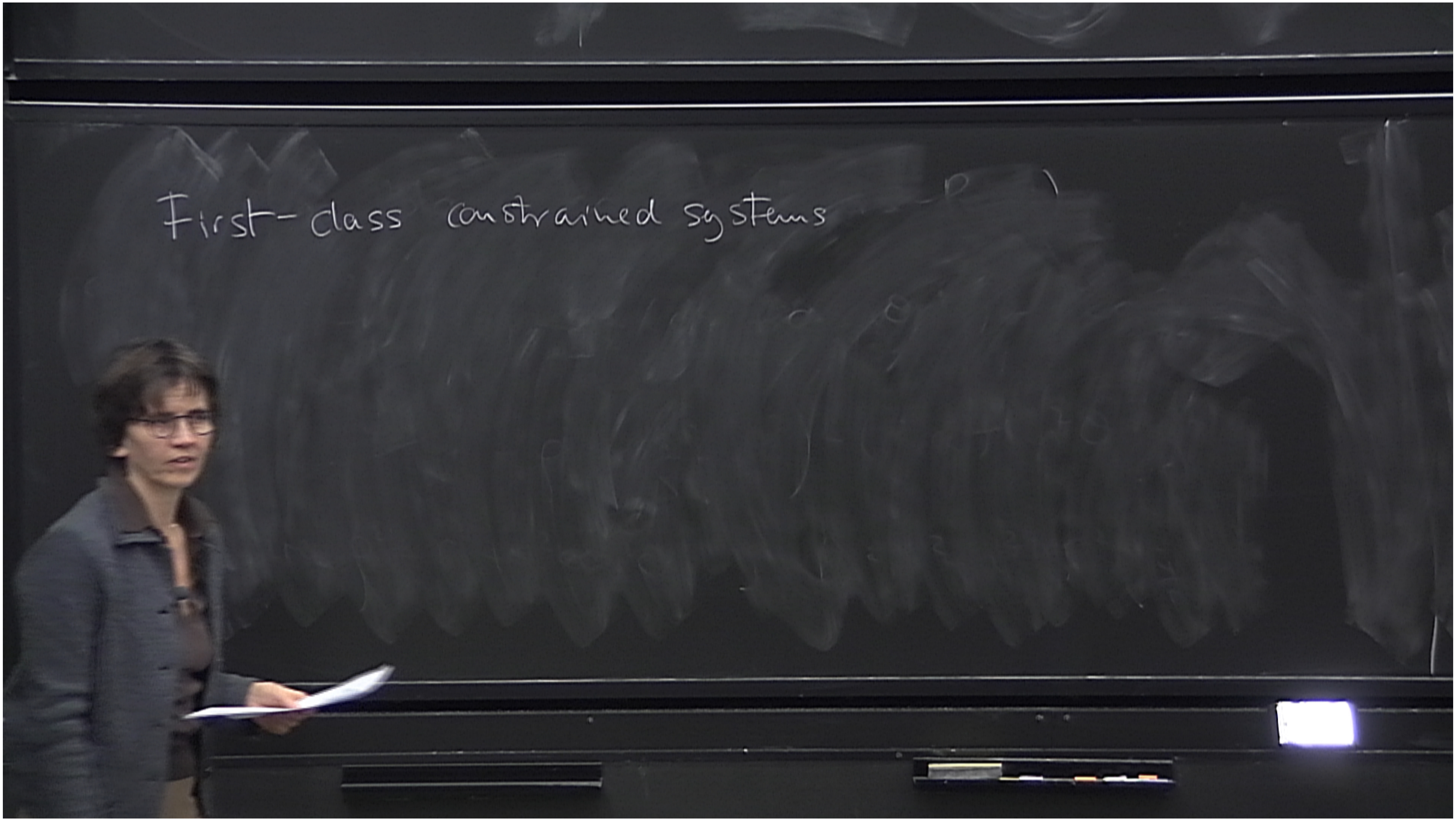
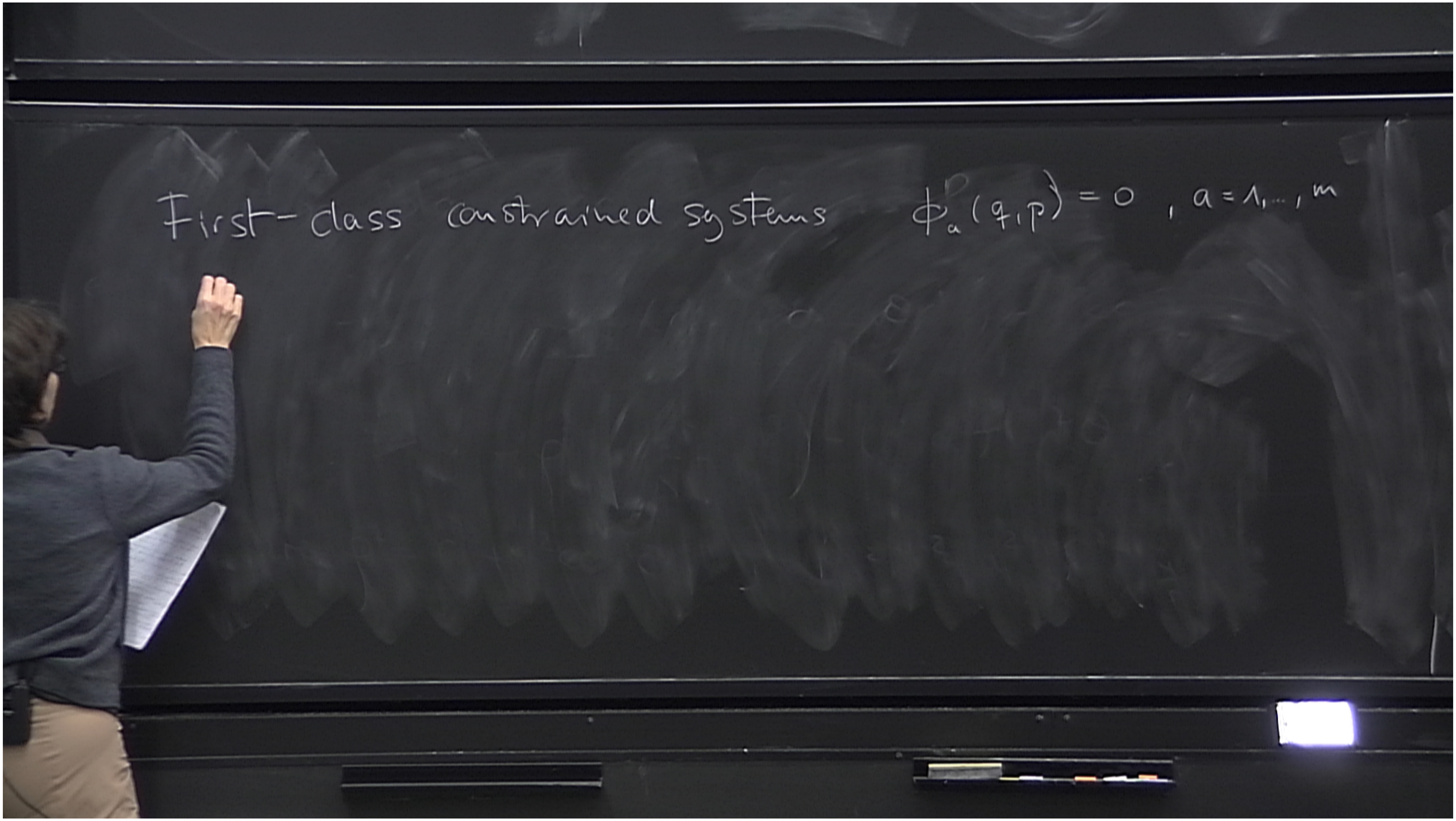
Title: Quantum Gravity (Review) - Lecture 8

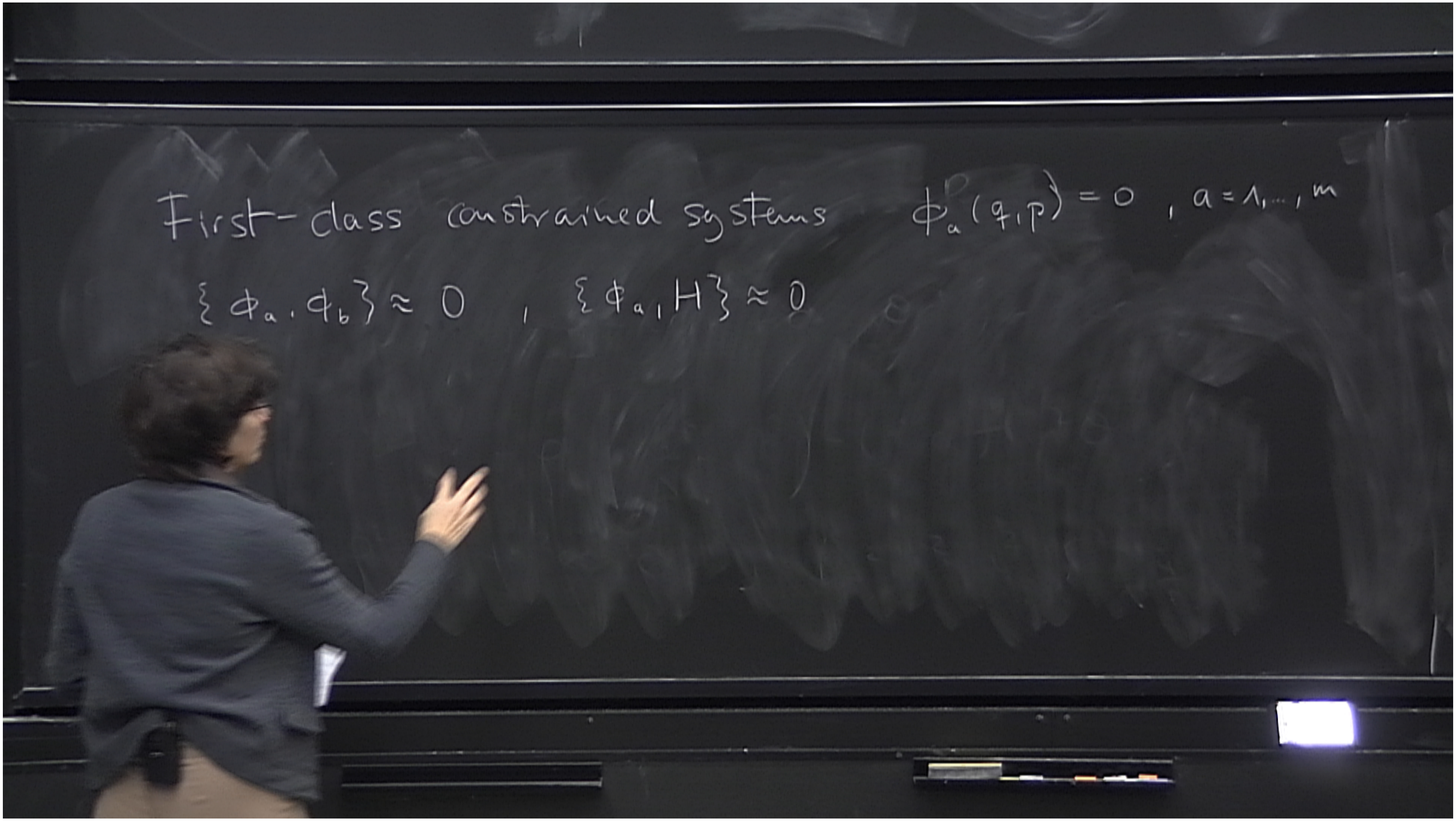
Date: Feb 01, 2012 10:15 AM

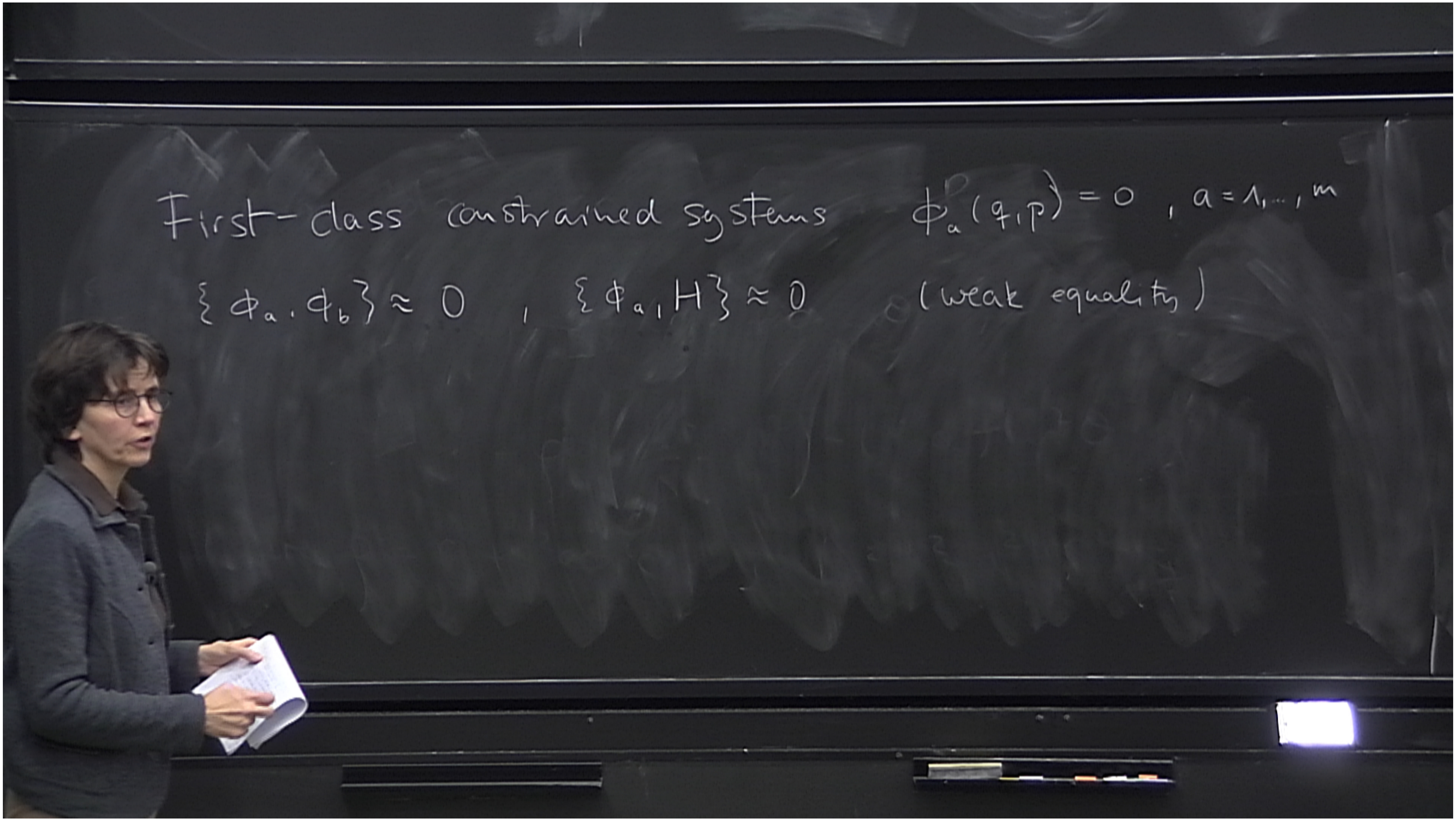
URL: http://pirsa.org/12020000

Abstract:


First-class constrained systems


First-class constrained systems $\phi_a(q,p) = 0$, $a = 1,\dots,m$


First-class constrained systems $\phi_a(q,p) = 0$, $a = 1, \dots, m$
$\{\phi_a, \phi_b\} \approx 0$, $\{\phi_a, H\} \approx 0$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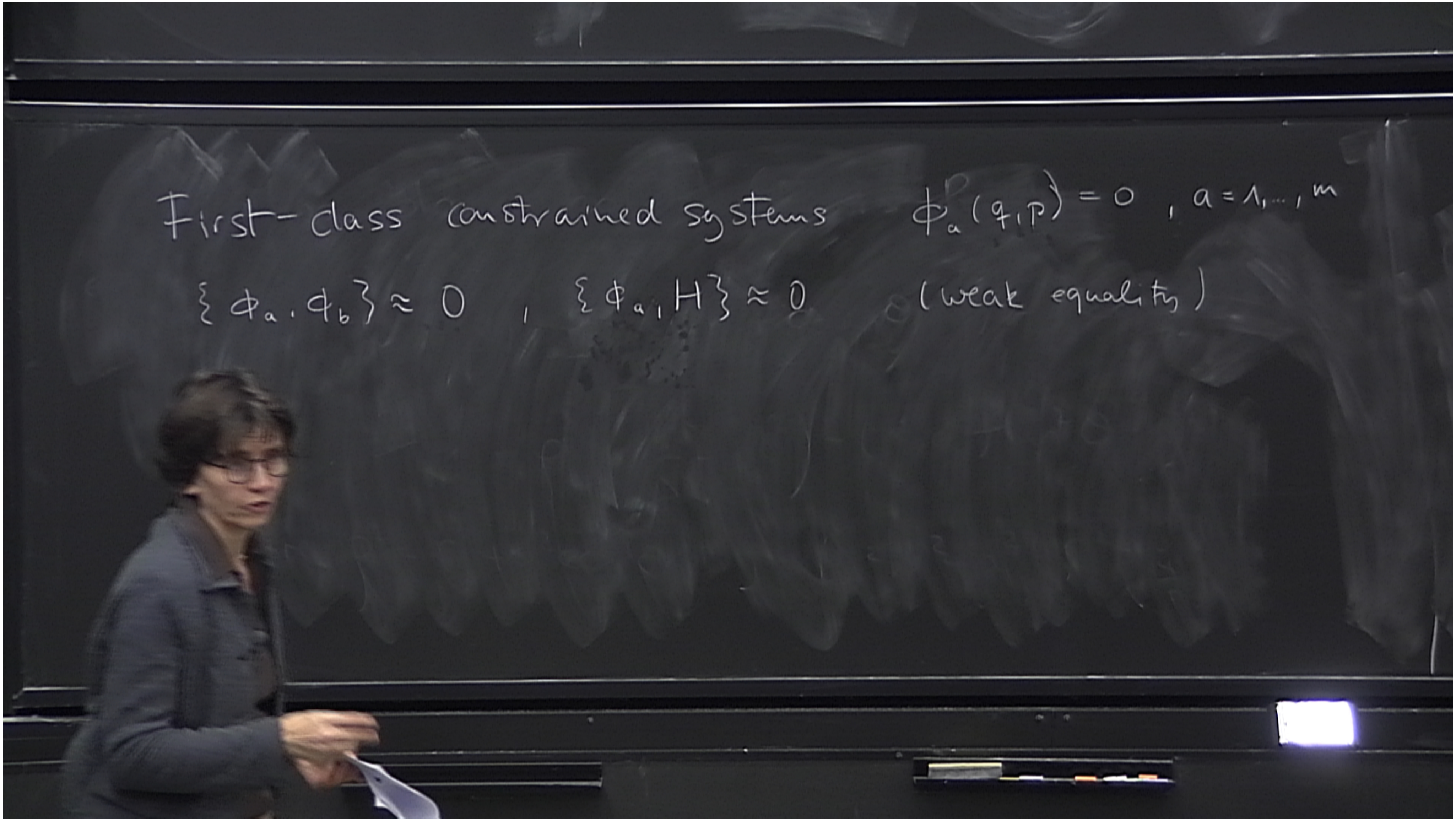
$\{\phi_a, \phi_b\} \approx 0$, $\{\phi_a, H\} \approx 0$ (weak equality)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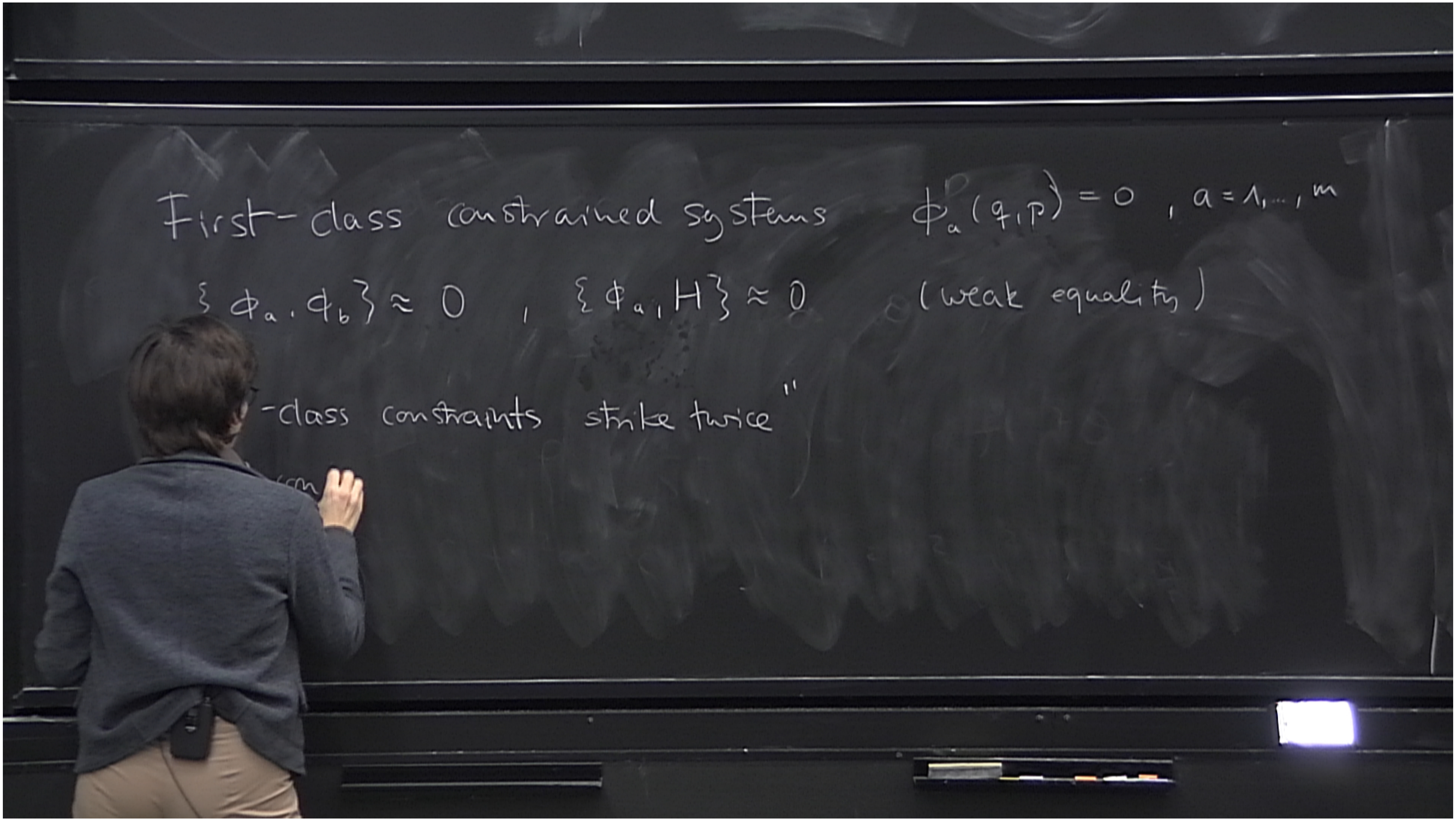
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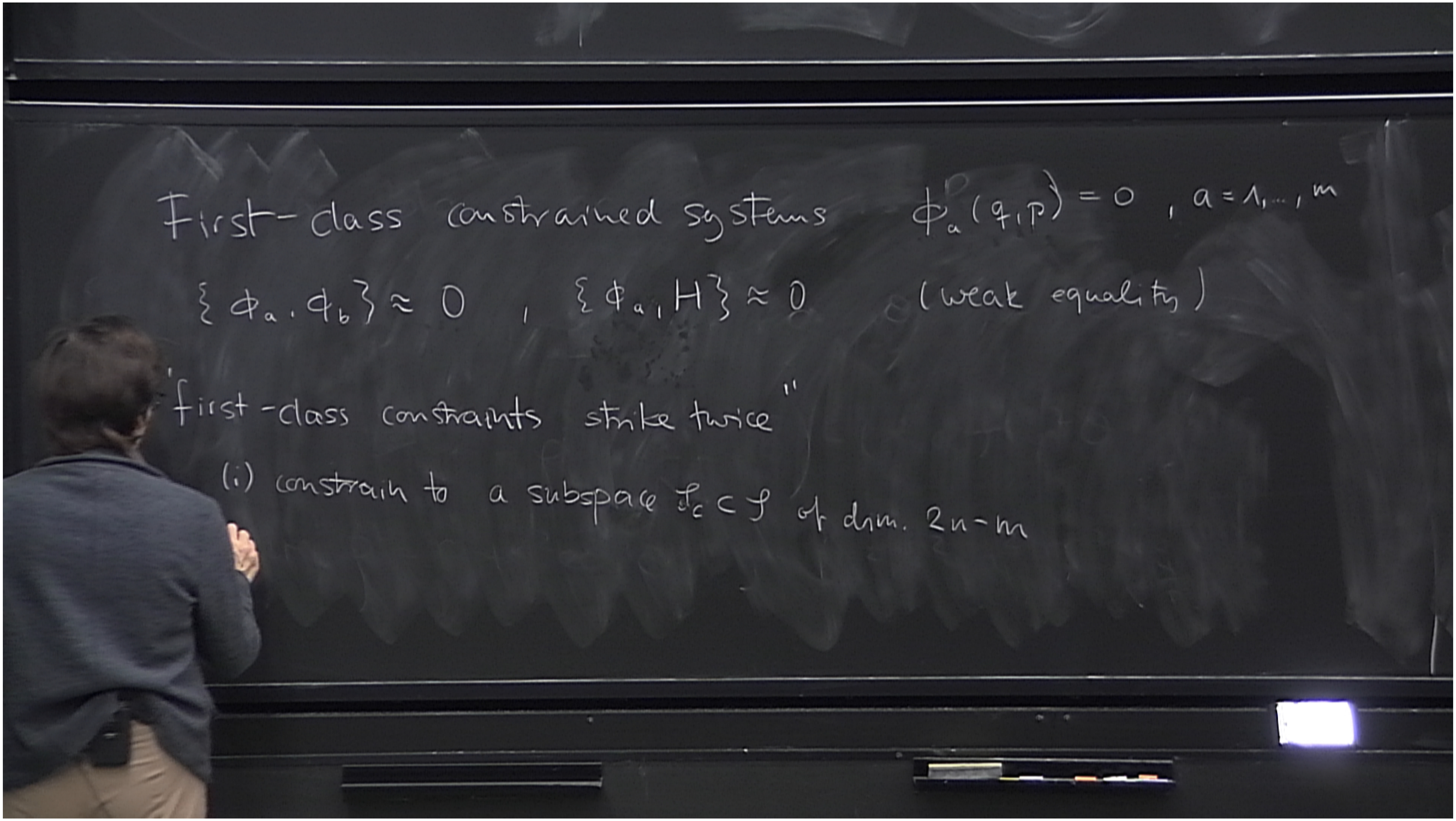
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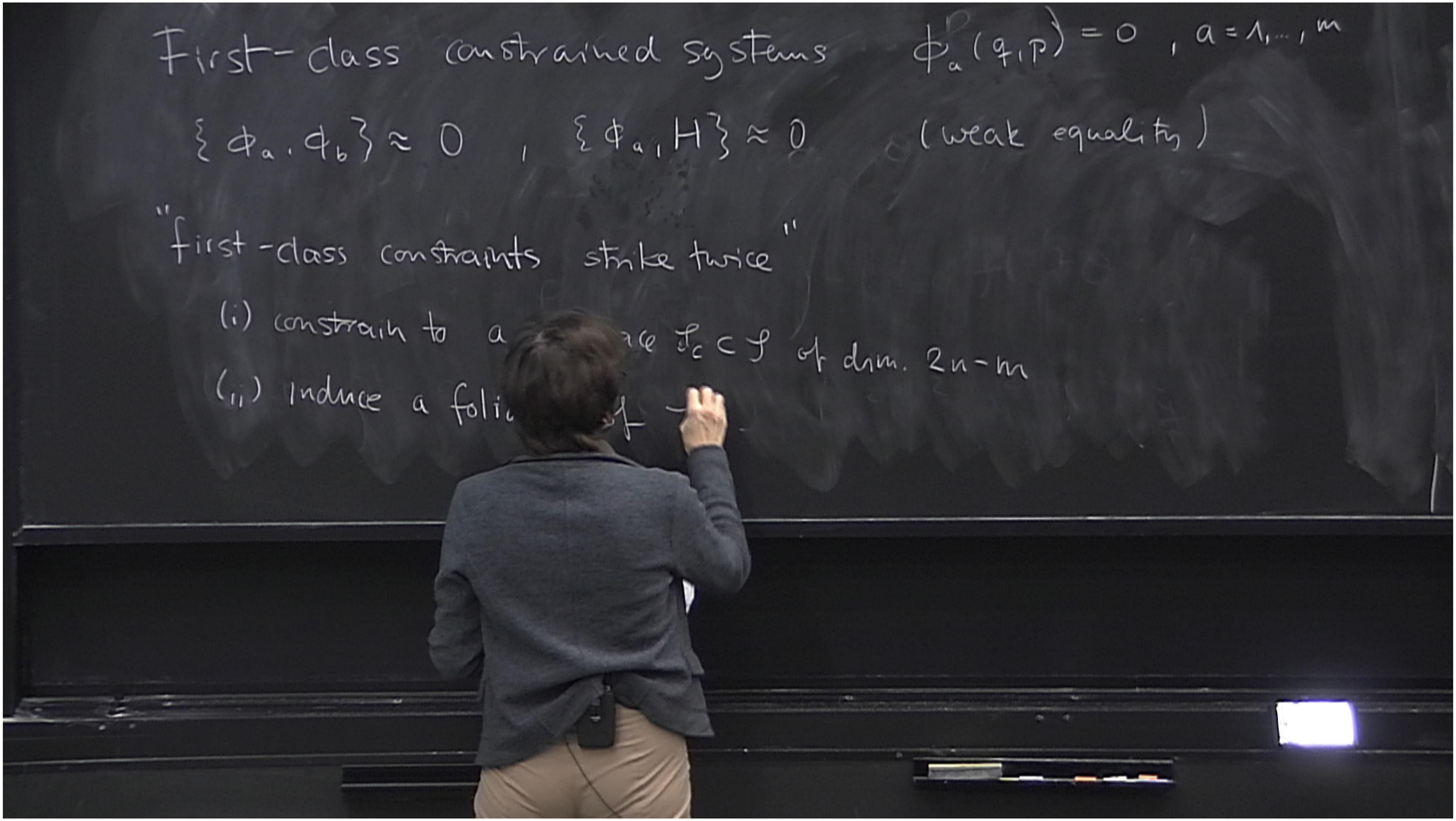
"first-class constraints strike twice"
(i) constrain to a subspace $\mathcal{S}_c \subset \mathcal{S}$ of dim. $2n-m$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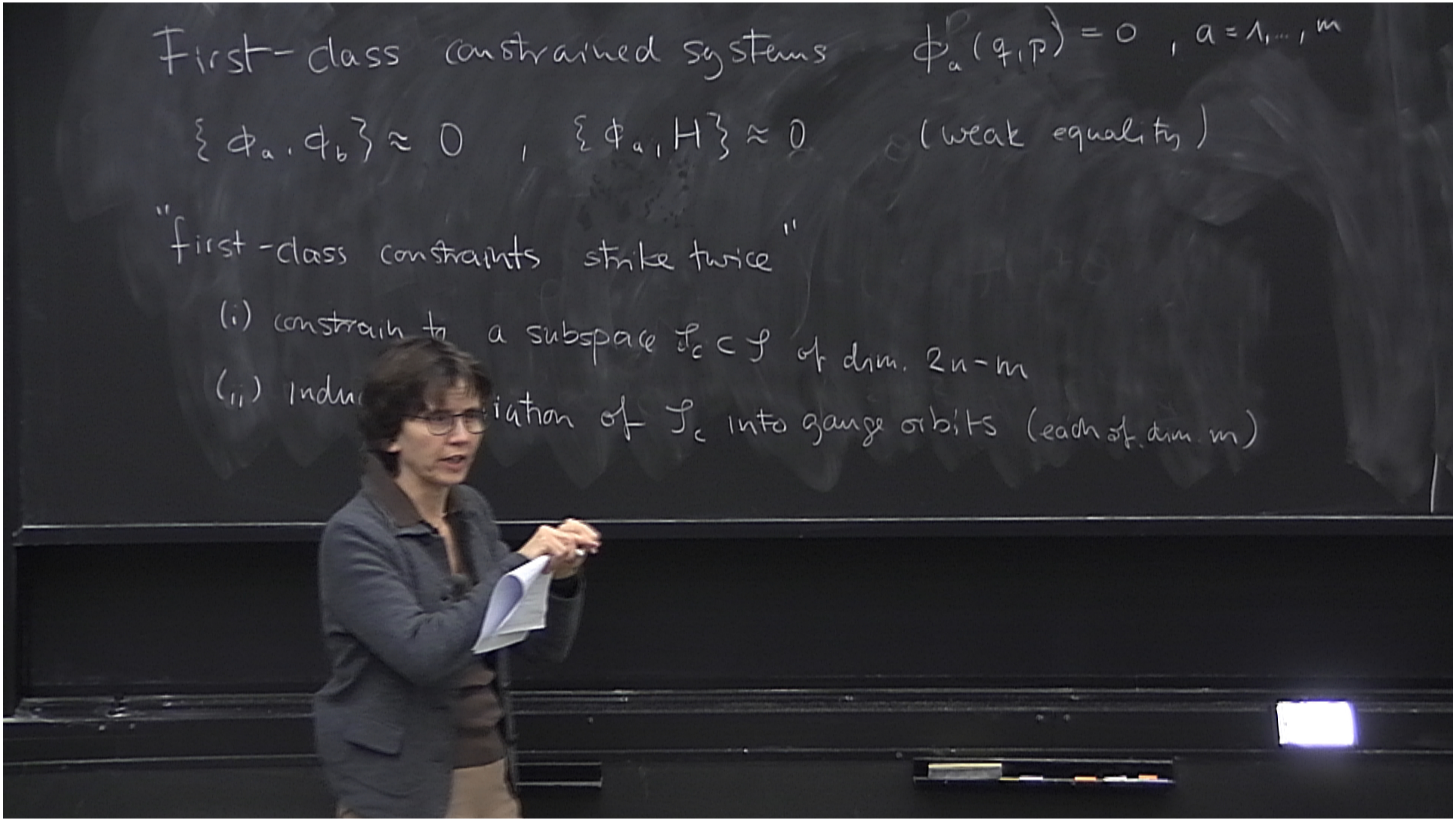
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(ii) indu
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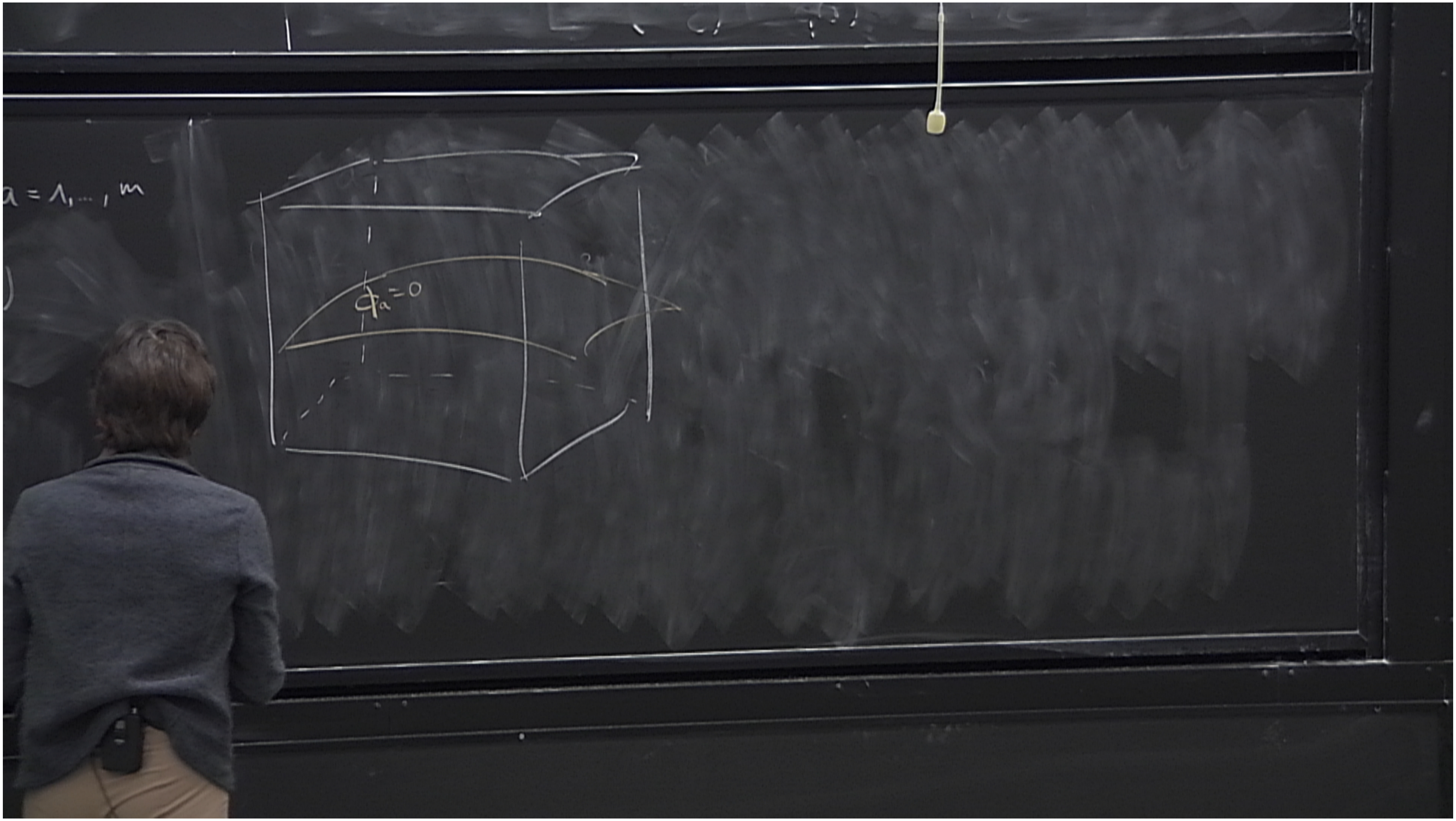
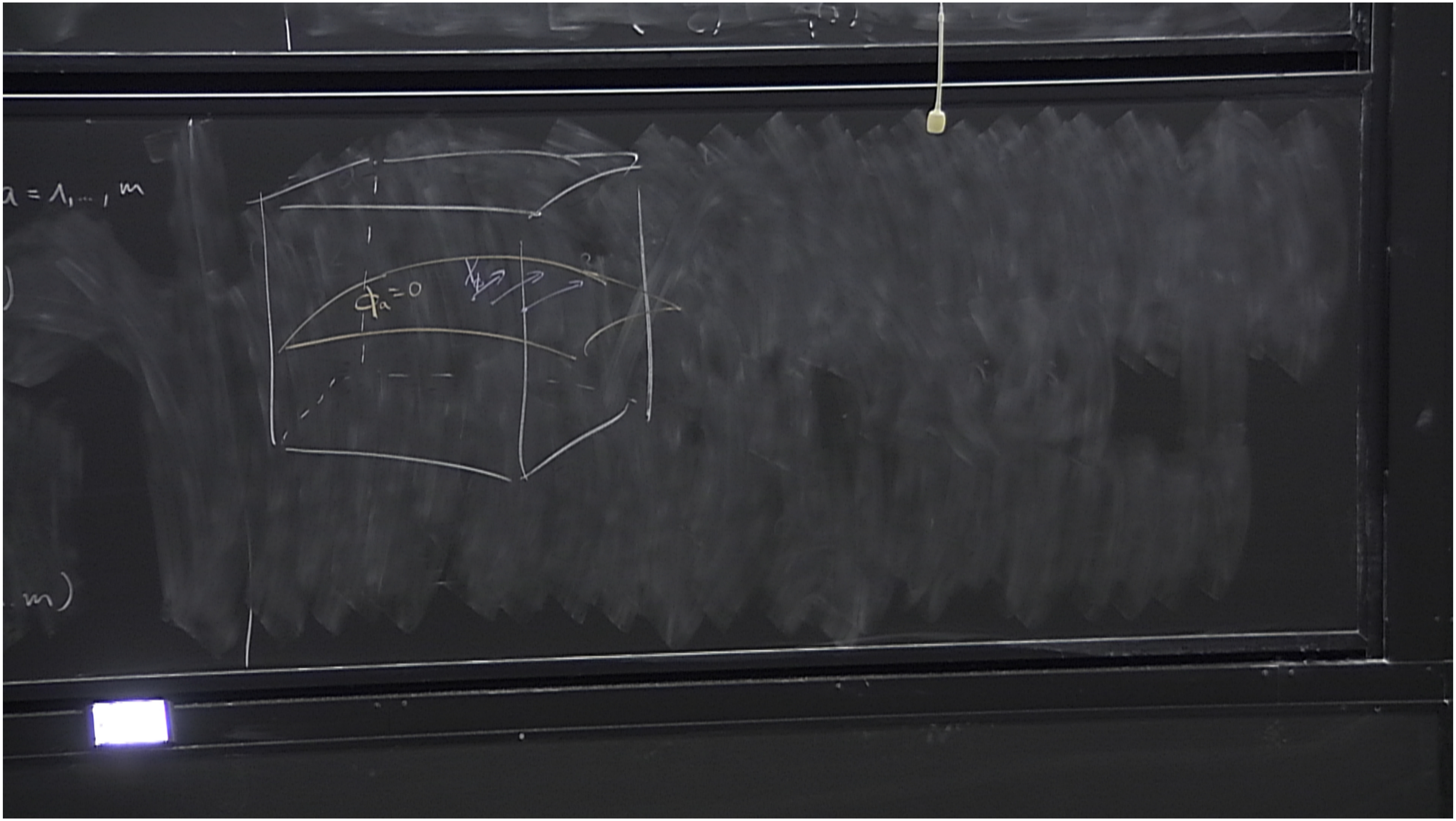
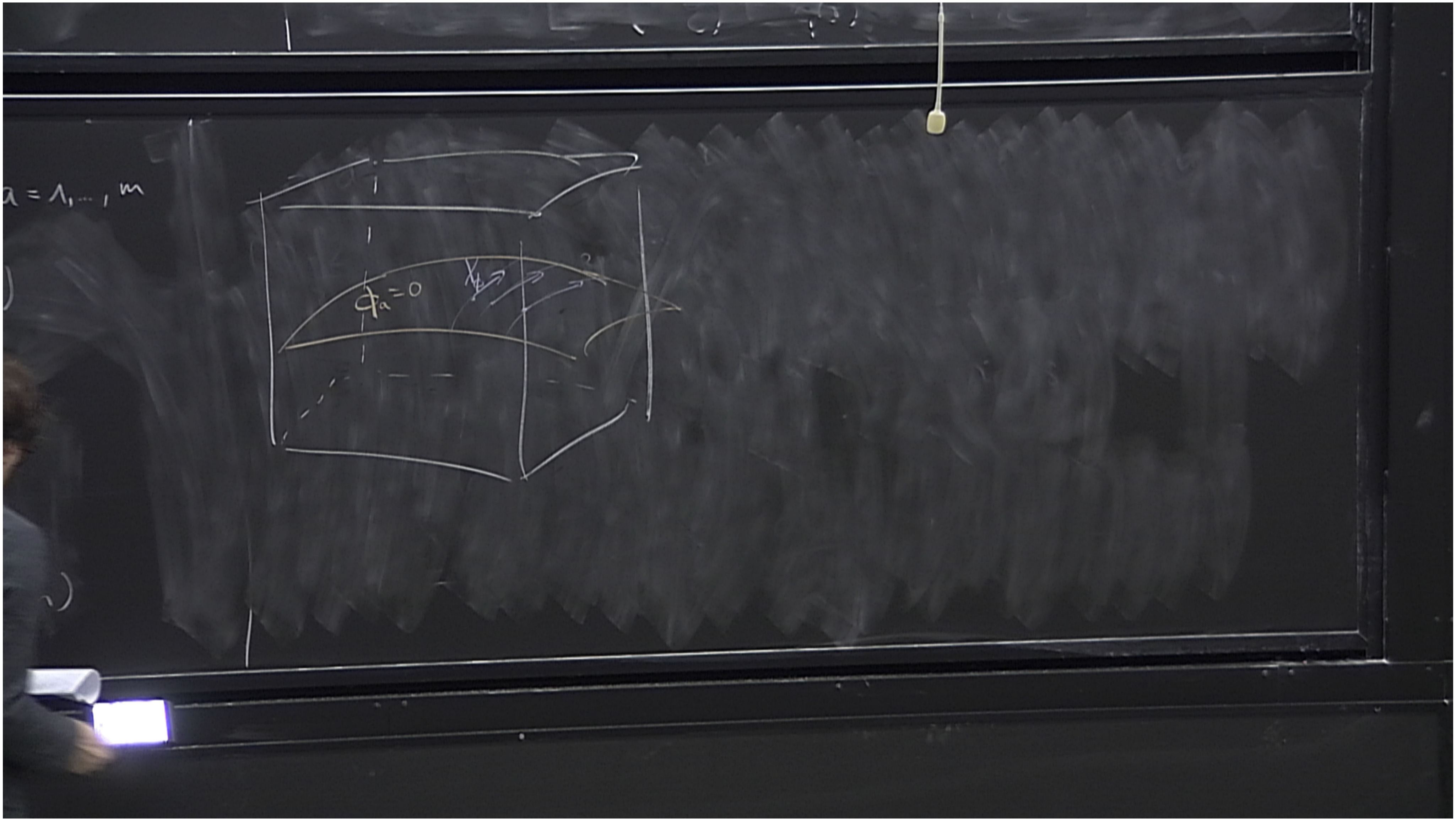
"first-class constraints strike twice"

(i) constrain to a subspace $\mathcal{S}_c \subset \mathcal{S}$ of dim. $2n-m$

(ii) induce a foliation of $\mathcal{S}_c$ into gauge orbits (each of dim. m)
(generated by the symplectic flow of the X_{ϕ_a})


a = 1, ..., m
q_a = 0


a = 1, ..., m
φa=0
m)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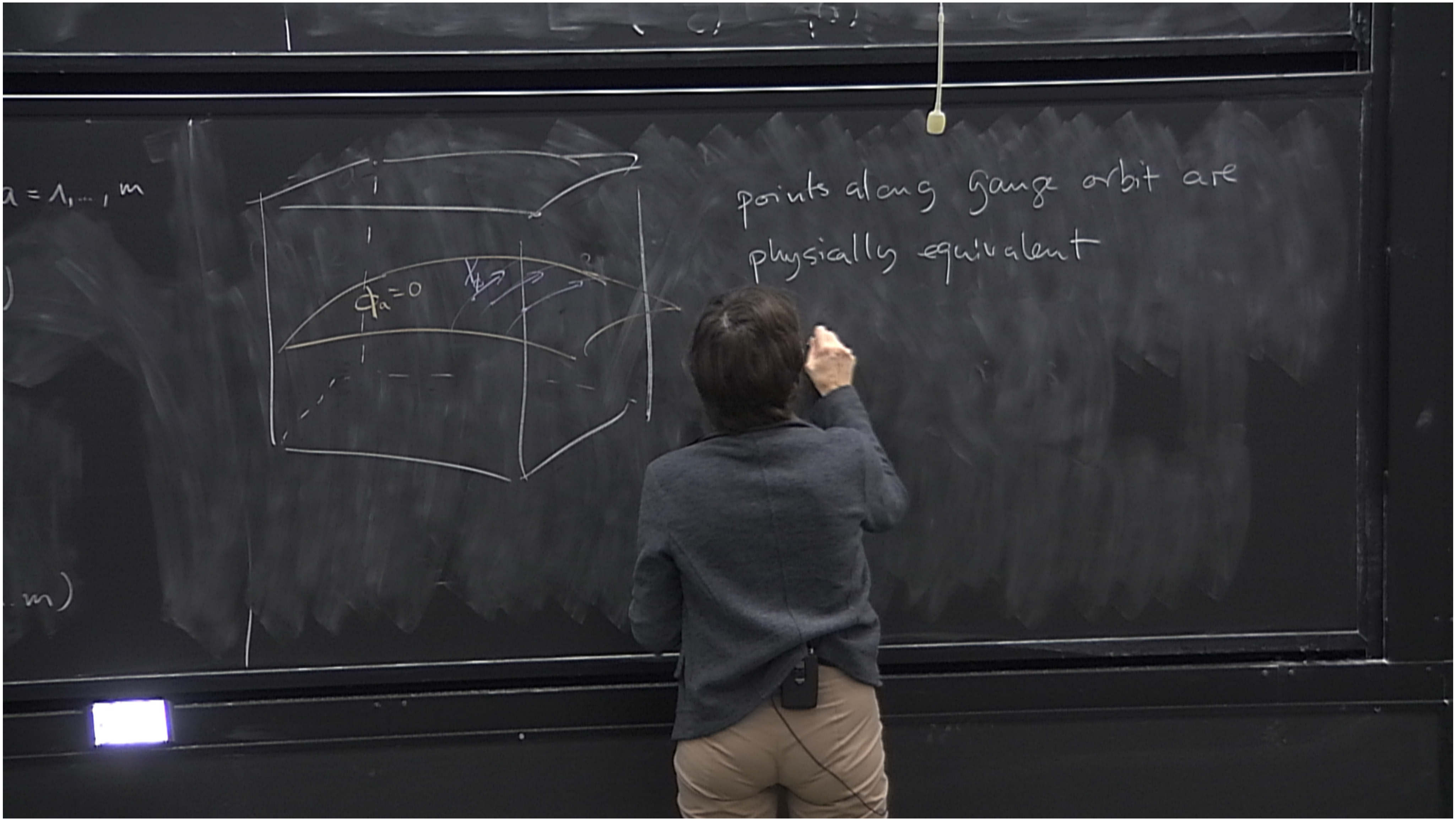
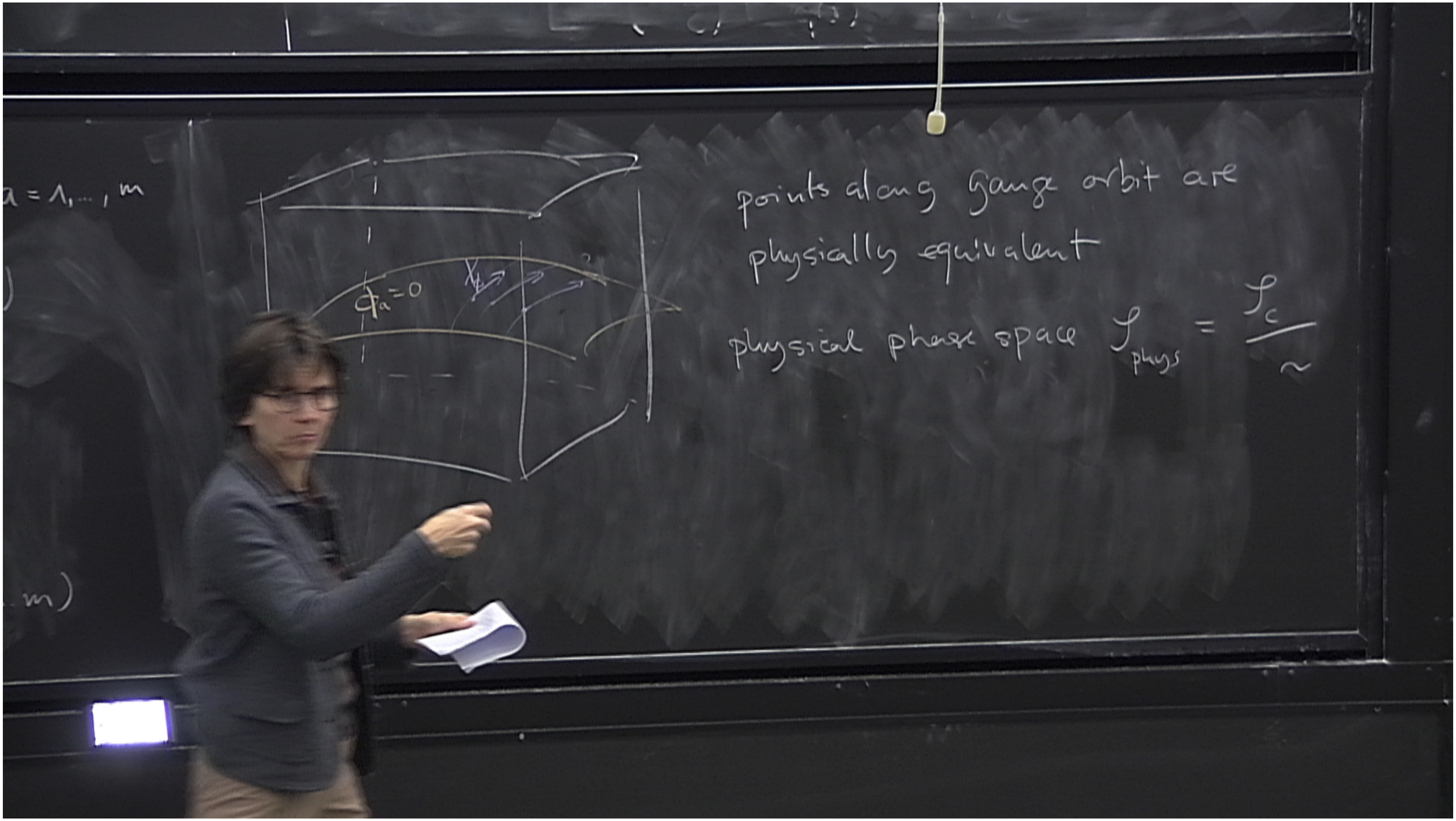
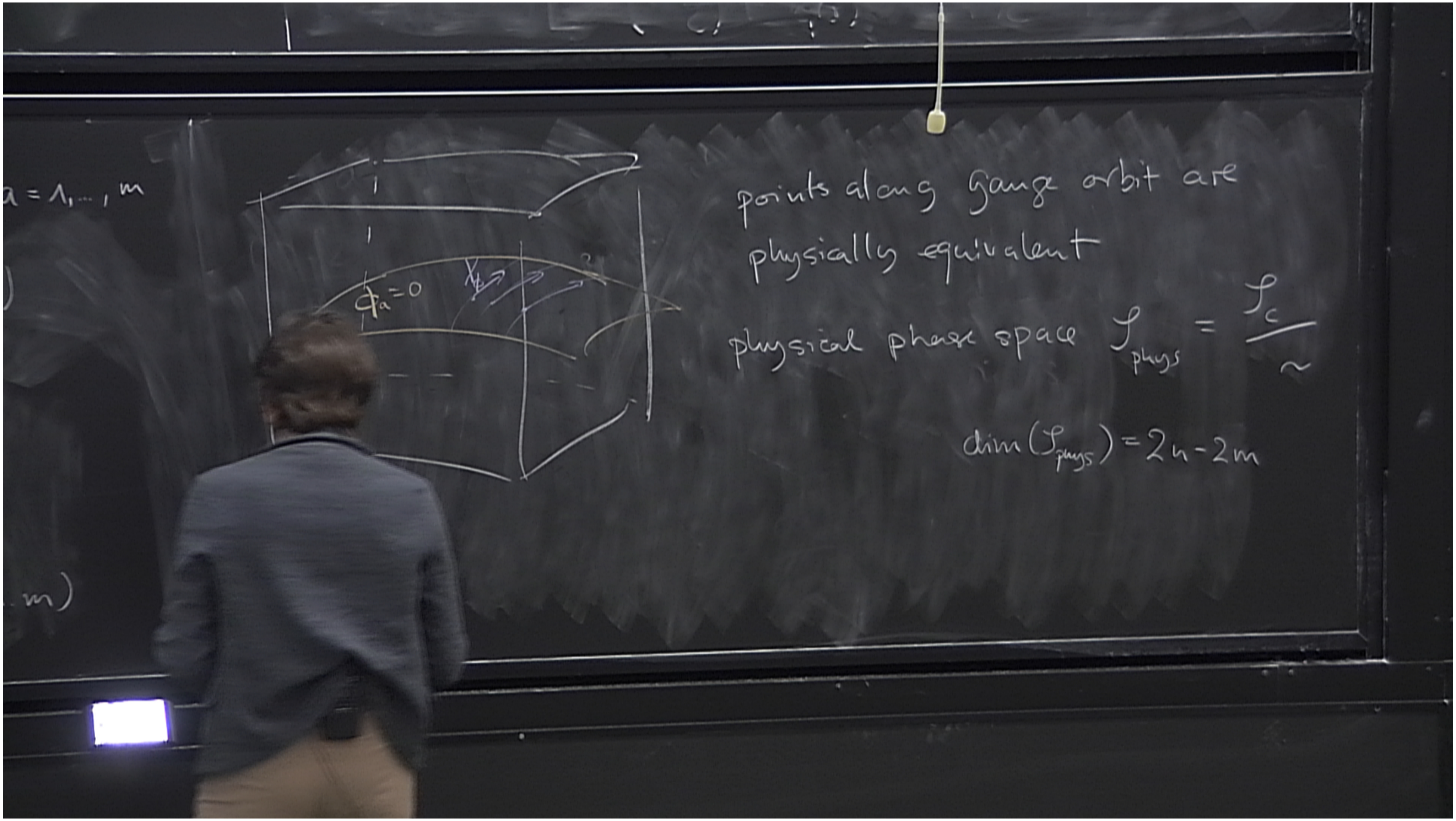
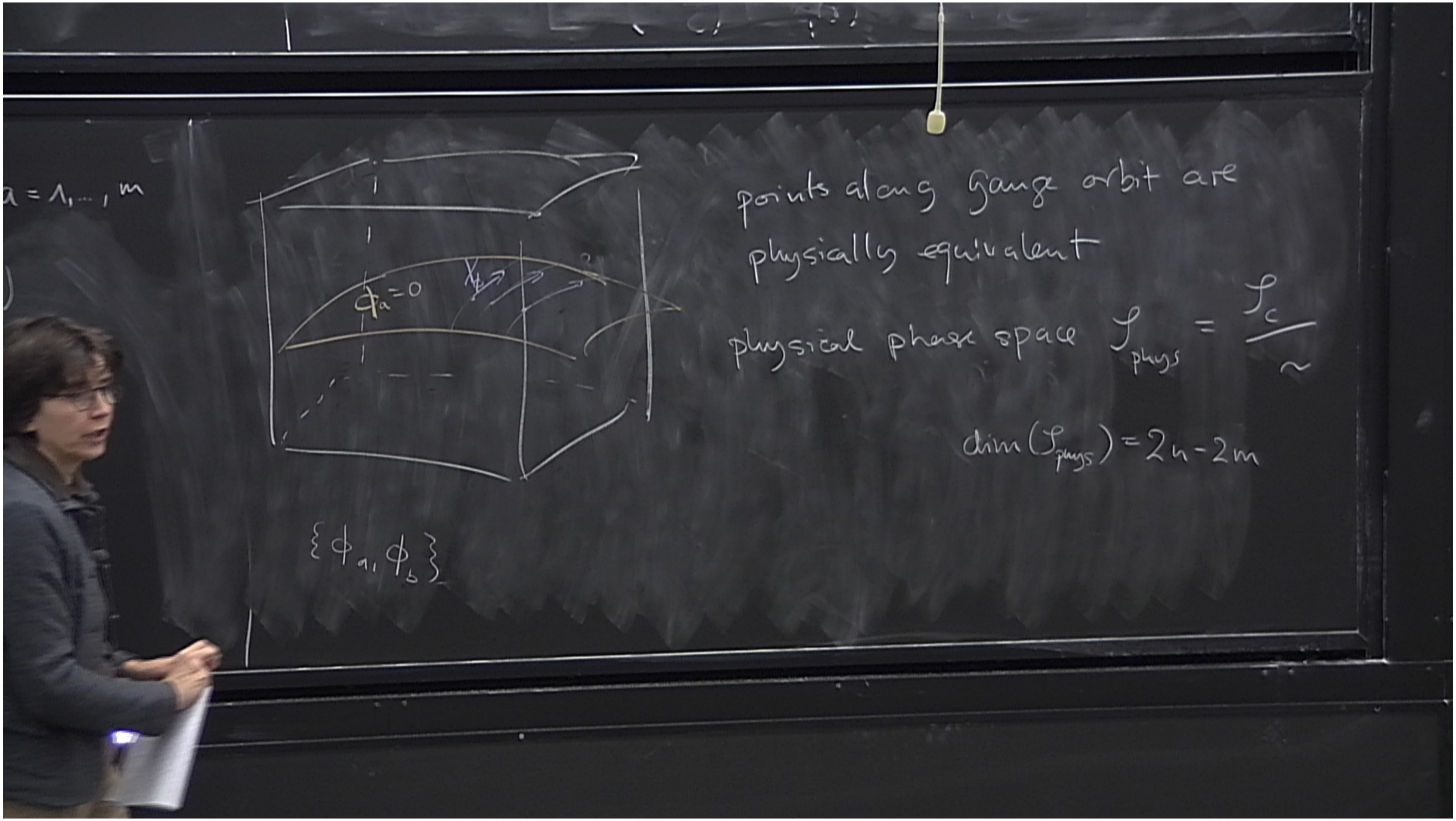
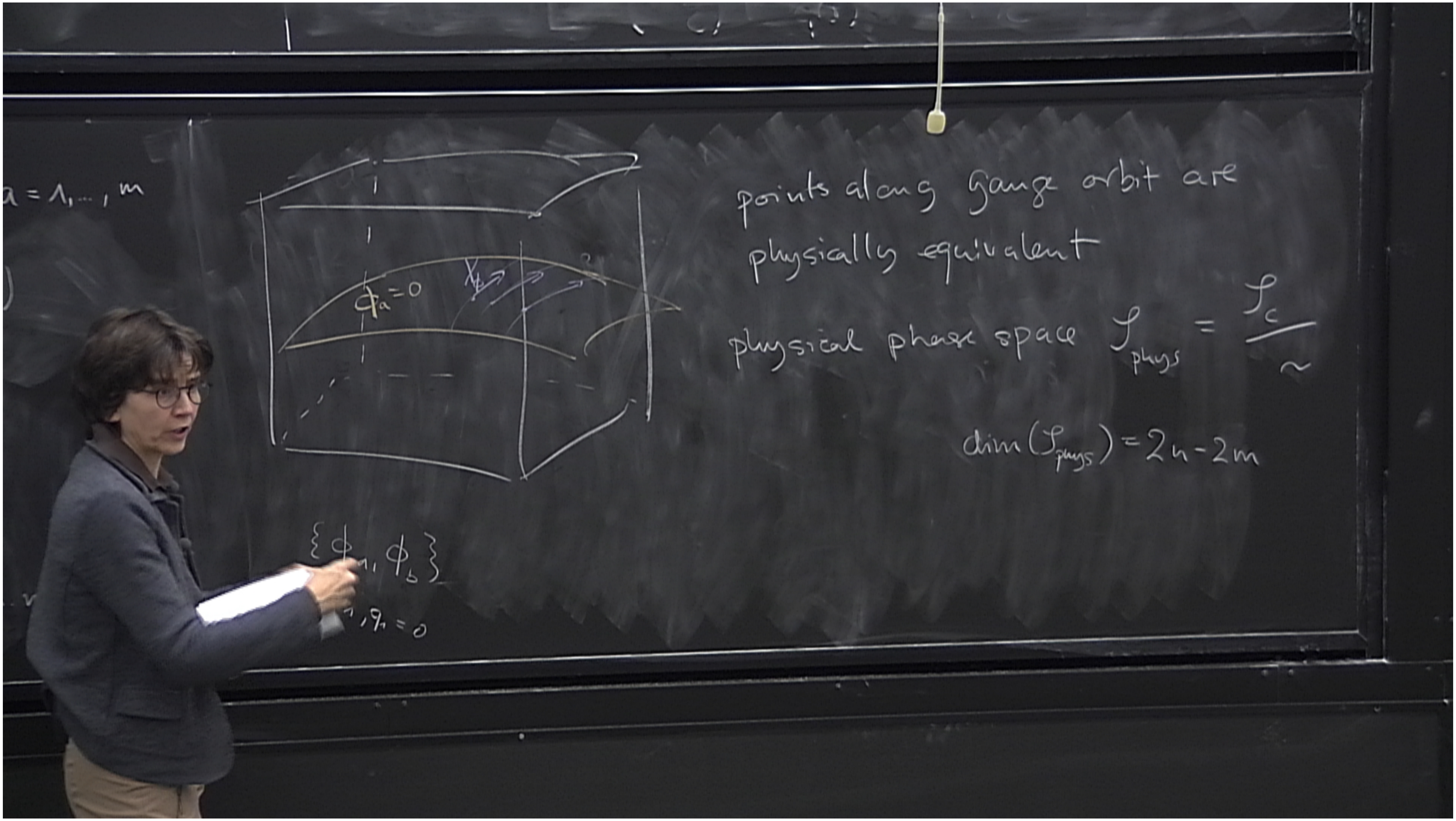

a = 1, ..., m
$\phi_a = 0$
points along gauge orbit are
physically equivalent


a = 1, ..., m
points along gauge orbit are
physically equivalent
physical phase space $S_{phys} = S_c / \sim$
$\phi_a = 0$
X_ϕ
m)


a = 1, ..., m
$\phi_a = 0$
points along gauge orbit are
physically equivalent
physical phase space $\Gamma_{phys} = \Gamma_c / \sim$
$\dim(\Gamma_{phys}) = 2n - 2m$
m)


a = 1, ..., m
φa = 0
points along gauge orbit are
physically equivalent
physical phase space Sphys = Sc / ~
dim(Sphys) = 2n - 2m
{φa, φb}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points along gauge orbit are
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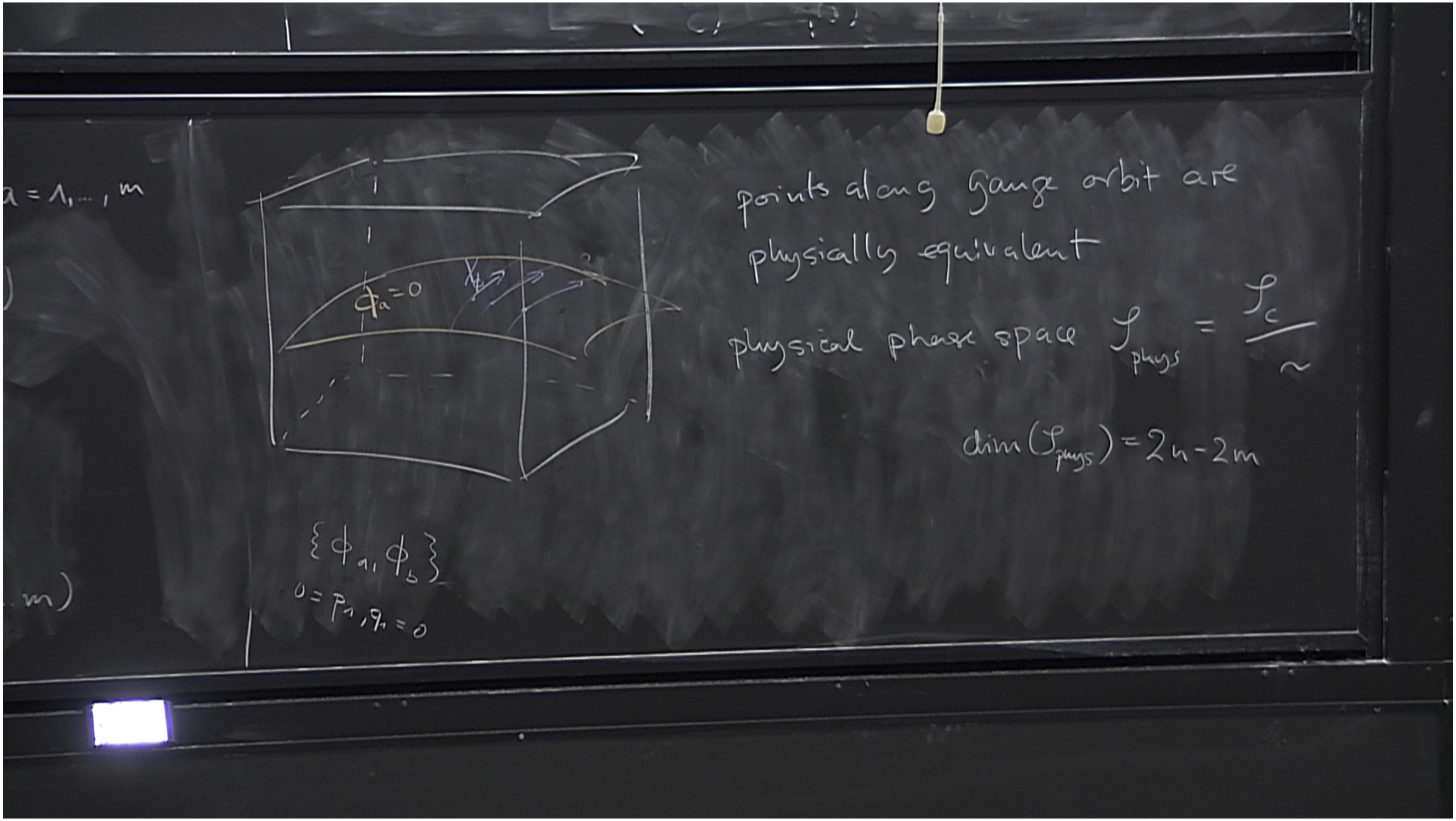
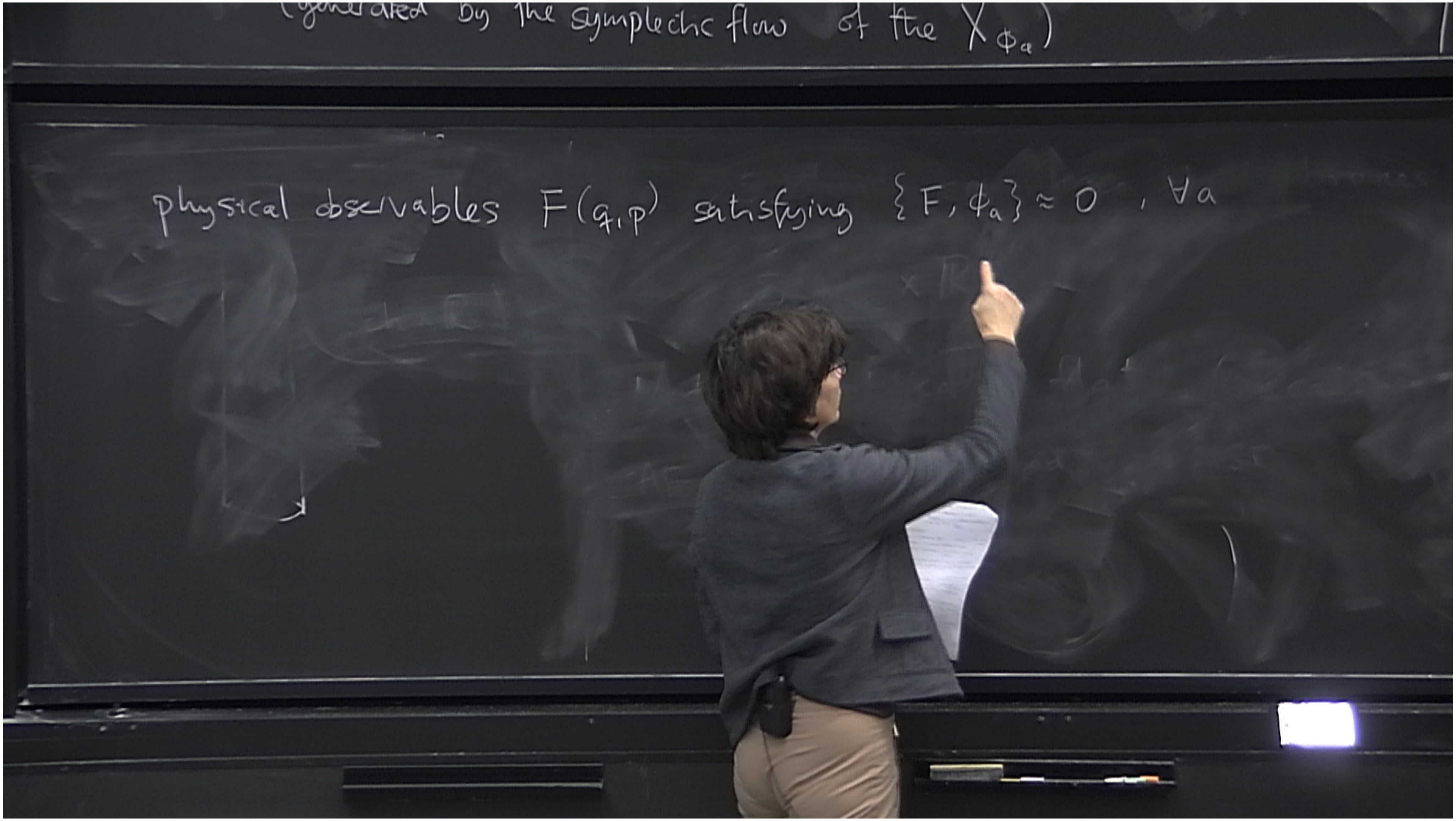
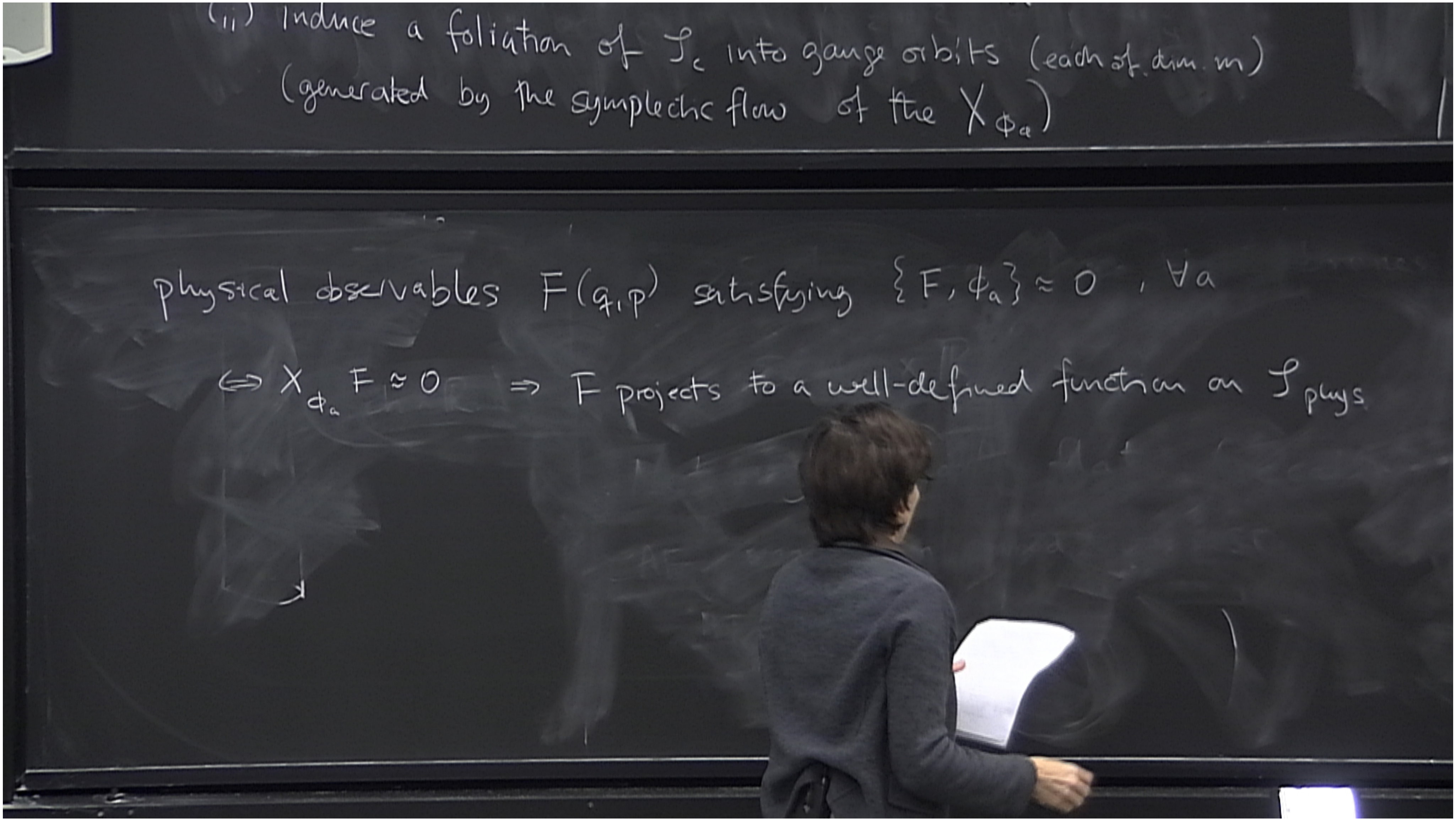

a = 1, …, m
$\phi_a = 0$
$\{\phi_a, \phi_b\}$
$0 = p_1, q_1 = 0$
points along gauge orbit are
physically equivalent
physical phase space $\Gamma_{phys} = \Gamma_c / \sim$
$\dim(\Gamma_{phys}) = 2n - 2m$


(generated by the symplectic flow of the X_{ϕ_a})
physical observables $F(q,p)$ satisfying $\{F, \phi_a\} \approx 0$, $\forall a$


(ii) induce a foliation of Γ_c into gauge orbits (each of dim. m)
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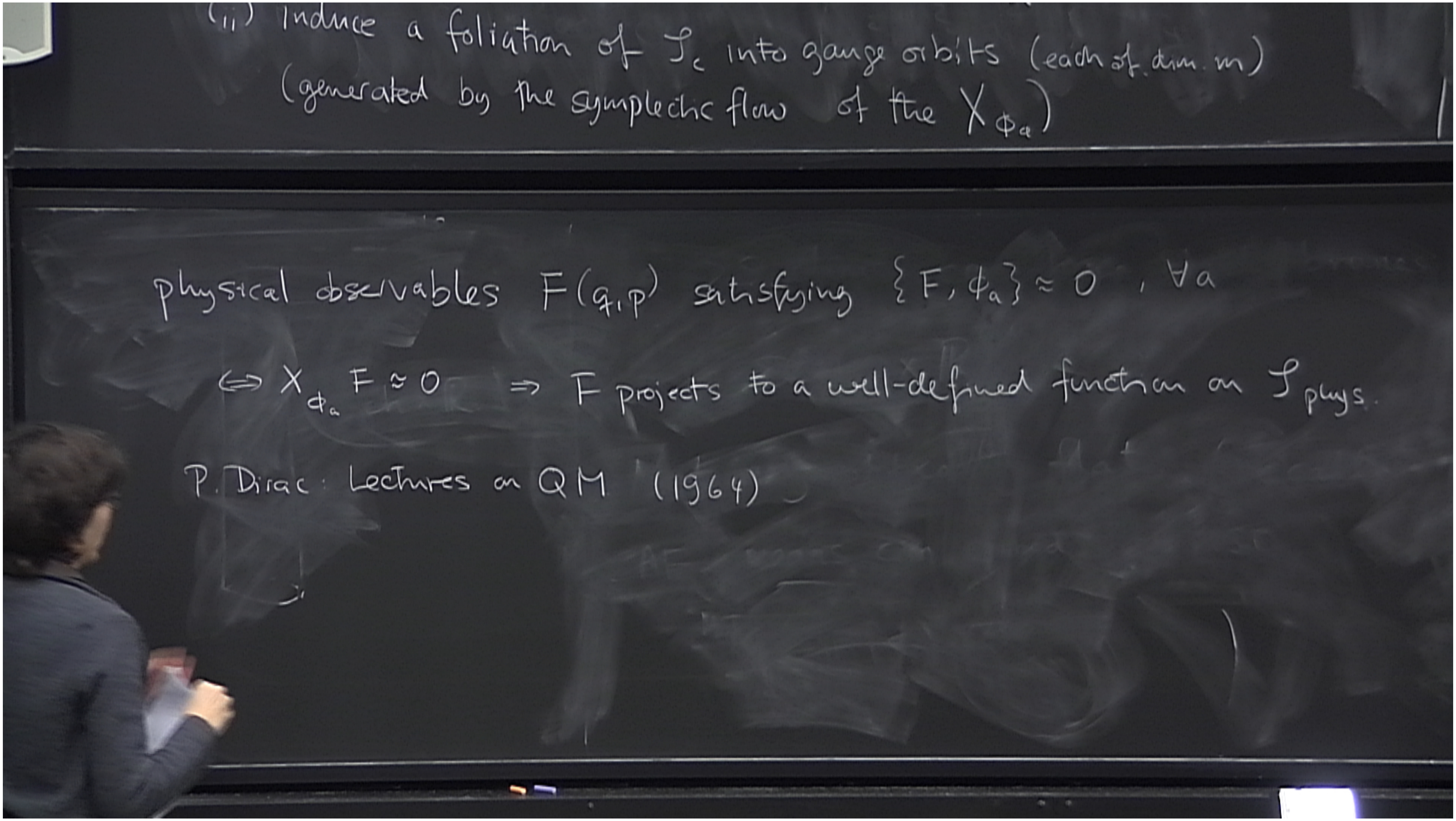
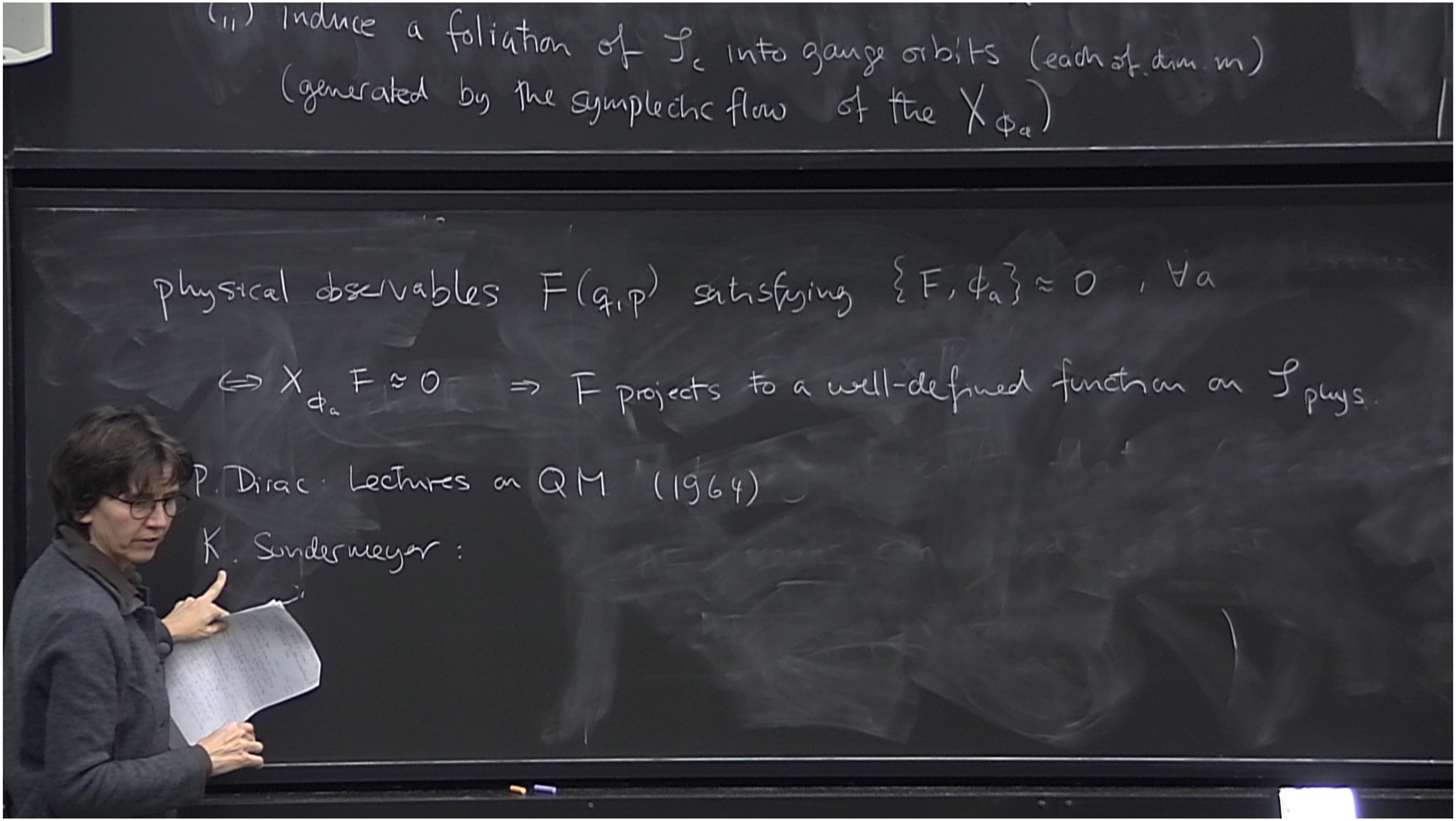
physical observables $F(q,p)$ satisfying $\{F, \phi_a\} \approx 0$, $\forall a$
$\Leftrightarrow X_{\phi_a} F \approx 0$ $\Rightarrow$ F projects to a well-defined function on Γ_{phys}


(ii) induce a foliation of $\mathcal{S}_c$ into gauge orbits (each of dim. m)
(generated by the symplectic flow of the X_{ϕ_a})
physical observables $F(q,p)$ satisfying $\{F, \phi_a\} \approx 0$, $\forall a$
$\Leftrightarrow X_{\phi_a} F \approx 0 \Rightarrow$ F projects to a well-defined function on $\mathcal{S}_{phys}$.
P. Dirac: Lectures on QM (1964)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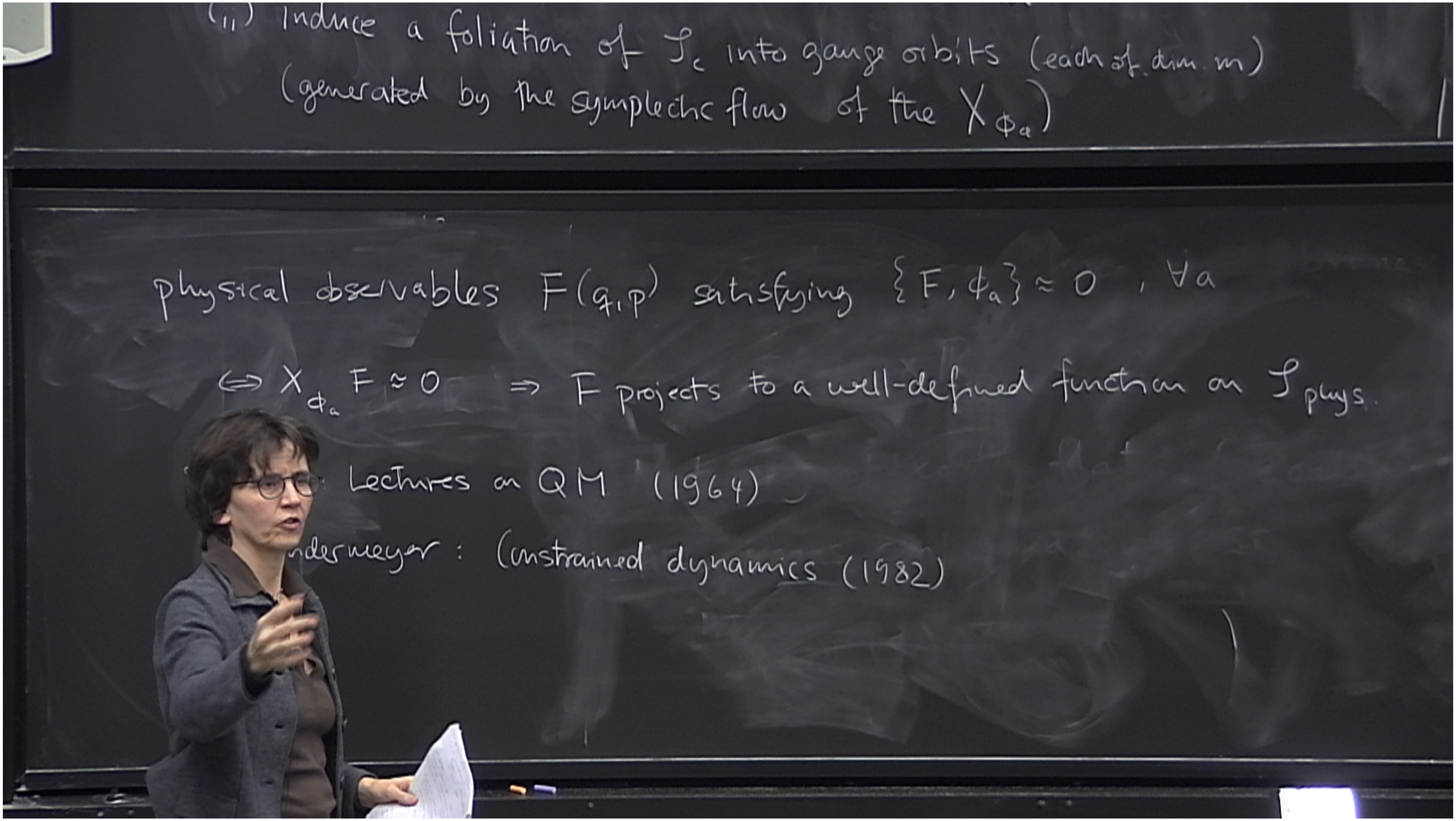
P. Dirac: Lectures on QM (1964)
K. Sundermeyer: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Lectures on QM (1964)
Constrained dynamics (1982)

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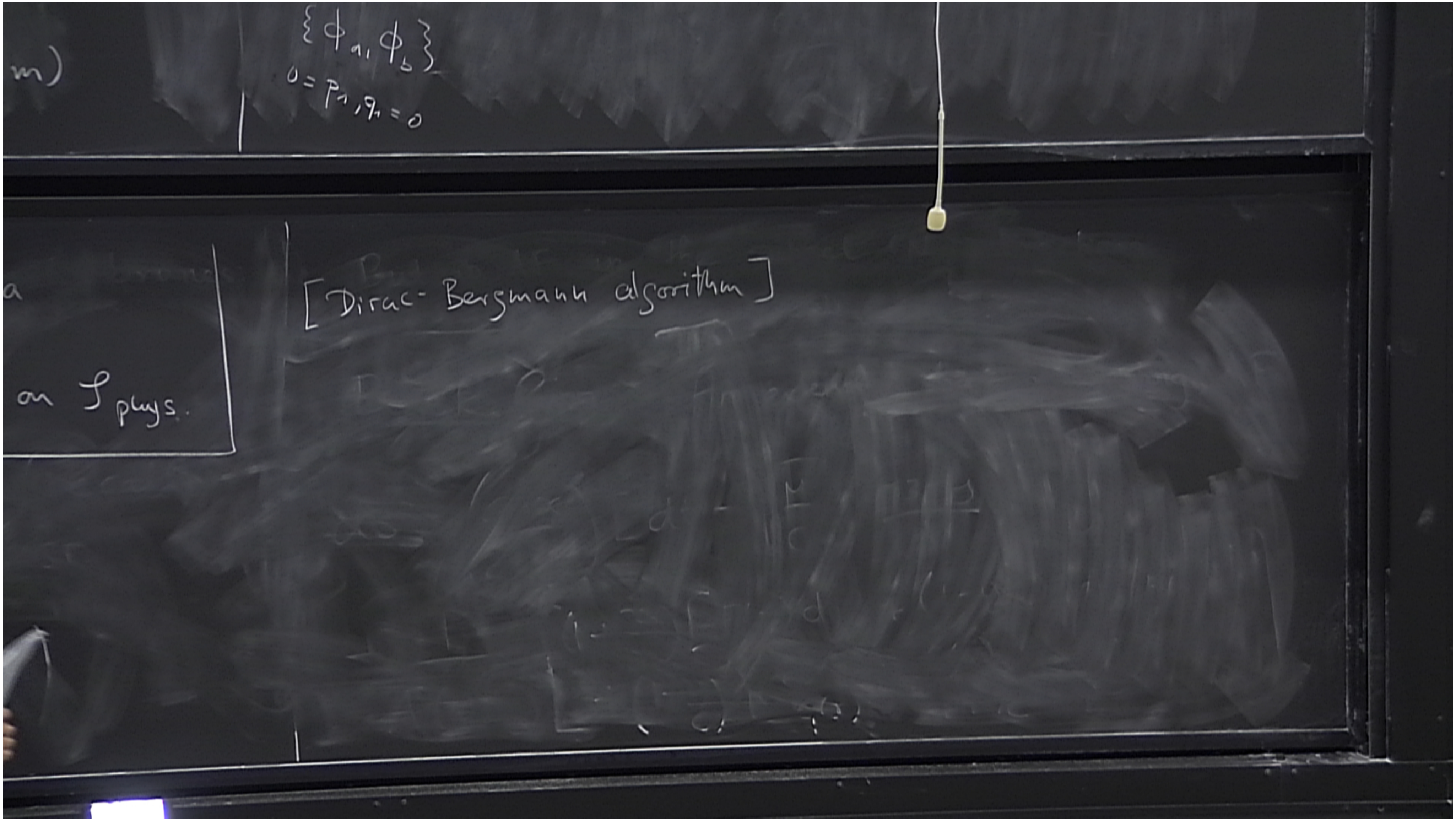
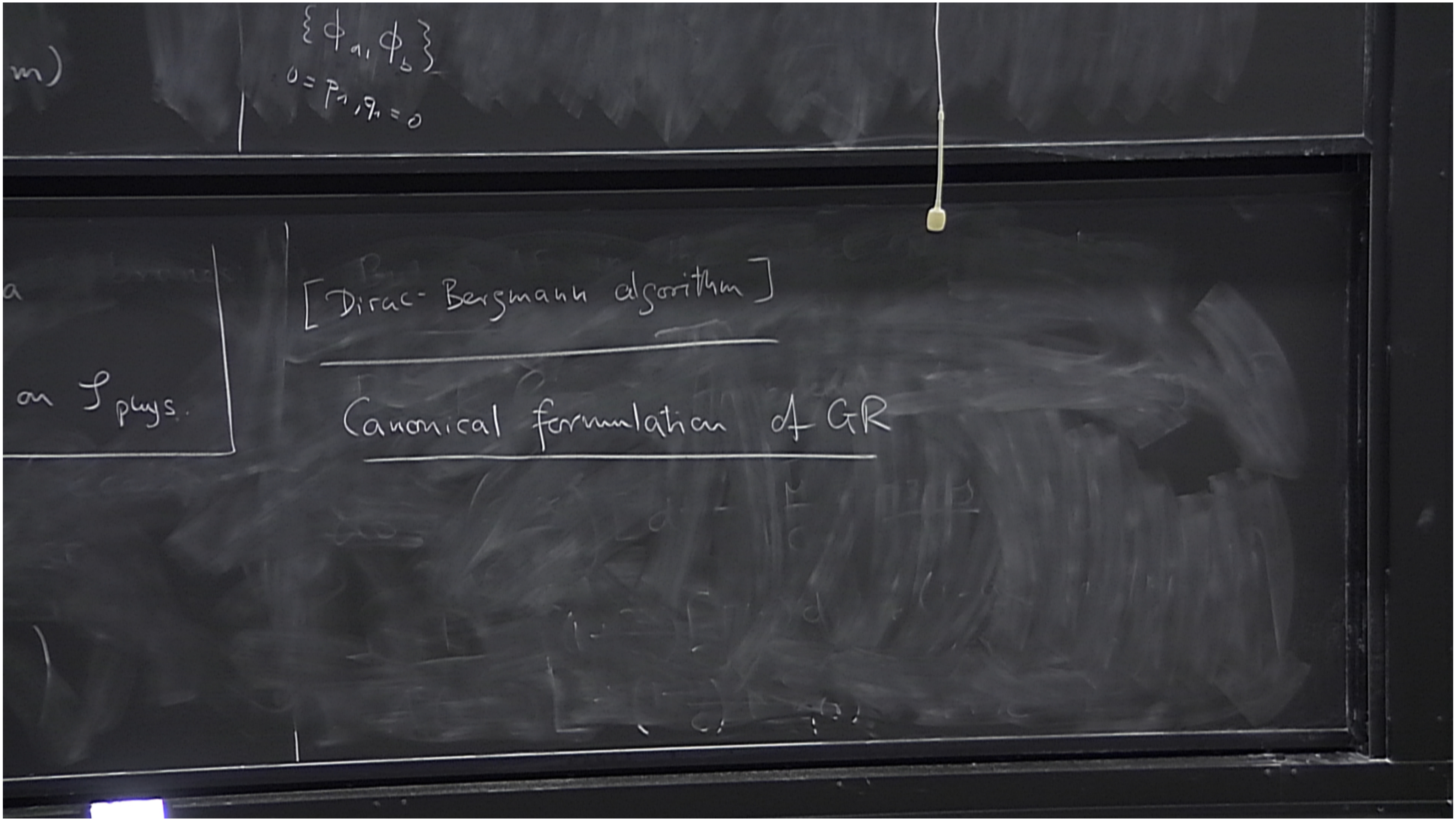
P. Dirac: Lectures on QM (1964)

K. Sundermeyer: Constrained dynamics (1982)

M. Henneaux & C. Teitelboim: Quantization of gauge systems (1994)


m)
{φ_a1, φ_b}
0 = p_1, q_1 = 0
on S_phys.
[Dirac-Bergmann algorithm]


{φa, φb}
0 = p1, q1 = 0
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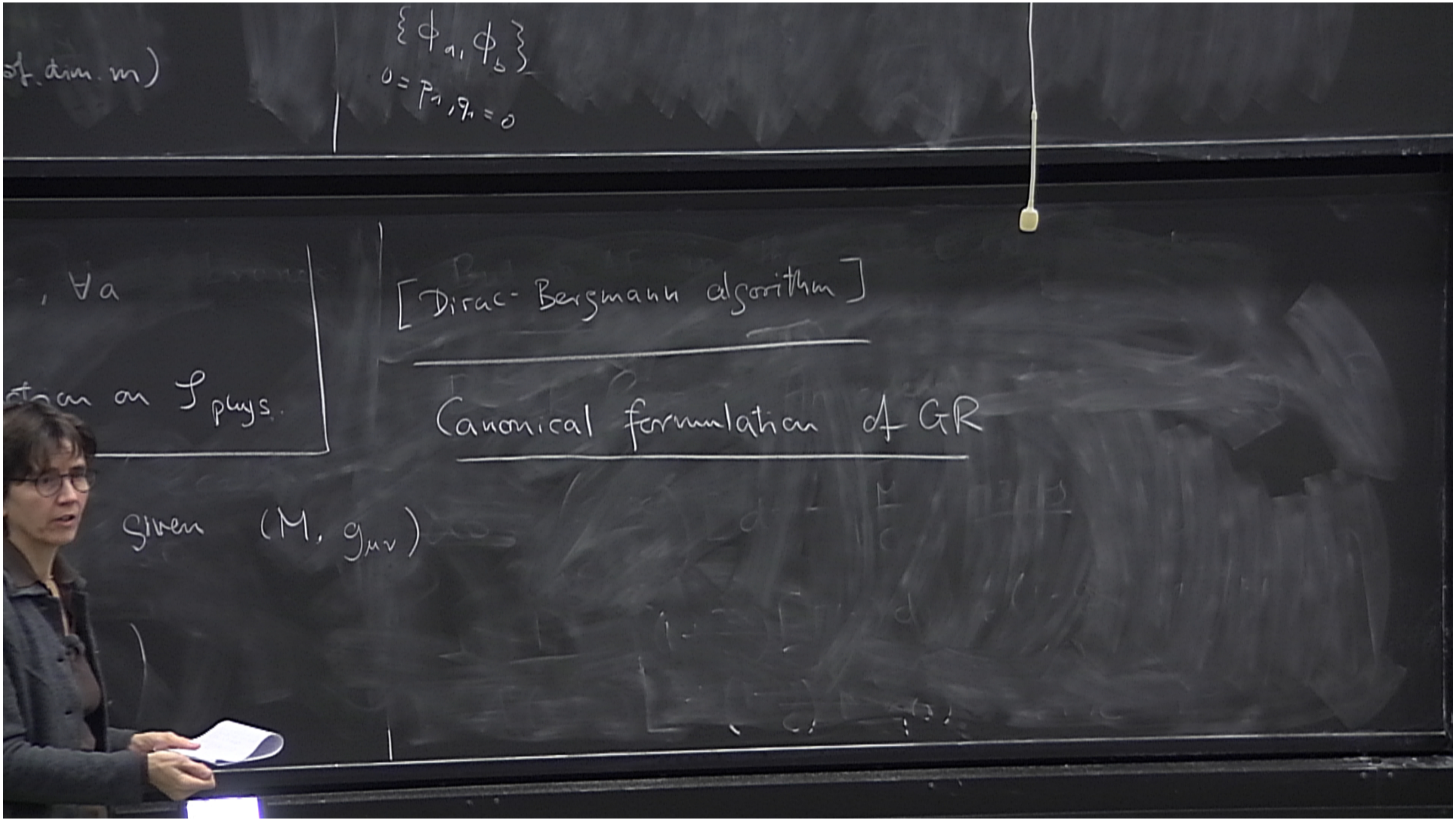
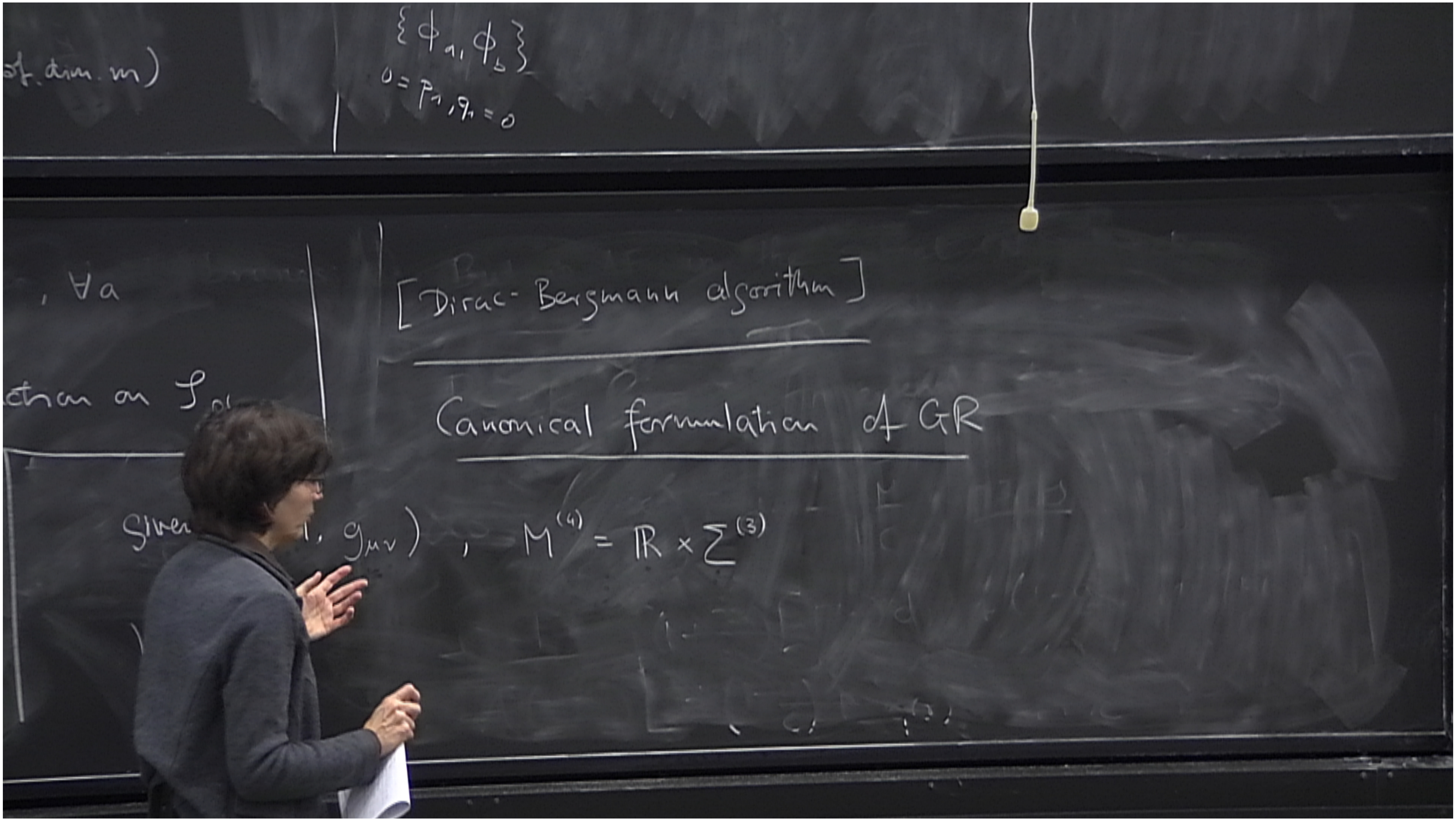
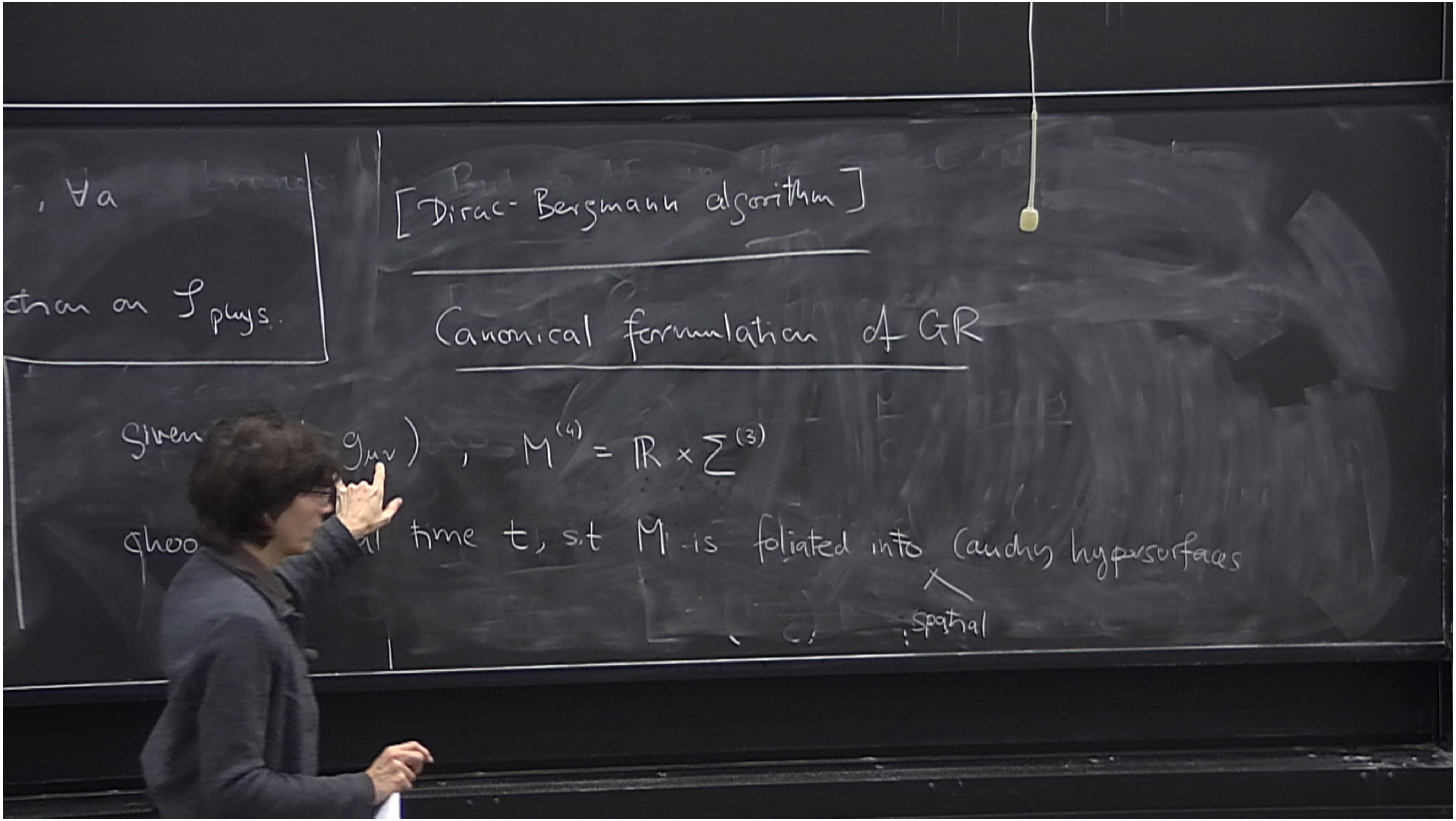
[Dirac-Bergmann algorithm]
Canonical formulation of GR


of dim. m)
{φ_a, φ_b}
0 = p_1, q_1 = 0
, ∀a
on S_phys.
[Dirac-Bergmann algorithm]
Canonical formulation of GR
given (M, g_μν)



[Dirac-Bergmann algorithm]
Canonical formulation of GR
$M^{(4)} = \mathbb{R} \times \Sigma^{(3)}$


, ∀a
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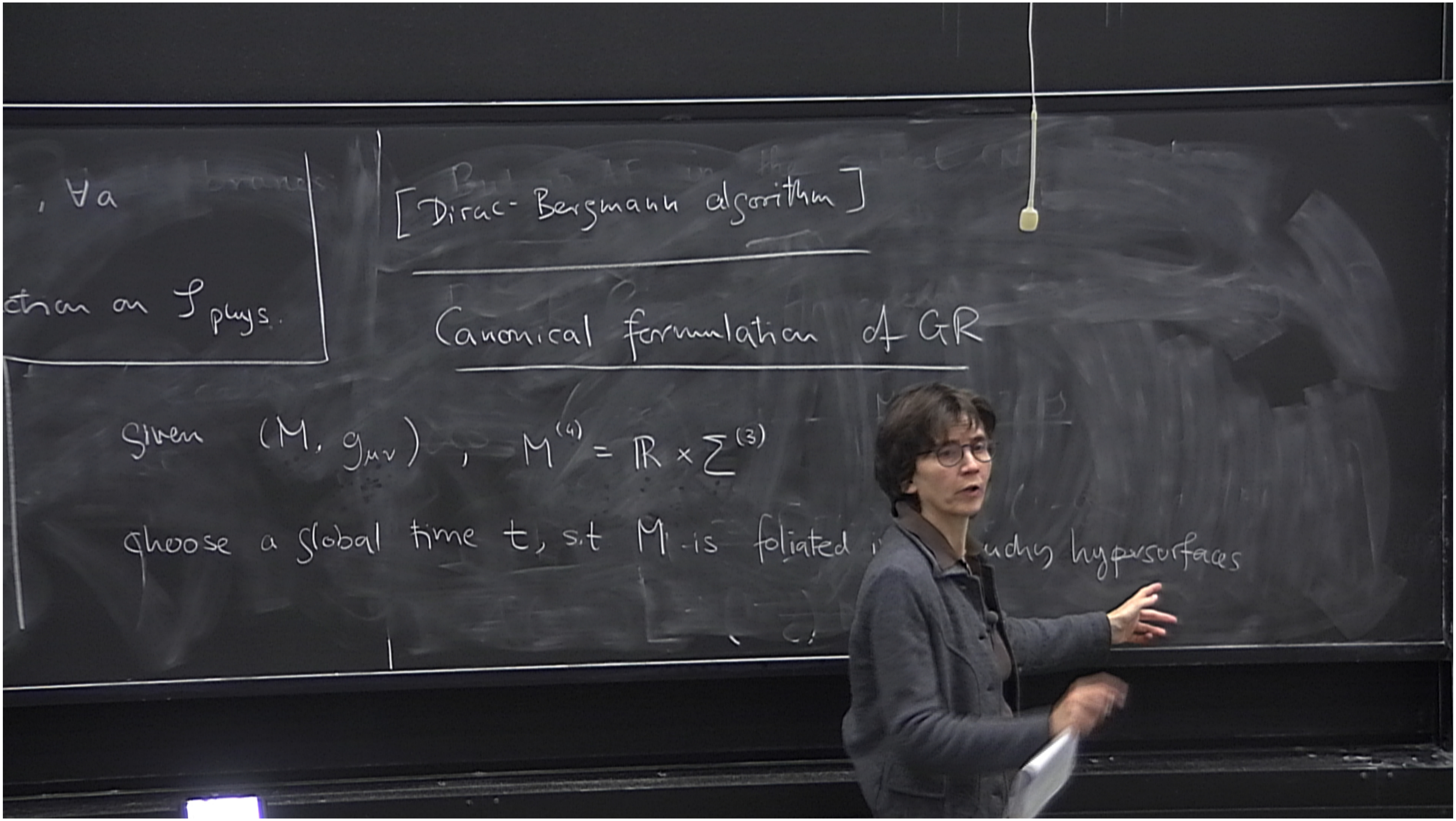
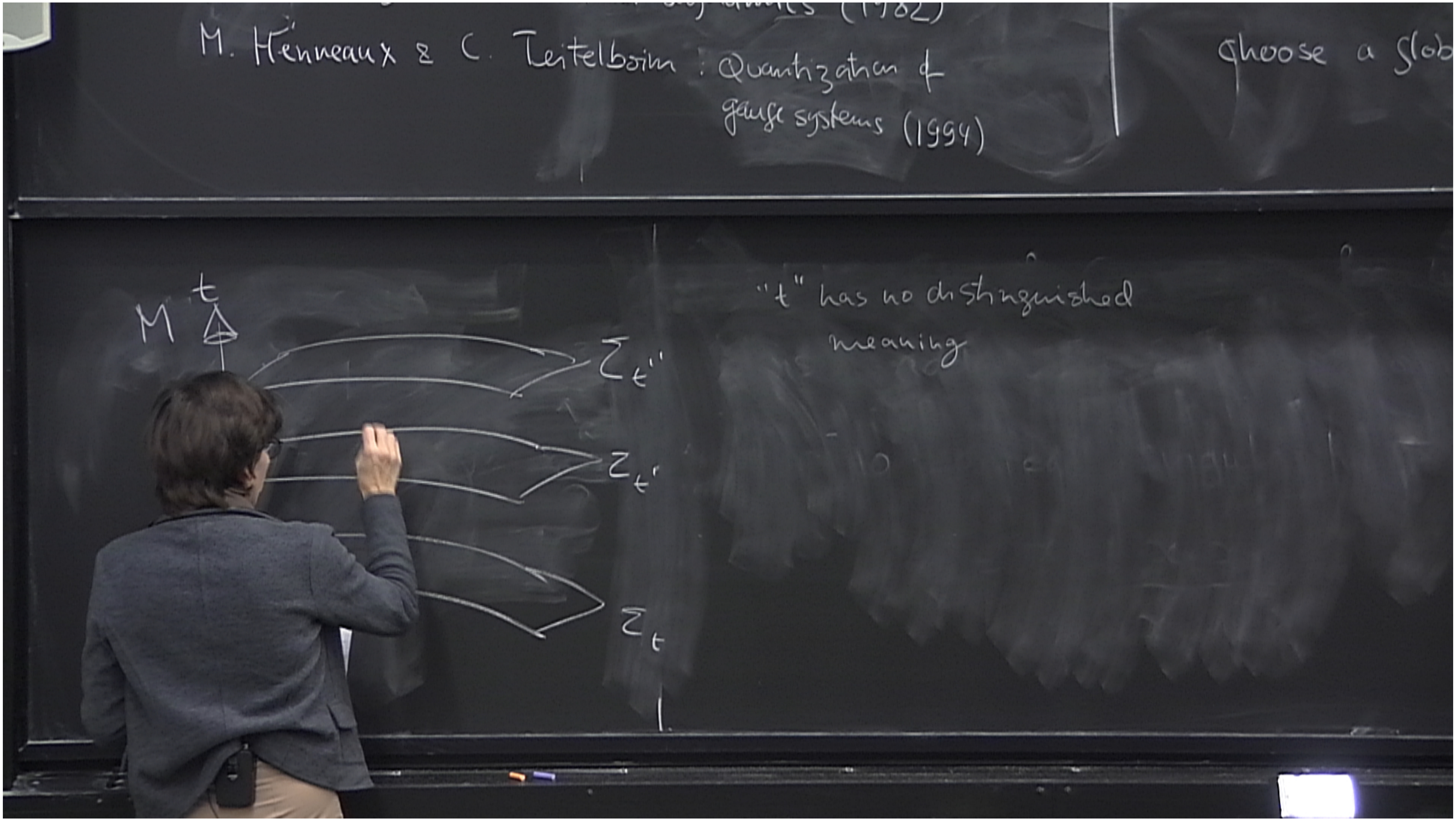
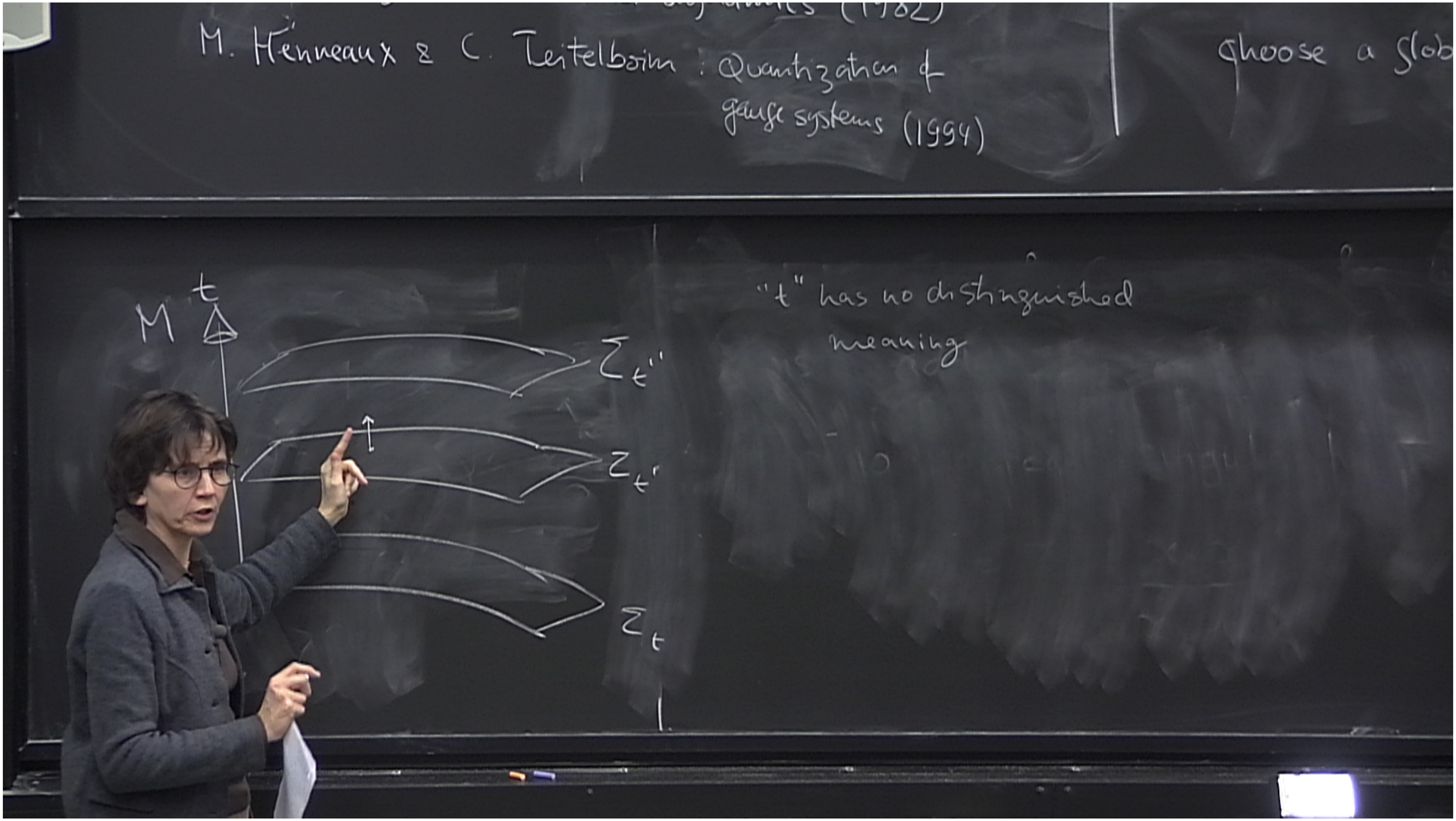
[Dirac-Bergmann algorithm]
Canonical formulation of GR
given ($g_{\mu\nu}$) , $M^{(4)} = \mathbb{R} \times \Sigma^{(3)}$
choo time t, s.t M is foliated into Cauchy hypersurfaces
spatial


[Dirac-Bergmann algorithm]
Canonical formulation of GR
given (M, $g_{\mu\nu}$) , $M^{(4)} = \mathbb{R} \times \Sigma^{(3)}$
choose a global time t, s.t M is foliated


M. Henneaux & C. Teitelboim : Quantization of gauge systems (1994)
choose a glob
M
t
Σ_{t''}
Σ_{t'}
Σ_t
"t" has no distinguished meaning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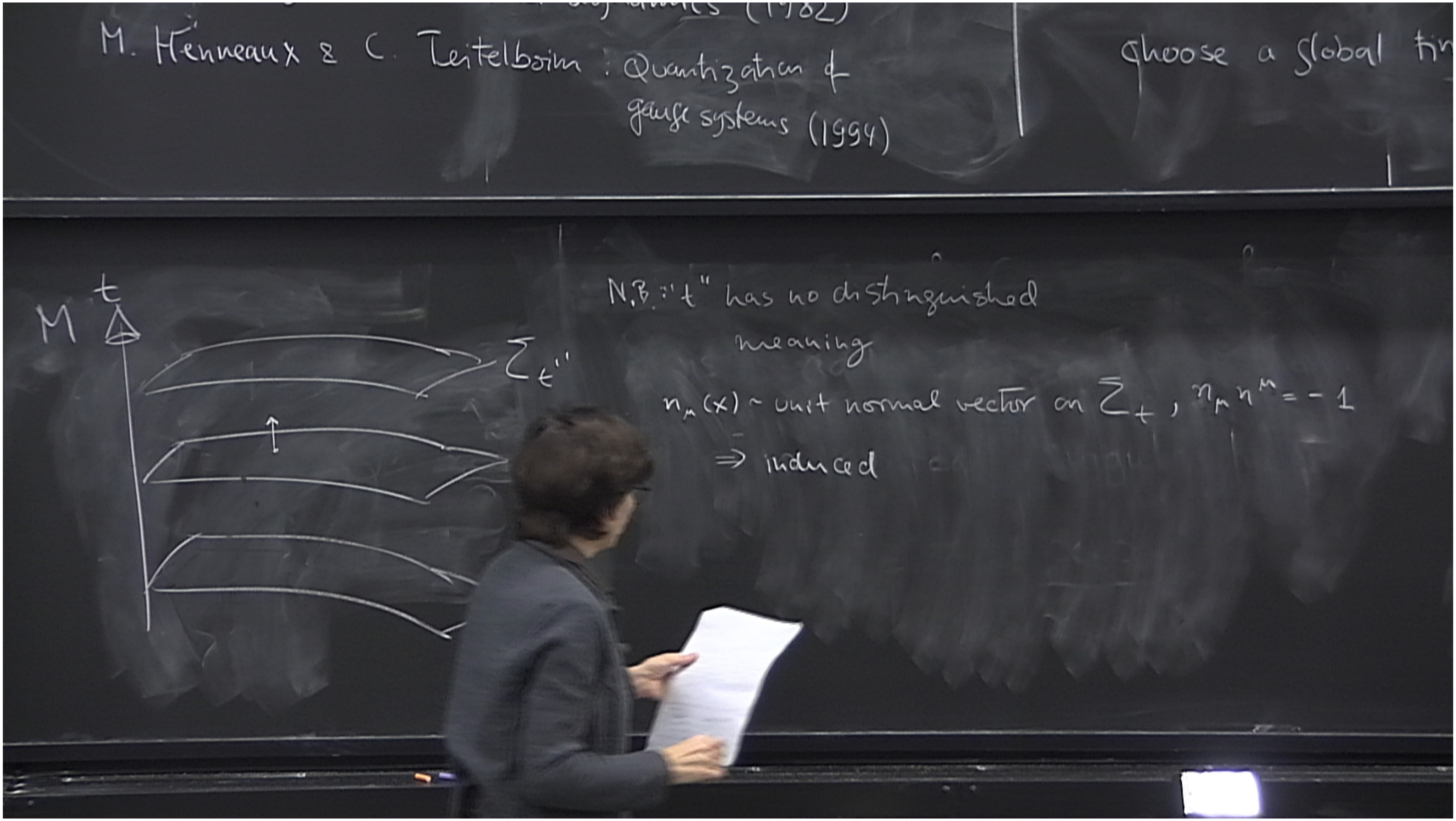
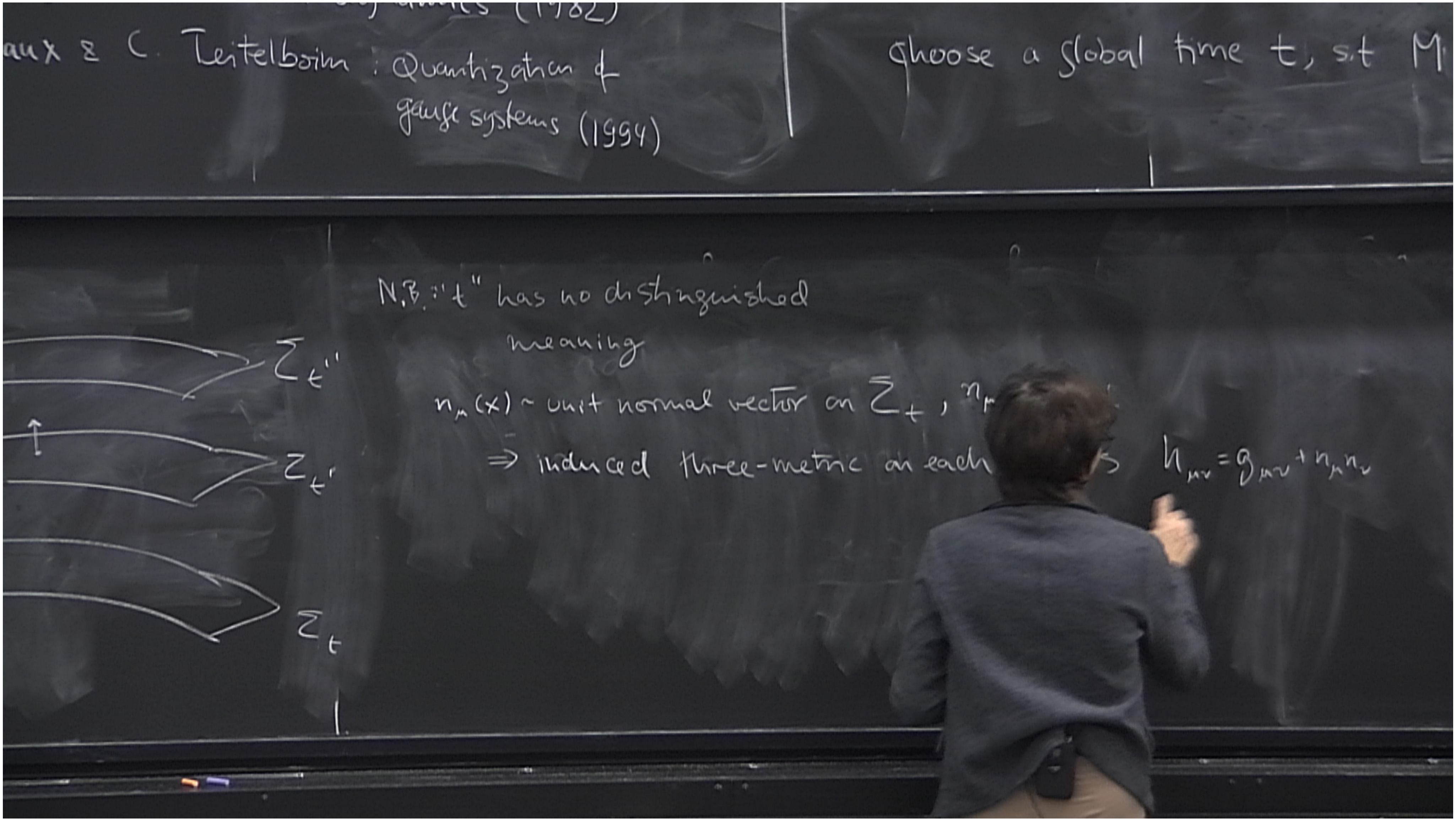
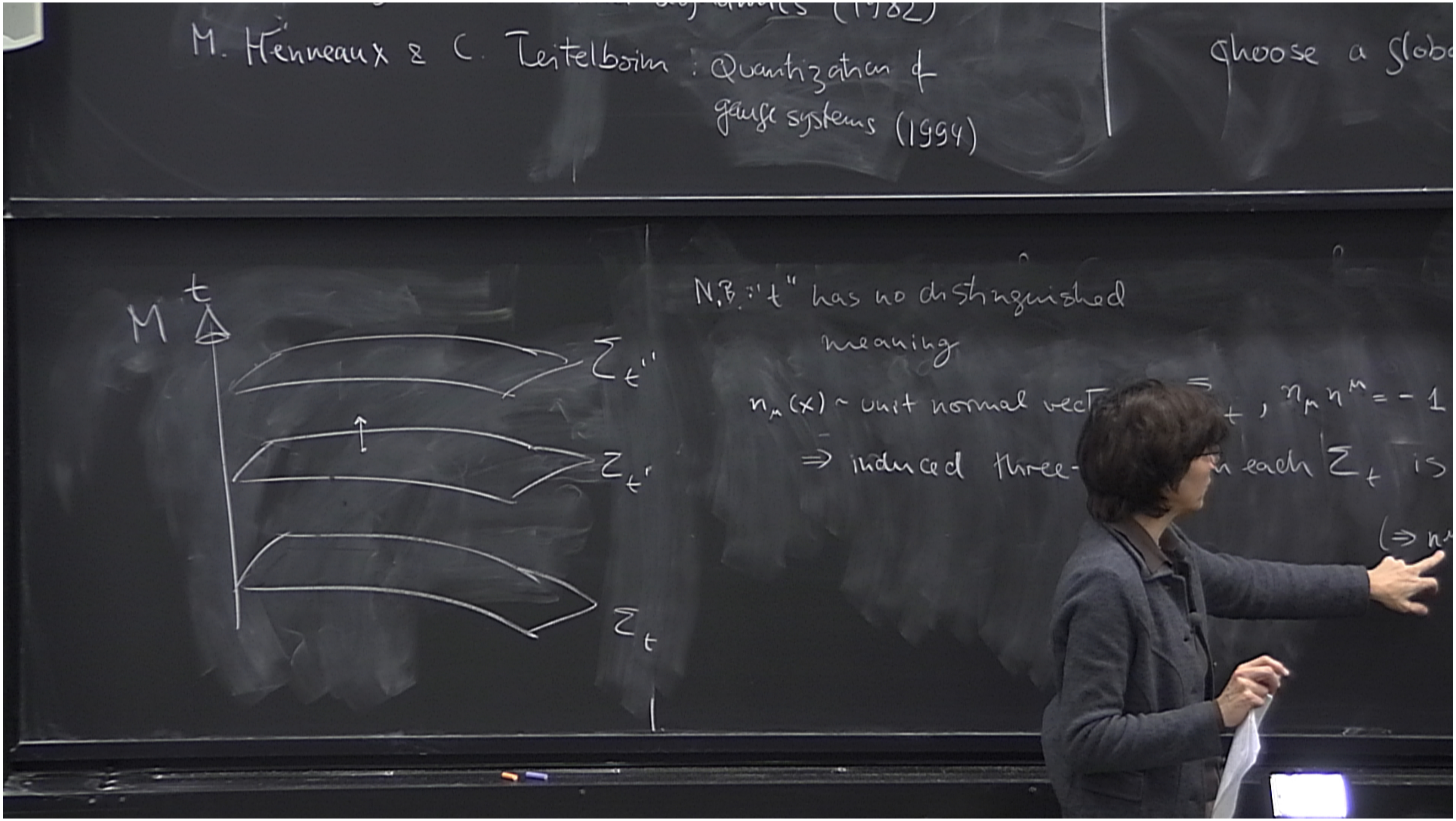

M. Henneaux & C. Teitelboim : Quantization of gauge systems (1994)
Choose a global
M
t
$\Sigma_{t''}$
N.B.: "t" has no distinguished meaning
$n_\mu(x)$ ~ unit normal vector on Σ_t , $n_\mu n^\mu = -1$
⇒ induced


aux & C. Teitelboim: Quantization of gauge systems (1994)
Choose a global time t, s.t
N.B.: "t" has no distinguished meaning
$n_\mu(x)$ ~ unit normal vector on Σ_t ,
⇒ induced three-metric on each
$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
$\Sigma_{t''}$
$\Sigma_{t'}$
Σ_t


M. Henneaux & C. Teitelboim : Quantization of gauge systems (1994)
Choose a glob
M
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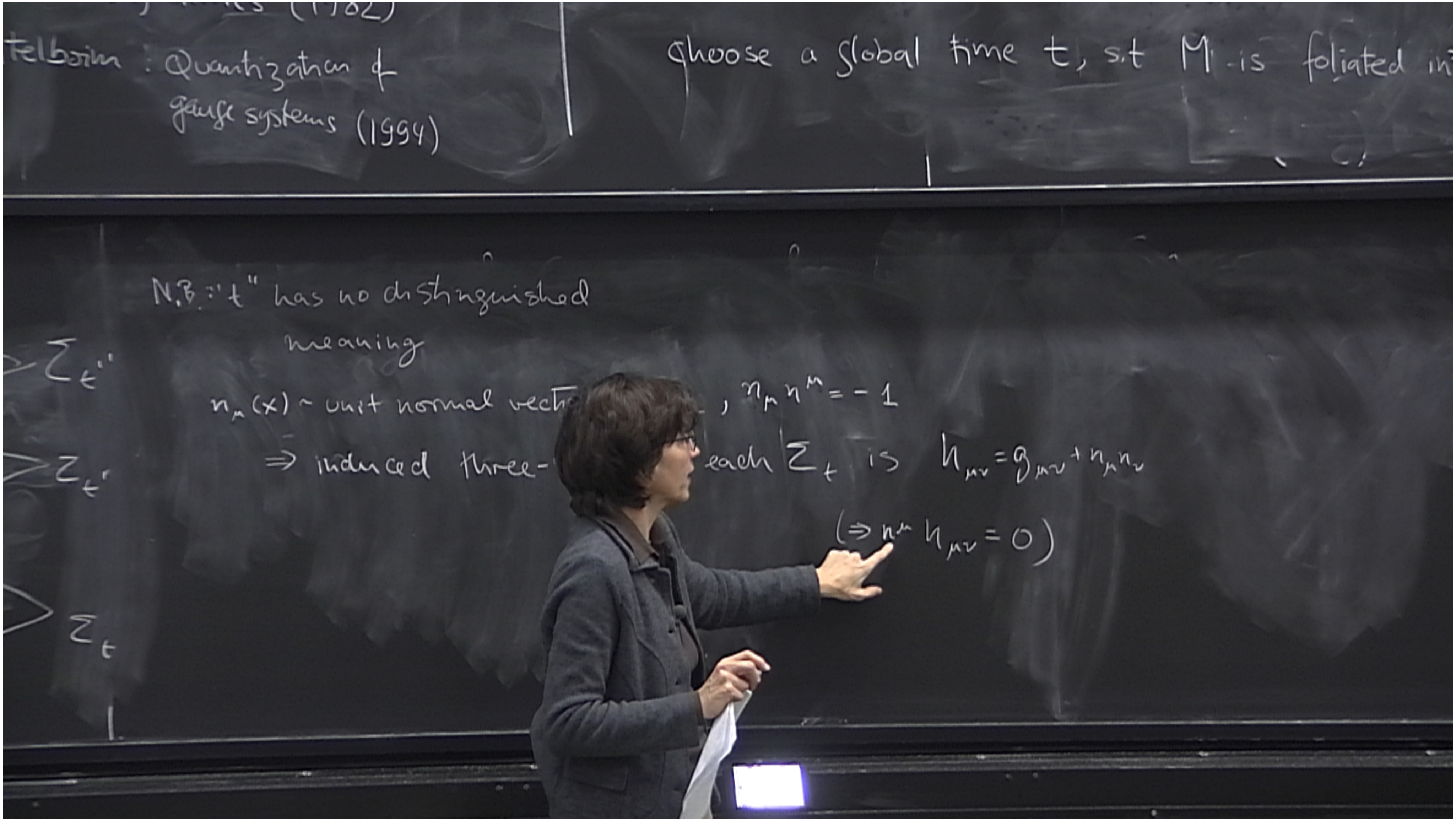
Quantization of gauge systems (1994)
Choose a global time t, s.t M is foliated in
N.B.: "t" has no distinguished meaning
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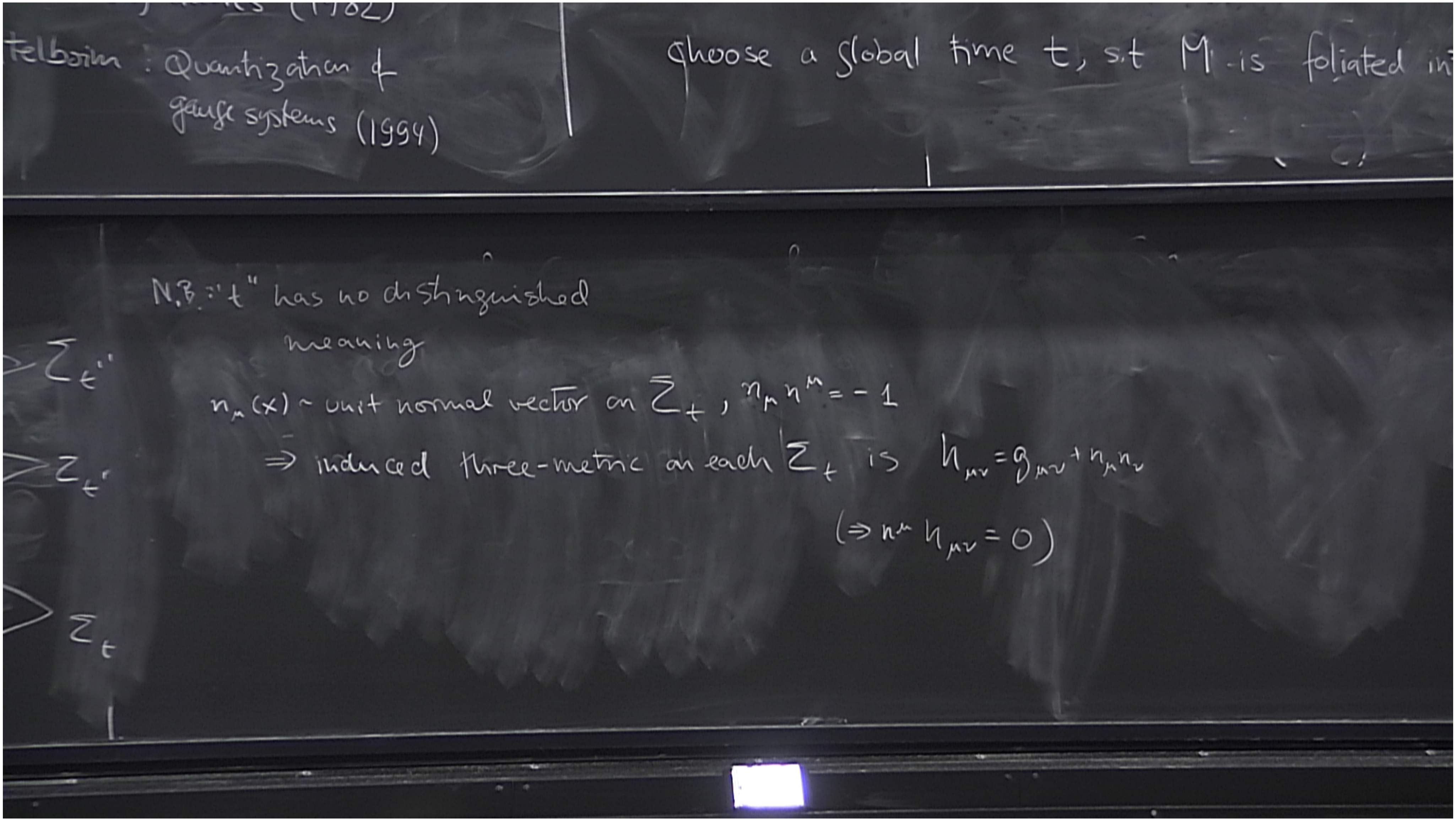
⇒ induced three-
each Σ_t is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
$(\Rightarrow n^\mu h_{\mu\nu} = 0)$
$\Sigma_{t''}$
$\Sigma_{t'}$
Σ_t


telboim : Quantization of
gauge systems (1994)
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N.B.: "t" has no distinguished
meaning
$\Sigma_{t''}$
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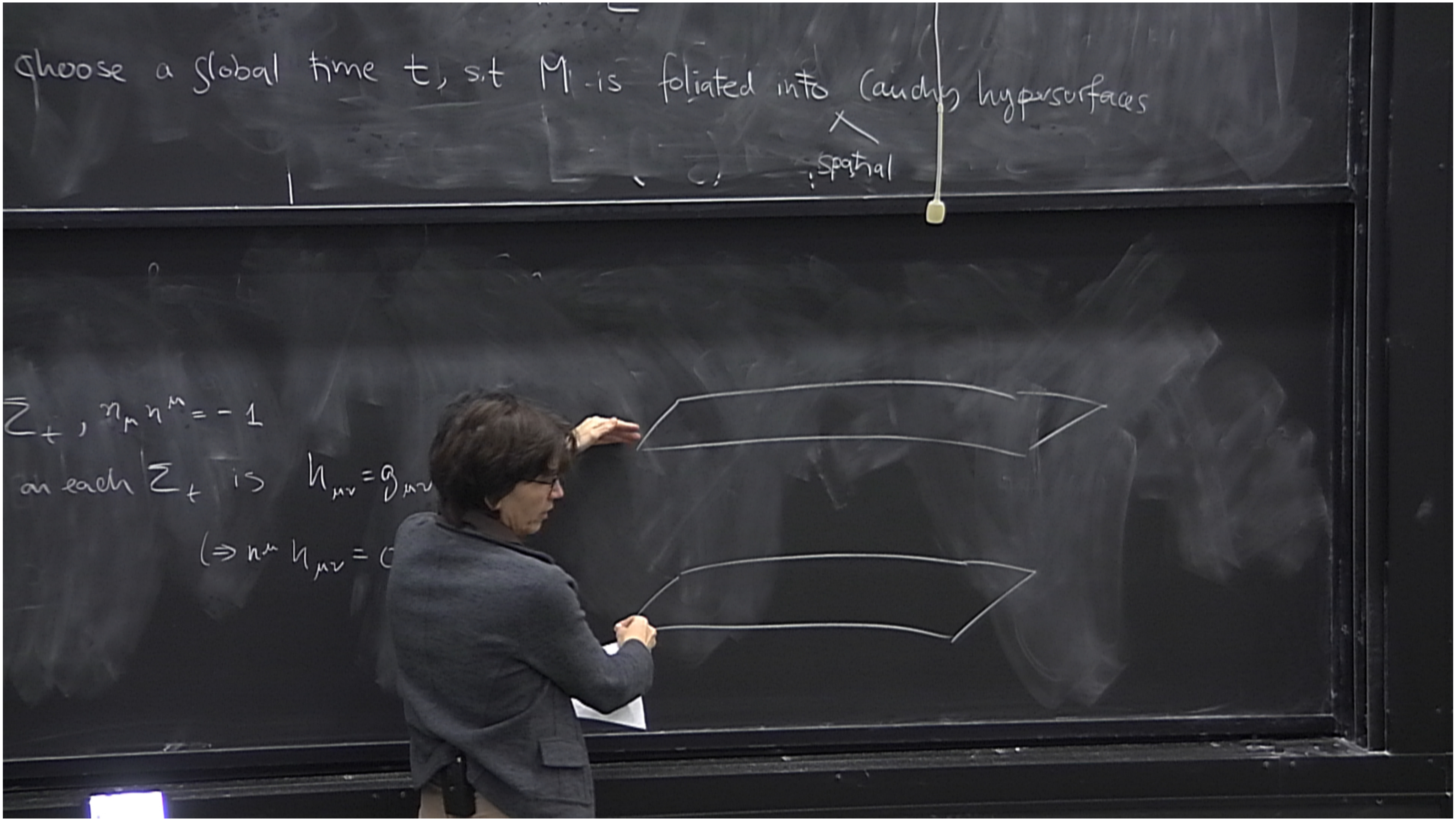
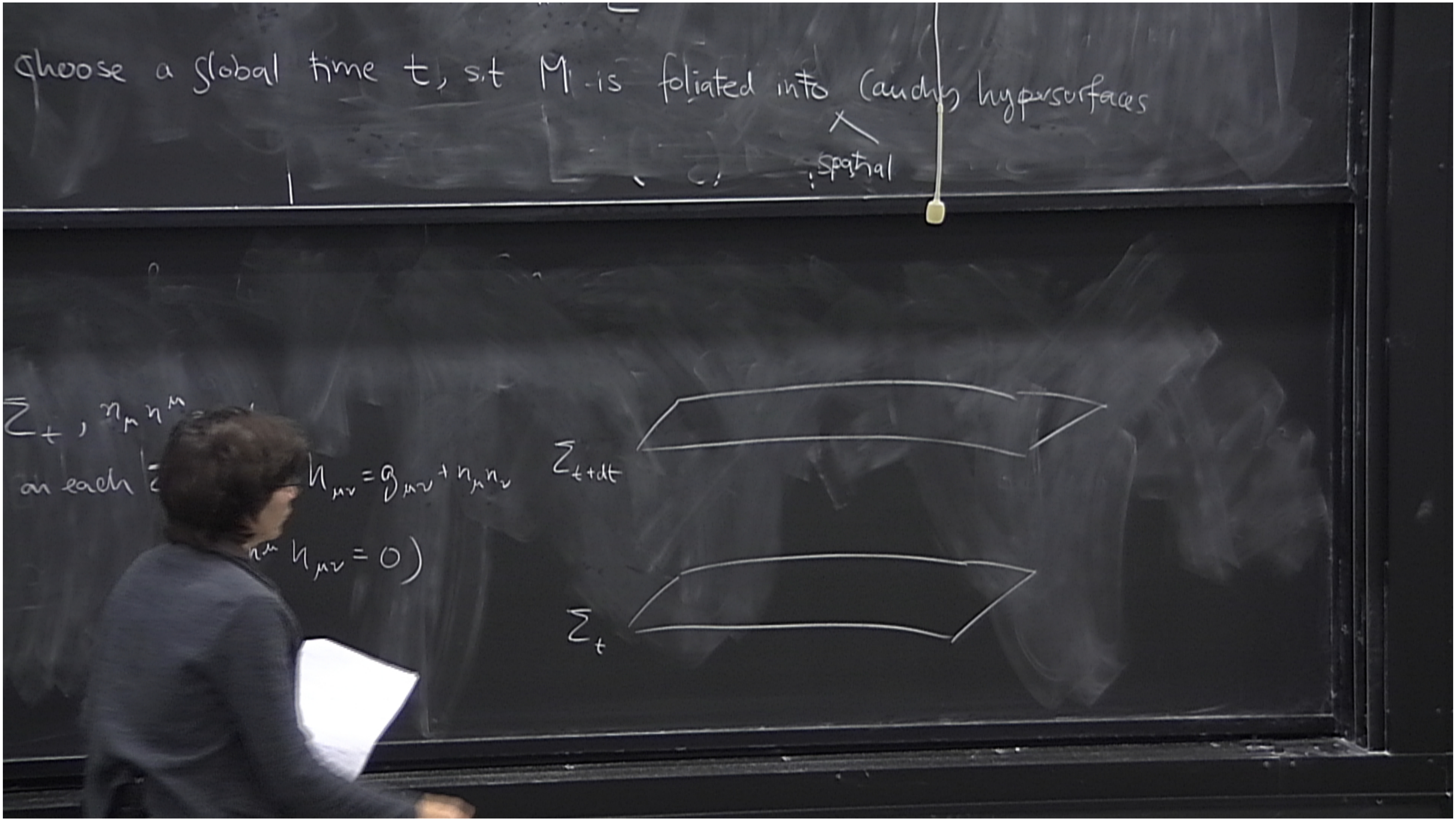
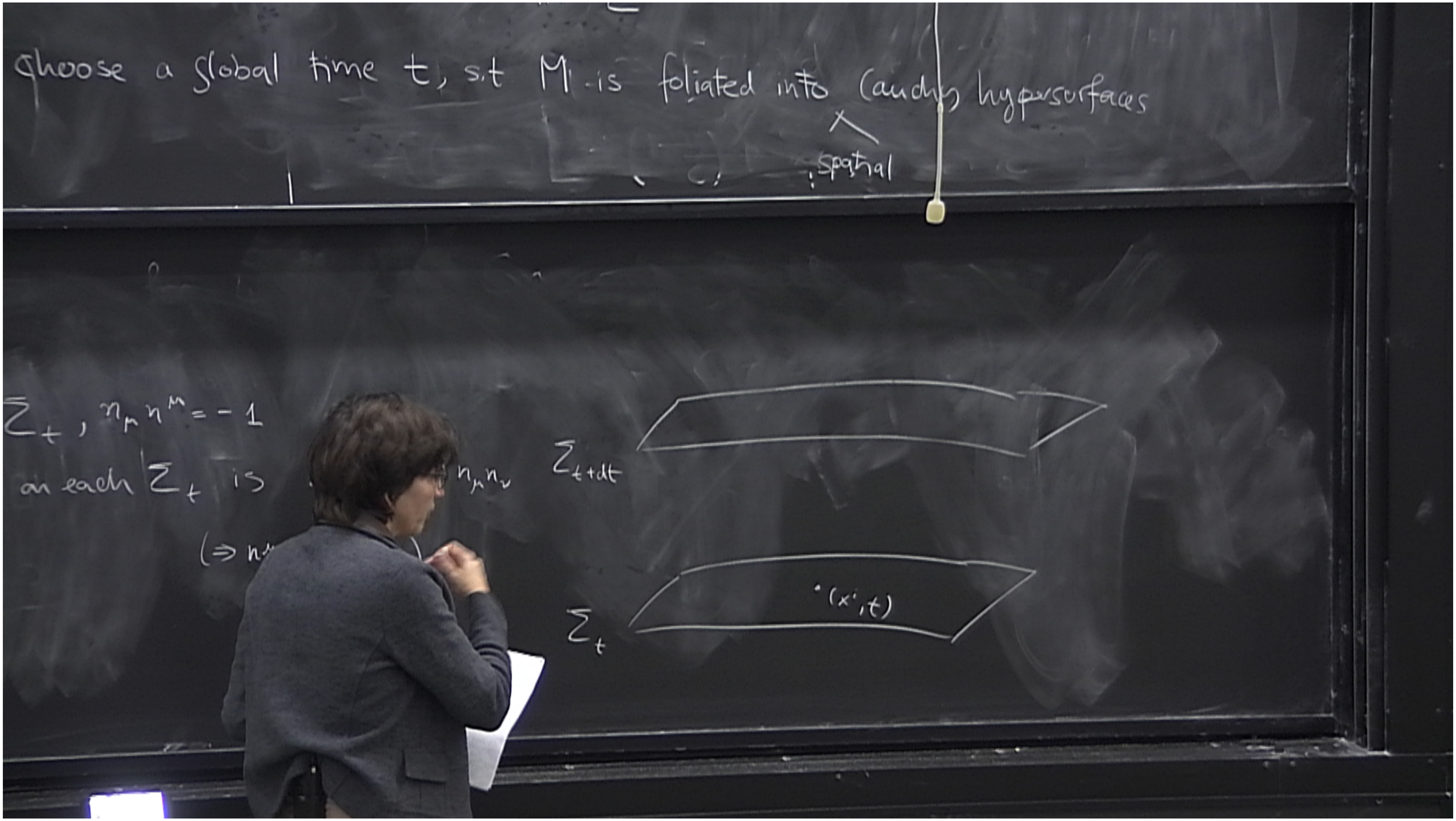
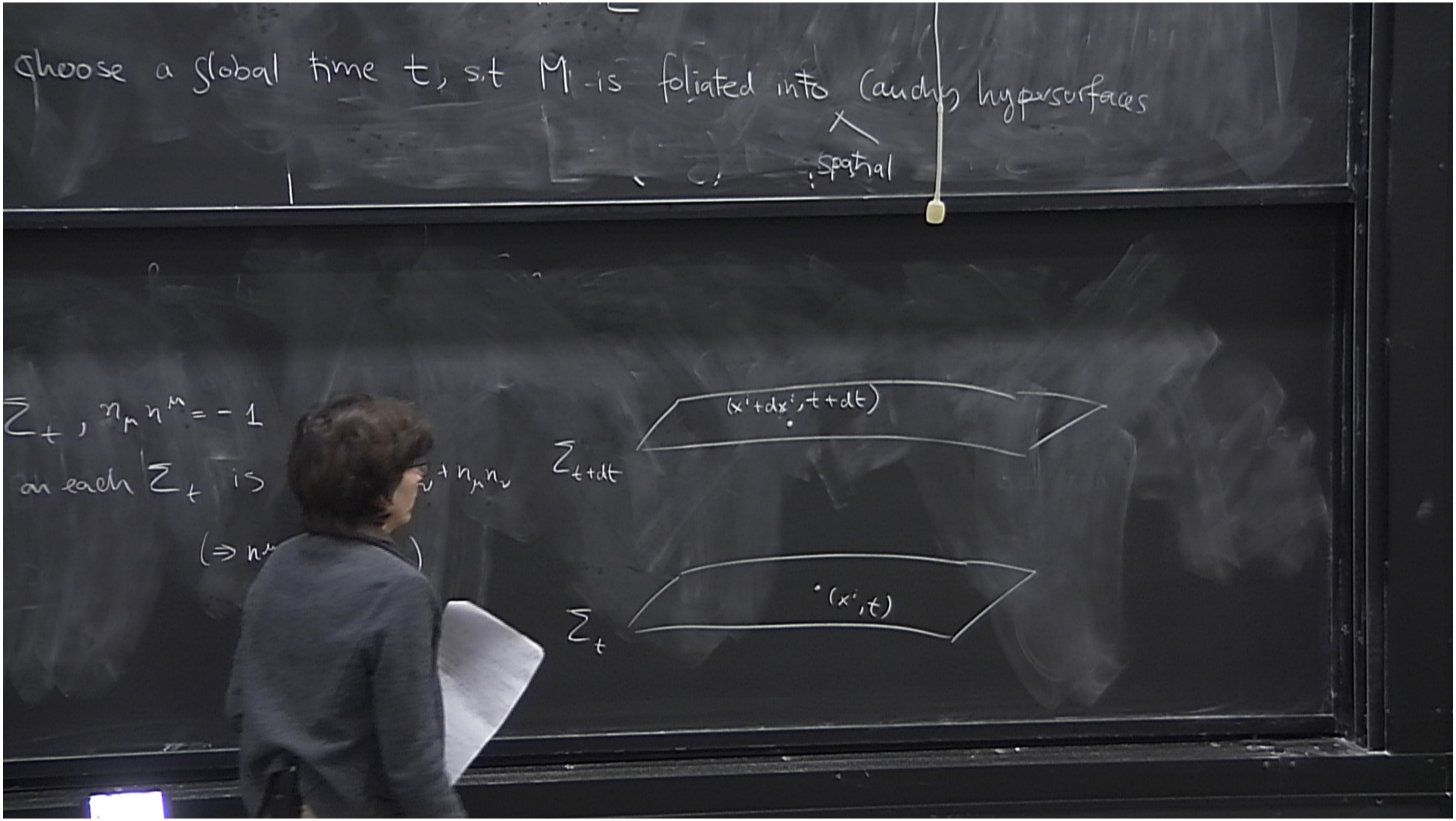
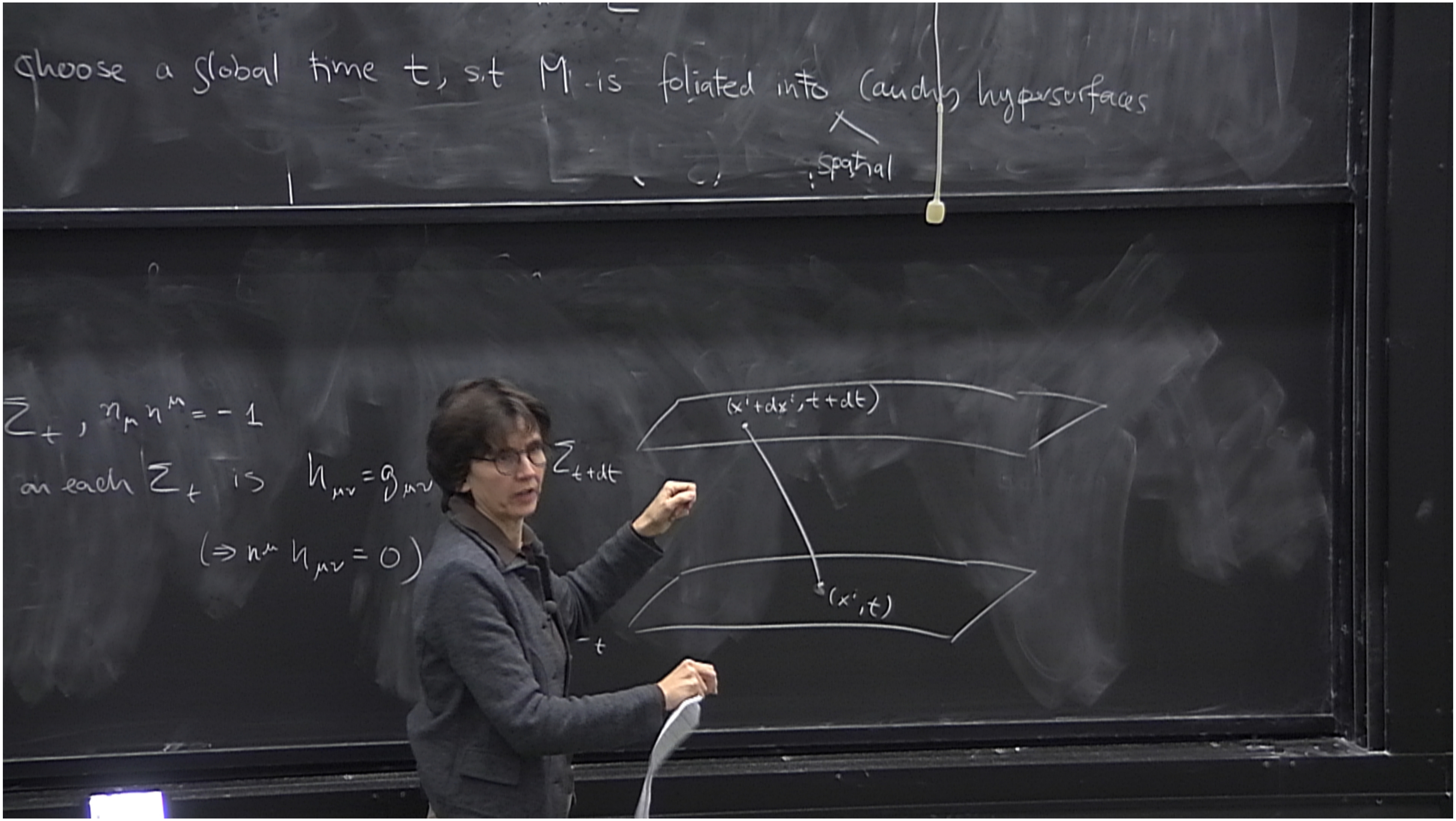
$\Sigma_{t'}$
⇒ induced three-metric on each Σ_t is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
$(\Rightarrow n^\mu h_{\mu\nu} = 0)$
Σ_t

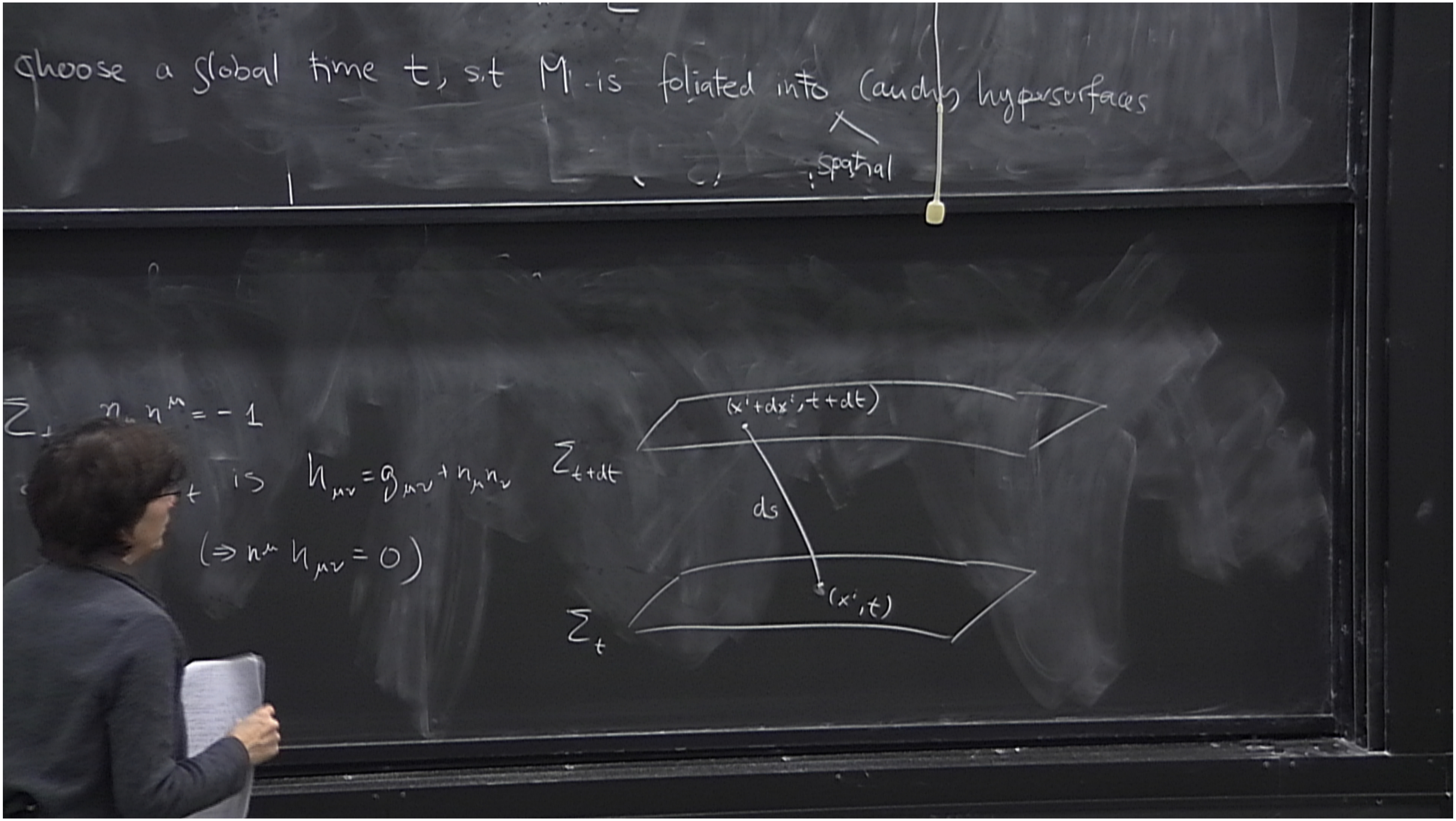

choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , n_μ n^μ = - 1
on each Σ_t is h_μν = g_μν
(⇒ n^μ h_μν =

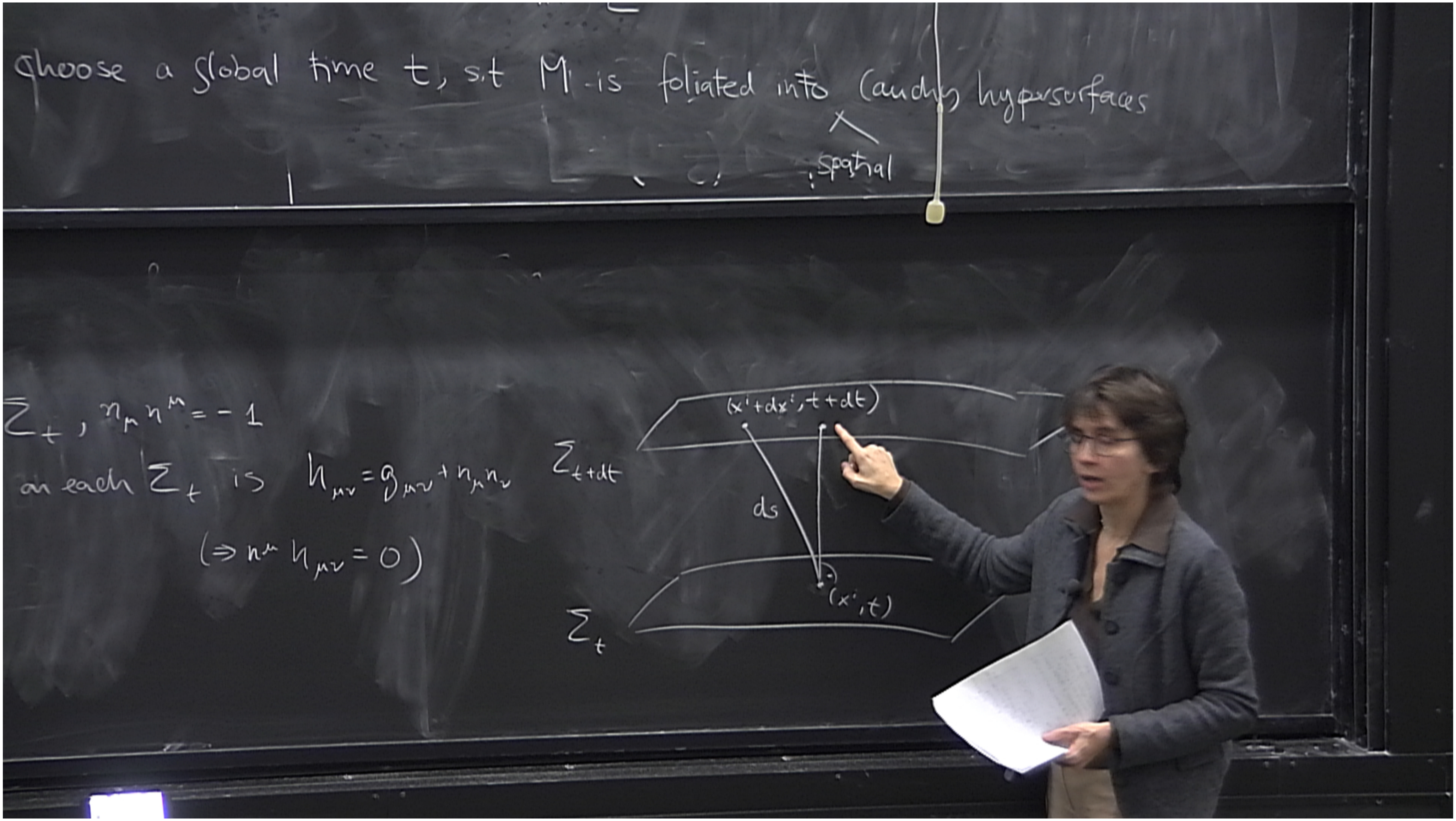

choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , $n_\mu n^\mu$
on each
$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
$n^\mu h_{\mu\nu} = 0$)
Σ_{t+dt}
Σ_t


choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , n_μ n^μ = - 1
on each Σ_t is
n_μ n_ν
Σ_t+dt
Σ_t
(x^i, t)


Choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , n_μ n^μ = - 1
on each Σ_t is
+ n_μ n_ν
(⇒
Σ_t+dt
(x^i+dx^i, t+dt)
(x^i, t)
Σ_t


choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σt , nμ n^μ = - 1
on each Σt is hμν = gμν
(⇒ n^μ hμν = 0)
Σt+dt
(x^i+dx^i, t+dt)
(x^i, t)


choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ n n^μ = - 1
t is h_μν = g_μν + n_μ n_ν
(⇒ n^μ h_μν = 0)
(x^i+dx^i, t+dt)
Σ_{t+dt}
ds
(x^i, t)
Σ_t


choose a global time t, s.t M is foliated into Cauchy hypersurfaces
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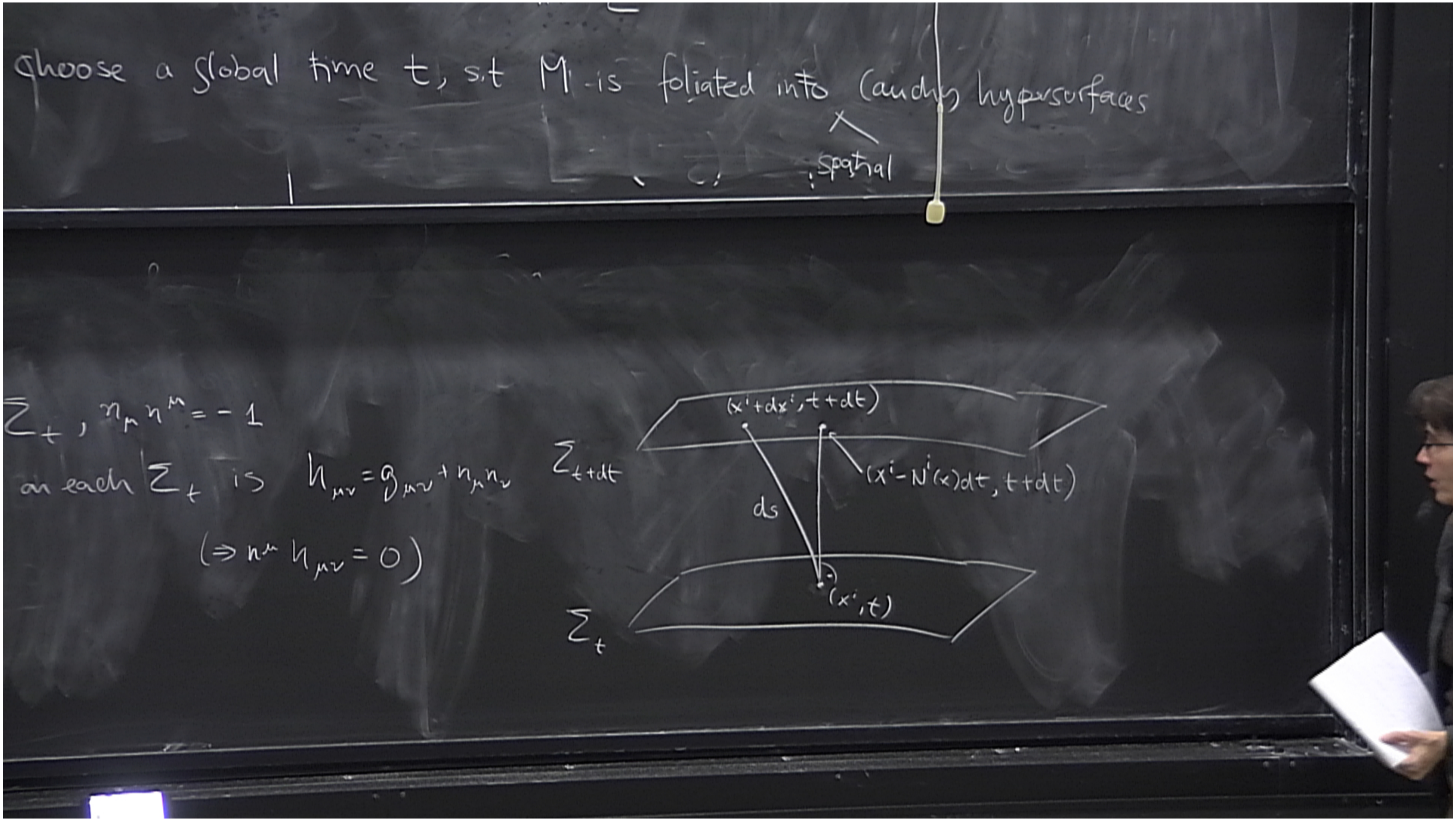
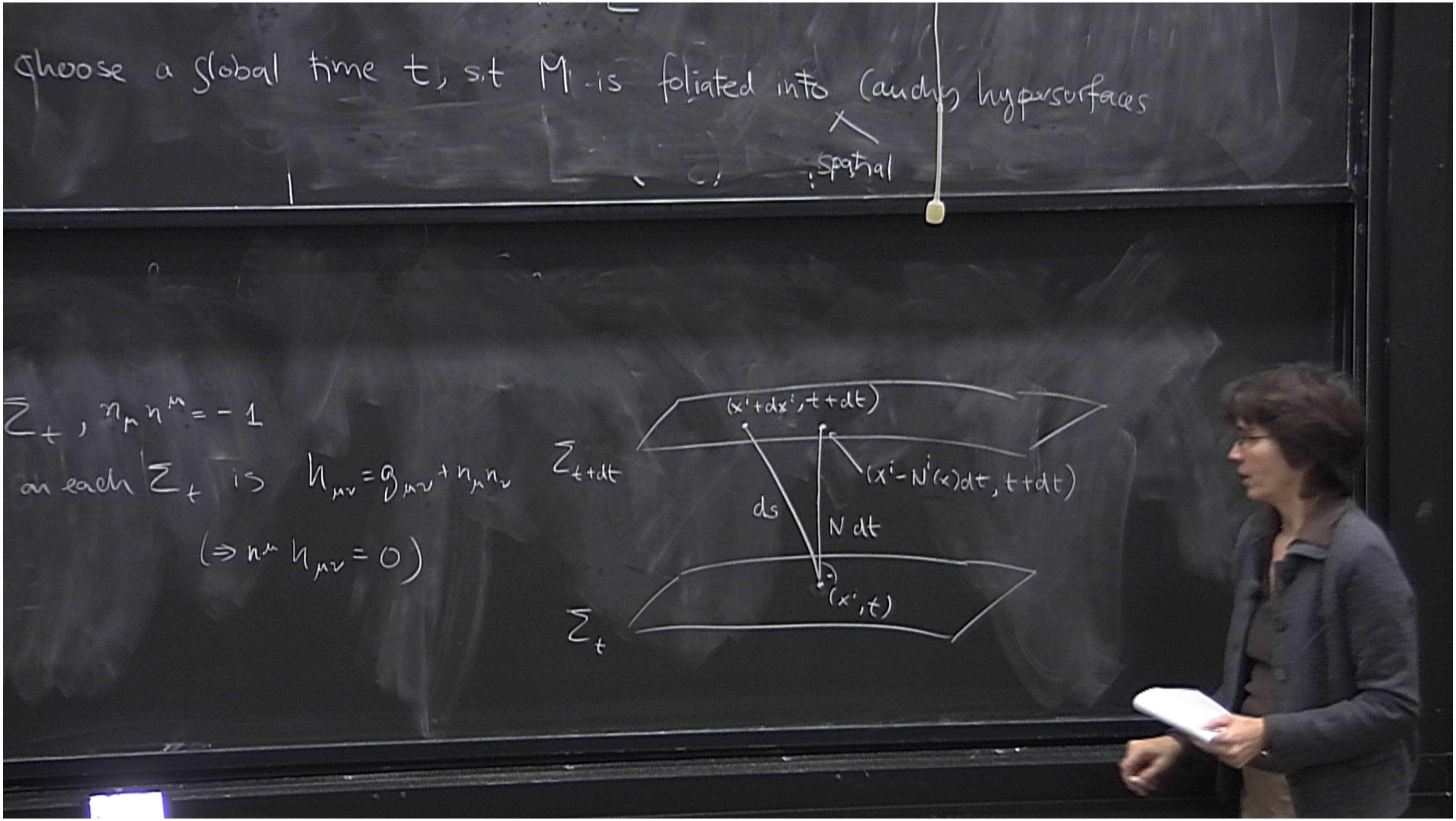
Σ_t , n_μ n^μ = - 1
on each Σ_t is h_μν = g_μν + n_μ n_ν
(⇒ n^μ h_μν = 0)
Σ_t+dt
(x^i+dx^i, t+dt)
ds
(x^i, t)
Σ_t


choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , $n_\mu n^\mu = -1$
on each Σ_t is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
$(\Rightarrow n^\mu h_{\mu\nu} = 0)$
Σ_{t+dt}
$(x^i + dx^i, t + dt)$
$(x^i - N^i(x)dt, t + dt)$
ds
(x^i, t)
Σ_t


Choose a global time t, s.t M is foliated into Cauchy hypersurfaces
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Σ_{t+dt}
$(x^i + dx^i, t+dt)$
$(x^i - N^i(x)dt, t+dt)$
ds
N dt
(x^i, t)
Σ_t

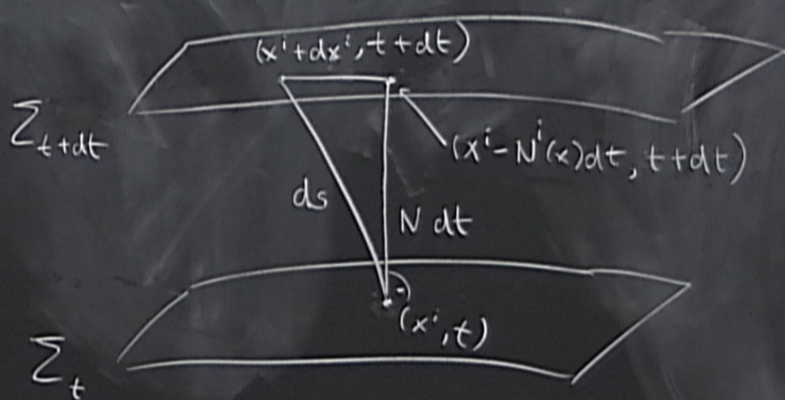
choose a global time t, s.t M is foliated into Cauchy hypersurfaces

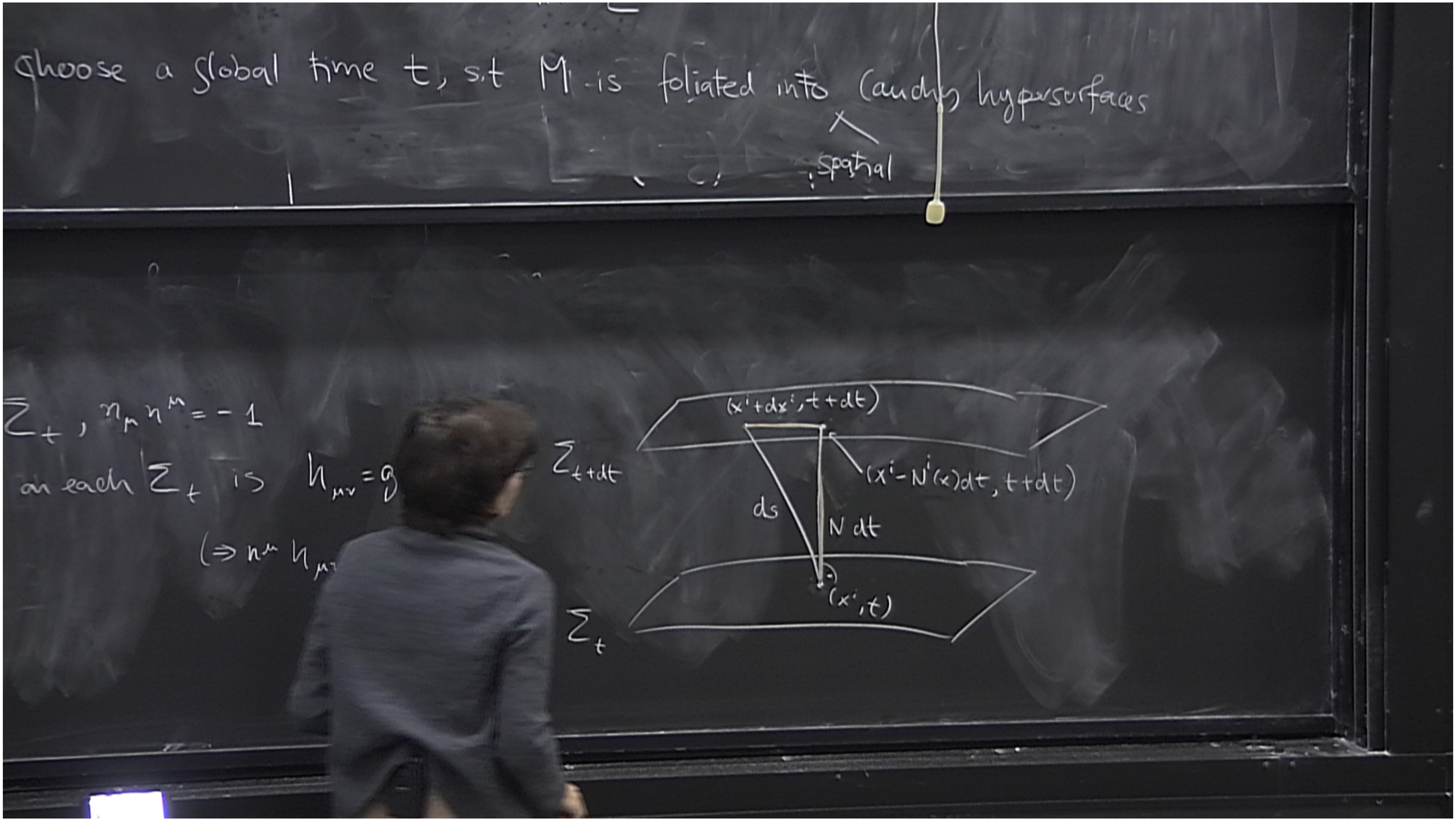
spatial

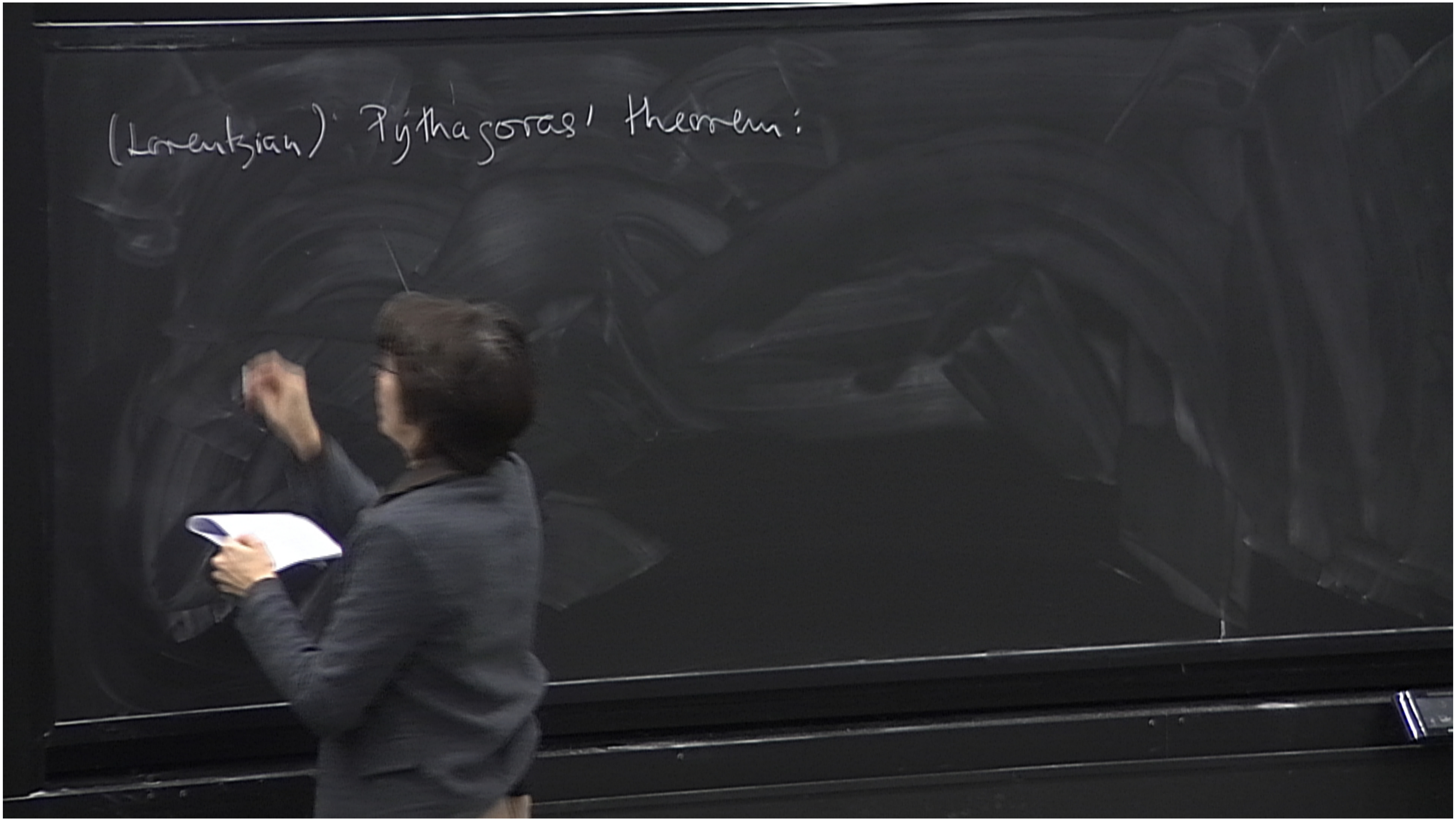
Σ_t, $n_\mu n^\mu = -1$

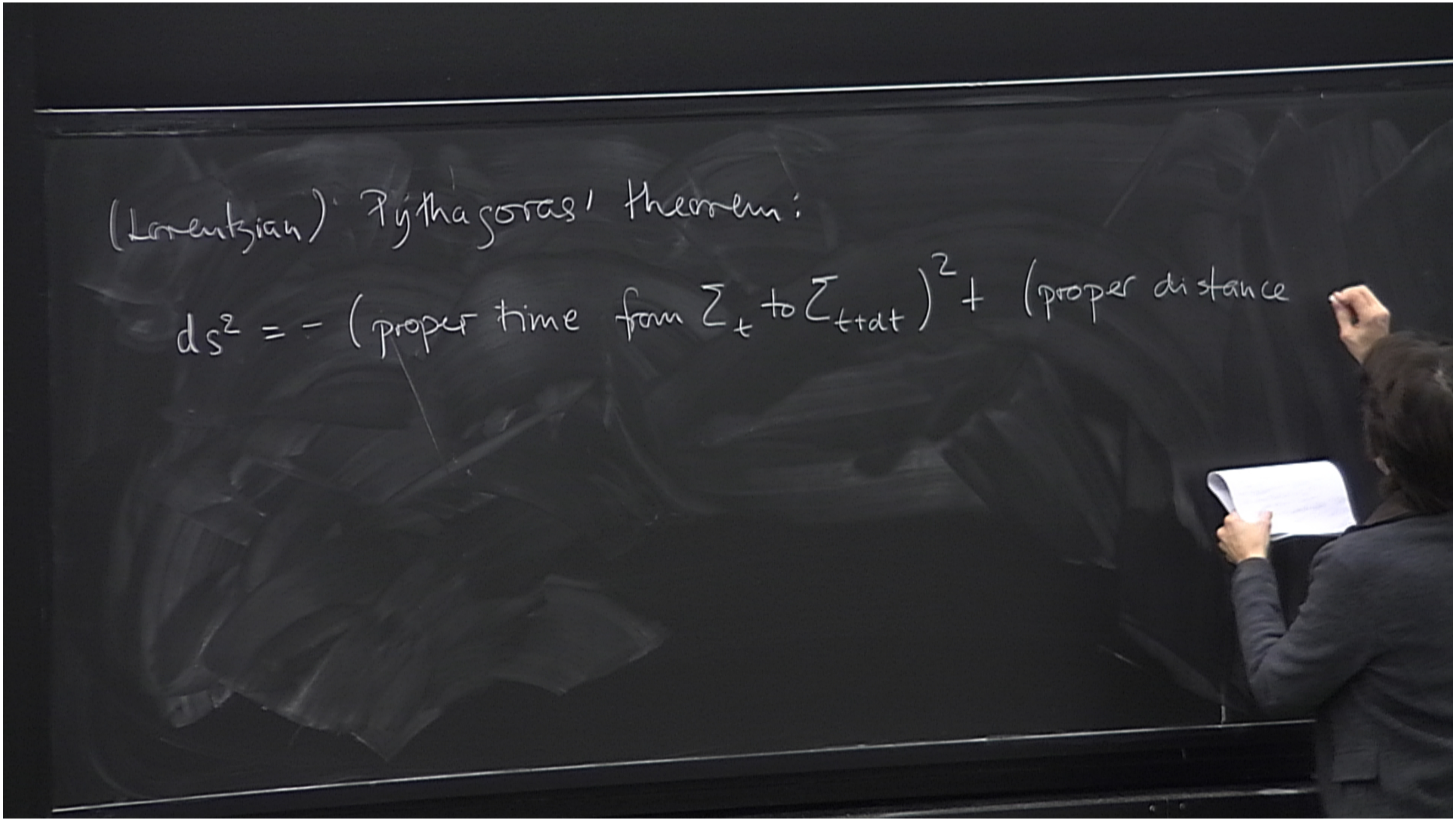
... Σ_t is $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

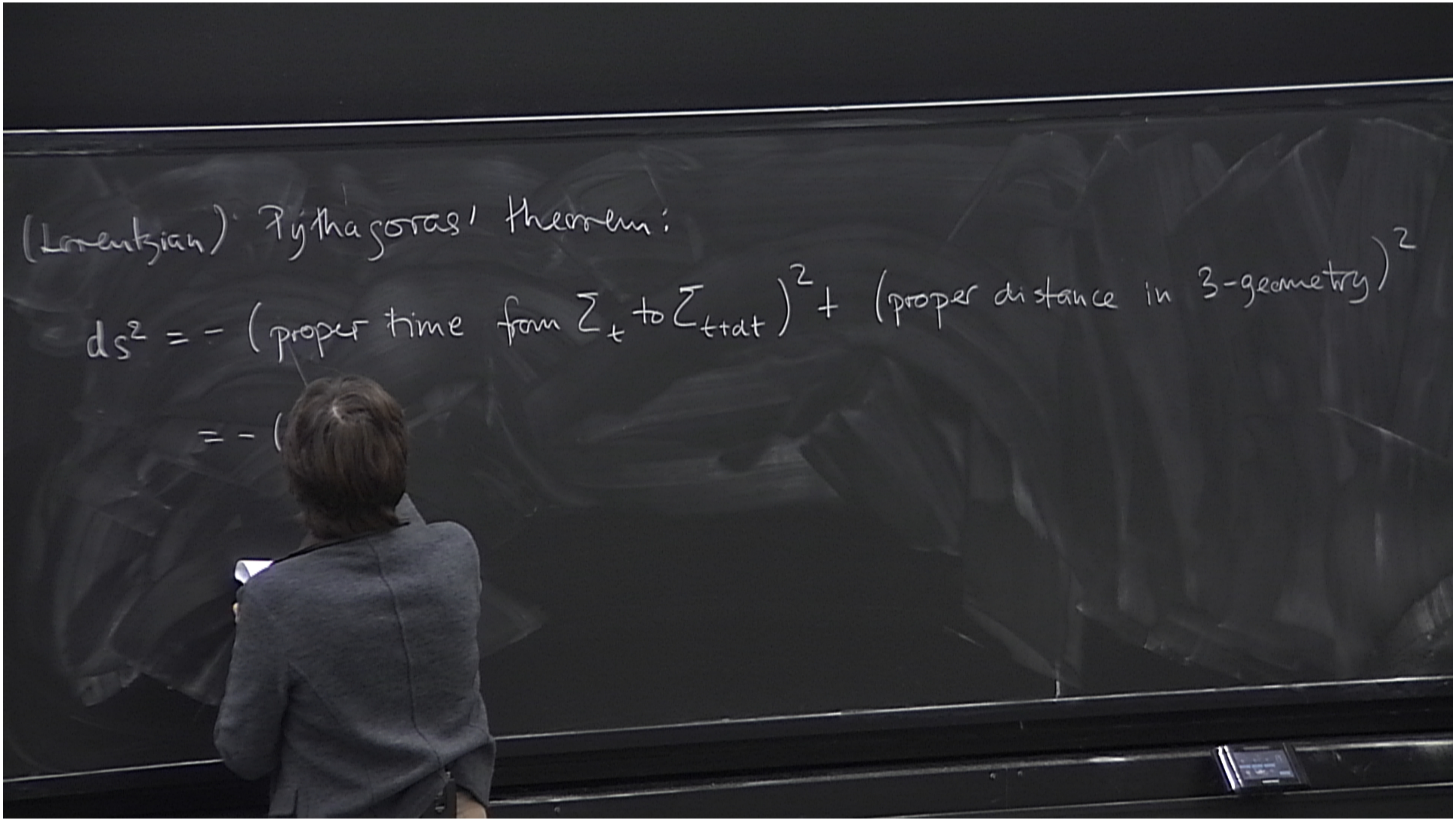
$(\Rightarrow n^\mu h_{\mu\nu} = 0)$

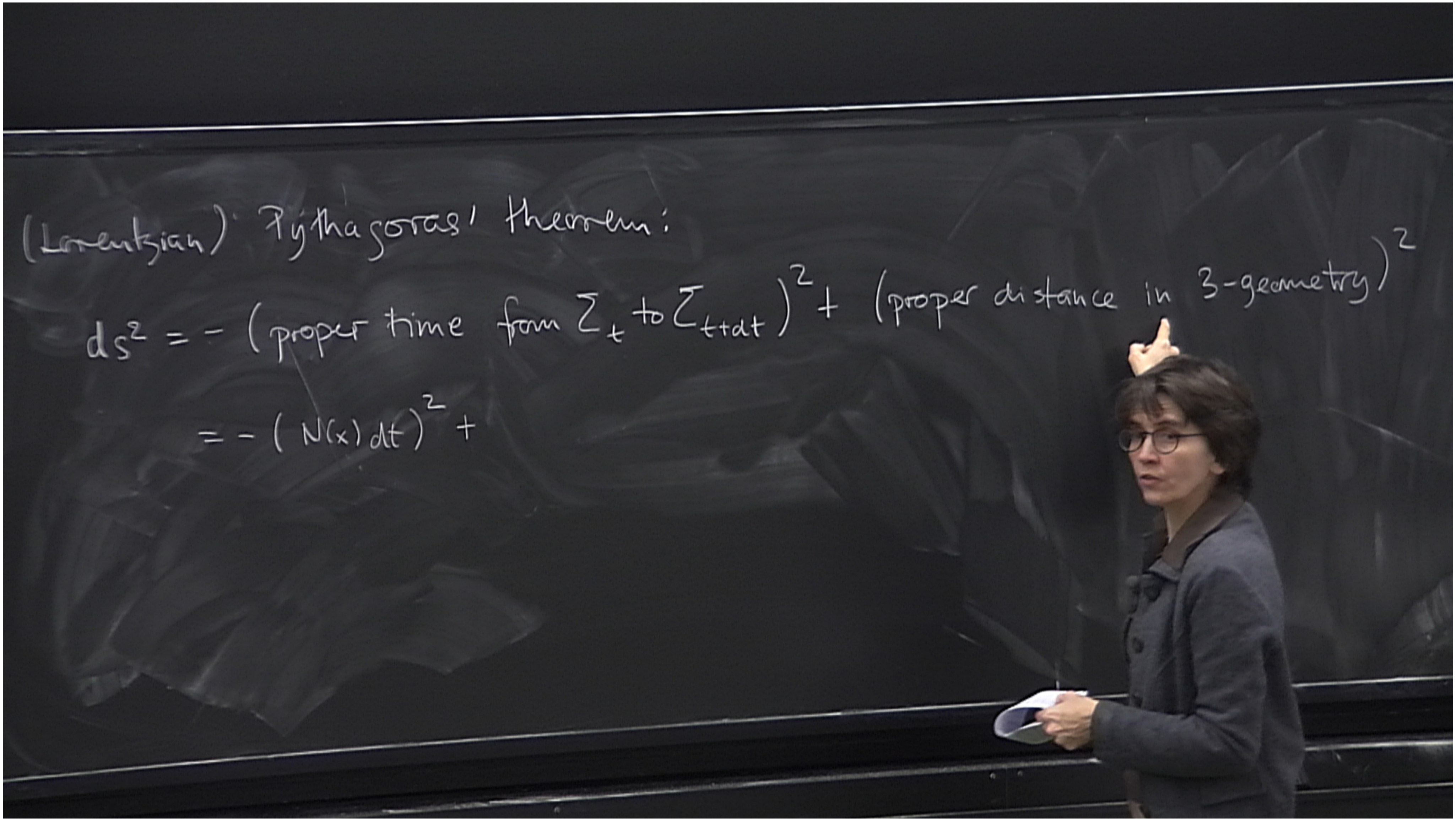


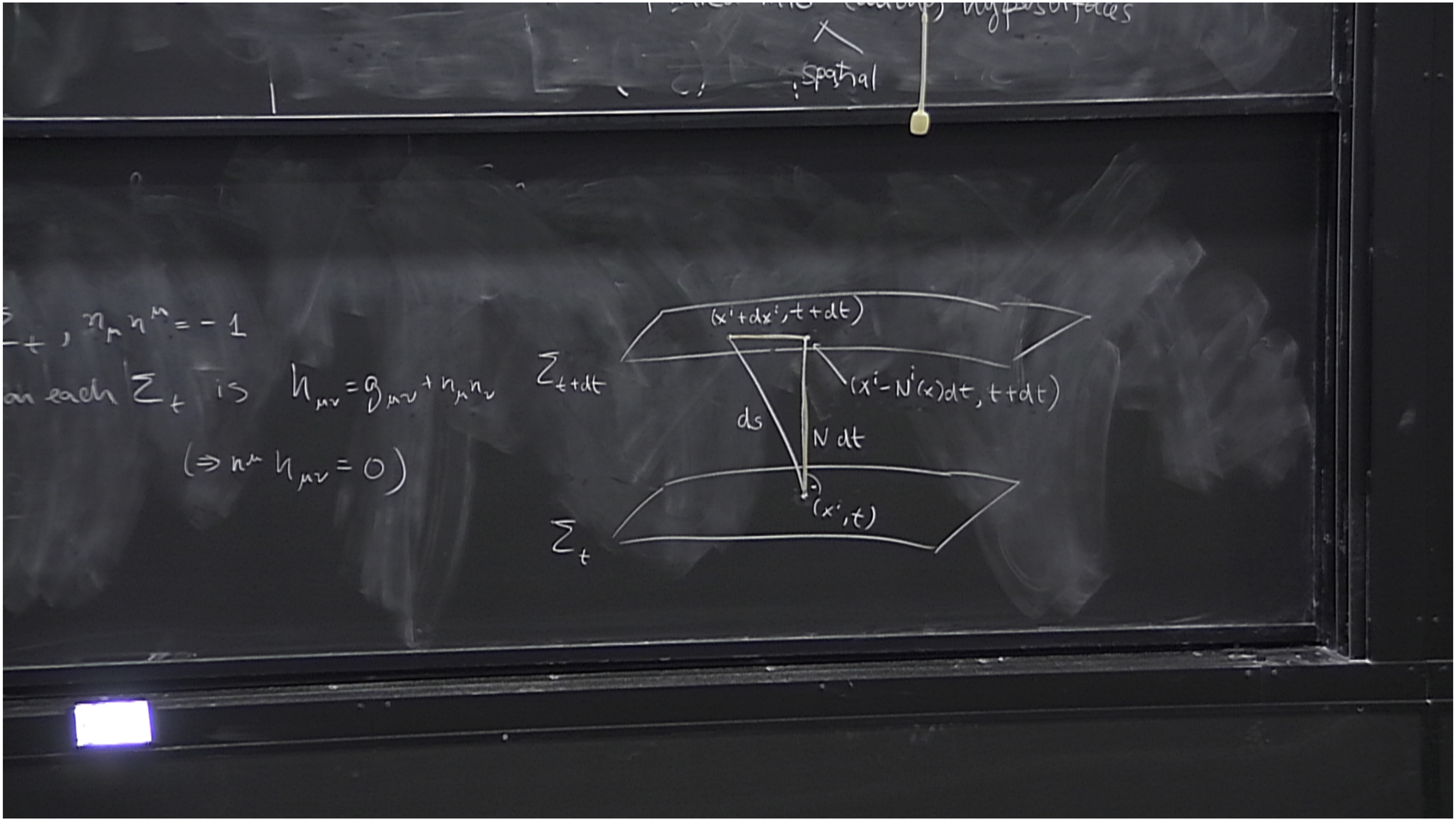

choose a global time t, s.t M is foliated into Cauchy hypersurfaces
spatial
Σ_t , n_μ n^μ = - 1
on each Σ_t is h_μν = g
(⇒ n^μ h_μ
Σ_t+dt
Σ_t
(x^i+dx^i, t+dt)
(x^i-N^i(x)dt, t+dt)
ds
N dt
(x^i, t)


(Lorentzian) Pythagoras' theorem:


(Lorentzian) Pythagoras' theorem:
ds² = − (proper time from Σt to Σt+dt)² + (proper distance


(Lorentzian) Pythagoras' theorem:
$ds^2 = -(\text{proper time from } \Sigma_t \text{ to } \Sigma_{t+dt})^2 + (\text{proper distance in 3-geometry})^2$
$= -($


(Lorentzian) Pythagoras' theorem:
$ds^2 = -(\text{proper time from } \Sigma_t \text{ to } \Sigma_{t+dt})^2 + (\text{proper distance in 3-geometry})^2$
$= -(N(x)\,dt)^2 +$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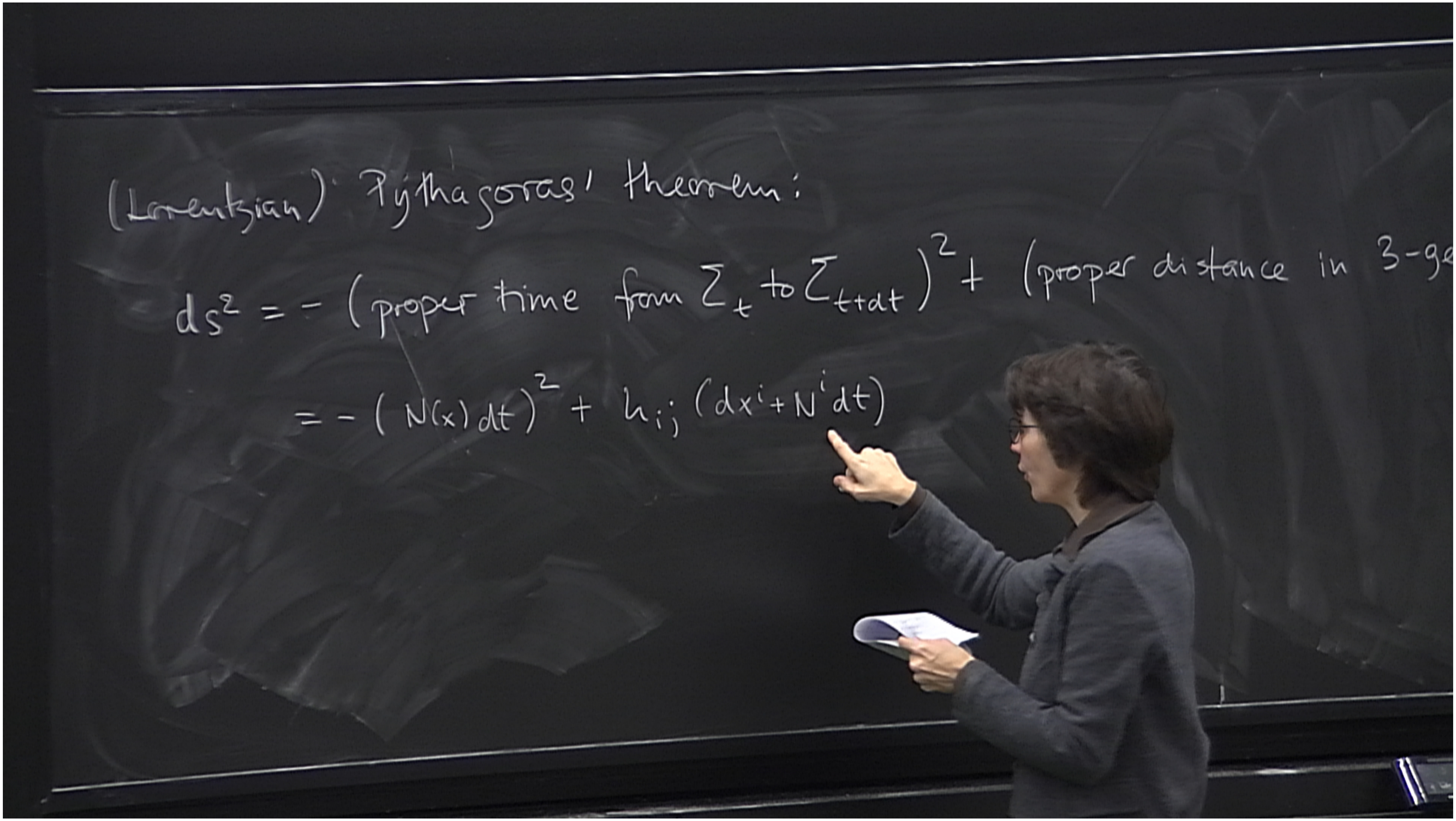
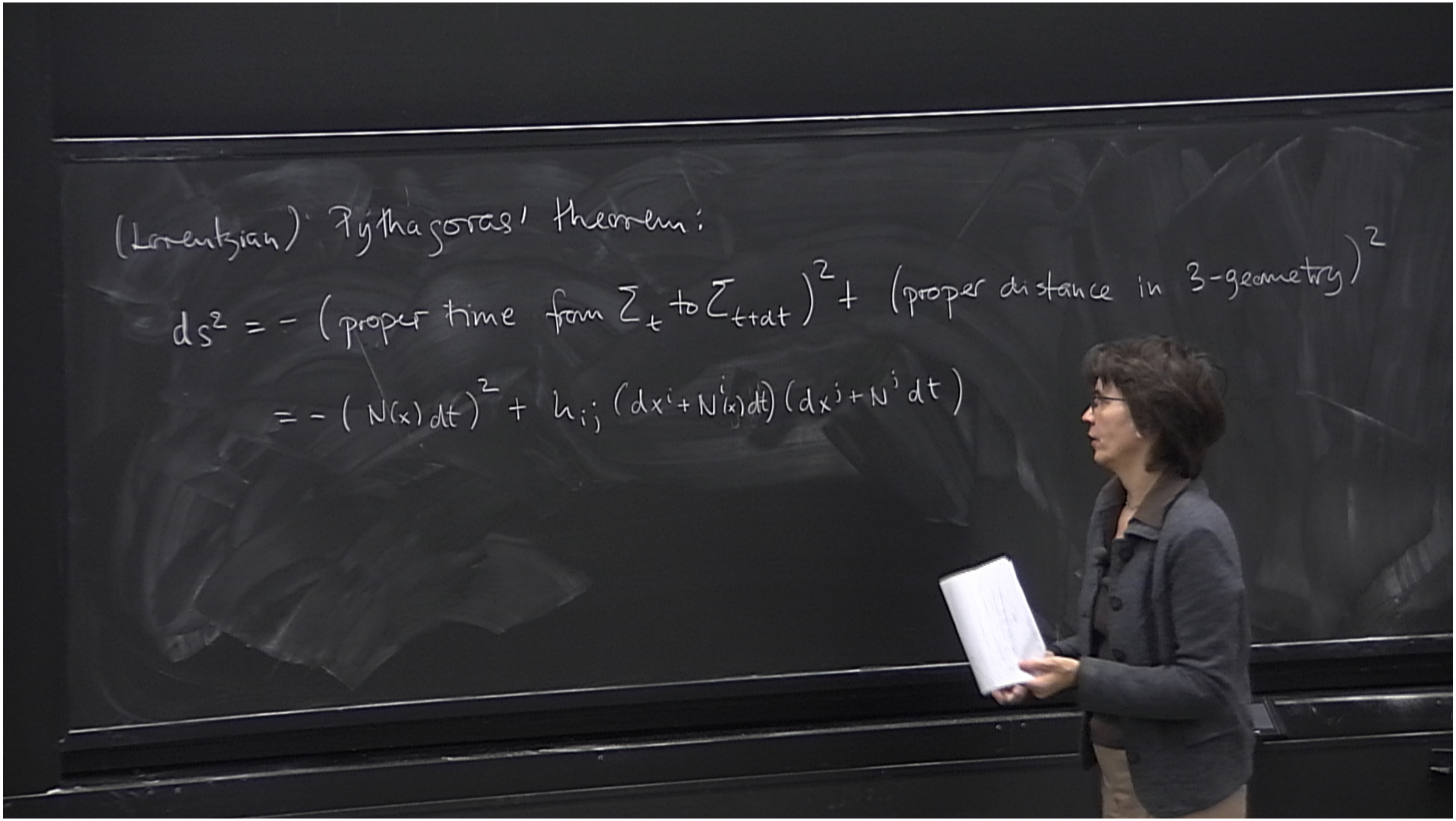
ds
N dt
(x^i, t)
Σ_t


(Lorentzian) Pythagoras' theorem:
ds² = − (proper time from Σt to Σt+dt)² + (proper distance in 3-ge
= − (N(x) dt)² + hij (dxi + Ni dt)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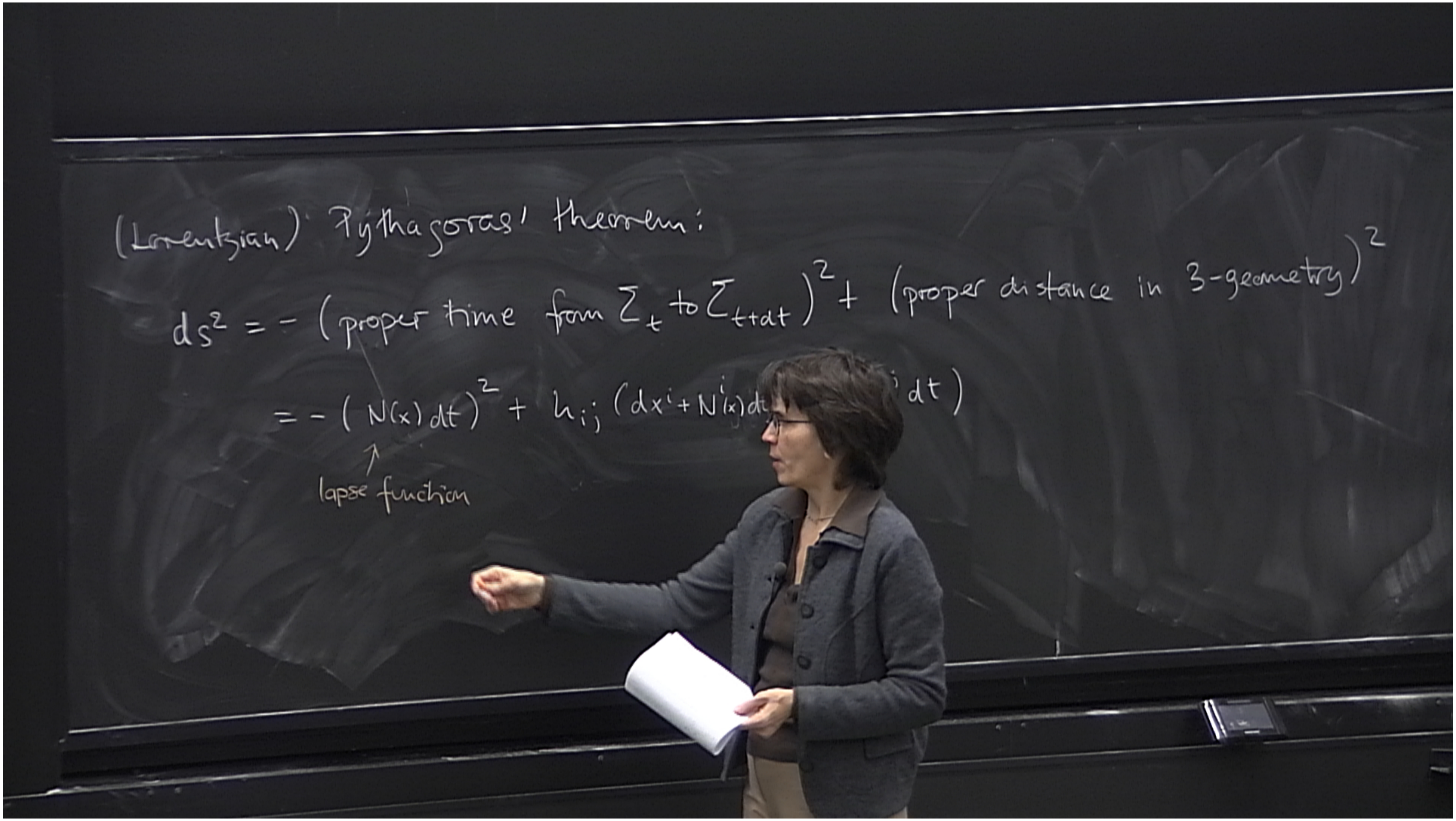
$= -(N(x)\,dt)^2 + h_{ij}(dx^i + N^i(x)dt)(dx^j + N^j dt)$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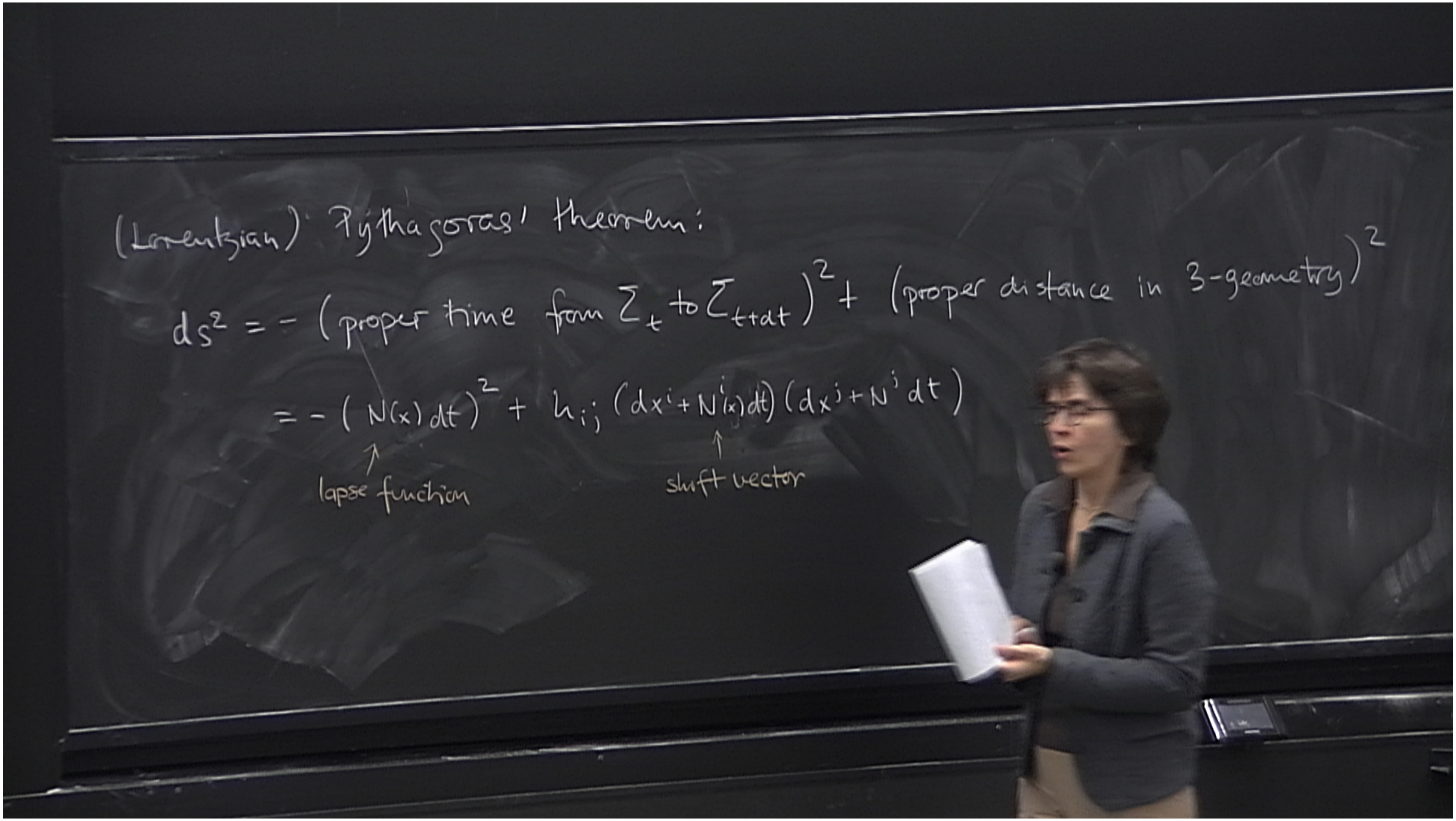
= − (N(x) dt)² + hij (dxi + Ni(x)d ... dt)
lapse function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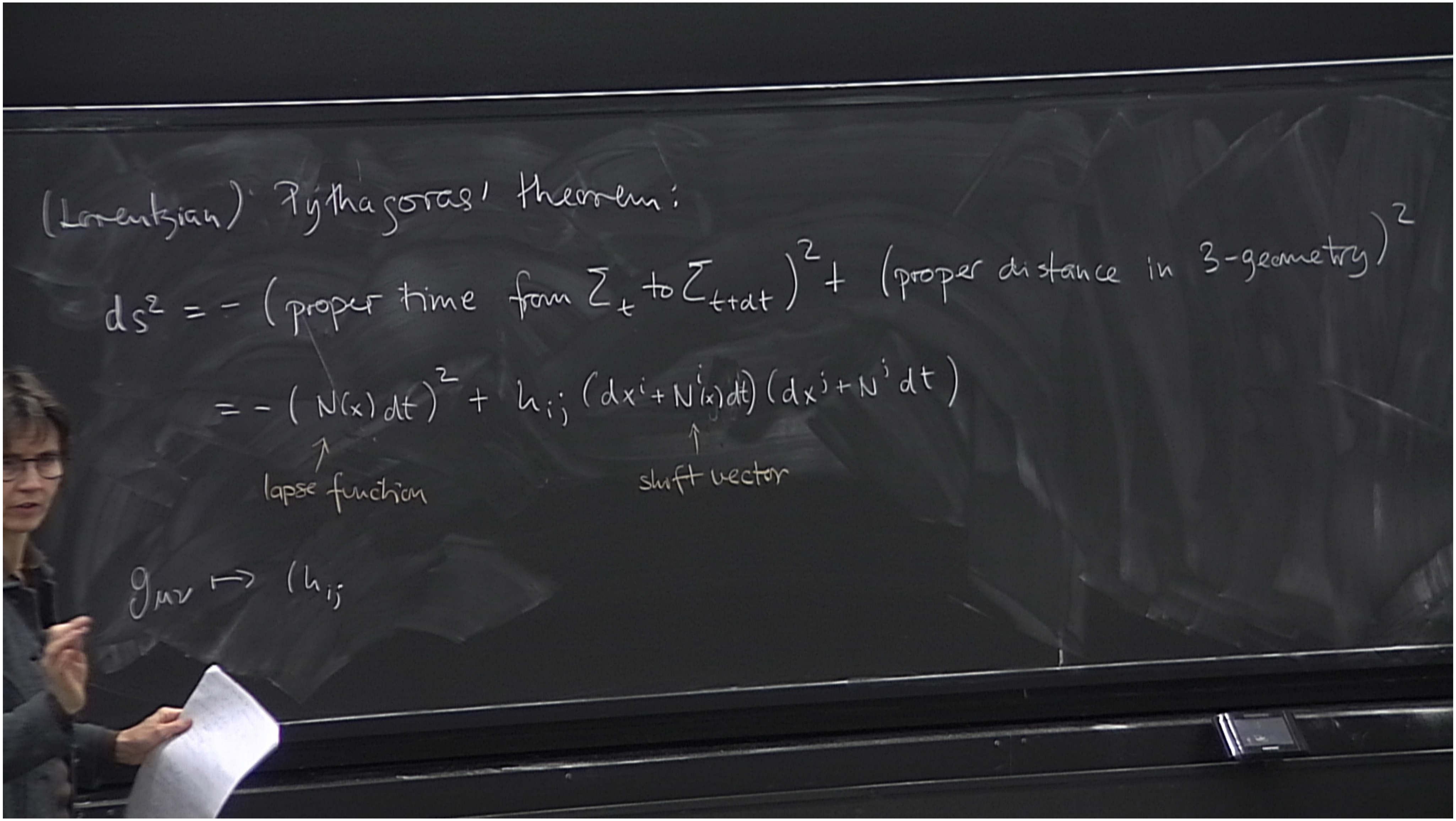
lapse function
shift vector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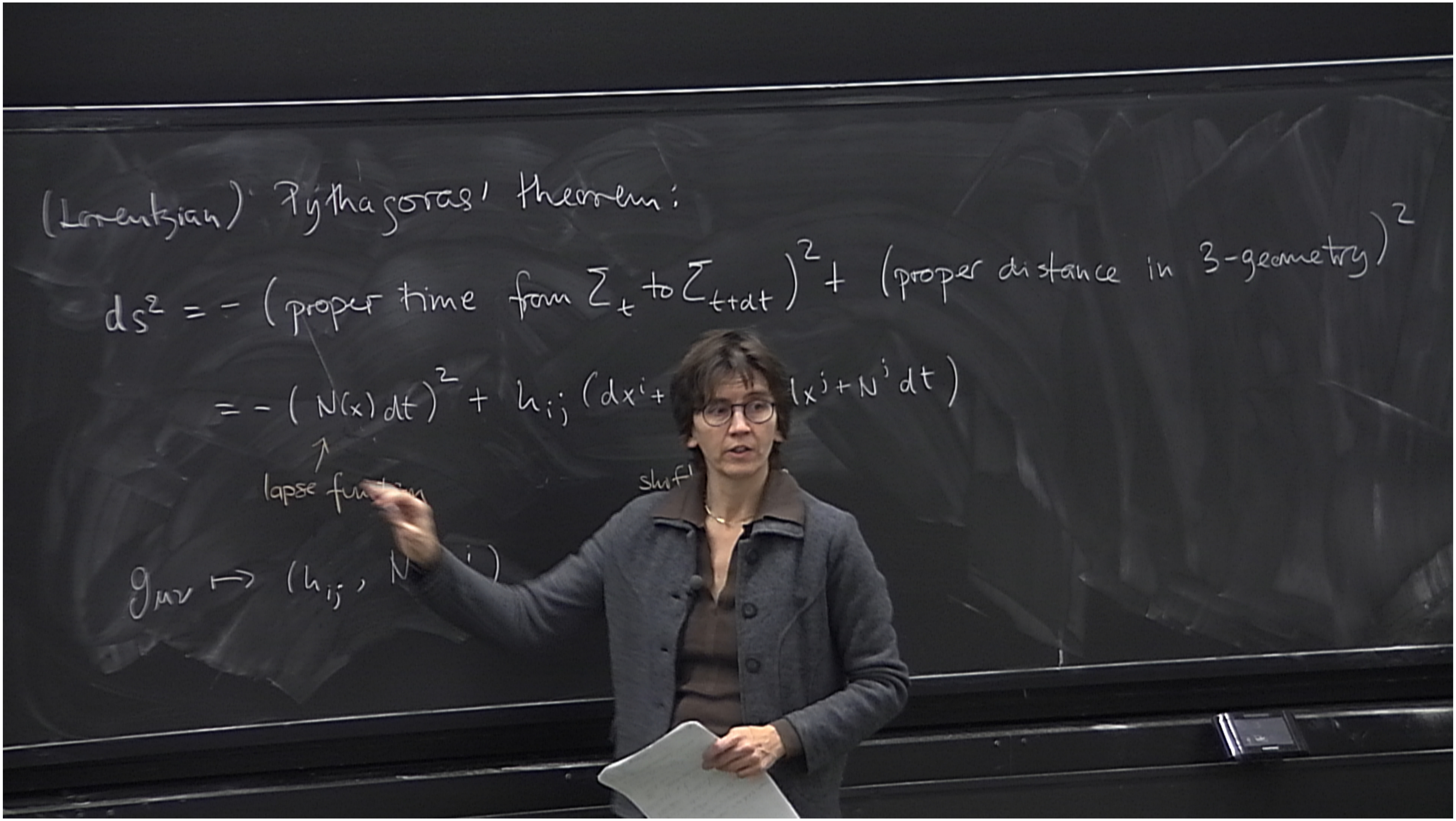
lapse function
shift vector
$g_{\mu\nu} \mapsto (h_{ij}$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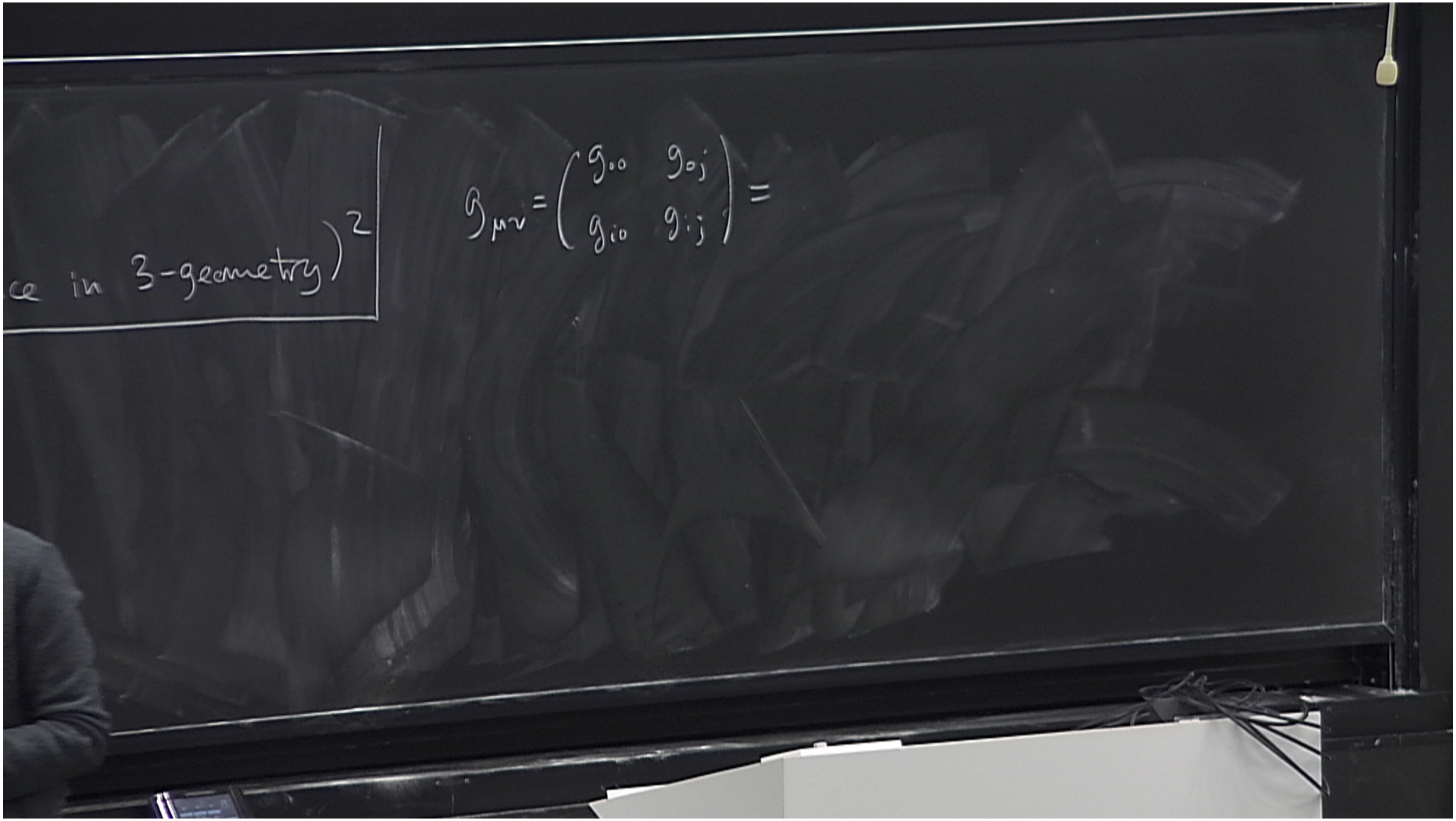
$$= -(N(x)\,dt)^2 + h_{ij}(dx^i + N^i(x)\,dt)(dx^j + N^j\,dt)$$

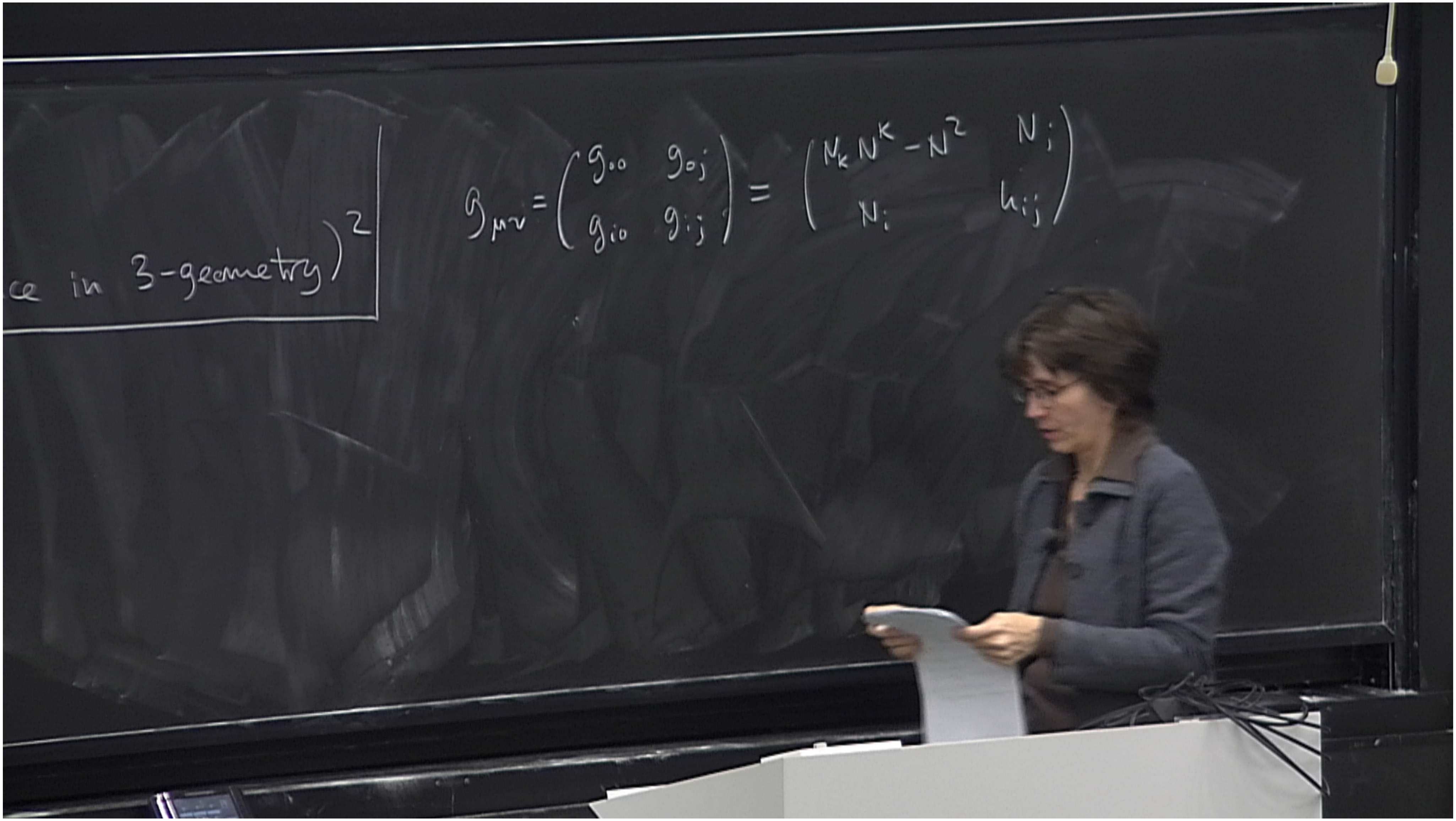
$N(x)$ ↑ lapse function

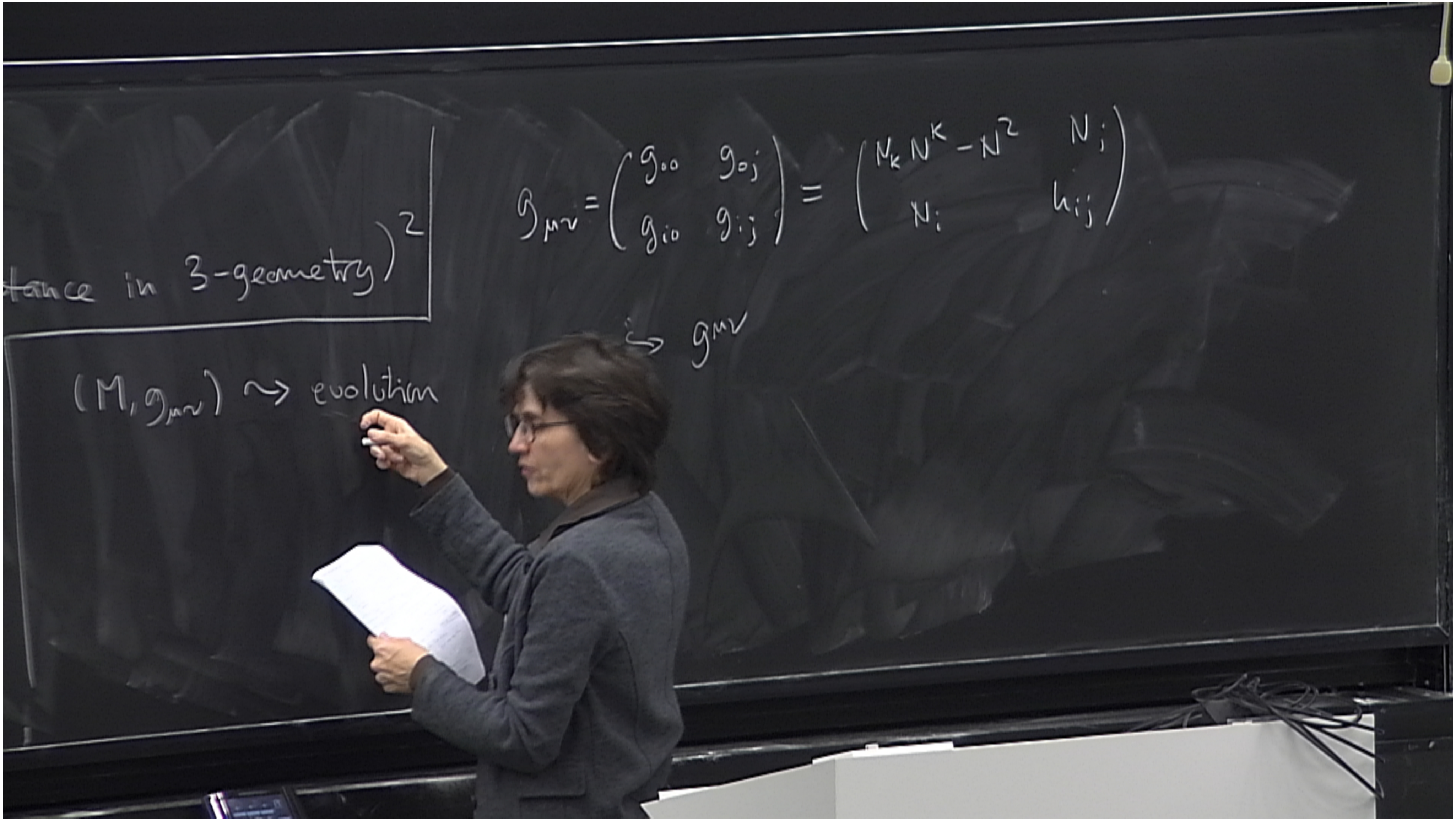
$N^i(x)$ ↑ shift vector

$$g_{\mu\nu} \mapsto (h_{ij},\ N,\ N^i)$$

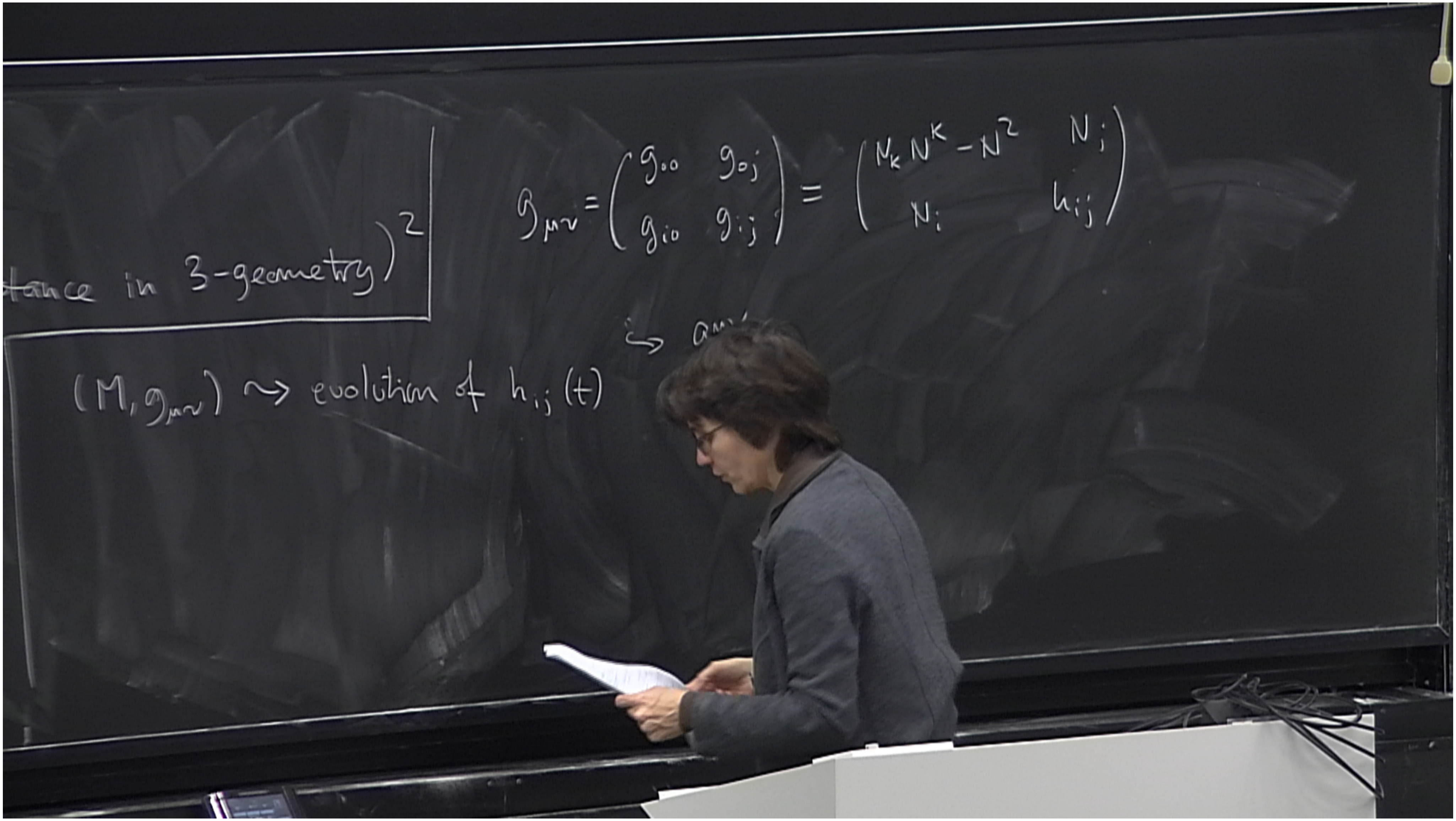
$$10 \qquad 6 \qquad 1 \qquad 3$$

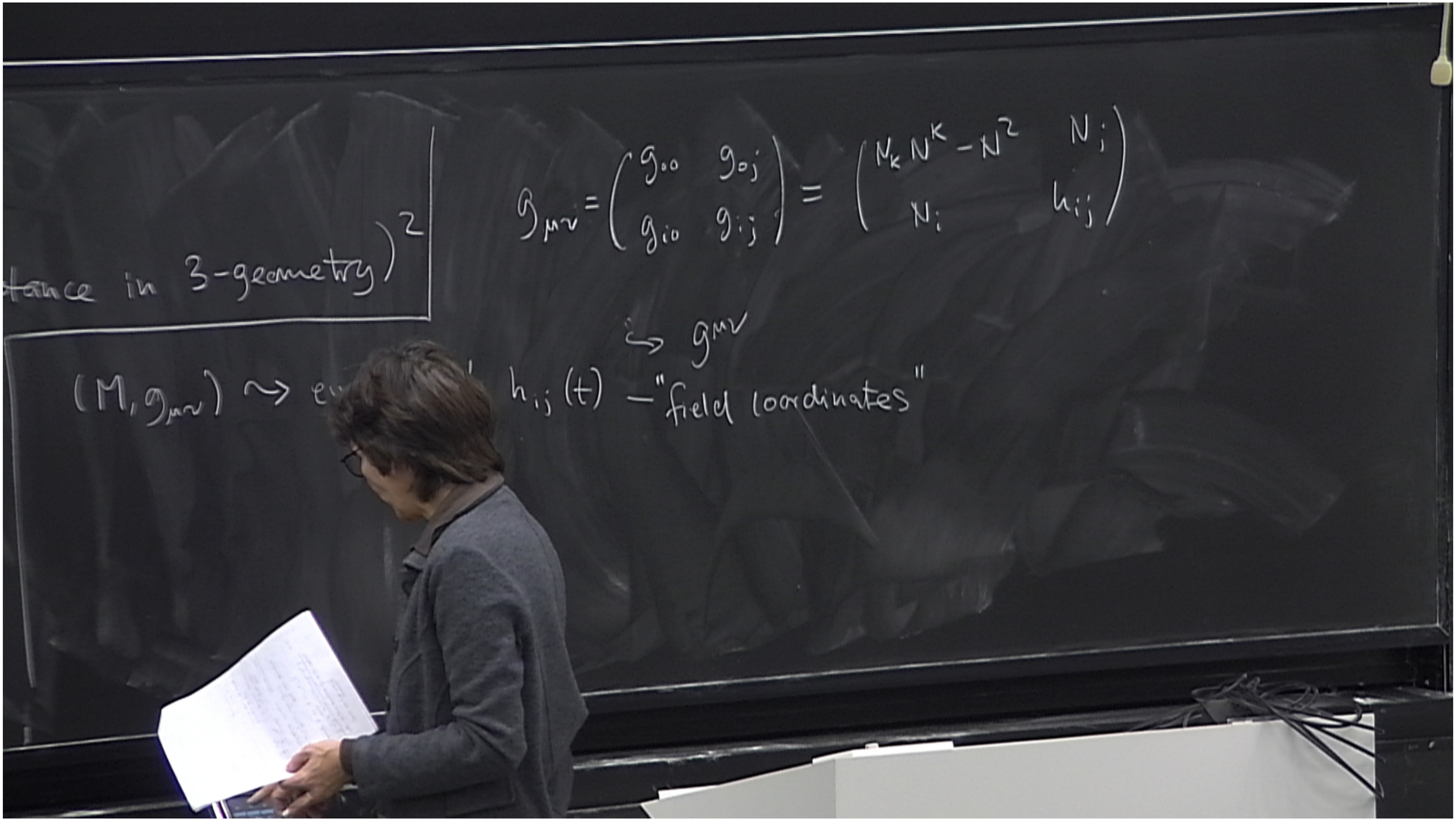

ce in 3-geometry$)^2$
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} =$

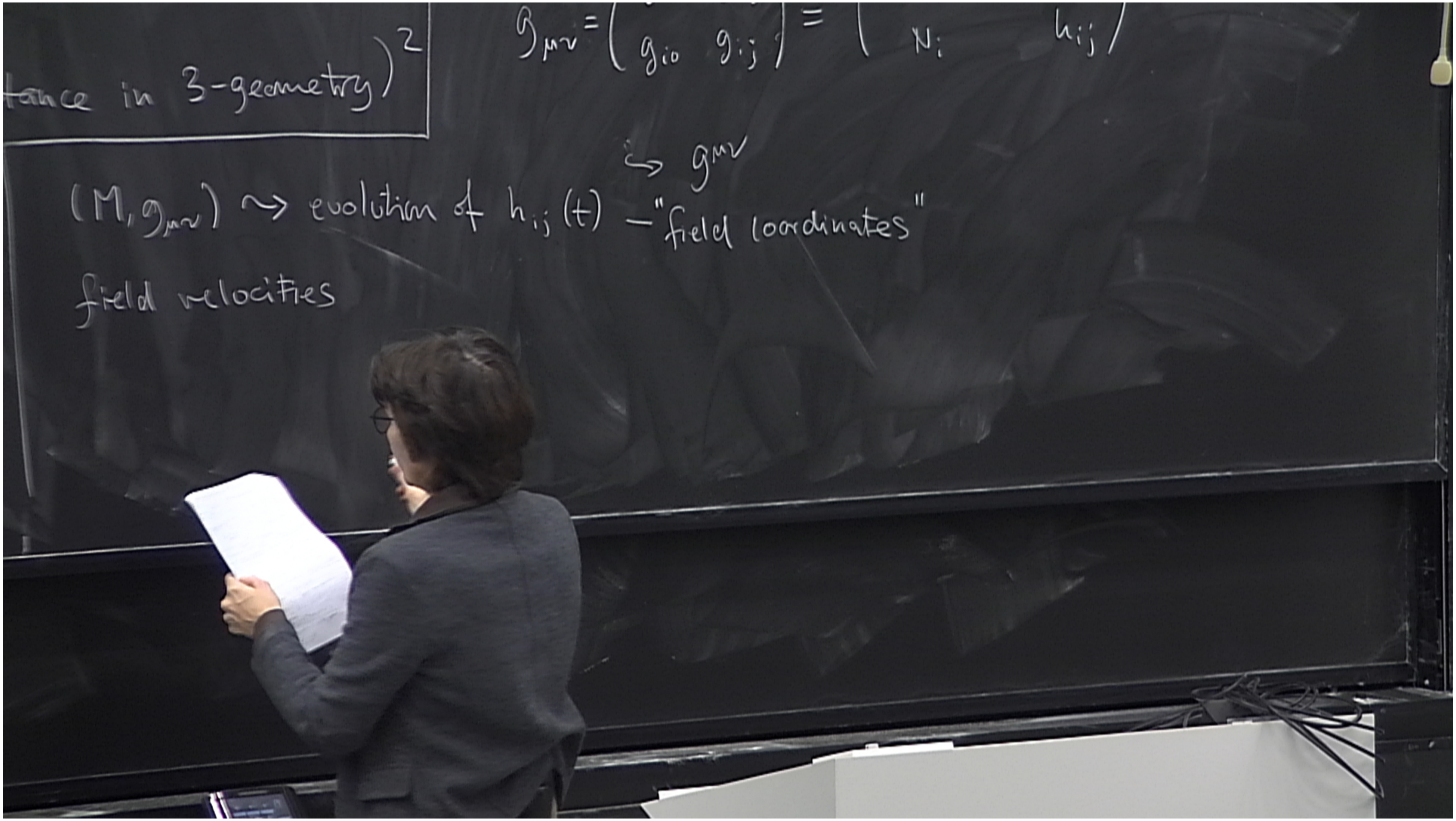

ce in 3-geometry)^2
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$

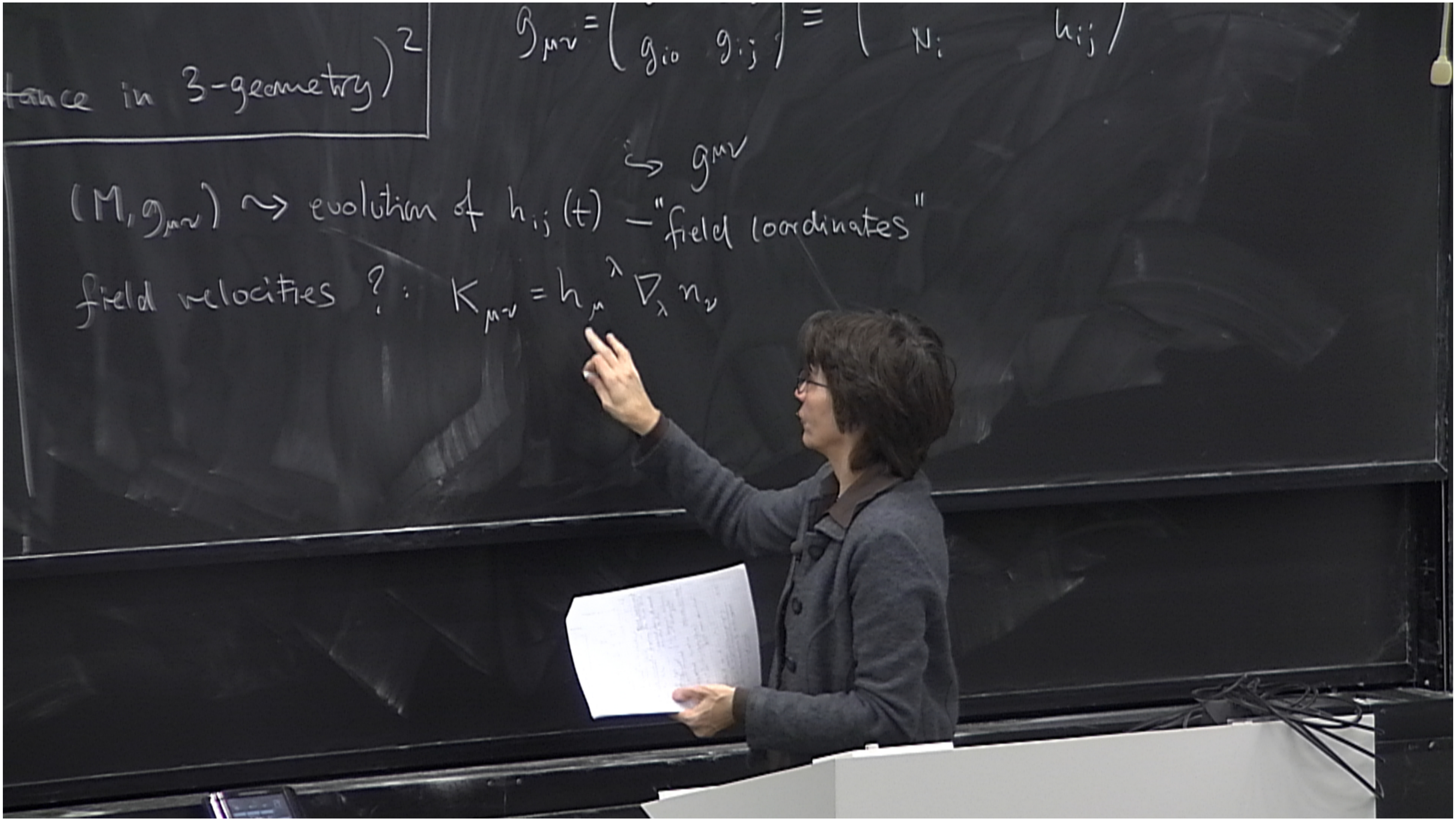


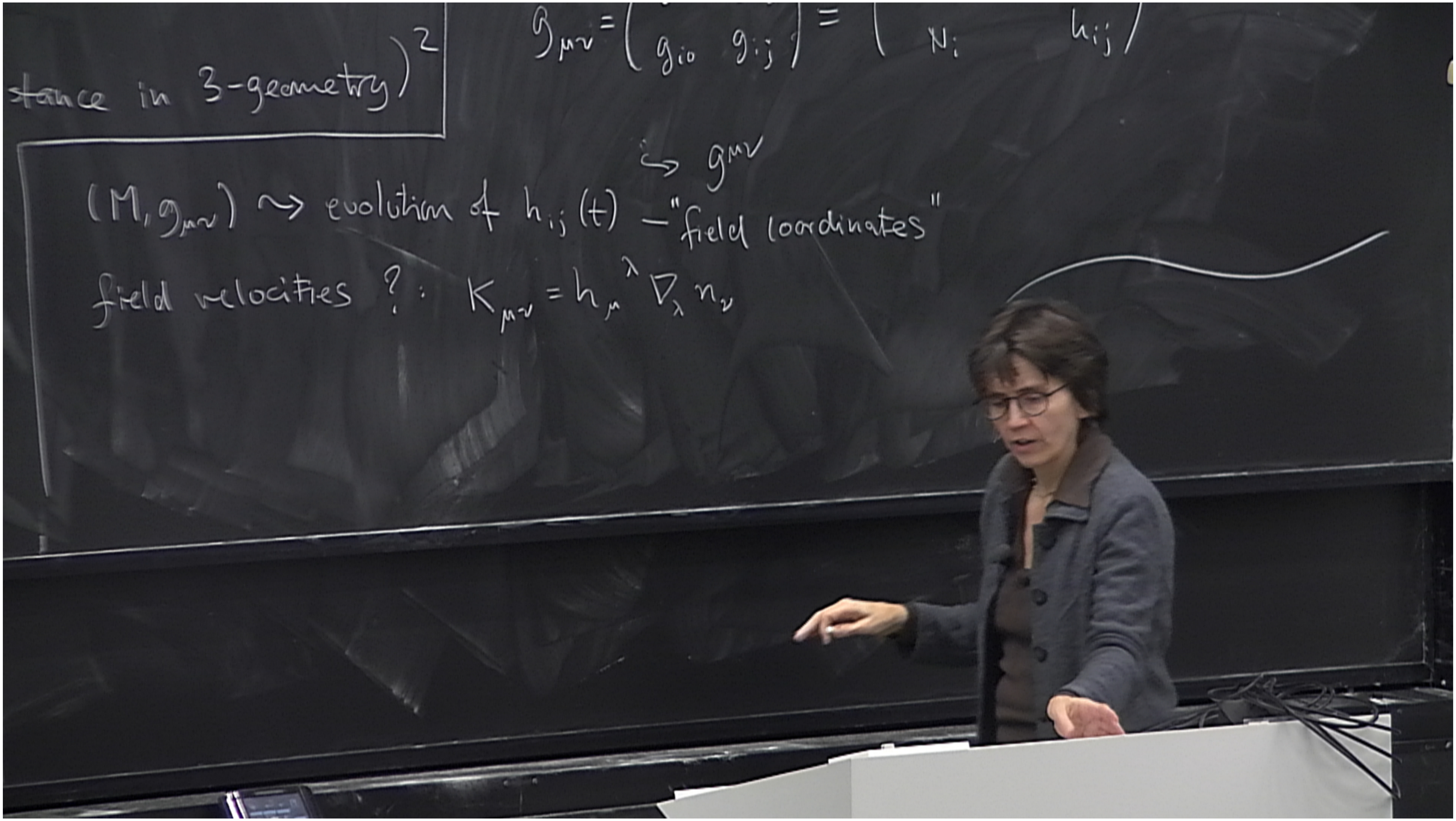
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$
tance in 3-geometry)^2
$\rightsquigarrow g^{\mu\nu}$
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution


tance in 3-geometry)^2
g_μν = (g_00 g_0j ; g_i0 g_ij) = (N_k N^k − N^2 N_j ; N_i h_ij)
(M, g_μν) ~> evolution of h_ij (t)

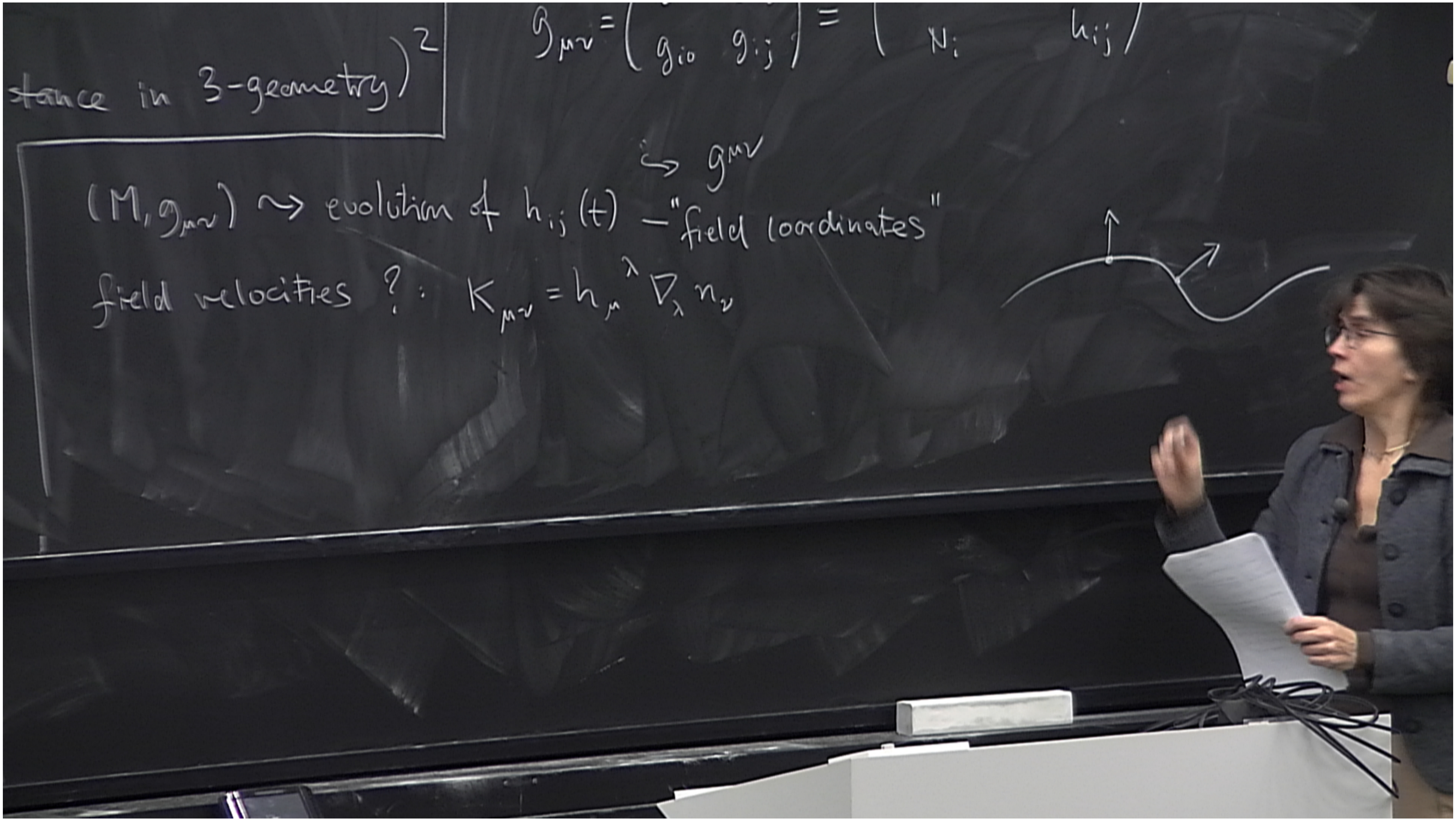

tance in 3-geometry)²
g_μν = (g_00 g_0j ; g_i0 g_ij) = (N_k N^k − N² N_j ; N_i h_ij)
⇝ g^μν
(M, g_μν) ⇝ e h_ij (t) — "field coordinates"


g_μν = (g_i0 g_ij) = (N_i h_ij)
tance in 3-geometry)^2
g^μν
(M, g_μν) ~> evolution of h_ij (t) — "field coordinates"
field velocities


tance in 3-geometry)²
gμν = (gi0 gij) = (Ni hij)
↪ g^μν
(M, gμν) ~> evolution of hij (t) – "field coordinates"
field velocities ? : Kμν = hμ^λ ∇λ nν



$g_{\mu\nu} = \begin{pmatrix} & \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} & \\ N_i & h_{ij} \end{pmatrix}$
stance in 3-geometry$)^2$
$\hookrightarrow g^{\mu\nu}$
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ – "field coordinates"
field velocities ? : $K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$


g_μν = (g_i0 g_ij) = (N_i h_ij)
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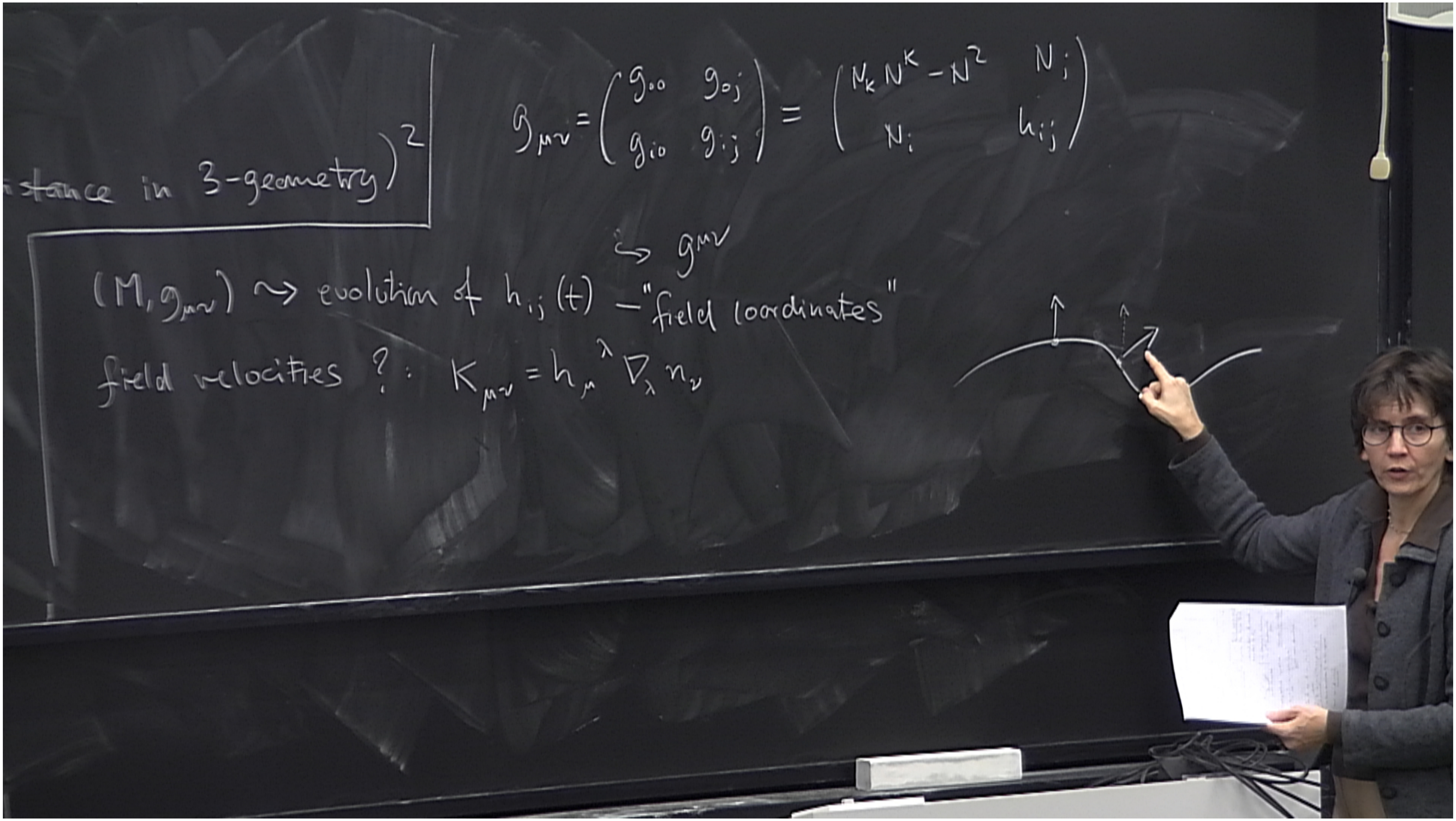
istance in 3-geometry$)^2$
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$
$\hookrightarrow g^{\mu\nu}$
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ – "field coordinates"
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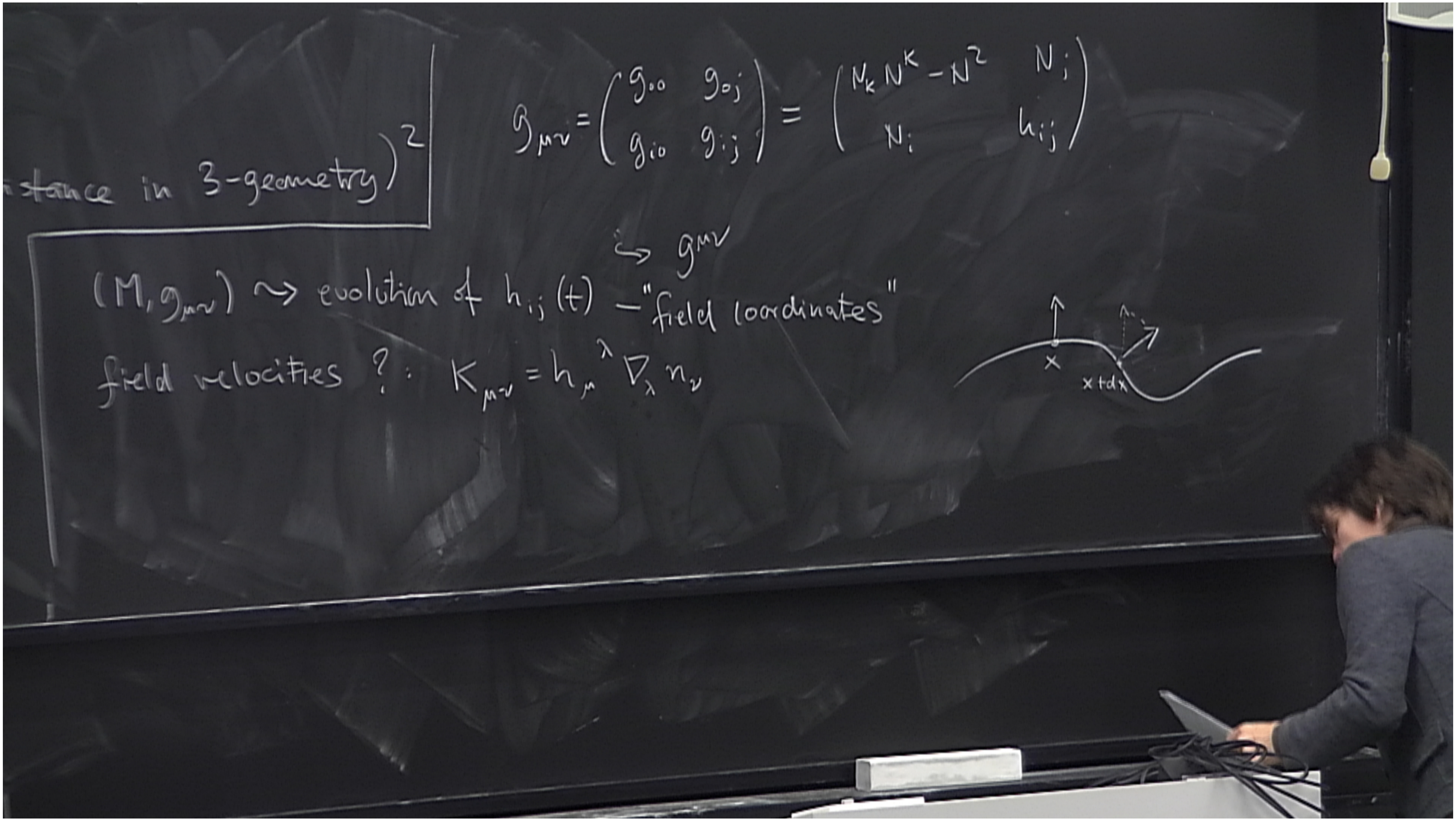
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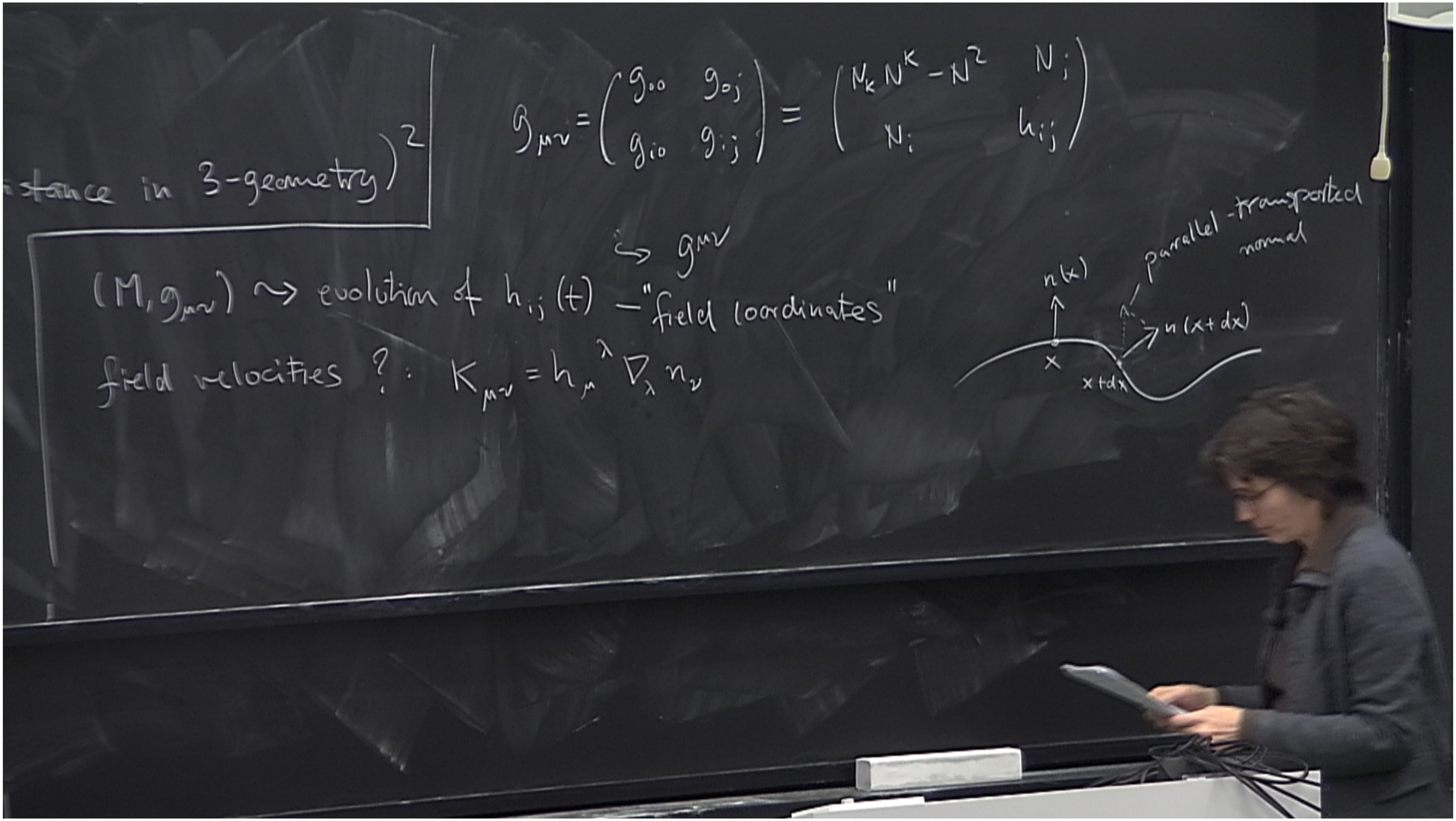
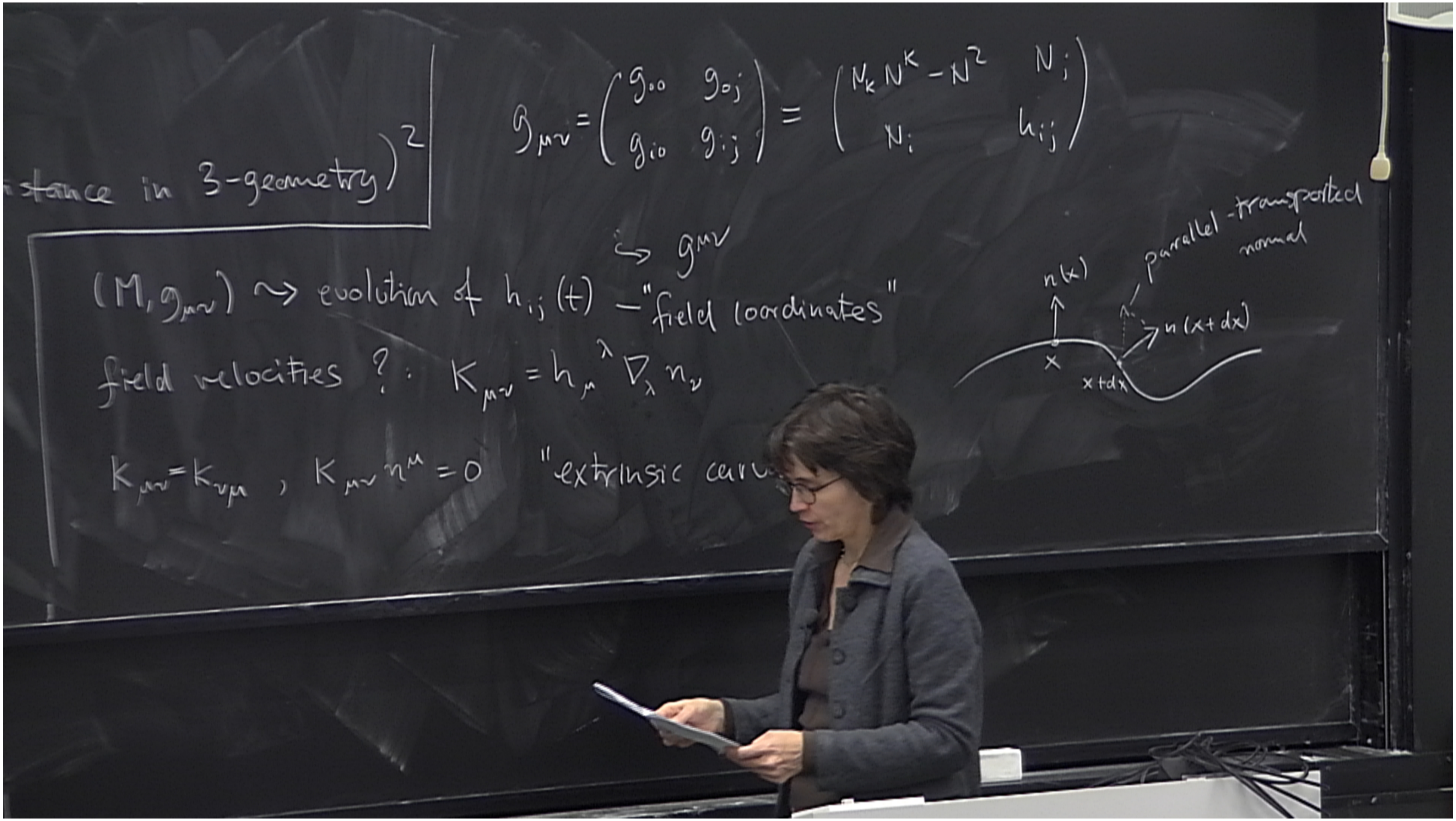
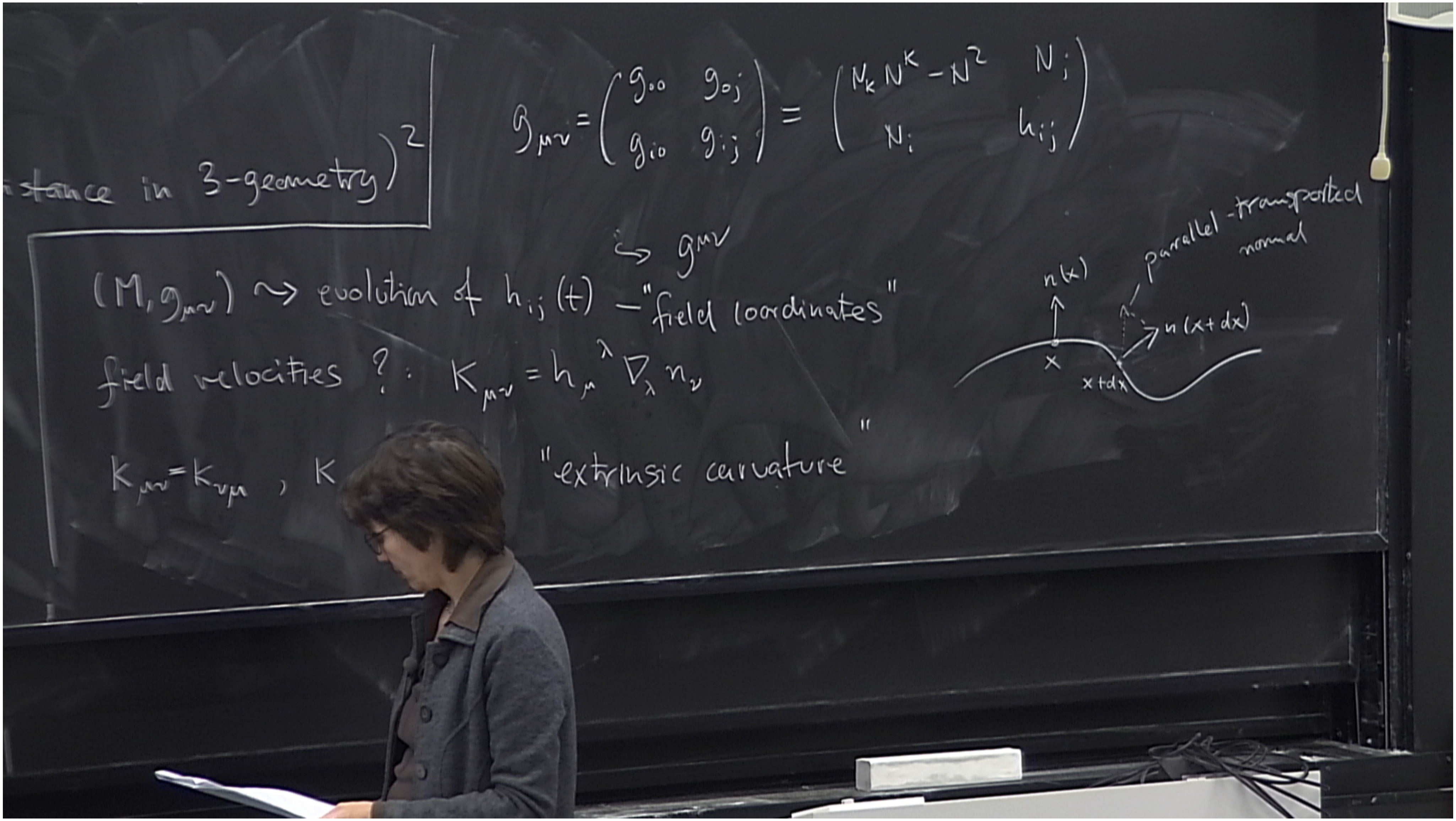
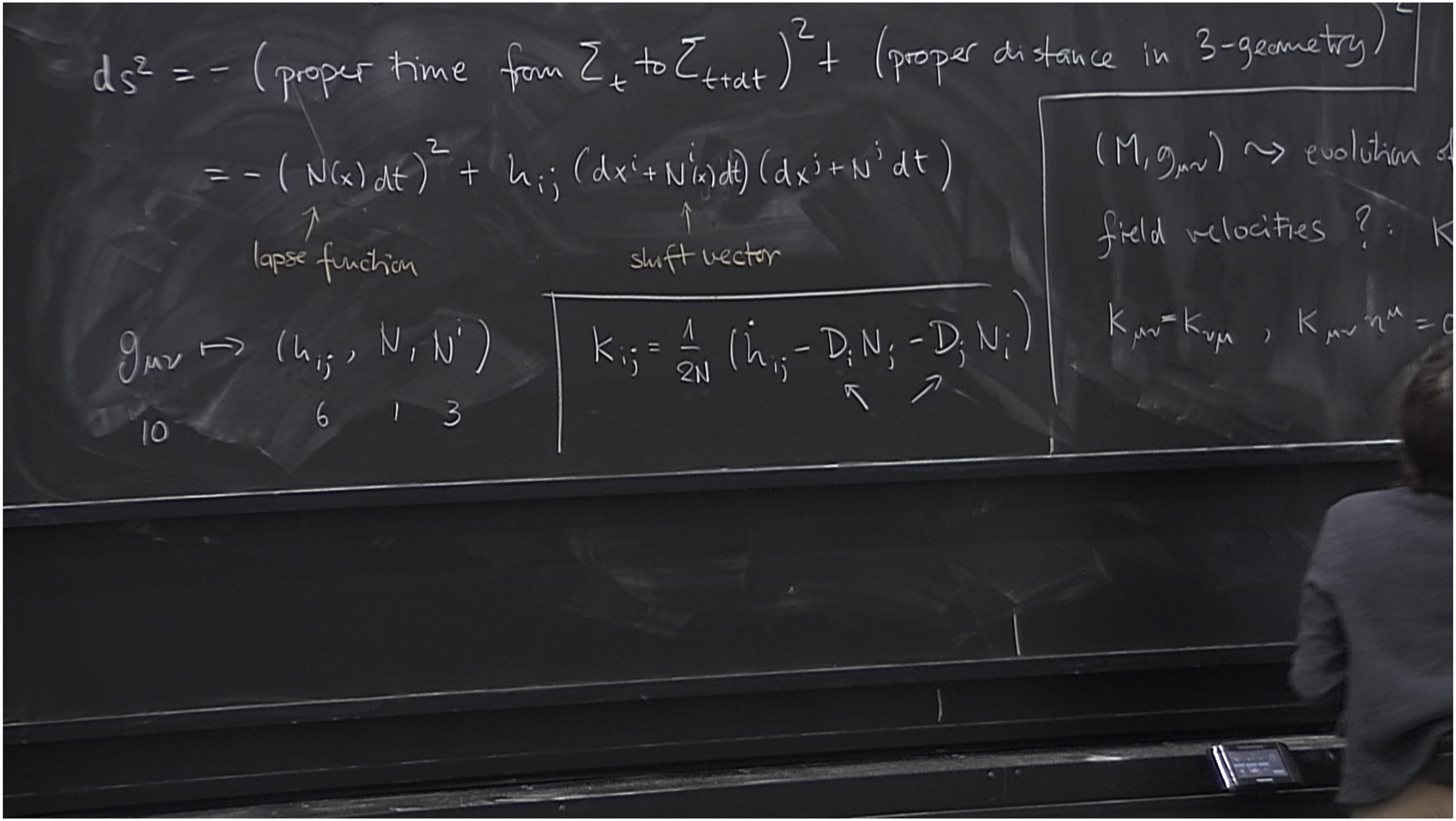
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ – "field coordinates"
field velocities ?: $K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
x
x+dx

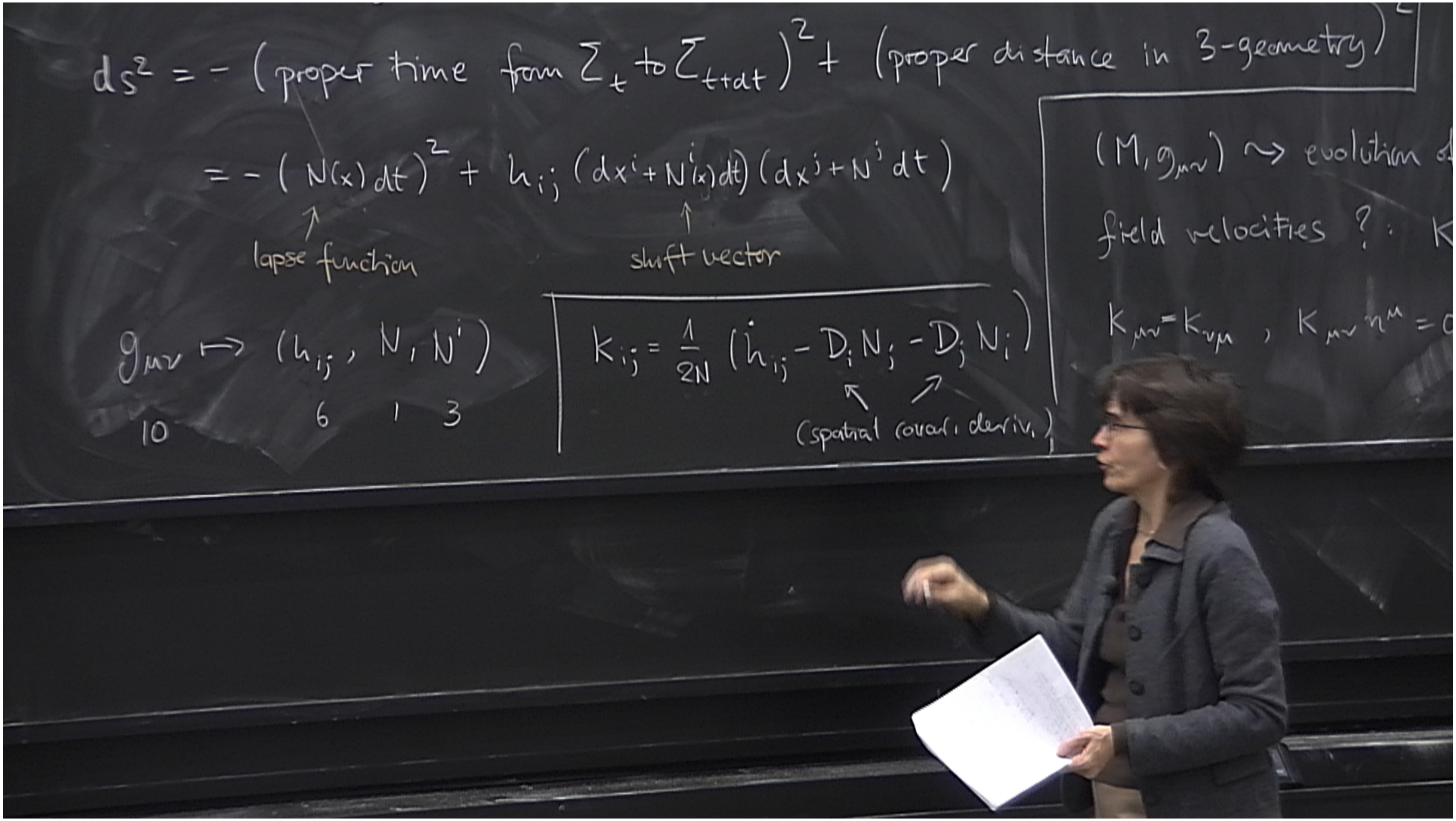

$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$
istance in 3-geometry)^2
$\hookrightarrow g^{\mu\nu}$
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ — "field coordinates"
field velocities ?: $K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
parallel-transported normal
n(x)
n(x+dx)
x
x+dx

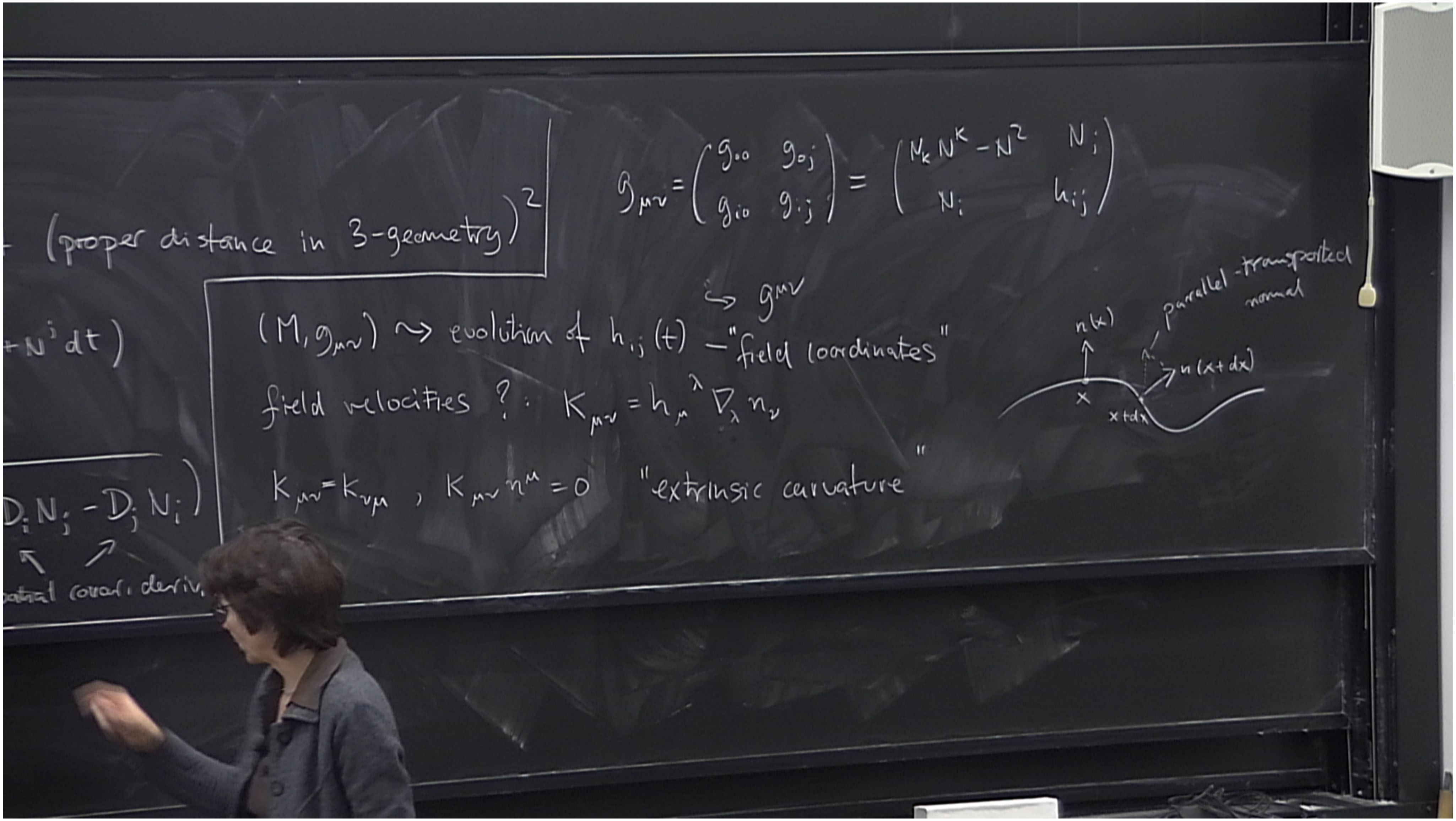


$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$
istance in 3-geometry)²
$\hookrightarrow g^{\mu\nu}$
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ – "field coordinates"
field velocities ? : $K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
$K_{\mu\nu} = K_{\nu\mu}$, $K_{\mu\nu} n^\mu = 0$ "extrinsic curv
parallel-transported normal
n(x)
n(x+dx)
x
x+dx


g_μν = (g_00 g_0j ; g_i0 g_ij) = (N_k N^k − N^2 N_j ; N_i h_ij)
istance in 3-geometry)^2
g^μν
(M, g_μν) ~> evolution of h_ij (t) — "field coordinates"
field velocities ? : K_μν = h_μ^λ ∇_λ n_ν
K_μν = K_νμ , K
"extrinsic curvature"
n(x)
parallel-transported normal
n(x+dx)
x
x+dx


ds² = − (proper time from Σ_t to Σ_{t+dt})² + (proper distance in 3-geometry)²
= − (N(x) dt)² + h_ij (dx^i + N^i(x) dt) (dx^j + N^j dt)
lapse function
shift vector
g_μν ↦ (h_ij , N, N^i)
10 6 1 3
K_ij = 1/2N (ḣ_ij − D_i N_j − D_j N_i)
(M, g_μν) ⇝ evolution o
field velocities ? : K
K_μν = K_νμ , K_μν n^μ = 0


$ds^2 = -(\text{proper time from } \Sigma_t \text{ to } \Sigma_{t+dt})^2 + (\text{proper distance in 3-geometry})^2$
$= -(N(x)\,dt)^2 + h_{ij}(dx^i + N^i(x)dt)(dx^j + N^j dt)$
lapse function
shift vector
$g_{\mu\nu} \mapsto (h_{ij}, N, N^i)$
10 6 1 3
$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i)$
(spatial covar. deriv.)
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of
field velocities ?
$K_{\mu\nu} = K_{\nu\mu}$, $K_{\mu\nu} n^\mu =$


(proper distance in 3-geometry)^2
$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{pmatrix} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}$
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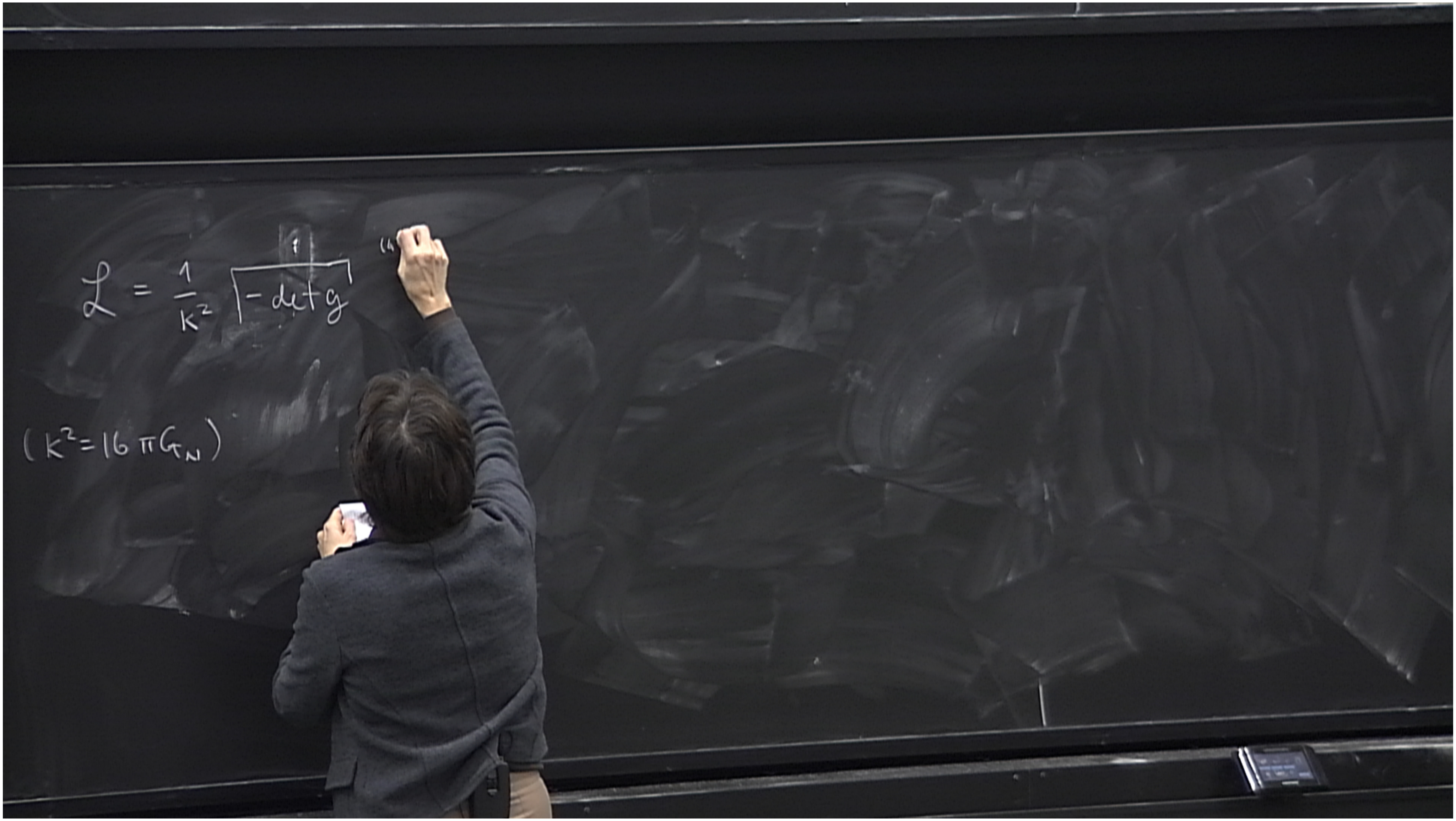
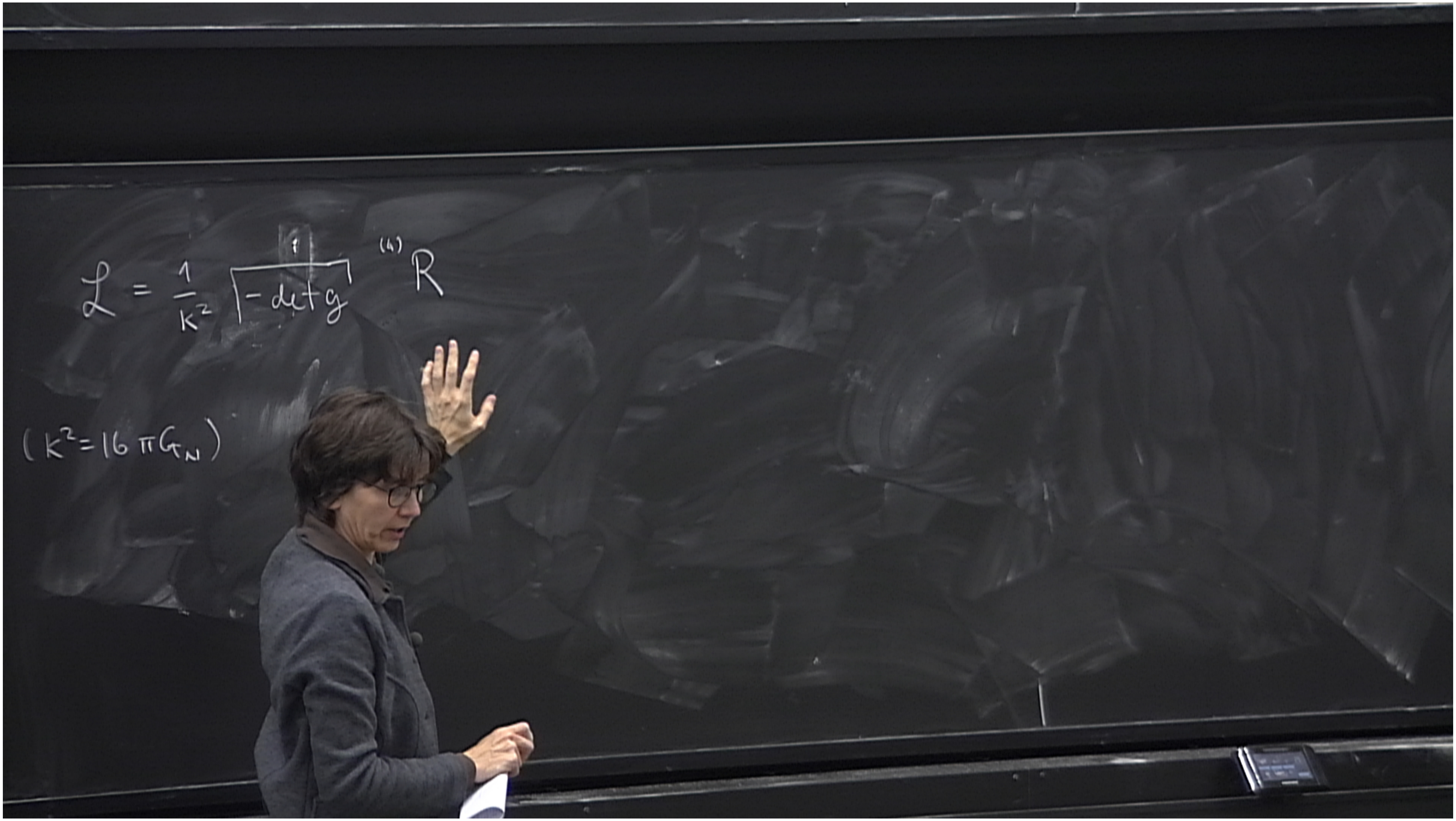
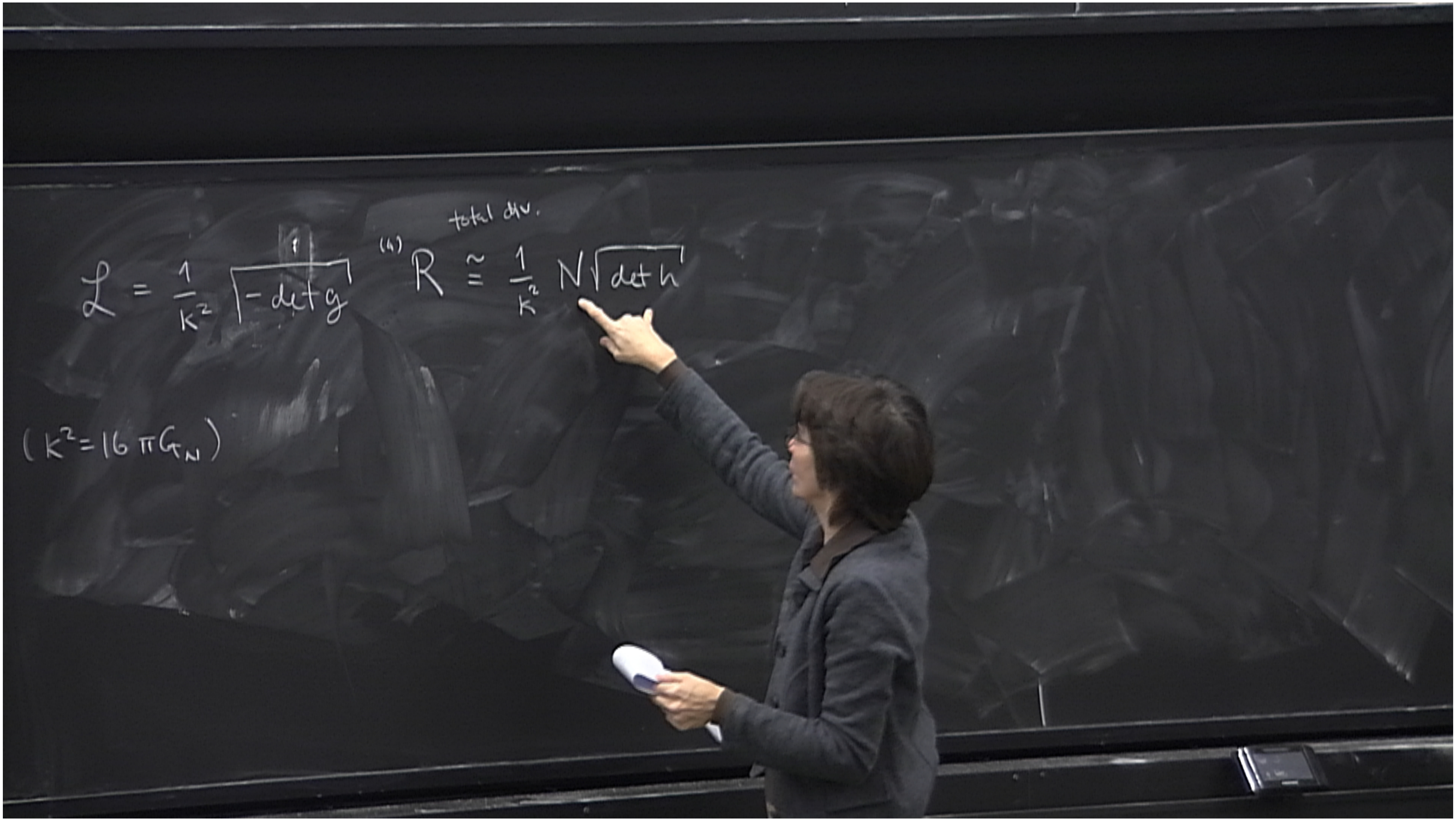
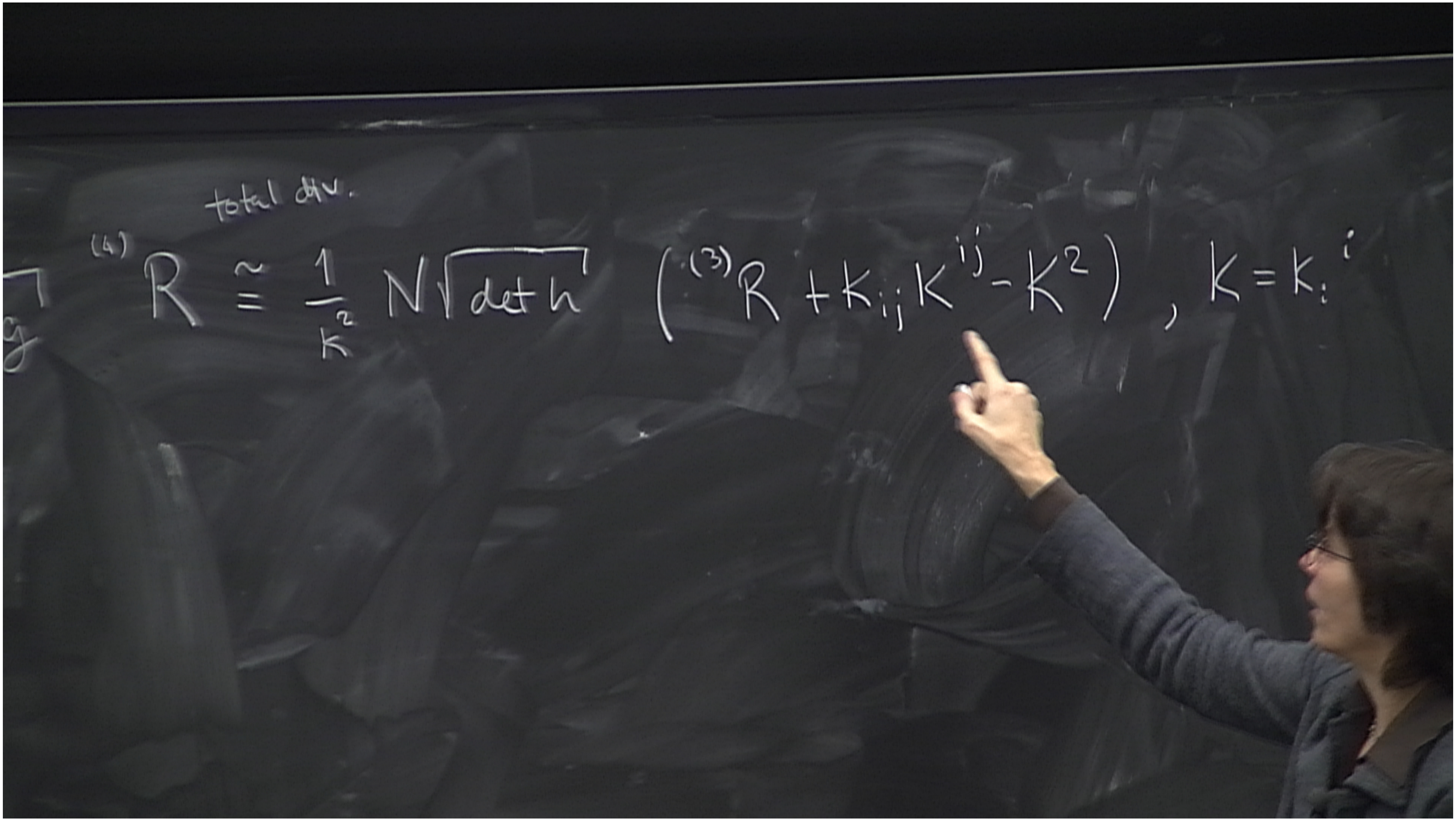
$(M, g_{\mu\nu}) \rightsquigarrow$ evolution of $h_{ij}(t)$ — "field coordinates"
field velocities ?: $K_{\mu\nu} = h_\mu{}^\lambda \nabla_\lambda n_\nu$
$K_{\mu\nu} = K_{\nu\mu}$, $K_{\mu\nu} n^\mu = 0$ "extrinsic curvature"
parallel-transported normal
n(x)
n(x+dx)
x
x+dx
$+ N^j dt)$
$D_i N_j - D_j N_i)$
spatial covar. deriv.

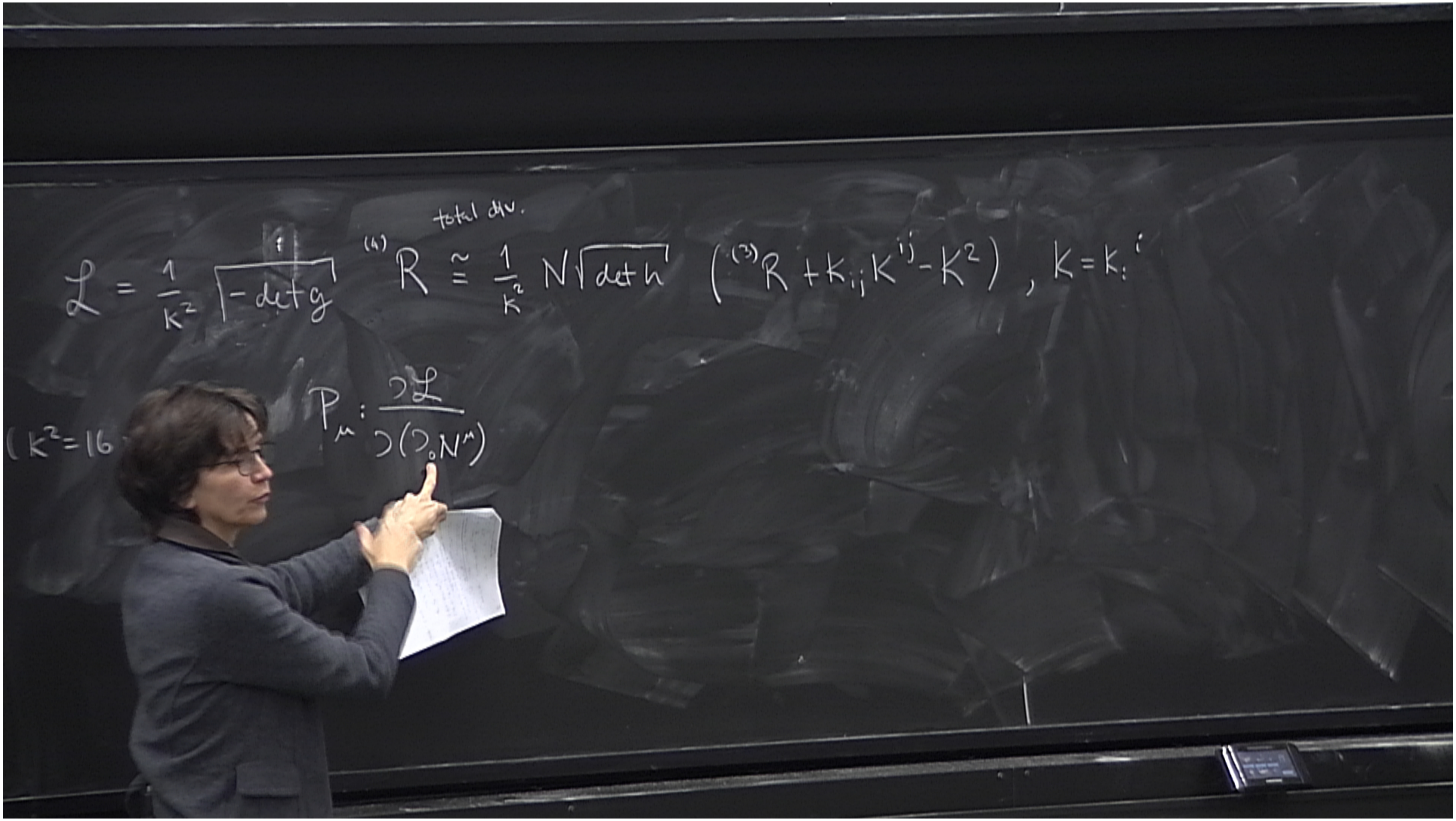

$\mathcal{L} = \frac{1}{K^2}$

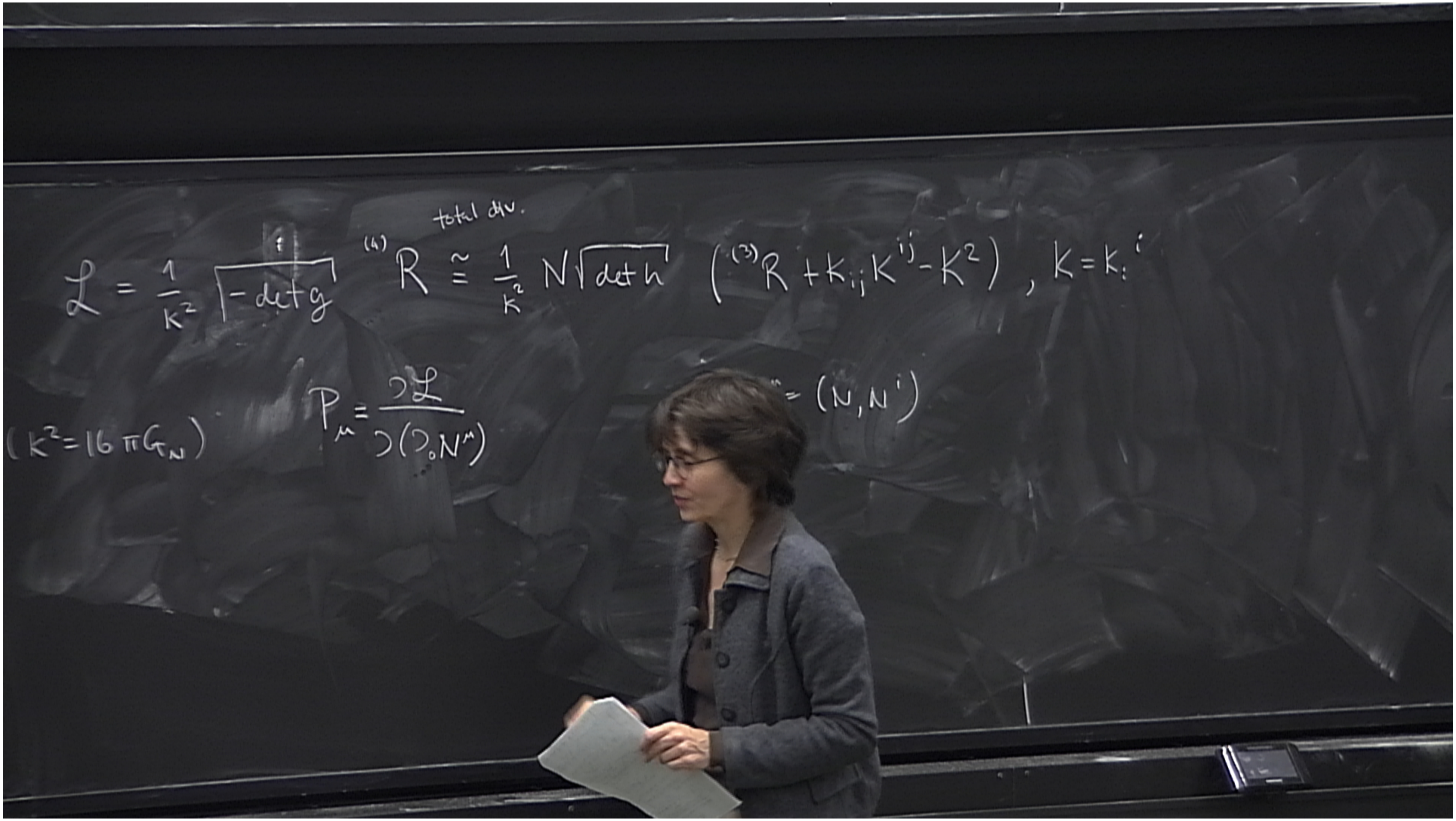

L = 1/k² √(-det g)
(k² = 16 π G_N)

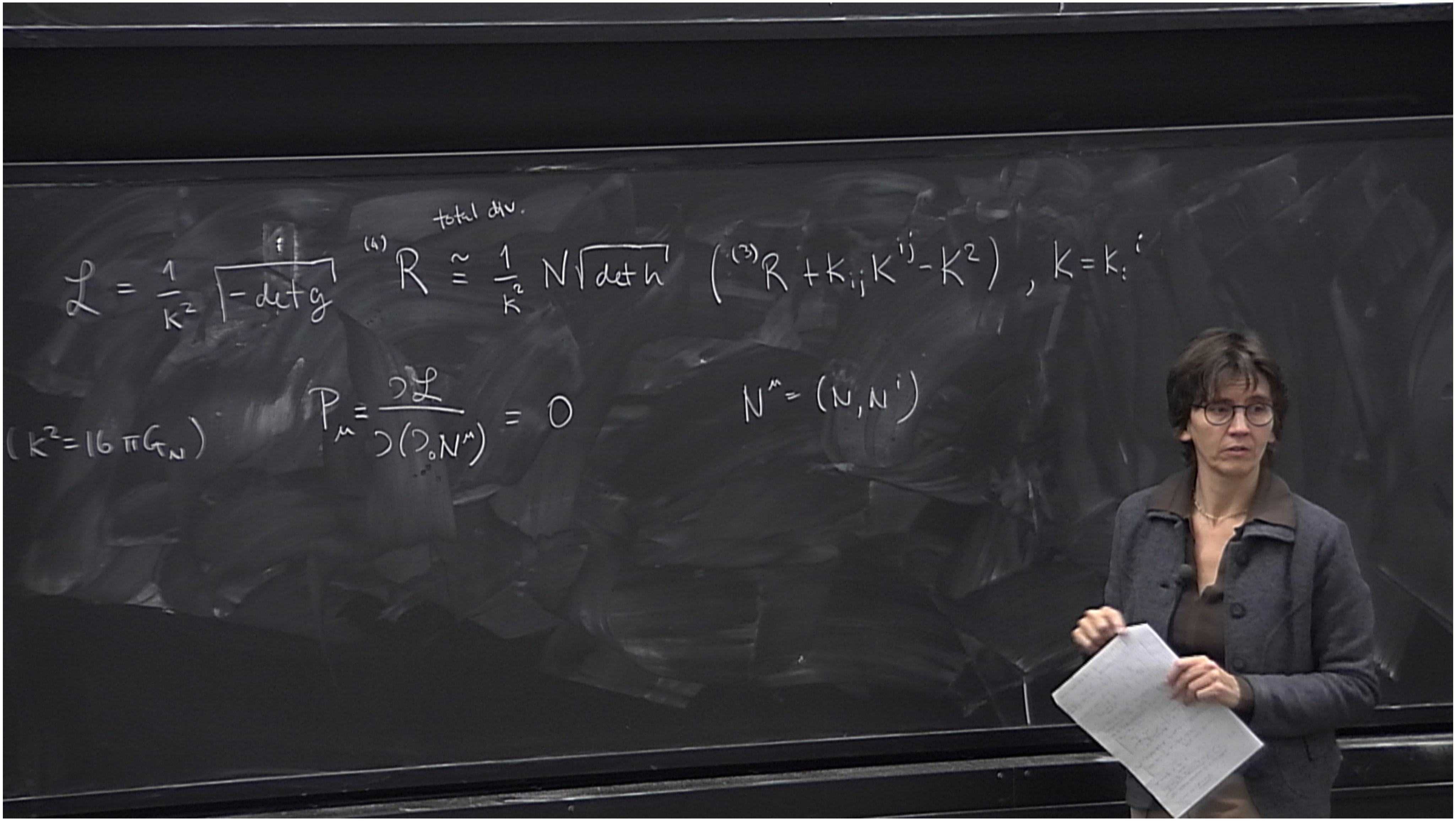

$\mathcal{L} = \frac{1}{k^2}\sqrt{-\det g}\;{}^{(4)}R$
$(k^2 = 16\pi G_N)$

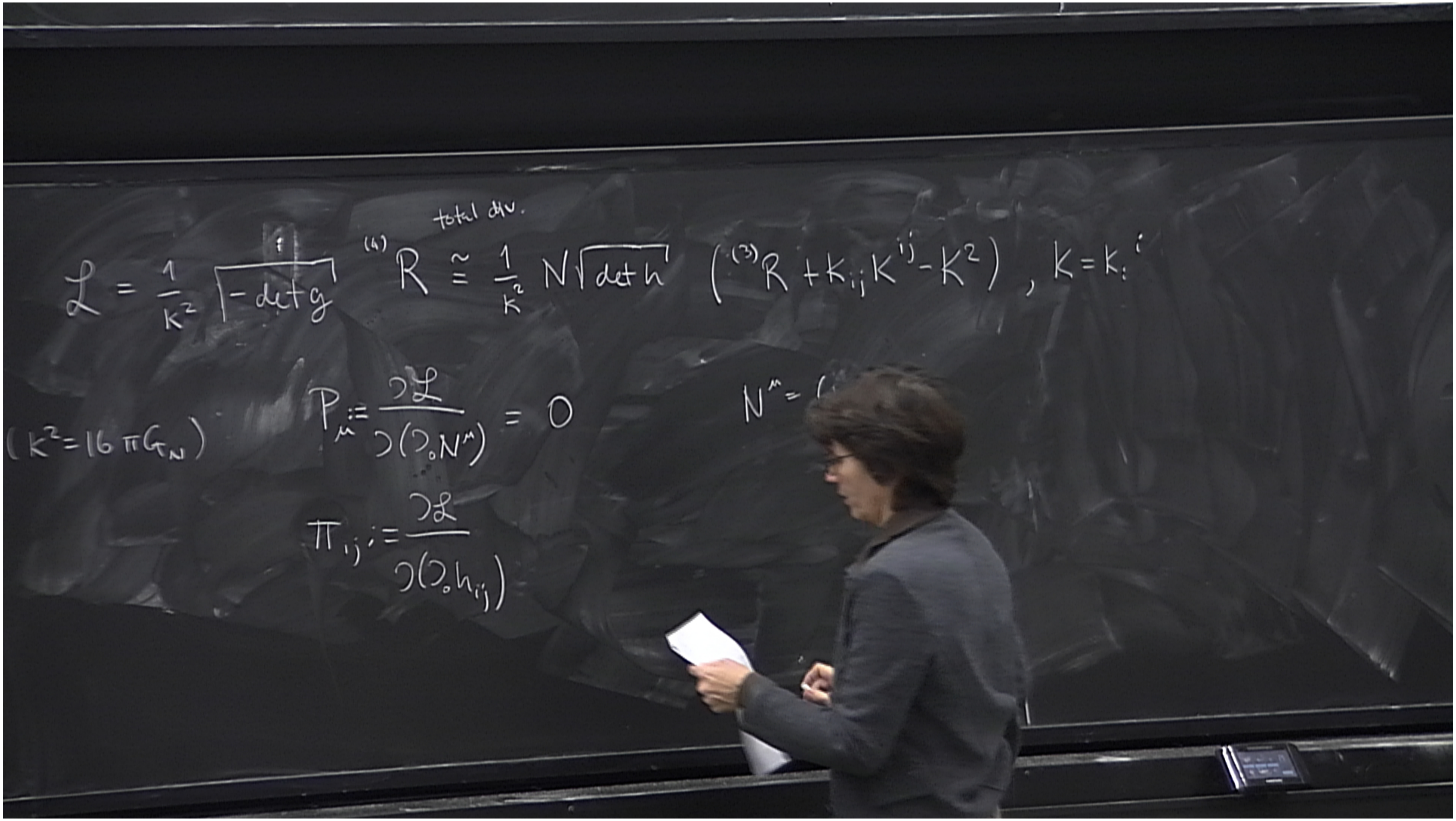

total div.
L = 1/k² √(-det g) (4)R ≃ 1/k² N √(det h)
(k² = 16 π G_N)

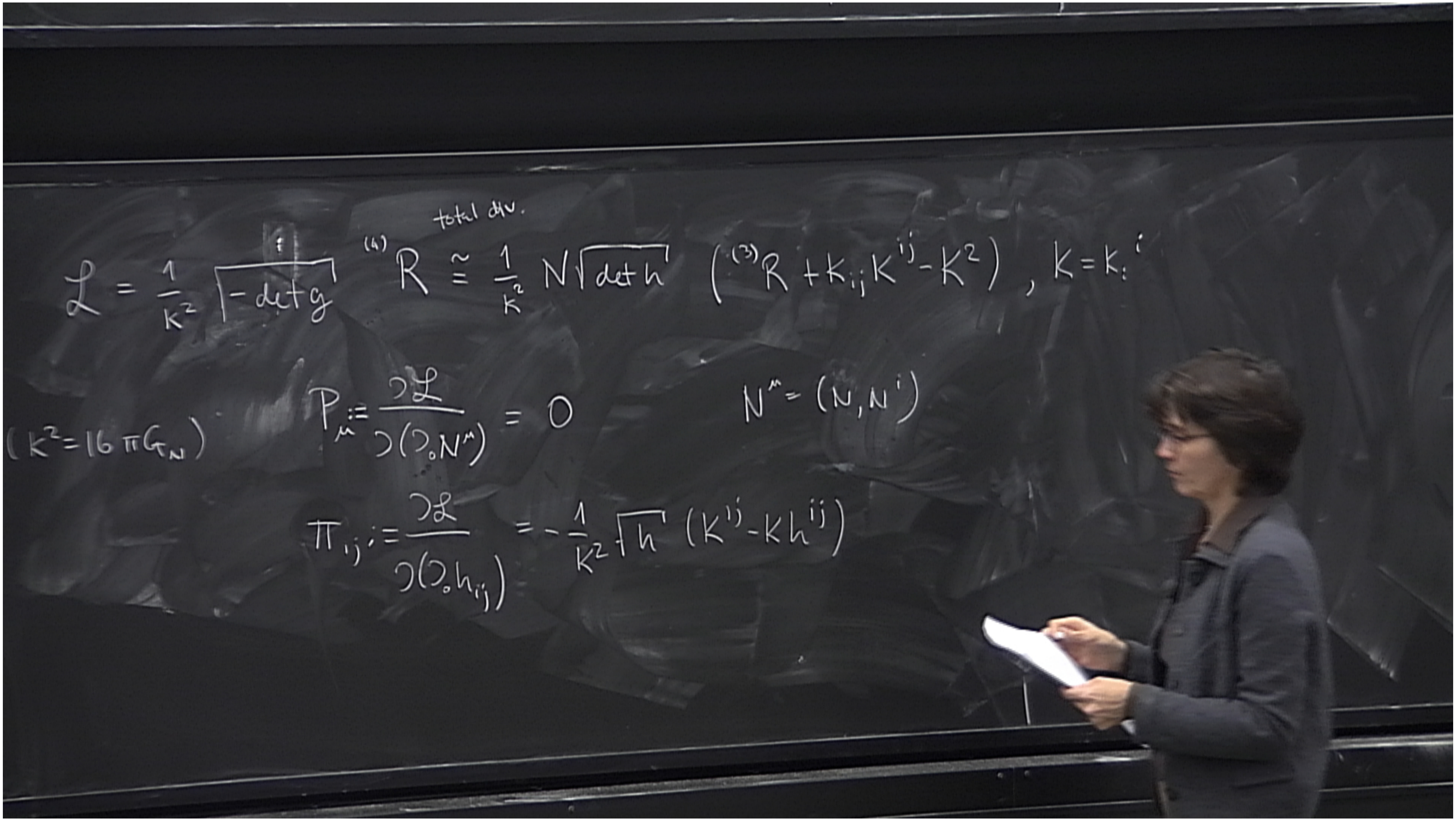

total div.
$^{(4)}R \stackrel{\sim}{=} \frac{1}{\kappa^2} N\sqrt{\det h} \left(^{(3)}R + K_{ij}K^{ij} - K^2 \right), \; K = K_i{}^i$

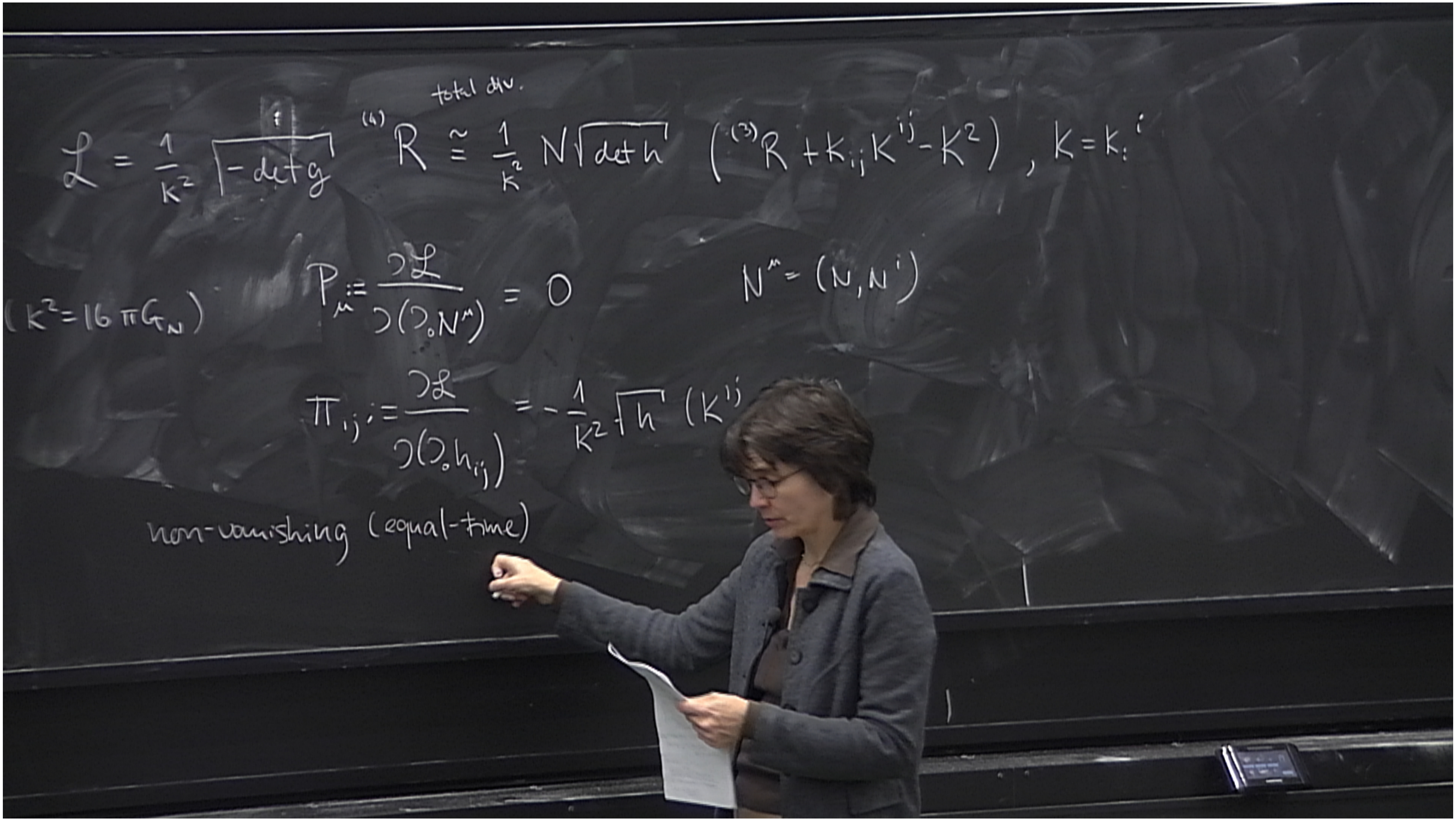

total div.
L = 1/k² √(-det g) ⁽⁴⁾R ≅ 1/k² N √(det h) (⁽³⁾R + K_ij K^ij - K²) , K = K_i^i
(k² = 16
P_μ : ∂L / ∂(∂₀N^μ)

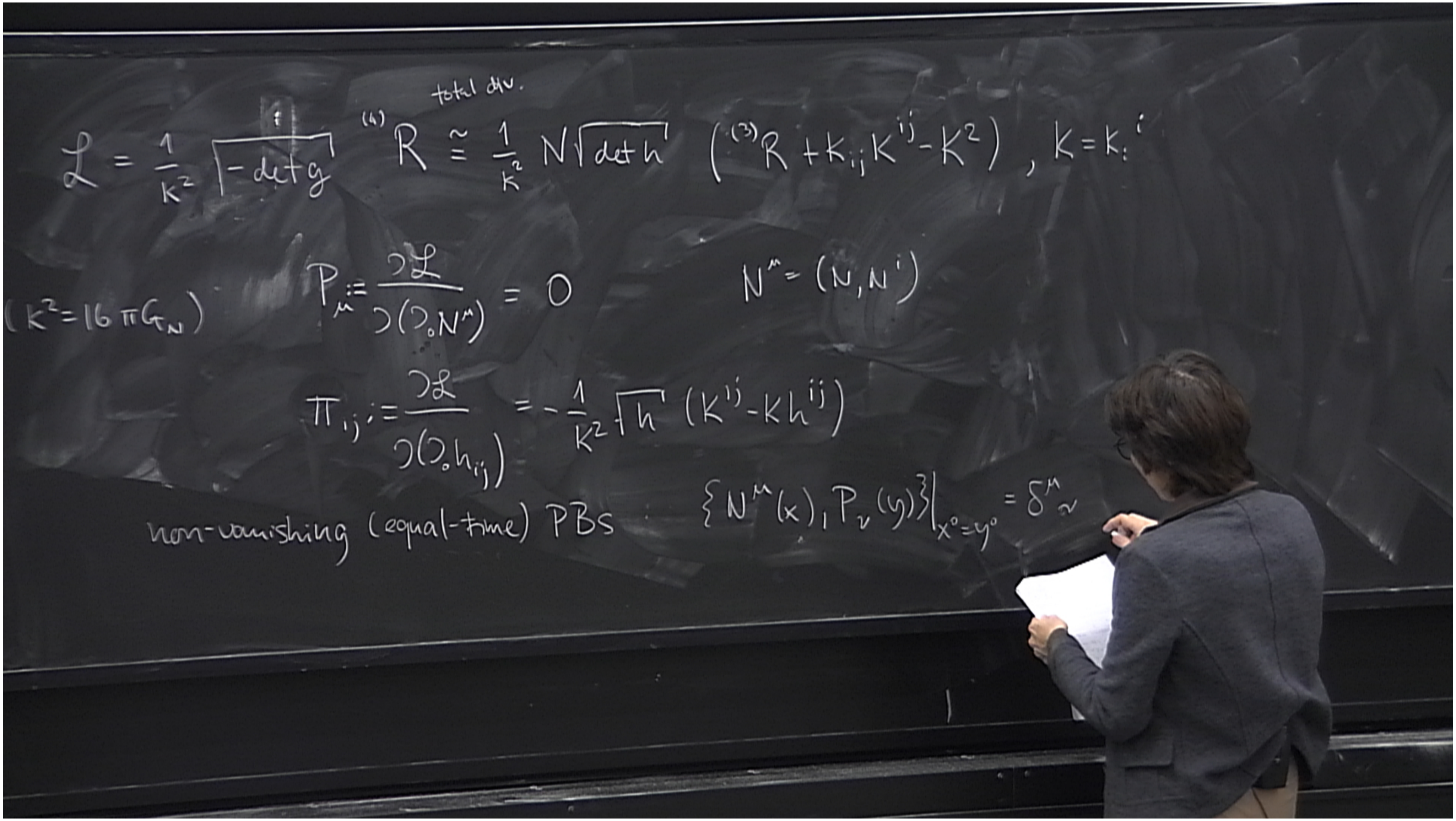

total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \simeq \frac{1}{\kappa^2} N\sqrt{\det h}\ \left({}^{(3)}R + K_{ij}K^{ij} - K^2\right),\ K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)}$
$= (N, N^i)$


total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \simeq \frac{1}{\kappa^2} N\sqrt{\det h}\ ({}^{(3)}R + K_{ij}K^{ij} - K^2)$, $K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
$N^\mu = (N, N^i)$


total div.
L = 1/κ² √(-det g) ⁽⁴⁾R ≃ 1/κ² N √(det h) (⁽³⁾R + K_ij K^ij - K²) , K = K_i^i
(κ² = 16 π G_N)
P_μ := ∂L / ∂(∂₀N^μ) = 0
N^μ = (
π_ij := ∂L / ∂(∂₀h_ij)


total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \simeq \frac{1}{\kappa^2} N\sqrt{\det h}\,\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$, $K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu := \frac{\partial\mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
$N^\mu = (N, N^i)$
$\pi_{ij} := \frac{\partial\mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij} - K h^{ij})$


total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\;{}^{(4)}R \cong \frac{1}{\kappa^2} N\sqrt{\det h}\,\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right), \quad K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
$N^\mu = (N, N^i)$
$\pi_{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij}$
non-vanishing (equal-time)


total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \simeq \frac{1}{\kappa^2} N \sqrt{\det h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$, $K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
$N^\mu = (N, N^i)$
$\pi_{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij} - K h^{ij})$
non-vanishing (equal-time) PBs : $\{N^\mu(x), P_\nu(y)\}\big|_{x^0=y^0} = \delta^\mu{}_\nu$

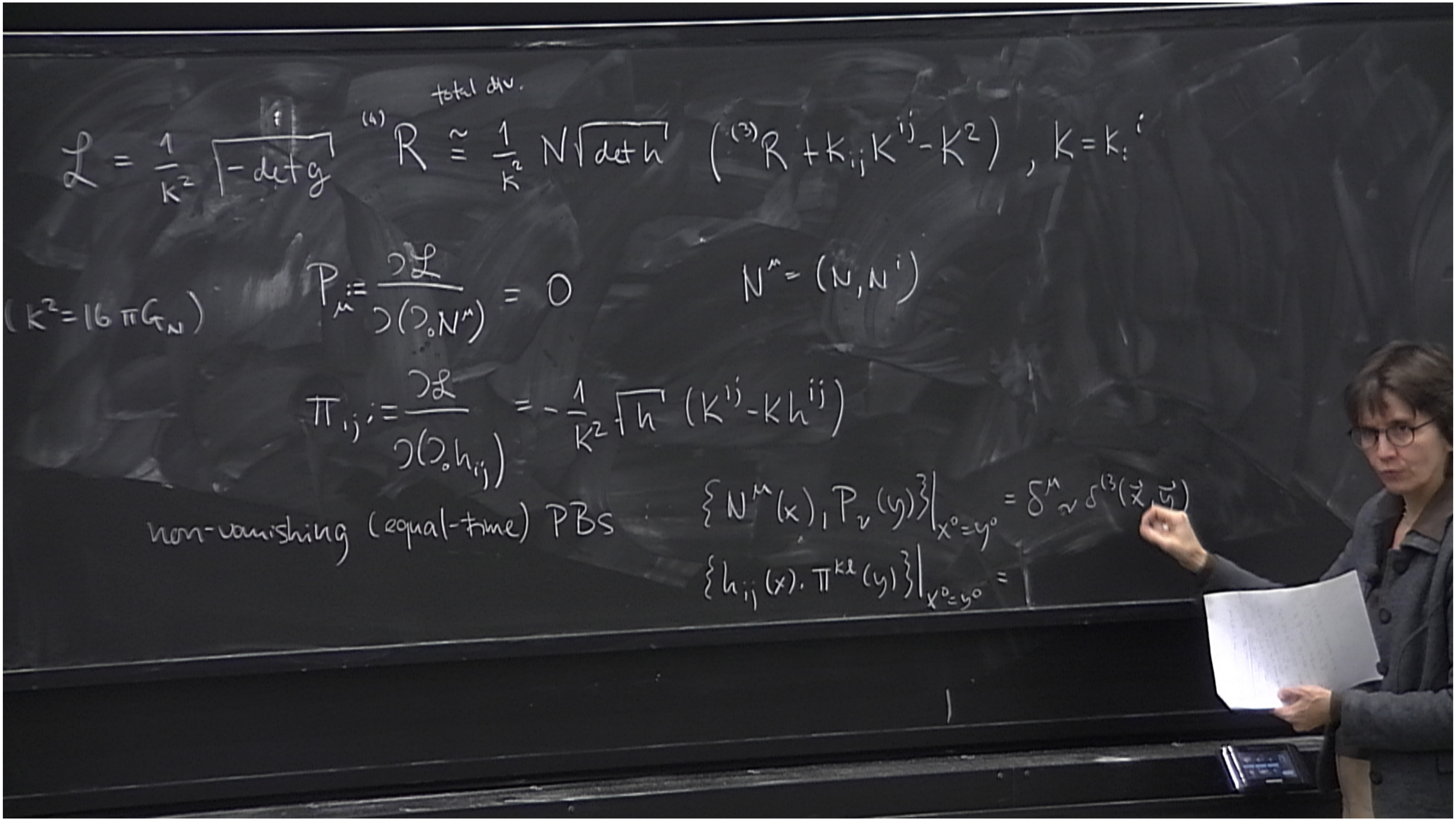
$$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\; {}^{(4)}R \overset{\text{total div.}}{\simeq} \frac{1}{\kappa^2} N\sqrt{\det h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right), \quad K = K_i{}^i$$

$$(\kappa^2 = 16\pi G_N)$$

$$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0 \qquad N^\mu = (N, N^i)$$

$$\pi_{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij} - K h^{ij})$$

non-vanishing (equal-time) PBs: $\{N^\mu(x), P_\nu(y)\}\big|_{x^0=y^0} = \delta^\mu_\nu\, \delta^{(3)}(\vec{x}, \vec{y})$


total div.
$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\ {}^{(4)}R \simeq \frac{1}{\kappa^2} N\sqrt{\det h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$, $K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
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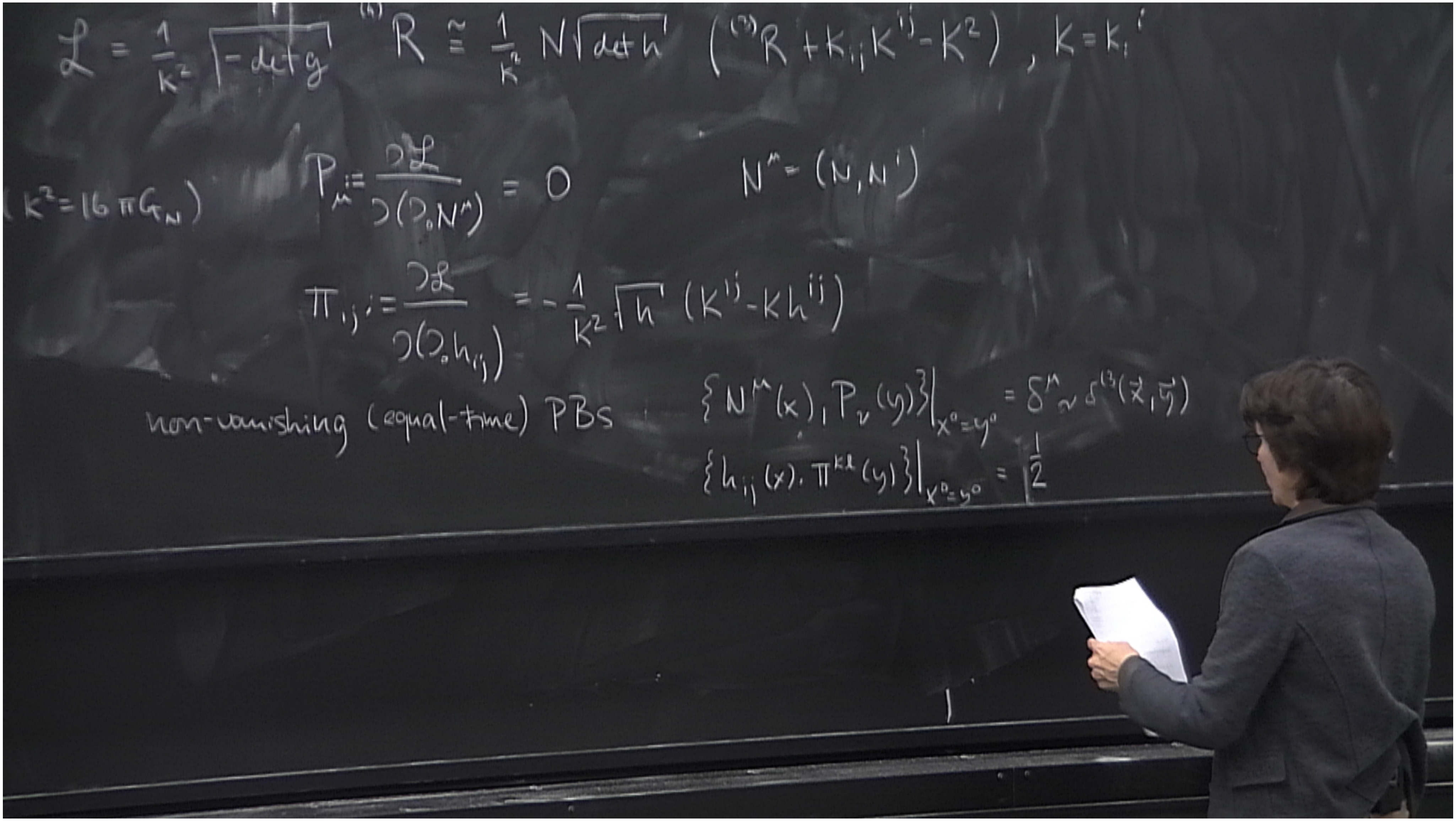
non-vanishing (equal-time) PBs :
$\{N^\mu(x), P_\nu(y)\}\big|_{x^0=y^0} = \delta^\mu_\nu\,\delta^{(3)}(\vec{x}, \vec{y})$
$\{h_{ij}(x), \pi^{kl}(y)\}\big|_{x^0=y^0} =$


$\mathcal{L} = \frac{1}{\kappa^2}\sqrt{-\det g}\,{}^{(4)}R \cong \frac{1}{\kappa^2} N\sqrt{\det h}\left({}^{(3)}R + K_{ij}K^{ij} - K^2\right)$, $K = K_i{}^i$
$(\kappa^2 = 16\pi G_N)$
$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0$
$N^\mu = (N, N^i)$
$\pi^{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij} - K h^{ij})$
non-vanishing (equal-time) PBs
$\{N^\mu(x), P_\nu(y)\}|_{x^0=y^0} = \delta^\mu_\nu\, \delta^{(3)}(\vec{x},\vec{y})$
$\{h_{ij}(x), \pi^{kl}(y)\}|_{x^0=y^0} = \frac{1}{2}$

$(\kappa^2 = 16\pi G_N)$

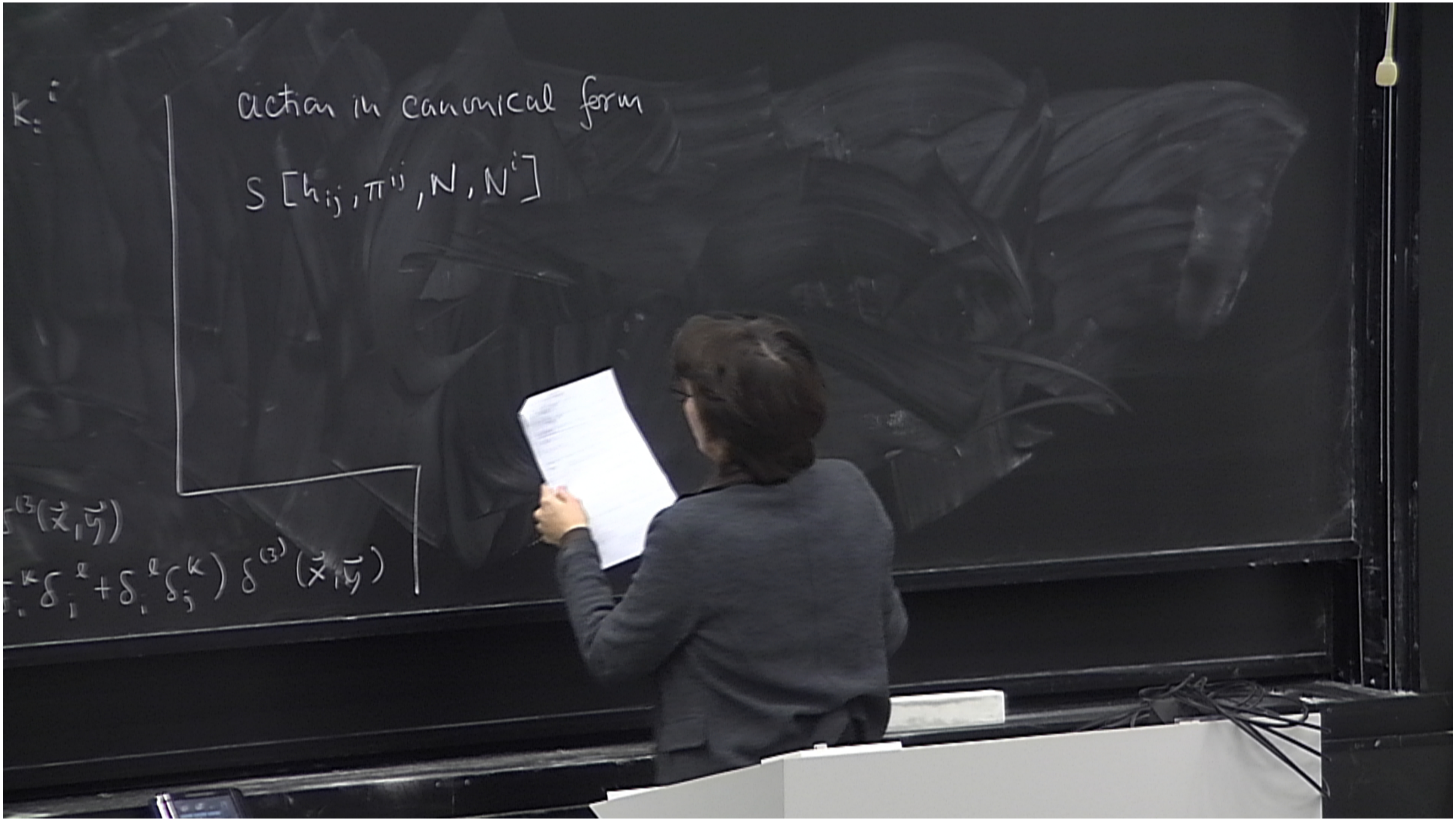
$$P_\mu := \frac{\partial \mathcal{L}}{\partial(\partial_0 N^\mu)} = 0 \qquad N^\mu = (N, N^i)$$

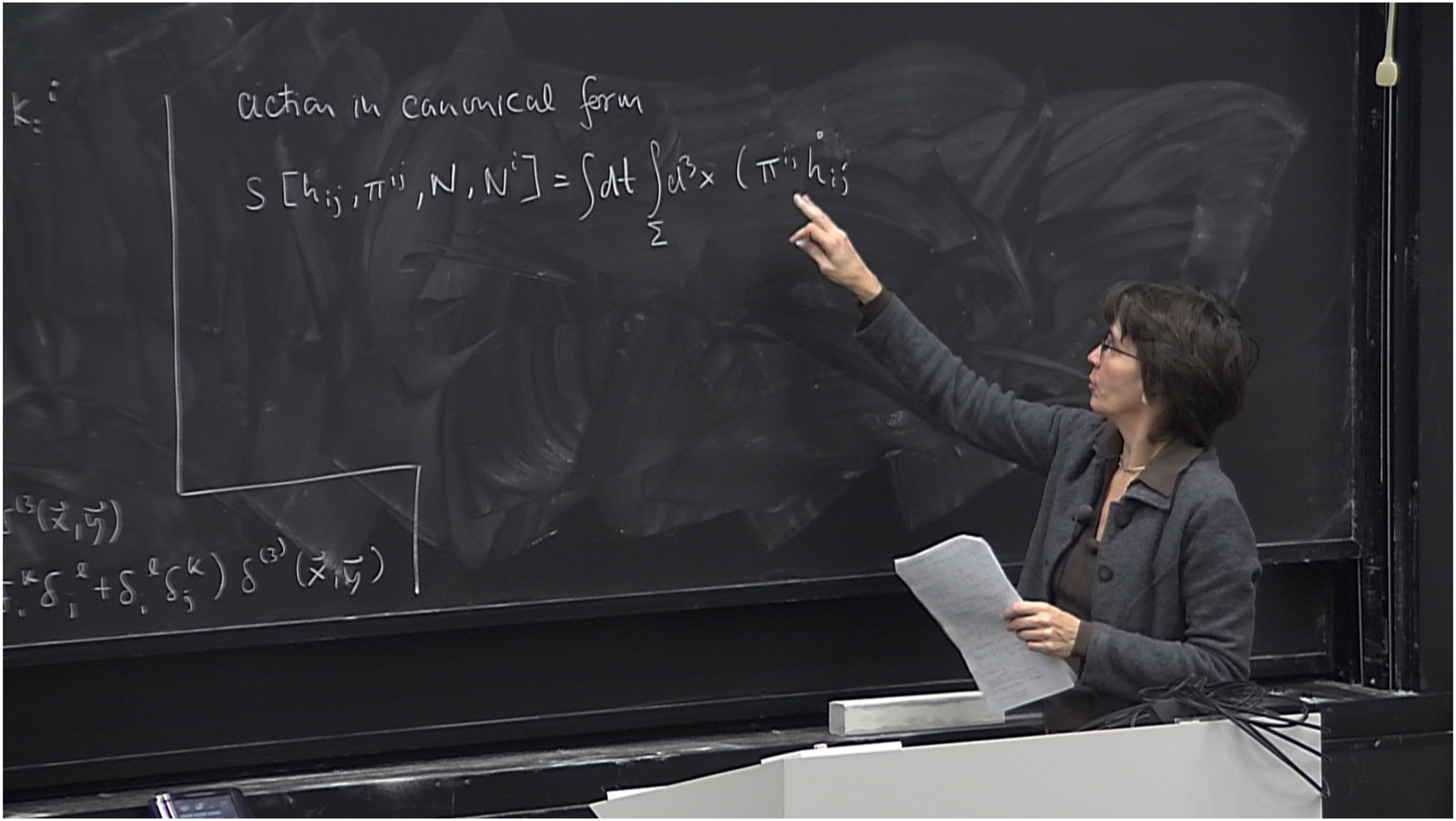
$$\pi^{ij} := \frac{\partial \mathcal{L}}{\partial(\partial_0 h_{ij})} = -\frac{1}{\kappa^2}\sqrt{h}\,(K^{ij} - K h^{ij})$$

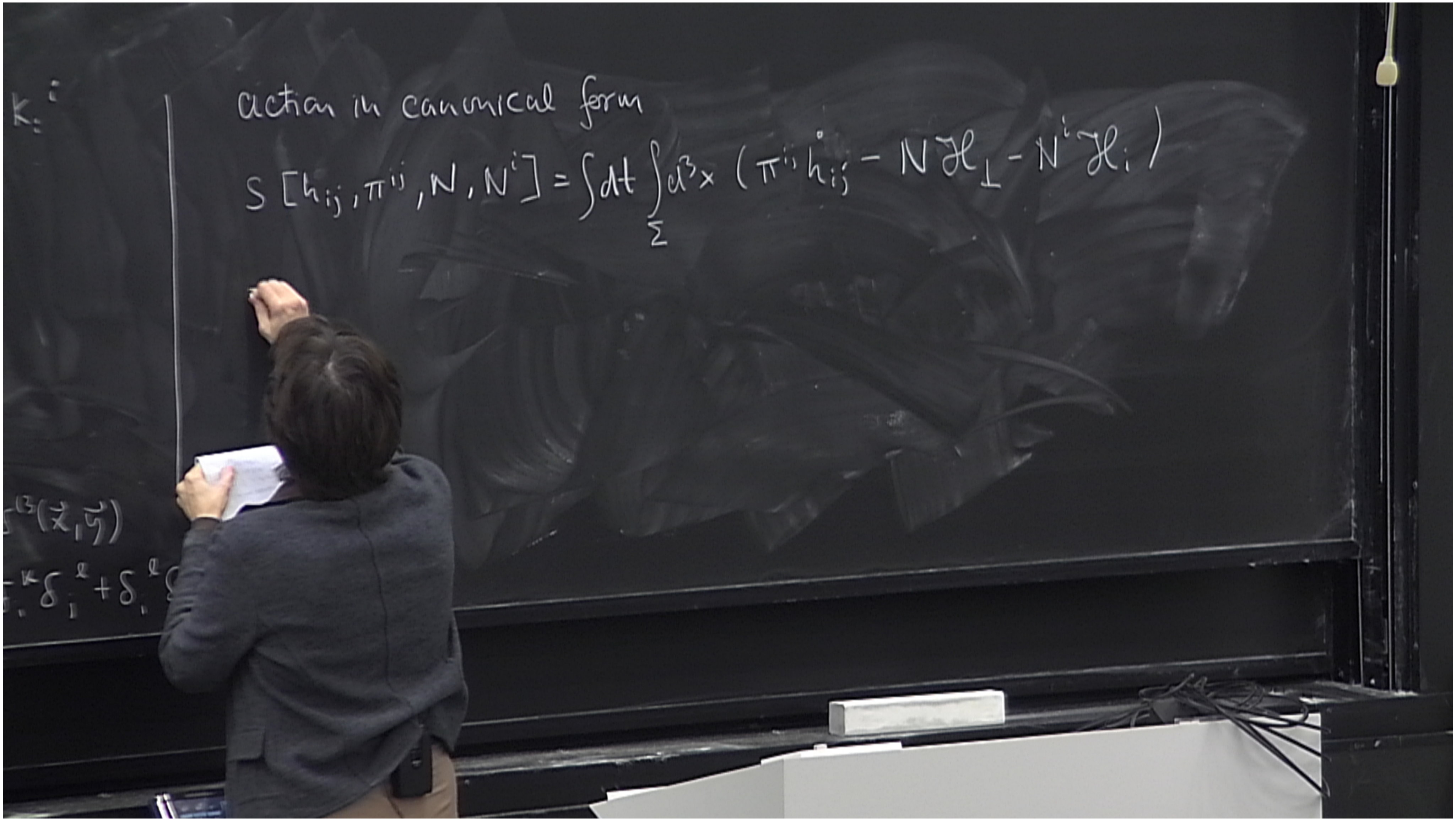
non-vanishing (equal-time) PBs:

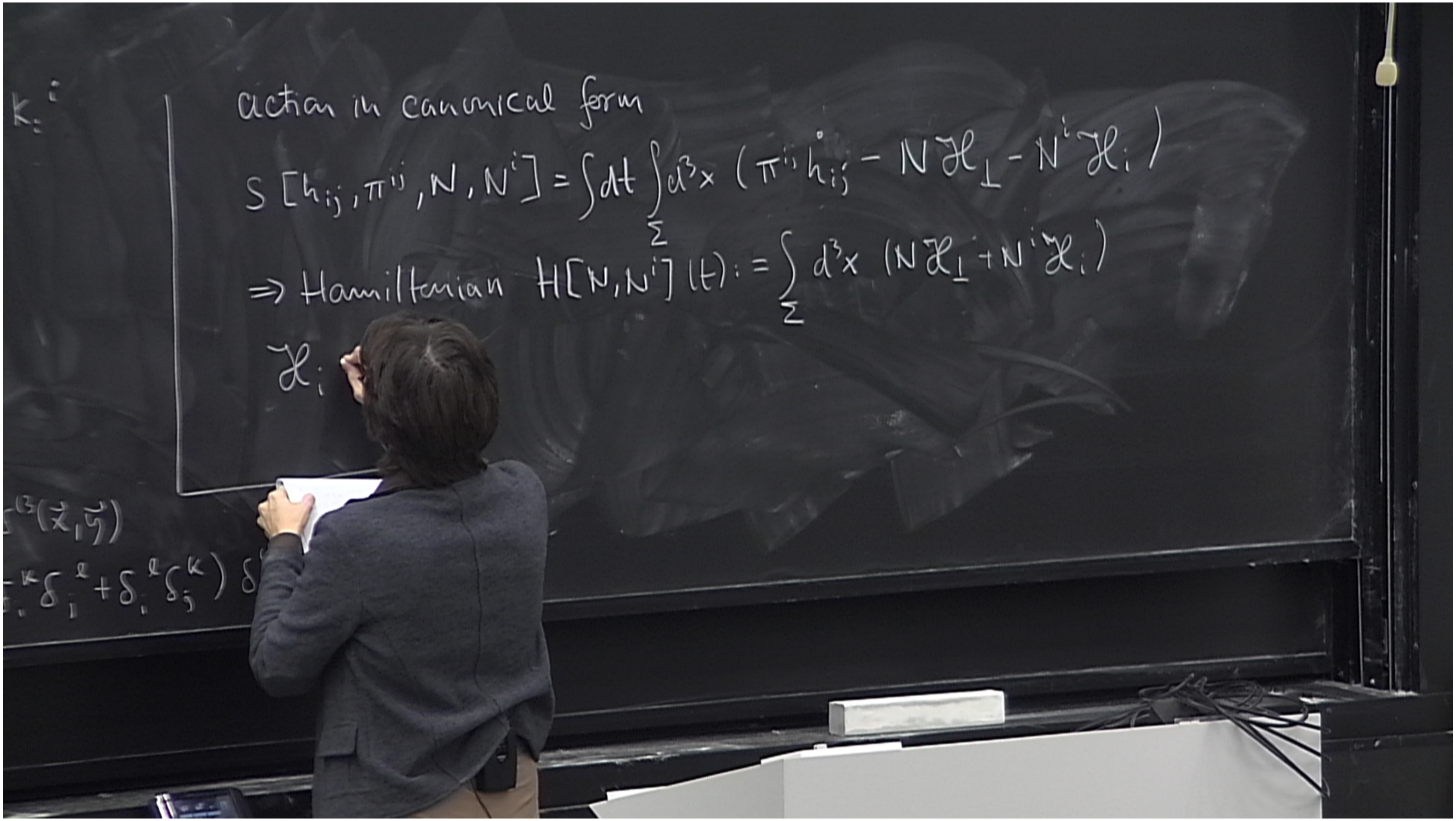
$$\{N^\mu(x), P_\nu(y)\}\big|_{x^0=y^0} = \delta^\mu_\nu\, \delta^{(3)}(\vec{x},\vec{y})$$

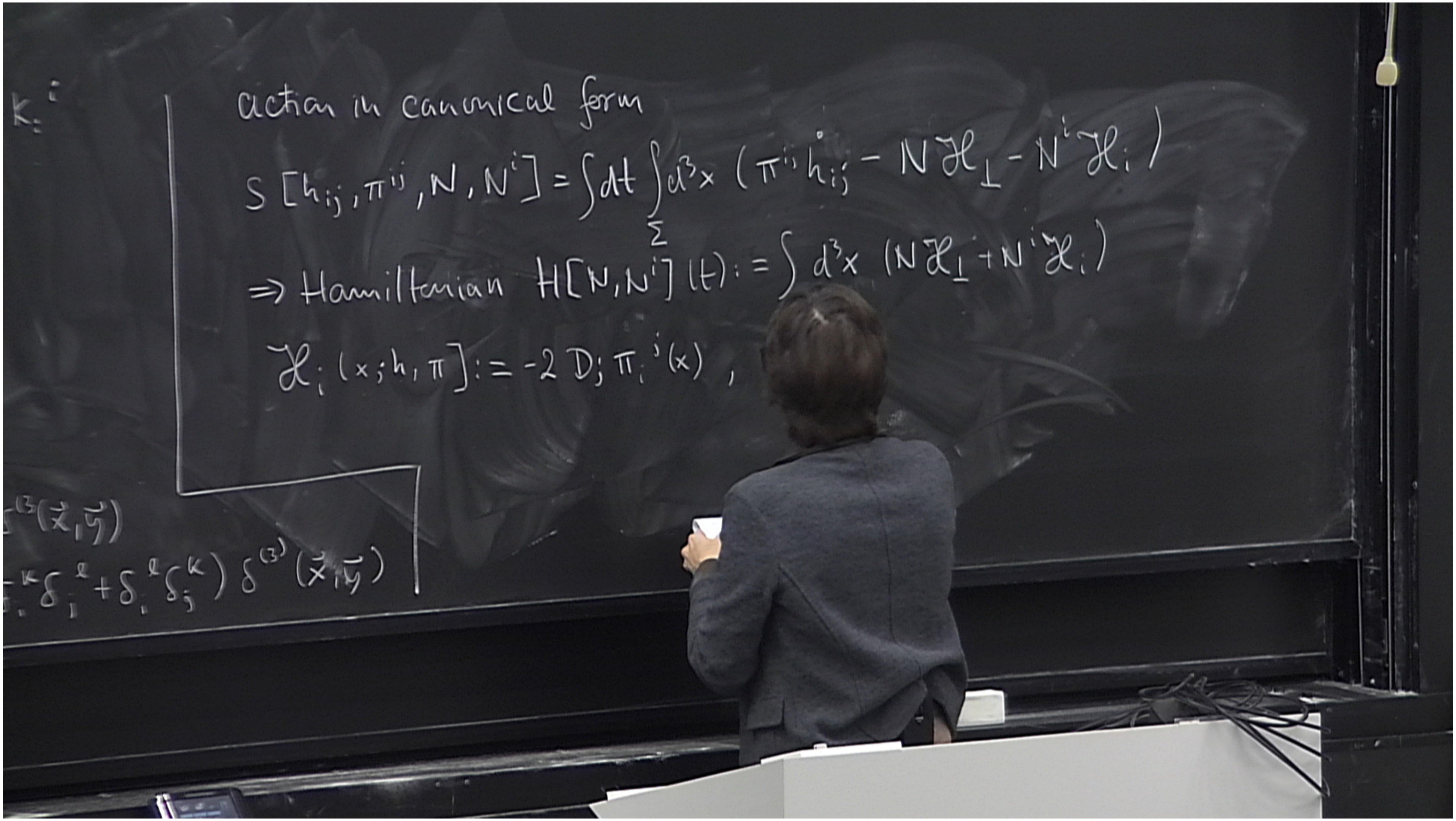
$$\{h_{ij}(x), \pi^{kl}(y)\}\big|_{x^0=y^0} = \frac{1}{2}\left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k\right)\delta^{(3)}(\vec{x},\vec{y})$$

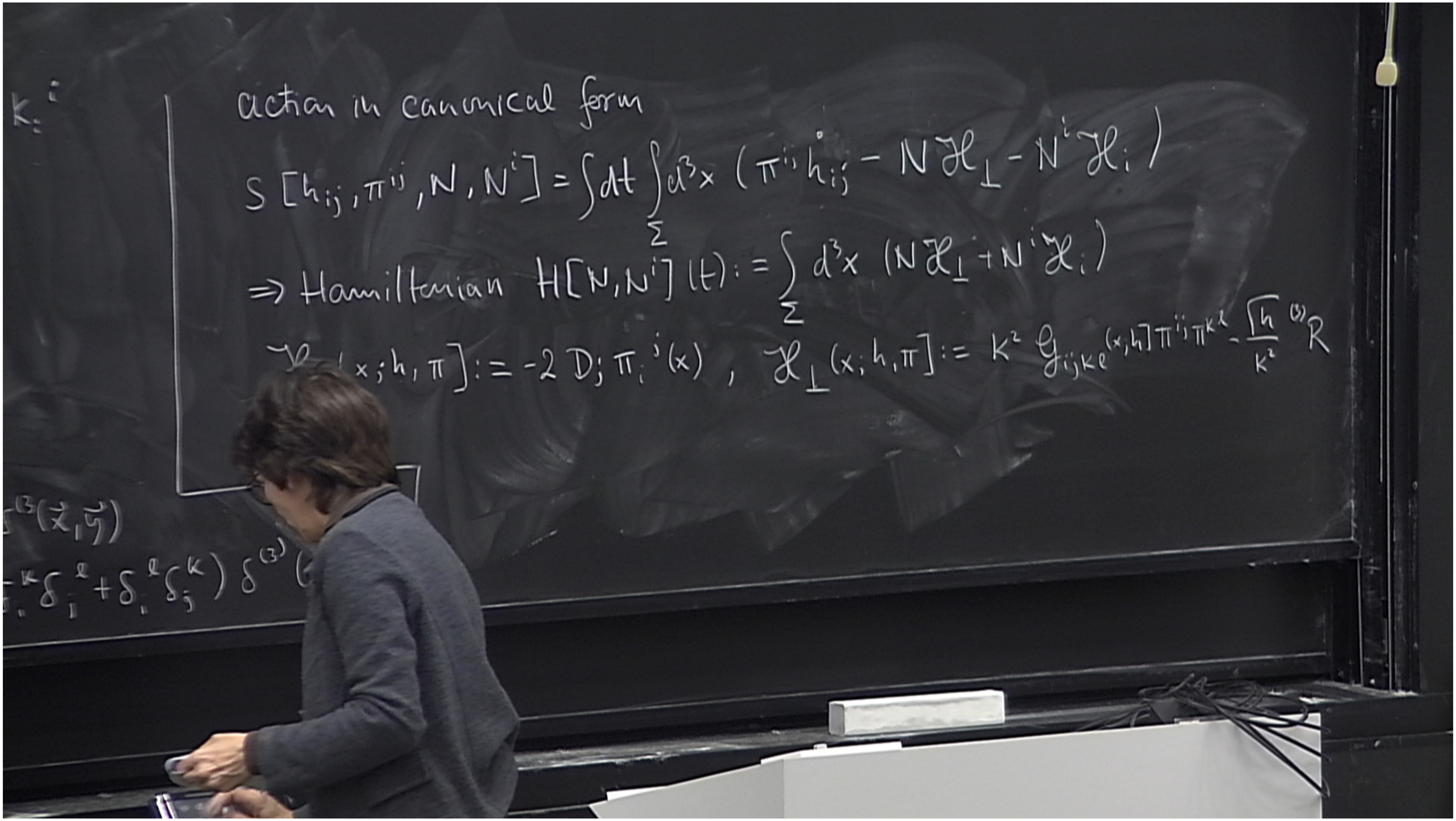

action in canonical form
S [h_ij , π^ij , N , N^i]

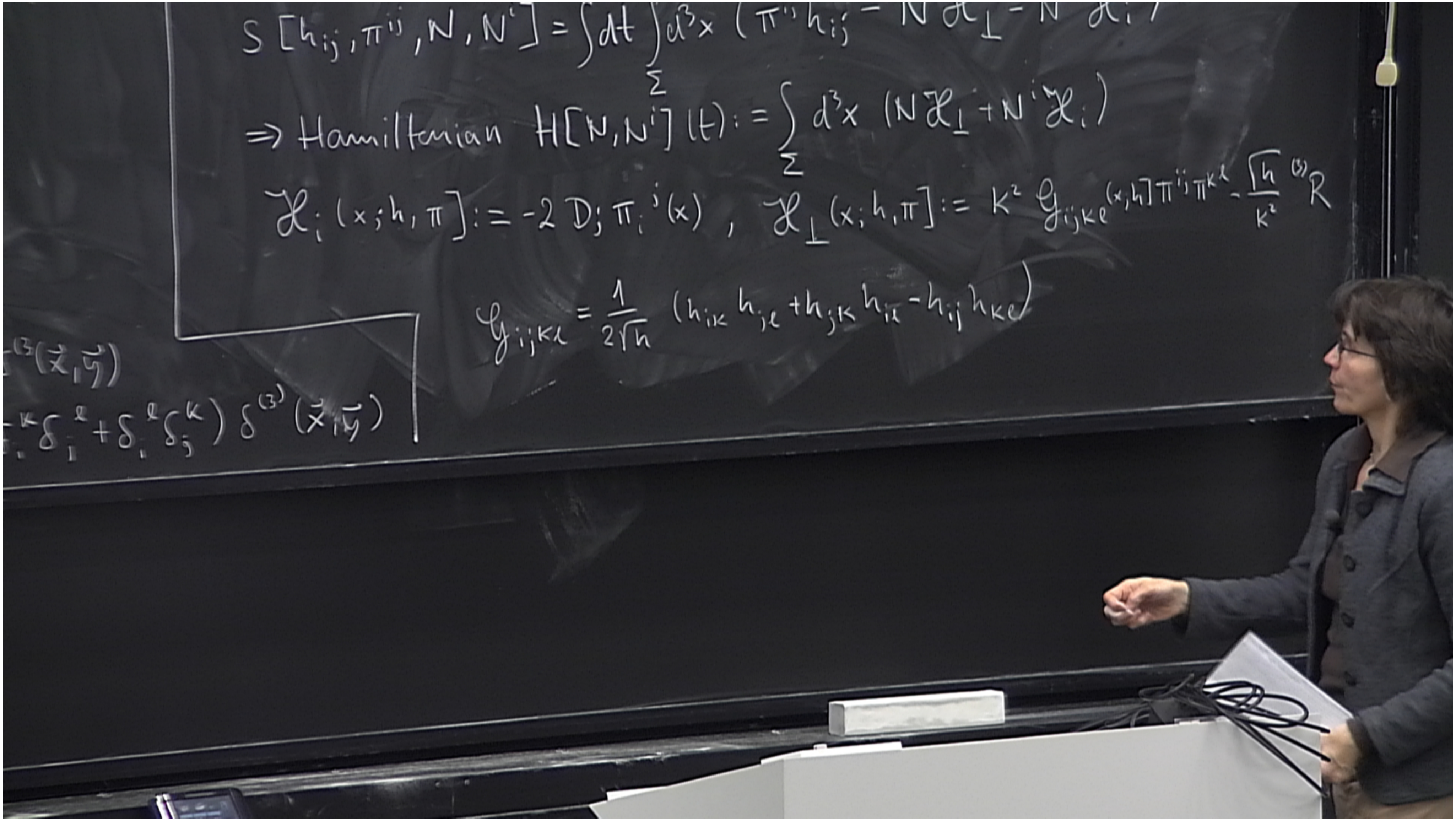

action in canonical form
$S[h_{ij}, \pi^{ij}, N, N^i] = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij}$


action in canonical form
$S[h_{ij}, \pi^{ij}, N, N^i] = \int dt \int_\Sigma d^3x \, (\pi^{ij} \dot{h}_{ij} - N \mathcal{H}_\perp - N^i \mathcal{H}_i)$


action in canonical form
S [h_ij, π^ij, N, N^i] = ∫dt ∫_Σ d^3x (π^ij ḣ_ij − N H_⊥ − N^i H_i)
⇒ Hamiltonian H[N, N^i] (t) := ∫_Σ d^3x (N H_⊥ + N^i H_i)
H_i


action in canonical form
S [h_ij , π^ij , N , N^i] = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i)
⇒ Hamiltonian H[N, N^i] (t) := ∫ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i (x; h, π] := −2 D_j π_i^j (x) ,


action in canonical form
S[h_ij, π^ij, N, N^i] = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i)
⇒ Hamiltonian H[N, N^i](t) := ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
(x; h, π] := −2 D_j π_i^j(x) , ℋ_⊥(x; h, π] := κ² G_ijkl(x; h] π^ij π^kl − √h/κ² ⁽³⁾R


S [h_ij , π^ij , N , N^i] = ∫dt ∫_Σ d³x (π^ij ḣ_ij − N ℋ_⊥ − N^i ℋ_i)
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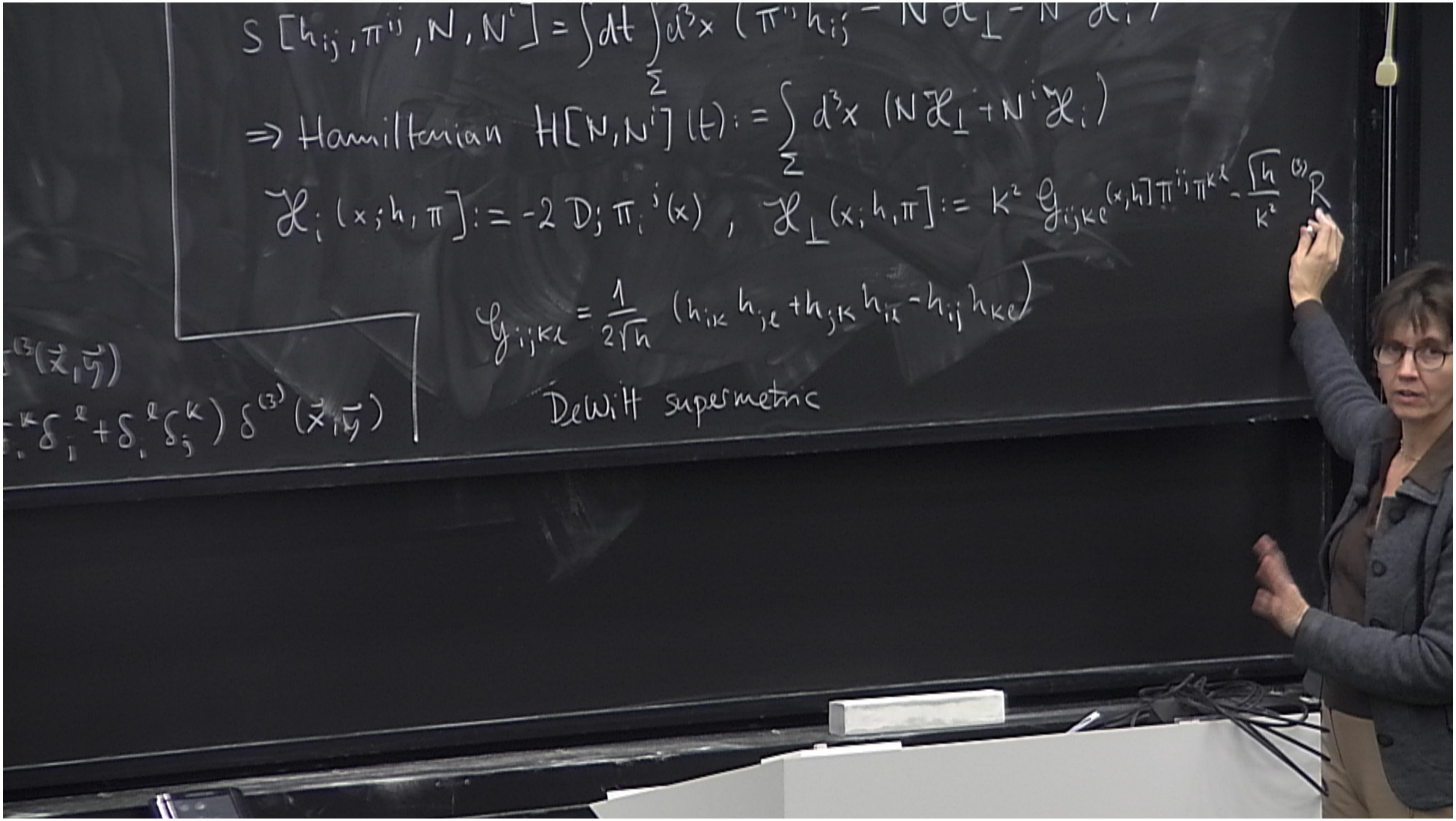
ℋ_i (x; h, π] := −2 D_j π_i^j (x) , ℋ_⊥ (x; h, π] := κ² G_ijkl (x; h] π^ij π^kl − √h/κ² ⁽³⁾R
G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il − h_ij h_kl)


⇒ Hamiltonian H[N,N^i](t) := ∫_Σ d³x (N ℋ_⊥ + N^i ℋ_i)
ℋ_i(x; h, π] := −2 D_j π_i^j(x) , ℋ_⊥(x; h, π] := κ² G_ijkl(x; h] π^ij π^kl − √h/κ² ⁽³⁾R
G_ijkl = 1/(2√h) (h_ik h_jl + h_jk h_il − h_ij h_kl)
DeWitt supermetric