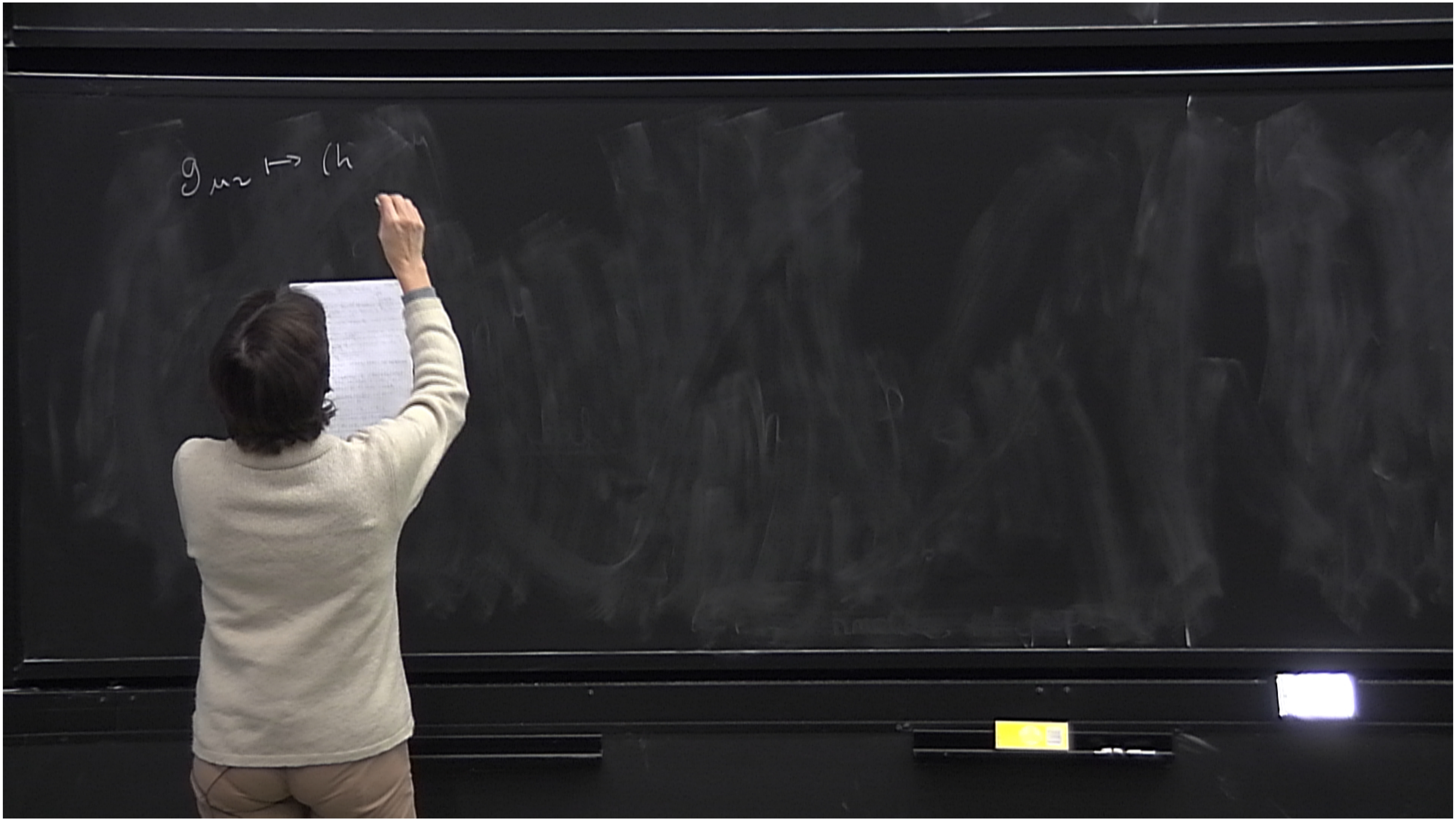
Title: Quantum Gravity (Review) - Lecture 9

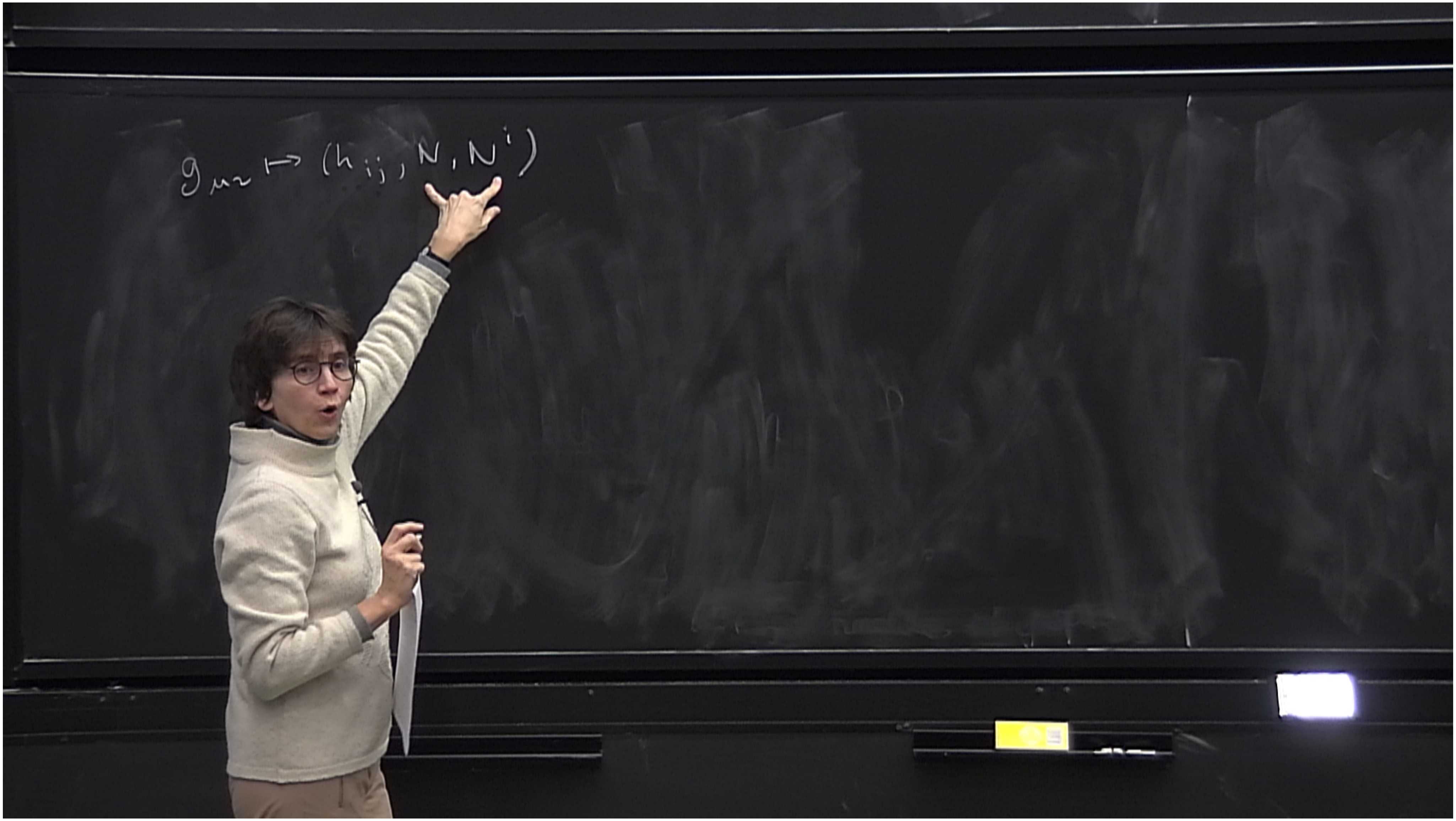
Date: Feb 02, 2012 10:15 AM

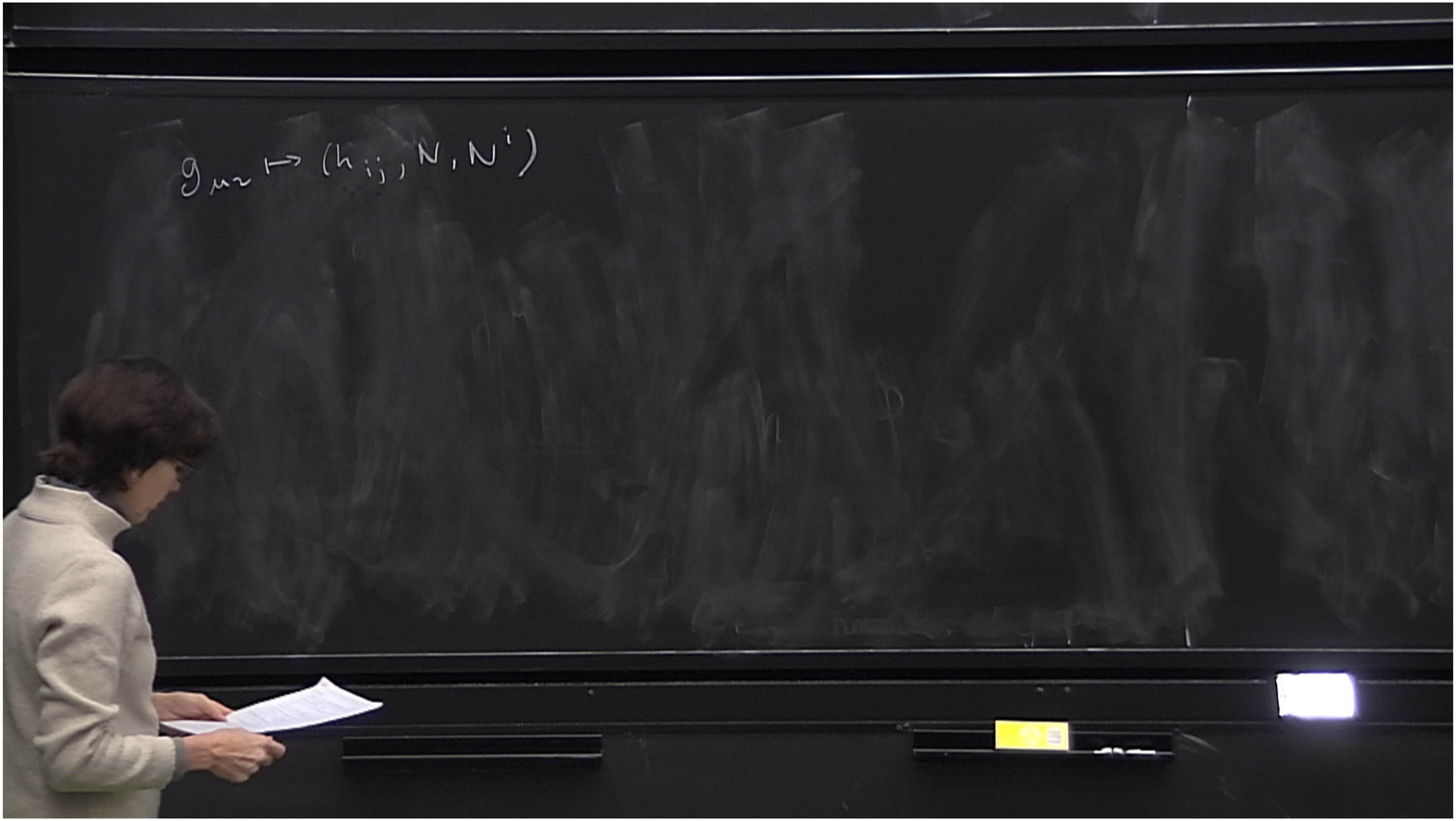
URL: http://pirsa.org/12020001

Abstract:


$g_{\mu\nu} \mapsto$


$g_{\mu\nu} \mapsto (h$


$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$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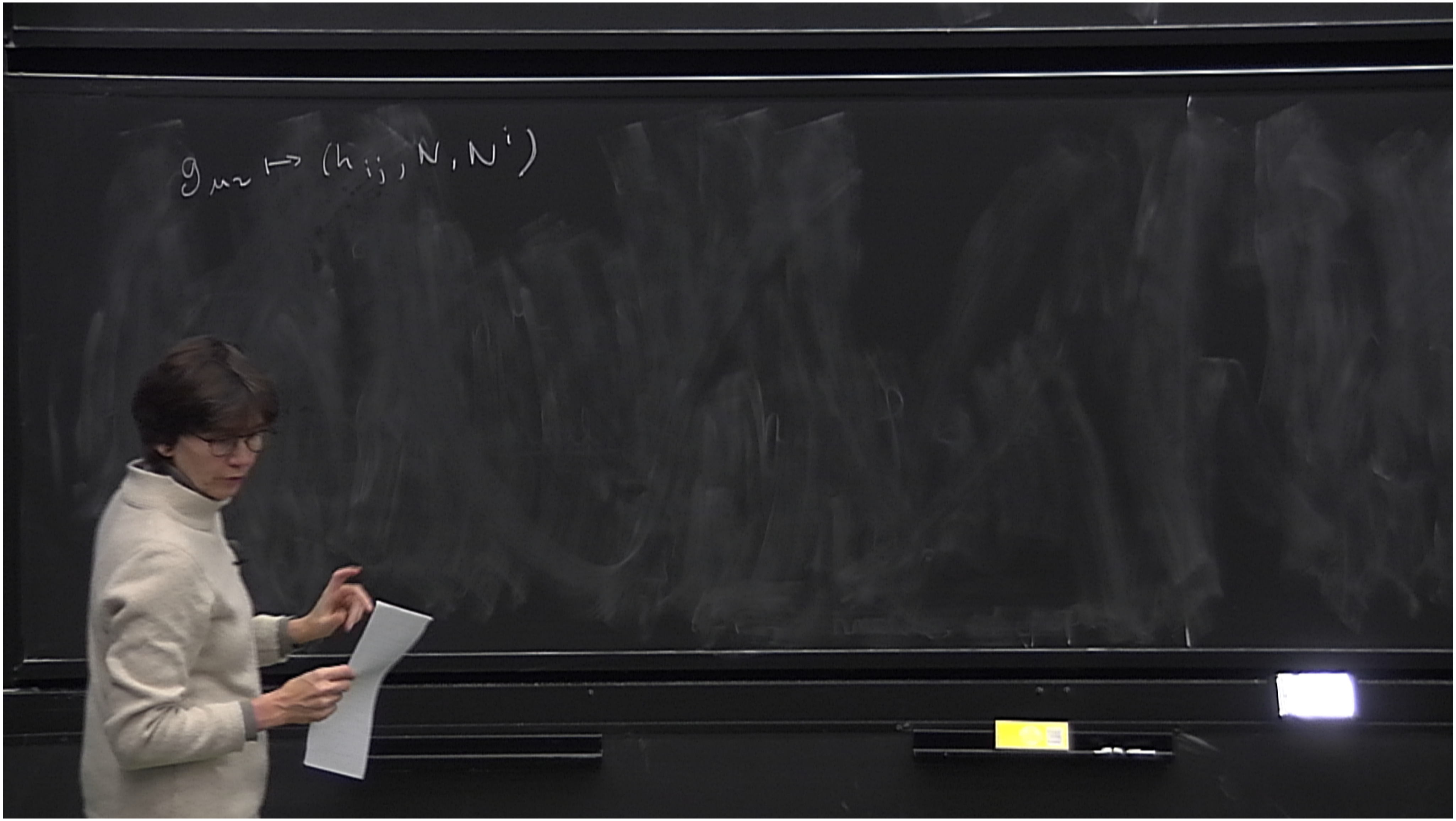
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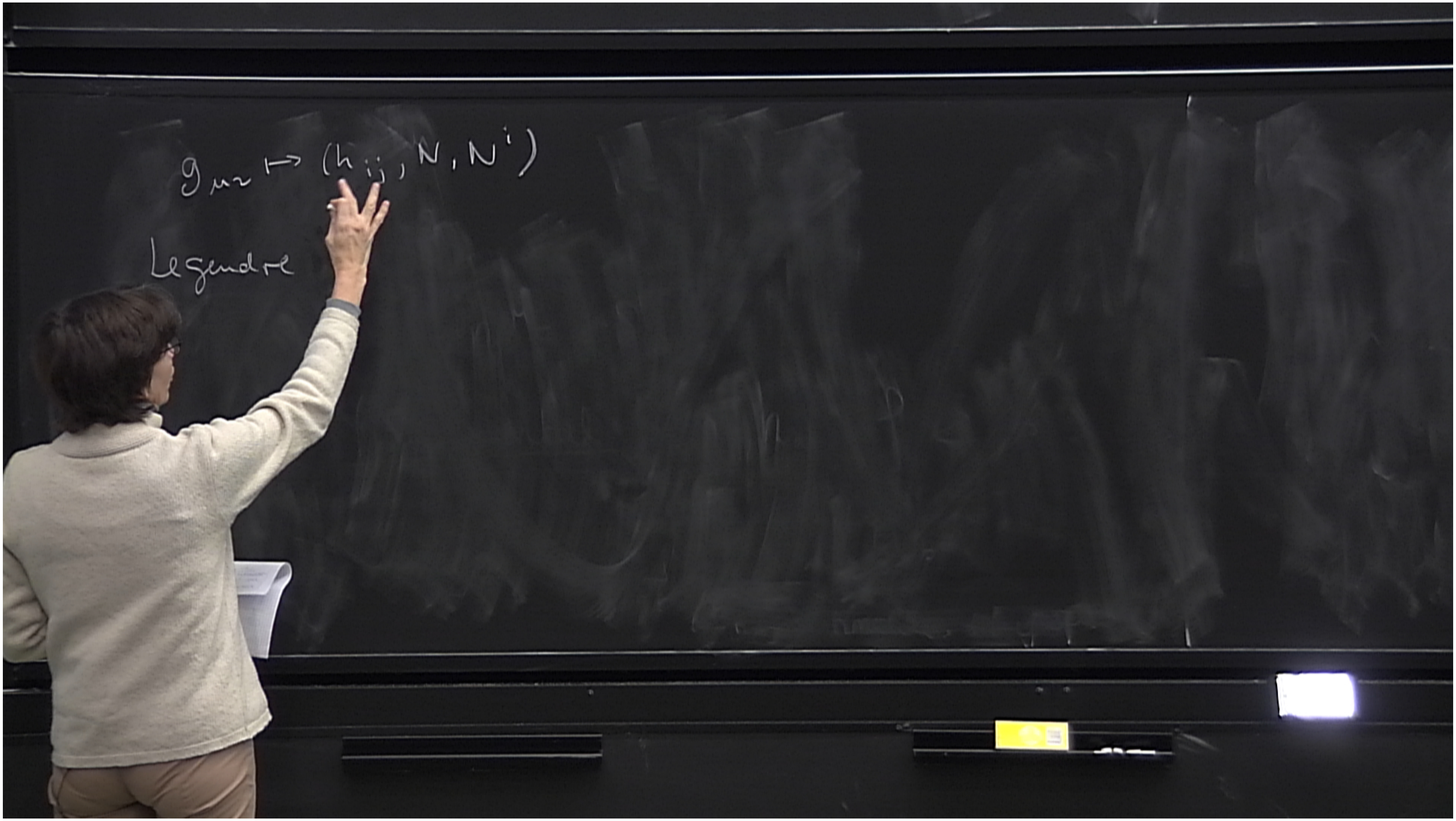
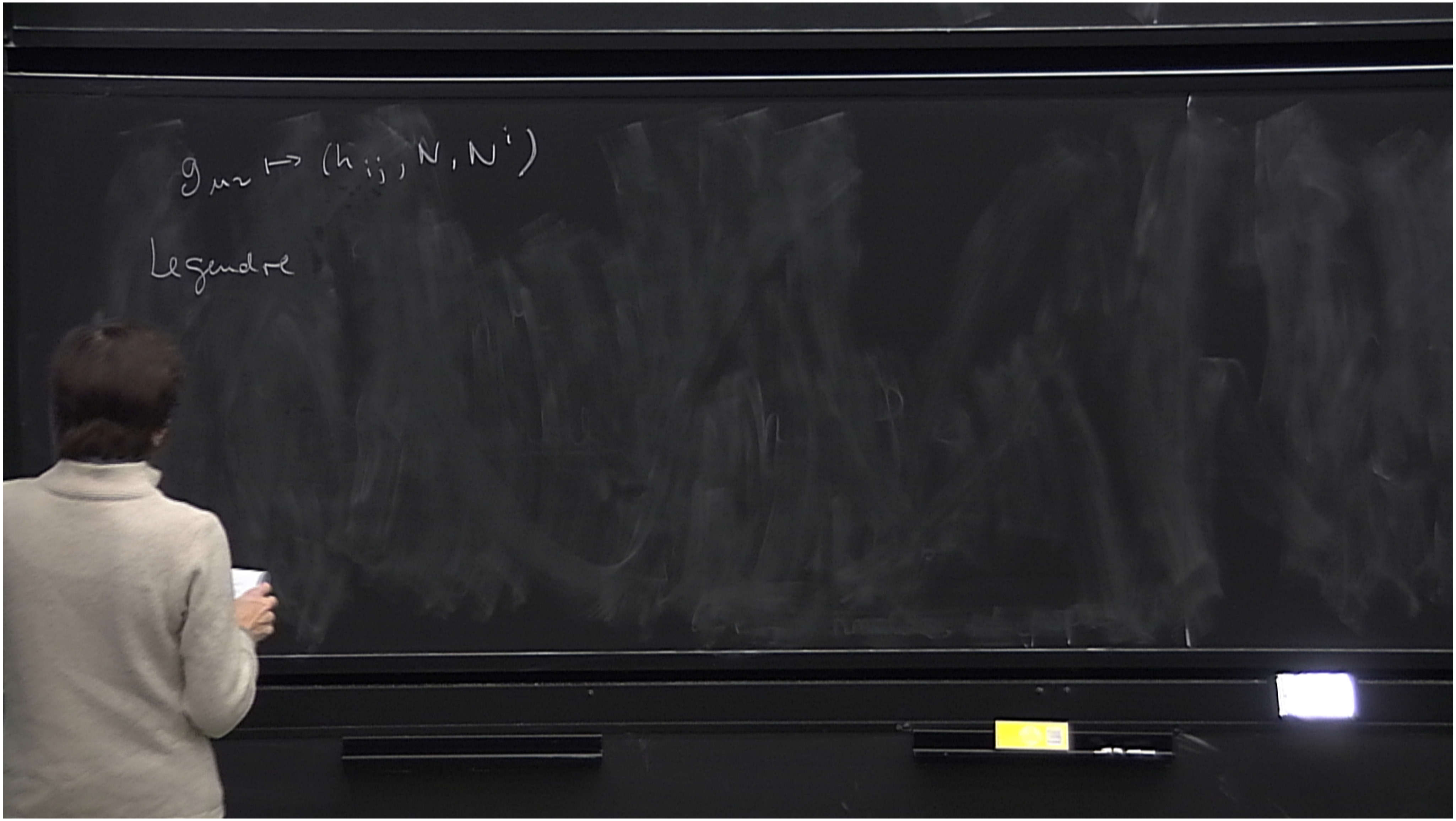

$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre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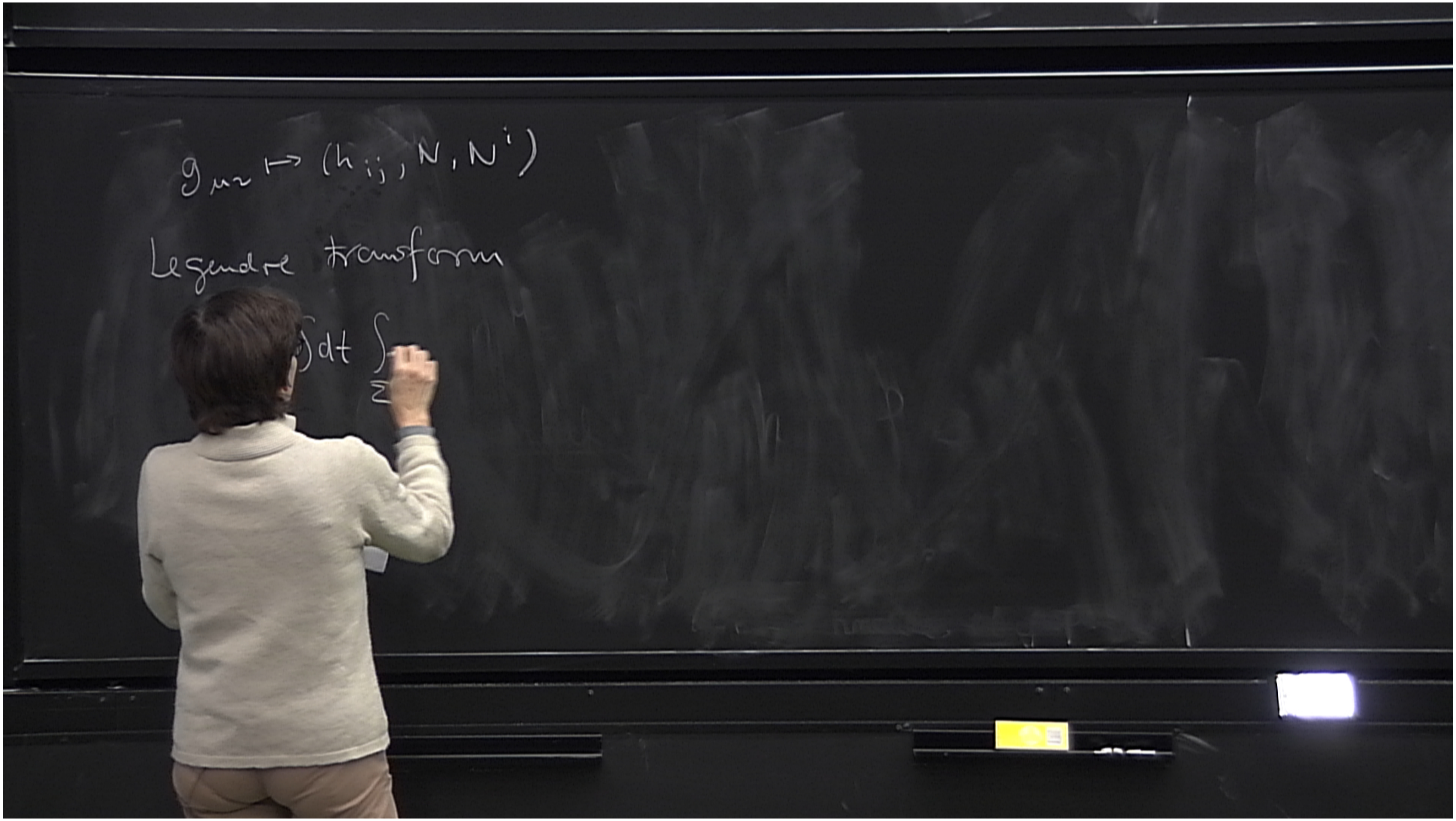
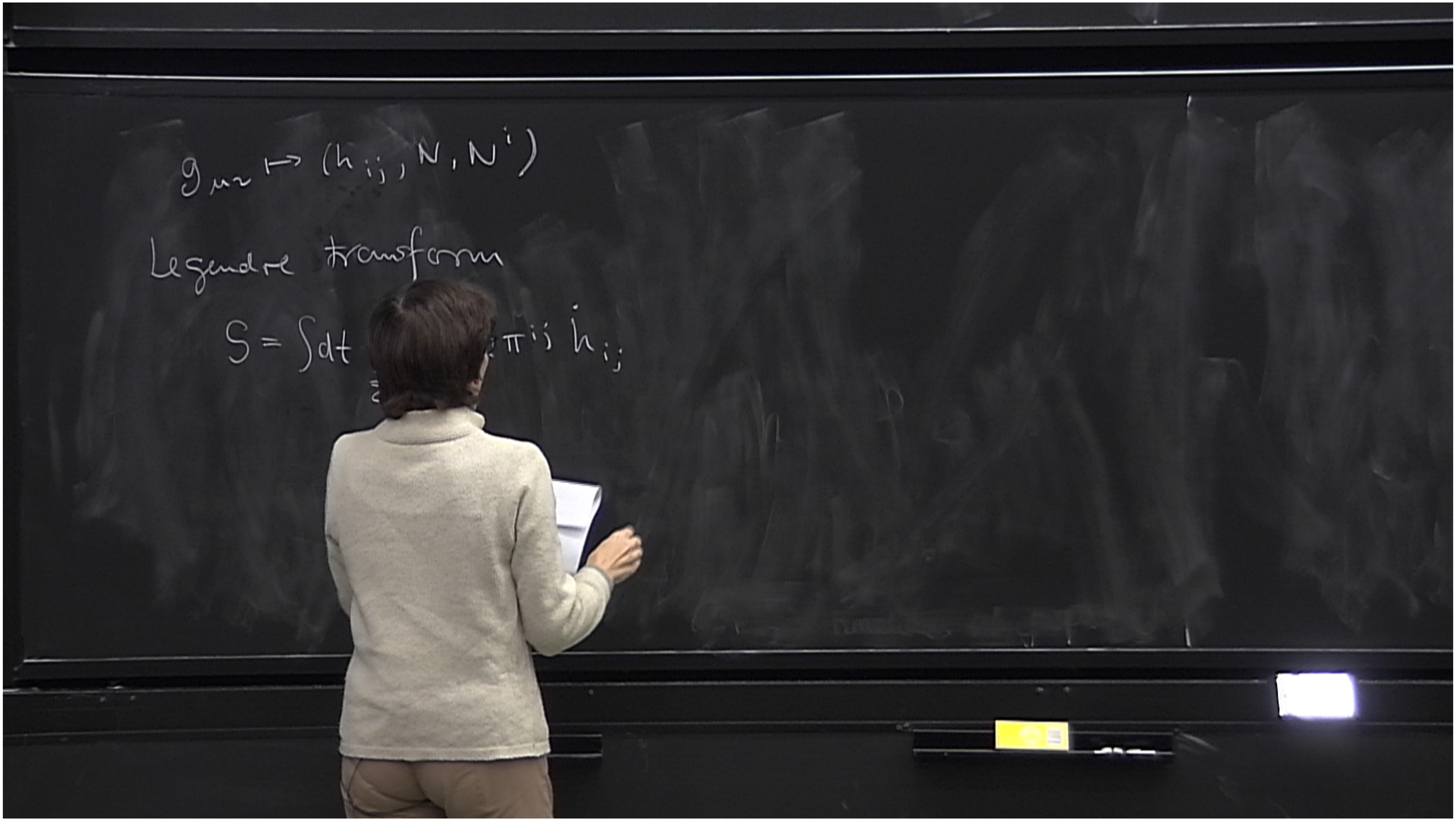
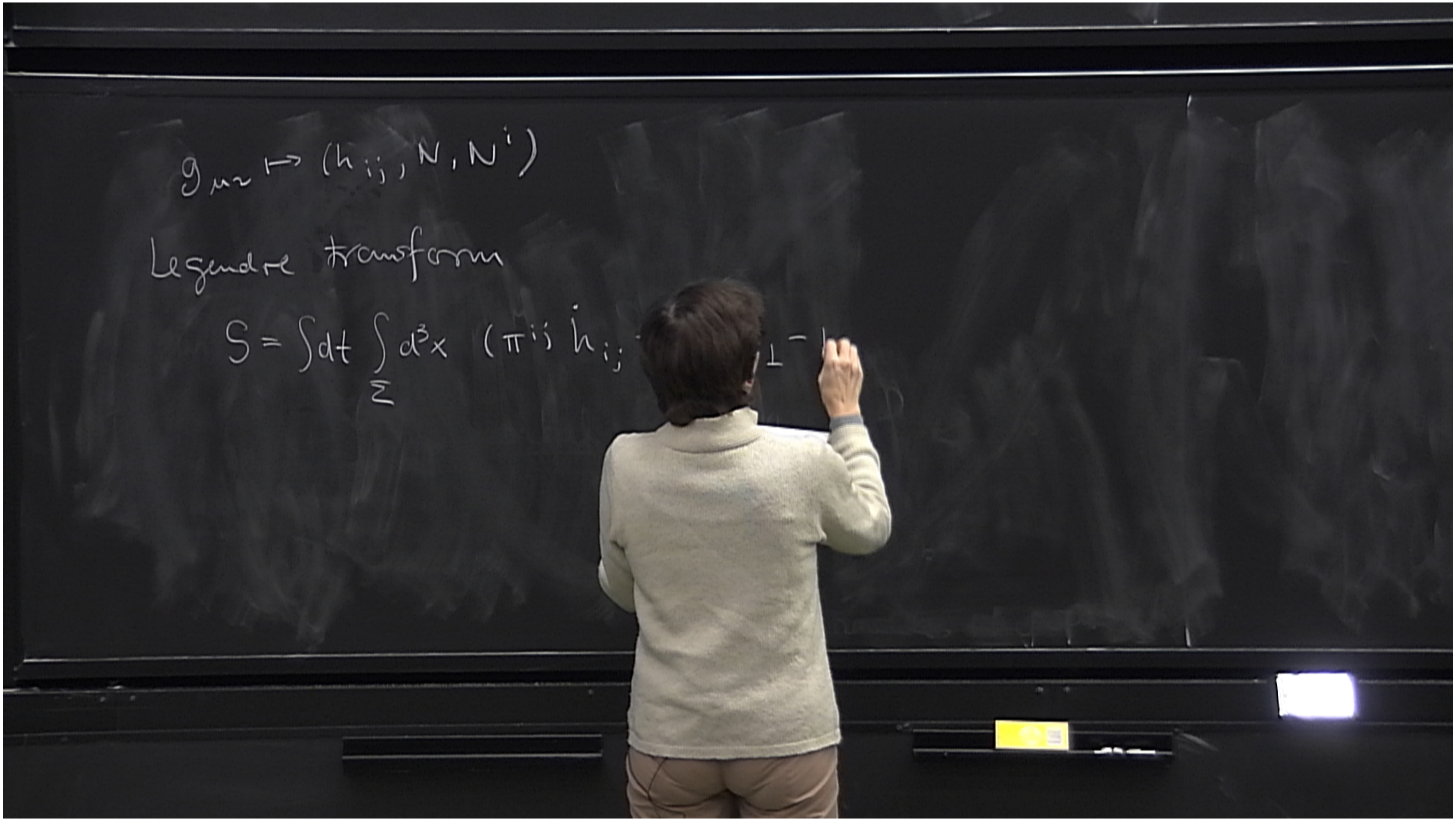
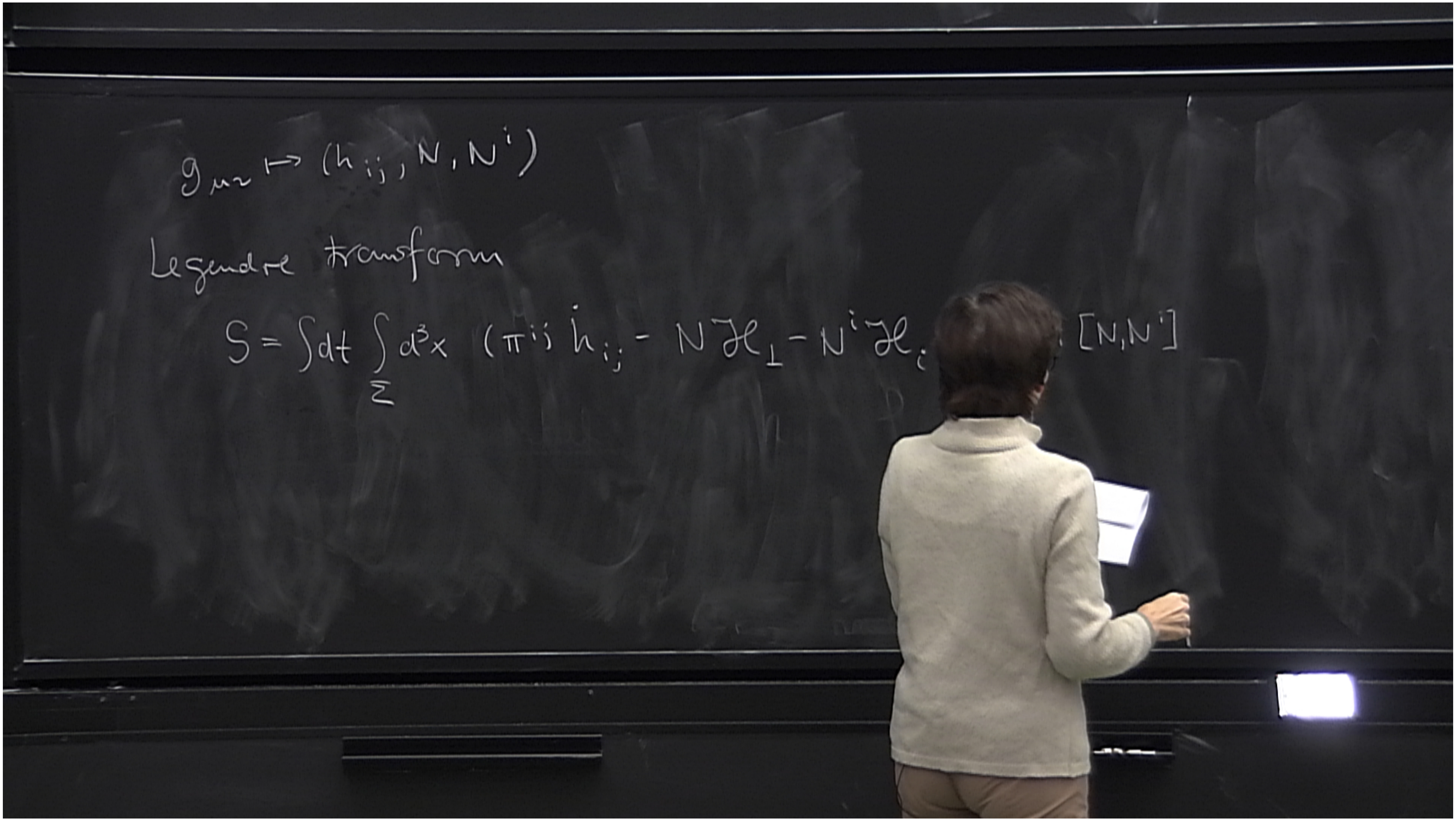
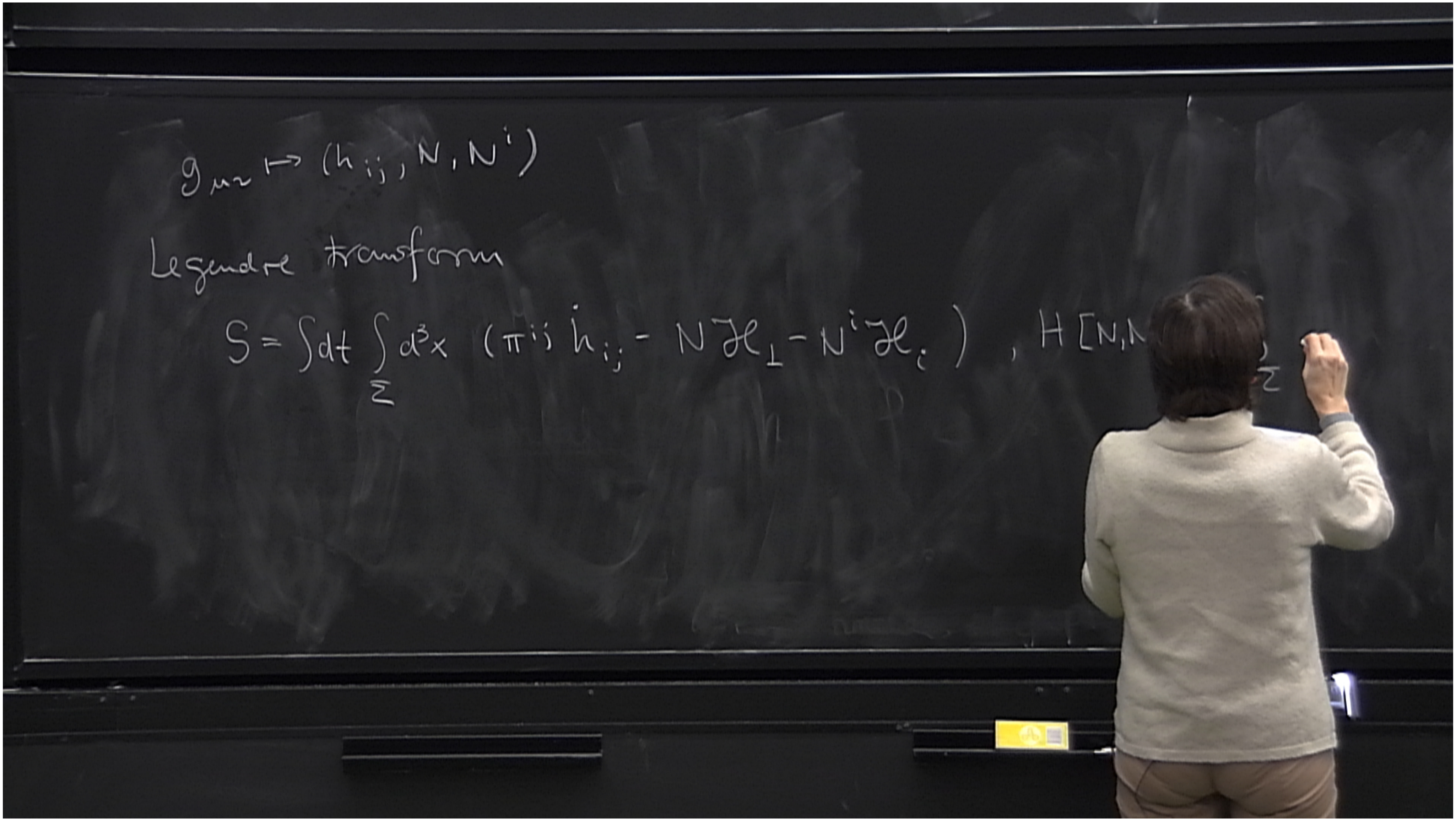

g_μν ↦ (h_ij, N, N^i)
Legendre transform
∫dt ∫_Σ


$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt$ ... $\pi^{ij} \dot{h}_{ij}$


$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \; (\pi^{ij} \dot{h}_{ij}$


$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i$
$[N, N^i]$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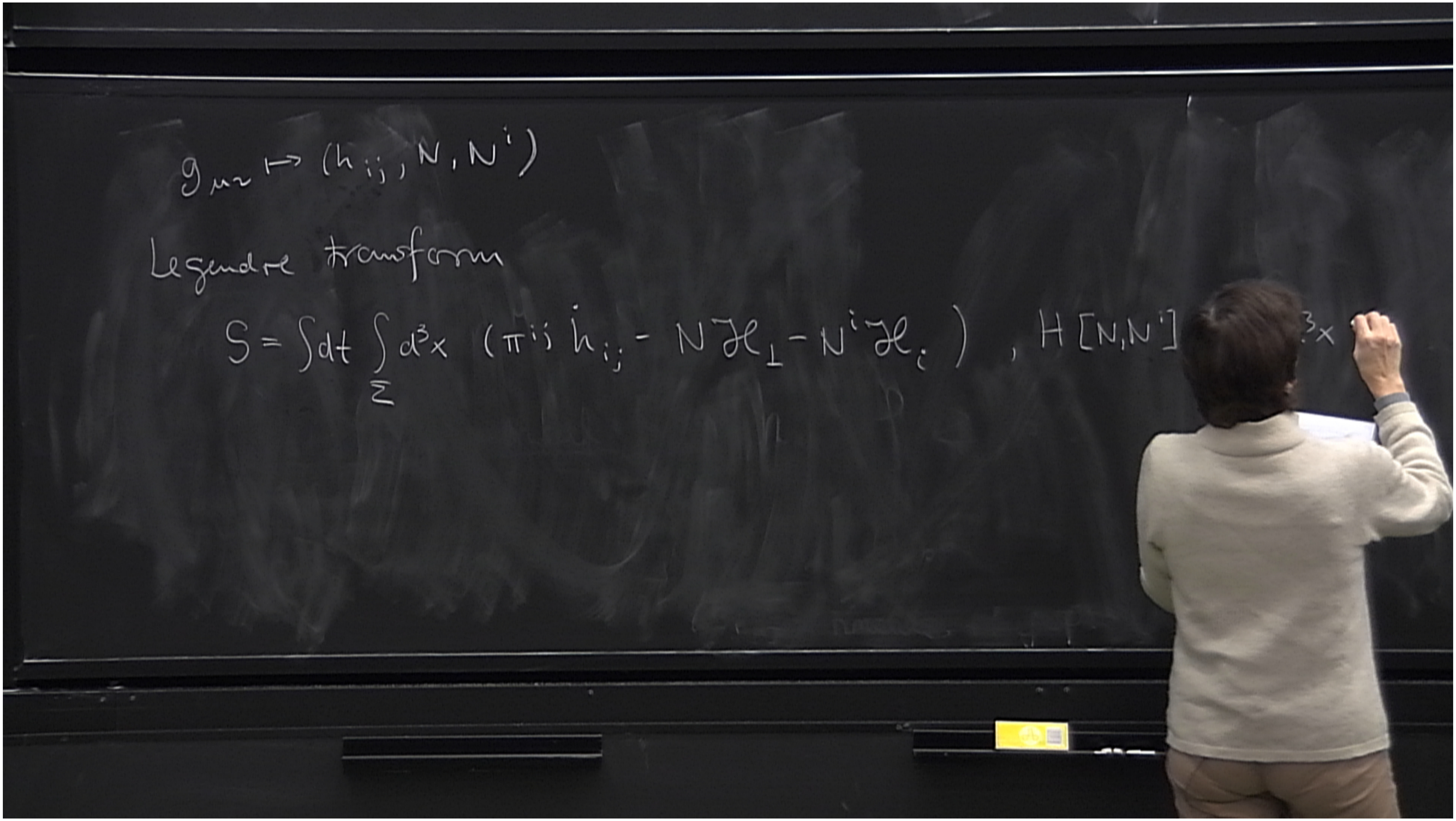
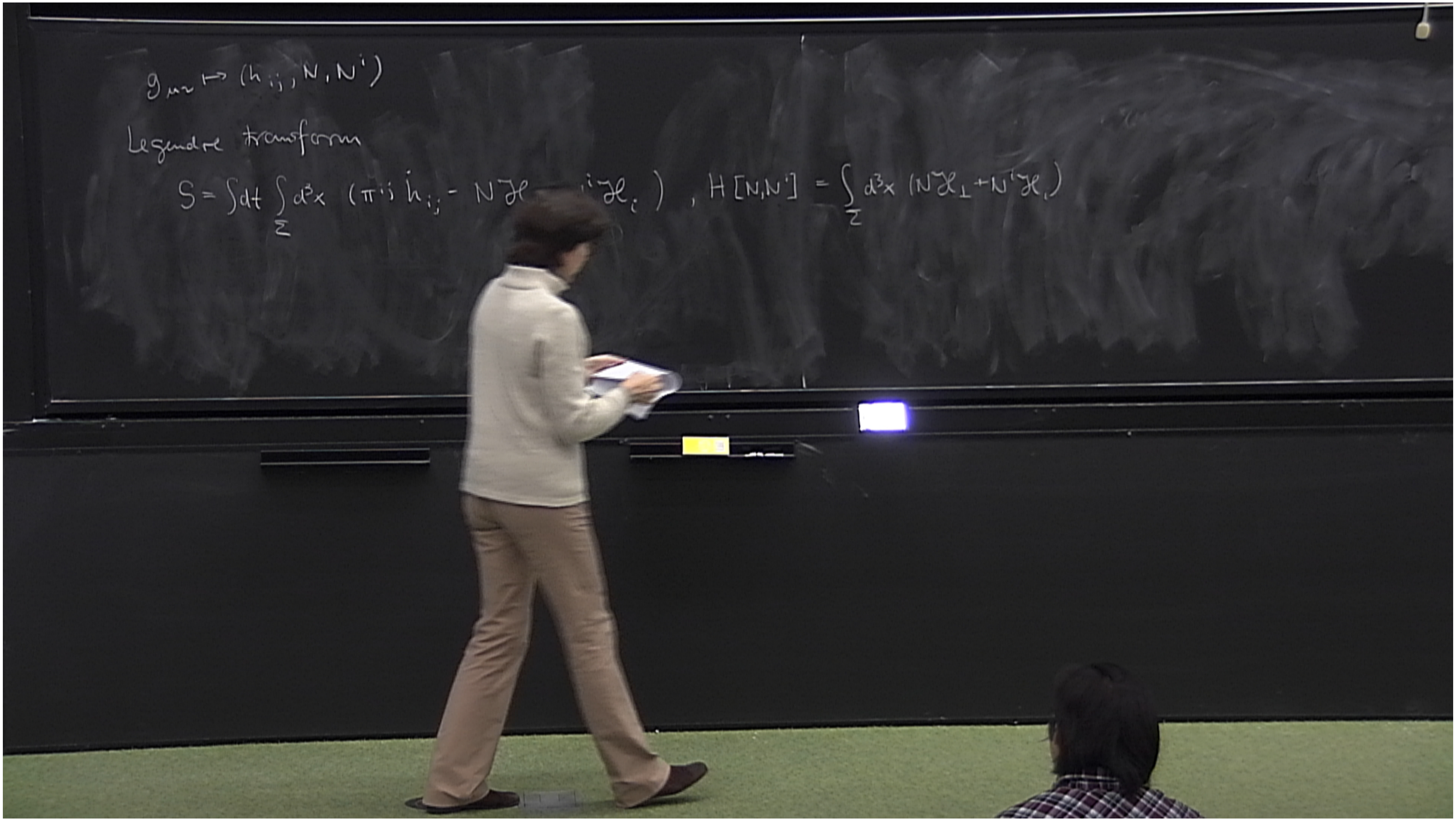
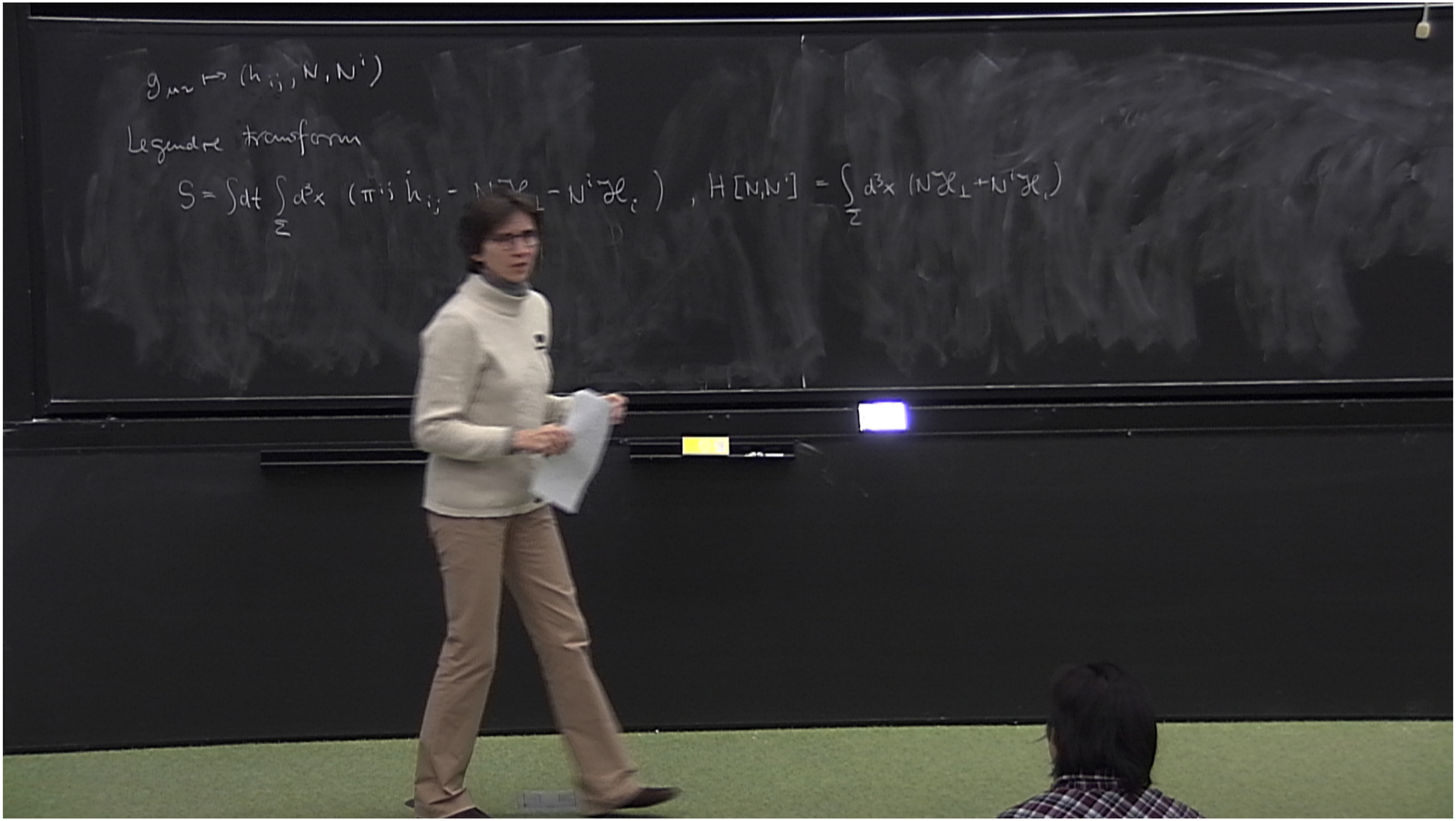
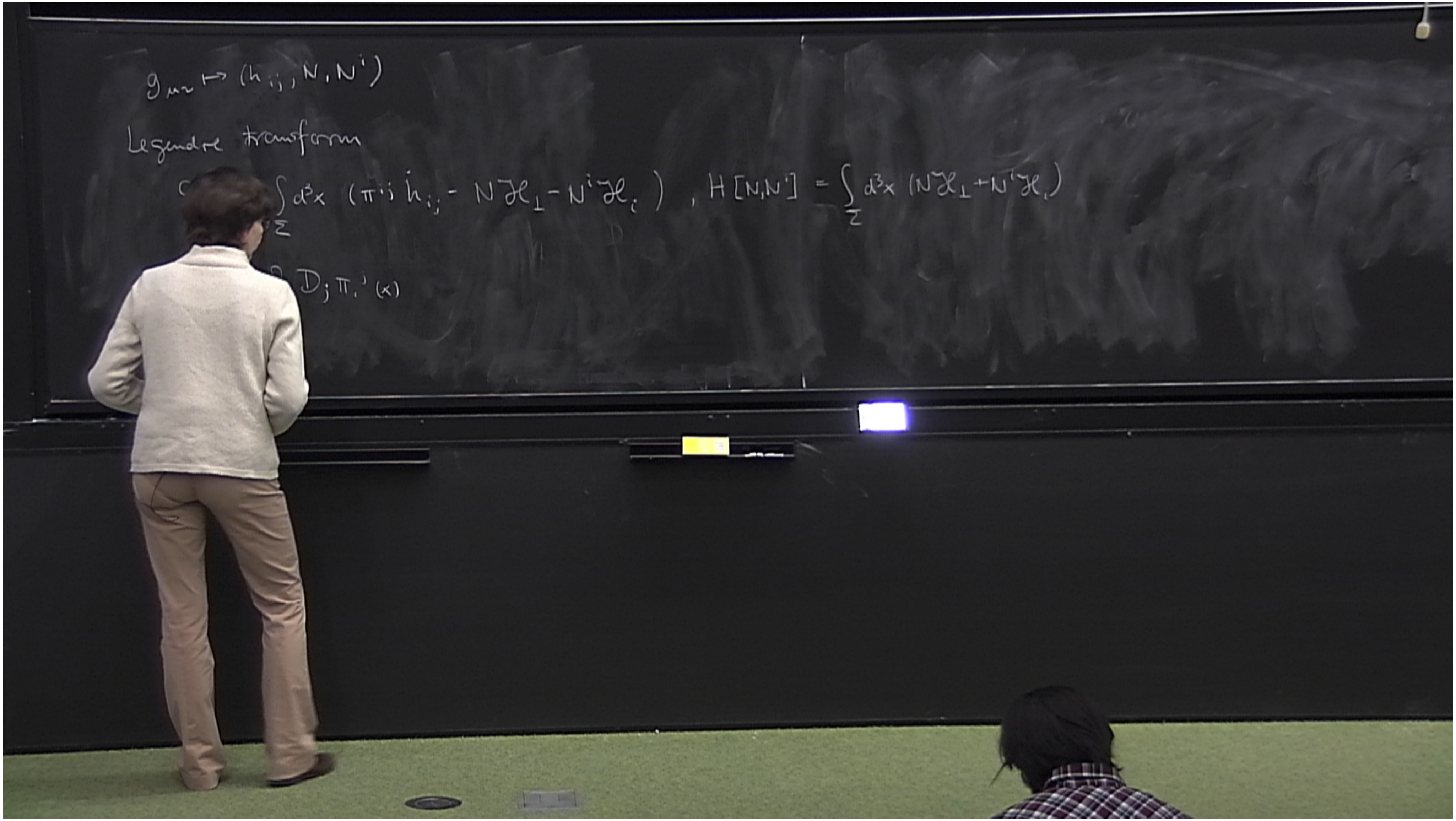
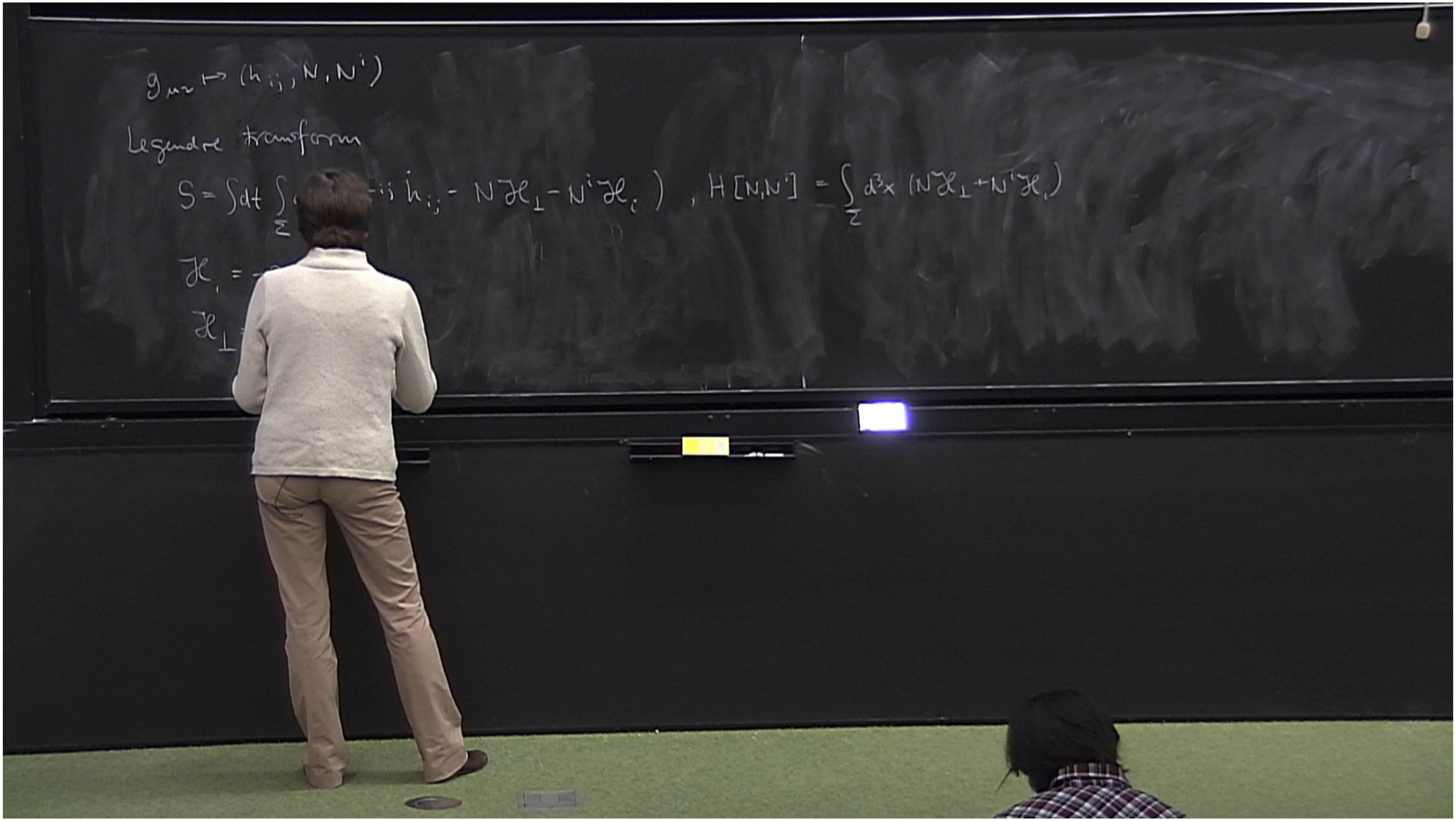
S = ∫dt ∫_Σ d³x (π^{ij} ḣ_{ij} − N ℋ_⊥ − N^i ℋ_i) , H [N,N

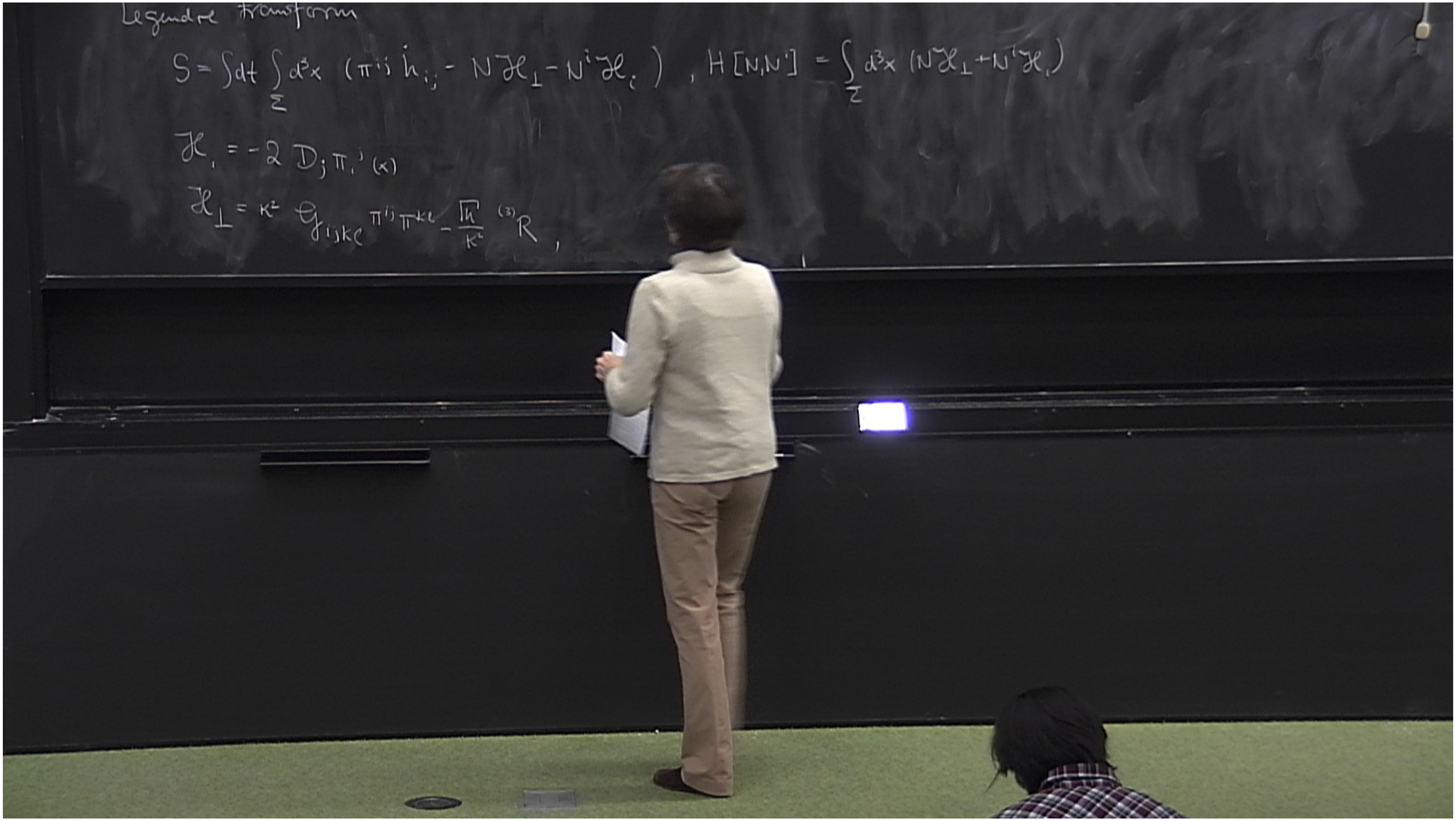

$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i)$, $H[N, N^i]$

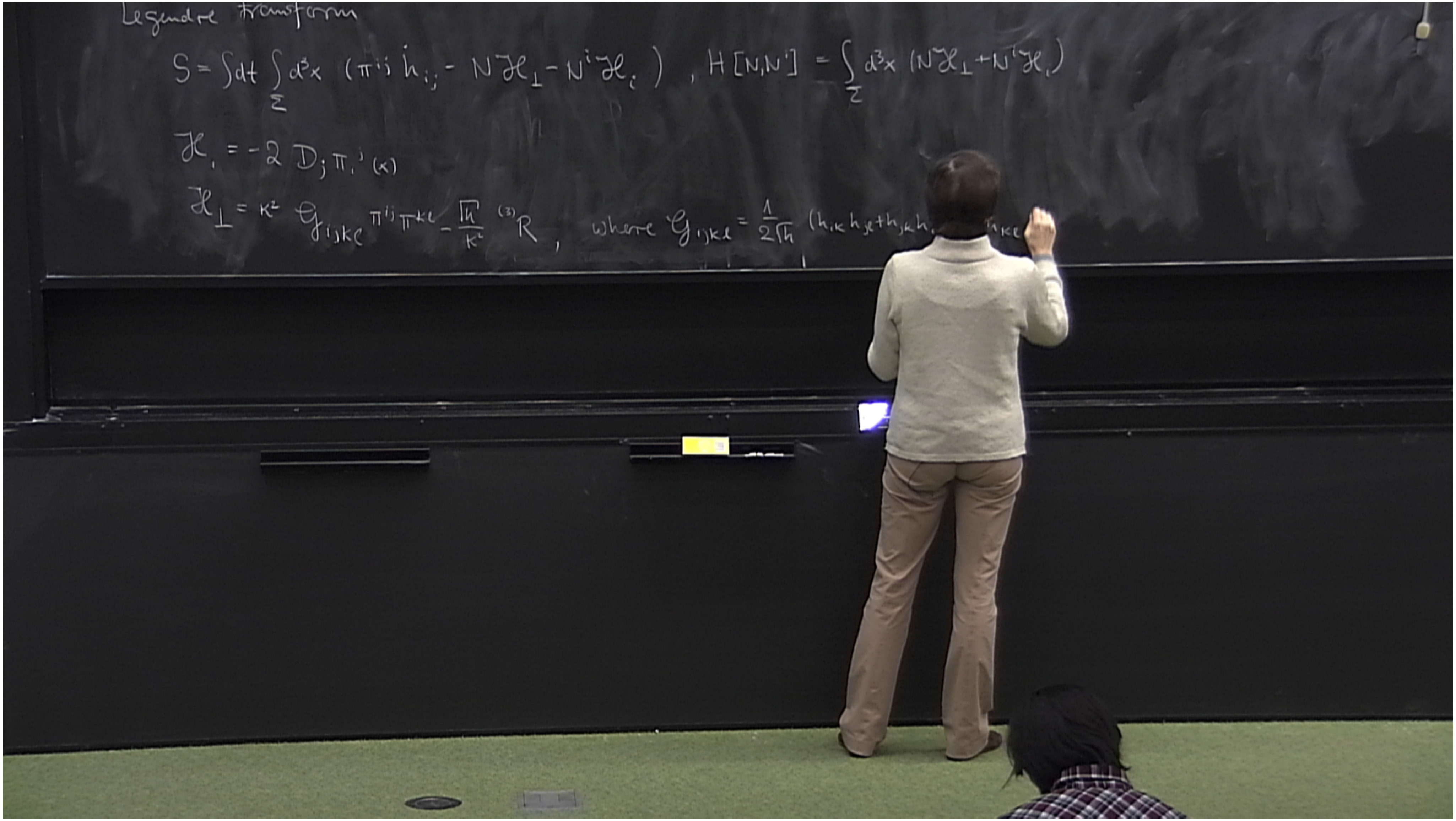

$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N\mathcal{H}$
$^i\mathcal{H}_i)$
$, H[N,N^i] = \int_\Sigma d^3x \, (N\mathcal{H}_\perp + N^i\mathcal{H}_i)$


$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - \ldots - N^i \mathcal{H}_i)$, $H[N,N^i] = \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$


g_μν ↦ (h_ij , N, N^i)
Legendre transform
∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i) , H[N,N^i] = ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
D_j π_i^j (x)


g_μν ↦ (h_ij, N, N^i)
Legendre transform
S = ∫dt ∫_Σ ḣ_ij − N ℋ_⊥ − N^i ℋ_i) , H [N, N^i] = ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i =
ℋ_⊥


Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i)$, $H[N, N^i] = \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$
$\mathcal{H}_i = -2 D_j \pi_i{}^j (x)$
$\mathcal{H}_\perp = \kappa^2 \mathcal{G}_{ijkl} \pi^{ij} \pi^{kl} - \frac{\sqrt{h}}{\kappa^2} {}^{(3)}R$,


Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i)$, $H[N, N^i] = \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$
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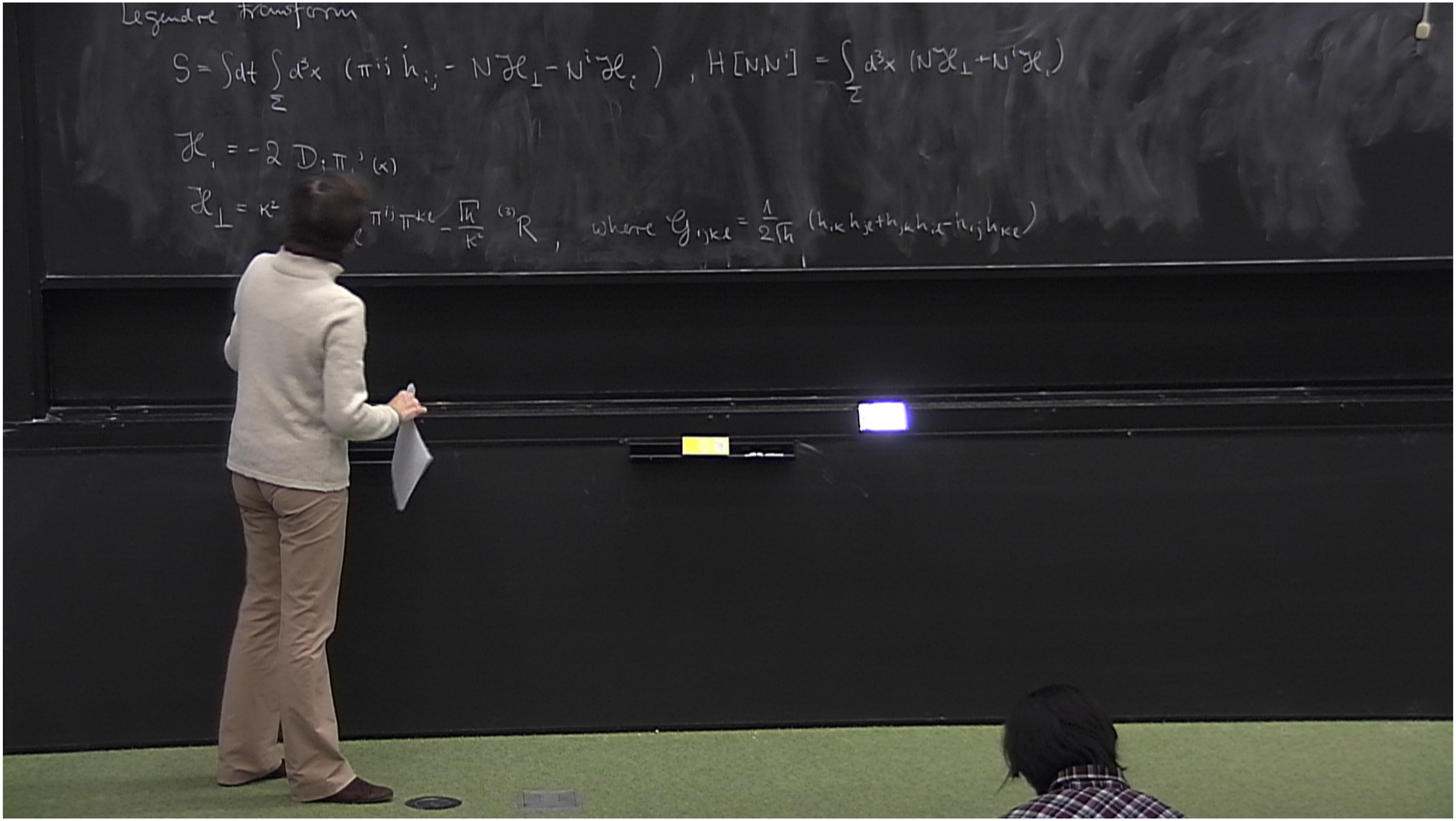
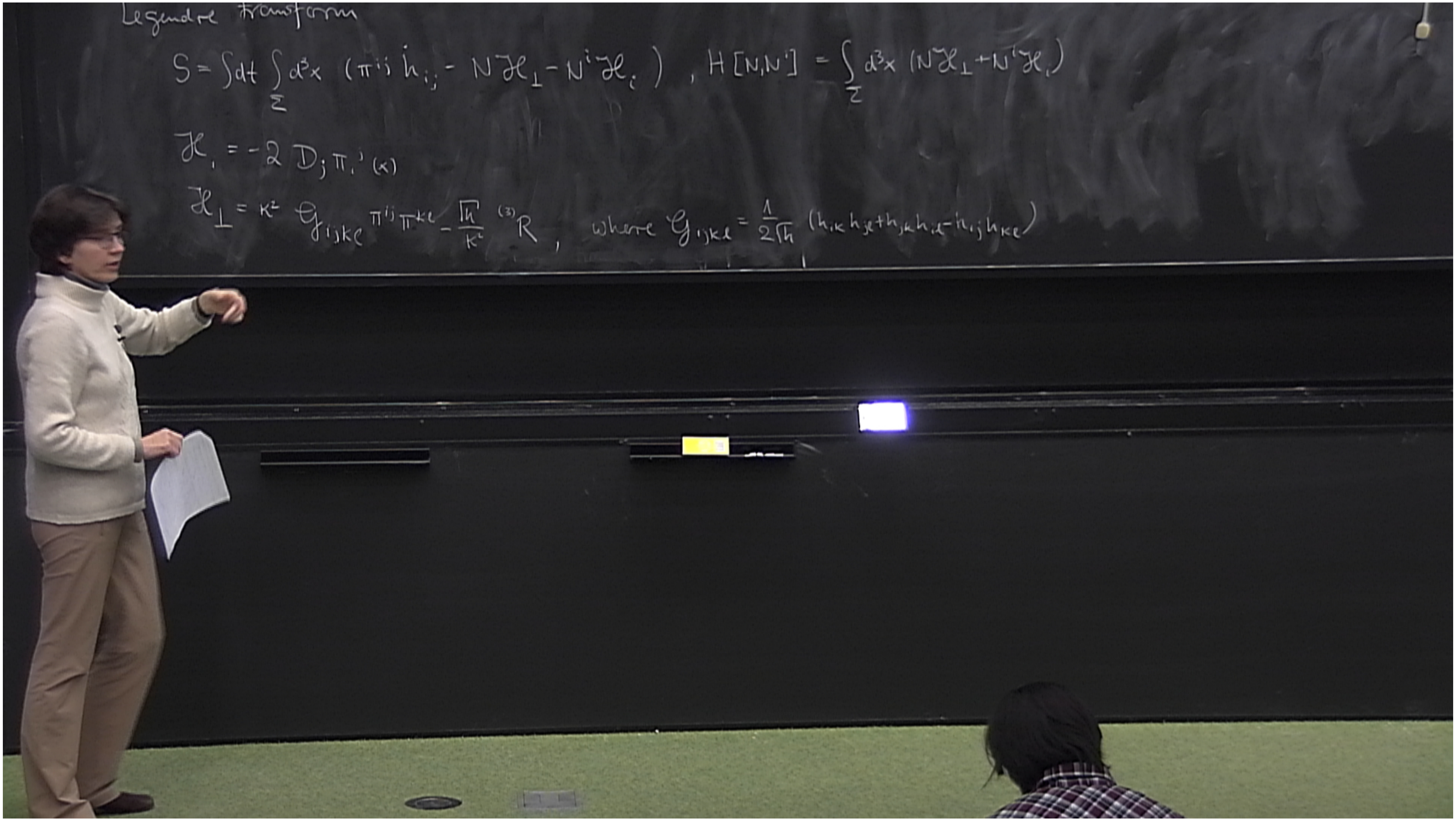
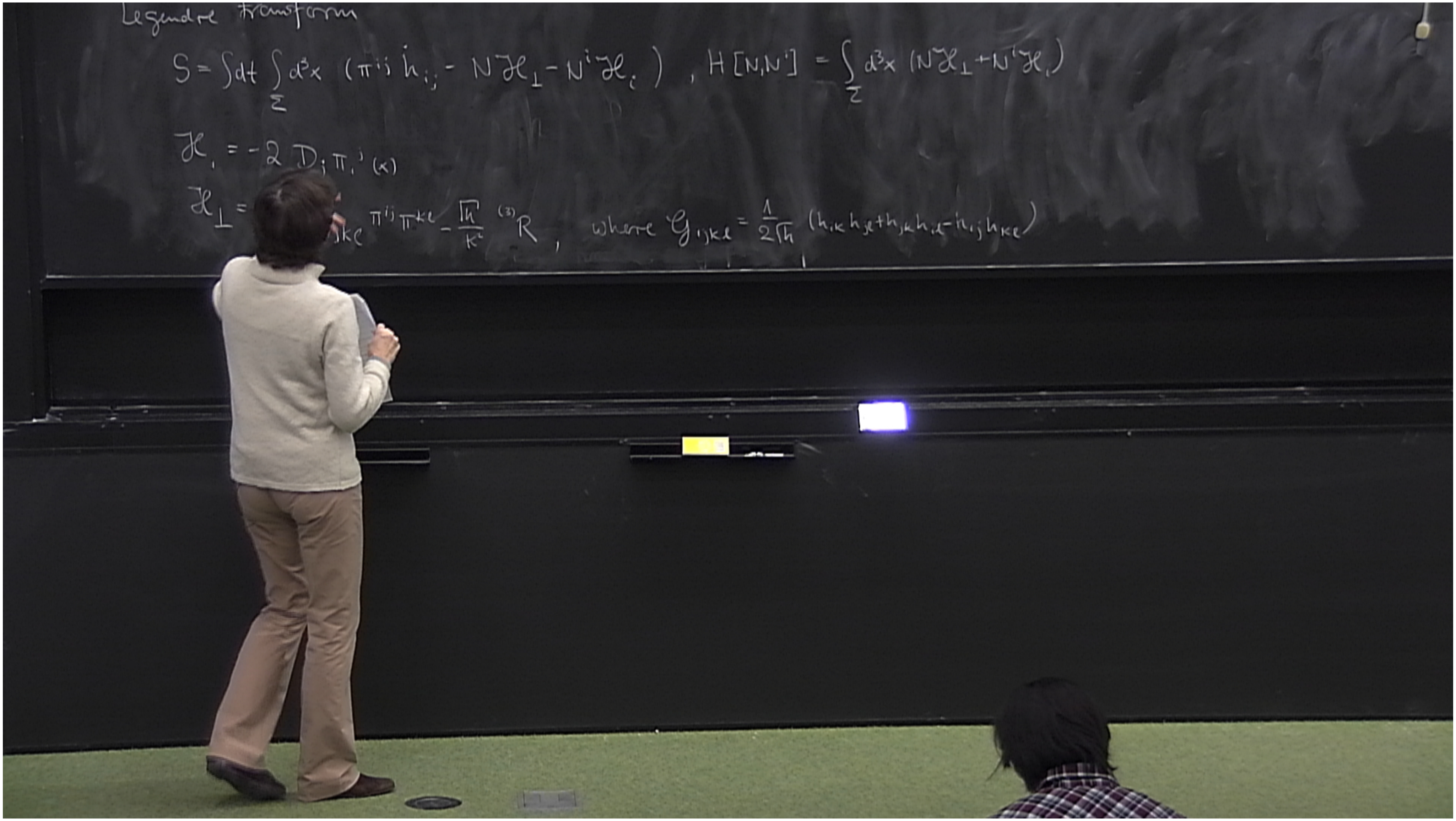
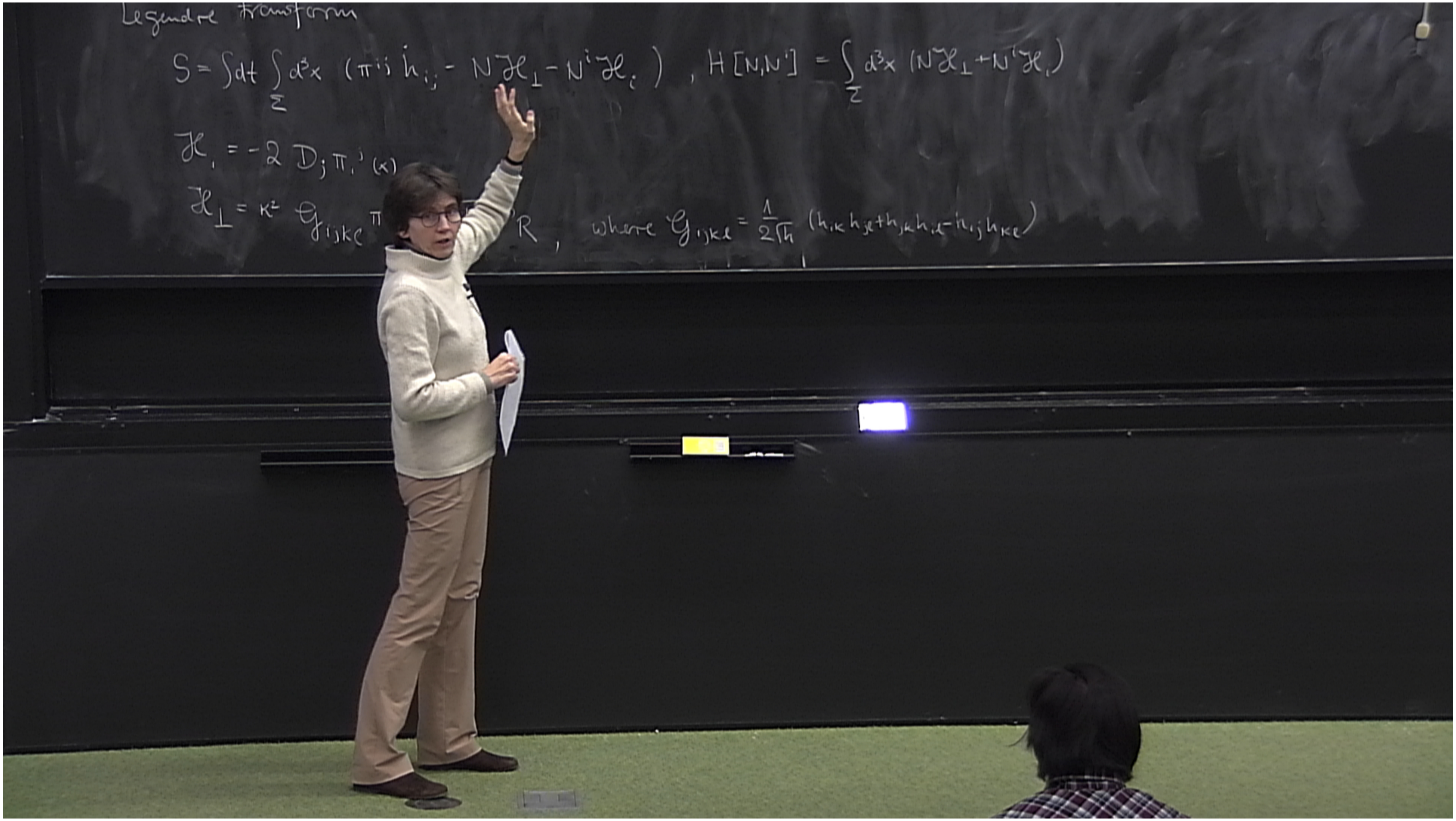
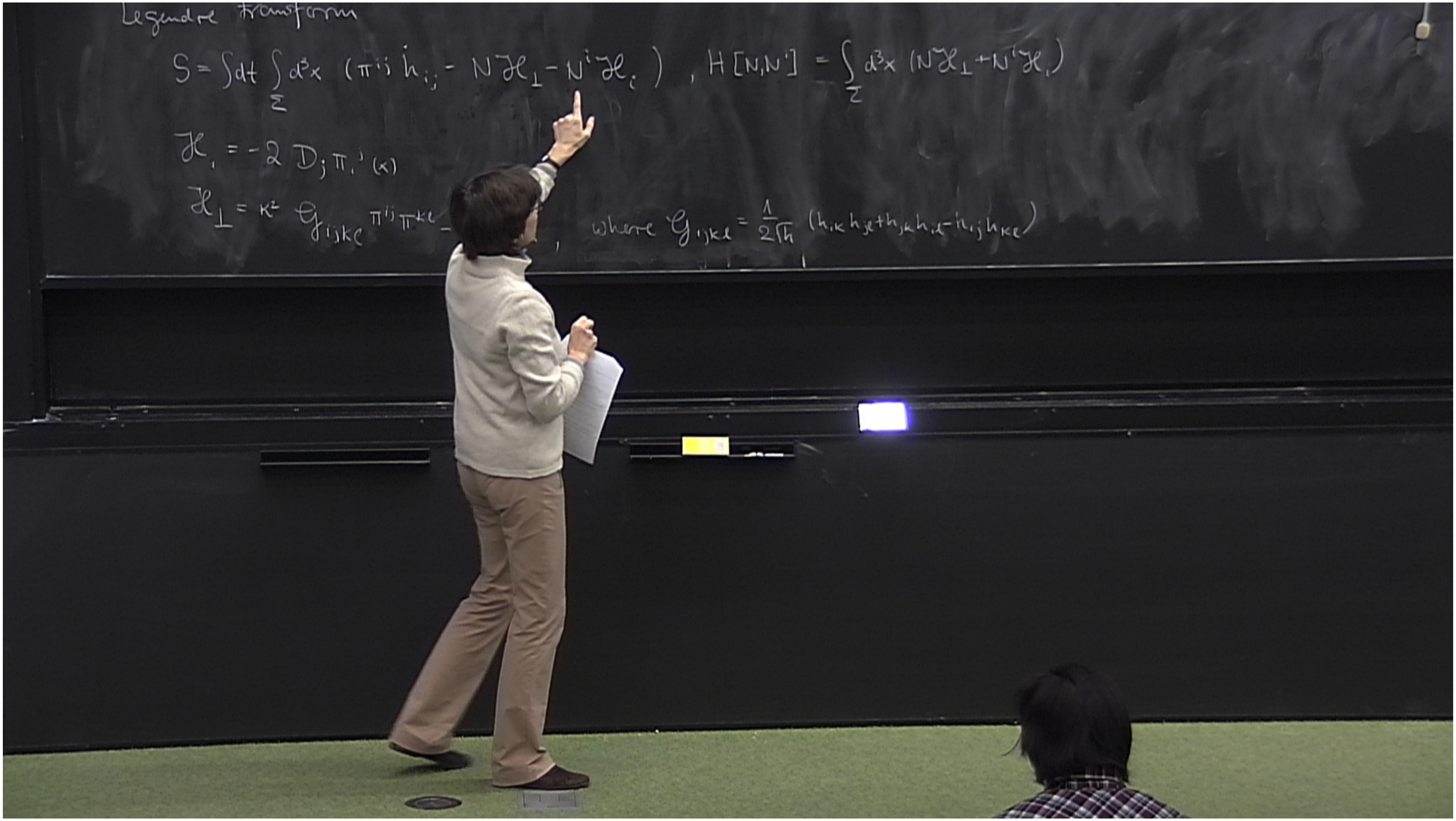
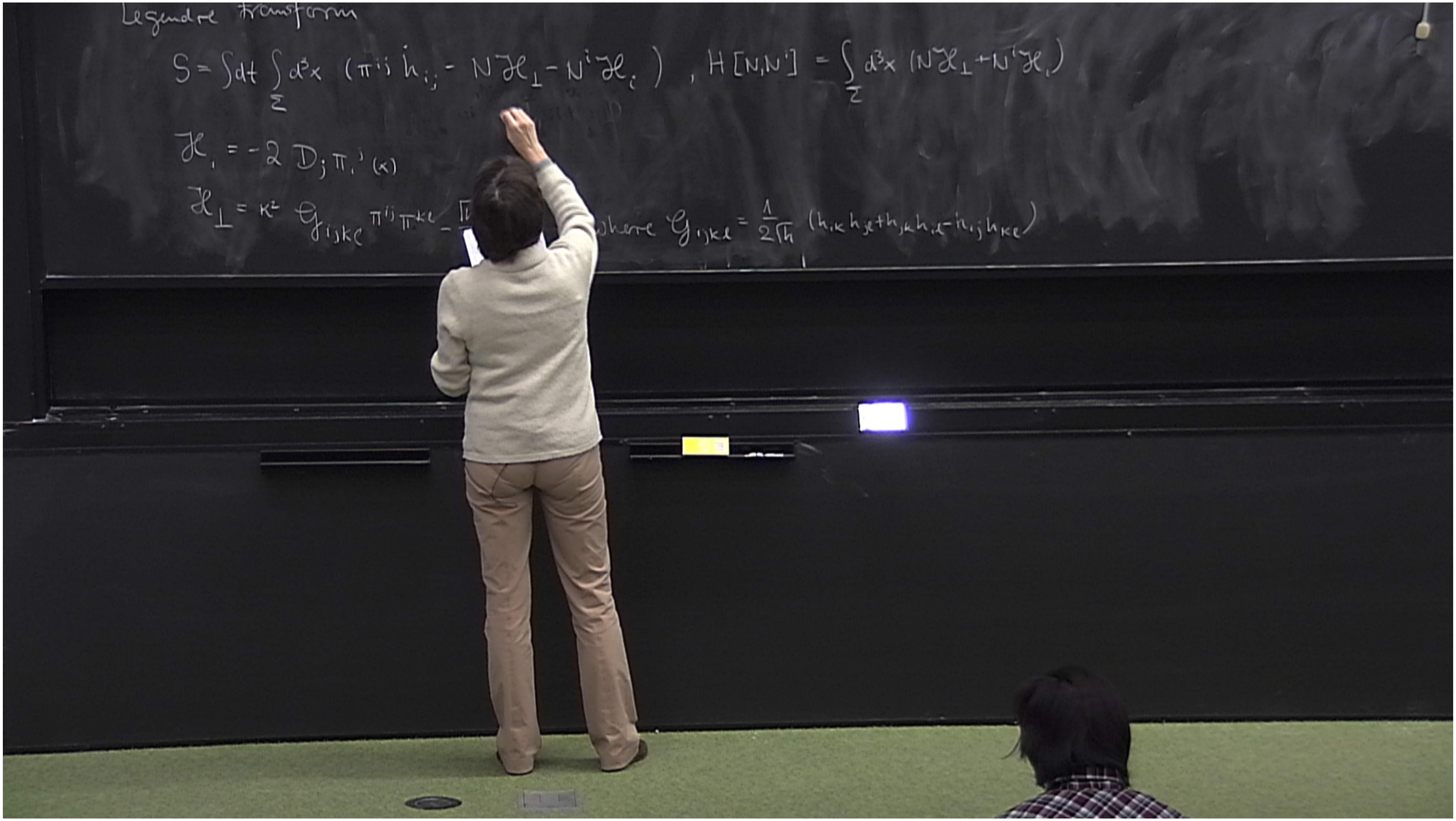

Legendre transform
S = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i) , H[N,N^i] = ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i = −2 D_j π_i^j (x)
where

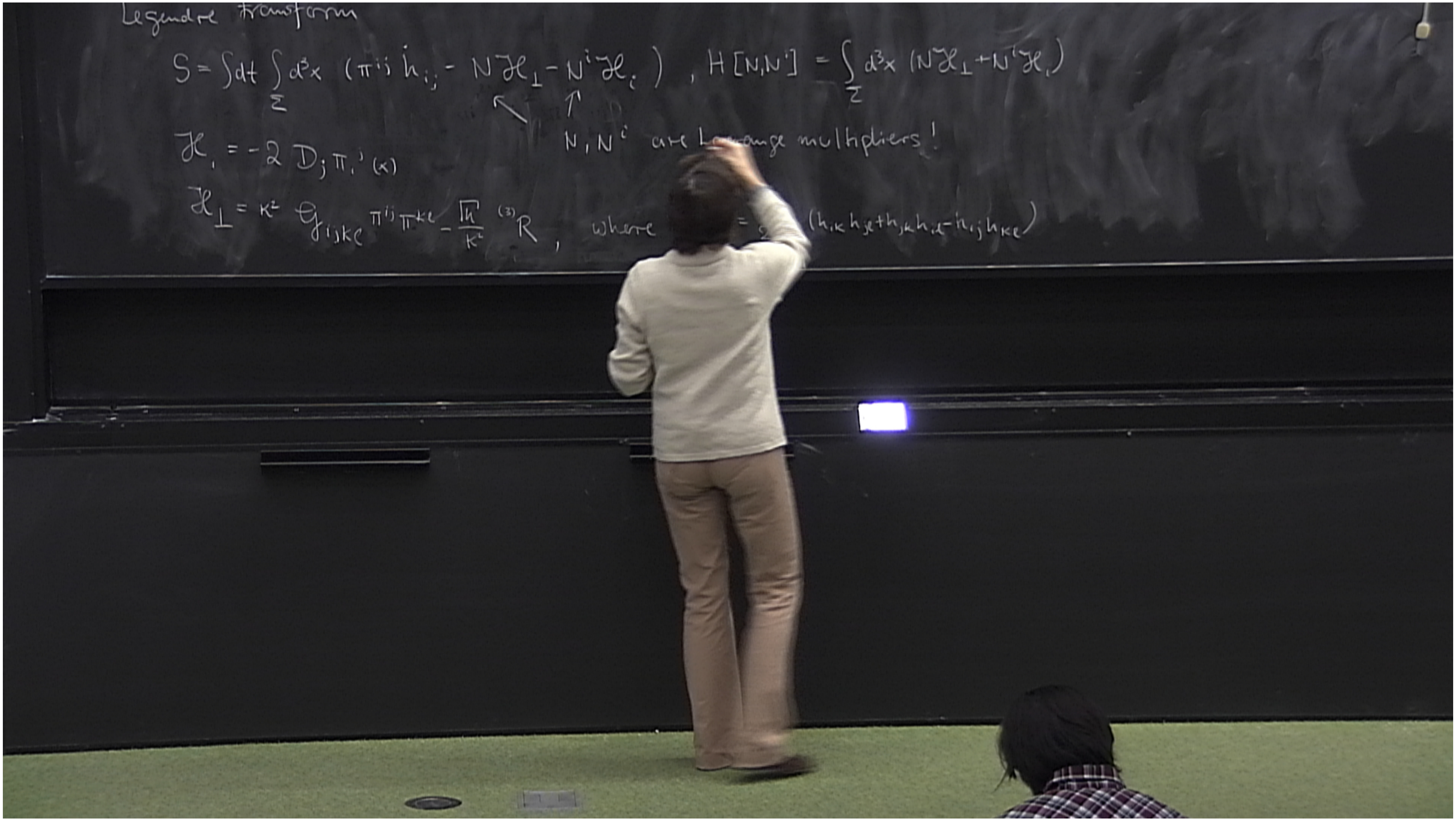

Legendre transform


Legendre transform
S = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i) , H[N,N^i] = ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i = −2 D_j π_i^j (x)
ℋ_⊥ = … π^ij π^kl − √h/κ² (3)R , where G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il − h_ij h_kl)

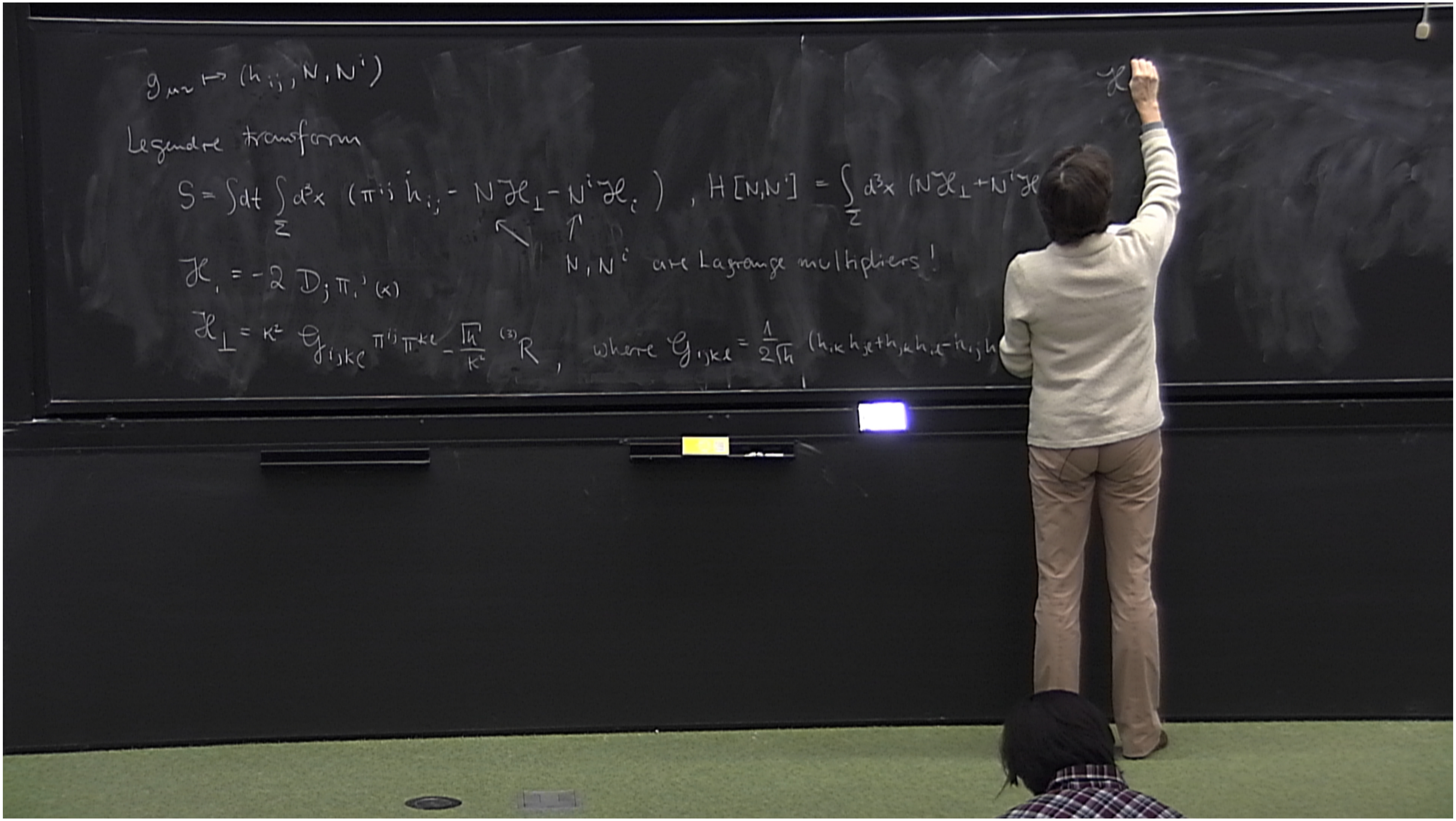

Legendre transform
S = ∫dt ∫Σ d³x (π^ij ḣ_ij − N ℋ⊥ − N^i ℋ_i) , H[N,N^i] = ∫Σ d³x (N ℋ⊥ + N^i ℋ_i)
ℋ_i = −2 D_j π_i^j (x)
where G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il − h_ij h_kl)

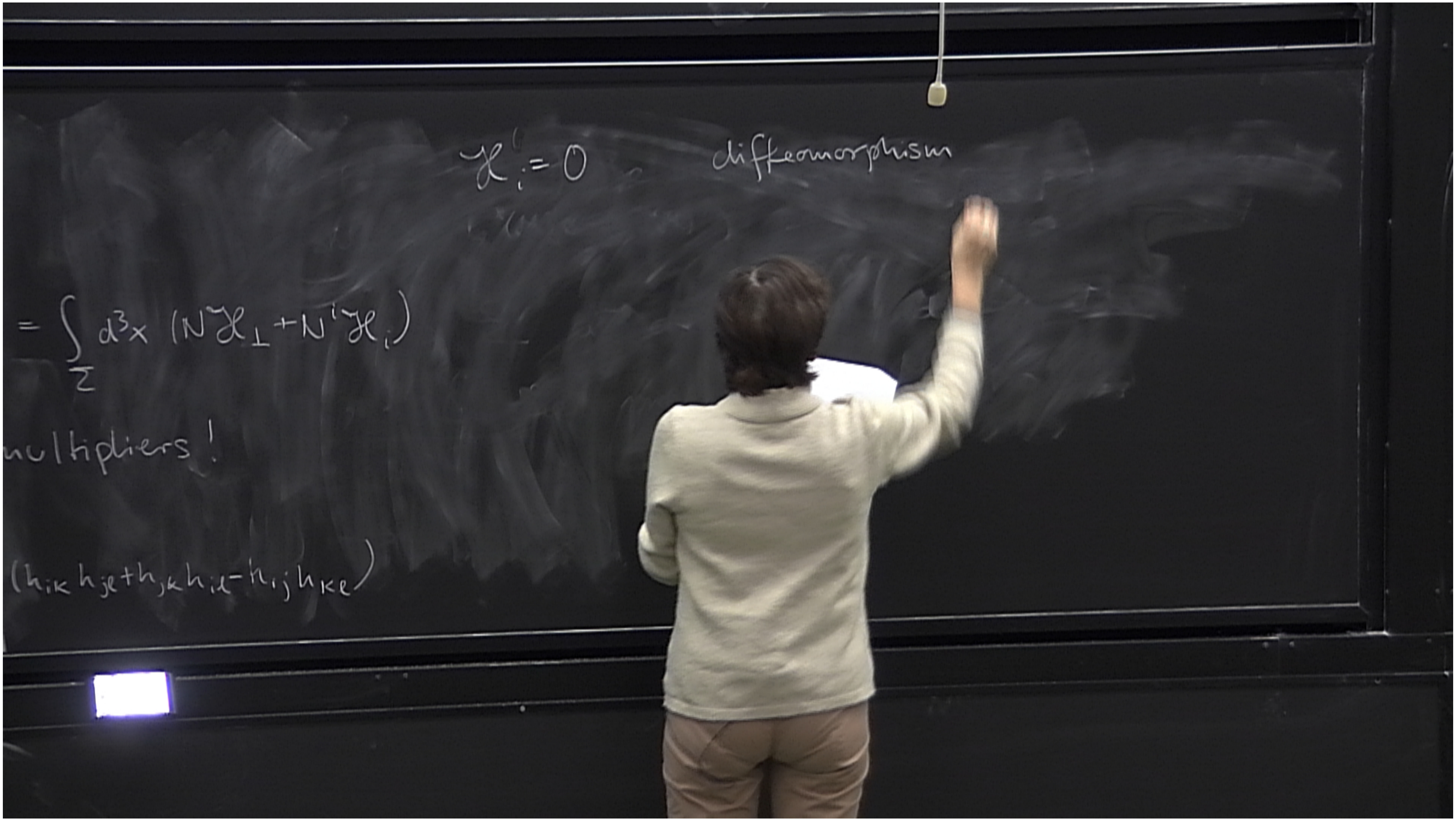

Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N\mathcal{H}_\perp - N^i \mathcal{H}_i)$, $H[N, N^i] = \int_\Sigma d^3x \, (N\mathcal{H}_\perp + N^i \mathcal{H}_i)$
$\mathcal{H}_i = -2 D_j \pi_i{}^j (x)$
$\mathcal{H}_\perp = \kappa^2 G_{ijkl} \pi^{ij} \pi^{kl}$
where $G_{ijkl} = \frac{1}{2\sqrt{h}} (h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h_{kl})$

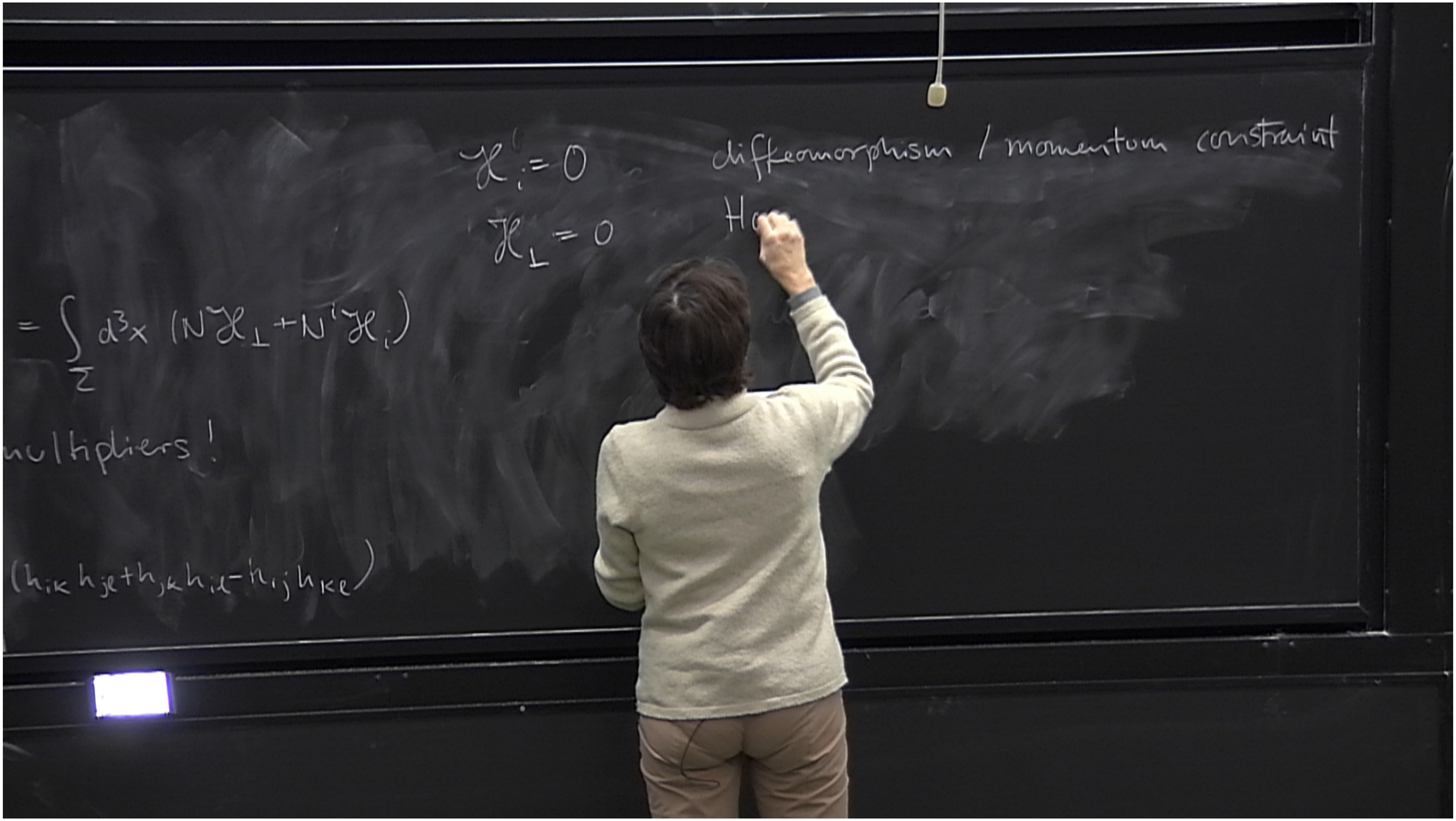

Legendre transform
S = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i) , H [N,N^i] = ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i = −2 D_j π_i^j (x)
where G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il − h_ij h_kl)

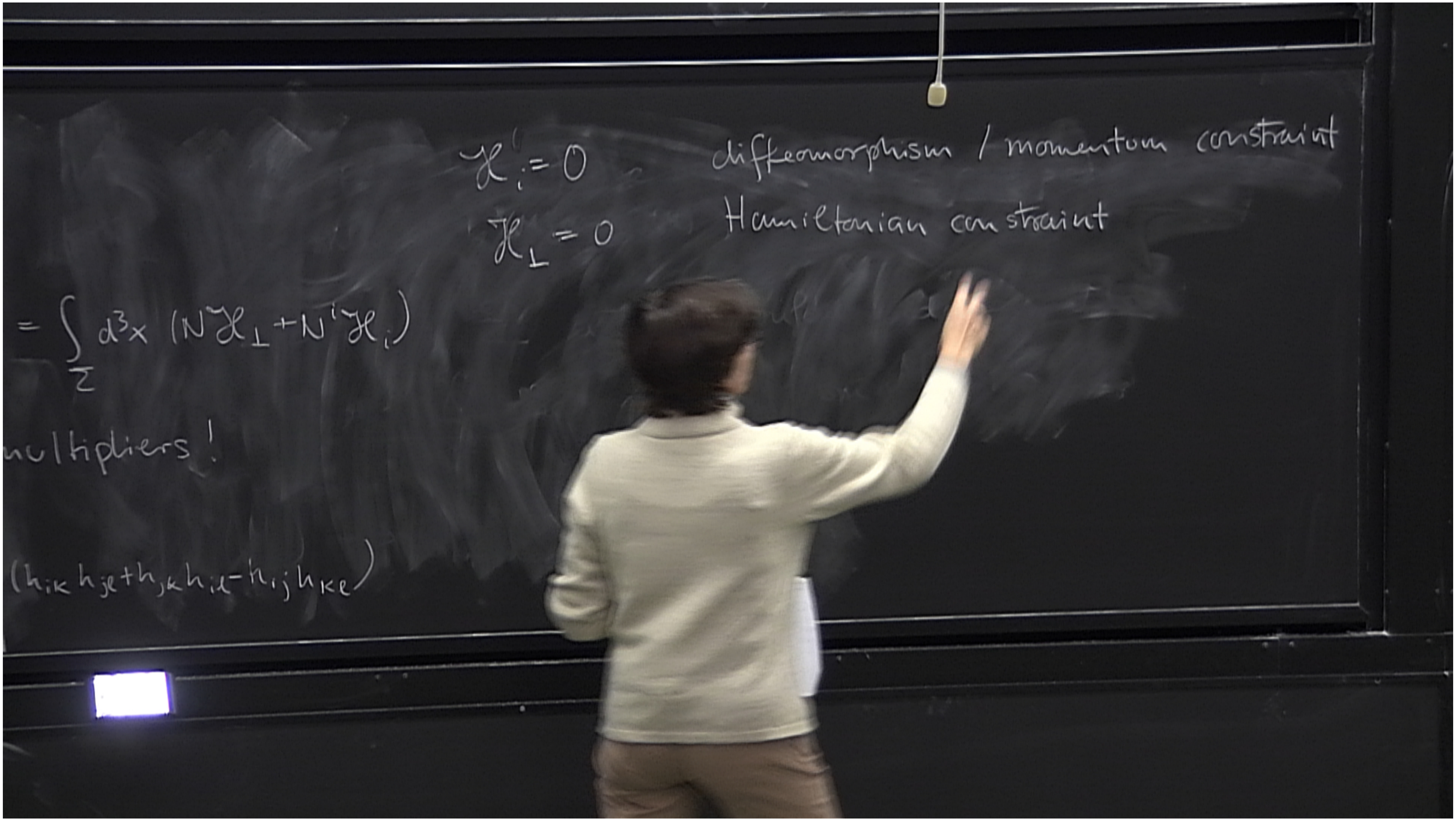


Legendre transform
S = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N H_⊥ − N^i H_i) , H[N,N^i] = ∫_Σ d³x (N H_⊥ + N^i H_i)
N, N^i are L ange multipliers!
H_i = −2 D_j π_i^j (x)
H_⊥ = κ² G_ijkl π^ij π^kl − √h/κ² ⁽³⁾R , where (h_ik h_jl + h_jk h_il − h_ij h_kl)

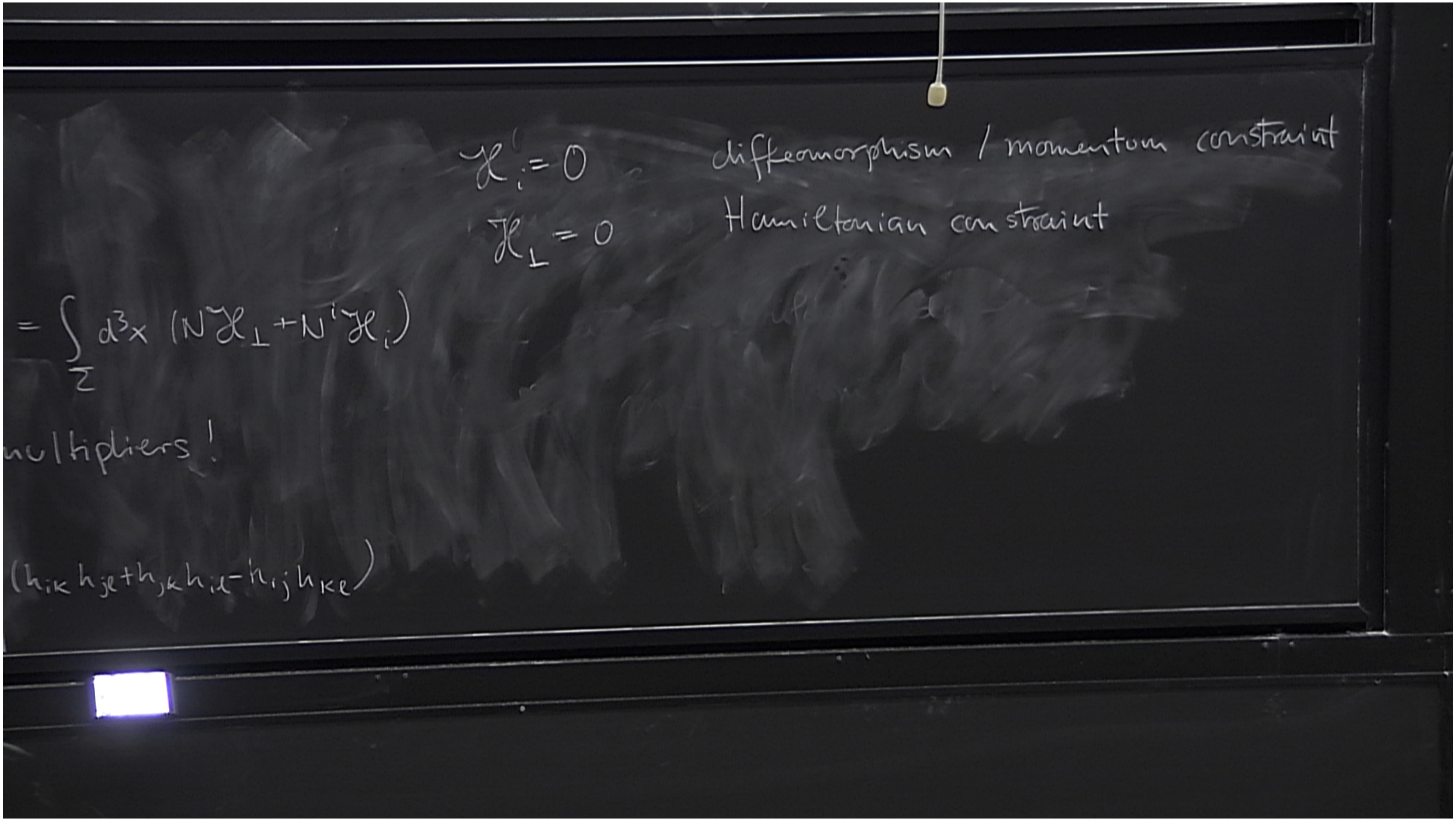

$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
Legendre transform
$S = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i)$, $H[N, N^i] = \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}$
N, N^i are Lagrange multipliers!
$\mathcal{H}_i = -2 D_j \pi_i{}^j (x)$
$\mathcal{H}_\perp = \kappa^2 \mathcal{G}_{ijkl} \pi^{ij} \pi^{kl} - \frac{\sqrt{h}}{\kappa^2} {}^{(3)}R$, where $\mathcal{G}_{ijkl} = \frac{1}{2\sqrt{h}} (h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h$
$\mathcal{H}$

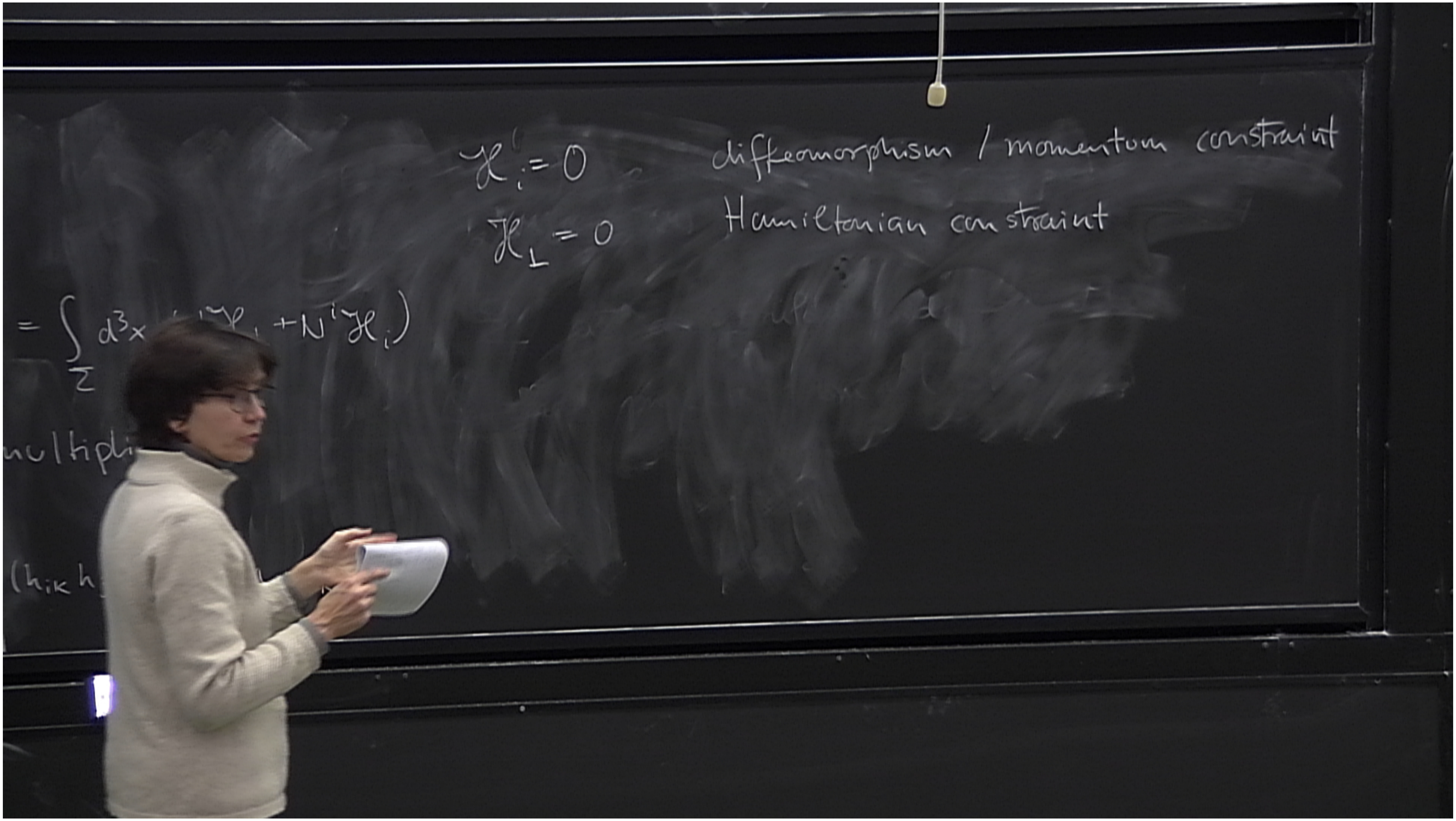

H_i = 0
diffeomorphism
= ∫ d³x (N H⊥ + N^i H_i)
Σ
ultipliers!
(h_ik h_jl + h_jk h_il − h_ij h_kl)

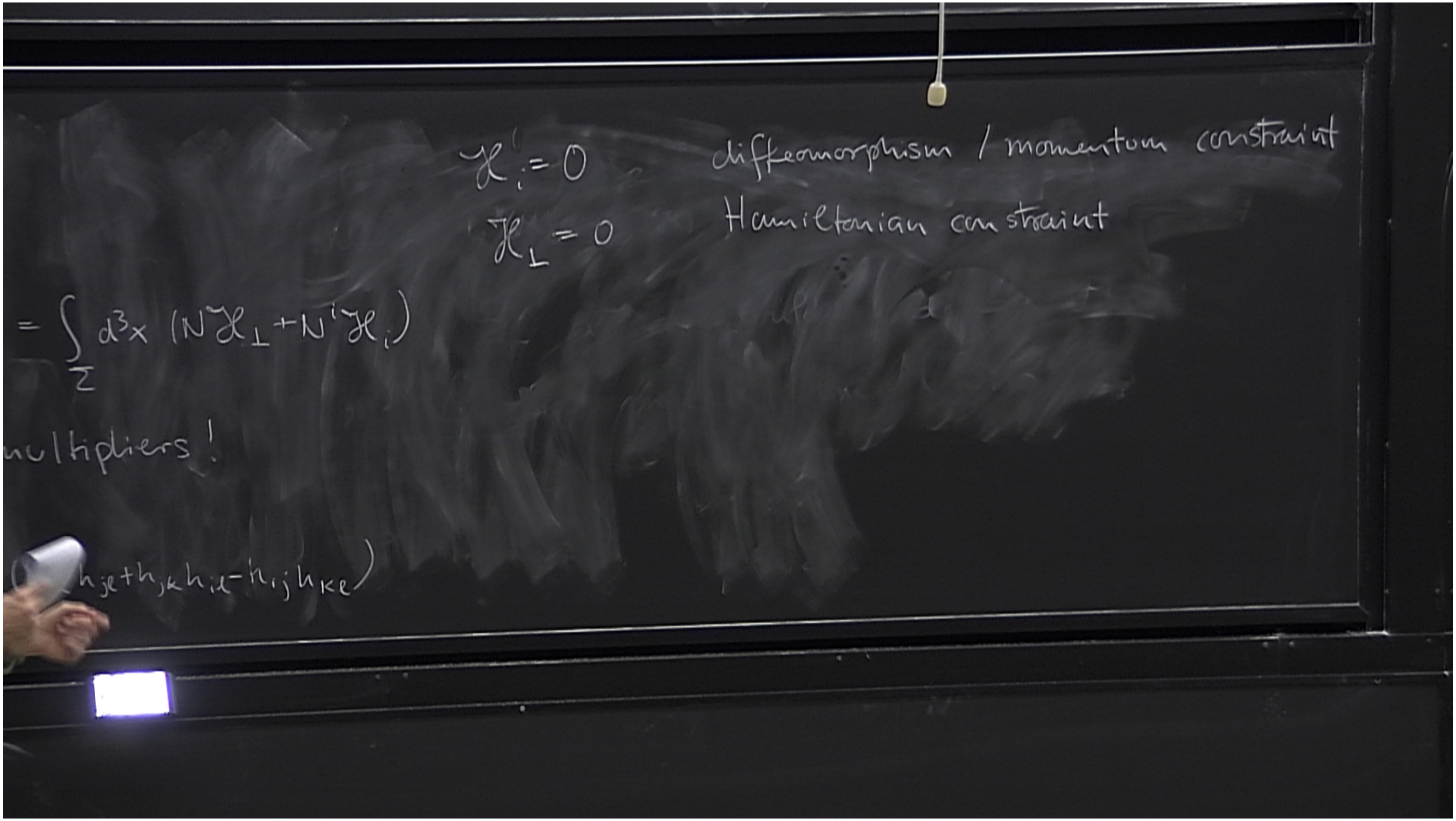

$\mathcal{H}_i = 0$
diffeomorphism / momentum constraint
$\mathcal{H}_\perp = 0$
$= \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$
multipliers!
$(h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h_{kl})$

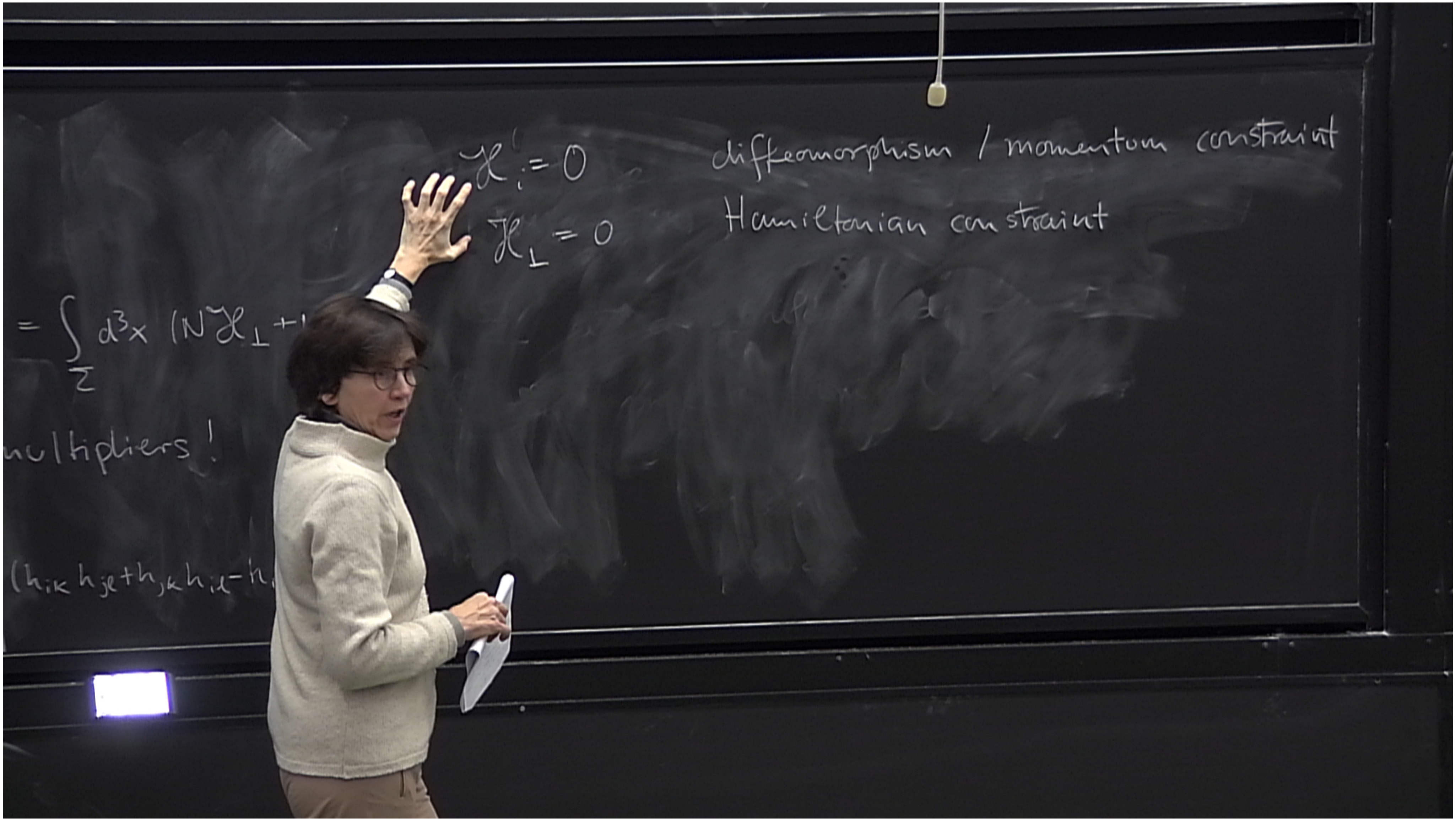


ℋ_i = 0
diffeomorphism / momentum constraint
ℋ_⊥ = 0
Hamiltonian constraint
= ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
multipliers!
(h_ik h_jl + h_jk h_il − h_ij h_kl)

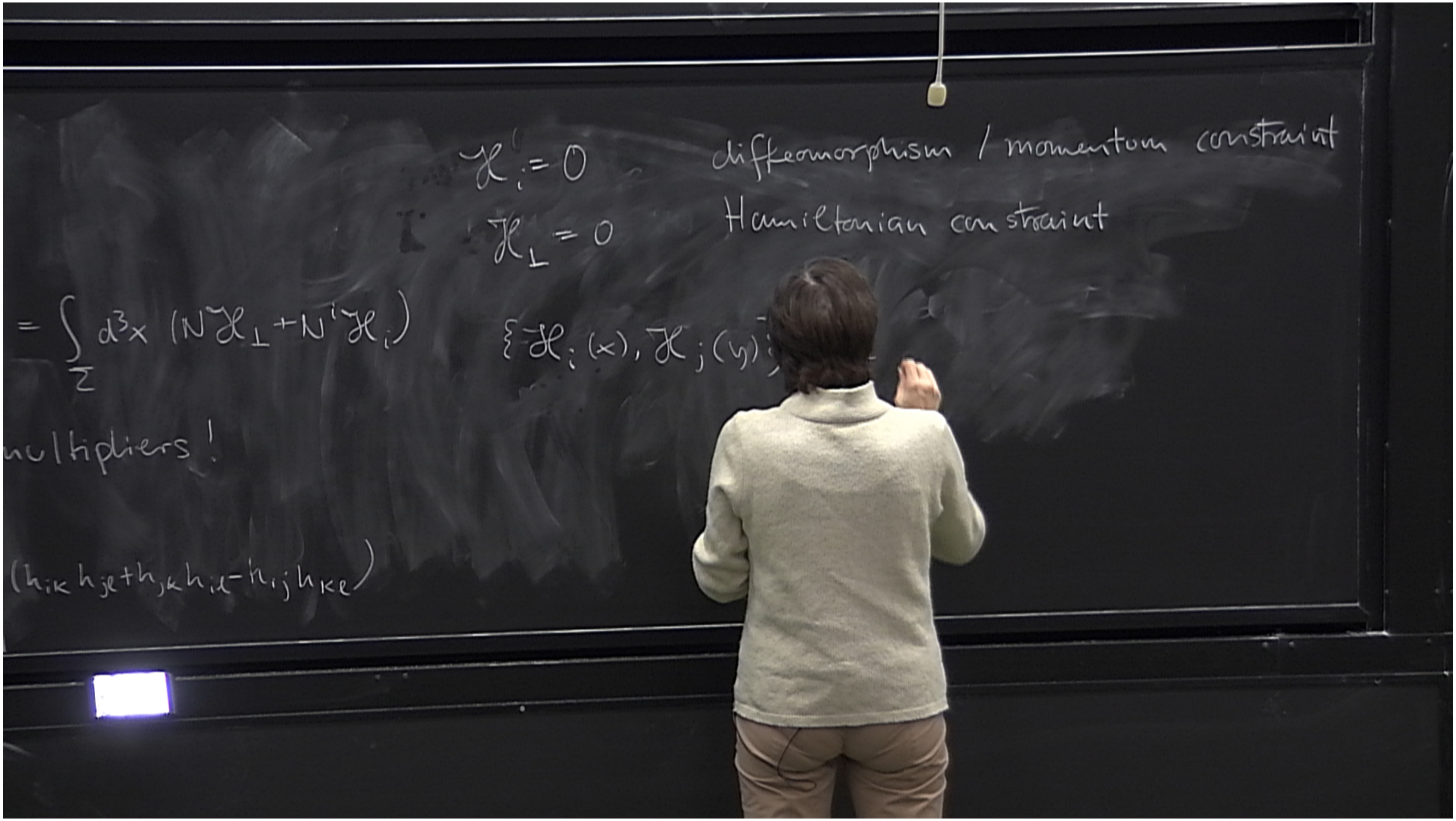

$\mathcal{H}_i = 0$ diffeomorphism / momentum constraint
$\mathcal{H}_\perp = 0$ Hamiltonian constraint
$= \int_\Sigma d^3x \, (N\mathcal{H}_\perp + N^i\mathcal{H}_i)$
ultipliers !
$(h_{ik}h_{jl} + h_{jk}h_{il} - h_{ij}h_{kl})$

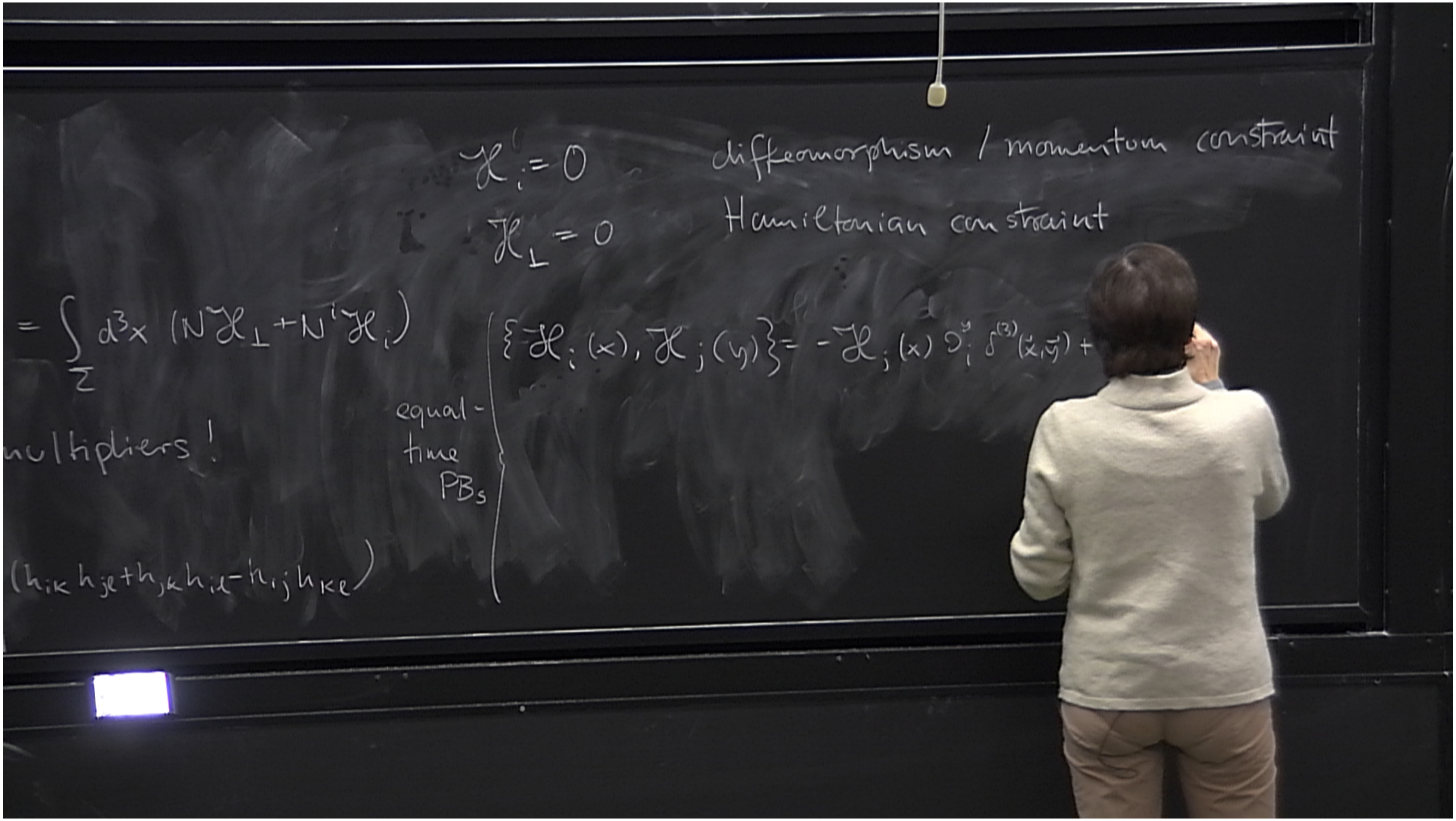

diffeomorphism / momentum constraint
Hamiltonian constraint

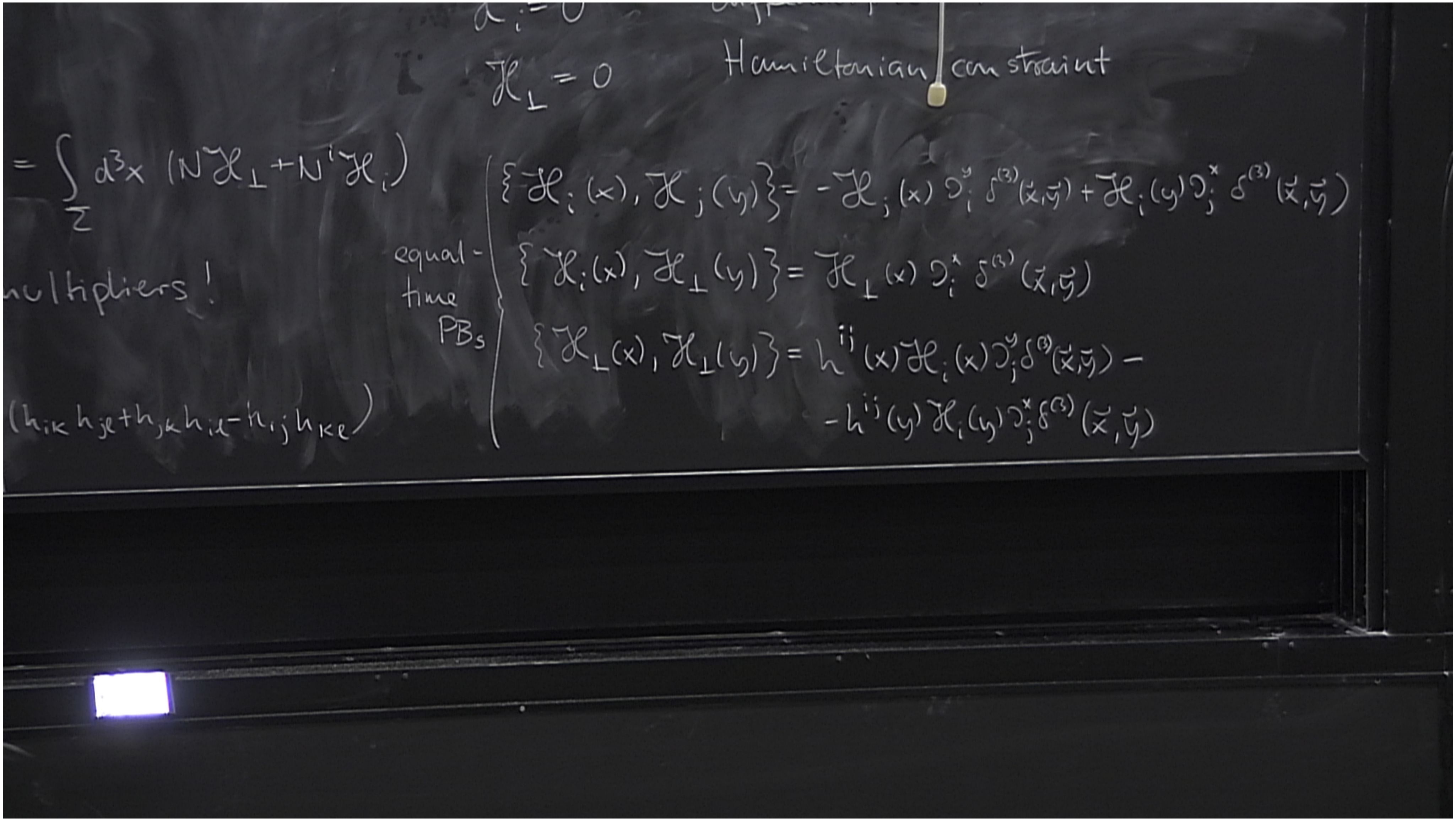

ℋ_i = 0 diffeomorphism / momentum constraint
ℋ_⊥ = 0 Hamiltonian constraint



$\mathcal{H}_i = 0$
diffeomorphism / momentum constraint
$\mathcal{H}_\perp = 0$
Hamiltonian constraint
$= \int_\Sigma d^3x \, (N \mathcal{H}_\perp +$
ultipliers!


$\mathcal{H}_i = 0$ diffeomorphism / momentum constraint
$\mathcal{H}_\perp = 0$ Hamiltonian constraint
$= \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$
$\{ \mathcal{H}_i(x), \mathcal{H}_j(y) \}$
ultipliers !
$(h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h_{kl})$


diffeomorphism / momentum constraint
Hamiltonian constraint
equal-
time
PBs
multipliers !



$\mathcal{H}_\perp = 0$
Hamiltonian constraint
$= \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$
ultipliers!
$(h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h_{kl})$
equal-time PBs
$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = -\mathcal{H}_j(x)\, \partial^y_i \delta^{(3)}(\vec{x},\vec{y}) + \mathcal{H}_i(y)\, \partial^x_j \delta^{(3)}(\vec{x},\vec{y})$
$\{\mathcal{H}_i(x), \mathcal{H}_\perp(y)\} = \mathcal{H}_\perp(x)\, \partial^x_i \delta^{(3)}(\vec{x},\vec{y})$
$\{\mathcal{H}_\perp(x), \mathcal{H}_\perp(y)\} = h^{ij}(x) \mathcal{H}_i(x)\, \partial^y_j \delta^{(3)}(\vec{x},\vec{y}) - h^{ij}(y) \mathcal{H}_i(y)\, \partial^x_j \delta^{(3)}(\vec{x},\vec{y})$

$\mathcal{H}_i = 0$

$\mathcal{H}_\perp = 0$ Hamiltonian constraint

Dirac algebra

$= \int_\Sigma d^3x \, (N \mathcal{H}_\perp + N^i \mathcal{H}_i)$

...multipliers!

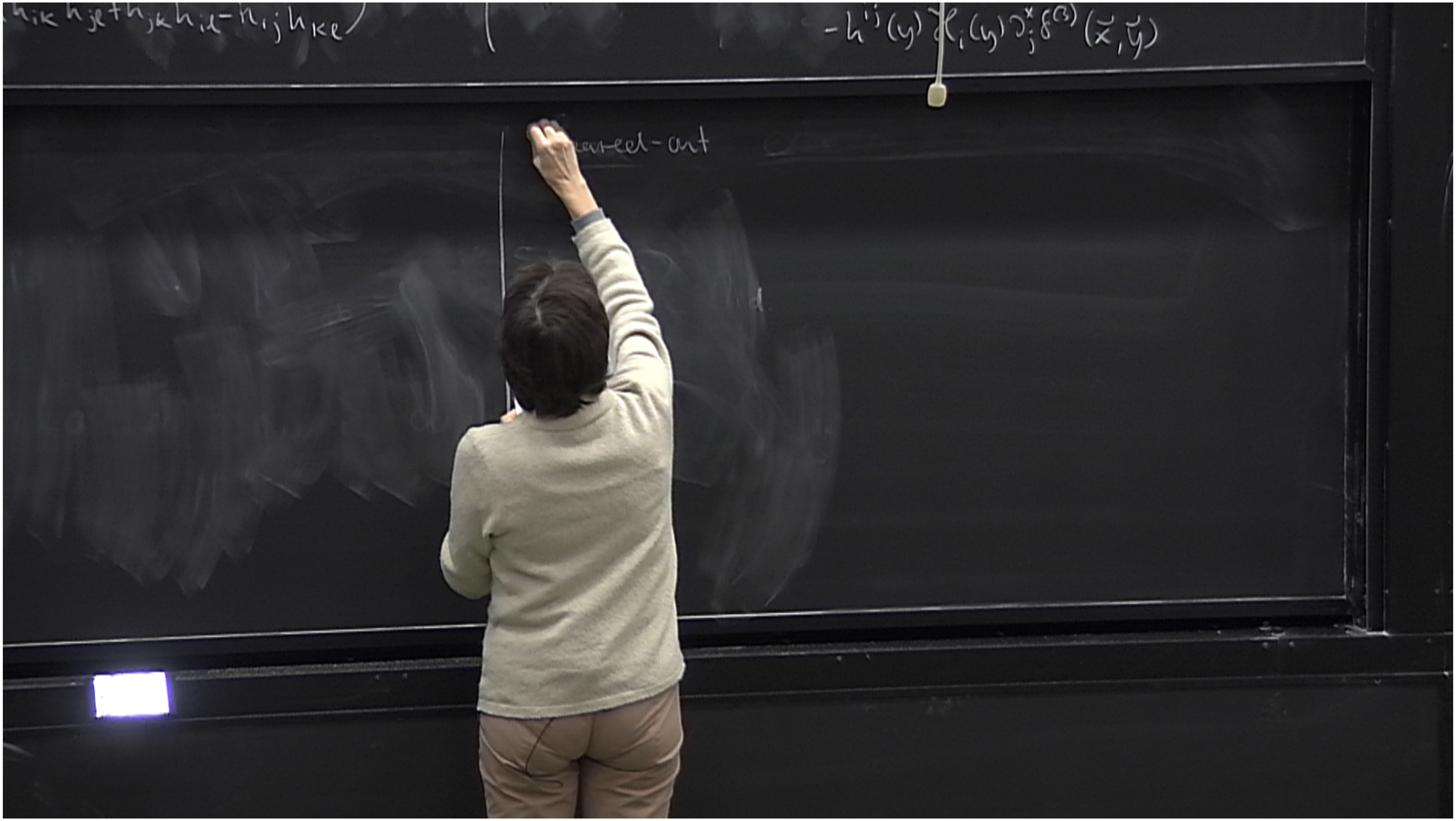
$(h_{ik} h_{jl} + h_{jk} h_{il} - h_{ij} h_{kl})$

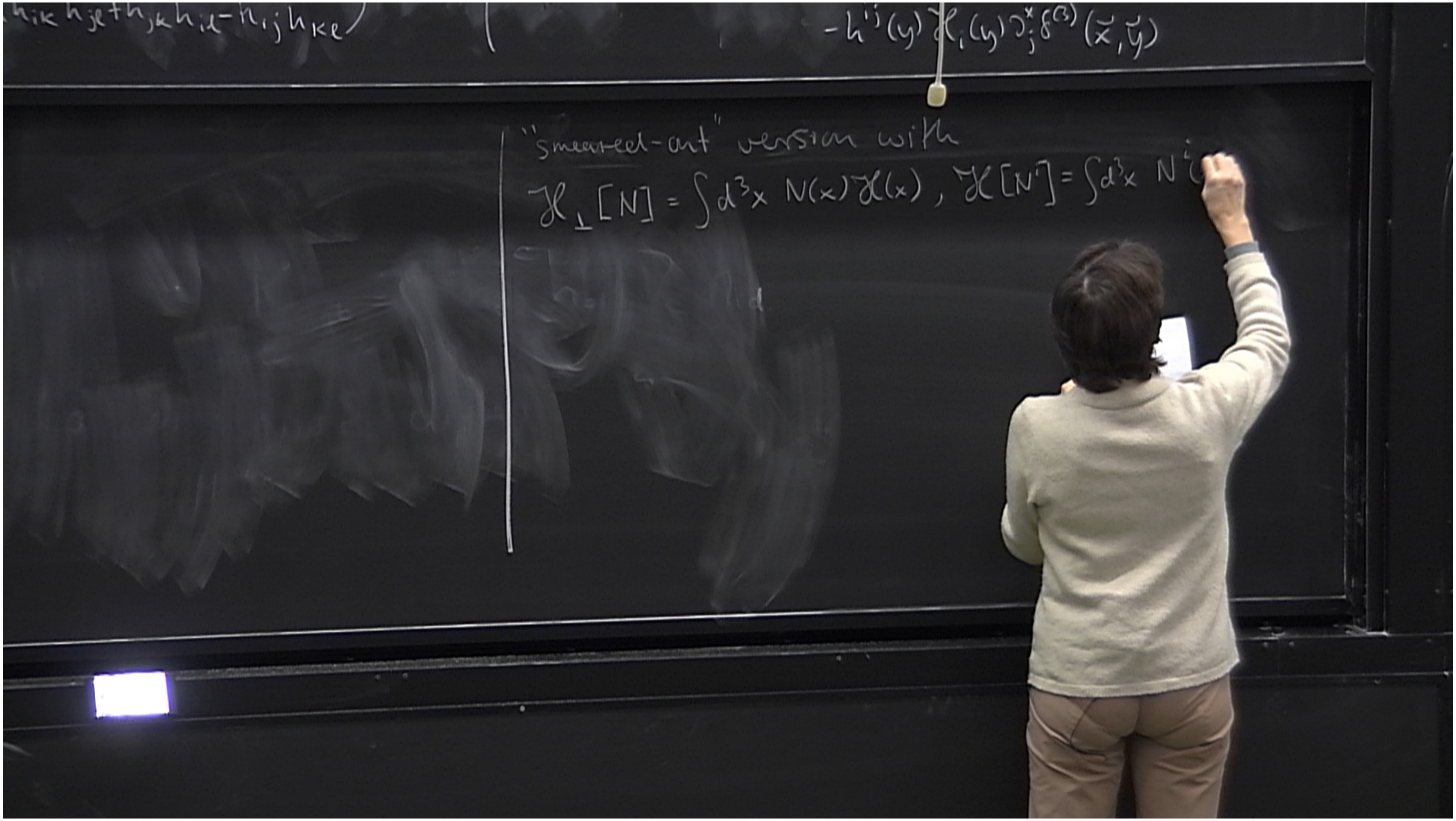
equal-time PBs

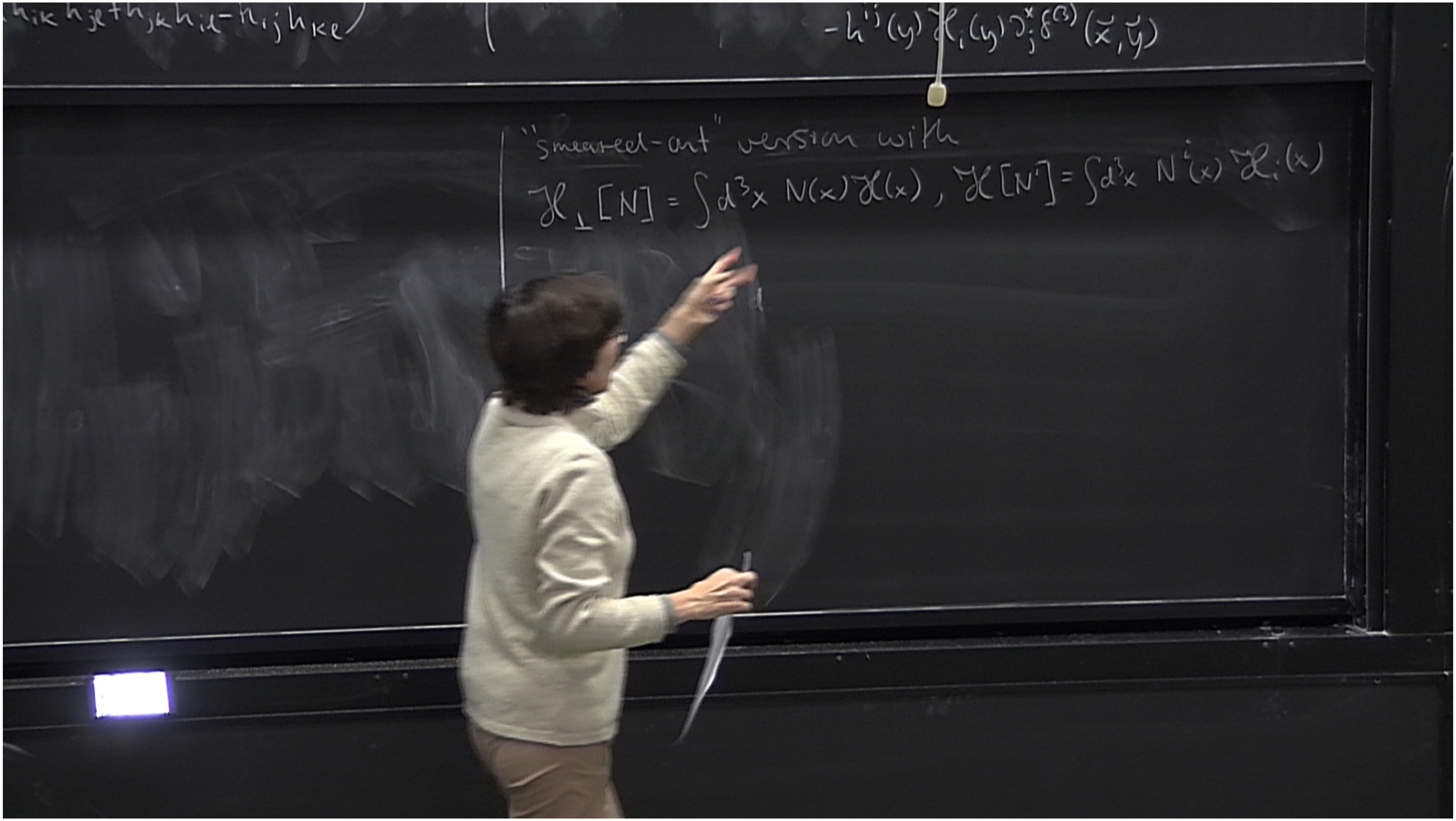
$$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = -\mathcal{H}_j(x)\, \partial_i^y \delta^{(3)}(\vec{x},\vec{y}) + \mathcal{H}_i(y)\, \partial_j^x \delta^{(3)}(\vec{x},\vec{y})$$

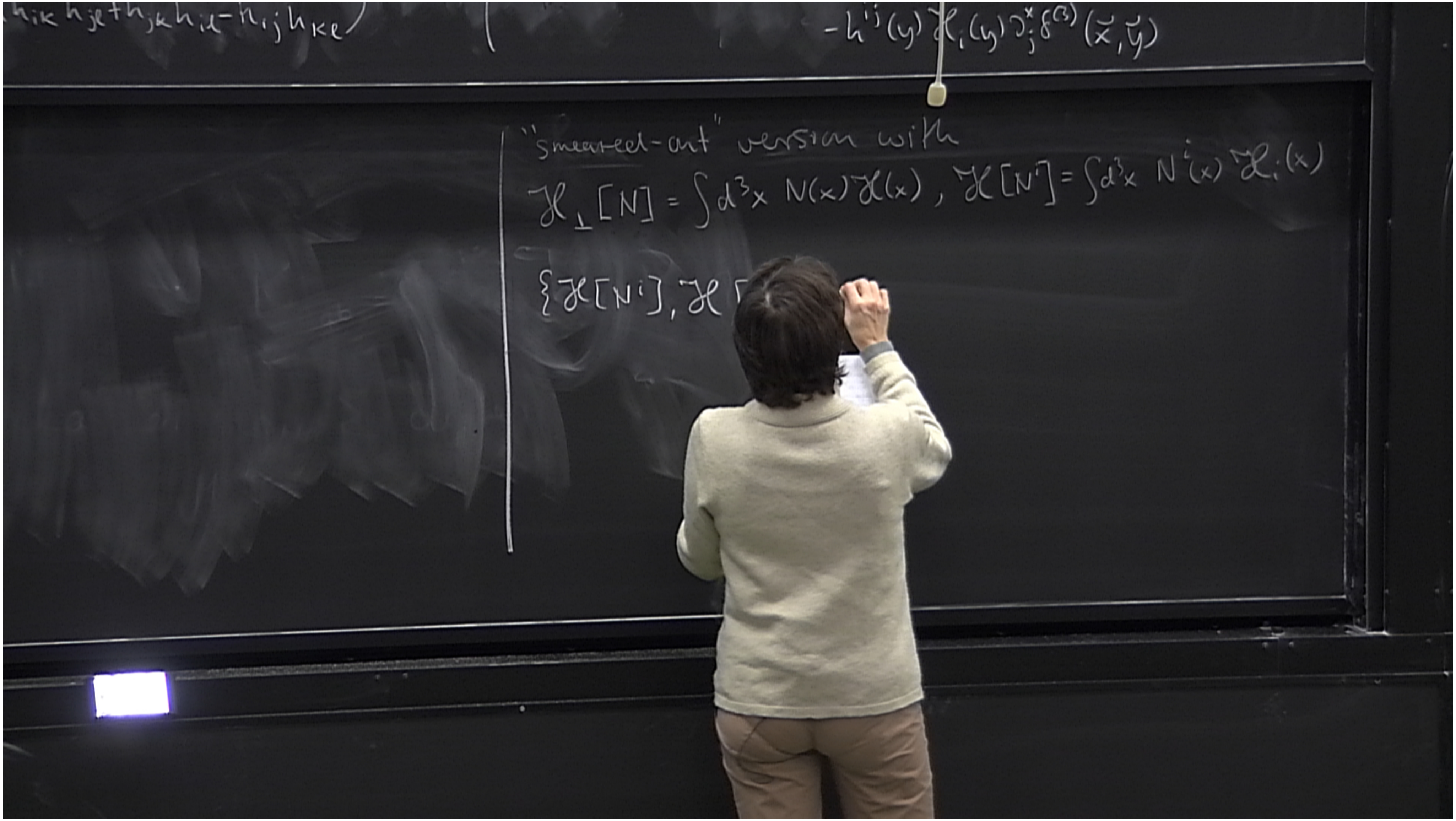
$$\{\mathcal{H}_i(x), \mathcal{H}_\perp(y)\} = \mathcal{H}_\perp(x)\, \partial_i^x \delta^{(3)}(\vec{x},\vec{y})$$

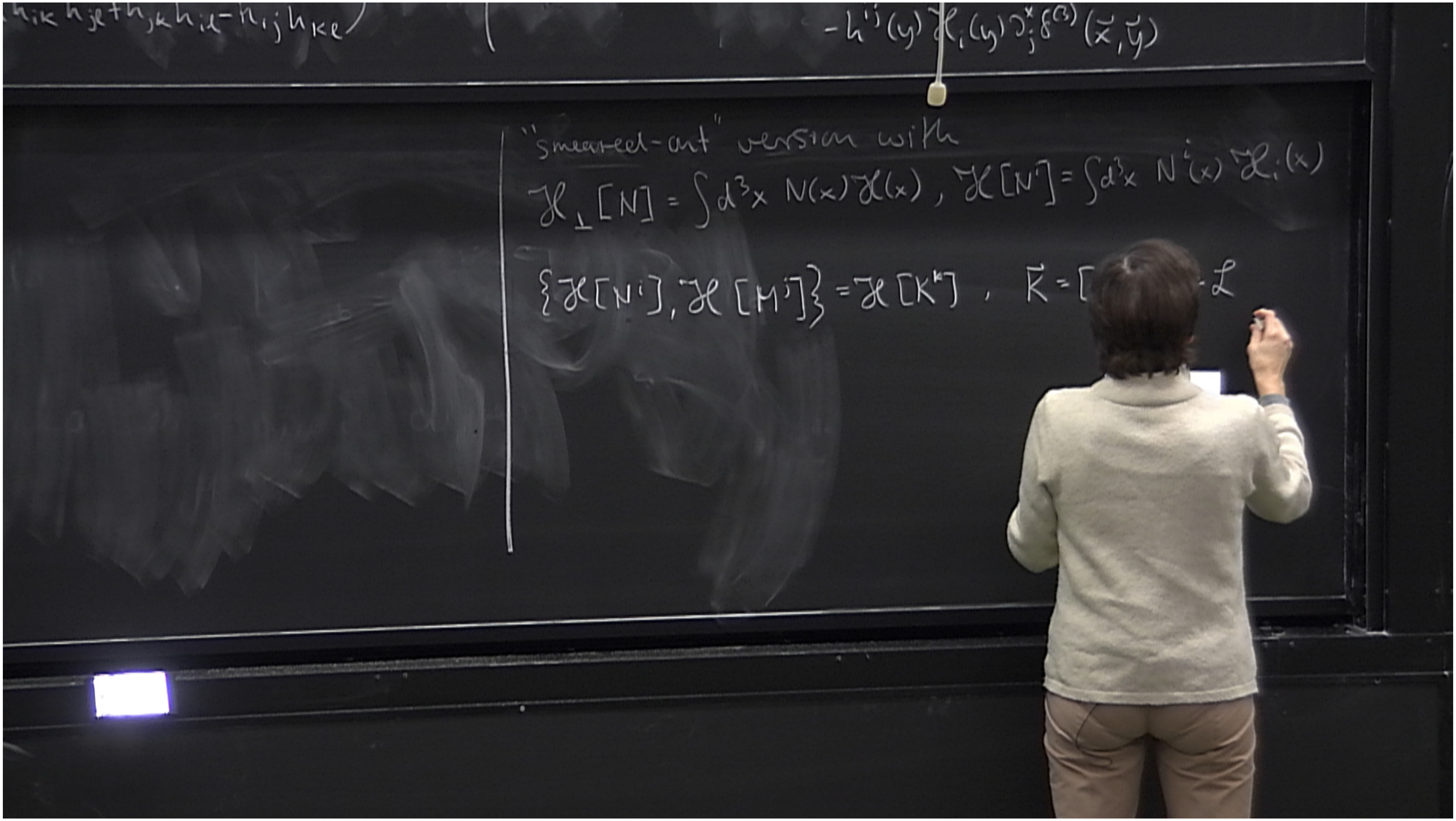
$$\{\mathcal{H}_\perp(x), \mathcal{H}_\perp(y)\} = h^{ij}(x)\mathcal{H}_i(x)\, \partial_j^y \delta^{(3)}(\vec{x},\vec{y}) - h^{ij}(y)\mathcal{H}_i(y)\, \partial_j^x \delta^{(3)}(\vec{x},\vec{y})$$

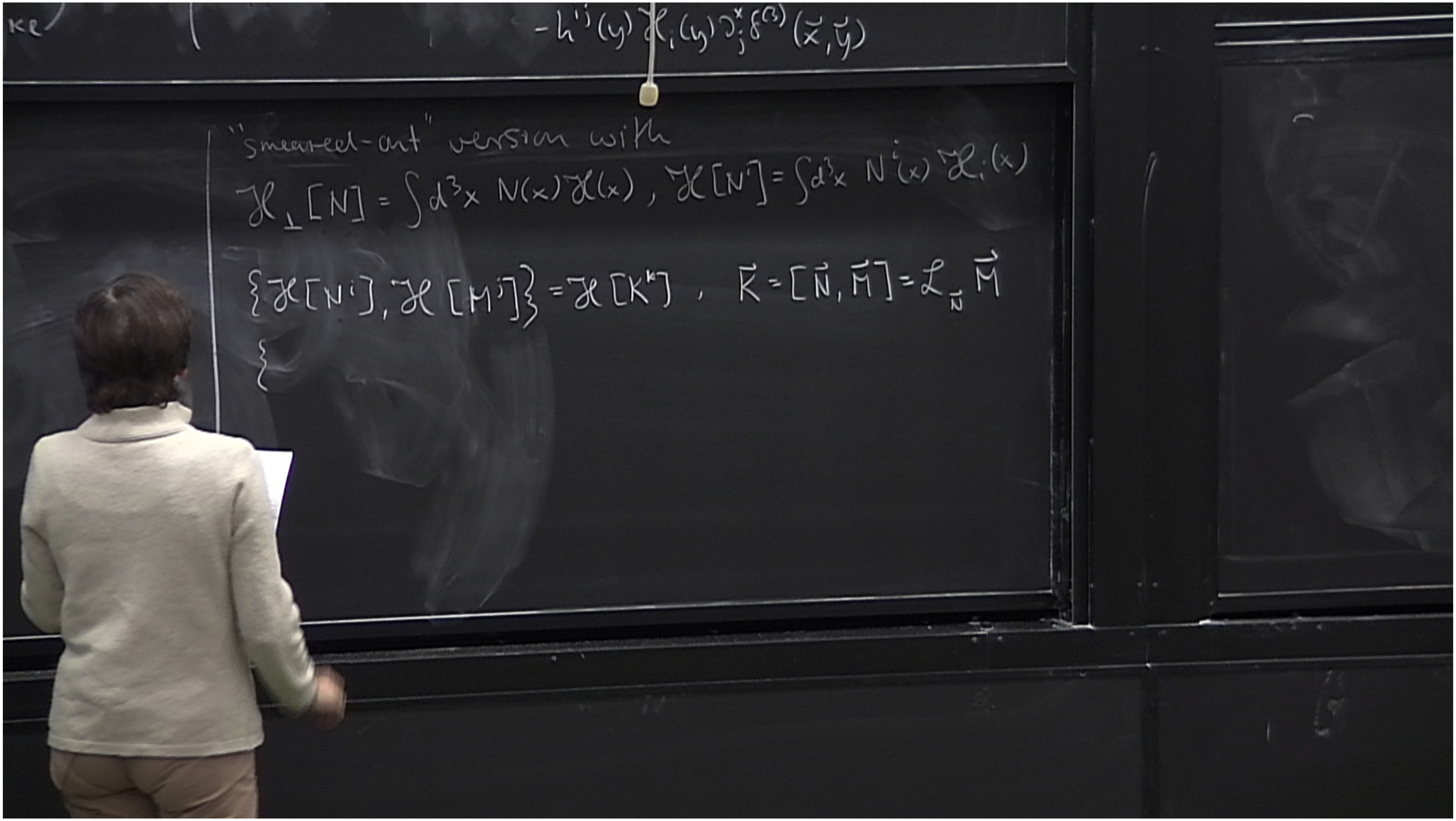


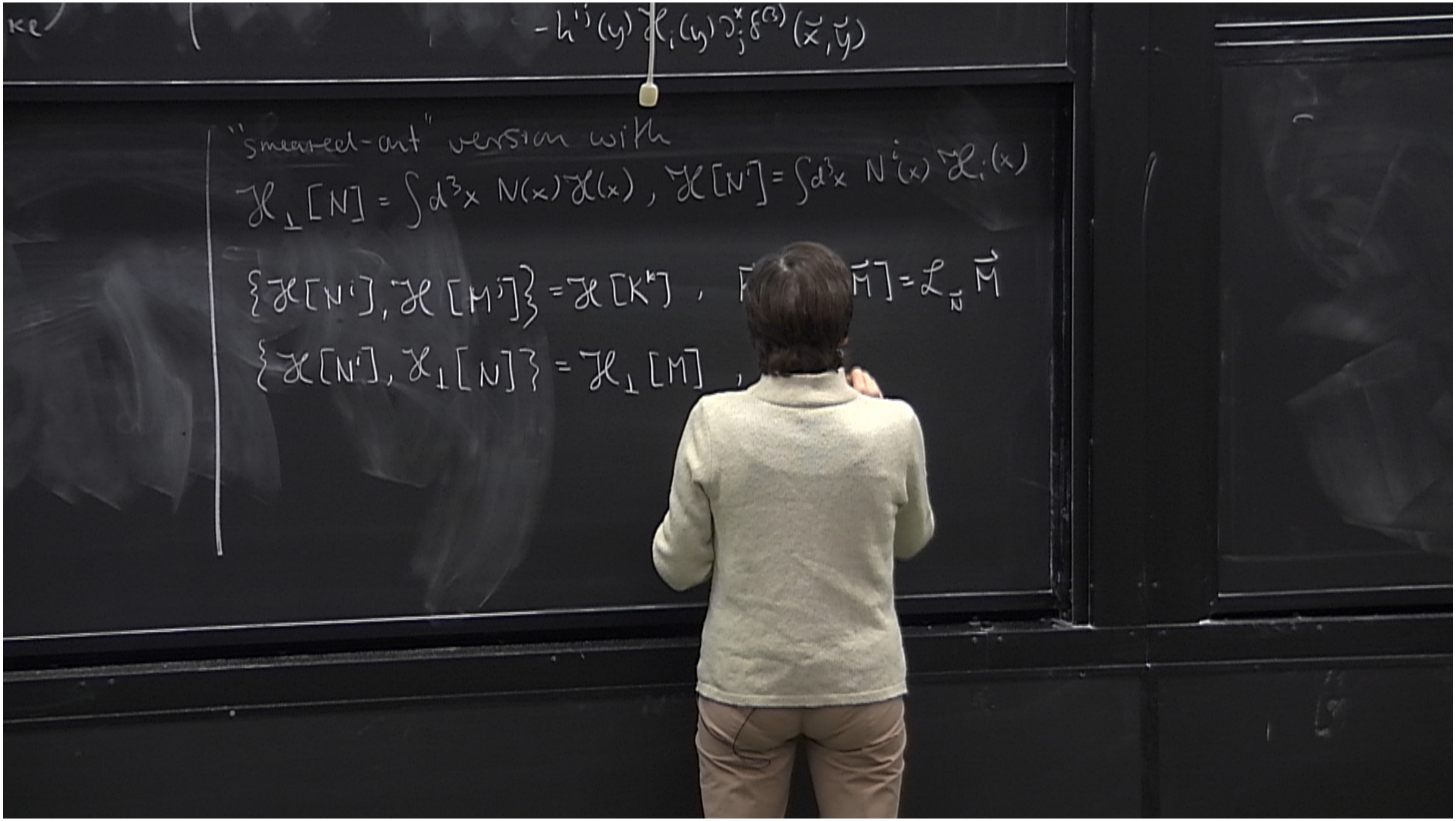

"smeared-out" version with
H⊥[N] = ∫d³x N(x) H(x) , H[N⃗] = ∫d³x N

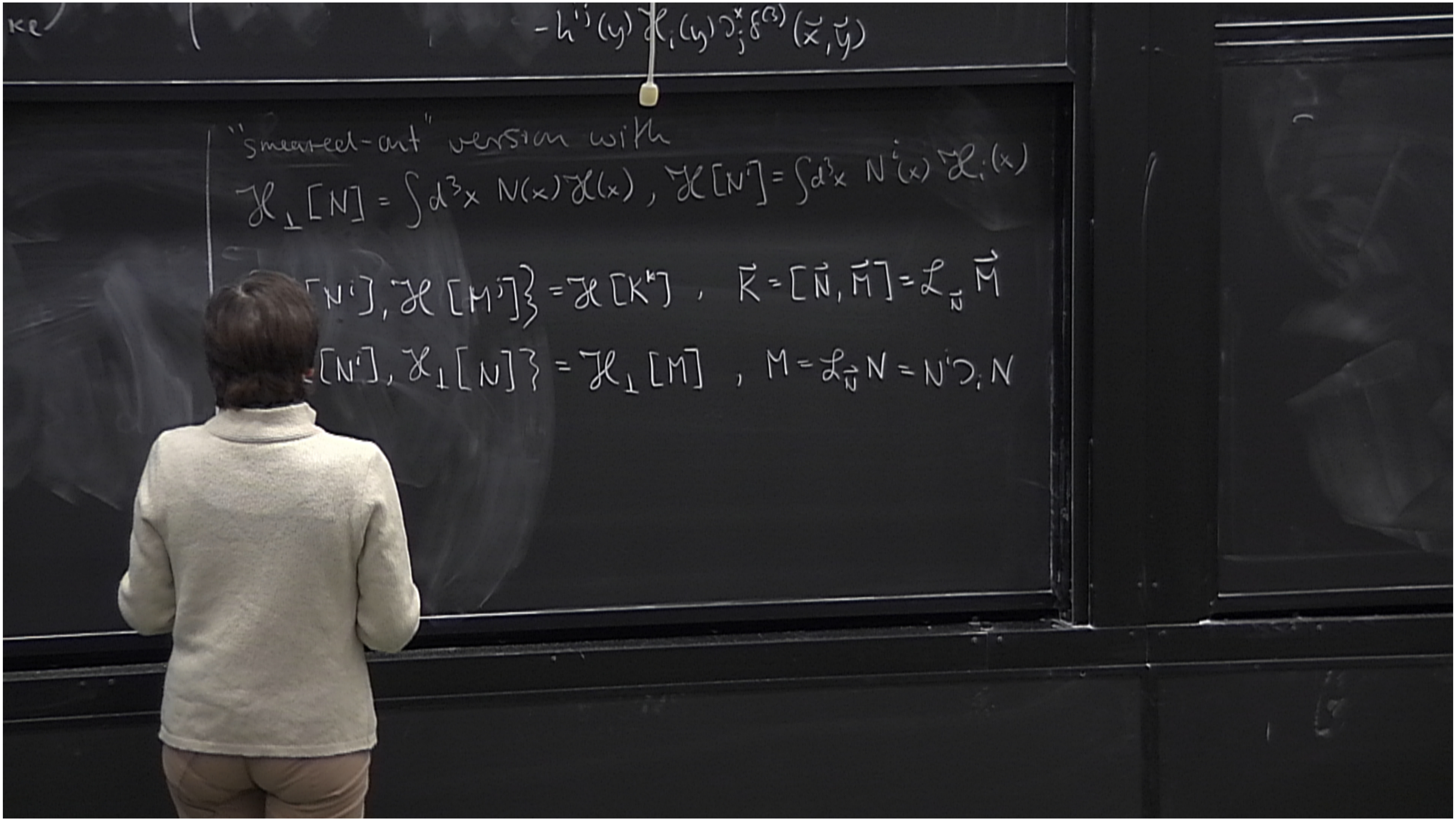

"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x \, N(x)\,\mathcal{H}(x)$, $\mathcal{H}[\vec{N}] = \int d^3x \, N^i(x)\,\mathcal{H}_i(x)$
$-h^{ij}(y)\,\mathcal{H}_i(y)\,\partial^x_j \delta^{(3)}(\vec{x},\vec{y})$

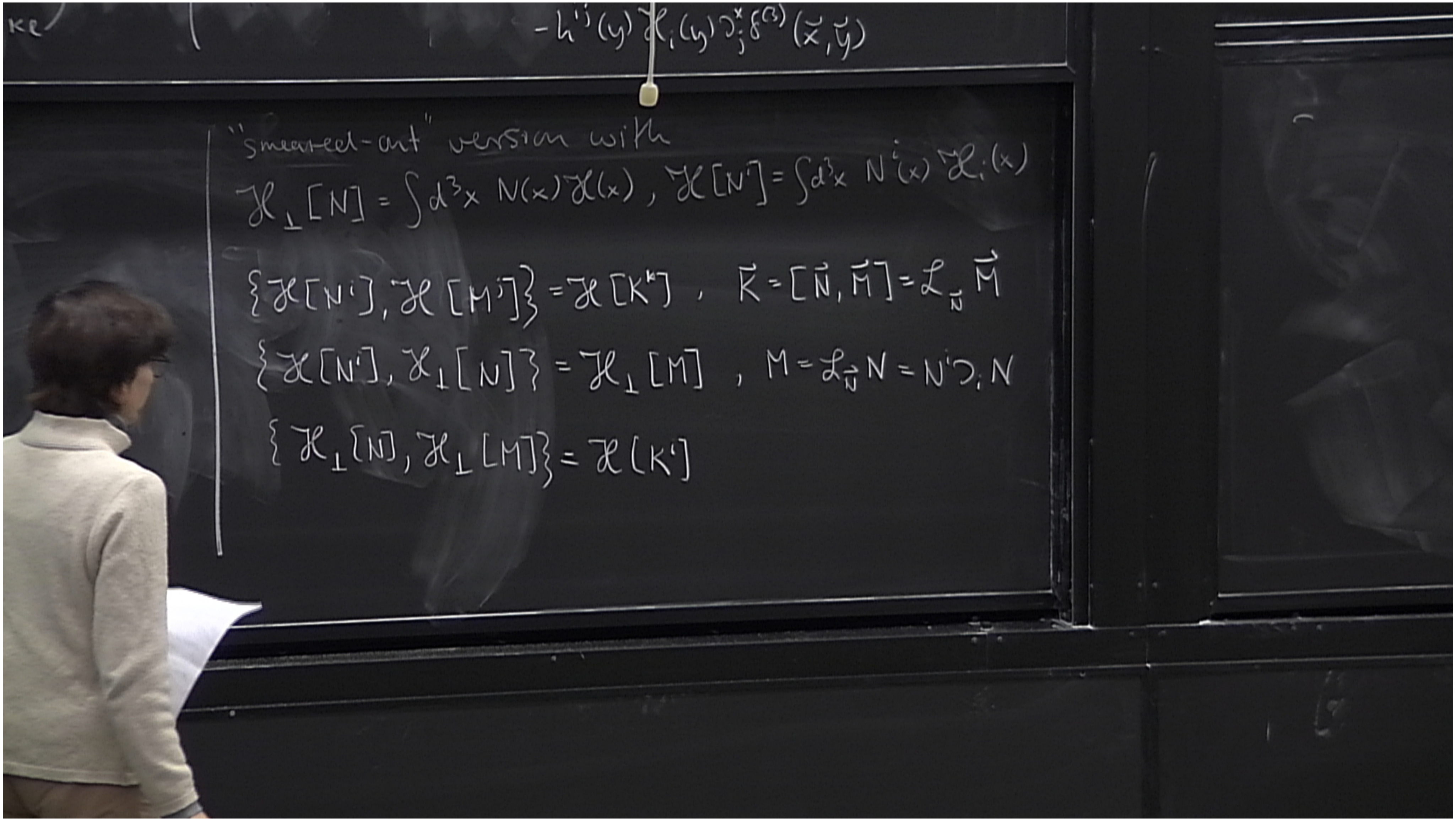

h_ik h_jl + h_jk h_il − h_ij h_kl)
− h^ij(y) H_i(y) ∂^x_j δ^(3)(x, y)
"smeared-out" version with
H_⊥[N] = ∫d³x N(x) H(x), H[N^i] = ∫d³x N^i(x) H_i(x)
{H[N^i], H[

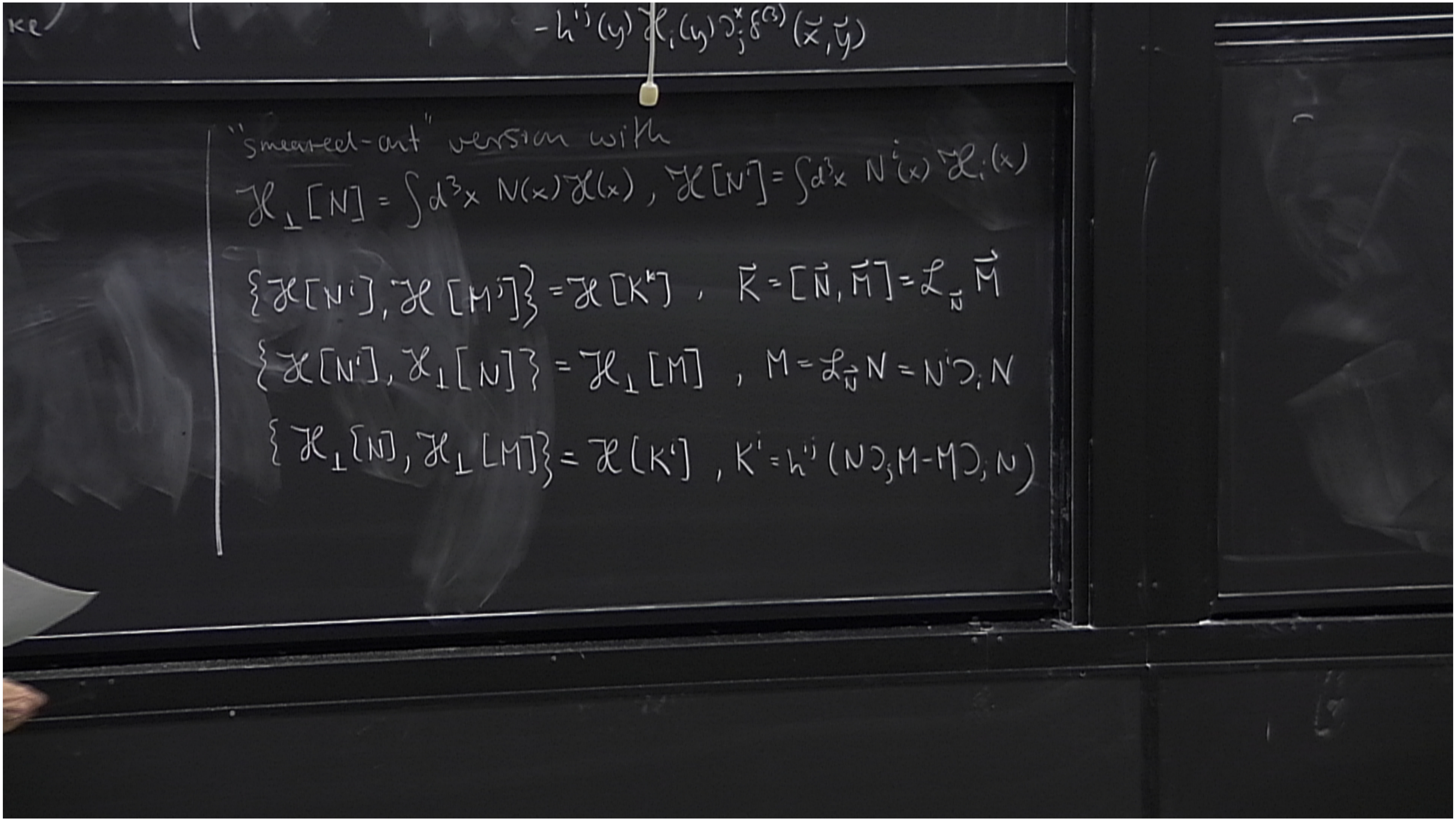

$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)$, $\mathcal{H}[\vec{N}] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]$

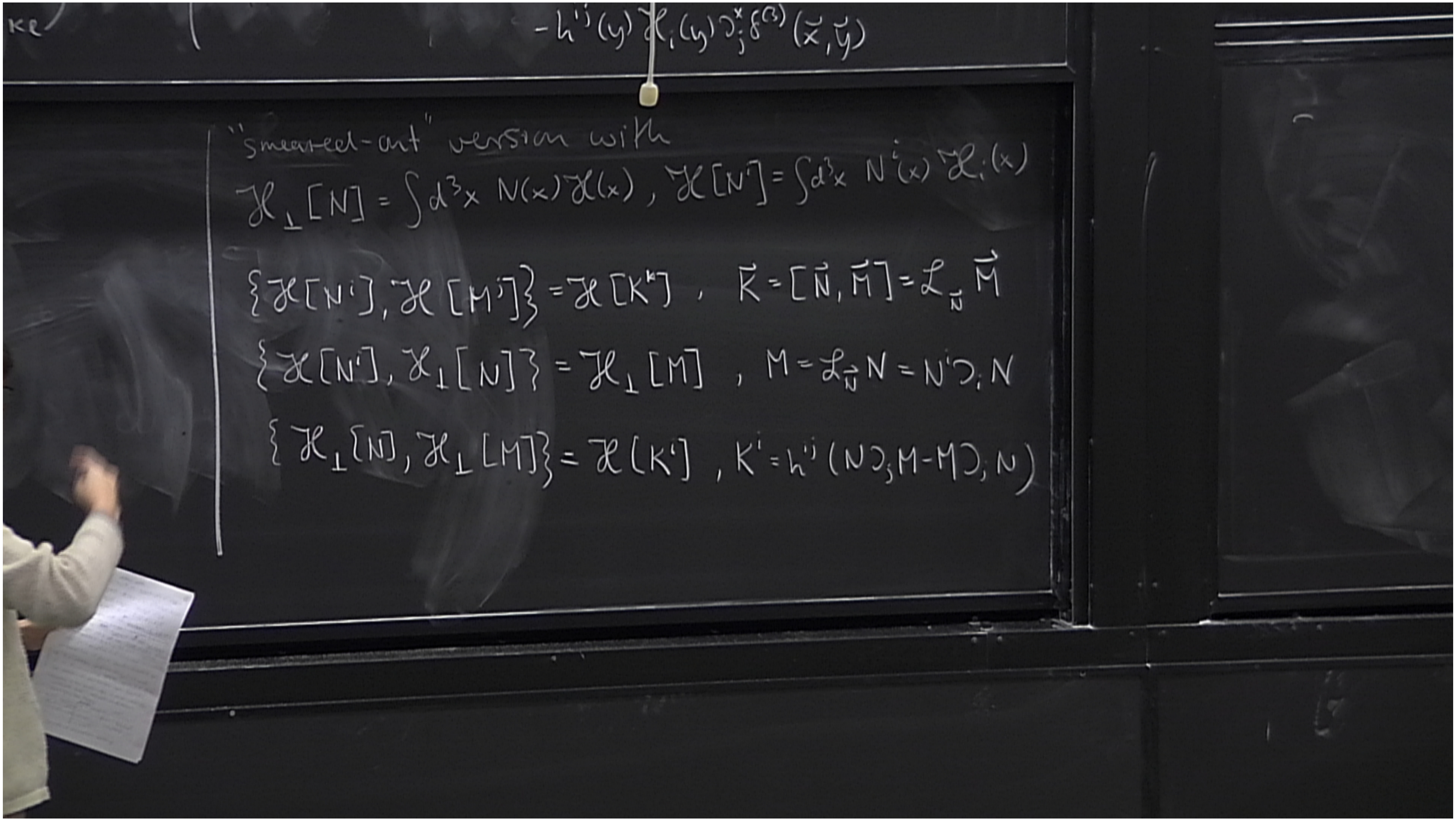

$-h^{ij}(y)\,\mathcal{H}_i(y)\,\partial^x_j\,\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\,\mathcal{H}(x)\,,\ \mathcal{H}[N^i] = \int d^3x\, N^i(x)\,\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]\,,\ \vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$

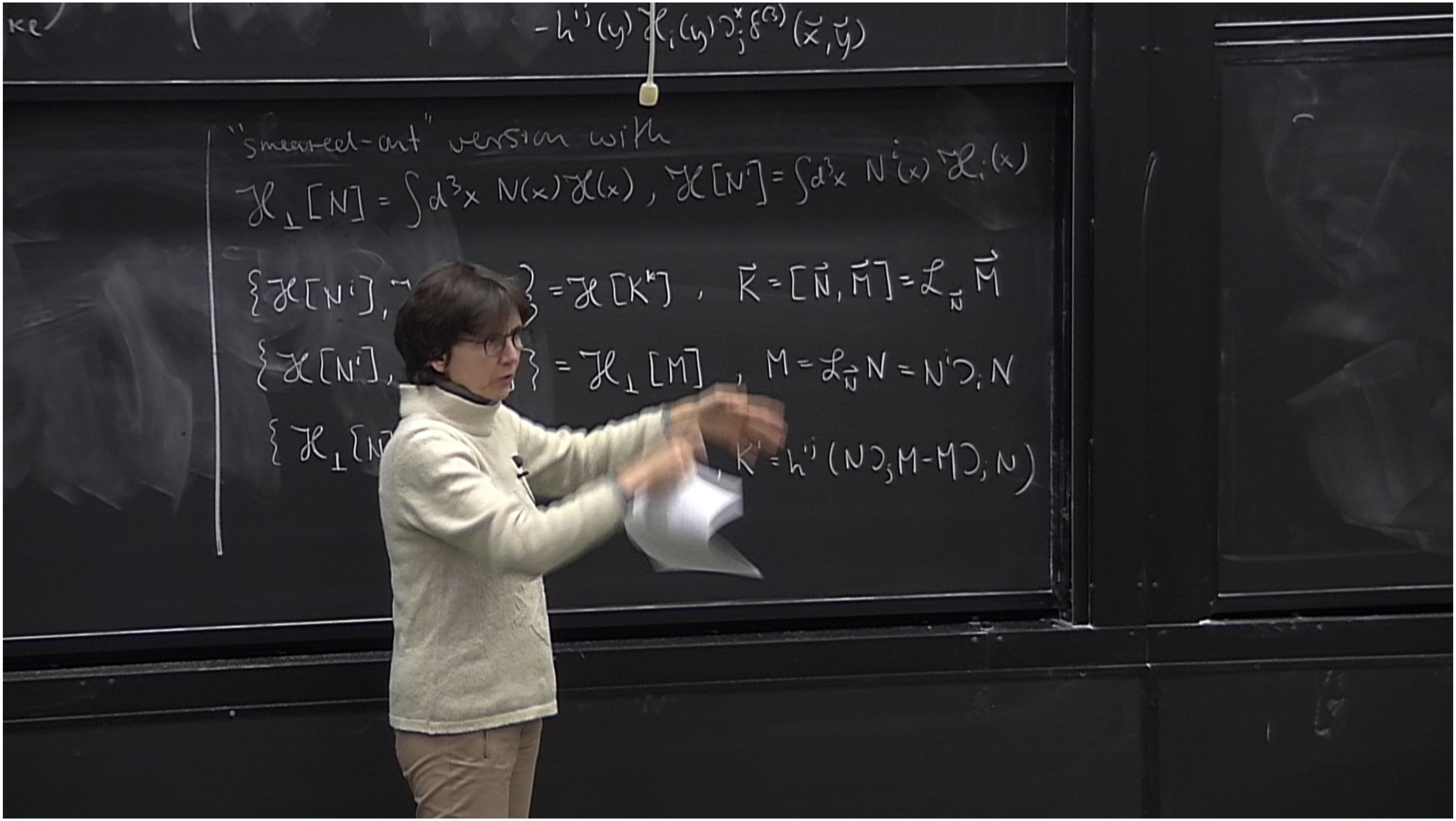

"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\,\mathcal{H}(x)\,,\quad \mathcal{H}[\vec{N}] = \int d^3x\, N^i(x)\,\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]\,,$
$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]\,,$

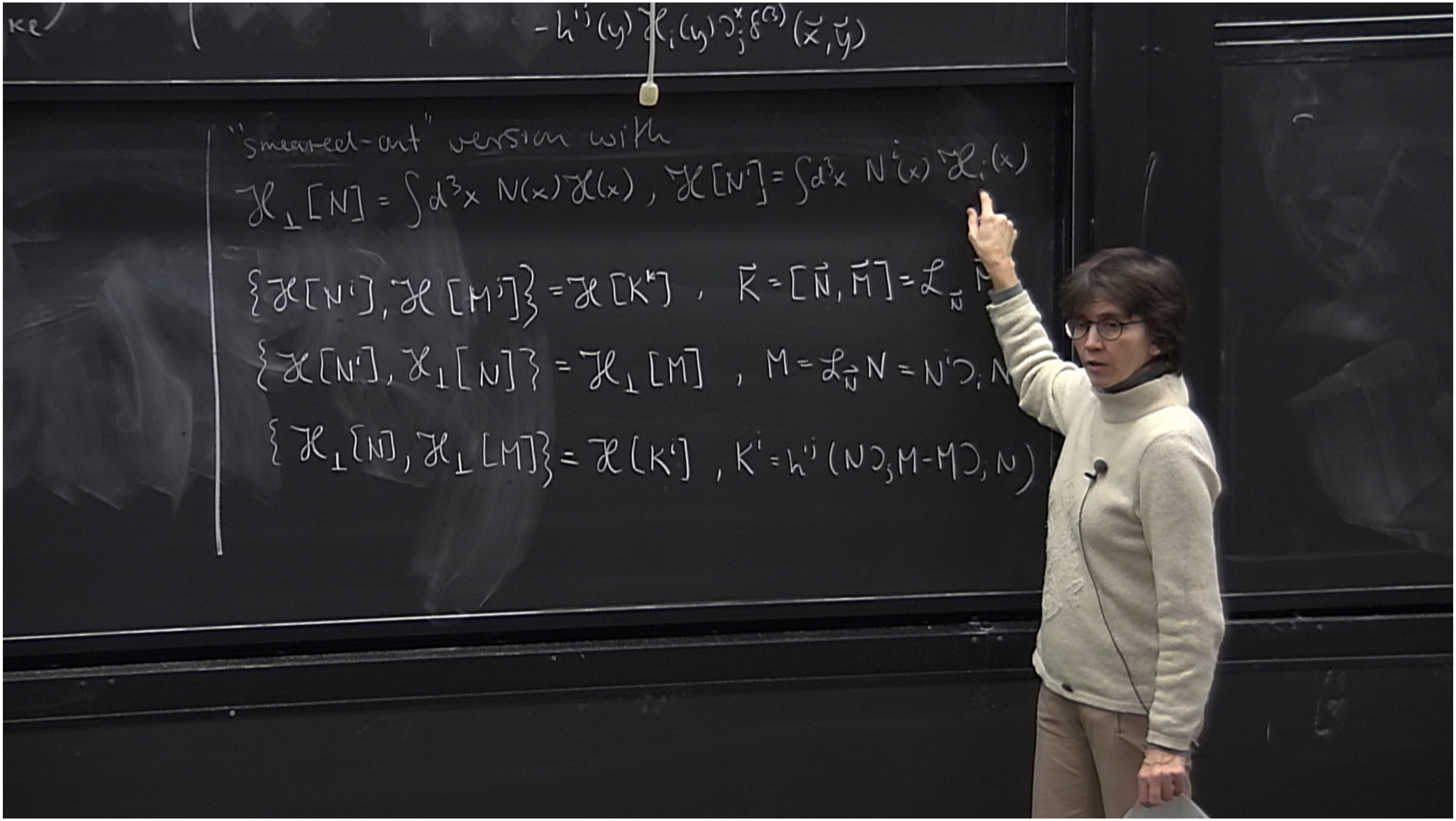

$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N]=\int d^3x\, N(x)\mathcal{H}(x)\,,\ \mathcal{H}[N^i]=\int d^3x\, N^i(x)\mathcal{H}_i(x)$
$[N^i],\mathcal{H}[M^j]\}=\mathcal{H}[K^k]\,,\quad \vec{K}=[\vec{N},\vec{M}]=\mathcal{L}_{\vec{N}}\vec{M}$
$[N^i],\mathcal{H}_\perp[N]\}=\mathcal{H}_\perp[M]\,,\quad M=\mathcal{L}_{\vec{N}}N=N^i\partial_i N$


$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)$, $\mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]$, $\vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$
$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]$, $M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$
$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}(K^i]$


$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)\,,\quad \mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i],\mathcal{H}[M^j]\} = \mathcal{H}[K^k]\,,\quad \vec{K} = [\vec{N},\vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$
$\{\mathcal{H}[N^i],\mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]\,,\quad M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$
$\{\mathcal{H}_\perp[N],\mathcal{H}_\perp[M]\} = \mathcal{H}[K^i]\,,\quad K^i = h^{ij}(N\partial_j M - M\partial_j N)$


$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)$, $\mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]$, $\vec{K} = [\vec{N},\vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$
$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]$, $M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$
$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}[K^i]$, $K^i = h^{ij}(N\partial_j M - M\partial_j N)$


"smeared-out" version with
ℋ⊥[N] = ∫d³x N(x) ℋ(x) , ℋ[N^i] = ∫d³x N^i(x) ℋ_i(x)
= ℋ[K^k] , K = [N, M] = L_N M
= ℋ⊥[M] , M = L_N N = N^i ∂_i N
K^i = h^ij (N∂_j M − M∂_j N)


$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)$, $\mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]$, $\vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$
$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]$, $M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$
$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}[K^i]$, $K^i = h^{ij}(N\partial_j M - M\partial_j N)$

$$-h^{ij}(y)\mathcal{H}_i(y)\partial^x_j\delta^{(3)}(\vec{x},\vec{y})$$

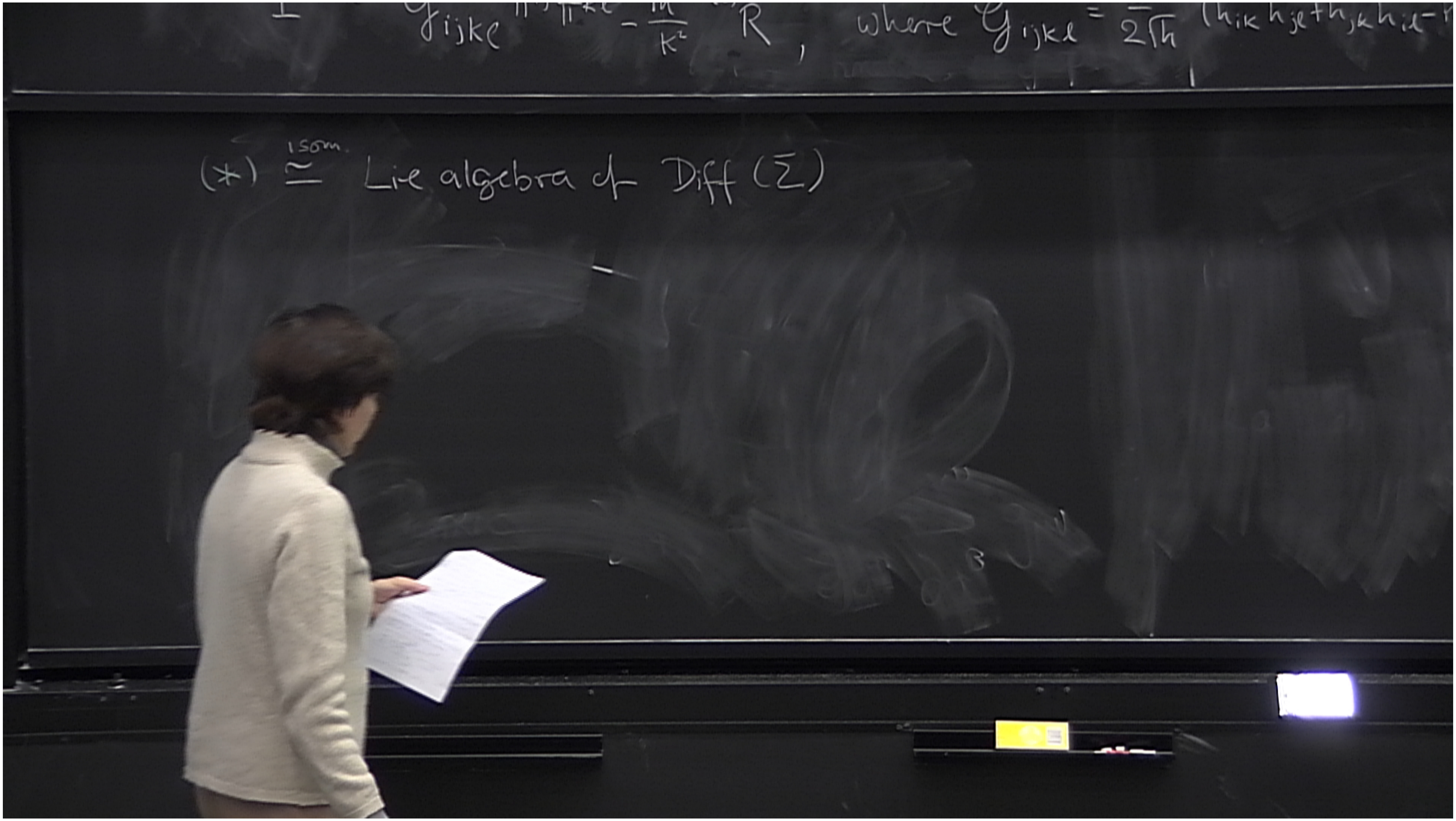
"smeared-out" version with

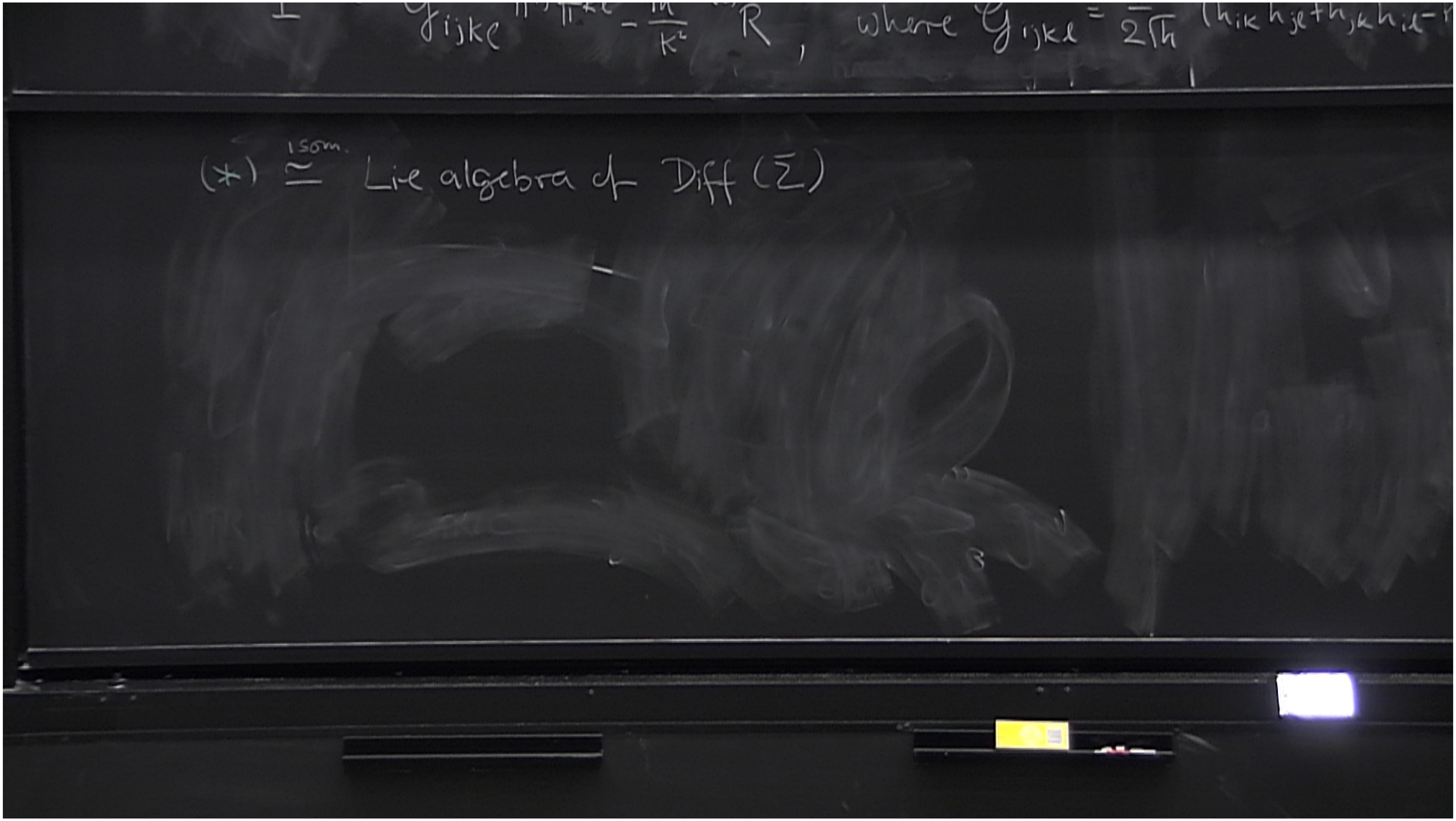
$$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x), \quad \mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$$

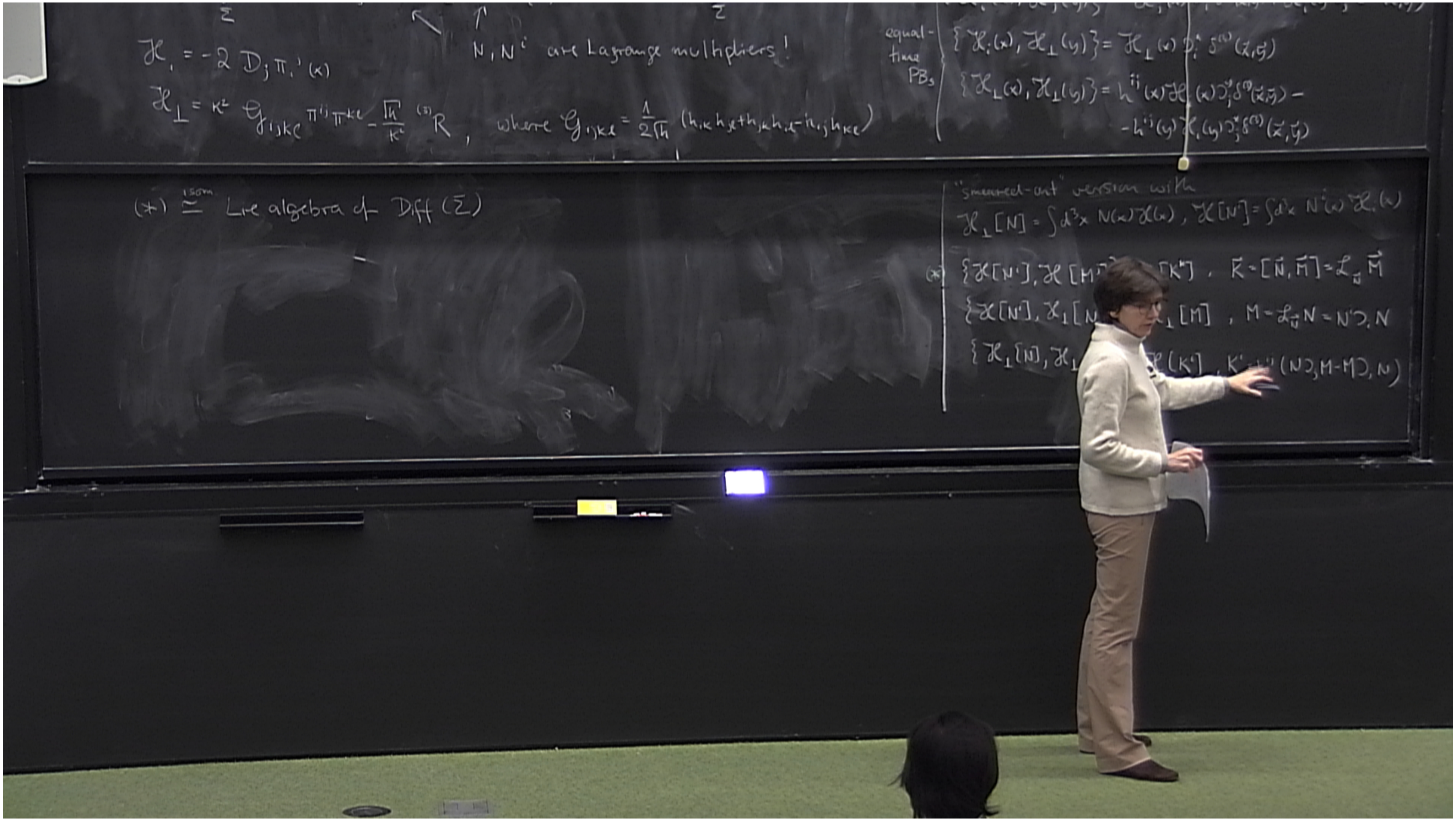
(*) $$\{\mathcal{H}[N^i],\mathcal{H}[M^j]\} = \mathcal{H}[K^k], \quad \vec{K} = [\vec{N},\vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$$

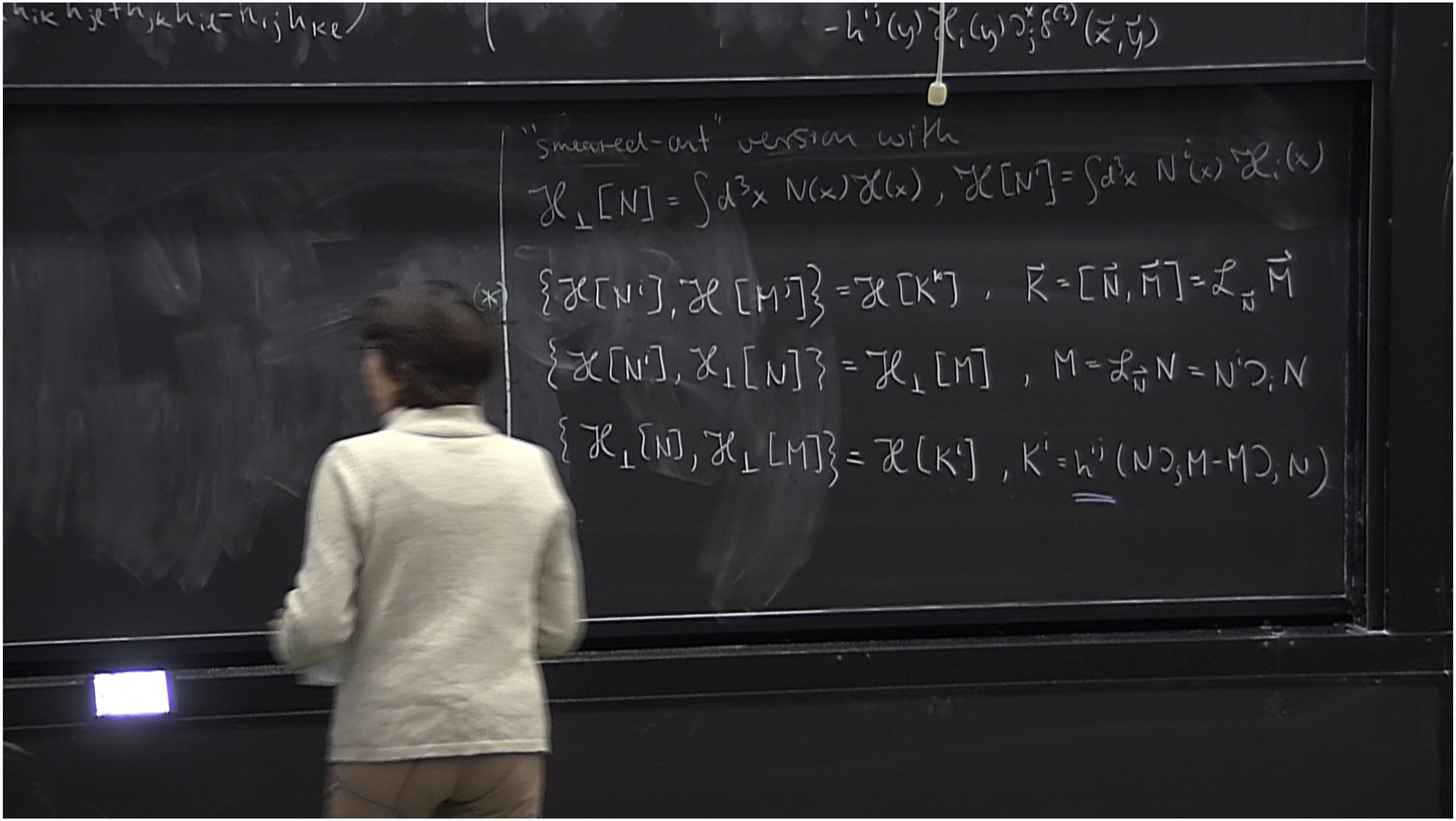
$$\{\mathcal{H}[N^i],\mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M], \quad M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$$

$$\{\mathcal{H}_\perp[N],\mathcal{H}_\perp[M]\} = \mathcal{H}[K^i], \quad K^i = h^{ij}(N\partial_j M - M\partial_j N)$$


, where G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il −
(*) ≅ (isom.) Lie algebra of Diff(Σ)


where $G_{ijkl} = \frac{1}{2\sqrt{h}}(h_{ik}h_{jl} + h_{jk}h_{il} -$
$(*) \overset{\text{isom.}}{\simeq}$ Lie algebra of $\mathrm{Diff}(\Sigma)$

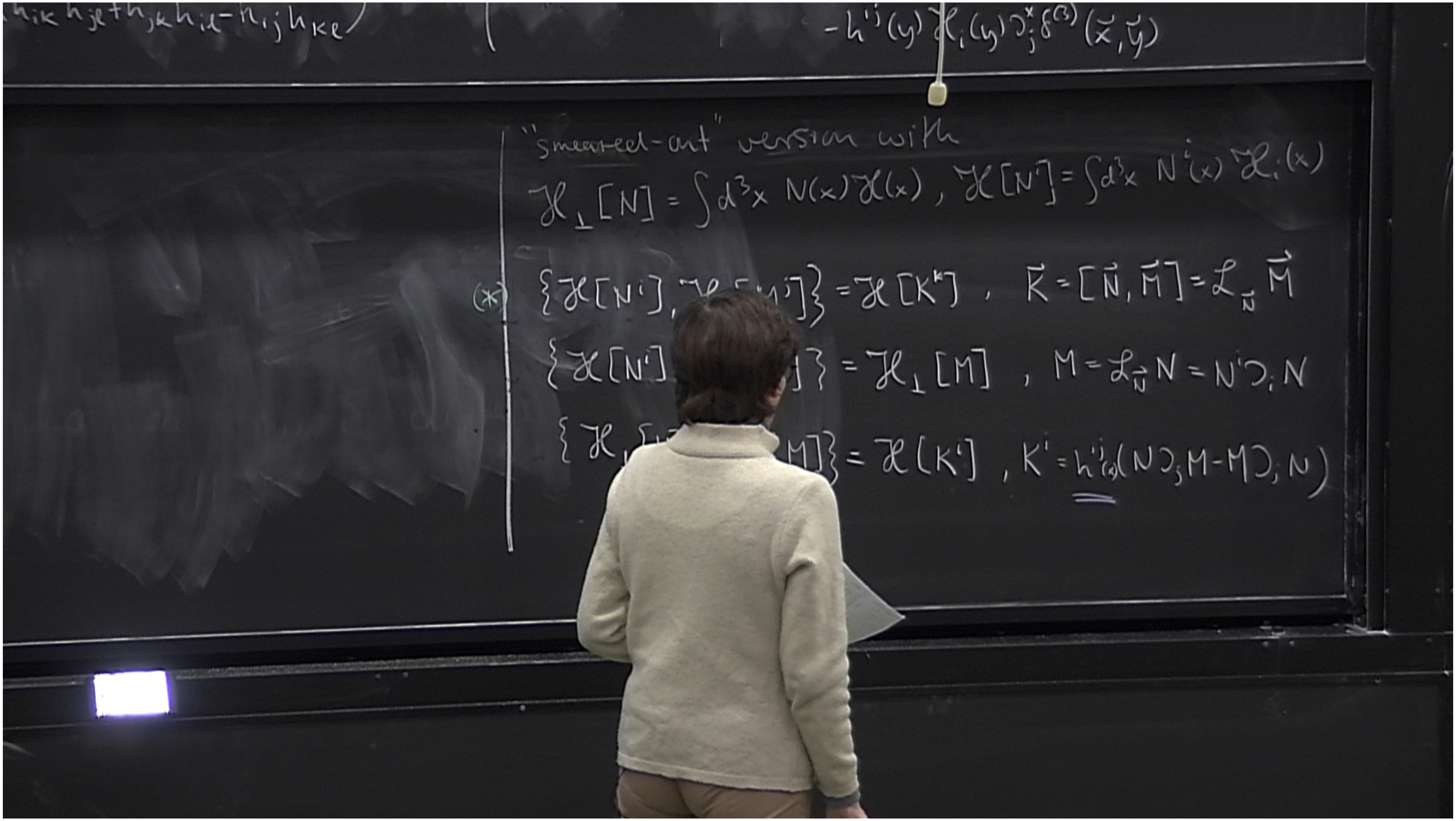

N, N^i are Lagrange multipliers!
where
(*) ≅ Lie algebra of Diff (Σ)
equal-time PBs
"smeared-out" version with


$h_{ik}h_{jl}+h_{jk}h_{il}-h_{ij}h_{kl}$
$-h^{ij}(y)\mathcal{H}_i(y)\partial_j^x\delta^{(3)}(\vec{x},\vec{y})$
"smeared-out" version with
$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\mathcal{H}(x)$, $\mathcal{H}[N^i] = \int d^3x\, N^i(x)\mathcal{H}_i(x)$
(*) $\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]$, $\vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$
$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]$, $M = \mathcal{L}_{\vec{N}}N = N^i\partial_i N$
$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}[K^i]$, $K^i = h^{ij}(N\partial_j M - M\partial_j N)$

$G_{ijkl}\pi^{ij}\pi^{kl} - \frac{h}{\kappa^2} R$, where $G_{ijkl} = \frac{1}{2\sqrt{h}}(h_{ik}h_{jl} + h_{jk}h_{il} - \ldots$

(*) $\overset{\text{isom.}}{\simeq}$ Lie algebra of Diff (Σ)

Dirac algebra $\not\simeq$ Diff (M)


"smeared-out" version with

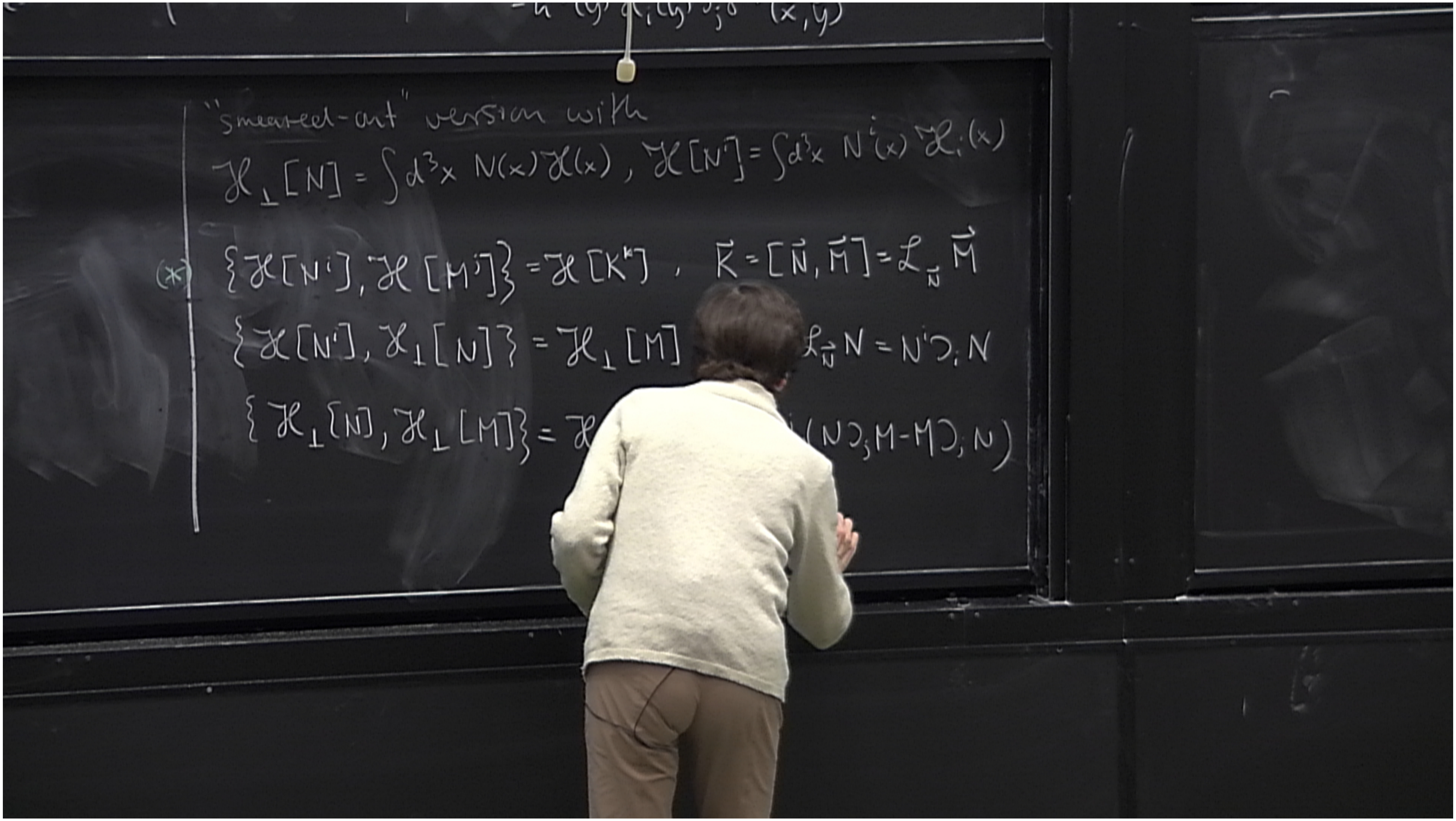
"smeared-out" version with

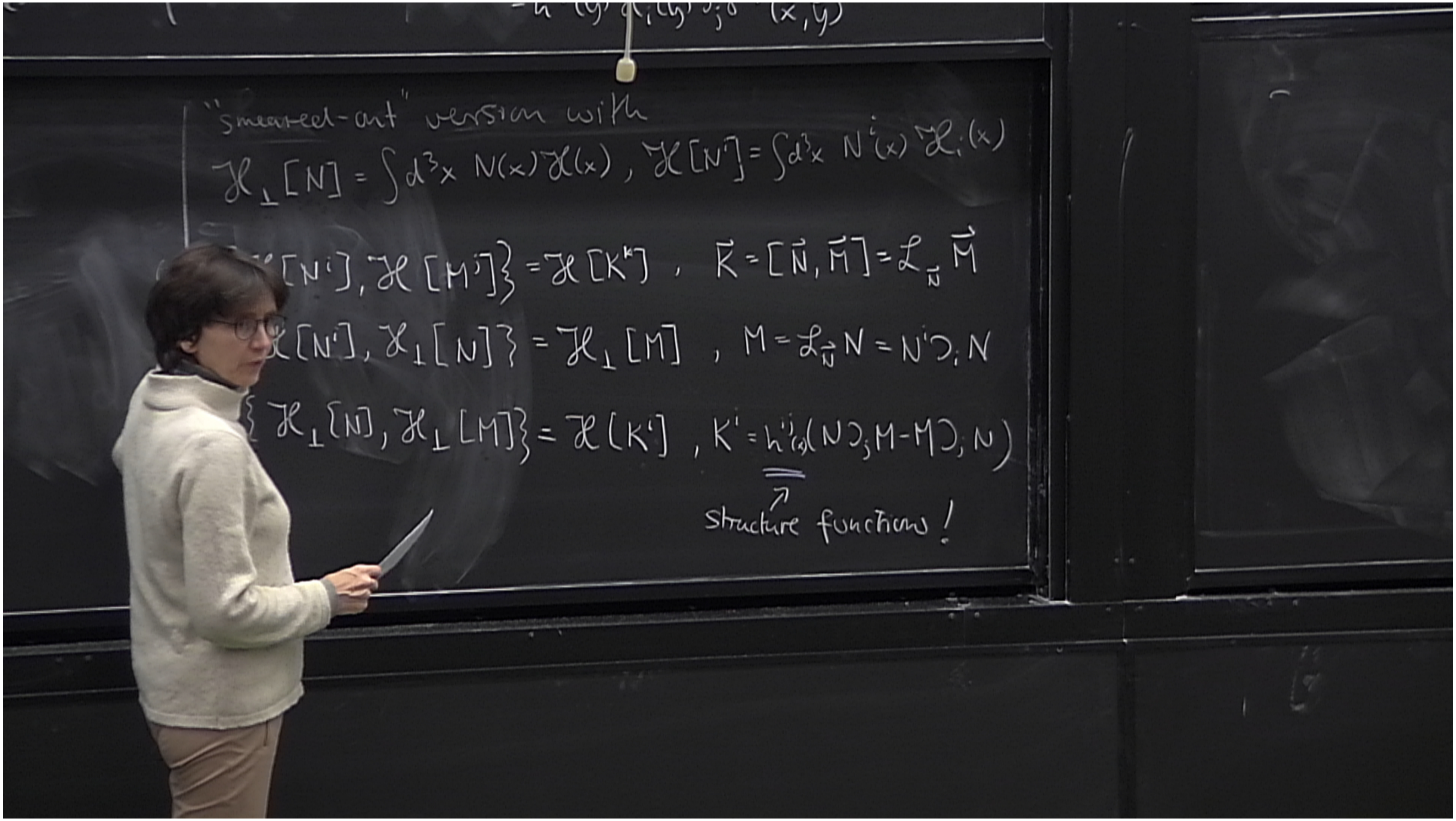
$$\mathcal{H}_\perp[N] = \int d^3x \, N(x)\mathcal{H}(x) , \quad \mathcal{H}[N^i] = \int d^3x \, N^i(x)\mathcal{H}_i(x)$$

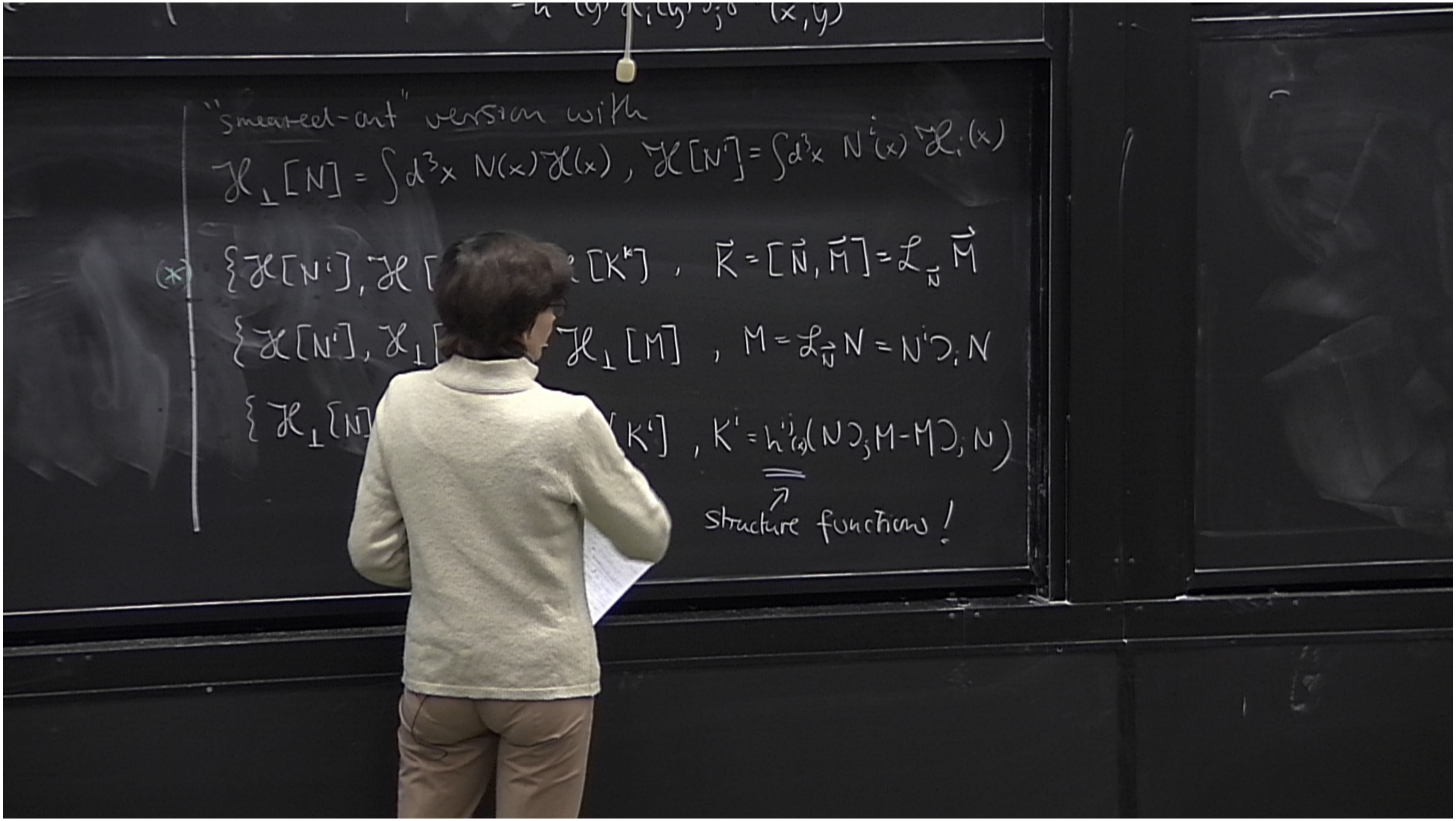
(*) $\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k] , \quad \vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}} \vec{M}$

$$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M] , \quad M = \mathcal{L}_{\vec{N}} N = N^i \partial_i N$$

$$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}[K^i] , \quad K^i = h^{ij}(x)(N\partial_j M - M\partial_j N)$$


"smeared-out" version with
ℋ⊥[N] = ∫d³x N(x) ℋ(x) , ℋ[N^i] = ∫d³x N^i(x) ℋ_i(x)
(*) {ℋ[N^i], ℋ[M^i]} = ℋ[K^k] , K = [N, M] = L_N M
{ℋ[N^i], ℋ⊥[N]} = ℋ⊥[M]
L_N N = N^i ∂_i N
{ℋ⊥[N], ℋ⊥[M]} =
(N∂_j M − M∂_j N)


"smeared-out" version with
H⊥[N] = ∫d³x N(x) H(x) , H[Nⁱ] = ∫d³x Nⁱ(x) Hᵢ(x)
{H[Nⁱ], H[Mʲ]} = H[Kᵏ] , K⃗ = [N⃗, M⃗] = L_N⃗ M⃗
{H[Nⁱ], H⊥[N]} = H⊥[M] , M = L_N⃗ N = Nⁱ∂ᵢN
{H⊥[N], H⊥[M]} = H[Kⁱ] , Kⁱ = hⁱʲ(x)(N∂ⱼM − M∂ⱼN)
structure functions!


"smeared-out" version with
H⊥[N] = ∫d³x N(x) H(x) , H[Nⁱ] = ∫d³x Nⁱ(x) Hᵢ(x)
(*) {H[Nⁱ], H[
[Kᵏ] , K = [N, M] = L_N M
{H[Nⁱ], H⊥[
H⊥[M] , M = L_N N = Nⁱ∂ᵢN
{H⊥[N]
[Kⁱ] , Kⁱ = hⁱʲ(x)(N∂ⱼM − M∂ⱼN)
structure functions !

"smeared-out" version with

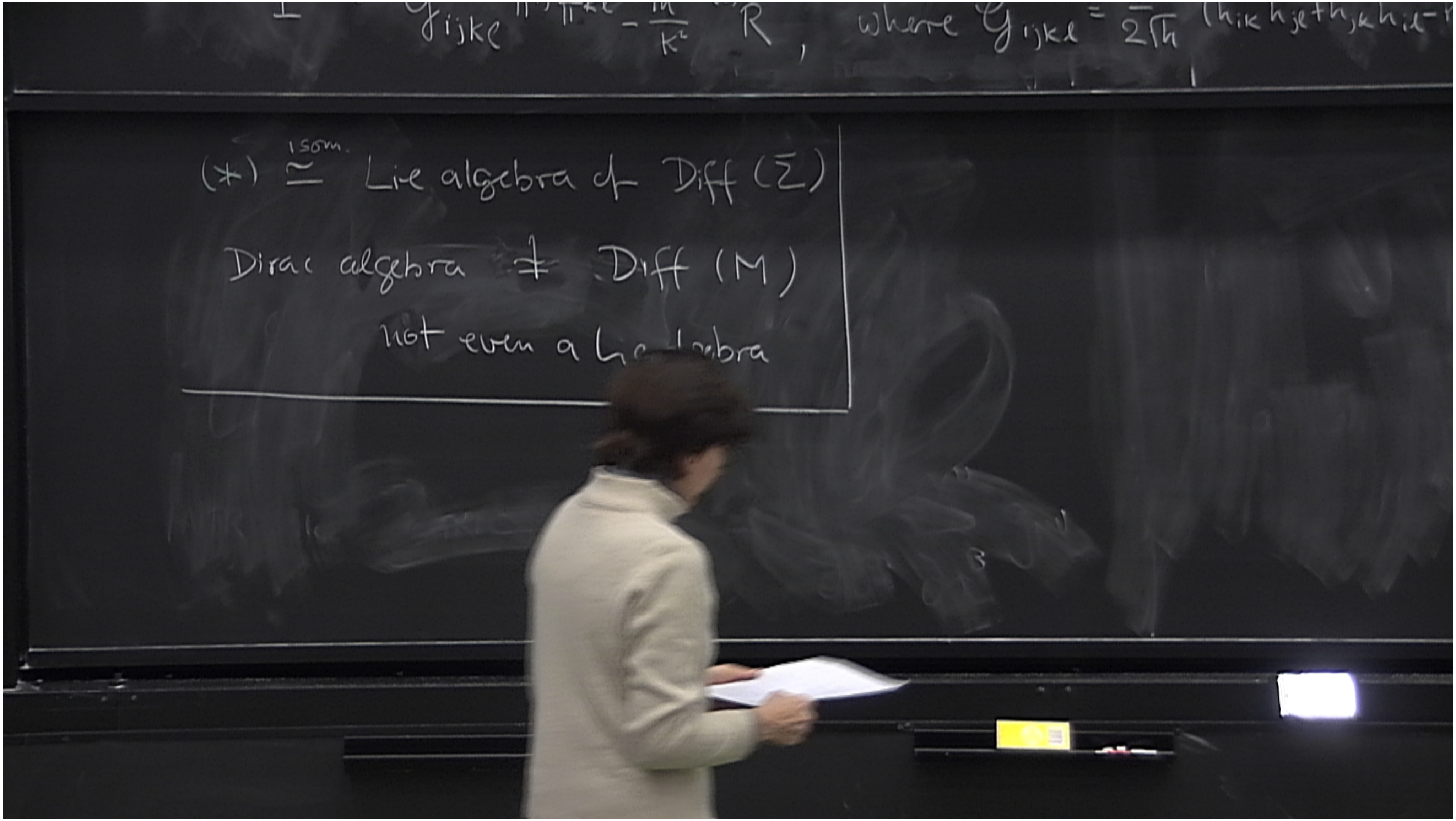
$$\mathcal{H}_\perp[N] = \int d^3x\, N(x)\,\mathcal{H}(x)\,, \quad \mathcal{H}[N^i] = \int d^3x\, N^i(x)\,\mathcal{H}_i(x)$$

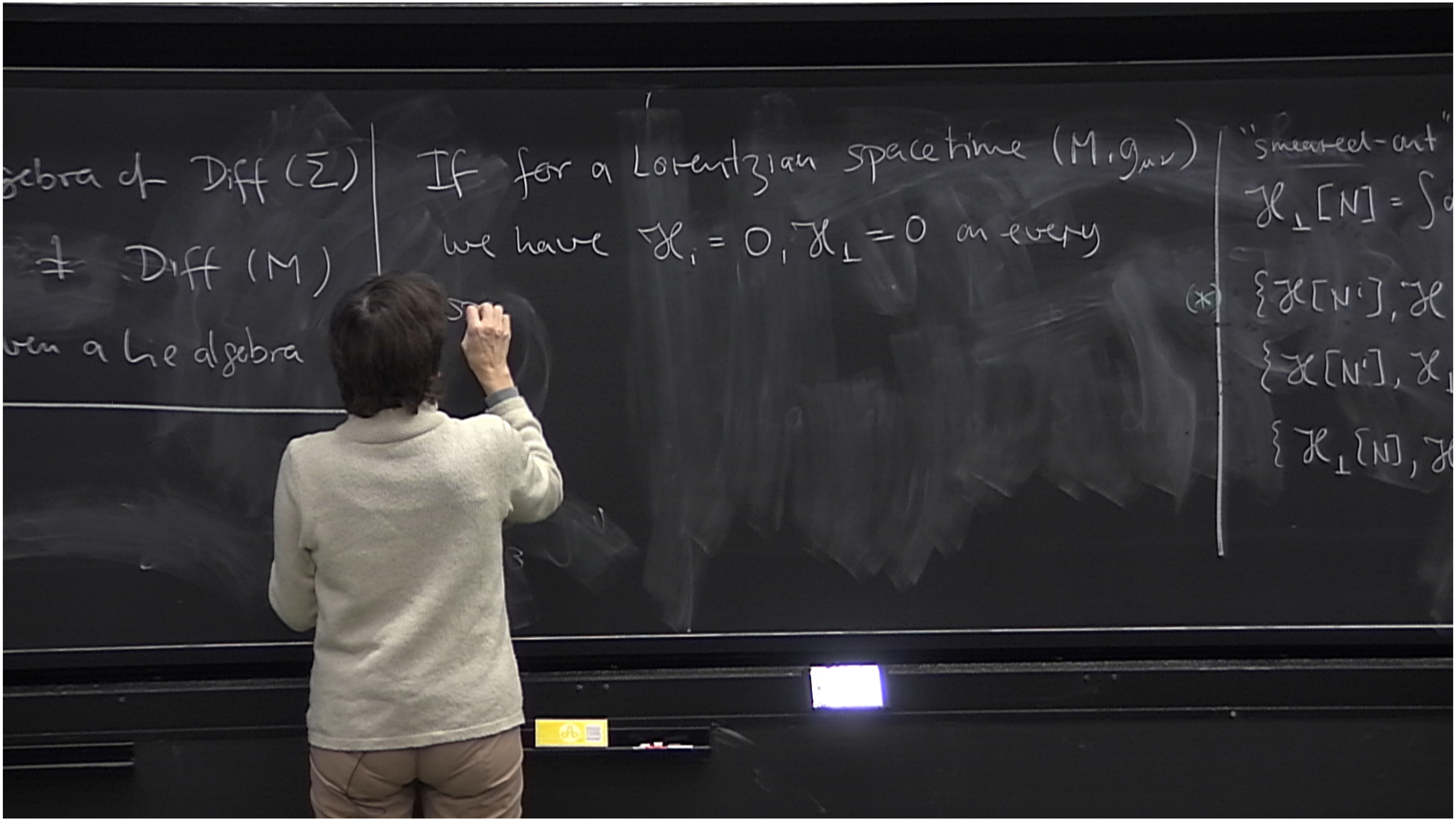
(*) $$\{\mathcal{H}[N^i], \mathcal{H}[M^j]\} = \mathcal{H}[K^k]\,, \quad \vec{K} = [\vec{N}, \vec{M}] = \mathcal{L}_{\vec{N}}\vec{M}$$

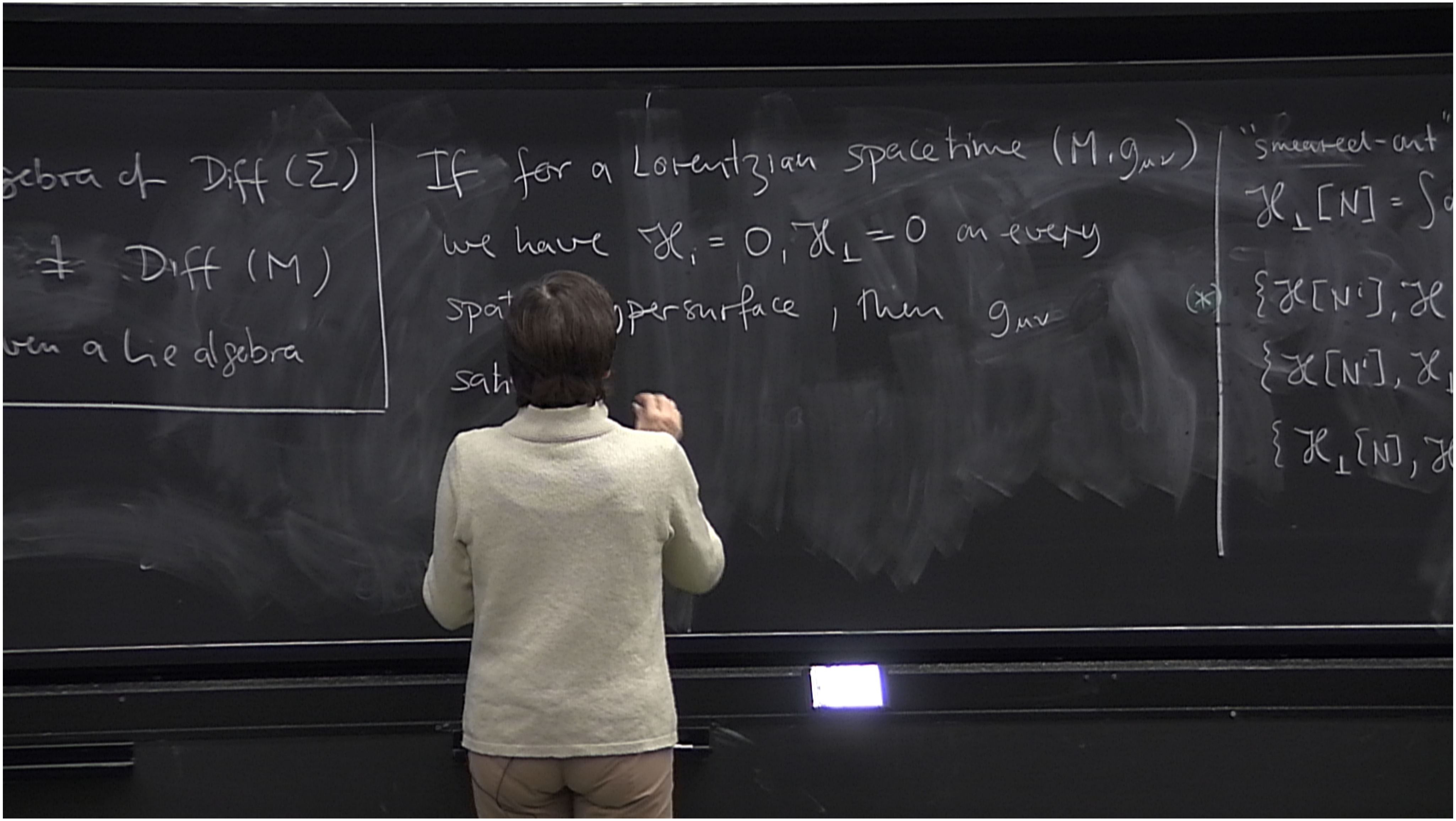
$$\{\mathcal{H}[N^i], \mathcal{H}_\perp[N]\} = \mathcal{H}_\perp[M]\,, \quad M = \mathcal{L}_{\vec{N}} N = N^i \partial_i N$$

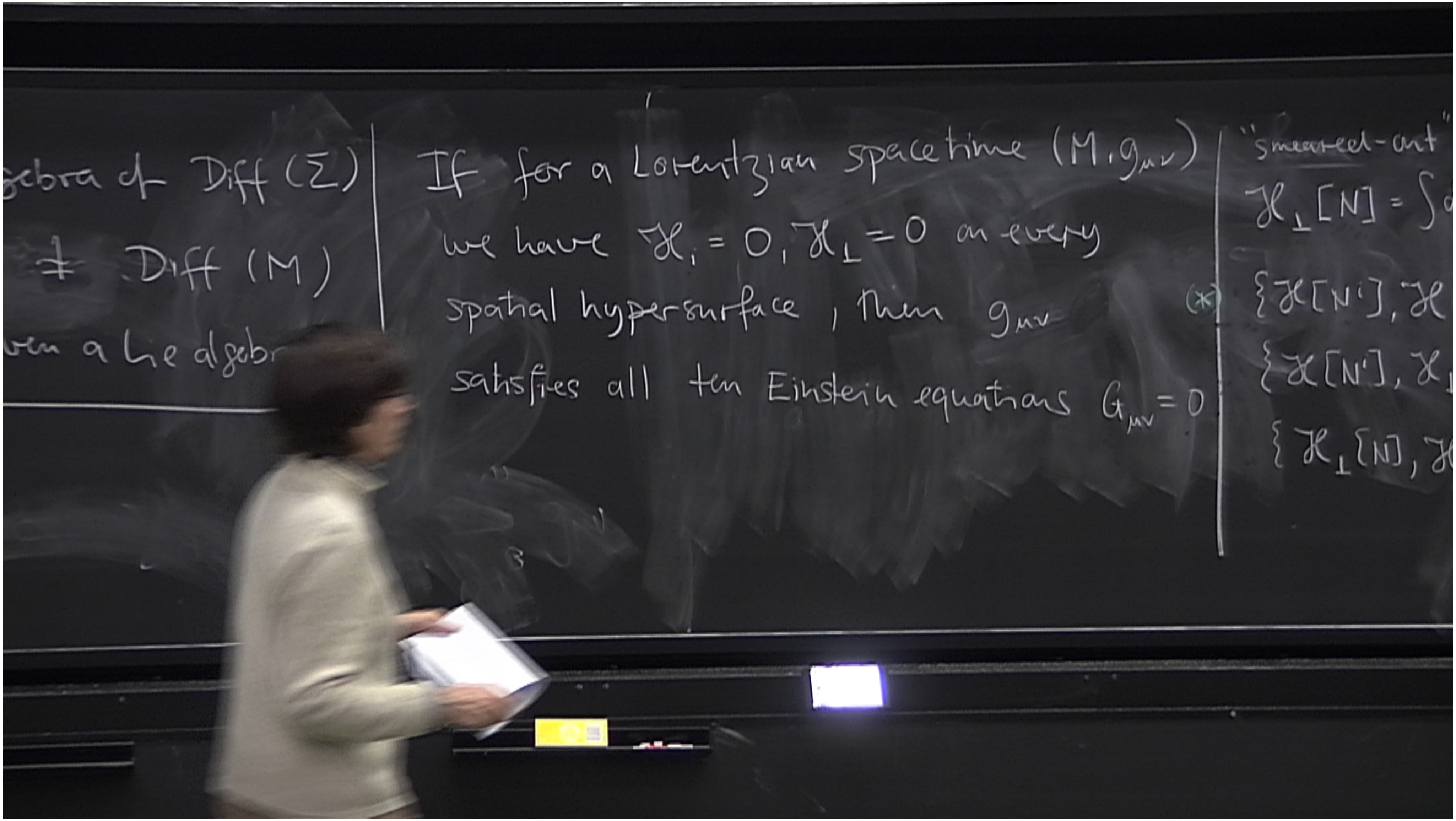
$$\{\mathcal{H}_\perp[N], \mathcal{H}_\perp[M]\} = \mathcal{H}[K^i]\,, \quad K^i = \underline{\underline{h^{ij}(x)}}(N\partial_j M - M\partial_j N)$$

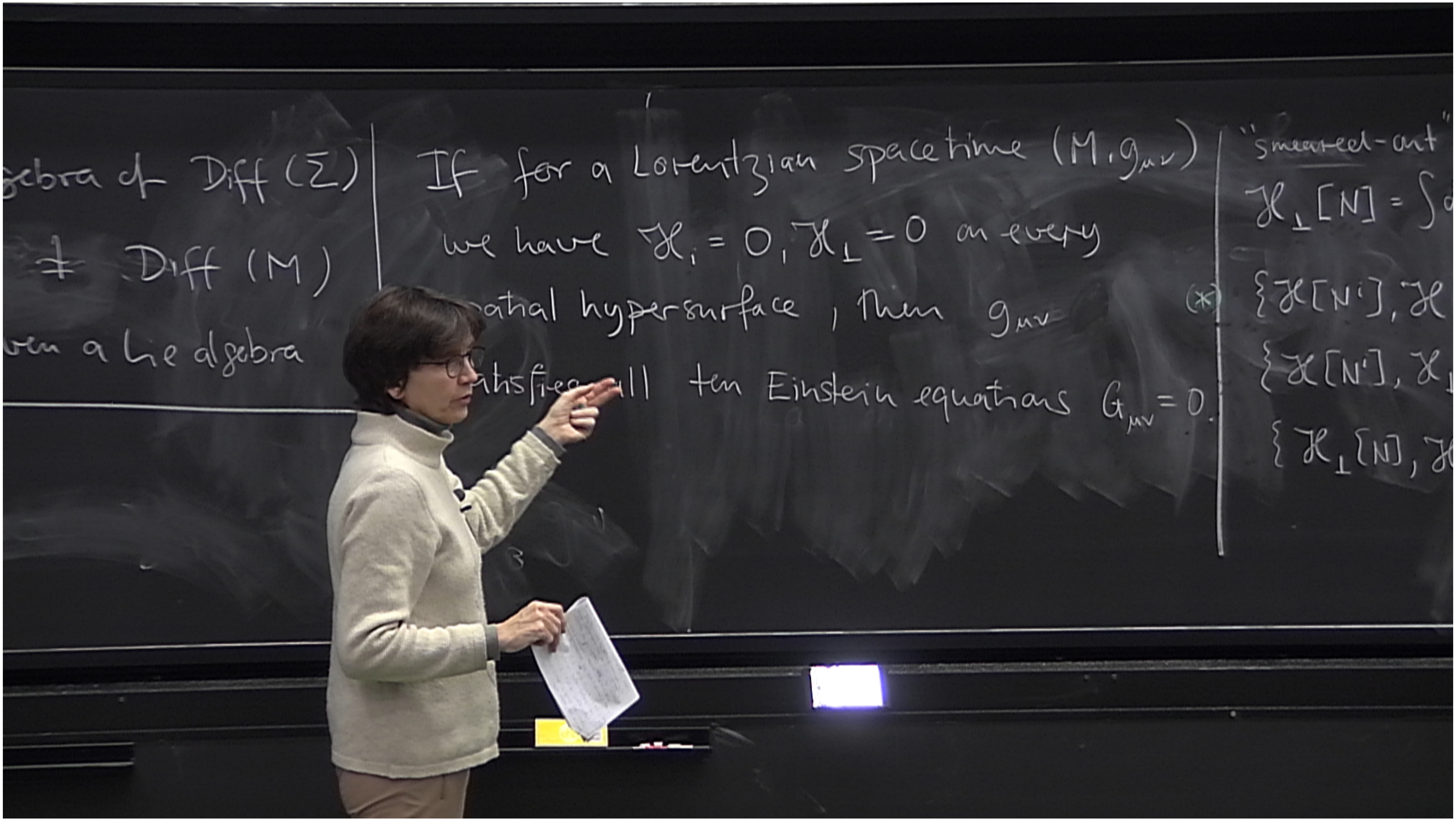
↗ structure functions!


(*) ≅ isom. Lie algebra of Diff (Σ)
Dirac algebra ≇ Diff (M)
not even a Li lgebra

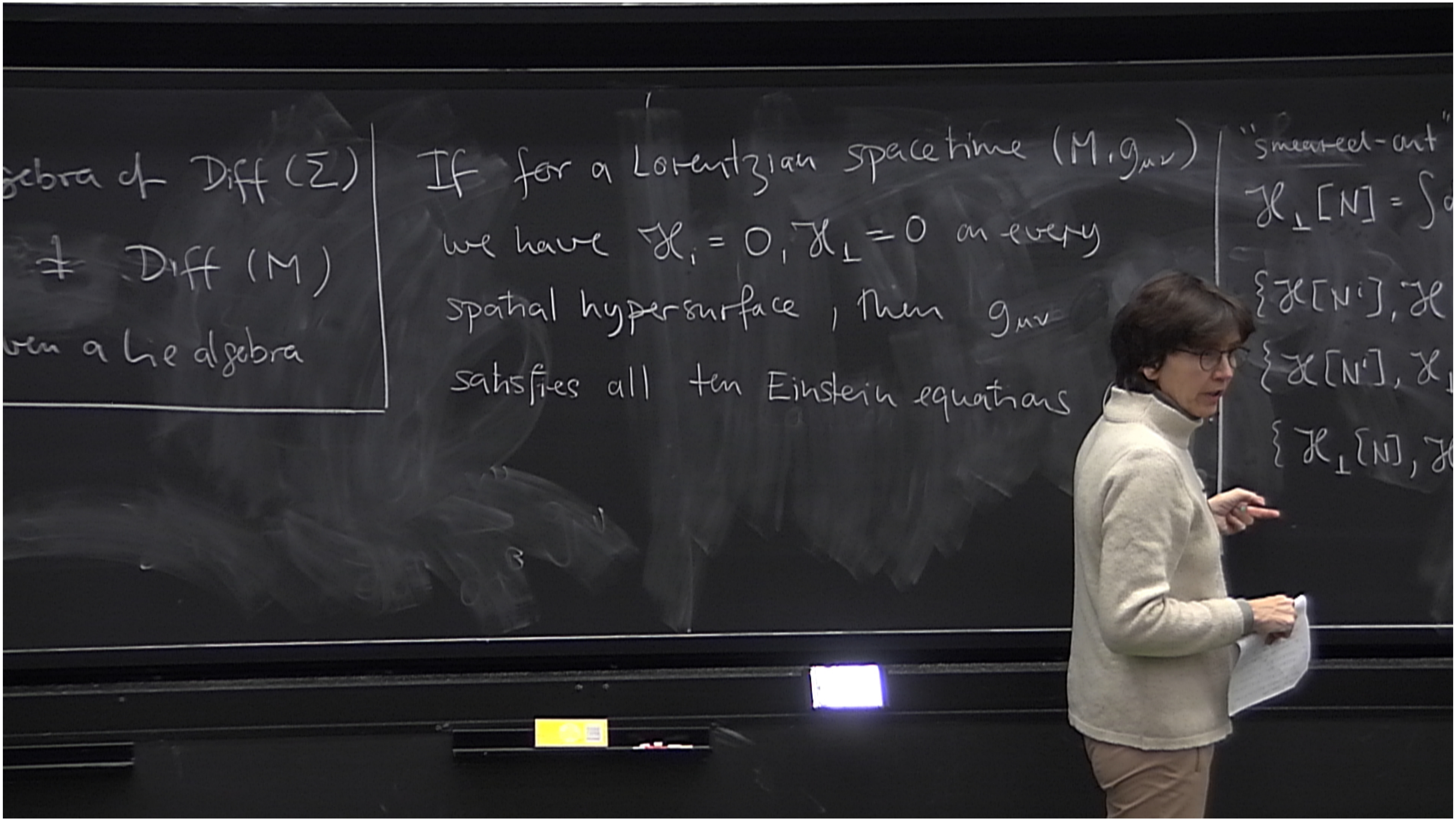

gebra of Diff (Σ)
≠ Diff (M)
ven a Lie algebra
If for a Lorentzian spacetime (M, g_μν)
we have H_i = 0, H_⊥ = 0 on every
"smeared-out"
H_⊥ [N] = ∫
(*)
{H[N'], H
{H[N'], H
{H_⊥[N], H

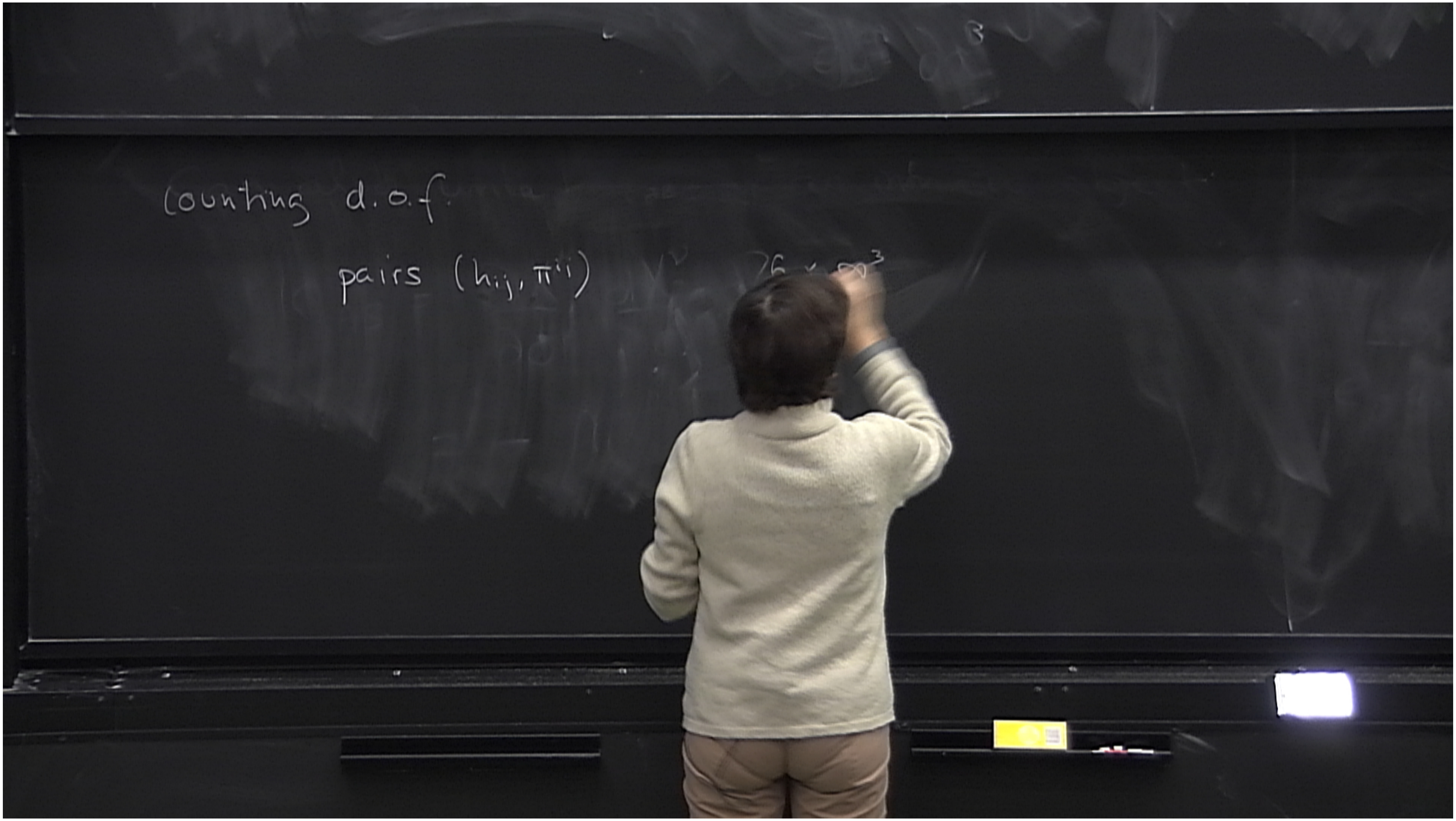

gebra of Diff (Σ)
≠ Diff (M)
ven a Lie algebra
If for a Lorentzian spacetime (M, g_μν)
we have ℋ_i = 0, ℋ_⊥ = 0 on every
spat persurface , then g_μν
sati
"smeared-out"
ℋ_⊥[N] = ∫
(*)
{ℋ[N'], ℋ
{ℋ[N'], ℋ
{ℋ_⊥[N], ℋ

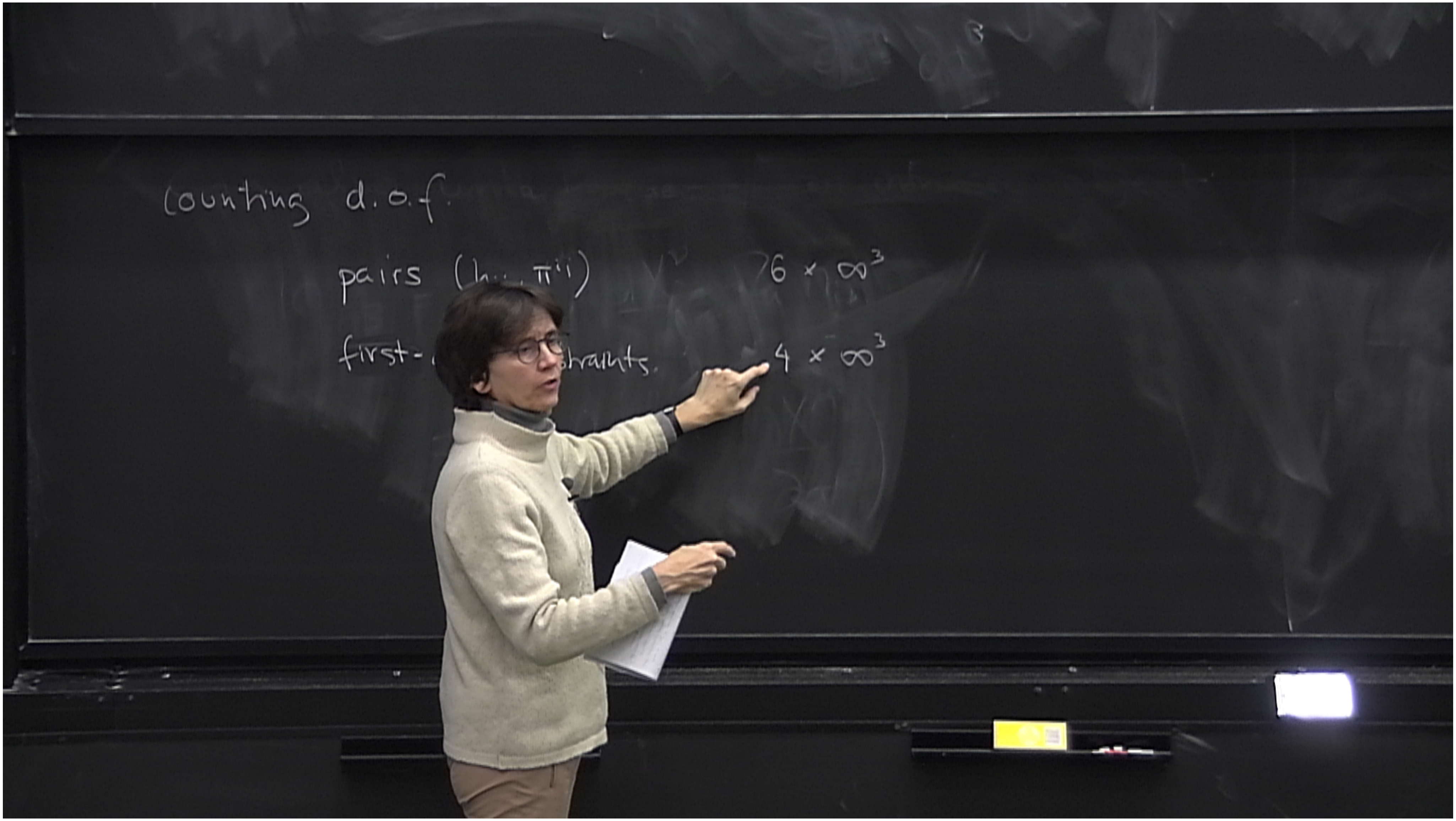

gebra of Diff(Σ)
≠ Diff (M)
If for a Lorentzian spacetime $(M, g_{\mu\nu})$
we have $\mathcal{H}_i = 0$, $\mathcal{H}_\perp = 0$ on every
spatial hypersurface, then $g_{\mu\nu}$
satisfies all ten Einstein equations $G_{\mu\nu} = 0$
"smeared-out"
$\mathcal{H}_\perp[N] =$
(*)

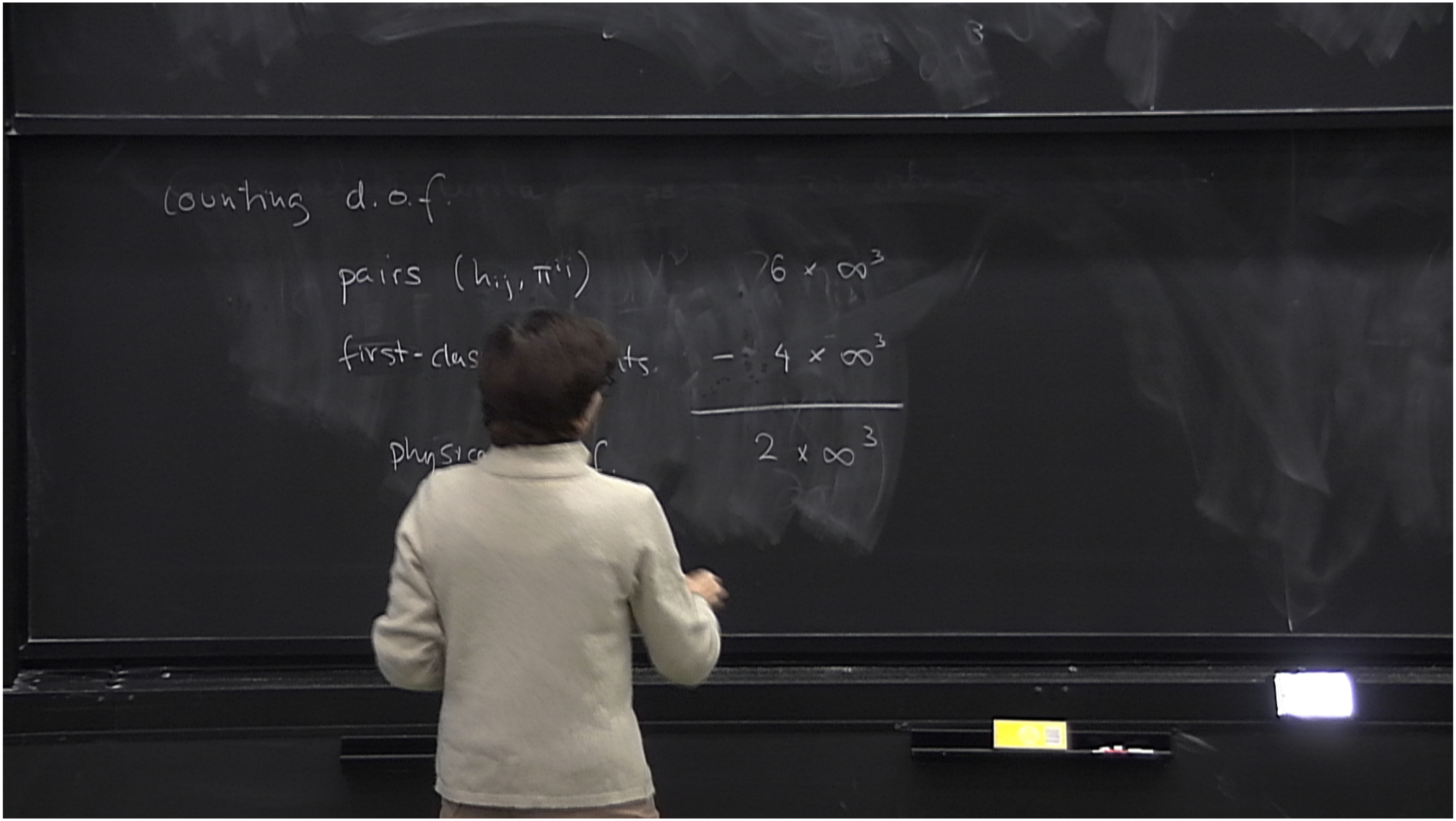


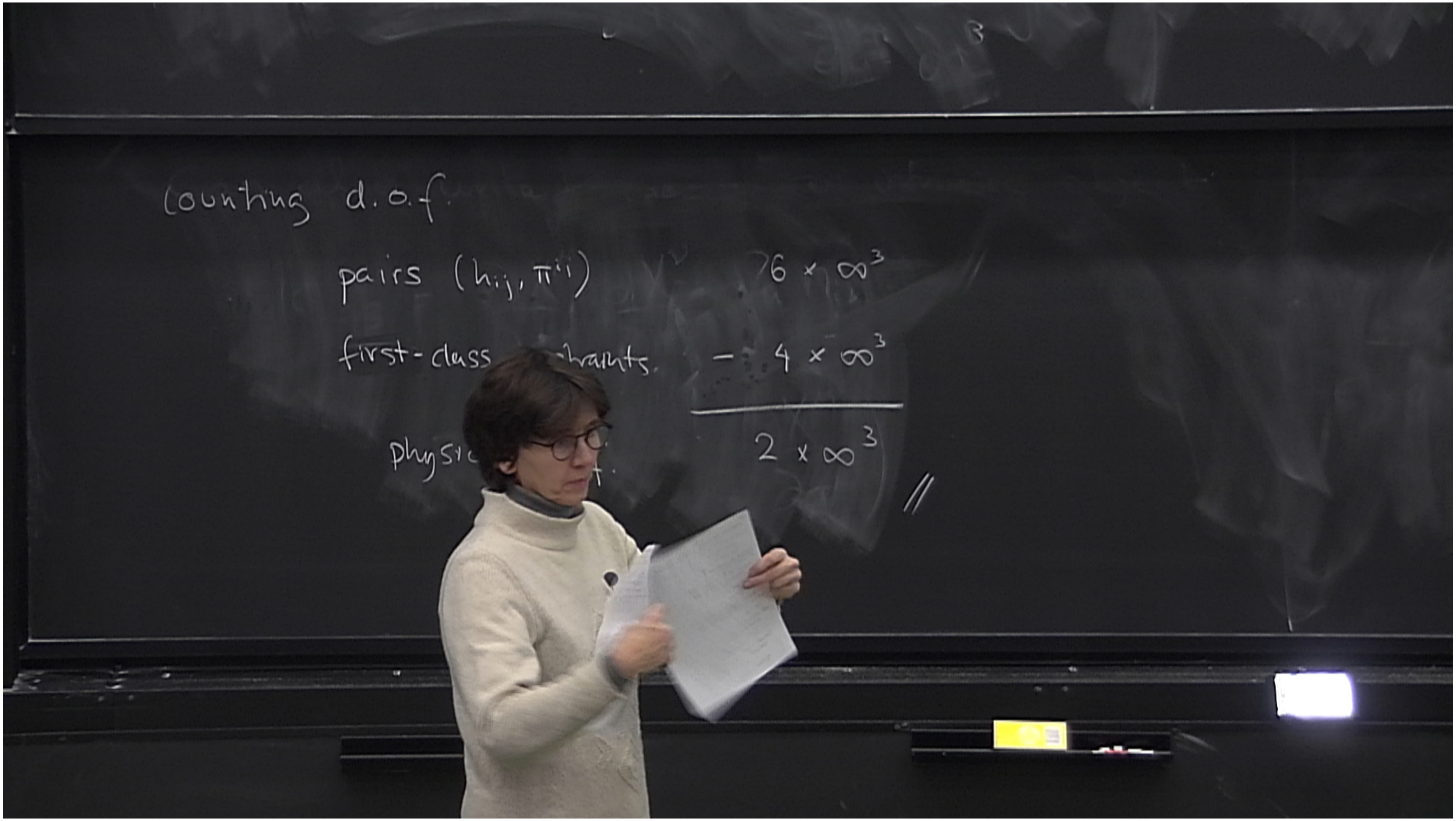
gebra of Diff (Σ)
≠ Diff (M)
ven a Lie algebra
If for a Lorentzian spacetime (M, $g_{\mu\nu}$)
we have $\mathcal{H}_i = 0$, $\mathcal{H}_\perp = 0$ on every
atial hypersurface, then $g_{\mu\nu}$
atisfies all ten Einstein equations $G_{\mu\nu} = 0$.
"smeared-out"
$\mathcal{H}_\perp[N] = \int$
(*)

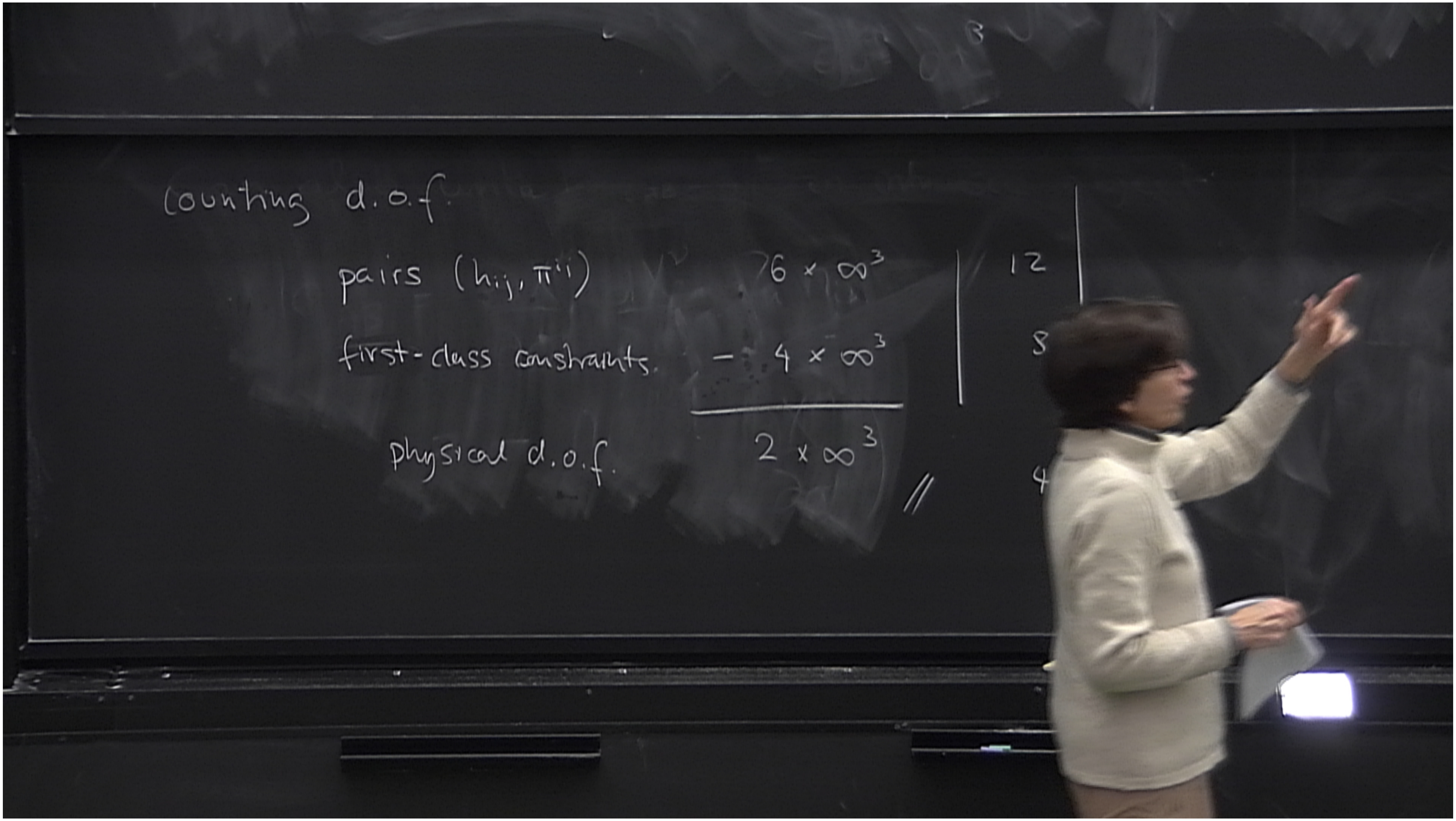

gebra of Diff (Σ)
≠ Diff (M)
ven a Lie algebra
If for a Lorentzian spacetime (M, g_μν)
we have ℋ_i = 0, ℋ_⊥ = 0 on every
spatial hypersurface, then g_μν
satisfies all ten Einstein equations
"smeared-out"
ℋ_⊥ [N] = ∫
{ℋ [N^i], ℋ
{ℋ [N^i], ℋ
{ℋ_⊥ [N], ℋ

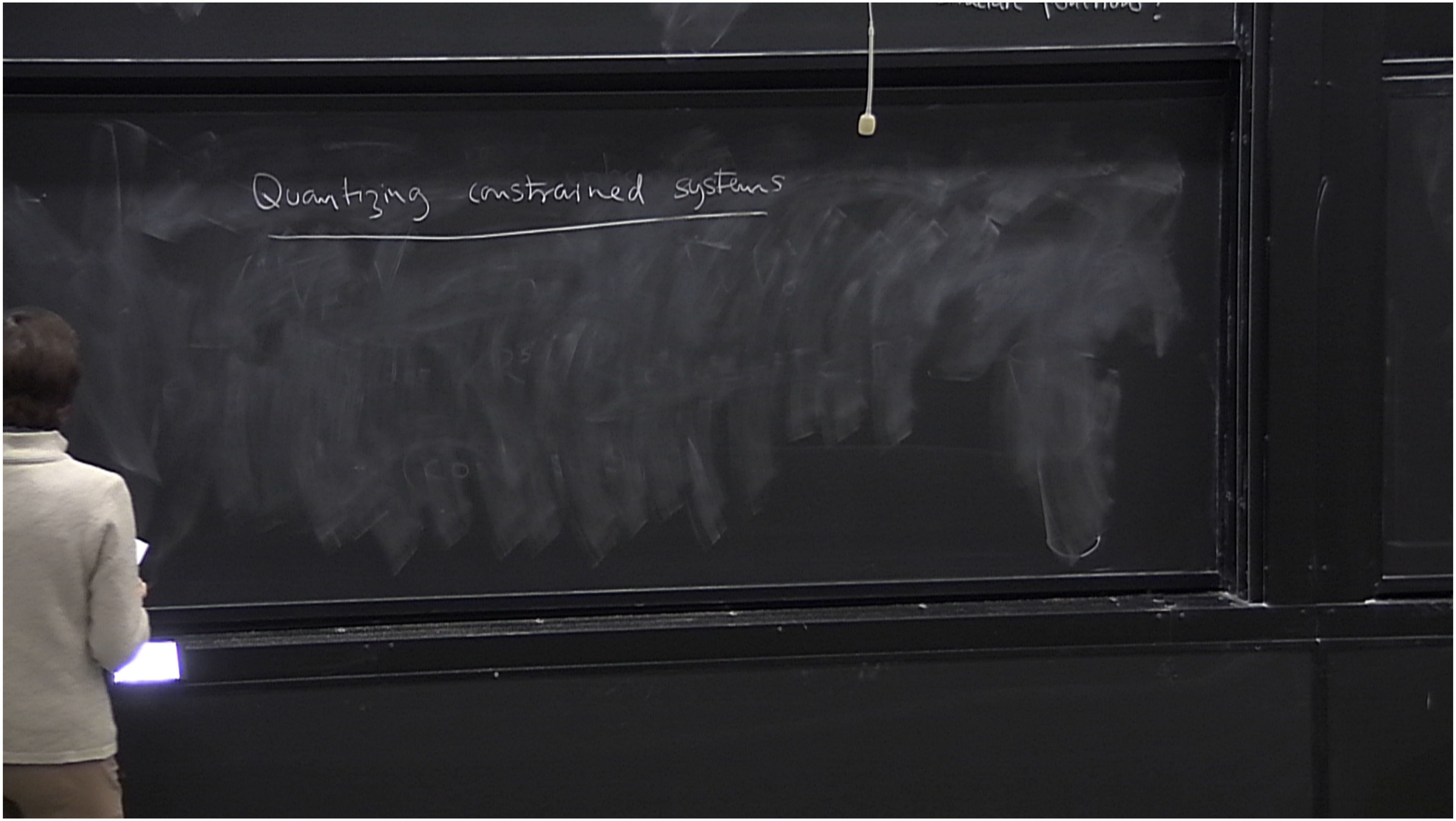

counting d.o.f.
pairs (hij, πij)

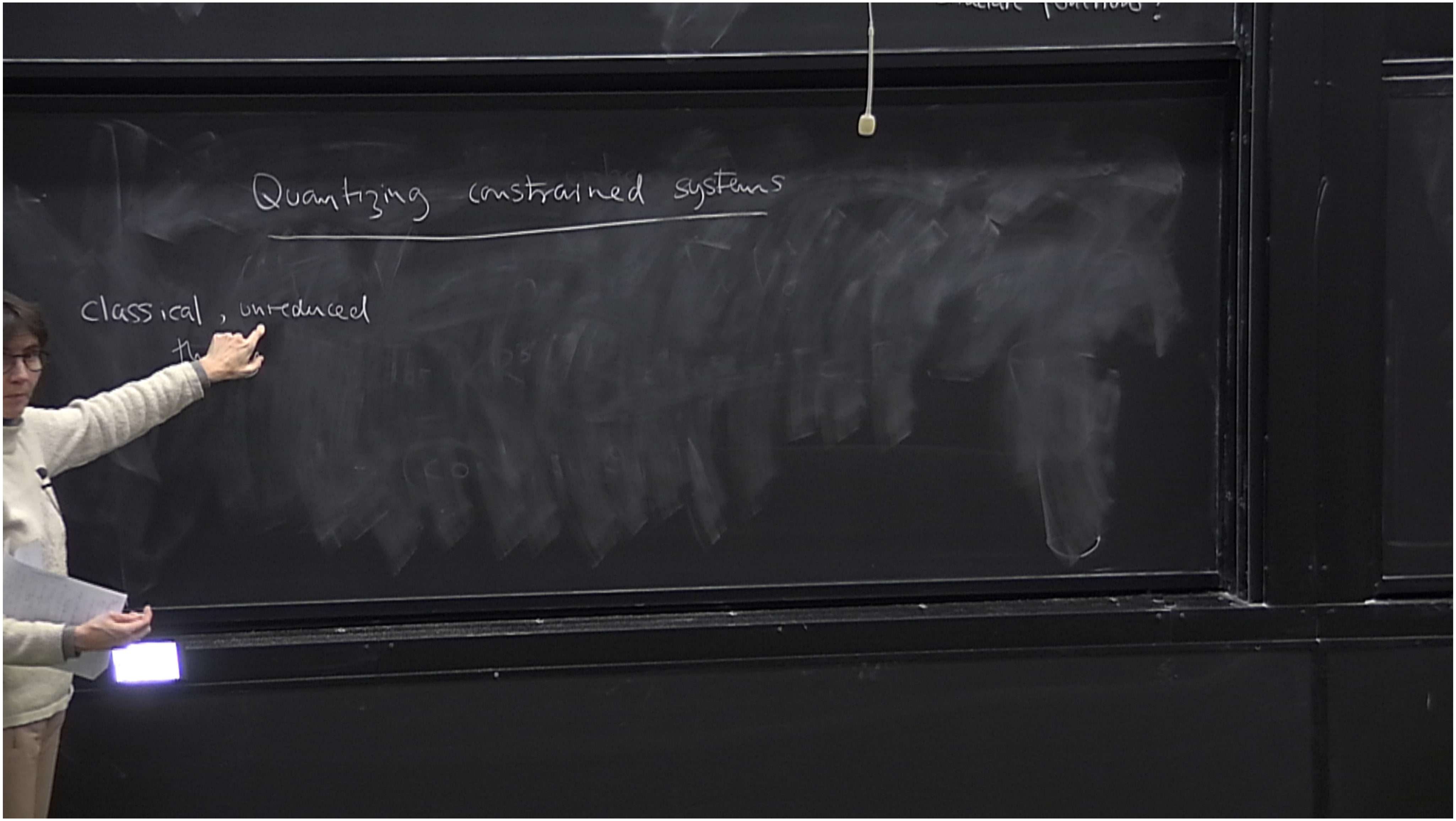

counting d.o.f.
pairs
6 × ∞3
4 × ∞3

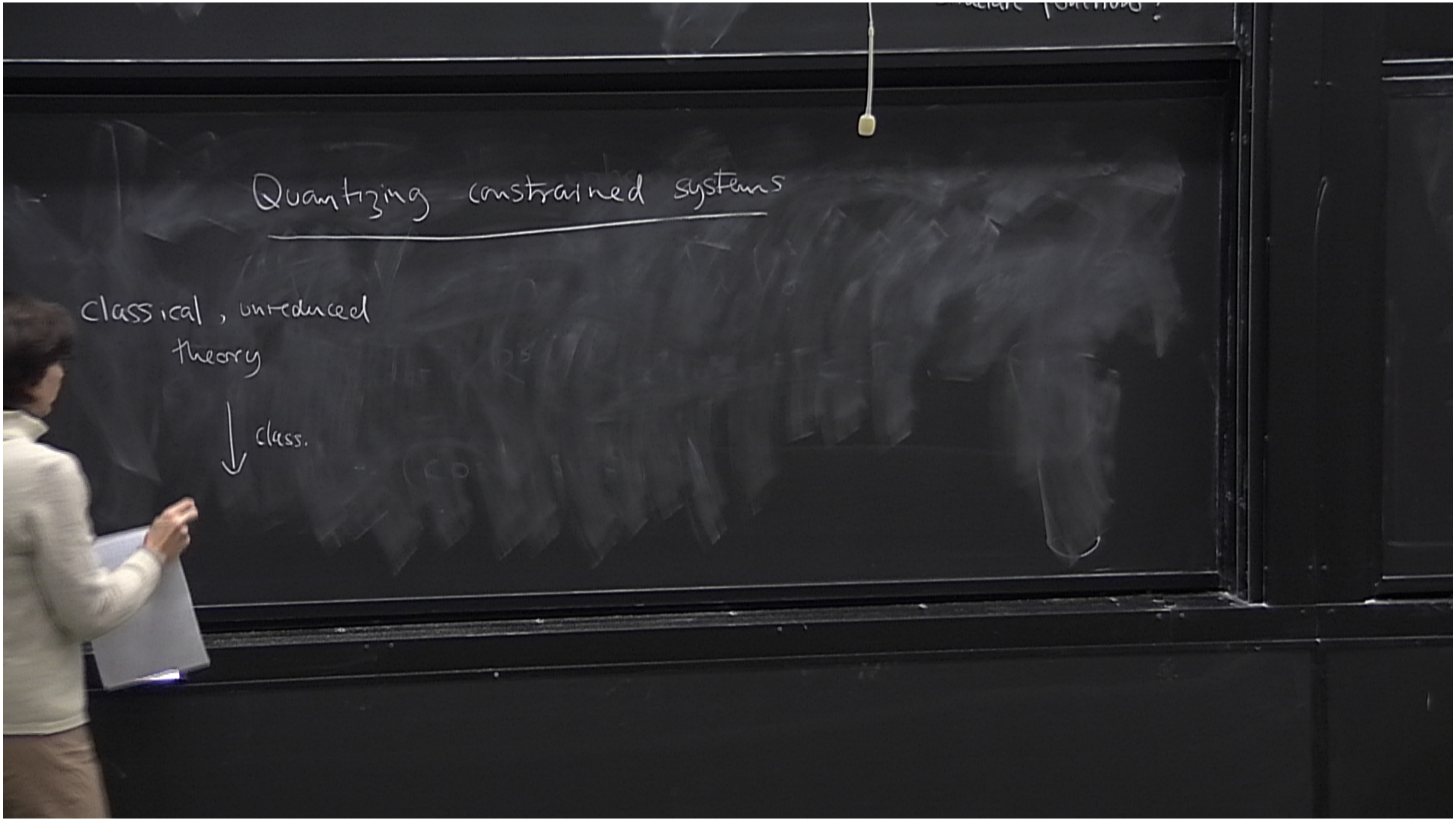

counting d.o.f.
pairs (h_{ij}, π^{ij}) $6 \times \infty^3$
first-clas ts. $- 4 \times \infty^3$
$2 \times \infty^3$

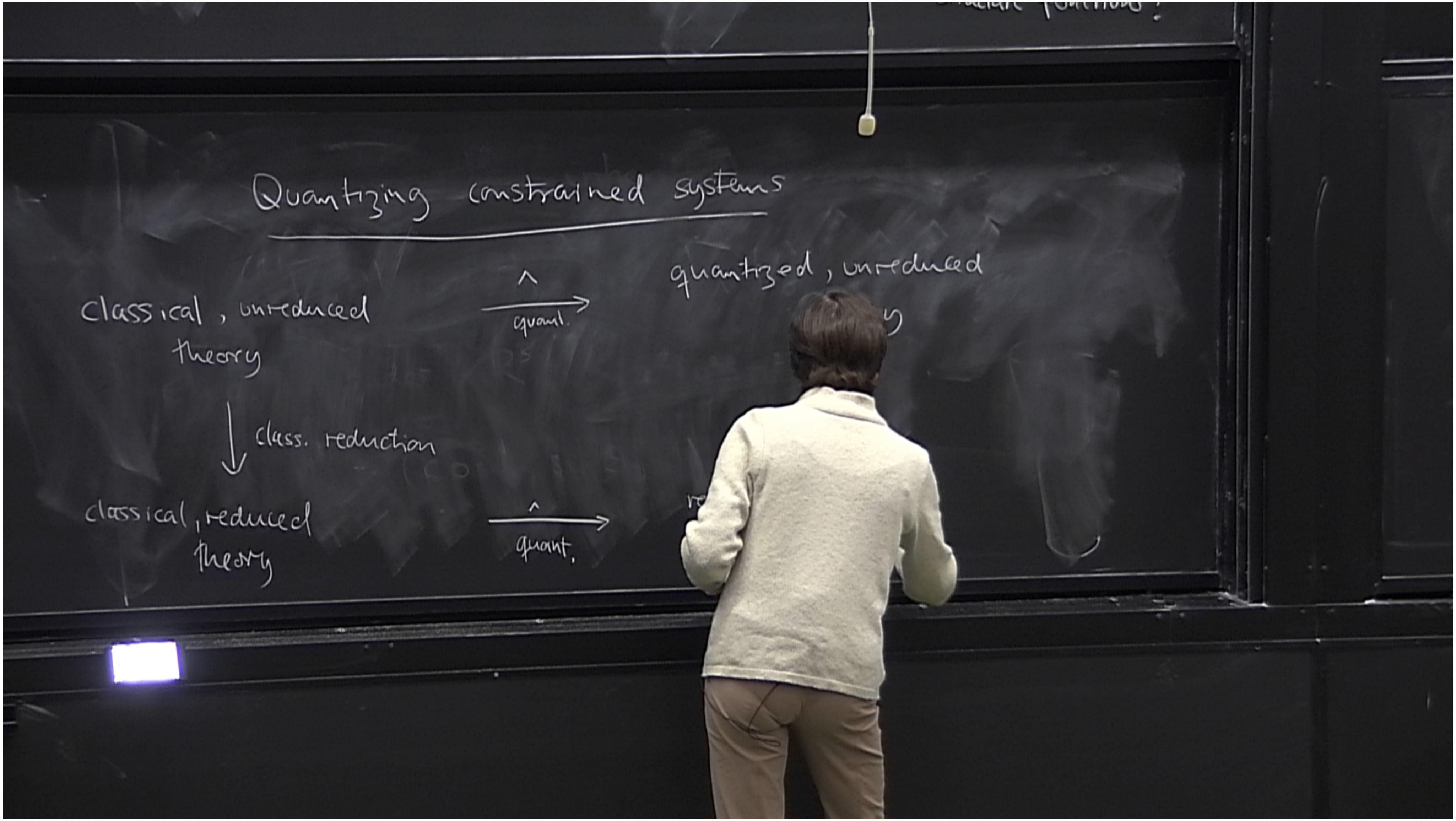

counting d.o.f.
pairs (h_ij, π^ij)
6 × ∞^3
first-class
4 × ∞^3
2 × ∞^3


counting d.o.f.
pairs (h_ij, π^ij) 6 × ∞^3
12
first-class constraints. − 4 × ∞^3
8
physical d.o.f. 2 × ∞^3
4


Quantizing constrained systems


Quantizing constrained systems
classical, unreduced


Quantizing constrained systems
classical, unreduced theory
class.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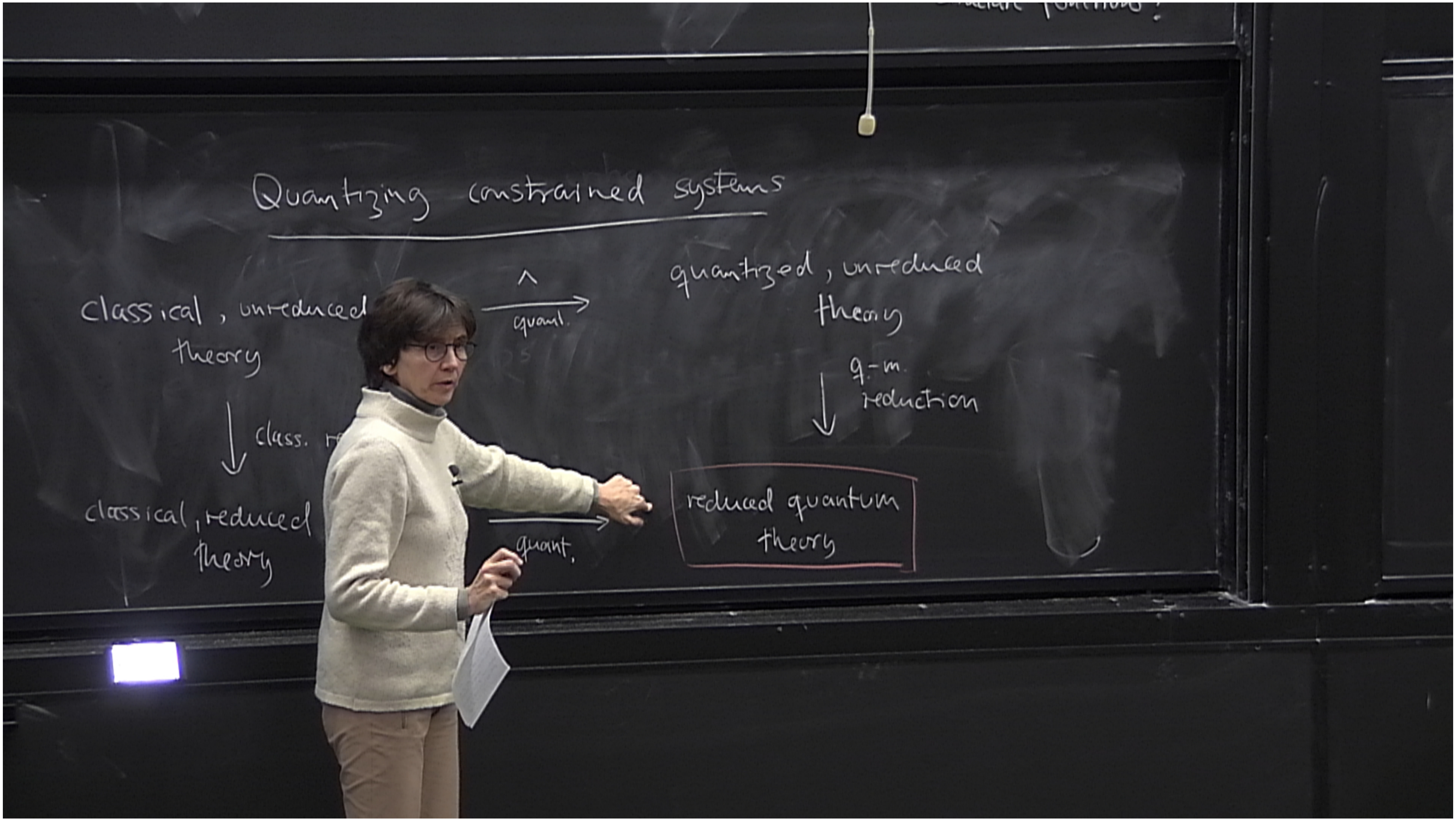

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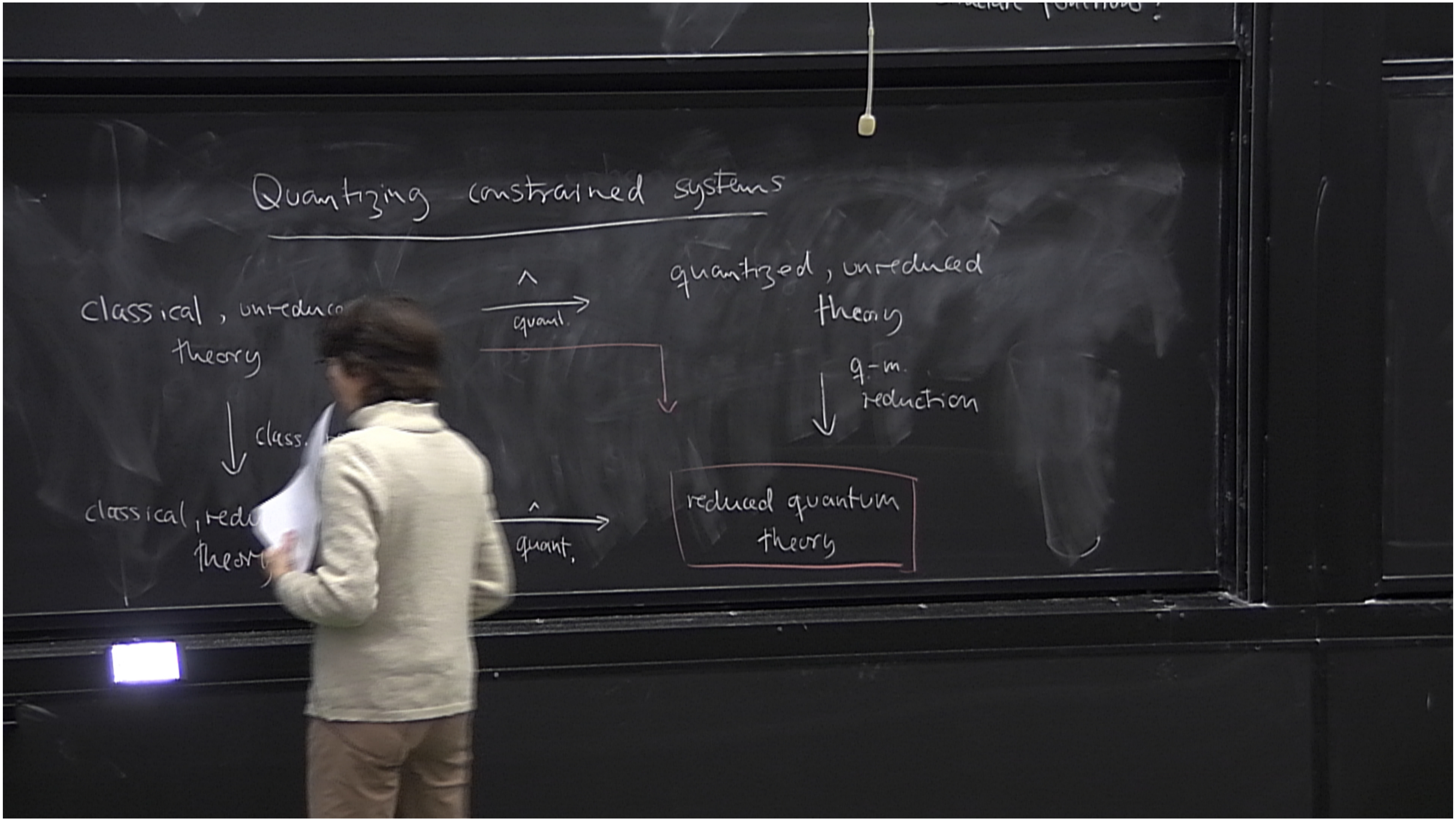

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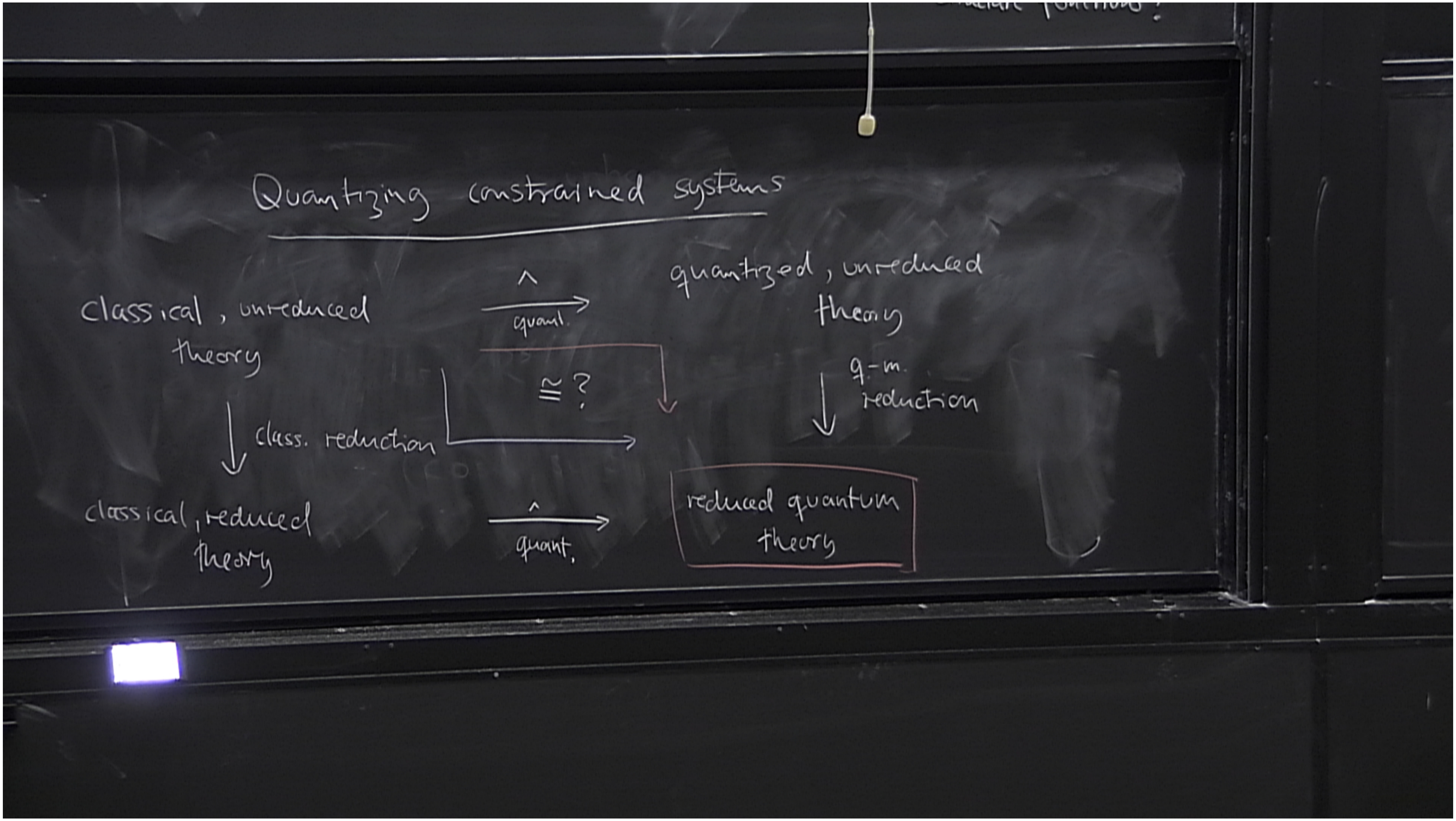
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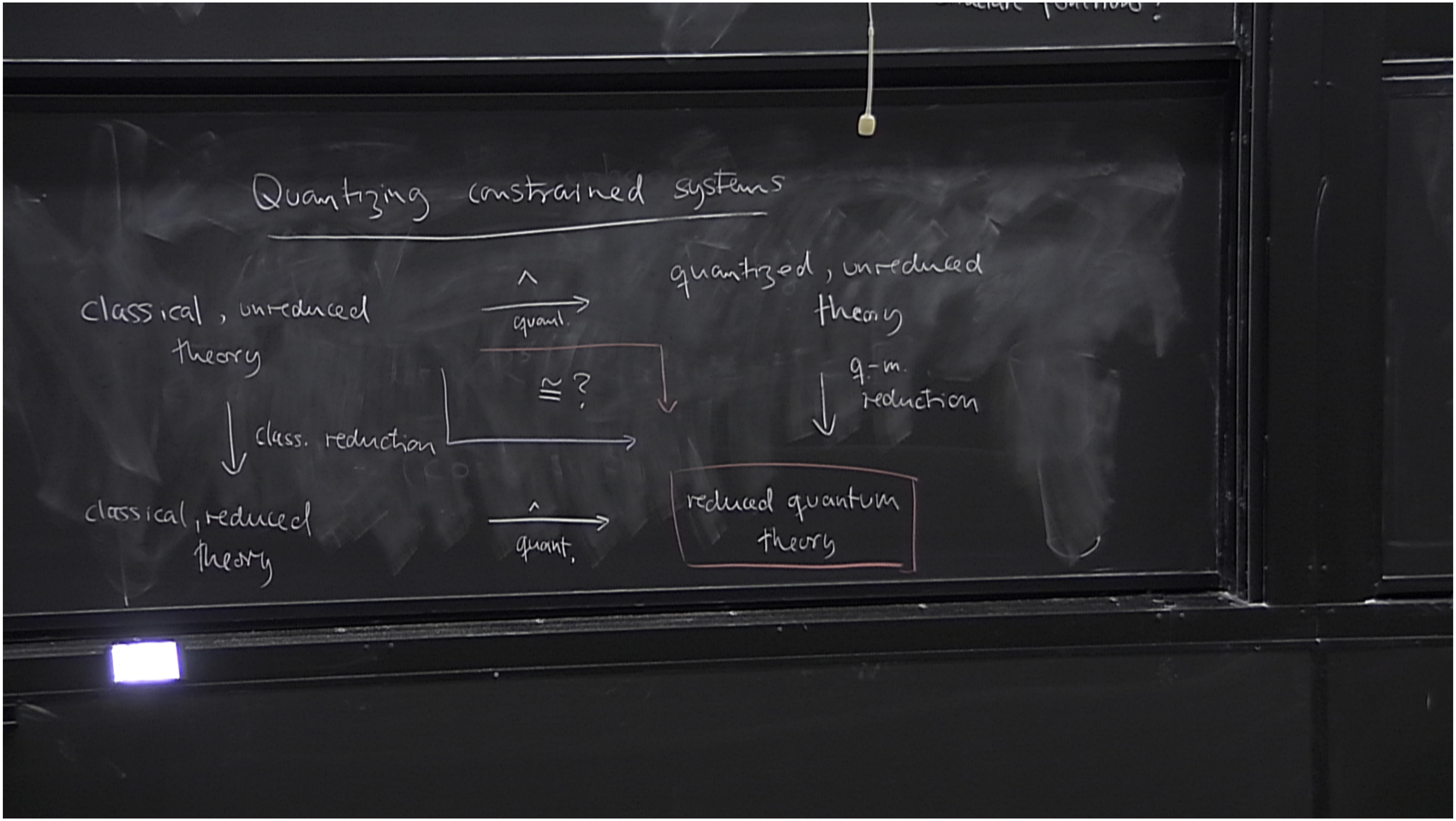
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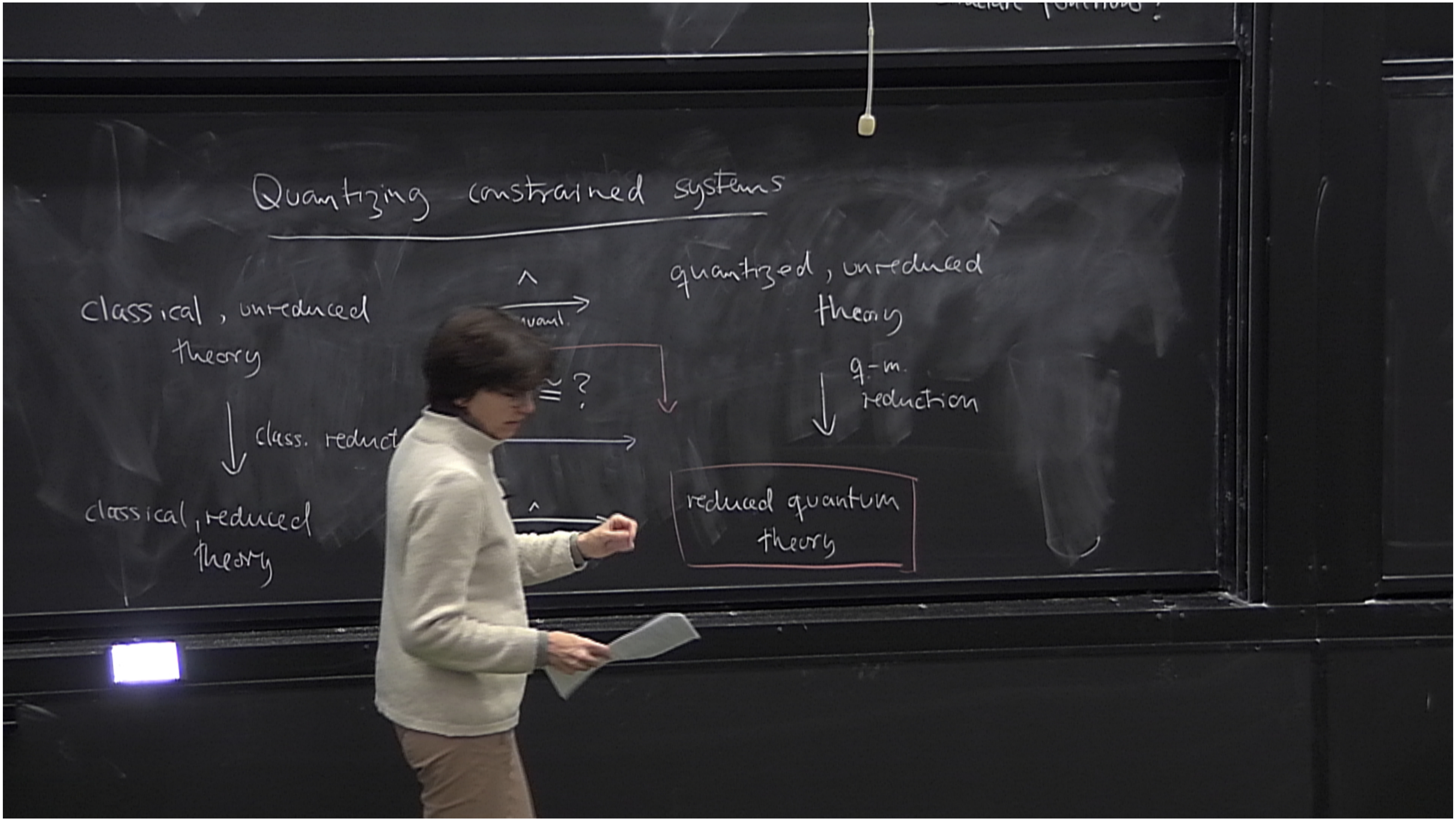
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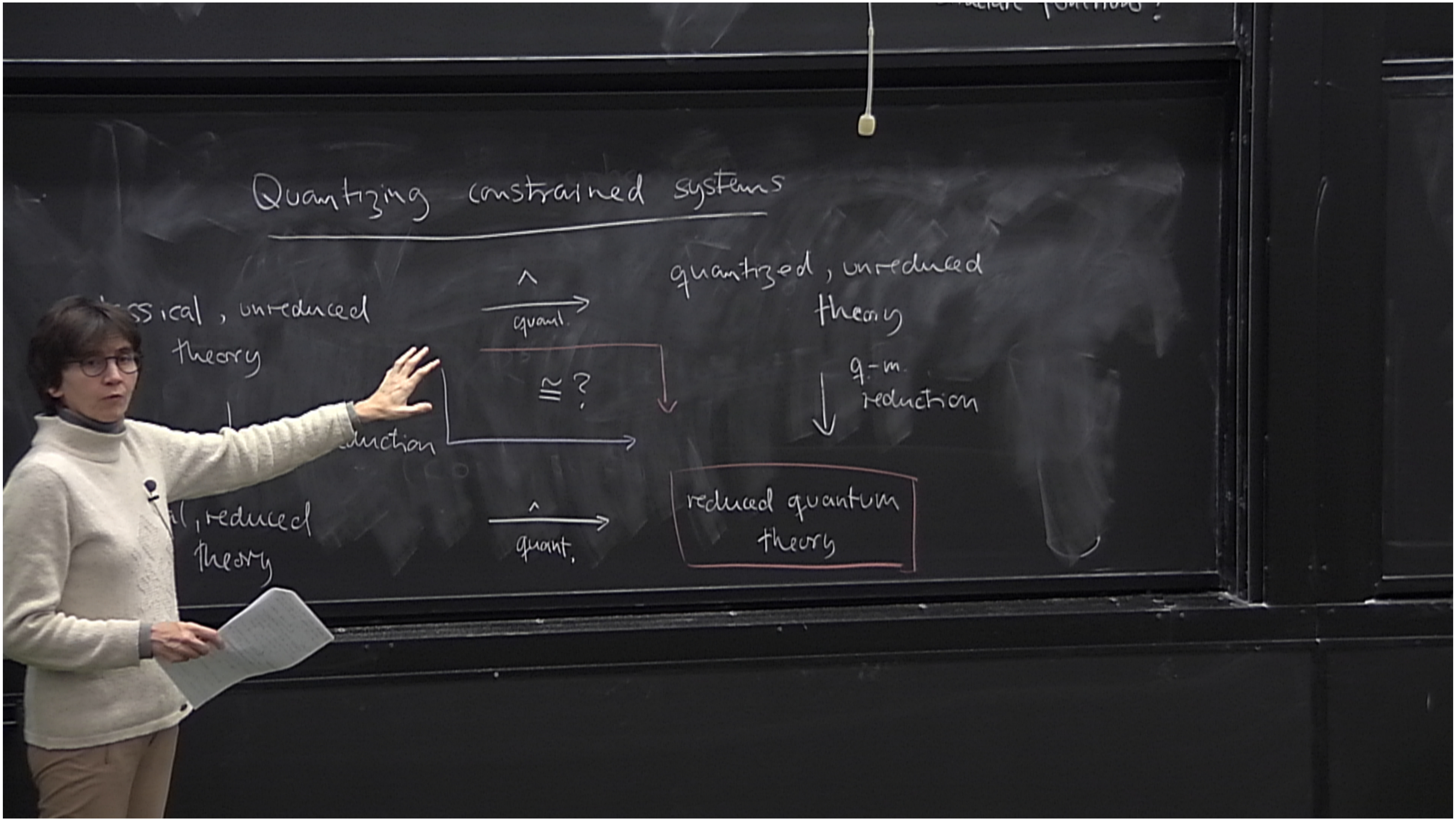
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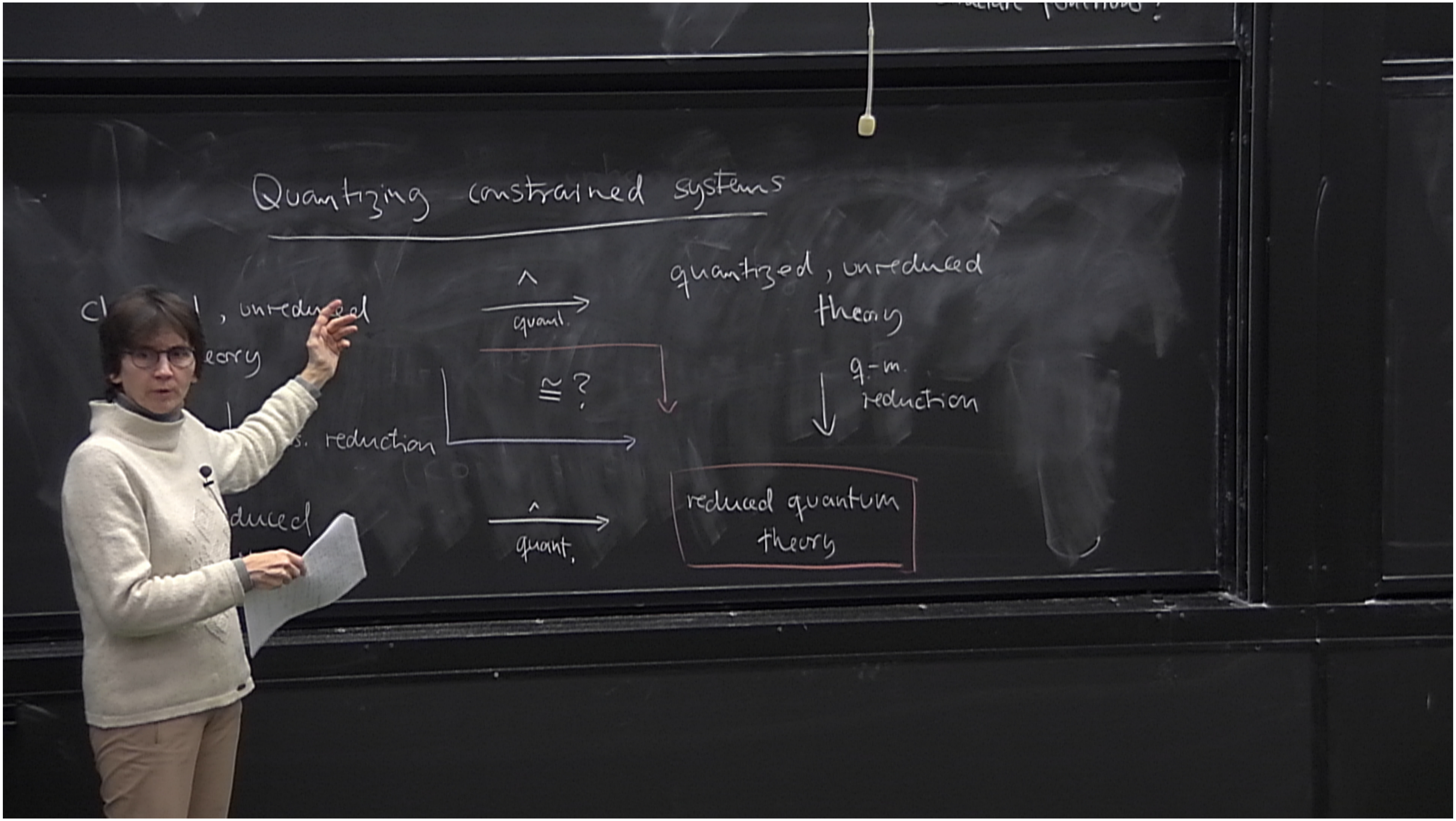
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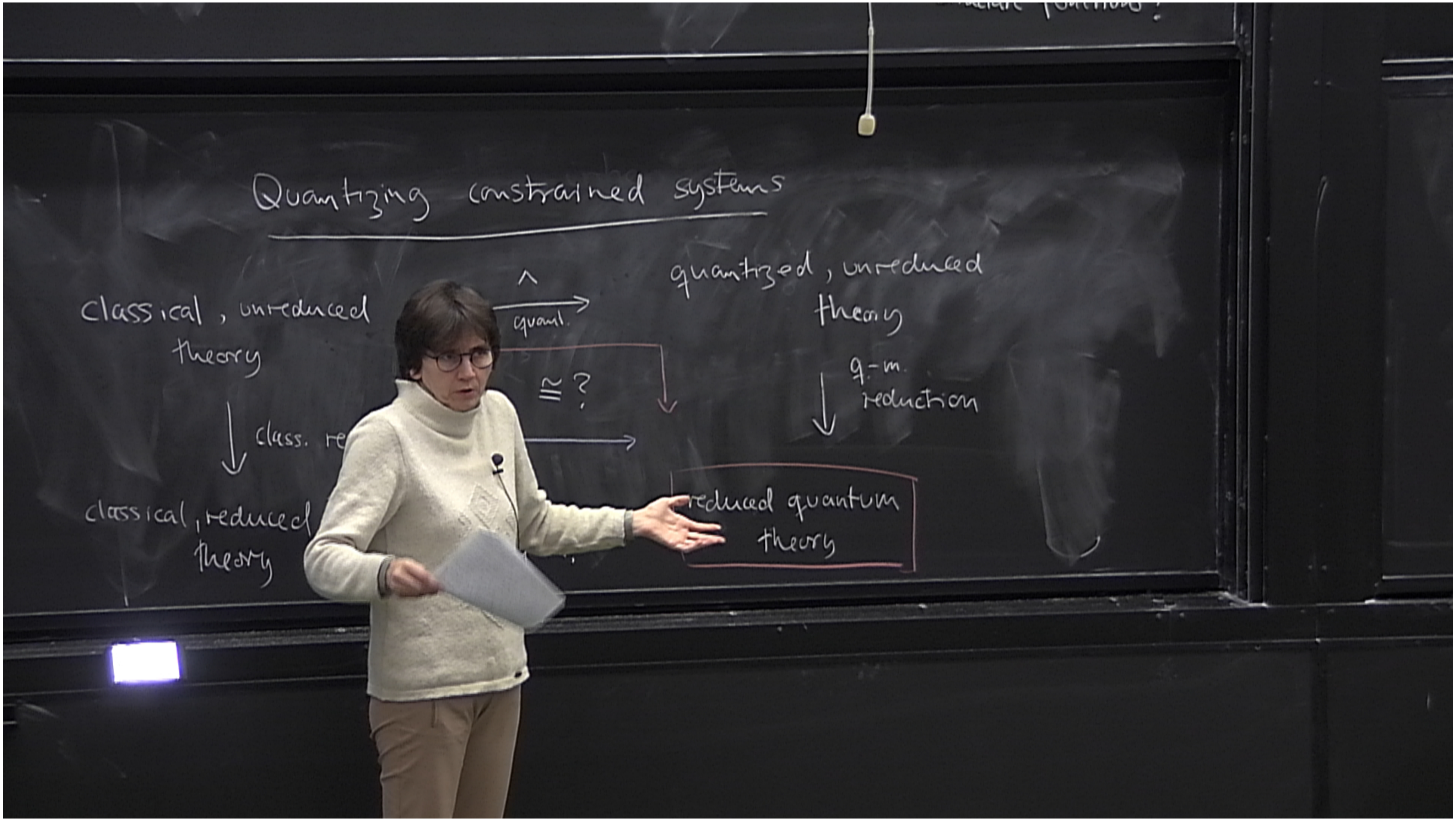
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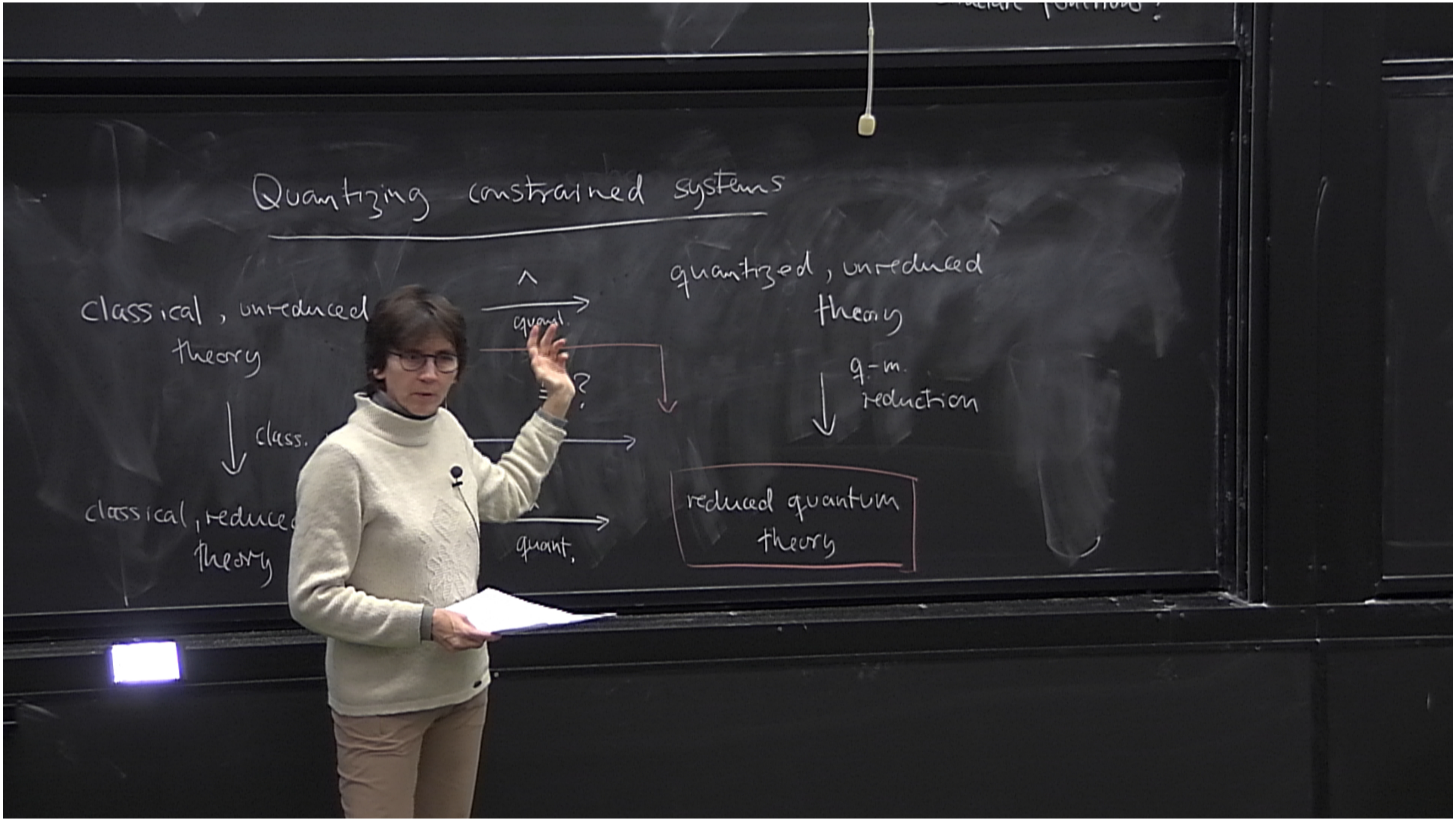
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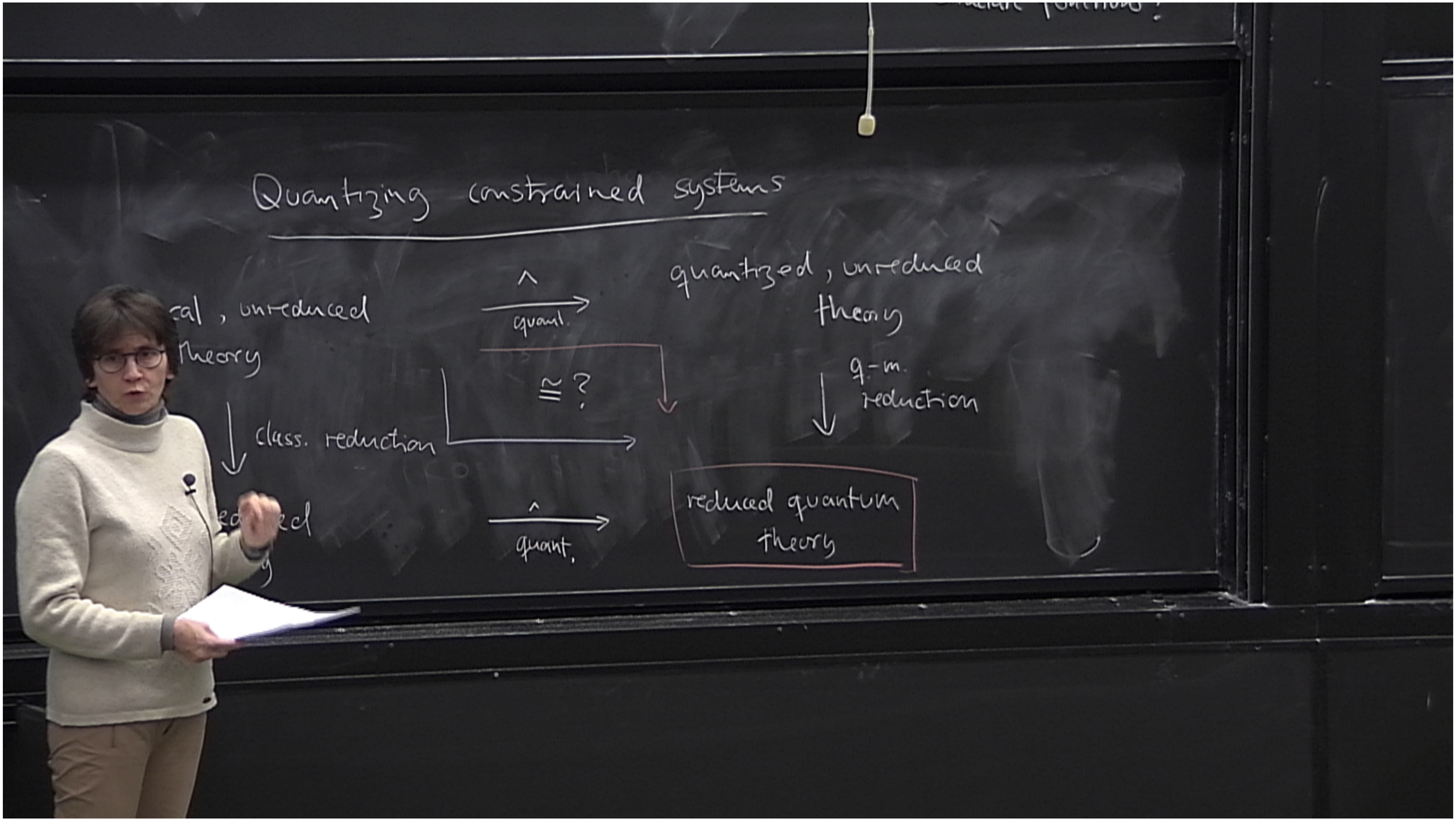
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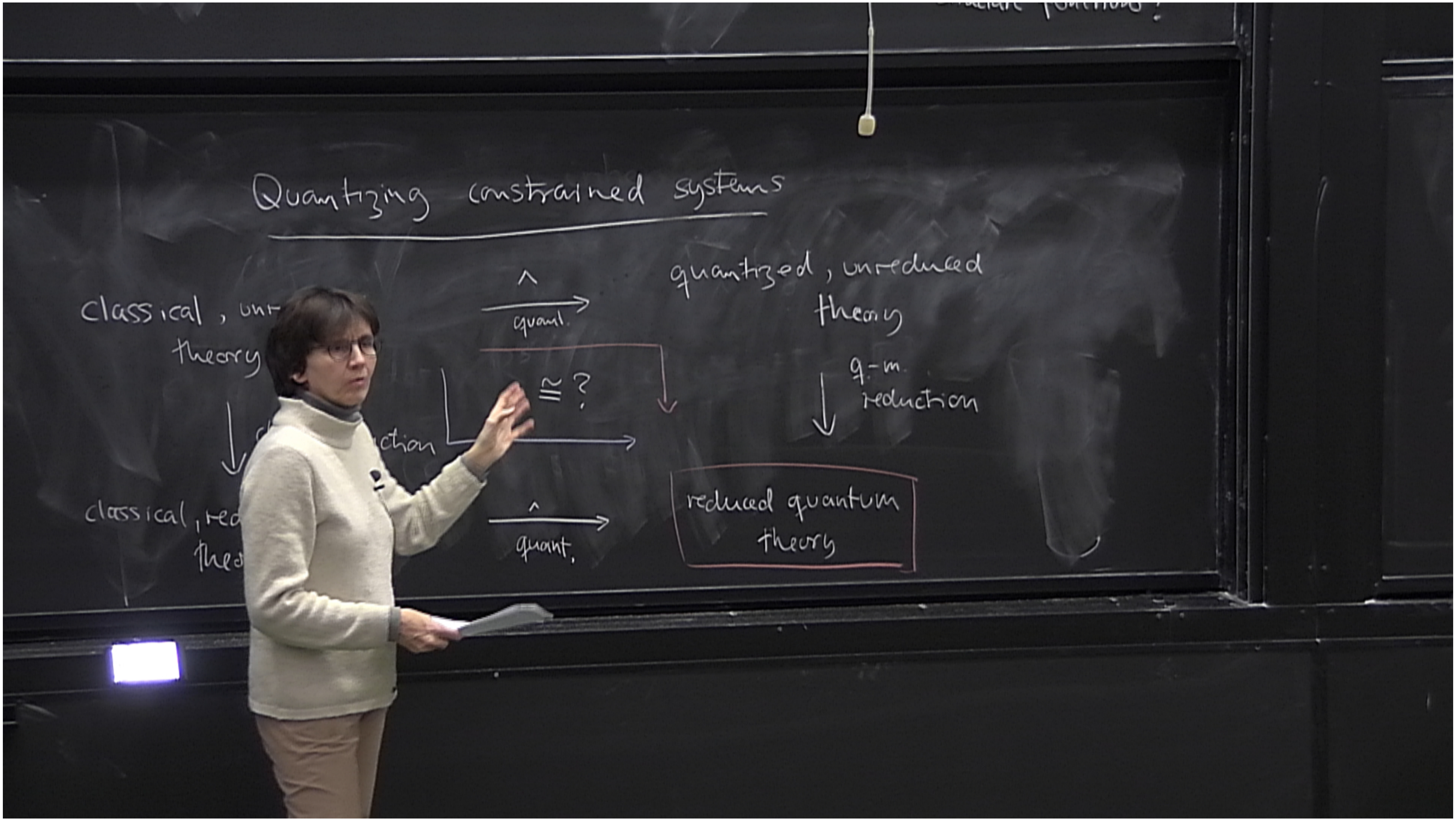
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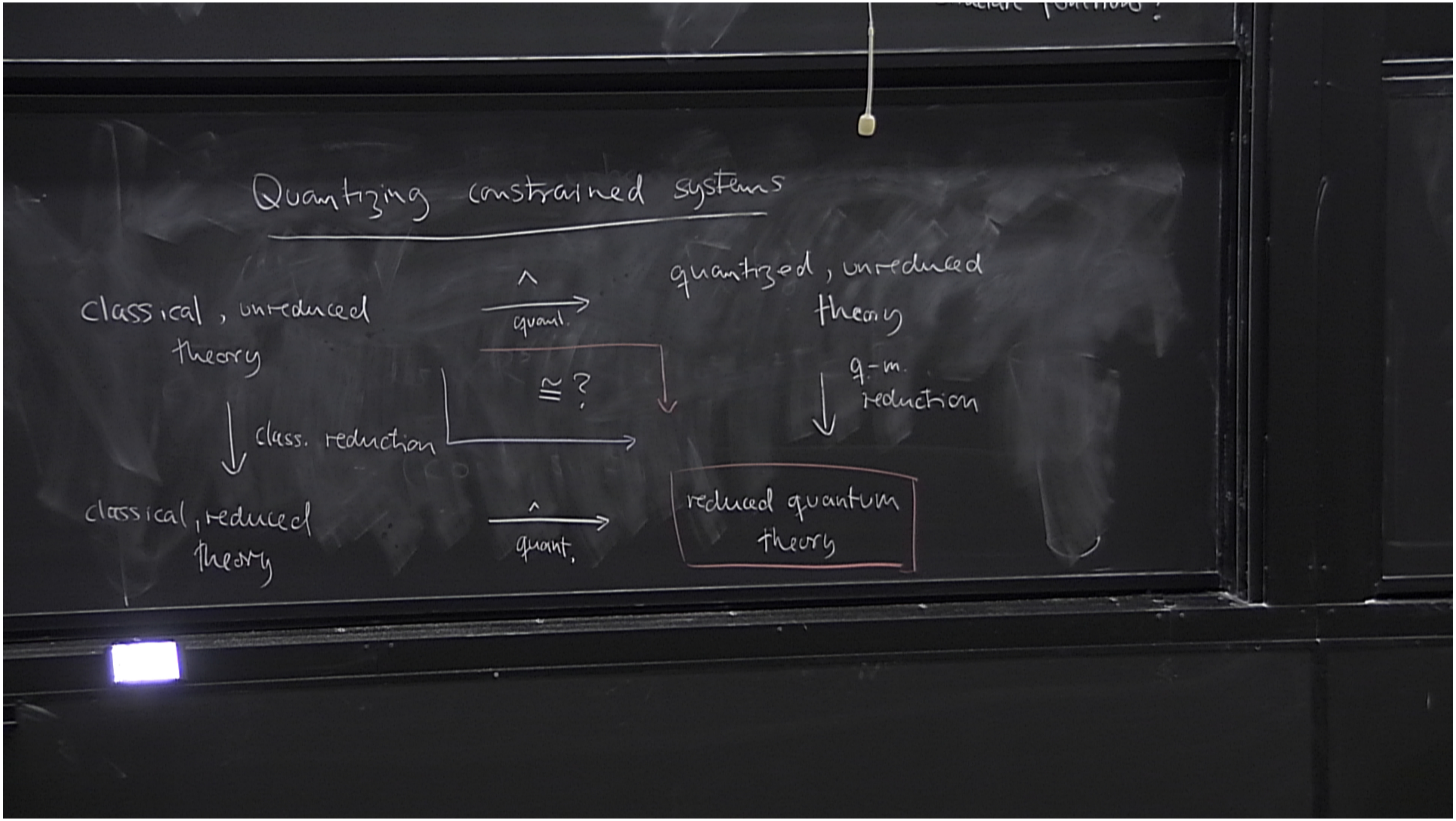
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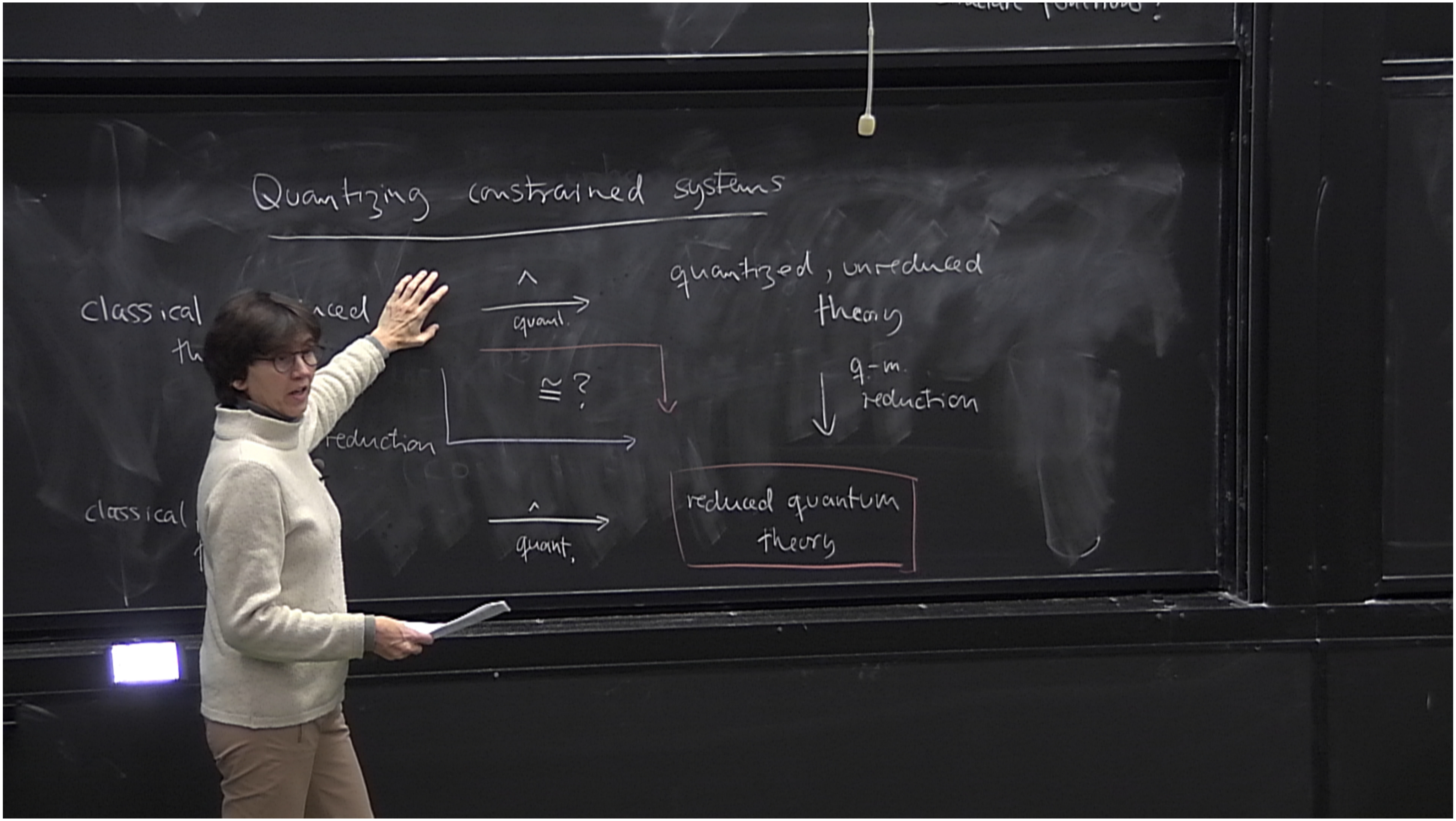
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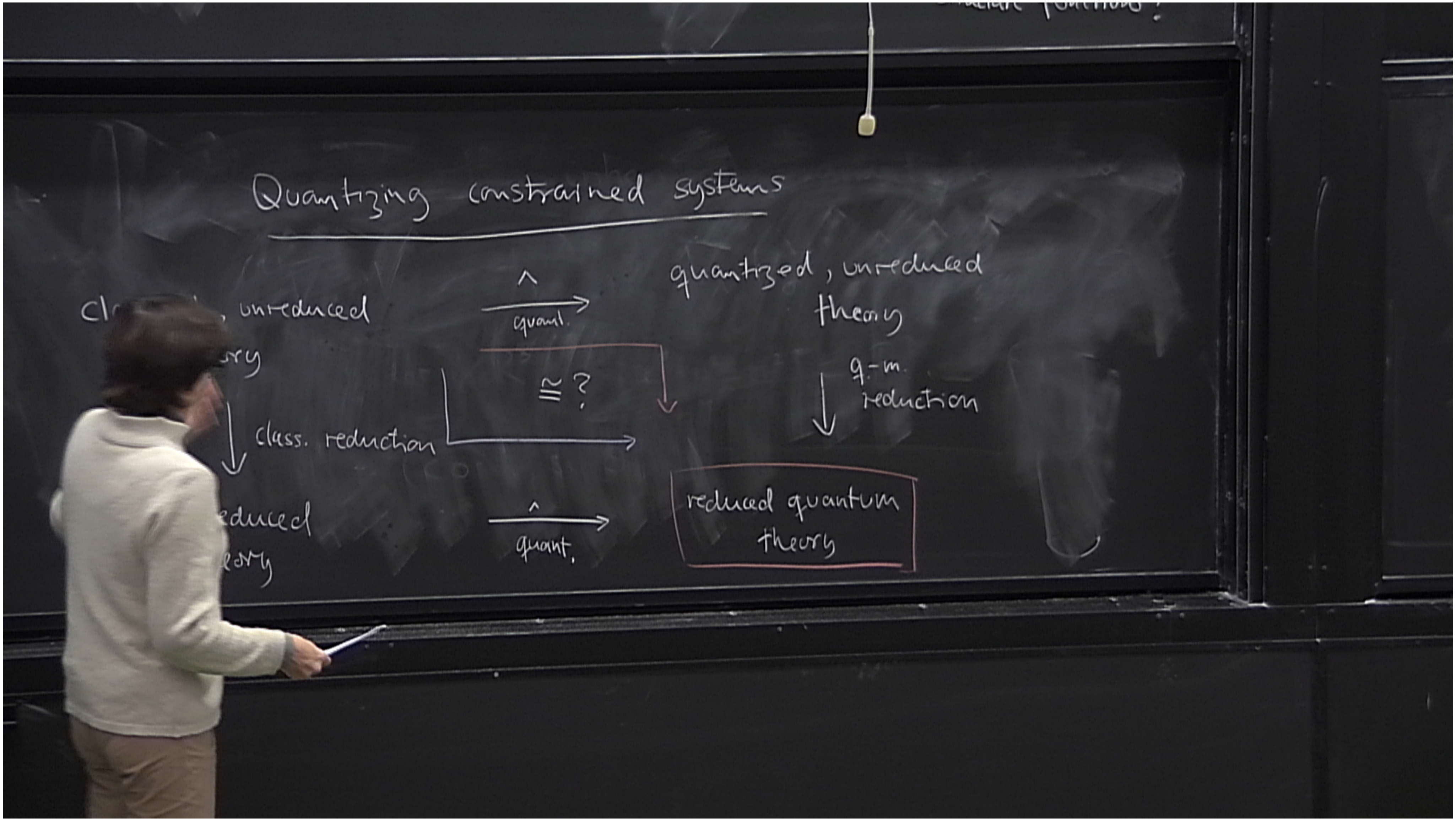

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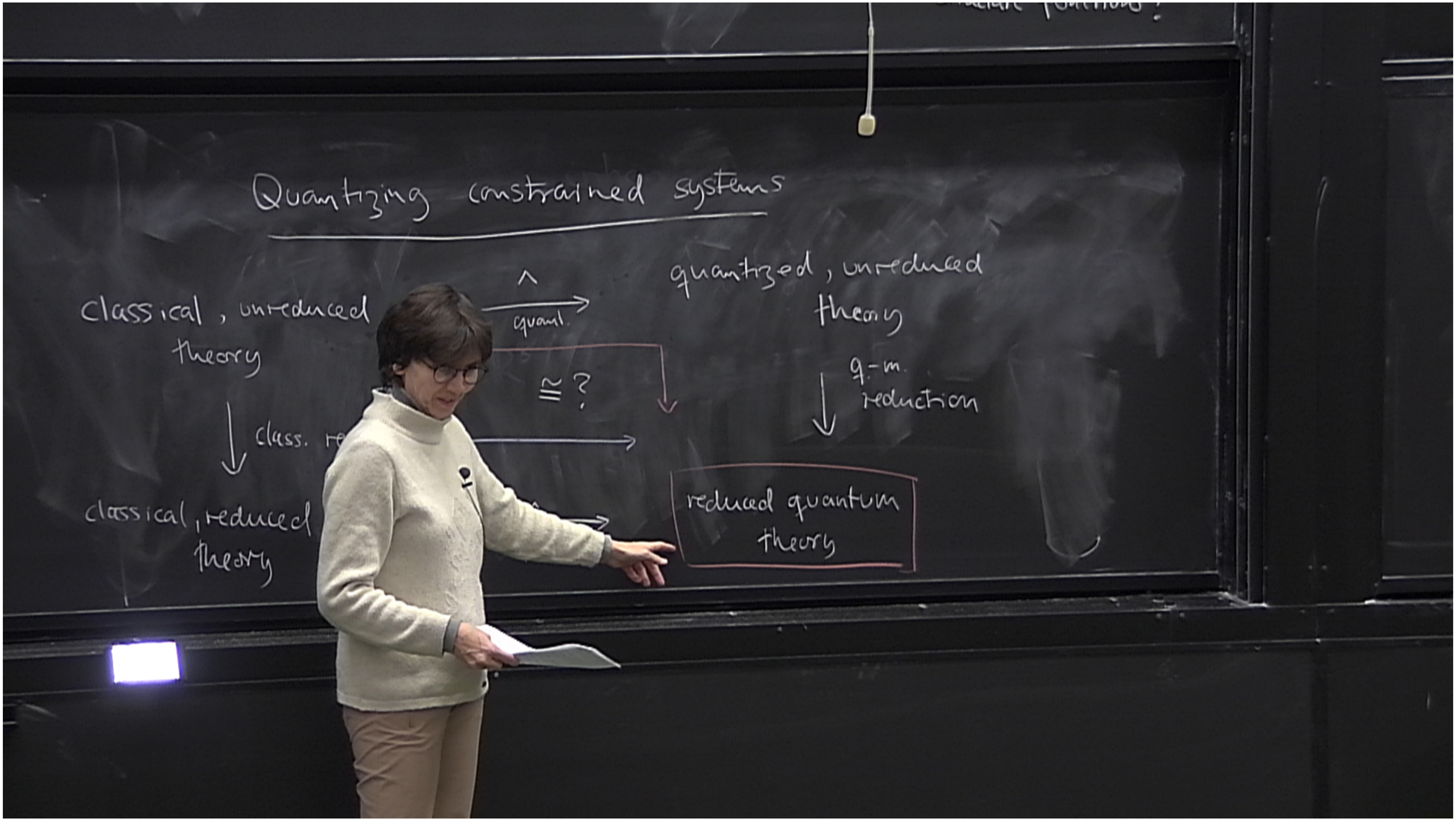
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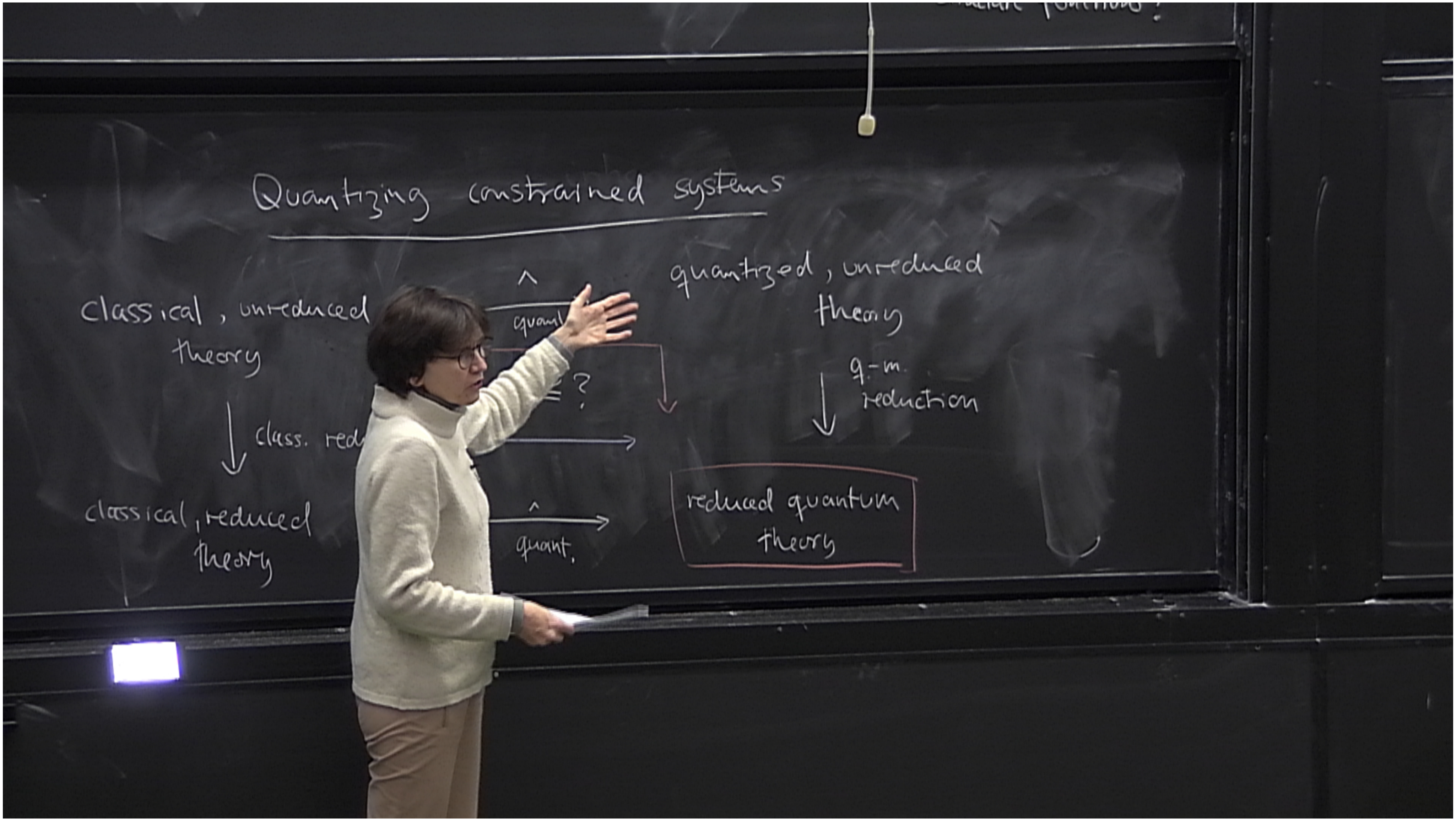
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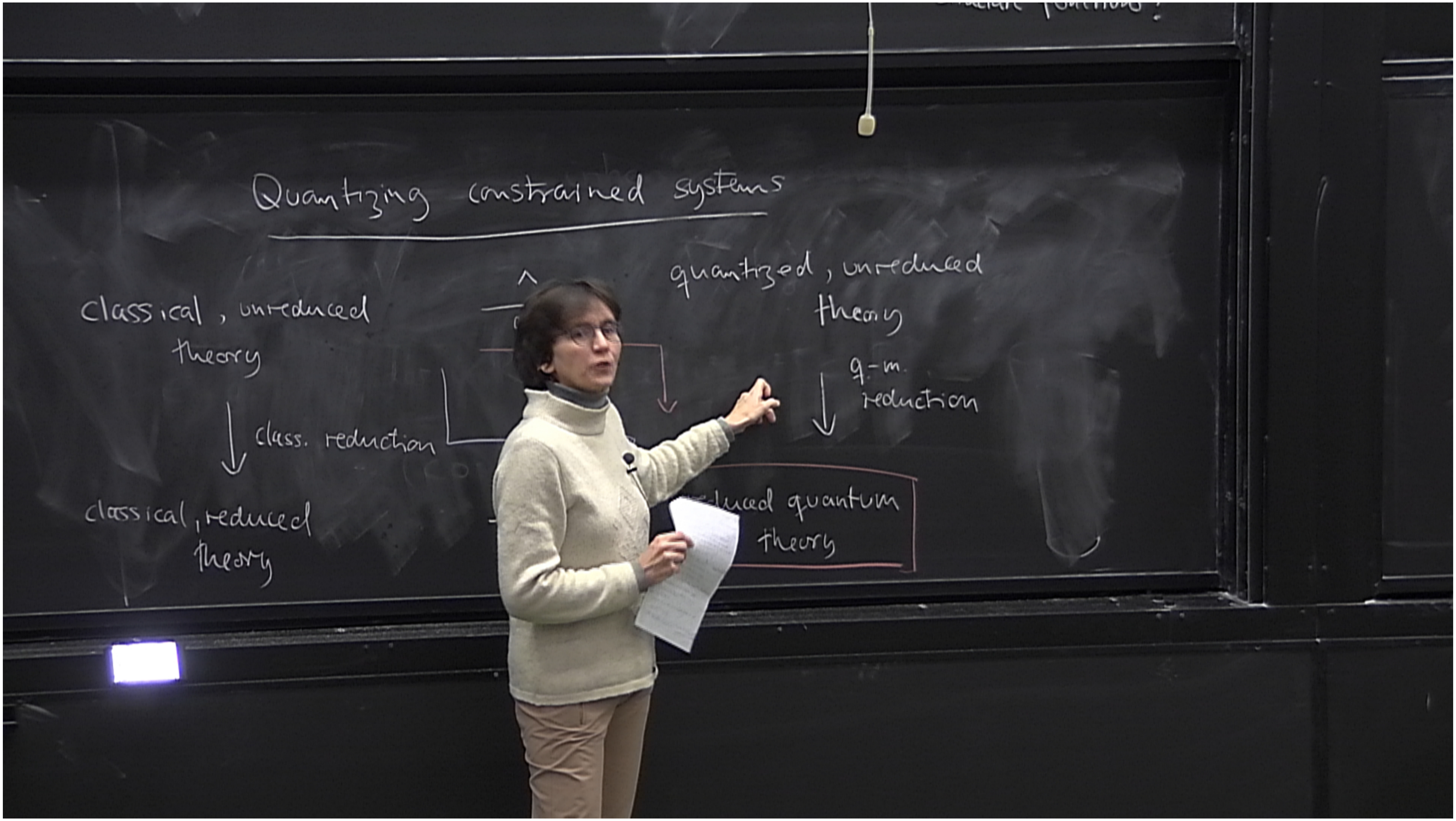
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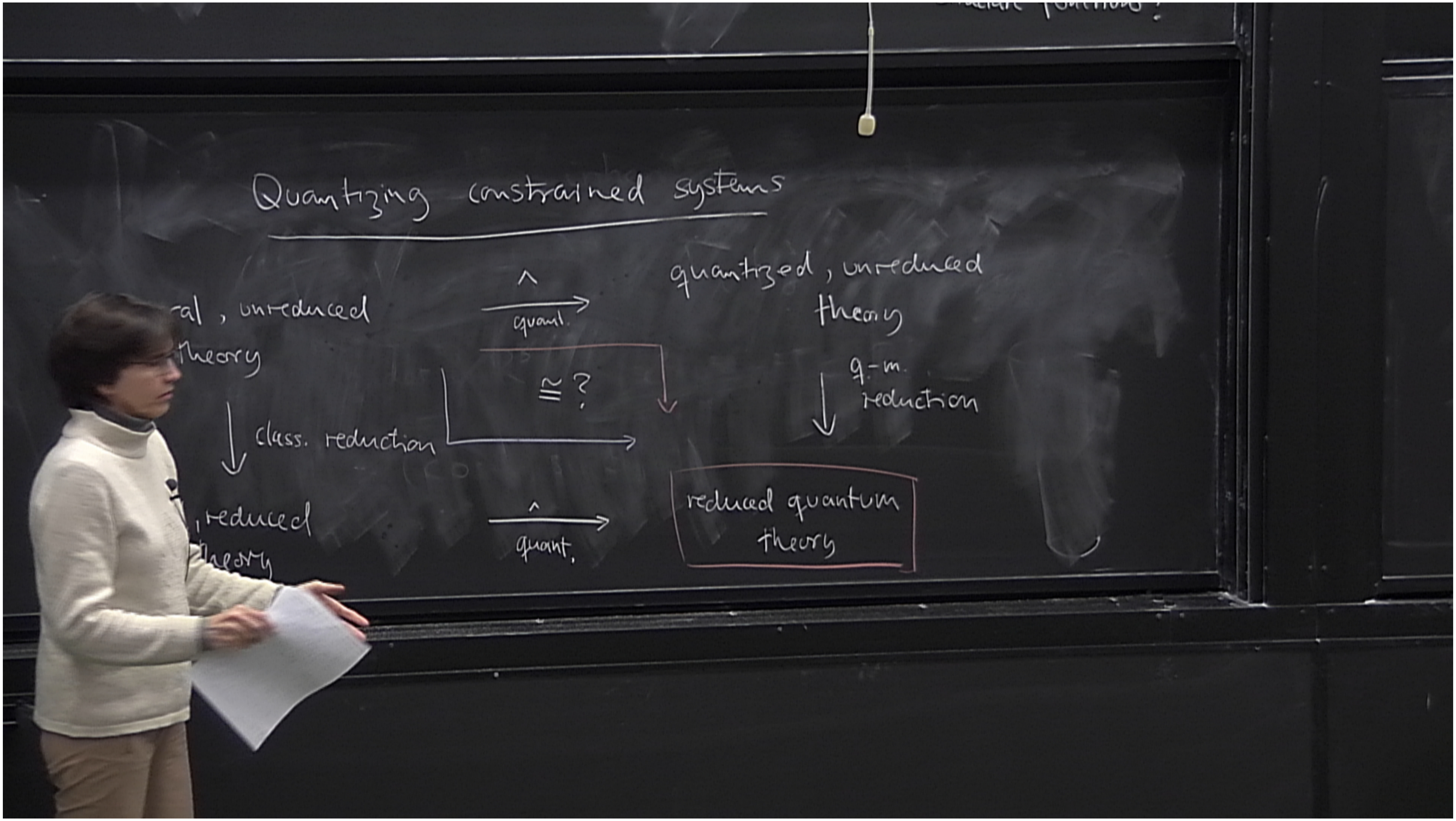
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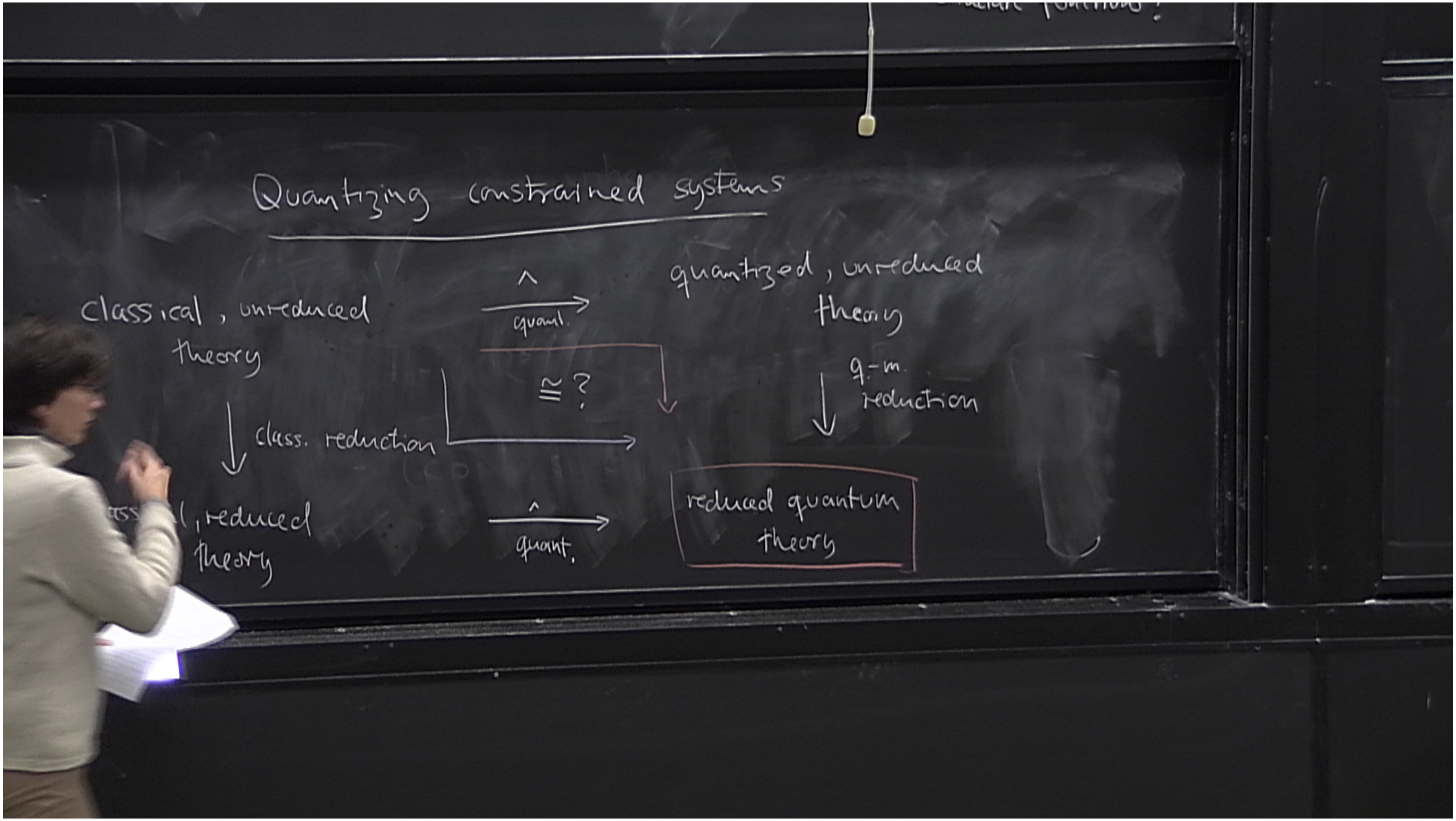
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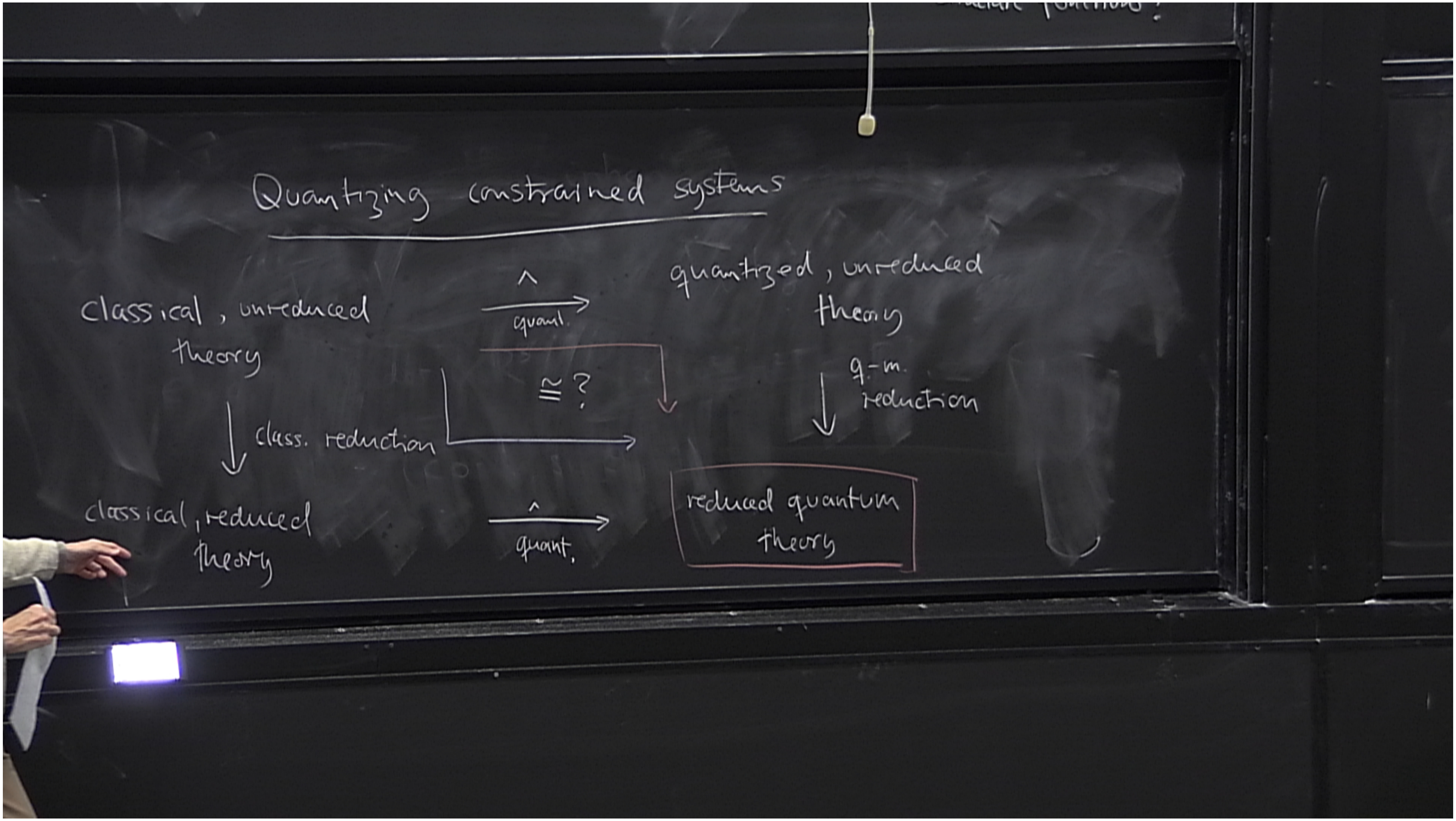
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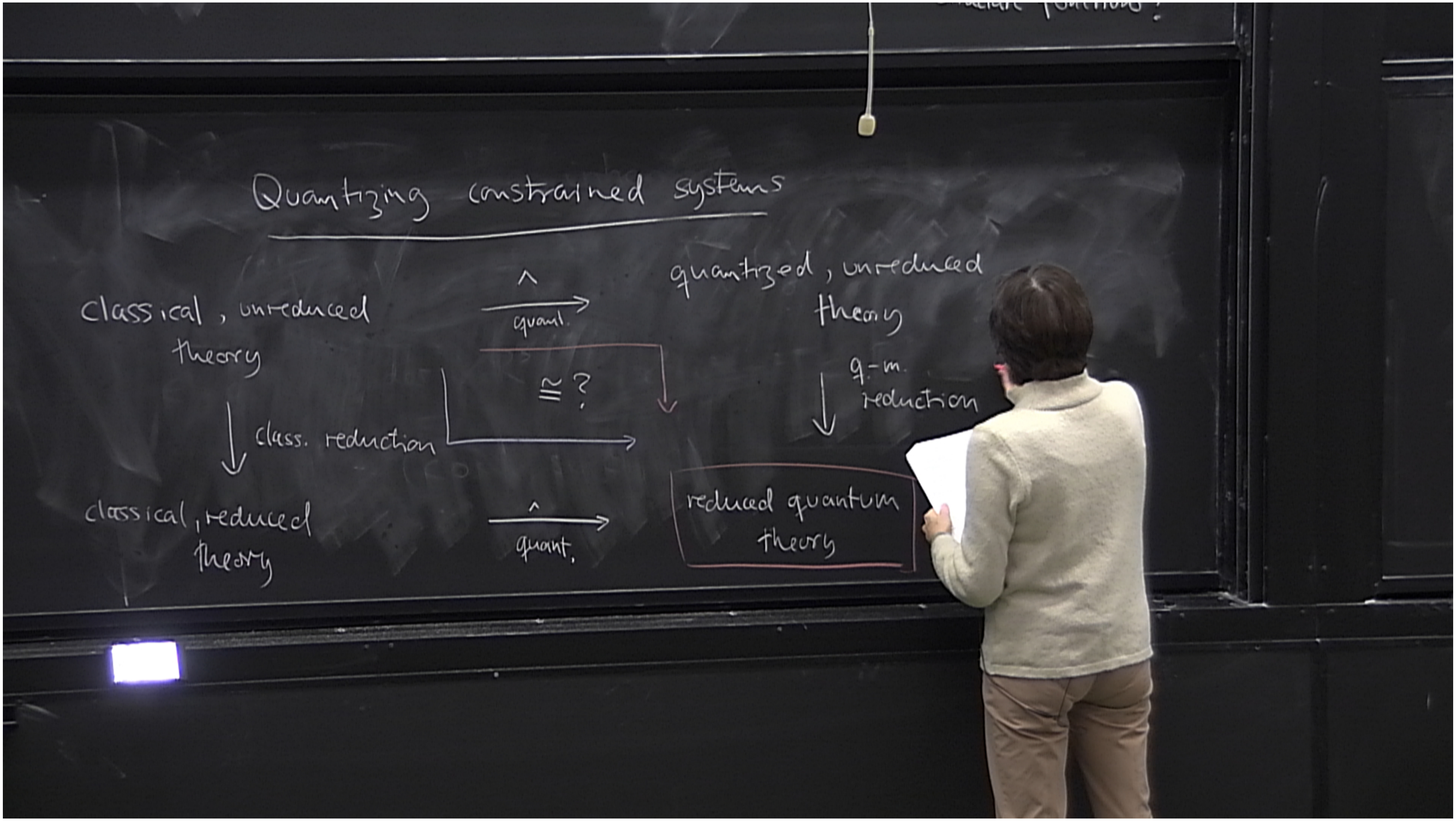
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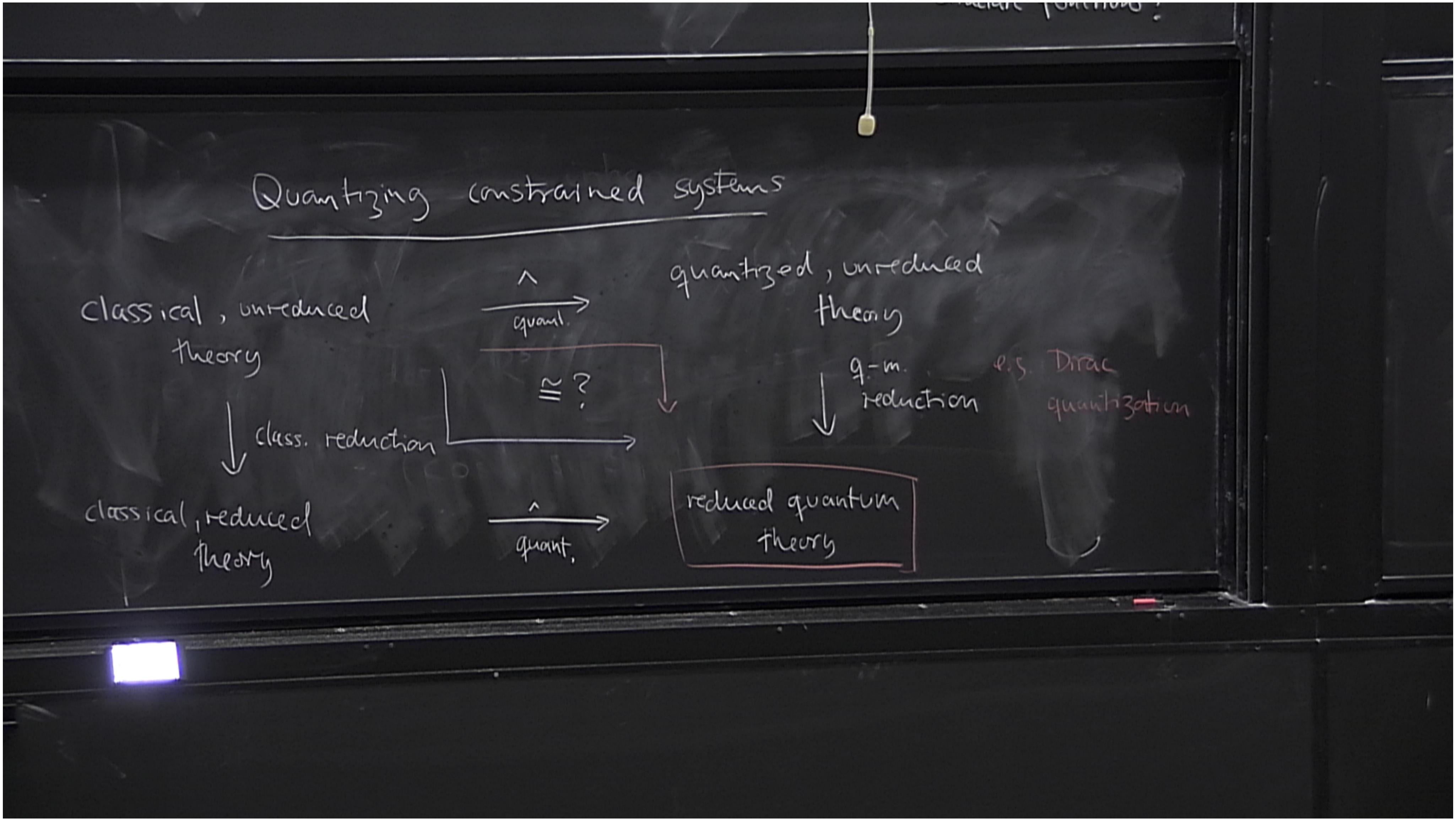
Quantizing constrained systems
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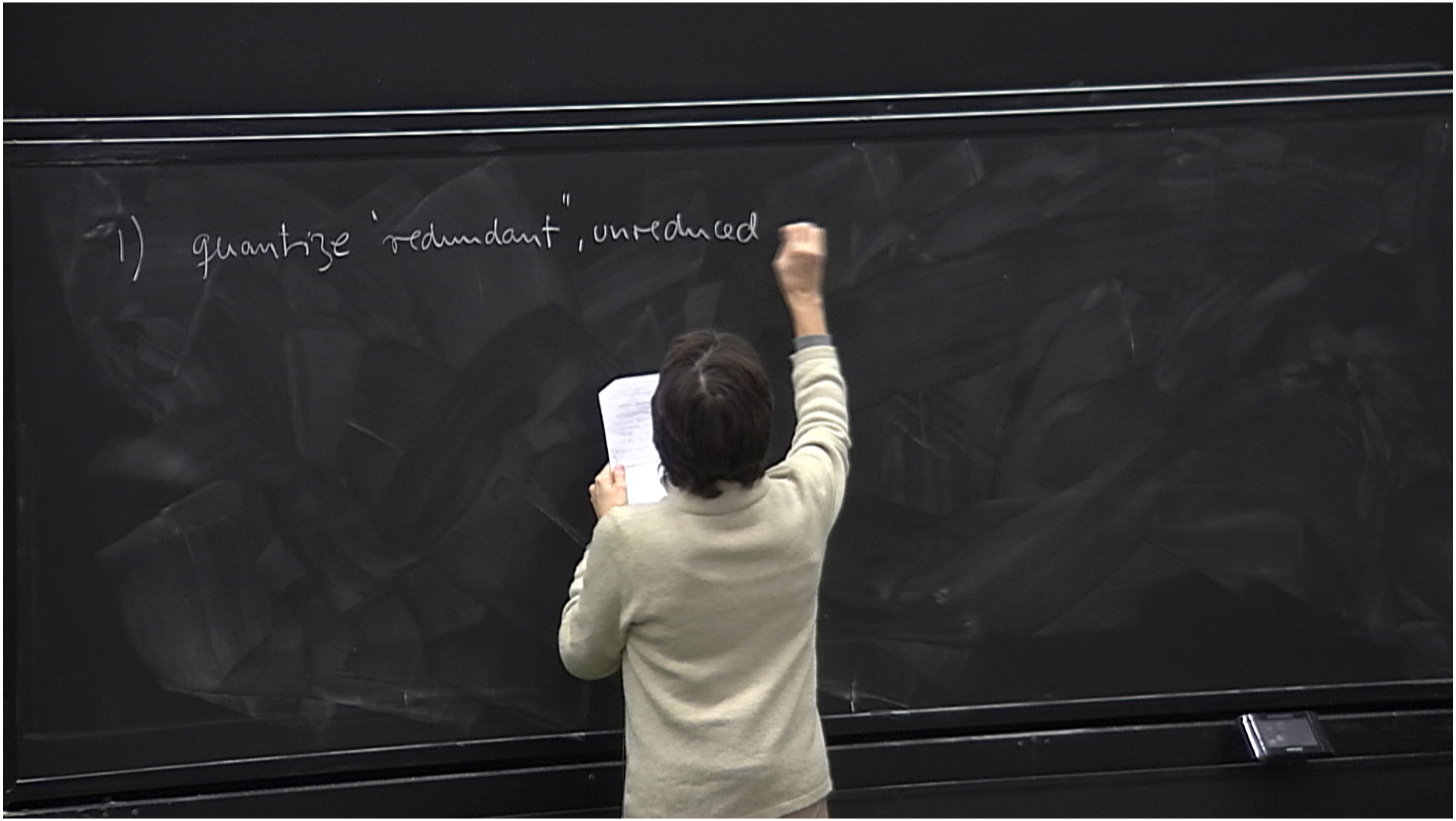
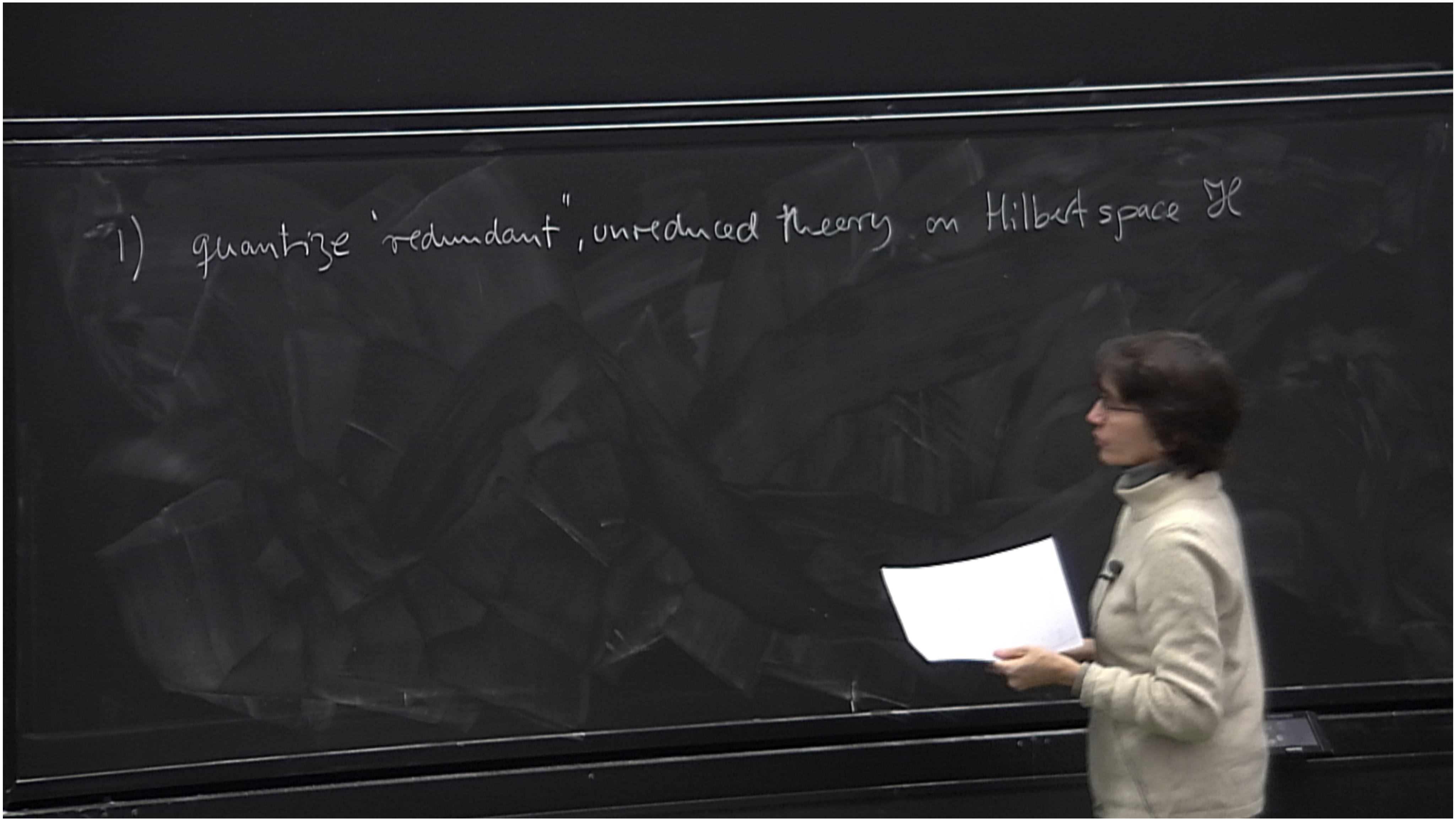
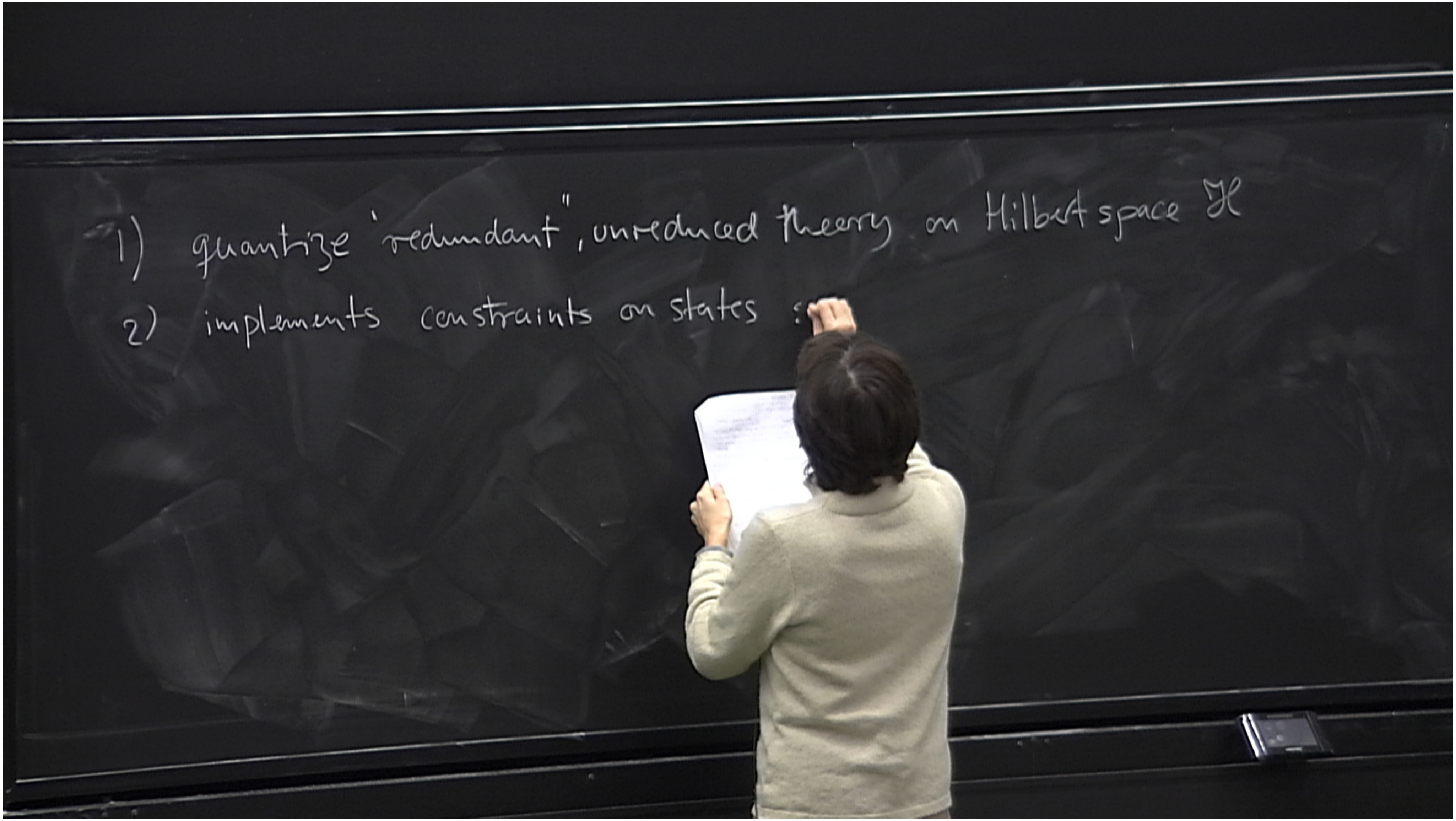
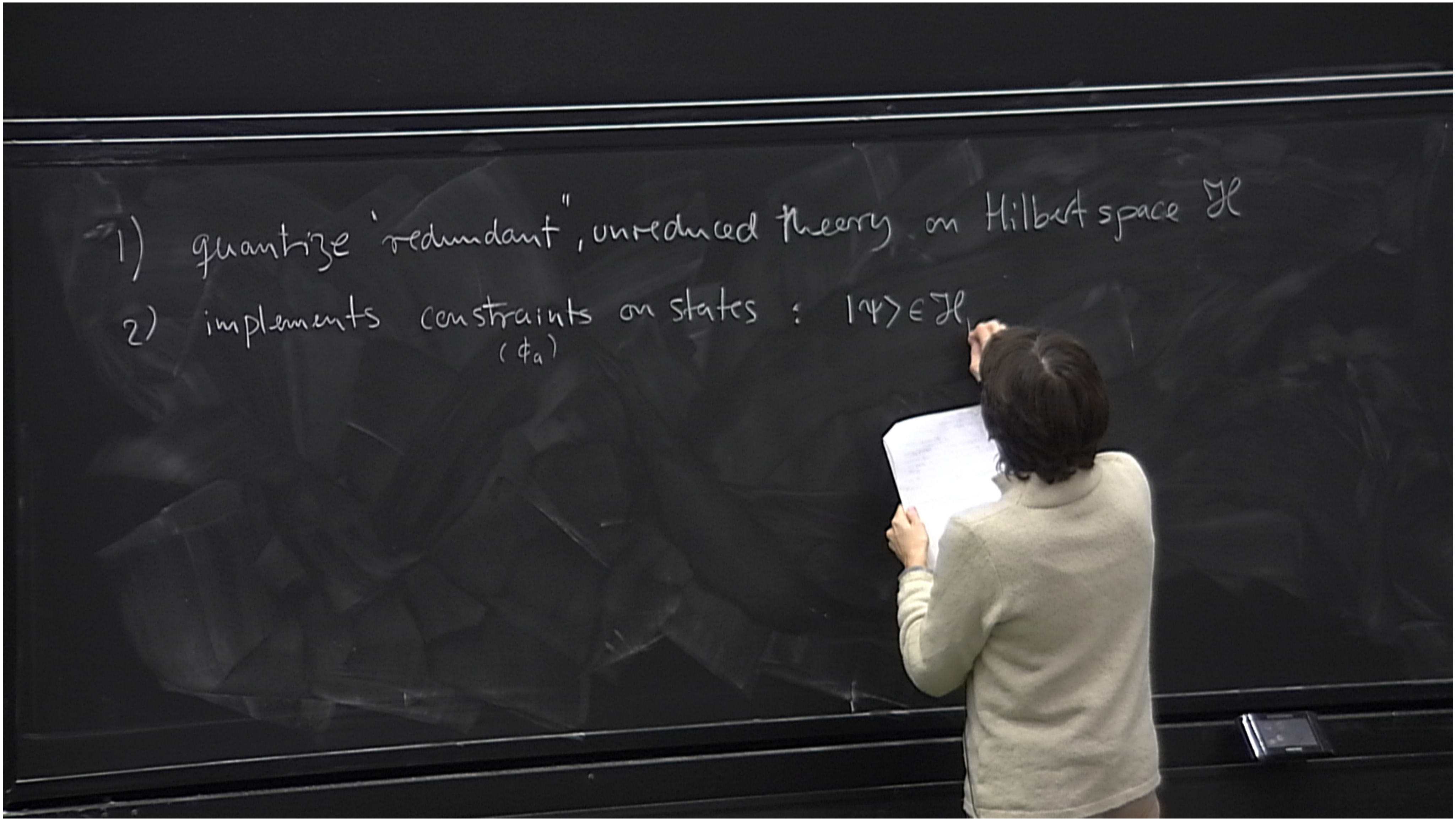
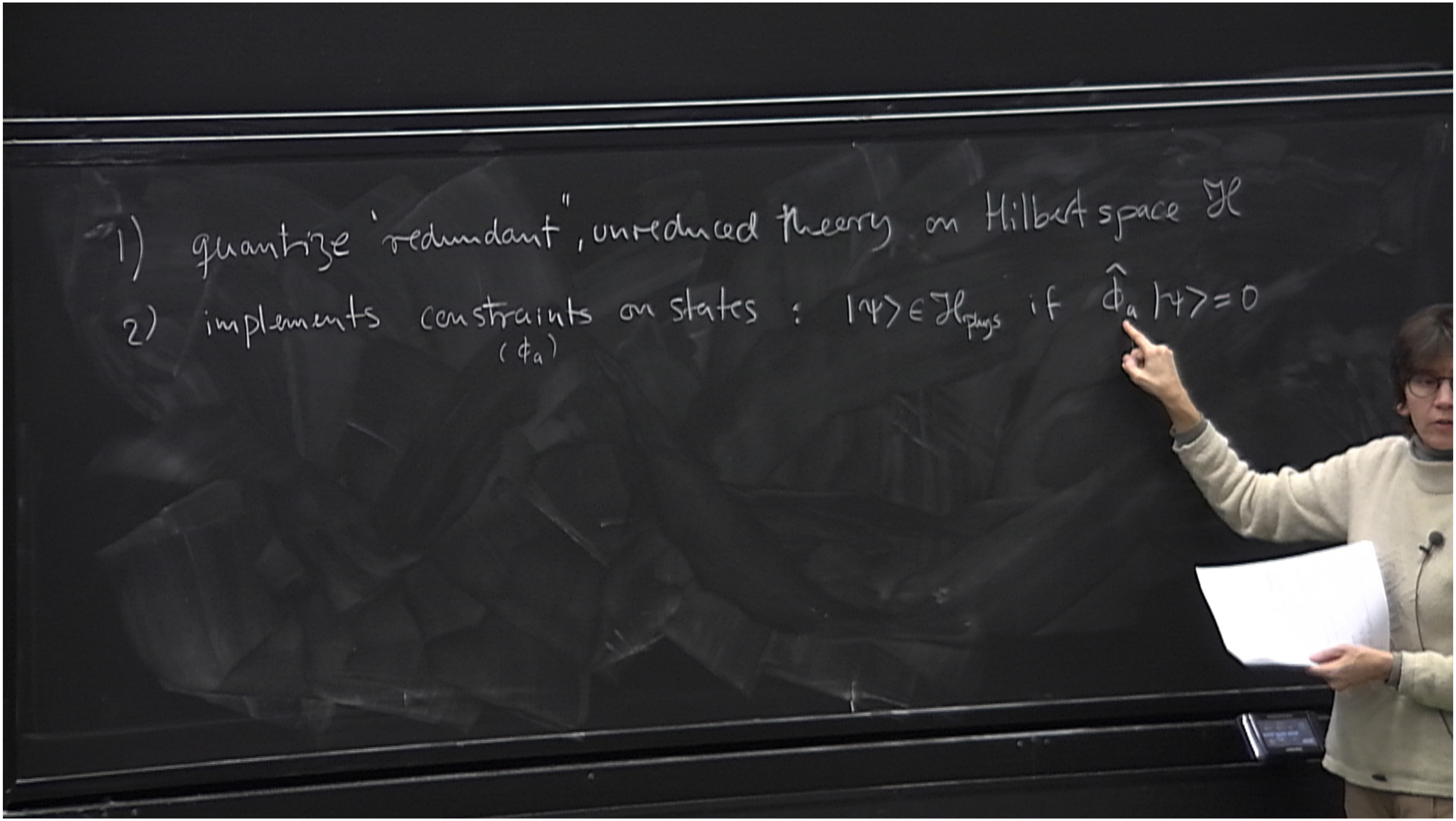
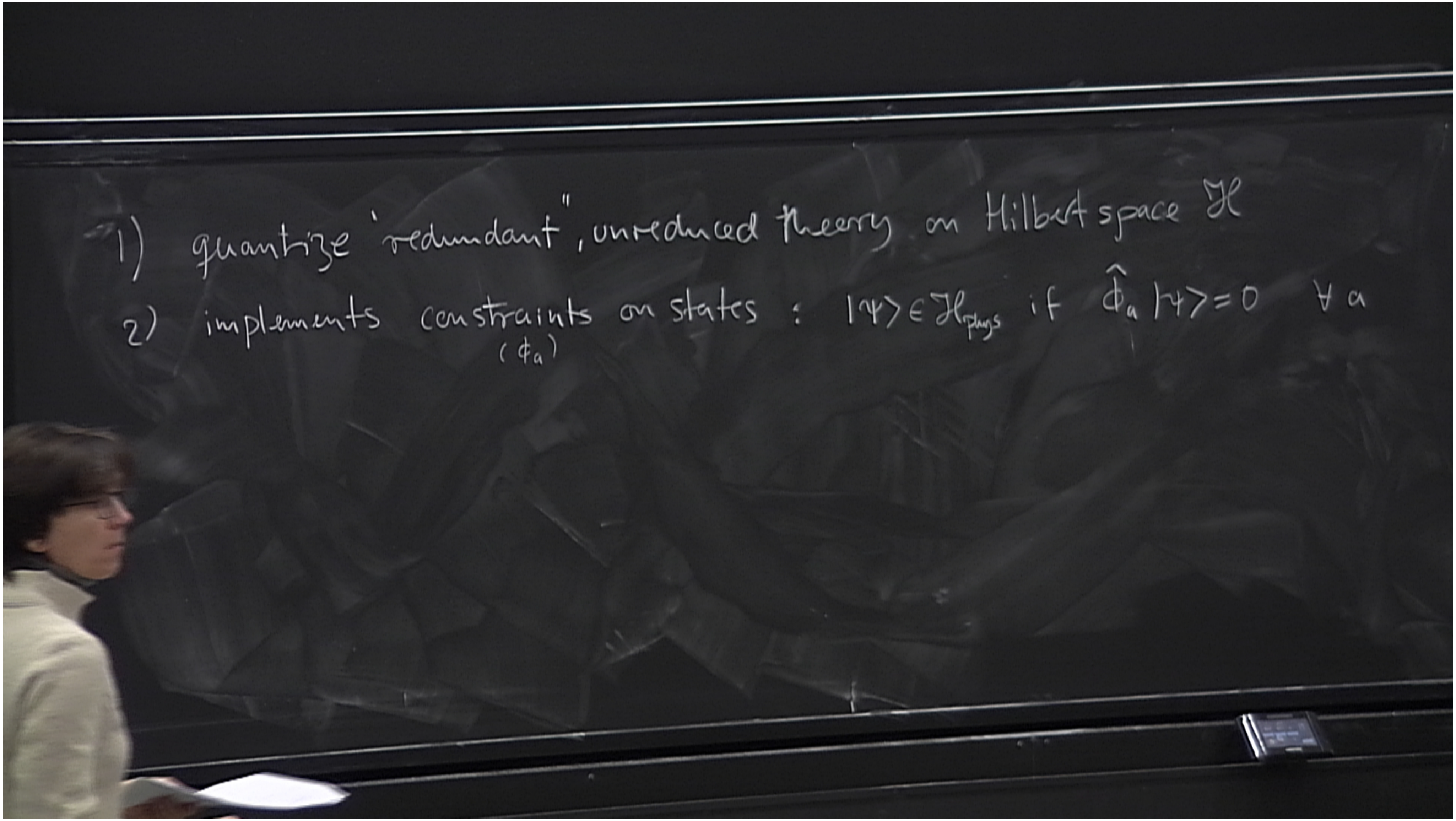

1) quantize "redundant", unreduced


1) quantize 'redundant", unreduced theory on Hilbert space ℋ


1) quantize 'redundant", unreduced theory on Hilbert space ℋ
2) implements constraints on states :


1) quantize "redundant", unreduced theerry on Hilbert space ℋ
2) implements constraints on states : |Ψ⟩ ∈ ℋ
(ϕa)


1) quantize "redundant", unreduced theory on Hilbert space $\mathcal{H}$
2) implements constraints on states : $|\Psi\rangle \in \mathcal{H}_{phys}$ if $\hat{\Phi}_a |\Psi\rangle = 0$
(ϕ_a)


1) quantize "redundant", unreduced theory on Hilbert space $\mathcal{H}$
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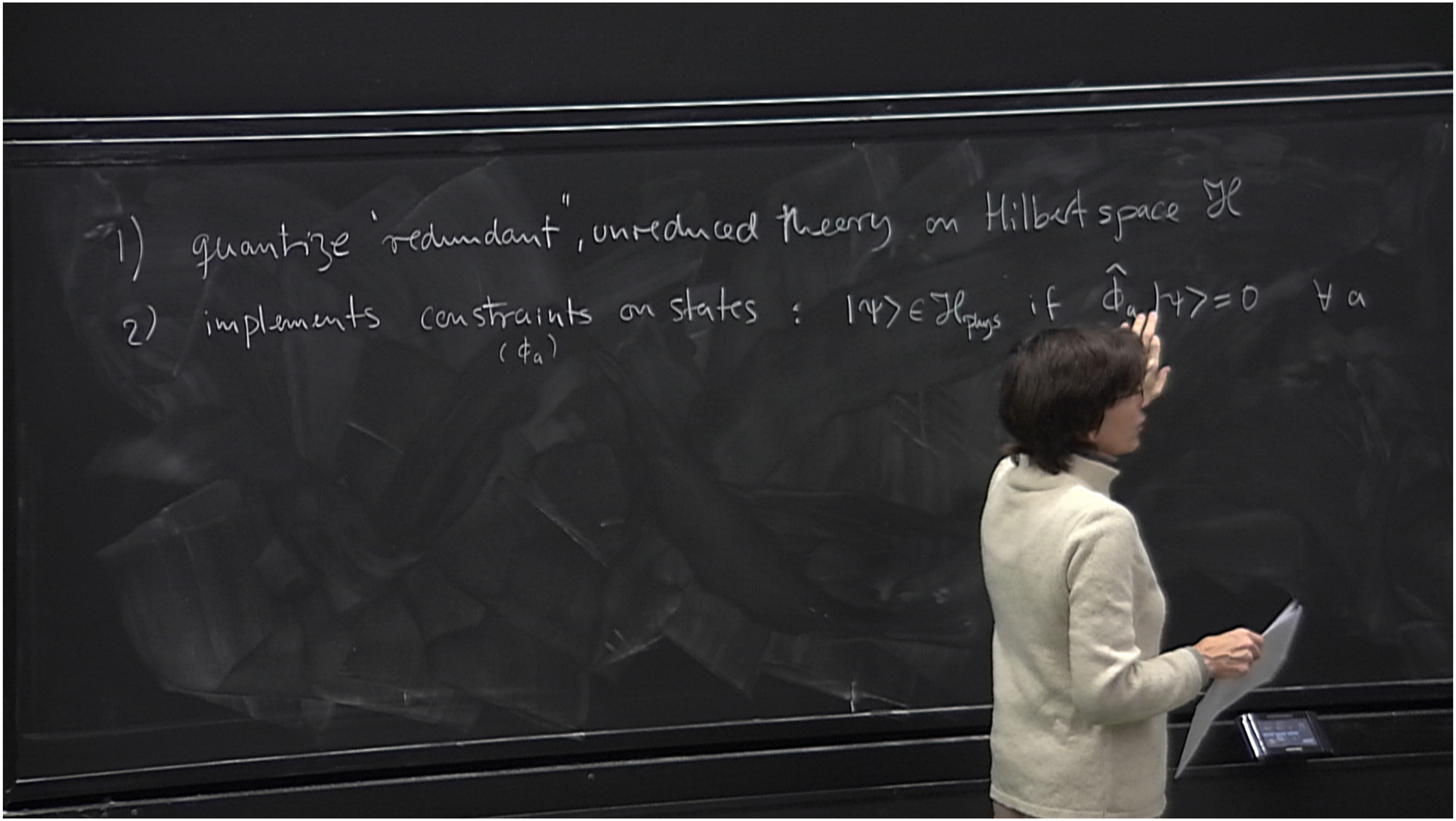
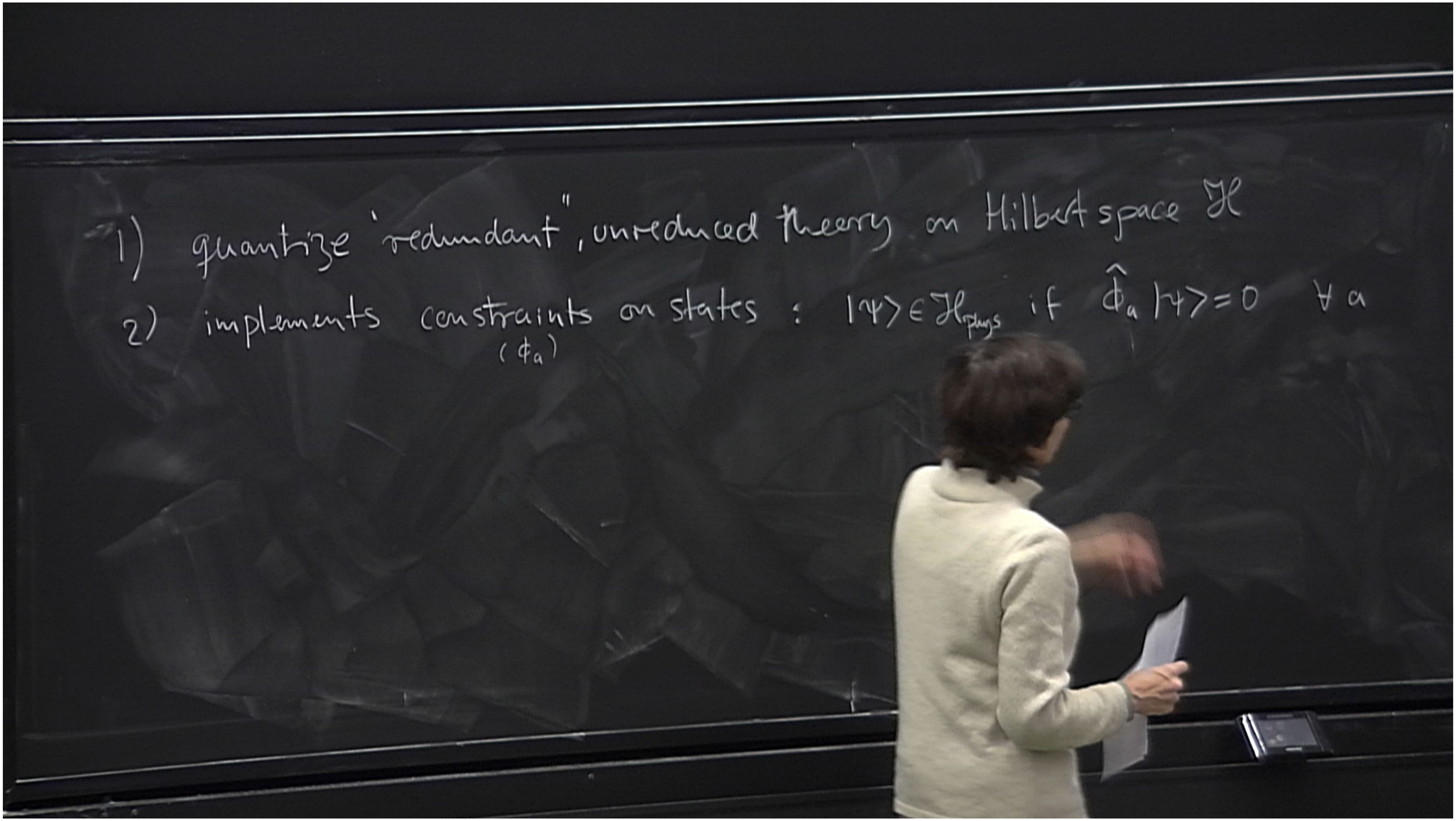
(ϕ_a)


1) quantize "redundant", unreduced theory on Hilbert space ℋ
2) implements constraints (ϕa) on states : |ψ> ∈ ℋphys if Φ̂a |ψ> = 0 ∀ a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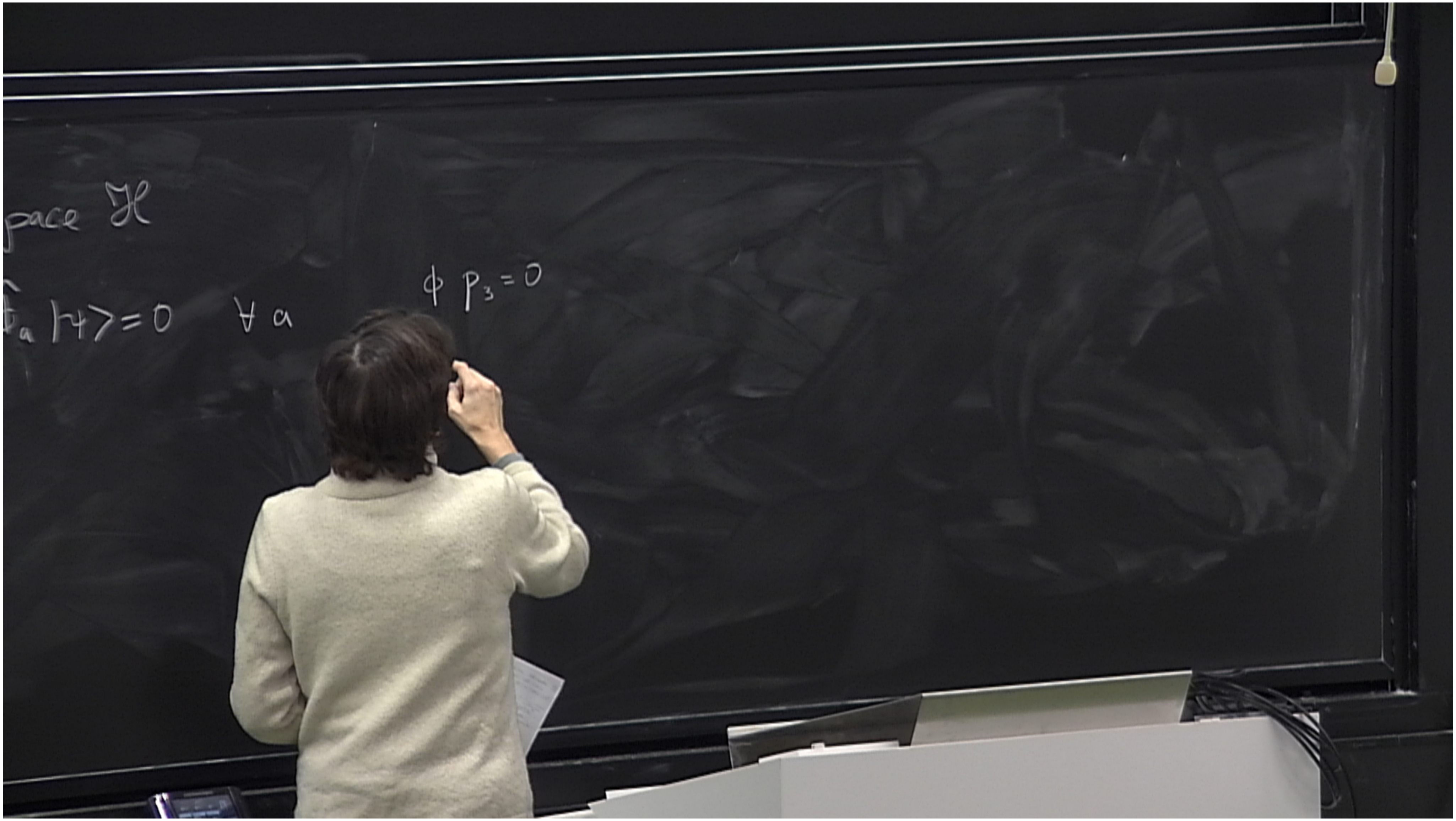
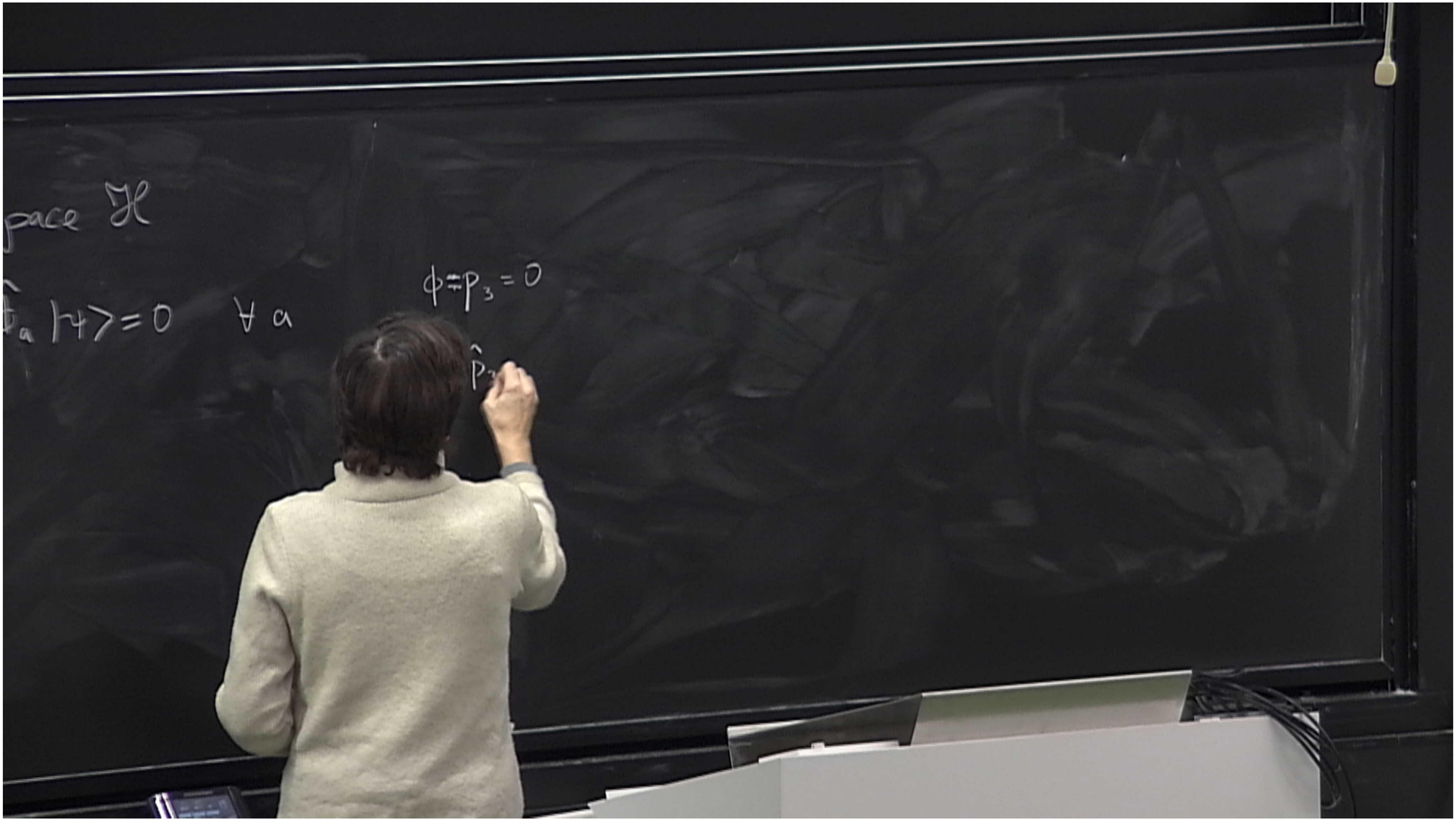
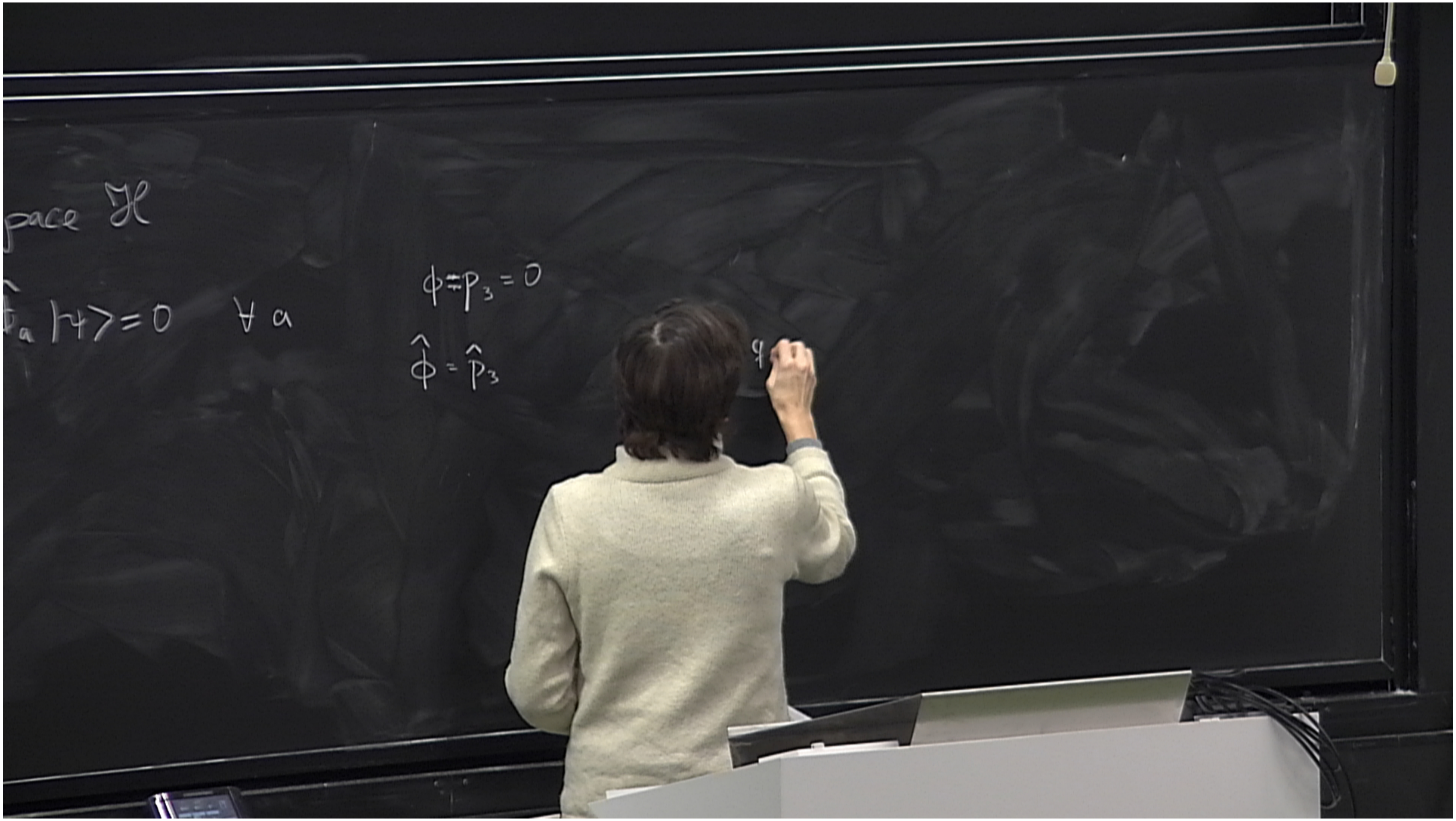
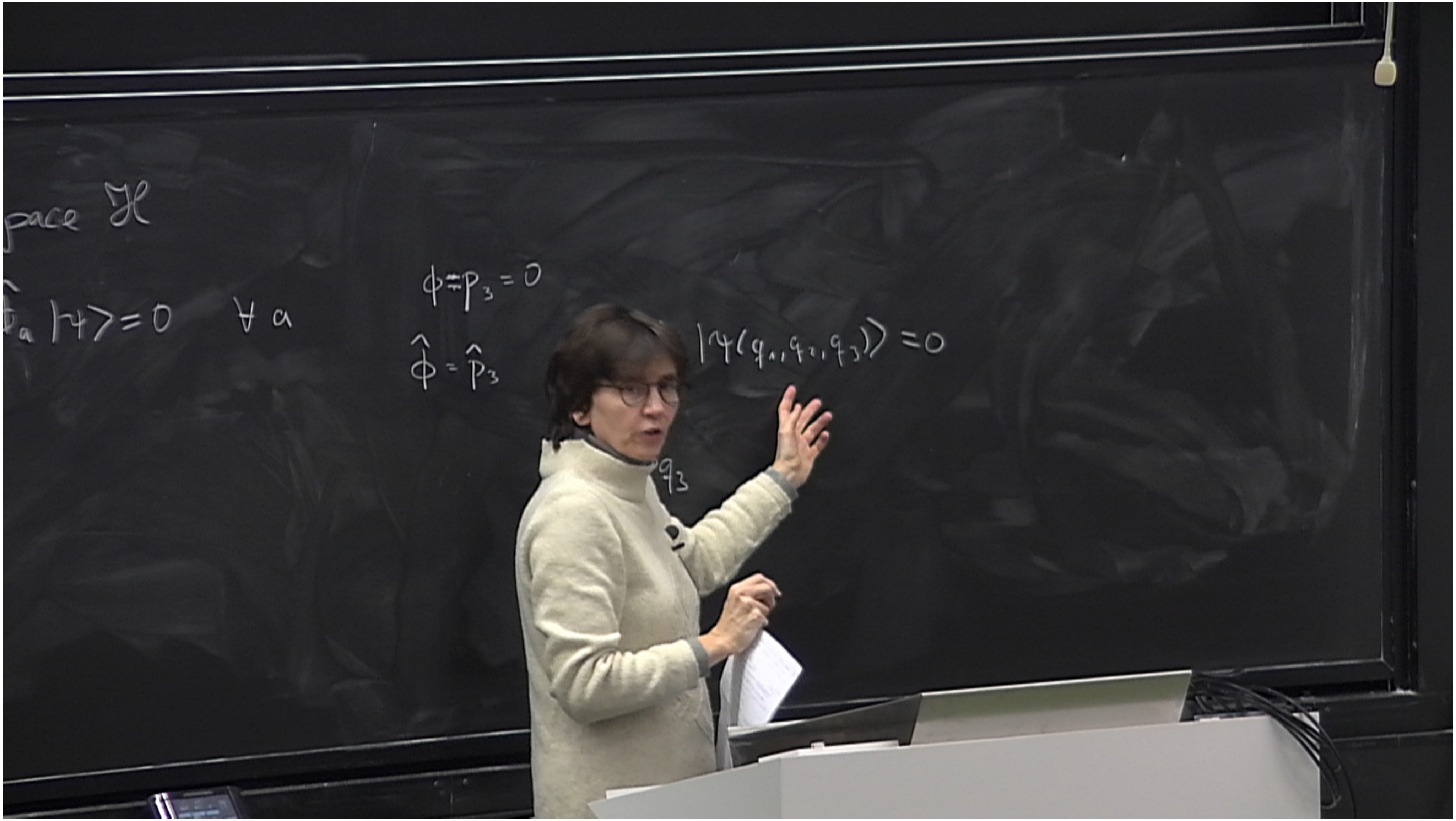
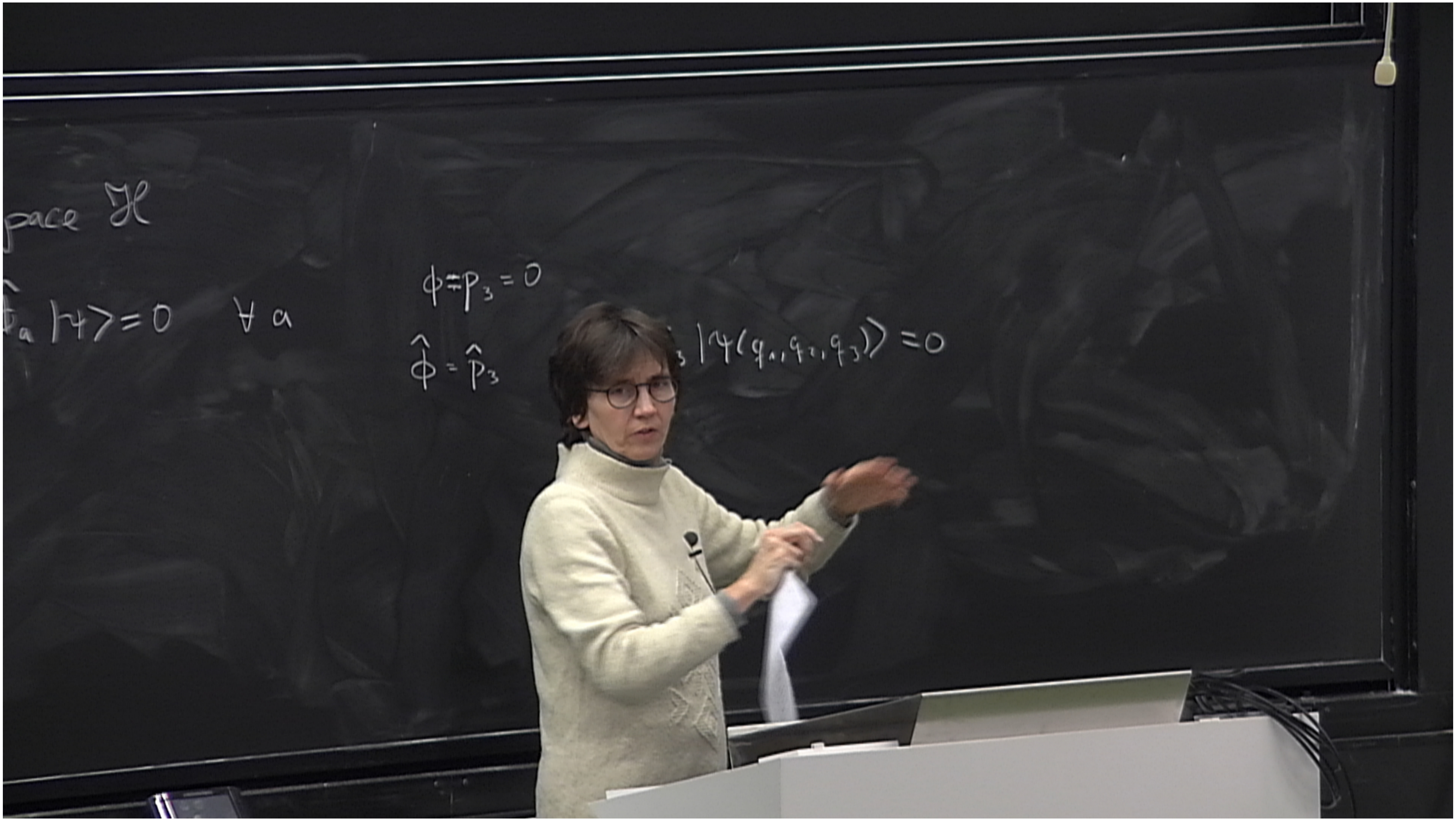

pace ℋ
|ψ> = 0 ∀ a
φ p3 = 0


pace ℋ
φ ≐ p₃ = 0
ℋ̂ₐ |ψ⟩ = 0   ∀ a
p̂₃


pace ℋ
φ̂a |ψ⟩ = 0   ∀ a
φ ≈ p3 = 0
φ̂ = p̂3


pace $\mathcal{H}$
$\hat{\phi}_a |\psi\rangle = 0 \quad \forall a$
$\phi = p_3 = 0$
$\hat{\phi} = \hat{p}_3$
$|\psi(q_1, q_2, q_3)\rangle = 0$
q_3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$\hat{\phi}_a |\psi\rangle = 0 \quad \forall a$
$\phi = p_3 = 0$
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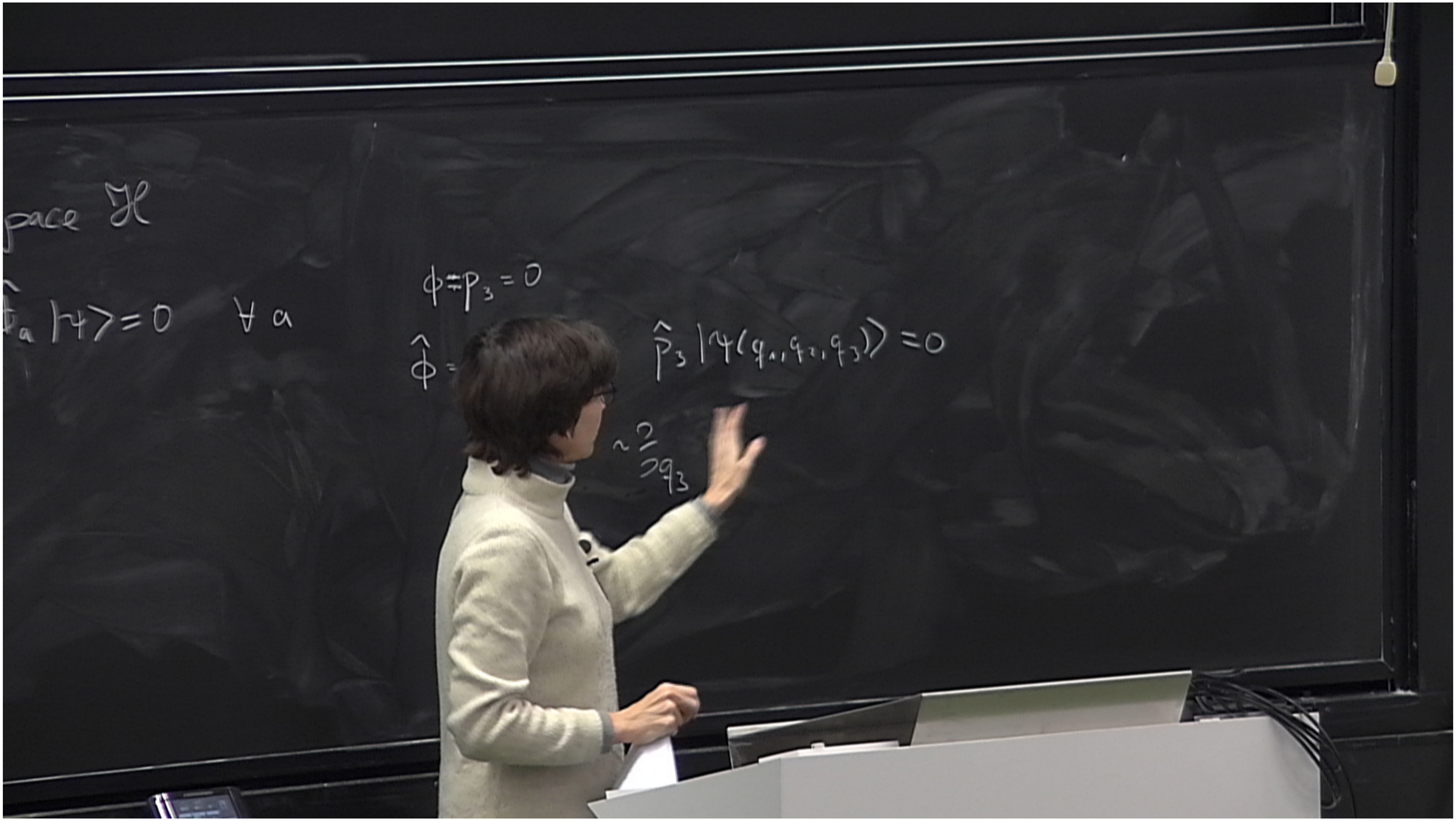
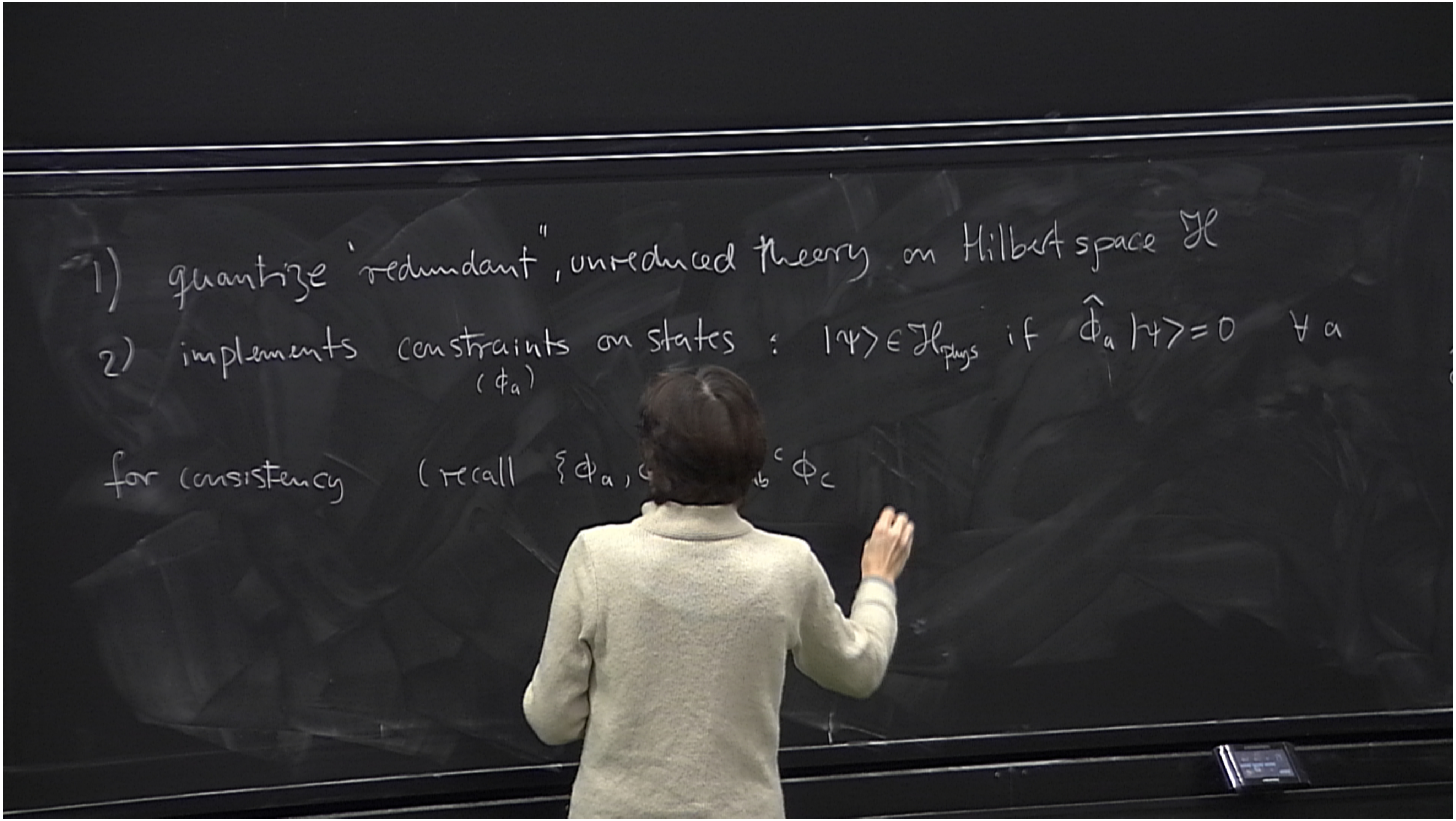
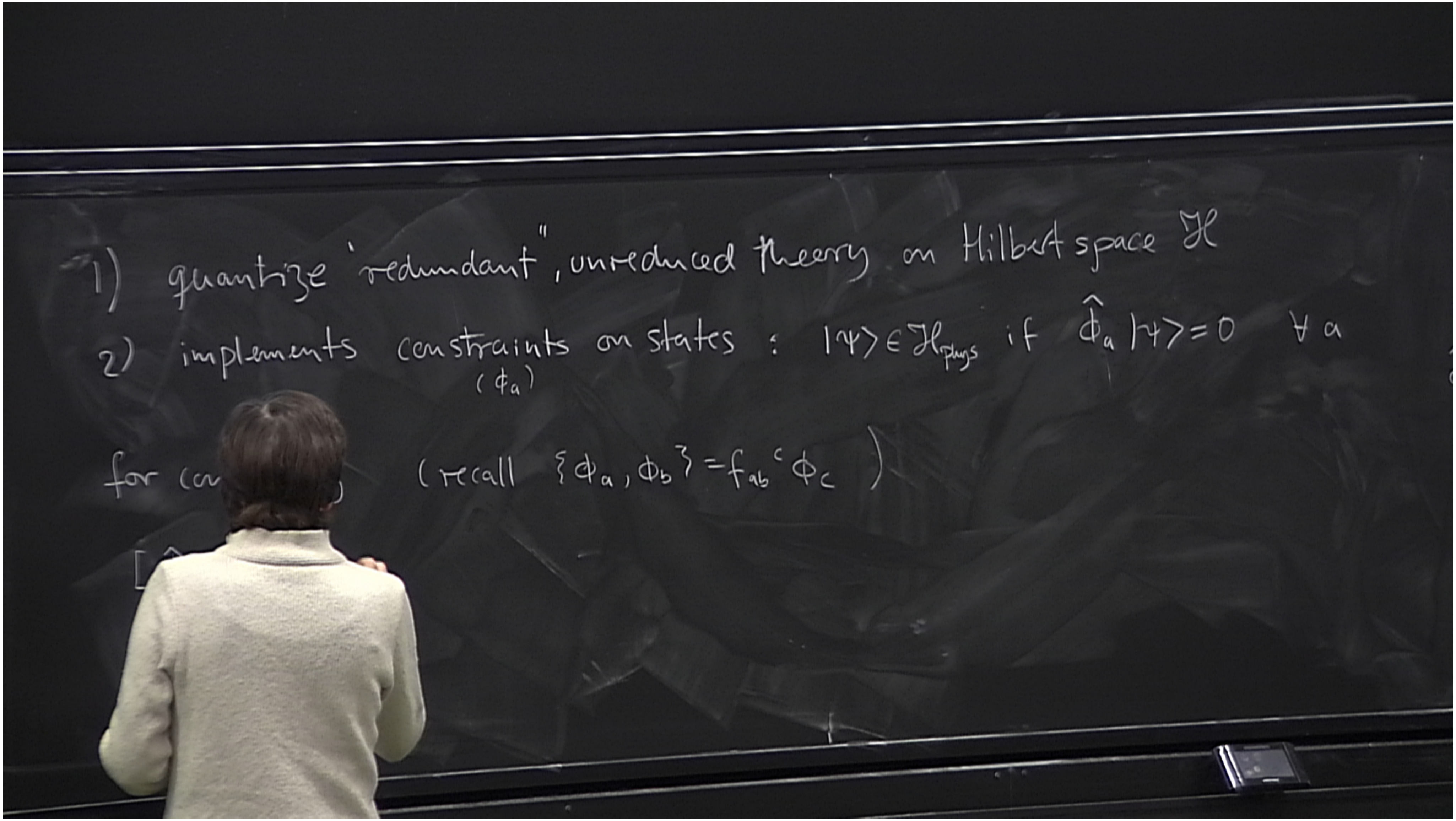
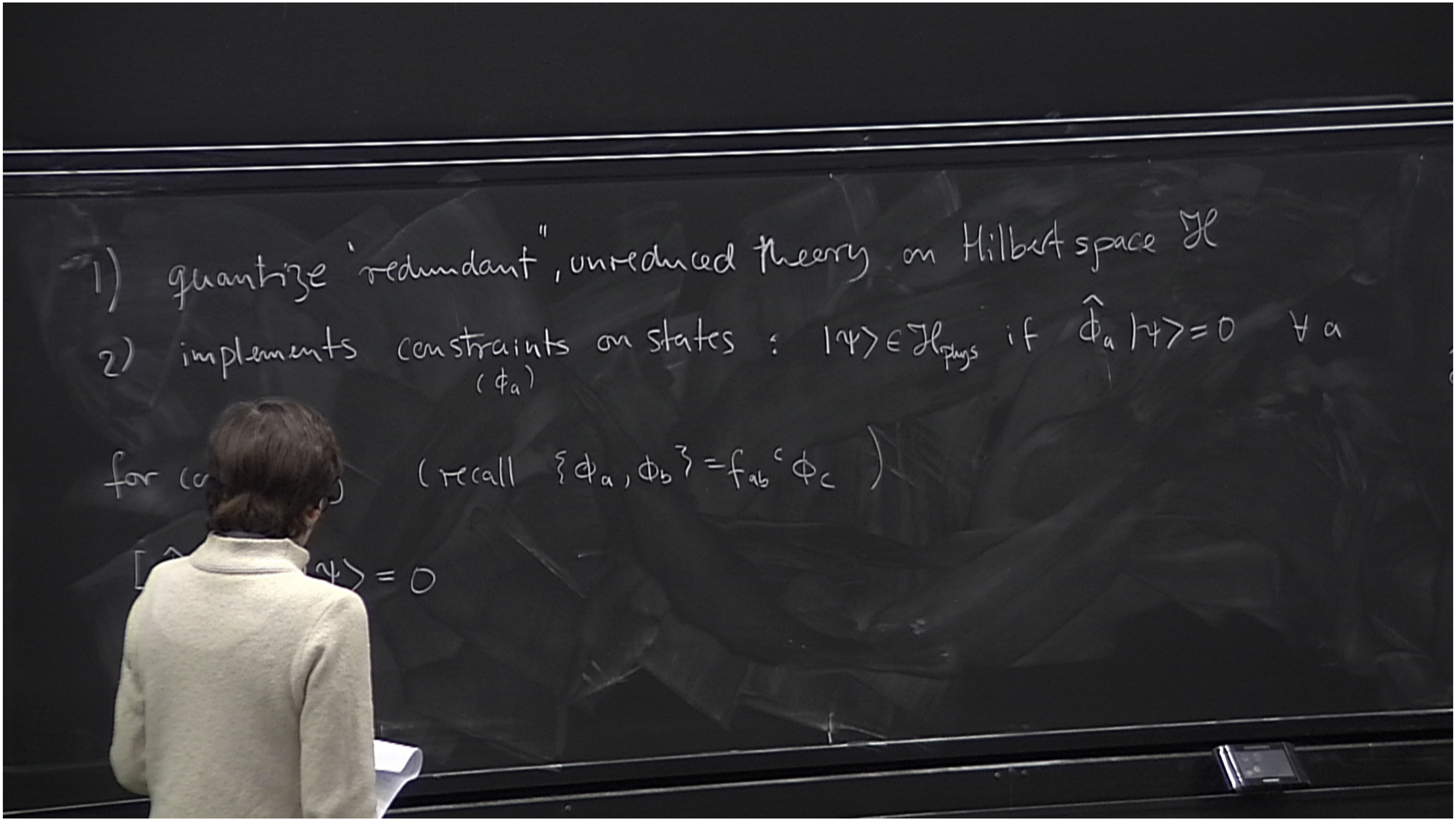
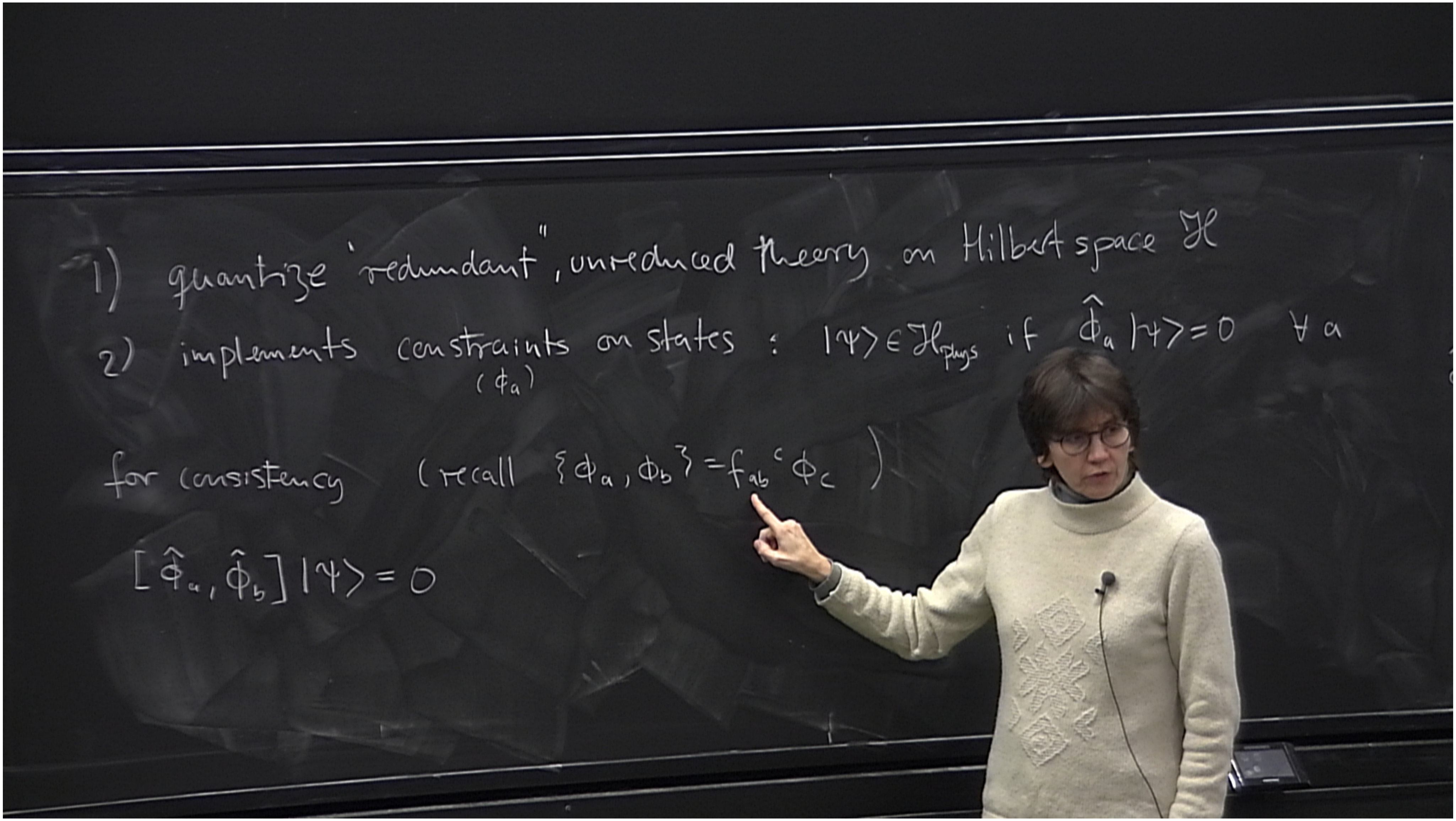
$|\psi(q_1, q_2, q_3)\rangle = 0$


pace ℋ
|ψ> = 0 ∀ a
ϕ = p₃ = 0
p̂₃ |ψ(q₁, q₂, q₃)> = 0
~ ∂/∂q₃


1) quantize "redundant", unreduced theory on Hilbert space H
2) implements constraints on states : |Ψ⟩ ∈ H_phys if Φ̂_a |Ψ⟩ = 0 ∀ a
(φ_a)
for consistency (recall {φ_a,
φ_c


1) quantize "redundant", unreduced theory on Hilbert space ℋ
2) implements constraints (φ_a) on states : |ψ⟩ ∈ ℋ_phys if Φ̂_a |ψ⟩ = 0 ∀ a
(recall {Φ_a, Φ_b} = f_ab^c Φ_c)


1) quantize "redundant", unreduced theory on Hilbert space $\mathcal{H}$
2) implements constraints on states : $|\psi\rangle \in \mathcal{H}_{phys}$ if $\hat{\Phi}_a |\psi\rangle = 0 \quad \forall a$
(ϕ_a)
for (c (recall $\{\Phi_a, \Phi_b\} = f_{ab}{}^c \Phi_c$)
$\psi\rangle = 0$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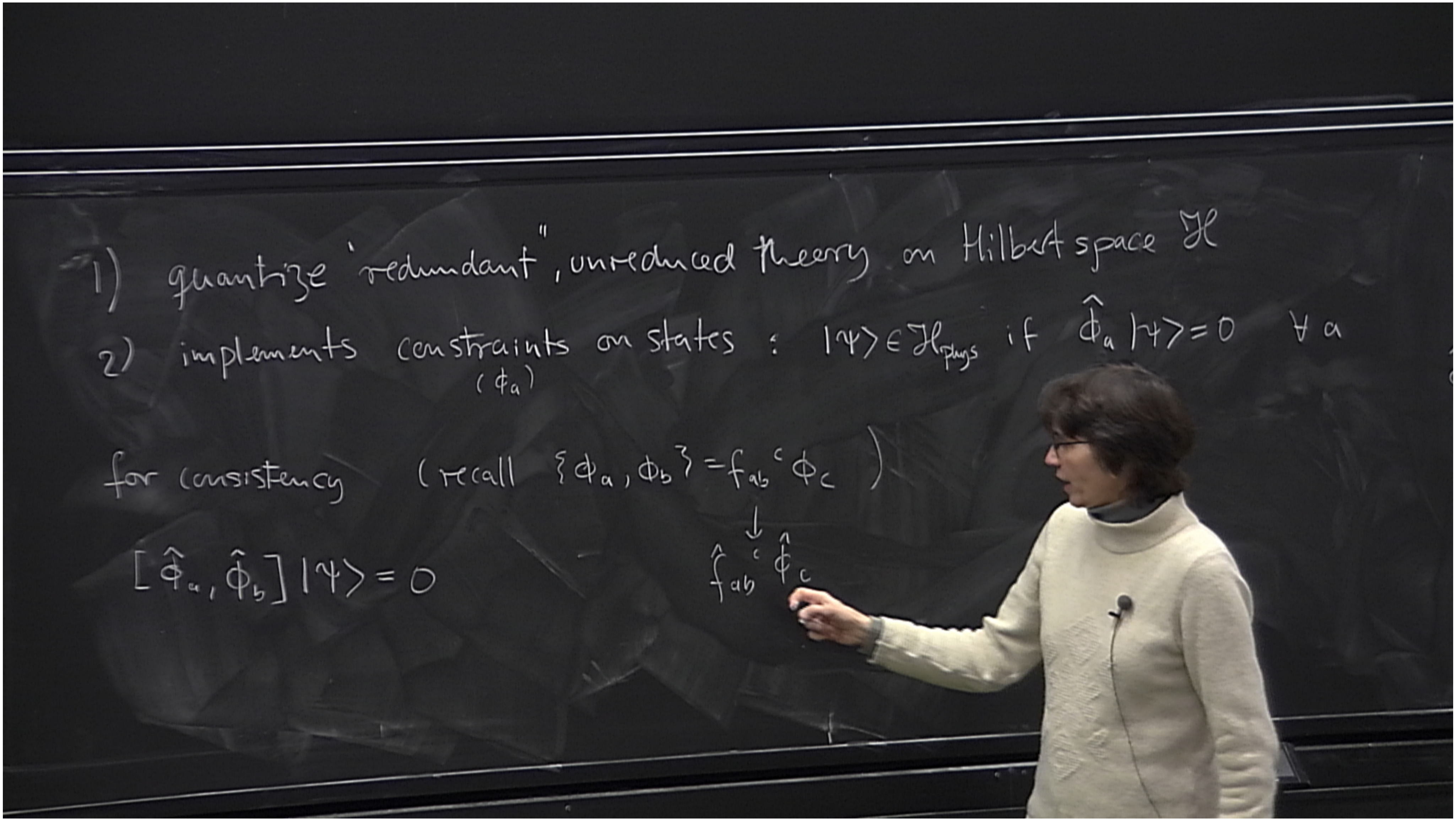
(ϕ_a)
for consistency (recall $\{\Phi_a, \Phi_b\} = f_{ab}{}^{c} \Phi_c$)
$[\hat{\Phi}_a, \hat{\Phi}_b] |\psi\rangle = 0$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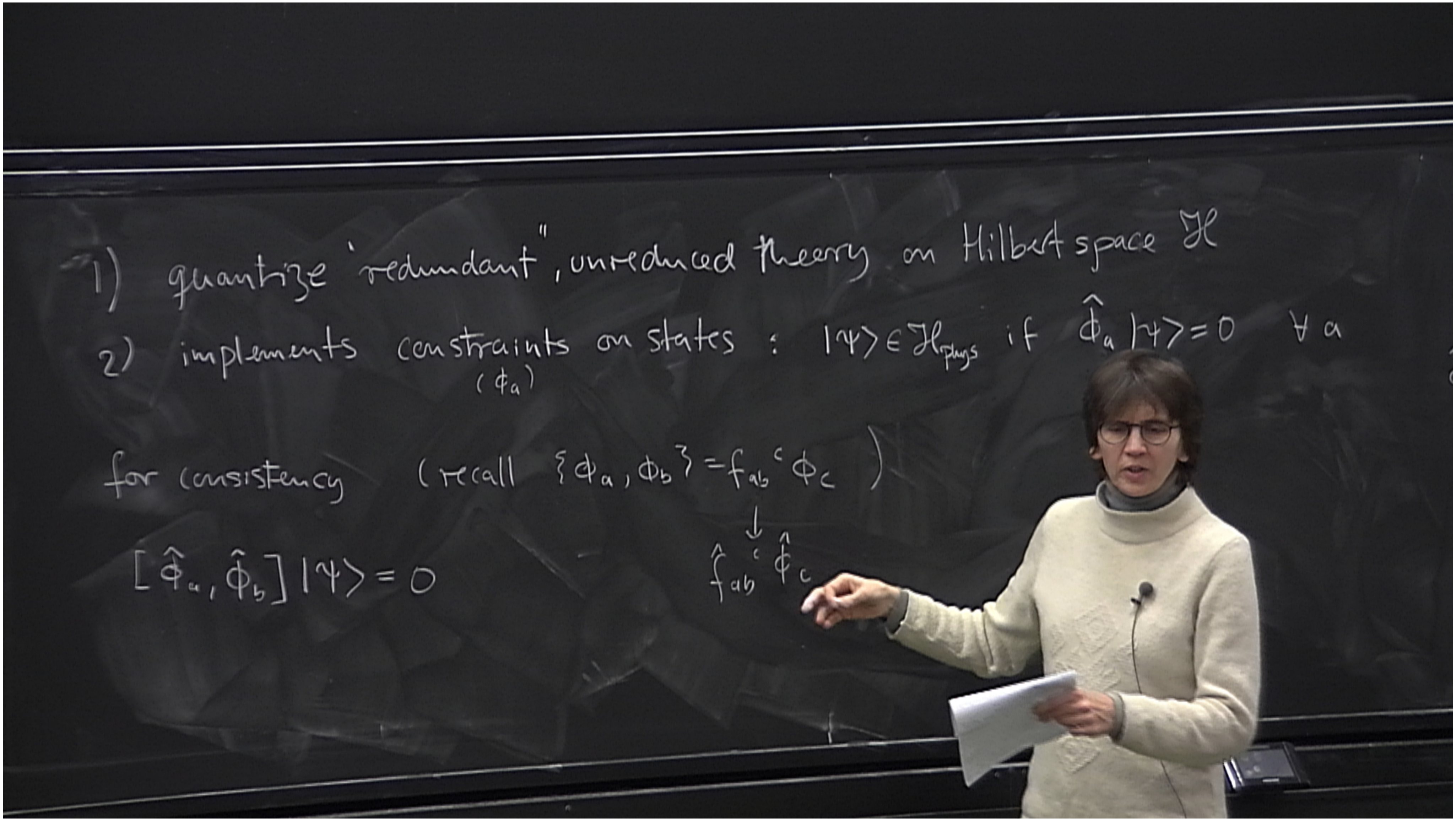
for consistency (recall $\{\Phi_a, \Phi_b\} = f_{ab}{}^c \Phi_c$)
$[\hat{\Phi}_a, \hat{\Phi}_b] |\psi\rangle = 0$
$\hat{f}_{ab}{}^c \hat{\Phi}_c$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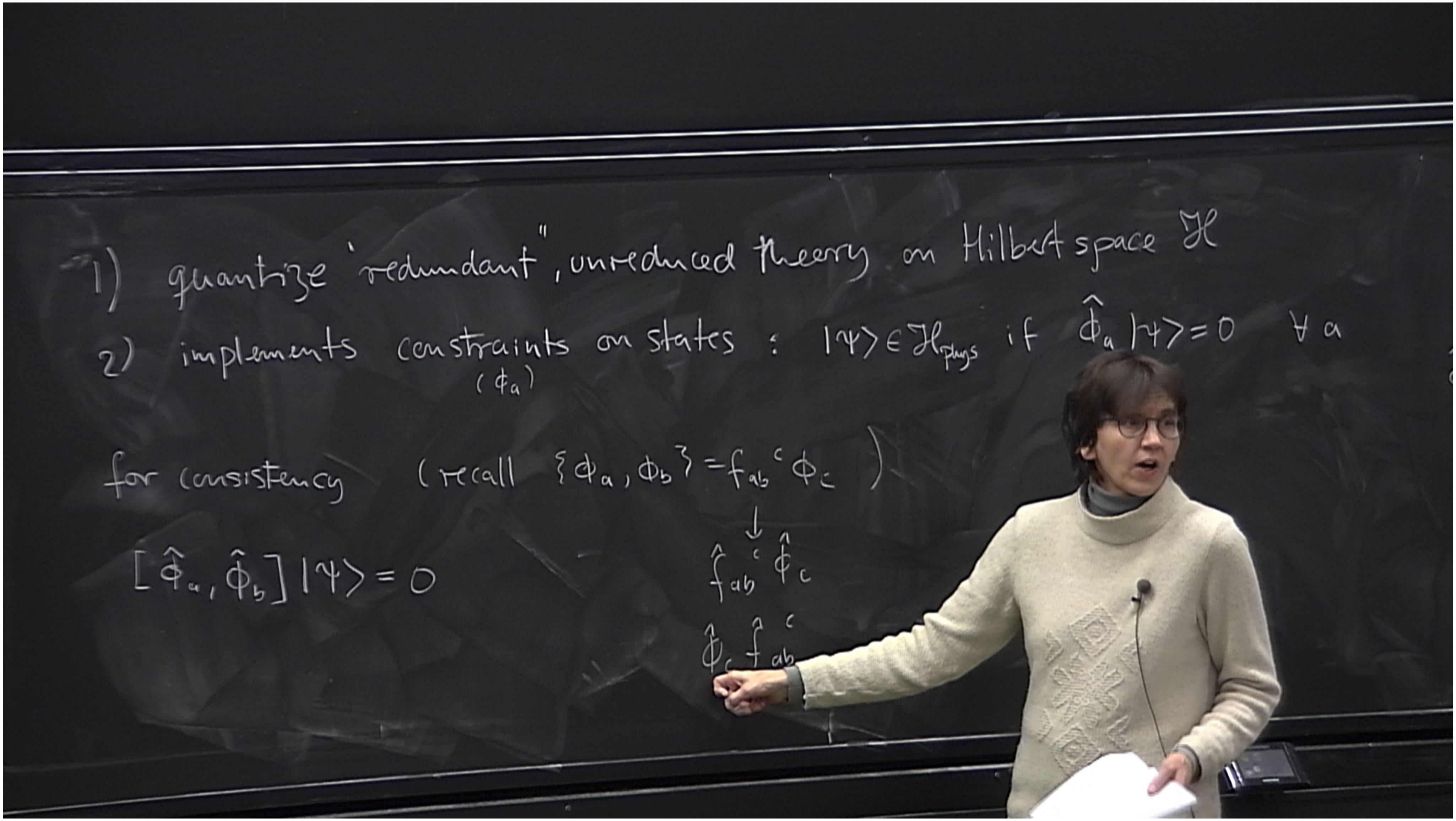
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for consistency (recall $\{\Phi_a, \Phi_b\} = f_{ab}{}^c \Phi_c$)
$[\hat{\Phi}_a, \hat{\Phi}_b] |\psi\rangle = 0$
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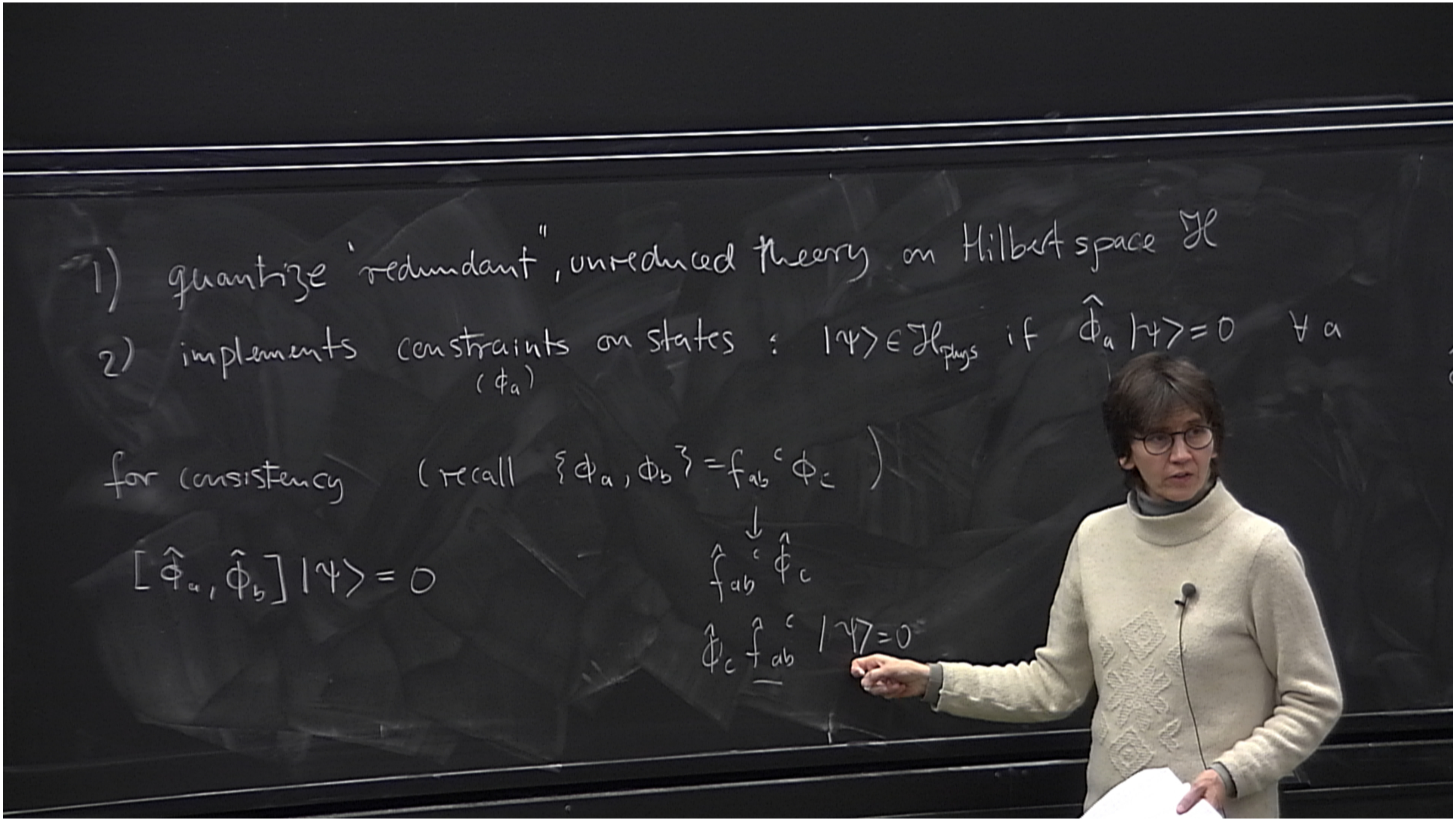
$\hat{\Phi}_c \hat{f}_{ab}{}^c$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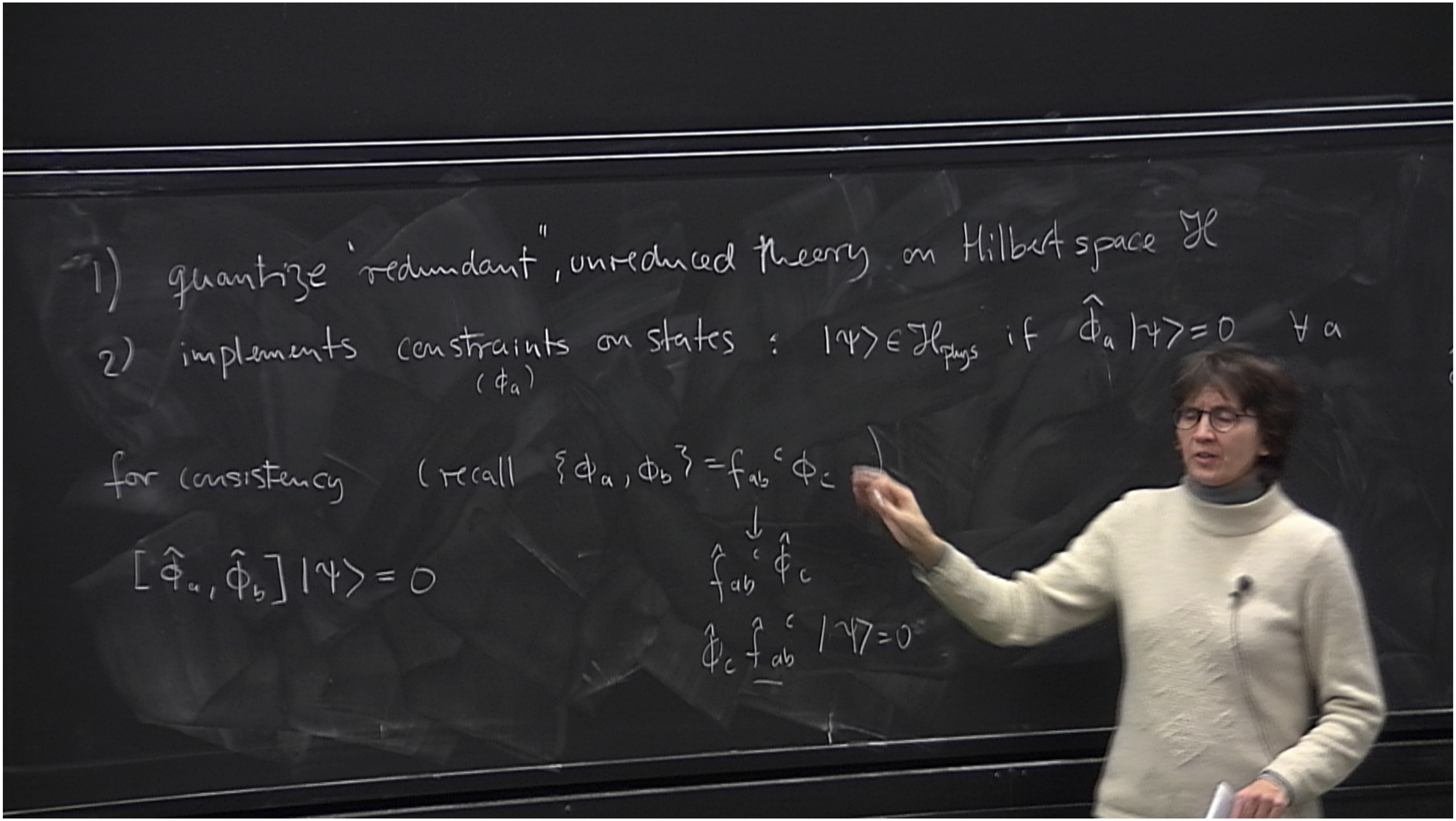
for consistency (recall $\{\Phi_a, \Phi_b\} = f_{ab}{}^c \Phi_c$)
$[\hat{\Phi}_a, \hat{\Phi}_b] |\psi\rangle = 0$
$\hat{f}_{ab}{}^c \hat{\Phi}_c$
$\hat{\Phi}_c \hat{f}_{ab}{}^c |\psi\rangle = 0$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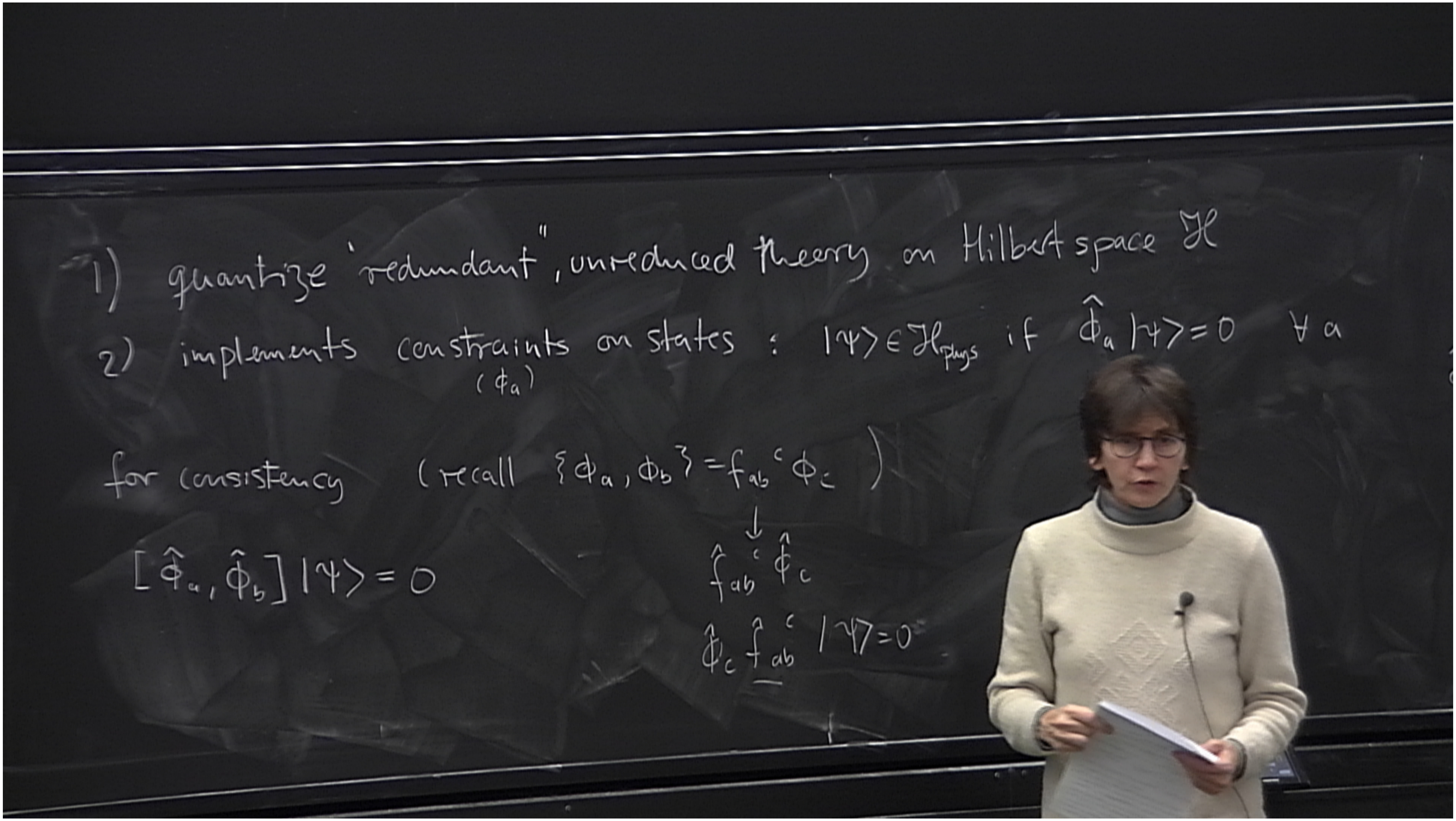
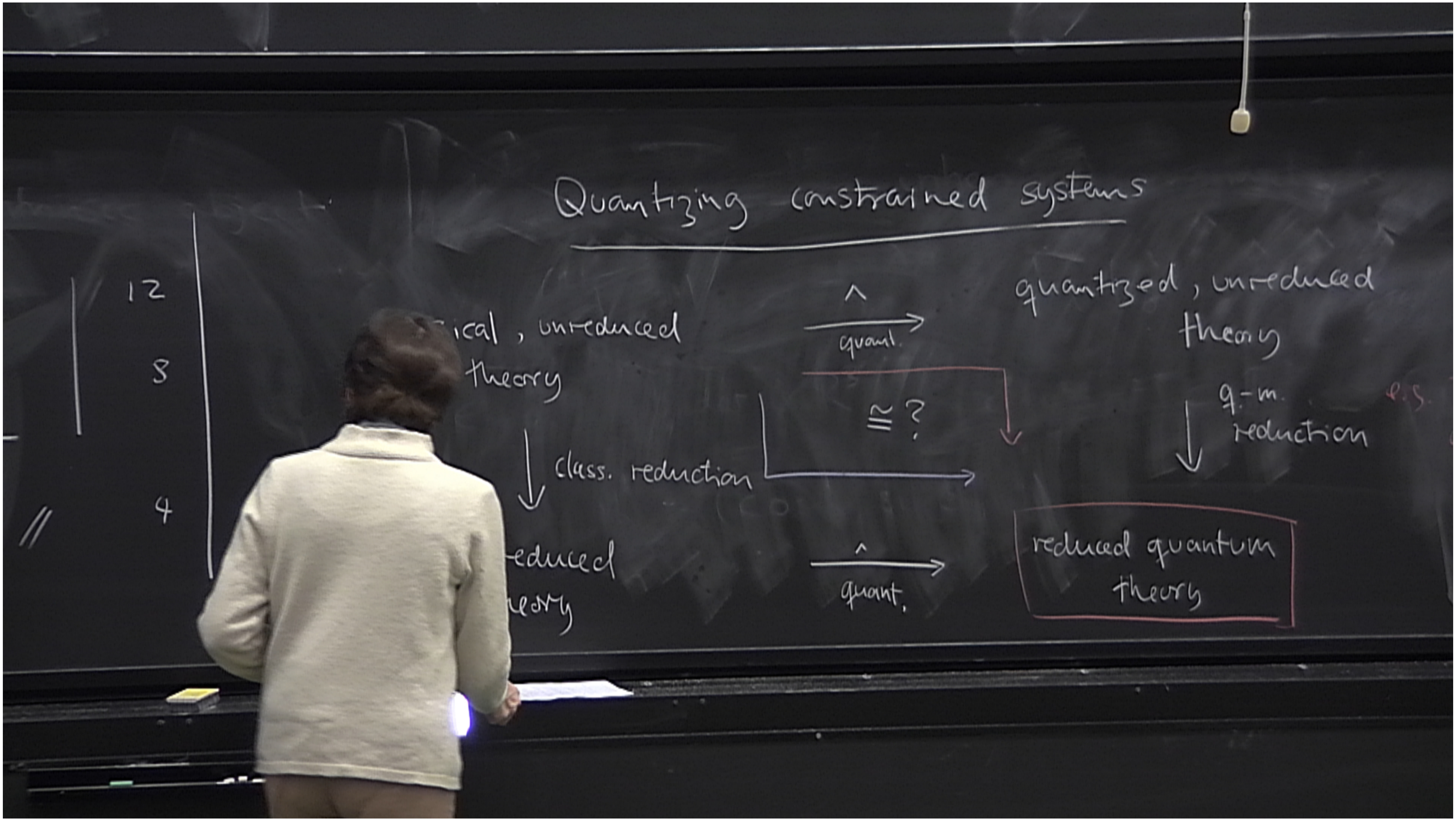
(φ_a)
for consistency (recall {Φ_a , Φ_b} = f_ab^c Φ_c)
[Φ̂_a , Φ̂_b] |ψ> = 0
f̂_ab^c Φ̂_c
Φ̂_c f̂_ab^c |ψ> = 0


1) quantize "redundant", unreduced theory on Hilbert space $\mathcal{H}$
2) implements constraints (ϕ_a) on states : $|\psi\rangle \in \mathcal{H}_{phys}$ if $\hat{\phi}_a |\psi\rangle = 0 \quad \forall a$
for consistency (recall $\{\phi_a, \phi_b\} = f_{ab}{}^c \phi_c$)
$[\hat{\phi}_a, \hat{\phi}_b] |\psi\rangle = 0$
$\hat{f}_{ab}{}^c \hat{\phi}_c$
$\hat{\phi}_c \hat{f}_{ab}{}^c |\psi\rangle = 0$


Quantizing constrained systems
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