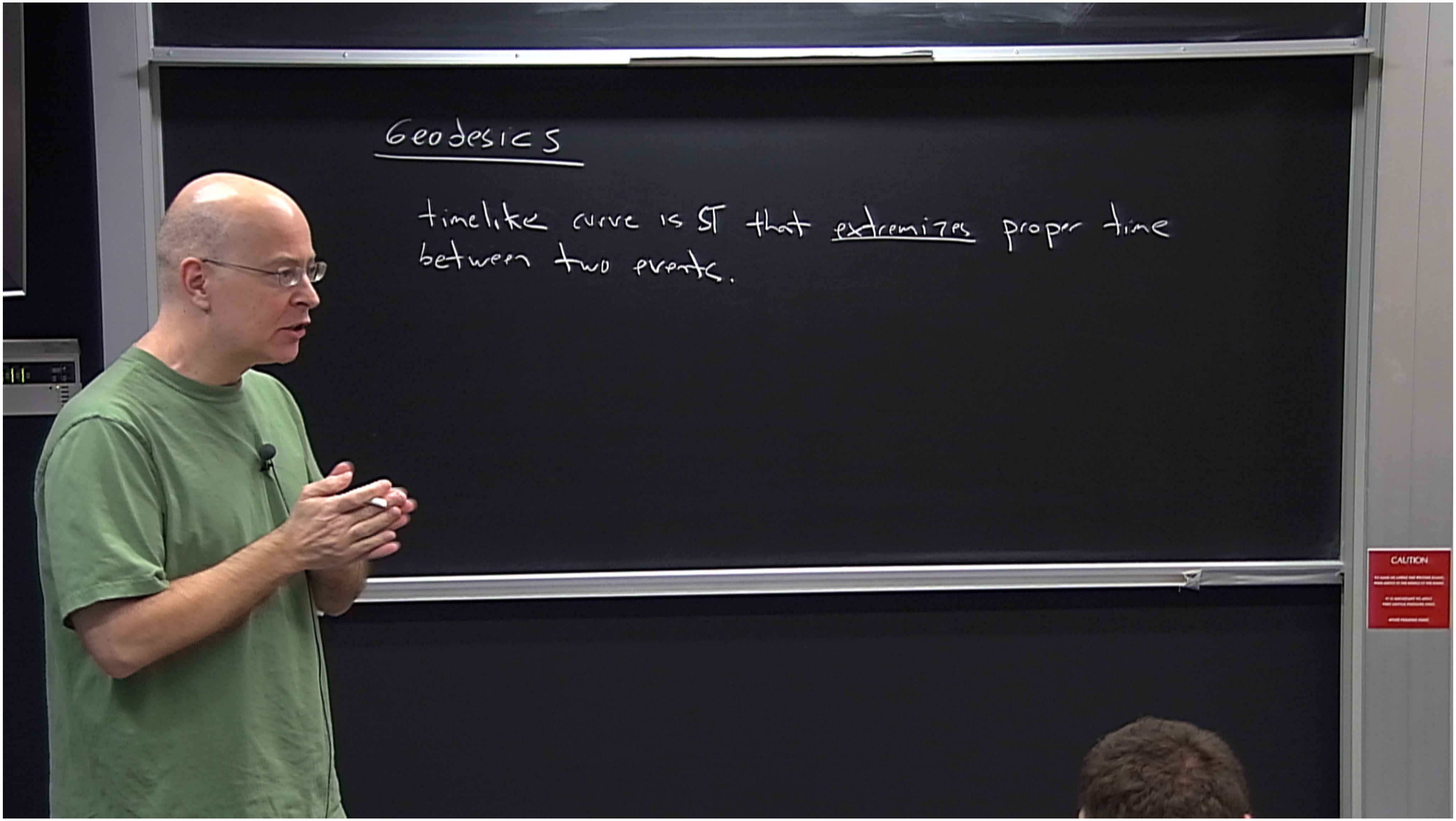
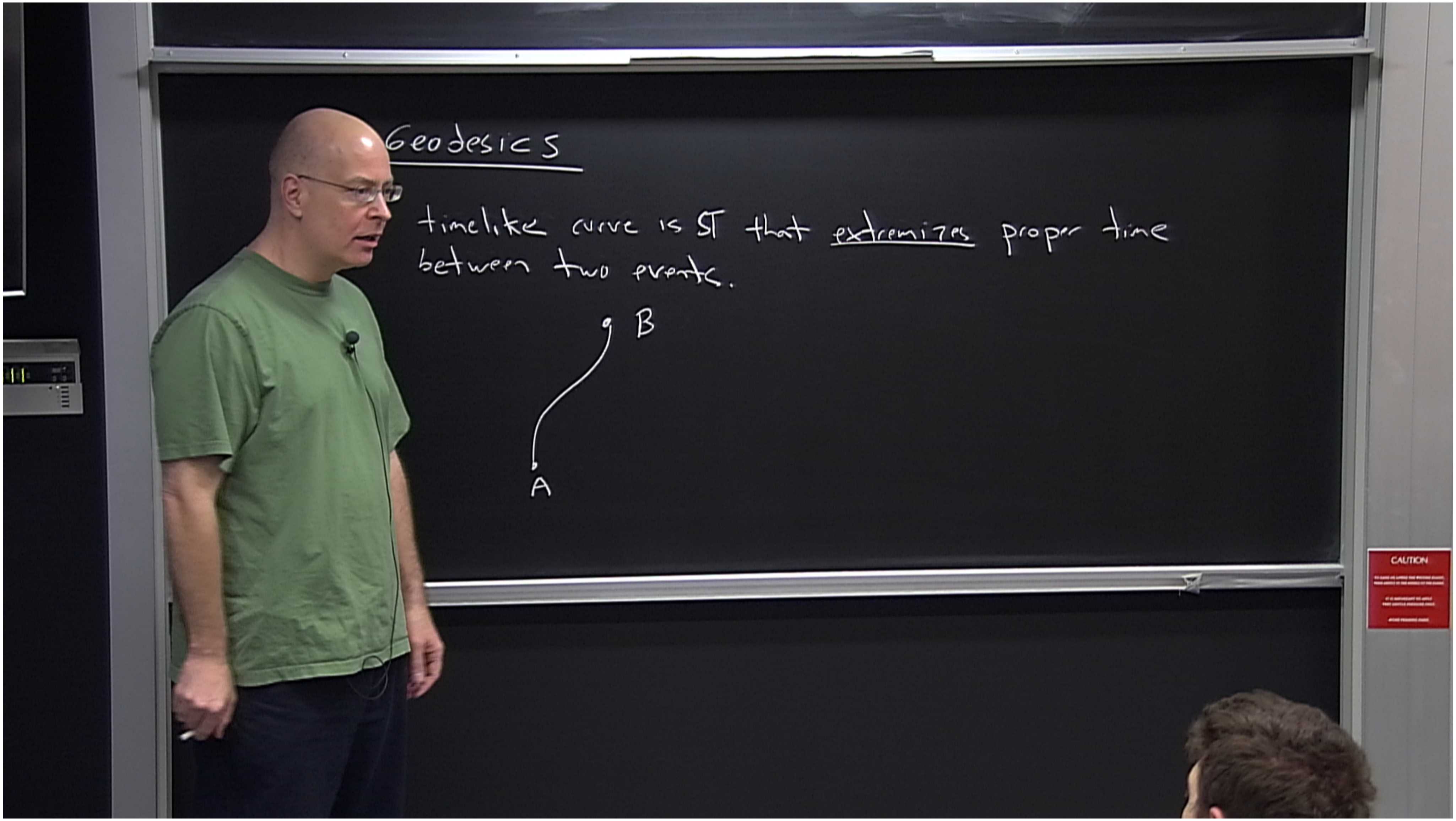
Title: Advanced General Relativity - Lecture 2

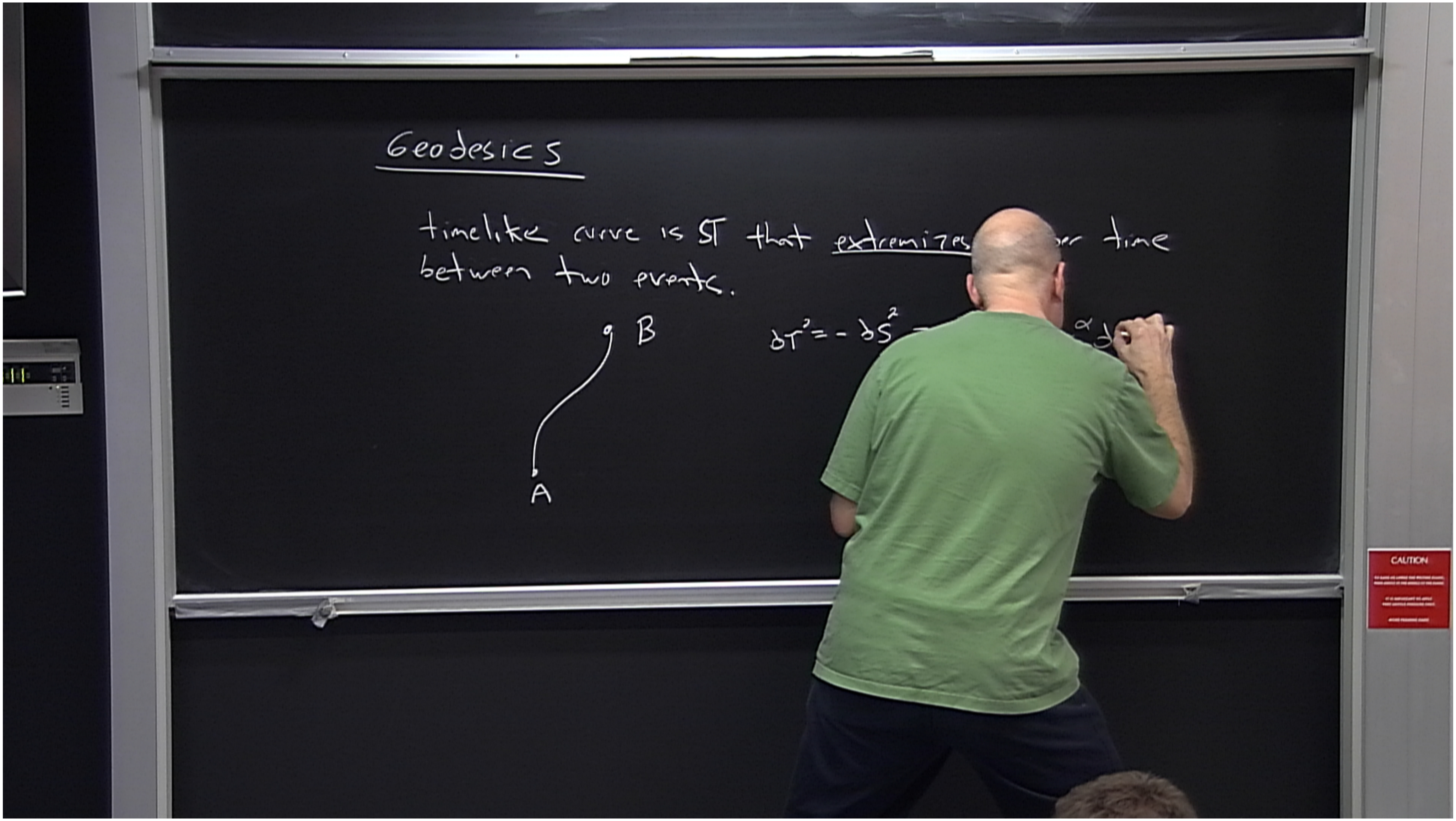
Date: Jan 11, 2012 03:30 PM

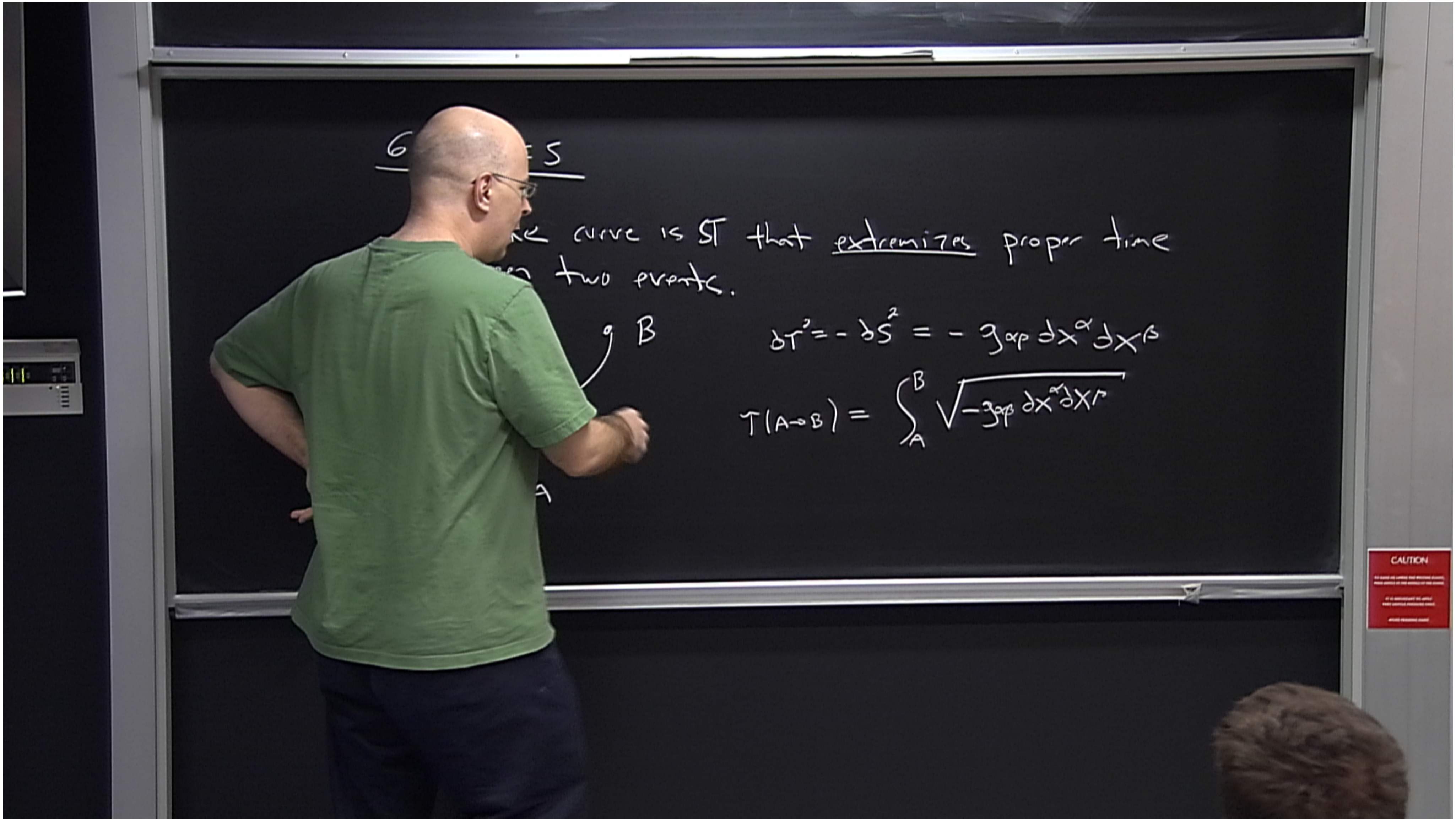
URL: http://pirsa.org/12010154

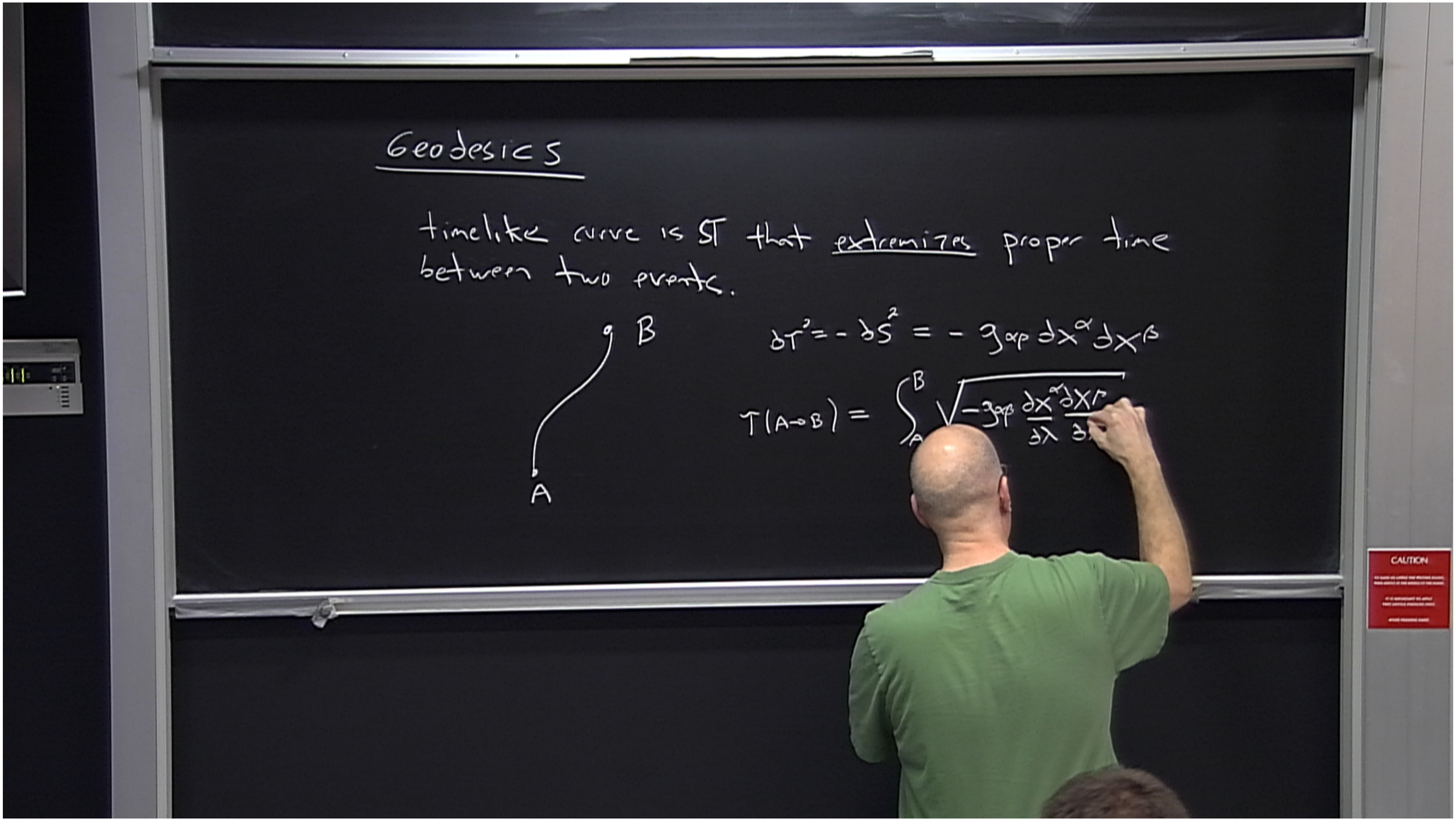
Abstract:

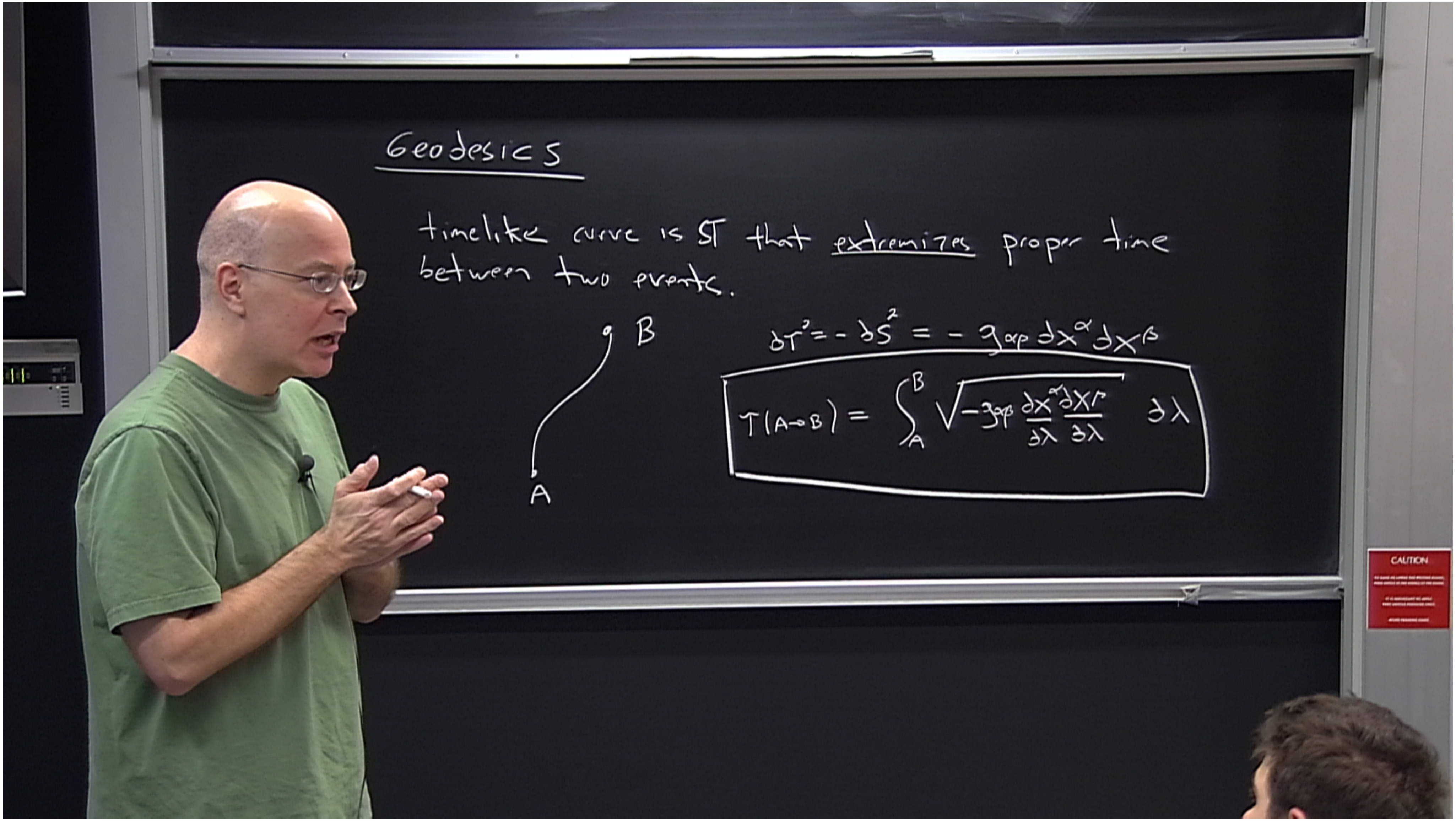

Geodesics
timelike curve is ST that extremizes proper time
between two events.
CAUTION


Geodesics
timelike curve is ST that extremizes proper time between two events.
B
A
CAUTION

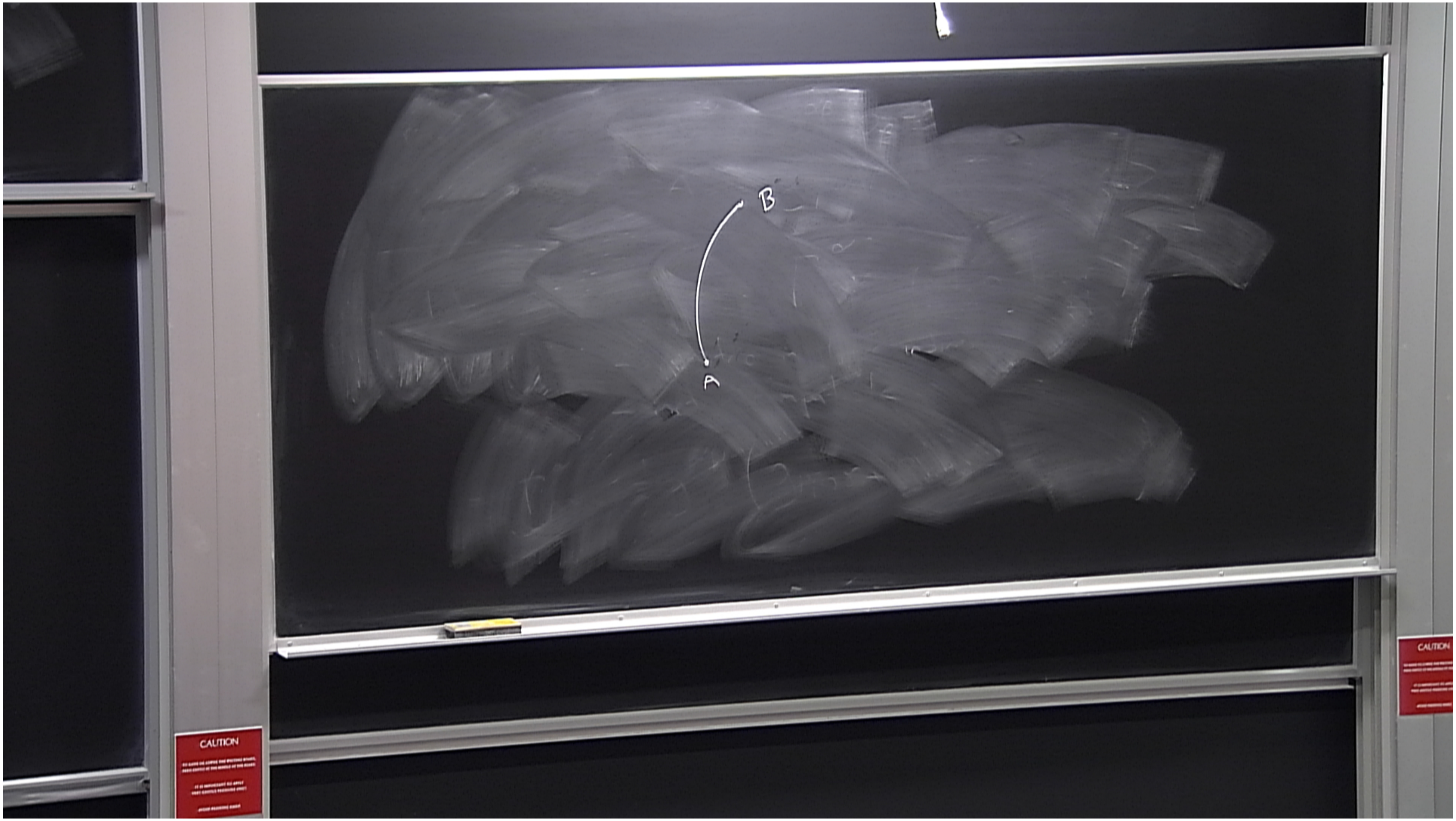

Geodesics
timelike curve is ST that extremizes
time
between two events.
B
A
$d\tau^2 = -ds^2$
CAUTION

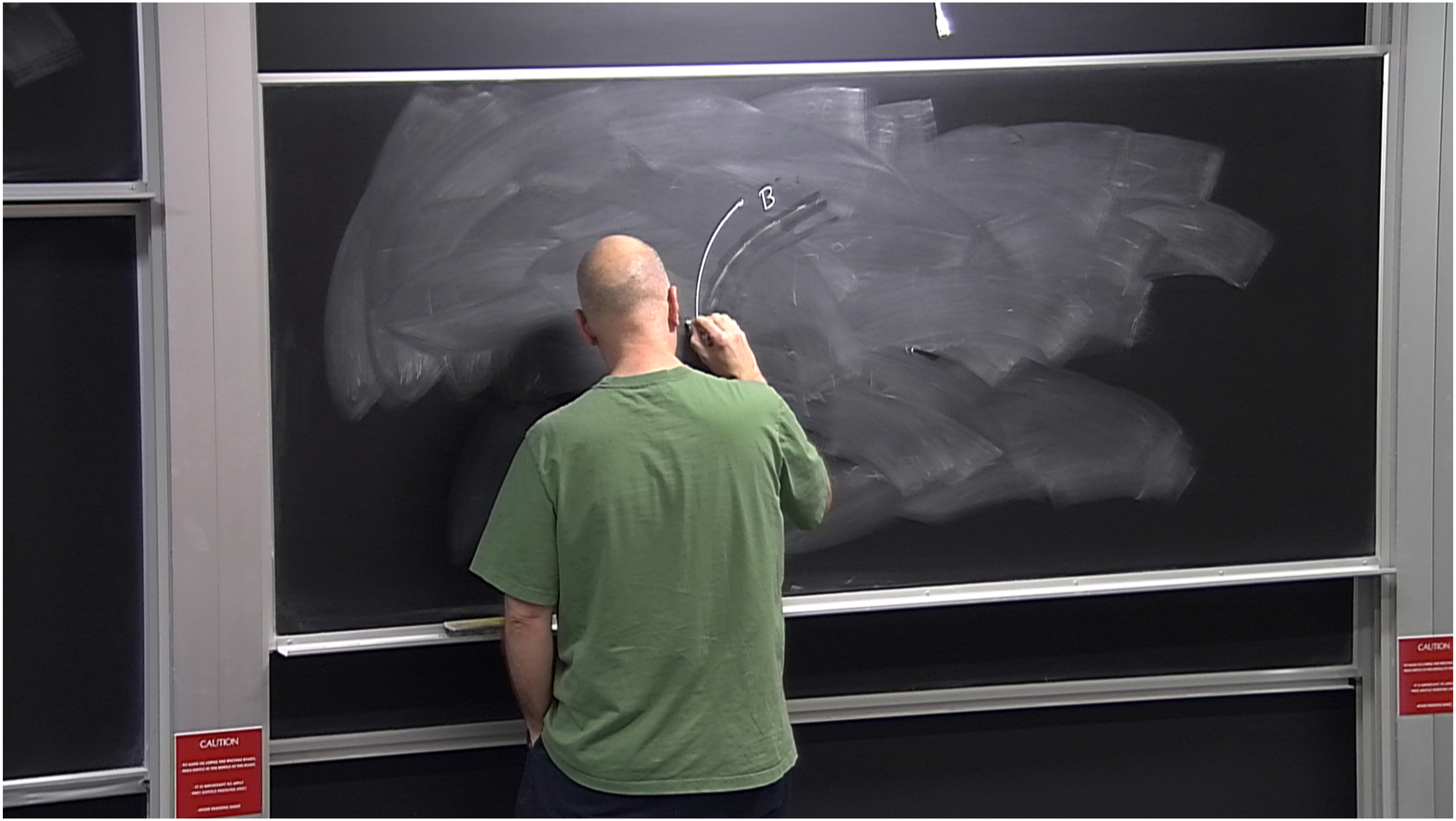

curve is ST that extremizes proper time
two events.
B
A
dτ² = - ds² = - gαβ dxα dxβ
τ(A→B) = ∫A B √(-gαβ dxα dxβ)
CAUTION

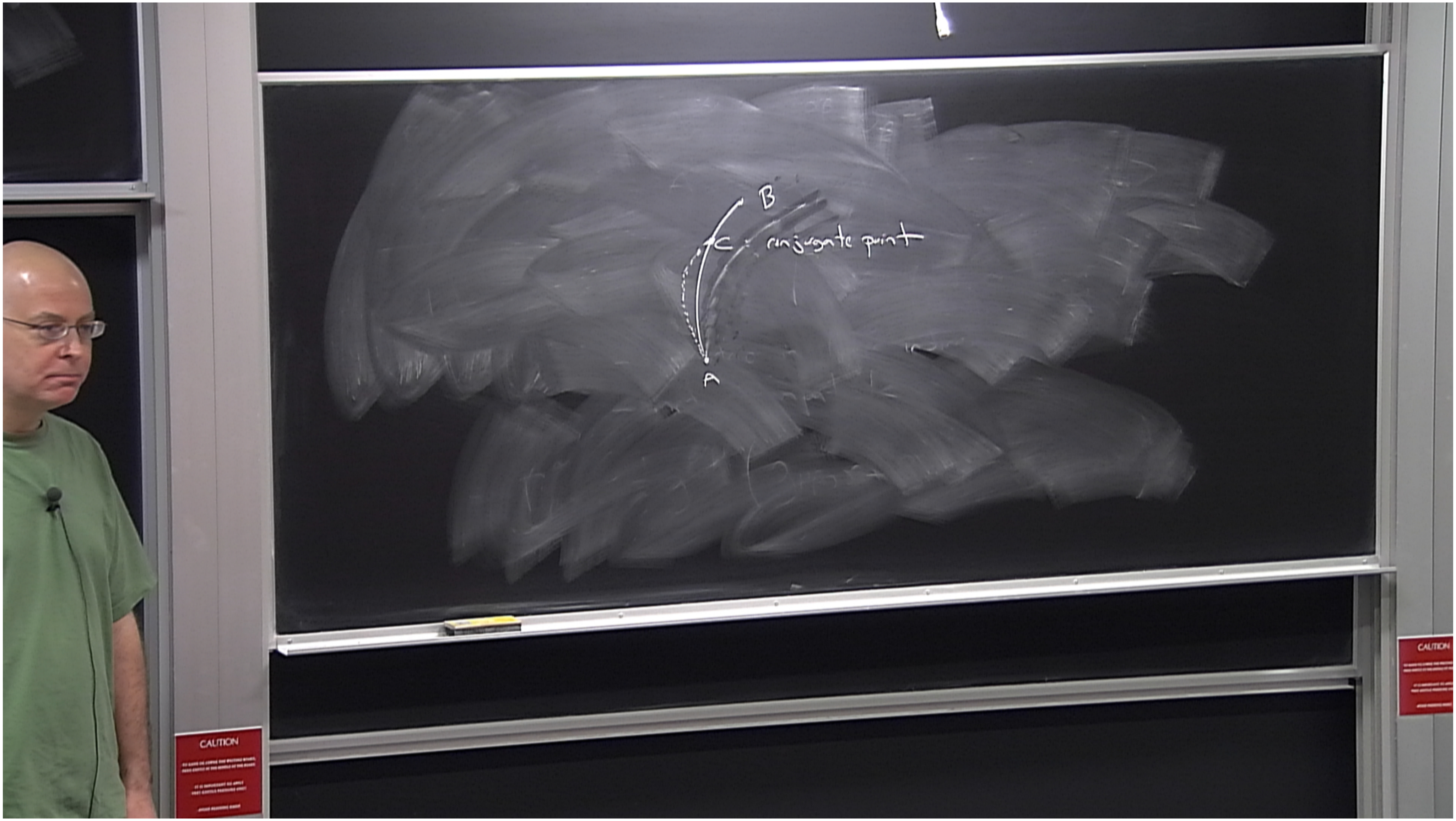

Geodesics
timelike curve is ST that extremizes proper time
between two events.
B
A
dτ² = - ds² = - g_αβ dx^α dx^β
τ(A→B) = ∫_A^B √(-g_αβ dx^α/dλ dx^β/dλ)

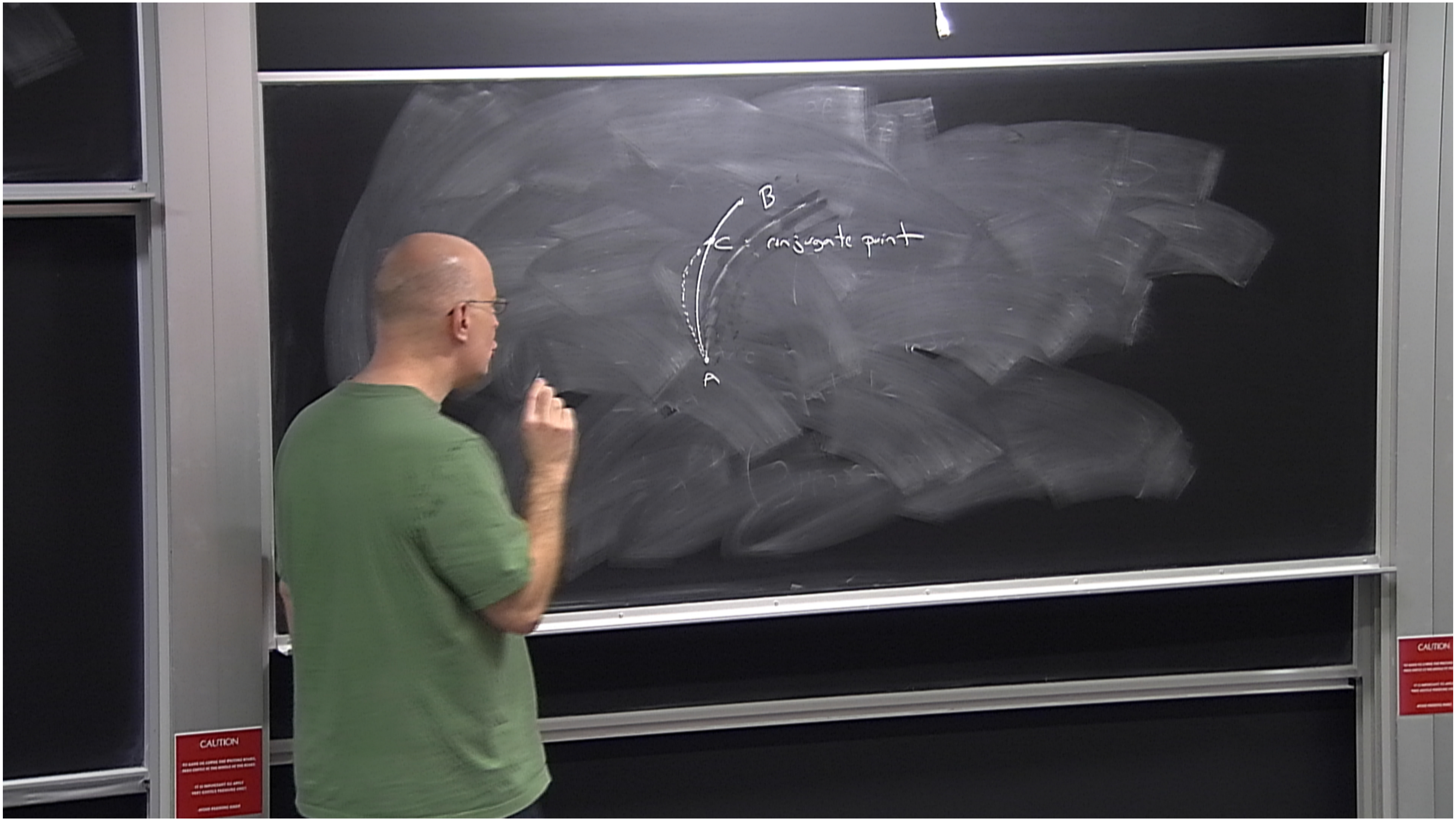


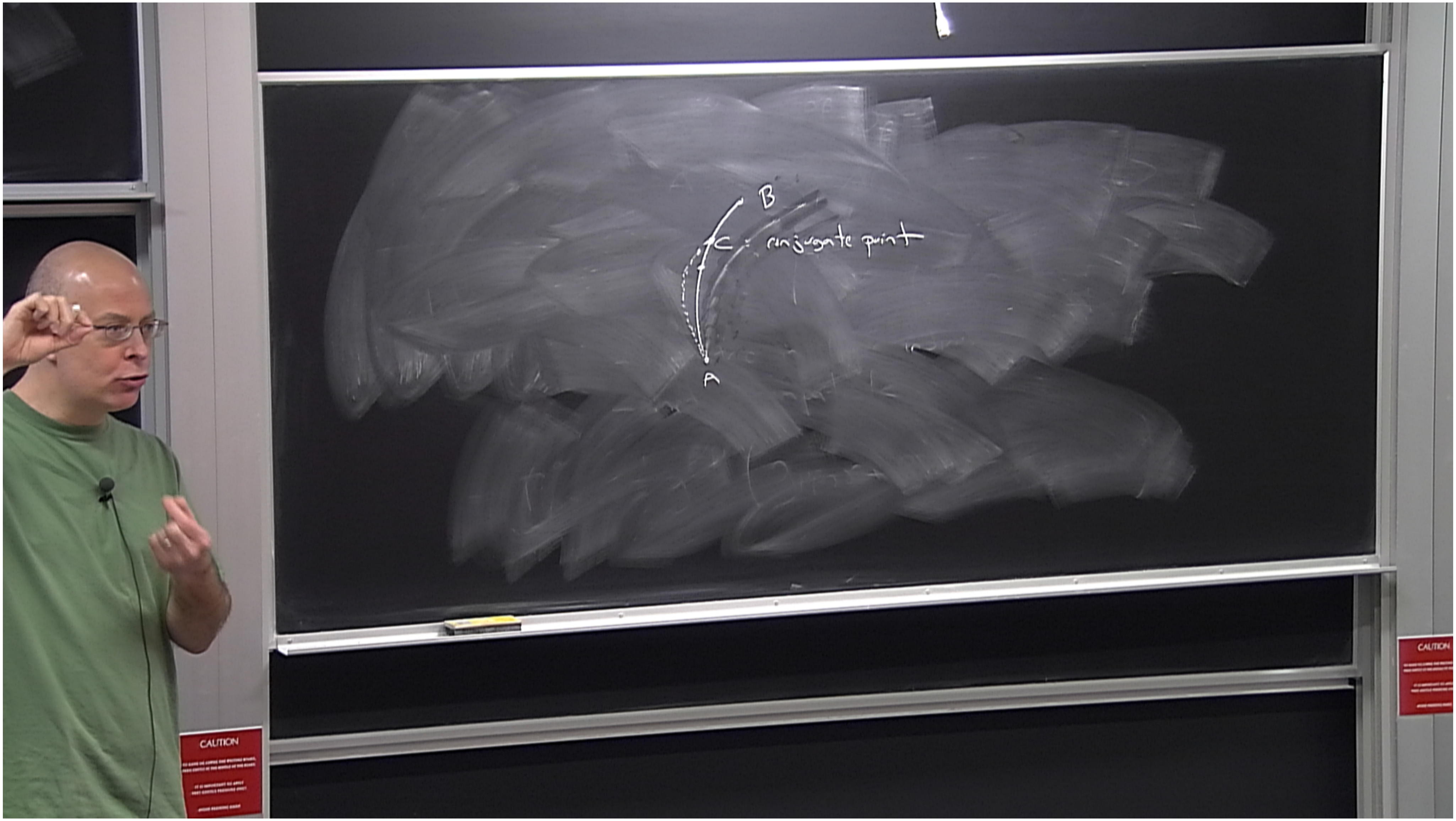
Geodesics
timelike curve is ST that extremizes proper time between two events.
B
A
$d\tau^2 = -ds^2 = -g_{\alpha\beta}\,dx^\alpha dx^\beta$
$\tau(A \to B) = \int_A^B \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}}\, d\lambda$
CAUTION

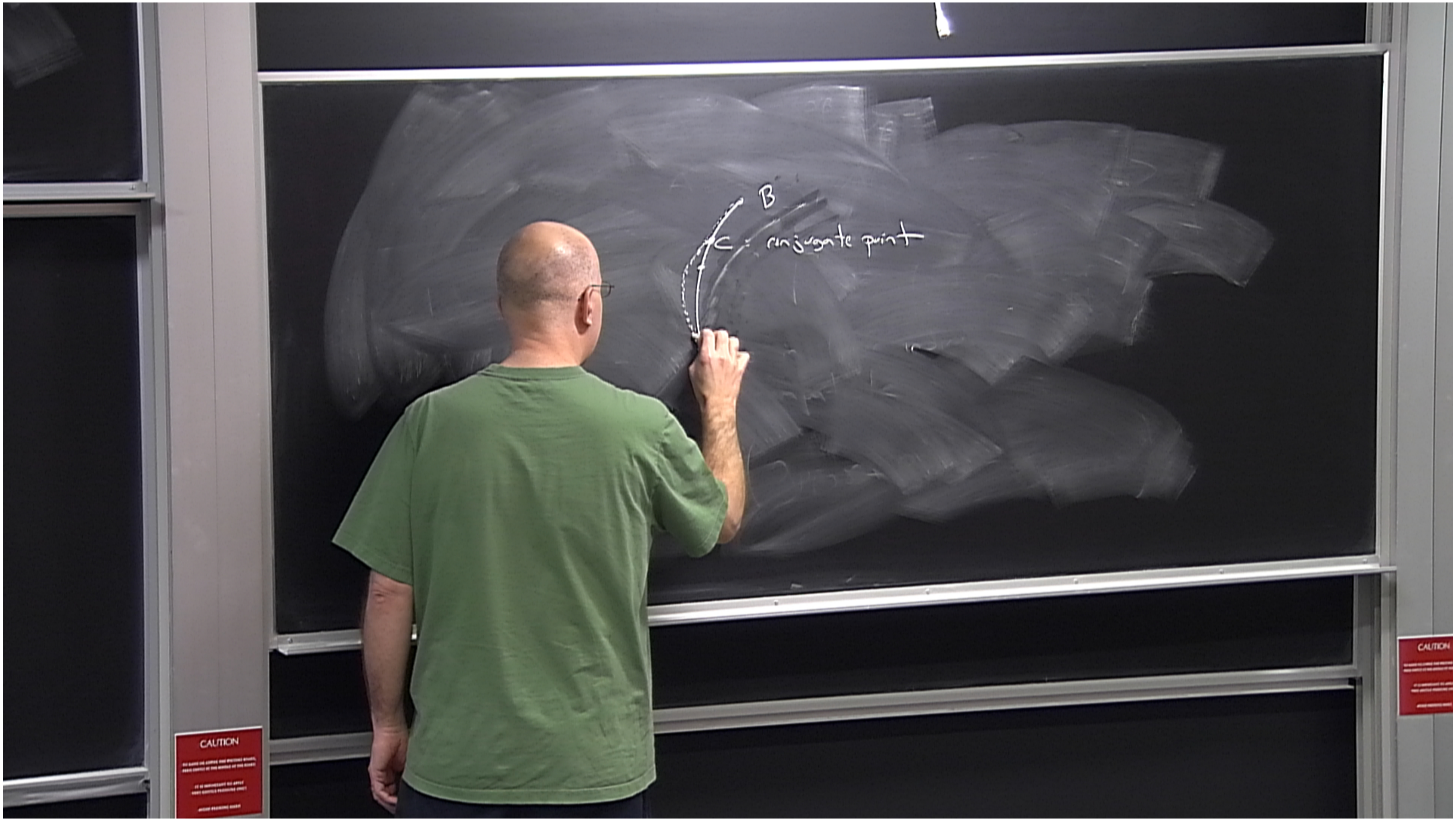

B
A
CAUTION

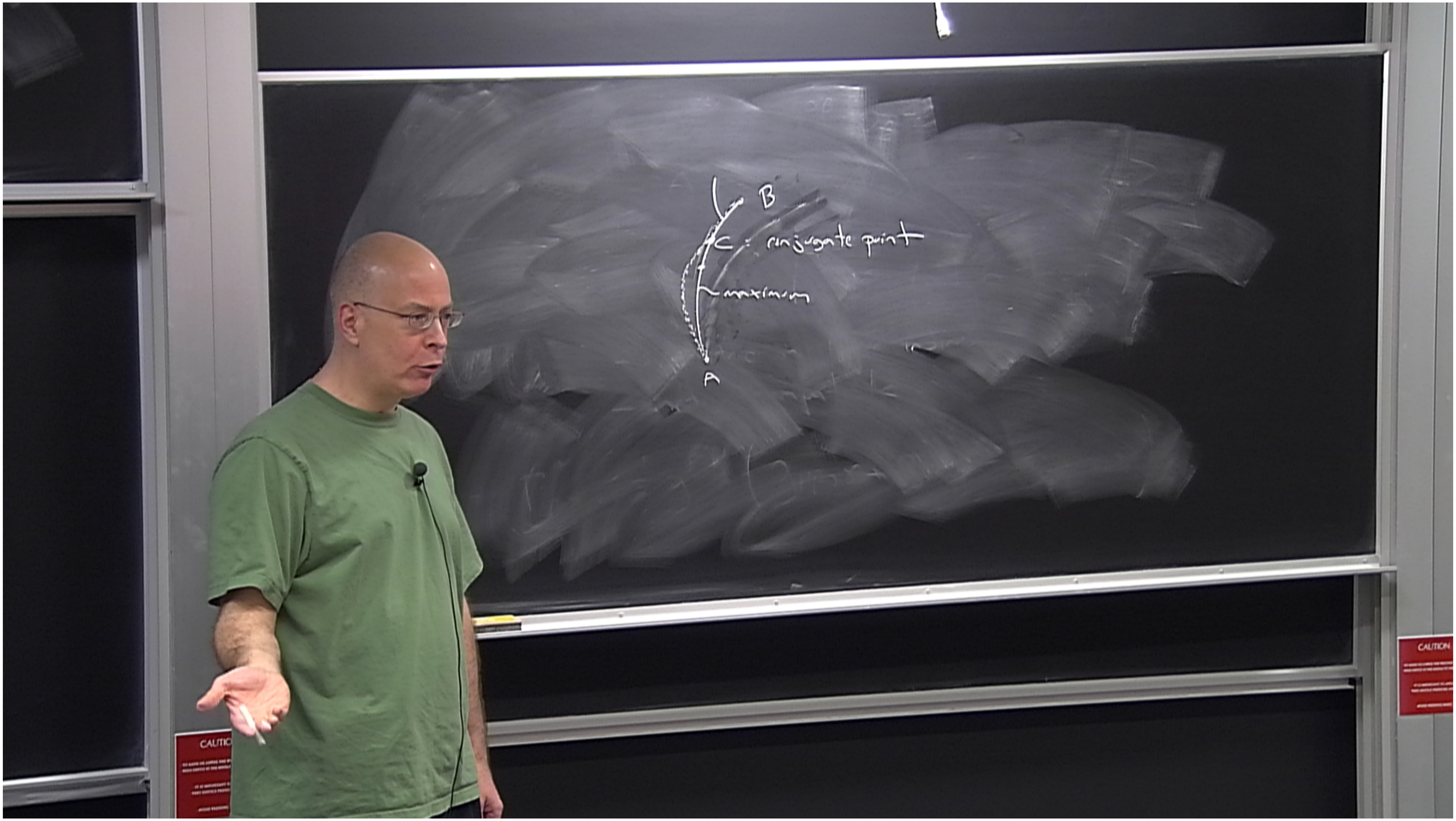

B

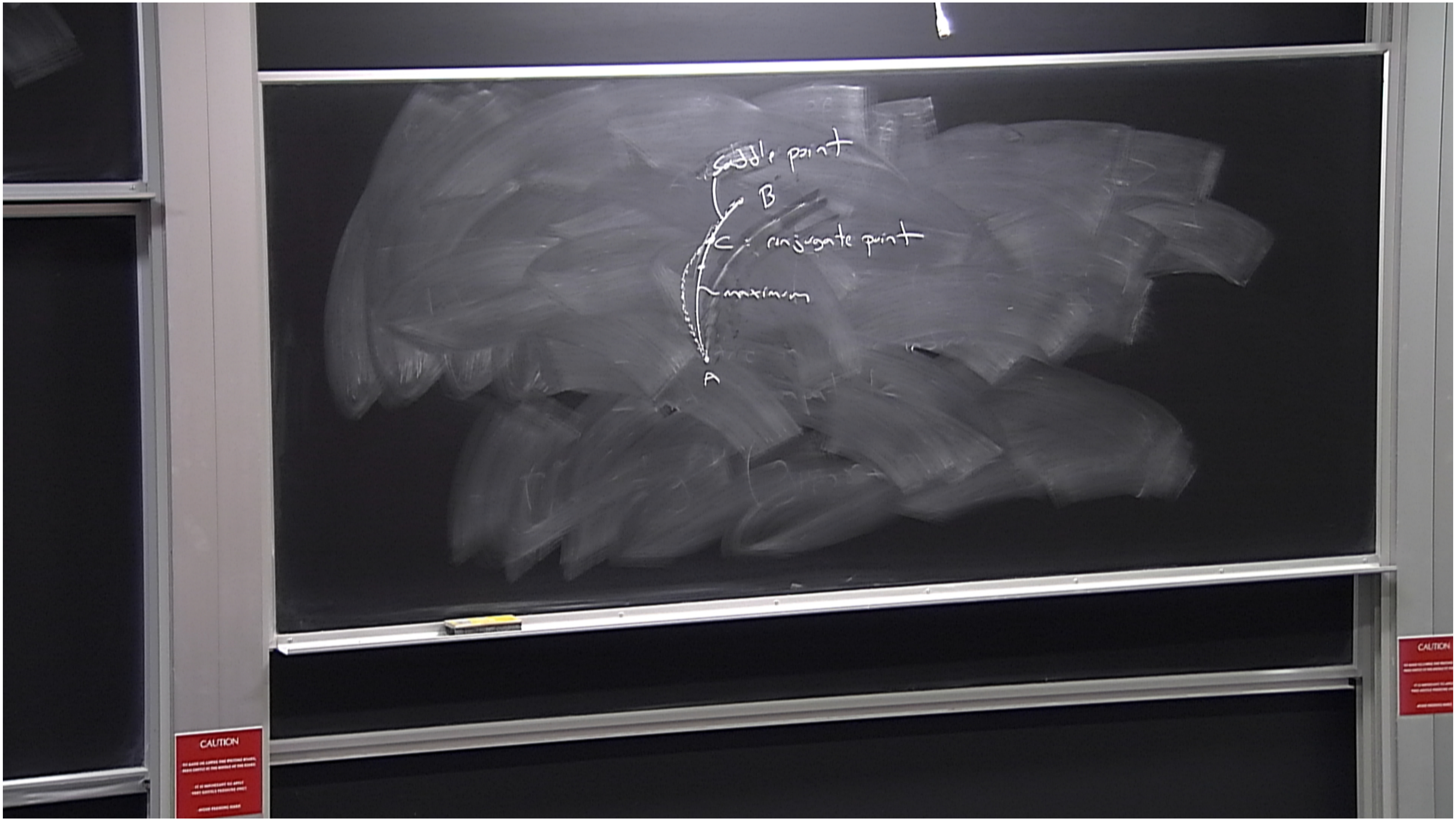

B
C : conjugate point
A

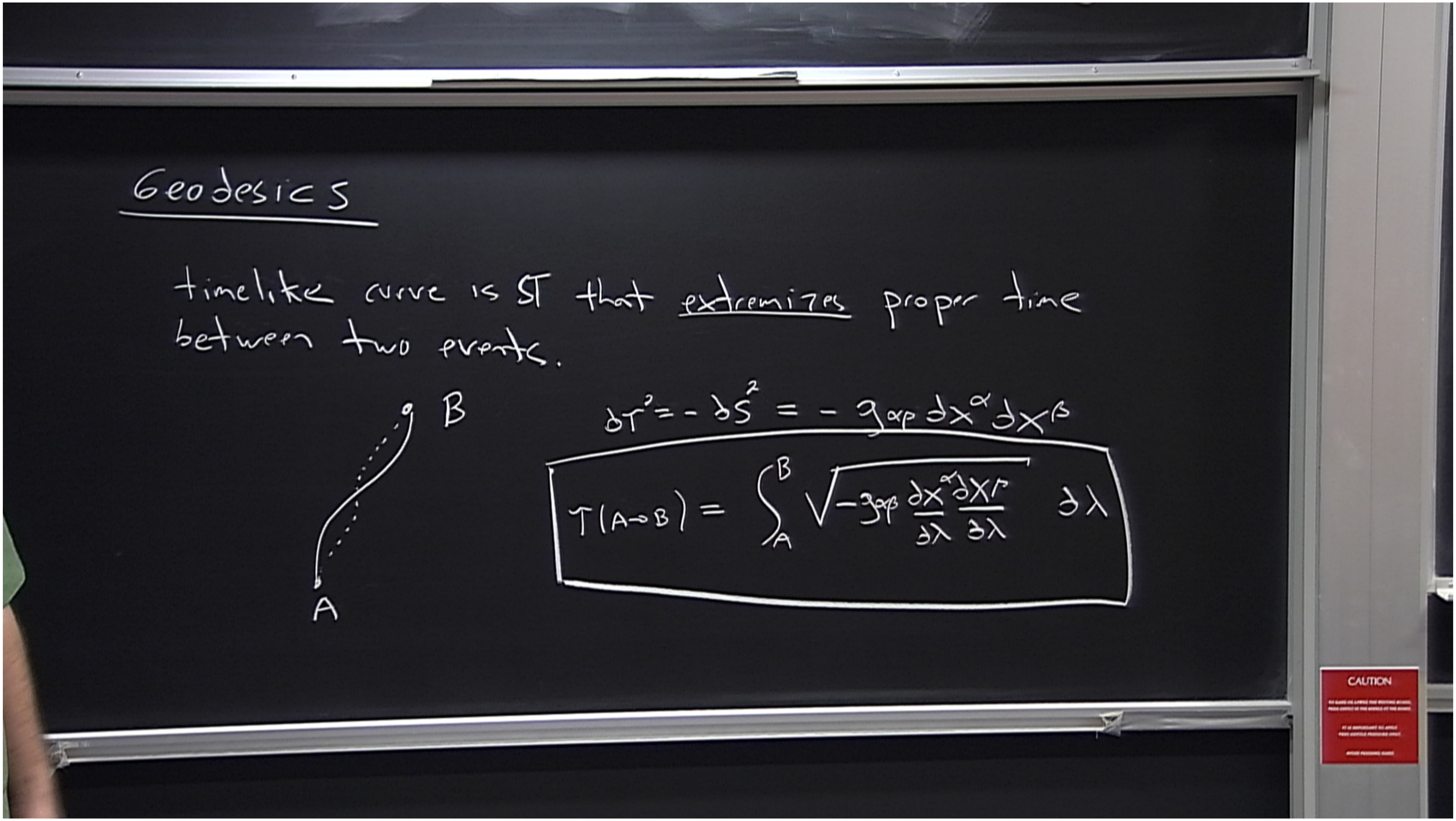

B
C : conjugate point
A


B
C : conjugate point
A


B
C : conjugate point
CAUTION


B
C : conjugate point
maximum
A


saddle point
B
C : conjugate point
maximum
A


Geodesics
timelike curve is ST that extremizes proper time
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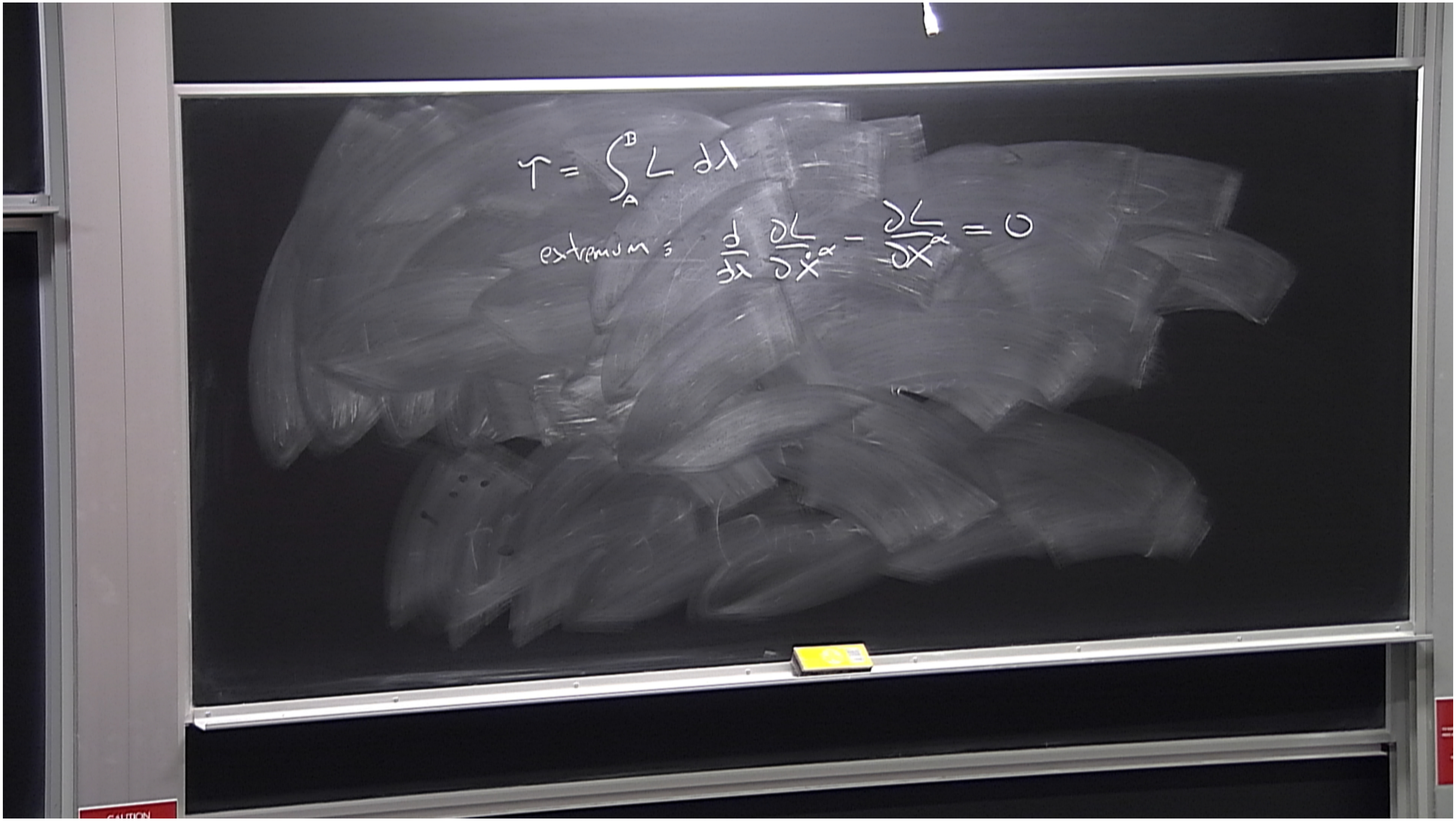
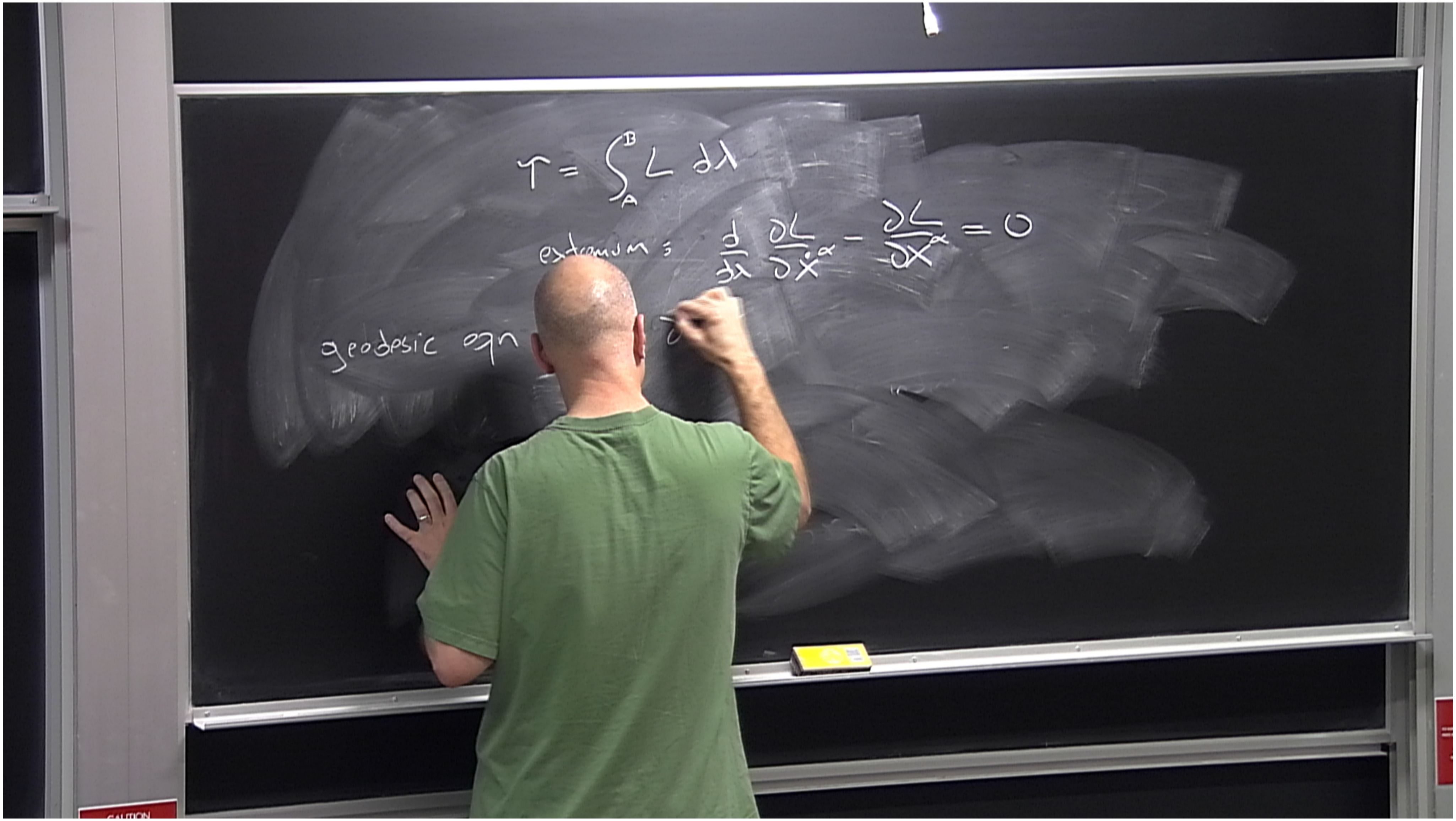
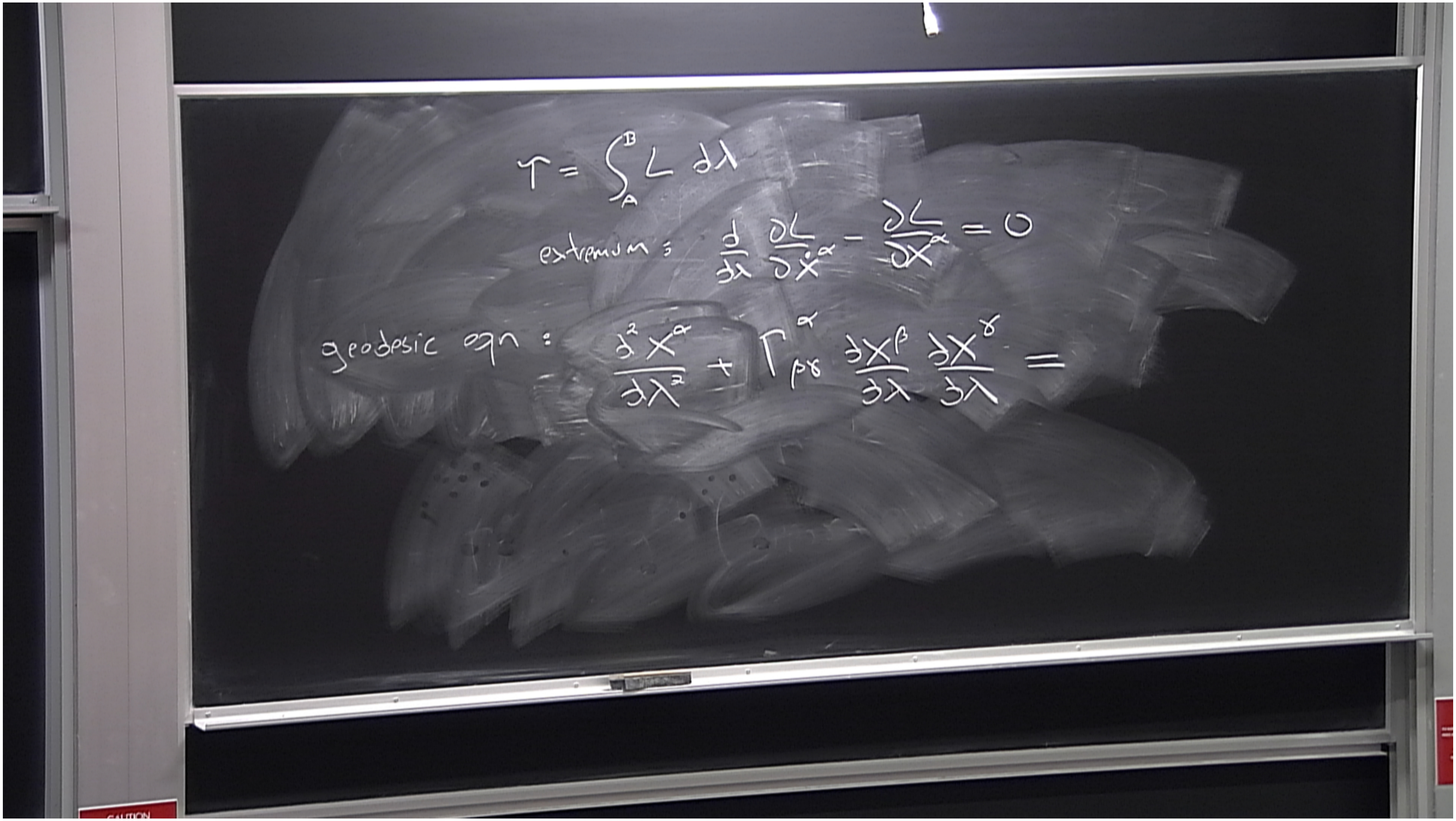
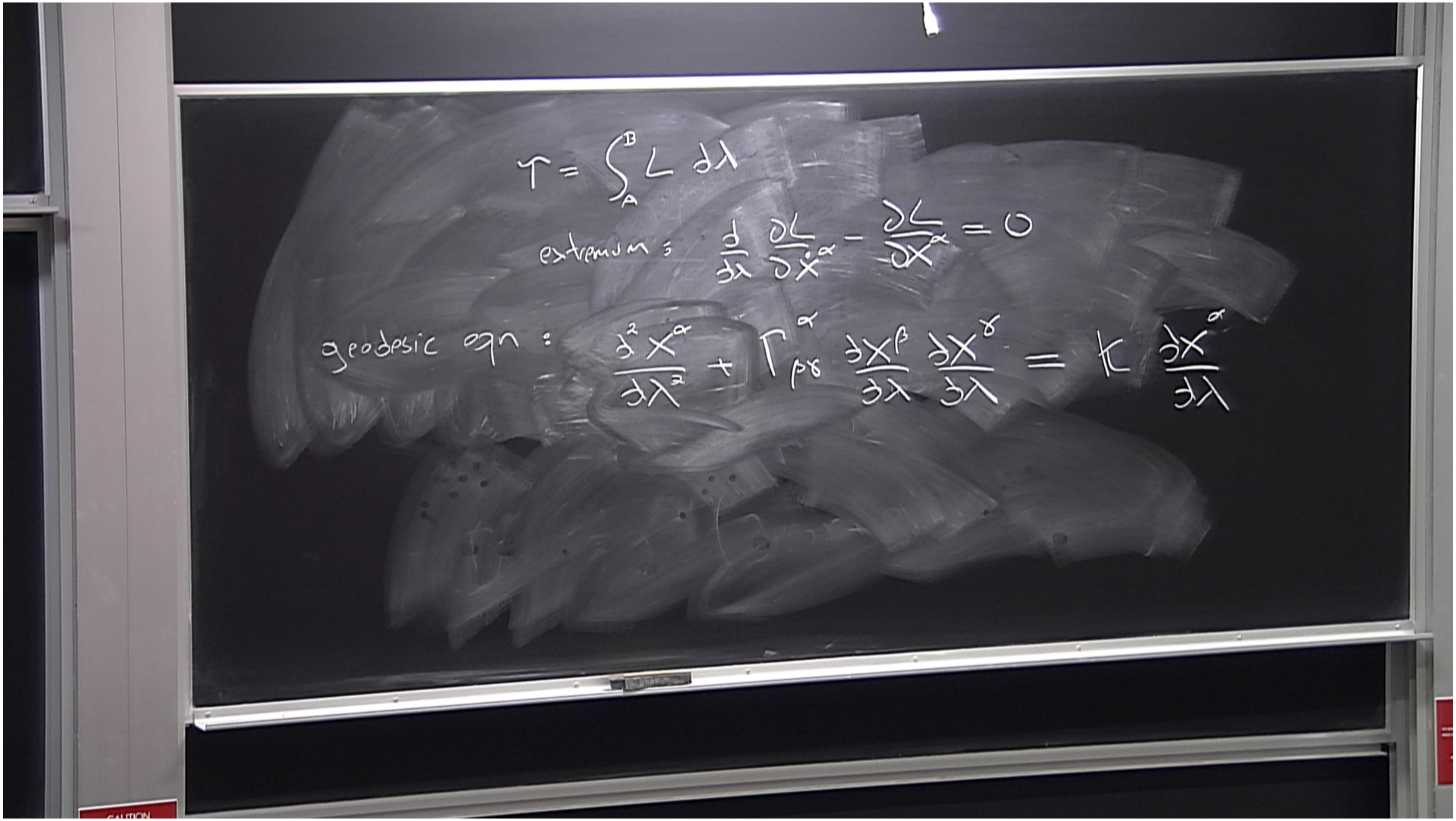
B
A
$d\tau^2 = -ds^2 = -g_{\alpha\beta}\, dx^\alpha dx^\beta$
$\tau(A\to B) = \int_A^B \sqrt{-g_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}}\, d\lambda$
CAUTION

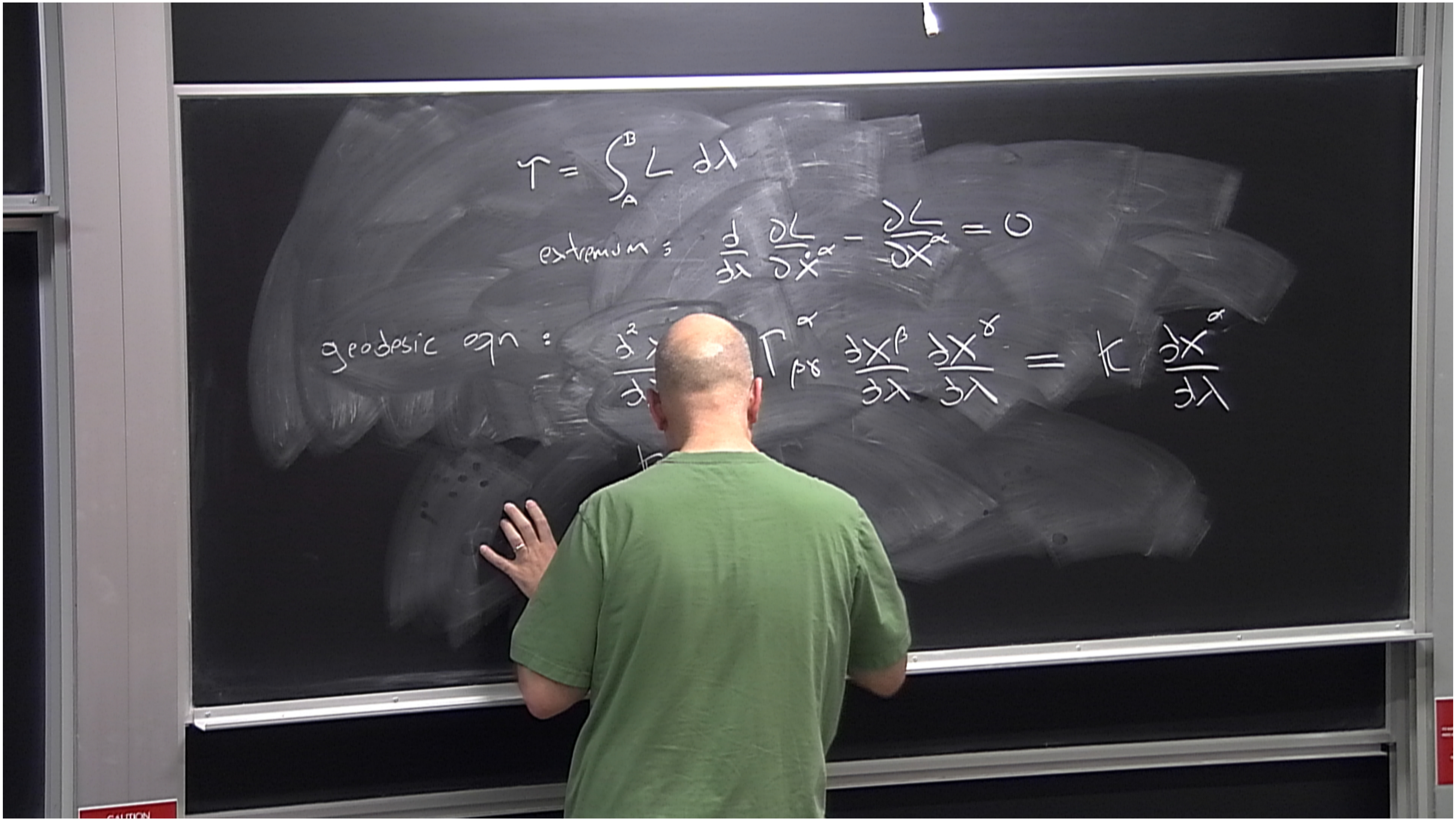

$\tau = \int_A^B L \, d\lambda$
extremum: $\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$


τ = ∫_A^B L dλ
extremum: d/dλ ∂L/∂ẋ^α − ∂L/∂x^α = 0
geodesic eqn


$\tau = \int_A^B L \, d\lambda$
extremum: $\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$
geodesic eqn: $\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} =$



$\tau = \int_A^B L \, d\lambda$
extremum: $\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$
geodesic eqn: $\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda} = k \frac{dx^\alpha}{d\lambda}$


$\tau = \int_A^B L \, d\lambda$
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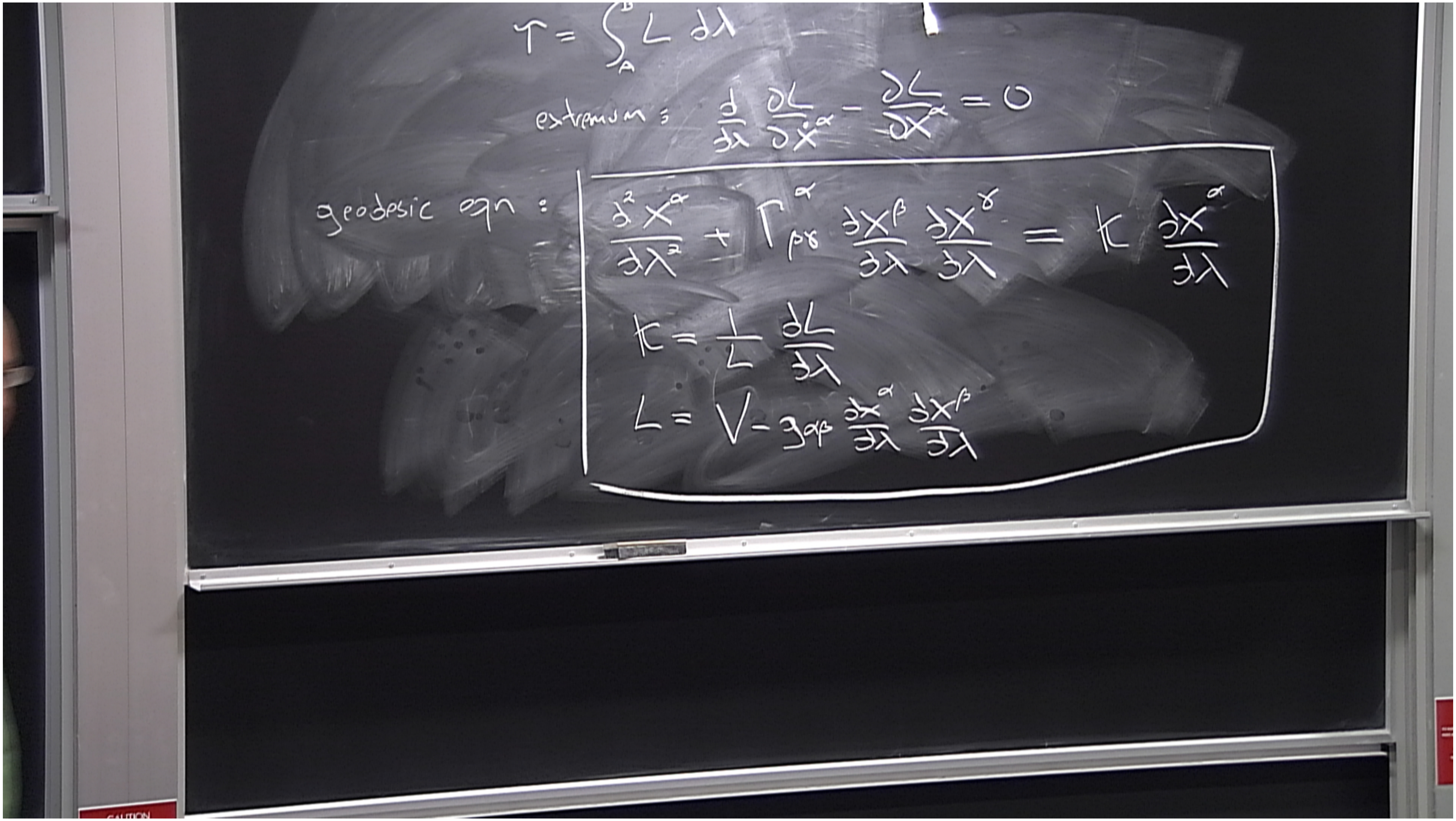
geodesic eqn :


$\tau = \int_A^B L \, d\lambda$
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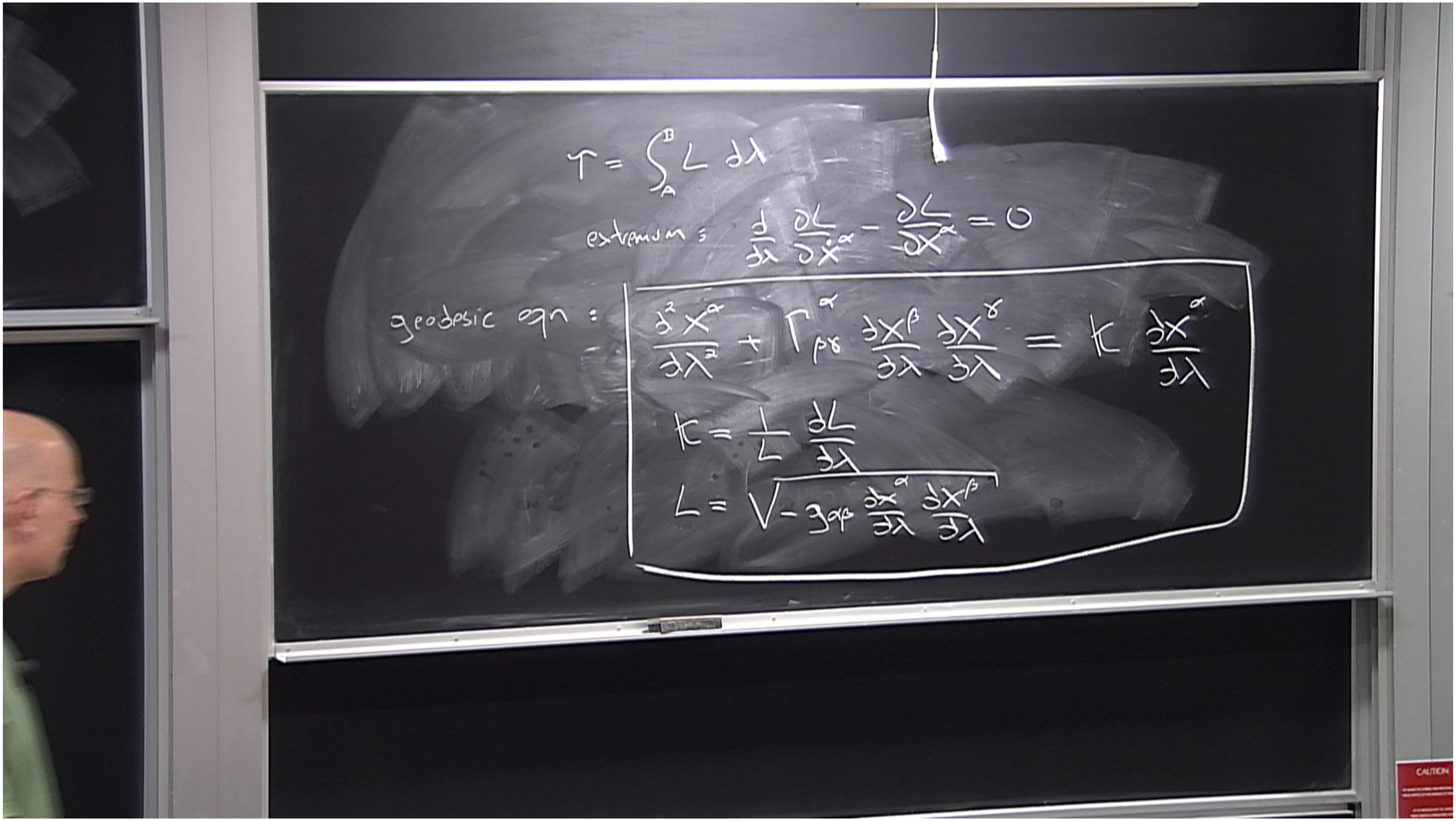
geodesic eqn: $\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = \kappa \frac{dx^\alpha}{d\lambda}$
$\kappa = \frac{1}{L} \frac{dL}{d\lambda}$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda}}$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extremum: $\frac{d}{d\lambda}\frac{\partial L}{\partial \dot{x}^\alpha} - \frac{\partial L}{\partial x^\alpha} = 0$
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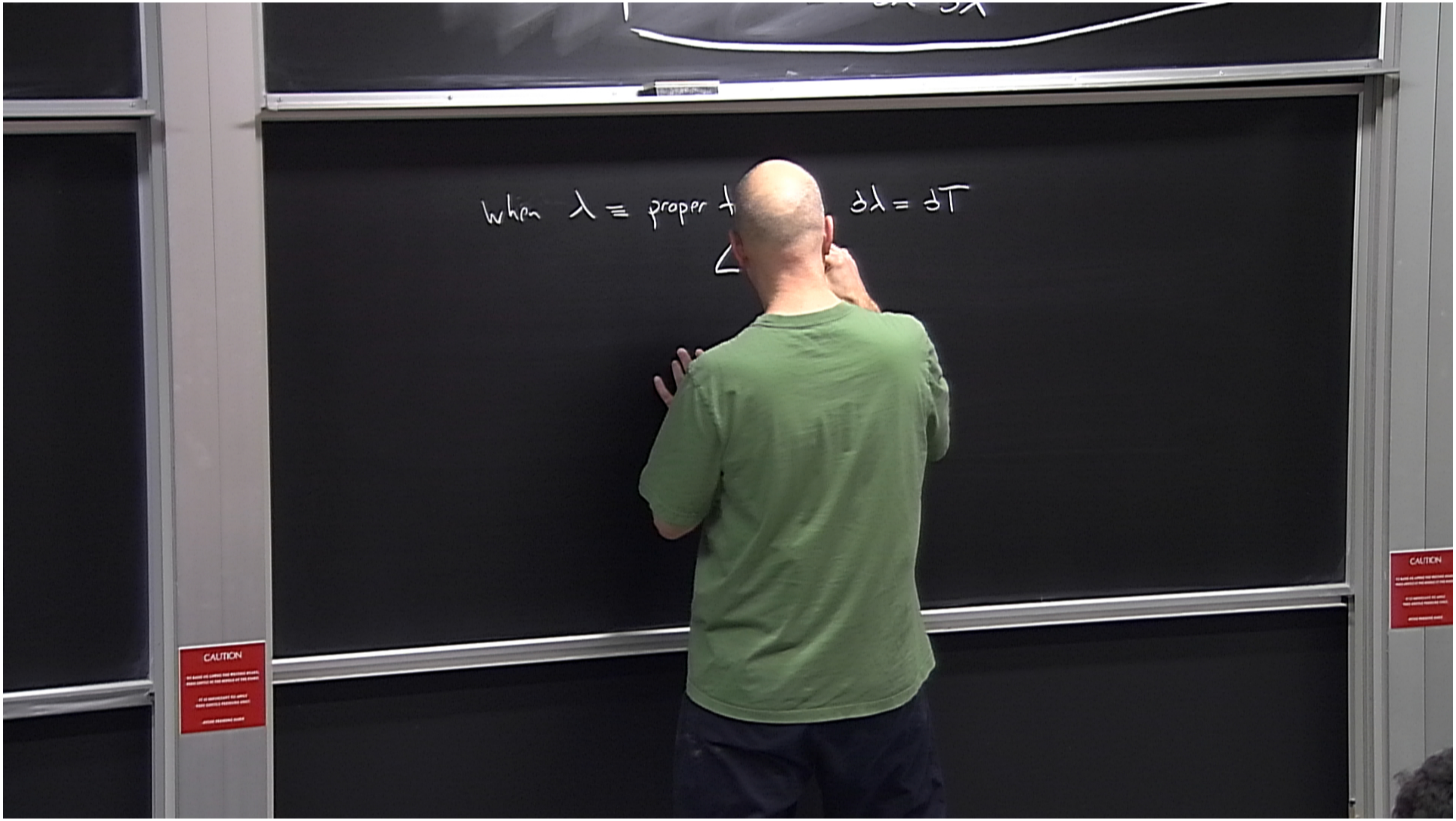
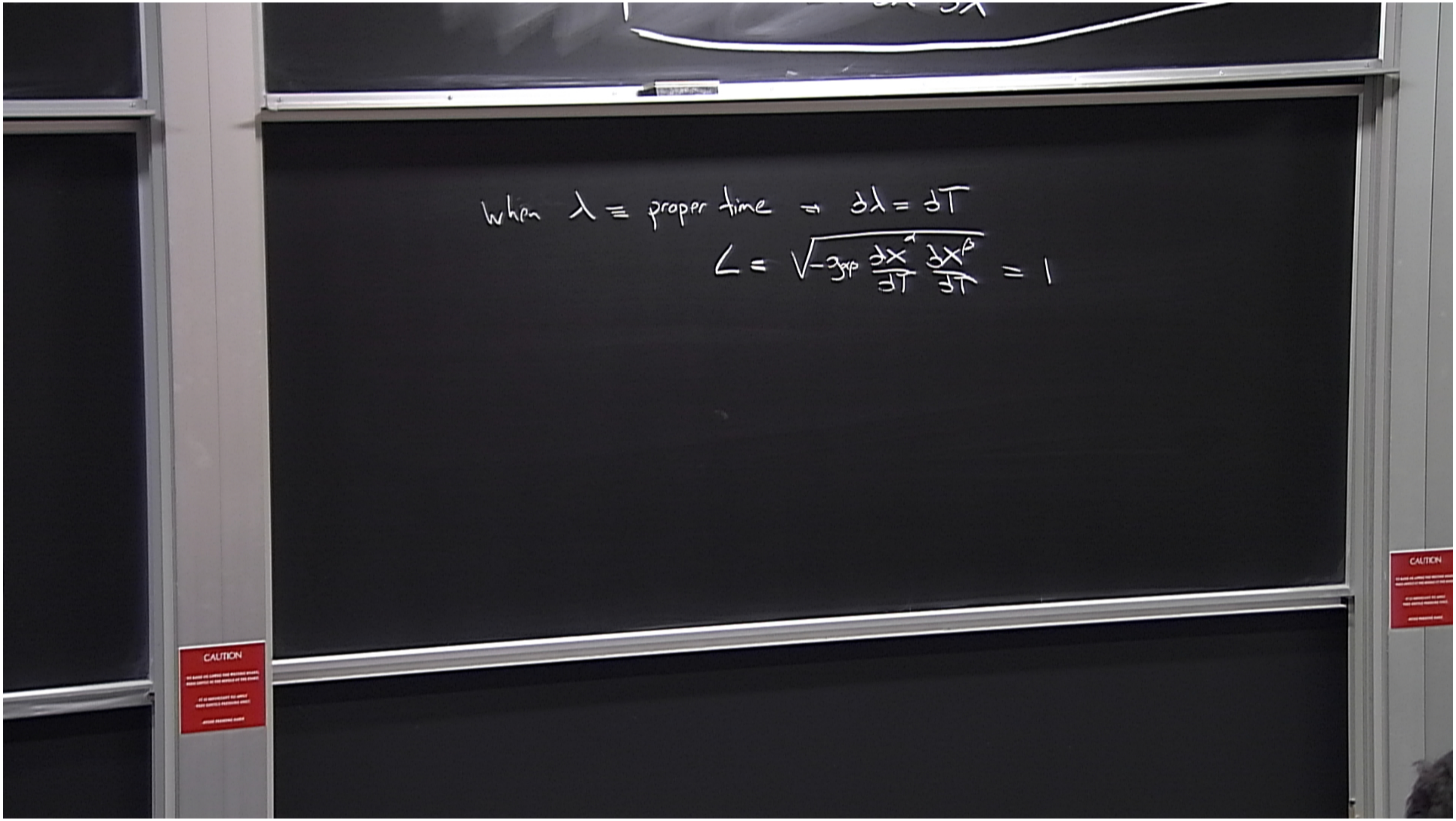
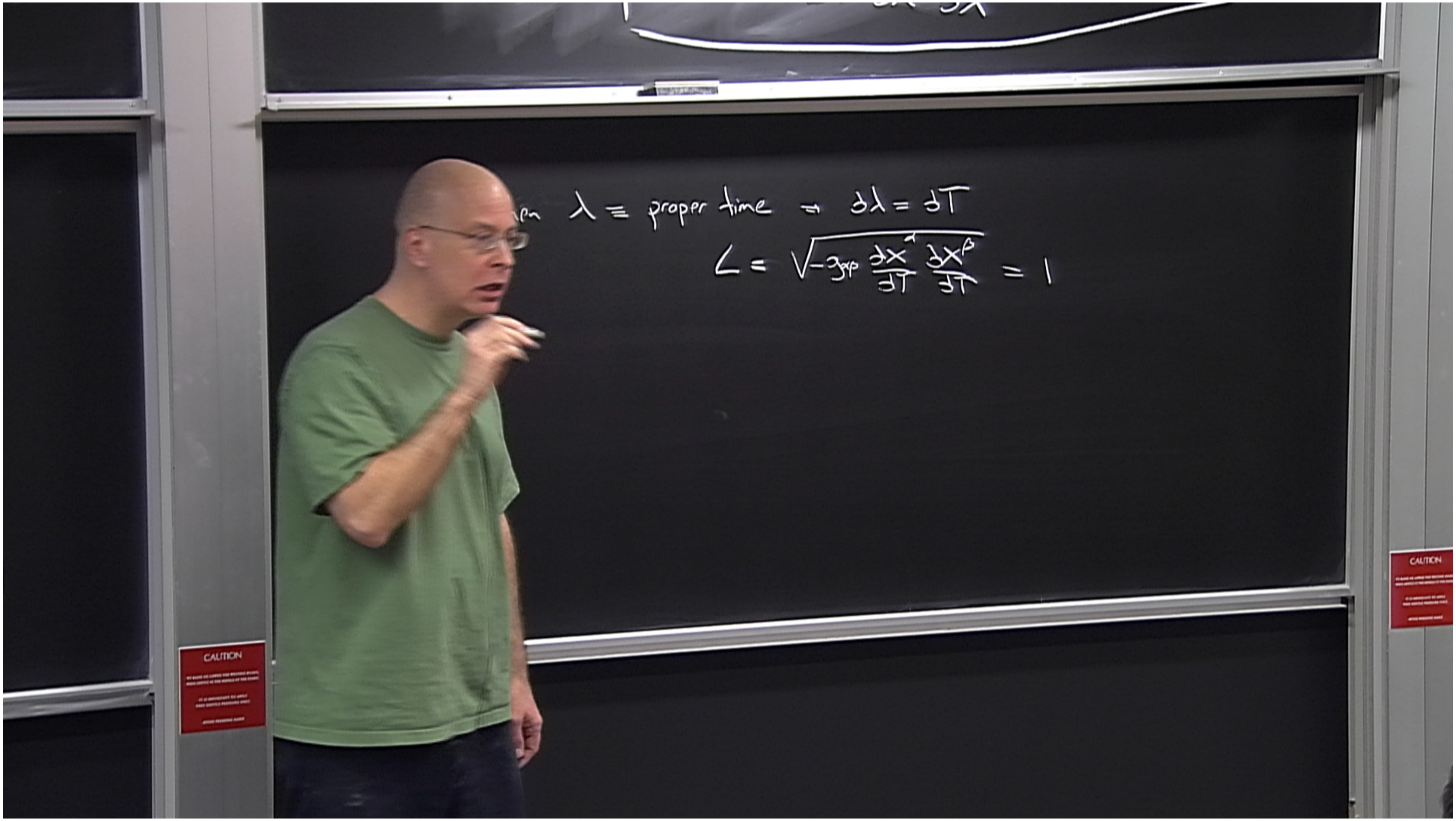
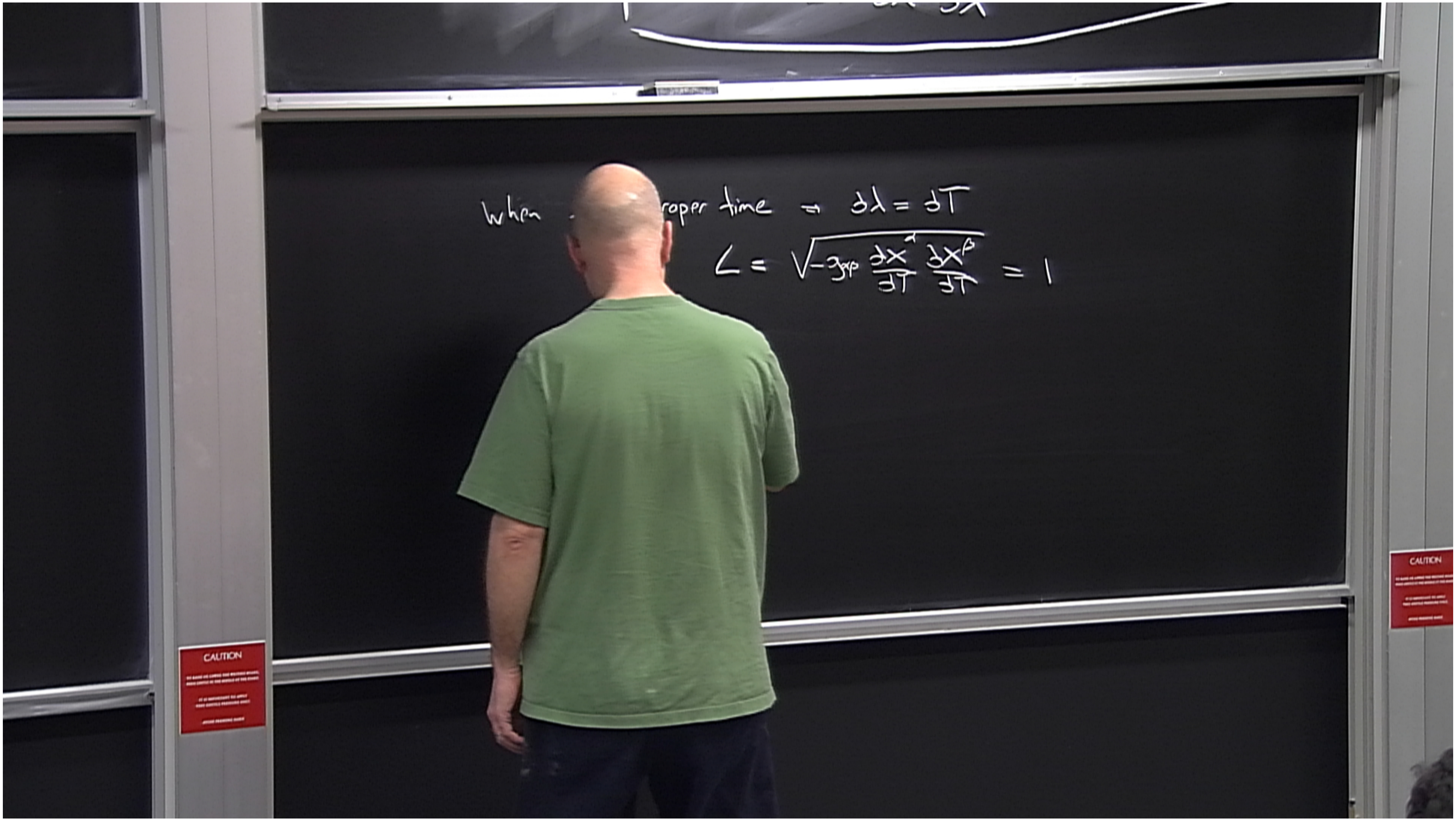
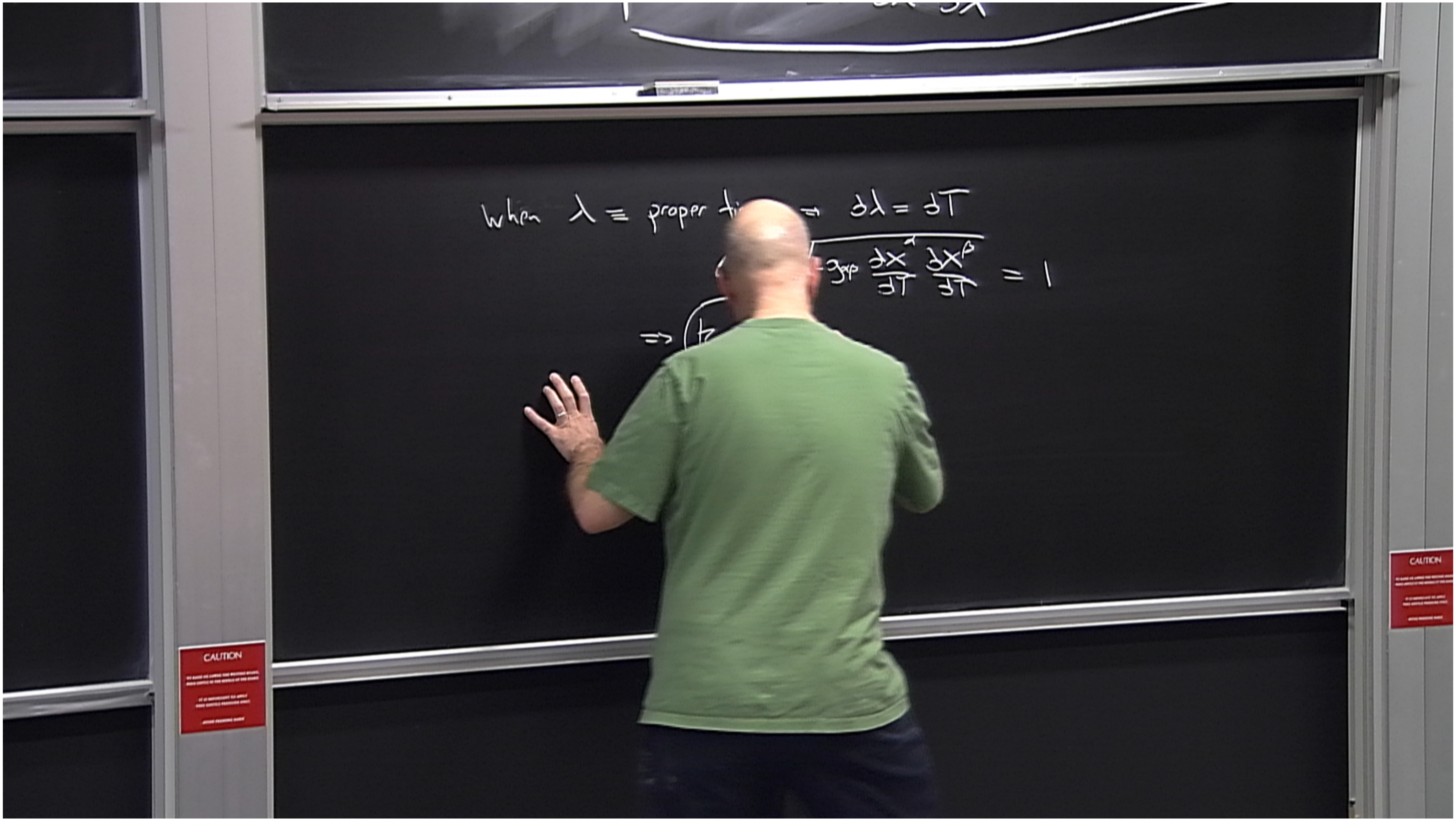
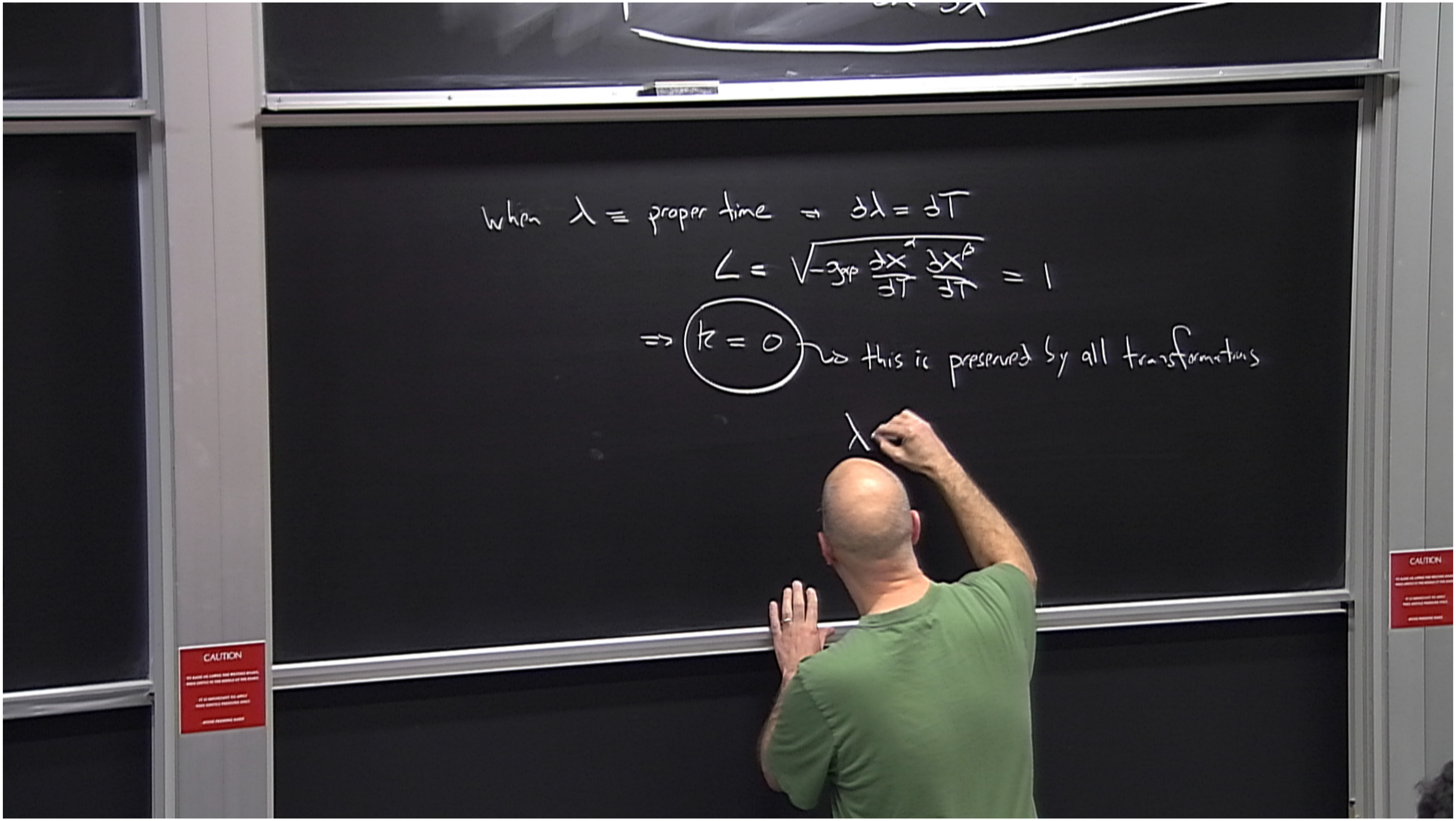

when λ ≡ proper t
dλ = dT

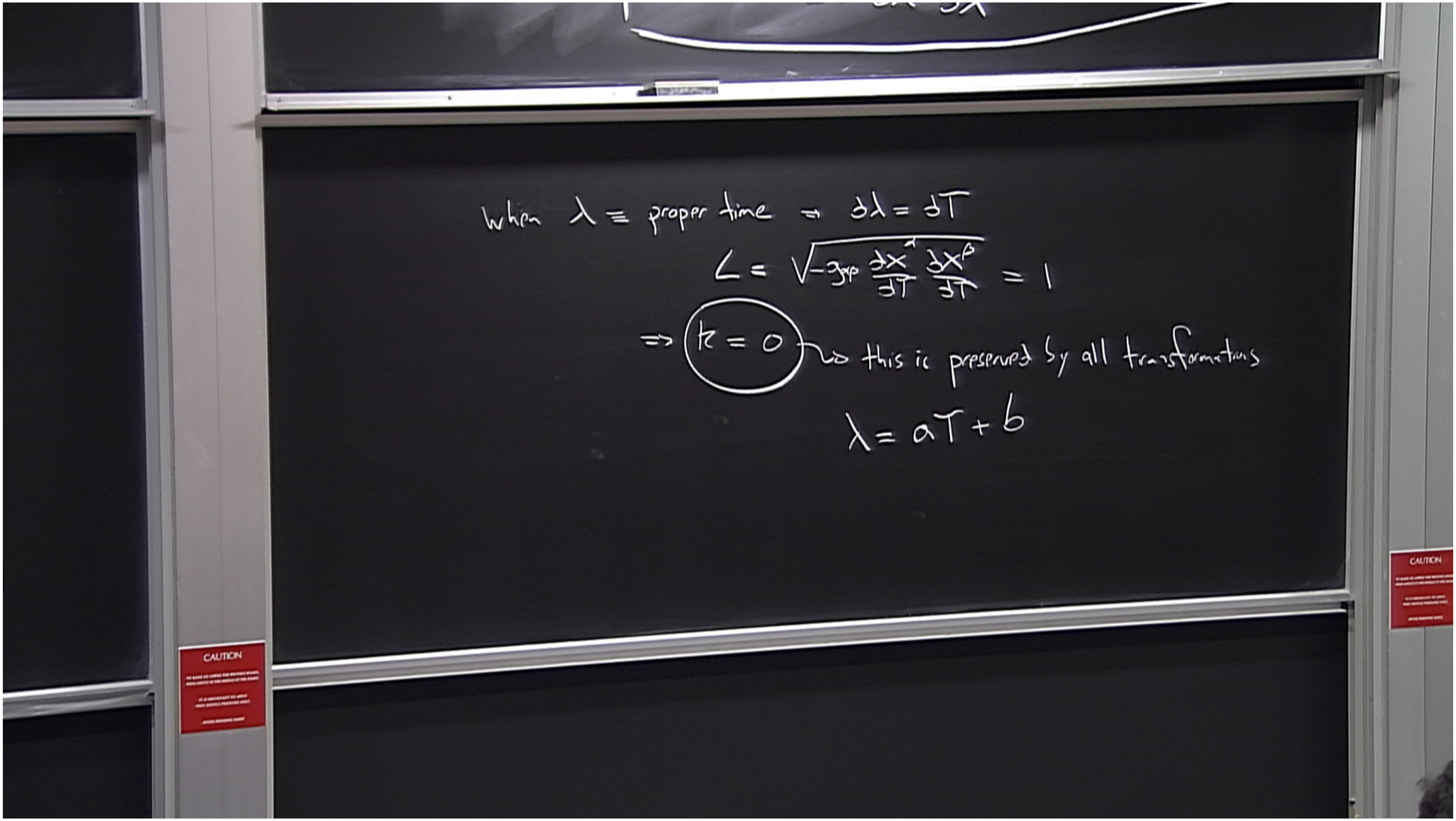

when $\lambda \equiv$ proper time $\Rightarrow d\lambda = d\tau$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}} = 1$


λ ≡ proper time ⇒ dλ = dτ
L = √(−g_{αβ} dx^α/dτ dx^β/dτ) = 1
CAUTION
CAUTION

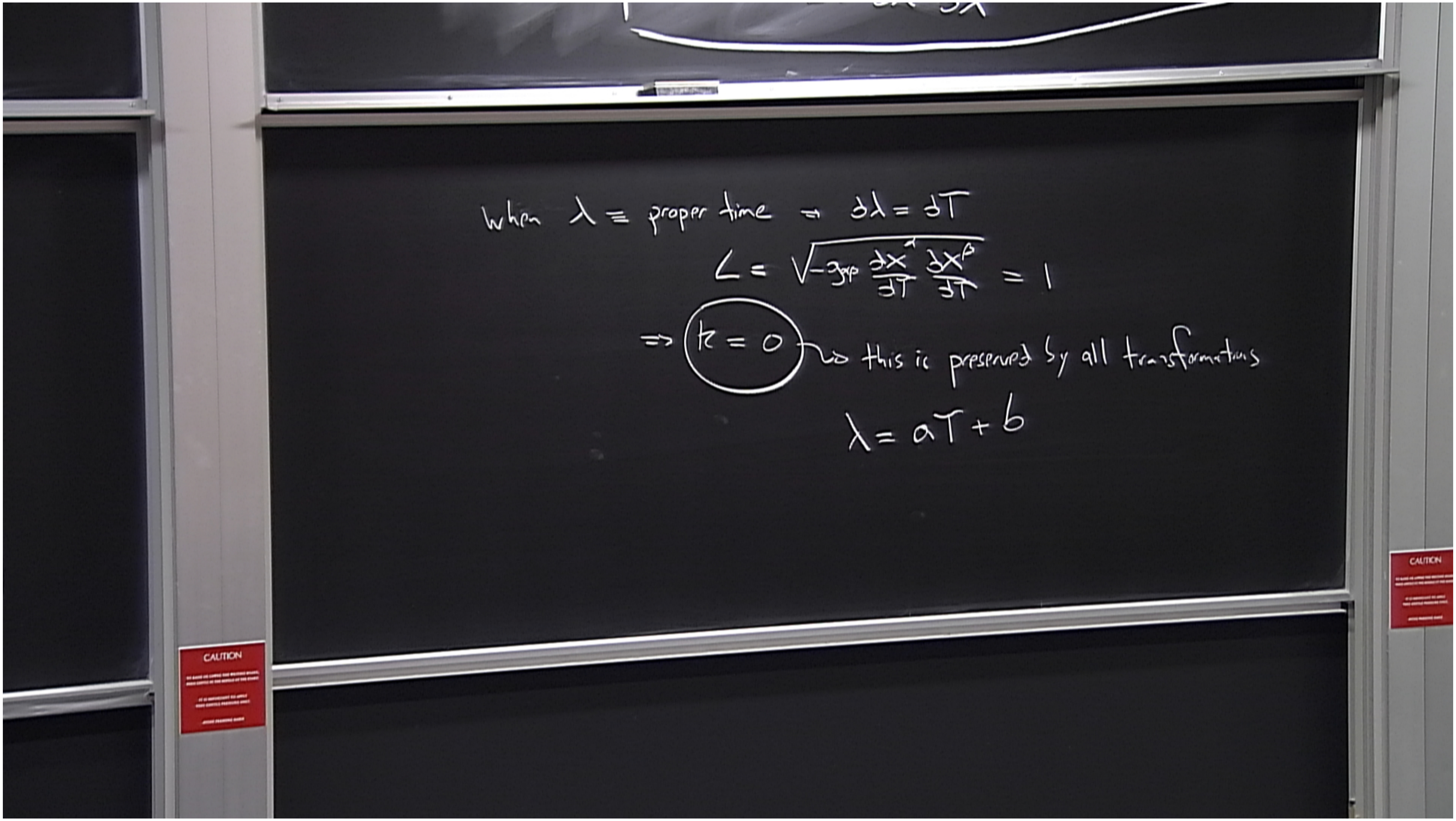

When
roper time ⇒ dλ = dτ
L = √(−g_{αβ} dx^α/dτ dx^β/dτ) = 1
CAUTION
CAUTION

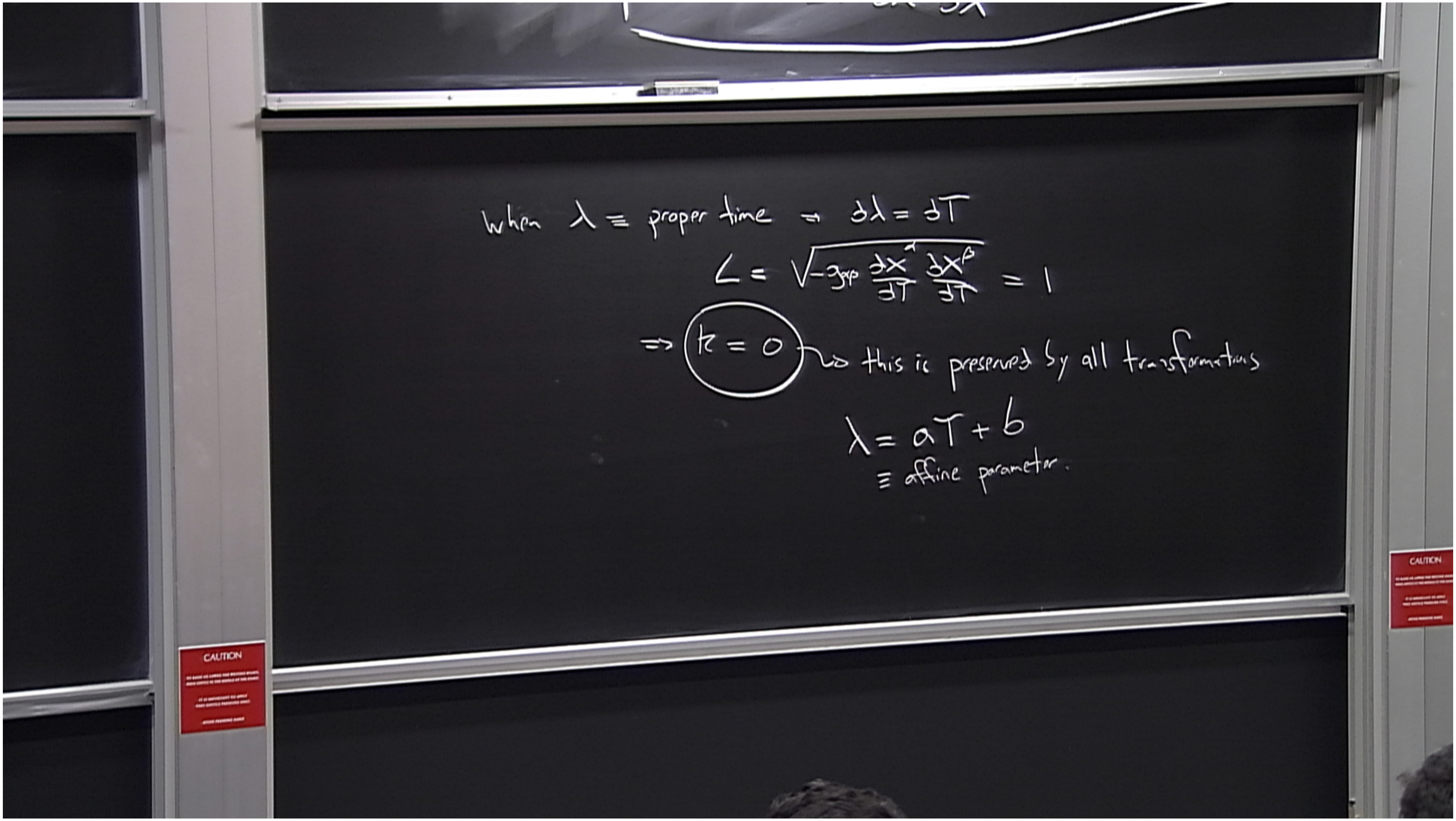

When λ ≡ proper ti
⇒ dλ = dτ
= 1
CAUTION

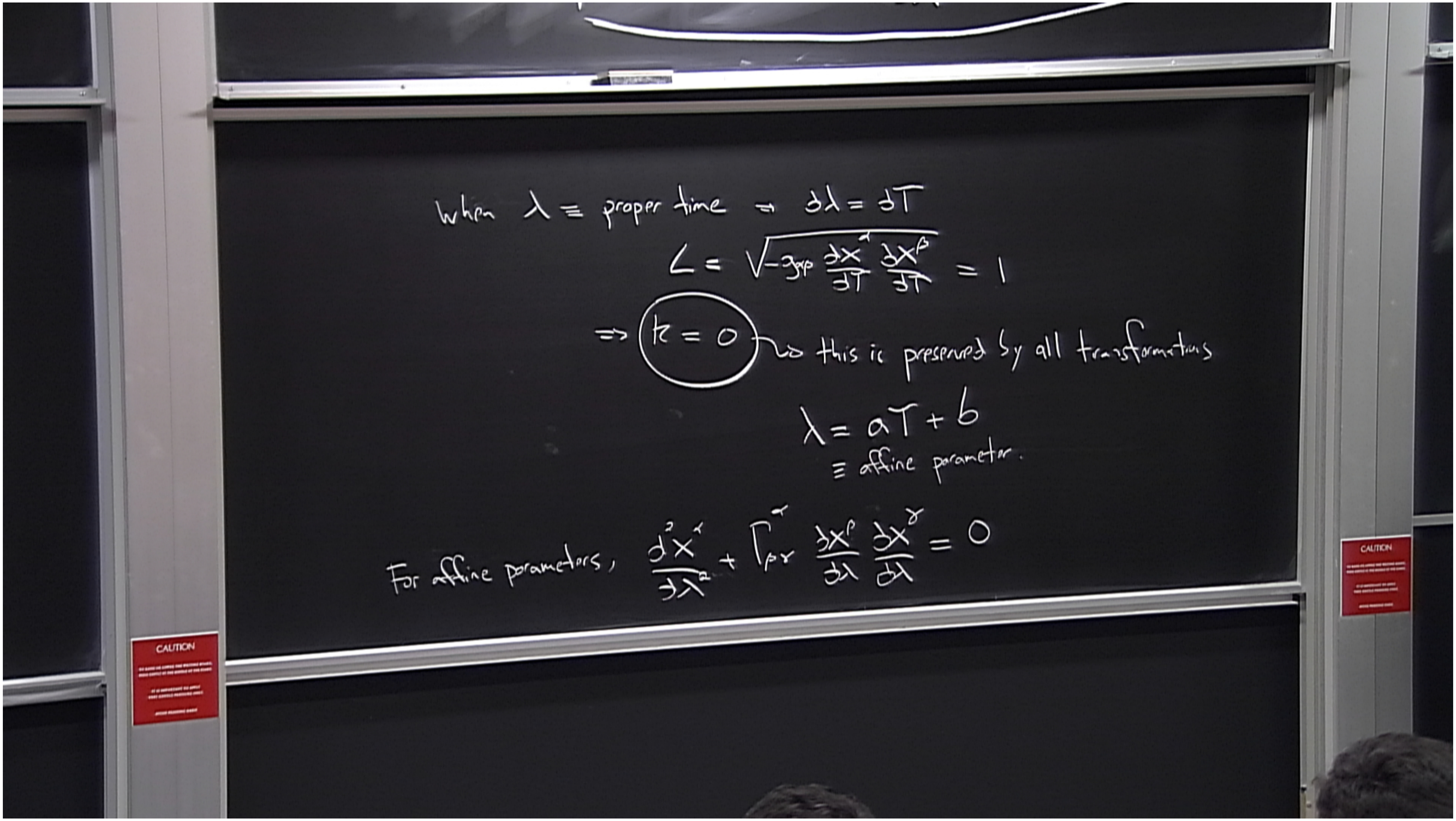

when $\lambda \equiv$ proper time $\Rightarrow d\lambda = d\tau$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}} = 1$
$\Rightarrow k = 0$ ⤳ this is preserved by all transformations
λ
CAUTION
CAUTION

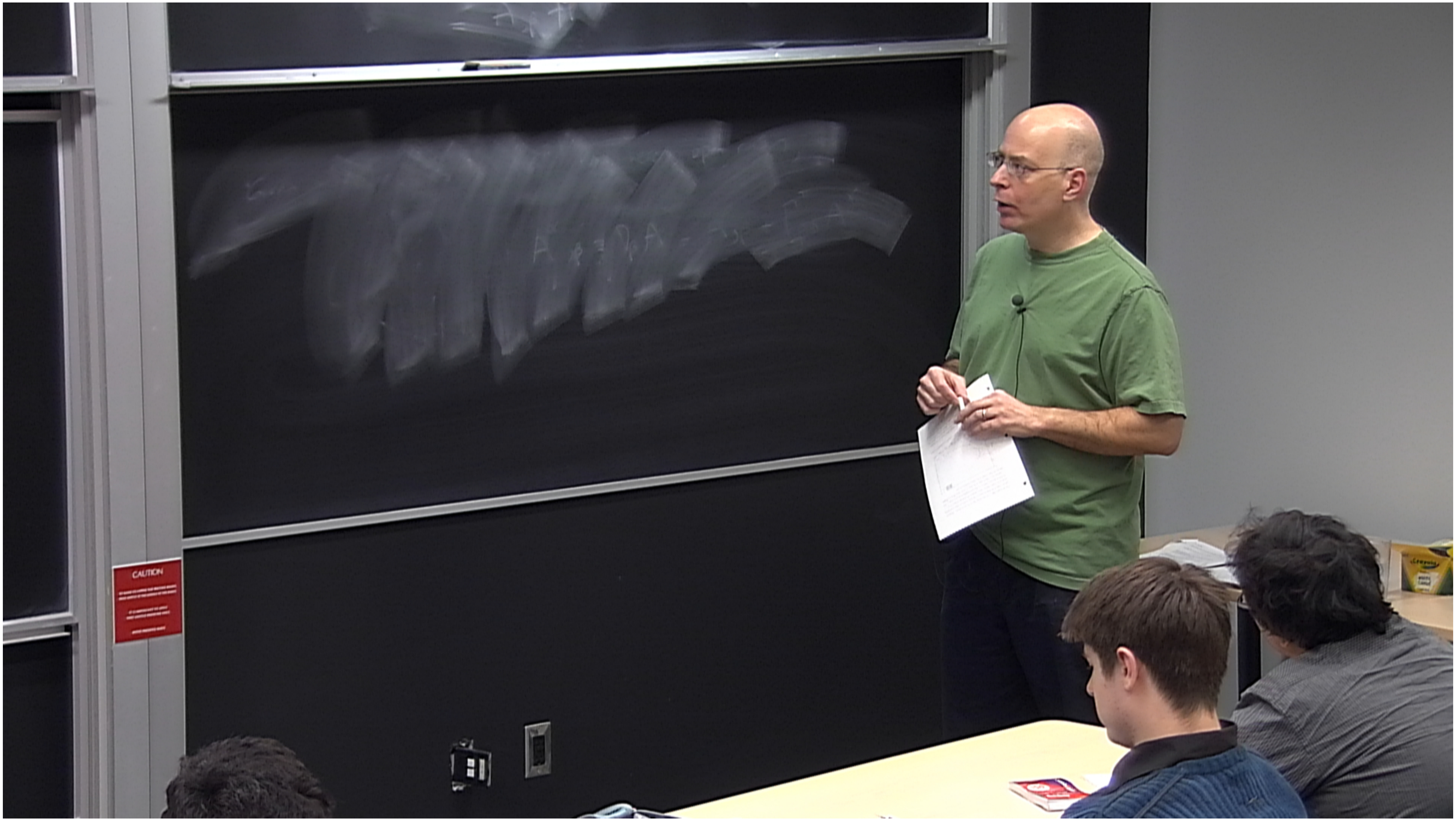


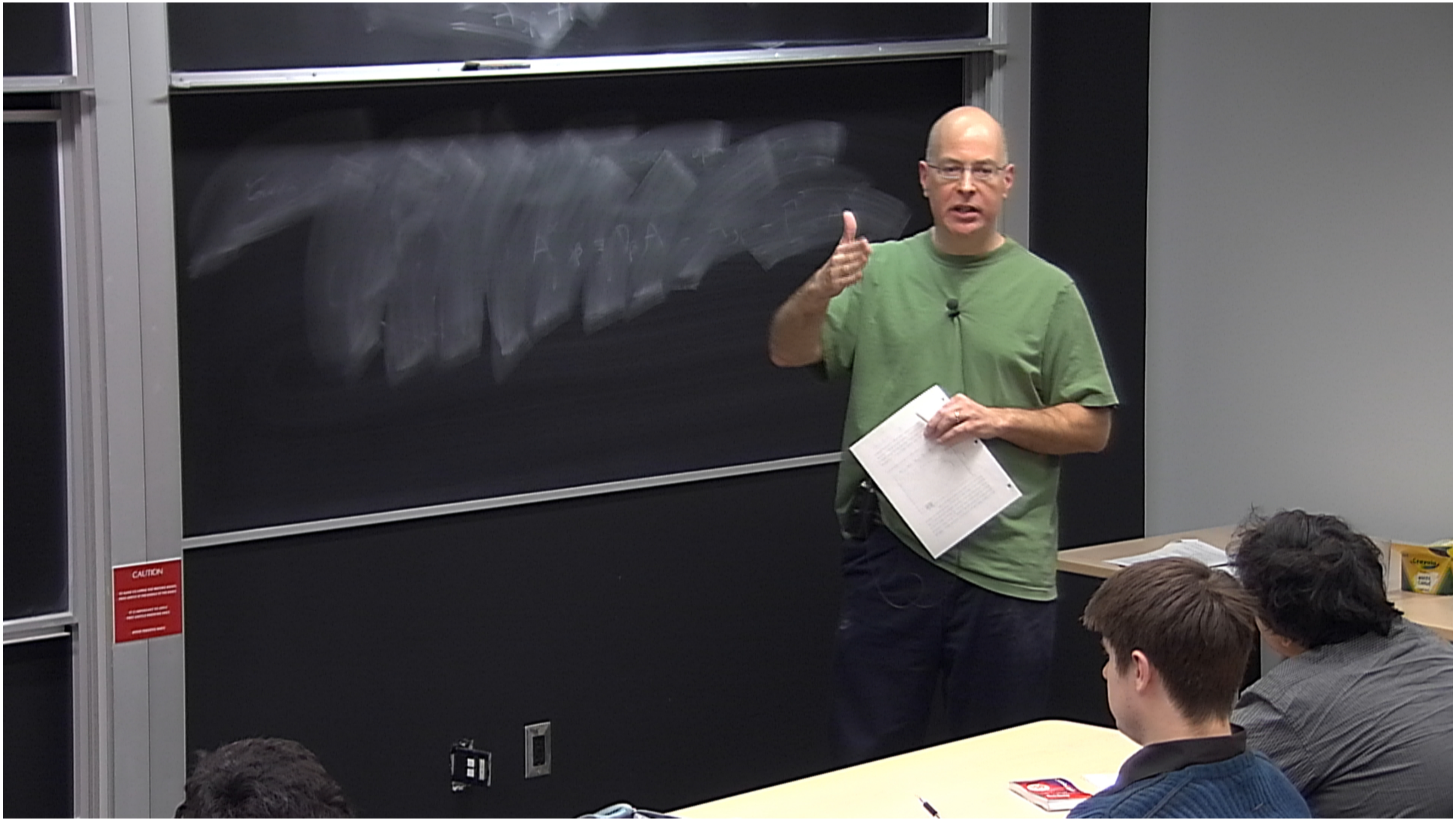
when $\lambda \equiv$ proper time $\Rightarrow d\lambda = d\tau$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}} = 1$
$\Rightarrow$ $k = 0$ $\rightsquigarrow$ this is preserved by all transformations
$\lambda = a\tau + b$
CAUTION

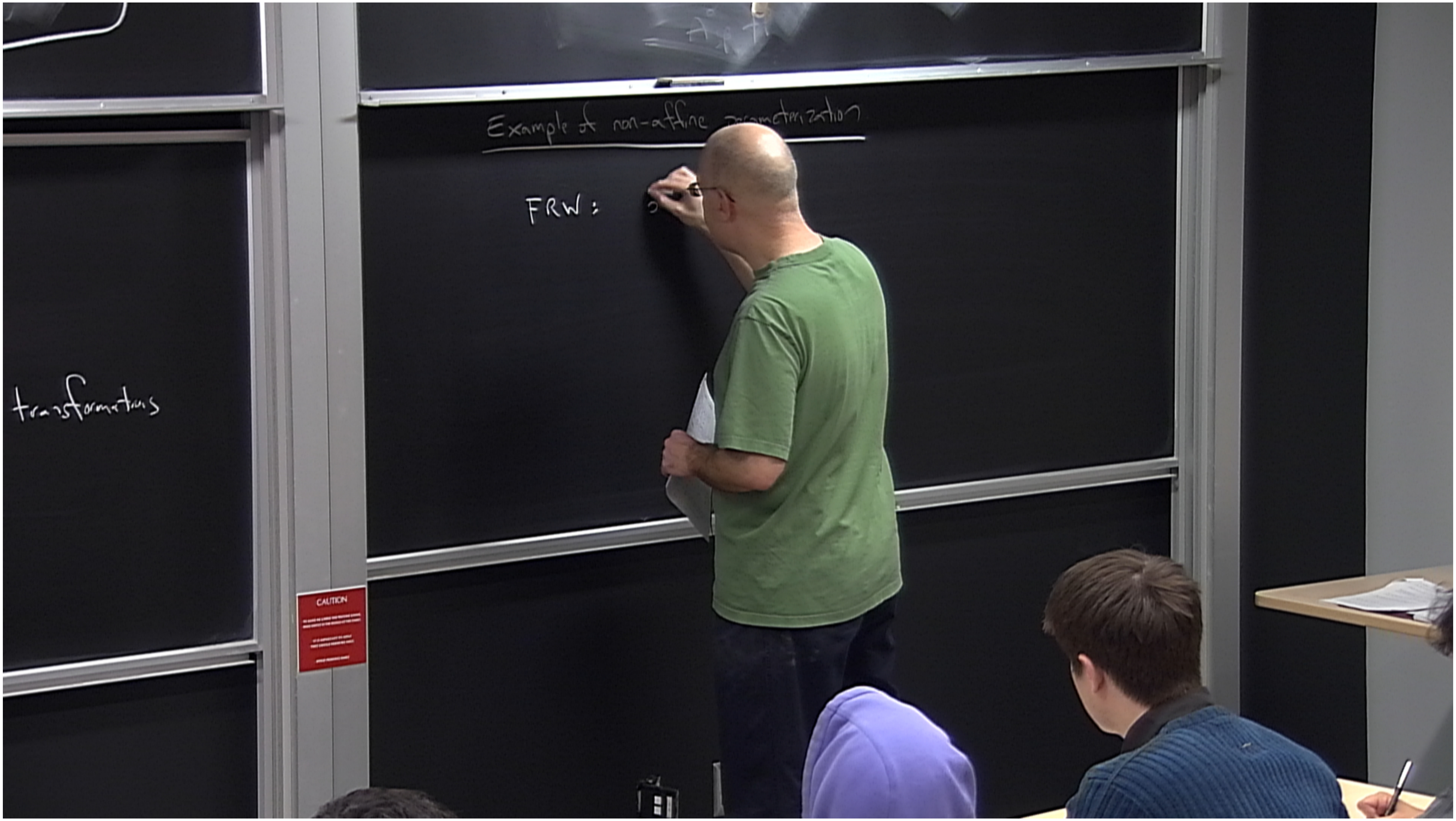

When $\lambda \equiv$ proper time $\Rightarrow d\lambda = d\tau$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}} = 1$
$\Rightarrow k = 0$ this is preserved by all transformations
$\lambda = a\tau + b$

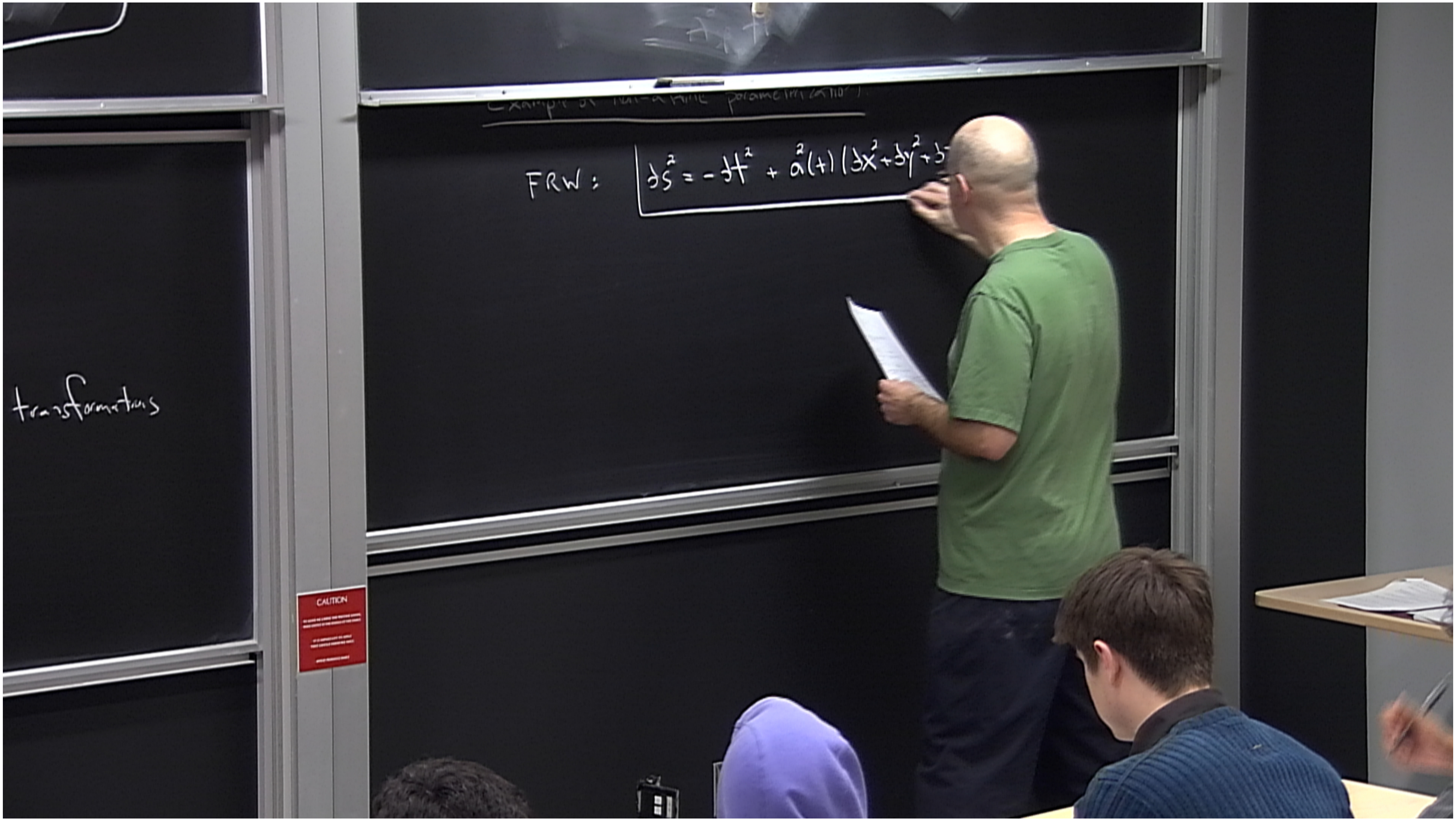

when λ ≡ proper time ⇒ dλ = dτ
L = √(−g_αβ (dx^α/dτ)(dx^β/dτ)) = 1
⇒ k = 0 ⇝ this is preserved by all transformations
λ = aτ + b
≡ affine parameter.
CAUTION
CAUTION


When $\lambda \equiv$ proper time $\Rightarrow d\lambda = d\tau$
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}} = 1$
$\Rightarrow k = 0$ this is preserved by all transformations
$\lambda = a\tau + b$
$\equiv$ affine parameter.
For affine parameters, $\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = 0$

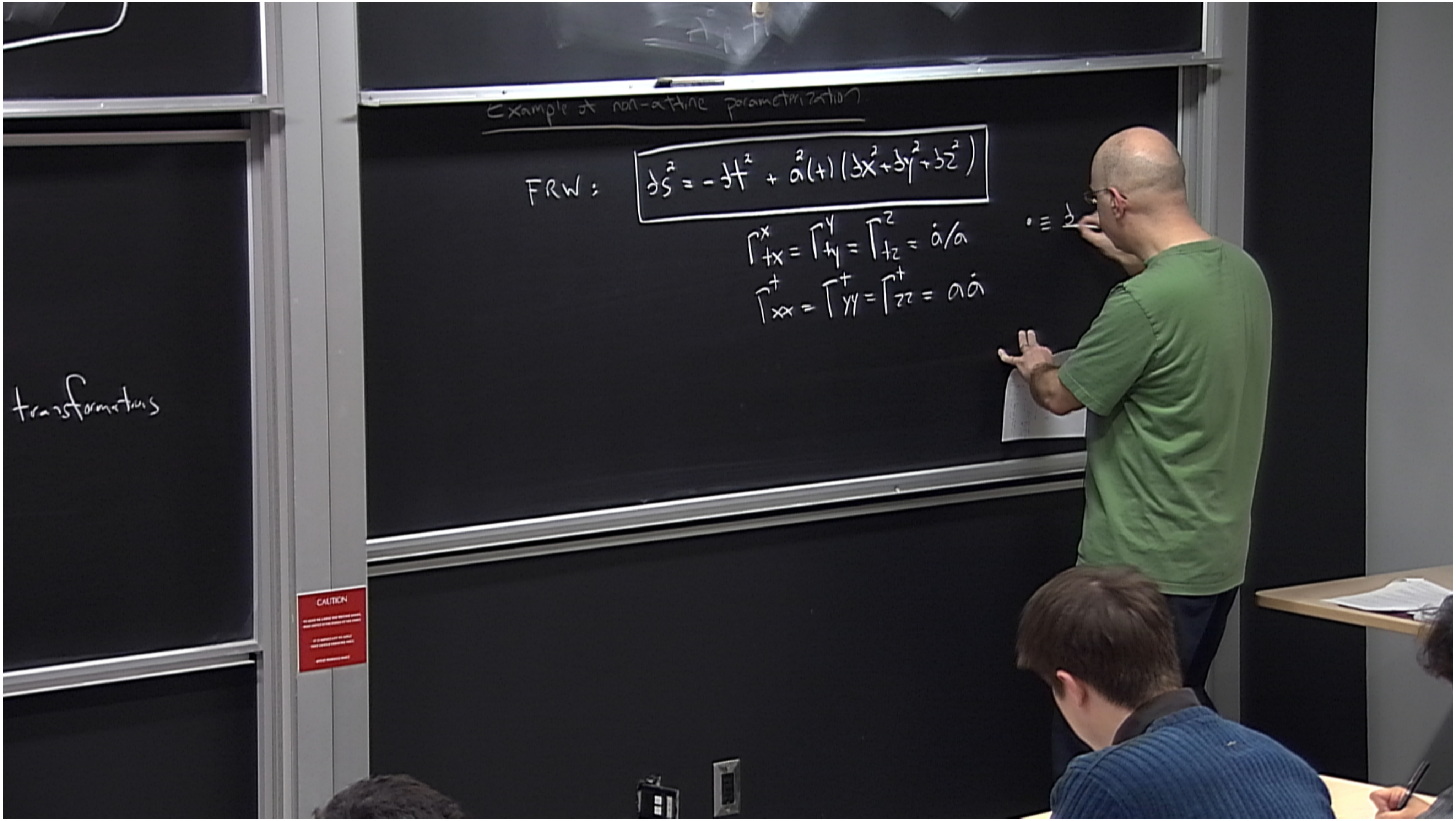

CAUTION

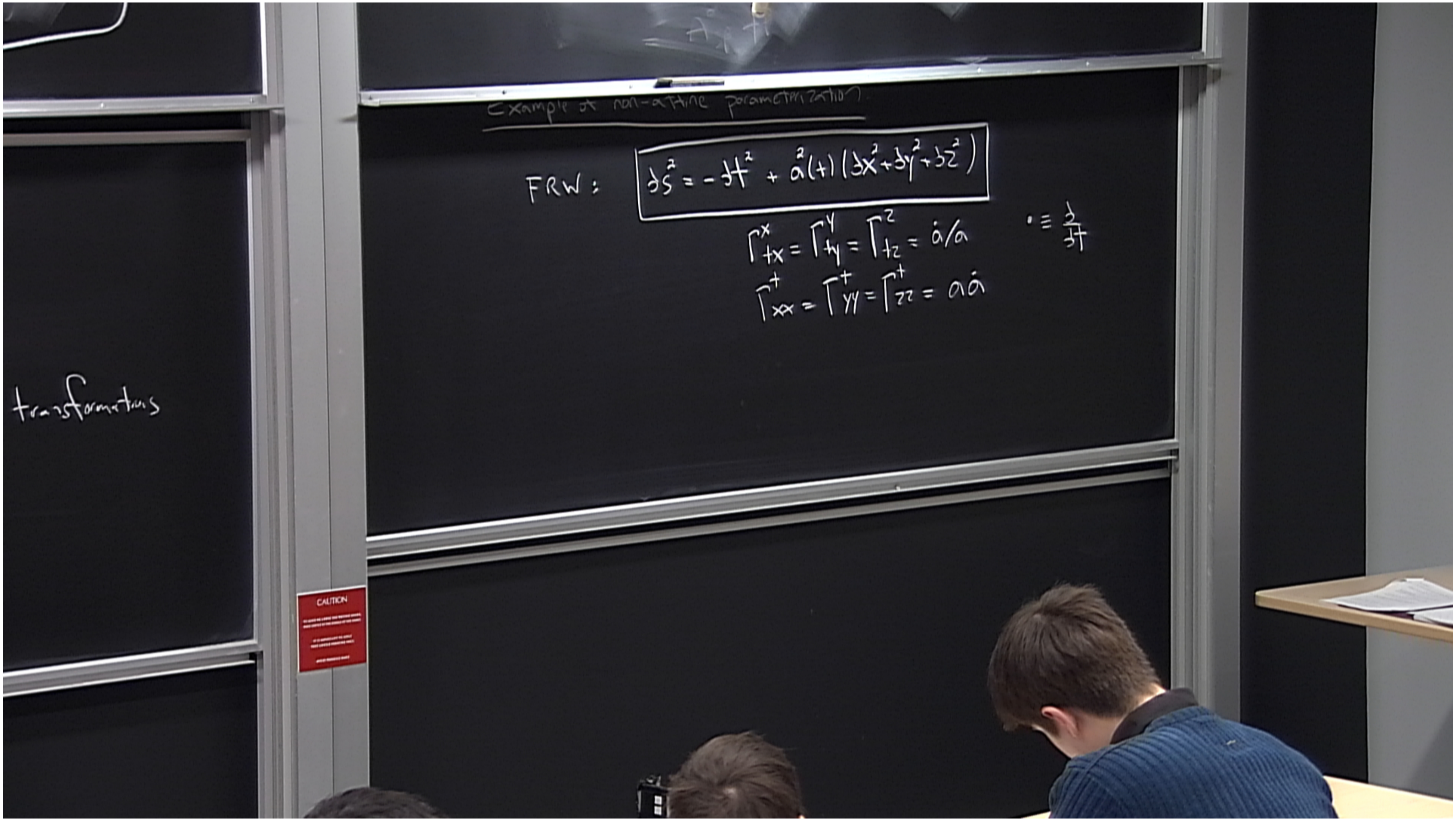


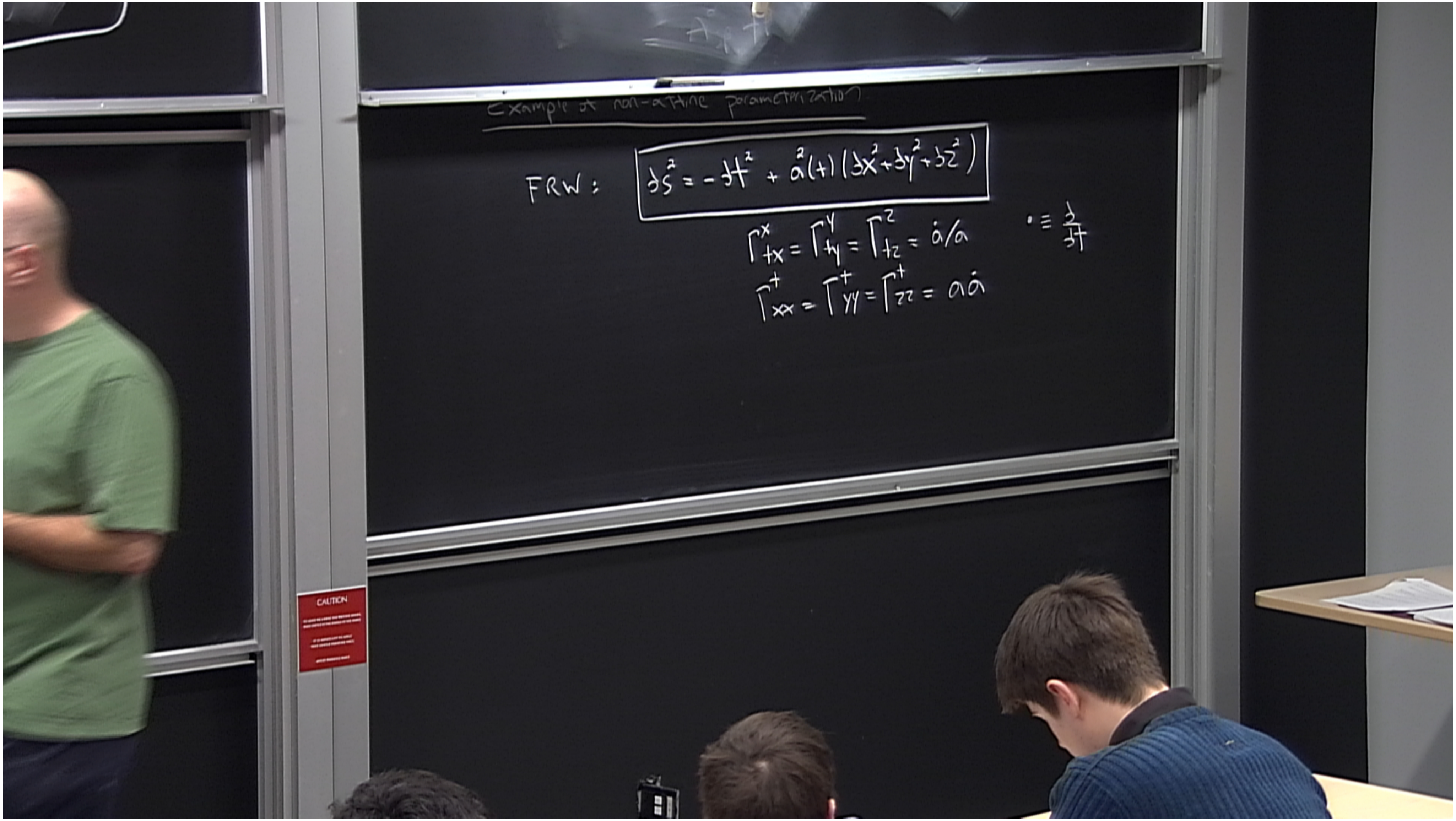

Example of non-affine parametrization
FRW :
transformations
CAUTION



FRW :
transformations


Example of non-affine parametrization
FRW: $ds^2 = -dt^2 + a^2(t)(dx^2+dy^2+dz^2)$
$\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \dot{a}/a$
$\Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}$
transformations
CAUTION


Example of non-affine parametrization
FRW: ds² = −dt² + a²(t)(dx² + dy² + dz²)
Γˣ_tx = Γʸ_ty = Γᶻ_tz = ȧ/a
Γᵗ_xx = Γᵗ_yy = Γᵗ_zz = aȧ
• ≡ d/dt
transformations


Example of non-affine parametrization
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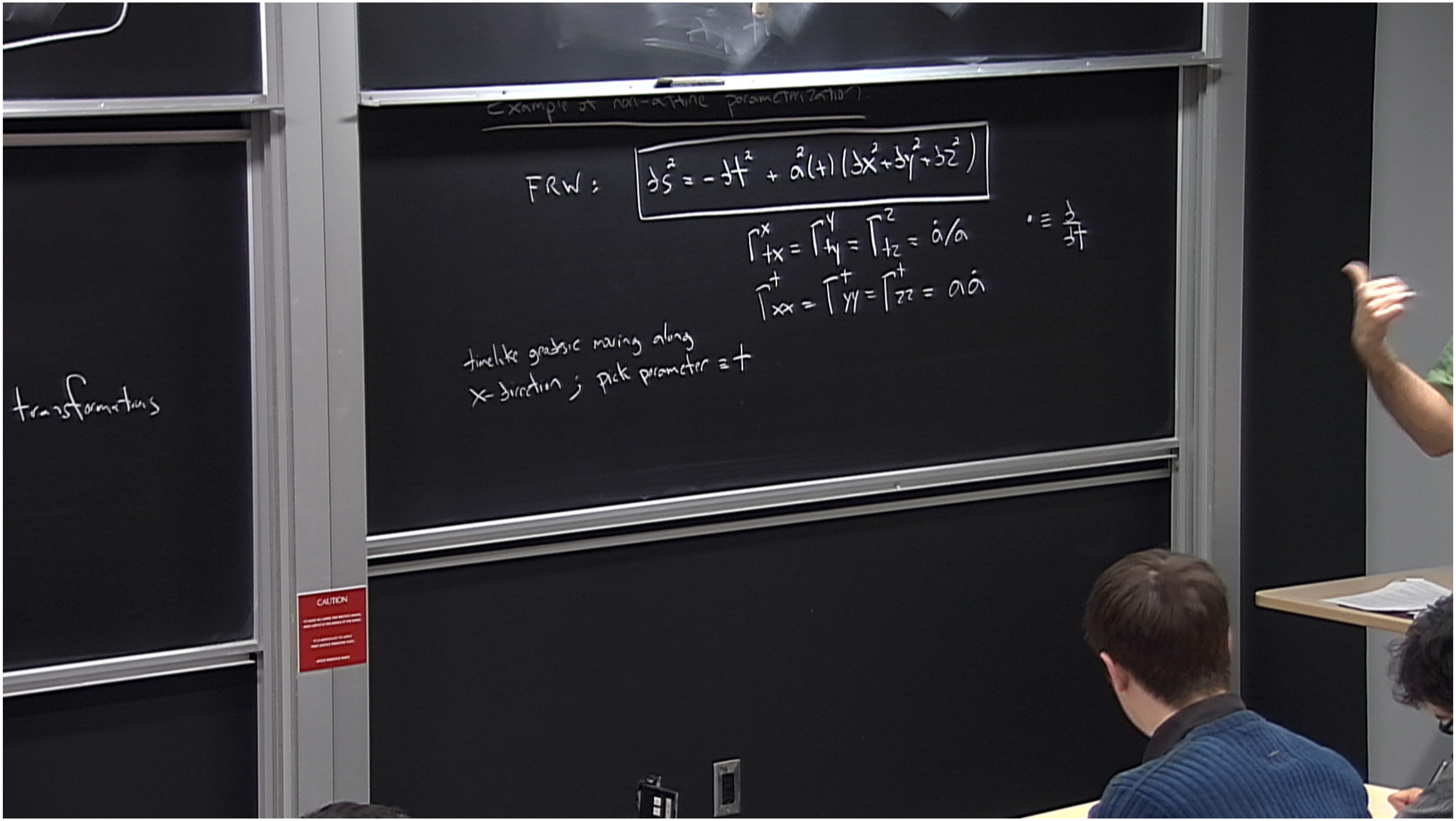
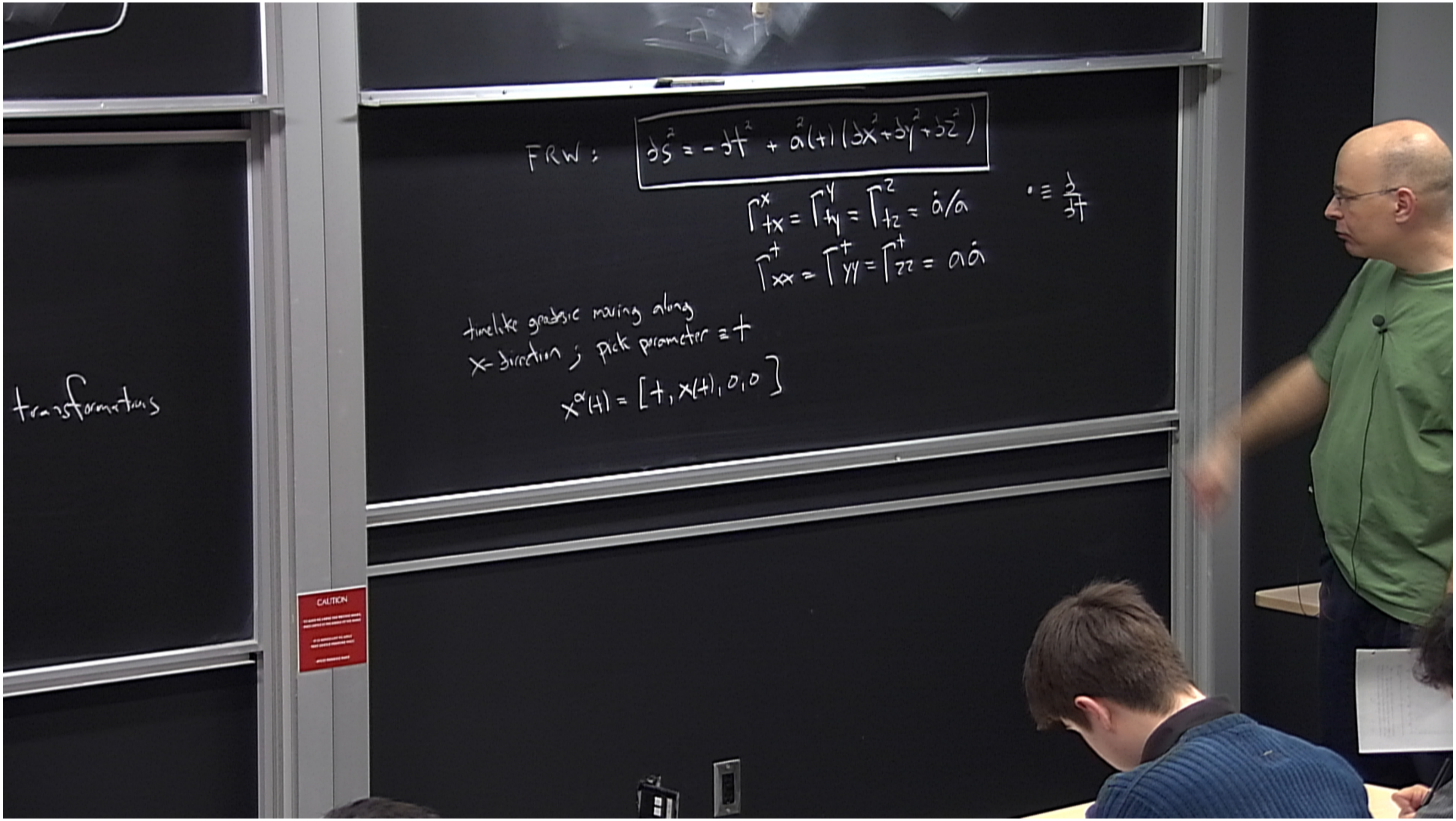
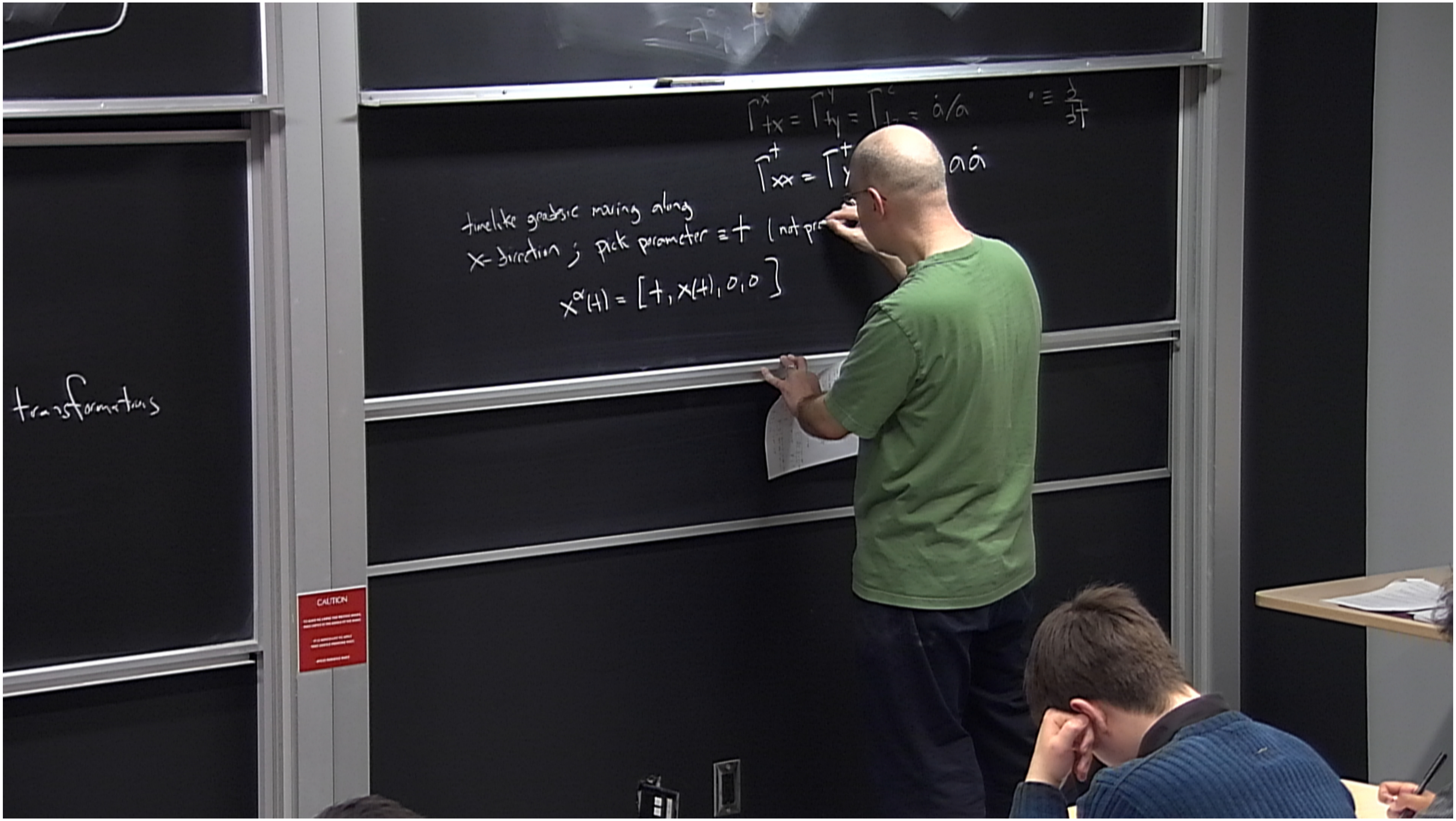
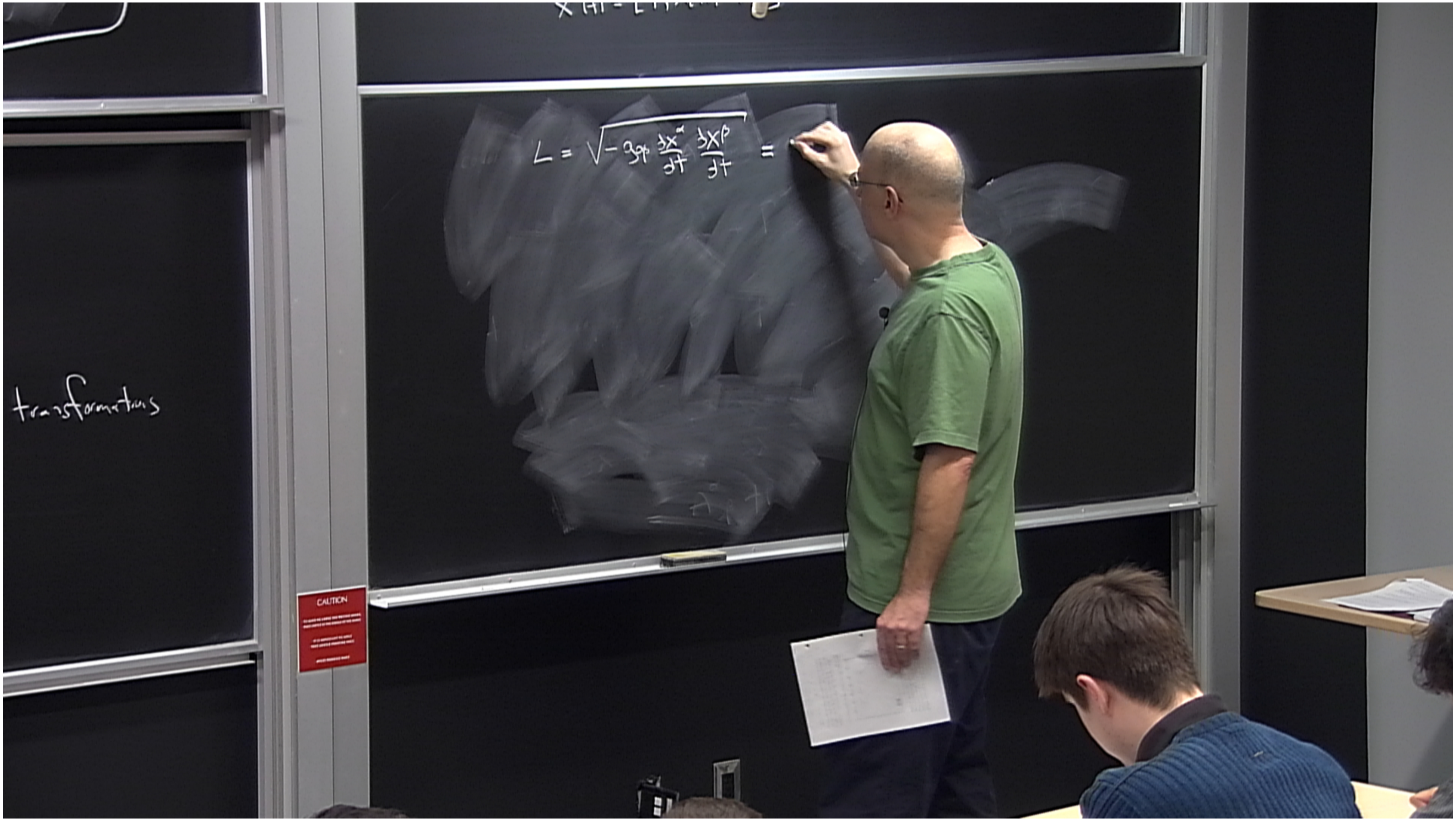
$\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \dot{a}/a$
$\Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}$
$\dot{} \equiv \frac{d}{dt}$
CAUTION

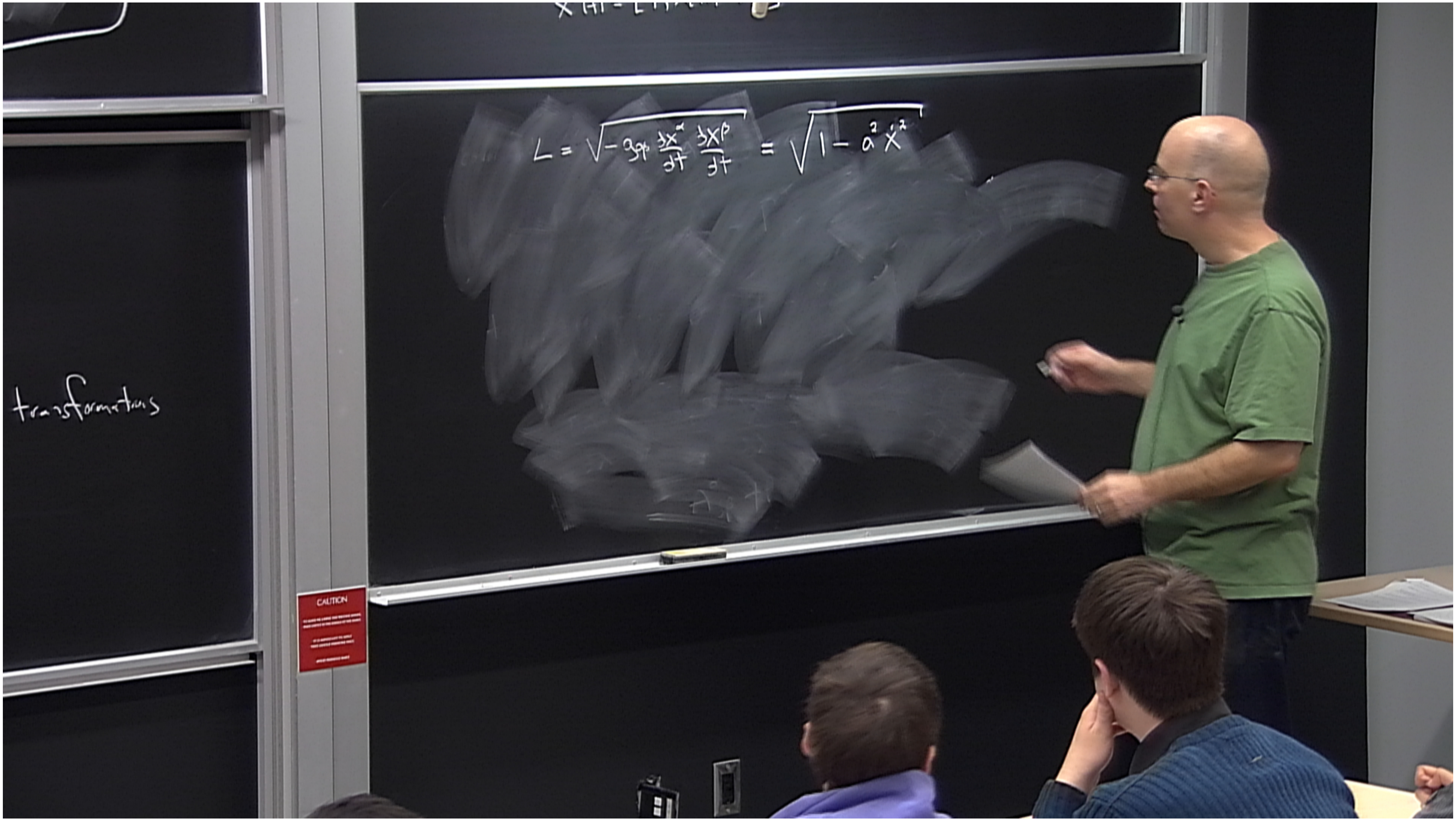

Example of non-affine parametrization.
FRW: $ds^2 = -dt^2 + a^2(t)(dx^2+dy^2+dz^2)$
$\Gamma^x_{tx} = \Gamma^y_{ty} = \Gamma^z_{tz} = \dot{a}/a$
$\dot{} \equiv \frac{d}{dt}$
$\Gamma^t_{xx} = \Gamma^t_{yy} = \Gamma^t_{zz} = a\dot{a}$
timelike geodesic moving along x-direction; pick parameter = t
transformations
CAUTION


FRW: ds² = -dt² + a²(t)(dx²+dy²+dz²)
Γ^x_tx = Γ^y_ty = Γ^z_tz = ȧ/a
• ≡ d/dt
Γ^t_xx = Γ^t_yy = Γ^t_zz = aȧ
timelike geodesic moving along
x-direction; pick parameter = t
x^α(t) = [t, x(t), 0, 0]
transformations
CAUTION

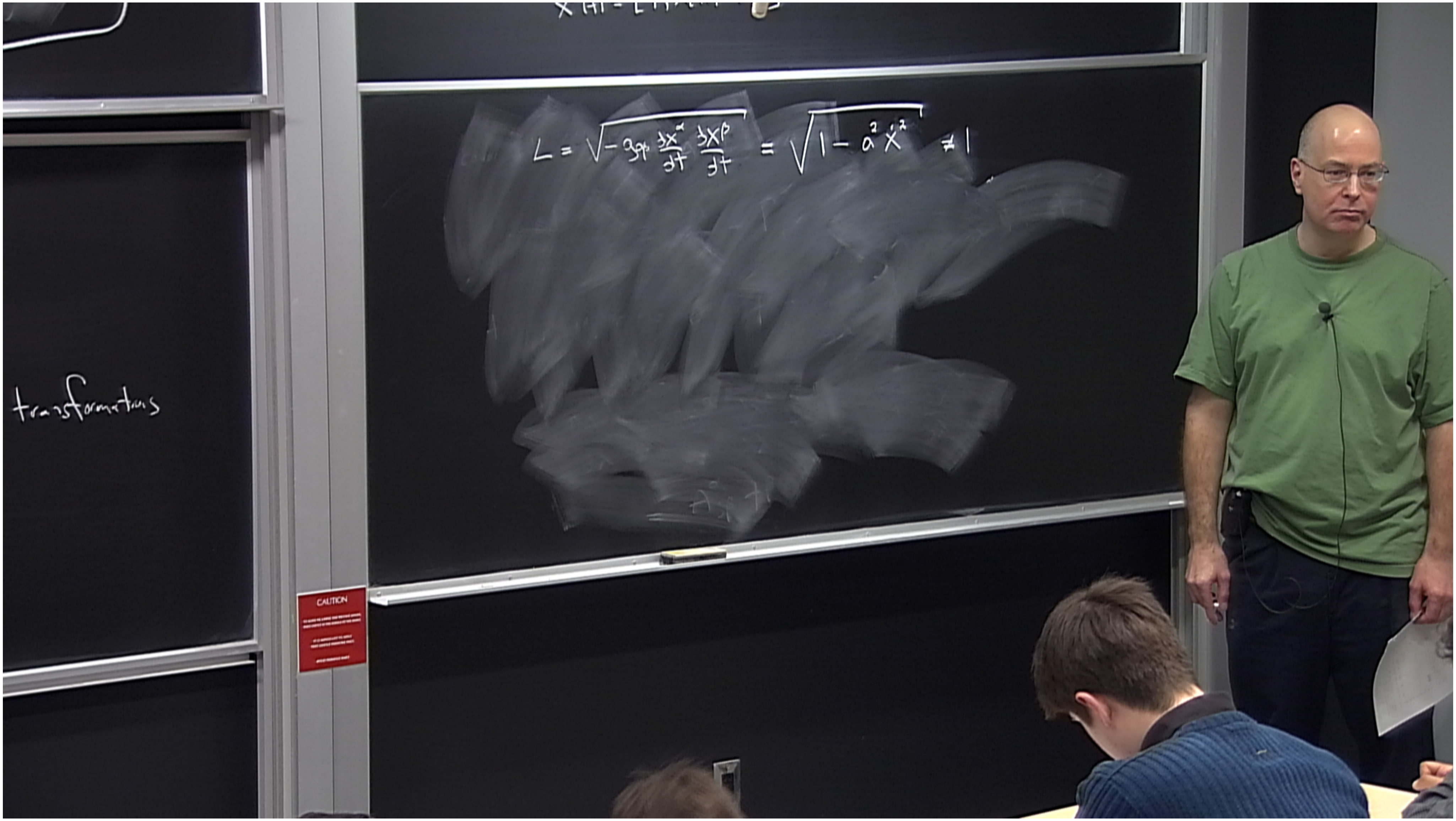

Γ^x_tx = Γ^y_ty = Γ^z_tz = ȧ/a
Γ^t_xx = Γ^t_yy
timelike geodesic moving along
x-direction ; pick parameter = t (not pr
x^α(t) = [t, x(t), 0, 0]
transformations
CAUTION

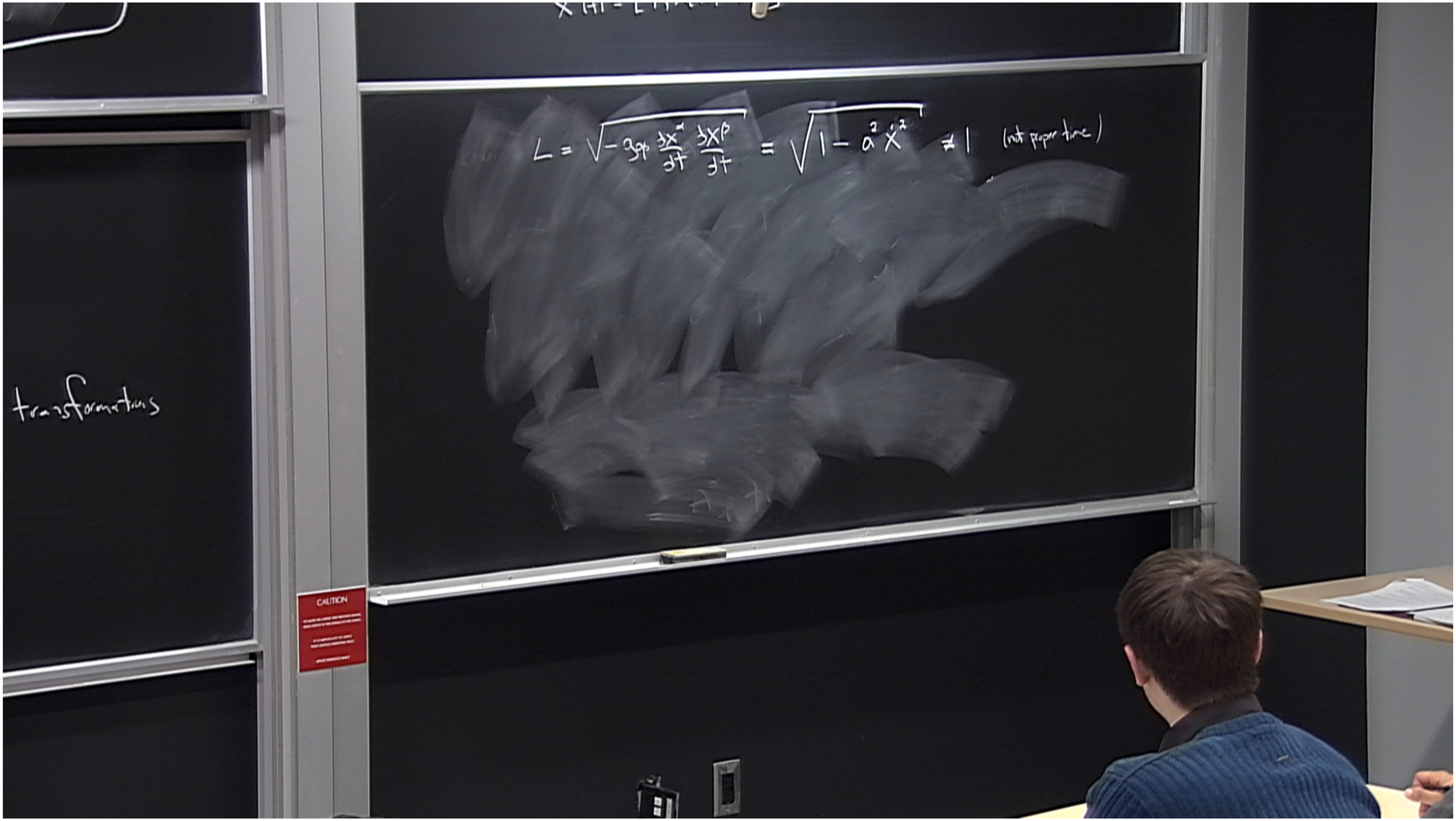


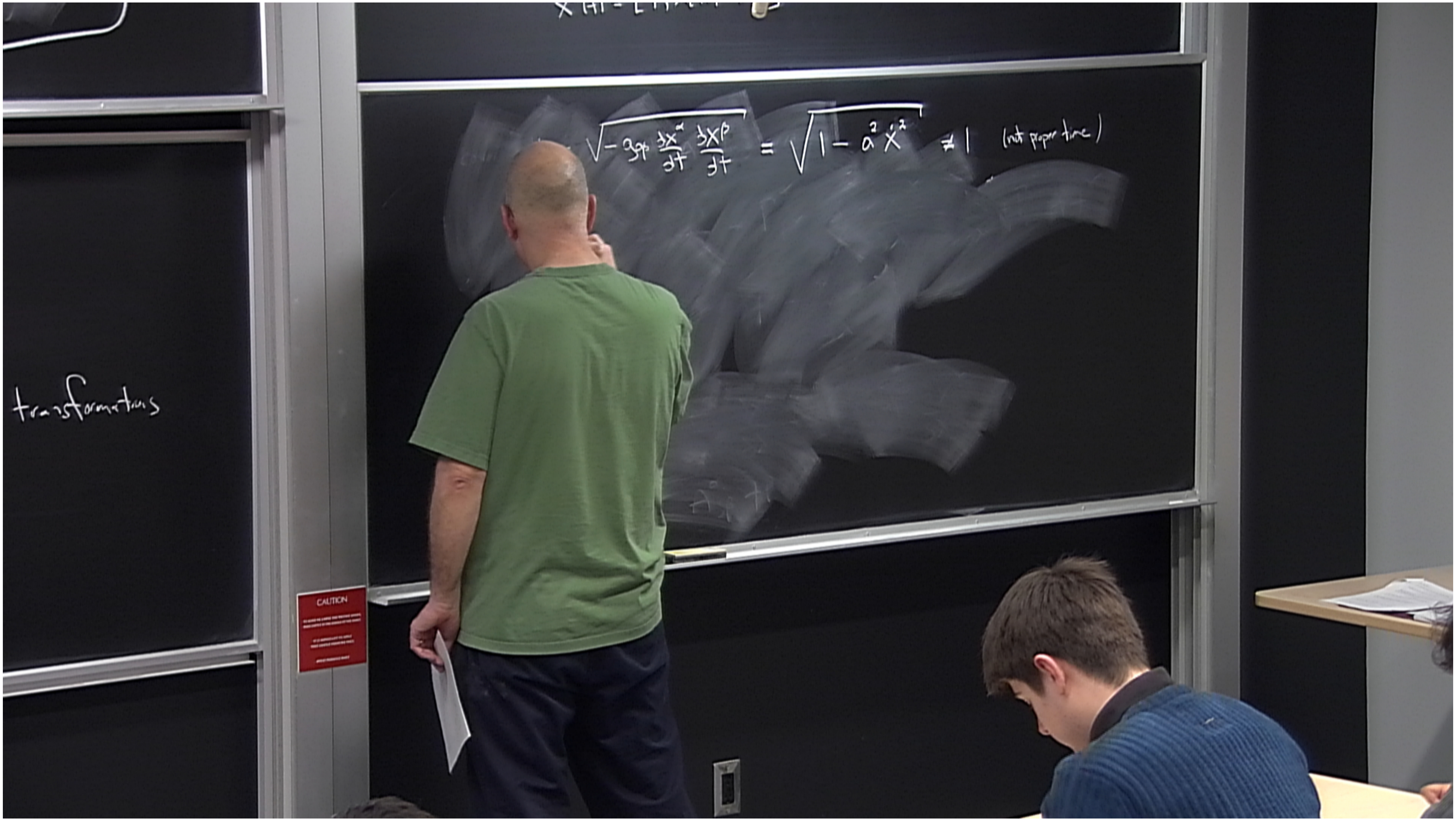
$L = \sqrt{-g_{\alpha\beta}\frac{\partial x^\alpha}{\partial t}\frac{\partial x^\beta}{\partial t}} =$
transformations
CAUTION

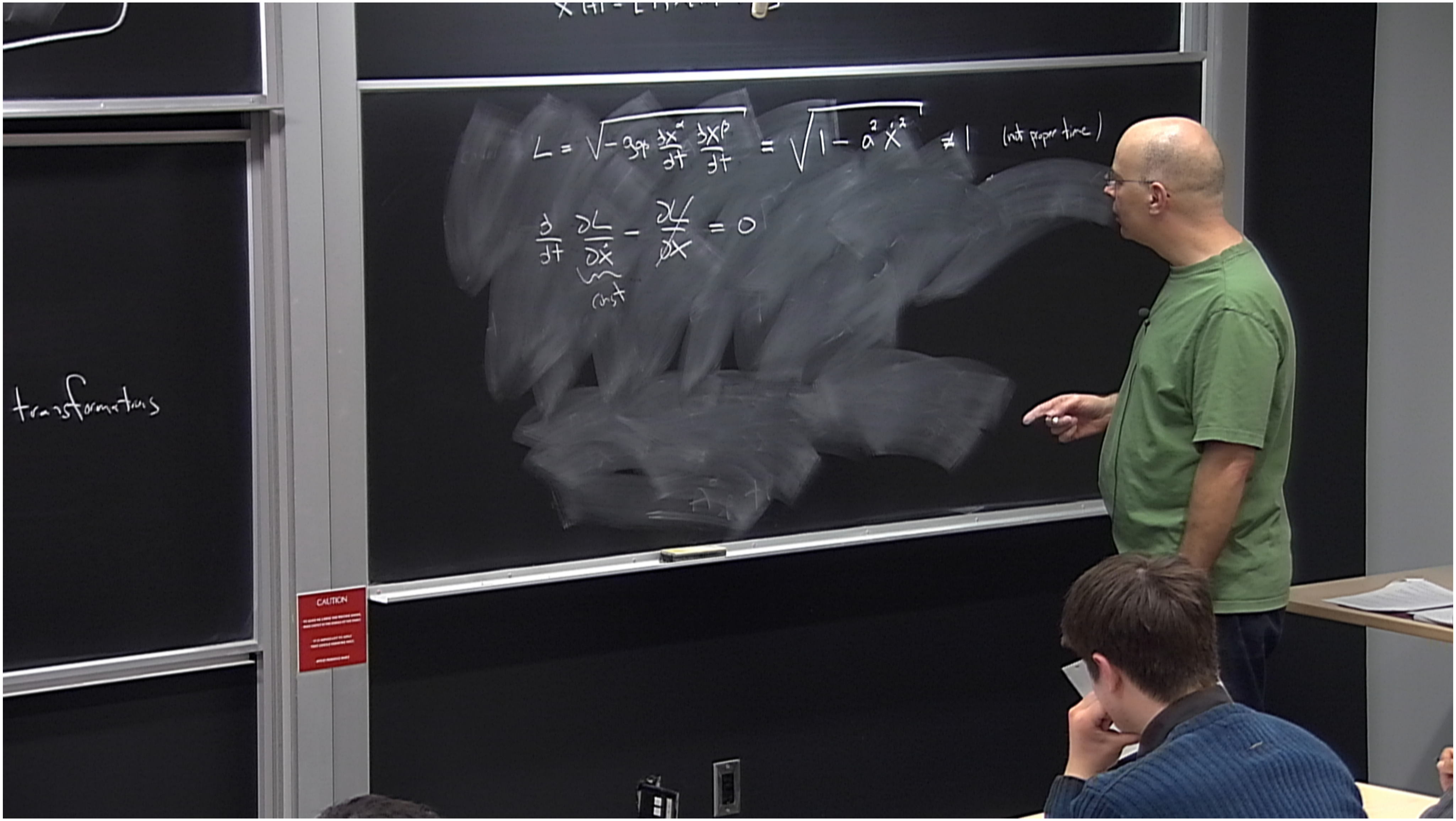


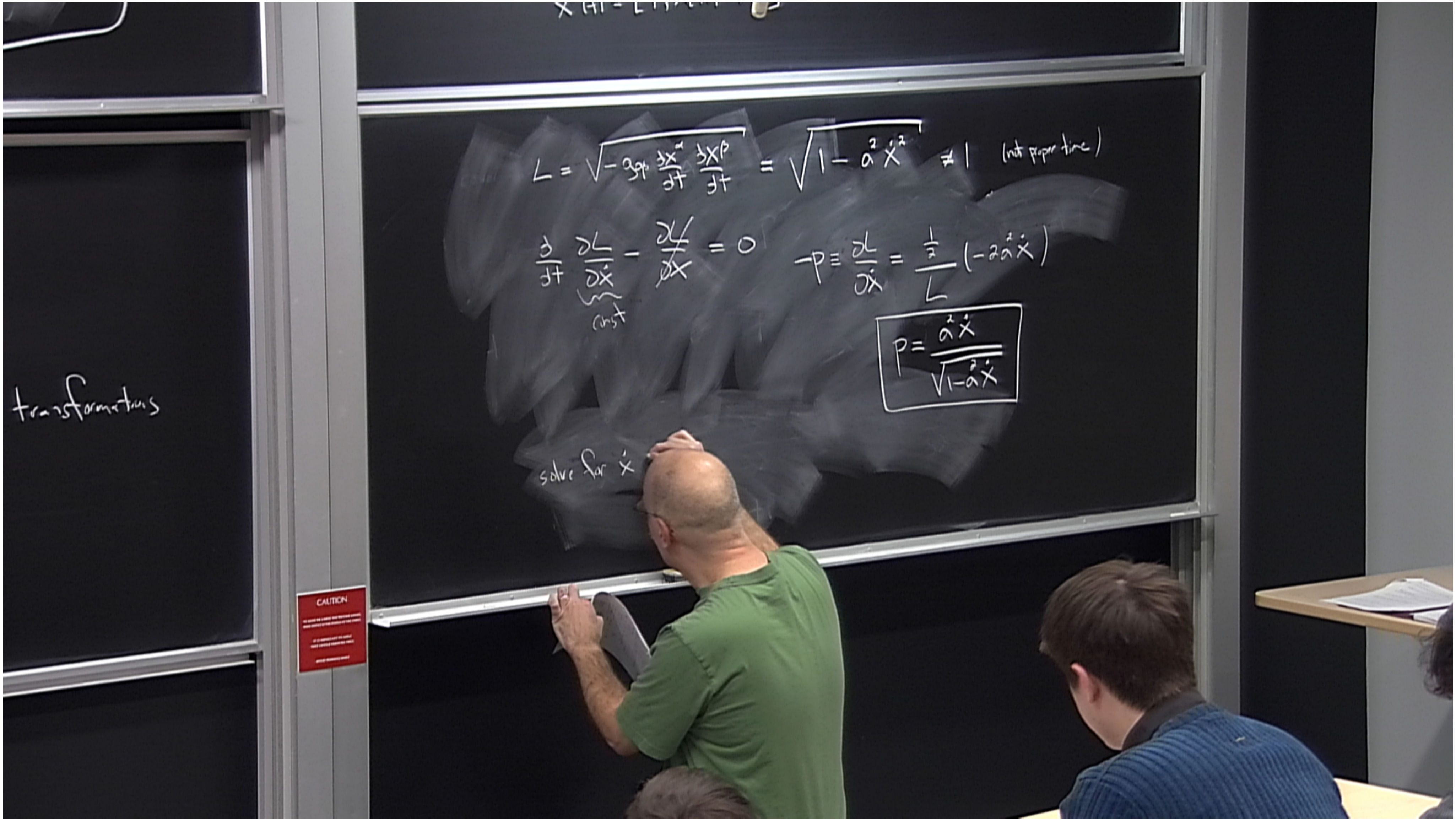
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}} = \sqrt{1 - a^2 \dot{x}^2}$
transformations
CAUTION


$L = \sqrt{-g_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}} = \sqrt{1-a^2 x'^2} \neq 1$
transformations
CAUTION

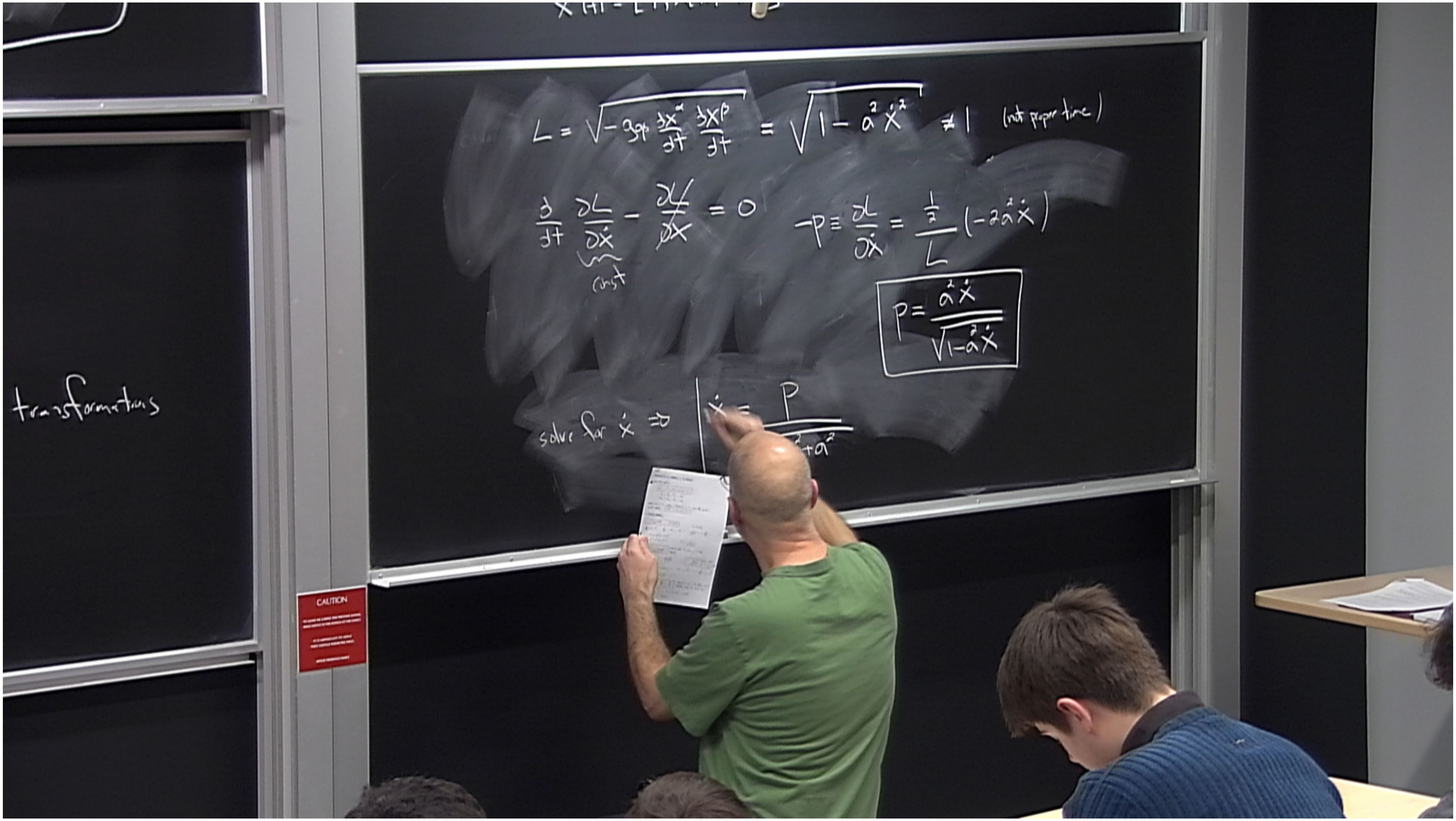

L = √(−g_{αβ} ∂X^α/∂t ∂X^β/∂t) = √(1 − a² X'²) ≠ 1 (not proper time)
transformations
CAUTION

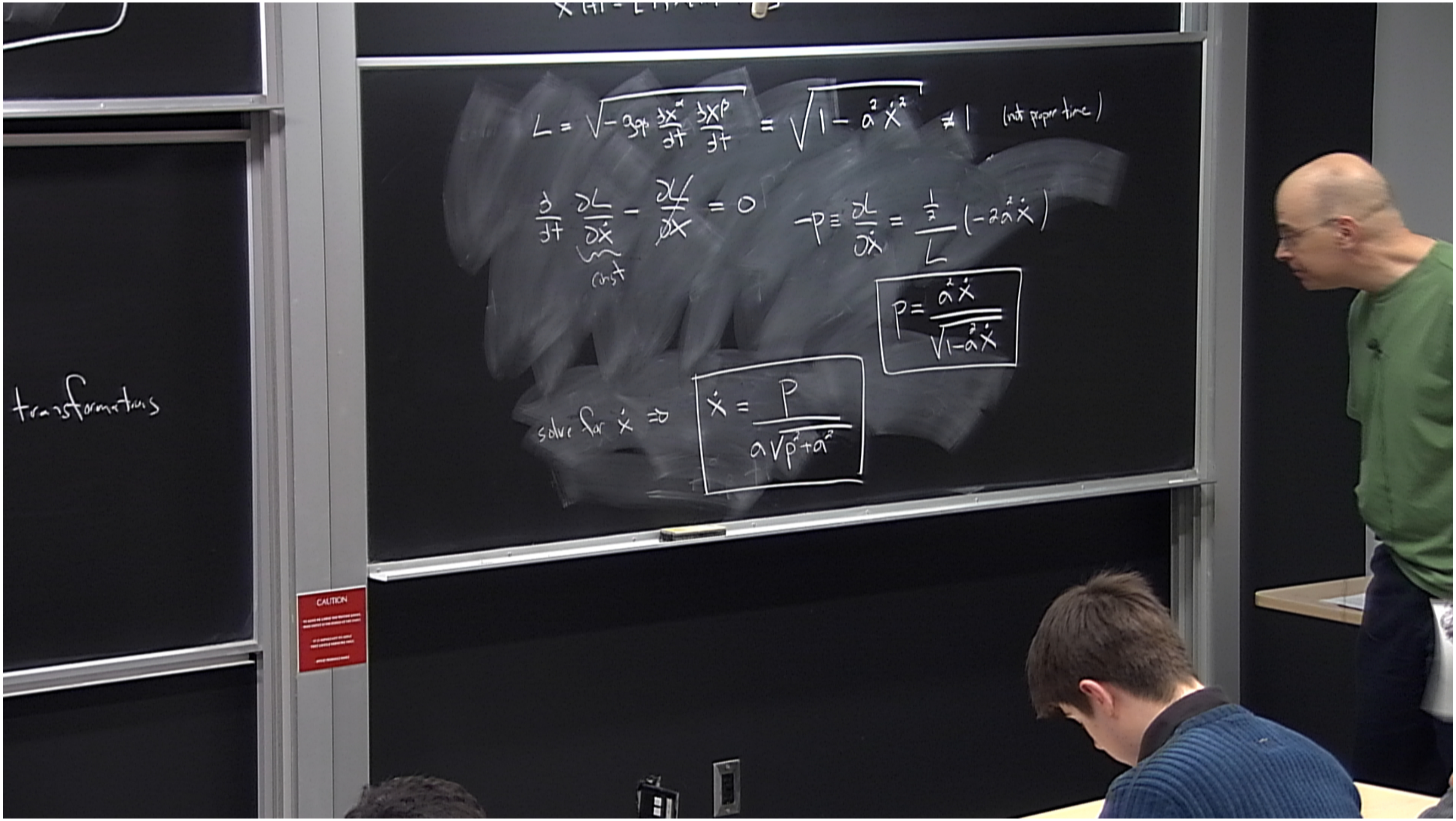

√(-g_{αβ} ∂x^α/∂t ∂x^β/∂t) = √(1 - a²ẋ²) ≠ 1 (not proper time)
transformations

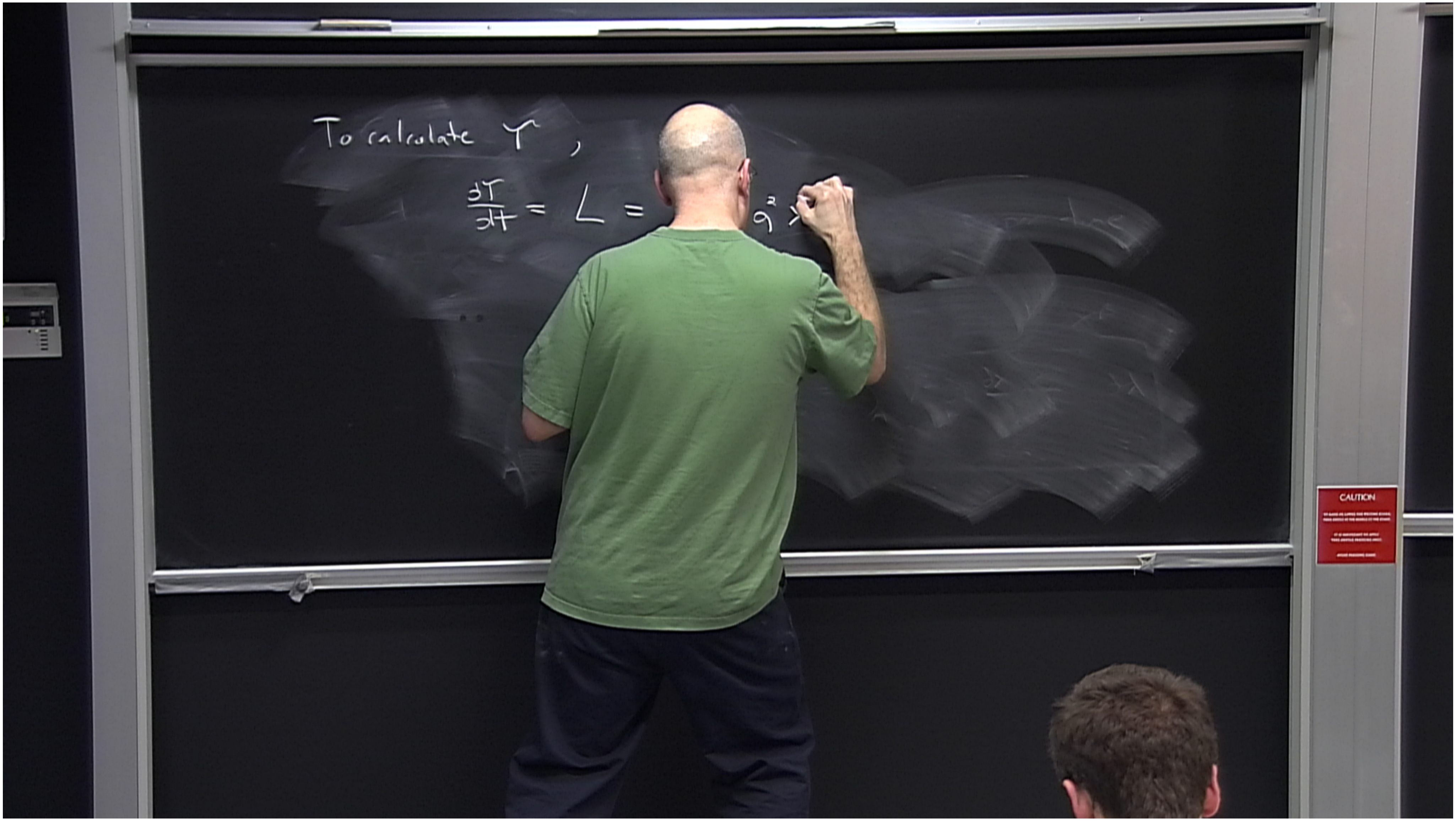

L = √(−g_{αβ} (∂x^α/∂t)(∂x^β/∂t)) = √(1 − a² ẋ²) ≠ 1 (not proper time)
∂/∂t ∂L/∂ẋ − ∂L/∂x = 0
const
transformations
CAUTION

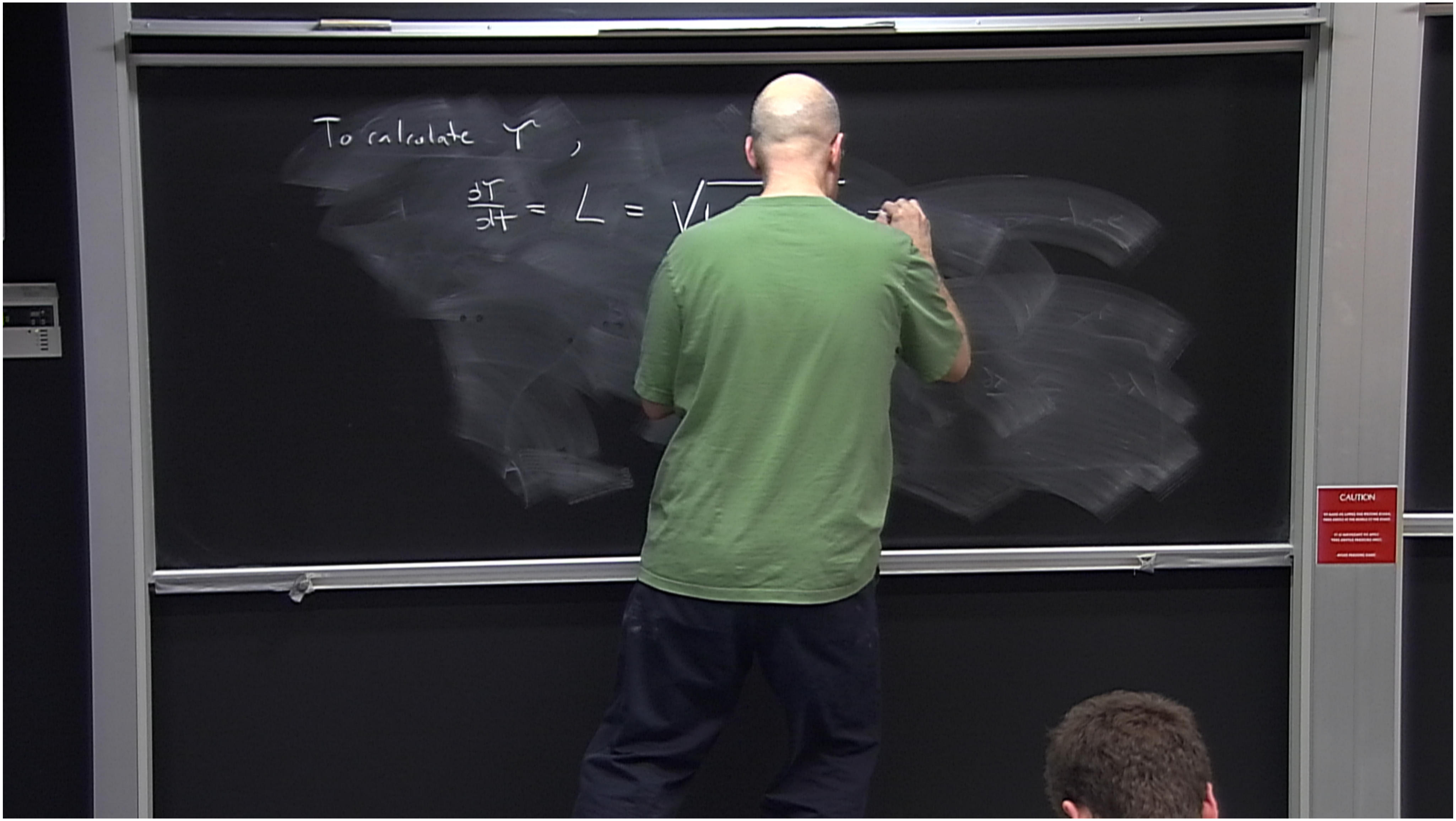


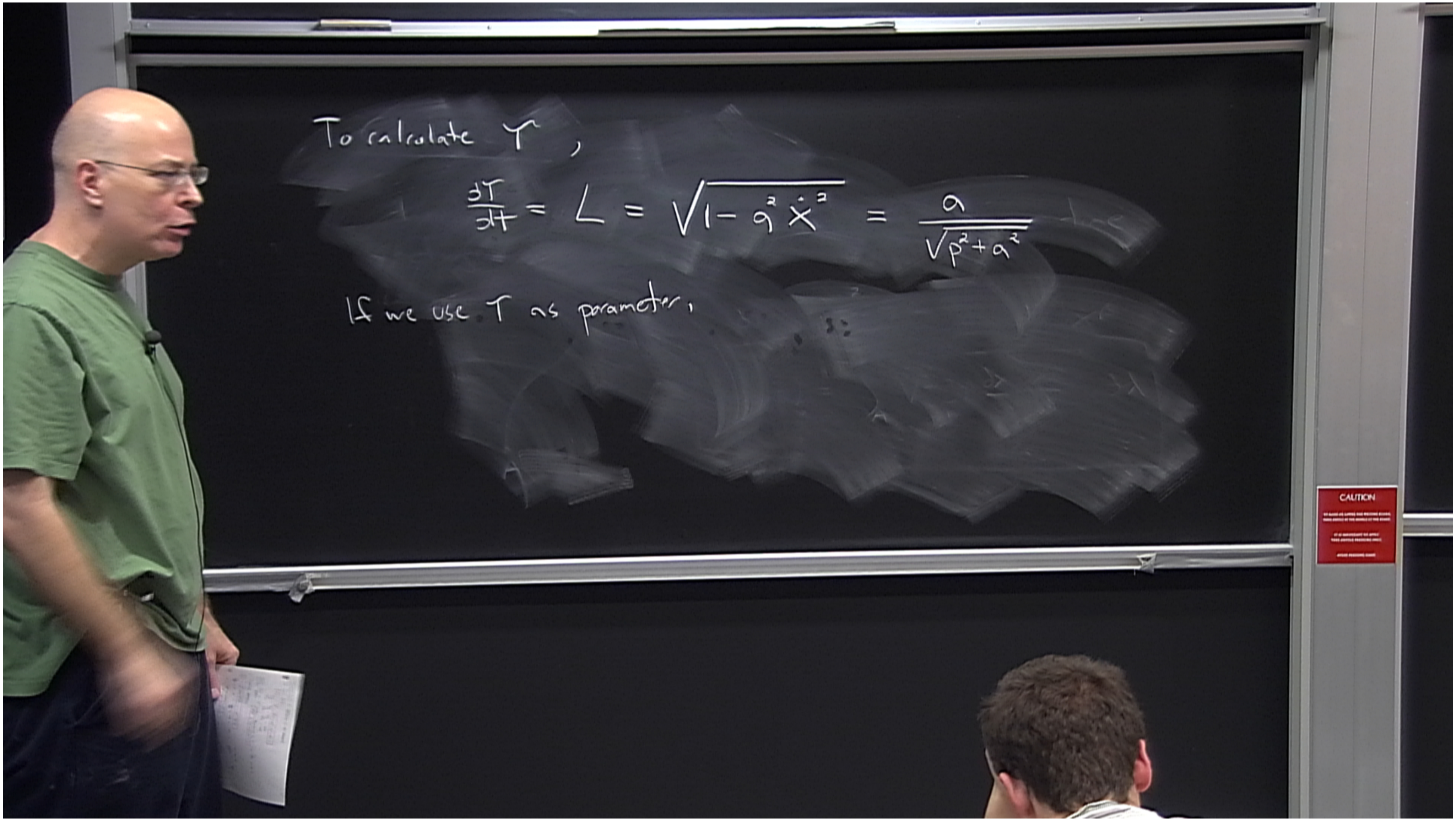
transformations
$L = \sqrt{-g_{\alpha\beta} \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}} = \sqrt{1 - a^2 \dot{x}^2} \neq 1$ (not proper time)
$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$
const
$-p \equiv \frac{\partial L}{\partial \dot{x}} = \frac{\frac{1}{2}}{L}(-2a^2\dot{x})$
$p = \frac{a^2 \dot{x}}{\sqrt{1 - a^2 \dot{x}}}$
solve for $\dot{x}$
CAUTION

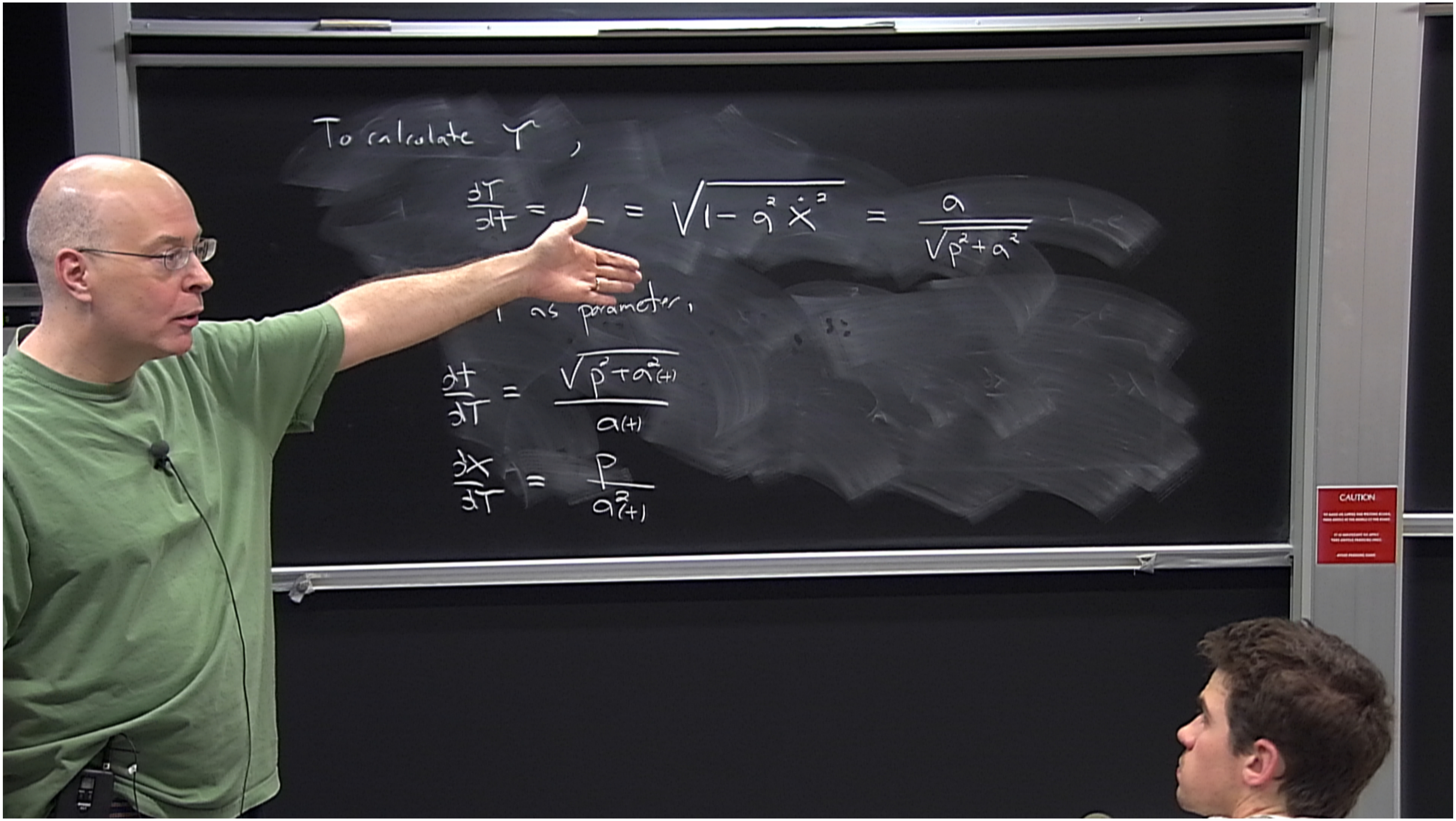

L = √(-g_{αβ} dx^α/dt dx^β/dt) = √(1 - a²ẋ²) ≠ 1 (not proper time)
d/dt ∂L/∂ẋ - ∂L/∂x = 0
const
-p ≡ ∂L/∂ẋ = (1/2)/L (-2a²ẋ)
p = a²ẋ / √(1-a²ẋ)
solve for ẋ ⇒ ẋ = p
transformations
CAUTION

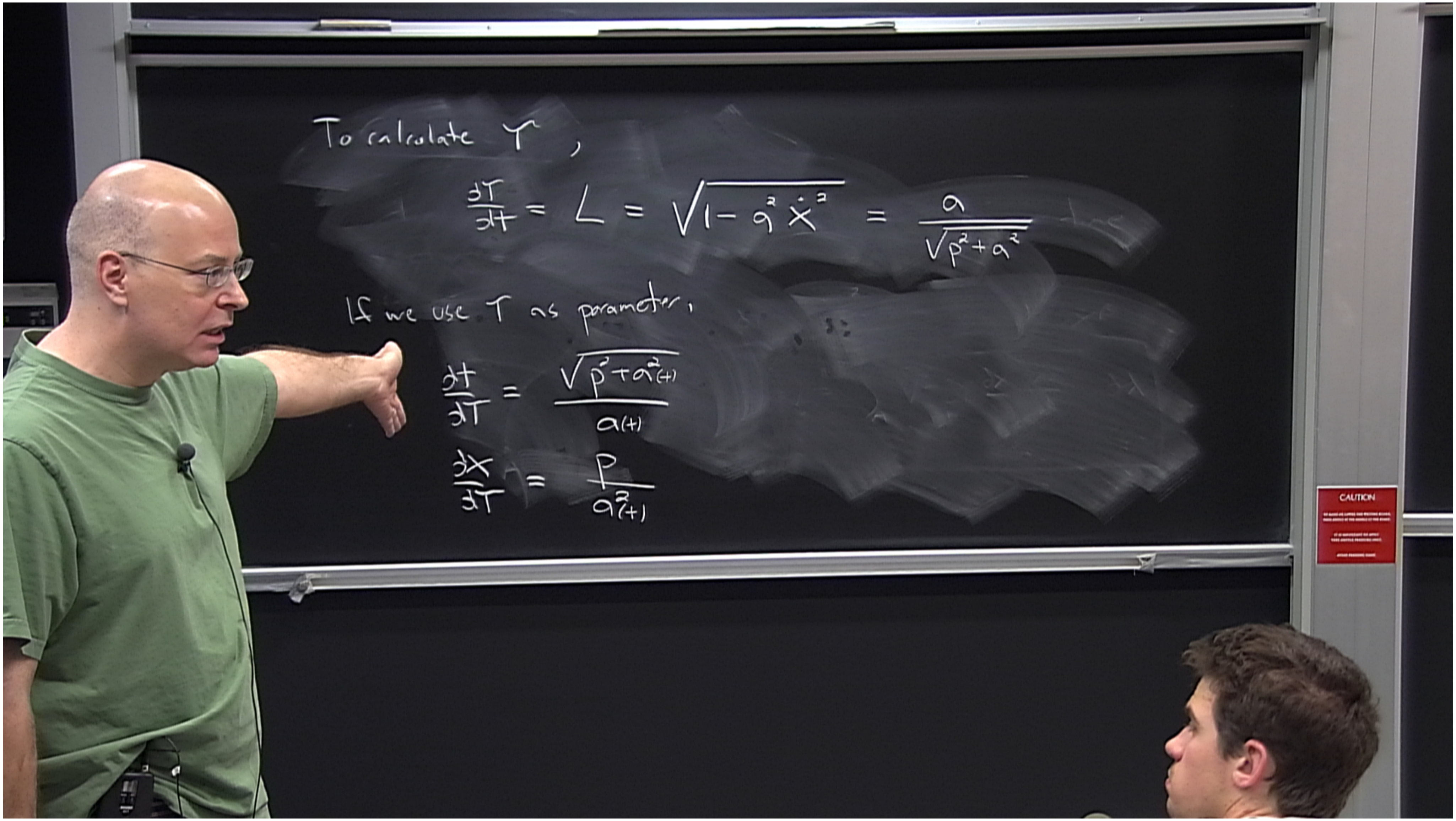

transformations
(not proper time)
const
solve for ẋ ⇒
CAUTION

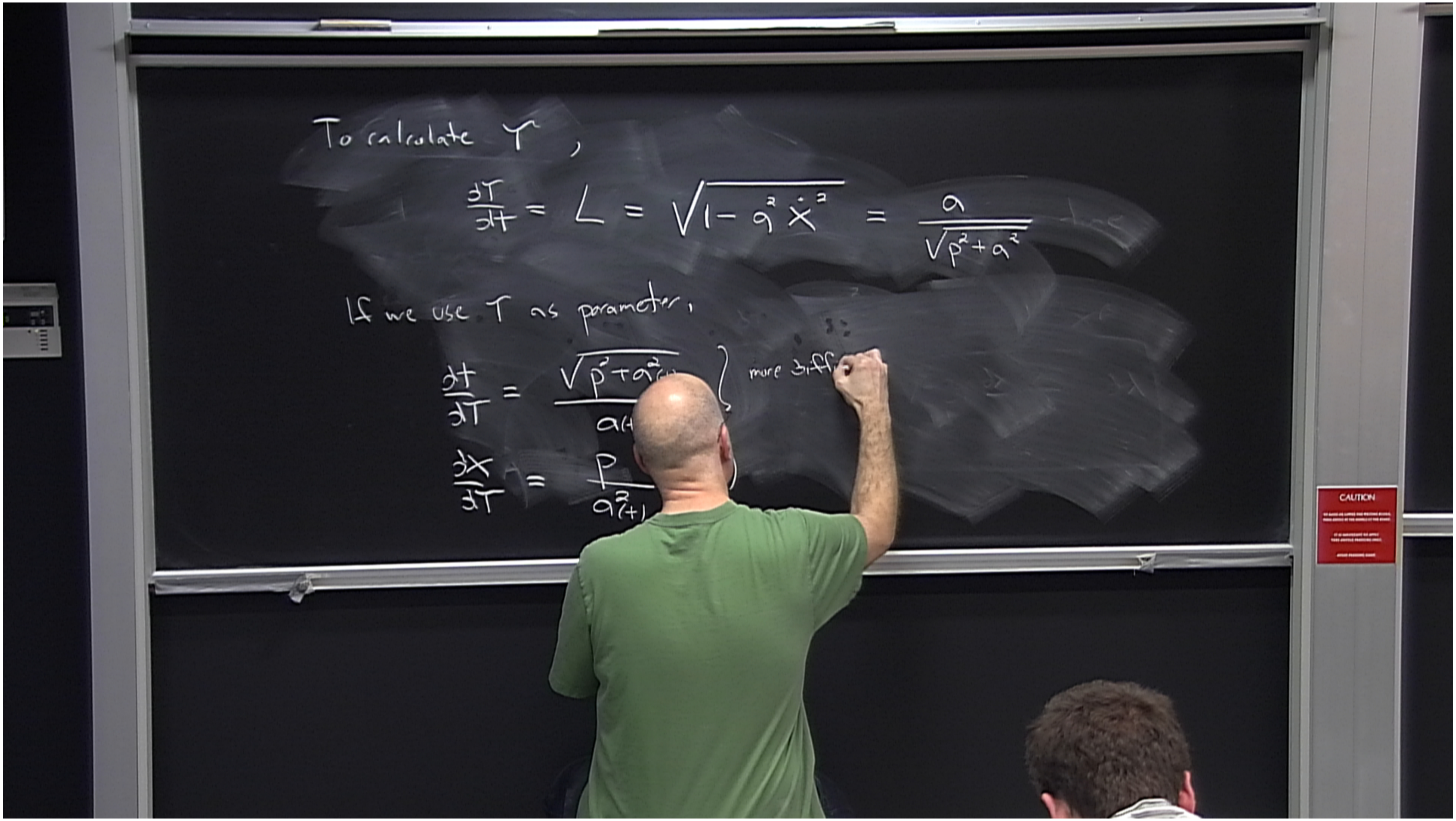

To calculate Υ ,
dΥ/dt = L =

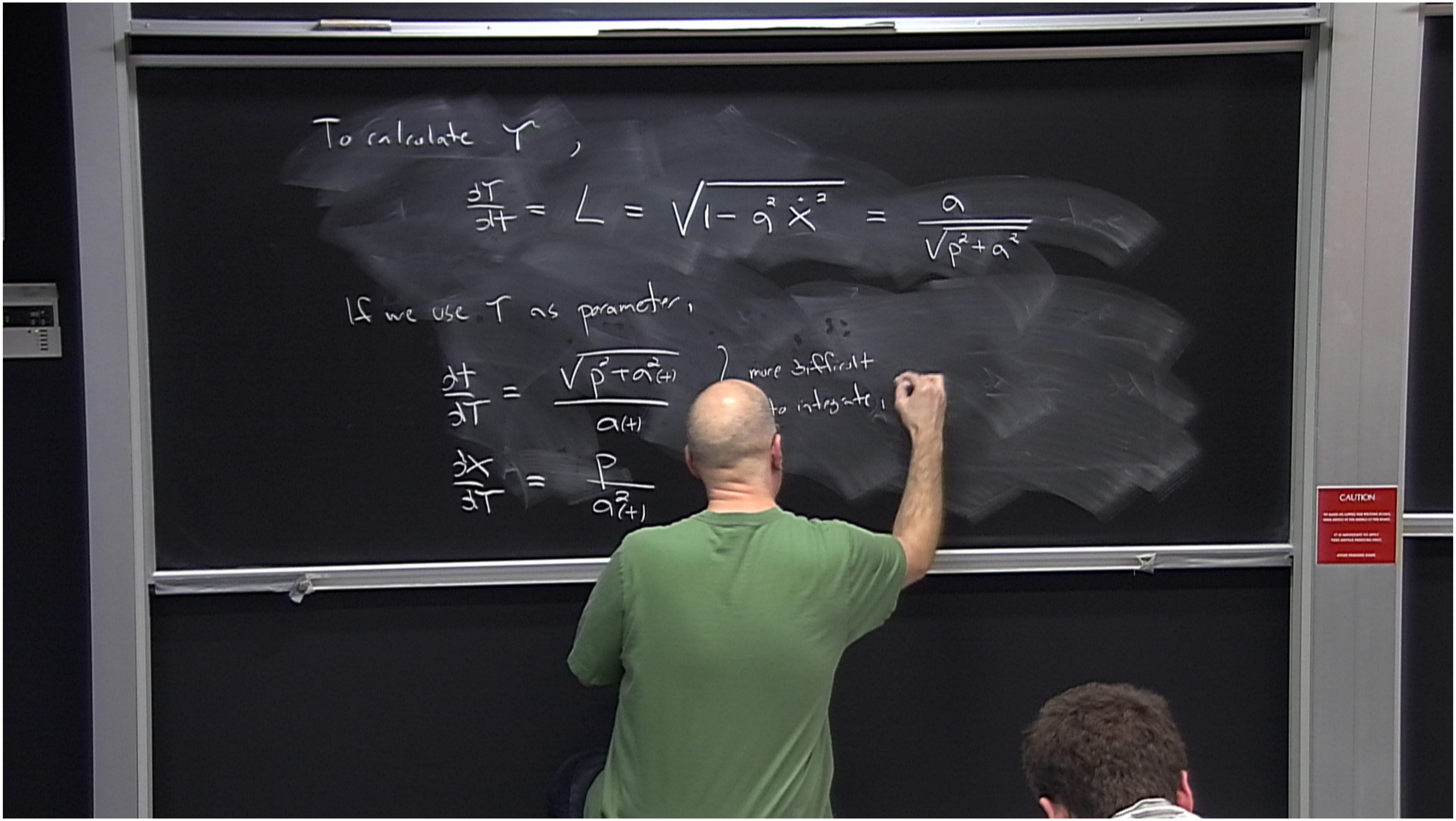

To calculate Υ,
dΥ/dt = L =

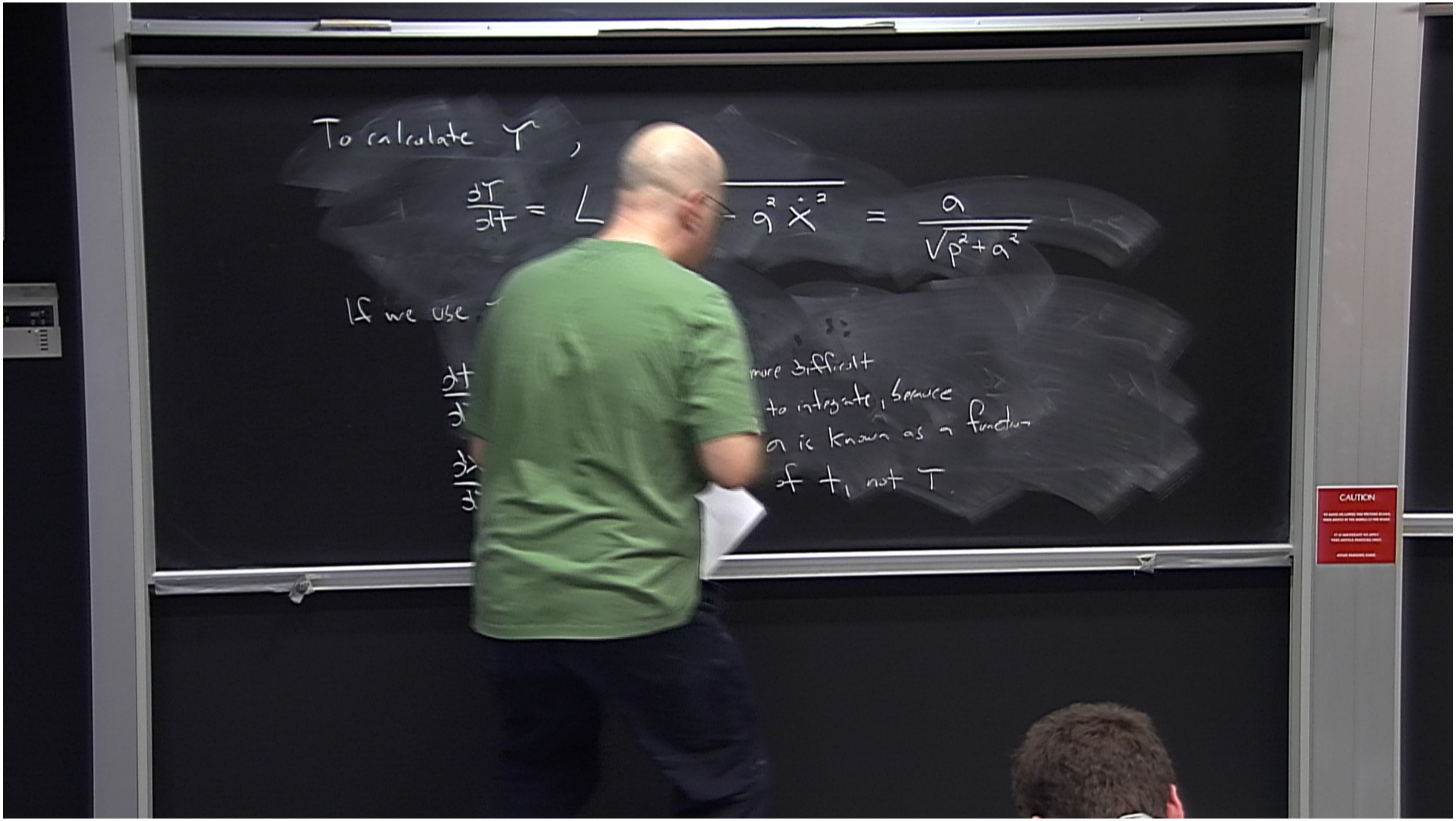

To calculate τ,
dτ/dt = L = √(1 − a² ẋ²) = a/√(p² + a²)
If we use τ as parameter,
CAUTION

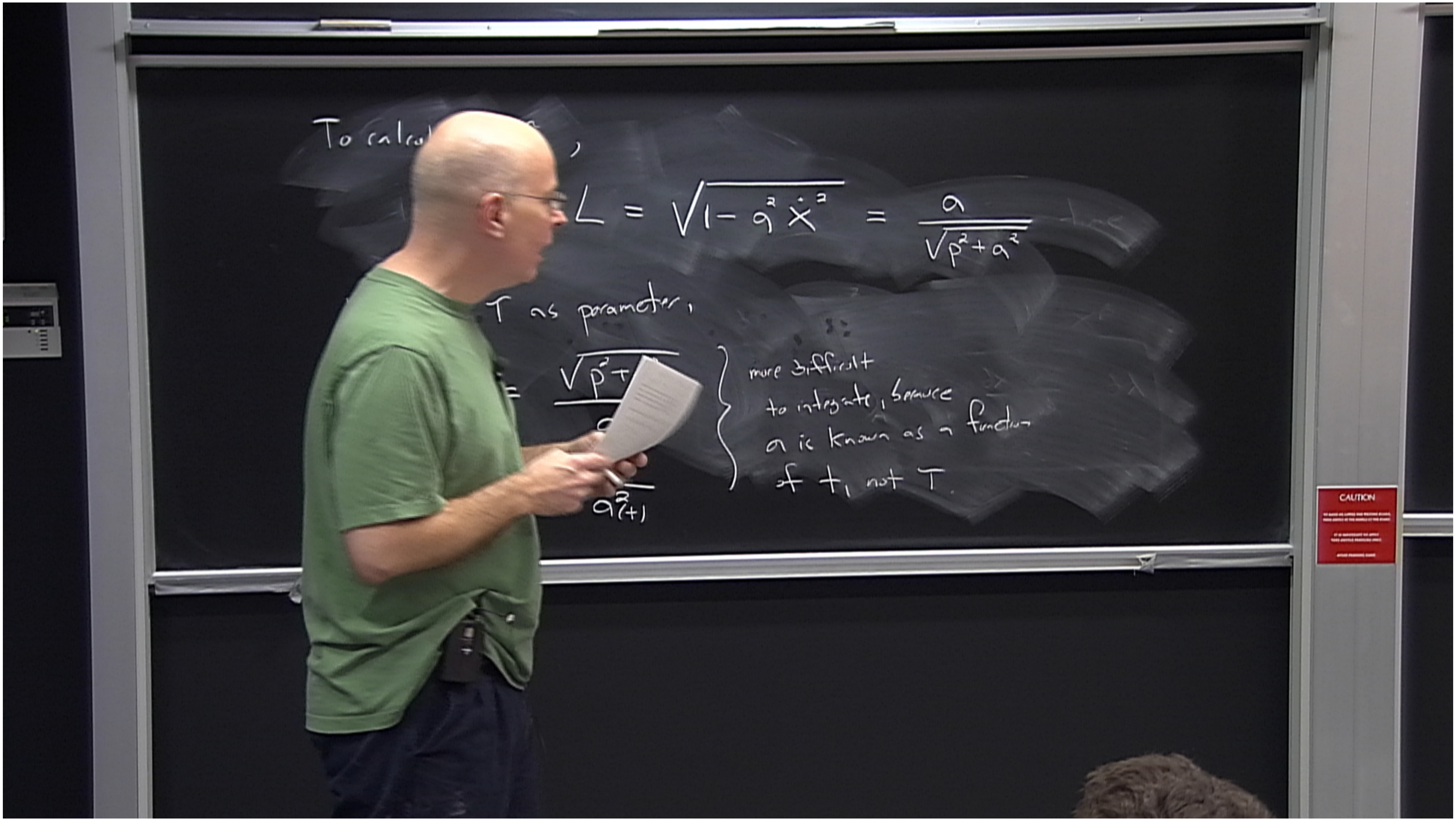

To calculate τ,
dτ/dt = 1/γ = √(1 − a² ẋ²) = a/√(p² + a²)
as parameter,
dt/dτ = √(p² + a²(t)) / a(t)
dx/dτ = p / a²(t)
CAUTION


To calculate τ,
$\frac{d\tau}{dt} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$
If we use τ as parameter,
$\frac{dt}{d\tau} = \frac{\sqrt{p^2 + a^2(t)}}{a(t)}$
$\frac{dx}{d\tau} = \frac{p}{a^2(t)}$
CAUTION

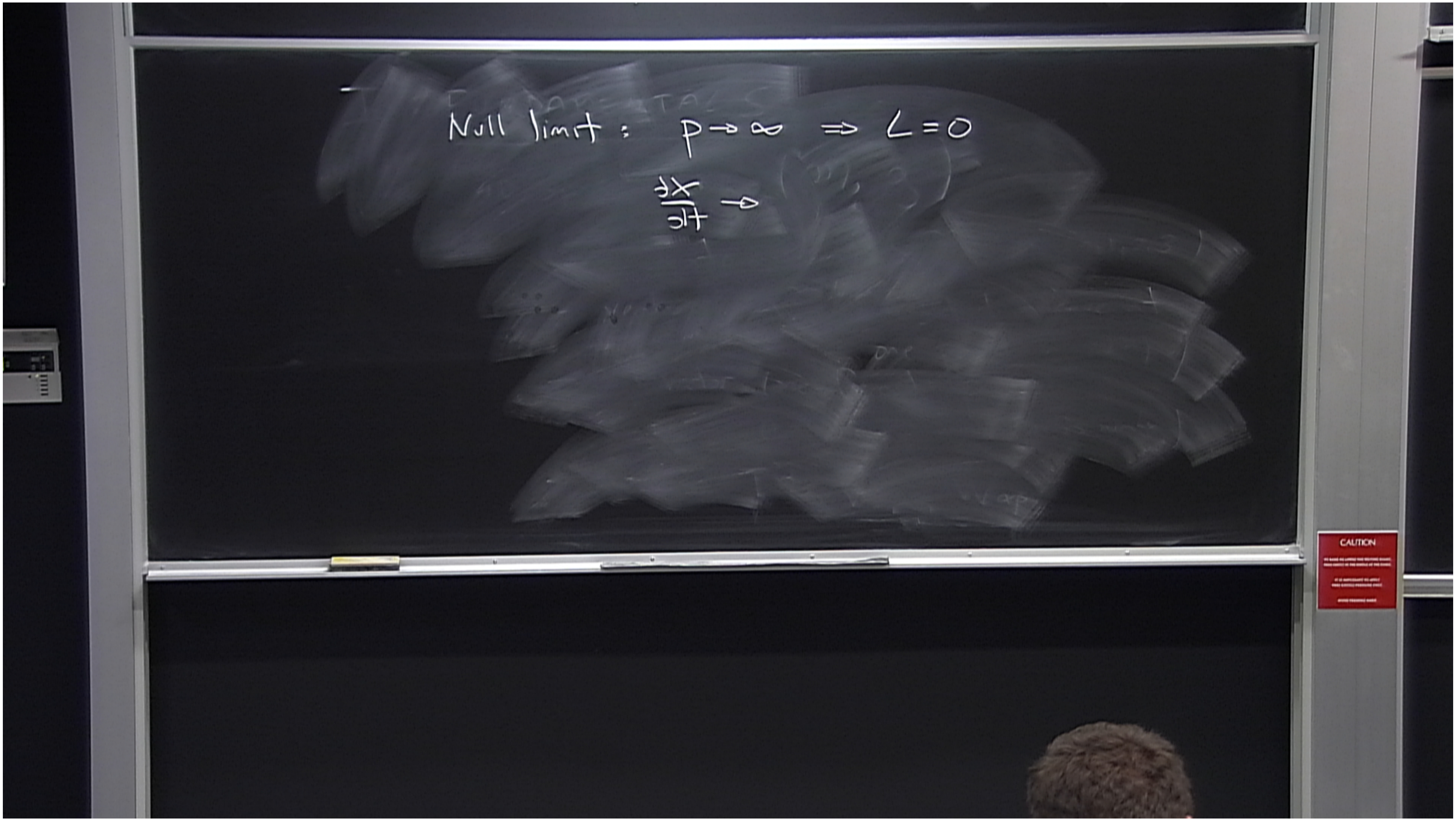

To calculate τ,
dτ/dt = L = √(1 − a² ẋ²) = a/√(p² + a²)
If we use τ as parameter,
dt/dτ = √(p² + a²)/a
dx/dτ = p/a²
more diff

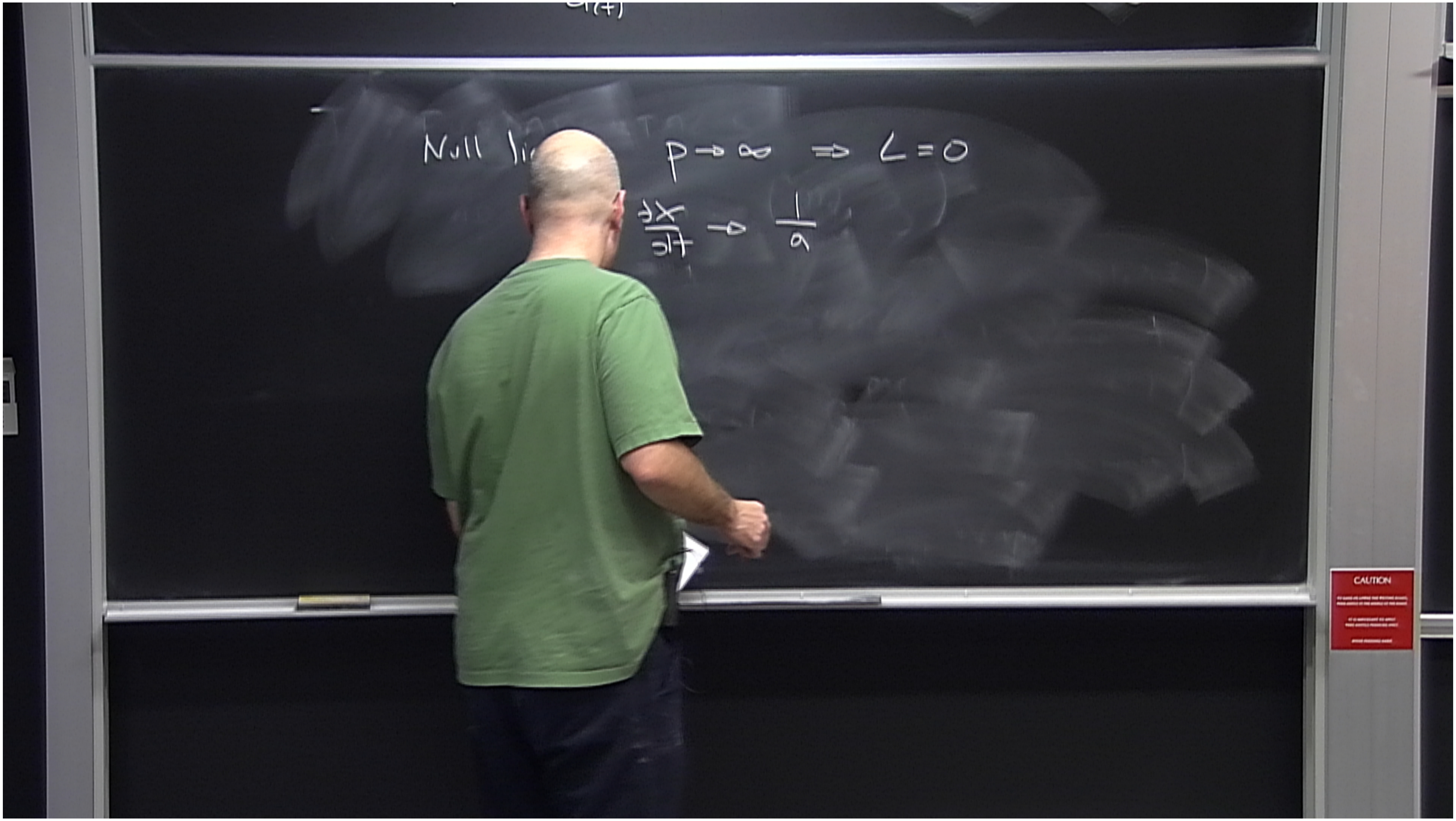

To calculate τ,
$\frac{d\tau}{dt} = L = \sqrt{1 - a^2 \dot{x}^2} = \frac{a}{\sqrt{p^2 + a^2}}$
If we use τ as parameter,
$\frac{dt}{d\tau} = \frac{\sqrt{p^2 + a^2(t)}}{a(t)}$
more difficult to integrate.
$\frac{dx}{d\tau} = \frac{p}{a^2(t)}$
CAUTION

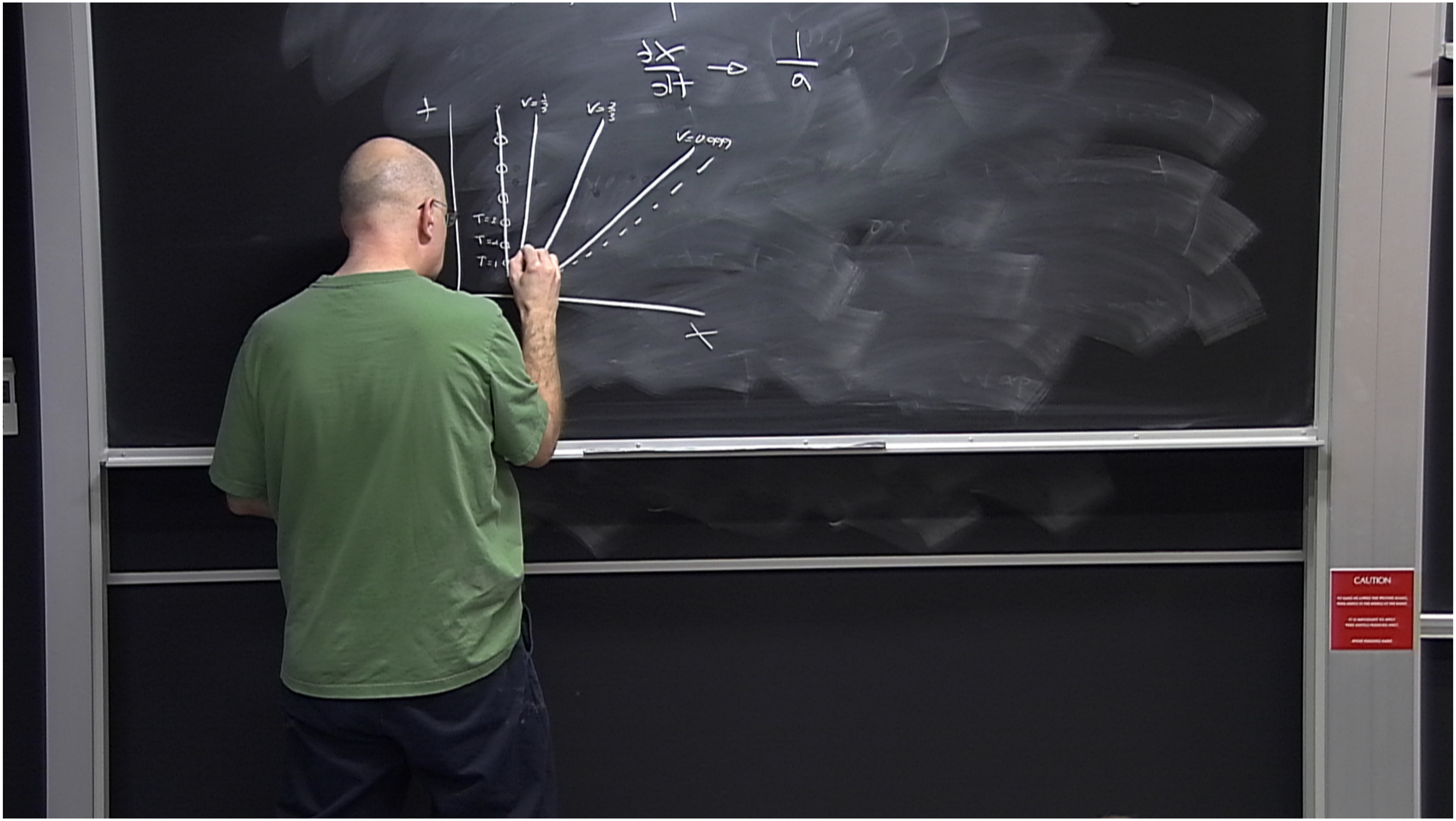

To calculate τ ,
If we use
more difficult
to integrate, because
a is known as a function
of t, not τ.
CAUTION

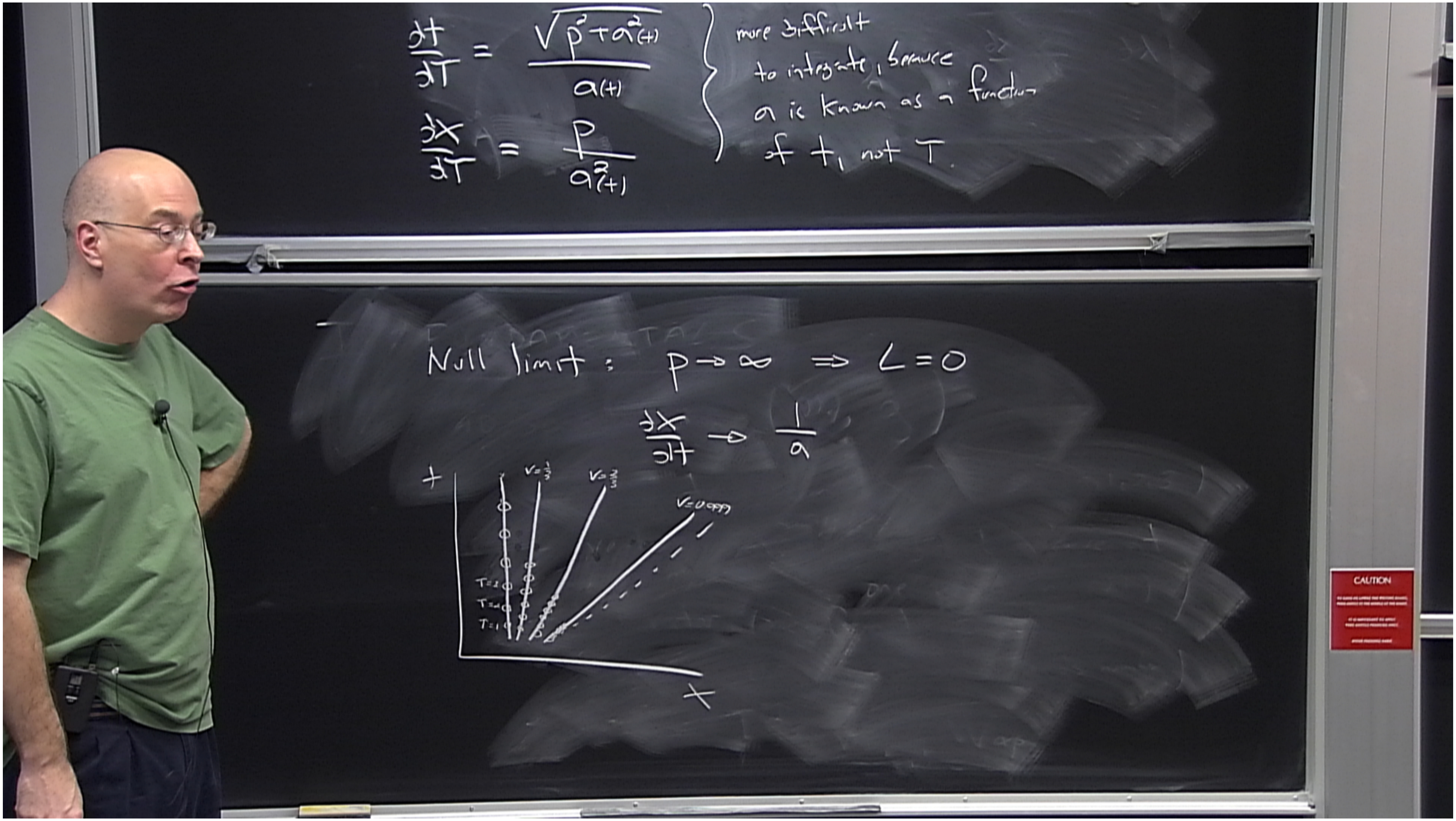


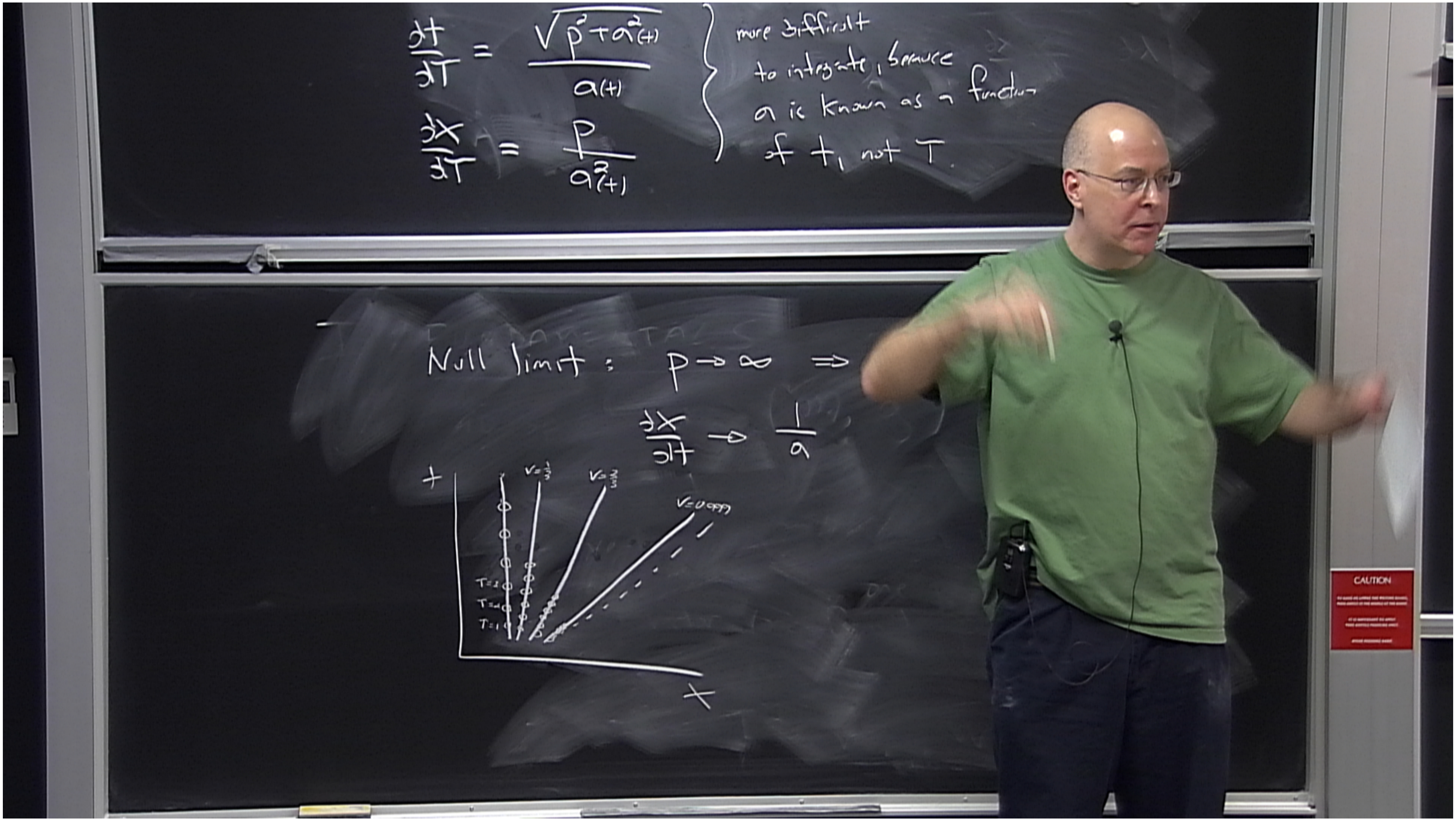
To calcul
L = √(1 − a²ẋ²) = a/√(p²+a²)
T as parameter,
more difficult
to integrate, because
a is known as a function
of t, not T.

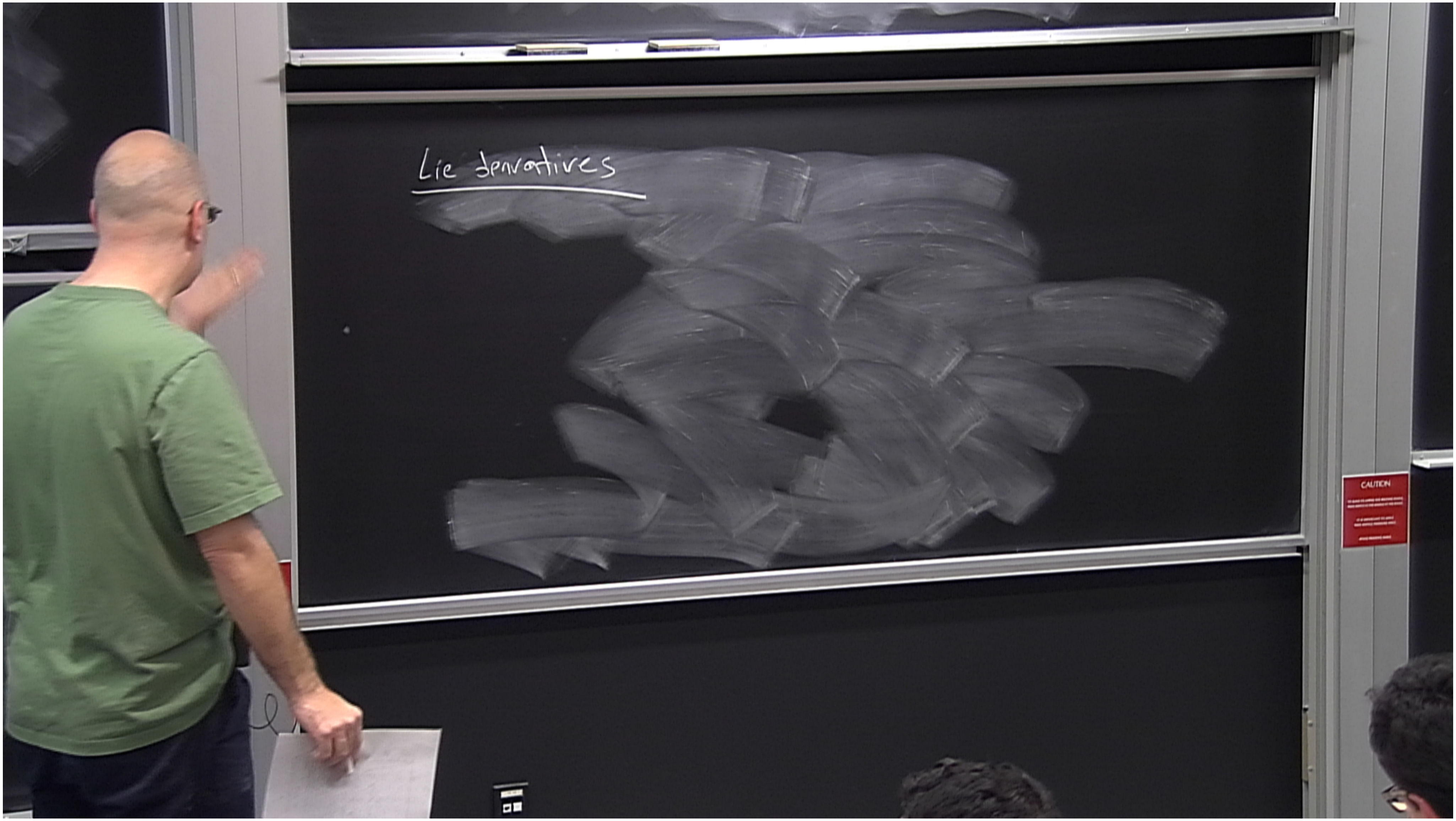

Null limit: $p \to \infty \Rightarrow L = 0$
$\frac{dx}{dt} \to$

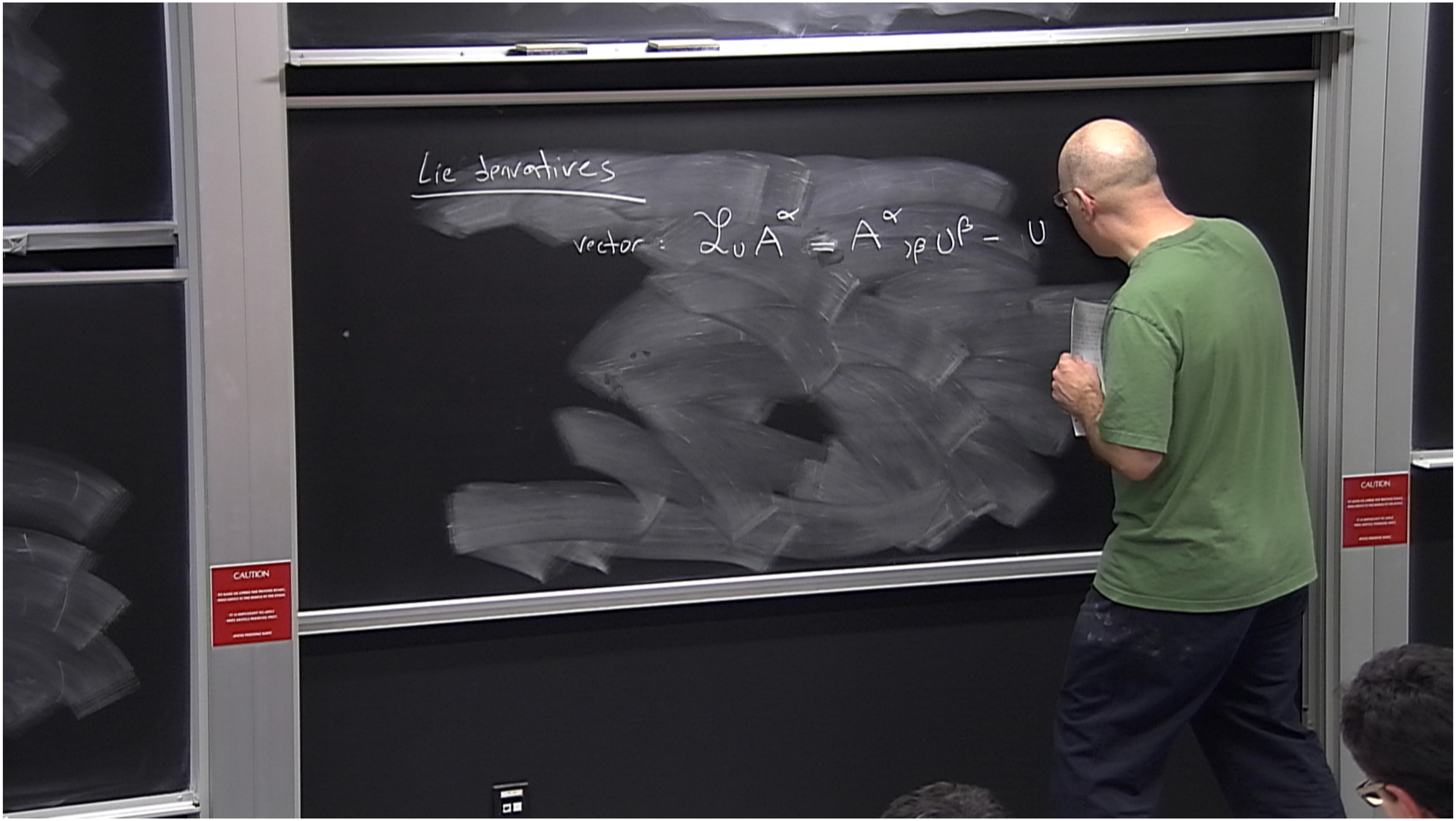

Null li
p → ∞ ⇒ L = 0
∂x/∂t → 1/a
CAUTION

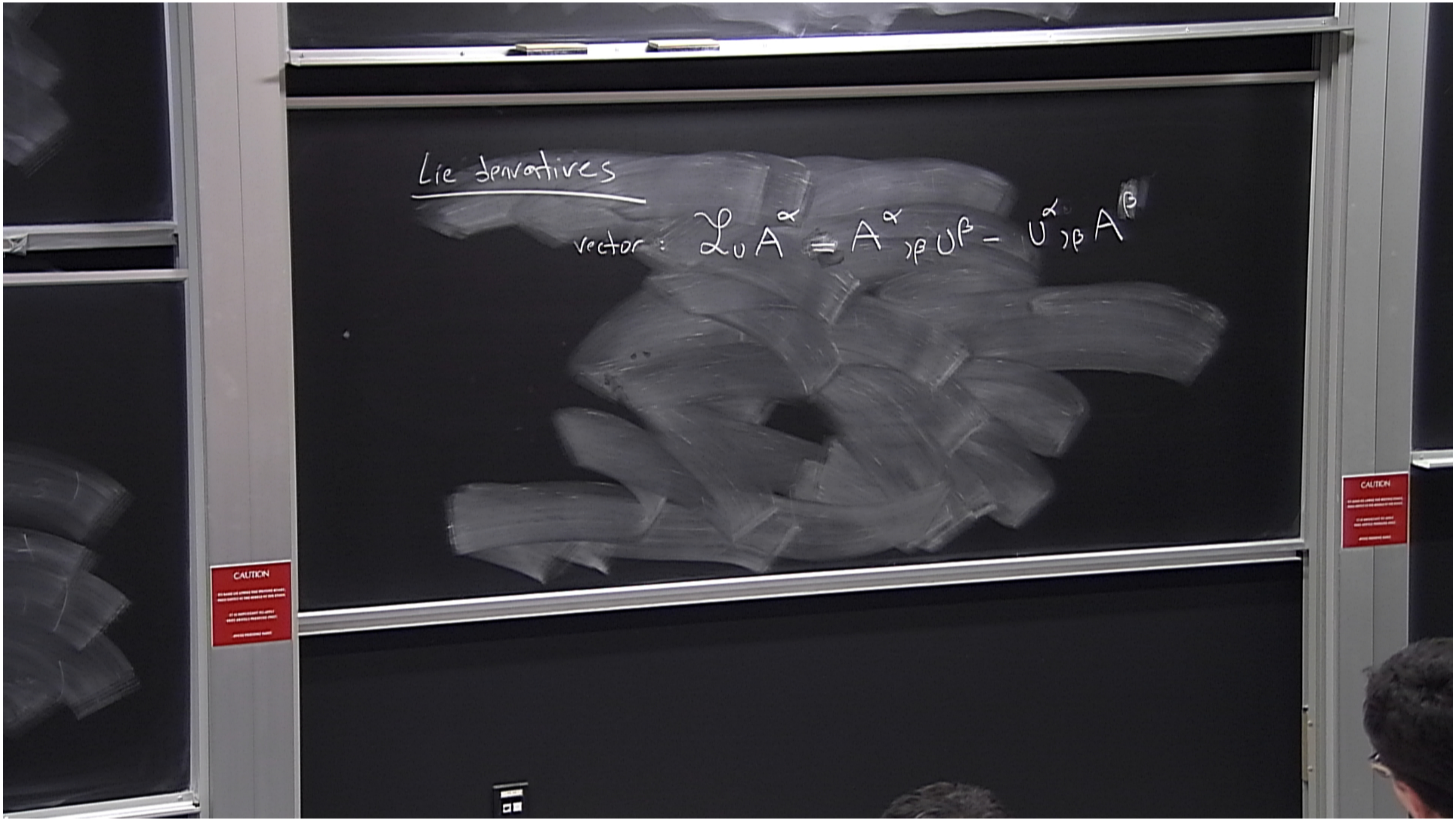

dx/dt → 1/a
t
V=1/3
V=2/3
V=0.999
x

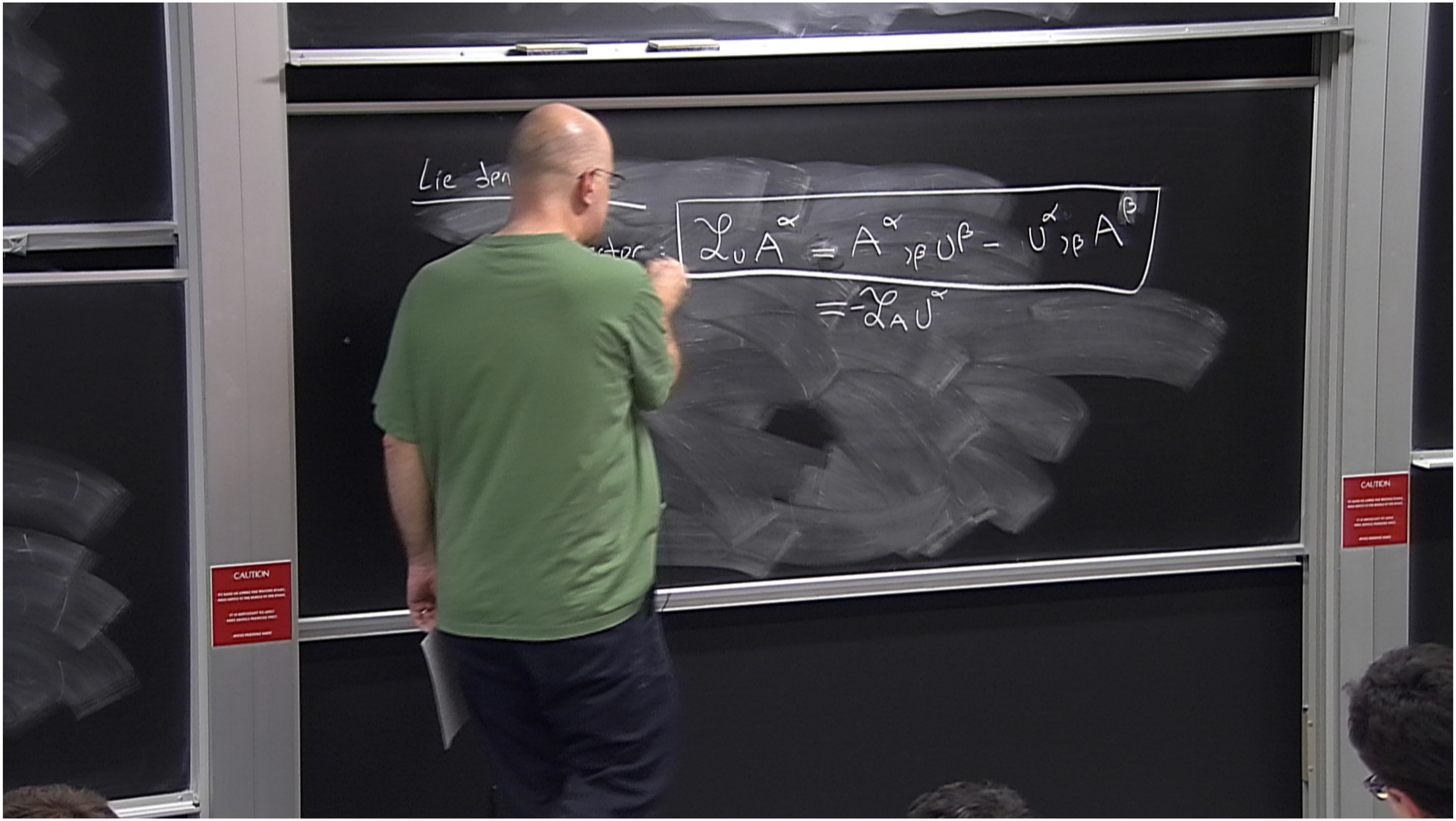

$\frac{dt}{dT} = \frac{\sqrt{p^2 + a^2(t)}}{a(t)}$
$\frac{dx}{dT} = \frac{p}{a^2(t)}$
more difficult to integrate, because a is known as a function of t, not T.
Null limit: $p \to \infty \Rightarrow L = 0$
$\frac{dx}{dt} \to \frac{1}{a}$
t
x
v=0.999

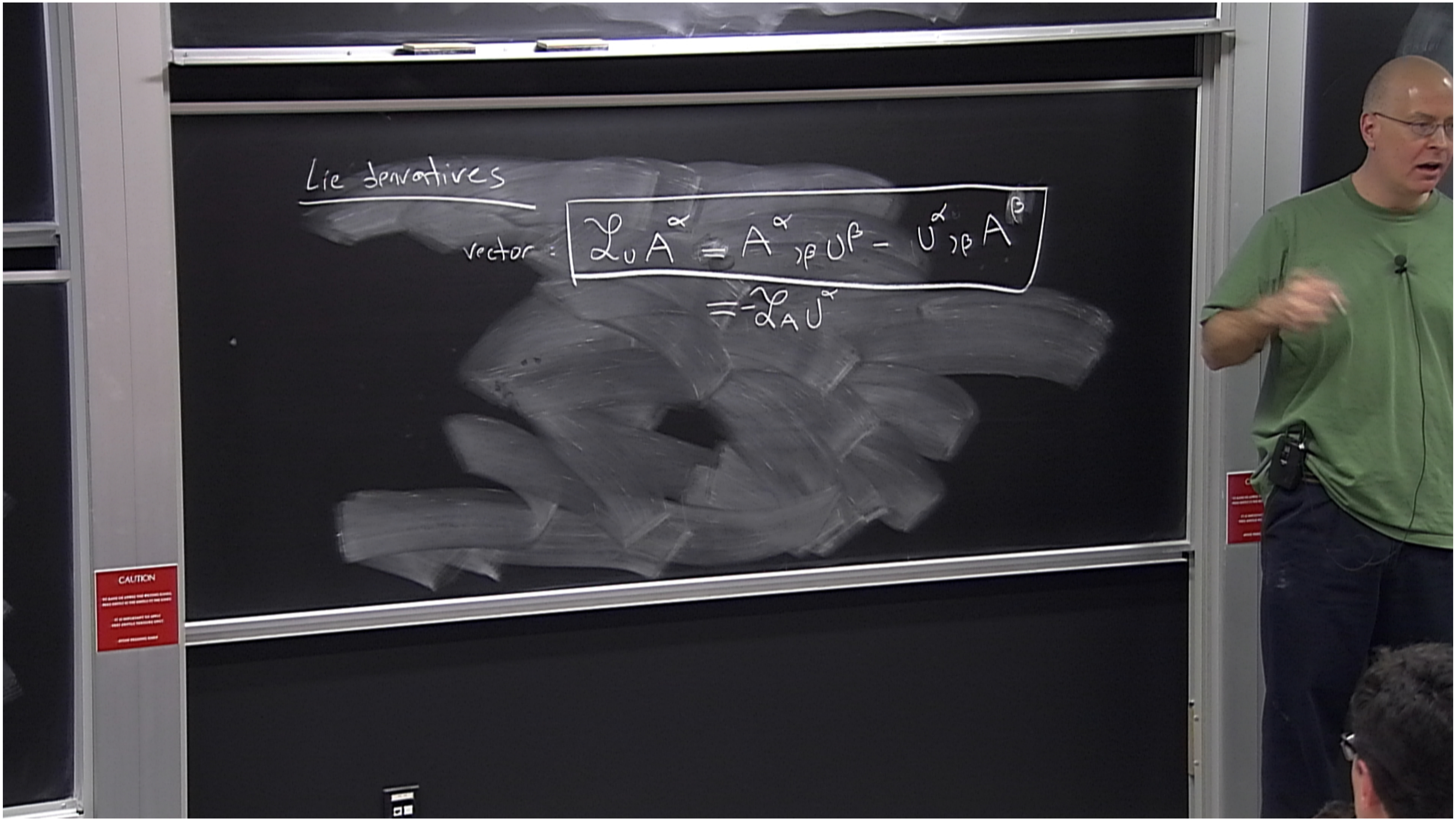

dt/dT = √(p² + a²(t)) / a(t)
dx/dT = p / a²(t)
more difficult to integrate, because a is known as a function of t, not T.
Null limit: p → ∞ ⇒
dx/dt → 1/a
t
x
T=3
T=2
T=1
v=1/3
v=2/3
v=0.999

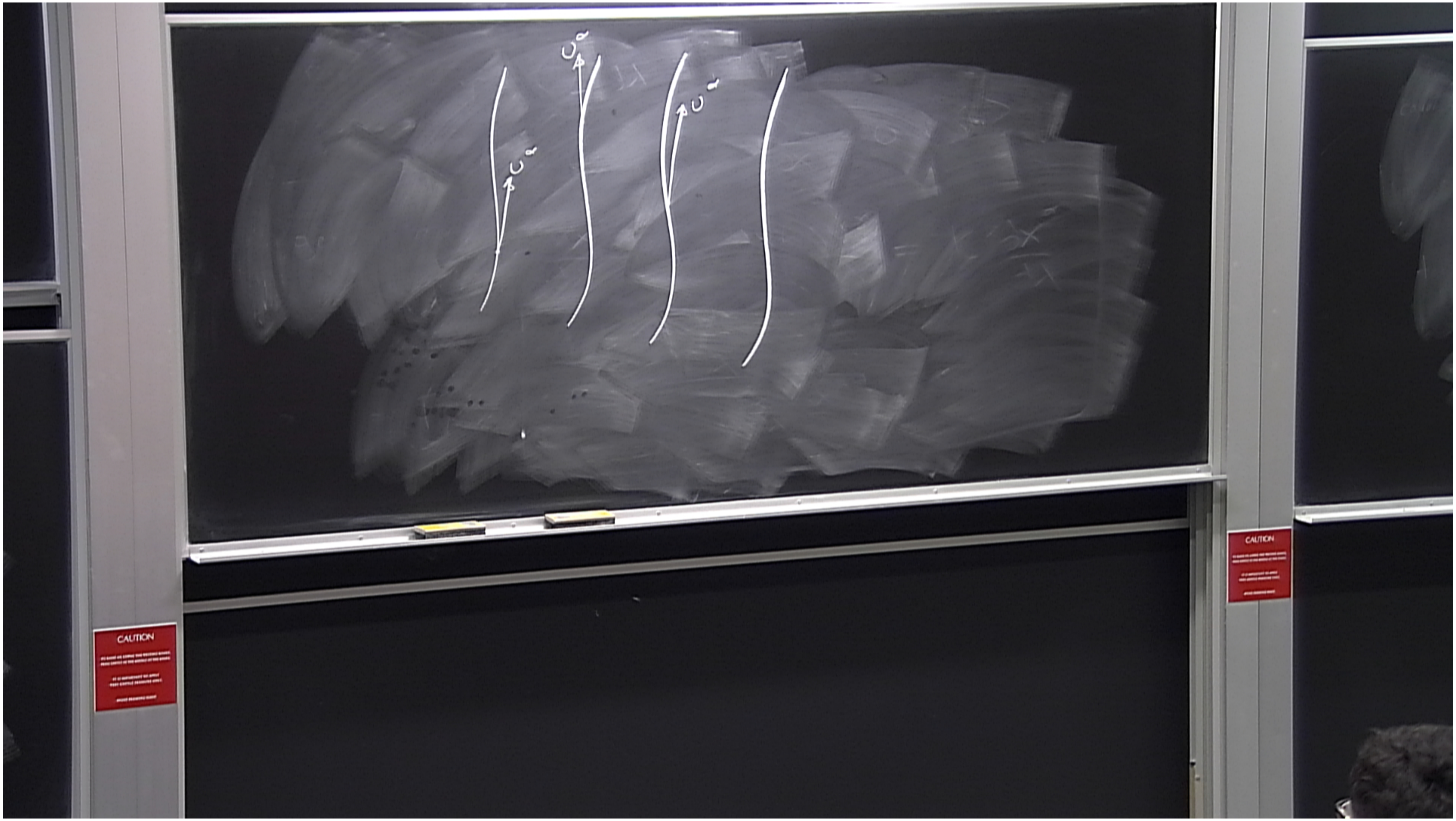

Lie derivatives
CAUTION

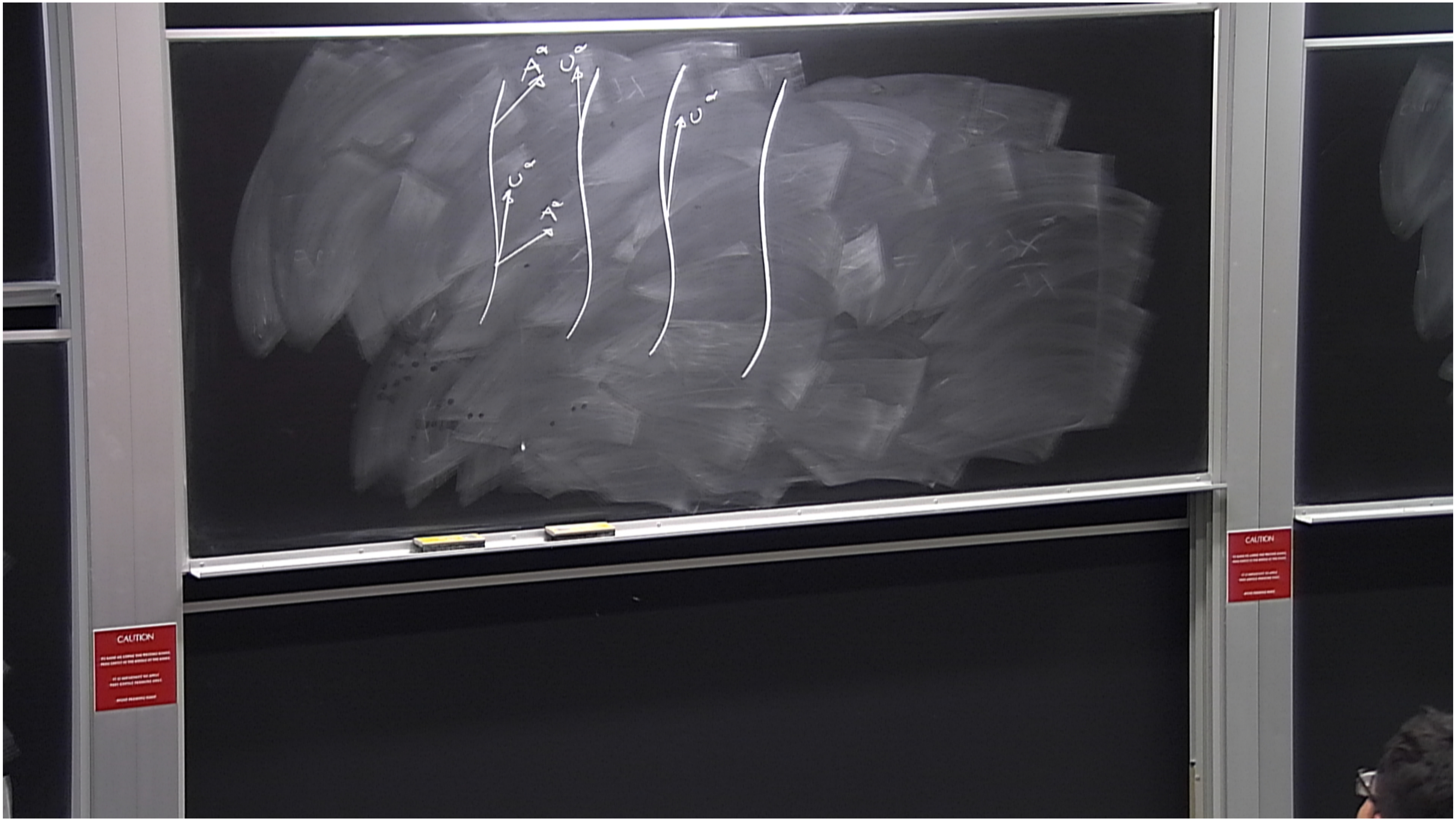

Lie derivatives
vector : $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U$
CAUTION

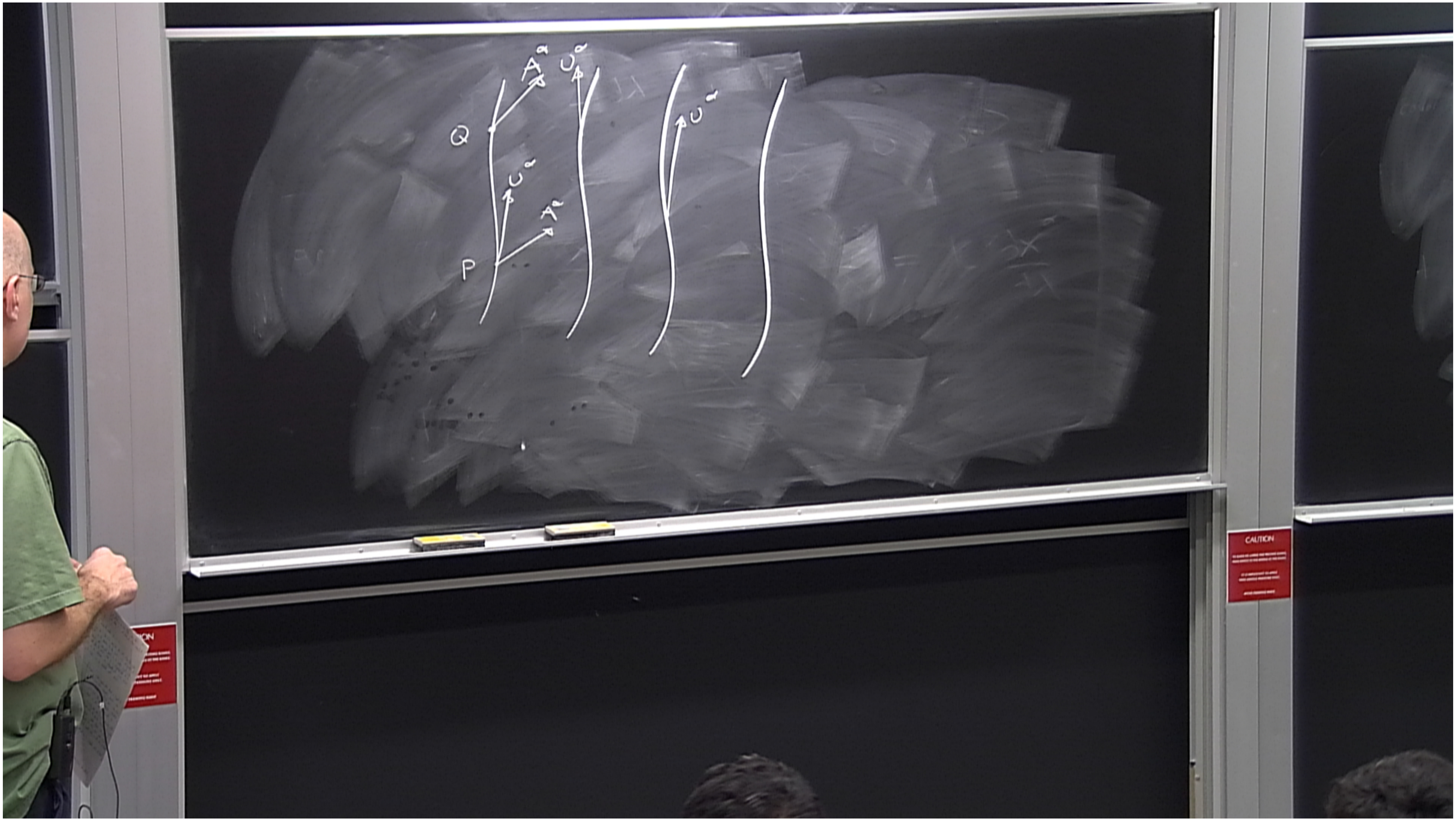

Lie derivatives
vector : $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
CAUTION


Lie der
$\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
$= -\mathcal{L}_A U^\alpha$
CAUTION
CAUTION

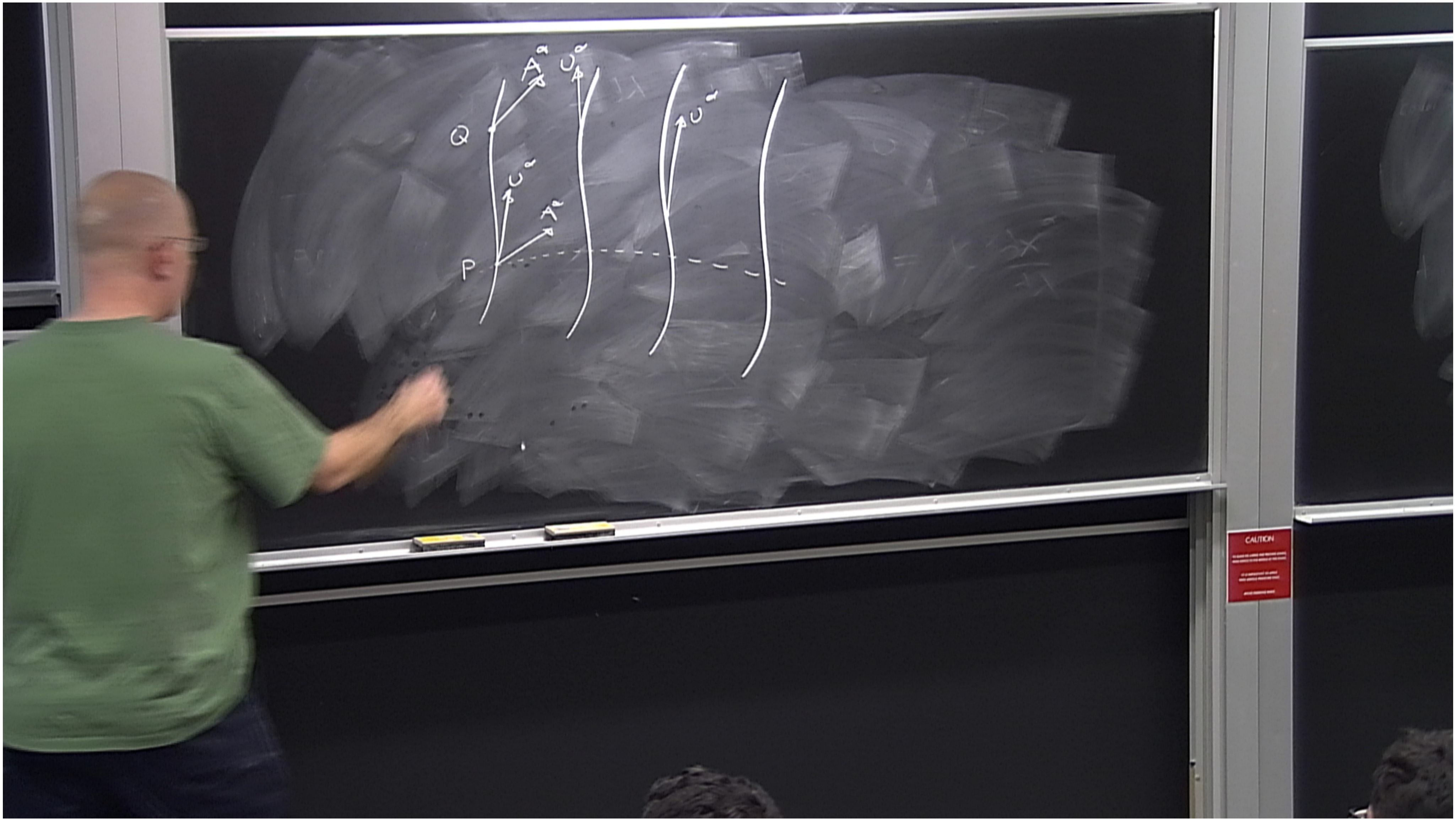

Lie derivatives
vector: $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
$= -\mathcal{L}_A U^\alpha$
CAUTION


CAUTION
CAUTION

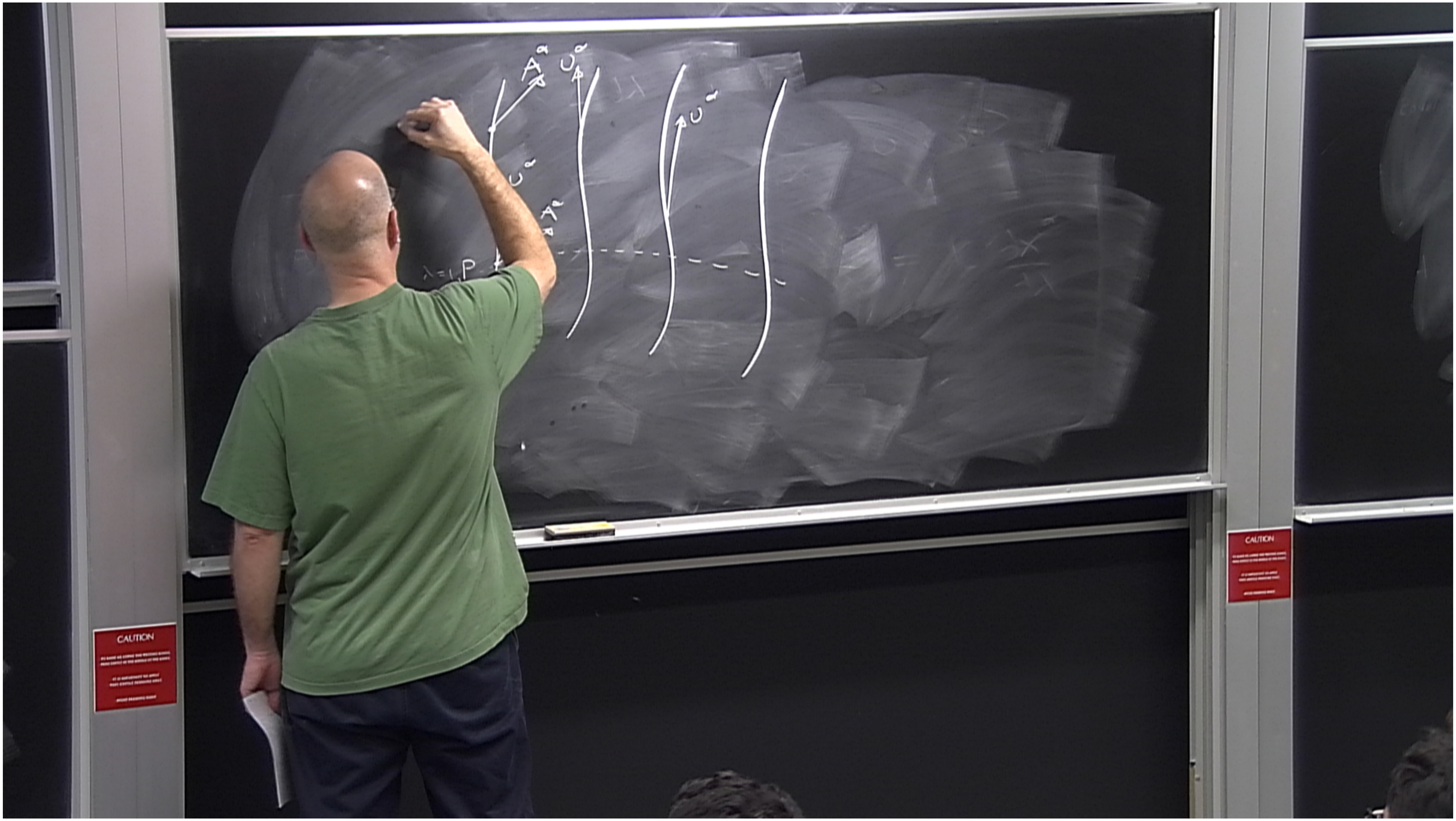


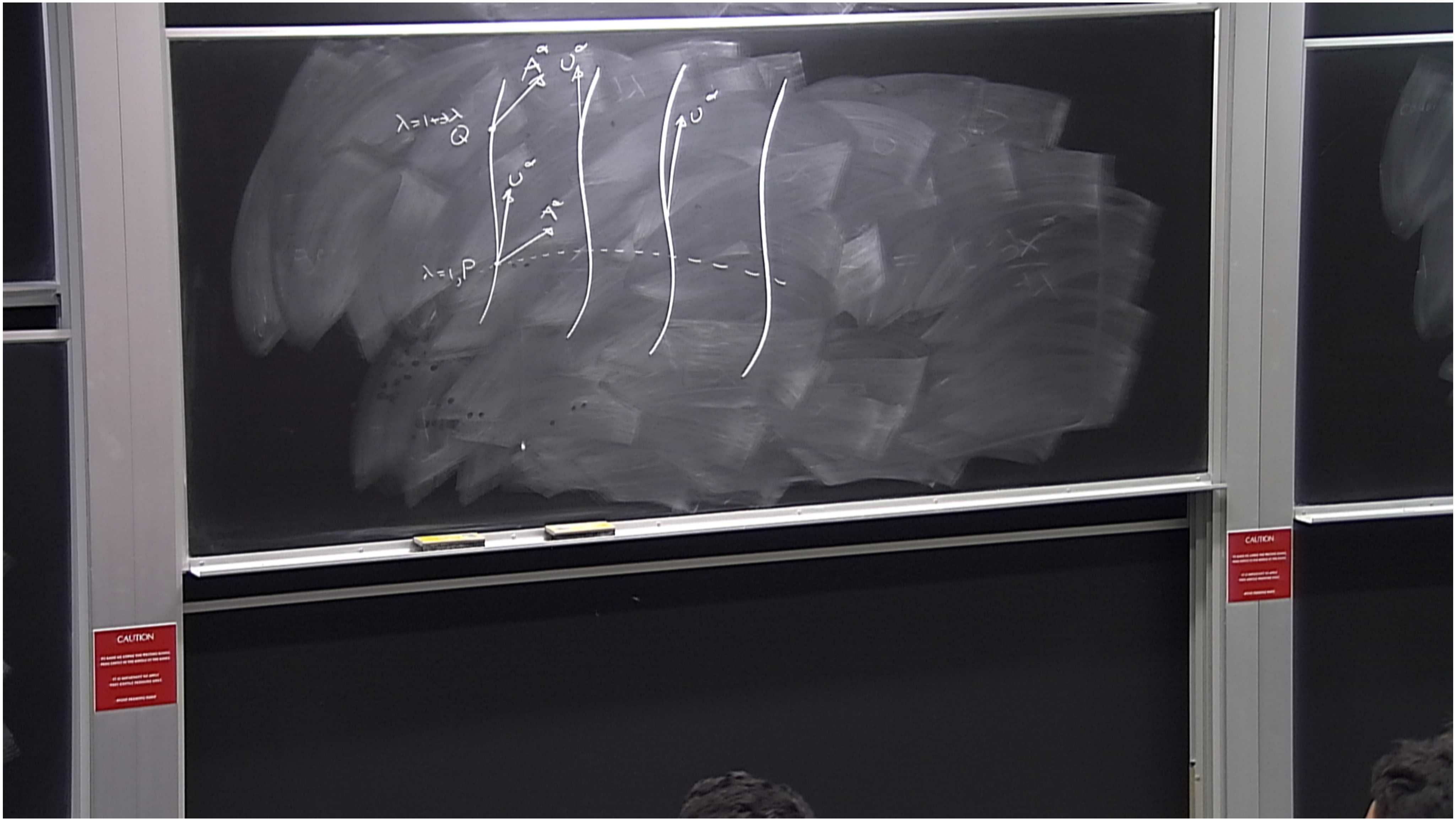


Aα
Uα
Q
Uα
Aα
P
Uα

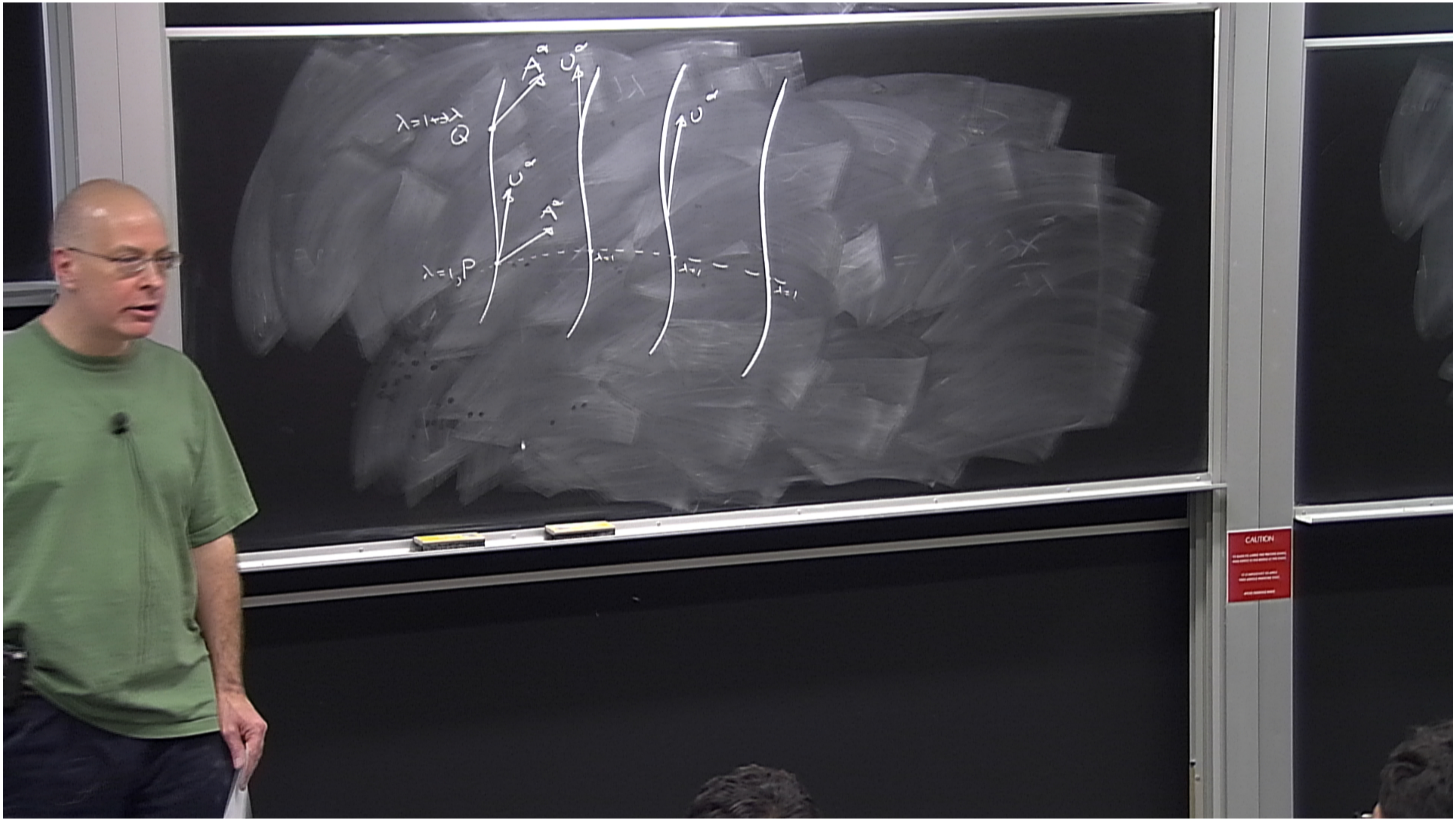


A^α
U^α
Q
U^α
A^α
P
U^α

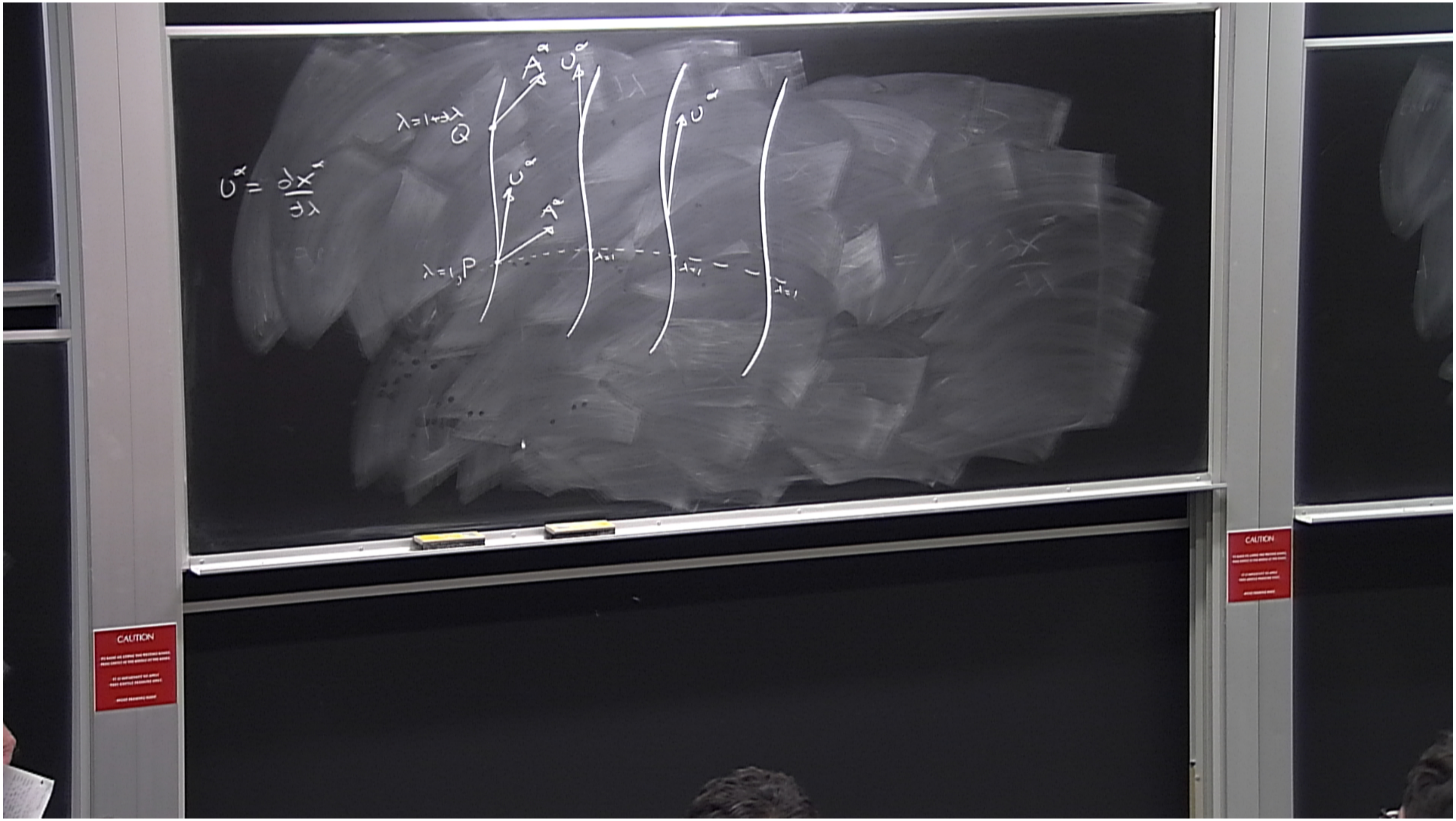


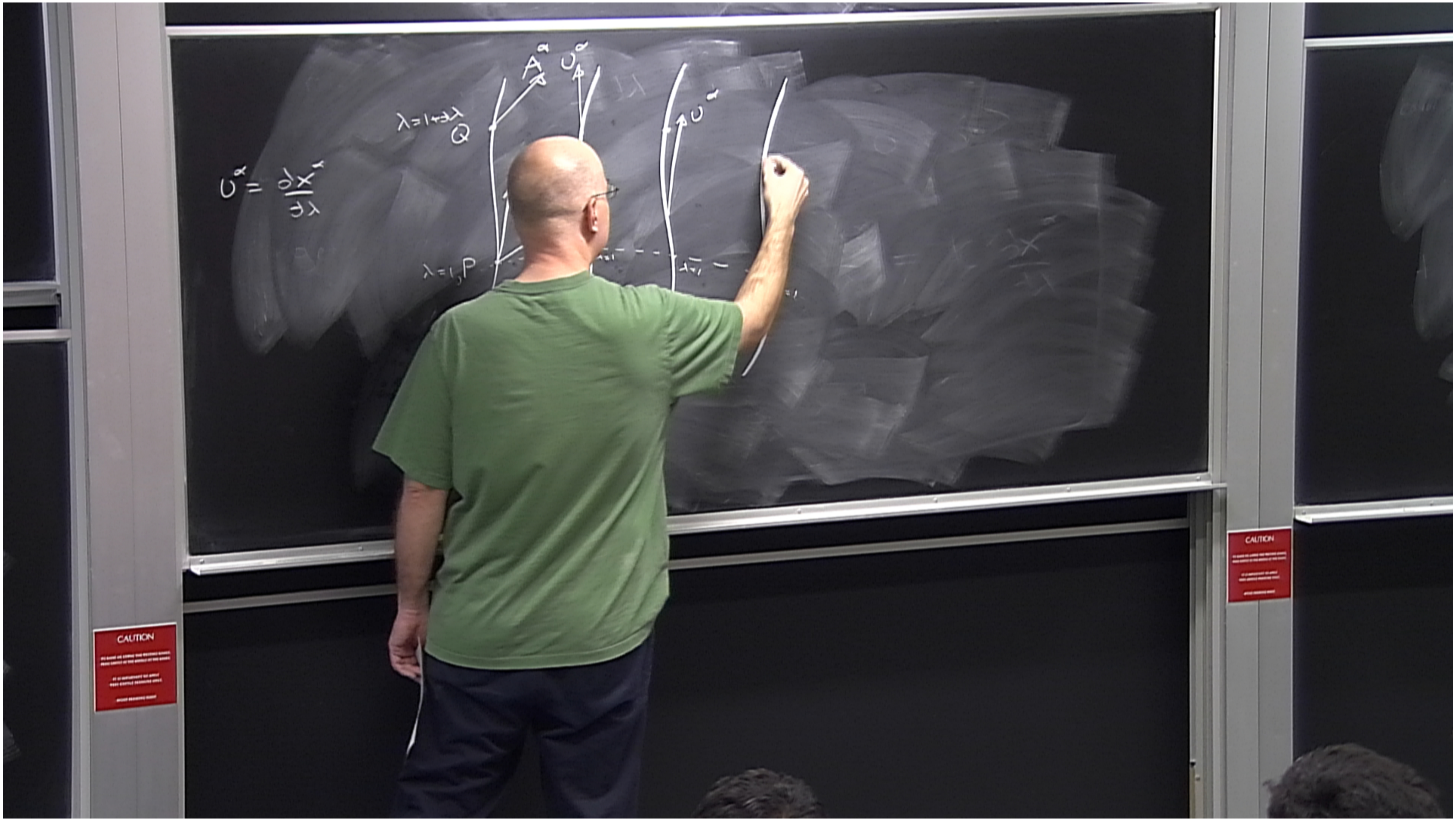


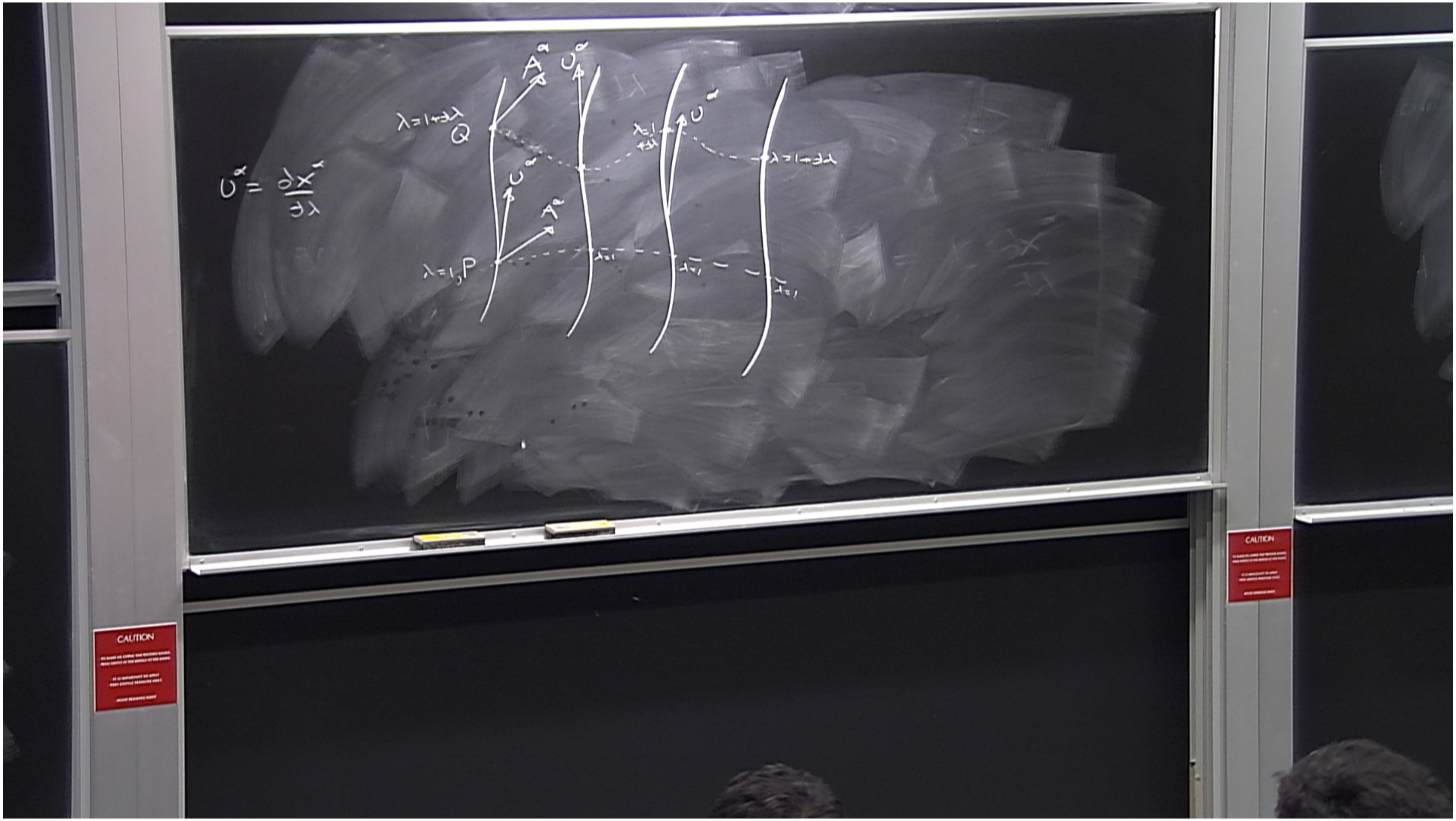
A^α
U^α
$\lambda = 1 + \delta\lambda$
Q
U^α
U^α
A^α
$\lambda = 1, P$



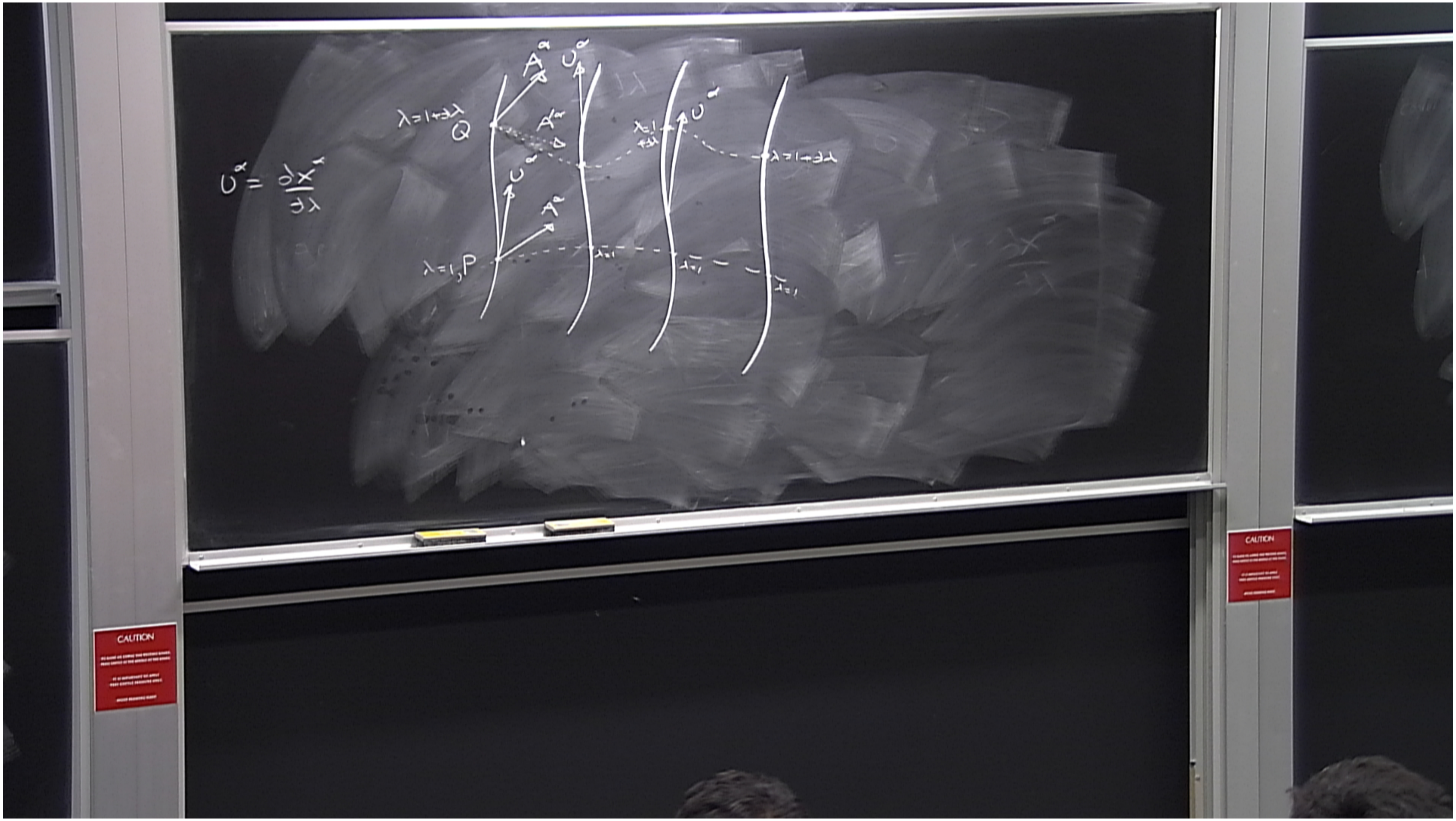
A^α
U^α
$\lambda = 1 + \Delta\lambda$
Q
U^α
A^α
U^α
$\lambda = 1$, P
$\lambda = 1$
$\lambda = 1$
$\lambda = 1$

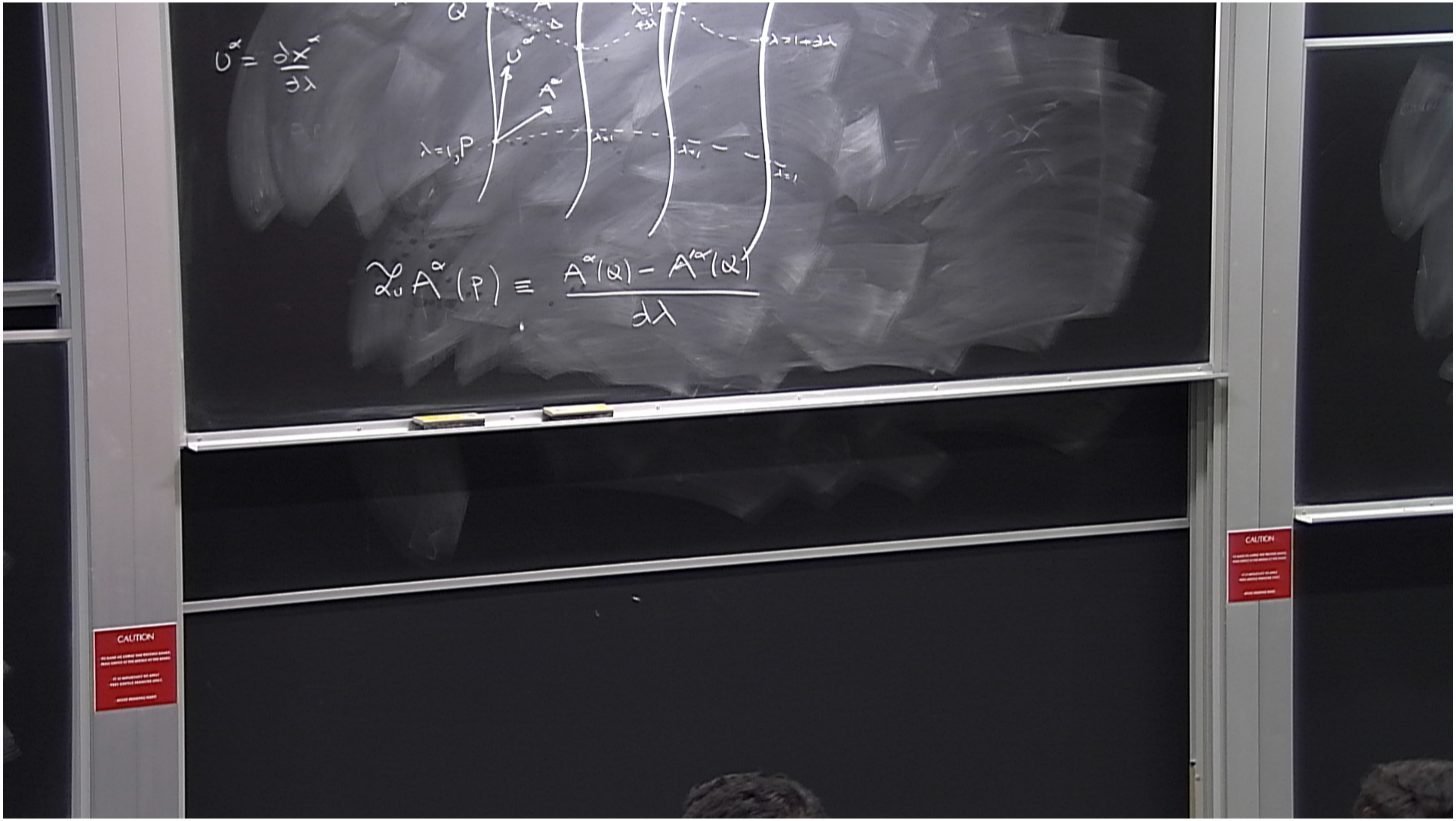

U^α = dx^α/dλ
λ=1+Δλ
Q
A^α
U^α
λ=1, P
CAUTION
CAUTION

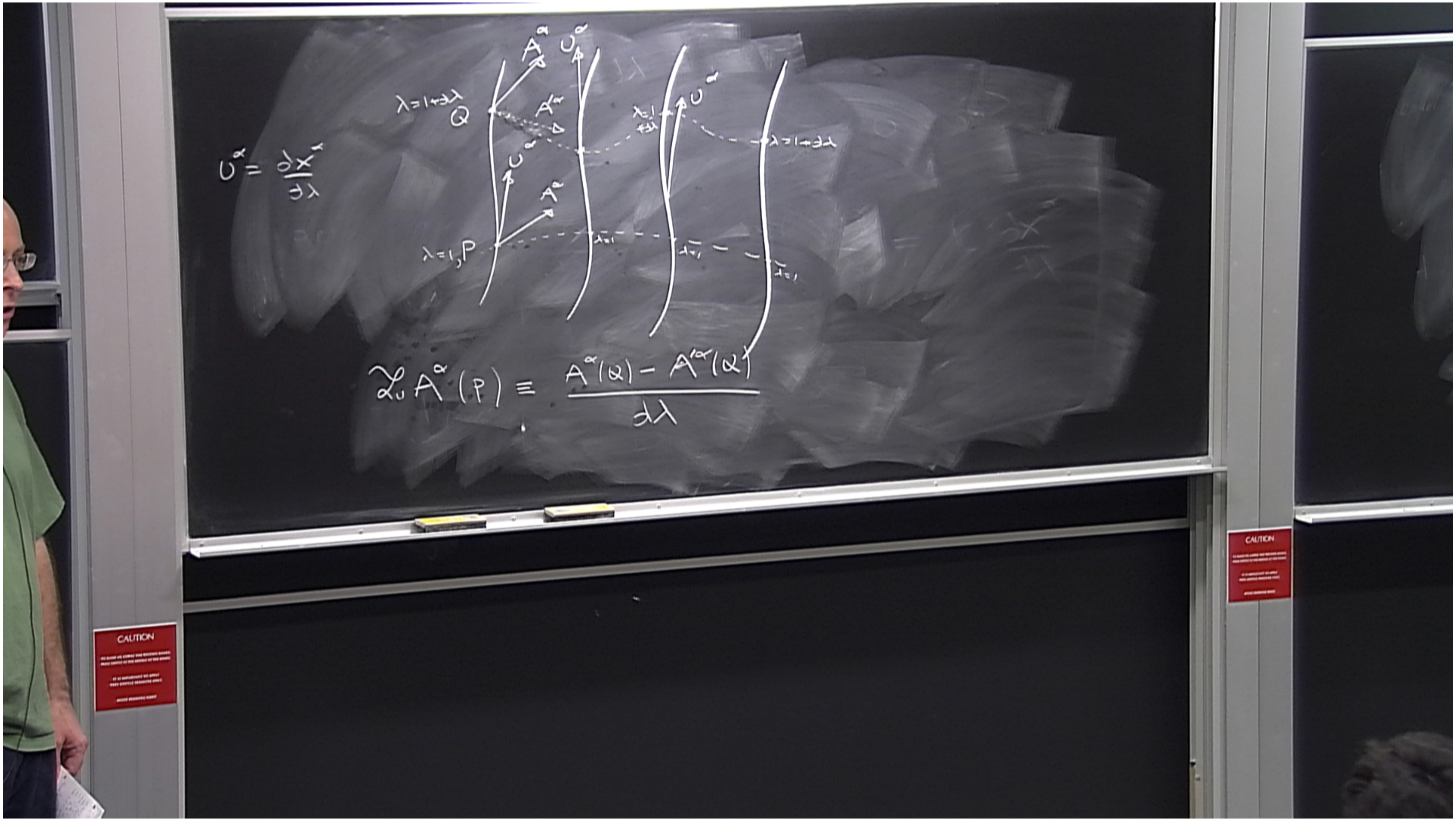

CAUTION

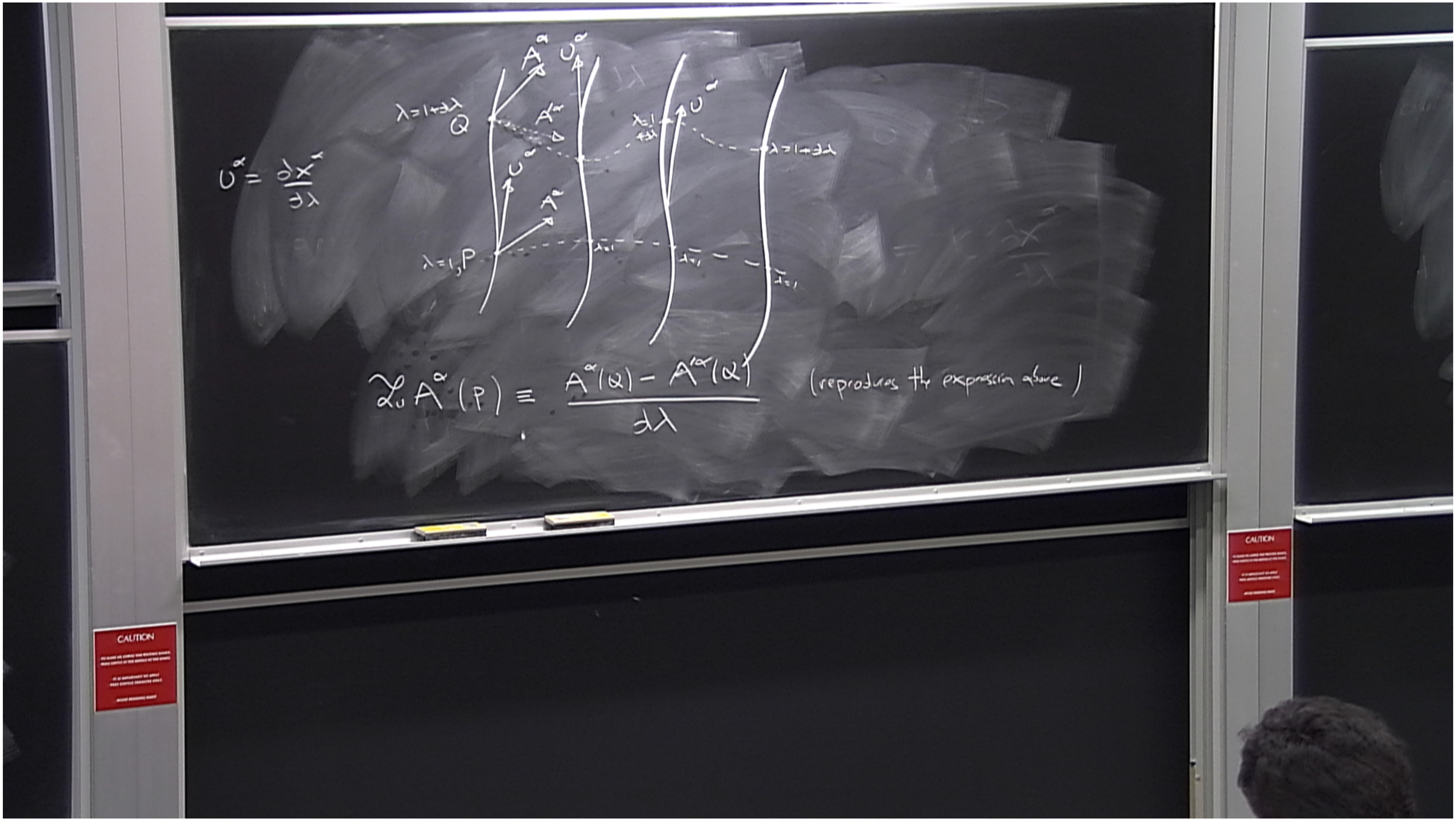


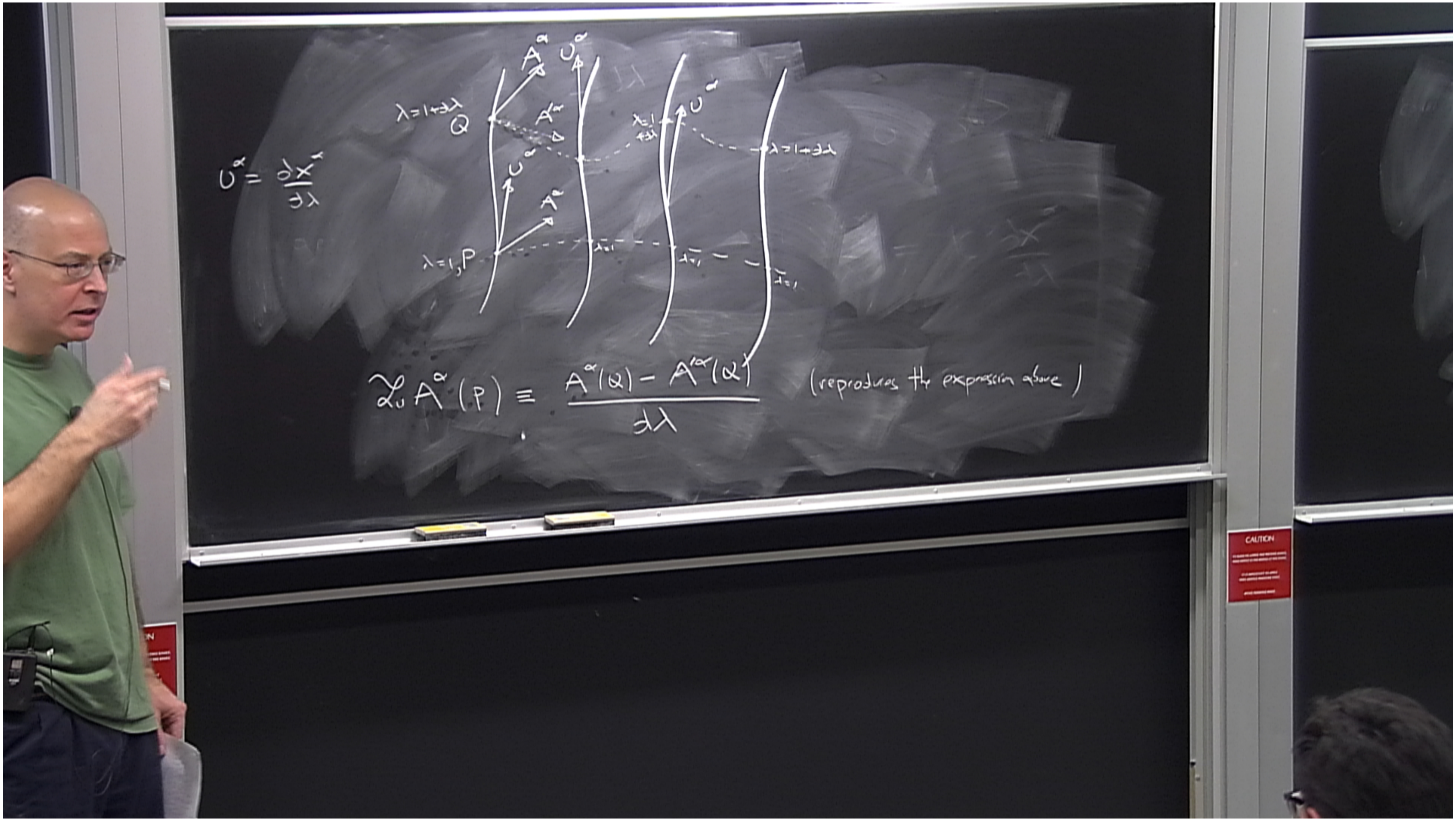
U^α = ∂x^α / ∂λ
λ=1+∂λ
Q
A^α
U^α
λ=1, P
λ=1+∂λ
CAUTION

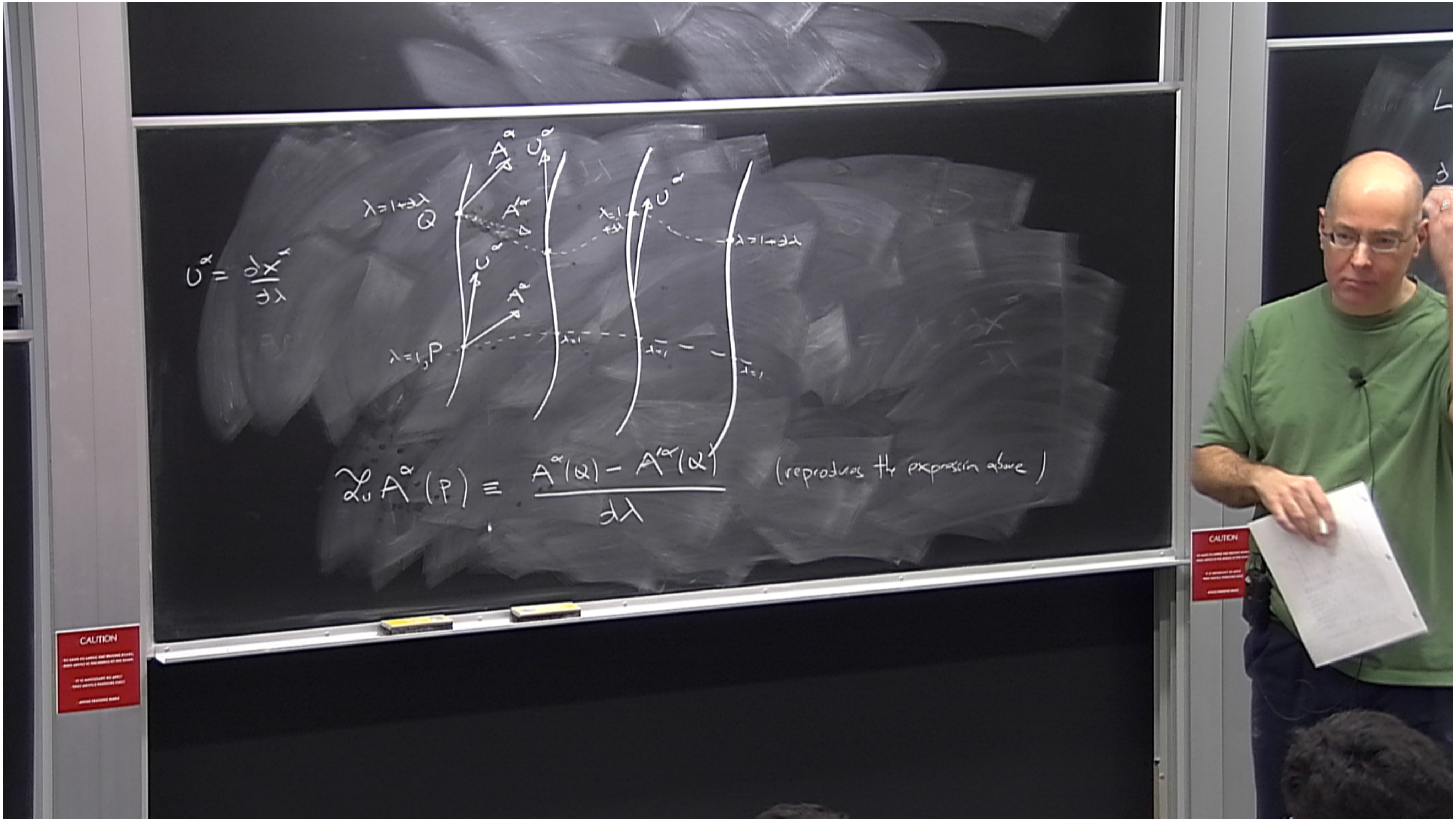

U^α = ∂x^α/∂λ
λ=1+∂λ
Q
λ=1, P
A^α
U^α

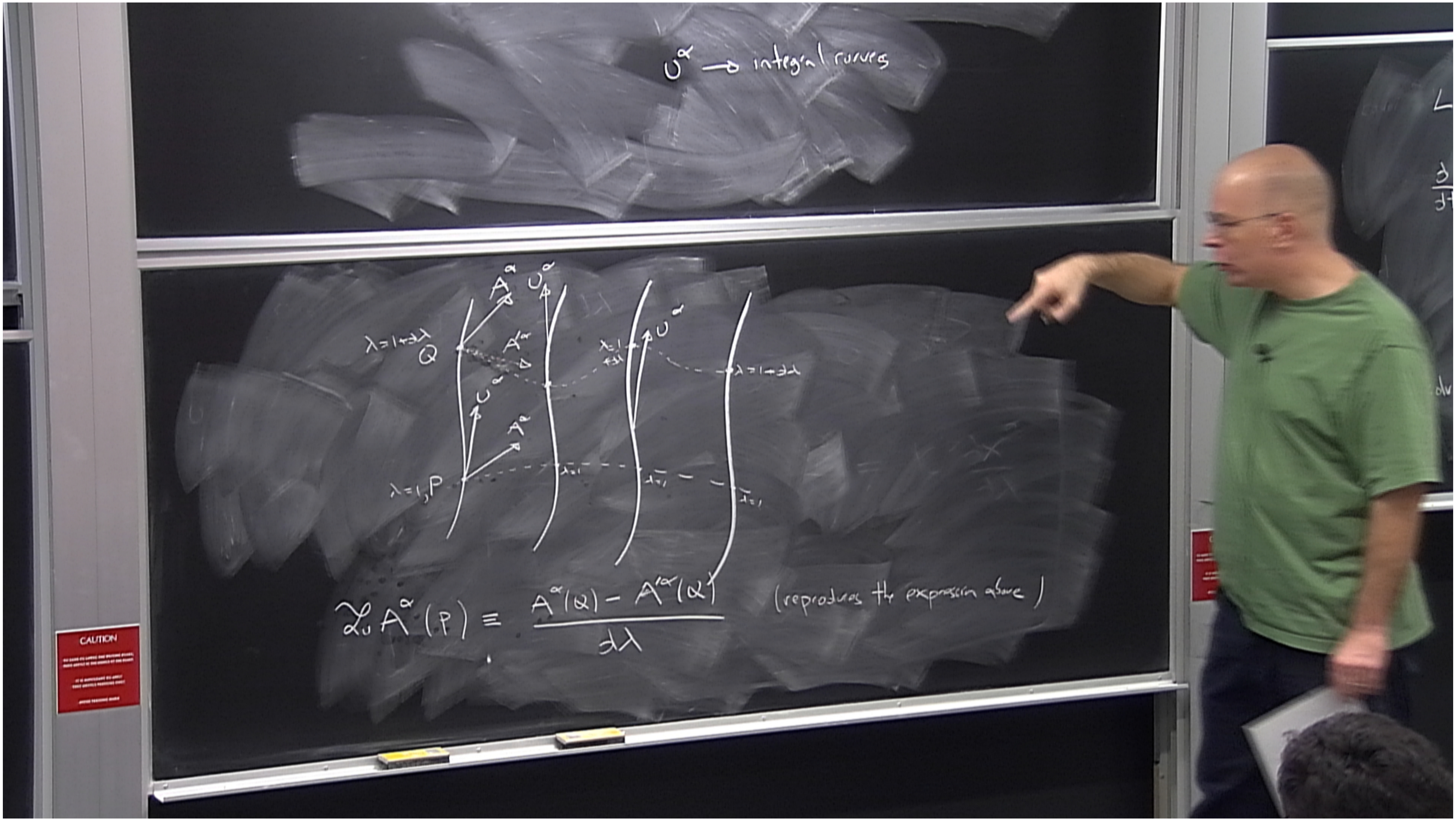

U^α = ∂x^α/∂λ
Q
U^α
A^α
λ=1, P
λ=1+∂λ
£_U A^α(P) ≡ (A^α(Q) − A'^α(Q))/∂λ

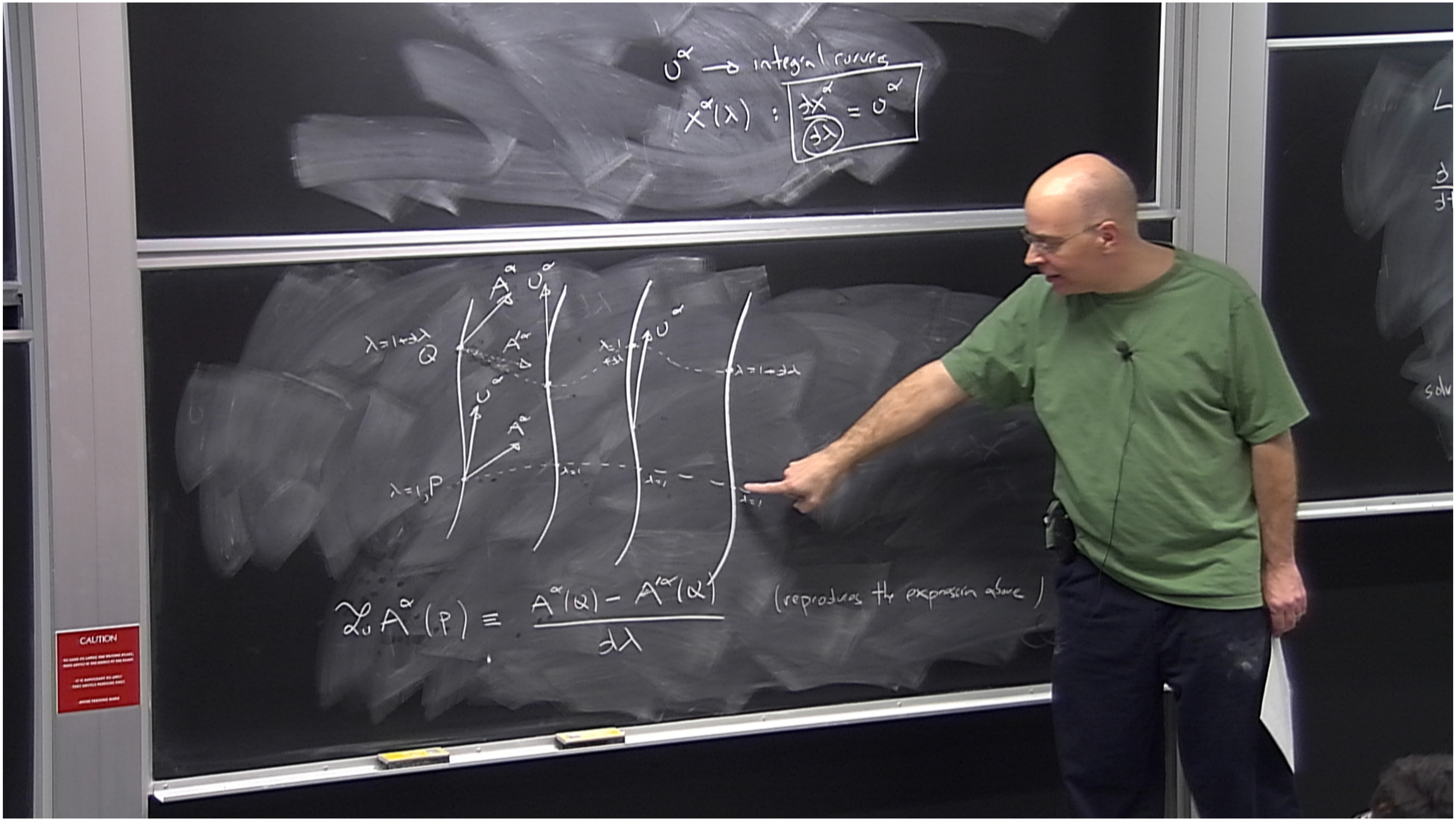

U^α = ∂x^α/∂λ
λ=1+∂λ
Q
A^α
U^α
A'^α
λ=1, P
ℒ_U A^α (P) ≡ (A^α(Q) − A'^α(Q))/∂λ
CAUTION


U^α = ∂x^α / ∂λ
λ=1+∂λ Q
λ=1, P
A^α
U^α
λ=1+∂λ
£_U A^α(P) ≡ (A^α(Q) − A'^α(Q)) / ∂λ
(reproduces the expression above)
CAUTION

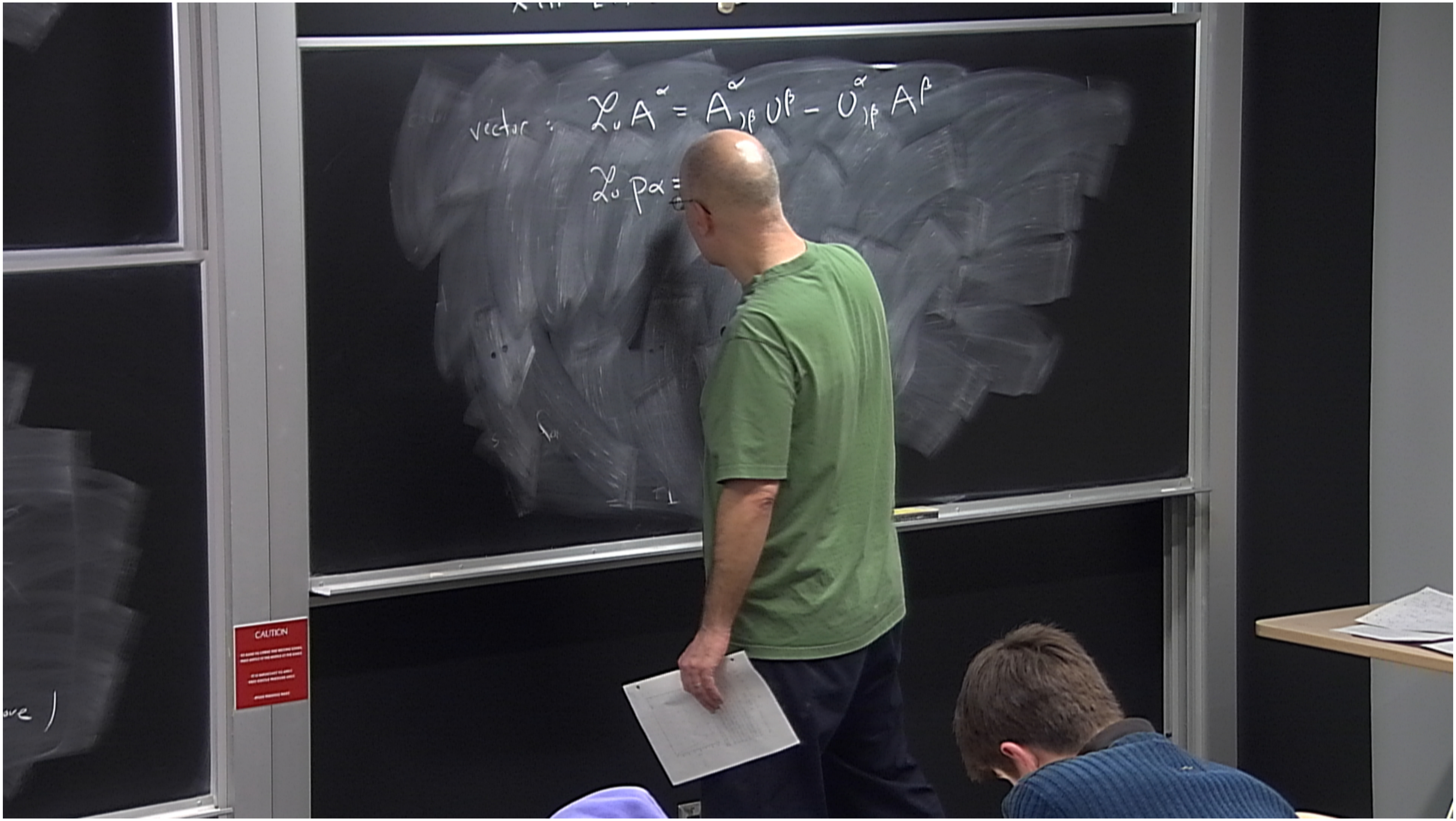

U^α = ∂x^α/∂λ
λ=1+∂λ
Q
λ=1, P
A^α
U^α
λ=1+∂λ
λ=1
£_U A^α (P) ≡ (A^α(Q) − A'^α(Q)) / ∂λ
(reproduces the expression above)
CAUTION

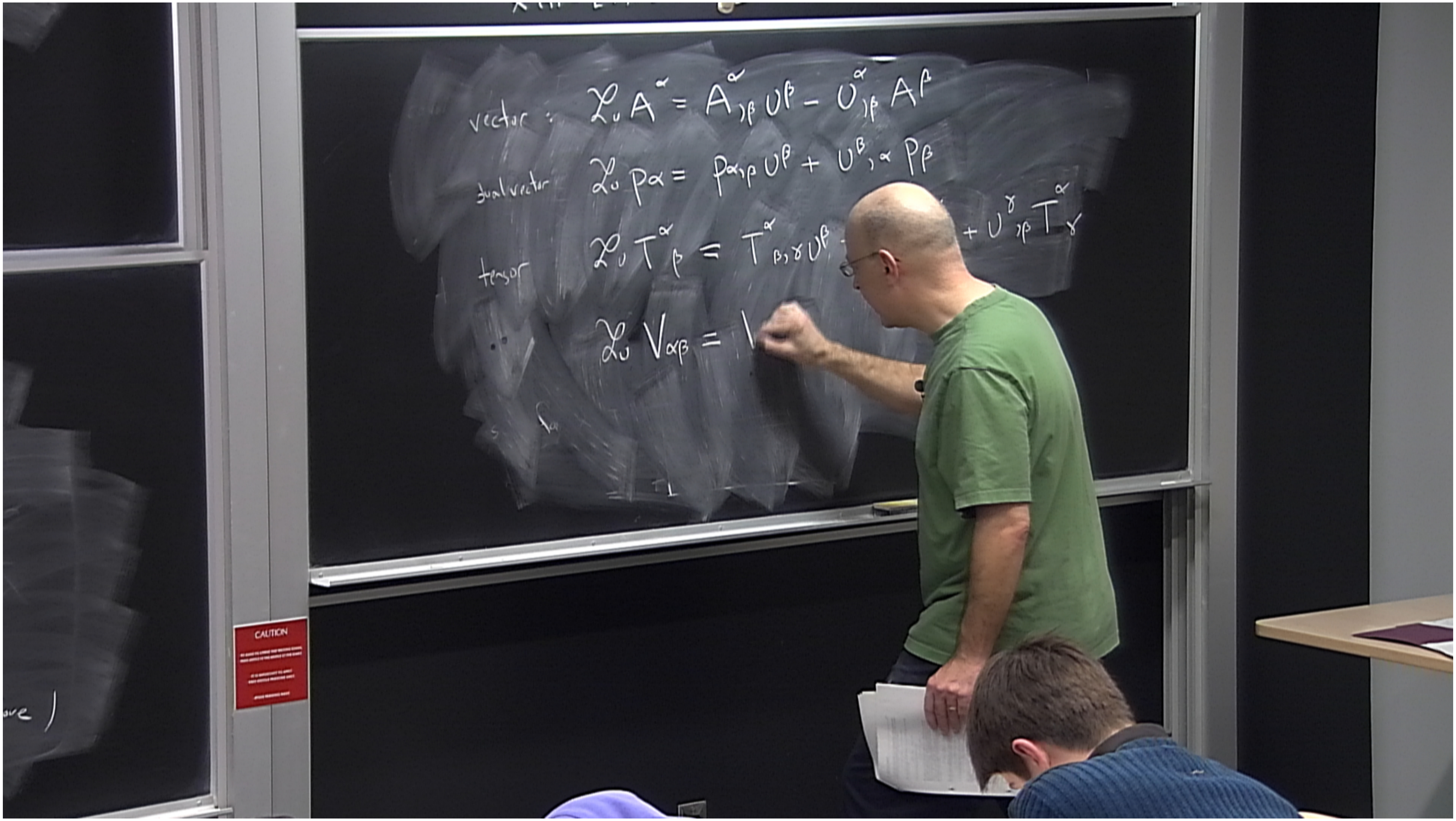

U^α = dx^α/dλ
λ=1+dλ Q
λ=1, P
£_U A^α(P) ≡ (A^α(Q) − A'^α(Q))/dλ
(reproduces the expression above)
CAUTION
CAUTION

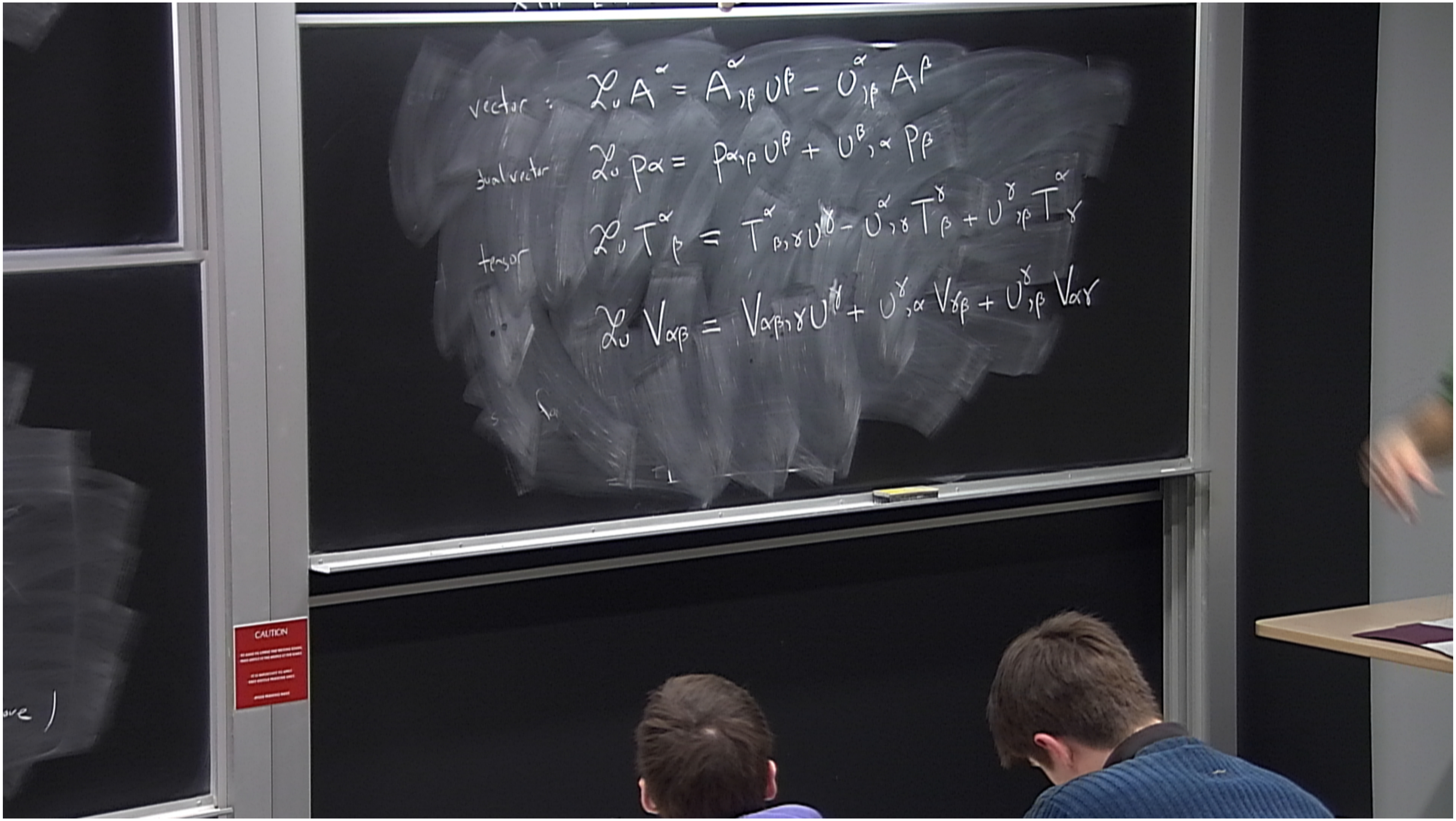

U^α → integral curves
λ=1+dλ
Q
λ=1, P
λ=1+dλ
(reproduces the expression above)
$\mathcal{L}_U A^\alpha(P) \equiv \frac{A^\alpha(Q) - A'^\alpha(Q)}{d\lambda}$

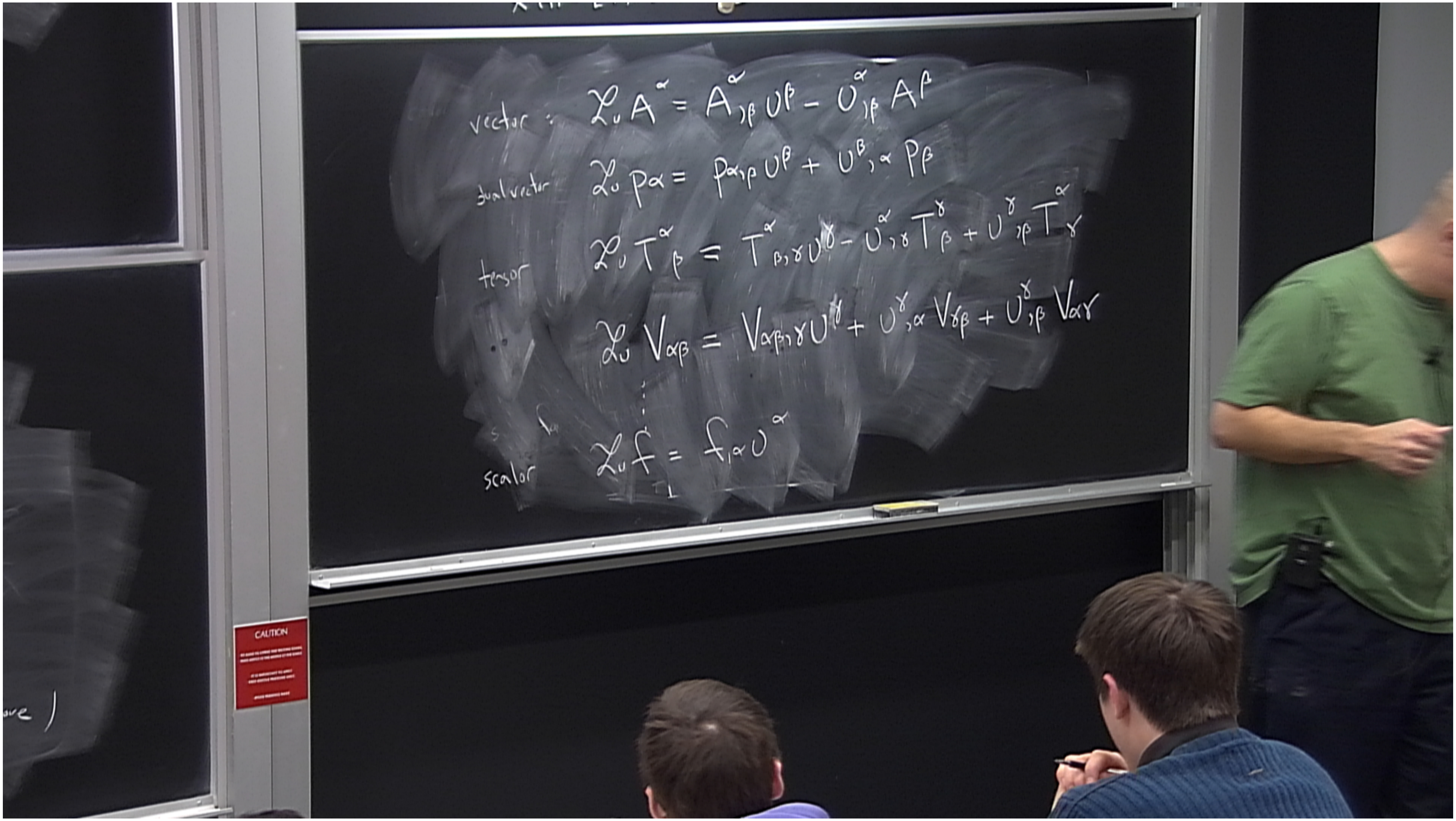


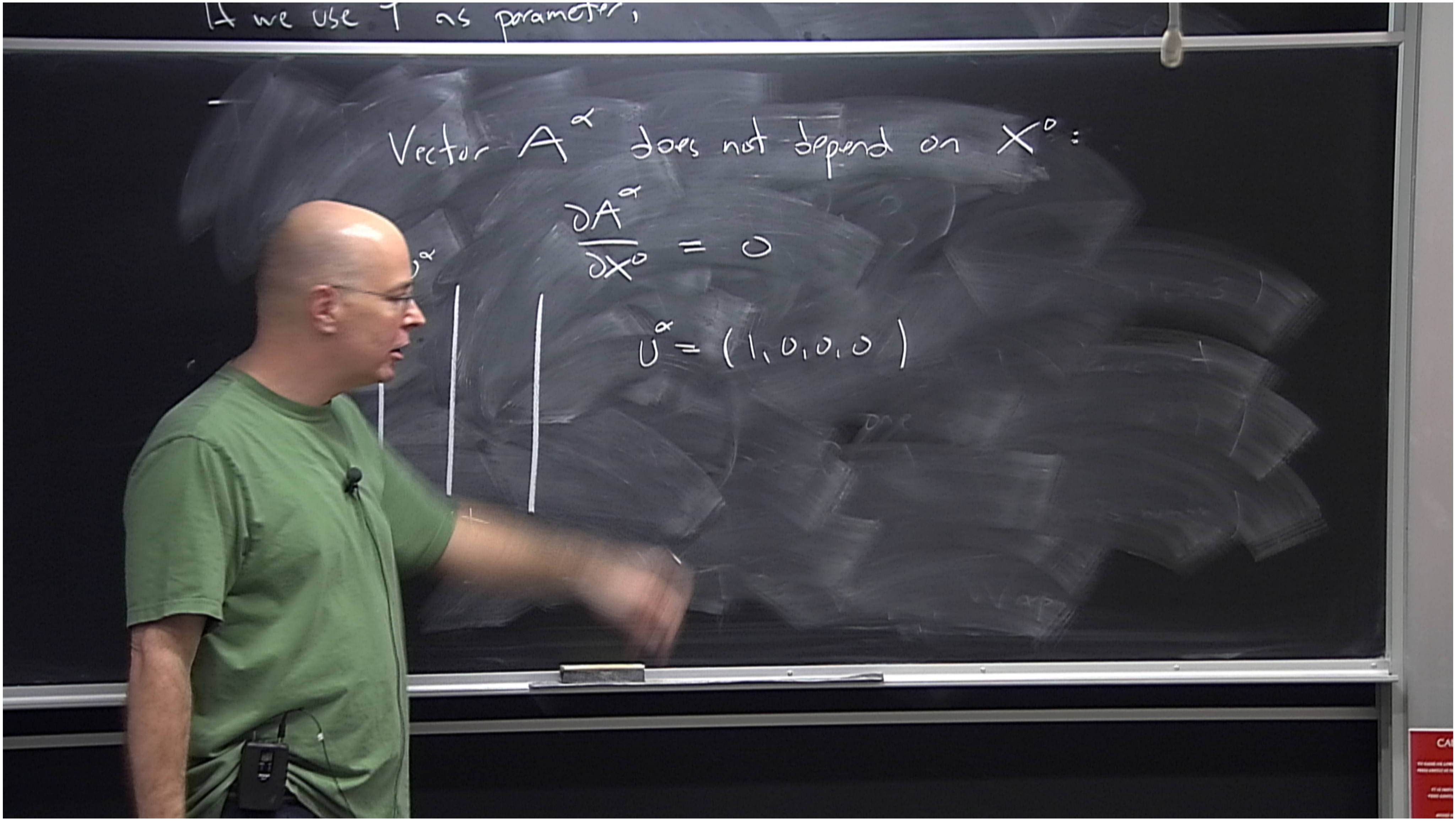
$$\mathcal{L}_U A^\alpha(P) \equiv \frac{A^\alpha(Q) - A'^\alpha(Q)}{d\lambda}$$

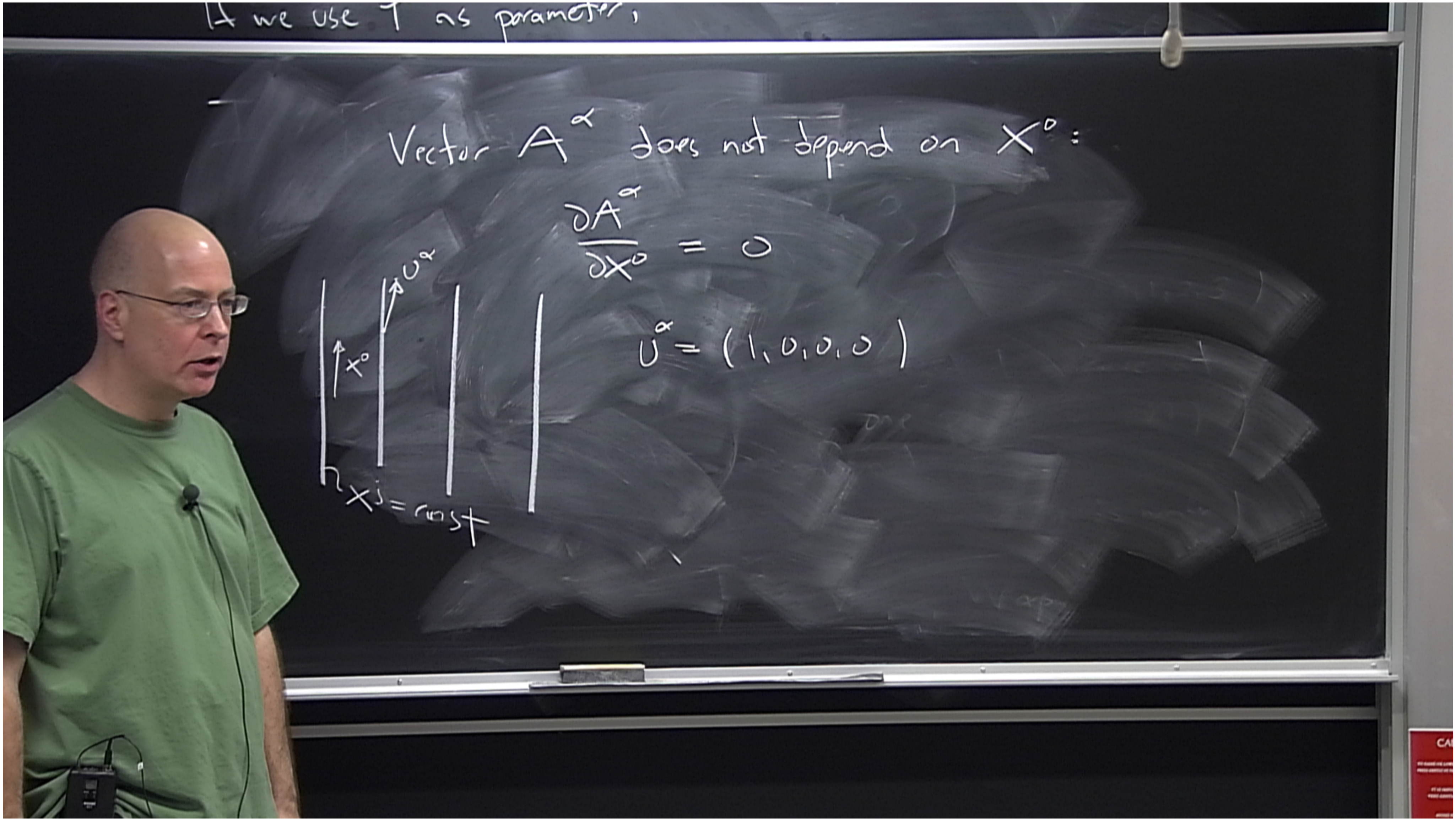

vector : $\mathcal{L}_u A^\alpha = A^\alpha_{;\beta} u^\beta - u^\alpha_{;\beta} A^\beta$
$\mathcal{L}_u p_\alpha =$
CAUTION

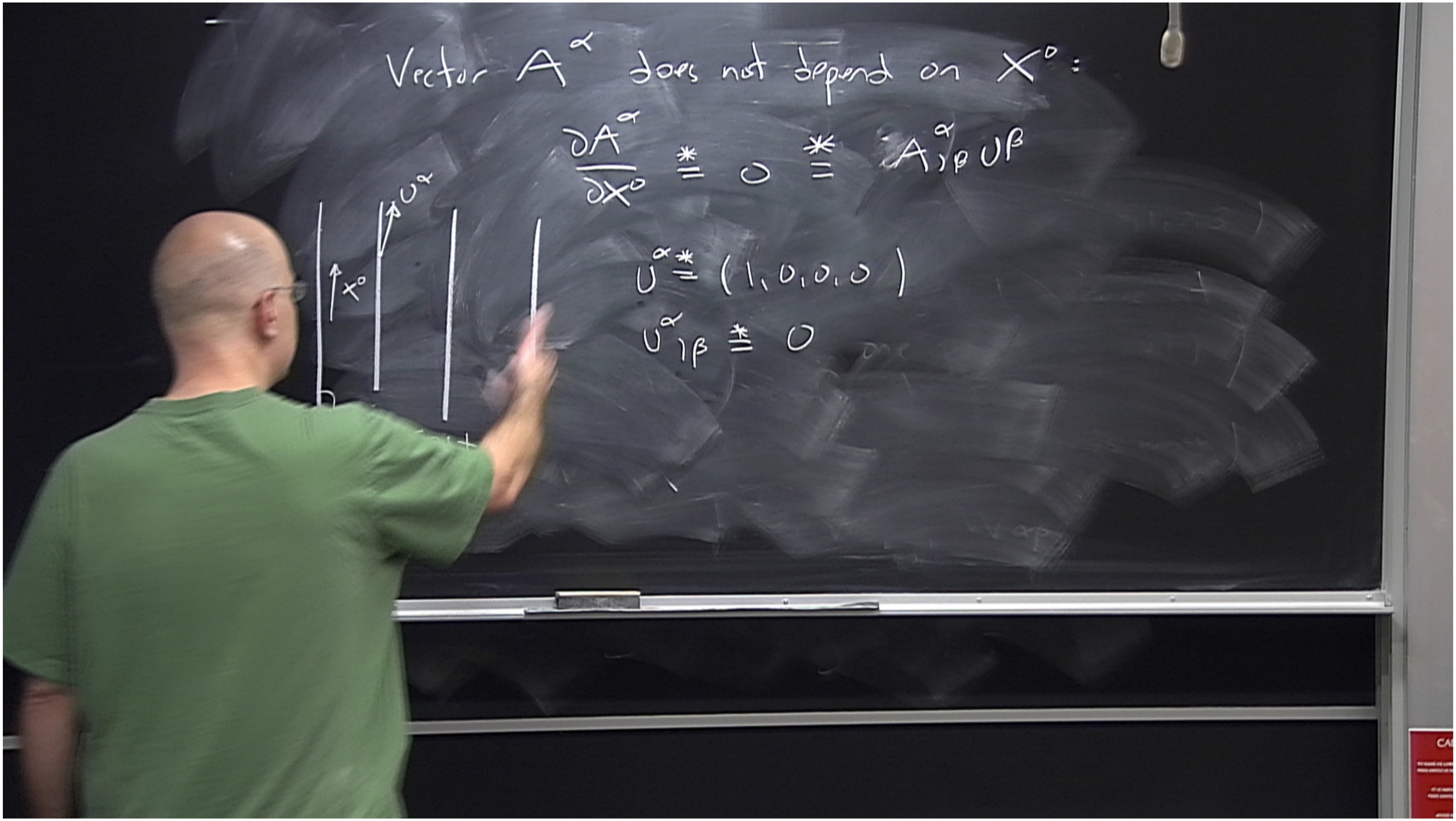

vector : $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
dual vector $\mathcal{L}_U p_\alpha = p_{\alpha,\beta} U^\beta + U^\beta{}_{,\alpha} p_\beta$
tensor $\mathcal{L}_U T^\alpha{}_\beta = T^\alpha{}_{\beta,\gamma} U^\beta$
$+ U^\gamma{}_{,\beta} T^\alpha{}_\gamma$
$\mathcal{L}_U V_{\alpha\beta} =$
CAUTION

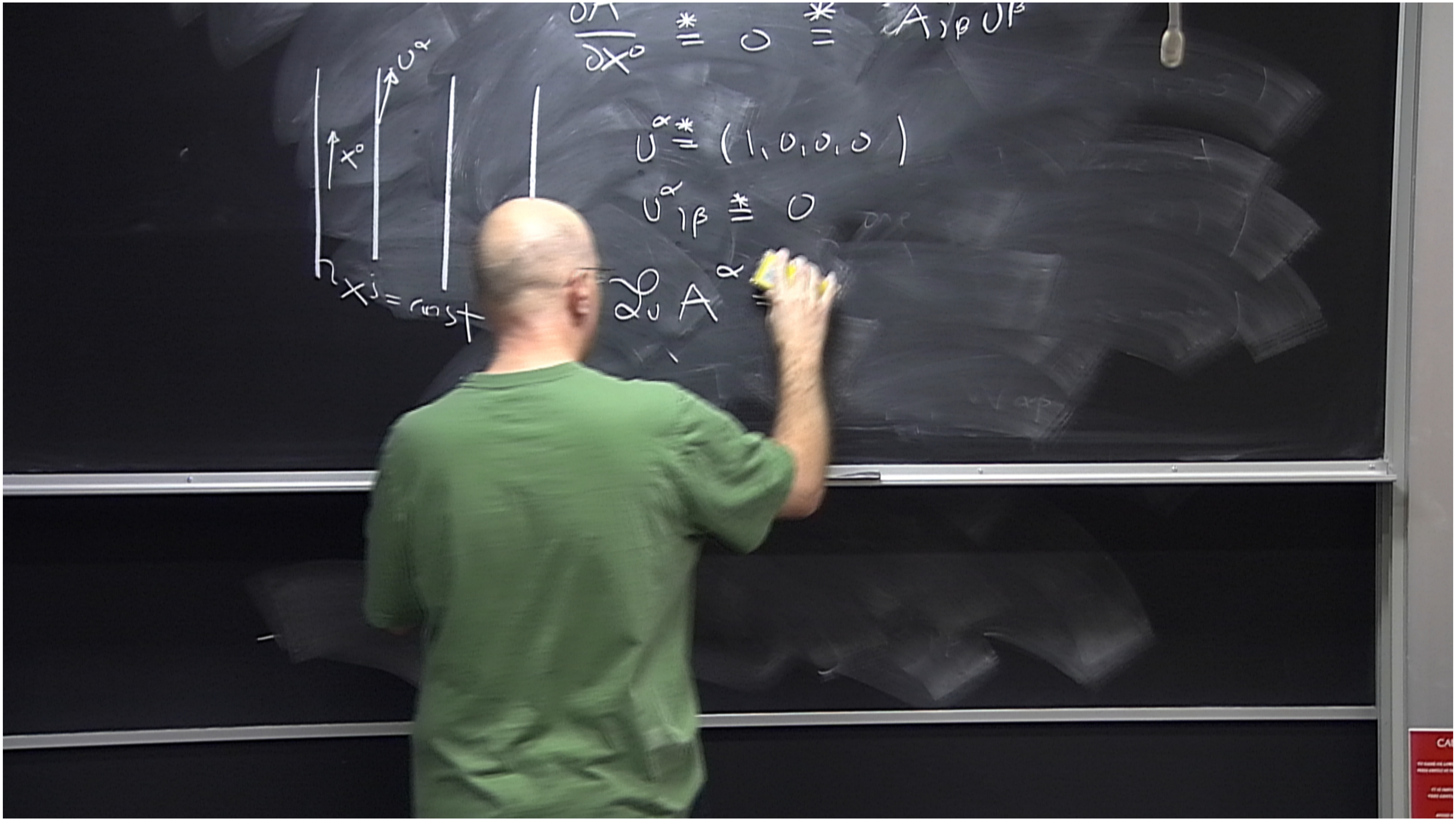

vector : $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
dual vector $\mathcal{L}_U p_\alpha = p_{\alpha,\beta} U^\beta + U^\beta{}_{,\alpha} p_\beta$
tensor $\mathcal{L}_U T^\alpha{}_\beta = T^\alpha{}_{\beta,\gamma} U^\gamma - U^\alpha{}_{,\gamma} T^\gamma{}_\beta + U^\gamma{}_{,\beta} T^\alpha{}_\gamma$
$\mathcal{L}_U V_{\alpha\beta} = V_{\alpha\beta,\gamma} U^\gamma + U^\gamma{}_{,\alpha} V_{\gamma\beta} + U^\gamma{}_{,\beta} V_{\alpha\gamma}$
CAUTION


vector : $\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
dual vector $\mathcal{L}_U p_\alpha = p_{\alpha,\beta} U^\beta + U^\beta{}_{,\alpha} p_\beta$
tensor $\mathcal{L}_U T^\alpha{}_\beta = T^\alpha{}_{\beta,\gamma} U^\gamma - U^\alpha{}_{,\gamma} T^\gamma{}_\beta + U^\gamma{}_{,\beta} T^\alpha{}_\gamma$
$\mathcal{L}_U V_{\alpha\beta} = V_{\alpha\beta,\gamma} U^\gamma + U^\gamma{}_{,\alpha} V_{\gamma\beta} + U^\gamma{}_{,\beta} V_{\alpha\gamma}$
scalar $\mathcal{L}_U f = f_{,\alpha} U^\alpha$
CAUTION

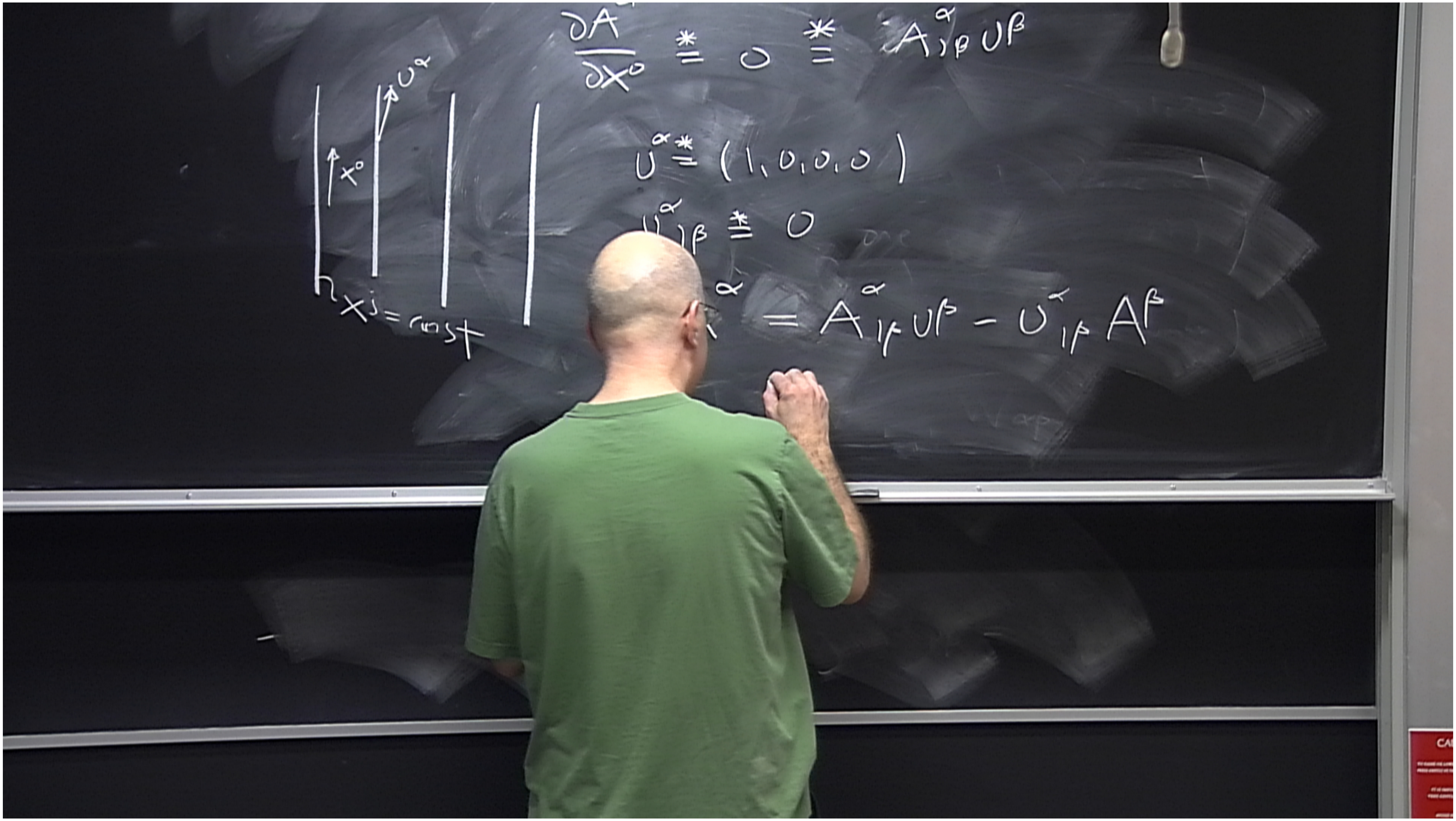

If we use τ as parameter,
Vector A^α does not depend on X^0 :
∂A^α/∂X^0 = 0
U^α = (1,0,0,0)

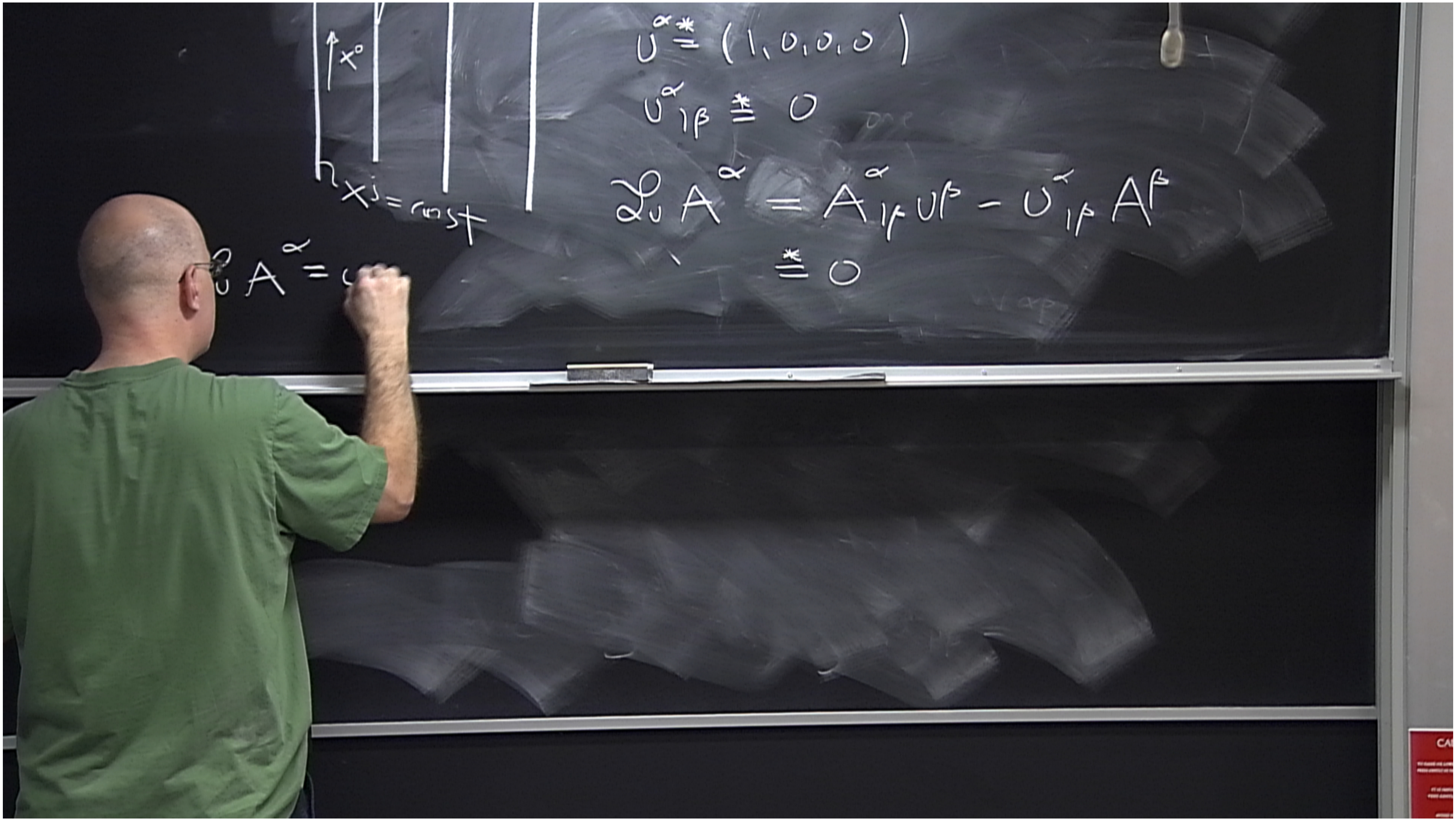

If we use τ as parameter,
Vector A^α does not depend on X^0:
∂A^α/∂x^0 = 0
U^α = (1, 0, 0, 0)
U^α
x^0
x^i = const

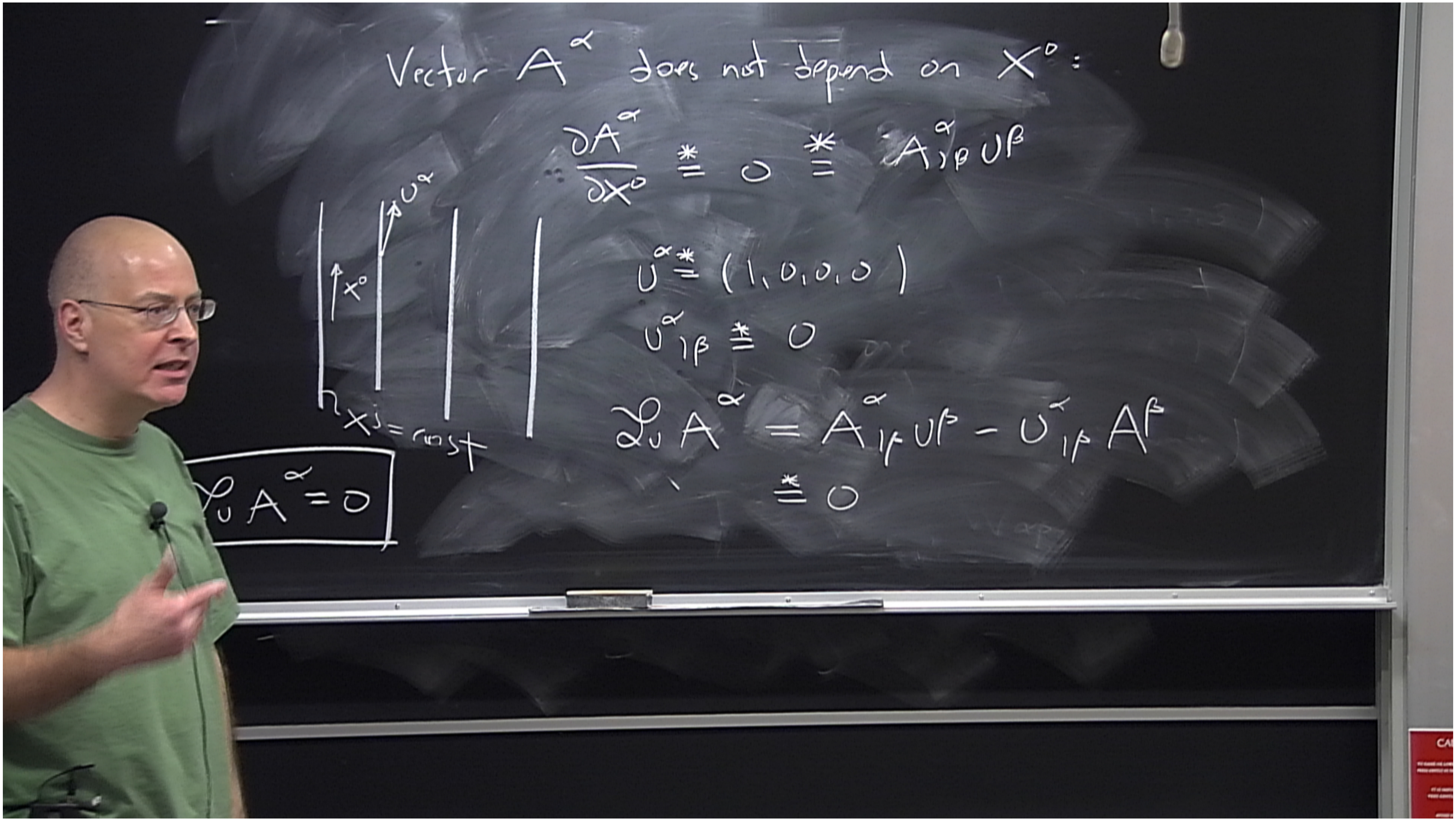

Vector A^α does not depend on X^0:
$\frac{\partial A^\alpha}{\partial X^0} \overset{*}{=} 0 \overset{*}{=} A^\alpha{}_{;\beta} U^\beta$
U^α
X^0
$U^\alpha \overset{*}{=} (1,0,0,0)$
$U^\alpha{}_{;\beta} \overset{*}{=} 0$

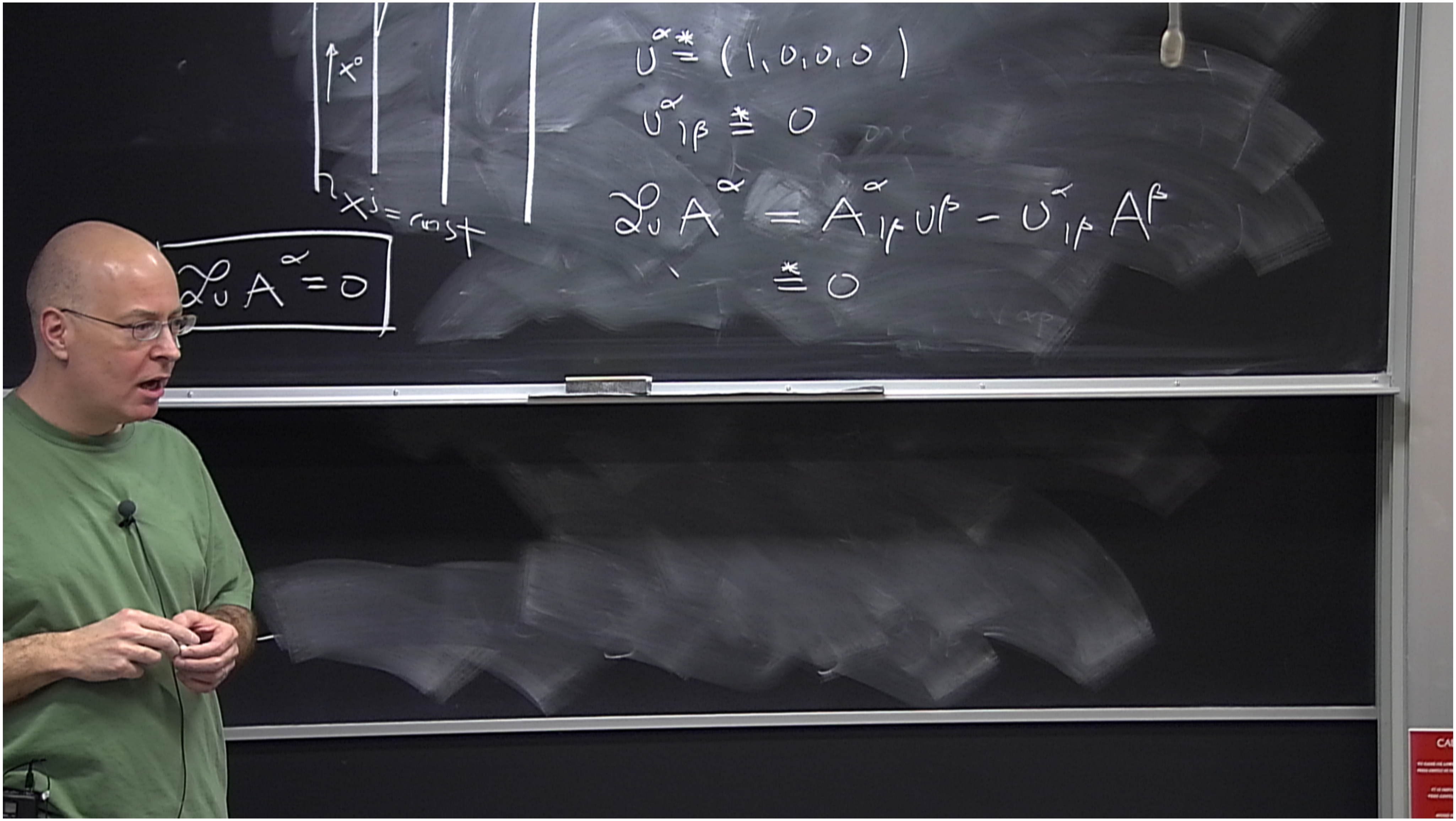


x^0
x^j = const
U^α
U^α ≐ (1, 0, 0, 0)
U^α_{;β} ≐ 0


∂A / ∂x⁰ ≛ 0 ≛ A;β Uβ
Uα
x⁰
xʲ = const
Uα ≛ (1, 0, 0, 0)
Uα;β ≛ 0
= Aα;β Uβ − Uα;β Aβ


x^0
$U^\alpha \overset{*}{=} (1,0,0,0)$
$U^\alpha{}_{|\beta} \overset{*}{=} 0$
$x^j = \text{const}$
$\mathcal{L}_U A^\alpha = A^\alpha{}_{|\beta} U^\beta - U^\alpha{}_{|\beta} A^\beta$
$\overset{*}{=} 0$


Vector A^α does not depend on x^0:
$\frac{\partial A^\alpha}{\partial x^0} \overset{*}{=} 0 \overset{*}{=} A^\alpha{}_{,\beta} U^\beta$
$U^\alpha \overset{*}{=} (1,0,0,0)$
$U^\alpha{}_{,\beta} \overset{*}{=} 0$
$\mathcal{L}_U A^\alpha = A^\alpha{}_{,\beta} U^\beta - U^\alpha{}_{,\beta} A^\beta$
$\overset{*}{=} 0$
U^α
x^0
$x^j = \text{const}$
$\mathcal{L}_U A^\alpha = 0$



x^0
$x^j = \text{const}$
$\mathcal{L}_U A^\alpha = 0$
$U^\alpha \overset{*}{=} (1,0,0,0)$
$U^\alpha{}_{|\beta} \overset{*}{=} 0$
$\mathcal{L}_U A^\alpha = A^\alpha{}_{|\beta} U^\beta - U^\alpha{}_{|\beta} A^\beta$
$\overset{*}{=} 0$