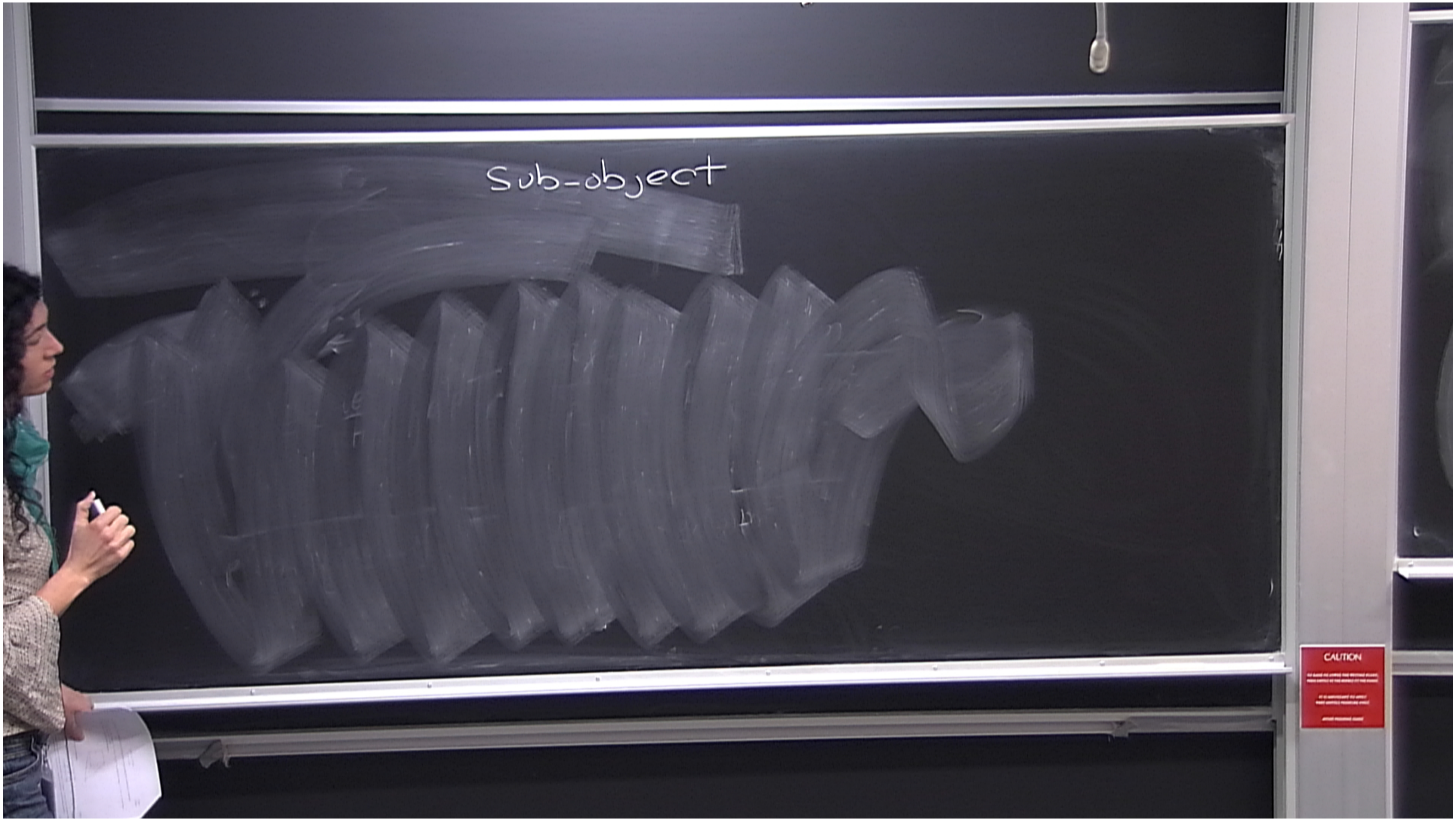
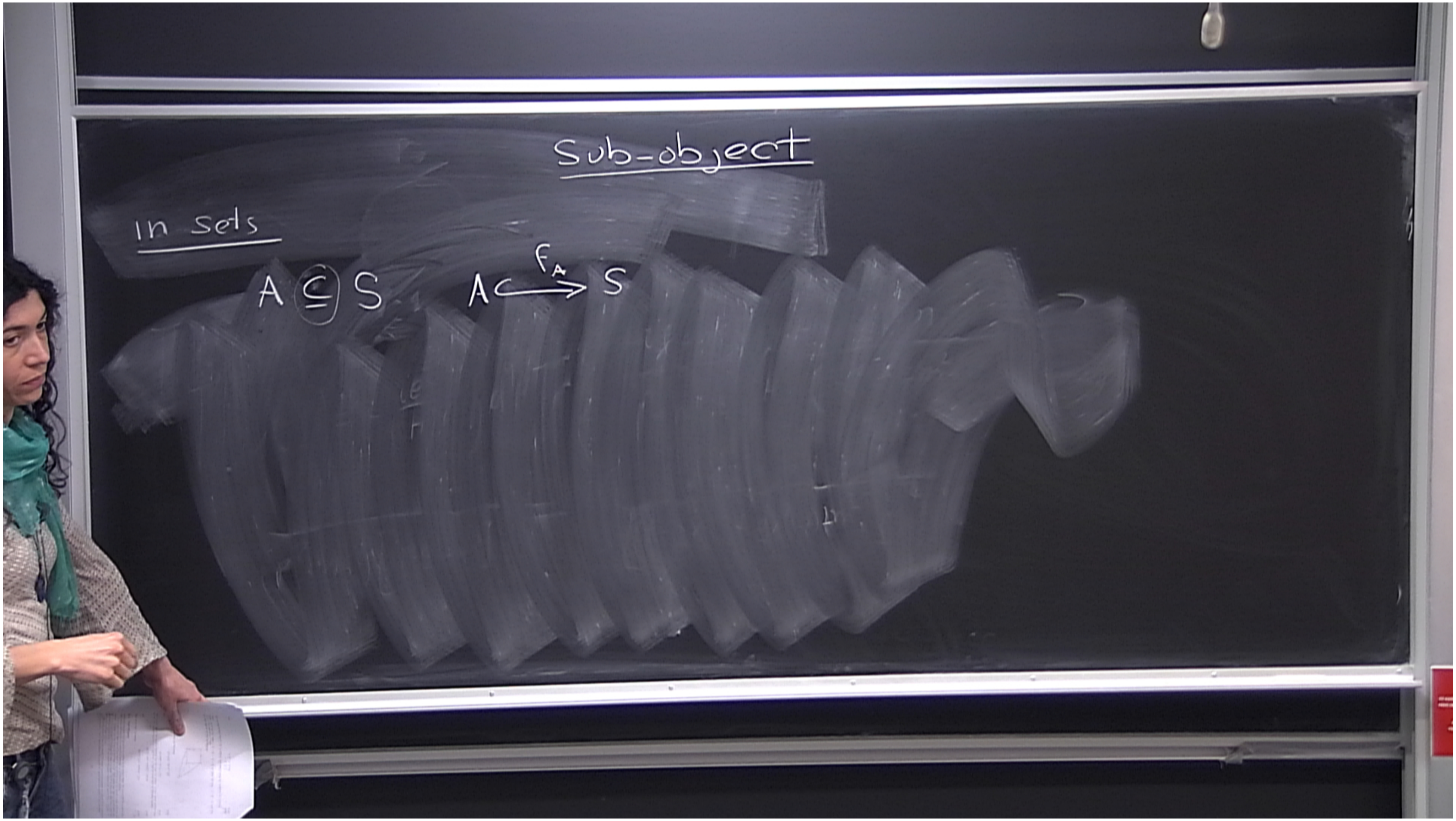
Title: Topos Quantum Physics - Lecture 6

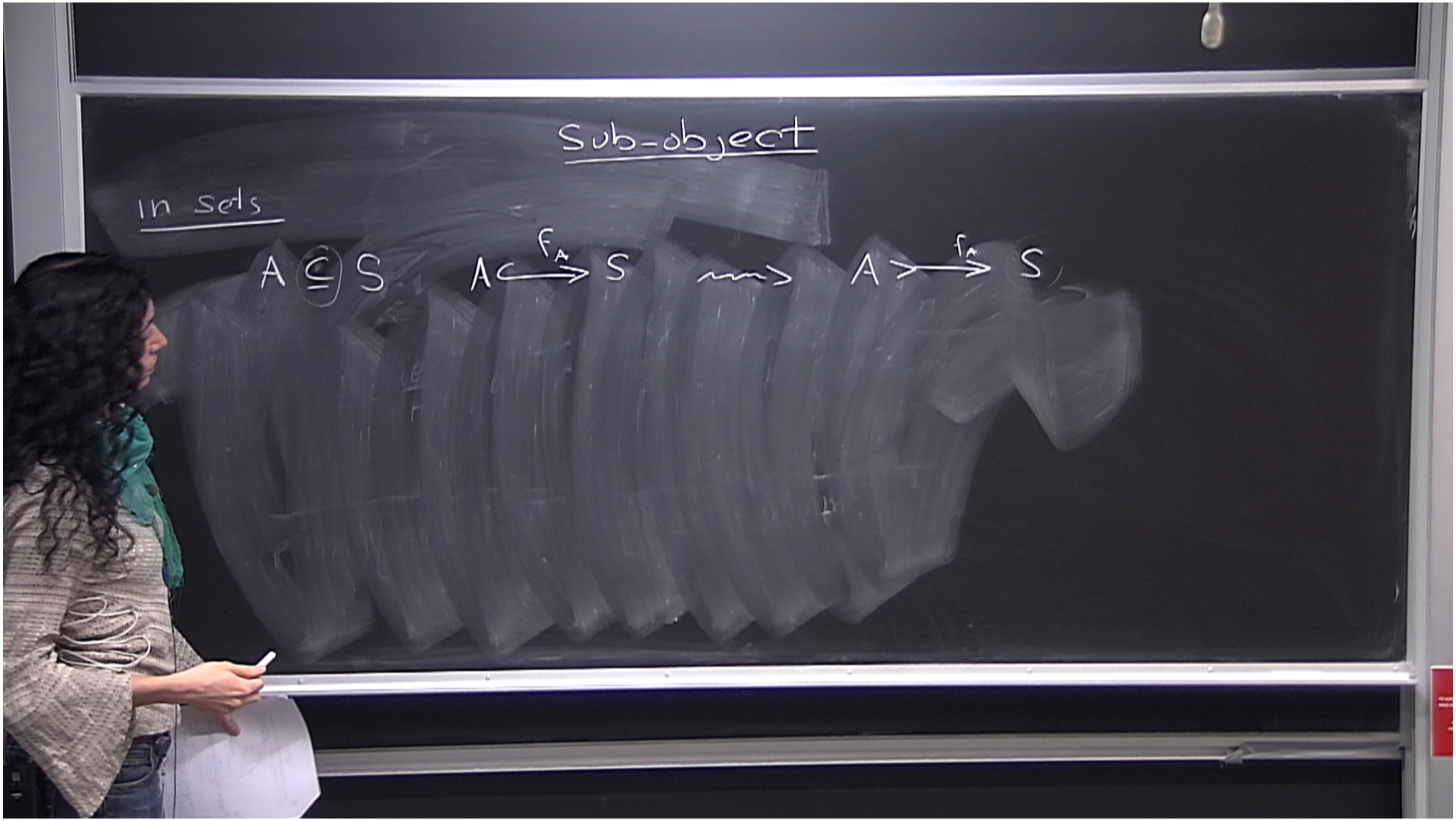
Date: Jan 20, 2012 01:30 PM

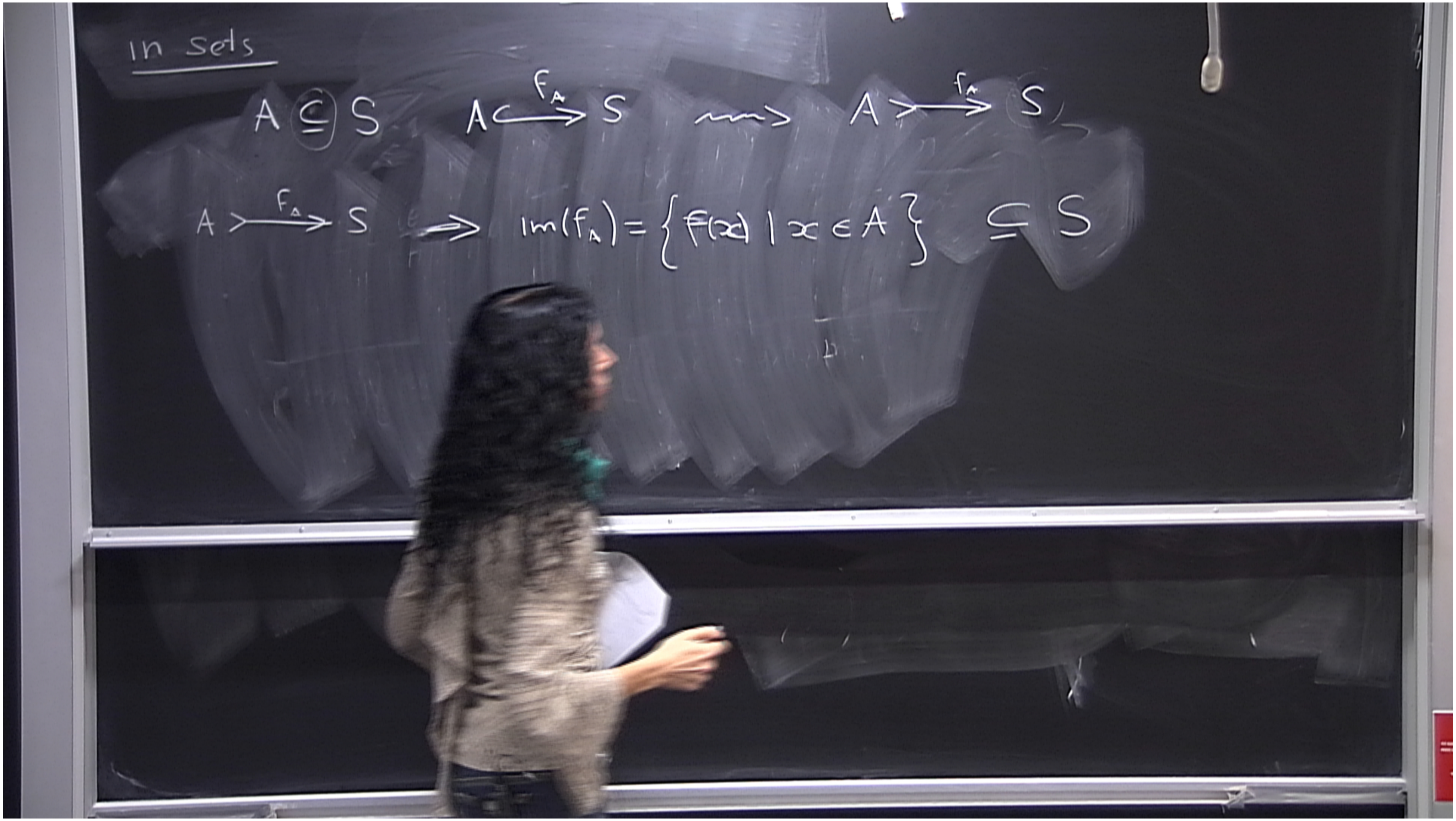
URL: http://pirsa.org/12010149

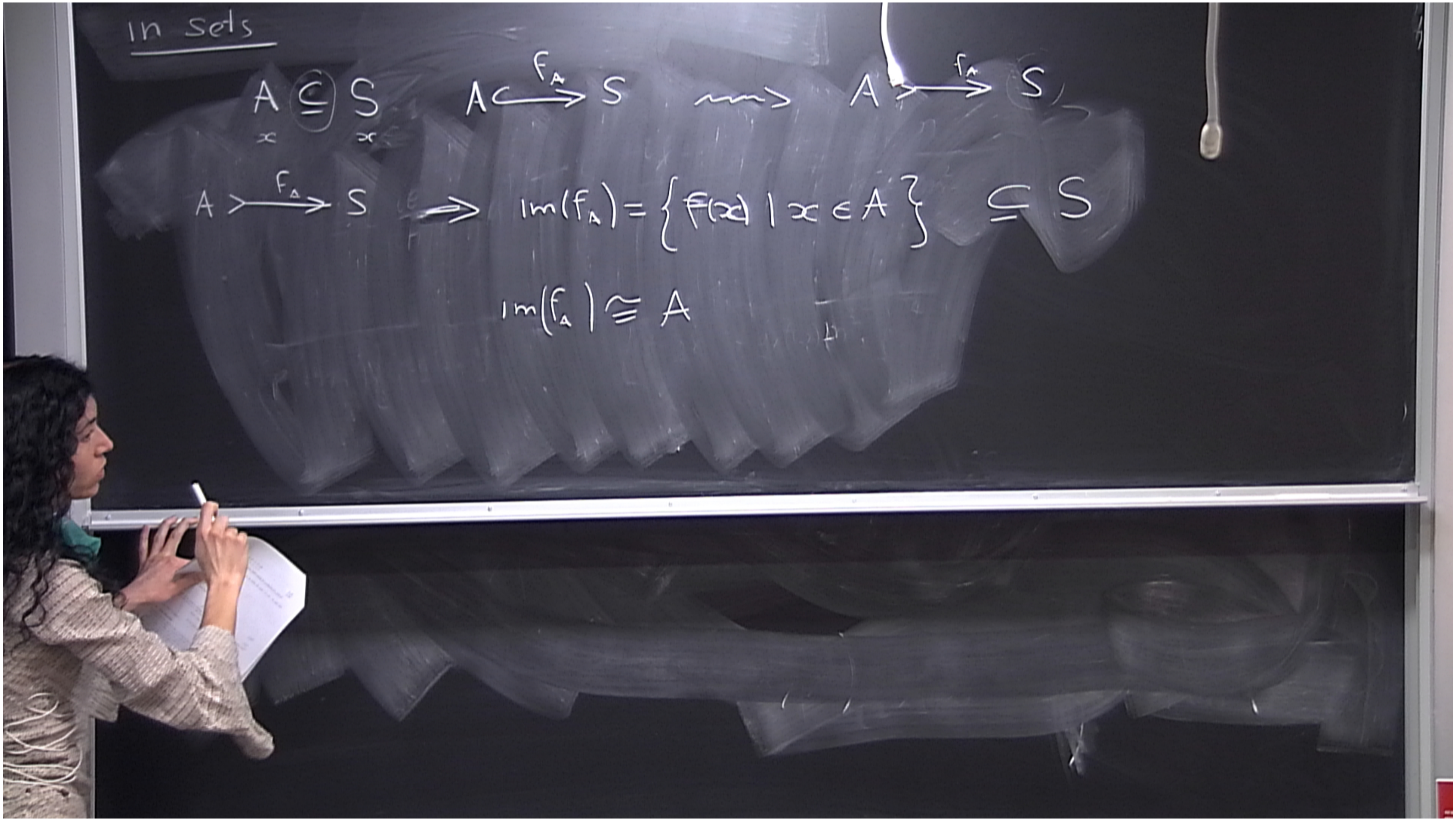
Abstract:

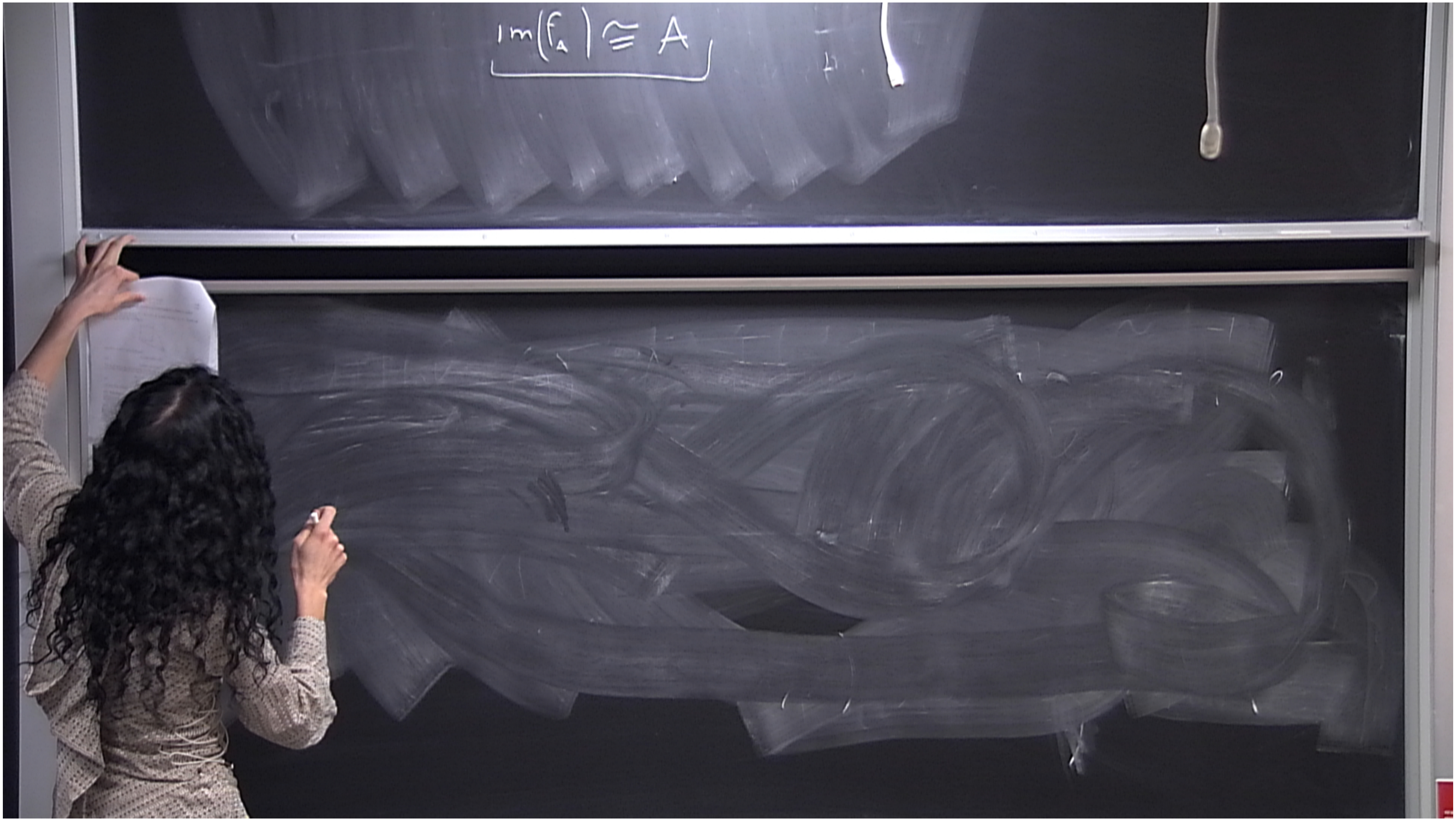

Sub-object
CAUTION


Sub-object
In sets
A ⊆ S
A ↪ S
f_A

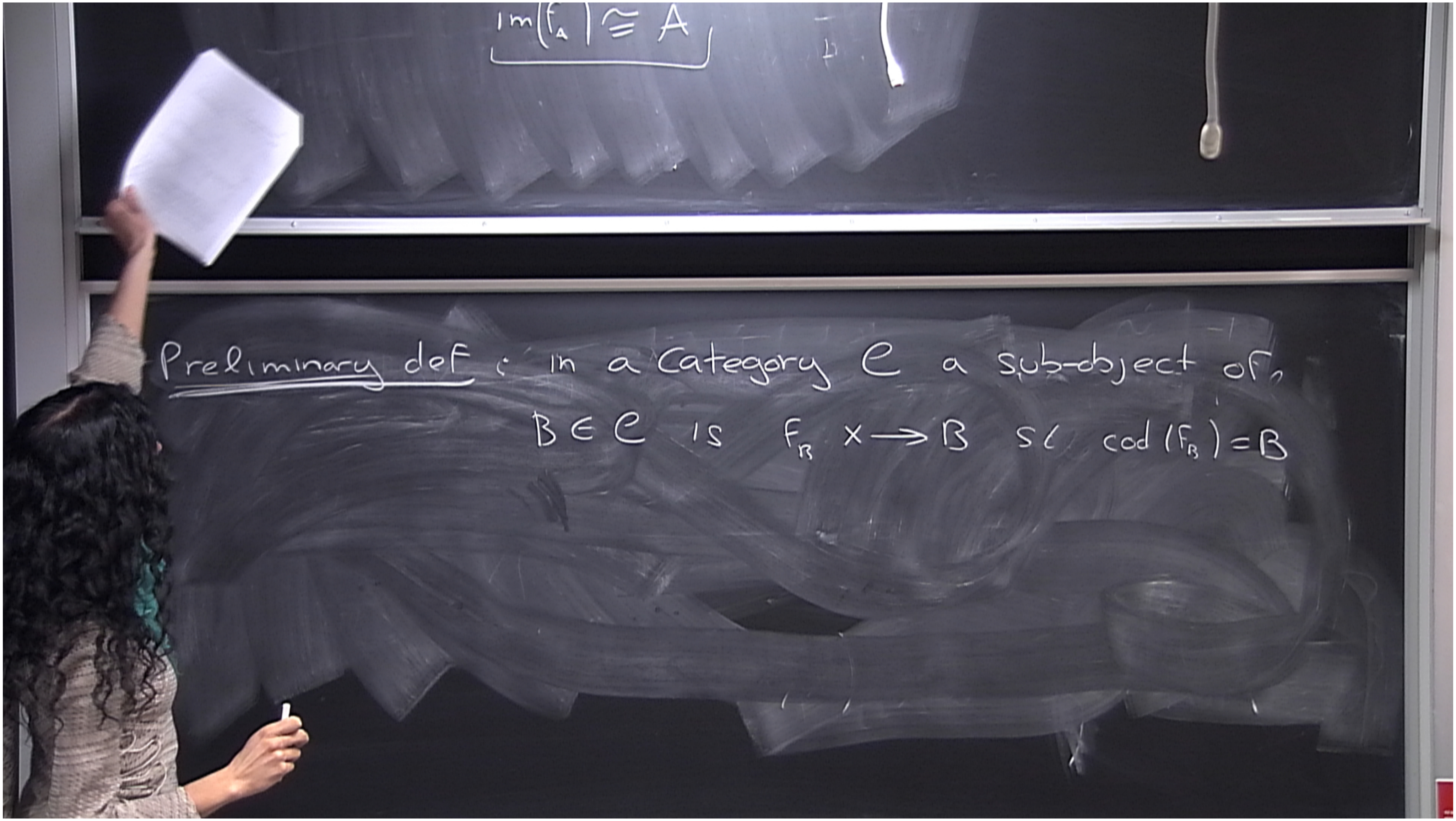

Sub-object
In sets
A ⊆ S
A ↪ S (f_A)
A ↣ S (f_A)

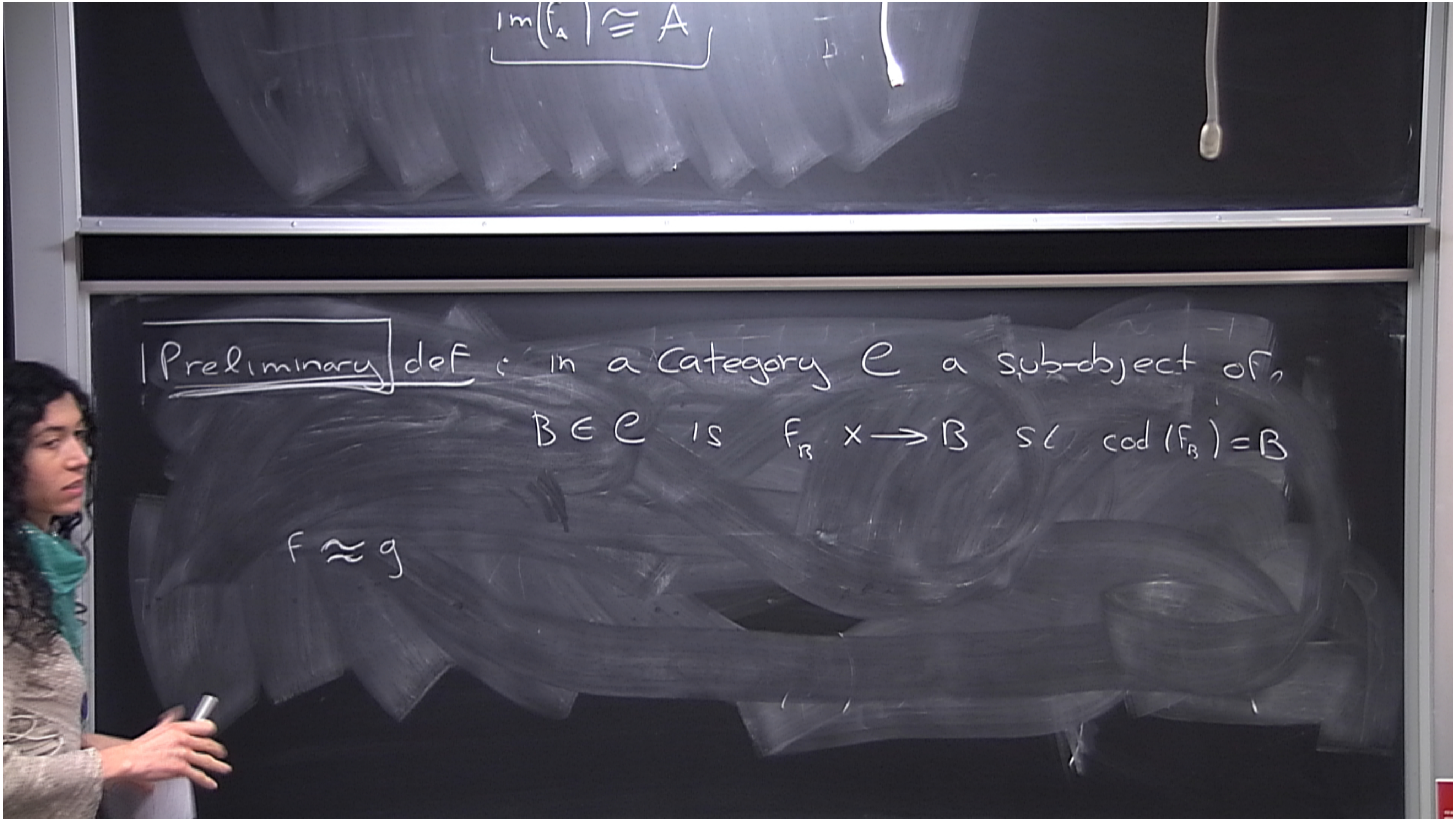

in sets
A ⊆ S
f_A
A ↪ S
A ↣ S
A ↣ S
im(f_A) = { f(x) | x ∈ A } ⊆ S

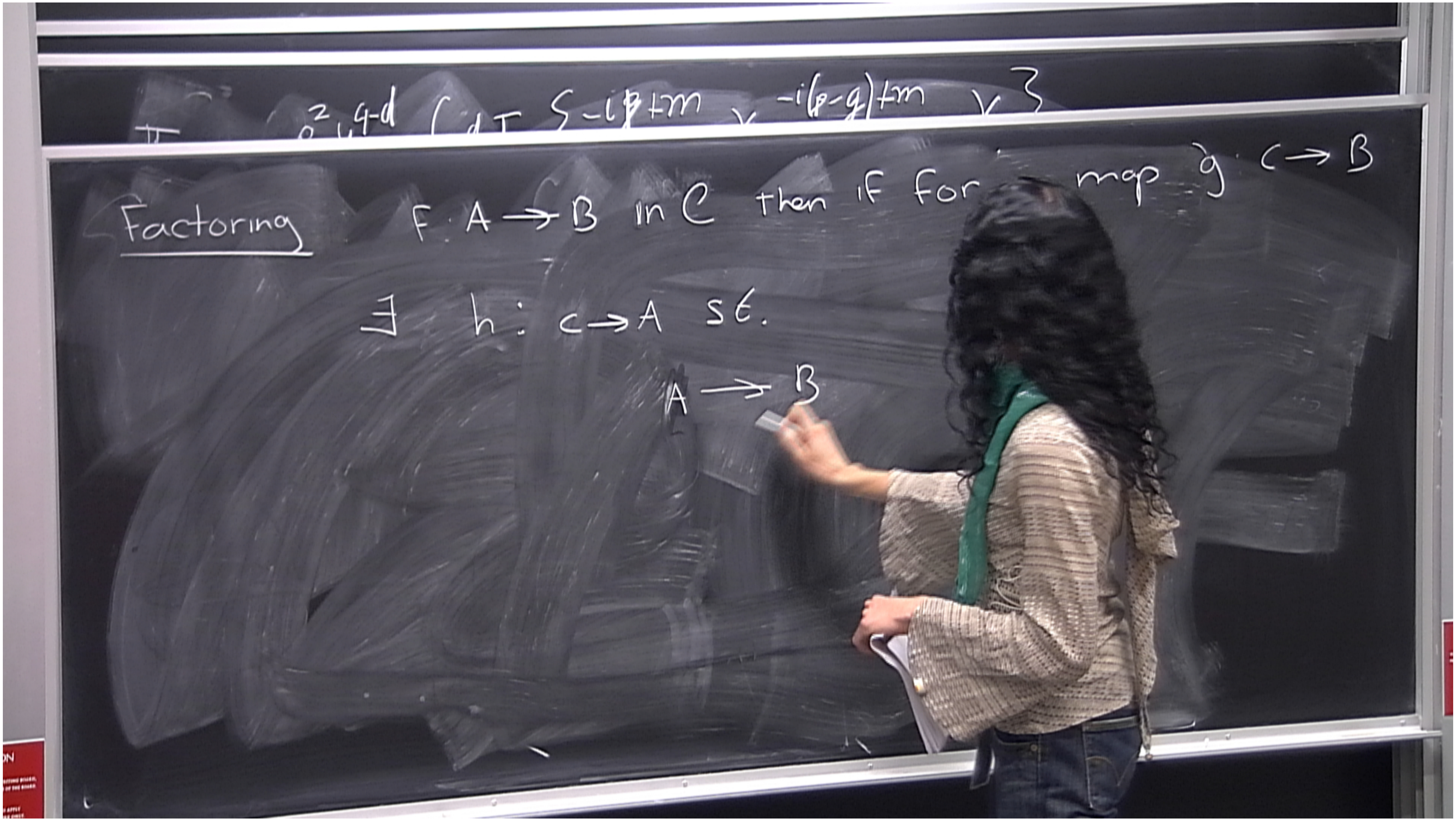

in sets
A ⊆ S
A ↪ S
f_A
A ↣ S
f_A
A ↣ S ⇒ im(f_A) = {f(x) | x ∈ A} ⊆ S
f_A
im(f_A) ≅ A

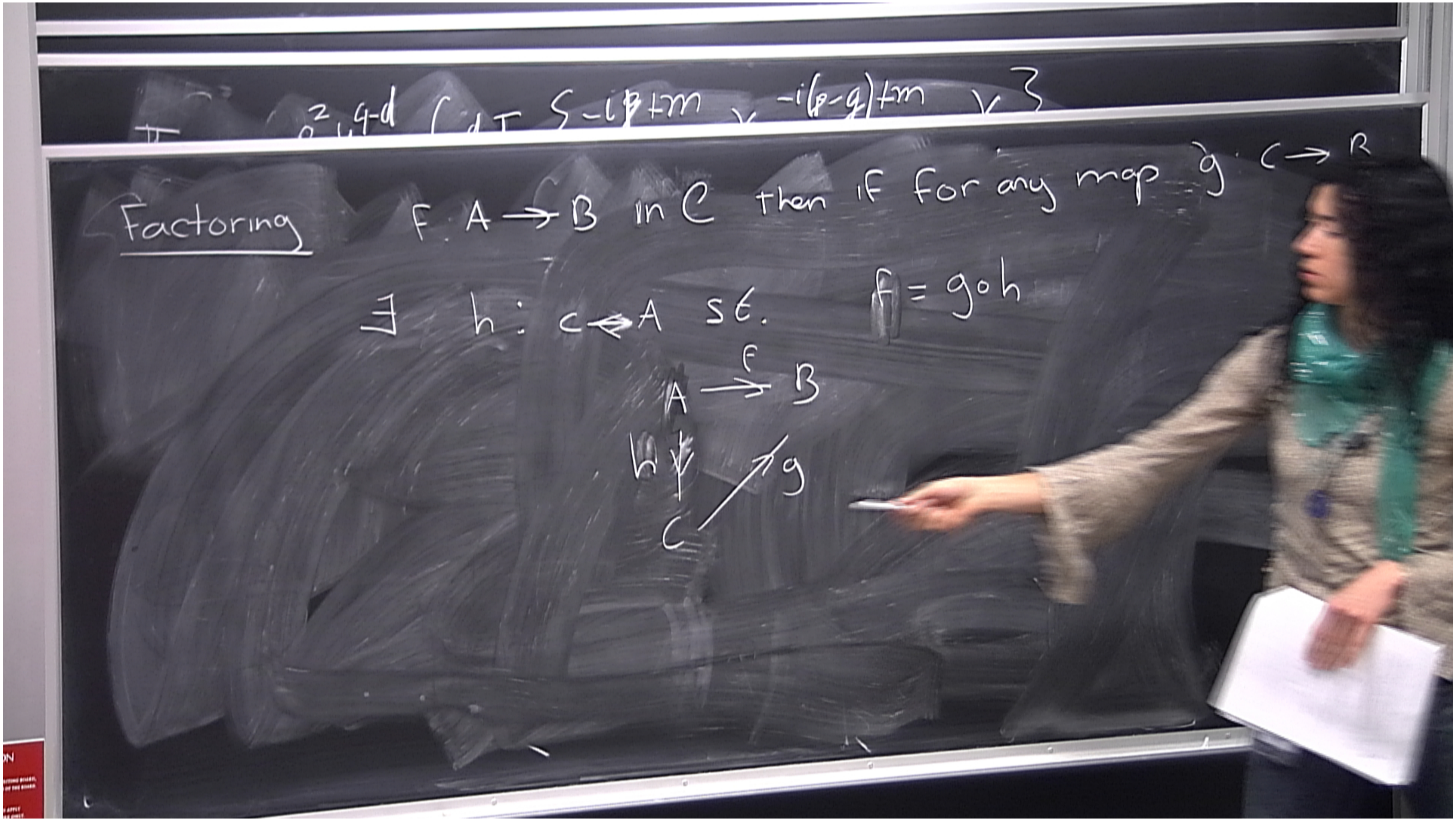


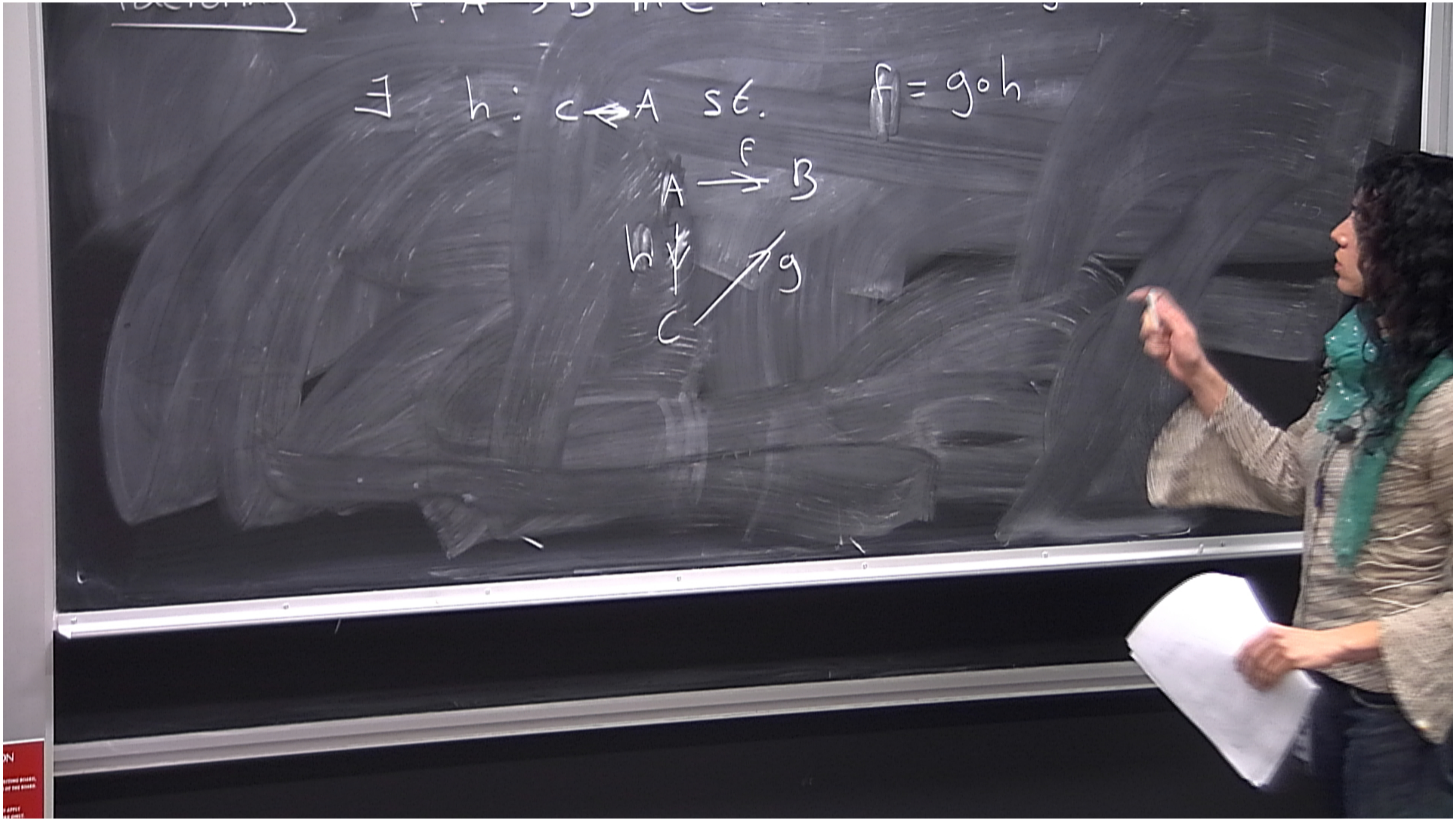
$Im(f_A) \cong A$

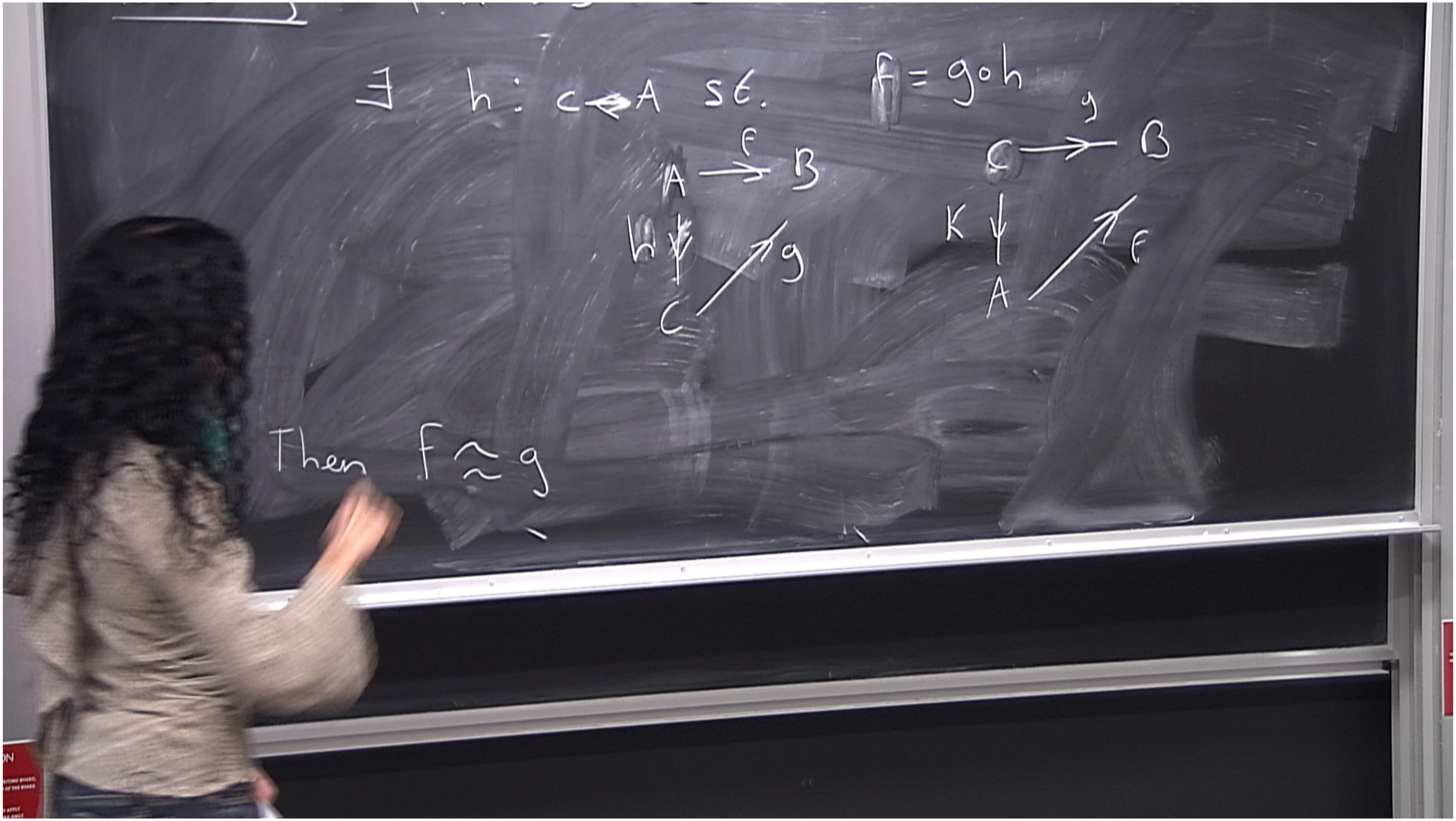

Im(f_A) ≅ A
Preliminary def: In a category C a sub-object of B ∈ C is f_B X → B s.t. cod(f_B) = B

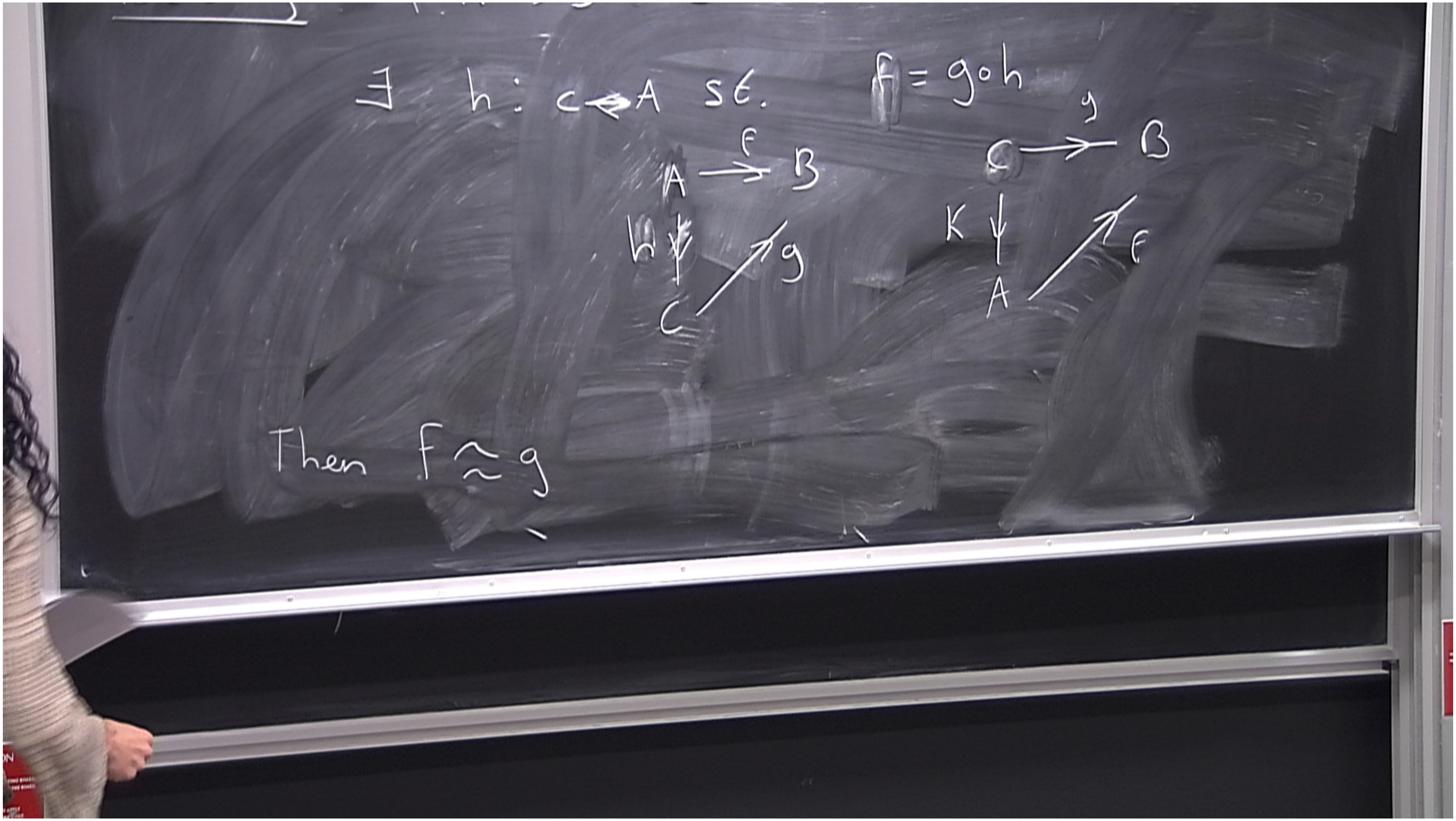

Im(f_A) ≅ A
Preliminary def : in a category C a sub-object of
B ∈ C is f_B X → B st cod(f_B) = B
f ≈ g

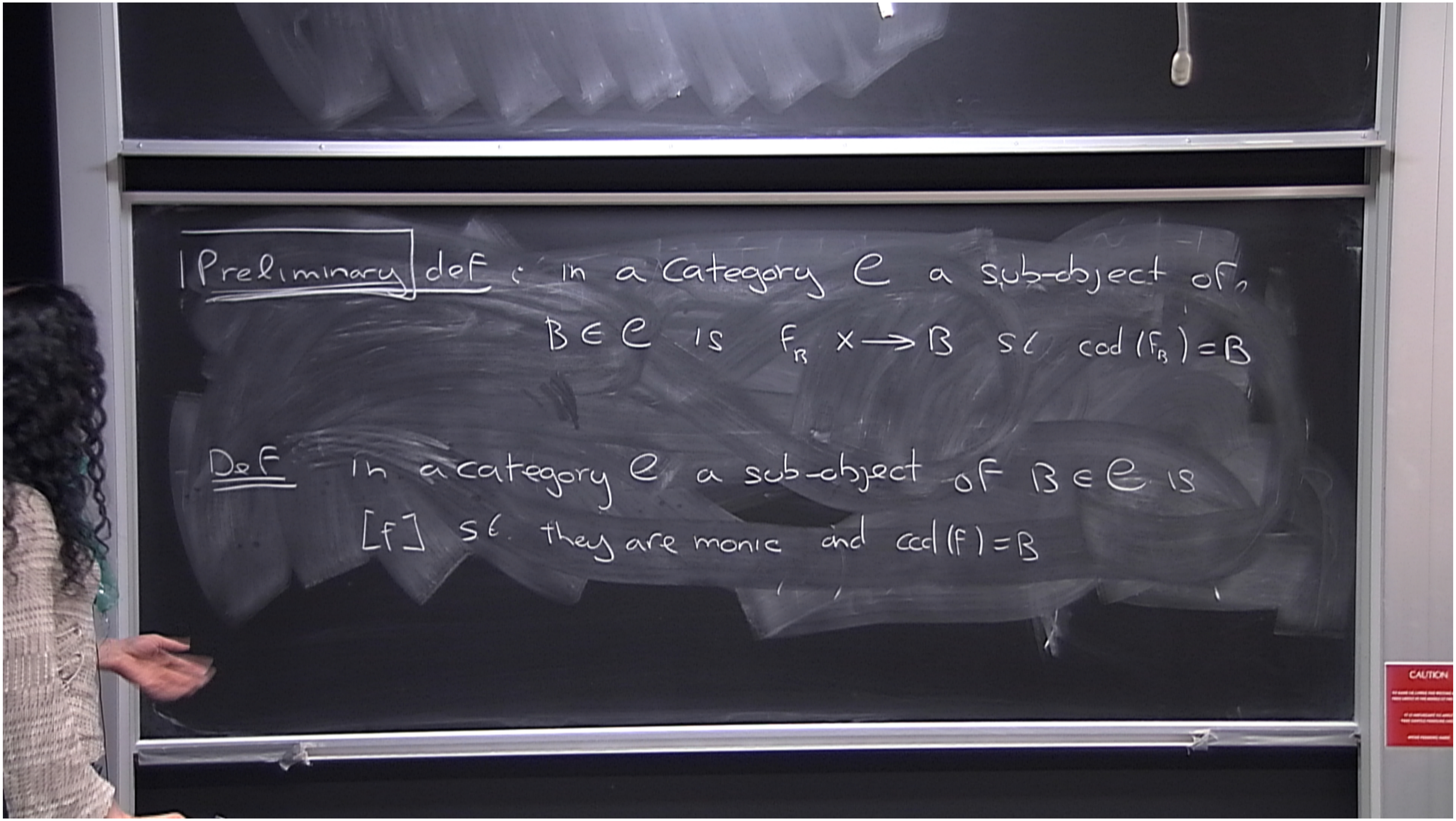

Factoring
f: A → B in C then if for map g: C → B
∃ h: C → A st.
A → B

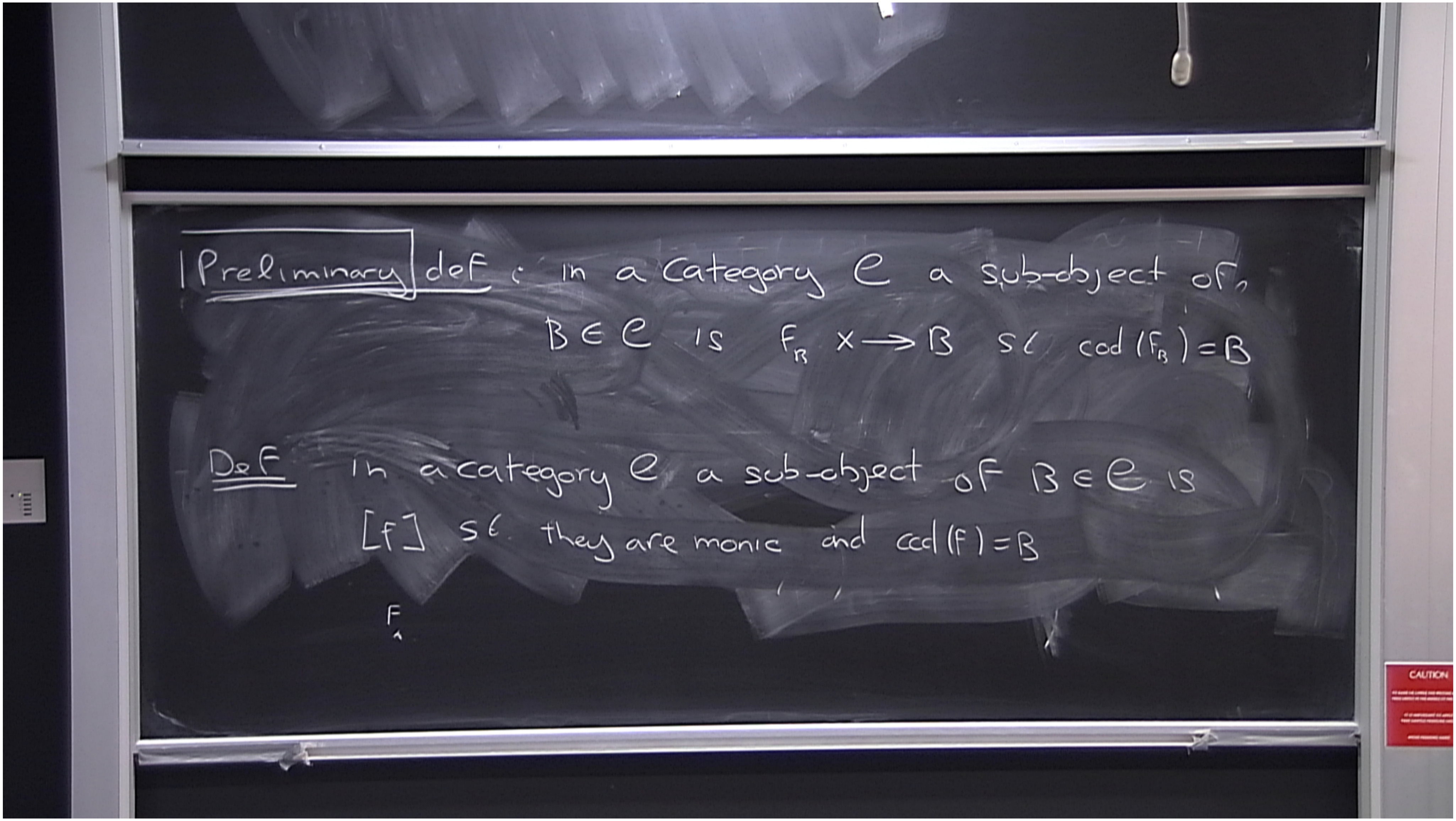

Factoring
f: A → B in C then if for any map g: C → B
∃ h: C → A s.t. f = g∘h
A
f
B
h
g
C

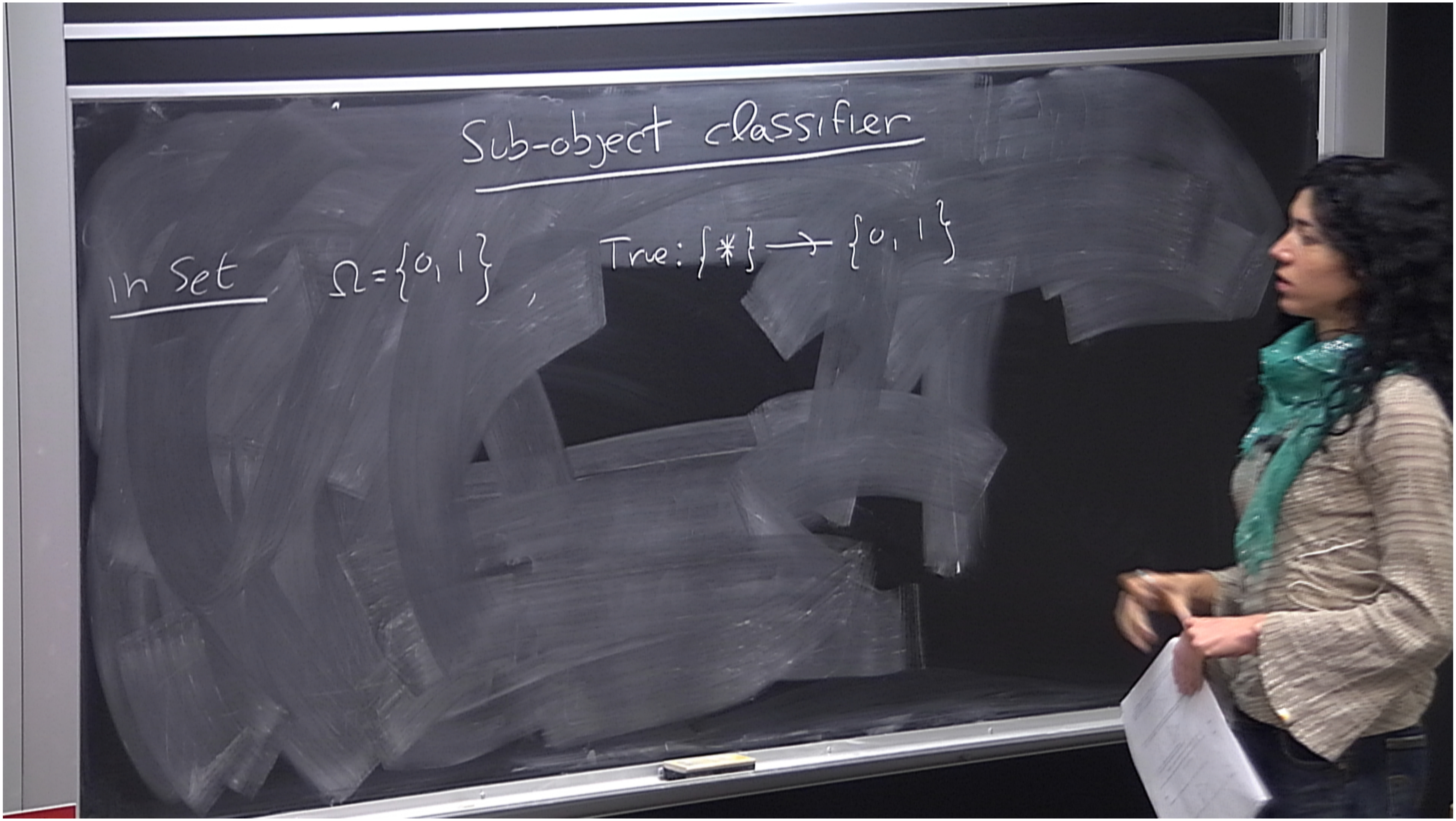

∃ h : C → A s.t. f = g∘h
A
f
B
h
g
C

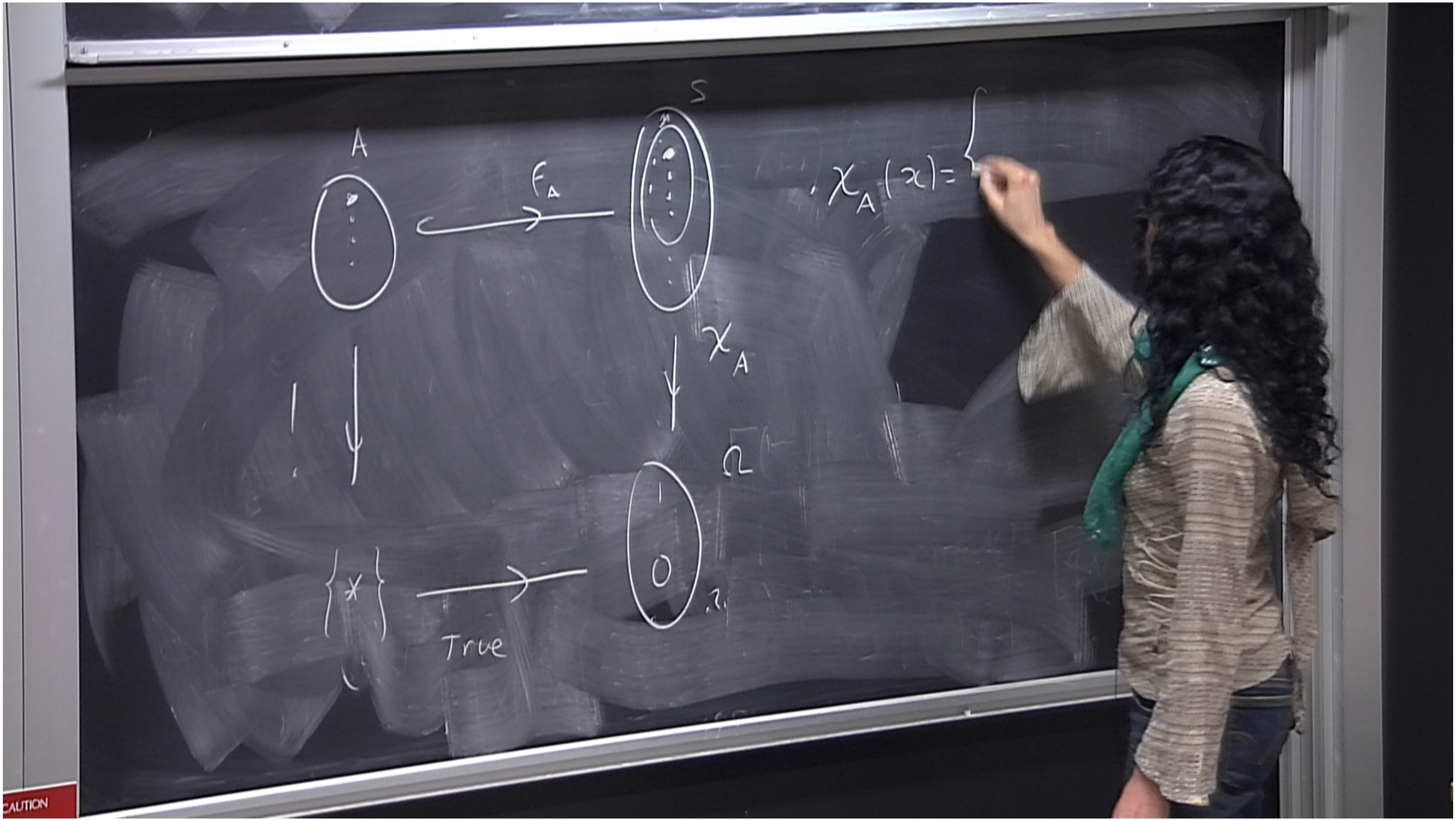

∃ h: C → A s.t. f = g∘h
A → B
f
h
C
g
C → B
g
K
A
f
Then f ≃ g

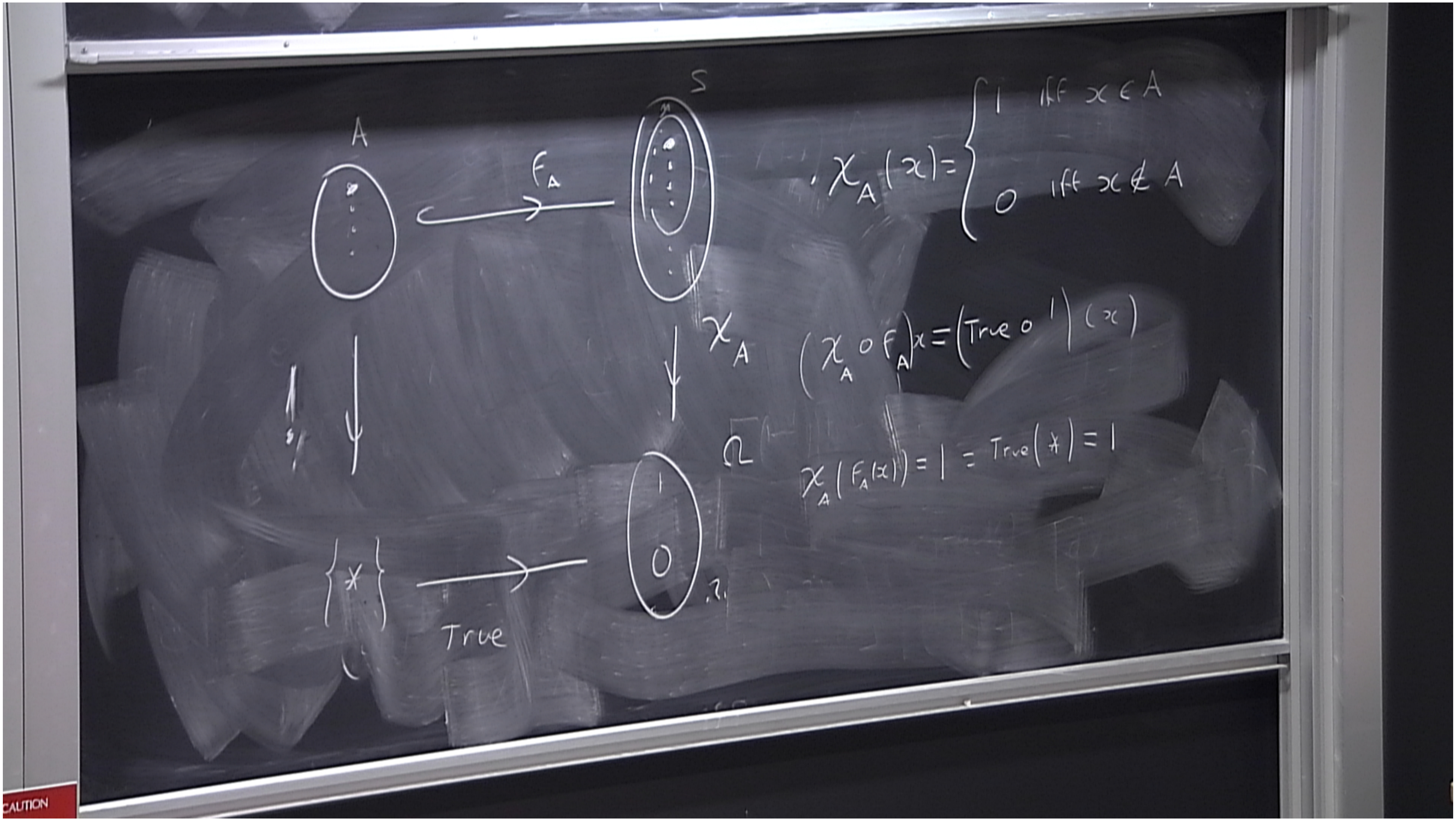

∃ h: C → A s.t. f = g∘h
f
A → B
h
g
C
g
C → B
K
f
A
Then f ≃ g

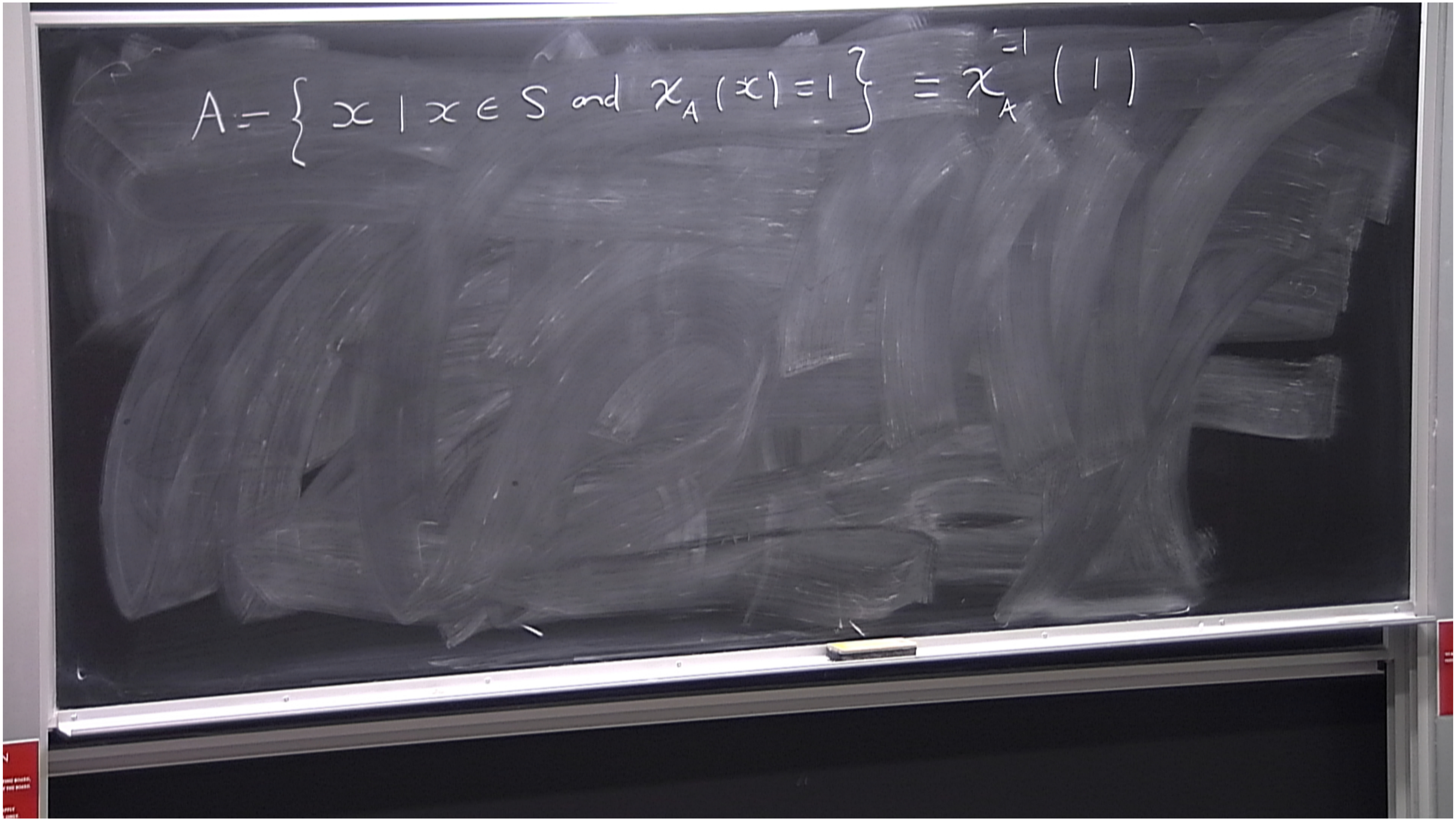

Preliminary def: In a category 𝒞 a sub-object of
B ∈ 𝒞 is f_B X ⟶ B s.t. cod(f_B) = B
Def In a category 𝒞 a sub-object of B ∈ 𝒞 is
[f] s.t. they are monic and cod(f) = B

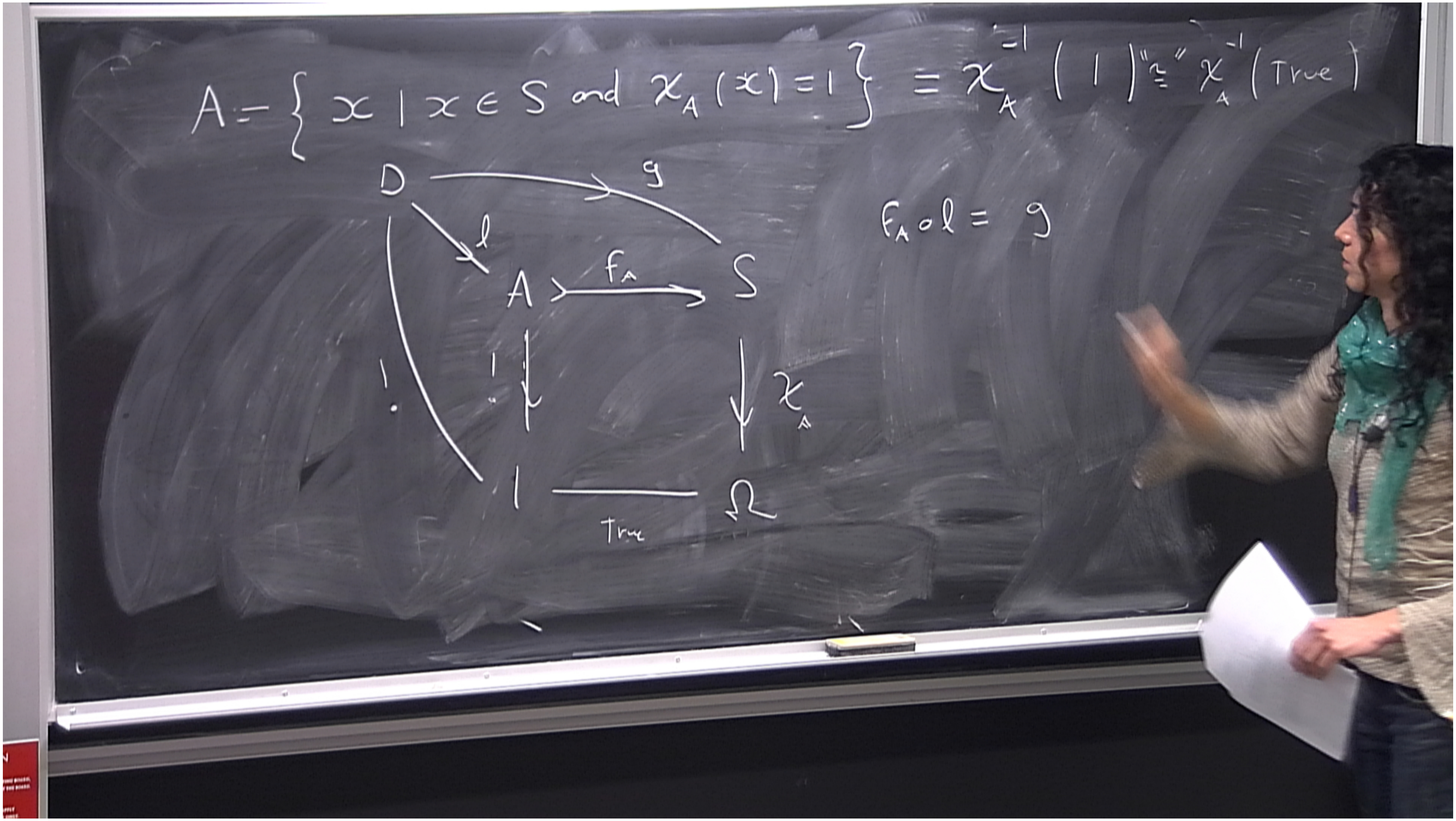

Preliminary def: In a Category 𝒞 a sub-object of
B ∈ 𝒞 is f_B X → B s.t. cod(f_B) = B
Def In a category 𝒞 a sub-object of B ∈ 𝒞 is
[f] s.t. they are monic and cod(f) = B

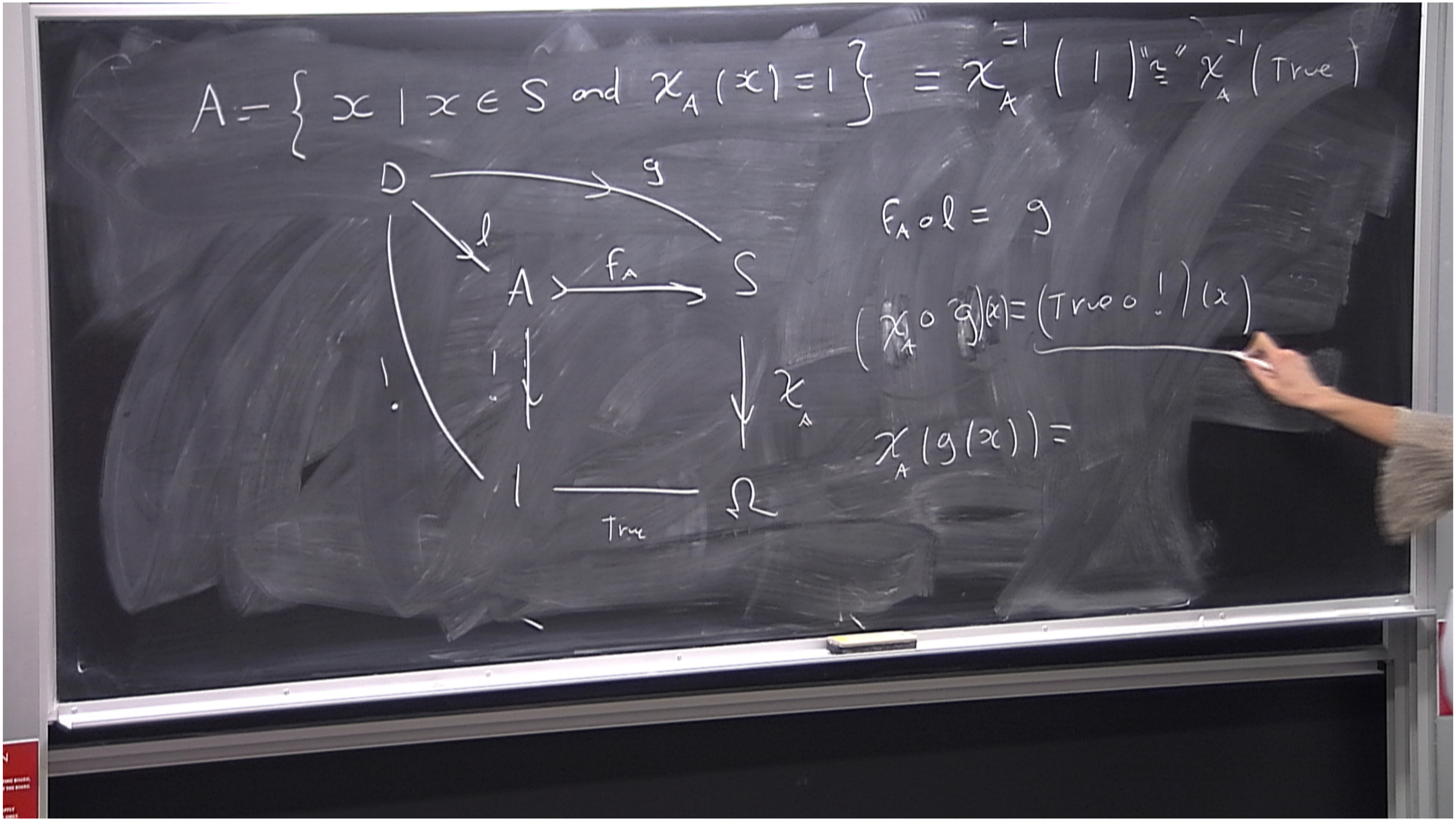

Sub-object classifier
in Set
Ω={0, 1},
True: {*} → {0, 1}

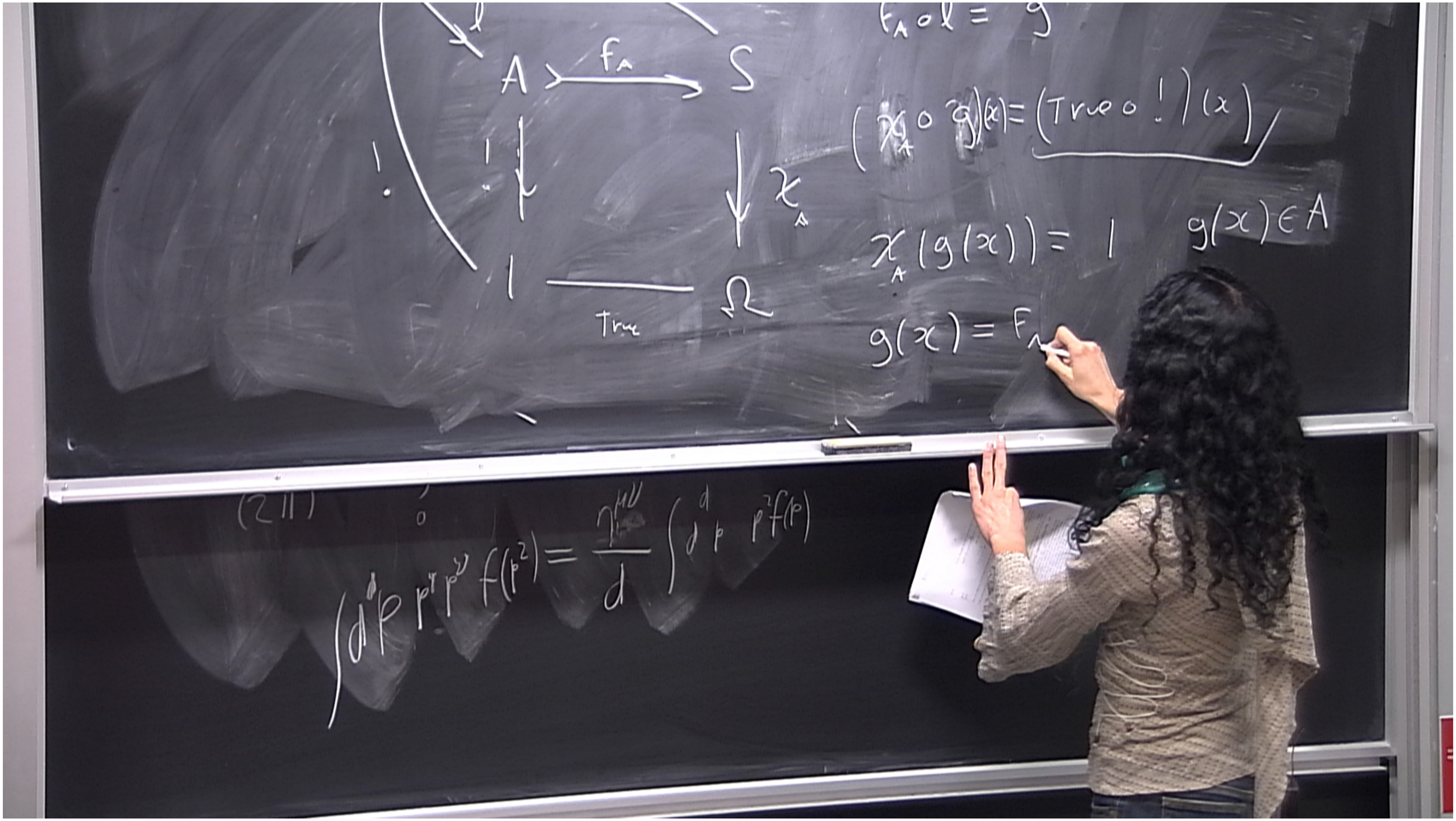

A
S
f A
χ A
χ A (x) =
!
Ω
1
0
{*}
True
CAUTION


A
S
f_A
χ_A(x) = { 1 iff x ∈ A ; 0 iff x ∉ A
χ_A
(χ_A ∘ f_A)x = (True ∘ !)(x)
Ω
χ_A(f_A(x)) = 1 = True(*) = 1
{*}
True
CAUTION

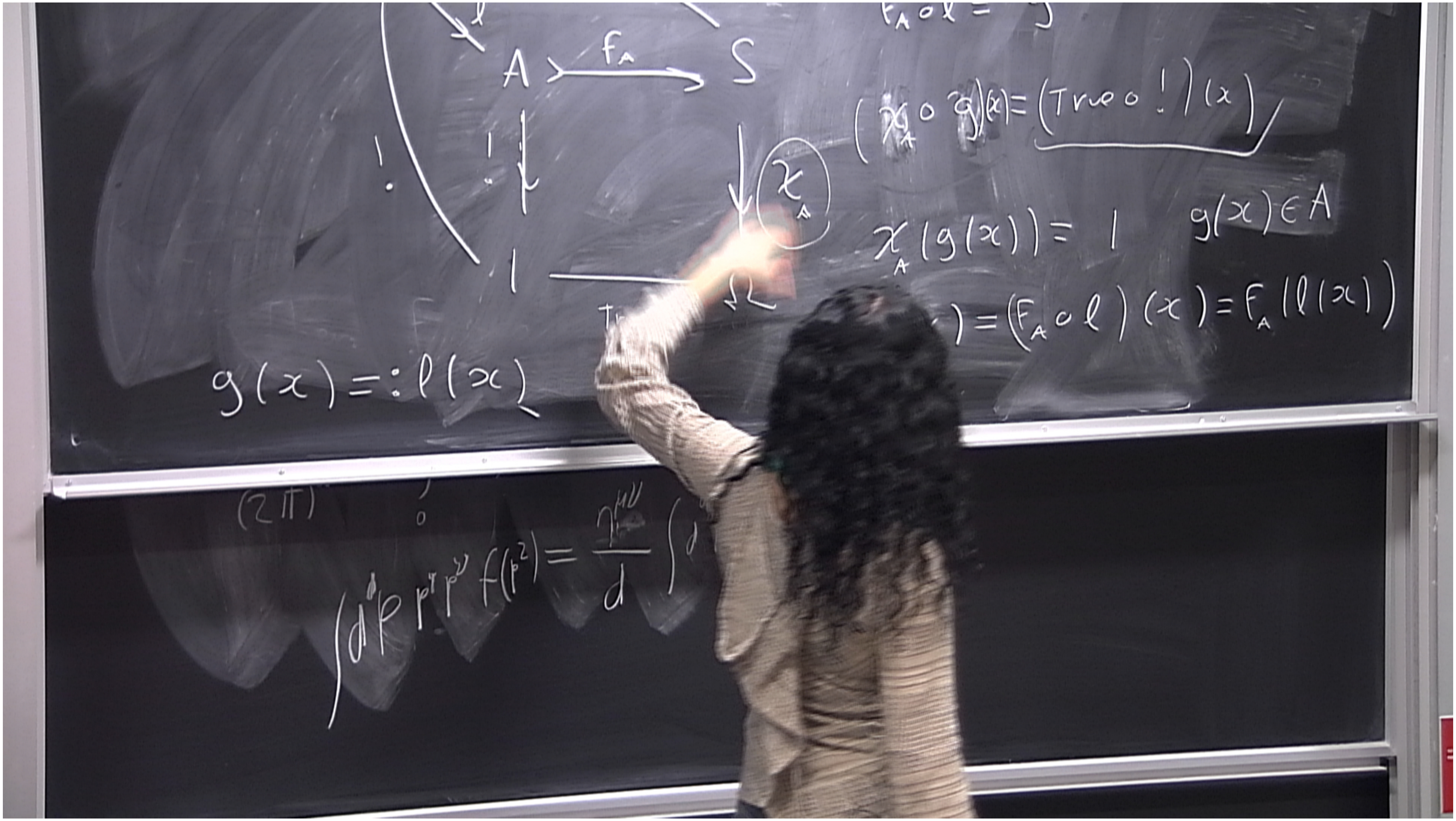

$A := \{ x \mid x \in S \text{ and } \chi_A(x) = 1 \} = \chi_A^{-1}(1)$

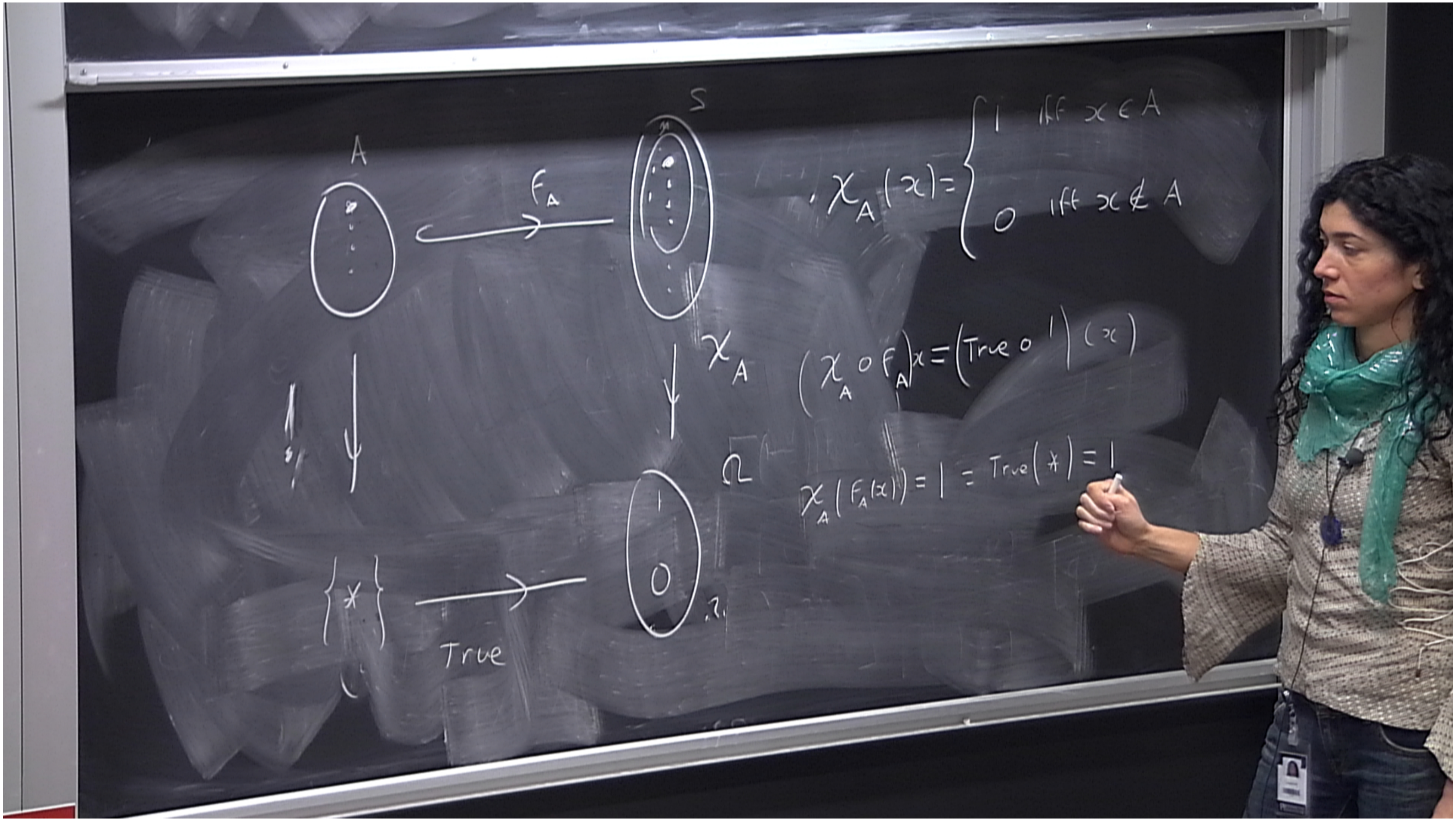

A := { x | x ∈ S and χ_A(x) = 1 } = χ_A^-1 (1) "≅" χ_A^-1 (True)
D
g
l
A
f_A
S
!
χ_A
1
True
Ω
f_A ∘ l = g

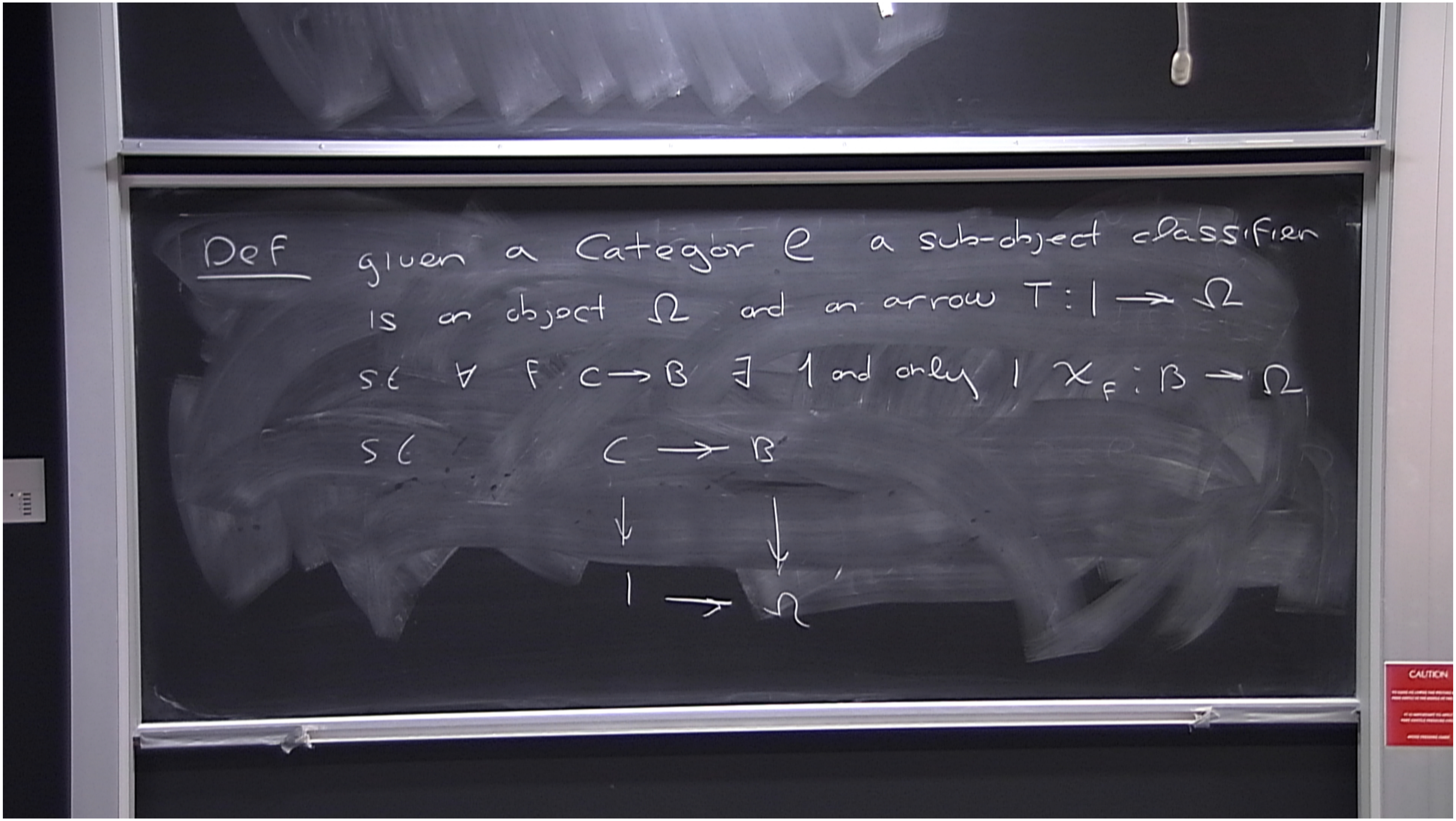

A = { x | x ∈ S and χ_A(x) = 1 } = χ_A^{-1}(1) "≅" χ_A^{-1}(True)
D
g
l
A
f_A
S
!
χ_A
1
True
Ω
f_A ∘ l = g
(χ_A ∘ g)(x) = (True ∘ !)(x)
χ_A(g(x)) =

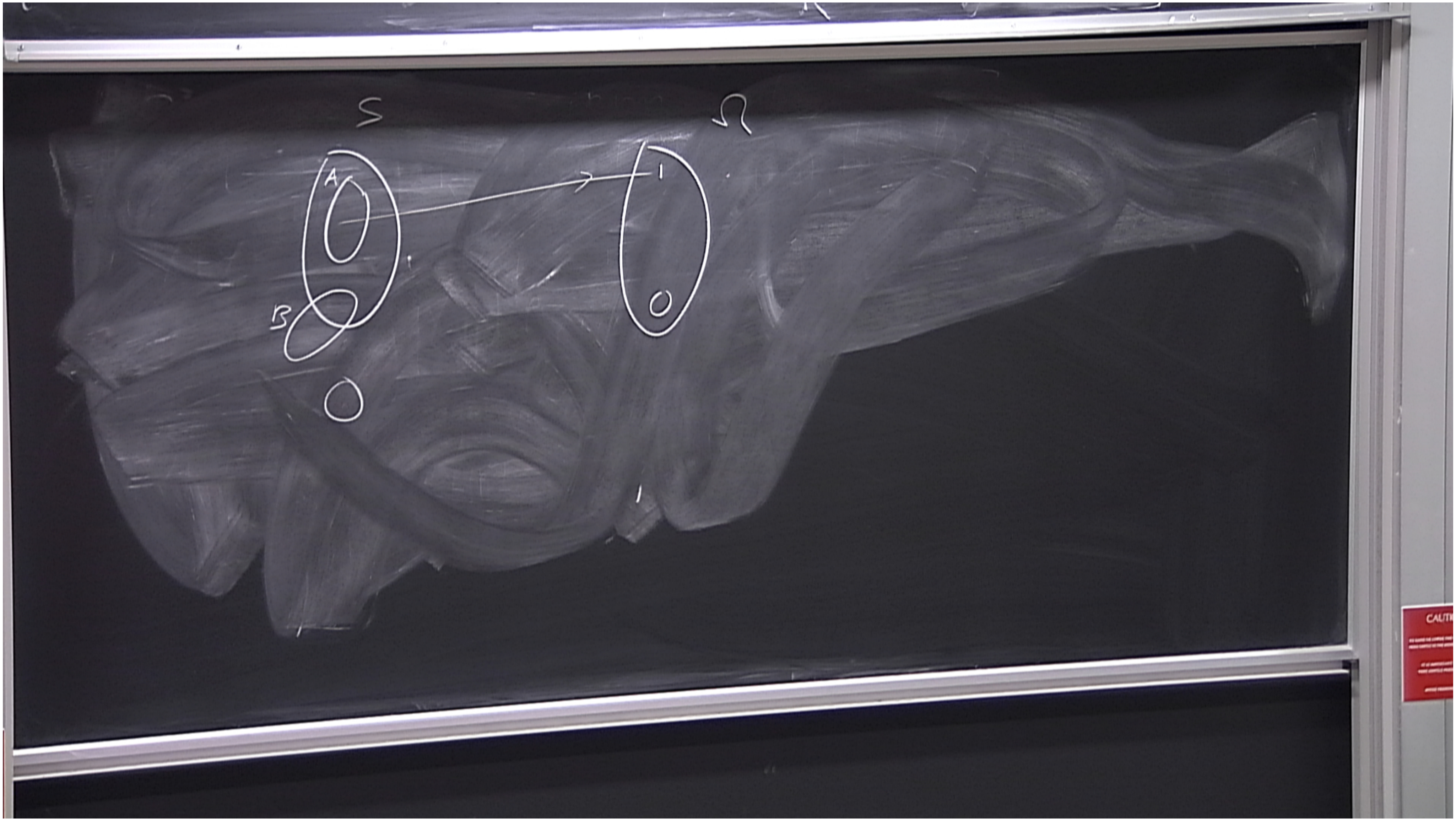


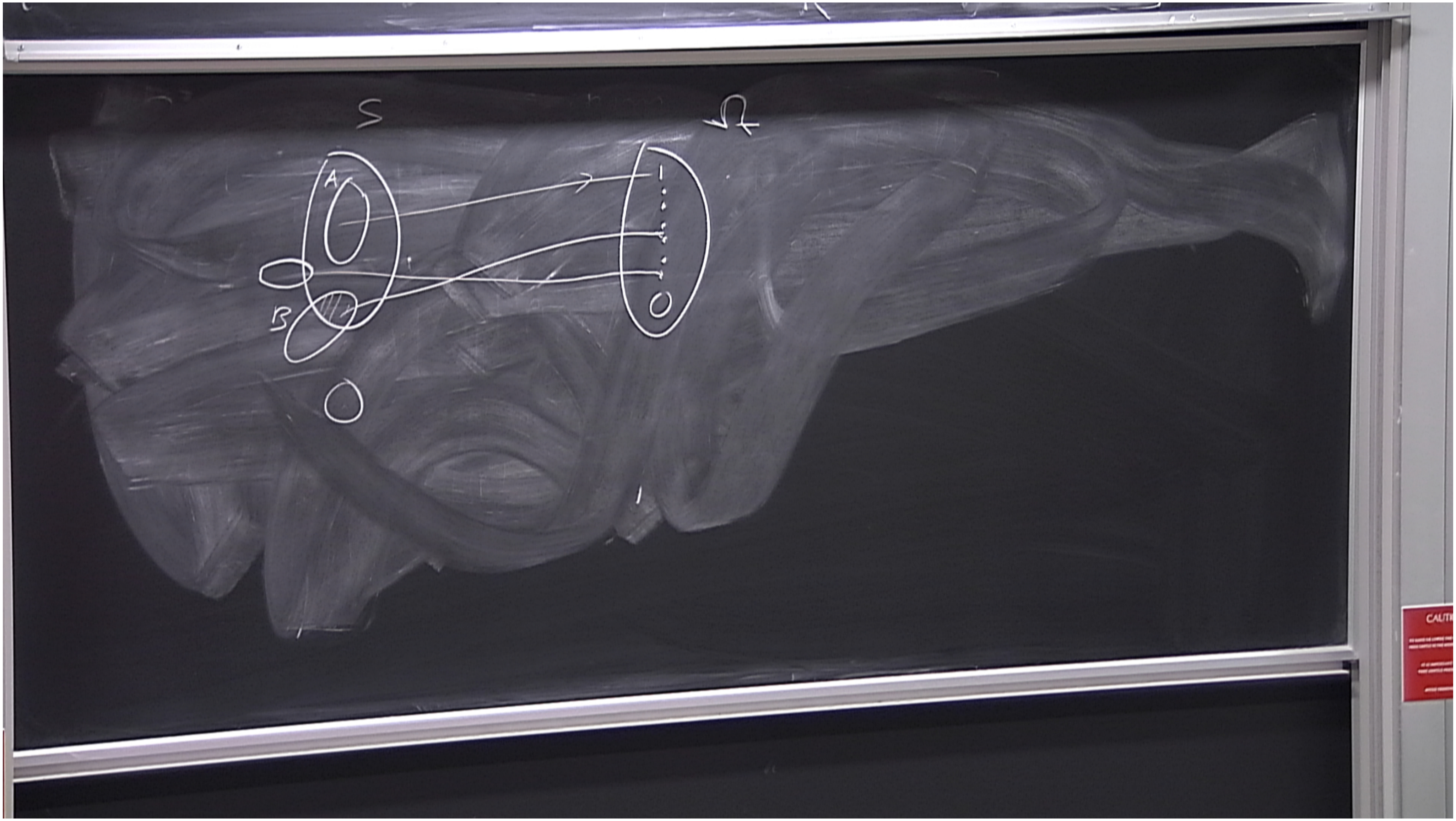
$f_A \circ l = g$
$A \rightarrowtail S$
f_A
χ_A
1
Ω
True
$(\chi_A \circ g)(x) = (True \circ !)(x)$
$\chi_A(g(x)) = 1$
$g(x) \in A$
$g(x) = f_A$

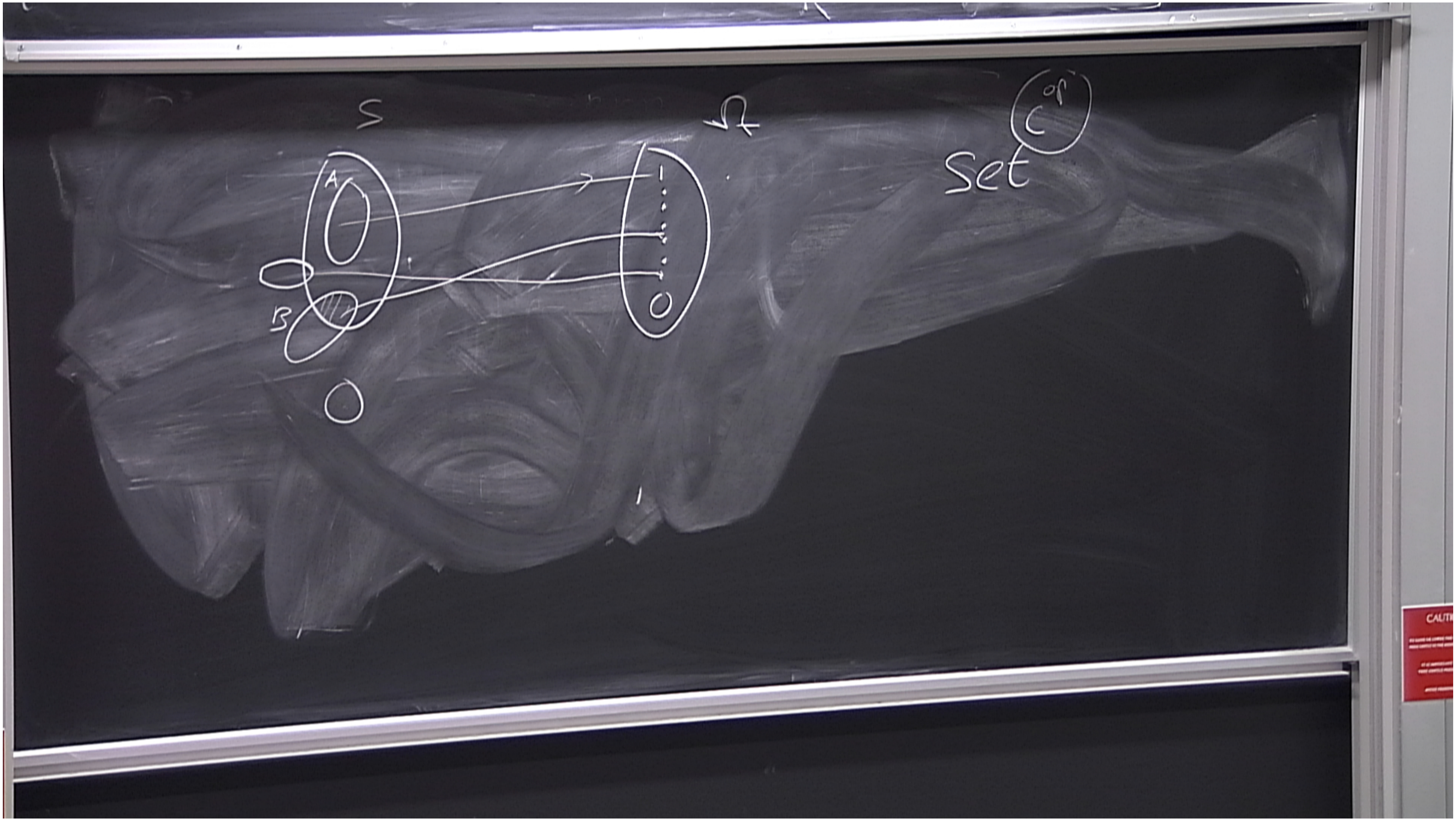

A
f_A
S
χ_A
(χ_A ∘ g)(x) = (True ∘ !)(x)
χ_A(g(x)) = 1
g(x) ∈ A
) = (f_A ∘ ℓ)(x) = f_A(ℓ(x))
g(x) =: ℓ(x)


A
S
f_A
$\chi_A(x) = \begin{cases} 1 & \text{iff } x \in A \\ 0 & \text{iff } x \notin A \end{cases}$
χ_A
$(\chi_A \circ f_A)x = (\text{True} \circ !)(x)$
Ω
$\chi_A(f_A(x)) = 1 = \text{True}(*) = 1$
$\{*\}$
True
1
0

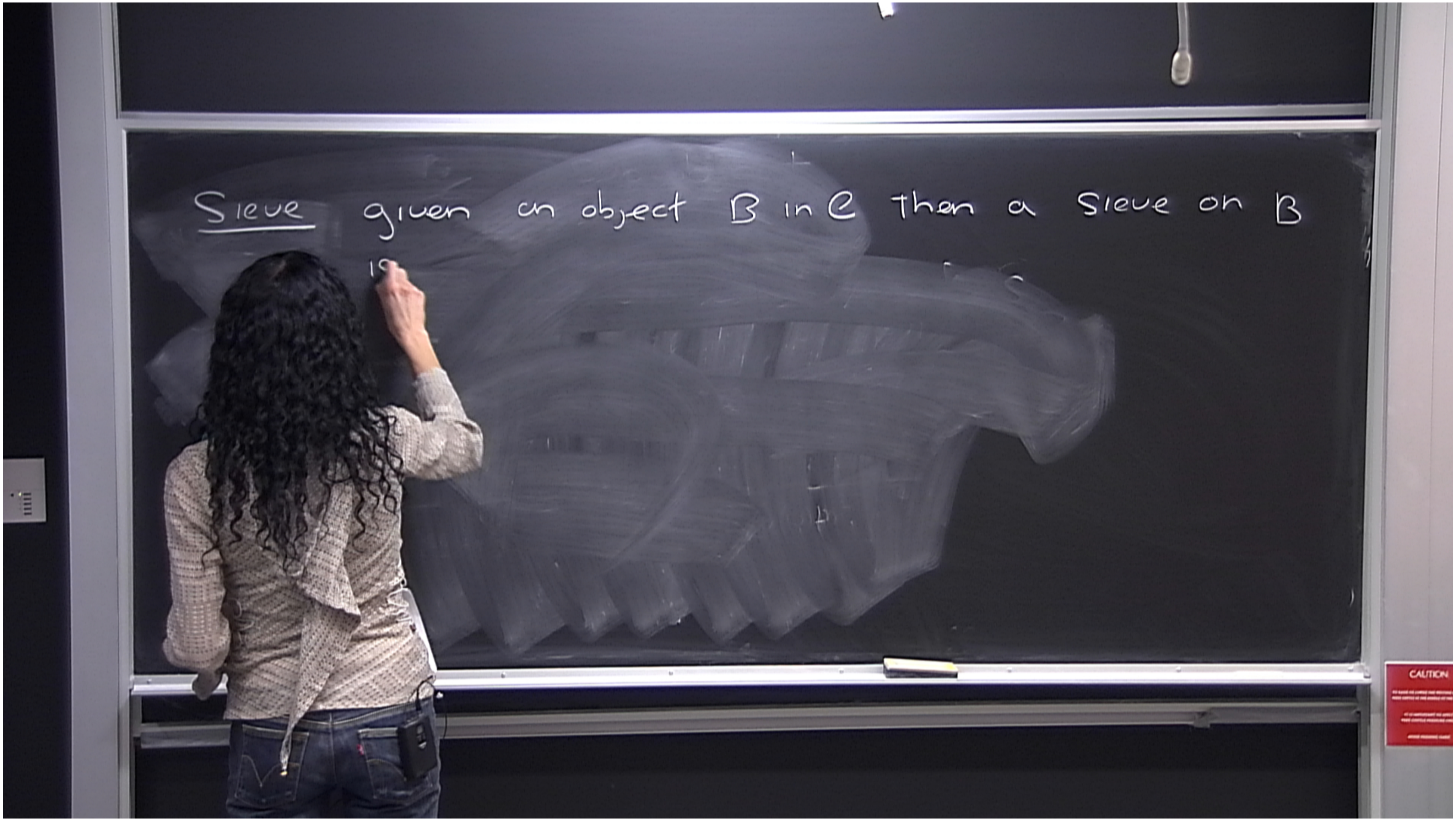

Def given a Categor 𝒞 a sub-object classifier
is an object Ω and an arrow T : 1 → Ω
st ∀ f : C → B ∃ 1 and only 1 χf : B → Ω
st
C → B
1 → Ω

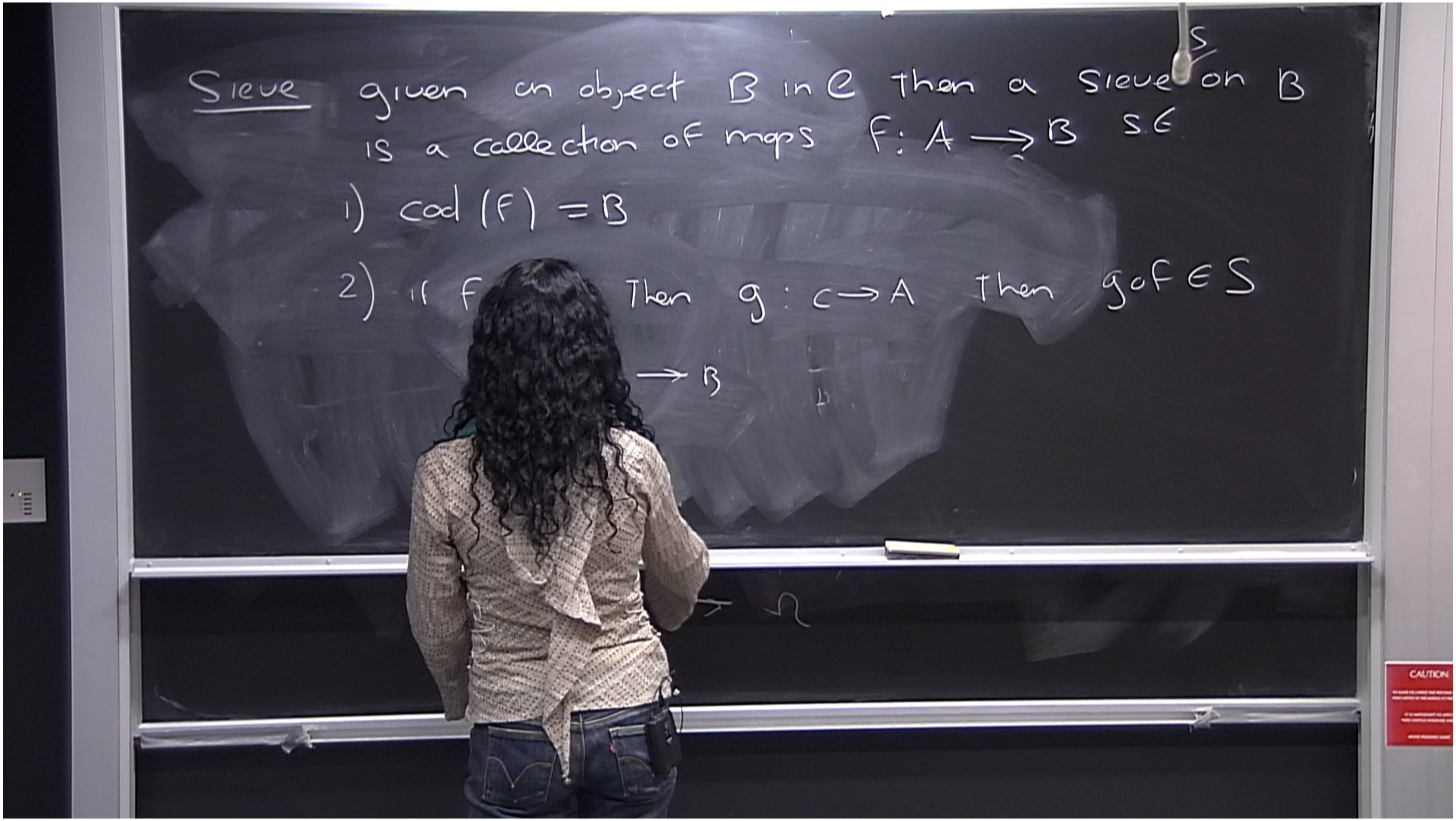

S
Ω
A
B
1
0


S
Ω
A
B

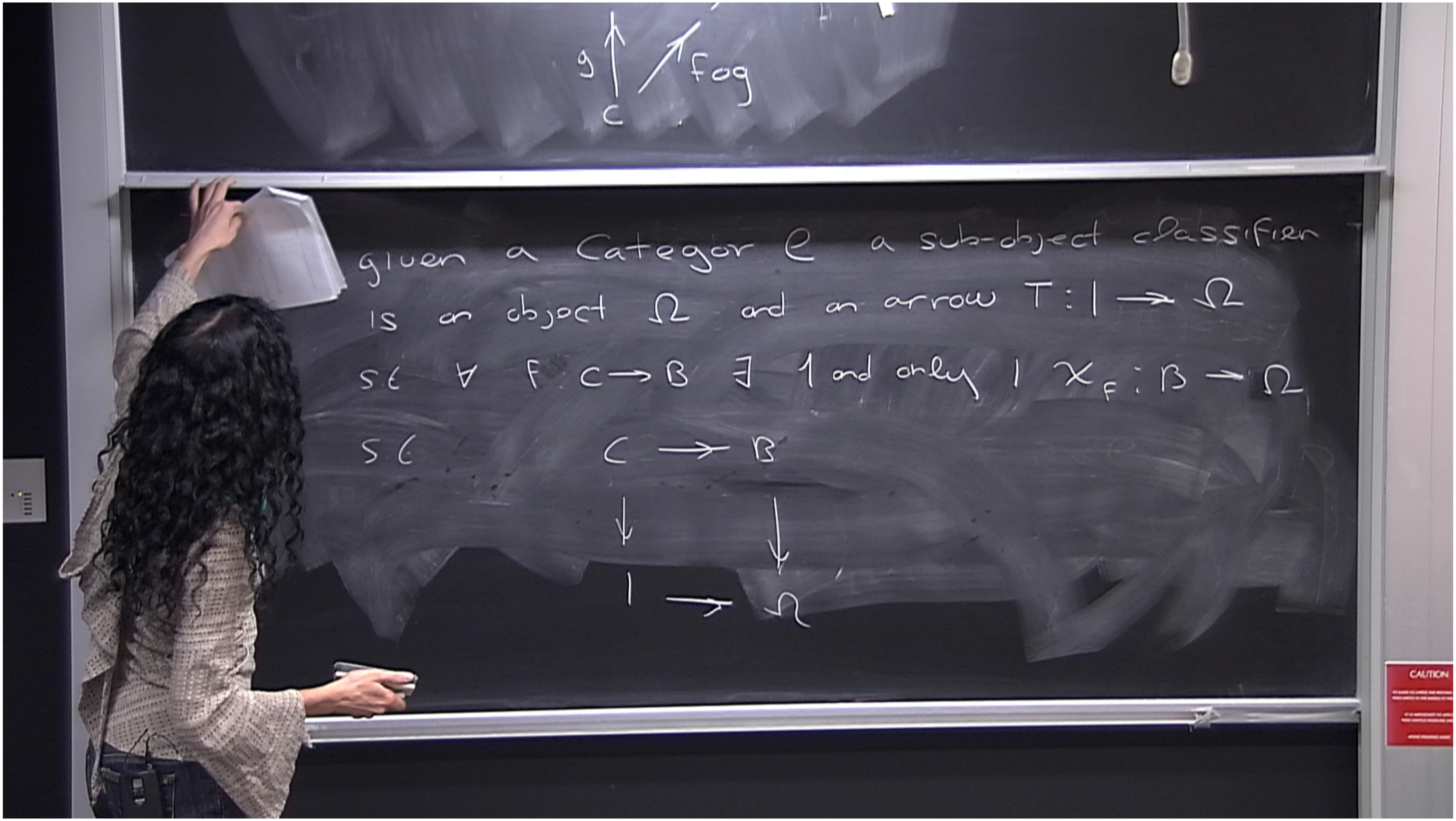

S
Ω
A
B
1
0
Set
C^{op}

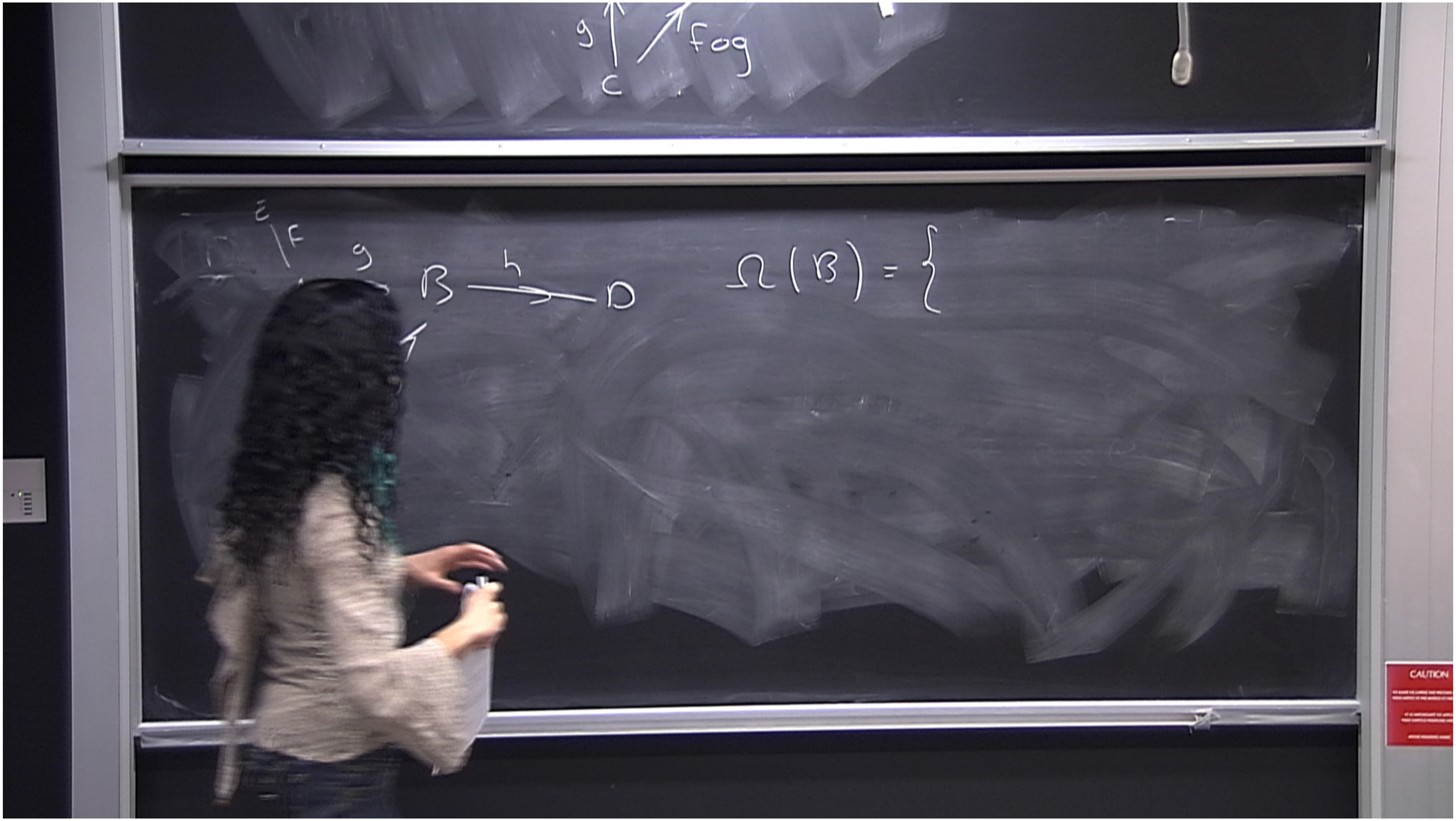

Sieve

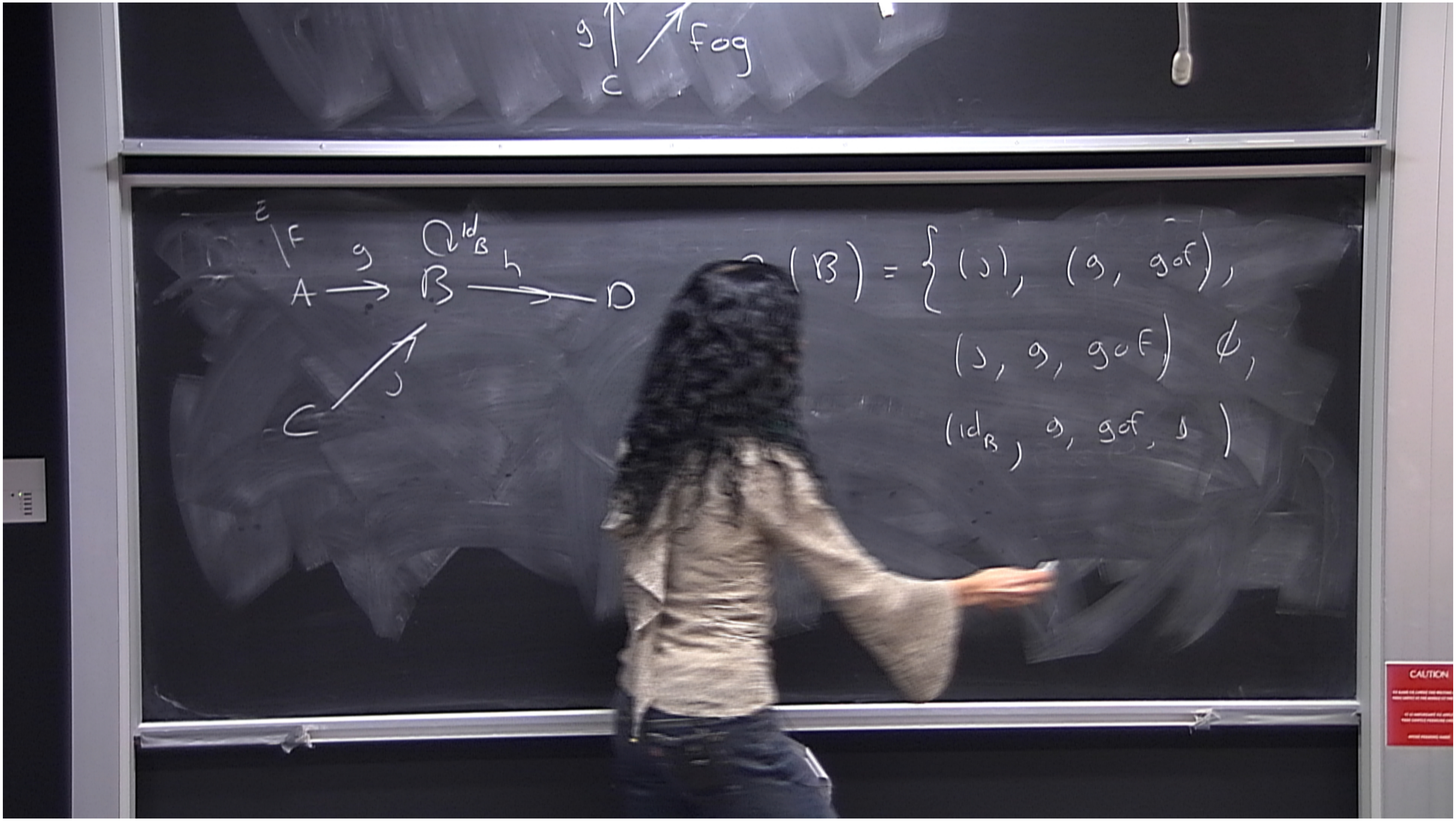

Sieve given an object B in C then a sieve on B
is

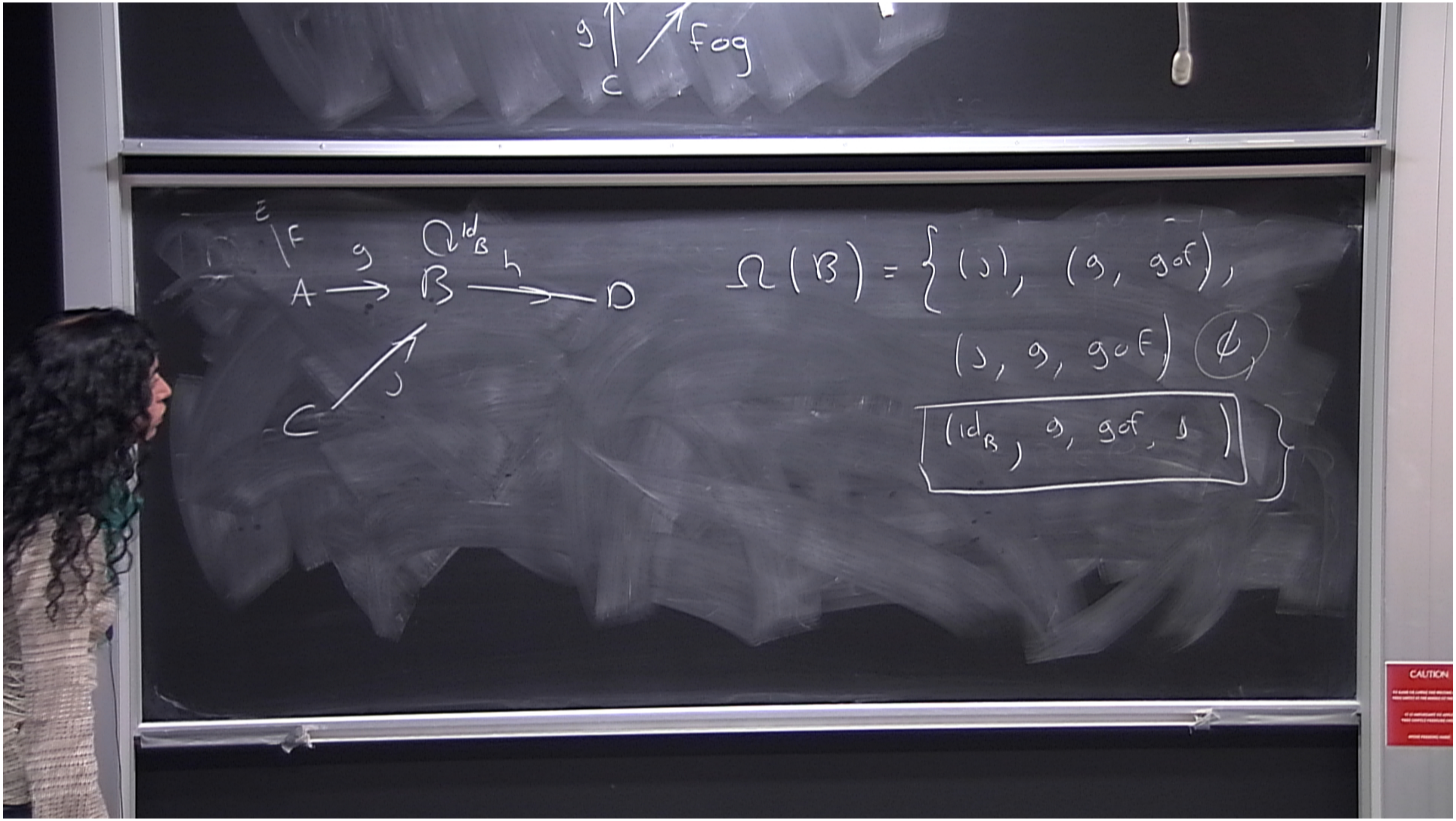


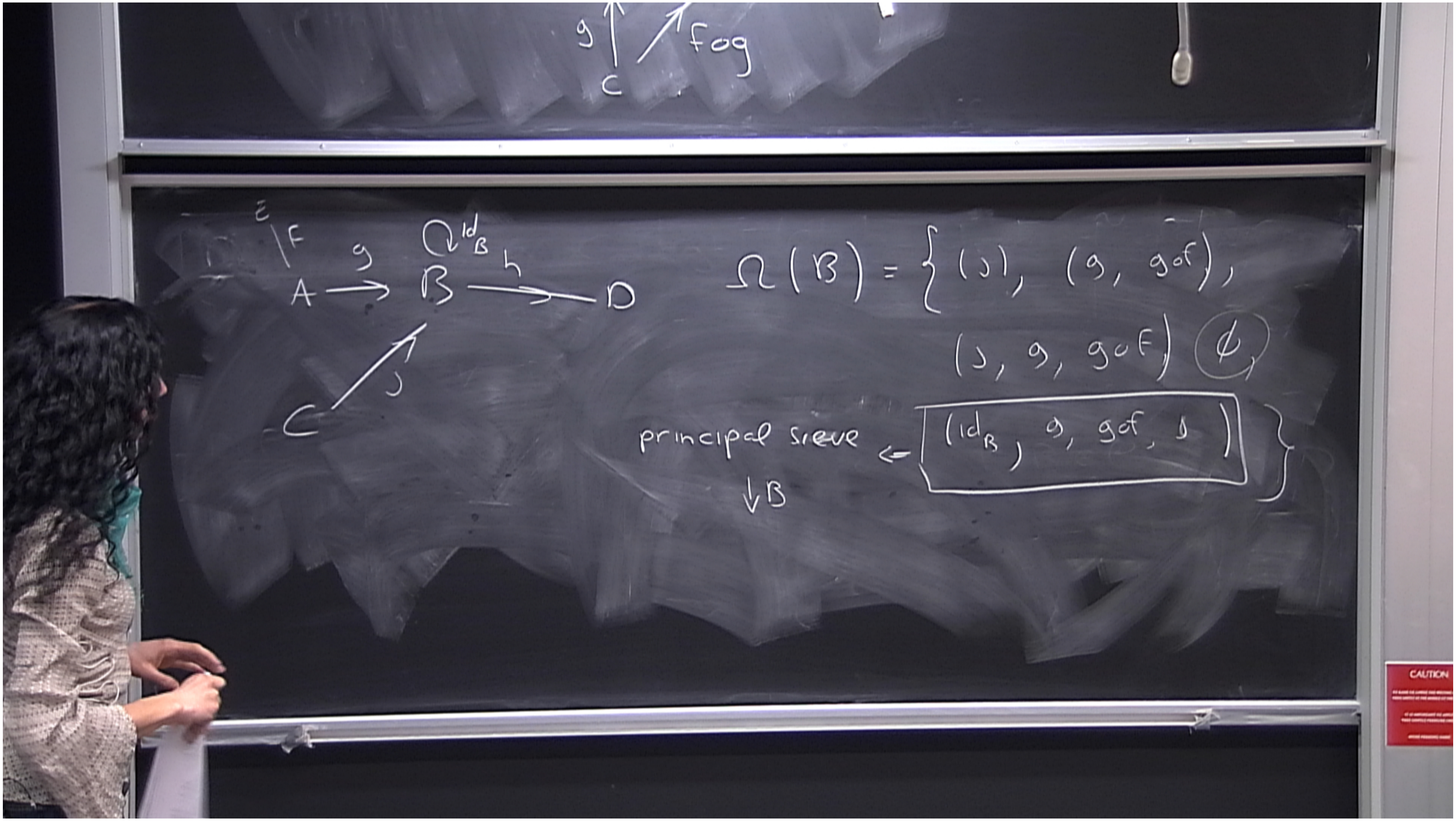
S
Sieve given an object B in C then a sieve on B
is a collection of maps f: A → B s.t.
1) cod(f) = B
2) if f
Then g: C → A then g∘f ∈ S

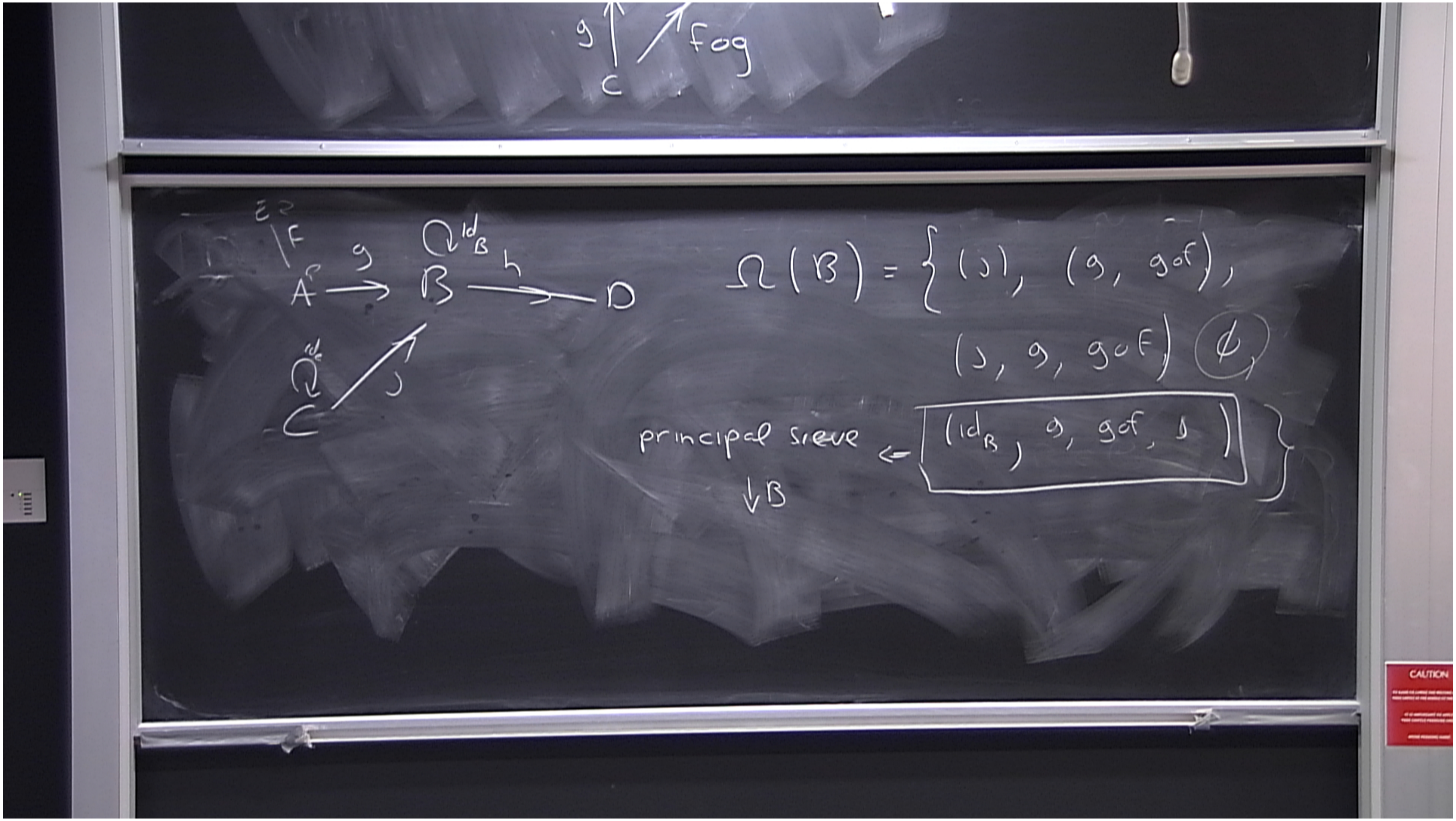

g
fog
C
given a Categor C a sub-object classifier
is an object Ω and an arrow T : 1 → Ω
st ∀ f : C → B ∃ 1 and only 1 χf : B → Ω
st C → B
1 → Ω
CAUTION

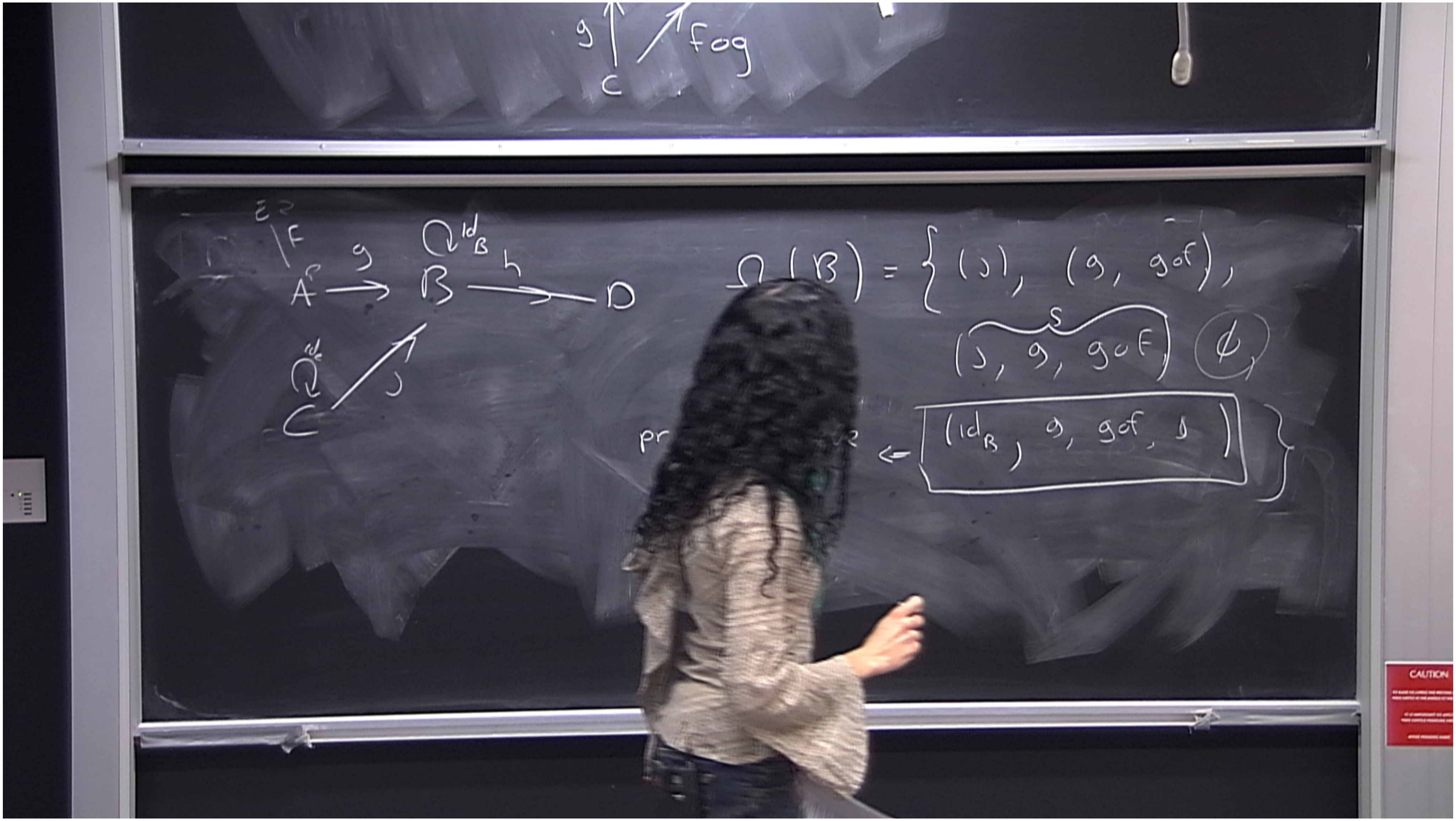

g
C
fog
F
g
B
h
D
$\Omega(B) = \{$
CAUTION

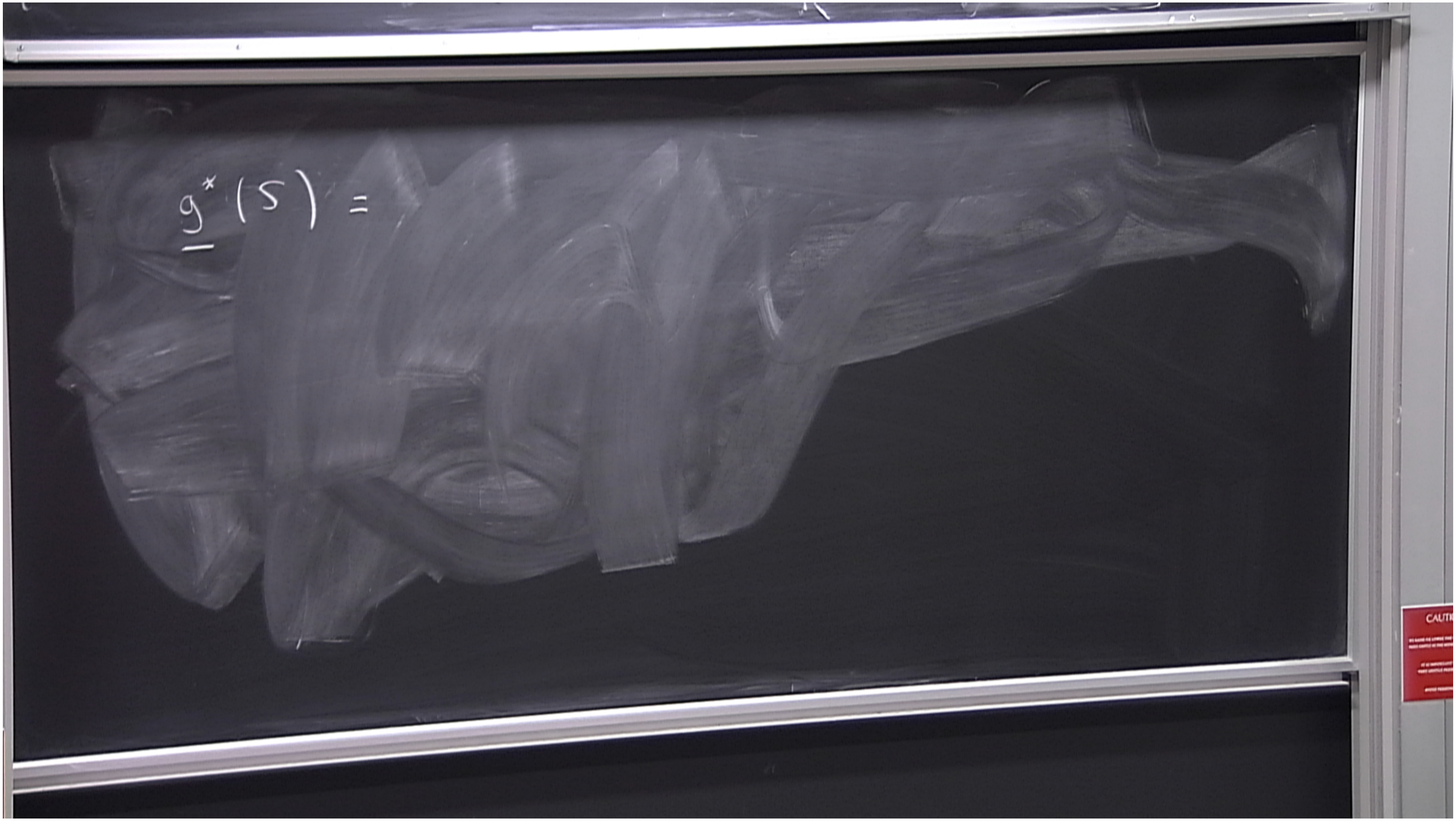

fog
A → B → D
(B) = { (j), (g, gof),
(j, g, gof) ∅,
(id_B, g, gof, j))

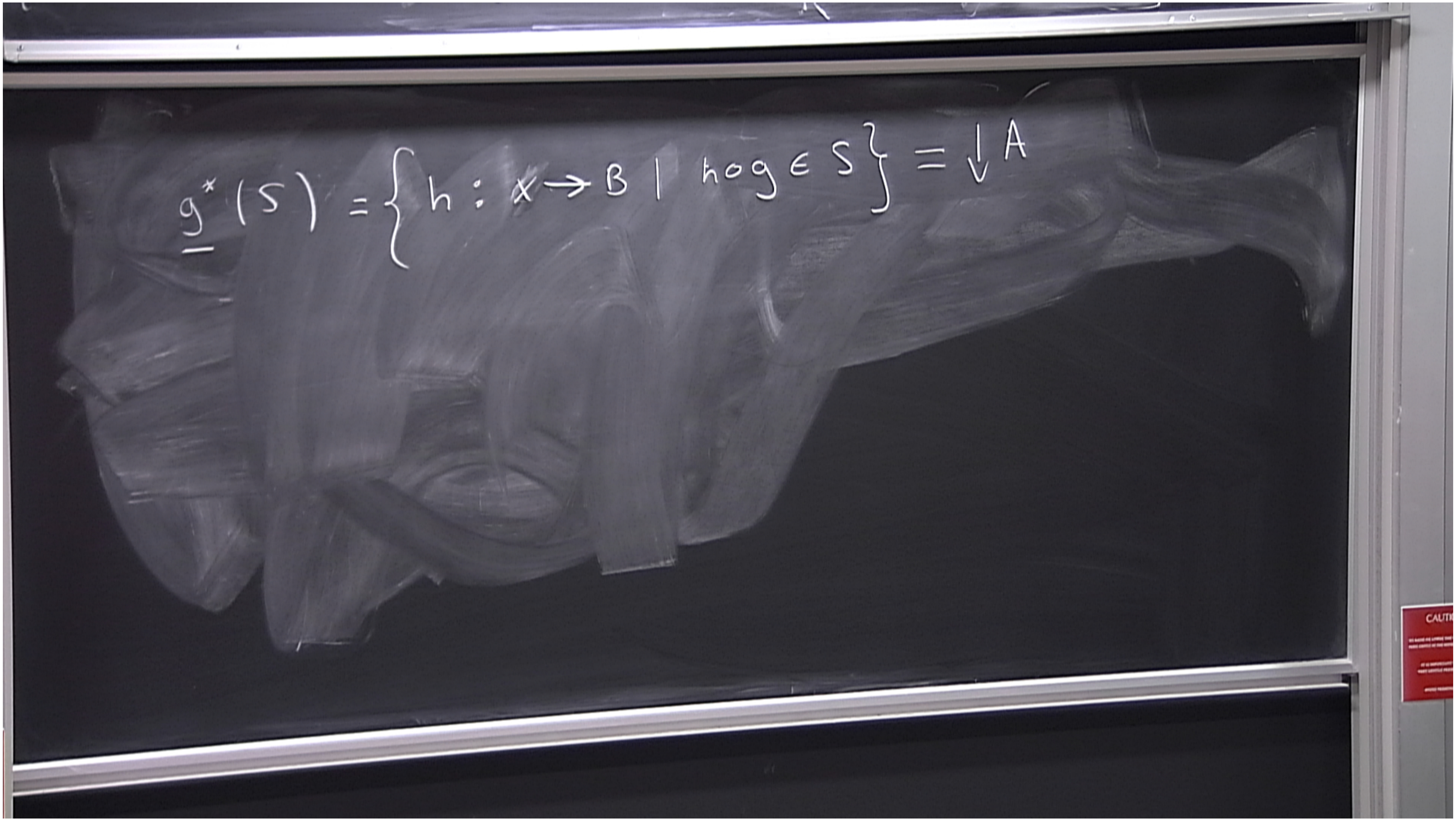

f∘g
A → B → D
Ω(B) = { (j), (g, g∘f),
(j, g, g∘f), ∅,
(id_B, g, g∘f, j) }

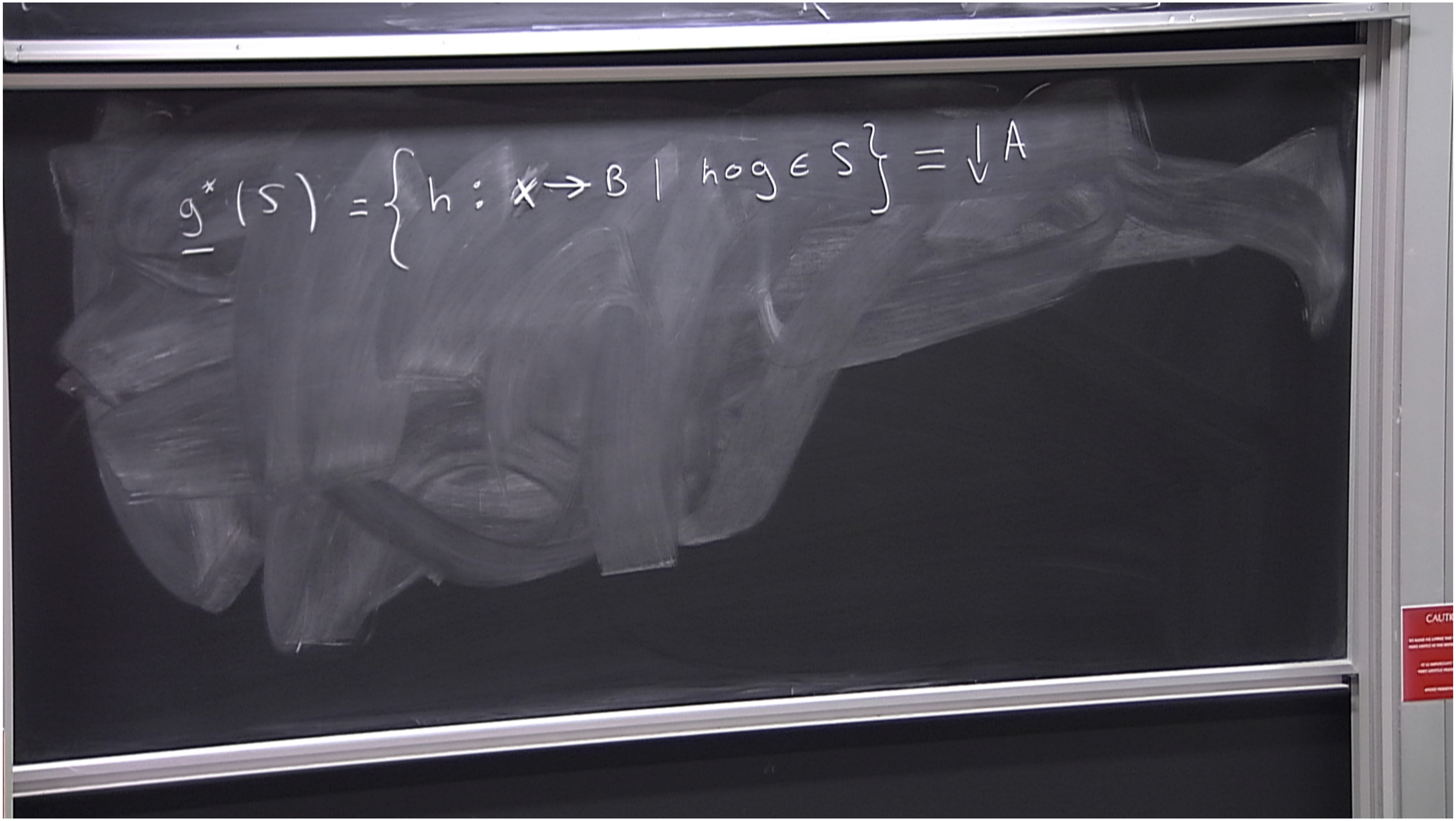

fog
A → B → D
principal sieve
Ω(B) = {(j), (g, gof),
(j, g, gof) ∅,
(id_B, g, gof, j)}
↓B
CAUTION

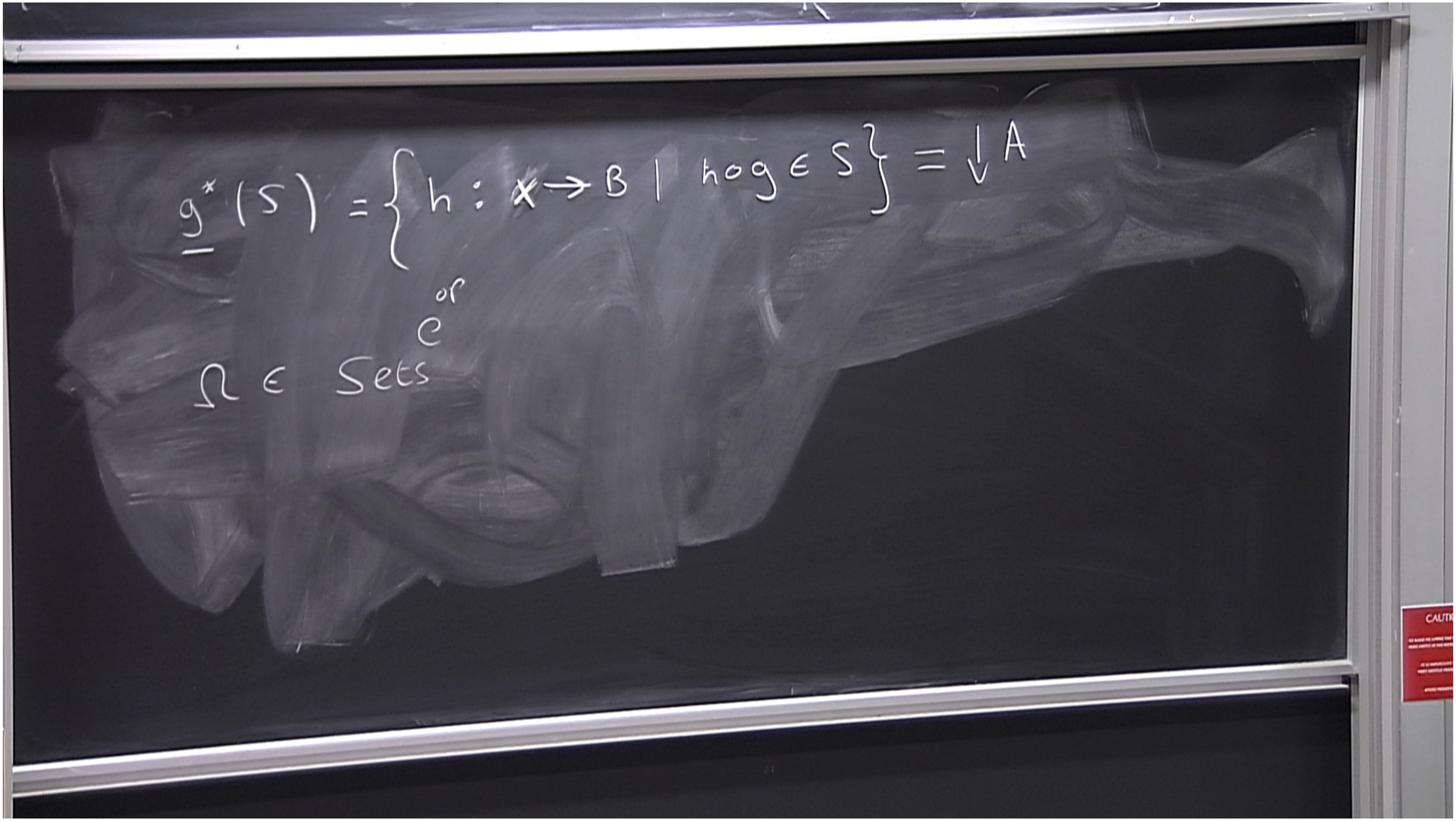

g
fog
C
A
B
D
h
idB
Ω(B) = {(j), (g, g∘f),
(j, g, g∘f) ∅,
(idB, g, g∘f, j)}
principal sieve
↓B

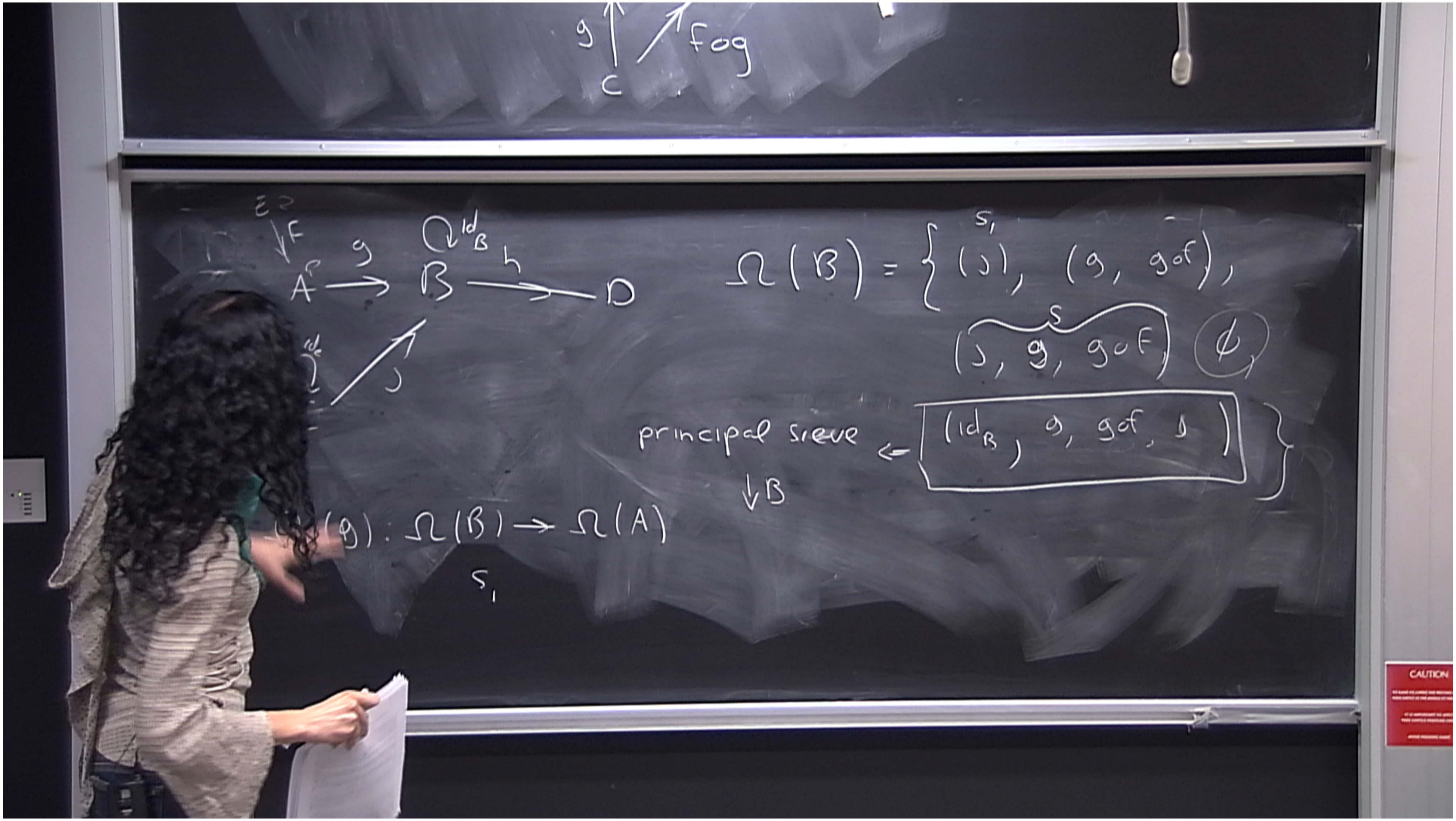

fog
$\Omega(B) = \{ (j), (g, g \circ f),$
$(j, g, g \circ f)$
$\emptyset$
$(id_B, g, g \circ f, j)$

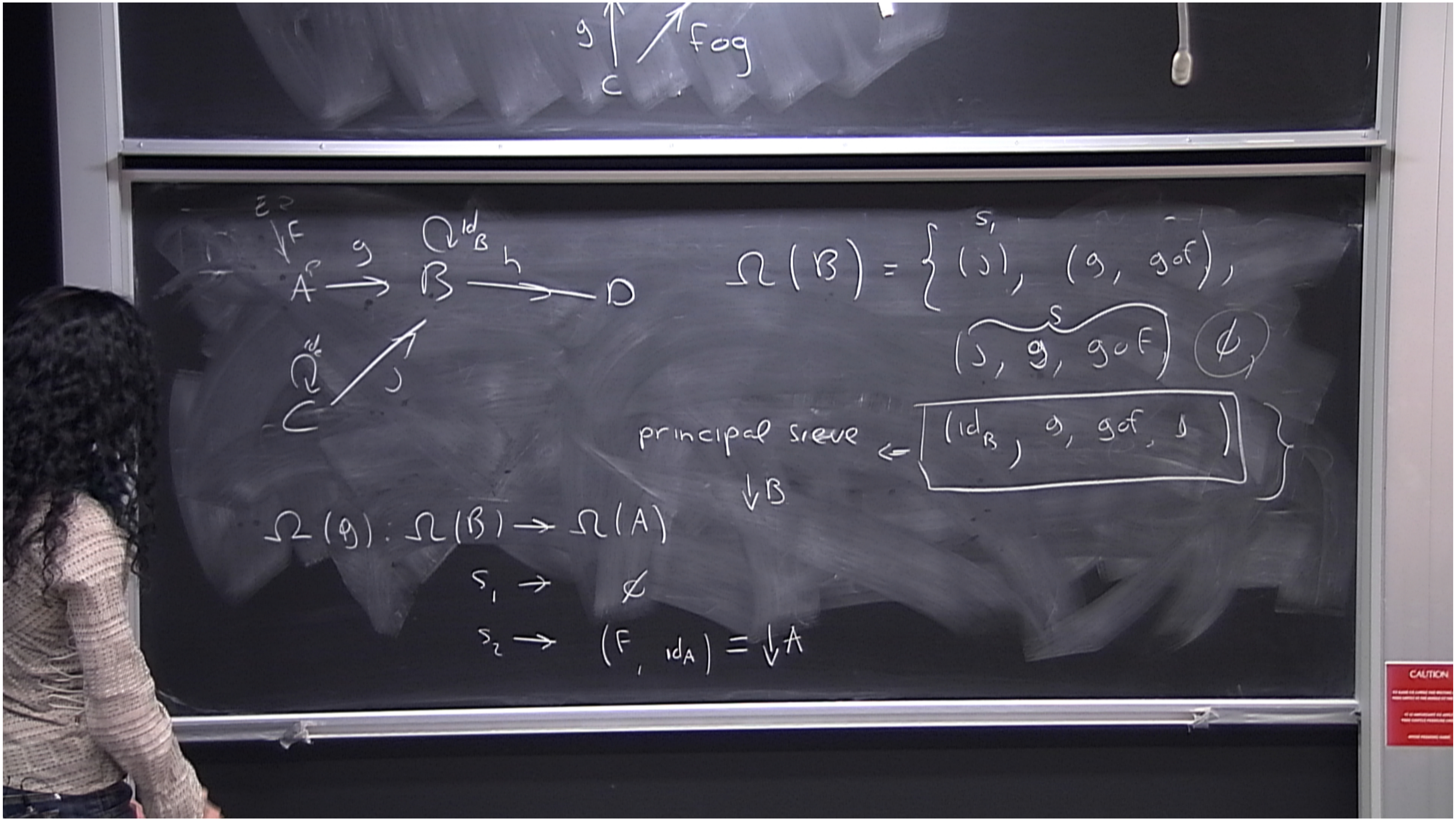

$\underline{g}^*(s) =$

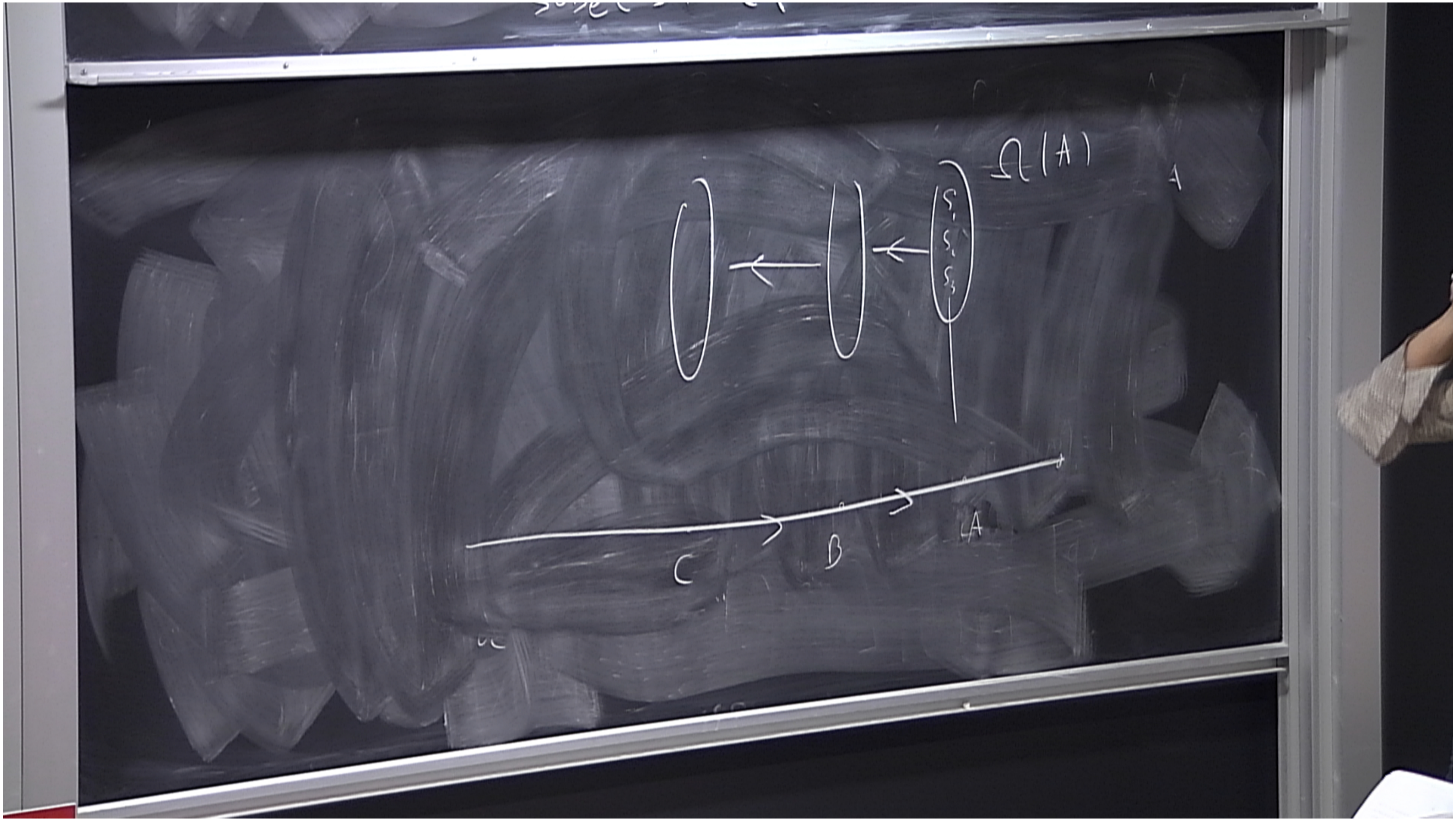

g*(S) = {h : X → B | h∘g ∈ S} = ↓A


$\underline{g}^*(S) = \{ h : X \to B \mid h \circ g \in S \} = \downarrow A$

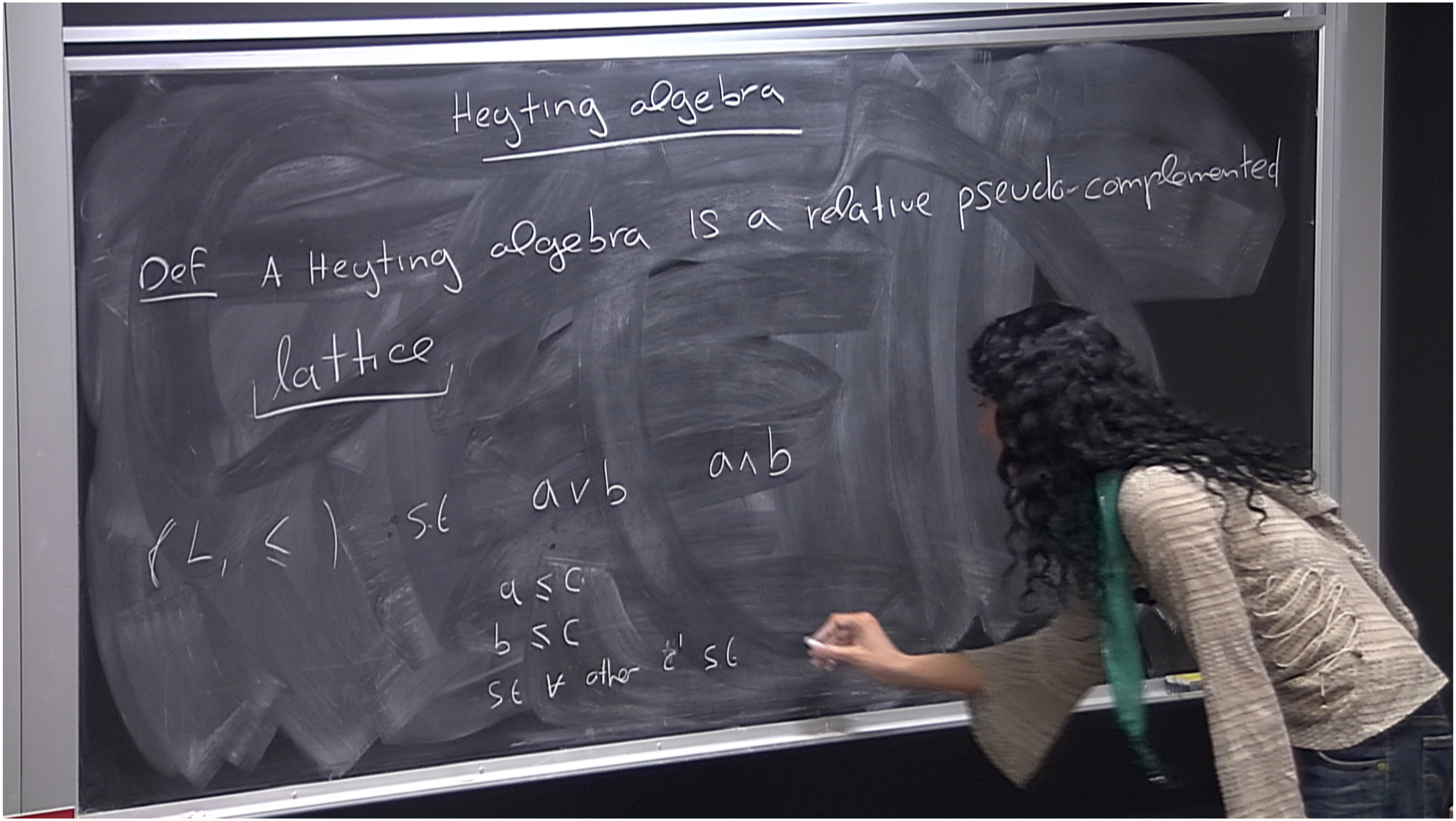

g* (S) = { h : X → B | h∘g ∈ S } = ↓A
Ω ∈ Sets^{C^op}

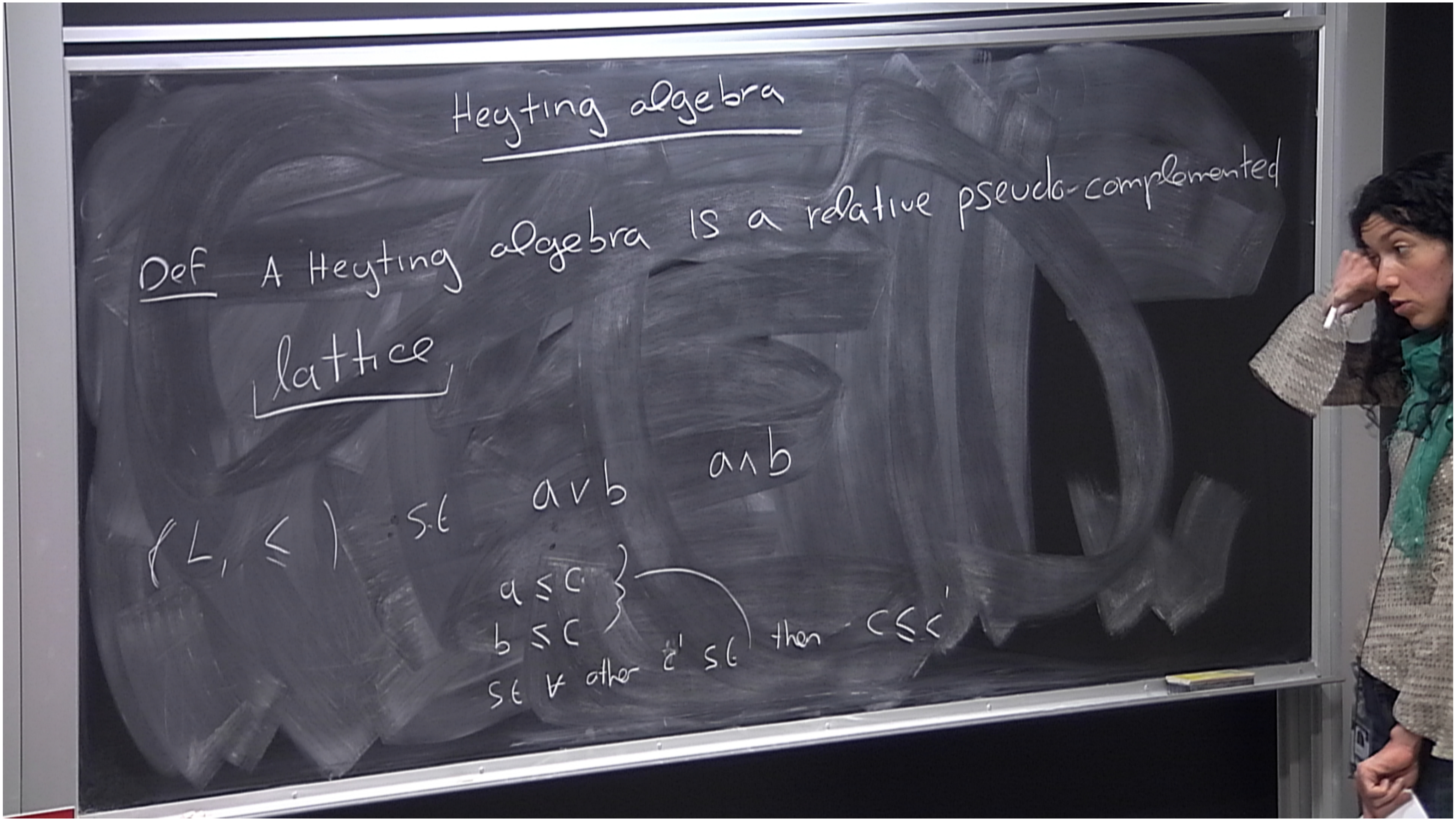

fog
principal sieve

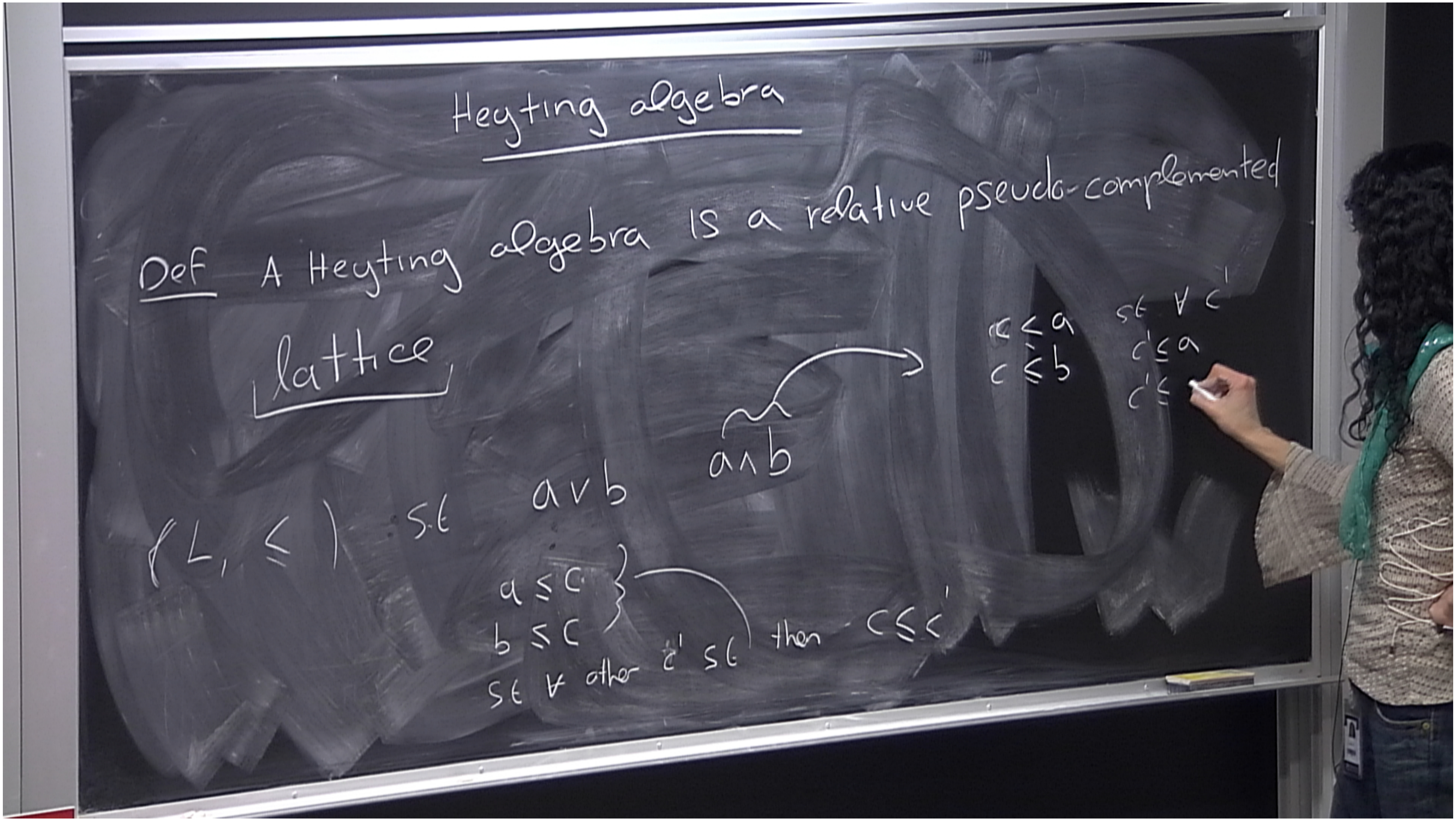

fog
Ω(B) = { (j), (g, gof),
(j, g, gof) ∅,
principal sieve
(id_B, g, gof, j)
↓B
Ω(g): Ω(B) → Ω(A)
s₁ → ∅
s₂ → (F, id_A) = ↓A
CAUTION

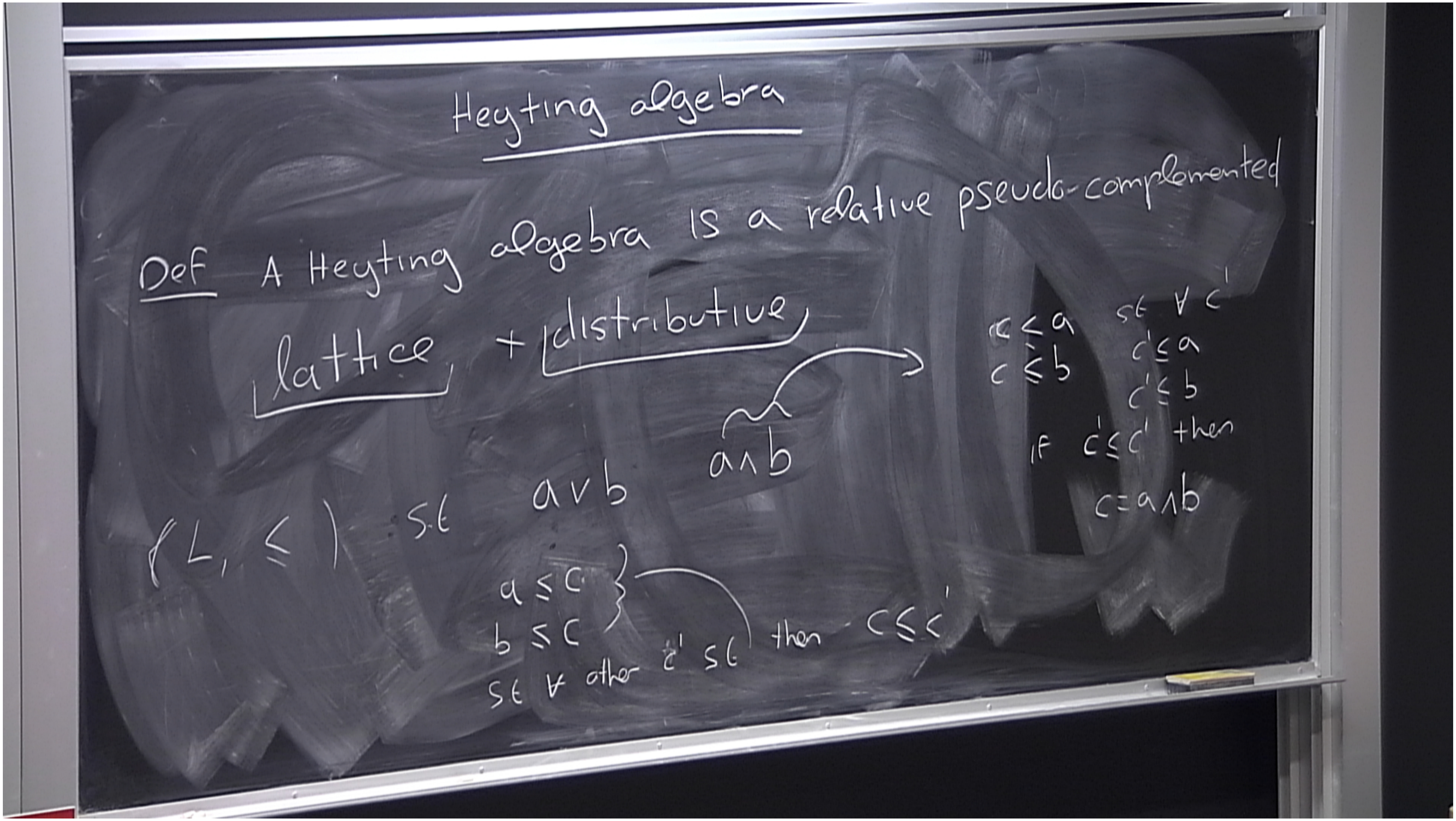


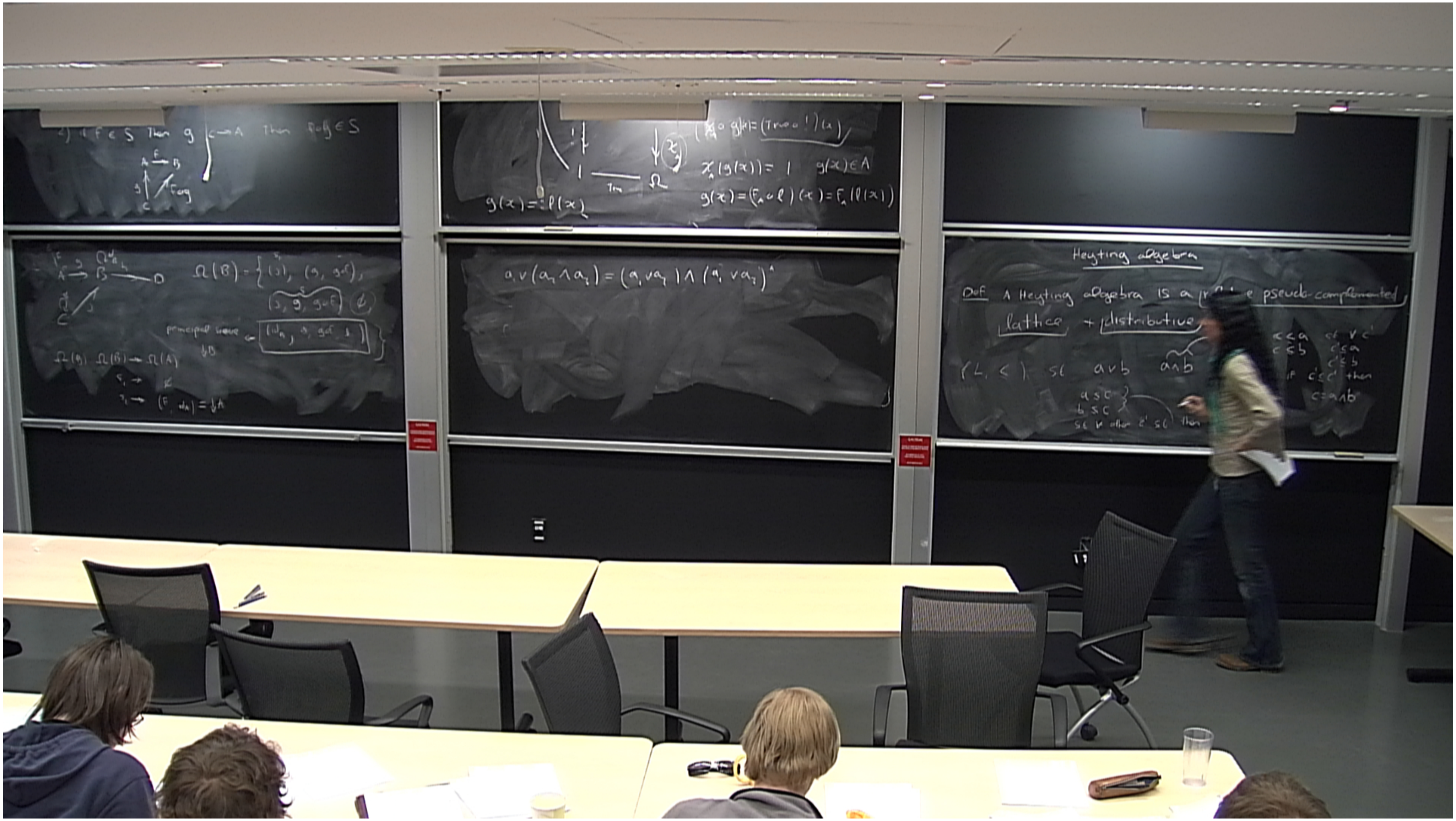
Ω(A)
A
C
B
A

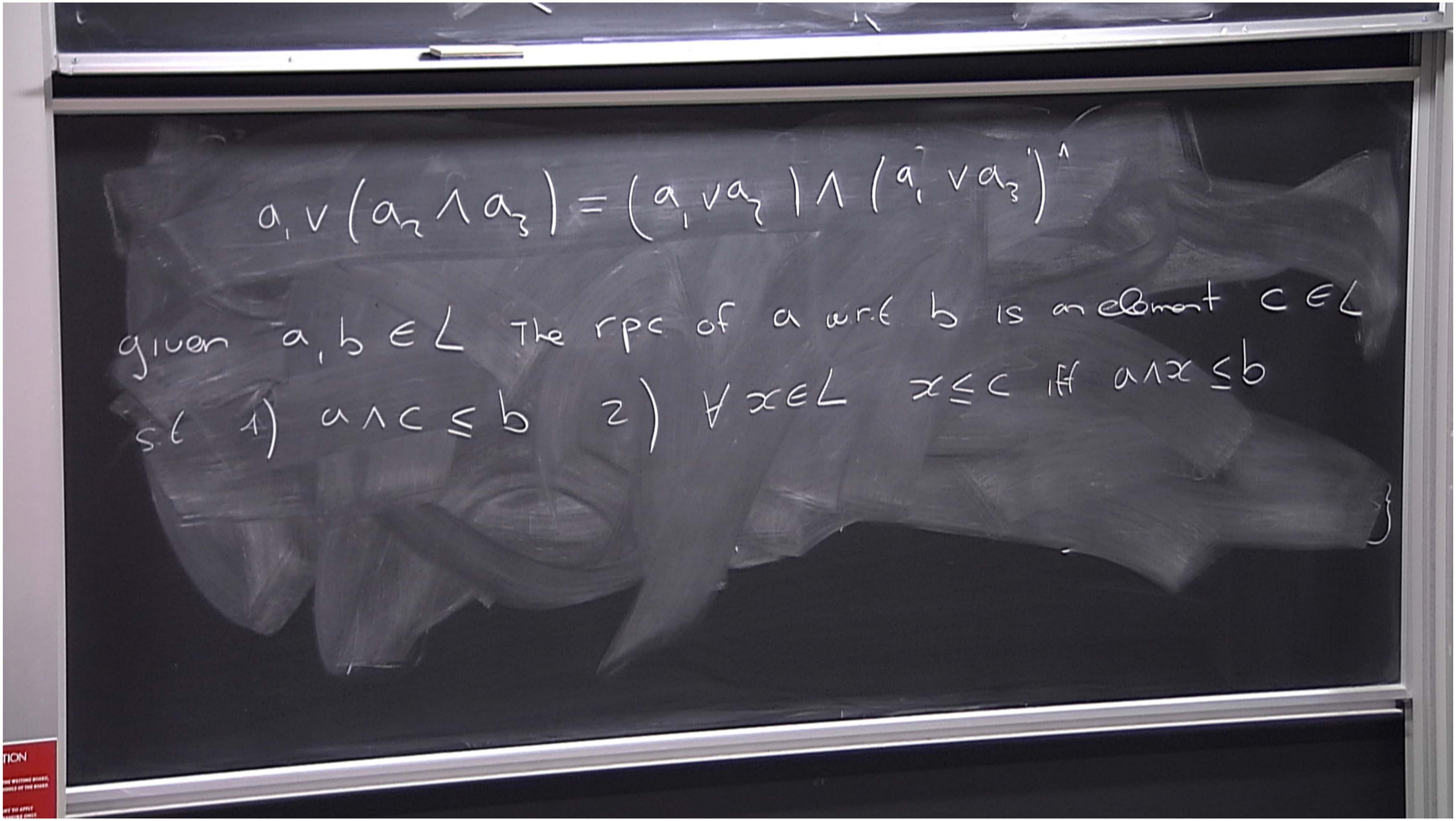

Heyting algebra
Def A Heyting algebra is a relative pseudo-complemented lattice
(L, ≤) s.t. a∨b a∧b
a ≤ c
b ≤ c
s.t ∀ other c' s.t


Heyting algebra
Def A Heyting algebra is a relative pseudo-complemented
lattice
(L, ≤) s.t.
a∨b
a∧b
a ≤ c
b ≤ c
s.t ∀ other c' s.t
then c ≤ c'


Heyting algebra
Def A Heyting algebra is a relative pseudo-complemented
lattice
(L, ≤) s.t.
a∨b
a∧b
a ≤ c
b ≤ c
s.t. ∀ other c' s.t.
then c ≤ c'
c ≤ a
c ≤ b
s.t. ∀ c'
c' ≤ a


Heyting algebra
Def A Heyting algebra is a relative pseudo-complemented
lattice + distributive
(L, ≤) s.t a∨b a∧b
a ≤ c
b ≤ c
s.t ∀ other c' s.t then c ≤ c'
c ≤ a s.t ∀ c'
c ≤ b c' ≤ a
c' ≤ b
if c' ≤ c' then
c = a∧b


Heyting algebra
lattice
distributive


a₁ v (a₂ ∧ a₃) = (a₁ v a₂) ∧ (a₁ v a₃)
given a, b ∈ L The rpc of a w.r.t b is an element c ∈ L
s.t 1) a ∧ c ≤ b 2) ∀ x ∈ L x ≤ c iff a ∧ x ≤ b

$$a_1 \vee (a_2 \wedge a_3) = (a_1 \vee a_2) \wedge (a_1 \vee a_3)$$

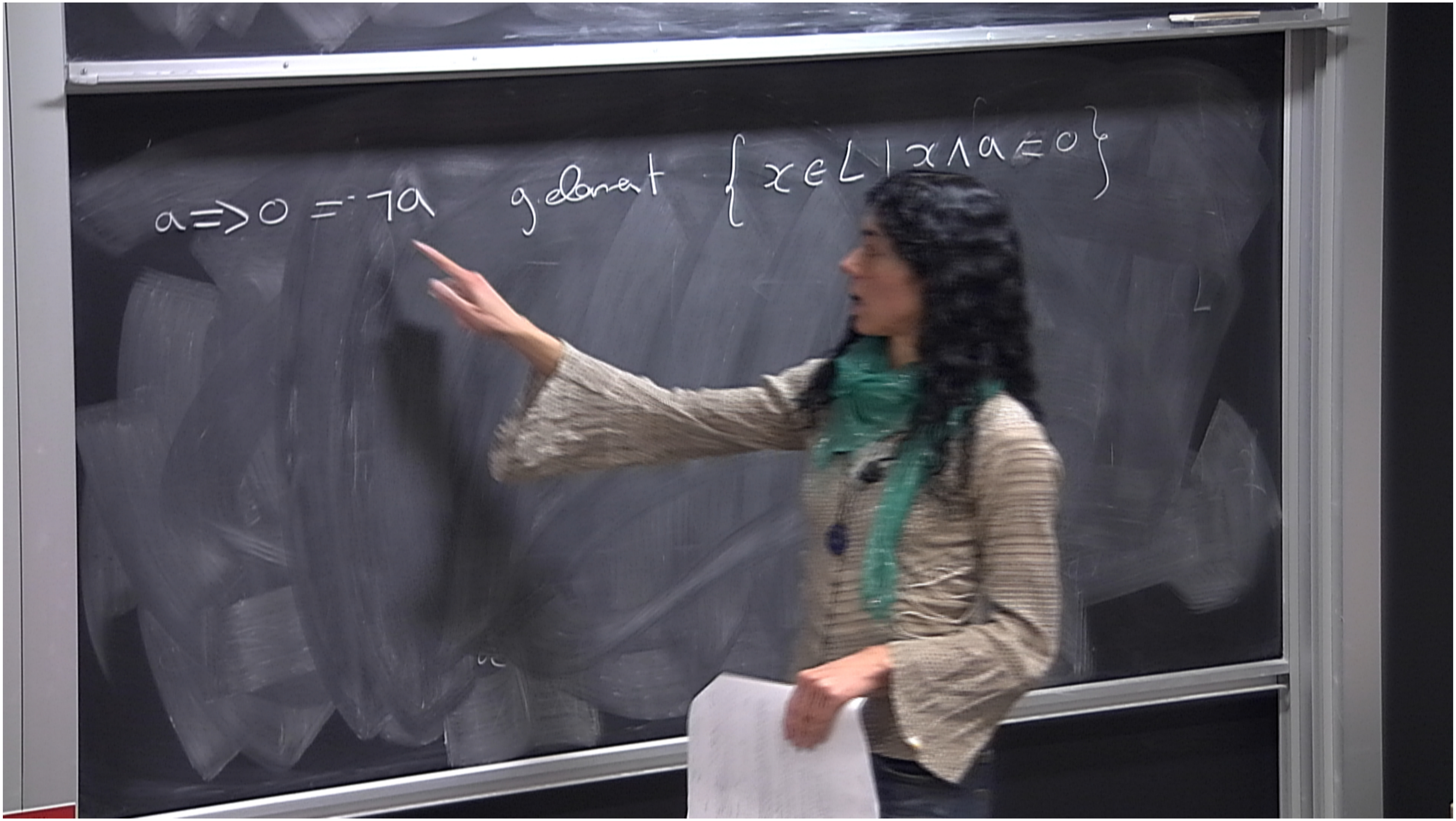
($\leq$)

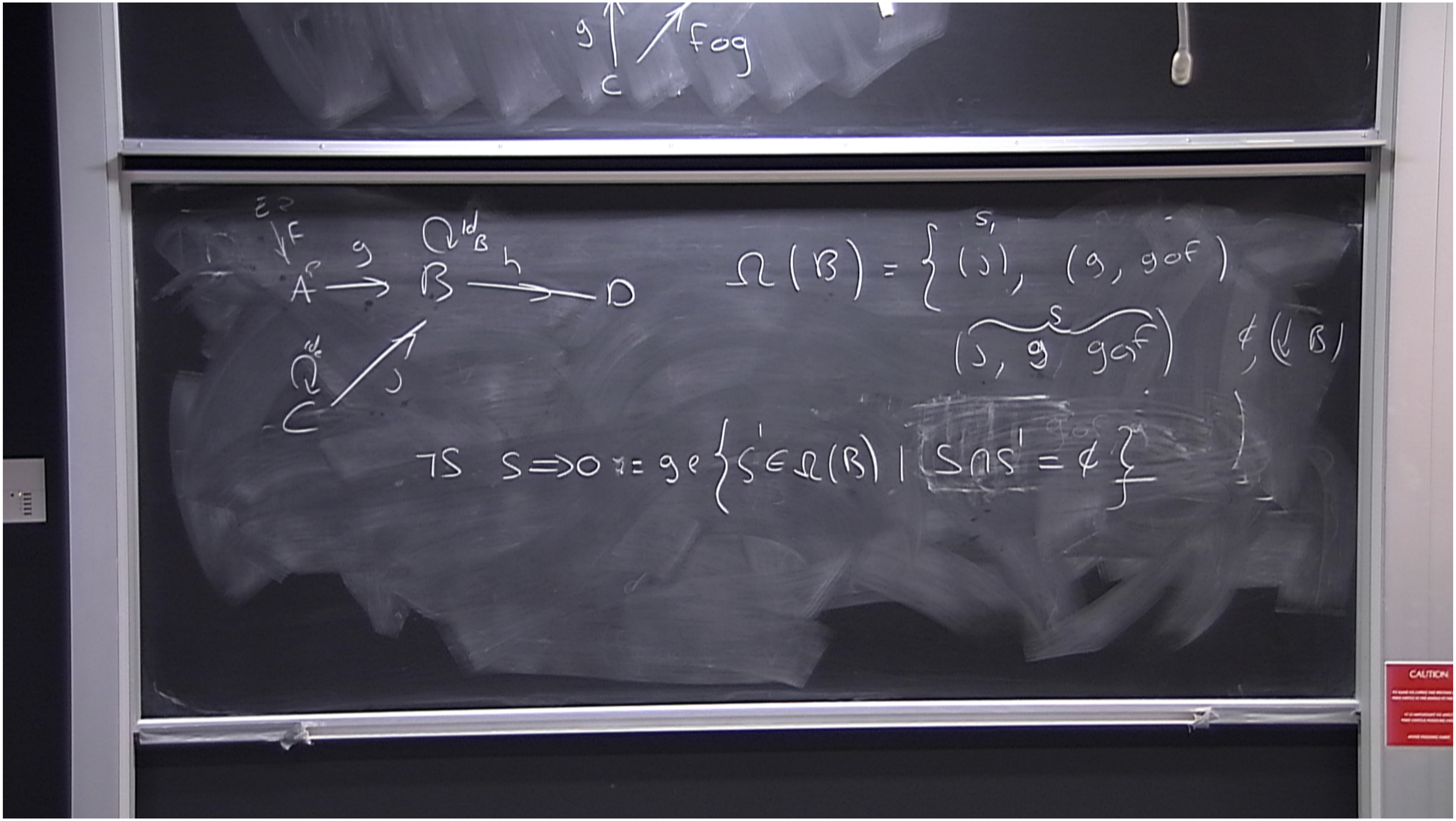
given $a, b \in L$ The rpc of a w.r.t. b is an element $c \in L$

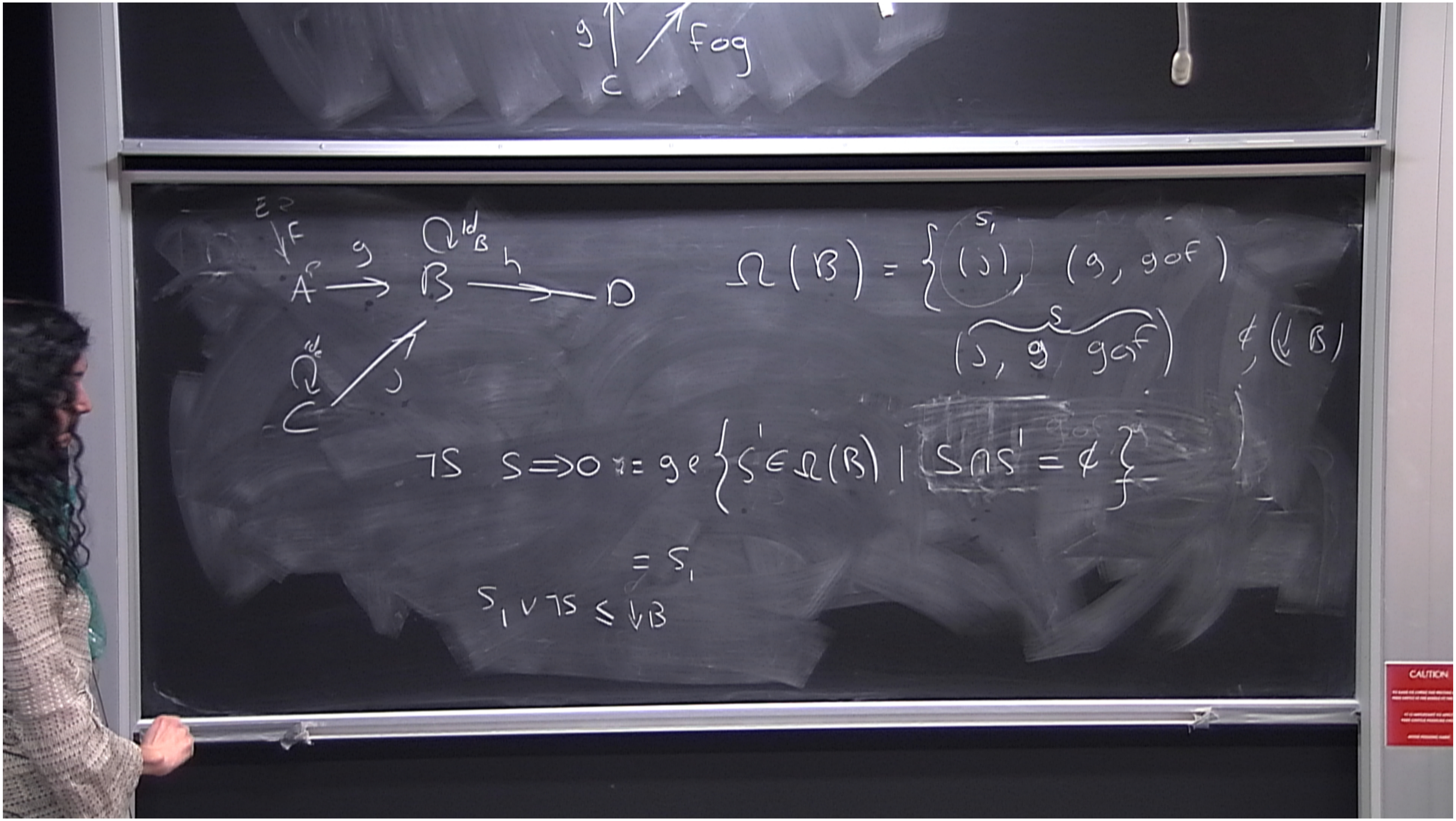
s.t. 1) $a \wedge c \leq b$ 2) $\forall x \in L$ $x \leq c$ iff $a \wedge x \leq b$

c is the greates element of $\{x \in L \mid x \wedge a \leq b\}$

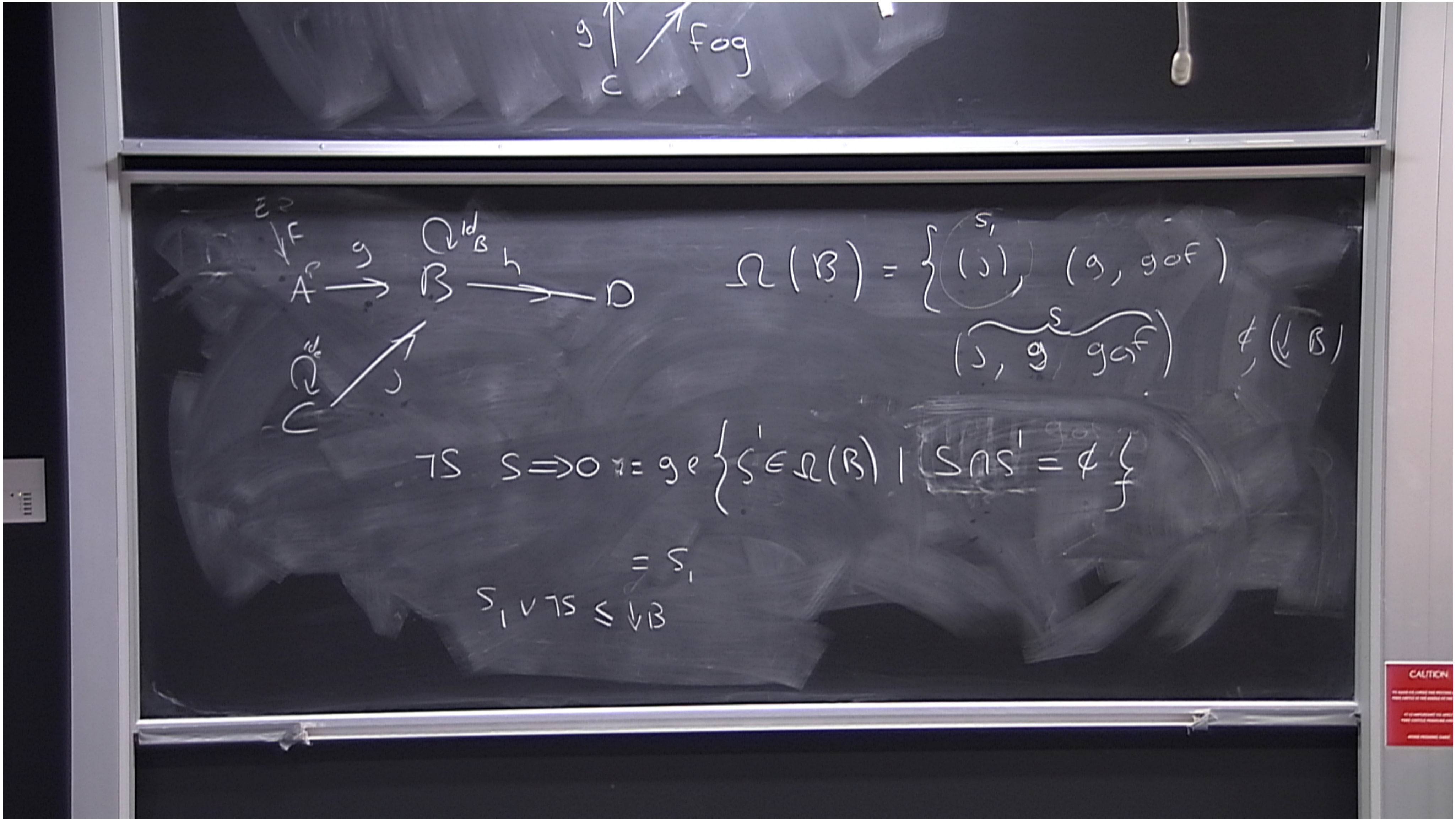
$c : (a \Rightarrow b)$

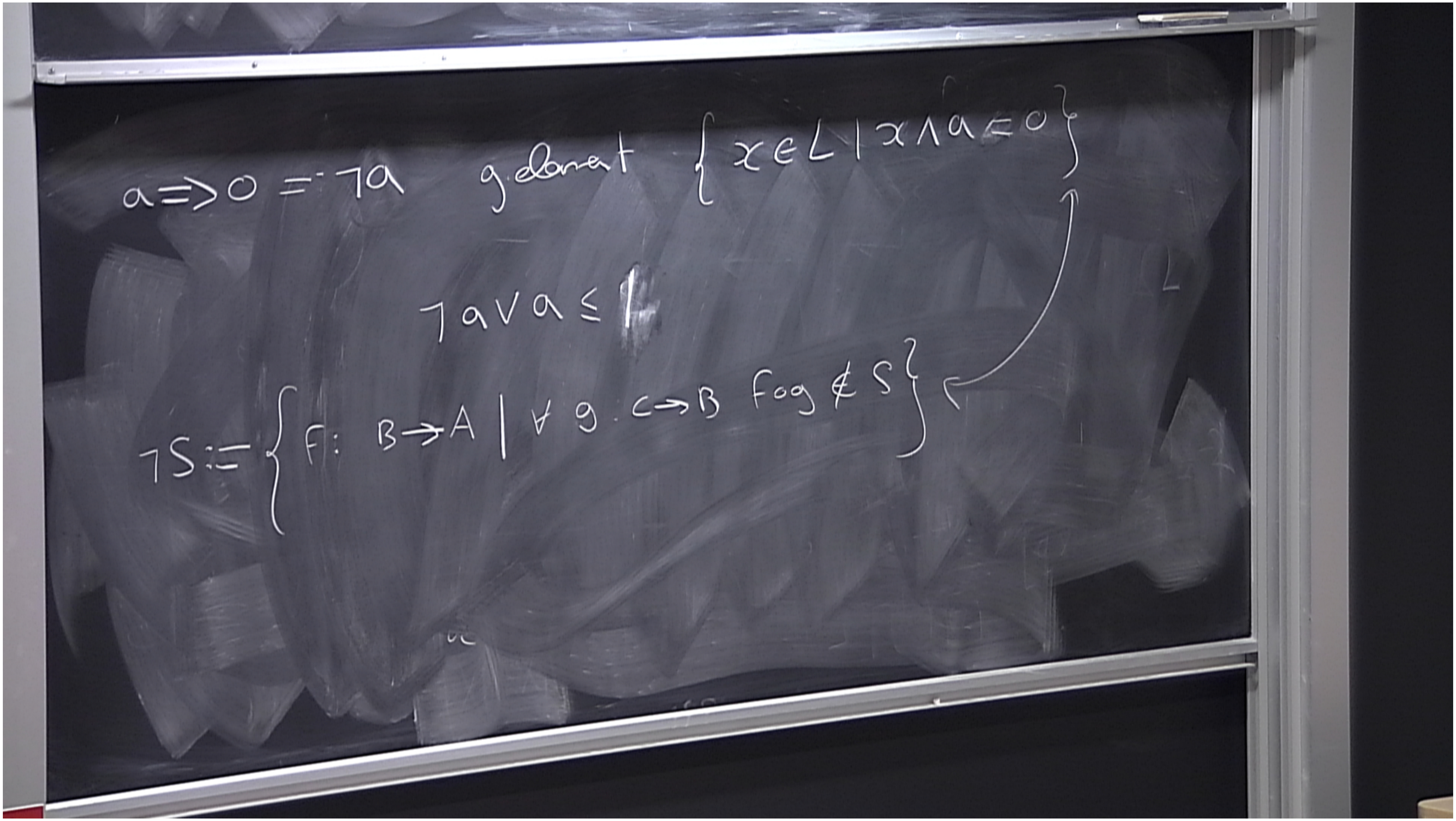

a⇒0 = ¬a
{x∈L | x∧a ≤ 0}

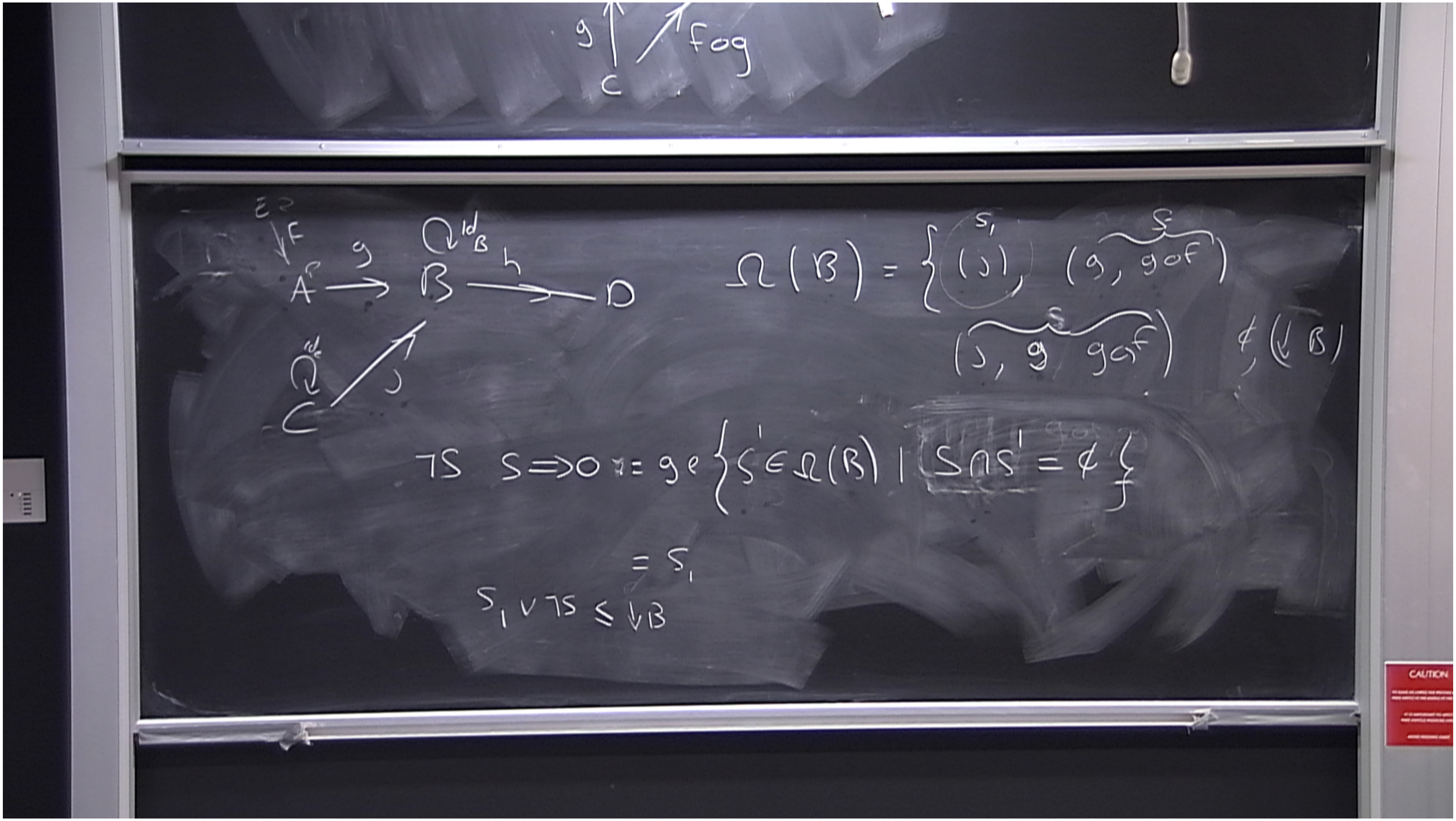

fog
Ω(B) = {(j), (g, gof)
(j, g, gof)
{S' ∈ Ω(B) | S∩S' = ∅}
CAUTION



g
fog
c
A
B
D
C
h
Ω(B) = {(j), (g, gof)
(j, g, gof)
S₁ ∨ ¬S ≤ ↓B
= S₁
CAUTION


fog
Ω(B) = {(J), (g, gof)
(J, g, gof) ∉ (↓B)
¬S S ⇒ 0 := ge {S' ∈ Ω(B) | S ∩ S' = ∅ }
= S₁
S₁ ∨ ¬S ≤ ↓B
CAUTION


a ⇒ 0 = ¬a   g. element {x ∈ L | x ∧ a ≤ 0}
¬a ∨ a ≤ 1
¬S := {f: B → A | ∀ g: C → B f∘g ∉ S}


fog
Ω(B) = {(1), (g, gof)
(1, g, gof)}
¬S S ⇒ 0 := {S' ∈ Ω(B) | S ∩ S' = ∅}
= S1
S1 ∨ ¬S ≤ ↓B