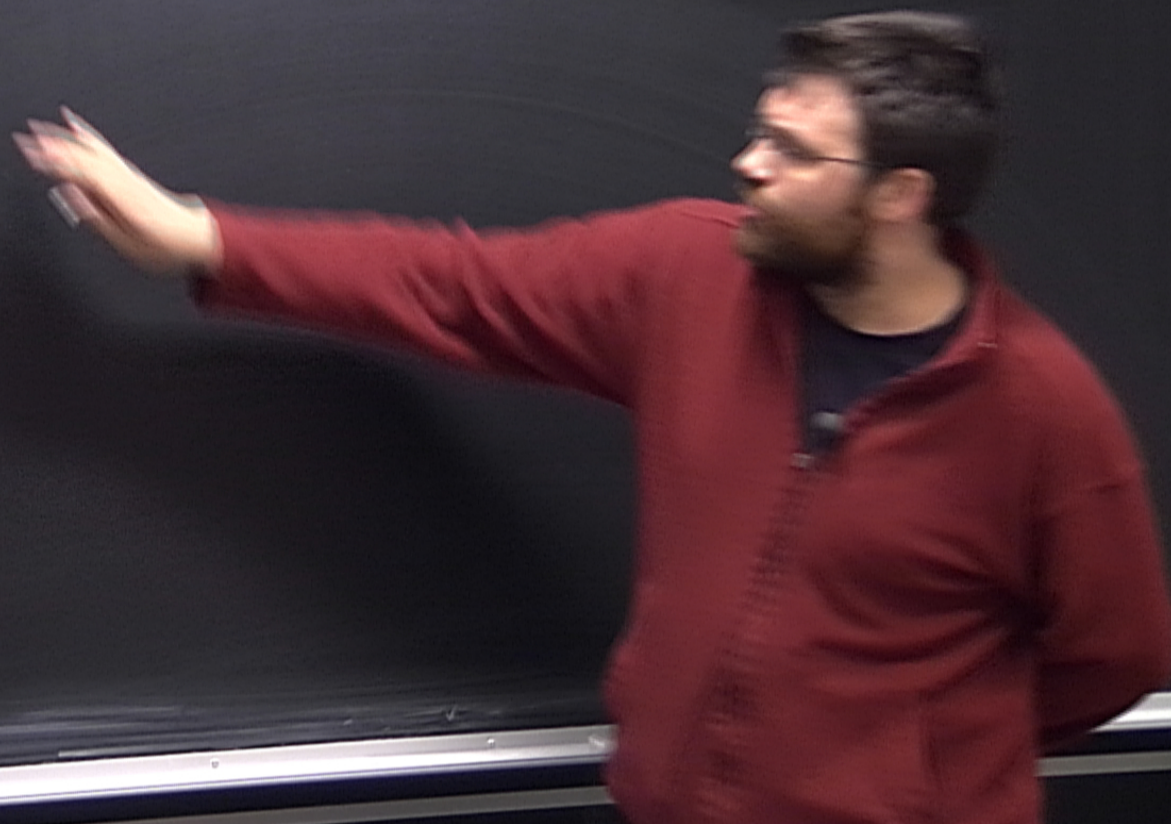


Title: Advanced Quantum Field Theory - Lecture 3

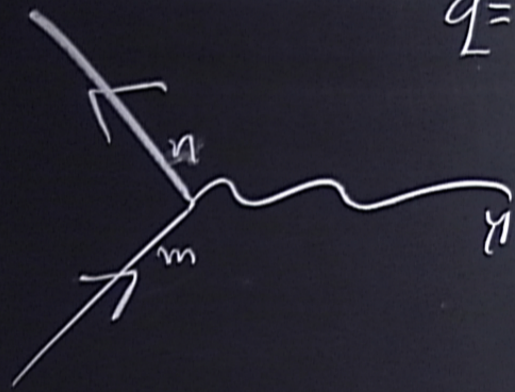
Date: Jan 17, 2012 09:30 AM

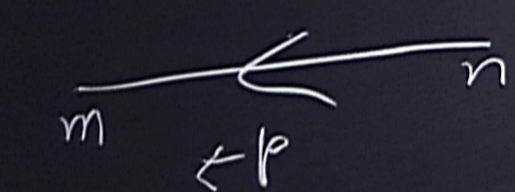
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Abstract:

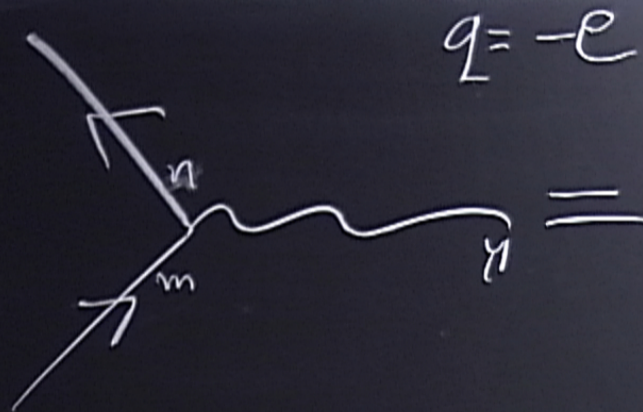


$q = -e$



$$\gamma = -g(\gamma^\mu)_{mn} (2\pi)^4 \delta^4(\epsilon p)$$


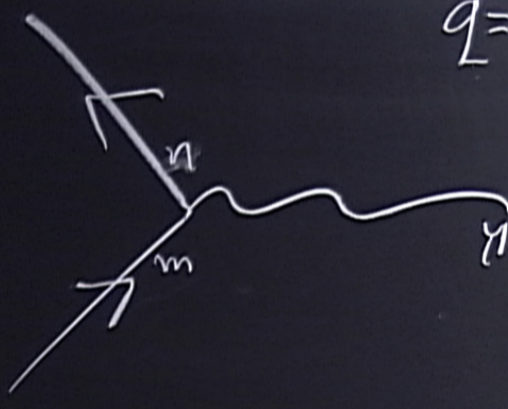
$$n = \frac{-i}{(2\pi)^4}$$

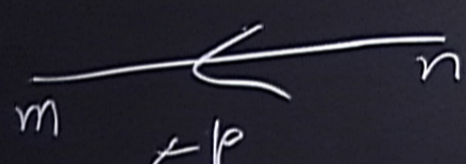


$$\gamma = -g(\gamma^\mu)_{mn} (2\pi)^4 \delta^4(\epsilon p)$$

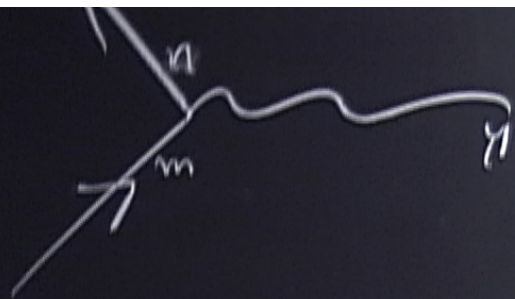
$$n = \frac{-i}{(2\pi)^4} \left[\frac{-i \cancel{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

$q = -e$

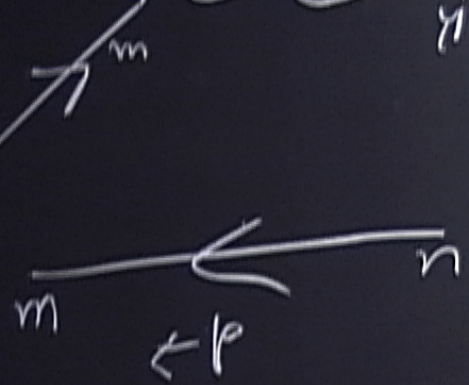


$$\gamma = -2(\gamma^\mu)_{mn} (2\pi)^4 \delta^4(\epsilon p)$$


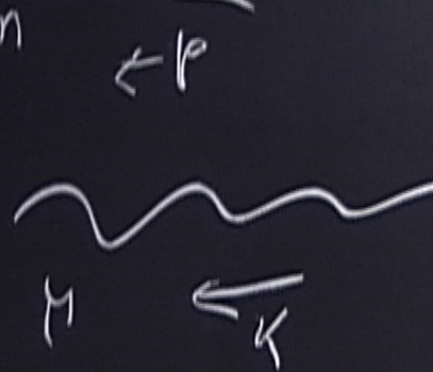
$$n = \frac{-i}{(2\pi)^4} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$



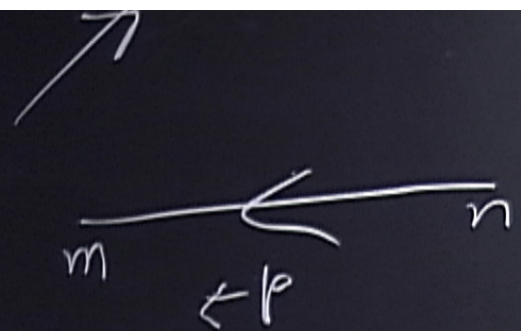
$$\gamma = -g(\gamma^\mu)_{mn} (2\pi)^4 \delta(\dots)$$



$$n = \frac{-i}{(2\pi)^4} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

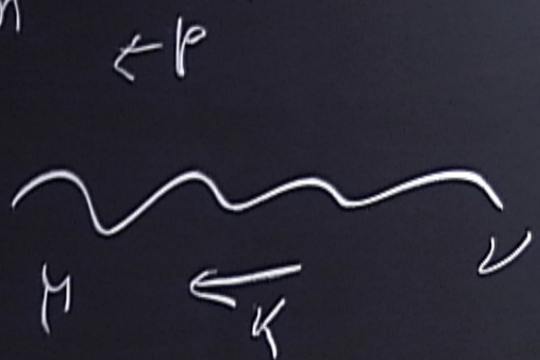


$$\nu = \frac{-i}{(2\pi)^4} \left[\frac{\eta_{\mu\nu} + \alpha_\mu K_\nu + \alpha_\nu K_\mu}{K^2 - i\epsilon} \right]$$



A horizontal line with an arrow pointing to the left. The left end is labeled 'm' and the right end is labeled 'n'. Below the line, between the two labels, is the expression $\not{p}$.

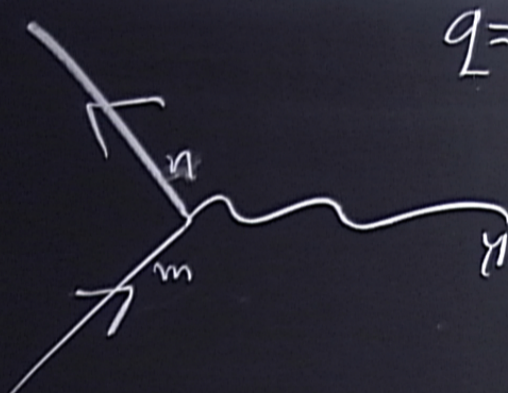
$$n = \frac{-i}{(2\pi)^4} \left[\frac{\not{p}}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

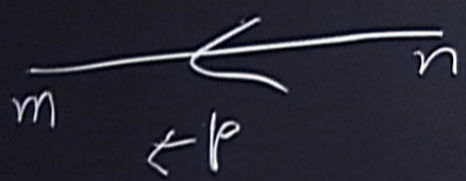


A horizontal wavy line. The left end is labeled 'H' and the right end is labeled 'V'. Below the line, between the two labels, is the expression $\not{k}$.

$$= \frac{-i}{(2\pi)^4} \left[\frac{\eta_{\mu\nu} + \alpha_\mu K_\nu + \alpha_\nu K_\mu}{k^2 - i\epsilon} \right]$$

$q = -e$


 $= -2(\gamma^\mu)_{mn} (2\pi)^4 \delta^4(\epsilon p)$


 $= \frac{-i}{(2\pi)^4} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$


 $= \frac{-i}{(2\pi)^4} \left[\frac{\eta_{\mu\nu} + \alpha_p K_\nu + \alpha_\nu K_\mu}{k^2 - i\epsilon} \right]$

$$V^{(r)}_{mn} (2\pi)^4 \delta^4(\varepsilon p)$$

$$\left[\frac{-i p + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

$$\frac{-i}{\pi/4} \left[\frac{\eta_{\mu\nu} + \alpha_\mu K_\nu + \alpha_\nu K_\mu}{k^2 - i\epsilon} \right]$$

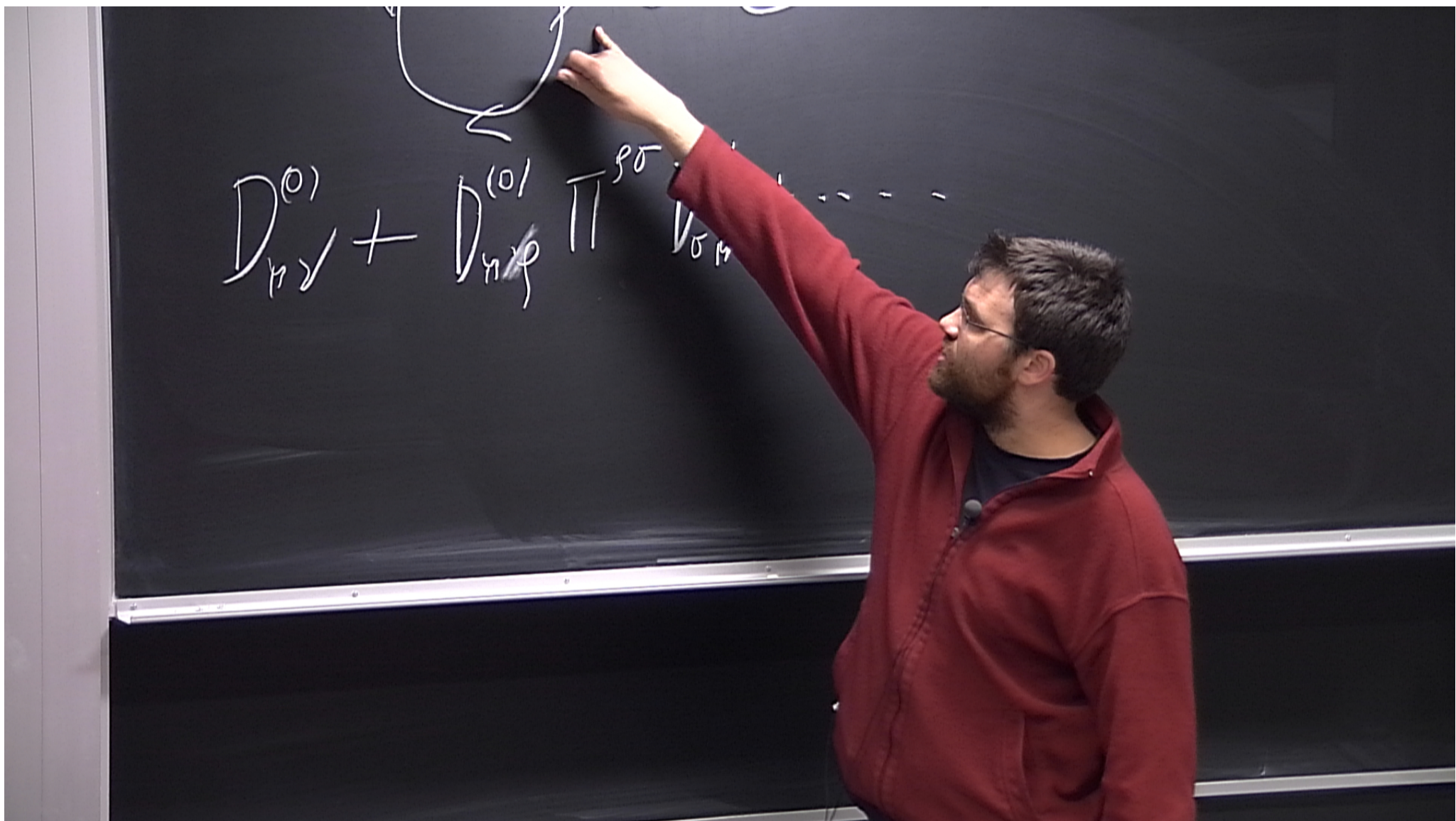
$$\alpha (\partial_\mu A^\mu)^2$$

$$i \gamma^{\mu} \delta^{\mu\nu} (2\pi)^4 \delta^4(\varepsilon p)$$

$$\left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

$$\frac{-i}{(2\pi)^4} \left[\frac{\eta_{\mu\nu} + \alpha_p K_{\mu} + \alpha_{\nu} K_{\mu}}{k^2 - i\epsilon} \right]$$

$$\frac{\alpha (\partial_{\mu} A^{\mu})^2}{(\alpha A^{\mu})^2} (?)$$



E = # external lines
 I = # internal lines
 L = # loops
 V = # vertices

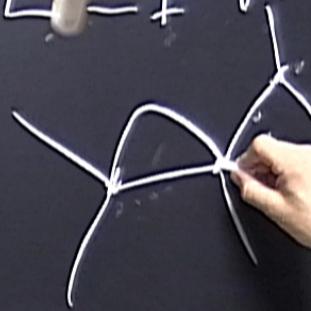
$$\begin{aligned}
 E &= \# \text{ external lines} \\
 I &= \# \text{ internal lines} \\
 L &= \# \text{ loops} \\
 V &= \# \text{ vertices}
 \end{aligned}
 \left[\frac{1}{(2\pi)^4} \right]^I \cdot (2\pi)^L \left[e \cdot (2\pi)^4 \right]^V \cdot (\text{some loop thing})$$

$$\int \frac{(2\pi^2)}{K^3} dK$$

$L = \# \text{ loops}$
 $V = \# \text{ vertices}$

$$I = \left(\frac{2\pi^2}{k} \right) k^3 dk$$

$$L = I - V + 1$$

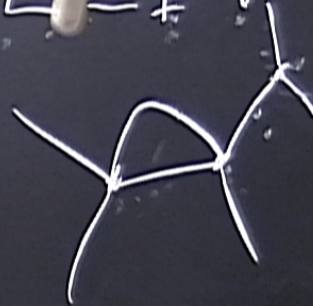


$$L = \# \text{ loops}$$

$$V = \# \text{ vertices}$$

$$I = (2\pi^2) \int K^3 dk$$

$$L = I - V + 1$$



$E = \# \text{ external lines}$
 $I = \# \text{ internal lines}$
 $L = \# \text{ loops}$
 $V = \# \text{ vertices}$

$$\left[\frac{1}{(2\pi)^4} \right]^I \cdot (2\pi)^3 \cdot \left[e \cdot (2\pi)^4 \right]^V \cdot (\text{some loop thing})$$

$$L = I - V + 1$$

$$\sum_i n_i V_i = 2I + E$$

$$= (2\pi^2) K^3 dK$$

$E = \# \text{ external lines}$
 $I = \# \text{ internal lines}$
 $L = \# \text{ loops}$
 $V = \# \text{ vertices}$

$$\left[\frac{1}{(2\pi)^4} \right]^I \cdot (2\pi)^2 \left[e \cdot (2\pi)^4 \right]^V \cdot (\text{some loop thing})$$

$$L = I - V + 1$$

$$I = E + 3L - 3$$

$$V = E + 2L - 2$$

$$= (2\pi^2) K^3 dK \quad (3V) = \sum_i n_i V_i = 2I + E$$

$$\text{graph } \mathcal{L} \left(\frac{1}{(2\pi)^4} \right) \quad E+3L-3 \quad \cdot (2\pi^2)^L [e \cdot (2\pi)^4]^L$$

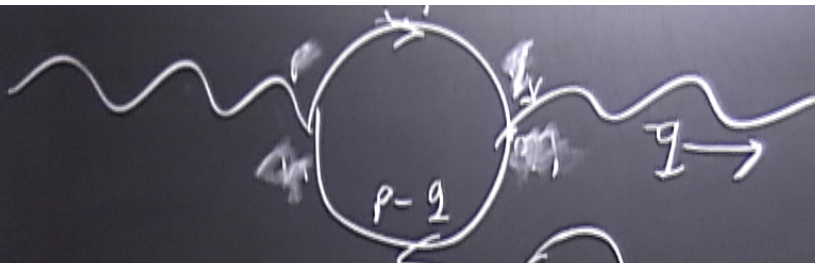
graph $\mathcal{L}$

$$\text{graph} \propto e^{E/(8\pi)} \cdot \left(\frac{e^2}{8\pi} \right)^L \cdot (\text{stuff})$$

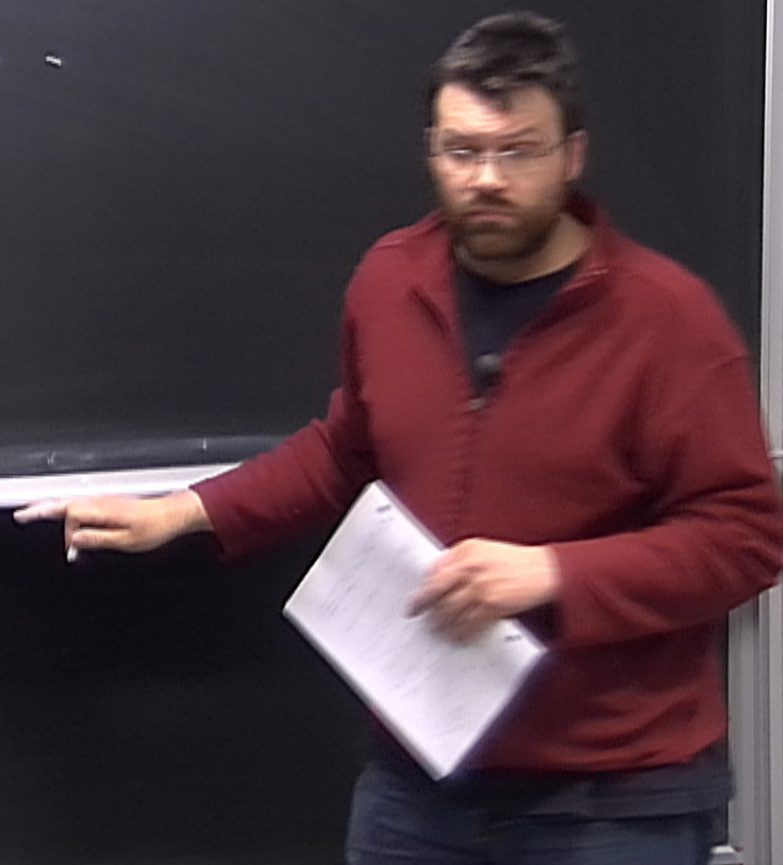
$\nwarrow$ $\sim 1.16 \cdot 10^{-3}$

$$i(2\pi)^4 \Pi_{\mu\nu} = -$$

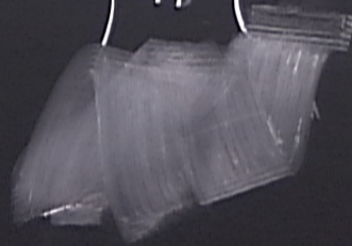
$$i \cdot (2\pi)^4 \Pi_{\mu\nu} = - \int \text{Tr} \left\{ \frac{-i}{(2\pi)^4} \not{p} \cdot e \gamma_\mu (2\pi)^4 \cdot \frac{-i}{(2\pi)^4} \not{p-2} \right\}$$



$$D_{\mu\nu}^{(0)} + D_{\mu\nu}^{(1)} \left(\Pi^{\mu\nu} \right) D_{\mu\nu}^{(0)} + \dots$$



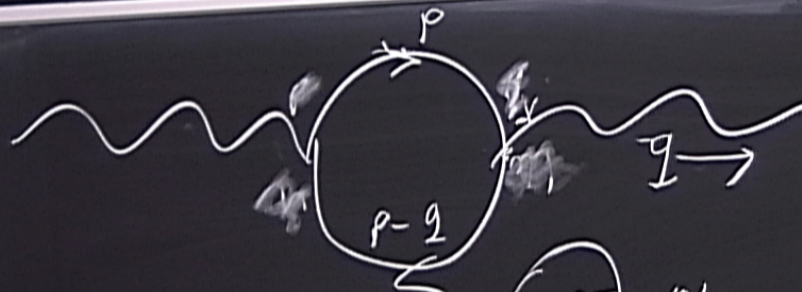
$$i \cdot (2\pi)^4 \Pi_{\mu\nu} = - \int d^4p \left\{ \frac{-i}{(2\pi)^4} \uparrow \delta(p) \cdot e\gamma_\mu(2\pi)^4 \cdot \frac{-i}{(2\pi)^4} \delta(p_2) \cdot e\gamma_\nu(2\pi)^4 \right\}$$



CAUTION
DO NOT TOUCH THE BOARD OR THE BOARD OR THE BOARD

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DO NOT TOUCH THE BOARD OR THE BOARD OR THE BOARD

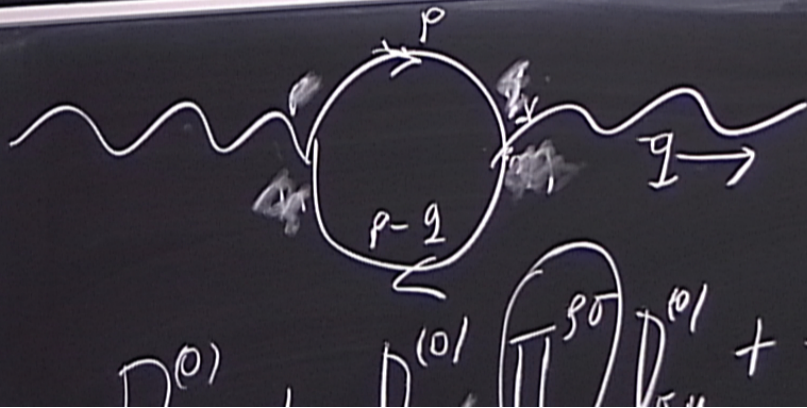
$$\Pi_{\mu\nu}(q) = \frac{-ie}{(2\pi)^4} \int d^4p \text{Tr} \left[\frac{\not{p} + m - i\epsilon}{p^2 + m^2 - i\epsilon} \gamma_\mu \frac{\not{p} - \not{q} + m - i\epsilon}{(p-q)^2 + m^2 - i\epsilon} \gamma_\nu \right]$$



$$D_{\mu\nu}^{(0)} + D_{\mu\nu}^{(1)} \left(\Pi^{(0)} \right) D_{\mu\nu}^{(1)} + \dots$$

$$i \cdot (2\pi)^4 \Pi_{\mu\nu} = - \int d^4p \text{Tr} \left\{ \frac{-i}{(2\pi)^4} \gamma_\mu \cdot e \gamma_\nu(p) \cdot \frac{-i}{(2\pi)^4} \gamma_\nu(p-q) \cdot e \gamma_\mu(p-q) \right\}$$

$$\Pi_{\mu\nu}(q) = \frac{-ie^2}{(2\pi)^4} \int d^4p \text{Tr} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \gamma_\mu \cdot \frac{-i \not{p-q} + m}{(p-q)^2 + m^2 - i\epsilon} \gamma_\nu \right]$$



Feynman Reparametrization

$$\int_0^1 \frac{dx}{[Ax + B(1-x)]^2} = \frac{1}{A-B} \int_B^A \frac{du}{u^2} = \frac{1}{AB}$$

$$\frac{1}{(p^2+m^2)(q^2+m^2)} = \int_0^1 \frac{dx}{[(p^2+m^2)(1-x) + (q^2+m^2)x]^2}$$

$$= \int \frac{d^4 p}{[(p-qx)^2 + q^2 x(1-x) + m^2]^2}$$

$$q = -e$$

$$\gamma = -2\gamma_{mn}^* (2\pi)^4 \delta^4(\epsilon p)$$

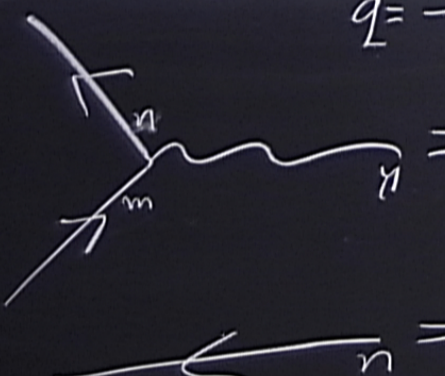
$$= \frac{-i}{(2\pi)^4} \left[\frac{-i p + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

$$\left[\right] = \int \frac{(\alpha(p) A^\mu)^2}{(\alpha(p) A^\mu)^2} (?)$$

$$\frac{1}{(p^2+m^2)((p-q)^2+m^2)} = \int_0^1 \frac{dx}{[(p^2+m^2)(1-x) + (p-q)^2+m^2)x]^2}$$

$$= \int d^4p \int_0^1 \frac{dx}{[(p-qx)^2 + q^2x(1-x) + m^2]^2}$$

$q = -e$



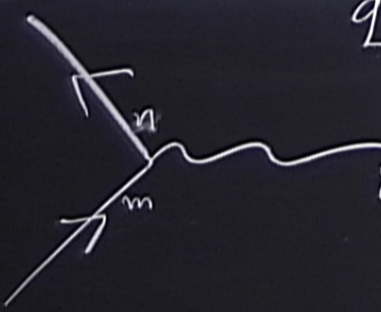
$$= -2\gamma_{mn}^{\mu} (2\pi)^4 \delta^4(\epsilon p)$$

$$= \frac{-i}{(2\pi)^4} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn} \leftarrow [] = \int (p) \frac{(\alpha \not{p}_1 A^{\mu})^2 (?) }{(\alpha \not{p}_1 A^{\mu})^2}$$

$$\frac{1}{(p^2+m^2)((p-q)^2+m^2)} = \int_0^1 \frac{dx}{[(p^2+m^2)(1-x) + (p-q)^2+m^2)x]^2}$$

$$= \int d^4p \int_0^1 \frac{dx}{[(p-qx)^2 + q^2x(1-x) + m^2]^2}$$

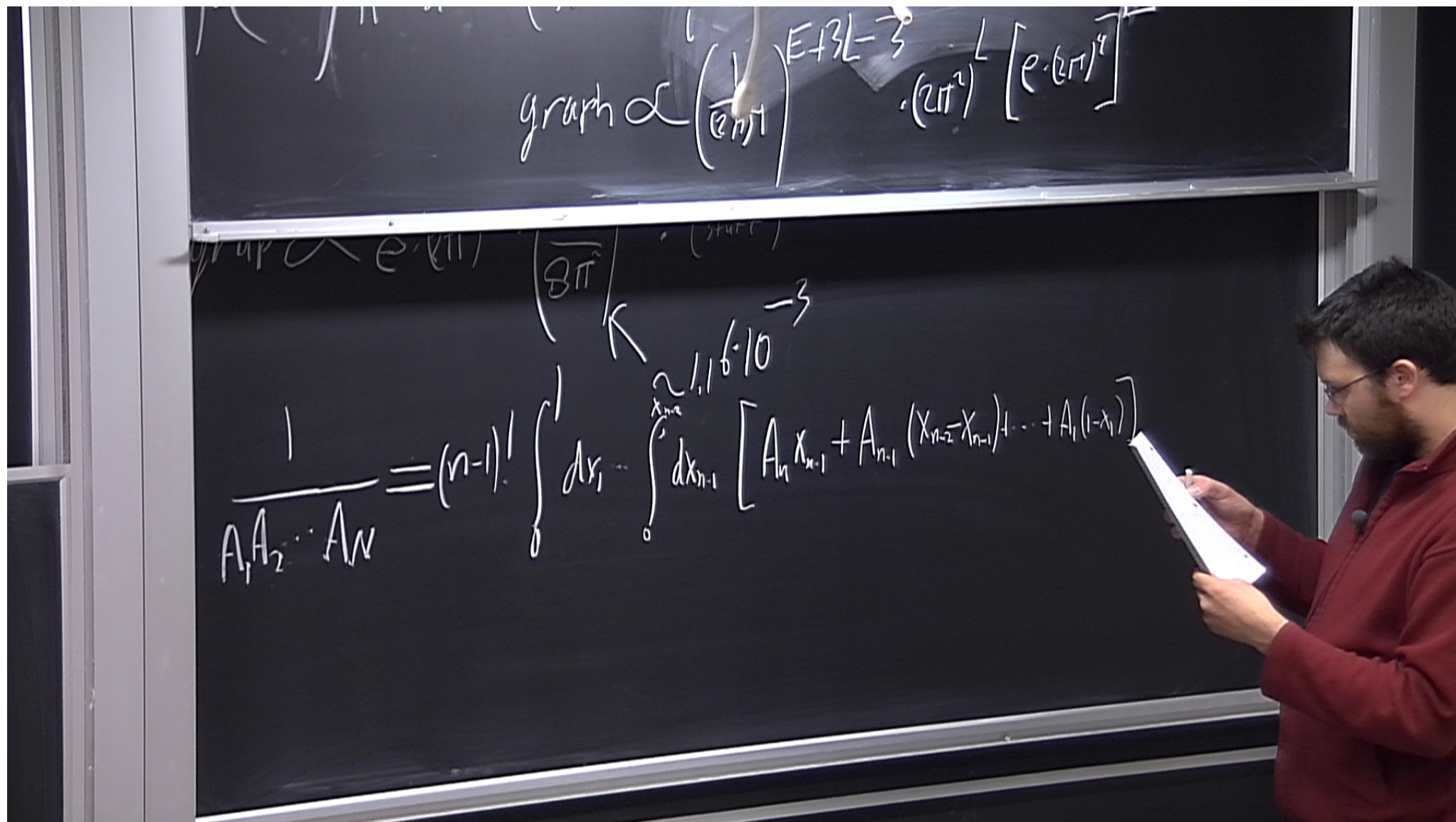
$q = -e$



$$= -2\gamma_{mn}^{\mu} (2\pi)^4 \delta^4(\epsilon p)$$

$$= \frac{-i}{(2\pi)^4} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \right]_{mn}$$

$$\left[\right] = \int (p) \frac{(\alpha \not{p} A^{\mu})^2 (?) }{(\alpha \not{p} A^{\mu})^2}$$



$$\text{graph } \mathcal{L}\left(\frac{1}{(2\pi)^4}\right)^{E+3L-3} \cdot (2\pi^2)^L [e \cdot (2\pi)^4]^L$$

$$\frac{1}{A_1 A_2 \cdots A_N} = (n-1)! \int_0^1 dx_1 \cdots \int_0^{x_{n-2}} dx_{n-1} \left[A_n x_{n-1} + A_{n-1} (x_{n-2} - x_{n-1}) + \cdots + A_1 (1 - x_1) \right]$$

$\left(\frac{8\pi}{K} \right) \sim 1.16 \cdot 10^{-3}$

$$\text{graph} \propto \left(\frac{1}{(2\pi)^4} \right)^{E+3L-3} \cdot (2\pi^2)^L [e \cdot (2\pi)^4]^L$$

$$\text{graph} \propto e^{-(2\pi)^4} \cdot \left(\frac{1}{8\pi^2} \right)^L \cdot (\text{stuff})$$

$$\frac{1}{A_1 A_2 \cdots A_N} = (n-1)! \int_0^1 dx_1 \cdots \int_0^{x_{n-2}} dx_{n-1} \left[A_n x_{n-1} + A_{n-1} (x_{n-2} - x_{n-1}) + \cdots + A_1 (1 - x_1) \right]^{-n}$$

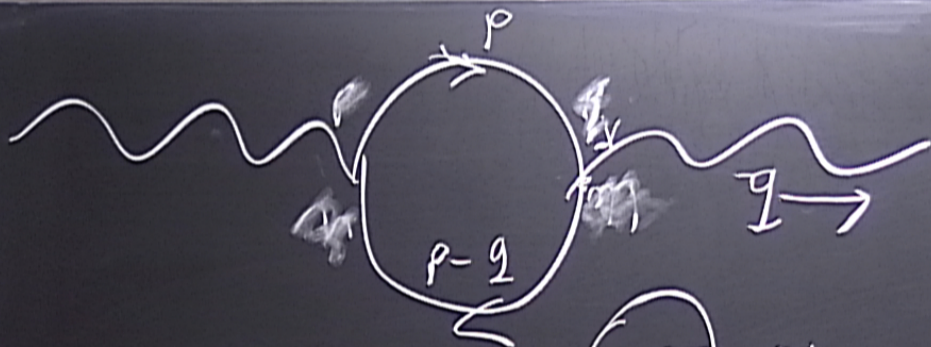
$\sim 1.16 \cdot 10^{-3}$

$$\text{graph} \propto \left(\frac{1}{(2\pi)^4} \right)^{E+3L-3} \cdot (2\pi^2)^L [e \cdot (2\pi)^4]^L$$

$$\text{graph} \propto e^{-(2\pi)^4} \cdot \left(\frac{1}{8\pi^2} \right)^L \cdot (\text{stuff})$$

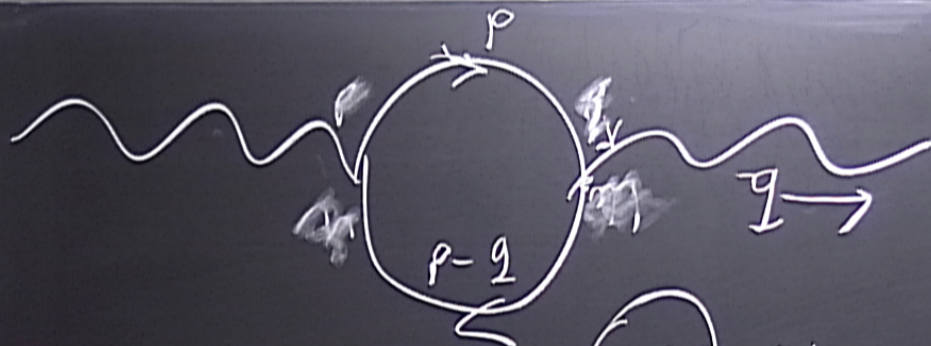
$$\frac{1}{A_1 A_2 \cdots A_N} = (n-1)! \int_0^1 dx_1 \cdots \int_0^{x_{n-2}} dx_{n-1} \left[A_n x_{n-1} + A_{n-1} (x_{n-2} - x_{n-1}) + \cdots + A_1 (1 - x_1) \right]^{-n}$$

$\sim 1.16 \cdot 10^{-3}$



$$\int \frac{d^4 q}{(2\pi)^4}$$

$$D_{\mu\nu}^{(0)} + D_{\mu\nu}^{(0)} \left(\Pi^{\rho\sigma} \right) D_{\sigma\mu}^{(0)} + \dots$$



$$\int \frac{d^4 q}{(2\pi)^4} \text{BAR}$$

$$D_{\mu\nu}^{(0)} + D_{\mu\nu}^{(0)} \left(\frac{1}{i} \Pi^{\rho\sigma} \right) D_{\sigma\mu}^{(0)} + \dots$$

$$\gamma \rightarrow e, \mu, \tau$$

$$i \cdot (2\pi)^4 \Pi_{\mu\nu} = - \int d^4p \text{Tr} \left\{ \frac{-i}{(2\pi)^4} \gamma_\mu \cdot e \gamma_\nu (2\pi)^4 \cdot \frac{-i}{(2\pi)^4} \gamma_\nu \cdot e \gamma_\mu (2\pi)^4 \right\}$$

$$\Pi_{\mu\nu}(q) = \frac{-ie^2}{(2\pi)^4} \int d^4p \text{Tr} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \cdot \gamma_\mu \cdot \frac{-i(\not{p} - \not{q}) + m}{(p - q)^2 + m^2} \right]$$

CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
IF YOU HAVE TO TOUCH THE BOARD OR THE BOARDER
PLEASE ASK THE LECTURER FIRST

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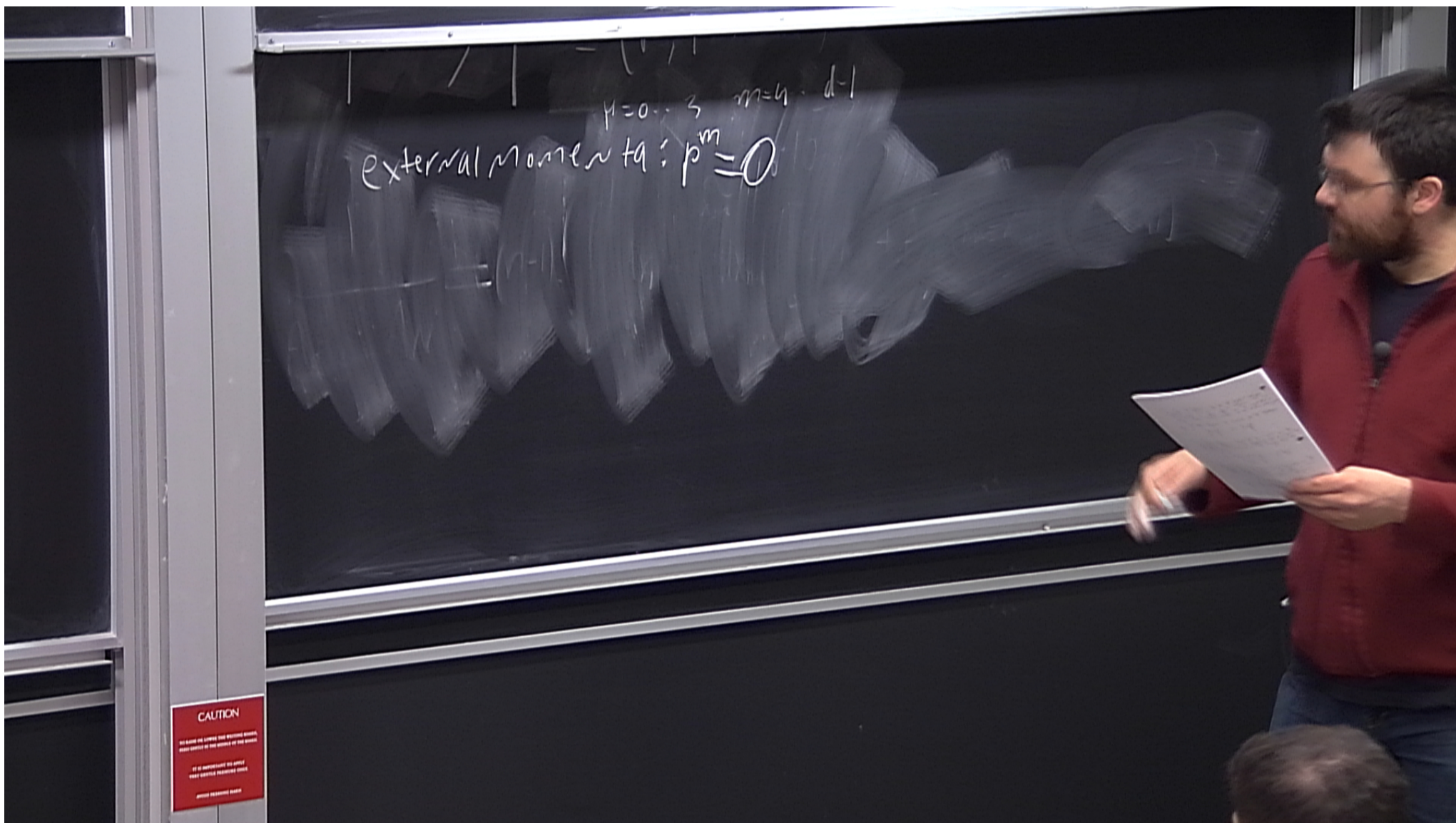
$$\Pi_{\mu\nu}(q) = \frac{-ie^2}{(2\pi)^4} \int d^4p \operatorname{Tr} \left[\frac{-i \not{p} + m}{p^2 + m^2 - i\epsilon} \gamma_\mu \cdot \frac{-i(\not{p} - \not{q}) + m}{(p-q)^2 + m^2 - i\epsilon} \gamma_\nu \right] \sim$$

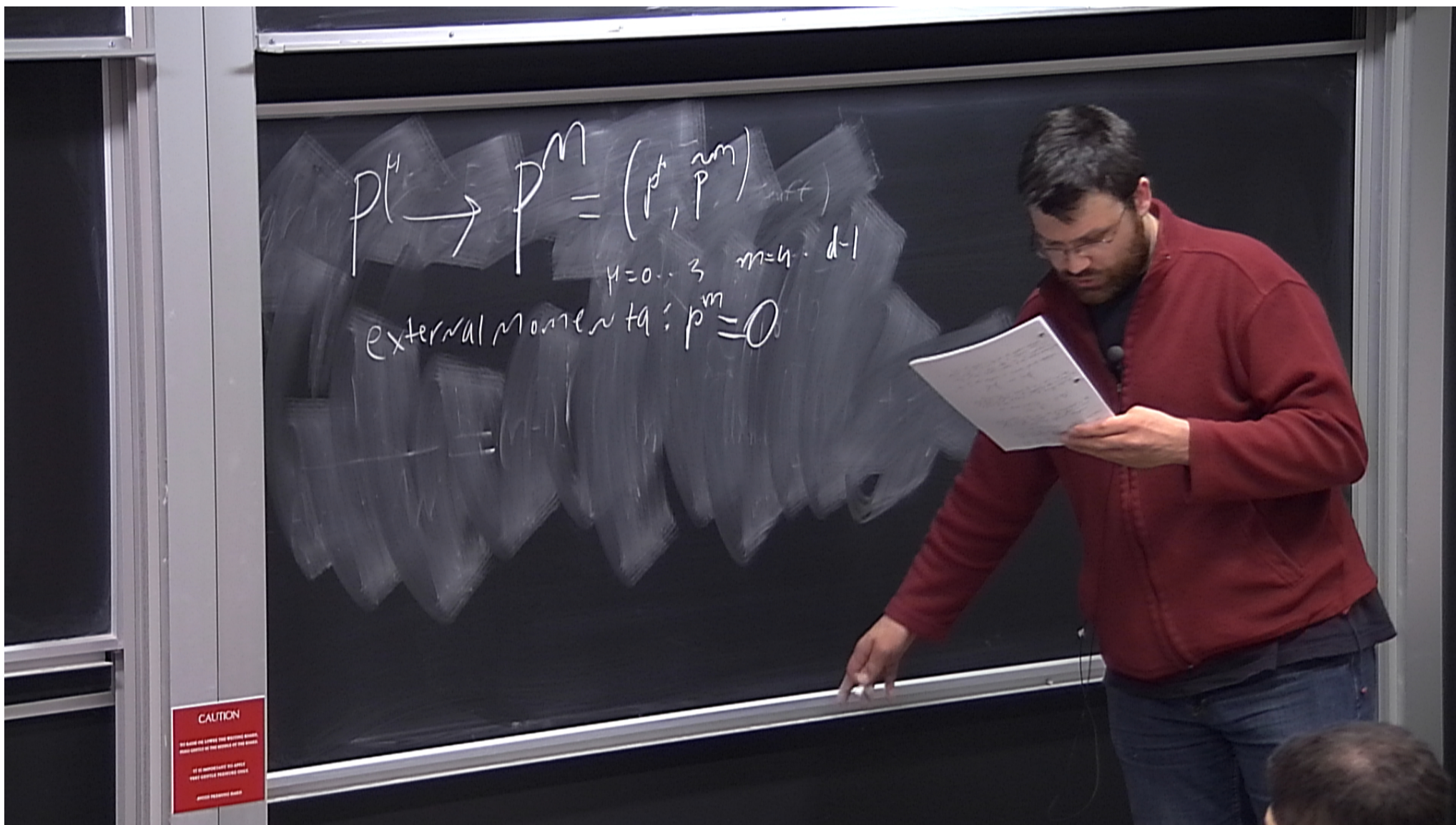
$$\rightarrow \int d^3p \, p \cdot \left(\frac{1}{r}\right)^2$$

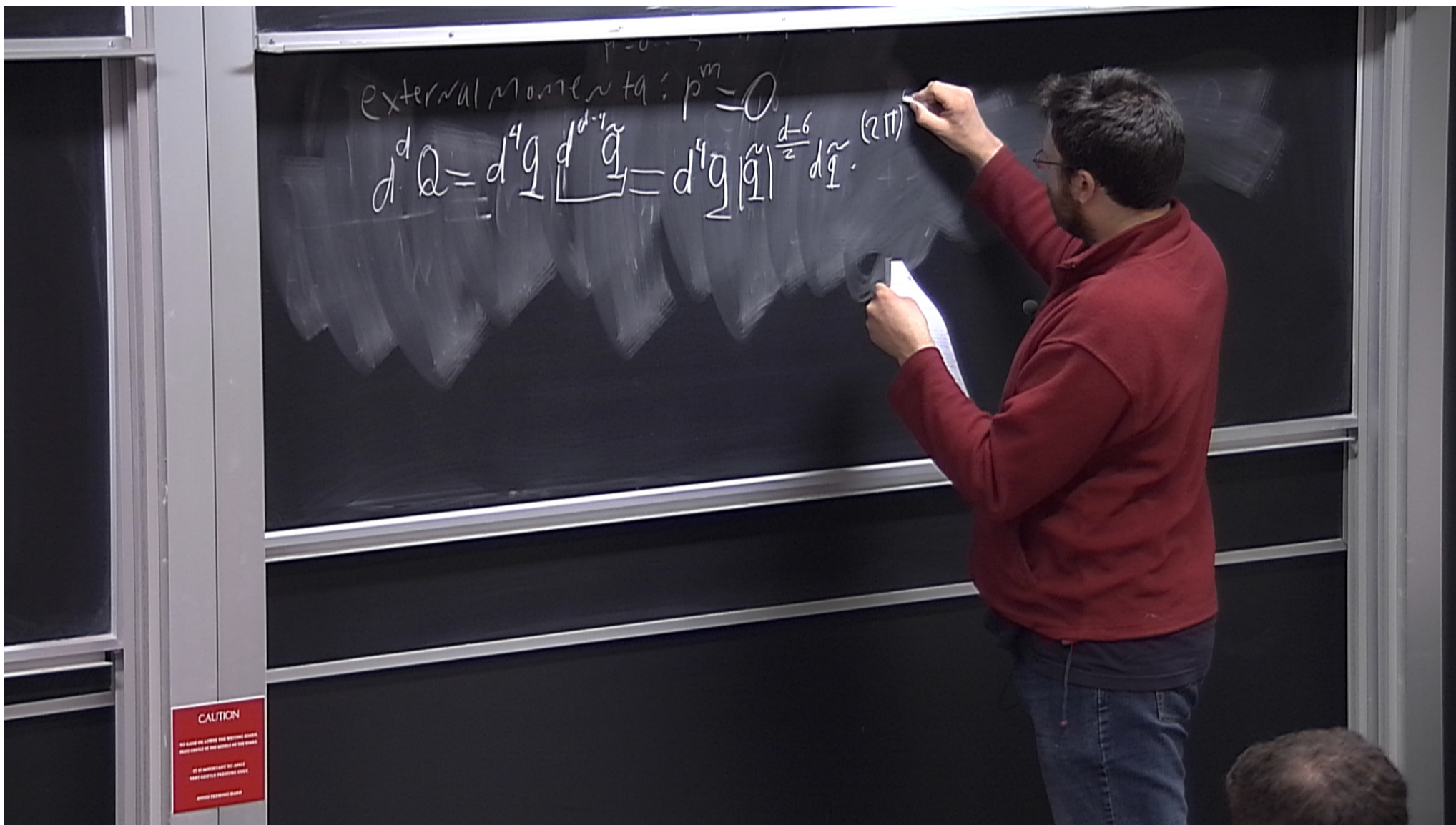
$$\int d^3p \rightarrow p^2 dp \left(\frac{1}{r}\right)^2$$

CAUTION
DO NOT TOUCH THE SURFACE OF THE BOARD
OR THE BOARD WILL BE DAMAGED

CAUTION
DO NOT TOUCH THE SURFACE OF THE BOARD
OR THE BOARD WILL BE DAMAGED







$E = \# \text{ external lines}$
 $I = \# \text{ internal lines}$
 $L = \# \text{ loops}$
 $V = \# \text{ vertices}$

$$\left[\frac{1}{(2\pi)^4} \right]^I \cdot (2\pi)^L \left[e \cdot (2\pi)^4 \right]^V \cdot (\text{some loop thing})$$

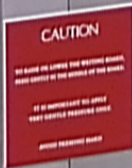
$$L = I - V + 1$$

$$I = E + 3L - 3$$

$$V = E + 2L - 2$$

$$\sum n_i v_i = 2I + E$$

$$(3V) = \sum n_i v_i = 2I + E$$



$$P \rightarrow P = (p^\mu, \tilde{p}^m)$$

$$\mu = 0 \dots 3 \quad m = 4 \dots d-1$$

external momenta: $p^m = 0$

$$d^d Q = d^4 q \underbrace{d^{d-4} \tilde{q}}_{= d^4 \tilde{q}} = d^4 q \underbrace{|\tilde{q}|^{\frac{d-4}{2}} d\tilde{q}}_{\frac{(2\pi)^{d-4}}{\Gamma(\frac{d-4}{2})}}$$

$$p^\mu \rightarrow p^m = (p^\mu, \tilde{p}^m)$$

$$\mu=0\dots 3 \quad m=4 \dots d-1$$

external momenta: $\tilde{p}^m = 0$

$$d^d Q = d^4 q \underbrace{d^{d-4} \tilde{q}}_{\tilde{q}} = d^4 q \underbrace{(\tilde{q})^{\frac{d-6}{2}}}_{\tilde{q}} d\tilde{q} \cdot \frac{(2\pi)^{(d-4)/2}}{\Gamma(\frac{d-4}{2})}$$

... (some 10

$$p^\mu \rightarrow p^m = (p^\mu, \tilde{p}^m)$$

$$\mu=0\dots 3, m=4\dots d-1$$

external momenta: $\tilde{p}^m = 0$

$$d^d Q = d^4 q \underbrace{d^{d-4} \tilde{q}}_{= d^4 \tilde{q}} = d^4 q \underbrace{d^{d-4} \tilde{q}}_{= d^4 \tilde{q}} \cdot \frac{(2\pi)^{(d-4)/2}}{\Gamma(\frac{d-4}{2})}$$

