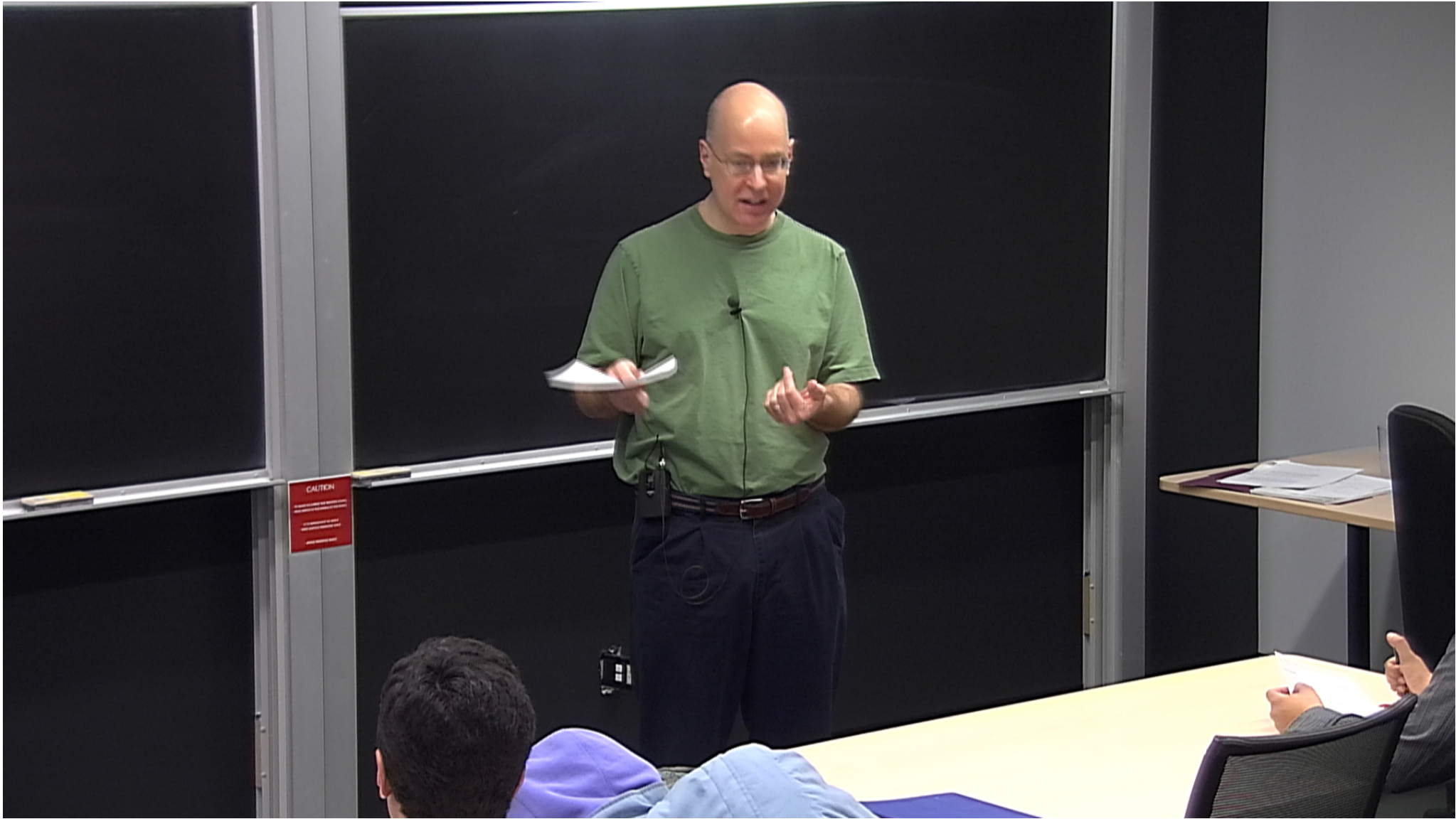


Title: Advanced General Relativity - Lecture 1

Date: Jan 11, 2012 10:00 AM

URL: <http://pirsa.org/12010151>

Abstract:



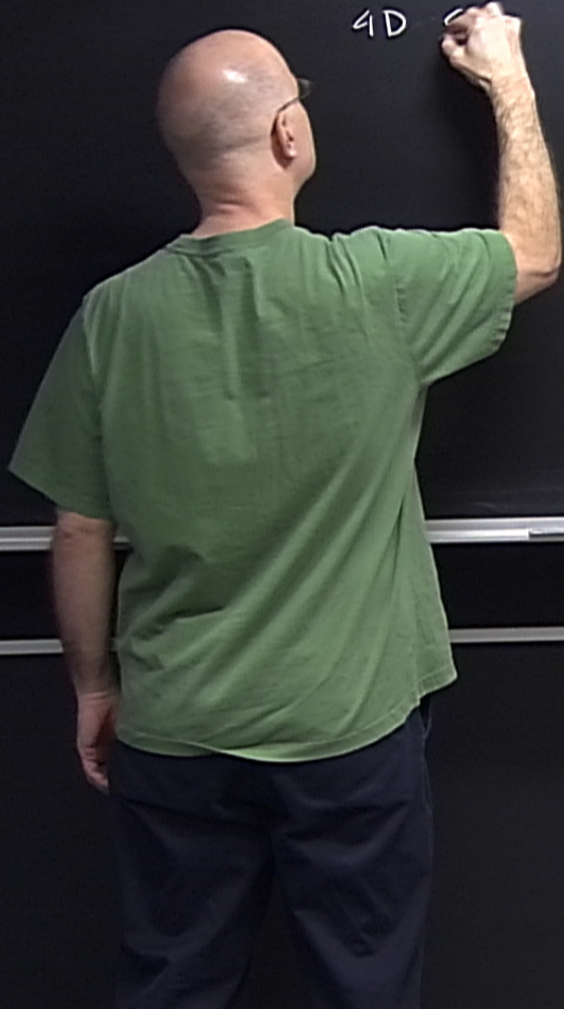
I : FUNDAMENTALS



CAUTION
DO NOT USE LIFELINES AND RESCUE SWAYS
UNLESS ADVISED BY THE OFFICER OF THE BOARD
IF IN DUBIOUSLY USE ANY
FROM OFFICIALS INSTRUCTED BY THE
BOARD MEMBER ONLY

I : FUNDAMENTALS

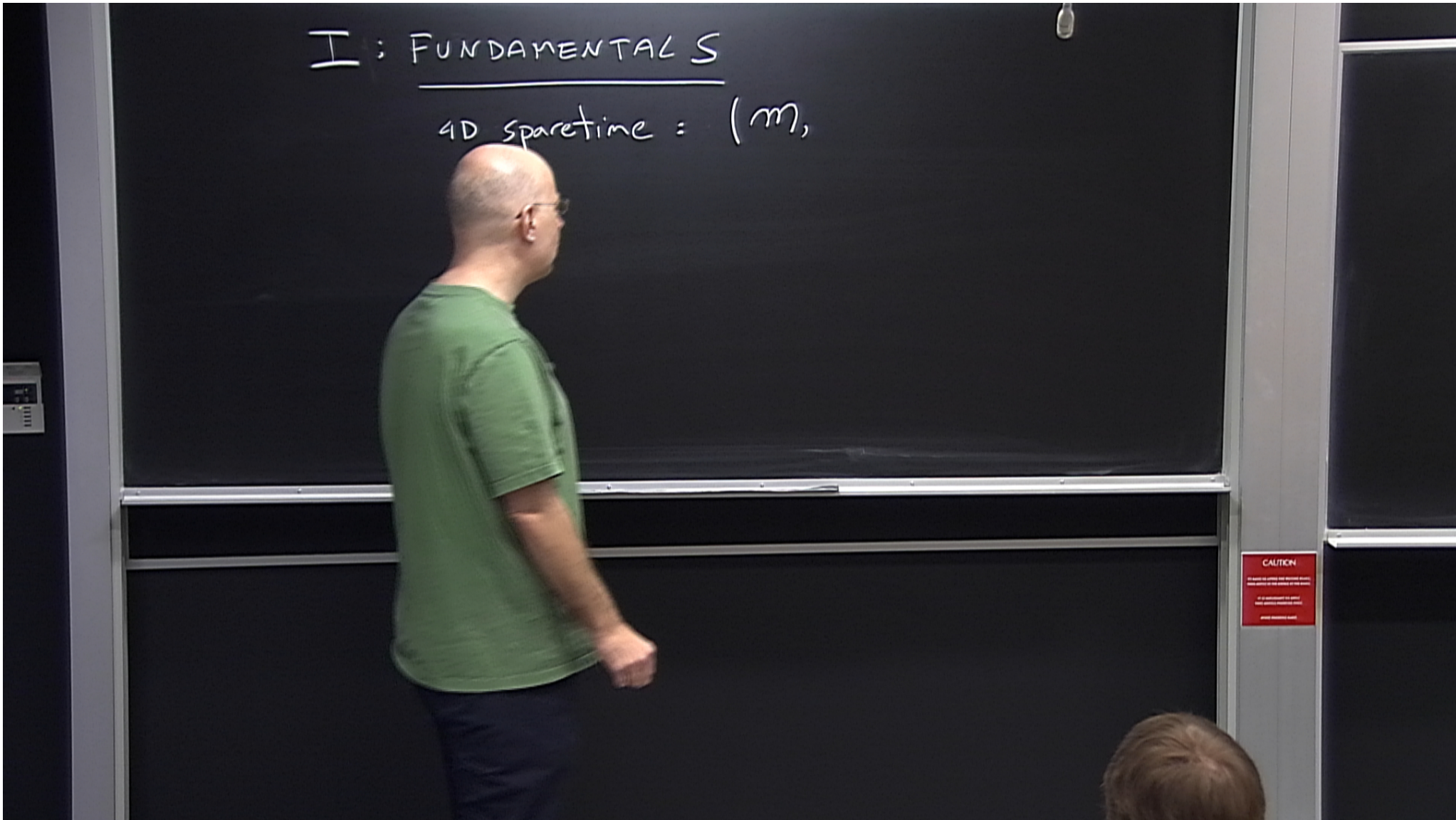
4D



CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE SURROUNDING AREA
AS IT MAY BE HOT OR DAMAGED
BY THE BOARD SURFACE.

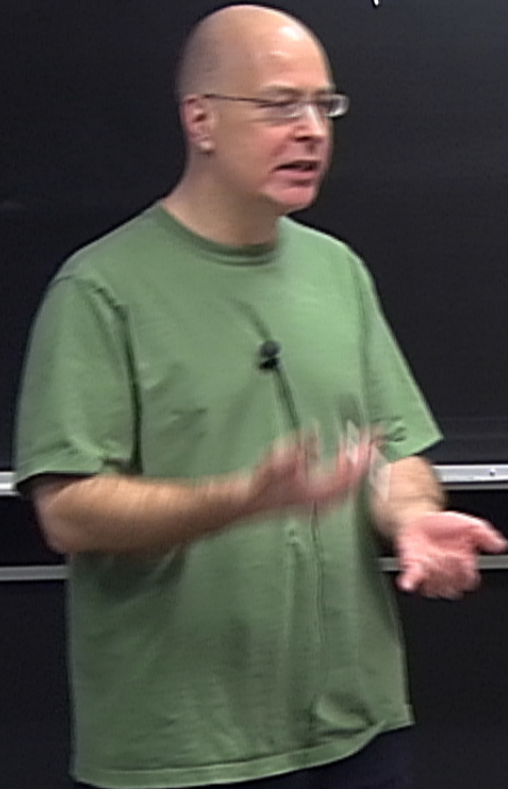
I : FUNDAMENTALS

4D spacetime = $(m,$



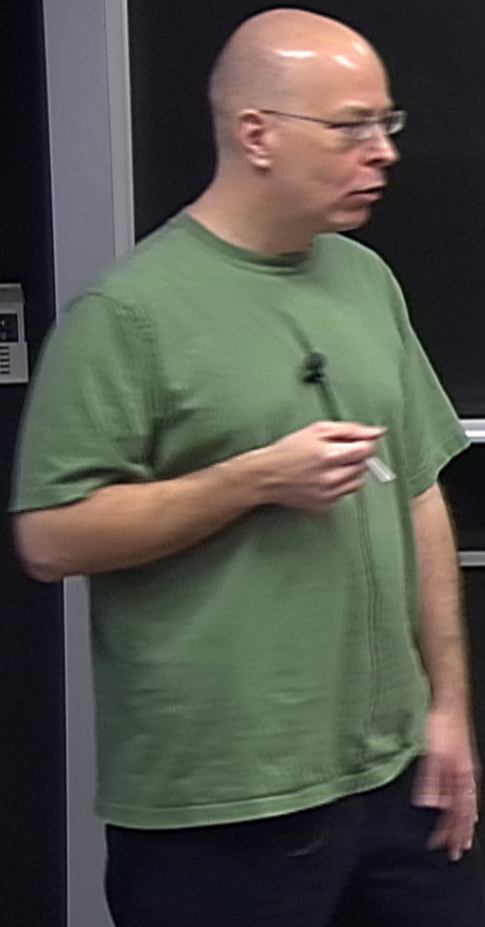
I : FUNDAMENTALS

4D spacetime = (m, g)



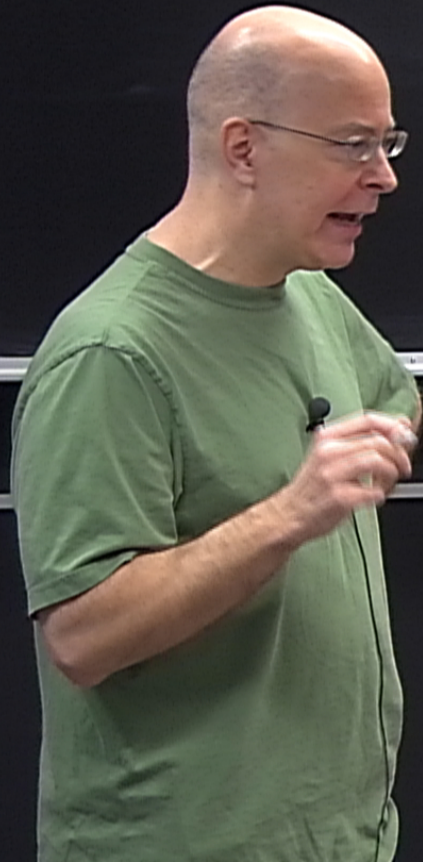
I : FUNDAMENTALS

4D spacetime = (m, g)

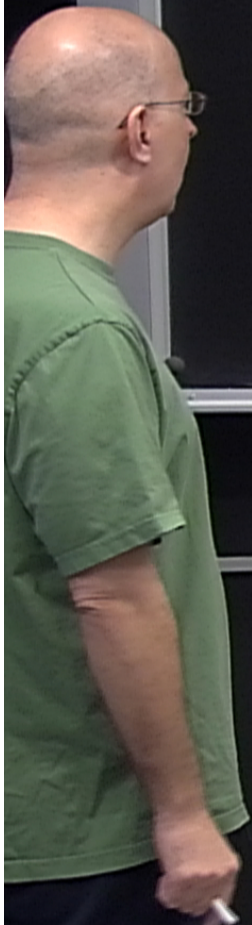


I : FUNDAMENTALS

4D spacetime = (m, g)



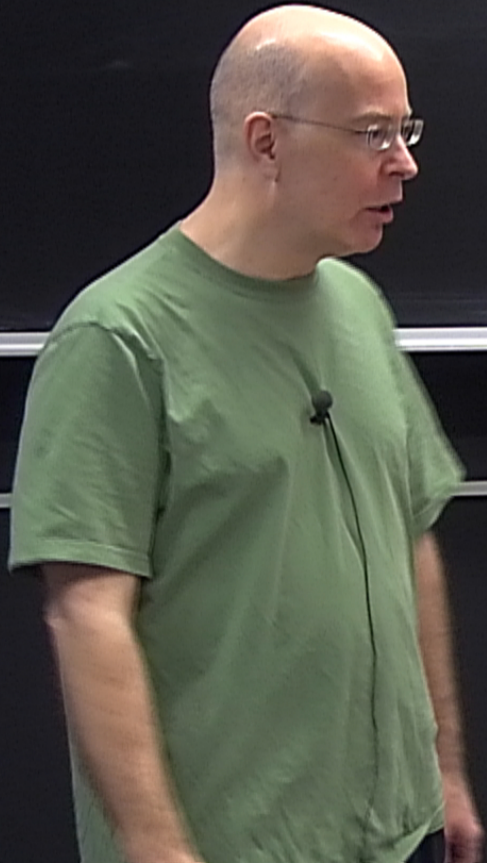
1.5. POSITION AND TIME
4D spacetime = (m, g)



CAUTION
DO NOT TOUCH THE BOARD
IF YOU TOUCH THE BOARD
YOU WILL BE ELECTRICALLY SHOCKED
PLEASE DO NOT TOUCH THE BOARD

I. FUNDAMENTALS

4D spacetime = (m, g)
coordinate system x^α

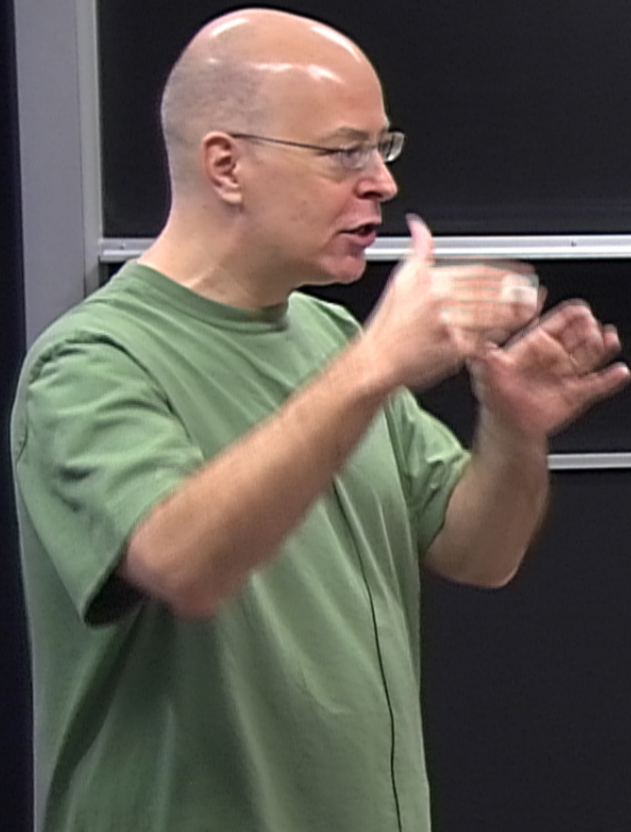


4D spacetime = (m, g)
coordinate system x^α

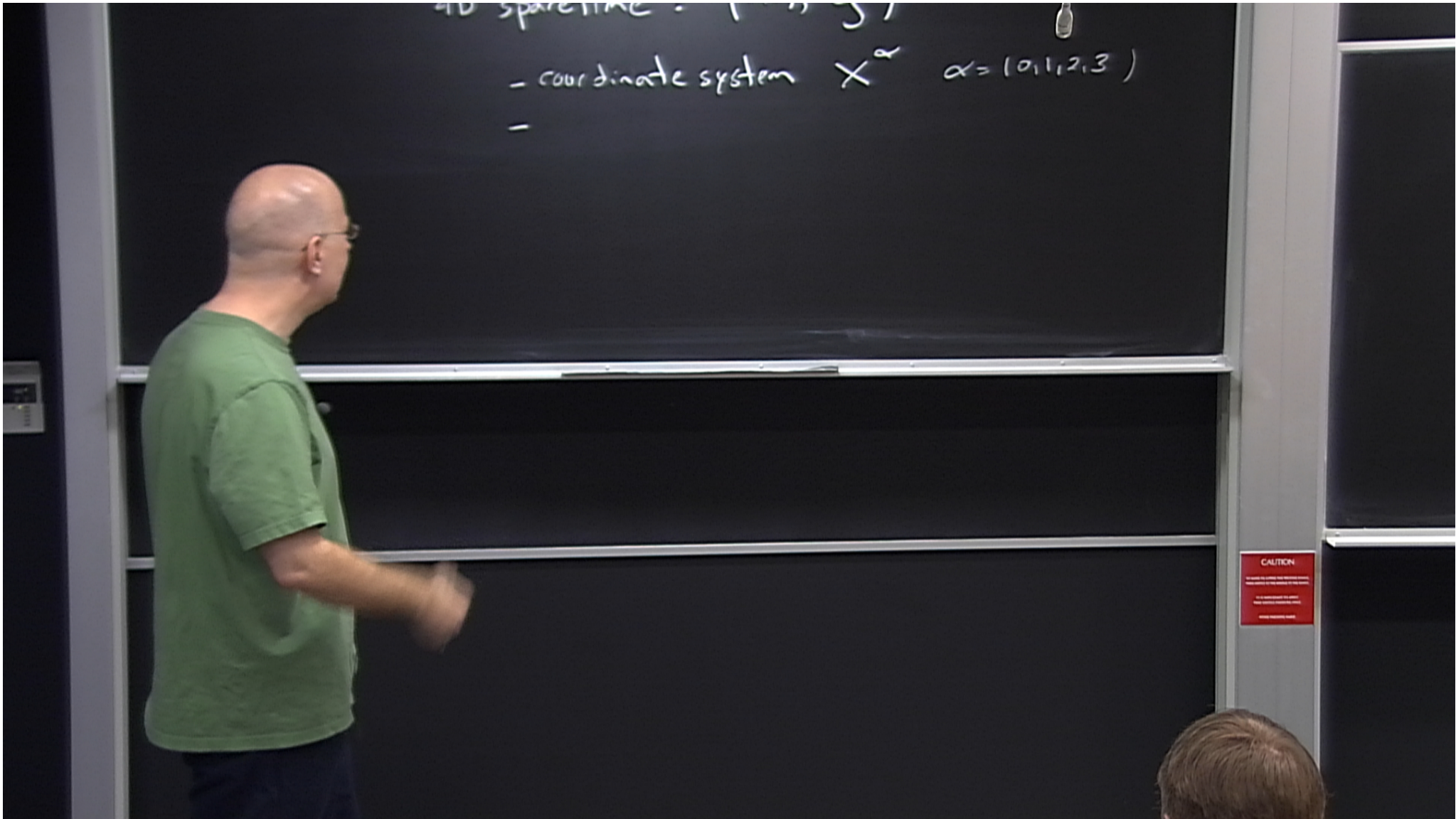


CAUTION
DO NOT TOUCH THE BOARD
IF YOU TOUCH THE BOARD
YOU WILL BE ELECTRICALLY SHOCKED.

4D spacetime = (m, g)
coordinate system x^α $\alpha = (0, 1, 2, 3)$



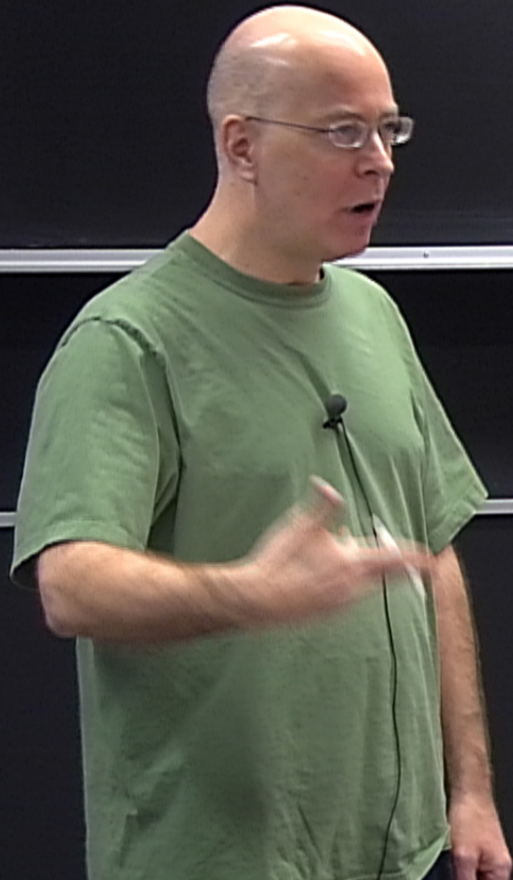
CAUTION
DO NOT TOUCH THE BOARD
IF A CAUTION TAG IS
PRESENT, PLEASE DO NOT
TOUCH THE BOARD.



4D spacetime = (t, x, y, z)

- coordinate system X^α $\alpha = (0, 1, 2, 3)$

- vectors A^α

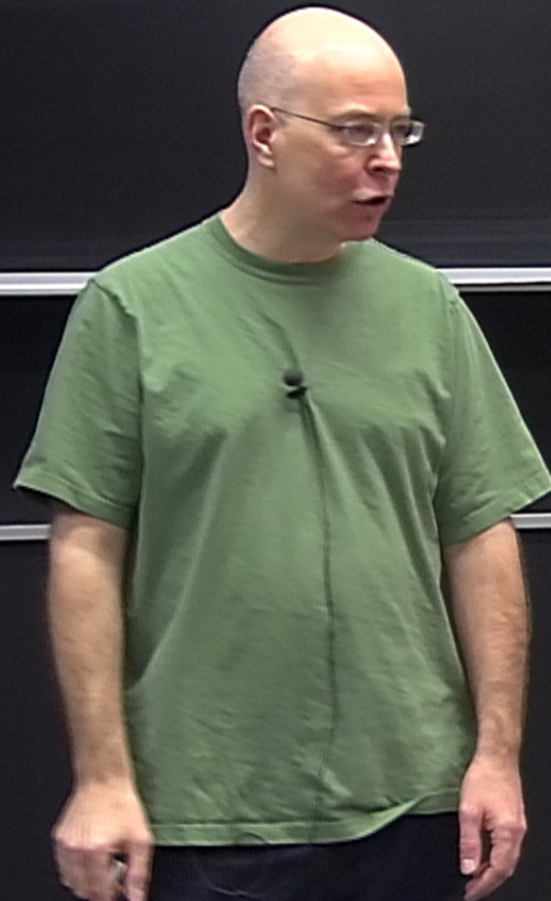


CAUTION
DO NOT TOUCH THE BOARD WHEN
IT IS HOT TO THE TOUCH
OR YOU WILL BE BURNED.
IF YOU TOUCH THE BOARD
YOU WILL BE BURNED.
PLEASE BE CAREFUL.

4D spacetime = (t, x, y, z)

- coordinate system x^α $\alpha = (0, 1, 2, 3)$

- vectors A^α

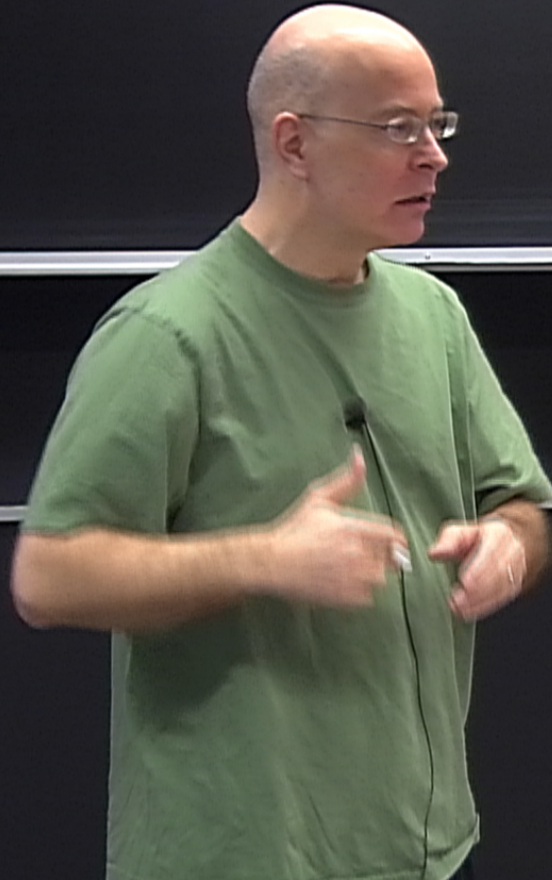


CAUTION
DO NOT TOUCH THE BOARD
IF IT IS HOT TO THE TOUCH
IT IS HOT TO THE TOUCH
DO NOT TOUCH THE BOARD

4D spacetime = (t, x, y, z)

- coordinate system x^α $\alpha = (0, 1, 2, 3)$

- vectors A^α



CAUTION
DO NOT TOUCH THE BOARD
IF YOU TOUCH THE BOARD
YOU WILL BE ELECTRICALLY SHOCKED.
PLEASE REMEMBER THIS.

4D spacetime = (t, x, y, z)

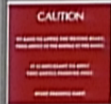
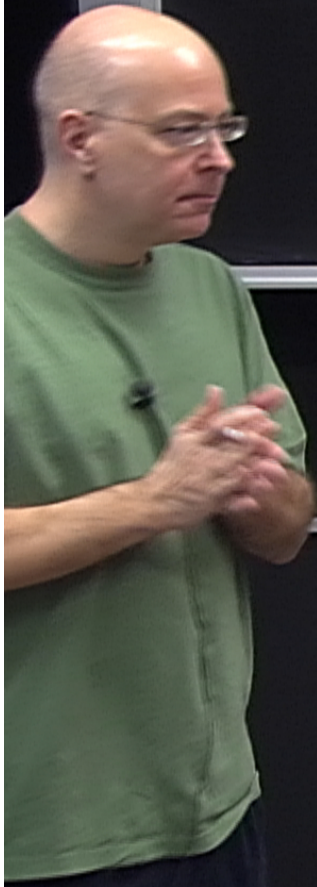
- coordinate system x^α $\alpha = (0, 1, 2, 3)$

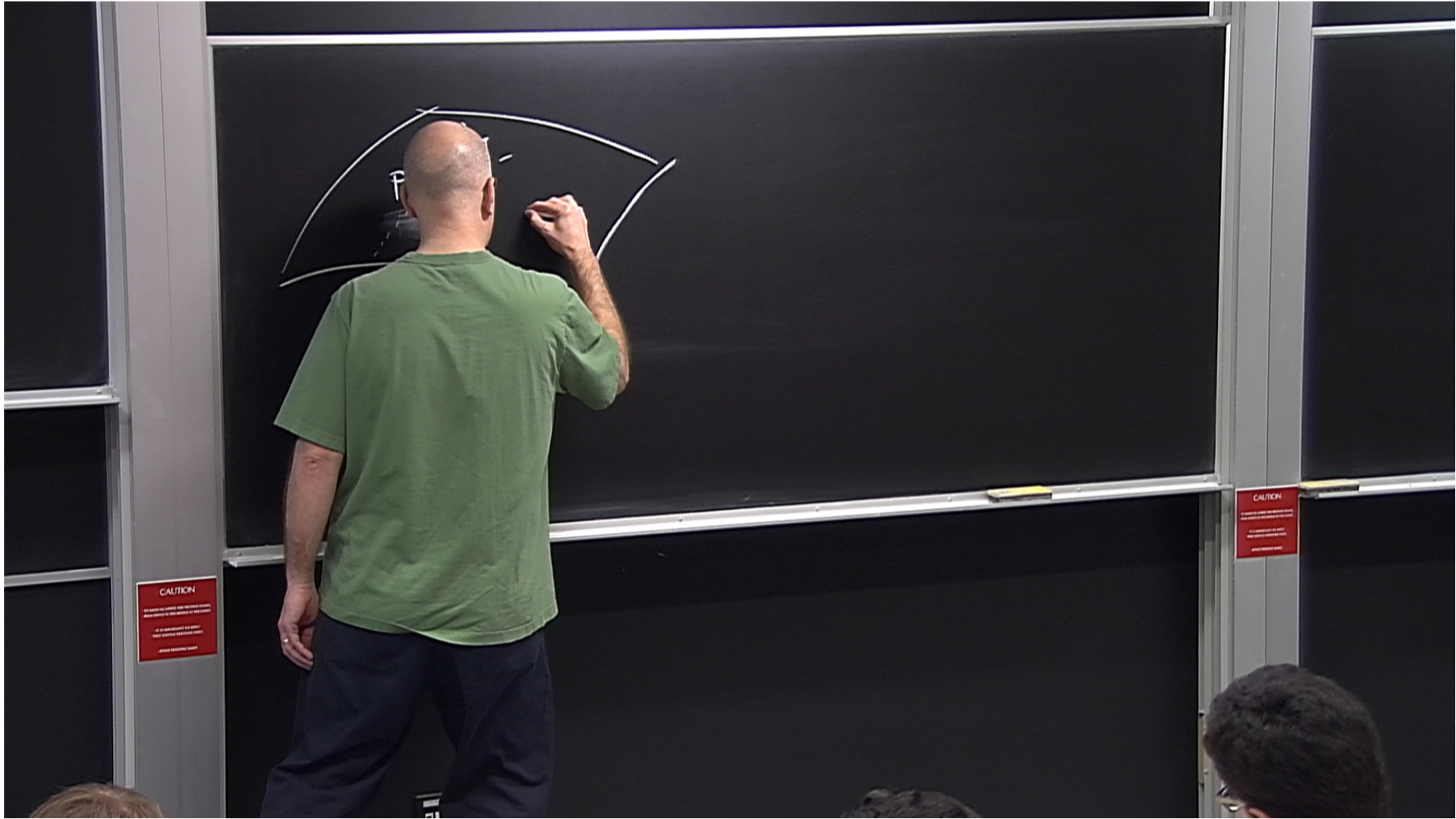
- vectors A^α (tangent to a curve α)

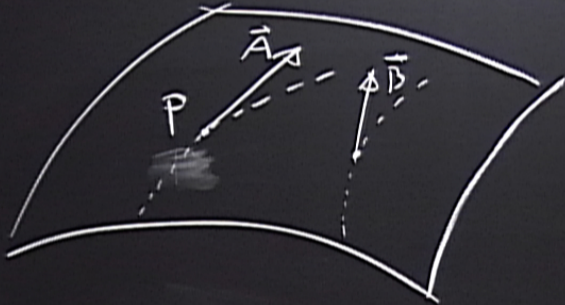
4D spacetime = (M, g)

- coordinate system x^α $\alpha = (0, 1, 2, 3)$

- vectors A^α (tangent to a curve at one point on M)

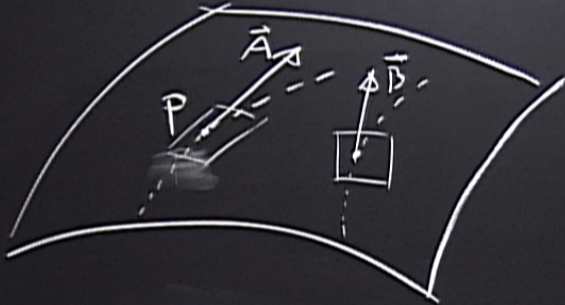






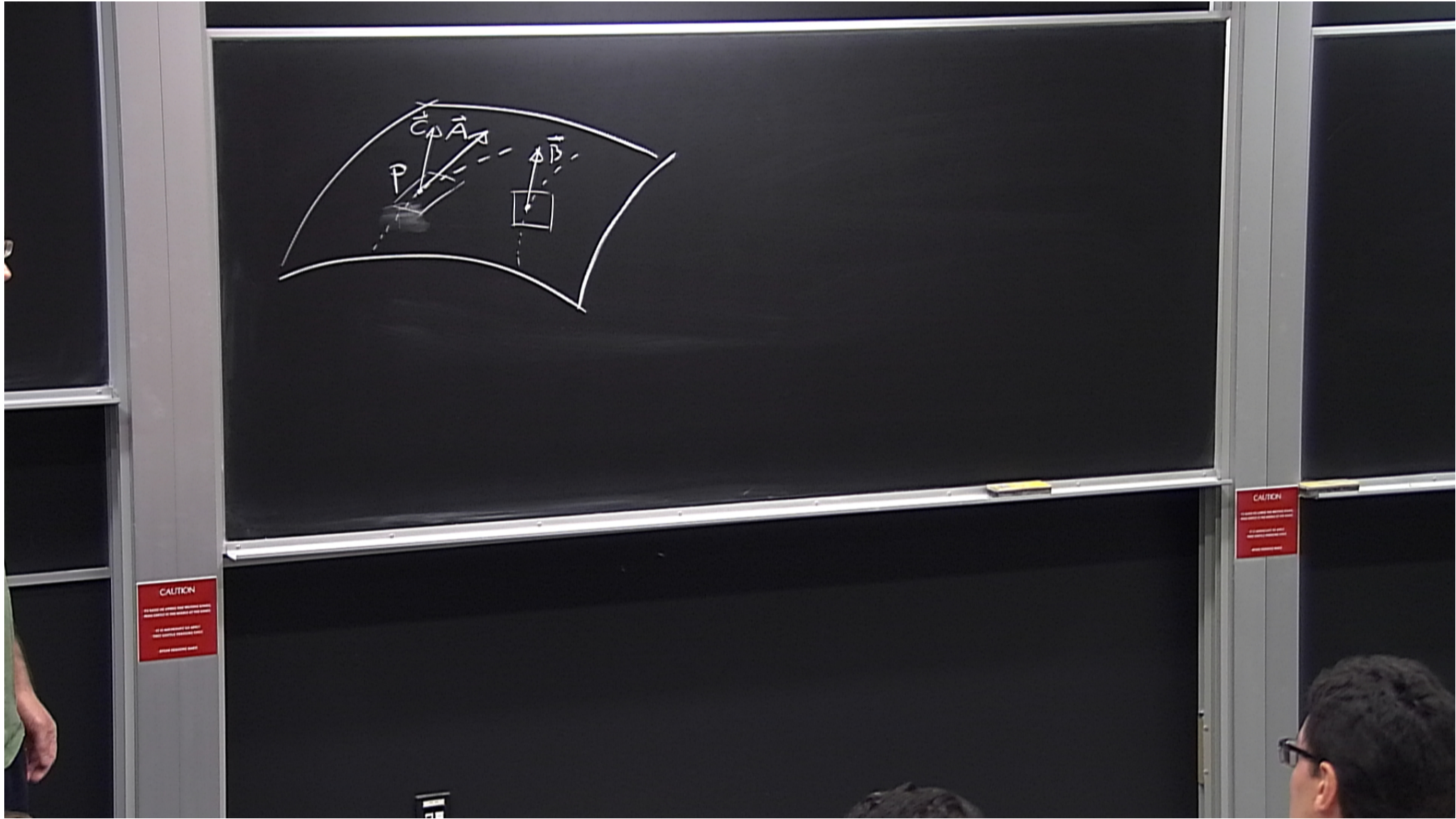
CAUTION
DO NOT USE FORCE OR PRESSURE TO
REMOVE OR DISPLACE THE BOARD
IF IT IS NECESSARY TO
DO SO, CONTACT THE
FACULTY ASSISTANT

CAUTION
DO NOT USE FORCE OR PRESSURE TO
REMOVE OR DISPLACE THE BOARD
IF IT IS NECESSARY TO
DO SO, CONTACT THE
FACULTY ASSISTANT



CAUTION
DO NOT TOUCH THE SURFACE OF THE LENS
OR THE SURFACE OF THE OBJECT
OR THE SURFACE OF THE IMAGE
OR THE SURFACE OF THE OBJECT

CAUTION
DO NOT TOUCH THE SURFACE OF THE LENS
OR THE SURFACE OF THE OBJECT
OR THE SURFACE OF THE IMAGE
OR THE SURFACE OF THE OBJECT

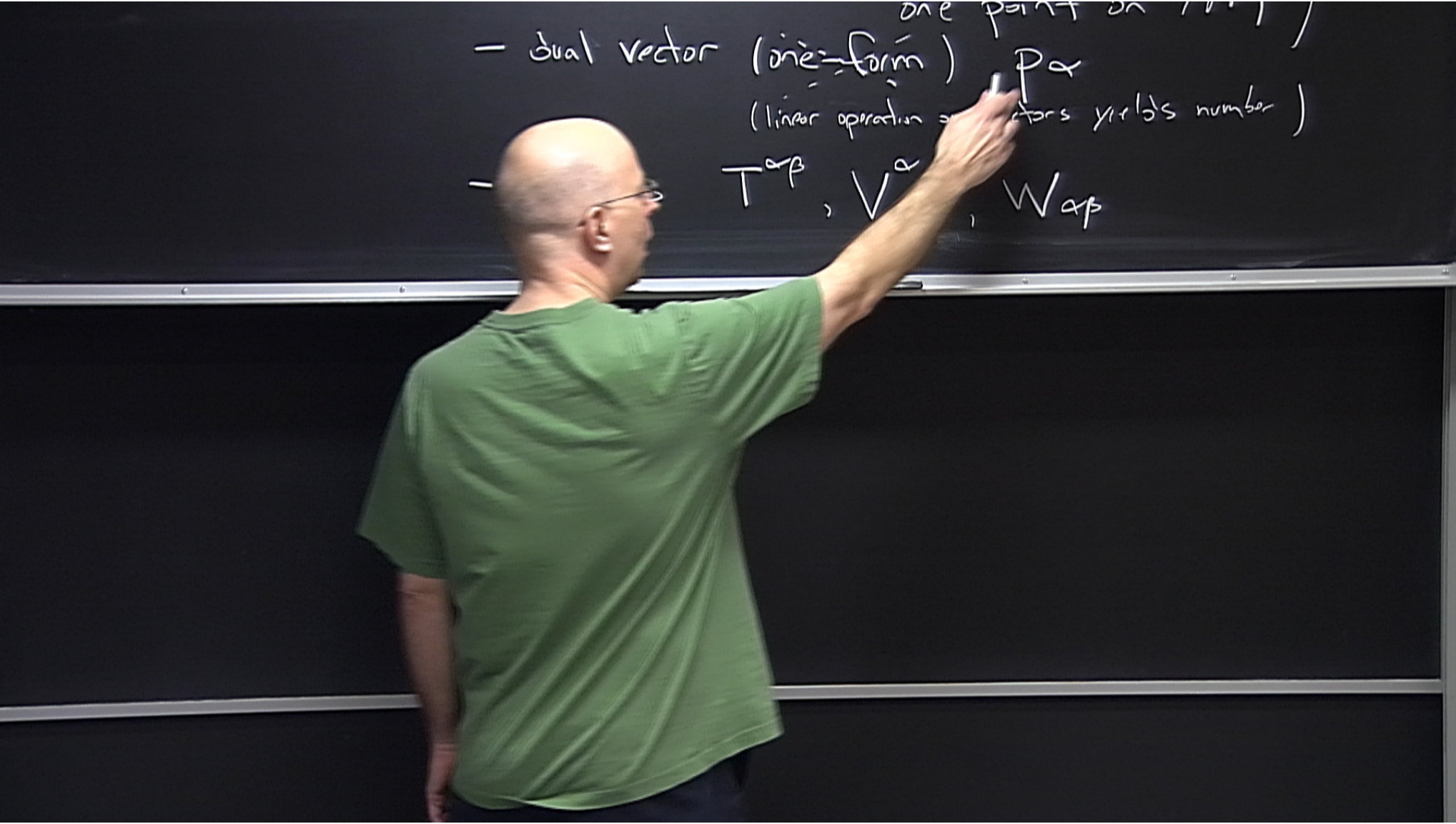


CAUTION
DO NOT ATTEMPT TO REMOVE COVER
OR TO OPERATE THE UNIT
UNLESS YOU ARE
A QUALIFIED SERVICE TECHNICIAN.
PLEASE CONSULT OWNER'S
MANUAL.

CAUTION
DO NOT ATTEMPT TO REMOVE COVER
OR TO OPERATE THE UNIT
UNLESS YOU ARE
A QUALIFIED SERVICE TECHNICIAN.
PLEASE CONSULT OWNER'S
MANUAL.

- coordinate system X^α ($\alpha = (0, 1, 2, 3)$)
- vectors A^α (tangent to a curve at one point on M)
- dual vector (one-form) p_α

- vectors A^α (tangent to a curve at one point on M)
- dual vector (one-form) p_α (operation on vectors yields number)
- tensors \otimes, \vee



one point on $(\mathbb{R}^n)^*$
- dual vector (one-form) ρ_α
(linear operation on vectors yields number)
- $T_{\alpha\beta}$, V^α , $W_{\alpha\beta}$

one point on $(1,0,0)$
- dual vector (one-form) p_α
(linear operation on vectors yields number)
- tensors $T^{\alpha\beta}$, V^α , $W_{\alpha\beta}$

- metric tensor $g_{\alpha\beta}$

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- tensor $\mathcal{g}_{\alpha\beta}$

$$ds^2 = \mathcal{g}_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- metric tensor $\gamma_{\alpha\beta}$

$$ds^2 = \gamma_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances
- describes gravity!

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances
- describes gravity!

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances
- describes gravity!

invers

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances
- describes gravity!

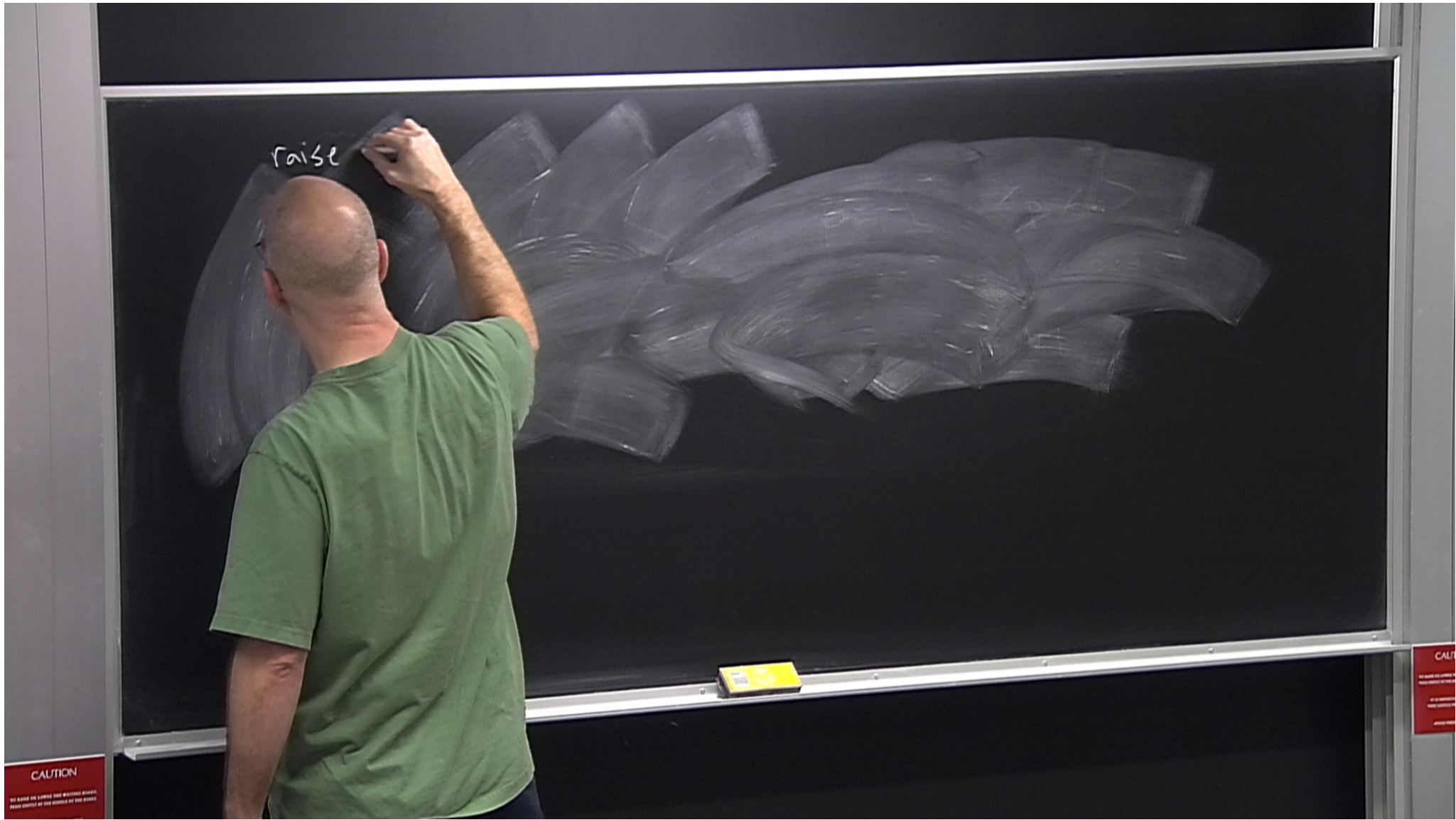
- inverse metric $g^{\alpha\beta}$

- metric tensor $g_{\alpha\beta}$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = \text{spacetime interval}$$

- converts coordinate differentials into physical distances
- describes gravity!

- inverse metric $g^{\alpha\beta}$: $g^{\alpha\mu} g_{\mu\beta} = \delta^\alpha_\beta$



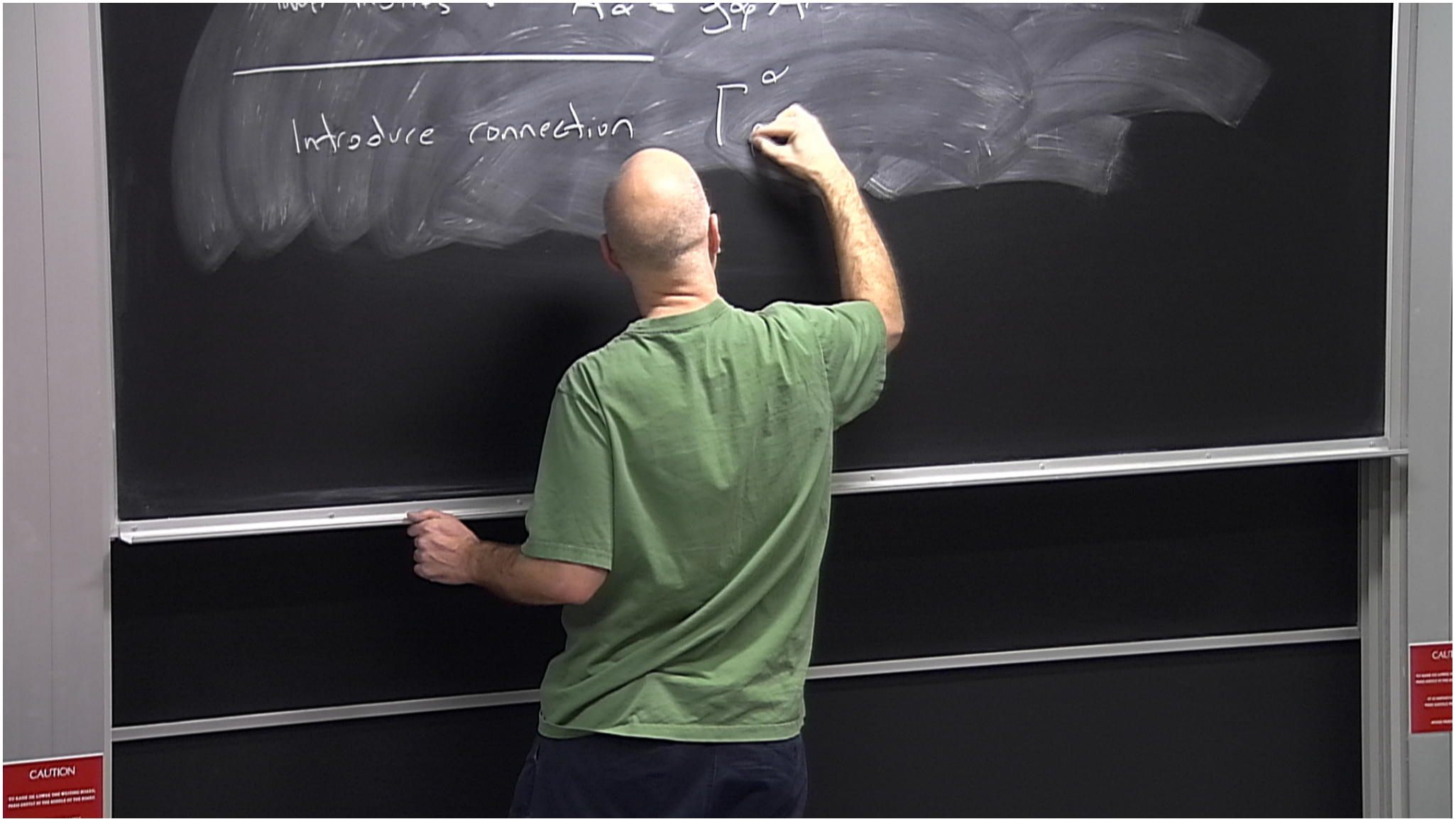
raise indices = $p^\alpha = g^{\alpha\beta} p_\beta$

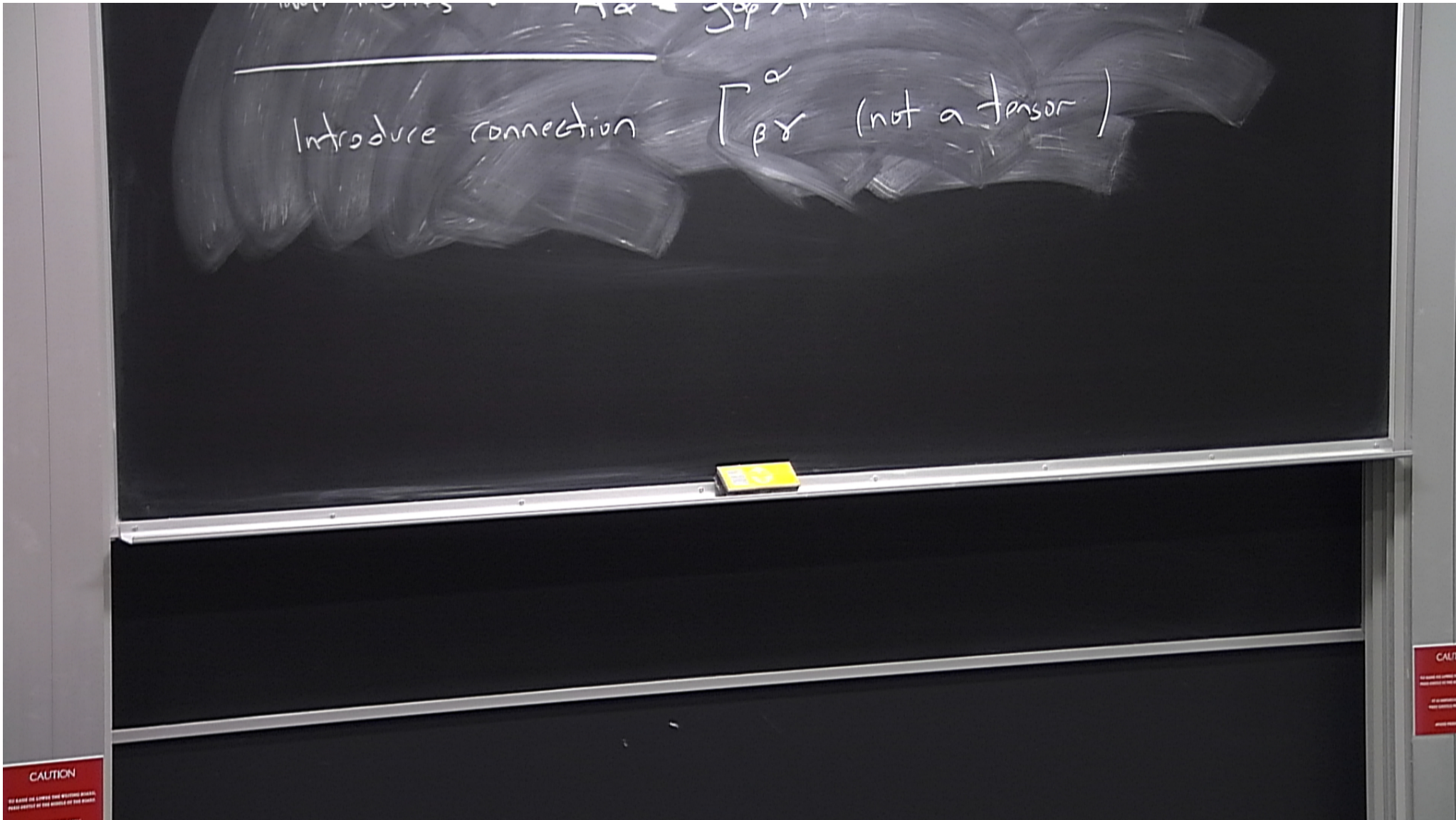


raise indices : $p^\alpha = g^{\alpha\beta} p_\beta$
lower indices : $A_\alpha = g_{\alpha\beta} A^\beta$

CAUTION

DO NOT STAND ON TOP OF THE BOARD.
READ CAREFULLY IN THE CENTER OF THE BOARD.

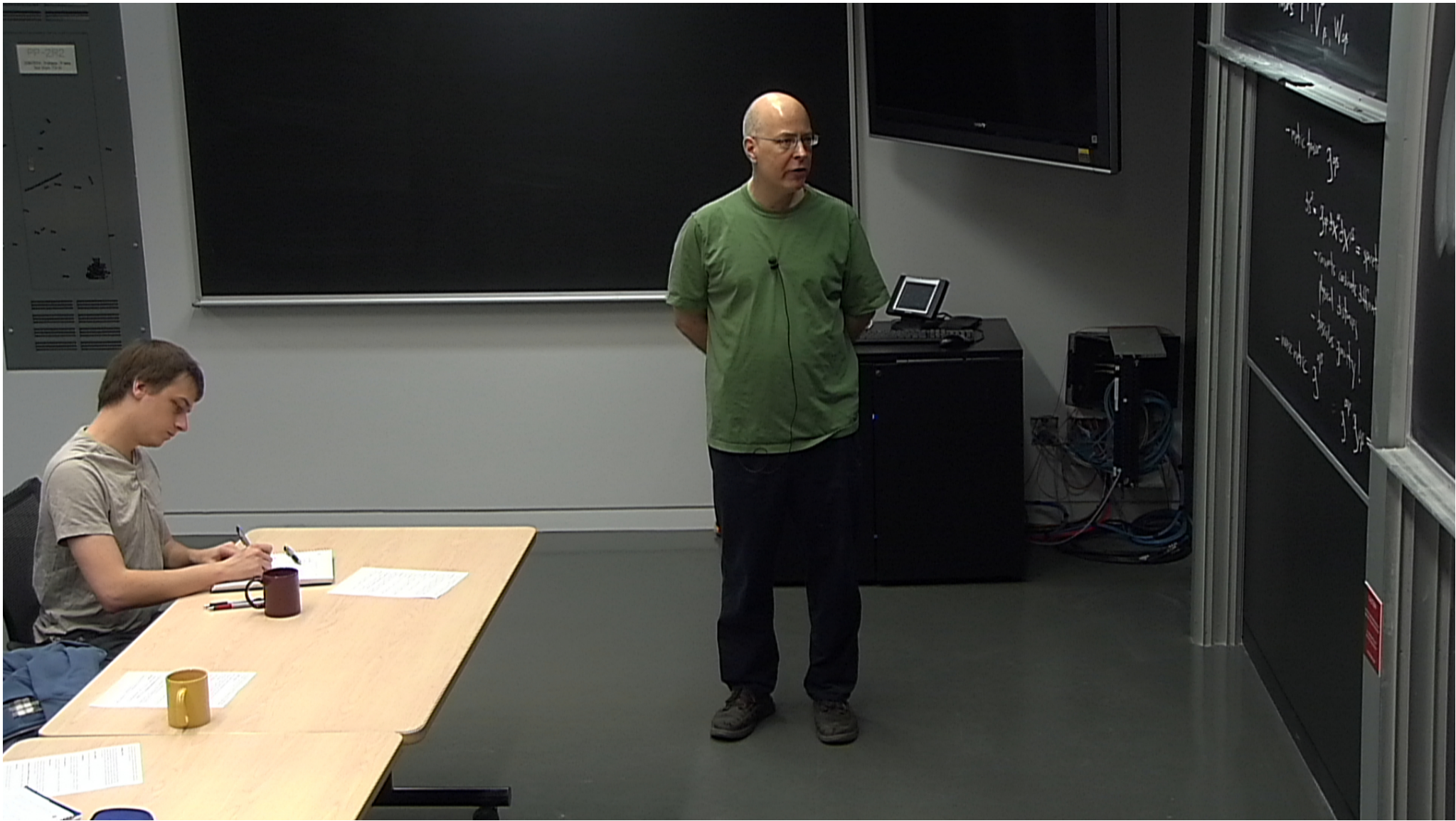


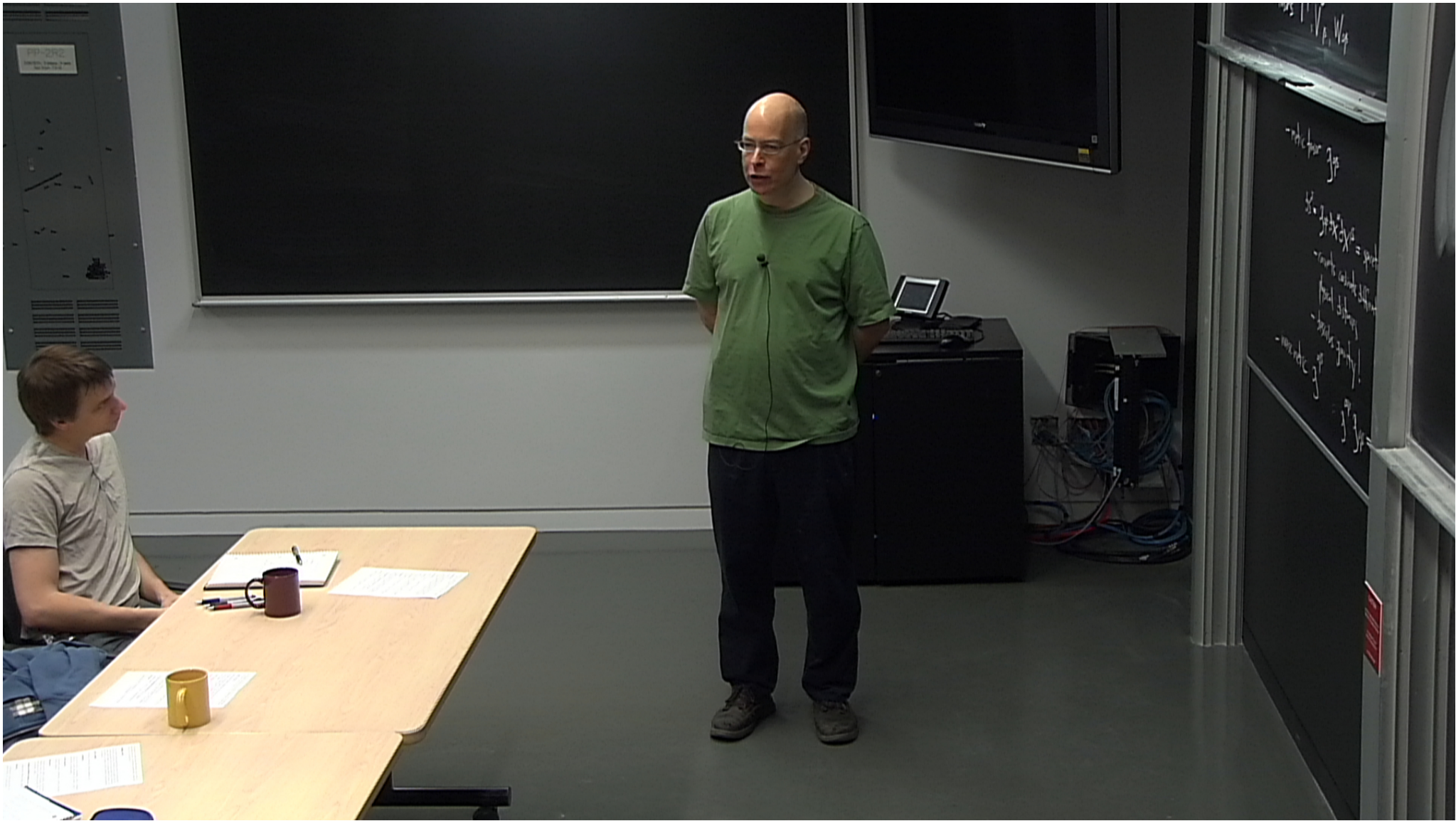


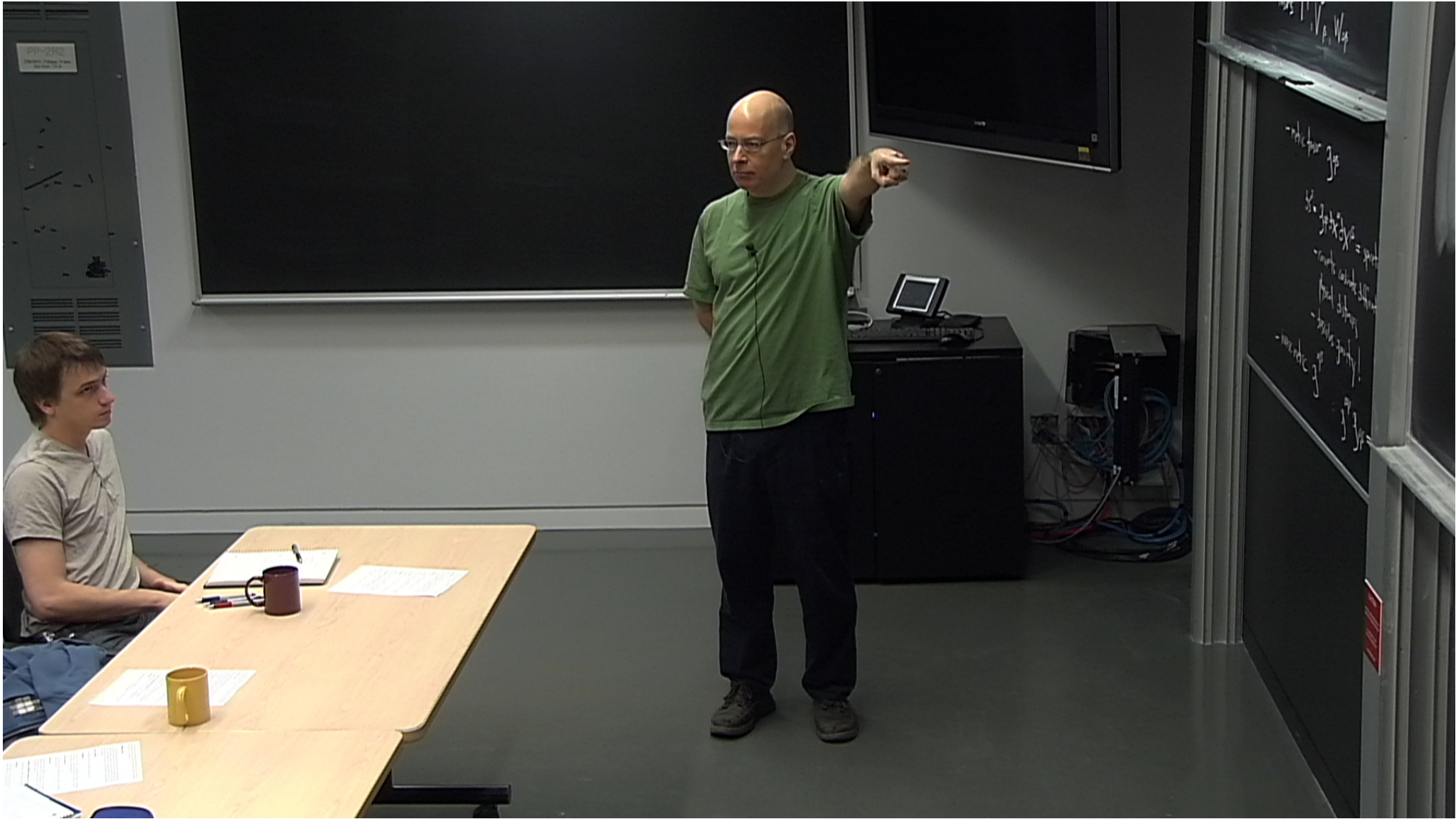
Introduce connection $\Gamma_{\beta\gamma}^{\alpha}$ (not a tensor)

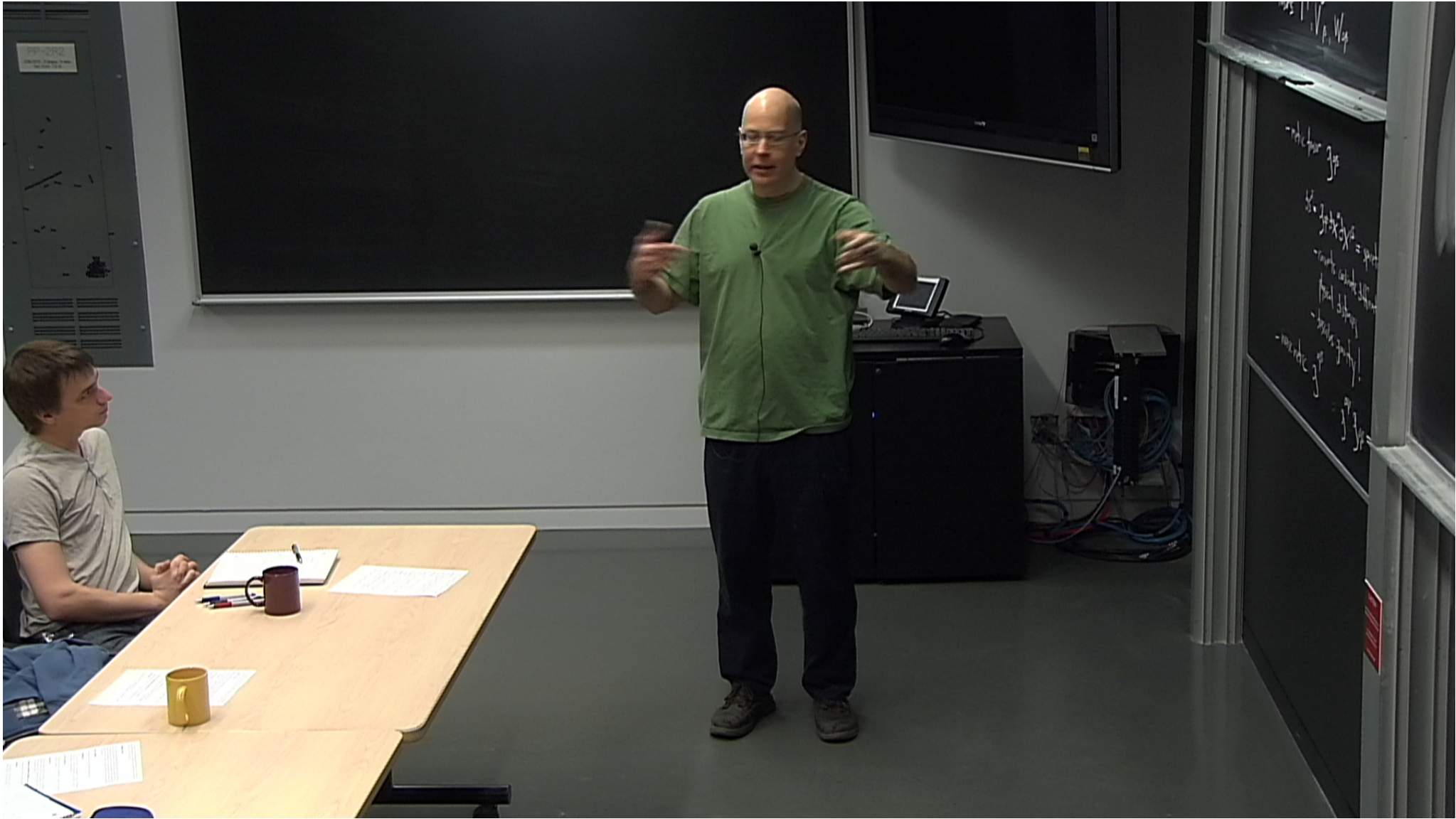
CAUTION

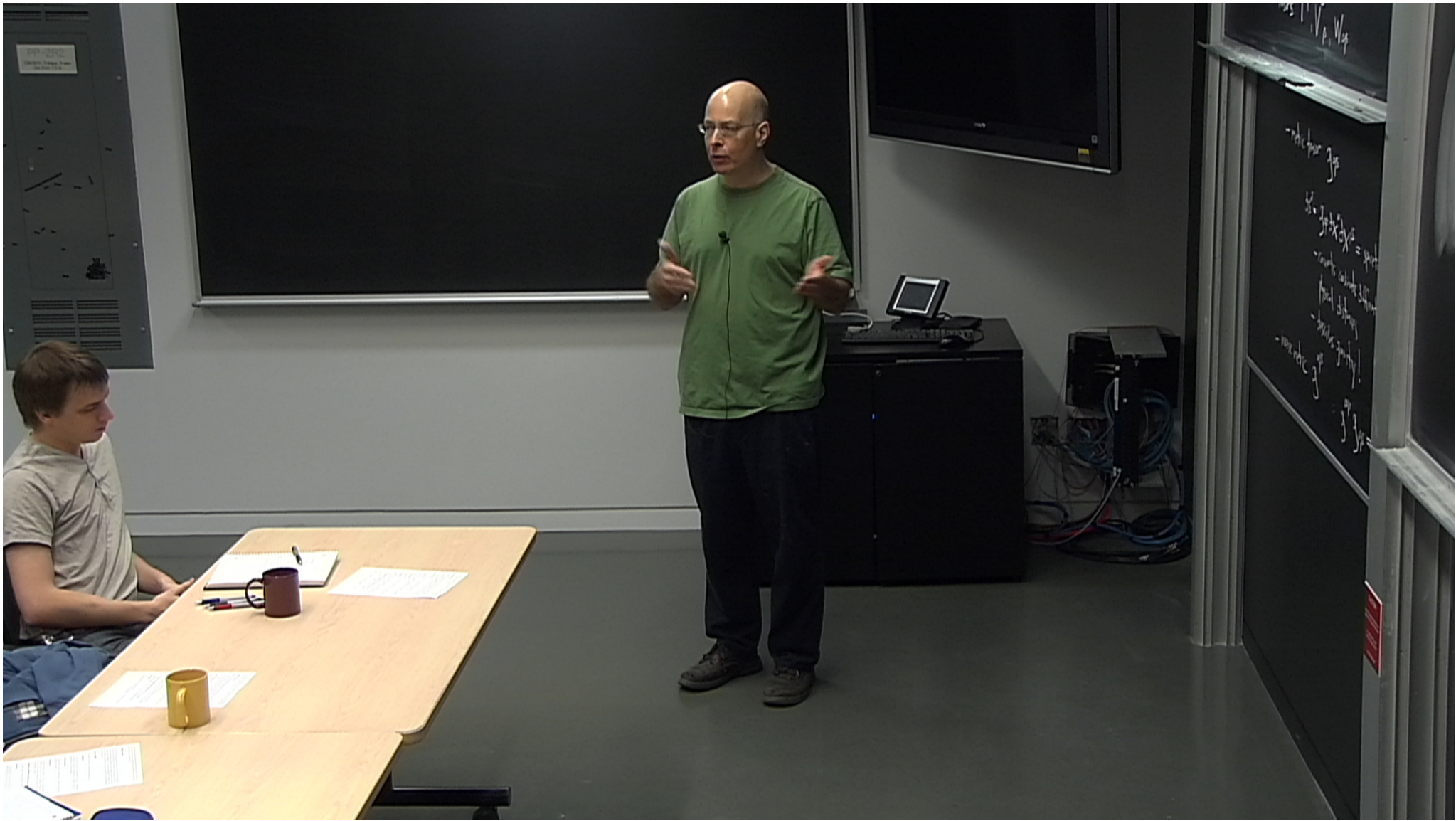
CAUTION

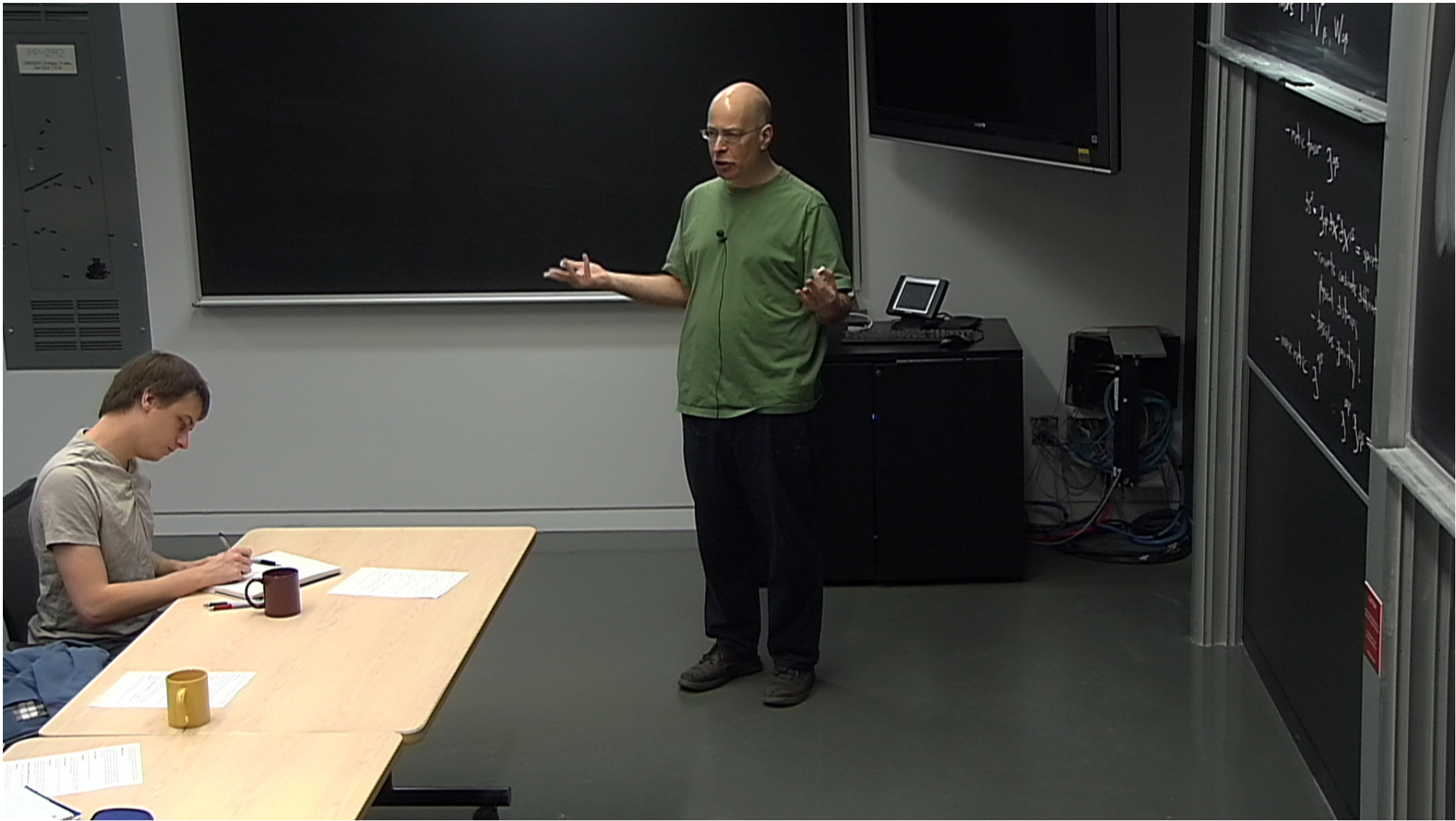


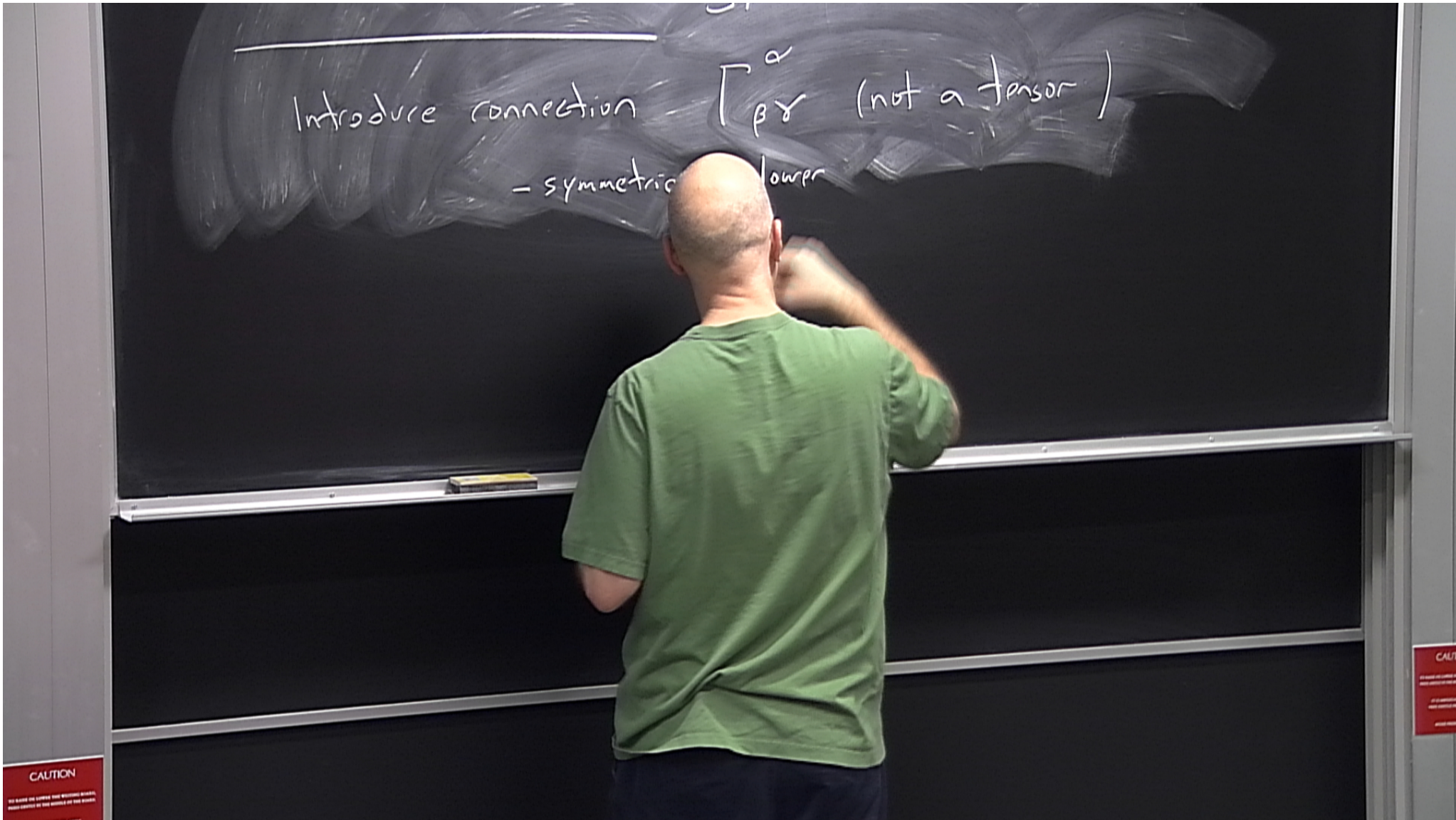


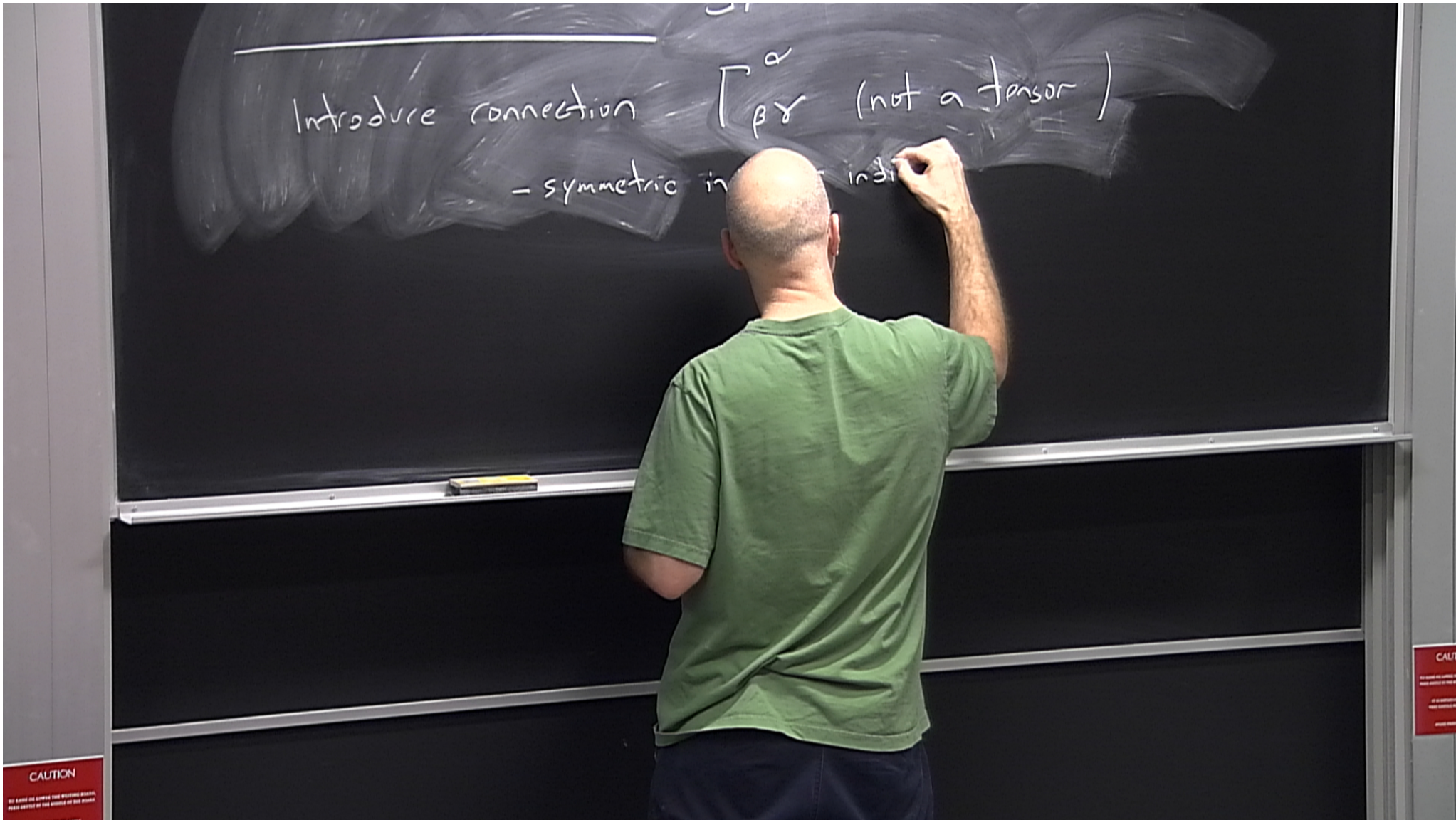


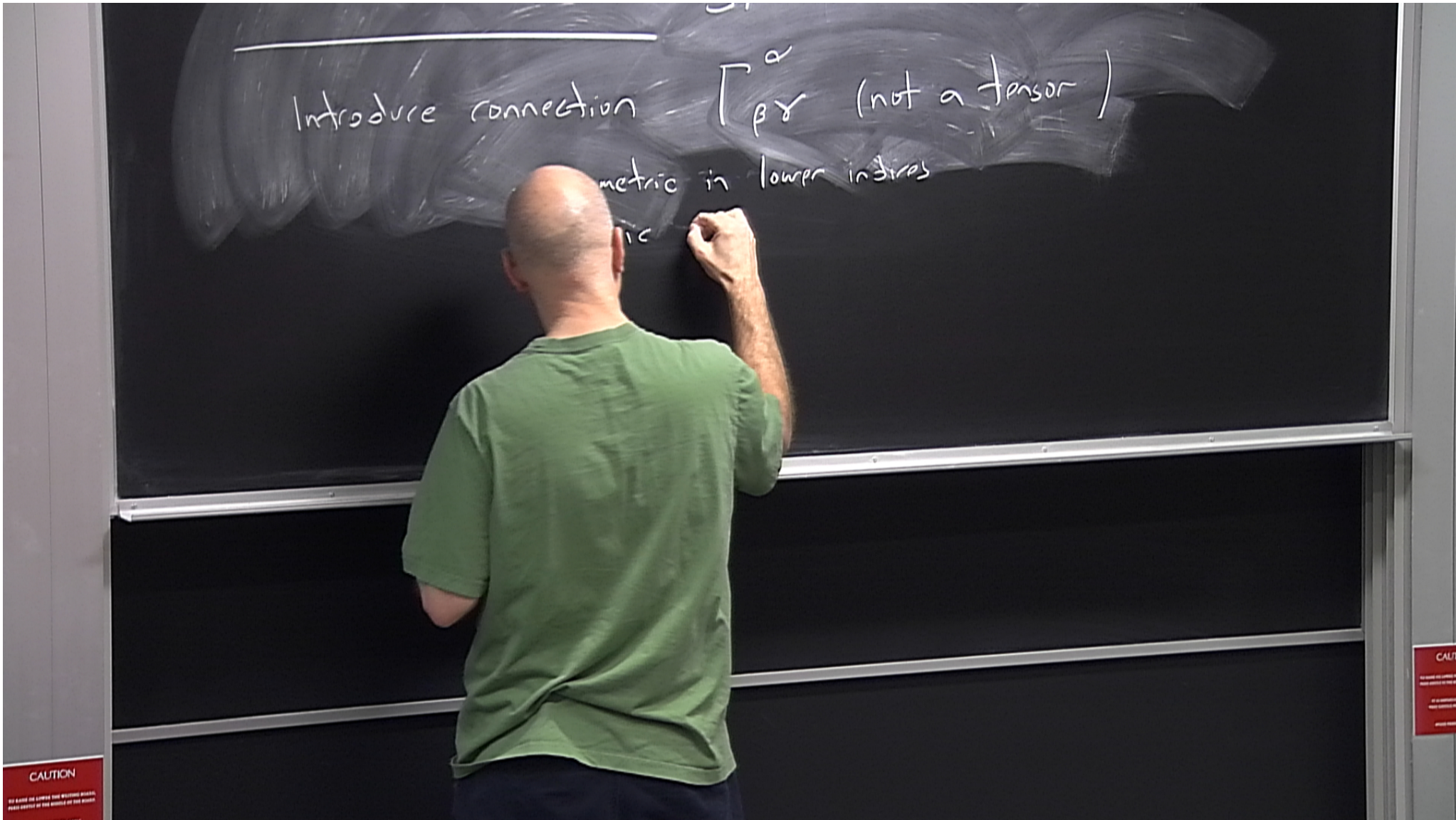






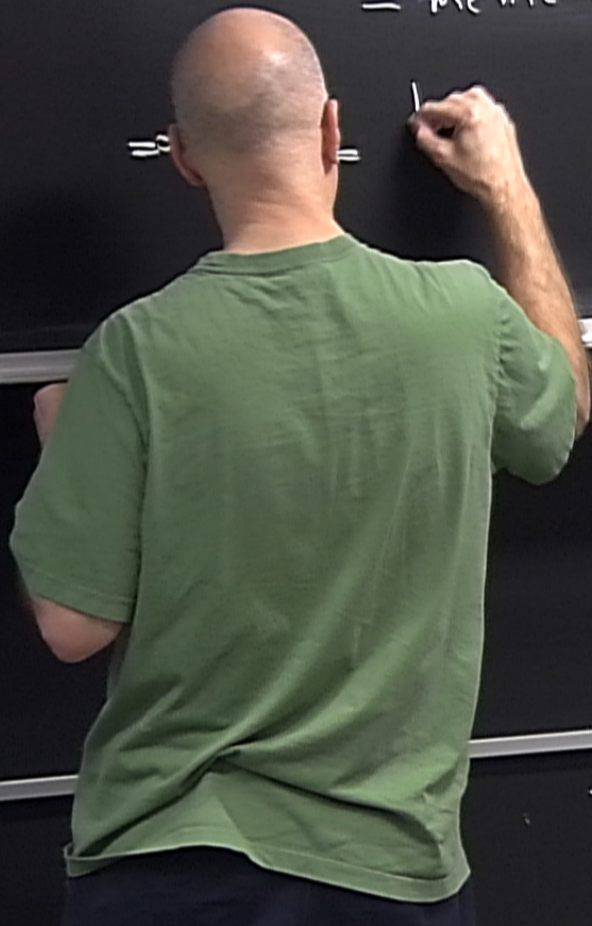






Introduce connection - $\Gamma_{\beta\gamma}$ (not a tensor)

- symmetric in lower indices
- metric compatible



Introduce connection - $\Gamma_{\beta\gamma}^{\alpha}$ (not a tensor)

- symmetric in lower indices
- metric compatible

$$\Rightarrow \Gamma_{\beta\gamma}^{\alpha} = \frac{1}{2} g^{\alpha\mu} (g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu})$$

CAUTION

Covariant differentiation (requires connection).

CAUTION

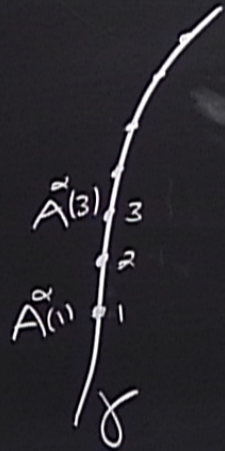
DO NOT TOUCH THE BOARD

PLEASE DO NOT TOUCH THE BOARD

CAUTION

Covariant differentiation (requires connection).

vector fields on a curve γ



differentiate A^{α} along the curve.

Parametric description of $\gamma = X^{\alpha}(\lambda)$
arbitrary parameter.

CAUTION

DO NOT TOUCH THE BOARD WHEN THE BOARD IS HOT.

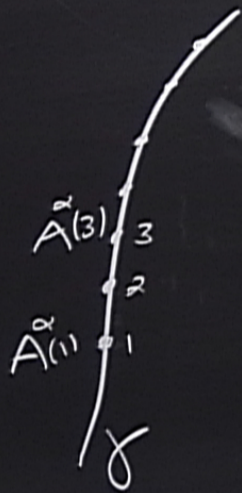
covariant derivative of A^α along curve:

$$\frac{DA^\alpha}{d\lambda} = \frac{\partial A^\alpha}{\partial \lambda}$$

CAUTION
DO NOT USE BOARD FOR OTHER PURPOSES
USE BOARD AS INTENDED BY THE MANUFACTURER

Covariant differentiation (requires connection).

vector fields on a curve γ



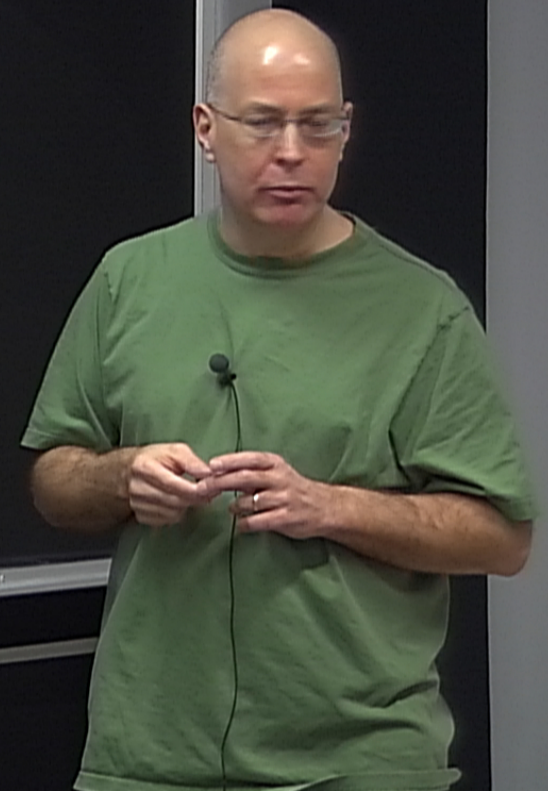
differentiate A^α along the curve

Parametric description of $\gamma = X^\alpha(\lambda)$
arbitrary parameter

vector: $A^\alpha(\lambda)$

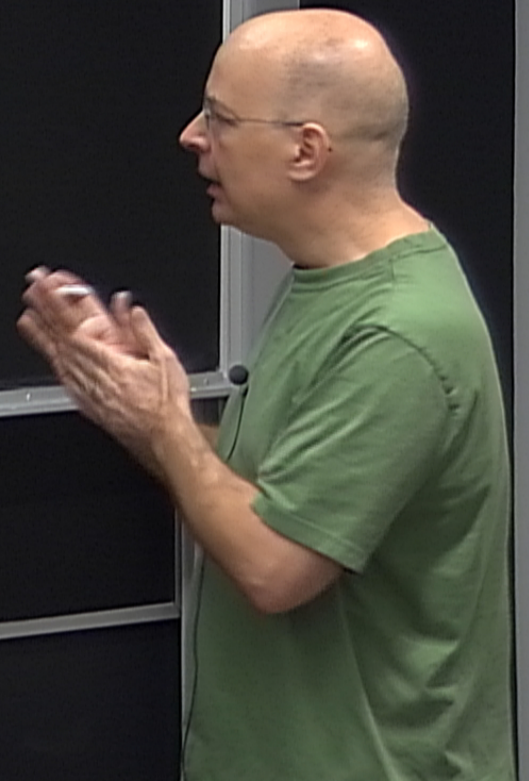
covariant derivative of A^α along curve:

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial X^\beta}{\partial \lambda} \frac{\partial X^\gamma}{\partial \lambda}$$



covariant derivative of A^α along u^μ :

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda}$$



covariant derivative of A^α along γ :

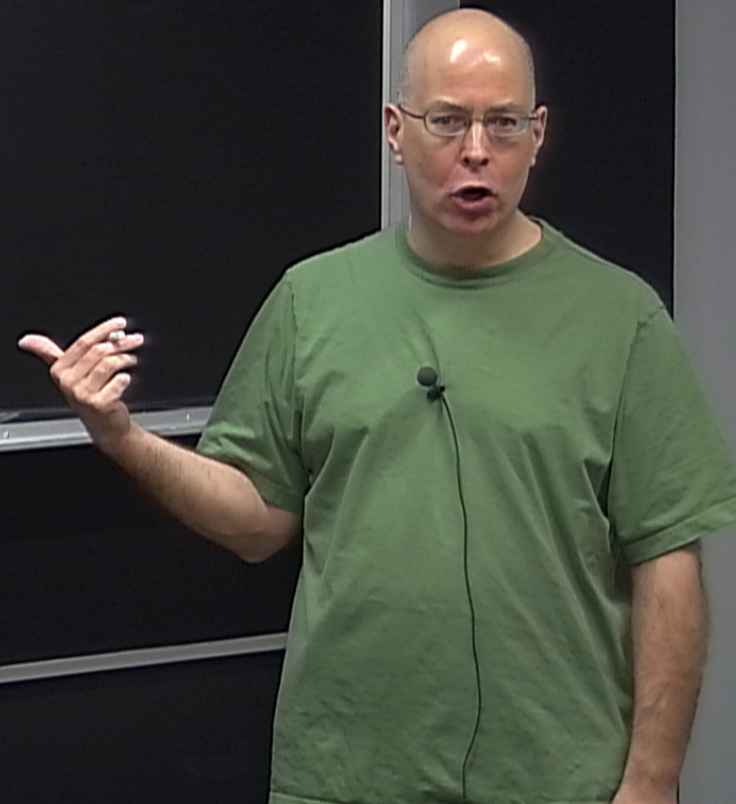
$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda}$$

tangent vector to $\gamma = \dot{}$

covariant derivative of A^α along γ :

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda}$$

tangent vector to $\gamma = \dot{x}^\alpha \equiv \frac{\partial x^\alpha}{\partial \lambda}$



covariant derivative of A^α along γ :

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma^\alpha_{\beta\gamma} \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda} = \frac{\partial A^\alpha}{\partial \lambda} +$$

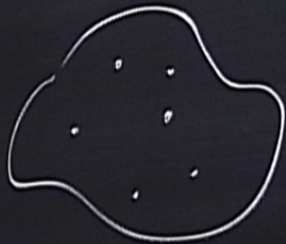
tangent vector to $\gamma = \dot{\gamma}^\alpha \equiv \frac{\partial x^\alpha}{\partial \lambda}$

covariant derivative of A^α along γ :

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda} = \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \dot{x}^\beta \dot{x}^\gamma$$

tangent vector to $\gamma = \dot{x}^\alpha \equiv \frac{\partial x^\alpha}{\partial \lambda}$

Given a vector field $\vec{A}(x^{\mu})$ everywhere in open region,



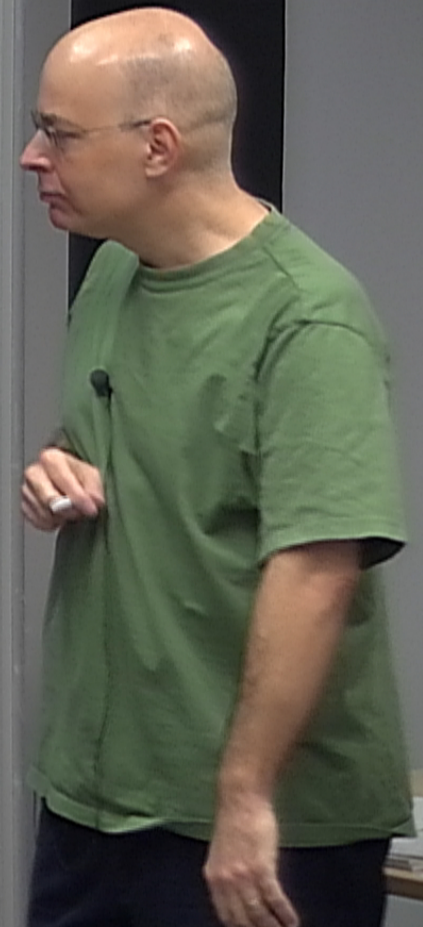
CAUTION
DO NOT TOUCH THE BOARD OR CHALK
OR MARKERS AT THE BOTTOM OF THE BOARD.
IF YOU NEED TO TOUCH THE BOARD
PLEASE ASK A TA OR TAUGHTER FIRST.

Given a vector field $\vec{A}(x^{\mu})$ everywhere in open region



CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER AT THE END OF THE BOARD.
IF YOU NEED TO TOUCH THE BOARD
PLEASE ASK A TEACHER OR TA

Given a vector field $\vec{A}(x^\mu)$ everywhere in open region,



CAUTION
DO NOT TOUCH THE BOARD OR CHALK
OR MARKERS AT THE BOTTOM OF THE BOARD.
DO NOT SMOKING OR DRINKING
OR EATING IN THE CLASSROOM.

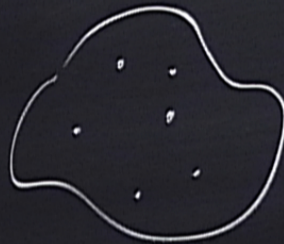
Given a vector field $A^\alpha(x^\mu)$ in open region,



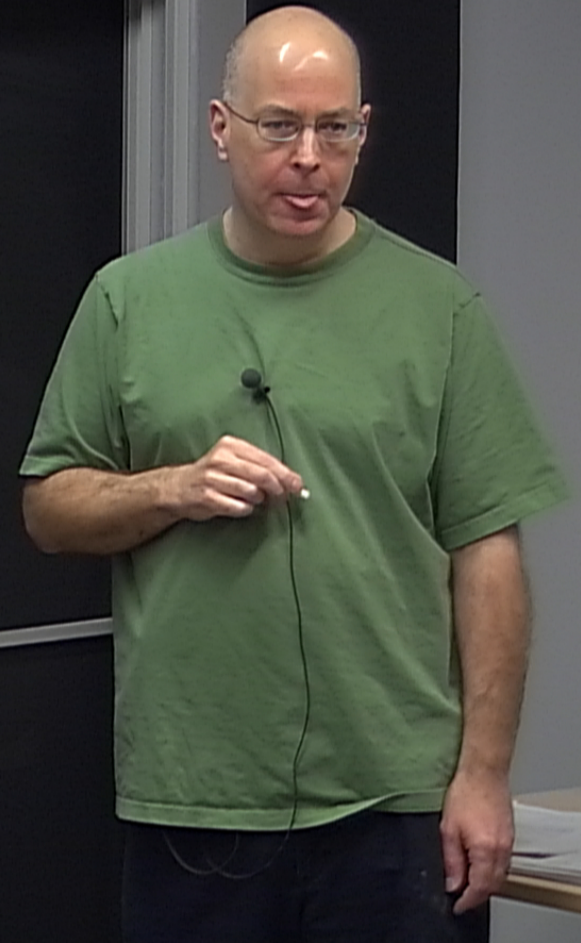
A^α

CAUTION
DO NOT TOUCH THE BOARD OR BOARDER
OR BOARDER AT THE BOTTOM OF THE BOARD

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,



$$A^\alpha_{;\beta}$$

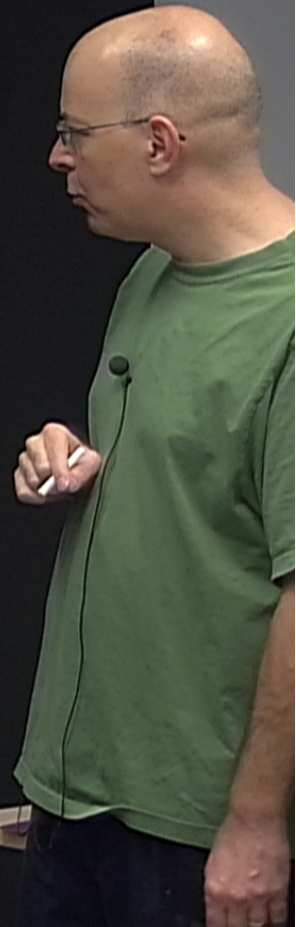


CAUTION
DO NOT TOUCH THE BOARD OR BOARDER
OR BOARDER AT THE BOTTOM OF THE BOARD

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,

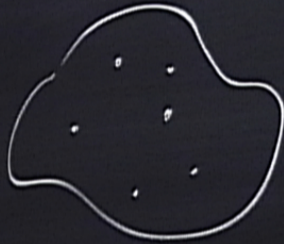


$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha$$



CAUTION
DO NOT TOUCH THE BOARD OR BOARDER
IF YOU ARE NOT A STUDENT OR TEACHER
IF YOU ARE A STUDENT OR TEACHER
PLEASE ASK THE INSTRUCTOR FOR PERMISSION

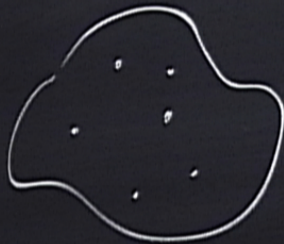
Given a vector field $A^\alpha(x^\mu)$ everywhere in open region



$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta}$$

CAUTION
DO NOT TOUCH THE BOARD OR THE MARKERS.
PLEASE RETURN TO THE BOARD AT THE END OF THE HOUR.
IF YOU NEED TO USE THE BOARD, PLEASE ASK THE TA FOR A MARKER.

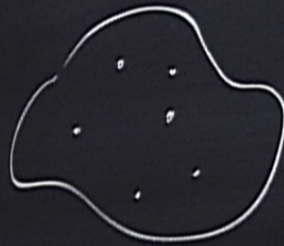
Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,



$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta} + \Gamma^\alpha{}_{\gamma\beta} A^\gamma$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER AT THE END OF THE BOARD.
IF YOU NEED TO TOUCH THE BOARD
PLEASE ASK THE BOARDER FIRST.

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,



$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{;\beta} + \Gamma^\alpha{}_{\beta\delta} A^\delta$$



CAUTION
DO NOT TOUCH THE BOARD OR THE MARKS,
WHEN WORKING AT THE BOARD OF THE BOARD.
IF YOU WANT TO USE THE BOARD,
PLEASE ASK THE ASSISTANT.

Given a vector field $A^\alpha(x^\mu)$ everywhere in open region,

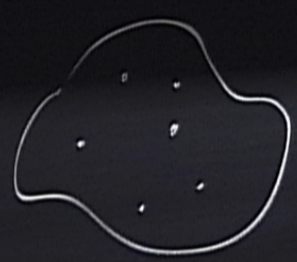


$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{;\beta} + \Gamma^\alpha{}_{\beta\delta} A^\delta$$

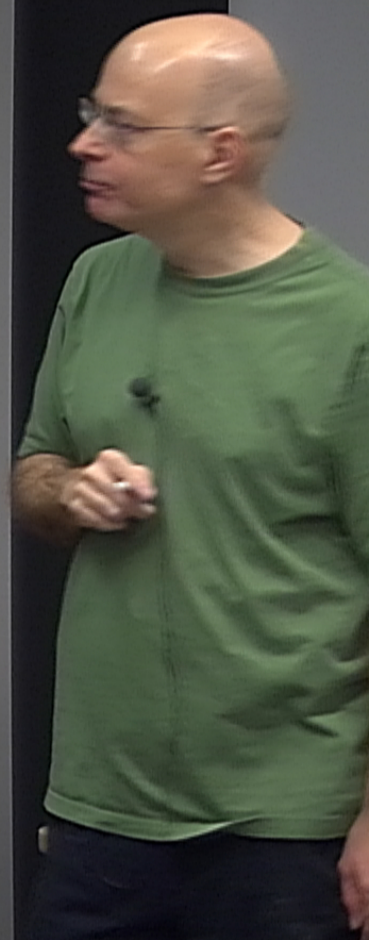


CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER AT THE BOTTOM OF THE BOARD.
IF YOU NEED TO TOUCH THE BOARD
PLEASE ASK THE BOARDER FIRST.

Given a vector field $A(x^\alpha)$ everywhere in open region,



$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha = A^\alpha{}_{,\beta} + \Gamma^\alpha{}_{\beta\delta} A^\delta$$



CAUTION
DO NOT TOUCH THE BOARD OR THE MARKERS, AND DO NOT TOUCH THE BOARD OR THE MARKERS.

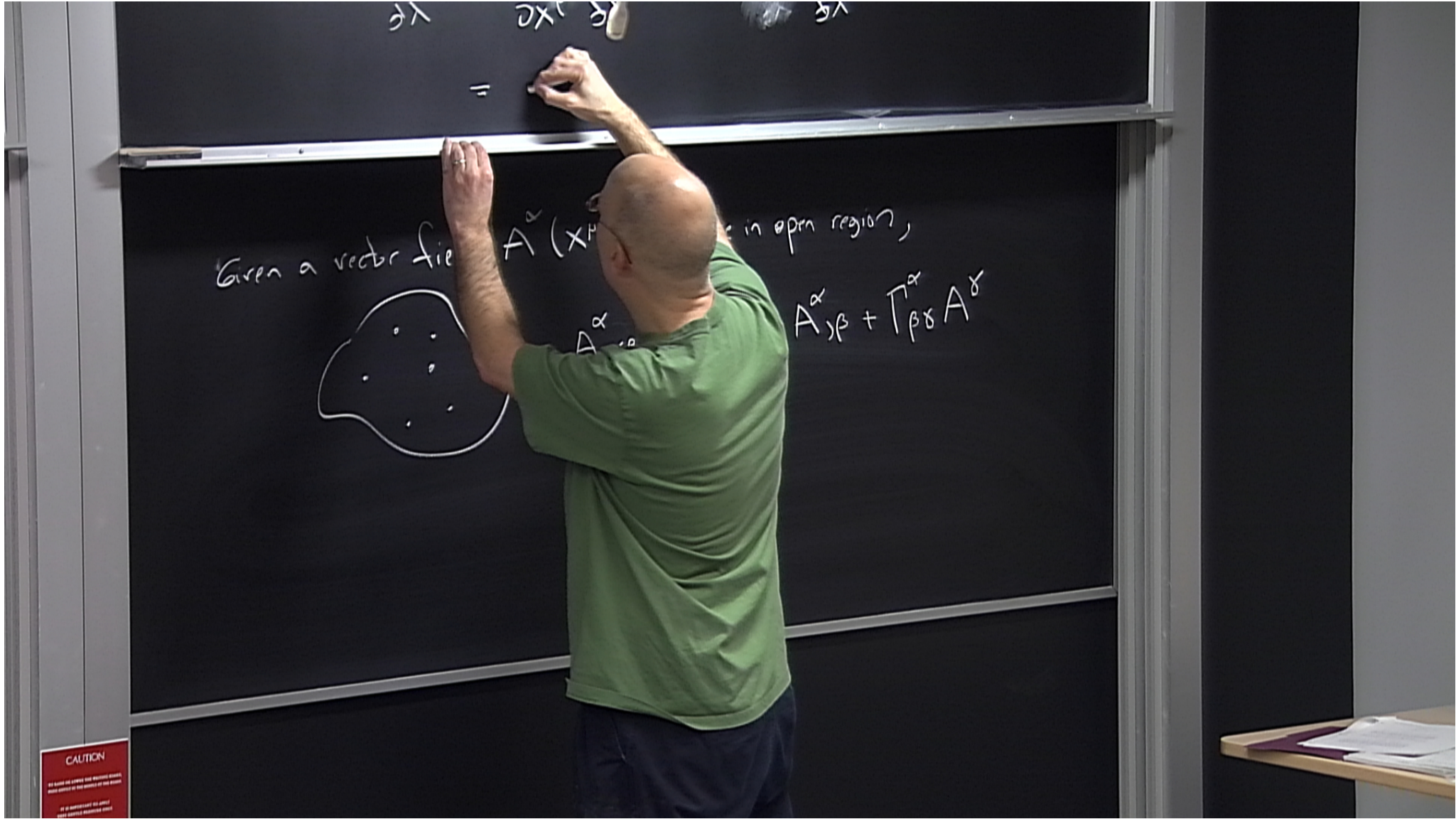
covariant derivative of A^α along γ :

$$\frac{DA^\alpha}{d\lambda} \equiv \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda} = \frac{\partial A^\alpha}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha A^\beta \frac{\partial x^\gamma}{\partial \lambda}$$

tangent vector to $\gamma = \dot{\gamma} \equiv \frac{\partial x^\alpha}{\partial \lambda}$

$$\frac{DA^\alpha}{d\lambda} = \frac{\partial A^\alpha}{\partial x^\beta} \frac{\partial x^\beta}{\partial \lambda} + \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \lambda} \frac{\partial x^\gamma}{\partial \lambda}$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER AT THE END OF THE BOARD.
IF IT APPEARS TO BE
HOT, PLEASE STOP.



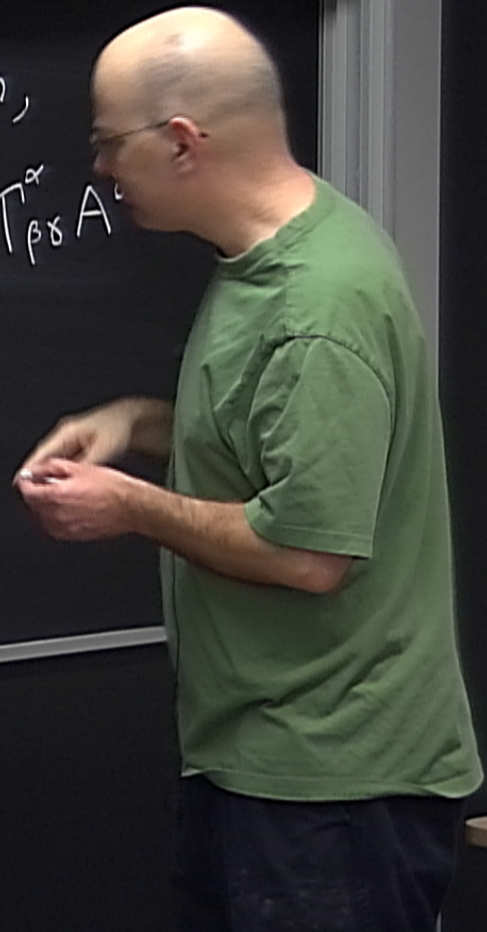
CAUTION
DO NOT TOUCH THE BOARD
OR THE BOARDER AT ANY TIME

$$= A^{\alpha}_{;\beta} + \Gamma^{\alpha}_{\beta\gamma} A^{\gamma}$$

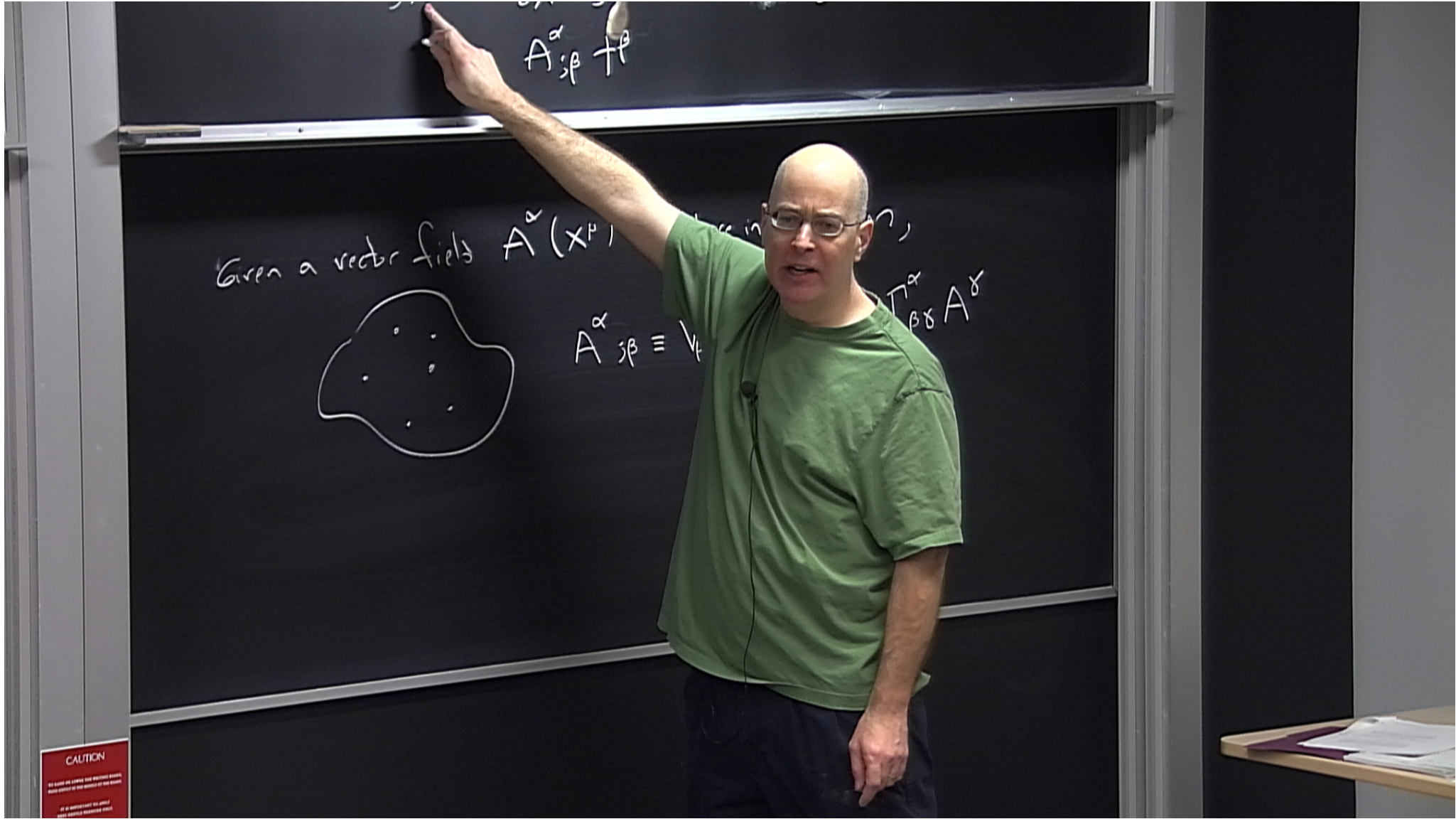
Given a vector field $A^{\alpha}(x^{\mu})$ everywhere in open region,



$$A^{\alpha}_{;\beta} \equiv \nabla_{\beta} A^{\alpha} = A^{\alpha}_{;\beta} + \Gamma^{\alpha}_{\beta\gamma} A^{\gamma}$$



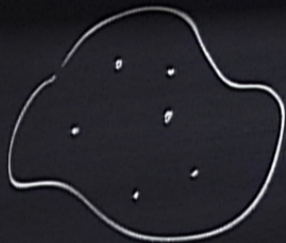
CAUTION
DO NOT TOUCH THE BOARD OR MARKERS
WHEN IN USE BY THE INSTRUCTOR
IF YOU NEED TO TOUCH THE BOARD
PLEASE ASK THE INSTRUCTOR FIRST



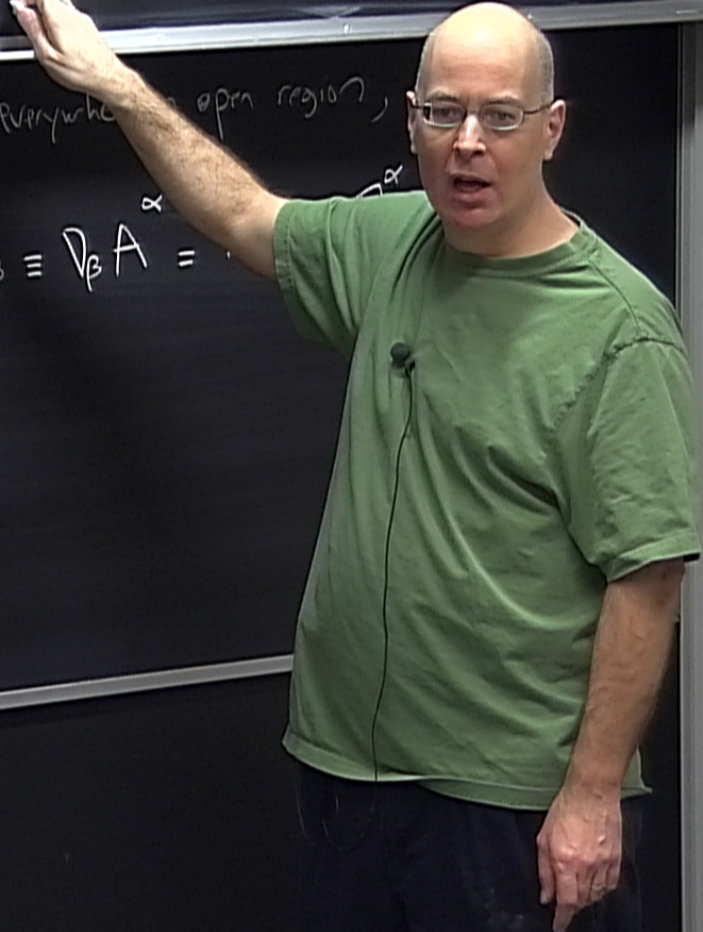
$$\frac{DA}{d\lambda} = \frac{\partial A^\alpha}{\partial x^\beta} \frac{dx^\beta}{d\lambda} + \Gamma_{\beta\gamma}^\alpha A^\beta \frac{dx^\gamma}{d\lambda}$$

$$= A^\alpha{}_{;\beta} \dot{x}^\beta$$

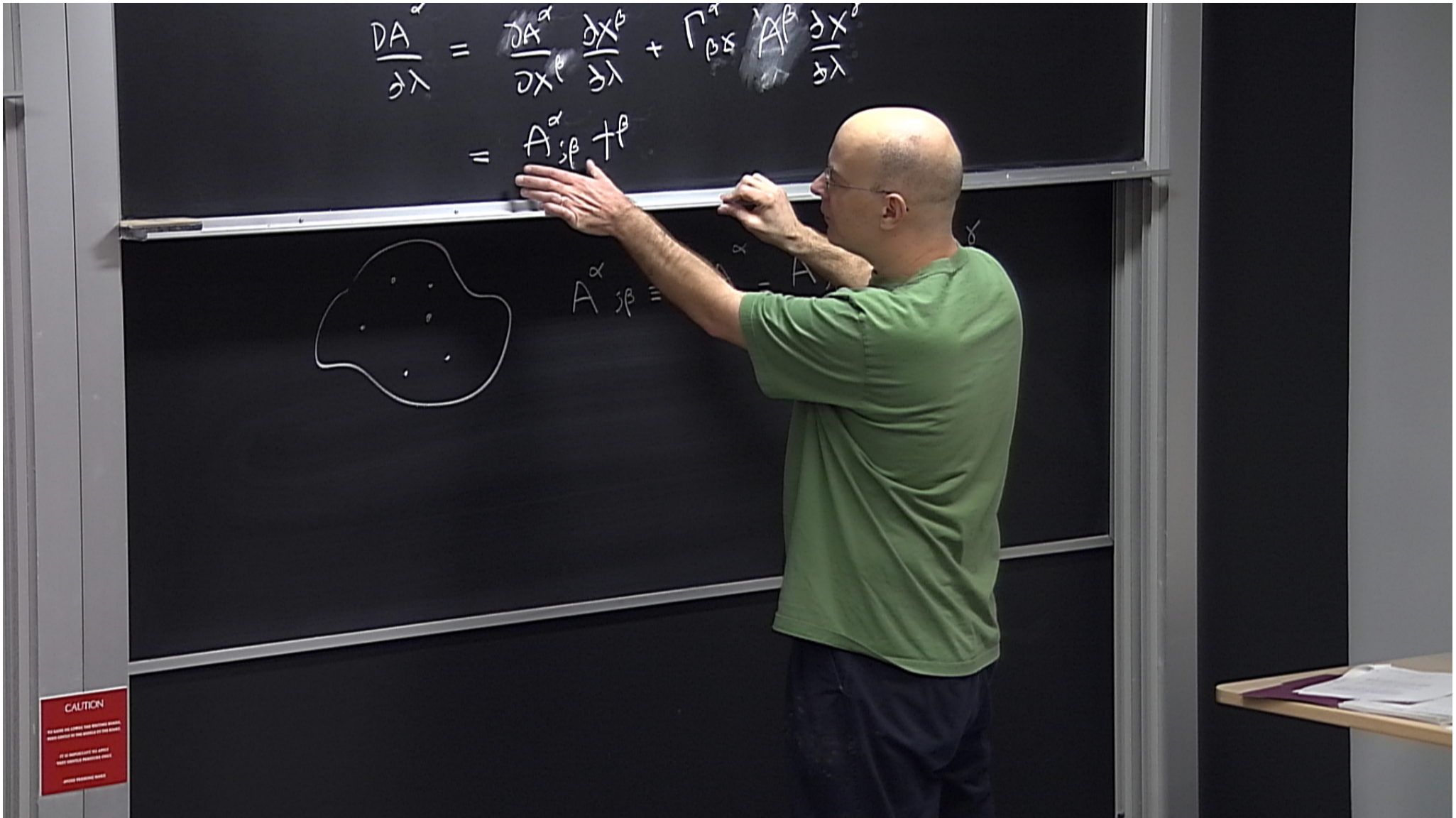
Given a vector field $A^\alpha(x^\mu)$ everywhere in an open region,

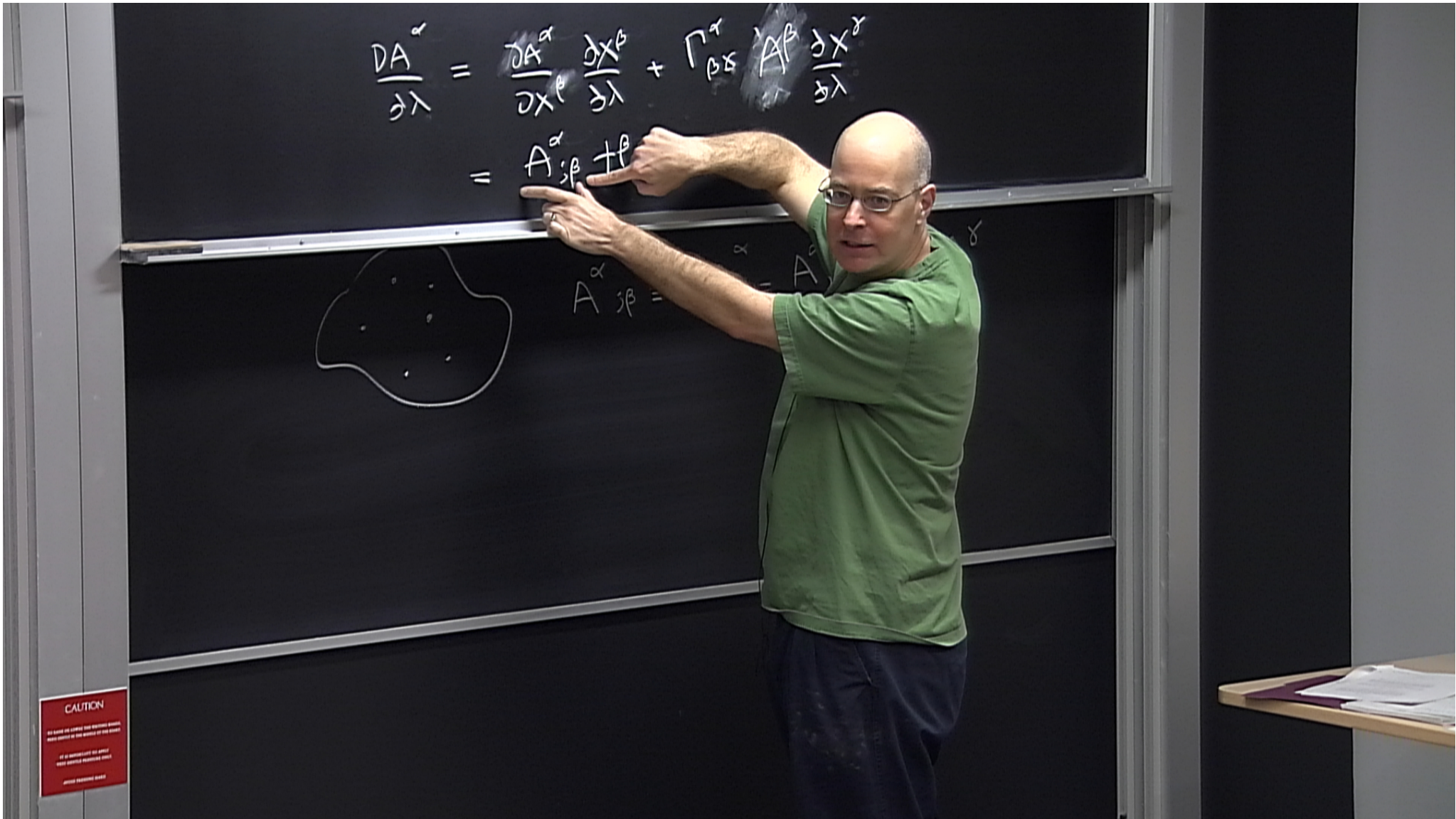


$$A^\alpha{}_{;\beta} \equiv \nabla_\beta A^\alpha =$$



CAUTION
DO NOT TOUCH THE BOARD OR MARKERS
OR ANYTHING ON THE BOARD OR THE BOARD



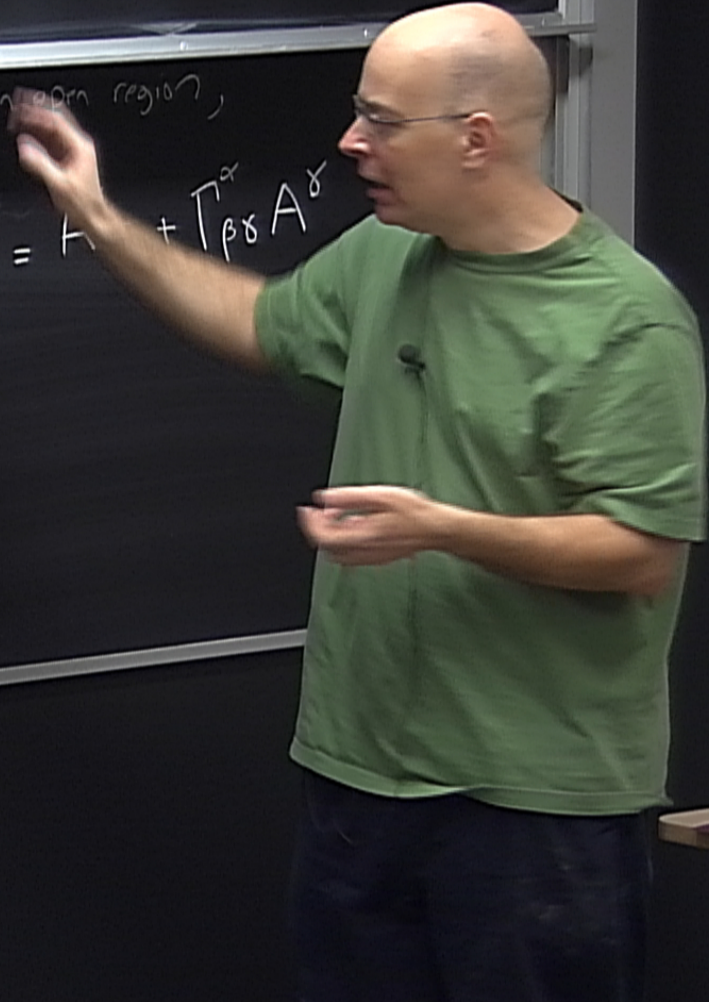


$$\partial_\lambda \partial x^\mu \partial \lambda = A^\alpha_{\beta\gamma} \partial^\beta \partial x^\mu \partial \lambda$$

Given a vector field $A(x^\mu)$ everywhere in an open region,



$$A^\alpha_{\beta\gamma} \equiv \nabla_\beta A^\alpha = \partial_\beta A^\alpha + \Gamma^\alpha_{\beta\delta} A^\delta$$



CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
IF YOU HAVE ANY QUESTIONS
PLEASE ASK THE LECTURER

tangent vector to $0 = f = \frac{\partial f}{\partial \lambda}$

$$\frac{DA^\alpha}{d\lambda} = \frac{\partial A^\alpha}{\partial x^\beta} \frac{dx^\beta}{d\lambda} + \Gamma_{\beta\gamma}^\alpha A^\beta \frac{dx^\gamma}{d\lambda}$$
$$= A^\alpha_{;\beta} \frac{dx^\beta}{d\lambda}$$

CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
IF YOU HAVE ANY QUESTIONS
PLEASE ASK THE LECTURER