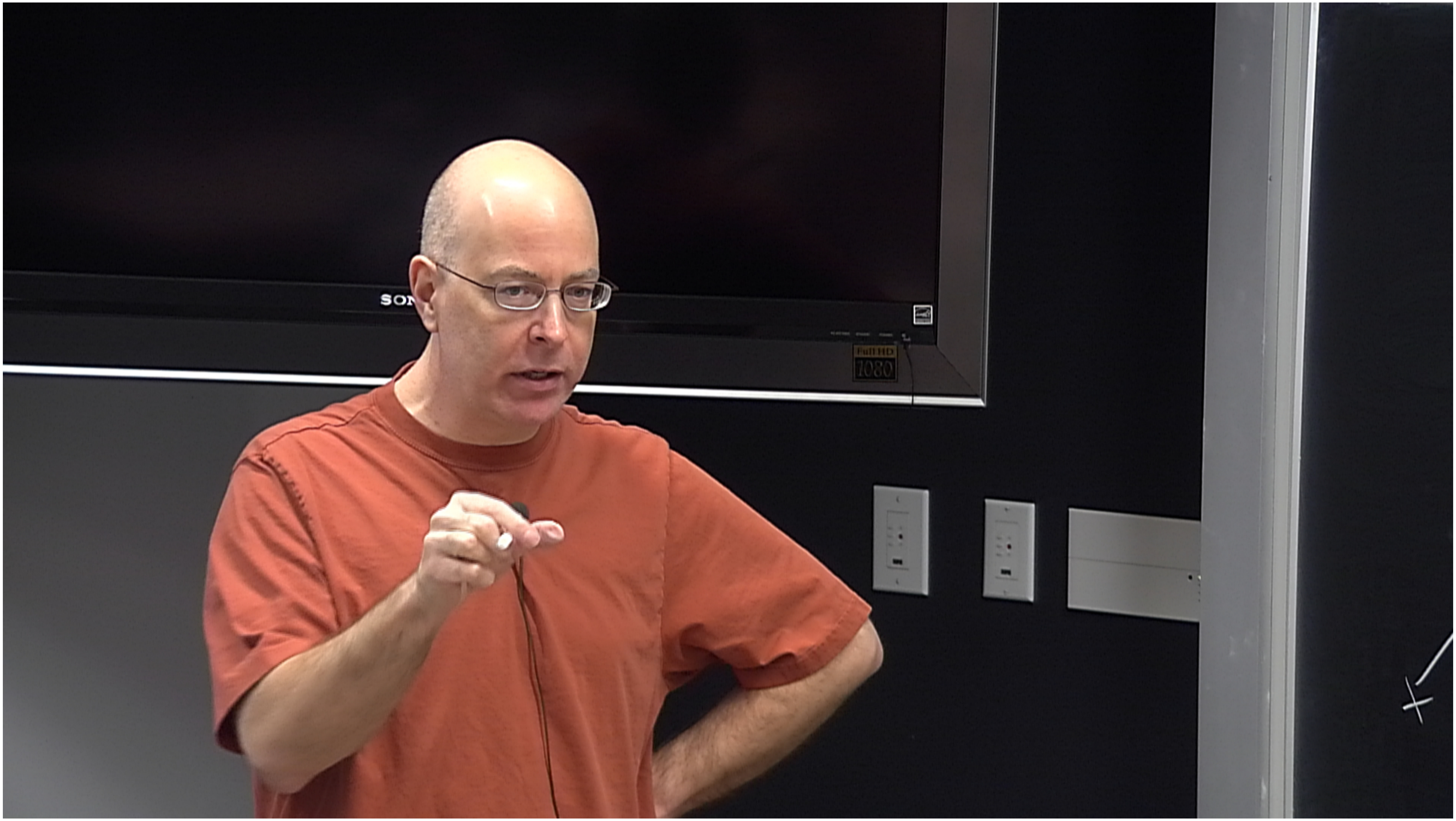
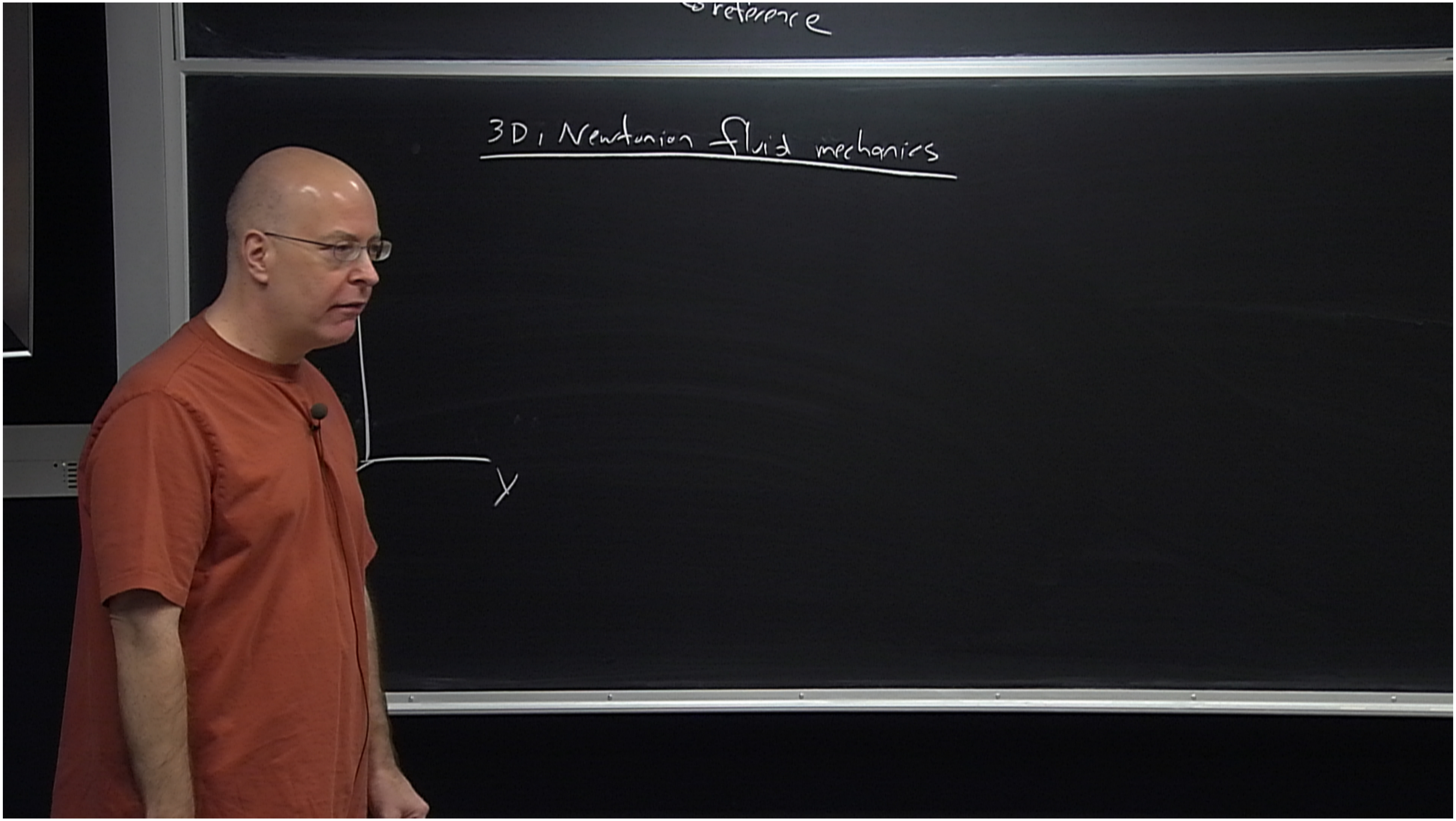
Title: Advanced General Relativity - Lecture 5

Date: Jan 25, 2012 10:00 AM

URL: http://pirsa.org/12010153

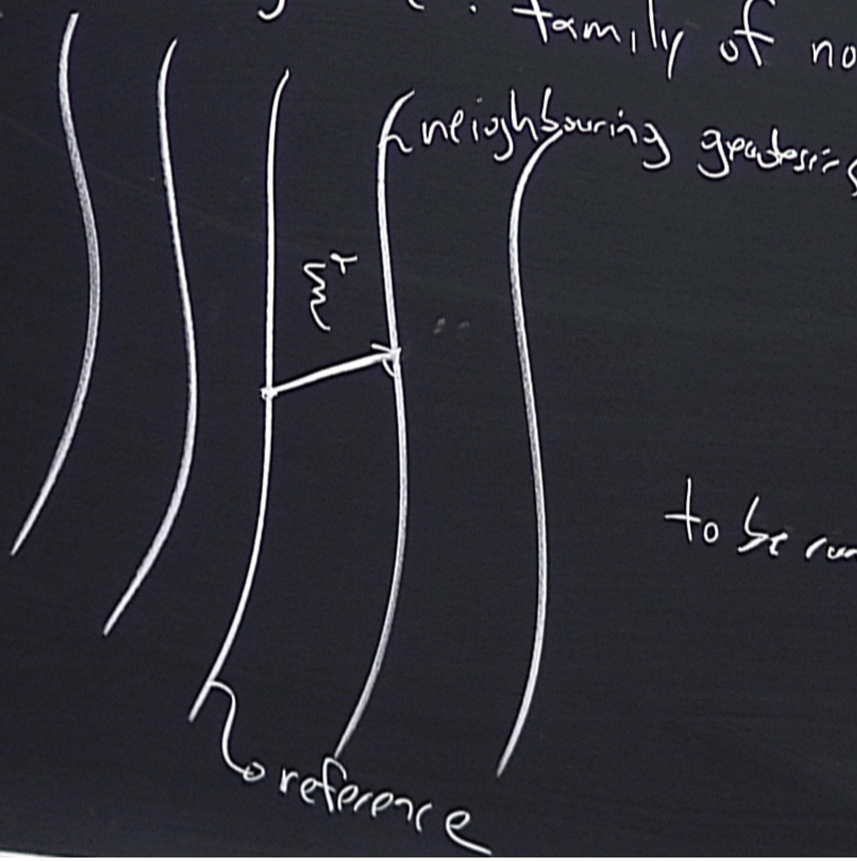
Abstract:



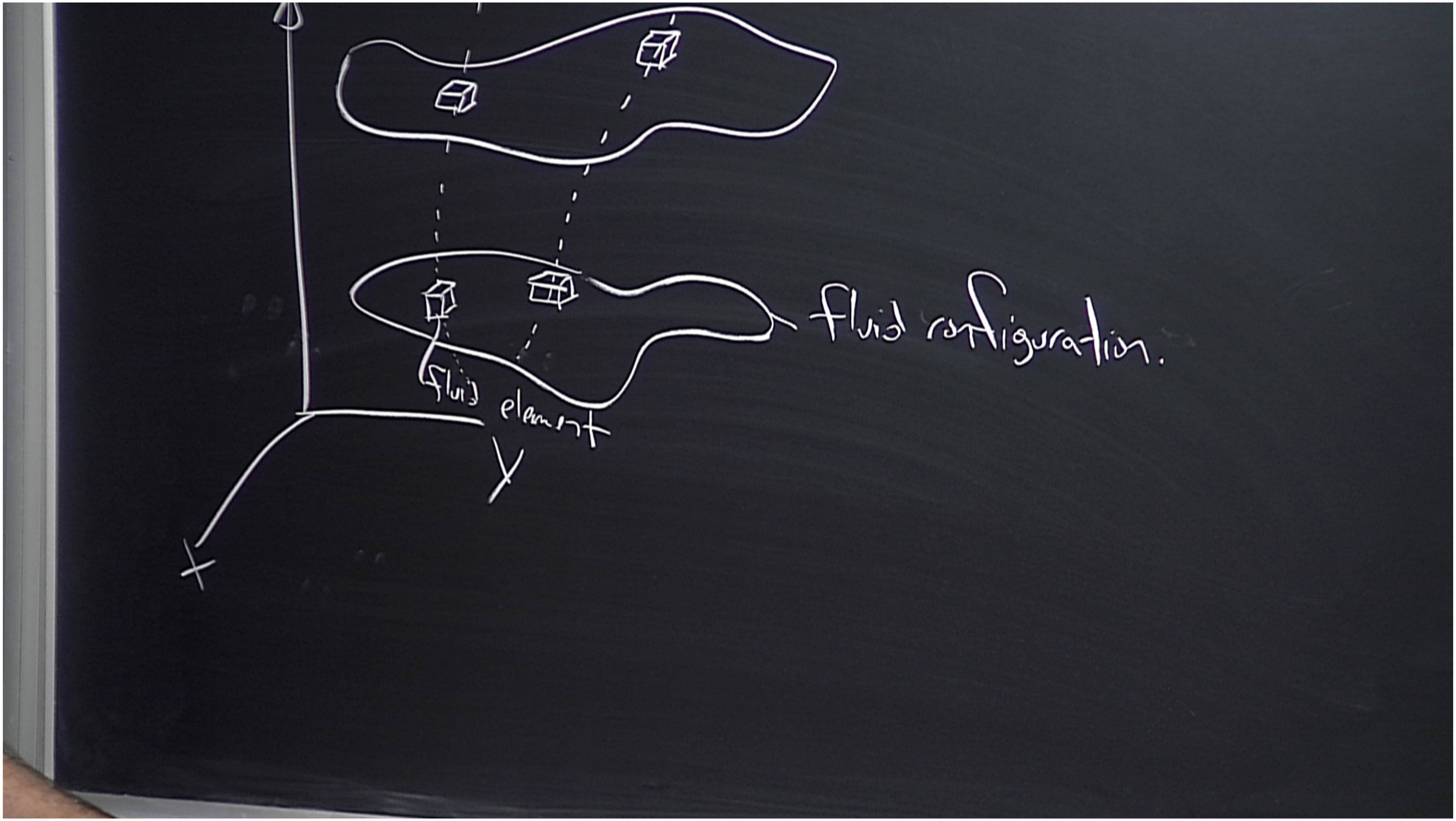

reference
3D, Newtonian fluid mechanics
y

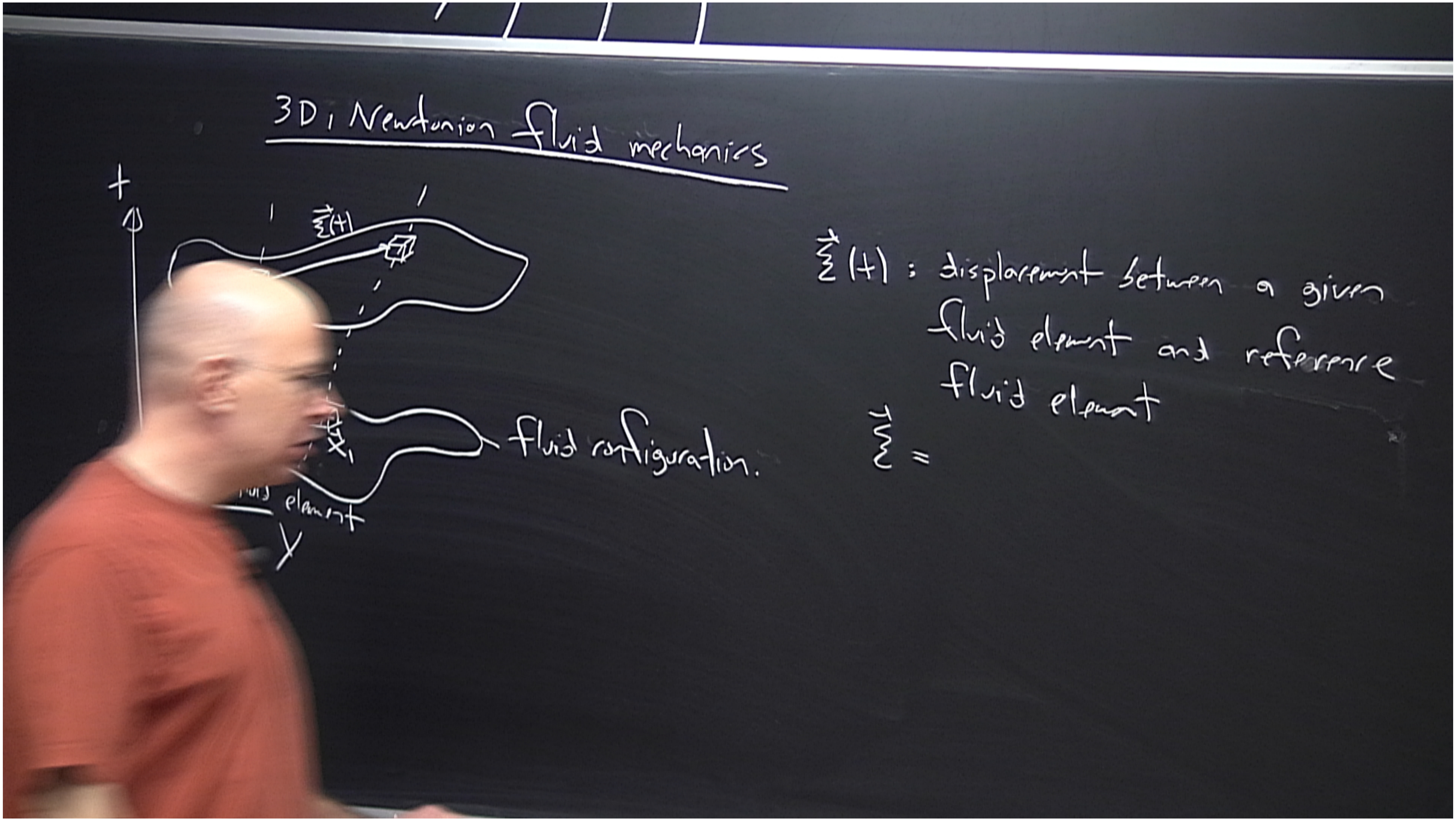
GEODESIC CONGRUENCES

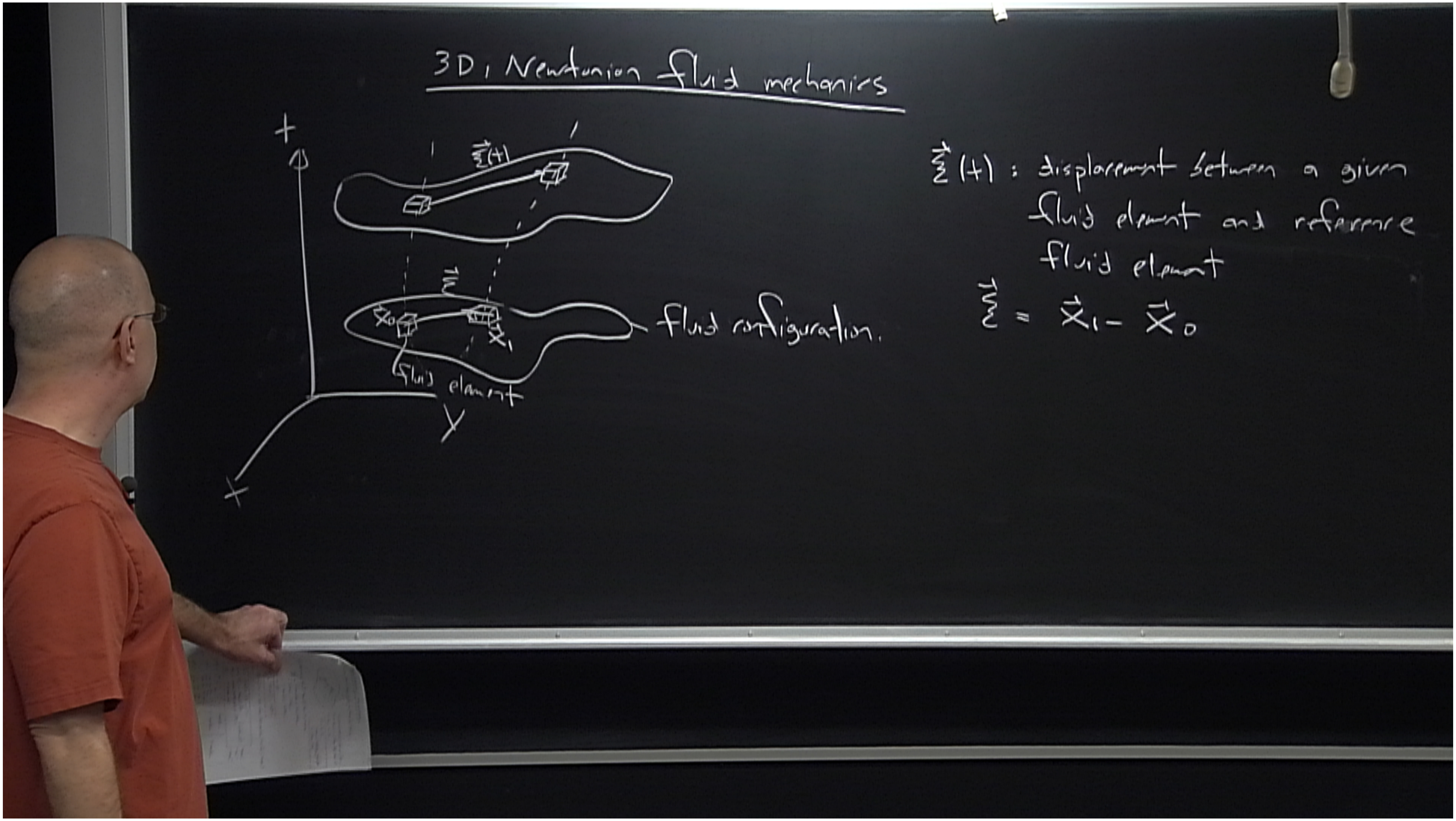
congruence : family of non-intersecting geodesics



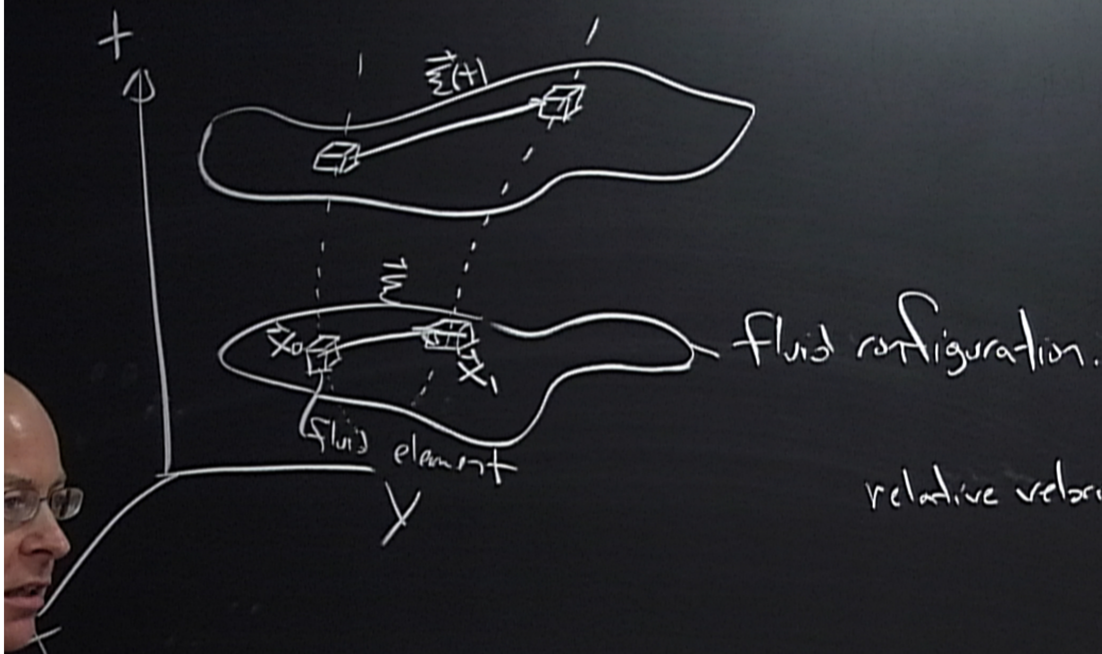
to be continued...


fluid configuration.
fluid element
X
Y


3D, Newtonian fluid mechanics
ξ(t)
X1
fluid configuration.
element
ξ(t) : displacement between a given fluid element and reference fluid element
ξ =


3D, Newtonian fluid mechanics
$\vec{\xi}(t)$
$\vec{\xi}$
$\vec{x}_0$
$\vec{x}_1$
fluid configuration.
fluid element
t
x
y
$\vec{\xi}(t)$: displacement between a given fluid element and reference fluid element
$\vec{\xi} = \vec{x}_1 - \vec{x}_0$

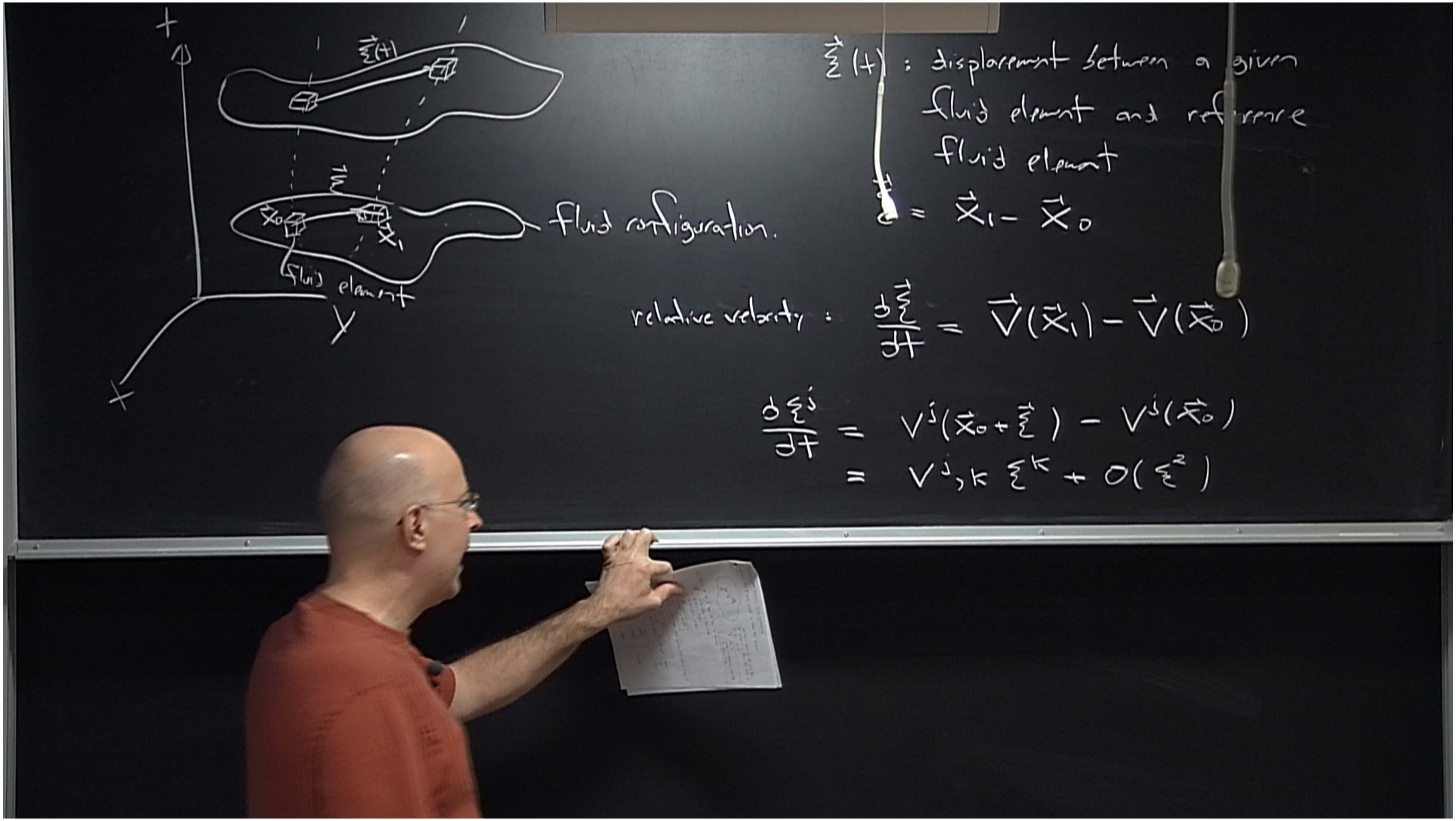
3D, Newtonian fluid mechanics

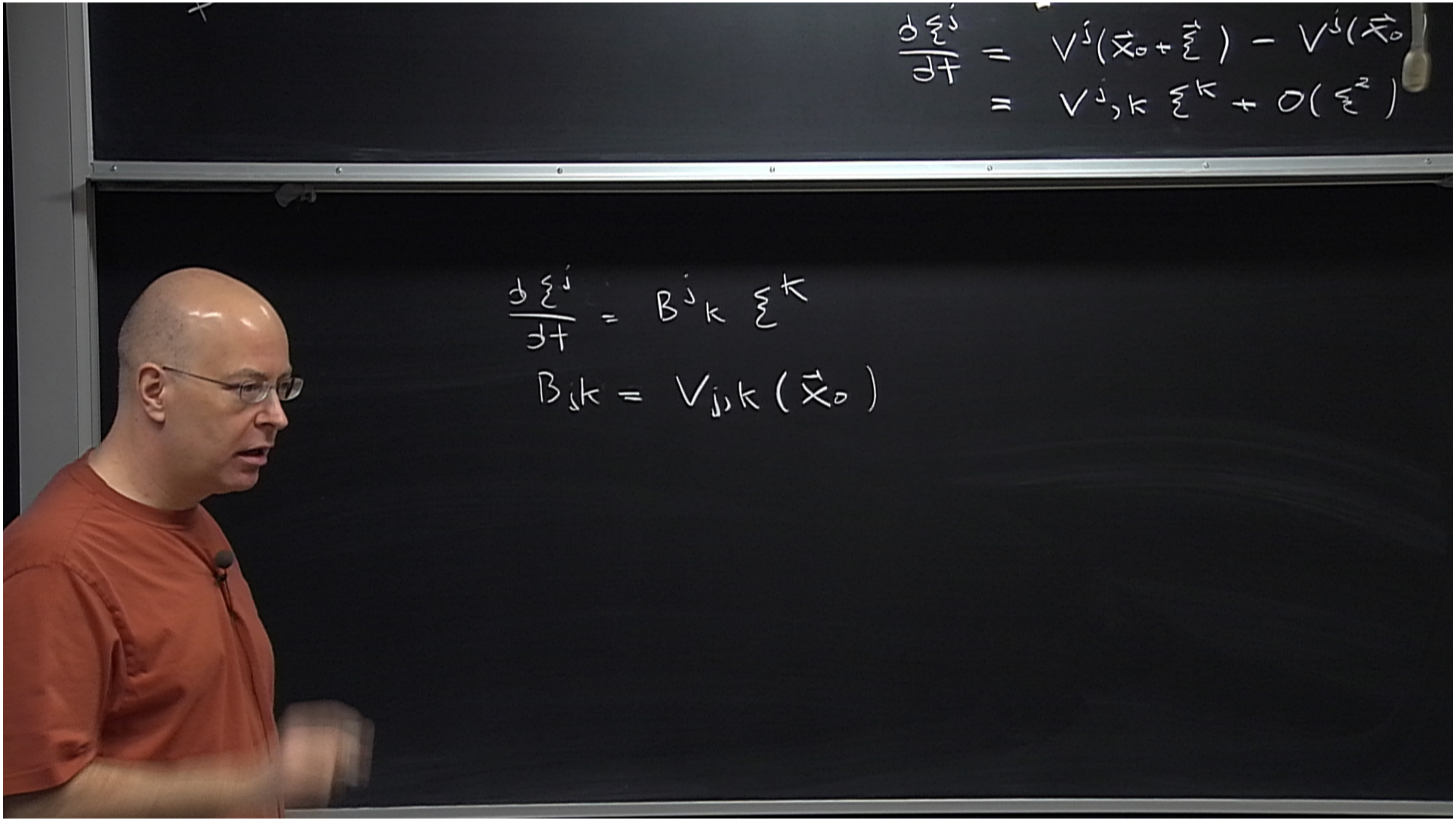


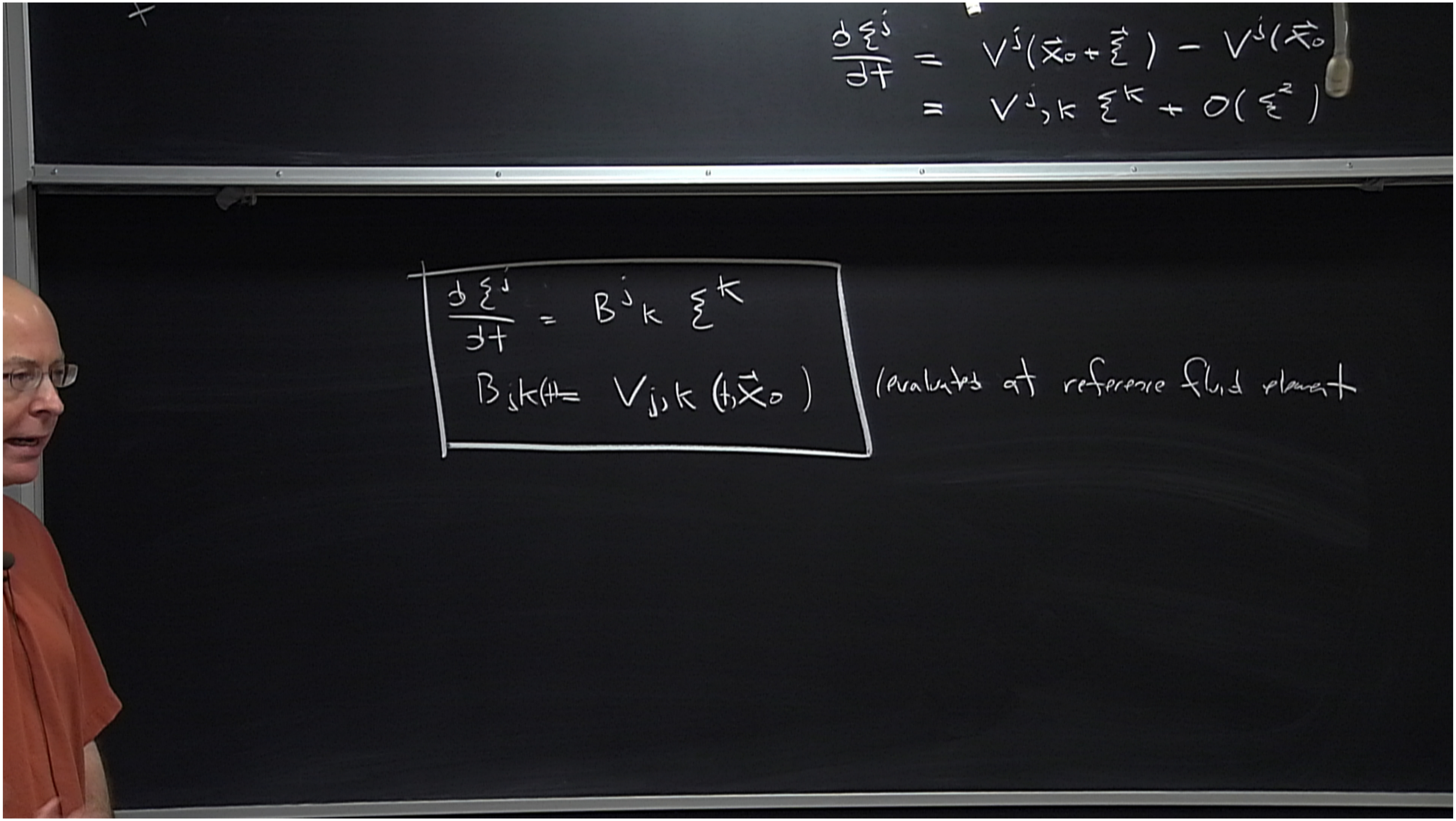
$\vec{\xi}(t)$: displacement between a given fluid element and reference fluid element

$$\vec{\xi} = \vec{x}_1 - \vec{x}_0$$

relative velocity : $$\frac{d\vec{\xi}}{dt} = \vec{v}(\vec{x}_1) - \vec{v}(\vec{x}_0)$$


fluid configuration.
fluid element
ξ(t) : displacement between a given fluid element and reference fluid element
ξ = x₁ − x₀
relative velocity : dξ/dt = V(x₁) − V(x₀)
dξʲ/dt = Vʲ(x₀ + ξ) − Vʲ(x₀)
= Vʲ,ₖ ξᵏ + O(ξ²)


$\frac{d\xi^j}{dt} = V^j(\vec{x}_0 + \vec{\xi}\,) - V^j(\vec{x}_0)$
$= V^j{}_{,k}\, \xi^k + O(\xi^2)$
$\frac{d\xi^j}{dt} = B^j{}_k\, \xi^k$
$B_{jk} = V_{j,k}(\vec{x}_0)$


$\frac{d\xi^j}{dt} = V^j(\vec{x}_0 + \vec{\xi}) - V^j(\vec{x}_0$
$= V^j{}_{,k}\, \xi^k + O(\xi^2)$
$\frac{d\xi^j}{dt} = B^j{}_k\, \xi^k$
$B_{jk}(t) = V_{j,k}(t, \vec{x}_0)$
(evaluated at reference fluid element

$$\frac{d\xi^j}{dt} = V^j(\vec{x}_0 + \vec{\xi}\,) - V^j(\vec{x}_0)$$
$$= V^j{}_{,k}\,\xi^k + O(\xi^2)$$

$$\boxed{\begin{aligned} \frac{d\xi^j}{dt} &= B^j{}_k\,\xi^k \\ B_{jk}(t) &= V_{j,k}(t, \vec{x}_0) \end{aligned}}$$

(evaluated at reference fluid element

matrix without symmetries $\rightarrow$ decompose it into irreducible pieces

$$B_{jk} =$$

$$\boxed{\frac{d\xi^j}{dt} = B^j{}_k\, \xi^k \qquad B_{jk}(t) = V_{j,k}(t, \vec{x}_0)}$$

(evaluated at reference fluid element

matrix without symmetries → decompose it into irreducible pieces

$$B_{jk} = \underbrace{\tfrac{1}{3}\delta_{jk}\,\Theta}_{\text{trace part}} + \underbrace{\sigma_{jk}}_{\substack{\text{symmetric}\\ \text{trace-free}}} + \underbrace{\omega_{jk}}_{\text{antisymmetric}}$$

$$\boxed{\frac{d\xi^j}{dt} = B^j{}_k\, \xi^k \qquad B_{jk}(t) = V_{j,k}(t,\vec{x}_0)}$$

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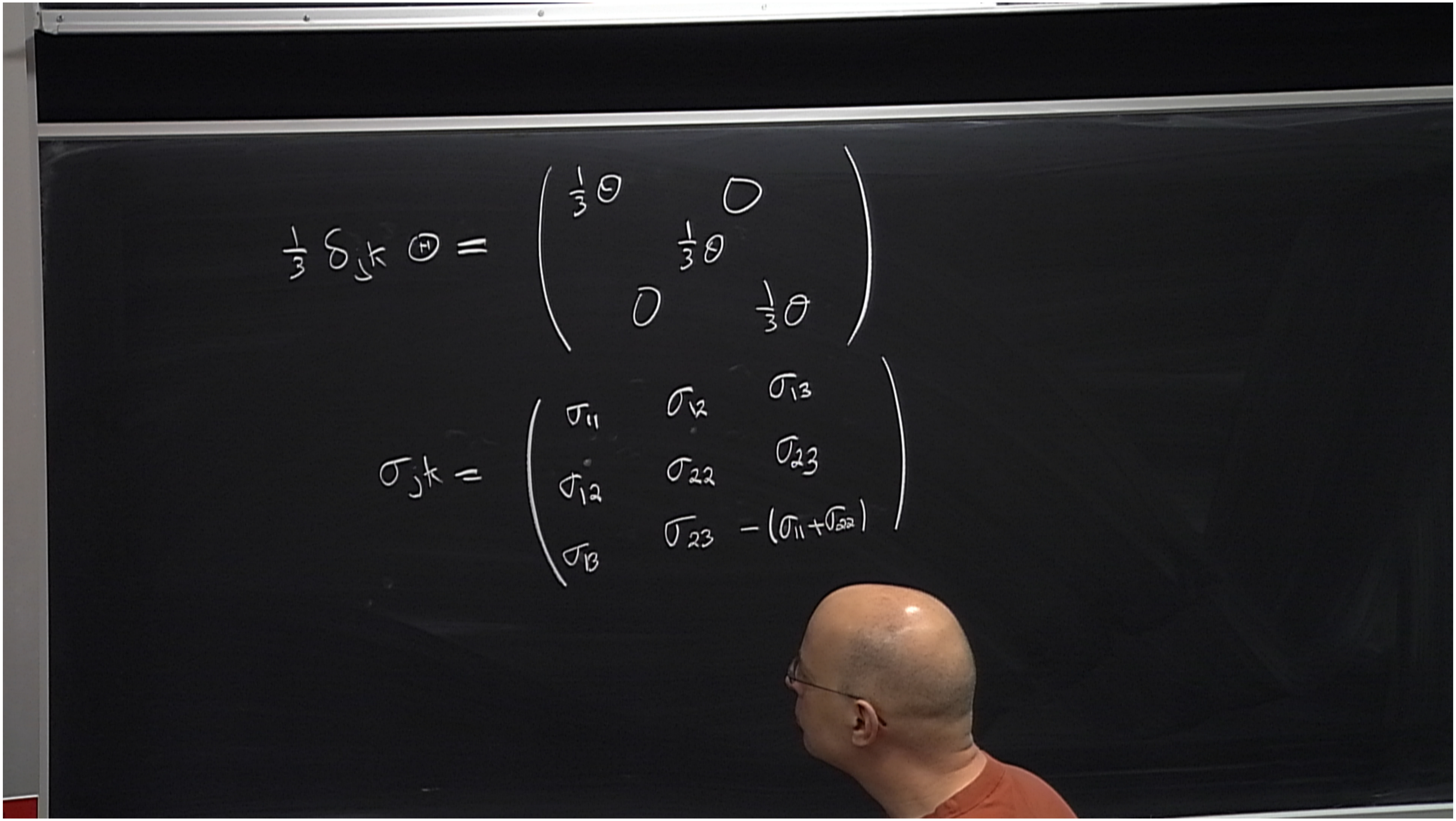
$$\Theta \equiv \delta^{jk} B_{jk} \qquad \sigma_{jk} = B_{(jk)} - \tfrac{1}{3}\delta_{jk}\,\Theta \;, \quad \omega_{jk} \equiv B_{[jk]}$$


$\frac{1}{3}\delta_{jk}\Theta = \begin{pmatrix} \frac{1}{3}\theta & & 0 \\ & \frac{1}{3}\theta & \\ 0 & & \frac{1}{3}\theta \end{pmatrix}$
$\sigma_{jk} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -(\sigma_{11}+\sigma_{22}) \end{pmatrix}$

$$\frac{1}{3}\delta_{jk}\Theta = \begin{pmatrix} \frac{1}{3}\Theta & & 0 \\ & \frac{1}{3}\Theta & \\ 0 & & \frac{1}{3}\Theta \end{pmatrix}$$

1 component

$$\sigma_{jk} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -(\sigma_{11}+\sigma_{22}) \end{pmatrix}$$

5 components

$$\omega_{jk} = \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{pmatrix}$$

$$\frac{1}{3}\delta_{jk}\Theta = \begin{pmatrix} \frac{1}{3}\Theta & & 0 \\ & \frac{1}{3}\Theta & \\ 0 & & \frac{1}{3}\Theta \end{pmatrix}$$

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5 components

$$W_{jk} = \begin{pmatrix} 0 & W_{12} & W_{13} \\ -W_{12} & 0 & W_{23} \\ -W_{13} & -W_{23} & 0 \end{pmatrix}$$

3 components

$$\boxed{\frac{d\xi^j}{dt} = B^j{}_k\, \xi^k \qquad B_{jk}(t) = V_{j,k}(t,\vec{x}_0)}$$

(evaluated at reference fluid element

matrix without symmetries → decompose it into irreducible pieces

$$\boxed{B_{jk} = \underbrace{\tfrac{1}{3}\delta_{jk}\Theta}_{\substack{\text{trace part} \\ \text{(expansion)}}} + \underbrace{\sigma_{jk}}_{\substack{\text{symmetric} \\ \text{trace-free} \\ \text{(shear)}}} + \underbrace{\omega_{jk}}_{\substack{\text{antisymmetric} \\ \text{(rotation)}}}}$$

$$\Theta \equiv \delta^{jk} B_{jk} \qquad \sigma_{jk} = B_{(jk)} - \tfrac{1}{3}\delta_{jk}\Theta \,, \quad \omega_{jk} \equiv B_{[jk]}$$

Expansion

$$\theta = \delta^{jk} V_{jk} = \vec{\nabla}\cdot\vec{V}$$ = fractional rate of expansion of volume of fluid elements

ω_{jk} ... $-\omega_{12}$ 0

$-\omega_{13}$ $-\omega_{23}$ 0

Expansion

$$\Theta = \delta^{jk} V_{j;k} = \vec{\nabla}\cdot\vec{V}$$ = fractional rate of expansion of volume of fluid elements

mass conservation :

$$\rho \equiv \frac{\delta m}{\delta V}$$ mass density

$$\rho\vec{V} \equiv$$ mass current density.

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

Expansion

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mass conservation: $\rho \equiv \dfrac{\delta m}{\delta V}$ mass density

$\rho \vec{v} \equiv$ mass current density.

$$\frac{\partial \rho}{\partial t} + \underbrace{\vec{\nabla} \cdot (\rho \vec{v})}_{\rho \vec{\nabla} \cdot \vec{v} + \vec{v} \cdot \vec{\nabla} \rho} = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho = - \rho \vec{\nabla} \cdot \vec{v}$$

σ_{12} σ_{13}

Expansion

$$\theta = \delta^{jk} V_{jk} = \vec{\nabla} \cdot \vec{V}$$ = fractional rate of expansion of volume of fluid elements

mass conservation :

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$$\rho \vec{V} \equiv \text{mass current density.}$$

$$\frac{\partial \rho}{\partial t} + \underbrace{\vec{\nabla} \cdot (\rho \vec{V})}_{\rho \vec{\nabla} \cdot \vec{V} + \vec{V} \cdot \vec{\nabla} \rho} = 0$$

$$\underbrace{\frac{\partial \rho}{\partial t} + \vec{V} \cdot \vec{\nabla} \rho}_{\frac{d\rho}{dt}} = - \rho \vec{\nabla} \cdot \vec{V}$$

σ_{12} σ_{13}

As we follow a given fluid element,

at t : $\rho(t, \vec{x})$

at $t+dt$: $\rho(t+dt, \vec{x}+d\vec{x})$

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at $t + dt$: $\rho(t+dt, \vec{x} + d\vec{x})$

total change $d\rho = \rho(t+dt, \vec{x}+d\vec{x}) - \rho(t, \vec{x}$

$= \frac{\partial \rho}{\partial t} dt + d\vec{x} \cdot \vec{\nabla} \rho + \cdots$

$\frac{\partial \rho}{}$

As we follow a given fluid element,

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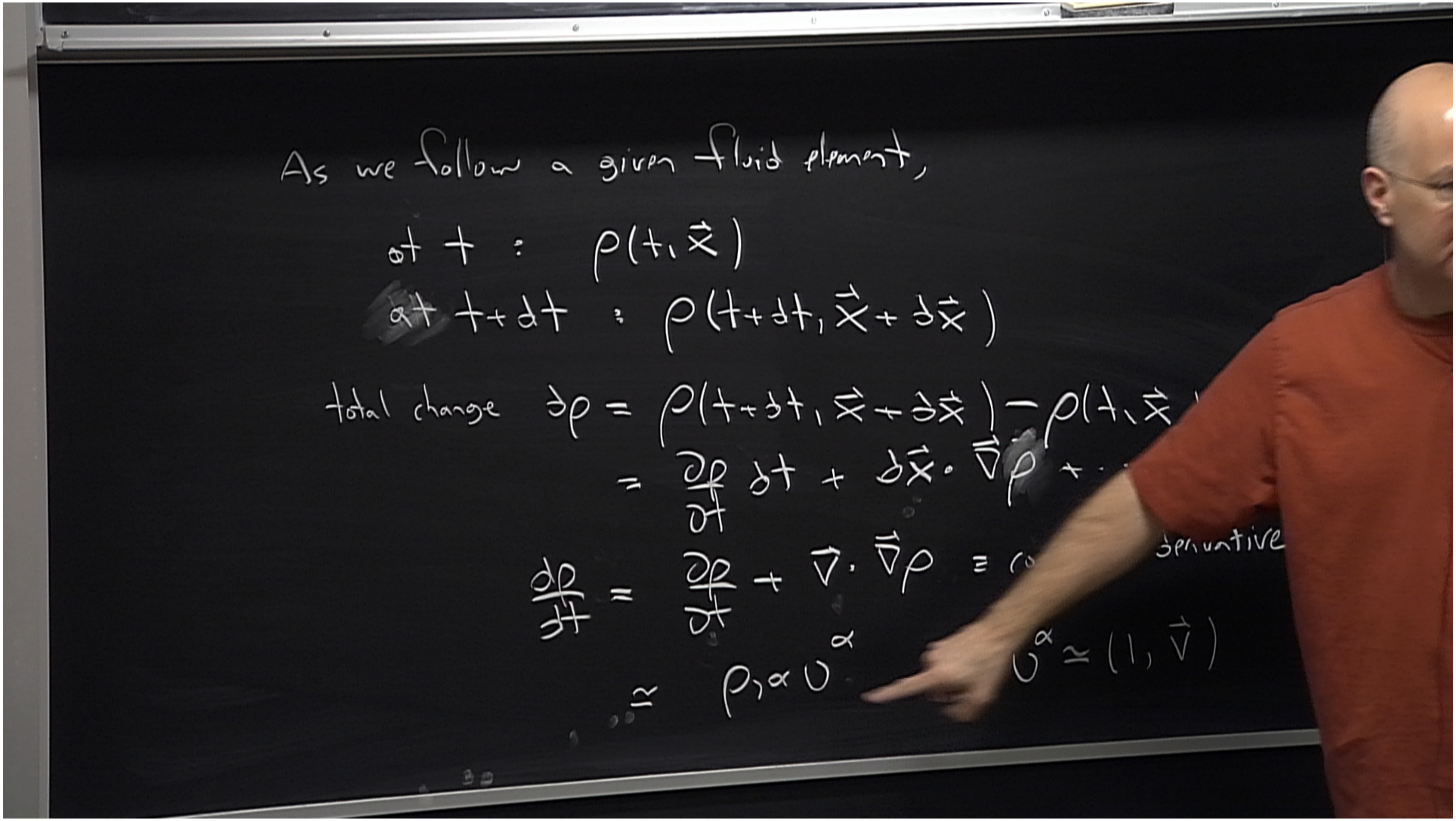
$$= \frac{\partial \rho}{\partial t}\, dt + d\vec{x} \cdot \vec{\nabla} \rho + \cdots$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho$$


As we follow a given fluid element,
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total change $d\rho = \rho(t+dt, \vec{x}+d\vec{x}) - \rho(t, \vec{x}$
$= \frac{\partial\rho}{\partial t} dt + d\vec{x} \cdot \vec{\nabla}\rho +$
$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \vec{v} \cdot \vec{\nabla}\rho =$
derivative
$\simeq \rho_{,\alpha} u^{\alpha}$
$u^{\alpha} \simeq (1, \vec{v})$

As we follow a given fluid element,

$$\text{at } t : \quad \rho(t, \vec{x})$$

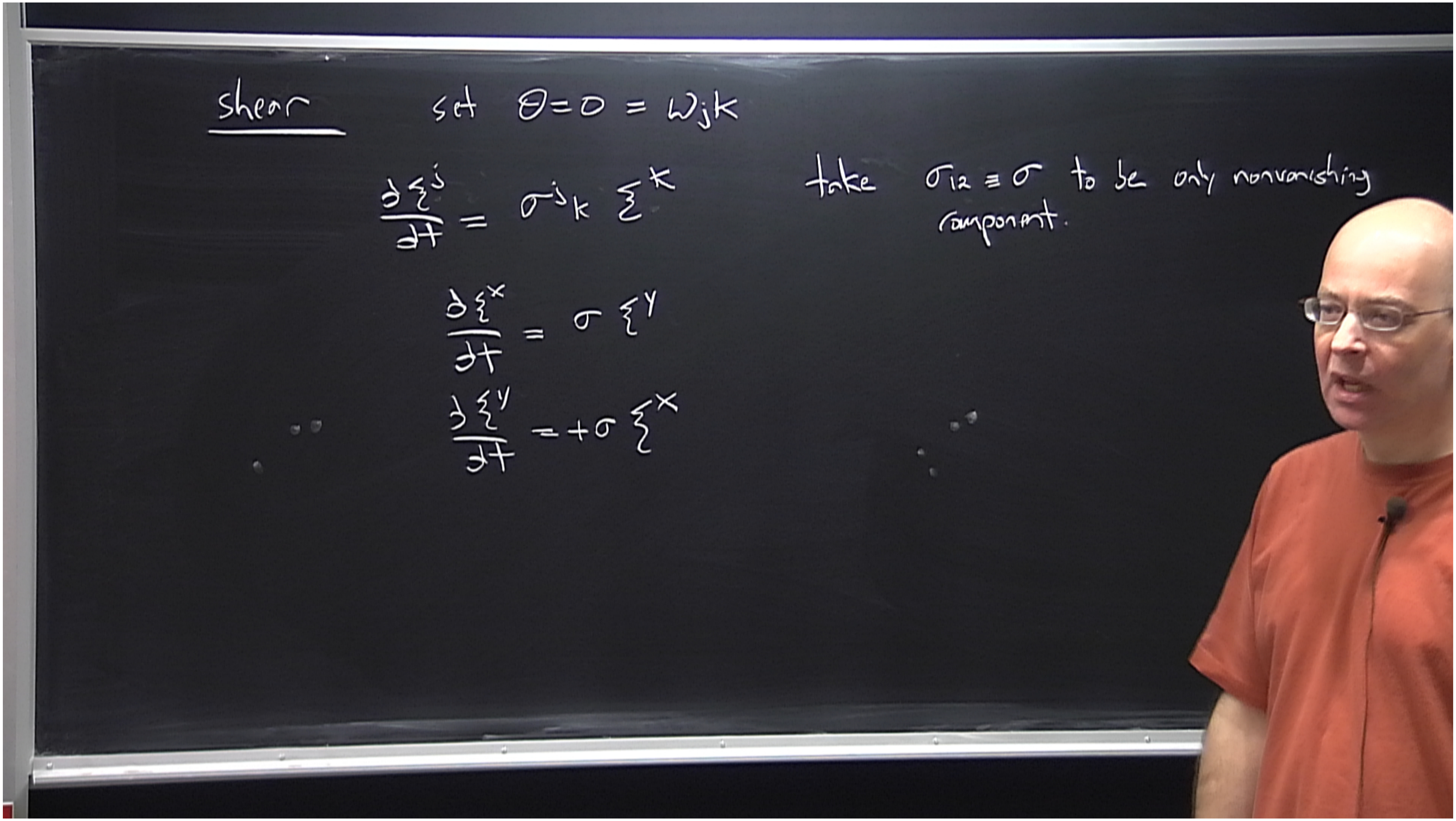
$$\text{at } t + dt : \quad \rho(t+dt, \vec{x} + d\vec{x})$$

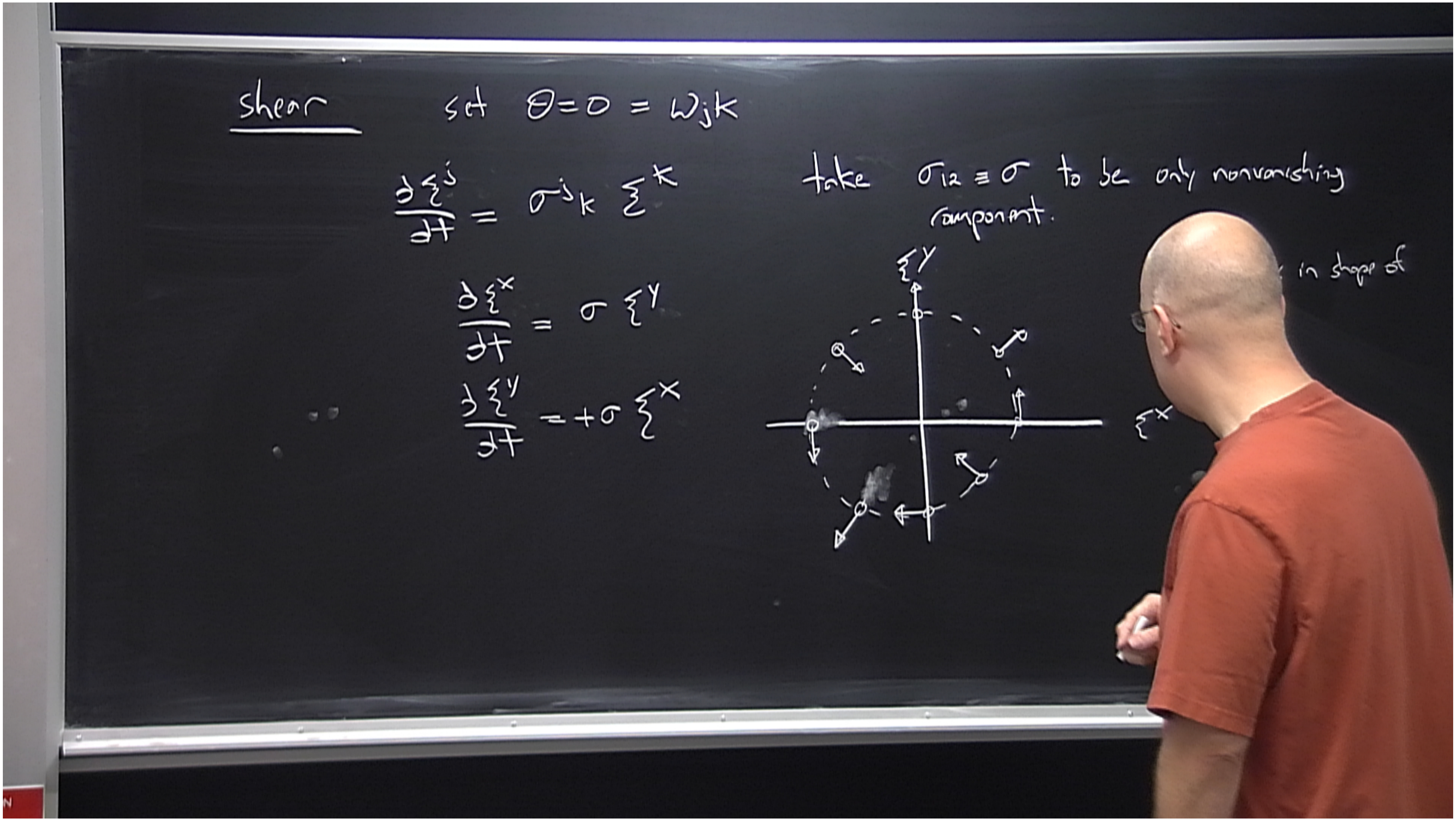
$$\text{total change} \quad d\rho = \rho(t+dt, \vec{x}+d\vec{x}) - \rho(t, \vec{x})$$

$$= \frac{\partial \rho}{\partial t} dt + d\vec{x} \cdot \vec{\nabla} \rho + \cdots$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho \equiv \text{convective derivative}$$

$$\simeq \rho_{,\alpha} u^{\alpha} \qquad u^{\alpha} \simeq (1, \vec{v})$$


shear
set $\theta = 0 = \omega_{jk}$
$\frac{\partial \xi^j}{\partial t} = \sigma^j{}_k \xi^k$
take $\sigma_{12} \equiv \sigma$ to be only nonvanishing component.
$\frac{\partial \xi^x}{\partial t} = \sigma \xi^y$
$\frac{\partial \xi^y}{\partial t} = +\sigma \xi^x$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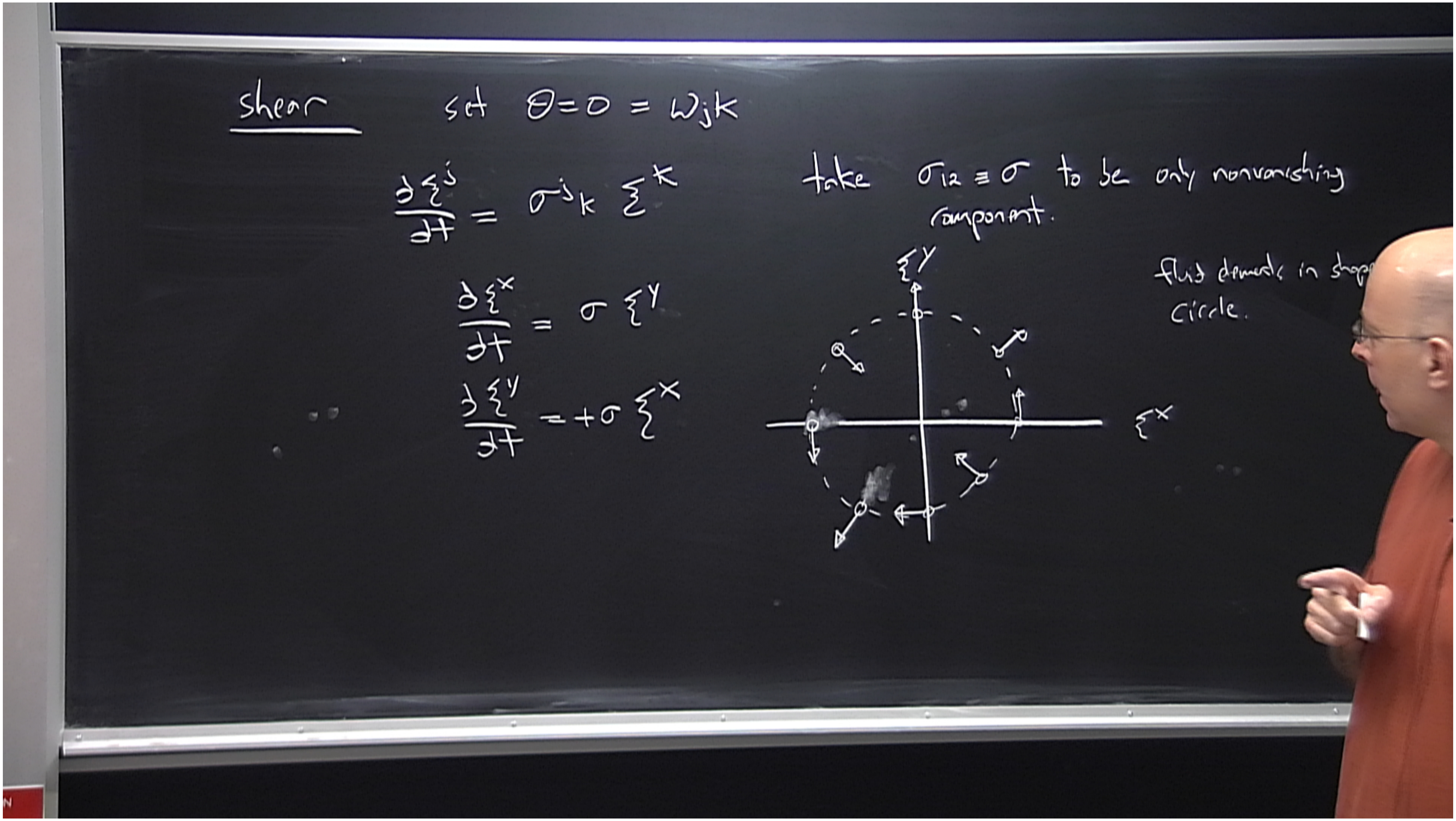
ξ^y
ξ^x
in shape of


shear
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$\frac{d\xi^j}{dt} = \sigma^j{}_k \xi^k$
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take $\sigma_{12} \equiv \sigma$ to be only nonvanishing component.
ξ^y
ξ^x
fluid elements in shape circle.

shear set $\theta = 0 = \omega_{jk}$

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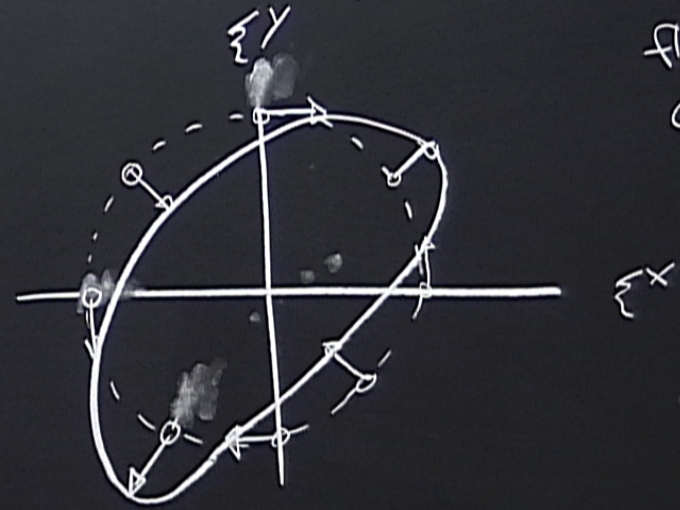

fluid elements in shape of circle.

shear

set $\theta = 0 = \omega$

$$\frac{d\xi^j}{dt} = \sigma^j{}_k\, \xi^k$$

take $\sigma_{12} \equiv \sigma$ to be only nonvanishing component.

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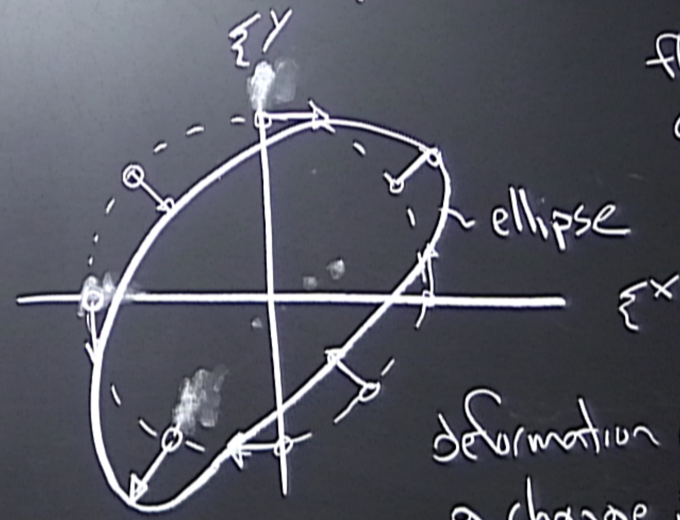
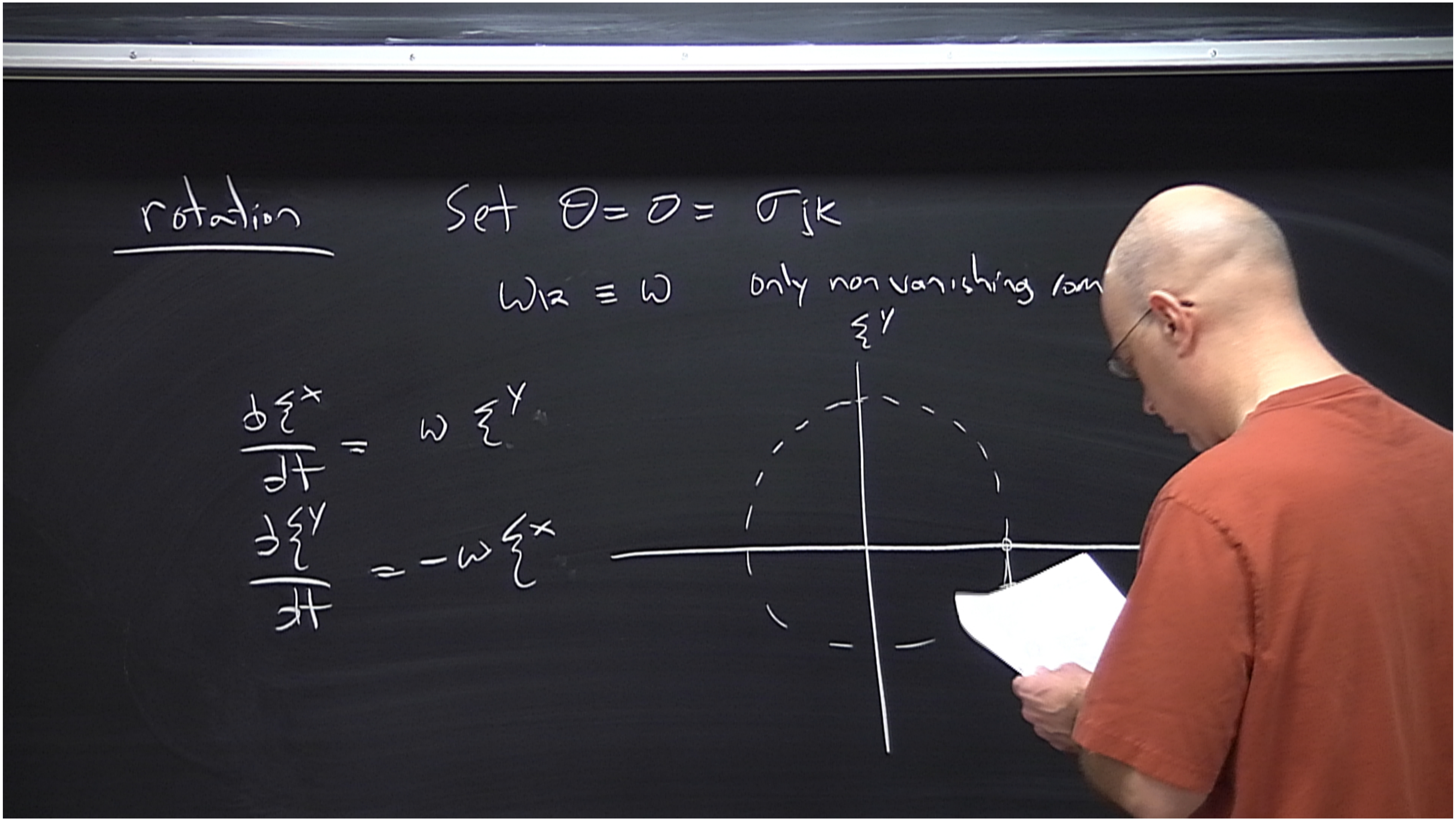
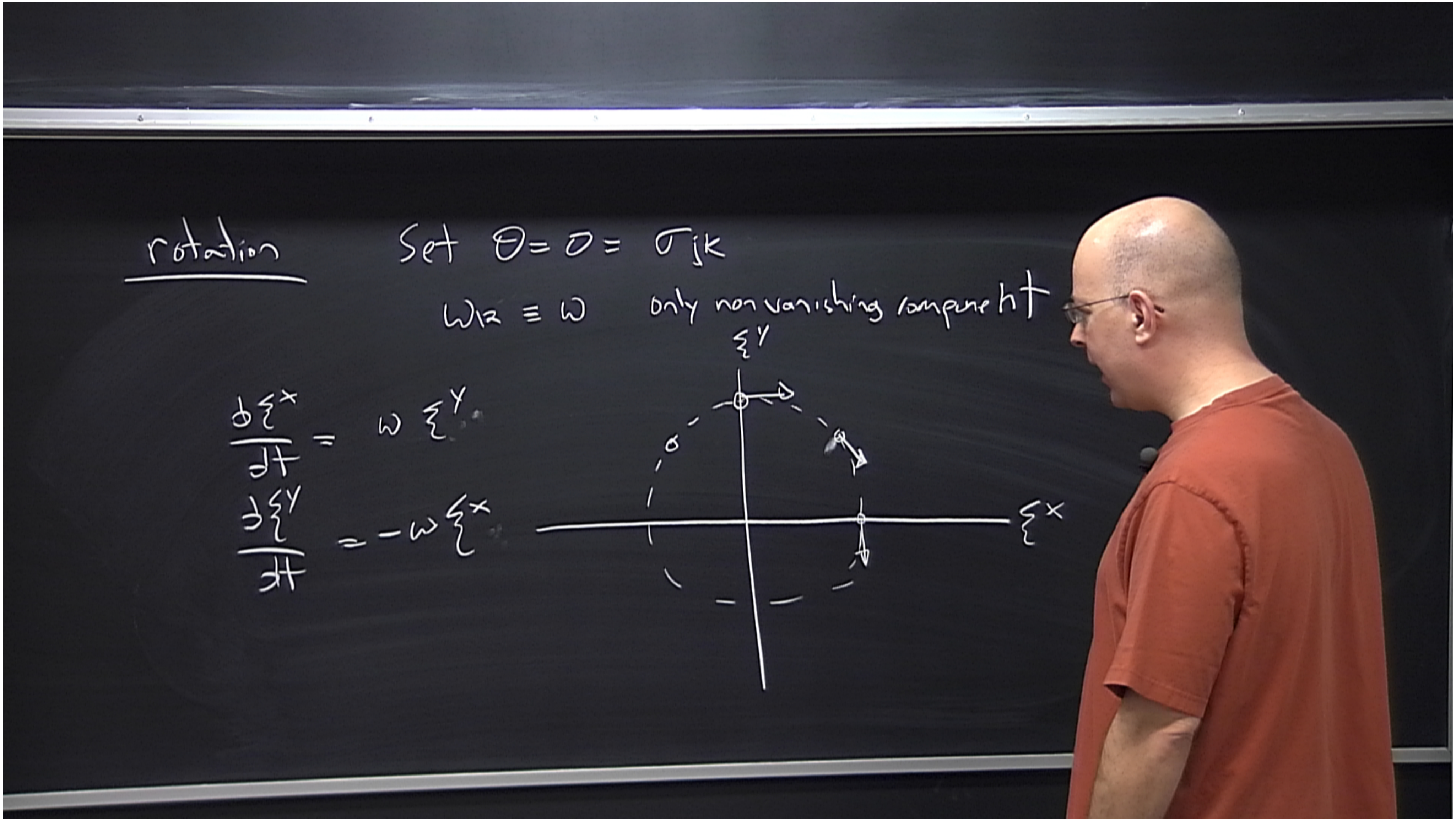


fluid elements in shape of circle.

deformation of shape, without a change in volume.


rotation
Set $\theta = \sigma = \sigma_{jk}$
$\omega_{12} \equiv \omega$
only nonvanishing com
ξ^y
$\frac{d\xi^x}{dt} = \omega \xi^y$
$\frac{d\xi^y}{dt} = -\omega \xi^x$


rotation
Set $\theta = 0 = \sigma_{jk}$
$\omega_{12} \equiv \omega$ only nonvanishing component
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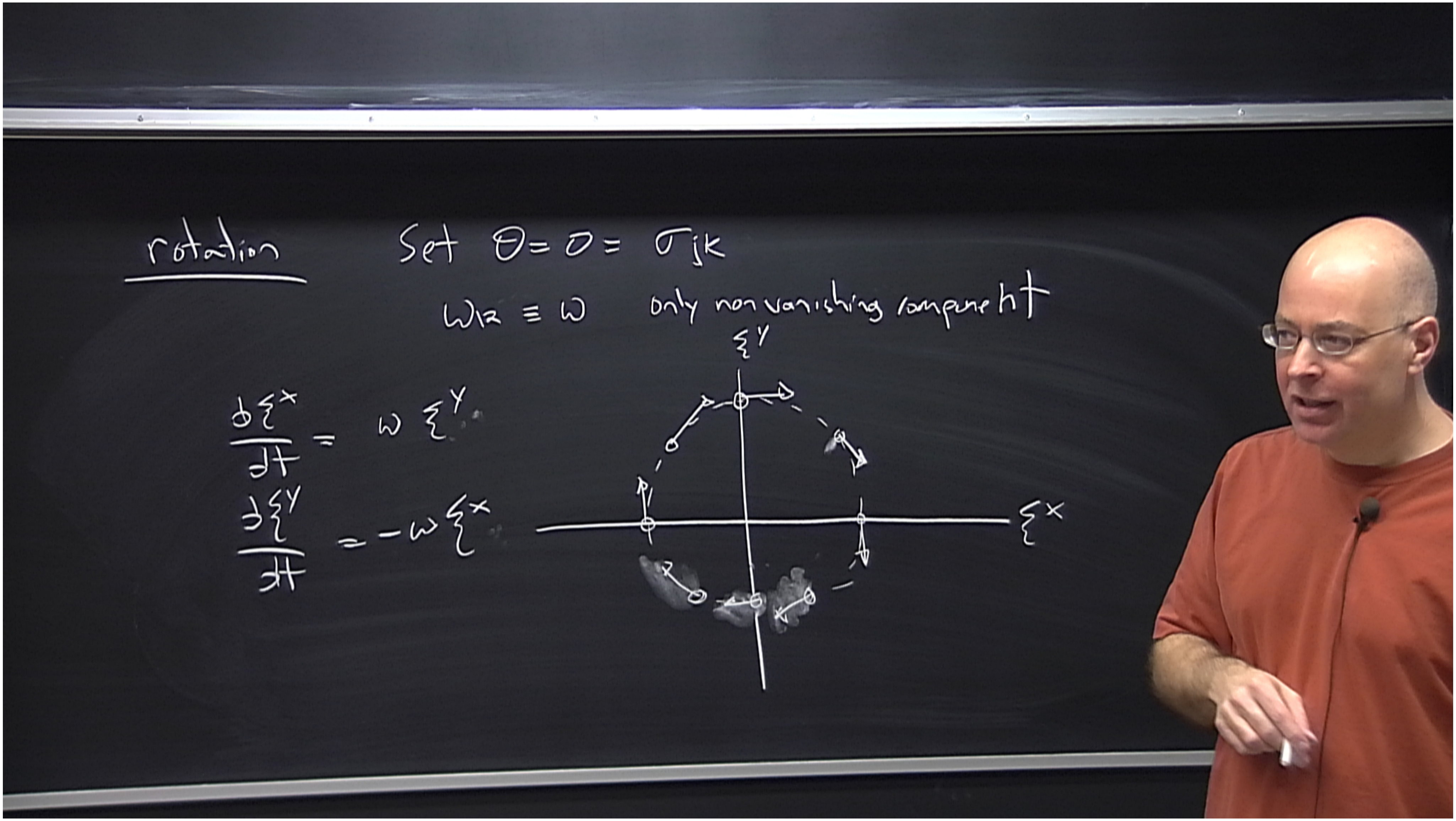
ξ^y
ξ^x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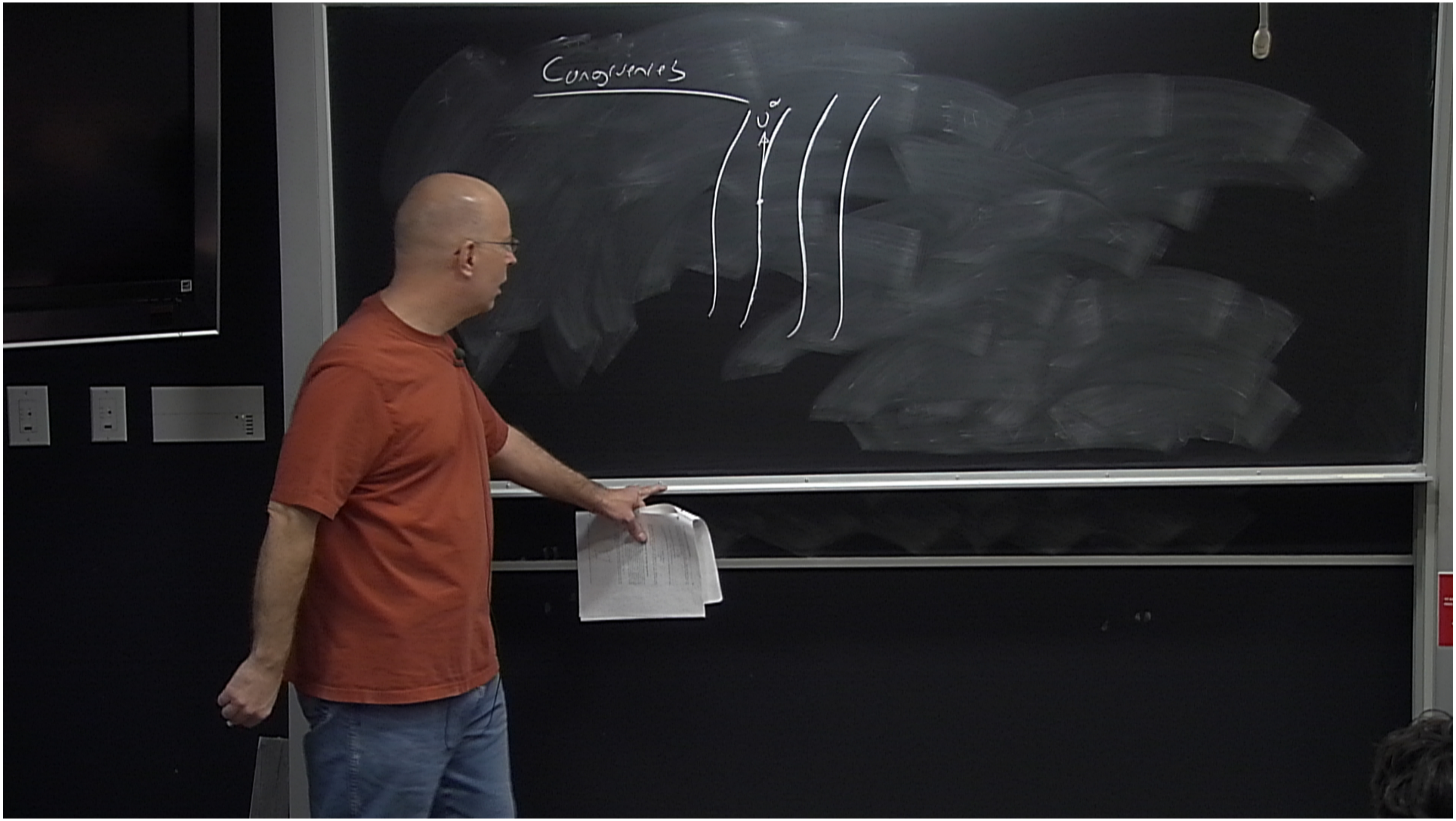
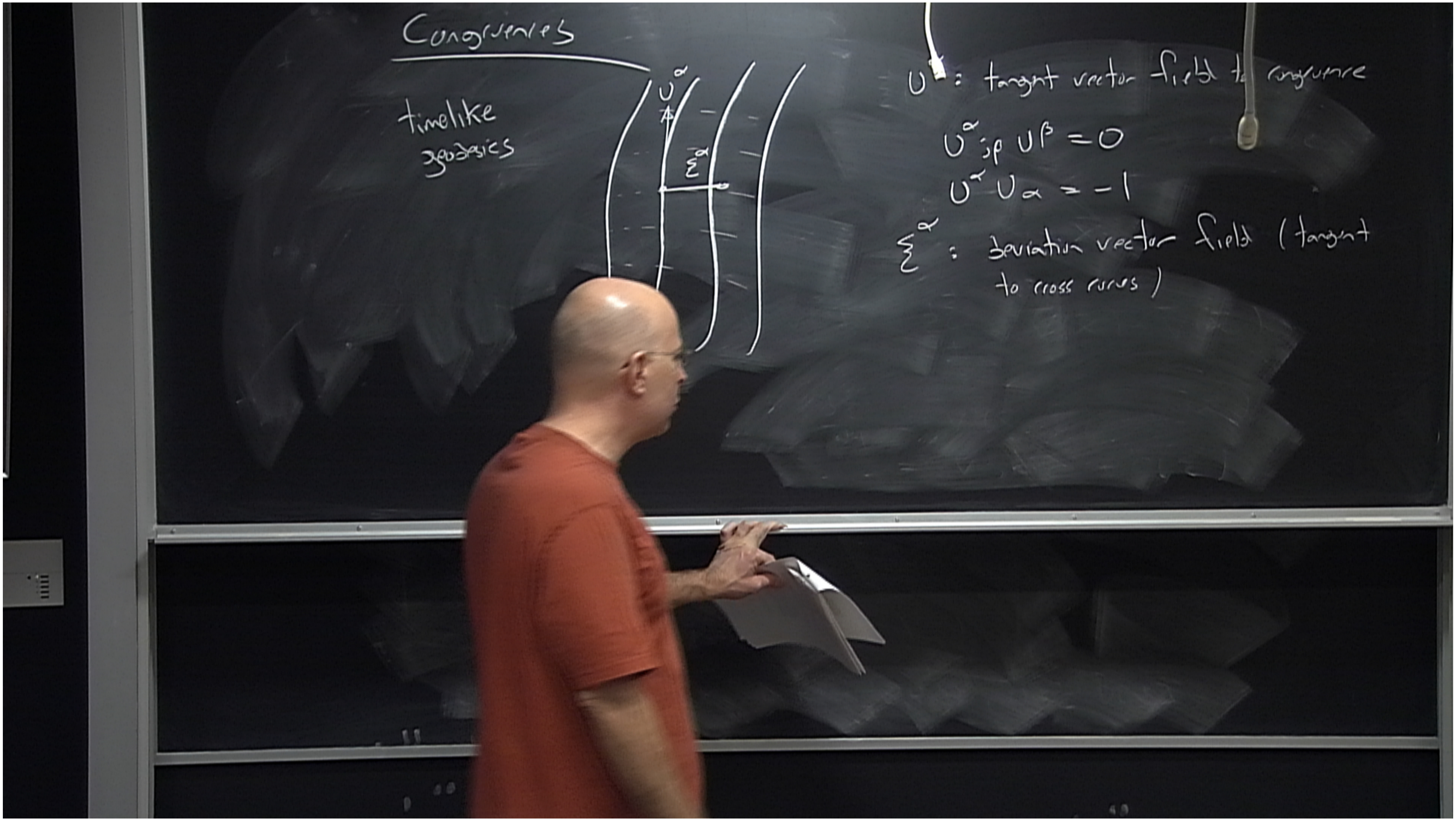
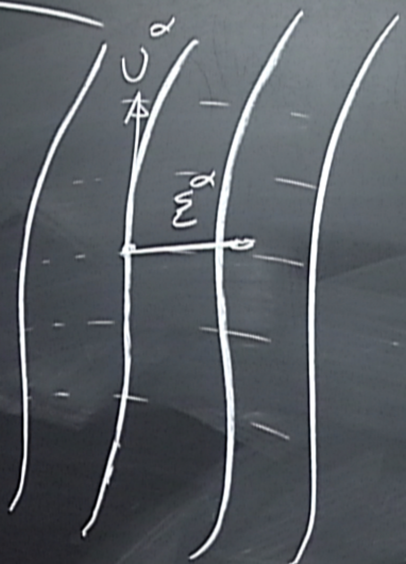
ξ^y
ξ^x


Congruences


Congruences
timelike geodesics
u^α
ξ^α
u^α : tangent vector field to congruence
$u^\alpha{}_{;\beta} u^\beta = 0$
$u^\alpha u_\alpha = -1$
ξ^α : deviation vector field (tangent to cross curves)

Congruences

timelike geodesics



u^α : tangent vector field to congruence

$$u^\alpha{}_{;\beta}\, u^\beta = 0$$

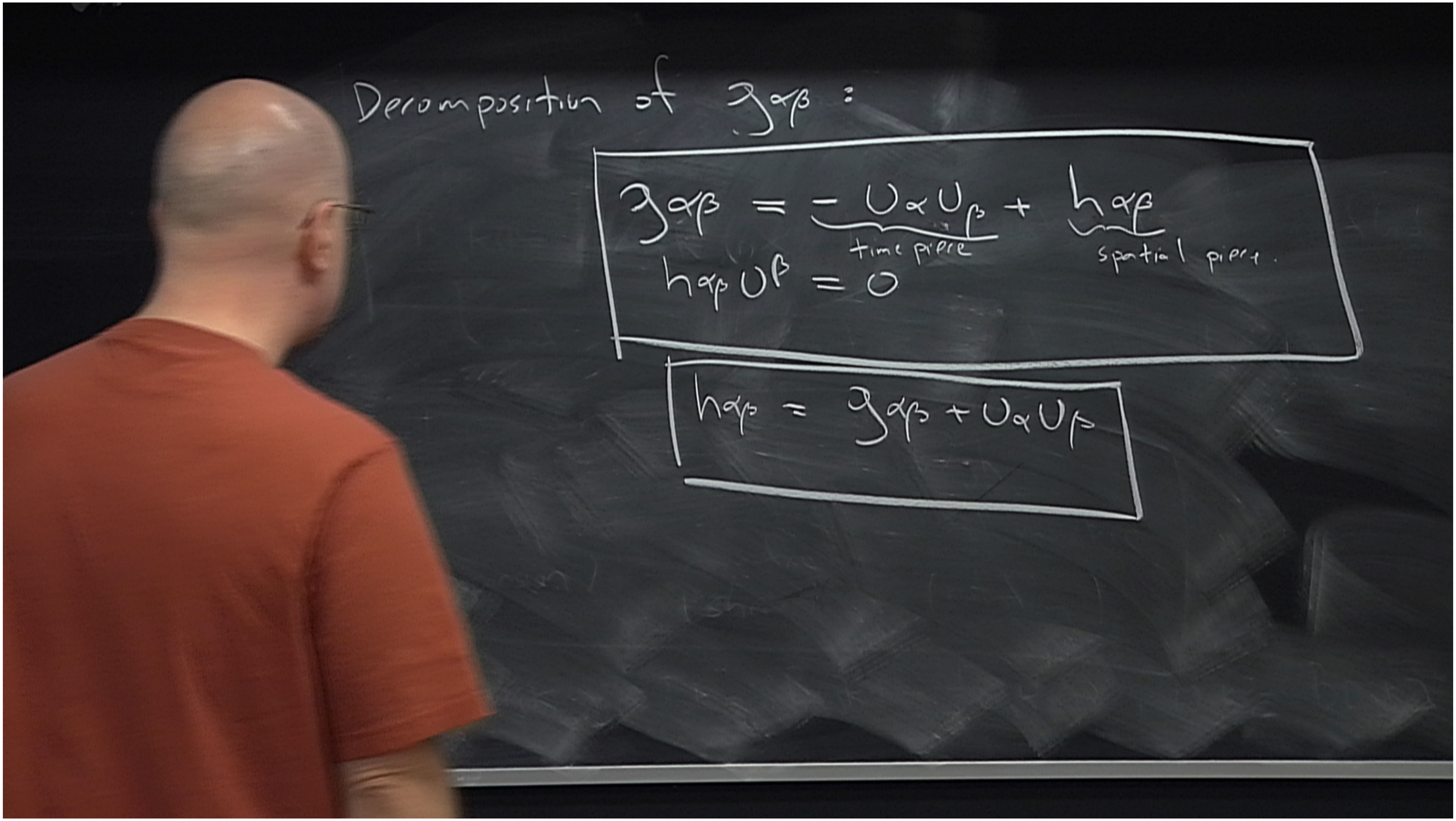
$$u^\alpha u_\alpha = -1$$

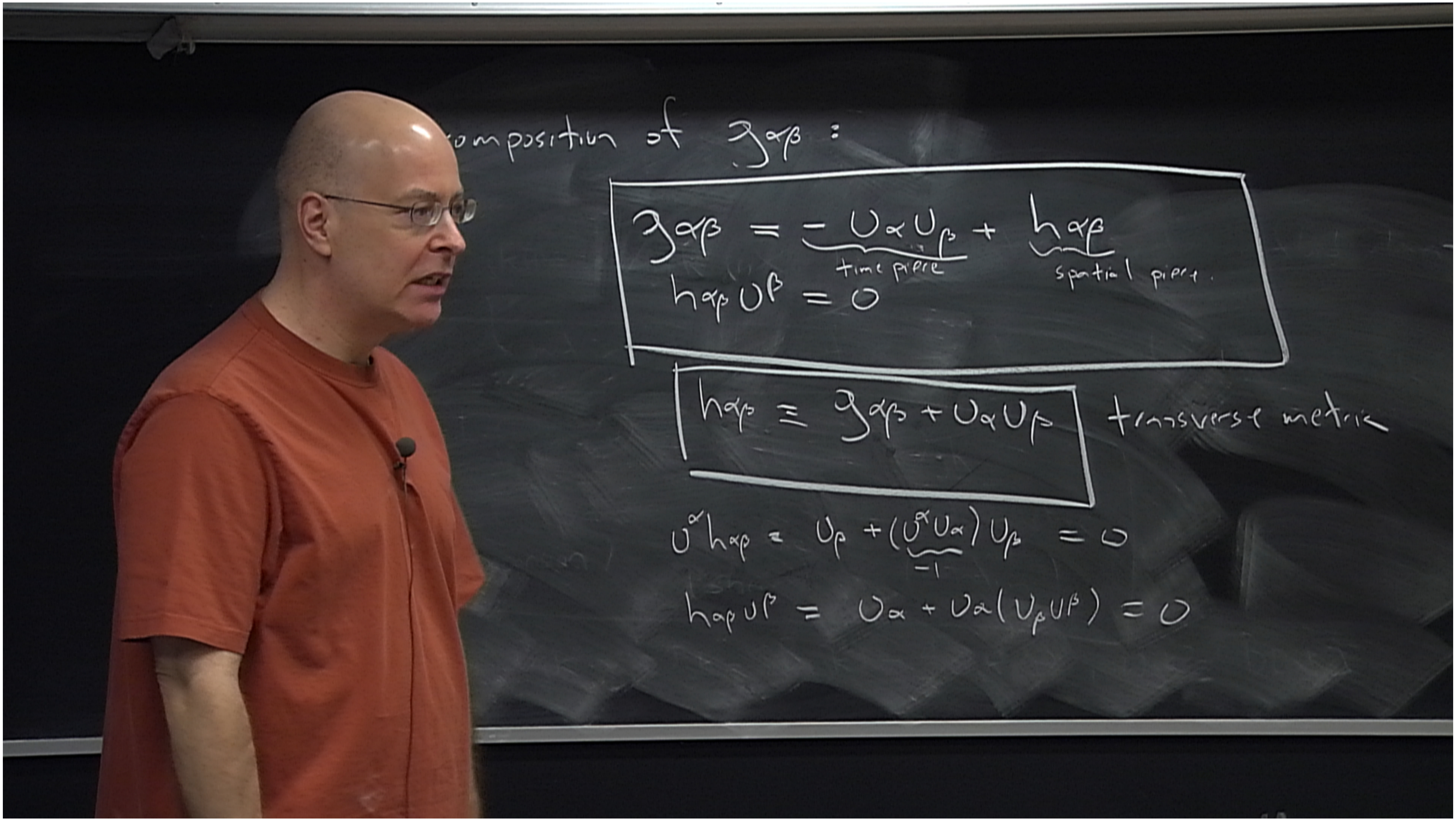
ξ^α : deviation vector field (tangent to cross curves)

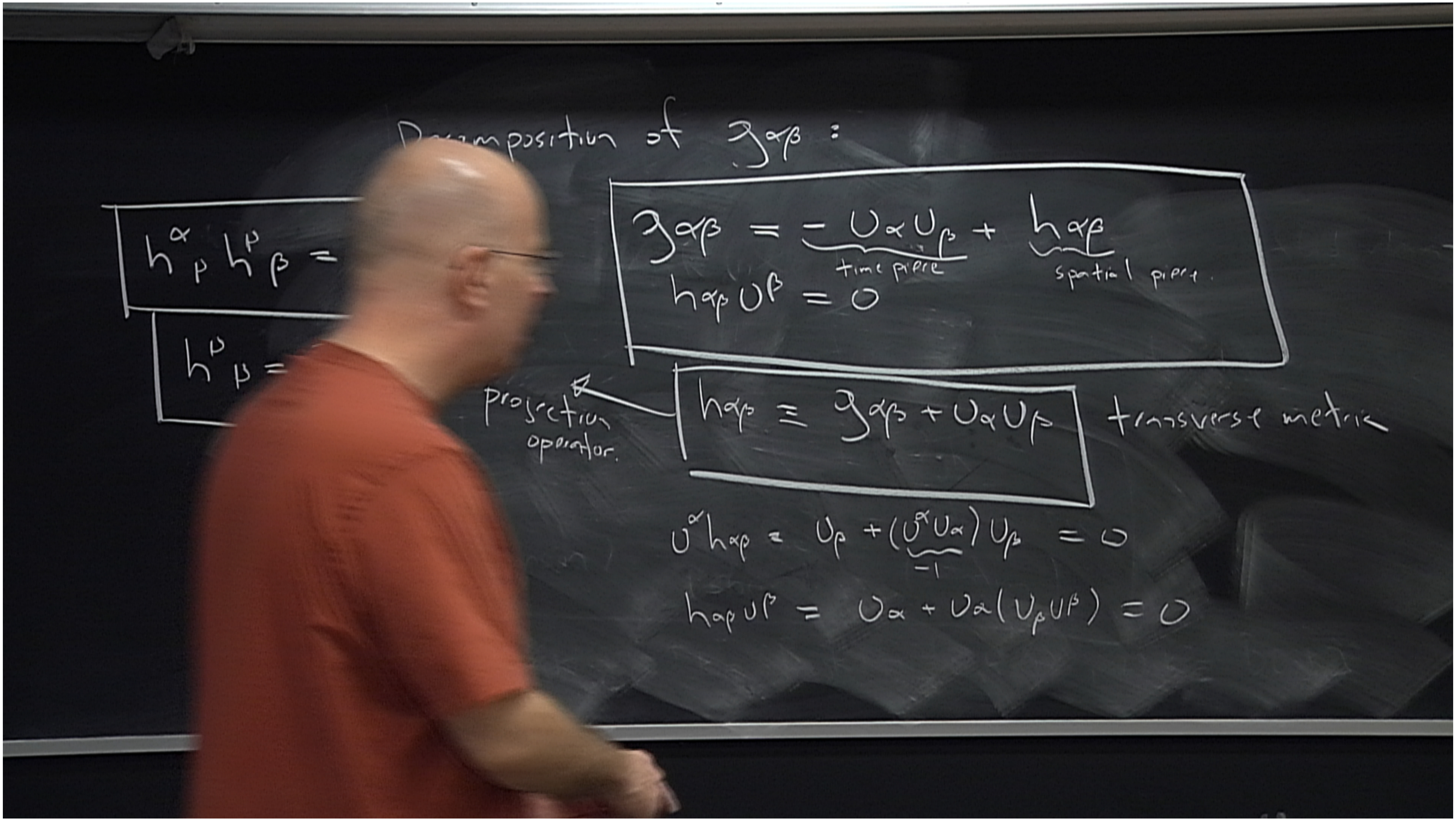
$$\mathcal{L}_u \xi^\alpha = \mathcal{L}_\xi u^\alpha$$

$$\Rightarrow \quad \xi^\alpha{}_{;\beta}\, u^\beta = u^\alpha{}_{;\beta}\, \xi^\beta$$

$$\xi^\alpha u_\alpha = 0$$


Decomposition of $g_{\alpha\beta}$:
$g_{\alpha\beta} = \underbrace{-U_\alpha U_\beta}_{\text{time piece}} + \underbrace{h_{\alpha\beta}}_{\text{spatial piece.}}$
$h_{\alpha\beta} U^\beta = 0$
$h_{\alpha\beta} = g_{\alpha\beta} + U_\alpha U_\beta$


omposition of $g_{\alpha\beta}$:
$g_{\alpha\beta} = \underbrace{-U_\alpha U_\beta}_{\text{time piece}} + \underbrace{h_{\alpha\beta}}_{\text{spatial piece.}}$
$h_{\alpha\beta}U^\beta = 0$
$h_{\alpha\beta} \equiv g_{\alpha\beta} + U_\alpha U_\beta$
transverse metric
$U^\alpha h_{\alpha\beta} = U_\beta + (\underbrace{U^\alpha U_\alpha}_{-1}) U_\beta = 0$
$h_{\alpha\beta}U^\beta = U_\alpha + U_\alpha(U_\beta U^\beta) = 0$



of $g_{\alpha\beta}$:
$g_{\alpha\beta} = \underbrace{-U_\alpha U_\beta}_{\text{time piece}} + \underbrace{h_{\alpha\beta}}_{\text{spatial piece}}$
$h_{\alpha\beta} U^\beta = 0$
$h_{\alpha\beta} \equiv g_{\alpha\beta} + U_\alpha U_\beta$
transverse metric
projection operator.
$U^\alpha h_{\alpha\beta} = U_\beta + (\underbrace{U^\alpha U_\alpha}_{-1}) U_\beta = 0$
$h_{\alpha\beta} U^\beta = U_\alpha + U_\alpha (U_\beta U^\beta) = 0$

$\xi^\alpha{}_{;\beta} U^\beta = U^\alpha{}_{;\beta} \xi^\beta$

…nsverse metric

$= 0$

In a local Lorentz frame momentarily comoving with reference geodesic,

$$U^\alpha \stackrel{*}{=} (1,0,0,0)$$

$$g_{\alpha\beta} \stackrel{*}{=} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix} \qquad h_{\alpha\beta} \stackrel{*}{=} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{h_{\alpha\beta} = g_{\alpha\beta} + U_\alpha U_\beta}$$

Behaviour of a neighbouring geodesic, relative to reference geodesic, is captured by

$$\xi^{\alpha}{}_{;\beta}\, u^{\beta} =$$

Behaviour of a neighbouring geodesic, relative to reference geodesic, is captured by

$$\boxed{\begin{gathered} \xi^{\alpha}{}_{;\beta}\, u^{\beta} = B^{\alpha}{}_{\beta}\, \xi^{\beta} \\ B_{\alpha\beta} \equiv u_{\alpha;\beta} \end{gathered}}$$

Behaviour of a neighbouring geodesic, relative to reference geodesic, is captured by

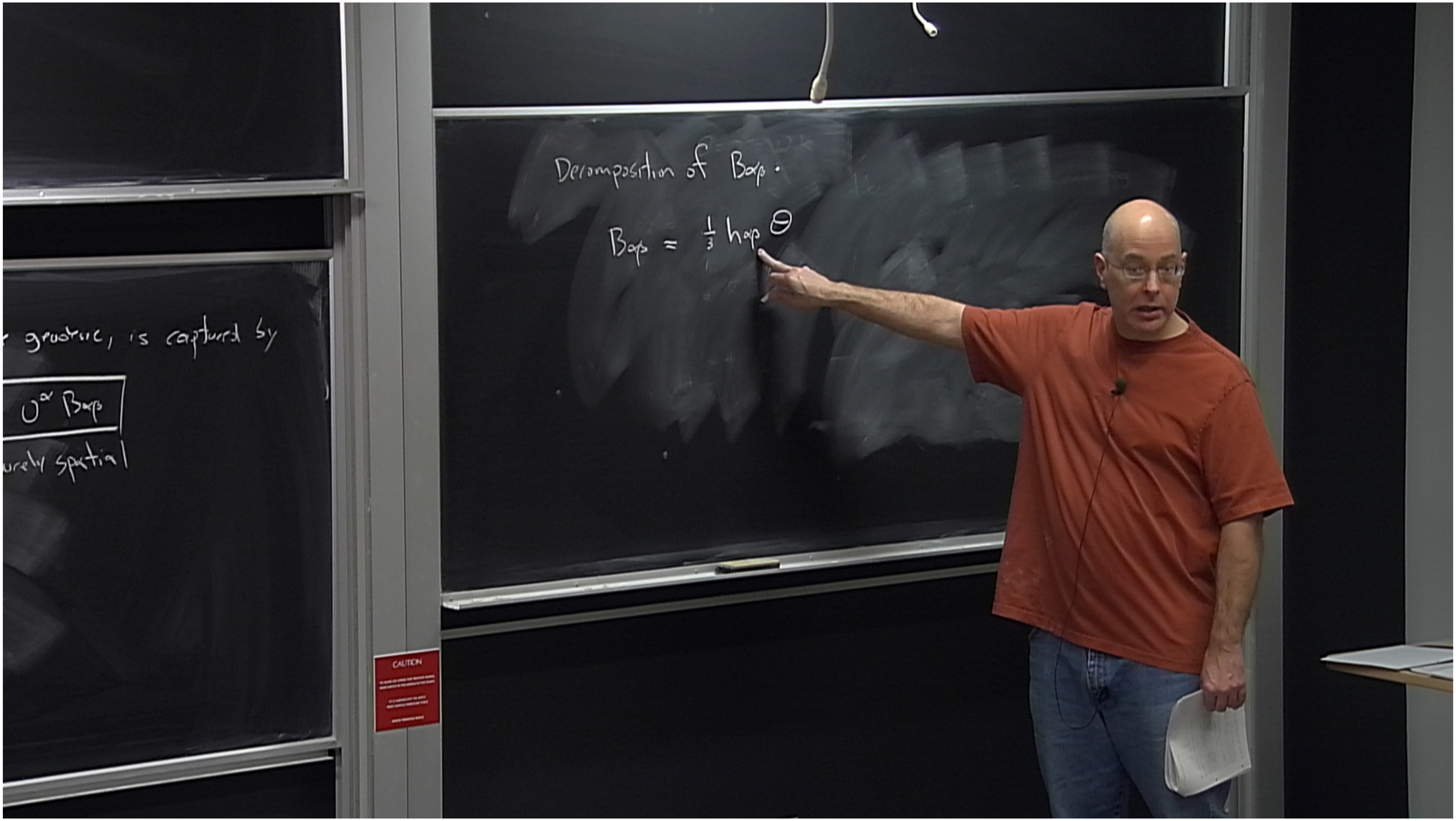
$$\boxed{\begin{aligned} \xi^{\alpha}{}_{;\beta} U^{\beta} &= B^{\alpha}{}_{\beta}\, \xi^{\beta} \\ B_{\alpha\beta} &\equiv U_{\alpha;\beta} \end{aligned}}$$

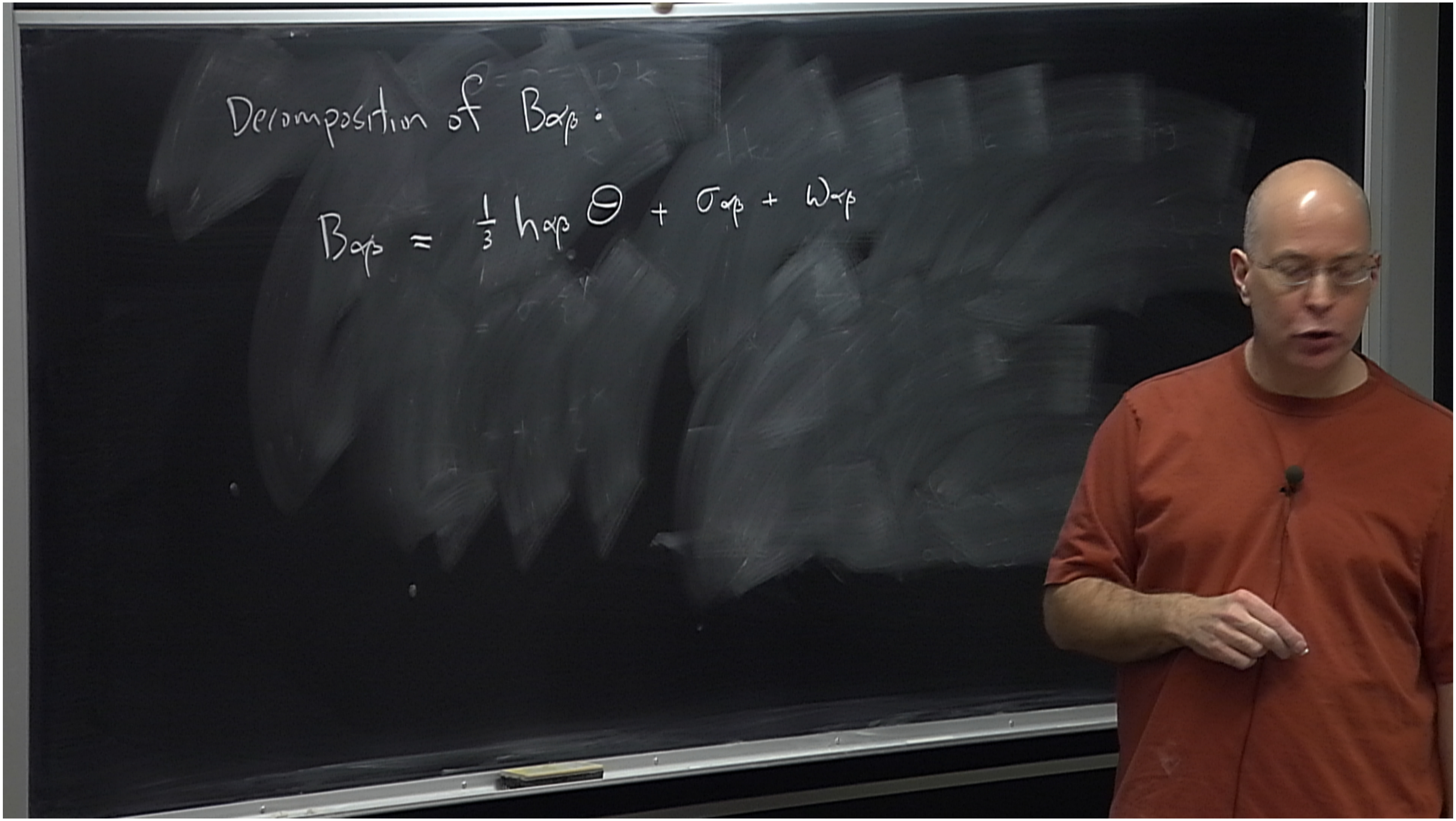
$$\boxed{B_{\alpha\beta} U^{\beta} = 0 = U^{\alpha} B_{\alpha\beta}}$$

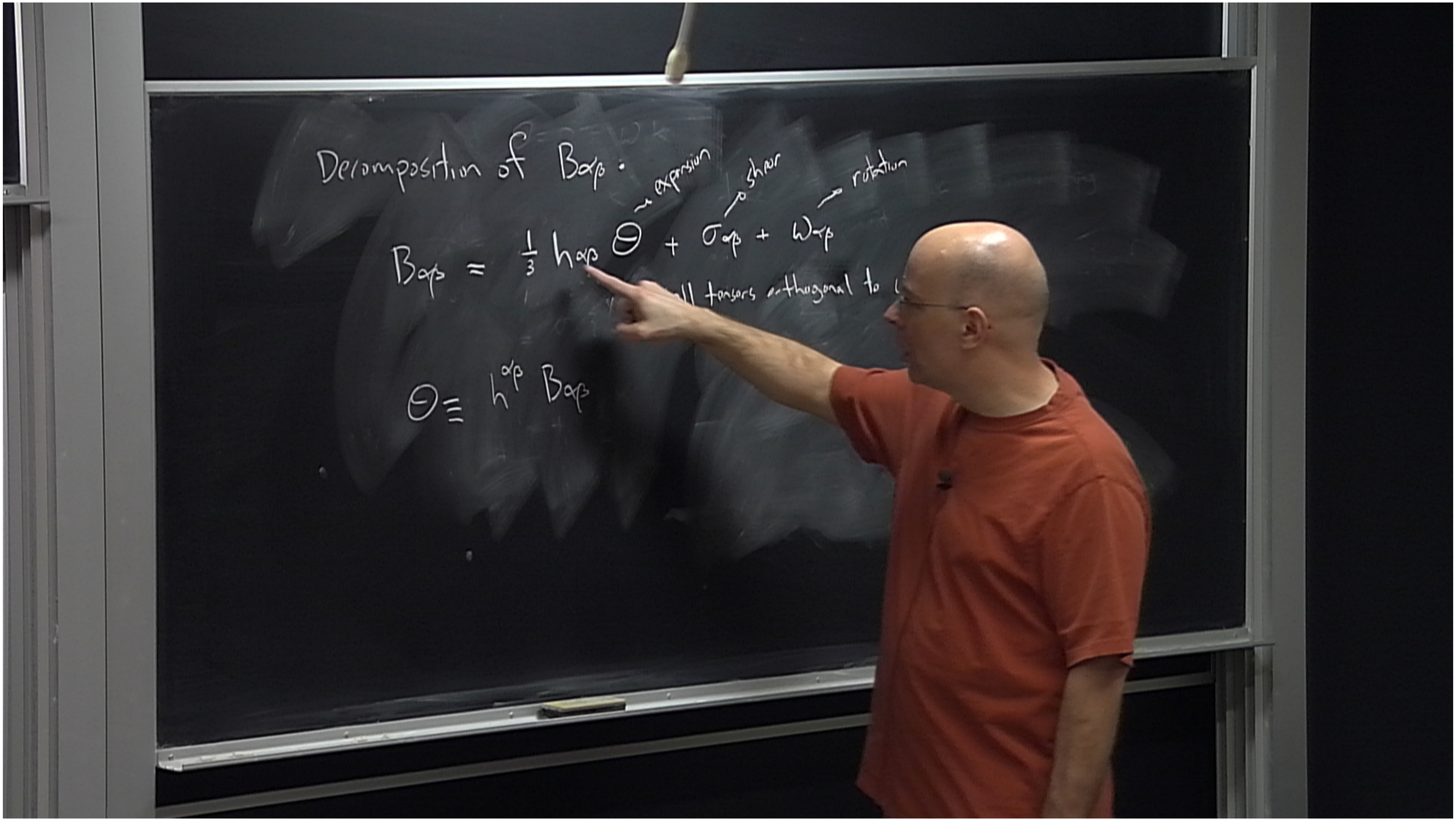
$B_{\alpha\beta}$ is purely spatial

$$B_{\alpha\beta} U^{\beta} = U_{\alpha;\beta} U^{\beta} = 0$$

$$U^{\alpha} B_{\alpha\beta} = U^{\alpha} U_{\alpha;\beta} = \tfrac{1}{2}\left(U^{\alpha} U_{\alpha}\right)_{;\beta} = 0$$


Decomposition of $B_{\alpha\beta}$:
$B_{\alpha\beta} = \frac{1}{3} h_{\alpha\beta}\,\theta$
geodesic, is captured by
$U^\alpha B_{\alpha\beta}$
spatial
CAUTION


Decomposition of $B_{\alpha\beta}$:
$B_{\alpha\beta} = \frac{1}{3} h_{\alpha\beta} \Theta + \sigma_{\alpha\beta} + \omega_{\alpha\beta}$


Decomposition of $B_{\alpha\beta}$:
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shear
rotation
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(all tensors orthogonal to u^α)

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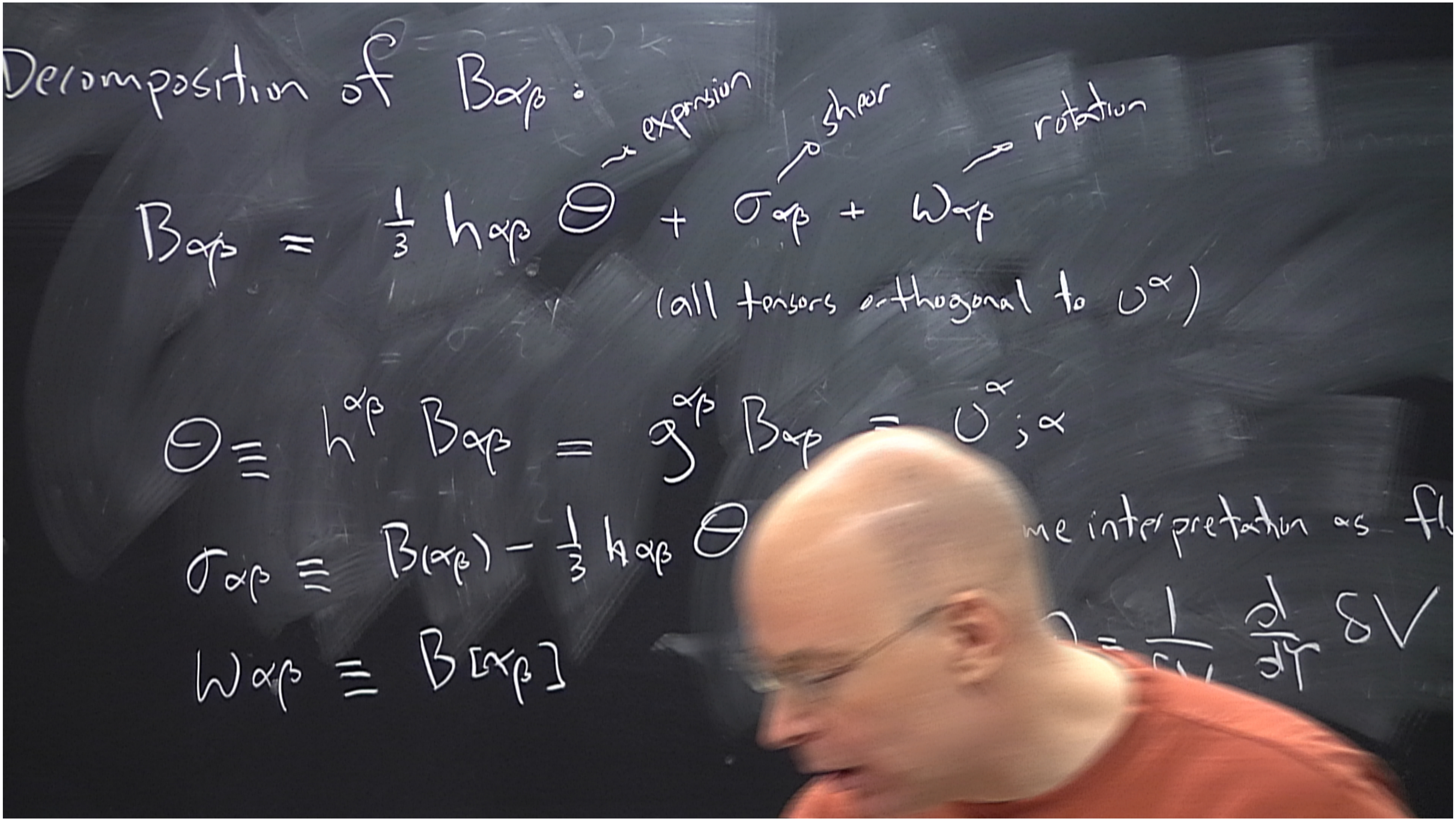
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δV

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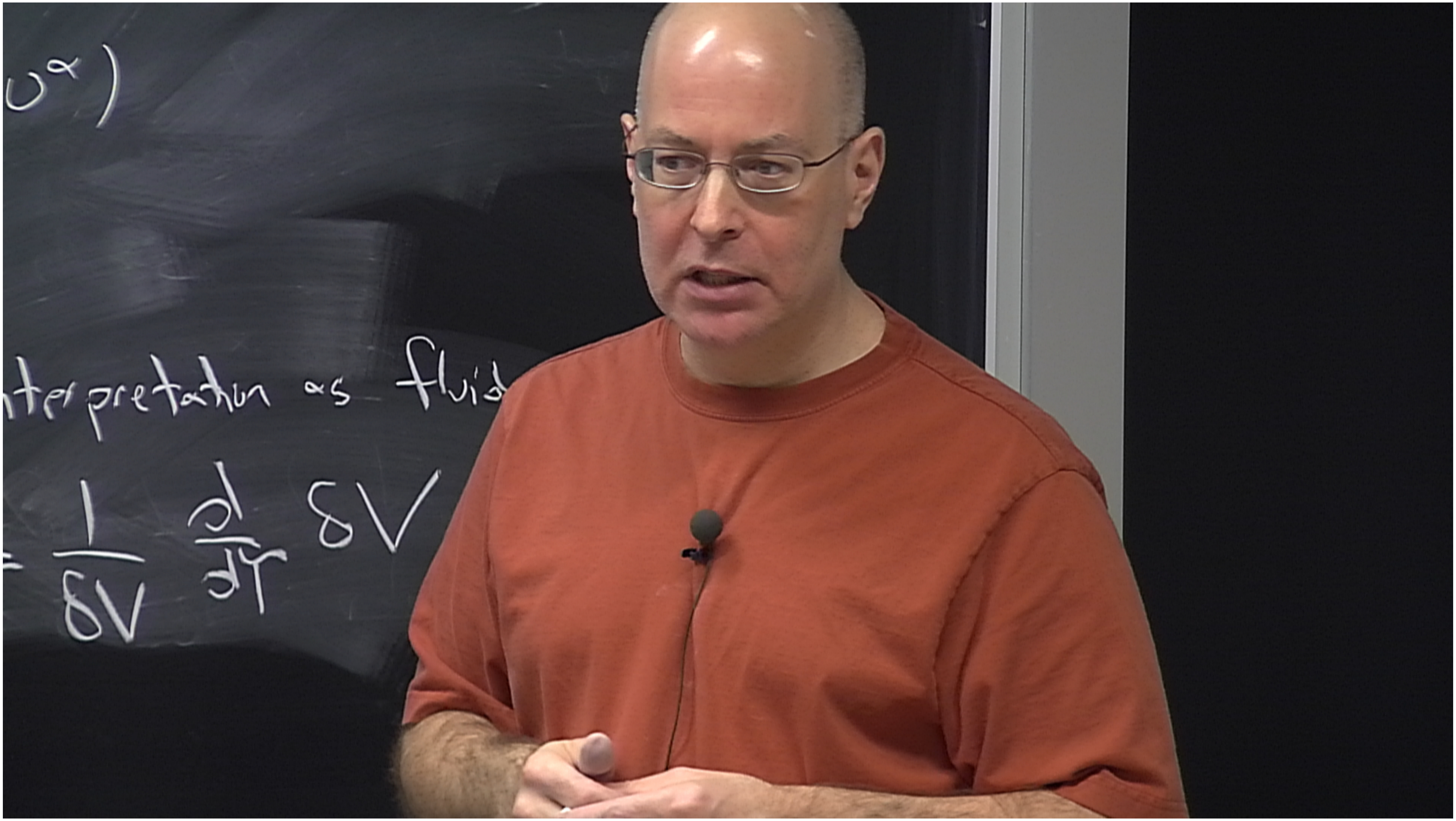

HW 1

1.13 : #3, #6, #9
2.5 : #1, #3

Feb 8