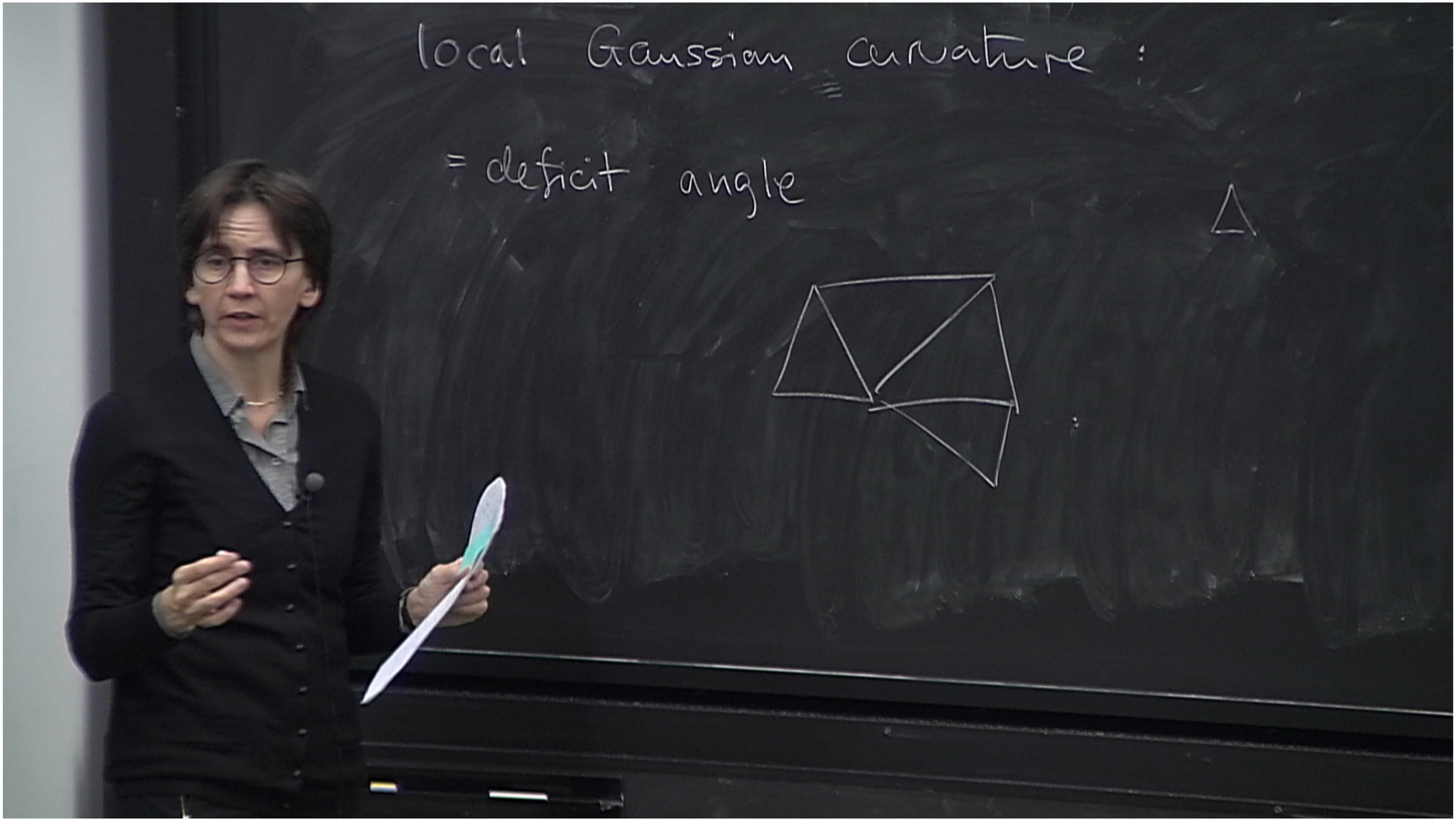
Title: Quantum Gravity (Review) - Lecture 13

Date: Feb 08, 2012 10:15 AM

URL: http://pirsa.org/12020007

Abstract:


local Gaussian curvature :
= deficit angle

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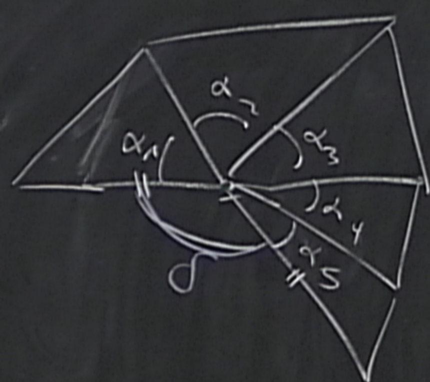
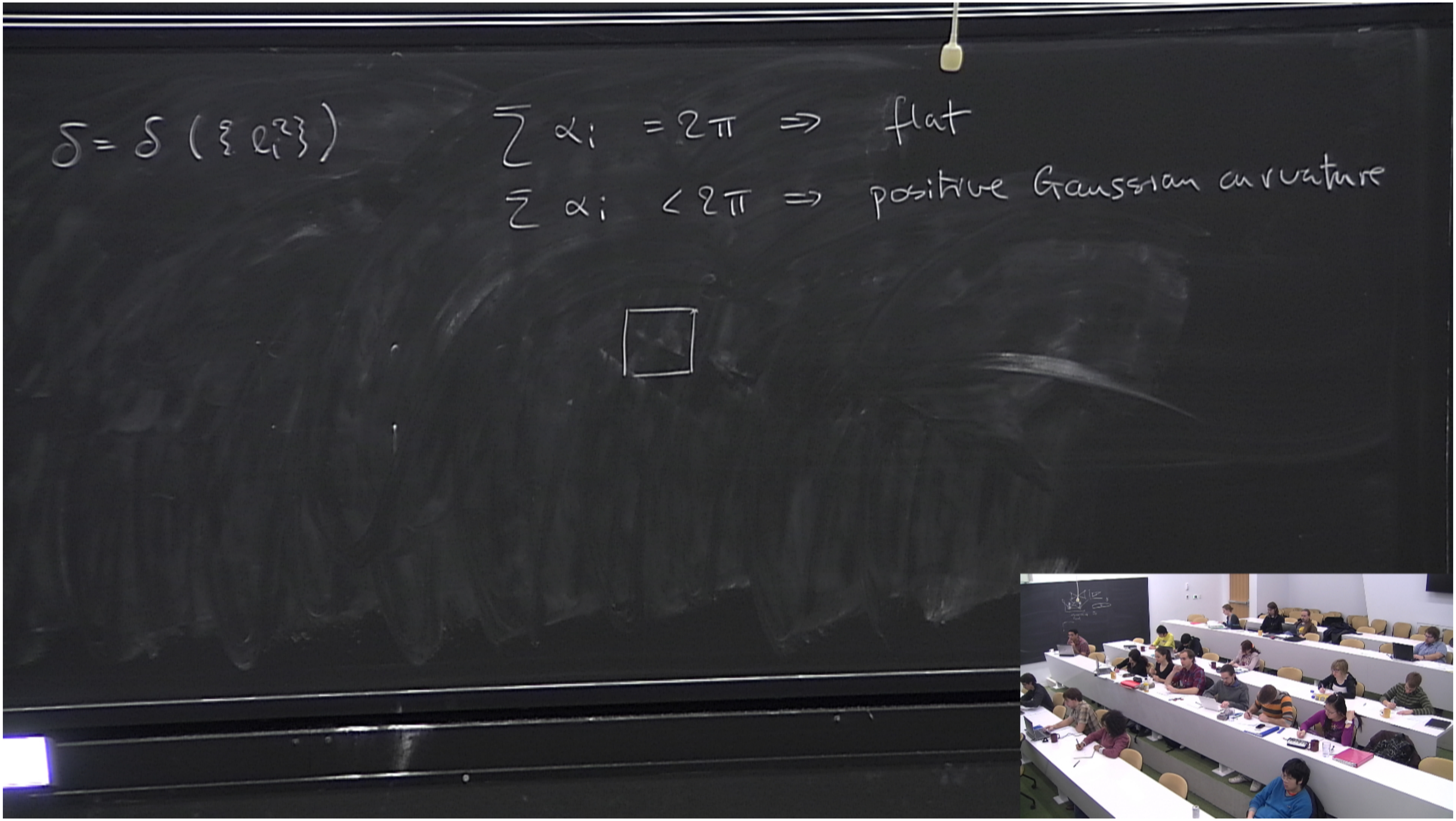
= deficit angle $\delta = 2\pi - \sum_{i \ni h} \alpha_i$

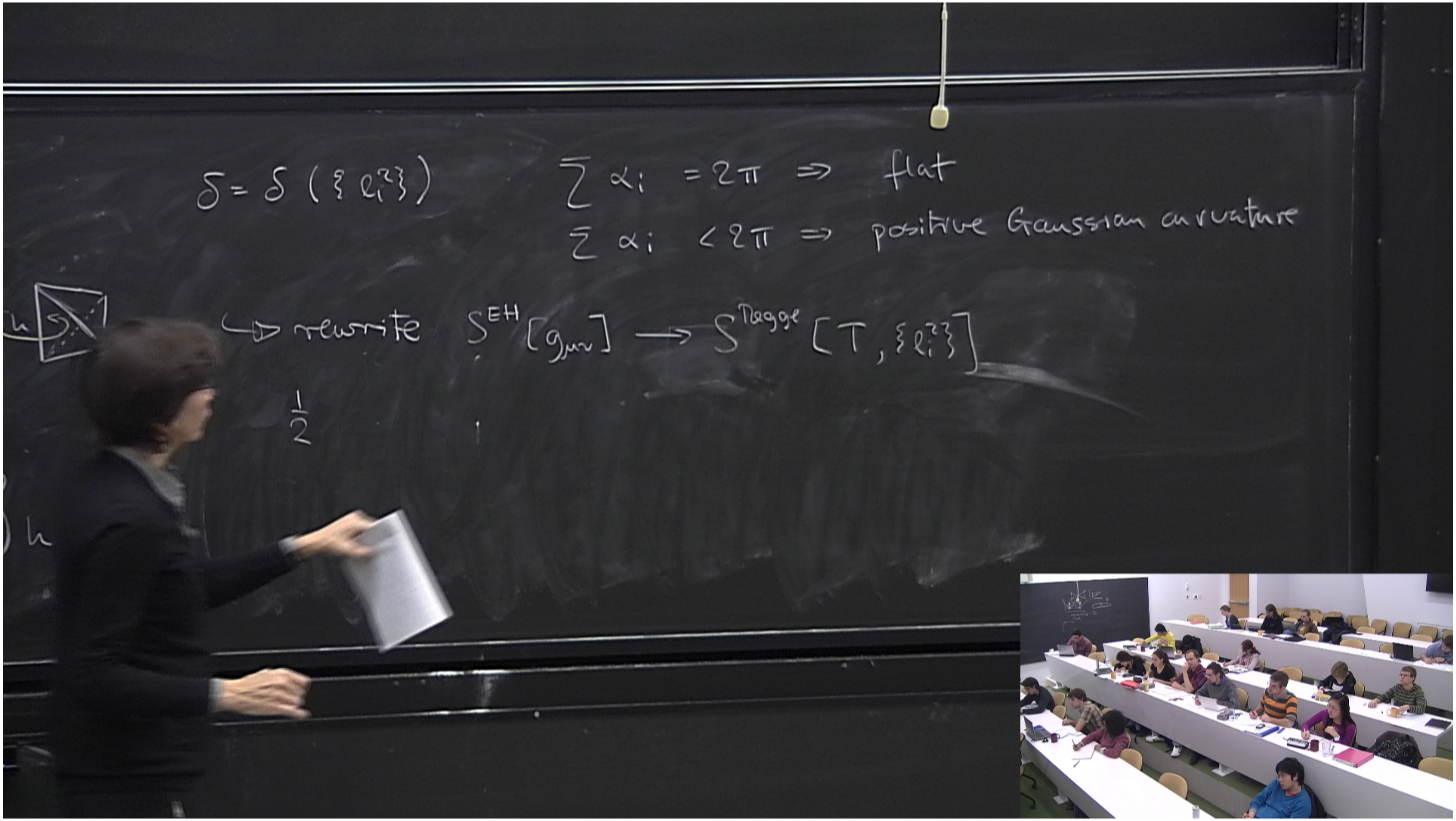
hinge

i labels d-simplices sharing a $(d-2)$ simplex ("hinge")

$\underline{d=2}$




$\delta = \delta(\{l_i^2\})$
$\sum \alpha_i = 2\pi \Rightarrow$ flat
$\sum \alpha_i < 2\pi \Rightarrow$ positive Gaussian curvature


δ = δ ({ l_i² })
Σ α_i = 2π ⇒ flat
Σ α_i < 2π ⇒ positive Gaussian curvature
⟺ rewrite S^EH [g_μν] ⟶ S^Regge [T, { l_i² }]
1/2

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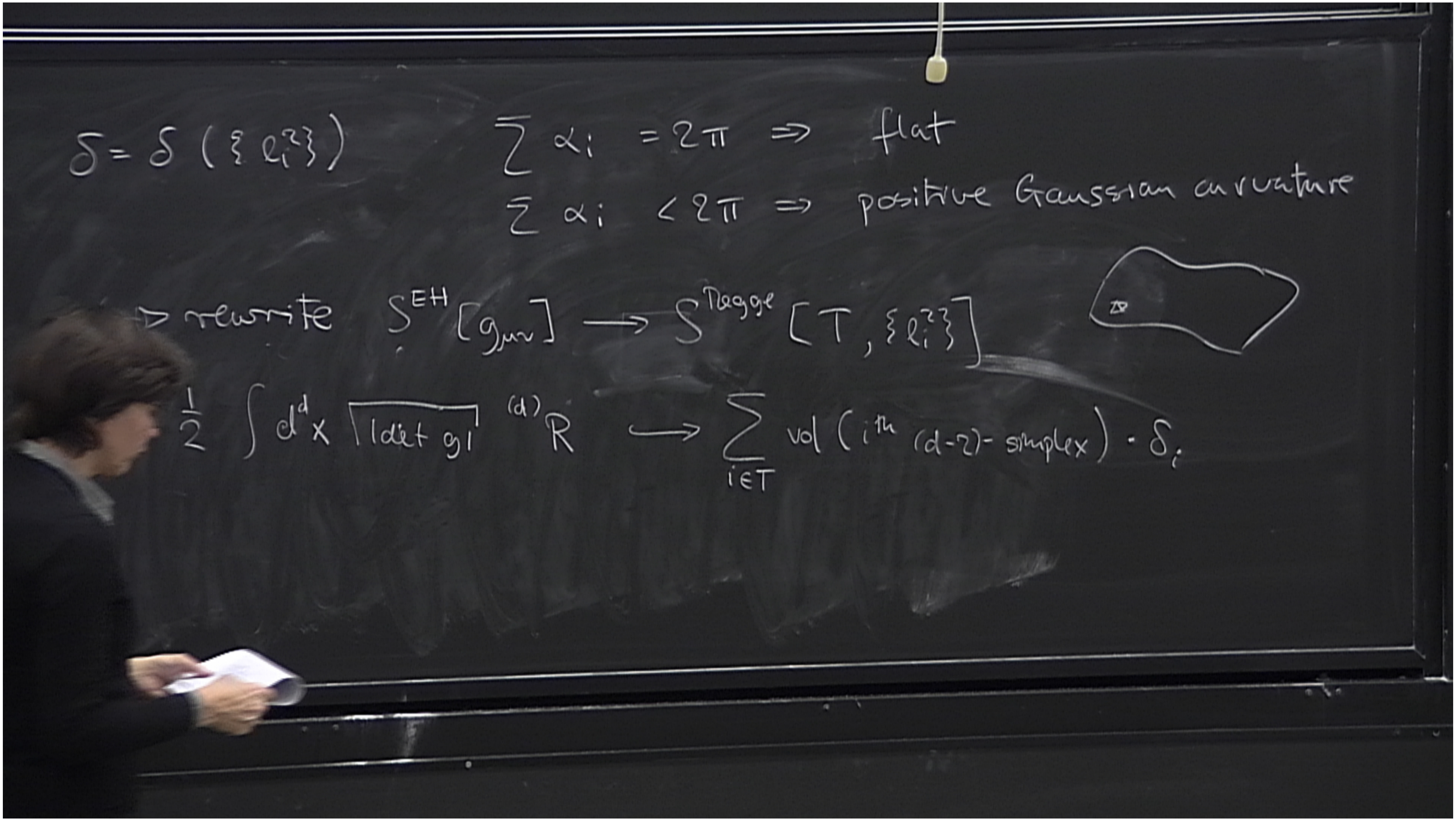
$\hookrightarrow$ rewrite $S^{EH}[g_{\mu\nu}] \longrightarrow S^{\text{Regge}}[T, \{l_i^2\}]$

$$\frac{1}{2}\int d^dx \sqrt{|\det g|}\; {}^{(d)}R \longrightarrow \sum_{i \in T} \text{vol}(i^{\text{th}}\ (d-2)\text{-simplex}) \cdot \delta_i$$



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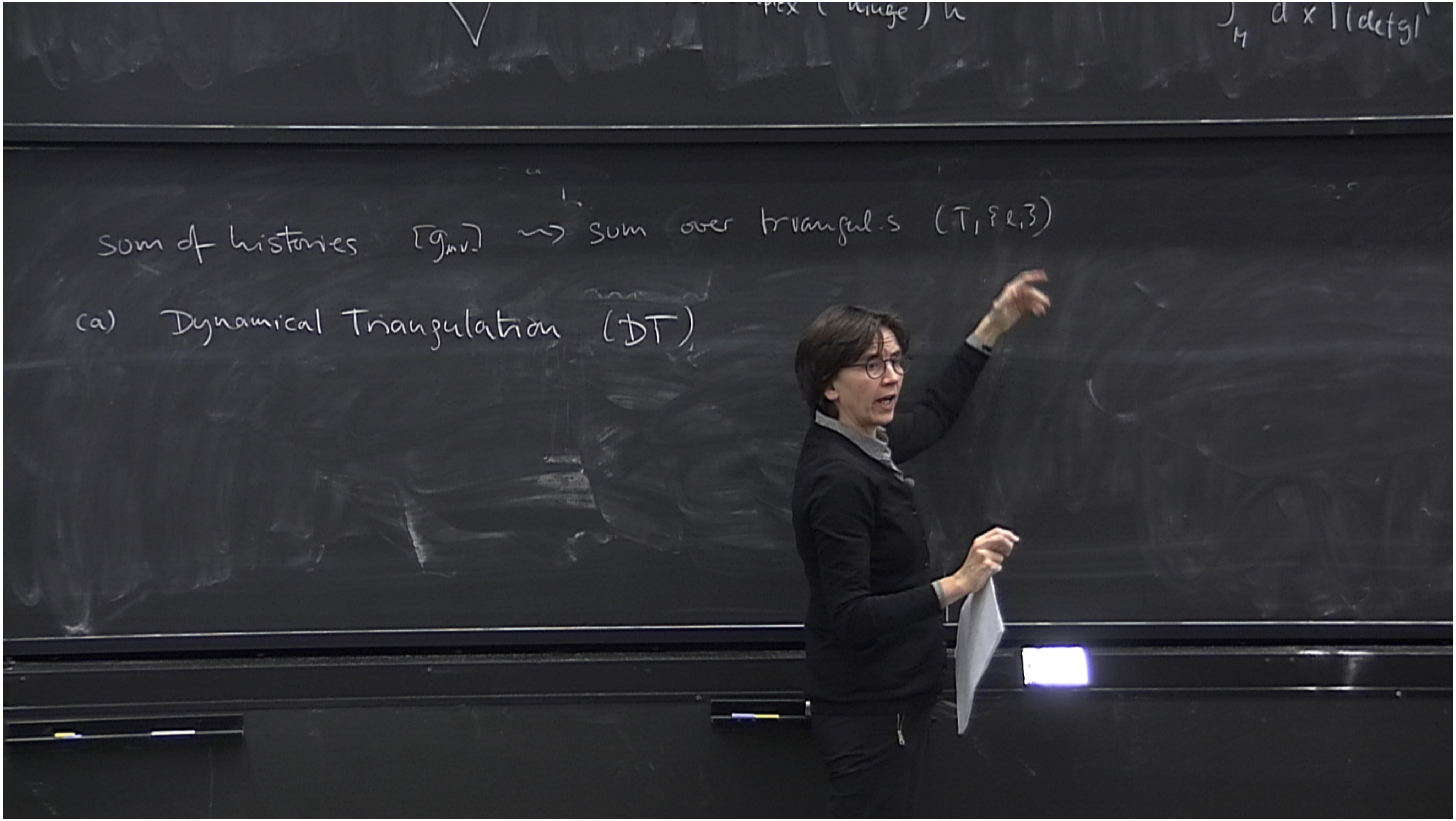
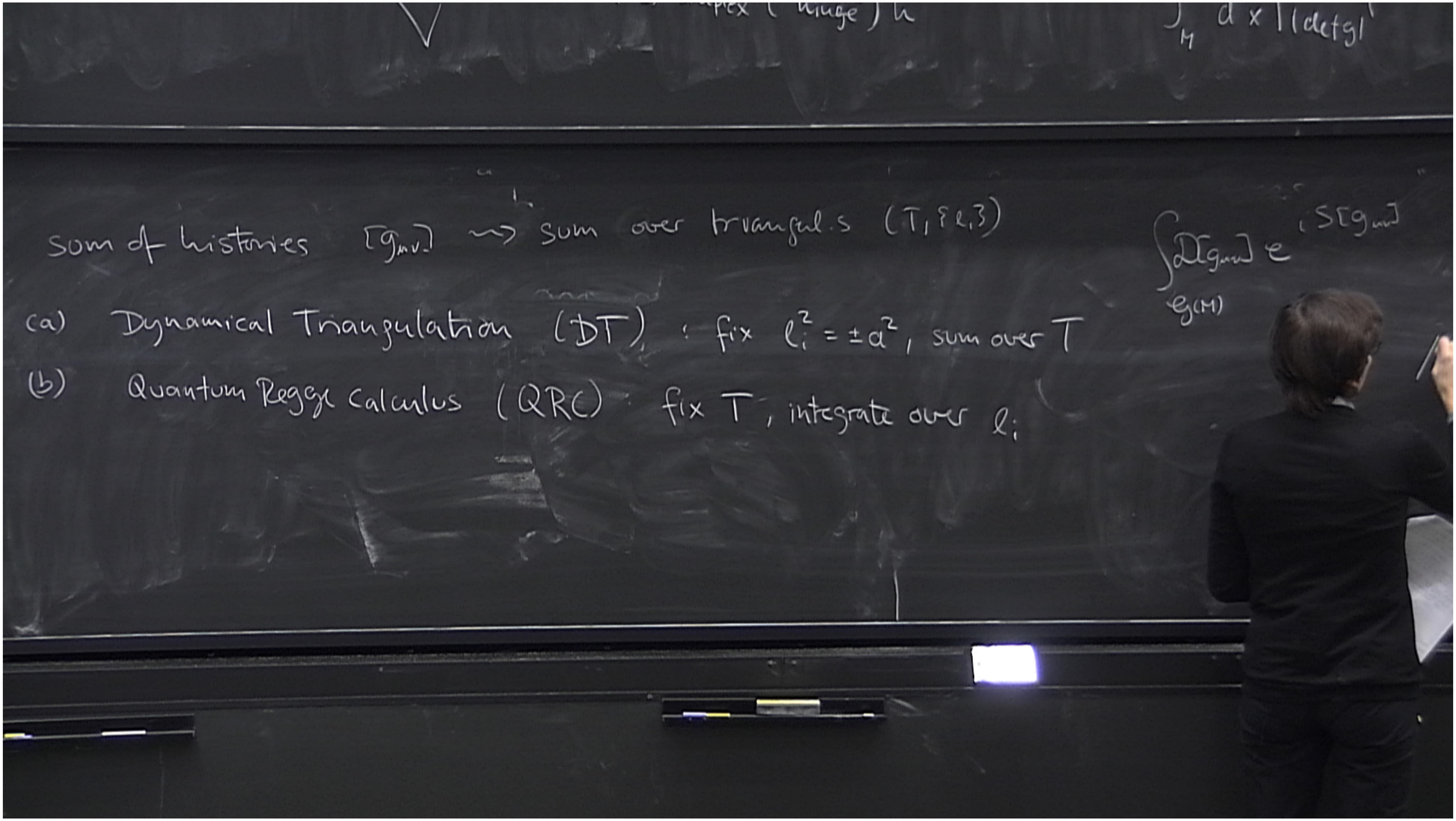
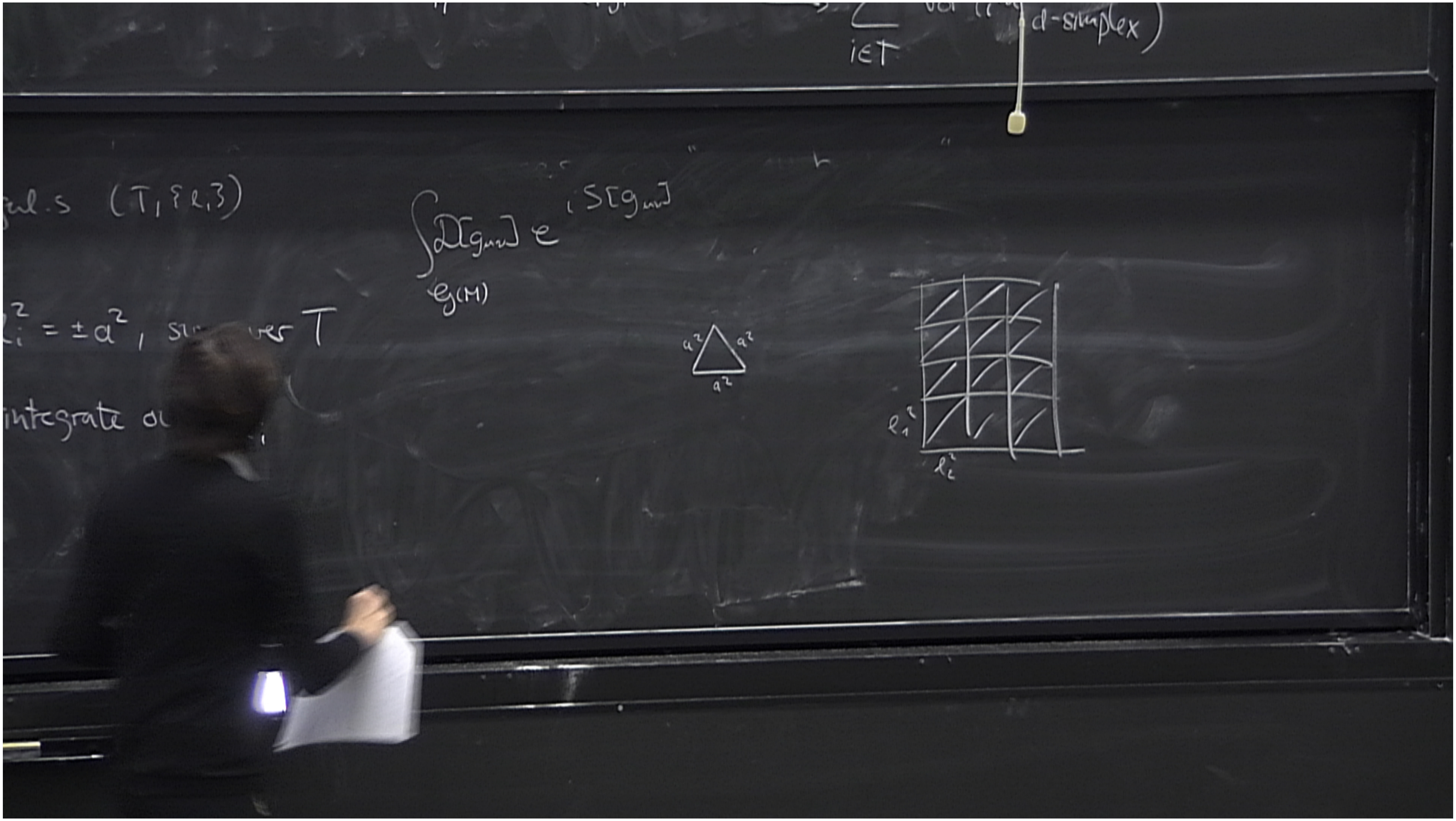
$\hookrightarrow$ rewrite $S^{EH}[g_{\mu\nu}] \longrightarrow S^{\text{Regge}}[T, \{l_i^2\}]$

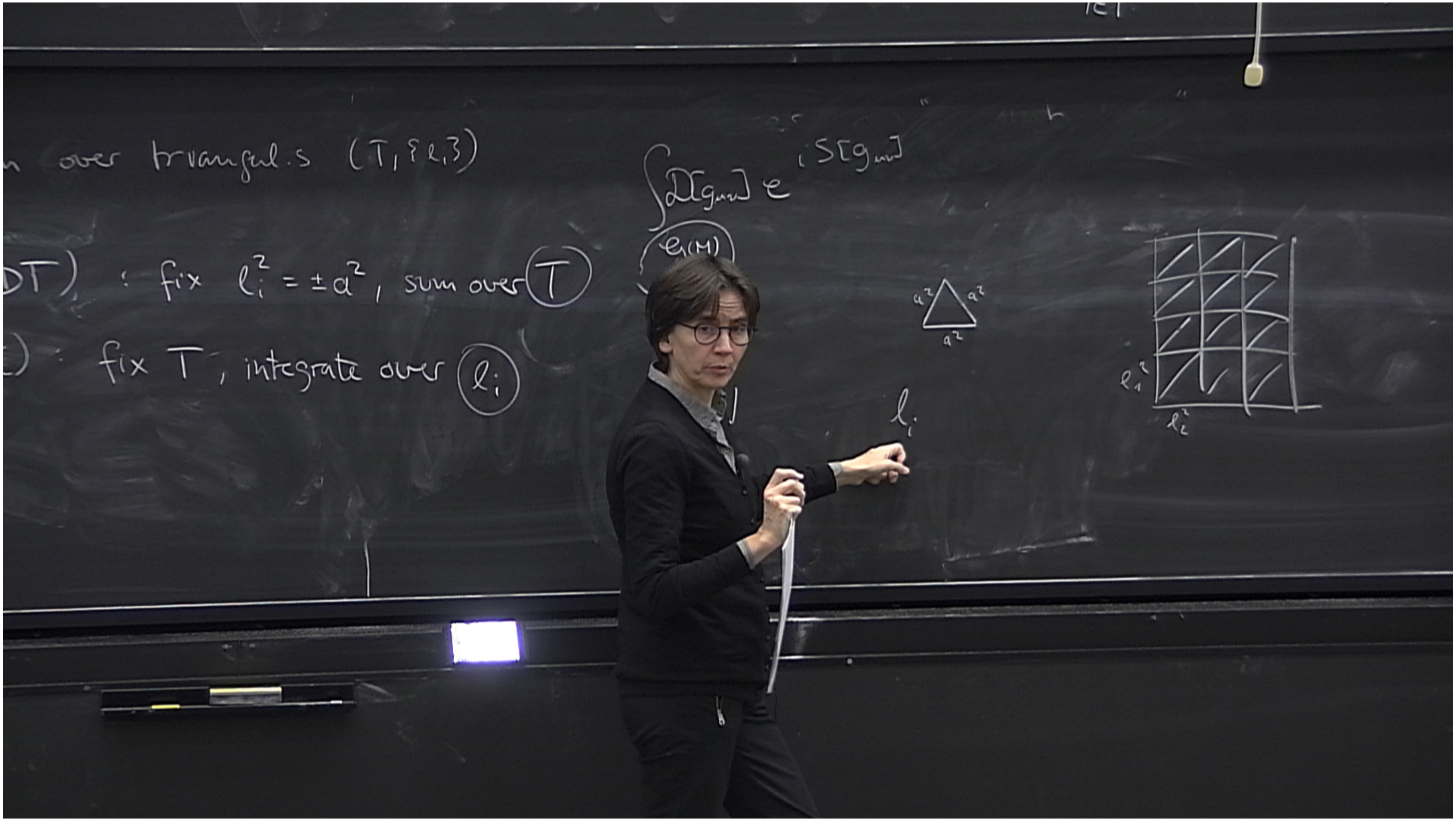
$$\frac{1}{2} \int_M d^dx \sqrt{|\det g|}\; {}^{(d)}R \longrightarrow \sum_{i \in T} \mathrm{vol}(i^{th}\ (d-2)\text{-simplex}) \cdot \delta_i$$

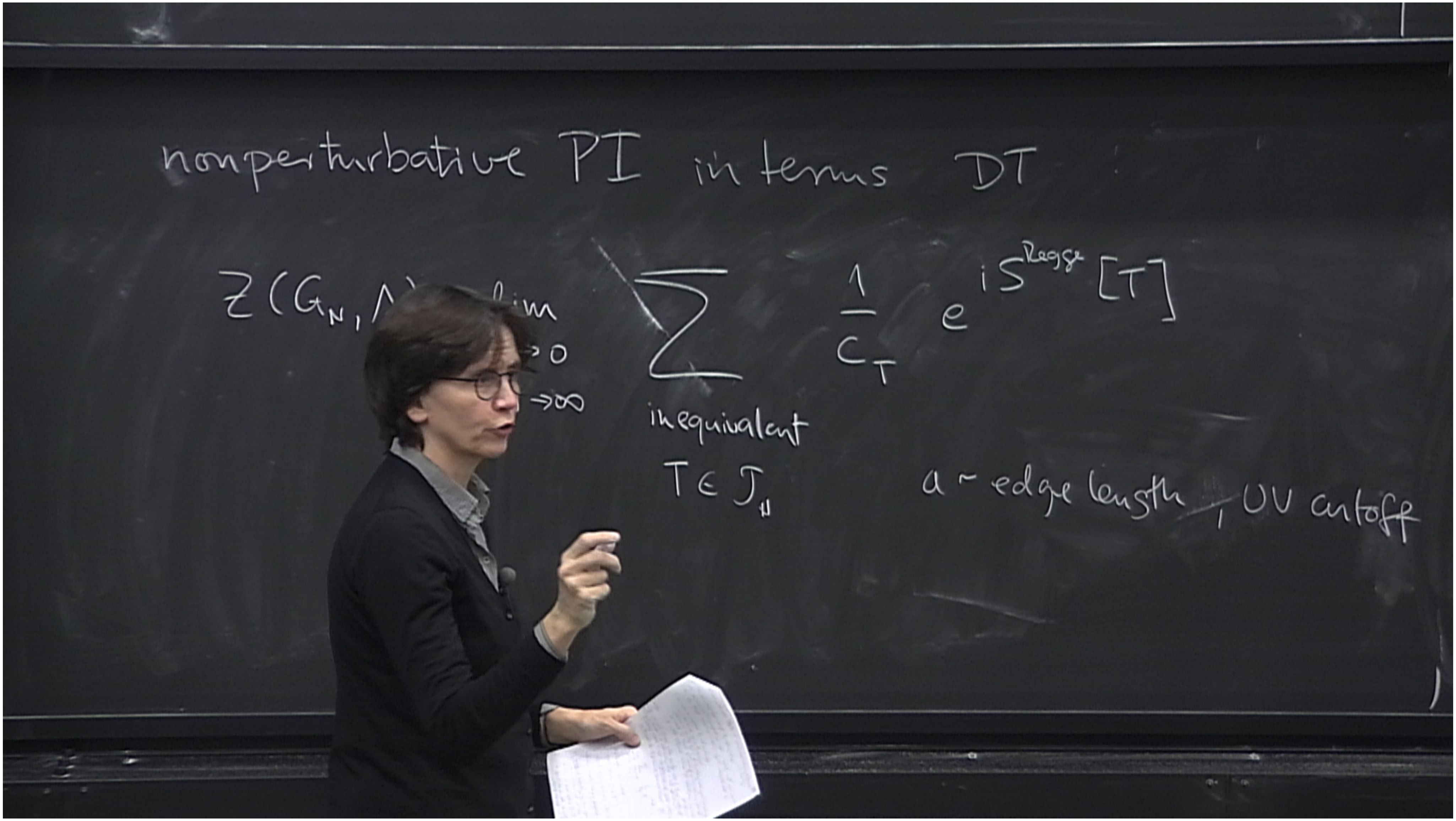
$$\int_M d^dx \sqrt{|\det g|} \hookrightarrow \sum_{i \in T} \mathrm{vol}(i^{th}\ d\text{-simplex})$$


sum of histories [g_μν] ⇝ sum over triangul.s (T, {l_i})
(a) Dynamical Triangulation (DT)


plex (hinge) h
sum of histories $[g_{\mu\nu}]$ ⇝ sum over triangul.s $(T, \{l_i\})$
$\int_{\mathcal{G}(M)} \mathcal{D}[g_{\mu\nu}]\, e^{iS[g_{\mu\nu}]}$
(a) Dynamical Triangulation (DT) : fix $l_i^2 = \pm a^2$, sum over T
(b) Quantum Regge Calculus (QRC) fix T, integrate over l_i


d-simplex)
i∈T
(T, {l_i})
∫D[g_μν] e^{iS[g_μν]}
G(M)
= ±a^2, sum over T
integrate
a^2
l_1^2
l_2^2


over triangul.s $(T, \{l_i\})$
$\int \mathcal{D}[g_{\mu\nu}]\, e^{iS[g_{\mu\nu}]}$
$\mathcal{G}(M)$
DT) : fix $l_i^2 = \pm a^2$, sum over T
fix T, integrate over l_i
a^2
l_i
l_1^2
l_2^2


nonperturbative PI in terms DT
Z(G_N, Λ
lim
→0
→∞
inequivalent
T ∈ J_N
1/C_T
e^{iS^{Regge}[T]}
a ~ edge length, UV cutoff

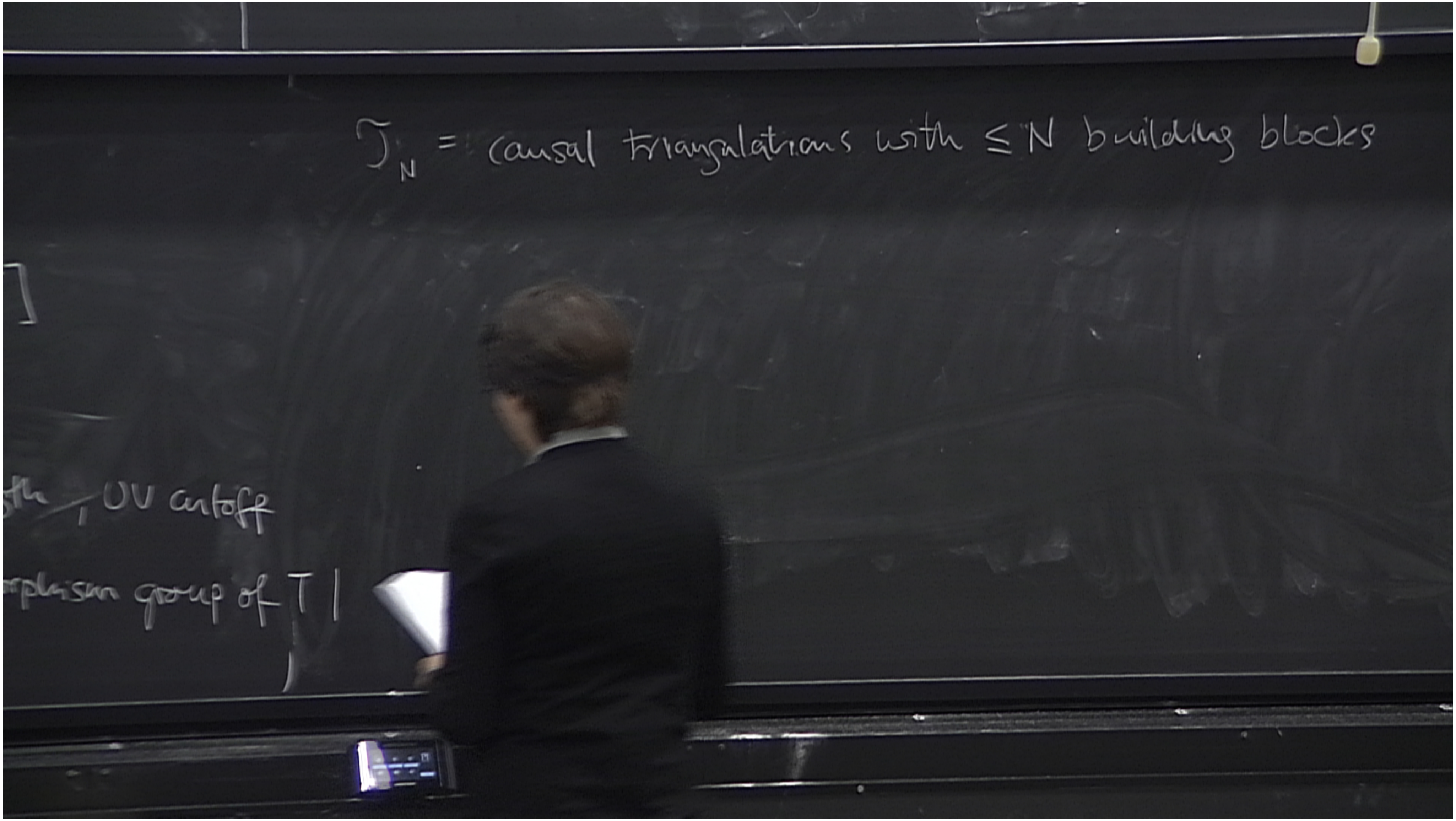
nonperturbative PI in terms DT

$\mathcal{T}_N$

$$Z(G_N, \Lambda) = \lim_{\substack{a \to 0 \\ N \to \infty}} \sum_{\substack{\text{inequivalent} \\ T \in \mathcal{T}_N}} \frac{1}{C_T} e^{i S^{\text{Regge}}[T]}$$

$a \sim$ edge length, UV cutoff

$C_T = |$automorphism group of $T|$


T_N = causal triangulations with ≤ N building blocks
UV cutoff
group of T

nonperturbative PI in terms DT

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continuum limit
keeping physical volume
$V_4 : a^d N$ fixed

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nonperturbative PI in terms DT

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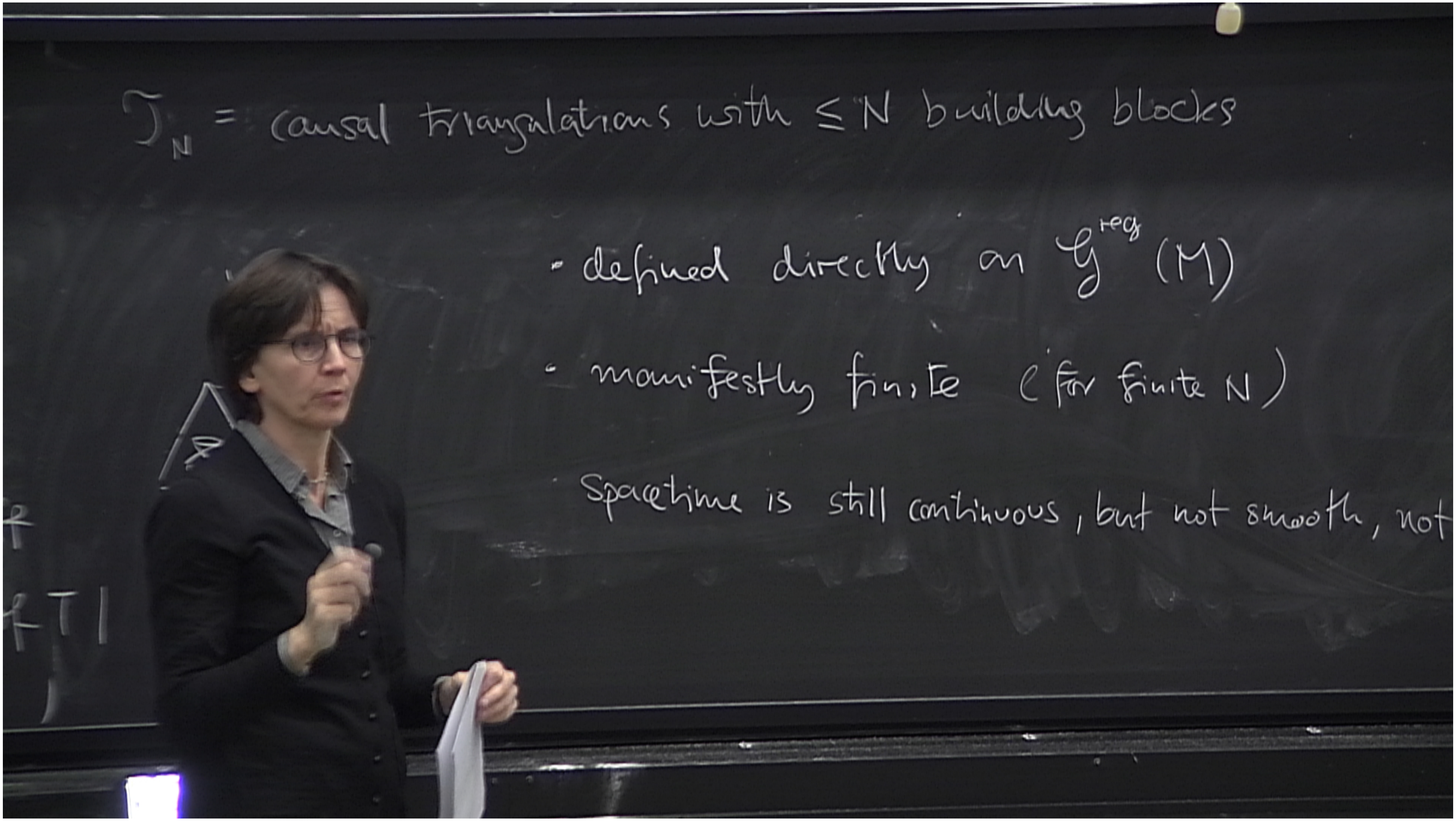
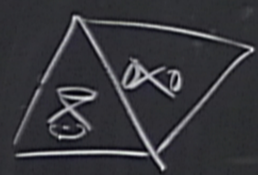

$\mathcal{T}_N$ = causal triangulations with $\leq N$ building blocks
· defined directly on $\mathcal{G}^{reg}(M)$
· manifestly finite (for finite N)
· Spacetime is still continuous, but not smooth, not

$\mathcal{T}_N$ = causal triangulations with $\leq N$ building blocks

$N = 10$



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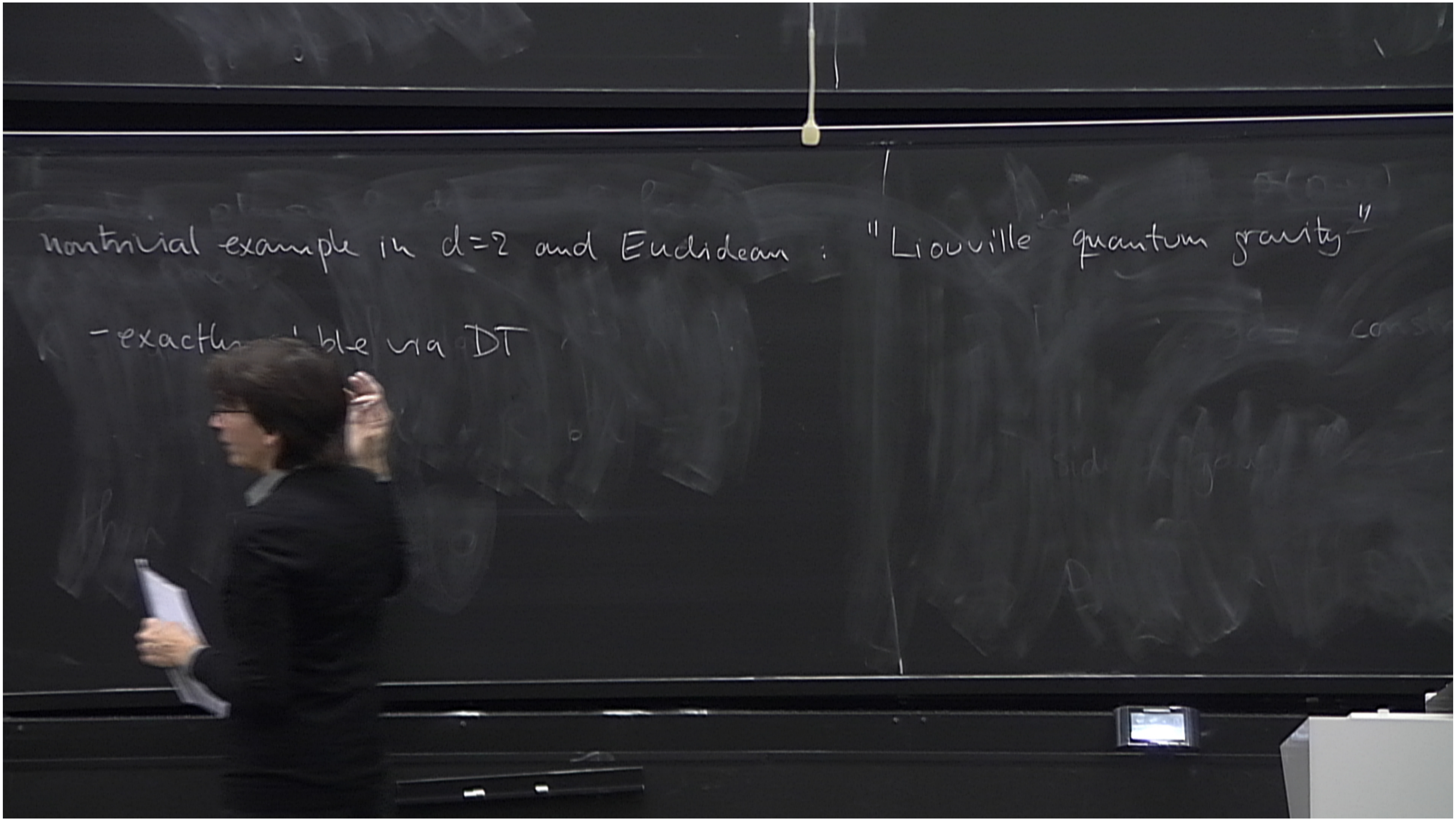
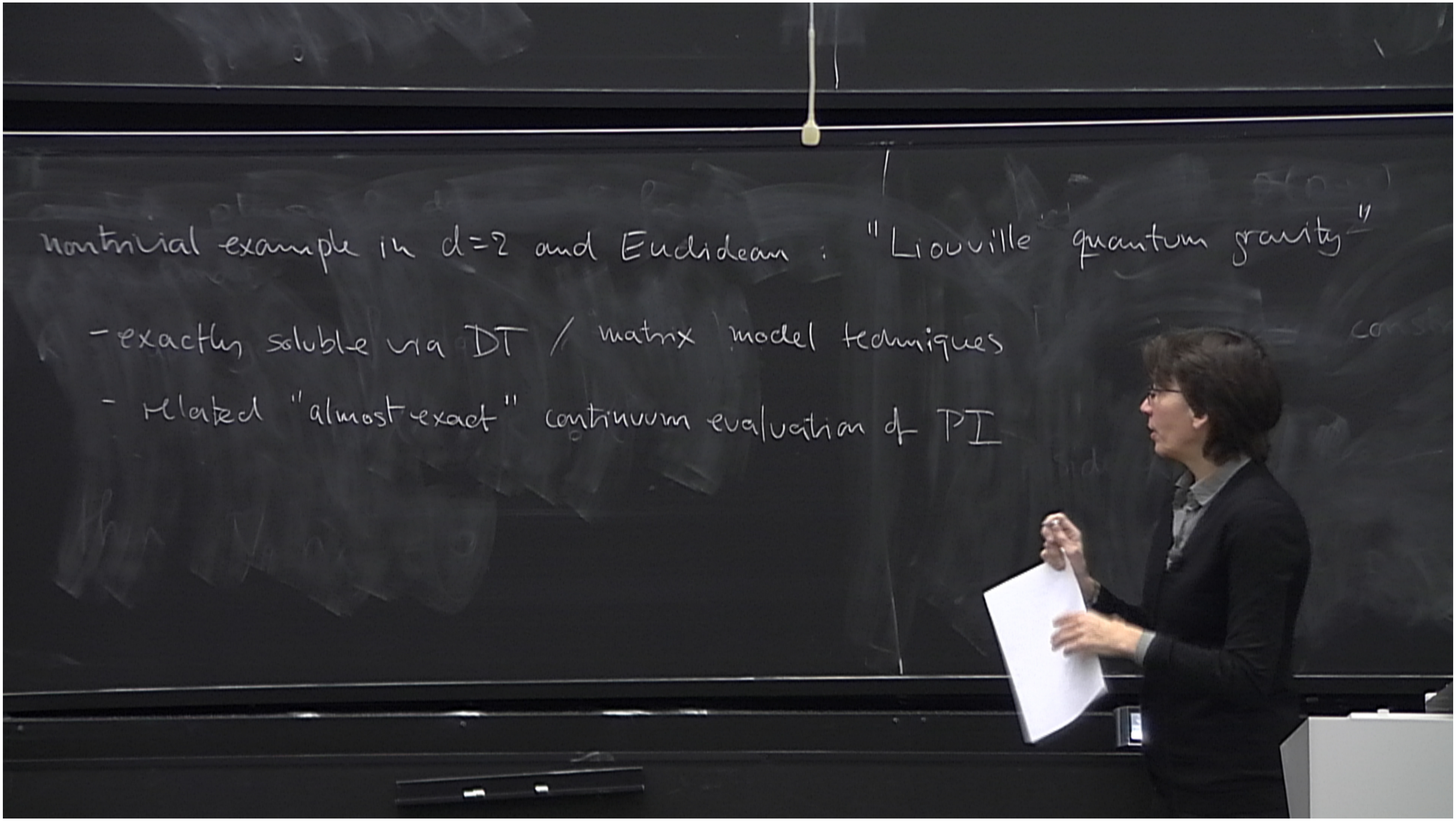
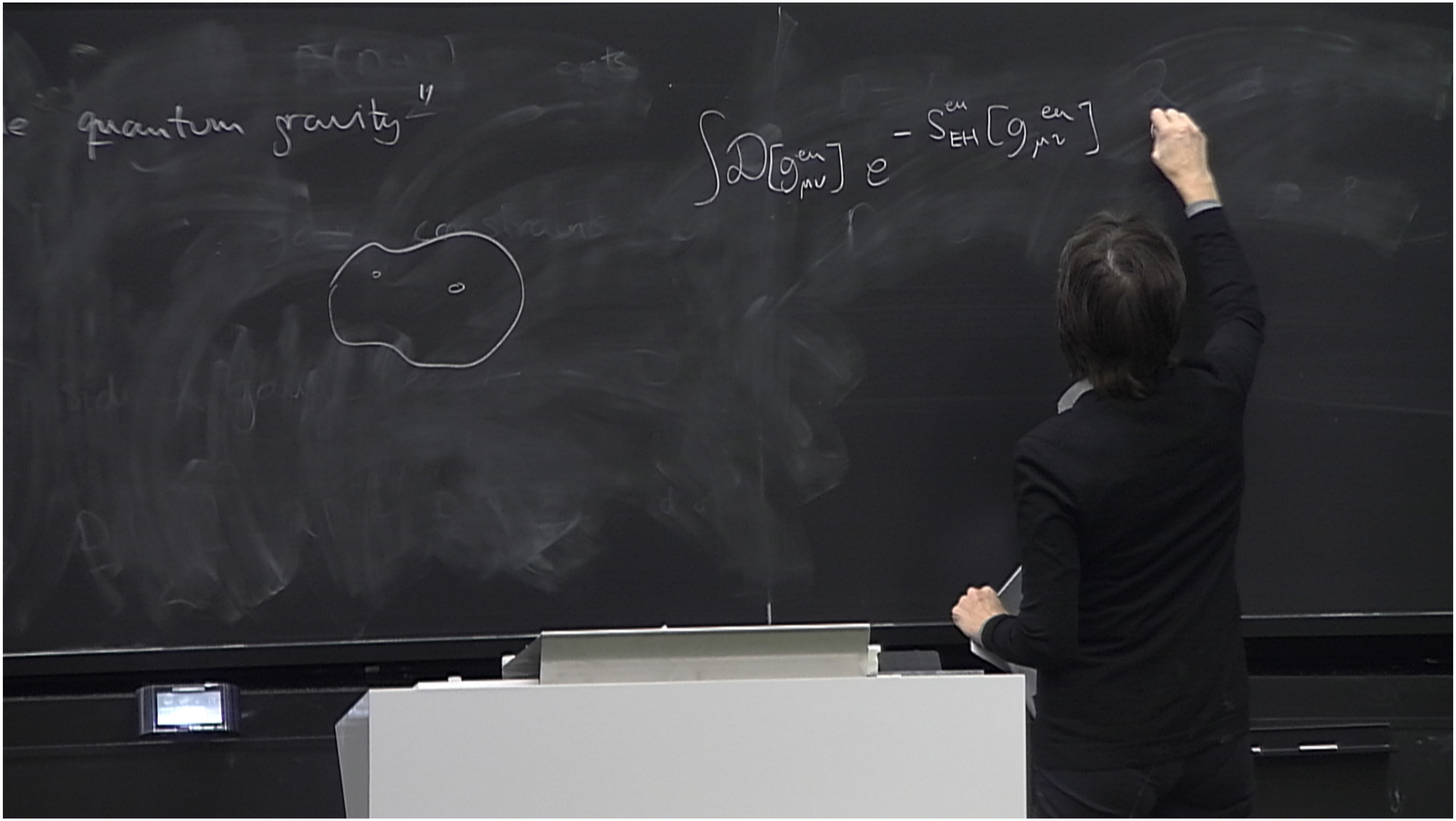
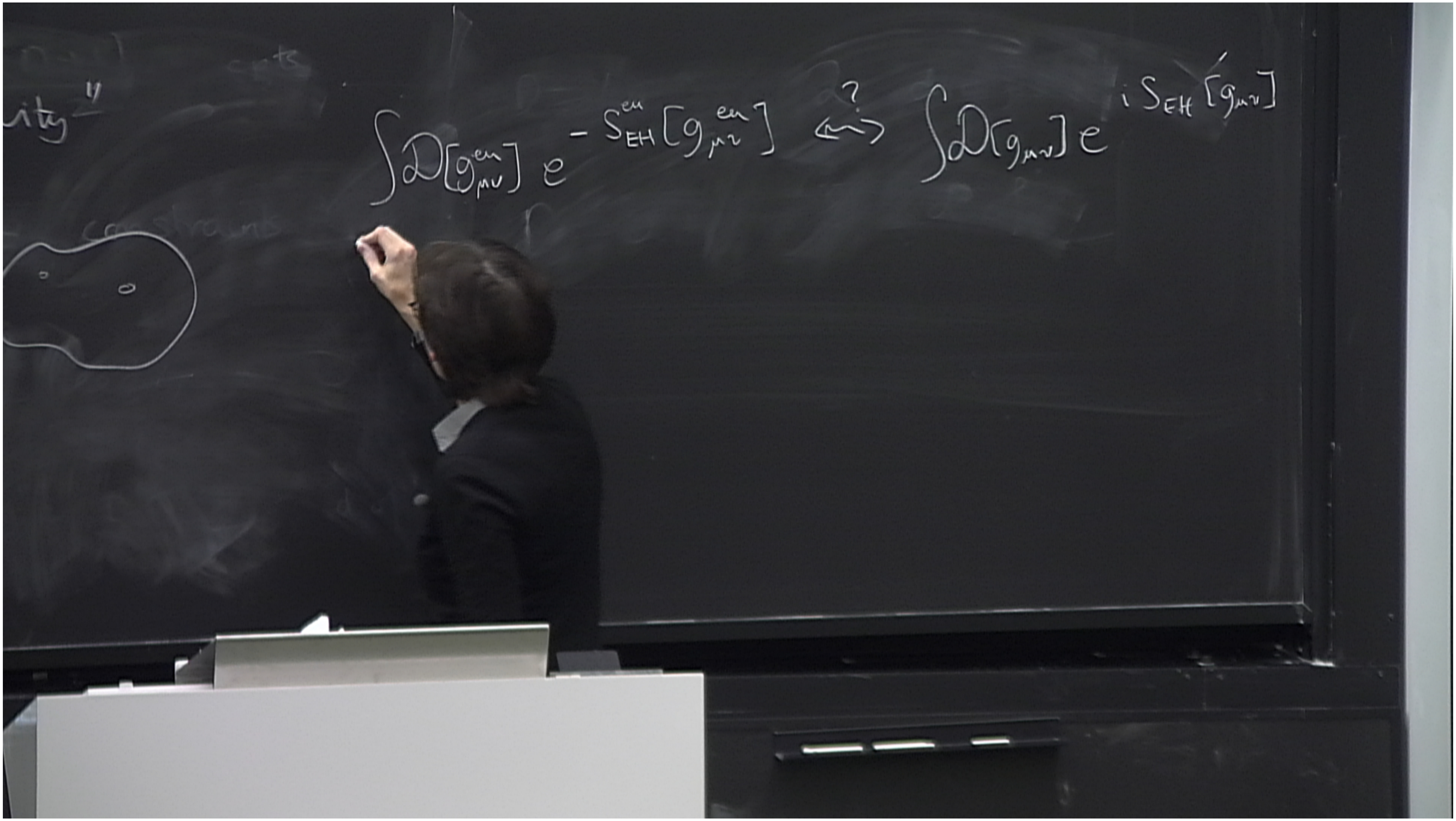
toff

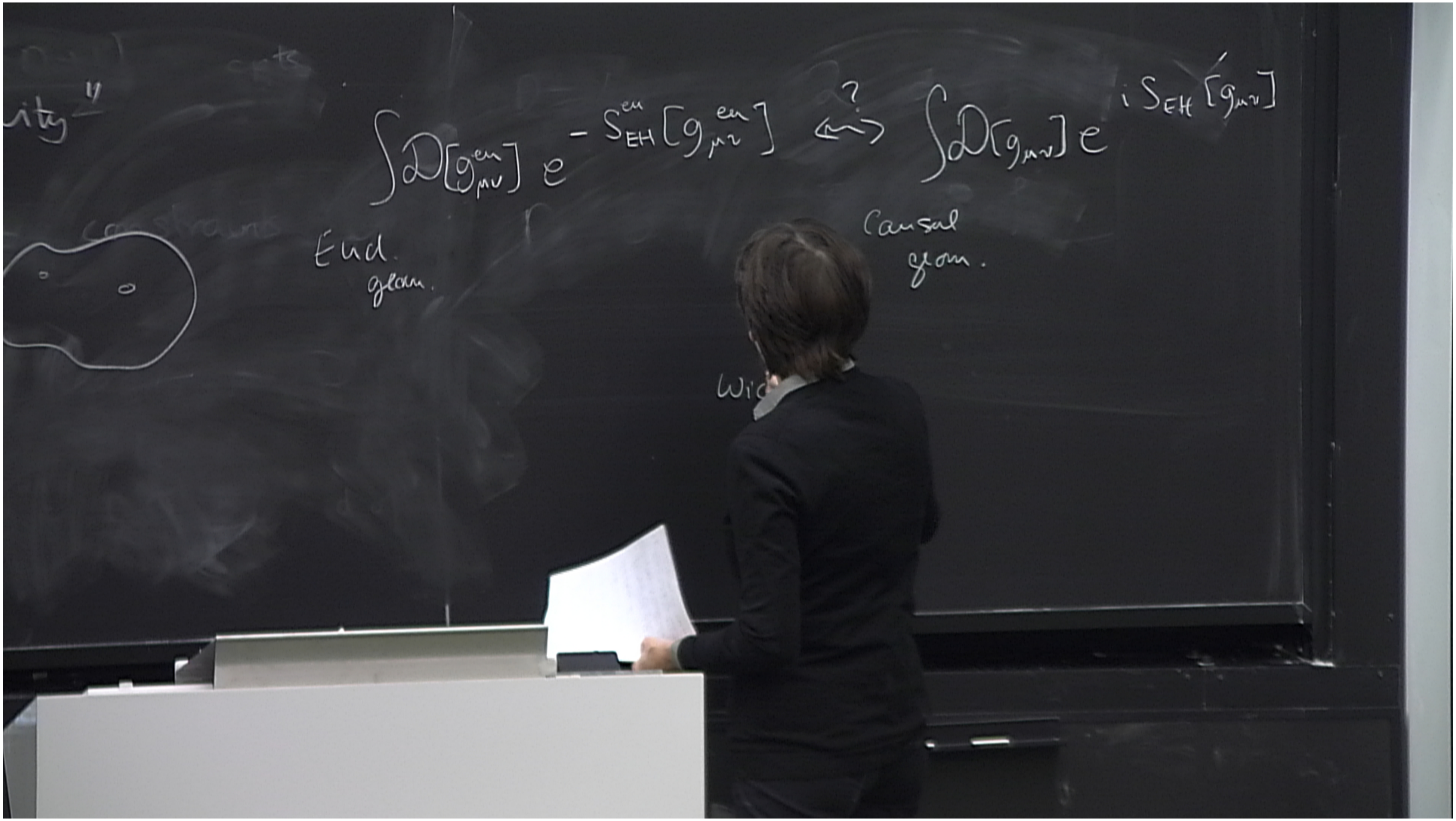
p of T |


nontrivial example in $d=2$ and Euclidean : "Liouville" quantum gravity"
- exactl ble via DT

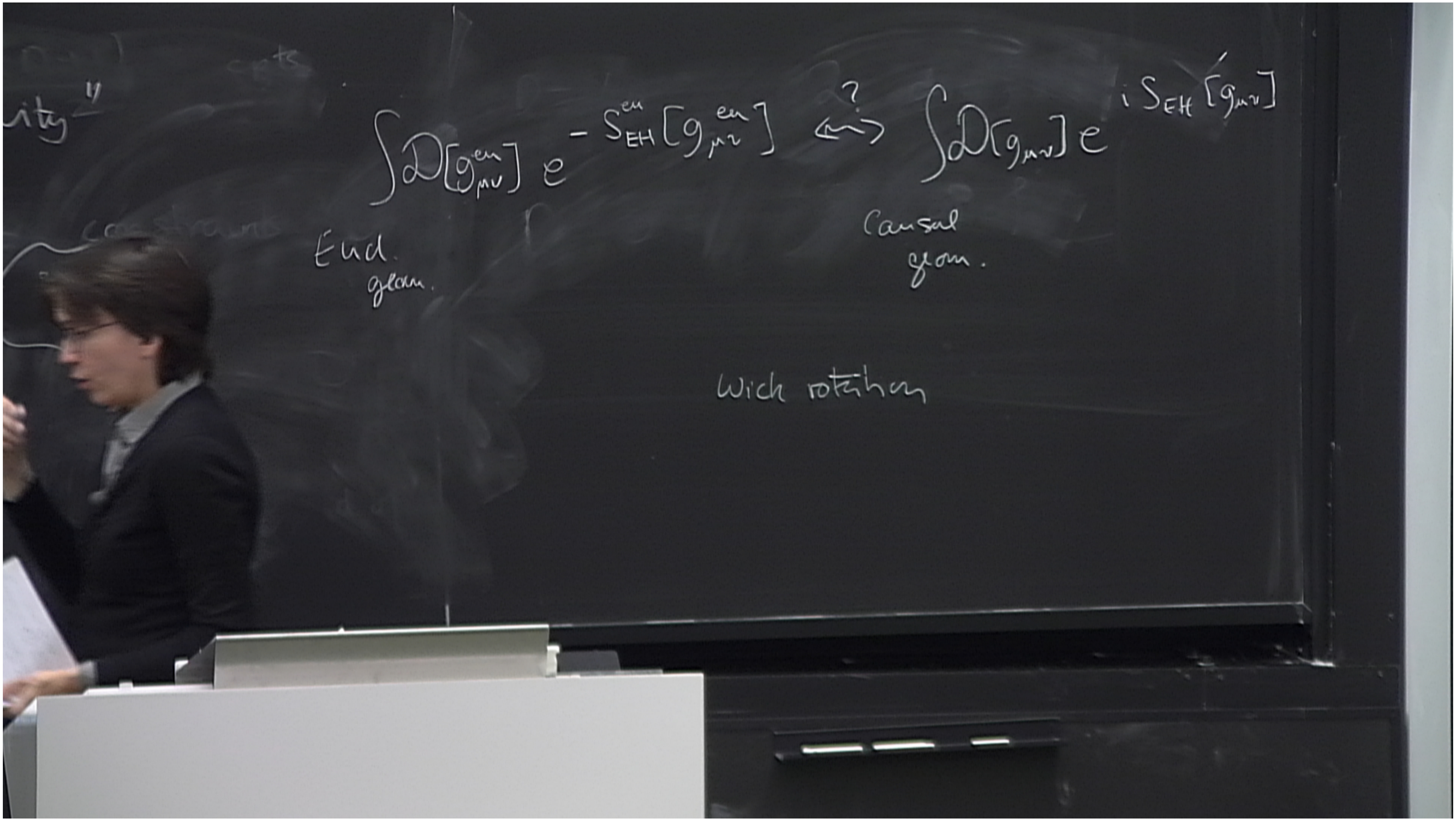

nontrivial example in d=2 and Euclidean : "Liouville quantum gravity"
- exactly soluble via DT / matrix model techniques
- related "almost exact" continuum evaluation of PI


"quantum gravity"
$\int \mathcal{D}[g^{eu}_{\mu\nu}]\, e^{-S^{eu}_{EH}[g^{eu}_{\mu\nu}]}$


ity"
∫D[g^{eu}_{μν}] e^{-S^{eu}_{EH}[g^{eu}_{μν}]} ⟷? ∫D[g_{μν}] e^{iS_{EH}[g_{μν}]}
constraints



$\int \mathcal{D}[g^{eu}_{\mu\nu}]\, e^{-S^{eu}_{EH}[g^{eu}_{\mu\nu}]} \overset{?}{\longleftrightarrow} \int \mathcal{D}[g_{\mu\nu}]\, e^{iS_{EH}[g_{\mu\nu}]}$
Eucl. geom.
Causal geom.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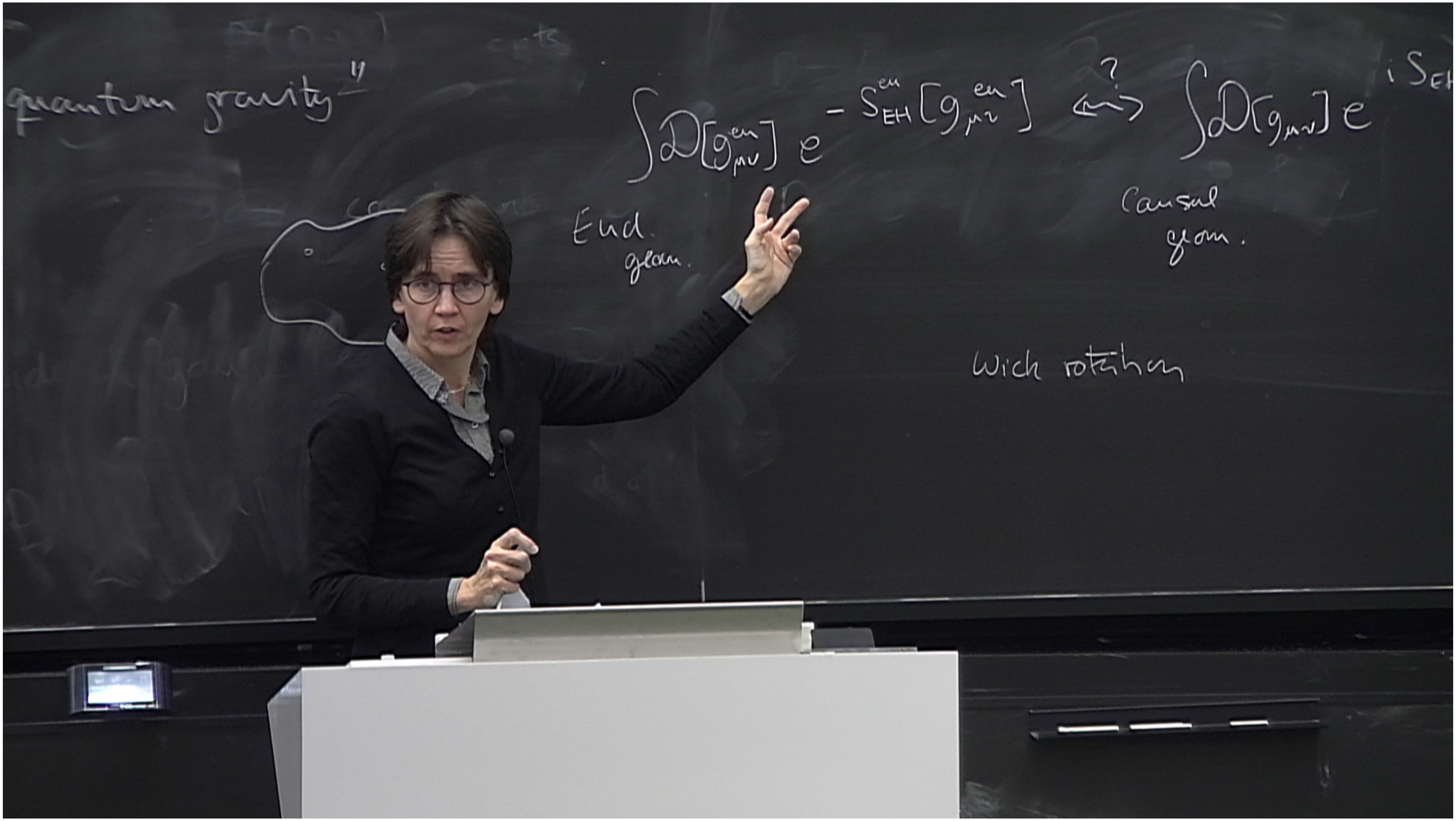
Eucl. geom.
Causal geom.
Wick rotation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