Title: Gravitational Physics (Review) - Lecture 15

Date: Feb 09, 2012 09:00 AM

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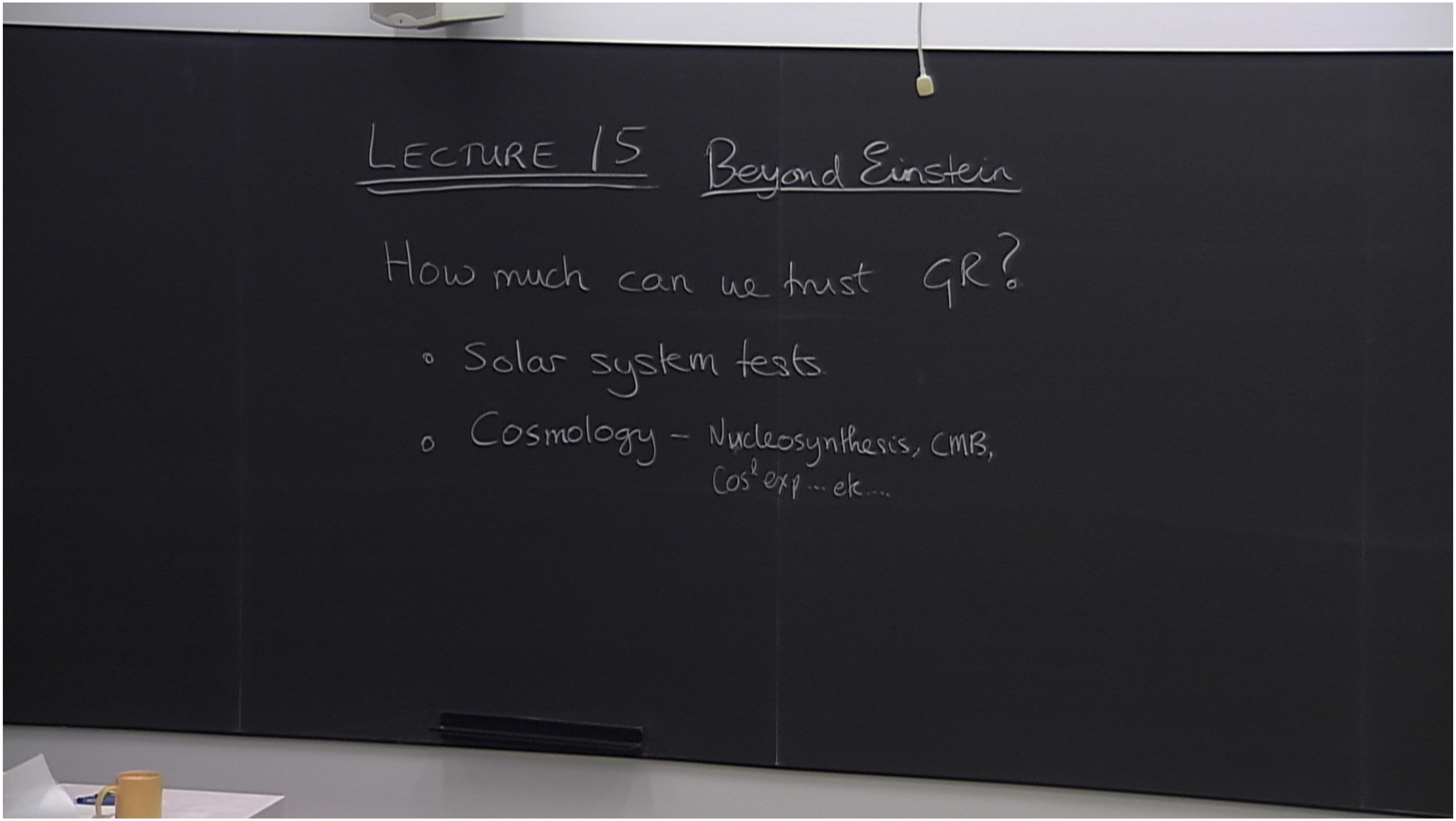
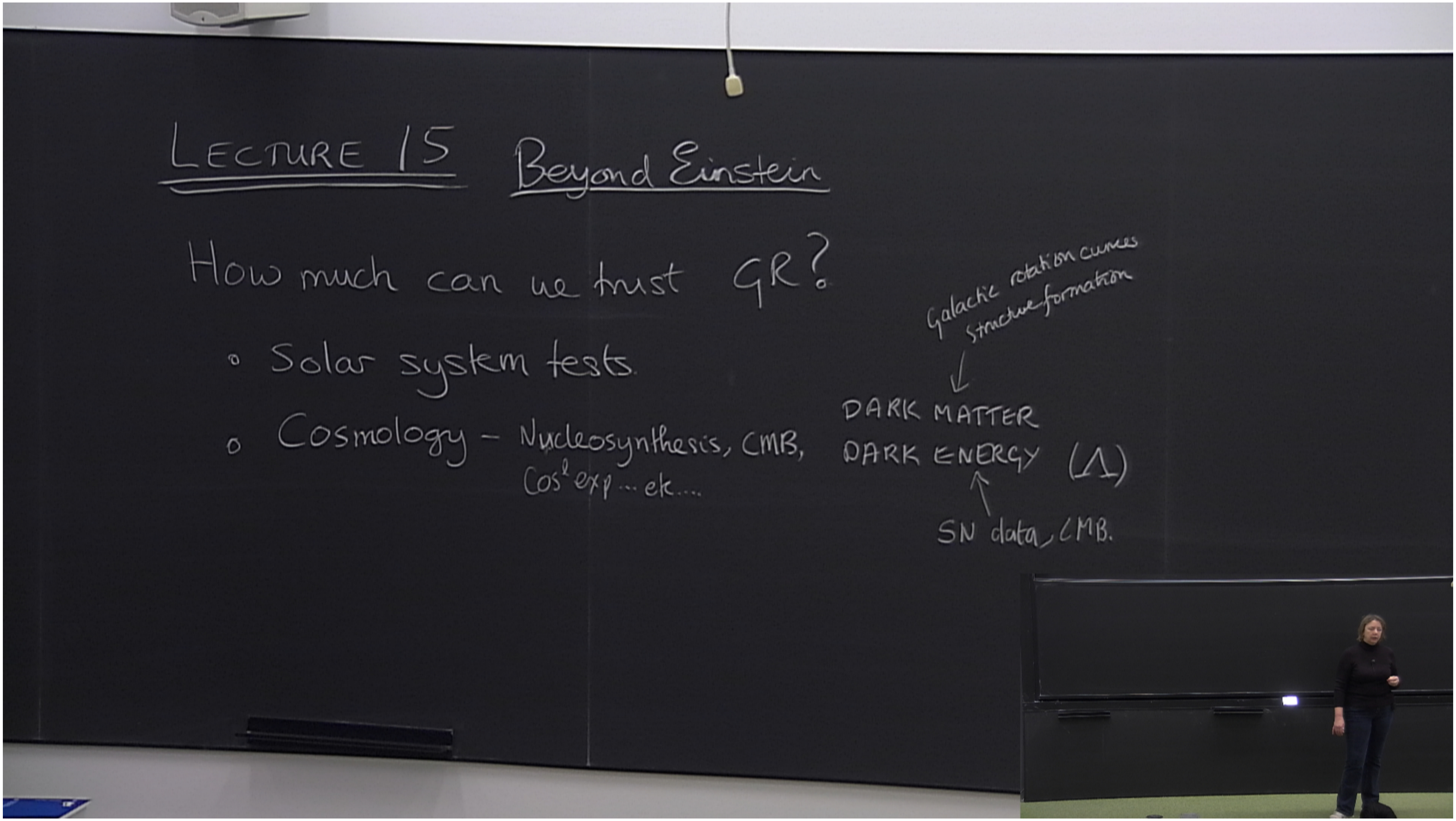
Abstract:

LECTURE 15 Beyond Einstein

How much can we trust GR?


LECTURE 15 Beyond Einstein
How much can we trust GR?
◦ Solar system tests
◦ Cosmology – Nucleosynthesis, CMB,
Cosl exp ... etc ...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How much can we trust GR?
◦ Solar system tests
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Galactic rotation curves
Structure formation
DARK MATTER
DARK ENERGY (Λ)
SN data, CMB.

LECTURE 15 Beyond Einstein

How much can we trust GR?

- Solar system tests
- Cosmology – Nucleosynthesis, CMB, Cos^l exp … etc….
  - Standard cosmological model needs extra ingredients

Galactic rotation curves
Structure formation

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DARK ENERGY (Λ)

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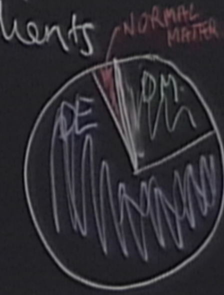

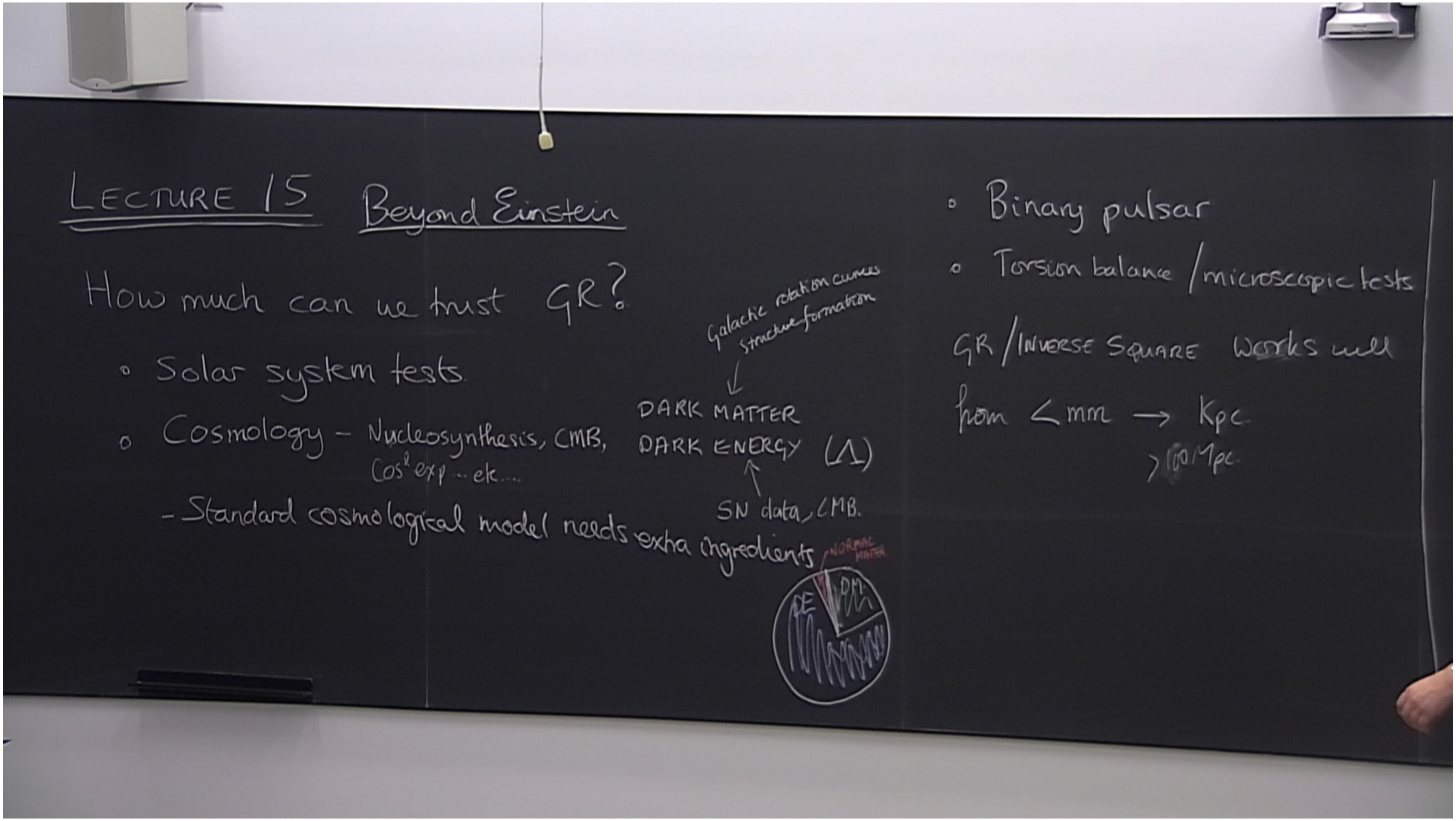
LECTURE 15
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How much can we trust GR?
◦ Solar system tests
◦ Cosmology – Nucleosynthesis, CMB, Cos[?] exp … etc….
– Standard cosmological model needs extra ingredients
Galactic rotation curves
Structure formation
DARK MATTER
DARK ENERGY (Λ)
SN data, CMB.
NORMAL MATTER
DE
DM
◦ Binary pulsar
◦ Torsion balance / microscopic tests
GR / INVERSE SQUARE works well
from < mm → Kpc
> 100 Mpc

sts.

l

Extra dimensions

Problems: moduli corresponding to low energy deformations of extra dims.

– KK theory: $g_{\mu\nu}$, A_μ, ϕ ← "breather mode" for internal dim.

Cosmologically ϕ evolves too much – need to stabilize internal dimensions

Large Extra Dimensions

Alternative to KK theory

$$\frac{1}{16\pi G_5}\int d^5x\sqrt{g_5}\,R_5 \xrightarrow[\int \text{ over } x^5]{} \frac{L}{8\pi G_5}\int d^4x\sqrt{g_4}\,R_4 + \ldots$$

Thus G_4 or $G_N = G_5/L$

or $M_p^2 = M_s^3 L$ $\qquad (M_D^{D-2} = 8\pi G_D)$

i.e. gravitational coupling in 4D is a derived quantity.

$$M_p^2 = M_D^{D-2} L^{D-4} \qquad L \sim \text{scale of internal manifold.}$$

$$= M_{(n+4)}^{n+2} L^{n}$$

Having L larger than Planck scale generates a hierarchy $M_p : M_D$

$$L_{KK} \sim \underbrace{\left(\frac{M_p}{M_D}\right)^{1+2/n}}\, M_p^{-1}$$

$$\left(\frac{10^{16}\,\mathrm{TeV}}{M_D}\right)^{1+2/n} L_p$$

$$\longrightarrow \left(\frac{\mathrm{TeV}}{M_D}\right)^{1+\frac{2}{n}} 10^{\frac{32}{n}-17}\ \mathrm{cm}$$

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$n=2$ $\qquad L \sim 1\,\mathrm{mm}$

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- we do not see these dimensions because we are confined on a brane.

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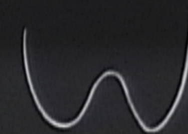
String theory has confined gauge theories on D-branes.

(M_D)

String theory h

Toy model: Rubakov + Shaposhnikov

Domain wall: $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{2}(\phi^2-\eta^2)^2$



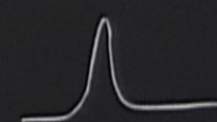
$\ominus$ $\oplus$.

$\phi = \eta \tanh \sqrt{\lambda}\,\eta z$.

$\longleftrightarrow$

localized $w \sim 1/\sqrt{\lambda}\eta$

$E \sim \eta^4 \operatorname{sech}^4 \sqrt{\lambda}\,\eta z$



brane

ring theory has confined gauge theories on D-branes.

Now couple to fermion.

$$\mathcal{L}_{\phi\Psi} = i\bar{\Psi}\Gamma^a \nabla_a \Psi - g\phi\bar{\Psi}\Psi$$

Look for soln $\Psi = \psi_0 \xi(z)$ $\quad \psi_0$ 4D chiral spinor $\boxed{i\gamma^5\psi_0 = -\psi_0}$

$$\Rightarrow \; -\xi' = g\eta \tanh\sqrt{\lambda}\eta z \; \xi$$

$$\Rightarrow \; \xi = (\mathrm{sech}\sqrt{\lambda}\eta z)^{g/\sqrt{\lambda}}$$

(M_D)

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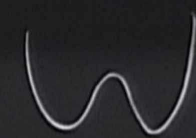

⊖ ⊕.

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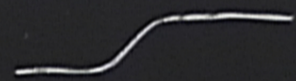

$\longleftrightarrow$

$E \sim \eta^4 \operatorname{sech}^4 \sqrt{\lambda}\eta z$

localised w ~ $1/\sqrt{\lambda}\eta$

Now co

Look

$\Rightarrow$

In background of kink, Ψ condenses into this nontrivial soln. Low energy excitations satisfy

$$\not{\partial}\psi_0 = 0 \qquad - \text{massless 4D fermion.}$$

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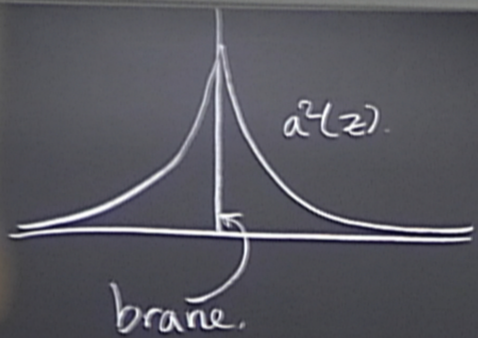
Gravity? In RS model gravity has an interesting behaviour.

$$ds^2 = e^{-2k|z|}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2$$



Do grav^l pert. th. $\quad \eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$.

Lichnerowicz:

$$a^{-2}\left[a^2\left[a^{-2}h^\lambda_\lambda\right]'\right]' = -\frac{2}{3}\delta(z)\, a^{-2}\, 8\pi G T^\lambda_\lambda$$

$$\left[a^{-2}\left(h^\lambda_{\lambda,\mu} - h^\lambda_{\mu,\lambda}\right)\right]' = 0.$$

$$-a^{-2}\left[a^4\left(a^{-2}h_{\mu\nu}\right)'\right]' + a^{-2}\partial^2 h_{\mu\nu} - 2a^2\,\bar{h}^\lambda{}_{(\mu,\nu)\lambda} - aa'\eta_{\mu\nu}\left(a^{-2}h^\lambda_\lambda\right)'$$

$$= -16\pi G\,\delta(z)\left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$$

Note $-T/3$ in last eqn - this has the tensor structure of 5D (or 4D massive) graviton.

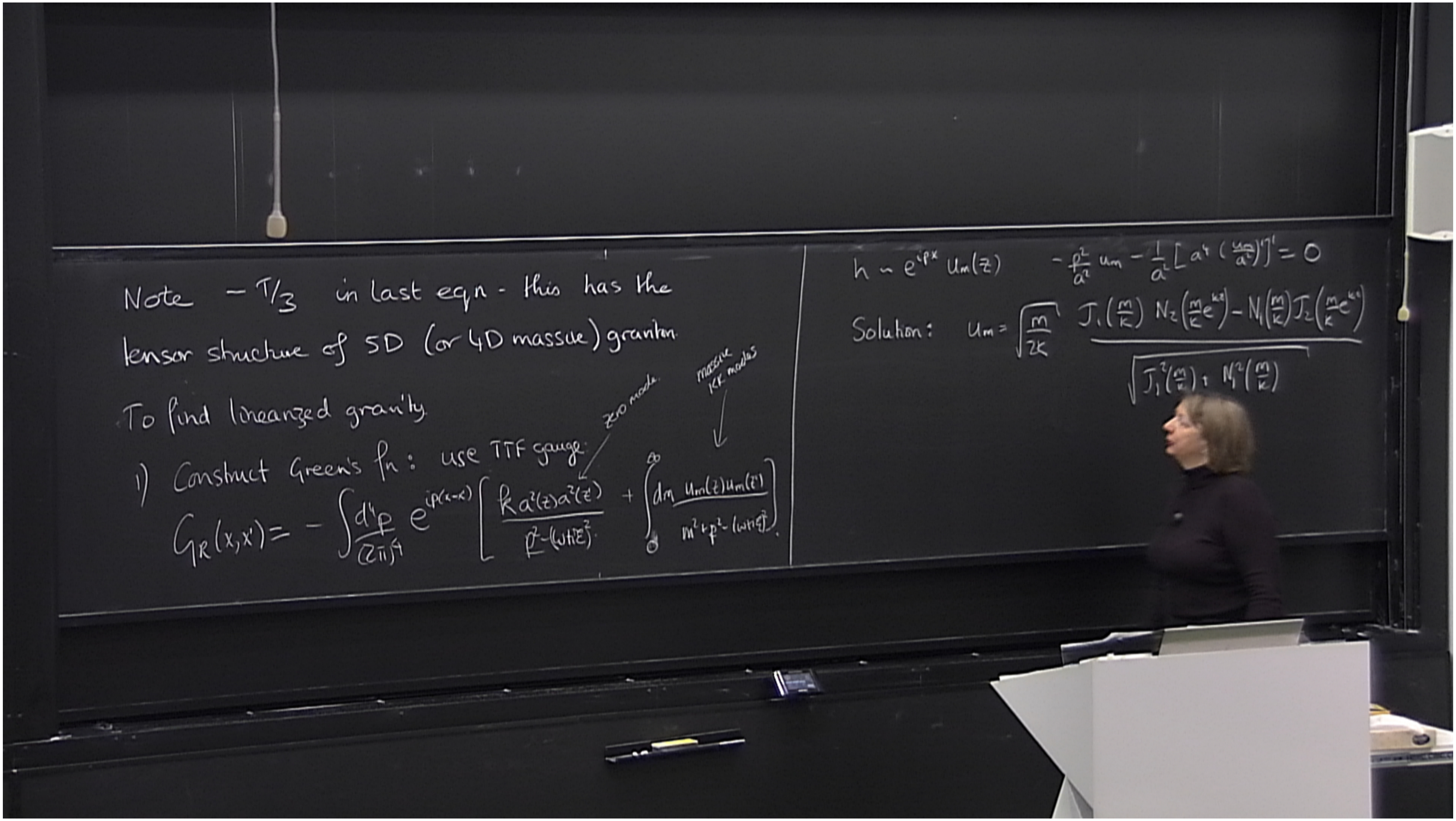
To find linearized gravity.

1) Construct Green's fn: use TTF gauge:

zero mode.

massive KK modes

$$G_R(x,x') = -\int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \left[\frac{k\, a^2(z) a^2(z')}{p^2 - (\omega + i\varepsilon)^2} + \int_0^\infty dm \frac{u_m(z) u_m(z')}{m^2 + p^2 - (\omega + i\varepsilon)^2} \right].$$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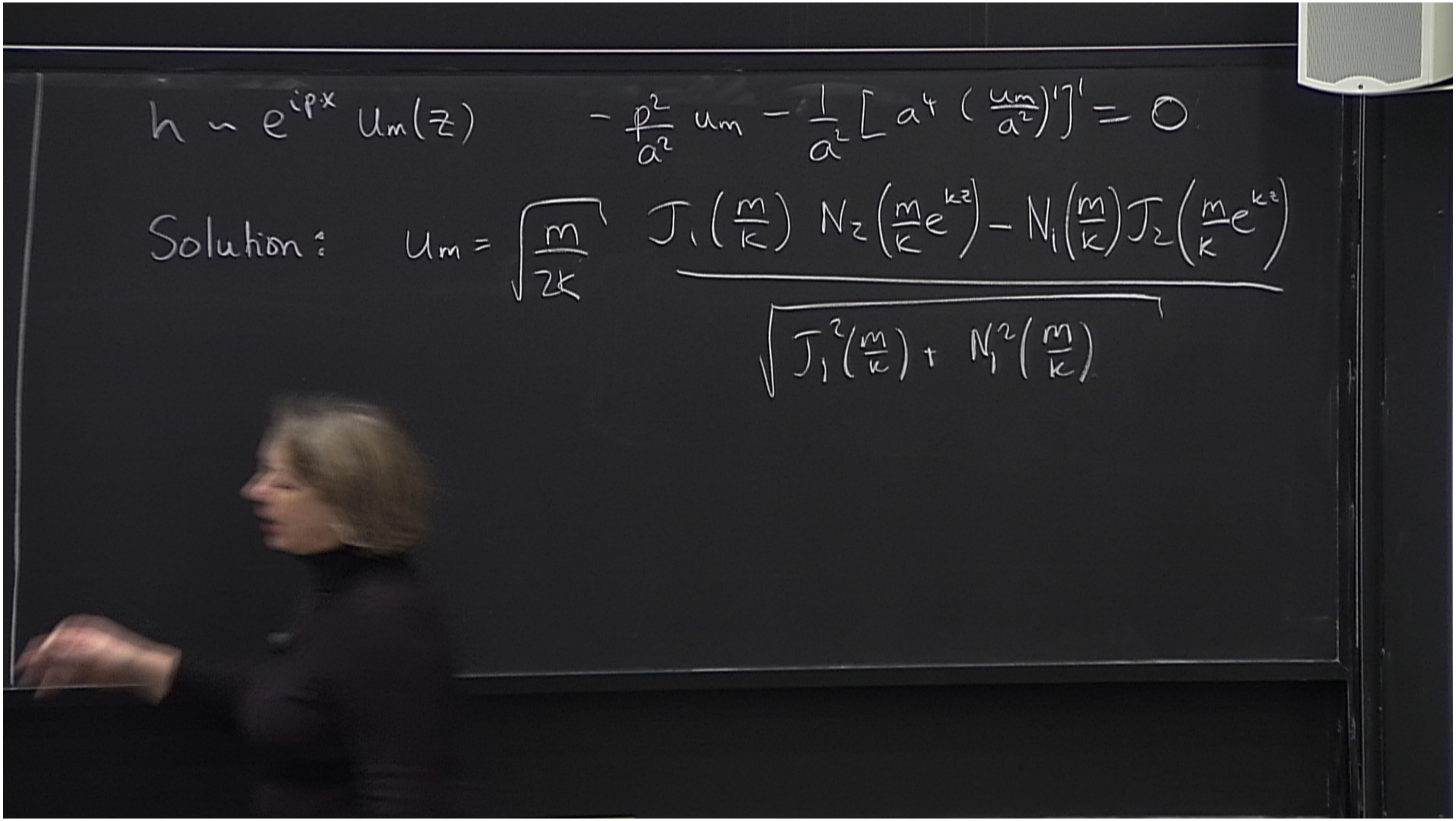
$h \sim e^{ipx} u_m(z)$
$-\frac{p^2}{a^2} u_m - \frac{1}{a^2}\left[a^4 \left(\frac{u_m}{a^2}\right)'\right]' = 0$
Solution: $u_m = \sqrt{\frac{m}{2k}} \frac{J_1\left(\frac{m}{k}\right) N_2\left(\frac{m}{k}e^{kz}\right) - N_1\left(\frac{m}{k}\right) J_2\left(\frac{m}{k}e^{kz}\right)}{\sqrt{J_1^2\left(\frac{m}{k}\right) + N_1^2\left(\frac{m}{k}\right)}}$



$h \sim e^{ip\cdot x} u_m(z)$
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c.f. KK which has discrete modes $e^{2\pi i x^5 n/L}$

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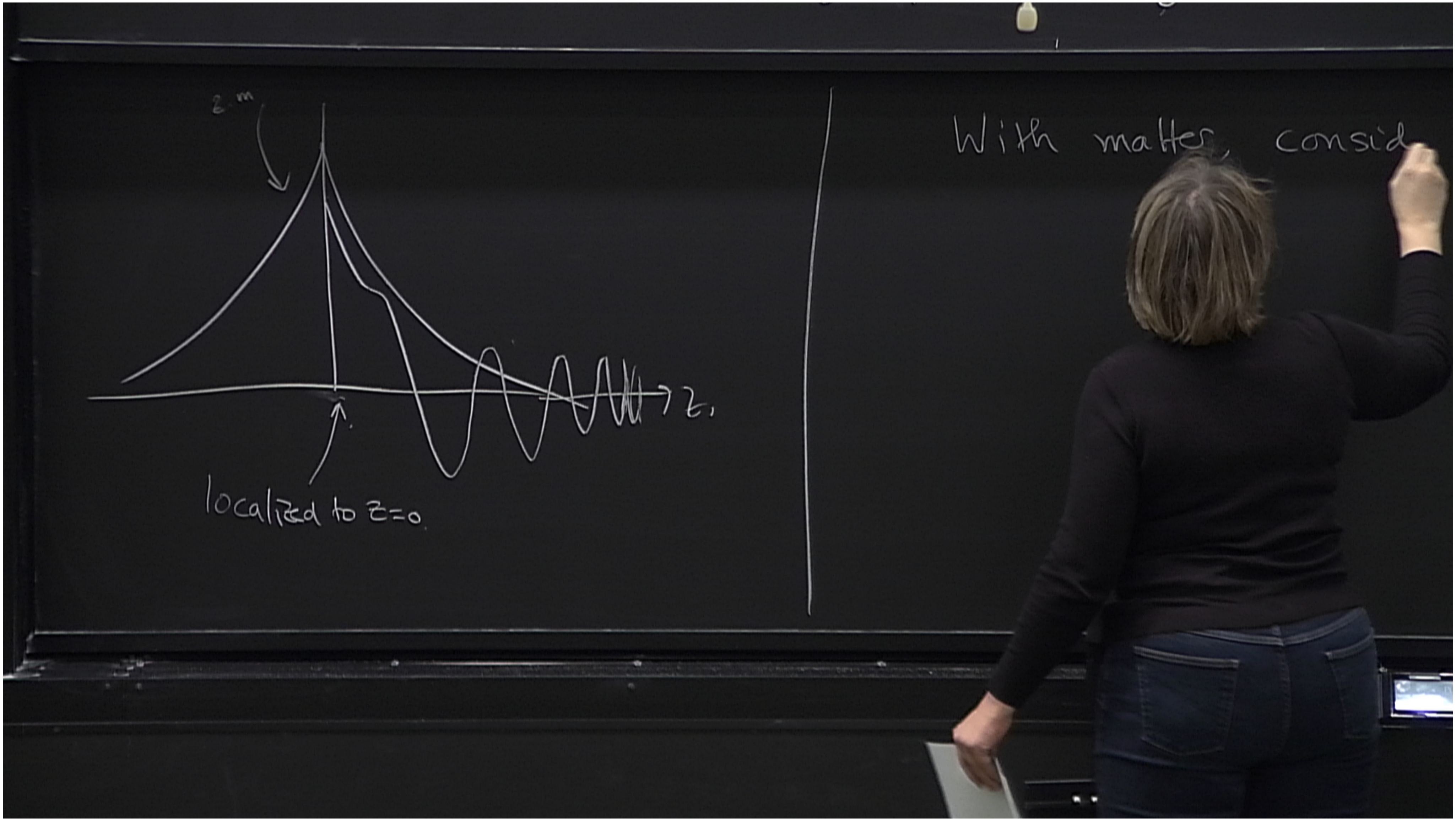
cf. KK which has discrete modes $e^{2\pi i x^5 n/L}$

Here have a continuum



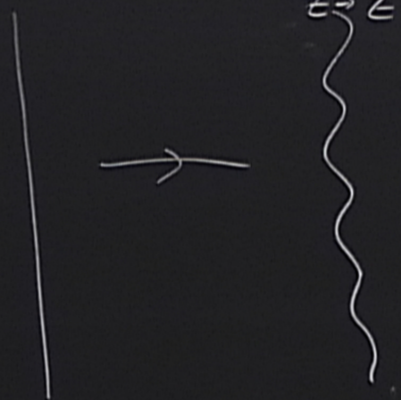
ε.m
localized to z=0
z
With matter, consid

$p^2-(\omega+i\epsilon)^2$. $m^2+p^2-(\omega+i\epsilon)^2$.

With matter, consider gauge more carefully.

GN gauge transfm: $\xi_z = f(x)$ $\quad \xi_\mu = -\frac{f_{,\mu}}{2k} + a^2 b_\mu(x)$ $\quad$ 4D gauge transfm

$z \to z + f$.



$$h_{\mu\nu} = -\frac{f_{,\mu\nu}}{k} + 2kfa^2\eta_{\mu\nu} + 2a^2 b_{(\mu,\nu)}$$

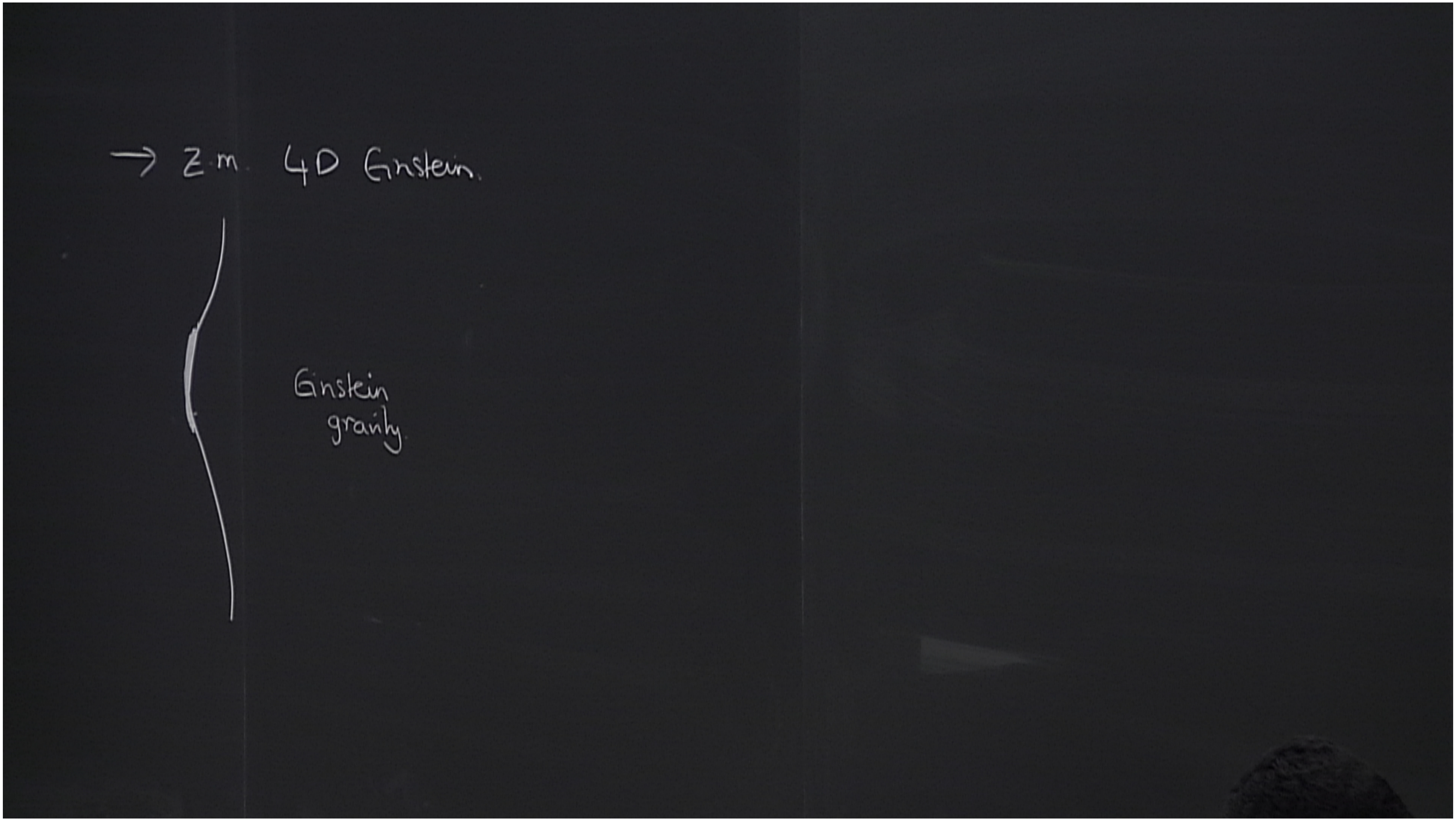
$$\mathrm{Lich}_{zz.} \rightarrow \quad \partial^2 f = -8\pi G\, T/6$$

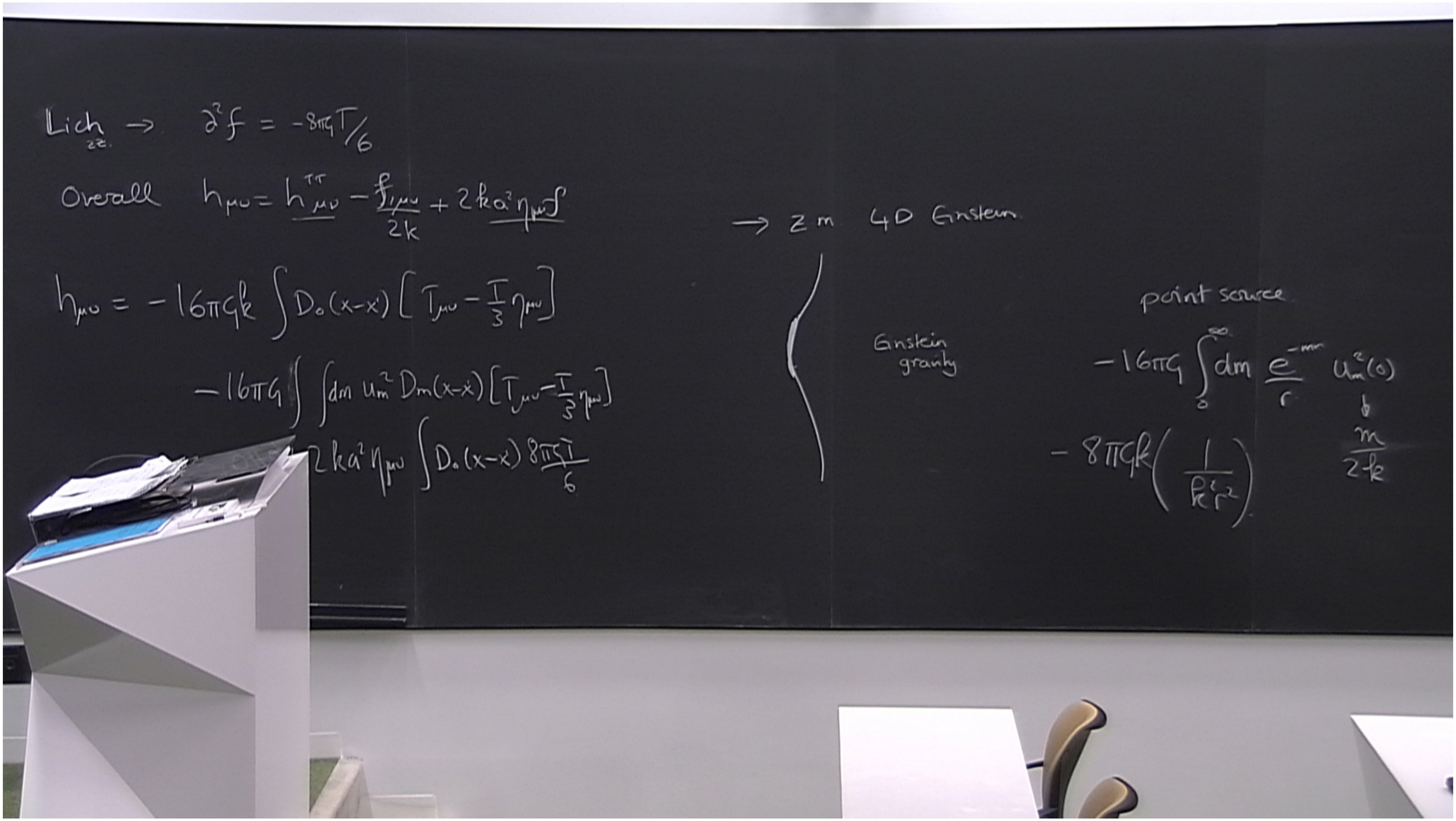
Overall $\quad h_{\mu\nu} = \underline{h^{TT}_{\mu\nu}} - \frac{f_{,\mu\nu}}{2k} + \underline{2ka^2\eta_{\mu\nu} f}$

$$h_{\mu\nu} = -16\pi G k \int D_0(x-x')\left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$$

$$-16\pi G \int \int dm\, u_m^2\, D_m(x-x')\left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$$

$$2ka^2\eta_{\mu\nu}\int D_0(x-x')\, \frac{8\pi G T}{6}$$


→ 2m. 4D Einstein.
Einstein
gravity.


$\text{Lich}_{zz} \to \partial^2 f = -8\pi G\, T/6$
Overall $h_{\mu\nu} = h^{TT}_{\mu\nu} - \frac{f_{,\mu\nu}}{2k} + 2k a^2 \eta_{\mu\nu} f$
$h_{\mu\nu} = -16\pi G k \int D_0(x-x') \left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$
$-16\pi G \int \int dm\, u_m^2 D_m(x-x') \left[T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right]$
$2ka^2 \eta_{\mu\nu} \int D_0(x-x')\, \frac{8\pi G T}{6}$
$\to$ 2m 4D Einstein
Einstein gravity
point source
$-16\pi G \int_0^\infty dm\, \frac{e^{-mr}}{r}\, u_m^2(0)$
$\frac{m}{2k}$
$-8\pi G k \left(\frac{1}{k^2 r^2}\right)$