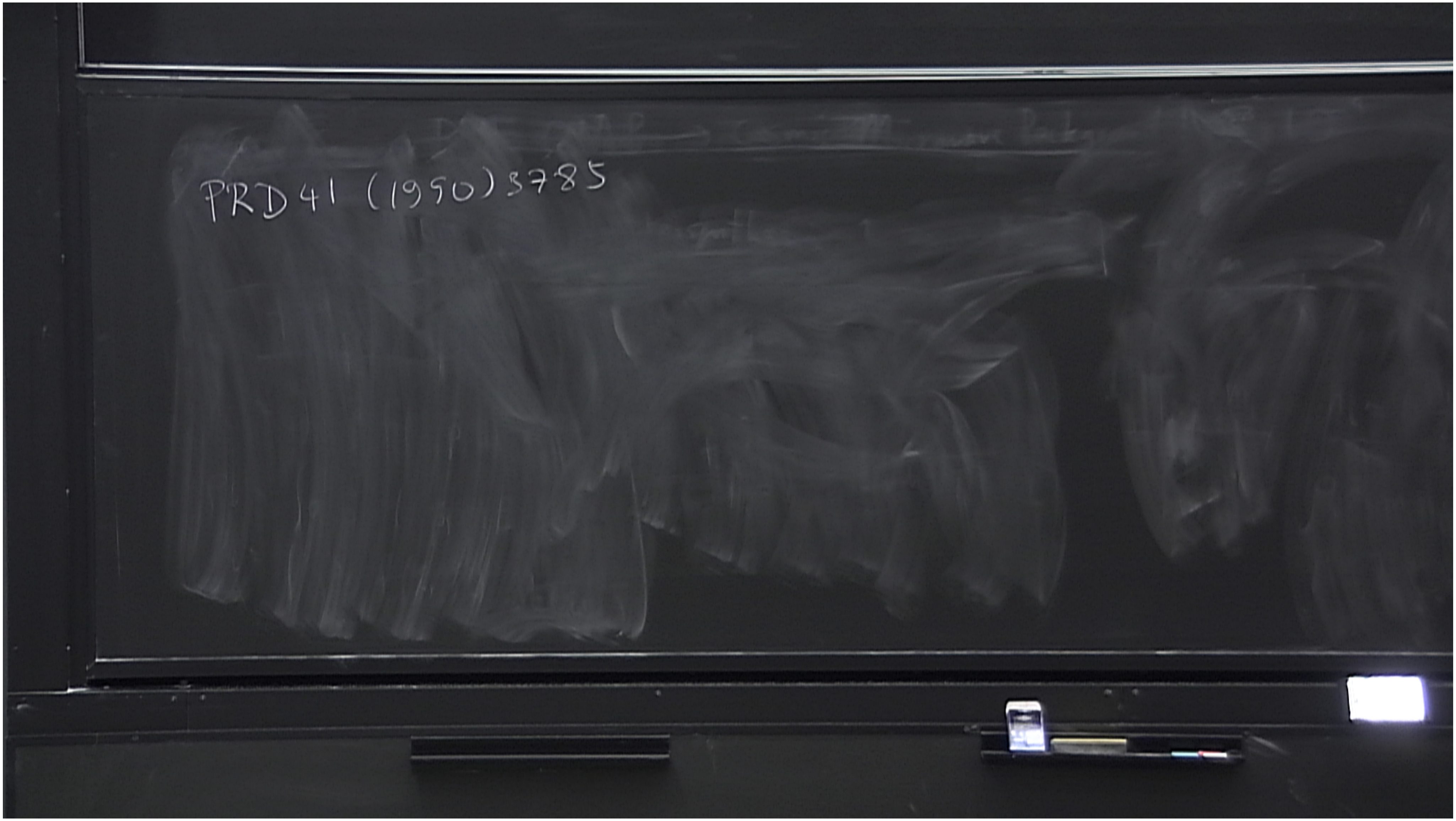
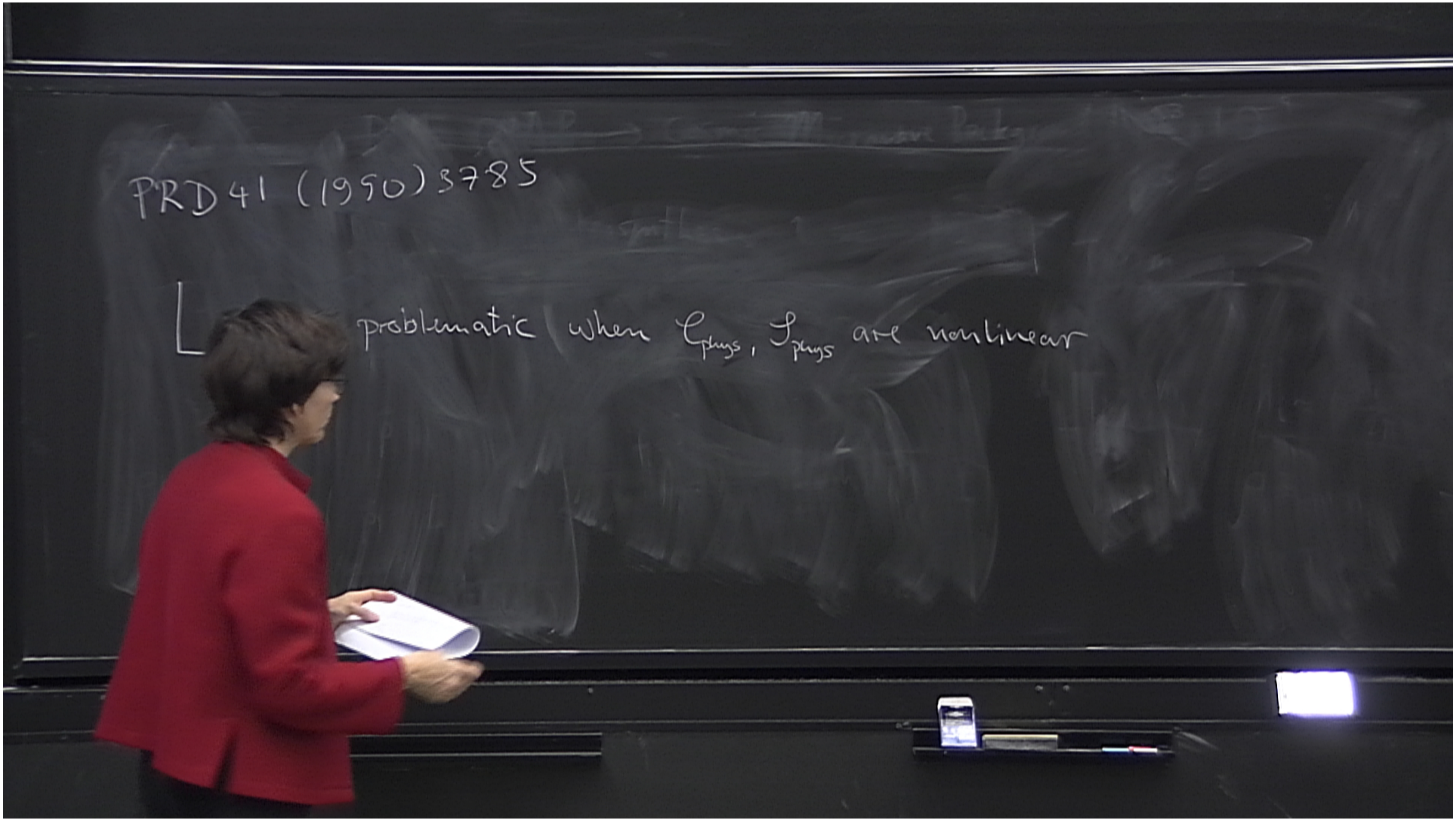
Title: Quantum Gravity (Review) - Lecture 10

Date: Feb 03, 2012 10:15 AM

URL: http://pirsa.org/12020002

Abstract:


PRD 41 (1990) 3785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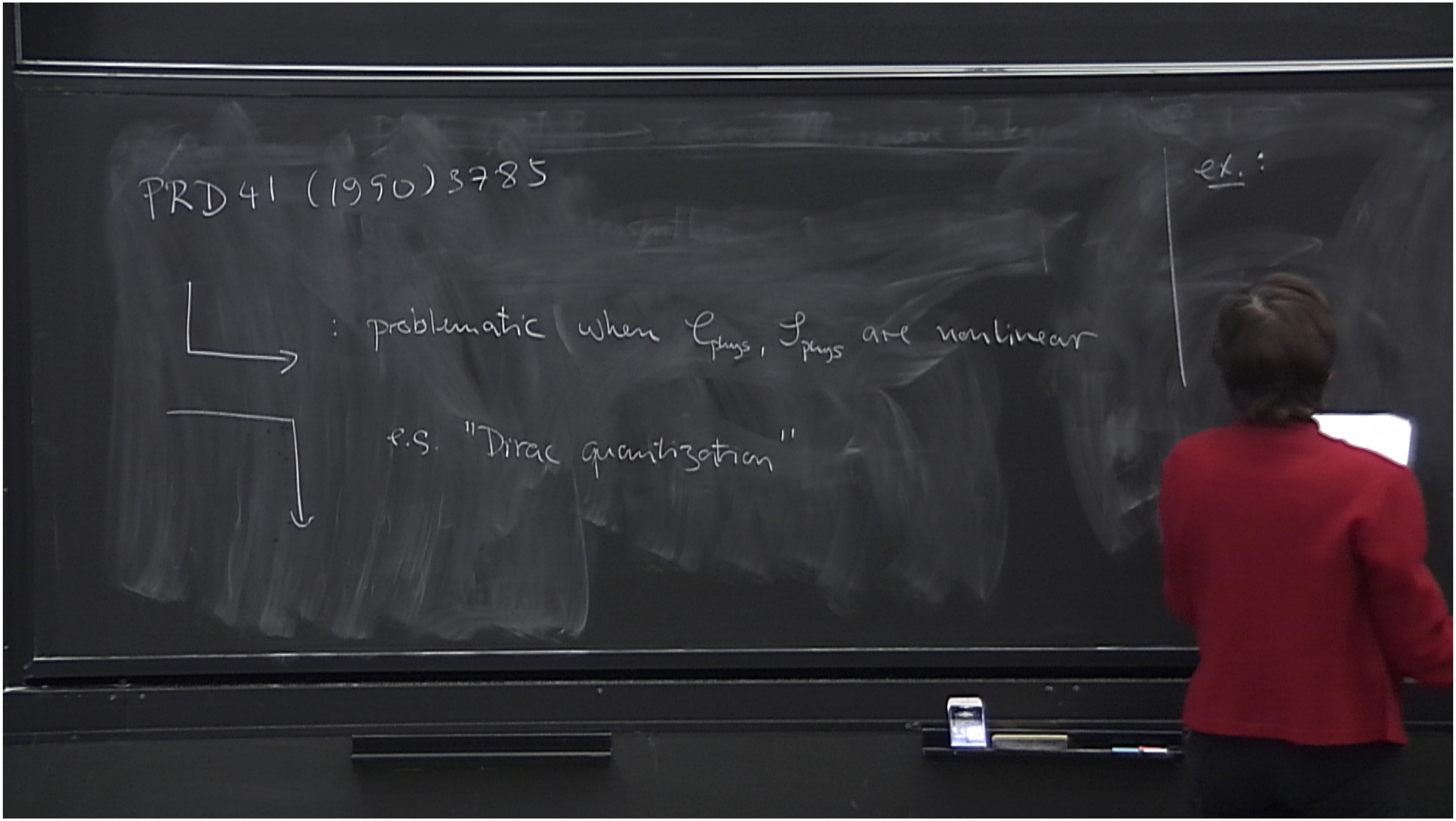
problematic when C_{phys}, S_{phys} are nonlinear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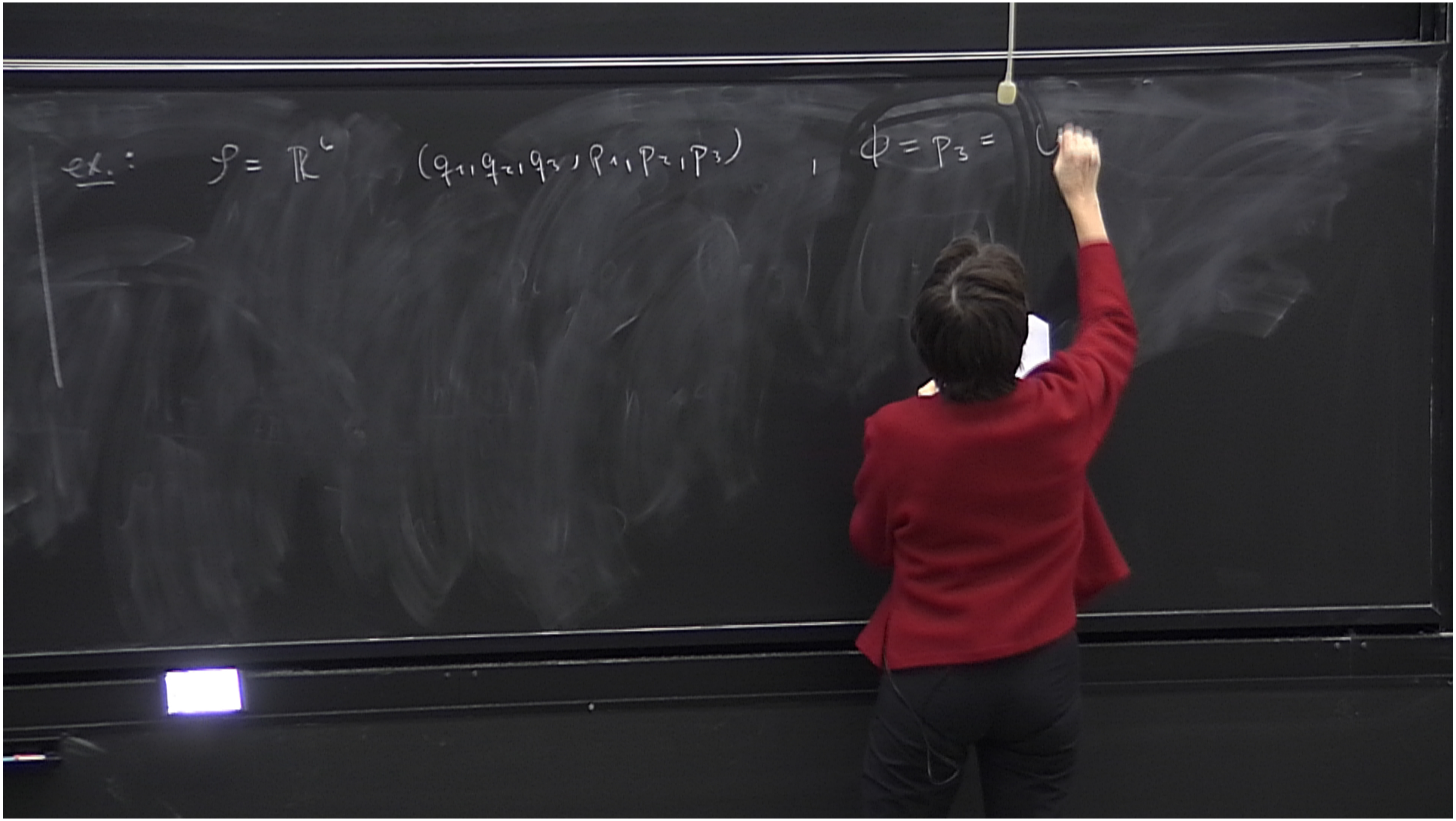
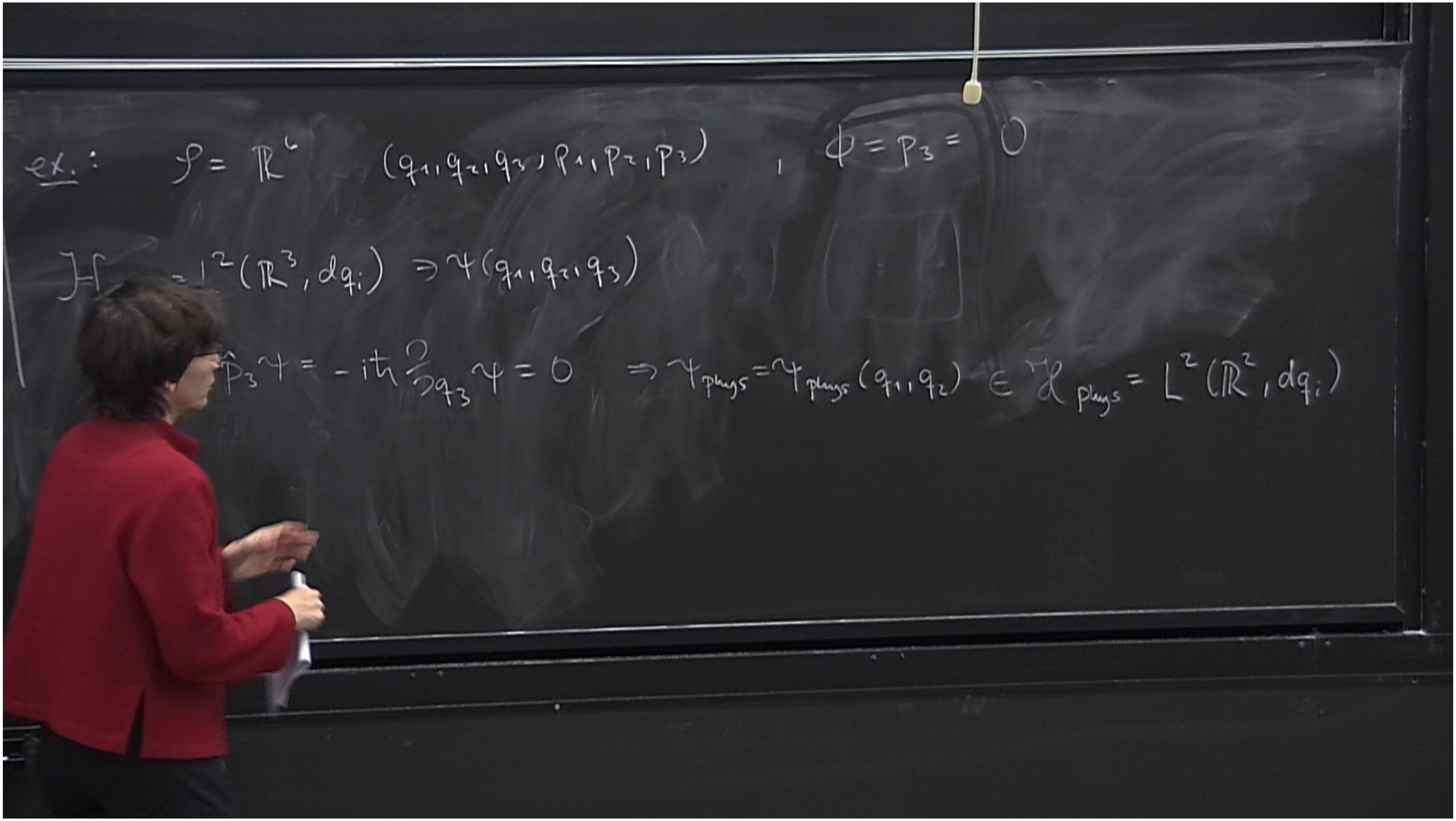
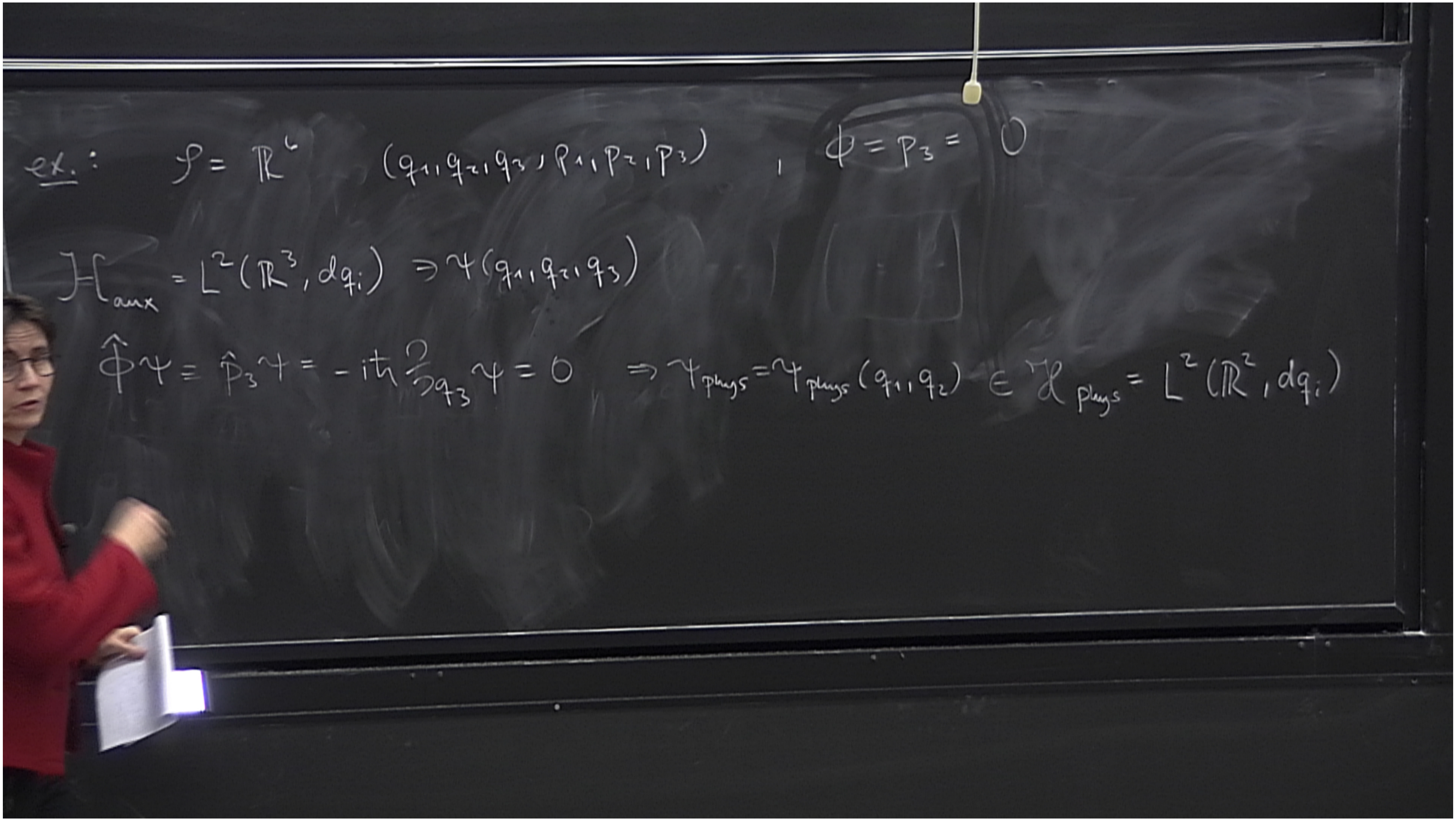
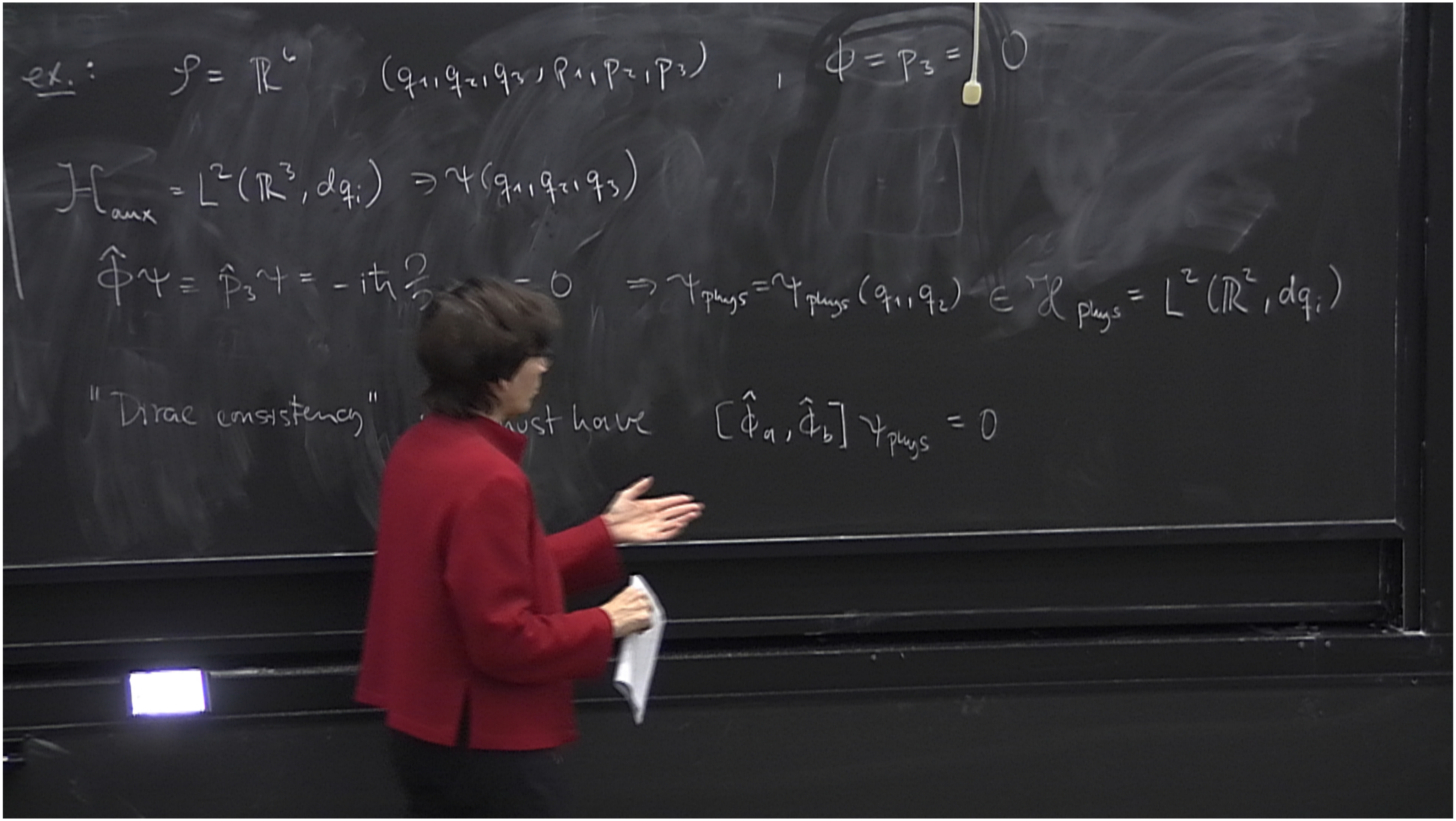
ex.:
: problematic when $\mathcal{C}_{phys}$, $\mathcal{S}_{phys}$ are nonlinear
e.g. "Dirac quantization"


ex.: $\mathcal{S} = \mathbb{R}^6$ $(q_1, q_2, q_3, p_1, p_2, p_3)$, $\phi = p_3 =$


ex.: $\mathcal{P} = \mathbb{R}^6 \quad (q_1, q_2, q_3, p_1, p_2, p_3) \quad , \quad \phi = p_3 = 0$
$\mathcal{H} = L^2(\mathbb{R}^3, dq_i) \ni \psi(q_1, q_2, q_3)$
$\hat{p}_3 \psi = -i\hbar \frac{\partial}{\partial q_3} \psi = 0 \Rightarrow \psi_{phys} = \psi_{phys}(q_1, q_2) \in \mathcal{H}_{phys} = L^2(\mathbb{R}^2, dq_i)$


ex.: $\mathcal{S} = \mathbb{R}^6$ $(q_1, q_2, q_3, p_1, p_2, p_3)$, $\phi = p_3 = 0$
$\mathcal{H}_{aux} = L^2(\mathbb{R}^3, dq_i) \ni \psi(q_1, q_2, q_3)$
$\hat{\phi}\psi = \hat{p}_3\psi = -i\hbar \frac{\partial}{\partial q_3}\psi = 0 \Rightarrow \psi_{phys} = \psi_{phys}(q_1, q_2) \in \mathcal{H}_{phys} = L^2(\mathbb{R}^2, dq_i)$



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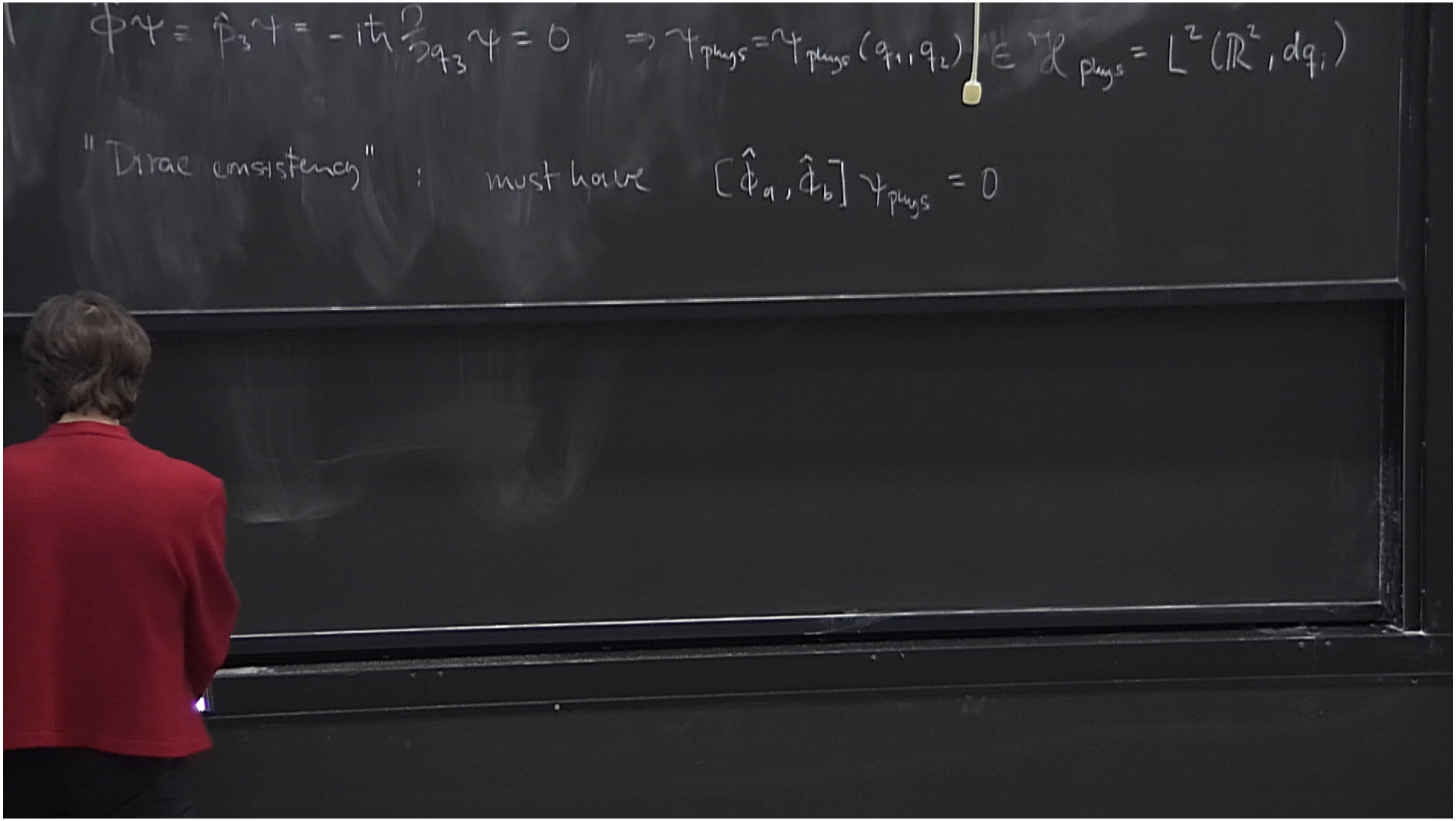
$\mathcal{H}_{aux} = L^2(\mathbb{R}^3, dq_i) \ni \psi(q_1, q_2, q_3)$
$\hat{\phi}\psi = \hat{p}_3\psi = -i\hbar\frac{\partial}{\partial}$
$= 0 \Rightarrow \psi_{phys} = \psi_{phys}(q_1, q_2) \in \mathcal{H}_{phys} = L^2(\mathbb{R}^2, dq_i)$
"Dirac consistency"
must have $[\hat{\Phi}_a, \hat{\Phi}_b]\psi_{phys} = 0$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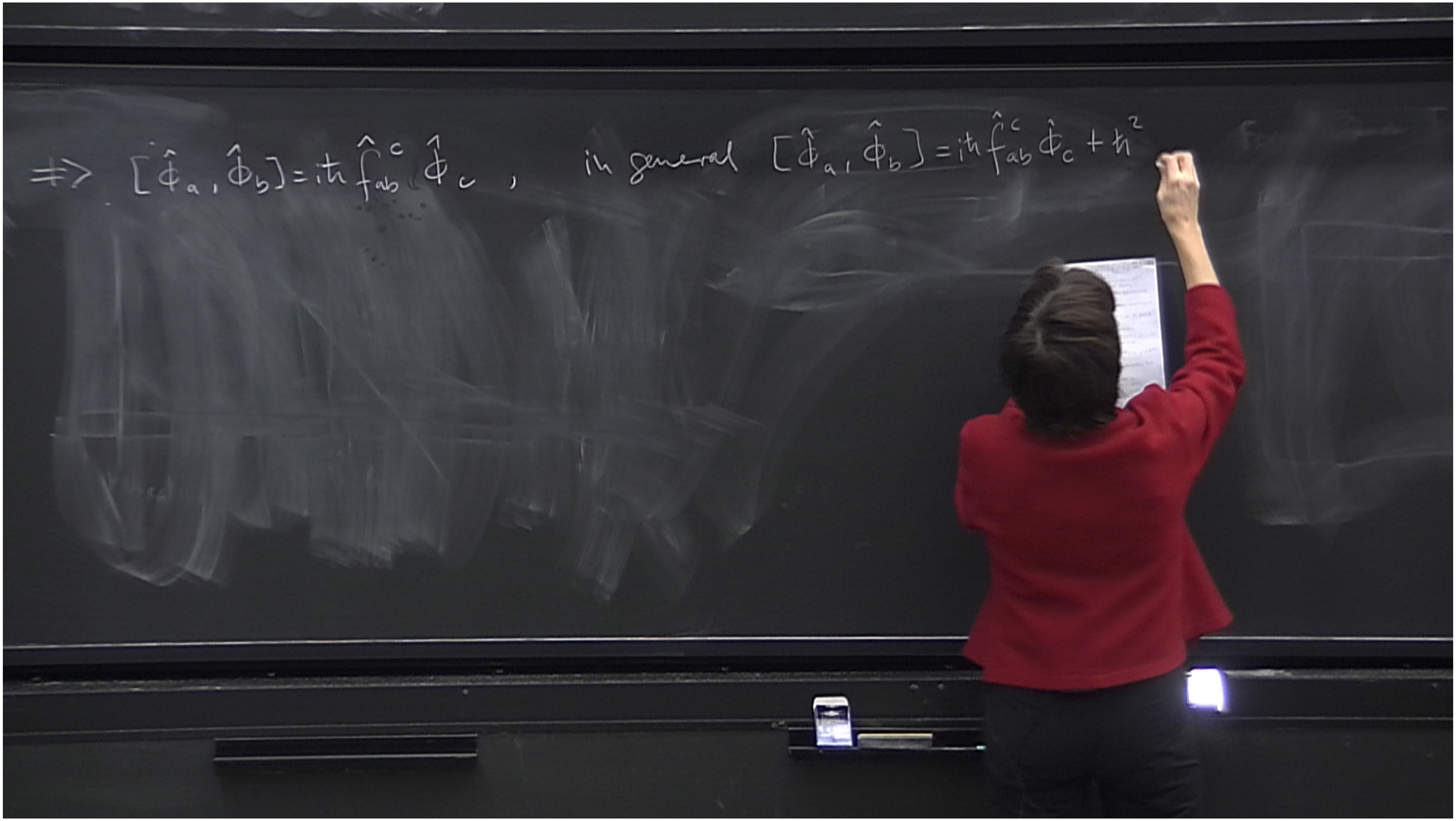
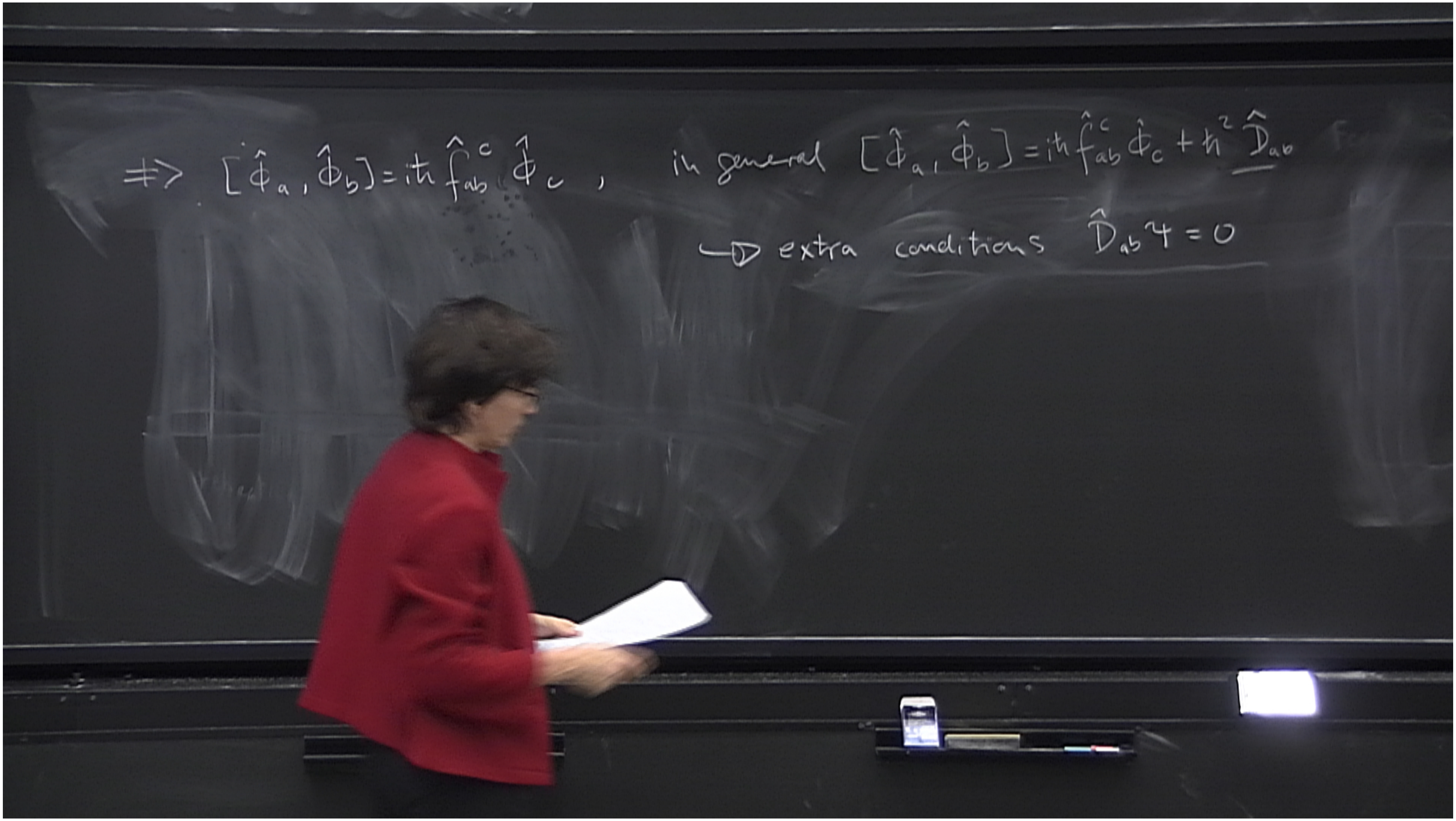
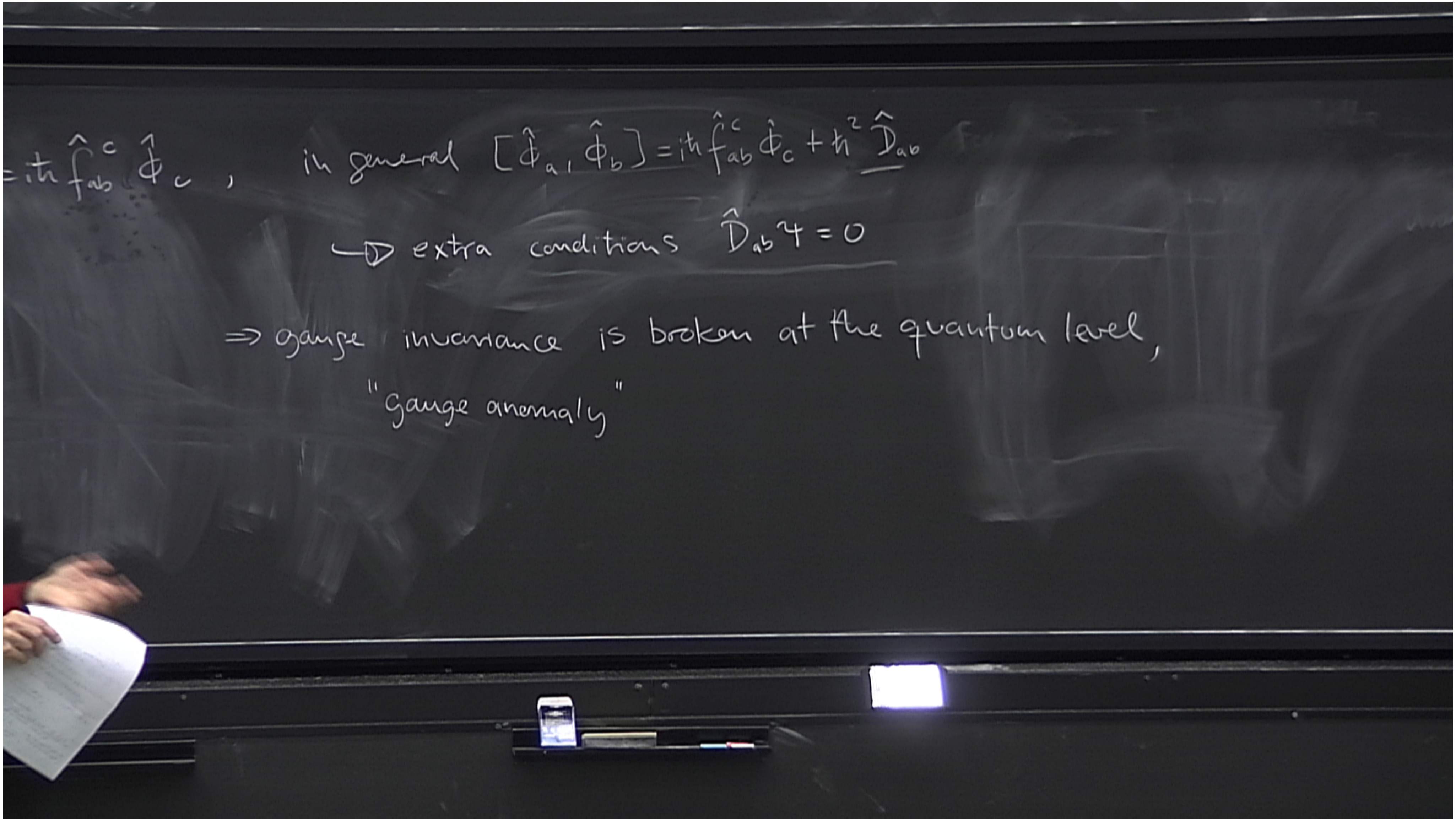
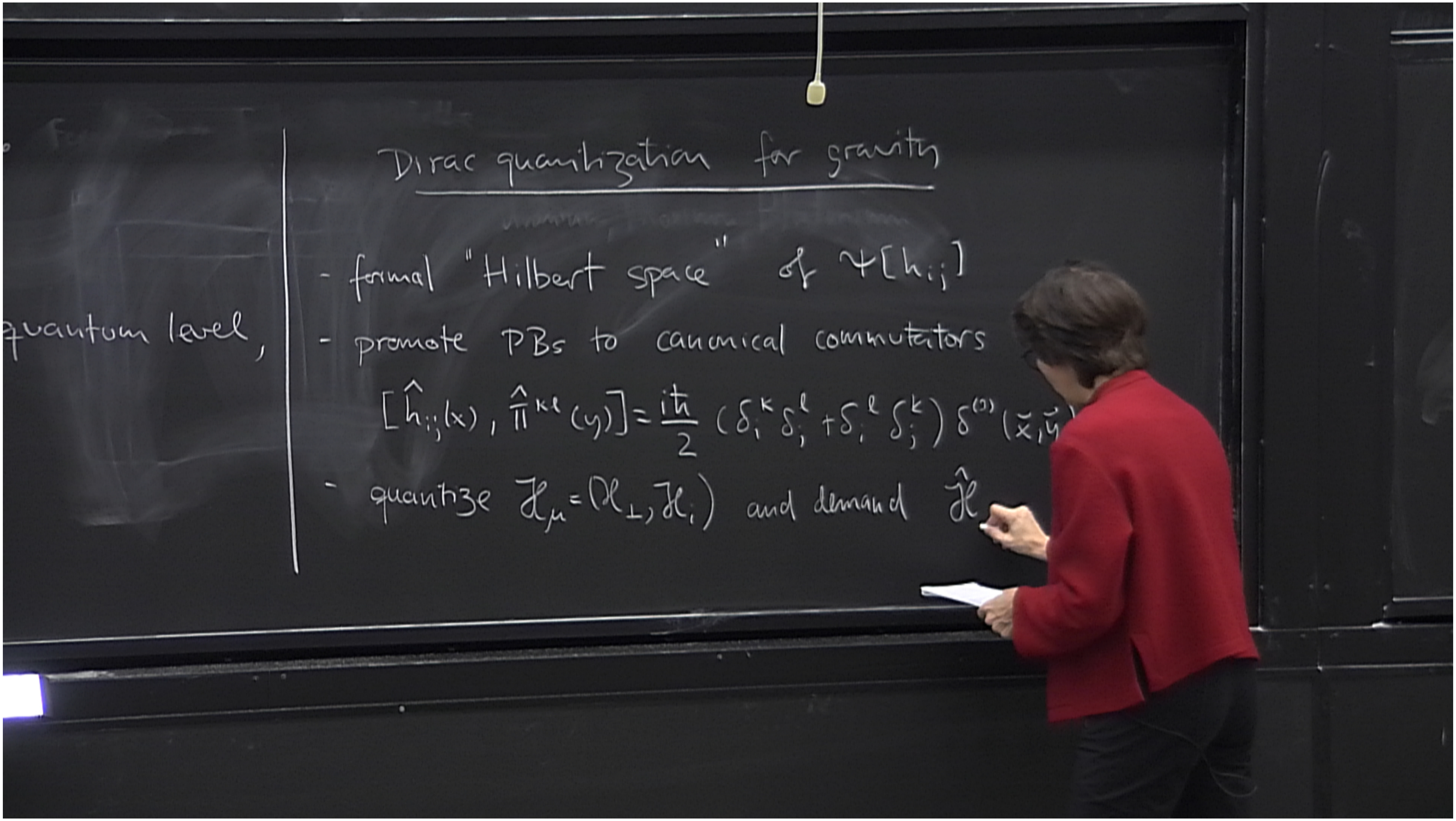
"Dirac consistency" : must have $[\hat{\Phi}_a, \hat{\Phi}_b]\psi_{phys} = 0$


$\Rightarrow [\hat{\Phi}_a, \hat{\Phi}_b] = i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c$,
in general $[\hat{\Phi}_a, \hat{\Phi}_b] = i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c + \hbar^2$


$\Rightarrow$ $[\hat{\Phi}_a, \hat{\Phi}_b] = i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c$,
in general $[\hat{\Phi}_a, \hat{\Phi}_b] = i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c + \hbar^2 \hat{D}_{ab}$
$\rightarrow$ extra conditions $\hat{D}_{ab}\Psi = 0$


$= i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c$, in general $[\hat{\Phi}_a, \hat{\Phi}_b] = i\hbar \hat{f}_{ab}^{\ c} \hat{\Phi}_c + \hbar^2 \hat{D}_{ab}$
$\rightarrow$ extra conditions $\hat{D}_{ab} \Psi = 0$
$\Rightarrow$ gauge invariance is broken at the quantum level,
"gauge anomaly"



quantum level,
Dirac quantization for gravity
- formal "Hilbert space" of $\Psi[h_{ij}]$
- promote PBs to canonical commutators
$[\hat{h}_{ij}(x), \hat{\pi}^{kl}(y)] = \frac{i\hbar}{2} (\delta_i^k \delta_j^l + \delta_i^l \delta_j^k) \delta^{(3)}(\vec{x}, \vec{y})$
- quantize $\mathcal{H}_\mu = (\mathcal{H}_\perp, \mathcal{H}_i)$ and demand $\hat{\mathcal{H}}$

quantum level,

Dirac quantization for gravity

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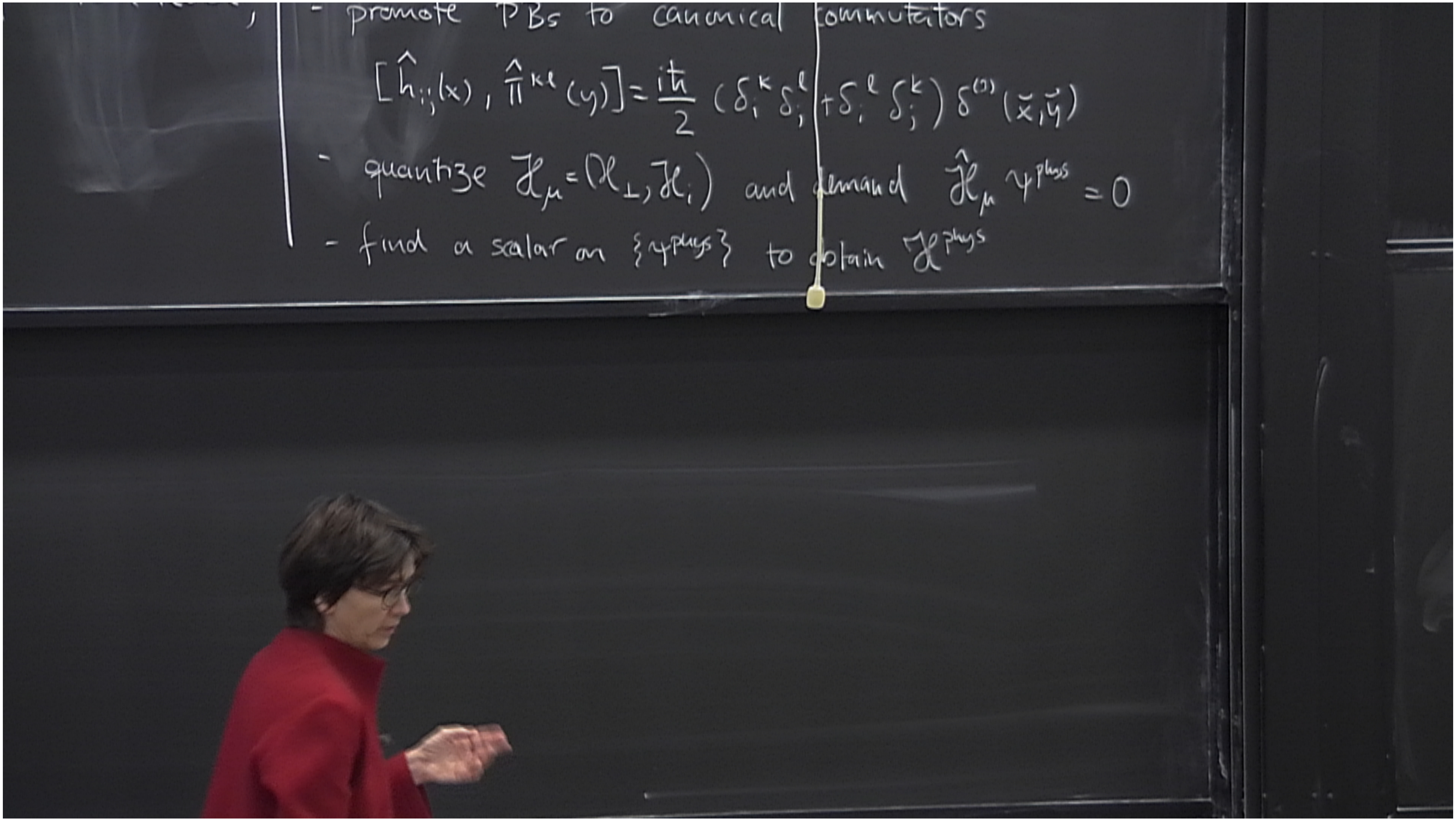
- quantize $\mathcal{H}_\mu = (\mathcal{H}_\perp, \mathcal{H}_i)$ and demand $\hat{\mathcal{H}}_\mu \Psi^{phys} = 0$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- find a scalar on $\{\psi^{phys}\}$ to obtain $\mathcal{H}^{phys}$

Dirac quantization for gravity

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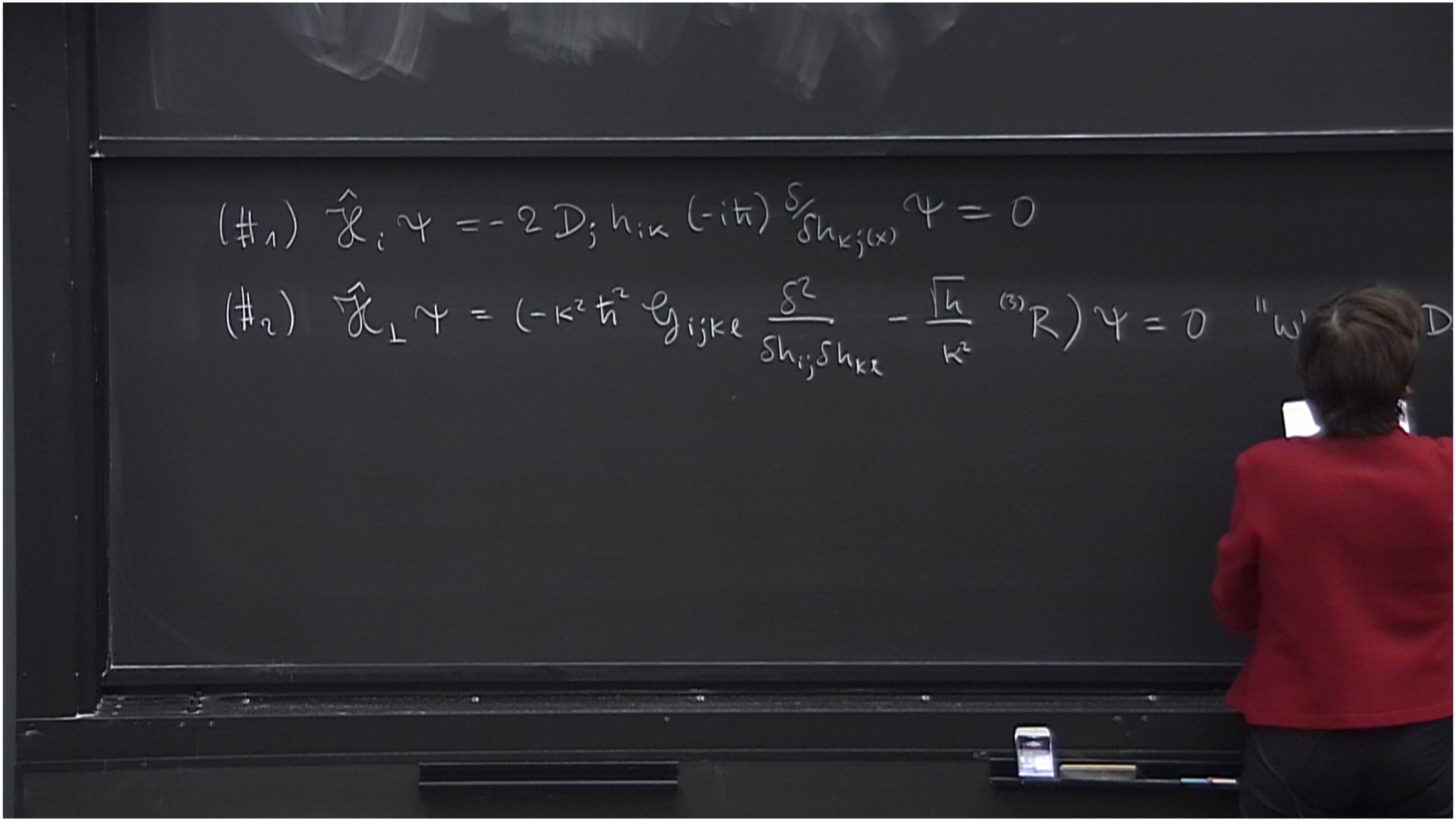


(#1) $\hat{\mathcal{H}}_i \Psi = -2 D_j h_{ik} (-i\hbar) \frac{\delta}{\delta h_{kj}(x)} \Psi = 0$
(#2) $\hat{\mathcal{H}}_\perp \Psi = \left(-\kappa^2 \hbar^2 \mathcal{G}_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} - \frac{\sqrt{h}}{\kappa^2} {}^{(3)}R\right) \Psi = 0$

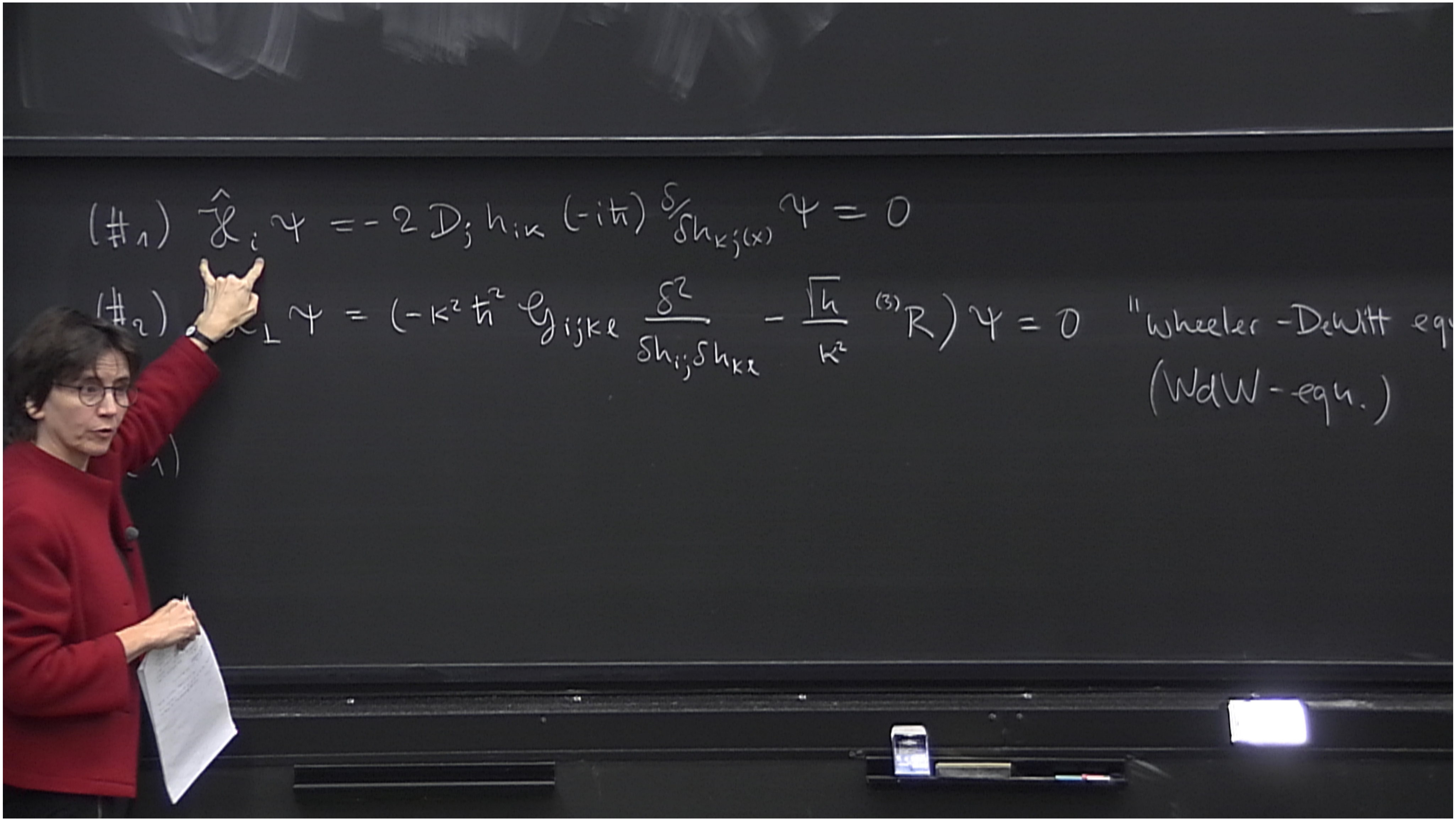
- find a scalar on $\{\Psi_{phys}$

$$(-i\hbar)\frac{\delta}{\delta h_{kj}(x)}\Psi = 0$$

$$\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \frac{\sqrt{h}}{\kappa^2}\,{}^{(3)}R\Big)\Psi = 0$$

"Wheeler-DeWitt equation"

(WdW-equ.)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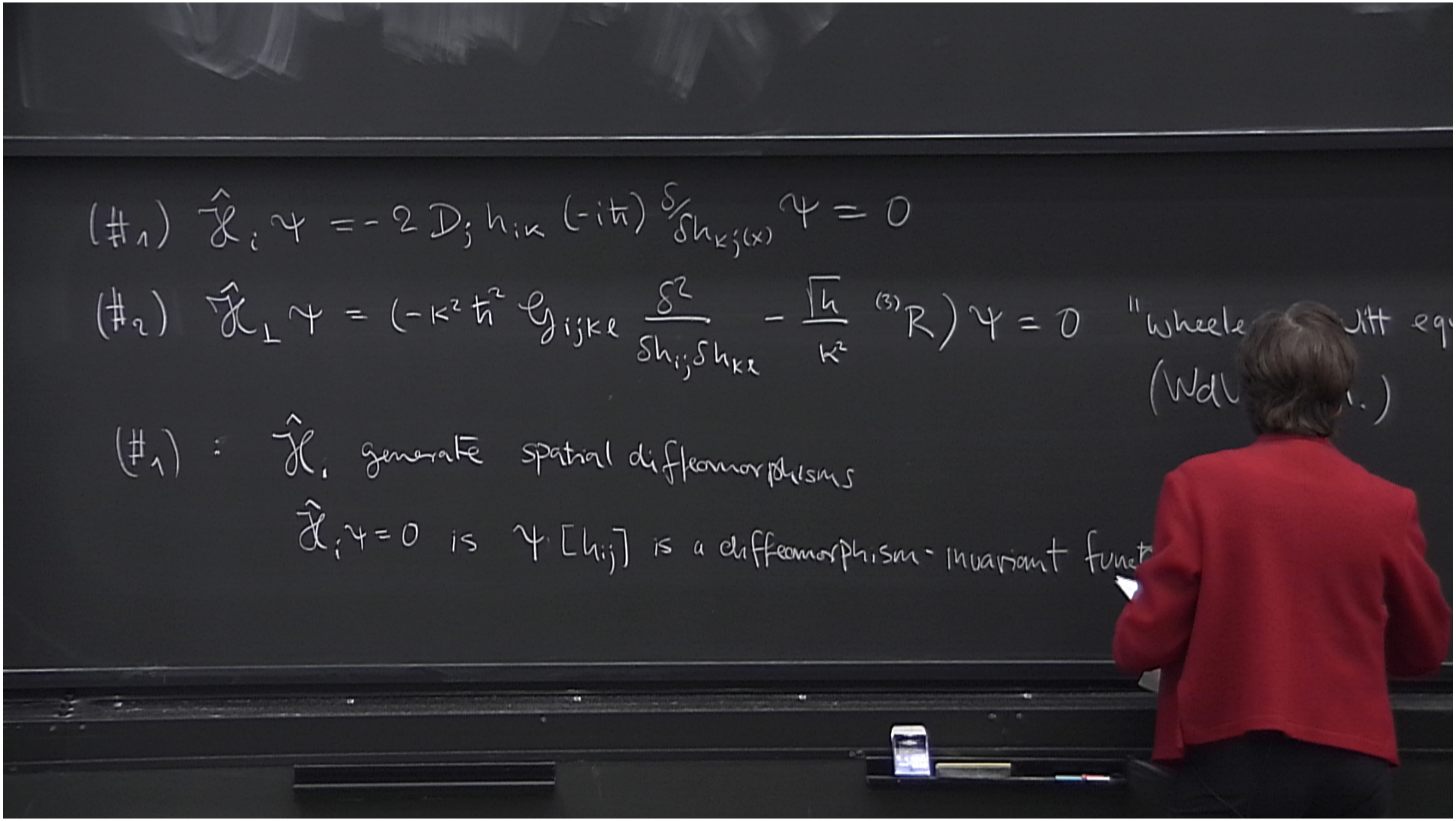
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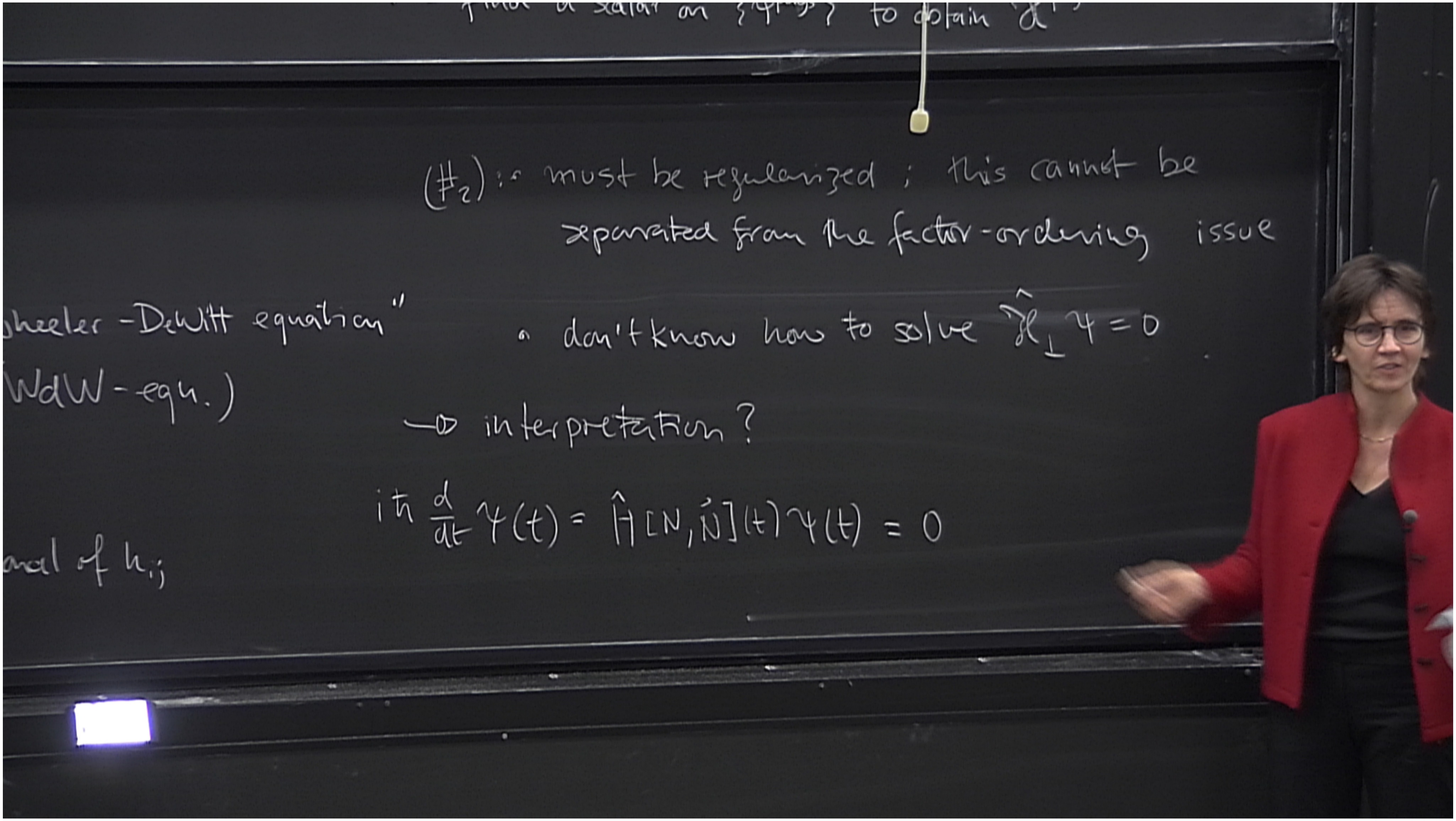
itt eq
(WdV
.)
(#1) : $\hat{\mathcal{H}}_i$ generate spatial diffeomorphisms
$\hat{\mathcal{H}}_i \Psi = 0$ is $\Psi[h_{ij}]$ is a diffeomorphism-invariant func


to obtain
($\#_2$): must be regularized ; this cannot be
separated from the factor-ordering issue
heeler-DeWitt equation"
WdW-eqn.)
don't know how to solve $\hat{\mathcal{H}}_\perp \Psi = 0$
→ interpretation?
$i\hbar \frac{d}{dt} \Psi(t) = \hat{H}[N, \vec{N}](t) \Psi(t) = 0$
onal of h_{ij}

a scalar on ... to obtain ...

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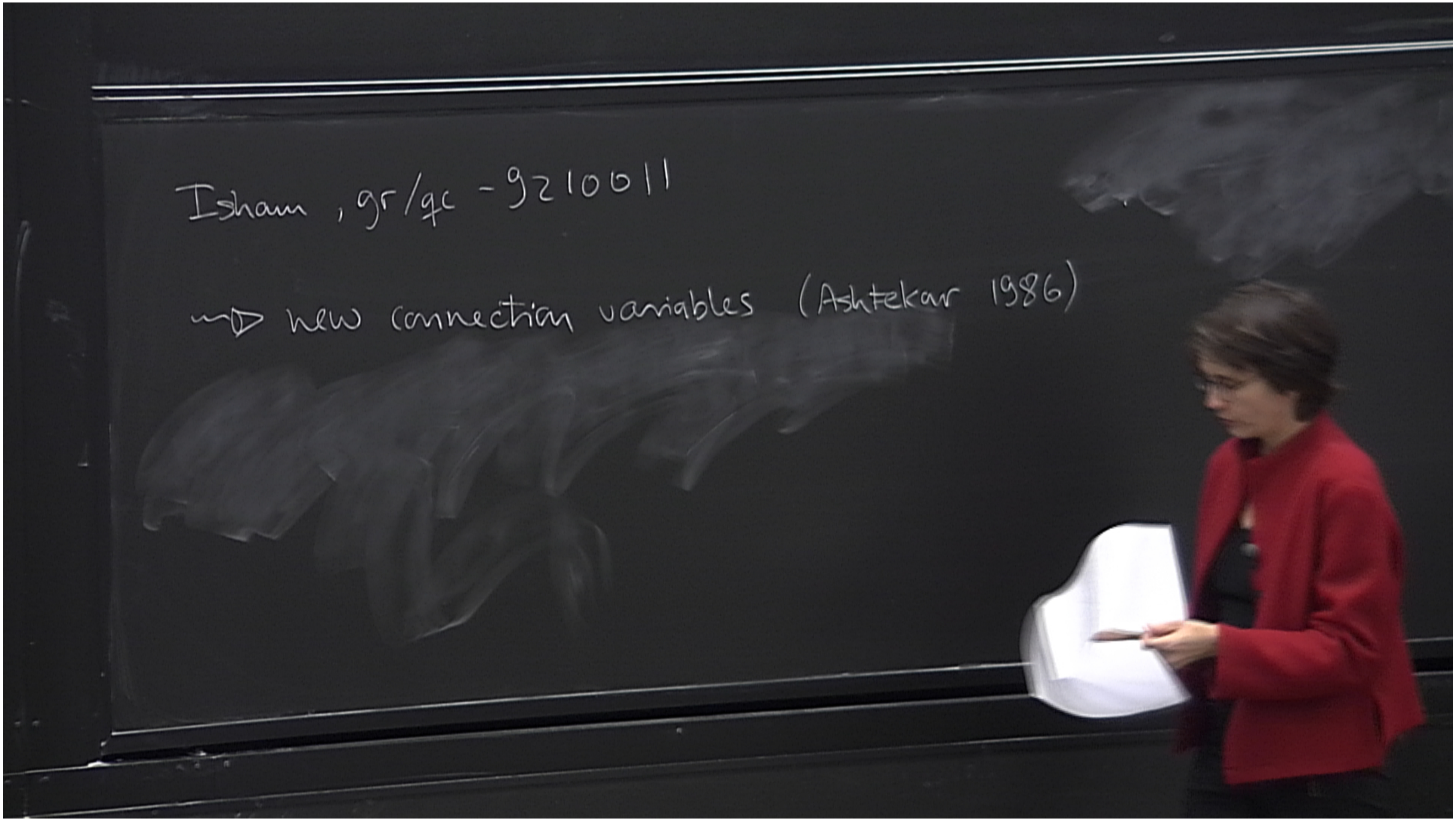
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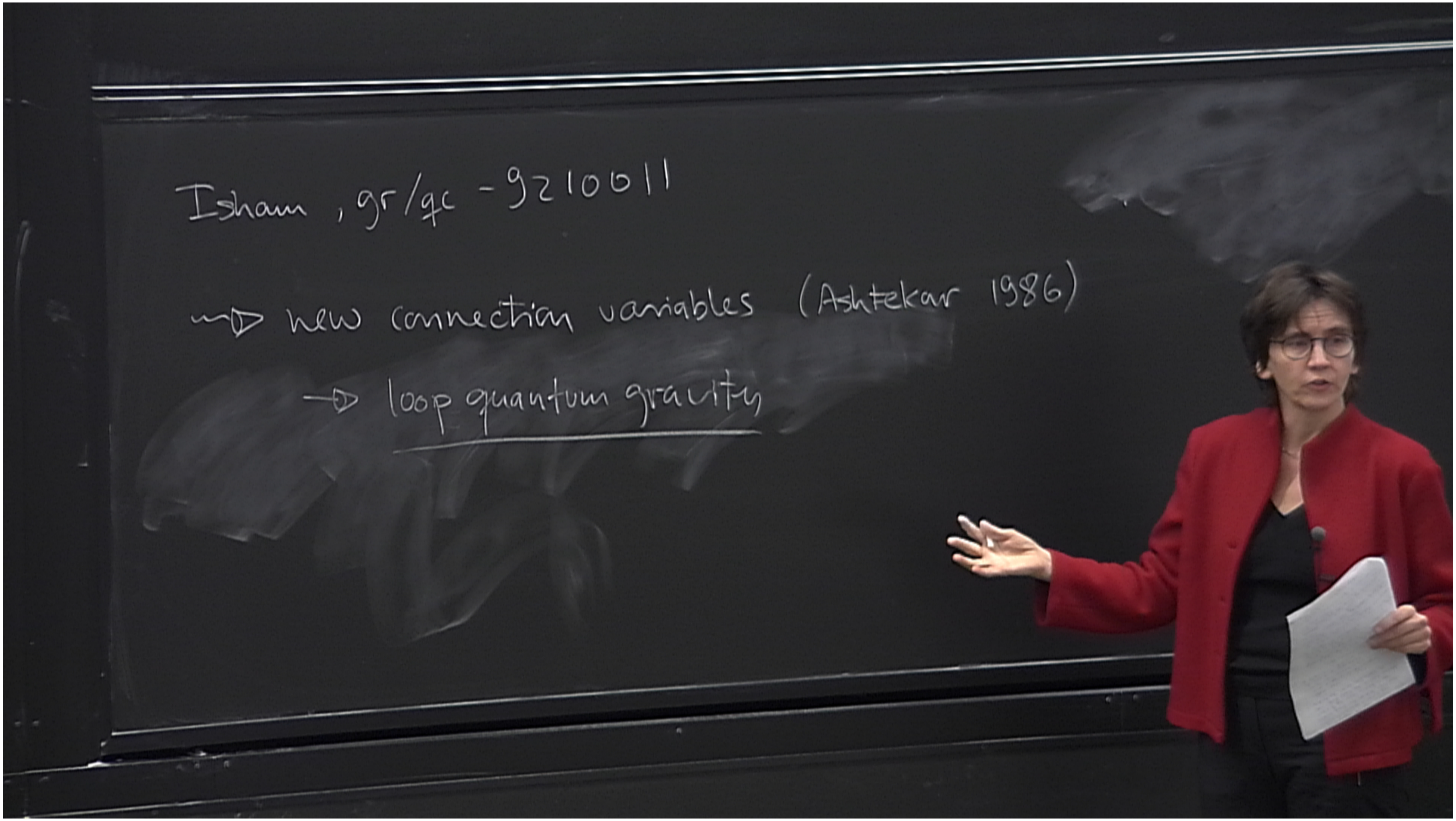
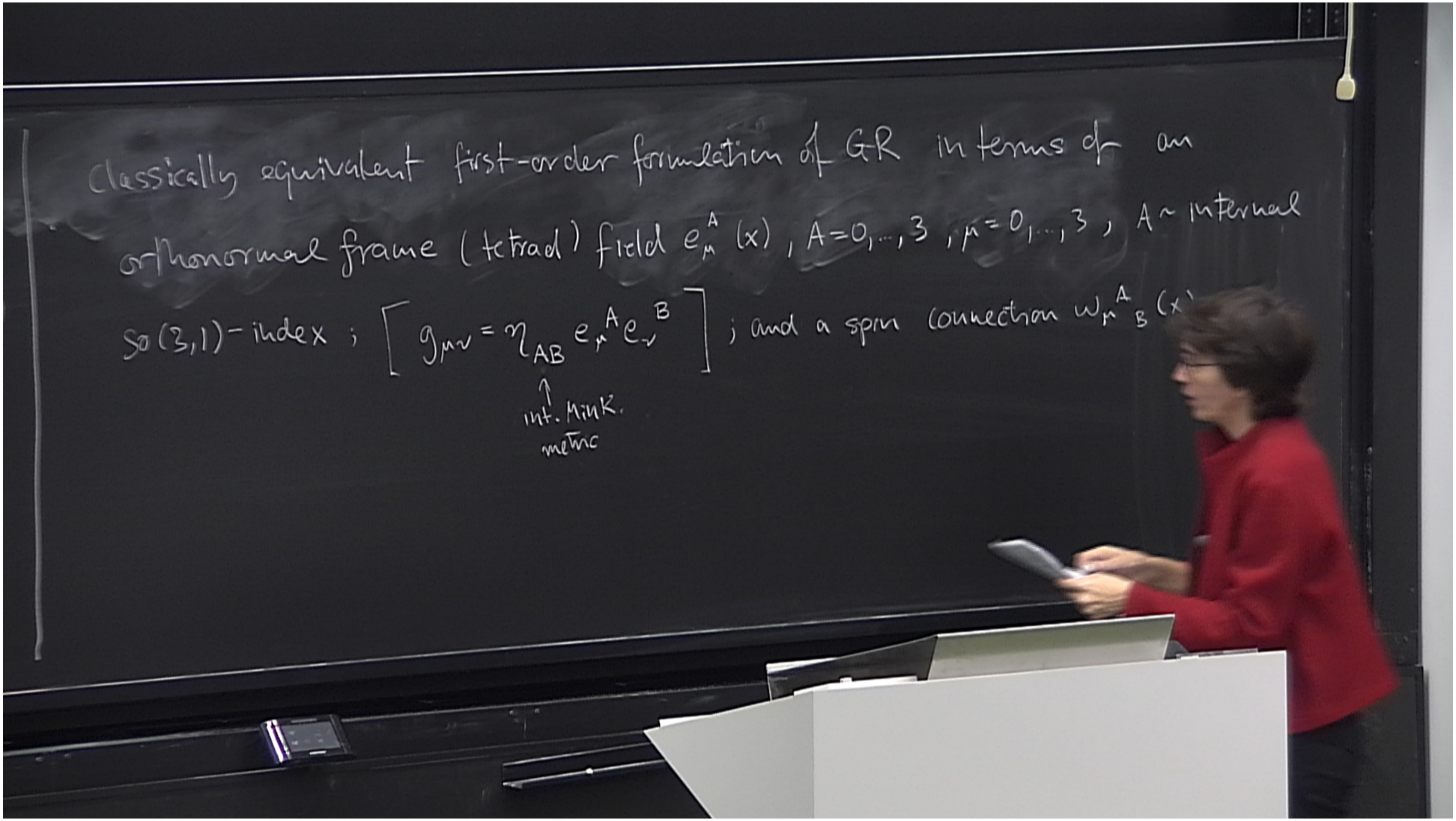
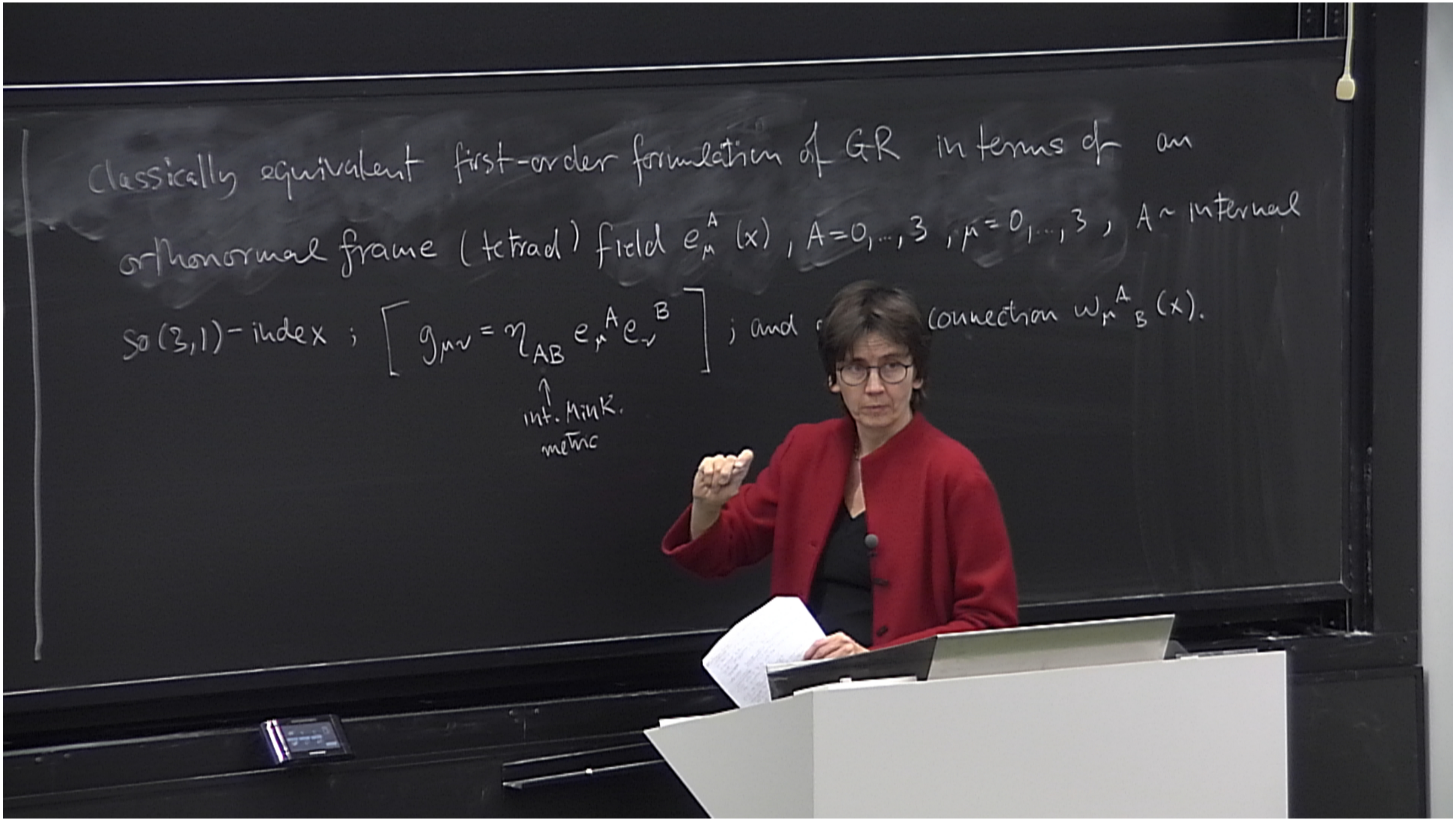
⇝ new connection variables (Ashtekar 1986)


Isham, gr/qc-9210011
new connection variables (Ashtekar 1986)
loop quantum gravity


Classically equivalent first-order formulation of GR in terms of an
orthonormal frame (tetrad) field $e_\mu^A(x)$, $A=0,\dots,3$, $\mu=0,\dots,3$, $A \sim$ internal
so(3,1)-index ; [$g_{\mu\nu} = \eta_{AB}\, e_\mu{}^A e_\nu{}^B$] ; and a spin connection $\omega_\mu{}^A{}_B(x)$
int. Mink. metric



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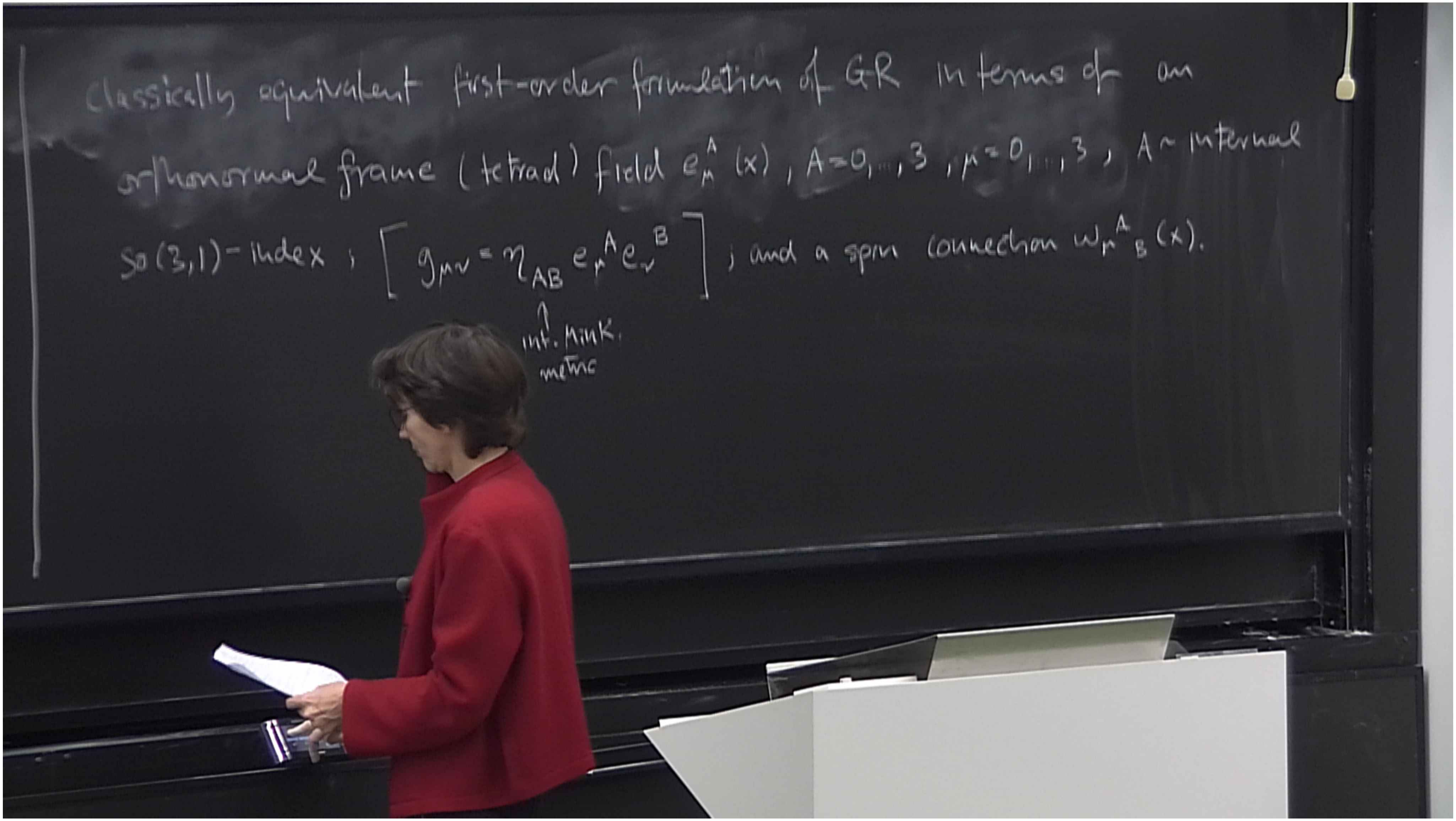
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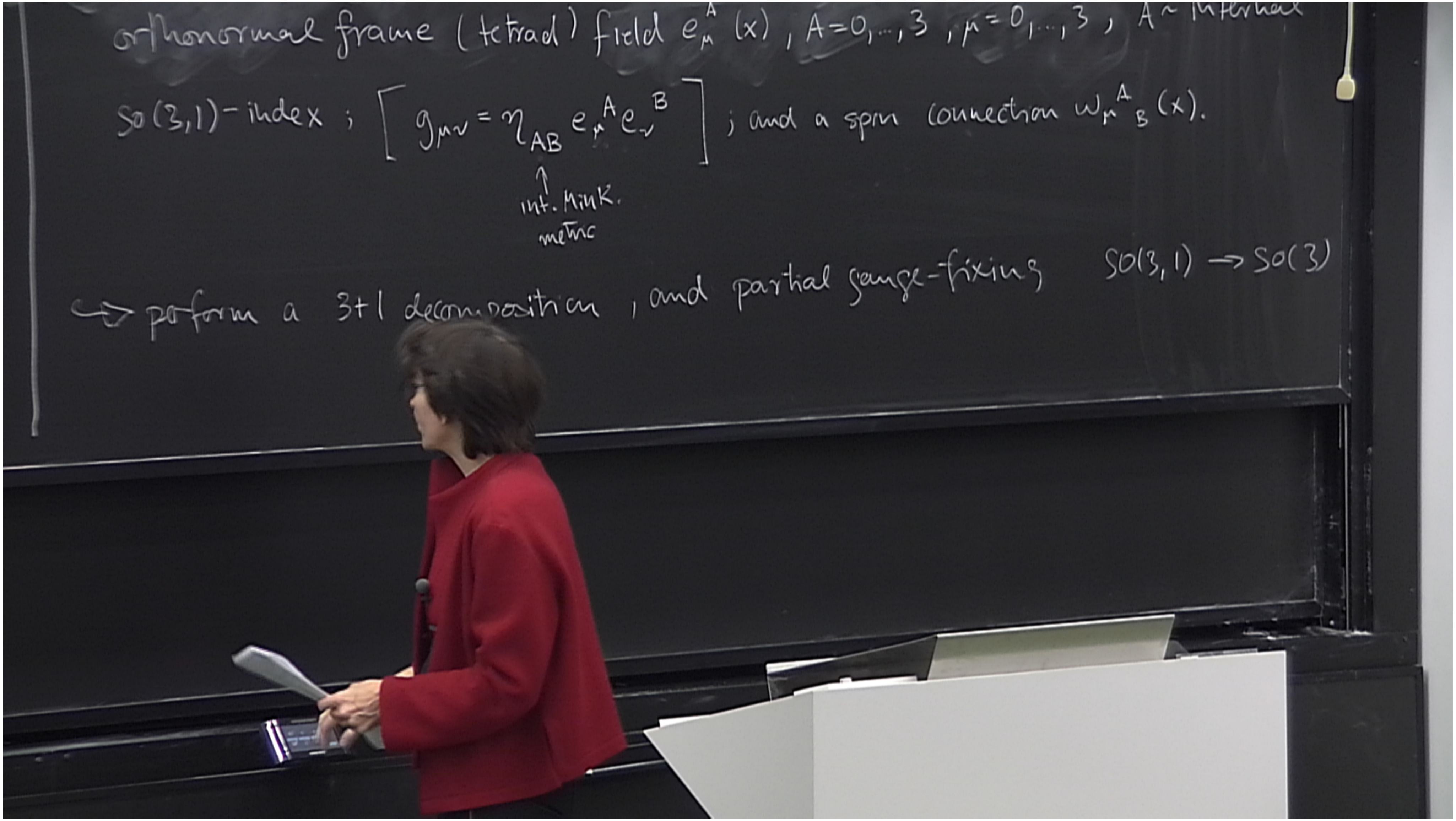
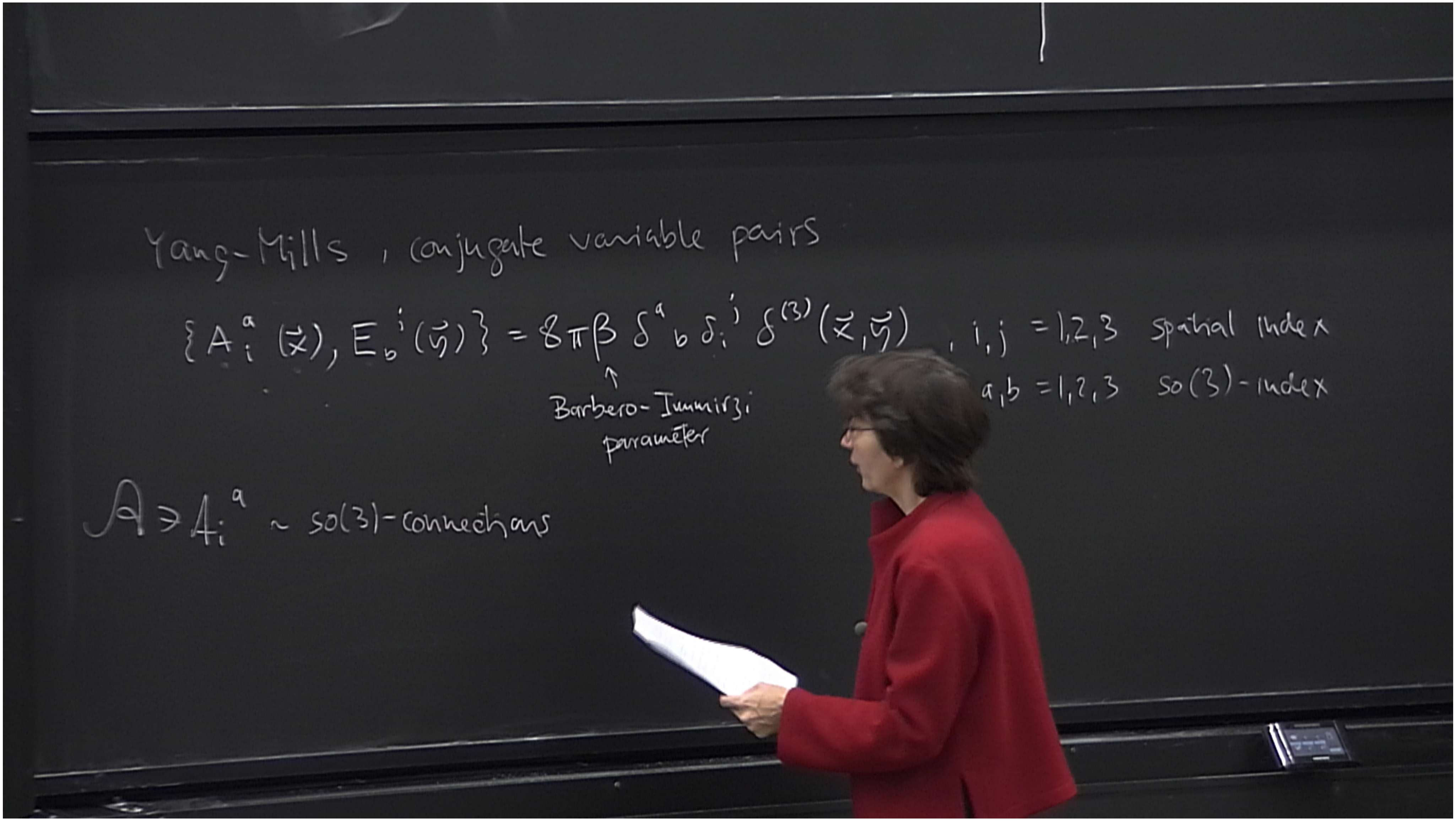

orthonormal frame (tetrad) field $e_\mu^A(x)$, $A=0,\dots,3$, $\mu=0,\dots,3$, $A \sim$ internal
$SO(3,1)$-index; $[g_{\mu\nu} = \eta_{AB}\, e_\mu{}^A e_\nu{}^B]$; and a spin connection $\omega_\mu{}^A{}_B(x)$.
int. Mink. metric
$\Rightarrow$ perform a 3+1 decomposition, and partial gauge-fixing $SO(3,1) \to SO(3)$


Yang-Mills , conjugate variable pairs
$\{A_i^a(\vec{x}), E_b^{\,i}(\vec{y})\} = 8\pi\beta\, \delta^a{}_b\, \delta_i^{\,j}\, \delta^{(3)}(\vec{x},\vec{y})$
, i, j = 1,2,3 spatial index
a,b = 1,2,3 so(3)-index
Barbero-Immirzi parameter
$\mathcal{A} \ni A_i^a$ ~ so(3)-connections

Yang-Mills, conjugate variable pairs

$$\{A_i^a(\vec{x}), E_b^j(\vec{y})\} = 8\pi\underbrace{\beta}_{\text{Barbero-Immirzi parameter}}\,\delta^a{}_b\,\delta_i{}^j\,\delta^{(3)}(\vec{x},\vec{y})$$

, $i,j = 1,2,3$ spatial index

$a,b = 1,2,3$ so(3)-index

$\mathcal{A} \ni A_i^a$ ~ so(3)-connections, $E_b^j = \sqrt{h}\, e_b^j$ "densitized triad"

associated curvature $F_{ij}^a = 2G_N\,\partial_{[i}A_{j]}^a + G_N^2\,\epsilon_{abc}\,A_i^b A_j^c$