

Title: Advanced General Relativity - Lecture 2

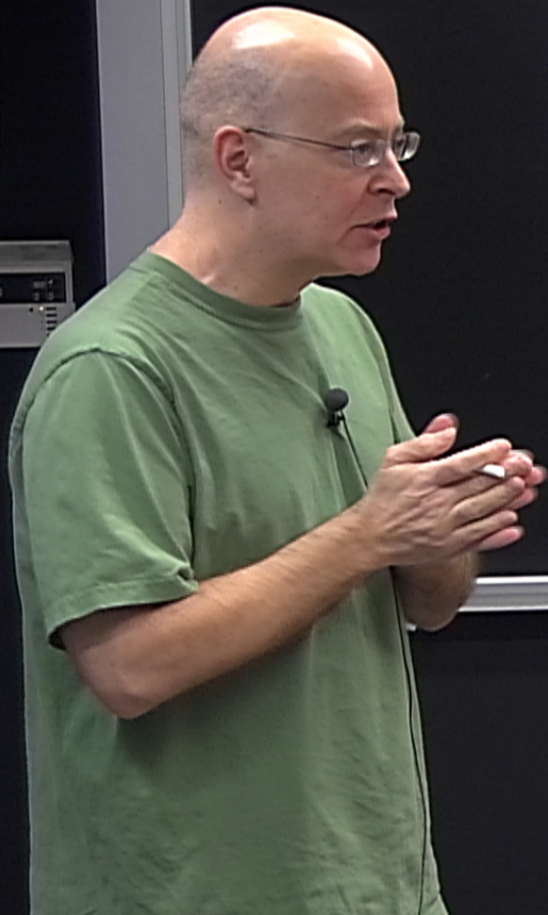
Date: Jan 11, 2012 03:30 PM

URL: <http://pirsa.org/12010154>

Abstract:

## Geodesics

timelike curve is ST that extremizes proper time between two events.



CAUTION  
Do not touch the screen or the board.  
All equipment is under  
high voltage and should  
not be touched.

## Geodesics

timelike curve is ST that extremizes proper time between two events.



# Geodesics

timelike curve is ST that extremizes time between two events.



$$\delta T^2 = -\delta S^2 \propto \delta \int$$

6 ES

the curve is ST that extremizes proper time  
between two events.

$$\delta T^2 = -\delta S^2 = -g_{\alpha\beta} dx^\alpha dx^\beta$$

$$T(A \rightarrow B) = \int_A^B \sqrt{-g_{\alpha\beta} dx^\alpha dx^\beta}$$

CAUTION  
ALL STUDENTS MUST WEAR SEATBELTS  
WHEN SEATED AT THE FRONT OF THE CLASS  
AT ALL TIMES  
WHILE THE CLASS IS IN SESSION

# Geodesics

timelike curve is ST that extremizes proper time between two events.

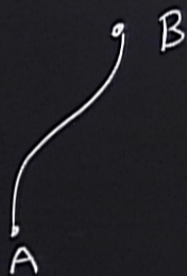


$$\Delta T^2 = -\Delta S^2 = -g_{\alpha\beta} dx^\alpha dx^\beta$$

$$T(A \rightarrow B) = \int_A^B \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}} d\lambda$$

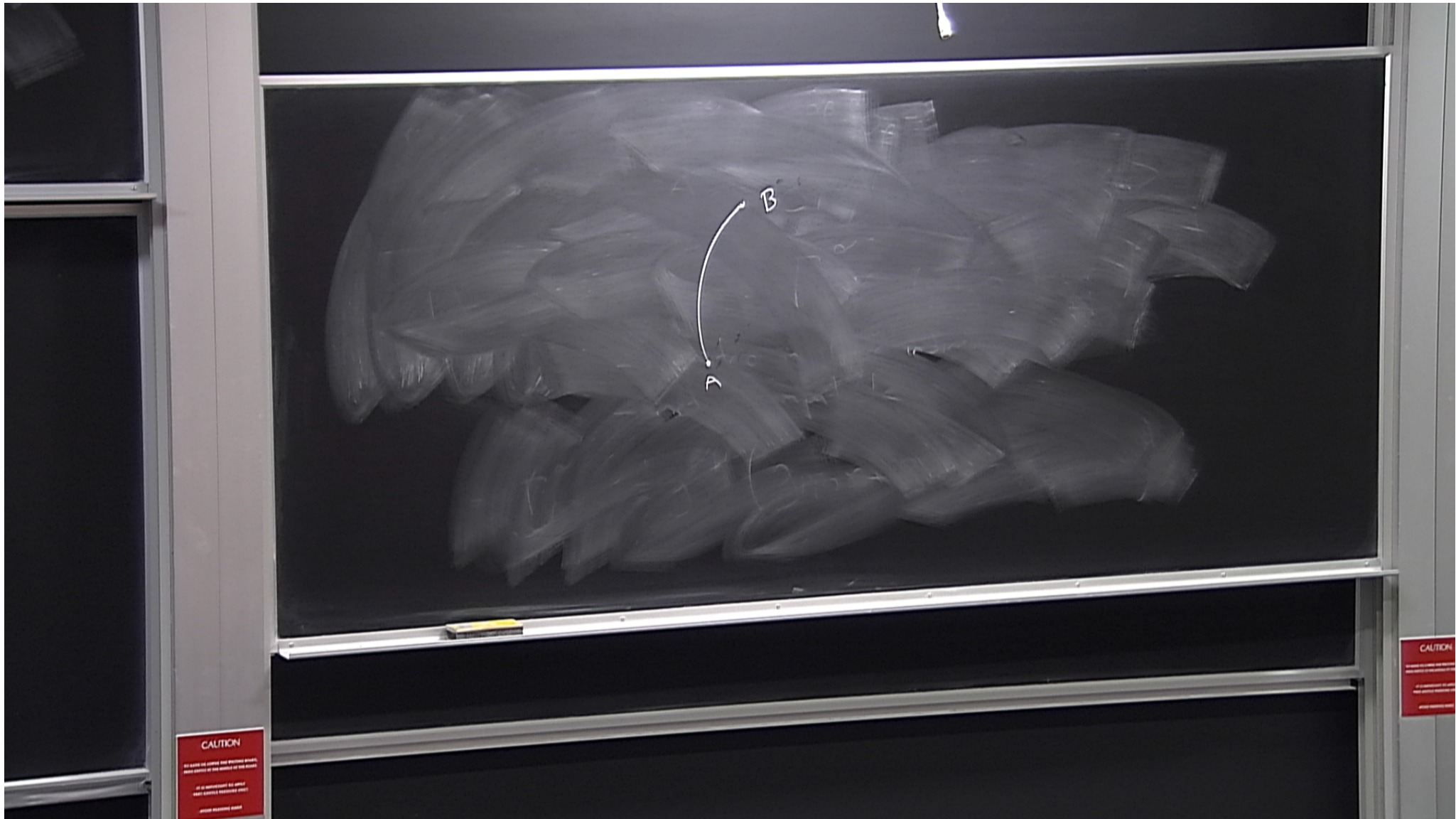
## Geodesics

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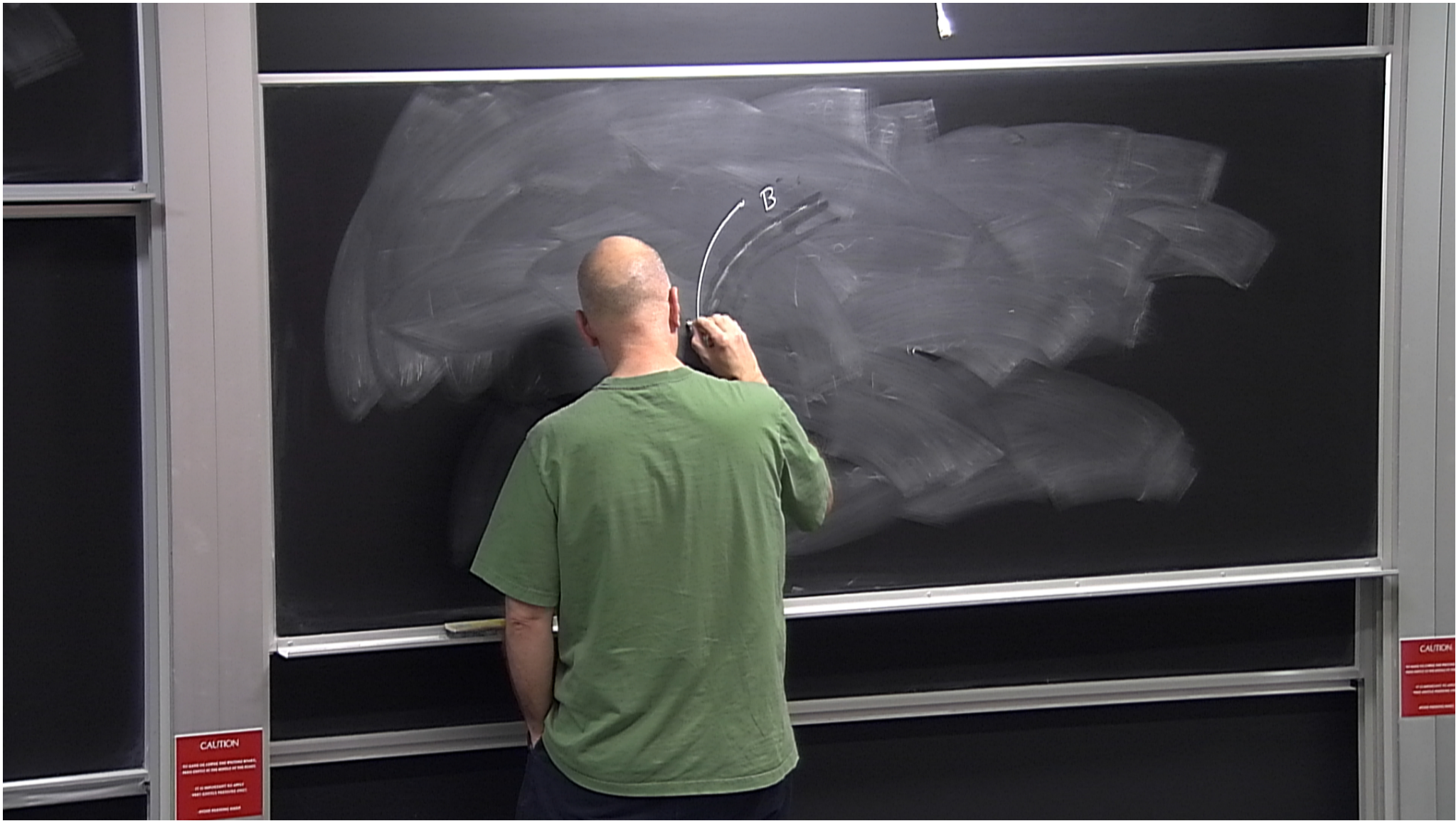


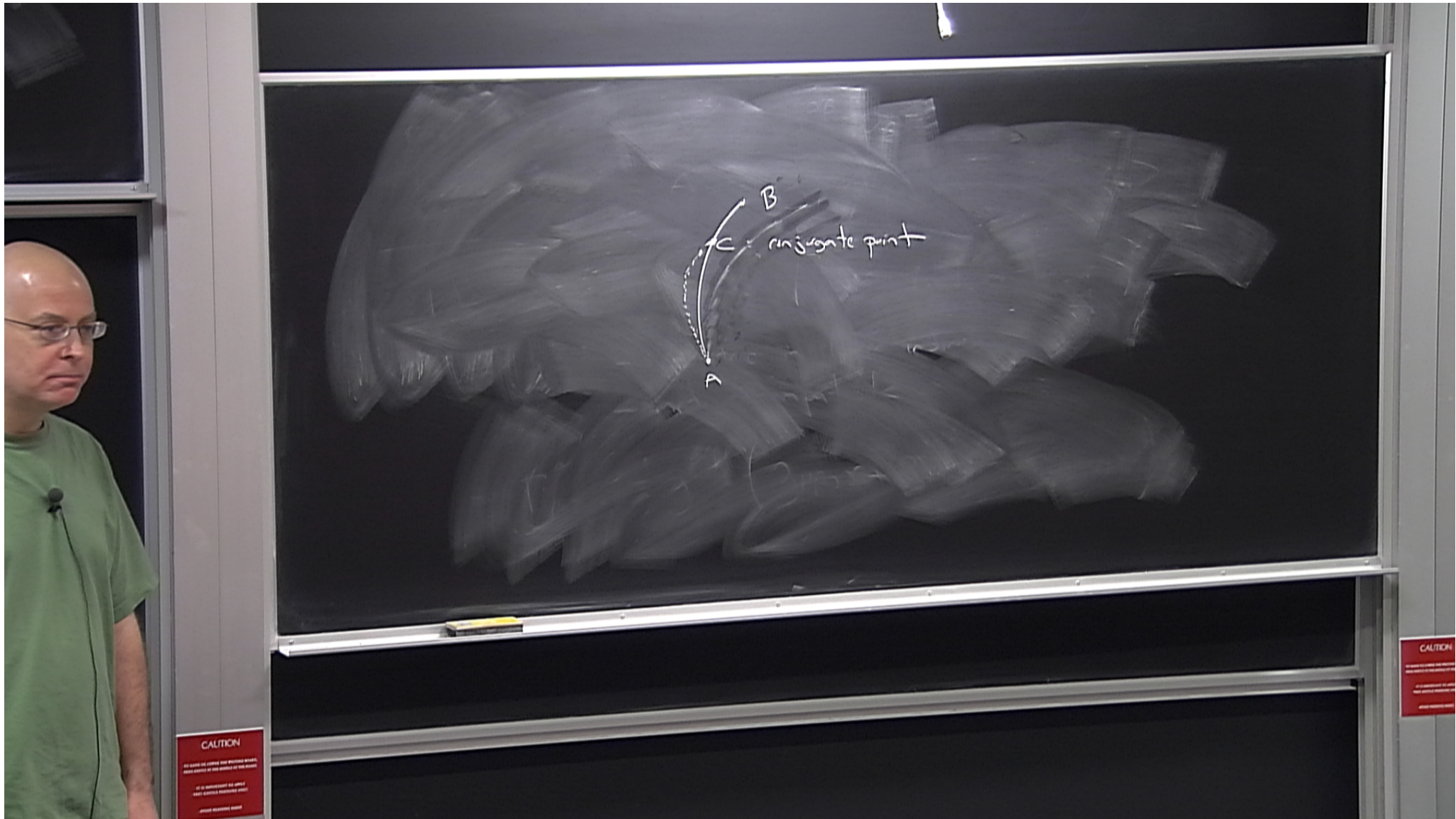
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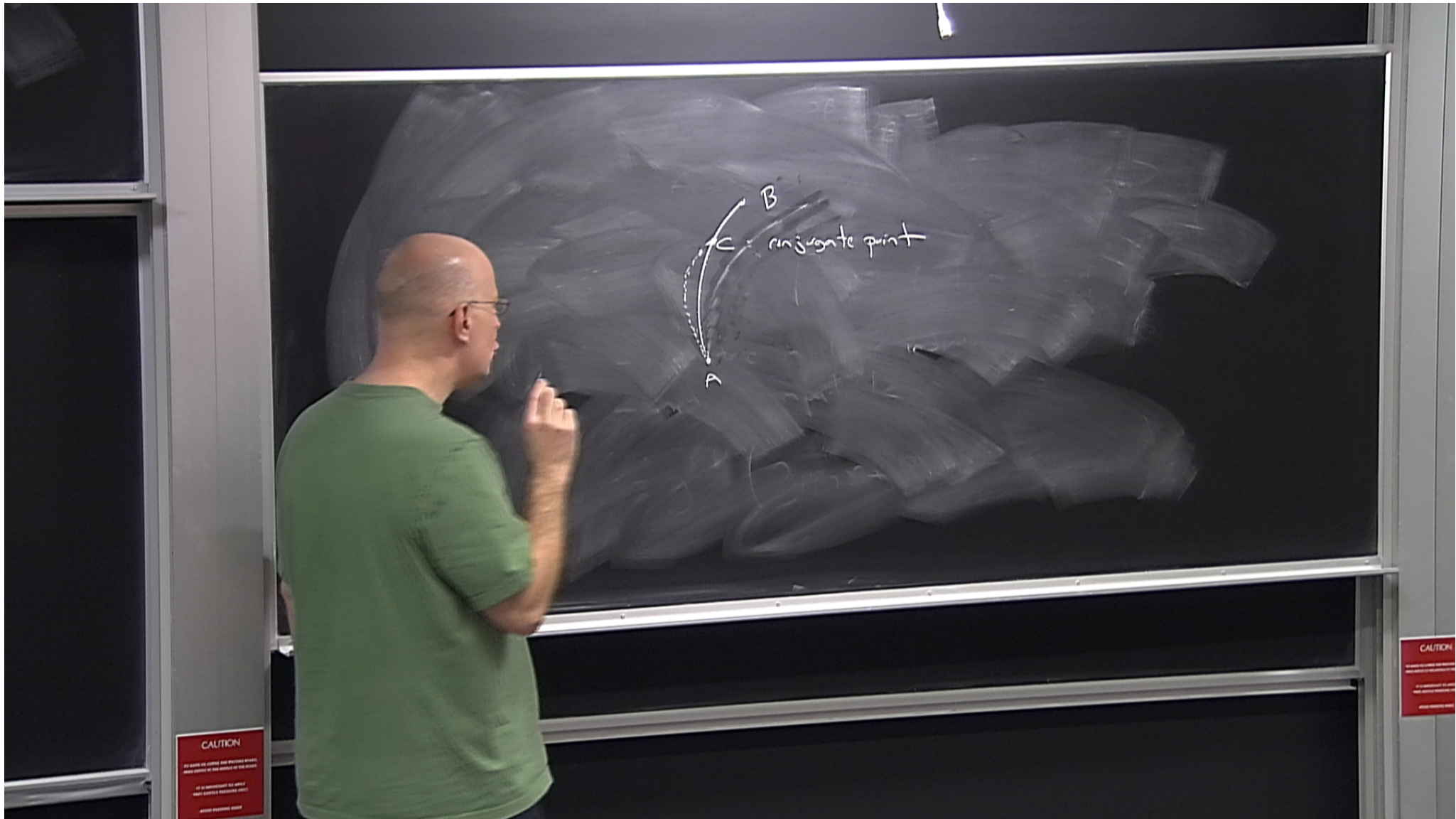
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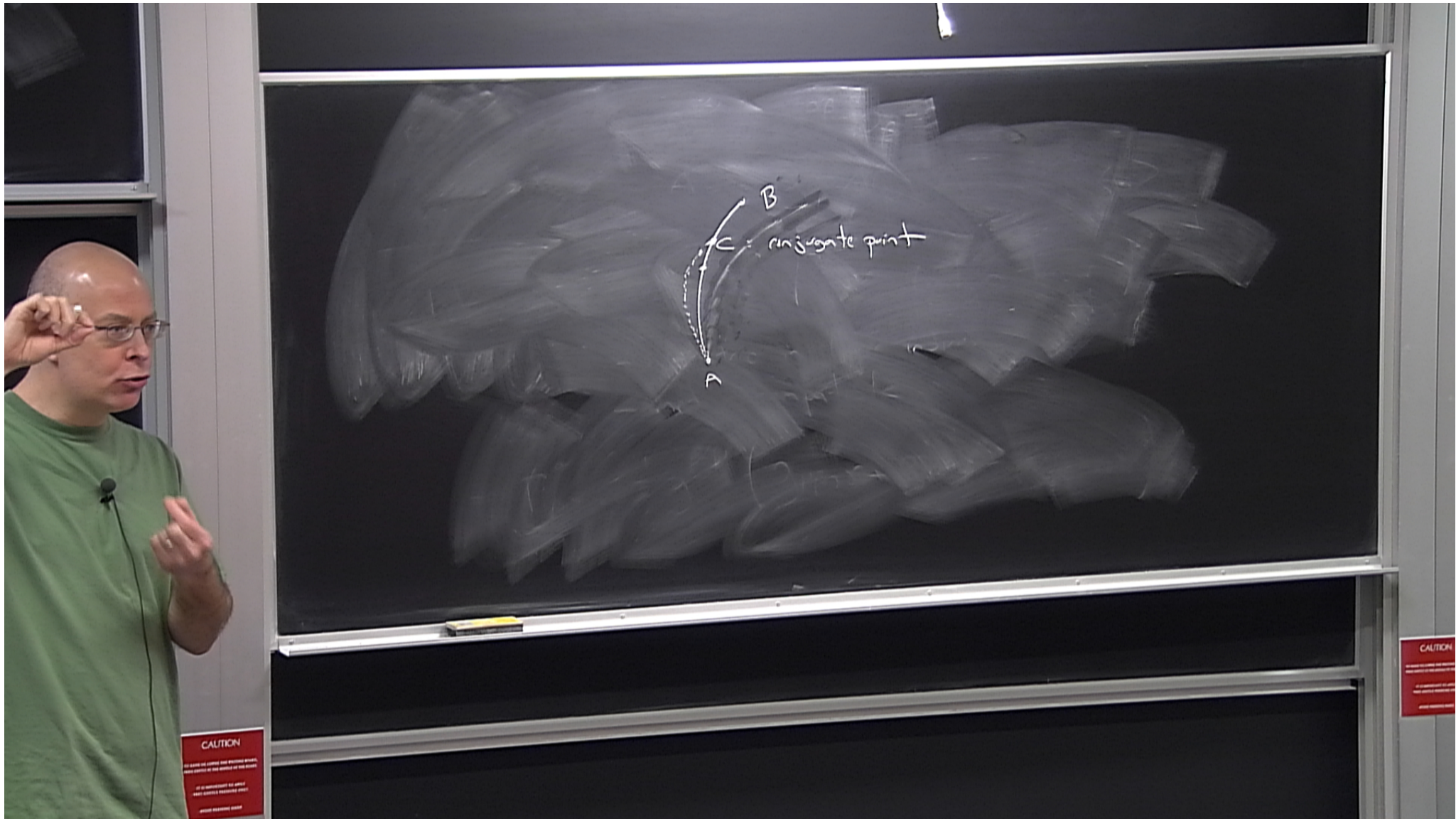


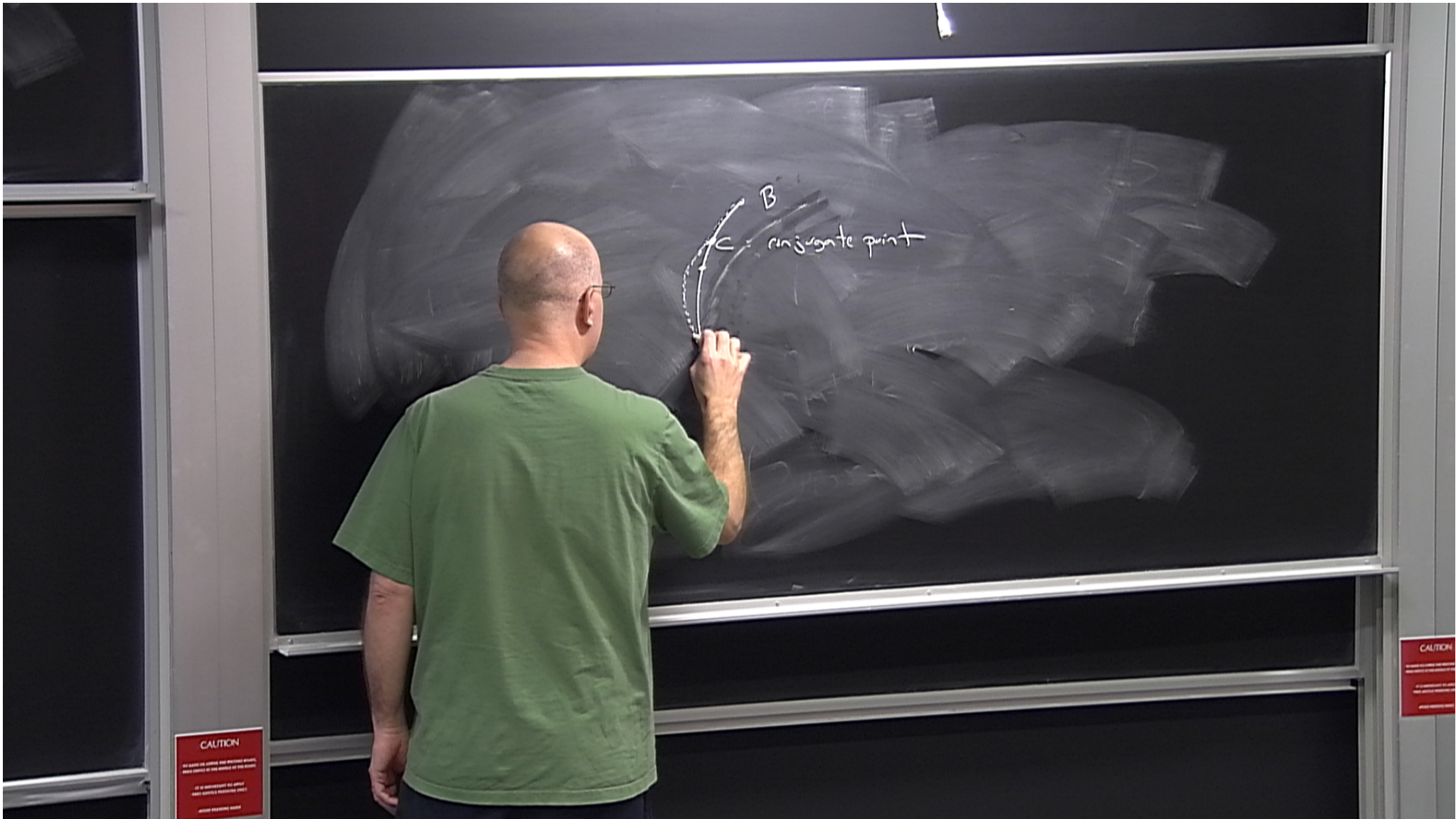


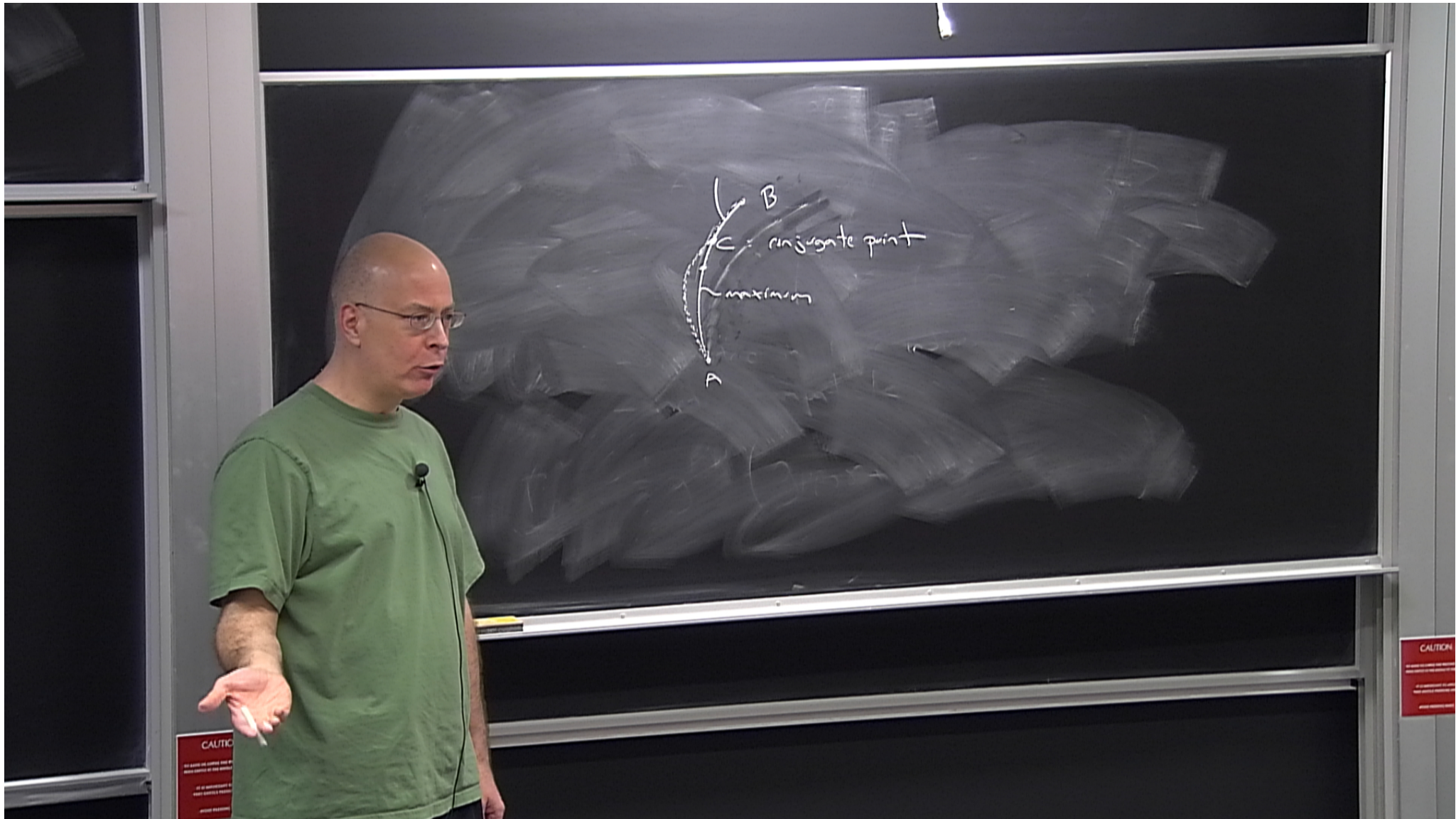


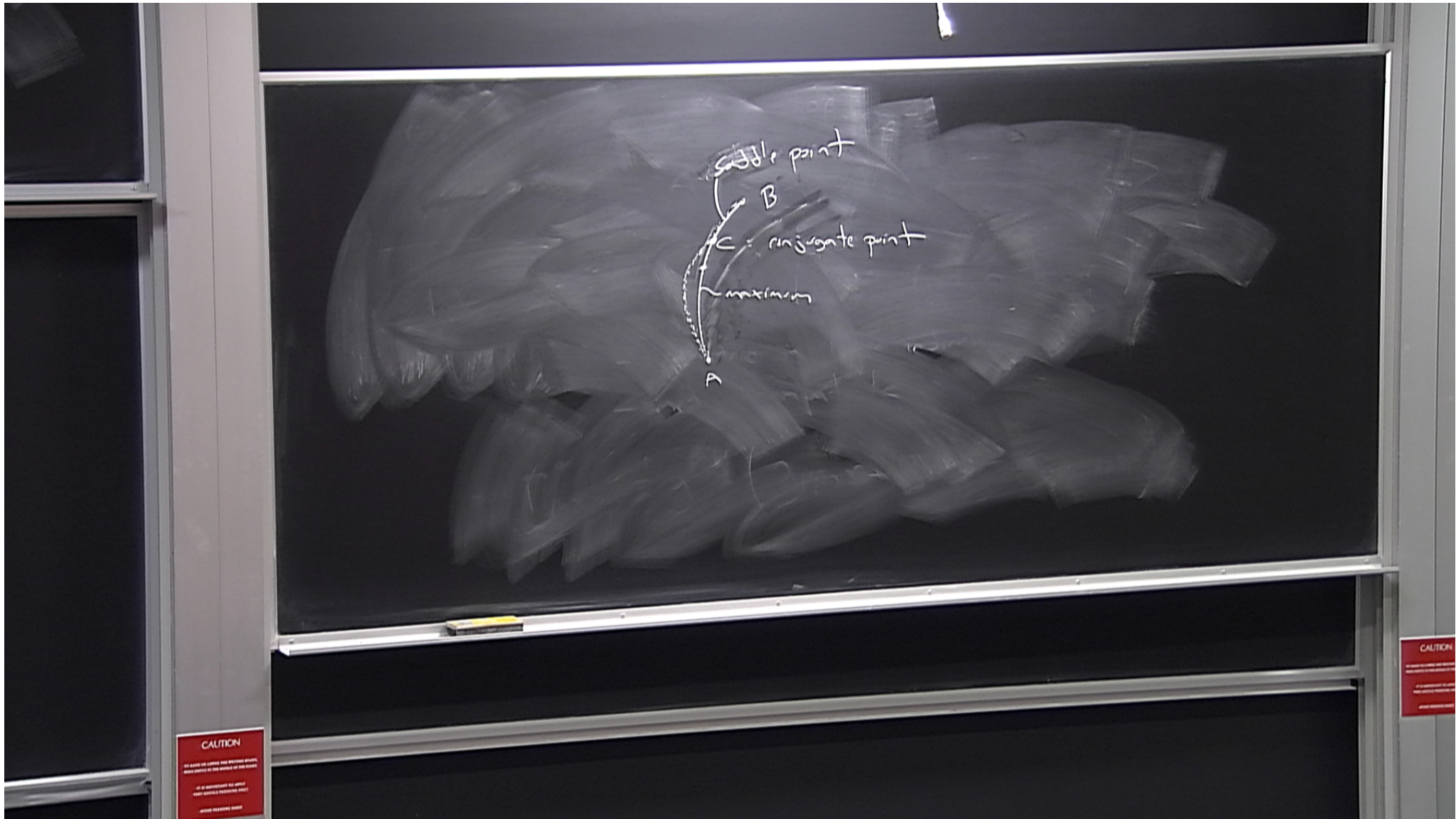












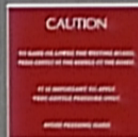
# Geodesics

timelike curve is ST that extremizes proper time between two events.



$$\delta T^2 = -\delta S^2 = -g_{\alpha\beta} dx^\alpha dx^\beta$$

$$T(A \rightarrow B) = \int_A^B \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}} d\lambda$$





$$T = \int_A^B L dx$$

extremum  $\Rightarrow \frac{d}{dx} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$

$$T = \int_A^B L d\lambda$$

$$\text{extremum} \Rightarrow \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$$

geodesic eqn

$$T = \int_A^B L d\lambda$$

$$\text{extremum} : \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{X}^\alpha} - \frac{\partial L}{\partial X^\alpha} = 0$$

$$\text{geodesic eqn} : \frac{d^2 X^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{\partial X^\beta}{\partial \lambda} \frac{\partial X^\gamma}{\partial \lambda} = 0$$

$$T = \int_A^B L d\lambda$$

$$\text{extremum} : \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$$

$$\text{geodesic eqn} : \frac{d^2 x^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda} = K \frac{\partial x^\alpha}{\partial \lambda}$$

$$\Upsilon = \int_A^B L d\lambda$$

$$\text{extremum} = \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{X}^\alpha} - \frac{\partial L}{\partial X^\alpha} = 0$$

$$\text{geodesic eqn} = \frac{d^2 X^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{\partial X^\beta}{\partial \lambda} \frac{\partial X^\gamma}{\partial \lambda} = \kappa \frac{\partial X^\alpha}{\partial \lambda}$$

$$\tau = \int_a^b L d\lambda$$

$$\text{extremum} = \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$$

geodesic eqn :

$$\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = \kappa \frac{dx^\alpha}{d\lambda}$$

$$\kappa = \frac{1}{L} \frac{dL}{d\lambda}$$

$$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}}$$

$$\Upsilon = \int_A^B L d\lambda$$

$$\text{extremum} : \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$$

geodesic eqn :

$$\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma_{\rho\sigma}^\alpha \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = \kappa \frac{dx^\alpha}{d\lambda}$$

$$\kappa = \frac{1}{L} \frac{dL}{d\lambda}$$

$$L = \sqrt{-g_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda}}$$

When  $\lambda \equiv$  proper  $\uparrow$   $\Delta\lambda = \Delta T$

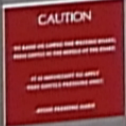
CAUTION  
DO NOT USE LENSES OR BINOCULARS BECAUSE THEY CONCENTRATE THE RAYS OF THE BEAM.  
IT IS ESSENTIAL TO WEAR PROPERLY FITTING EYE PROTECTION.  
PLEASE REMAIN SEATED.

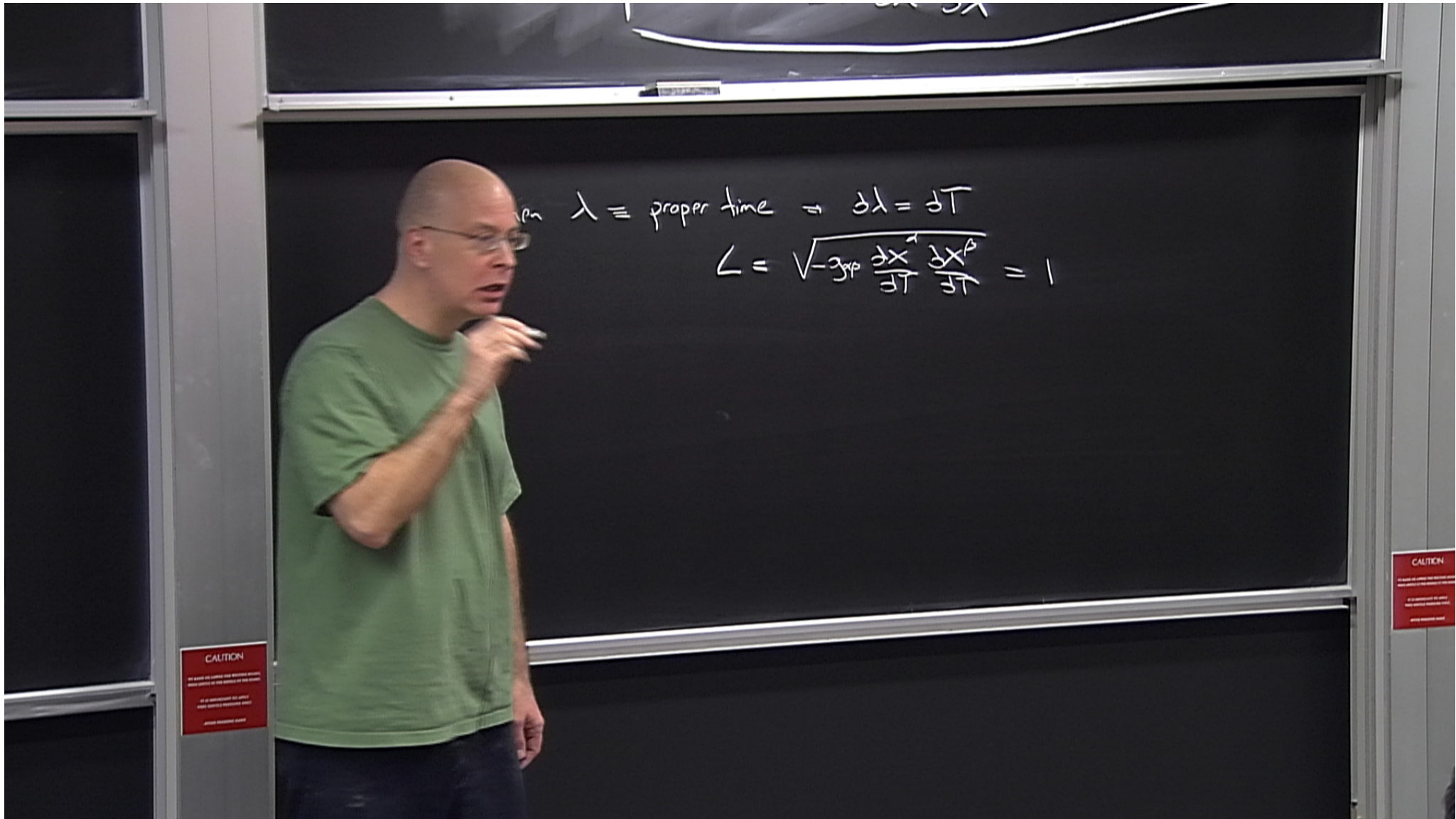
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When  $\lambda \equiv$  proper time  $\Rightarrow d\lambda = d\tau$

$$\mathcal{L} = \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} = 1$$





When proper time  $\Rightarrow d\lambda = dT$

$$\angle = \sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial T} \frac{\partial x^\nu}{\partial T}} = 1$$

When  $\lambda \equiv$  proper time  $\Rightarrow d\lambda = d\tau$

$$g_{\mu\nu} \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \tau} = 1$$

$\Rightarrow$  (circled  $\tau$ )

CAUTION  
DO NOT USE LIFELINE AND RESCUE SIGNALS  
UNLESS ADVISED BY THE INSTRUCTOR OF THE BOARD.

IT IS ESSENTIAL TO WEAR  
YOUR SAFETY HELMET AT ALL TIMES.  
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When  $\lambda \equiv$  proper time  $\Rightarrow d\lambda = d\tau$

$$L = \sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \tau}} = 1$$

$\Rightarrow k = 0$   $\rightarrow$  this is preserved by all transformations

$\lambda$

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$$\lambda = a\tau + b$$

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$\equiv$  affine parameter.



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$\equiv$  affine parameter.

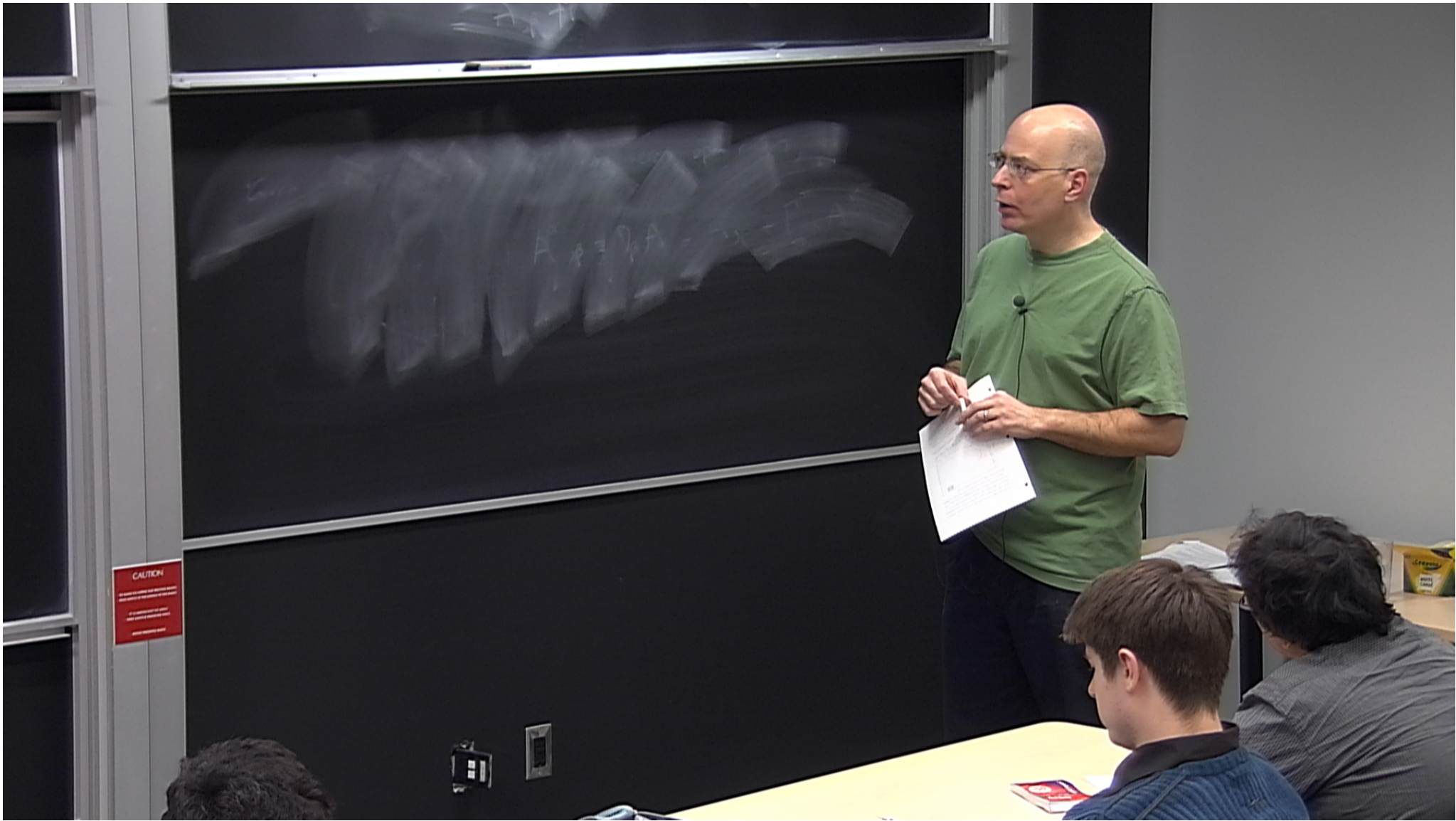
For affine parameters,  $\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0$

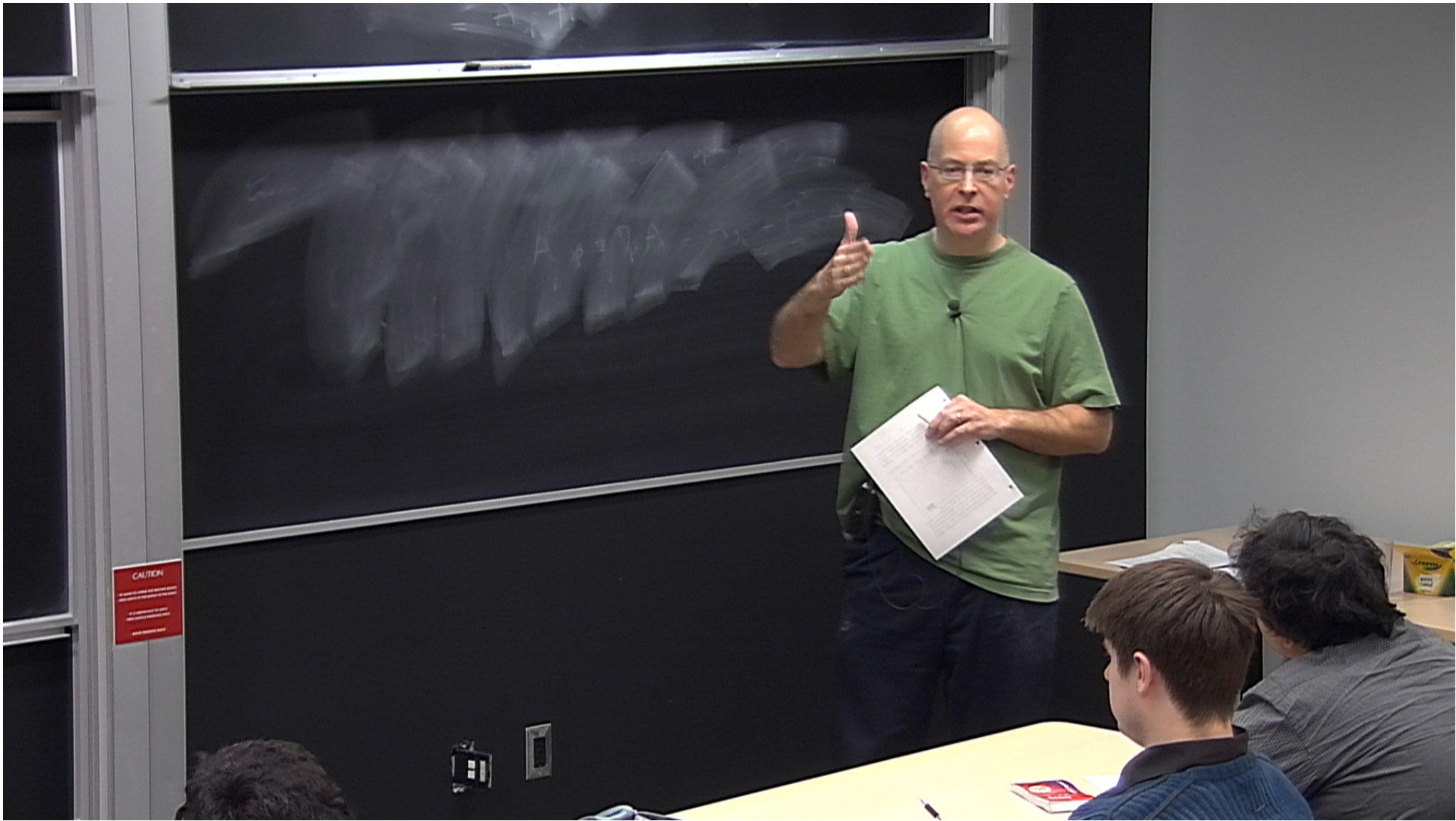
CAUTION

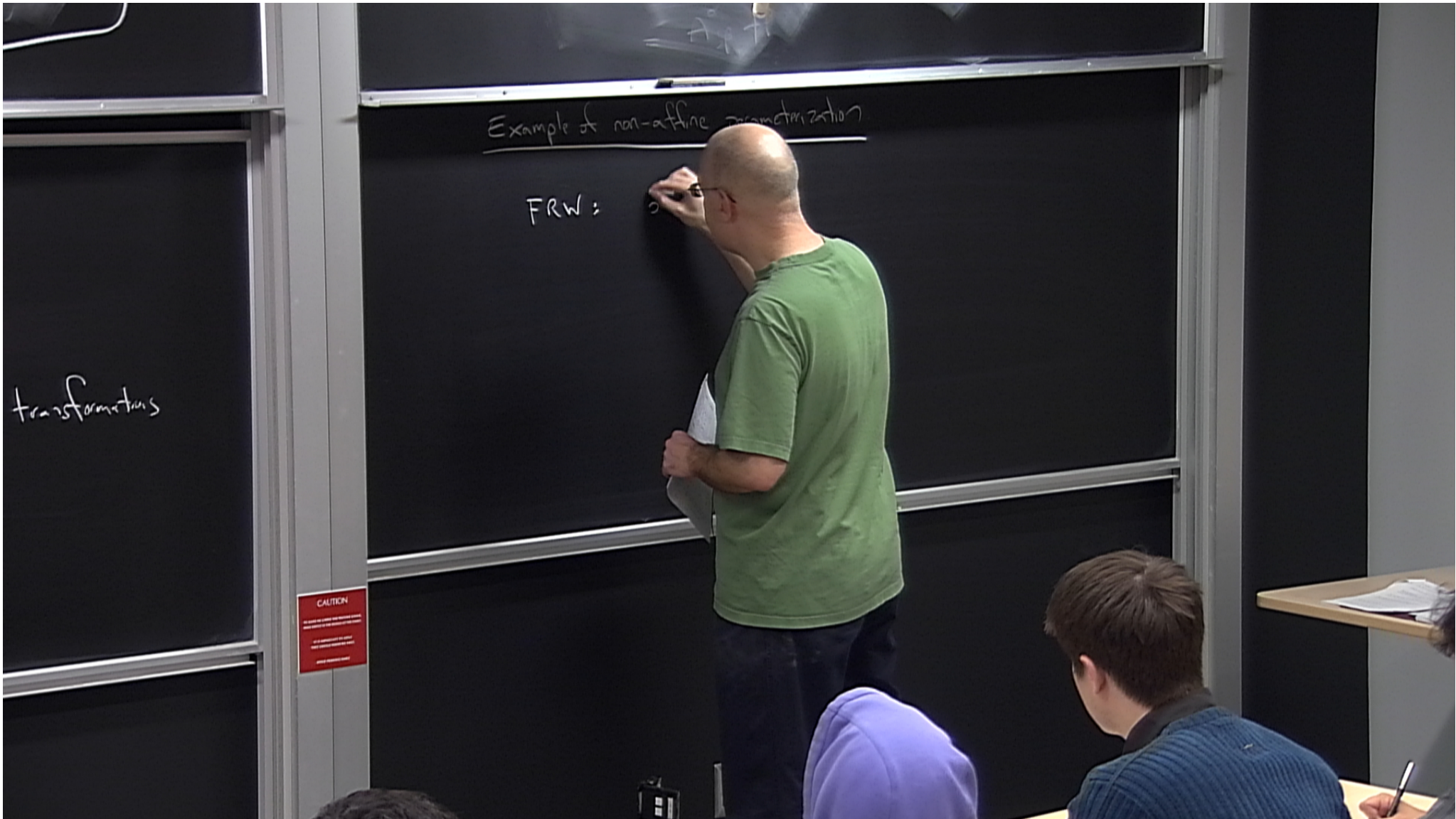
DO NOT RE-ENTER THE ROOM UNTIL YOU ARE TOLD TO DO SO.  
IF YOU ARE TOLD TO RE-ENTER THE ROOM, PLEASE FOLLOW THE INSTRUCTIONS OF THE STAFF.

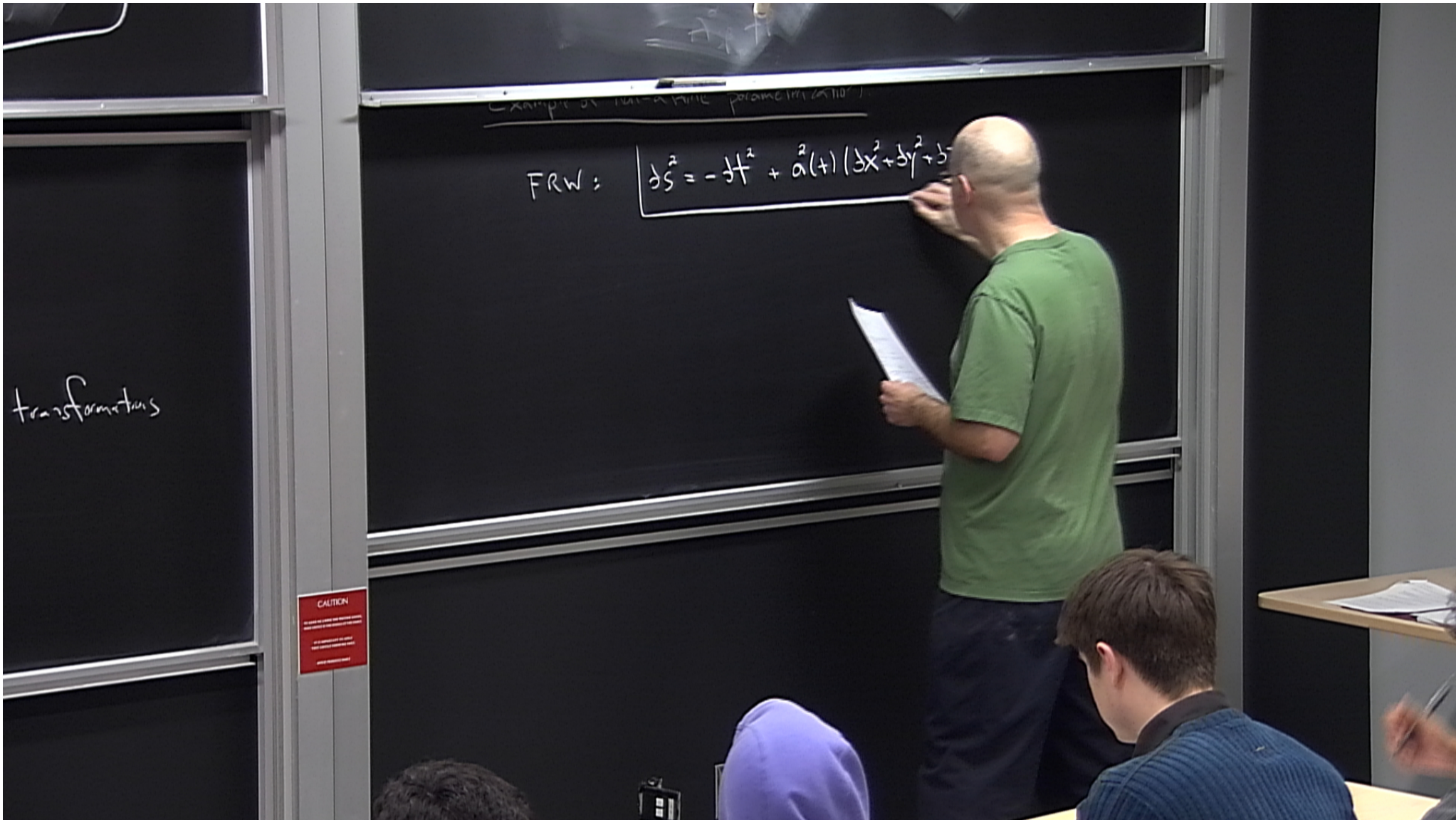
CAUTION

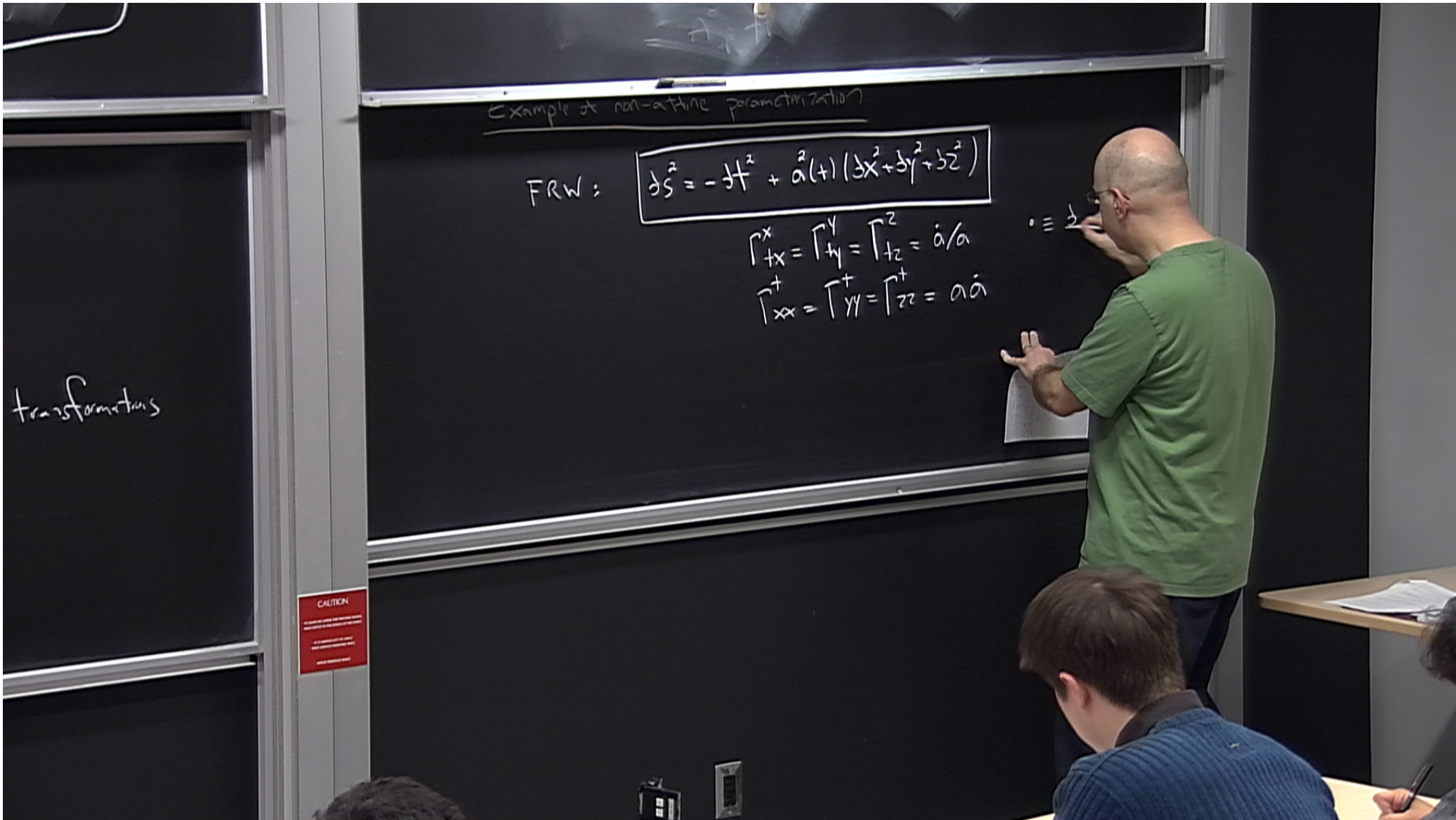
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Example of non-affine parametrization

$$\text{FRW: } \boxed{ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)}$$

$$\begin{aligned} \Gamma_{tx}^x &= \Gamma_{ty}^y = \Gamma_{tz}^z = \dot{a}/a & \bullet & \equiv \frac{\dot{a}}{a} \\ \Gamma_{xx}^+ &= \Gamma_{yy}^+ = \Gamma_{zz}^+ = a\dot{a} \end{aligned}$$

transformations

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD ERASER

Example of non-affine parametrization

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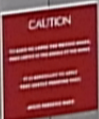
$$\Gamma_{tx}^x = \Gamma_{ty}^y = \Gamma_{tz}^z = \dot{a}/a$$

$$\equiv \frac{\dot{a}}{a}$$

$$\Gamma_{xx}^t = \Gamma_{yy}^t = \Gamma_{zz}^t = a\dot{a}$$

timelike geodesic moving along  
x-direction; pick parameter = t

transformations

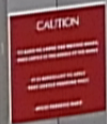


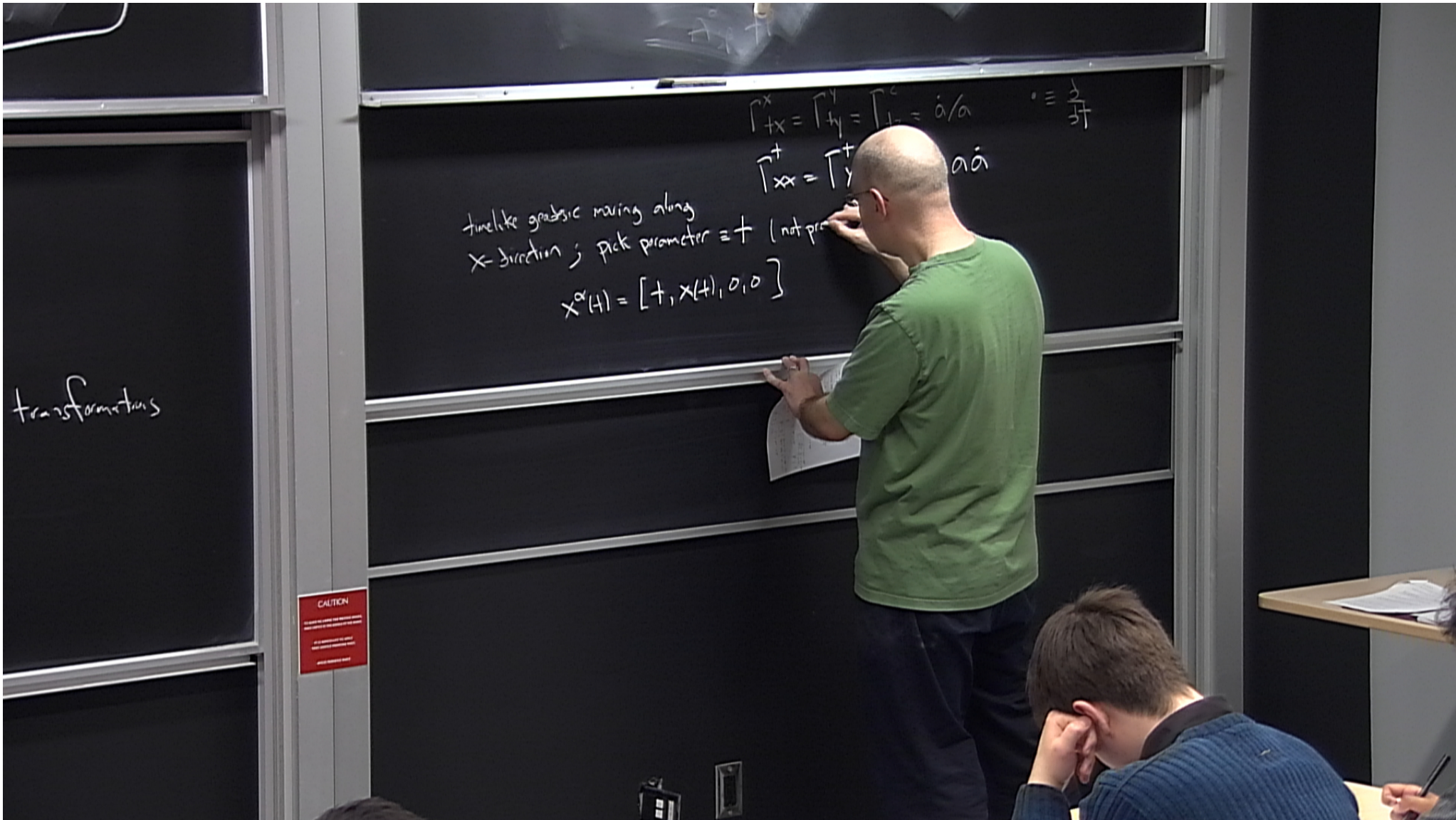
transformations

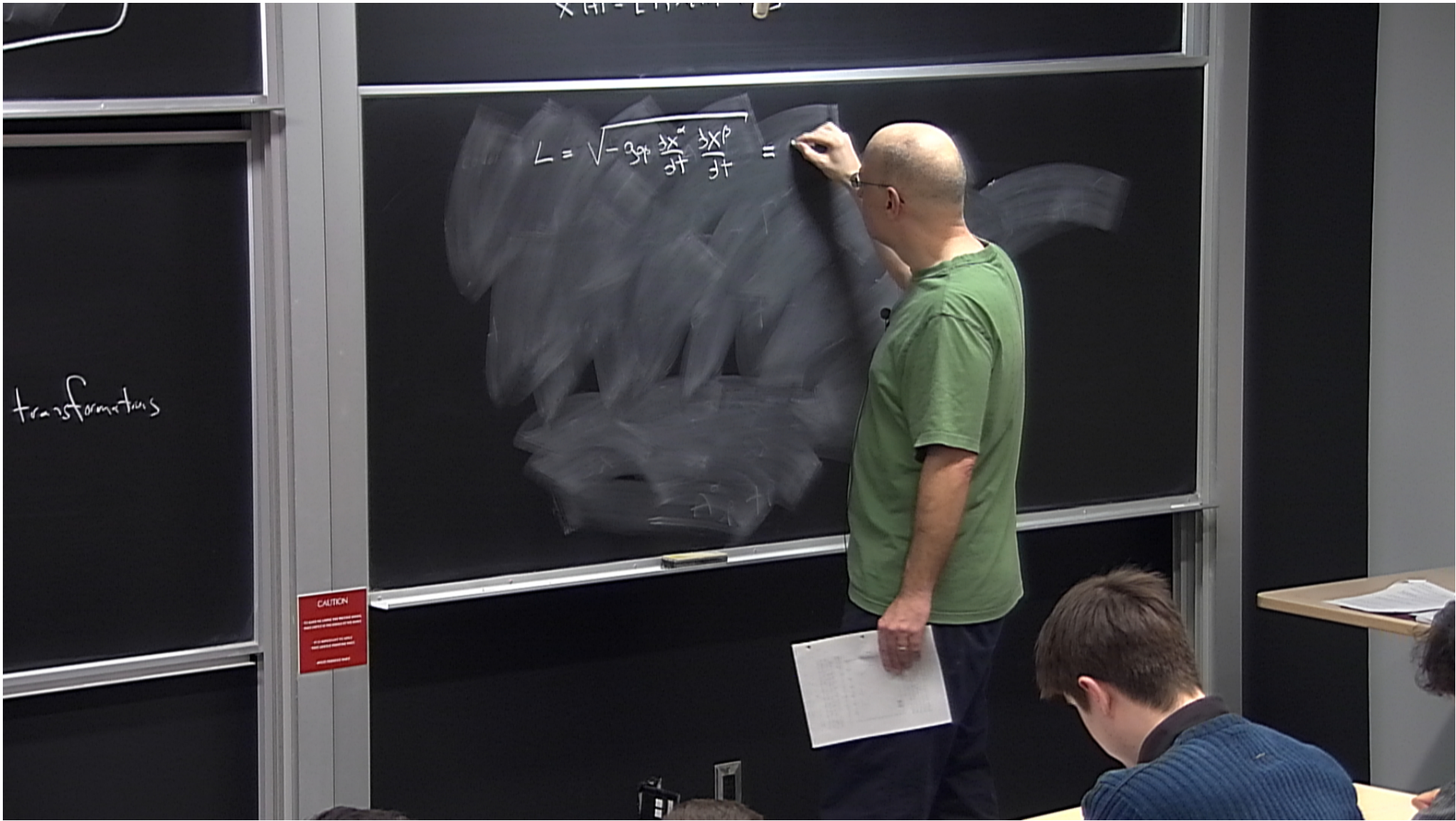
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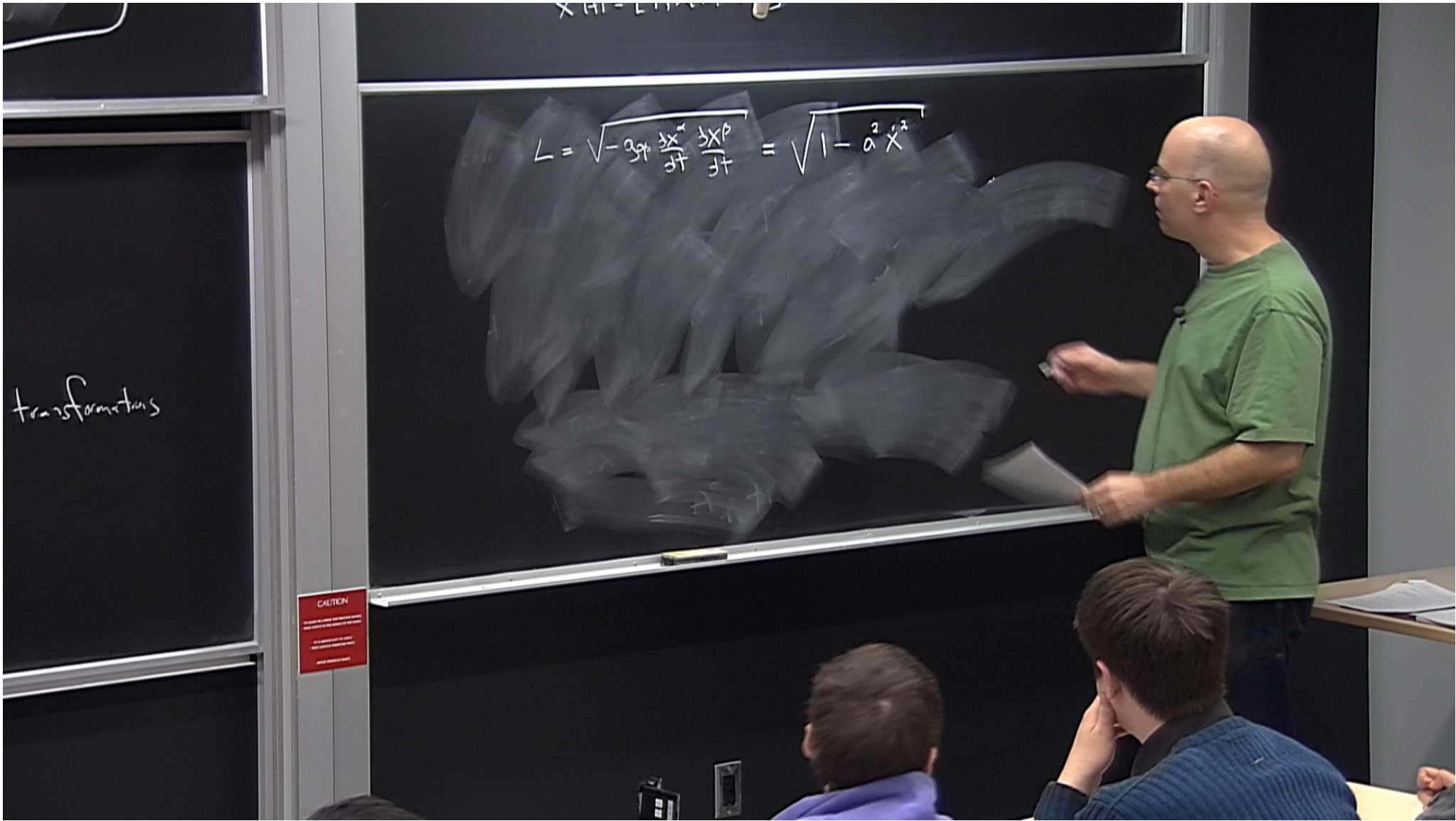
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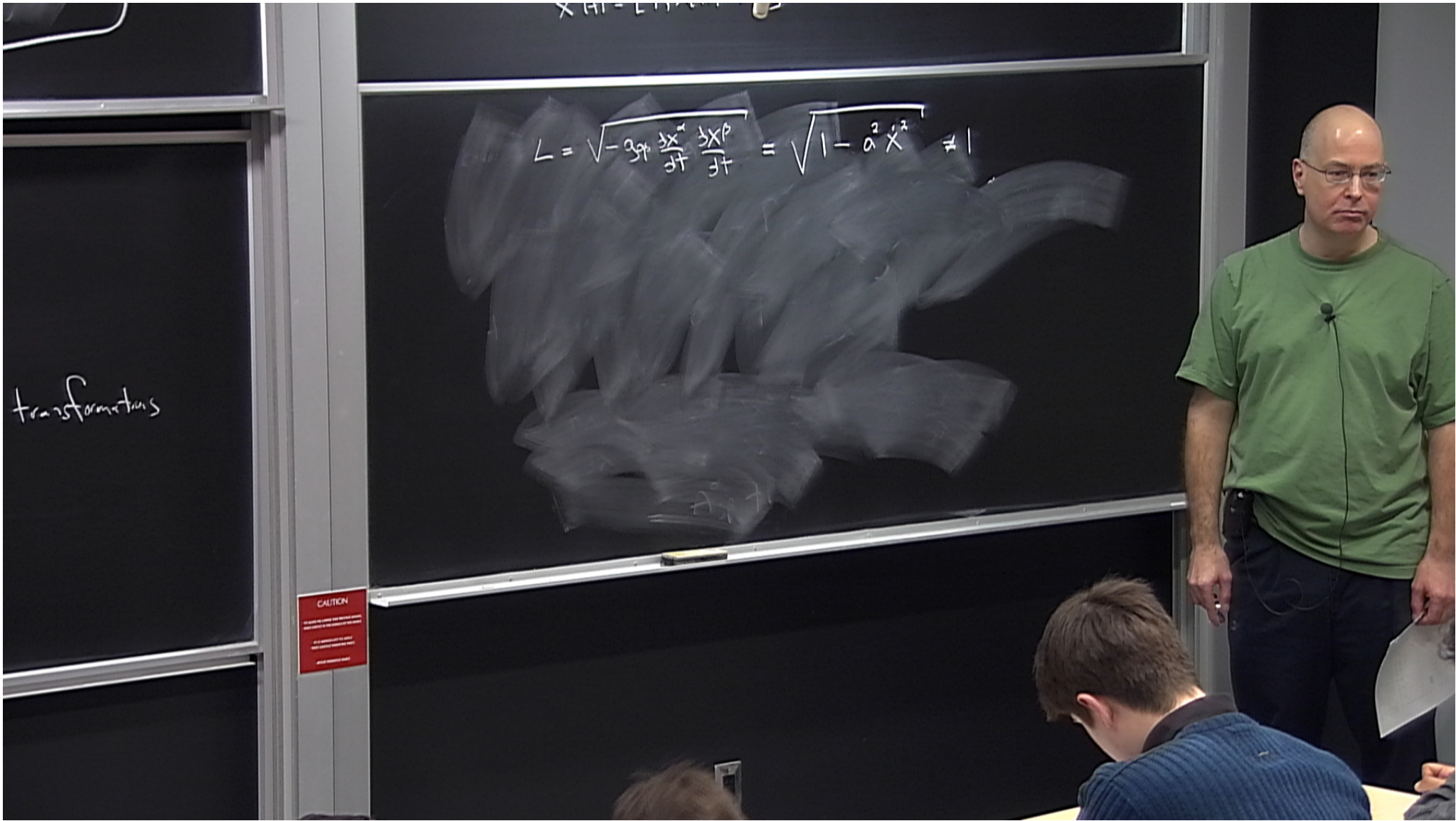
timelike geodesic moving along  
x-direction; pick parameter  $\equiv t$   
 $x^\alpha(t) = [t, x(t), 0, 0]$

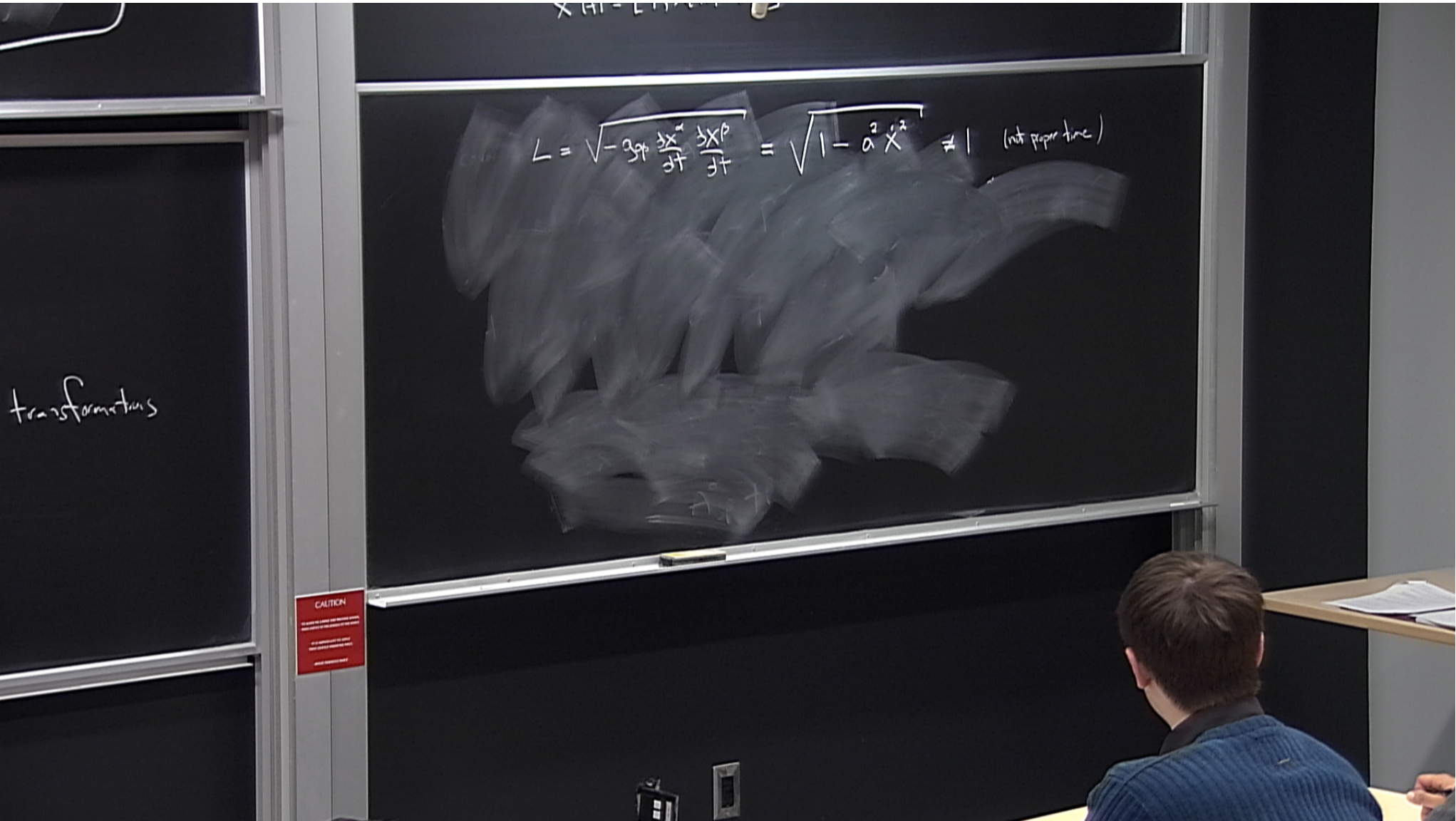


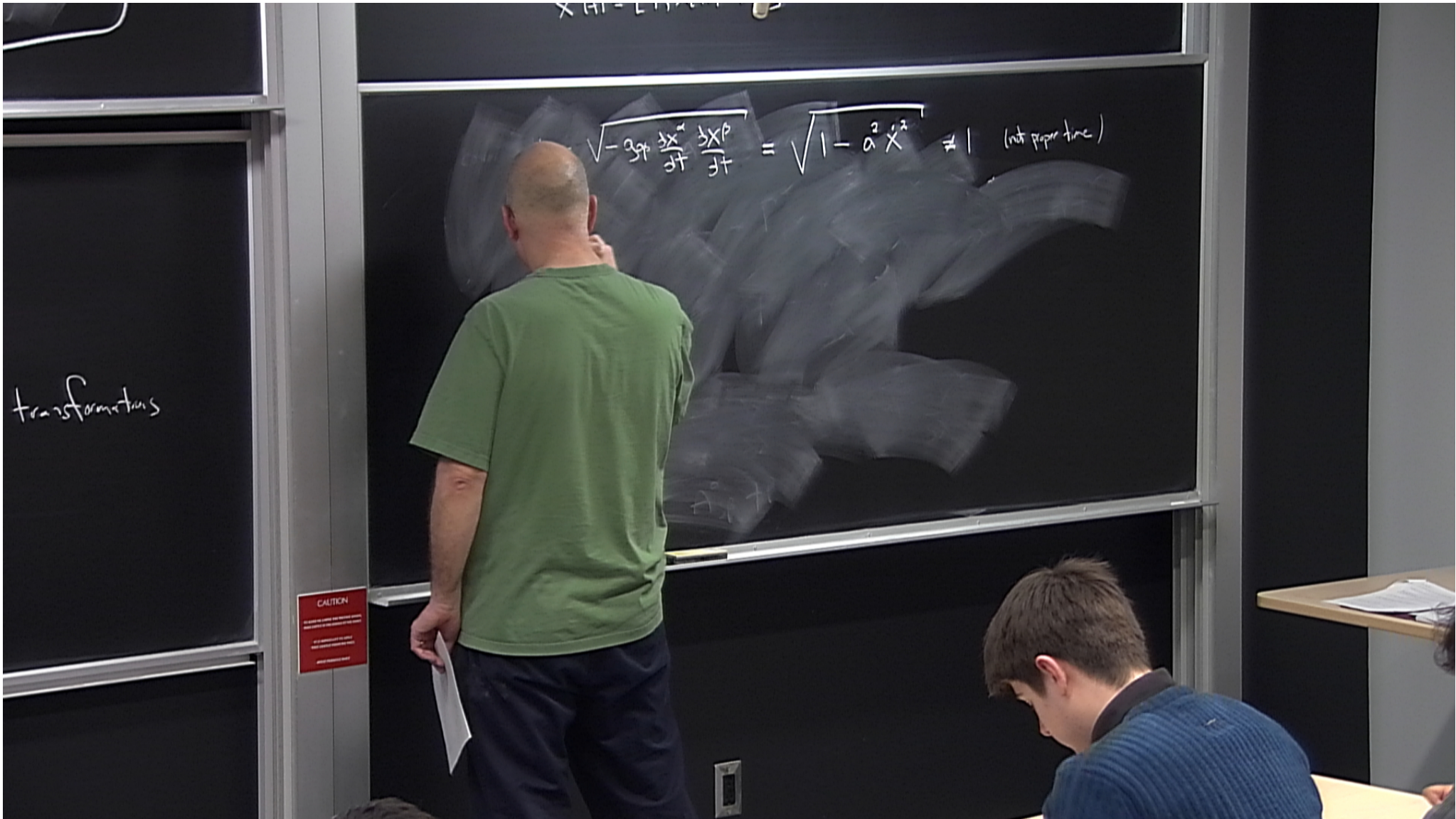




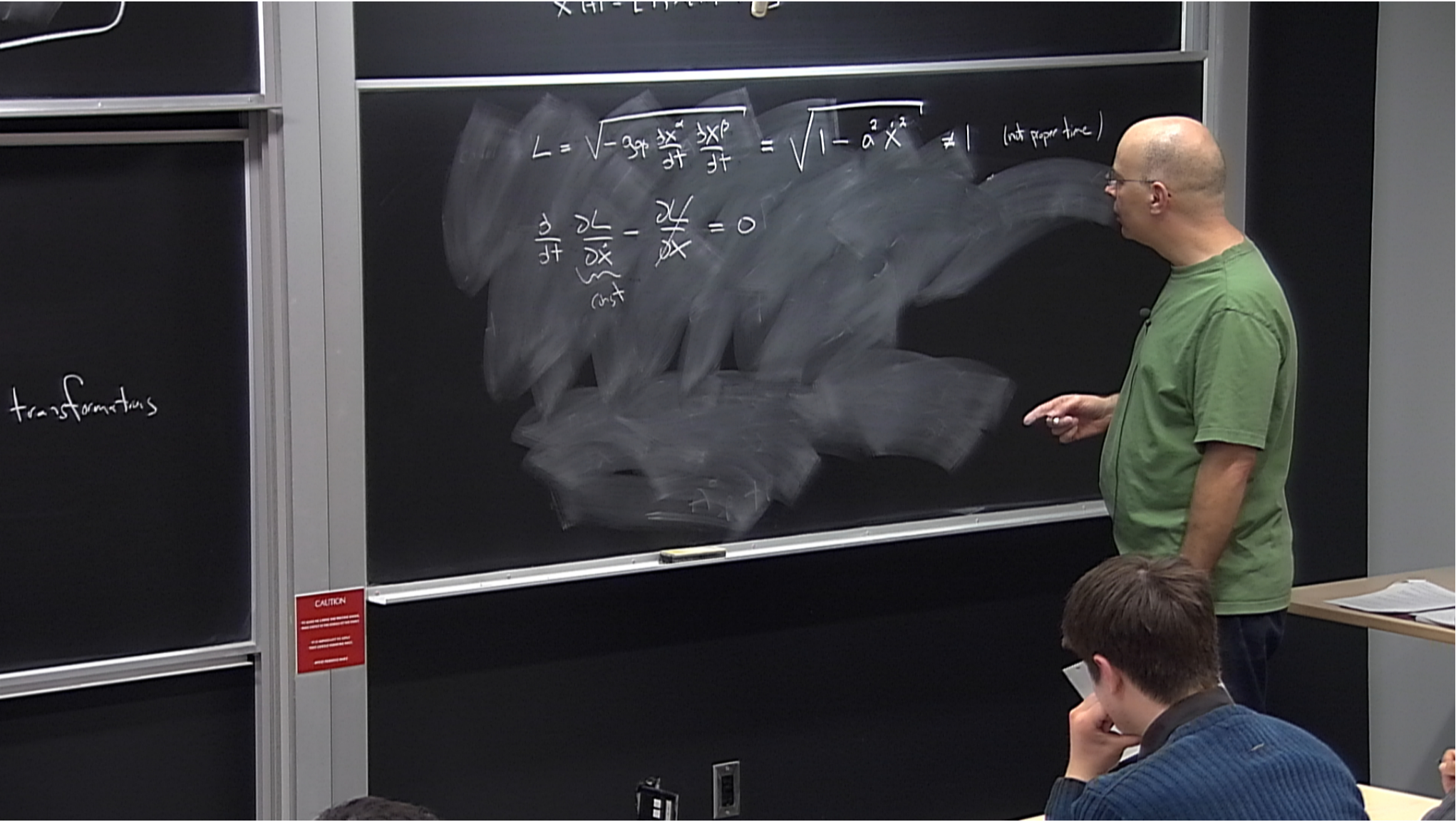












transformations

transformations

$$L = \sqrt{-2\gamma \frac{\partial X}{\partial t} \frac{\partial X P}{\partial t}} = \sqrt{1 - a^2 \dot{x}^2} \neq 1 \text{ (not proper time)}$$

$$\frac{\partial}{\partial t} \underbrace{\frac{\partial L}{\partial \dot{x}}}_{\text{const}} - \frac{\partial L}{\partial x} = 0 \quad -p \equiv \frac{\partial L}{\partial \dot{x}} = \frac{1}{L} (-2a^2 \dot{x})$$

$$p = \frac{a^2 \dot{x}}{\sqrt{1 - a^2 \dot{x}^2}}$$

solve for  $\dot{x}$

CAUTION  
DO NOT TOUCH THE BOARD WHEN IT IS BEING USED BY ANOTHER PERSON

transformations

$$L = \sqrt{-g_{ij} \frac{\partial x^i}{\partial t} \frac{\partial x^j}{\partial t}} = \sqrt{1 - a^2 \dot{x}^2} \neq 1 \quad (\text{not proper time})$$

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \quad -p \equiv \frac{\partial L}{\partial \dot{x}} = \frac{1}{L} (-2a^2 \dot{x})$$

$$p = \frac{a^2 \dot{x}}{\sqrt{1 - a^2 \dot{x}^2}}$$

solve for  $\dot{x} \Rightarrow \dot{x} = \frac{p}{\sqrt{1 + a^2 p^2}}$

CAUTION

transformations

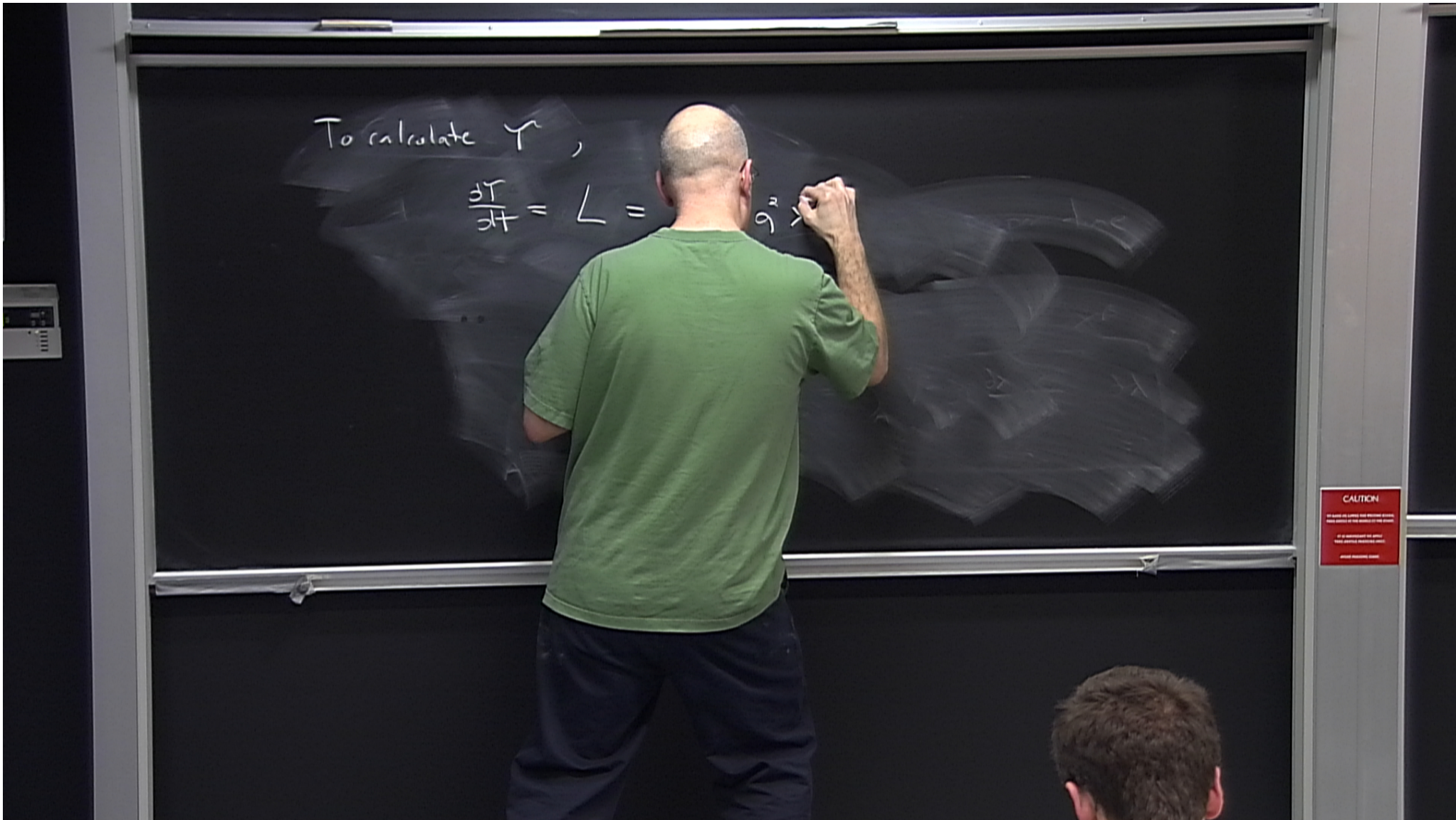
$$L = \sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial t} \frac{\partial x^\nu}{\partial t}} = \sqrt{1 - a^2 \dot{x}^2} \neq 1 \quad (\text{not proper time})$$

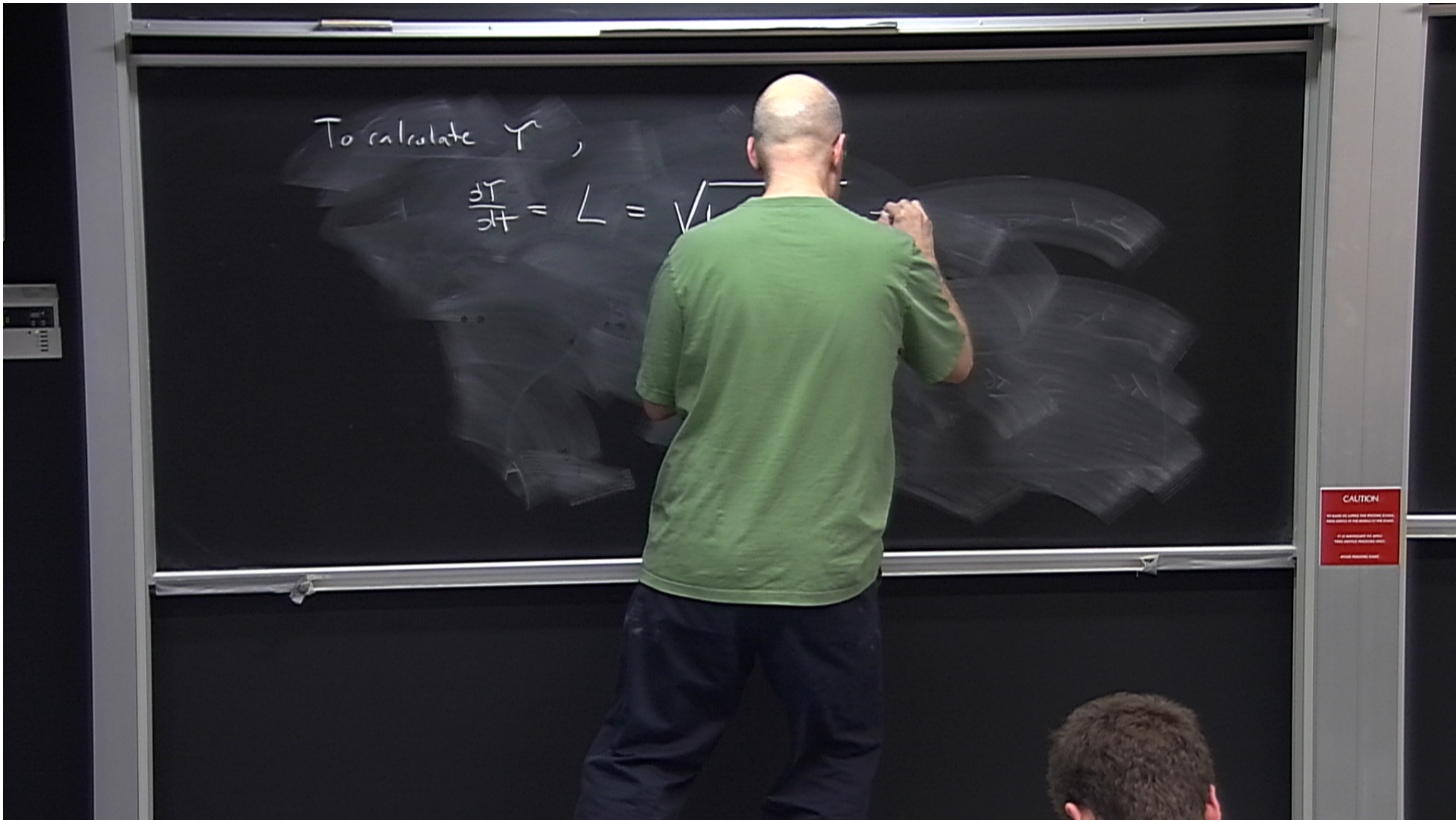
$$\frac{\partial}{\partial t} \underbrace{\frac{\partial L}{\partial \dot{x}}}_{\text{const}} - \frac{\partial L}{\partial x} = 0 \quad -p \equiv \frac{\partial L}{\partial \dot{x}} = \frac{1}{L} (-2a^2 \dot{x})$$

$$p = \frac{a^2 \dot{x}}{\sqrt{1 - a^2 \dot{x}^2}}$$

solve for  $\dot{x} \Rightarrow$  
$$\dot{x} = \frac{p}{a\sqrt{p^2 + a^2}}$$

CAUTION





To calculate  $\Upsilon$ ,

$$\frac{dT}{dt} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

If we use  $T$  as parameter,

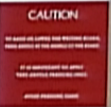
To calculate  $\gamma$ ,

$$\frac{dt}{dT} = \gamma = \frac{1}{\sqrt{1 - a^2 \dot{x}^2}} = \frac{a}{\sqrt{p^2 + a^2}}$$

$p$  as parameter,

$$\frac{dt}{dT} = \frac{\sqrt{p^2 + a^2}}{a(t)}$$

$$\frac{dx}{dT} = \frac{p}{a^2(t)}$$





To calculate  $\Upsilon$ ,

$$\frac{dt}{dT} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

If we use  $T$  as parameter,

$$\frac{dt}{dT} = \frac{\sqrt{p^2 + a^2}}{a(t)}$$

$$\frac{dx}{dT} = \frac{p}{a^2(t)}$$

CAUTION  
Do not touch the screen or the board.  
If it is necessary to touch the screen, please use the touch screen.  
Please do not touch the screen.

To calculate  $\Upsilon$ ,

$$\frac{dt}{dT} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

If we use  $T$  as parameter,

$$\frac{dt}{dT} = \frac{\sqrt{p^2 + a^2}}{a}$$
$$\frac{dx}{dT} = \frac{p}{a\sqrt{p^2 + a^2}}$$

} more diff

To calculate  $\Upsilon$ ,

$$\frac{\partial T}{\partial t} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

If we use  $T$  as parameter,

$$\frac{\partial t}{\partial T} = \frac{\sqrt{p^2 + a^2}}{a(t)}$$

} more difficult to integrate,

$$\frac{\partial x}{\partial T} = \frac{p}{a^2(t)}$$

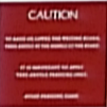
To calculate  $\Upsilon$ ,

$$\frac{\partial T}{\partial t} = \sqrt{g^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

If we use

$$\frac{\partial T}{\partial t}$$
$$\frac{\partial \Upsilon}{\partial t}$$

more difficult  
to integrate, because  
 $a$  is known as a function  
of  $t$ , not  $T$ .



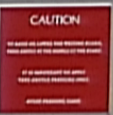
To calculate

$$L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$$

T as parameter,

$$\frac{\sqrt{p^2 + a^2}}{a^2(t)}$$

} more difficult  
to integrate, because  
a is known as a function  
of t, not T.



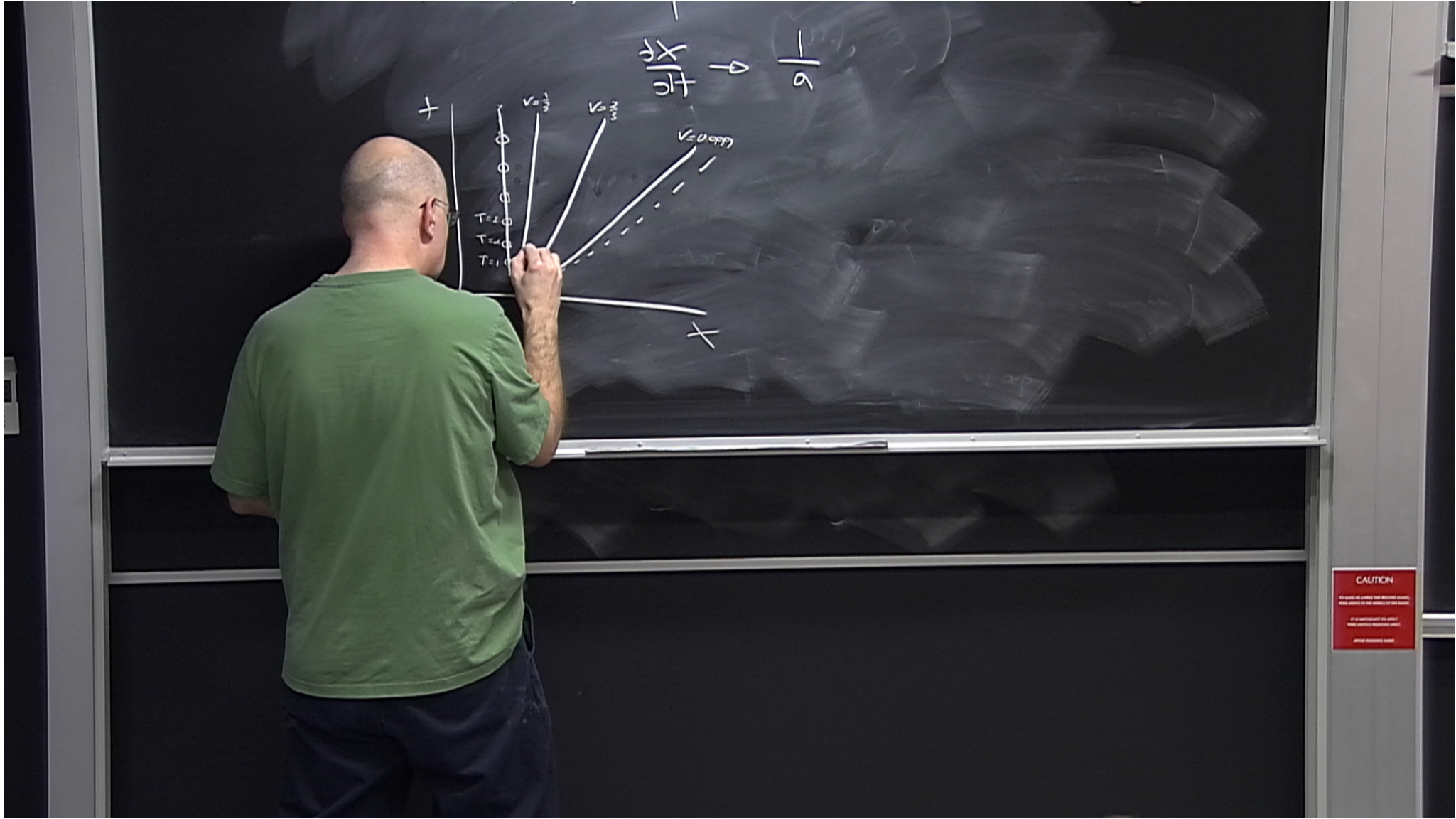
Null limit:  $p \rightarrow \infty \Rightarrow L=0$

$$\frac{dx}{dt} \rightarrow$$

Null 1

$$p \rightarrow 8 \Rightarrow L=0$$

$$\frac{X}{H} \rightarrow \frac{-1}{a}$$







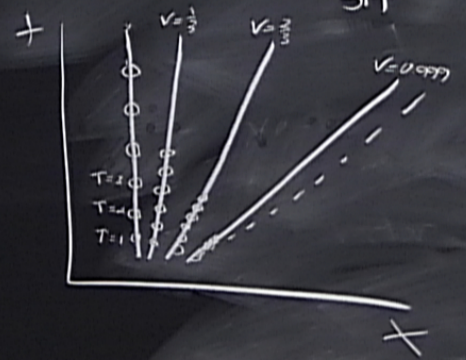
$$\frac{dt}{dT} = \frac{\sqrt{p^2 + a^2(t)}}{a(t)}$$

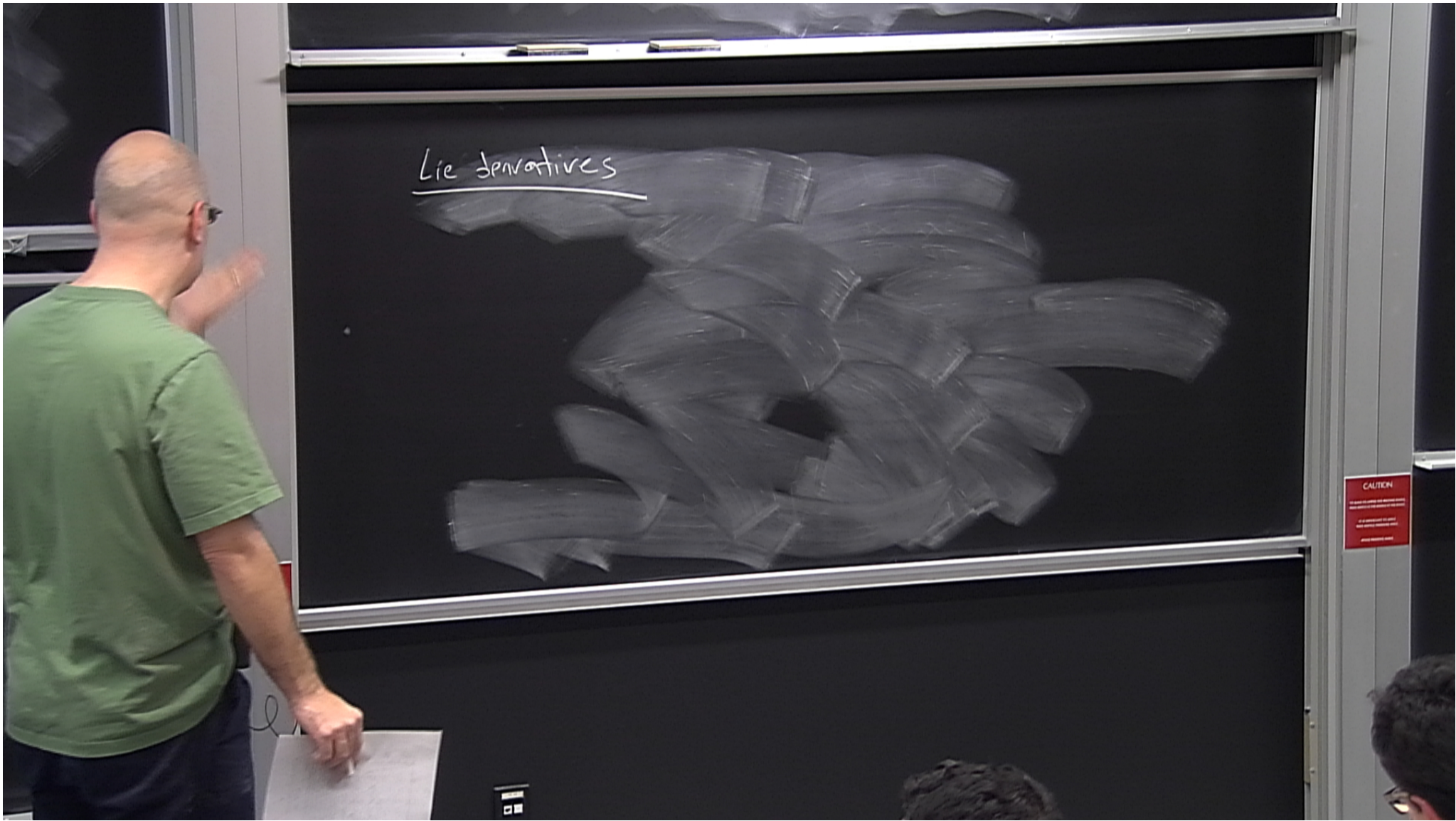
$$\frac{dx}{dT} = \frac{p}{a^2(t)}$$

more difficult to integrate, because  $a$  is known as a function of  $t$ , not  $T$ .

Null limit:  $p \rightarrow \infty \Rightarrow$

$$\frac{dx}{dT} \rightarrow \frac{1}{a}$$





Lie derivatives

vector:  $\mathcal{L}_U A^\alpha = A^\alpha_{;\beta} U^\beta - U$

CAUTION  
DO NOT TOUCH THE BOARD  
OR THE BOARDER'S HEAD  
OR THE BOARDER'S HEAD

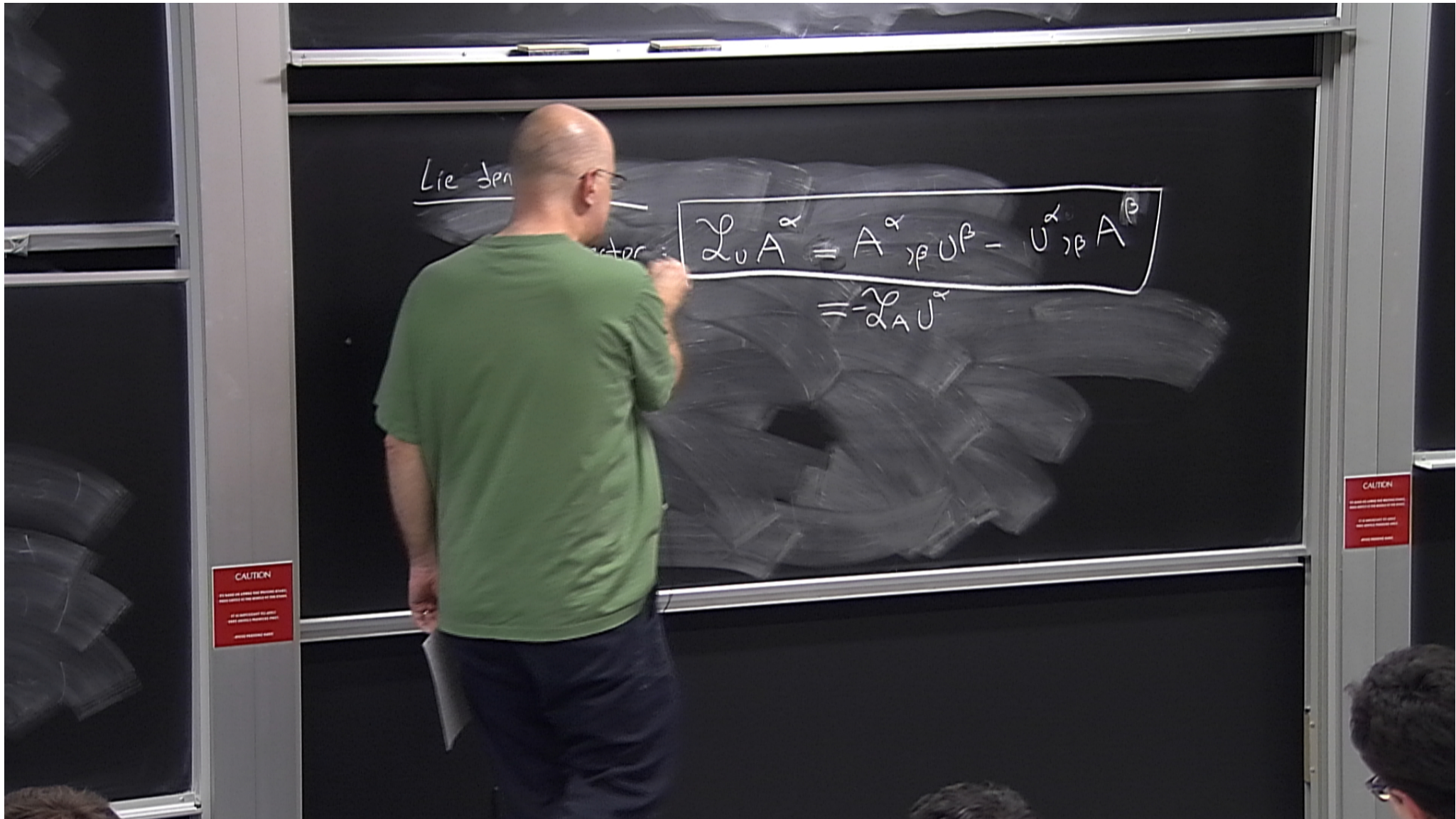
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Lie derivatives

vector:  $\mathcal{L}_U A^\alpha = A^\alpha{}_{;\beta} U^\beta - U^\alpha{}_{;\beta} A^\beta$

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OR THE BOARDER'S  
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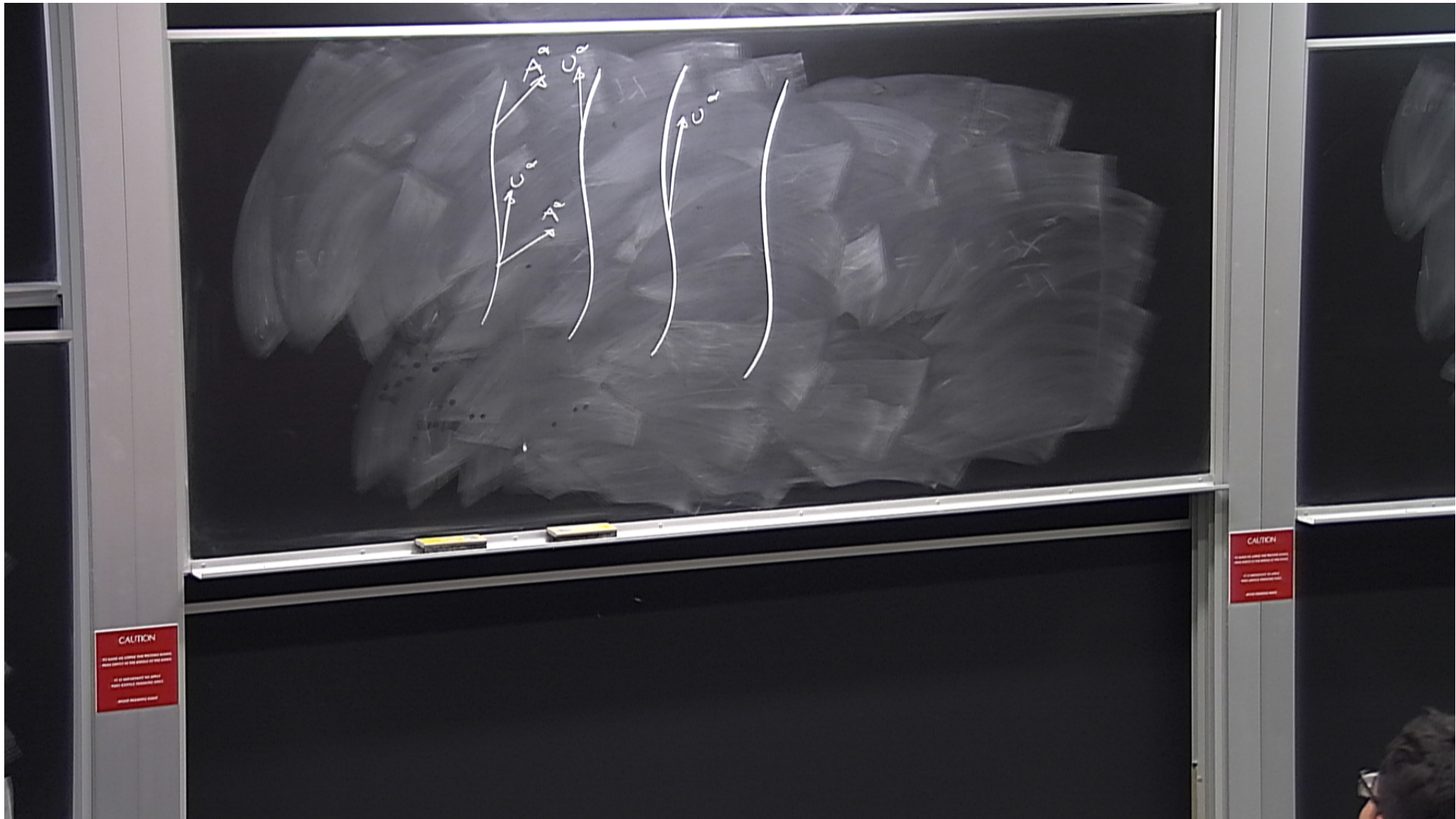
Lie derivatives

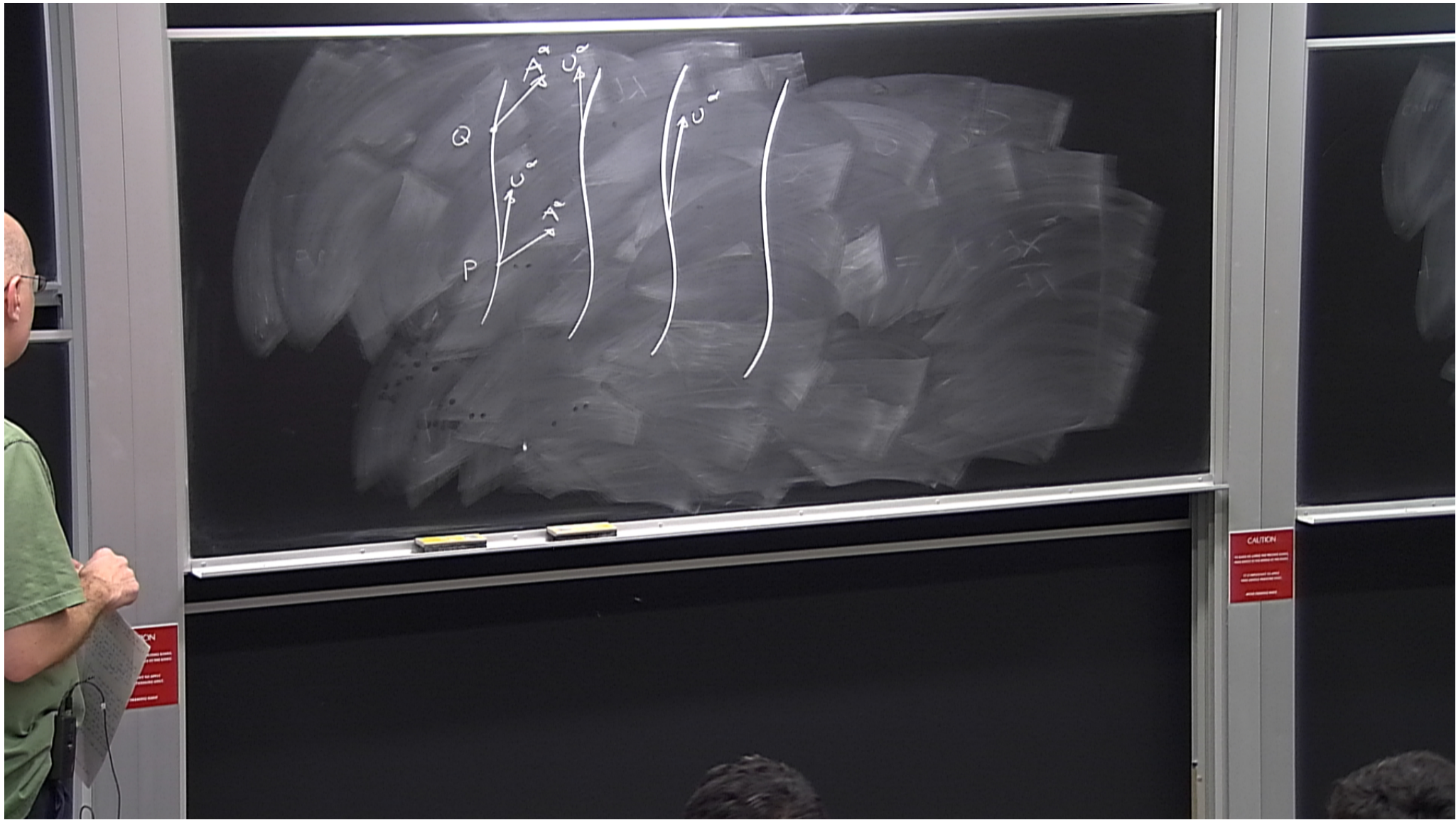
vector:  $\mathcal{L}_U A^\alpha = A^\alpha{}_{;\beta} U^\beta - U^\alpha{}_{;\beta} A^\beta$   
 $= -\mathcal{L}_A U^\alpha$

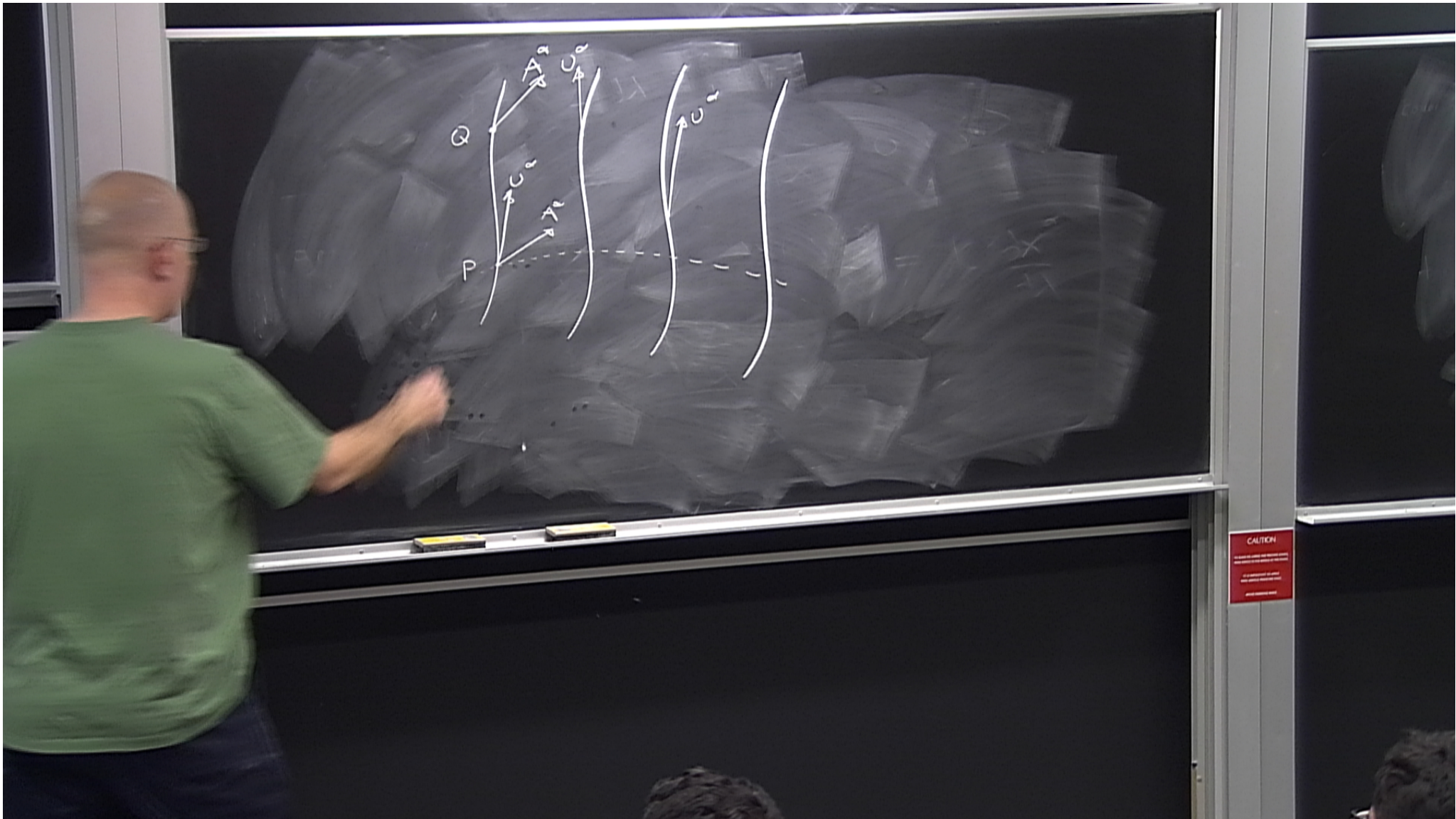




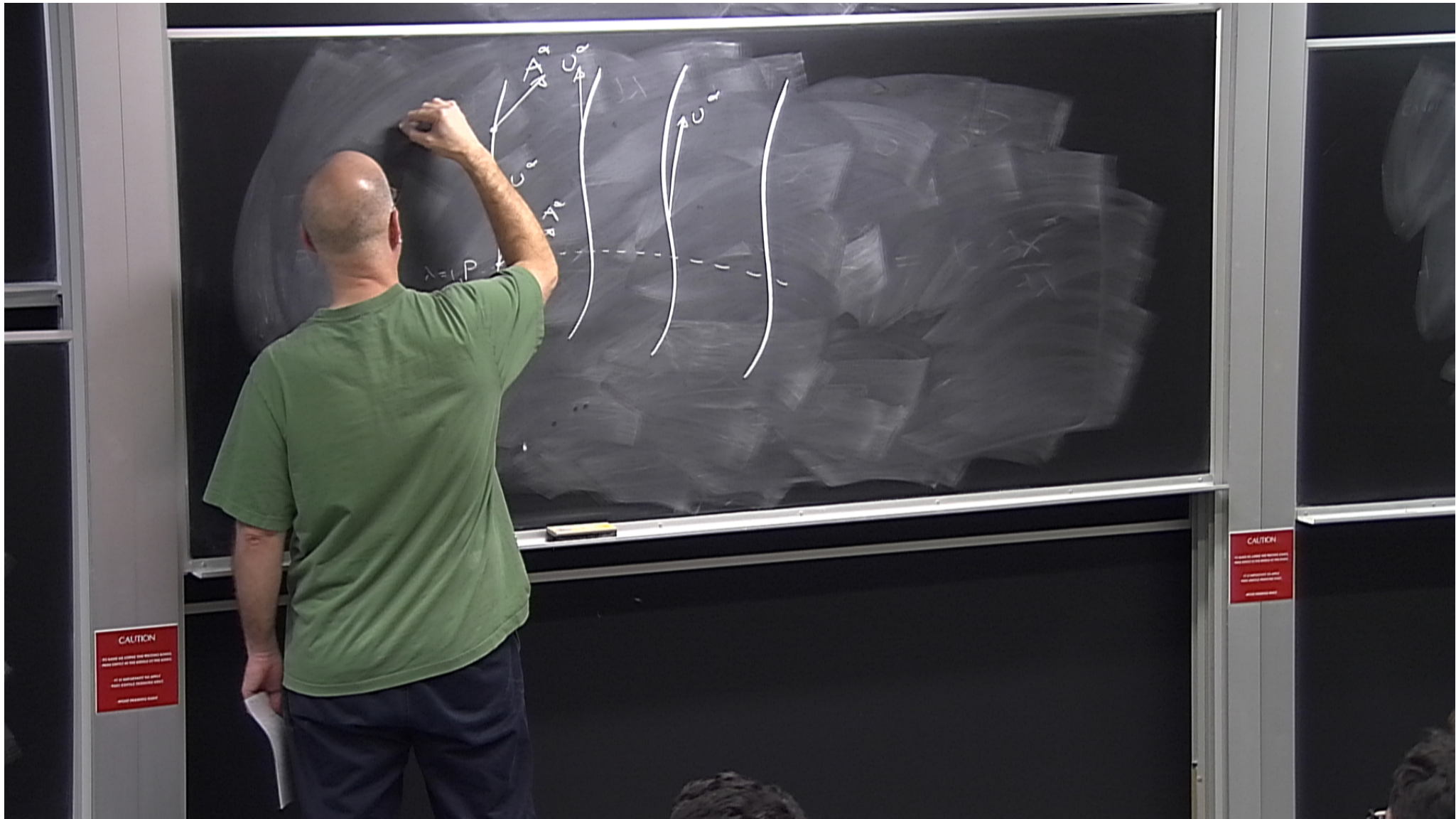


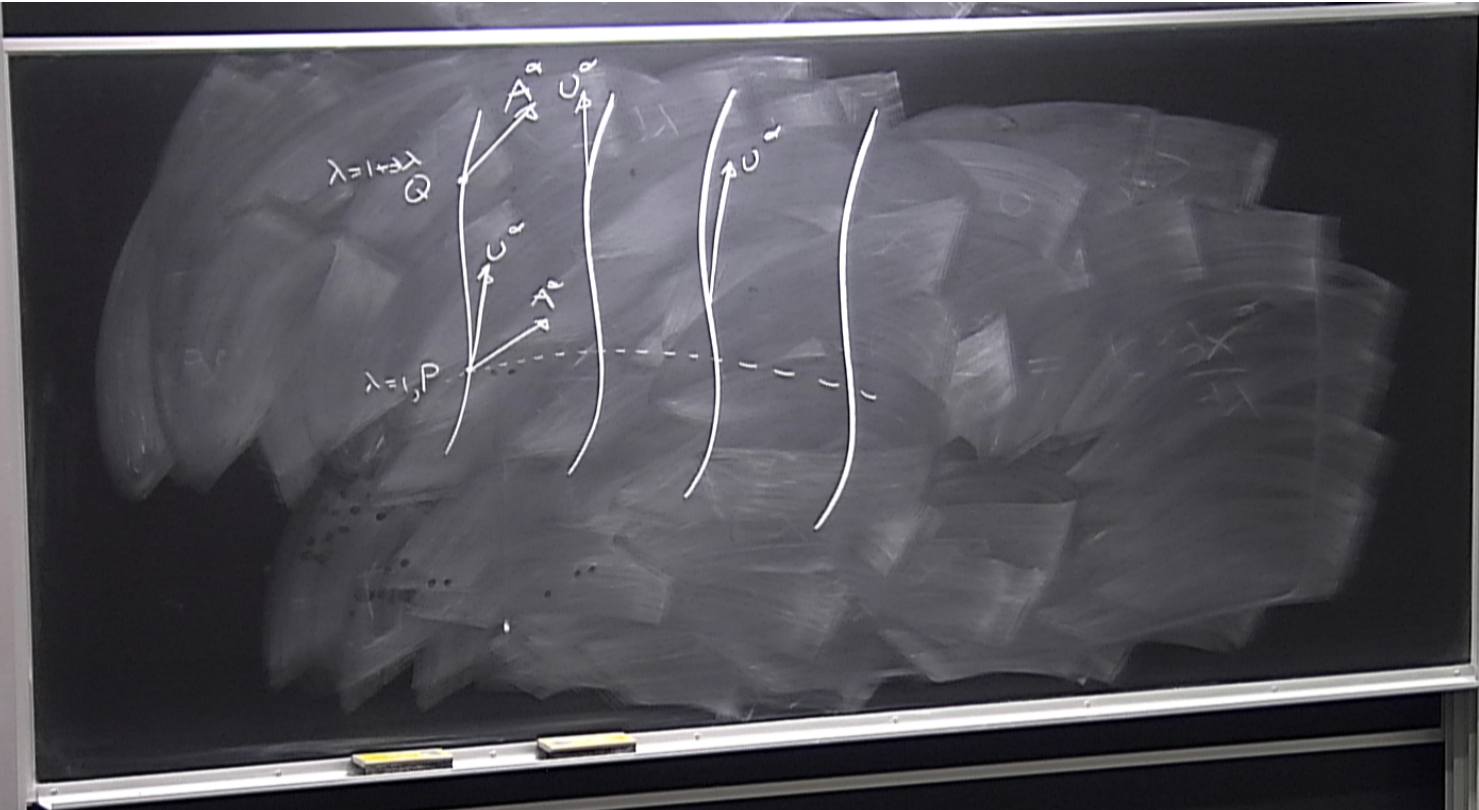






CAUTION  
DO NOT TOUCH THE BOARD  
IF YOU NEED TO CLEAN  
USE THE ERASER  
DO NOT WRITE ON THE BOARD

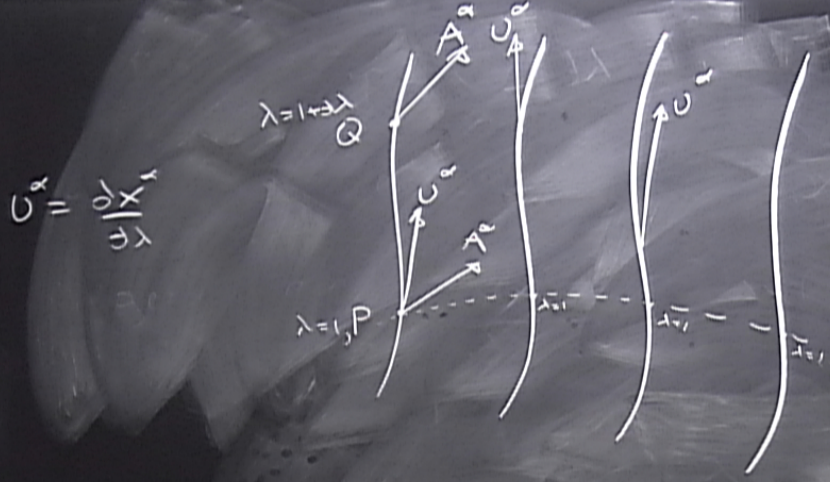




**CAUTION**  
 ALL SURFACES ARE HOT AND SHOULD BE HANDLED WITH CARE.  
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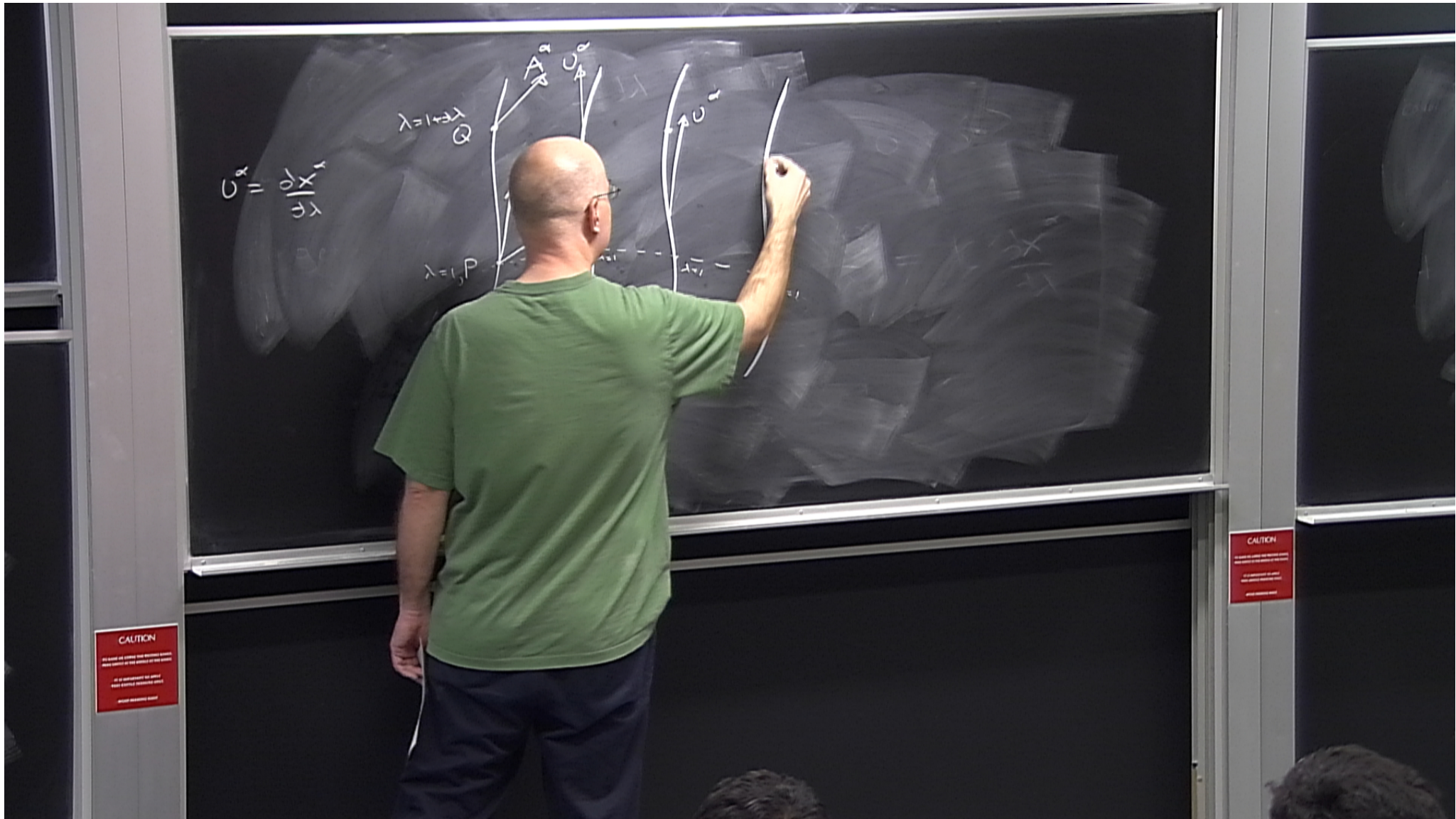
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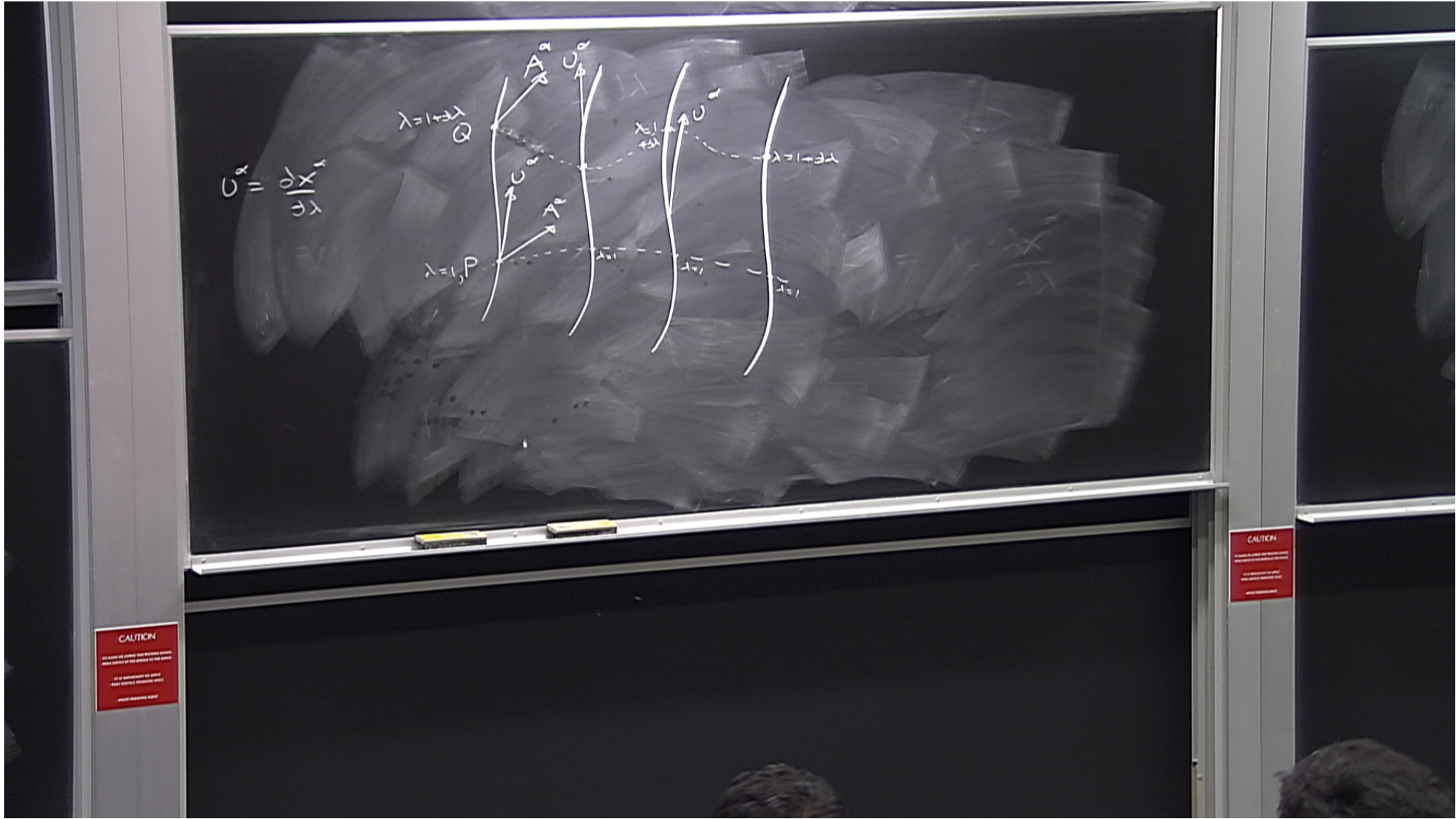


CAUTION

CAUTION



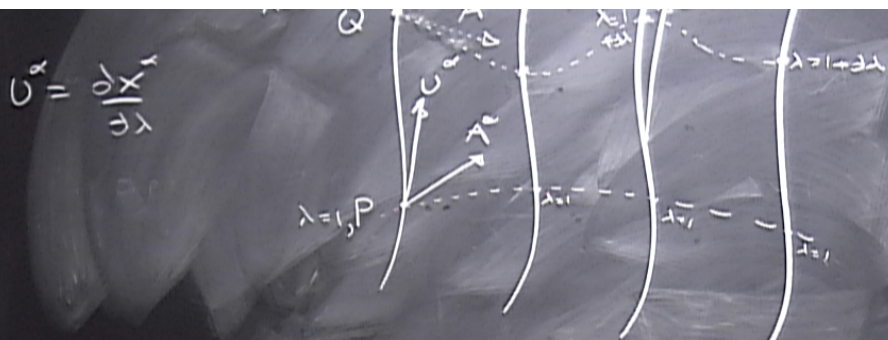




CAUTION  
No leaning on tables and no reaching across  
other people at the tables or the boards.  
All participants are asked  
to remain seated at all  
times.  
Thank you for your attention.

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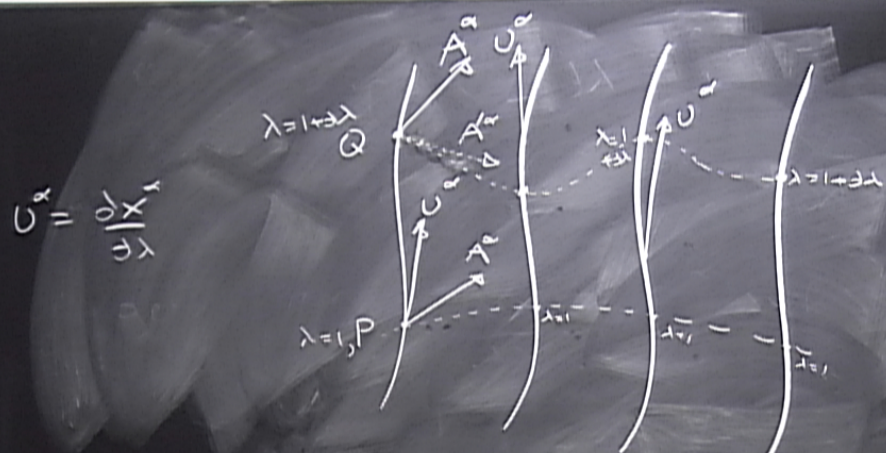




$$\frac{d}{d\lambda} A^\alpha(P) \equiv \frac{A^\alpha(Q) - A^{\alpha}(Q)}{d\lambda}$$

CAUTION  
 Do not lean on the chalkboard panel  
 when seated at the desk or the bench.  
 All measurements are subject  
 to a certain uncertainty.  
 Please observe safety.

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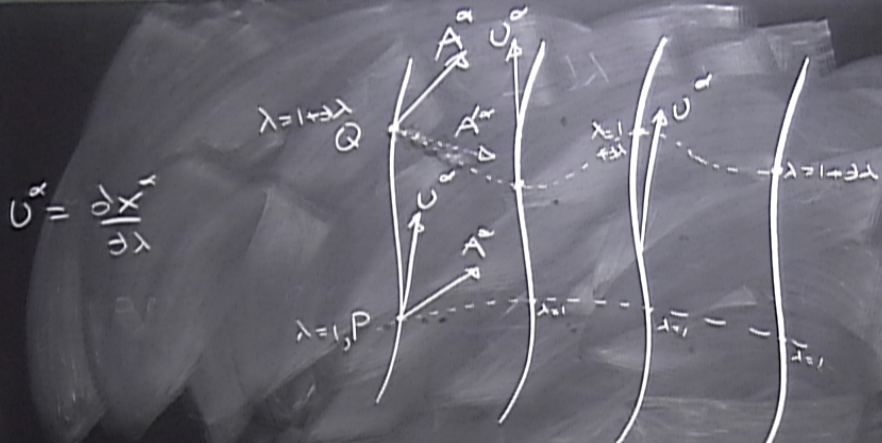


$$U^\alpha = \frac{\partial X^\alpha}{\partial \lambda}$$

$$\frac{\partial A^\alpha(P)}{\partial \lambda} \equiv \frac{A^\alpha(Q) - A^\alpha(P)}{\Delta \lambda}$$

CAUTION  
Do not touch the control panel  
or the control panel or the control panel  
or the control panel or the control panel  
or the control panel or the control panel

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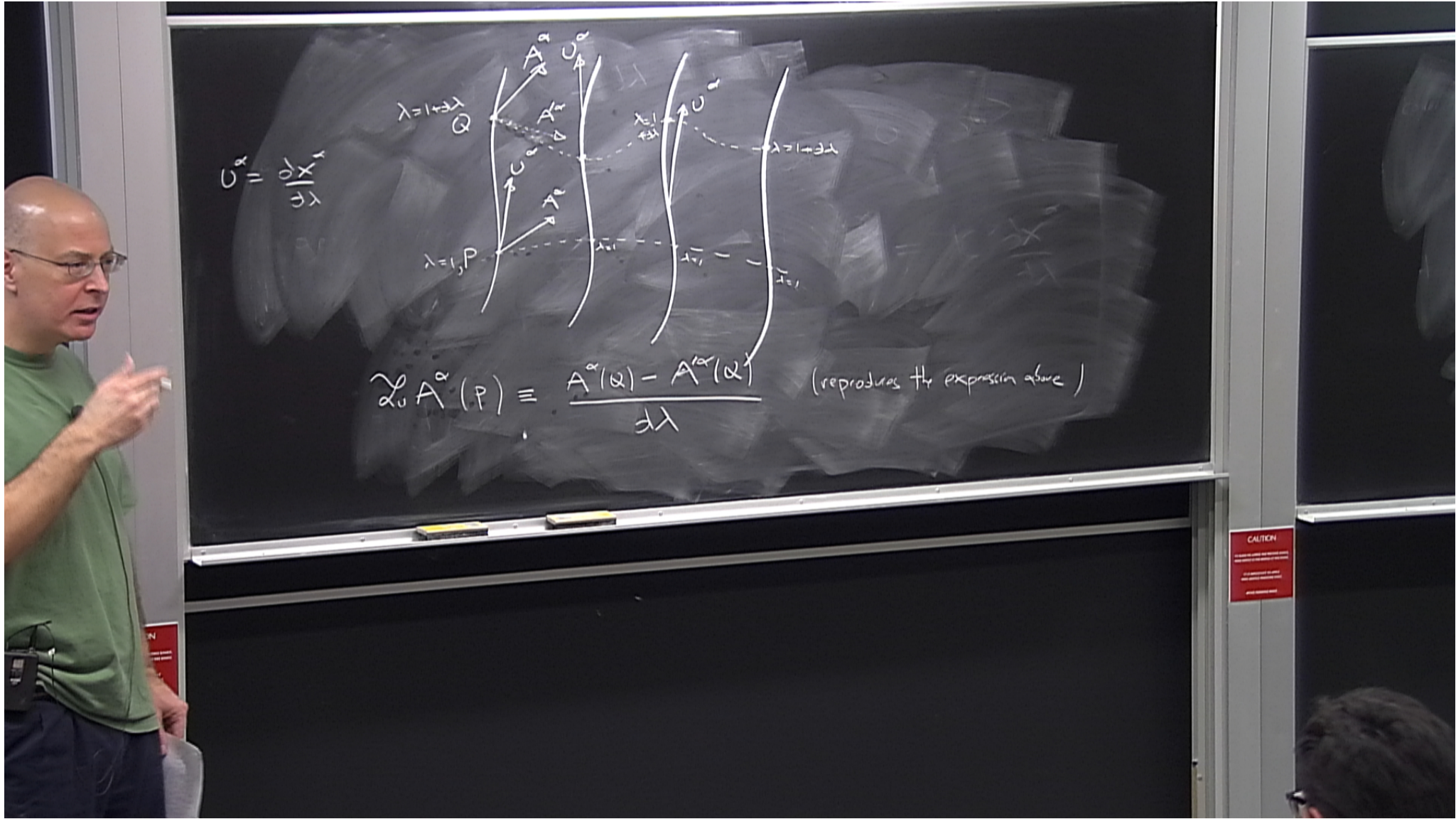


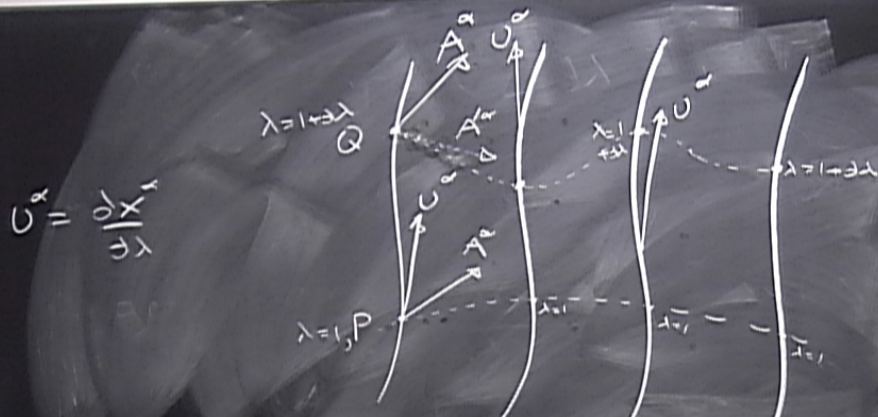
$$U^\alpha = \frac{\partial \chi_\lambda}{\partial \lambda}$$

$$\frac{d}{d\lambda} A^\alpha(P) \equiv \frac{A^\alpha(Q) - A^{\alpha\prime}(Q)}{\Delta\lambda} \quad (\text{reproduces the expression above})$$

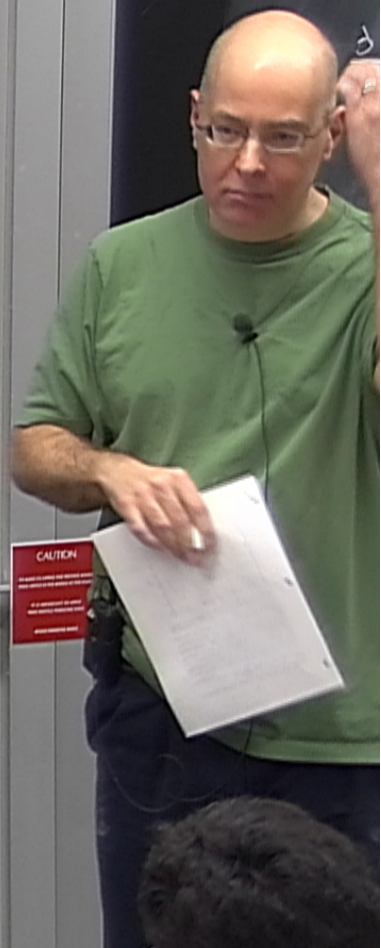
CAUTION  
 Do not touch the board when the board is in use.  
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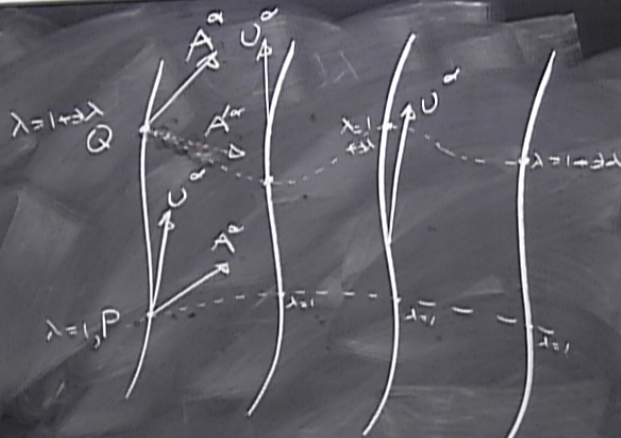




$$\lim_{\Delta\lambda} A^\alpha(P) \equiv \frac{A^\alpha(Q) - A^\alpha(Q')}{\Delta\lambda} \quad (\text{reproduces the expression above})$$



$U^\alpha \rightarrow$  integral curves



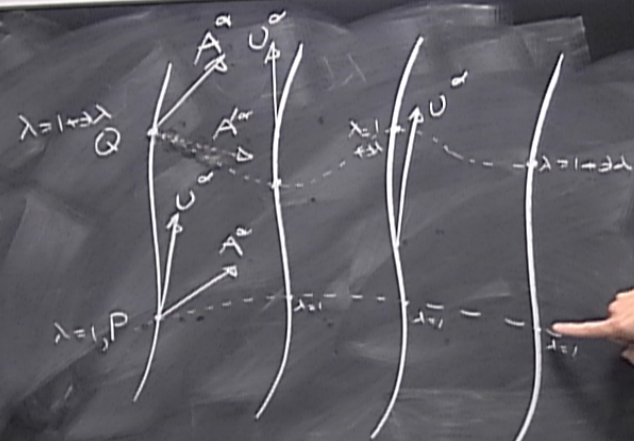
$$\oint_{\gamma} A^\alpha(P) \equiv \frac{A^\alpha(Q) - A^\alpha(Q')}{\delta \lambda} \quad (\text{reproduces the expression above})$$

CAUTION  
DO NOT TOUCH THE BOARD  
OR THE SURFACE OF THE BOARD  
OR THE SURFACE OF THE BOARD  
OR THE SURFACE OF THE BOARD



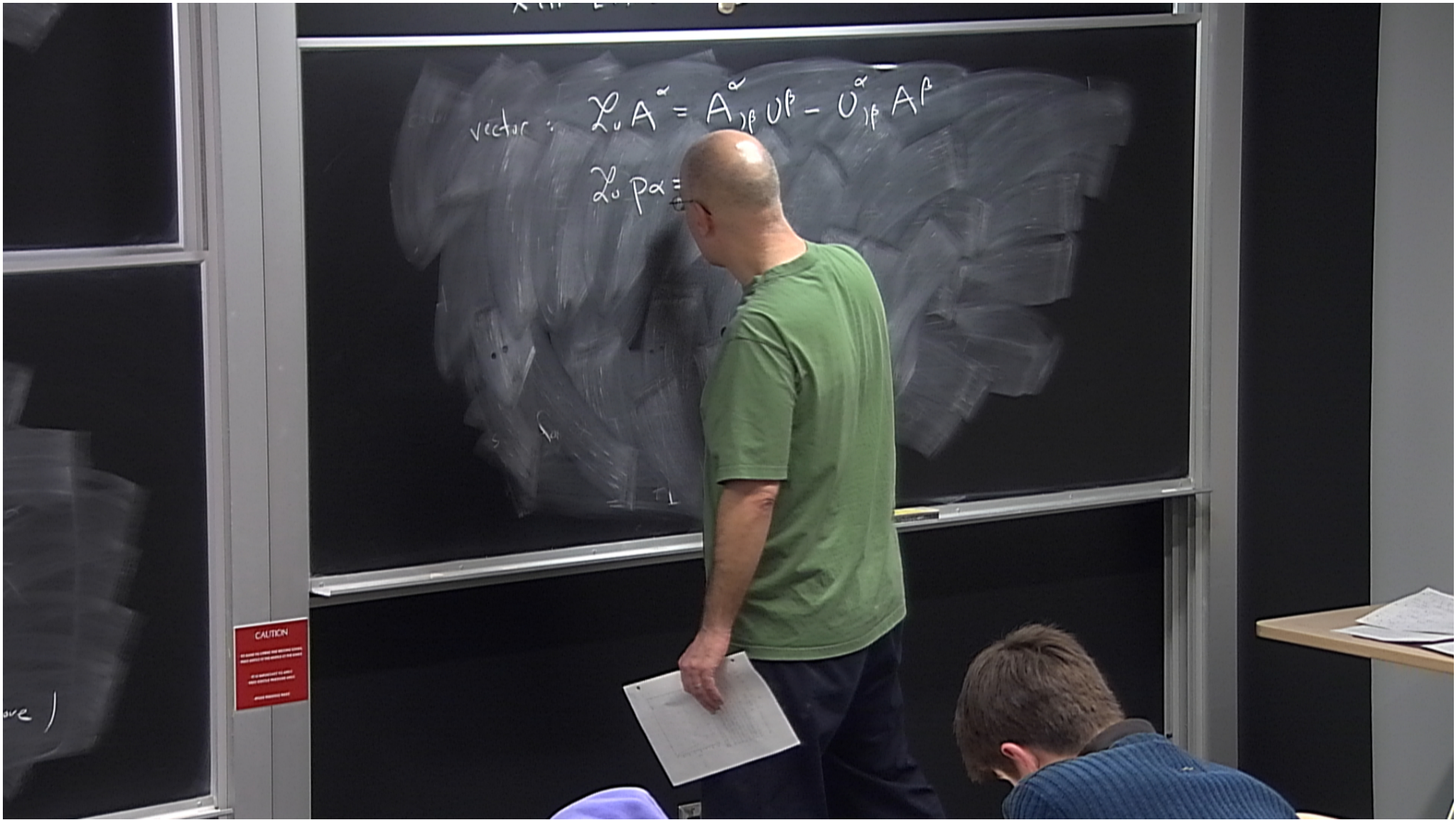
$U^\alpha \rightarrow$  integral curves

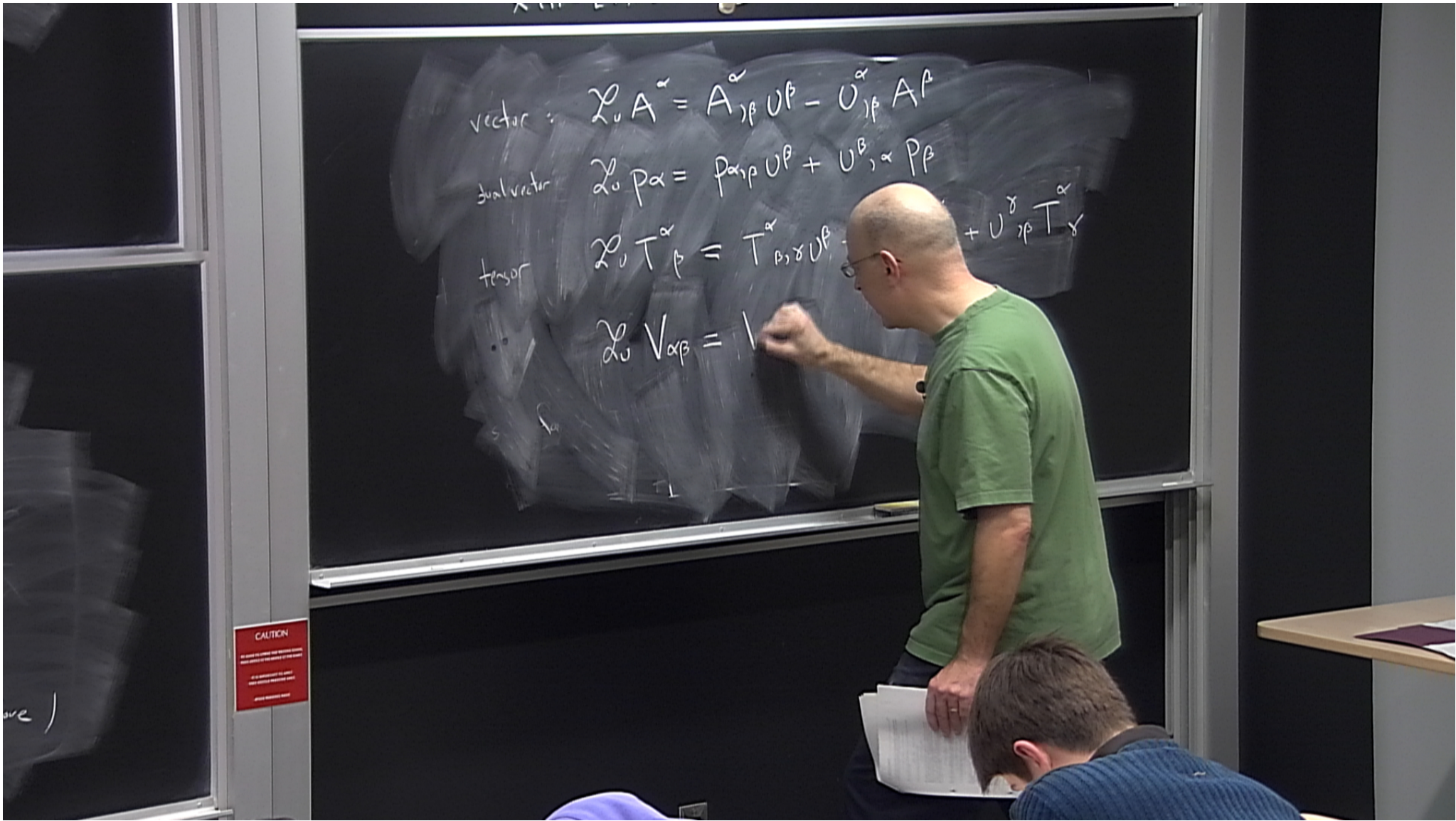
$$X^\alpha(\lambda) : \begin{cases} \frac{dX^\alpha}{d\lambda} = U^\alpha \\ \text{---} \end{cases}$$



$$\oint_{\alpha} A^\alpha(P) \equiv \frac{A^\alpha(Q) - A^\alpha(Q')}{d\lambda} \quad (\text{reproduces the expression above})$$

CAUTION  
 ALL USERS ARE ADVISED THAT THIS BOARD IS NOT TO BE USED FOR ANY PURPOSES OTHER THAN THAT OF A CHALKBOARD.  
 IT IS PROHIBITED TO WRITE ANYTHING ON THIS BOARD EXCEPT FOR CHALK.





vector:  $\sum_{\beta} A^{\alpha} = A^{\alpha}_{\beta} U^{\beta} - U^{\alpha}_{\beta} A^{\beta}$

dual vector:  $\sum_{\beta} p_{\alpha} = p_{\alpha\beta} U^{\beta} + U^{\beta}_{\alpha} p_{\beta}$

tensor:  $\sum_{\beta} T^{\alpha}_{\beta} = T^{\alpha}_{\beta\gamma} U^{\beta} + U^{\delta}_{\beta} T^{\alpha}_{\gamma}$

$\sum_{\alpha\beta} V_{\alpha\beta} = 1$

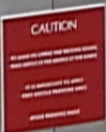
CAUTION  
DO NOT TOUCH THE BOARD  
OR THE BOARDER'S HEAD  
WHILE WORKING ON IT

vector:  $\mathcal{L}_U A^\alpha = A^\alpha_{;\beta} U^\beta - U^\alpha_{;\beta} A^\beta$

dual vector:  $\mathcal{L}_U p_\alpha = p_{\alpha;\beta} U^\beta + U^\beta_{;\alpha} p_\beta$

tensor:  $\mathcal{L}_U T^\alpha_\beta = T^\alpha_{\beta;\gamma} U^\gamma - U^\alpha_{;\gamma} T^\gamma_\beta + U^\gamma_{;\beta} T^\alpha_\gamma$

$$\mathcal{L}_U V_{\alpha\beta} = V_{\alpha\beta;\gamma} U^\gamma + U^\gamma_{;\alpha} V_{\gamma\beta} + U^\gamma_{;\beta} V_{\alpha\gamma}$$



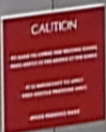
vector:  $\mathcal{L}_U A^\alpha = A^\alpha_{;\beta} U^\beta - U^\alpha_{;\beta} A^\beta$

dual vector:  $\mathcal{L}_U p_\alpha = p_{\alpha;\beta} U^\beta + U^\beta_{;\alpha} p_\beta$

tensor:  $\mathcal{L}_U T^\alpha_\beta = T^\alpha_{\beta;\gamma} U^\gamma - U^\alpha_{;\gamma} T^\gamma_\beta + U^\gamma_{;\beta} T^\alpha_\gamma$

$$\mathcal{L}_U V_{\alpha\beta} = V_{\alpha\beta;\gamma} U^\gamma + U^\gamma_{;\alpha} V_{\gamma\beta} + U^\gamma_{;\beta} V_{\alpha\gamma}$$

scalar:  $\mathcal{L}_U f = f_{;\alpha} U^\alpha$



If we use  $\tau$  as parameter,

Vector  $A^\alpha$  does not depend on  $X^0$ :

$$\frac{\partial A^\alpha}{\partial X^0} = 0$$

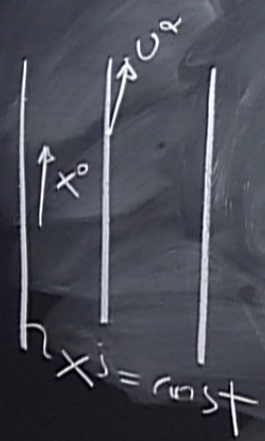
$$U^\alpha = (1, 0, 0, 0)$$

If we use  $\tau$  as parameter,

Vector  $A^\alpha$  does not depend on  $X^0$ :

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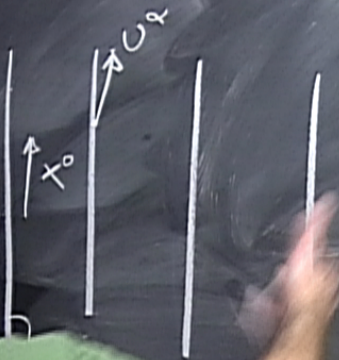


Vector  $A^\alpha$  does not depend on  $x^0$ :

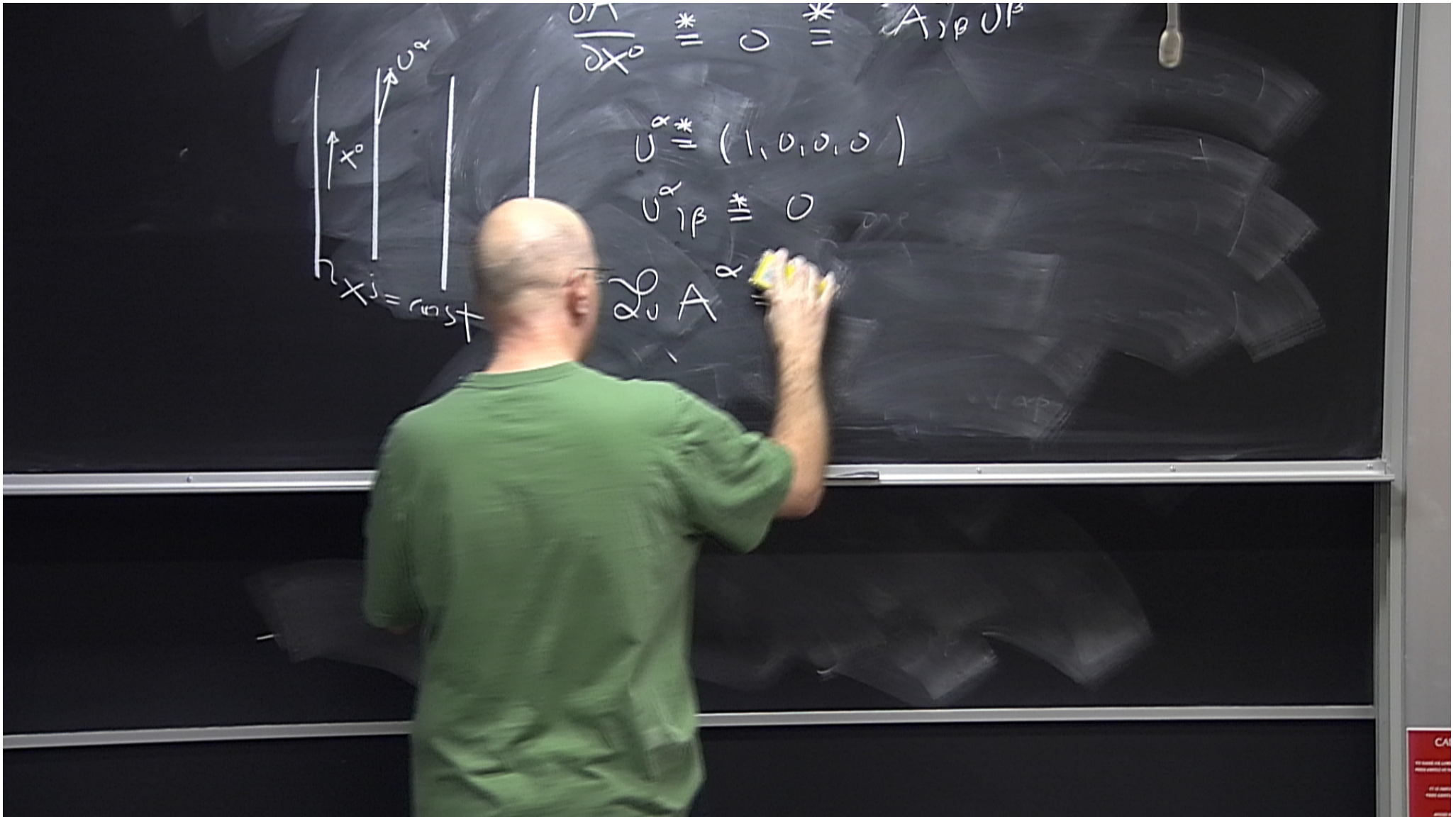
$$\frac{\partial A^\alpha}{\partial x^0} \stackrel{*}{=} 0 \stackrel{*}{=} A^\alpha_{;\beta} U^\beta$$

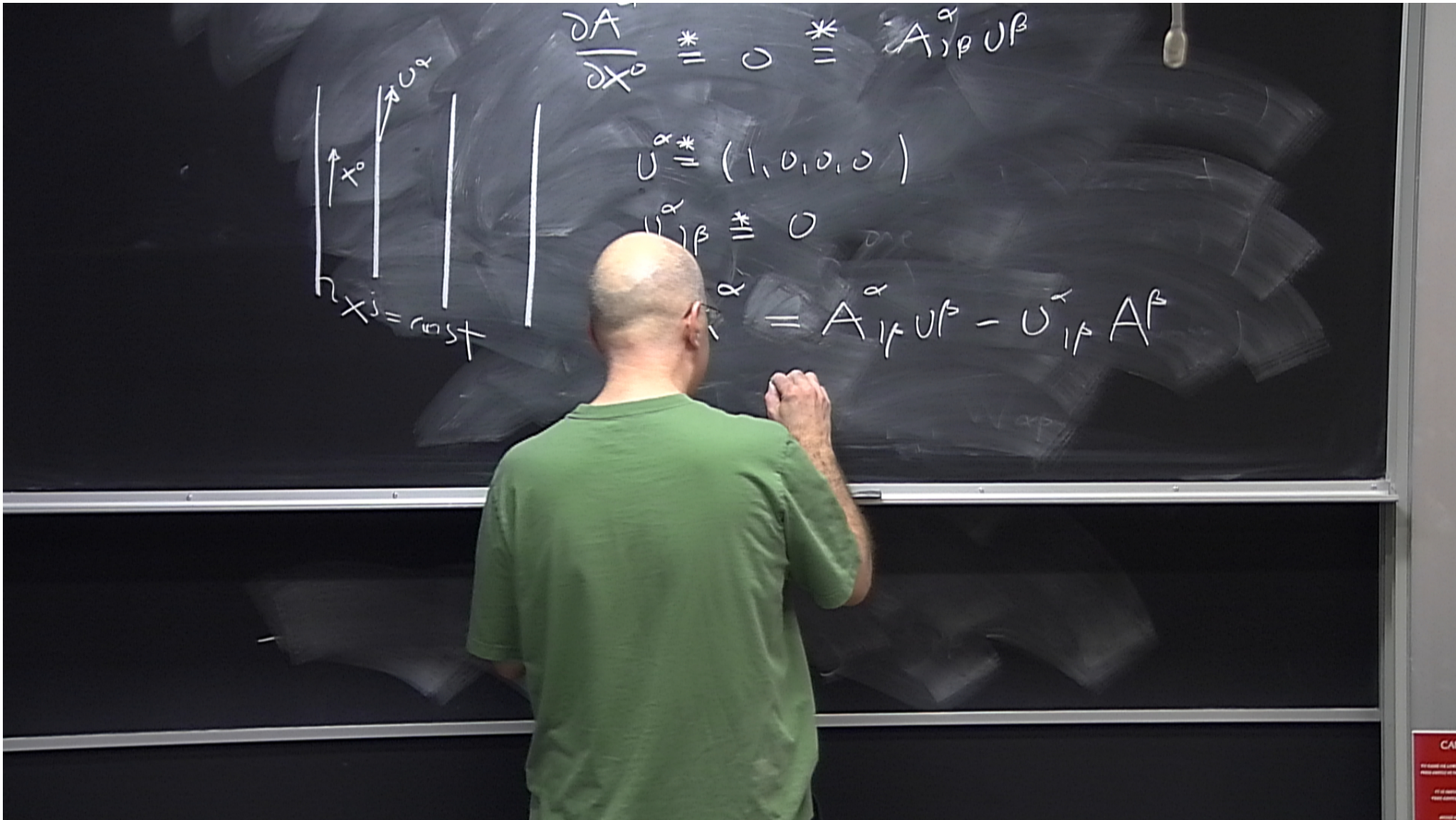
$$U^{\alpha*} = (1, 0, 0, 0)$$

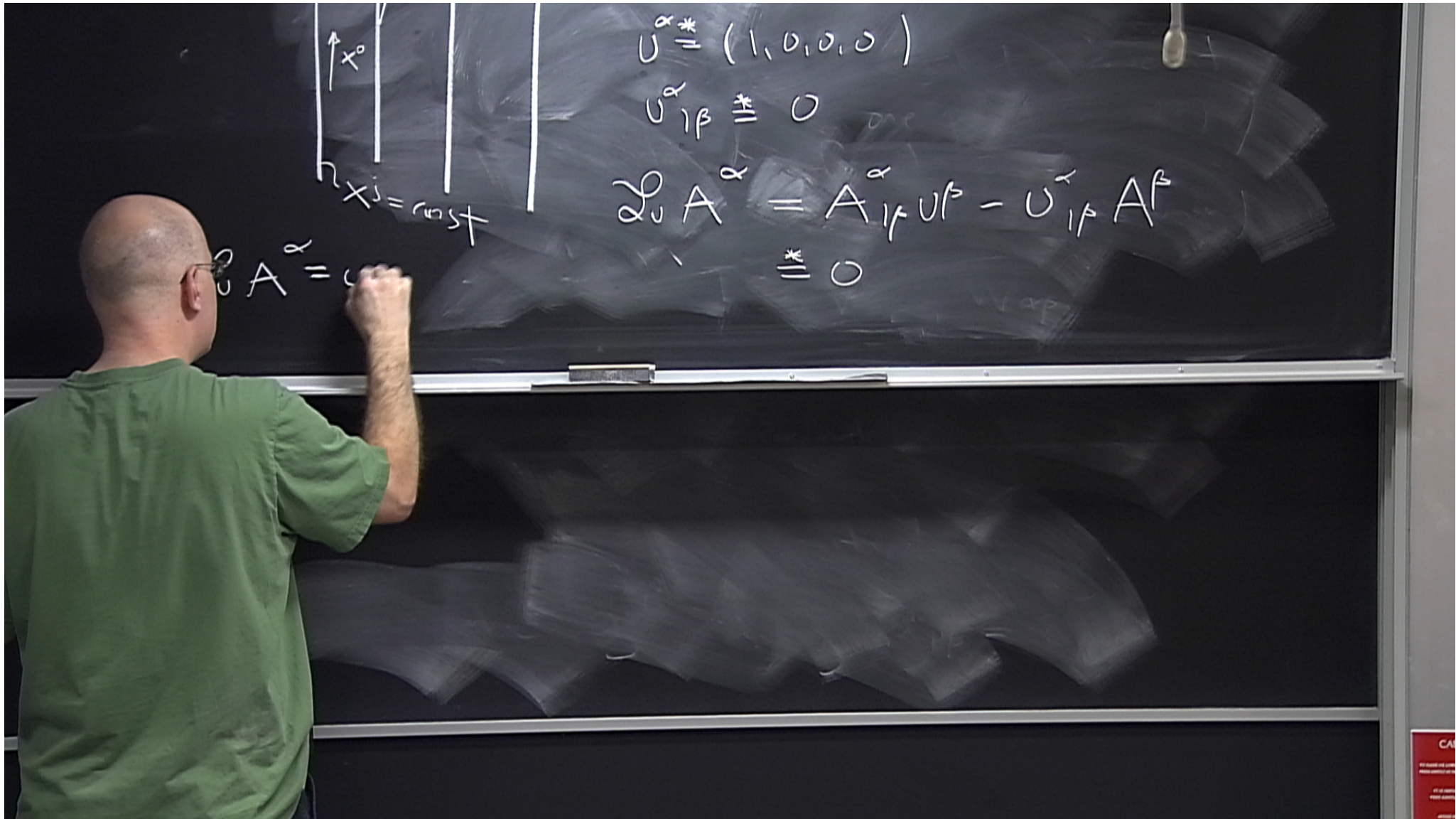
$$U^\alpha_{;\beta} \stackrel{*}{=} 0$$





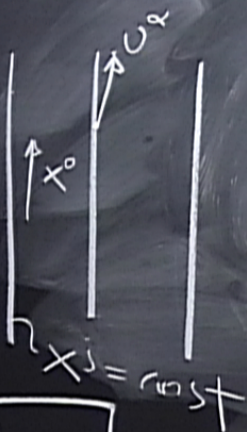






Vector  $A^\alpha$  does not depend on  $x^0$ :

$$\frac{\partial A^\alpha}{\partial x^0} \stackrel{*}{=} 0 \stackrel{*}{=} A_{\beta}^{\alpha} U^{\beta}$$



$$U^{\alpha*} = (1, 0, 0, 0)$$

$$U_{\beta}^{\alpha} \stackrel{*}{=} 0$$

$$\mathcal{L}_U A^\alpha = A_{\beta}^{\alpha} U^{\beta} - U_{\beta}^{\alpha} A^{\beta} \stackrel{*}{=} 0$$

$$\mathcal{L}_U A^\alpha = 0$$

$$\mathcal{L}_0 A^\alpha = 0$$

$x_0$   
 $x_j = \text{const}$

$$U^{\alpha*} = (1, 0, 0, 0)$$

$$U_{1\beta}^\alpha \stackrel{*}{=} 0$$

$$\mathcal{L}_0 A^\alpha = A_{1\beta}^\alpha U_{1\beta}^\alpha - U_{1\beta}^\alpha A^\beta \stackrel{*}{=} 0$$