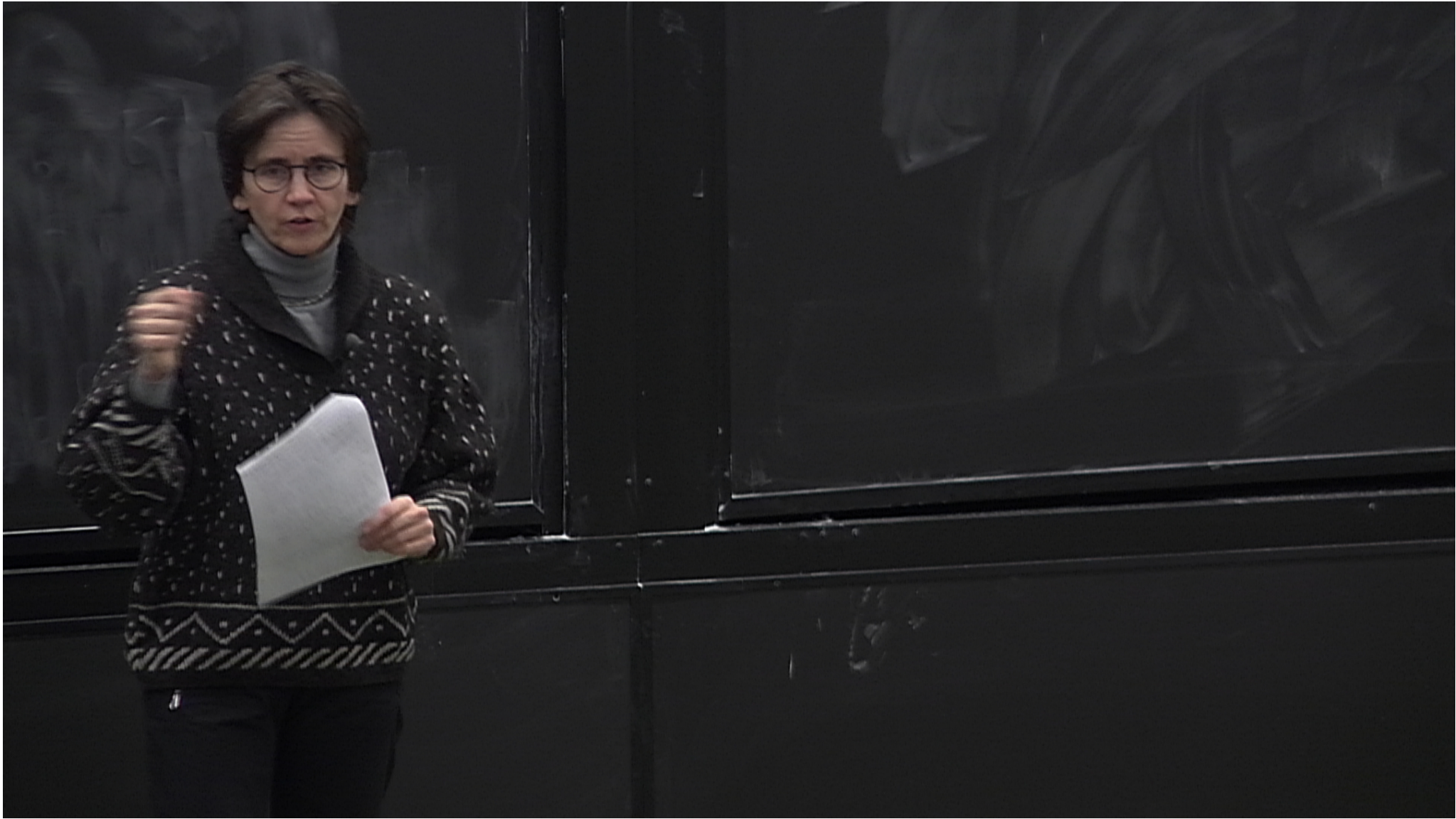


Title: Quantum Gravity (Review) - Lecture 11

Date: Feb 06, 2012 10:15 AM

URL: <http://pirsa.org/12020005>

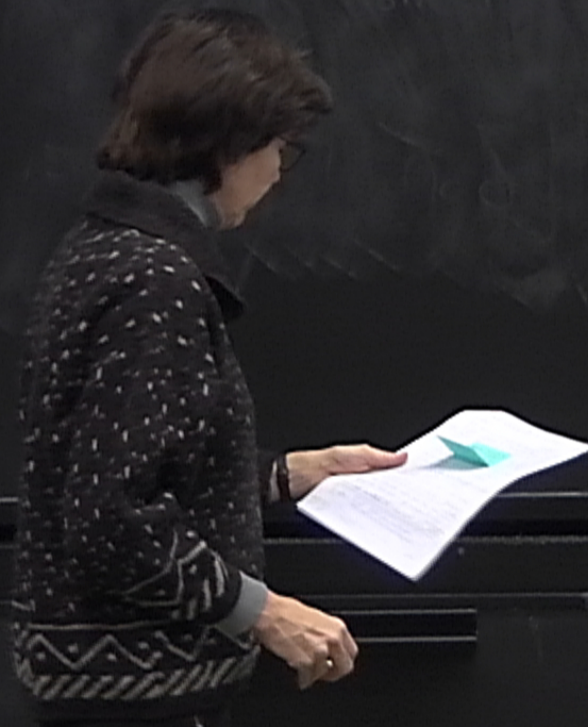
Abstract:





$$(A; (x), E_3)$$

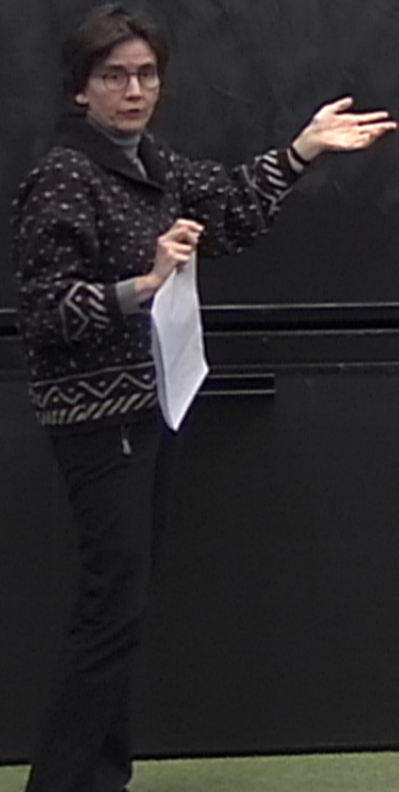
$$(A_i^a(x), E_b^j(x))$$





$$(A_i^a(x), E_b^j(x)) \quad ; \quad \mathcal{L}_\alpha = \mathcal{D}_i E_a^i + G_N \epsilon_{abc} A_i^b E^{ci} = \mathcal{D}_i E_a^i = 0$$

$$(A_i^a(x), E_a^i(x)) : \mathcal{L}_a = \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 \quad (\text{"Gauss's constraint"})$$





$$(A_i^a(x), E_a^i(x)) : \mathcal{G}_a = \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 \quad (\text{"Gauss's constraint"})$$

$$\mathcal{X}_i = F_{ij} E^j = 0 \quad (\text{momentum constraints})$$

$$(A_i^a(x), E_j^b(x)) : \mathcal{L}_a = \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 \quad (\text{"Gauss's constraint"})$$

$$\mathcal{X}_i = F_{ij}^a E_a^i = 0 \quad (\text{momentum constraints})$$

$$\frac{1}{2} \frac{\epsilon^{abc} F_{ic}^a}{|\det E_a^i|} E_a^i E_b^j - \frac{\beta^2 + 1}{\beta^2 |\det E_a^i|} E_a^i E_b^j (G_{ab} A_i^a - \Gamma_i^a) (G_{ab} A_j^b - \Gamma_j^b)$$

$\Gamma = \Gamma(E)$  - so(3)-connection



$$\begin{aligned}
 (A_i^a(x), E_j^b(x)) : \mathcal{G}_a &= \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^{ci} = \mathcal{D}_i E_a^i = 0 && \text{("Gauss's constraint")} \\
 \mathcal{X}_i &= F_{ij}^a E_a^i = 0 && \text{(momentum constraints)} \\
 \mathcal{X}_\perp &= \frac{F_{ic}^a E_a^i E_b^j}{|\det E_a^i|} - \frac{\beta^2 + 1}{\beta^2 |\det E_a^i|} E_a^i E_b^j (G_{ab} A_c^c - \Gamma_c^a) (G_{ab} A_c^c - \Gamma_c^b) && \\
 &&& \Gamma = \Gamma(E) \text{ - so(3)-connection}
 \end{aligned}$$

$$\begin{aligned}
 (A_i^a(x), E_j^b(x)) : \mathcal{L}_a &= \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 && \text{("Gauss's constraint")} \\
 \mathcal{X}_i &= F_{ij}^a E_a^i = 0 && \text{(momentum constraints)} \\
 \mathcal{X}_1 &= \frac{1}{2} \frac{\epsilon^{abc} F_{ij}^a E_b^i E_c^j}{|\det E|} - \frac{\beta^2 + 1}{\beta^2 |\det E|} E_a^i E_b^j (G_{ab} A_i^a - \Gamma_i^a) (G_{ab} A_j^b - \Gamma_j^b) \\
 &&& \Gamma = \Gamma(E) \text{ - so(3)-connection}
 \end{aligned}$$



$$\begin{aligned}
 (A_i^a(x), E_b^j(x)) : \mathcal{L}_a &= \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^{ci} = \mathcal{D}_i E_a^i = 0 && \text{("Gauss's constraint")} \\
 \mathcal{X}_i &= F_{ij}^a E_a^i = 0 && \text{(momentum constraints)} \\
 \mathcal{X}_1 &= \frac{1}{2} \frac{\epsilon^{abc} F_{ab}^c}{\sqrt{|\det E^a|}} E_b^j - \frac{\beta^2 + 1}{\beta^2 \sqrt{|\det E^a|}} E_a^i E_b^j (G_{ab} A_i^a - \Gamma_i^a) (G_{ab} A_j^b - \Gamma_j^b) \\
 &&& \Gamma = \Gamma(E) \text{ - } \text{so}(3)\text{-connection}
 \end{aligned}$$

$$\begin{aligned}
 (A_i^a(x), E_j^b(x)) : \mathcal{L}_a &= \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 && \text{("Gauss's constraint")} \\
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 & && \Gamma = \Gamma(E) \text{ - so(3)-connection} \\
 & && \beta = 1
 \end{aligned}$$



$$\begin{aligned}
 (A_i^a(x), E_j^b(x)) : \mathcal{G}_a &= \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^{ci} = \mathcal{D}_i E_a^i = 0 && \text{("Gauss's constraint")} \\
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 &&& \Gamma = \Gamma(E) \text{ - } so(3)\text{-connection}
 \end{aligned}$$

$$(A_i^a(x), E_j^b(x)) : \mathcal{G}_a = \mathcal{D}_i E_a^i + G_{ab} \epsilon^{abc} A_i^b E^c = \mathcal{D}_i E_a^i = 0 \quad (\text{"Gauss's constraint"})$$

$$\mathcal{X}_i = F_{ij}^a E_j^a = 0 \quad (\text{momentum constraints})$$

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$$\beta = 1$$

$$\Gamma = \Gamma(E) = \text{so}(3) \text{-connection}$$



$$D_i E_a^i = 0 \quad (\text{"Gauss constraint"})$$

(momentum constraints)

$$\frac{+1}{|\det E_a^i|} E_{[a}^i E_{b]}^j (G_N A_i^a - \Gamma_i^a) (G_N A_j^b - \Gamma_j^b)$$

$$\Gamma = \Gamma(E) \text{ - } so(3)\text{-connection}$$

algebra



$$D_i E_a^i = 0 \quad (\text{"Gauss' constraint"})$$

(momentum constraints)

$$\frac{+1}{|\det E_a^i|} E_{[a}^i E_{b]}^j (G_N A_i^a - \Gamma_i^a) (G_N A_j^b - \Gamma_j^b)$$

$$\Gamma = \Gamma(E) \text{ - } so(3)\text{-connection}$$

algebra closes under PBs



$(A_i^a(x), E_j^b(x))$

$$\begin{aligned}
 3 \times \infty \times \mathcal{G}_a &= \mathcal{D}_i E_a^i + G_{ij} \epsilon^{abc} A_i^a E^b E^c = \mathcal{D}_i E_a^i = 0 && \text{("Gauss' constraint")} \\
 3 & \mathcal{X}_i = F_{ij}^a E_a^i = 0 && \text{(momentum constraints)} \\
 1 & \mathcal{X}_1 = \frac{1}{2} \frac{\epsilon^{abc} F_{ij}^c}{|\det E_a^i|} E_a^i E_b^j - \frac{\beta^2 + 1}{\beta^2 |\det E_a^i|} E_a^i E_b^j (G_{ij} A_a^a - \Gamma_i^a) (G_{ij} A_b^b - \Gamma_j^b) && \\
 & && \Gamma = \Gamma(E) \text{ - } so(3)\text{-connection} \\
 & && \beta = i
 \end{aligned}$$

algebra closes under PBs

$\beta = i$

$\Gamma = \Gamma(E)$  - so(3)-connection

$$[\hat{A}_i^a(\vec{x}), \hat{E}_b^j(\vec{y})] = i\hbar 8\pi\beta \delta_b^a \delta_i^j \delta^{(3)}(\vec{x}, \vec{y})$$



$$[\hat{A}_i^a(\vec{x}), \hat{E}_b^j(\vec{y})] = i\hbar 8\pi\beta \delta_b^a \delta_i^j \delta^{(3)}(\vec{x}, \vec{y})$$

formally,  $\hat{A}_i^a(\vec{x}) \psi[A] = A_i^a(\vec{x}) \psi[A]$

$$\hat{E}_b^j(\vec{x}) \psi[A] = \frac{\hbar}{i} 8\pi\beta \delta_{A_b^j}(\vec{x}) \psi[A]$$



$$[\hat{A}_i^a(\vec{x}), \hat{E}_b^j(\vec{y})] = i\hbar 8\pi\beta \delta_b^a \delta_i^j \delta^{(3)}(\vec{x}, \vec{y})$$

$\hat{y}_a$

formally,  $\hat{A}_i^a(\vec{x}) \Psi[A] = A_i^a(\vec{x}) \Psi[A]$

$$\hat{E}_b^j(\vec{x}) \Psi[A] = \frac{\hbar}{i} 8\pi\beta \delta_{A_i^b}^j(\vec{x}) \Psi$$

$$\Psi[A] = \mathcal{H}^{aux}$$



$$[\hat{A}_a^\mu(\vec{x}), \hat{E}_b^\nu(\vec{y})] = i\hbar 8\pi\beta \delta_a^\nu \delta_b^\mu \delta^{(3)}(\vec{x}, \vec{y})$$

$$\hat{G}_a \Psi \propto D_a \delta / \delta A_a^\mu \Psi = 0$$

formally,  $\hat{A}_a^\mu(\vec{x}) \Psi[A] = A_a^\mu(\vec{x}) \Psi[A]$

$$\hat{E}_b^\nu(\vec{x}) \Psi[A] = \frac{\hbar}{i} 8\pi\beta \delta / \delta A_b^\nu(\vec{x}) \Psi[A]$$

$$\Psi[A] = \int \mathcal{H}^{aux}$$



$$[\hat{A}_i^a(\vec{x}), \hat{E}_j^b(\vec{y})] = i\hbar 8\pi\beta \delta_{ij}^a \delta^{(3)}(\vec{x}, \vec{y})$$

formally,  $\hat{A}_i^a(\vec{x}) \Psi[A] = A_i^a(\vec{x}) \Psi[A]$

$$\hat{E}_j^b(\vec{x}) \Psi[A] = \frac{\hbar}{i} 8\pi\beta \delta_{SA_j^b}(\vec{x}) \Psi[A]$$

$$\Psi[A] = \int \mathcal{D}A^a \dots$$

$\hat{G}_a \Psi \propto \mathcal{D} \delta / \delta A_i^a \Psi = 0$  is solved by functional



$\hat{G}_a \psi \propto D_i \delta / SA_i^a \psi = 0$  is solved by functionals which are invariant

CM

CM

d

M

[A]



$\hat{G}_a \psi \propto D_i \delta / SA_i^a \psi = 0$  is solved by functionals which are invariant  
under local gauge transformations  $g(x)$

[A]



$\hat{G}_a \psi \propto D_i \delta / \delta A_i^a \psi = 0$  is solved by functionals which are invariant  
under local gauge transformations  $g(x) \in G$  :  $A_i \rightarrow A_i^g = g^{-1} A_i g + g^{-1} \partial_i g$

[A]



$\hat{G}_a \psi \propto D_i \delta / \delta A_i^a \psi = 0$  is solved by functionals which are invariant  
 under local gauge transformations  $g(x) \in \begin{cases} SU(2) \\ SU(3) \end{cases} : A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

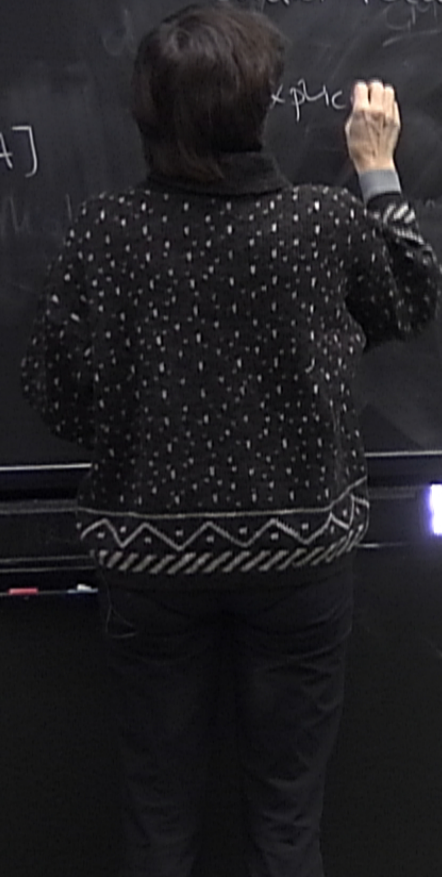
[A]



$\hat{g}_a \psi \propto D_i \delta / \delta A_i^a \psi = 0$  is solved by functionals which are invariant

under local gauge transformations  $g(x) \in \begin{cases} G^{SO(3)} \\ G^{SU(2)} \end{cases} : A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

[A]





$\hat{D}_a \psi \propto D_a \psi / SA_i^a \psi = 0$  is solved by functionals which are invariant

under local gauge transformations  $g(x) \in \{g^{SU(2)}, g^{SU(3)}\}$  :  $A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

→ explicitly gauge invariant functions

[A]



$\hat{D}_\mu \psi \propto D_\mu \psi / SA_i^a \psi = 0$  is solved by functionals which are invariant  
 under local gauge transformations  $g(x) \in \left( \begin{matrix} e^{i\theta(x)} \\ e^{i\theta(x)} \end{matrix} \right)$  :  $A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$   
 $\rightarrow$  explicitly gauge invariant functionals  $\left( \begin{matrix} \int \dots \\ \int \dots \end{matrix} \right)$  are the "Wilson loops"

$W[A]$



$\hat{D}_\mu \psi \propto D_\mu \psi / SA_i^a \psi = 0$  is solved by functionals which are invariant

under local gauge transformations  $g(x) \in \underbrace{(SU(3))}_{(SU(2))} : A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

→ explicitly gauge invariant functionals are the "Wilson loops"

[A]  $W_\gamma[A] = \text{Tr} \text{P exp} \int_\gamma A_\mu dx^\mu$

↑  
path ordered



$\hat{G}_a \Psi \propto \mathcal{D}_i \delta / \delta A_i^a \Psi = 0$  is solved by functionals which are invariant

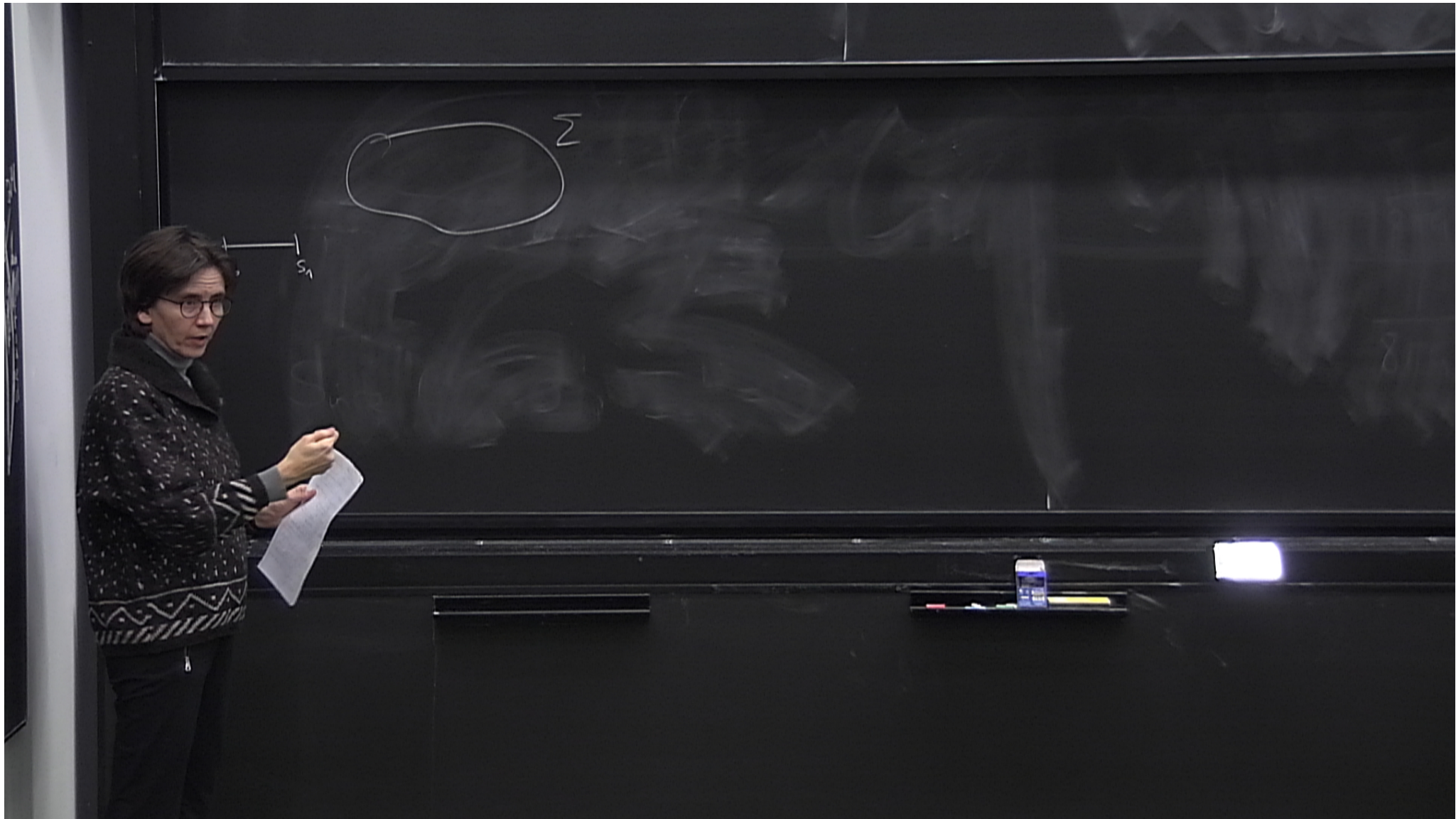
under local gauge transformations  $g(x) \in \left( \begin{matrix} e^{i\alpha(x)} \\ e^{i\alpha(x)} \end{matrix} \right)$   $A_i \mapsto A_i^g = g^{-1} A_i g - g^{-1} \partial_i g$

$\Rightarrow$  gauge invariant functionals are the "Wilson loops"

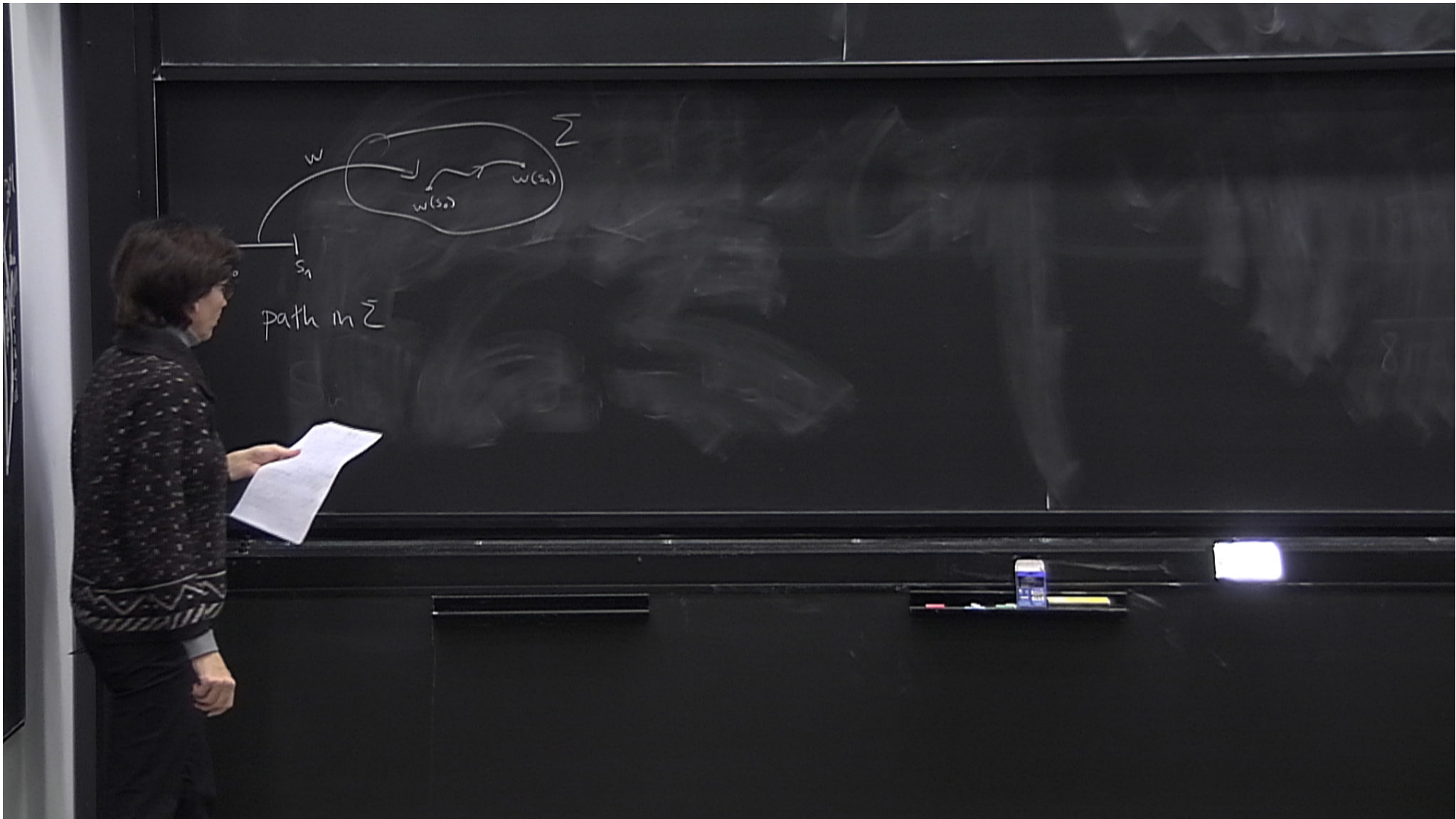
$$= \text{Tr} \mathcal{P} \exp \oint_{\gamma} A = \text{Tr} U_{\gamma}[A; s_0, s_0)$$

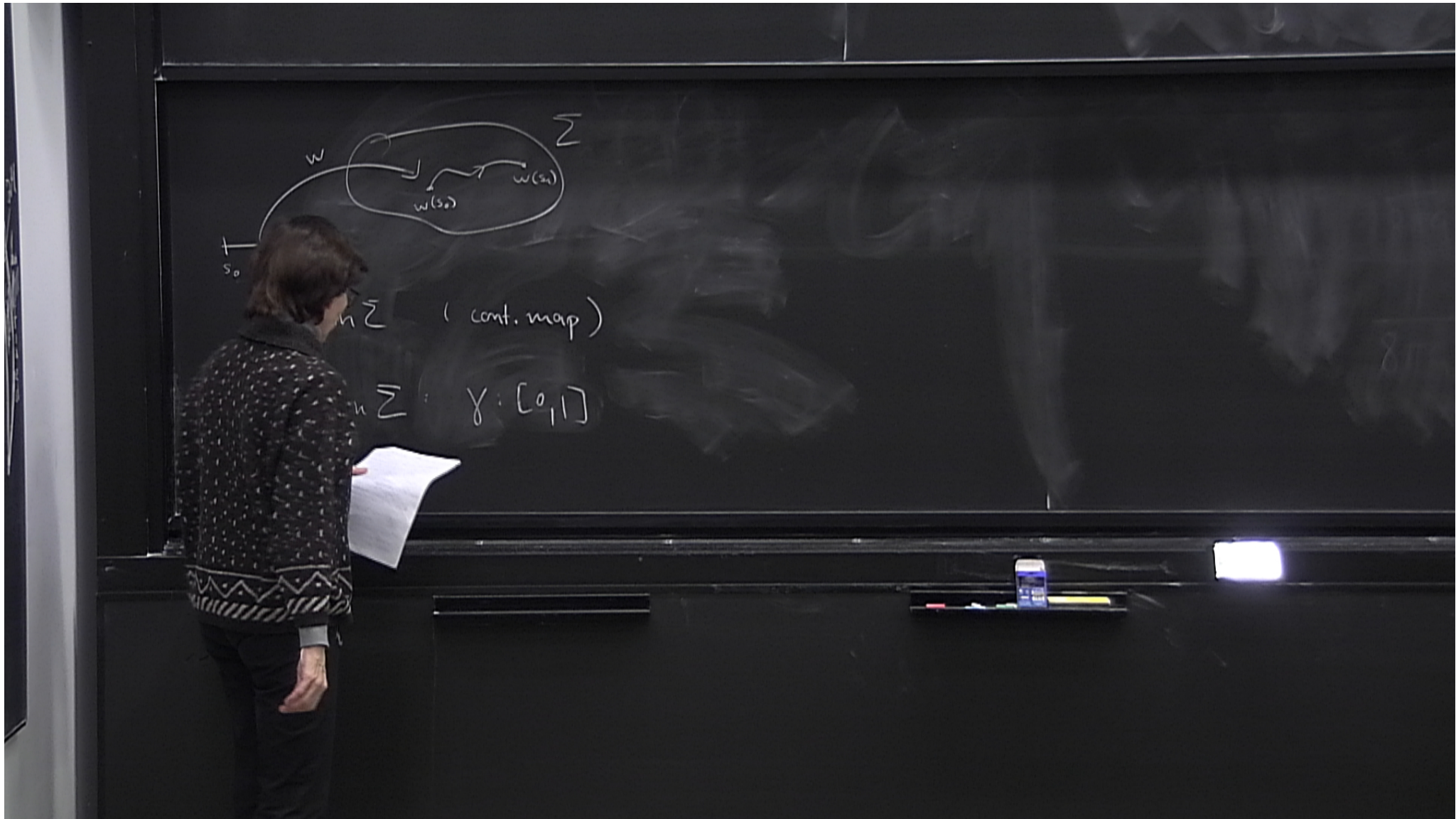
↑  
path ordering



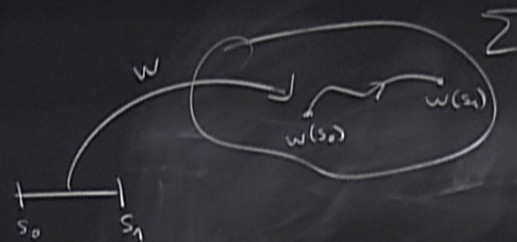












$$w: [s_1, s_2] \rightarrow \Sigma : s \mapsto w'(s)$$

path in  $\Sigma$  (cont. map)

loop in  $\Sigma$  :  $\gamma: [0, 1] \rightarrow \Sigma, s \mapsto \gamma'(s)$  with  $\gamma(0) = \gamma(1)$

A

sample ... A set

$$A \subset \mathcal{A} \quad , \quad A(x) = A_i^a(x) d$$



$$A \subset \mathcal{A} \quad , \quad A(x) = A_i^a(x) dx^i$$

1)  $A \subset \mathcal{A}$  ,  $A(x) = A_i(x) dx^i$



$$A \subset \mathcal{A} \quad , \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i \quad ,$$

$A \in \mathcal{A}$  ,  $A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i$  ,  $X_a \sim$  algebra generators ,  
in fundament



$A \in \mathcal{A}$  ,  $A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i$  ,  $X_a \sim$  algebra generators,  
in fundamental repr. of  
Lie group  $G$

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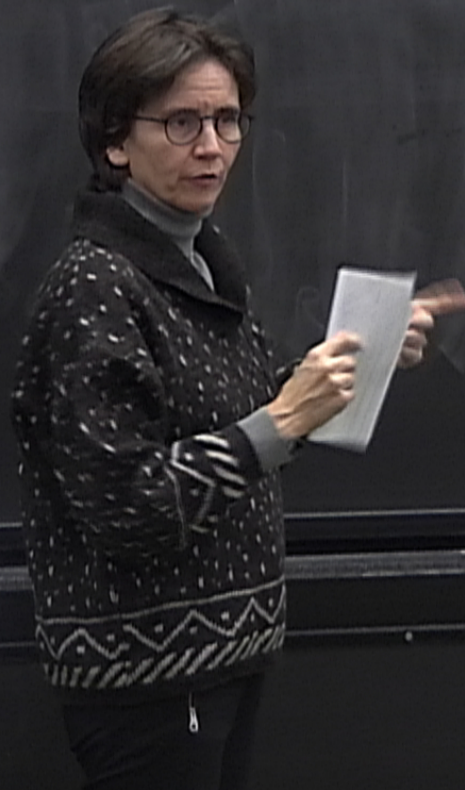


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$A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i$ ,  $X_a \sim$  algebra generators,  
in fundamental repr. of  
Lie group  $G$



saddle point  $\rightarrow$   $\lambda < 0$

$$A \in \mathfrak{A} \quad , \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i \quad , \quad X_a \sim \text{algebra generators,}$$

in fundamental repr. of  
Lie group  $G$

holonomy of a path  $w^M(s)$  with initial  
 $s_0$  and end point  $s_1$  is the solution of



$$A \in \mathcal{A}, \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i, \quad X_a \sim \text{algebra generators,}$$

in fundamental repr. of  
group  $G$

The holonomy of a path  $w^\mu(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

$$\frac{dU_w(s, s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1$$



$$A \in \mathcal{A}, \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i, \quad X_a \sim \text{algebra generators,}$$

in fundamental repr. of  
Lie group  $G$

The holonomy along a path  $w^M(s)$  with initial point  $s_0$  and final point  $s_1$  is the solution of

$$\frac{dU_w(s)}{ds} = - \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1, \quad \text{with } x = w(s), \text{ subject to the}$$

initial



$$A \in \mathcal{A}, \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i, \quad X_a \sim \text{algebra generators,} \\ \text{in fundamental repr. of Lie group } G$$

The holonomy of a path  $w^\mu(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

$$\frac{dU_w(s, s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1, \quad \text{with } x = w(s), \text{ subject to the}$$

1) initial condition  $U_w(s, s_0) = e \sim \text{unit el. in group } G$



$$A \in \mathcal{A} \quad , \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i \quad , \quad X_a \sim \text{algebra generators,} \\ \text{in fundamental repr. of} \\ \text{Lie group } G$$

The holonomy of a path  $w^M(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

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The holonomy of a path  $w^M(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

$$\frac{dU_w(s, s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1, \quad \text{with } x = w(s), \text{ subject to the}$$

1) initial condition  $U_w(s_1, s_0) = e \sim \text{unit el. in group } G$



$$A \in \mathcal{A}, \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i, \quad X_a \sim \text{algebra generators,} \\ \text{in fundamental repr. of} \\ \text{Lie group } G$$

The holonomy of a path  $w^\mu(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

$$\frac{dU_w(s, s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1, \quad \text{with } x = w(s), \text{ subject to the}$$

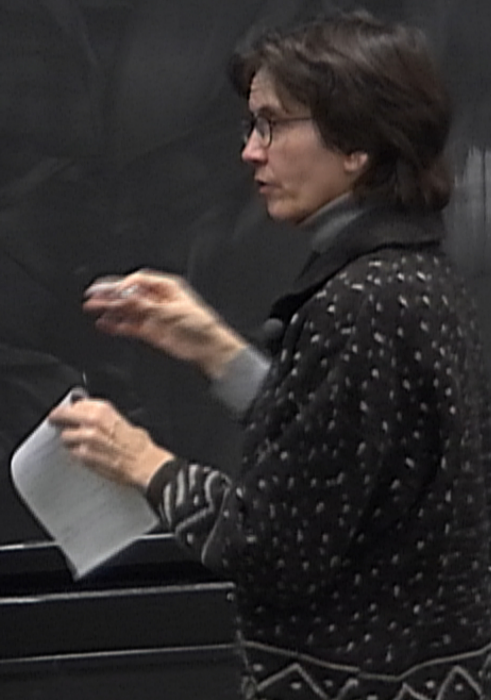
1) initial condition  $U_w(s_1, s_0) = e \sim \text{unit el. in group } G$



→ IDEA: choose



→ IDEA: choose Wilson loops  $W_\gamma[A] = \text{Tr} U_\gamma[A]$   
as "coordinates" on  $A$





→ IDEA: choose Wilson loops  $W_\gamma[A] = \text{Tr} U_\gamma[A]$   
as "coordinates" on  $A^{SU(2)} / \mathbb{C}P^1$





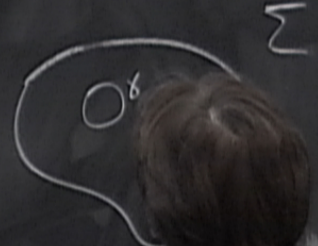
→ IDEA: choose Wilson loops  $W_\gamma[A] = \text{Tr} U_\gamma[A]$   
as "coordinates" on  $A^{SU(2)} / \mathfrak{g}^{SU(2)}$

(Rovelli, Smolin 1990)



→ IDEA: choose Wilson loops  $W_\gamma[A] = \text{Tr} U_\gamma[A]$   
as "coordinates" on  $A^{SU(2)} / \mathbb{Z}_2^{SU(2)}$

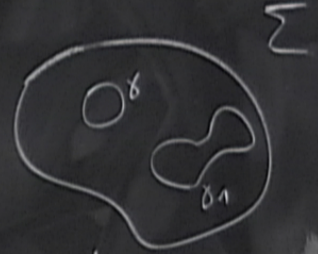
(Rovelli, Smolin 1990)





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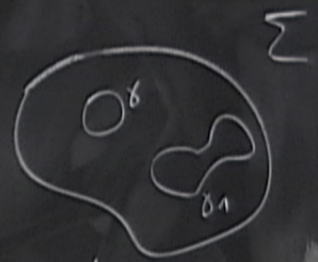
(Rovelli, Smolin 1990)





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(Witten, Smolin 1990)





saddle point  $\rightarrow$   $s_0$

$$\underline{A} \in \mathcal{A}, \quad A(x) = A_i(x) dx^i = A_i^a(x) X_a dx^i, \quad X_a \sim \text{algebra generators,}$$

in fundamental repr. of  
Lie group  $G$

The holonomy of a path  $w^\mu(s)$  with initial point  $s_0$  and end point  $s_1$  is the solution of

$$\frac{dU_w(s, s_0)}{ds} = A_i(x) \frac{dw^i}{ds} U_w(s, s_0), \quad s_0 \leq s \leq s_1, \quad \text{with } x = w(s), \text{ subject to the}$$

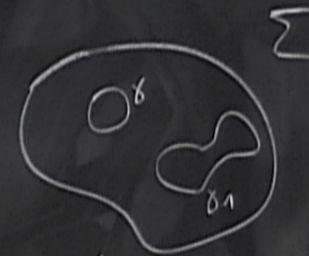
1) initial condition  $U_w(s_1, s_0) = e \sim \text{unit el. in group } G$



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(Rovelli, Smolin 1990)

$$\Psi[A] \sim \Psi[A^g]$$

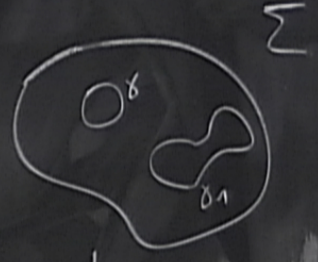




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$A(k)$  (Rovelli, Smolin 1990)

$$\Psi[A] \sim \Psi[A^g]$$

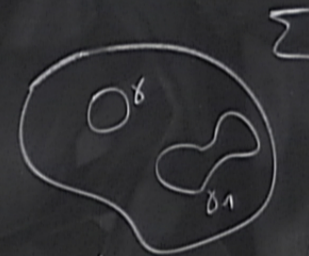




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AK (Rovelli, Smolin 1990)

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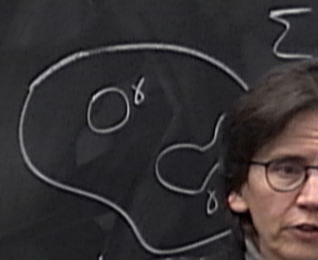




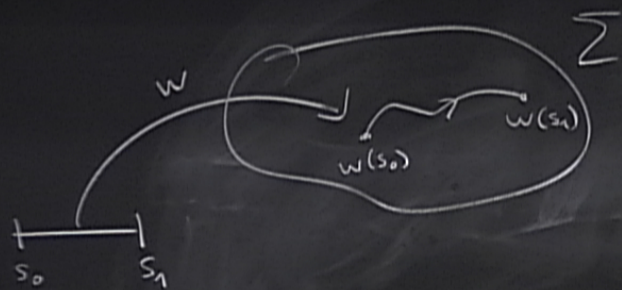
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AK (Rovelli, Smolin 1990)

$$\Psi[A] \sim \Psi[A^g]$$







$$w: [s_1, s_2] \rightarrow \Sigma : s \mapsto w'(s)$$

path in  $\Sigma$  (cont. map)

loop in  $\Sigma$  :  $\gamma: [0, 1] \rightarrow \Sigma$ ,  $s \mapsto \gamma'(s)$  with  $\gamma(0) = \gamma(1)$

A c A

The holonomy of point  $s_0$  and en

$$\frac{dU_w(s, s_0)}{ds} = A$$

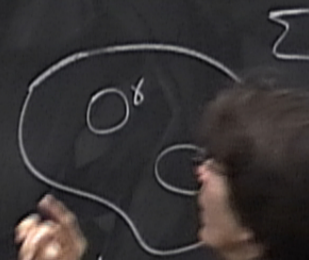
initial condition U



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AK (Rowell, Smolin 1990)

$$\Psi[A] \sim \Psi[A^g]$$

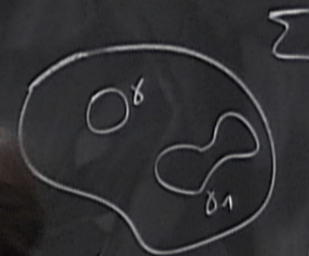




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Alk (Rovelli, Smolin 1990)

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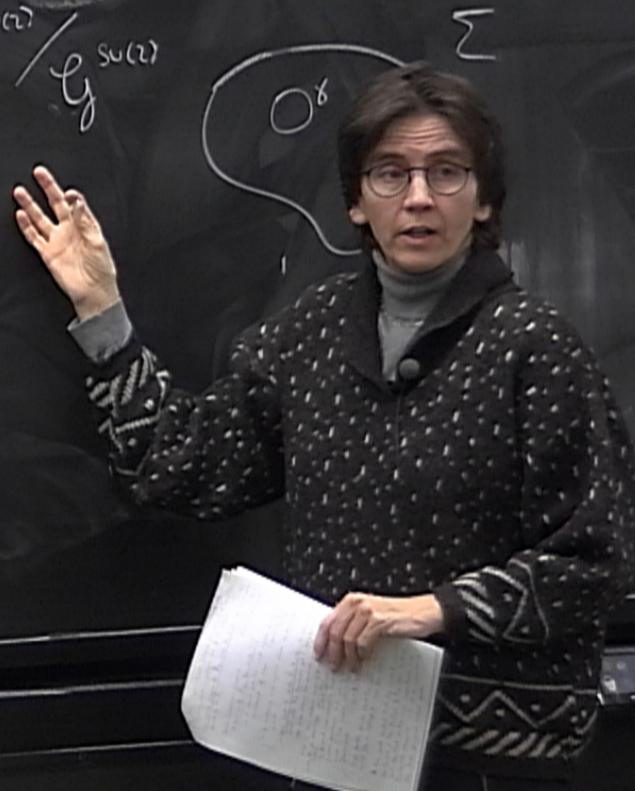




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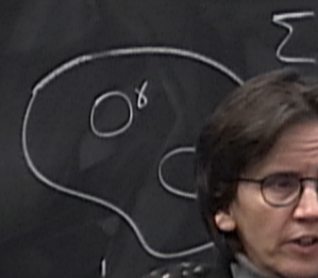




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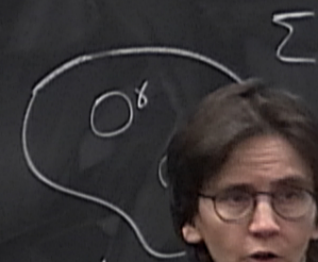




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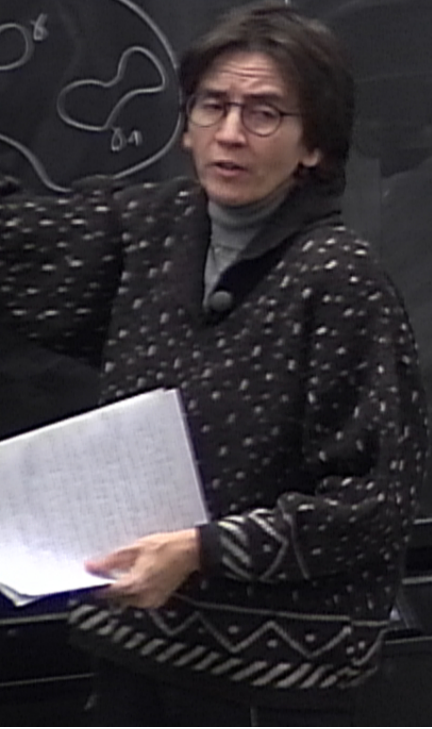




A]



CHOICE: promotes  $\{W_\gamma[A]\}$  (+ suitable momentum vars.)  
to well-defined, finite operators in the quantum theory.

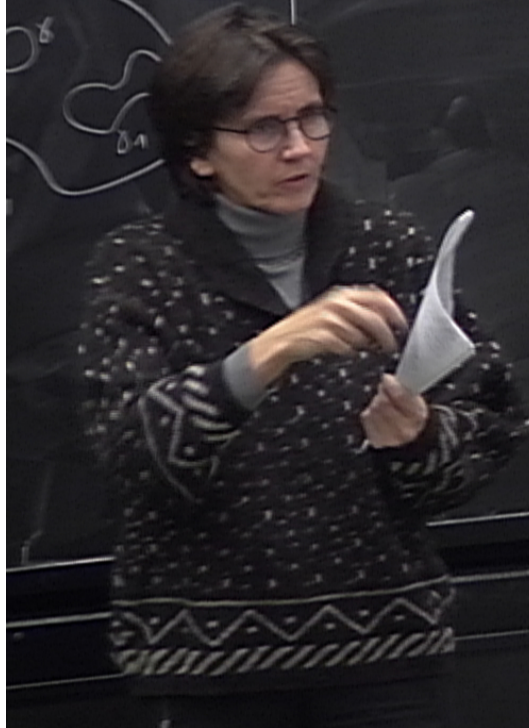




A]

CHOICE: promotes  $\{W_\alpha[A]\}$  (+ suitable momentum vars.)  
to well-defined, finite operators in the quantum theory.

$\Sigma$





A]

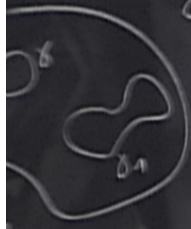


CHOICE: promotes  $\{W_g[A]\}$  (+ suitable momentum vars.)  
to well-defined, finite operators in the quantum theory.

→ a typical state has metric excited along



A]

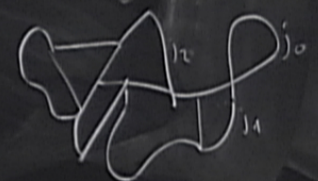


CHOICE: promotes  $\{W_g[A]\}$  (+ suitable momentum vars.)  
 to well-defined, finite operators in the quantum theory.

→ a typical state has metric excited  
 graphs in  $\Sigma$  e-dimensional curves /



A] CHOICE: promotes  $\{W_g[A]\}$  (+ suitable momentum vars.)  
 to well-defined, finite operators in the quantum theory.  
 state has metric excited along one-dimensional curves /  
 lines in  $\Sigma \rightsquigarrow$  "spinnetwork states"





well-defined Hilbert space  $\mathcal{H}^{\text{aux}} = L^2(\overline{A|e}, d\mu_{\text{ac}})$

↑ Ash



well-defined Hilbert space  $\mathcal{H}^{\text{aux}} = L^2(\overline{A/g}, d\mu_{\text{AL}})$   
↑ Ashtekar - Lewandowski