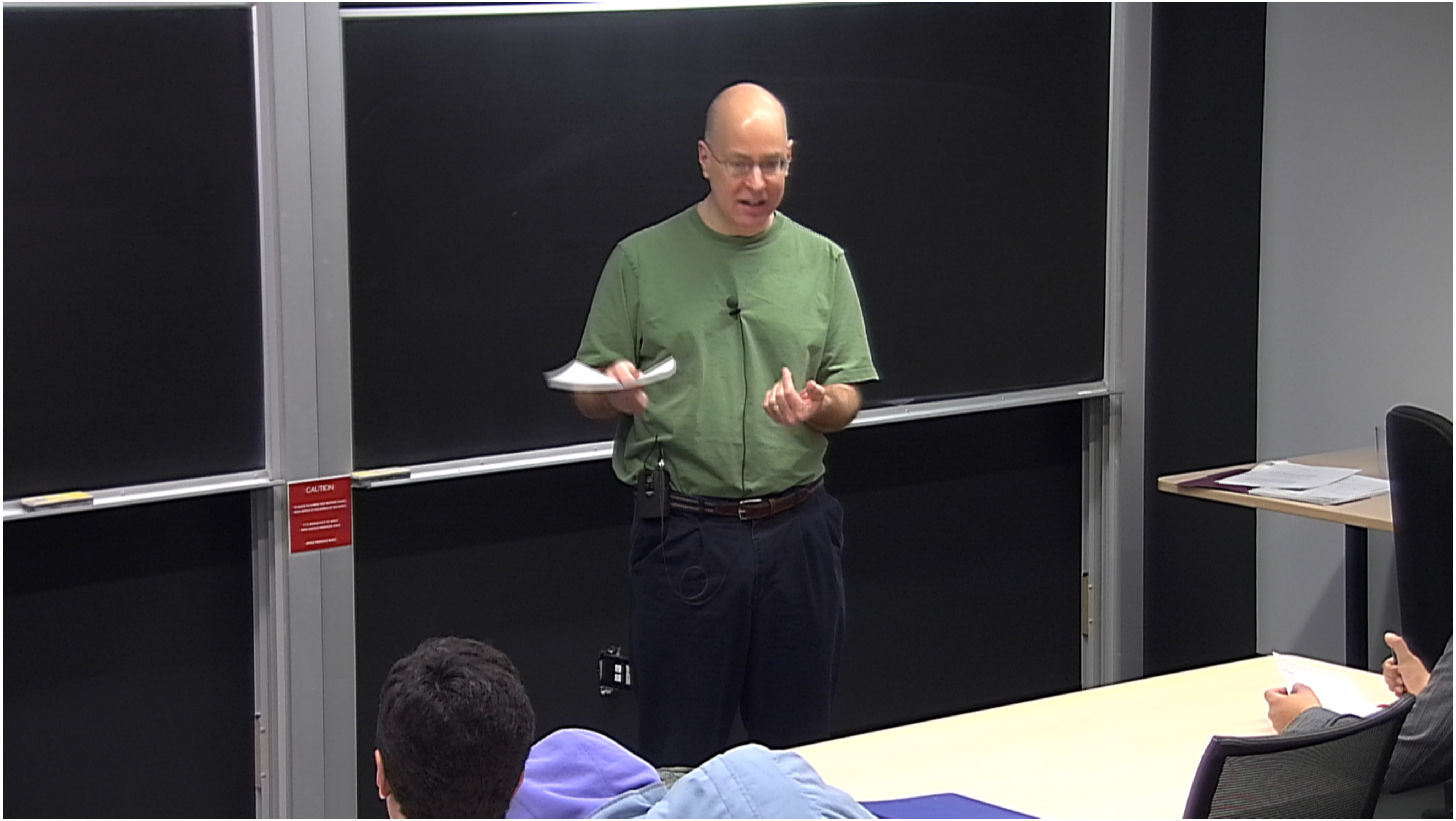
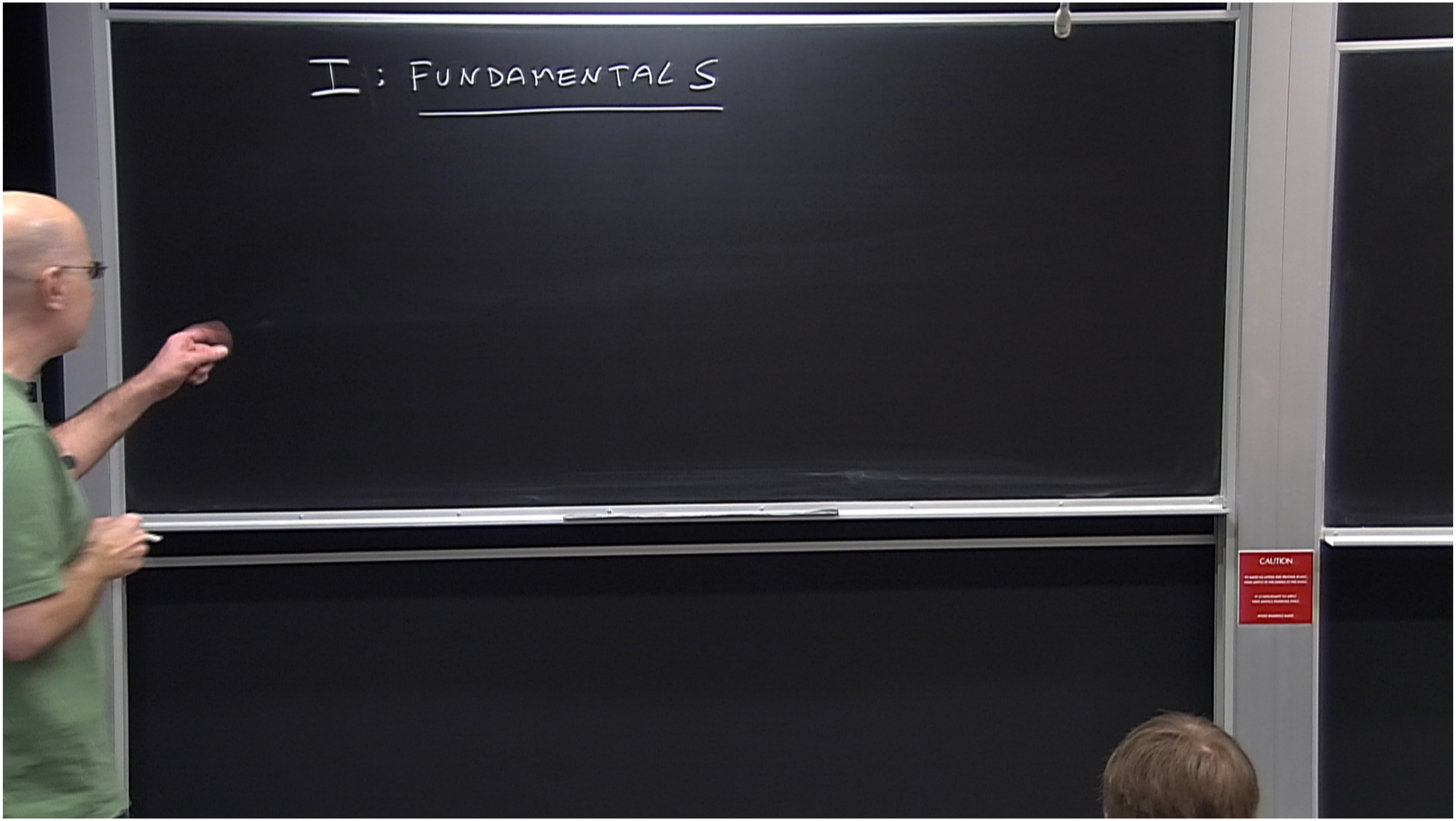
Title: Advanced General Relativity - Lecture 1

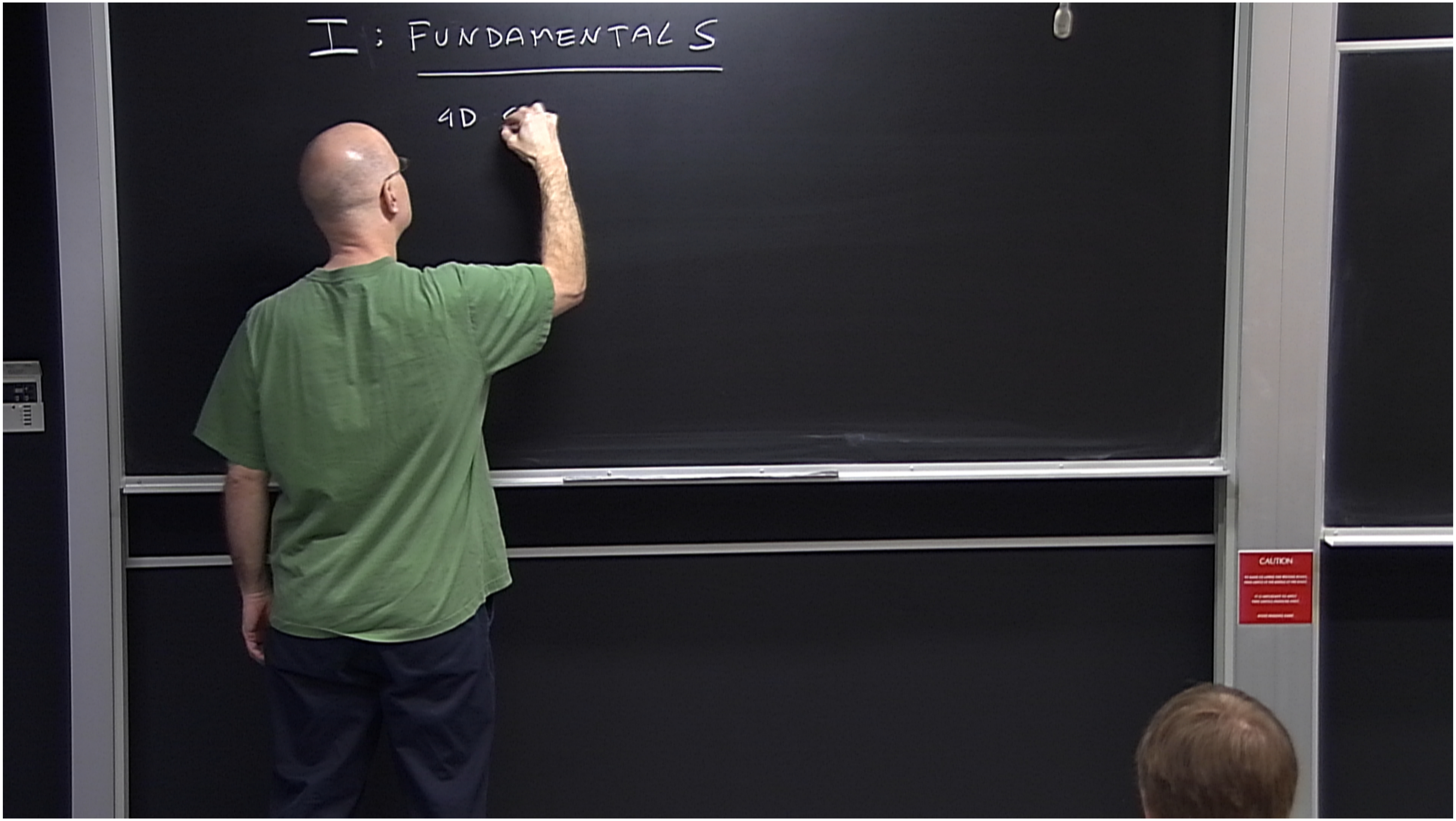
Date: Jan 11, 2012 10:00 AM

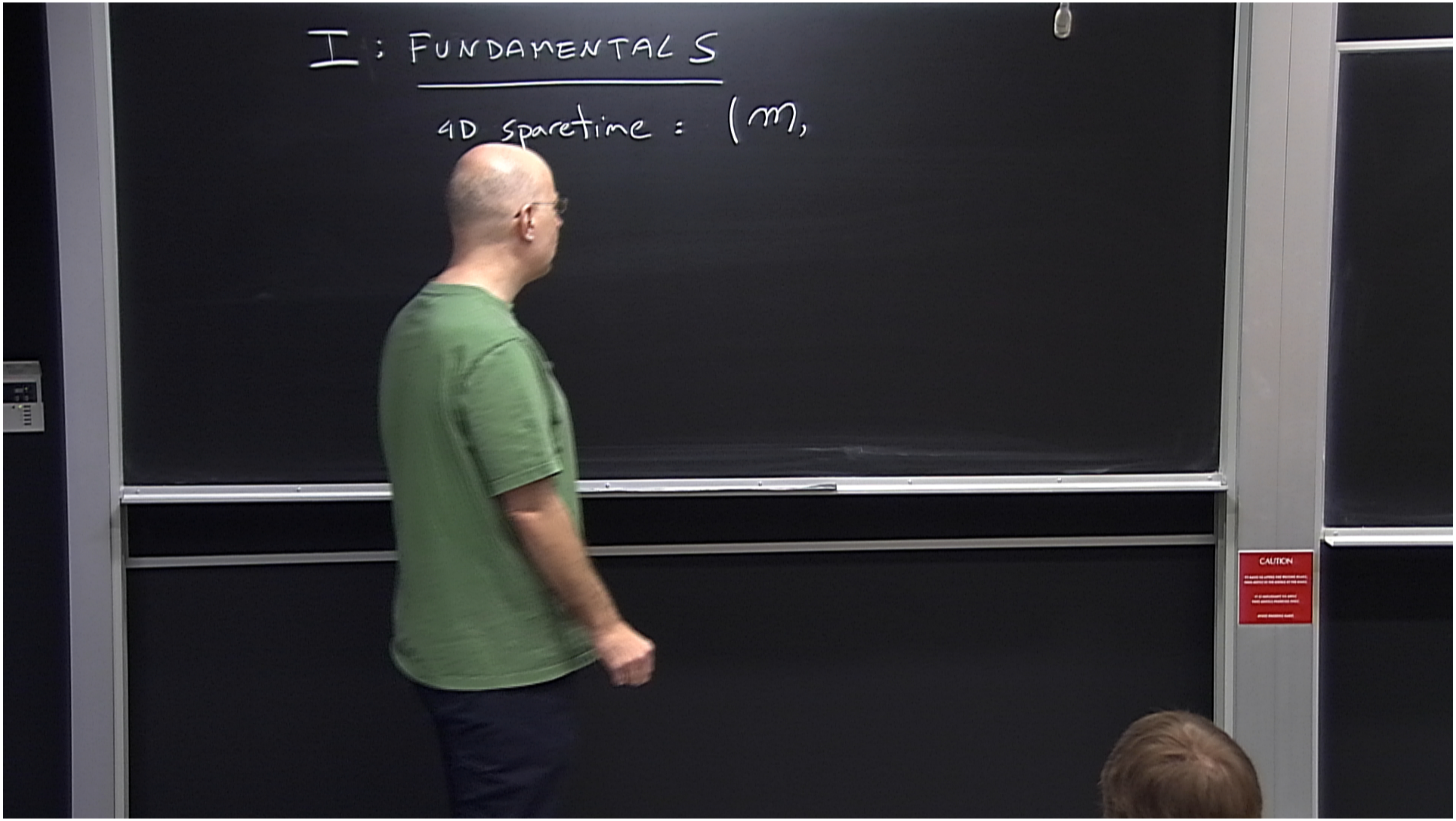
URL: http://pirsa.org/12010151

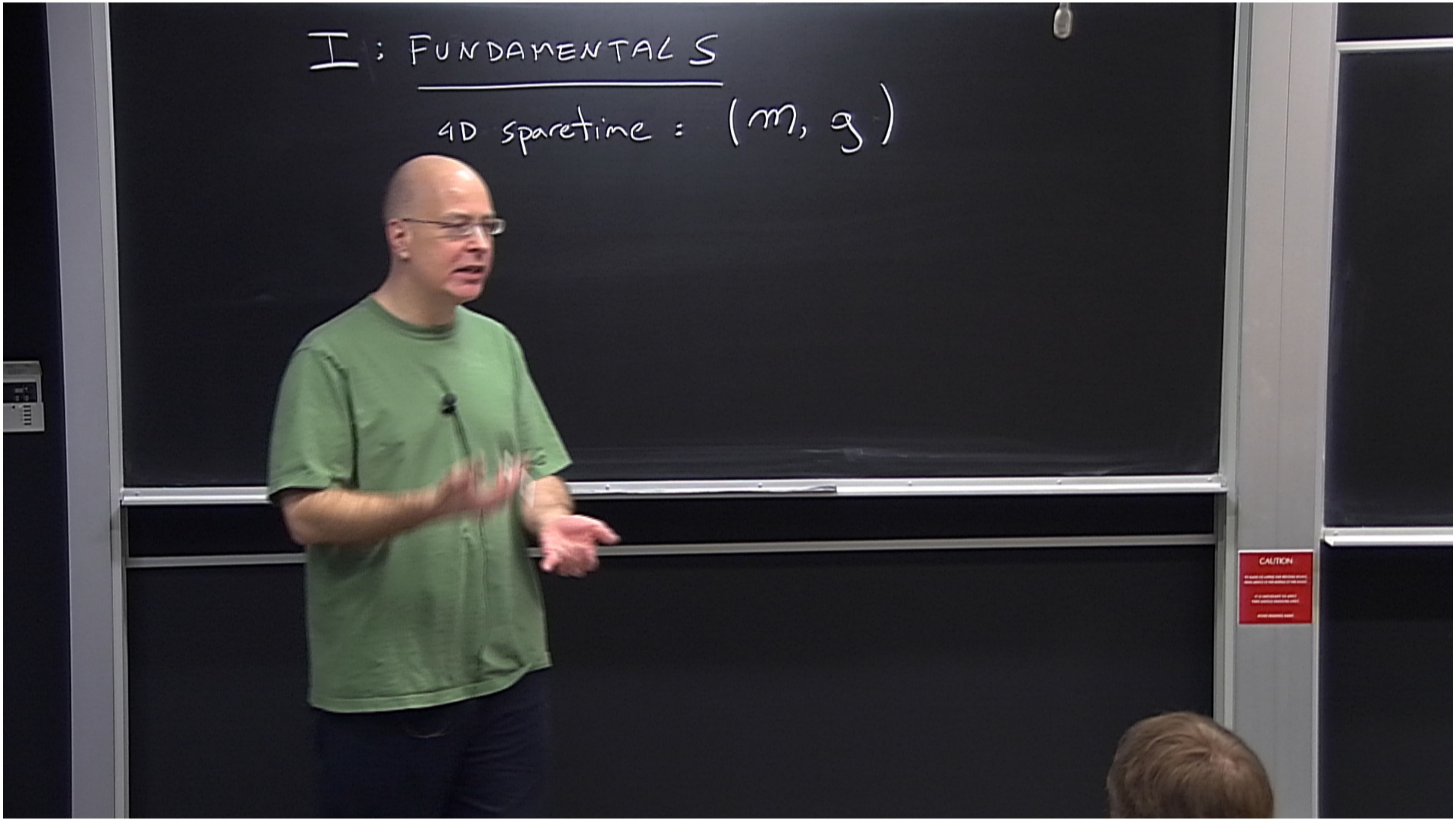
Abstract:

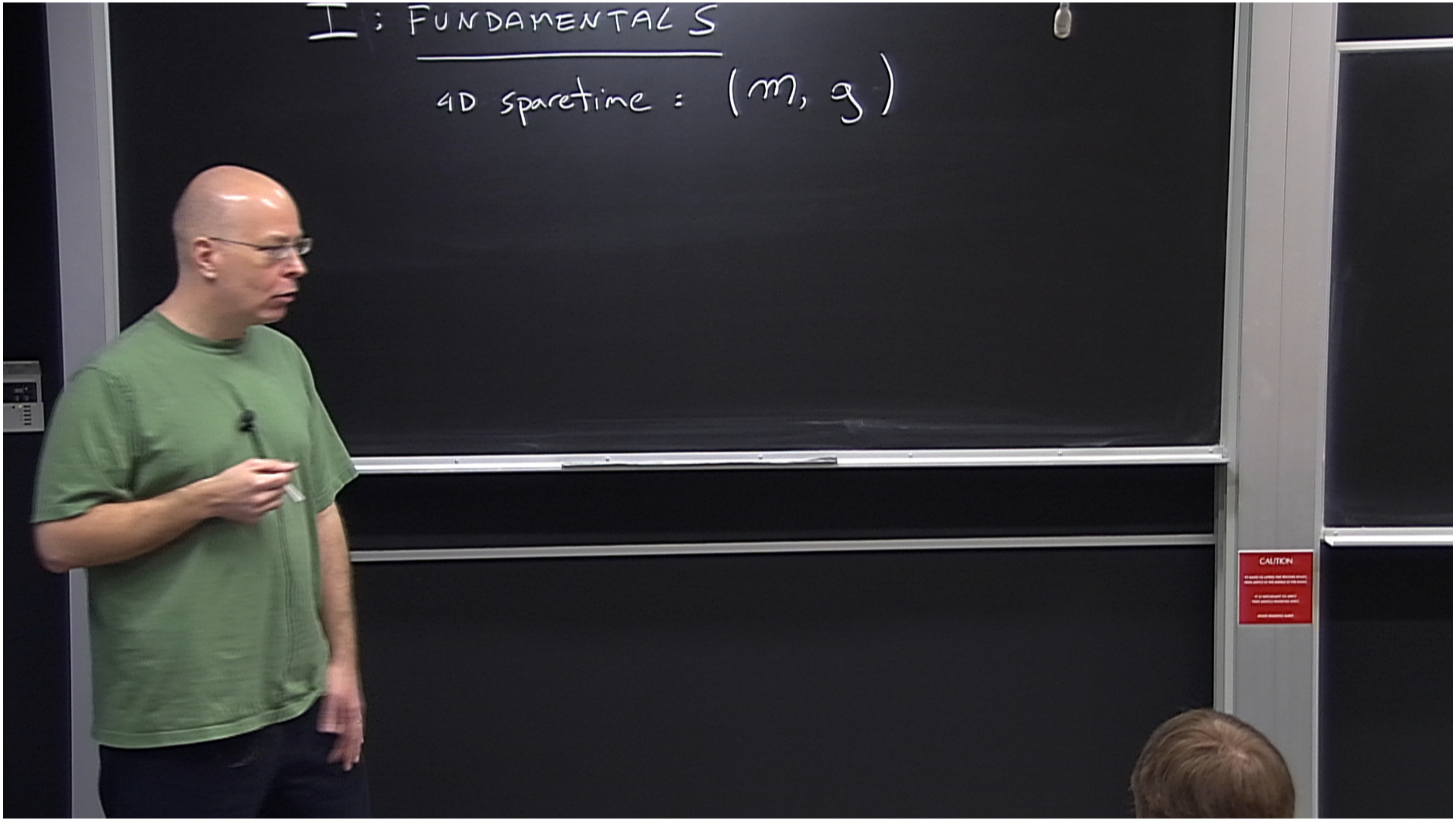

CAUTION

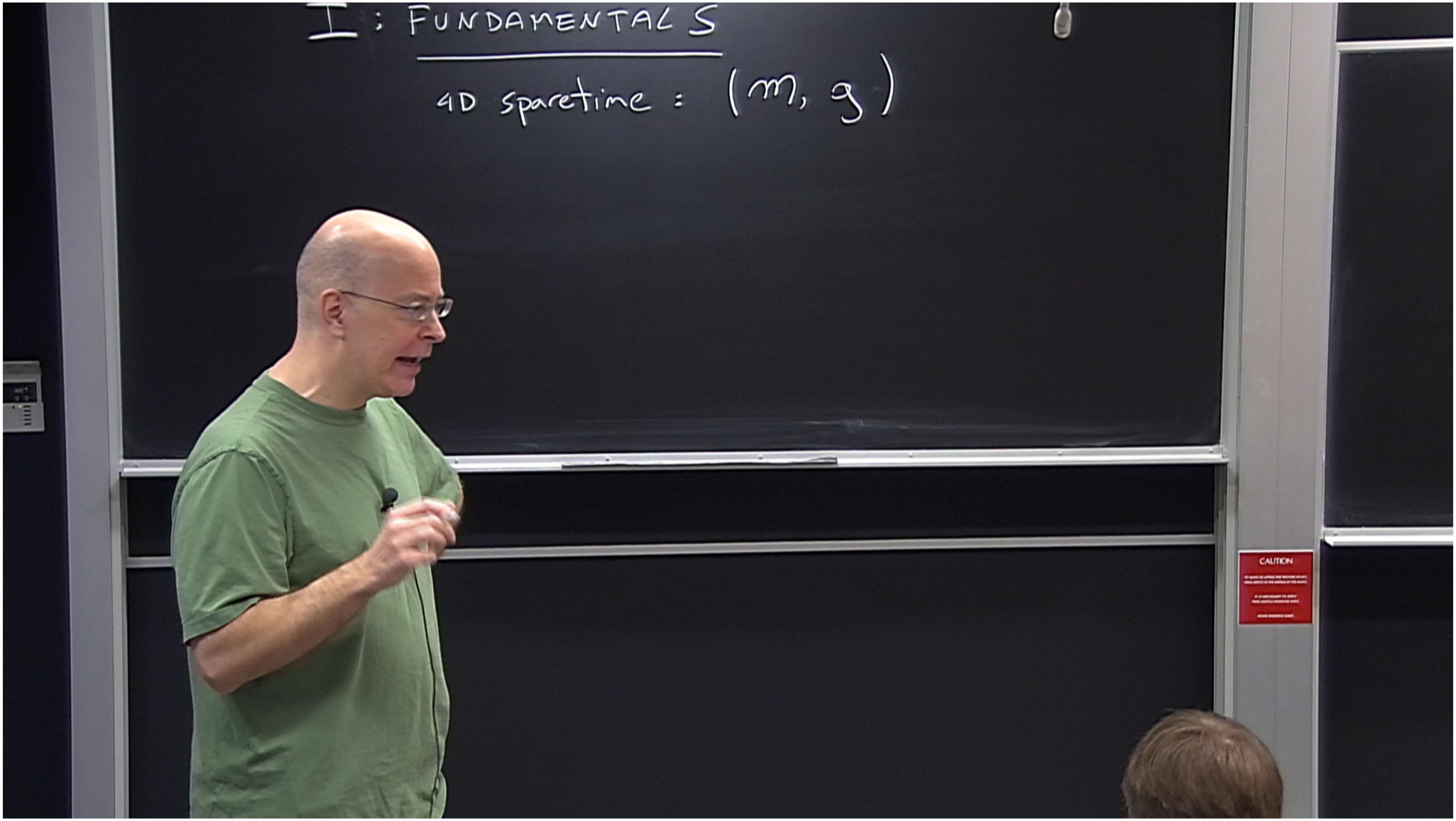

I : FUNDAMENTALS
CAUTION

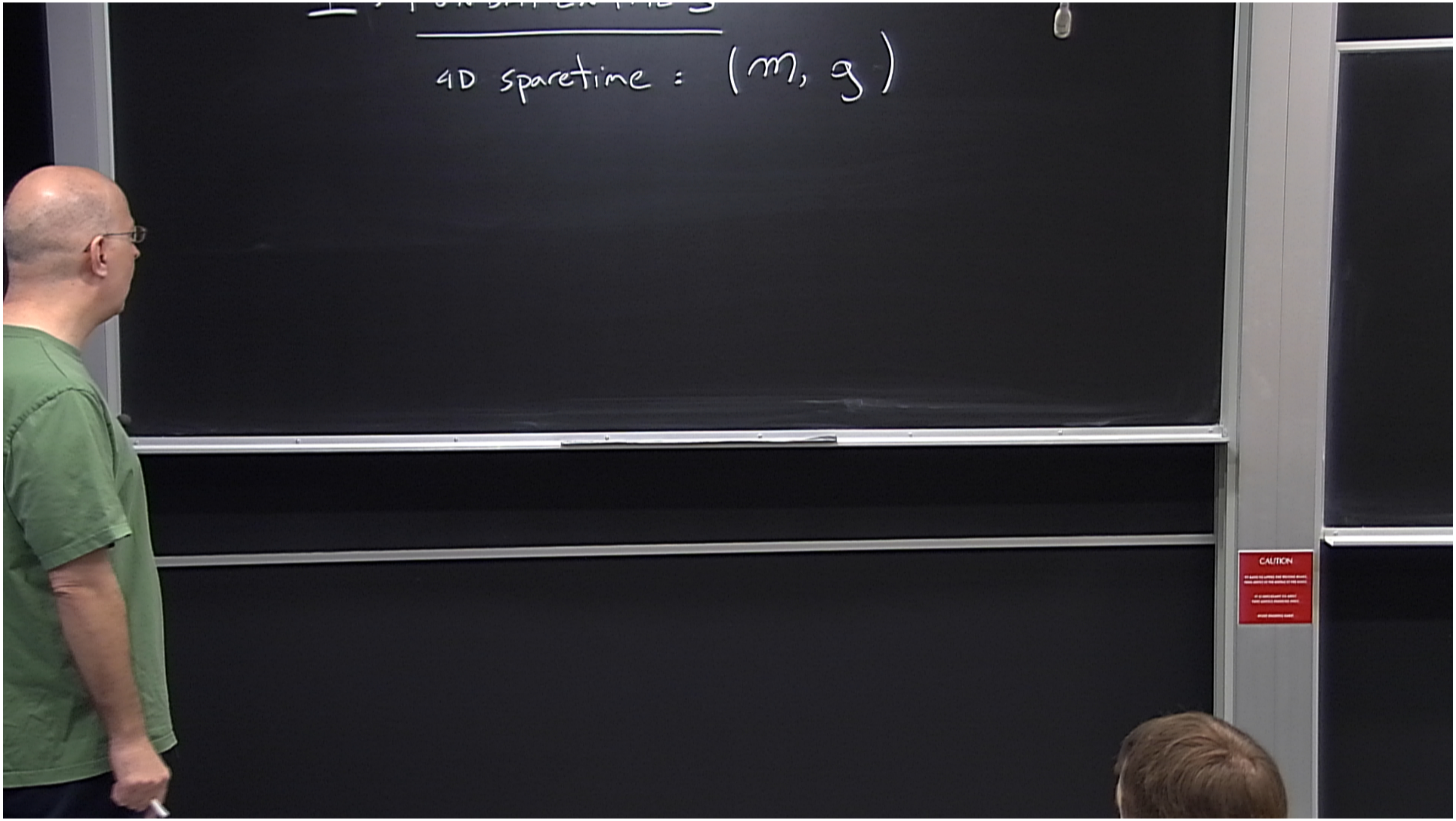

I : FUNDAMENTALS
4D
CAUTION

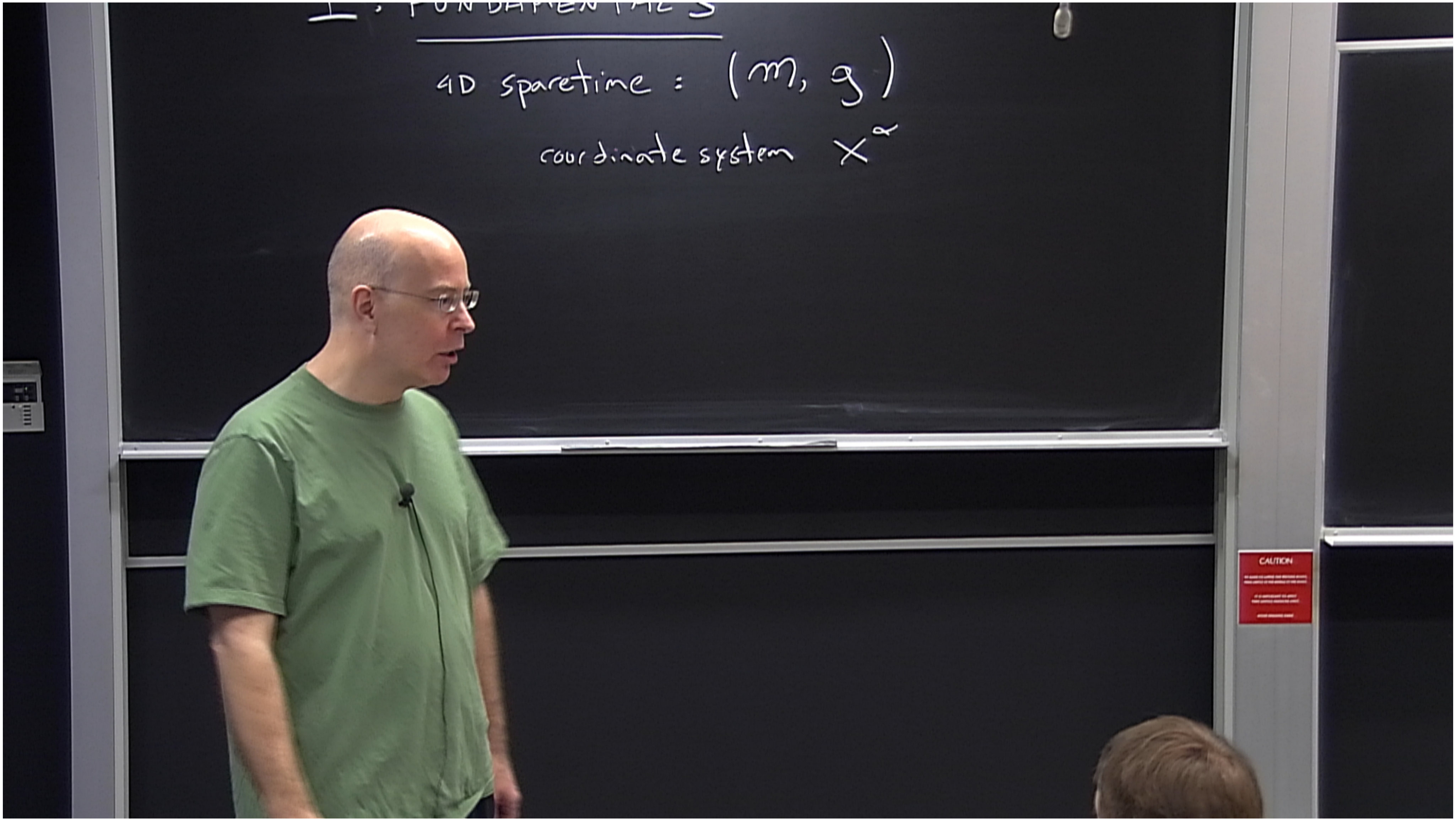

I : FUNDAMENTALS
4D spacetime : ($\mathcal{M}$,
CAUTION

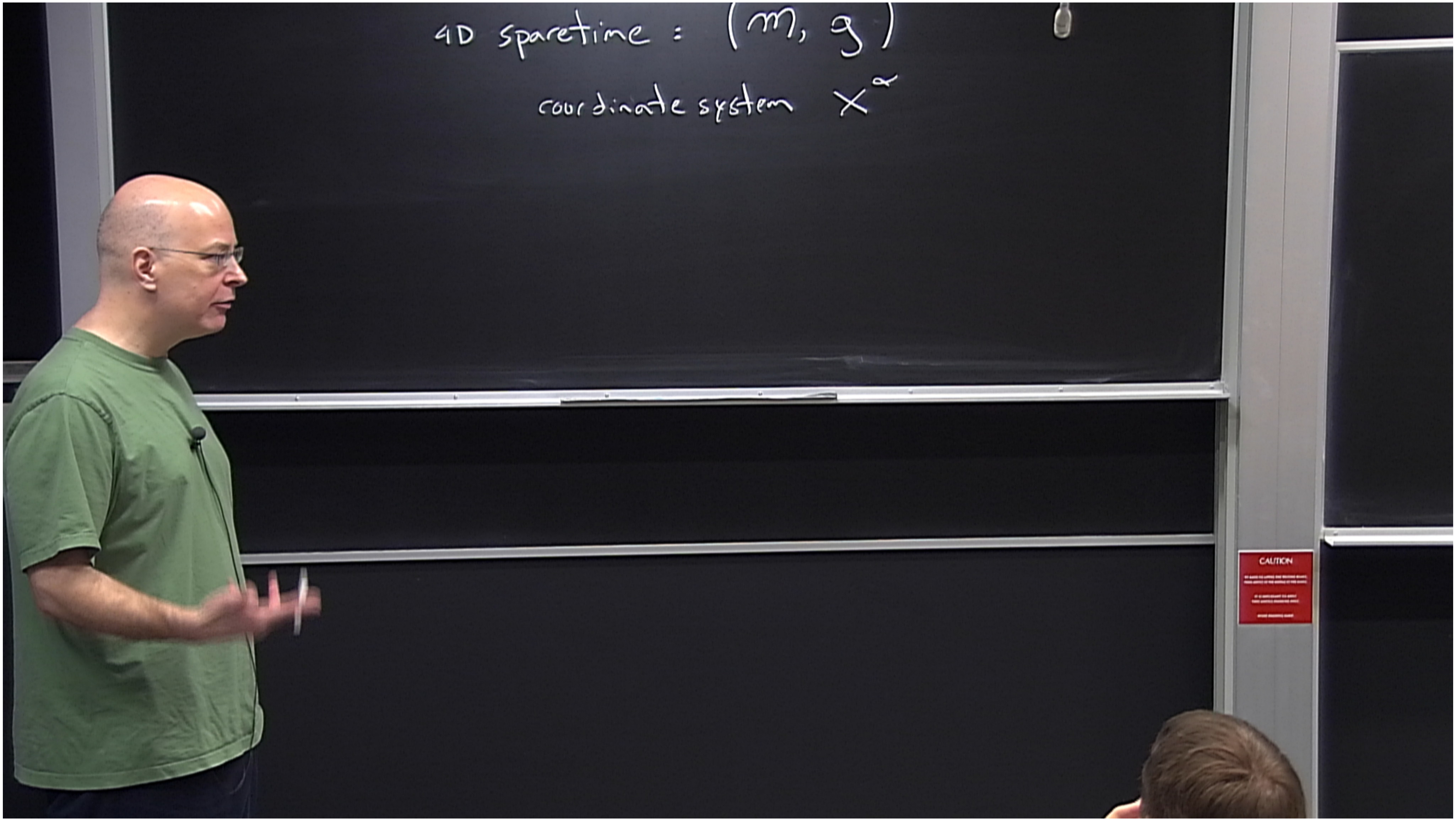

I : FUNDAMENTALS
4D spacetime : (M, g)
CAUTION

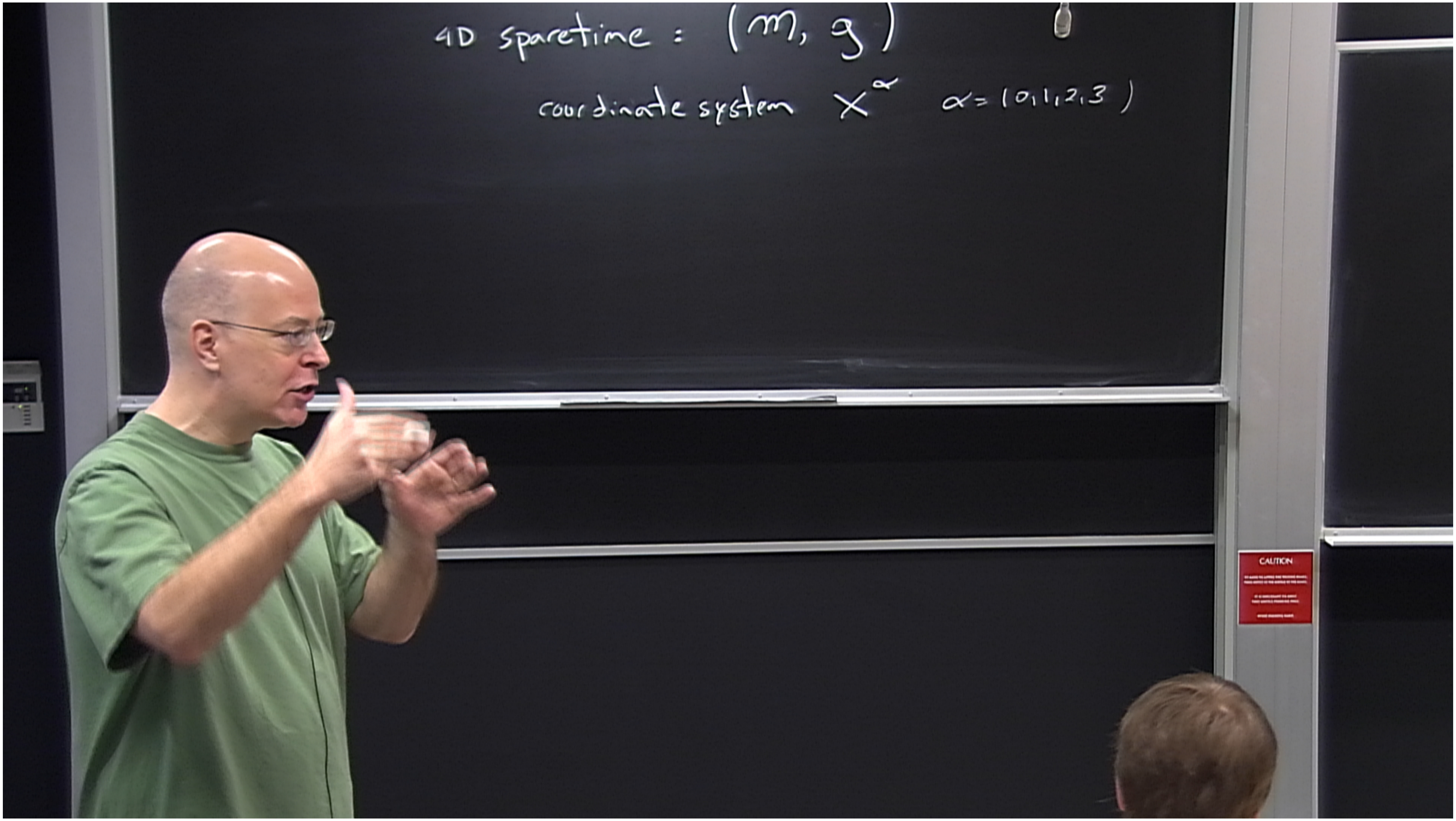

I : FUNDAMENTALS
4D spacetime : (M, g)
CAUTION

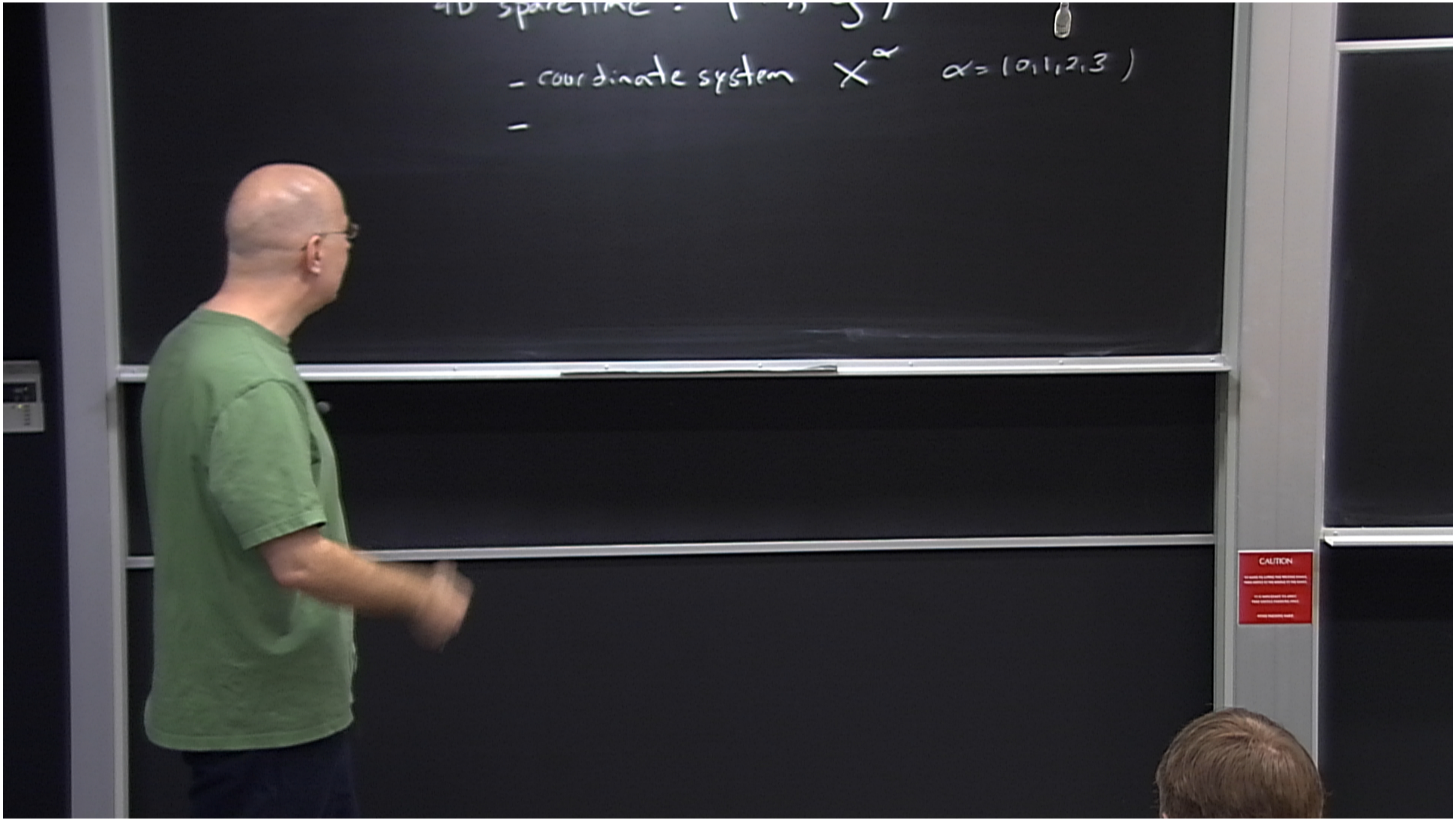

I : FUNDAMENTALS
4D spacetime : (M, g)
CAUTION

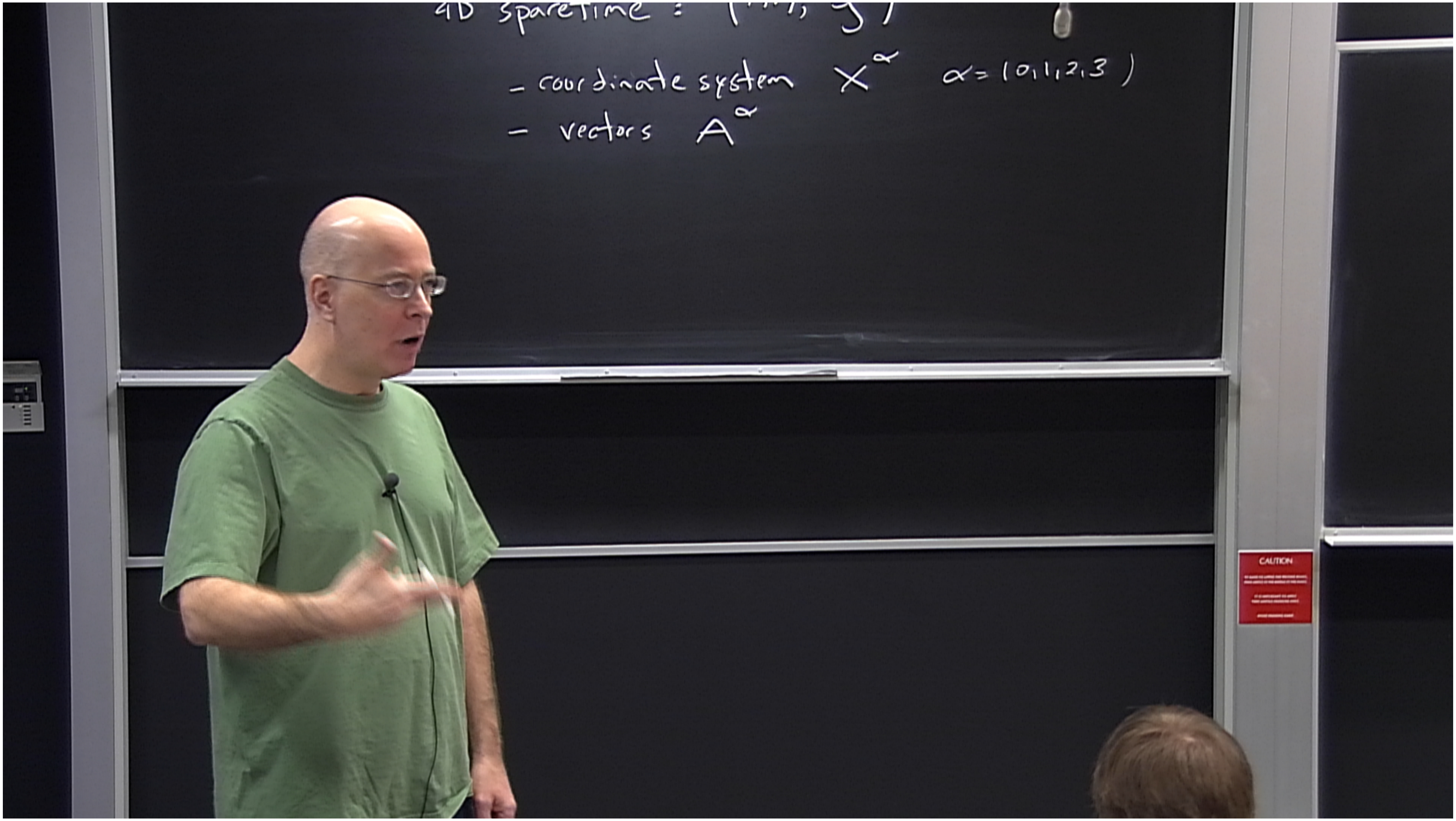

4D spacetime : (M, g)

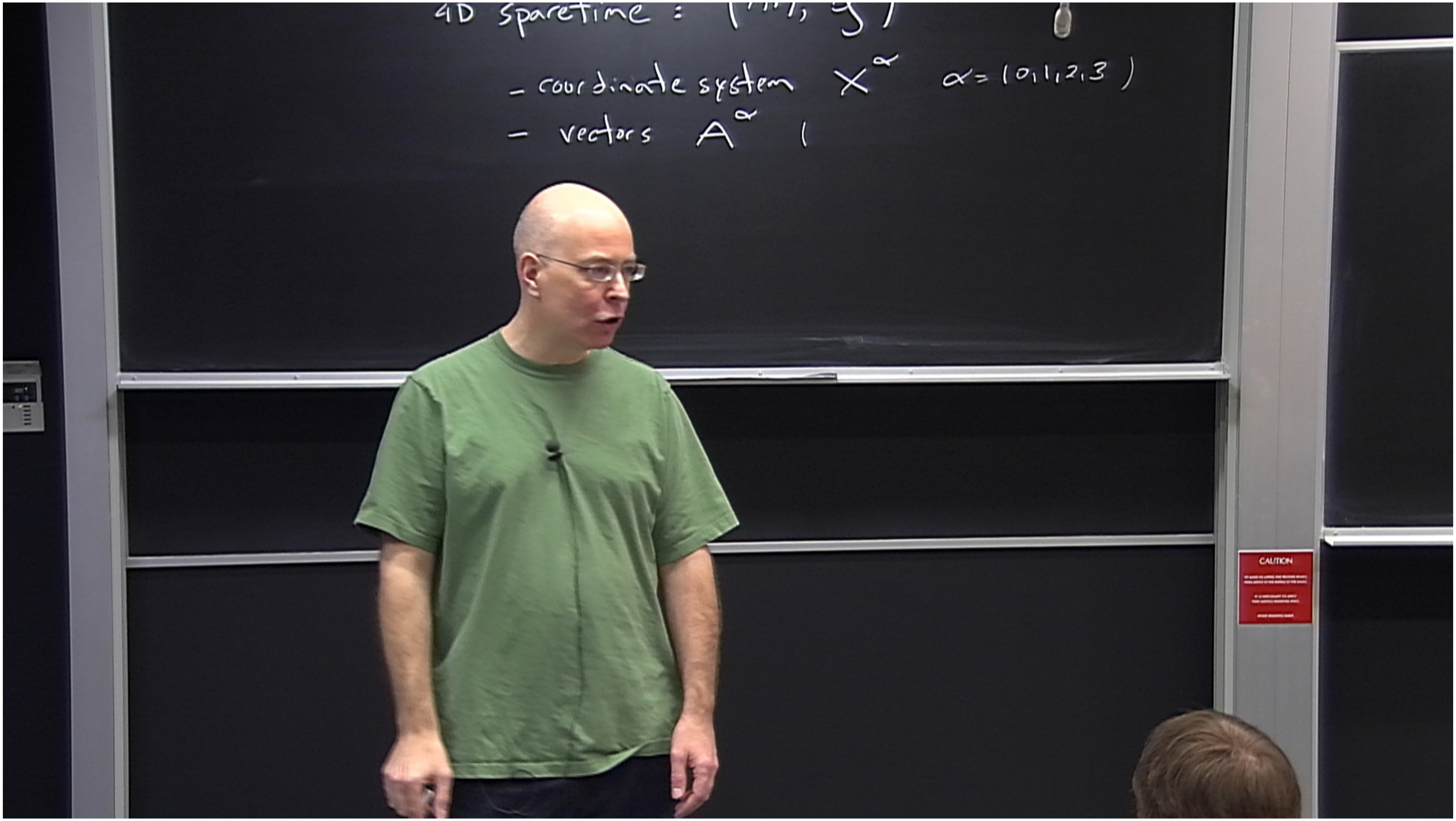

4D spacetime : (m, g)
coordinate system x^α
CAUTION

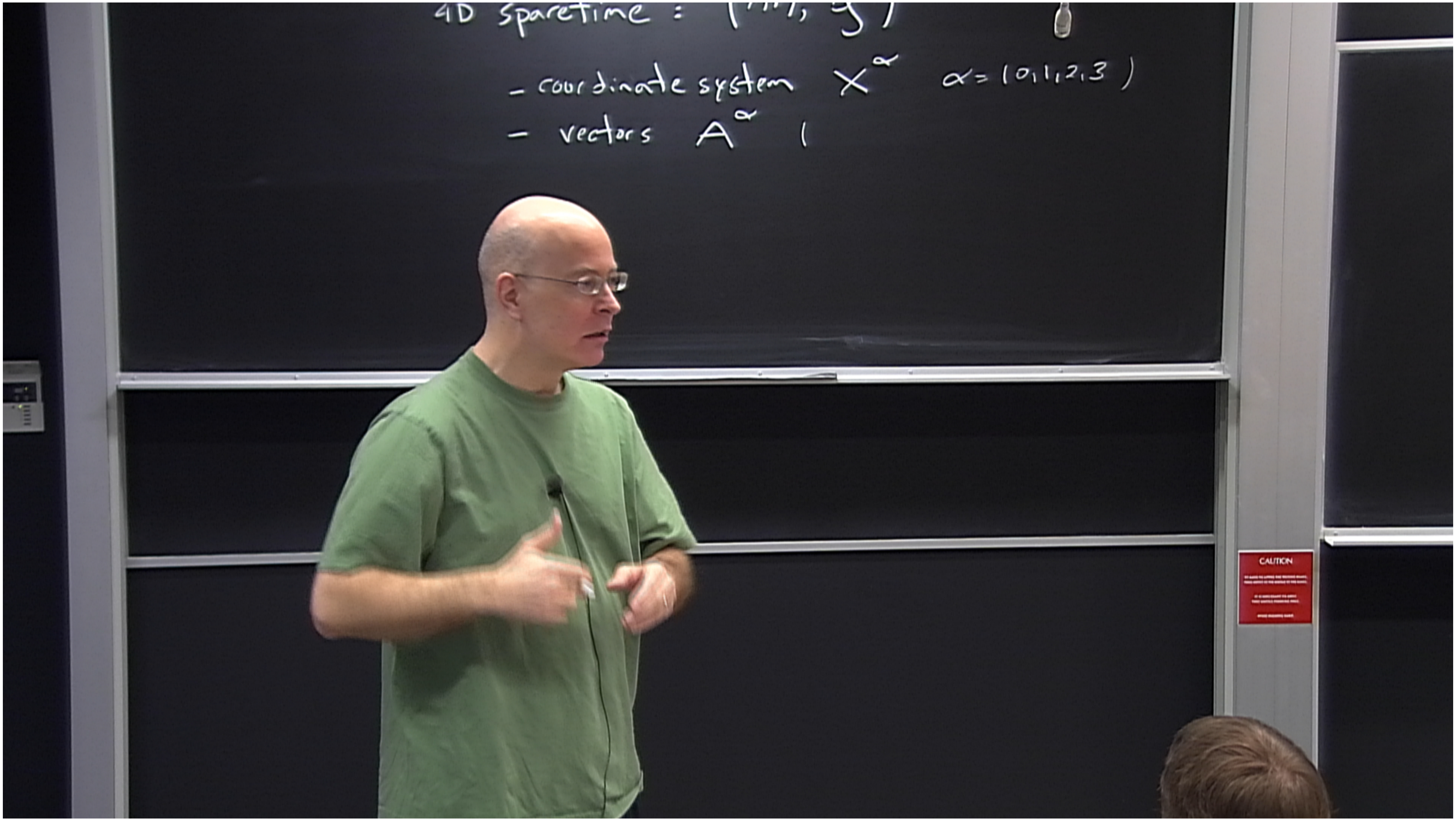

4D spacetime : (M, g)
coordinate system X^α
CAUTION

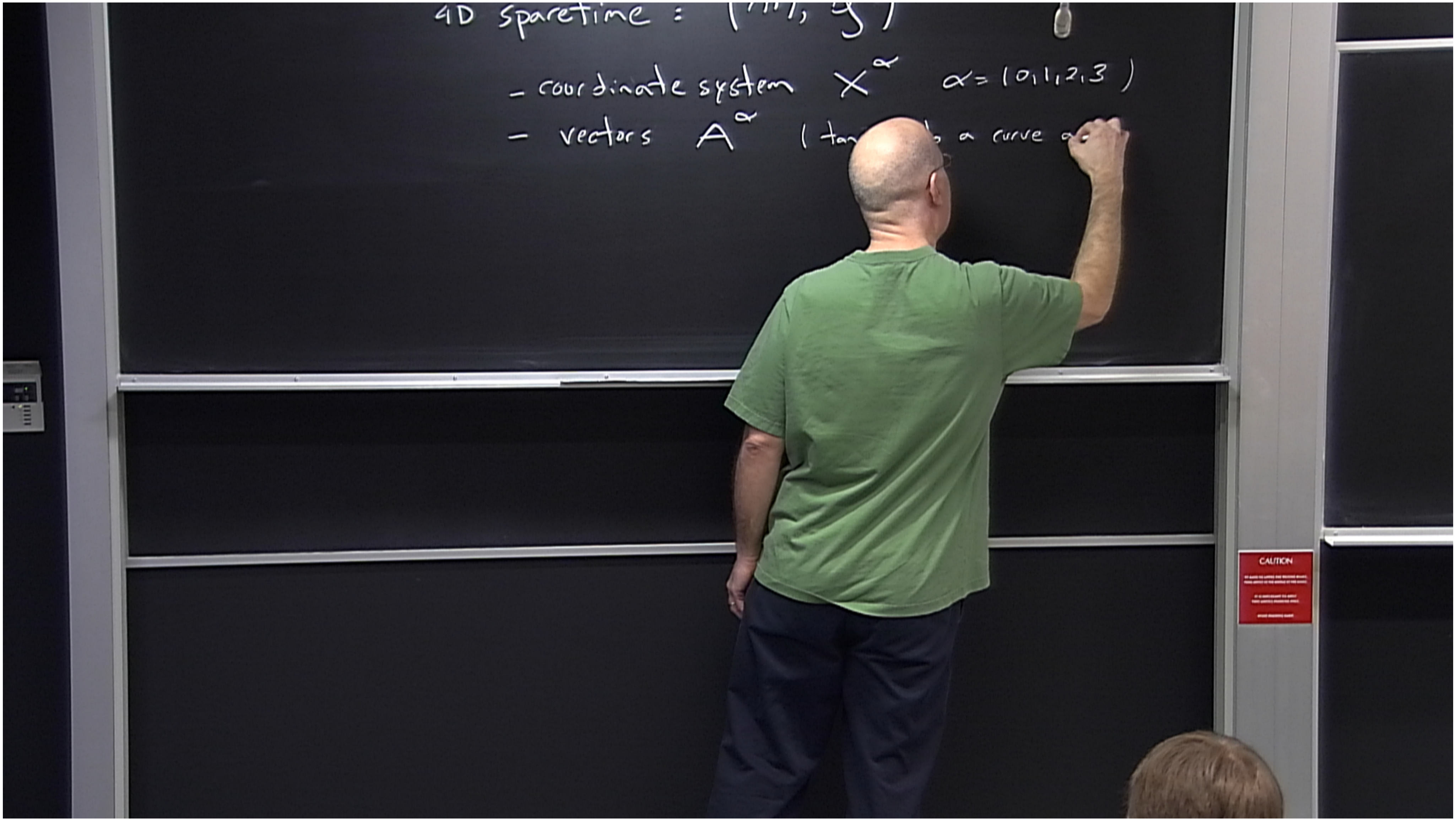

4D spacetime : (M, g)
coordinate system X^α α = (0,1,2,3)
CAUTION

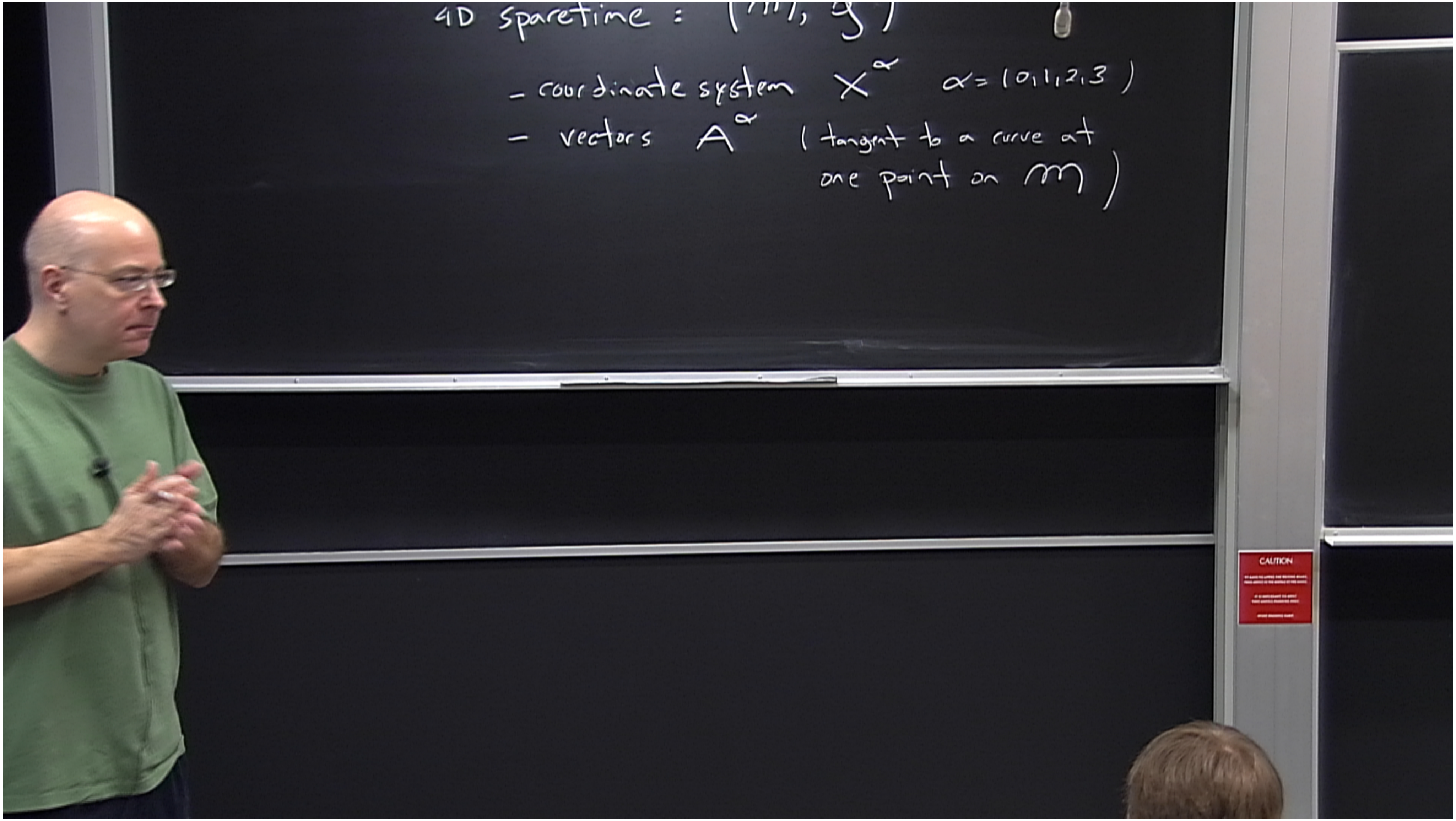

- coordinate system X^α $\alpha = (0,1,2,3)$
-

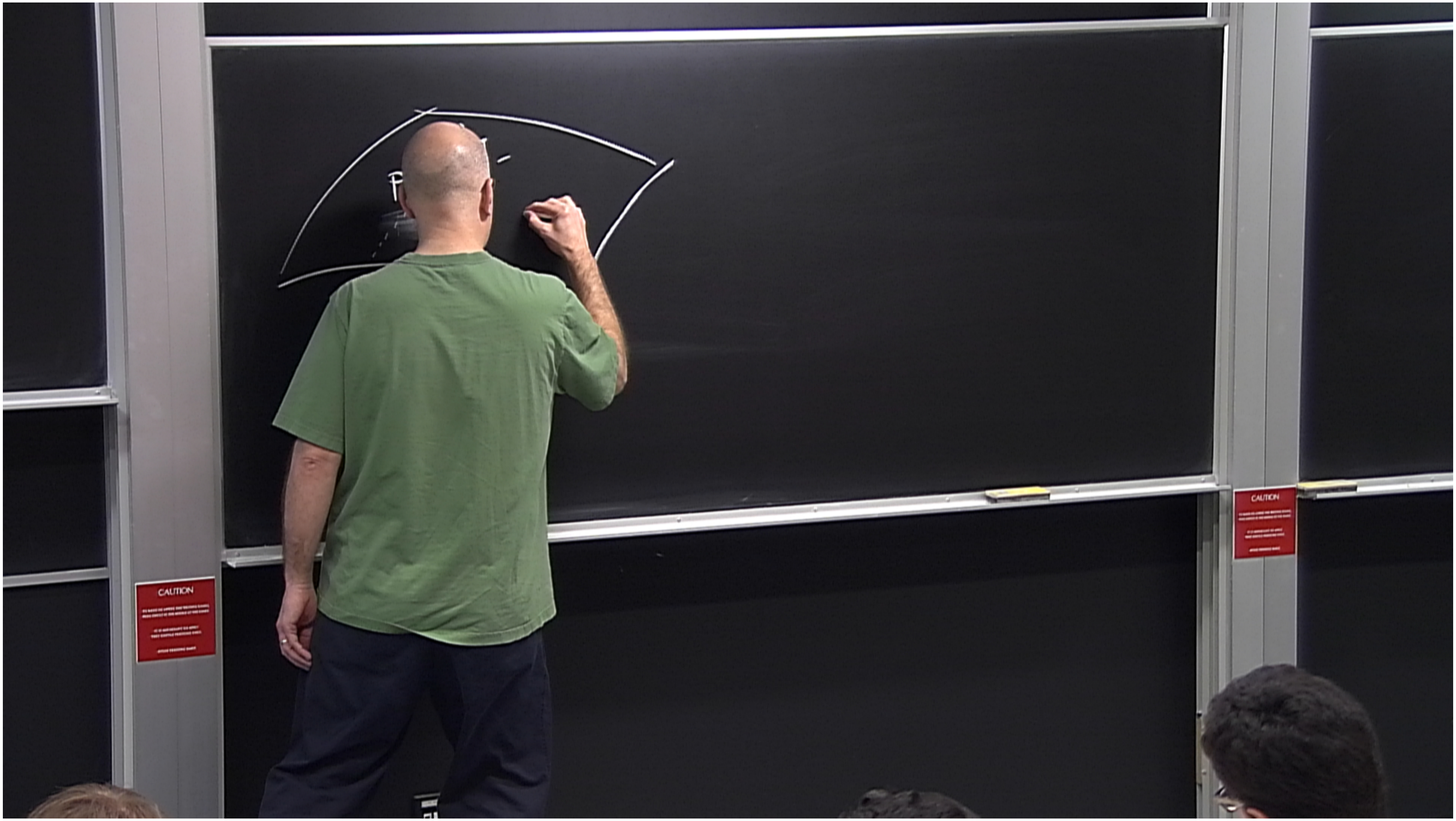

- coordinate system X^α α = (0,1,2,3)
- vectors A^α

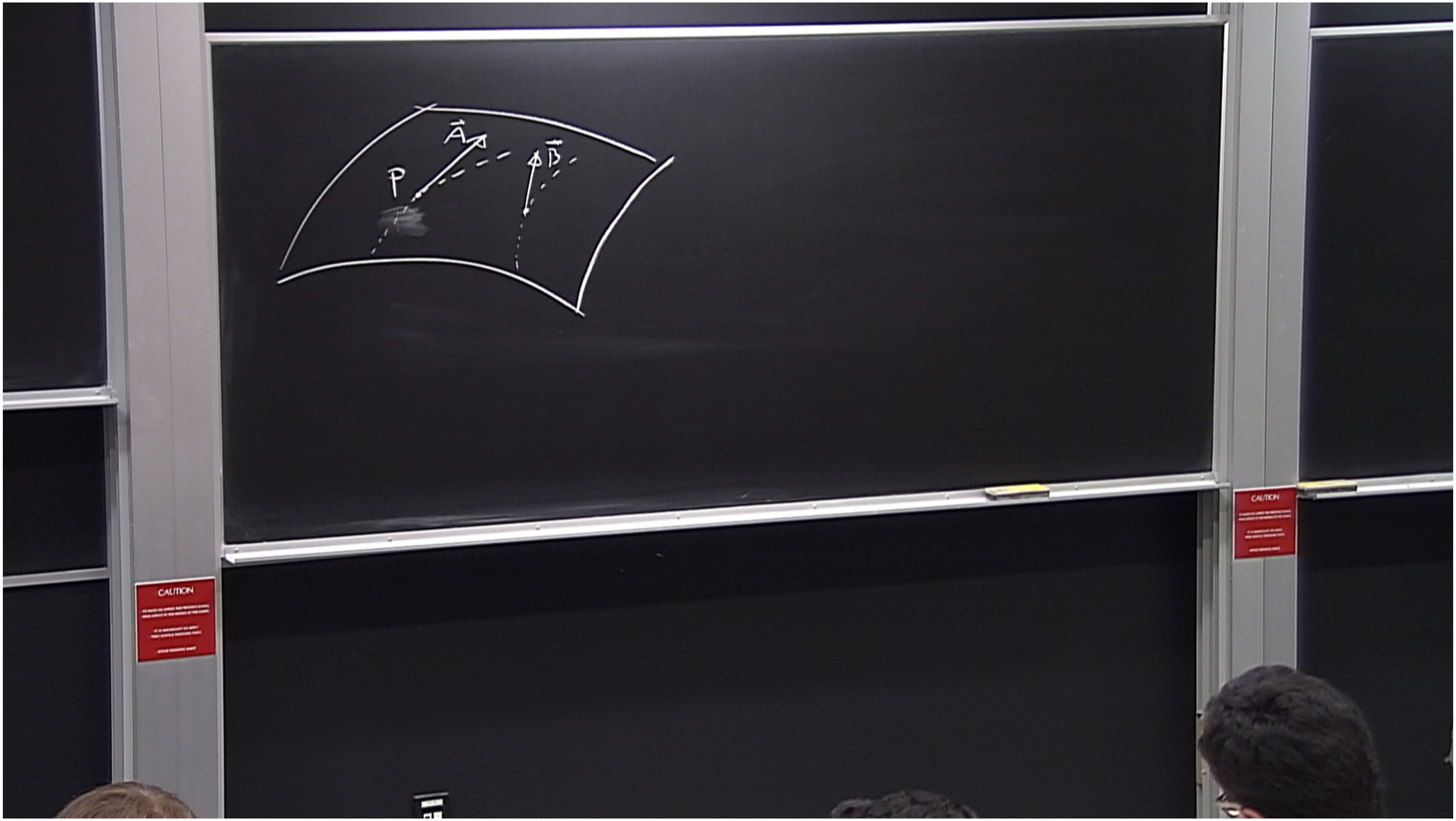

4D spacetime : (M, g)
- coordinate system X^α $\alpha = (0,1,2,3)$
- vectors A^α (

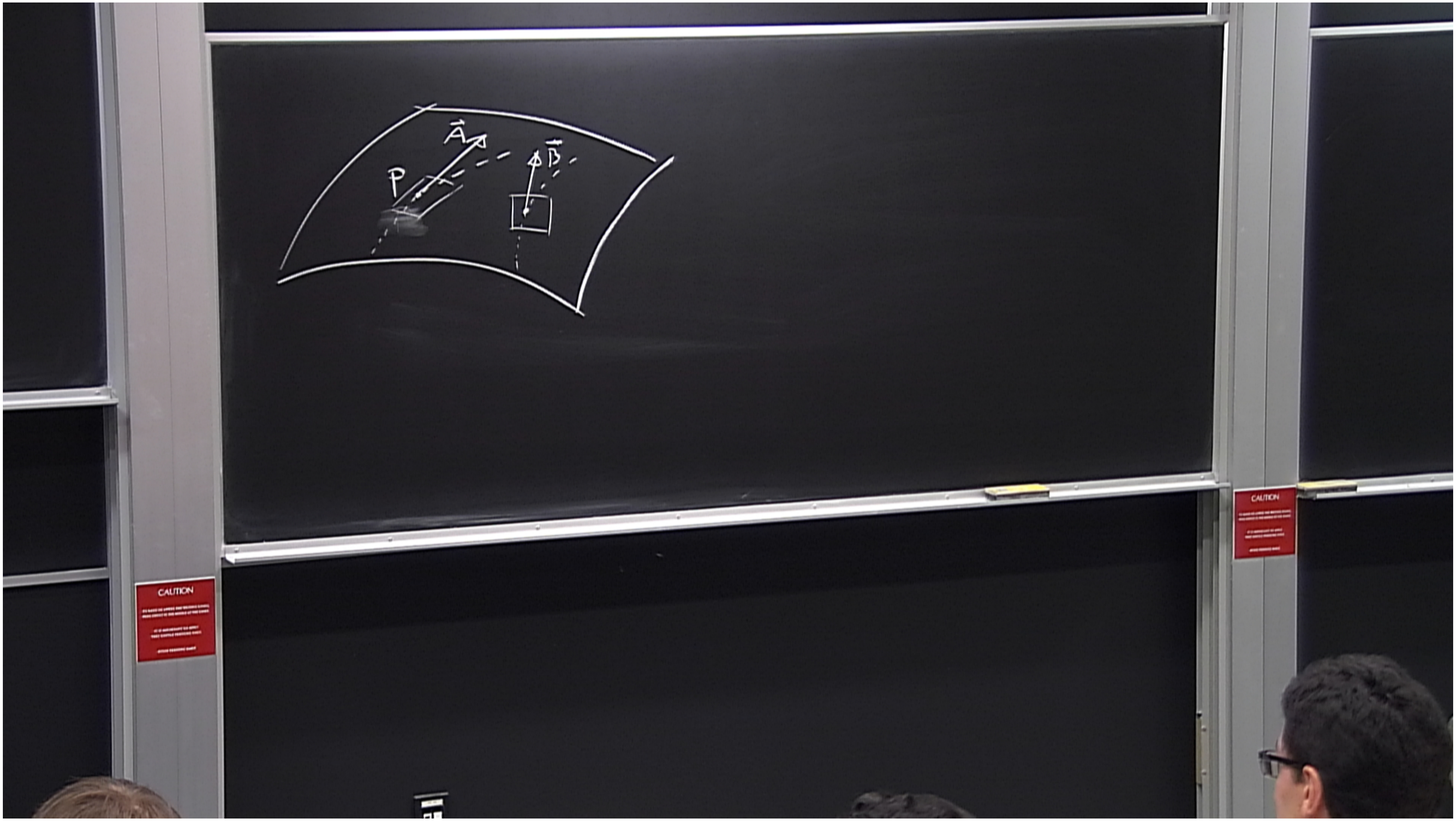

4D spacetime :
- coordinate system X^α α = (0,1,2,3)
- vectors A^α
CAUTION

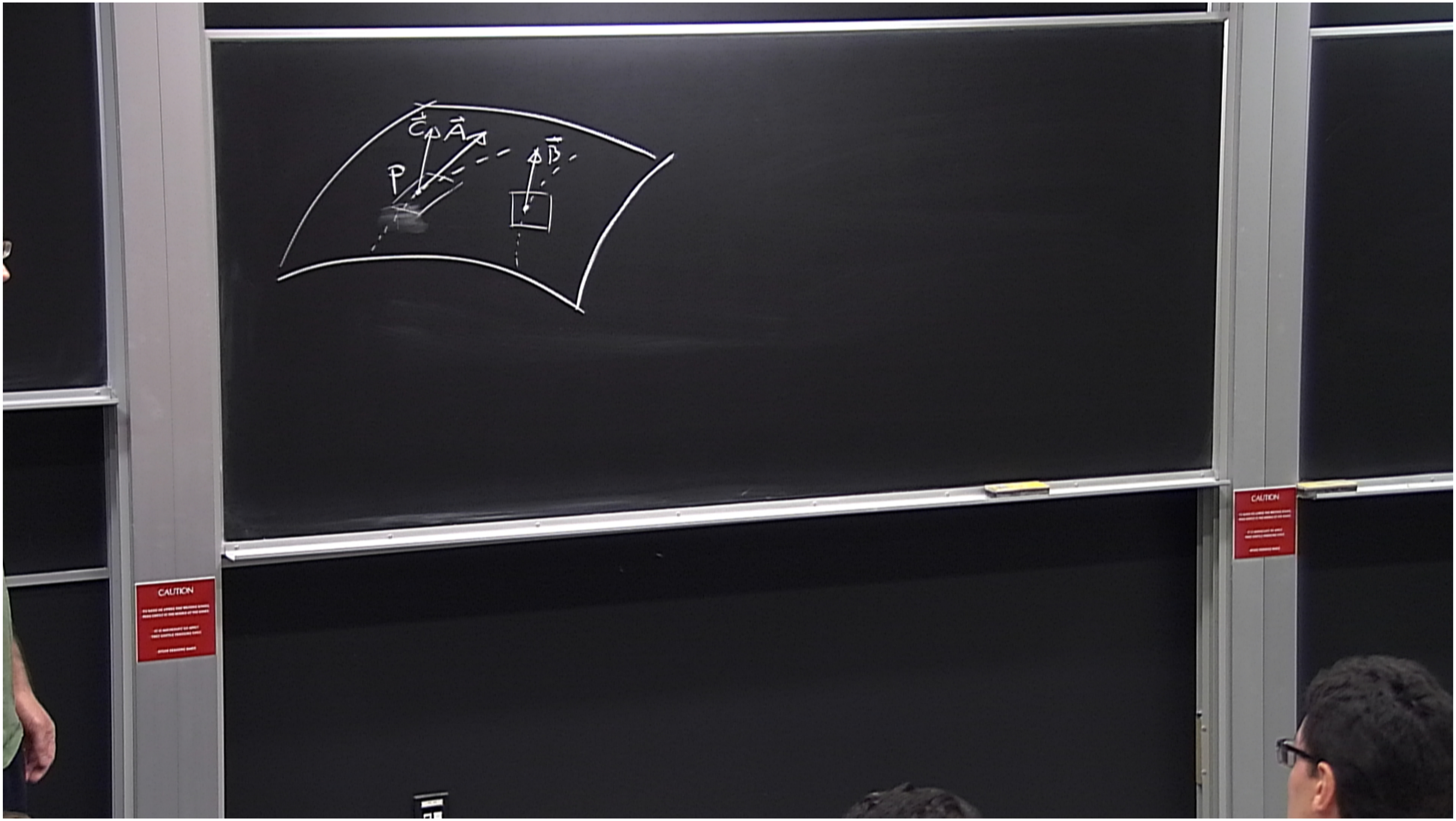

4D spacetime :
- coordinate system X^α α = (0,1,2,3)
- vectors A^α
a curve


4D spacetime :
- coordinate system x^α α = (0,1,2,3)
- vectors A^α (tangent to a curve at one point on M)
CAUTION



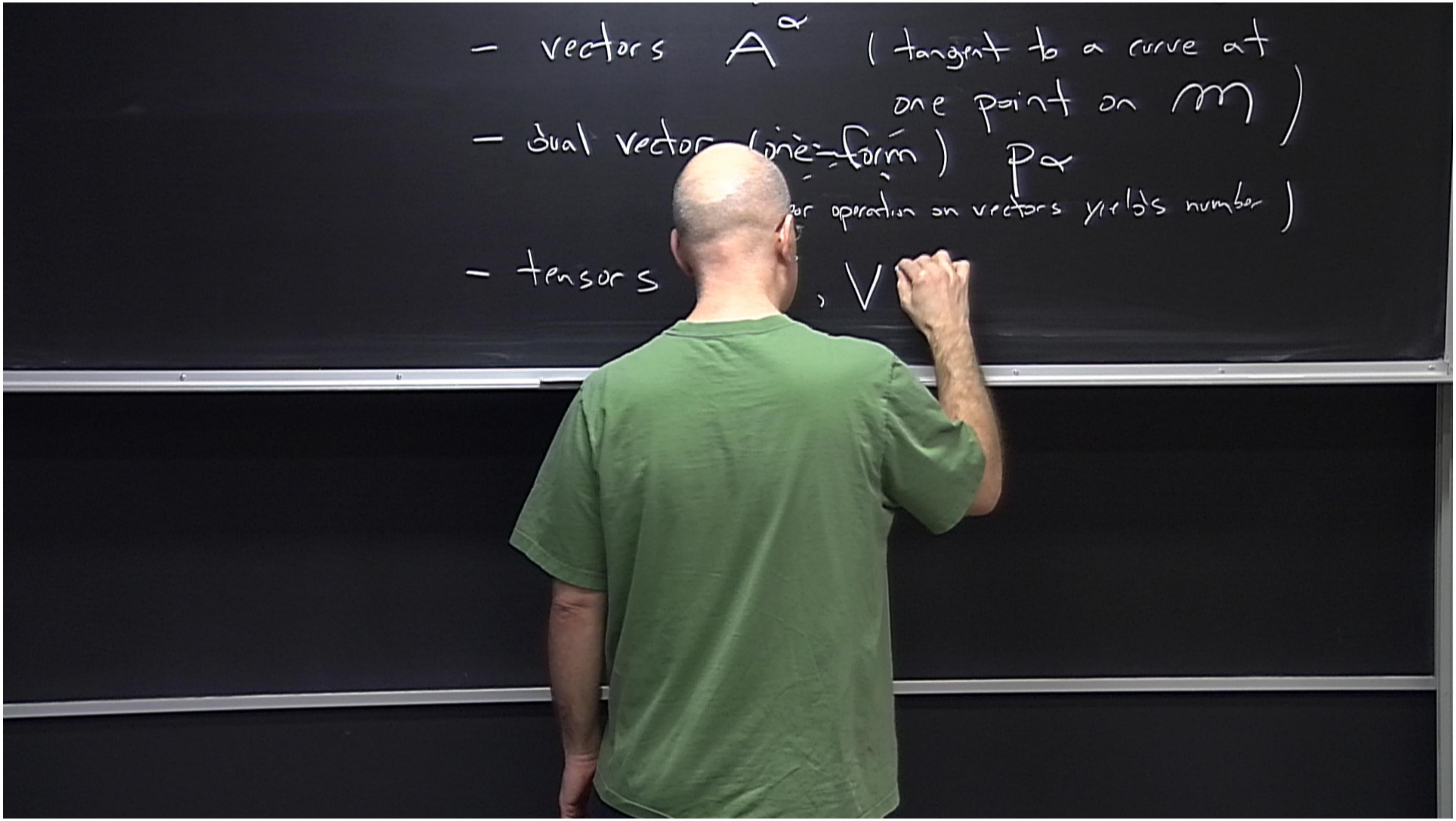

P
$\vec{A}$
$\vec{B}$

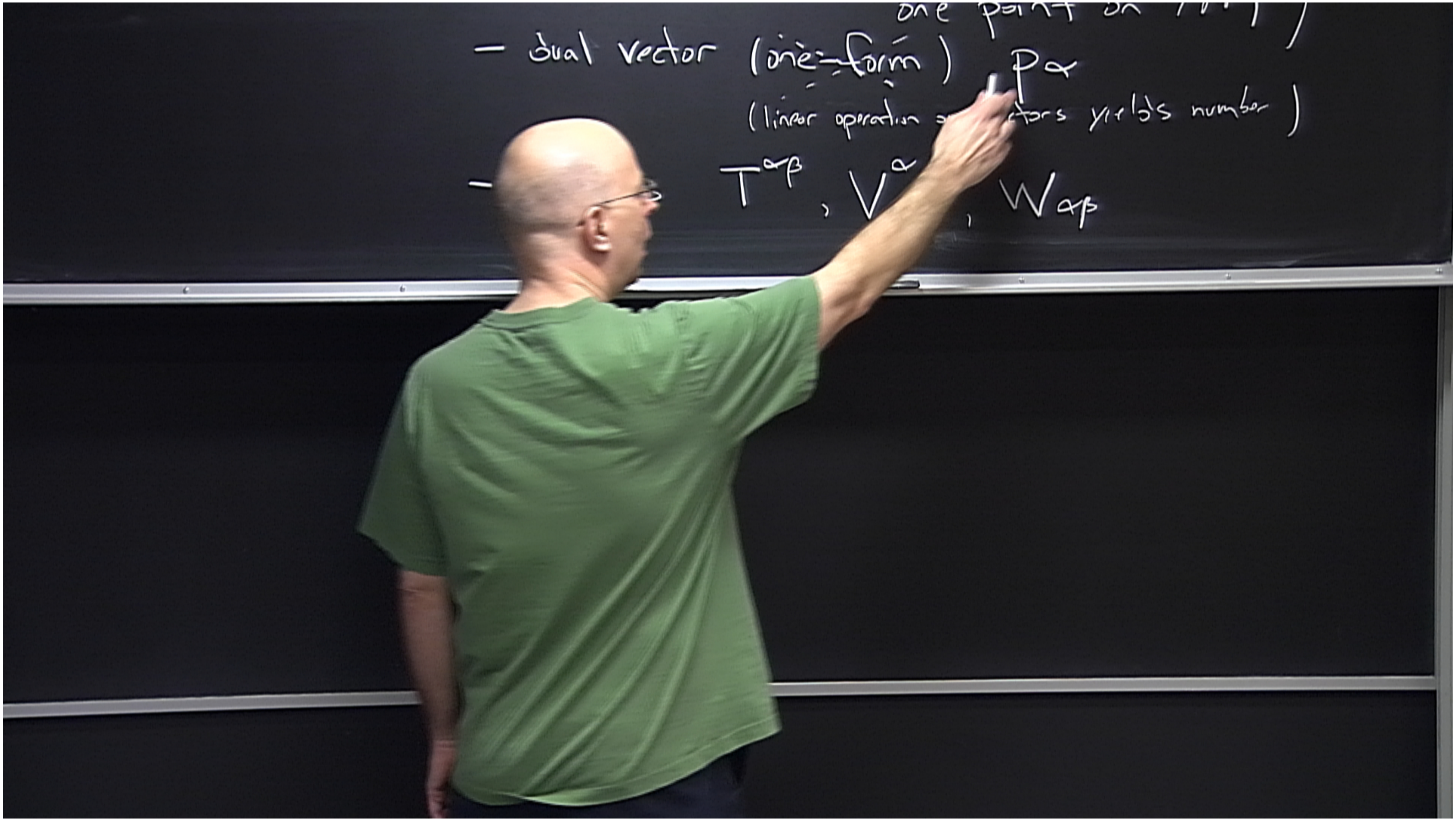

P
$\vec{A}$
$\vec{B}$
CAUTION
CAUTION

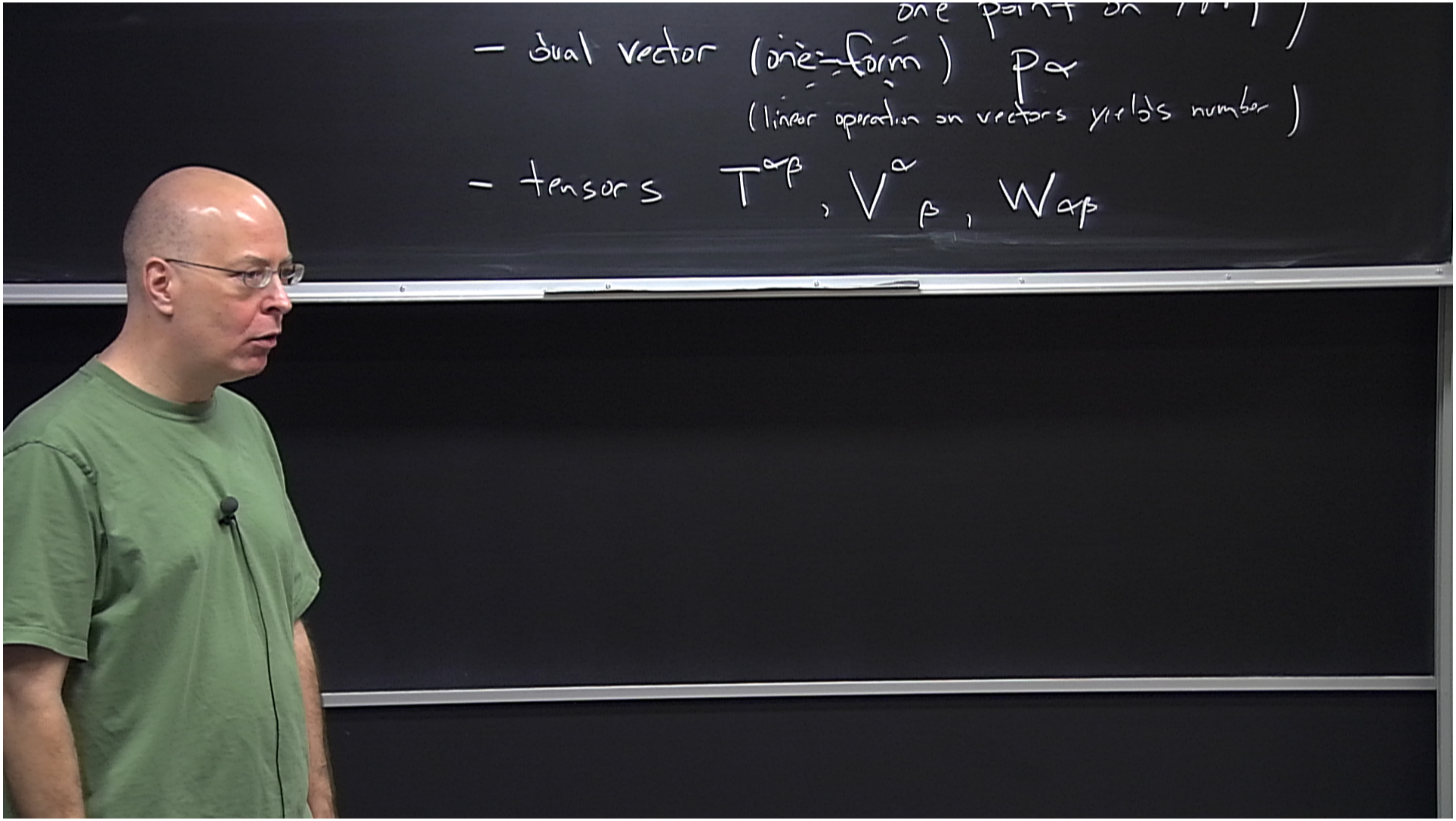


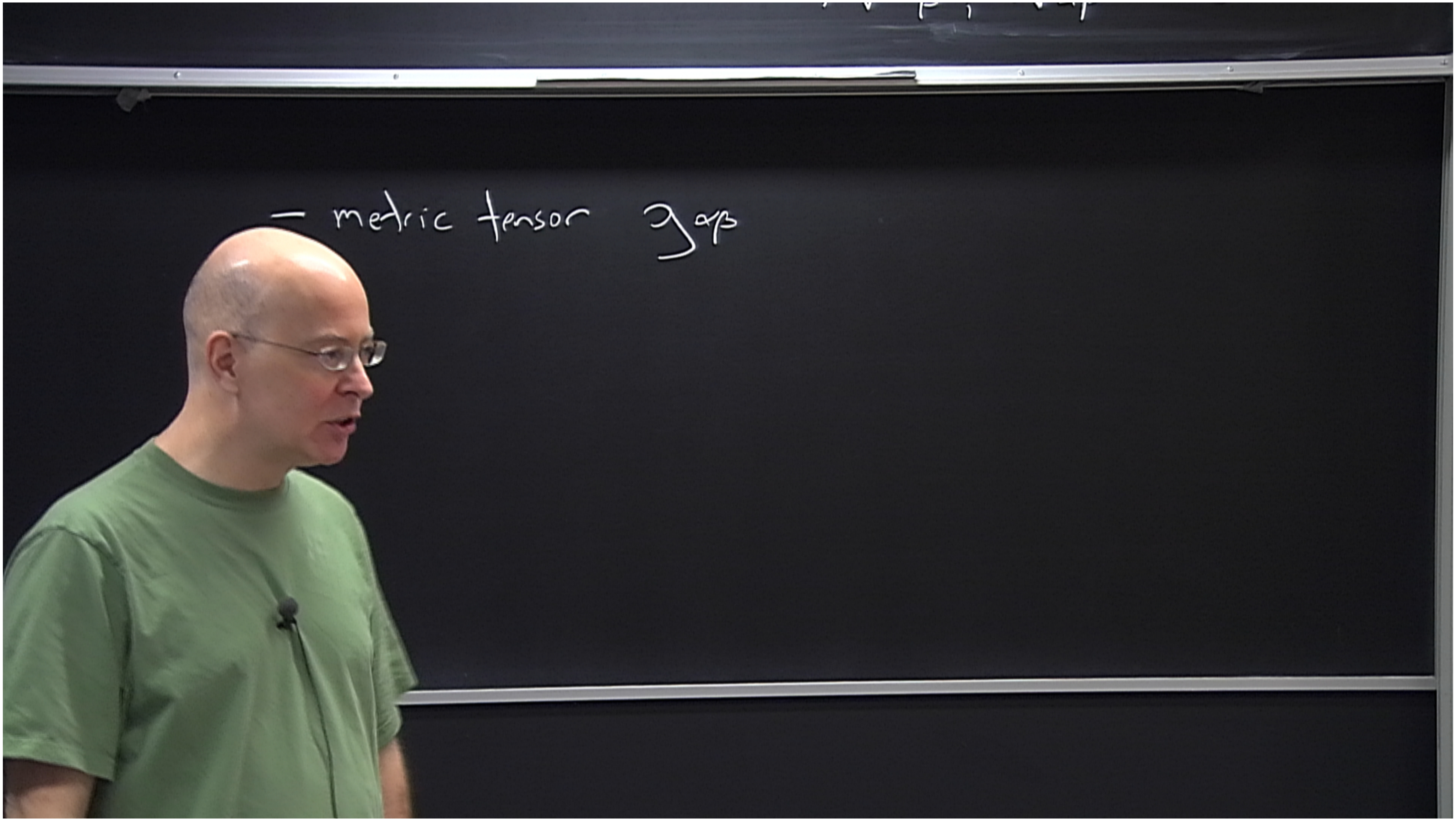
$\vec{C}$
$\vec{A}$
$\vec{B}$
P
CAUTION
CAUTION

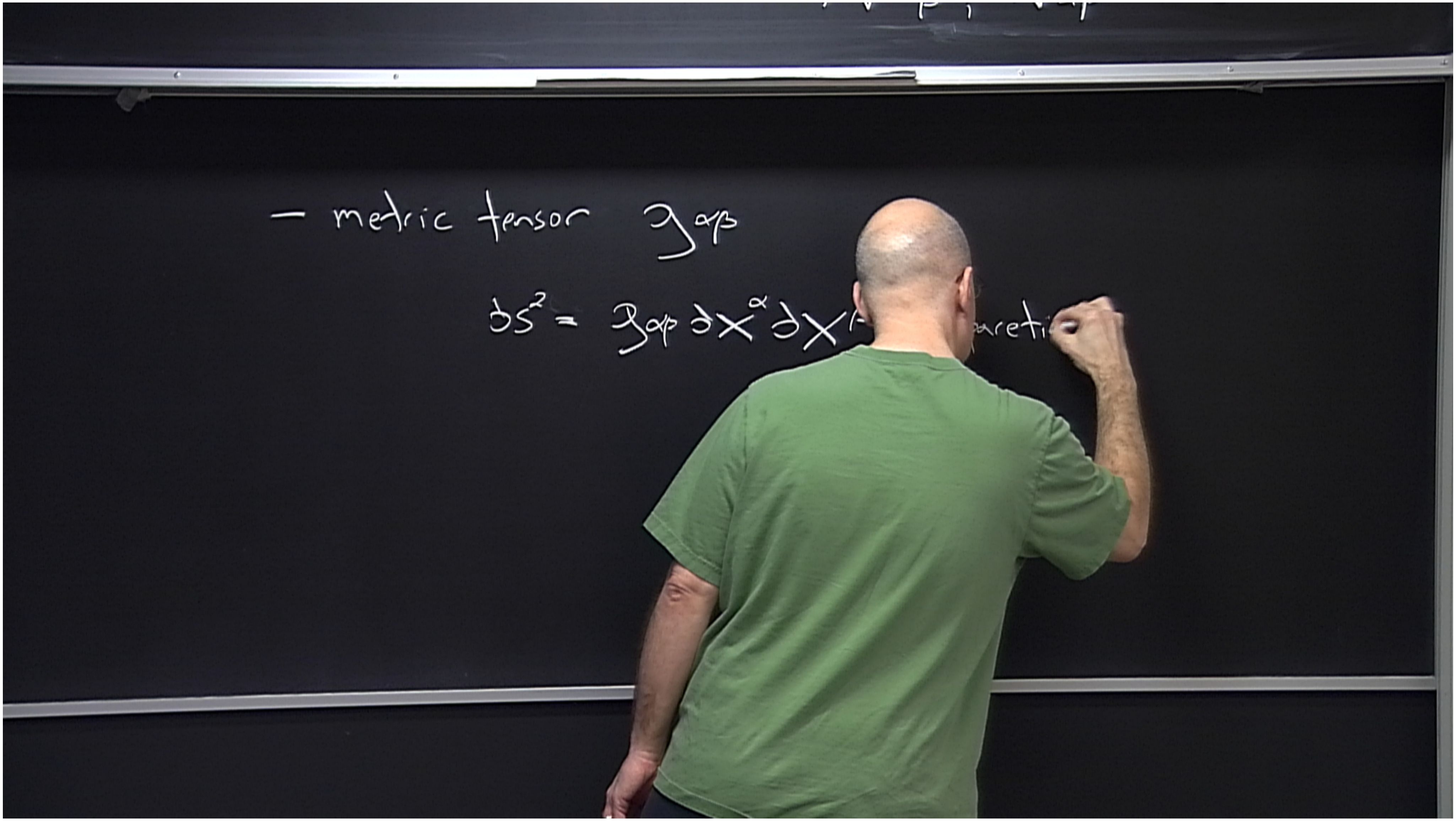
- coordinate system X^α $\alpha = (0, 1, 2, 3)$
- vectors A^α (tangent to a curve at one point on $\mathcal{M}$)
- dual vector (one-form) p_α

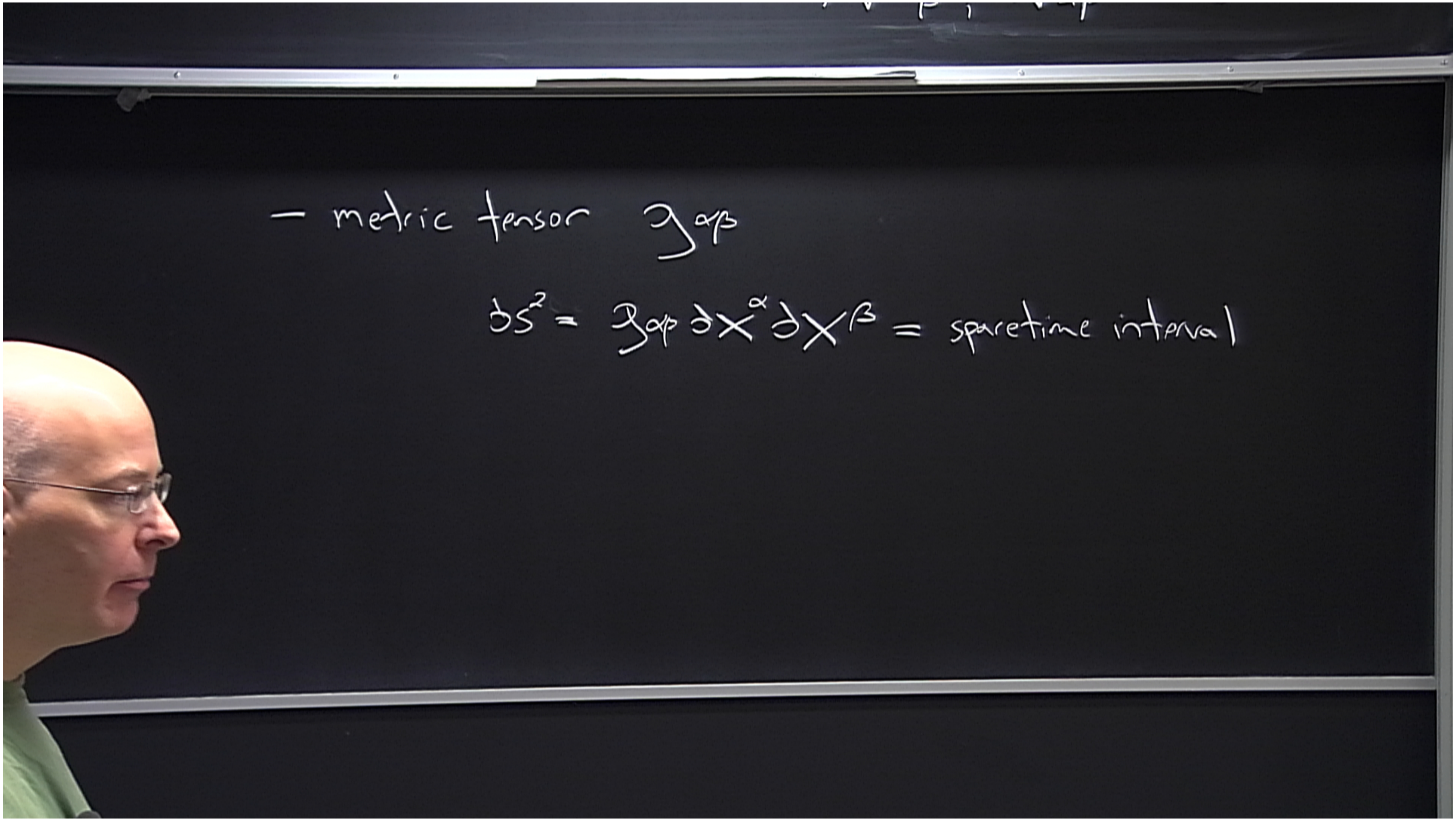

- vectors A^α (tangent to a curve at
one point on M)
- dual vector (one-form) p_α
operation on vectors yields number)
- tensors
, V

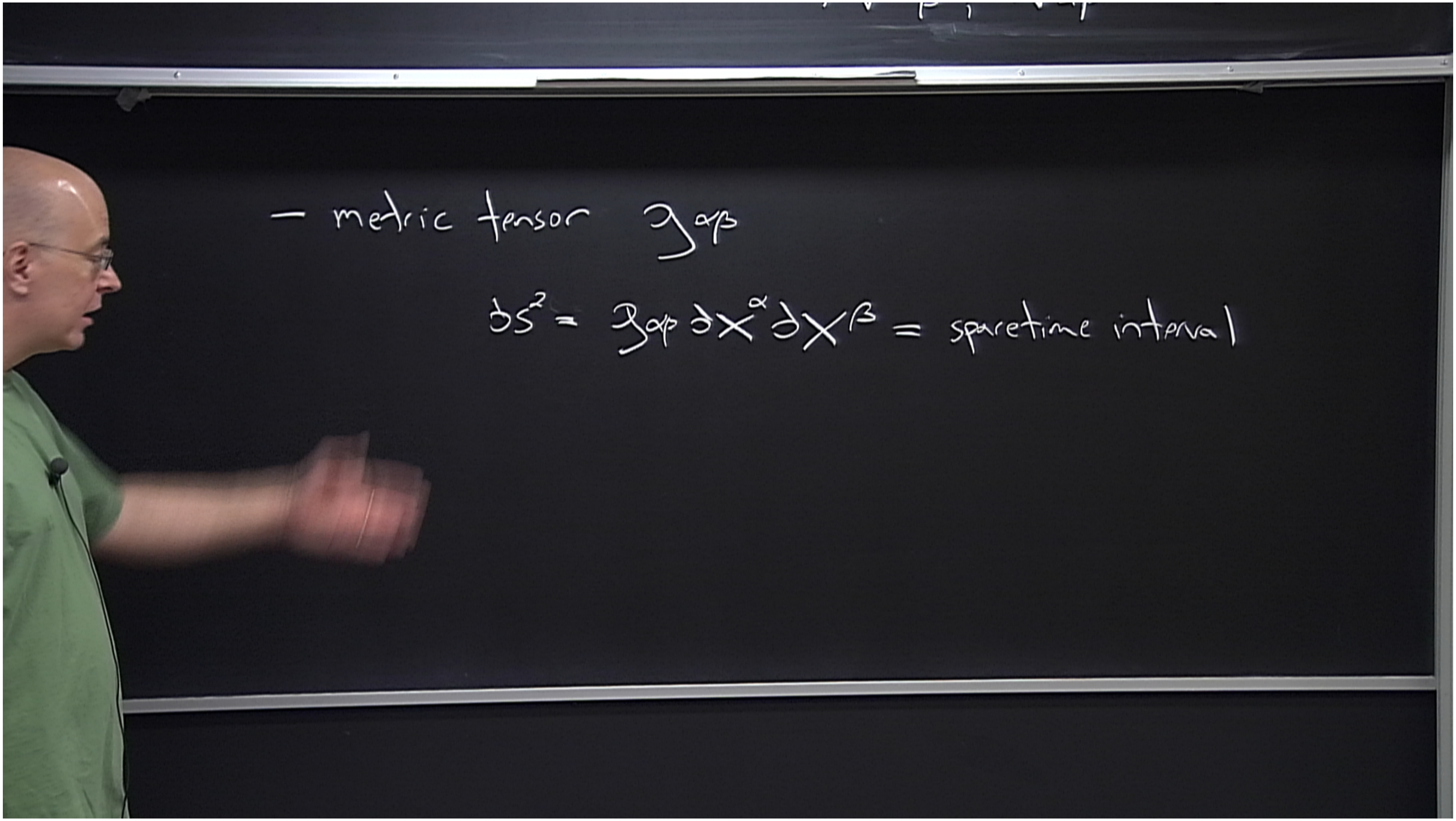

- dual vector (one-form) p_α
(linear operation on vectors yields number)
$T^{\alpha\beta}$, V^α, $W_{\alpha\beta}$

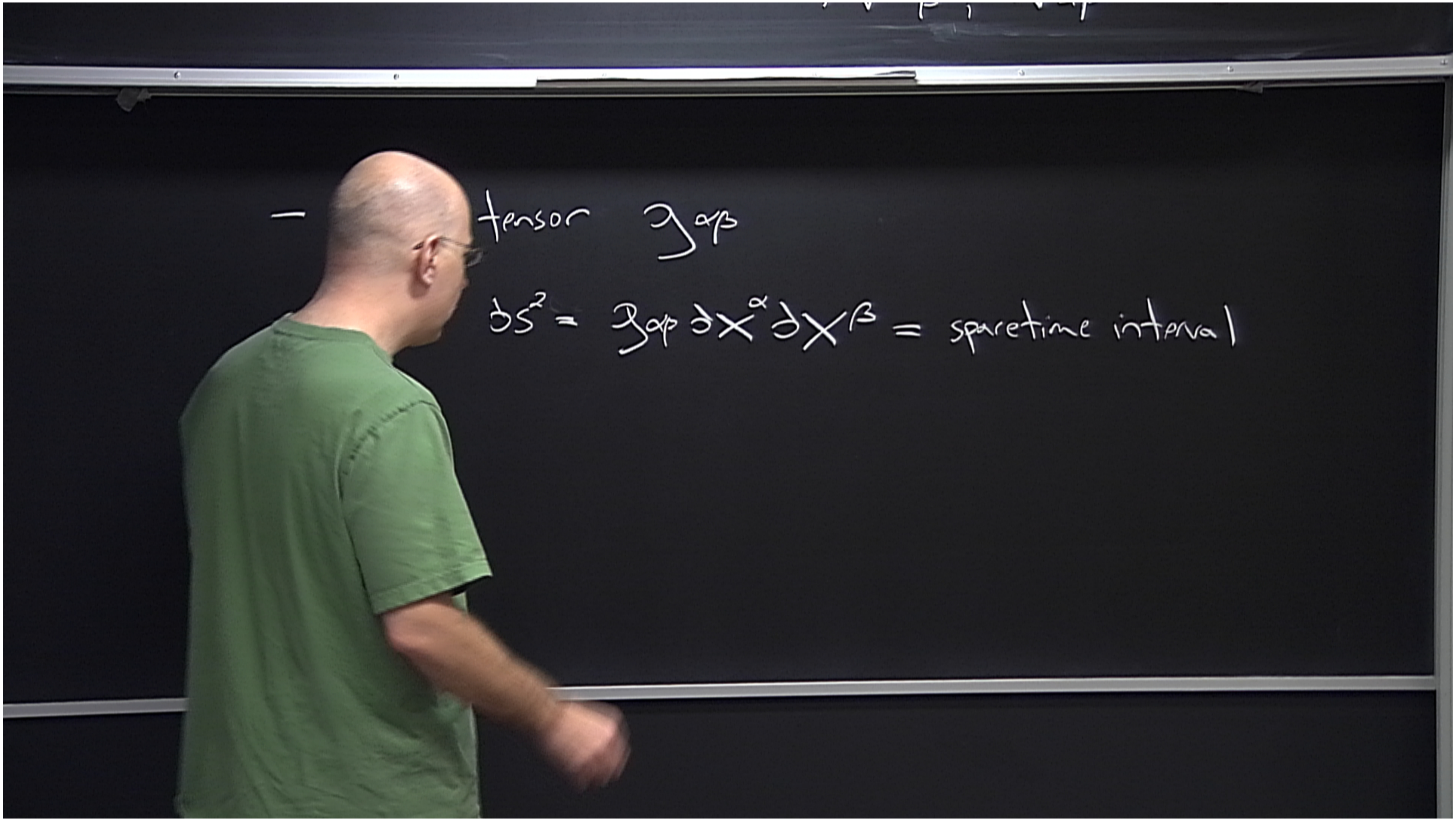

– dual vector (one-form) pα
(linear operation on vectors yields number)
– tensors Tαβ, Vαβ, Wαβ

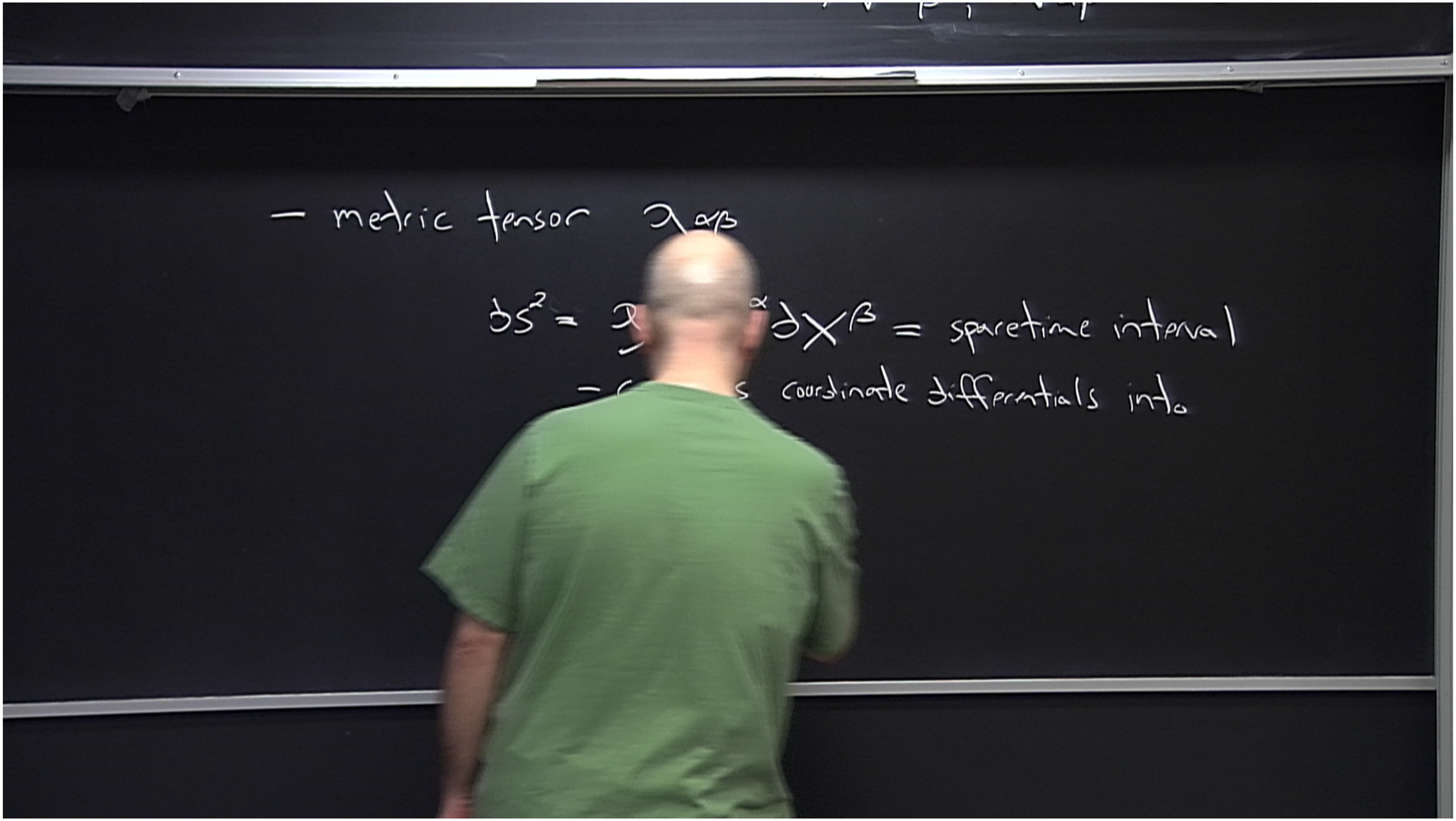

– metric tensor $g_{\alpha\beta}$

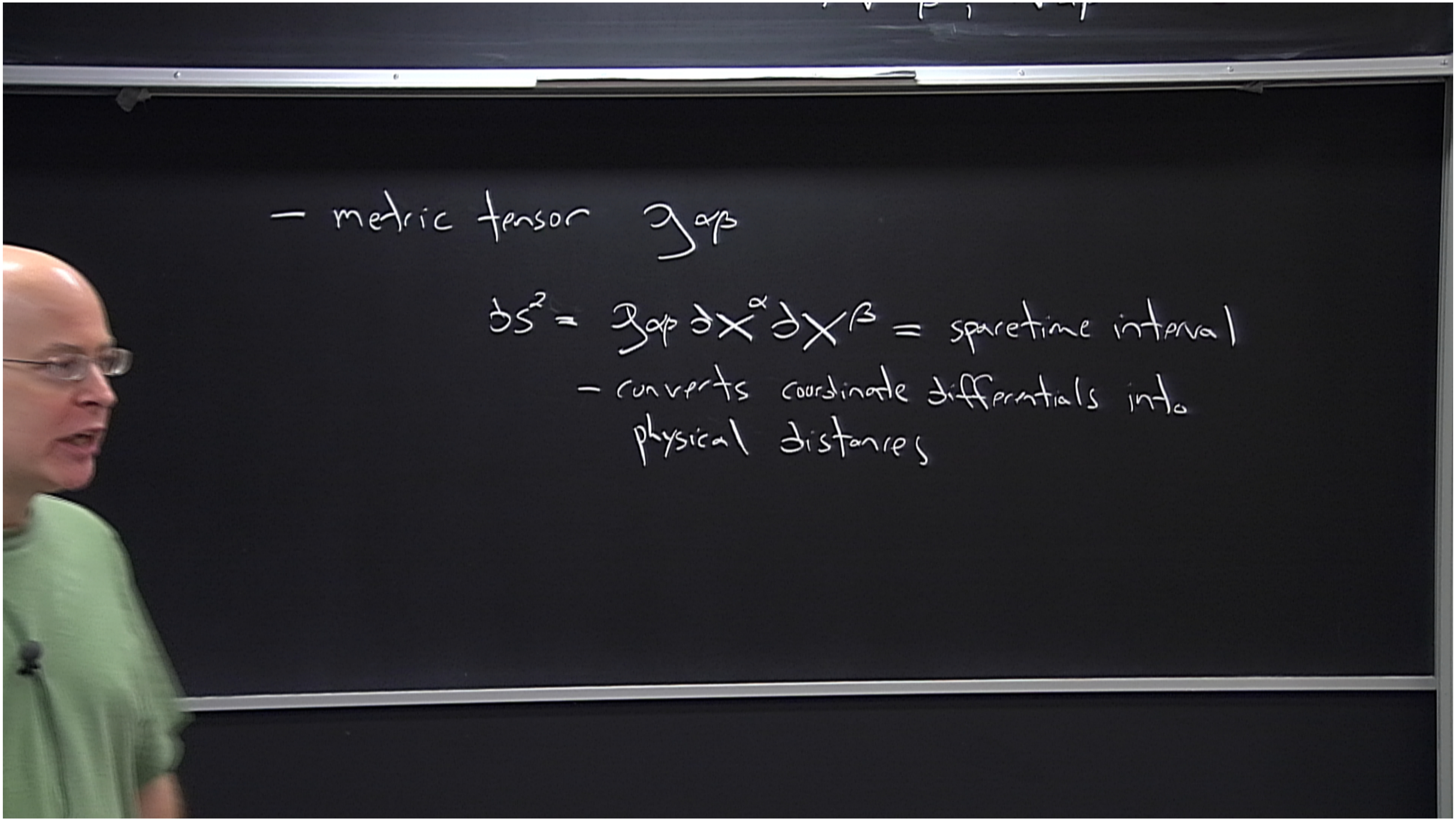

— metric tensor gαβ
ds² = gαβ dx^α dx^β

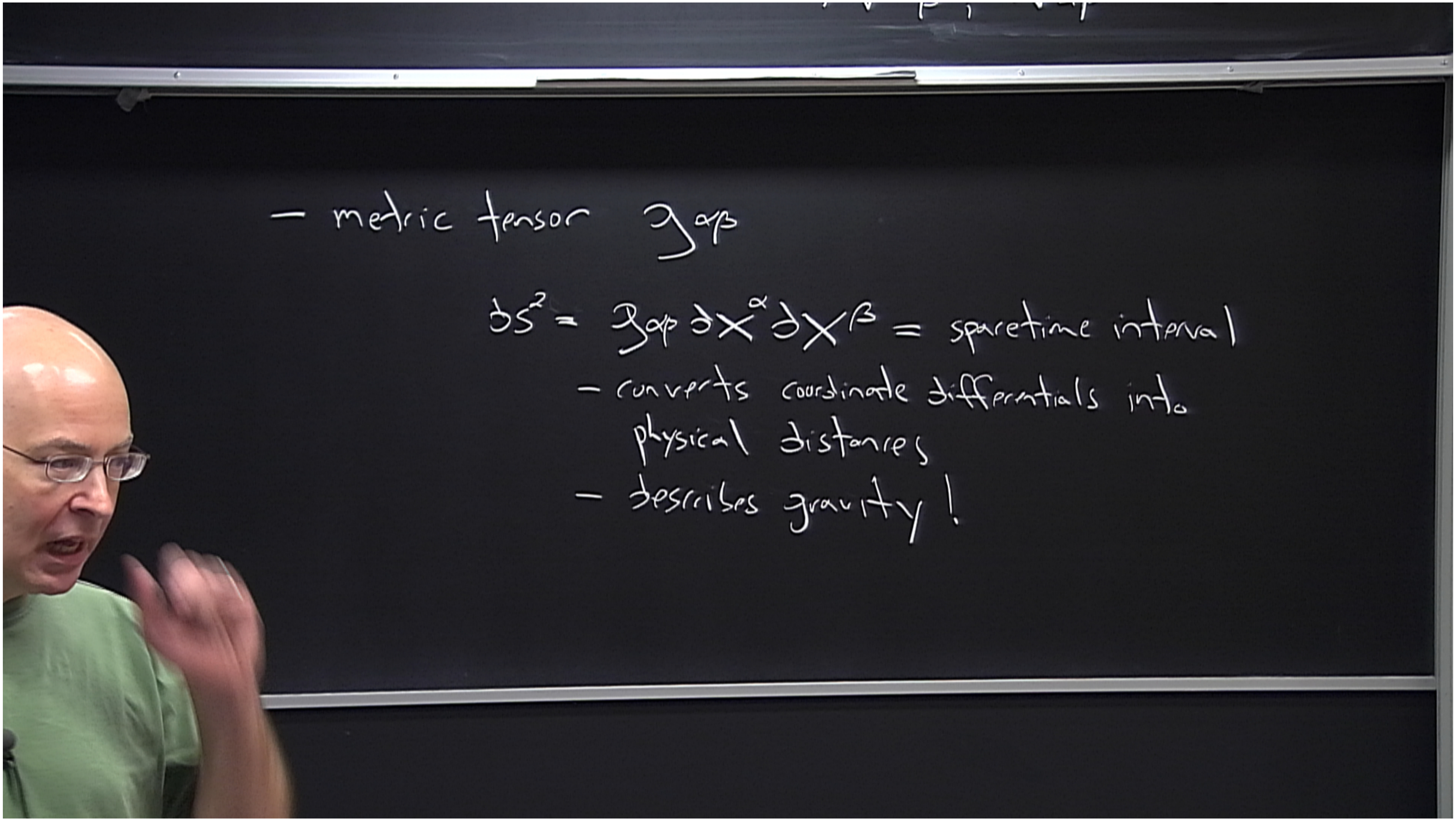

- metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ = spacetime interval

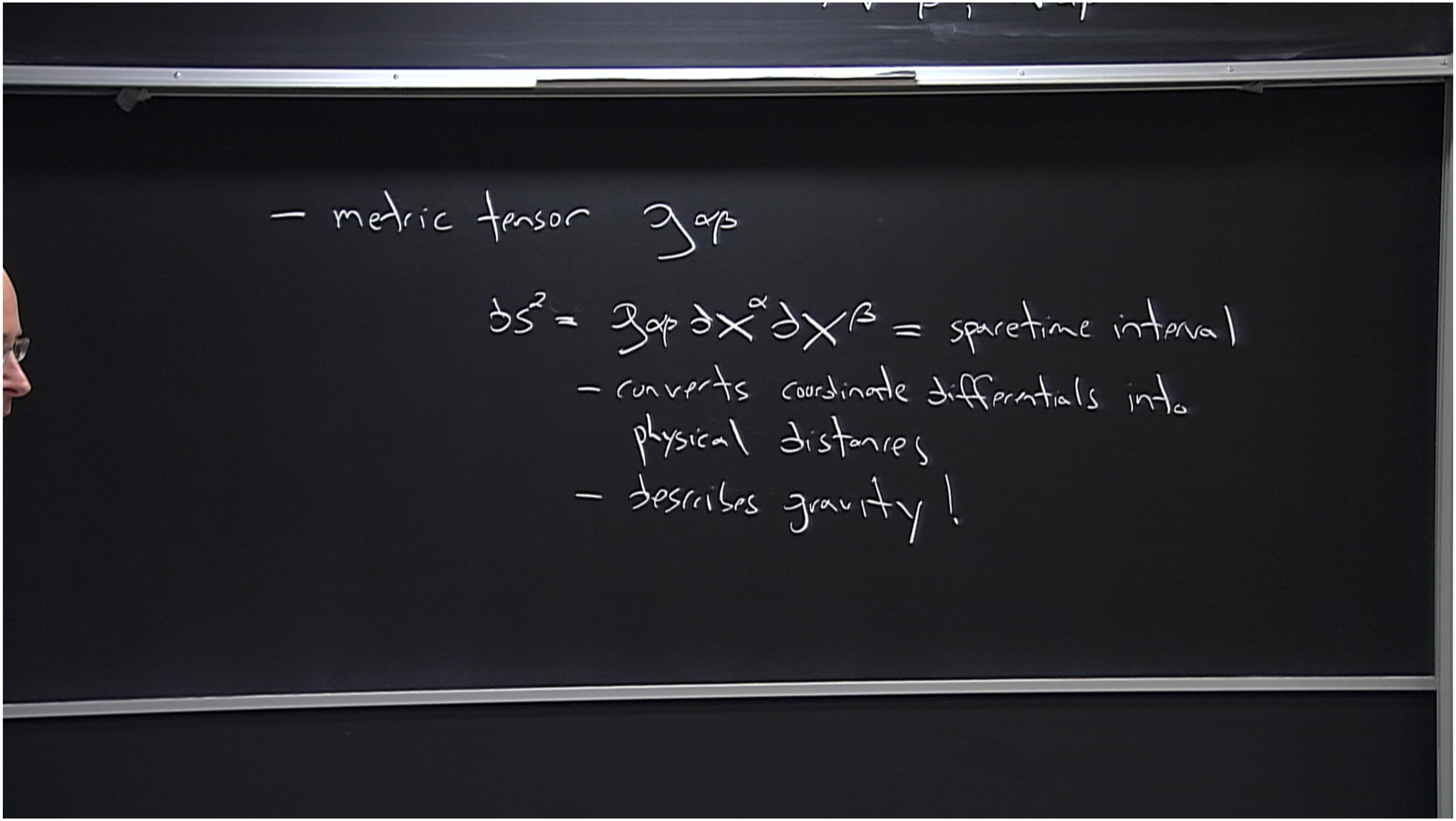

— metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ = spacetime interval

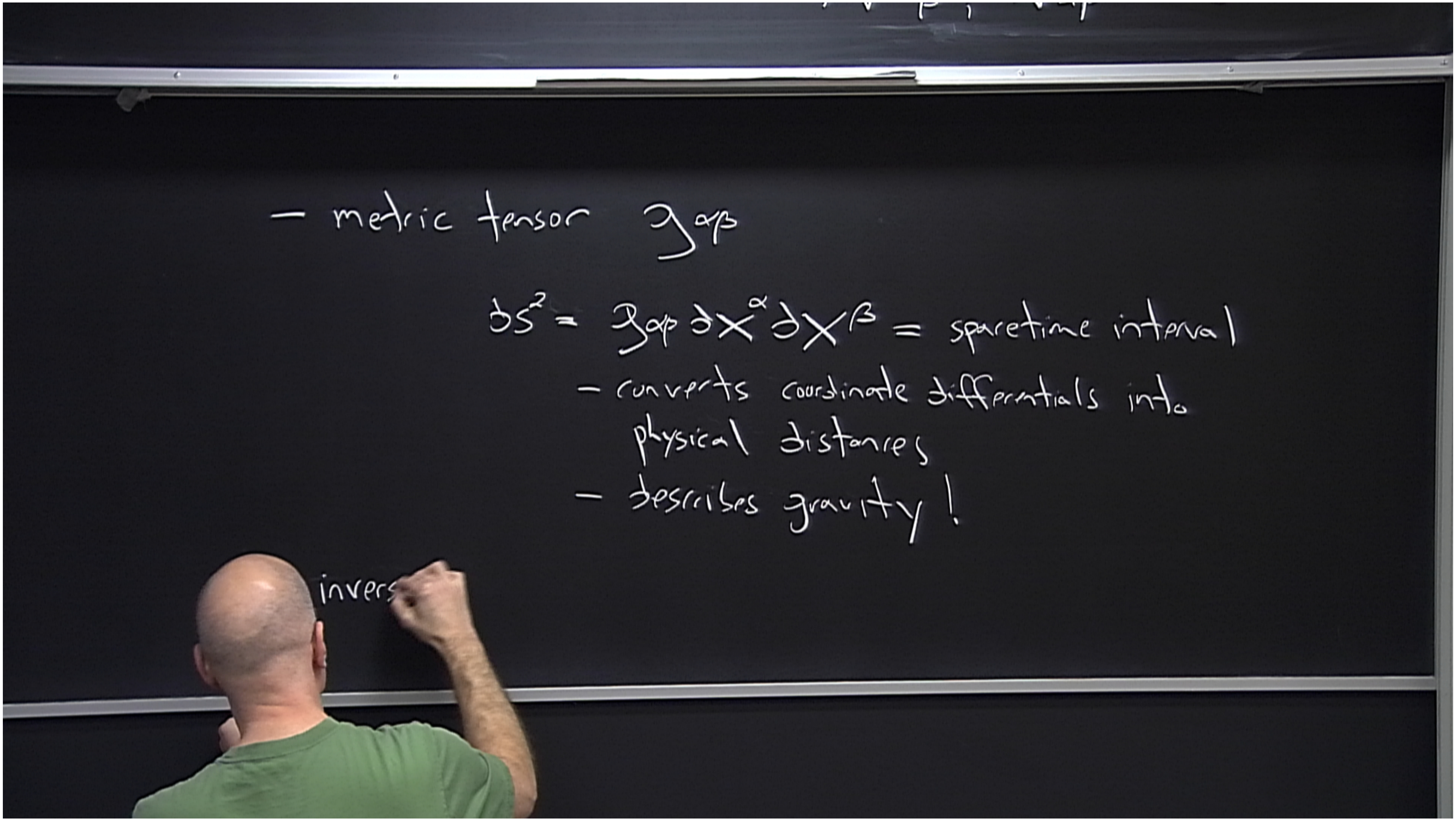

– tensor gαβ
ds² = gαβ dxα dxβ = spacetime interval


— metric tensor gαβ
dXβ = spacetime interval
coordinate differentials into


- metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ = spacetime interval
- converts coordinate differentials into
physical distances


- metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ = spacetime interval
- converts coordinate differentials into
physical distances
- describes gravity!


- metric tensor gαβ
ds² = gαβ dx^α dx^β = spacetime interval
- converts coordinate differentials into physical distances
- describes gravity!

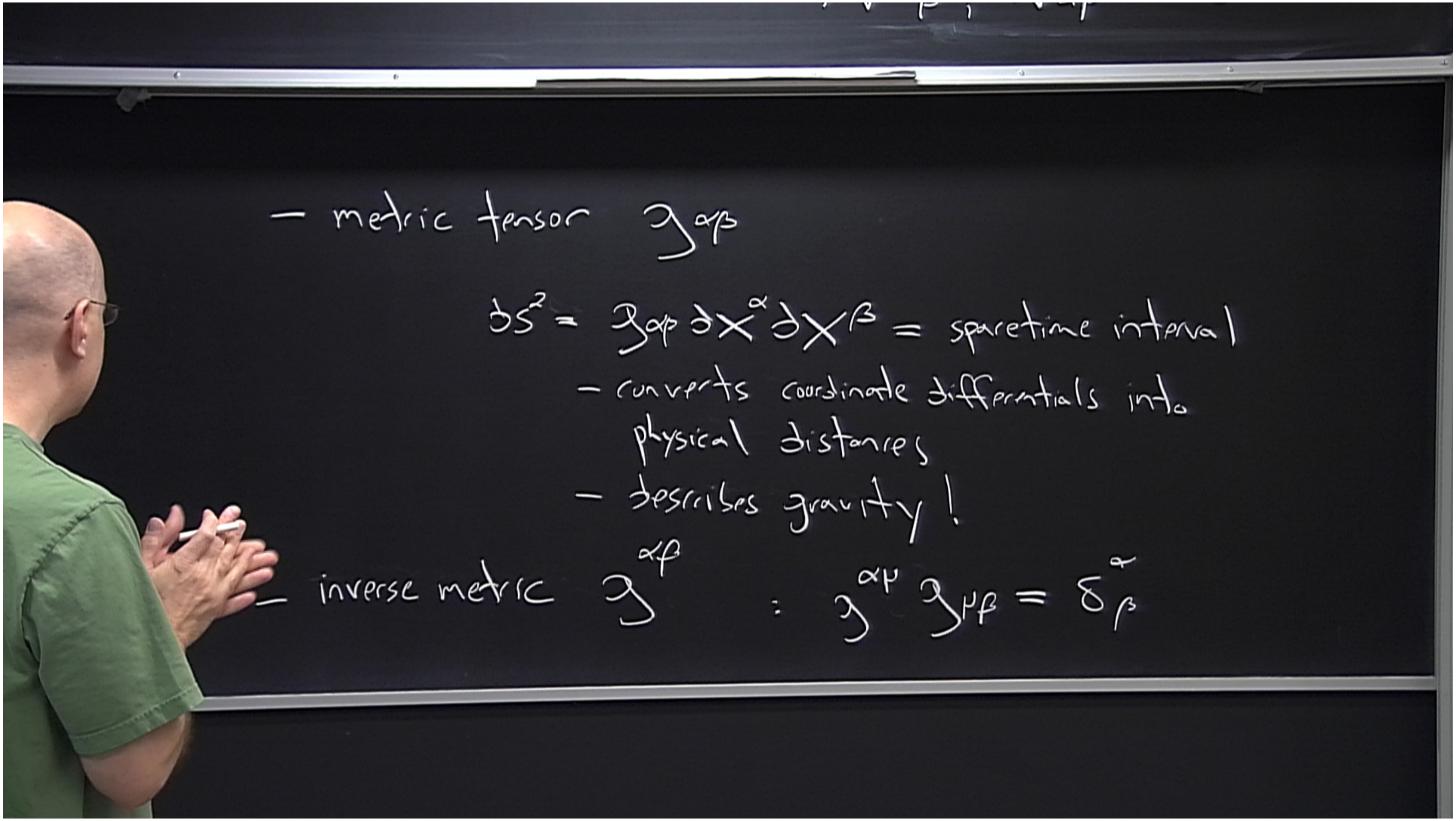

- metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ = spacetime interval
- converts coordinate differentials into physical distances
- describes gravity!

- metric tensor $g_{\alpha\beta}$

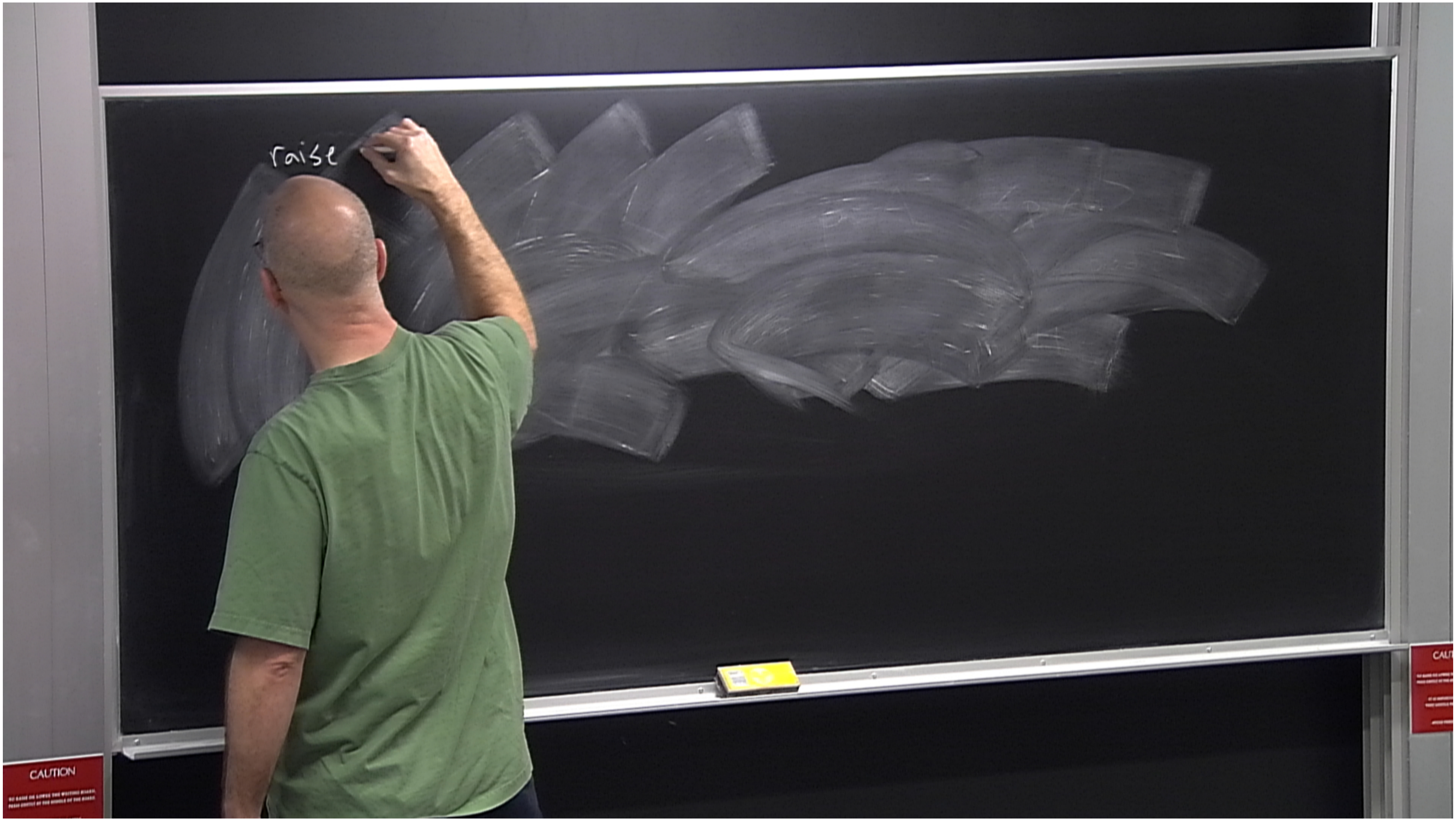
  $$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = \text{spacetime interval}$$

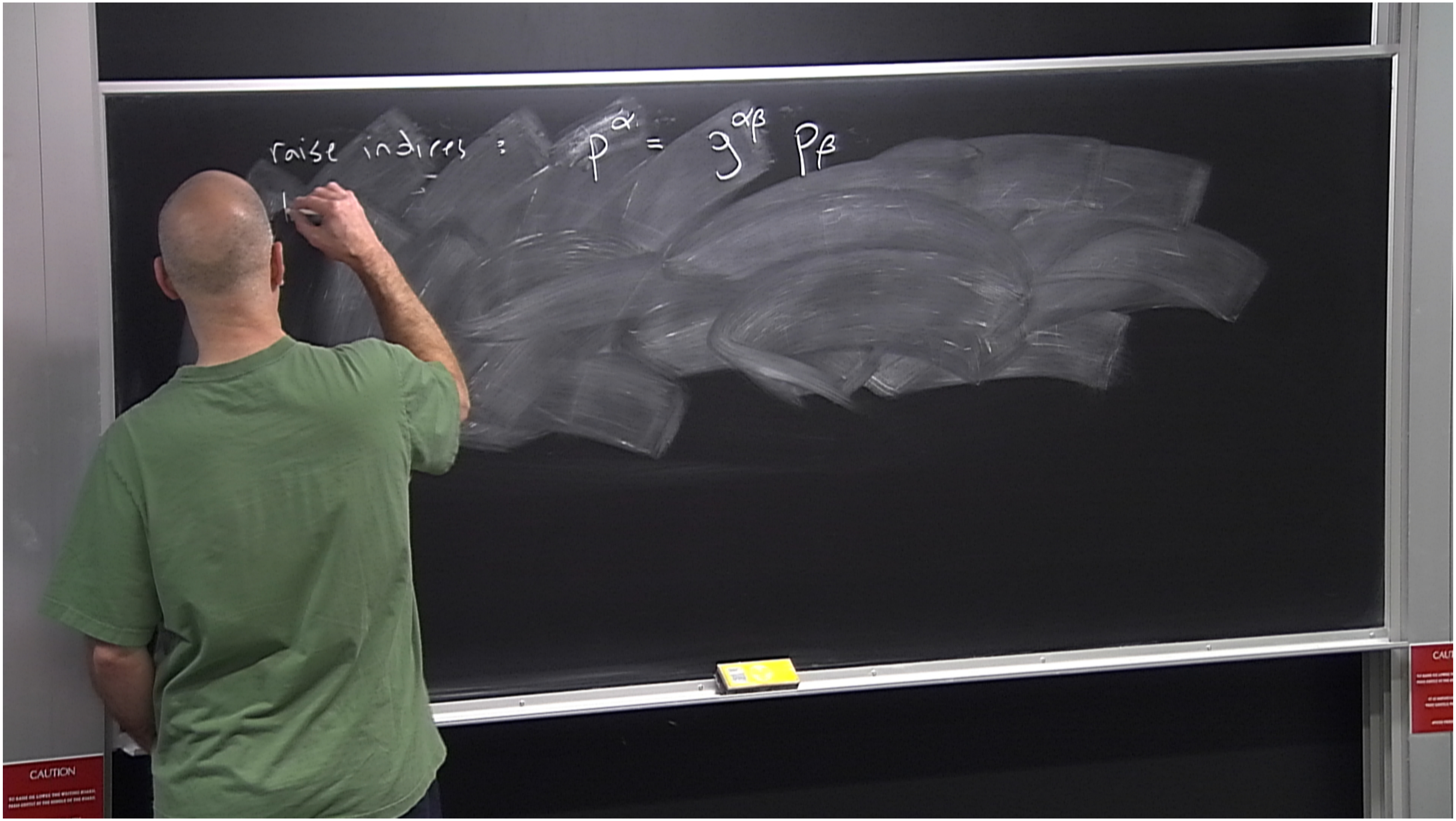
  - converts coordinate differentials into physical distances
  - describes gravity!

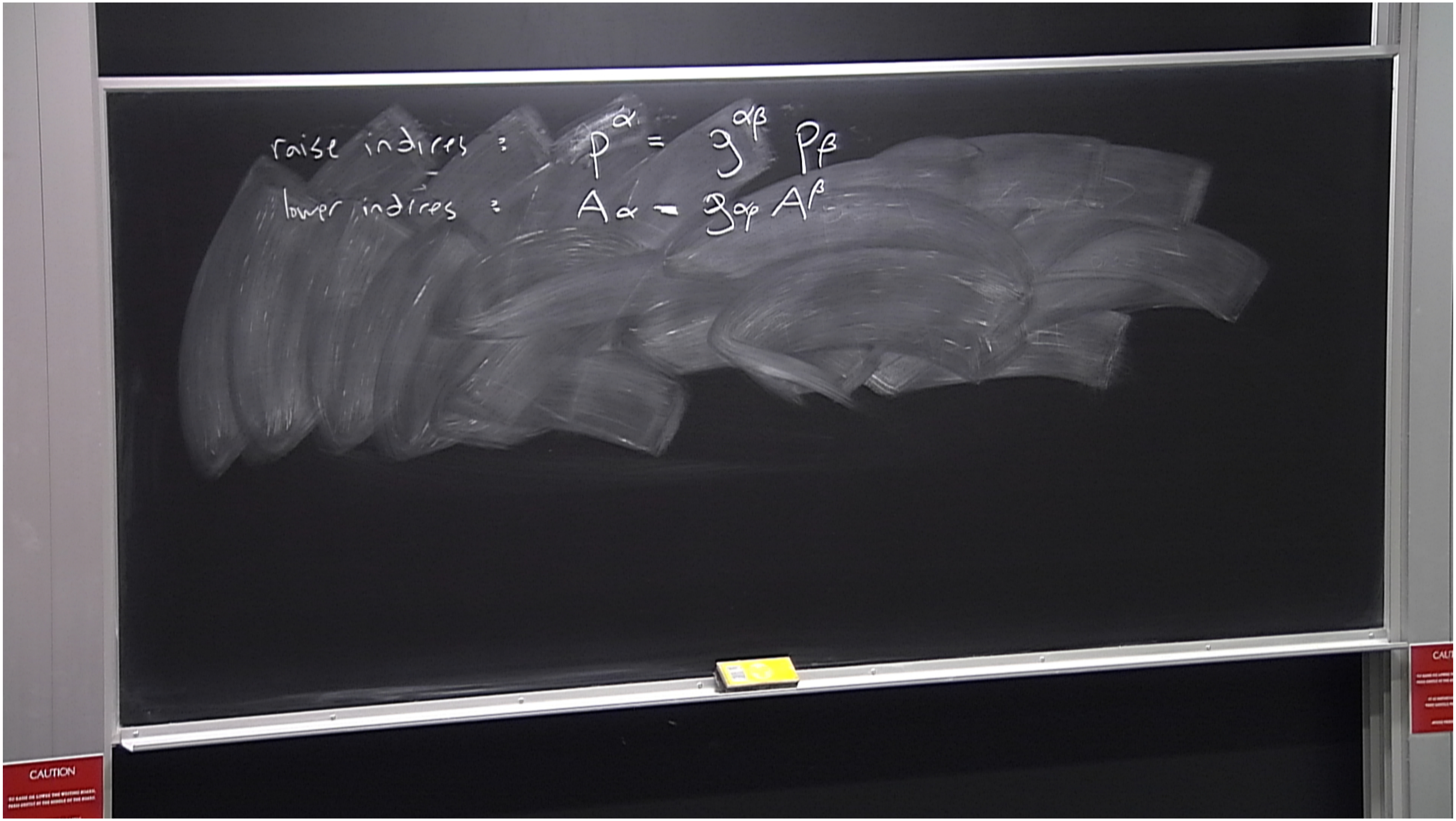
- inverse metric $g^{\alpha\beta}$

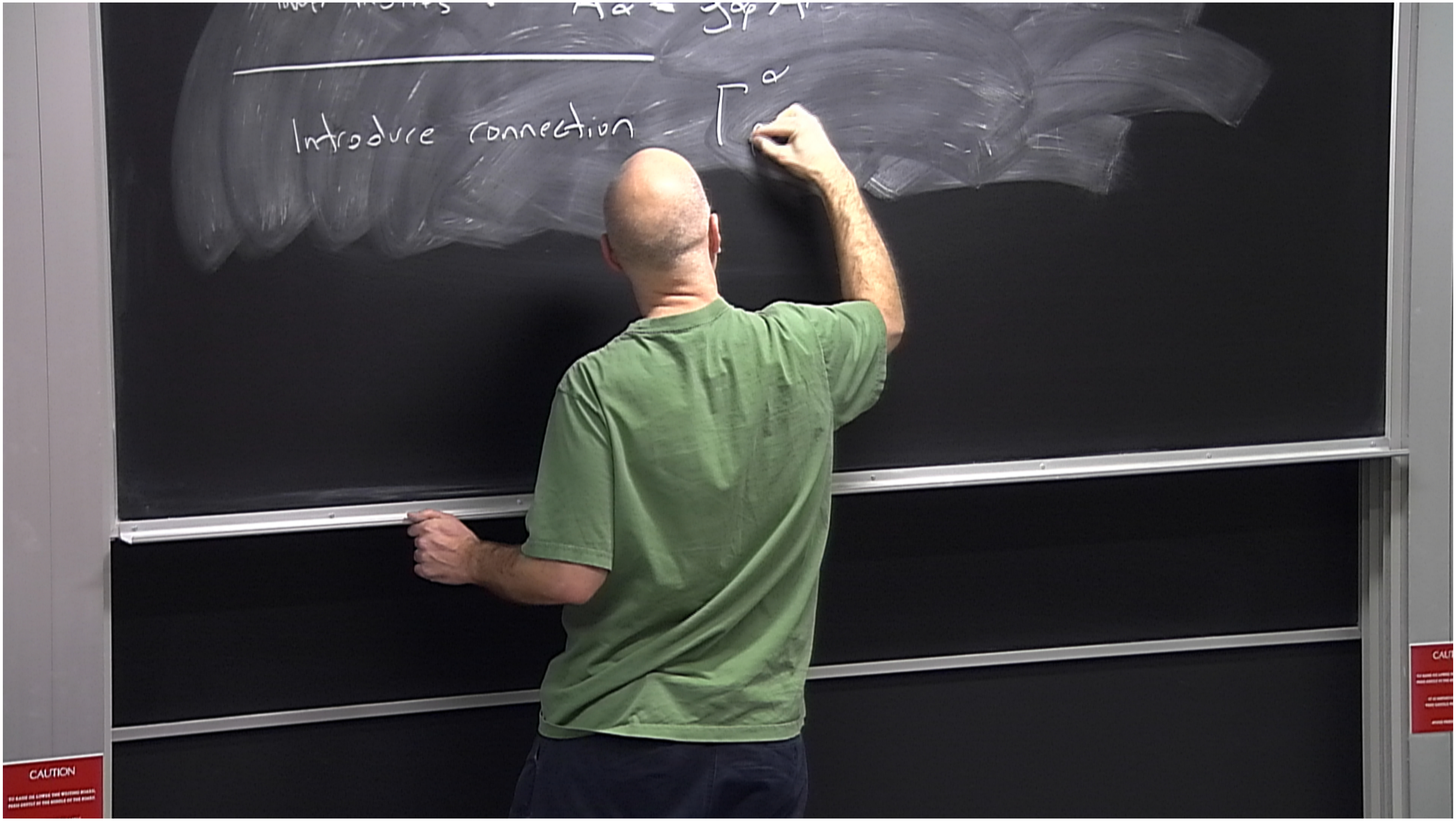


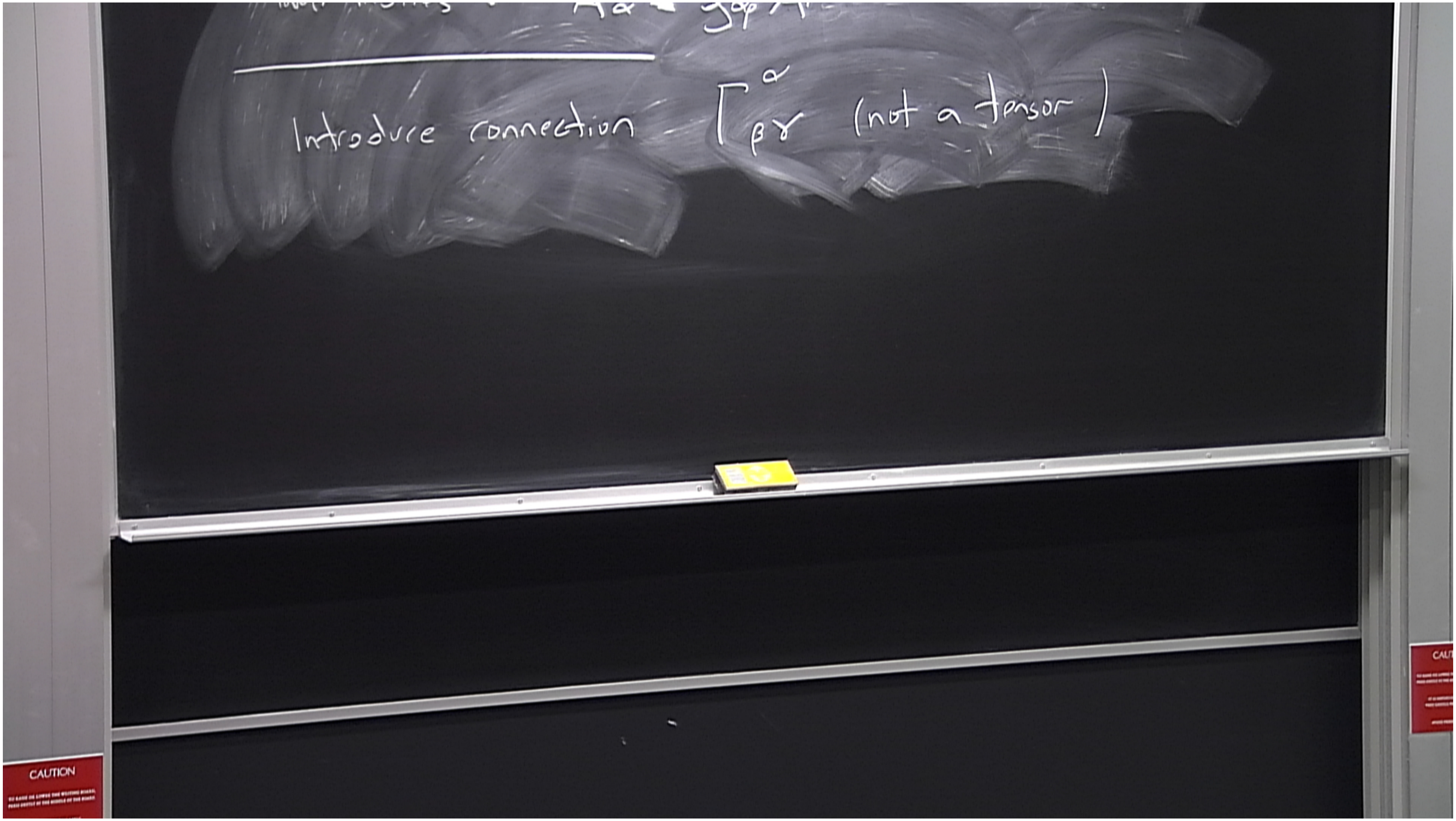
– metric tensor $g_{\alpha\beta}$
$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ = spacetime interval
– converts coordinate differentials into physical distances
– describes gravity!
– inverse metric $g^{\alpha\beta}$: $g^{\alpha\mu} g_{\mu\beta} = \delta^\alpha_\beta$

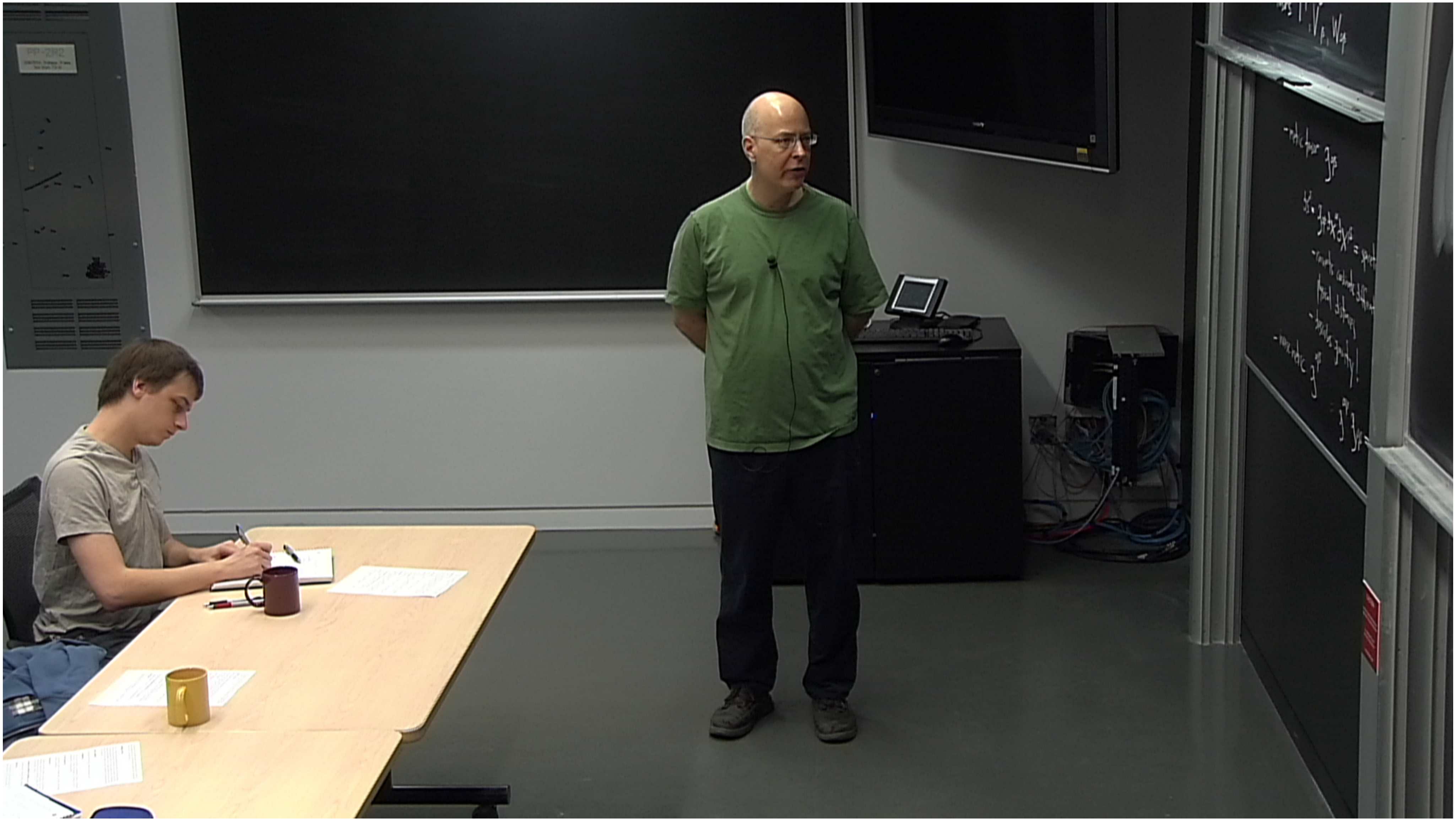

raise

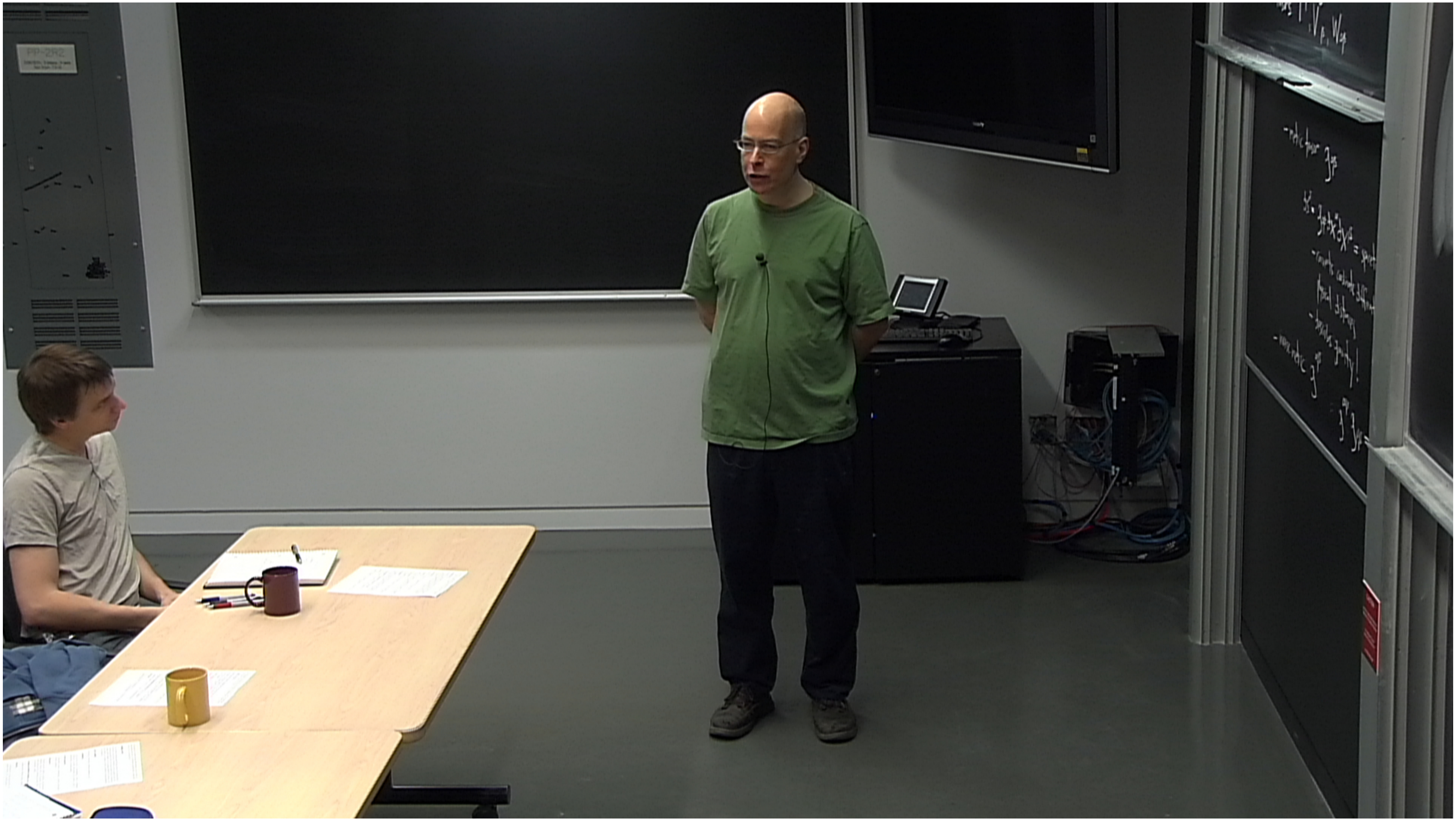

raise indices :
p^α = g^αβ p_β

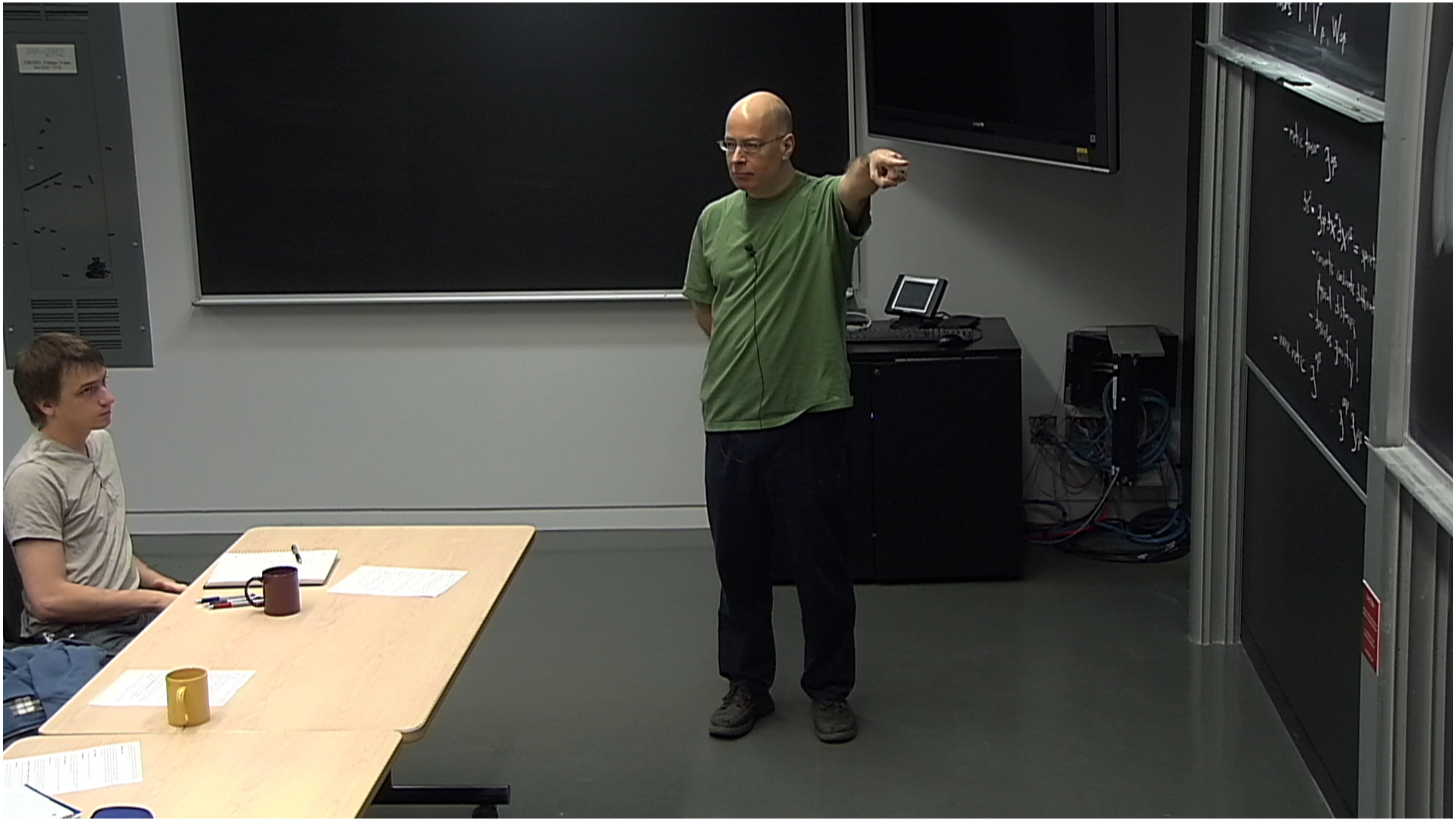

raise indices : $p^{\alpha} = g^{\alpha\beta} p_{\beta}$
lower indices : $A_{\alpha} = g_{\alpha\beta} A^{\beta}$

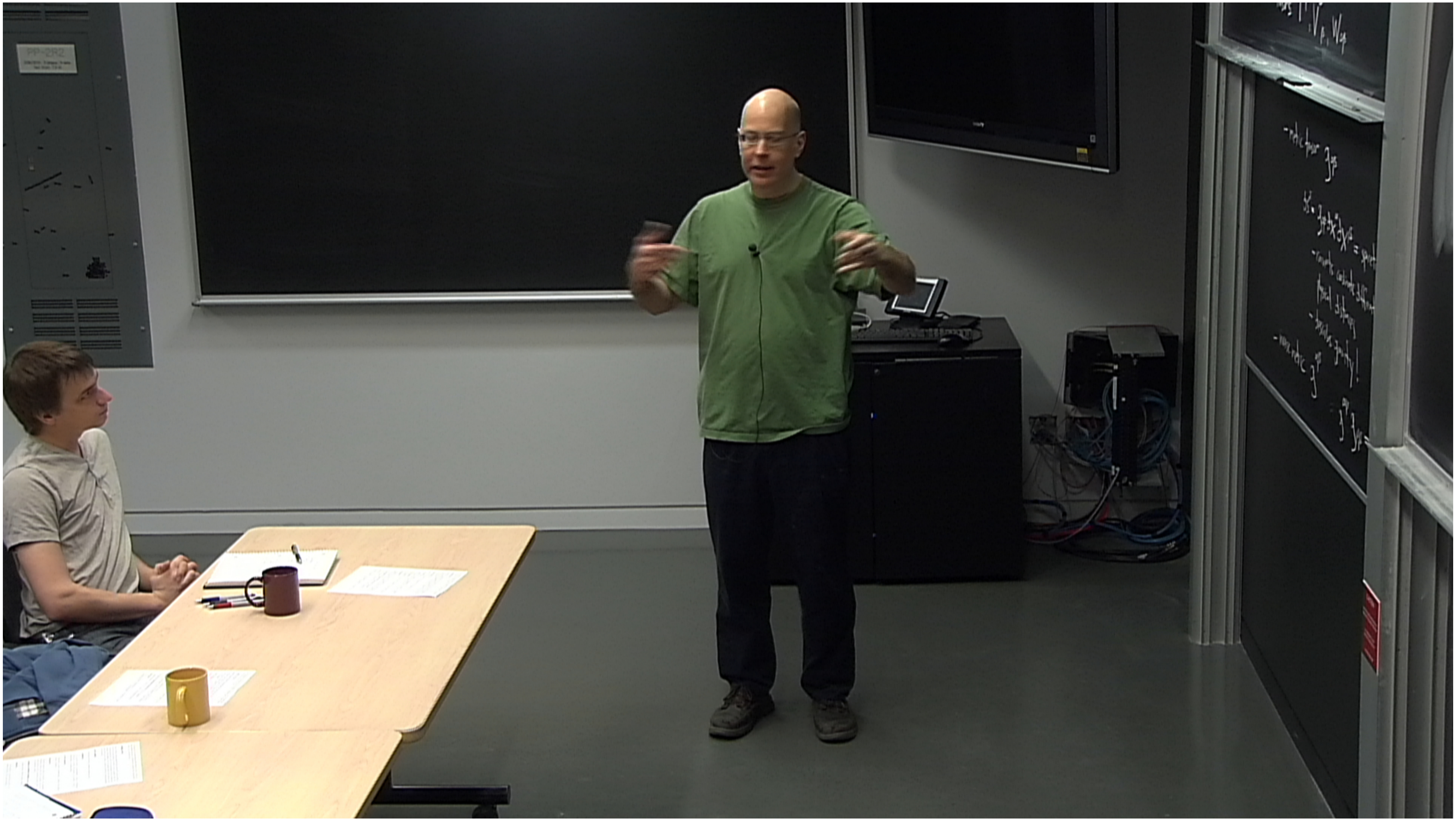

Introduce connection Γ^α
CAUTION

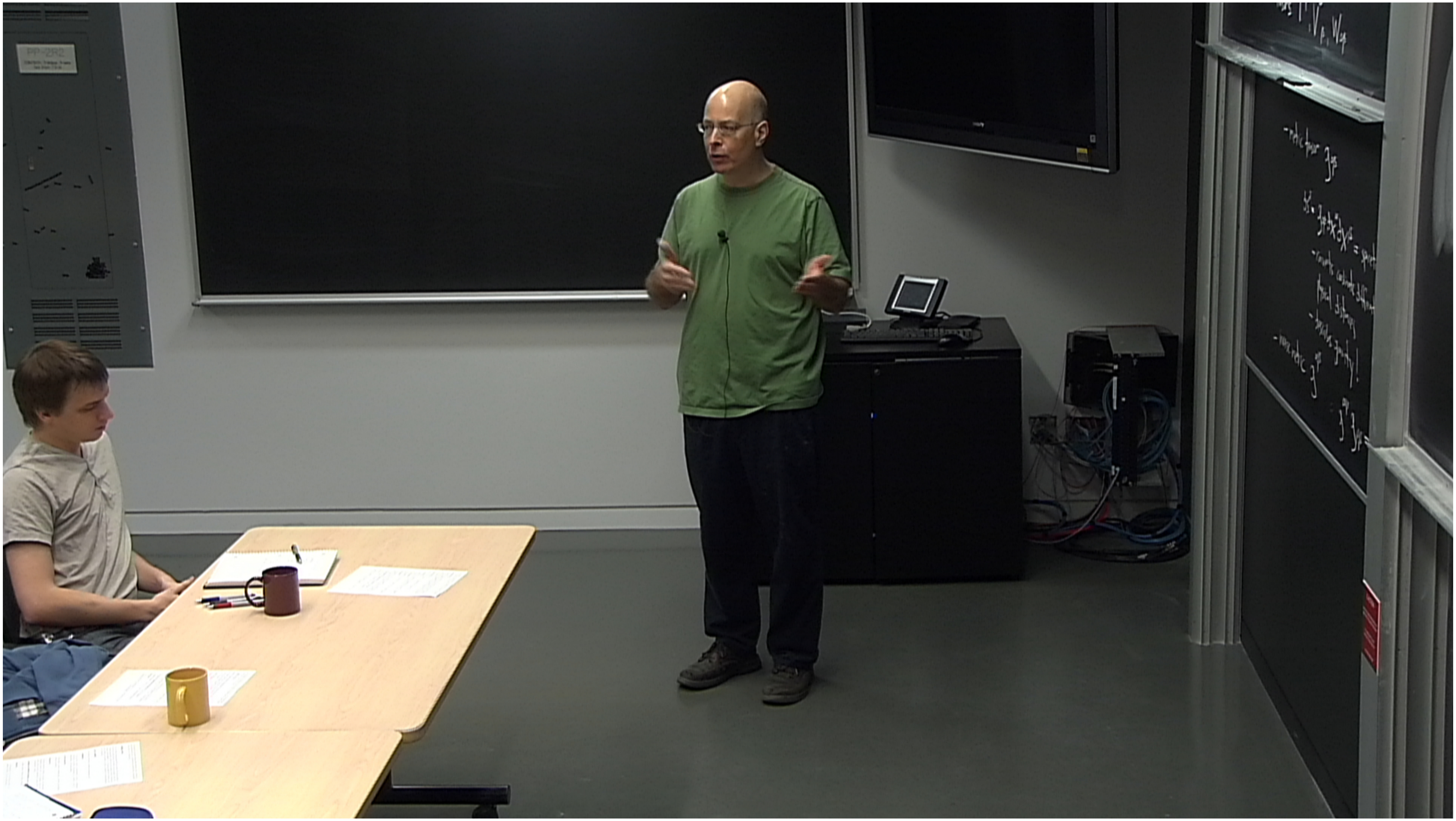

Introduce connection $\Gamma^{\alpha}_{\beta\gamma}$ (not a tensor)
CAUTION

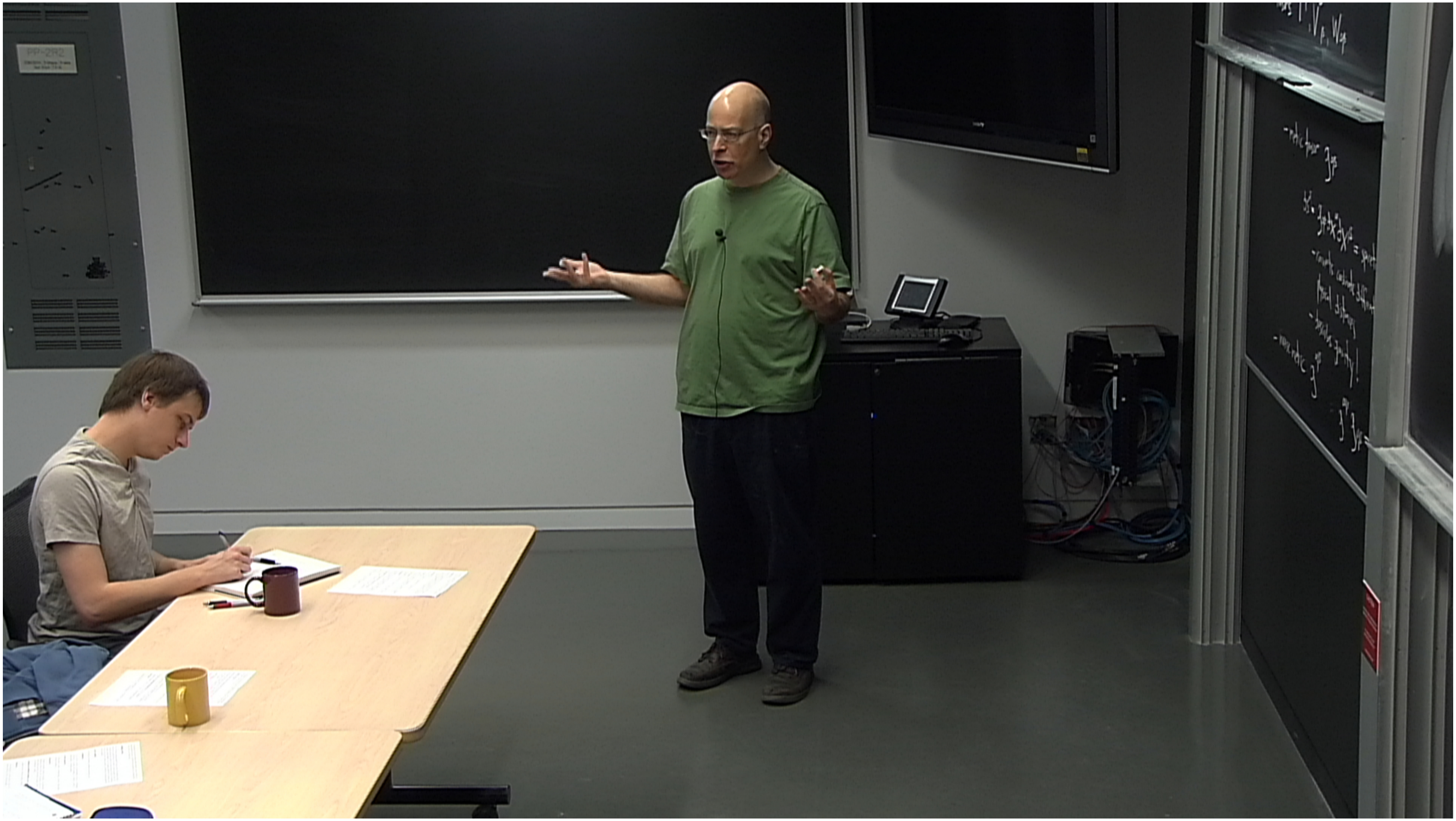


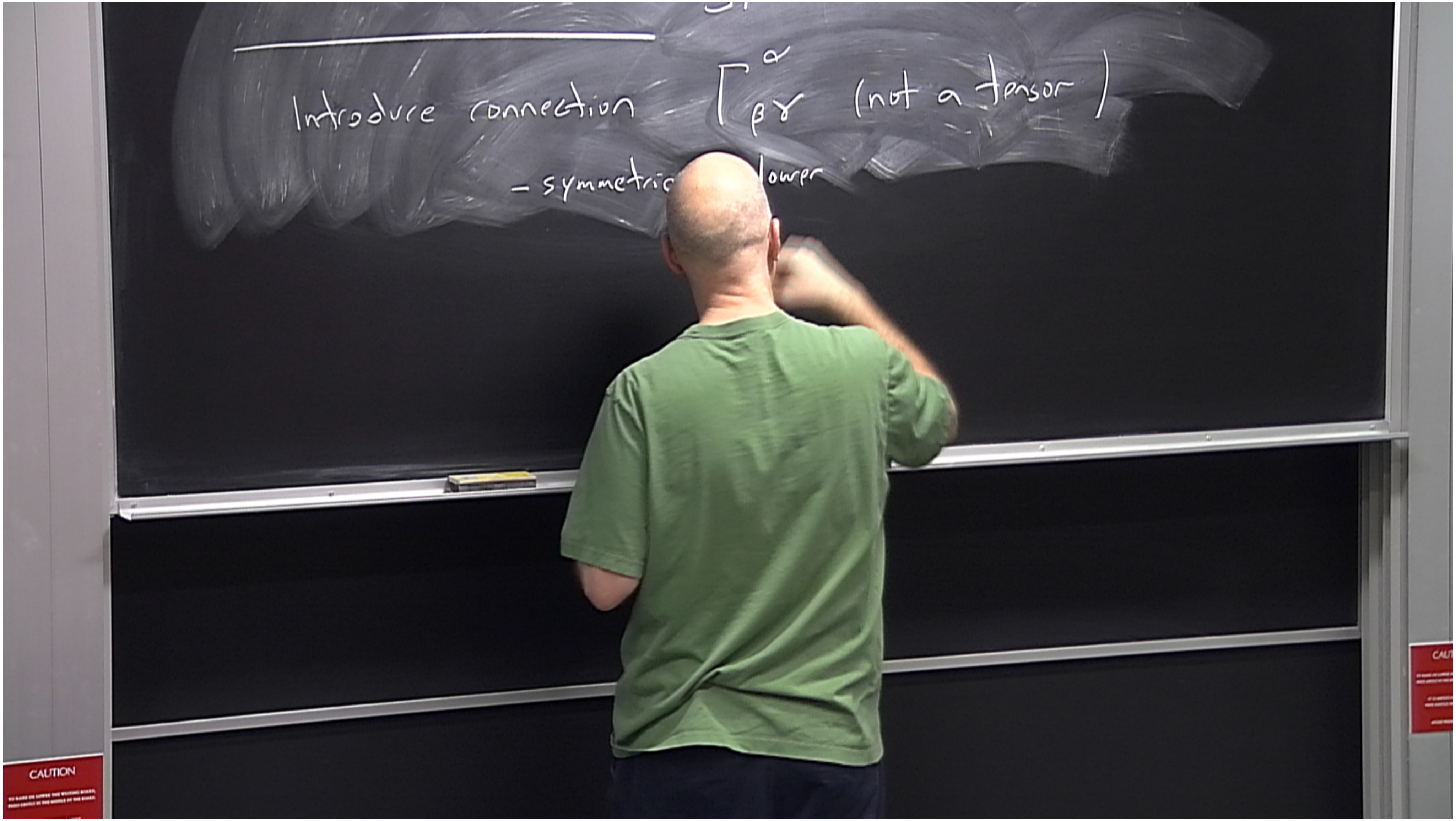


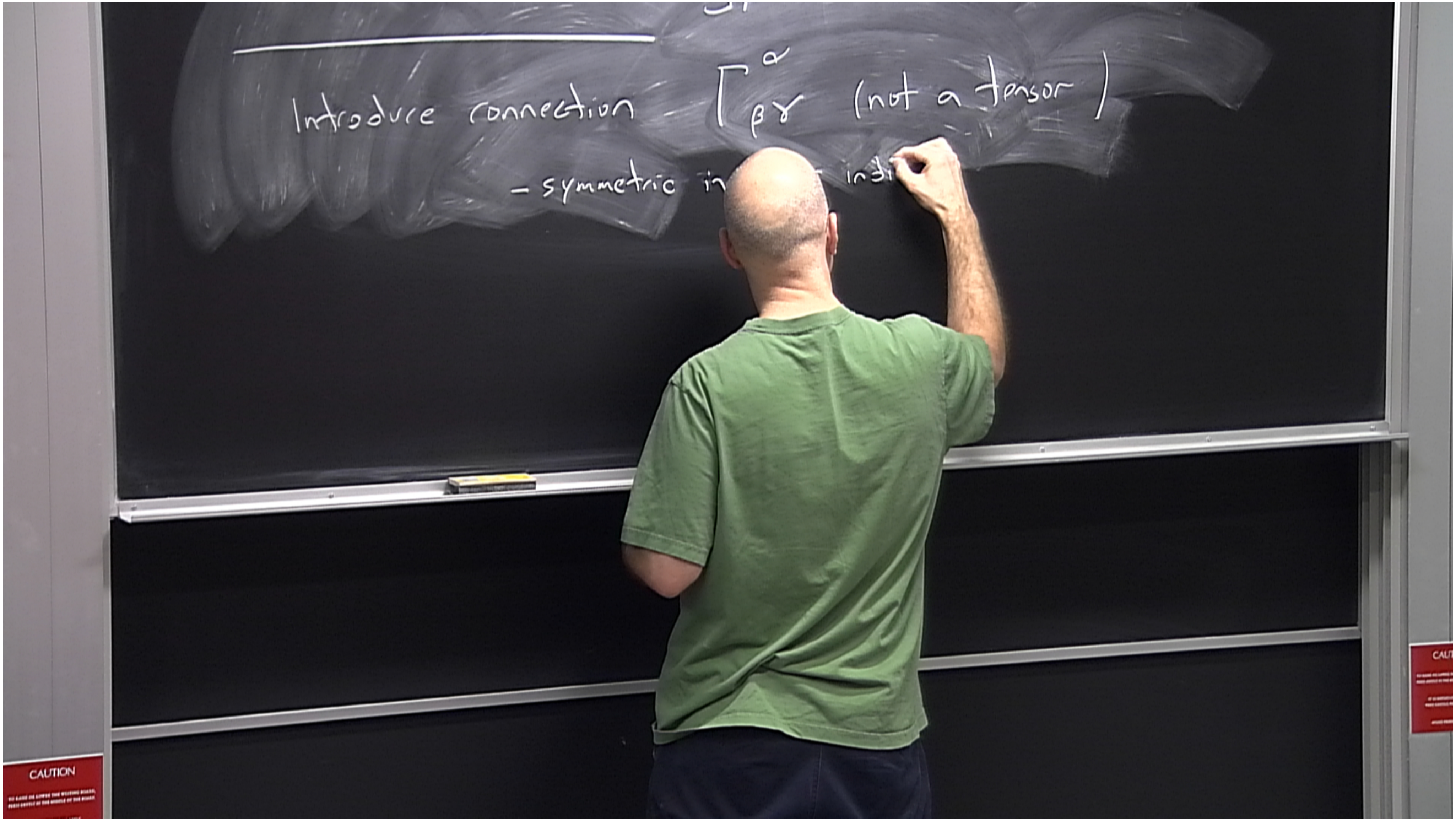


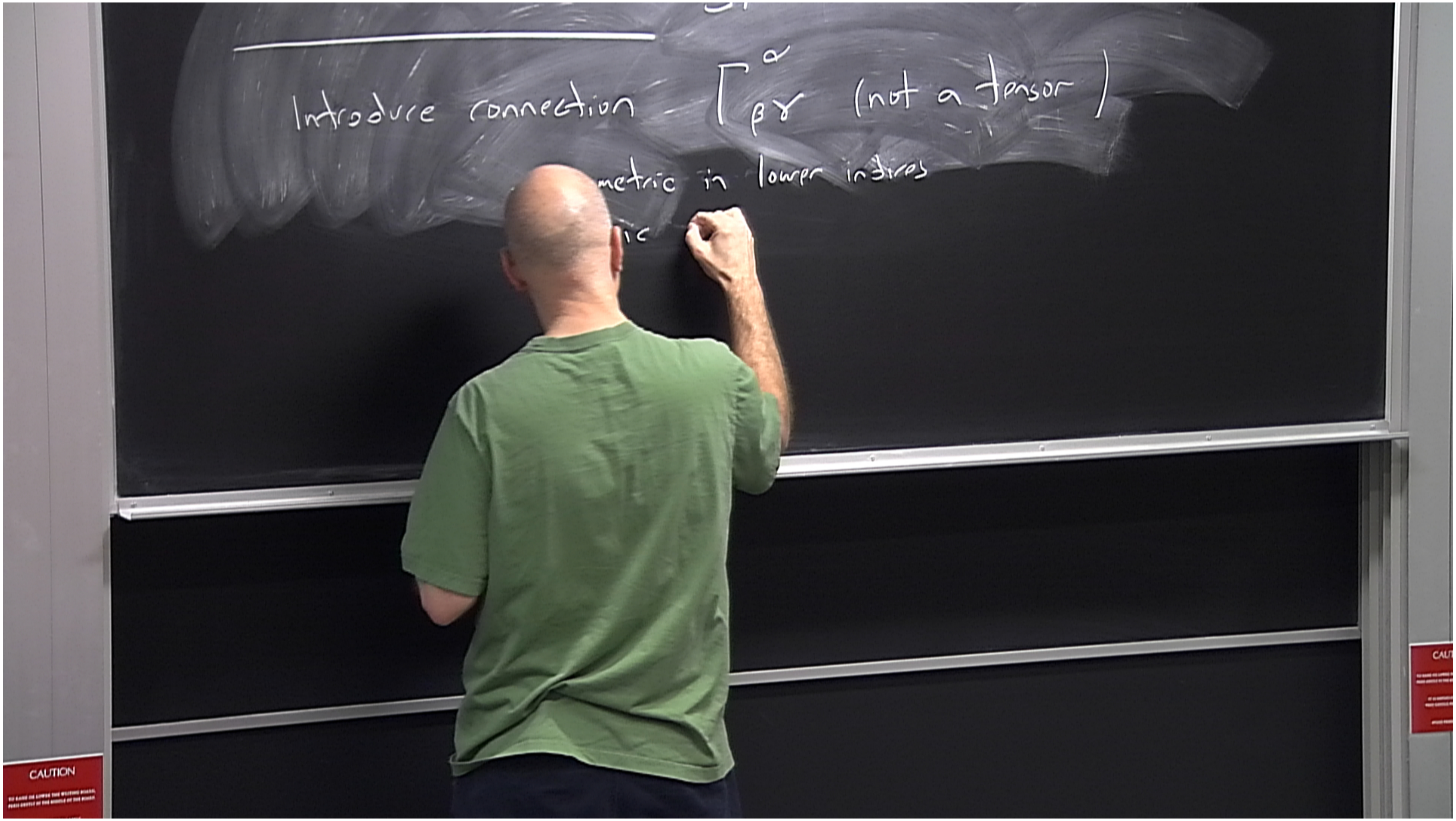


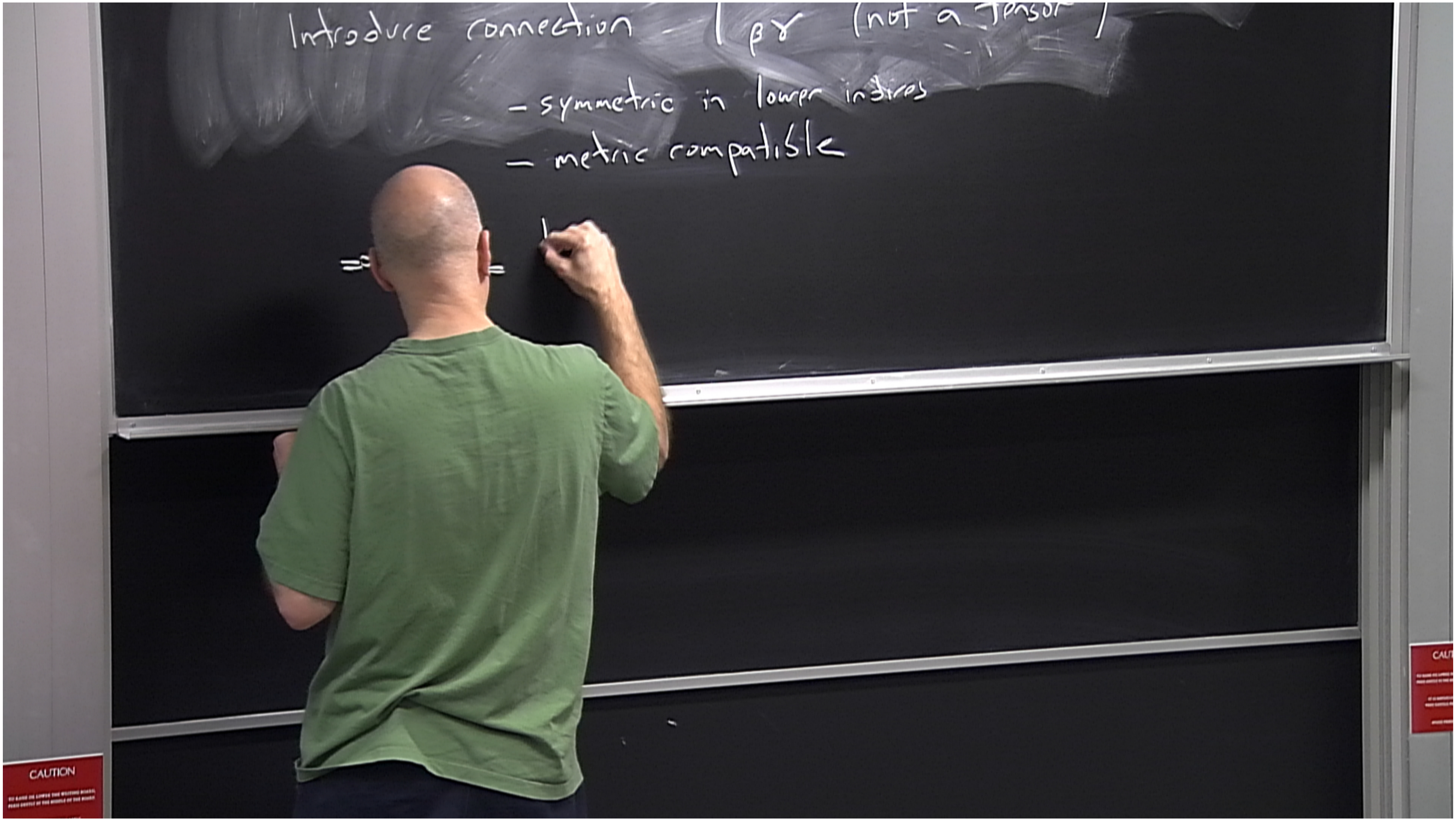


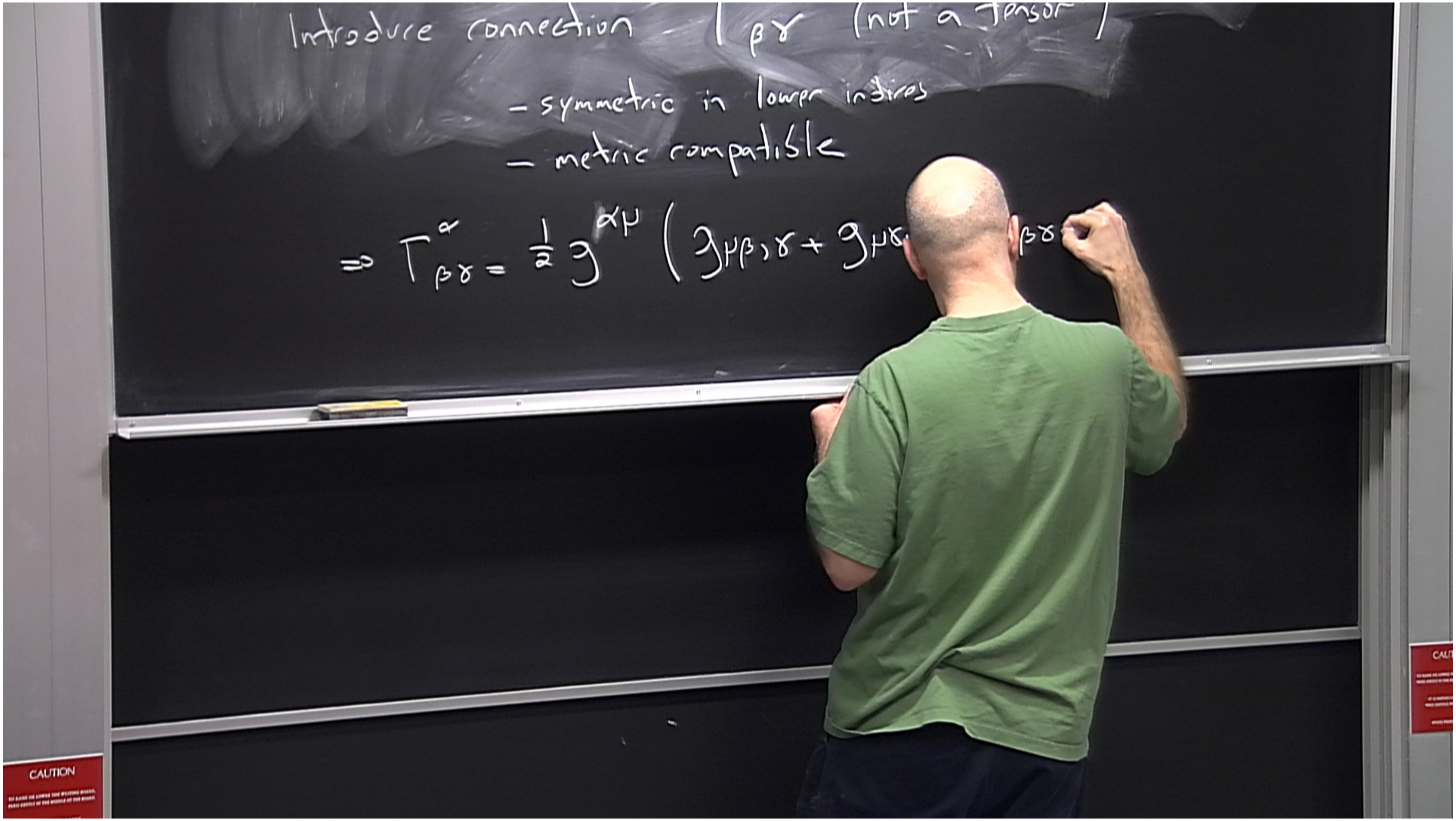



Introduce connection $\Gamma^{\alpha}_{\beta\gamma}$ (not a tensor)
- symmetric ... lower
CAUTION

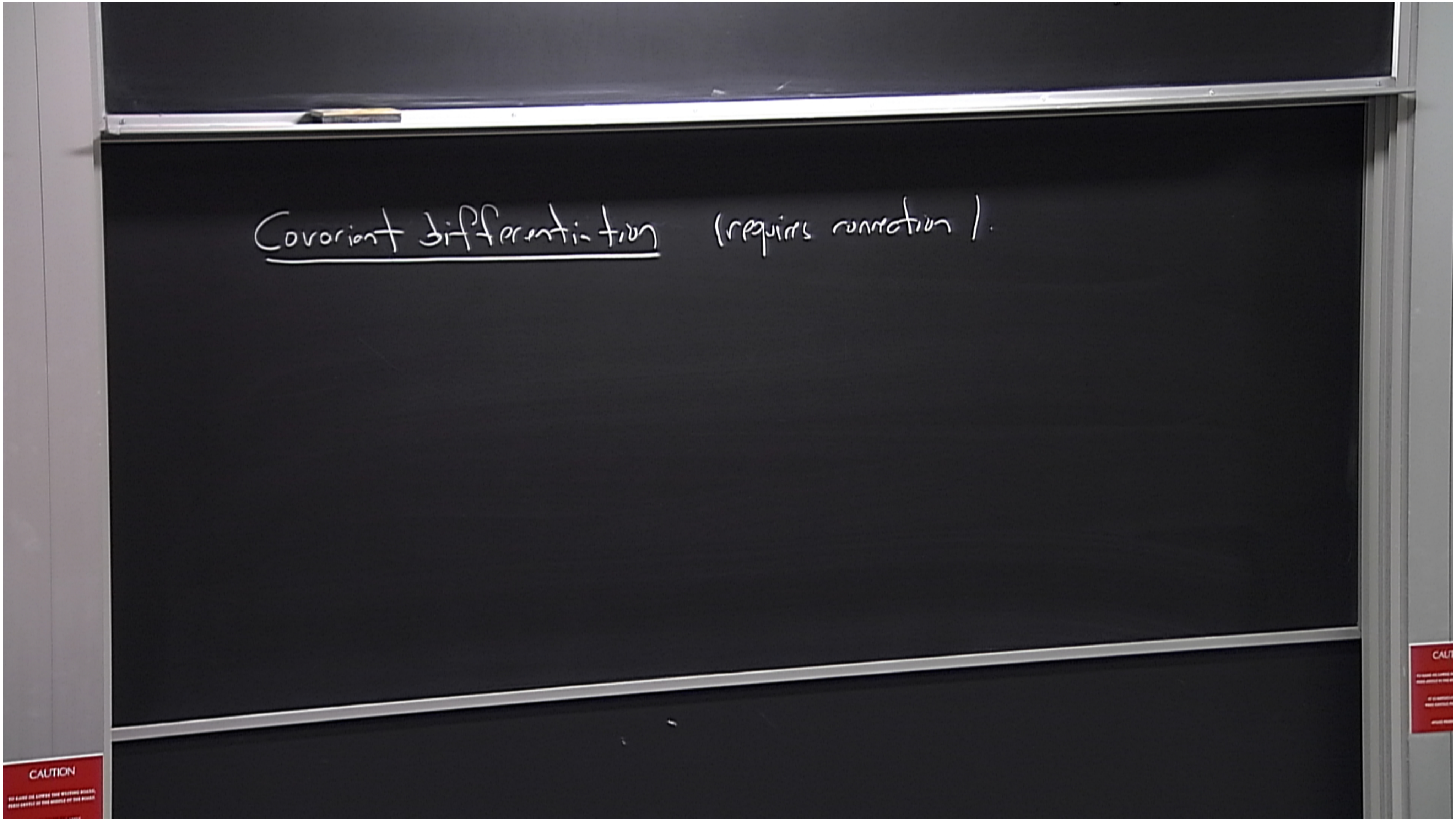

Introduce connection $\Gamma^{\alpha}_{\beta\gamma}$ (not a tensor)
- symmetric in
CAUTION

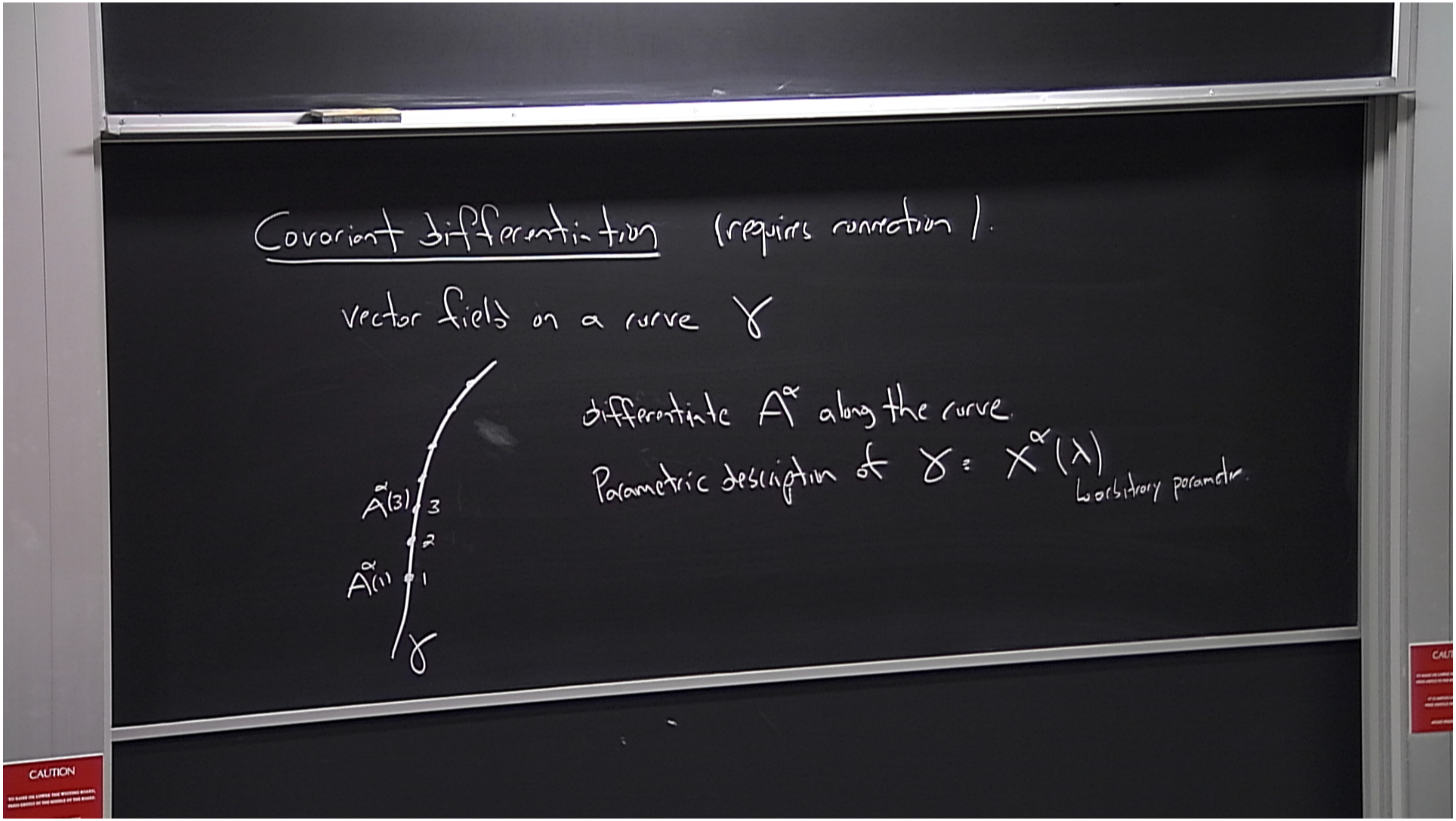

Introduce connection $\Gamma^{\alpha}_{\beta\gamma}$ (not a tensor)
metric in lower indires

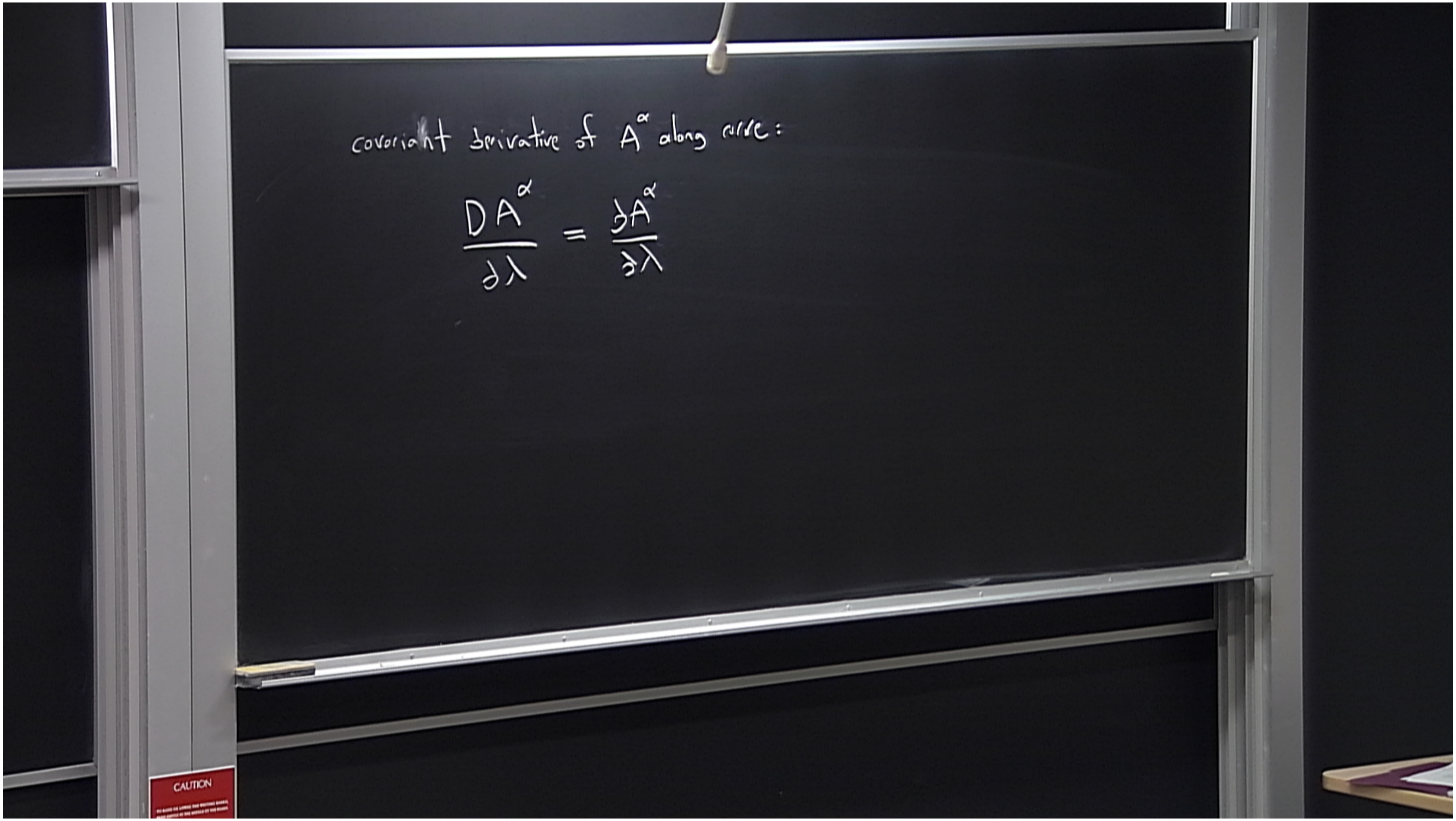

Introduce connection
(not a tensor)
- symmetric in lower indices
- metric compatible

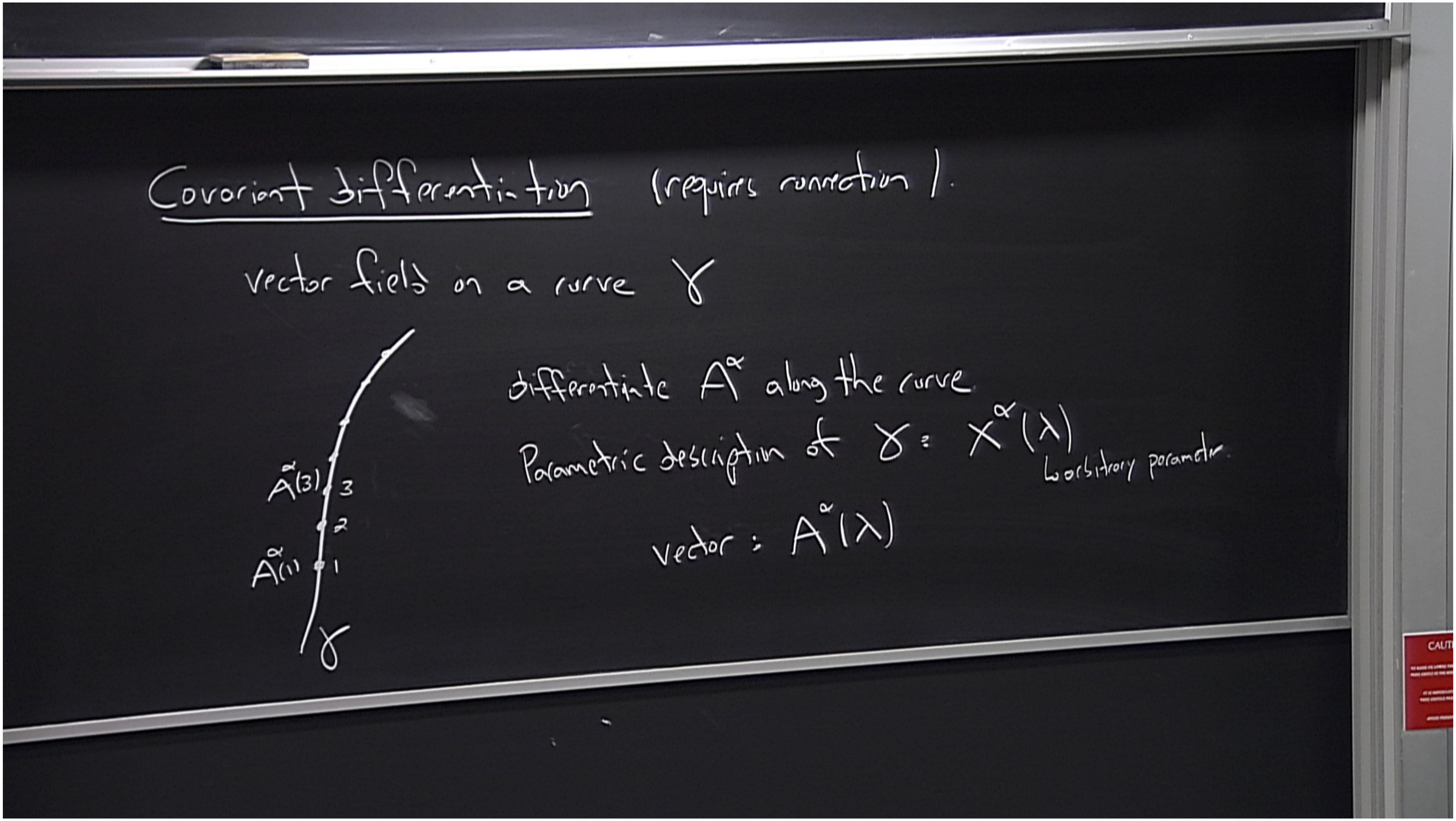


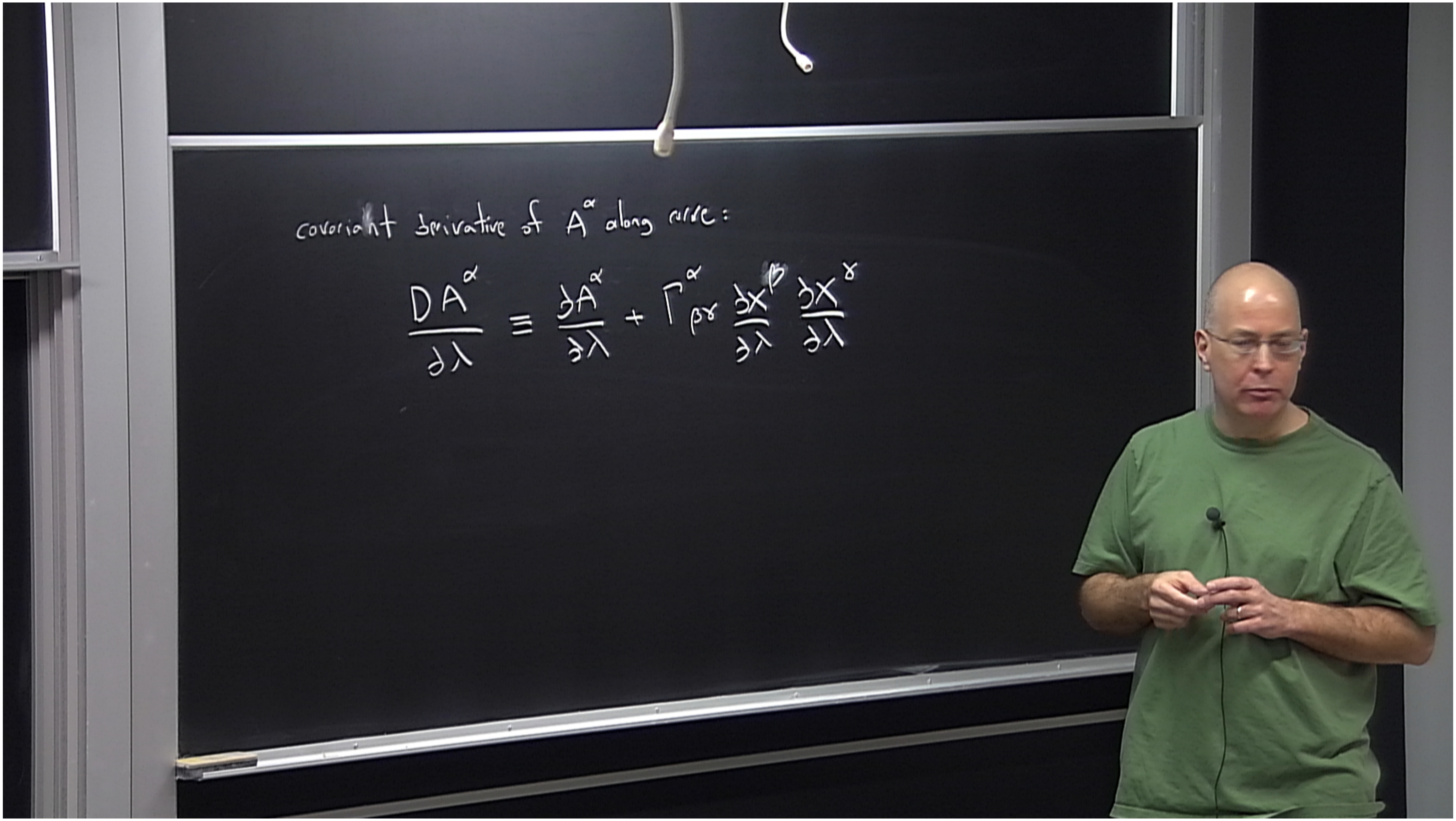
Introduce connection
(not a tensor)
- symmetric in lower indices
- metric compatible

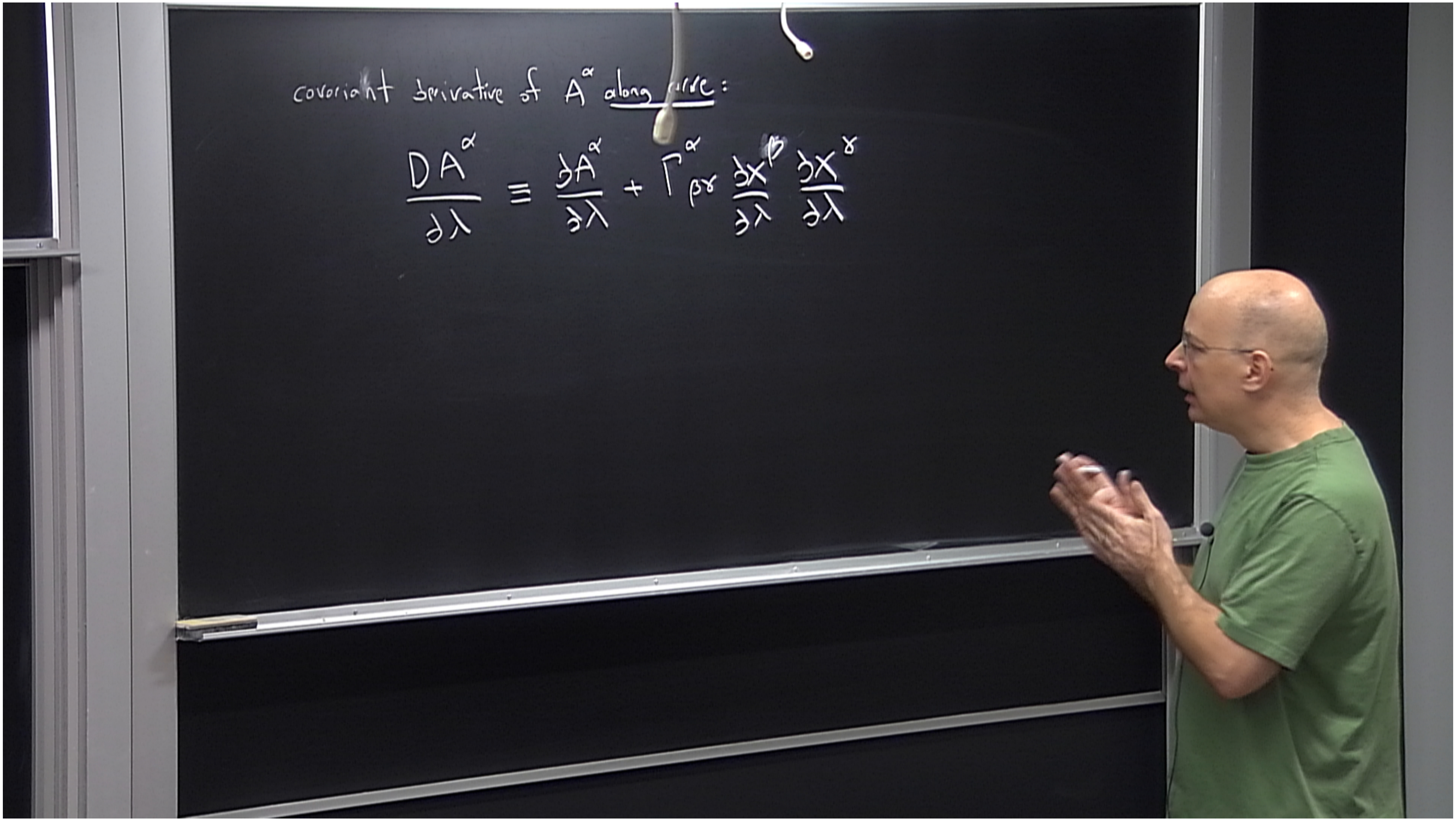

Covariant differentiation (requires connection).
CAUTION

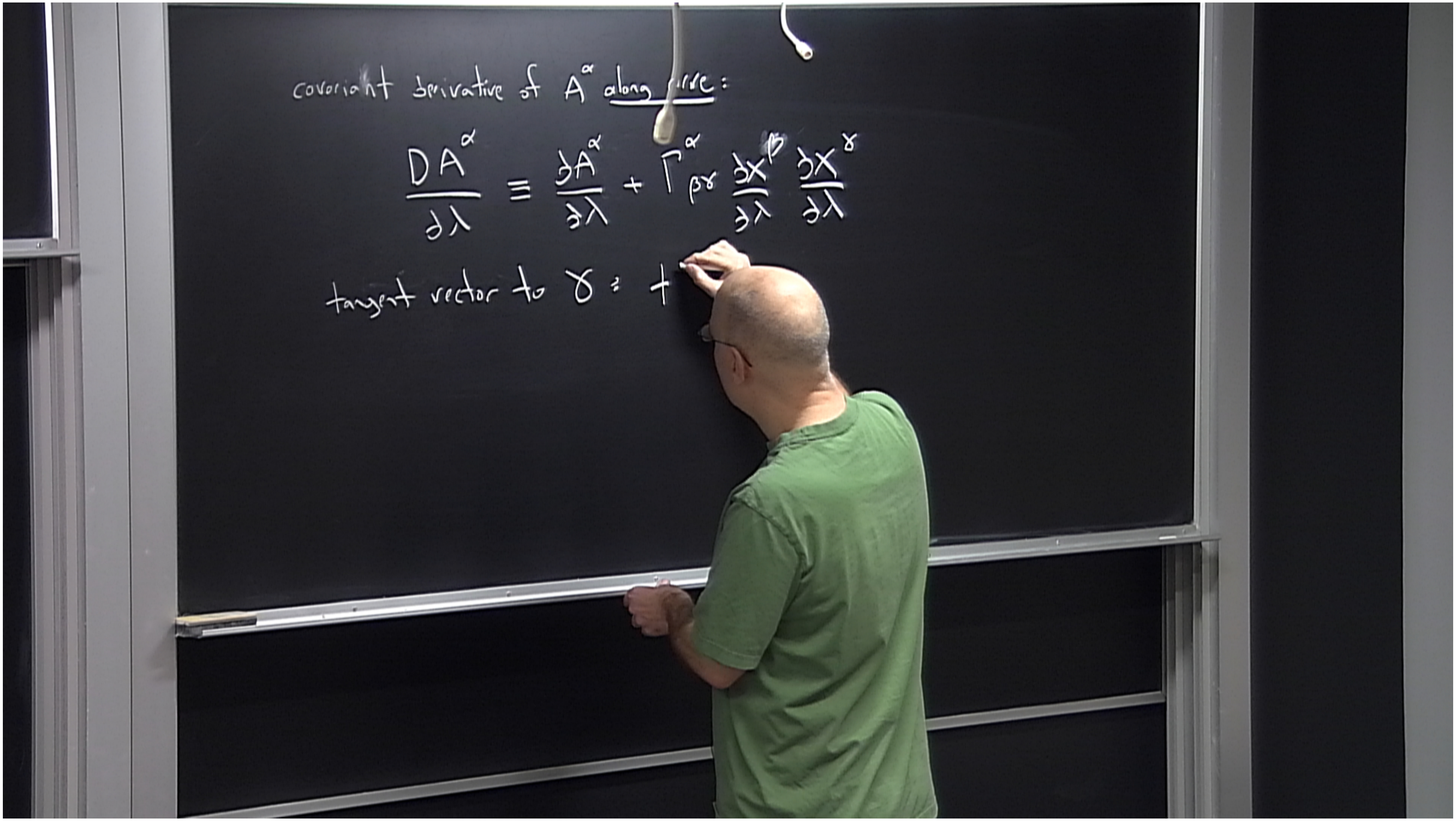

Covariant differentiation (requires connection).
vector fields on a curve γ
differentiate A^α along the curve
Parametric description of γ: x^α(λ)
arbitrary parameter
A^α(3)
A^α(1)
3
2
1
γ
CAUTION

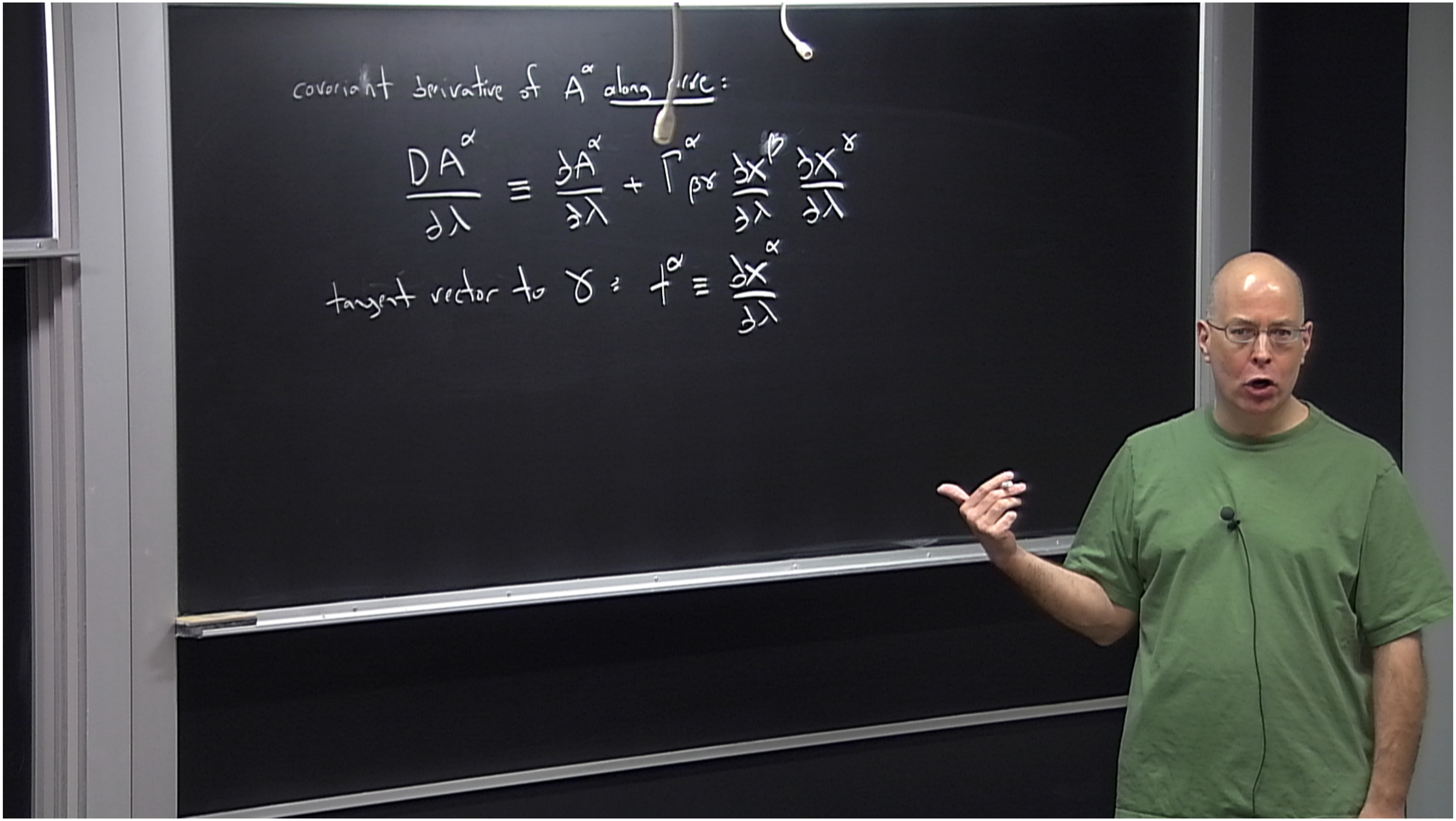

covariant derivative of A^α along curve:
$\frac{DA^\alpha}{\partial\lambda} = \frac{\partial A^\alpha}{\partial\lambda}$
CAUTION

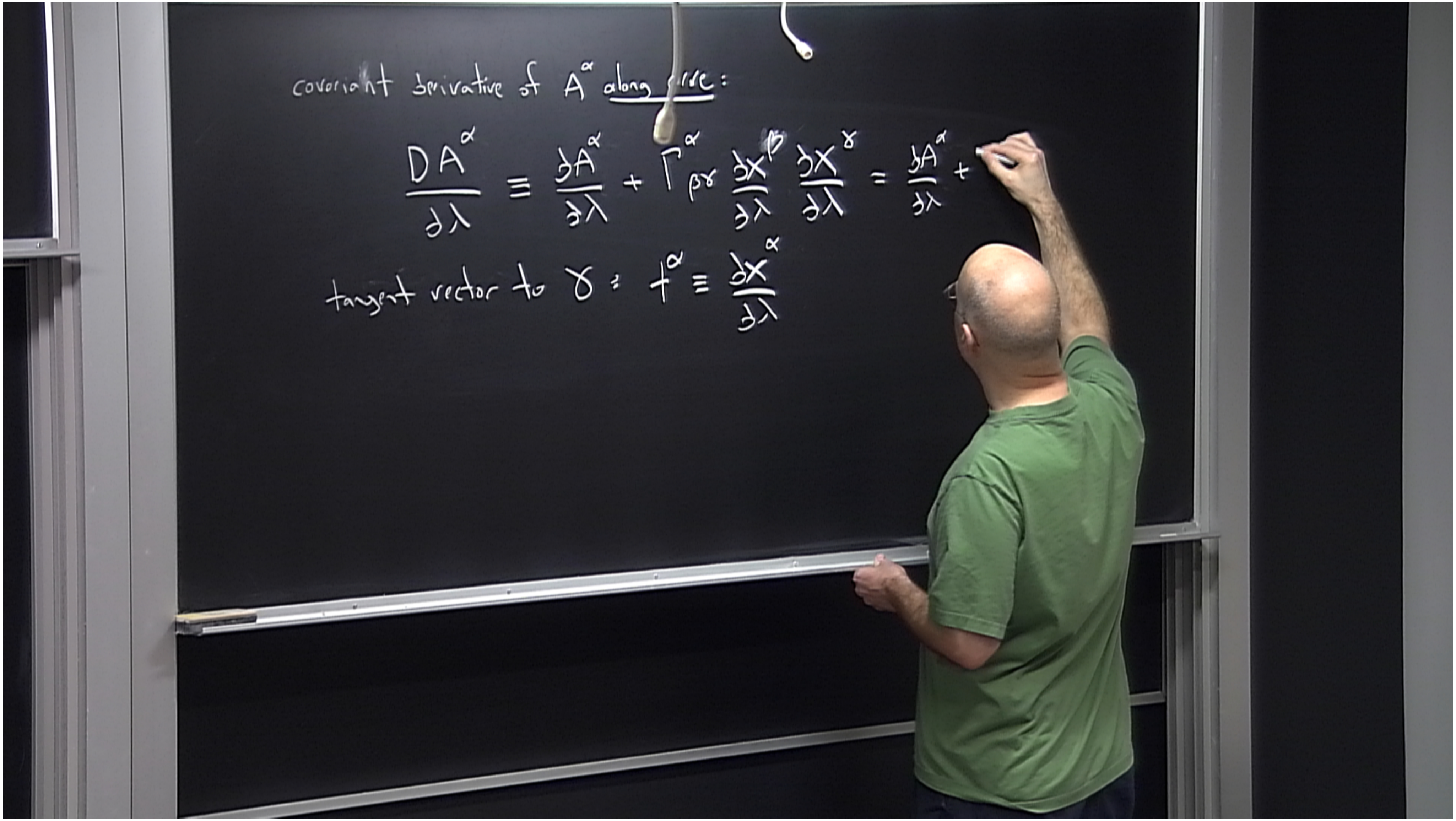

Covariant differentiation (requires connection).
vector fields on a curve γ
differentiate A^α along the curve
Parametric description of γ : x^α(λ)
arbitrary parameter
vector : A^α(λ)
A^α(3)
A^α(1)
3
2
1
γ

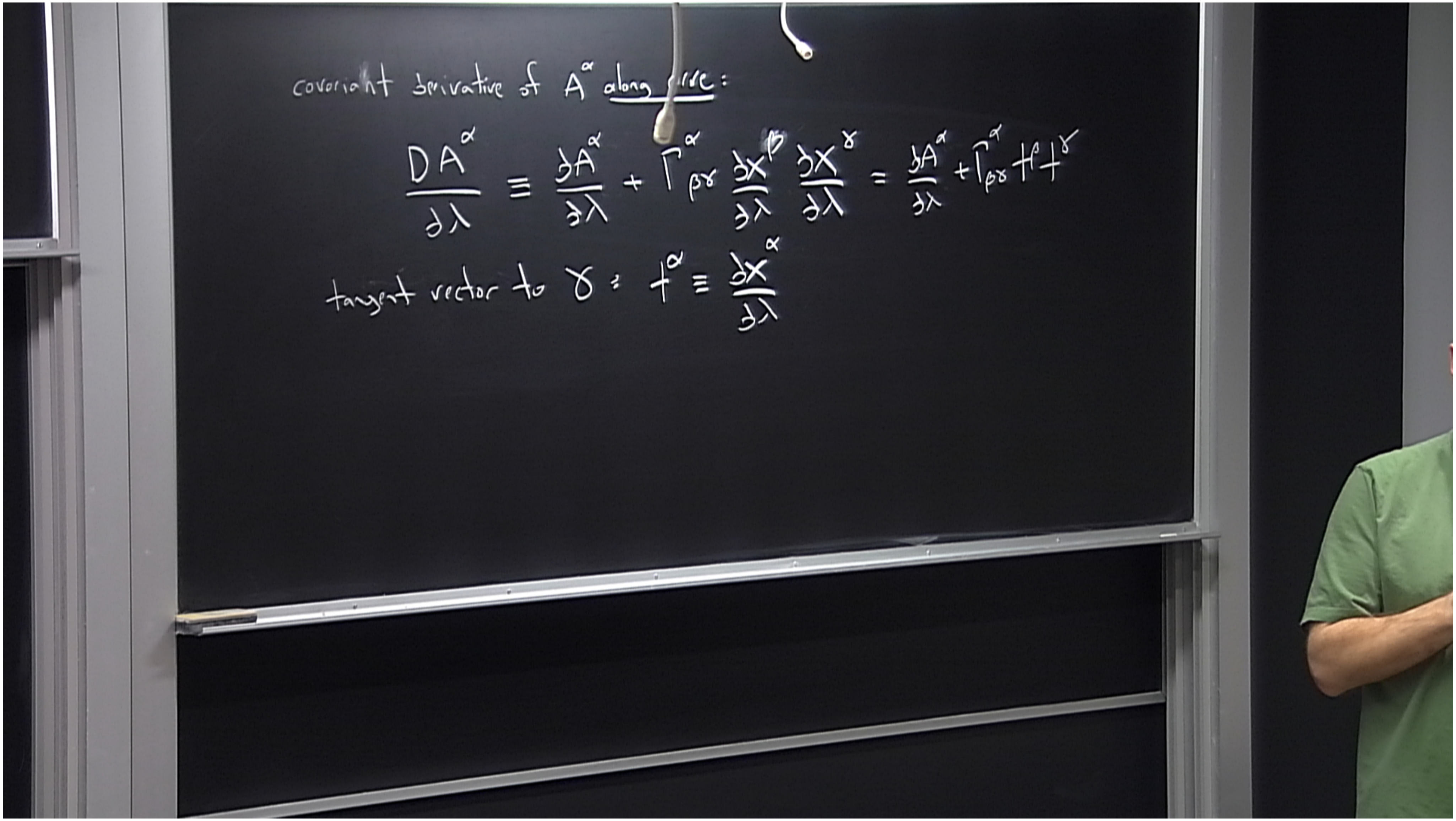

covariant derivative of A^α along curve:
$\frac{DA^\alpha}{\partial\lambda} \equiv \frac{\partial A^\alpha}{\partial\lambda} + \Gamma^\alpha_{\beta\gamma} \frac{\partial x^\beta}{\partial\lambda} \frac{\partial x^\gamma}{\partial\lambda}$

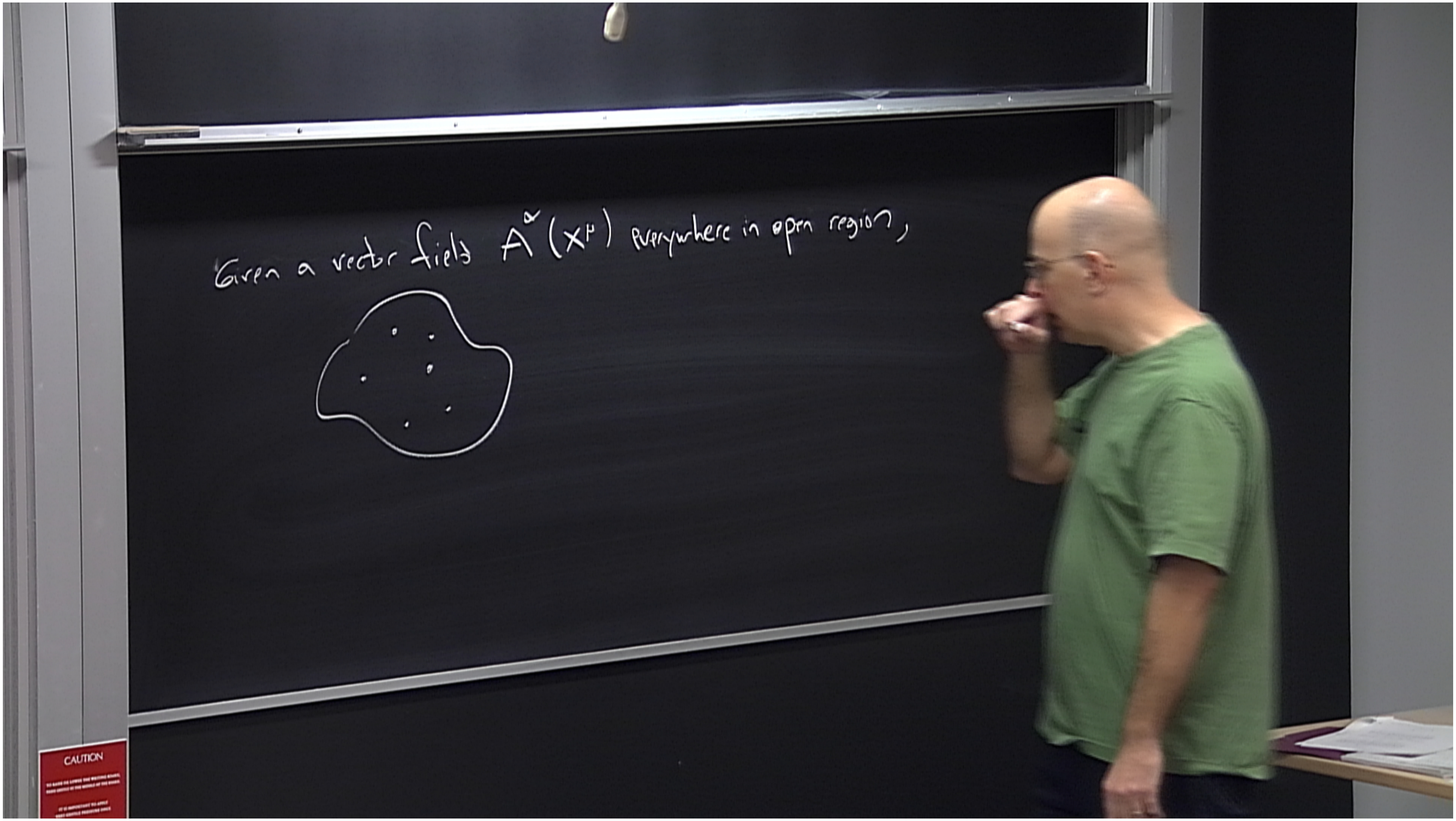

covariant derivative of A^α along curve:
$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma^\alpha_{\beta\gamma} \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda}$

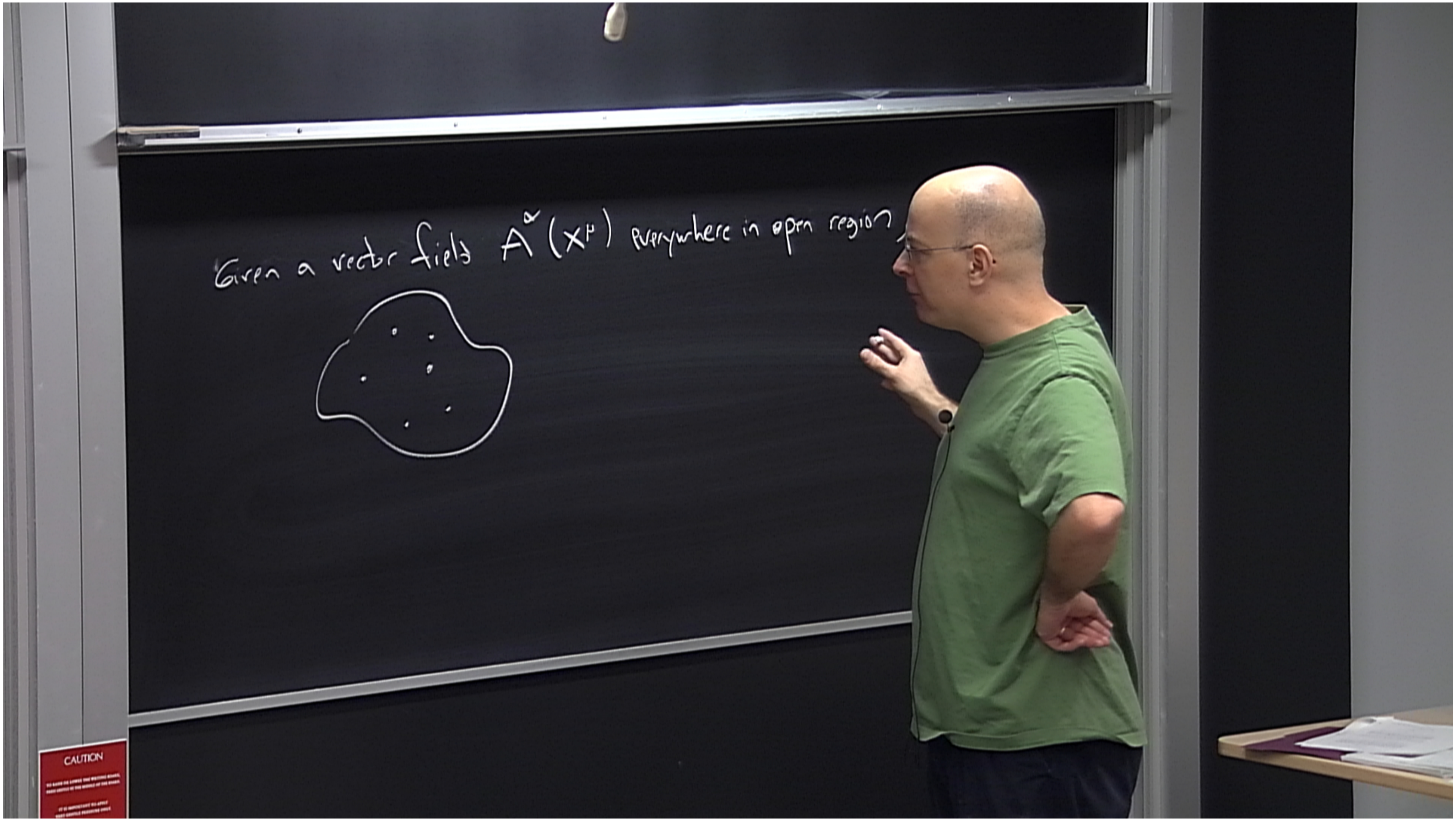

covariant derivative of A^α along curve:
DA^α/∂λ ≡ ∂A^α/∂λ + Γ^α_βγ ∂x^β/∂λ ∂x^γ/∂λ
tangent vector to γ = t

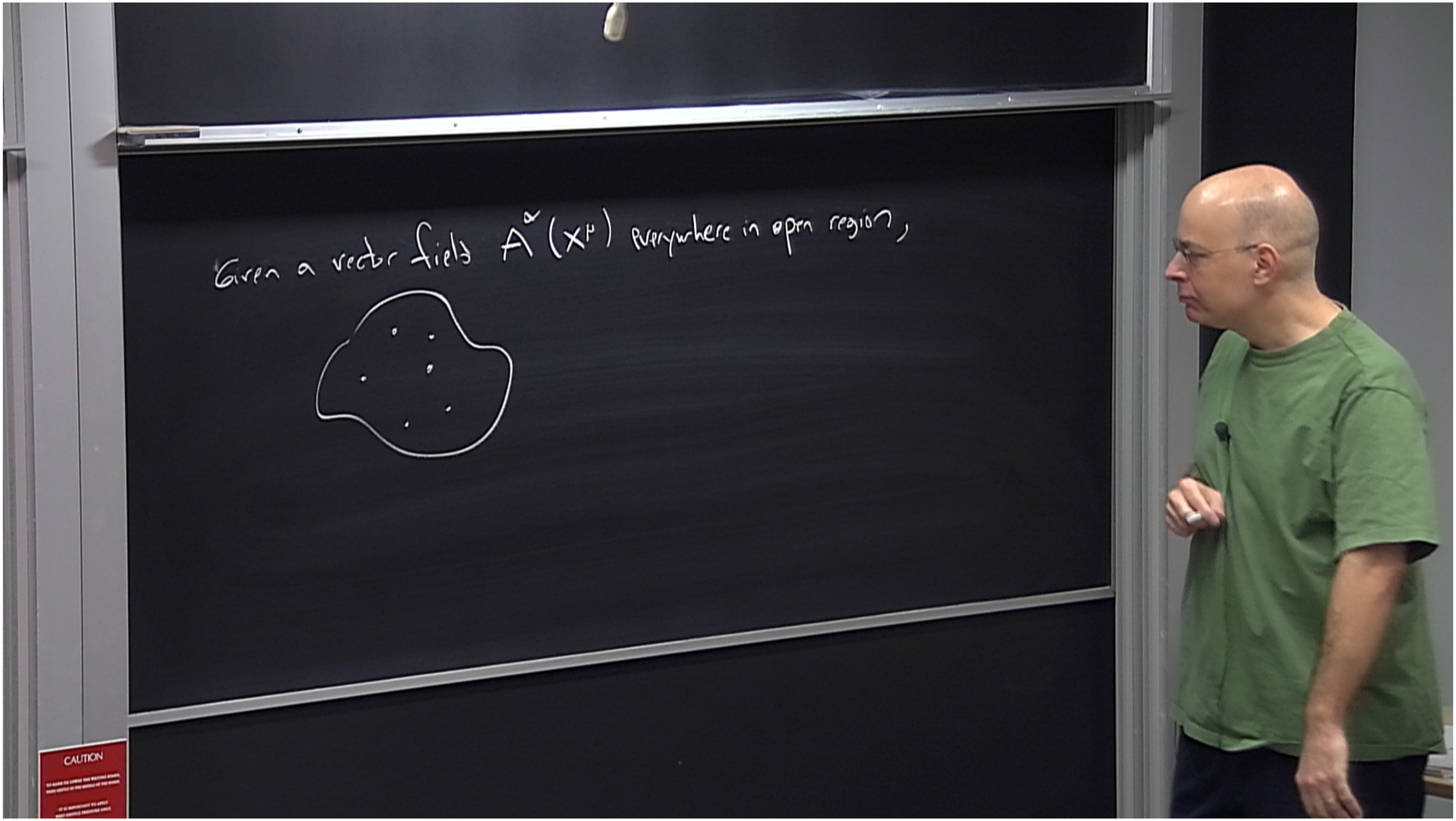

covariant derivative of A^α along curve:
DA^α/dλ ≡ dA^α/dλ + Γ^α_βγ dx^β/dλ dx^γ/dλ
tangent vector to γ: t^α ≡ dx^α/dλ

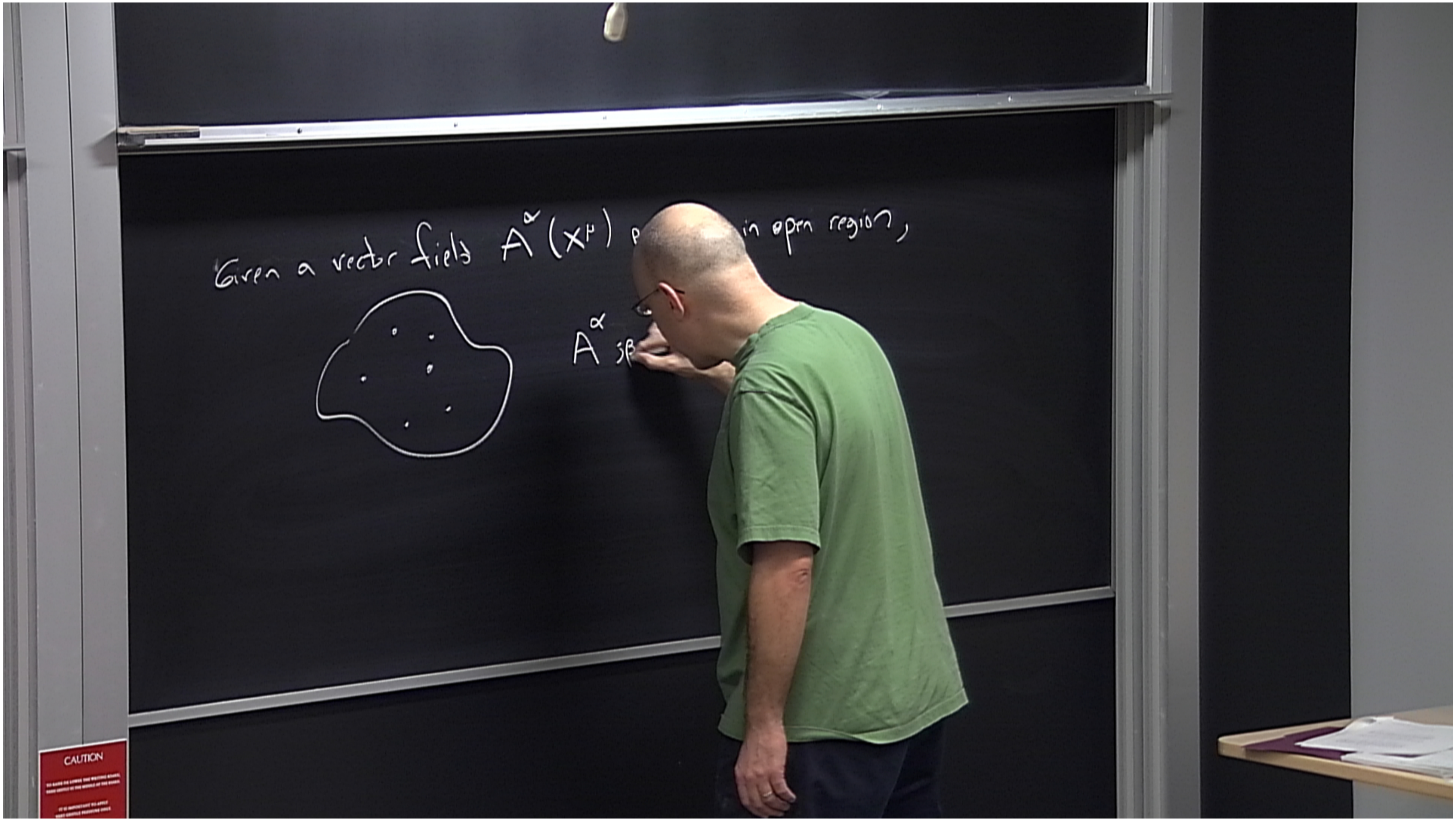

covariant derivative of A^α along curve:
DA^α/∂λ ≡ ∂A^α/∂λ + Γ^α_βγ ∂x^β/∂λ ∂x^γ/∂λ = ∂A^α/∂λ +
tangent vector to γ: t^α ≡ ∂x^α/∂λ

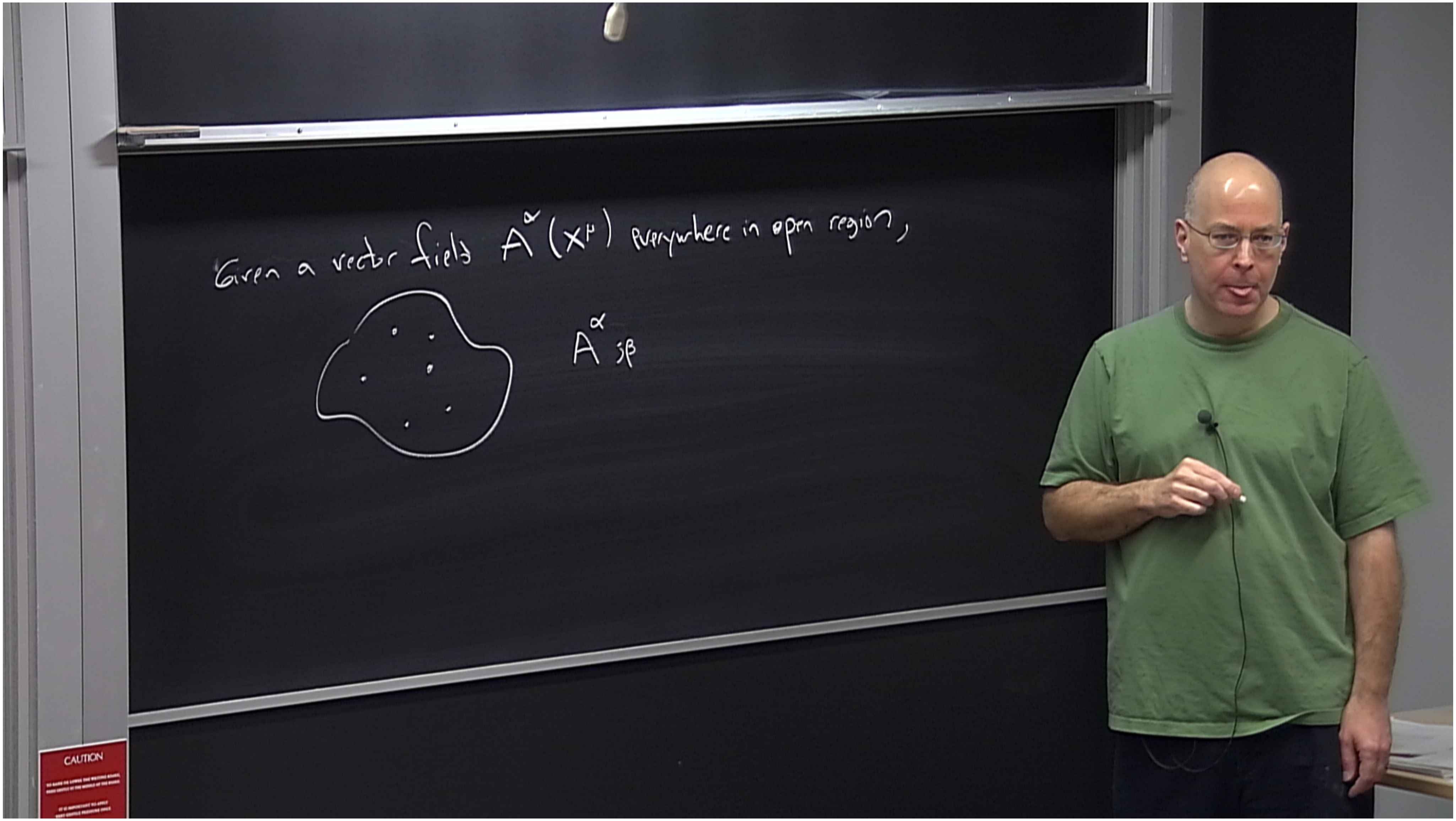

covariant derivative of A^α along curve:
$\frac{DA^\alpha}{\partial\lambda} \equiv \frac{\partial A^\alpha}{\partial\lambda} + \Gamma^\alpha_{\beta\gamma} \frac{\partial x^\beta}{\partial\lambda} \frac{\partial x^\gamma}{\partial\lambda} = \frac{\partial A^\alpha}{\partial\lambda} + \Gamma^\alpha_{\beta\gamma} t^\beta t^\gamma$
tangent vector to γ: $t^\alpha \equiv \frac{\partial x^\alpha}{\partial\lambda}$

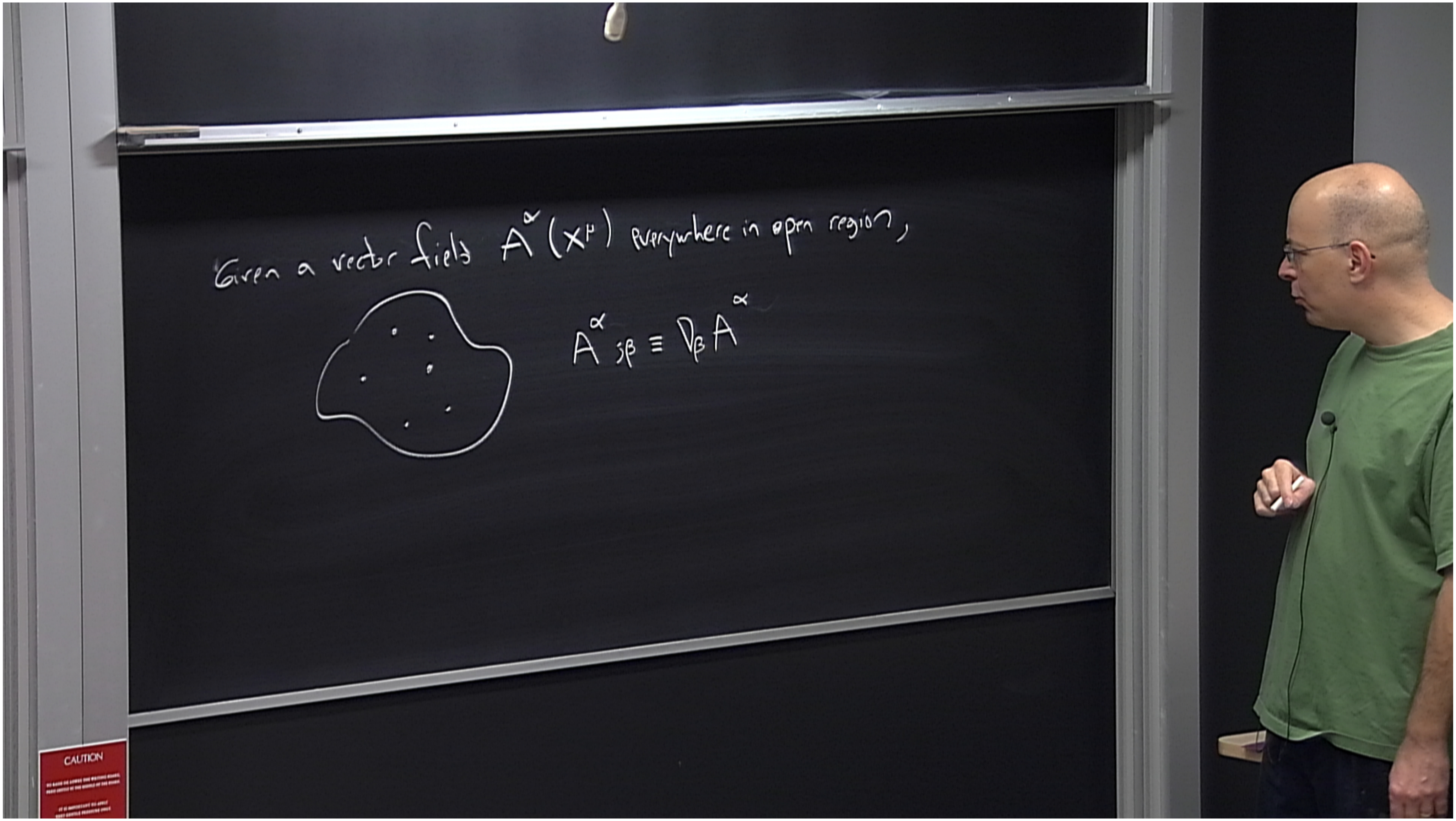

Given a vector field A^α (x^μ) everywhere in open region,
CAUTION

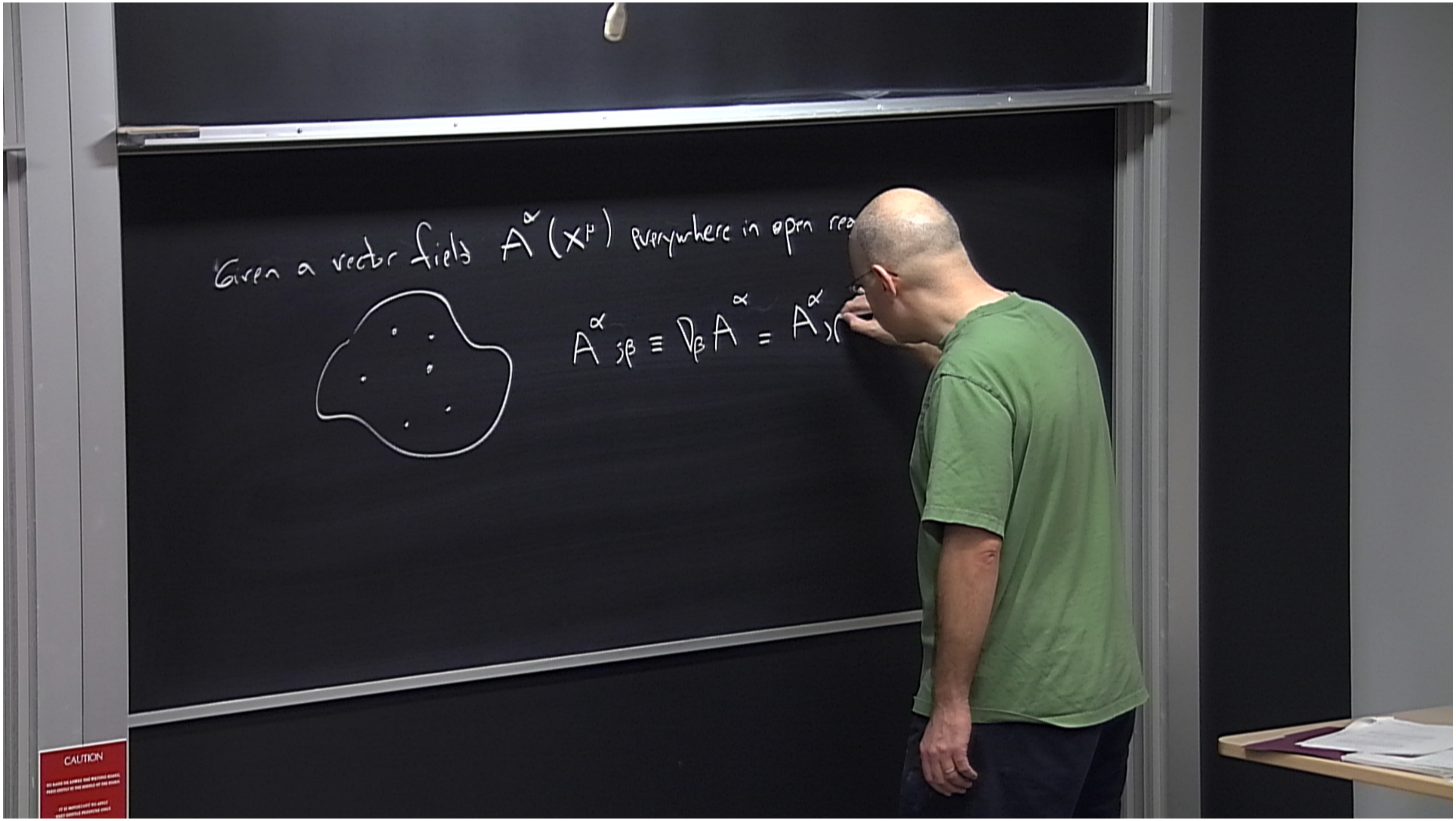

Given a vector field A^α (x^μ) everywhere in open region
CAUTION

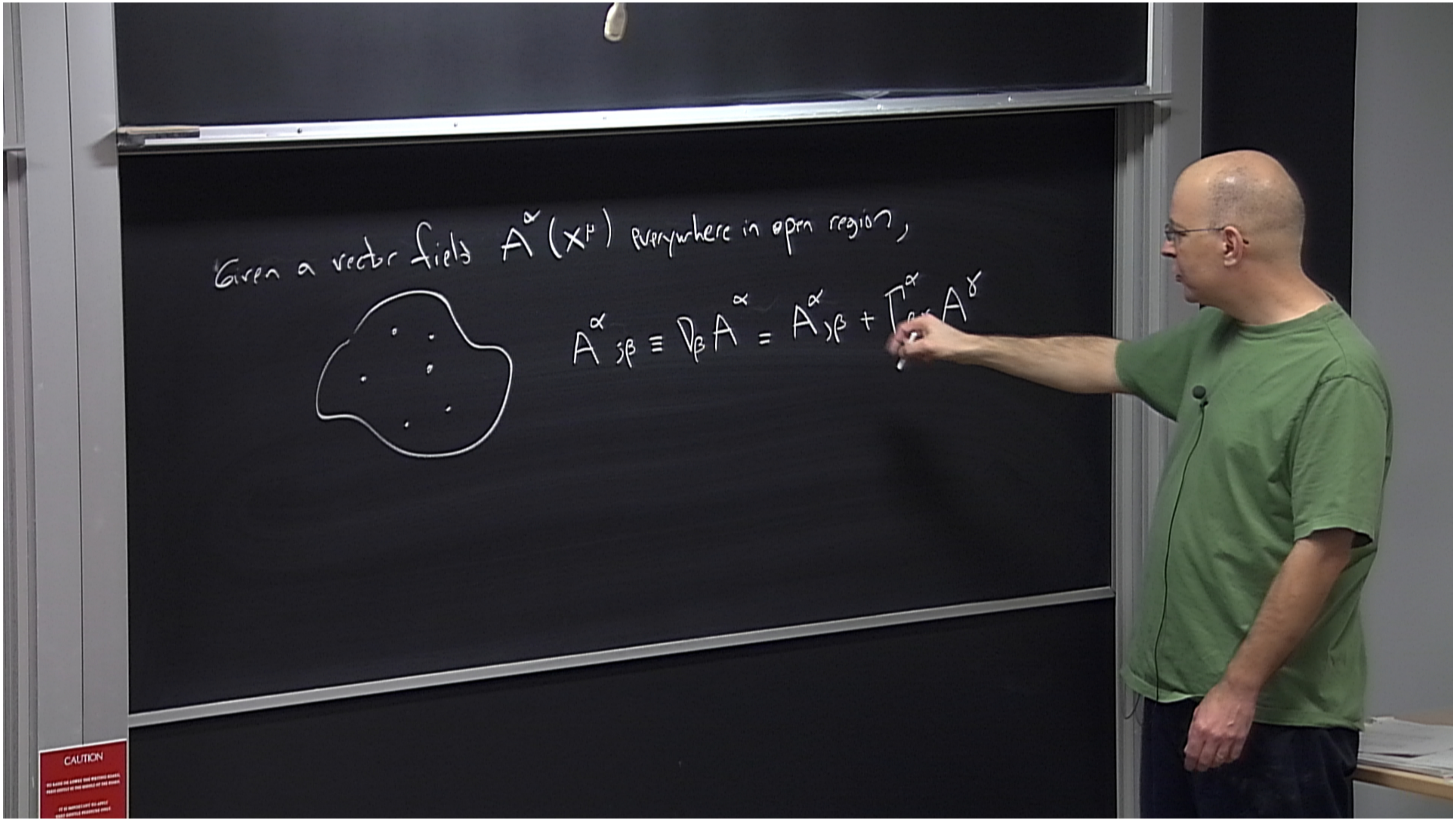

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
CAUTION

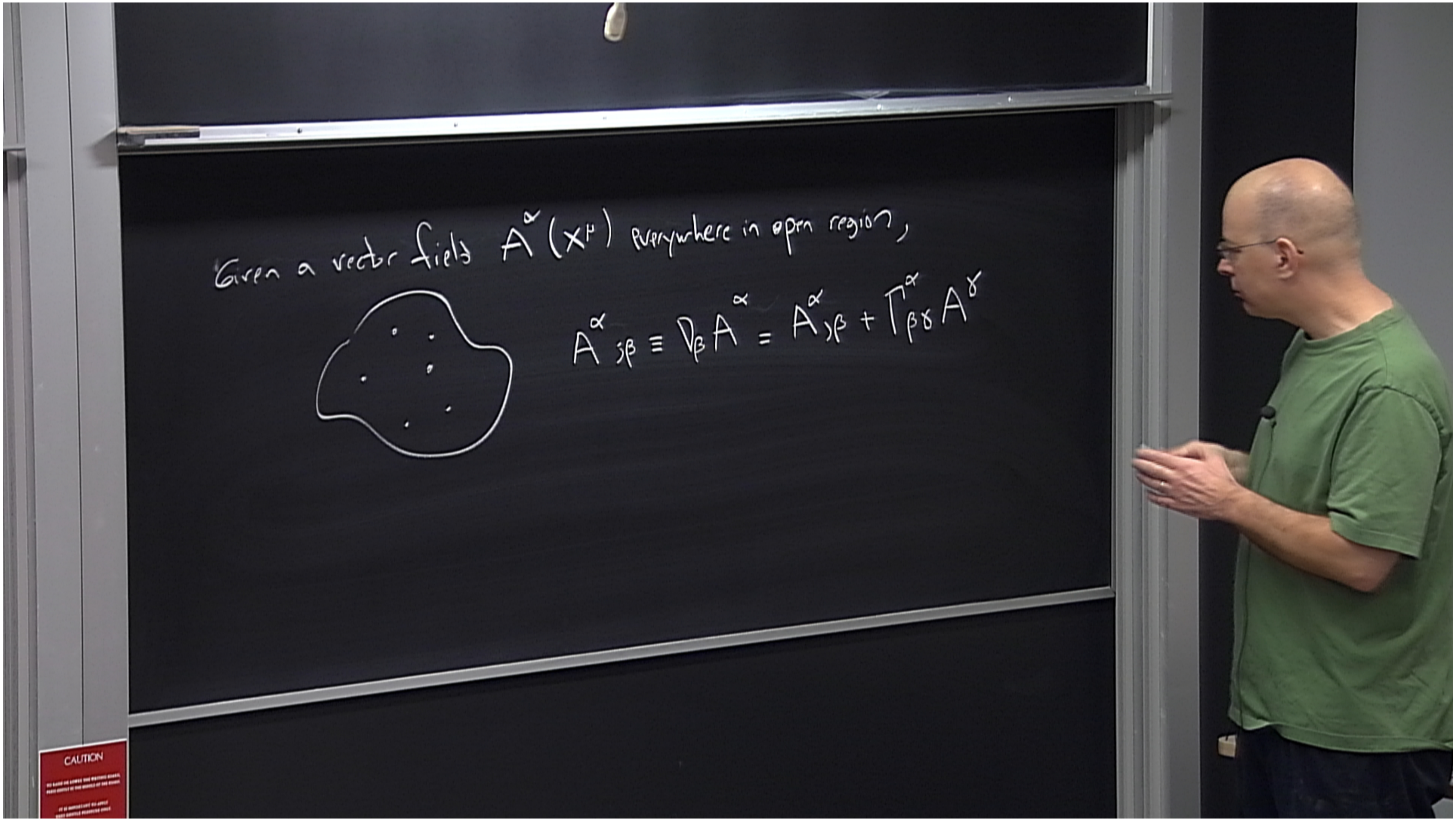

Given a vector field A^α (x^μ)
in open region,
CAUTION

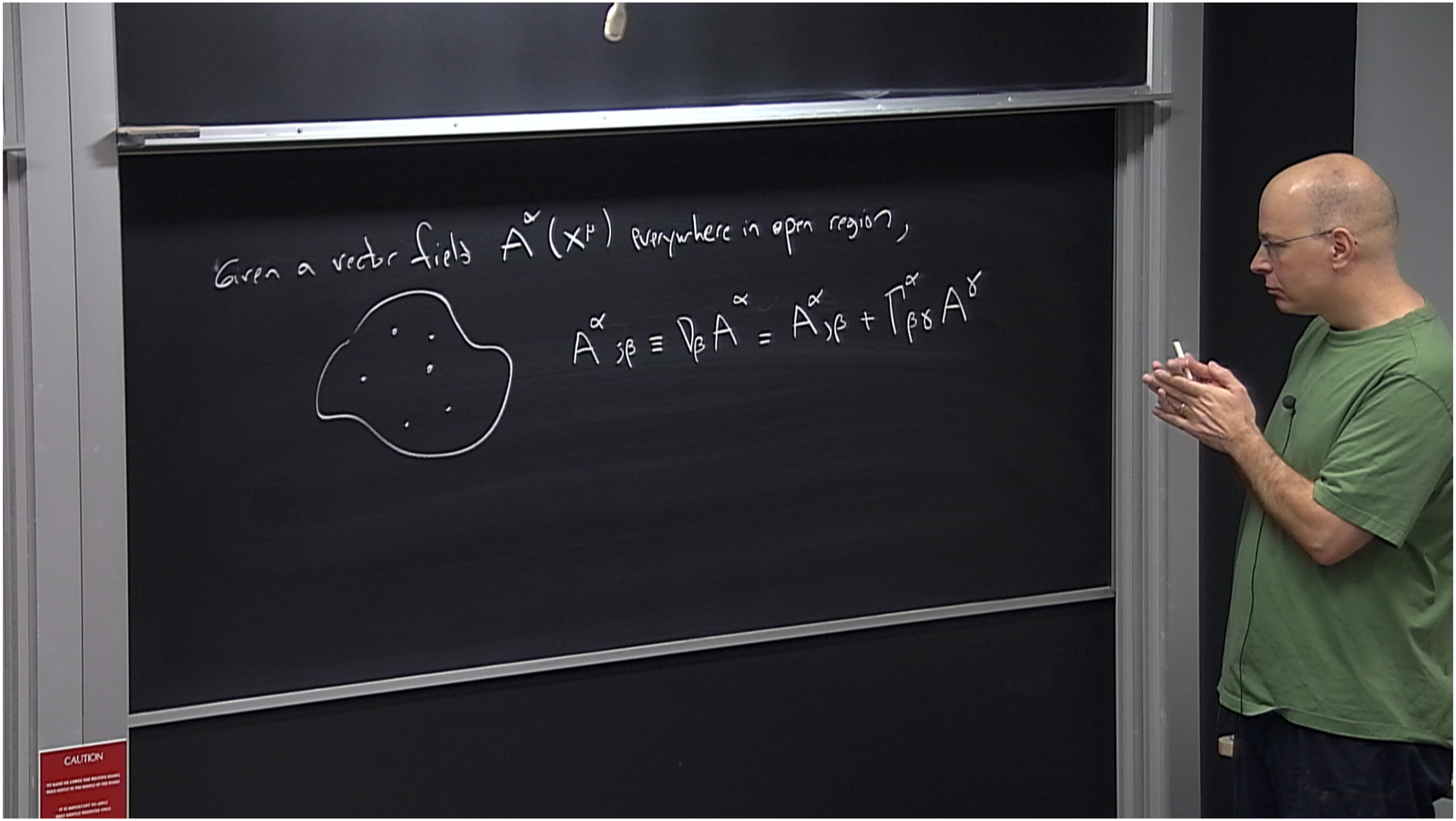

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
$A^\alpha{}_{;\beta}$
CAUTION

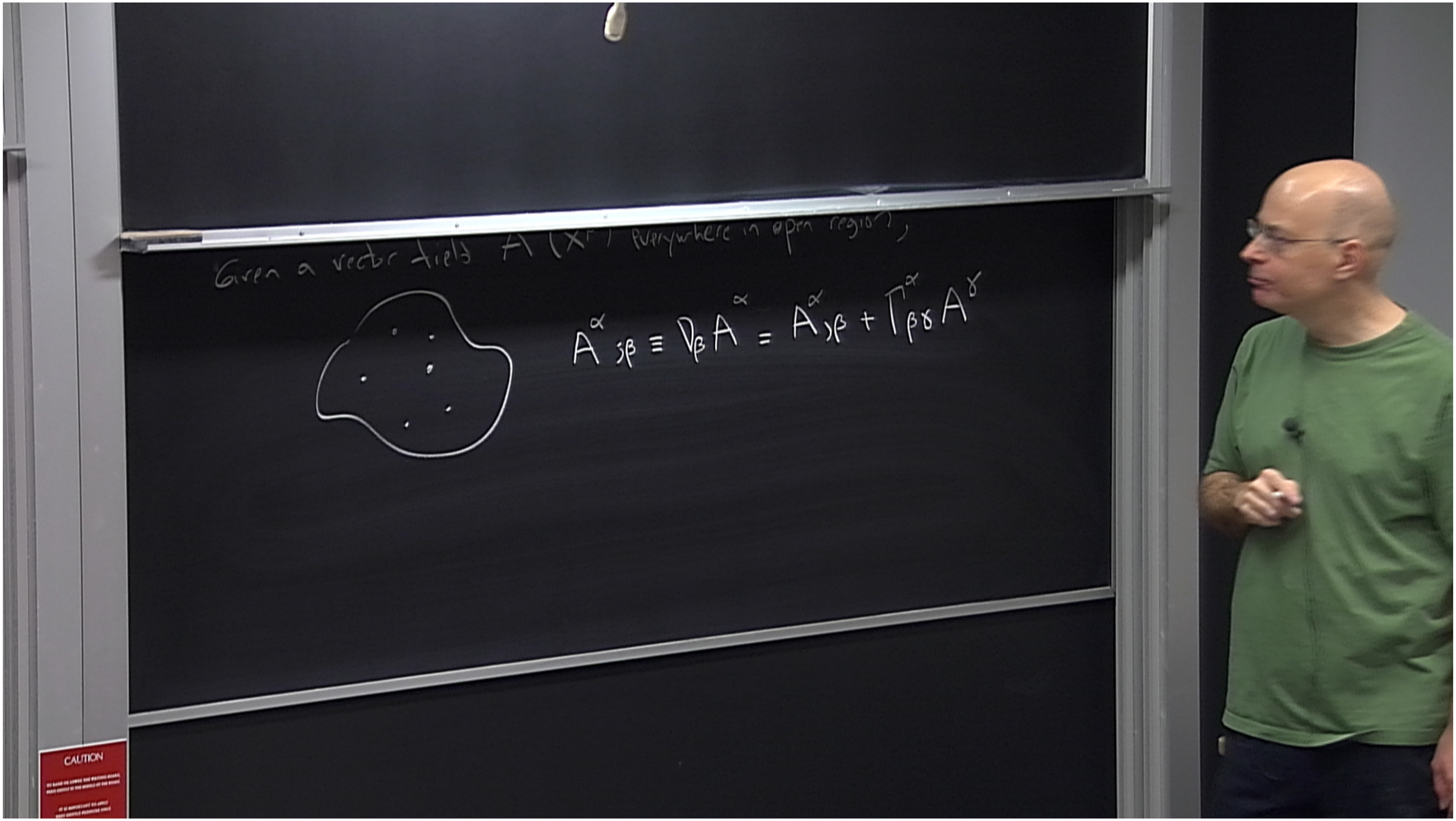

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha$
CAUTION

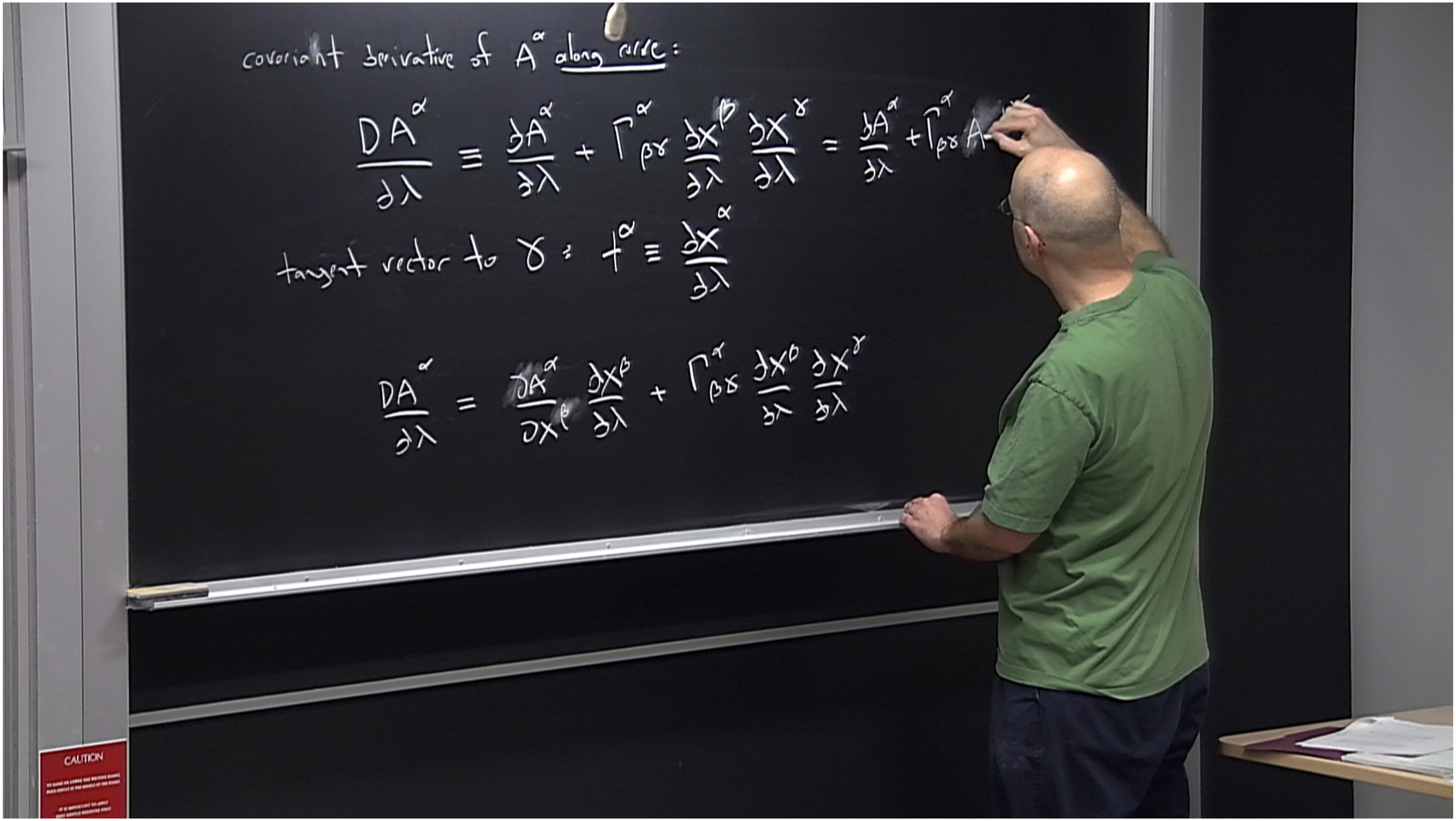

Given a vector field $A^\alpha(x^\mu)$ everywhere in open
$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,(}$
CAUTION

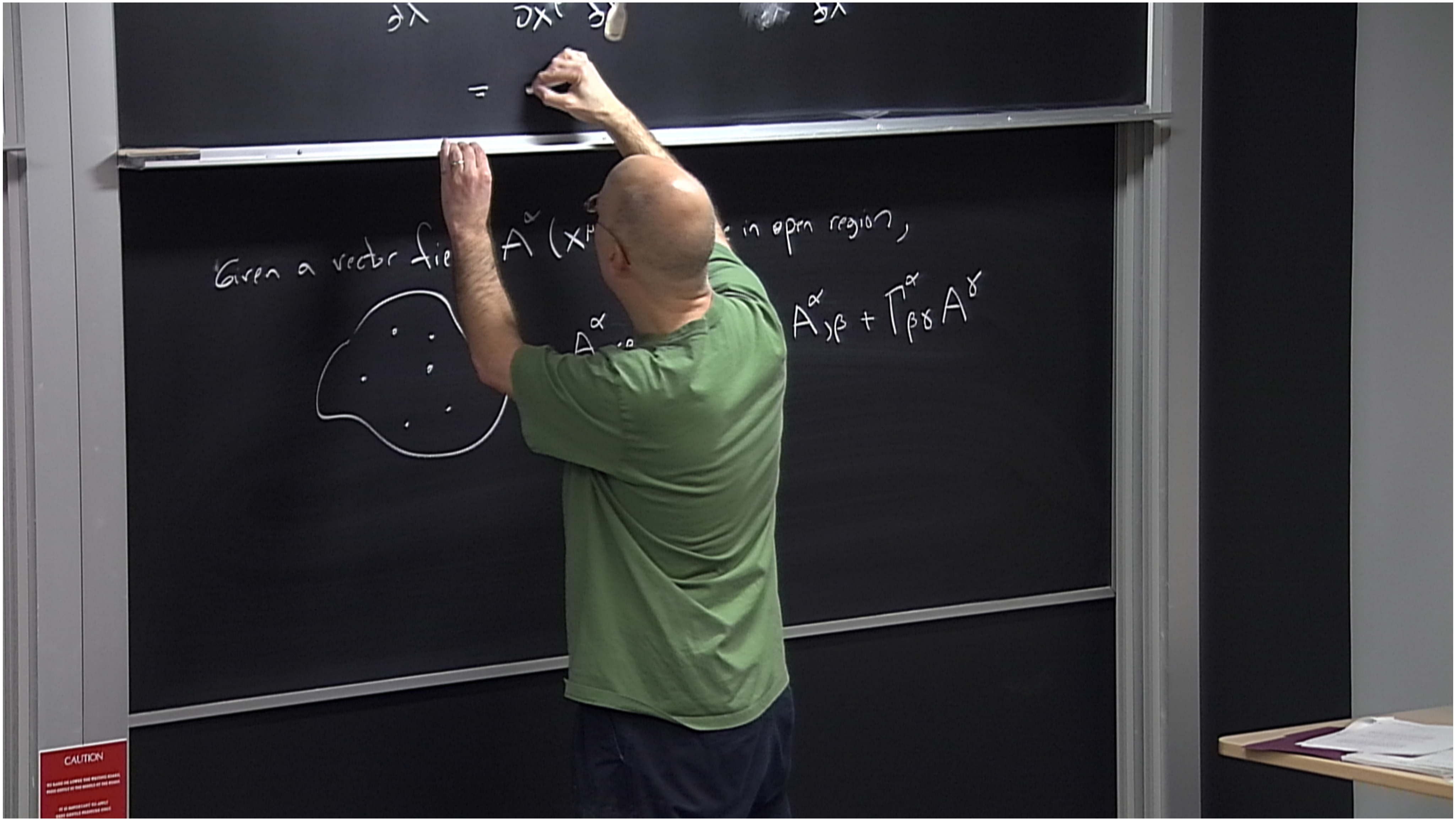

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta} + \Gamma^\alpha \; A^\gamma$

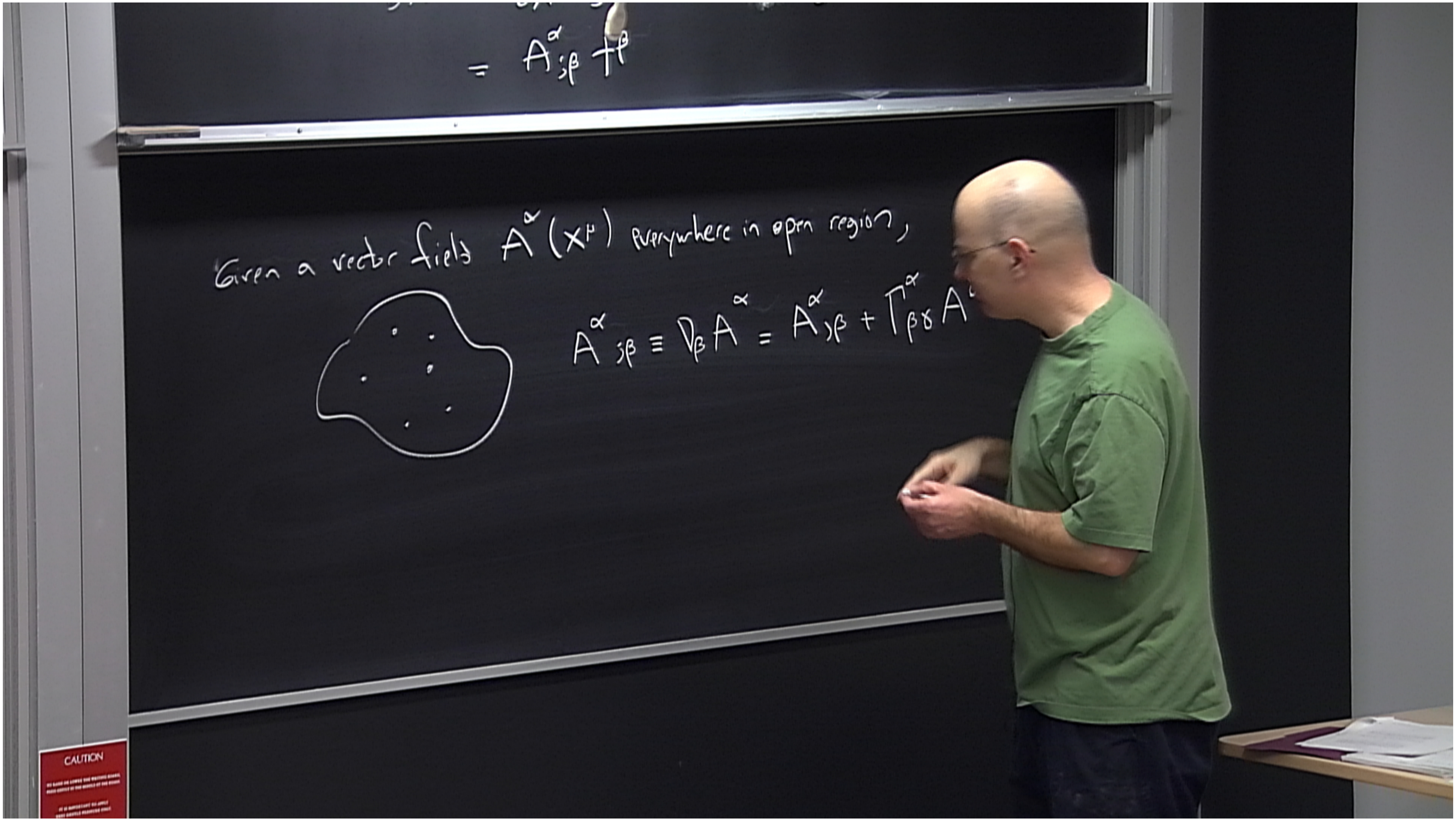

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta} + \Gamma^\alpha_{\beta\gamma} A^\gamma$
CAUTION

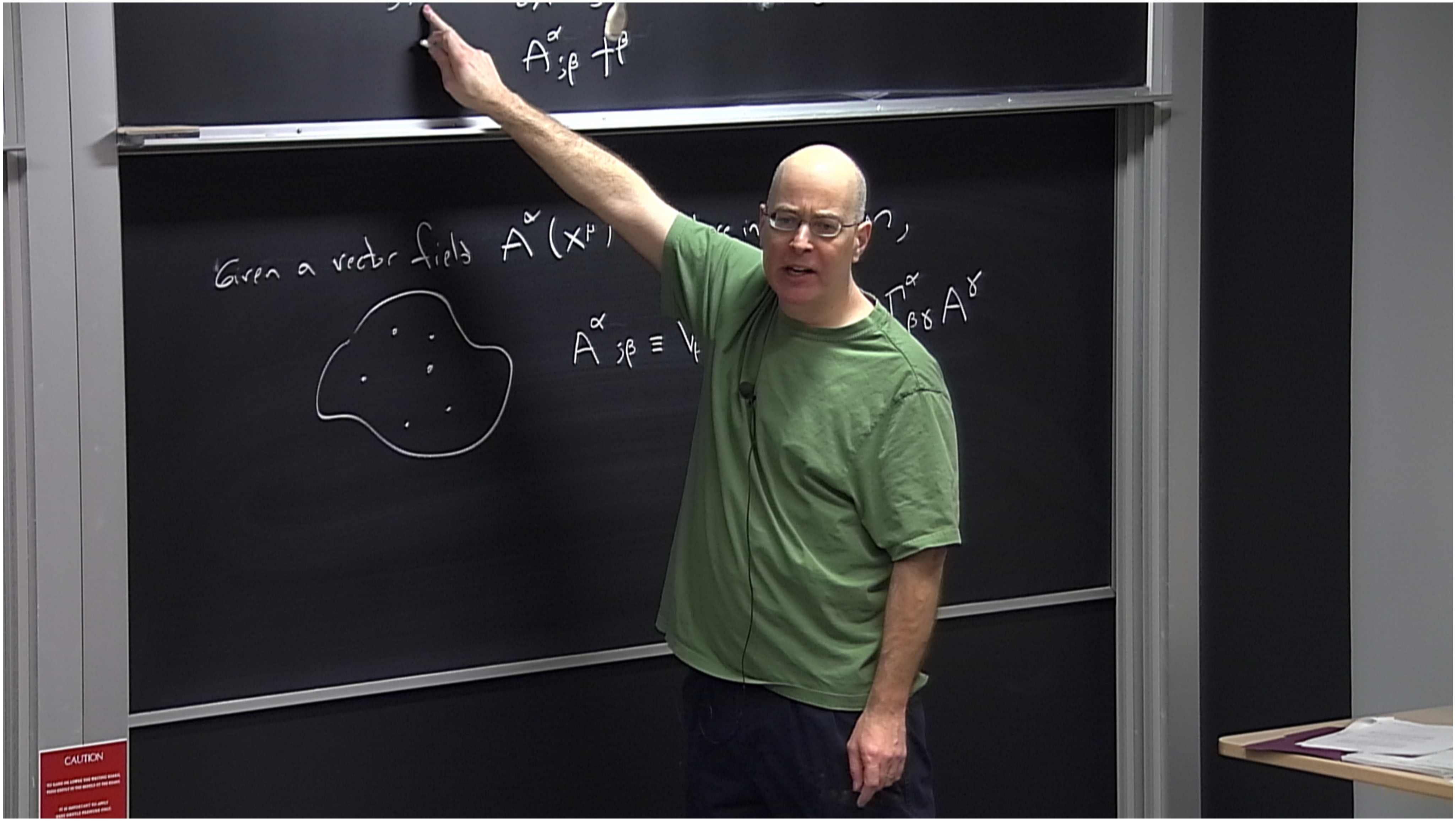

Given a vector field $A^{\alpha}(x^{\mu})$ everywhere in open region,
$A^{\alpha}{}_{;\beta} \equiv \nabla_{\beta} A^{\alpha} = A^{\alpha}{}_{,\beta} + \Gamma^{\alpha}_{\beta\gamma} A^{\gamma}$
CAUTION


Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,
$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta} + \Gamma^\alpha_{\beta\gamma} A^\gamma$
CAUTION

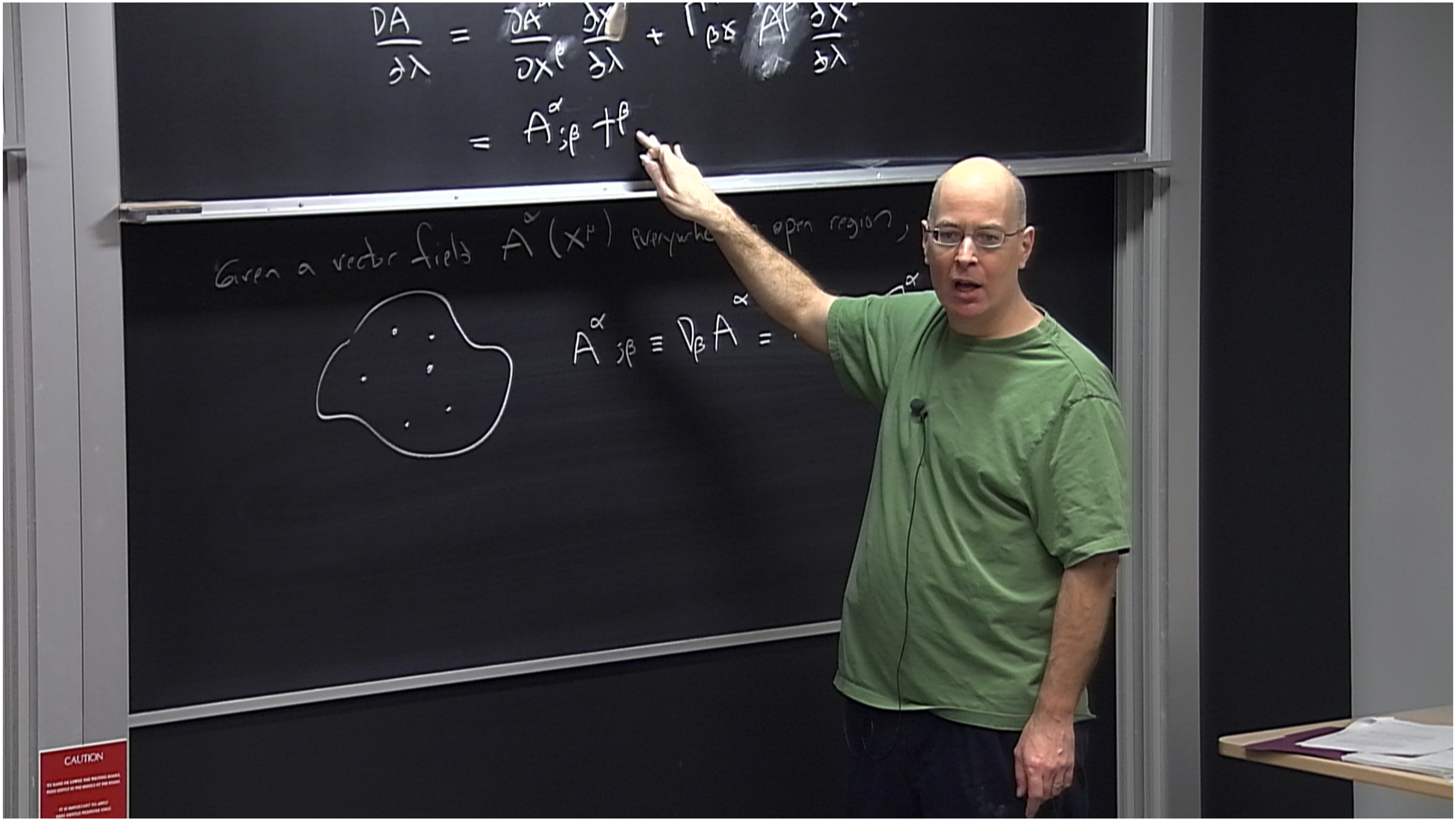

covariant derivative of A^α along curve:
DA^α/dλ ≡ ∂A^α/∂λ + Γ^α_βγ ∂x^β/∂λ ∂x^γ/∂λ = ∂A^α/∂λ + Γ^α_βγ A
tangent vector to γ: t^α ≡ ∂x^α/∂λ
DA^α/∂λ = ∂A^α/∂x^β ∂x^β/∂λ + Γ^α_βγ ∂x^β/∂λ ∂x^γ/∂λ
CAUTION

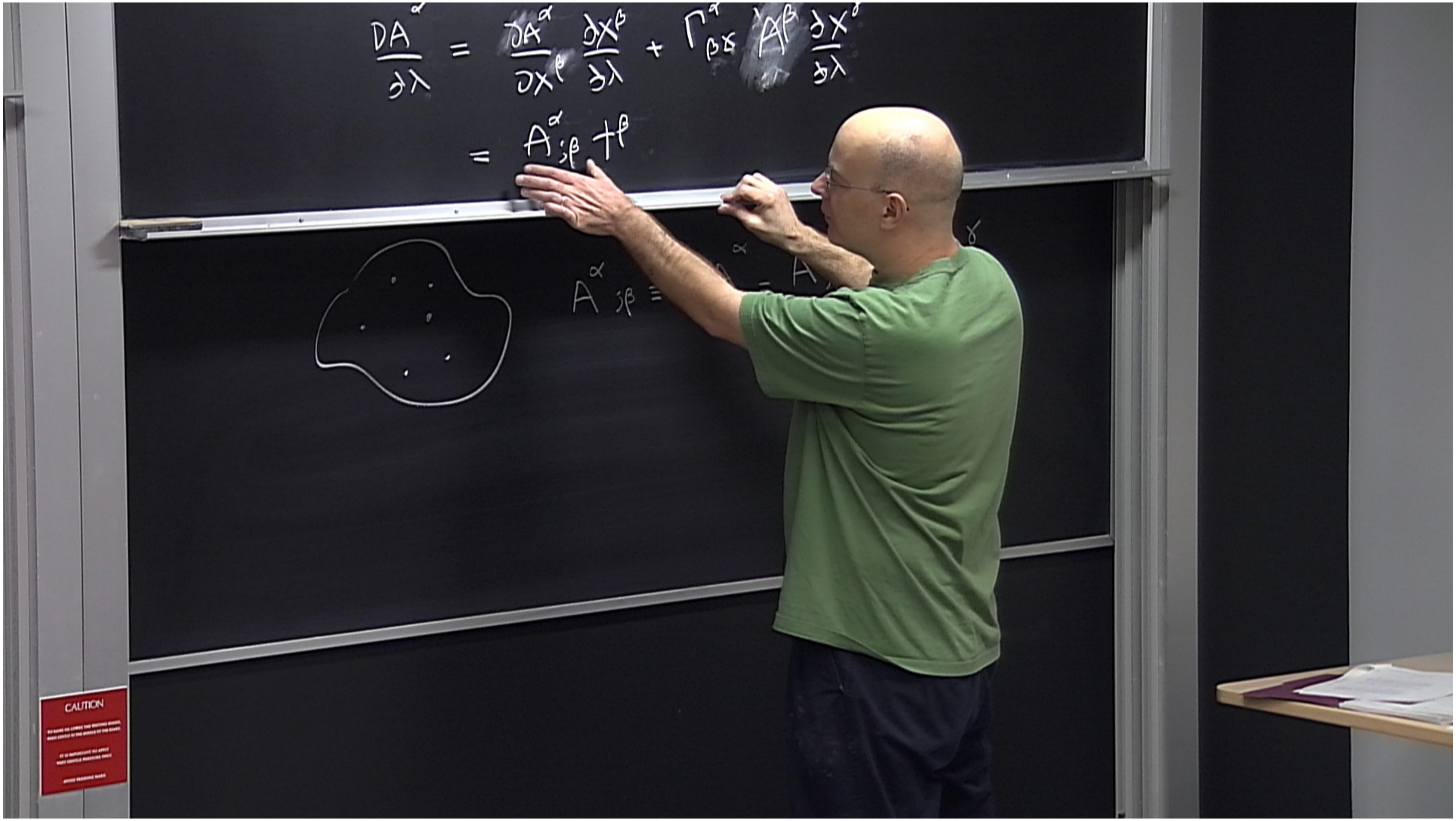

Given a vector fie
in open region,
CAUTION

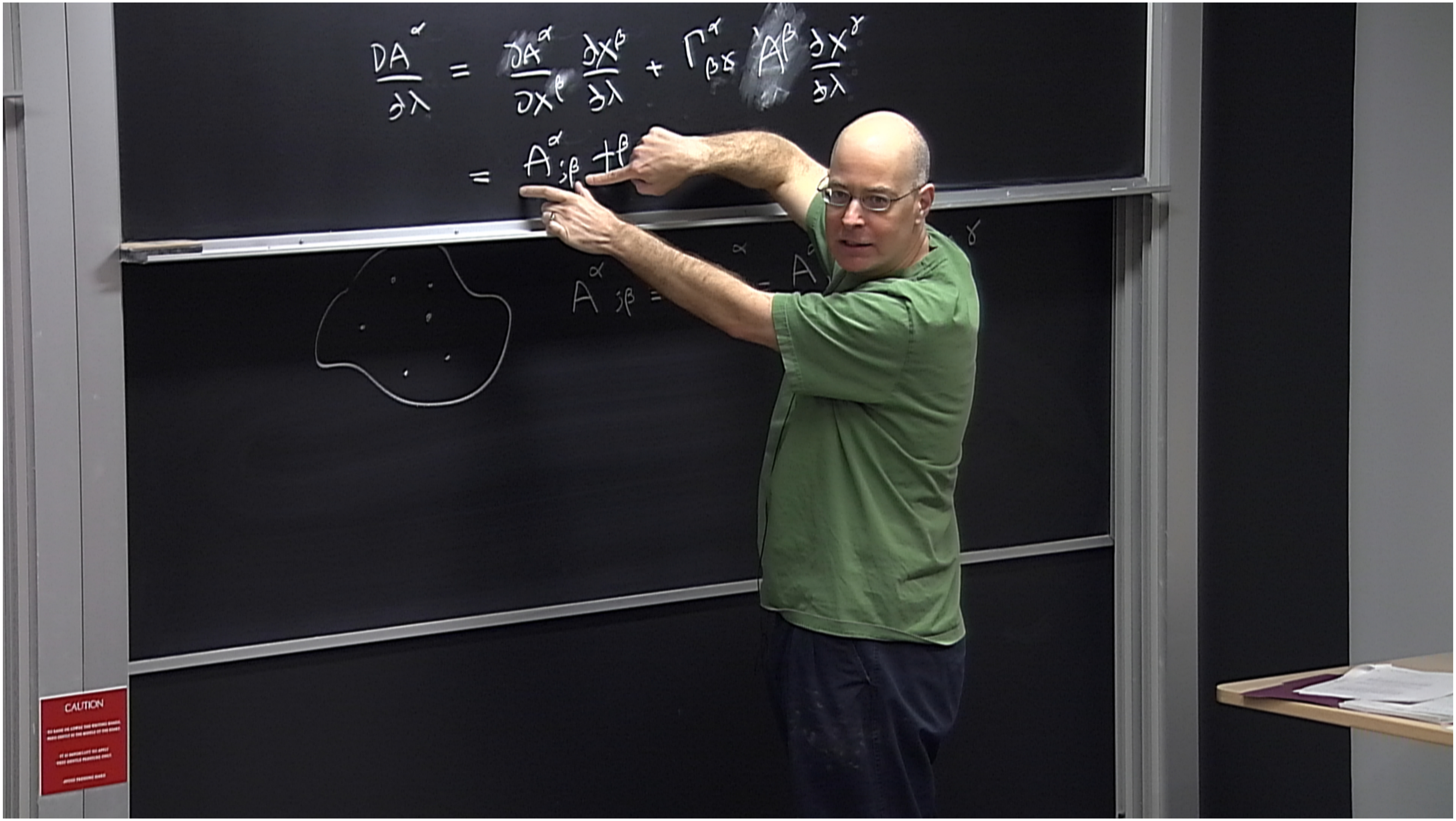

= A^α;β + Γ^β
Given a vector field A^α(x^μ) everywhere in open region,
A^α;β ≡ ∇_β A^α = A^α,β + Γ^α_βγ A
CAUTION

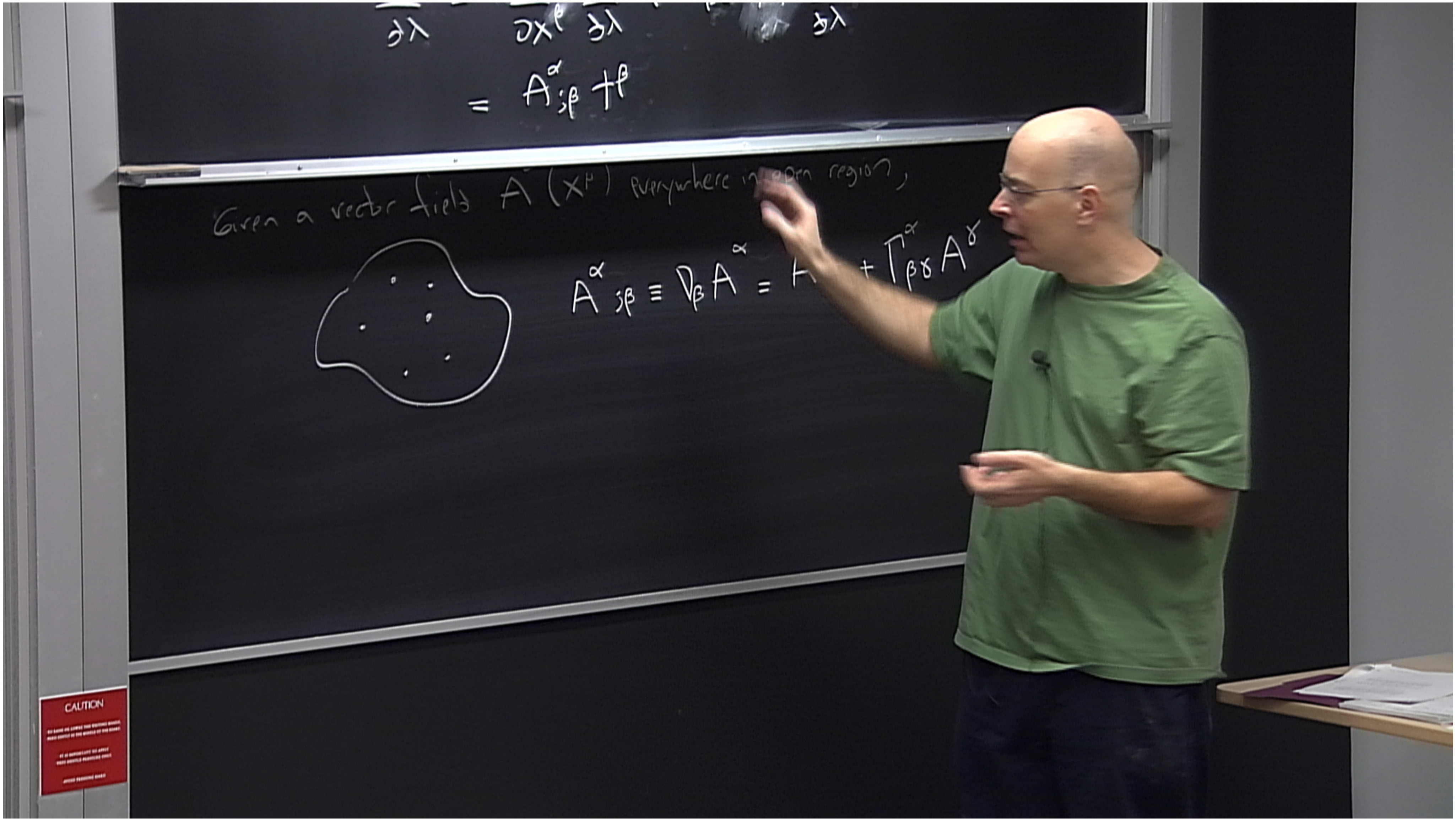


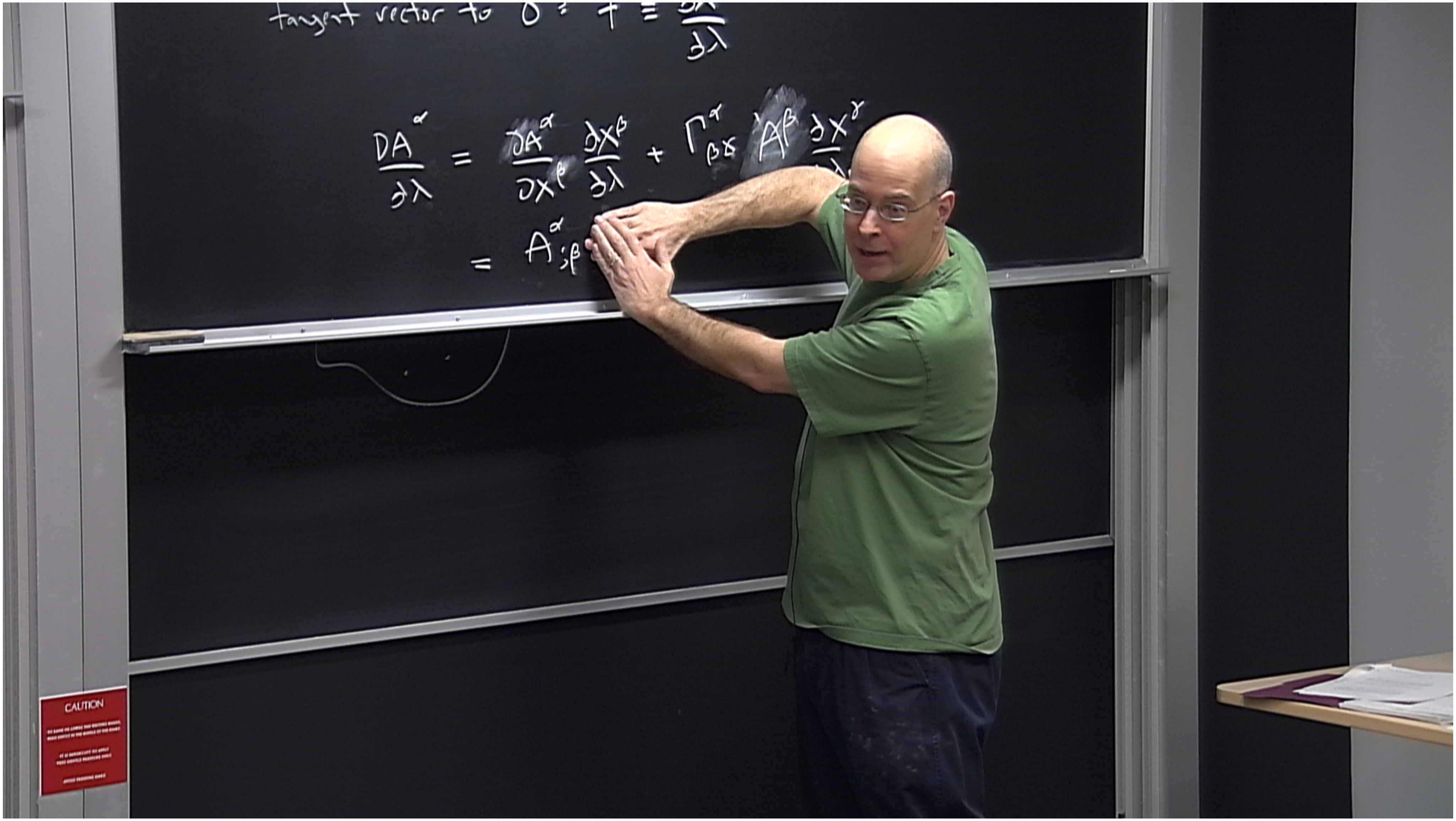
Given a vector field $A^\alpha(x^\mu)$
$A^\alpha{}_{;\beta} \equiv$
$\Gamma^\alpha_{\beta\gamma} A^\gamma$
$A^\alpha{}_{;\beta} t^\beta$
CAUTION


Given a vector field
open region,
CAUTION


$\frac{DA^\alpha}{d\lambda} = \frac{\partial A^\alpha}{\partial x^\beta}\frac{dx^\beta}{d\lambda} + \Gamma^\alpha_{\beta\gamma} A^\beta \frac{dx^\gamma}{d\lambda}$
$= A^\alpha_{;\beta} t^\beta$
CAUTION


CAUTION


= A^α_{;β} t^β
Given a vector field A(x^μ) everywhere in open region,
A^α_{;β} ≡ ∇_β A^α =
Γ^α_{βγ} A^γ
CAUTION


tangent vector to
CAUTION