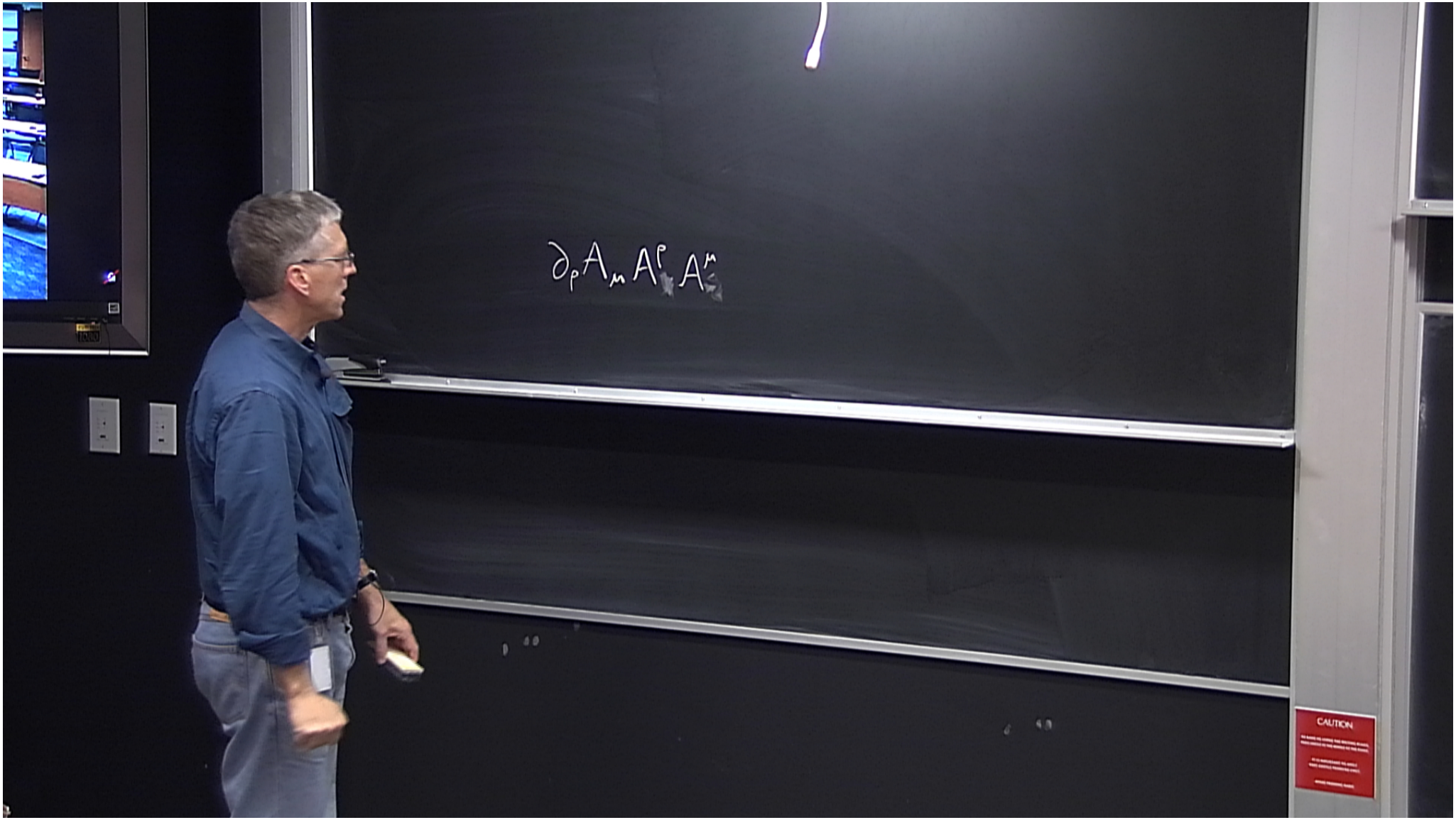
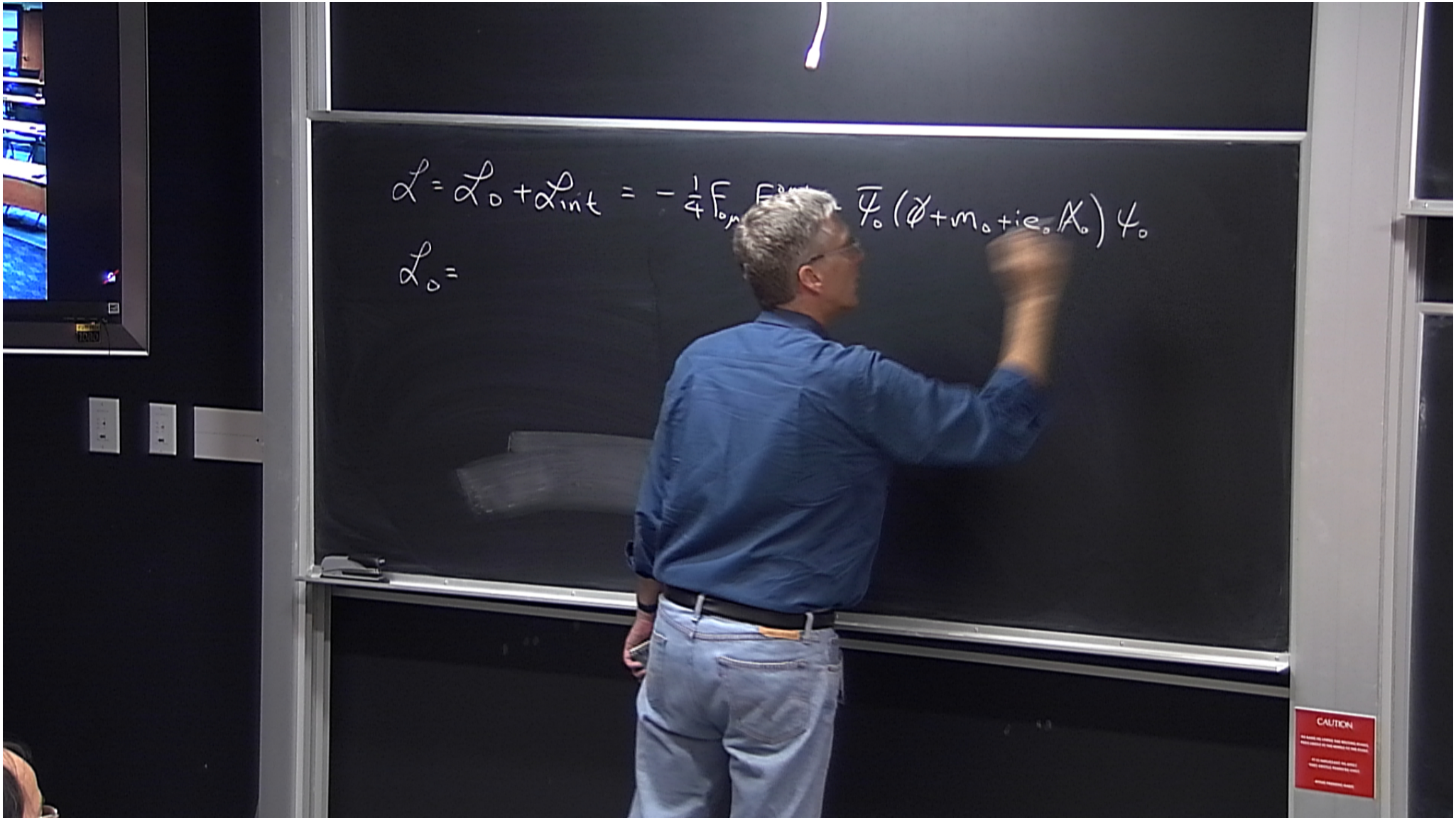
Title: Advanced Quantum Field Theory - Lecture 7

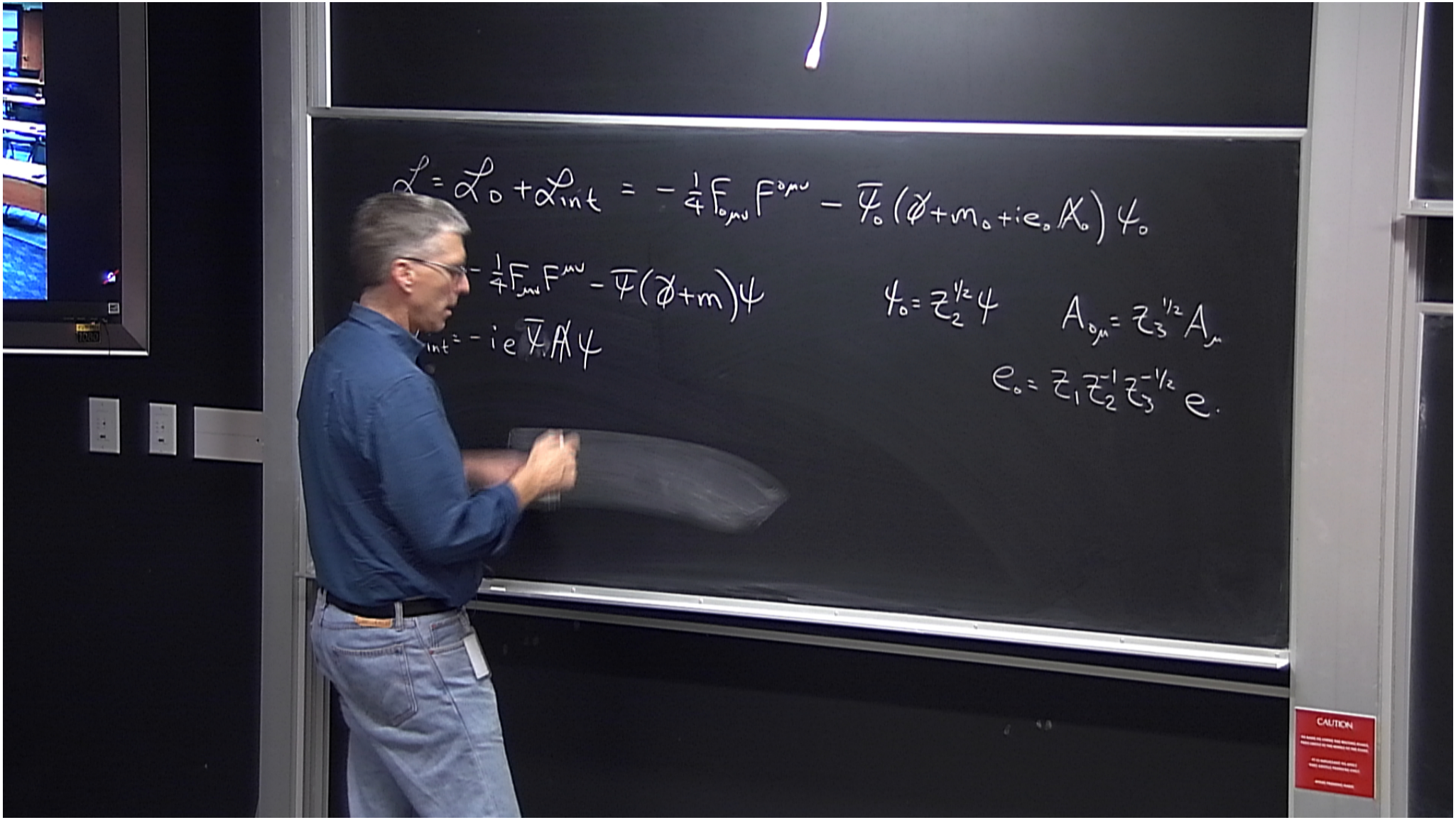
Date: Jan 31, 2012 09:30 AM

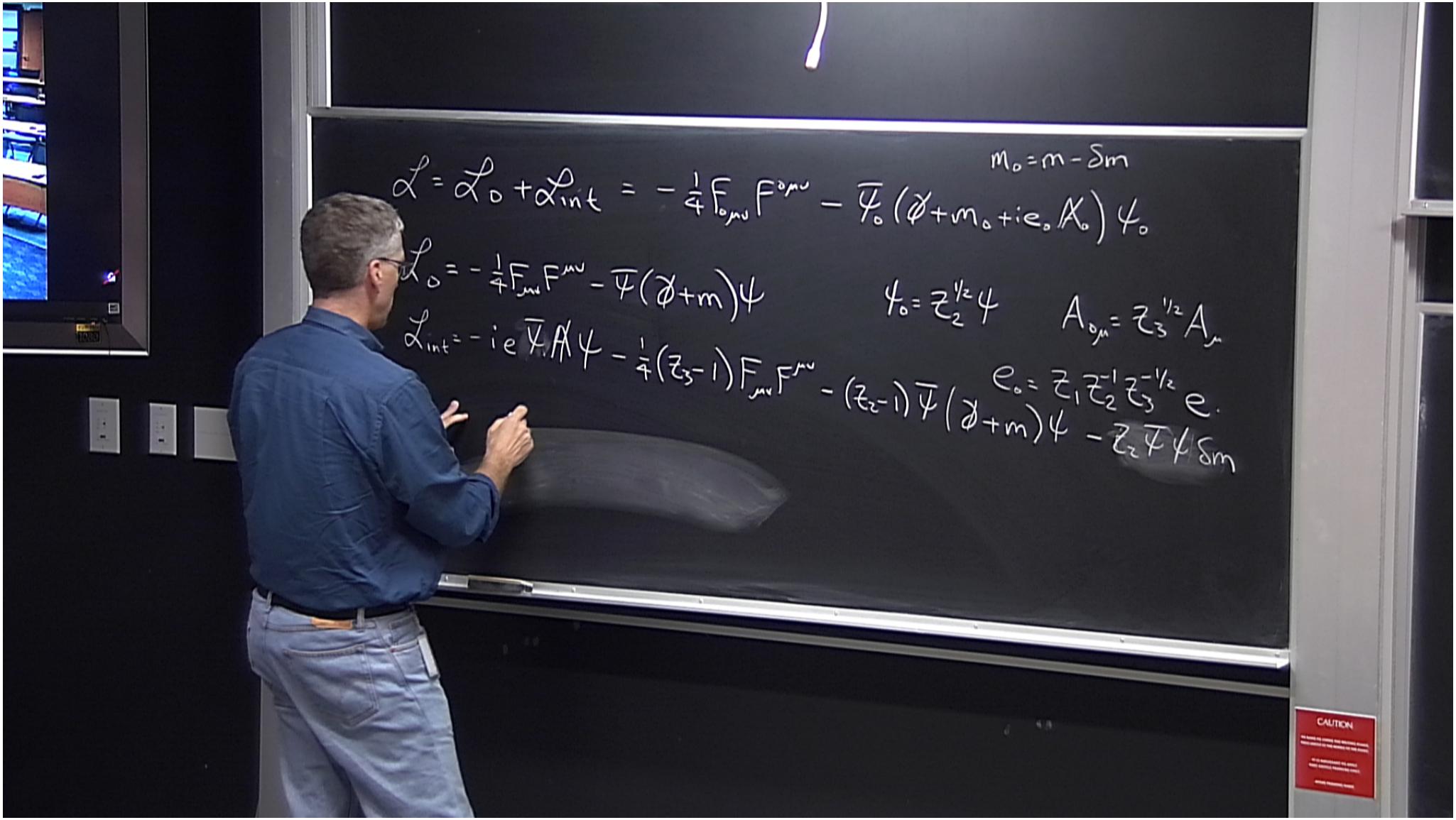
URL: http://pirsa.org/12010163

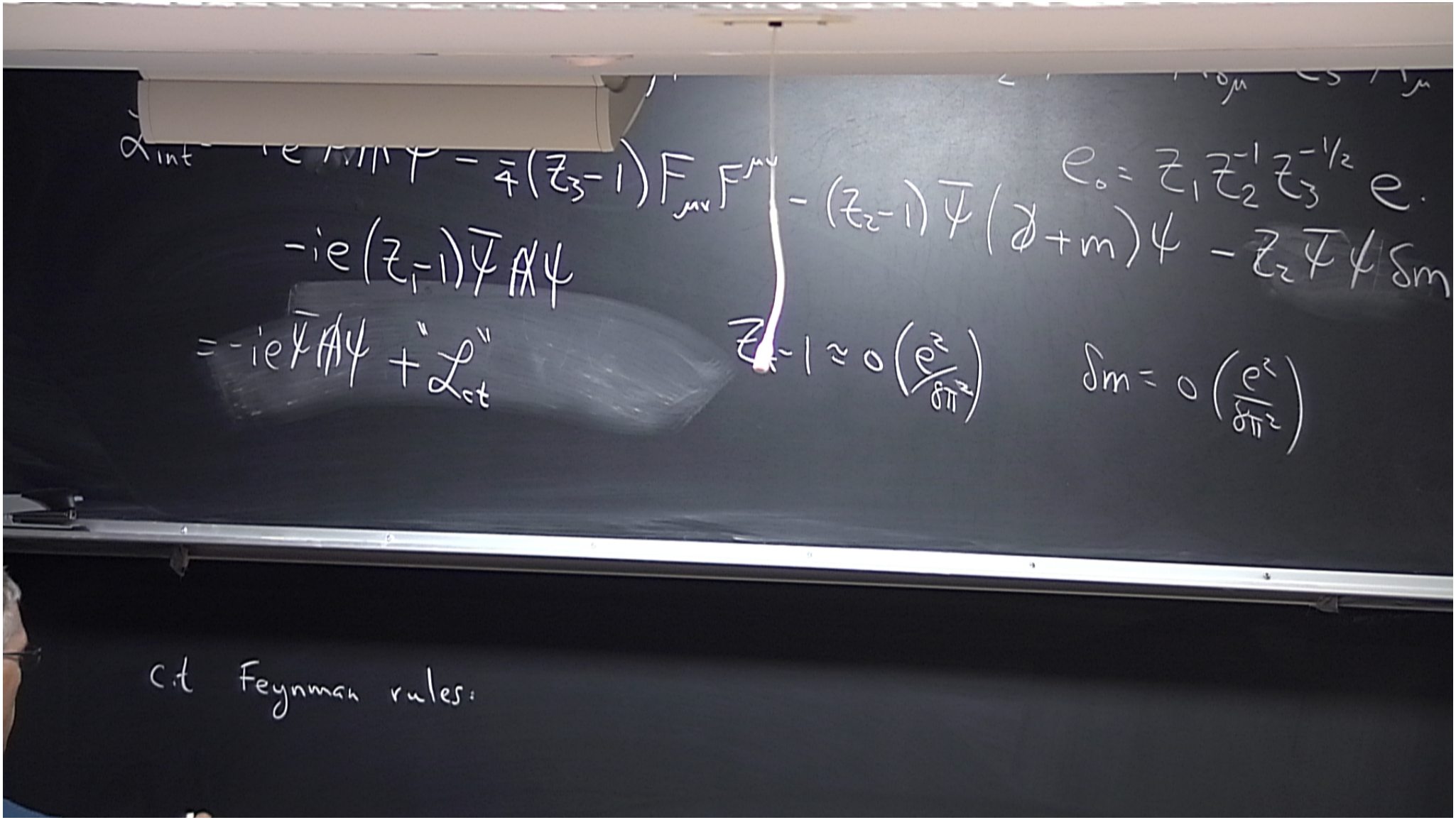
Abstract:

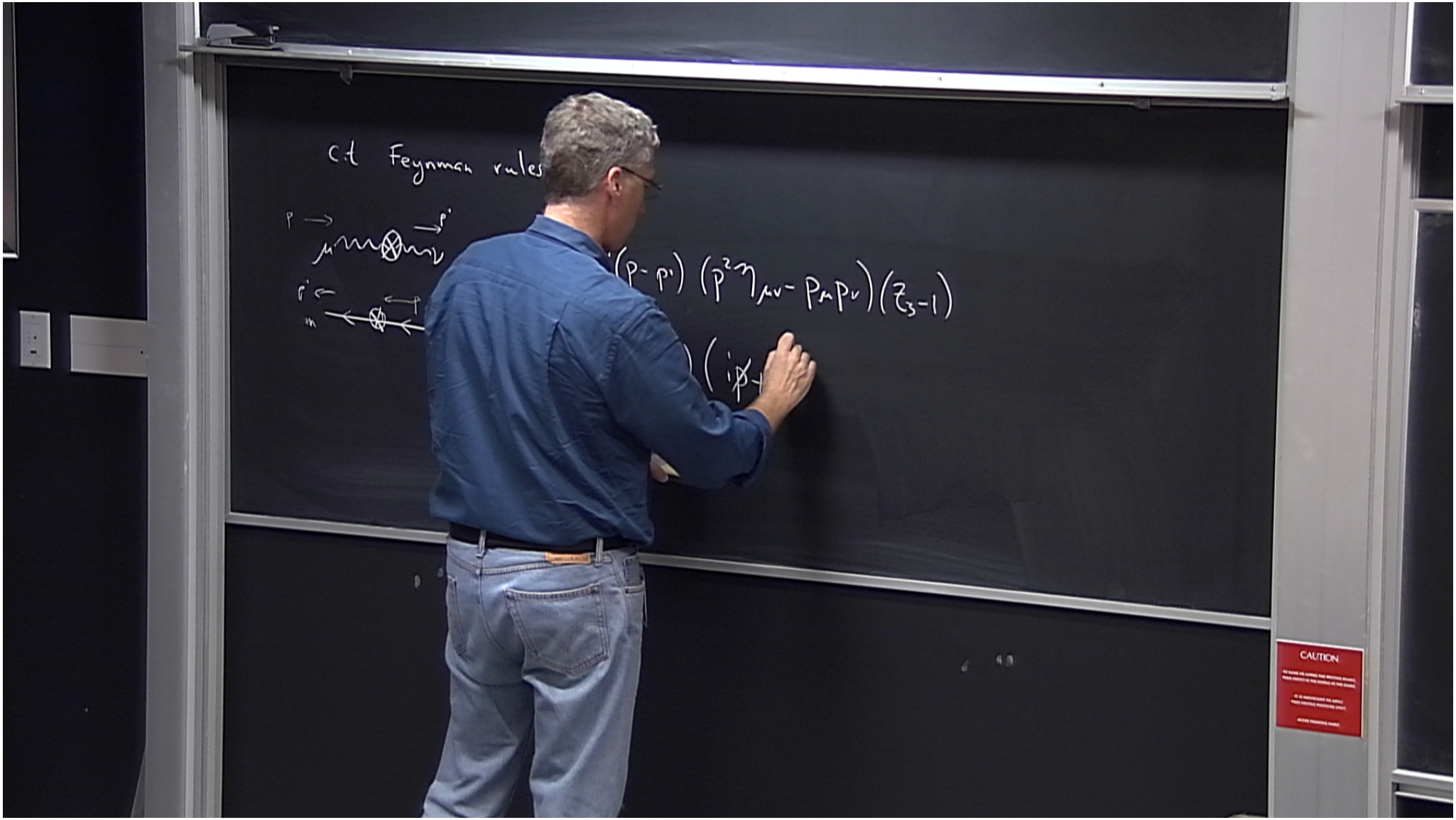

$\partial_\rho A_\mu A^\rho A^\mu$
CAUTION

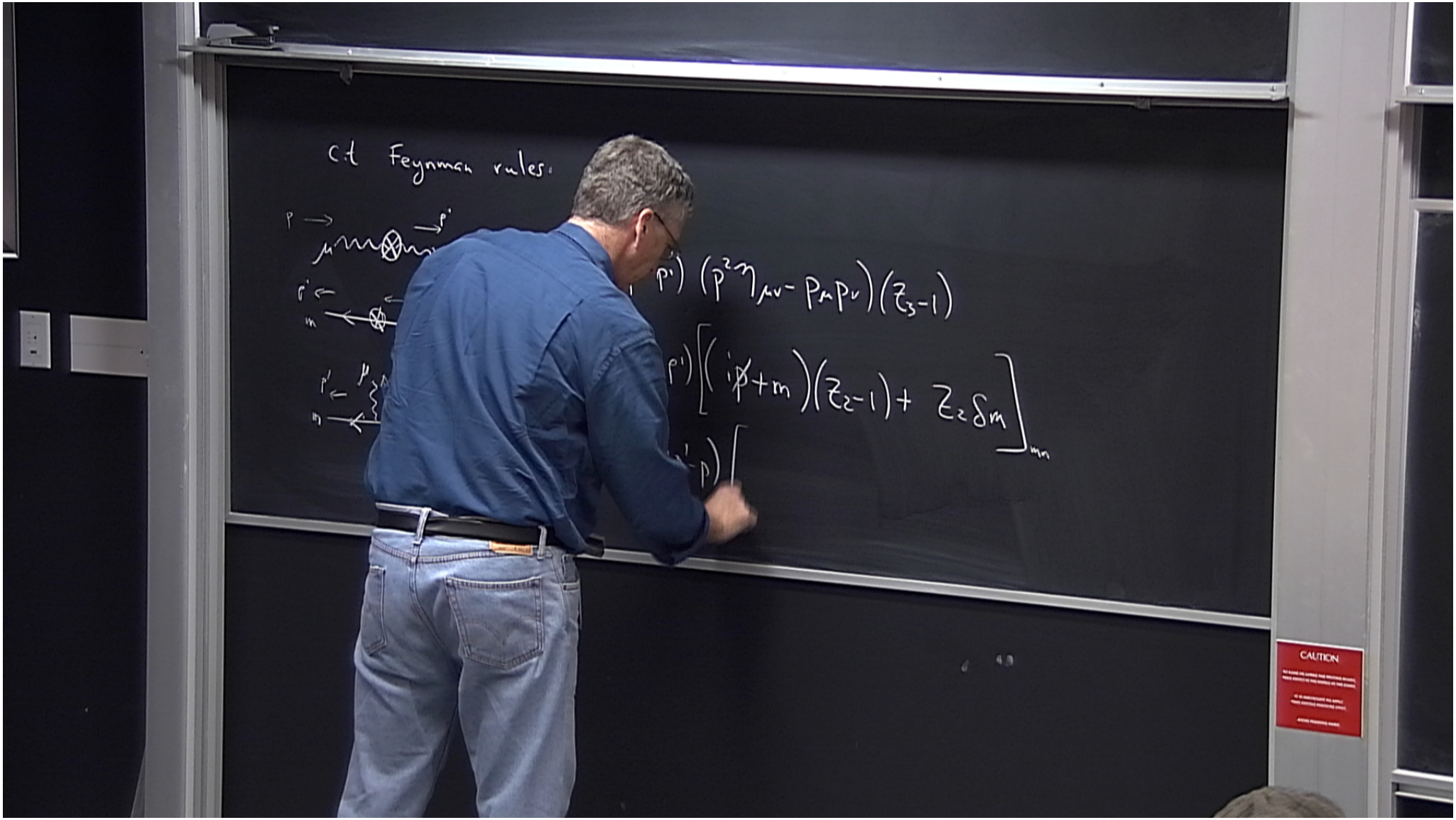

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + ie_0 \not{A}_0) \psi_0$
$\mathcal{L}_0 =$
CAUTION


$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + i e_0 \not{A}_0) \psi_0$
$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} (\not{\partial} + m) \psi$
$_{int} = -ie \bar{\psi} \not{A} \psi$
$\psi_0 = Z_2^{1/2} \psi$
$A_{0\mu} = Z_3^{1/2} A_\mu$
$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$.
CAUTION

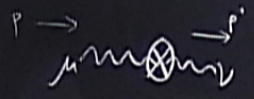

$m_0 = m - \delta m$
$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + i e_0 \not{A}_0) \psi_0$
$\mathcal{L}_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\psi}(\not{\partial} + m)\psi$
$\psi_0 = Z_2^{1/2} \psi$
$A_{0\mu} = Z_3^{1/2} A_\mu$
$\mathcal{L}_{int} = -ie \bar{\psi} \not{A} \psi - \frac{1}{4}(Z_3 - 1) F_{\mu\nu} F^{\mu\nu} - (Z_2 - 1)\bar{\psi}(\not{\partial} + m)\psi - Z_2 \bar{\psi} \psi \delta m$
$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$
CAUTION


$\mathcal{L}_{int} = -ie\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}(Z_3-1)F_{\mu\nu}F^{\mu\nu} - (Z_2-1)\bar{\psi}(\partial+m)\psi - Z_2\bar{\psi}\psi\delta m$
$-ie(Z_1-1)\bar{\psi}\gamma^\mu\psi A_\mu$
$= -ie\bar{\psi}\gamma^\mu\psi A_\mu + \mathcal{L}_{ct}$
$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$.
$Z_i - 1 \approx O\left(\frac{e^2}{8\pi^2}\right)$
$\delta m = O\left(\frac{e^2}{8\pi^2}\right)$
c.t Feynman rules:


c.t Feynman rules
p
p'
p'
m
$(p-p')\,(p^2\eta_{\mu\nu}-p_\mu p_\nu)(Z_3-1)$
CAUTION


c.t Feynman rules.
$(p^2\eta_{\mu\nu} - p_\mu p_\nu)(Z_3 - 1)$
$\left[(i\not{p} + m)(Z_2 - 1) + Z_2\delta_m\right]_{mn}$
CAUTION

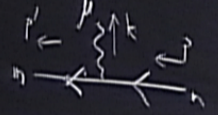
c.t Feynman rules.



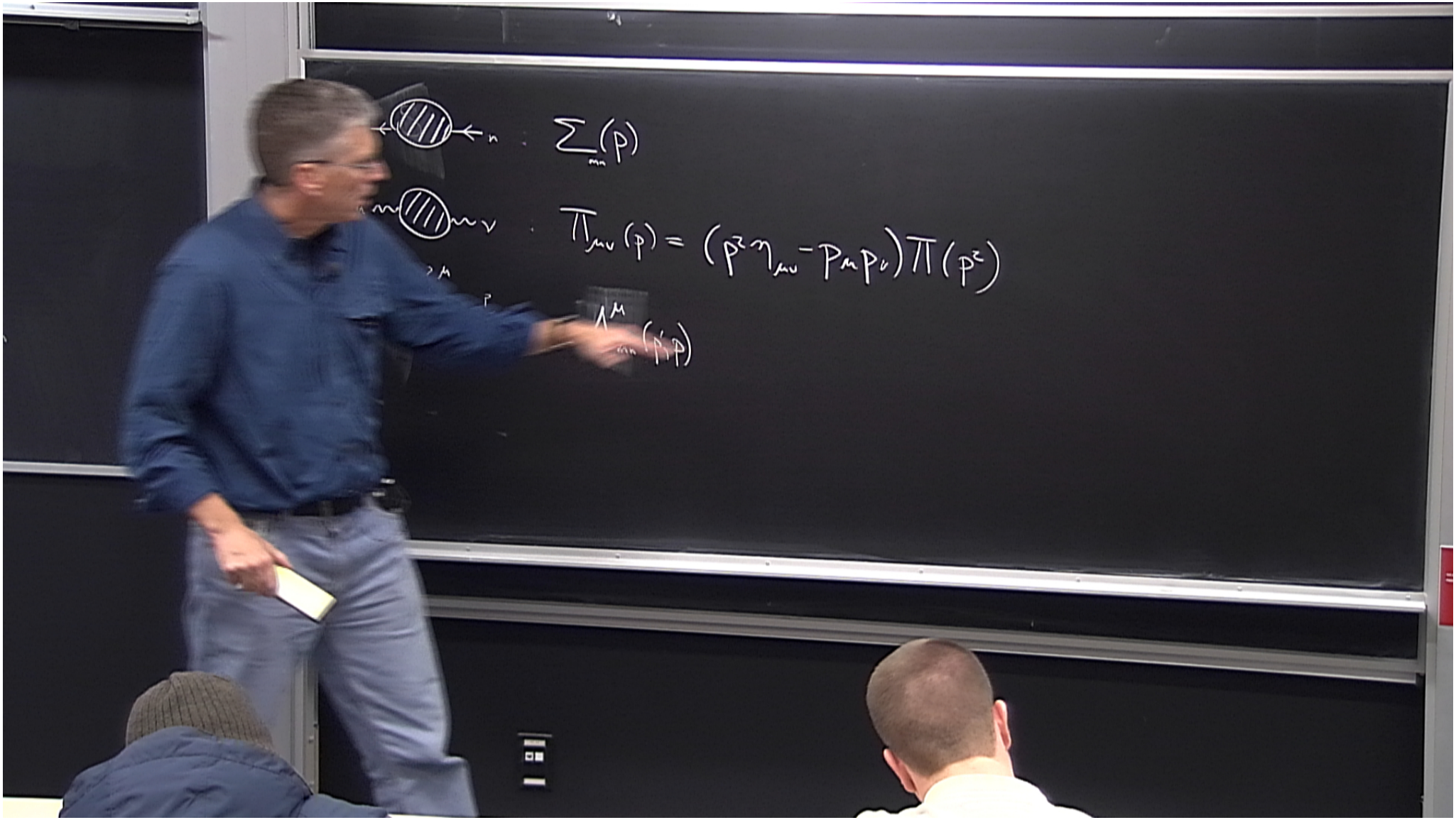
$$i(2\pi)^4 \delta^4(p-p')\,(p^2\eta_{\mu\nu} - p_\mu p_\nu)(Z_3-1)$$

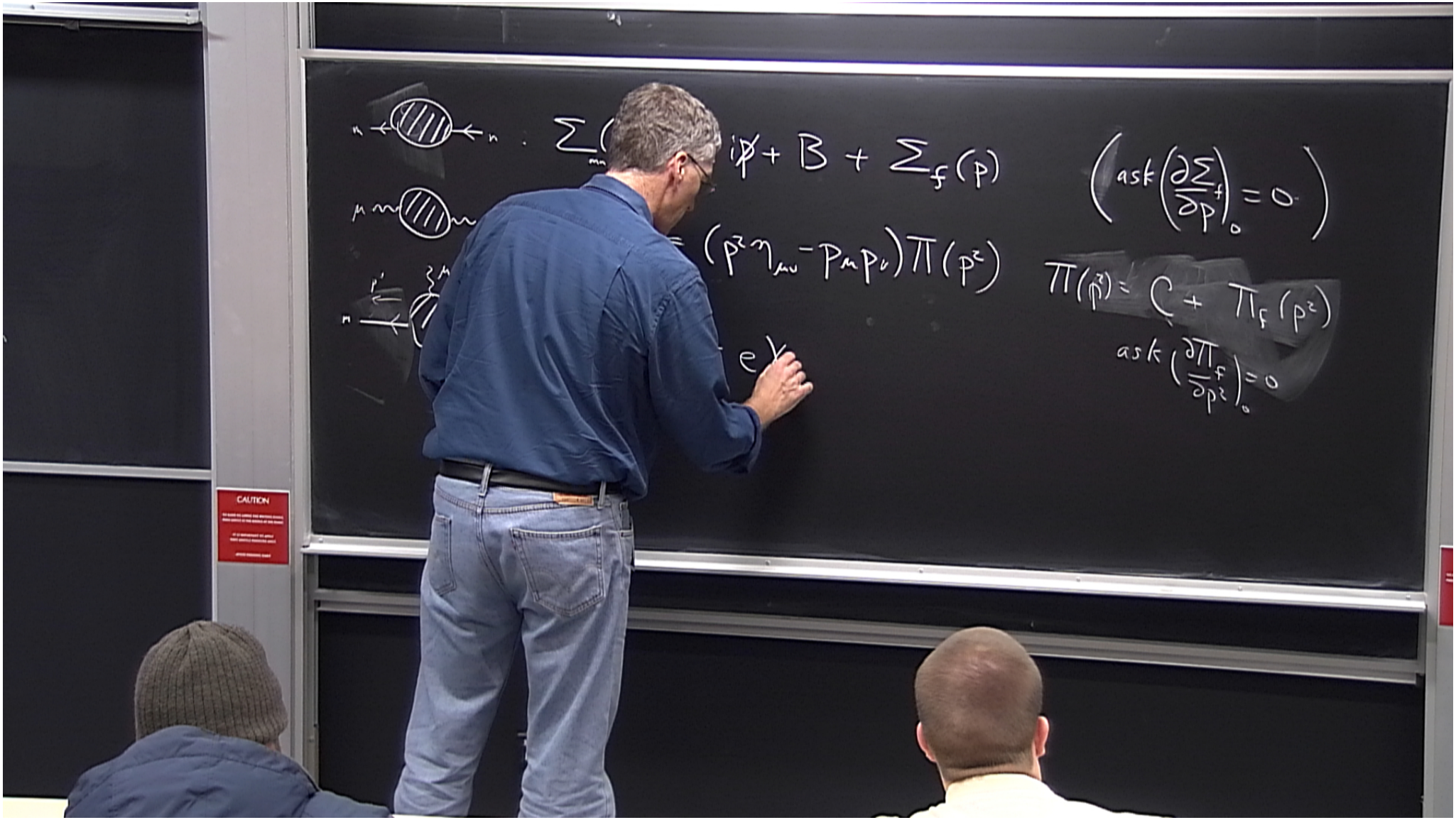


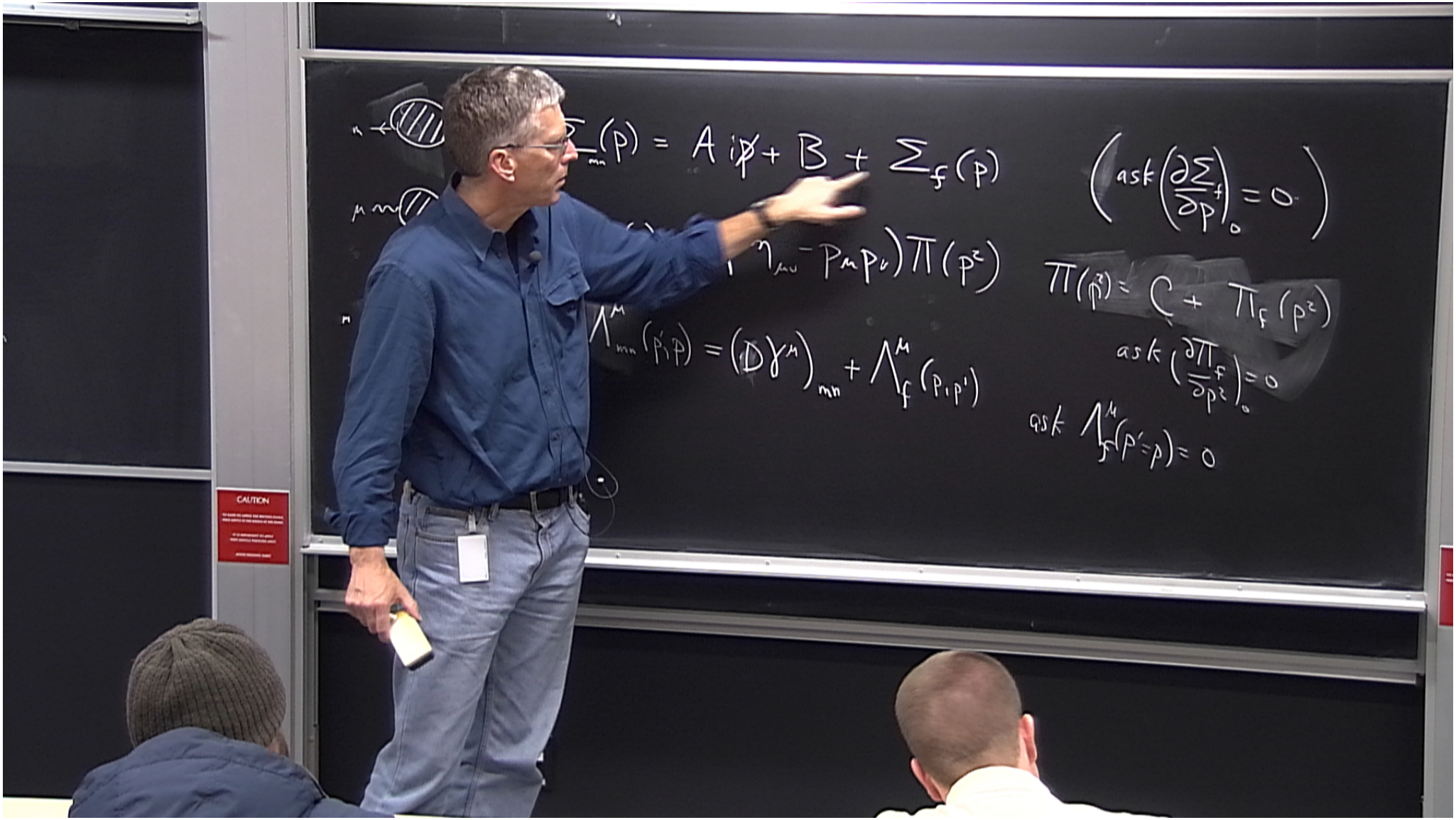
$$i(2\pi)^4 \delta^4(p-p')\left[(i\not{p}+m)(Z_2-1) + Z_2\delta m\right]_{mn}$$

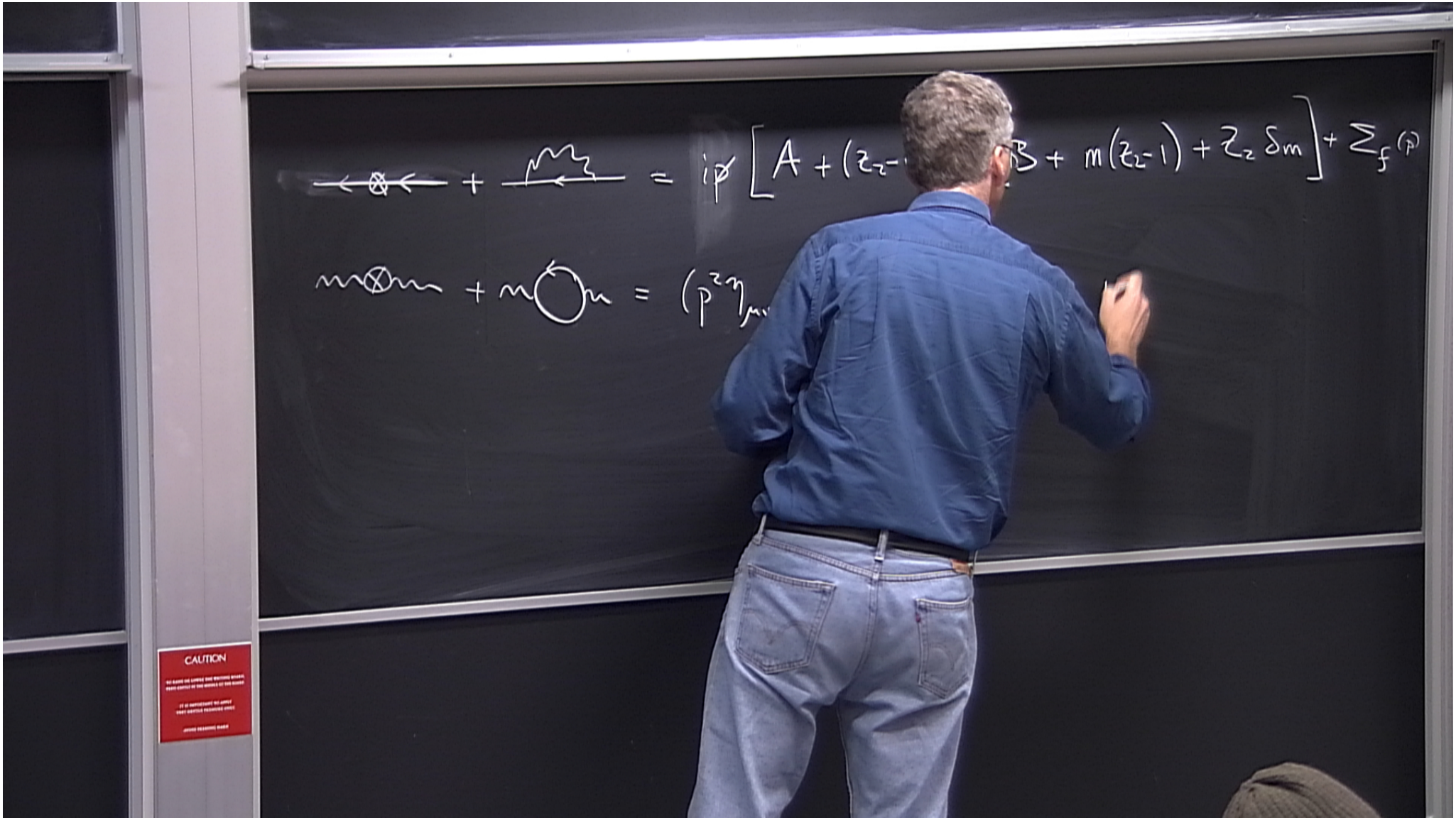


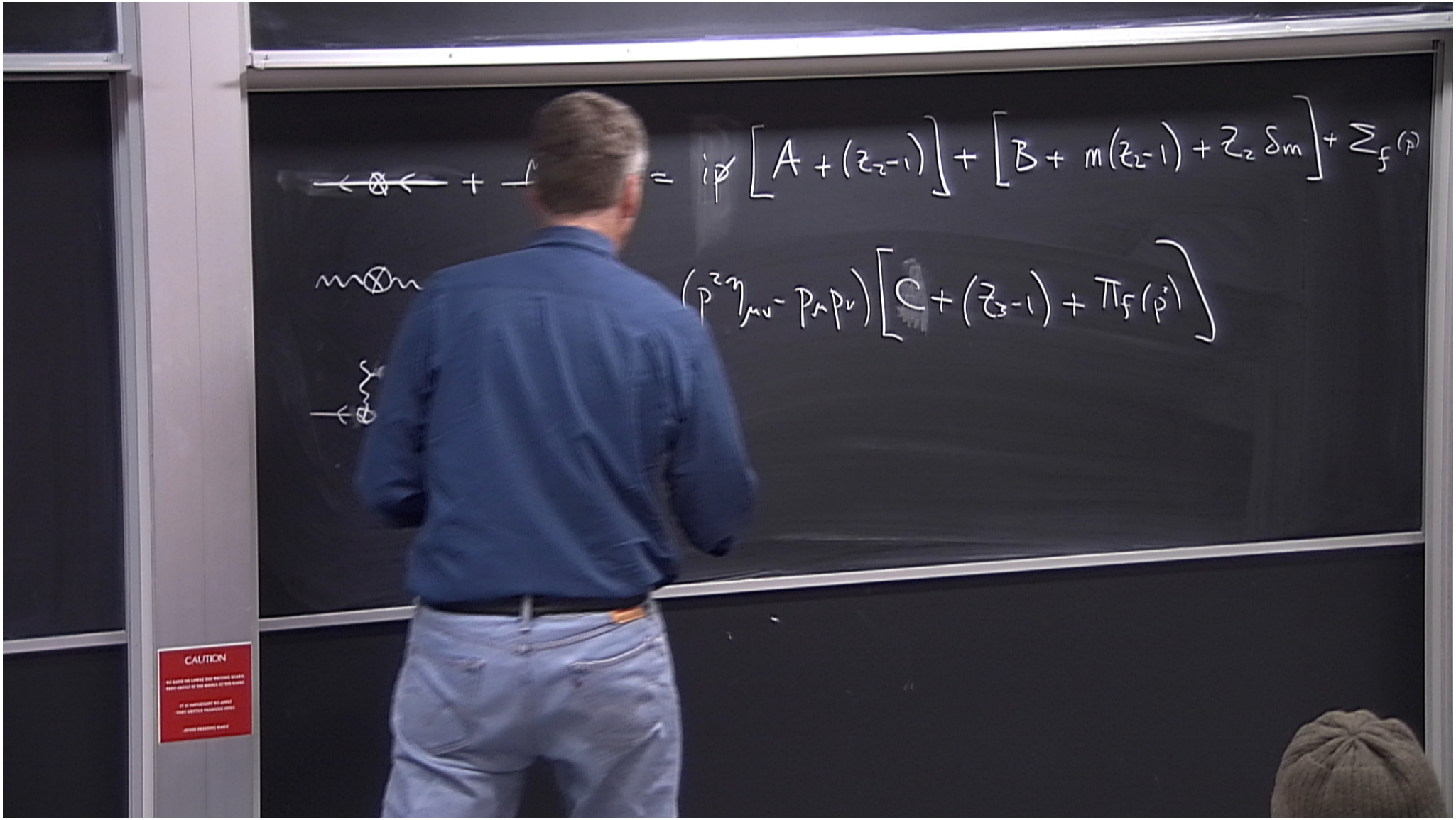
$$i(2\pi)^4 \delta^4(k+p'-p)\left[-ie(Z_1-1)\gamma^\mu\right]_{mn}$$

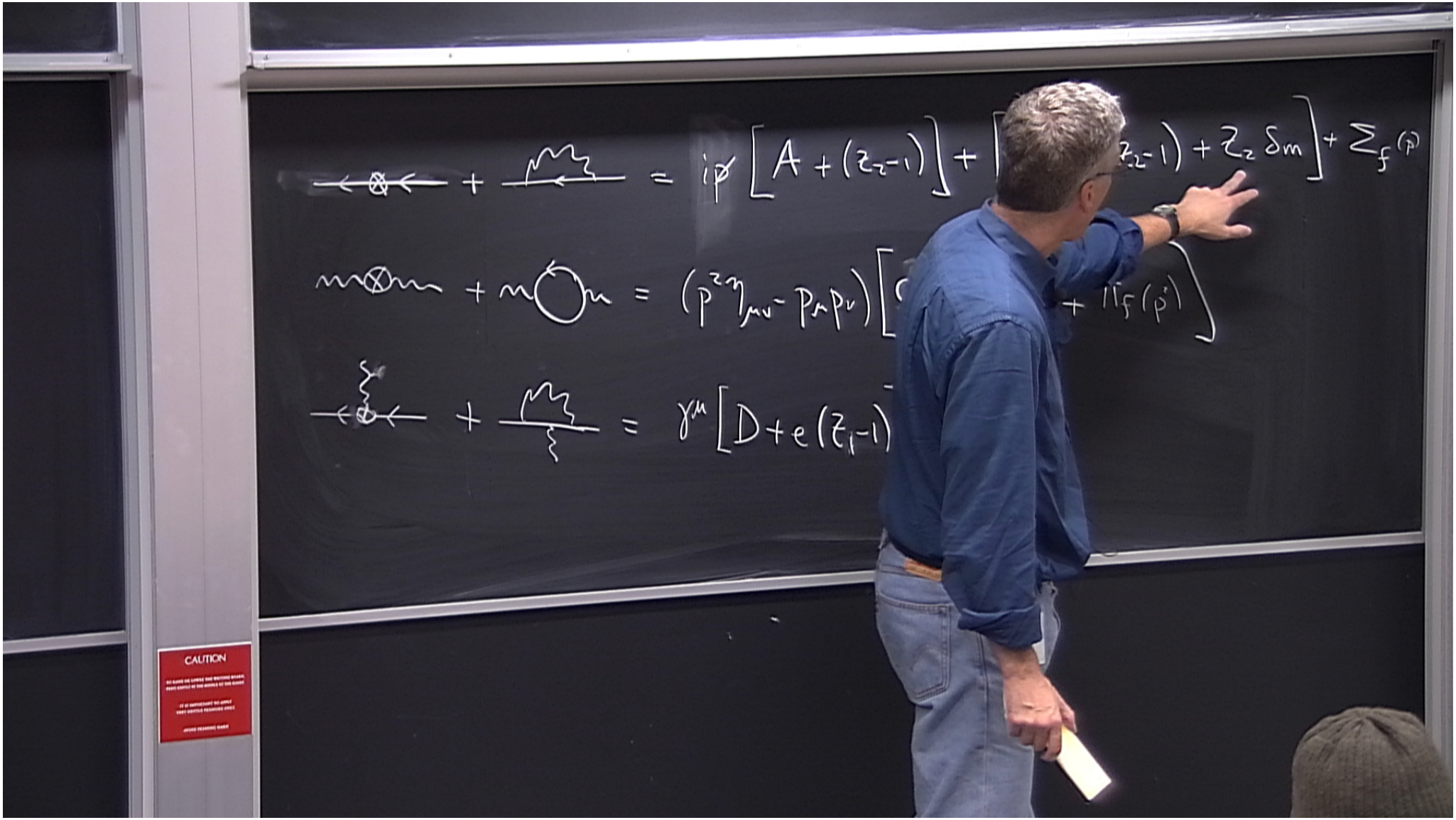

$\Sigma_{mn}(p)$
$\Pi_{\mu\nu}(p) = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$\Lambda^\mu_{mn}(p', p)$


$\Sigma($
$i\not{p} + B + \Sigma_f(p)$
$\left(\text{ask } \left(\frac{\partial \Sigma_f}{\partial p}\right)_0 = 0\right)$
$(p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$\Pi(p^2) = C + \Pi_f(p^2)$
$\text{ask } \left(\frac{\partial \Pi_f}{\partial p^2}\right)_0 = 0$
$e\gamma$
CAUTION


$\Sigma(p) = A\,i\not{p} + B + \Sigma_f(p)$
$\left(\text{ask } \left(\frac{\partial \Sigma_f}{\partial p}\right)_0 = 0\right)$
$\eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$\Pi(p^2) = C + \Pi_f(p^2)$
$\Lambda^\mu(p',p) = (D\gamma^\mu) + \Lambda_f^\mu(p',p)$
ask $\left(\frac{\partial \Pi_f}{\partial p^2}\right)_0 = 0$
ask $\Lambda_f^\mu(p'=p) = 0$
CAUTION


CAUTION


$= ip\left[A + (Z_2 - 1)\right] + \left[B + m(Z_2 - 1) + Z_2 \delta_m\right] + \Sigma_f(p)$
$(p^2 \eta_{\mu\nu} - p_\mu p_\nu)\left[C + (Z_3 - 1) + \Pi_f(p^2)\right]$
CAUTION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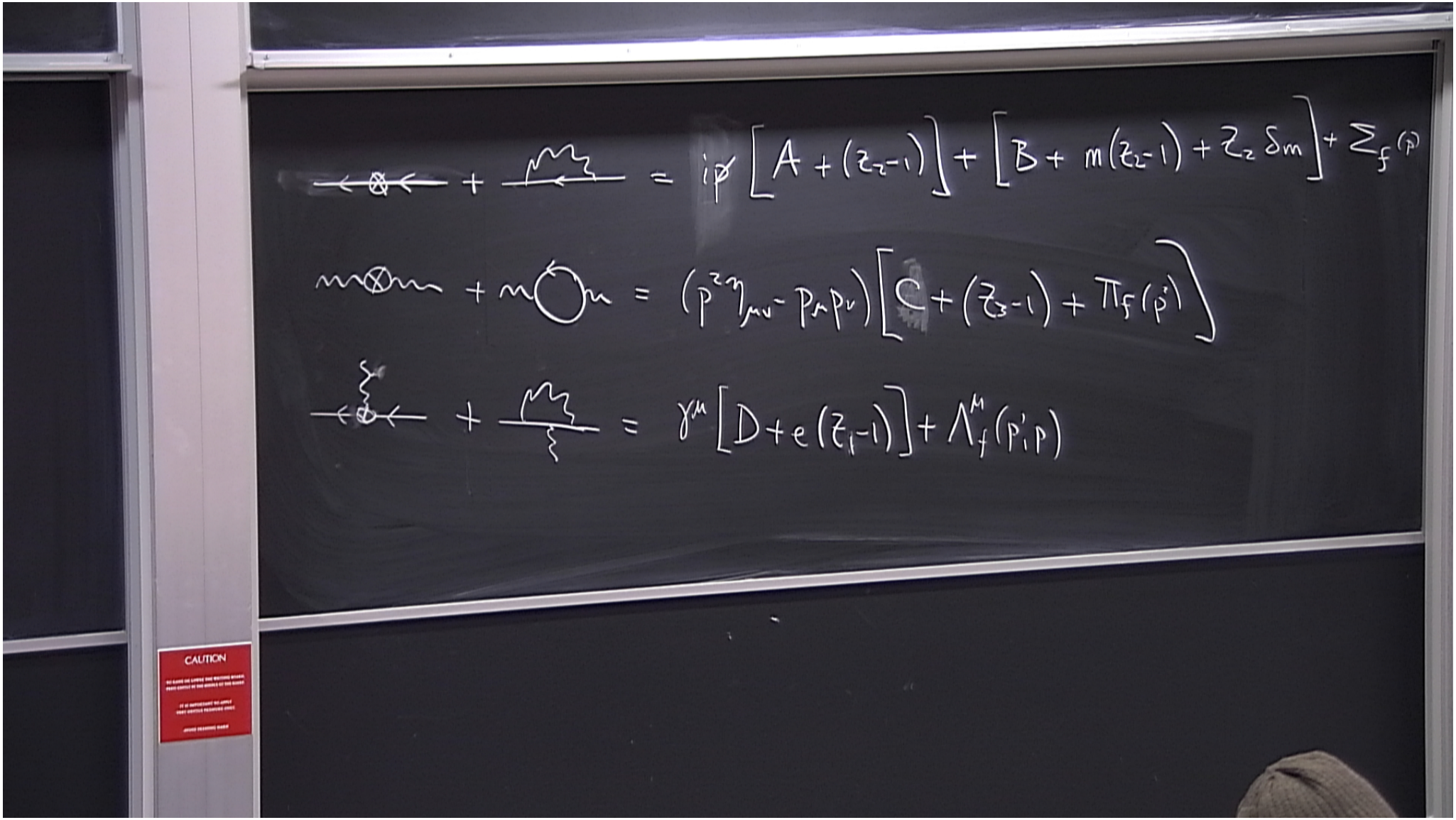
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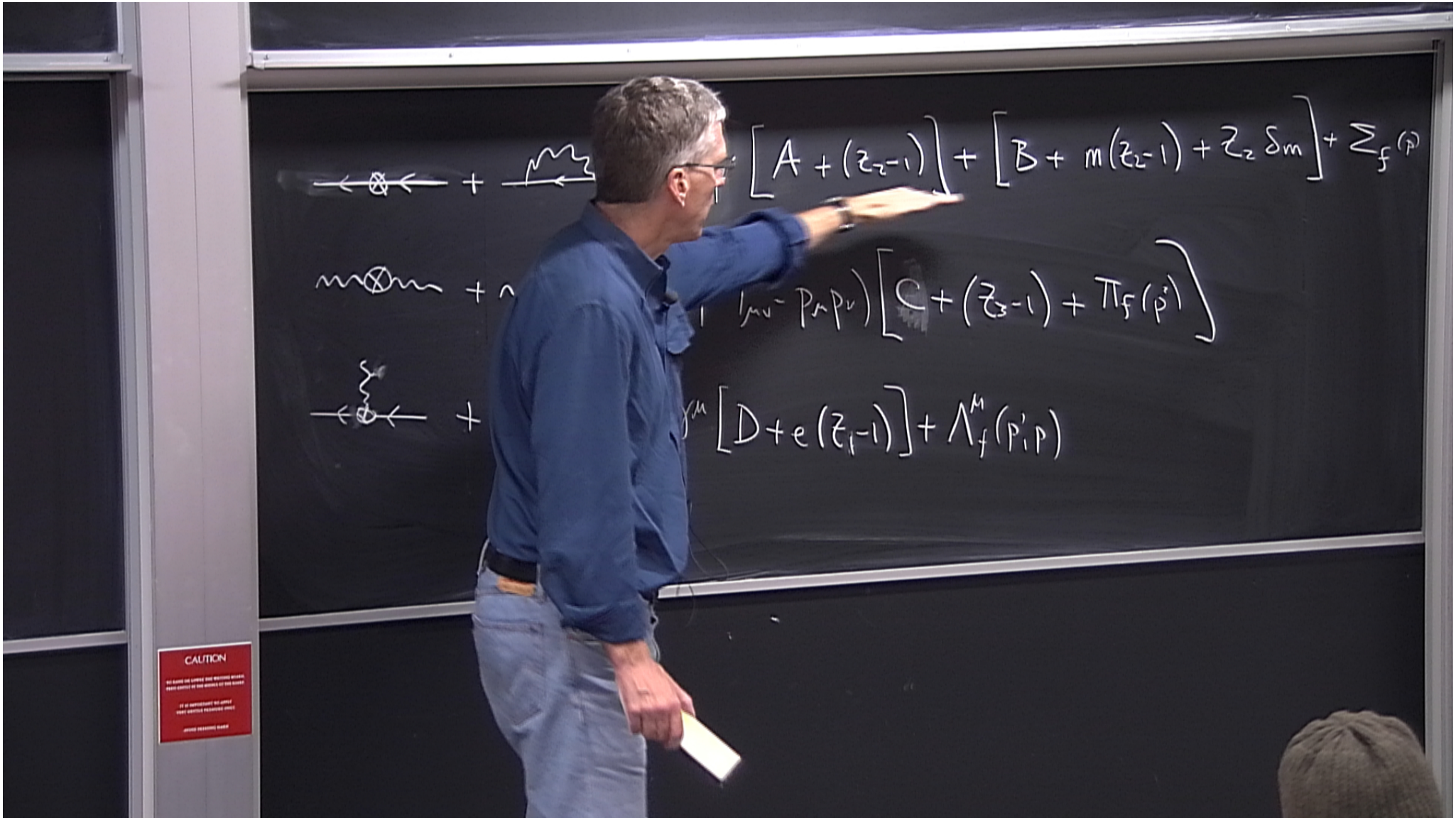
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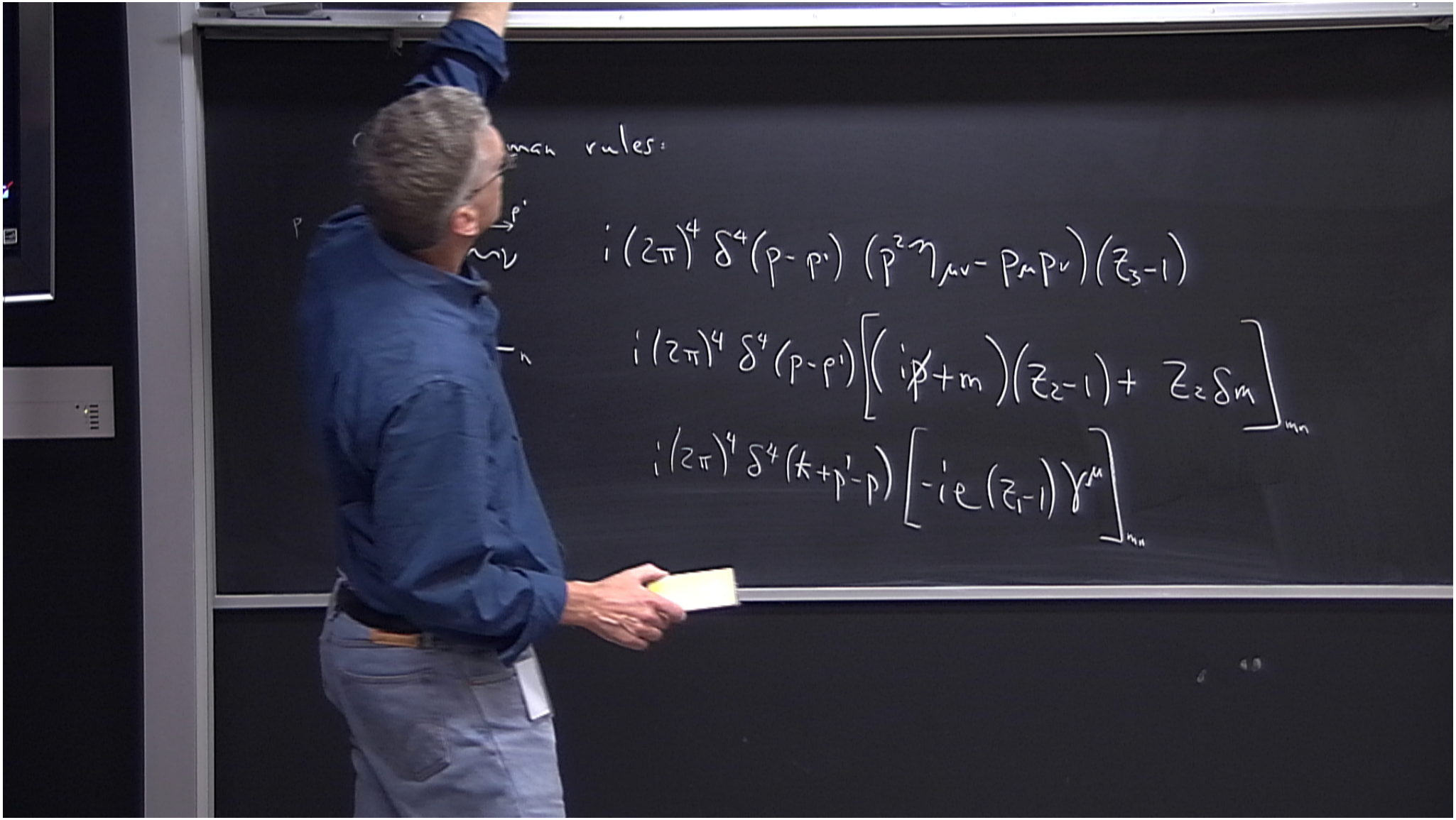
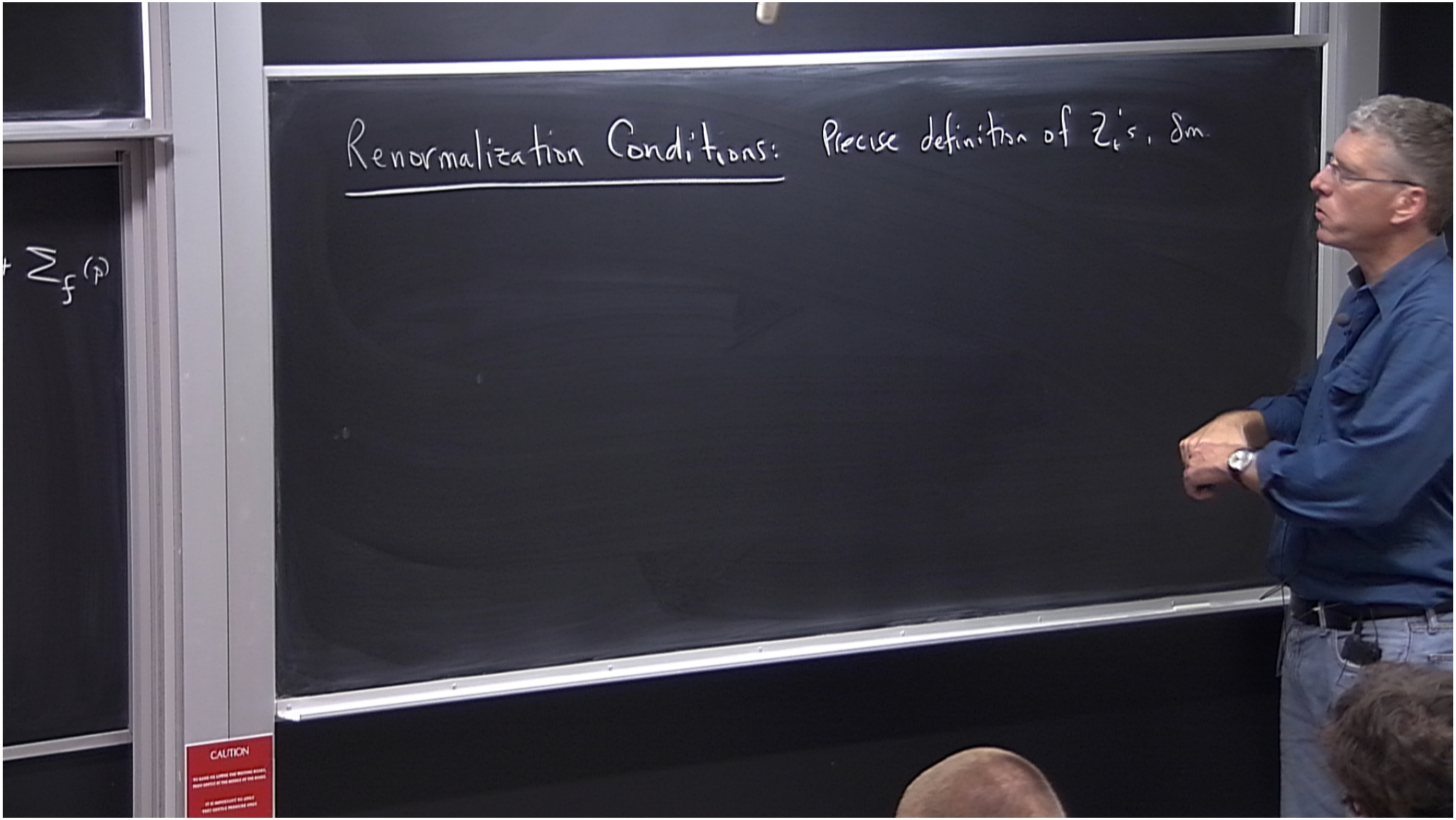
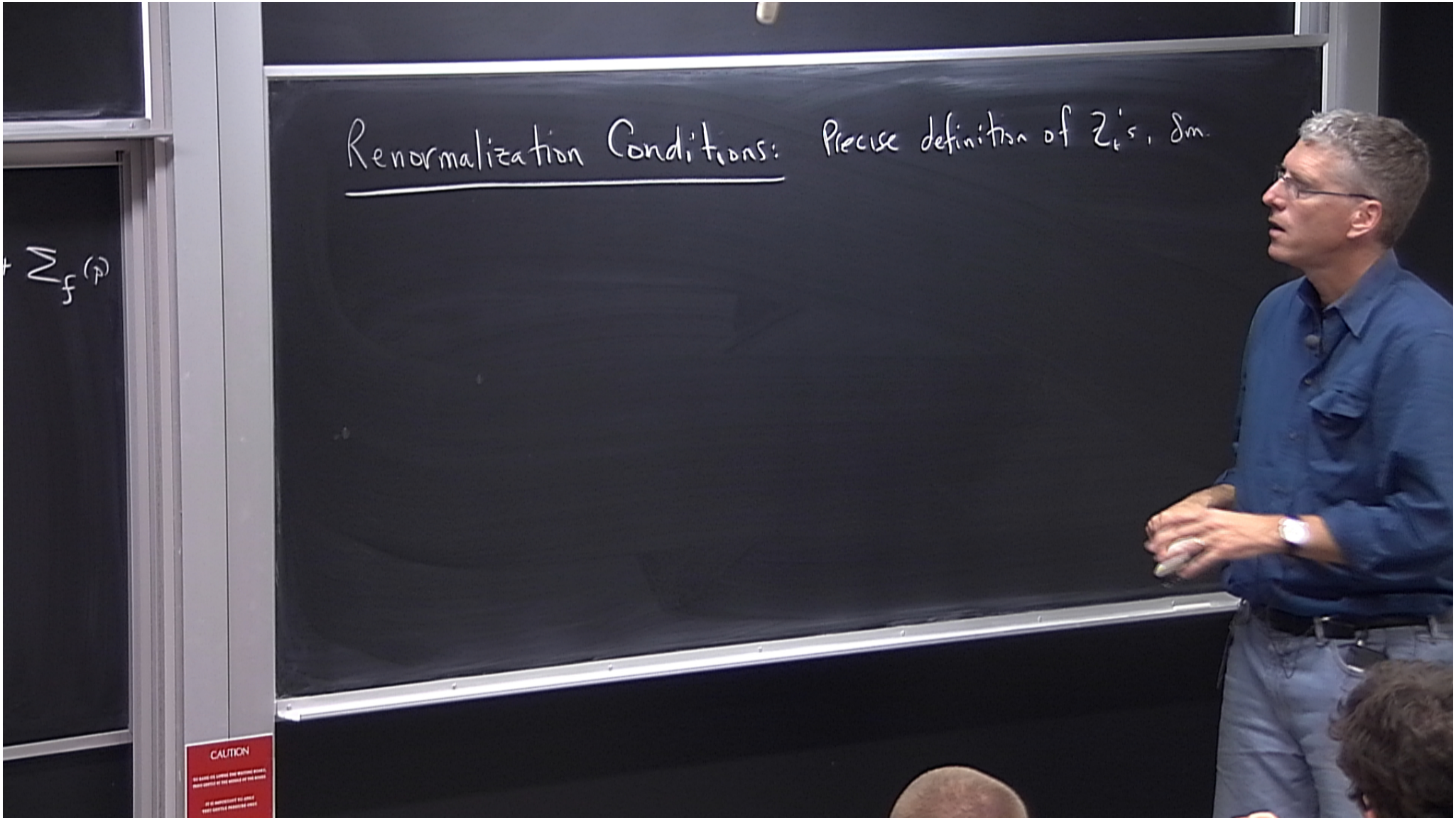
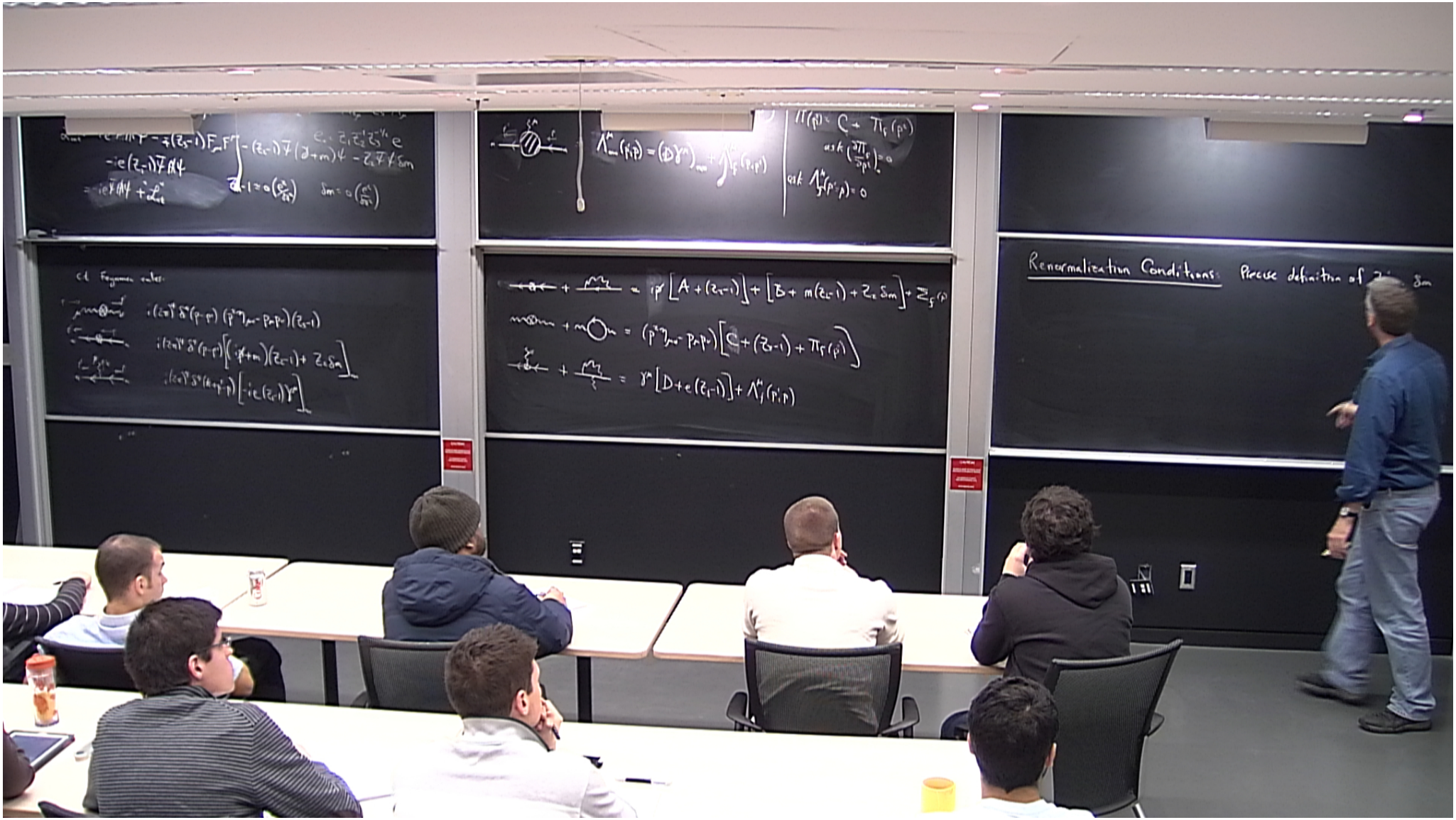
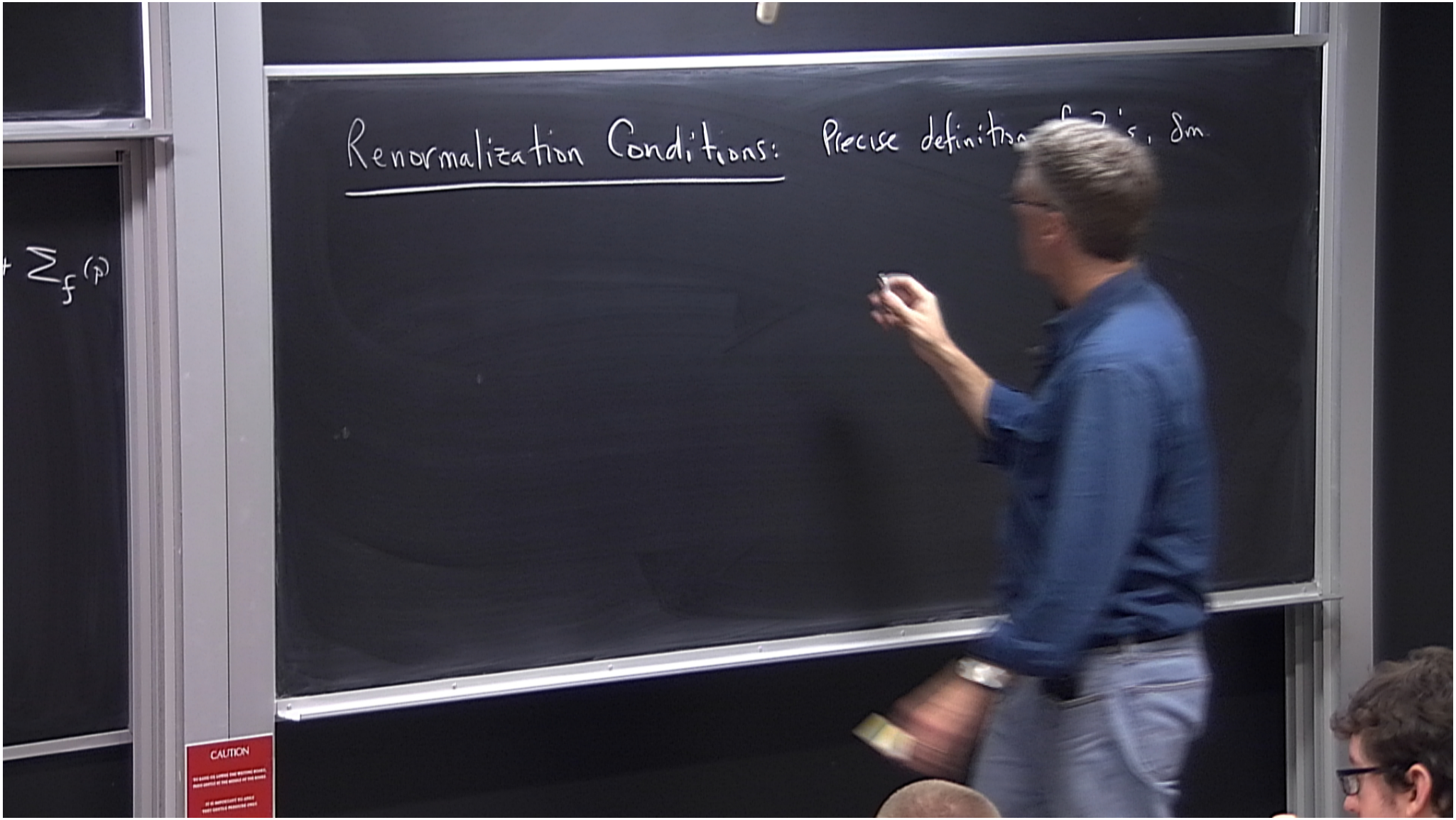
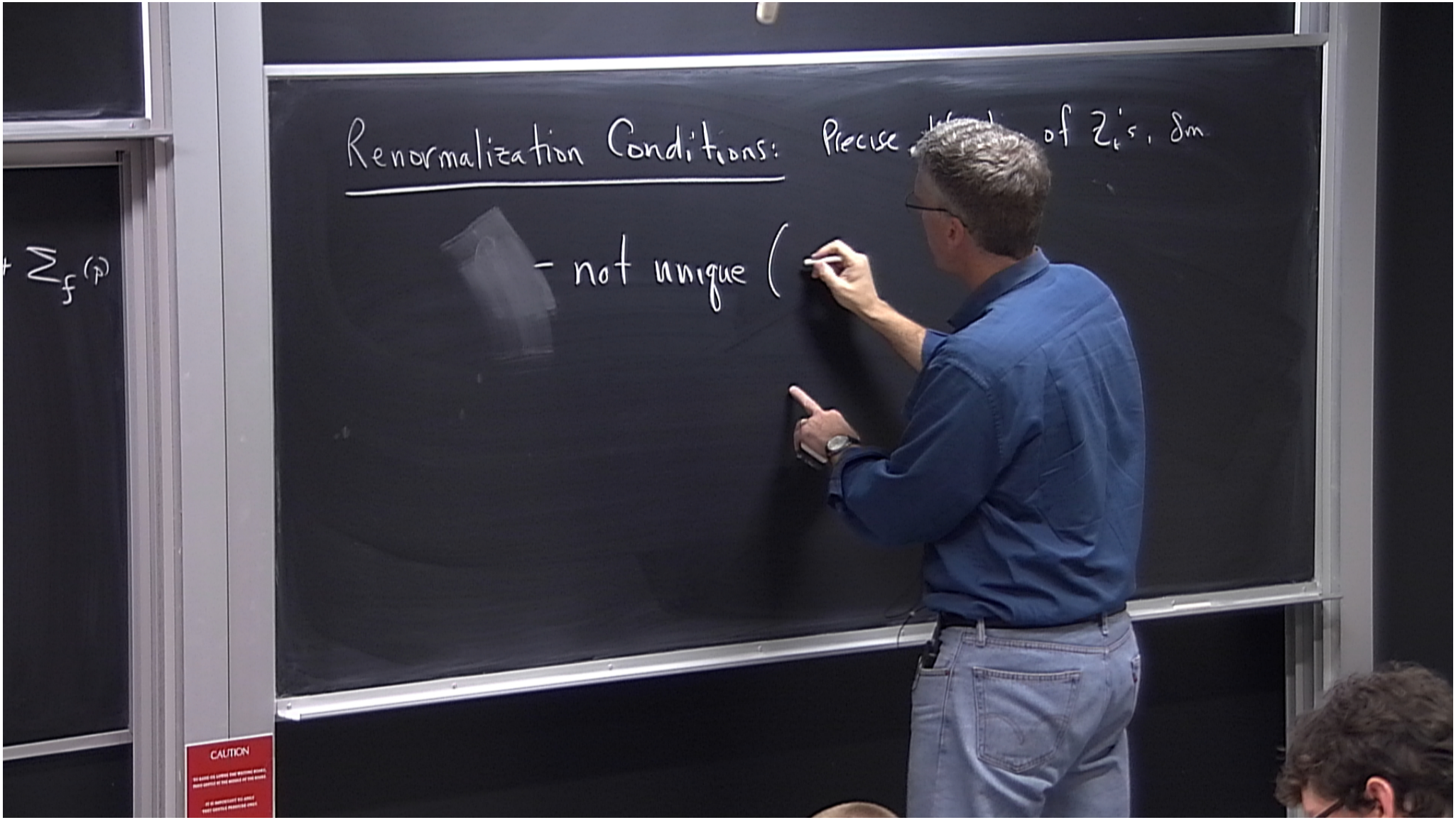

rules:
$i(2\pi)^4 \delta^4(p-p')\,(p^2\eta_{\mu\nu} - p_\mu p_\nu)(Z_3-1)$
$i(2\pi)^4 \delta^4(p-p')\left[(\not{p}+m)(Z_2-1) + Z_2\delta_m\right]_{mn}$
$i(2\pi)^4 \delta^4(k+p'-p)\left[-ie(Z_1-1)\gamma^\mu\right]_{mn}$

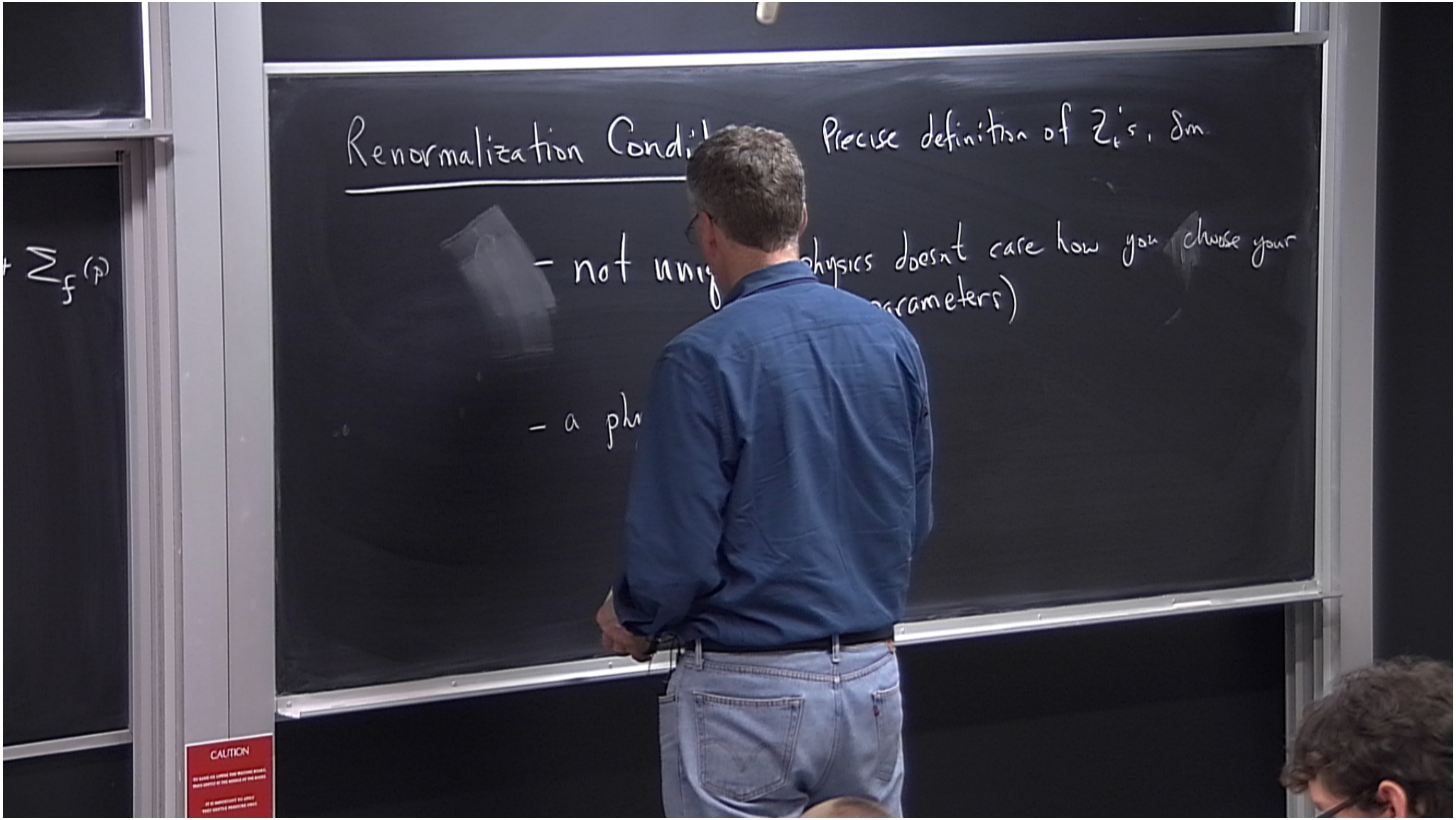

Renormalization Conditions: Precise definition of $Z_i$'s, $\delta m$
$+ \Sigma_f(\not{p})$
CAUTION

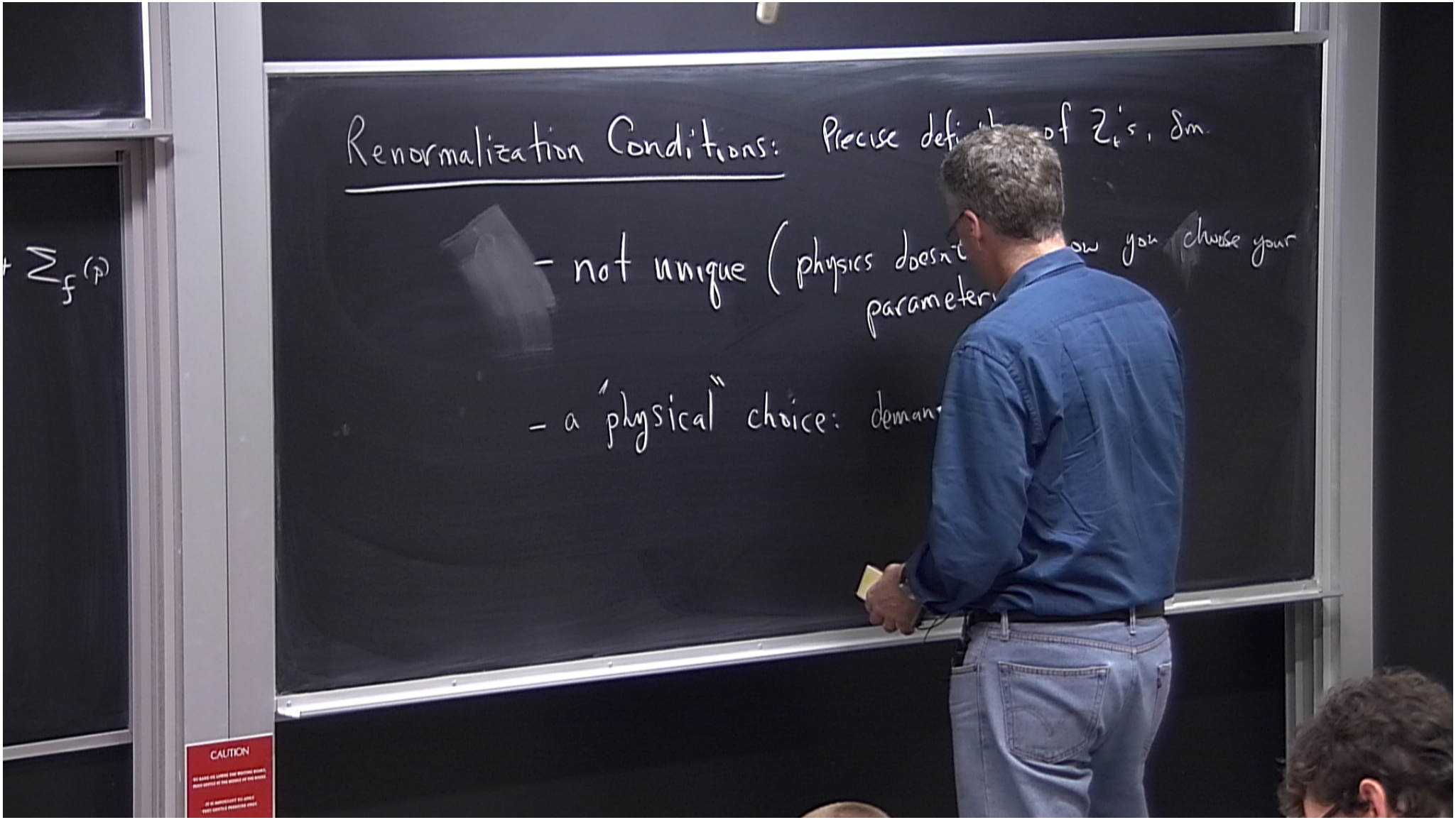

Renormalization Conditions: Precise definition of $Z_i$'s, $\delta m$
$+ \Sigma_f(\not{p})$
CAUTION

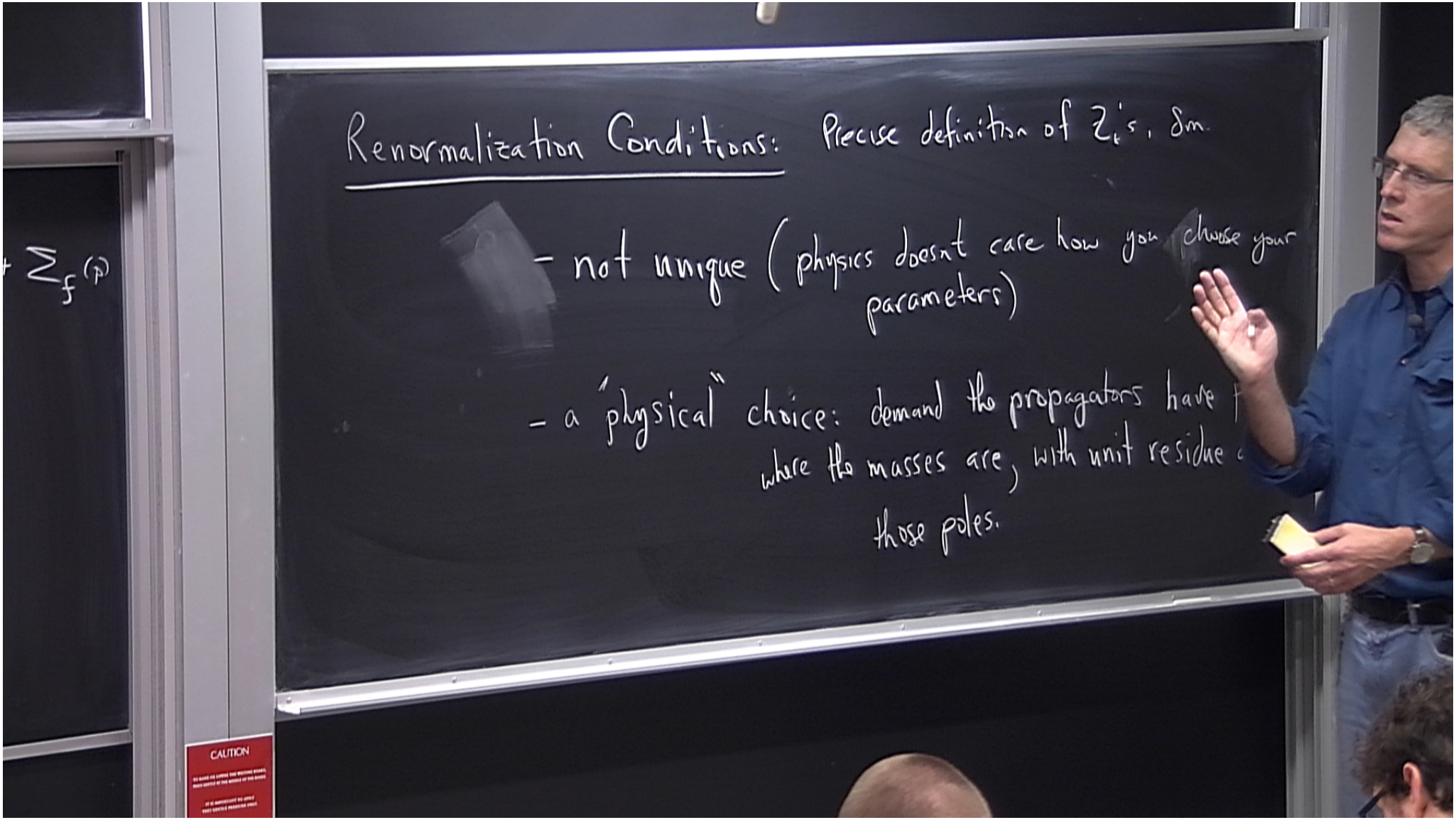

Renormalization Conditions: Precise definition of

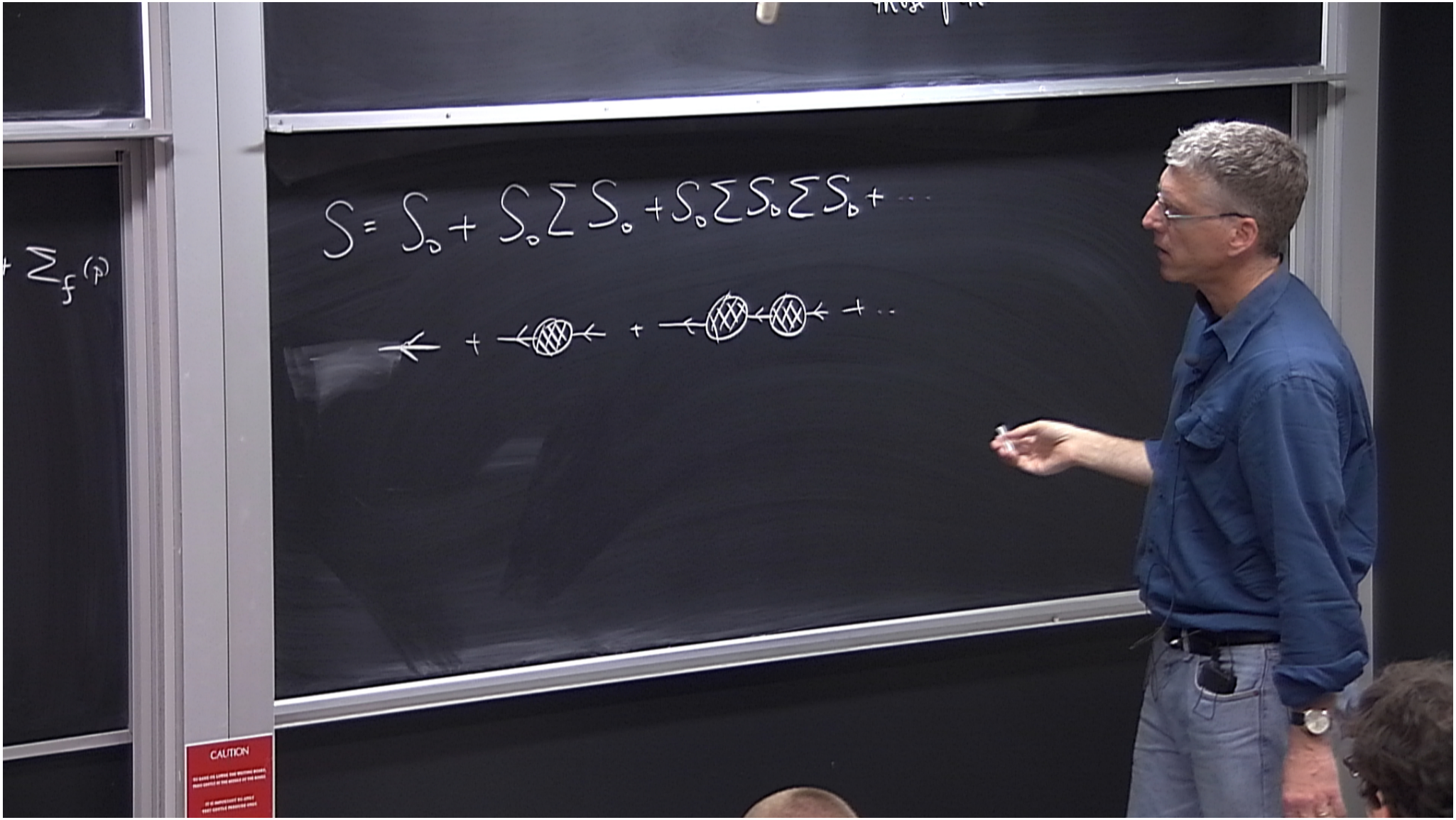

Renormalization Conditions:
CAUTION

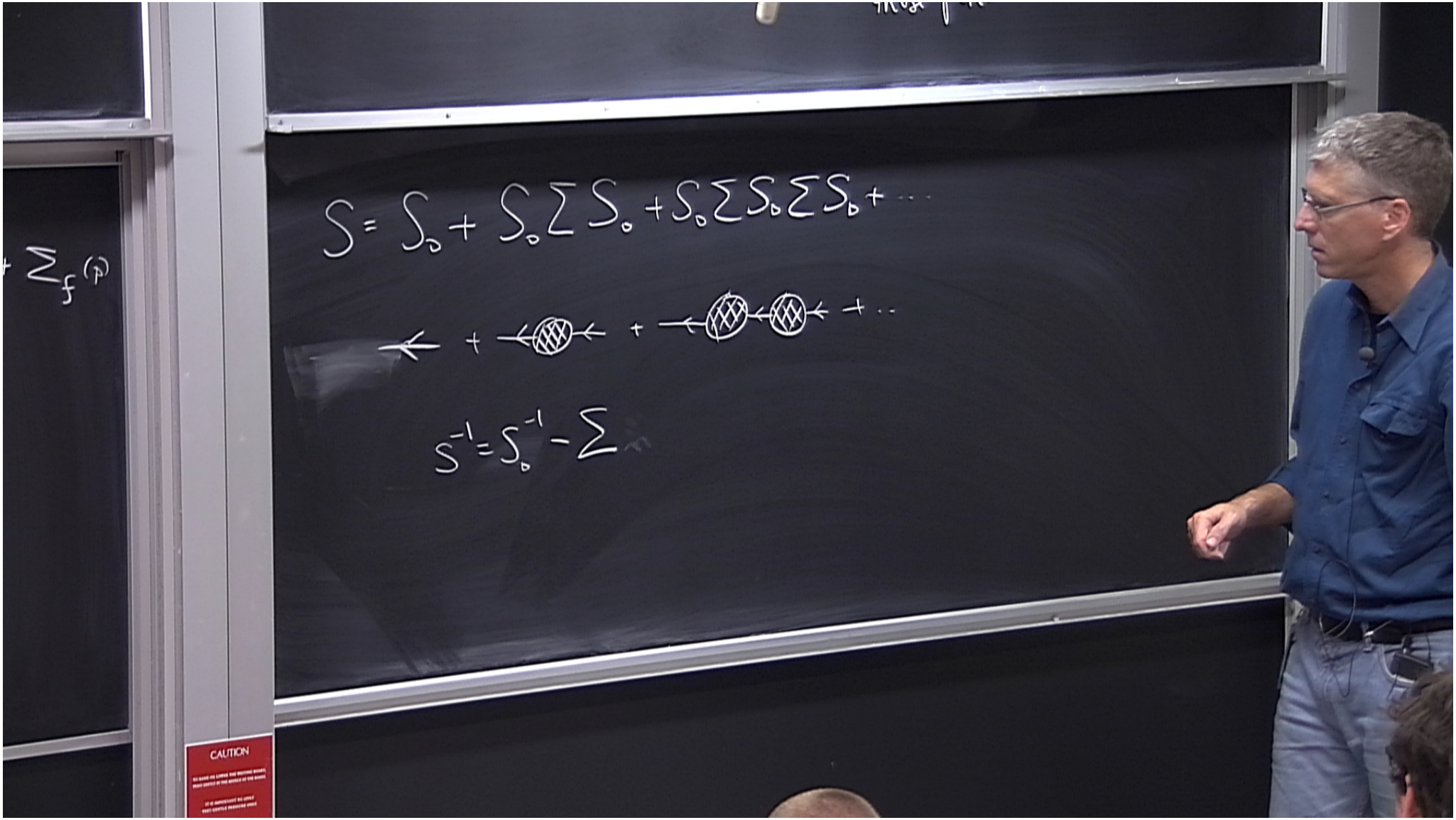

Renormalization Conditions:
- not unique (
CAUTION

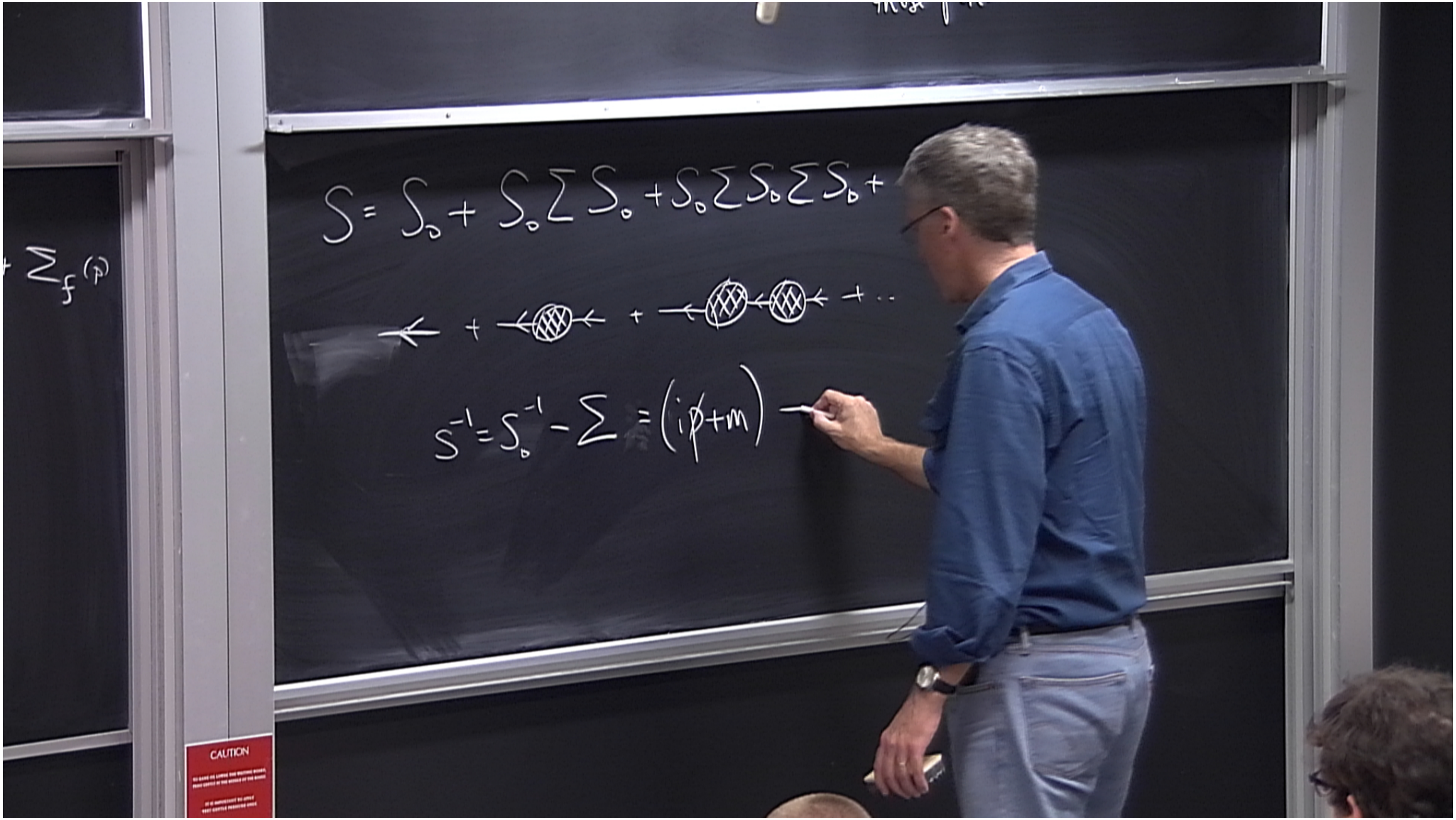

Renormalization Condi
Precise definition of Z_i's, δm
- not uni
physics doesnt care how you choose your
parameters)
- a ph
CAUTION

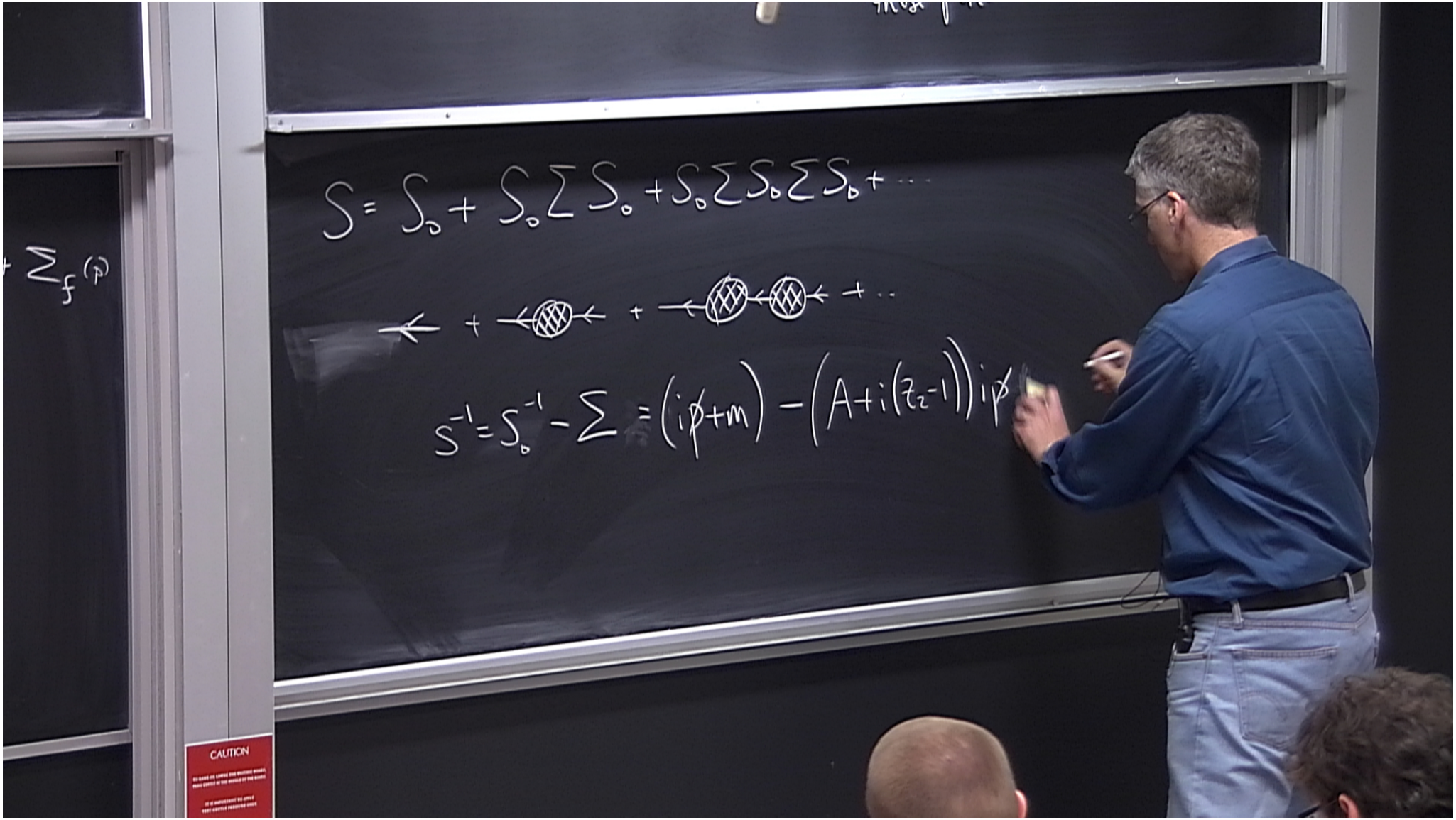

Renormalization Conditions: Precise defi of $Z_i$'s, $\delta m$
- not unique (physics doesn't ow you choose your
parameter
- a "physical" choice: deman
$+\Sigma_f(\not{p})$
CAUTION

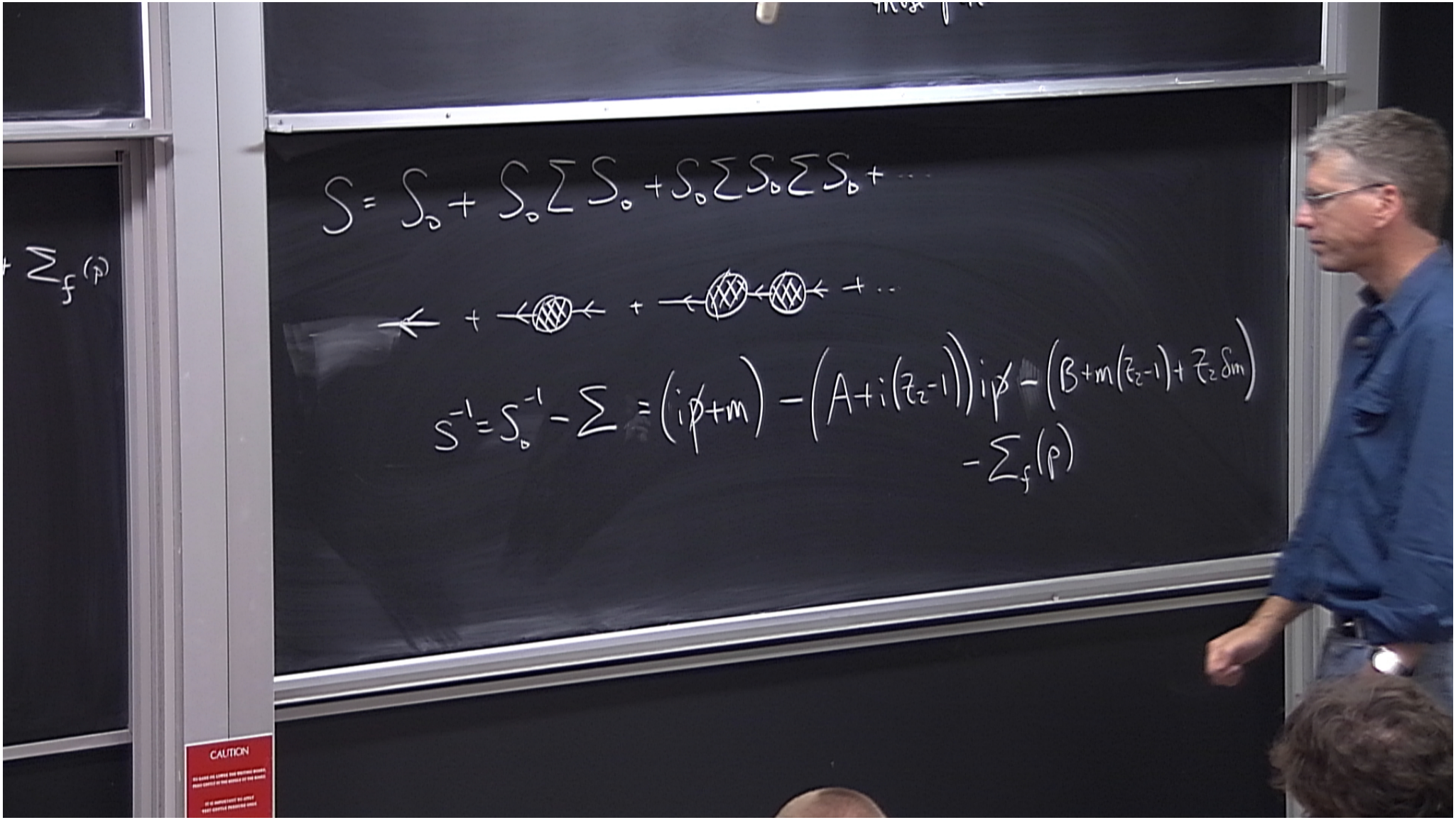

Renormalization Conditions: Precise definition of $Z_i$'s, $\delta m$
- not unique (physics doesn't care how you choose your parameters)
- a "physical" choice: demand the propagators have
where the masses are, with unit residue
those poles.
CAUTION

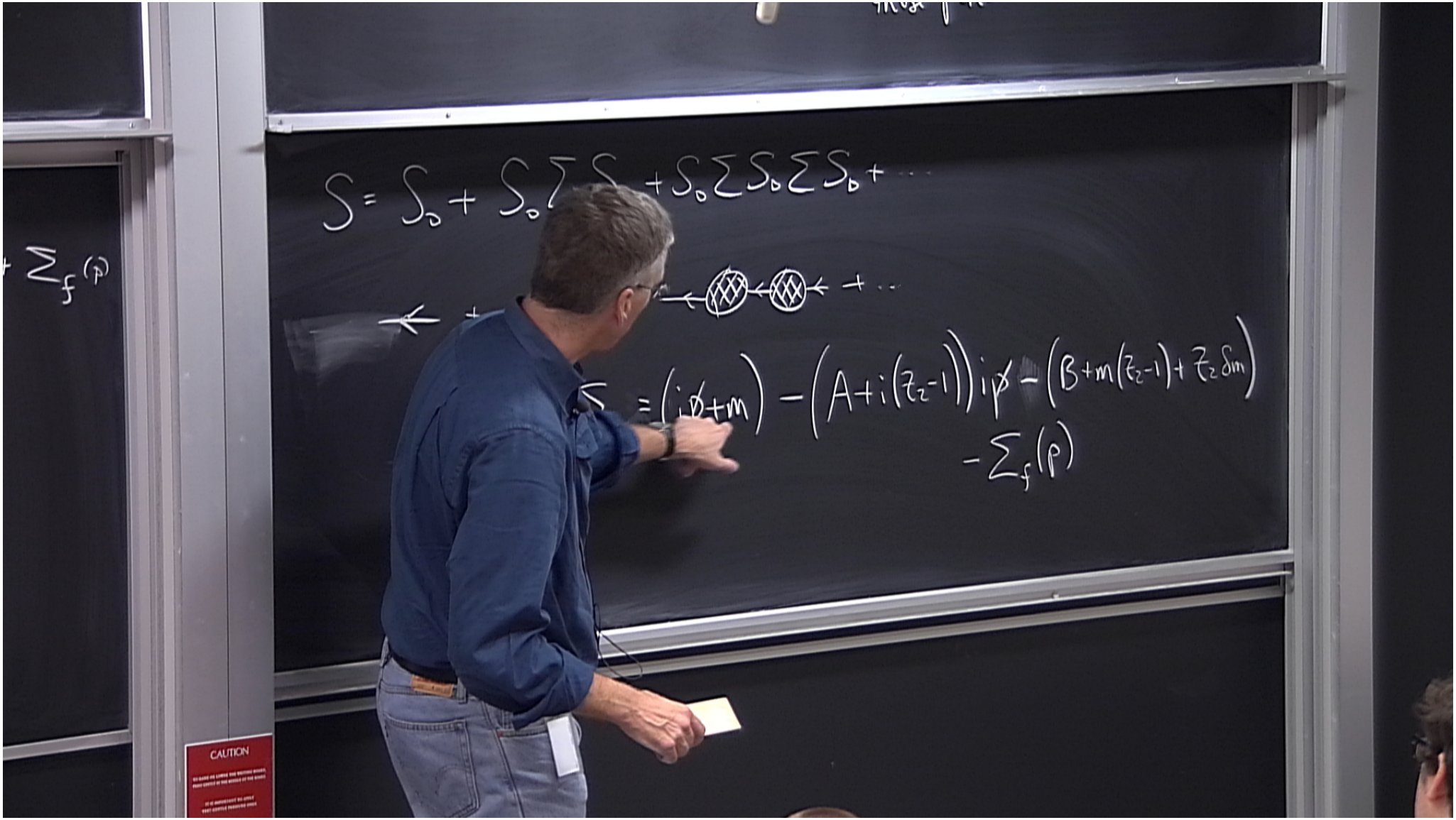

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
CAUTION

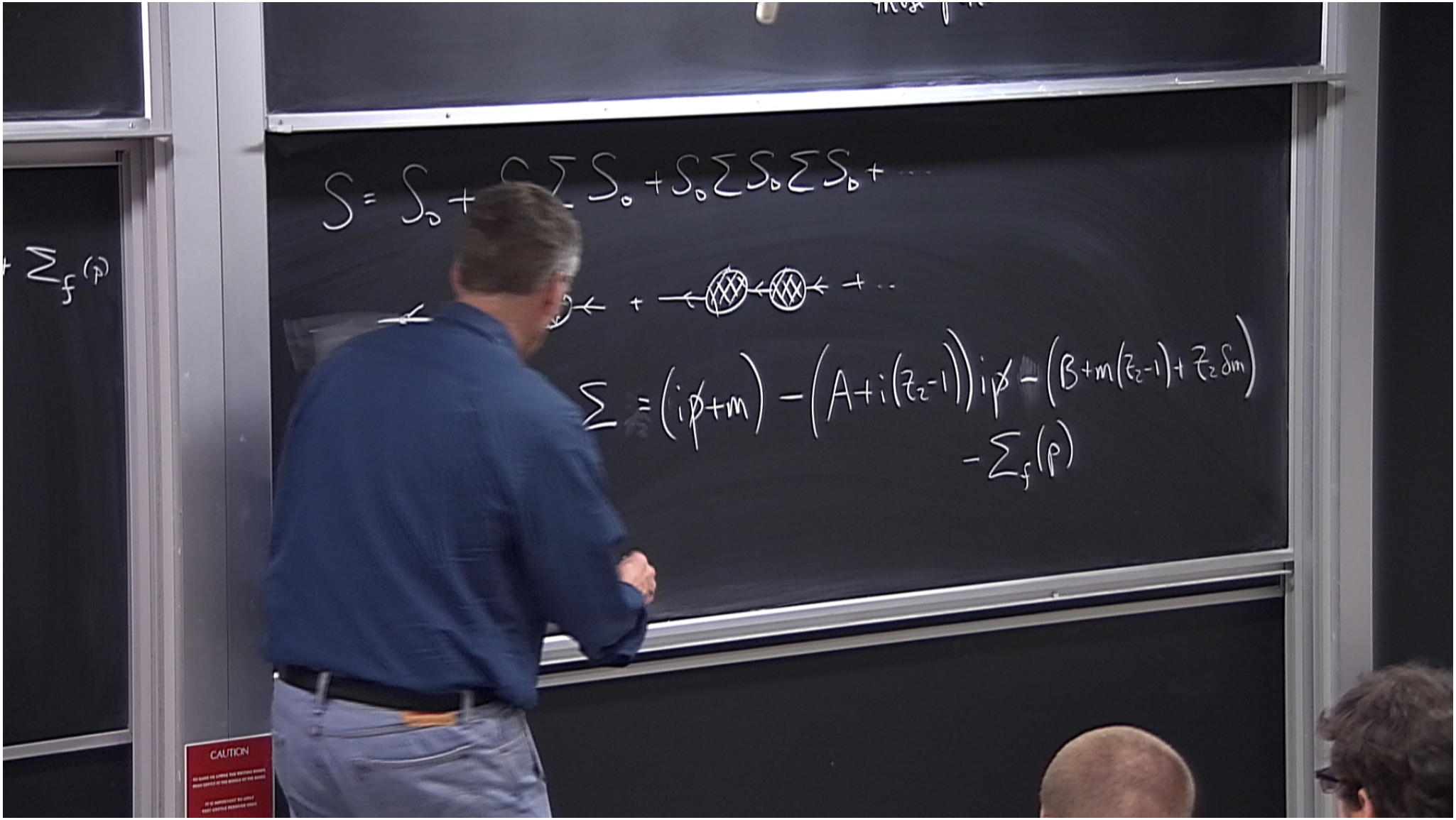

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$S^{-1} = S_0^{-1} - \Sigma$
CAUTION

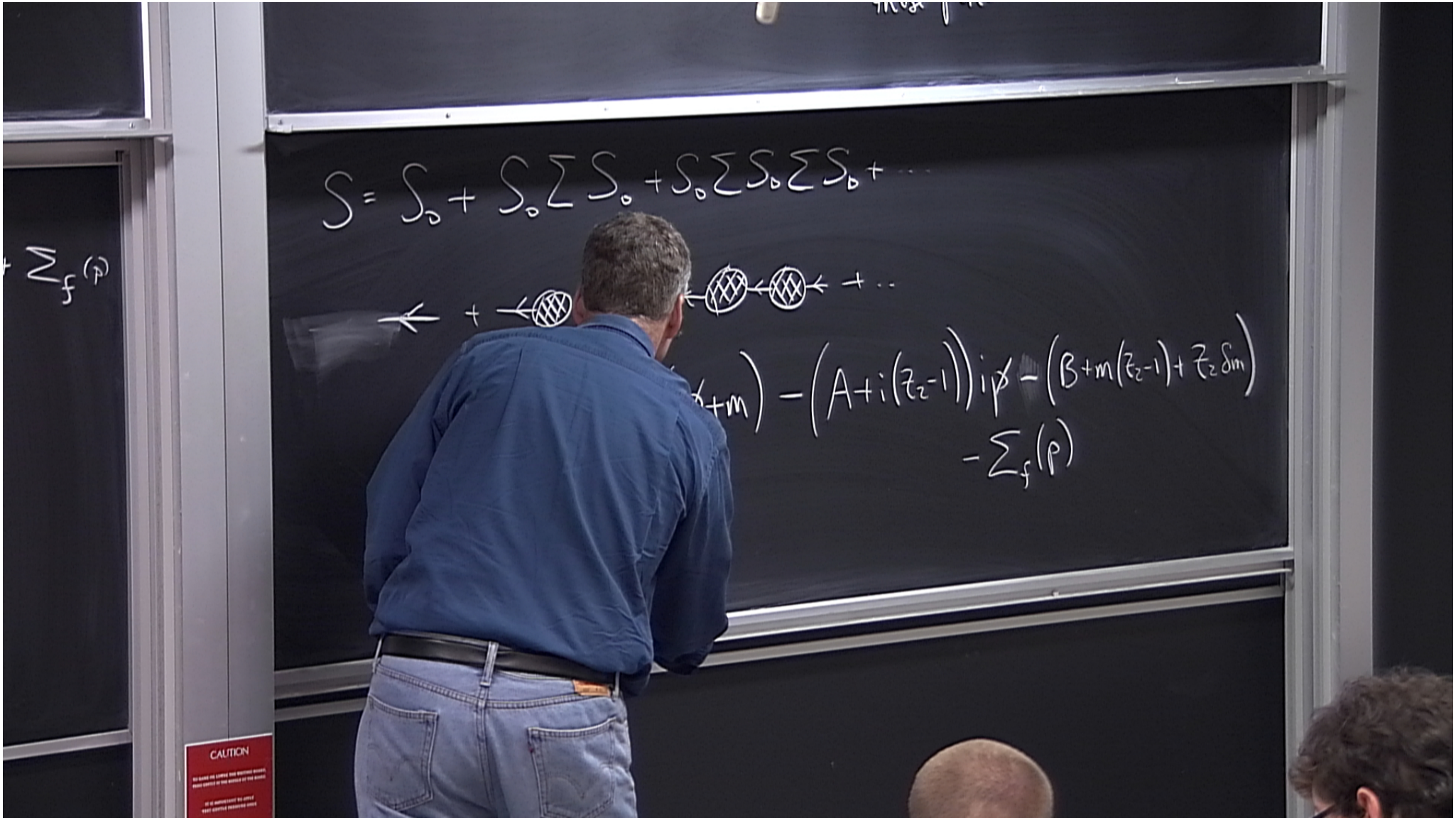

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 +$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m)$
CAUTION

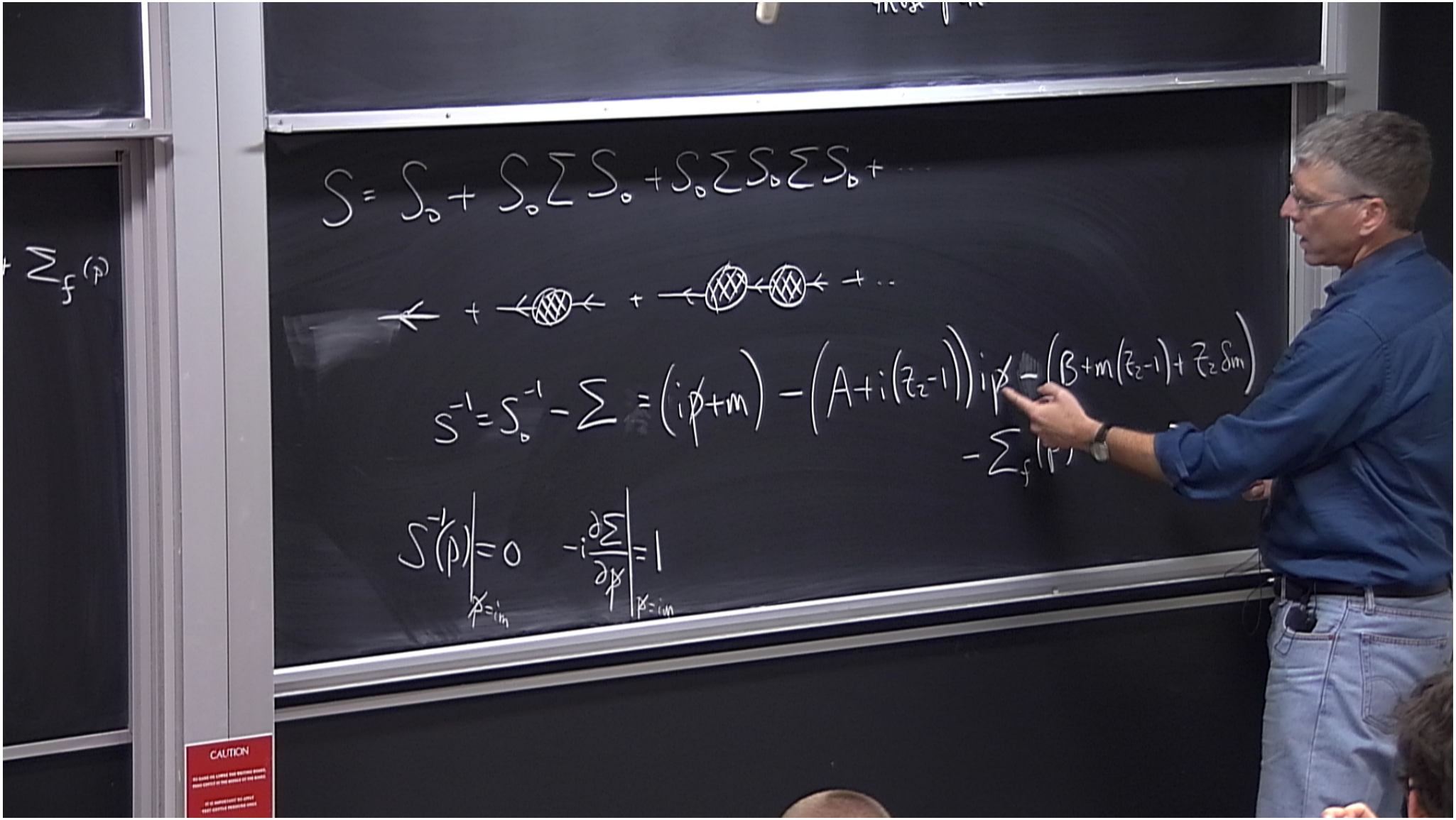

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - (A + i(z_2 - 1))i\not{p}$
CAUTION

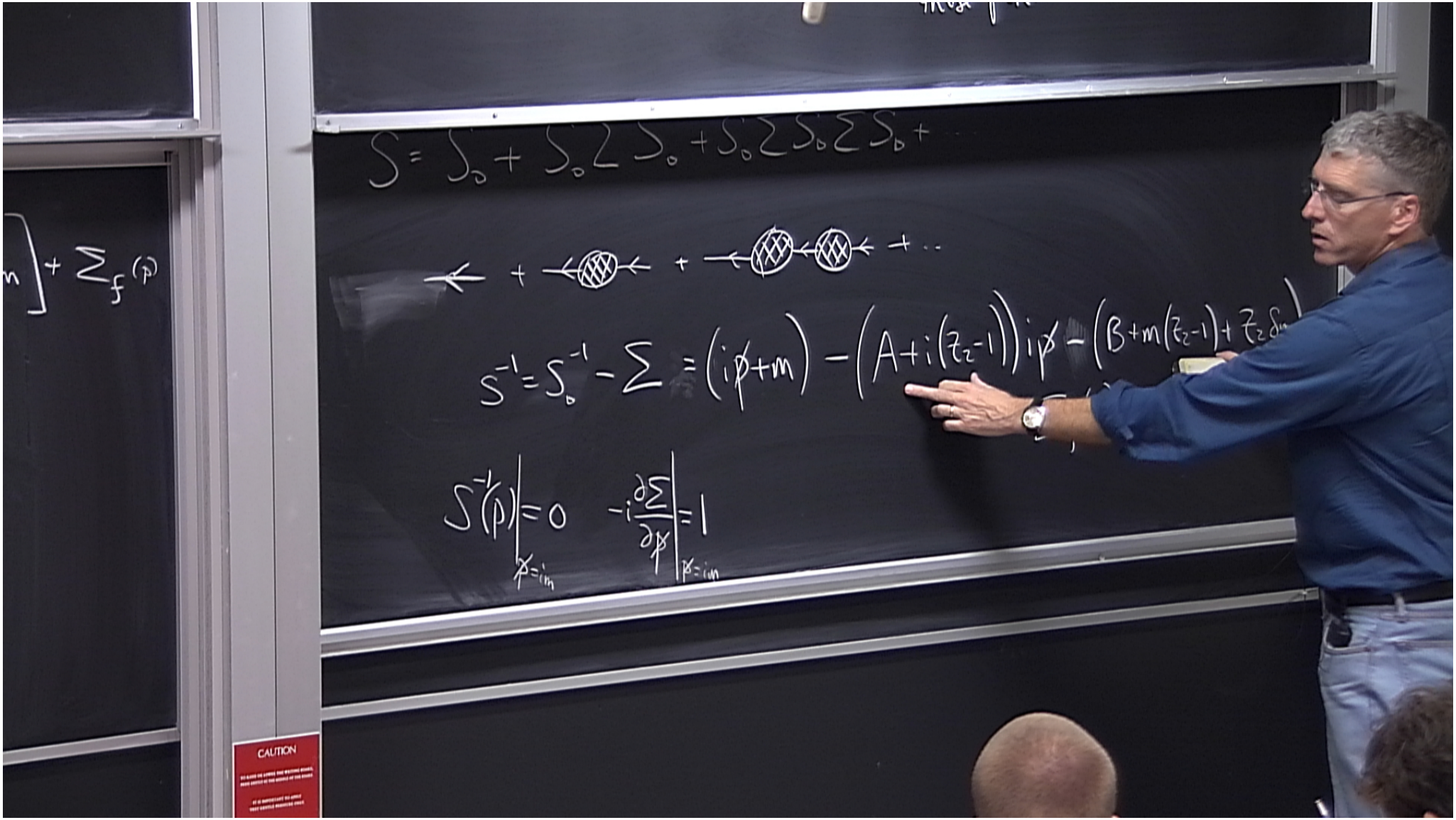

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta_m\right)$
$- \Sigma_f(p)$
CAUTION

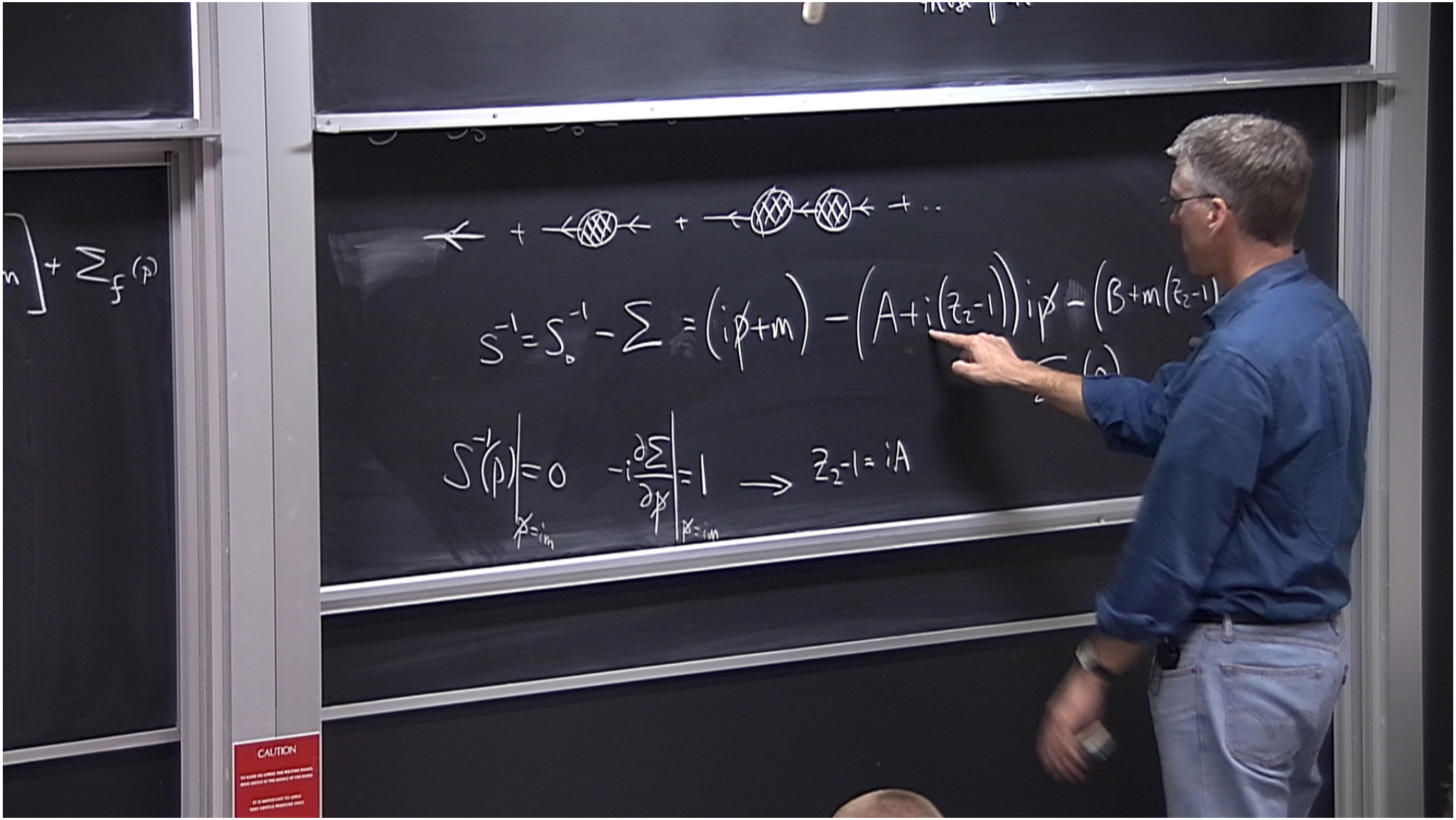

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$+ \Sigma_f(p)$
$= (i\not{p} + m) - (A + i(Z_2 - 1))\, i\not{p} - (B + m(Z_2 - 1) + Z_2 \delta_m)$
$- \Sigma_f(p)$
CAUTION

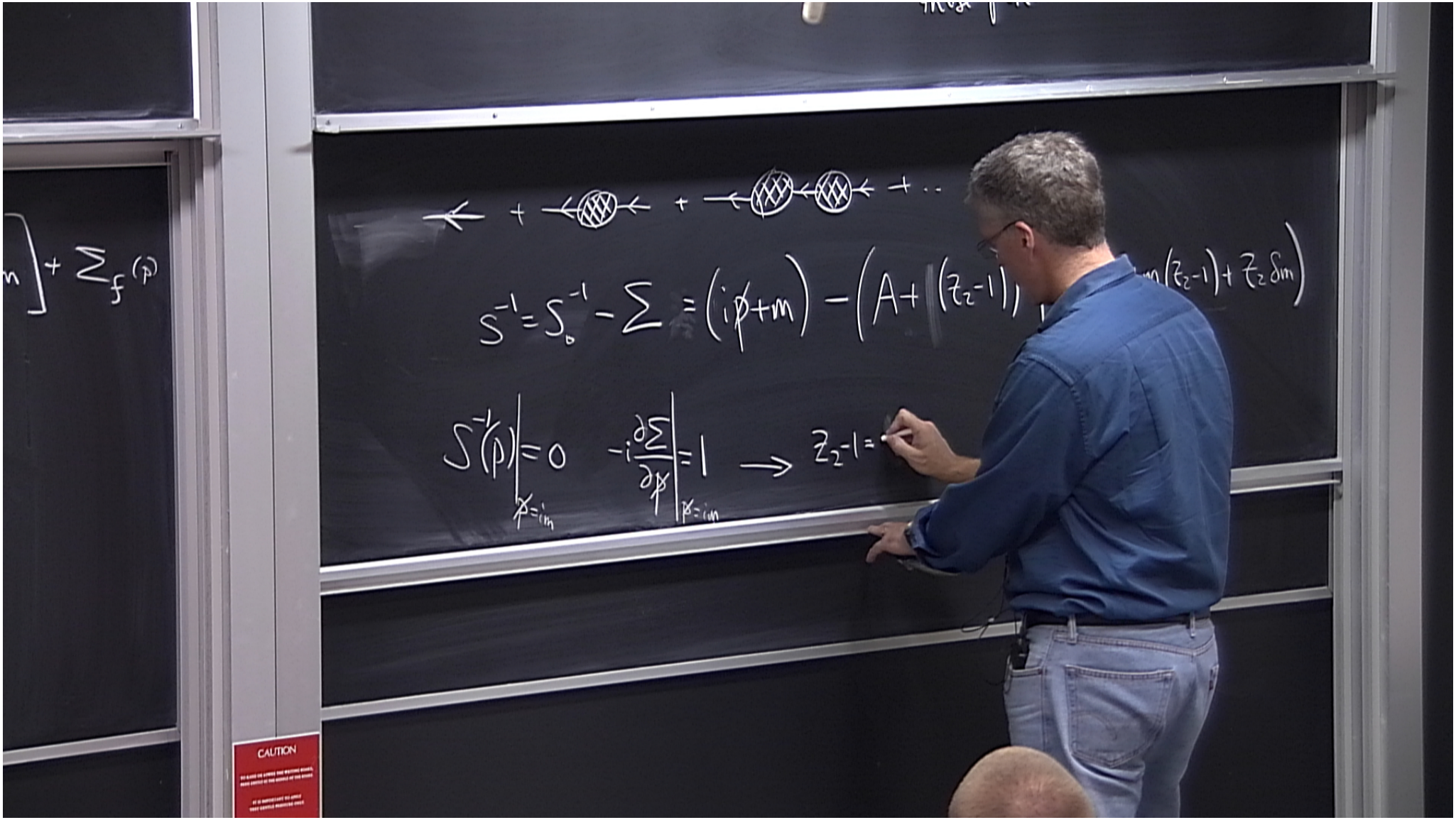

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$\Sigma = (i\not{p} + m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta_m\right)$
$- \Sigma_f(p)$
CAUTION

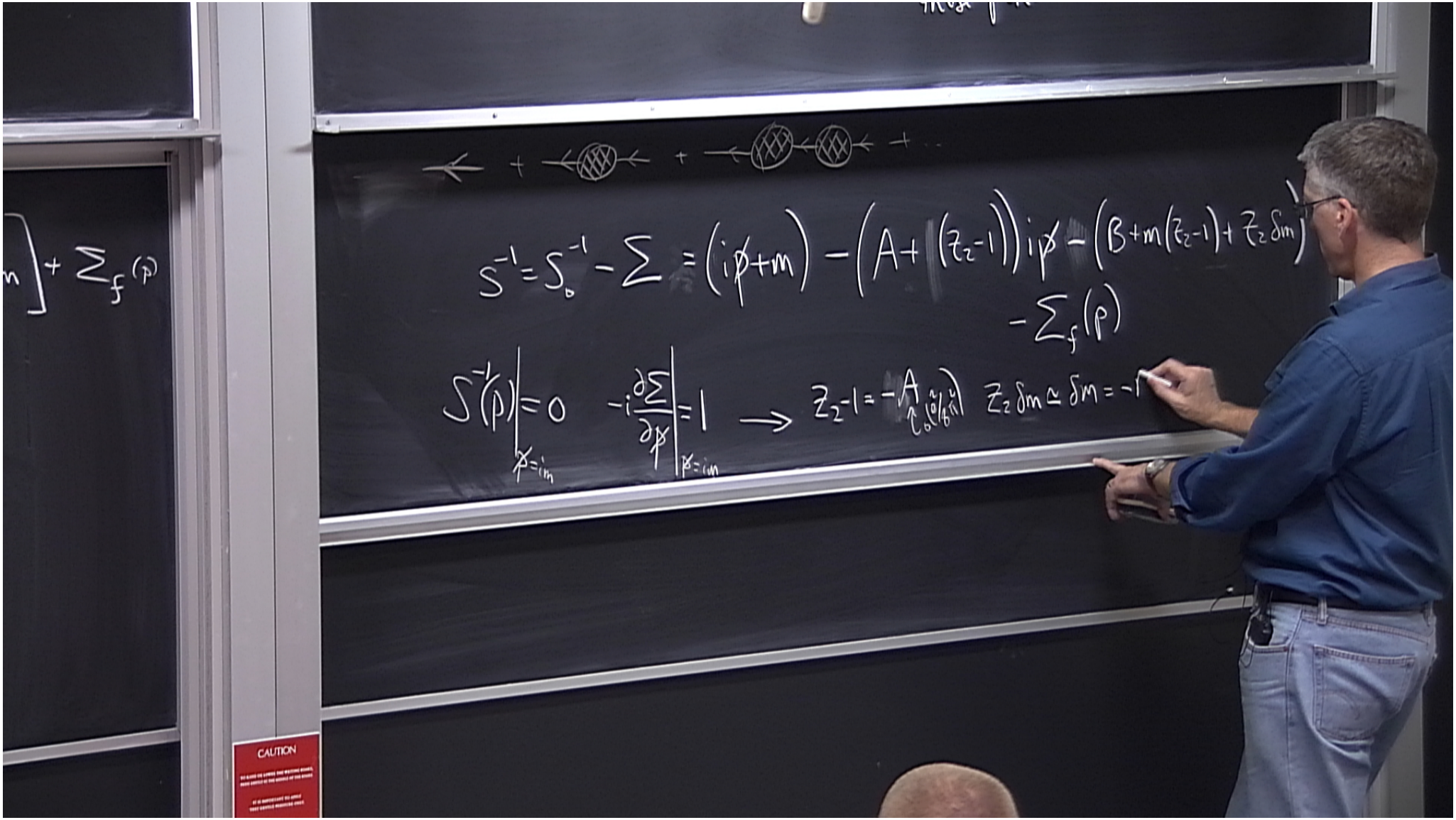

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$+ \cdots$
$+m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta m\right)$
$- \Sigma_f(p)$
$+ \Sigma_f(p)$
CAUTION

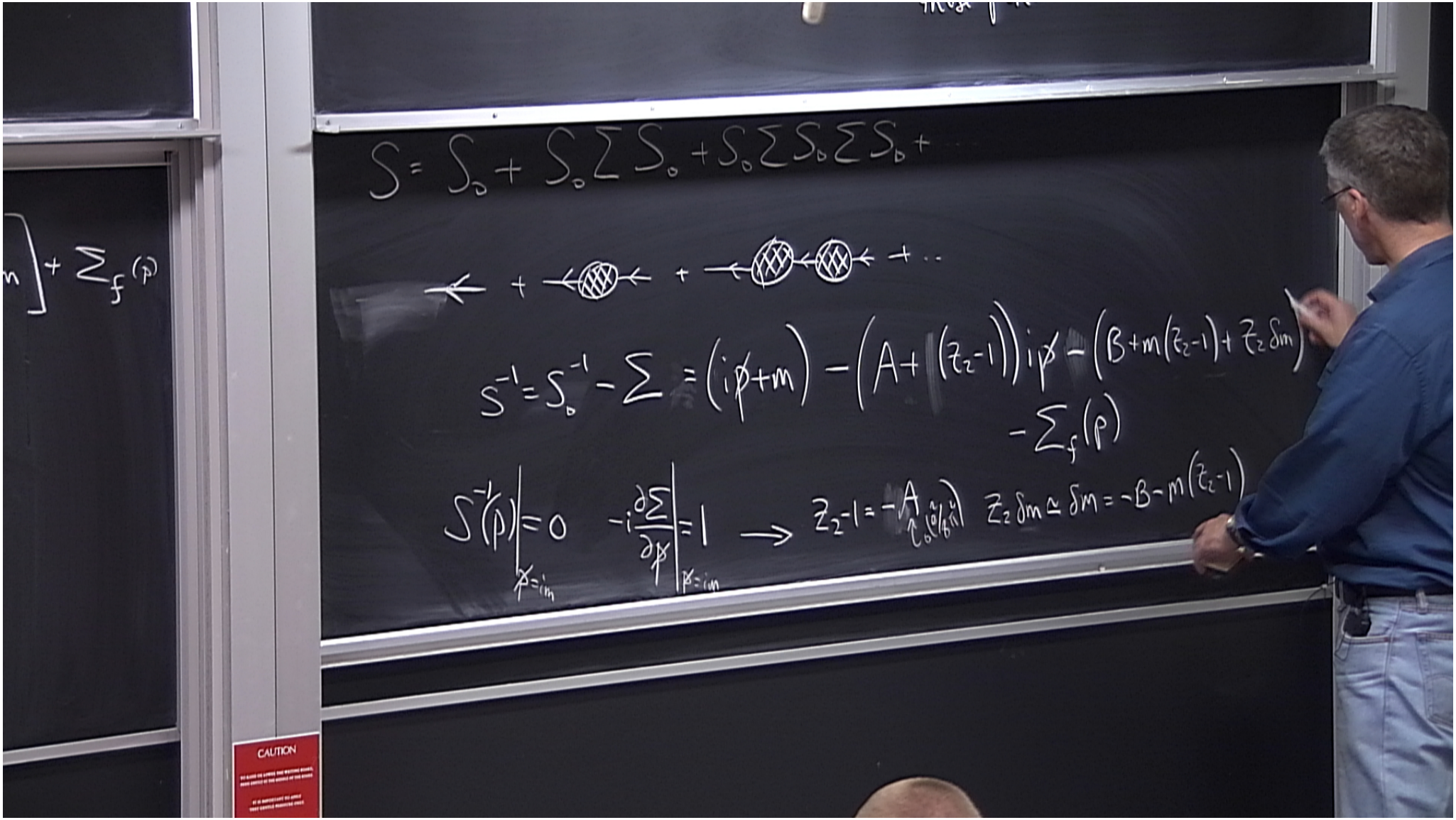

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta m\right)$
$- \Sigma_f(p)$
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1$
CAUTION

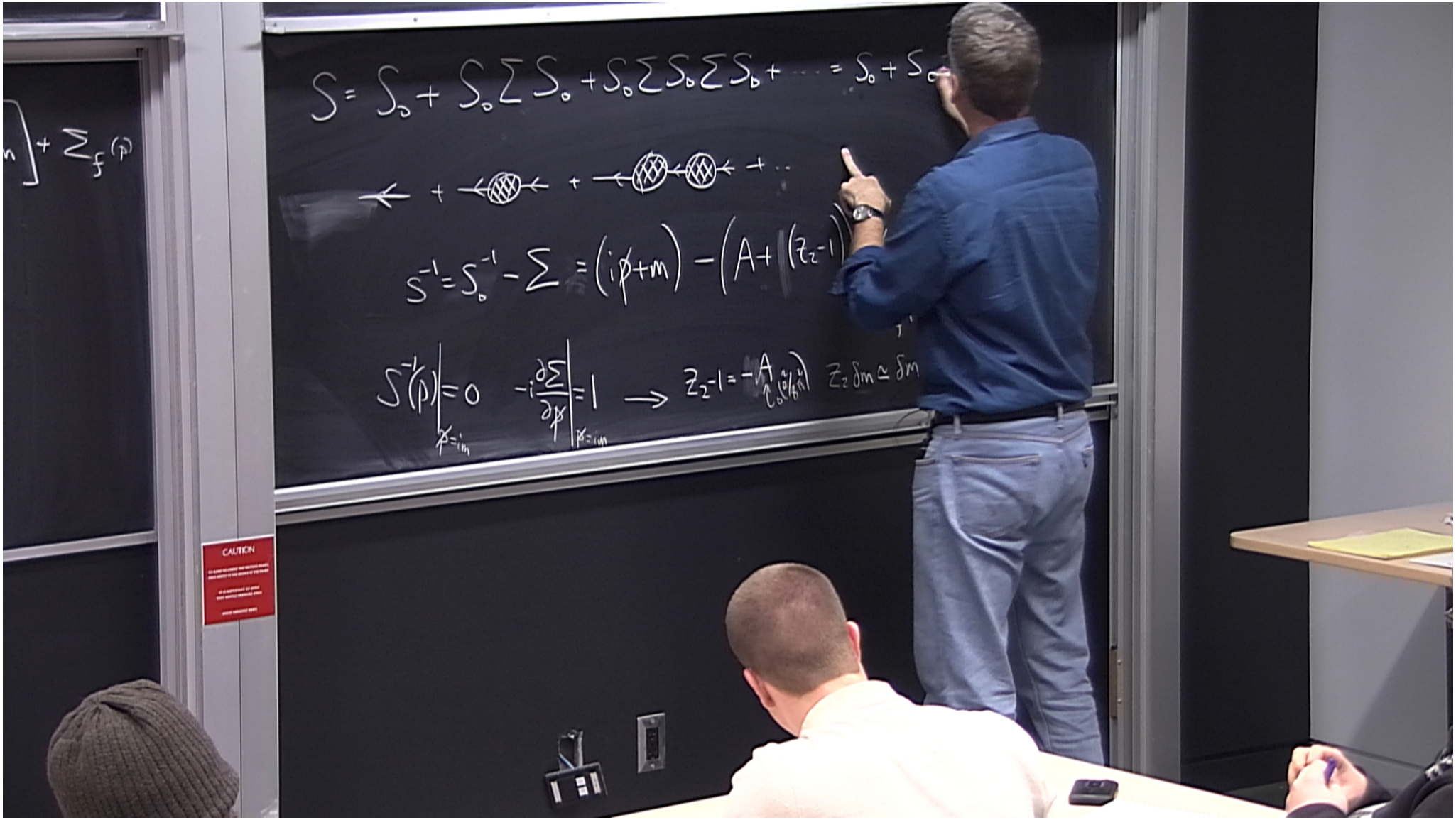

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 +$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta_m\right)$
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1$
CAUTION

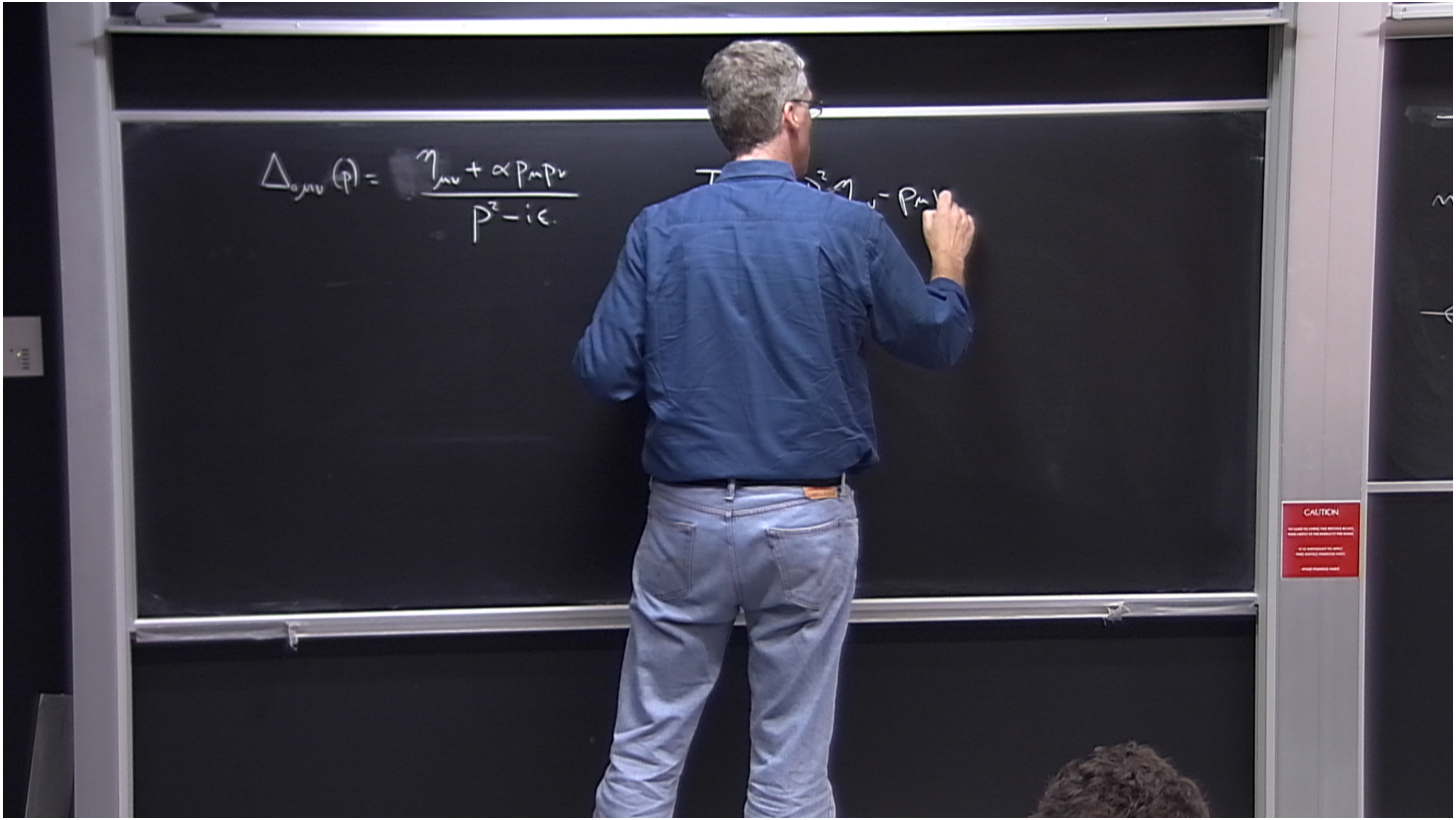

$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + i(Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1)\right)$
$S^{-1}(p)\Big|_{\not{p}=im} = 0 \quad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p}=im} = 1 \Rightarrow Z_2 - 1 = iA$
CAUTION

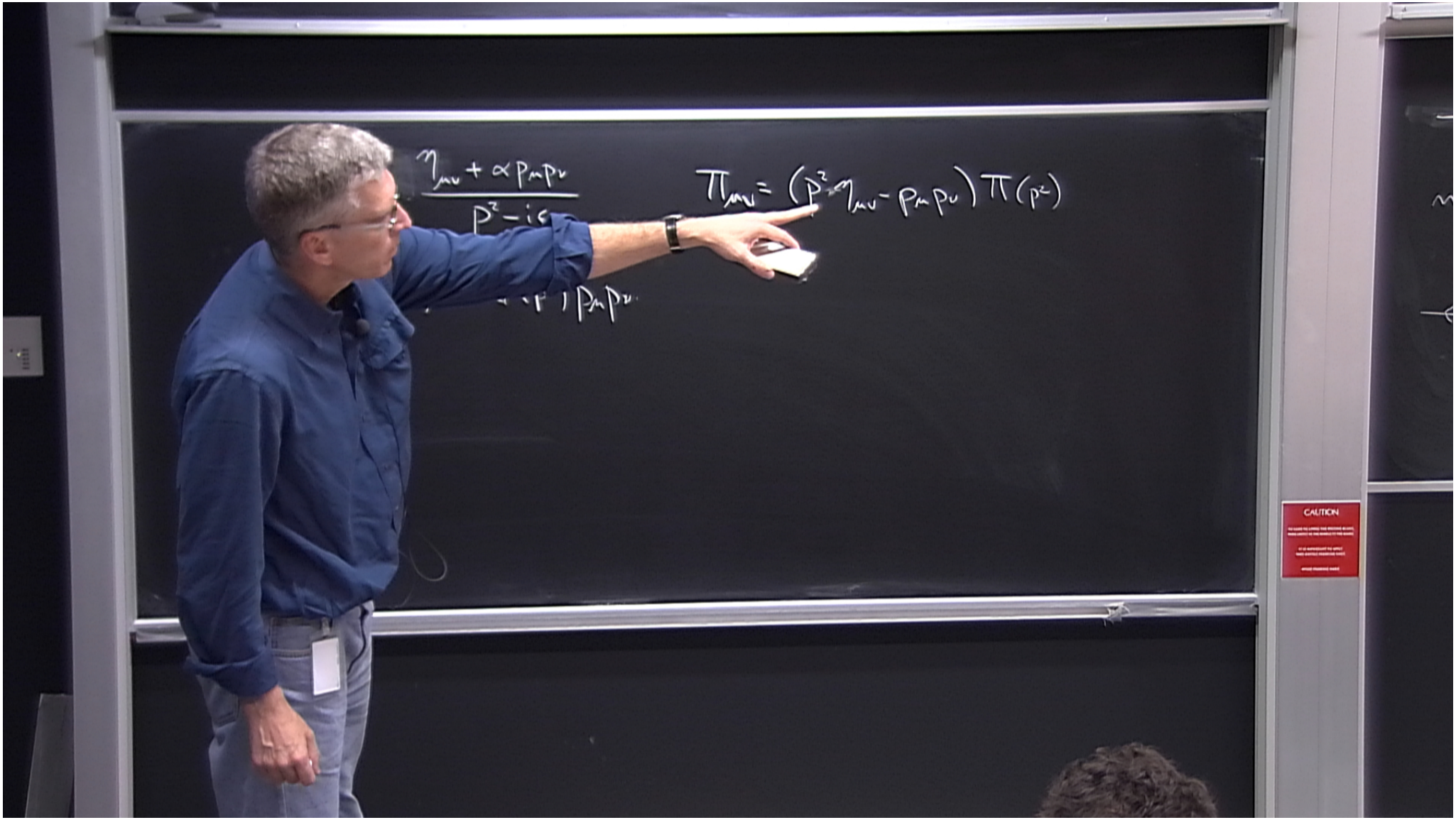

$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + (Z_2 - 1)\right)$
$S^{-1}(p)\big|_{\not{p}=im} = 0$
$-i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p}=im} = 1 \rightarrow$
$Z_2 - 1 =$
CAUTION

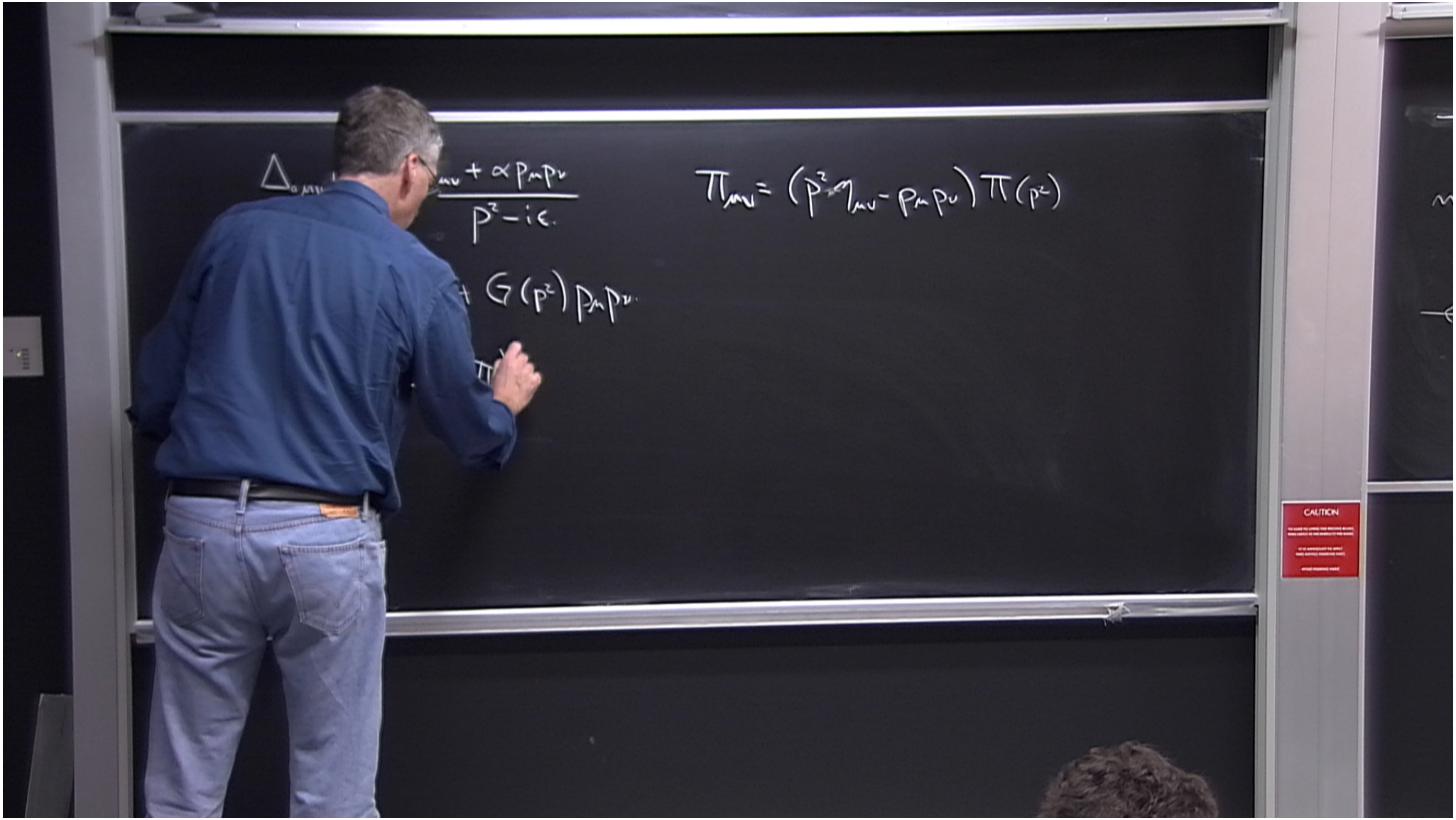

$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + (Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta m\right)$
$- \Sigma_f(p)$
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \quad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 - 1 = -A \quad Z_2 \delta m \simeq \delta m =$
CAUTION

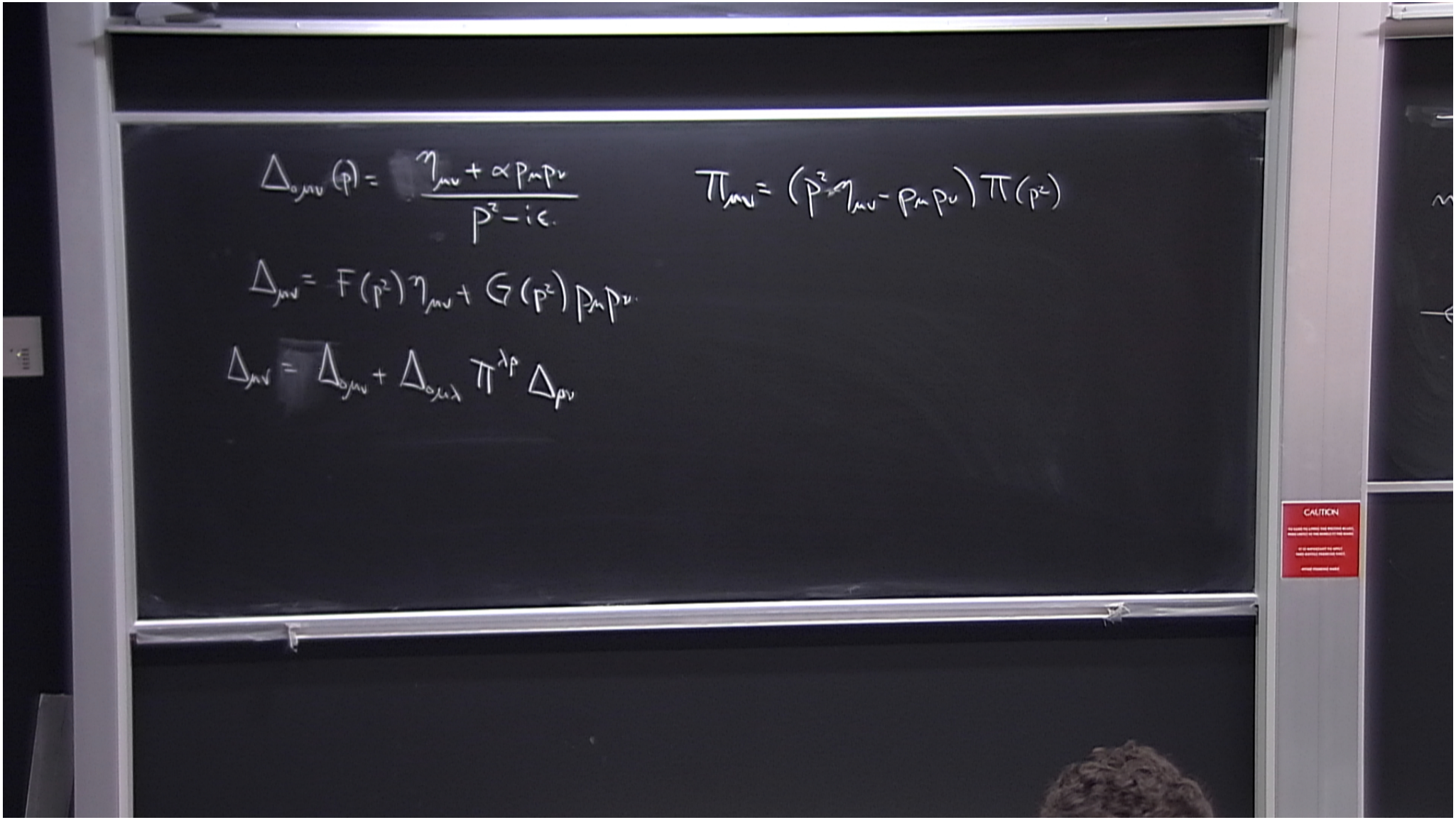

$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 +$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + (Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta m\right)$
$- \Sigma_f(p)$
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i \frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 - 1 = -A \qquad Z_2 \delta m \simeq \delta m = -B - m(Z_2 - 1)$
CAUTION

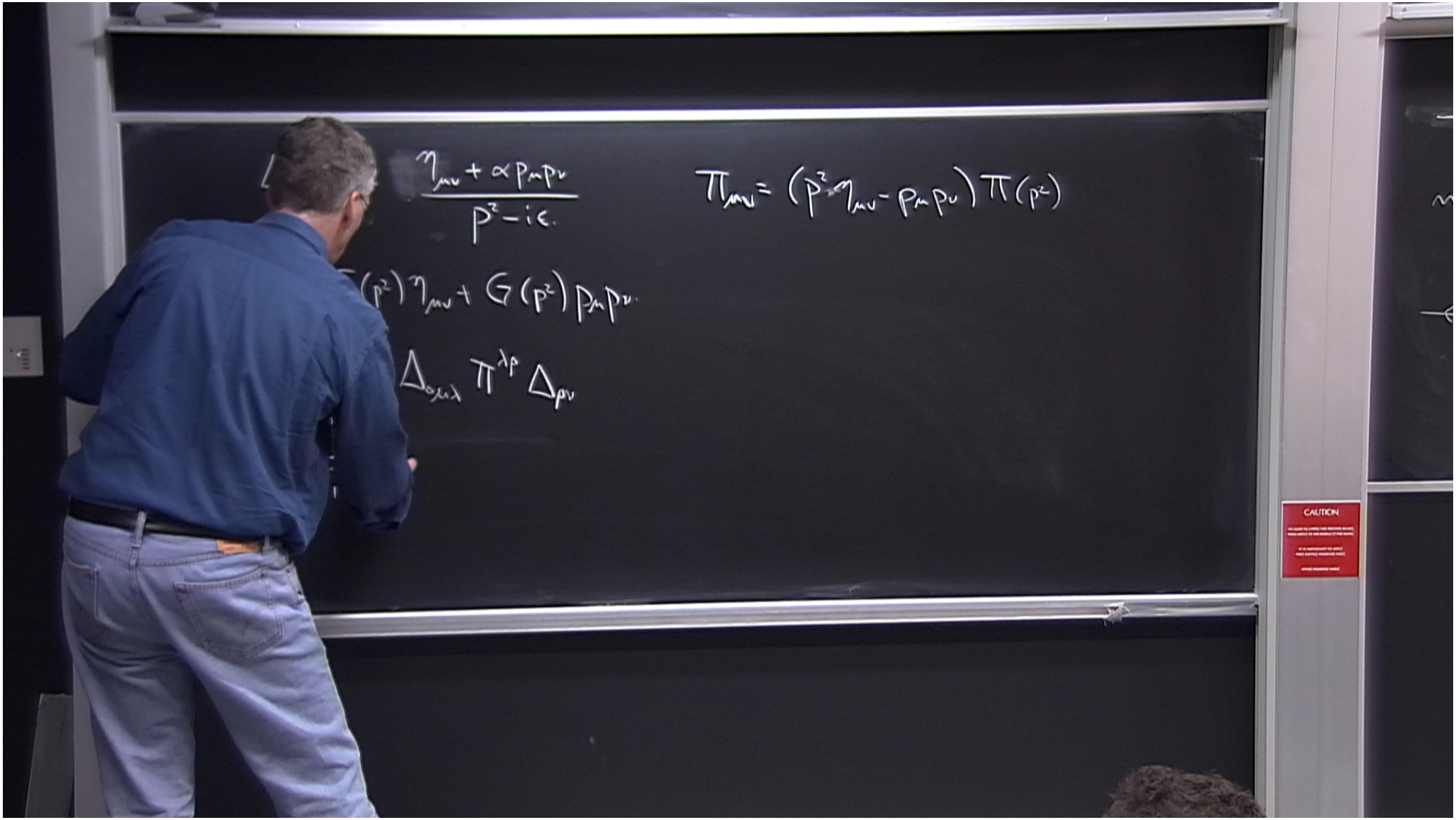

CAUTION

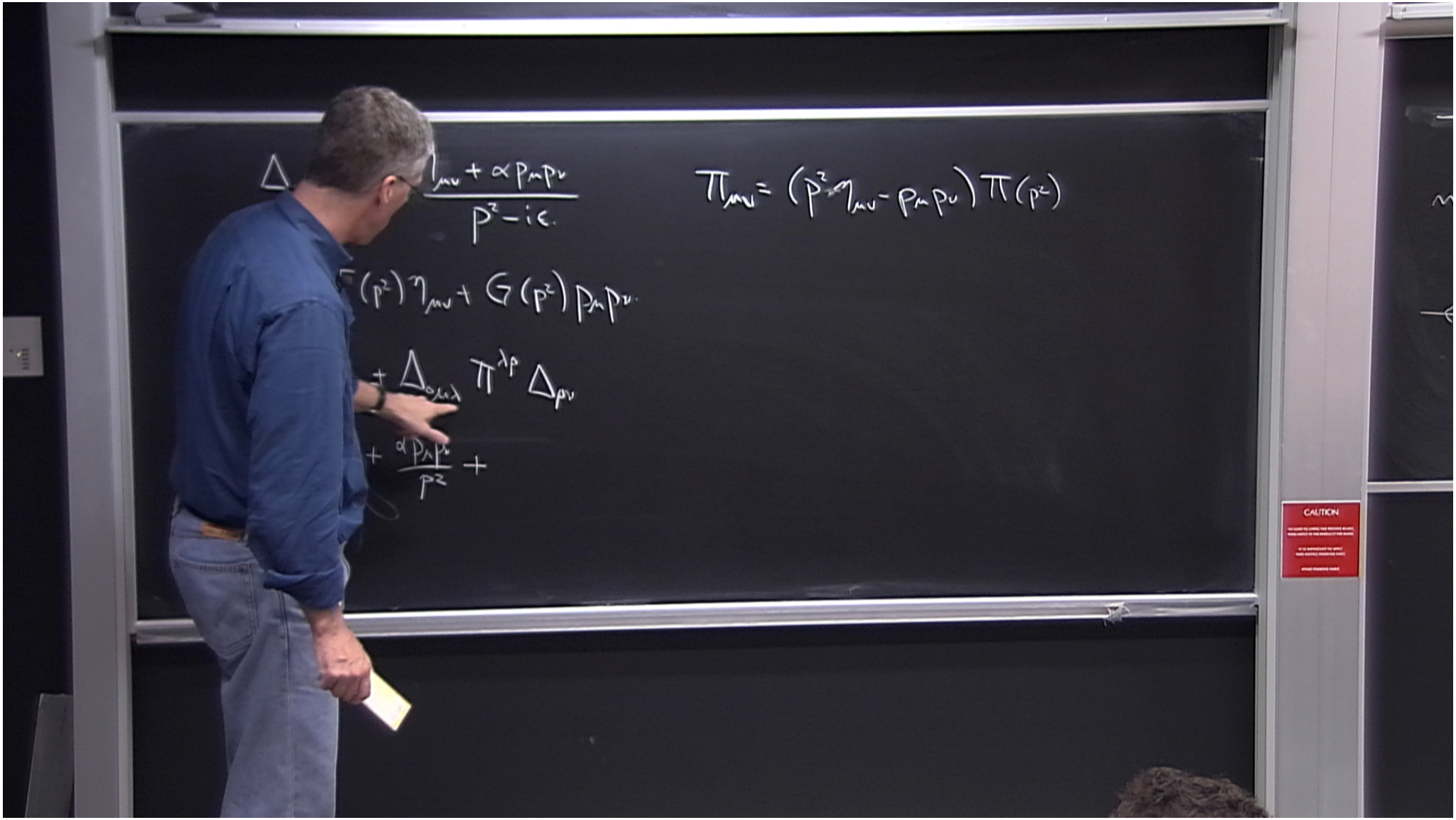

$\Delta_{\alpha\mu\nu}(p) = \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
CAUTION

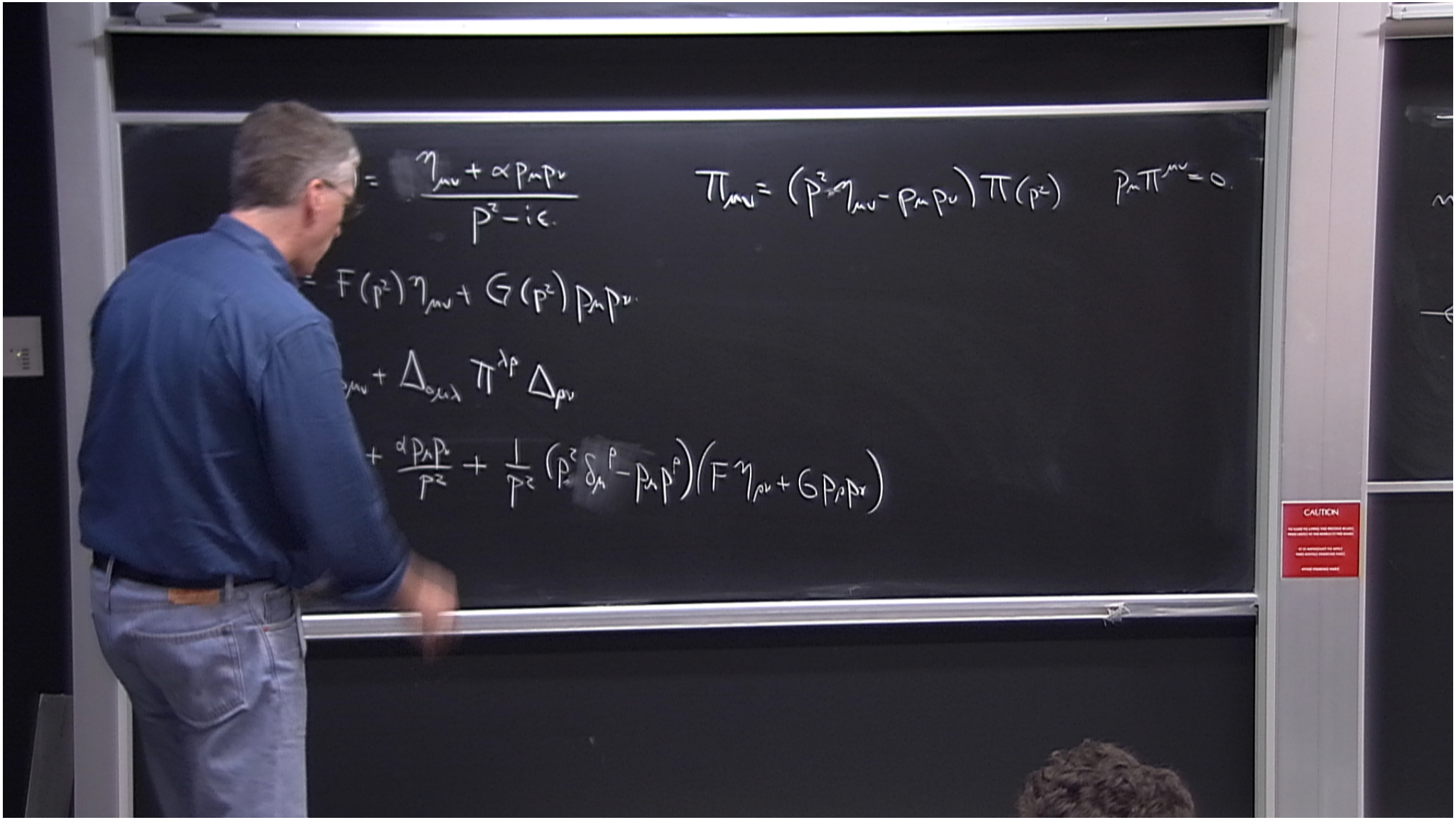

$\frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\,\Pi(p^2)$
CAUTION

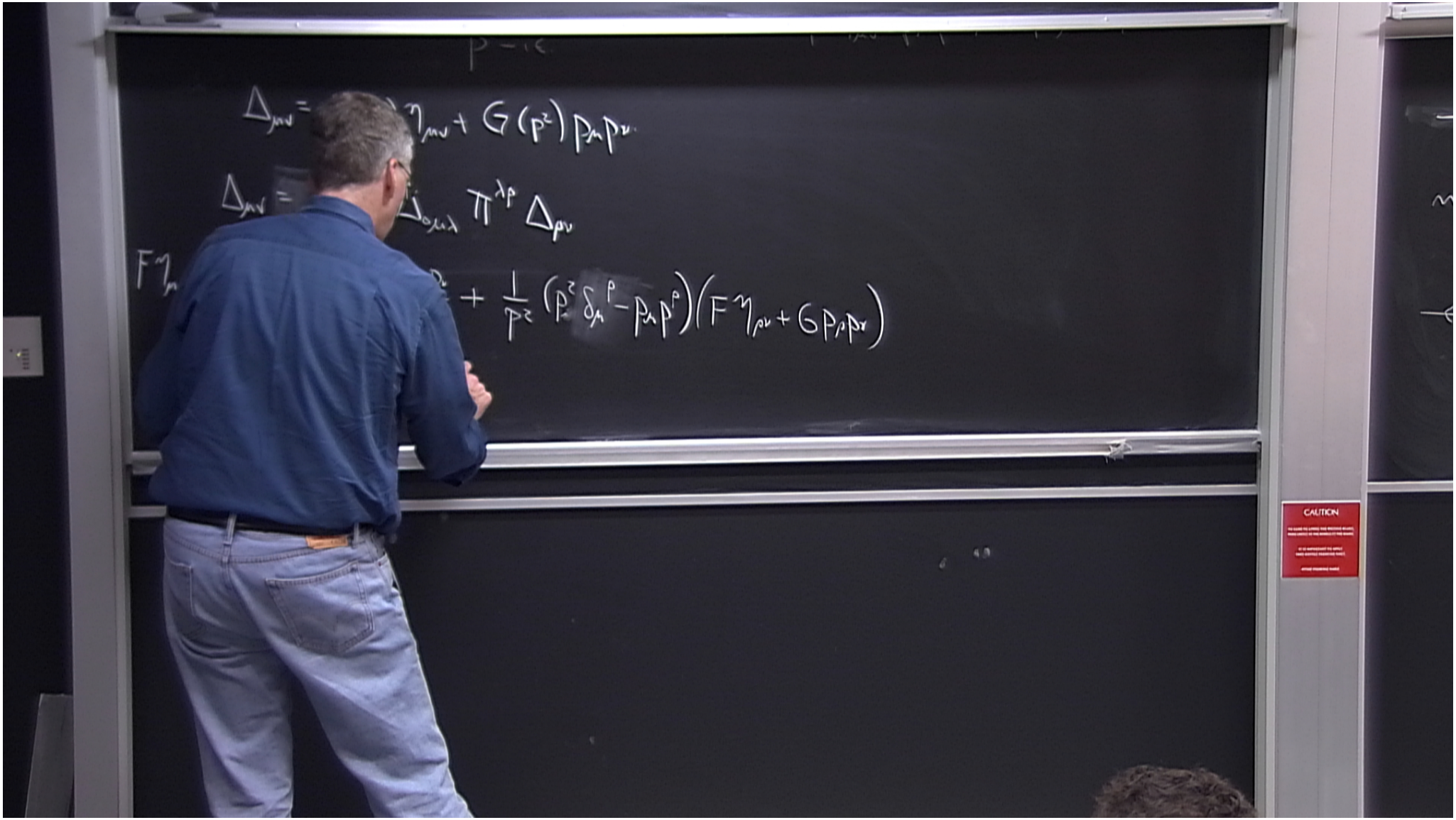

$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\,\Pi(p^2)$
$+ G(p^2)\, p_\mu p_\nu$
CAUTION

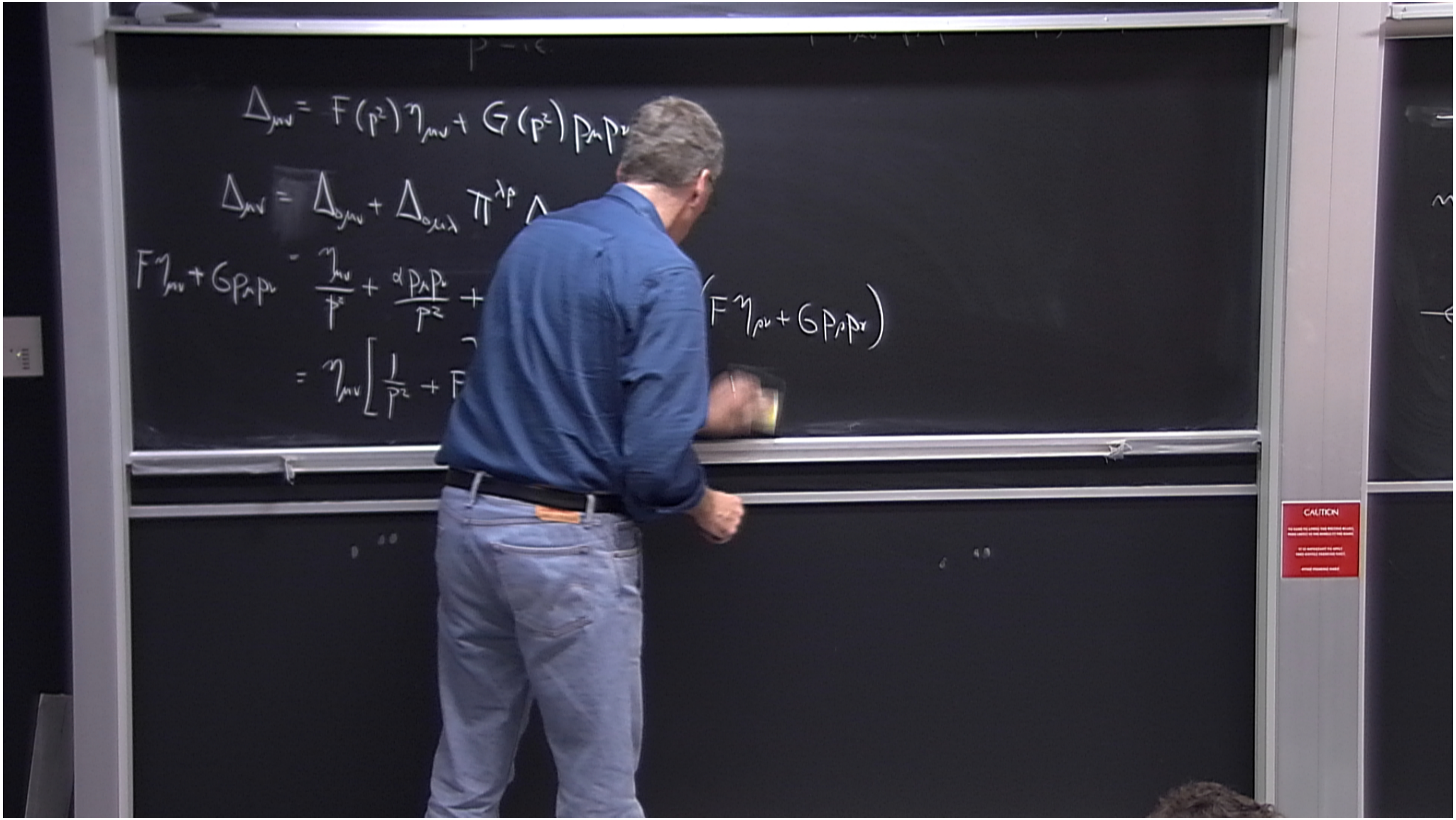

$\Delta_{0\mu\nu}(p) = \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$\Delta_{\mu\nu} = F(p^2)\eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$\Delta_{\mu\nu} = \Delta_{0\mu\nu} + \Delta_{0\mu\lambda}\Pi^{\lambda\rho}\Delta_{\rho\nu}$
CAUTION

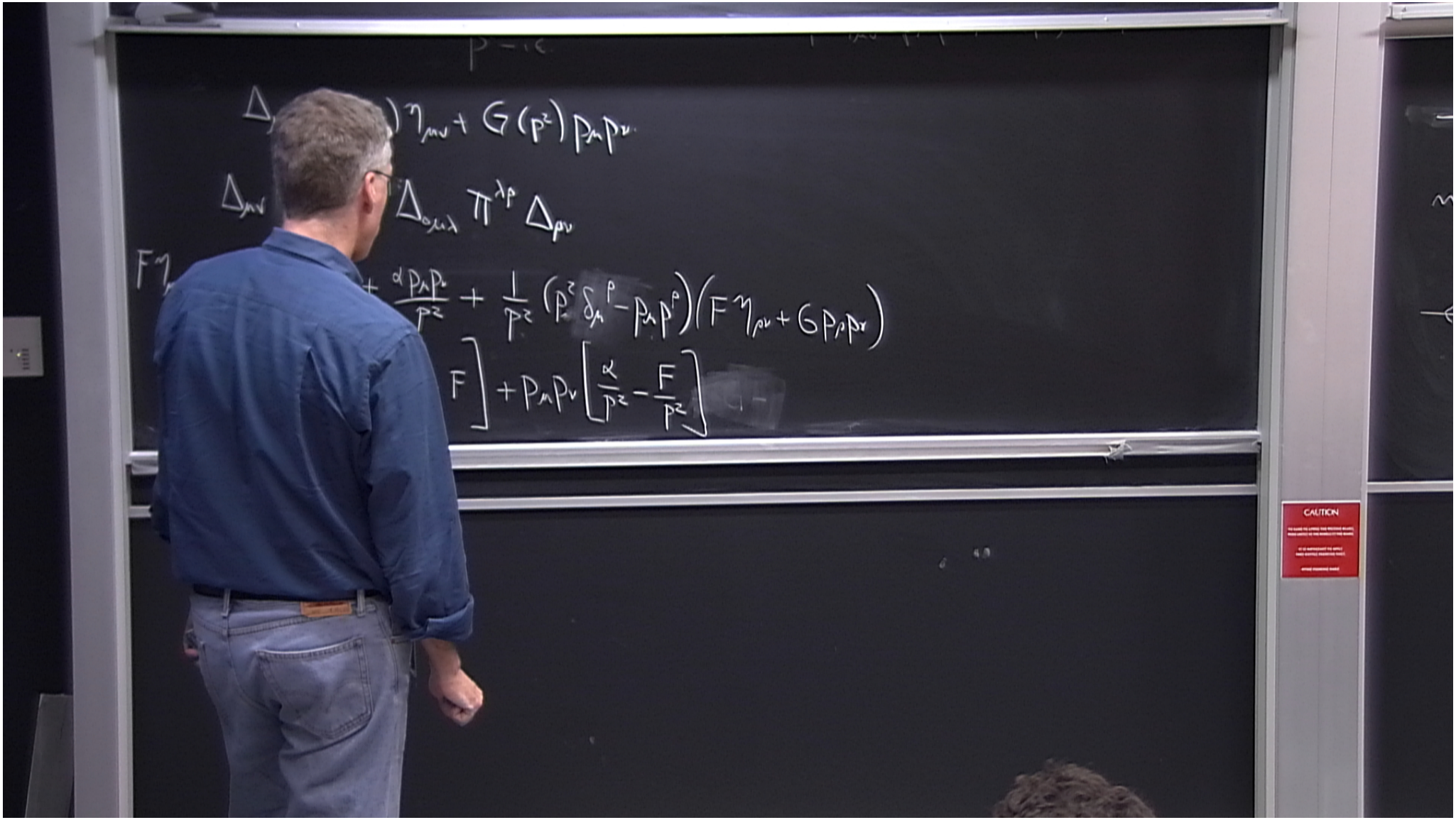

$\frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$(p^2)\eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$\Delta_{\alpha\mu\lambda}\, \pi^{\lambda\rho}\, \Delta_{\rho\nu}$
CAUTION

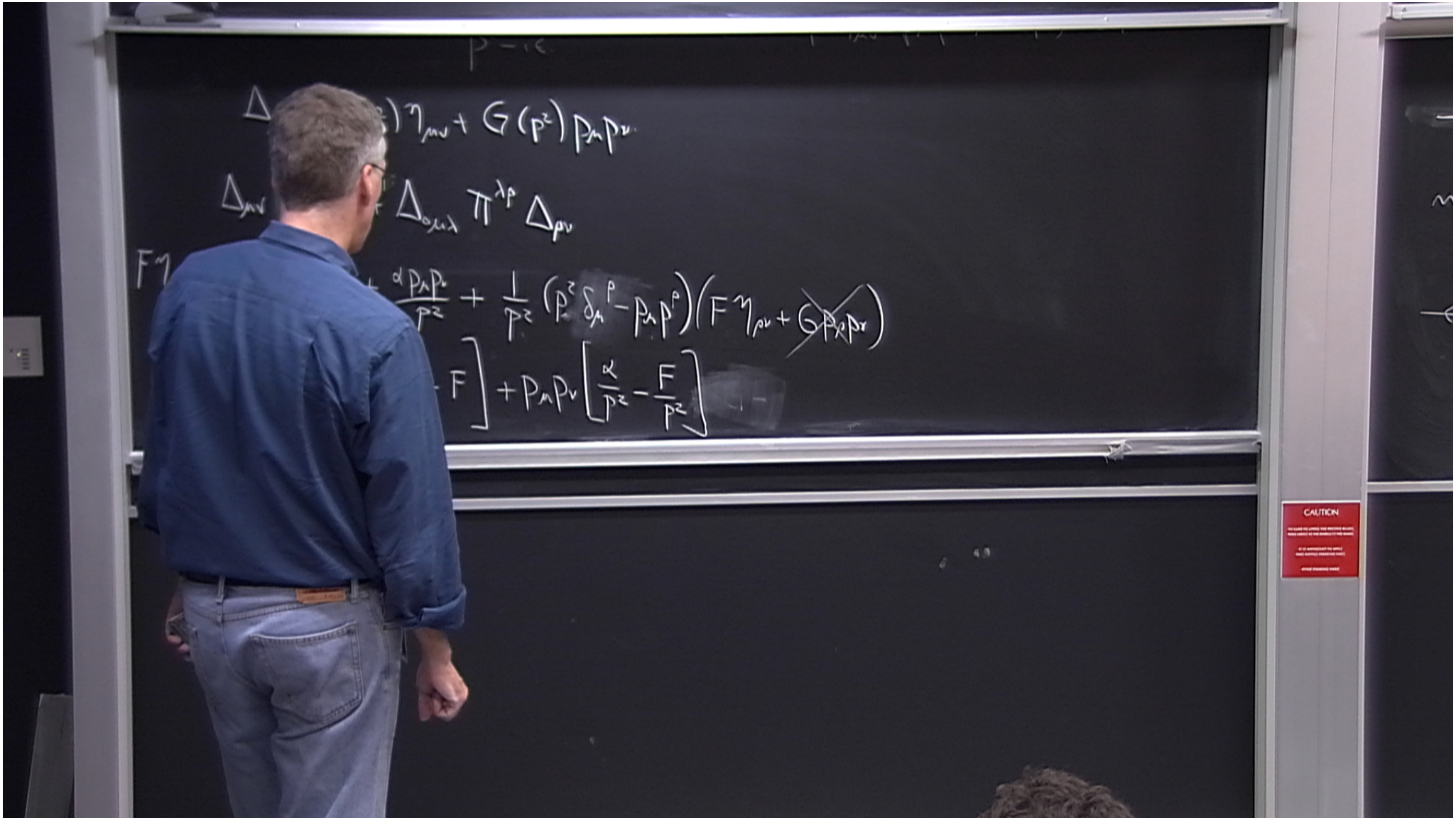

$\Delta \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$F(p^2)\eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$+ \Delta_{\alpha\mu\lambda} \Pi^{\lambda\rho} \Delta_{\rho\nu}$
$+ \frac{\alpha p_\lambda p_\nu}{p^2} +$
CAUTION

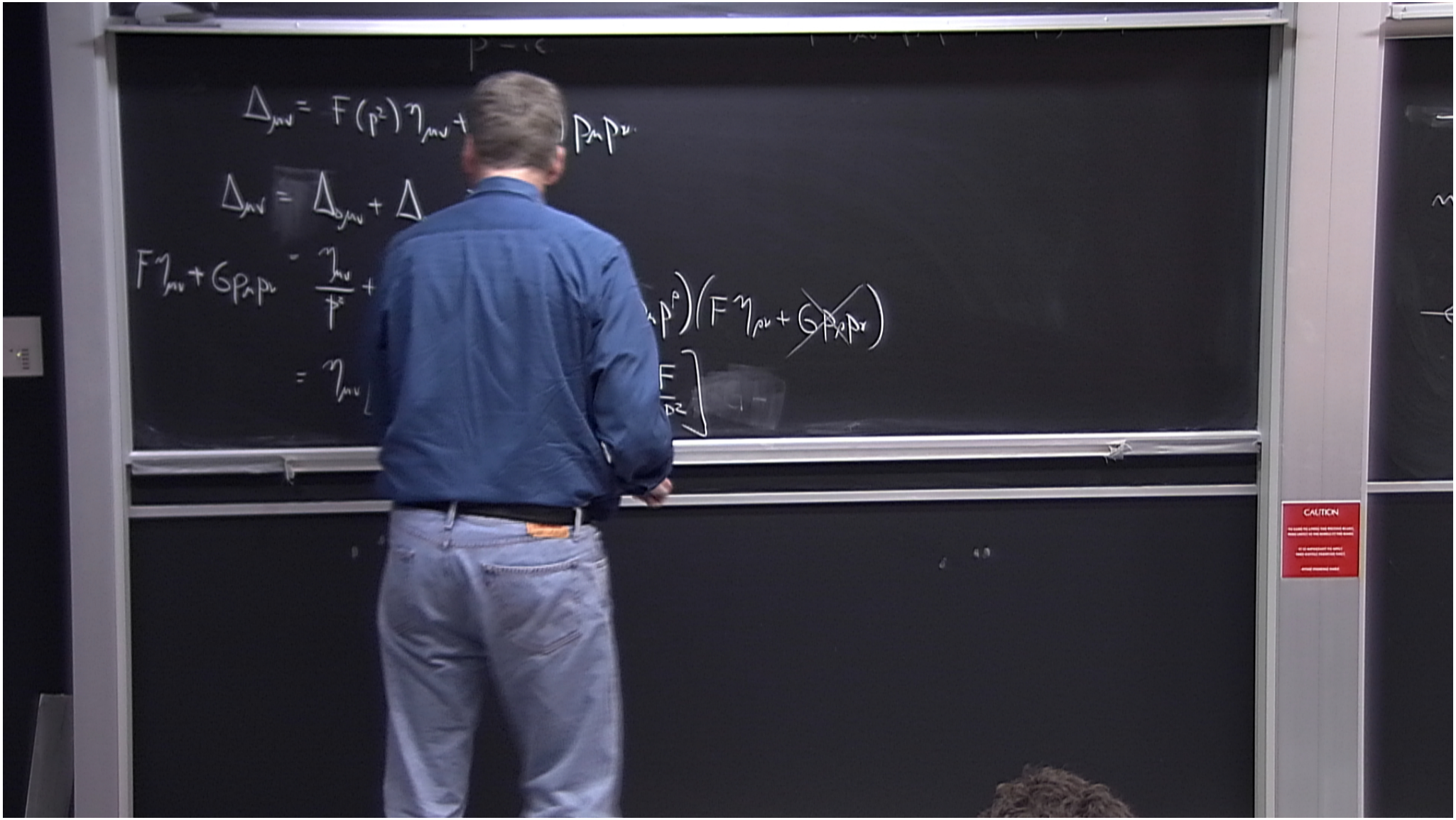

$= \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$p_\mu \Pi^{\mu\nu} = 0$
$= F(p^2)\eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$+ \Delta_{\alpha\mu\lambda} \Pi^{\lambda\rho} \Delta_{\rho\nu}$
$+ \frac{\alpha p_\lambda p_\nu}{p^2} + \frac{1}{p^2}(p^2 \delta_\lambda^{\ \rho} - p_\lambda p^\rho)(F\eta_{\rho\nu} + G p_\rho p_\nu)$
CAUTION

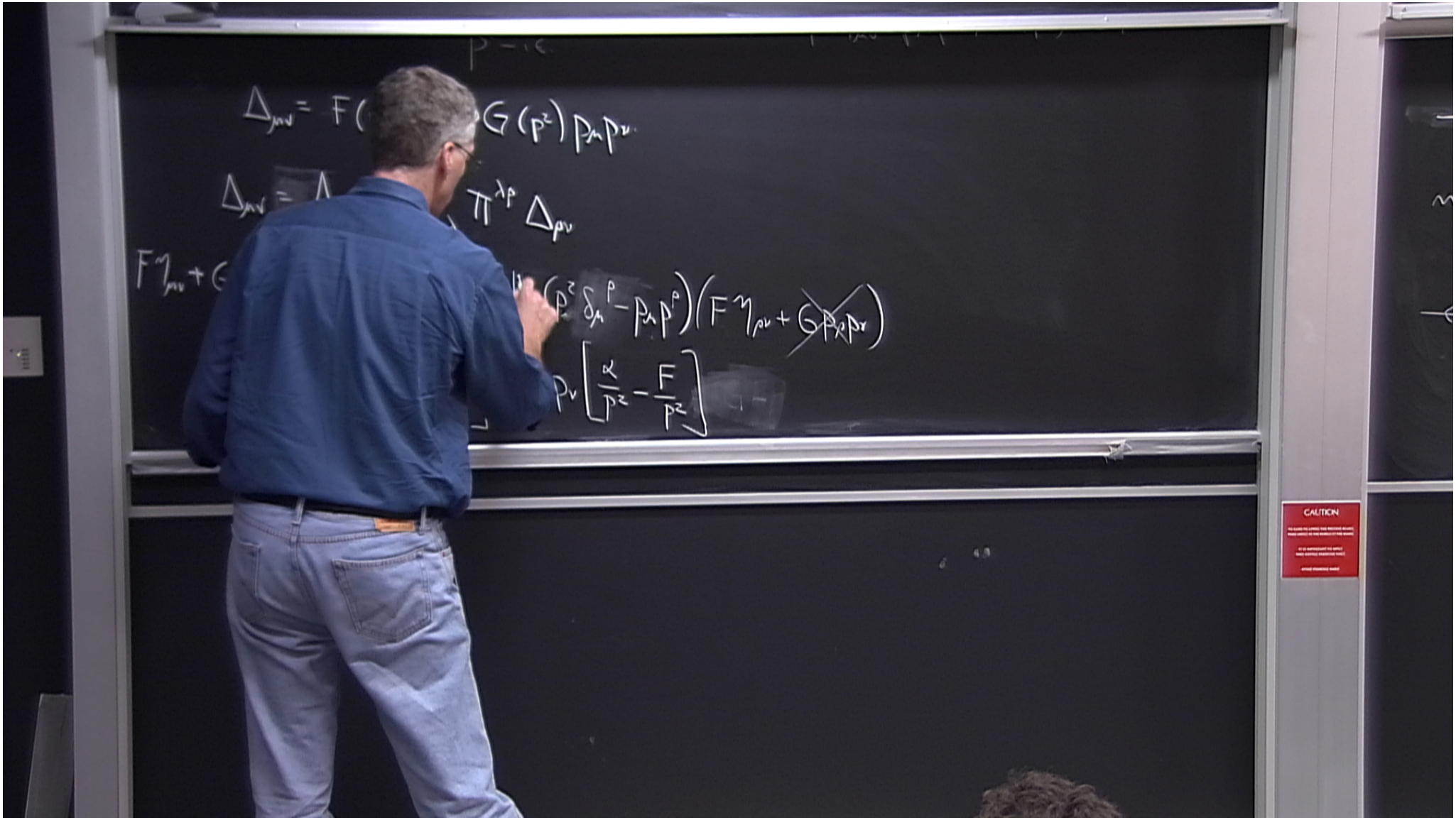

$\Delta_{\mu\nu} = \eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$\Delta_{\mu\nu} = \Delta_{\alpha\mu\lambda} \pi^{\lambda\rho} \Delta_{\rho\nu}$
$+ \frac{1}{p^2}(p^2 \delta_\mu^\rho - p_\mu p^\rho)(F \eta_{\rho\nu} + G p_\rho p_\nu)$
CAUTION

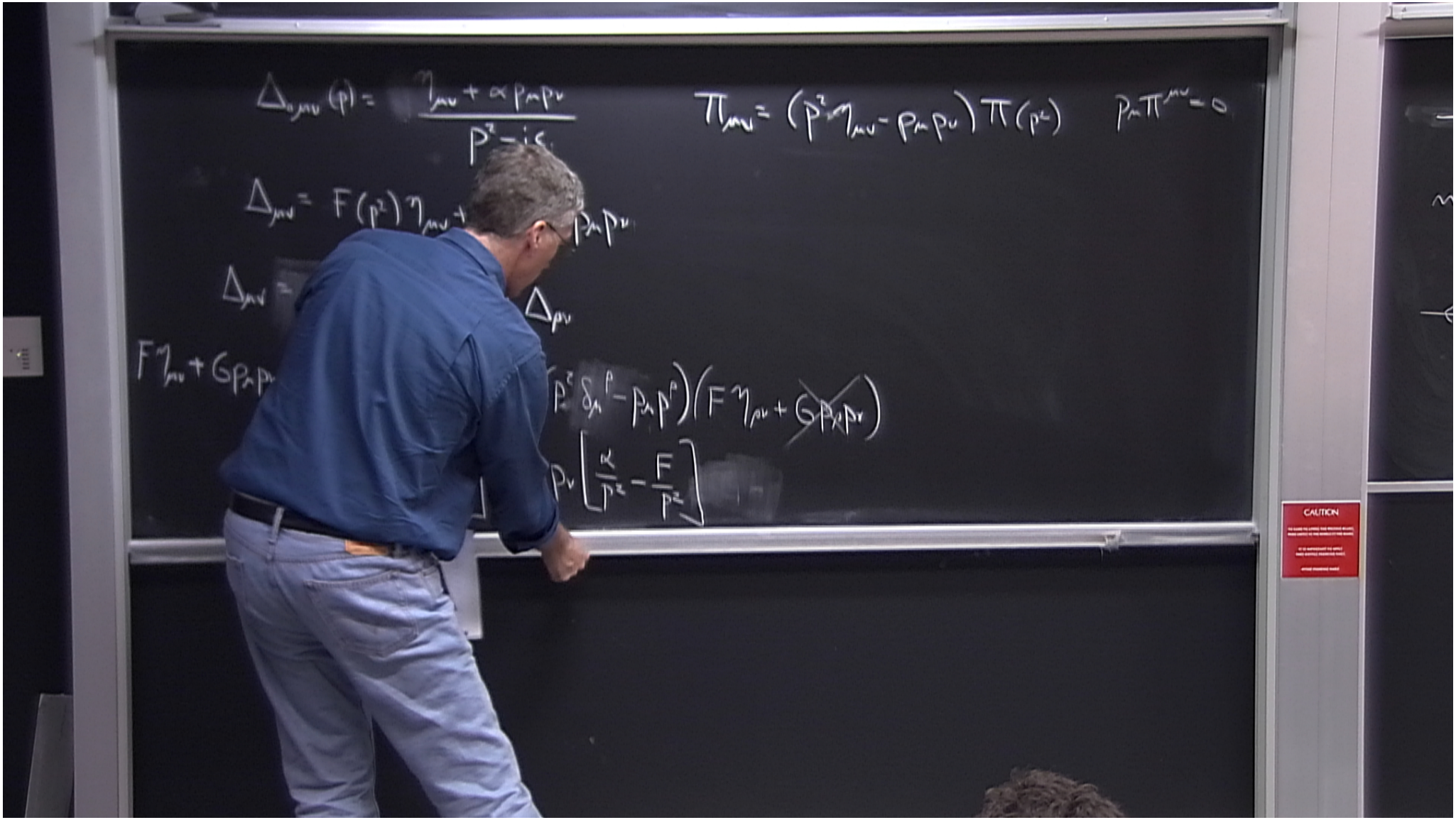

$\Delta_{\mu\nu} = F(p^2)\eta_{\mu\nu} + G(p^2)p_\mu p_\nu$
$\Delta_{\mu\nu} = \Delta_{0\mu\nu} + \Delta_{0\mu\lambda}\,\pi^{\lambda\rho}\,\Lambda$
$F\eta_{\mu\nu} + Gp_\mu p_\nu$
$\frac{\eta_{\mu\nu}}{p^2} + \frac{\alpha p_\mu p_\nu}{p^2} +$
$\left(F\eta_{\mu\nu} + Gp_\mu p_\nu\right)$
$= \eta_{\mu\nu}\left[\frac{1}{p^2} + F\right.$
CAUTION

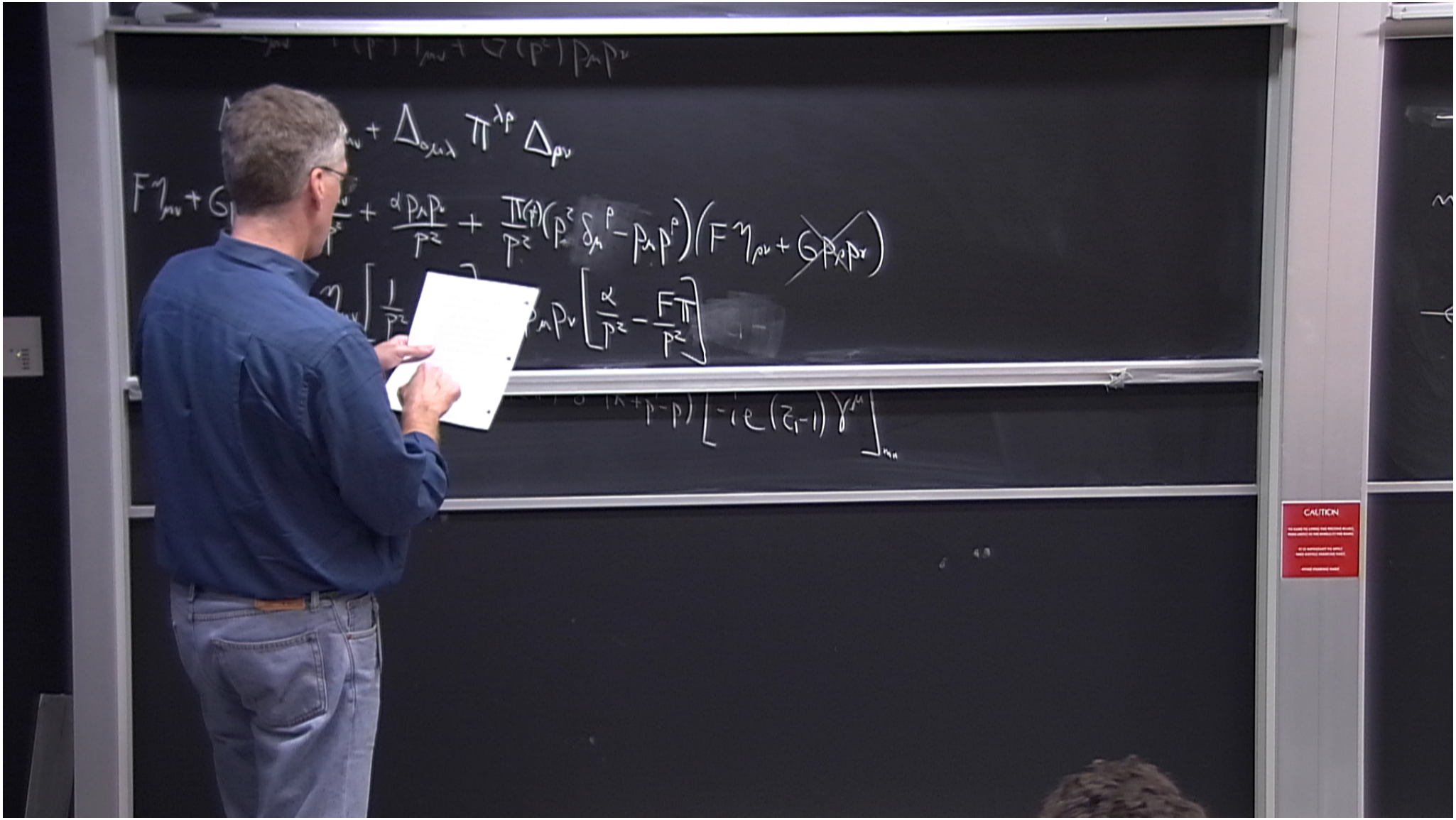

$G(p^2)\,p_\mu p_\nu$
$\Delta_{\alpha\mu\lambda}\,\pi^{\lambda\rho}\,\Delta_{\rho\nu}$
$\frac{\alpha\,p_\mu p_\nu}{p^2} + \frac{1}{p^2}\left(p^2\delta_\mu{}^\rho - p_\mu p^\rho\right)\left(F\eta_{\rho\nu} + G p_\rho p_\nu\right)$
$F\Big] + p_\mu p_\nu\left[\frac{\alpha}{p^2} - \frac{F}{p^2}\right]$
CAUTION

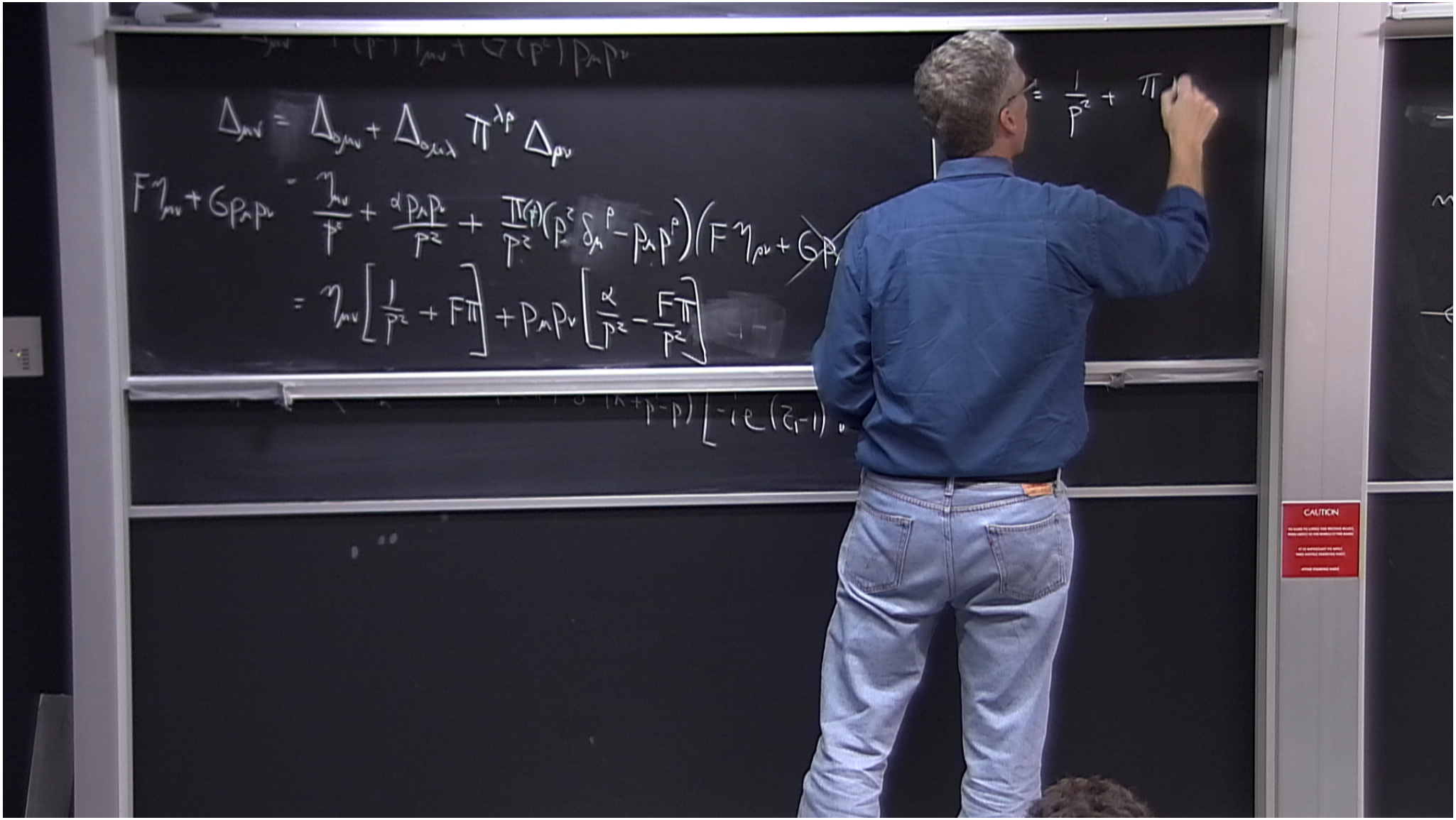

CAUTION

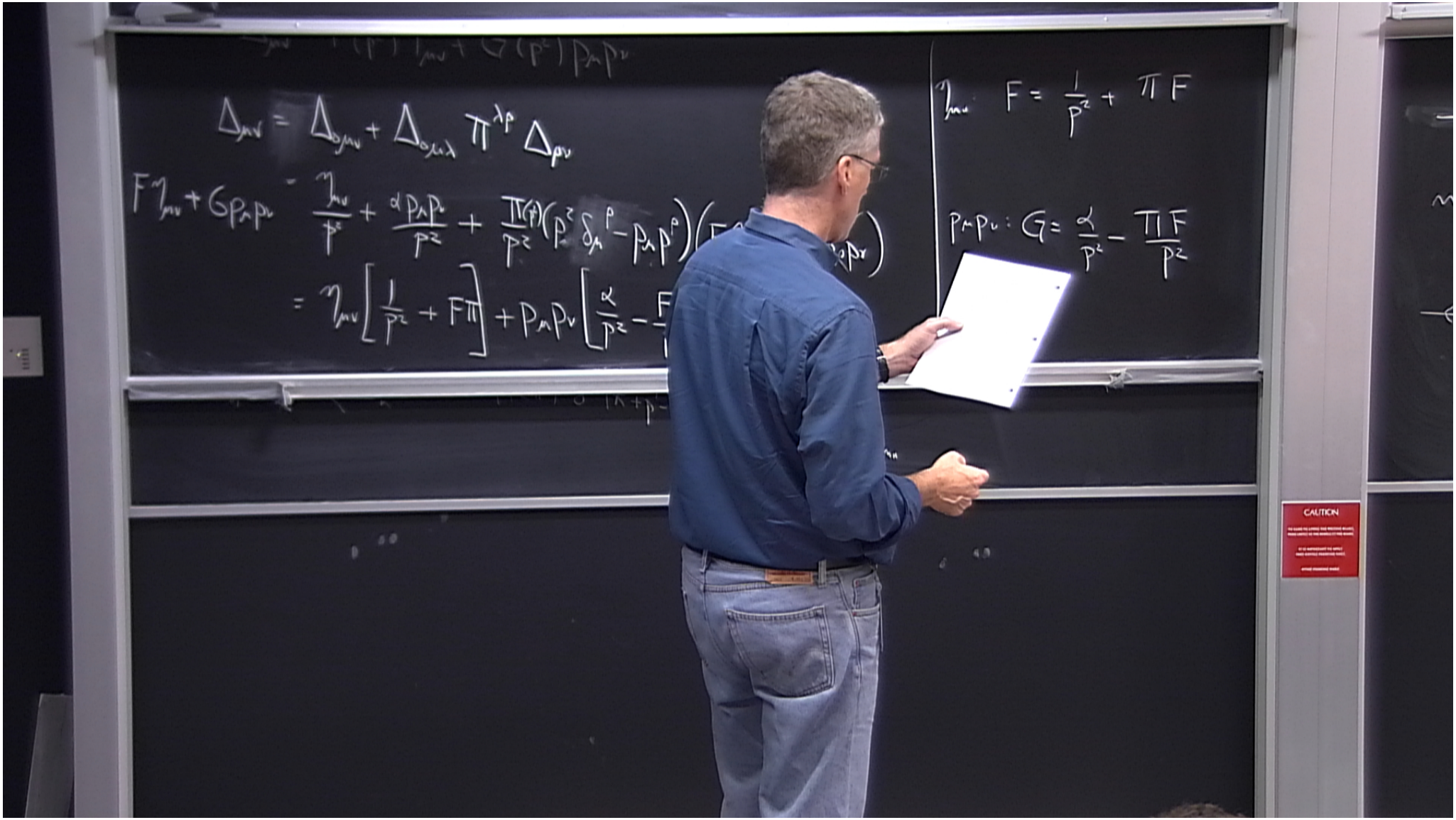

CAUTION

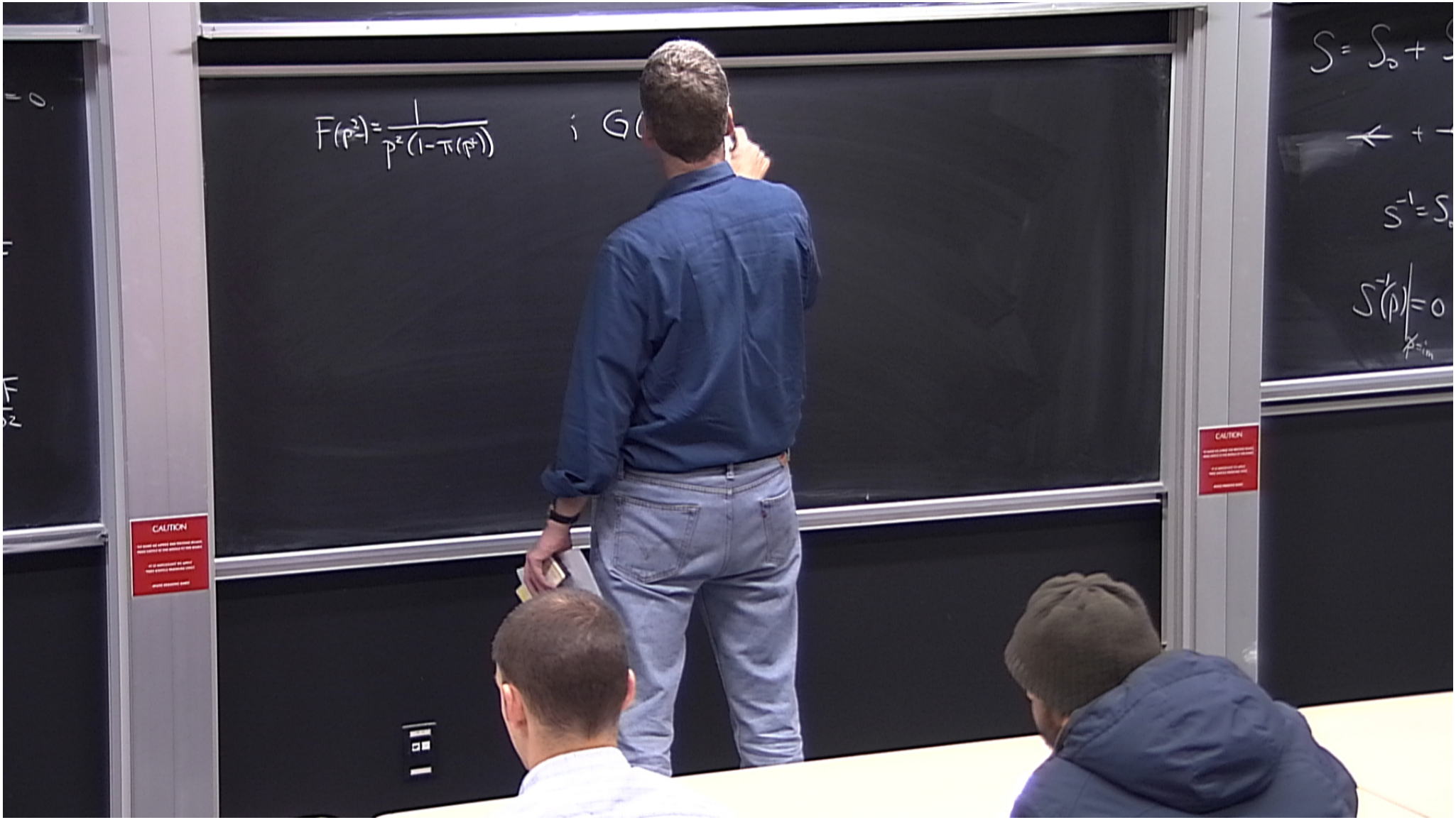

$\Delta_{\mu\nu} = F($
$G(p^2)\, p_\mu p_\nu$
$\Delta_{\mu\nu} = \Lambda$
$\pi^{\lambda\rho}\, \Delta_{\rho\nu}$
$F\eta_{\mu\nu} + G$
$(p^2 \delta_\mu^{\ \rho} - p_\mu p^\rho)(F\eta_{\rho\nu} + G p_\rho p_\nu)$
$p_\nu \left[\frac{\alpha}{p^2} - \frac{F}{p^2}\right]$
CAUTION

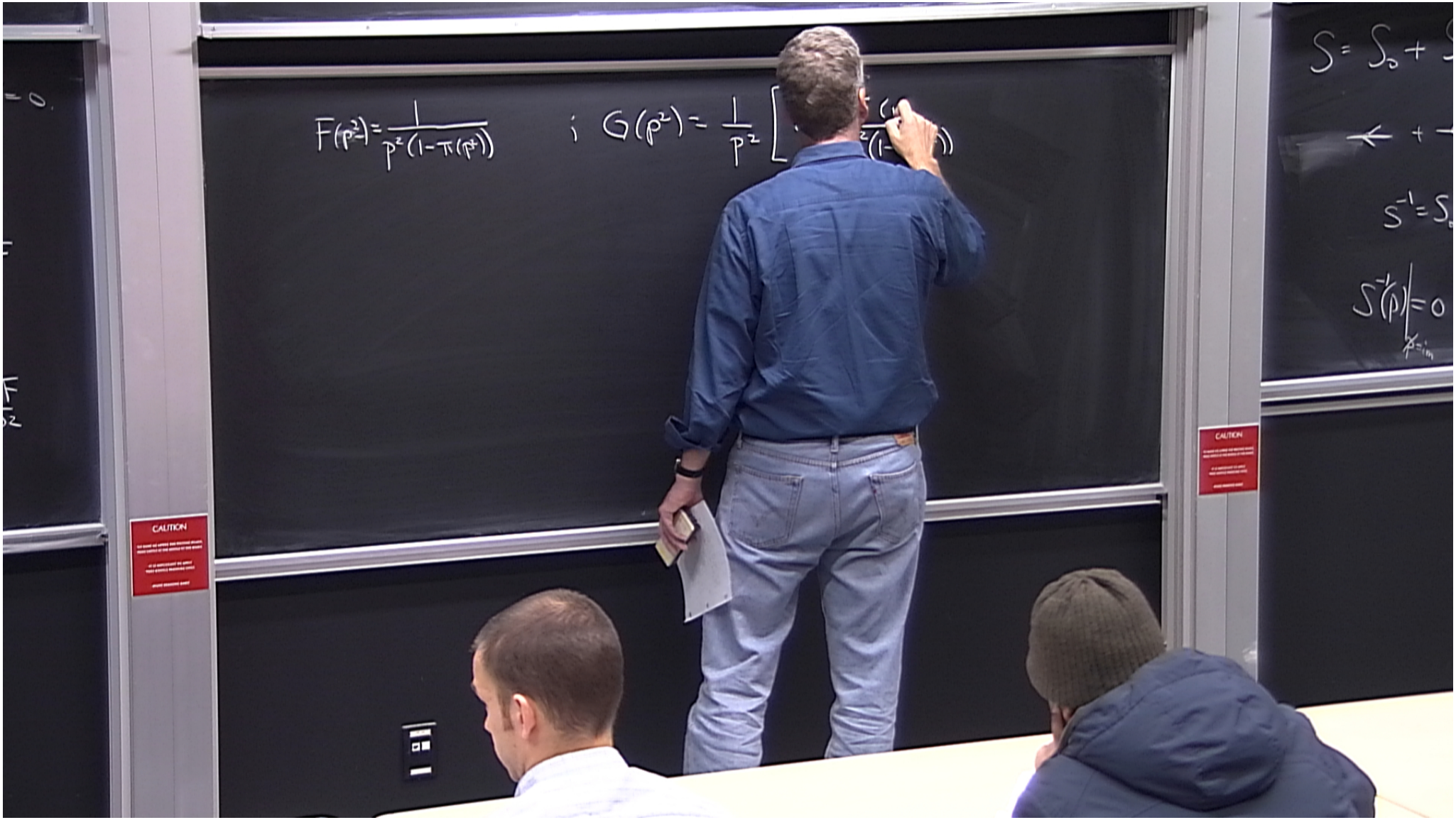

$\Delta_{\mu\nu}(p) = \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$
$p_\mu \Pi^{\mu\nu} = 0$
$\Delta_{\mu\nu} = F(p^2)\eta_{\mu\nu} +$
$\Delta_{\mu\nu} =$
$\Delta_{\rho\nu}$
$F\eta_{\mu\nu} + Gp_\mu p_\nu$
$(p^2 \delta_\mu^\rho - p_\mu p^\rho)(F\eta_{\rho\nu} + Gp_\rho p_\nu)$
$p_\nu \left[\frac{\alpha}{p^2} - \frac{F}{p^2}\right]$
CAUTION

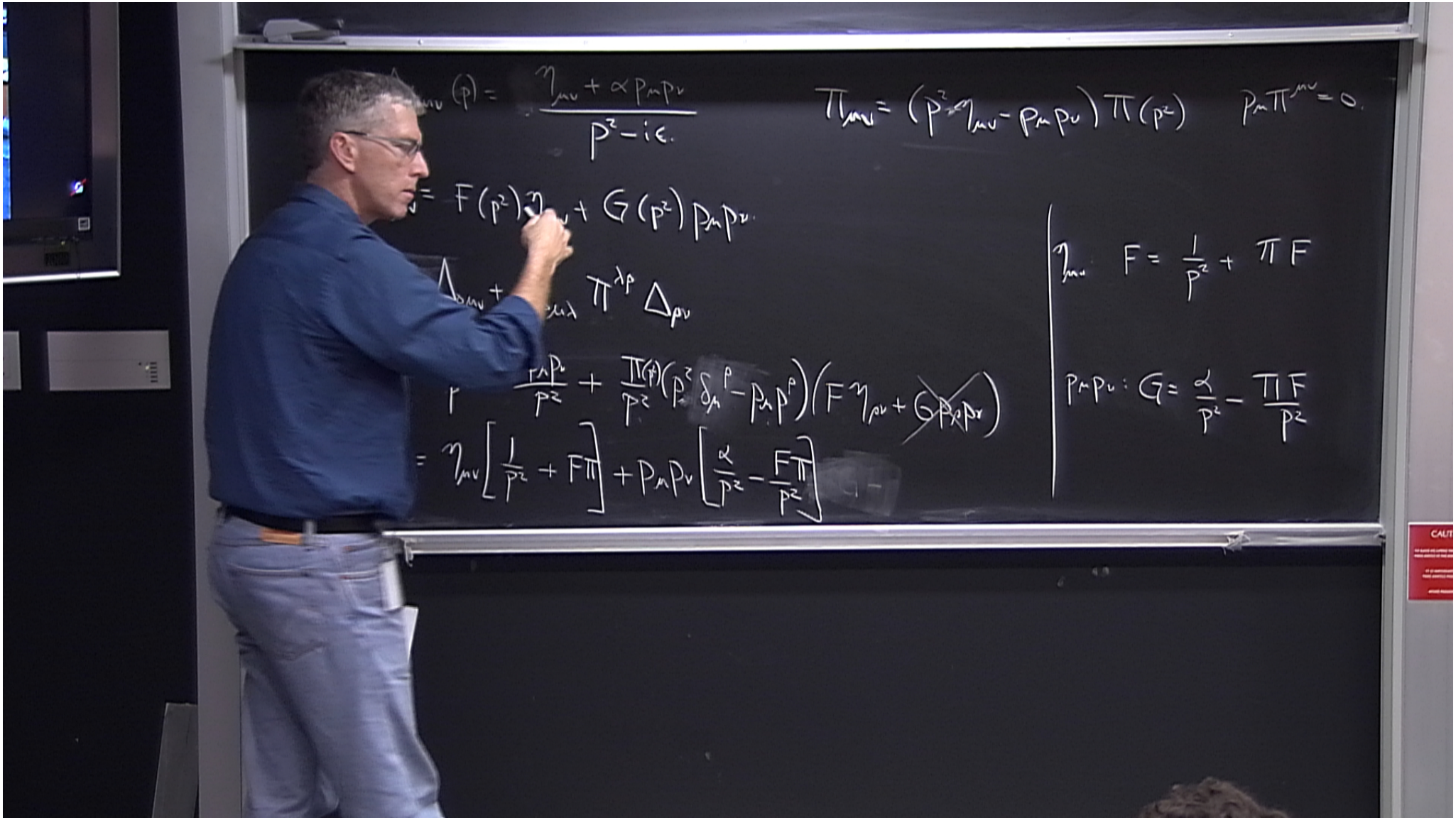

CAUTION

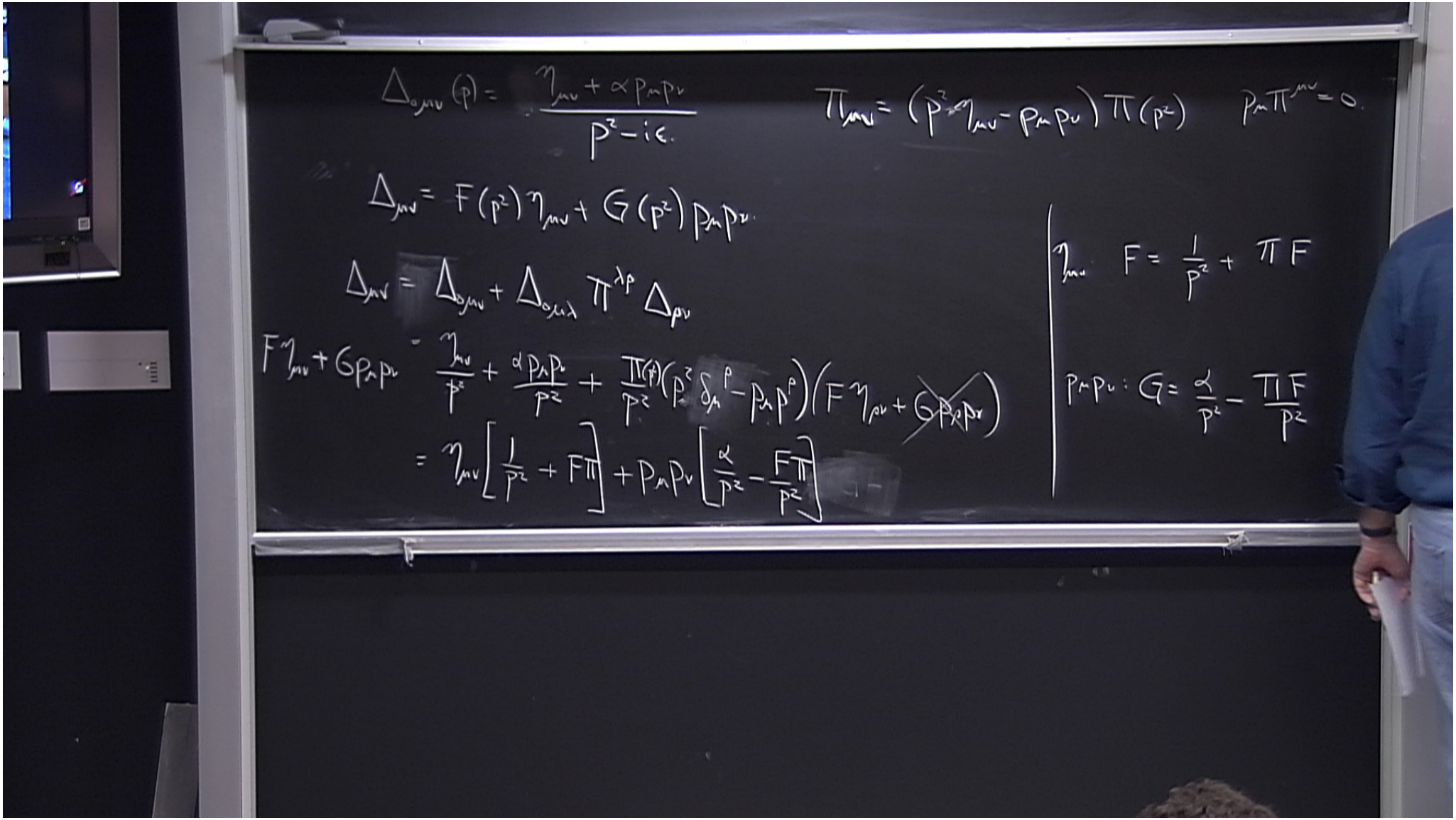

$\Delta_{\mu\nu} = \Delta_{0\mu\nu} + \Delta_{0\mu\lambda}\,\pi^{\lambda\rho}\,\Delta_{\rho\nu}$
$= \eta_{\mu\nu}\left[\frac{1}{p^2} + F\pi\right] + p_\mu p_\nu\left[\frac{\alpha}{p^2} - \frac{F\pi}{p^2}\right]$
CAUTION

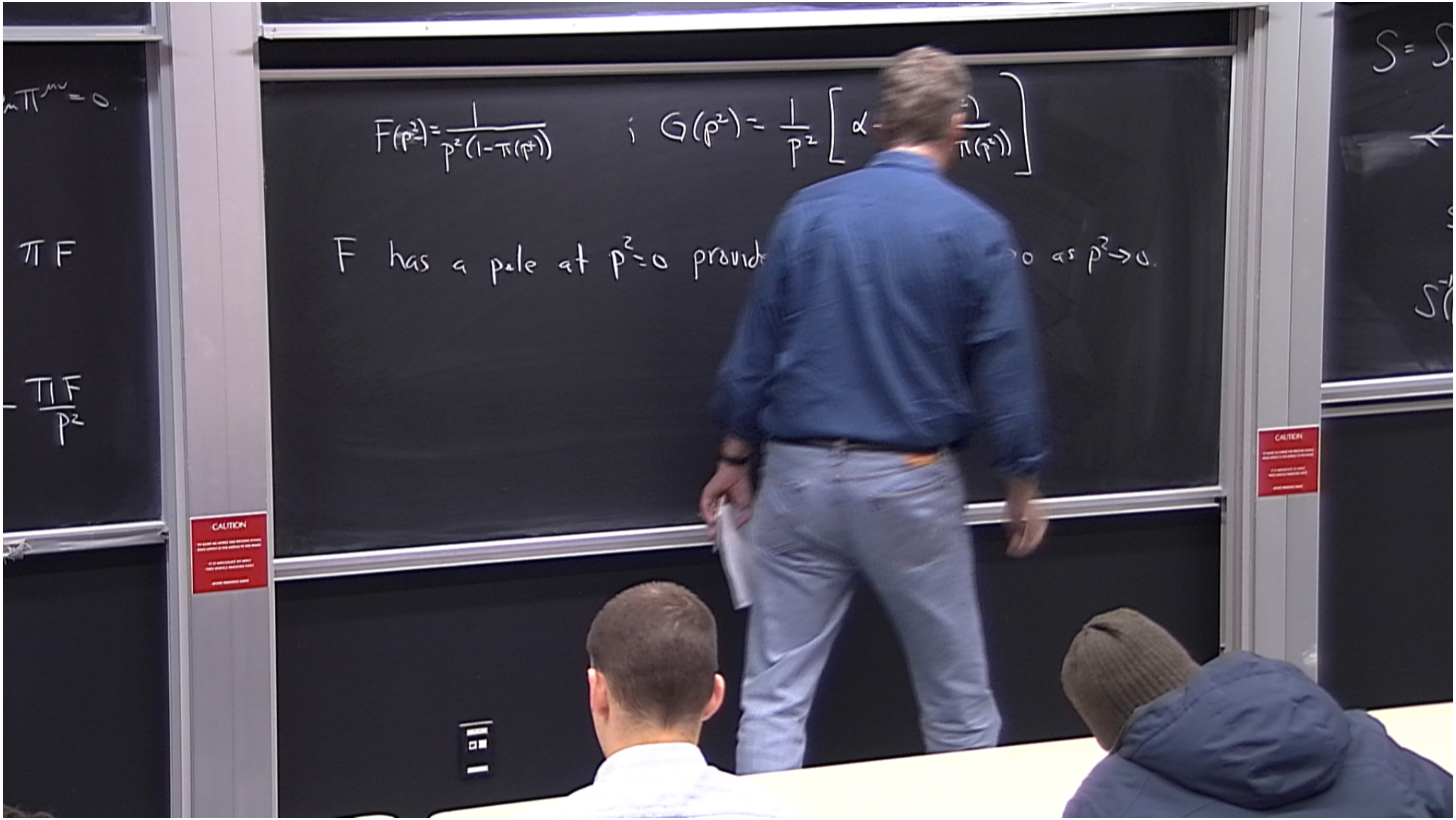

$\Delta_{\mu\nu} = \Delta_{0\mu\nu} + \Delta_{0\mu\lambda}\,\Pi^{\lambda\rho}\,\Delta_{\rho\nu}$
$F\eta_{\mu\nu} + G p_\mu p_\nu$
$= \eta_{\mu\nu}\left[\frac{1}{p^2} + F\Pi\right] + p_\mu p_\nu\left[\frac{\alpha}{p^2} -$
$\eta_{\mu\nu}: F = \frac{1}{p^2} + \Pi F$
$p_\mu p_\nu: G = \frac{\alpha}{p^2} - \frac{\Pi F}{p^2}$
CAUTION

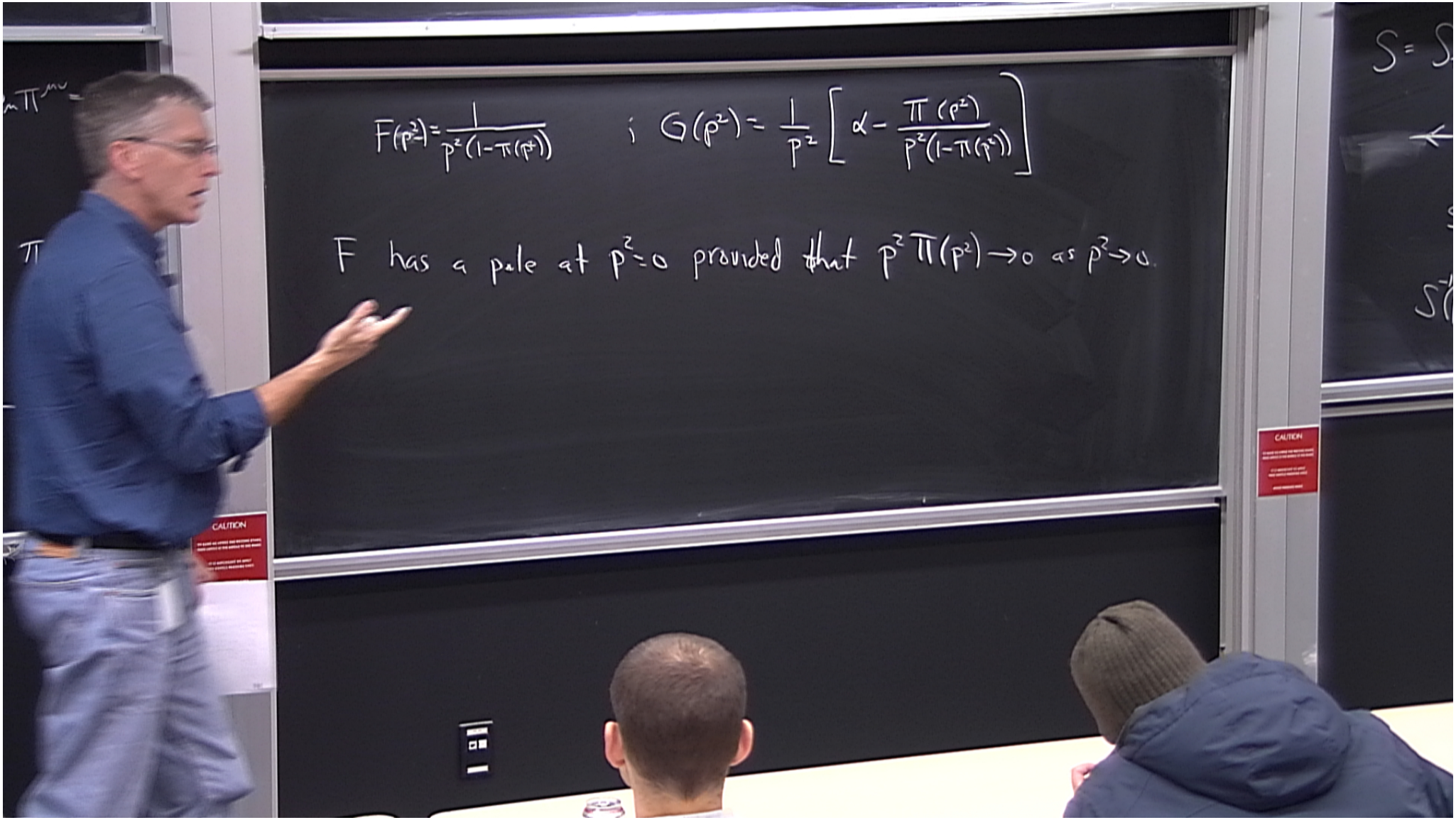

$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$
$S = S_0 +$
$S^{-1} = S$
$S^{-1}(p) = 0$
CAUTION
CAUTION

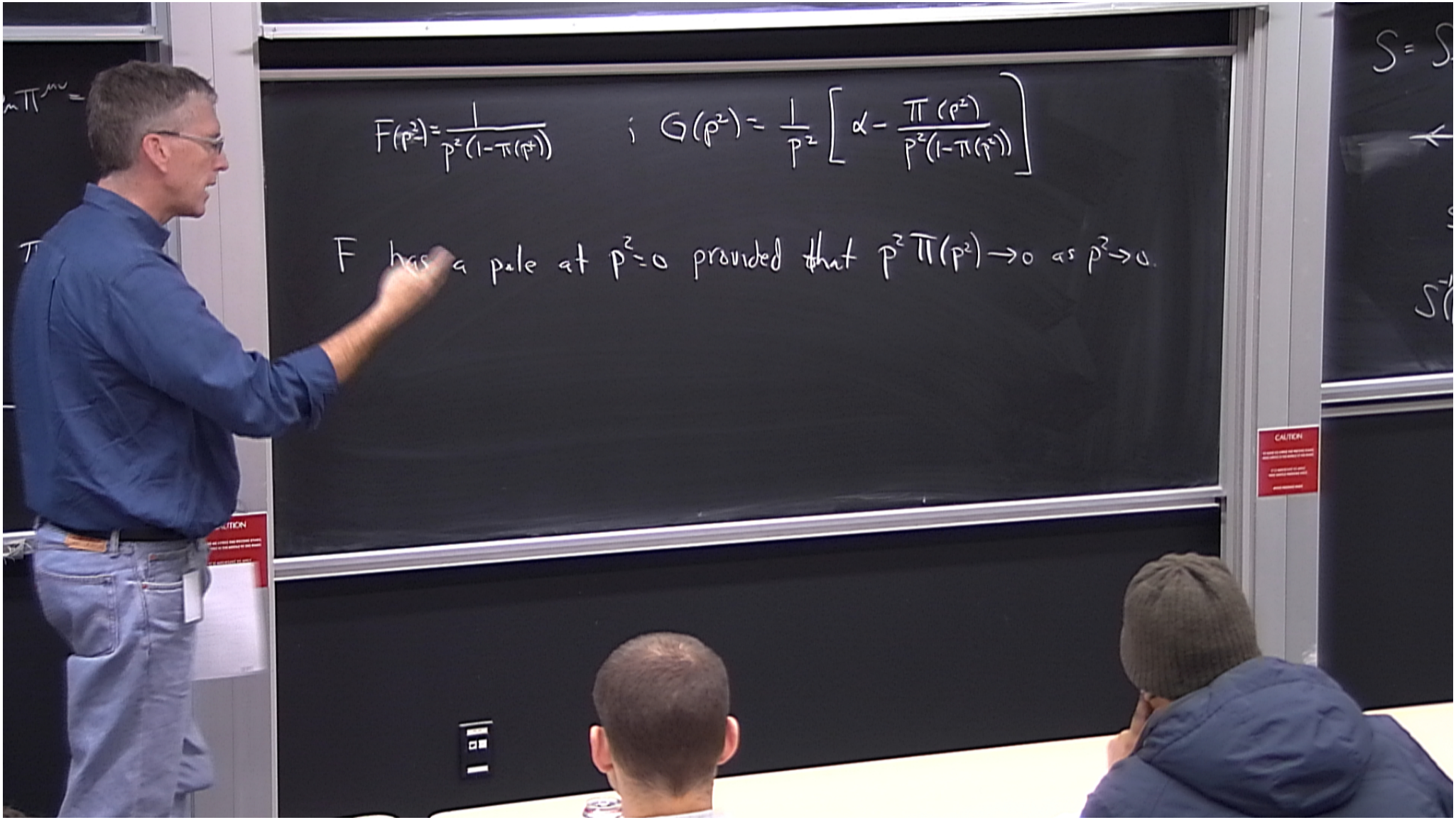

$F(p^2)=\frac{1}{p^2(1-\pi(p^2))}$
$G(p^2)=\frac{1}{p^2}$
$S=S_0+$
$S^{-1}=S_$
$S^{-1}(p)=0$
CAUTION
CAUTION

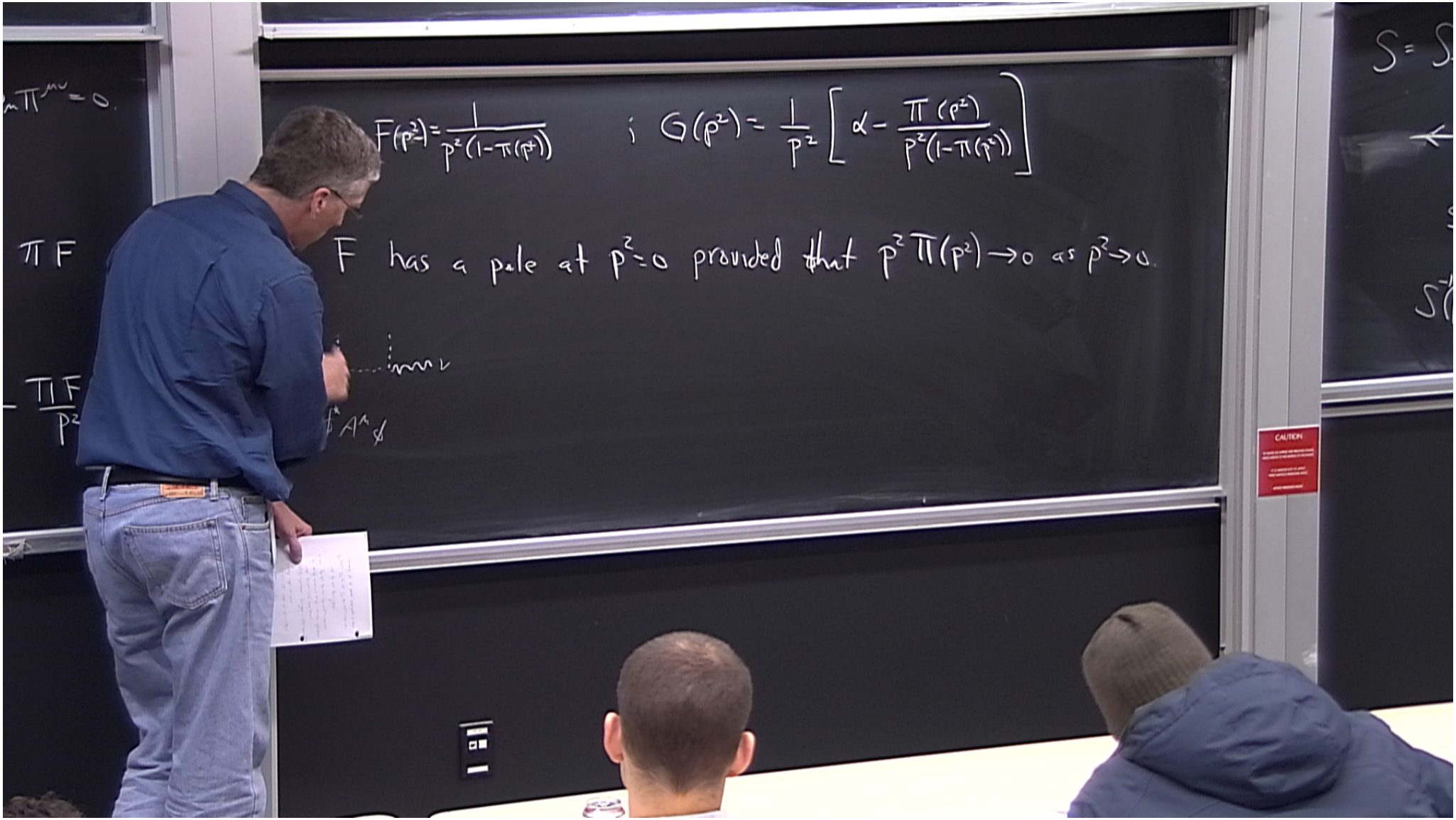

$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\,\Pi(p^2)$
$p_\mu \Pi^{\mu\nu} = 0$
$\eta_{\mu\nu}$ $F = \frac{1}{p^2} + \Pi F$
$p_\mu p_\nu$ : $G = \frac{\alpha}{p^2} - \frac{\Pi F}{p^2}$

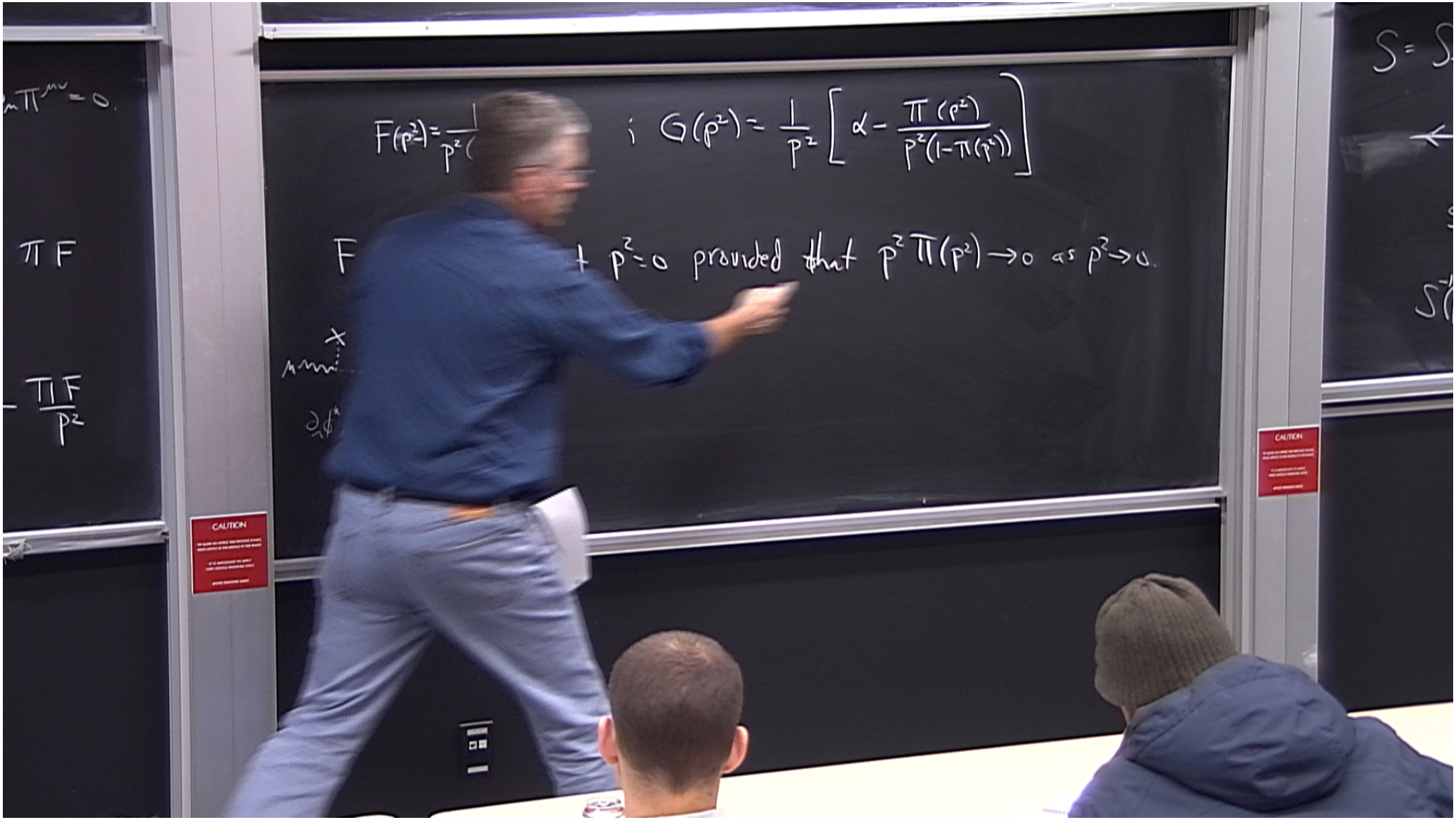

$\Delta_{0\mu\nu}(p) = \frac{\eta_{\mu\nu} + \alpha p_\mu p_\nu}{p^2 - i\epsilon}$
$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\Pi(p^2)$ $p_\mu \Pi^{\mu\nu} = 0$
$\Delta_{\mu\nu} = F(p^2)\eta_{\mu\nu} + G(p^2) p_\mu p_\nu$
$\Delta_{\mu\nu} = \Delta_{0\mu\nu} + \Delta_{0\mu\lambda}\Pi^{\lambda\rho}\Delta_{\rho\nu}$
$F\eta_{\mu\nu} + G p_\mu p_\nu = \frac{\eta_{\mu\nu}}{p^2} + \frac{\alpha p_\mu p_\nu}{p^2} + \frac{\Pi(p^2)}{p^2}(p^2 \delta_\mu^\rho - p_\mu p^\rho)(F\eta_{\rho\nu} + G p_\rho p_\nu)$
$= \eta_{\mu\nu}\left[\frac{1}{p^2} + F\Pi\right] + p_\mu p_\nu \left[\frac{\alpha}{p^2} - \frac{F\Pi}{p^2}\right]$
$\eta_{\mu\nu}$ $F = \frac{1}{p^2} + \Pi F$
$p_\mu p_\nu$ : $G = \frac{\alpha}{p^2} - \frac{\Pi F}{p^2}$


$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$
$G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{\pi(p^2)}\right]$
F has a pole at $p^2=0$
$\to 0$ as $p^2 \to 0$.
$\pi F$
$-\frac{\pi F}{p^2}$
CAUTION
CAUTION


$F(p^2) = \frac{1}{p^2(1-\Pi(p^2))}$ ; $G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\Pi(p^2)}{p^2(1-\Pi(p^2))}\right]$
F has a pole at $p^2=0$ provided that $p^2\Pi(p^2)\to 0$ as $p^2\to 0$.
CAUTION

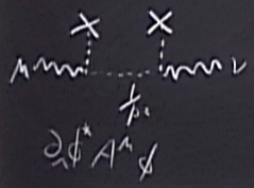

$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$ ; $G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{p^2(1-\pi(p^2))}\right]$
F has a pole at $p^2=0$ provided that $p^2\pi(p^2) \to 0$ as $p^2 \to 0$.
CAUTION

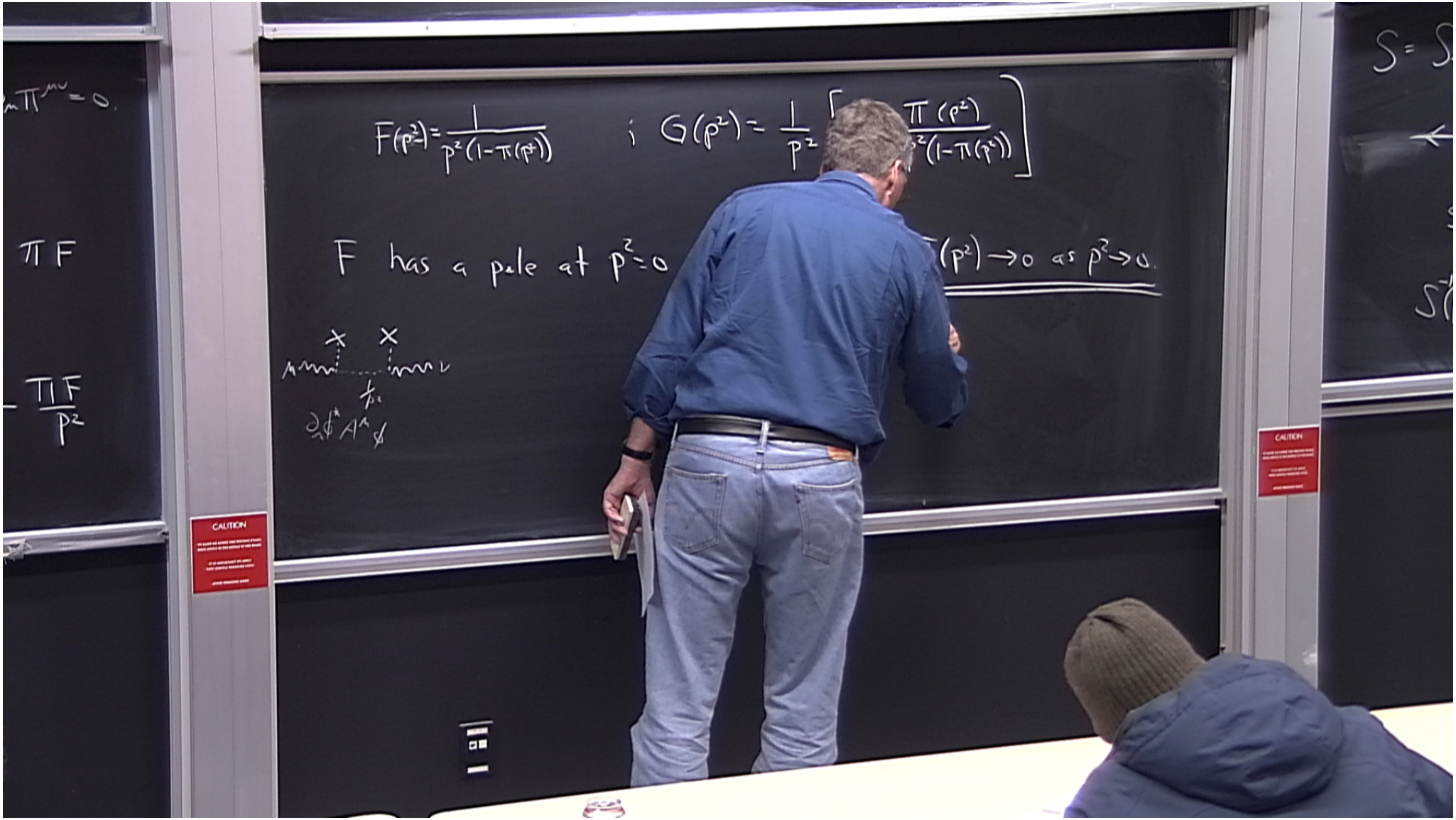

$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$ ; $G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{p^2(1-\pi(p^2))}\right]$
F has a pole at $p^2=0$ provided that $p^2\pi(p^2)\to 0$ as $p^2\to 0$.
CAUTION

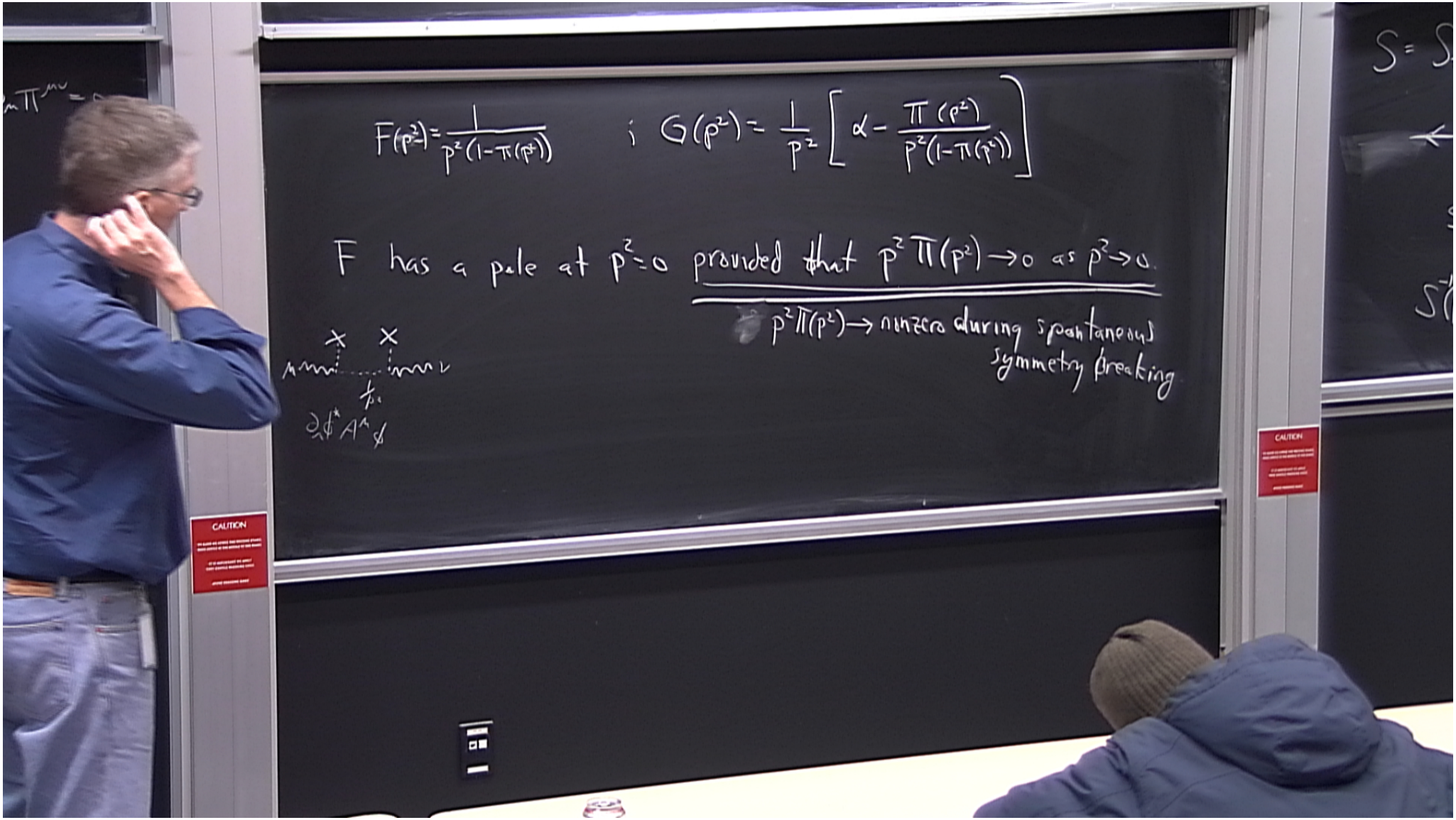

$F(p^2) = \frac{1}{p^2(}$
; $G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{p^2(1-\pi(p^2))}\right]$
$+ p^2 = 0$ provided that $p^2\pi(p^2) \to 0$ as $p^2 \to 0$.
$\pi F$
$\frac{\pi F}{p^2}$
CAUTION
CAUTION

$$F(p^2) = \frac{1}{p^2(1-\Pi(p^2))} \quad ; \quad G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\Pi(p^2)}{p^2(1-\Pi(p^2))}\right]$$

F has a pole at $p^2 = 0$ provided that $p^2\Pi(p^2) \to 0$ as $p^2 \to 0$.




$F(p^2) = \frac{1}{p^2(1-\Pi(p^2))}$
$G(p^2) = \frac{1}{p^2}$
F has a pole at $p^2 = 0$
$\Pi(p^2) \to 0$ as $p^2 \to 0$.
$\Pi F$
$\frac{\Pi F}{p^2}$
CAUTION


$F(p^2) = \frac{1}{p^2(1-\Pi(p^2))}$ ; $G(p^2) = \frac{1}{p^2}\left[\alpha - \frac{\Pi(p^2)}{p^2(1-\Pi(p^2))}\right]$
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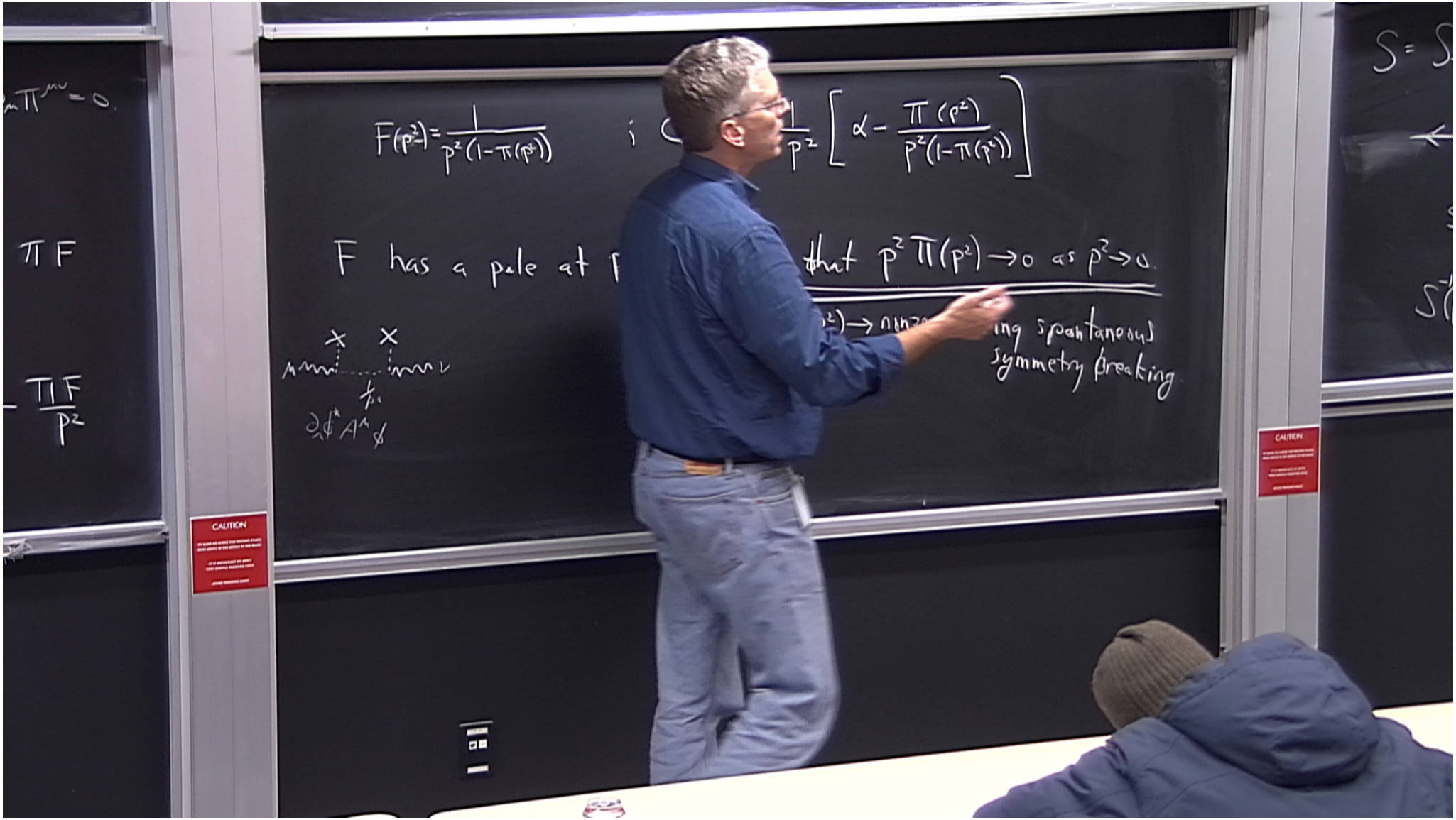
$p^2\Pi(p^2) \to$ nonzero during spontaneous symmetry breaking
CAUTION


$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$
$\frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{p^2(1-\pi(p^2))}\right]$
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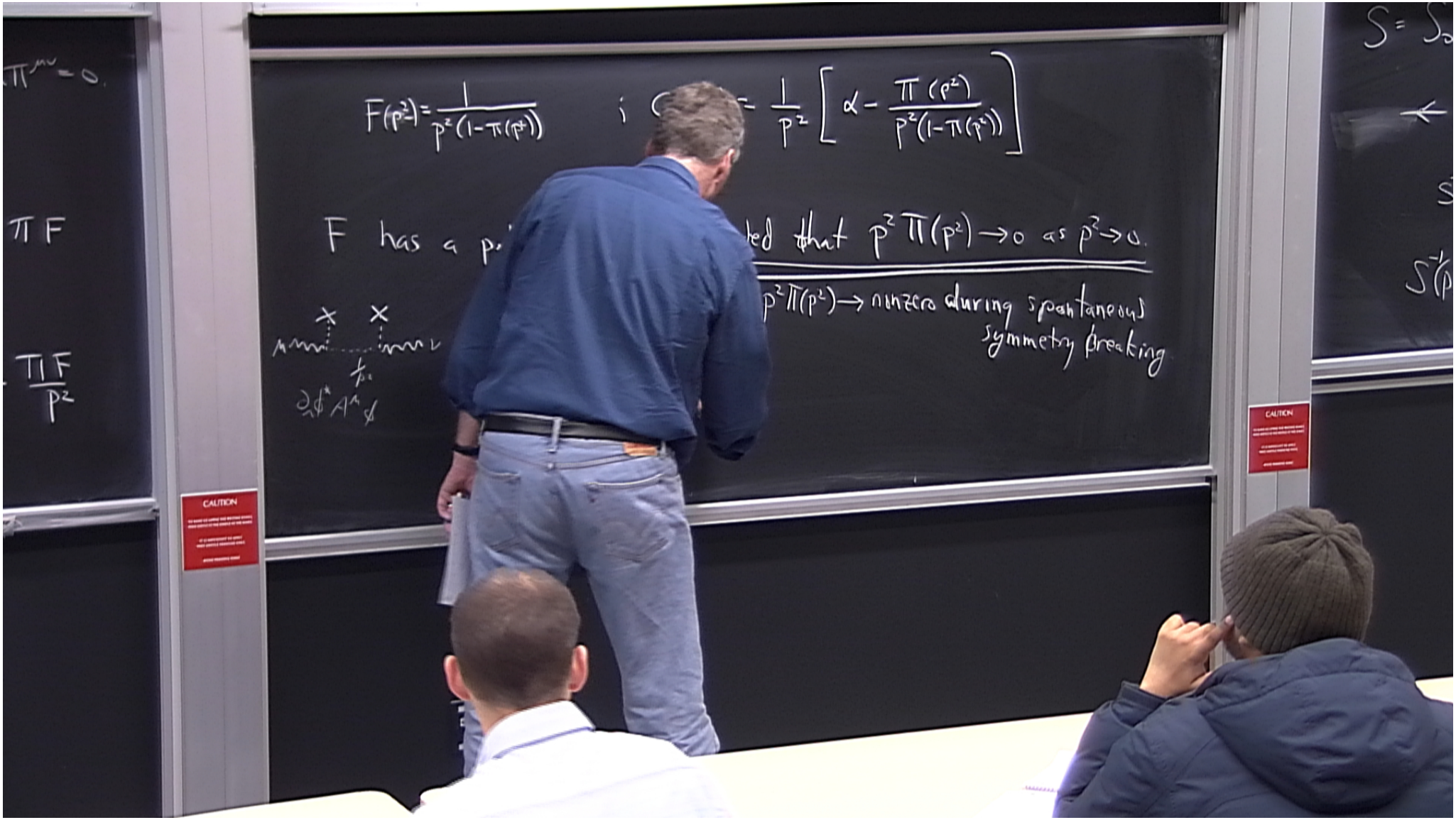
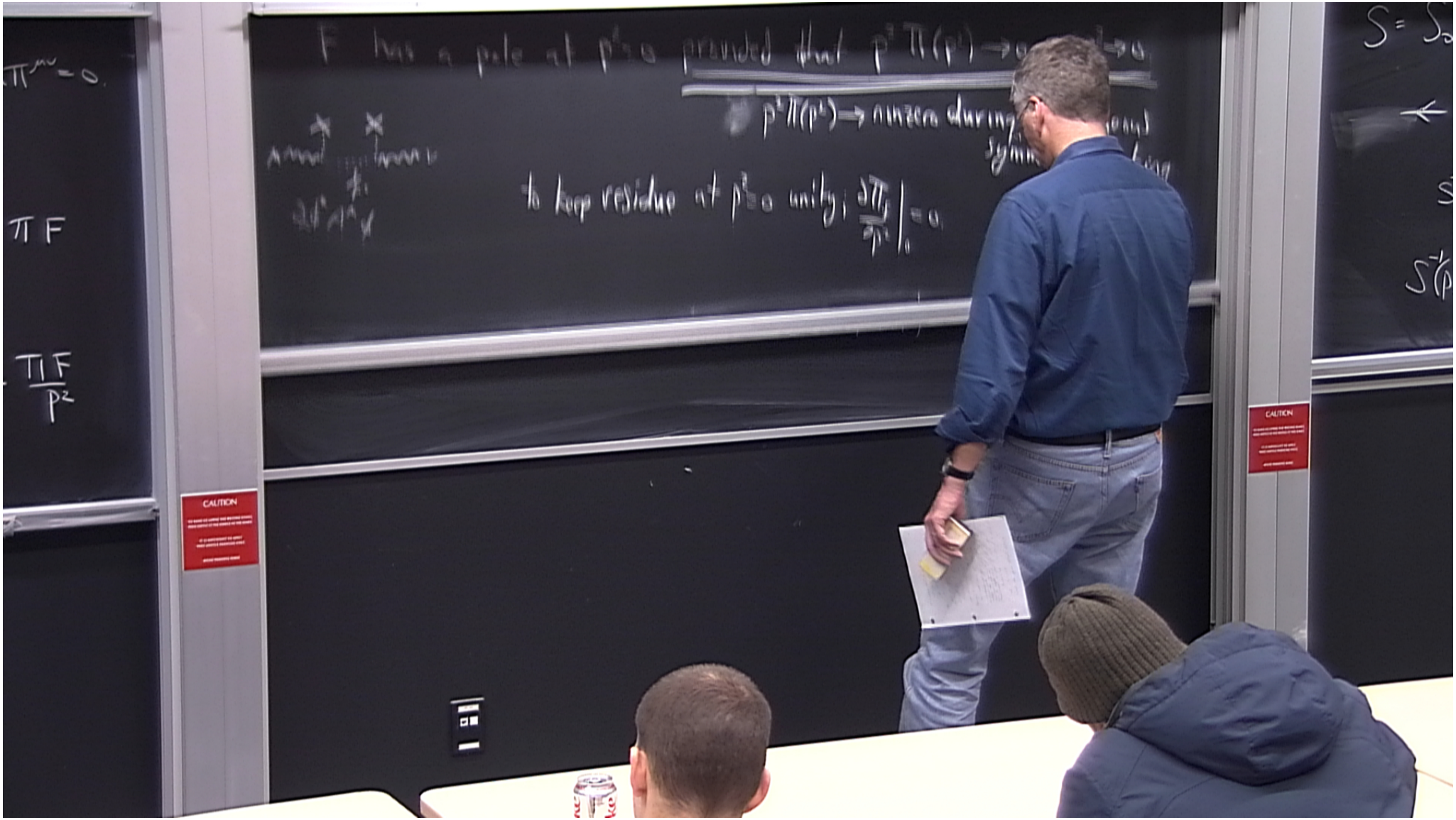
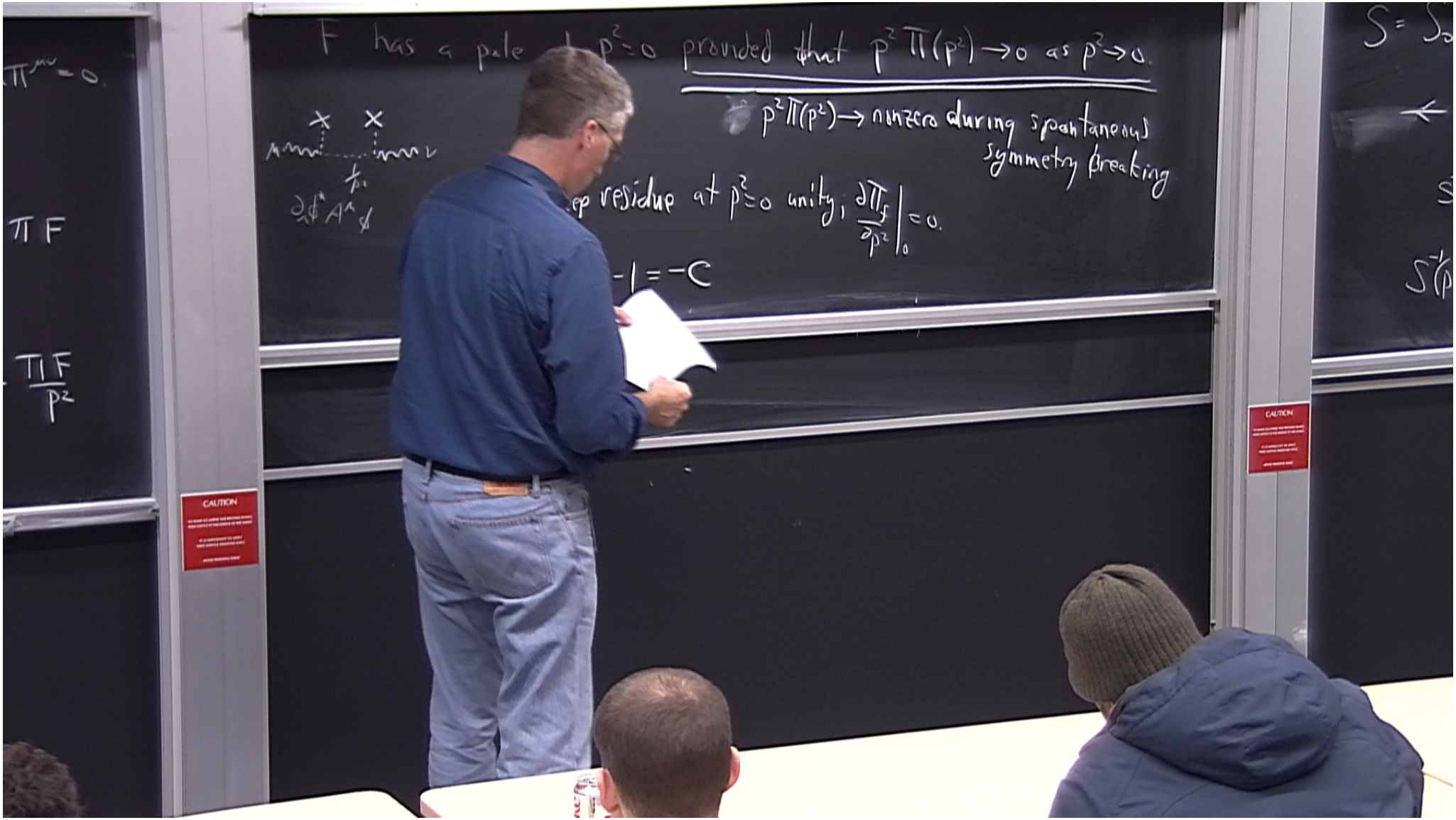
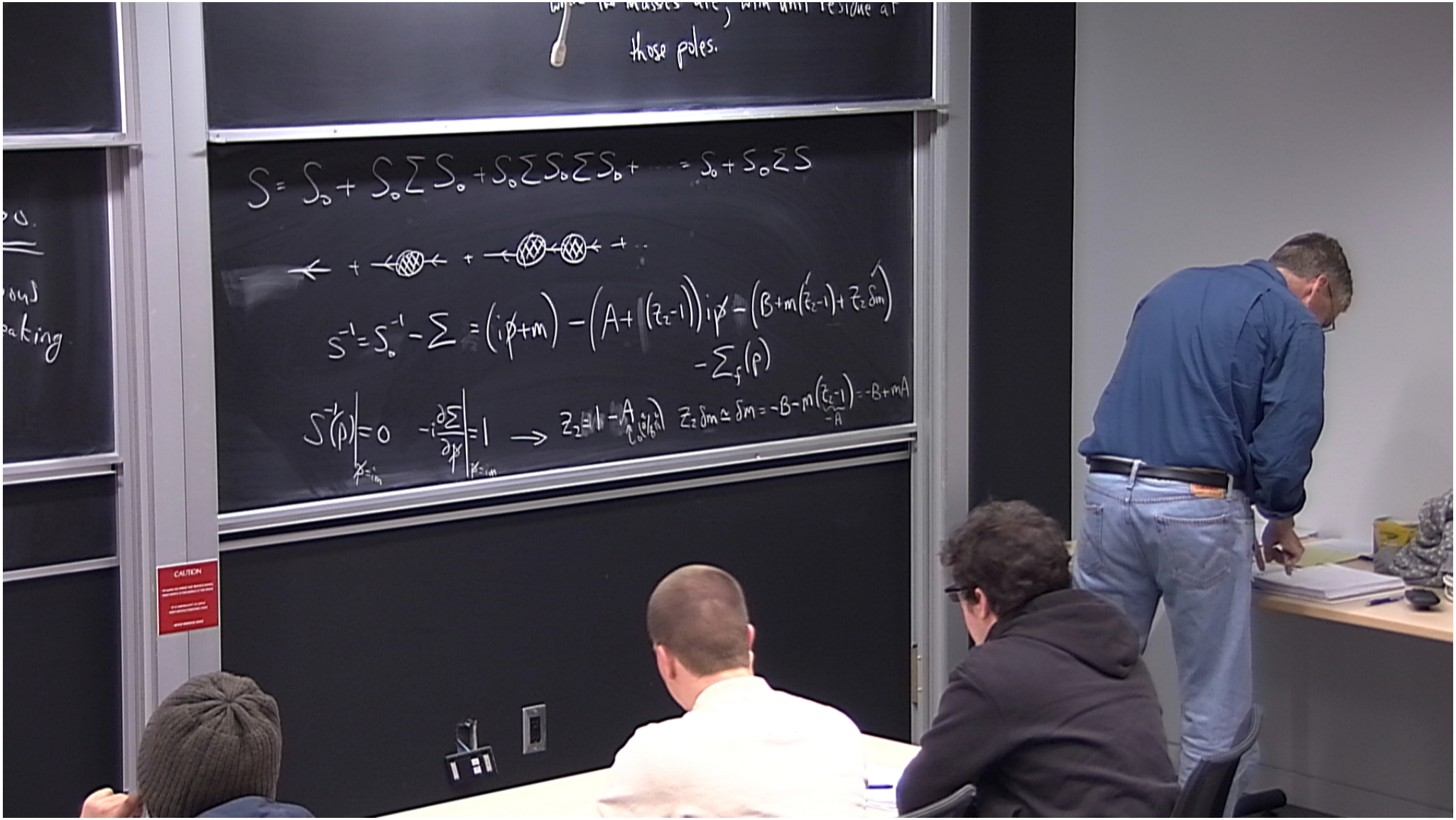
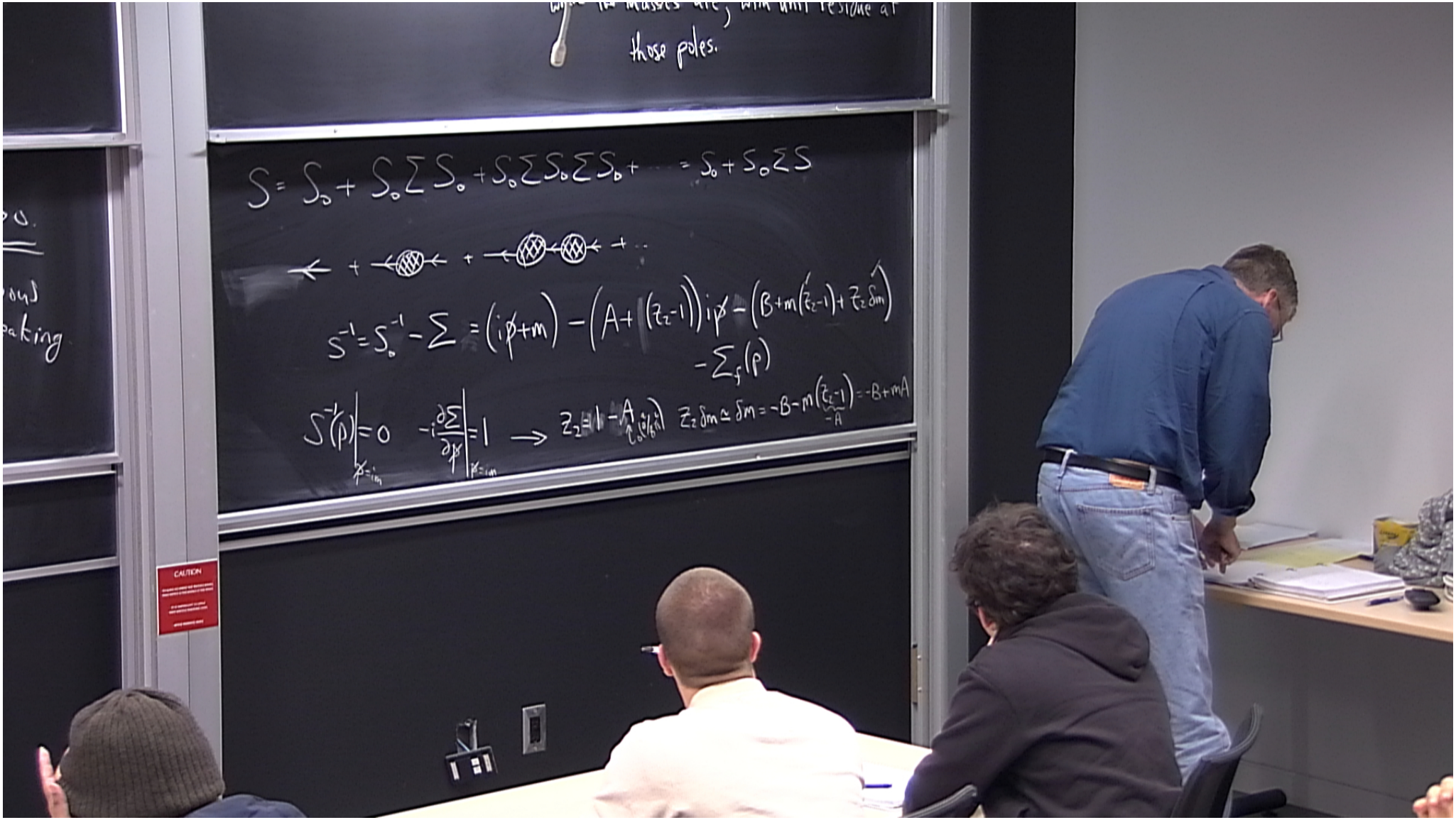
CAUTION


$F(p^2) = \frac{1}{p^2(1-\pi(p^2))}$
$= \frac{1}{p^2}\left[\alpha - \frac{\pi(p^2)}{p^2(1-\pi(p^2))}\right]$
F has a p
ed that $p^2\pi(p^2) \to 0$ as $p^2 \to 0$.
$p^2\pi(p^2) \to$ nonzero during spontaneous symmetry breaking
$\pi^{\mu\nu} = 0$
$\pi F$
$\frac{\pi F}{p^2}$
CAUTION


F has a pole at p²=0 provided that
to keep residue at p²=0 unity;
CAUTION
CAUTION


F has a pole at p²=0 provided that p²Π(p²)→0 as p²→0.
p²Π(p²)→ nonzero during spontaneous symmetry breaking
residue at p²=0 unity, ∂Π_F/∂p² |_0 = 0.
-1 = -C
CAUTION


those poles.
$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \ldots = S_0 + S_0 \Sigma S$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A + (Z_2 - 1)\right) i\not{p} - \left(B + m(Z_2 - 1) + Z_2 \delta m\right) - \Sigma_f(p)$
$S^{-1}(p)\big|_{\not{p} = im} = 0 \quad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 = 1 - A \quad Z_2 \delta m \simeq \delta m = -B - m\underbrace{(Z_2 - 1)}_{-A} = -B + mA$
CAUTION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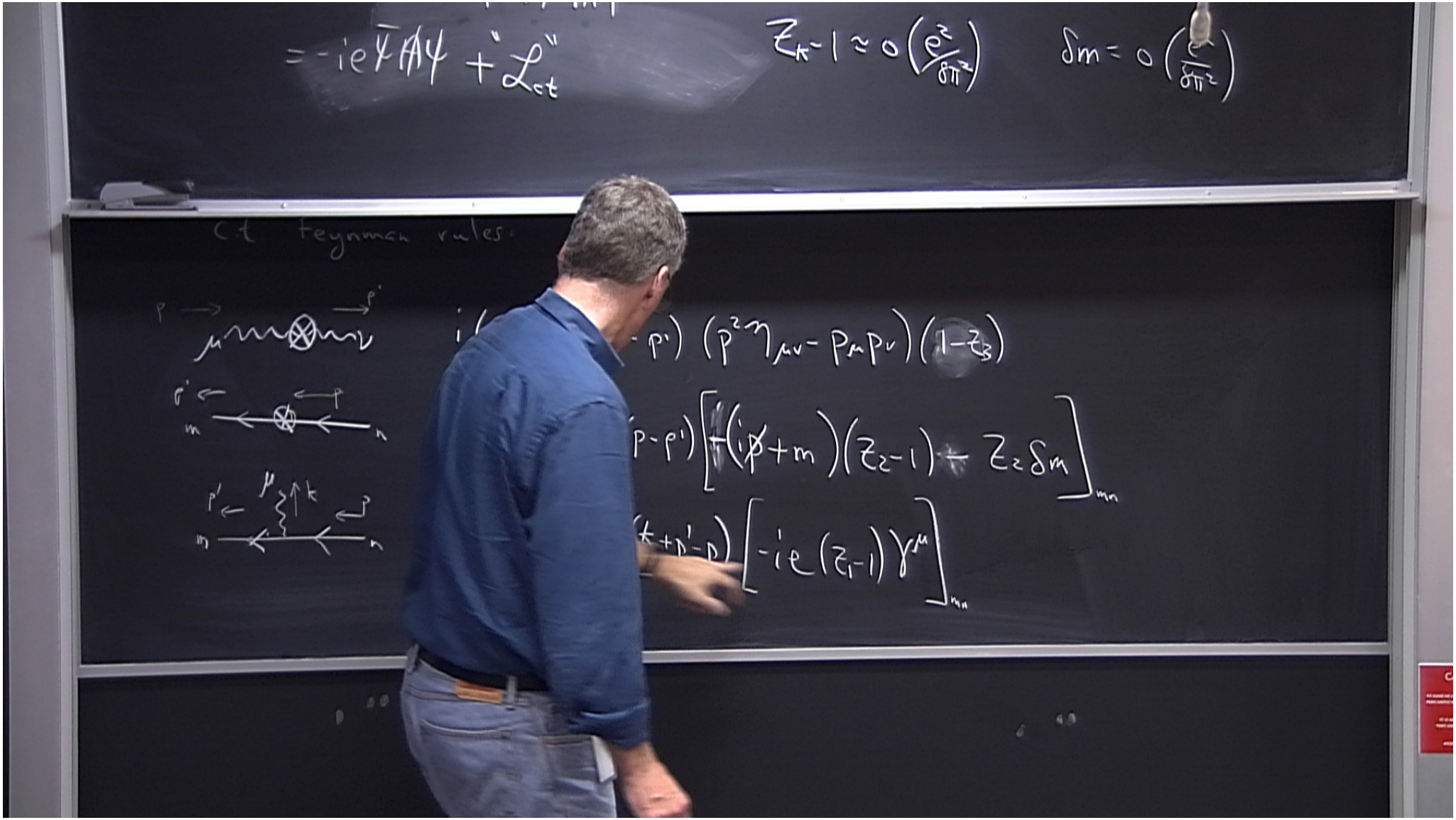
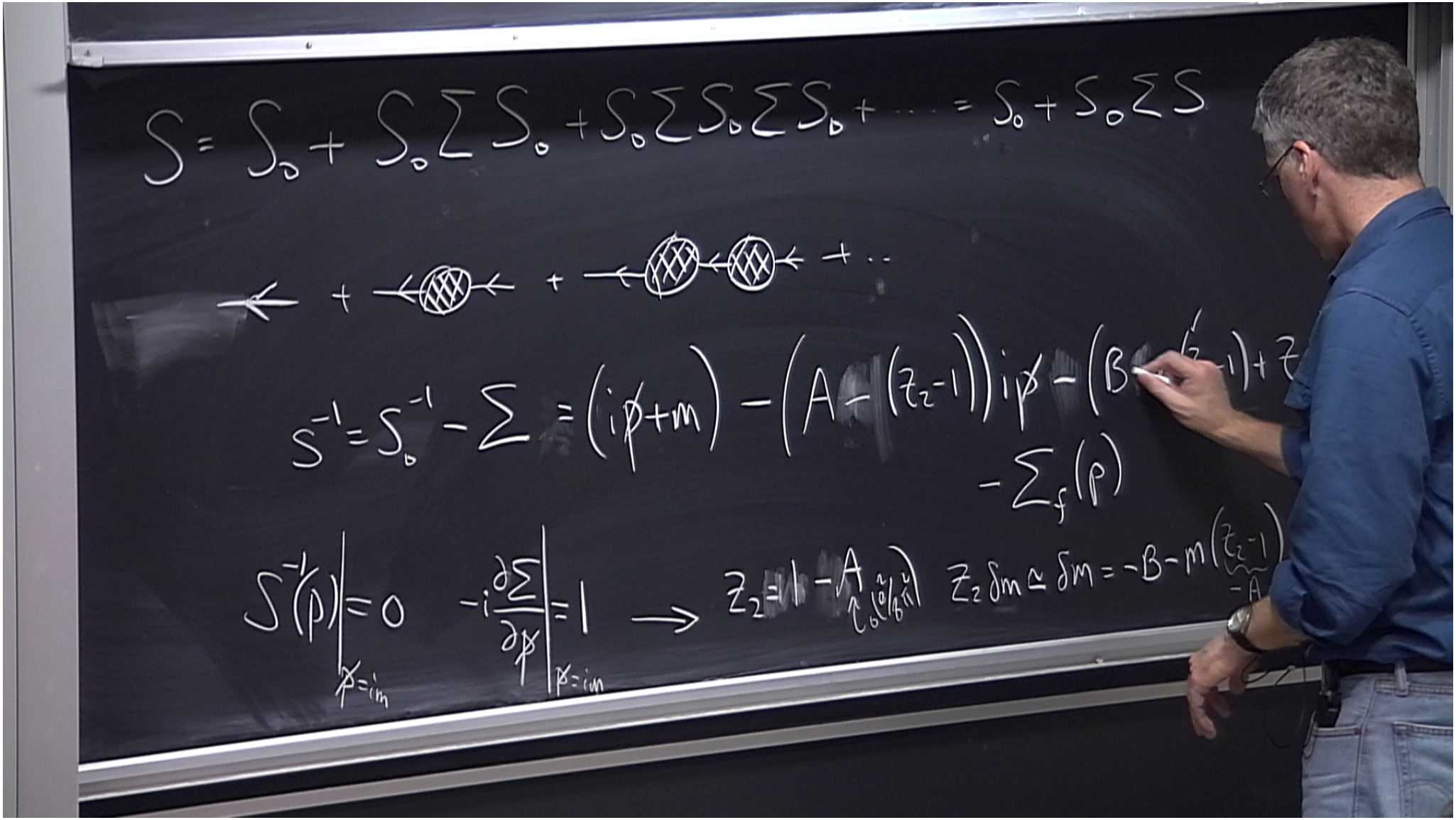
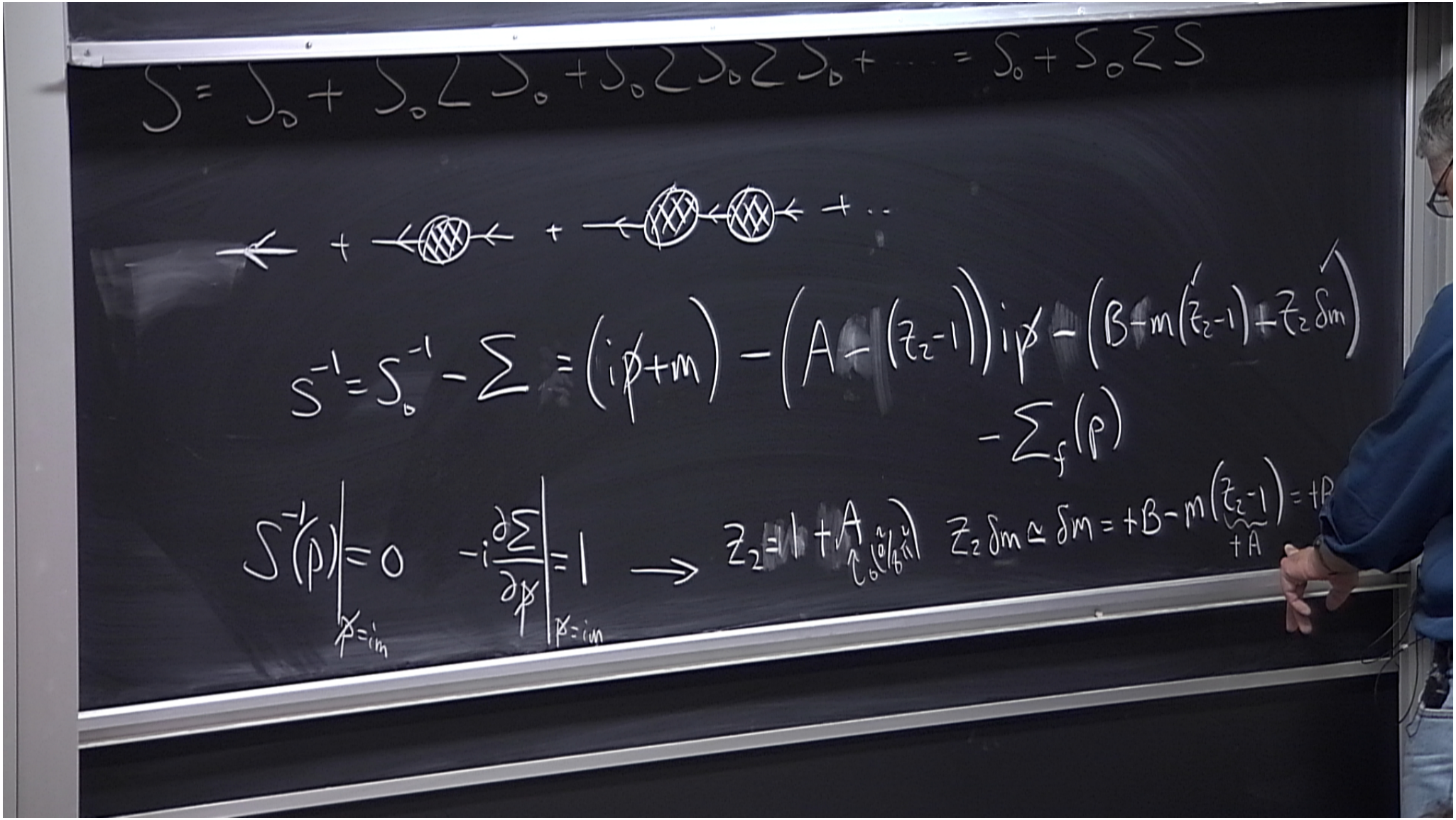
CAUTION


$= -ie\bar{\psi}A\psi + \mathcal{L}_{ct}$
$Z_\pi - 1 \approx O\left(\frac{e^2}{8\pi^2}\right)$
$\delta m = O\left(\frac{e^2}{8\pi^2}\right)$
c.t. feynman rules:
$(p^2\eta_{\mu\nu} - p_\mu p_\nu)(1 - Z_3)$
$(p - p')\left[(ip\!\!\!/ + m)(Z_2 - 1) - Z_2\delta m\right]_{mn}$
$(k + p' - p)\left[-ie(Z_1 - 1)\gamma^\mu\right]_{mn}$


$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \ldots = S_0 + S_0 \Sigma S$
$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A - (Z_2 - 1)\right) i\not{p} - \left(B - \frac{\delta m}{m}(Z_2 - 1) + Z_2 \delta m\right)$
$- \Sigma_f(p)$
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 = 1 - A \quad Z_2 \delta m \simeq \delta m = -B - m\left(\underbrace{Z_2 - 1}_{-A}\right)$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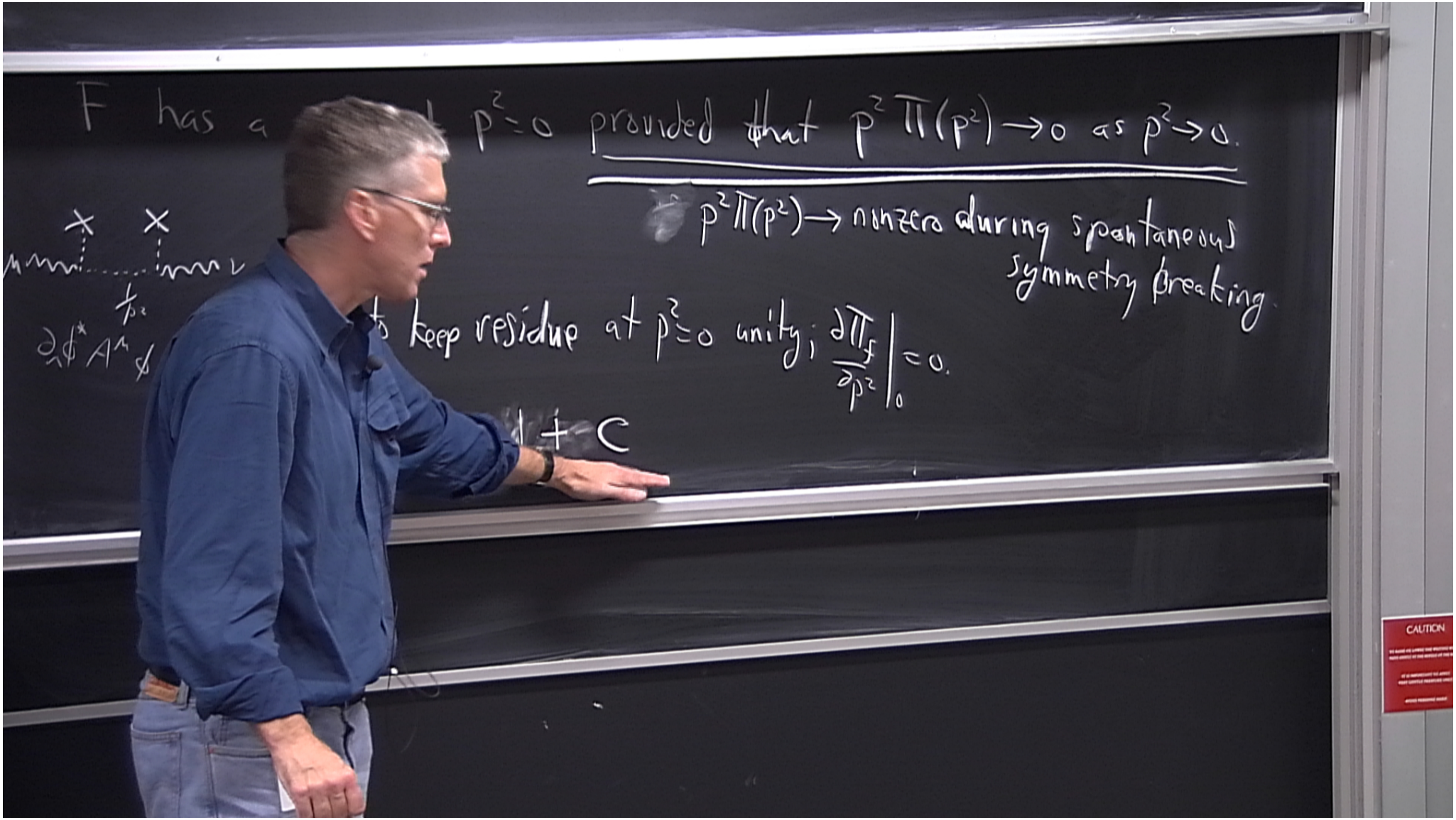
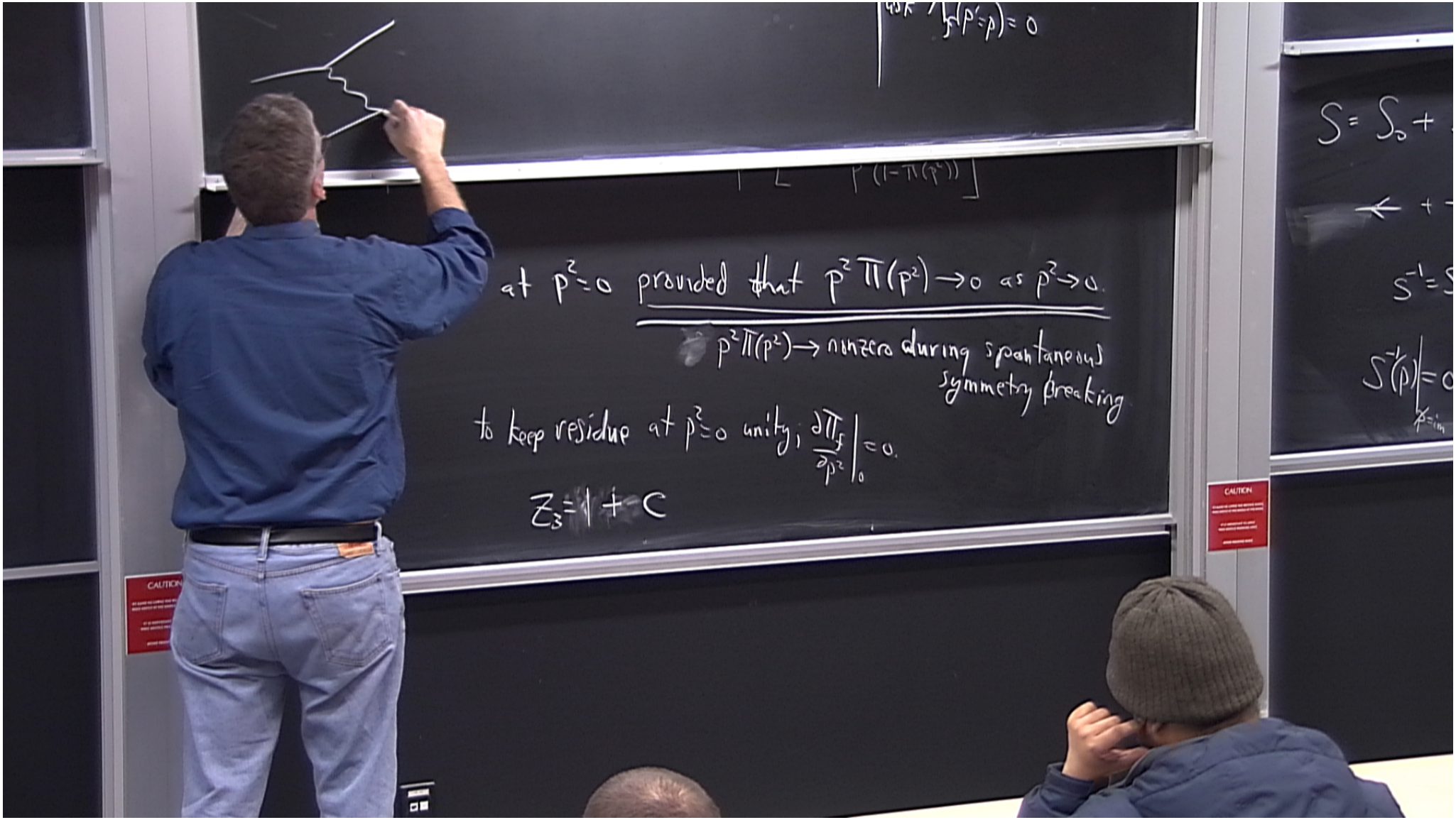
$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i \frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 = 1 + A \qquad Z_2 \delta m \simeq \delta m = +B - m\underbrace{(Z_2 - 1)}_{+A} =$


F has a
$p^2=0$ provided that $p^2\Pi(p^2)\to 0$ as $p^2\to 0$.
$p^2\Pi(p^2)\to$ nonzero during spontaneous symmetry breaking.
keep residue at $p^2=0$ unity; $\frac{\partial\Pi_F}{\partial p^2}\Big|_0=0$.
$1+C$
CAUTION


at $p^2=0$ provided that $p^2\Pi(p^2)\to 0$ as $p^2\to 0$.
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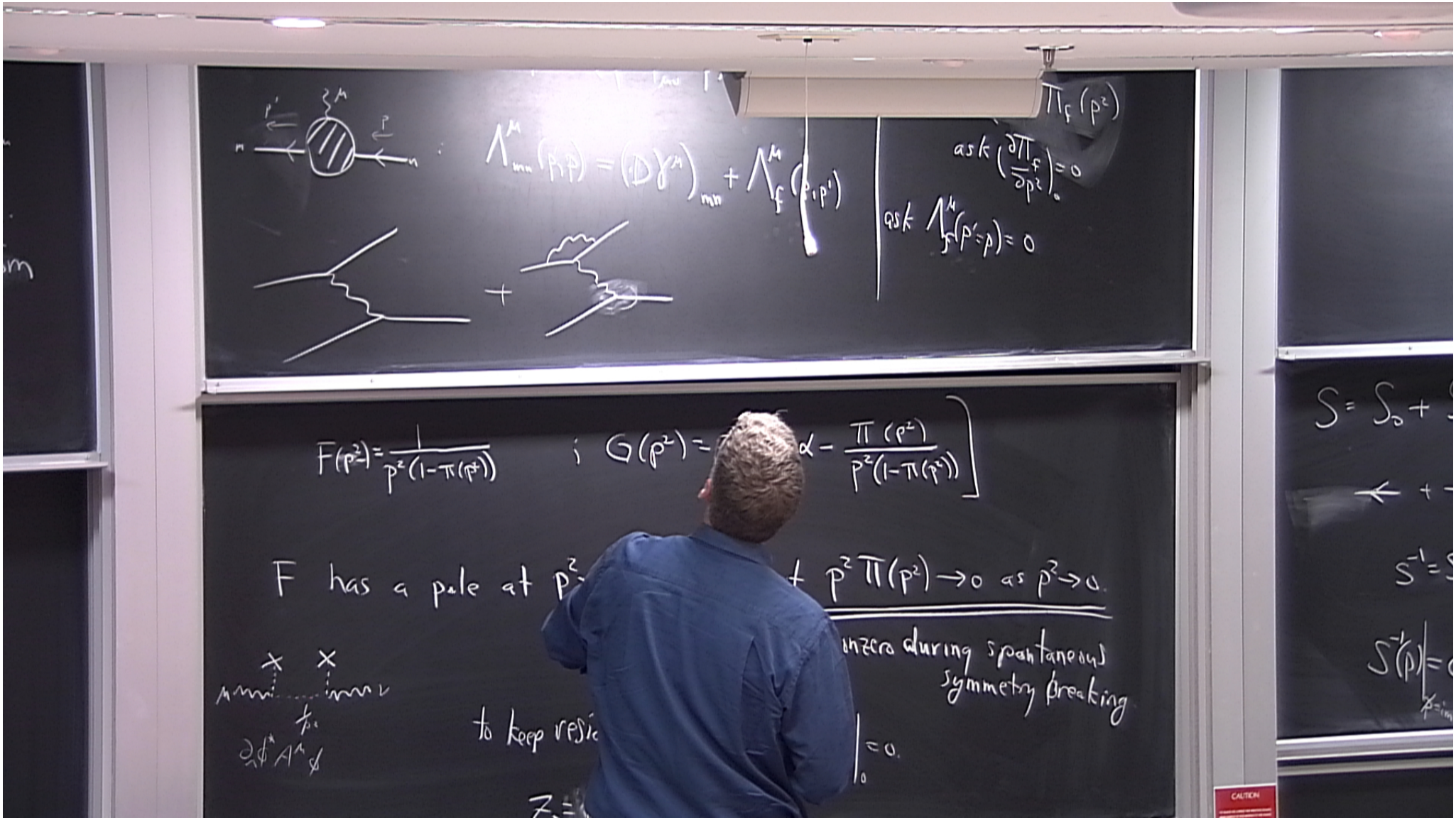
to keep residue at $p^2=0$ unity, $\left.\frac{\partial\Pi_s}{\partial p^2}\right|_0=0$.
$Z_3=1+C$
$S=S_0+$
$S^{-1}=$
$S^{-1}(p)|=0$
CAUTION
CAUTION


$\Lambda^\mu_{mn}(p,p) = (D\gamma^\mu)_{mn} + \Lambda^\mu_f(p,p')$
$\Pi_f(p^2)$
ask $\left(\frac{\partial \Pi_f}{\partial p^2}\right)_0 = 0$
ask $\Lambda^\mu_f(p'=p) = 0$
$F(p^2) = \frac{1}{p^2(1-\Pi(p^2))}$
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F has a pole at $p^2$
$+ p^2\Pi(p^2) \to 0$ as $p^2 \to 0$.
nzero during spontaneous
symmetry breaking
to keep resi
$= 0.$
$S = S_0 +$
$S^{-1} =$
$S^{-1}(p) =$
CAUTION

$+ \mathcal{L}_{ct}$

$\delta m = O\left(\frac{e^2}{8\pi^2}\right)$

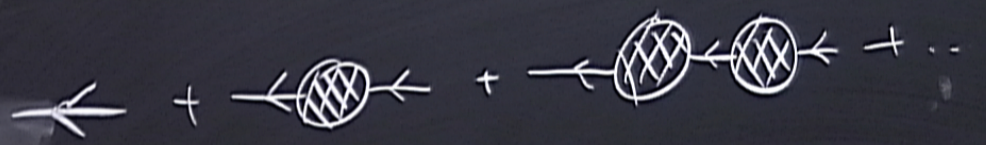
$$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$$

rules:

$$i(2\pi)^4 \delta^4(p-p')\,(p^2\eta_{\mu\nu} - p_\mu p_\nu)(1-Z_3)$$

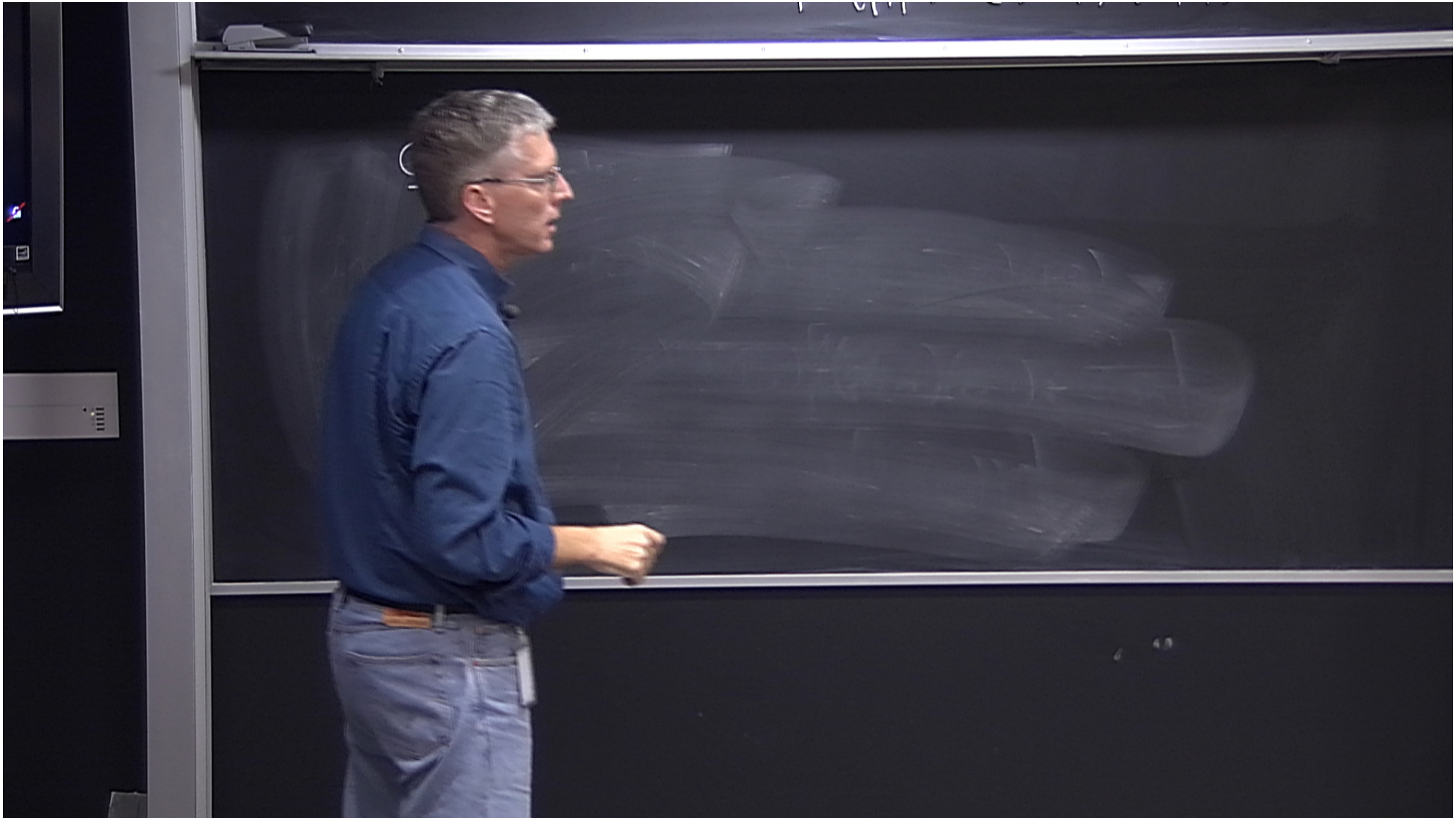
$$i(2\pi)^4 \delta^4(p-p')\left[-(i\not{p}+m)(Z_2-1) - Z_2\delta m\right]_{mn}$$

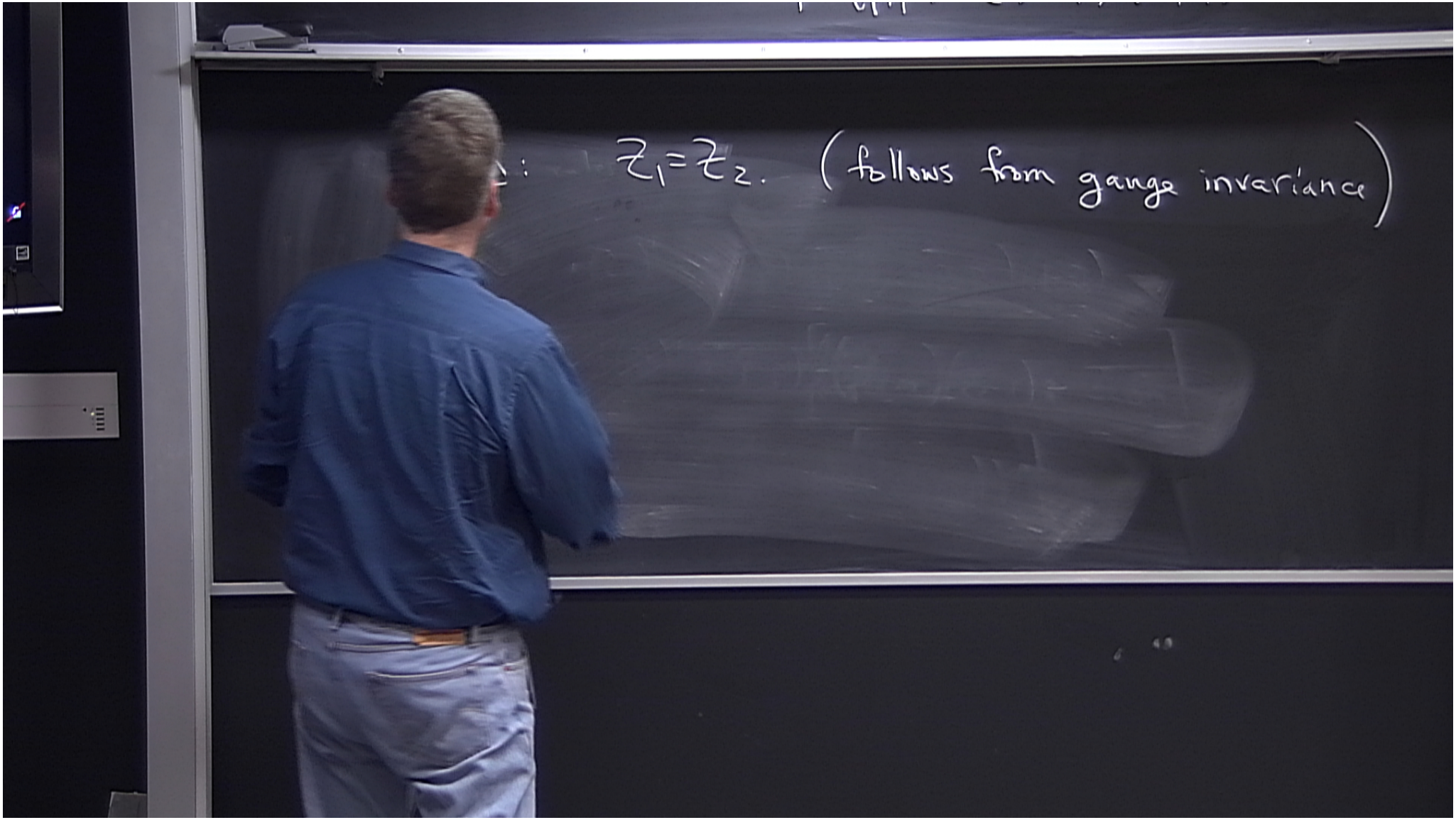
$$S = S_0 + S_0 \Sigma S_0 + S_0 \Sigma S_0 \Sigma S_0 + \cdots = S_0 + S_0 \Sigma S$$

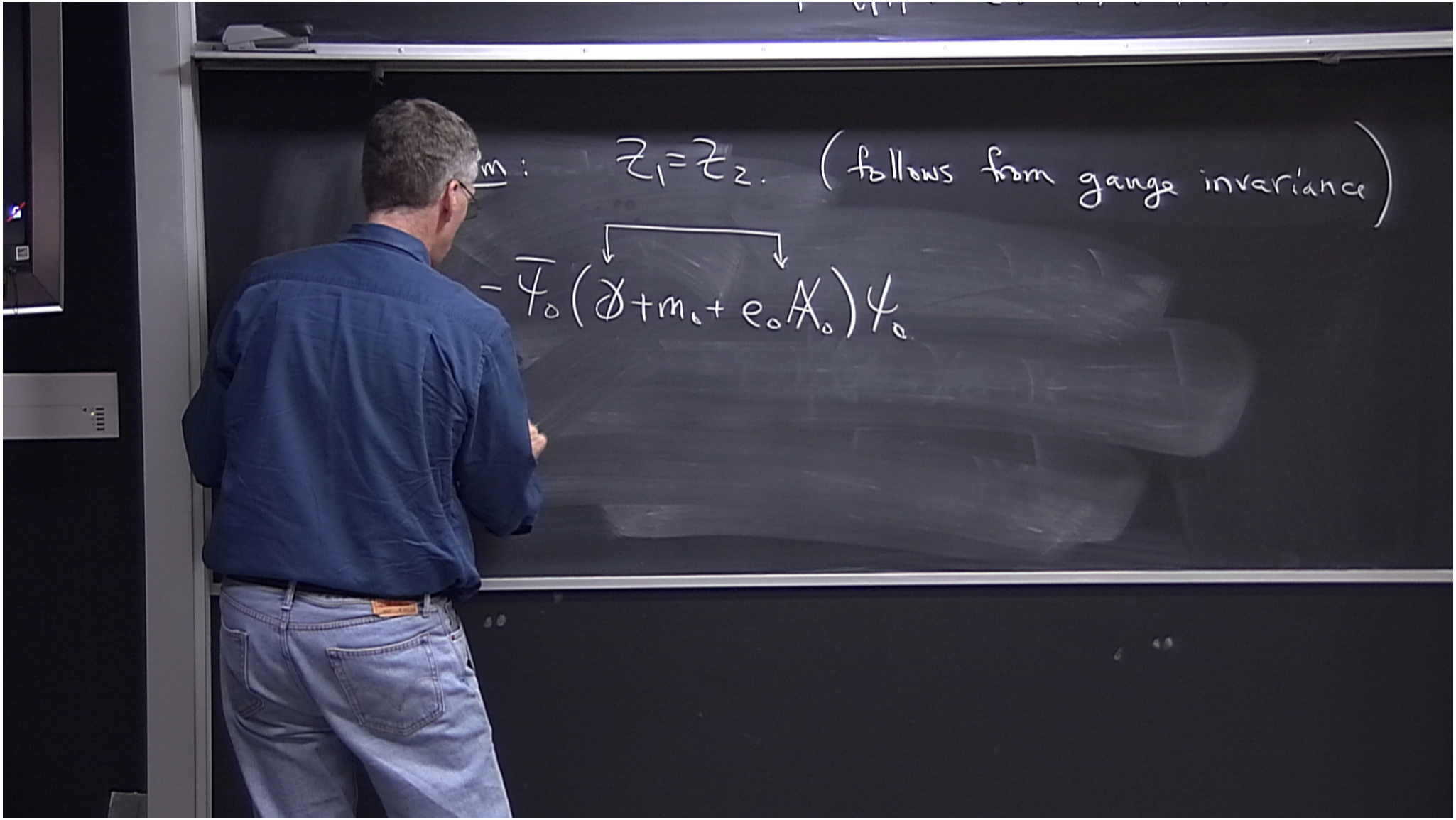


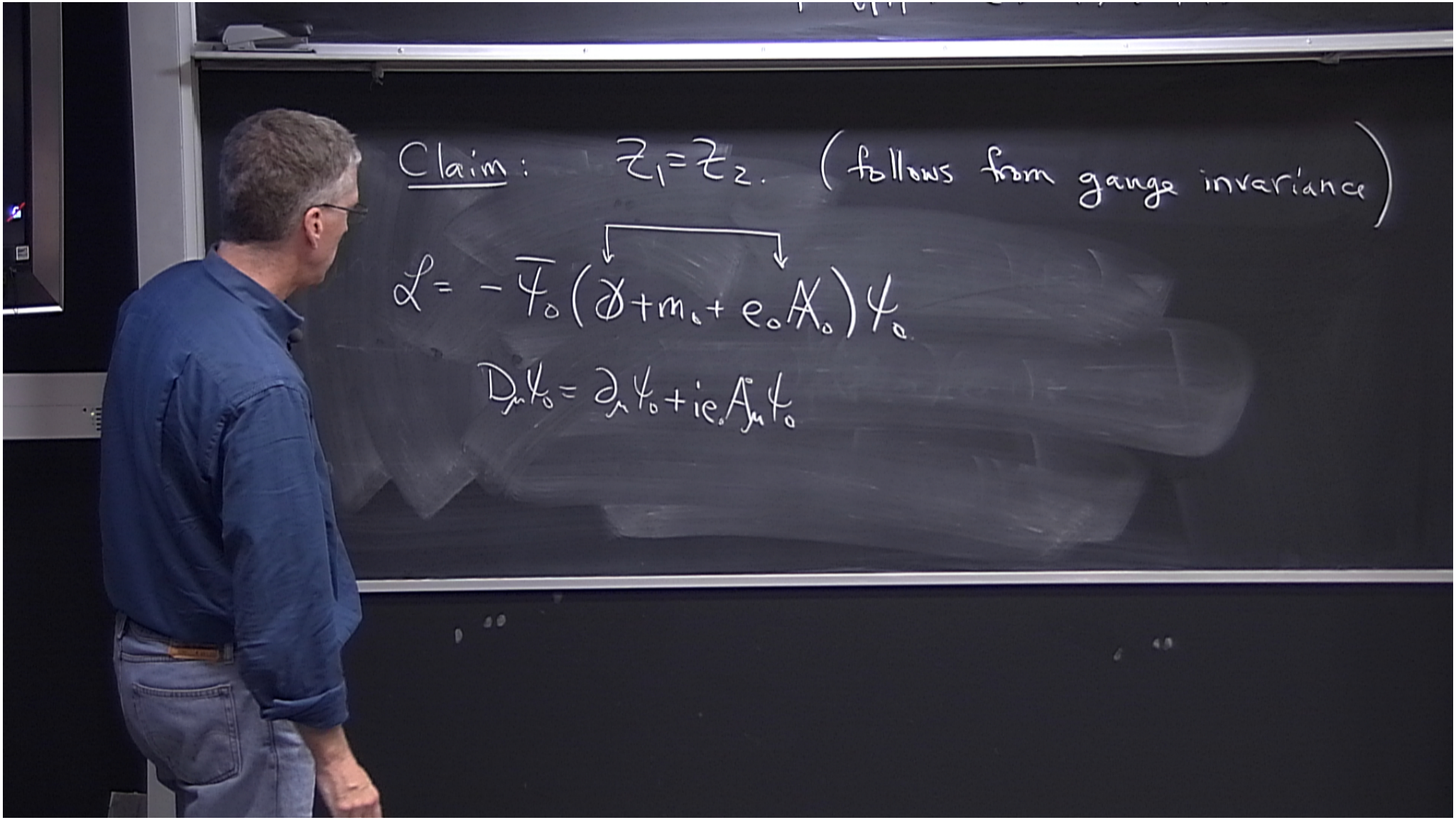
$$S^{-1} = S_0^{-1} - \Sigma = (i\not{p} + m) - \left(A - (Z_2 - 1)\right) i\not{p} - \left(B - m(Z_2 - 1) + Z_2 \delta m\right) - \Sigma_f(p)$$

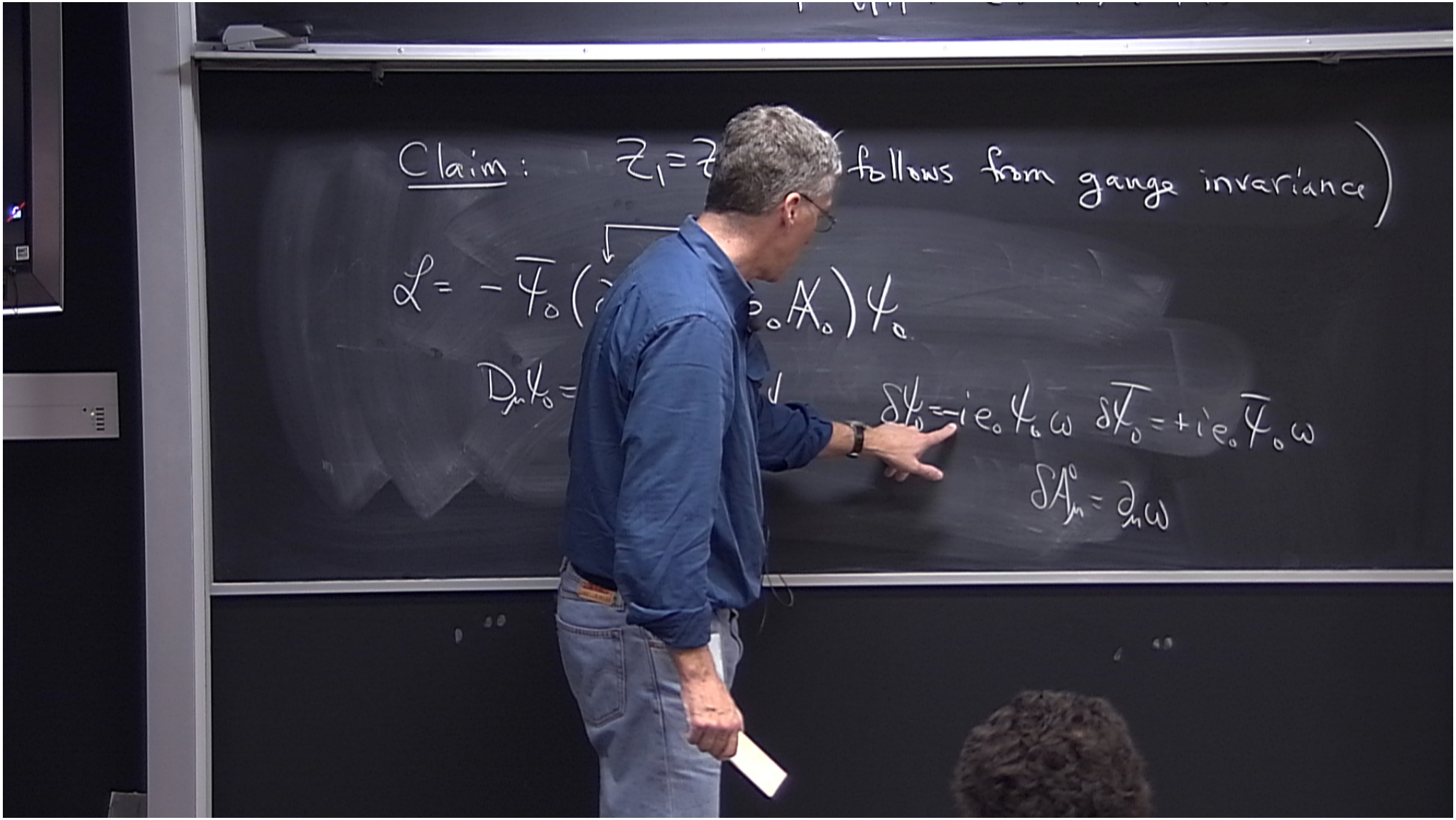
$$S^{-1}(p)\Big|_{\not{p} = im} = 0 \qquad -i\frac{\partial \Sigma}{\partial \not{p}}\Big|_{\not{p} = im} = 1 \longrightarrow Z_2 = 1 + A \qquad Z_2 \delta m \simeq \delta m = +B - m\underbrace{(Z_2 - 1)}_{+A} = +B - mA$$

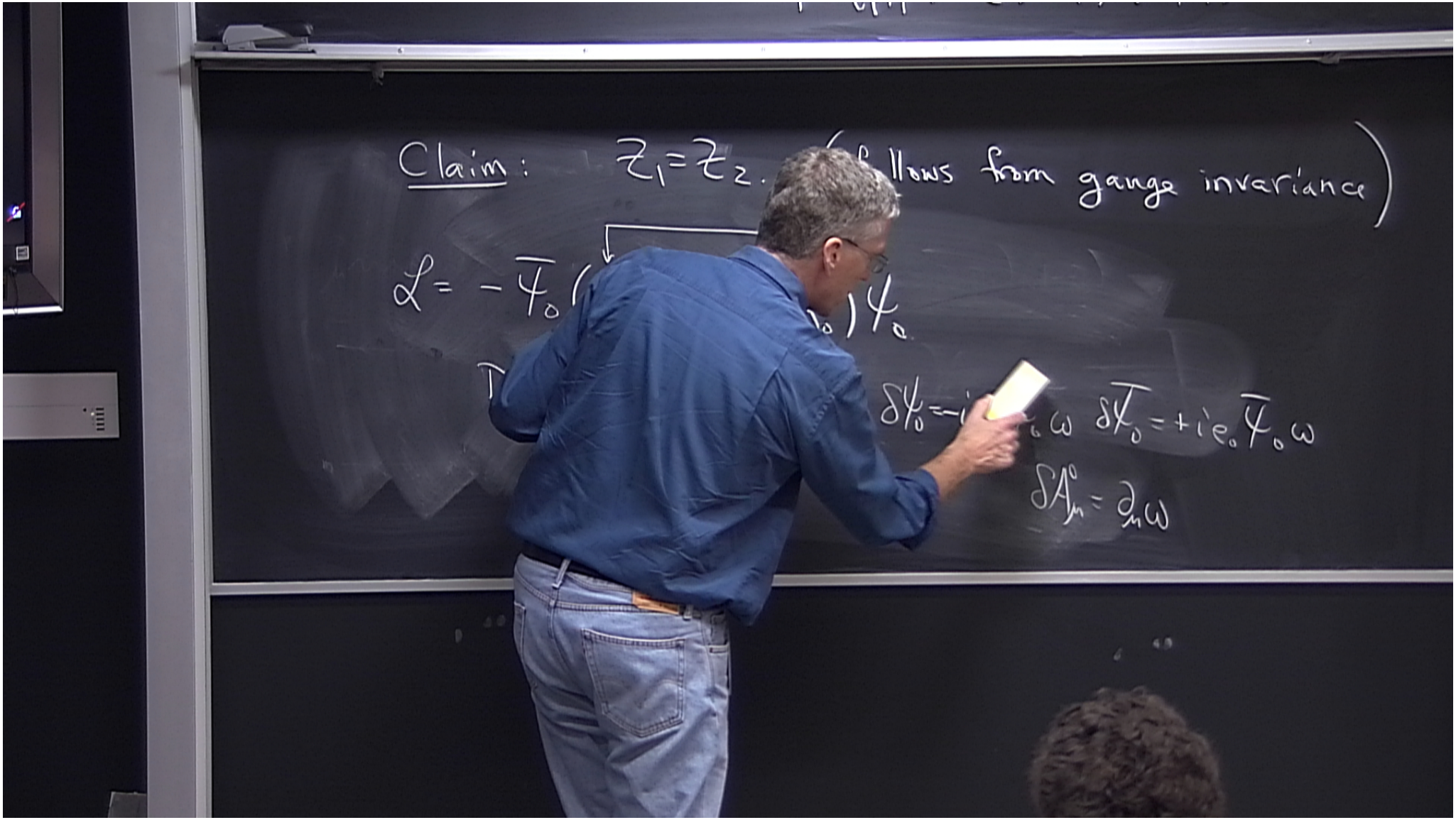


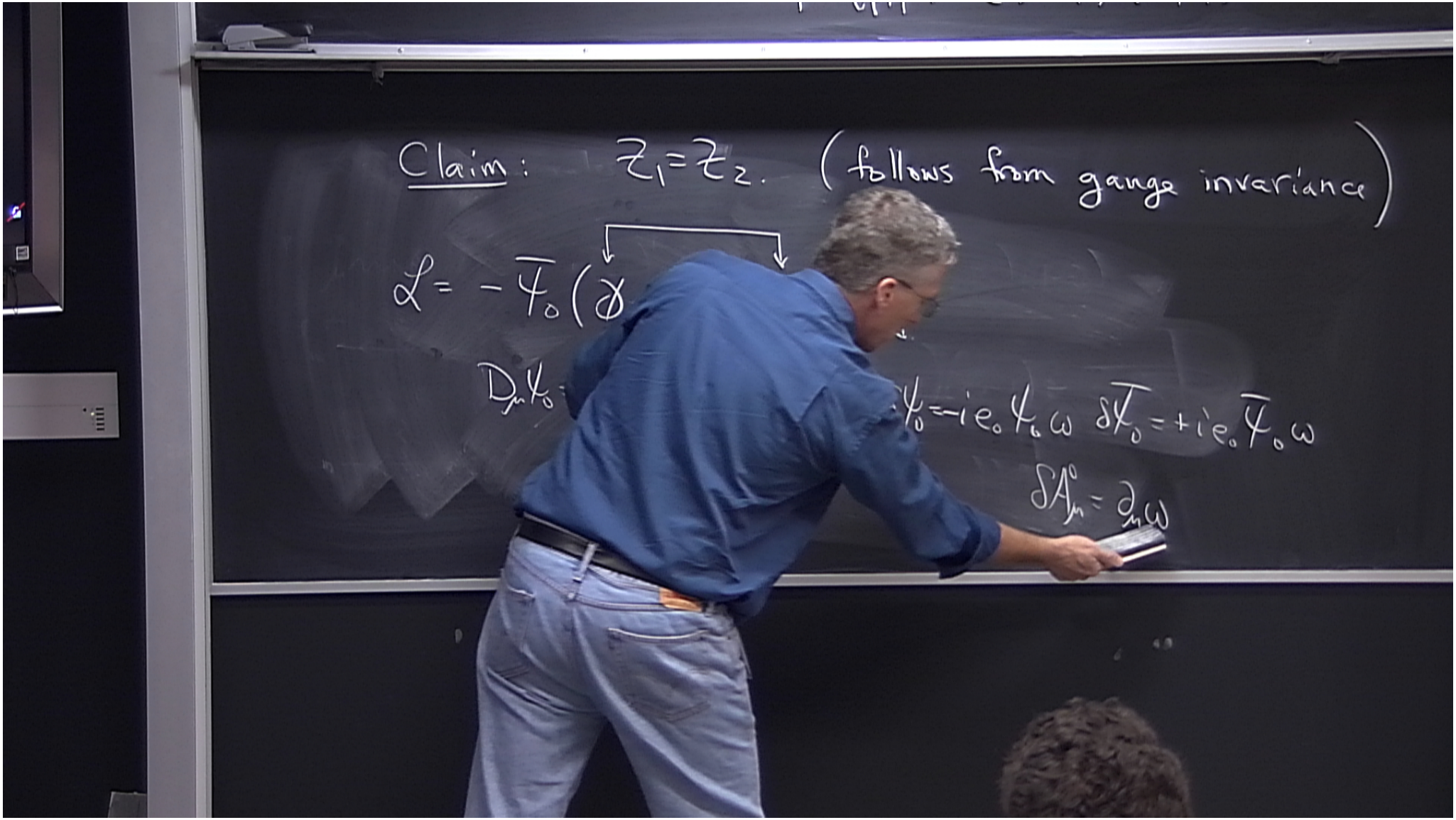

$Z_1 = Z_2$. (follows from gauge invariance)

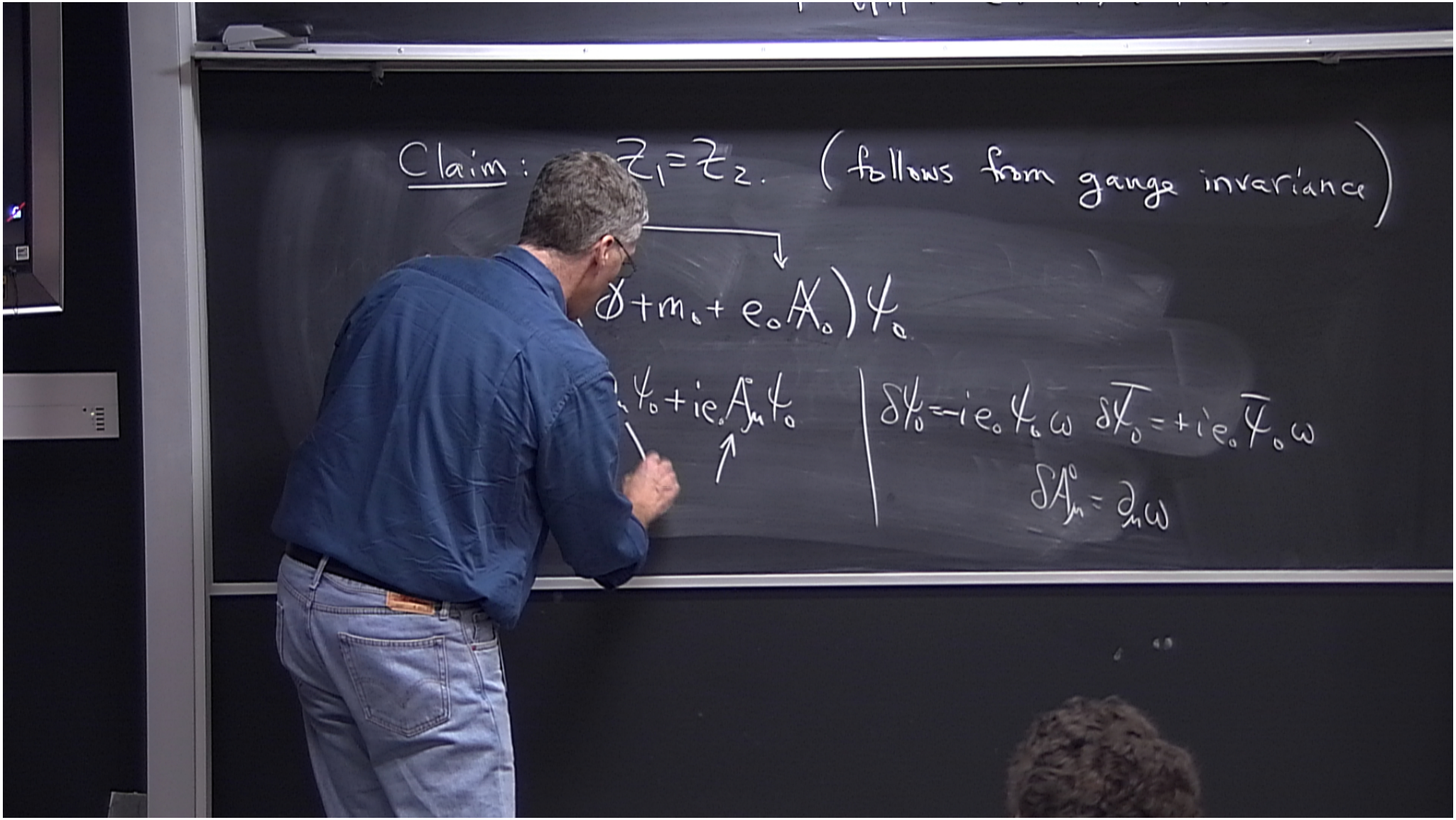

$Z_1 = Z_2$. (follows from gauge invariance)
$-\bar{\psi}_0(\not{\partial} + m_0 + e_0 \not{A}_0)\psi_0$

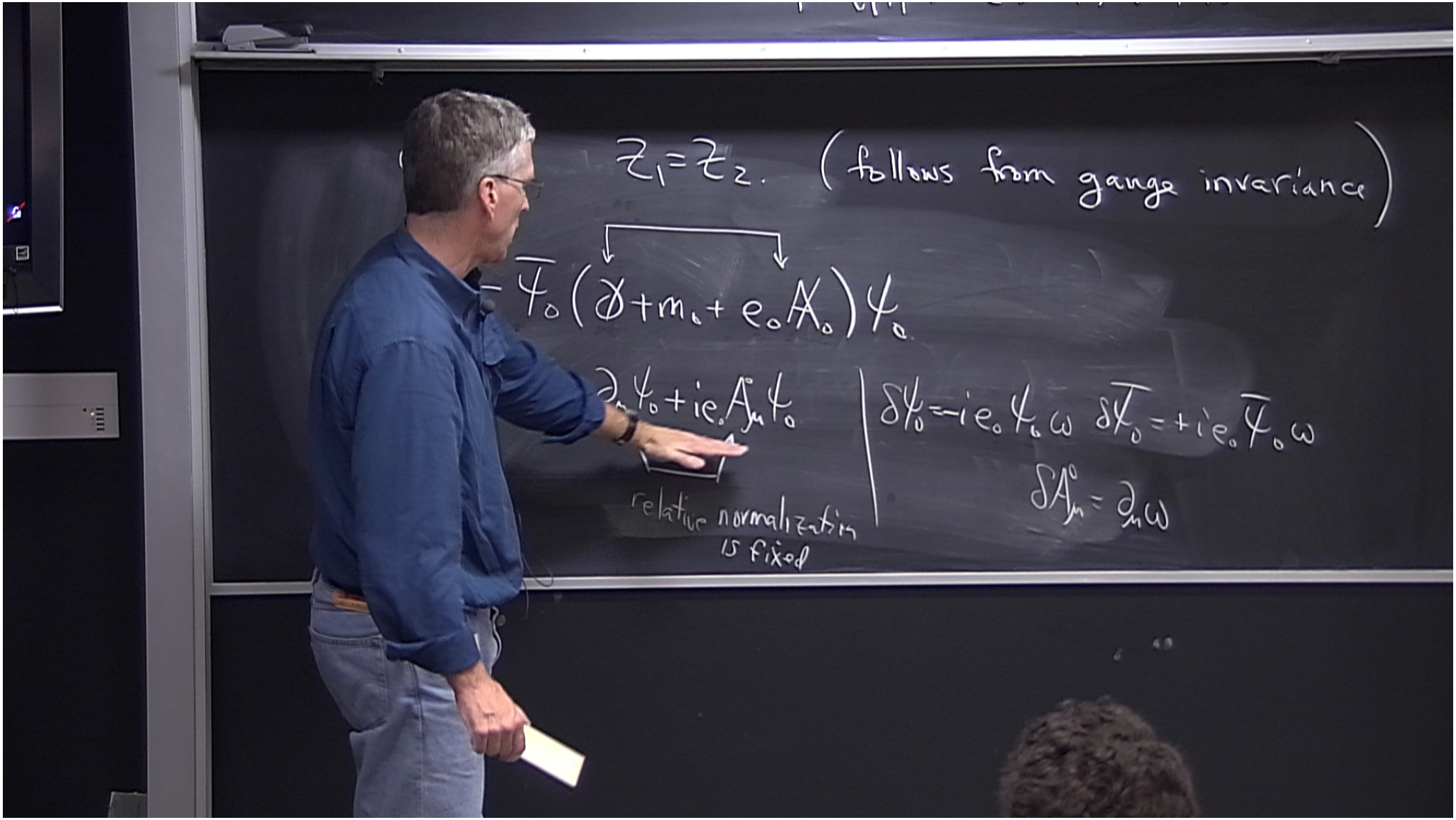

Claim: $Z_1 = Z_2$. (follows from gauge invariance)
$\mathcal{L} = -\bar{\psi}_0 (\not{\partial} + m_0 + e_0 \not{A}_0) \psi_0$
$D_\mu \psi_0 = \partial_\mu \psi_0 + i e_0 A_\mu \psi_0$

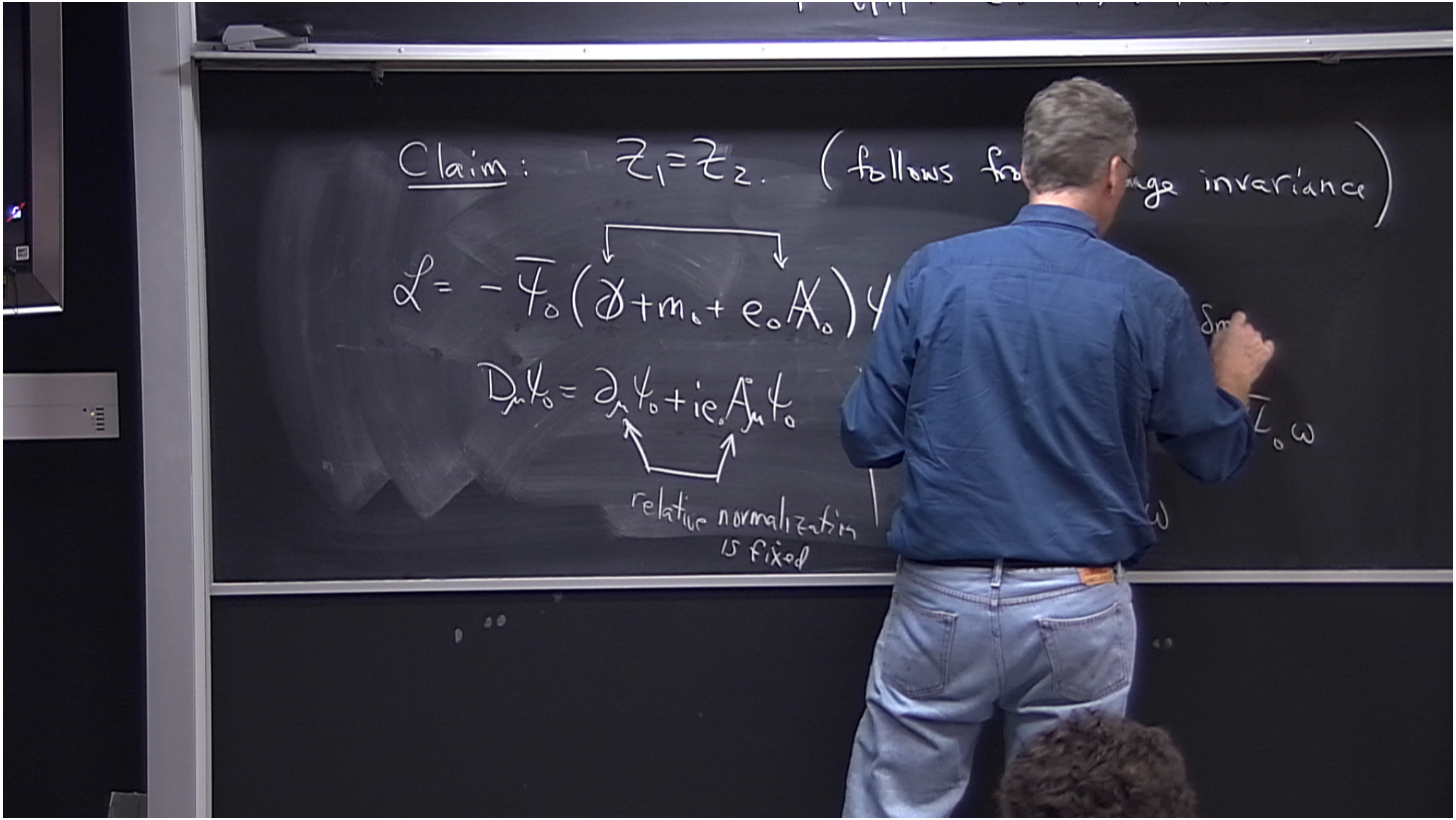

Claim: Z1 = Z
follows from gauge invariance)
δψ0 = -ie0 ψ0 ω δψ̄0 = +ie0 ψ̄0 ω
δA0μ = ∂μω


Claim: $Z_1 = Z_2$. (follows from gauge invariance)
$\mathcal{L} = -\bar{\psi}_0 ( \ldots ) \psi_0$
$\delta\psi_0 = -i \ldots \omega$
$\delta\bar{\psi}_0 = +ie_0\bar{\psi}_0\omega$
$\delta A^0_\mu = \partial_\mu \omega$


Claim: $Z_1 = Z_2$. (follows from gauge invariance)
$\mathcal{L} = -\bar{\psi}_0 ($
$\delta\psi_0 = -ie_0\psi_0\omega \quad \delta\bar{\psi}_0 = +ie_0\bar{\psi}_0\omega$
$\delta A_\mu^0 = \partial_\mu\omega$


Claim: $Z_1 = Z_2$. (follows from gauge invariance)
$\partial + m_0 + e_0 A_0) \psi_0$
$\psi_0 + ie_0 A_\mu \psi_0$
$\delta\psi_0 = -ie_0 \psi_0 \omega \quad \delta\bar{\psi}_0 = +ie_0 \bar{\psi}_0 \omega$
$\delta A_\mu = \partial_\mu \omega$


$Z_1 = Z_2$. (follows from gauge invariance)
$-\bar{\psi}_0(\partial\!\!\!/ + m_0 + e_0 A\!\!\!/_0)\psi_0$
$\partial_\mu \psi_0 + ie_0 A_\mu \psi_0$
relative normalization is fixed
$\delta\psi_0 = -ie_0 \psi_0 \omega \quad \delta\bar{\psi}_0 = +ie_0 \bar{\psi}_0 \omega$
$\delta A^0_\mu = \partial_\mu \omega$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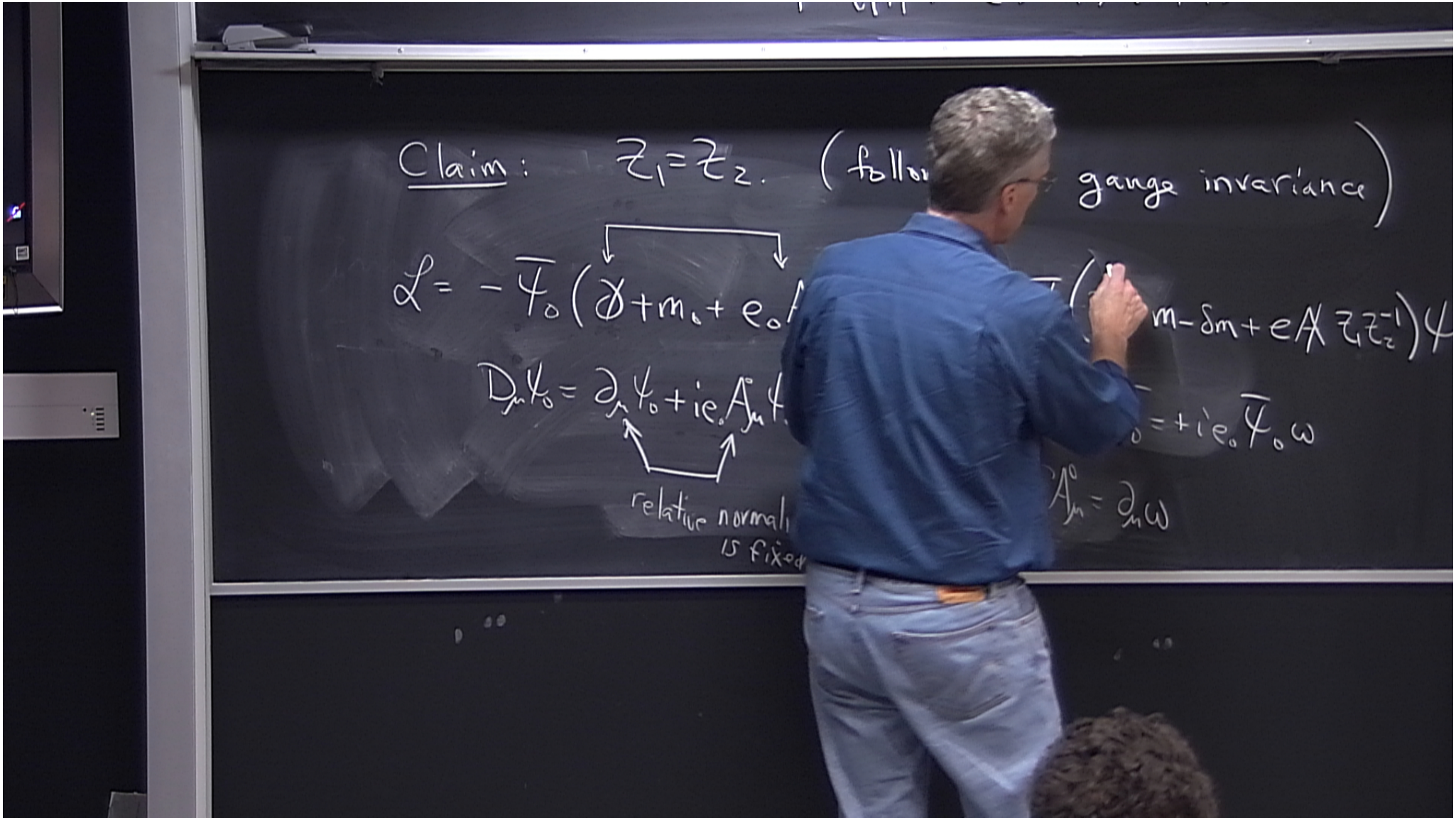
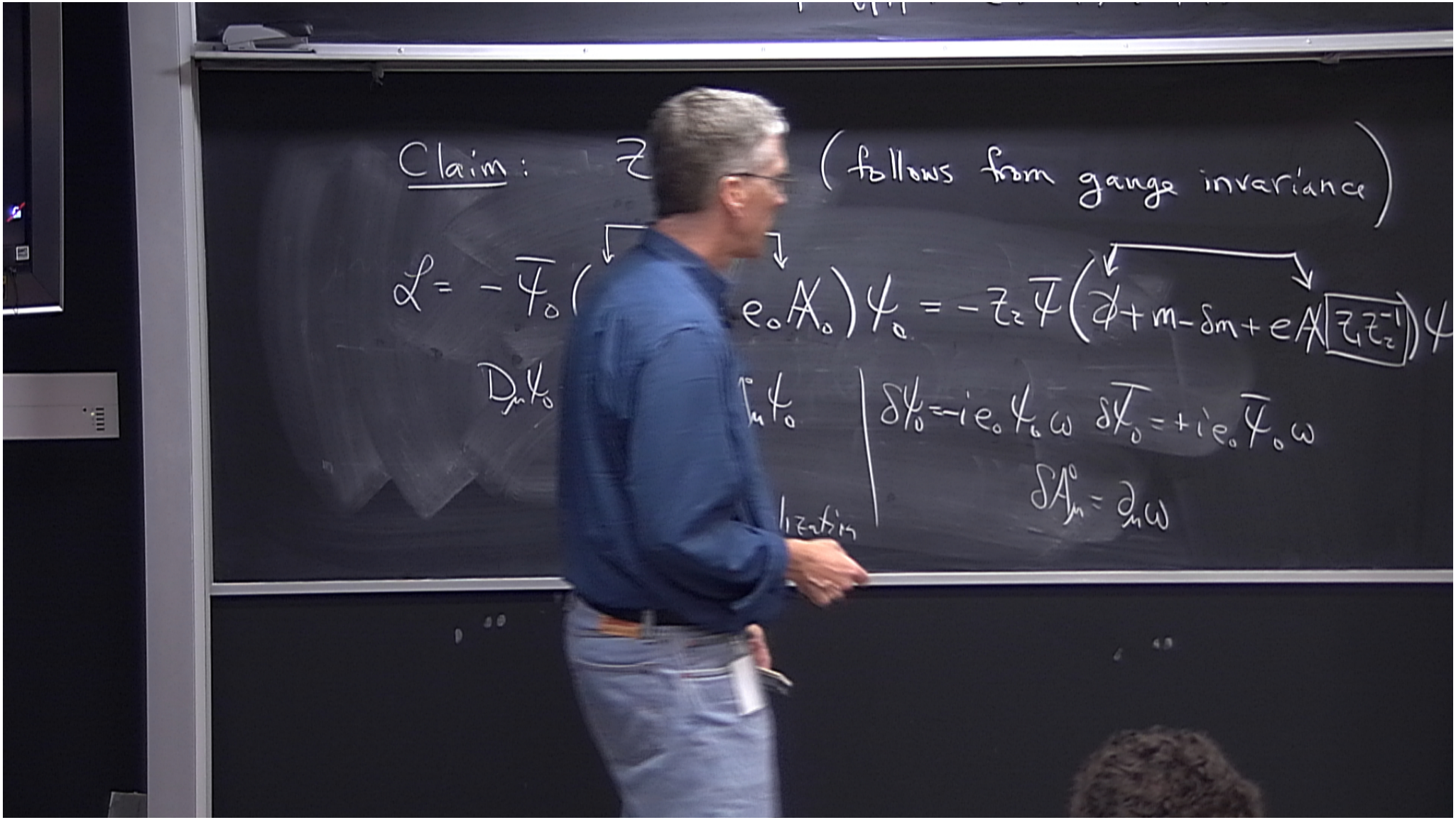

Claim: $Z_1 = Z_2$.
gauge invariance
$\mathcal{L} = -\bar{\psi}_0 (\not{\partial} + m_0 + e_0$
$m - \delta m + e \not{A} Z_1 Z_2^{-1}) \psi$
$D_\mu \psi_0 = \partial_\mu \psi_0 + i e_0 A_\mu$
$= + i e_0 \bar{\psi}_0 \omega$
$A^0_\mu = \partial_\mu \omega$
relative normal
is fixed


Claim:
(follows from gauge invariance
$\mathcal{L} = -\bar{\psi}_0 ( \quad e_0 A_0 ) \psi_0 = -Z_2 \bar{\psi} ( \partial + m - \delta m + e A Z_1 Z_2^{-1} ) \psi$
$D_\mu \psi_0$
$\Gamma_\mu \psi_0$
$\delta\psi_0 = -ie_0 \psi_0 \omega \quad \delta\bar{\psi}_0 = +ie_0 \bar{\psi}_0 \omega$
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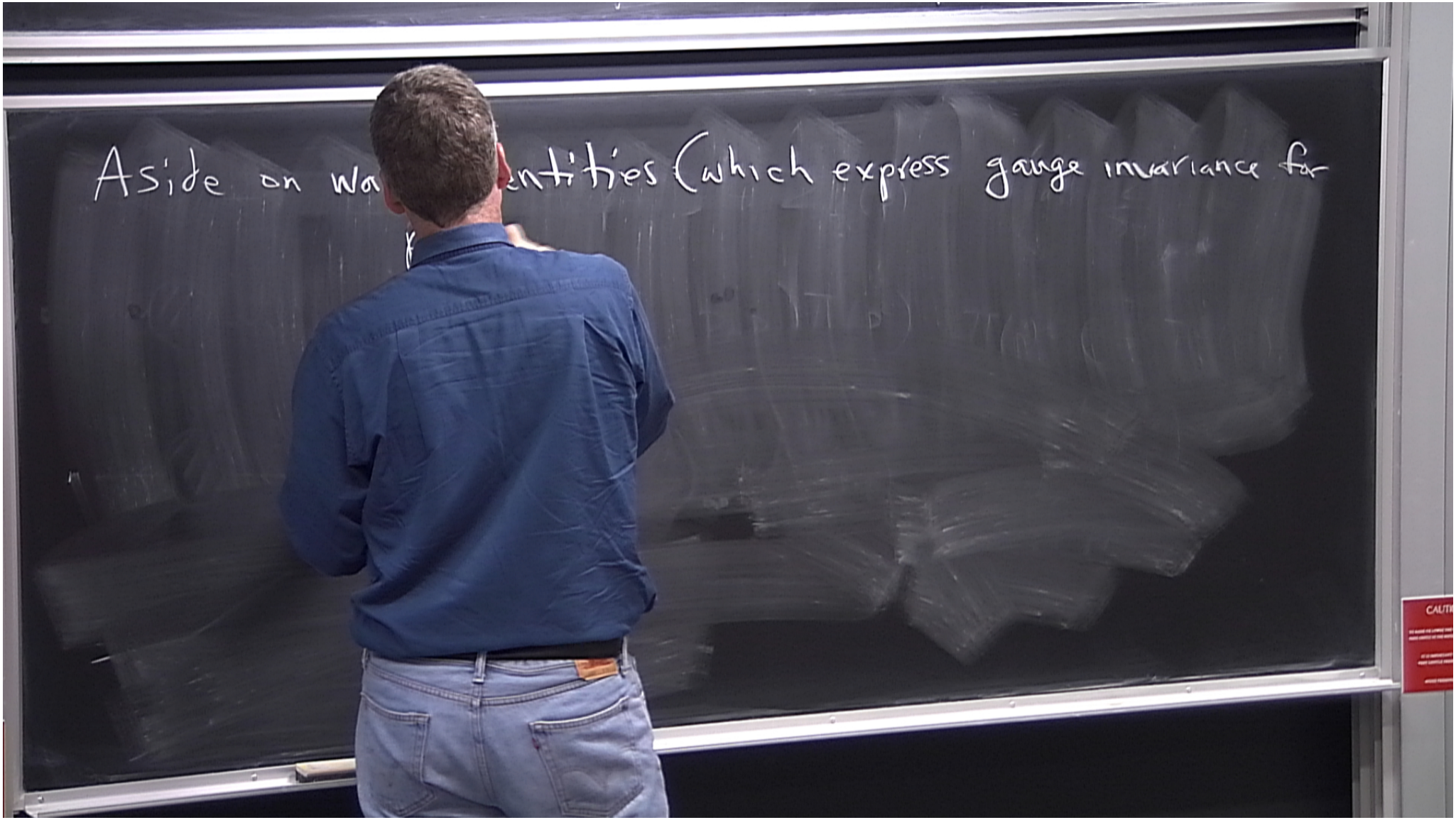
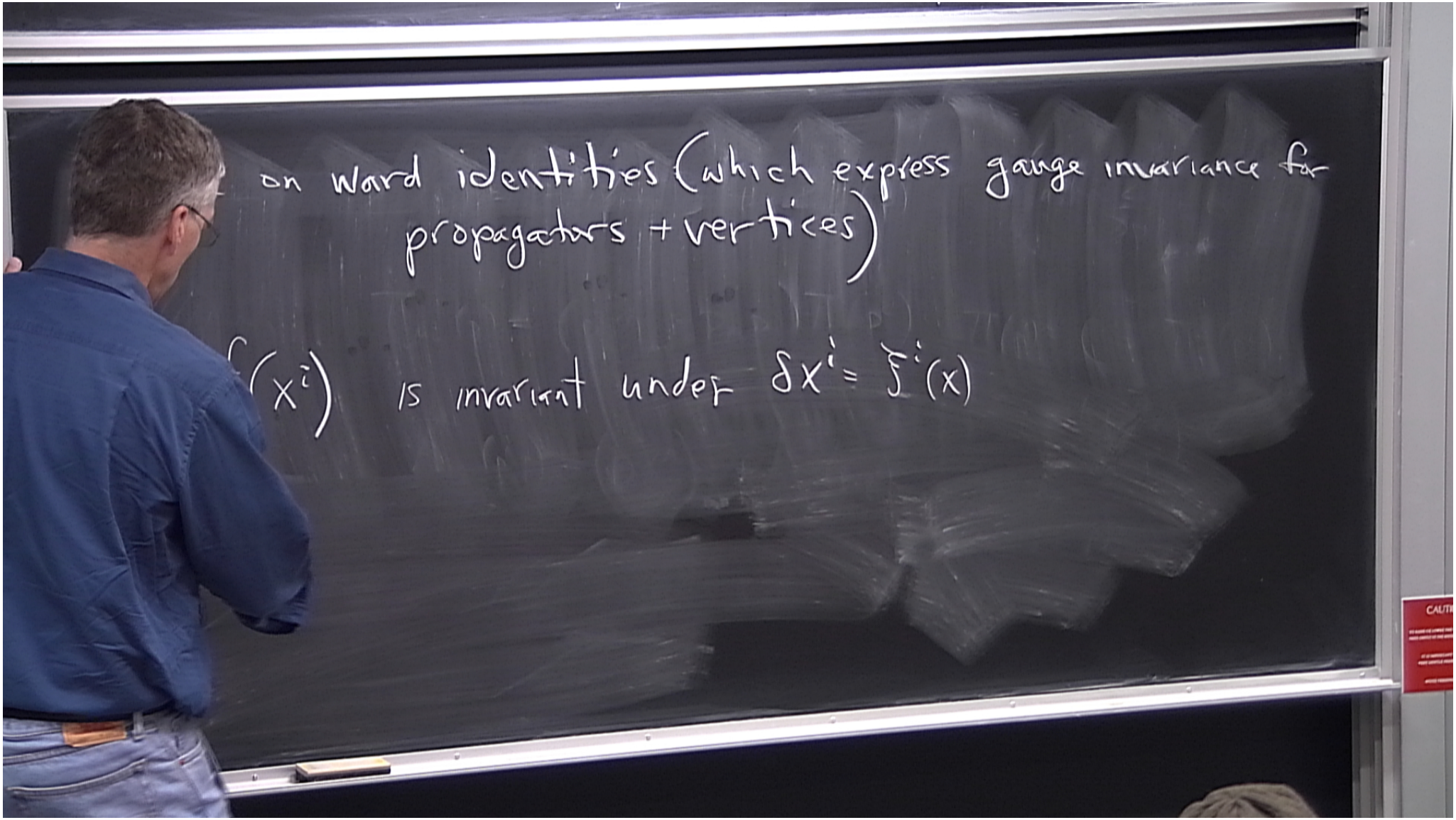

Aside on Wa entities (which express gauge invariance for

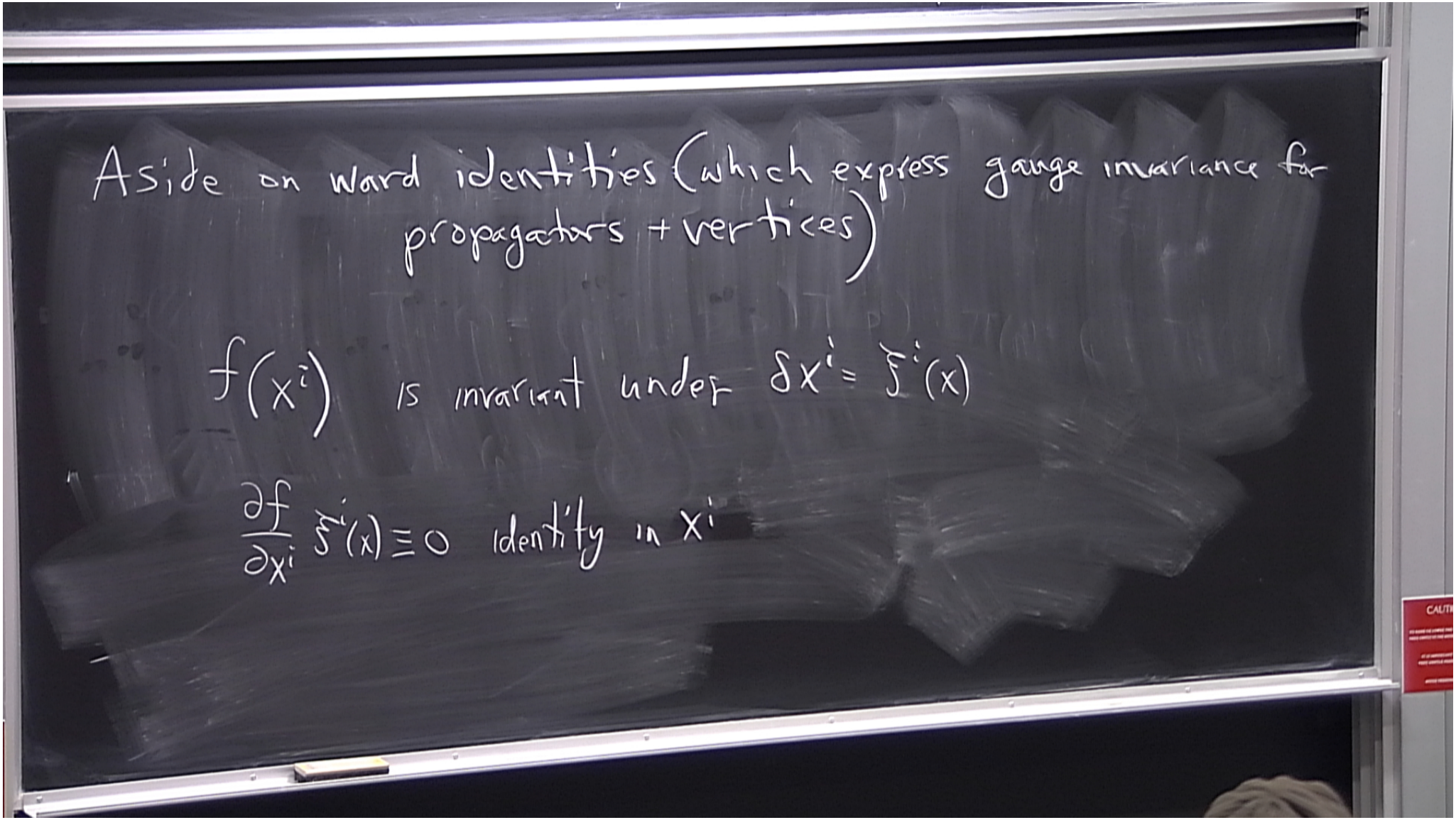
Aside on Ward identities (which express gauge invariance for propagators + vertices)


on Ward identities (which express gauge invariance for propagators + vertices)
$\Gamma(x^i)$ is invariant under $\delta x^i = \xi^i(x)$

Aside on Ward identities (which express gauge invariance for propagators + vertices)

$f(x^i)$ is invariant under $\delta x^i = \xi^i(x)$

$$\frac{\partial f}{\partial x^i}\xi^i(x) = 0$$

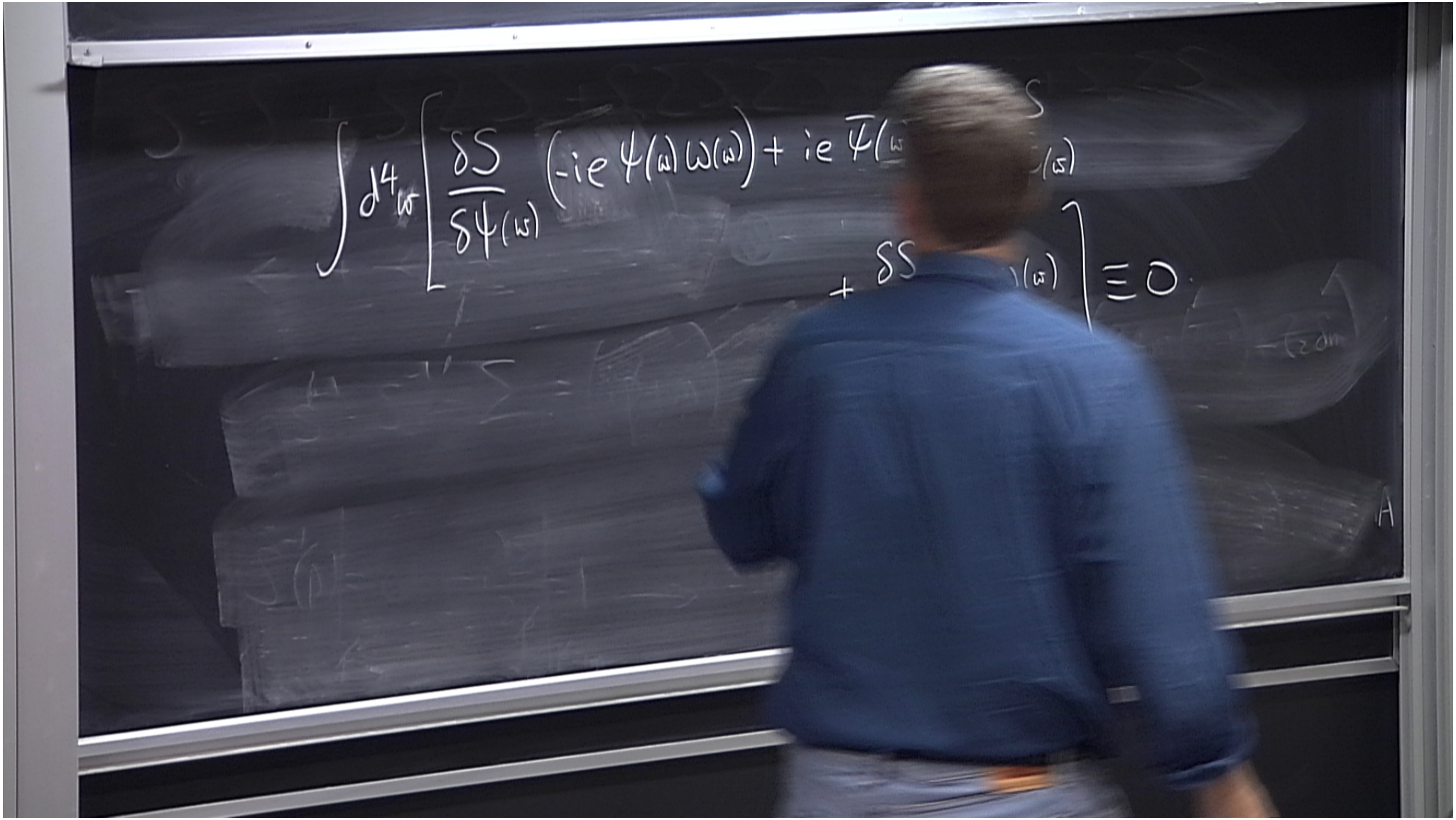

Aside on Ward identities (which express gauge invariance for propagators + vertices)
$f(x^i)$ is invariant under $\delta x^i = \xi^i(x)$
$\frac{\partial f}{\partial x^i}\xi^i(x) \equiv 0$ identity in $x^i$

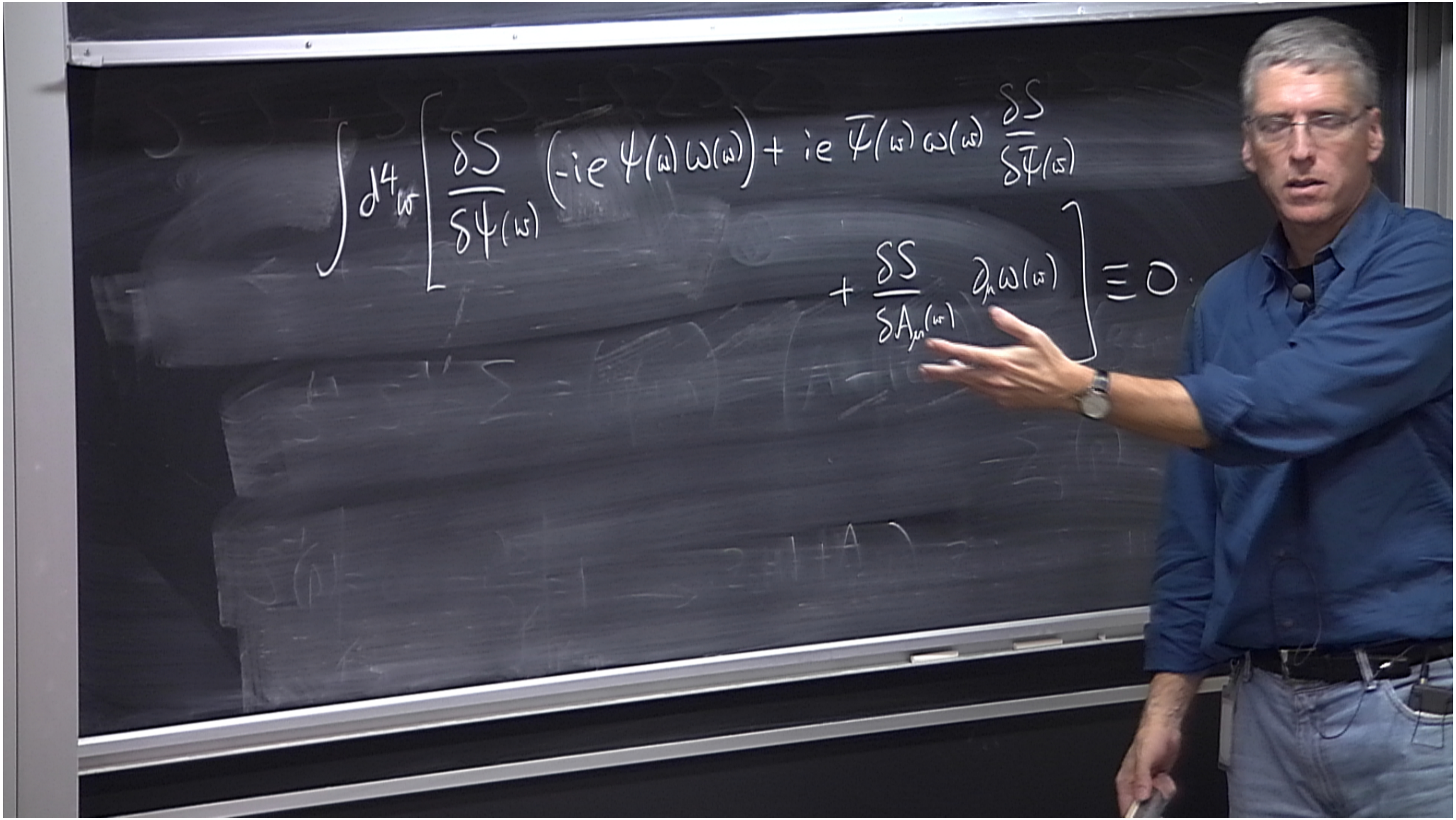
$f(x^i)$ is invariant under $\delta x^i = \xi^i(x)$

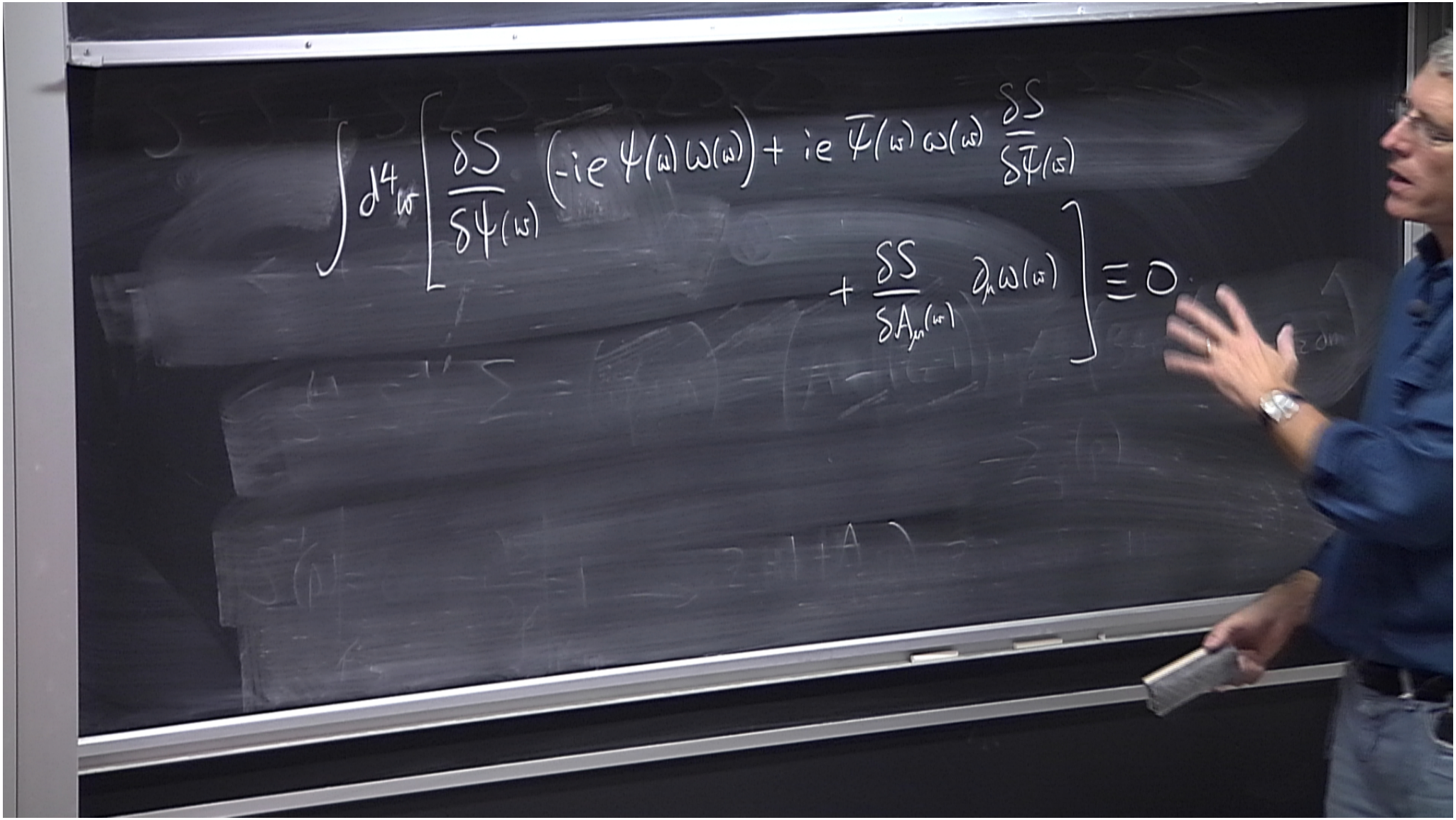
$$\frac{\partial f}{\partial x^i}\xi^i(x) \equiv 0 \quad \text{identify in } x^i$$

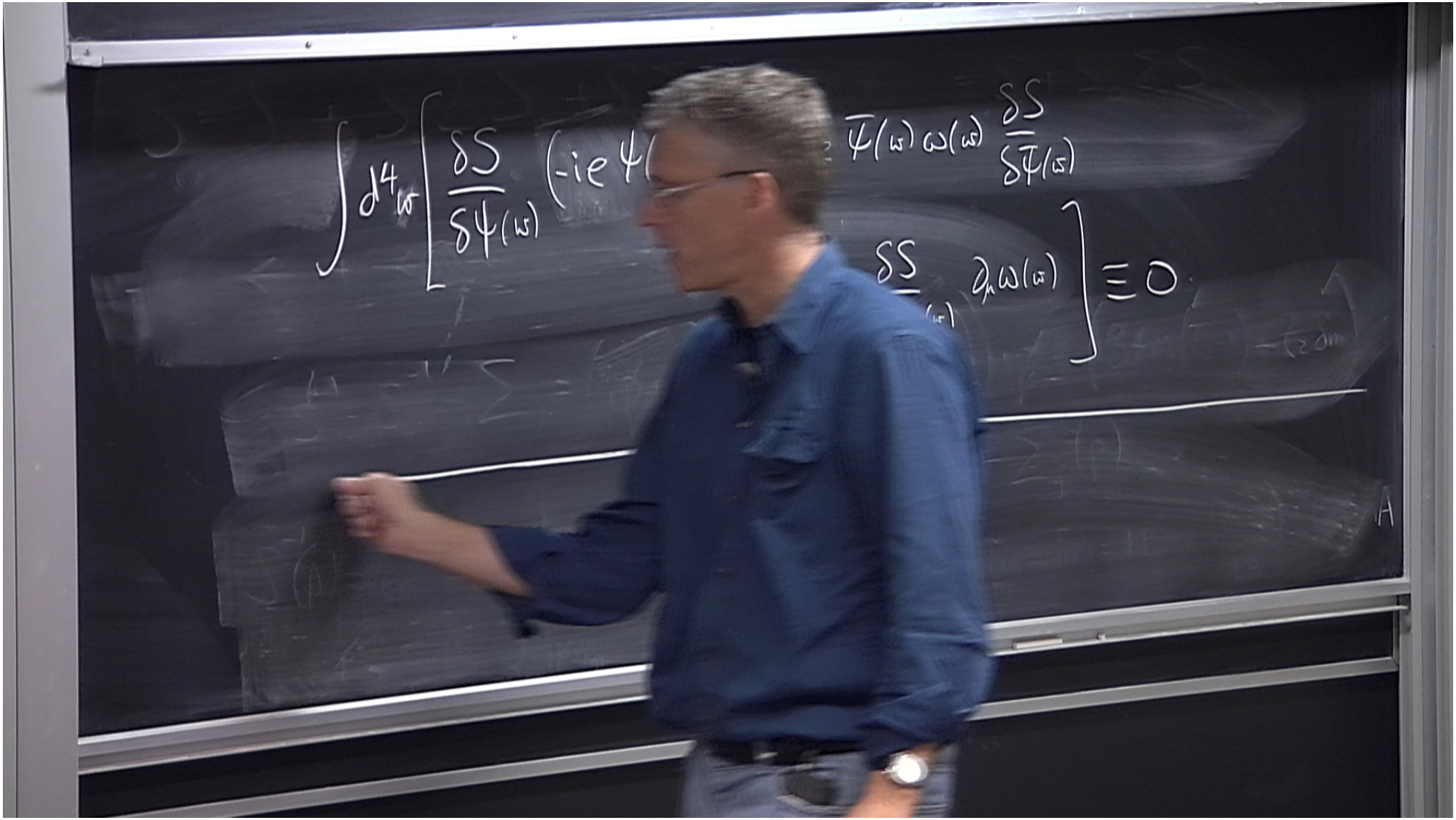
$$\longrightarrow \frac{\partial^2 f}{\partial x^j \partial x^i}\xi^i + \frac{\partial f}{\partial x^i}\frac{\partial \xi^i}{\partial x^j} = 0 \qquad \frac{\partial^3 f}{\partial x^k \partial x^j \partial x^i}\xi^i + \frac{\partial^2 f}{\partial x^i \partial x^i}\frac{\partial \xi^i}{\partial x^k} + \frac{\partial^2 f}{\partial x^k \partial x^i}\frac{\partial \xi^i}{\partial x^j} + \frac{\partial f}{\partial x^i}\frac{\partial^2 \xi^i}{\partial x^k \partial x^j}$$

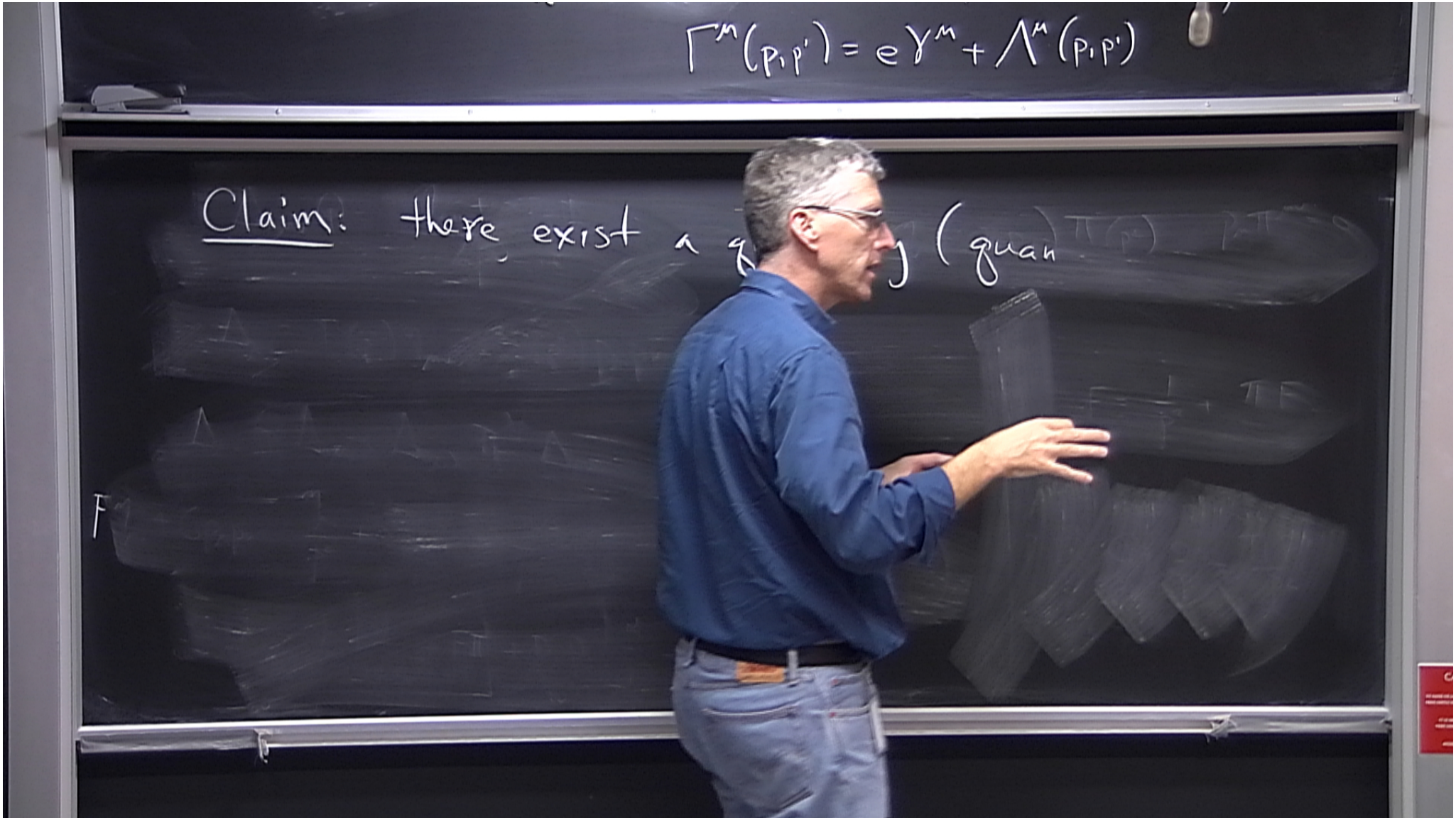
$$= \gamma^\mu\left[D + e(z_1 - 1)\right] + \Lambda_f^\mu(p', p)$$

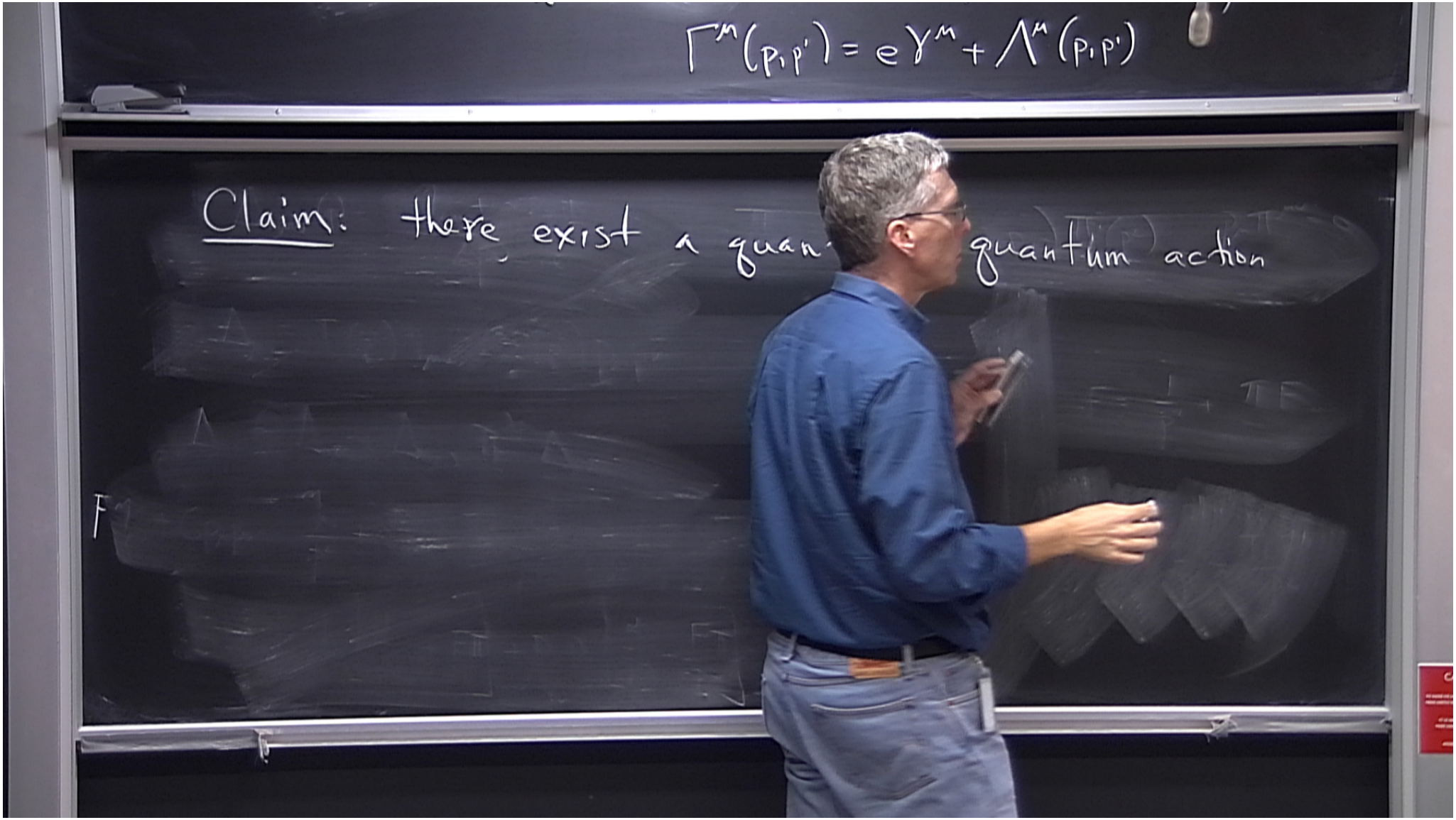

$\int d^4w \left[ \frac{\delta S}{\delta \psi(w)} (-ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w \quad S \quad (w) \right.$
$\left. + \delta S \quad (w) \right] \equiv 0$

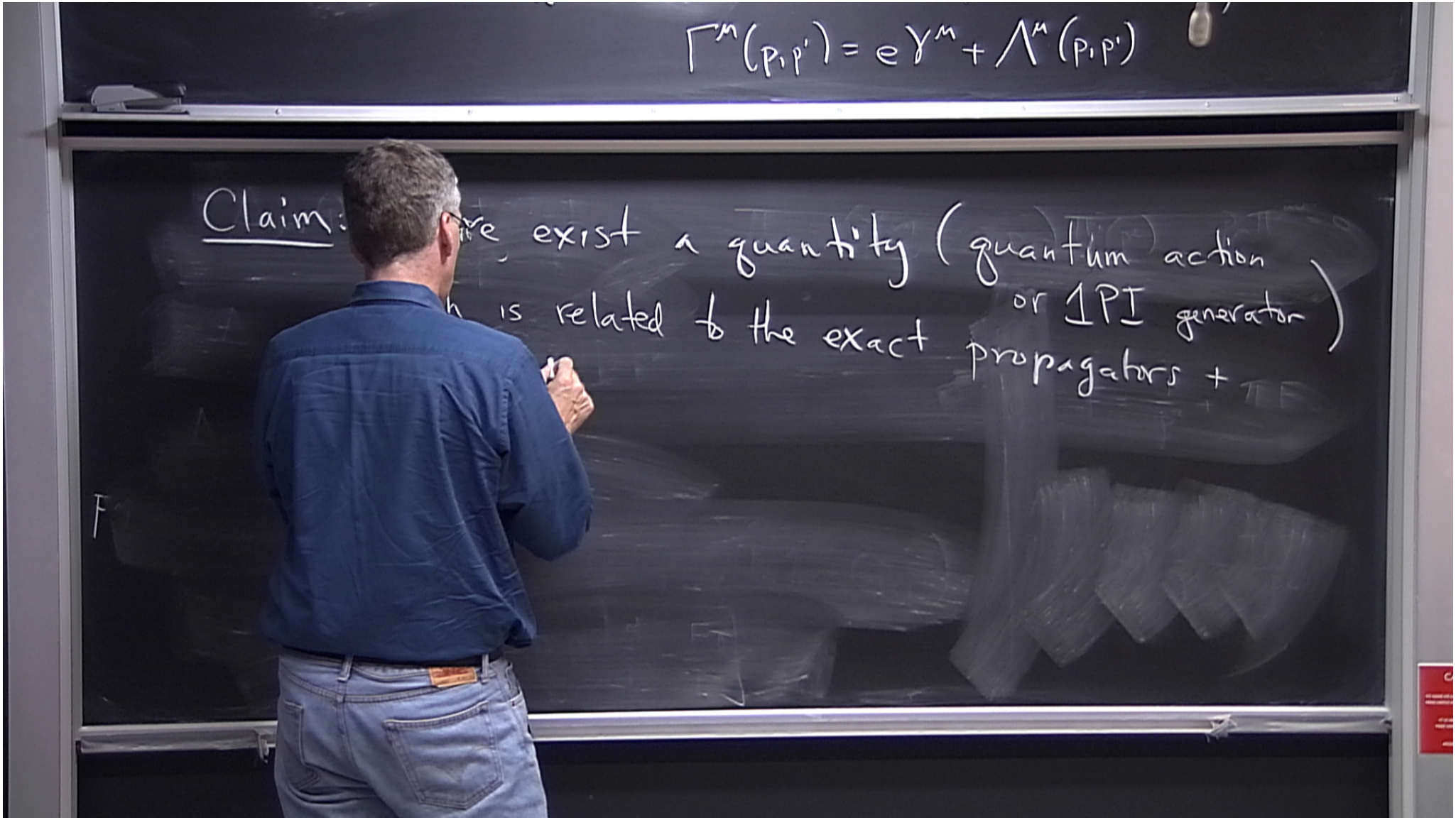

$\int d^4w \left[ \frac{\delta S}{\delta \psi(w)} (-ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w)\omega(w) \frac{\delta S}{\delta \bar{\psi}(w)} + \frac{\delta S}{\delta A_\mu(w)} \partial_\mu \omega(w) \right] \equiv 0$

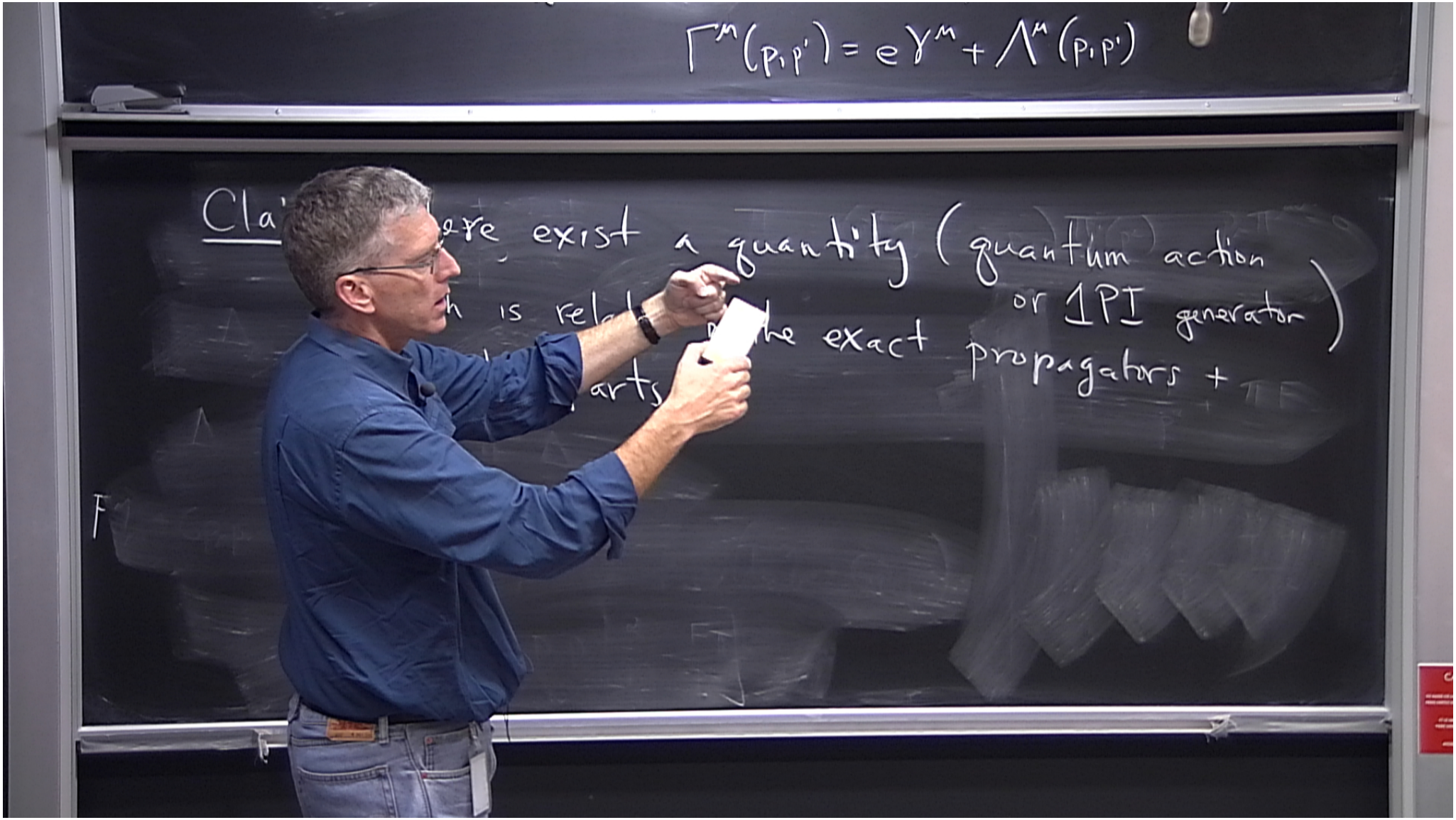

$\int d^4w \left[ \frac{\delta S}{\delta \psi(w)} (-ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w)\omega(w) \frac{\delta S}{\delta \bar{\psi}(w)} + \frac{\delta S}{\delta A_\mu(w)} \partial_\mu \omega(w) \right] \equiv 0$


$\int d^4w \left[ \frac{\delta S}{\delta \psi(w)} (-ie\psi($
$\bar{\psi}(w)\omega(w) \frac{\delta S}{\delta \bar{\psi}(w)}$
$\frac{\delta S}{\delta}$ $\partial_\mu \omega(w) \Big] \equiv 0$


$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim: there exist a


$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim: there exist a quan quantum action


$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim:
exist a quantity ( quantum action
or 1PI generator
is related to the exact propagators +


$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
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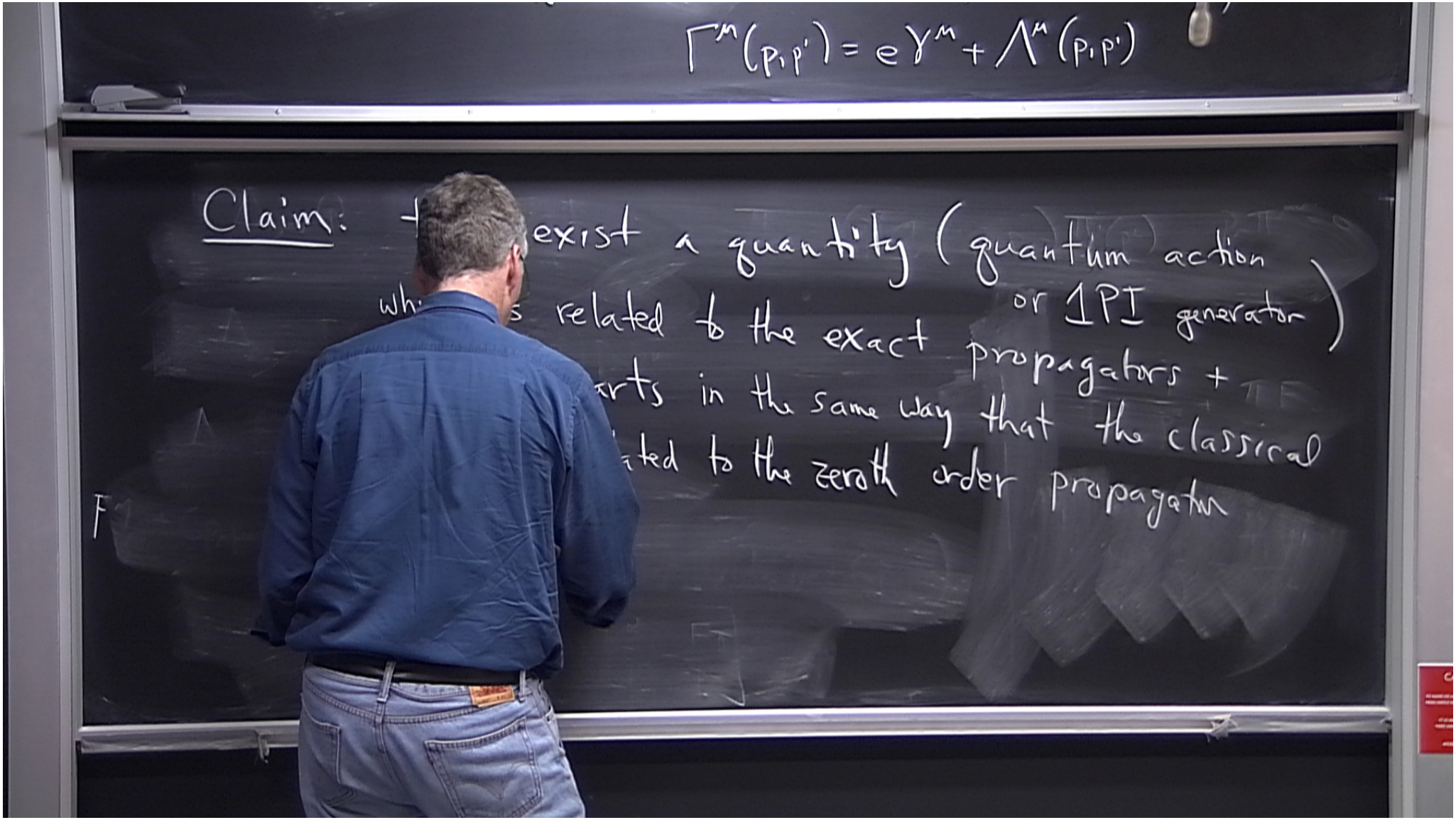
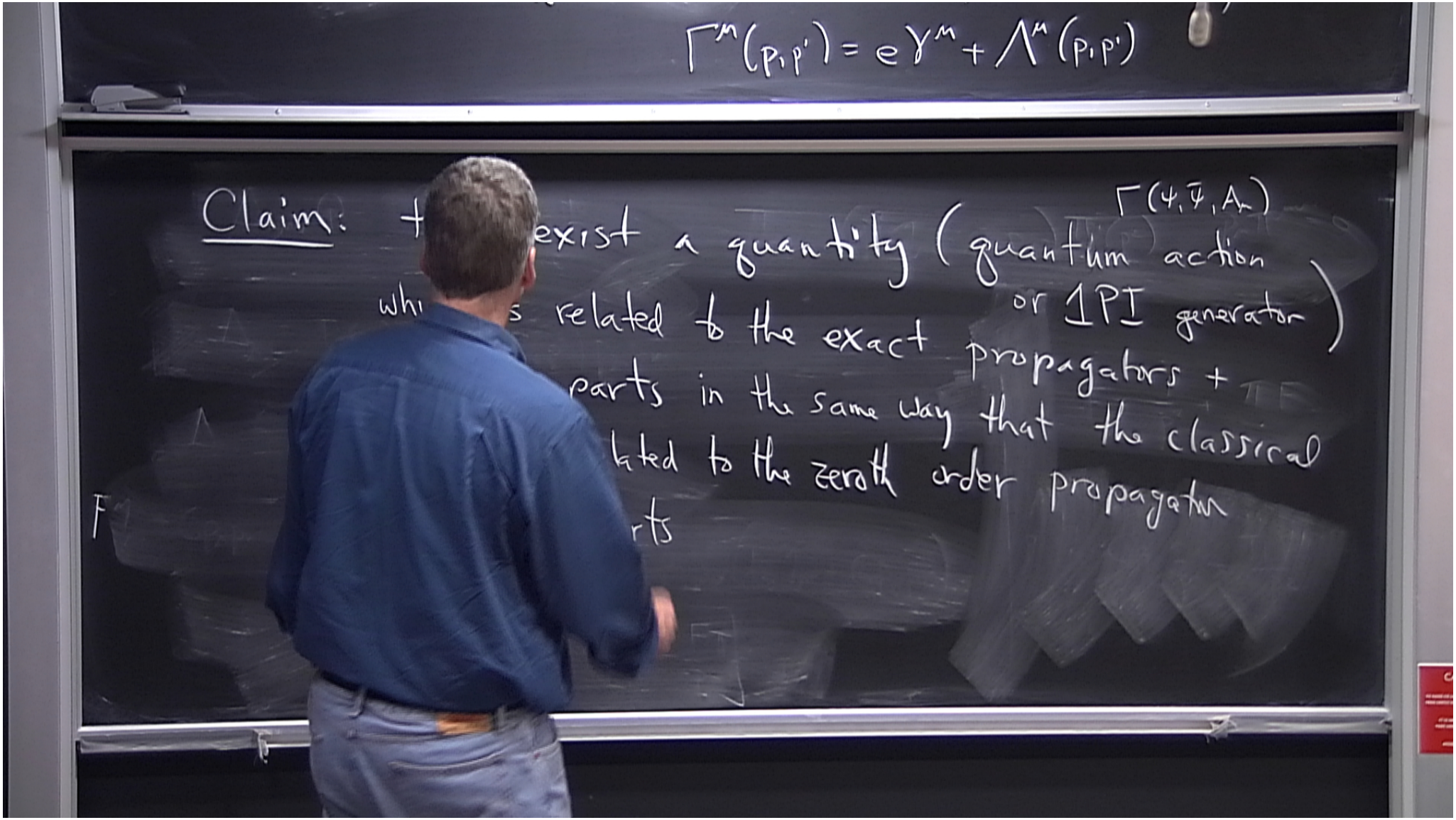
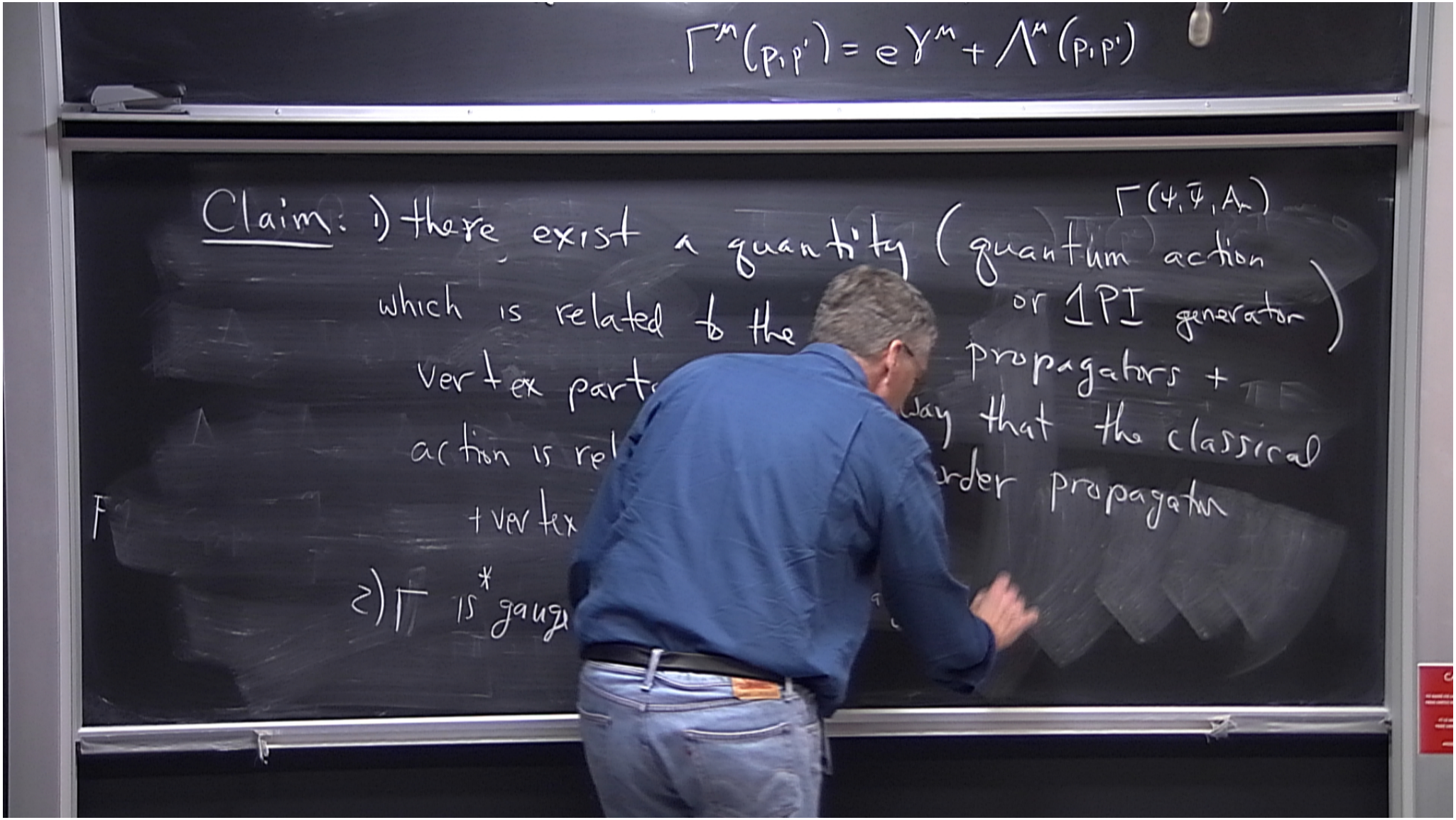
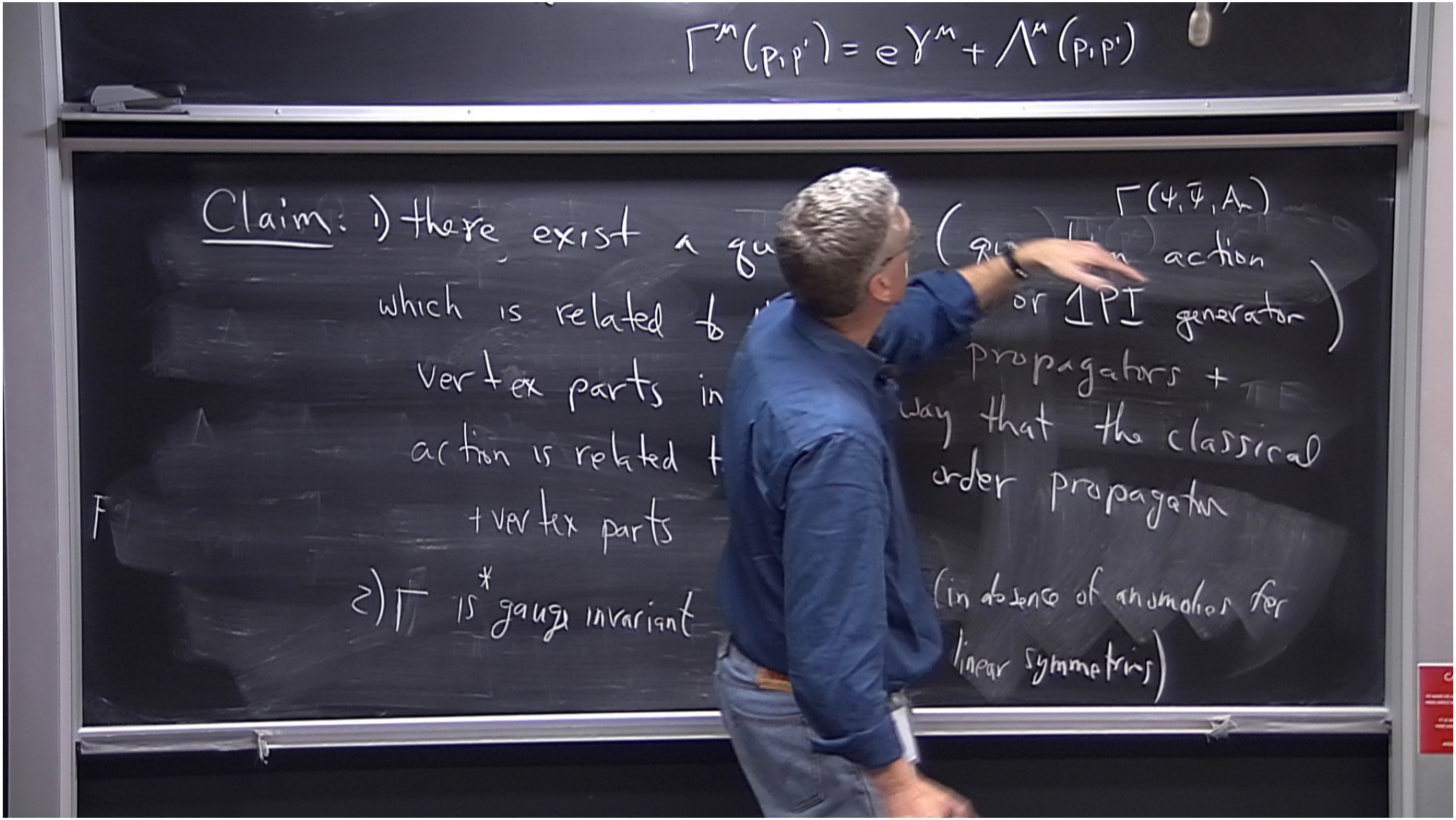
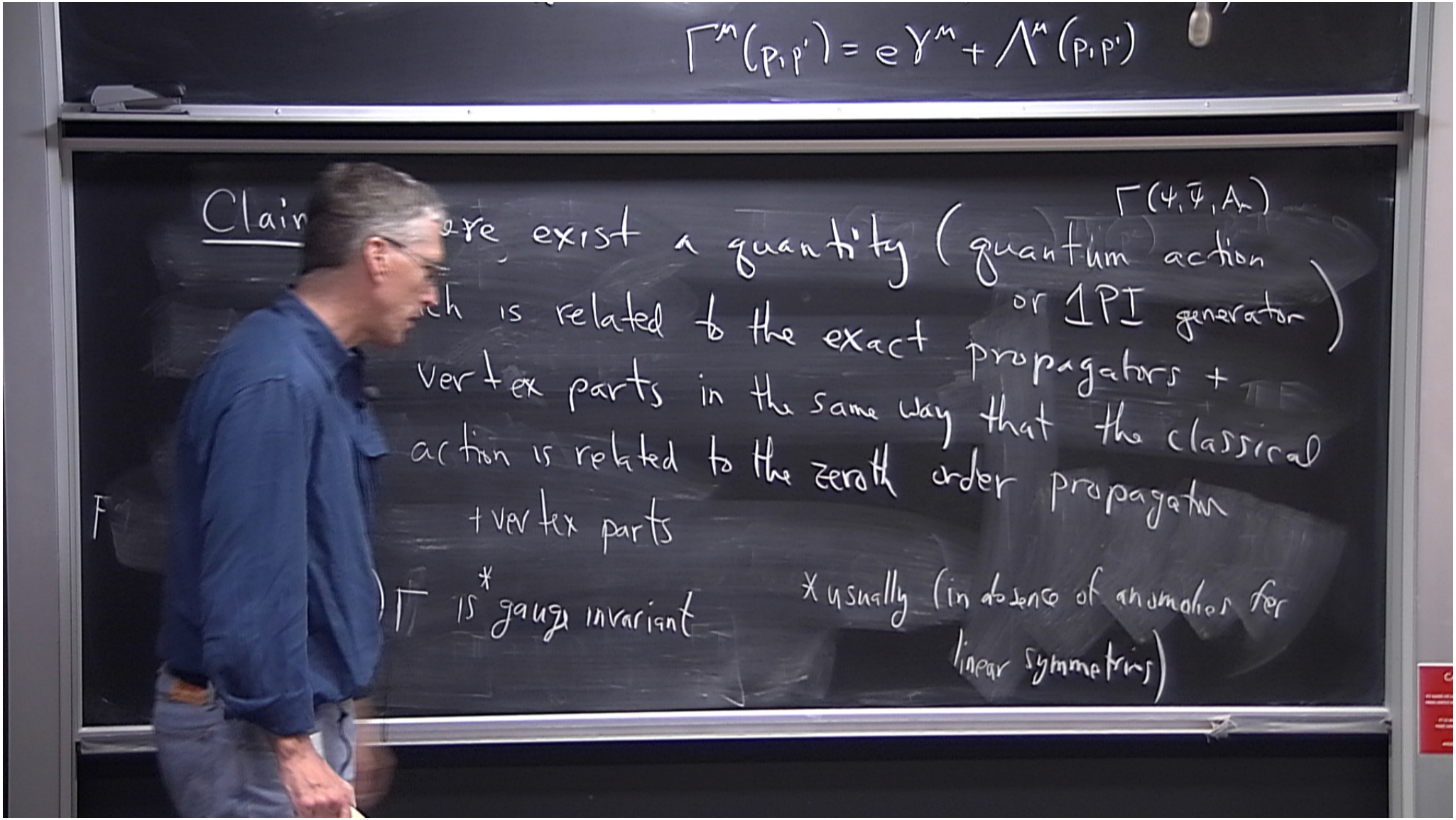
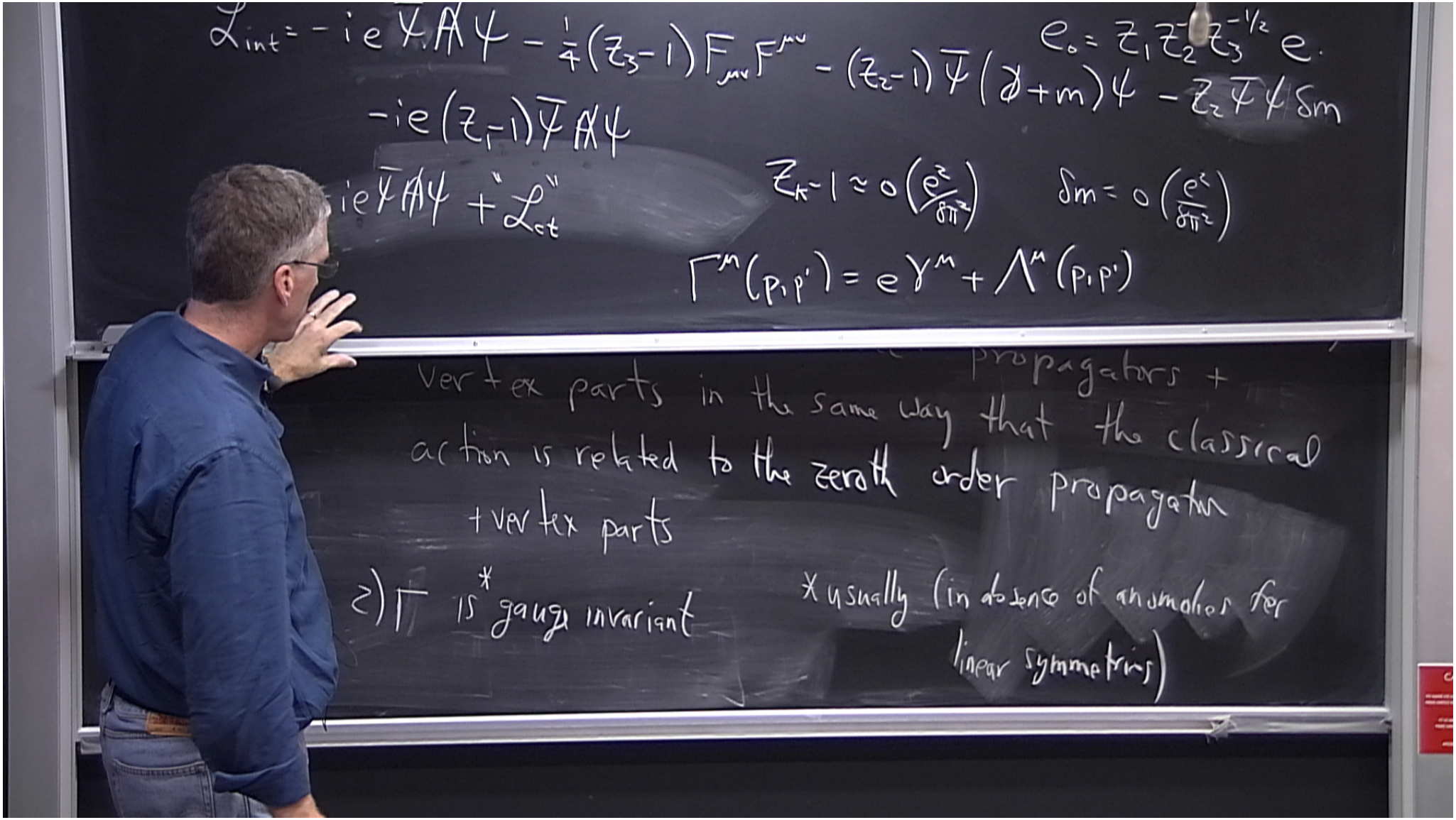

$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim:
exist a quantity (quantum action
or 1PI generator
related to the exact propagators +
in the same way that the classical
to the zeroth order propagator

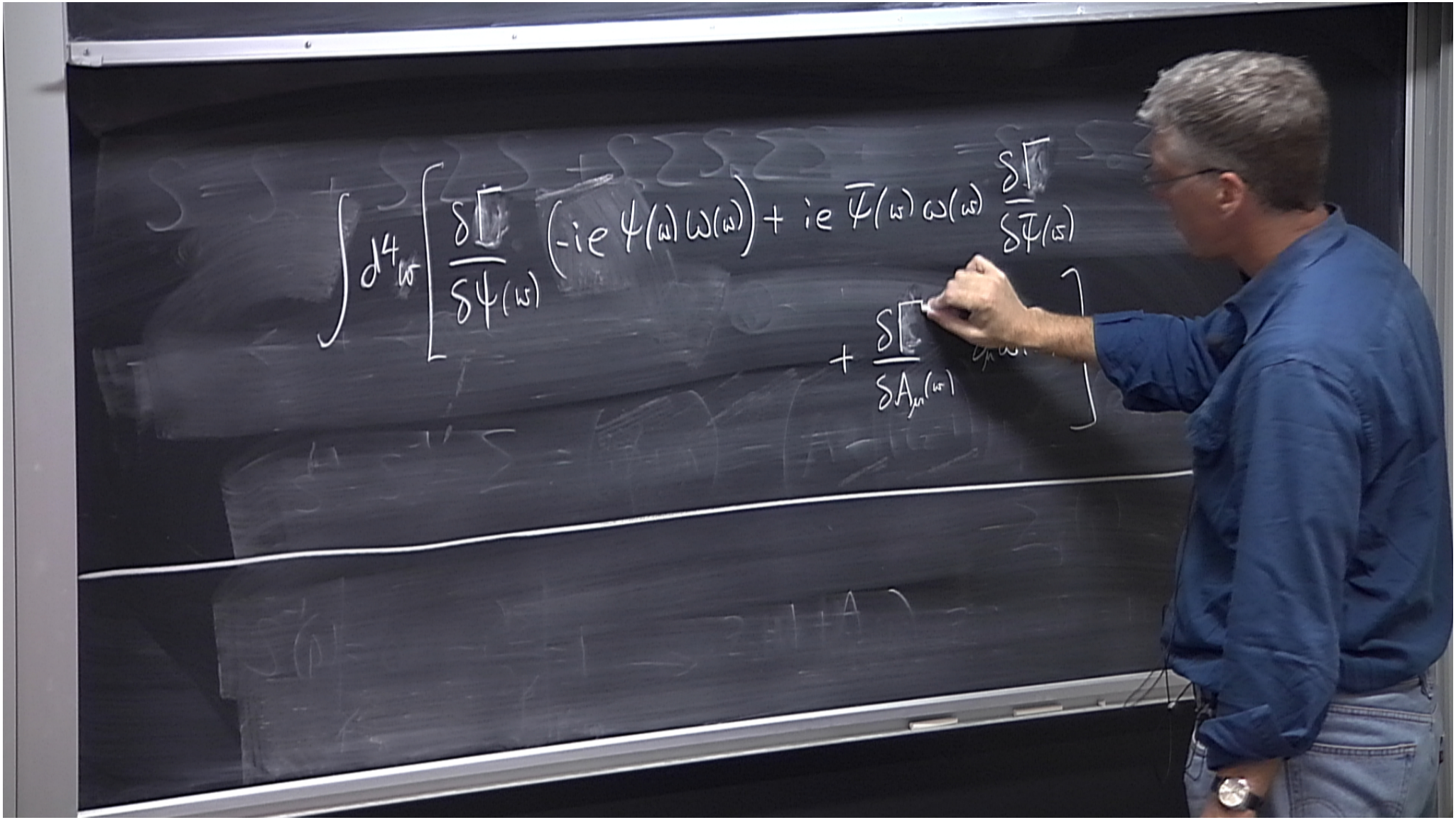

$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim:
exist a quantity ( quantum action
$\Gamma(\psi,\bar{\psi},A_\mu)$
or 1PI generator )
related to the exact propagators +
parts in the same way that the classical
lated to the zeroth order propagator
rts

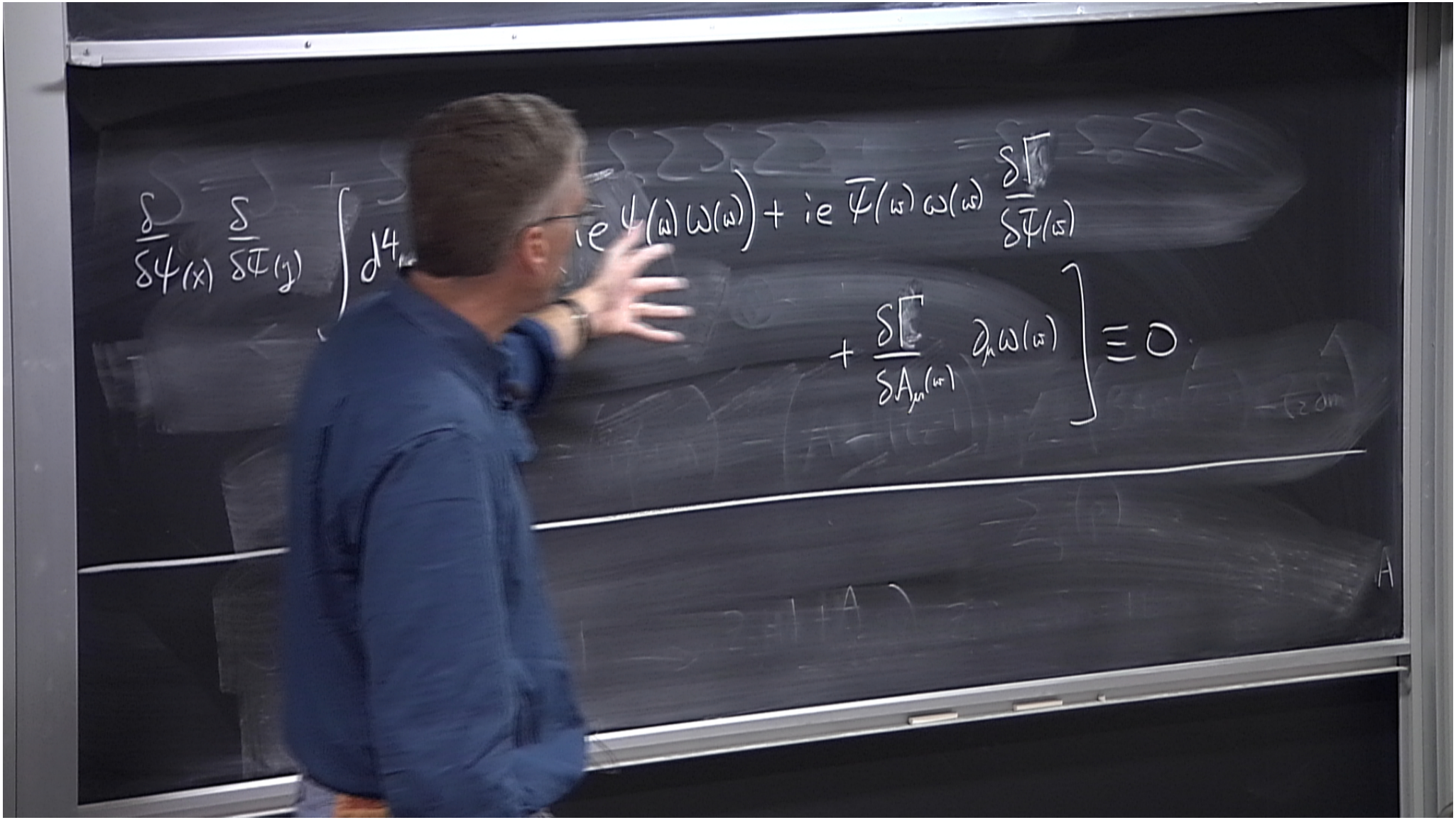

$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim: 1) there exist a quantity ($\Gamma(\psi,\bar{\psi},A_\mu)$ quantum action
which is related to the or 1PI generator
vertex part propagators +
action is rel way that the classical
+ vertex rder propagator
2) $\Gamma$ is* gaug

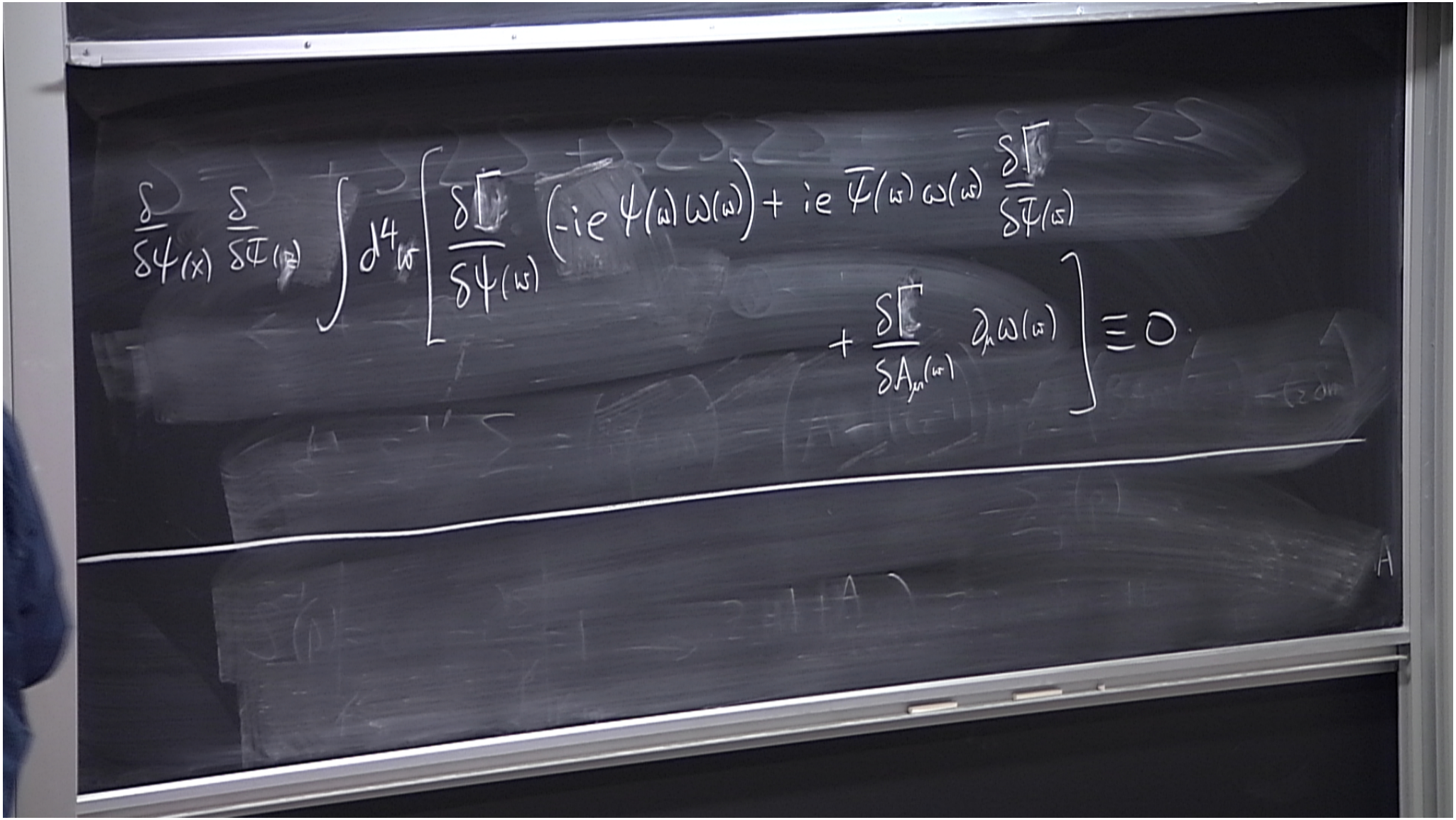

$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Claim: 1) there exist a gu
which is related to
vertex parts in
action is related t
+ vertex parts
2) $\Gamma$ is* gauge invariant
$\Gamma(\phi,\bar{\psi},A,\ldots)$
action
or 1PI generator
propagators +
way that the classical
order propagator
(in absence of anomalies for
linear symmetries)

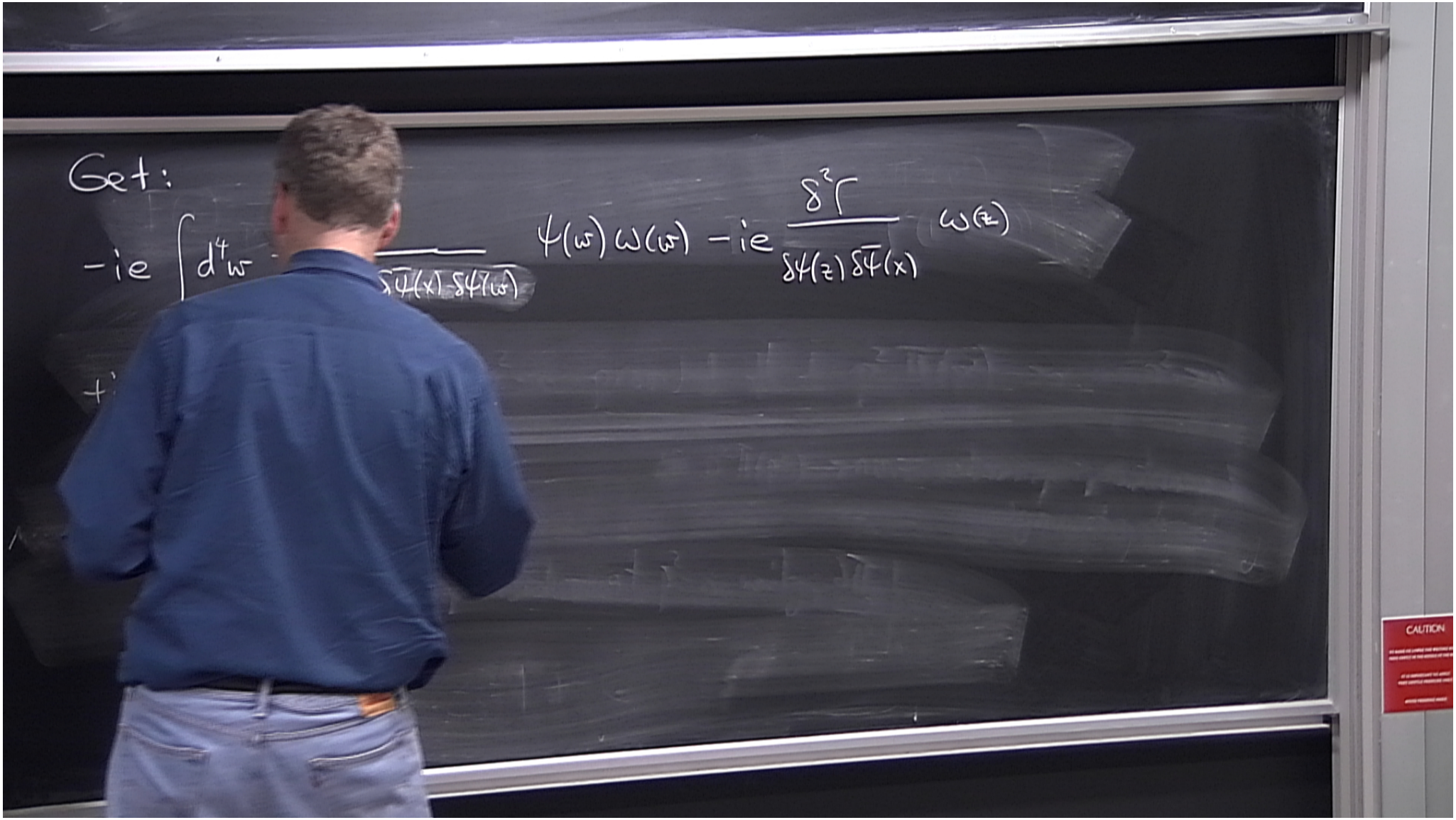

$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
Clair
re exist a quantity ( quantum action
$\Gamma(\psi,\bar{\psi},A_\mu)$
ch is related to the exact propagators +
or 1PI generator )
vertex parts in the same way that the classical
action is related to the zeroth order propagator
+ vertex parts
F
) $\Gamma$ is* gauge invariant
* usually (in absence of anomalies for
linear symmetries)

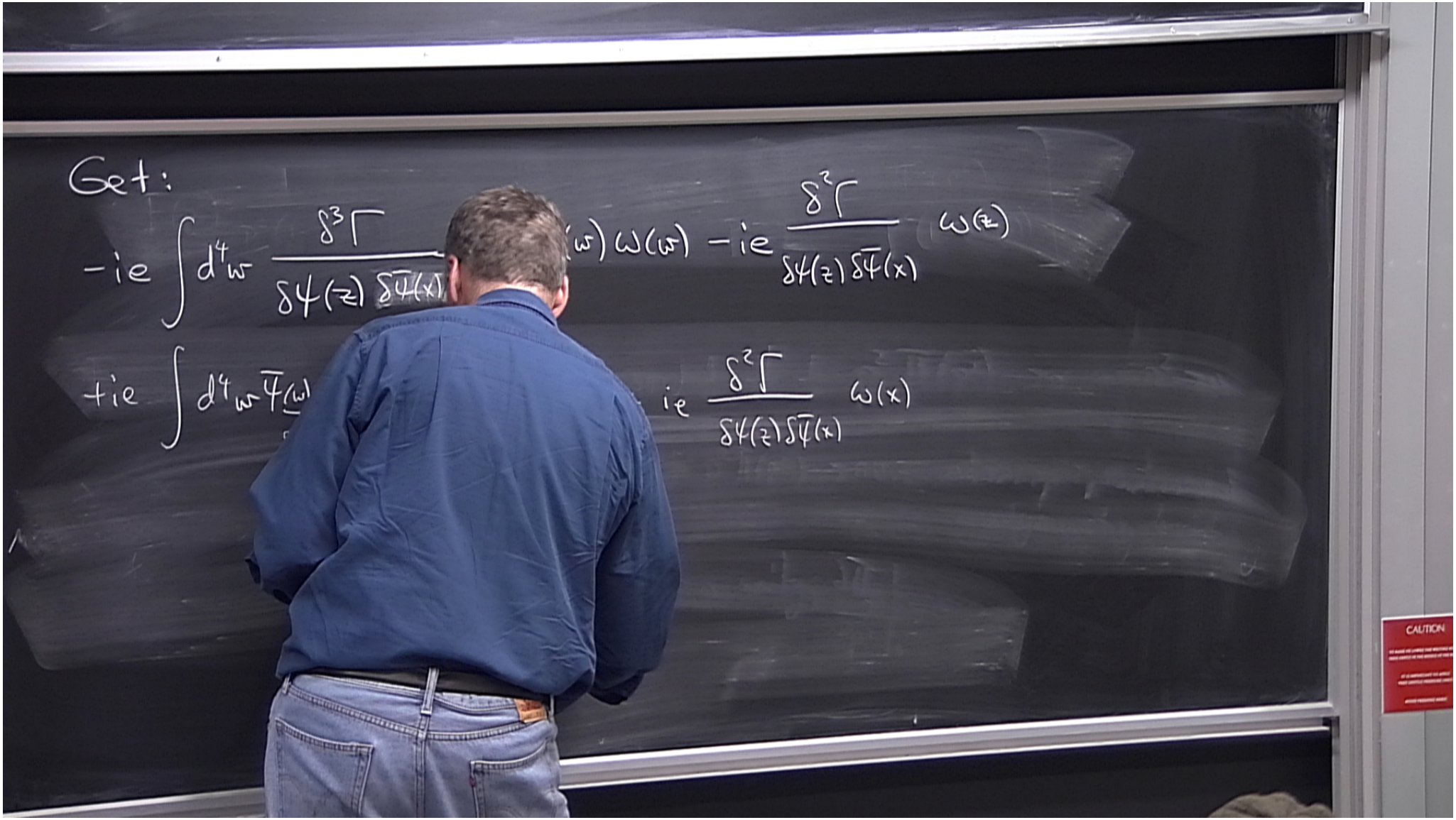

$\mathcal{L}_{int} = -ie\bar{\psi}A\psi - \frac{1}{4}(Z_3-1)F_{\mu\nu}F^{\mu\nu} - (Z_2-1)\bar{\psi}(\partial+m)\psi - Z_2\bar{\psi}\psi\delta m$
$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$
$-ie(Z_1-1)\bar{\psi}A\psi$
$ie\bar{\psi}A\psi + \mathcal{L}_{ct}$
$Z_k - 1 \approx O\left(\frac{e^2}{8\pi^2}\right)$
$\delta m = O\left(\frac{e^2}{8\pi^2}\right)$
$\Gamma^\mu(p,p') = e\gamma^\mu + \Lambda^\mu(p,p')$
propagators +
Vertex parts in the same way that the classical action is related to the zeroth order propagator
+ vertex parts
2) $\Gamma$ is* gauge invariant
* usually (in absence of anomalies for linear symmetries)

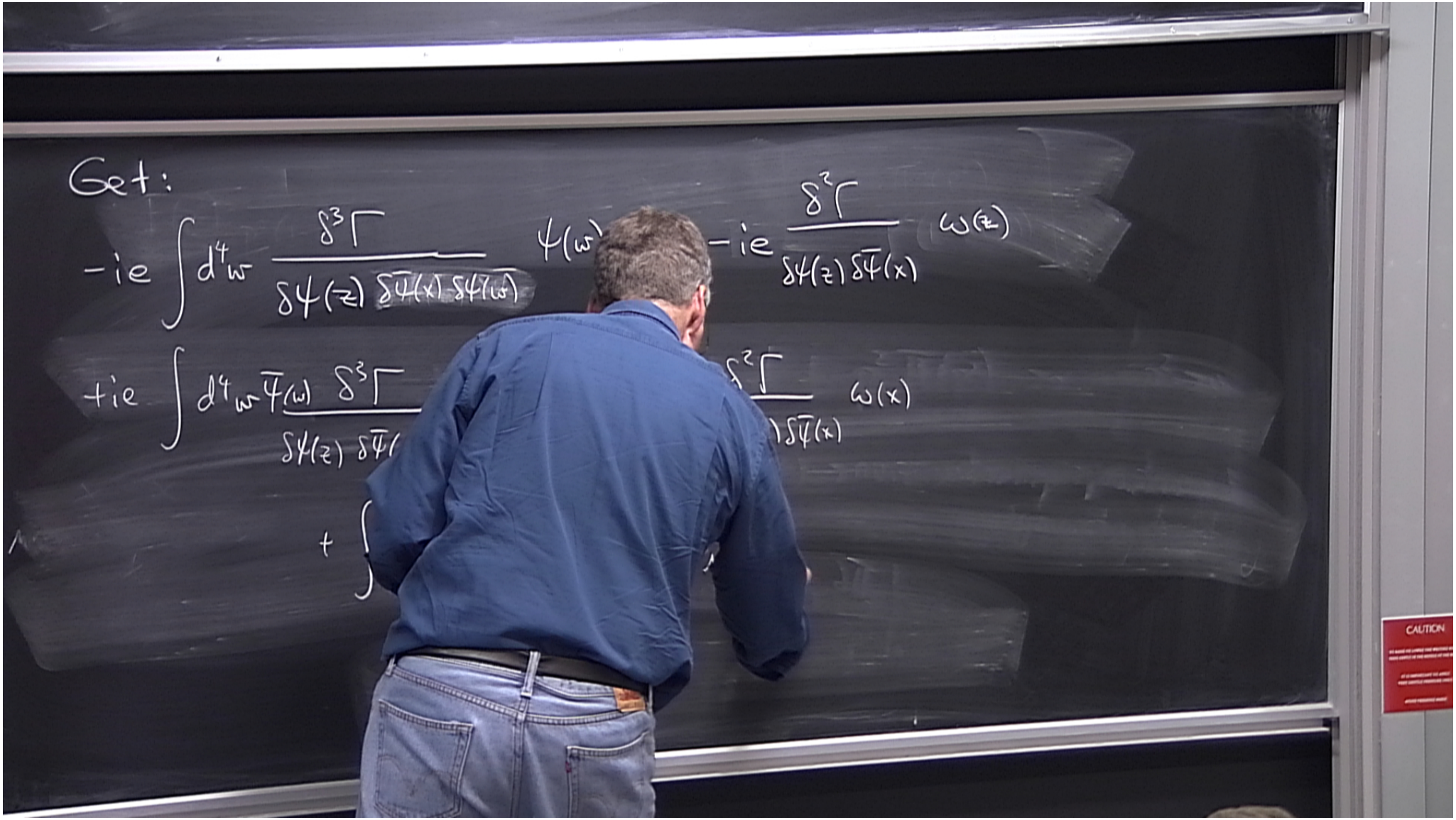

$\int d^4w \left[ \frac{\delta \Gamma}{\delta \psi(w)} (-ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w)\omega(w) \frac{\delta \Gamma}{\delta \bar{\psi}(w)} \right.$
$\left. + \frac{\delta \Gamma}{\delta A_\mu(w)} \right]$


$\frac{\delta}{\delta\psi(x)}\frac{\delta}{\delta\bar{\psi}(y)}\int d^4$
$ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w)\omega(w)\frac{\delta\Gamma}{\delta\psi(w)}$
$+\frac{\delta\Gamma}{\delta A_\mu(w)}\partial_\mu\omega(w)\Big] \equiv 0$


$\frac{\delta}{\delta\psi(x)}\frac{\delta}{\delta\bar{\psi}(y)}\int d^4w\left[\frac{\delta\Gamma}{\delta\psi(w)}\left(-ie\,\psi(w)\omega(w)\right)+ie\,\bar{\psi}(w)\omega(w)\frac{\delta\Gamma}{\delta\bar{\psi}(w)}\right.$
$\left.+\frac{\delta\Gamma}{\delta A_\mu(w)}\partial_\mu\omega(w)\right]\equiv 0$


Get:
$-ie\int d^4w$
$\delta\bar{\psi}(x)\,\delta\psi(w)$
$\psi(w)\,\omega(w) - ie\,\dfrac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(z)$
CAUTION


Get:
$-ie\int d^4w \frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}$
$\omega(w)\,\omega(w)$
$-ie\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(z)$
$+ie\int d^4w\,\bar{\psi}(w)$
$ie\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(x)$
CAUTION

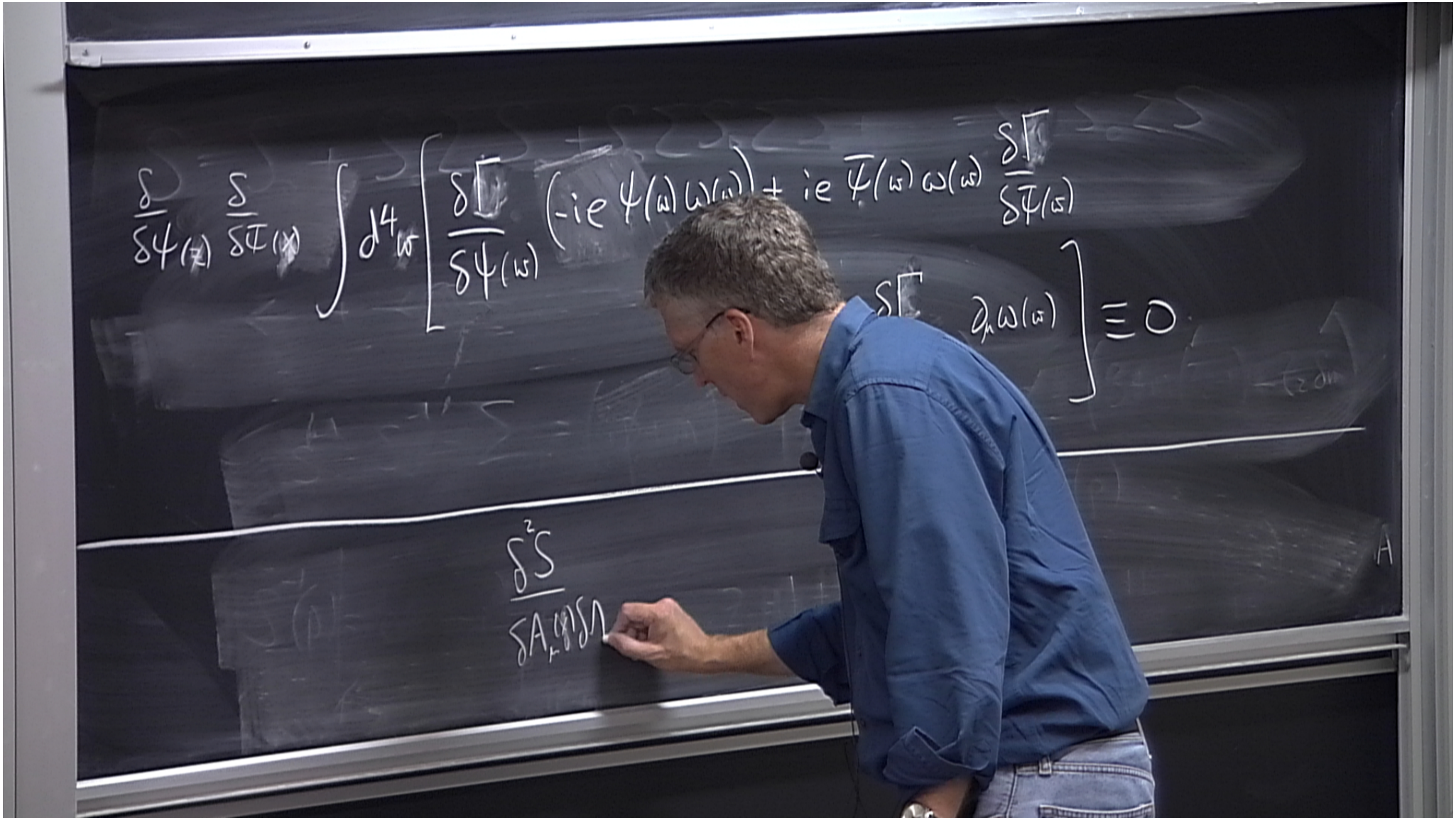

Get:
$-ie\int d^4w\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)\,\delta\psi(w)}\,\psi(w)$
$-ie\,\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)}\,\omega(z)$
$+ie\int d^4w\,\bar\psi(w)\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar\psi($
$\frac{\delta^2\Gamma}{\delta\psi(x)}\,\omega(x)$
CAUTION

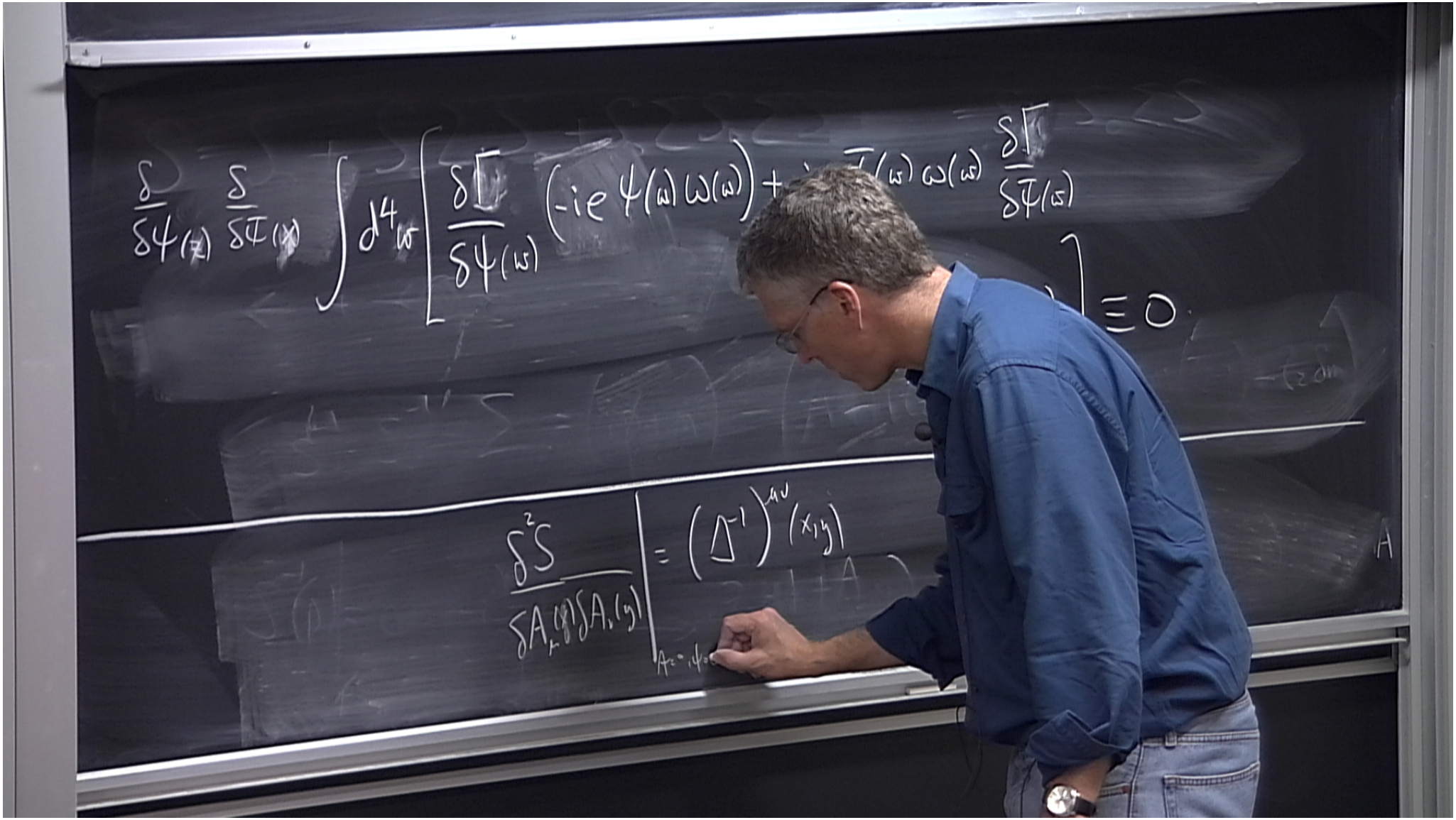
Get:

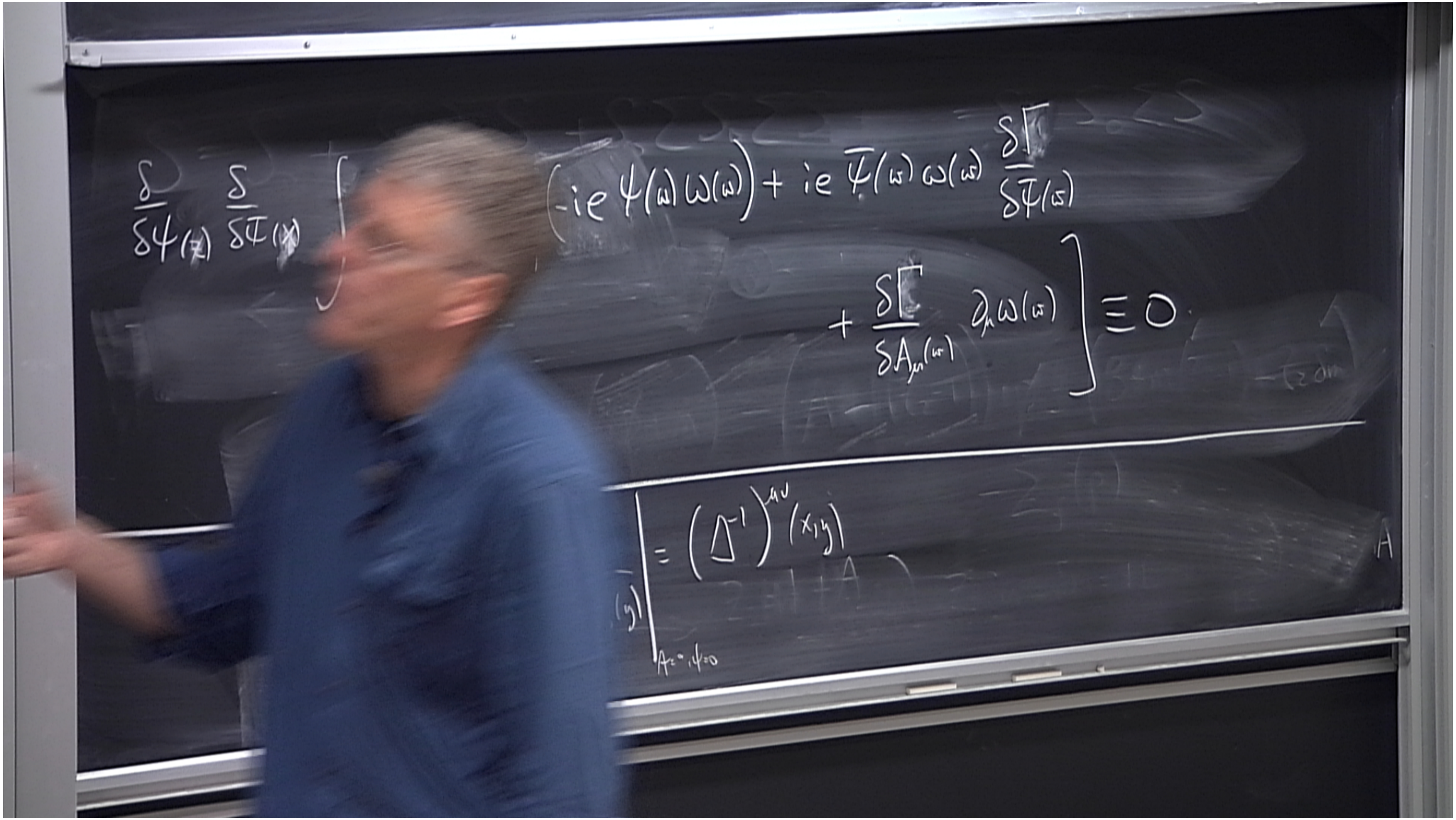
$$-ie\int d^4w\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)\,\delta\psi(w)}\,\psi(w)\,\omega(w) - ie\,\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)}\,\omega(z)$$

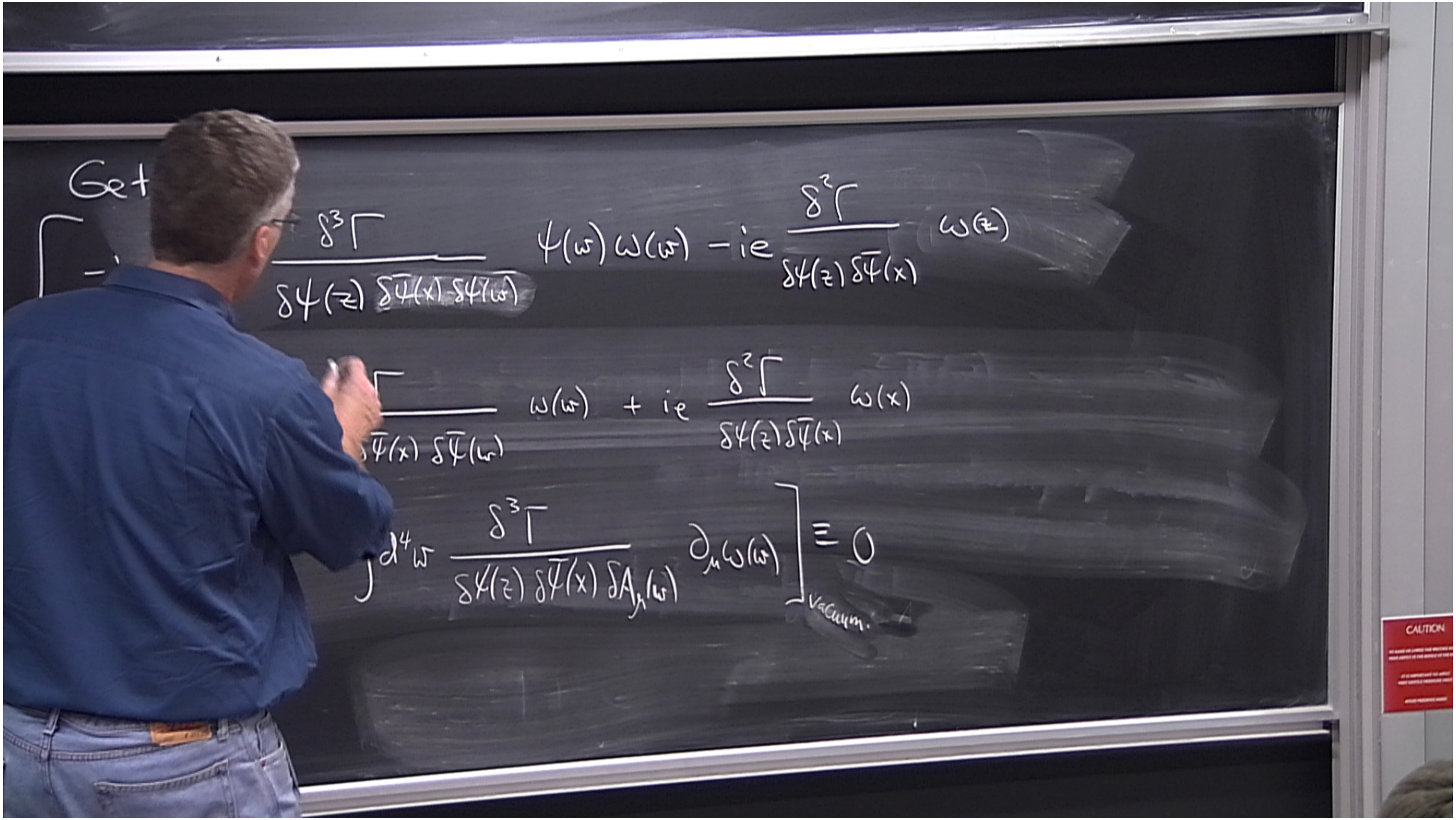
$$+ie\int d^4w\,\bar\psi(w)\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)\,\delta\bar\psi(w)}\,\omega(w) + ie\,\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)}\,\omega(x)$$

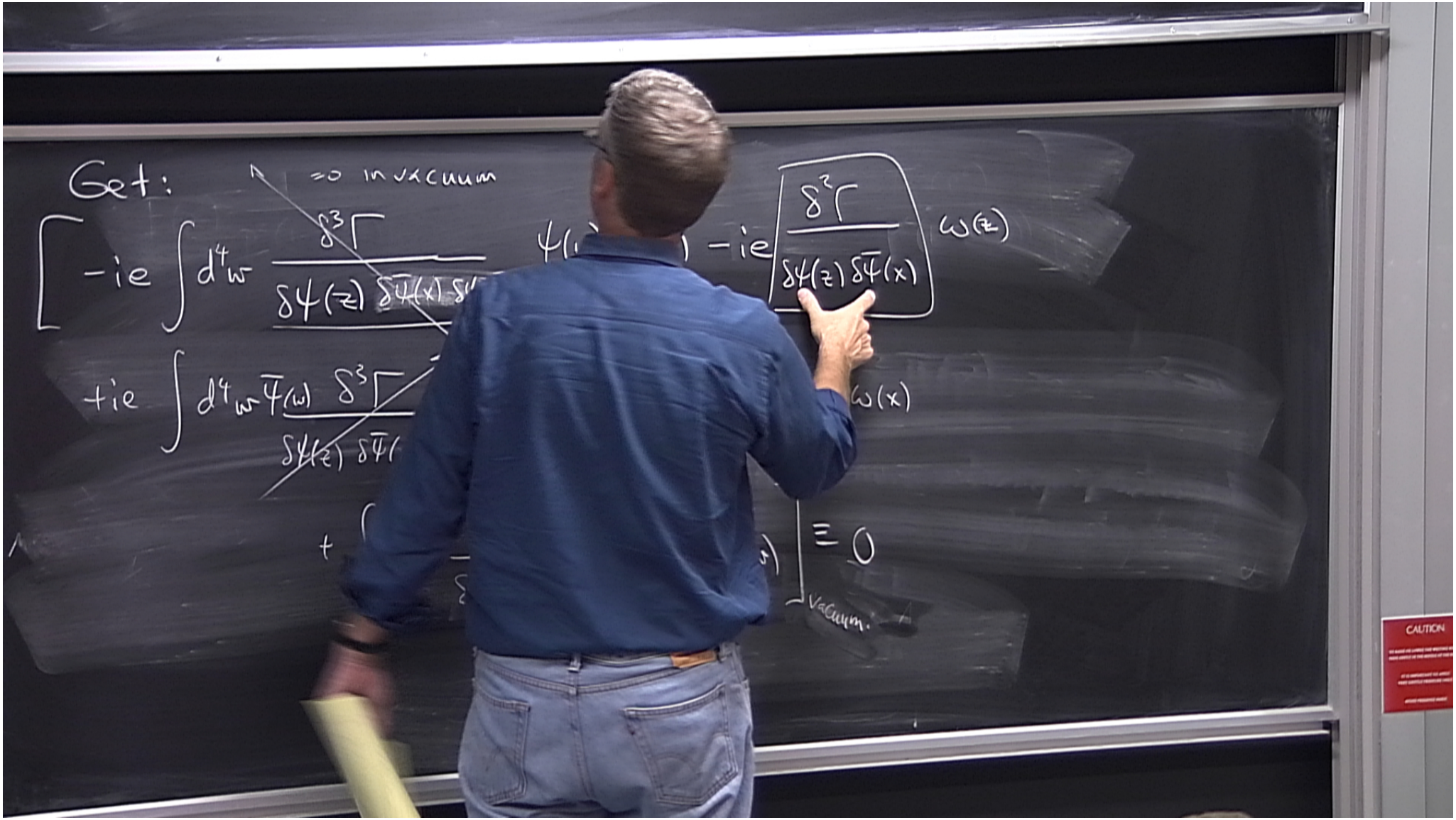
$$+\int d^4w\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar\psi(x)\,\delta A_\mu(w)}\,\partial_\mu\omega(w) \equiv 0$$

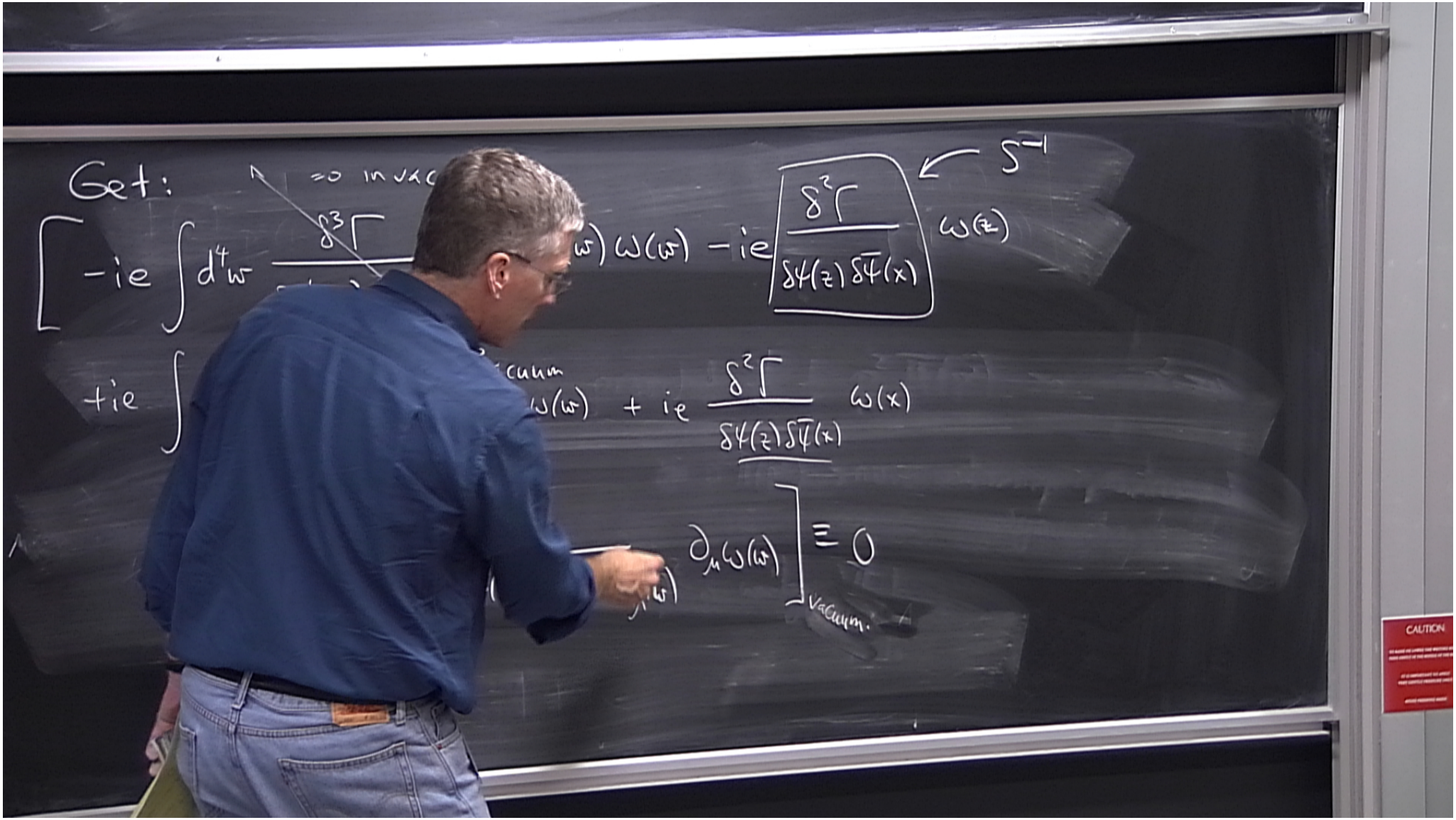

$\frac{\delta}{\delta\psi(x)}\frac{\delta}{\delta\bar{\psi}(x)}\int d^4w\left[\frac{\delta\Gamma}{\delta\psi(w)}(-ie\,\psi(w)\omega(w)) + ie\,\bar{\psi}(w)\omega(w)\frac{\delta\Gamma}{\delta\bar{\psi}(w)}\right.$
$\left.\frac{\delta\Gamma}{\delta A_\mu(w)}\partial_\mu\omega(w)\right] \equiv 0$
$\frac{\delta^2 S}{\delta A_\mu(y)\delta\Lambda}$

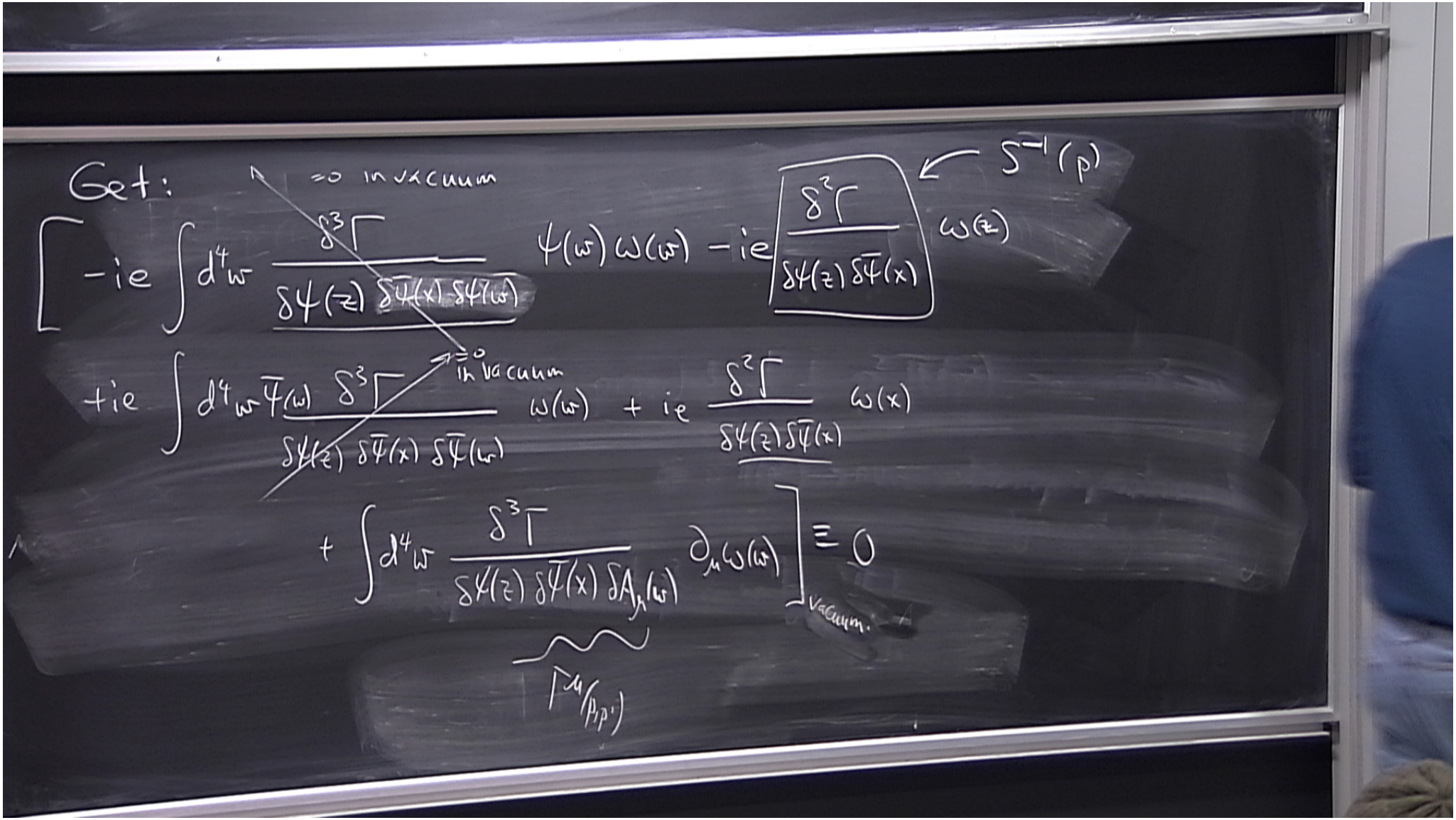

$\frac{\delta}{\delta\psi(x)}\frac{\delta}{\delta\bar{\psi}(x)}\int d^4w\left[\frac{\delta\Gamma}{\delta\psi(w)}(-ie\,\psi(w)\omega(w)) + \bar{\psi}(w)\omega(w)\frac{\delta\Gamma}{\delta\bar{\psi}(w)}\right] \equiv 0$
$\frac{\delta^2 S}{\delta A_\mu(x)\delta A_\nu(y)} = (\Delta^{-1})^{\mu\nu}(x,y)$

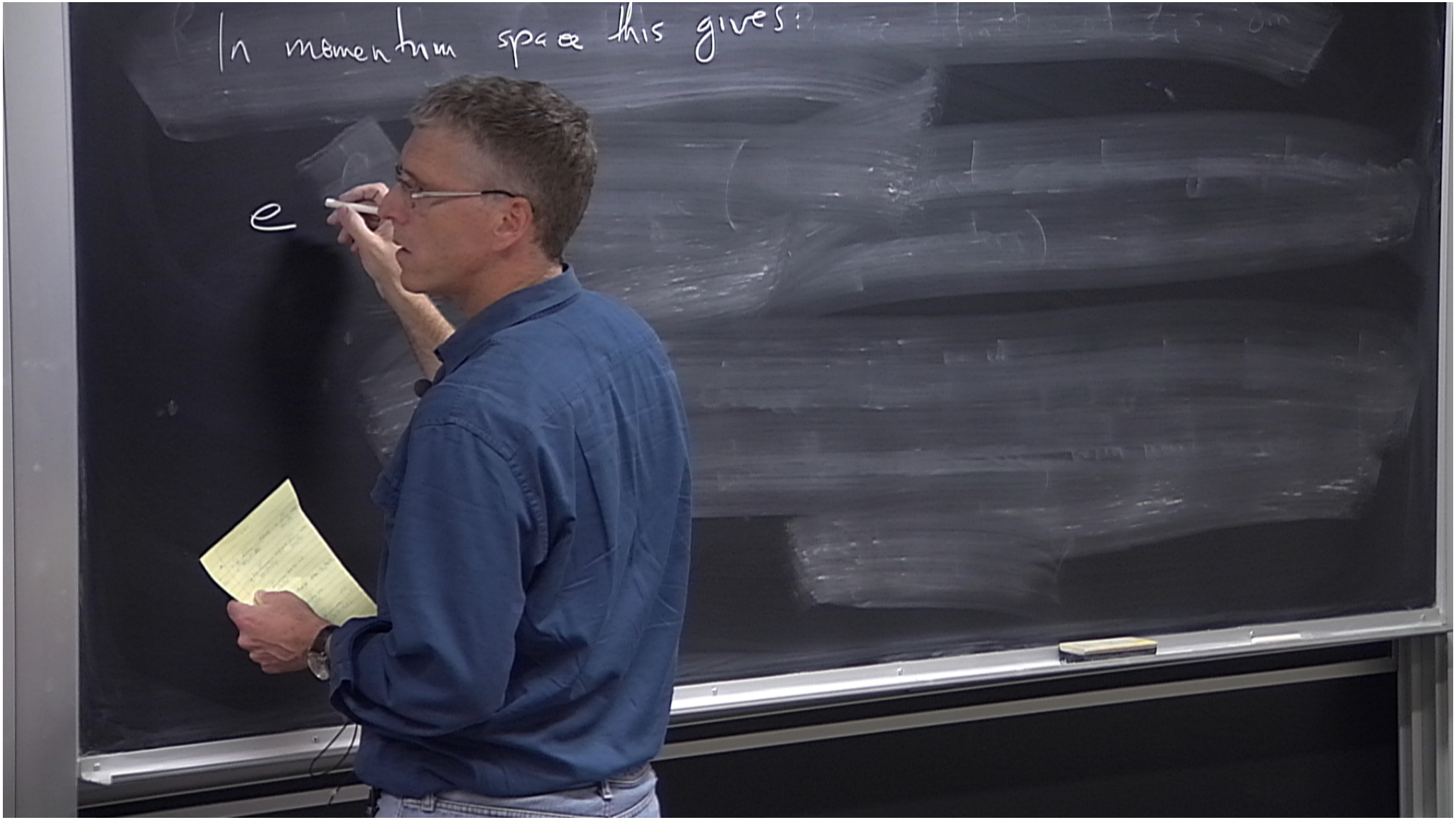

$\frac{\delta}{\delta\psi(x)}\frac{\delta}{\delta\bar{\psi}(x)}\int\left(-ie\,\psi(\omega)\omega(\omega)\right)+ie\,\bar{\psi}(\omega)\omega(\omega)\frac{\delta\Gamma}{\delta\bar{\psi}(\omega)}$
$+\frac{\delta\Gamma}{\delta A_\mu(\omega)}\partial_\mu\omega(\omega)\Big]\equiv 0$
$=\left(\Delta^{-1}\right)^{\mu\nu}(x,y)$
$A=\psi=0$

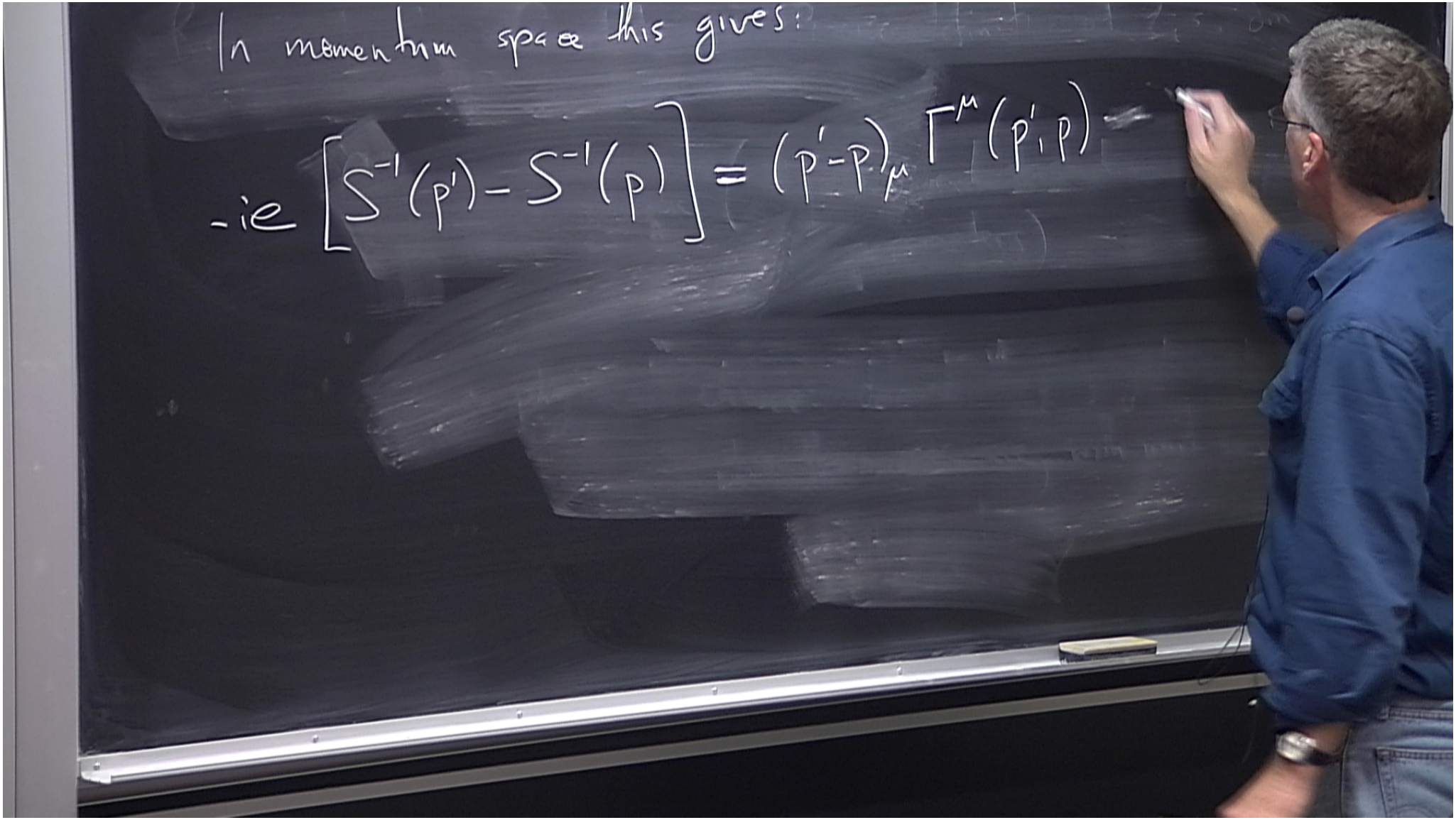

Get
$\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)\,\delta\psi(w)}\,\psi(w)\,\omega(w) - ie\,\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(z)$
$\frac{\Gamma}{\bar{\psi}(x)\,\delta\bar{\psi}(w)}\,\omega(w) + ie\,\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(x)$
$\int d^4w\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)\,\delta A_\mu(w)}\,\partial_\mu\omega(w)\Big]_{\text{vacuum}} \equiv 0$
CAUTION

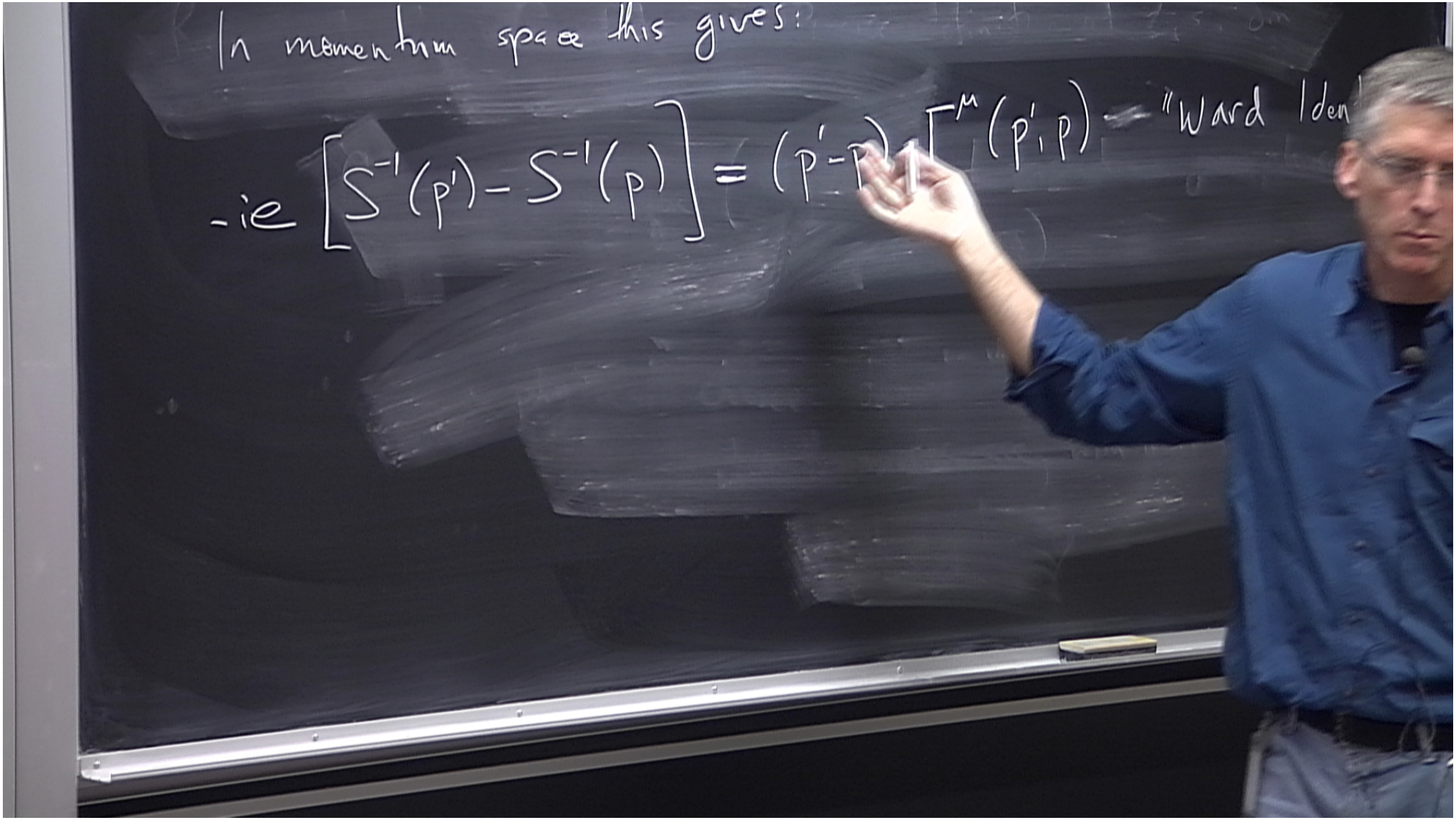

Get:
=0 in vacuum
$\left[-ie\int d^4w \frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)\,\delta}\right.$
$-ie\frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\omega(z)$
$+ie\int d^4w\,\bar{\psi}(w)\,\frac{\delta^3\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(}$
$\omega(x)$
$\left.\right]=0$
vacuum.
CAUTION


Get:
=0 in vac
$S^{-1}$
$-ie \int d^4w \frac{\delta^3\Gamma}{}$
$w)\,\omega(w) - ie \frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(z)$
$+ie \int$
cuum
$\omega(w) + ie \frac{\delta^2\Gamma}{\delta\psi(z)\,\delta\bar{\psi}(x)}\,\omega(x)$
$\partial_\mu \omega(w)\Big] \equiv 0$
vacuum.
CAUTION


Get:
=0 in vacuum
$\left[-ie\int d^4w \frac{\delta^3\Gamma}{\delta\psi(z)\delta\bar{\psi}(x)\delta\psi(w)}\psi(w)\omega(w) - ie\frac{\delta^2\Gamma}{\delta\psi(z)\delta\bar{\psi}(x)}\omega(z)\right.$
$S^{-1}(p)$
$+ie\int d^4w\,\bar{\psi}(w)\frac{\delta^3\Gamma}{\delta\psi(z)\delta\bar{\psi}(x)\delta\bar{\psi}(w)}\omega(w) + ie\frac{\delta^2\Gamma}{\delta\psi(z)\delta\bar{\psi}(x)}\omega(x)$
$=0$ in vacuum
$\left.+\int d^4w \frac{\delta^3\Gamma}{\delta\psi(z)\delta\bar{\psi}(x)\delta A_\mu(w)}\partial_\mu\omega(w)\right]_{vacuum} \equiv 0$
$\Gamma^\mu(p,p')$


In momentum space this gives:


In momentum space this gives:
$-ie\left[S^{-1}(p') - S^{-1}(p)\right] = (p'-p)_\mu \Gamma^\mu(p',p)$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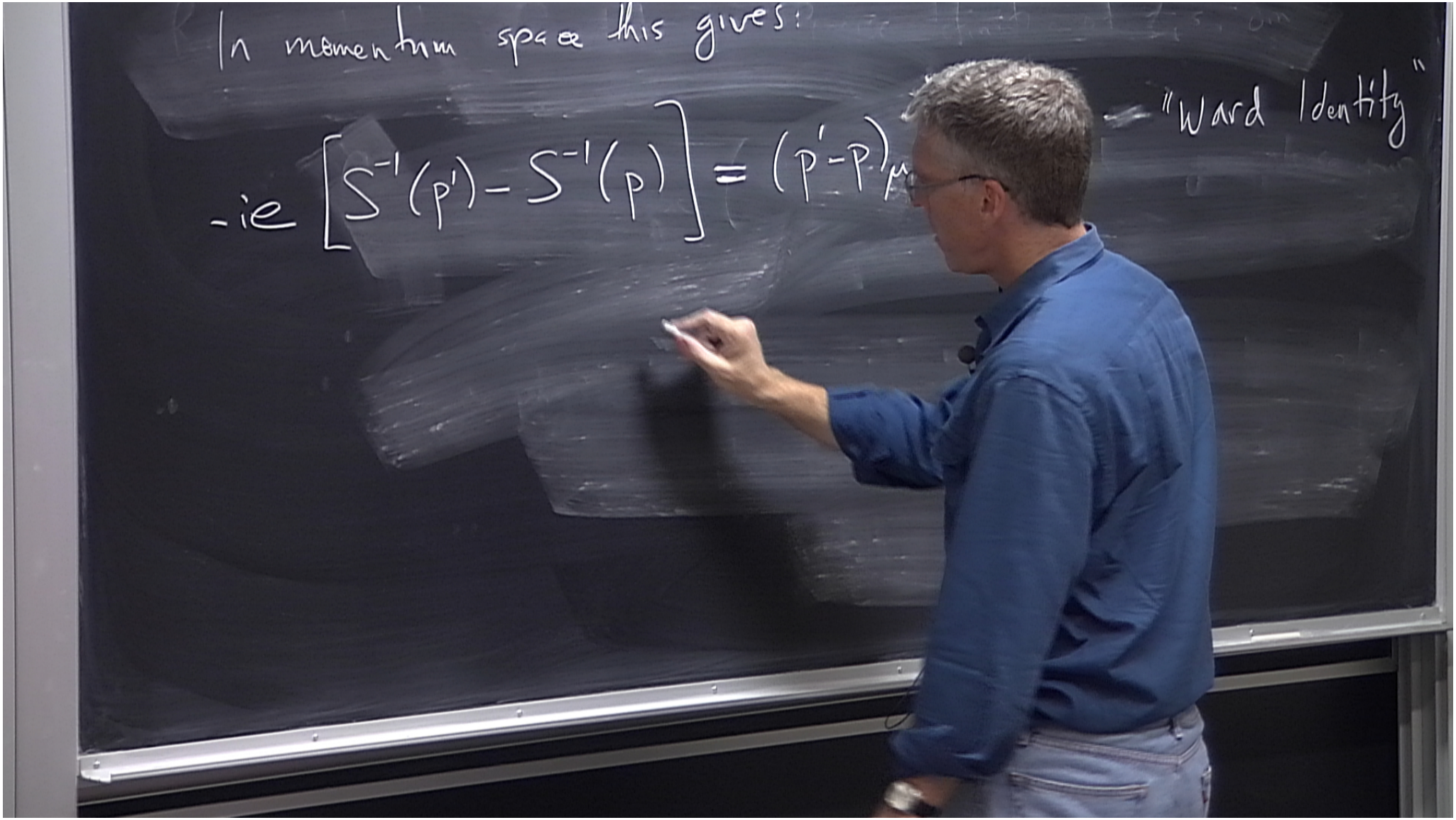
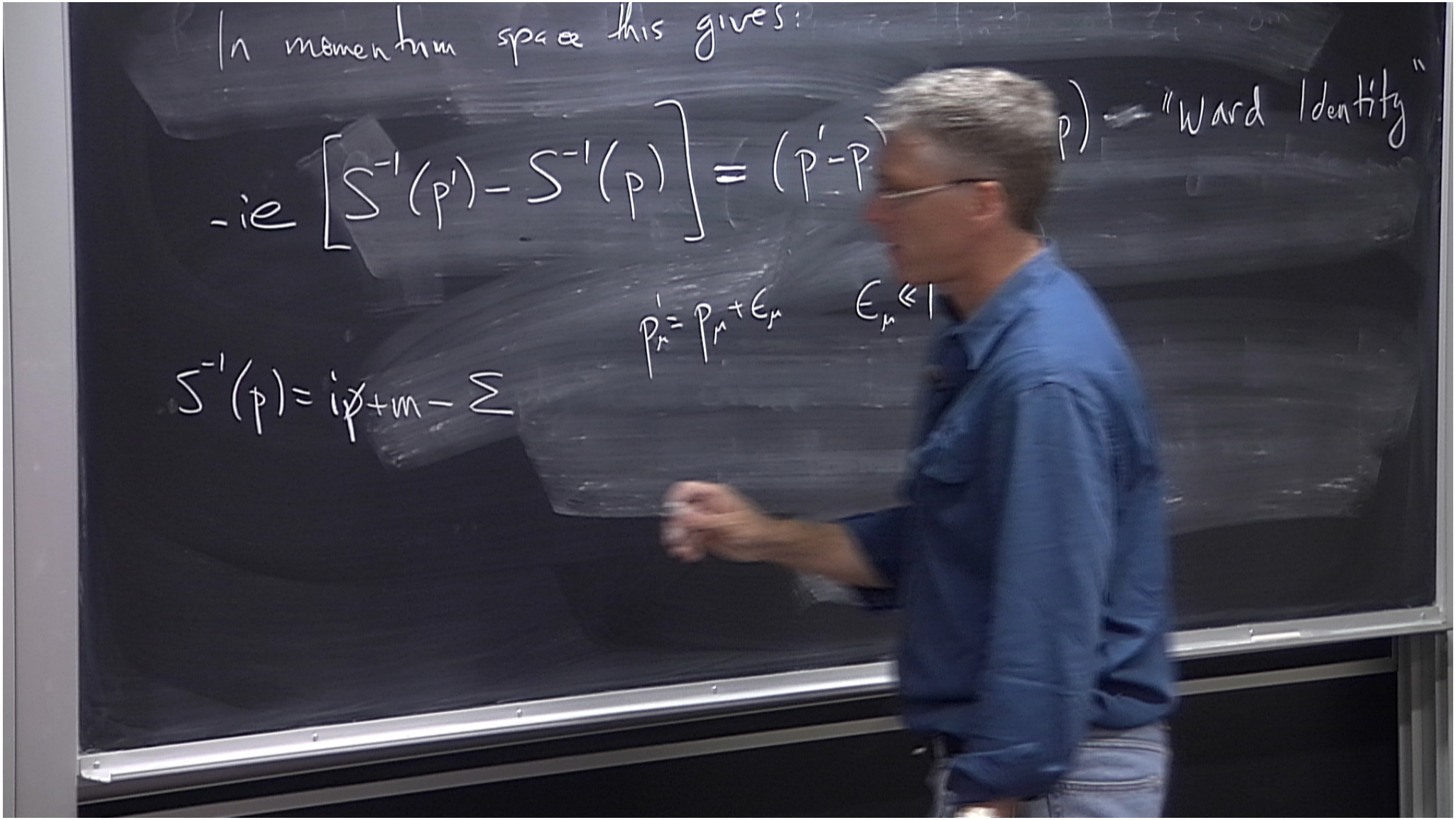
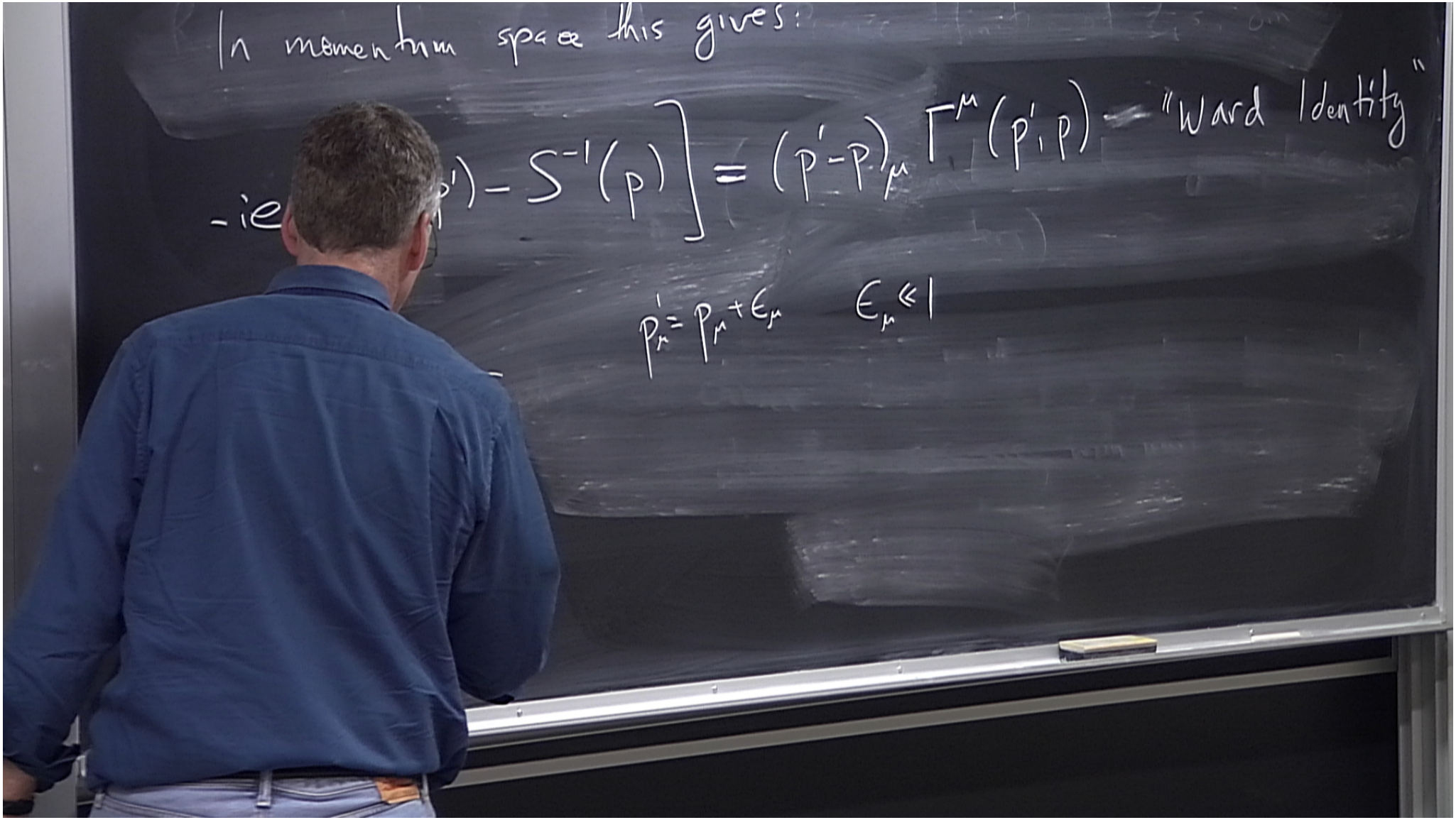
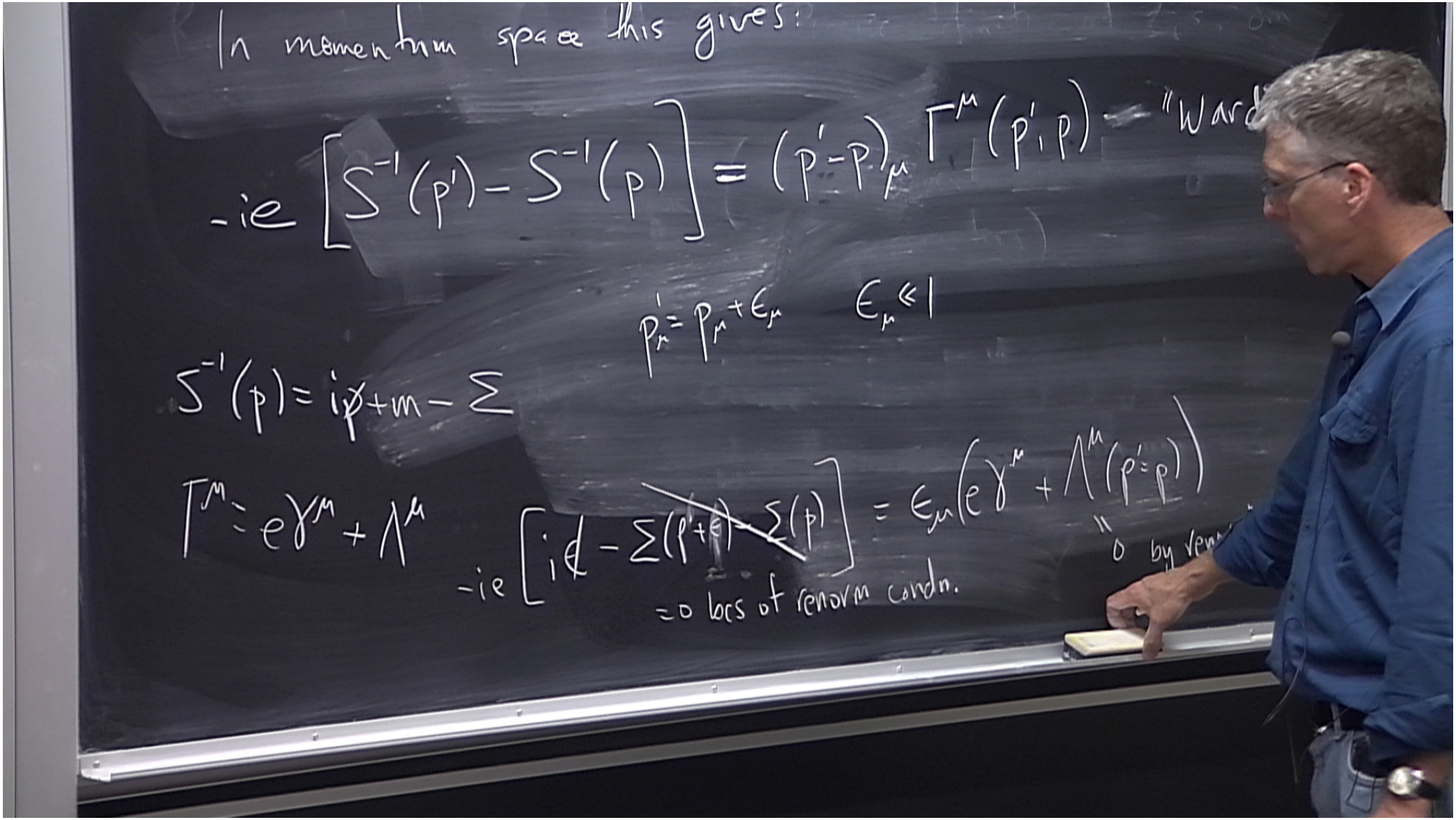
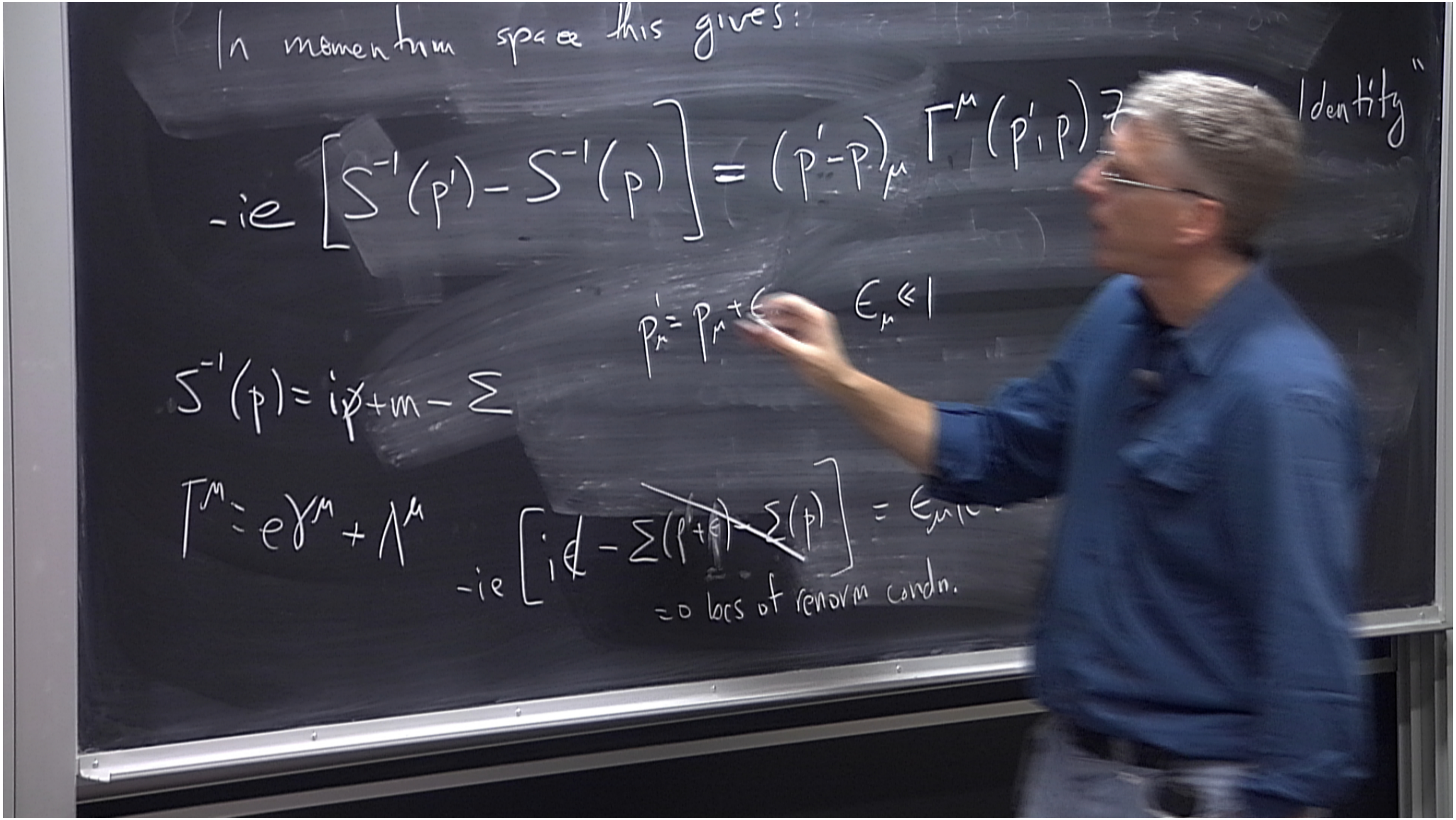
"Ward Iden


In momentum space this gives:
$-ie\left[S^{-1}(p') - S^{-1}(p)\right] = (p'-p)_\mu$
"Ward Identity"


In momentum space this gives:
$-ie\left[S^{-1}(p')-S^{-1}(p)\right]=(p'-p$
$p)$
"Ward Identity"
$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll 1$
$S^{-1}(p) = i\not{p} + m - \Sigma$


In momentum space this gives:
$-ie\left[\Gamma^{\mu}(p',p) - S^{-1}(p)\right] = (p'-p)_{\mu}\,\Gamma^{\mu}(p',p)$
"Ward Identity"
$p'_{\mu} = p_{\mu} + \epsilon_{\mu} \quad \epsilon_{\mu} \ll 1$


In momentum space this gives:
$-ie\left[S^{-1}(p')-S^{-1}(p)\right]=(p'-p)_\mu \Gamma^\mu(p',p)$
"Ward
$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll 1$
$S^{-1}(p)=i\not{p}+m-\Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ie\left[i\not{\epsilon}-\Sigma(p'+\epsilon)-\Sigma(p)\right]=\epsilon_\mu\left(e\gamma^\mu+\Lambda^\mu(p',p)\right)$
=0 bcs of renorm condn.
" 0 by


In momentum space this gives:
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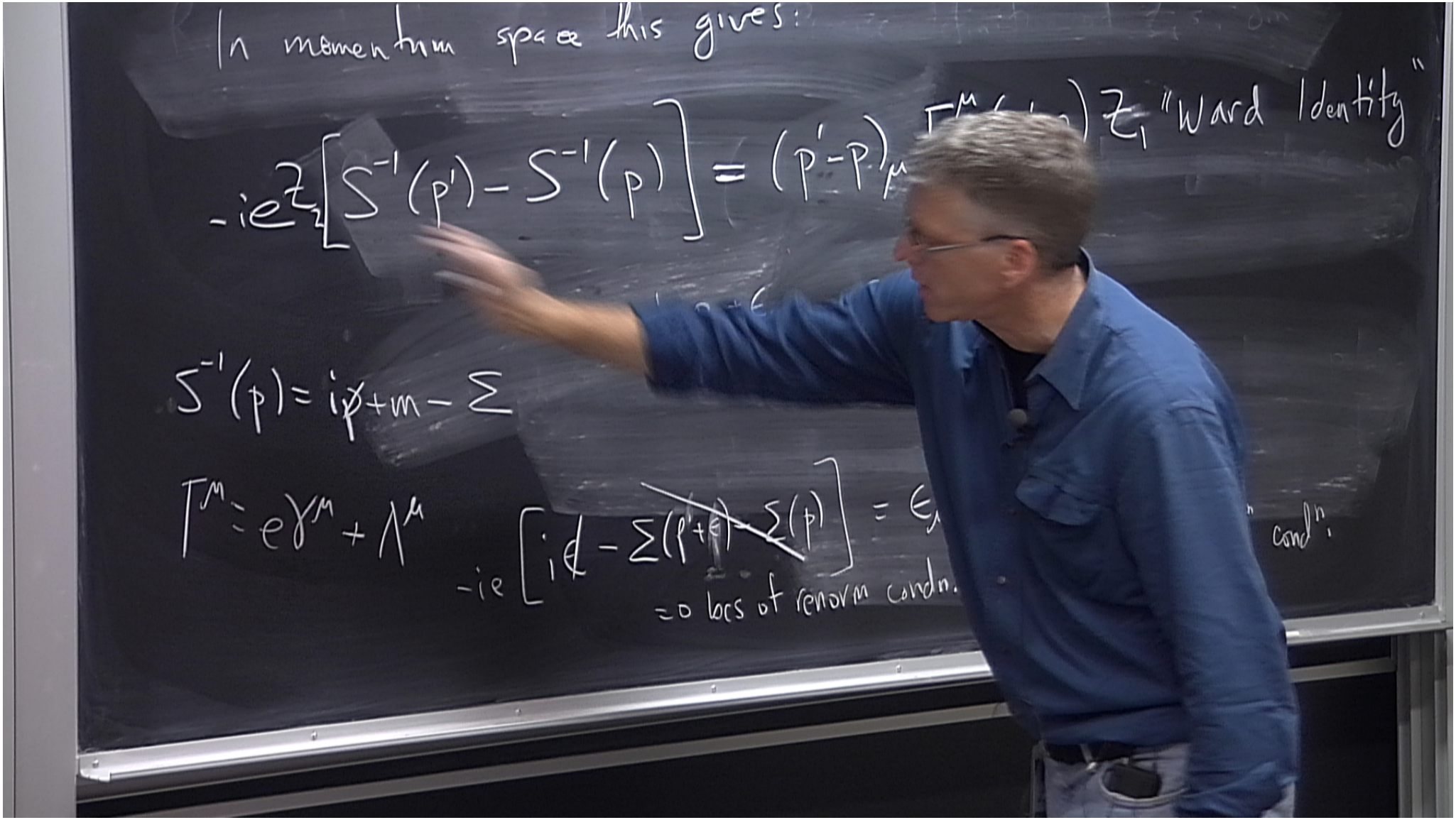
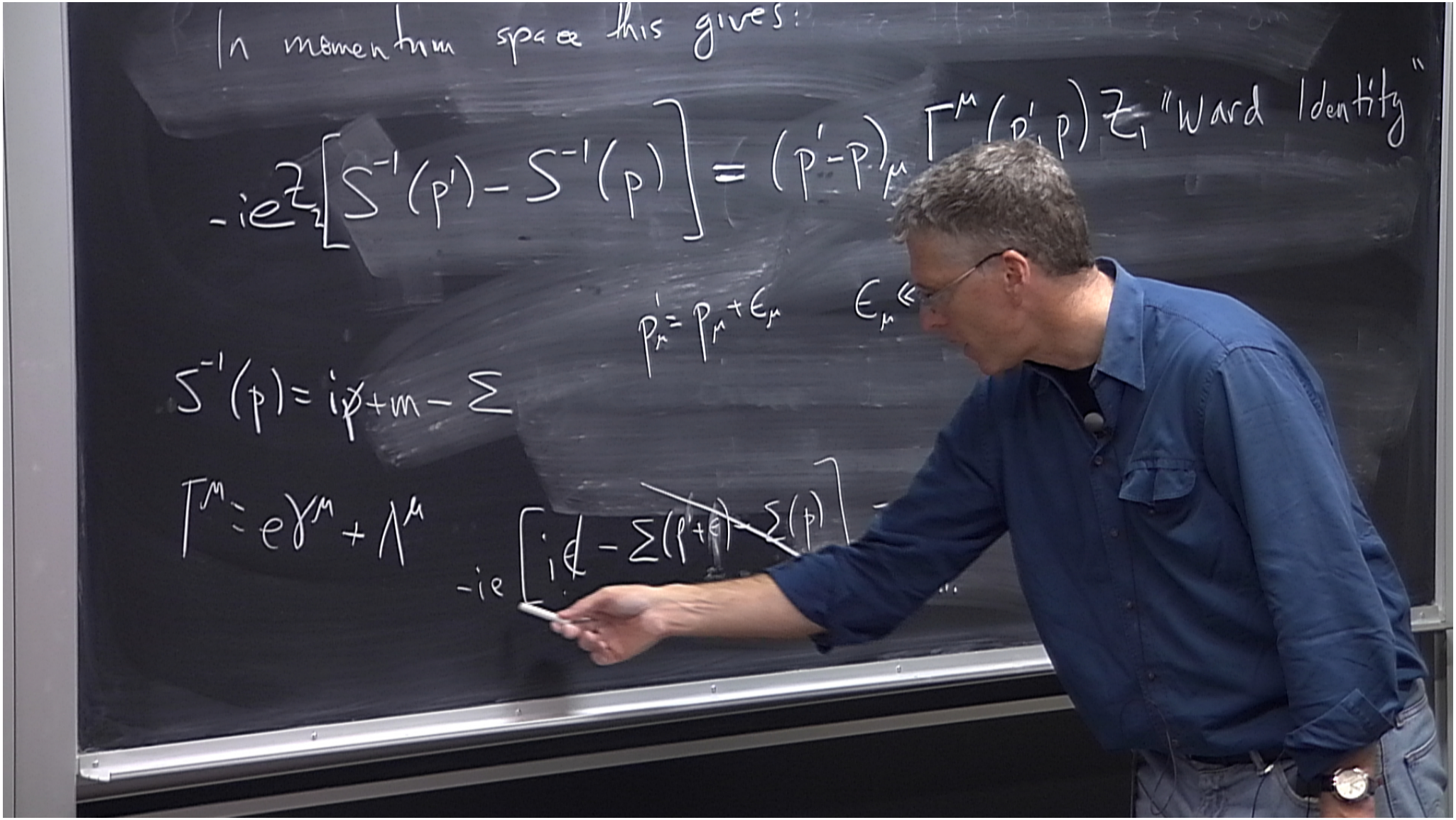
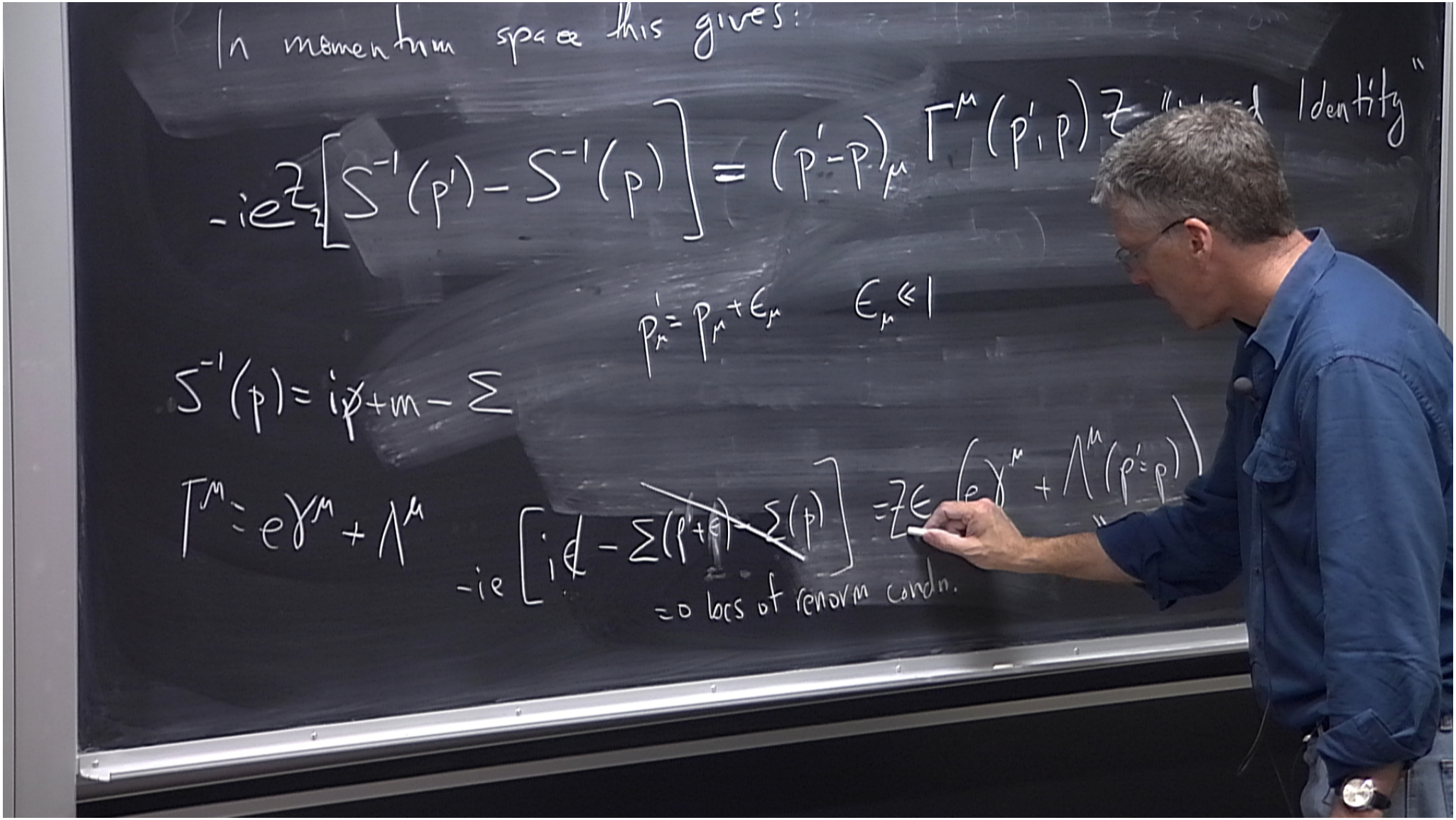
$-ie\left[i\not{\epsilon}-\Sigma(p'+\epsilon)-\Sigma(p)\right]=\epsilon_\mu(\ldots$
$\Rightarrow$ loss of renorm condn.


In momentum space this gives:
$-ie Z_2 \left[ S^{-1}(p') - S^{-1}(p) \right] = (p'-p)_\mu$
"Ward Identity"
$S^{-1}(p) = i\not{p} + m - \Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ie \left[ i\not{\epsilon} - \Sigma(p'+\epsilon) - \Sigma(p) \right] =$
=0 bcs of renorm condn.
cond$^n$.


In momentum space this gives:
$-ie Z_2 \left[ S^{-1}(p') - S^{-1}(p) \right] = (p'-p)_\mu \Gamma^\mu(p',p) Z_1$ "Ward Identity"
$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll$
$S^{-1}(p) = i\not{p} + m - \Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ie \left[ i\not{\epsilon} - \Sigma(p'+\epsilon) - \Sigma(p) \right]$


In momentum space this gives:
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$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll 1$
$S^{-1}(p) = i\not{p} + m - \Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ie \left[ i\not{\epsilon} - \Sigma(p'+\epsilon) - \Sigma(p) \right] = Z\epsilon \left( e\gamma^\mu + \Lambda^\mu(p',p) \right)$
$= 0$ bcs of renorm condn.

In momentum space this gives:

$$-ieZ_2\left[S^{-1}(p') - S^{-1}(p)\right] = (p'-p)_\mu \Gamma^\mu(p',p) Z_1 \quad \text{"Ward Identity"}$$

$$p'_\mu = p_\mu + \epsilon_\mu \qquad \epsilon_\mu \ll 1$$

$$S^{-1}(p) = i\not{p} + m - \Sigma$$

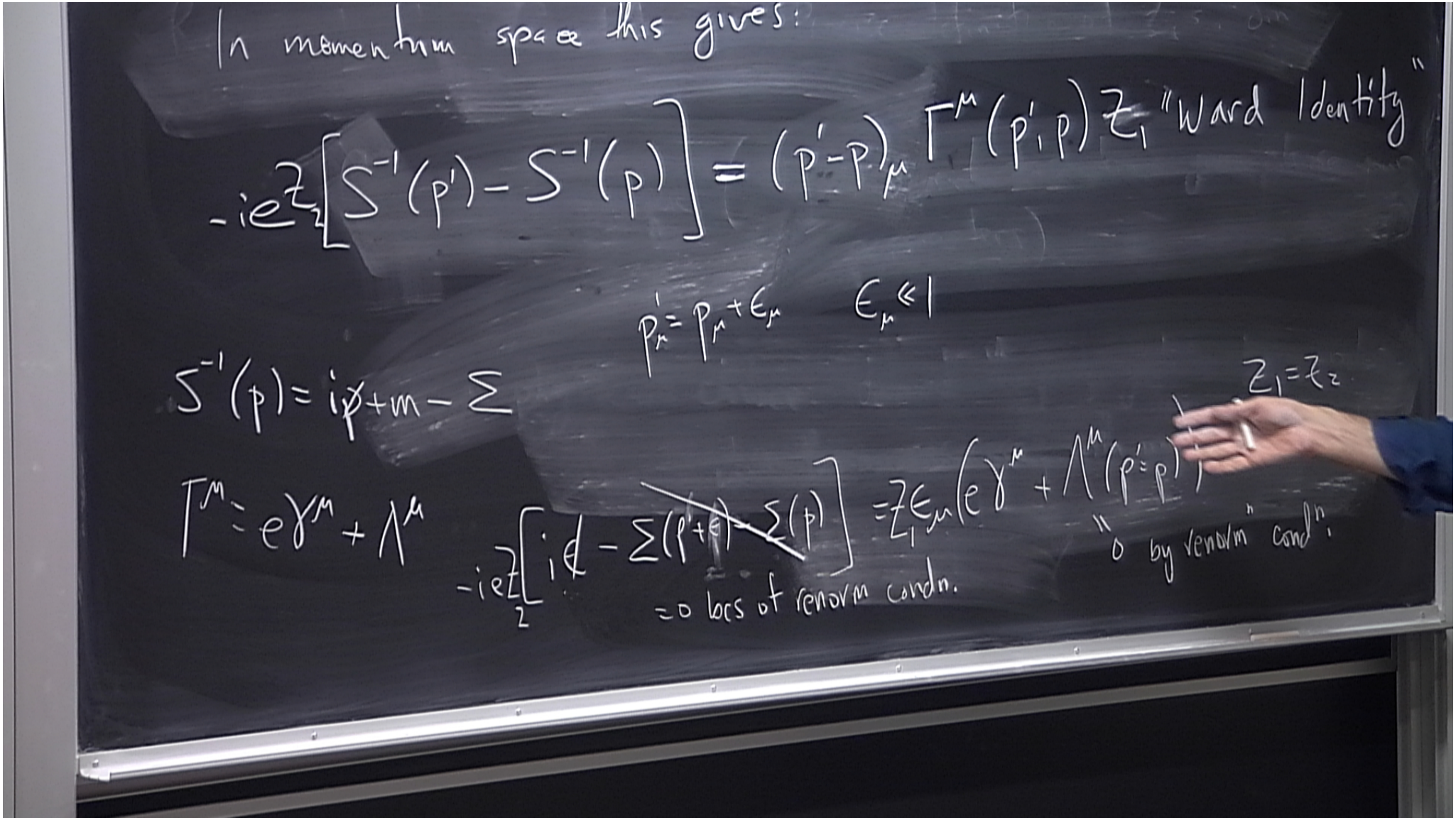
$$Z_1 = Z_2$$

$$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$$

$$-ieZ_2\left[i\not{\epsilon} - \cancel{\Sigma(p'+\epsilon) - \Sigma(p)}\right] = Z_1\epsilon_\mu\left(e\gamma^\mu + \Lambda^\mu(p'=p)\right)$$

$= 0$ bcs of renorm condn.

"0 by renorm" condn.


In momentum space this gives:
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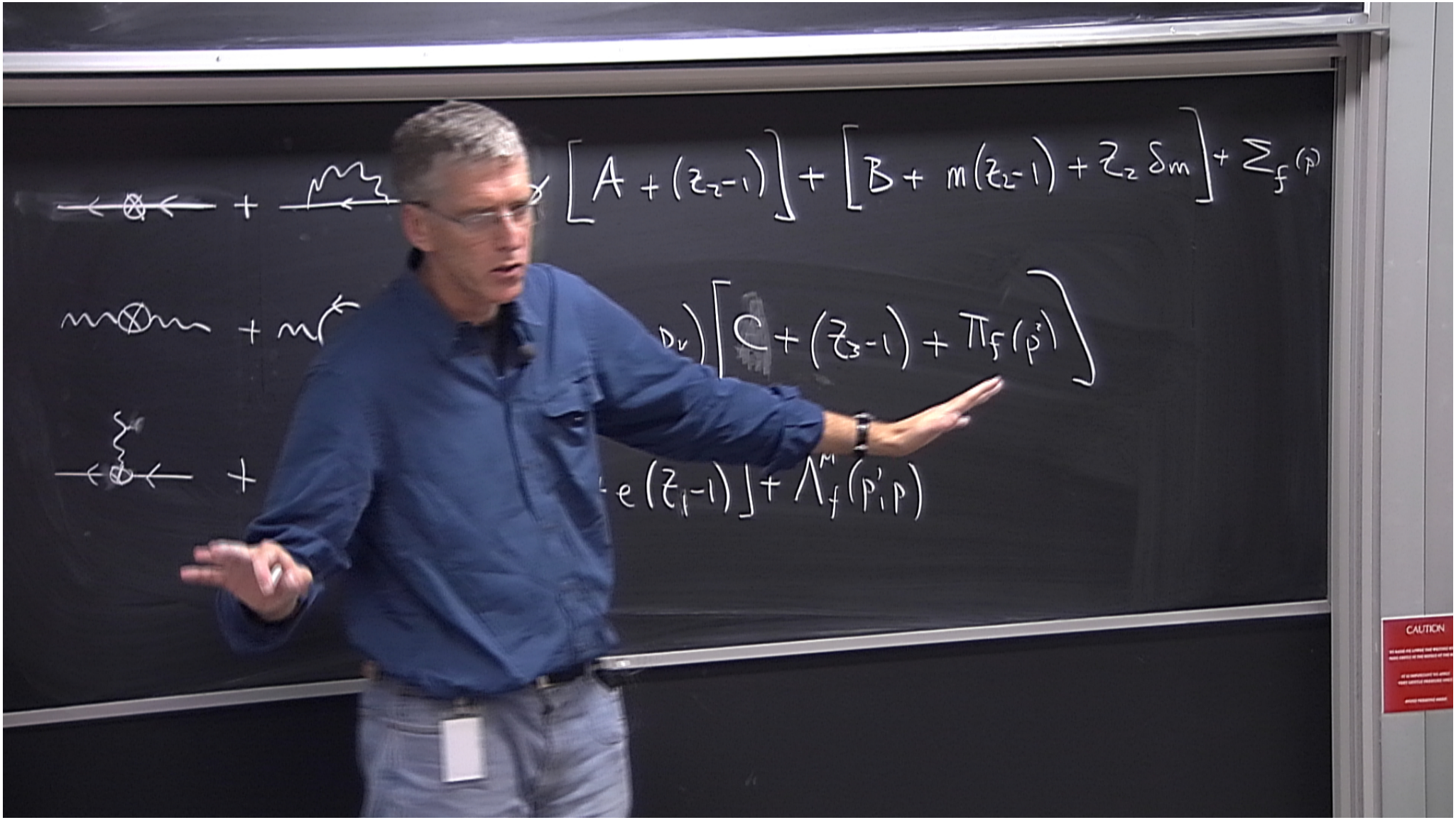
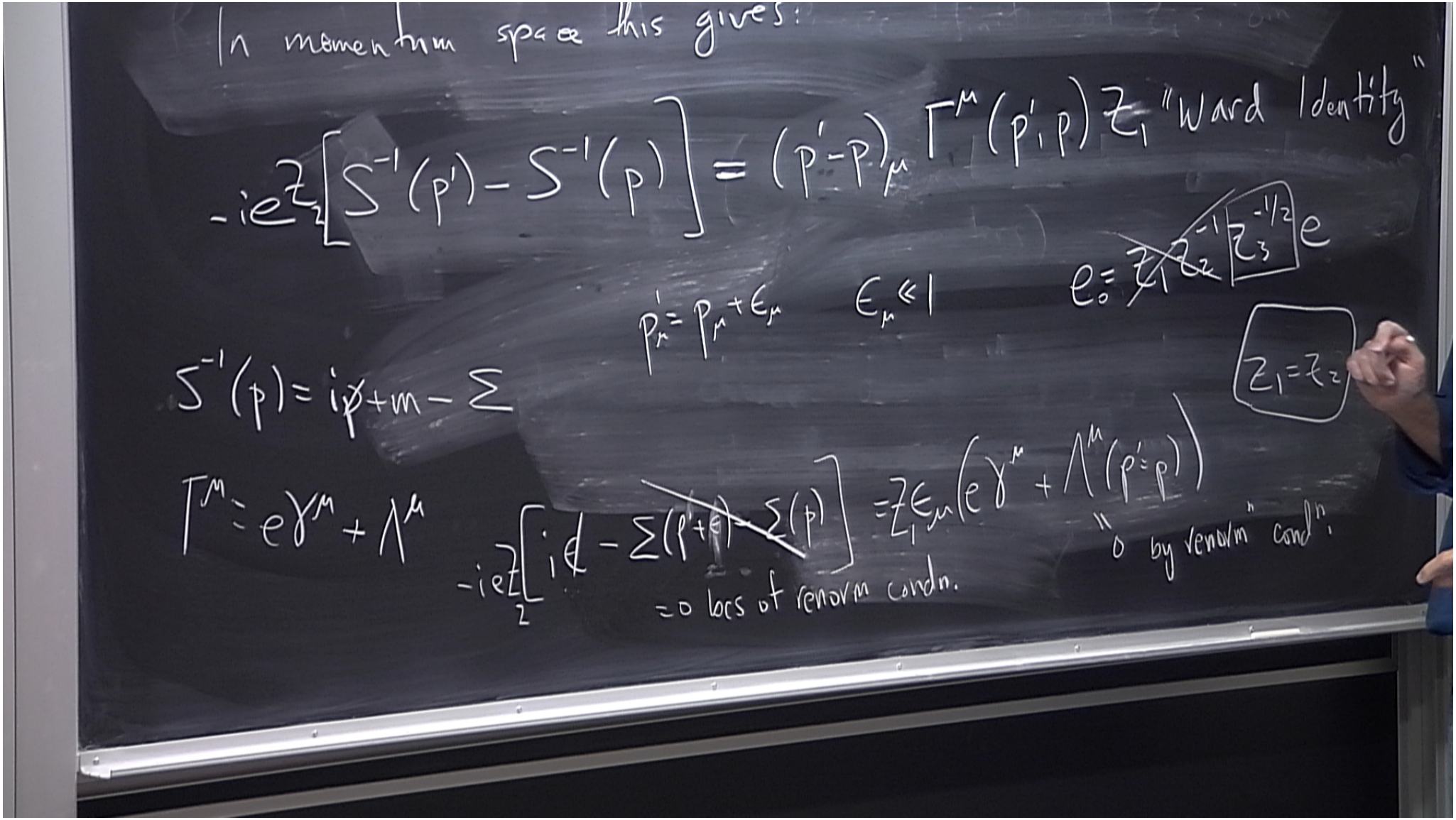
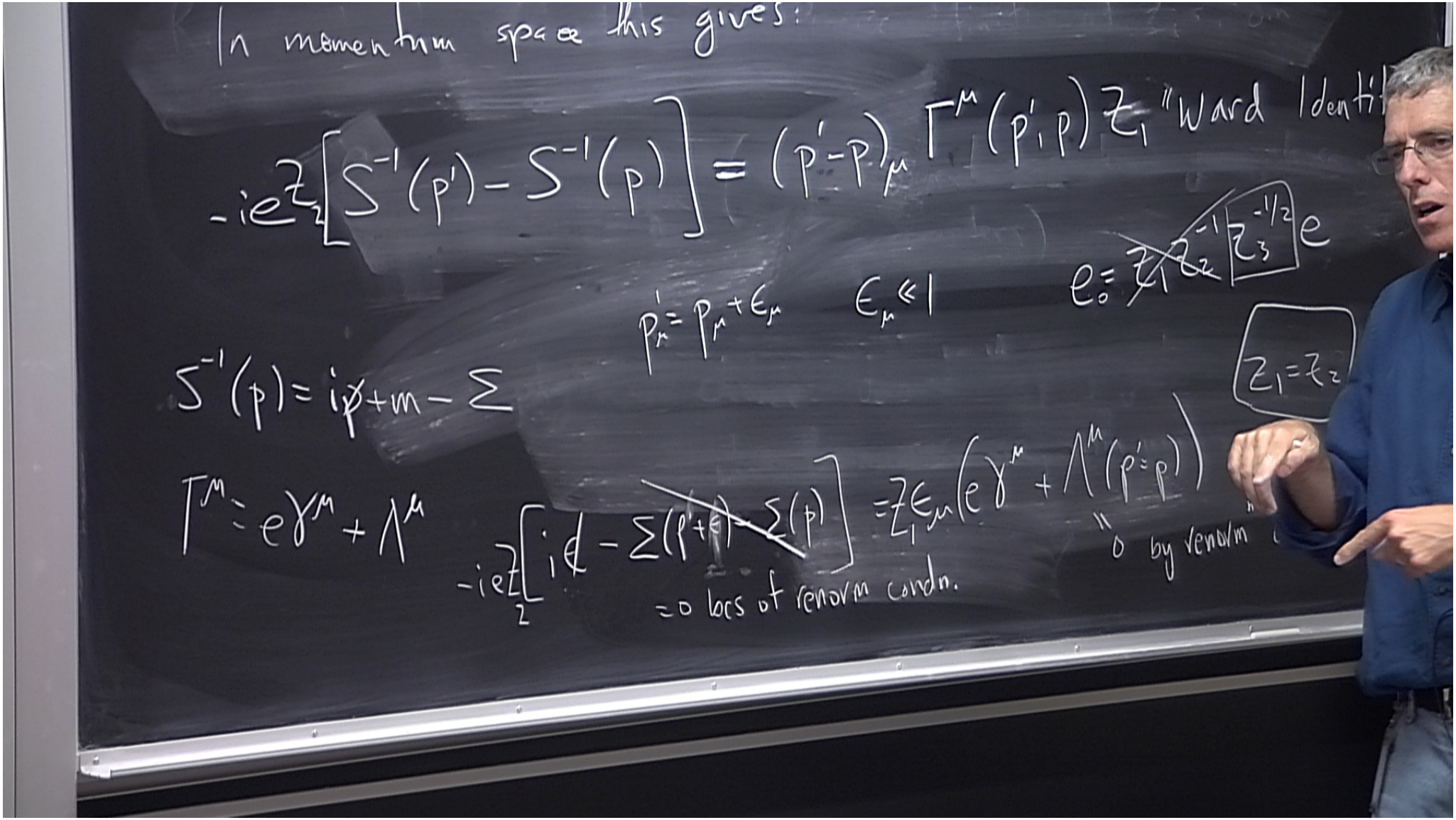
$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll 1$
$S^{-1}(p) = i\not{p} + m - \Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$Z_1 = Z_2$
$-ieZ_2\left[i\not{\epsilon} - \Sigma(p'+\epsilon) - \Sigma(p)\right] = Z_1 \epsilon_\mu \left(e\gamma^\mu + \Lambda^\mu(p',p)\right)$
$=0$ bcs of renorm condn.
$0$ by renorm" condn.


$\left[A + (Z_2 - 1)\right] + \left[B + m(Z_2 - 1) + Z_2 \delta_m\right] + \Sigma_f(p)$
$\left[C + (Z_3 - 1) + \Pi_f(p^2)\right]$
$-e(Z_1 - 1)] + \Lambda^{\mu}_f(p', p)$
CAUTION


In momentum space this gives:
$-ieZ_2\left[S^{-1}(p') - S^{-1}(p)\right] = (p'-p)_\mu \Gamma^\mu(p',p) Z_1$ "Ward Identity"
$p'_\mu = p_\mu + \epsilon_\mu \quad \epsilon_\mu \ll 1$
$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$
$Z_1 = Z_2$
$S^{-1}(p) = i\not{p} + m - \Sigma$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ieZ_2\left[i\not{\epsilon} - \Sigma(p'+\epsilon) - \Sigma(p)\right] = Z_1 \epsilon_\mu \left(e\gamma^\mu + \Lambda^\mu(p',p)\right)$
$= 0$ bcs of renorm condn.
$= 0$ by renorm condn.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$Z_1 = Z_2$
$\Gamma^\mu = e\gamma^\mu + \Lambda^\mu$
$-ie Z_2 \left[ i\not{\epsilon} - \Sigma(p+\epsilon) - \Sigma(p) \right] = Z_1 \epsilon_\mu \left( e\gamma^\mu + \Lambda^\mu(p',p) \right)$
$= 0$ by renorm
$= 0$ bcs of renorm condn.