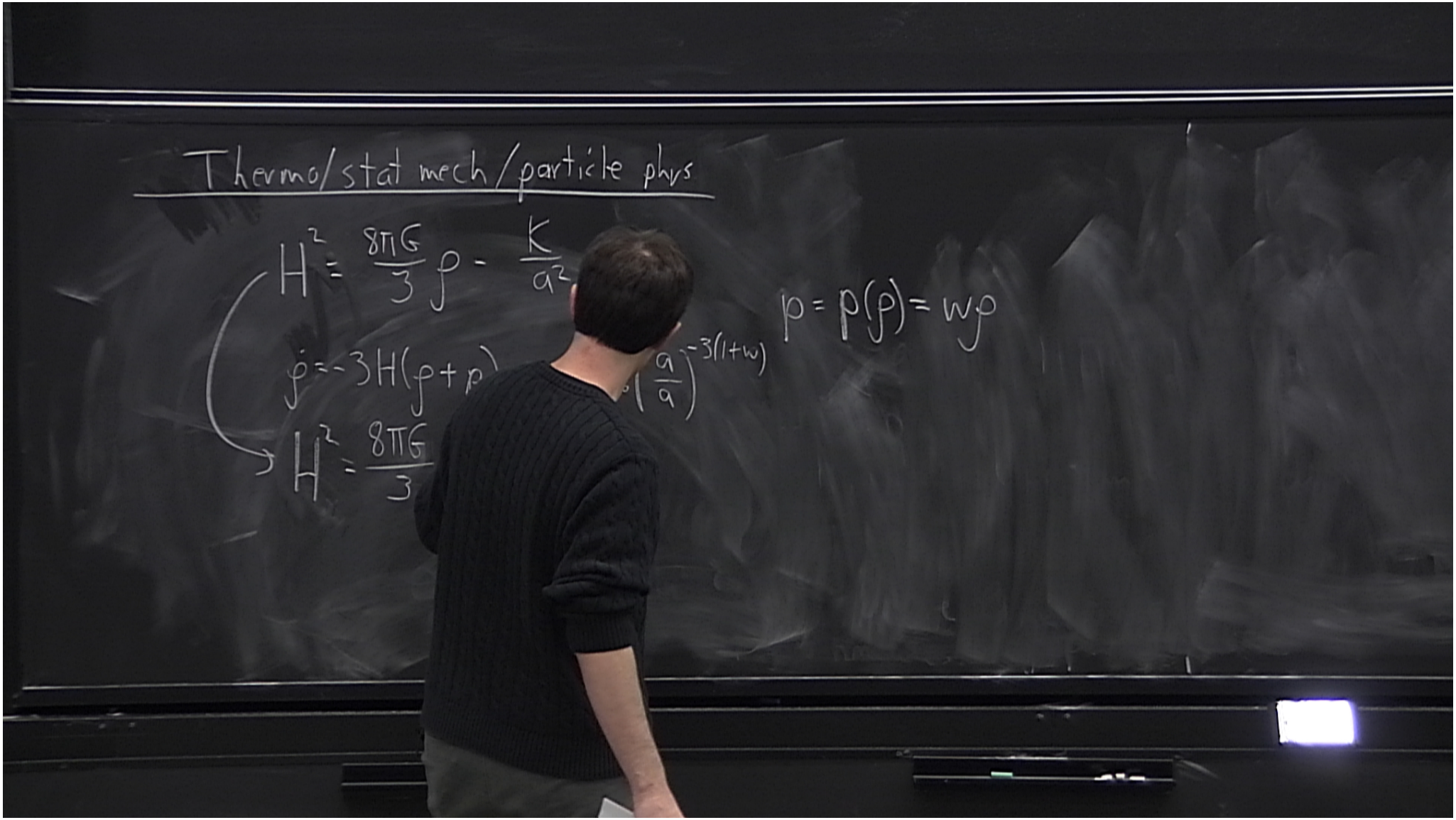
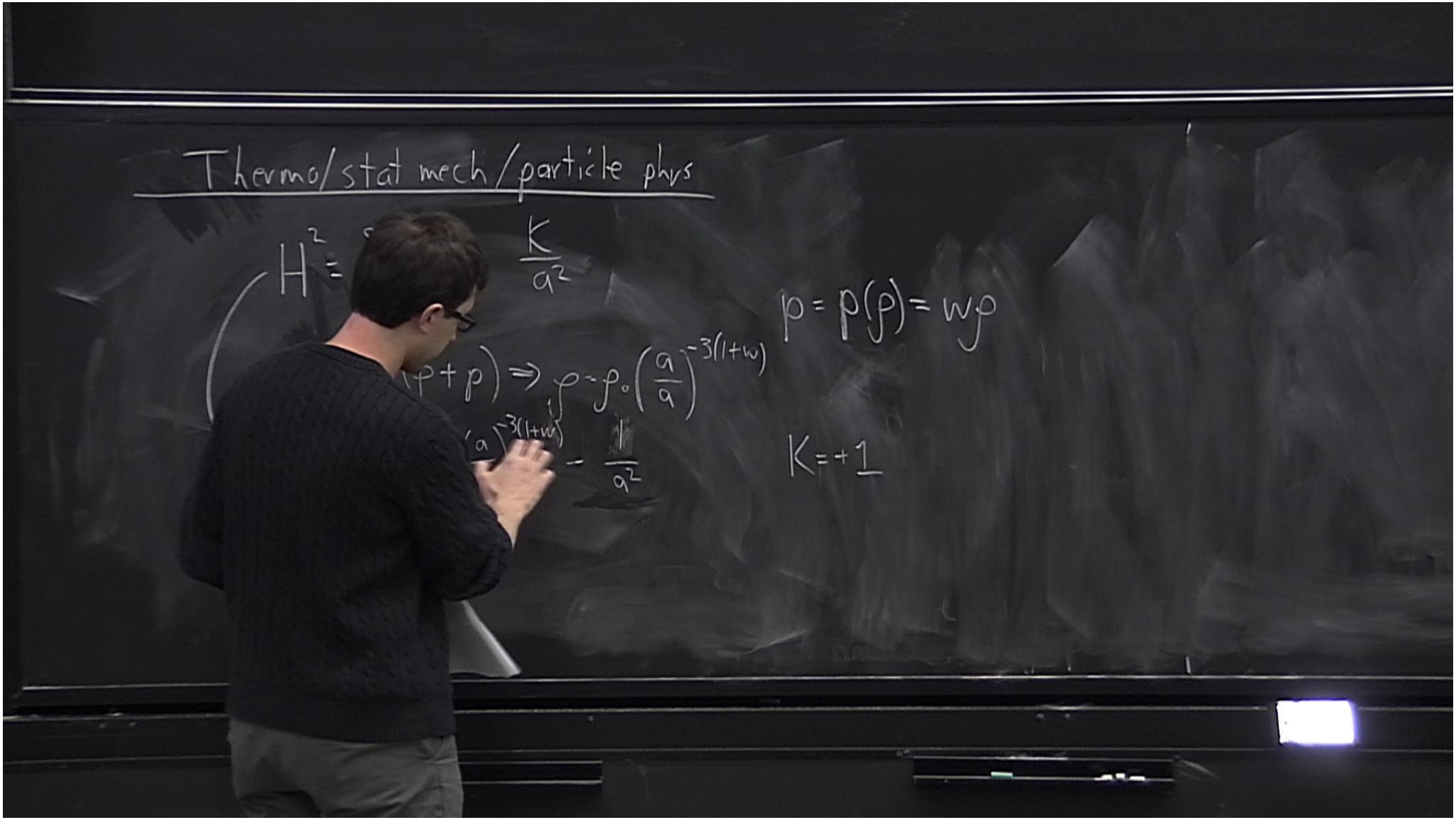
Title: Cosmology (Review) - Lecture 7

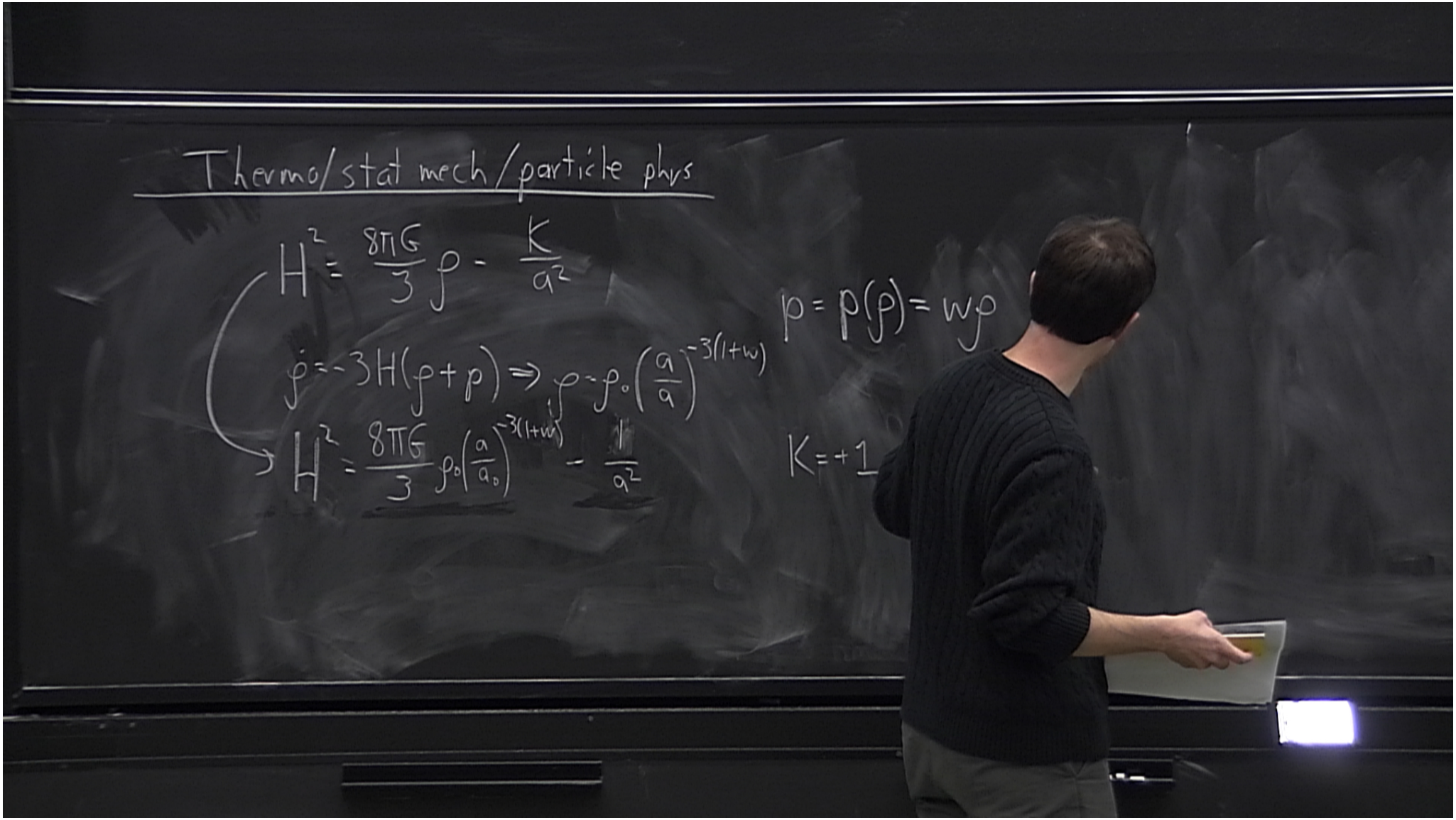
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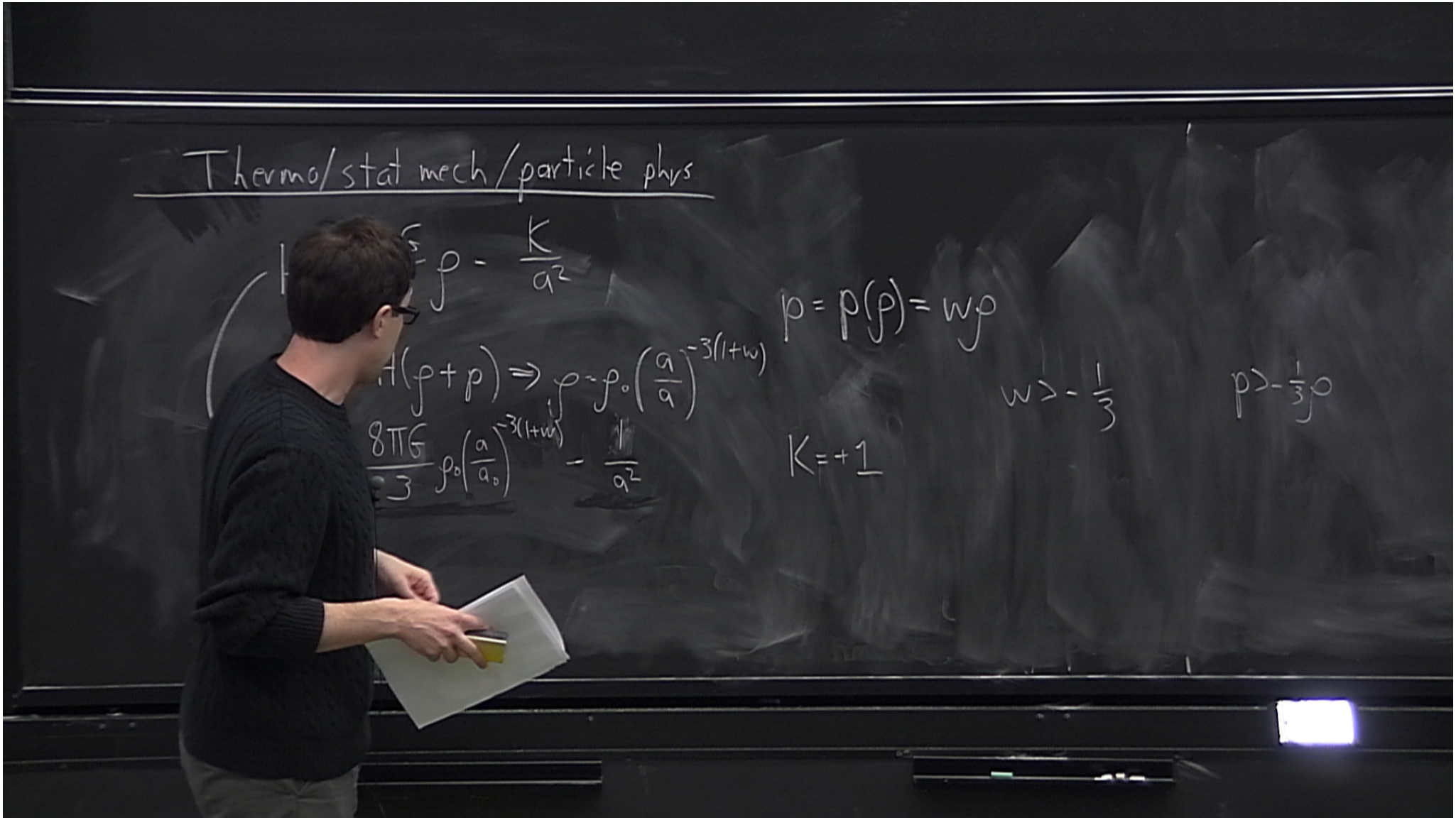
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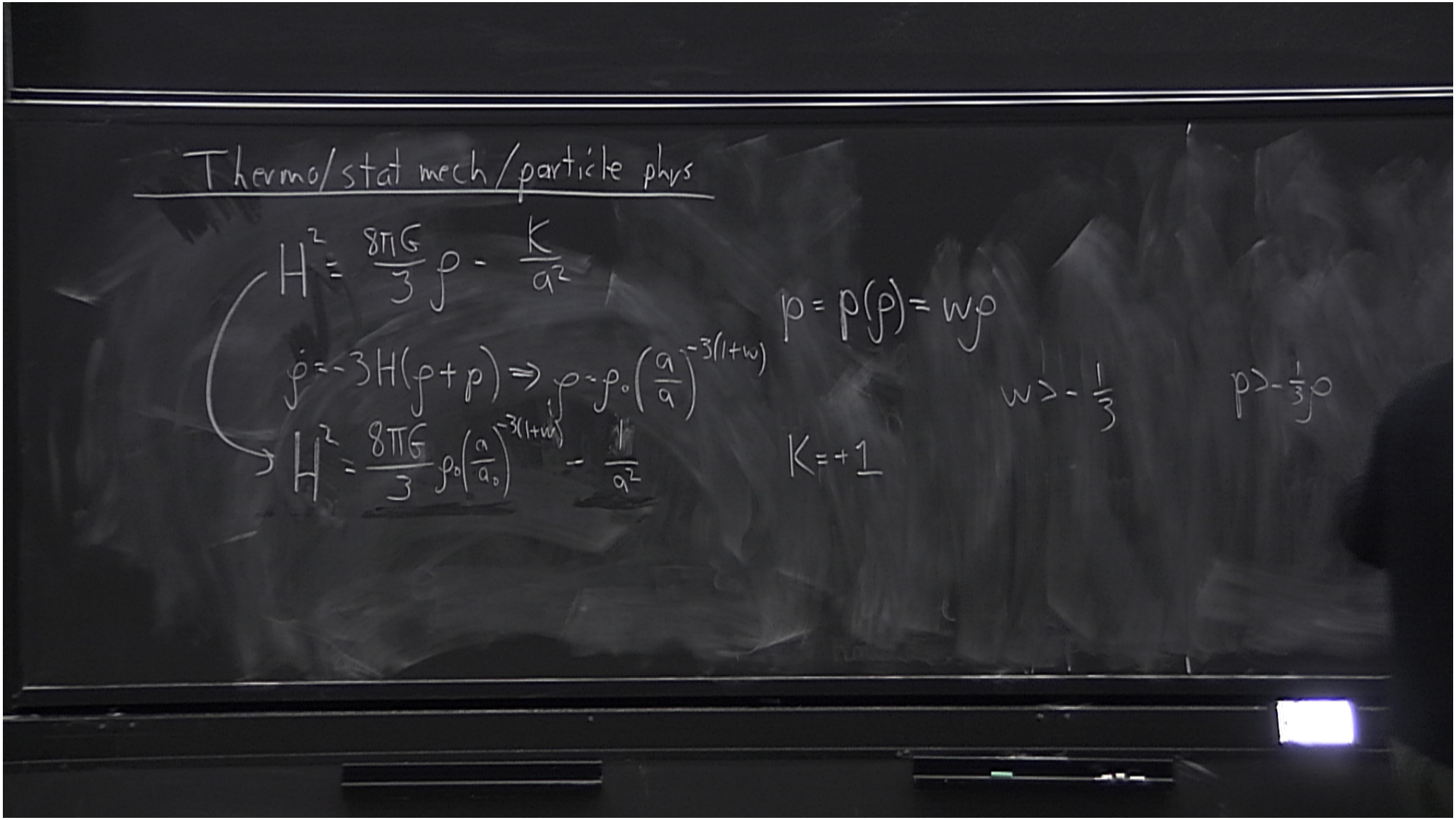
Abstract:


Thermo/stat mech/particle phys
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p)$
$\rho = \rho(\rho) = w\rho$
$\left(\frac{a}{a}\right)^{-3(1+w)}$
$H^2 = \frac{8\pi G}{3}$

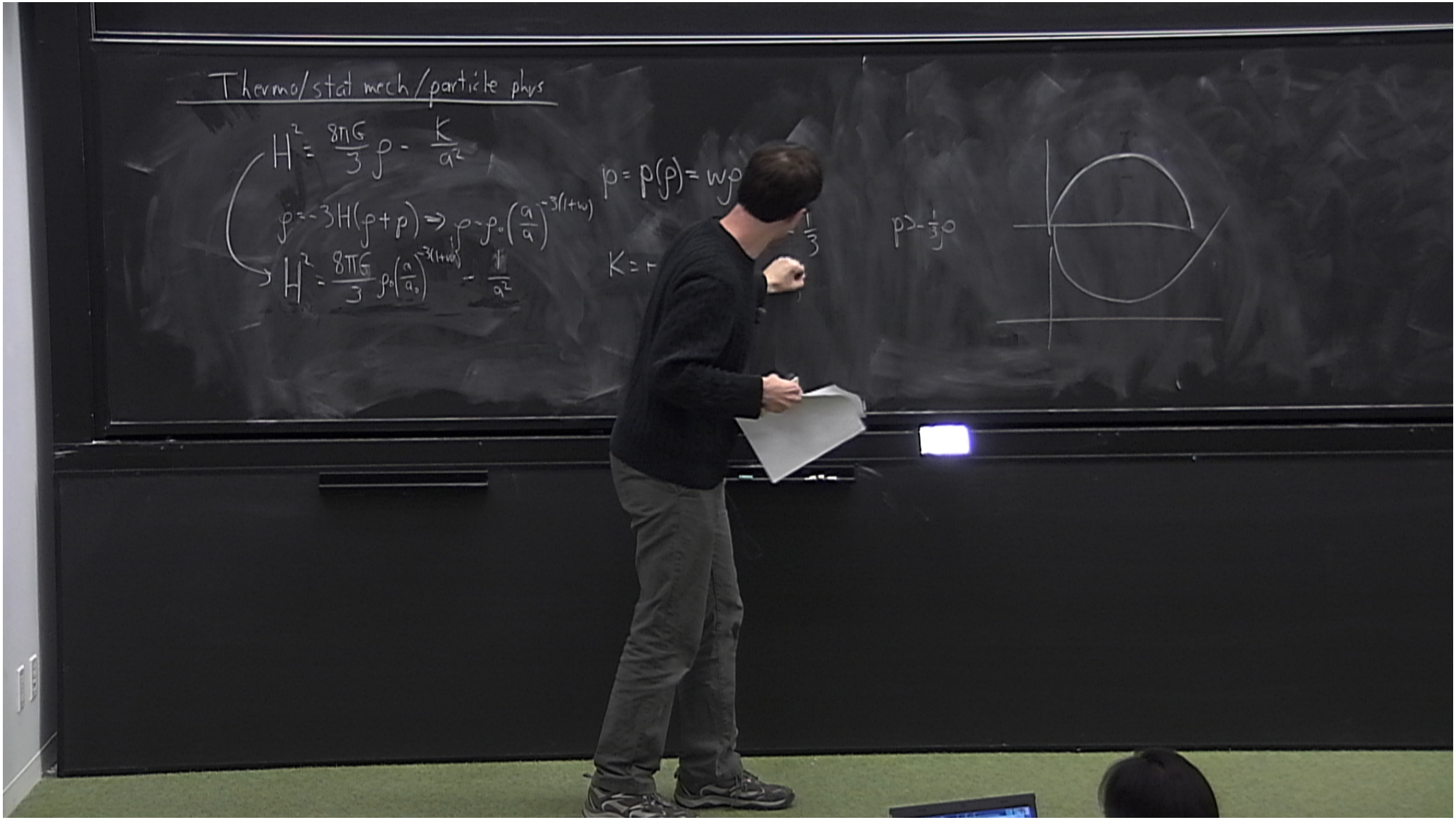

Thermo/stat mech/particle phys
$H^2 =$
$\frac{K}{a^2}$
$p = p(\rho) = w\rho$
$+p) \Rightarrow \rho = \rho_0 \left(\frac{a}{a}\right)^{-3(1+w)}$
$\left(\frac{a}{a}\right)^{-3(1+w)}$
$- \frac{1}{a^2}$
$K = +1$


Thermo/stat mech/particle phys
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p) \Rightarrow \rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$
$H^2 = \frac{8\pi G}{3}\rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$p = p(\rho) = w\rho$
$K = +1$


Thermo/stat mech/particle phys
$\rho - \frac{K}{a^2}$
$p = p(\rho) = w\rho$
$H(\rho+p) \Rightarrow \rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$
$w > -\frac{1}{3}$
$p > -\frac{1}{3}\rho$
$\frac{8\pi G}{3} \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{1}{a^2}$
$K = +1$

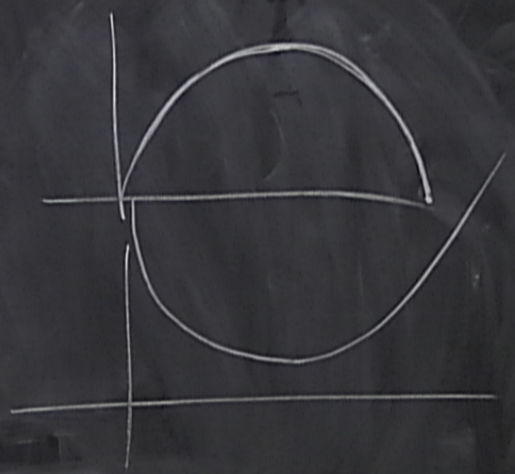

Thermo/stat mech/particle phys
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p) \Rightarrow \rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$
$H^2 = \frac{8\pi G}{3}\rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$p = p(\rho) = w\rho$
$w > -\frac{1}{3}$
$p > -\frac{1}{3}\rho$
$K = +1$


$p > -\frac{1}{3}\rho$


Thermo/stat mech/particle phys
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot{\rho} = -3H(\rho + p) \Rightarrow \rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$
$H^2 = \frac{8\pi G}{3}\rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} - \frac{K}{a^2}$
$p = p(\rho) = w\rho$
$K = +$
$-\frac{1}{3}$
$p > -\frac{1}{3}\rho$

$$H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$$

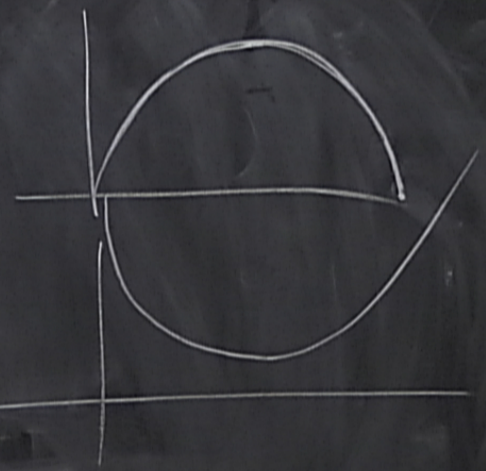
$p > -\frac{1}{3}\rho$

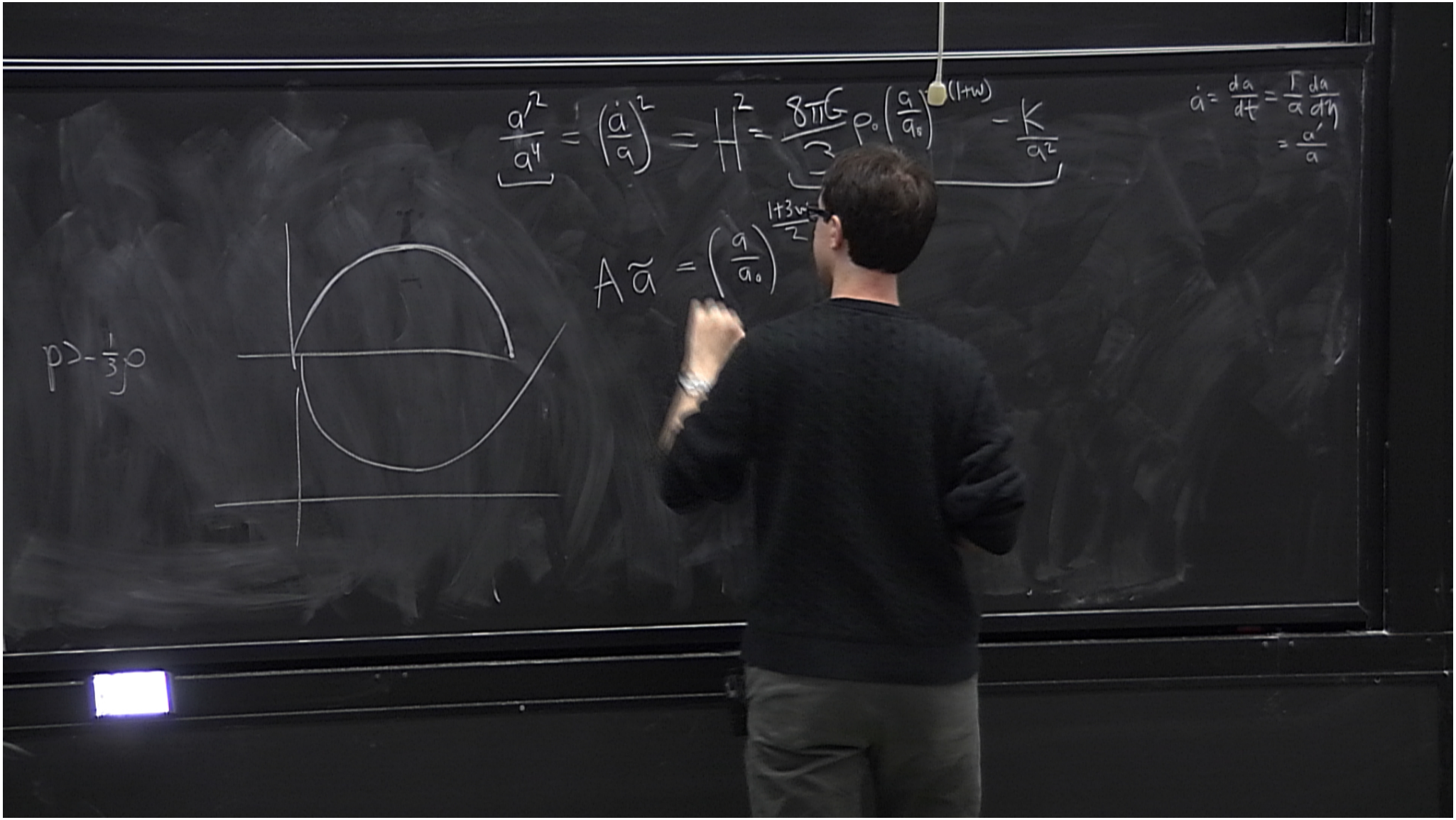


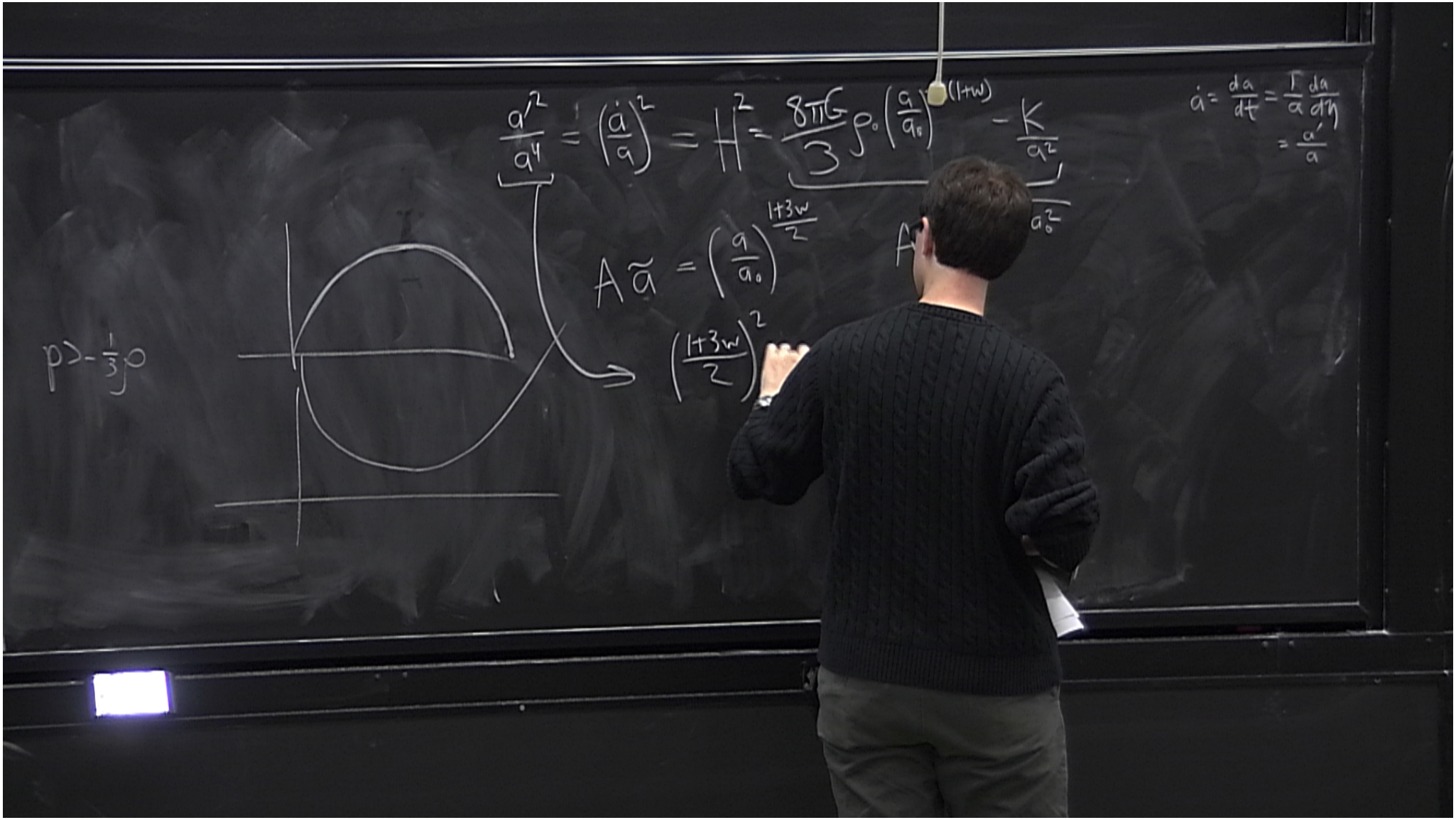
$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$$

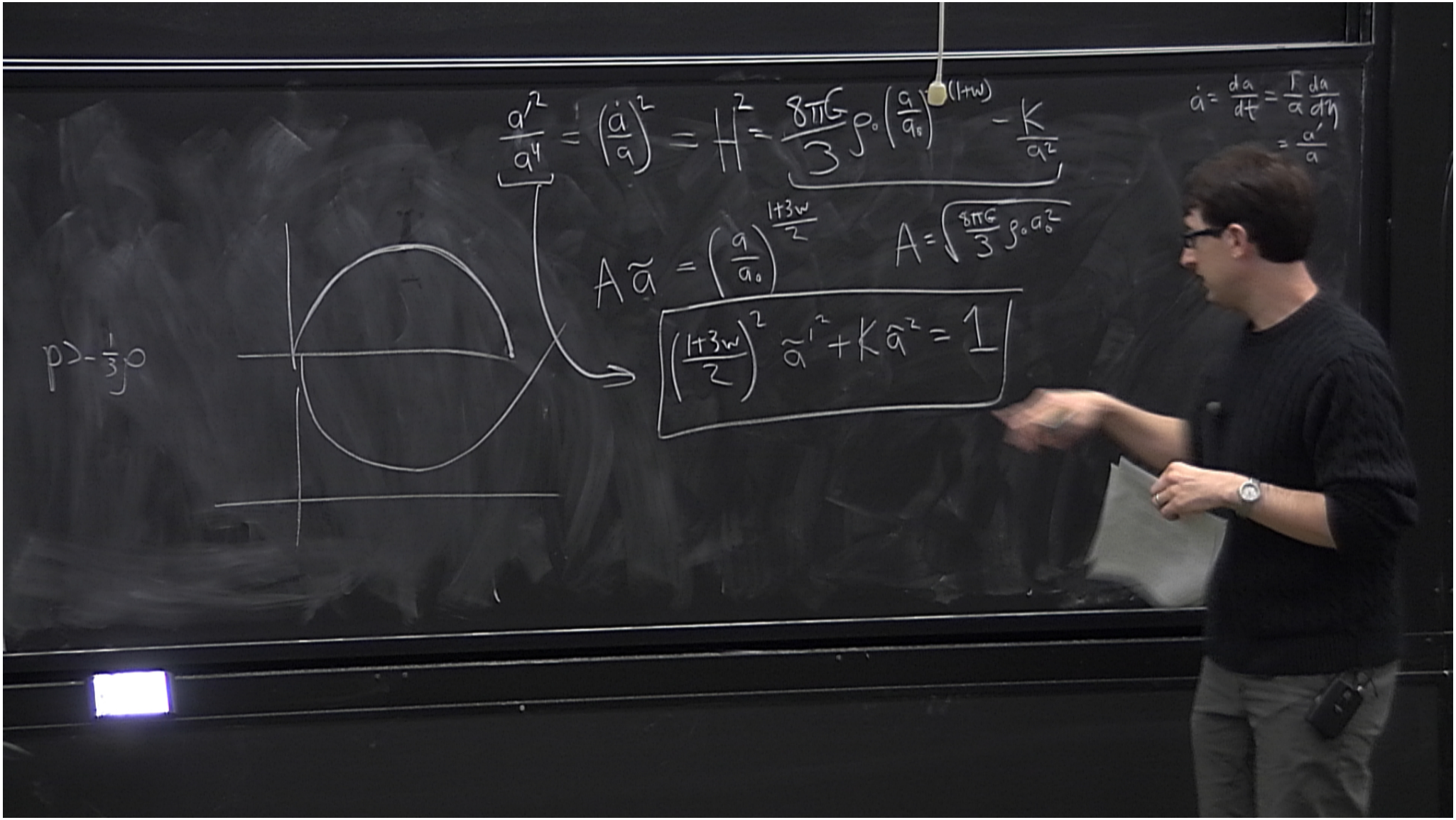
$$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$$

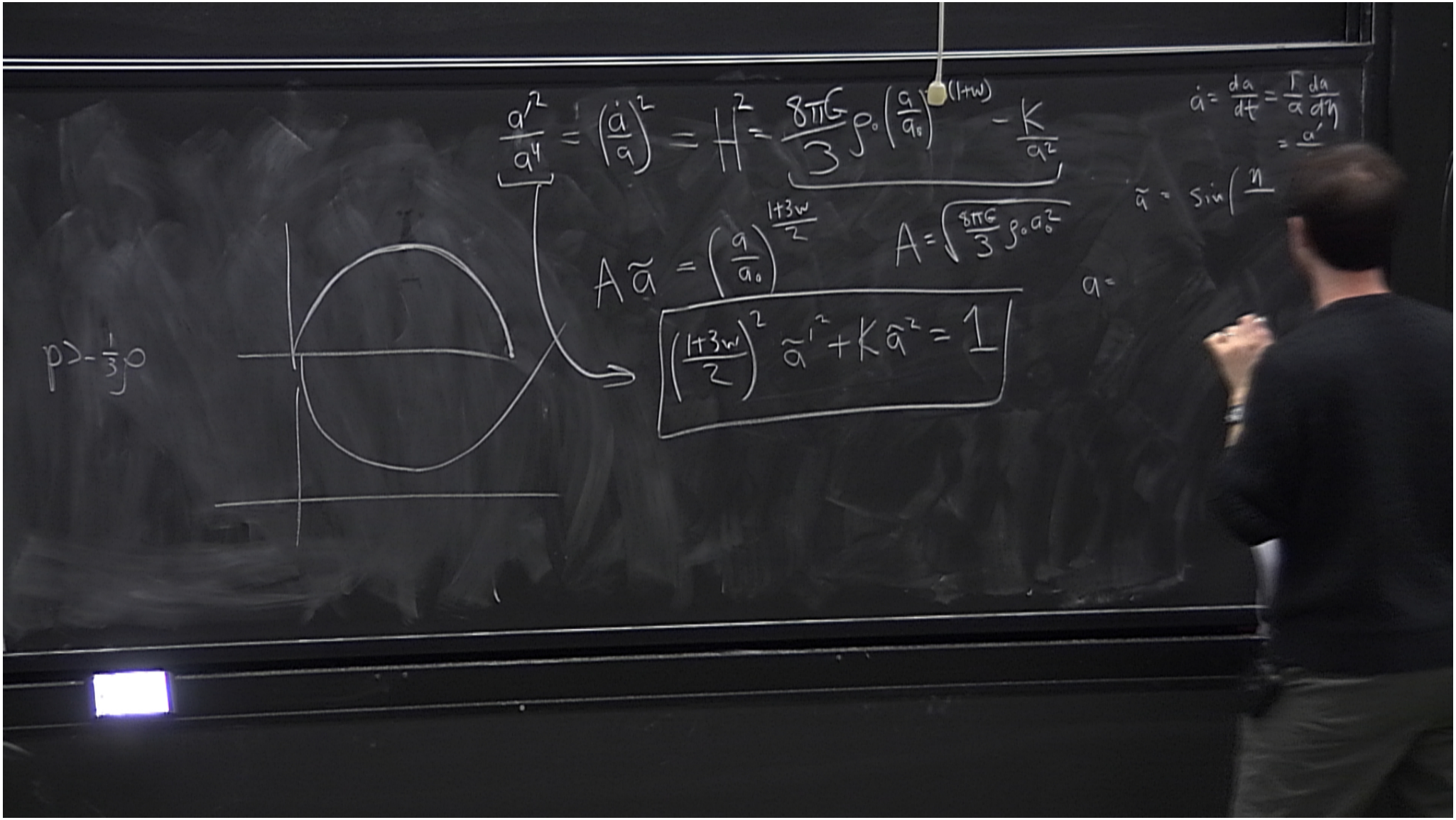
$p > -\frac{1}{3}\rho$

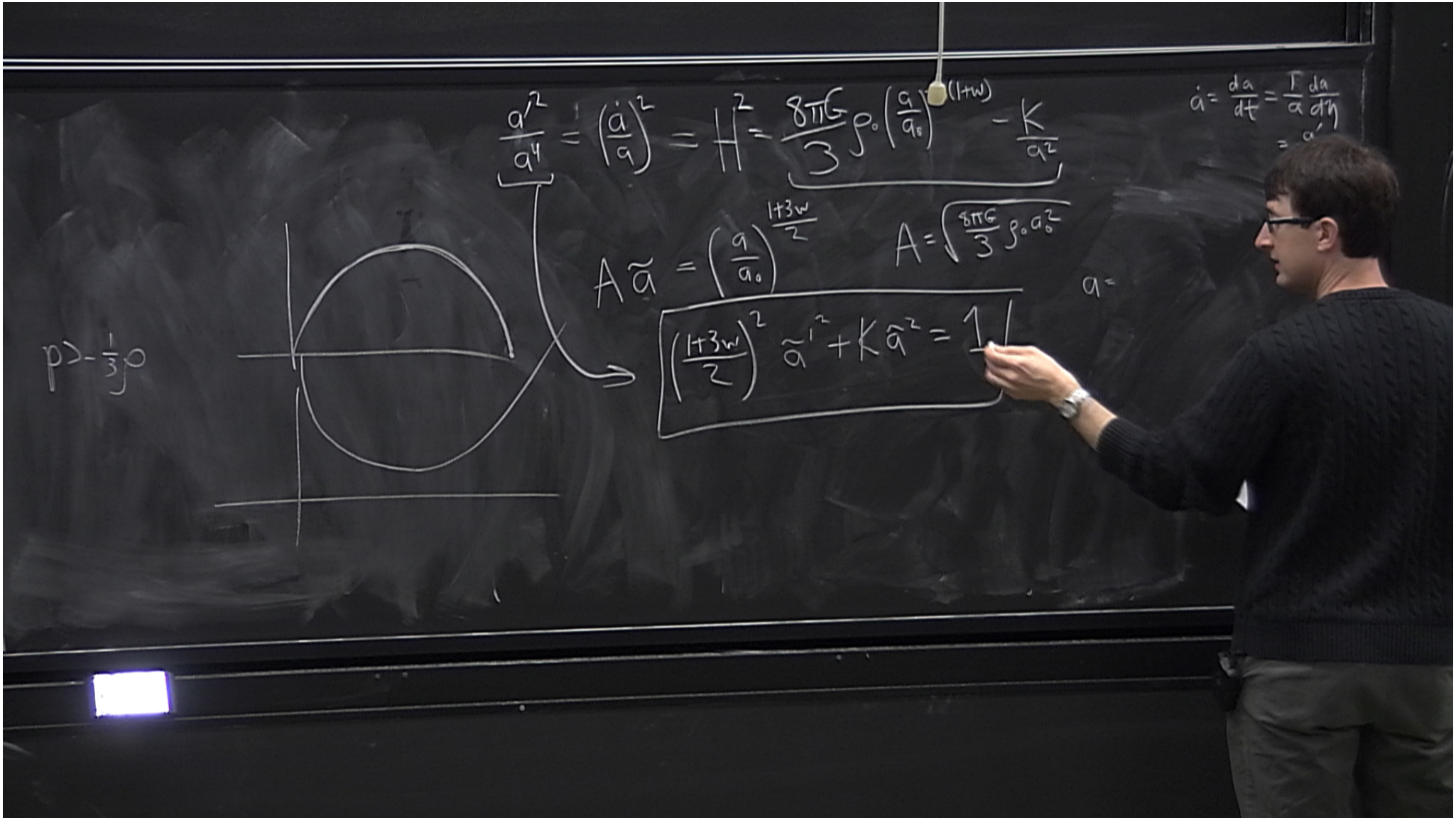


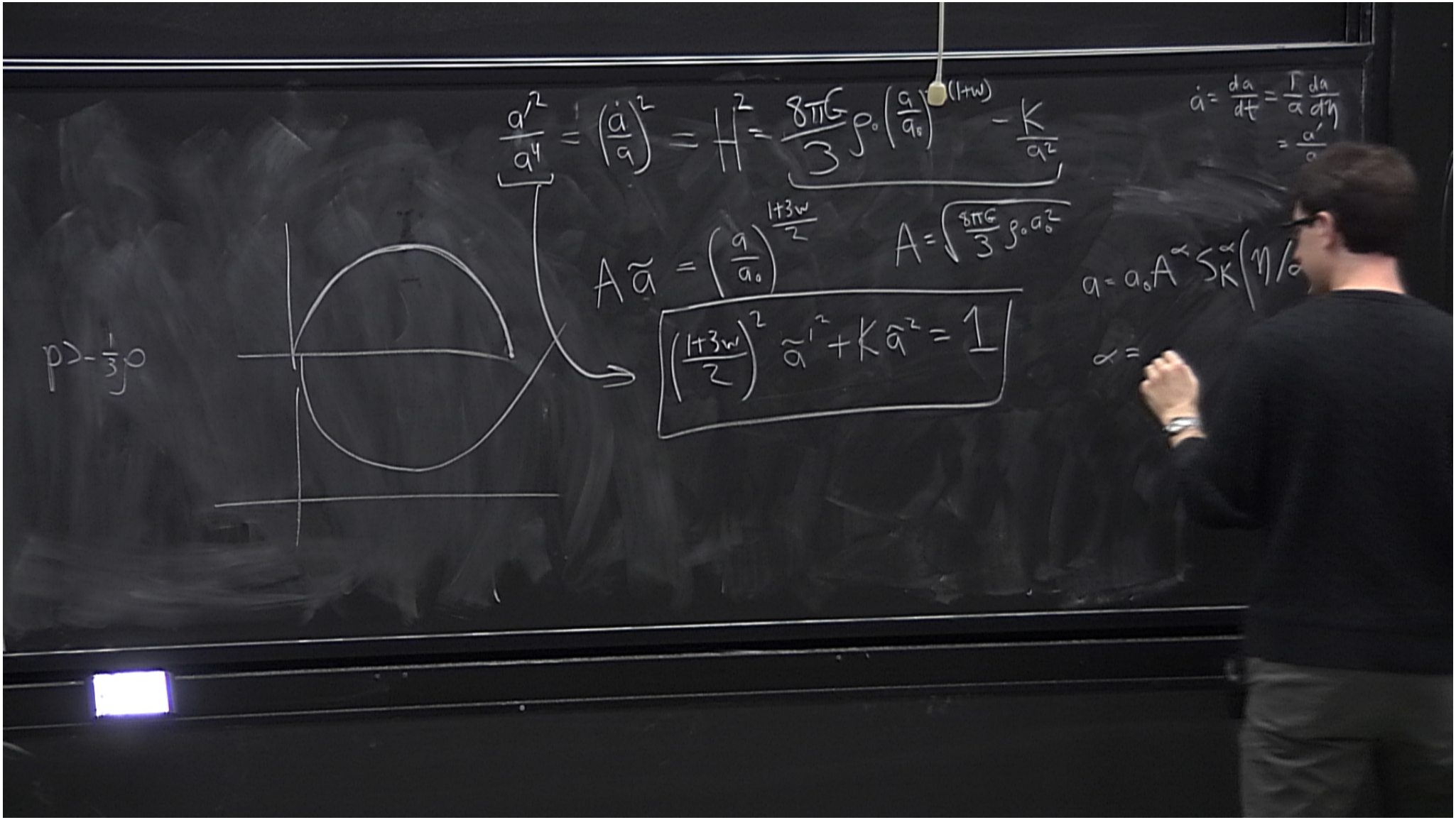

$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$
$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$p > -\frac{1}{3}\rho$


$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
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$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$\left(\frac{1+3w}{2}\right)^2$
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$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$
$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$
$\left(\frac{1+3w}{2}\right)^2 \tilde{a}'^2 + K\tilde{a}^2 = 1$
$p > -\frac{1}{3}\rho$


$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta}$
$= a'$
$\tilde{a} = \sin\left(\frac{\eta}{}\right.$
$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$
$a =$
$\left(\frac{1+3w}{2}\right)^2 \tilde{a}'^2 + K\tilde{a}^2 = 1$
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$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
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$A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$
$\left(\frac{1+3w}{2}\right)^2 \tilde{a}'^2 + K\tilde{a}^2 = 1$
$a = a_0 A^{\alpha} S_K^{\alpha}(\eta/a$
$\alpha =$
$p > -\frac{1}{3}\rho$

$$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$$

$$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$$

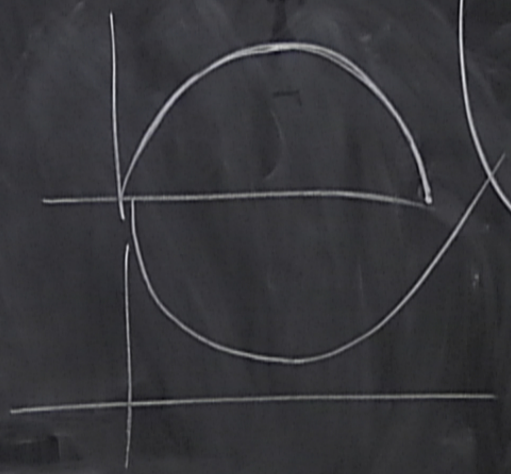
$$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}} \qquad A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$$

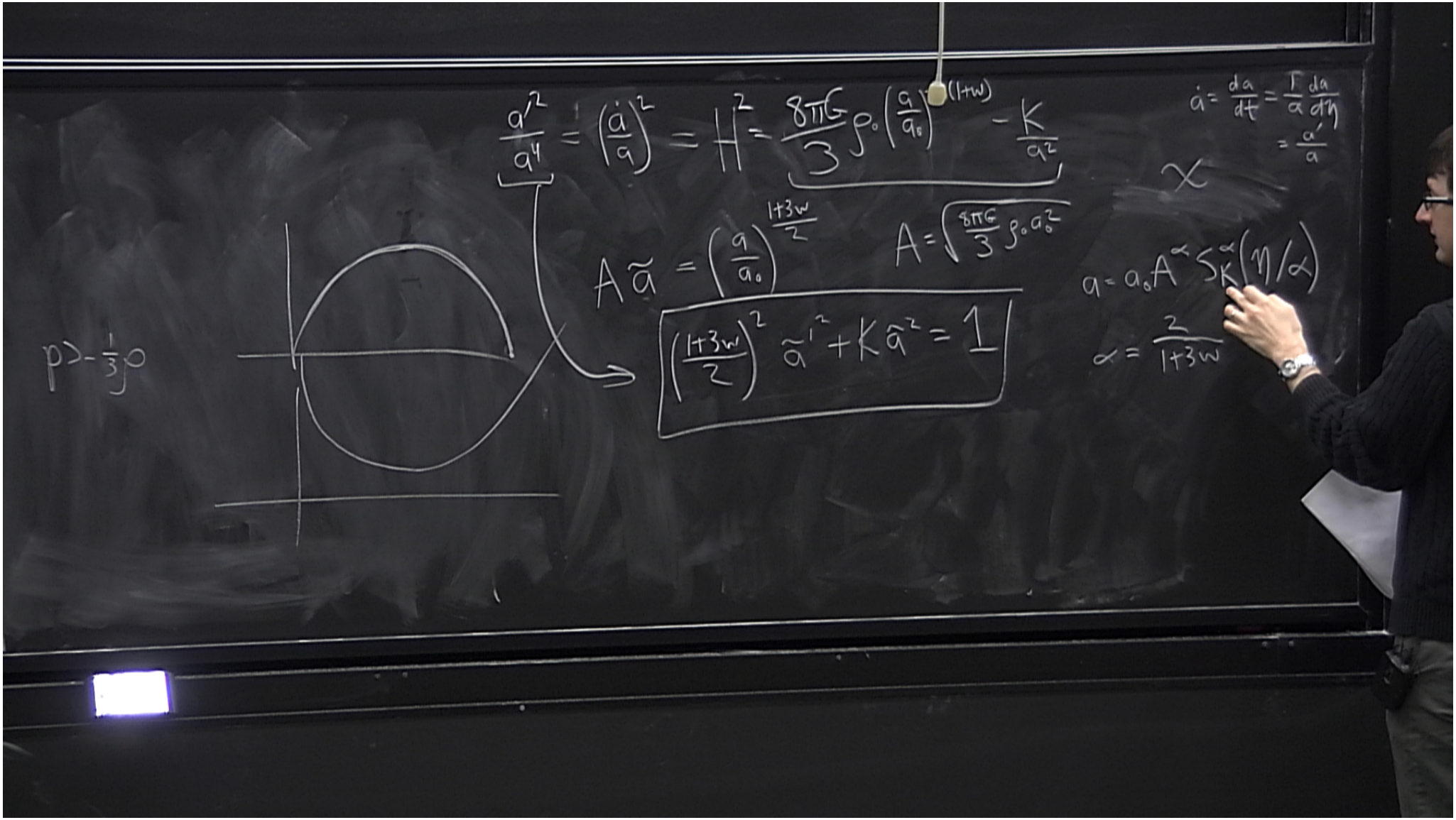
$$\boxed{\left(\frac{1+3w}{2}\right)^2 \tilde{a}'^2 + K\tilde{a}^2 = 1}$$

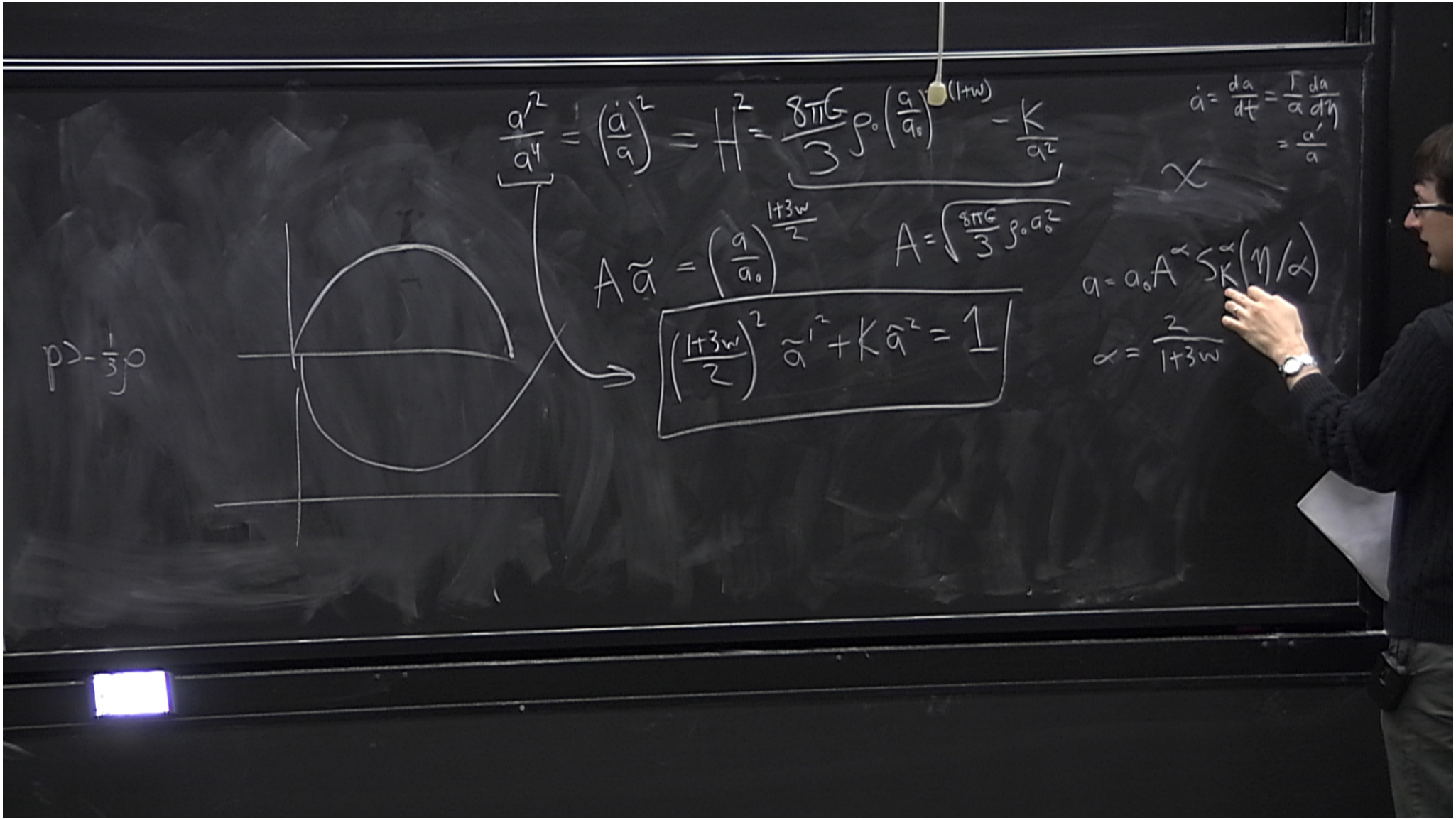
$$a = a_0 A^{\alpha} S_K^{\alpha}(\eta/\alpha)$$

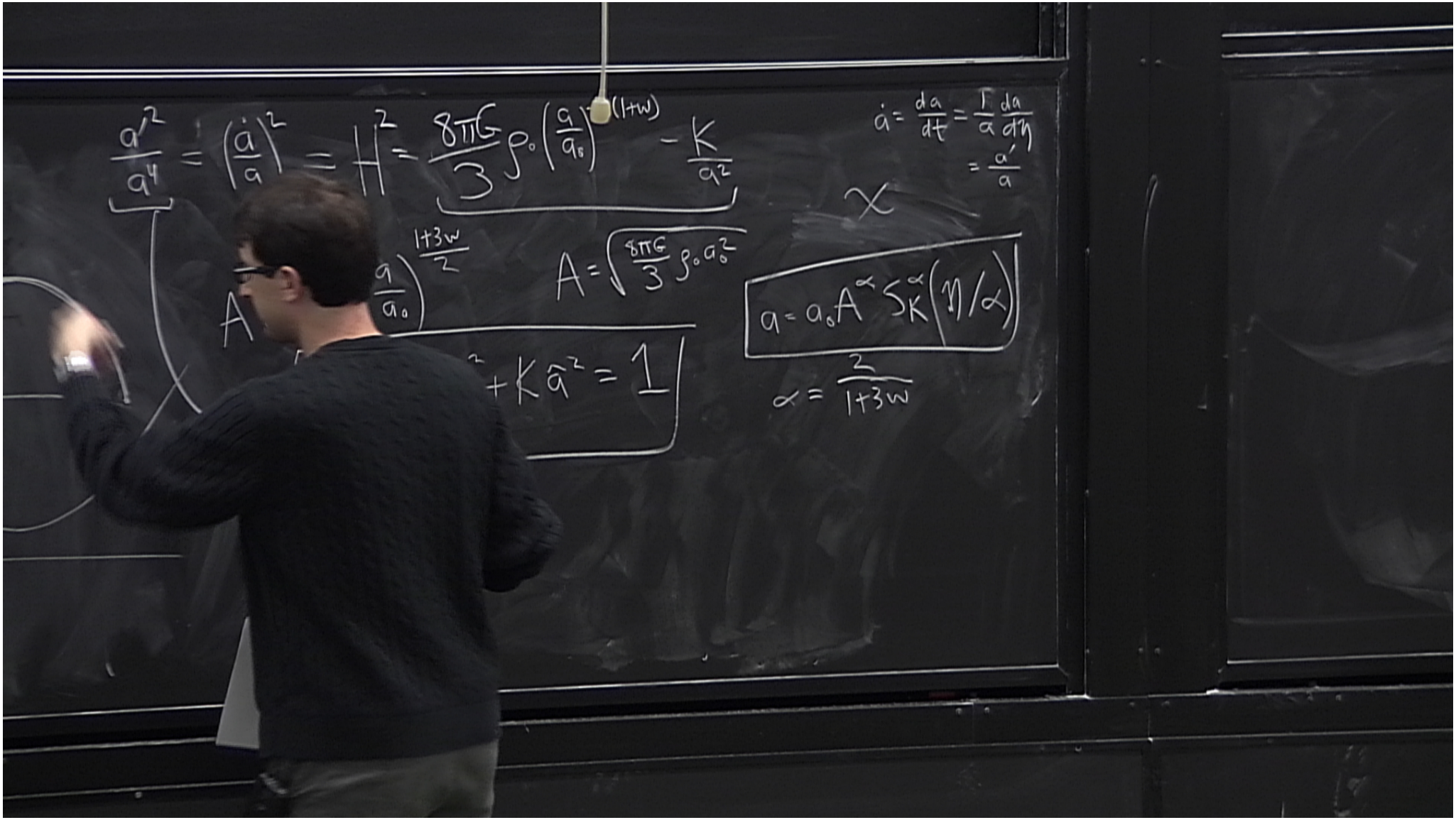
$$\alpha = \frac{2}{1+3w}$$

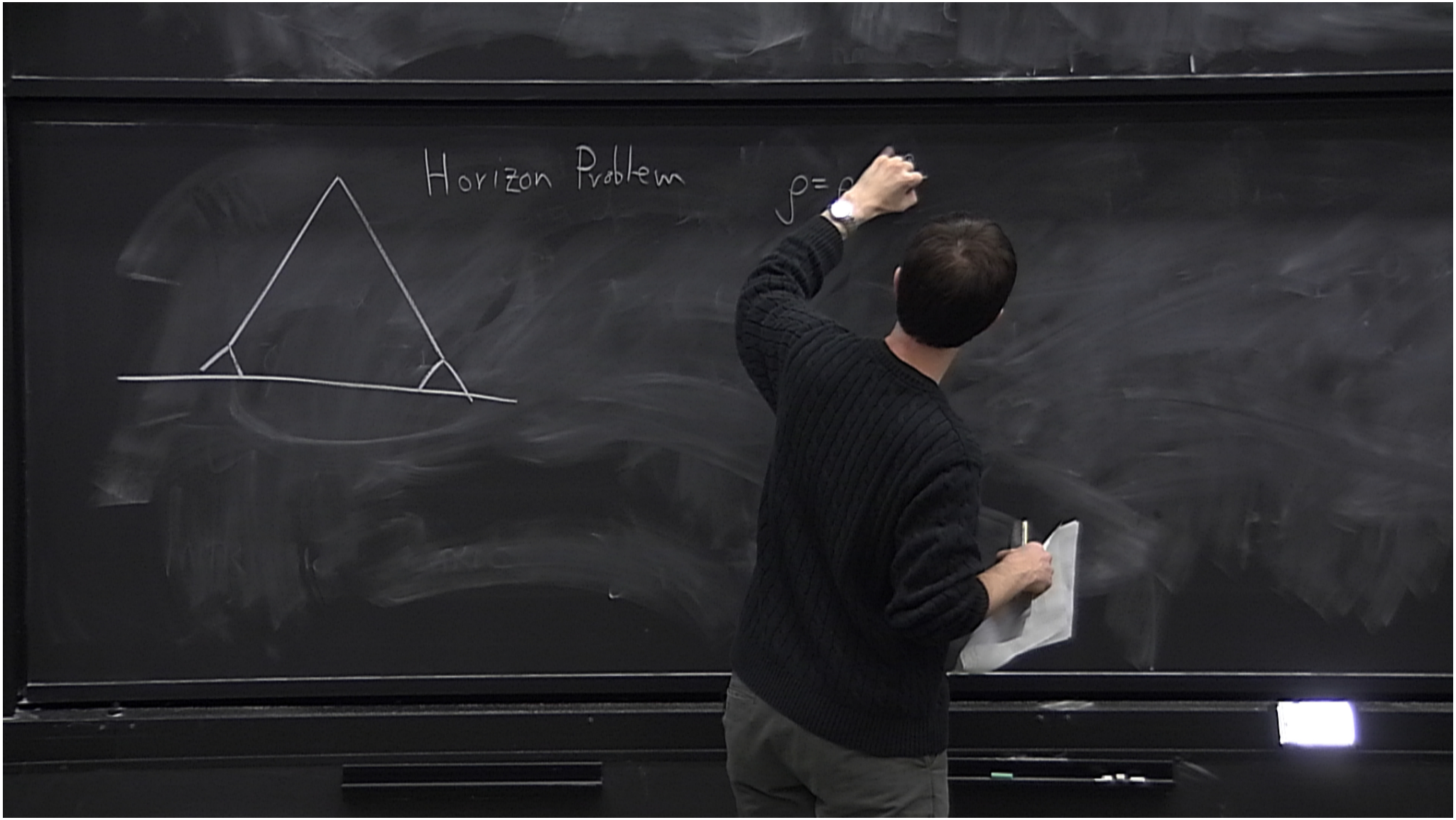
$$p > -\frac{1}{3}\rho$$




$\frac{a'^2}{a^4} = \left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_0\left(\frac{a}{a_0}\right)^{-(1+w)} - \frac{K}{a^2}$
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$\alpha = \frac{2}{1+3w}$
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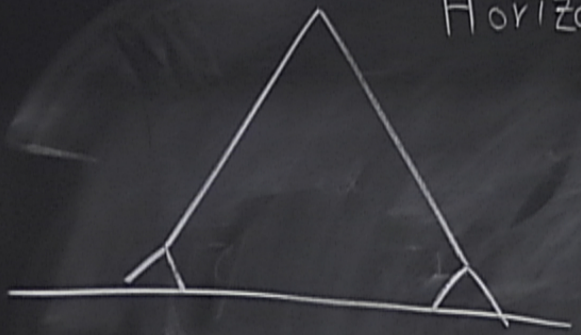

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$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$
χ
$A\tilde{a} = \left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$
$a = a_0 A^{\alpha} S_K^{\alpha}(\eta/\alpha)$
$\left(\frac{1+3w}{2}\right)^2 \tilde{a}'^2 + K\tilde{a}^2 = 1$
$\alpha = \frac{2}{1+3w}$
$p > -\frac{1}{3}\rho$

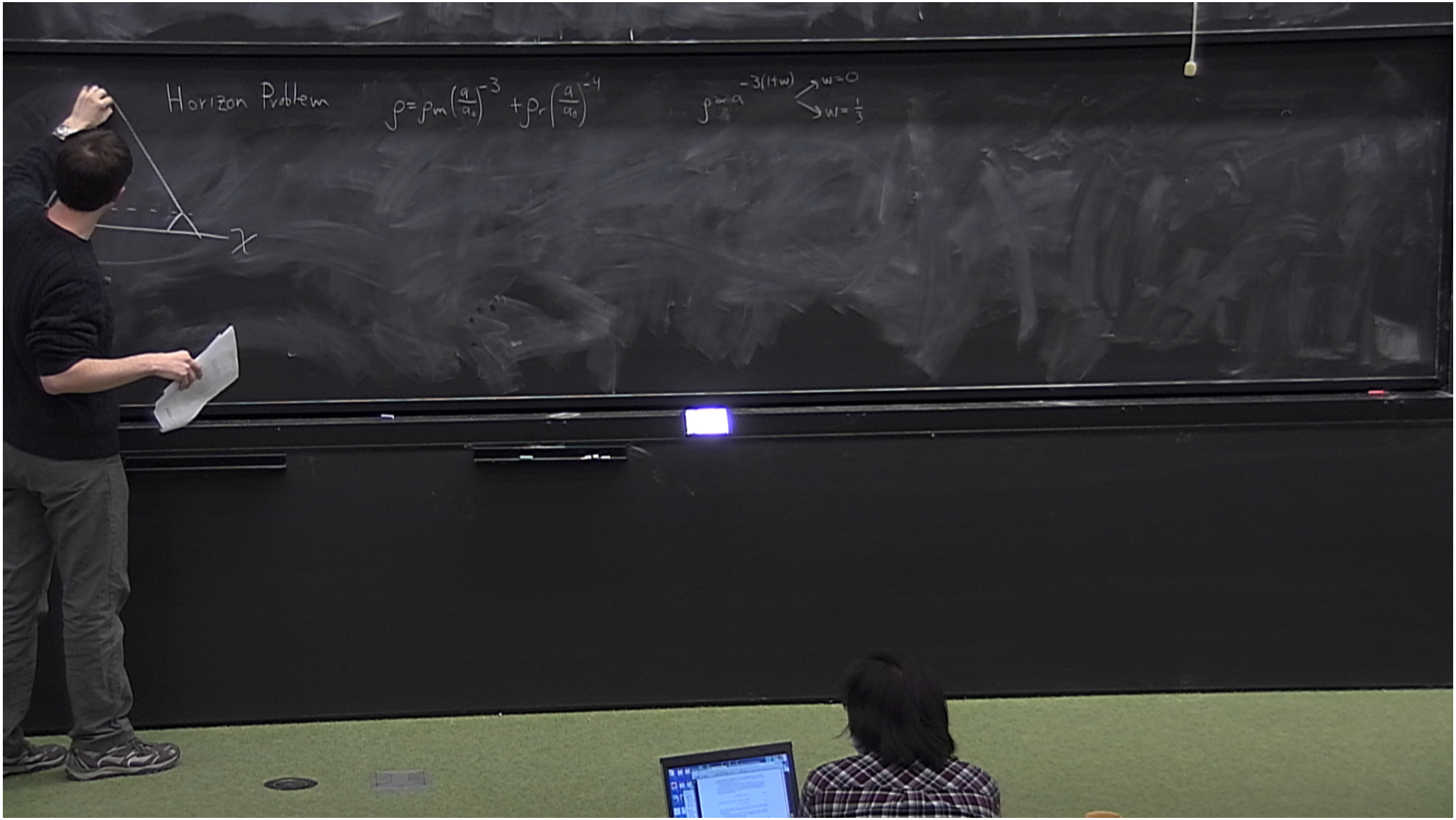

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$\dot{a} = \frac{da}{dt} = \frac{1}{a}\frac{da}{d\eta} = \frac{a'}{a}$
χ
$\left(\frac{a}{a_0}\right)^{\frac{1+3w}{2}}$
$A = \sqrt{\frac{8\pi G}{3}\rho_0 a_0^2}$
$a = a_0 A^{\alpha} S_K^{\alpha}(\eta/\alpha)$
$+Ka^2 = 1$
$\alpha = \frac{2}{1+3w}$

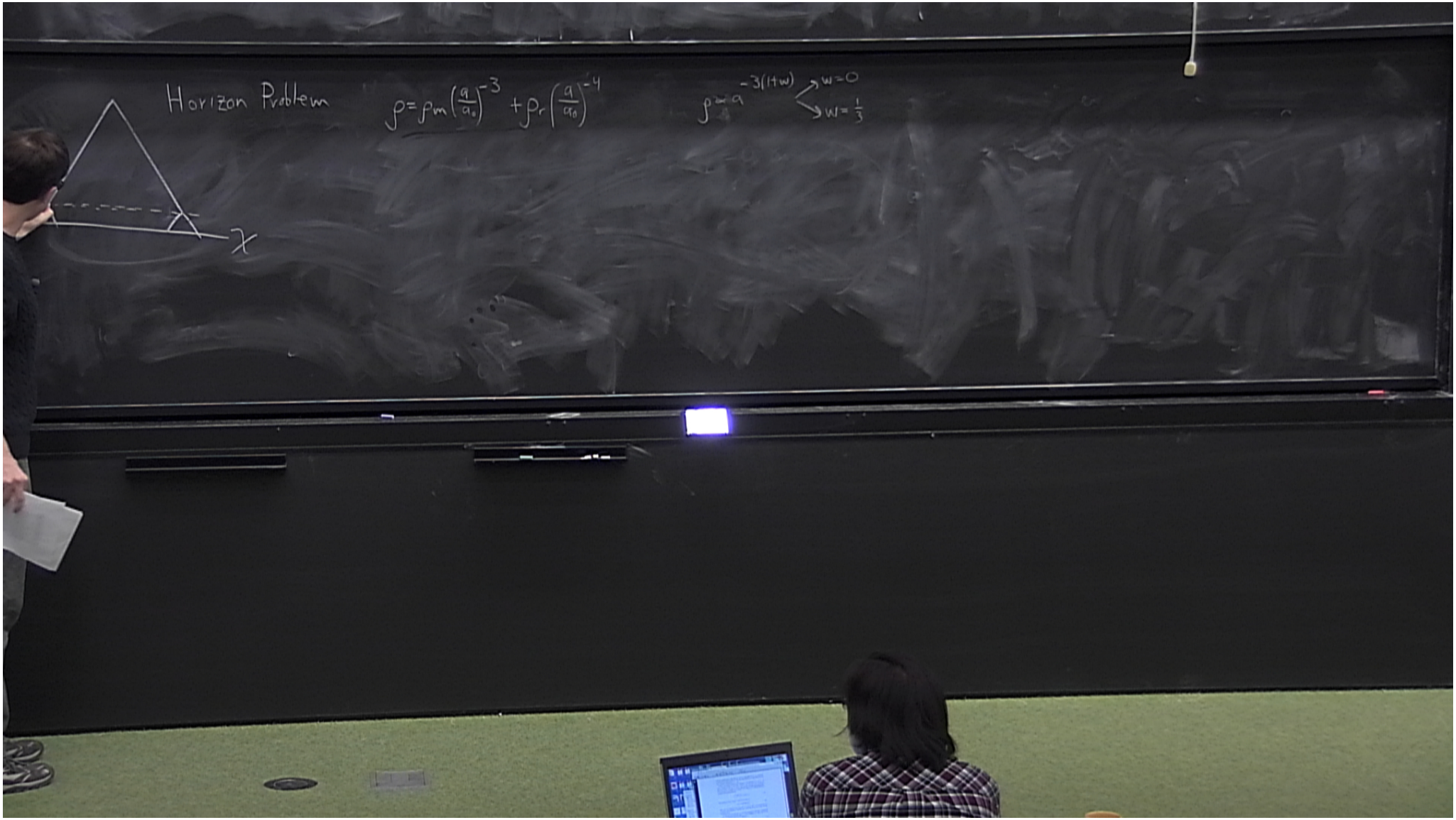

Horizon Problem

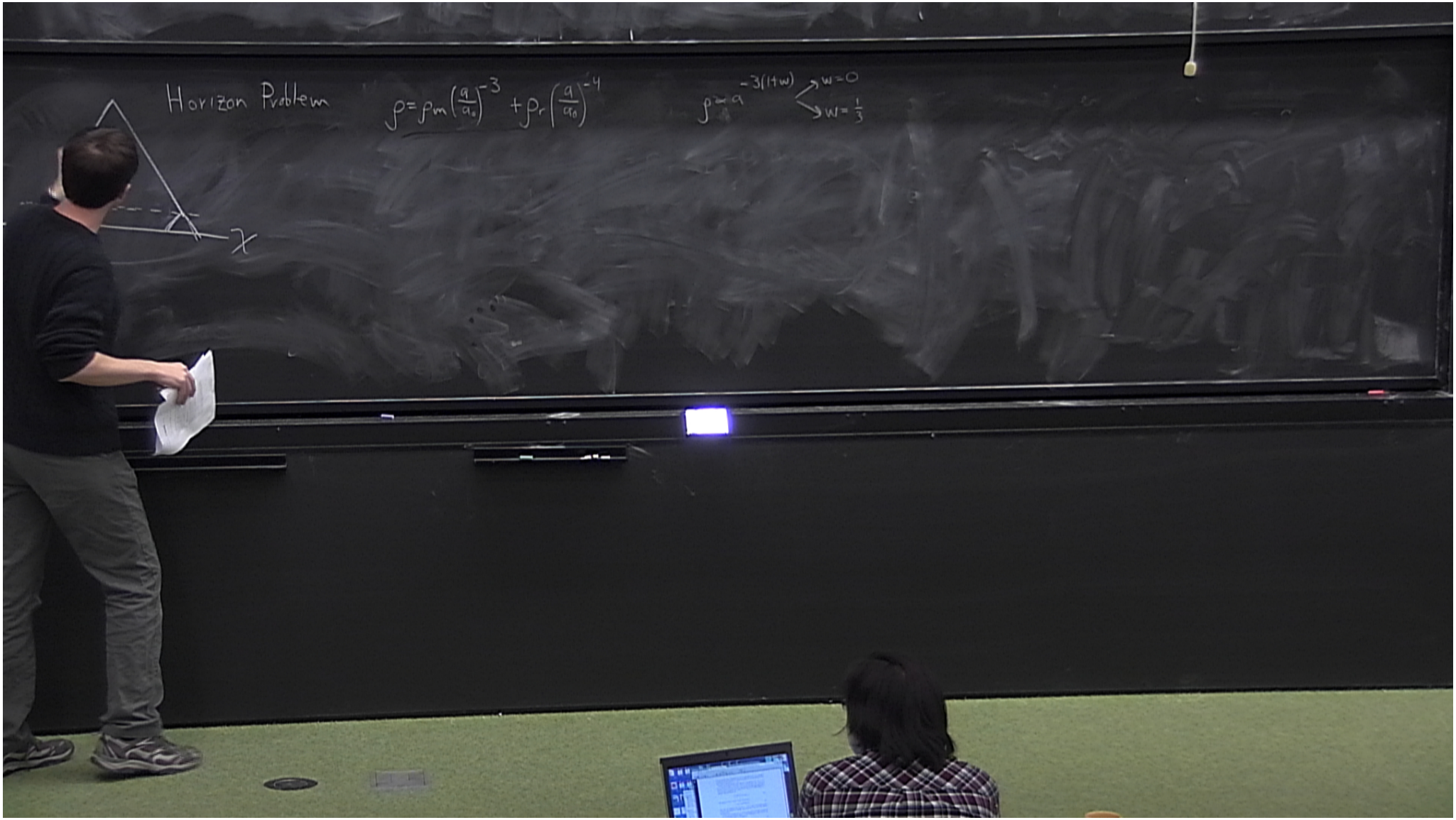
Horizon Problem

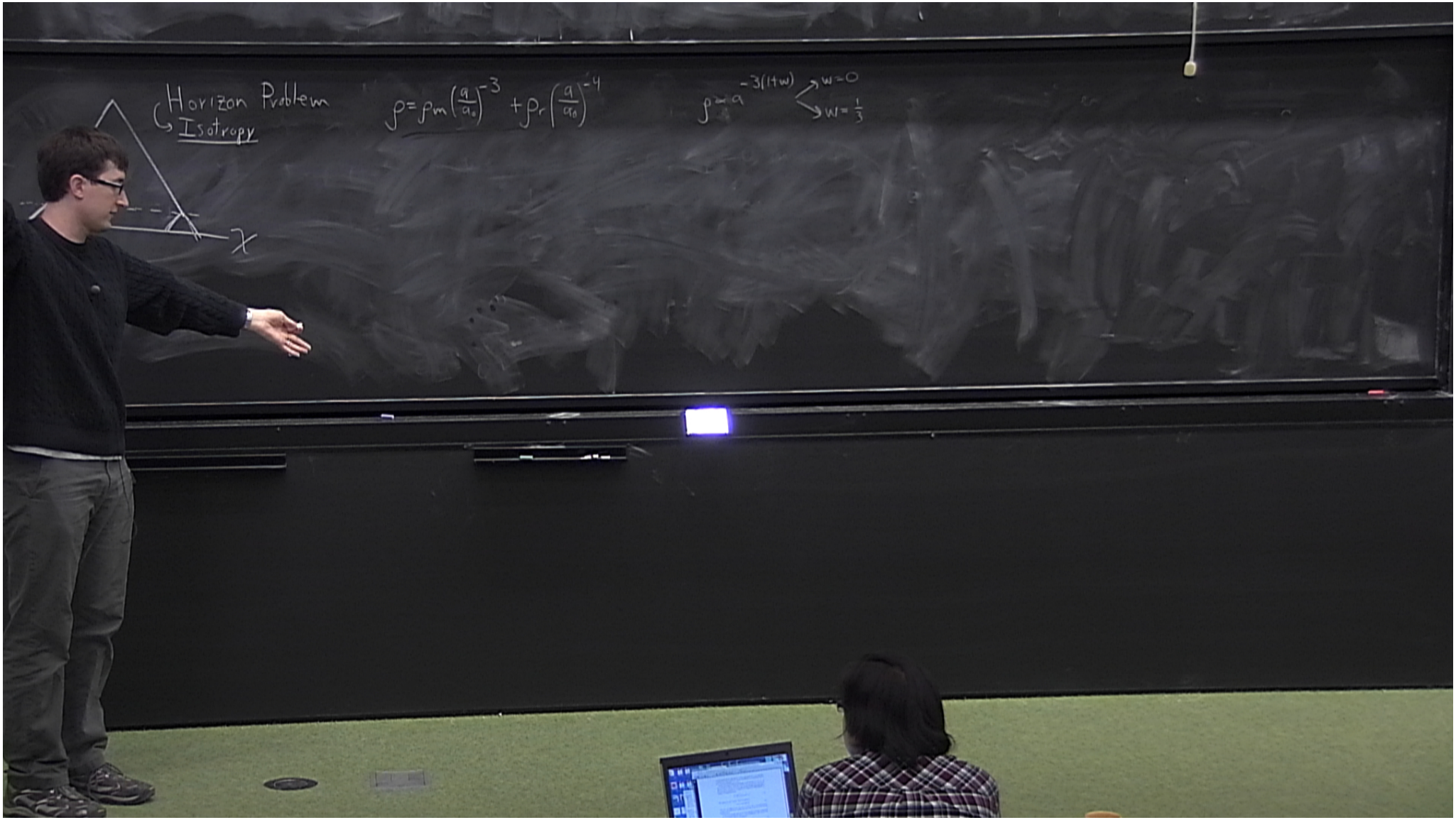
$$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$$

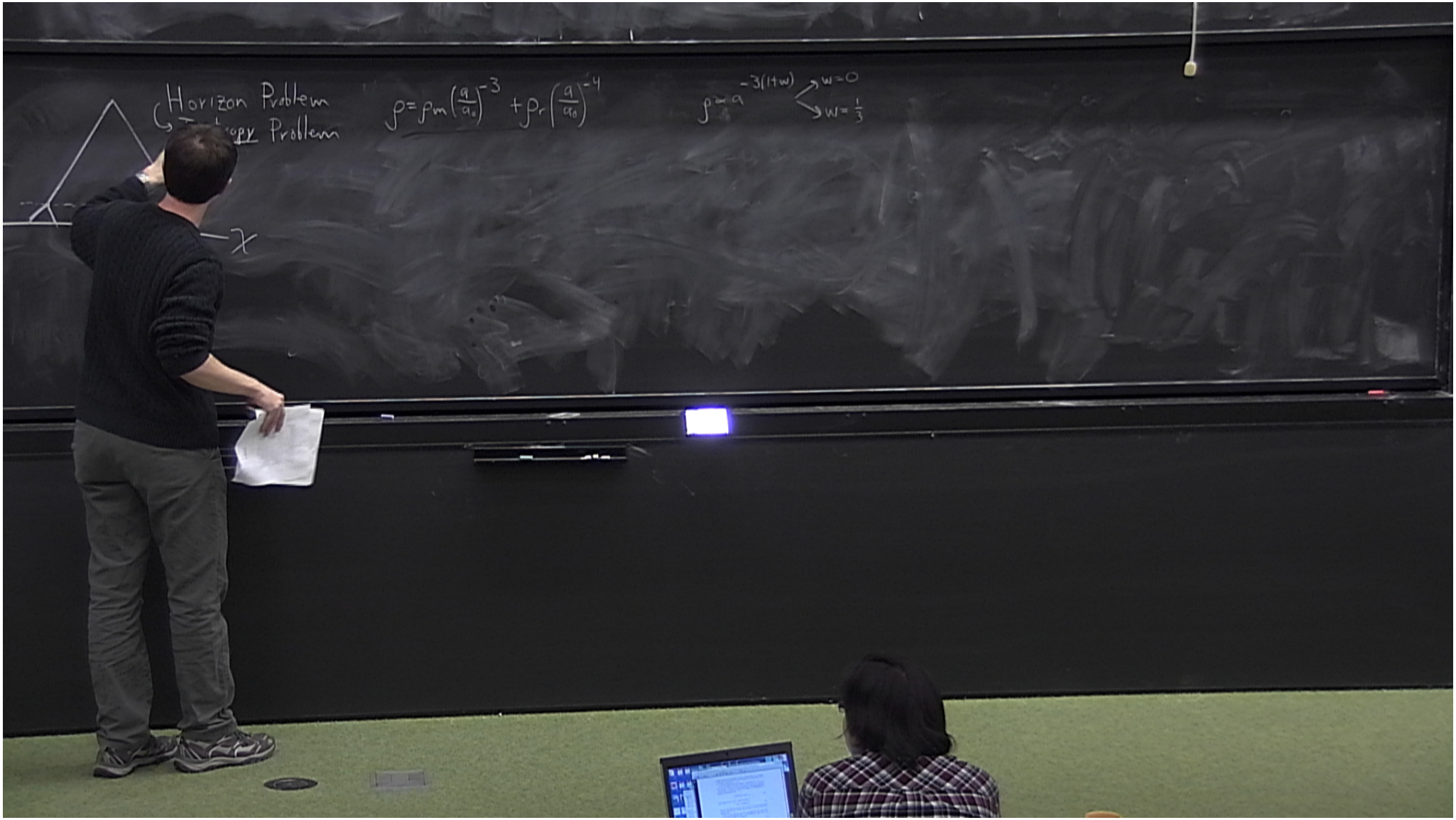


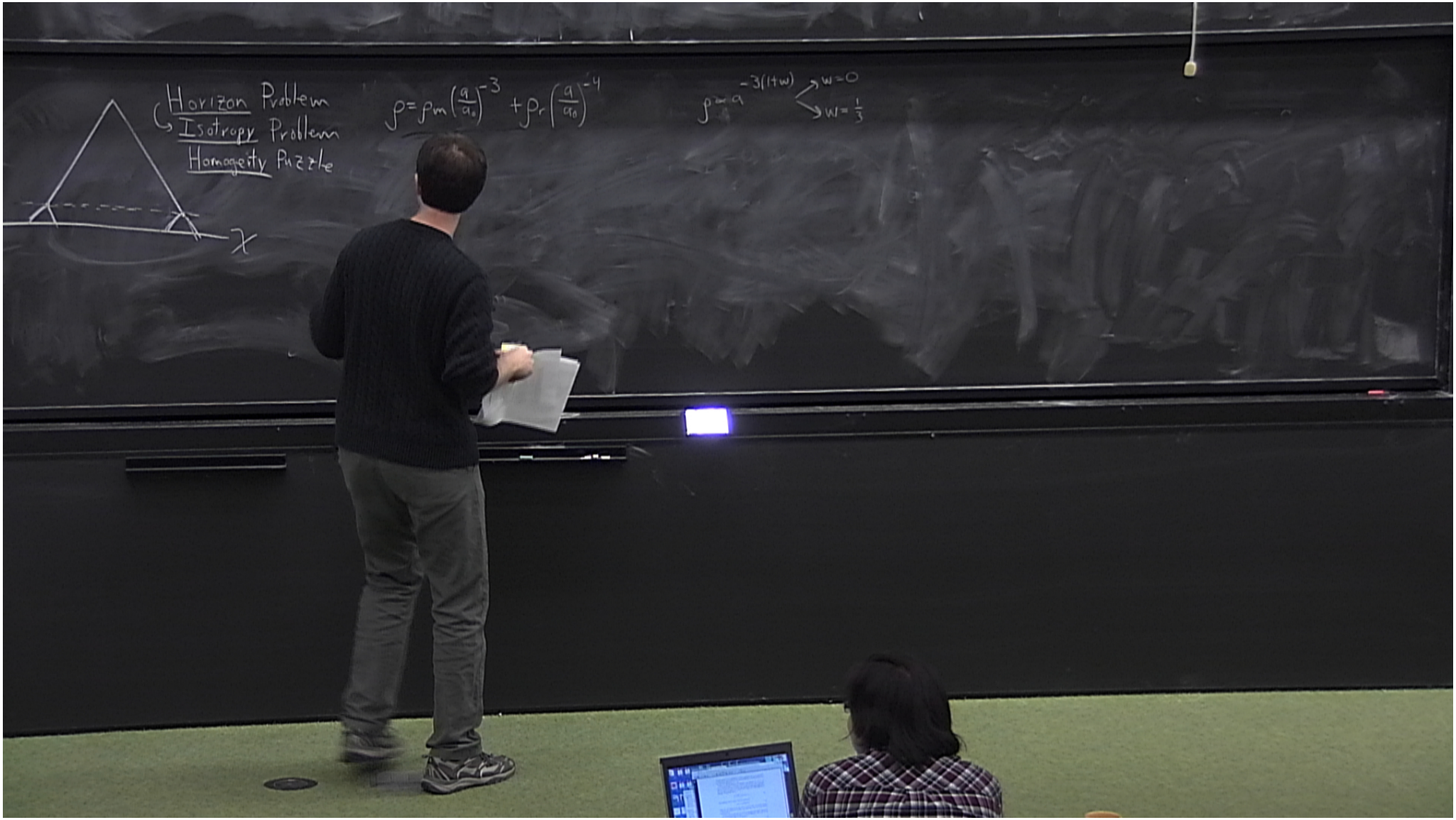

Horizon Problem
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
χ

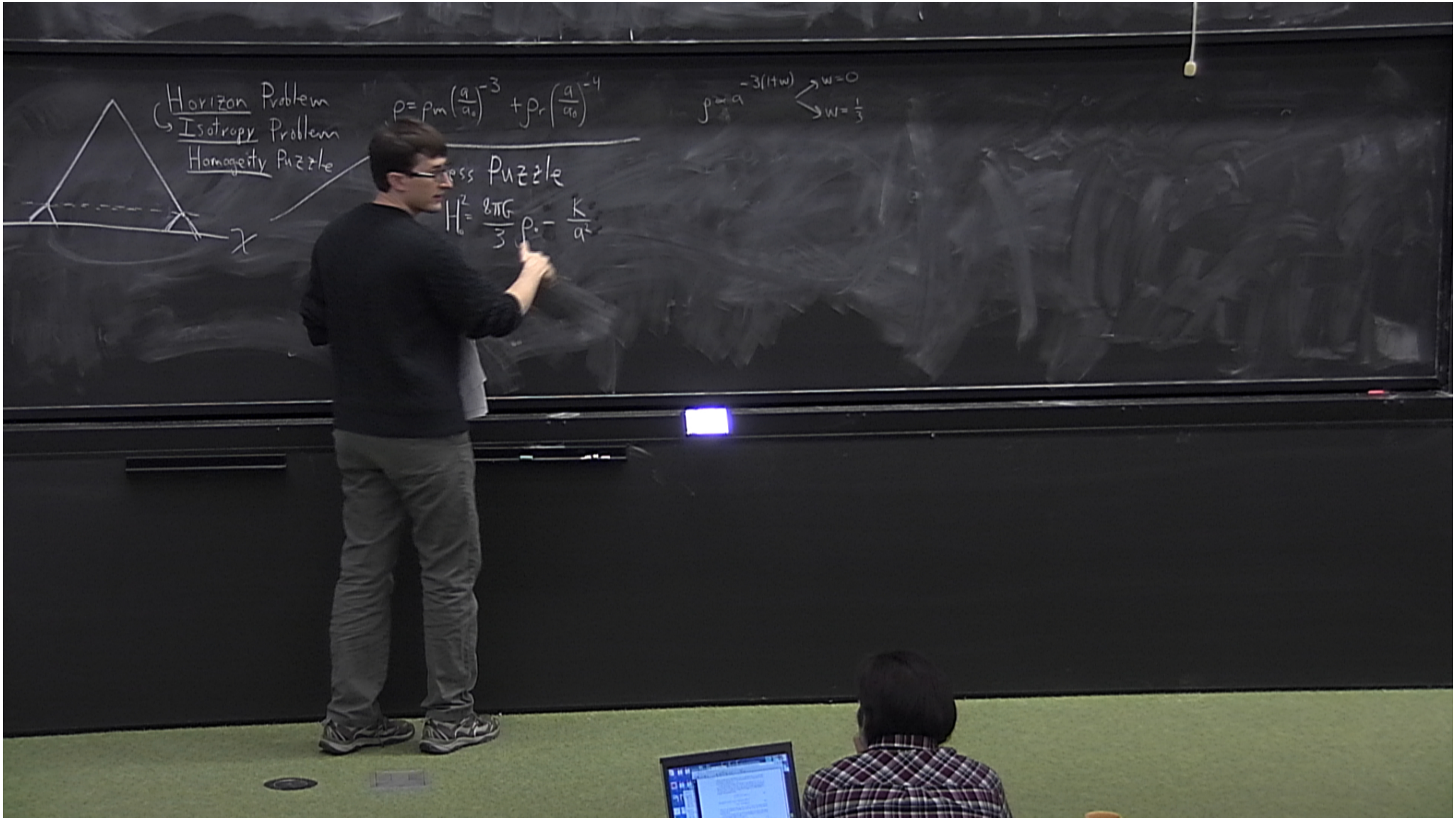

Horizon Problem
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χ

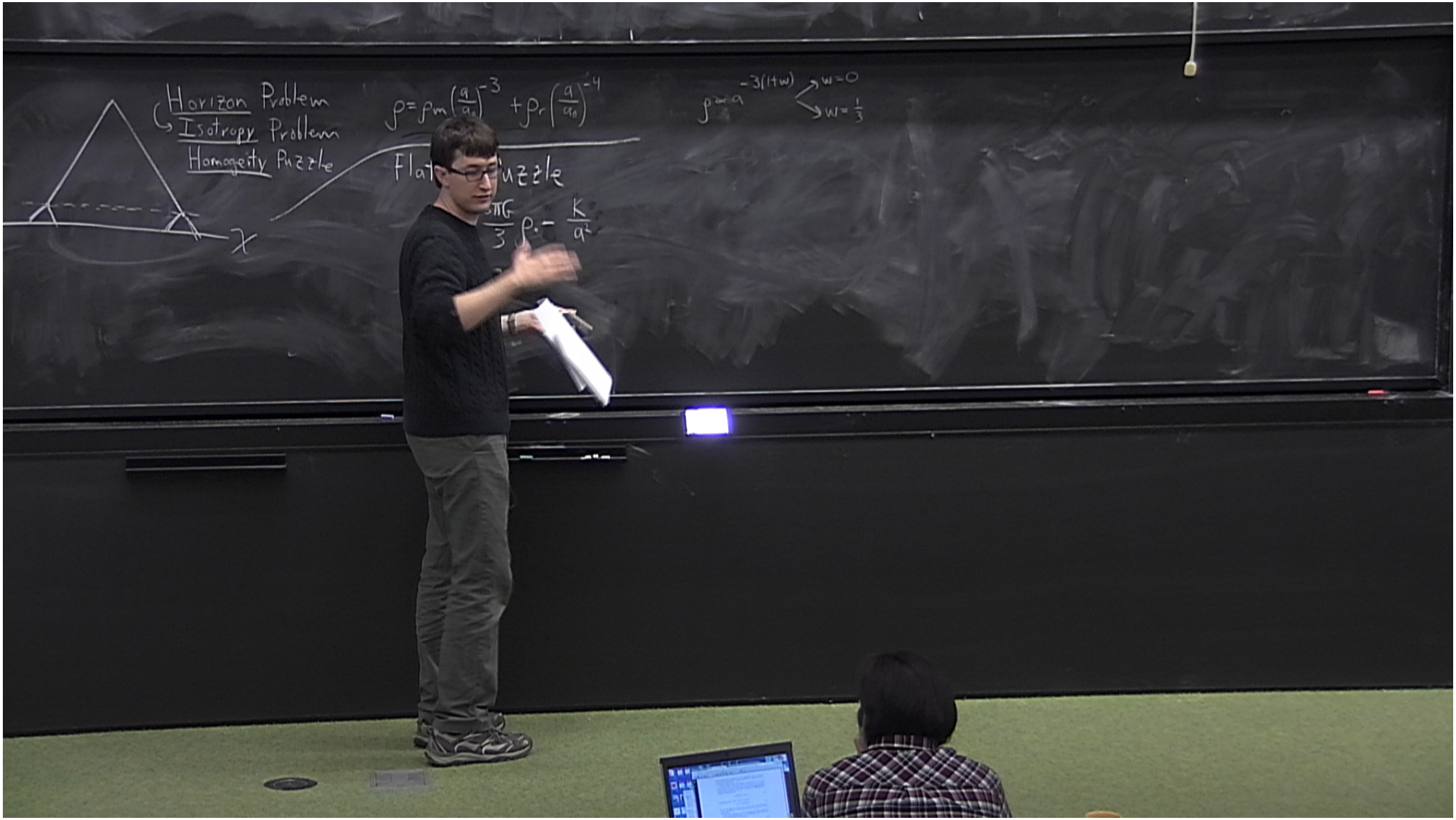

Horizon Problem
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
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χ

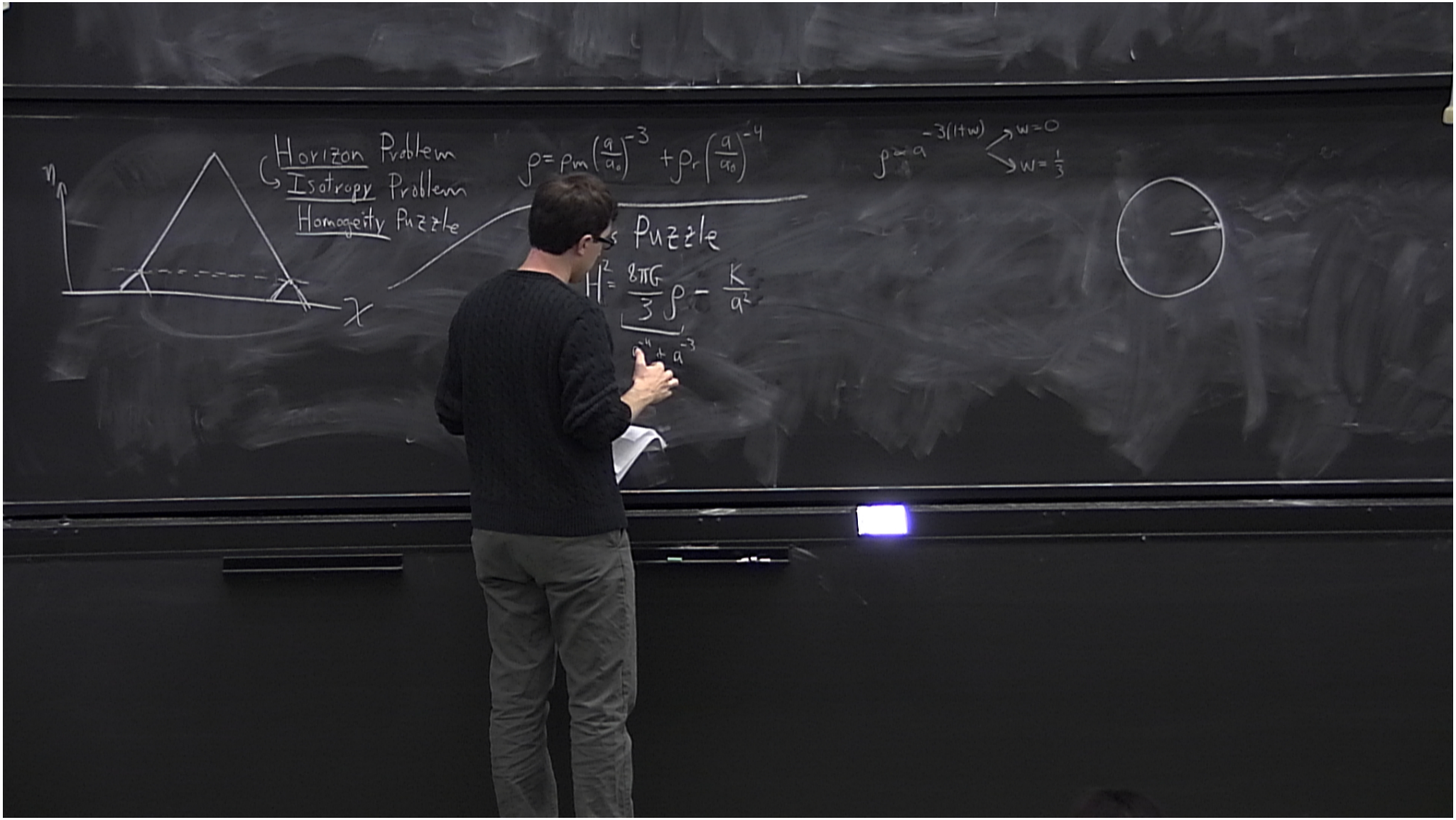

Horizon Problem
Isotropy
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
x

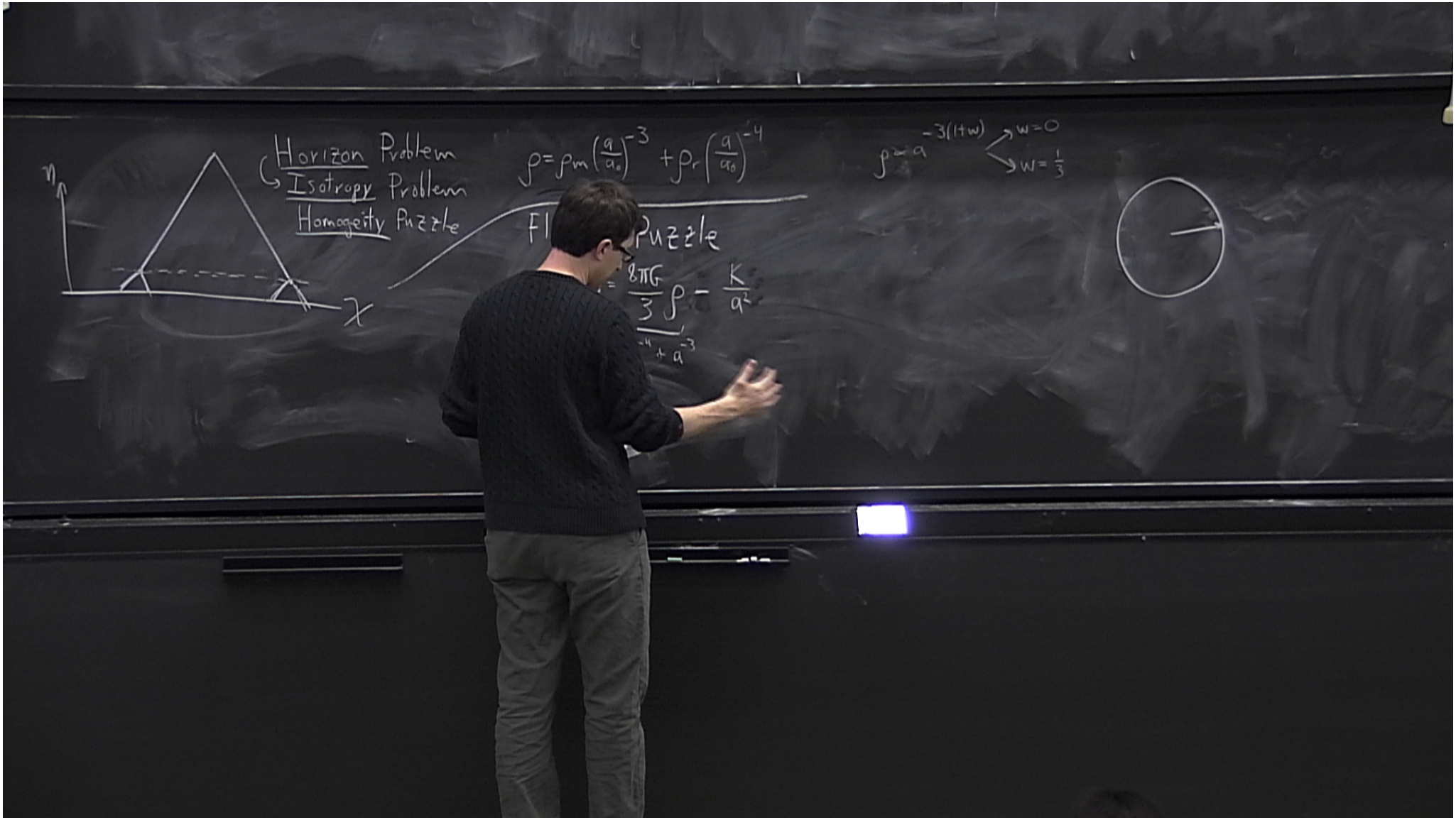

Horizon Problem
Problem
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
χ

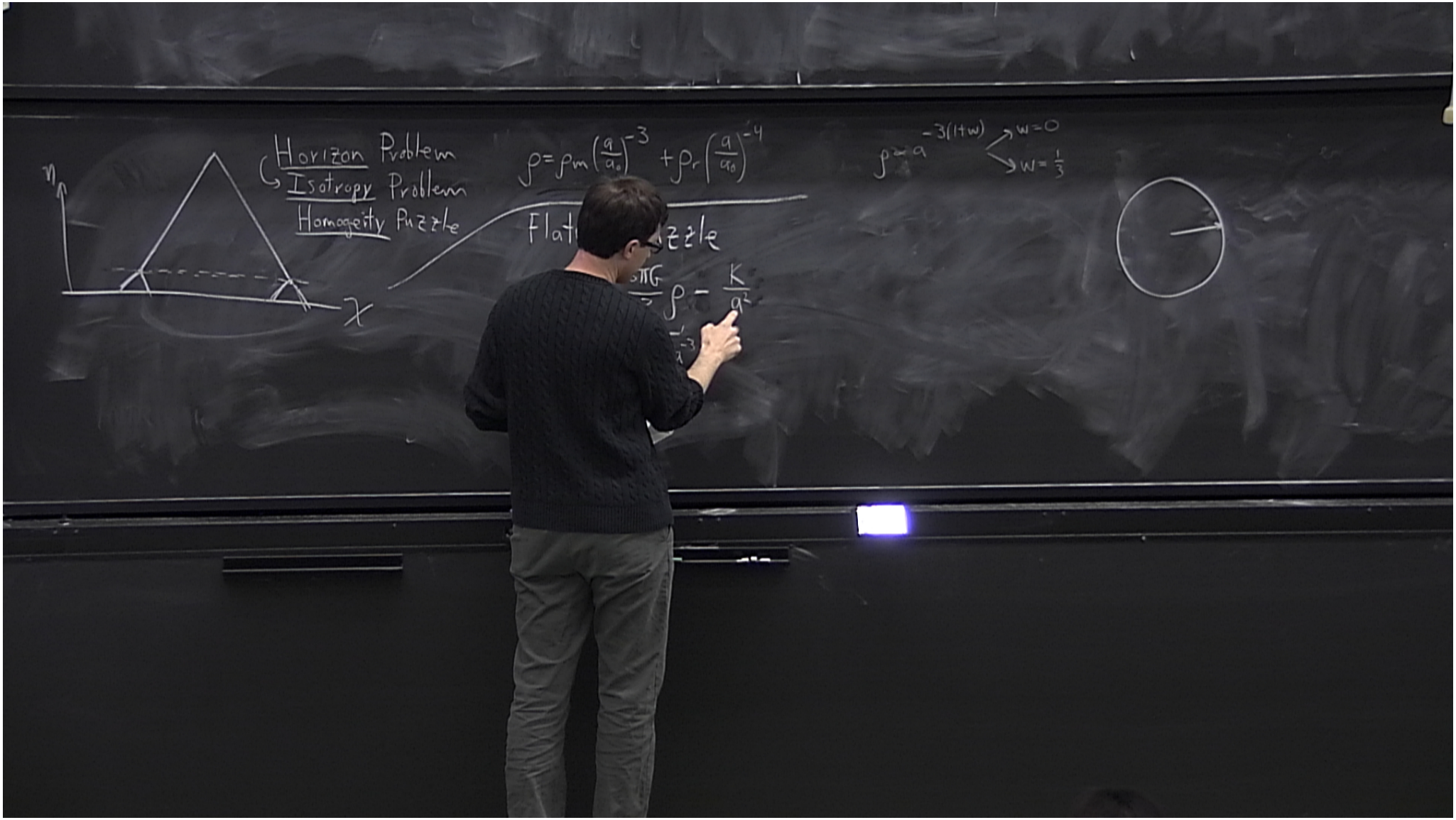

Horizon Problem
Isotropy Problem
Homogeneity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
χ

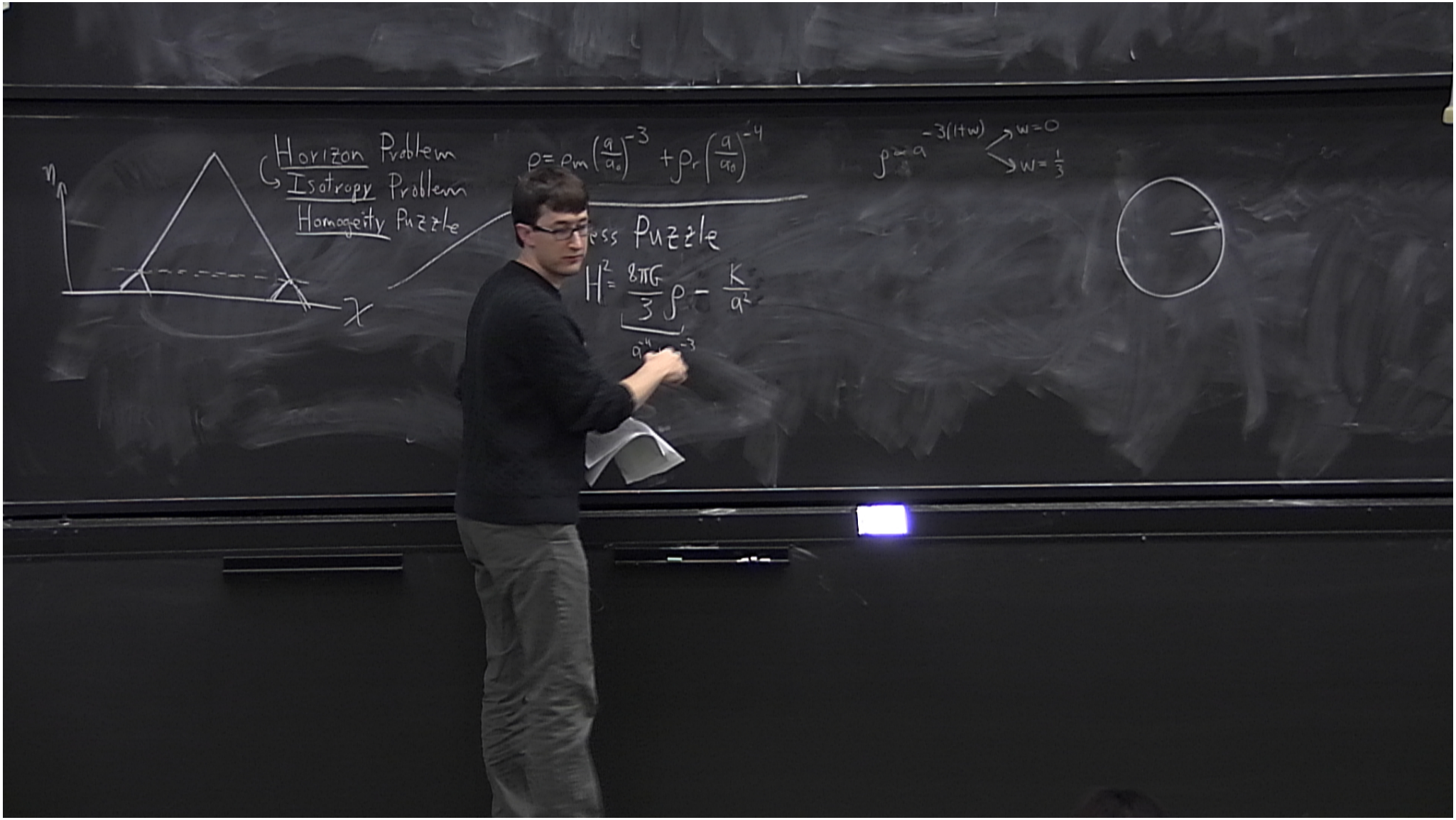

Horizon Problem
Isotropy Problem
Homogeneity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
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$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$

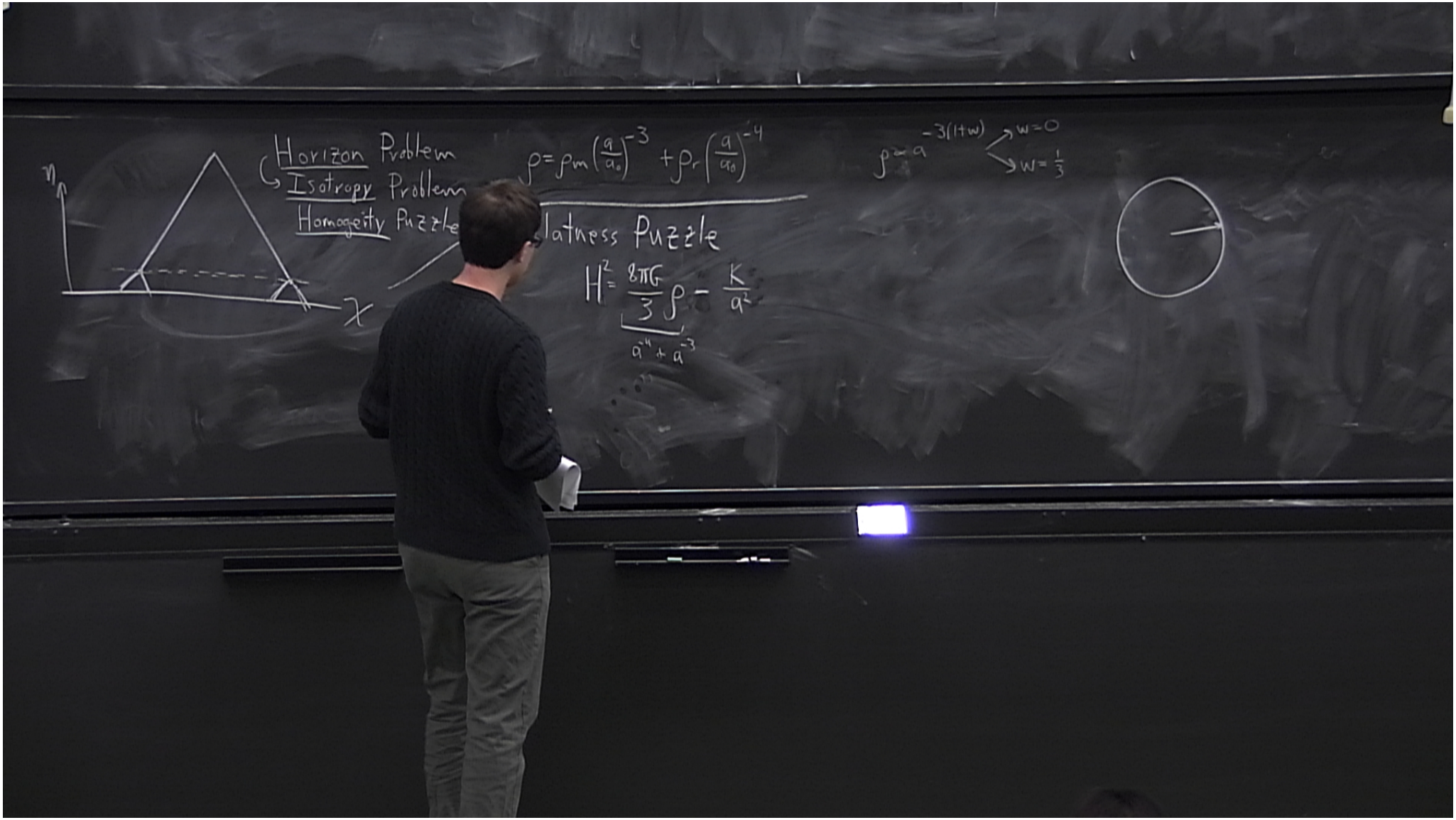

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Flat
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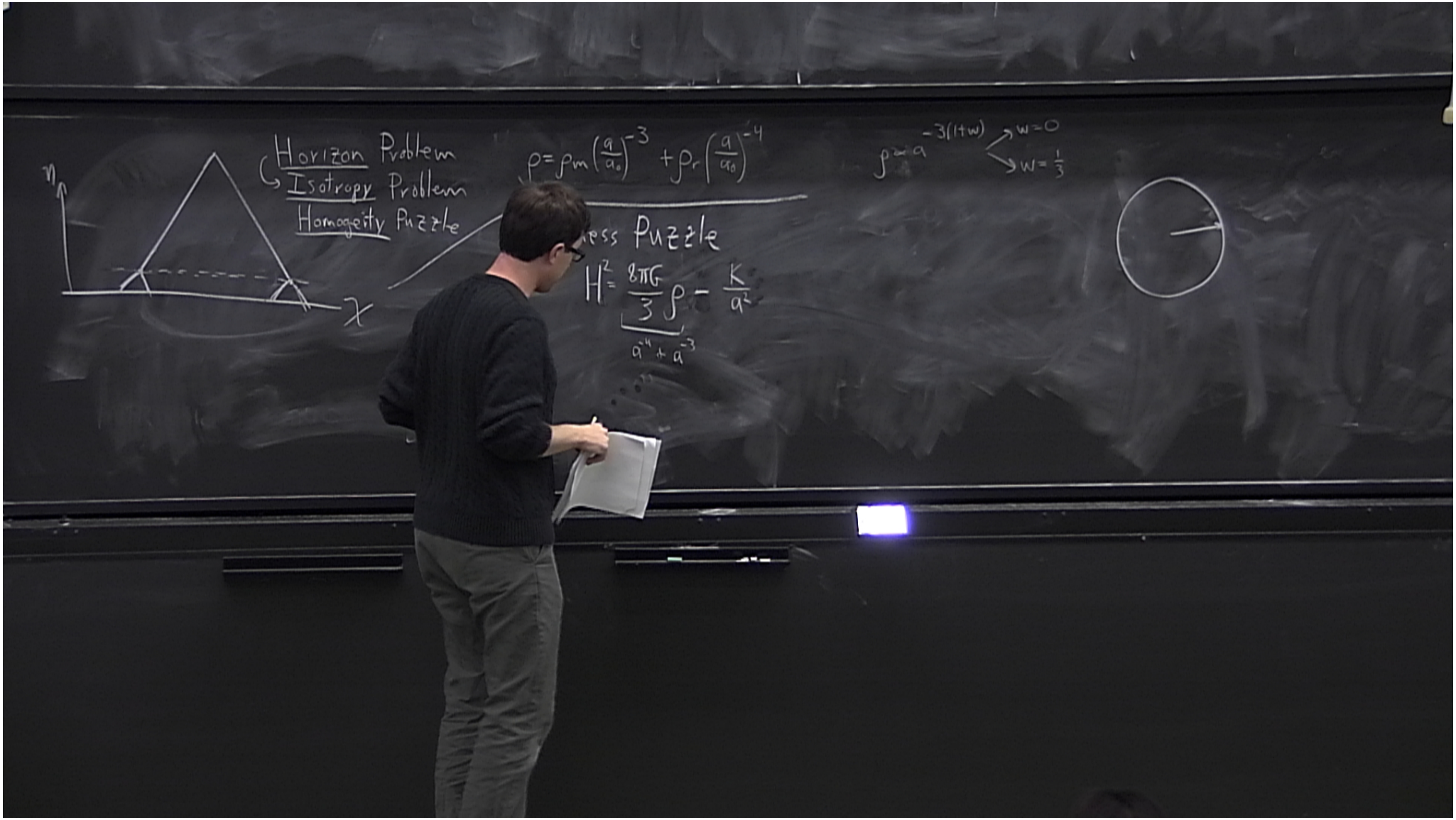

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Puzzle
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$a^{-4} + a^{-3}$
η
χ

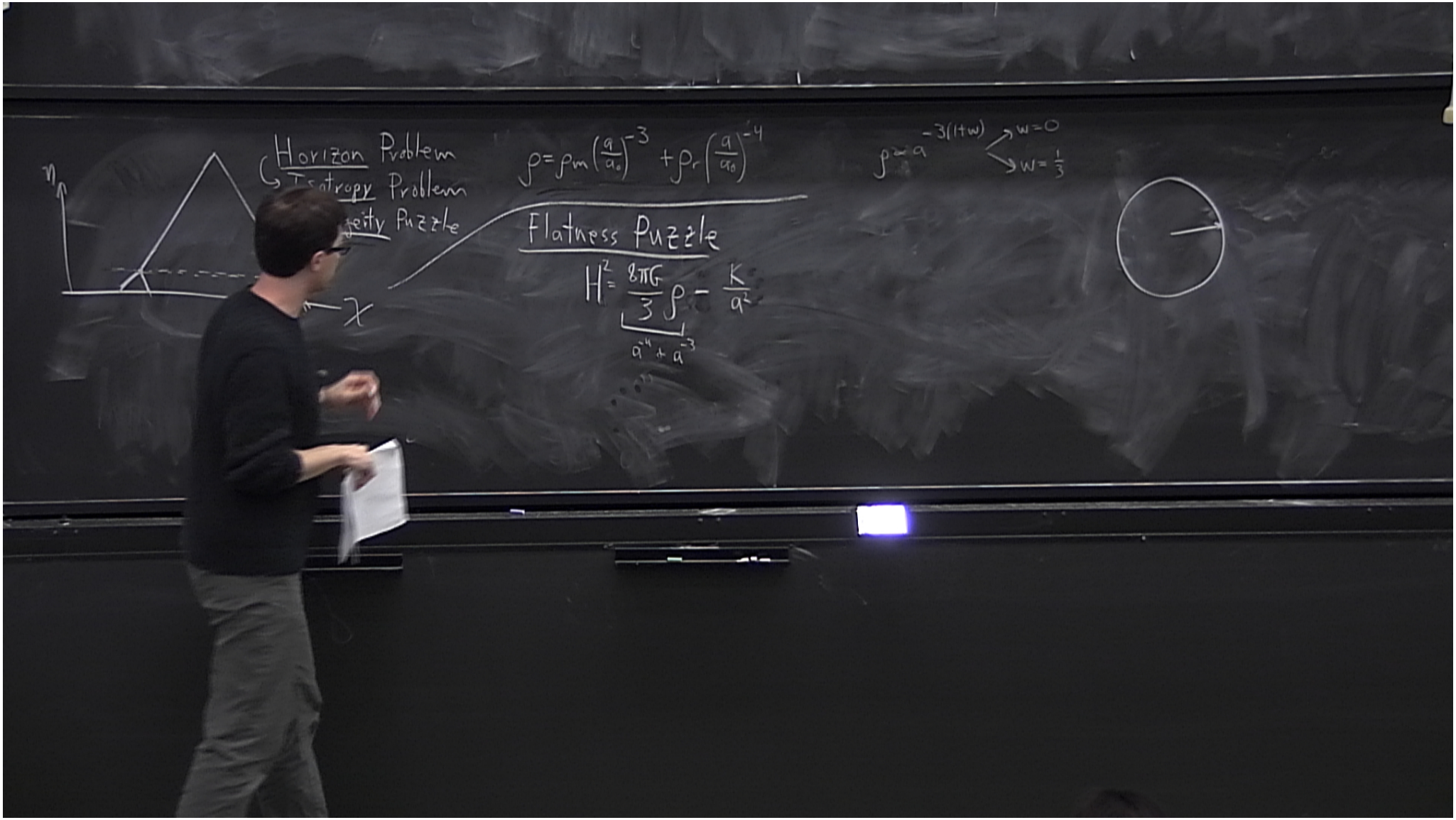

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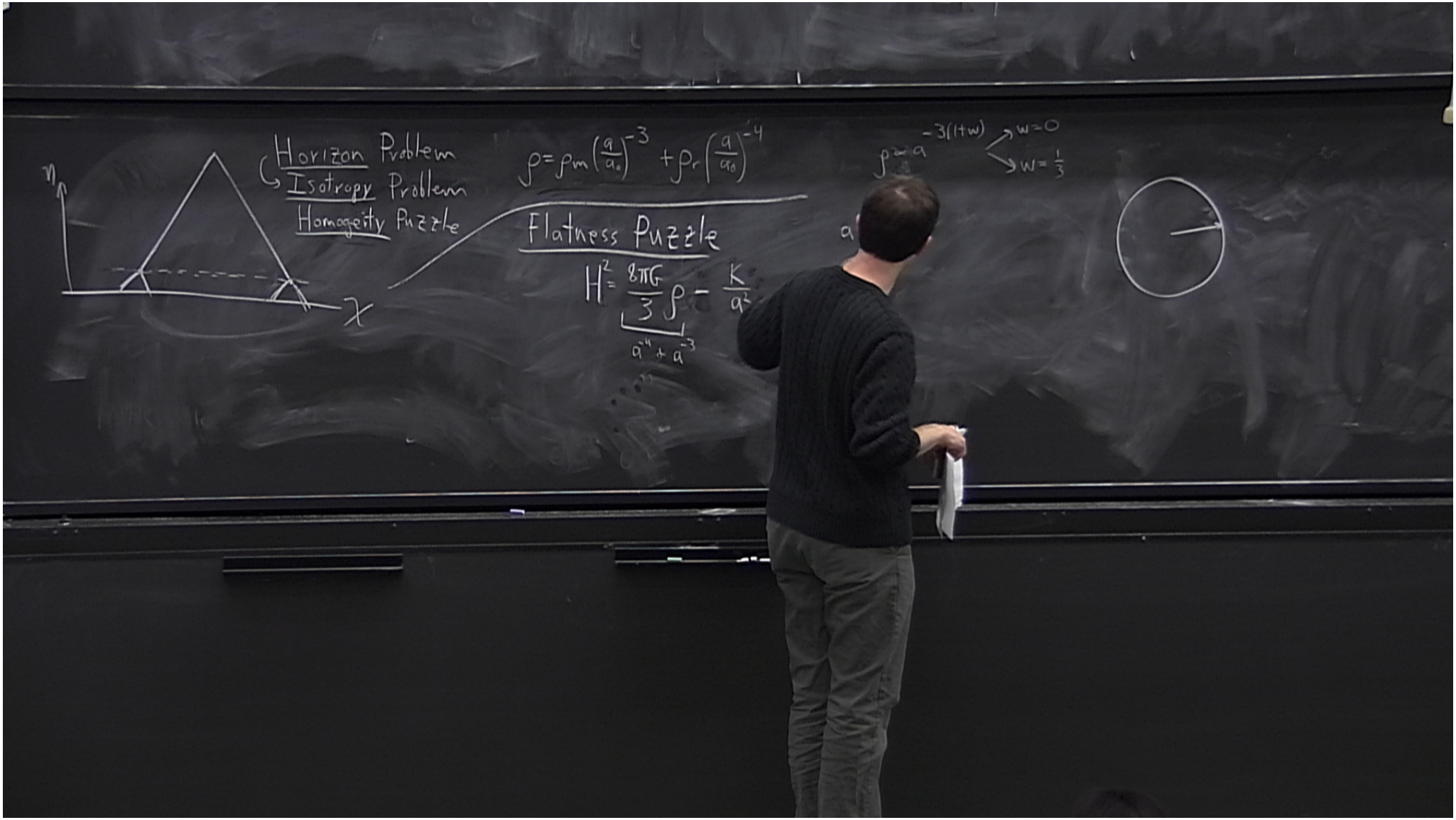

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η
χ
$\rho - \frac{K}{a^2}$

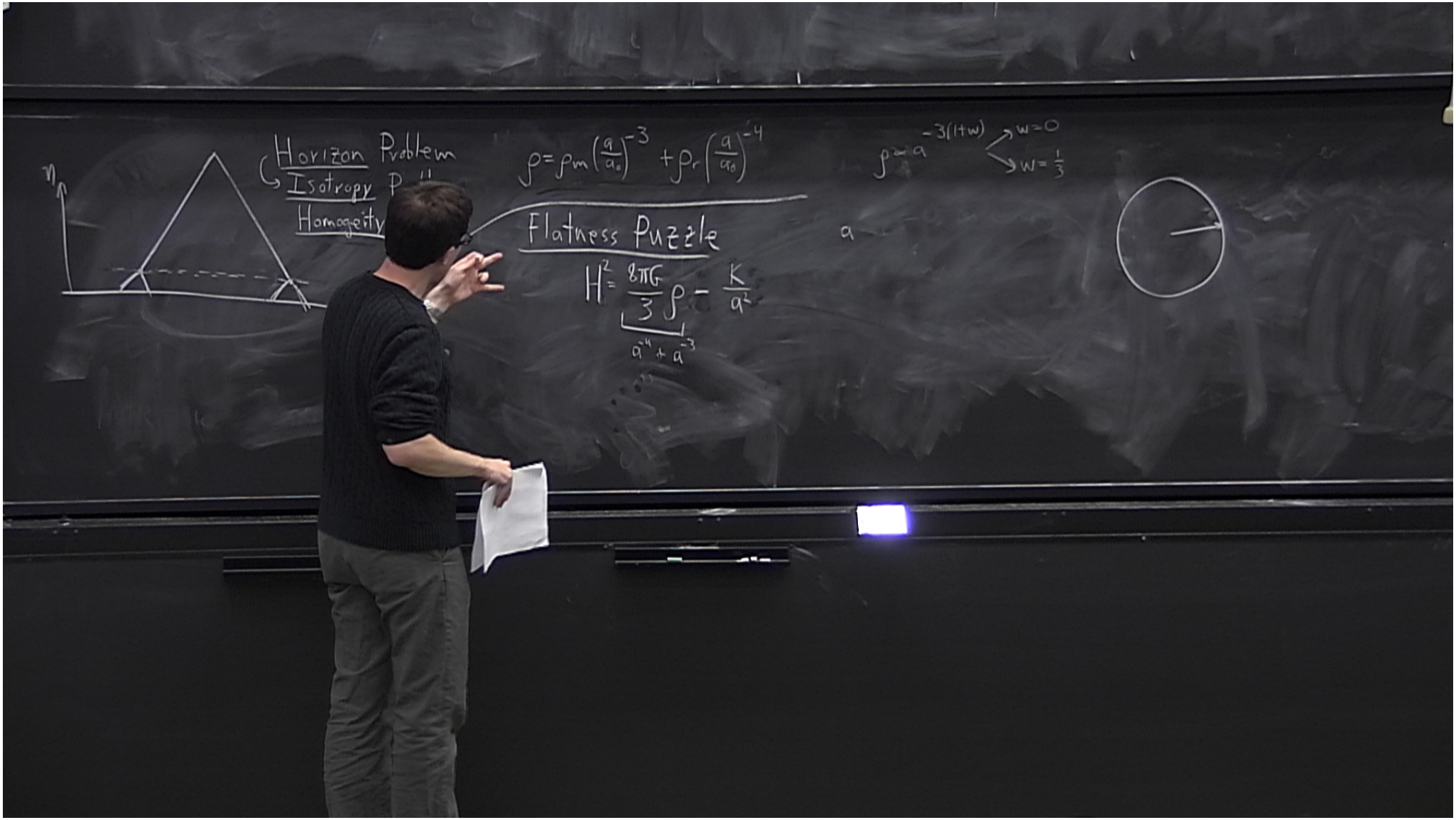

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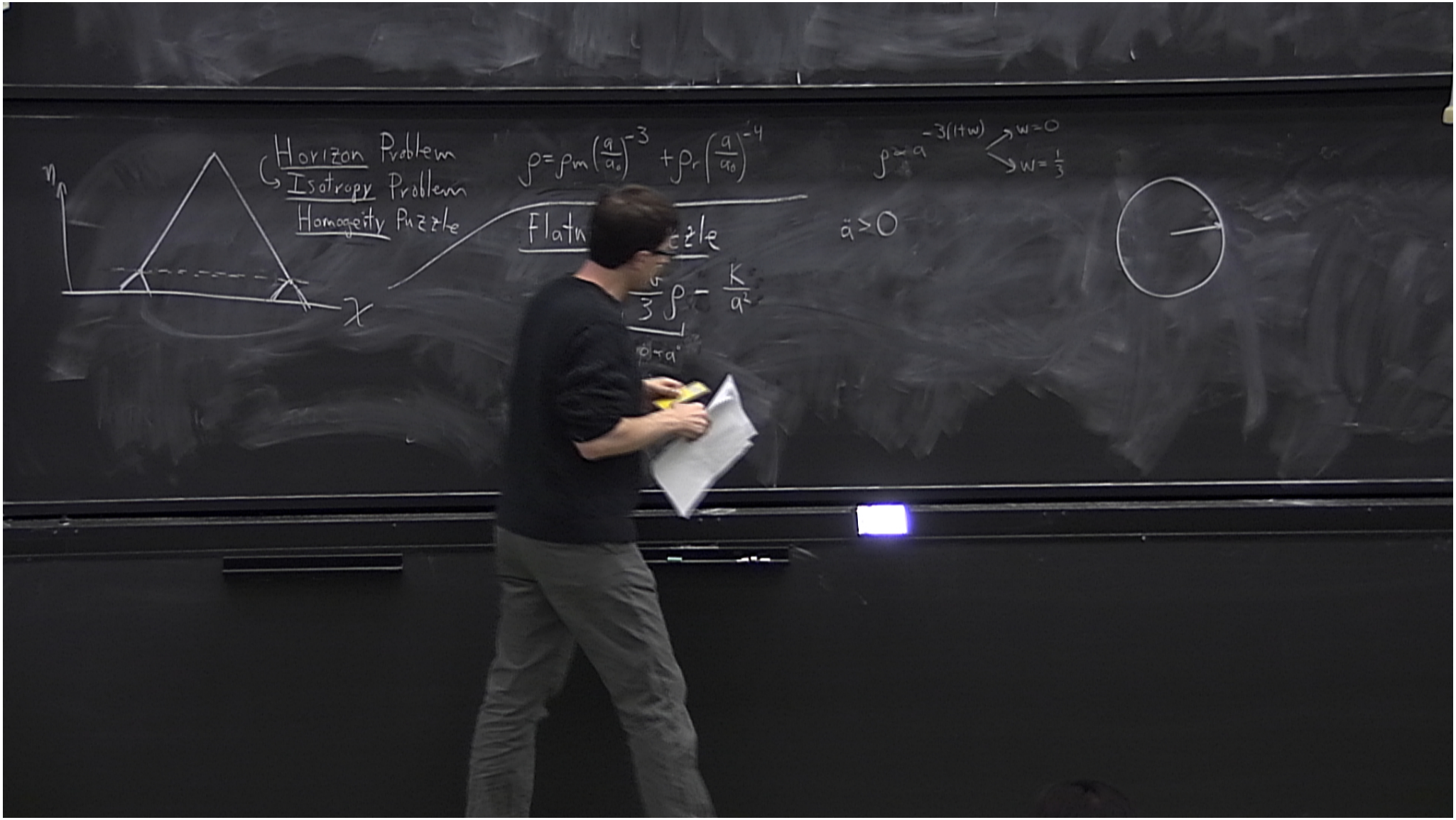

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χ

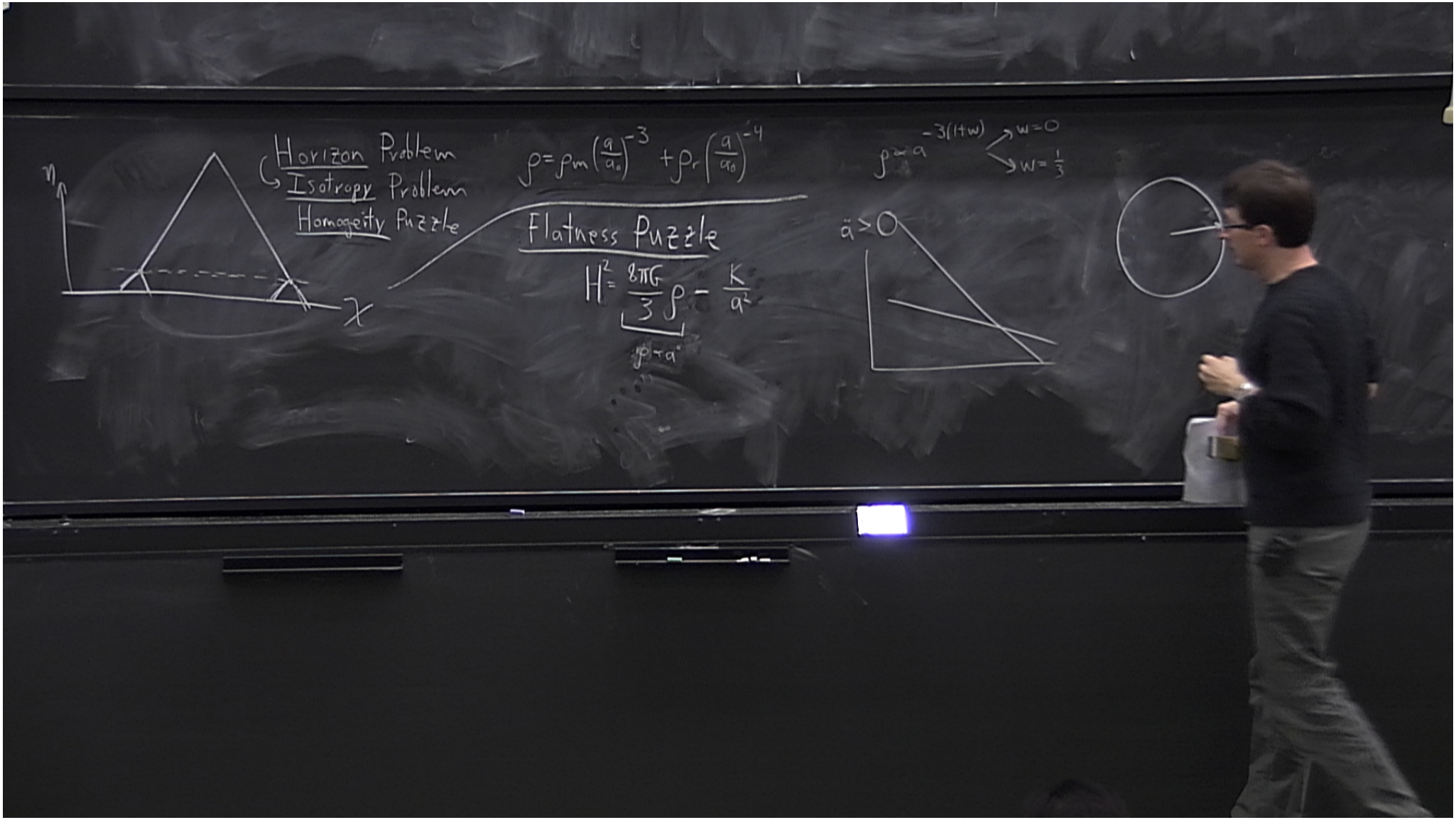

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Homogeity Puzzle
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$a^{-4} + a^{-3}$
η
x

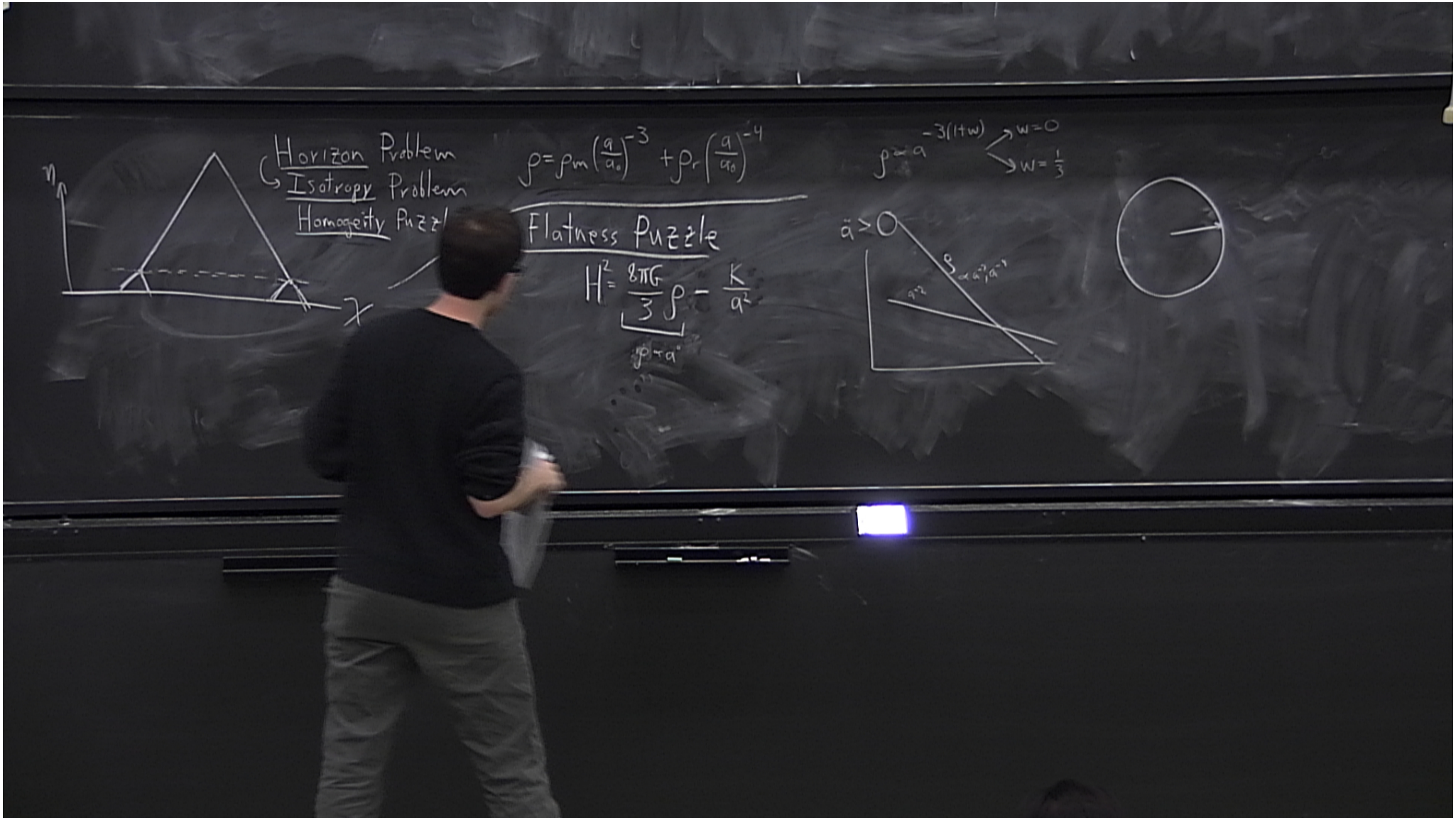

Horizon Problem
Isotropy Problem
Flatness Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \sim a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$a^{-4} + a^{-3}$

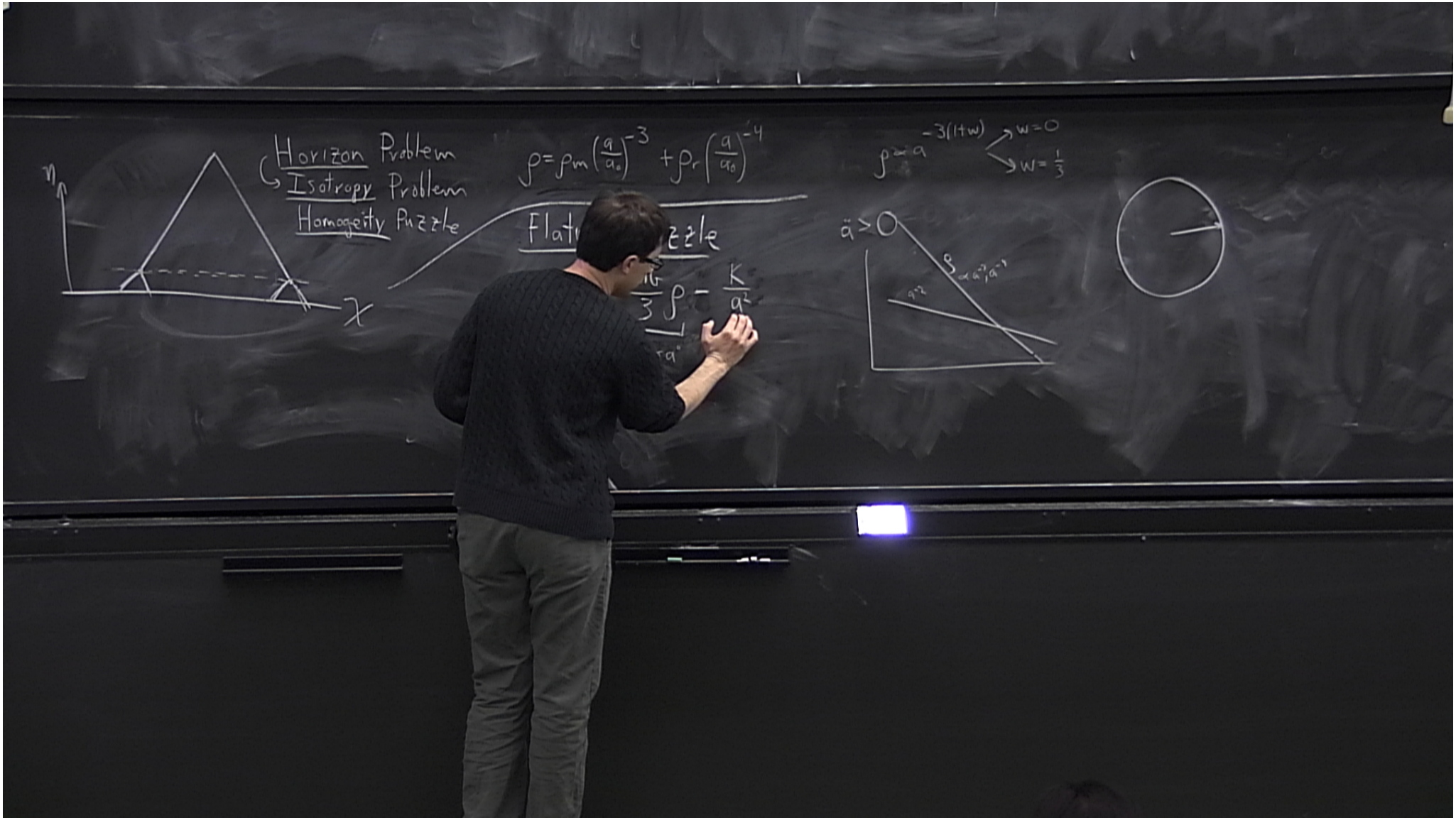

Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
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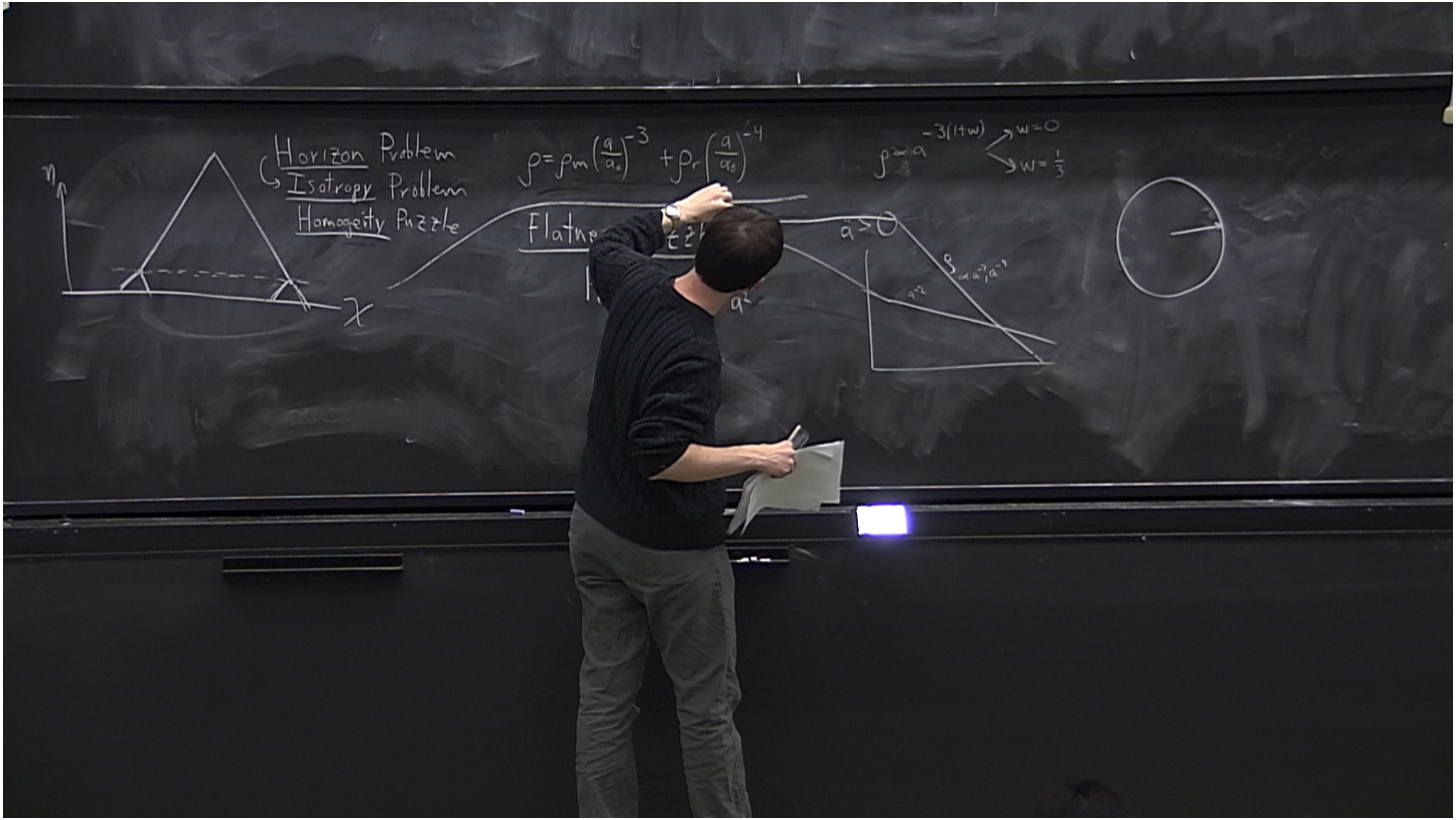

Horizon Problem
Isotropy
Homogeneity
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \sim a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
Flatness Puzzle
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$a^{-4} + a^{-3}$
a

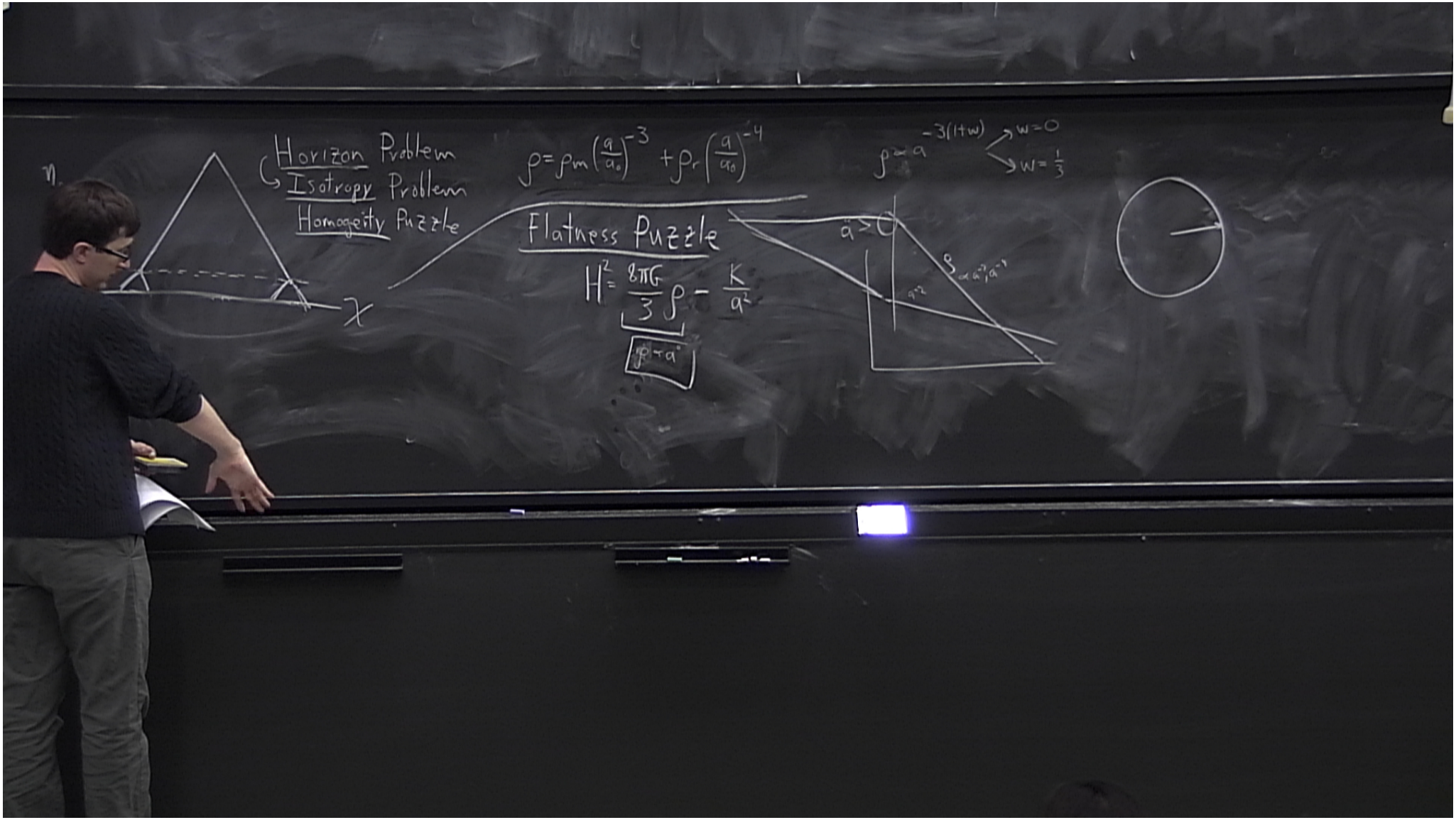

Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \sim a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
$\ddot{a} > 0$
$\frac{8\pi G}{3}\rho - \frac{K}{a^2}$
η
χ

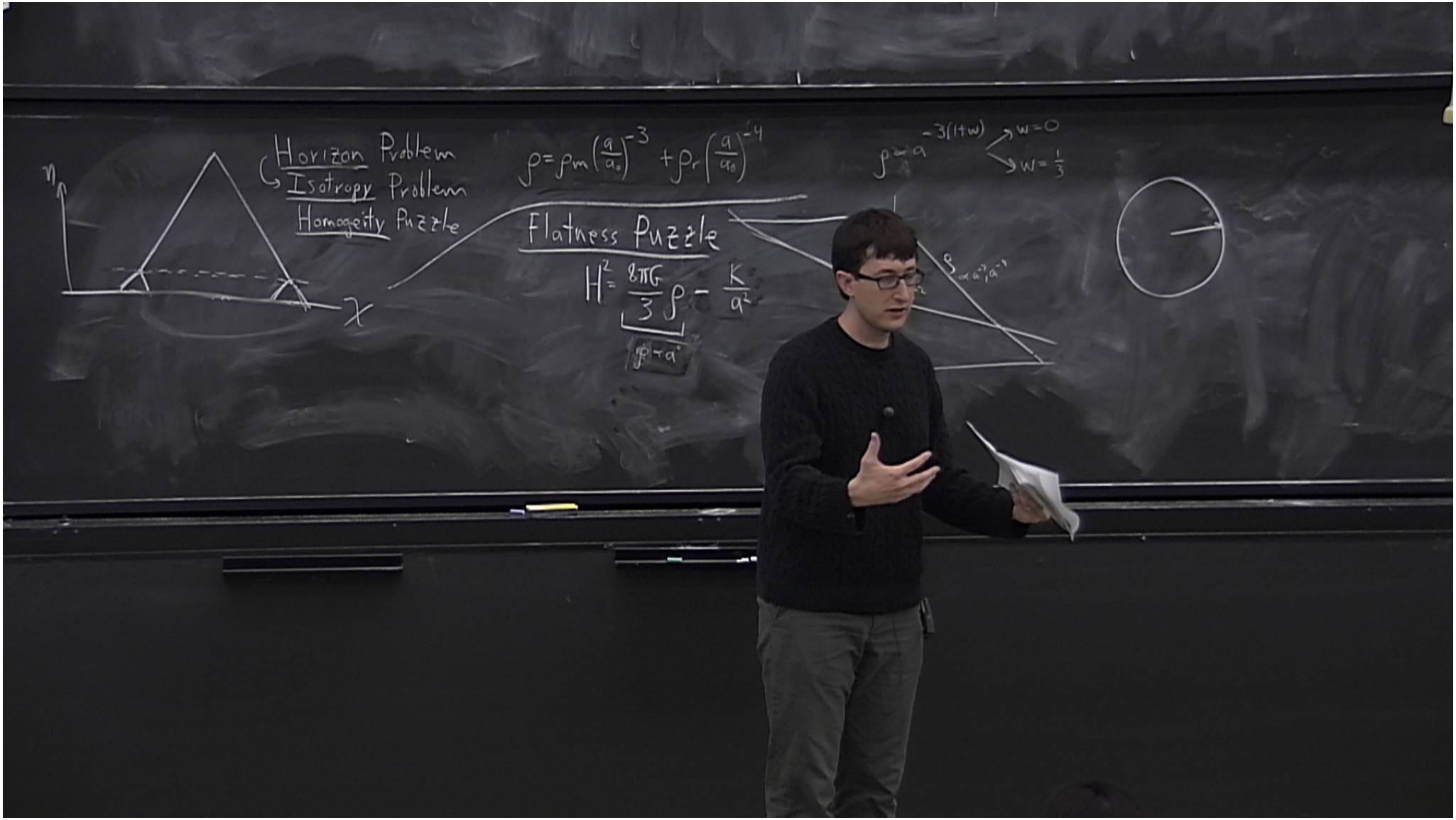

Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
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Flatness Puzzle
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\rho \sim a^{-4}$
$\ddot{a} > 0$

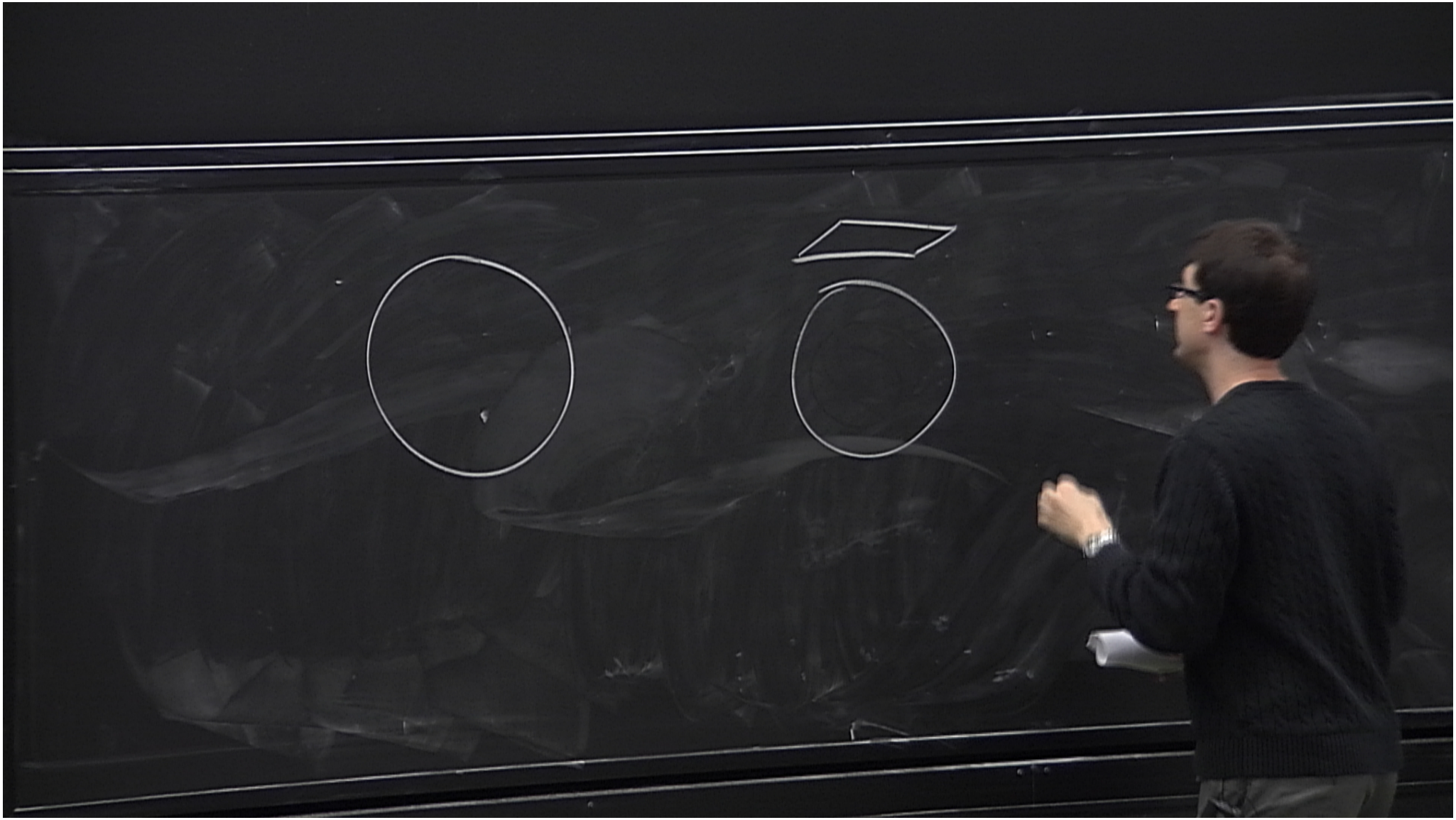

Horizon Problem
Isotropy Problem
Homogeity Puzz
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
Flatness Puzzle
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
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$w = 0$
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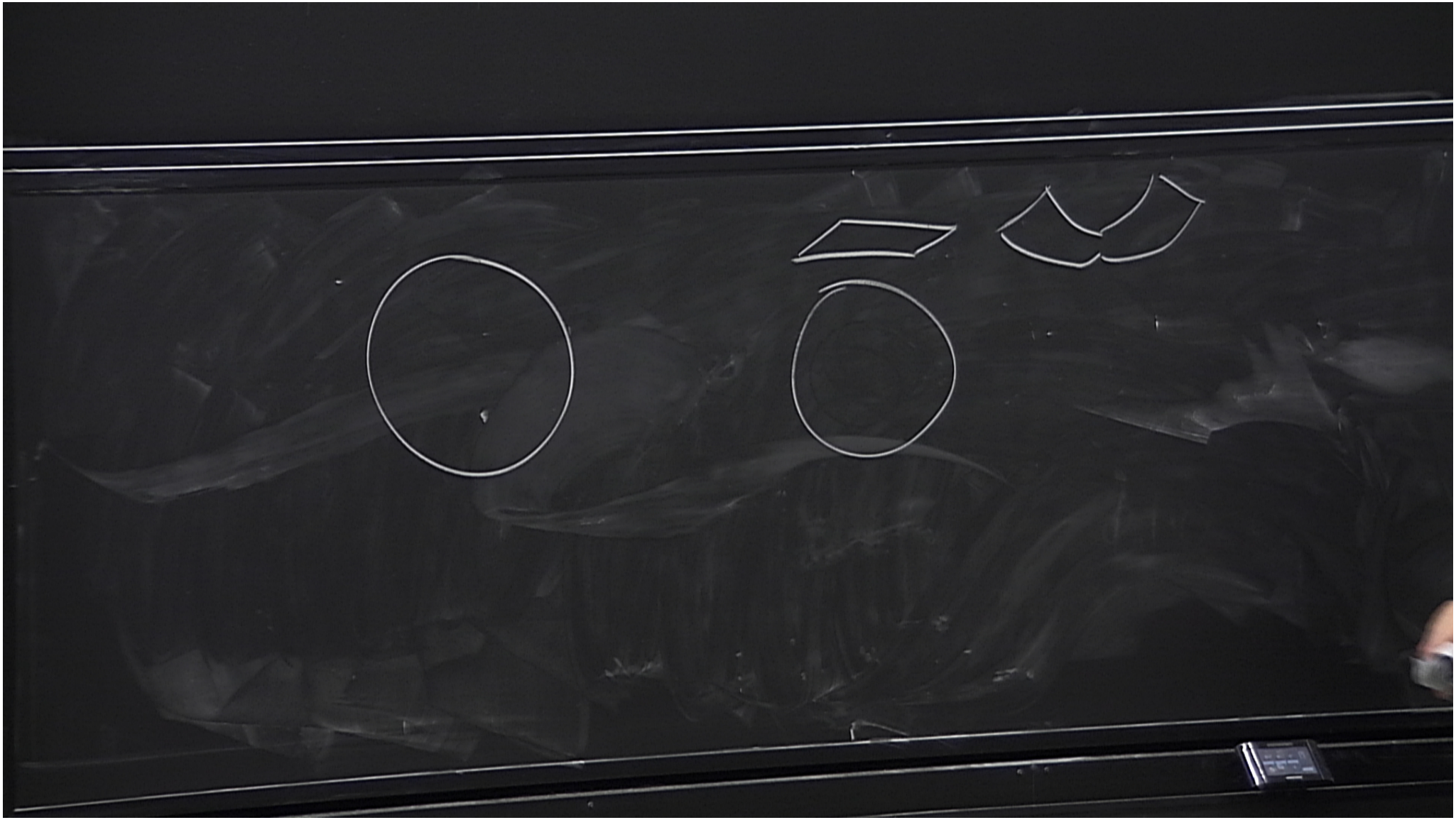

Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
$\ddot{a} > 0$
η
χ

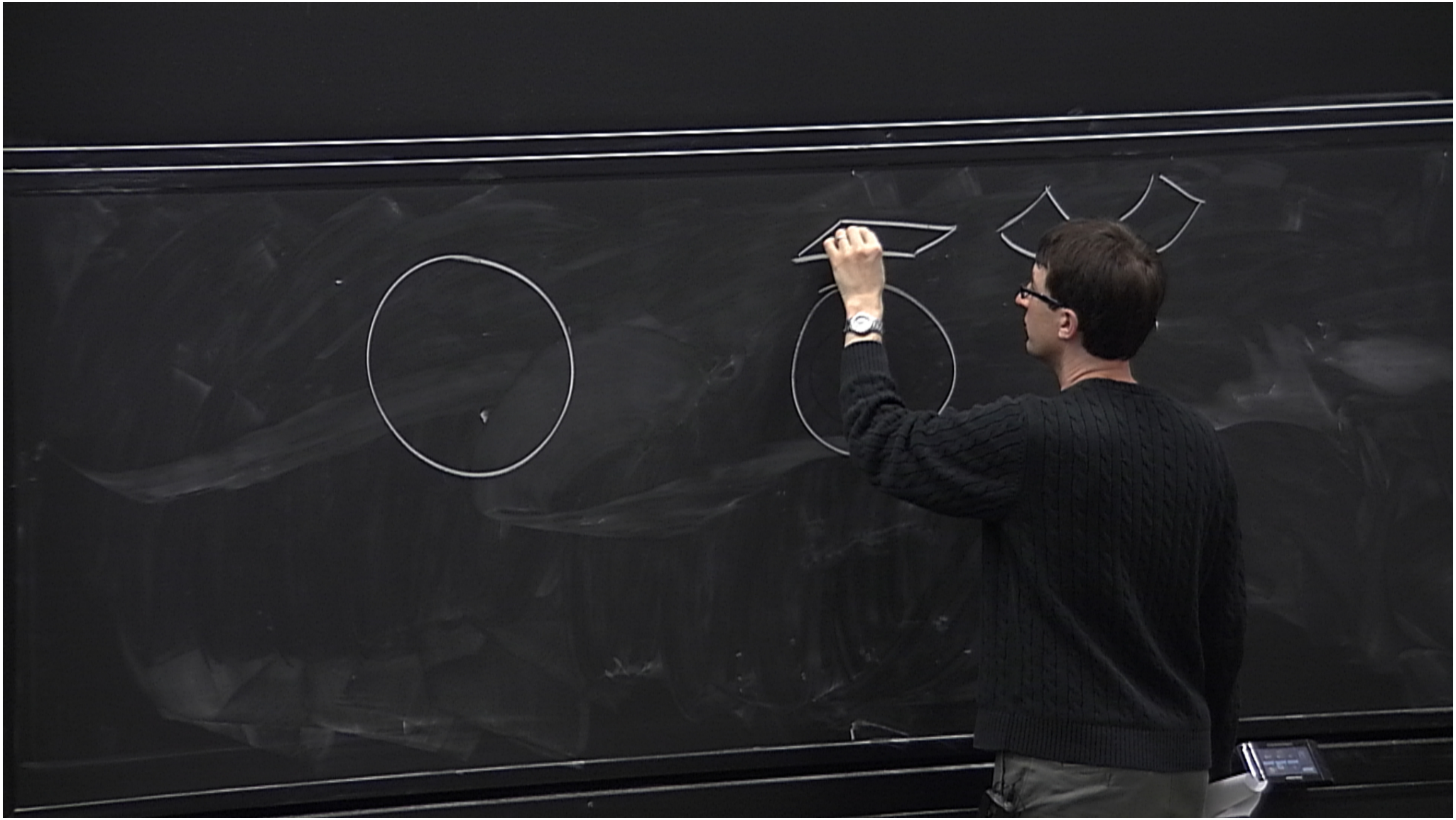

Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
$\rho \sim a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
η
χ
$\ddot{a} > 0$
a^2

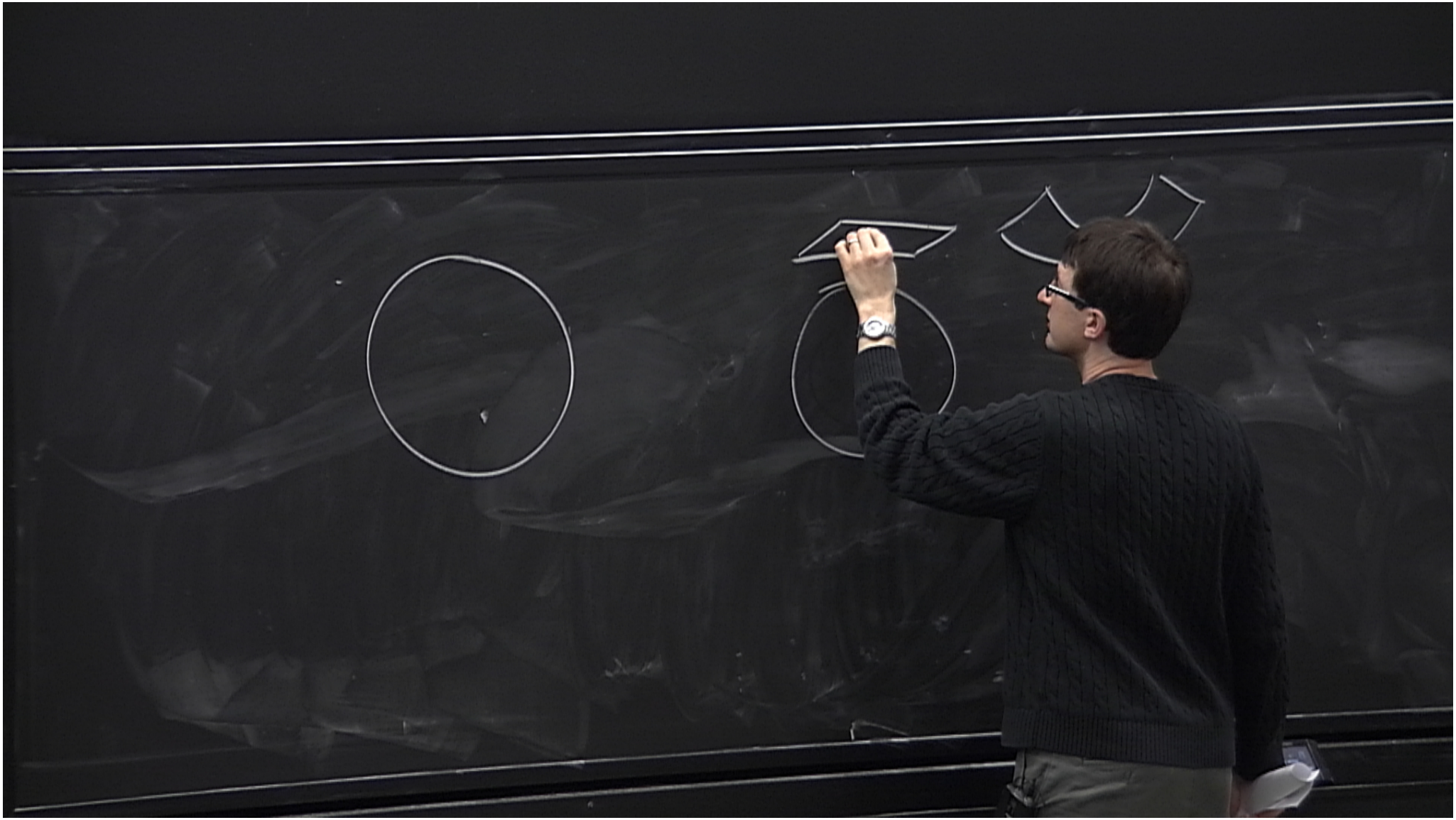

η
χ
Horizon Problem
Isotropy Problem
Homogeity Puzzle
$\rho = \rho_m \left(\frac{a}{a_0}\right)^{-3} + \rho_r \left(\frac{a}{a_0}\right)^{-4}$
Flatness Puzzle
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\rho \propto a$
$\rho \propto a^{-3(1+w)}$
$w = 0$
$w = \frac{1}{3}$
$\ddot{a} > 0$
$\rho \propto a^{-2}, a^{-3}$
a^{-2}

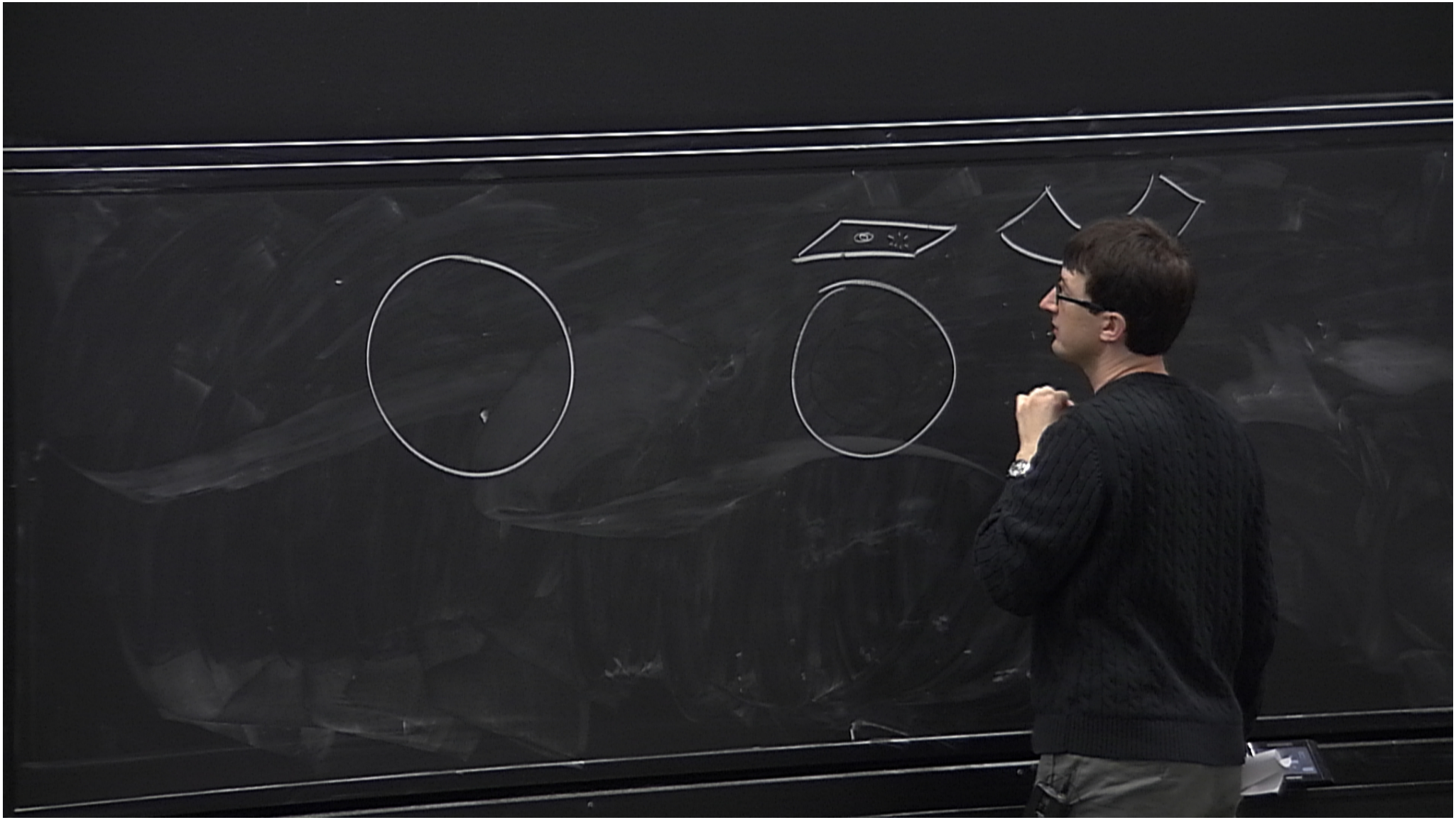

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η
χ

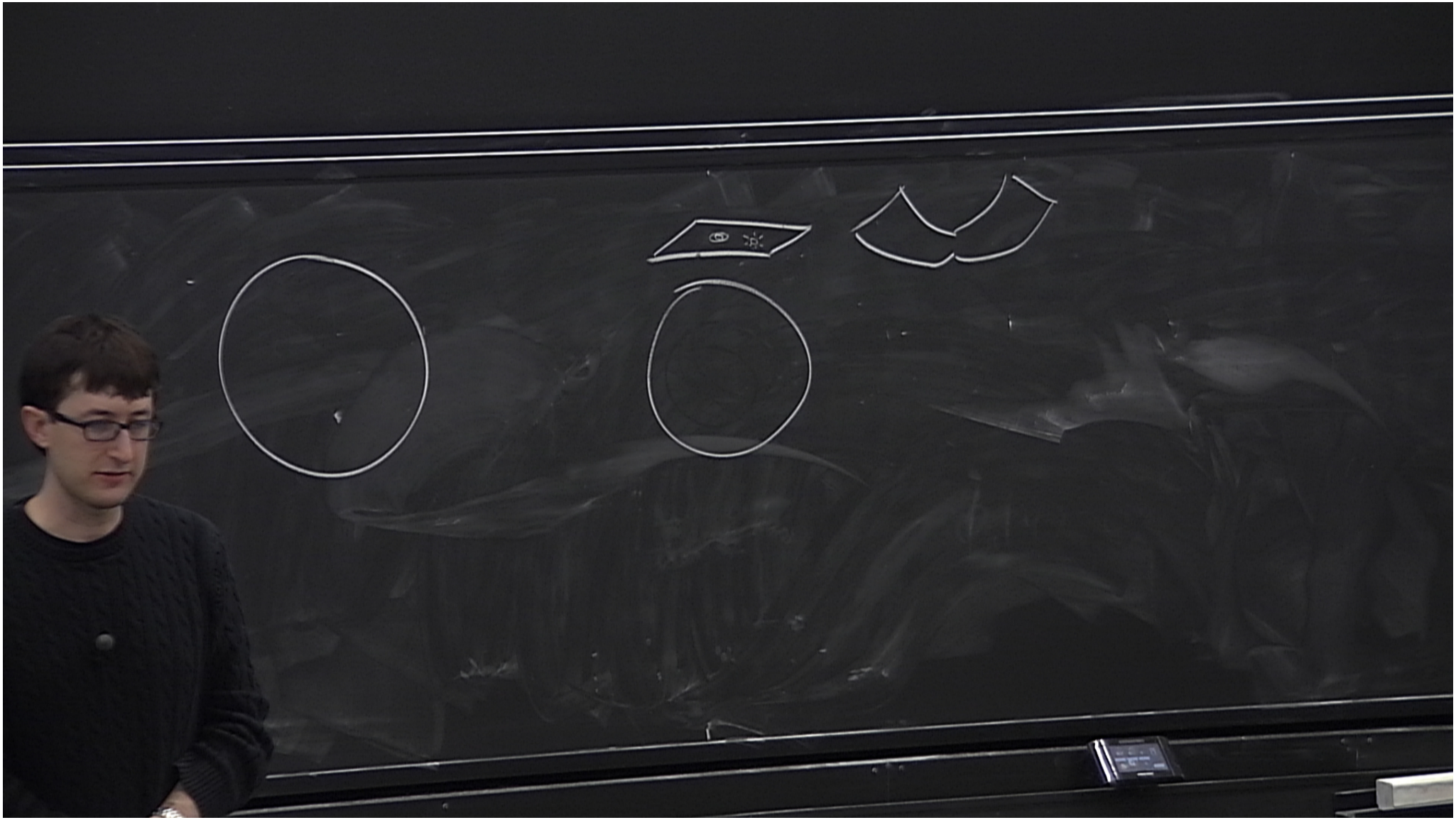




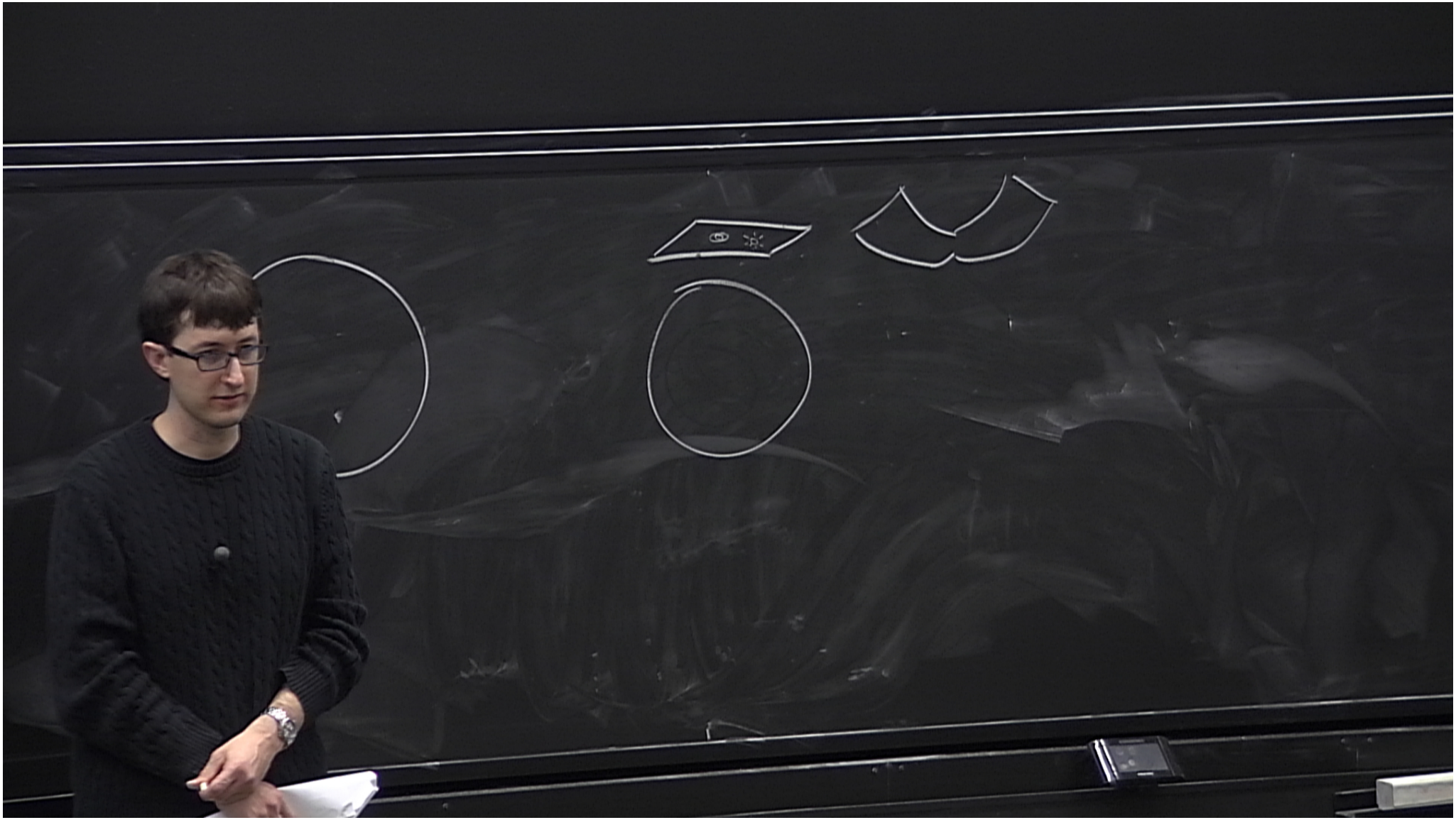


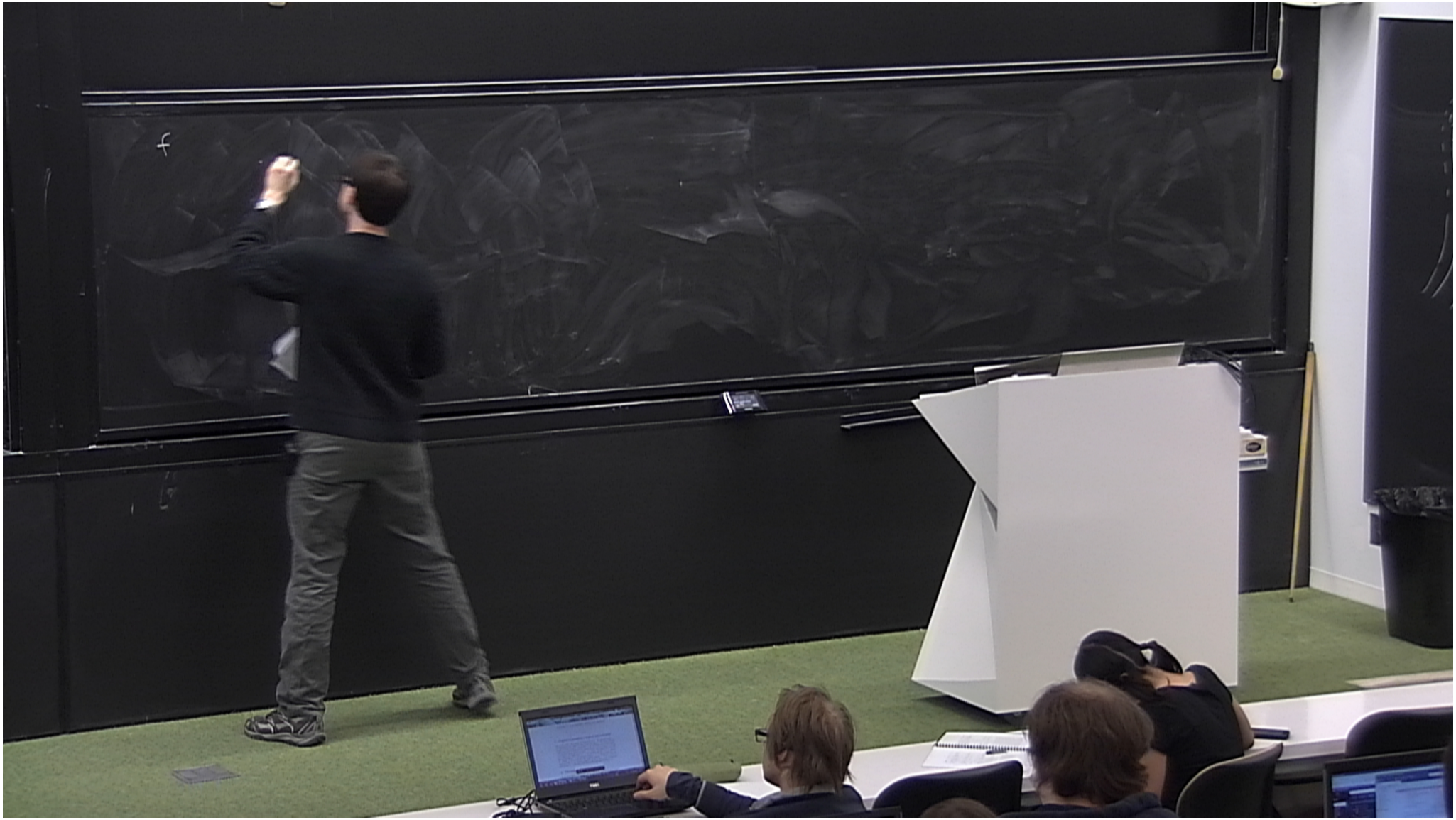




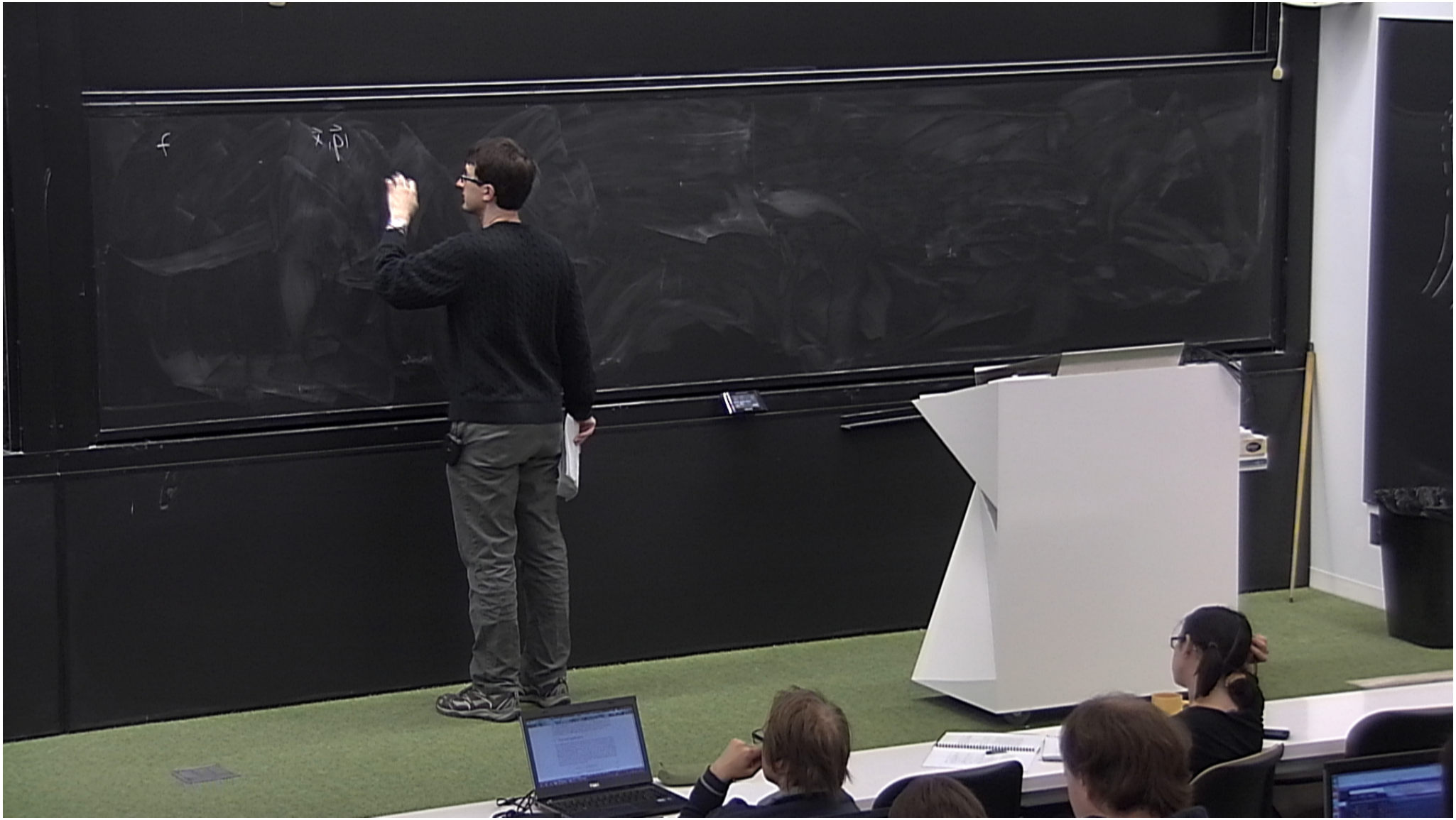


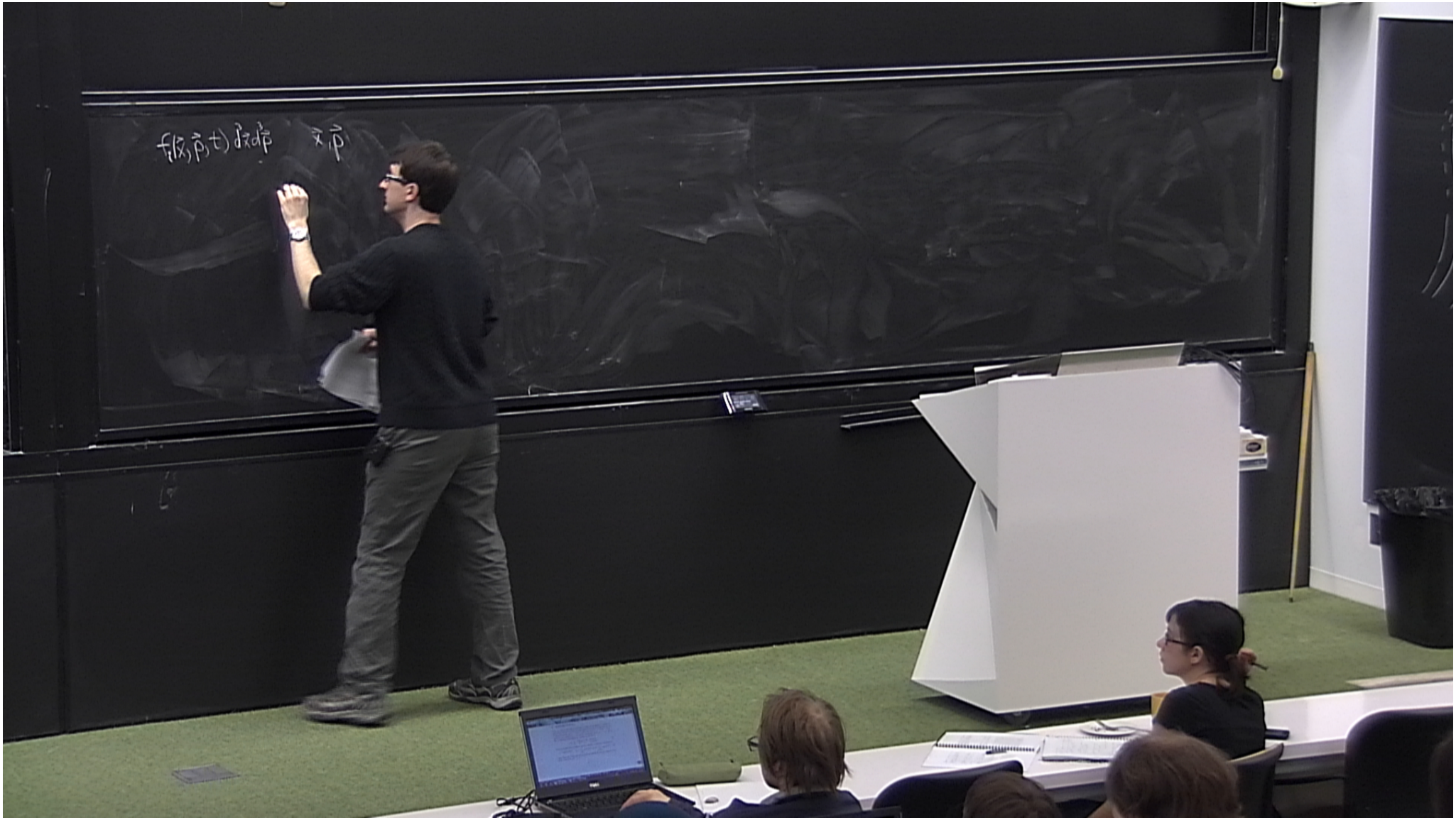






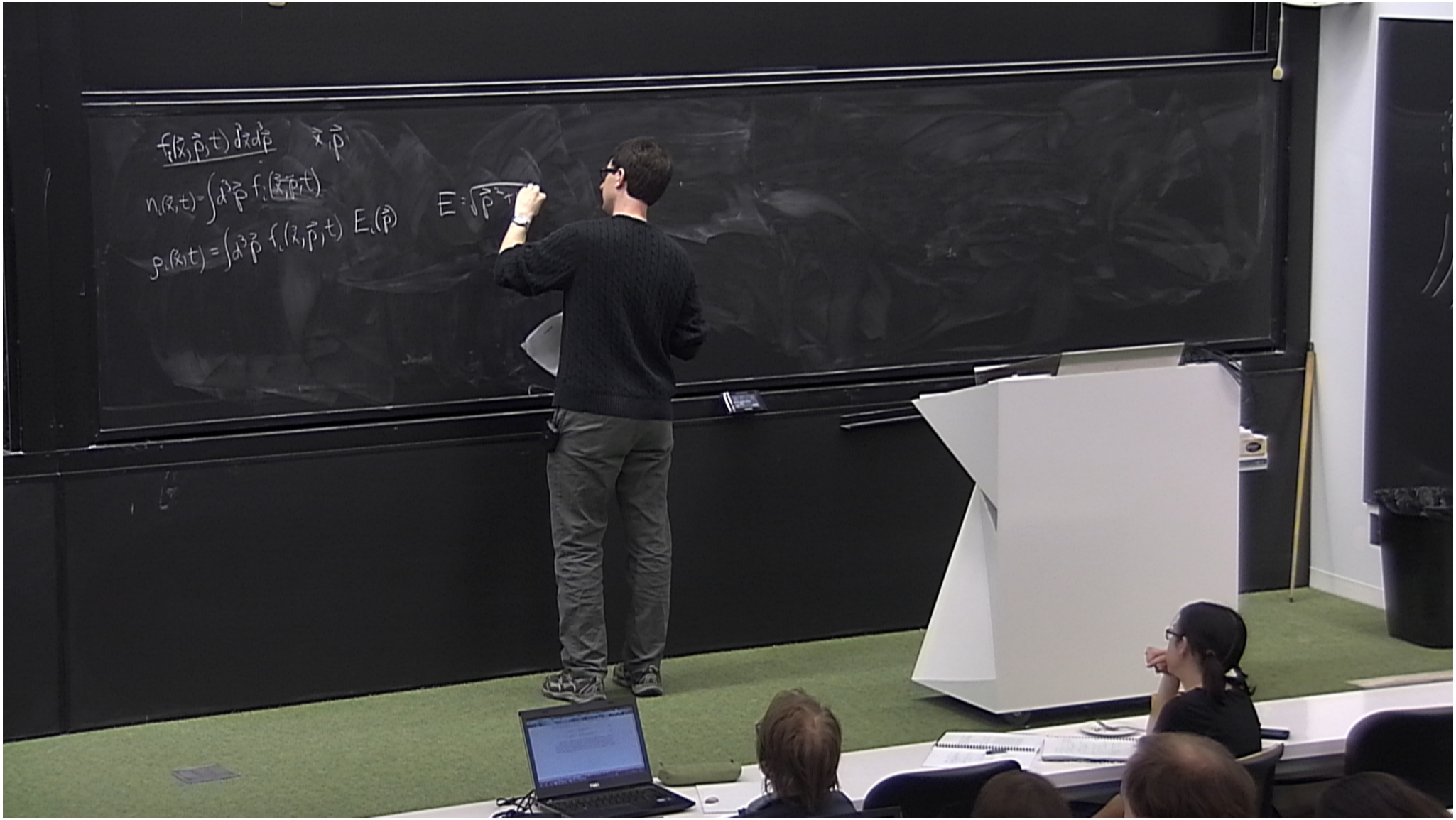


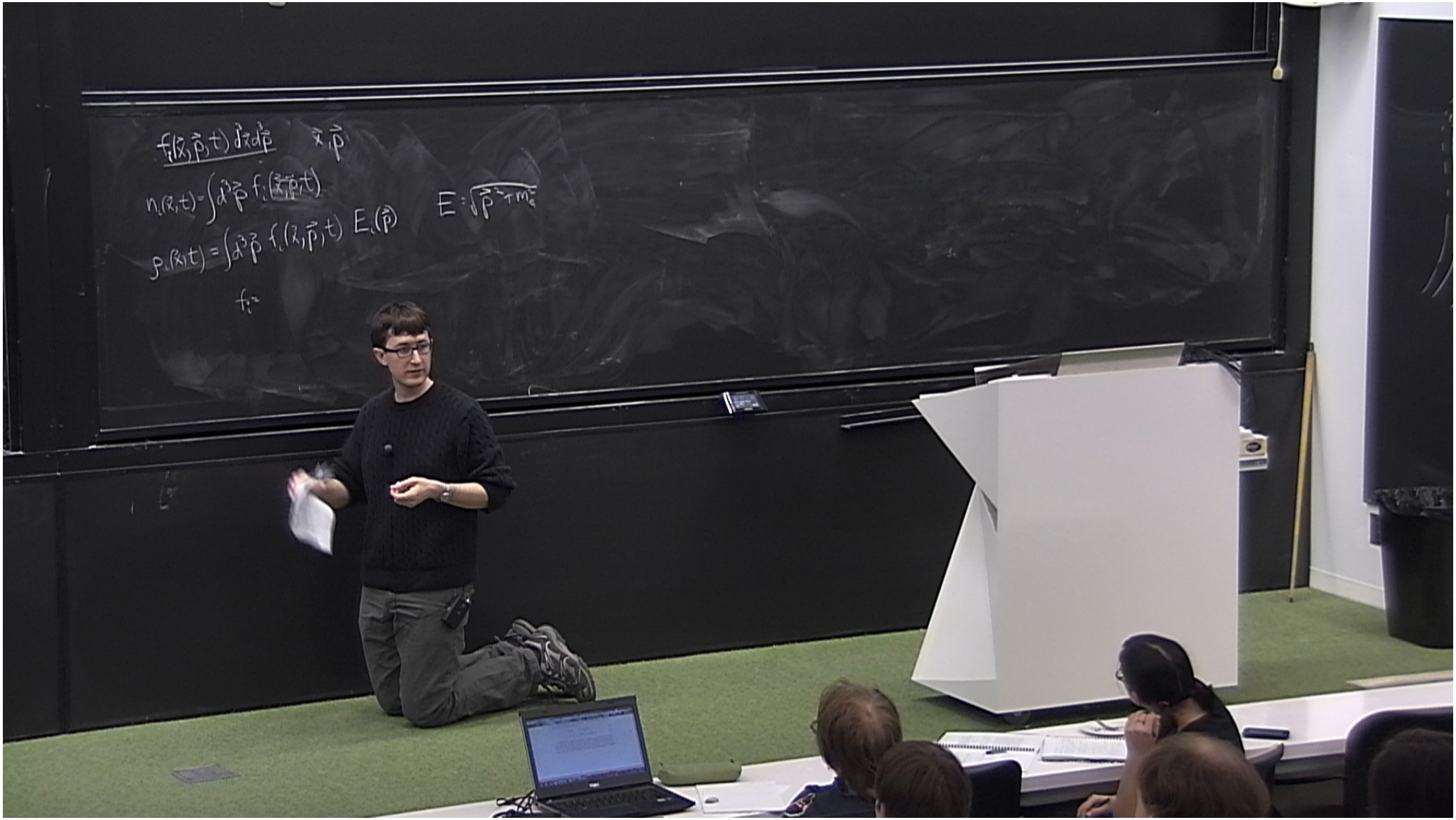


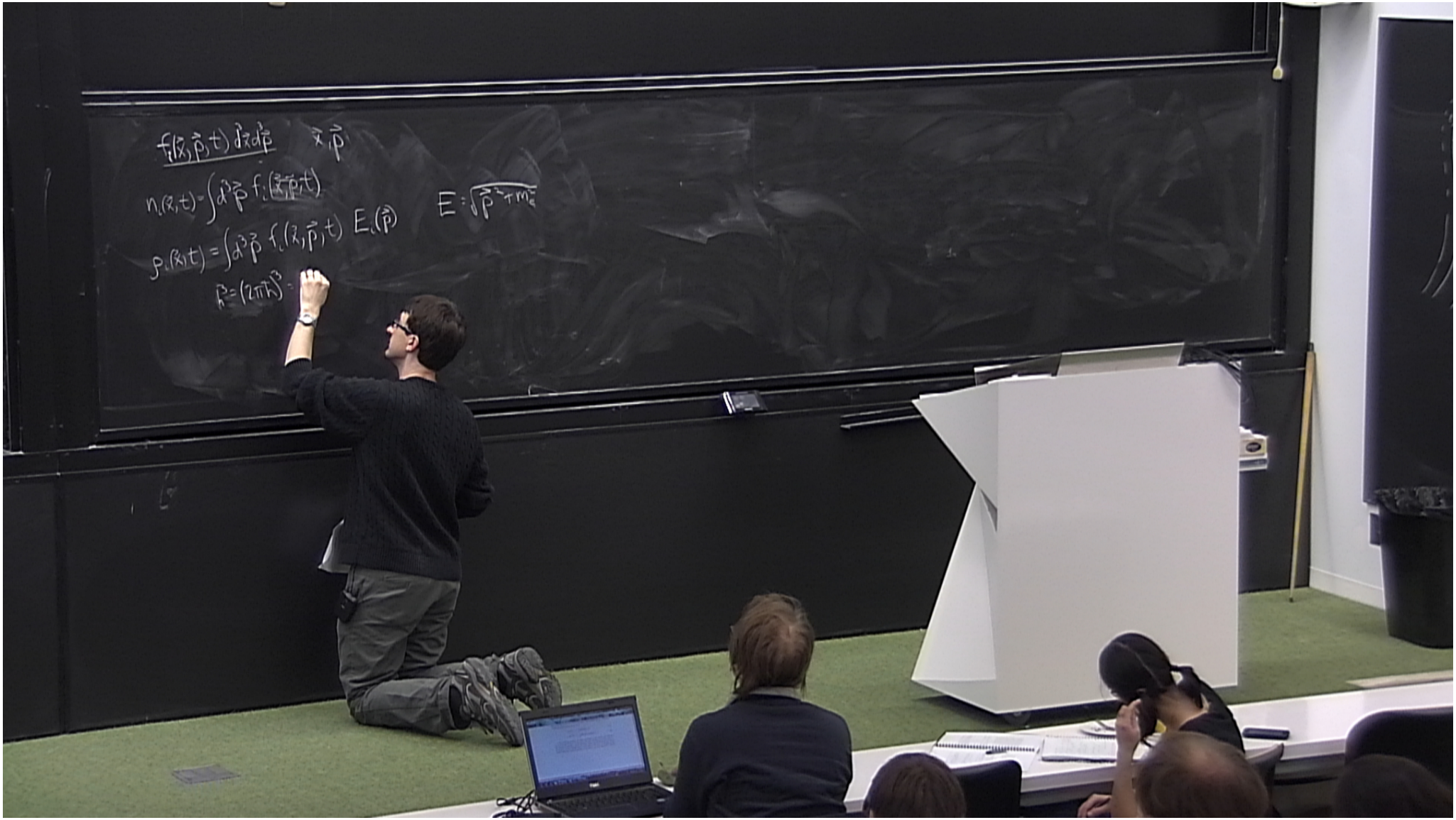

f(x,p,t) dx dp
x,p

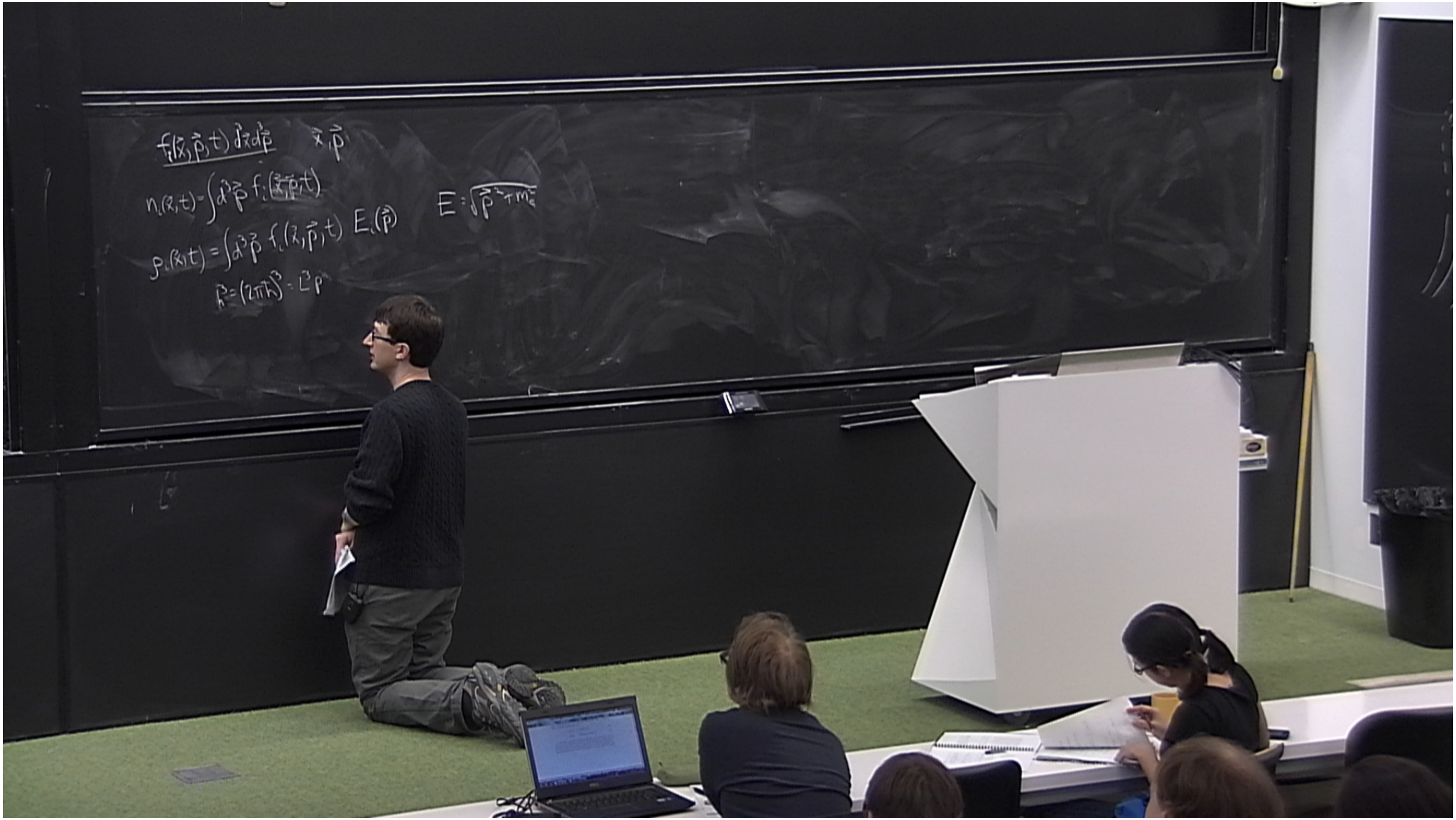


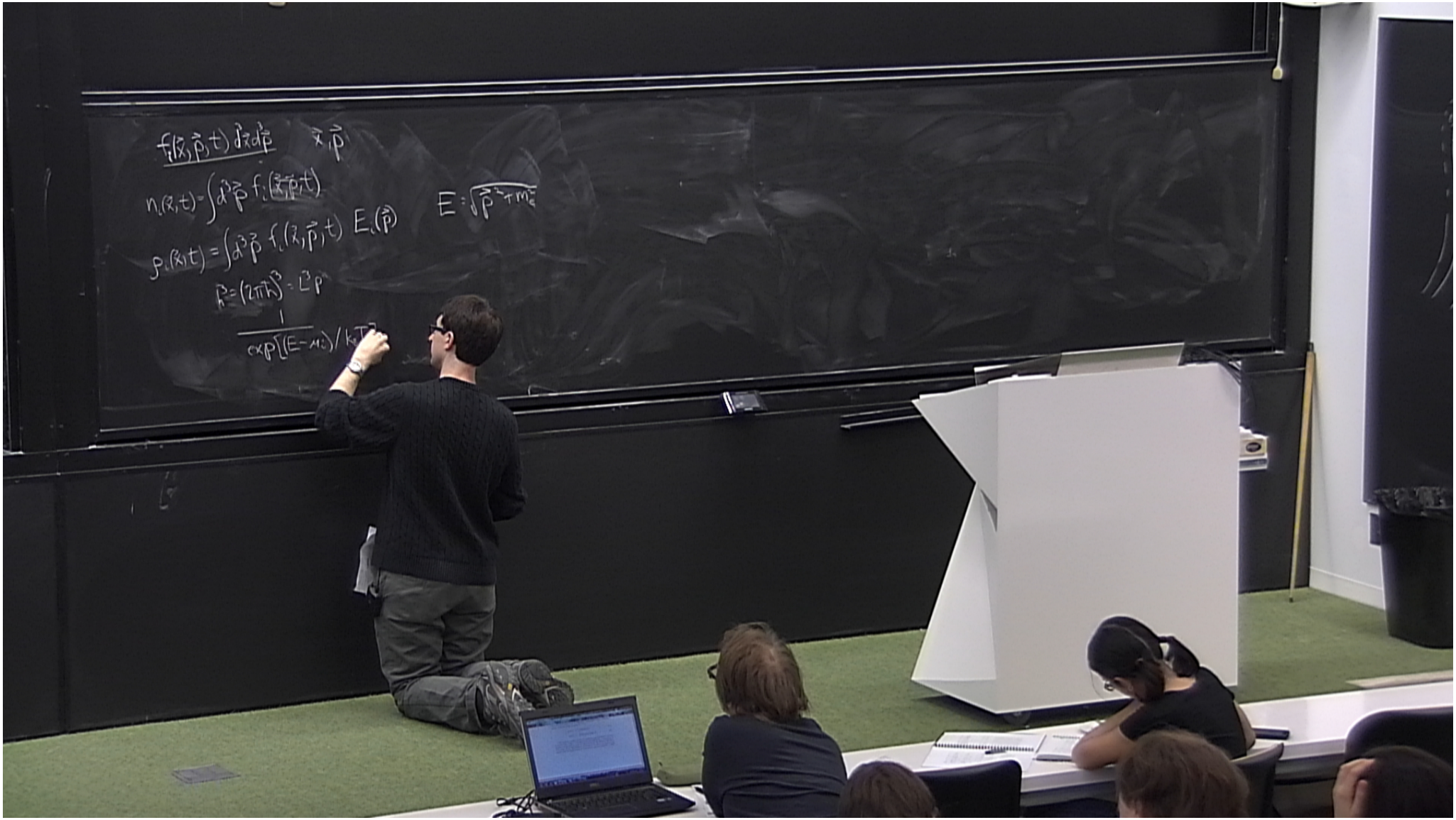

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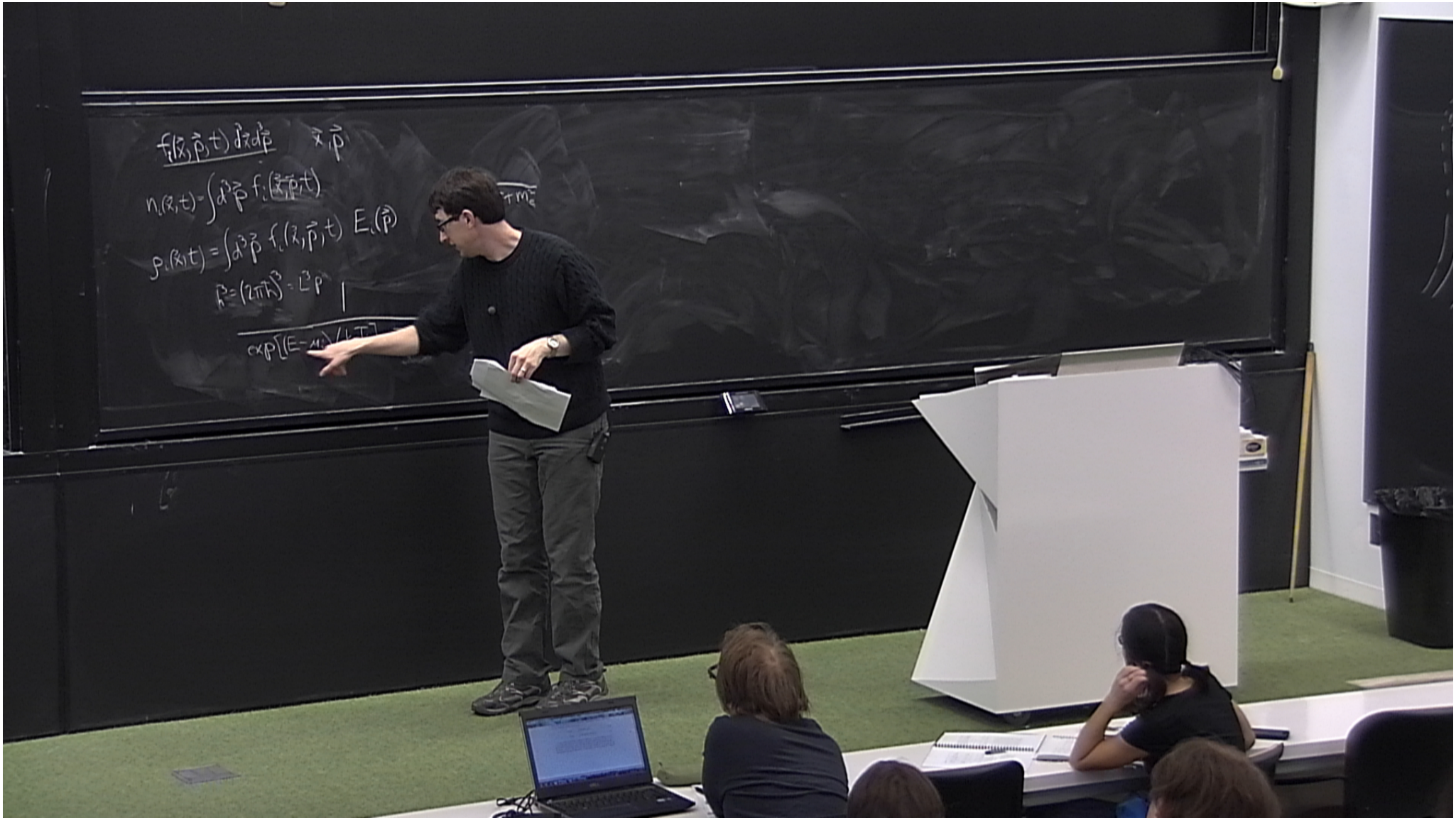

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 +$


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$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m_i^2}$
$f_i =$

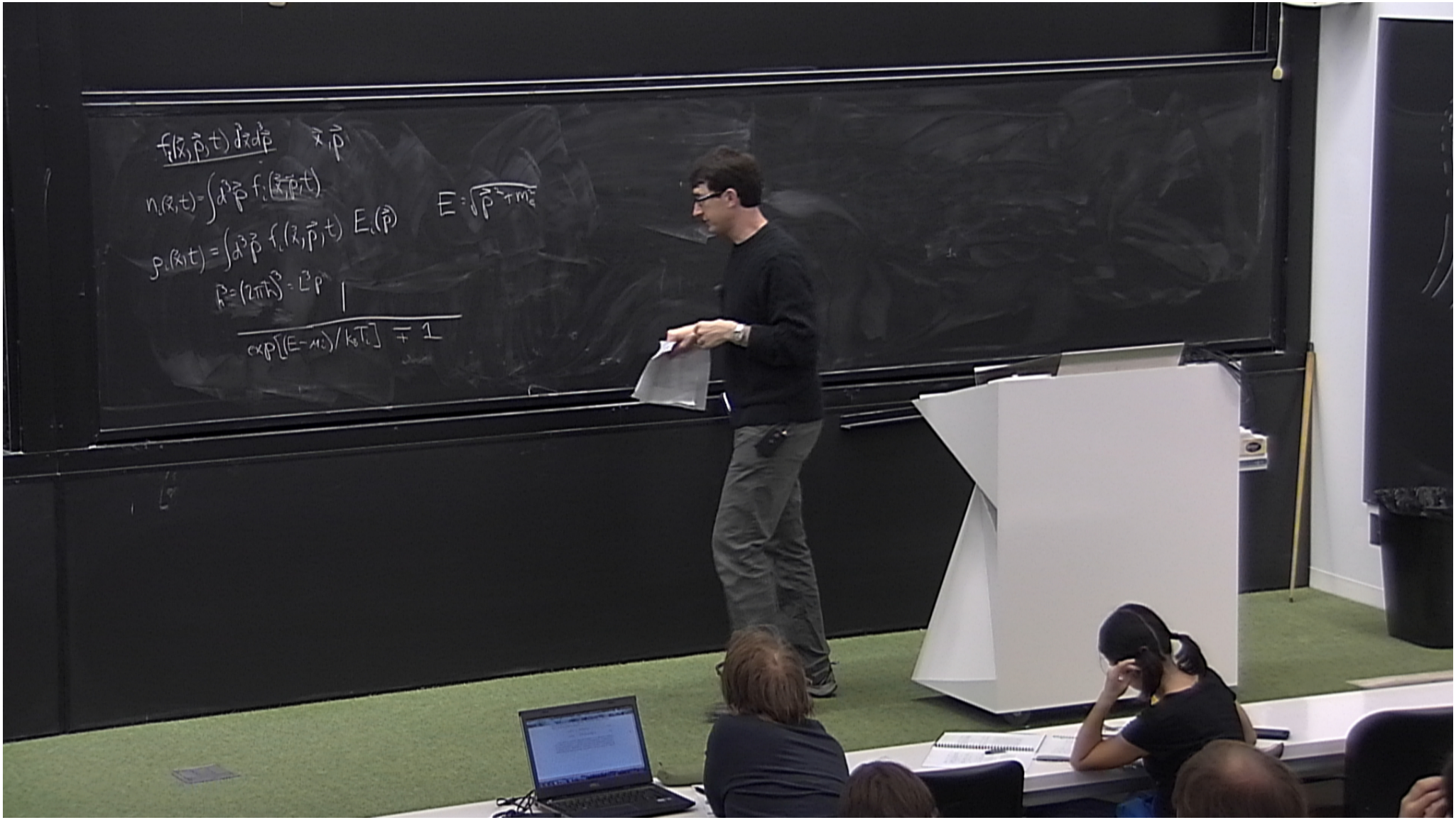

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$E = \sqrt{\vec{p}^2 + m_i^2}$
$h^3 = (2\pi\hbar)^3$

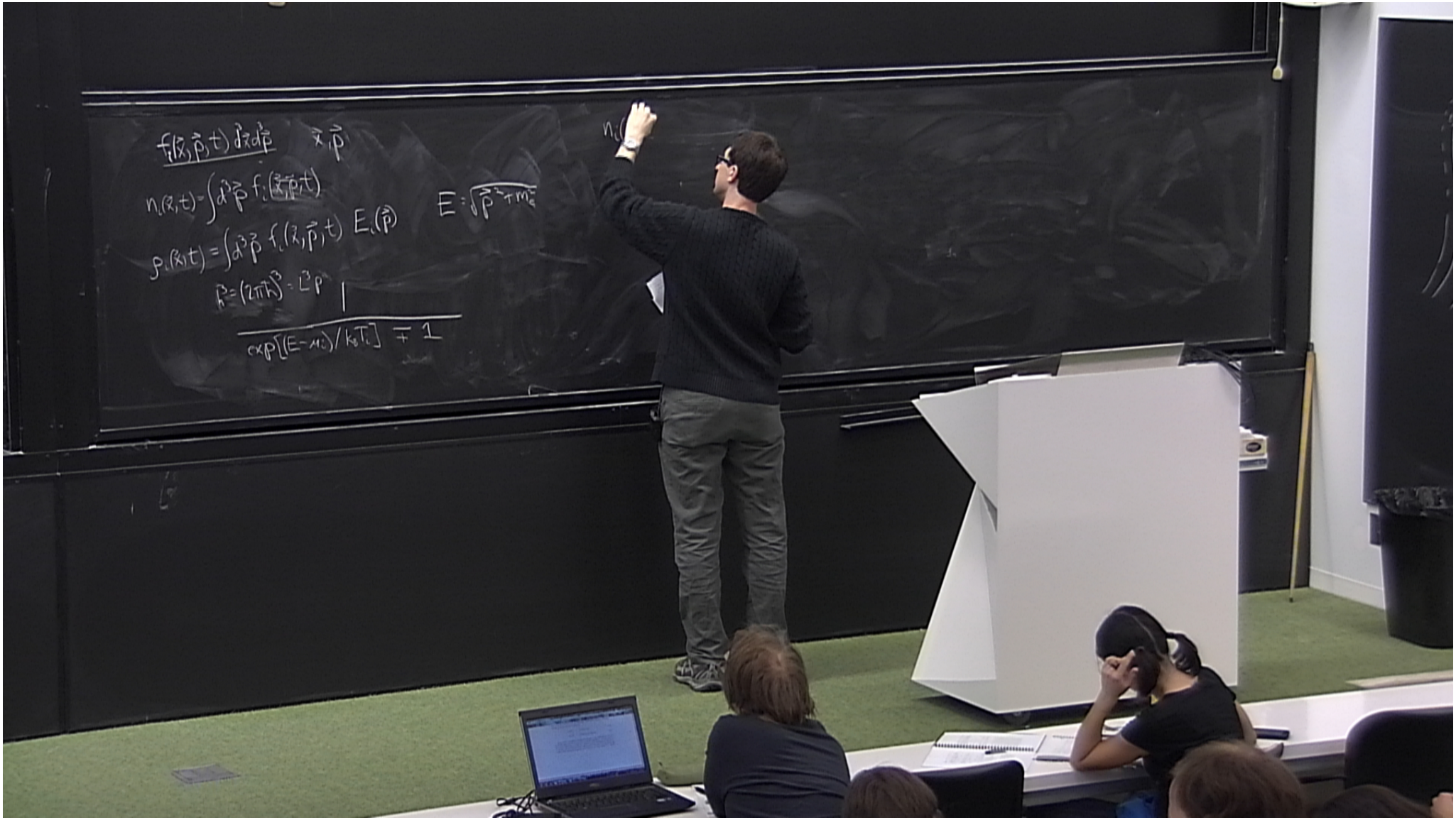

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$E = \sqrt{\vec{p}^2 + m^2}$
$h^3 = (2\pi\hbar)^3 = L^3 p$

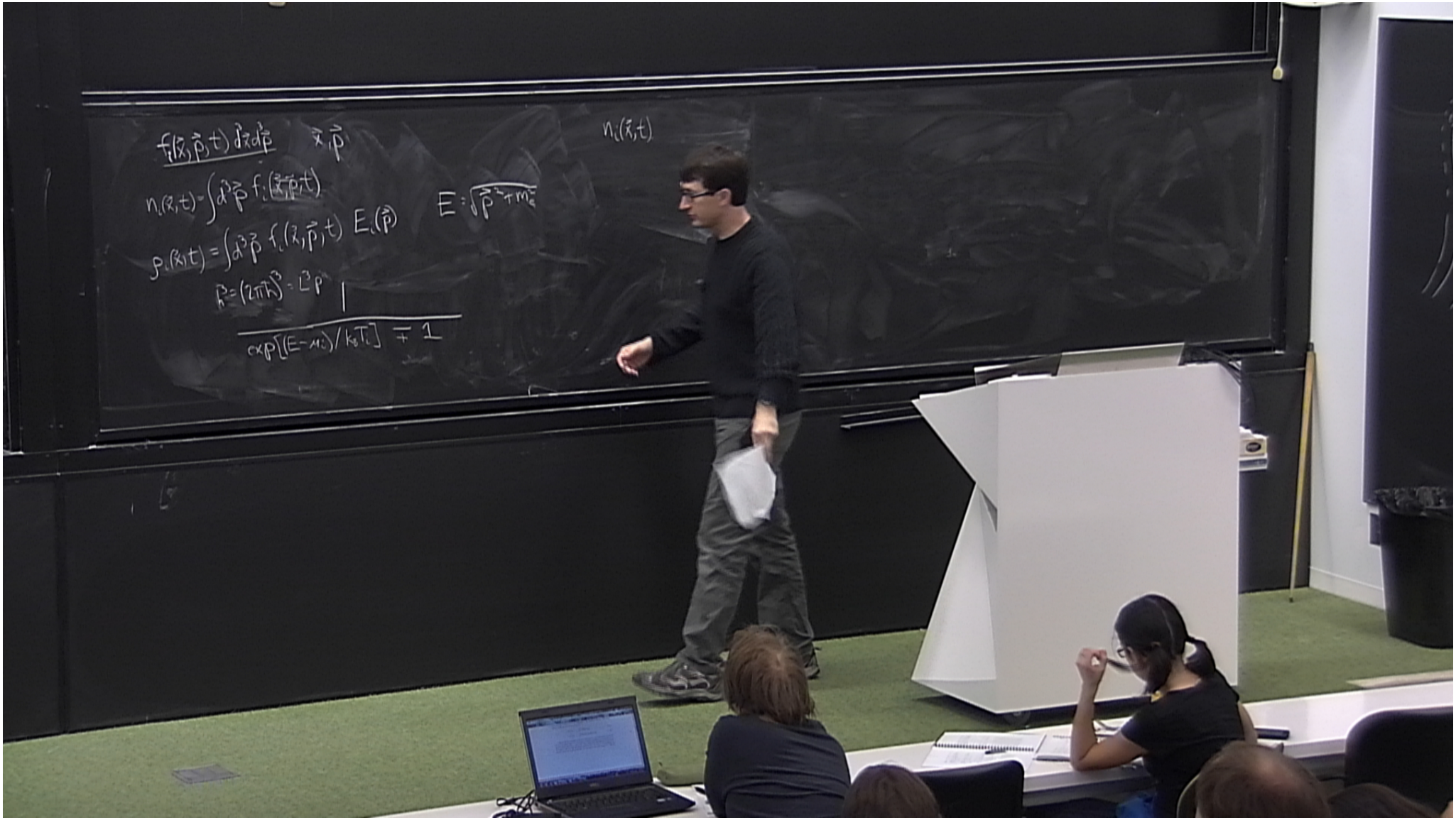

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$\vec{x},\vec{p}$
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$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m^2}$
$\hbar = (2\pi\hbar)^3 = L^3 p$
$\frac{1}{\exp[(E-\mu_i)/k_B T]}$

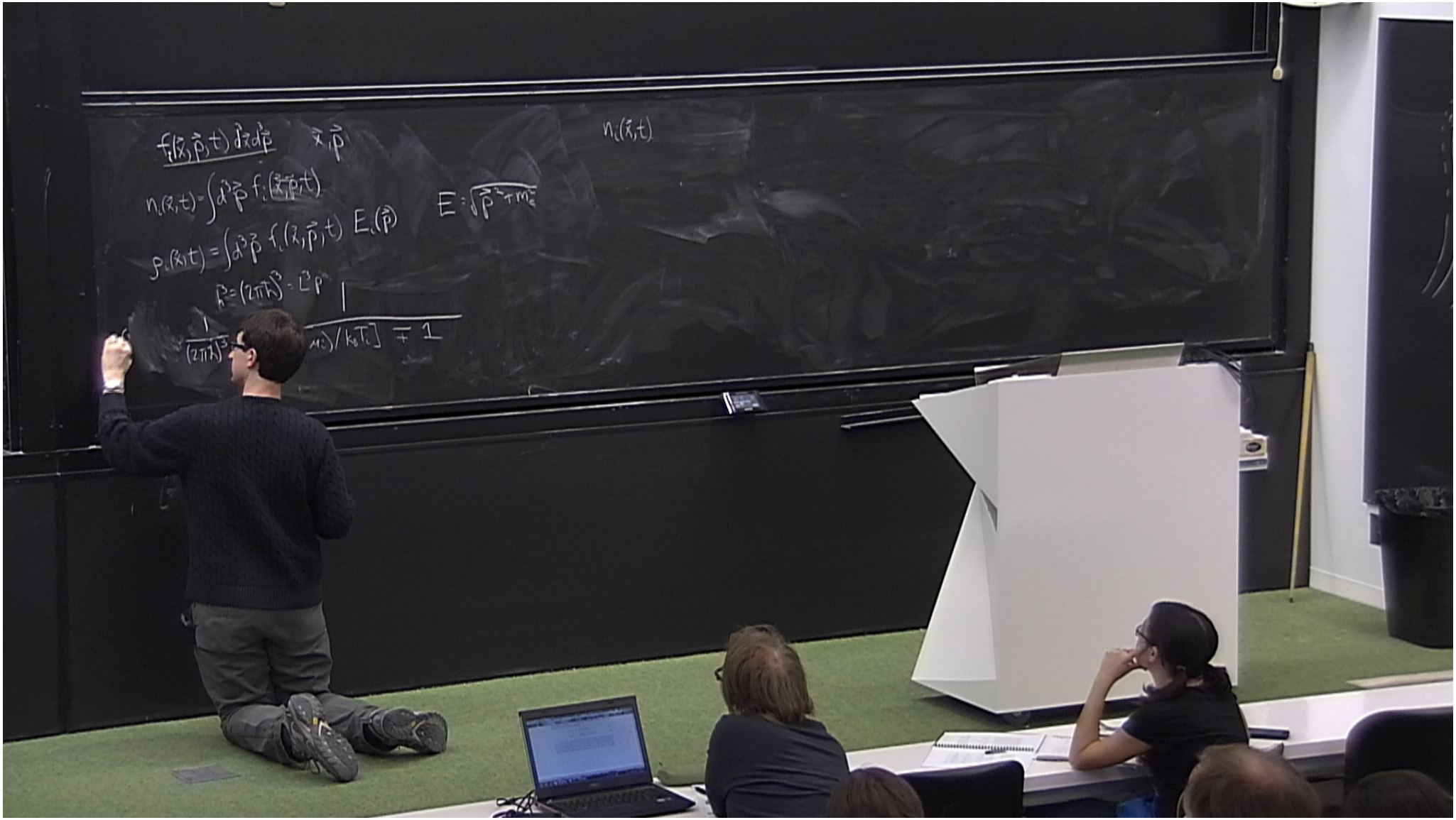

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$\vec{p} = (2\pi\hbar)^3 = L^3 p$

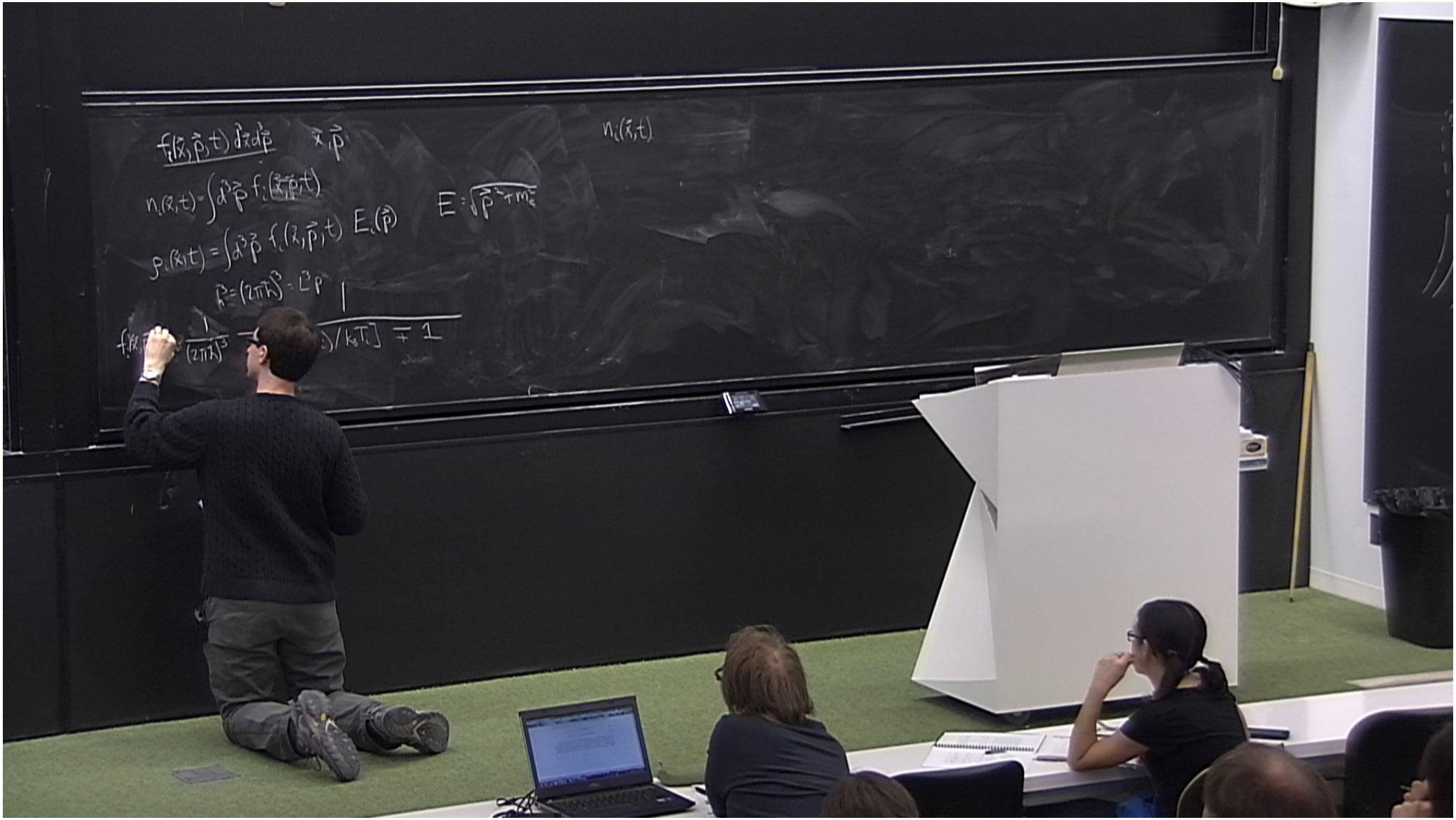

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$\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m^2}$
$\hbar = (2\pi\hbar)^3 = L^3 p$
$\exp[(E-\mu_i)/k_B T_i] \mp 1$

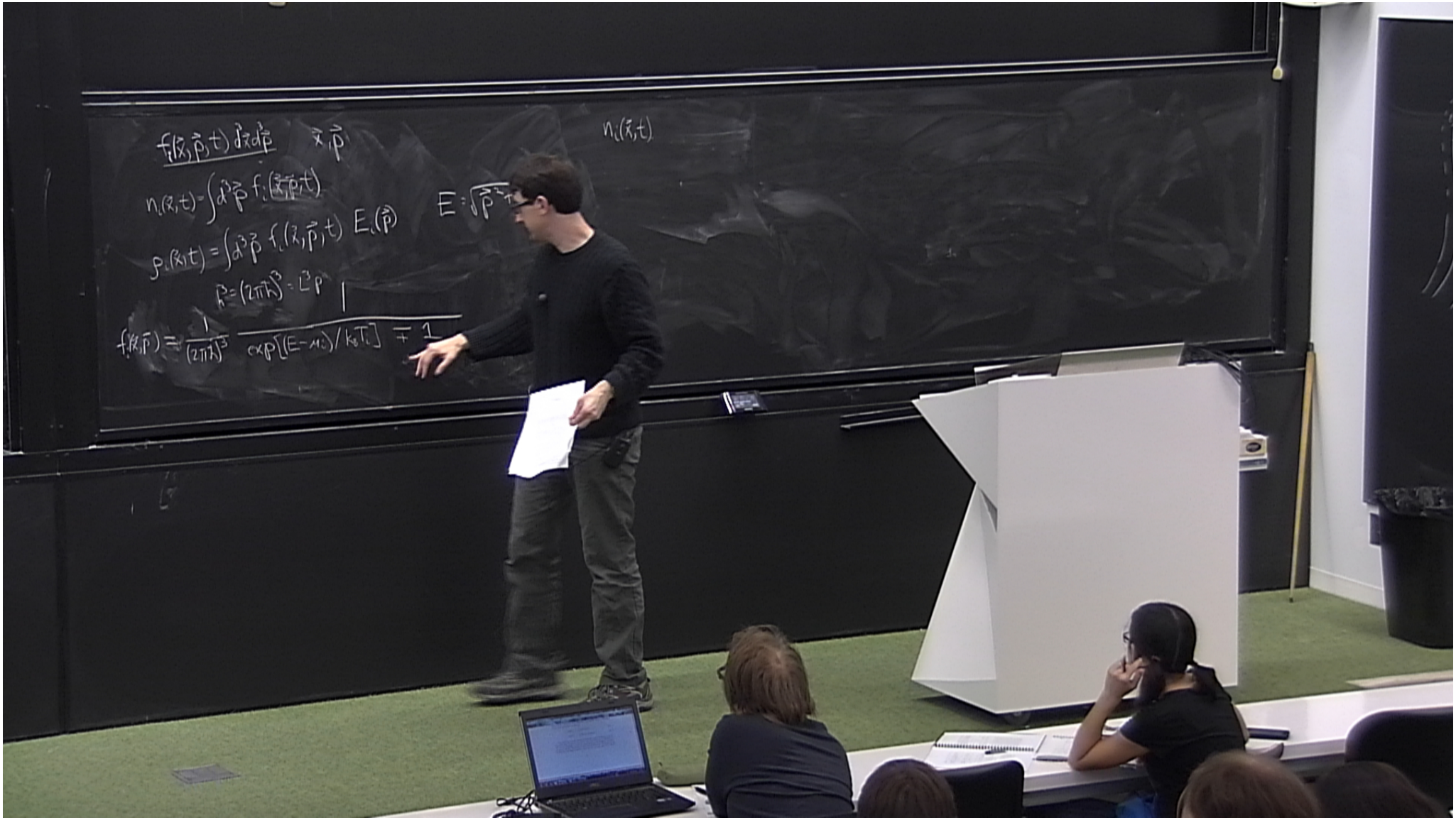

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m^2}$
$h^3 = (2\pi\hbar)^3 = L^3 p$
$\exp[(E-\mu_i)/k_B T_i] \mp 1$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m^2}$
$\hat{p} = (2\pi\hbar)^3 = L^3 p$
$\exp[(E-\mu_i)/k_B T_i] \mp 1$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$n_i(\vec{x},t)$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m_i^2}$
$h^3 = (2\pi\hbar)^3 = L^3 p^3$
$\exp[(E-\mu_i)/k_B T_i] \mp 1$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$n_i(\vec{x},t)$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$E = \sqrt{\vec{p}^2 + m_i^2}$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$\vec{p} = (2\pi\hbar)^3 \cdot \vec{l}\,\vec{p}$
$\frac{1}{(2\pi\hbar)^3}$
$m_i)/k_B T_i] \mp 1$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$E = \sqrt{\vec{p}^2 + m^2}$
$n_i(\vec{x},t)$
$\vec{p} = (2\pi\hbar)^3 = L^3 p$
$\frac{1}{(2\pi\hbar)^3}$
$/k_B T_i] \mp 1$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$f(\vec{x},\vec{p}) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_B T_i] \mp 1}$
$n_i(\vec{x},t)$

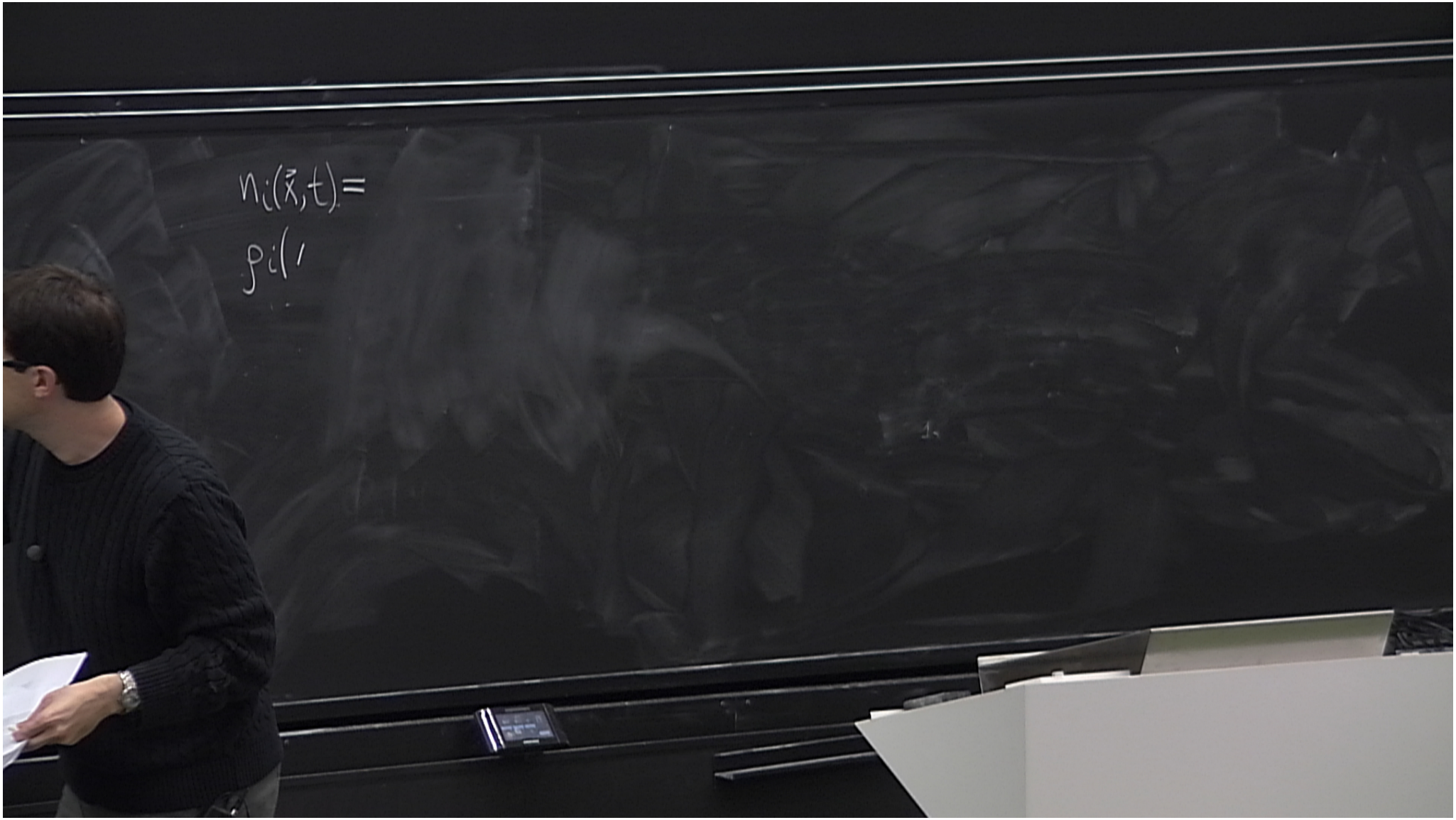
$f_i(\vec{x},\vec{p},t)\, d^3\vec{x}\, d^3\vec{p}$ $\vec{x},\vec{p}$ $n_i(\vec{x},t)$

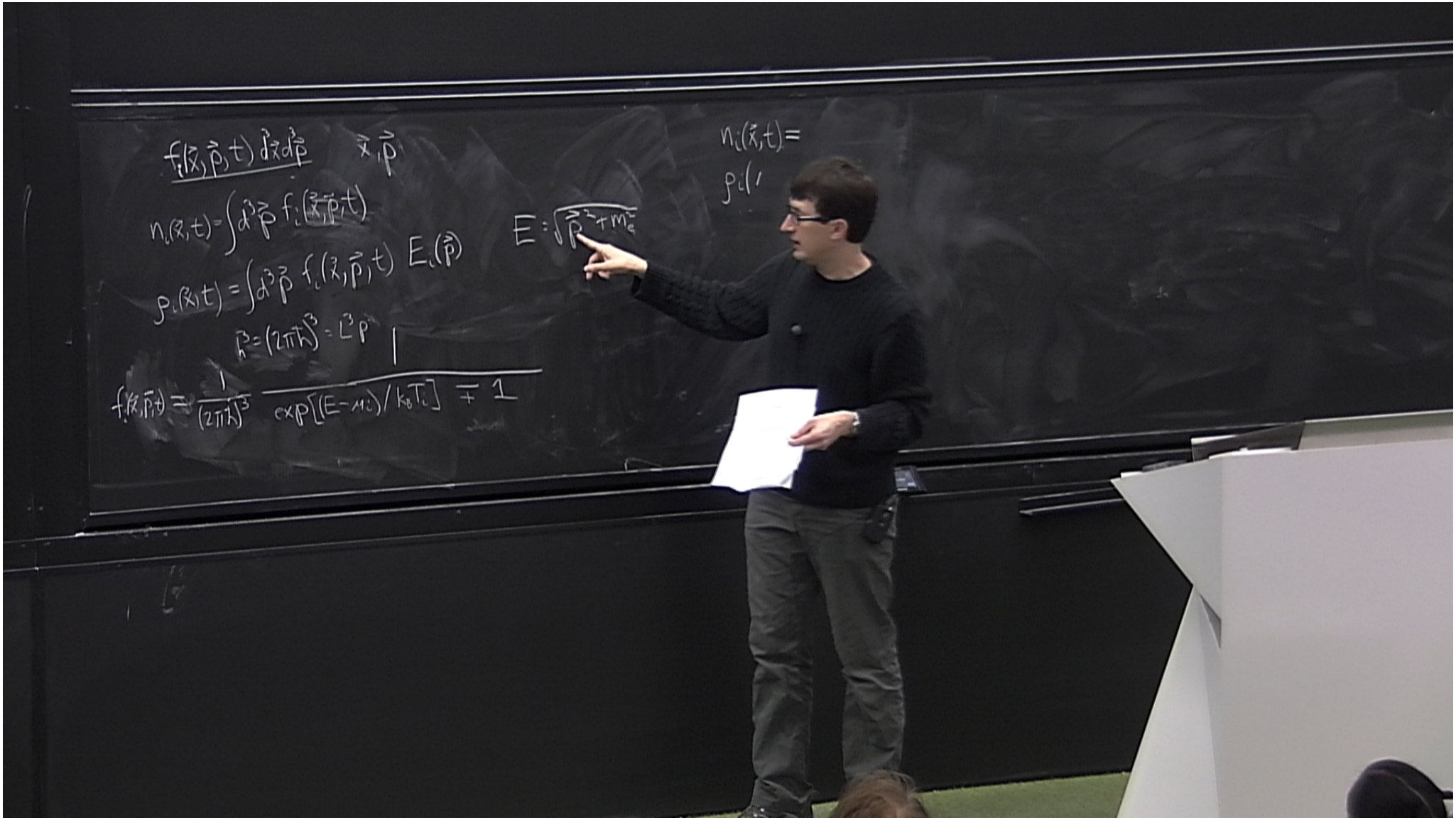
$$n_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)$$

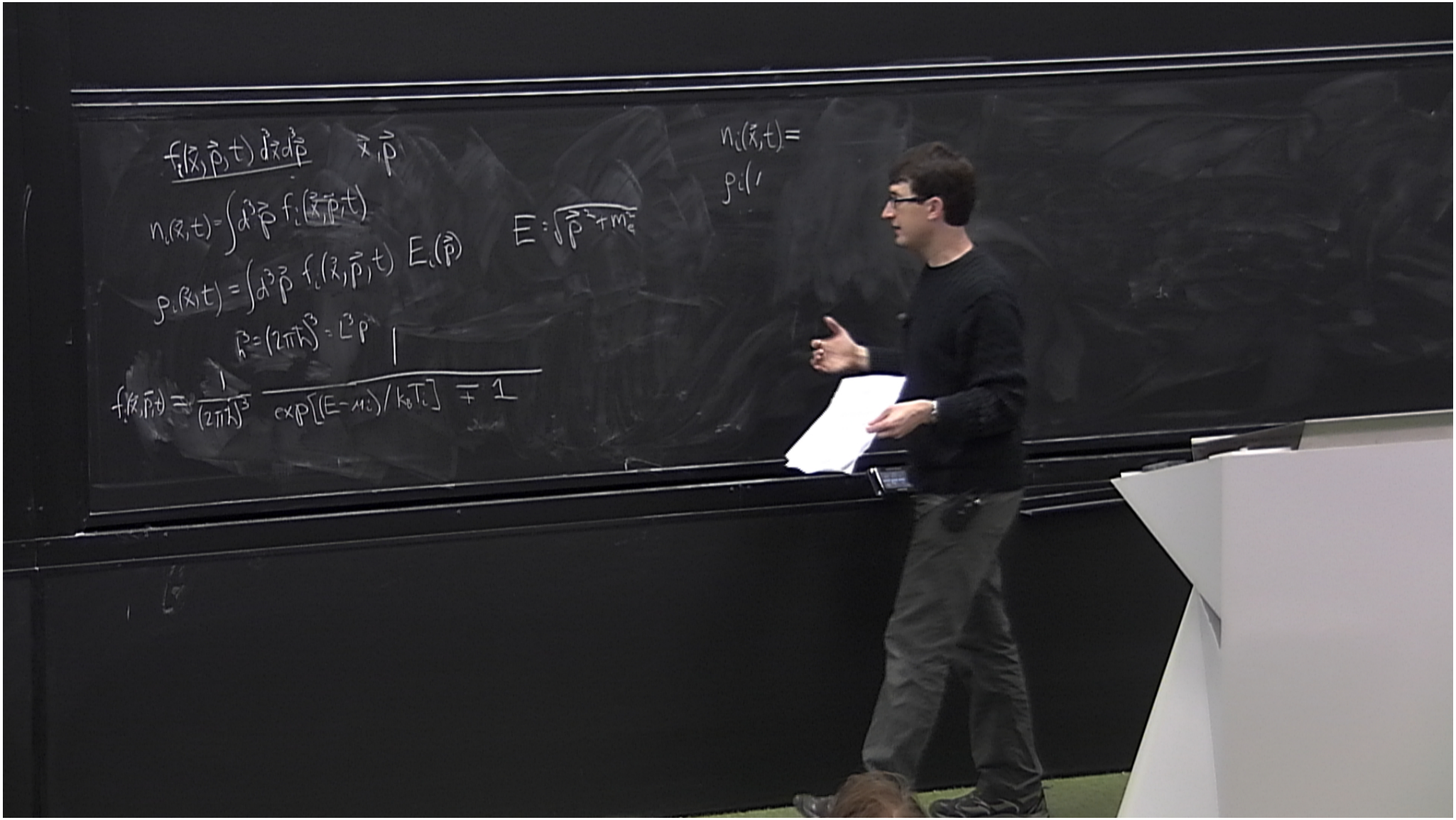
$$p_i(\vec{x},t) = \int d^3\vec{p}\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p}) \qquad E = \sqrt{\vec{p}^2 + m_e^2}$$

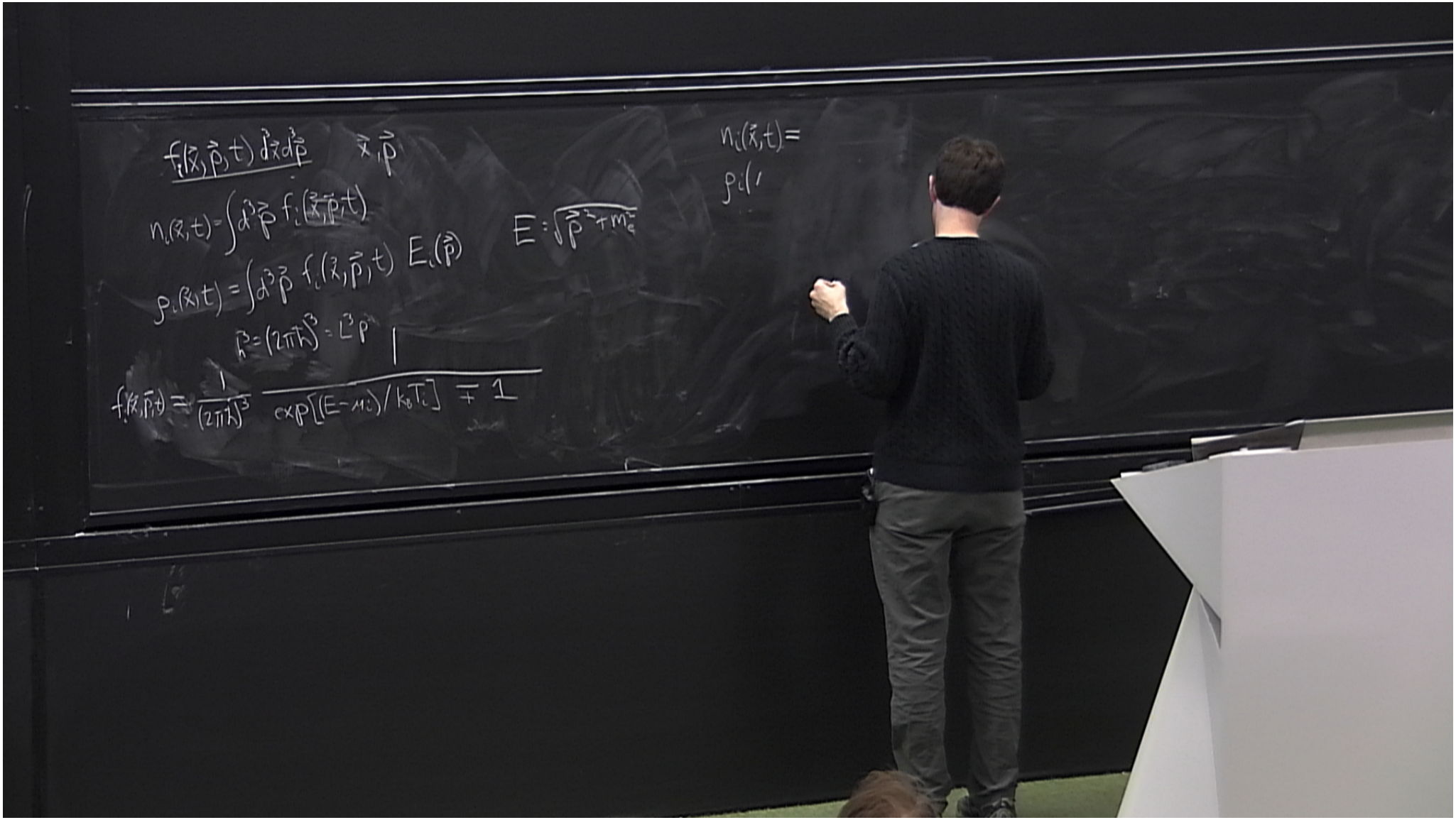
$$h^3 = (2\pi\hbar)^3 = L^3 p^3$$

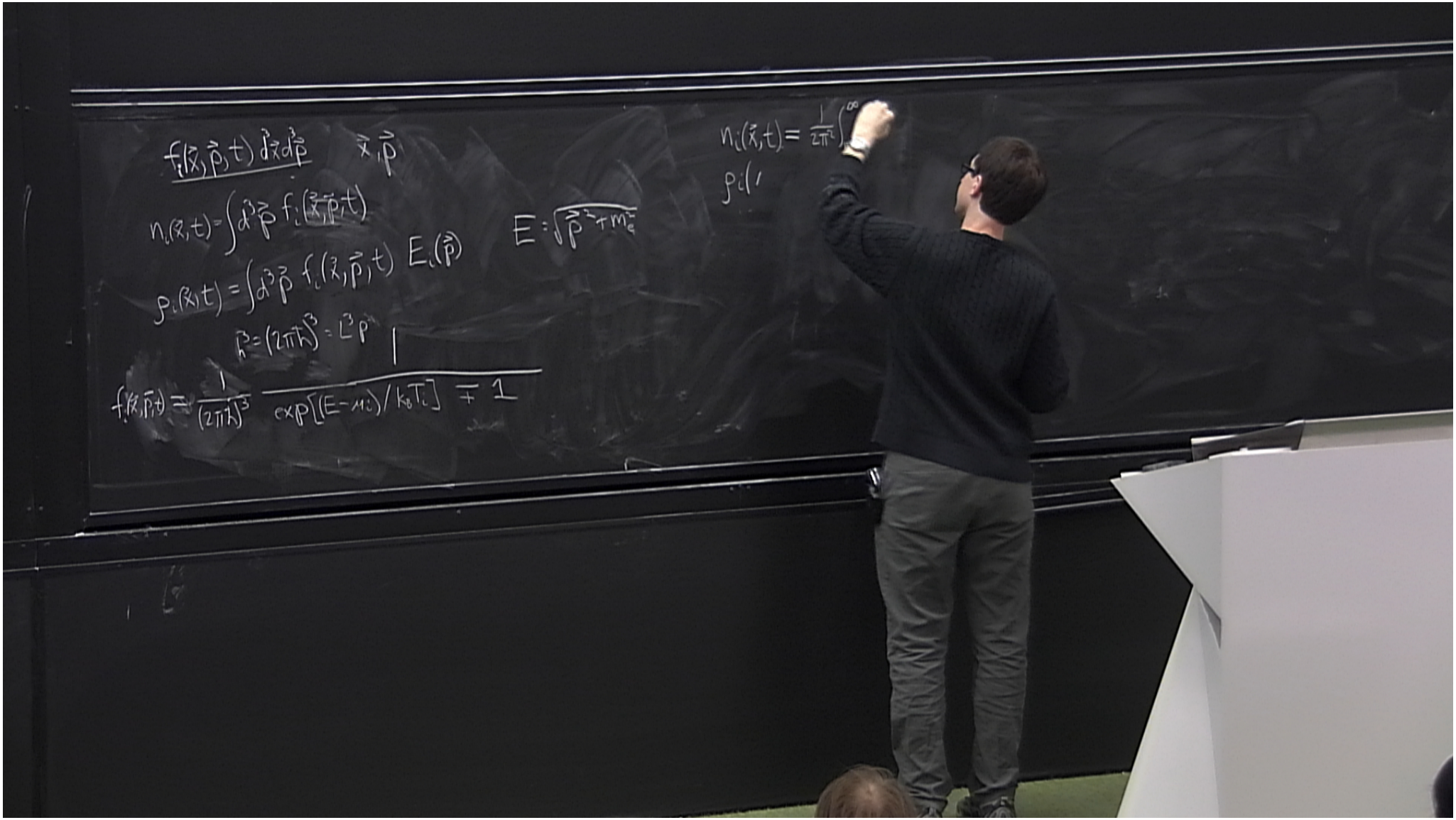
$$f_i(\vec{x},\vec{p},t) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp\left[(E-\mu_i)/k_B T_i\right] \mp 1}$$

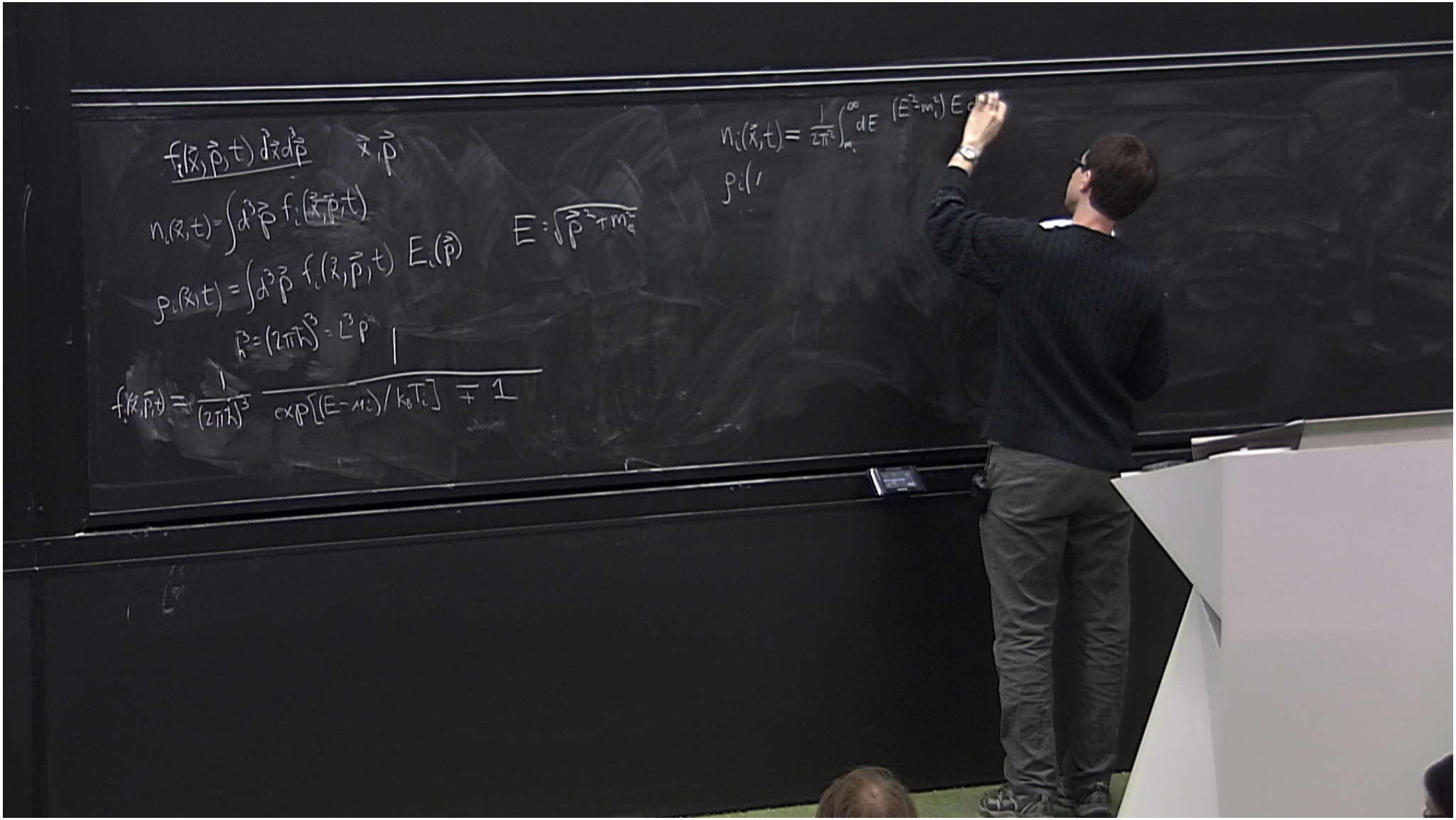

$n_i(\vec{x},t) =$
$p_i($

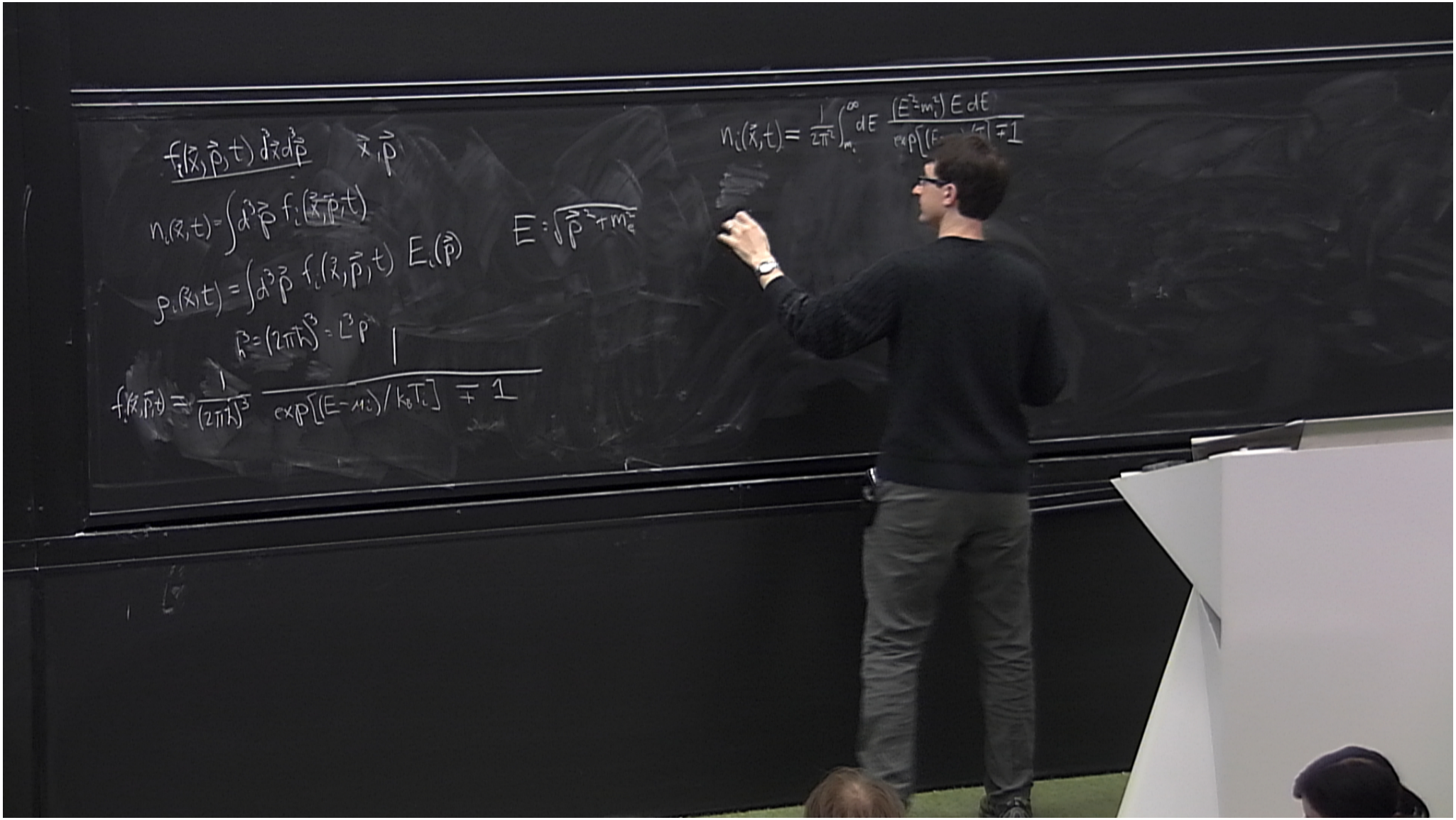

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E=\sqrt{\vec{p}^2+m_i^2}$
$h^3=(2\pi\hbar)^3=L^3p$
$f_i(\vec{x},\vec{p},t)=\frac{1}{(2\pi\hbar)^3}\frac{1}{\exp[(E-\mu_i)/k_BT_i]\mp 1}$
$n_i(\vec{x},t)=$
$\rho_i($

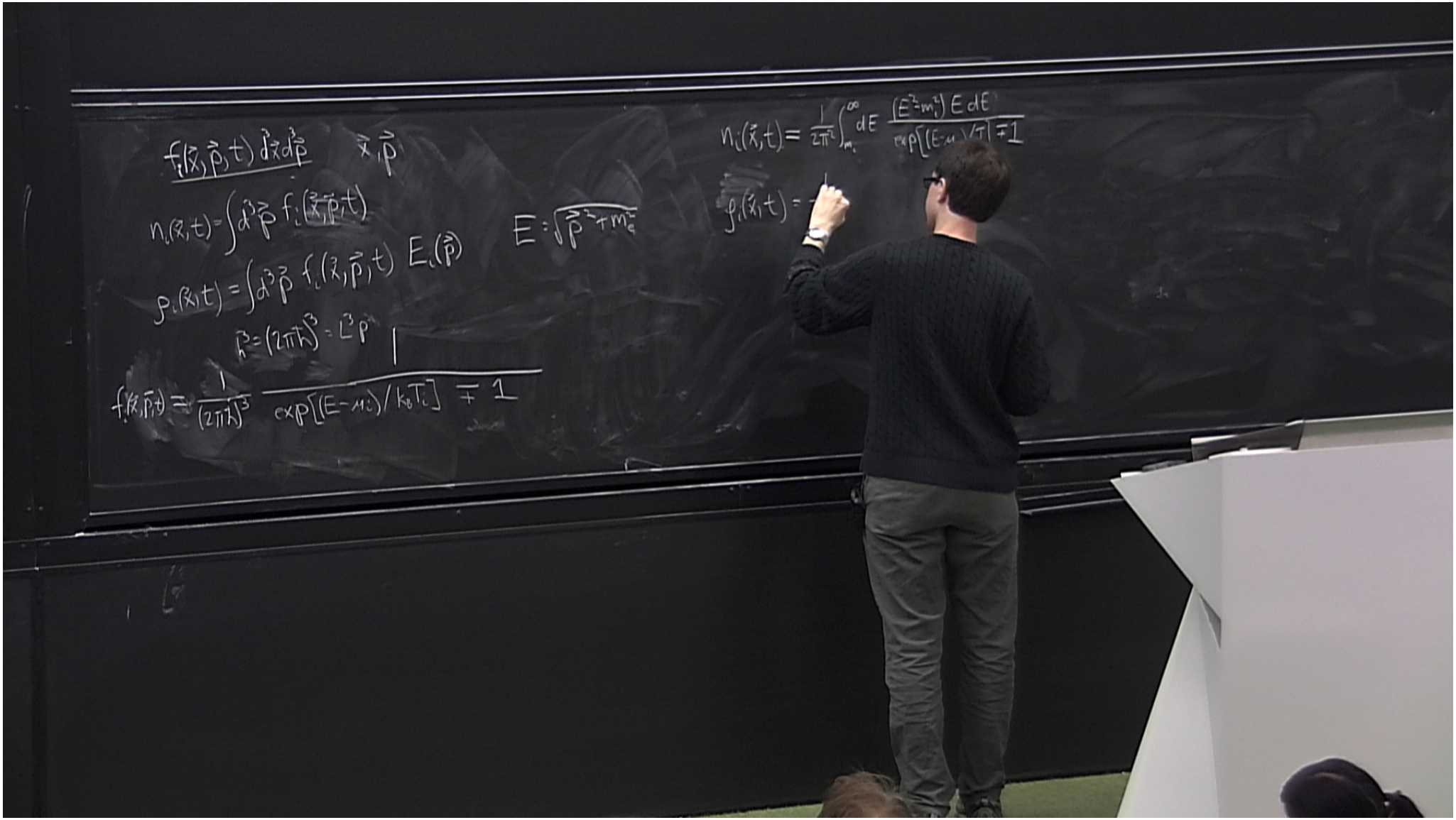

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$E = \sqrt{\vec{p}^2 + m_i^2}$
$h^3 = (2\pi\hbar)^3 = L^3 p$
$f_i(\vec{x},\vec{p},t) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_B T_i] \mp 1}$
$n_i(\vec{x},t) =$
$\rho_i($

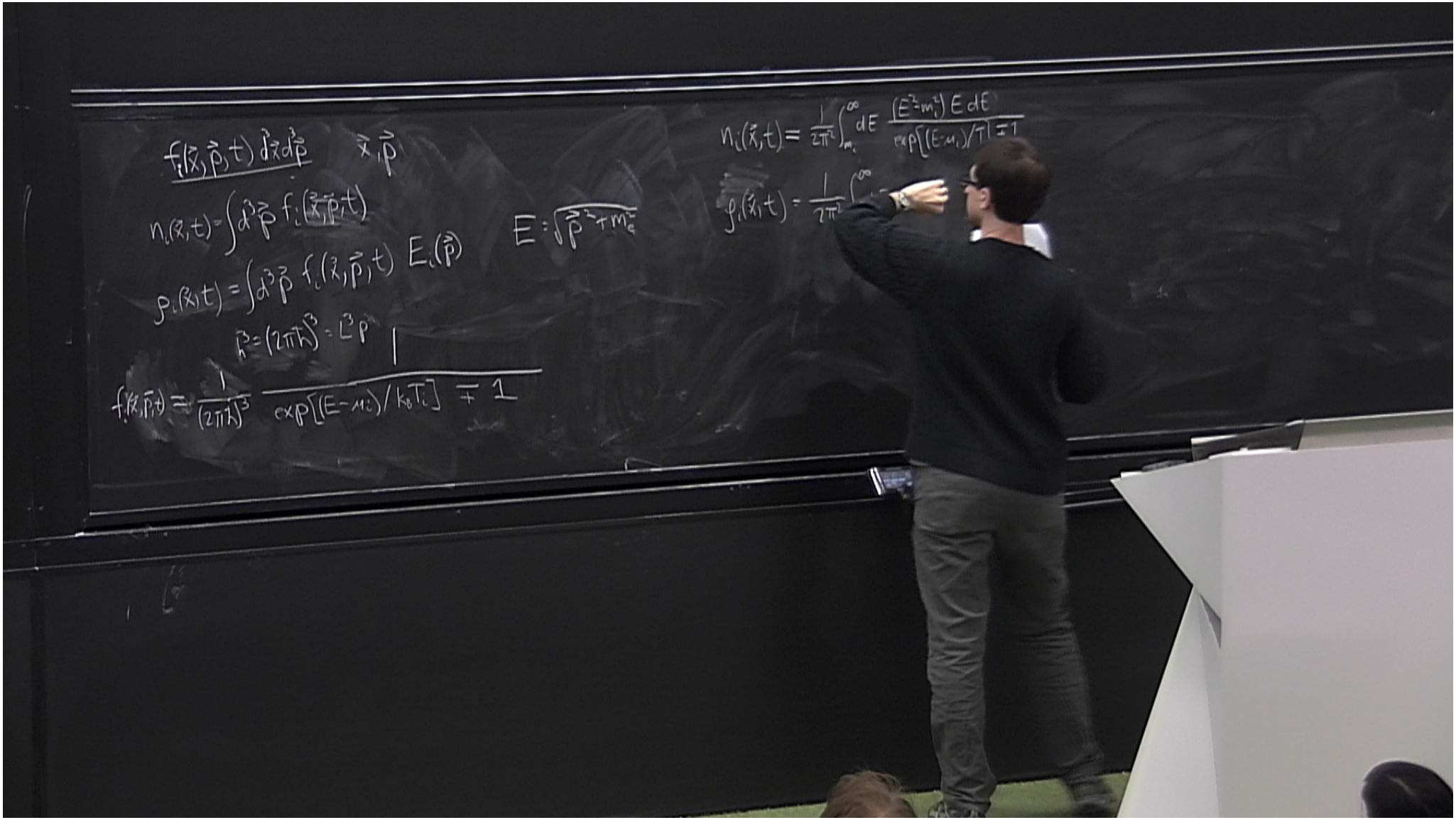

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$n_i(\vec{x},t) =$
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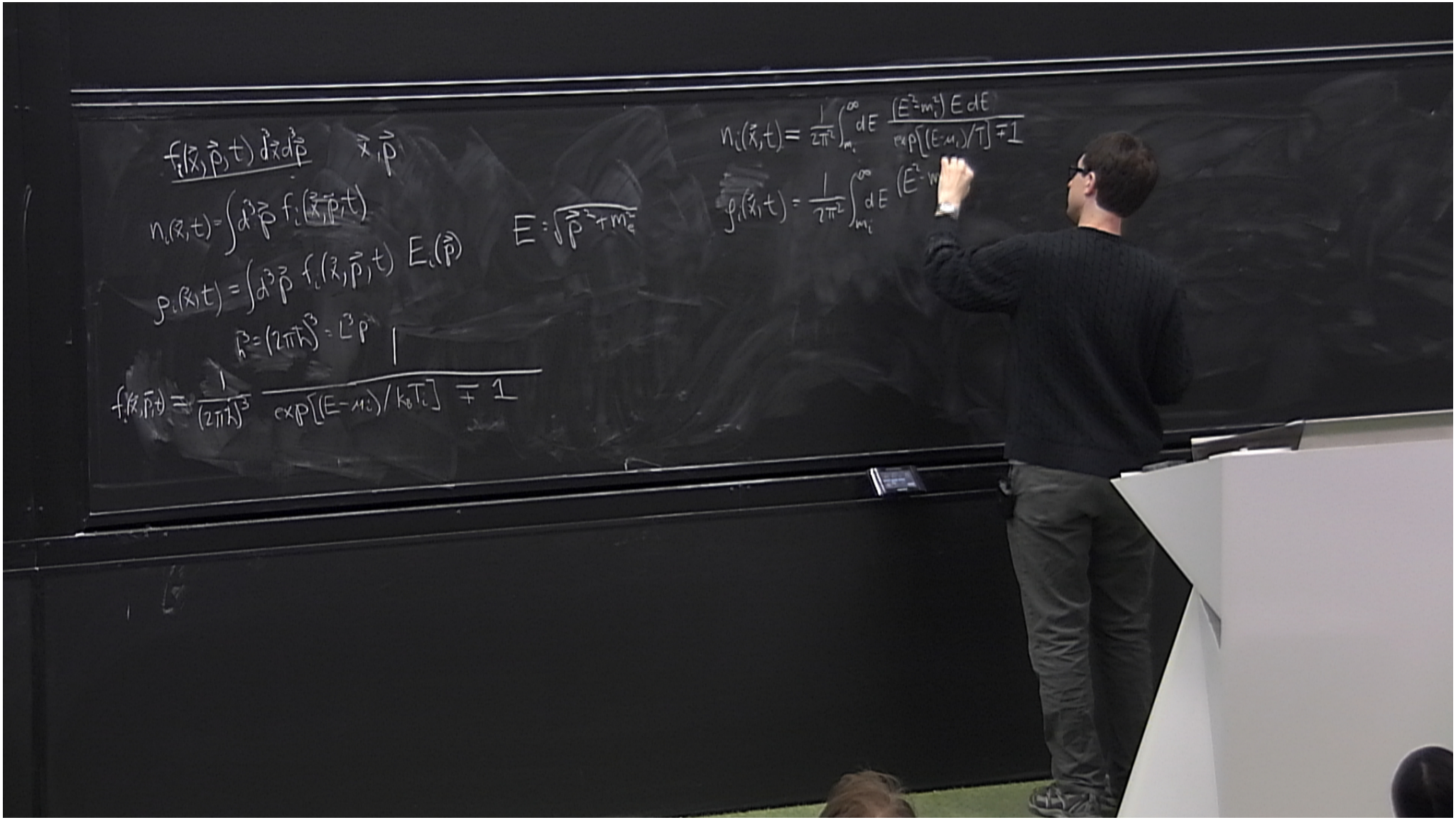

$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$ $E=\sqrt{\vec{p}^2+m_i^2}$
$h^3=(2\pi\hbar)^3=L^3p$
$f_i(\vec{x},\vec{p},t)=\frac{1}{(2\pi\hbar)^3}\frac{1}{\exp[(E-\mu_i)/k_BT_i]\mp 1}$
$n_i(\vec{x},t)=\frac{1}{2\pi^2}\int_0^\infty$
$\rho_i($

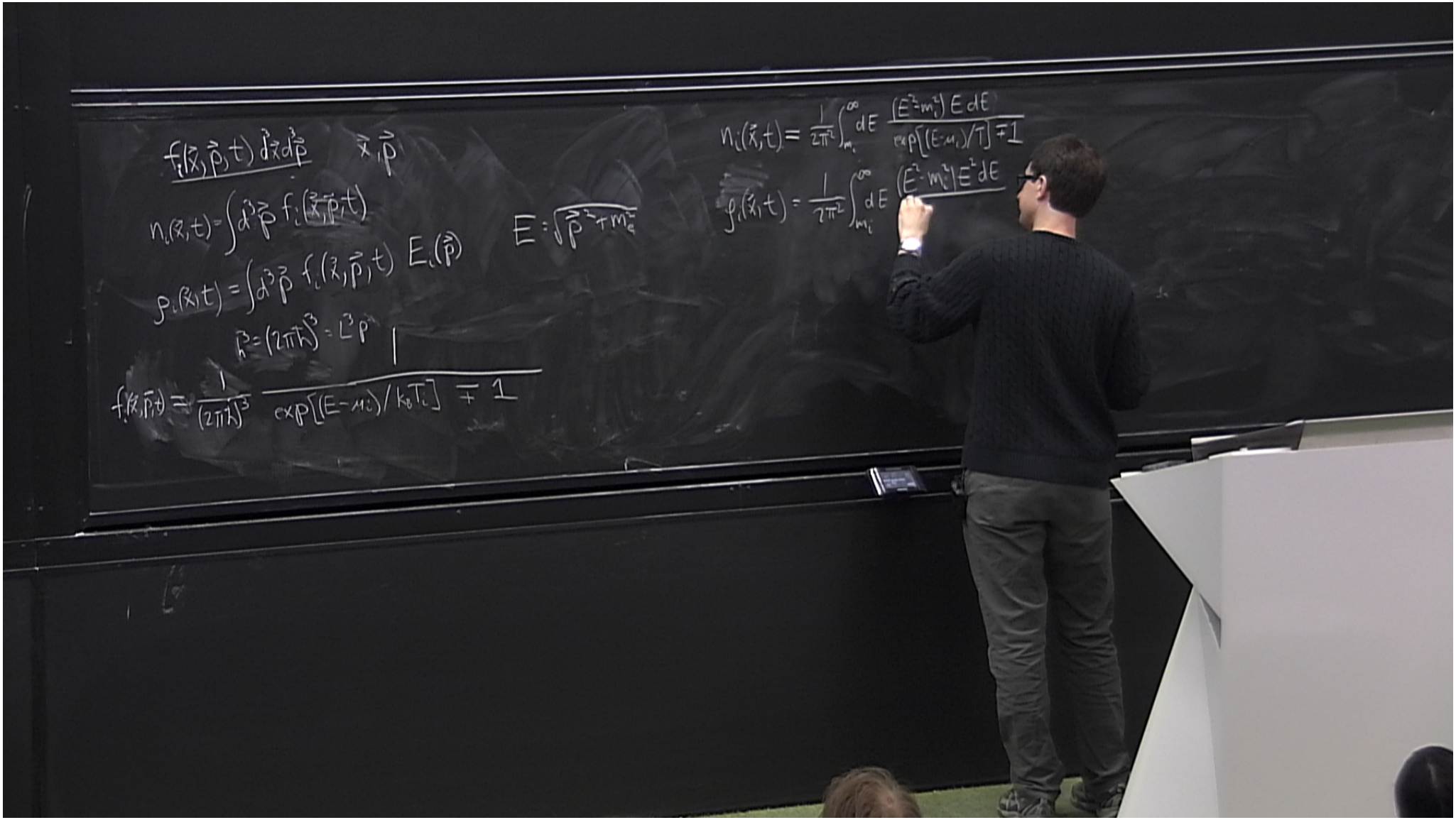

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$h^3 = (2\pi\hbar)^3 = L^3 p^3$
$f_i(\vec{x},\vec{p},t) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_B T_i] \mp 1}$
$n_i(\vec{x},t) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} dE\, (E^2 - m_i^2)\, E$

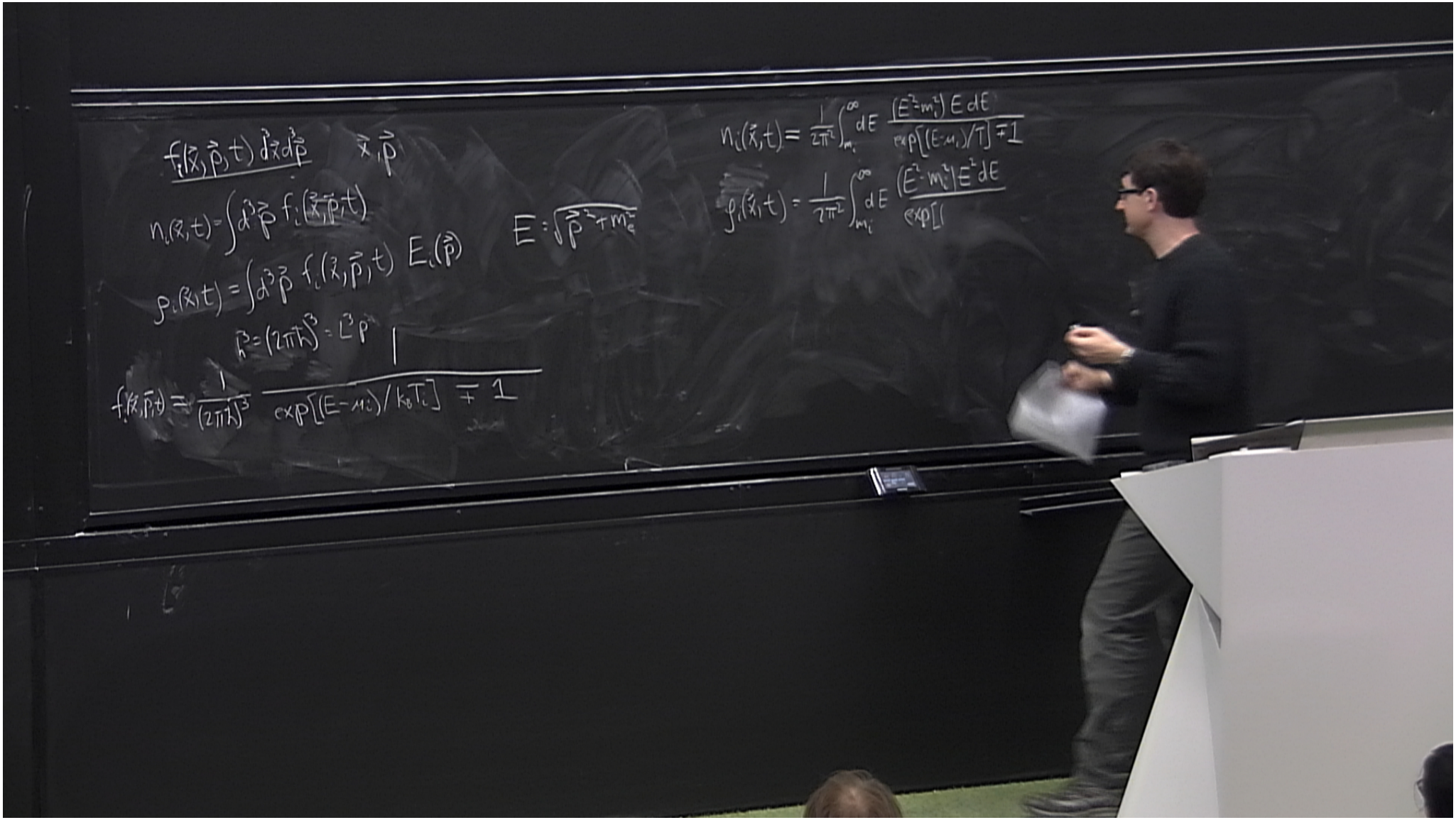

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
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$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\, E\, dE}{\exp[(E \quad)/T] \mp 1}$

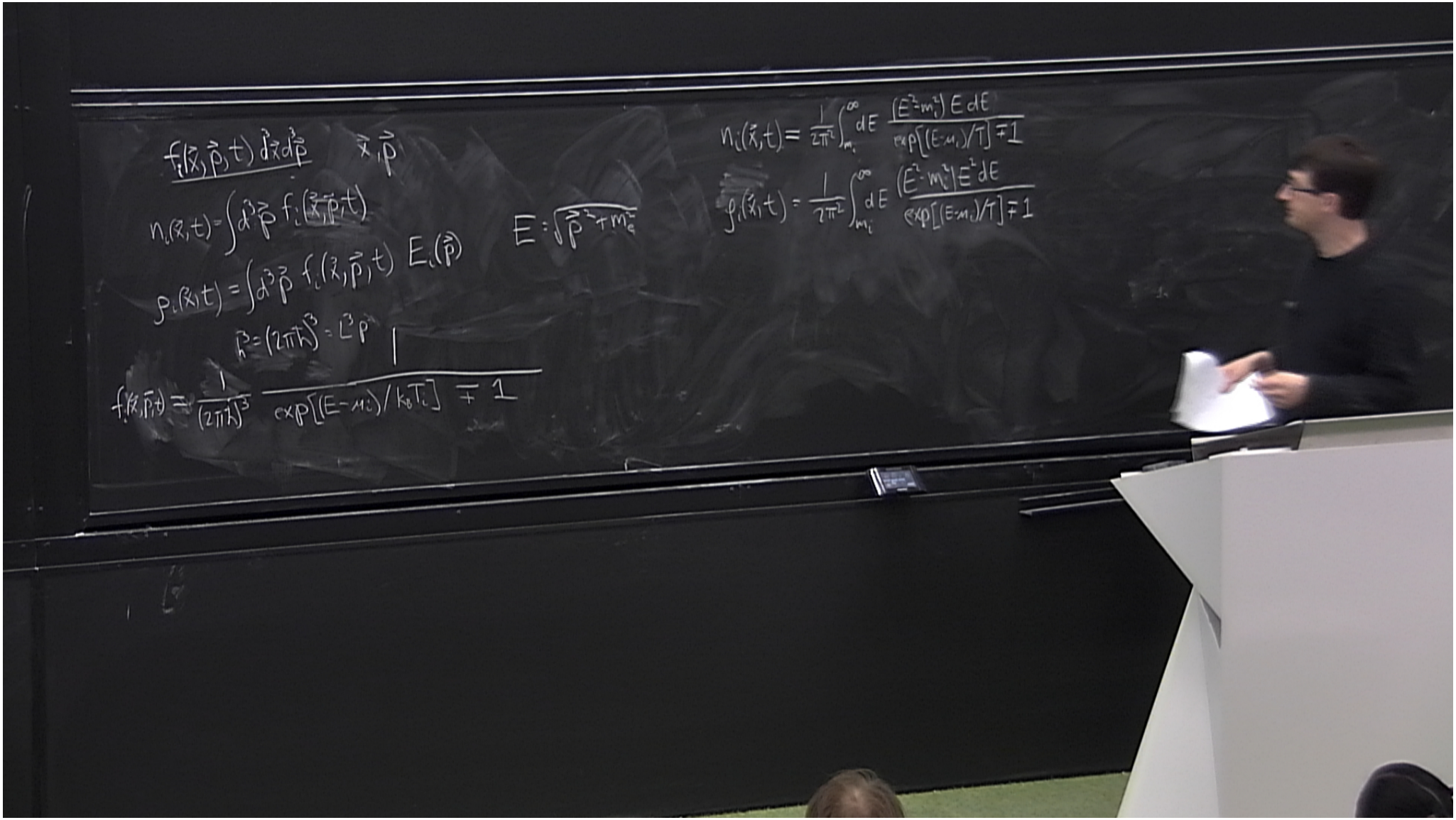

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$n_i(\vec{x},t) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} dE \frac{(E^2 - m_i^2)\, E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x},t) =$

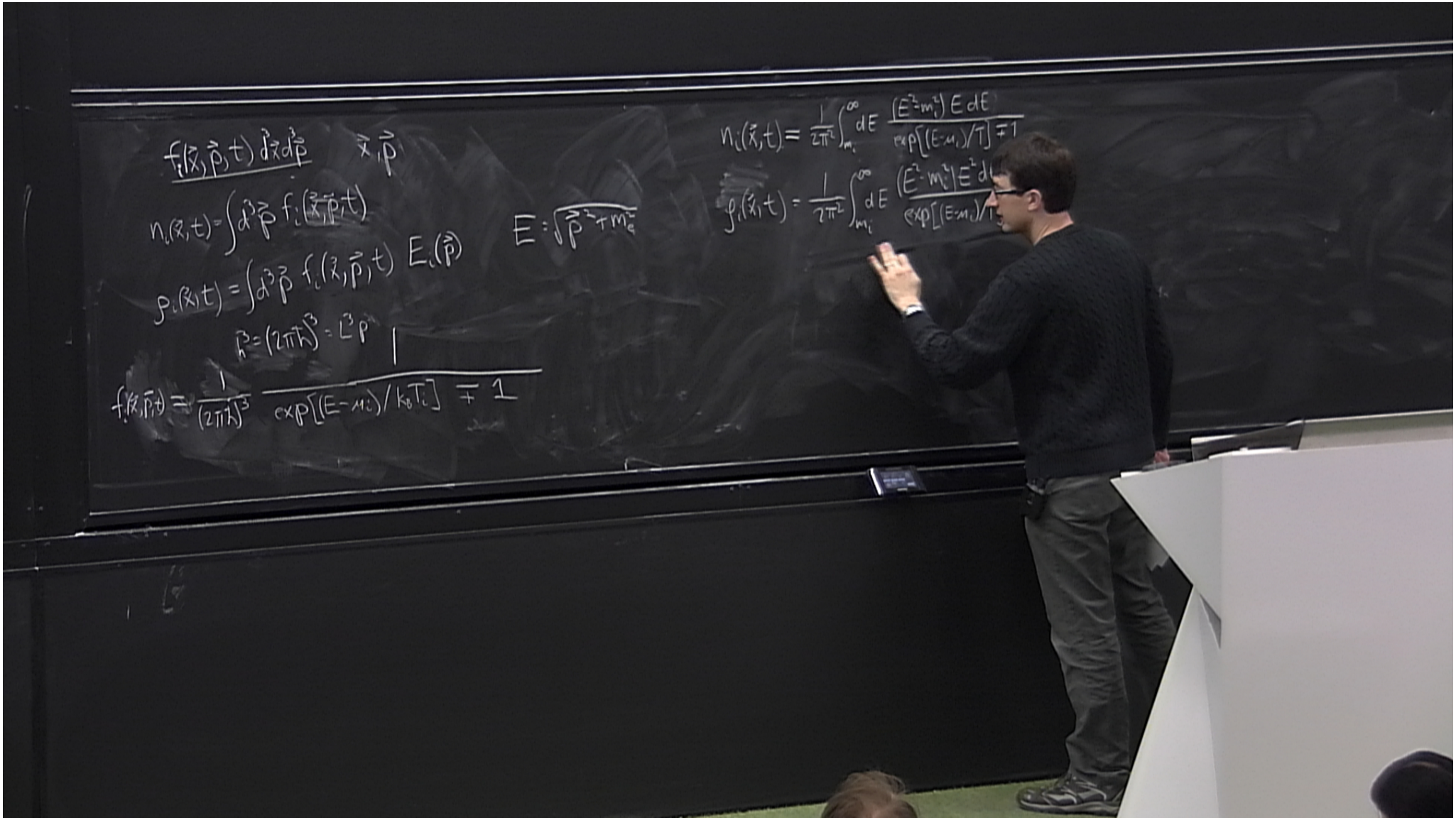

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$h^3 = (2\pi\hbar)^3 = L^3 p$
$f_i(\vec{x},\vec{p},t) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_B T_i] \mp 1}$
$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_e}^{\infty} dE \frac{(E^2-m_e^2)\, E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int^{\infty}$

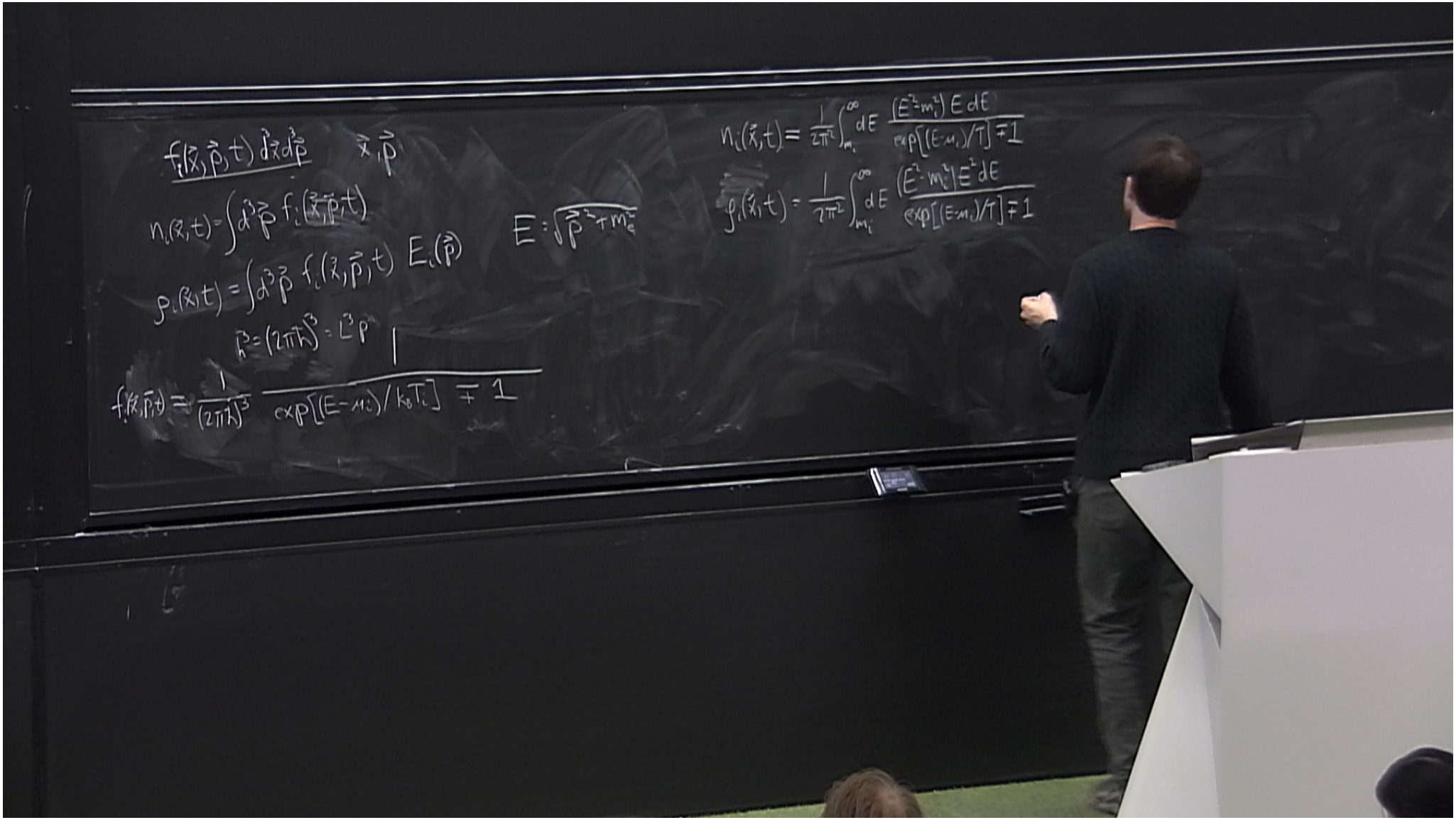

$f_i(\vec{x},\vec{p},t)\, d^3x\, d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\, E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE\, (E^2-m$

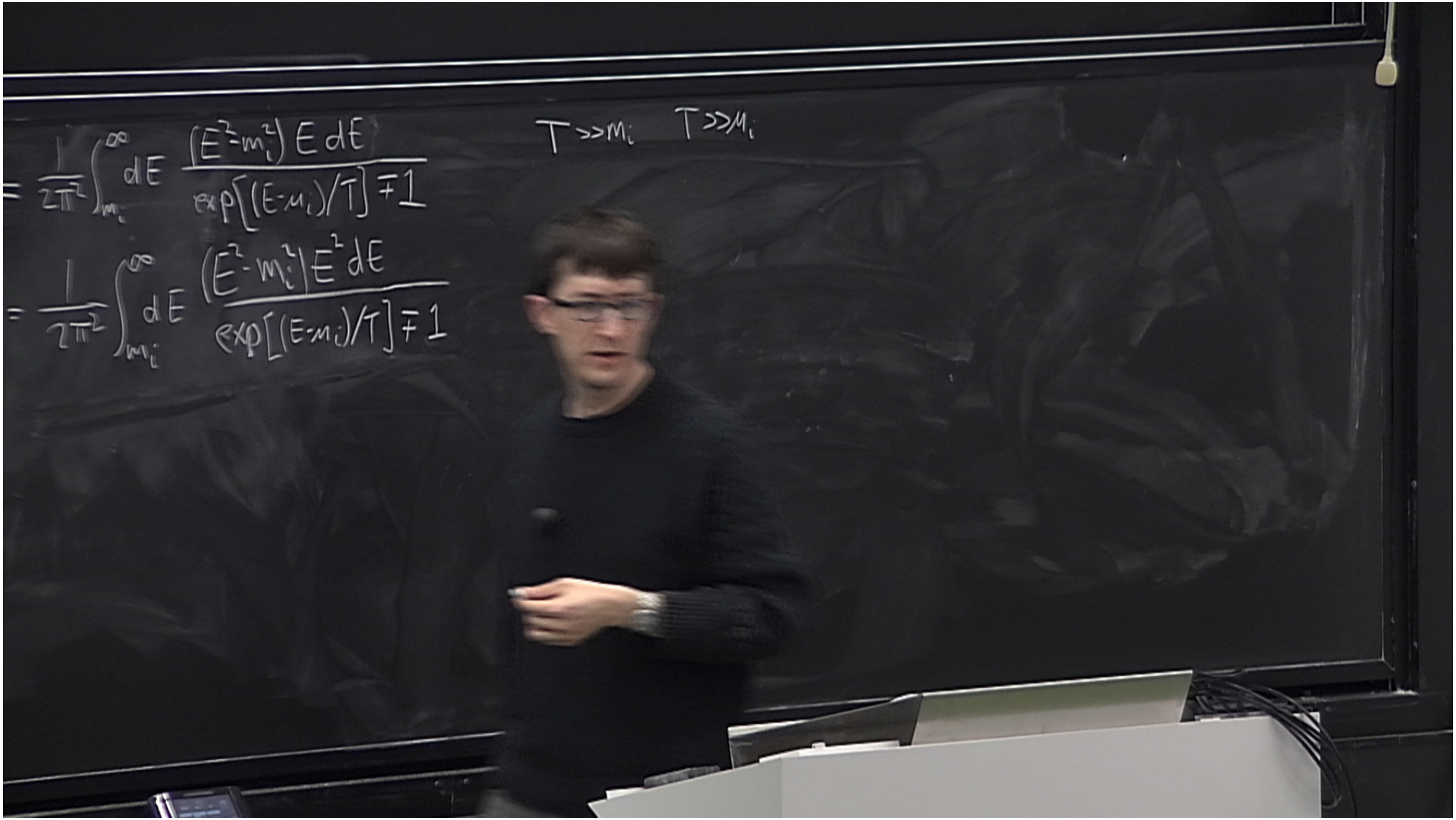

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p \quad \vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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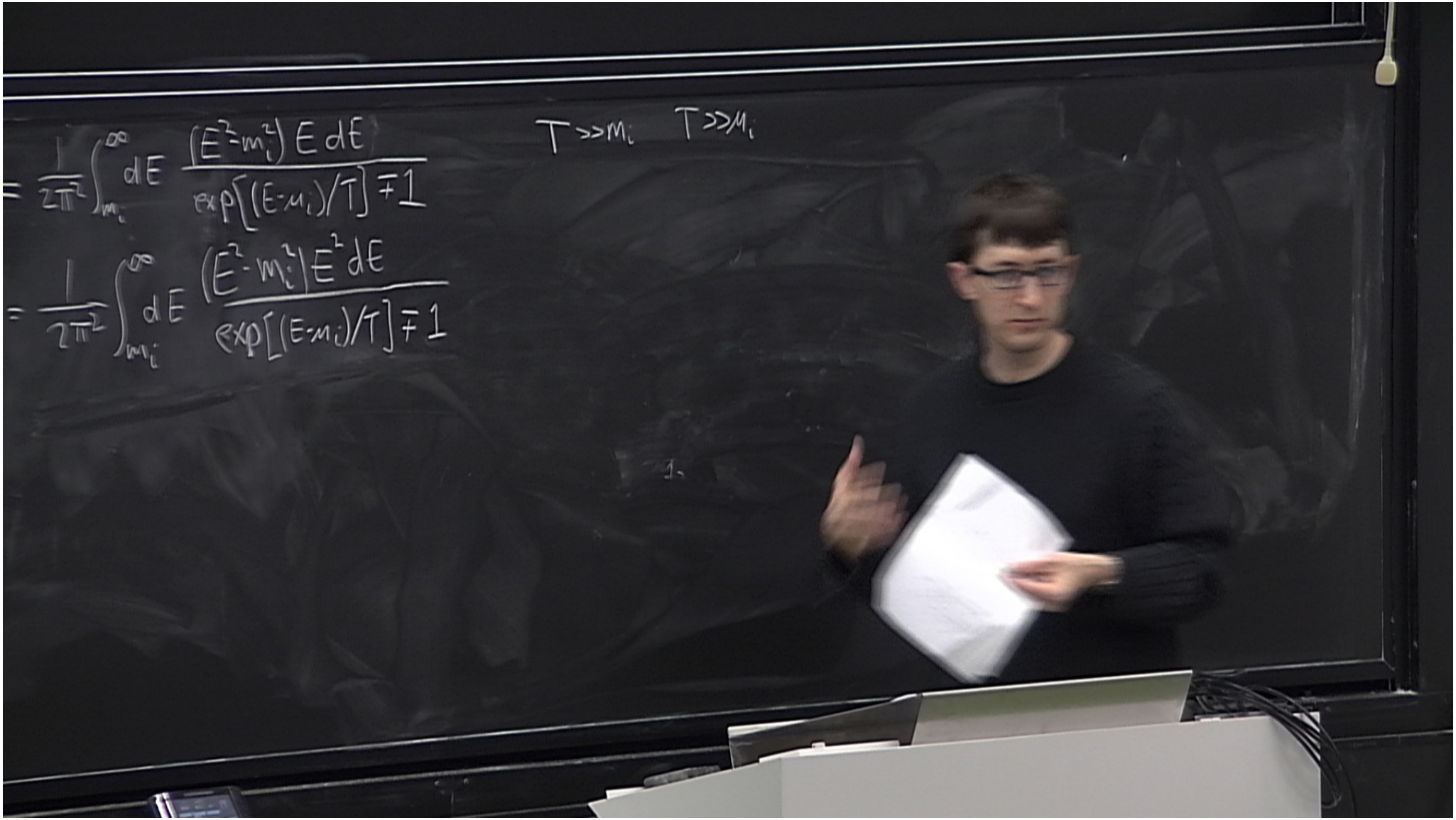

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
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$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\, E^2 dE}{\exp[($

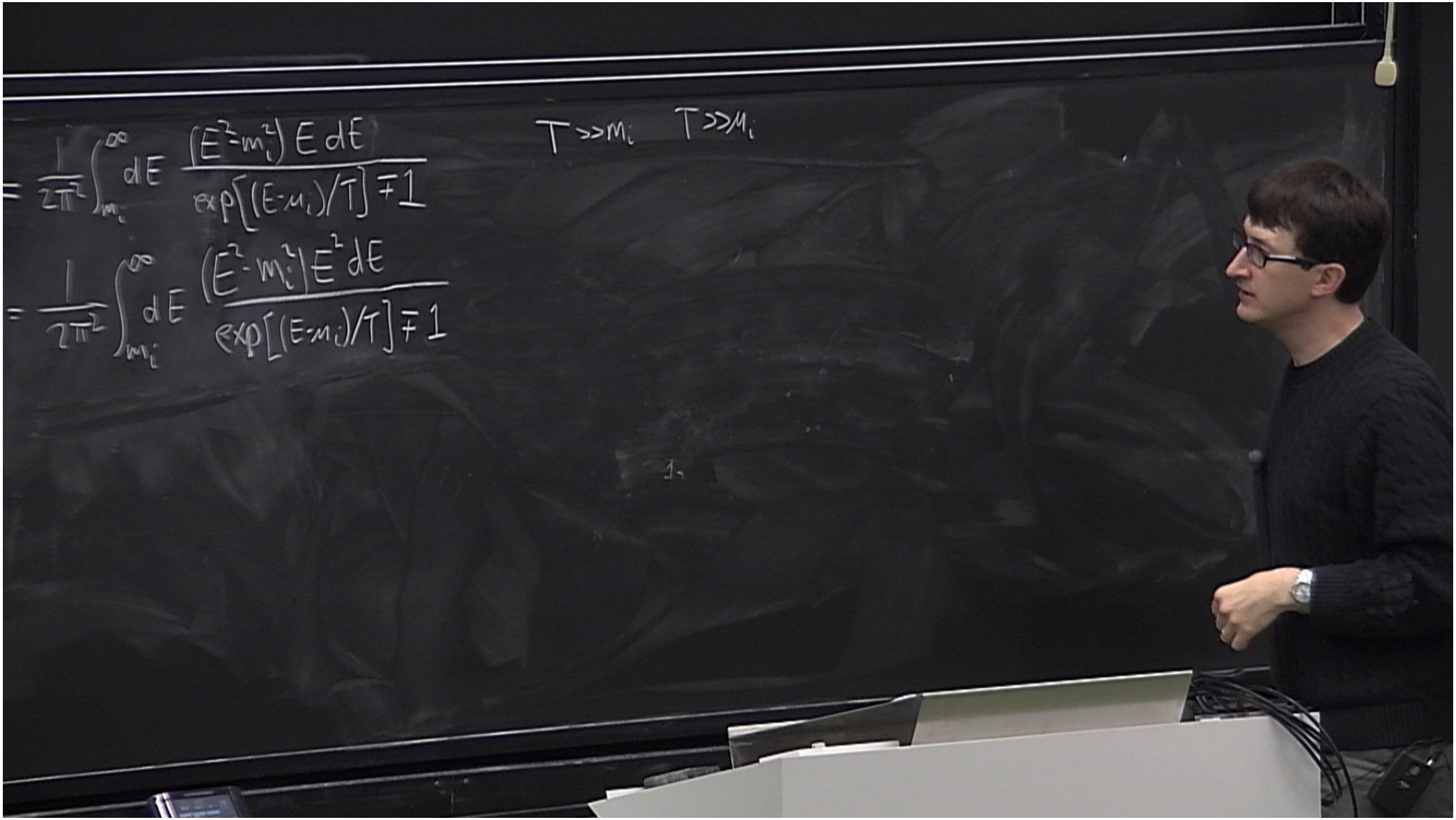

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p \quad \vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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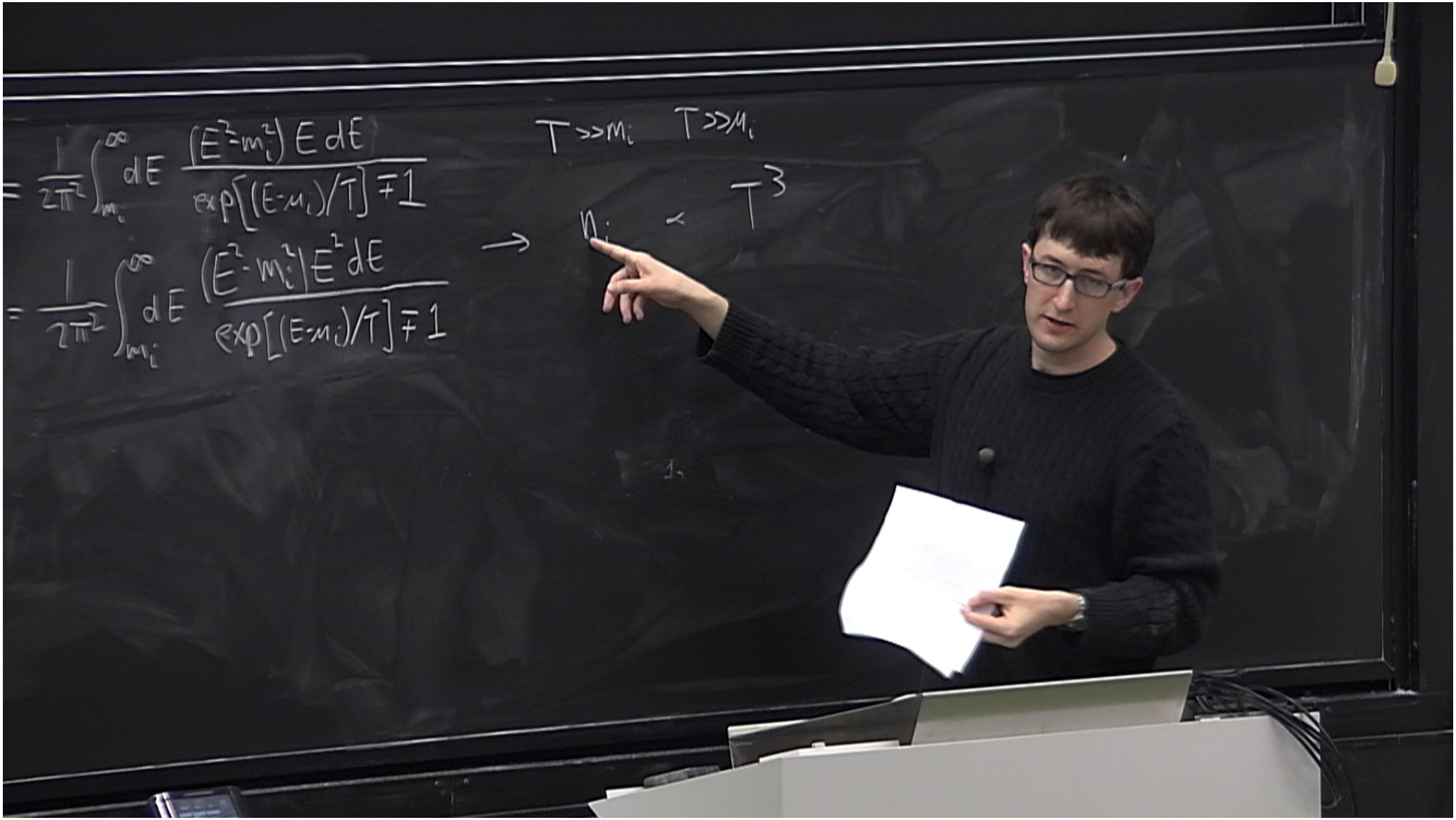

$f_i(\vec{x},\vec{p},t)\, d^3x\, d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
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$h^3 = (2\pi\hbar)^3 = L^3 p$
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$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E^2 dE}{\exp[(E-\mu_i)/T] \mp 1}$

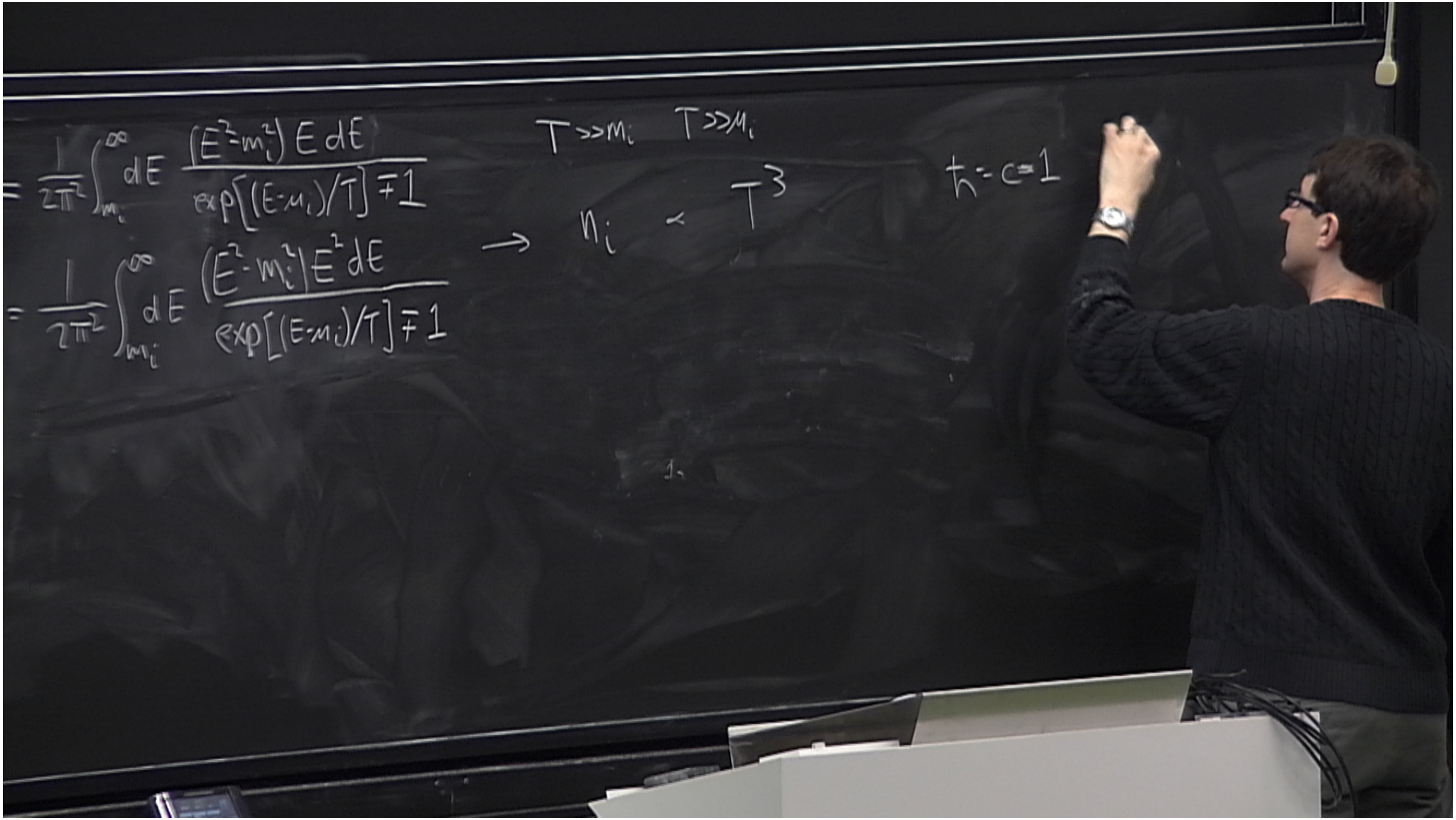

$f_i(\vec{x},\vec{p},t)\, d^3x d^3p$ $\vec{x},\vec{p}$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$ $E = \sqrt{\vec{p}^2 + m_i^2}$
$h^3 = (2\pi\hbar)^3 = L^3 p$
$f_i(\vec{x},\vec{p},t) = \frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_B T_i] \mp 1}$
$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE\, \frac{(E^2-m_i^2)\, E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE\, \frac{(E^2-m_i^2)\, E^2 dE}{\exp[(E-\mu_i)/T] \mp 1}$

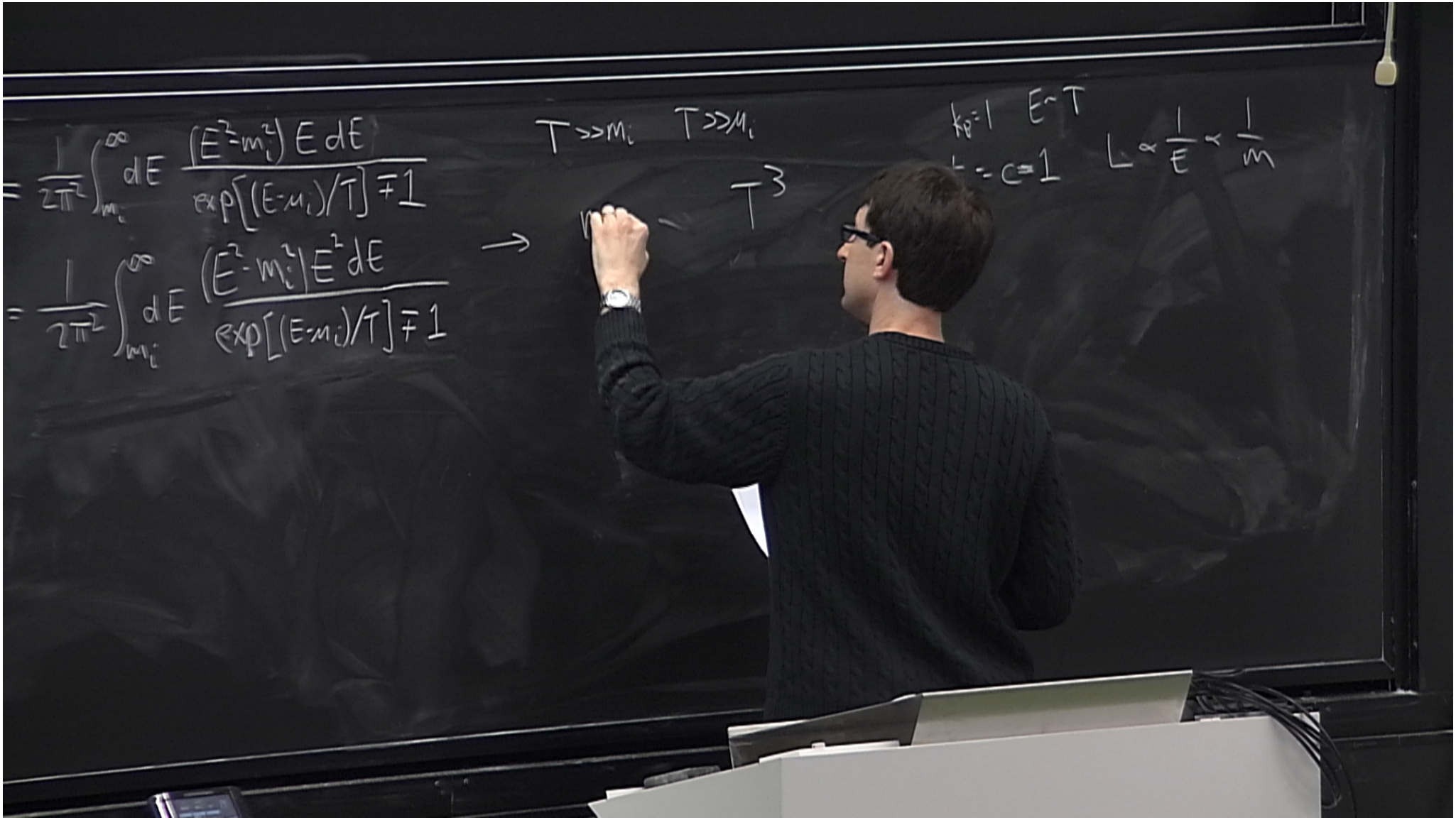

$= \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\,E\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
$= \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\,E^2\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
$T \gg m_i$ $T \gg \mu_i$

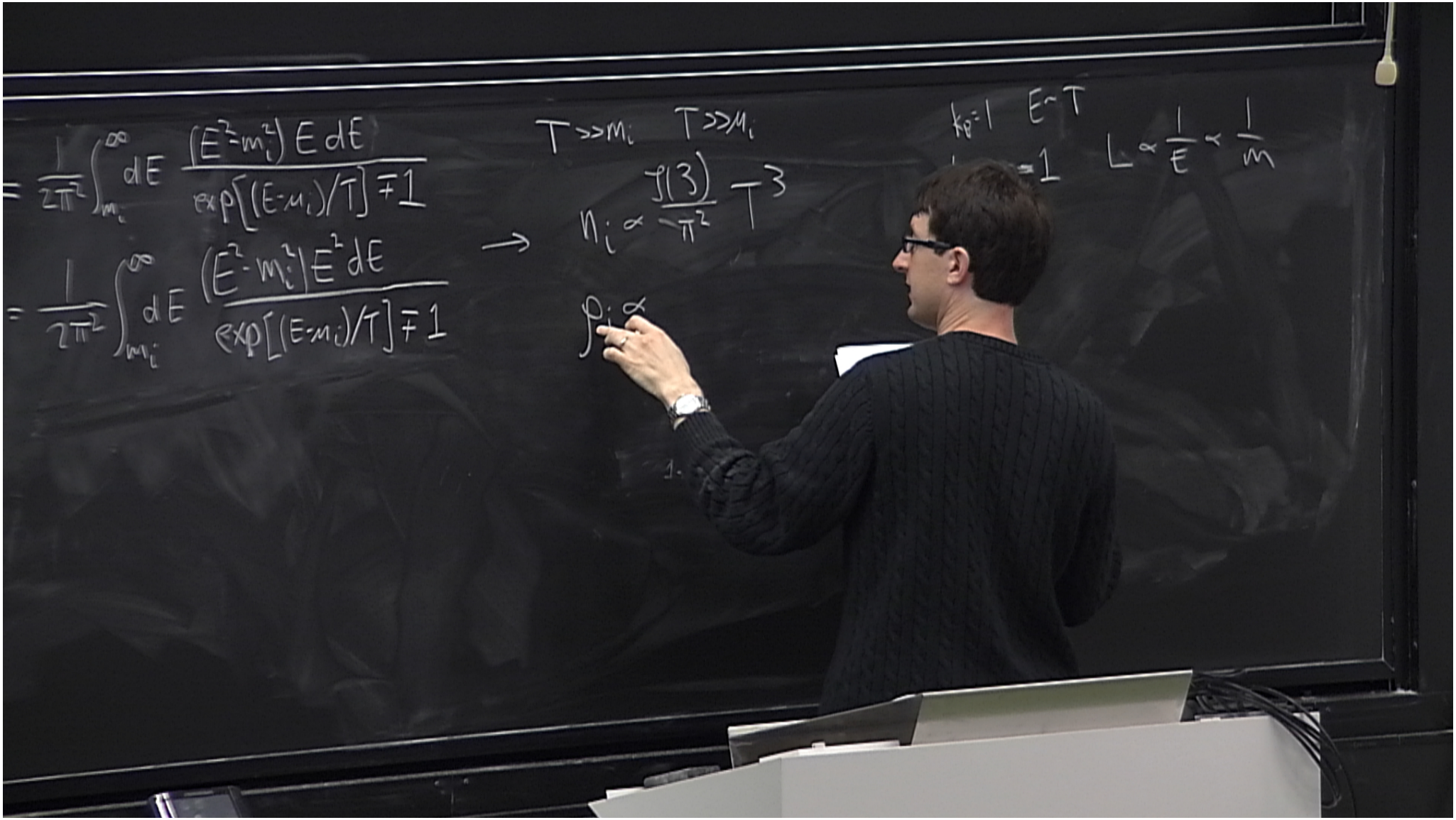

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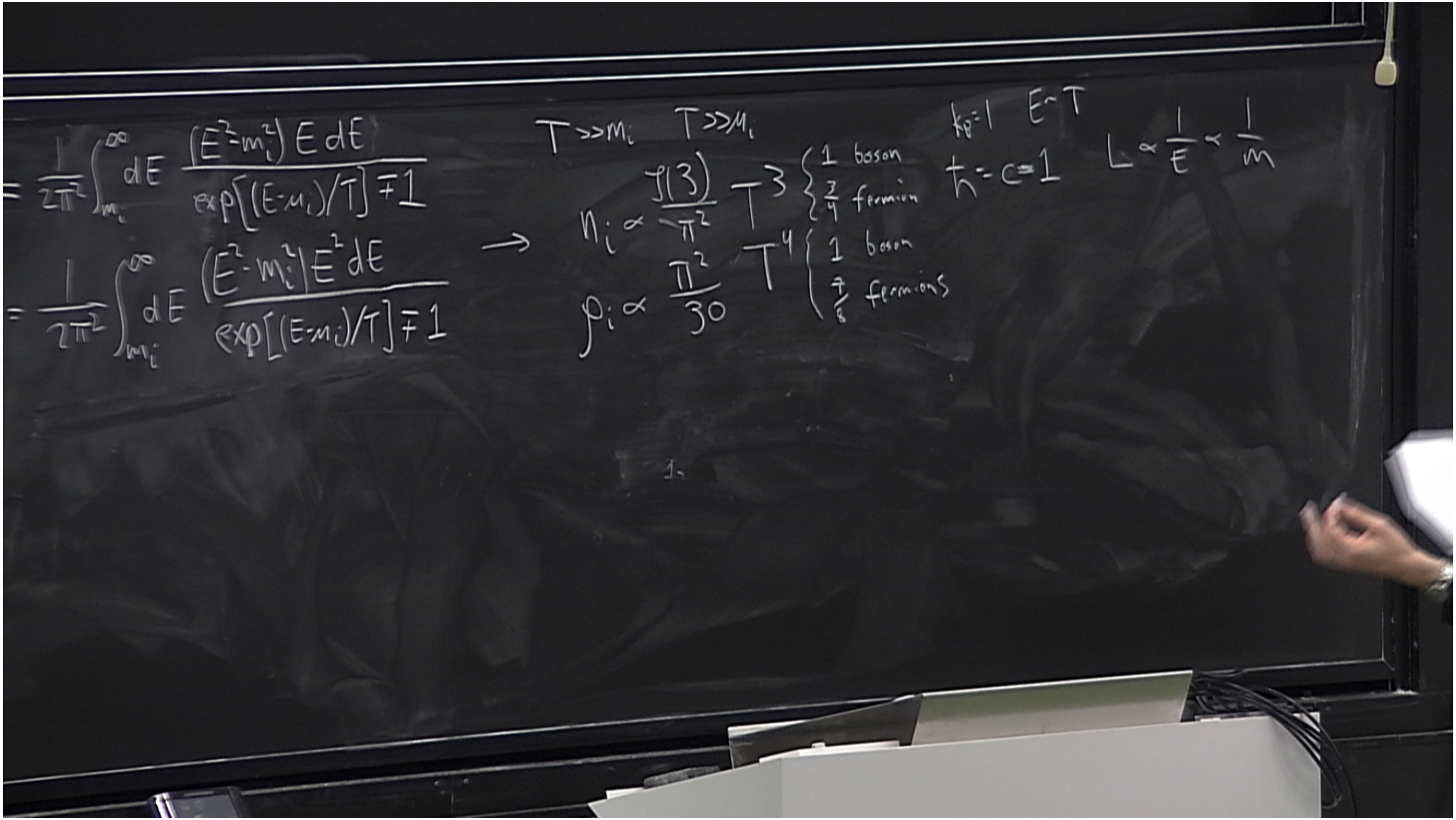

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1.

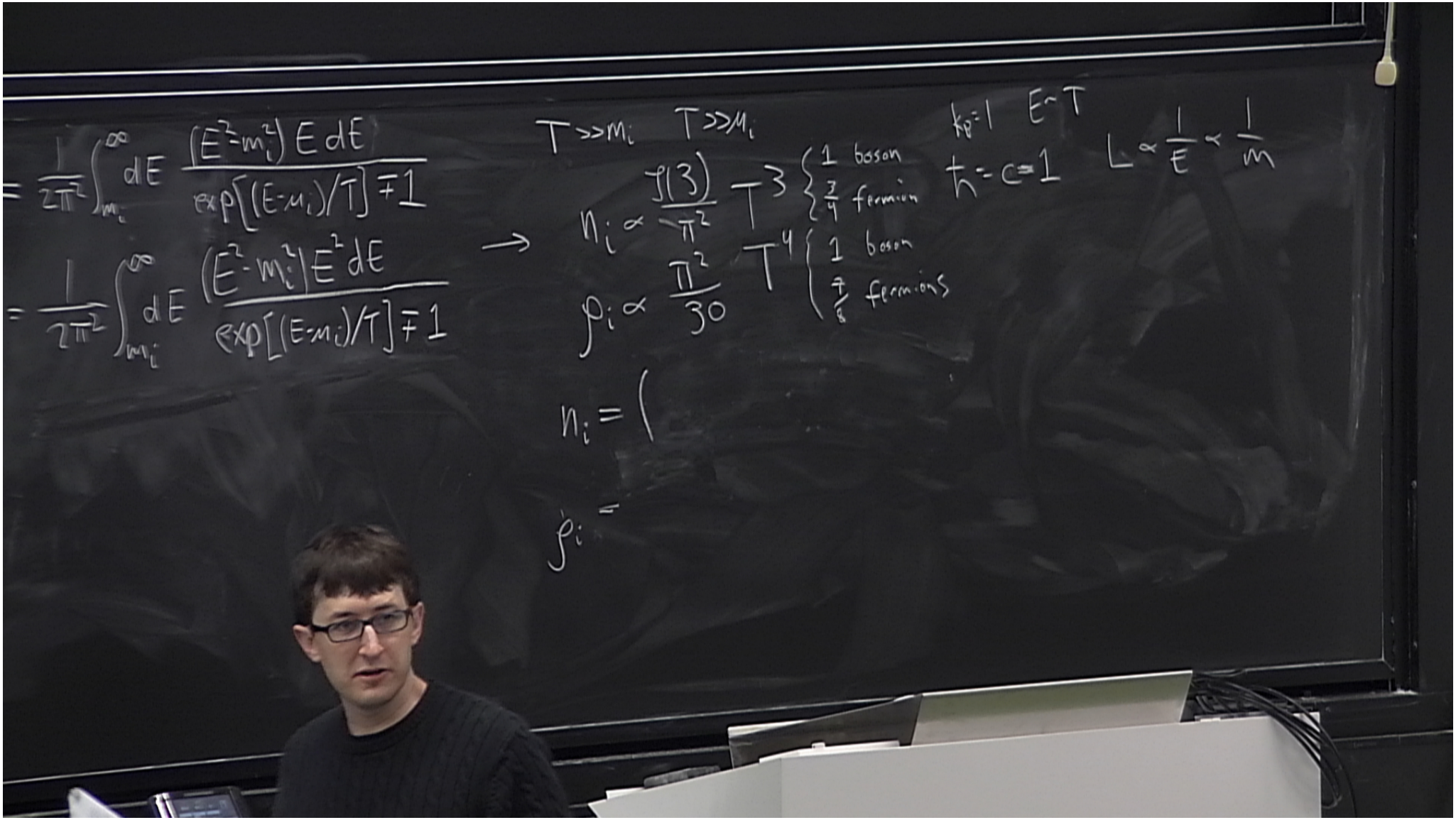

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$T \gg m_i \quad T \gg \mu_i$
$\rightarrow \quad n_i \propto T^3$

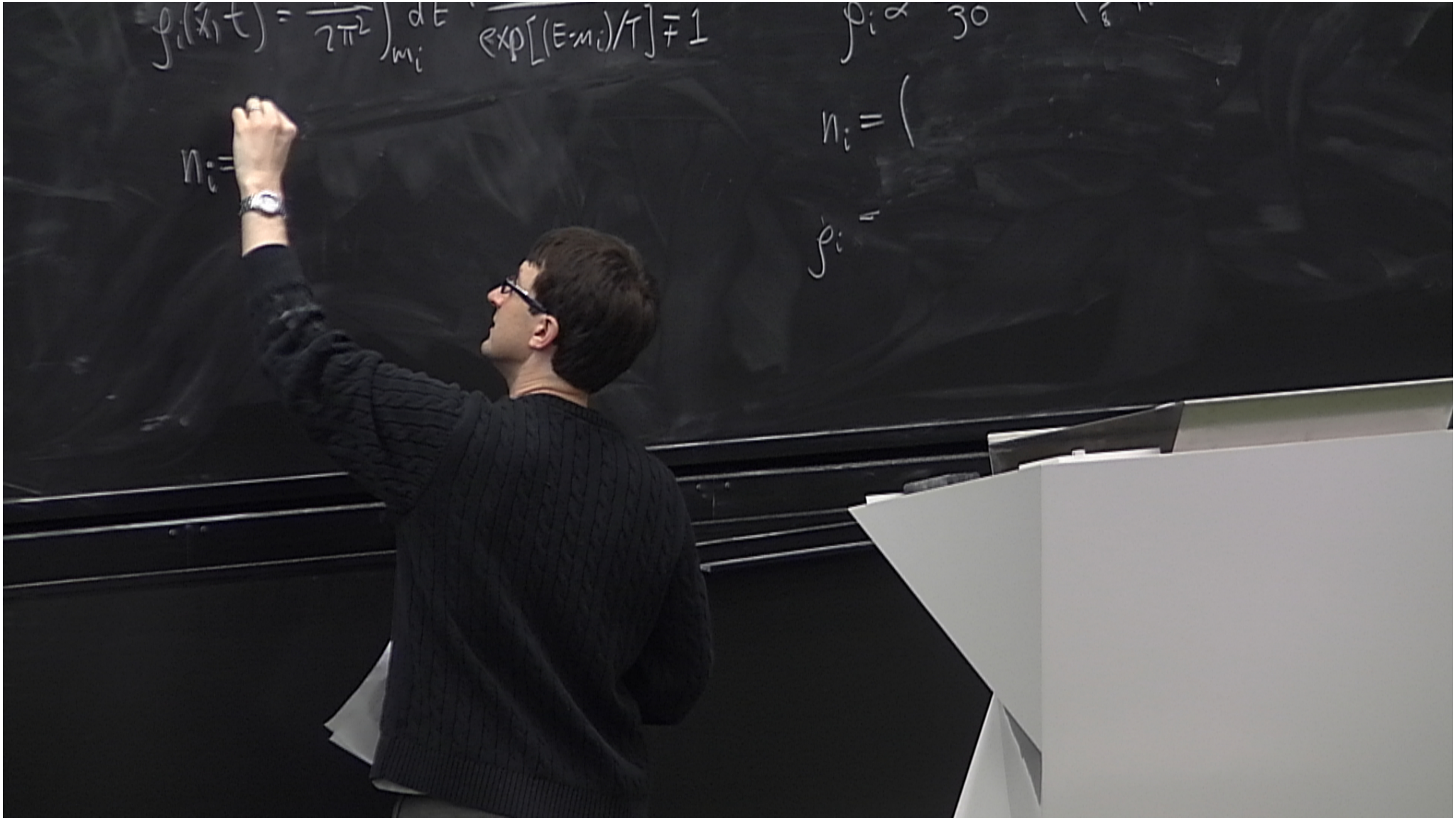

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$\hbar = c = 1$

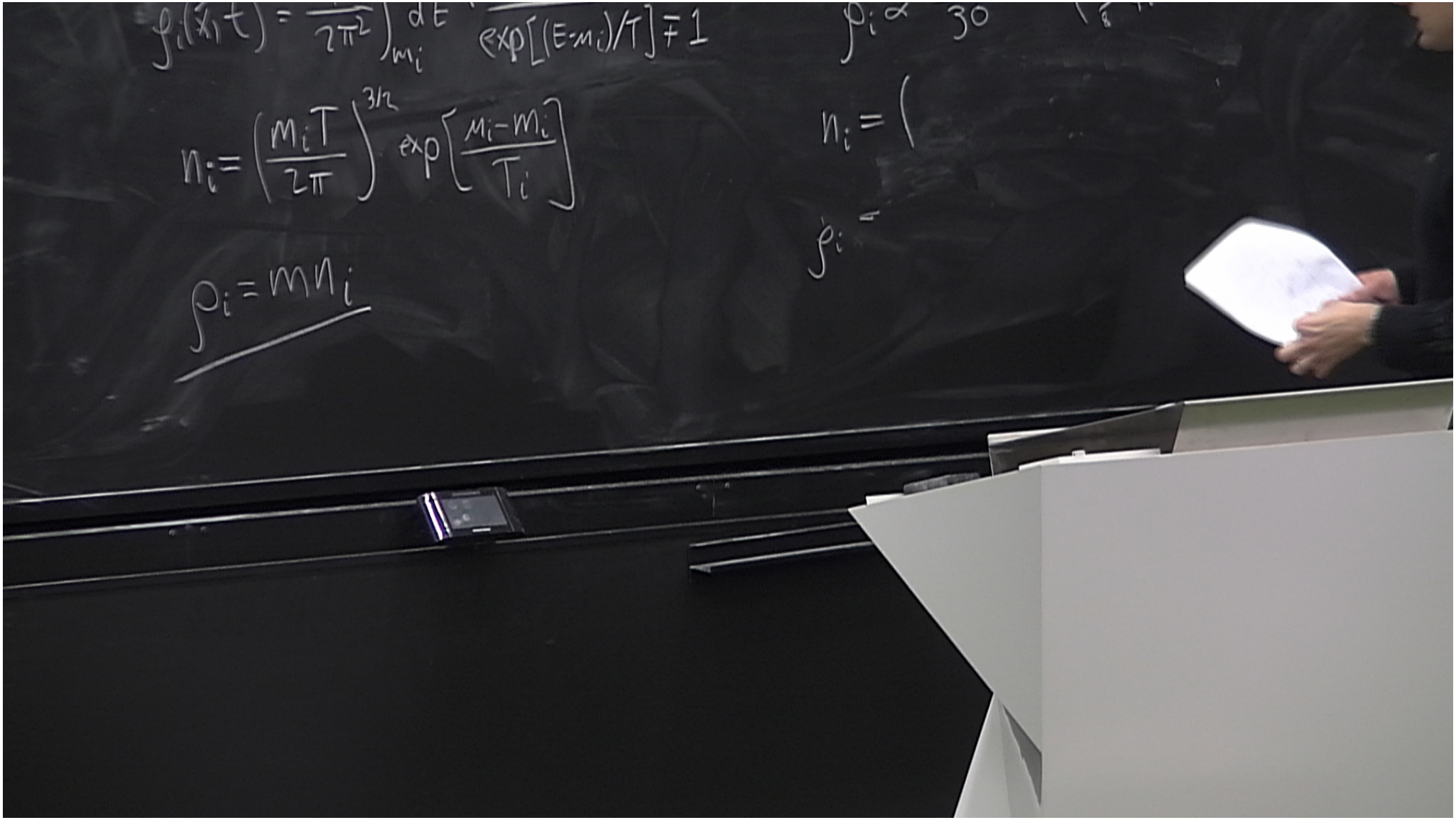

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$\rightarrow$
$T \gg m_i \quad T \gg \mu_i$
T^3
$k_B = 1 \quad E \sim T$
$\hbar = c = 1 \quad L \propto \frac{1}{E} \propto \frac{1}{m}$

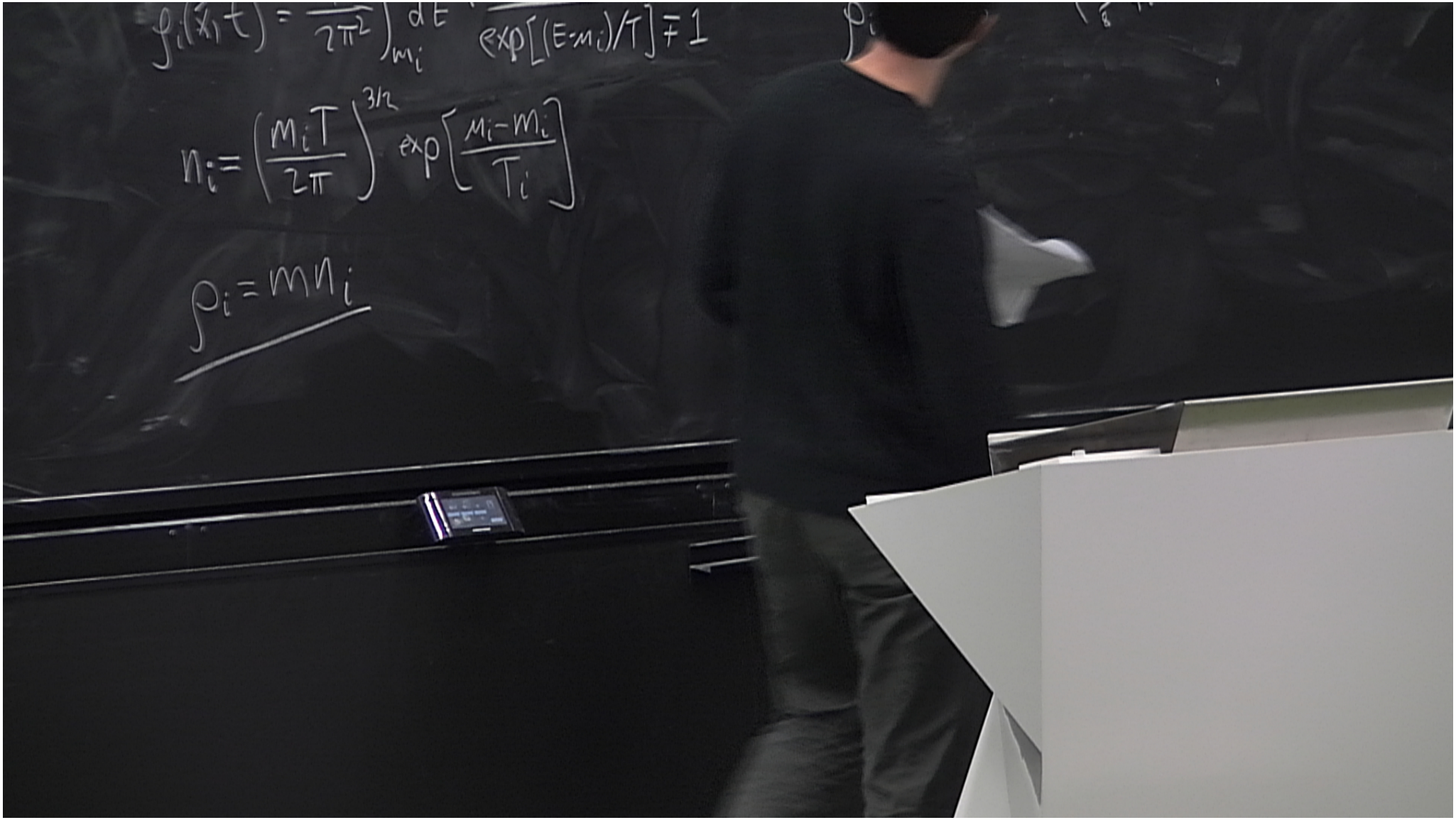

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$T \gg m_i \quad T \gg \mu_i$
$\rightarrow$
$n_i \propto \frac{\zeta(3)}{\pi^2} T^3$
$\rho_i \propto$
$k_B = 1 \quad E \sim T$
$= 1 \quad L \propto \frac{1}{E} \propto \frac{1}{m}$

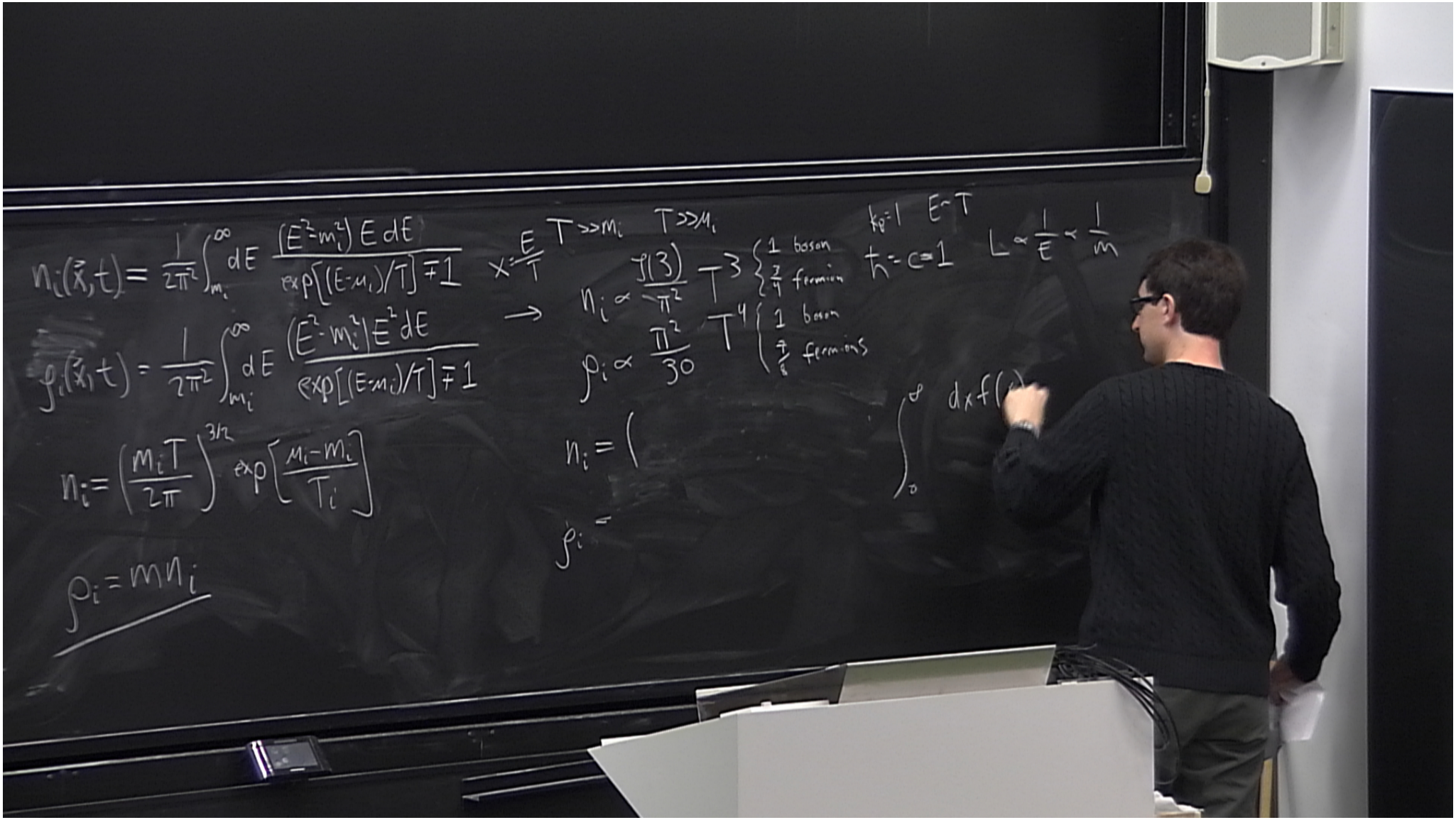

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$= \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\,E^2\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rightarrow$
$T \gg m_i \quad T \gg \mu_i$
$n_i \propto \frac{\zeta(3)}{\pi^2} T^3$ { 1 boson, $\frac{3}{4}$ fermion
$\rho_i \propto \frac{\pi^2}{30} T^4$ { 1 boson, $\frac{7}{8}$ fermions
$k_B = 1 \quad E \sim T$
$\hbar = c = 1 \quad L \propto \frac{1}{E} \propto \frac{1}{m}$

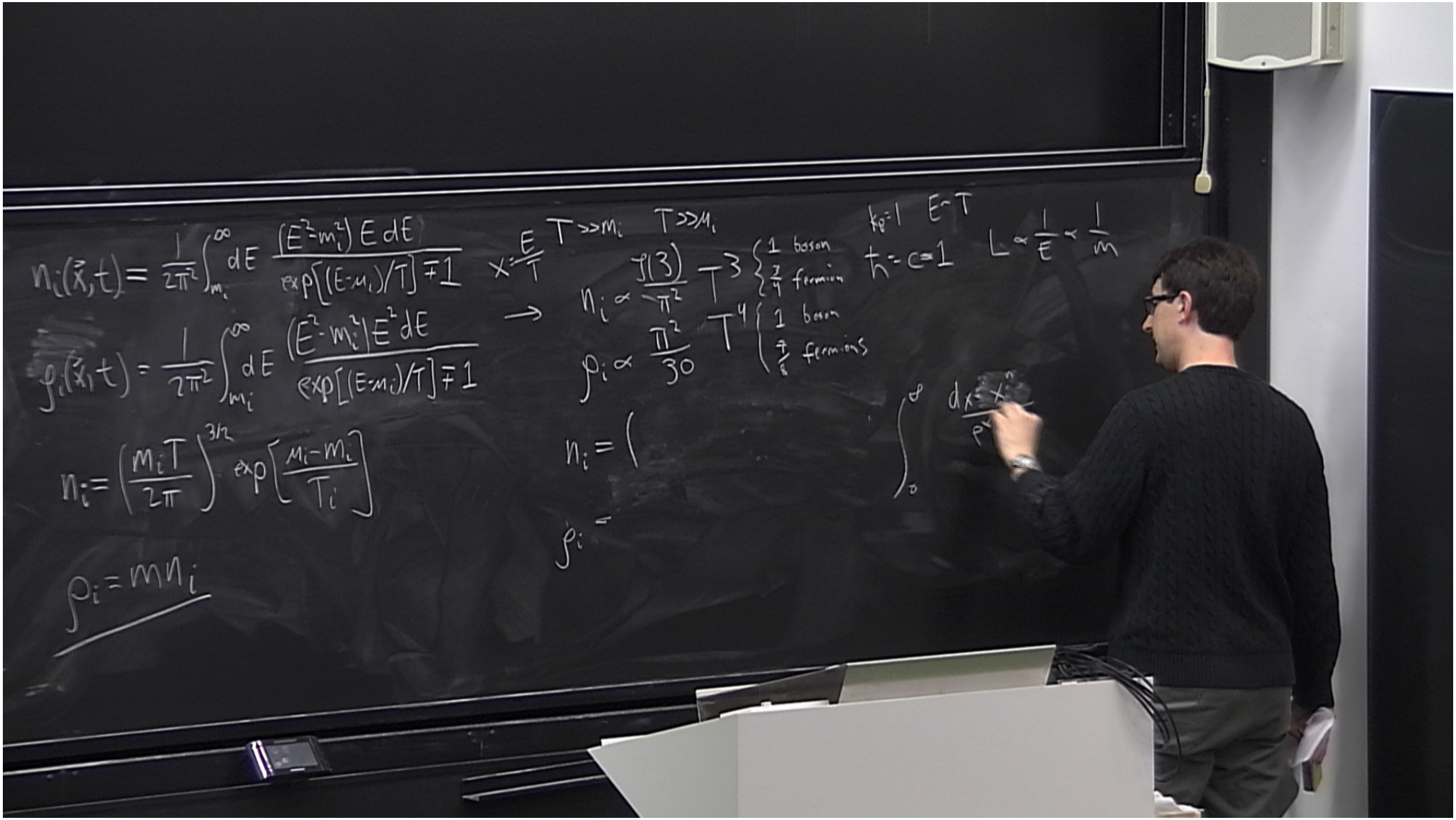

$= \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)\,E\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
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$n_i = ($
$\rho_i =$
$k_B = 1 \quad E \sim T$
$\hbar = c = 1$
$L \propto \frac{1}{E} \propto \frac{1}{m}$

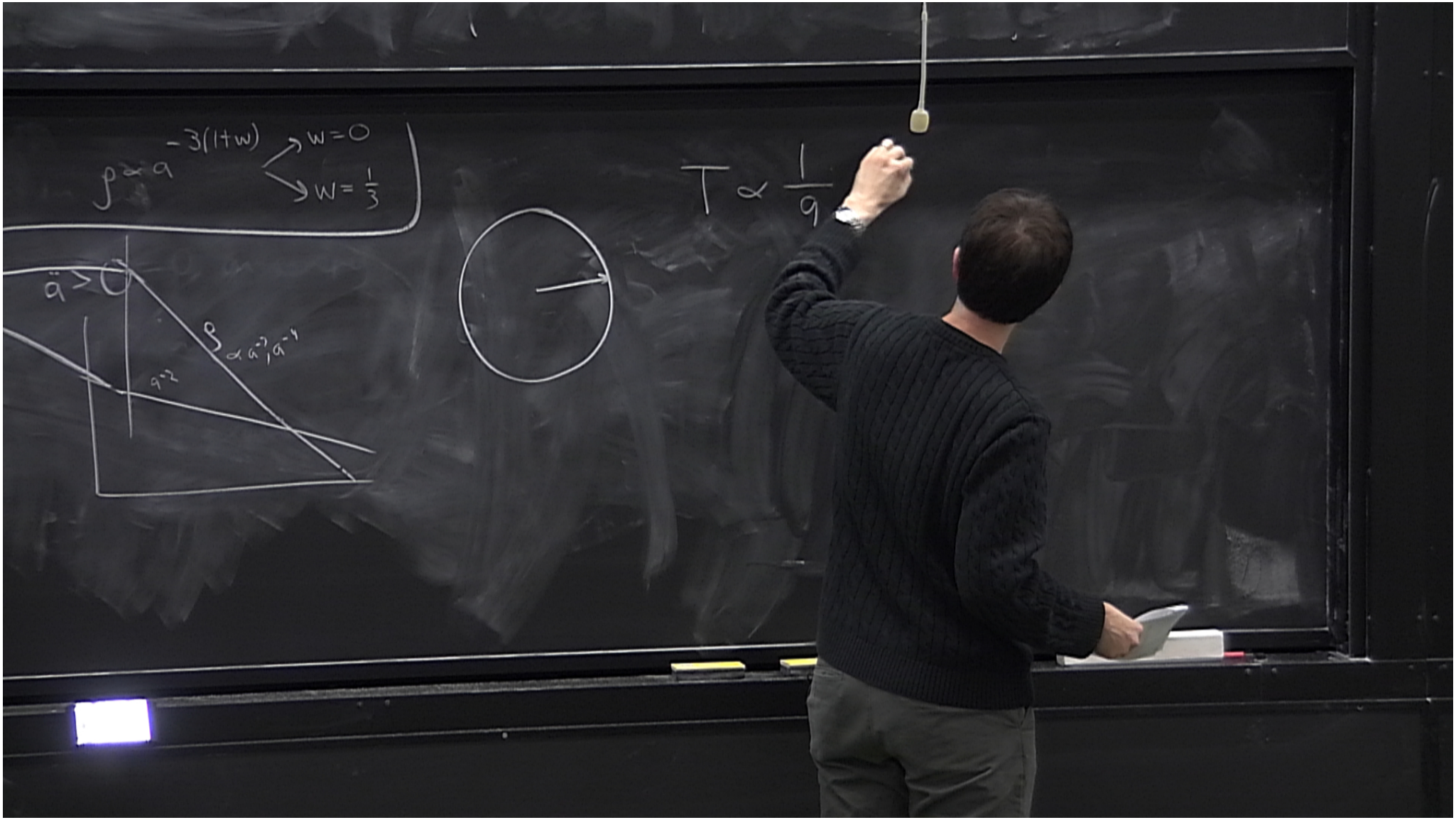

$n_i =$
$n_i =$
$\rho_i \propto 30$
$\rho_i =$

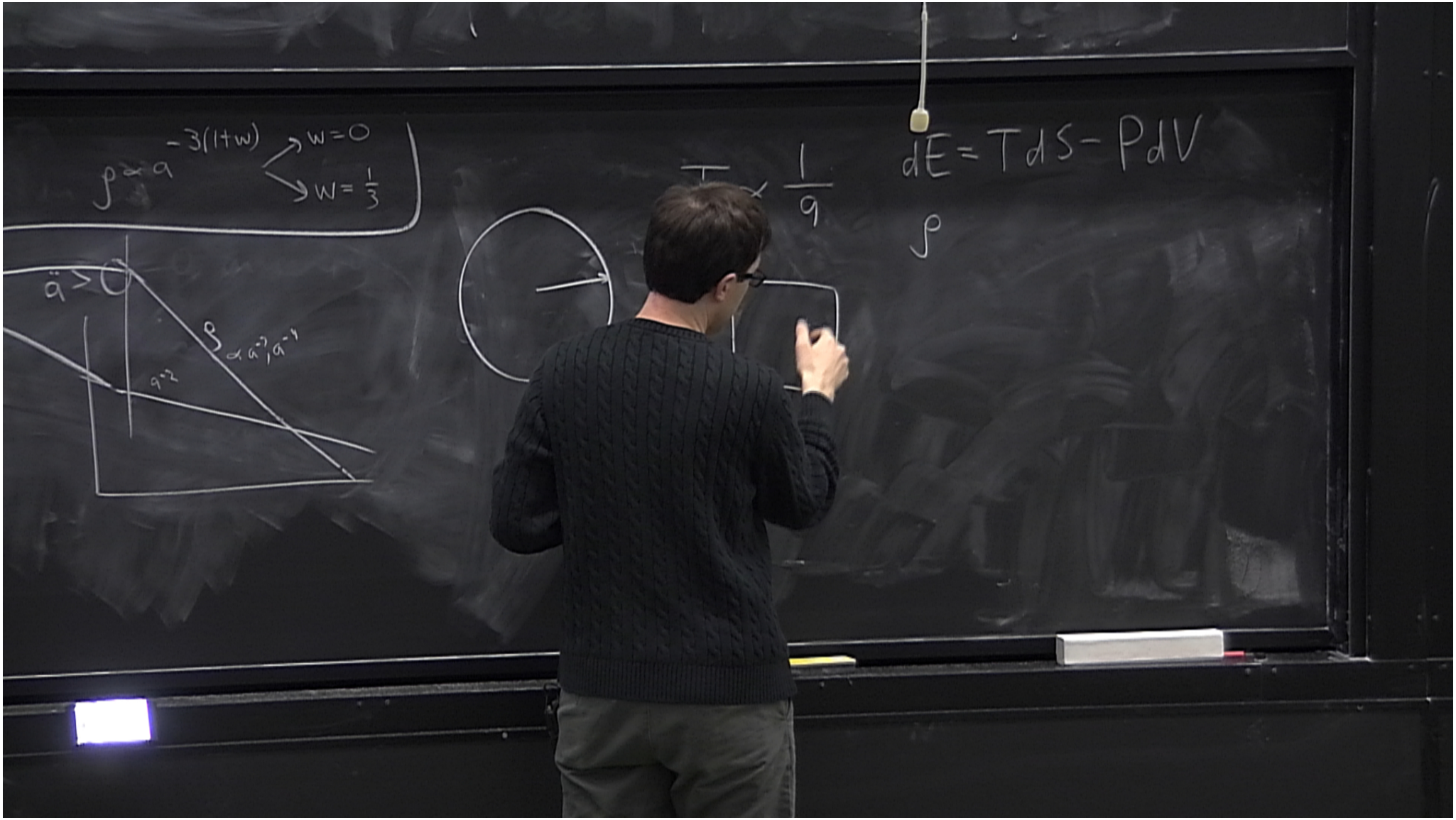

$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T_i}\right]$
$\rho_i = m n_i$

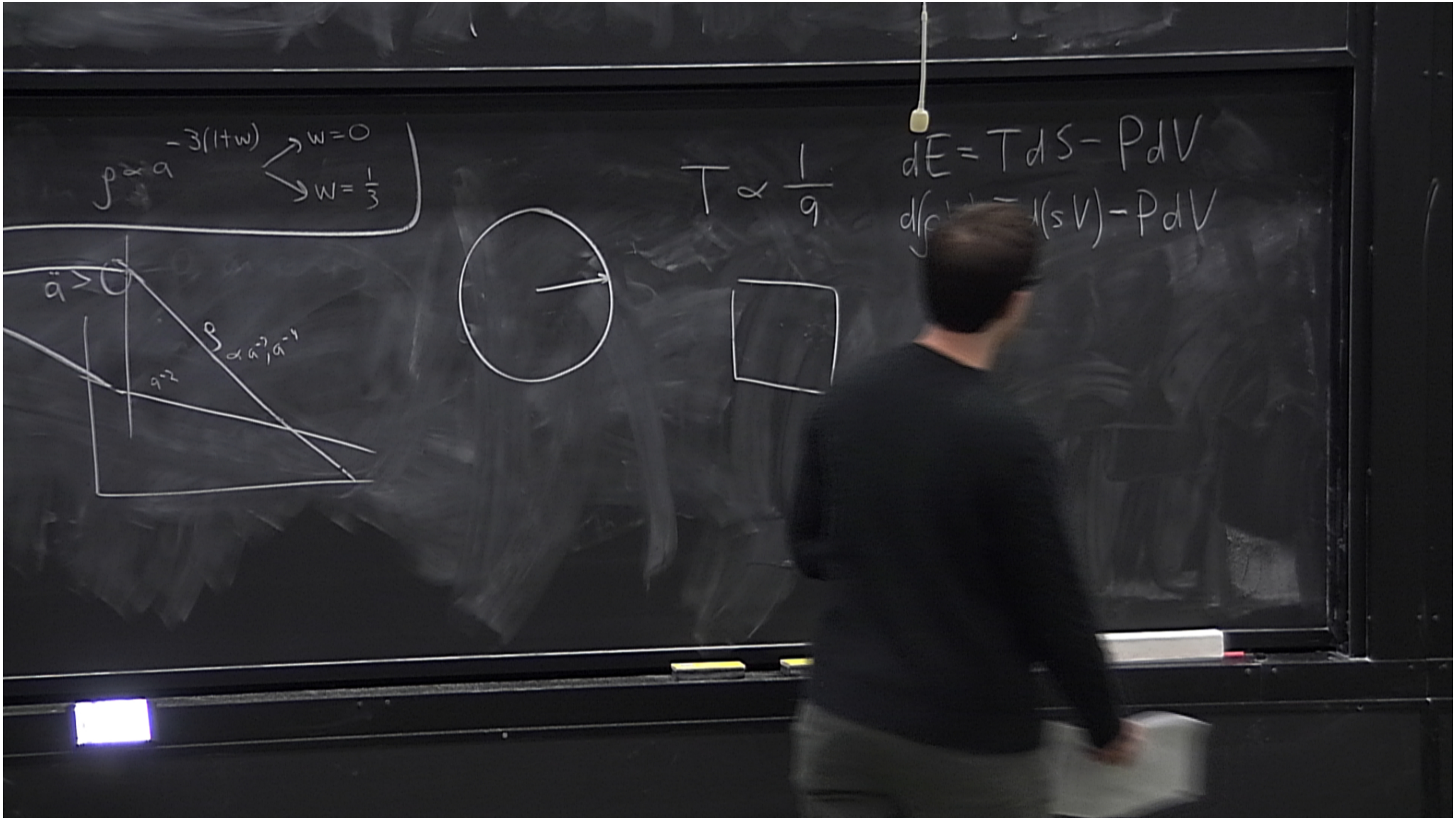

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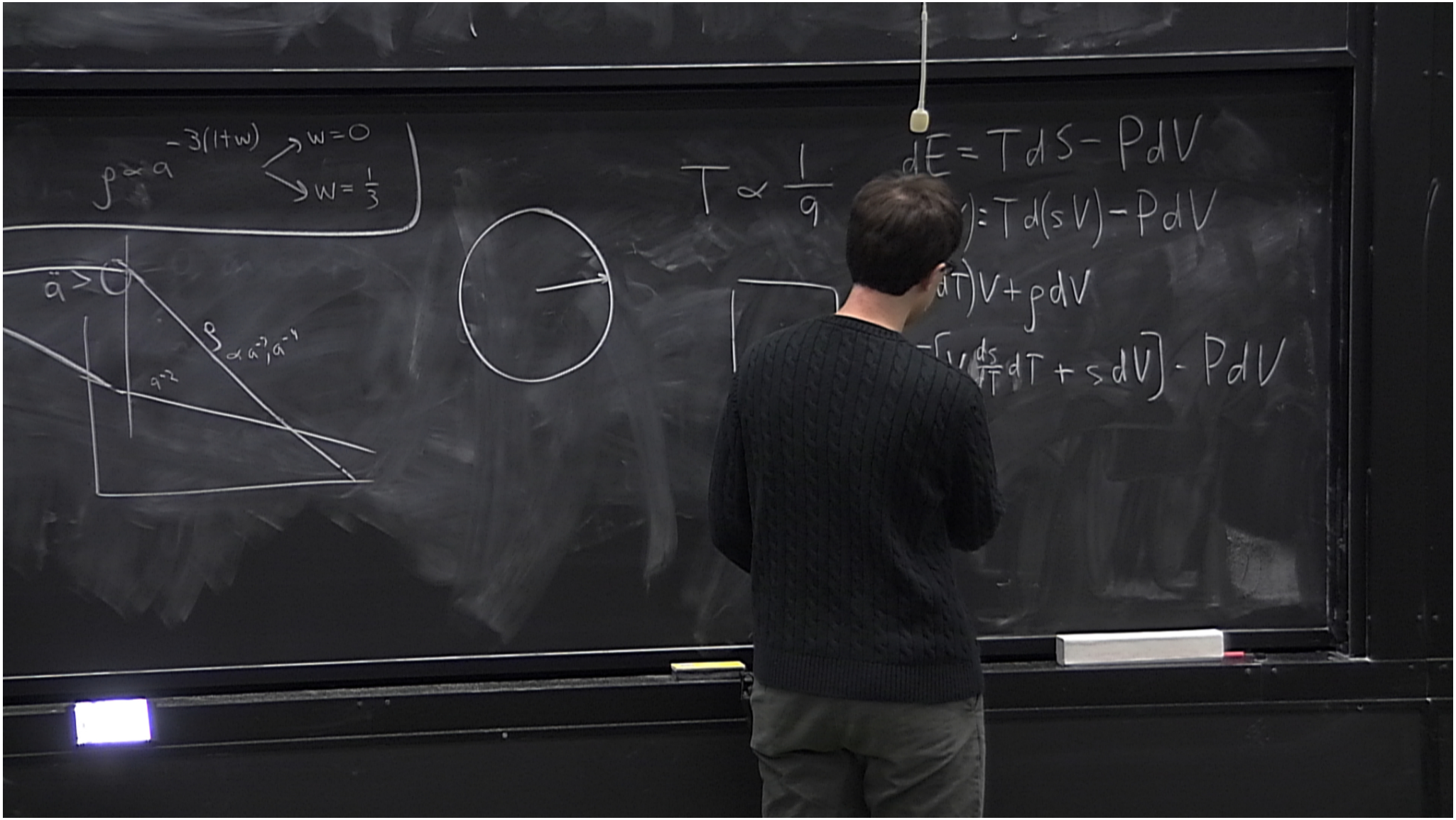

$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$x = \frac{E}{T}$
$T \gg m_i \quad T \gg \mu_i$
$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E^2 dE}{\exp[(E-\mu_i)/T] \mp 1}$
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$\rho_i = m n_i$
$n_i \propto \frac{\zeta(3)}{\pi^2} T^3 \begin{cases} 1 \text{ boson} \\ \frac{3}{4} \text{ fermion} \end{cases}$
$\rho_i \propto \frac{\pi^2}{30} T^4 \begin{cases} 1 \text{ boson} \\ \frac{7}{8} \text{ fermions} \end{cases}$
$k_B = 1 \quad E \sim T$
$\hbar = c = 1 \quad L \propto \frac{1}{E} \propto \frac{1}{m}$
$\int_0^{\infty} dx f($

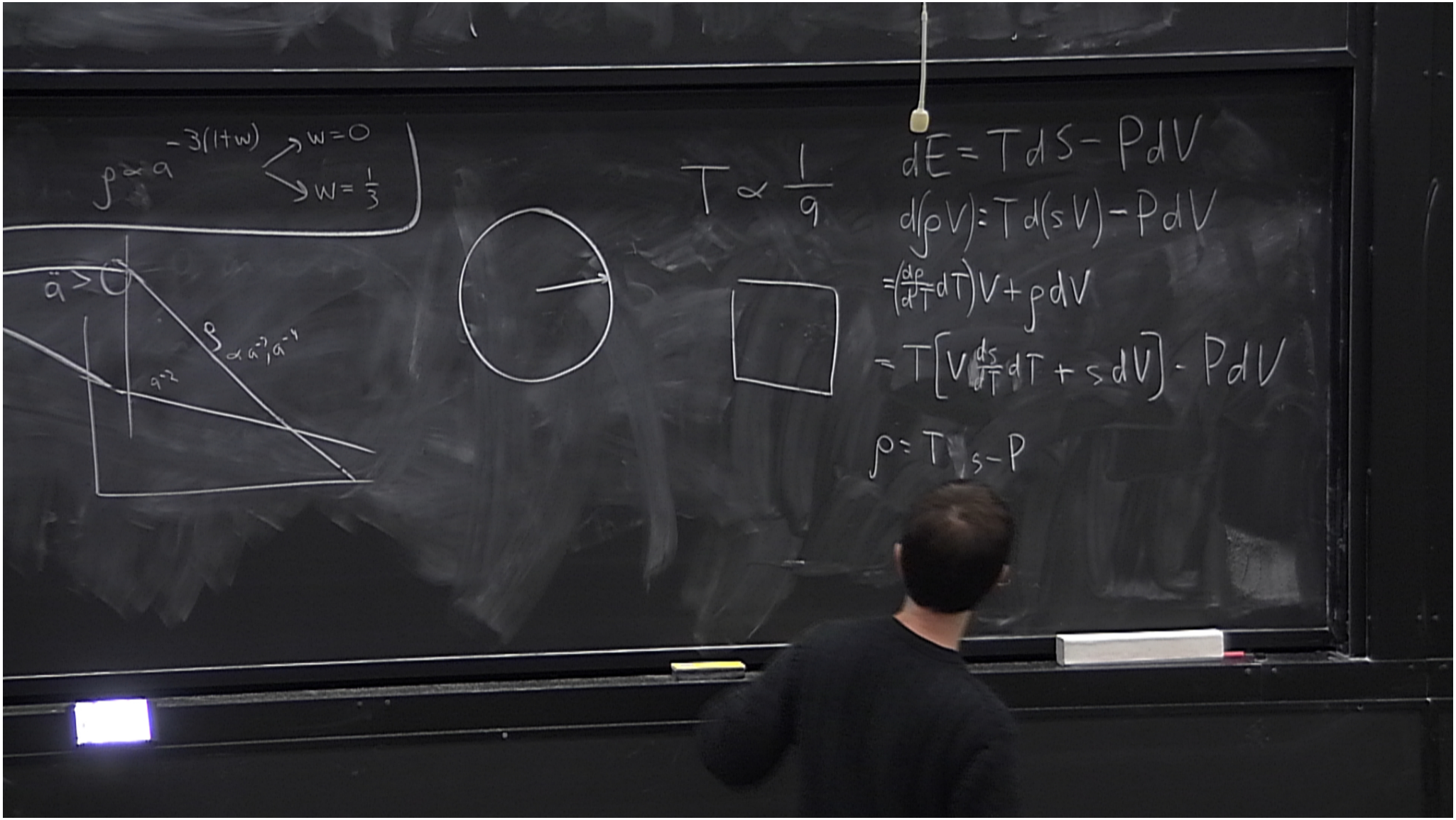

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$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T_i}\right]$
$n_i =$
$\rho_i =$
$\int_0^{\infty} dx \frac{x^n}{e^x}$
$\rho_i = m n_i$


$\rho \propto a^{-3(1+w)}$
$w=0$
$w=\frac{1}{3}$
$T \propto \frac{1}{a}$
$\ddot{a}>0$
$\rho_{\propto a^{-2}, a^{-4}}$
a^{-2}


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$dT)V + pdV$
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$= \left(\frac{d\rho}{dT} dT\right) V + \rho dV$
$= T\left[V \frac{ds}{dT} dT + s dV\right] - PdV$
$\rho = Ts - P$

$\rho \propto a^{-3(1+w)}$ → $w=0$, $w=\frac{1}{3}$

$\ddot{a} > 0$

$\rho \propto a^{-3}, a^{-4}$

a^{-2}

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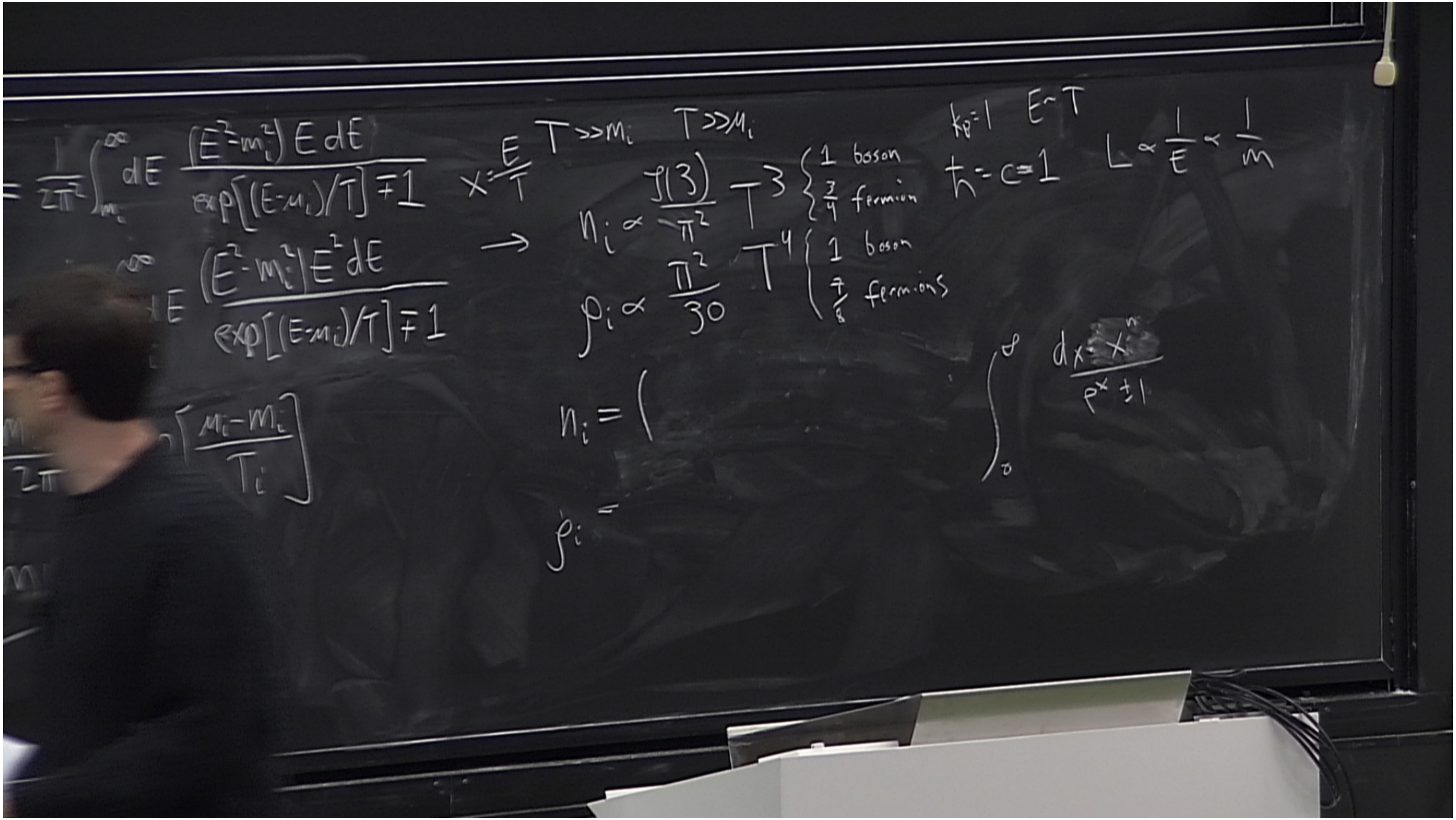
$dE = TdS - PdV$

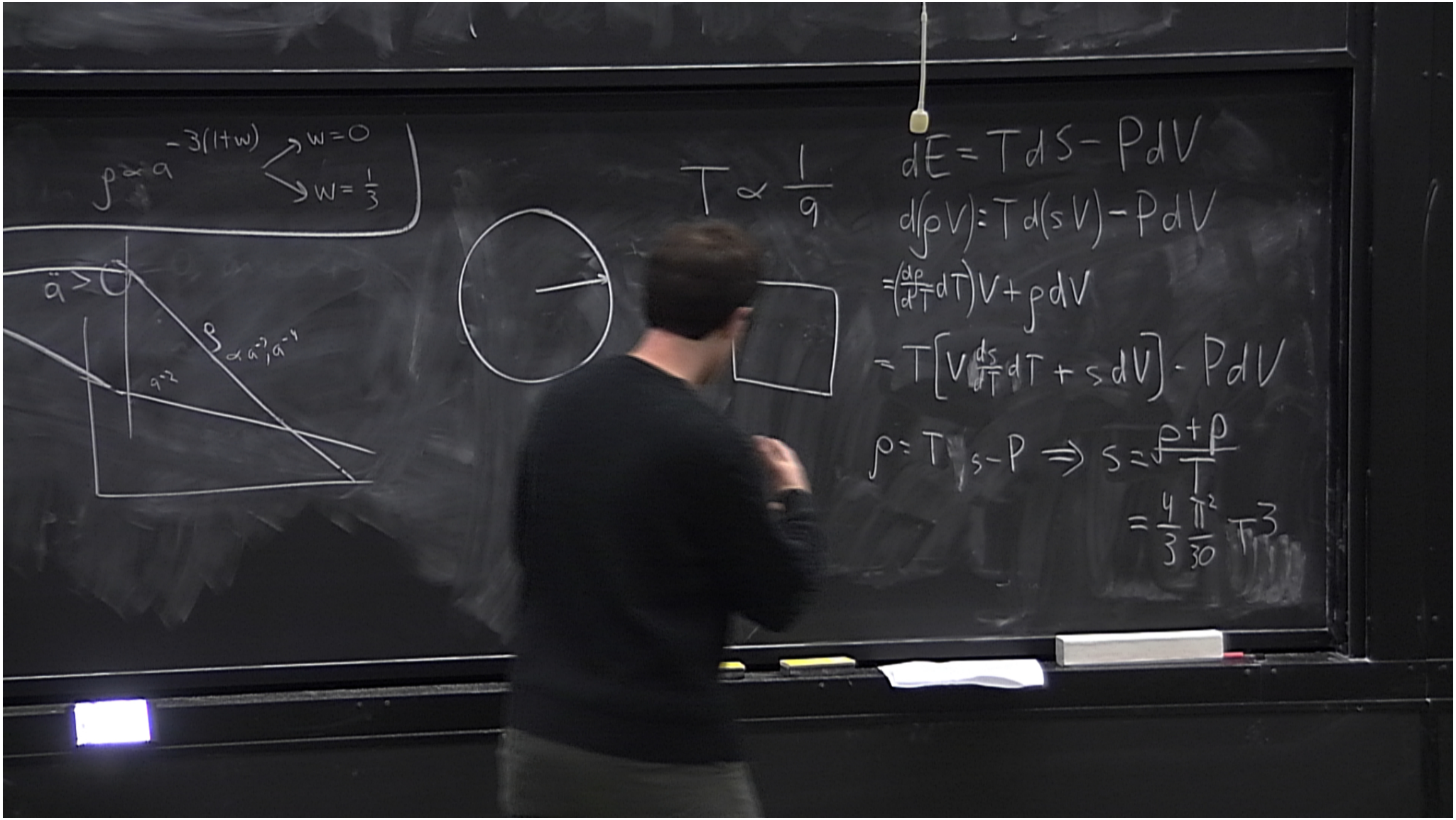
$d(\rho V) = Td(sV) - PdV$

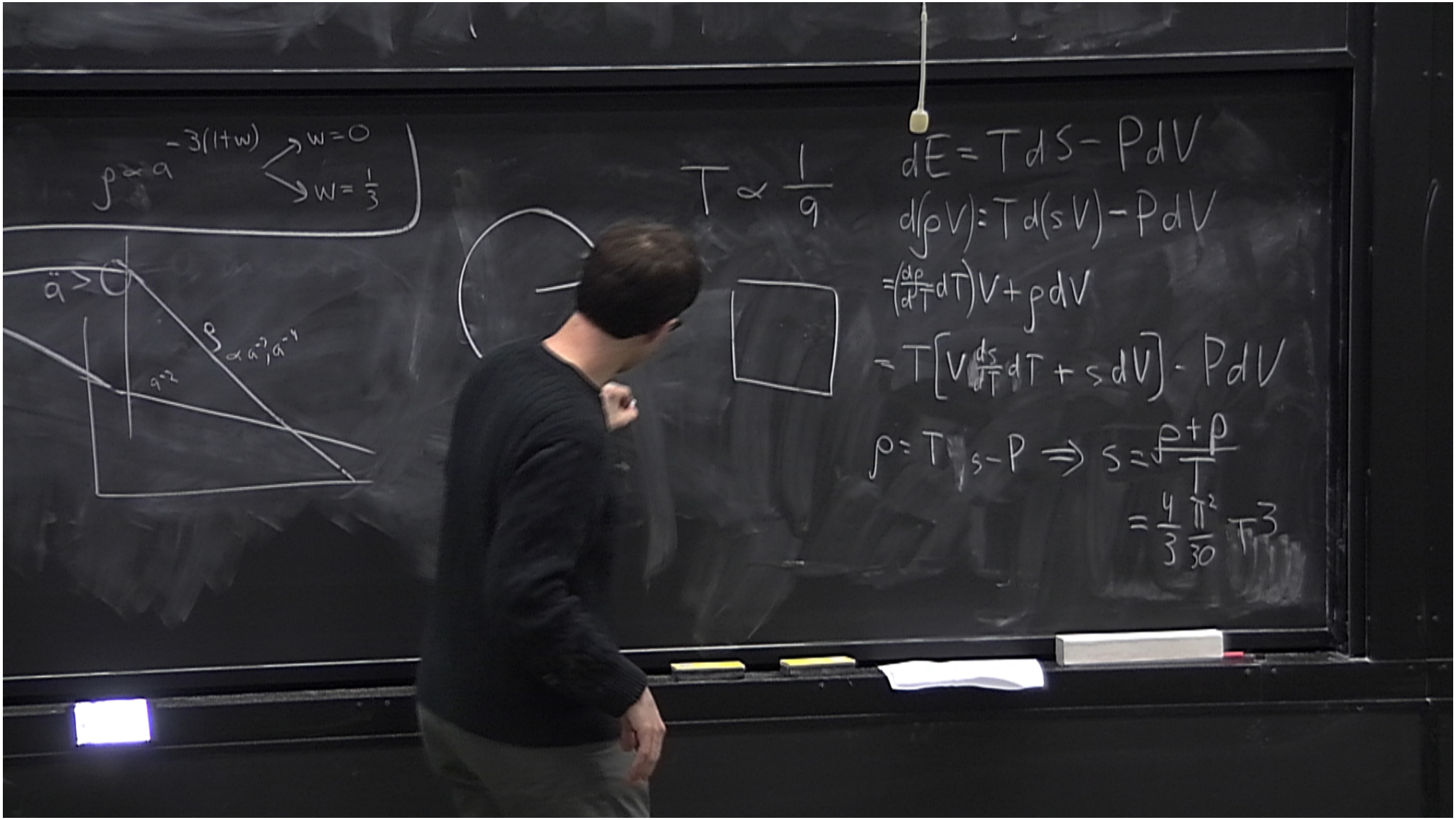
$= \left(\frac{d\rho}{dT} dT\right) V + \rho dV$

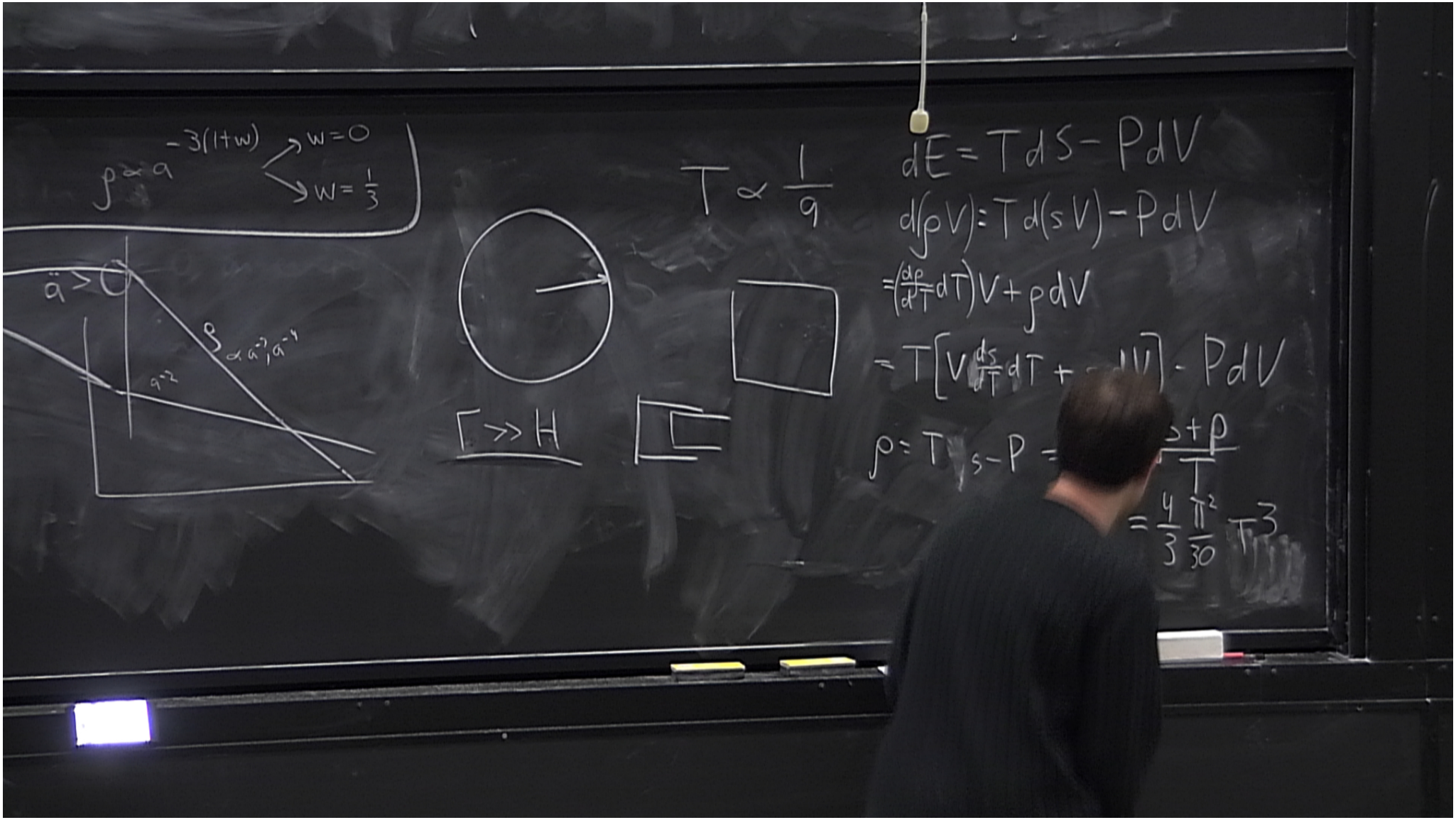
$= T\left[V \frac{ds}{dT} dT + s dV\right] - PdV$

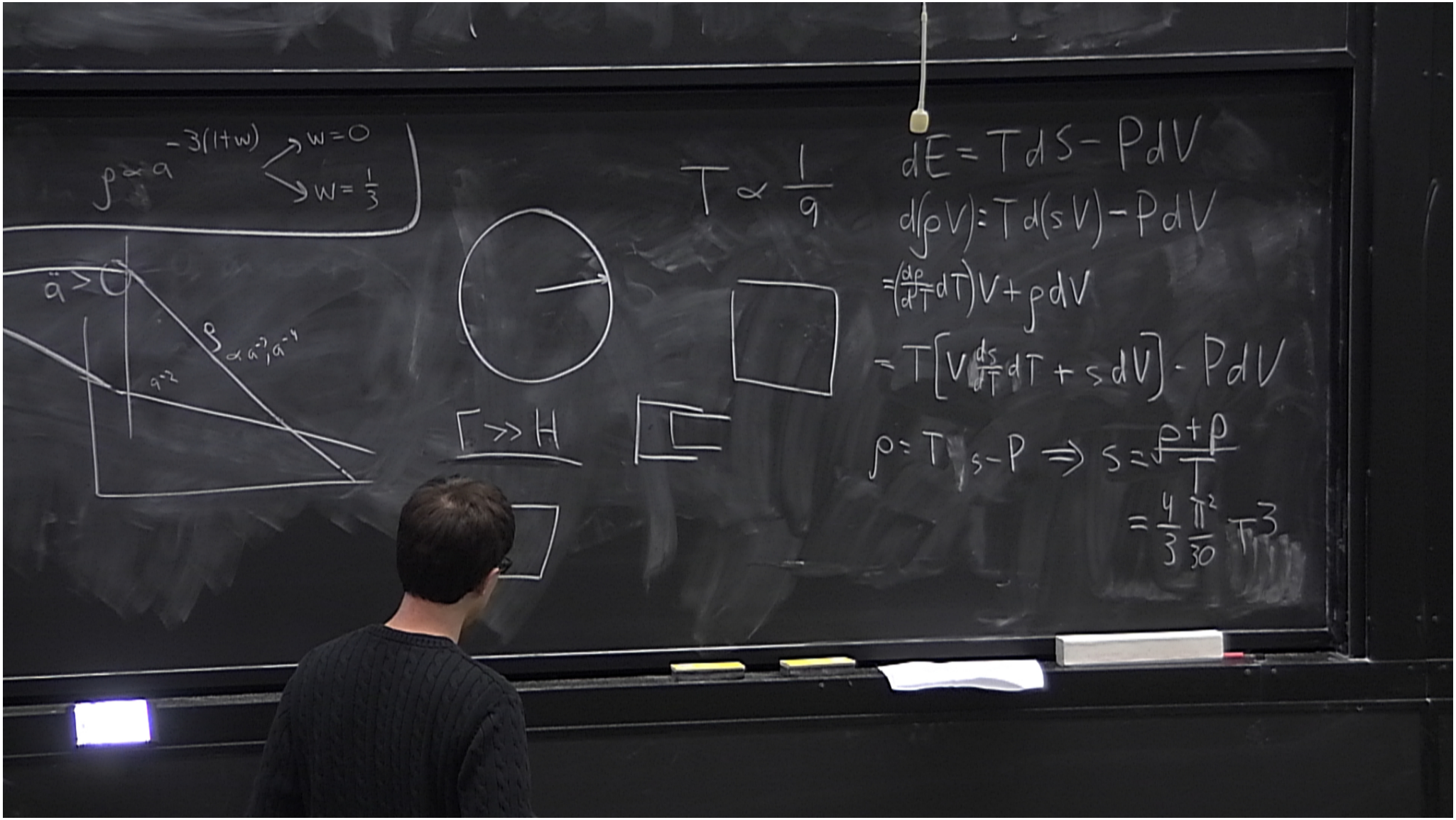
$\rho = Ts - P \Rightarrow s = \frac{\rho + P}{T}$

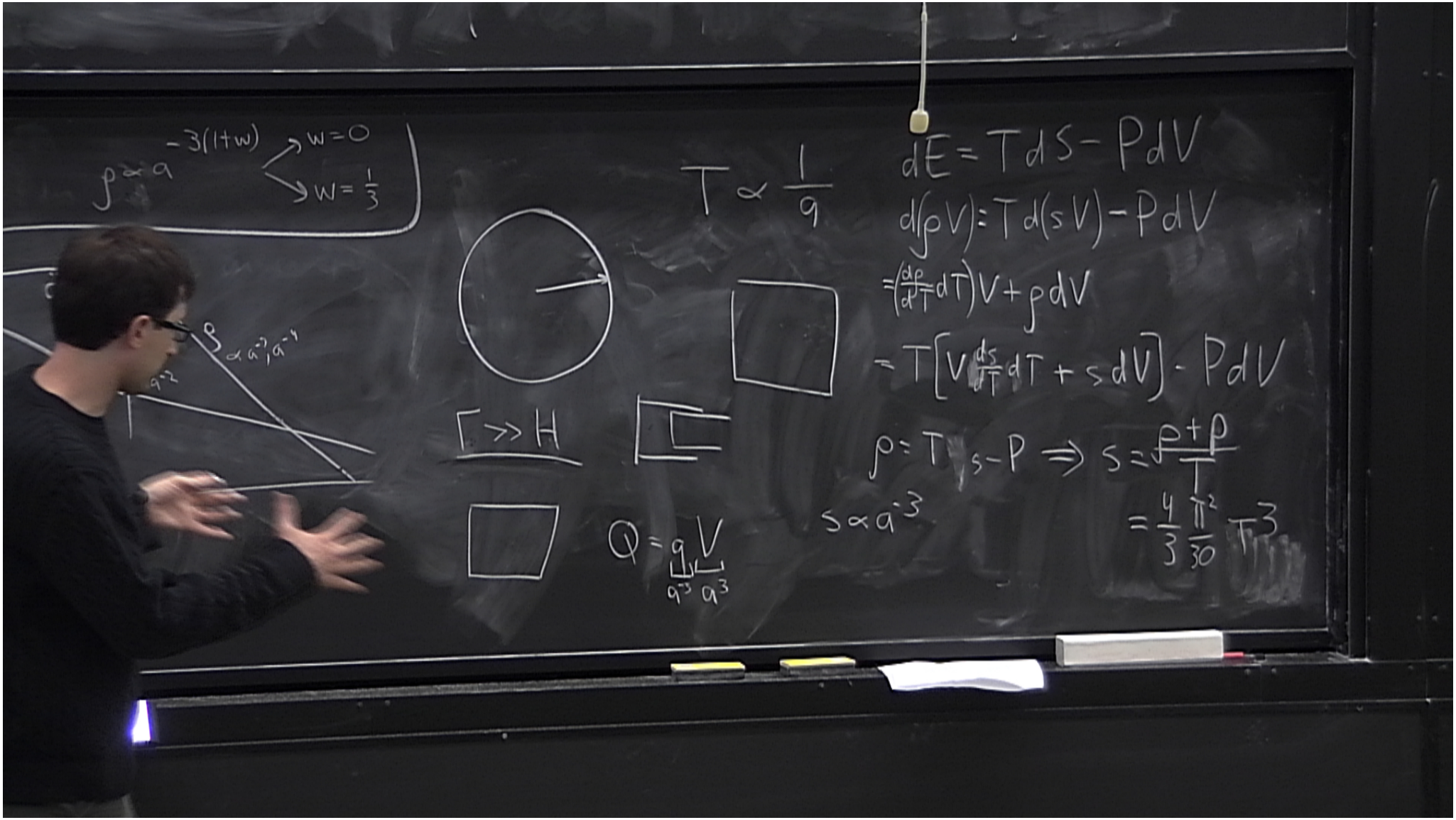

$\frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{\frac{1}{2}} E\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
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$\rho_i \propto \frac{\pi^2}{30} T^4$ $\begin{cases} 1 \text{ boson} \\ \frac{7}{8} \text{ fermions} \end{cases}$
$k_B = 1$ $E \sim T$
$\hbar = c = 1$
$L \propto \frac{1}{E} \propto \frac{1}{m}$
$\int_0^\infty \frac{dx\, x^n}{e^x \pm 1}$
$\left[\frac{\mu_i - m_i}{T_i}\right]$
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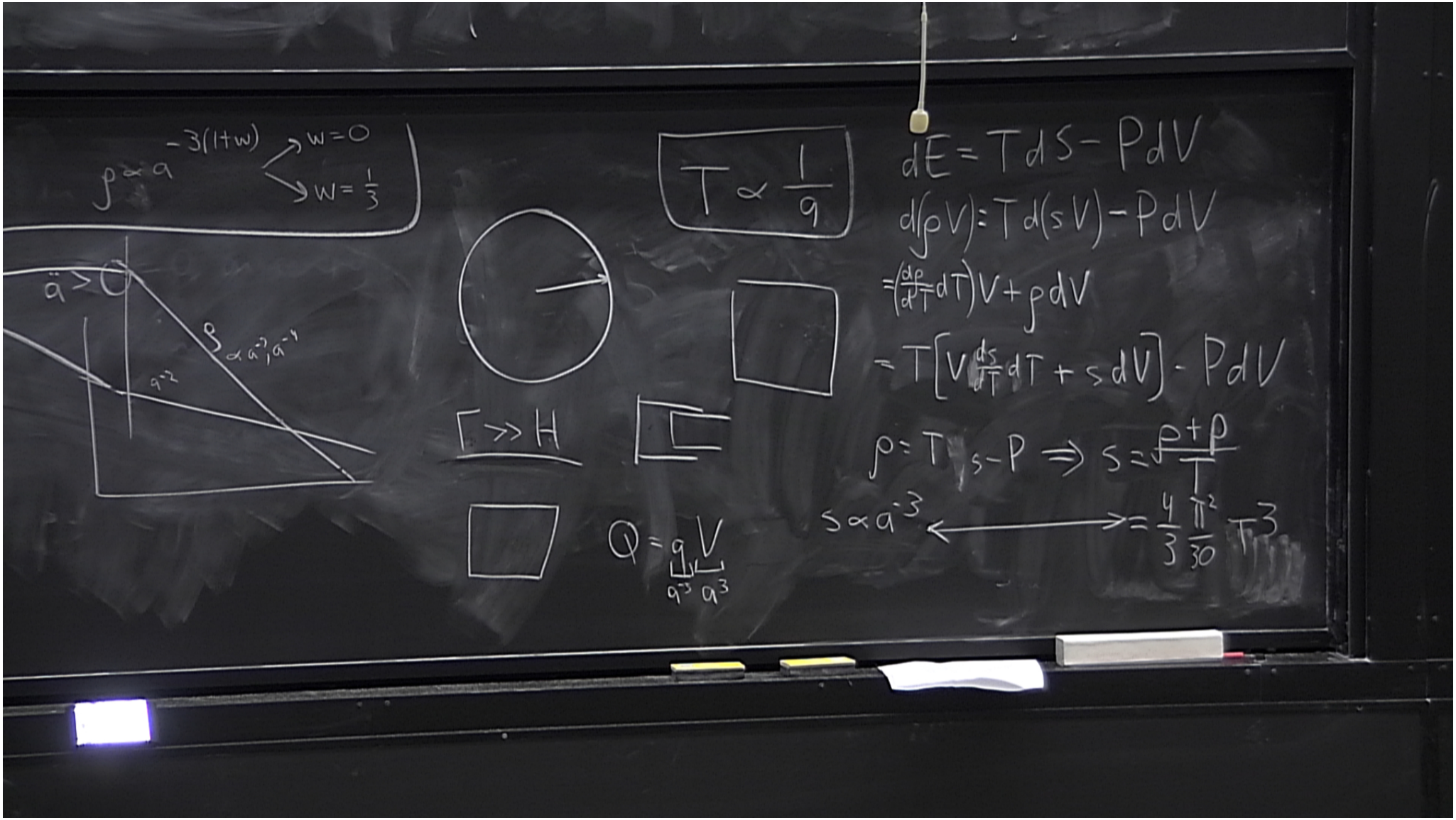

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$= \left(\frac{d\rho}{dT} dT\right) V + \rho dV$
$= T\left[V \frac{ds}{dT} dT + s dV\right] - PdV$
$\rho = Ts - P \Rightarrow s = \frac{\rho + P}{T}$
$= \frac{4}{3} \frac{\pi^2}{30} T^3$

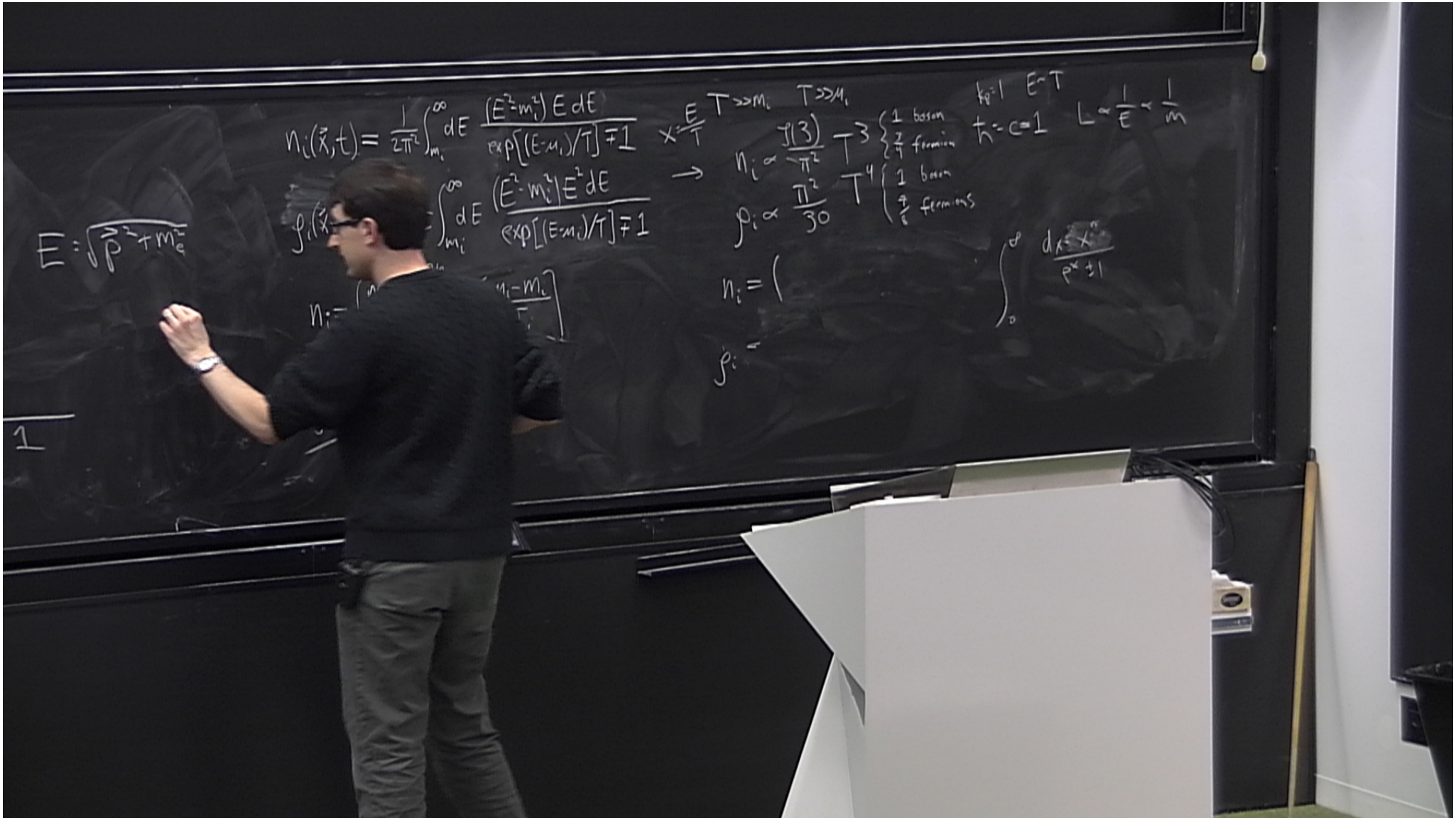

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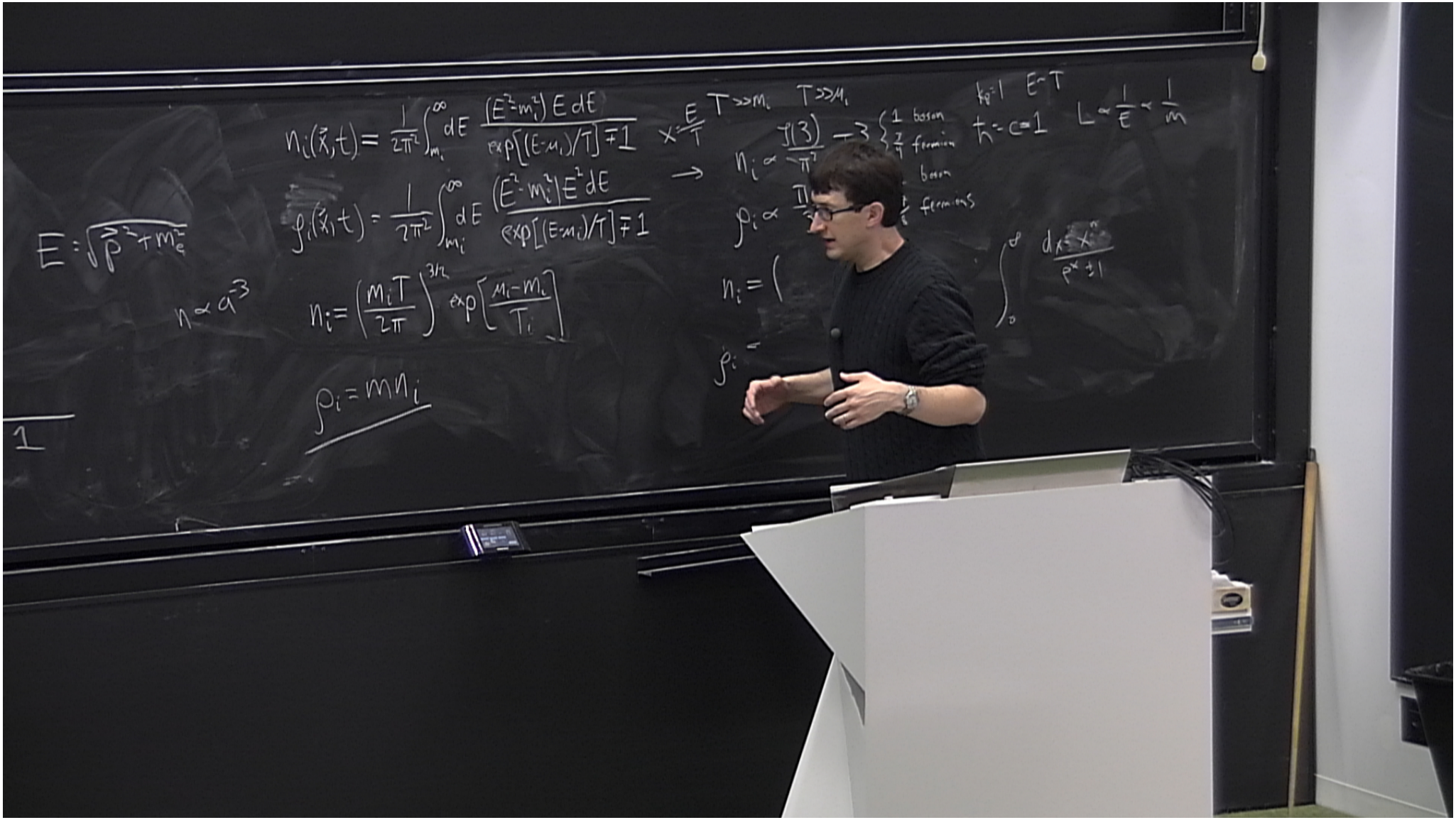

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$\ddot{a} > 0$
$\rho \propto a^{-3}, a^{-4}$
a^{-3}
$\Gamma \gg H$
$\rho = Ts - P =$
$= \frac{4}{3}\frac{\pi^2}{30} T^3$

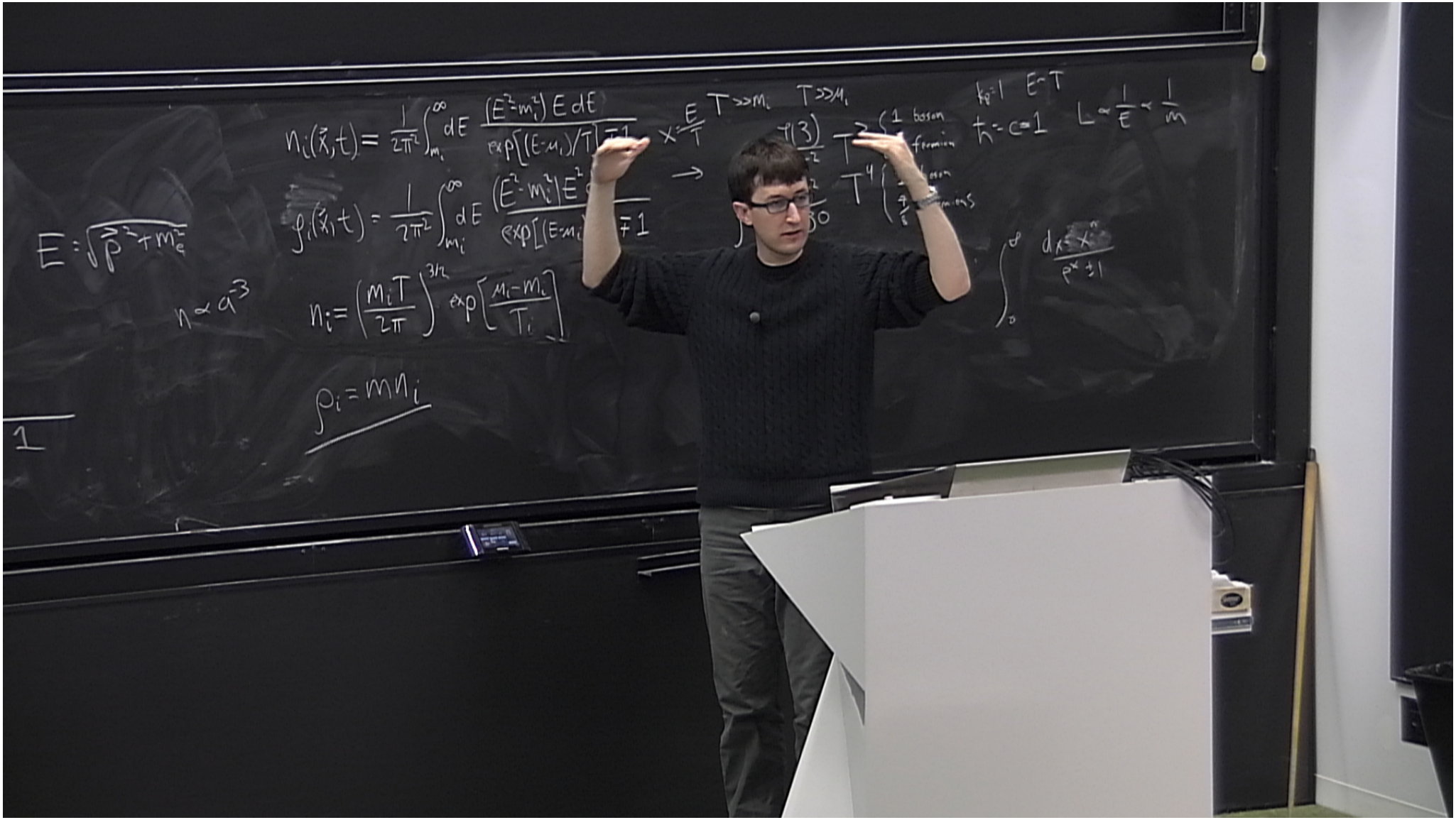

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$\Gamma \gg H$
$T \propto \frac{1}{a}$
$dE = TdS - PdV$
$d(\rho V) = Td(sV) - PdV$
$= \left(\frac{d\rho}{dT} dT\right) V + \rho dV$
$= T\left[V \frac{ds}{dT} dT + s dV\right] - PdV$
$\rho = Ts - P \Rightarrow s = \frac{\rho + P}{T}$
$= \frac{4}{3} \frac{\pi^2}{30} T^3$

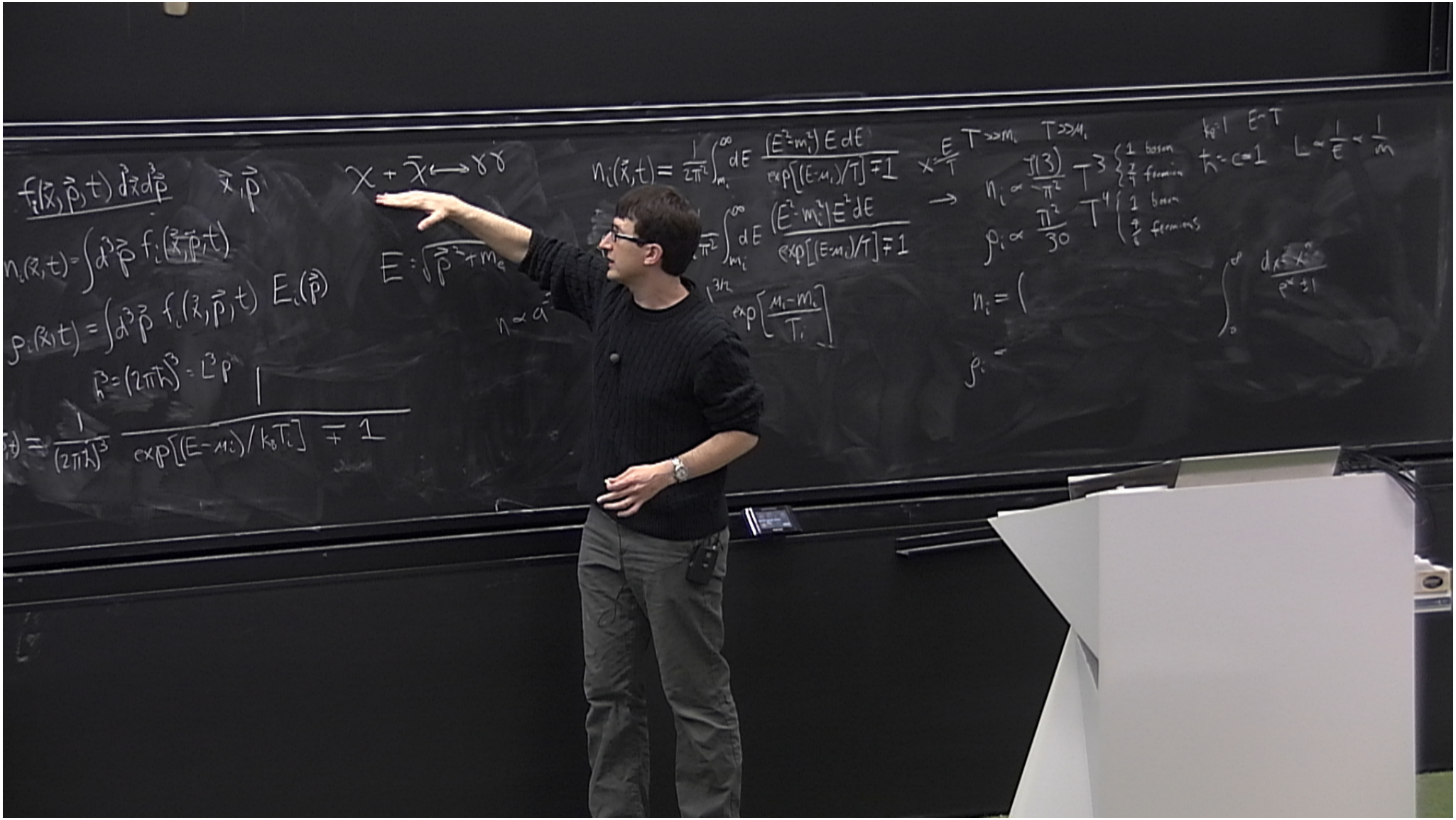

$\rho \propto a^{-3(1+w)}$
$w=0$
$w=\frac{1}{3}$
$T \propto \frac{1}{a}$
$dE = TdS - PdV$
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$s \propto a^{-3}$
$Q = qV$
a^{-3} a^{-3}
$\Gamma \gg H$

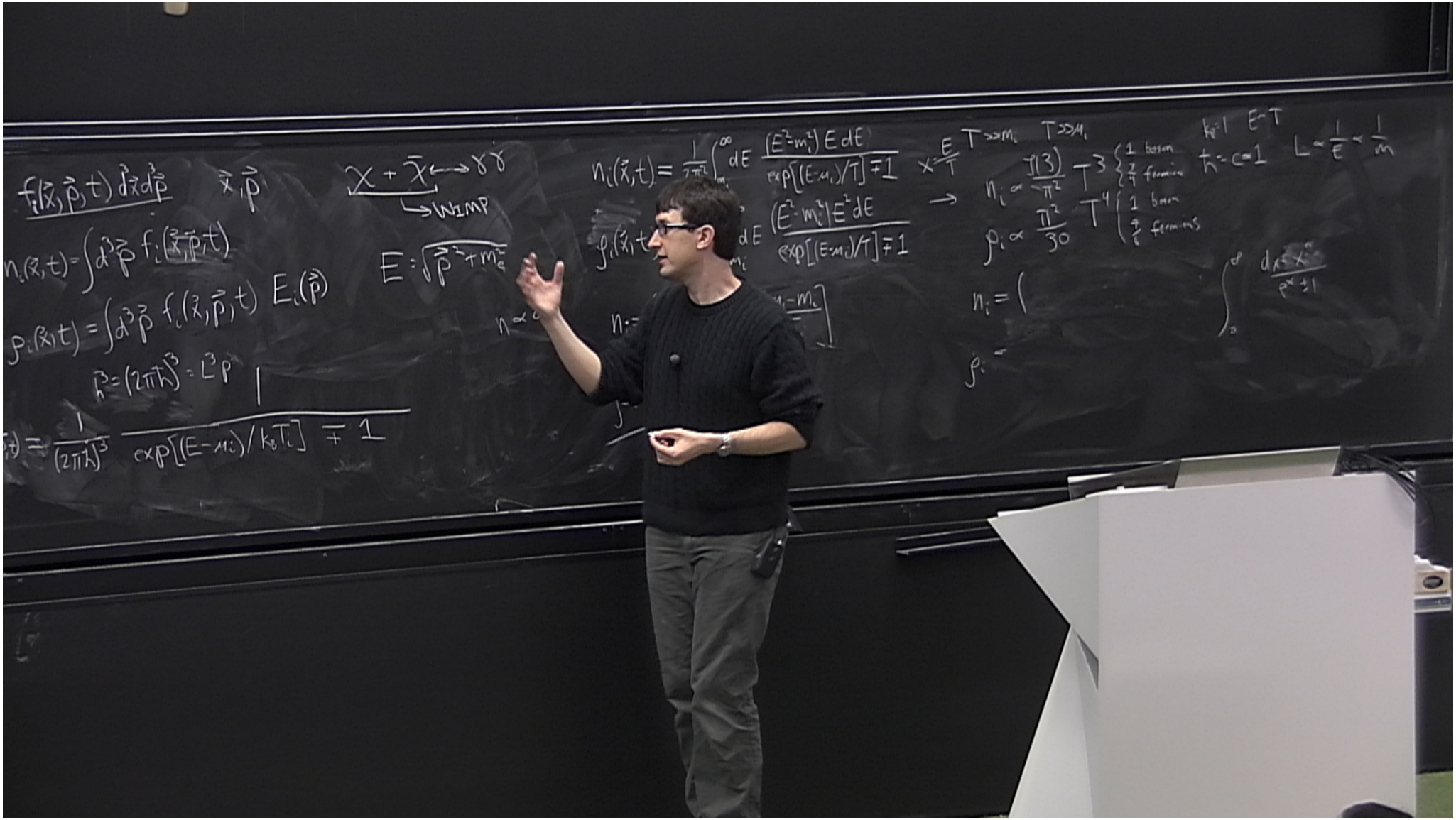

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$\ddot{a} > 0$
$\rho_{\propto a^{-3}, a^{-4}}$
a^{-2}
$\Gamma \gg H$
$\rho = Ts - P \Rightarrow s = \frac{\rho + P}{T}$
$s \propto a^{-3}$
$= \frac{4}{3} \frac{\pi^2}{30} T^3$
$Q = qV$
a^{-3} a^3


$E = \sqrt{\vec{p}^2 + m_e^2}$
$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{\frac{1}{2}} E\,dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\rho_i(\vec{x}$
$x = \frac{E}{T}$
$T \gg m_i \quad T \gg \mu_i$
$n_i \propto \frac{\zeta(3)}{\pi^2} T^3$
$\rho_i \propto \frac{\pi^2}{30} T^4$
1 boson
$\frac{7}{8}$ fermion
1 boson
$\frac{7}{8}$ fermions
$k_B = 1 \quad E \sim T$
$\hbar = c = 1 \quad L \propto \frac{1}{E} \propto \frac{1}{m}$
$\int_0^{\infty} \frac{dx\, x^n}{e^x \mp 1}$


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$\rho_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E^2 dE}{\exp[(E-\mu_i)/T] \mp 1}$
$x \equiv \frac{E}{T}$
$T \gg m_i \quad T \gg \mu_i$
$E = \sqrt{\vec{p}^2 + m_e^2}$
$n \propto a^{-3}$
$n_i = \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[\frac{\mu_i - m_i}{T_i}\right]$
$\rho_i = m n_i$
$k_B = 1 \quad E \sim T$
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1 boson
$\frac{7}{8}$ fermion
boson
$\frac{7}{8}$ fermions
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$\zeta(3)$
(1 boson
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$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$X + \bar{X} \longleftrightarrow \gamma\gamma$
$n_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t) = \int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$E = \sqrt{\vec{p}^2 + m_a}$
$n \propto a^{-3}$
$\vec{p} = (2\pi\hbar)^3 = L^3 p$
$\frac{1}{(2\pi\hbar)^3} \frac{1}{\exp[(E-\mu_i)/k_b T_i] \mp 1}$
$n_i(\vec{x},t) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\frac{1}{\pi^2}\int_{m_i}^{\infty} dE \frac{(E^2-m_i^2)^{1/2} E^2\, dE}{\exp[(E-\mu_i)/T] \mp 1}$
$\exp\left[\frac{\mu_i - m_i}{T_i}\right]$
$x = \frac{E}{T}$
$T \gg m_i$
$T \gg \mu_i$
$n_i \propto \frac{\zeta(3)}{\pi^2} T^3$
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1 boson
$\frac{7}{8}$ fermions
$k_b = 1$
$E \sim T$
$\hbar = c = 1$
$L \sim \frac{1}{E} \sim \frac{1}{m}$
$\int_0^{\infty} \frac{dx\, x^2}{e^x \pm 1}$


$f_i(\vec{x},\vec{p},t)\,d^3x\,d^3p$
$\vec{x},\vec{p}$
$n_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)$
$\rho_i(\vec{x},t)=\int d^3p\, f_i(\vec{x},\vec{p},t)\, E_i(\vec{p})$
$p=\frac{h}{\lambda}=(2\pi\hbar)^3=L^3p$
$\frac{1}{(2\pi\hbar)^3}\,\frac{1}{\exp[(E-\mu_i)/k_bT_i]\mp 1}$
$\chi+\bar{\chi}\longleftrightarrow\gamma\gamma$
WIMP
$E=\sqrt{\vec{p}^2+m_i^2}$
$n_i(\vec{x},t)=\frac{1}{\pi^2}\int_{m_i}^{\infty} dE\,\frac{(E^2-m_i^2)^{\frac{1}{2}}E\,dE}{\exp[(E-\mu_i)/T]\mp 1}$
$\rho_i(\vec{x},t)=\int dE\,\frac{(E^2-m_i^2)^{\frac{1}{2}}E^2\,dE}{\exp[(E-\mu_i)/T]\mp 1}$
$x=\frac{E}{T}$
$T\gg m_i$
$T\ll m_i$
$n_i\propto\frac{\zeta(3)}{\pi^2}T^3$ $\begin{cases}1 \text{ boson}\\ \frac{3}{4} \text{ fermion}\end{cases}$
$\rho_i\propto\frac{\pi^2}{30}T^4$ $\begin{cases}1 \text{ boson}\\ \frac{7}{8} \text{ fermions}\end{cases}$
$k_b=1$ $E\sim T$
$\hbar=c=1$
$L\sim\frac{1}{E}\sim\frac{1}{m}$
$\int_0^{\infty}\frac{dx\,x^n}{e^x\mp 1}$