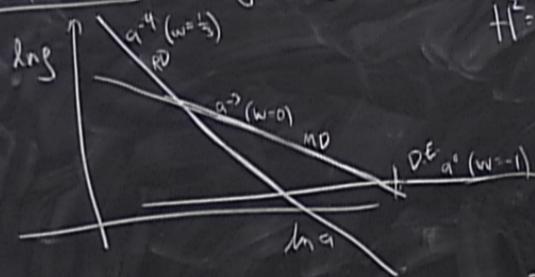
$$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit} |_0}$$

$$\Omega_{Baryons} = \frac{\rho_{baryon}}{\rho_{crit} |_0}$$

$$\Delta_R^2(k) = \frac{\Delta_R^2(k)}{k^3}$$

Lecture 6

$w = \frac{p}{p}$   $p \propto a^{-3(1+w)}$



$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(t) \propto \sum K_i(t_i/a_i)$   $\propto \frac{2}{H_{3W}}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2}$

$\rho > \rho_{crit} \quad K = +1$   
 $\rho < \rho_{crit} \quad K = -1$   
 $\rho = \rho_{crit} \quad K = 0$

$\rightarrow T = \frac{1}{a} \quad n_s = 1$

Lecture 7 Thermo/SM

$T \gg m \rightarrow n_i \propto T^3$   
 $\frac{\rho}{\rho} = \frac{1}{3} \sum \rho_i \propto T^4$   
 $\rho_i \propto T^4$

$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G}$

$0.7 \leftarrow \Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$

$0.25 \leftarrow \Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$

$\Delta_R^2(K) = \frac{\Delta_R^2(K_0)}{K_0} \left(\frac{K}{K_0}\right)^{n_s-1}$

$$\alpha = \frac{2}{1+3W}$$

$$f_{\text{crit}} = \frac{3H^2}{8\pi G_N}$$

$$p = (p_{\text{crit}})(1 \pm 0.01)$$

$p > p_{\text{crit}} \quad K = +1$   
 $p < p_{\text{crit}} \quad K = -1$   
 $p = p_{\text{crit}} \quad K = 0$

$$\Rightarrow T \propto \frac{1}{a}$$

$$\Delta_R^2(k) = \Delta_R^2(k_0) \left(\frac{k}{k_0}\right)^{n_s-1}$$

$n_s - 1 \approx 0$

$$\frac{p_{\text{crit}}}{p_0} = 0.05$$

## Lecture 7: Thermo/SM

$T \gg m \rightarrow n_i$   
 $\frac{p}{\rho} = \frac{1}{3} = w$

$$n_i(T_i, m_i, \mu_i)$$

$$p_i(\dots)$$

$$\rho_i(\dots)$$

$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i T)}{T}\right]$

$\rho_i = m_i n_i \quad p_i = T n_i$

$$\frac{p}{\rho} = 0 = w$$

Lecture 7 Thermo/SM

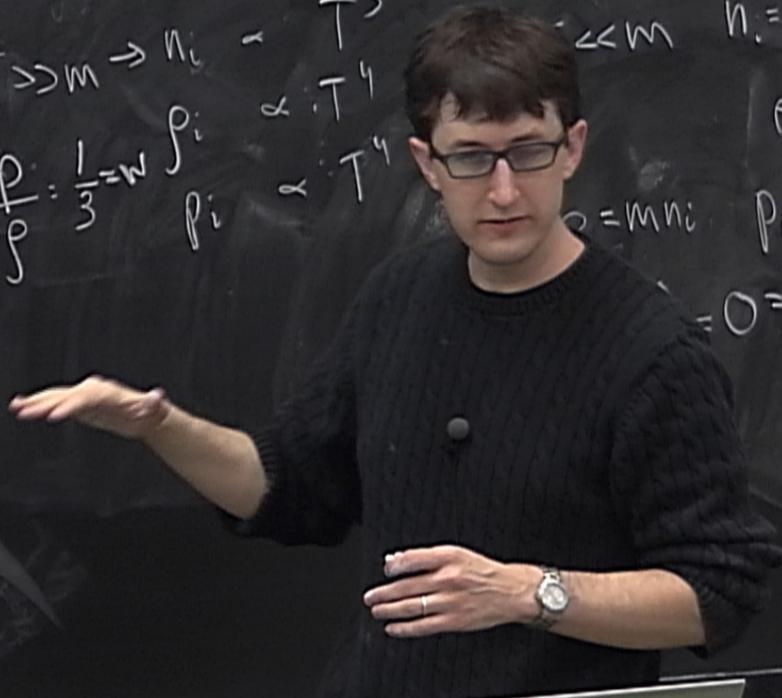
$n_i(T_i, m_i, \mu_i)$   
 $p_i(\dots)$   
 $p_i(\dots)$

$T \gg m \rightarrow n_i \propto T^3$   
 $\rho = \frac{1}{3} = w$   
 $p_i \propto T^4$   
 $p_i \propto T^4$

$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i c^2)}{T}\right]$   
 $\rho = m n_i$   
 $p_i = T n_i$   
 $\rho = 0 = w$

$\alpha = \frac{2}{1+3w}$   
 $f_{crit} = \frac{3H^2}{8\pi G_N}$   
 $\rho = (f_{crit})(1 \pm 0.01)$   
 $\rho > f_{crit} \quad K=+1$   
 $\rho < f_{crit} \quad K=-1$   
 $\rho = f_{crit} \quad K=0$   
 $\Rightarrow T \propto \frac{1}{a}$   
 $\Delta_R^2(k) = \Delta_R^2(k_0) \left(\frac{k}{k_0}\right)^{n_s-1}$   
 $n_s-1 \approx 0$

$\frac{b_{crit}}{f_{crit} |_0} = 0.05$



$$\alpha = \frac{2}{1+3W}$$

$$g_{\text{crit}} = \frac{3H^2}{8\pi G_N}$$

$$g = (g_{\text{crit}})(1 \pm 0.01)$$

$p > p_{\text{crit}} \quad K = +1$   
 $p < p_{\text{crit}} \quad K = -1$   
 $p = p_{\text{crit}} \quad K = 0$

$$\Rightarrow T \propto \frac{1}{a}$$

$$\Delta_R^2(k) = \Delta_R^2(k_0) \left(\frac{k}{k_0}\right)^{n_s-1}$$

$n_s - 1 \approx 0$

$$\frac{g_{\text{crit}}}{g_{\text{crit}0}} = 0.05$$

## Lecture 7: Thermo/SM

$T \gg m \rightarrow n_i$   
 $\frac{p}{\rho} = \frac{1}{3} = w$

$$n_i(T_i, m_i, \mu_i)$$

$$p_i(\dots)$$

$$\rho_i(\dots)$$

$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i - \mu_i)}{T}\right]$

$p_i = m_i n_i \quad \rho_i = T n_i$   
 $\frac{p}{\rho} = 0 = w$



Lecture 7: Thermo/SM

$\alpha = \frac{2}{1+3W}$

$f_{\text{crit}} = \frac{3H^2}{8\pi G_M}$

$\rho = (f_{\text{crit}})(1 \pm 0.01)$

$\Rightarrow T \propto \frac{1}{a}$

$\Delta_R^2(k) = \Delta_R^2(k_0) \left(\frac{k}{k_0}\right)^{n_s-1}$

$\frac{\rho_{\text{crit}}}{\rho_0} = 0.05$

$n_s = 0.9$

$T \gg m \Rightarrow$

$\frac{\rho}{\rho} = \frac{1}{3} = w$

$T^4$

$n_i(T_i, m_i, \mu_i)$

$p_i(\dots)$

$p_i(\dots)$

$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2}$

$\exp\left[-\frac{(m_i T)}{T}\right]$

$\rho_i = m n_i \quad p_i = T n_i$

$\frac{\rho}{\rho} = 0 = w$

$K = +1$

$K = -1$

$K = 0$

$\rho > f_{\text{crit}}$

$\rho < f_{\text{crit}}$

$\rho = f_{\text{crit}}$

Lecture 7 Thermo/SM

$\alpha = \frac{2}{1+3W}$

$f_{\text{crit}} = \frac{3H^2}{8\pi G_M}$

$p = (p_{\text{crit}})(1 \pm 0.01)$

$\Rightarrow T \propto \frac{1}{a}$

$\Delta_R^2(k) = \frac{\Delta_R^2(k_0)}{(k/k_0)^{n_s-1}}$

$n_s - 1 \approx 0$

$\frac{p_{\text{crit}}}{p_0} = 0.05$

$p > p_{\text{crit}} \quad K = +1$

$p < p_{\text{crit}} \quad K = -1$

$p = p_{\text{crit}} \quad K = 0$

$n_s \rightarrow m \rightarrow n_i \propto T^3$

$\frac{1}{3} = w \quad p_i \propto T^4$

$p_i \propto T^4$

$n_i(T_i, m_i, \mu_i)$

$p_i(\dots)$

$p_i(\dots)$

$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2}$

$\exp\left[-\frac{(m_i - \mu_i)}{T}\right]$

$p_i = m n_i \quad p_i = T n_i$

$\frac{p}{p_0} = 0 = w$

Lecture 7: Thermo/SM

$\alpha = \frac{2}{1+3W}$

$f_{crit} = \frac{3H^2}{8\pi G_N}$

$\rho = (f_{crit})(1 \pm 0.01)$

$\Rightarrow T \propto \frac{1}{a}$

$\Delta_R^2(k) = \Delta_R^2(k_0) \left(\frac{k}{k_0}\right)^{2K}$

$\frac{\rho_{crit}}{\rho_0} = 0.05$

$K = +1$

$K = -1$

$K = 0$

$T \gg m \rightarrow n_i \propto T^3$

$\frac{\rho}{\rho} = \frac{1}{3} = w$

$\rho_i \propto T^4$

$\rho_i \propto T^4$

$n_i(T_i, m_i, \mu_i)$

$\rho_i(\dots)$

$\rho_i(\dots)$

$T \ll m$

$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i c^2)}{T}\right]$

$\rho_i = m n_i$

$\rho_i = T n_i$

$\frac{\rho}{\rho} = 0 = w$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \quad a(t) \propto \sum_K (t/\alpha)^{\alpha} \quad \alpha = \frac{2}{1+3w}$$

$$\frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\rho = \rho_{crit} (1 \pm 0.01)$$

Lecture 7 Thermo/SM

$n_i = 0.917005$

$T \gg m \rightarrow n_i \propto T^3$

$T \ll m \rightarrow n_i \propto T^4$

$\frac{p}{\rho} = \frac{1}{3} = w$

$\rho_i \propto T^4$

$n_i(T_i, m_i, g_i)$

$\rho_i(\dots)$

$p_i(\dots)$

$T \ll m \quad n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_i T)}{T}\right]$

$\rho_i = m n_i \quad p_i = T n_i$

$\frac{p}{\rho} = 0 = w$

$\Delta_R^2(K) = \frac{\Delta_R^2(K_0)}{10^{2n_s}}$

$\left(\frac{K}{K_0}\right)^{n_s-1}$

$n_s = 1$

$n_s = 0$

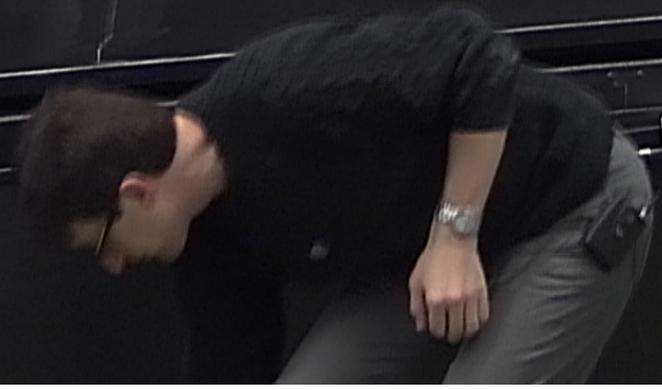
$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G}$

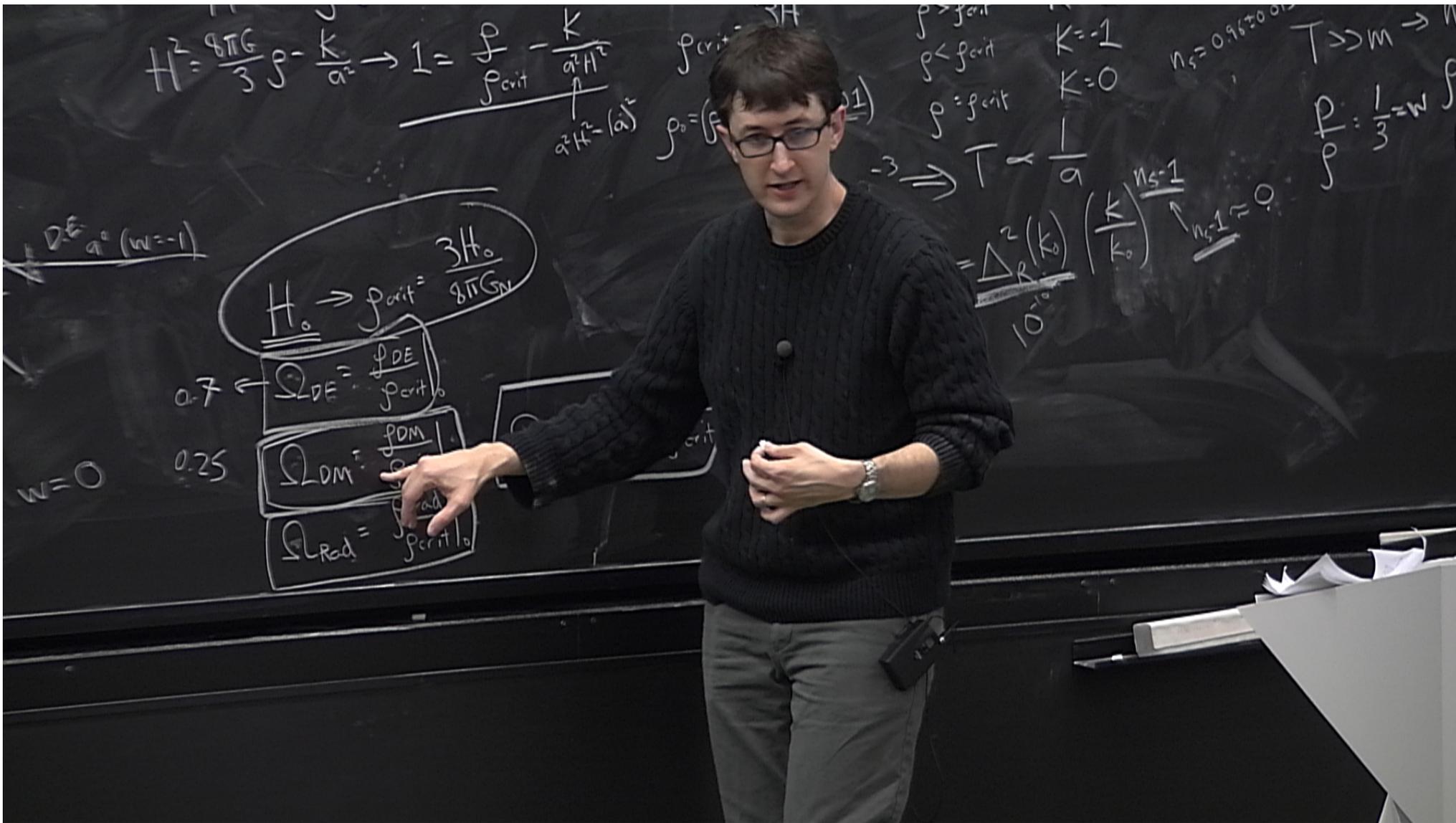
$\Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$

$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$

$\Omega_{rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$

$\Omega_{Baryons} = \frac{\rho_{baryon}}{\rho_{crit,0}} = 0.05$





$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H}{8\pi G_N}$$

$$\rho_0 = (\rho_{crit})(1 \pm 0.01)$$

$\rho > \rho_{crit}$   
 $\rho < \rho_{crit}$   
 $\rho = \rho_{crit}$

$K = -1$   
 $K = 0$

$$n_s = 0.96 \pm 0.01$$

$$\Rightarrow T \propto \frac{1}{a} \left( \frac{K}{K_0} \right)^{\frac{n_s-1}{2}}$$

$$\Delta_R^2(K) = \frac{\Delta_R^2(K_0)}{10^{\frac{n_s-1}{2} \log \left( \frac{K}{K_0} \right)}} \approx 10^{-\frac{n_s-1}{2} \log \left( \frac{K}{K_0} \right)}$$

$\leftarrow \rho_{DE} \propto a^w (w = -1)$

$w = 0$

$$H_0 \Rightarrow \rho_{crit} = \frac{3H_0}{8\pi G_N}$$

$$0.7 \leftarrow \Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$$

$$0.25 \leftarrow \Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$$

$$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$$

$$\Omega_{Baryons} = \frac{\rho_{baryon}}{\rho_{crit,0}} = 0.05$$

$$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2 H^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{a^2 H^2}$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\rho > \rho_{crit}$$

$$\rho < \rho_{crit}$$

$$\rho = \rho_{crit}$$

$$K = -1$$

$$K = 0$$

$$n_s = 0.96 \approx 0.96$$

$$T \gg m \rightarrow$$

$$\rho = \frac{1}{3} = w \rho$$

$$DE = a^0 (w = -1)$$

$$w = 0$$

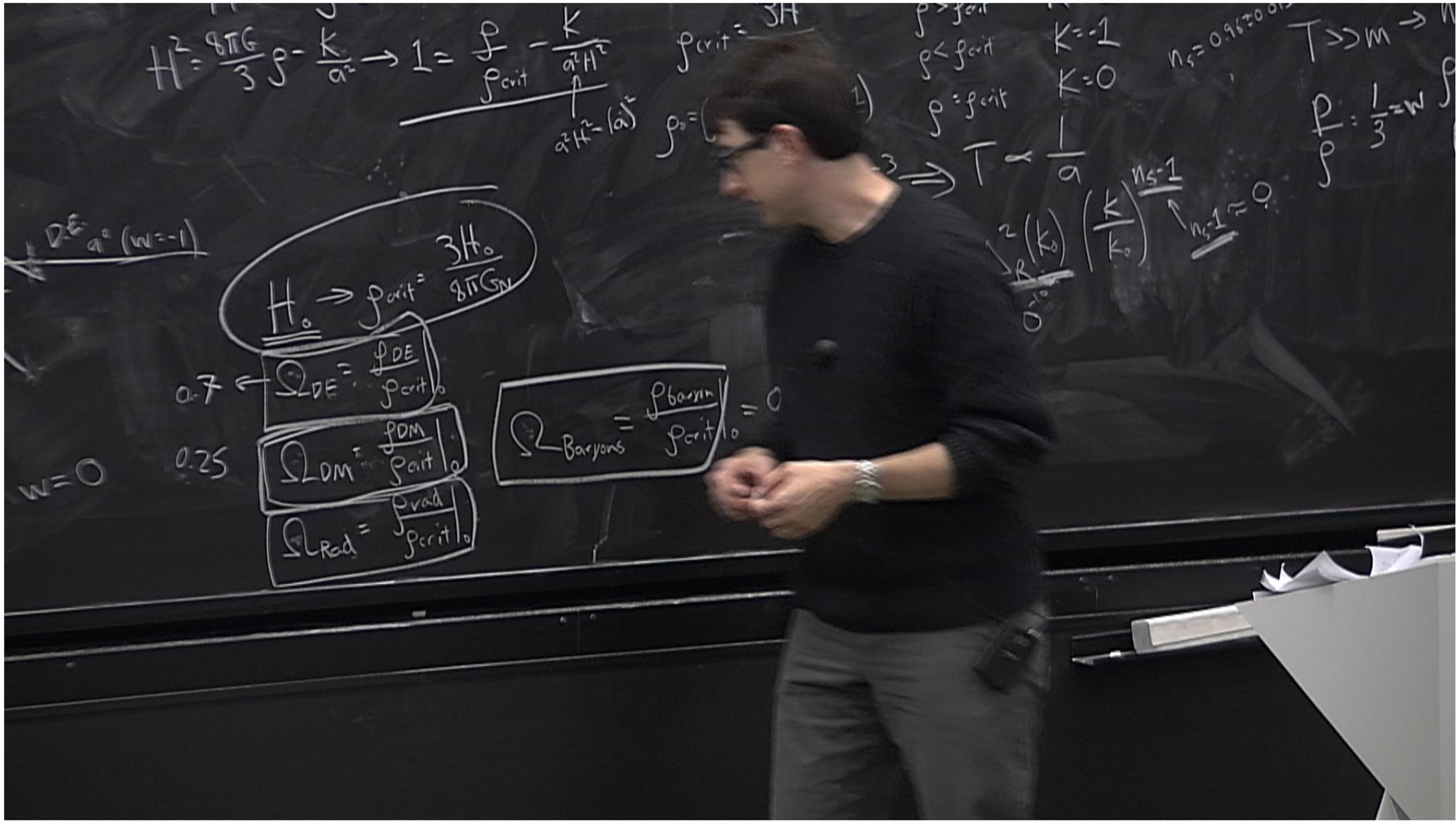
$H_0 \rightarrow \rho_{crit} = \frac{3H_0^2}{8\pi G}$

$0.7 \leftarrow \Omega_{DE} = \frac{\rho_{DE}}{\rho_{crit,0}}$

$0.25 \leftarrow \Omega_{DM} = \frac{\rho_{DM}}{\rho_{crit,0}}$

$\Omega_{Rad} = \frac{\rho_{rad}}{\rho_{crit,0}}$

$$\Omega_{Baryons} = \frac{\rho_{baryon}}{\rho_{crit,0}} = 0$$



Lecture 6

$w = \frac{p}{\rho}$   $\rho \propto a^{-3(1+w)}$

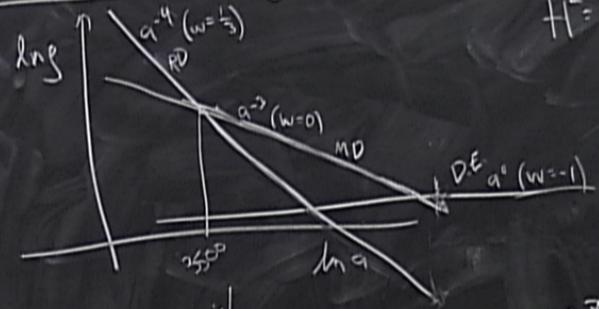
$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2}$   $a(\eta) \propto \int \frac{d\eta}{\sqrt{K(\eta/a)}}$   $\alpha = \frac{2}{1+3w}$

$H^2 = \frac{8\pi G}{3} \rho - \frac{K}{a^2} \rightarrow 1 = \frac{\rho}{\rho_{crit}} - \frac{K}{H^2 a^2}$   $\rho_{crit} = \frac{3H^2}{8\pi G}$

$\rho = \rho_{crit} (1 + 0.01)$

$\rho > \rho_{crit} \quad K = +1$   
 $\rho < \rho_{crit} \quad K = -1$   
 $\rho = \rho_{crit} \quad K = 0$

$n_s = 0.96 \pm 0.05$



Baryogenesis  
 Leptogenesis  
 Matter/anti-matter  $w=0$

$H_0 \rightarrow$

$0.7 \leftarrow \Omega_{DM}$

$0.25 \leftarrow \Omega_{Lensing}$

$\Omega_{Lensing} = 0.05$

$\Delta_R^2(k) = \frac{\Delta_R^2(k)}{k^3} \left( \frac{k}{k_s} \right)^{n_s-1}$

$n_s = 1 \rightarrow 0$