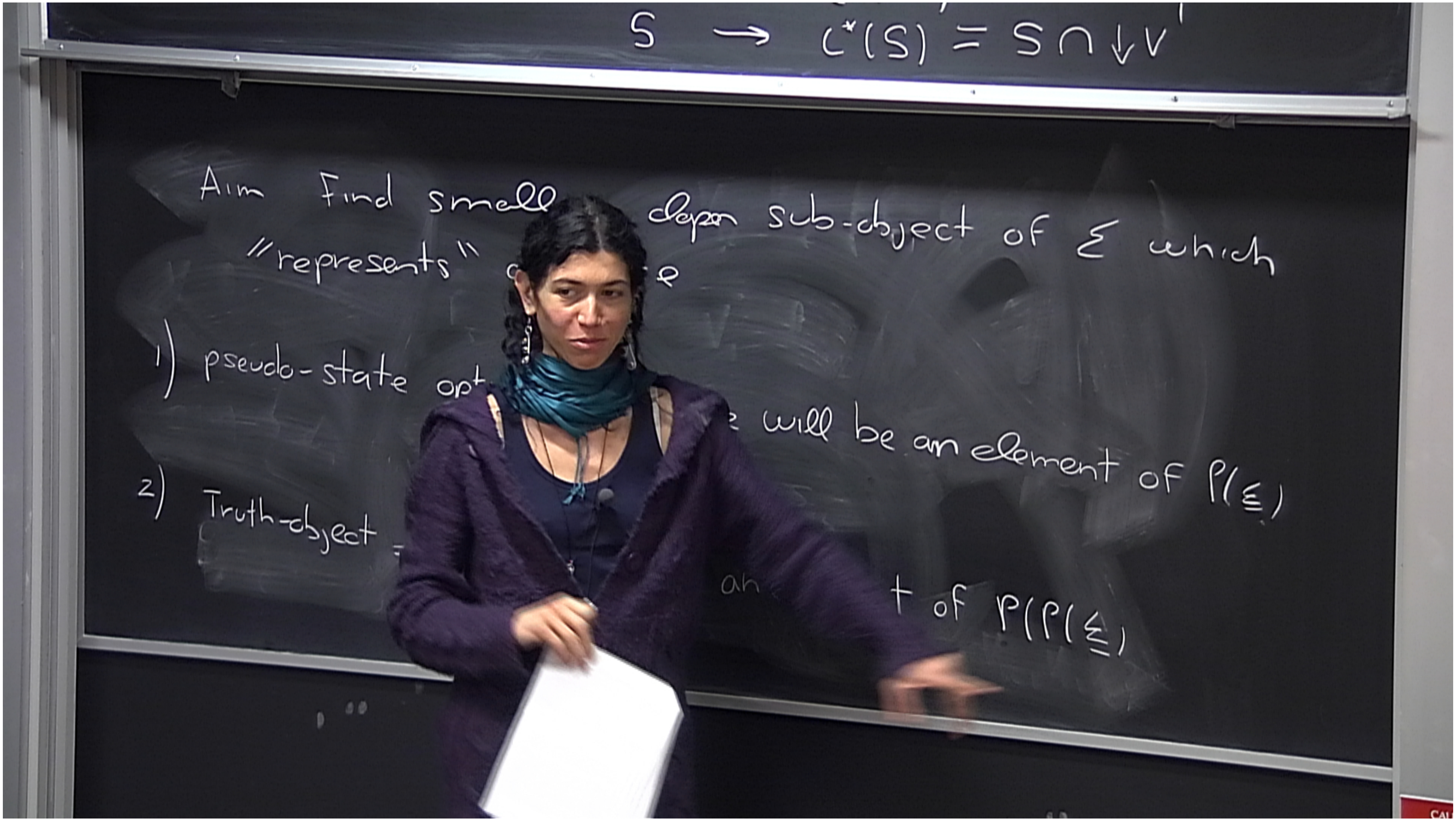
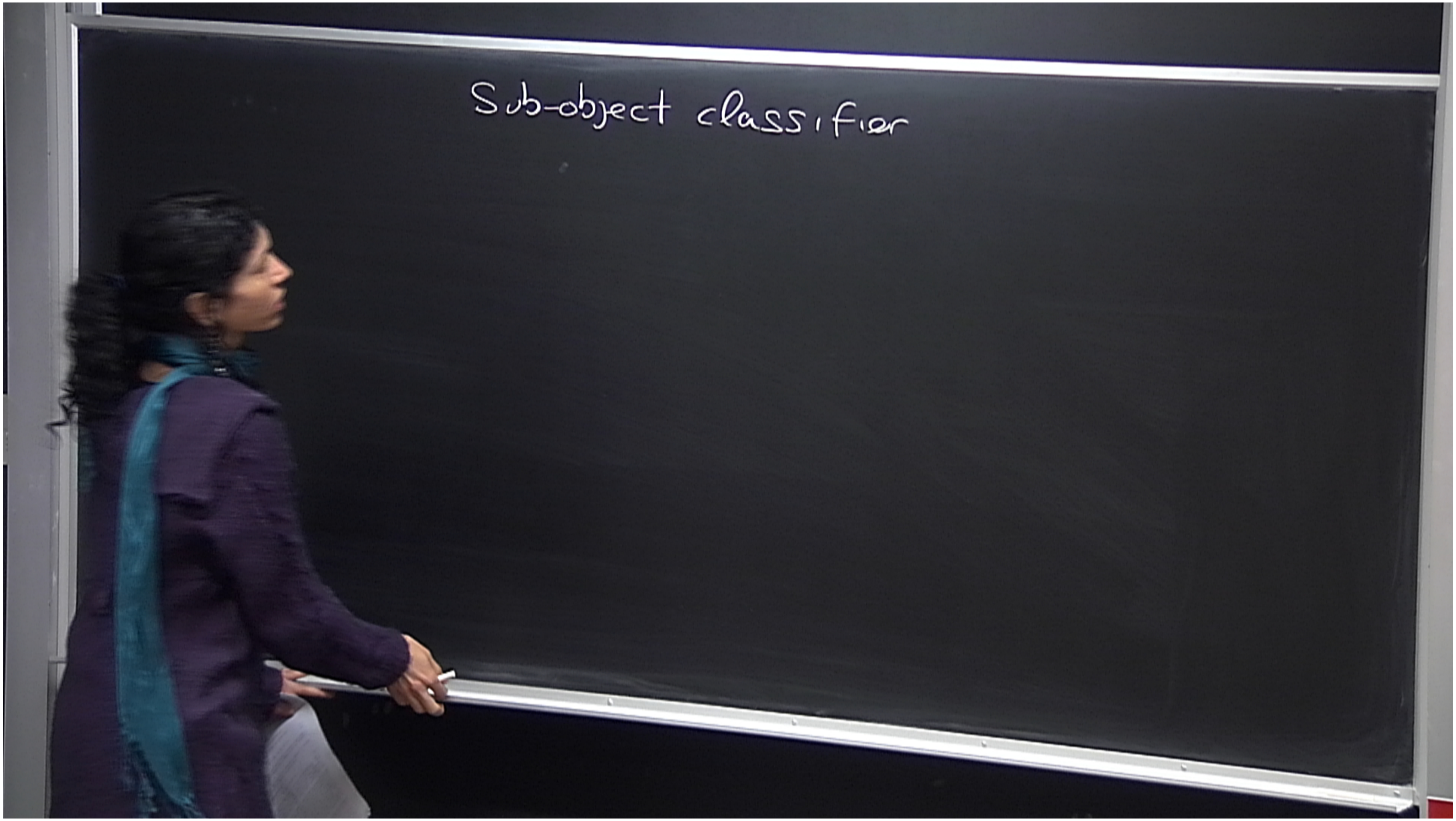
Title: Topos Quantum Physics - Lecture 10

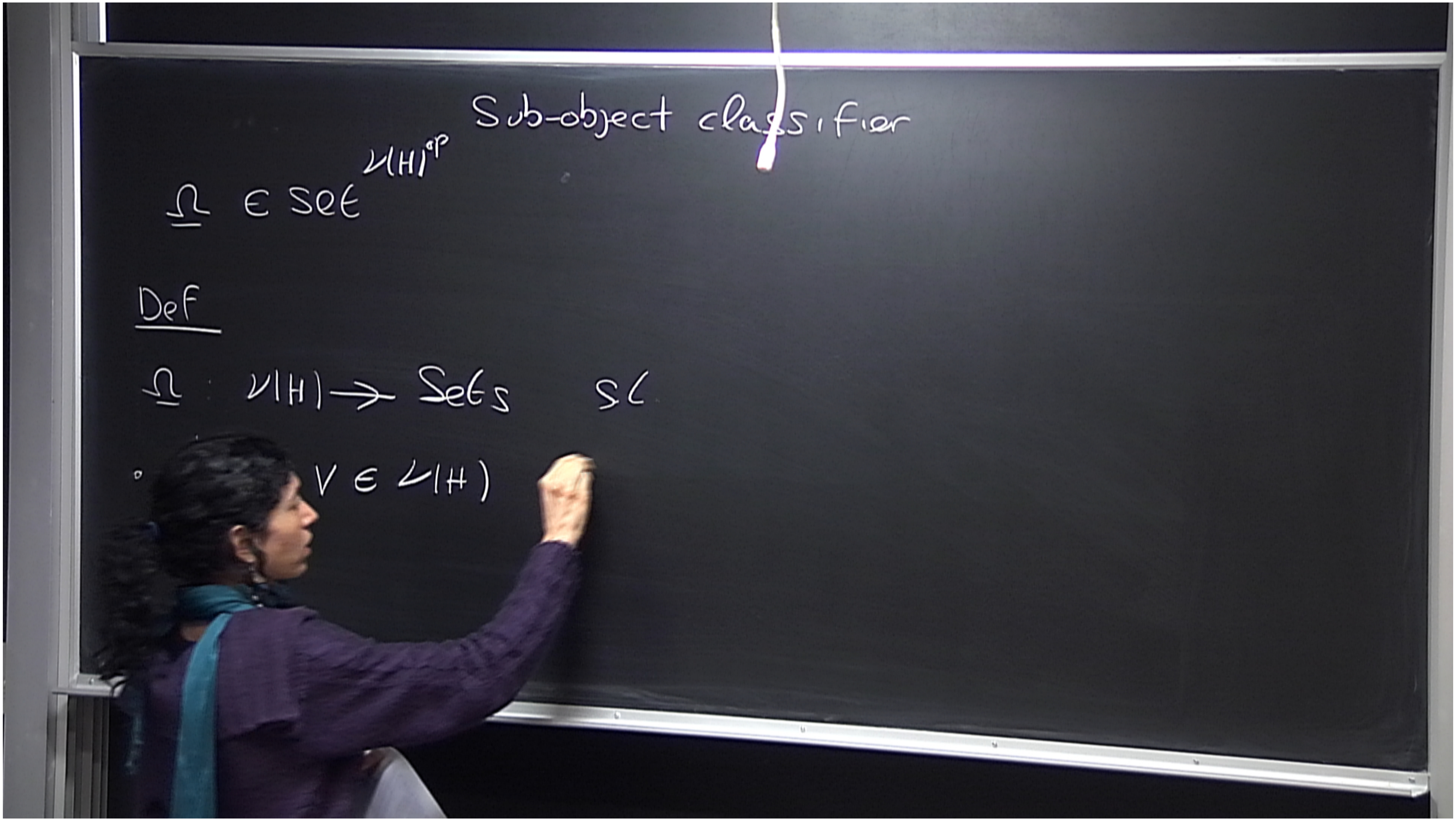
Date: Jan 30, 2012 01:30 PM

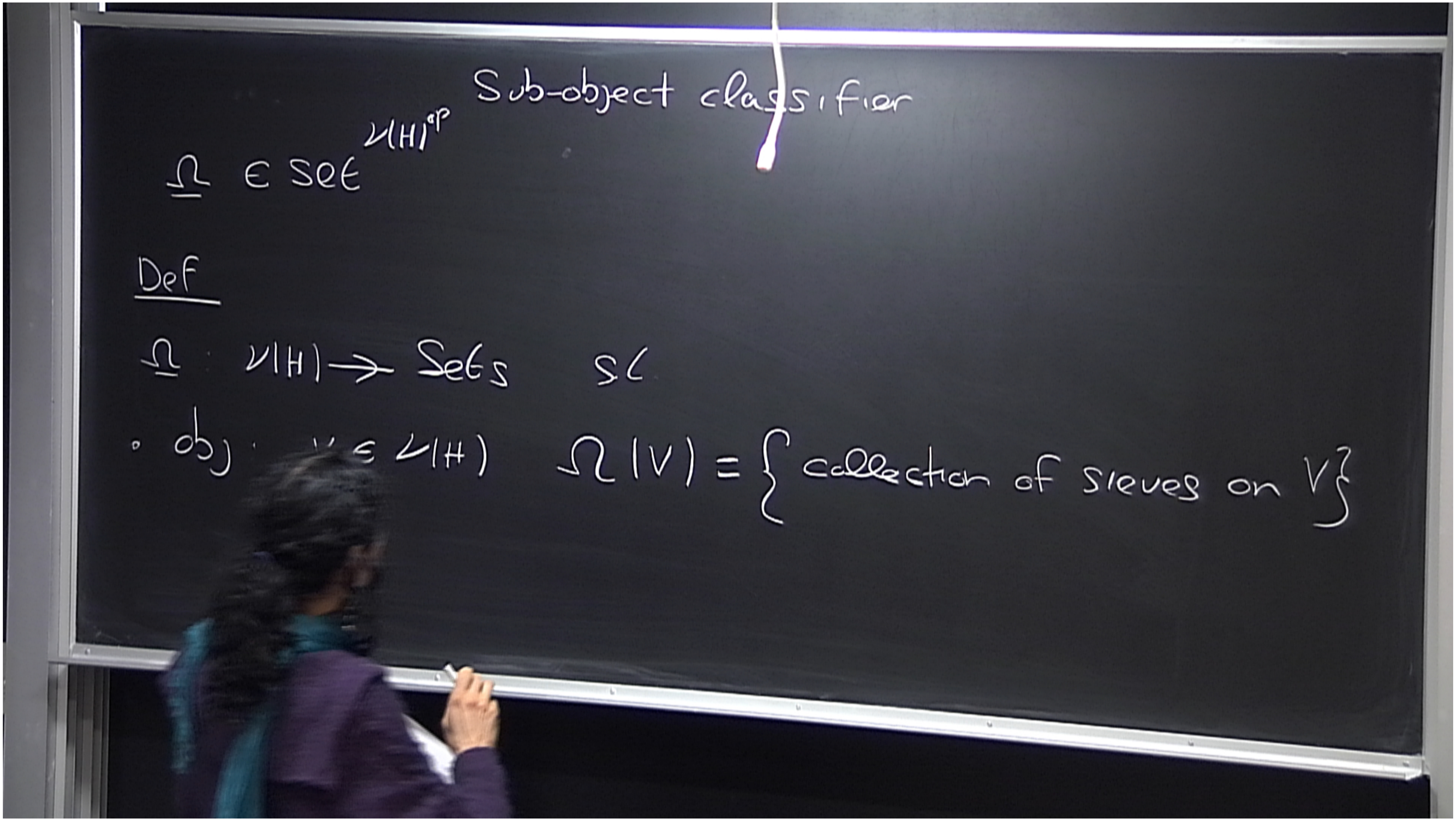
URL: http://pirsa.org/12010144

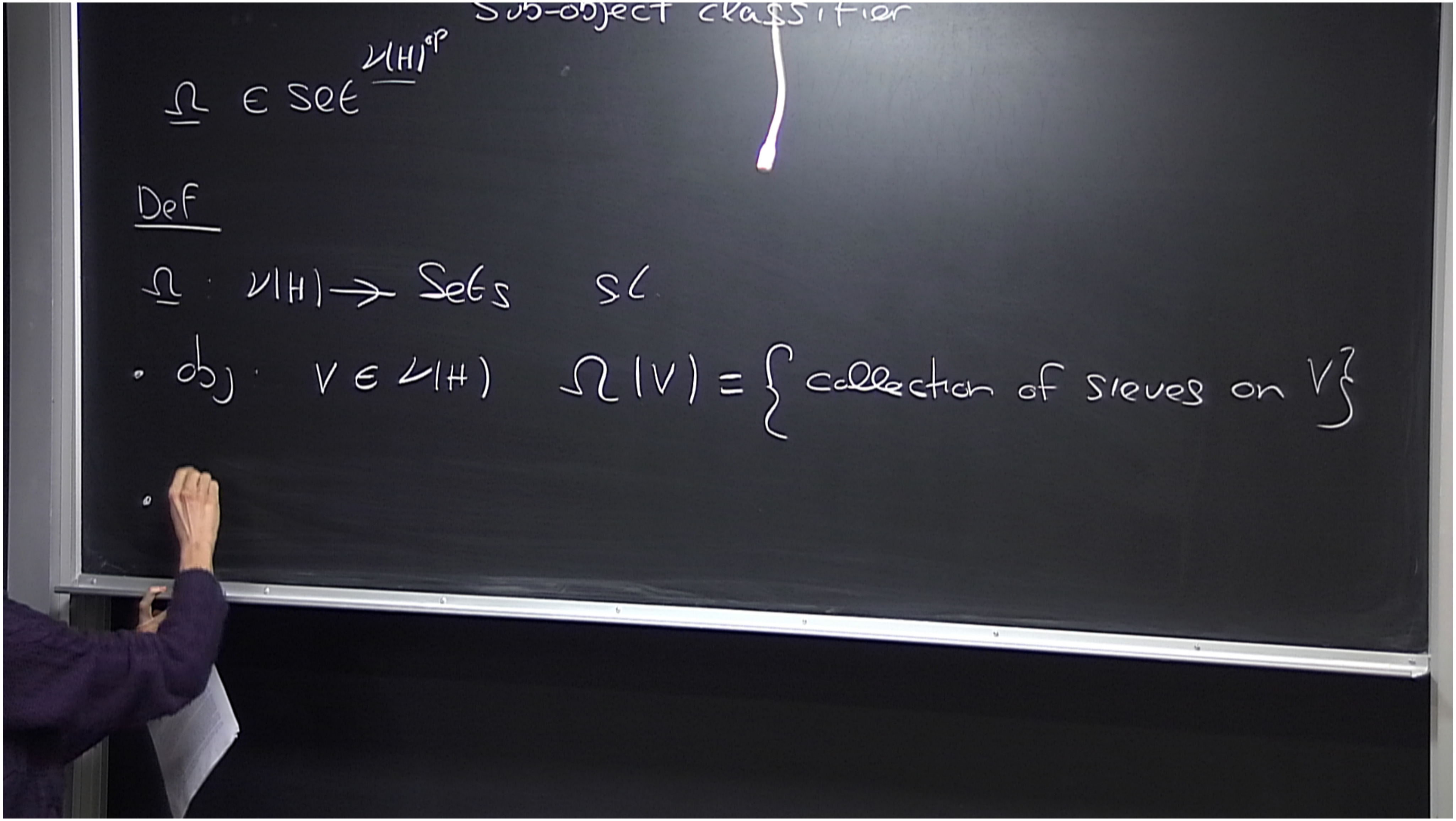
Abstract:


S → c*(S) = S ∩ ↓V
Aim Find small clopen sub-object of Σ which
"represents"
1) pseudo-state
will be an element of P(Σ)
2) Truth-object
of P(P(Σ)


Sub-object classifier


Sub-object classifier
Ω ∈ Set^V(H)^op
Def
Ω : V(H) → Sets s.t.
V ∈ V(H)


Sub-object classifier
Ω ∈ Set^{V(H)^op}
Def
Ω : V(H) → Sets s.t.
· obj: V ∈ V(H)   Ω(V) = {collection of sieves on V}


Sub-object classifier
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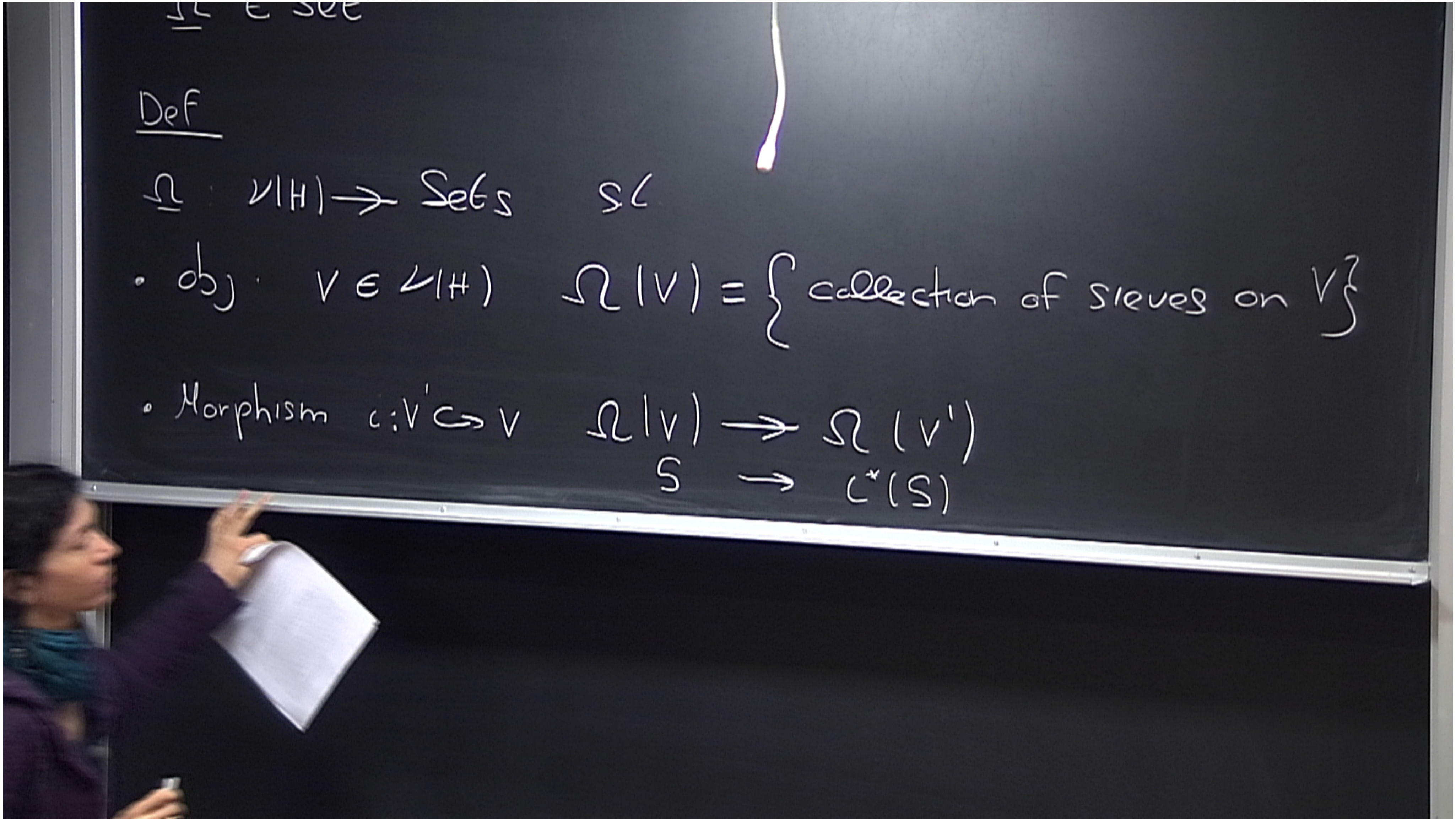
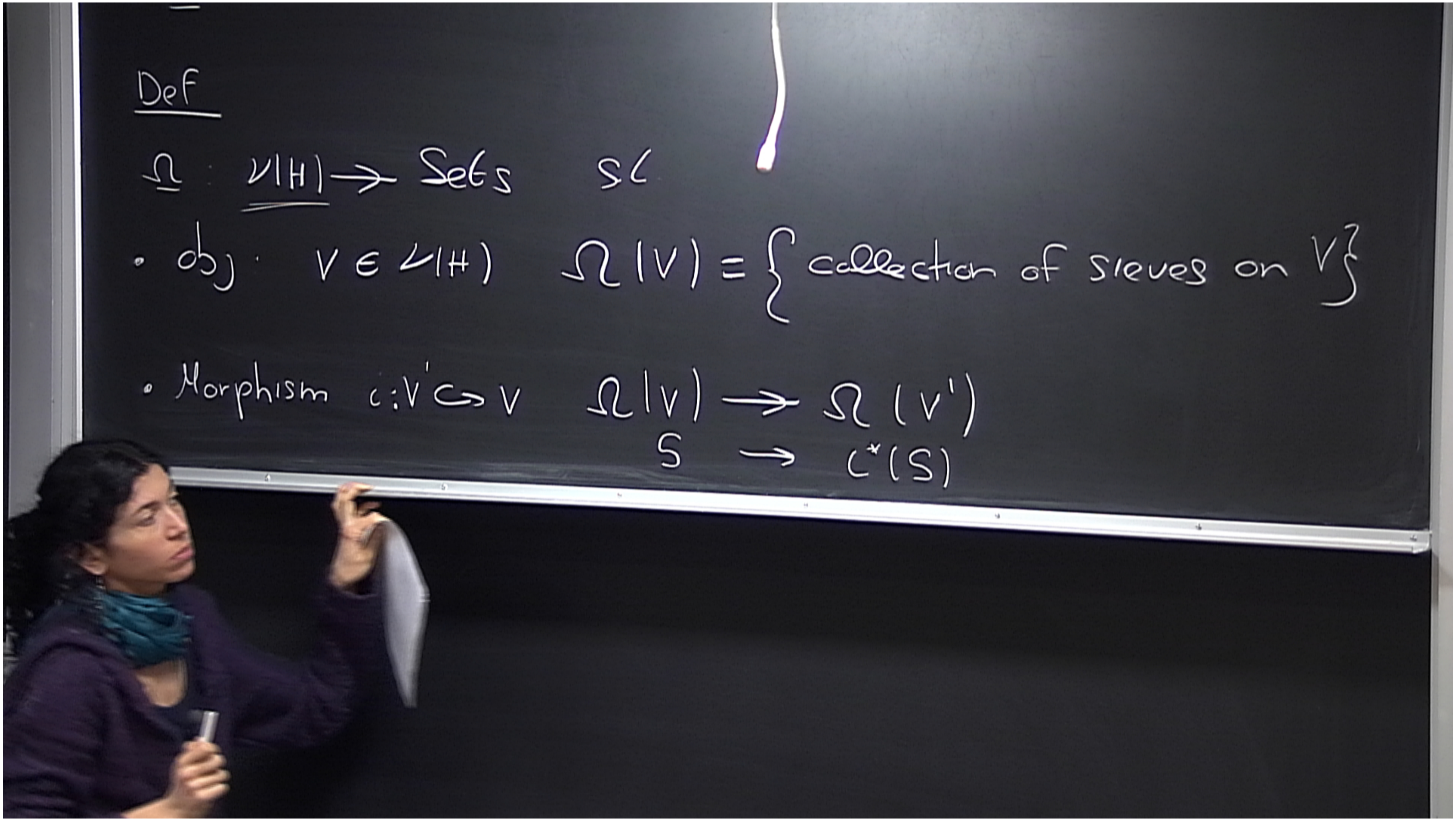

Def
Ω : V(H) → Sets s.t.
• obj: V ∈ V(H) Ω(V) = {collection of sieves on V}
• Morphism i: V' ↪ V Ω(V) → Ω(V')
S → i*(S)

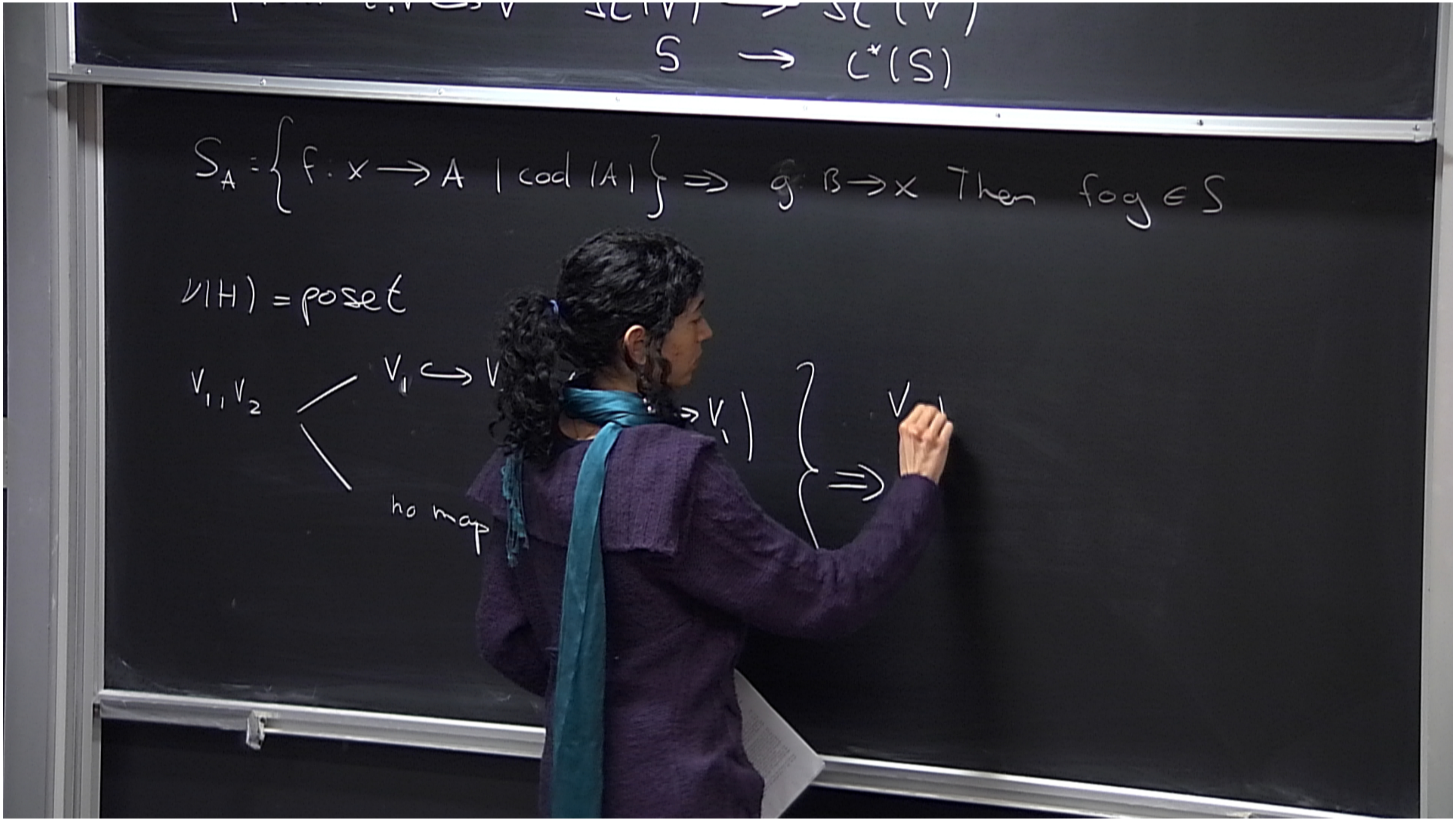

Def
Ω : V(H) → Sets s.t.
• obj: V ∈ V(H) Ω(V) = { collection of sieves on V }
• Morphism i: V' ↪ V Ω(V) → Ω(V')
S → i*(S)

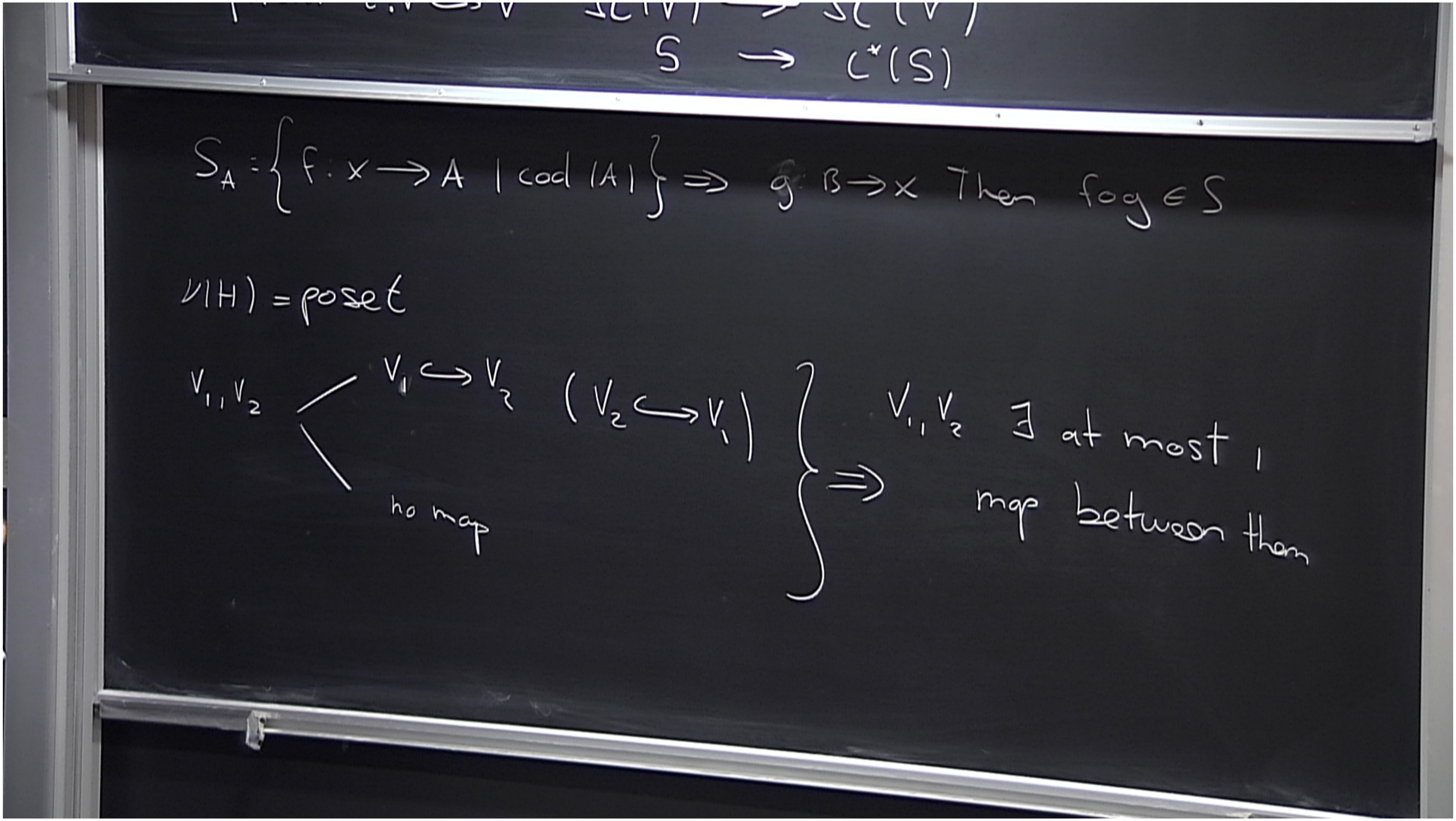
- obj: $V \in \mathcal{V}(H)$ $\quad \Omega(V) = \{\text{collection of sieves on } V\}$

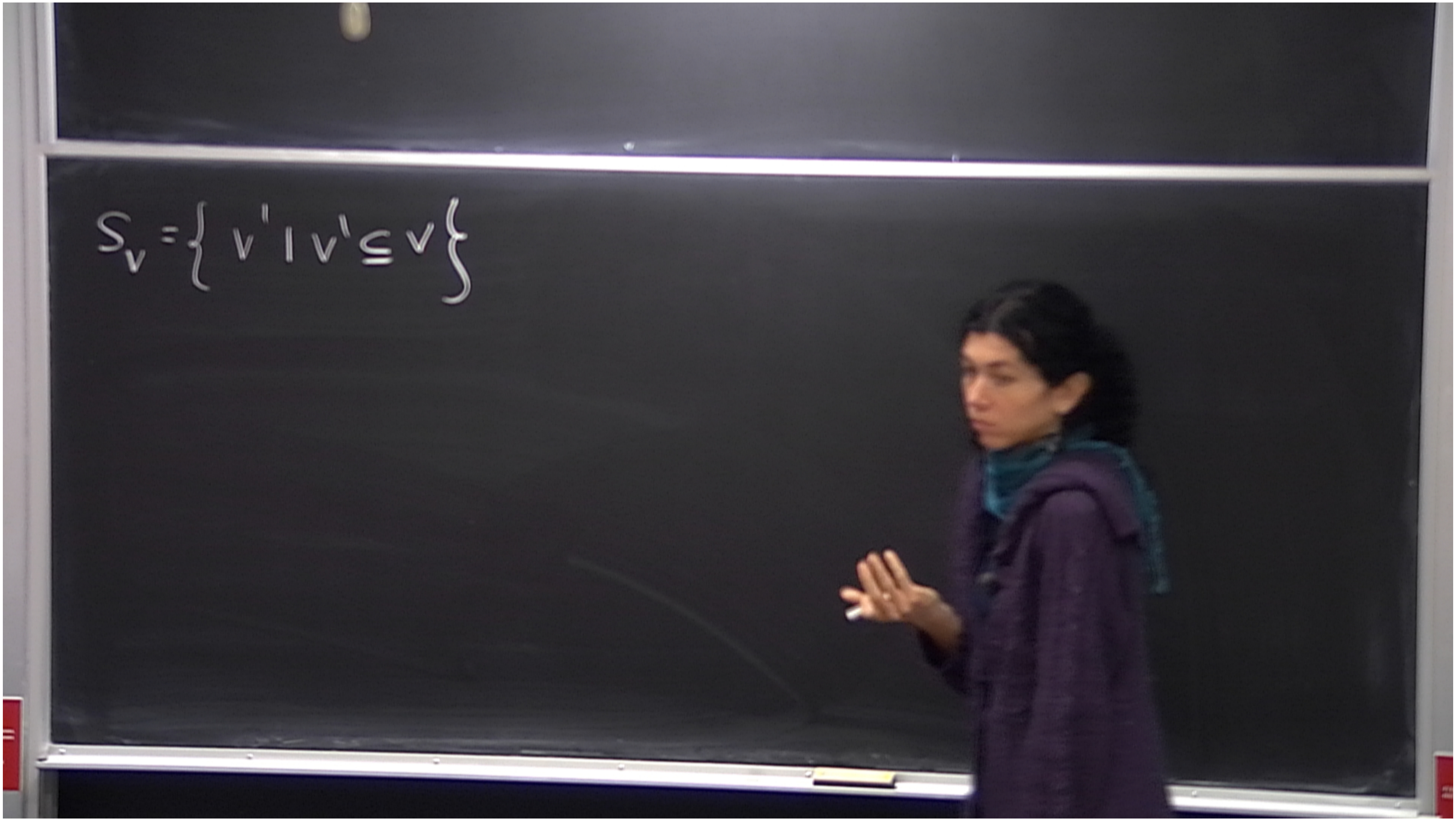
- Morphism $i: V' \hookrightarrow V$ $\quad \Omega(V) \rightarrow \Omega(V')$

$$S \rightarrow i^*(S)$$

$S_A := \{ f: X \rightarrow A \mid \text{cod}(A) \} \Rightarrow g: B \rightarrow X$ Then $f \circ g \in S$


S → C*(S)
S_A = { f : X → A | cod (A) } ⇒ g : B → X Then f∘g ∈ S
V(H) = poset
V_1, V_2
V_1 ↪ V
no map


S → C*(S)
S_A := { f : X → A | cod(A) } ⇒ g : B → X Then f∘g ∈ S
V(H) = poset
V_1, V_2
V_1 ↪ V_2 (V_2 ↪ V_1)
no map
⇒ V_1, V_2 ∃ at most 1 map between them

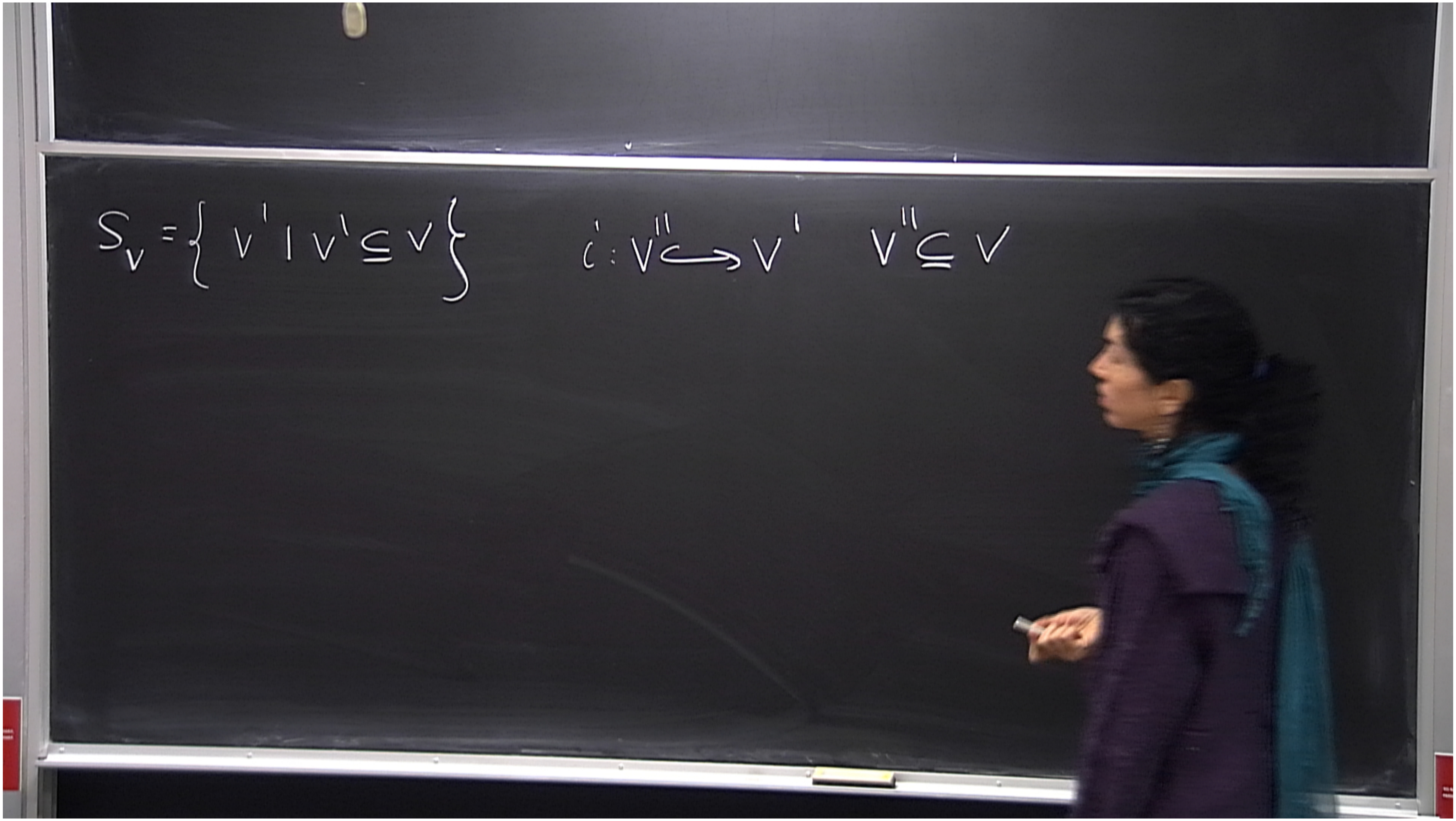

$S_v = \{ v' \mid v' \subseteq v \}$

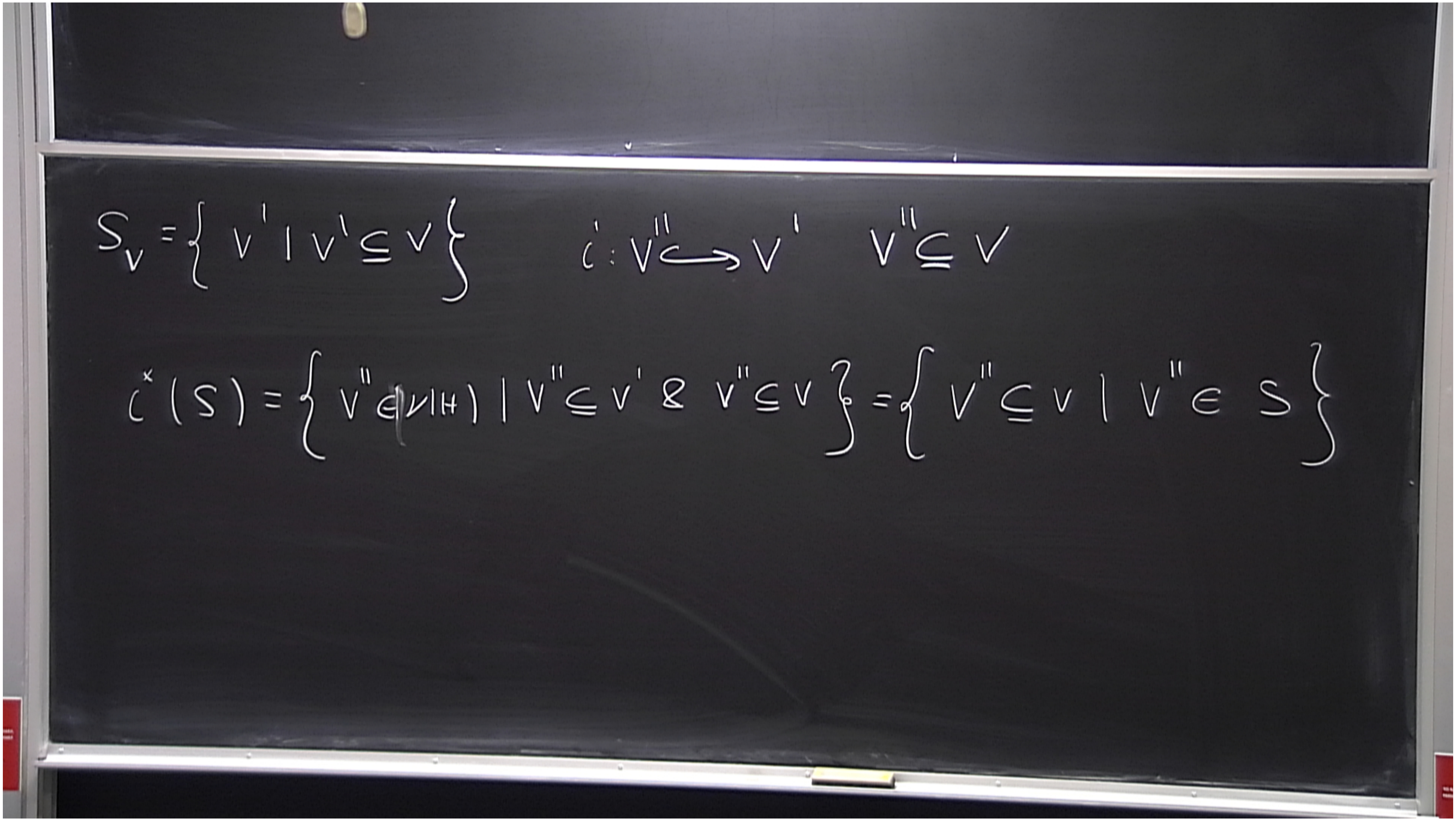
$S \to C^*(S)$

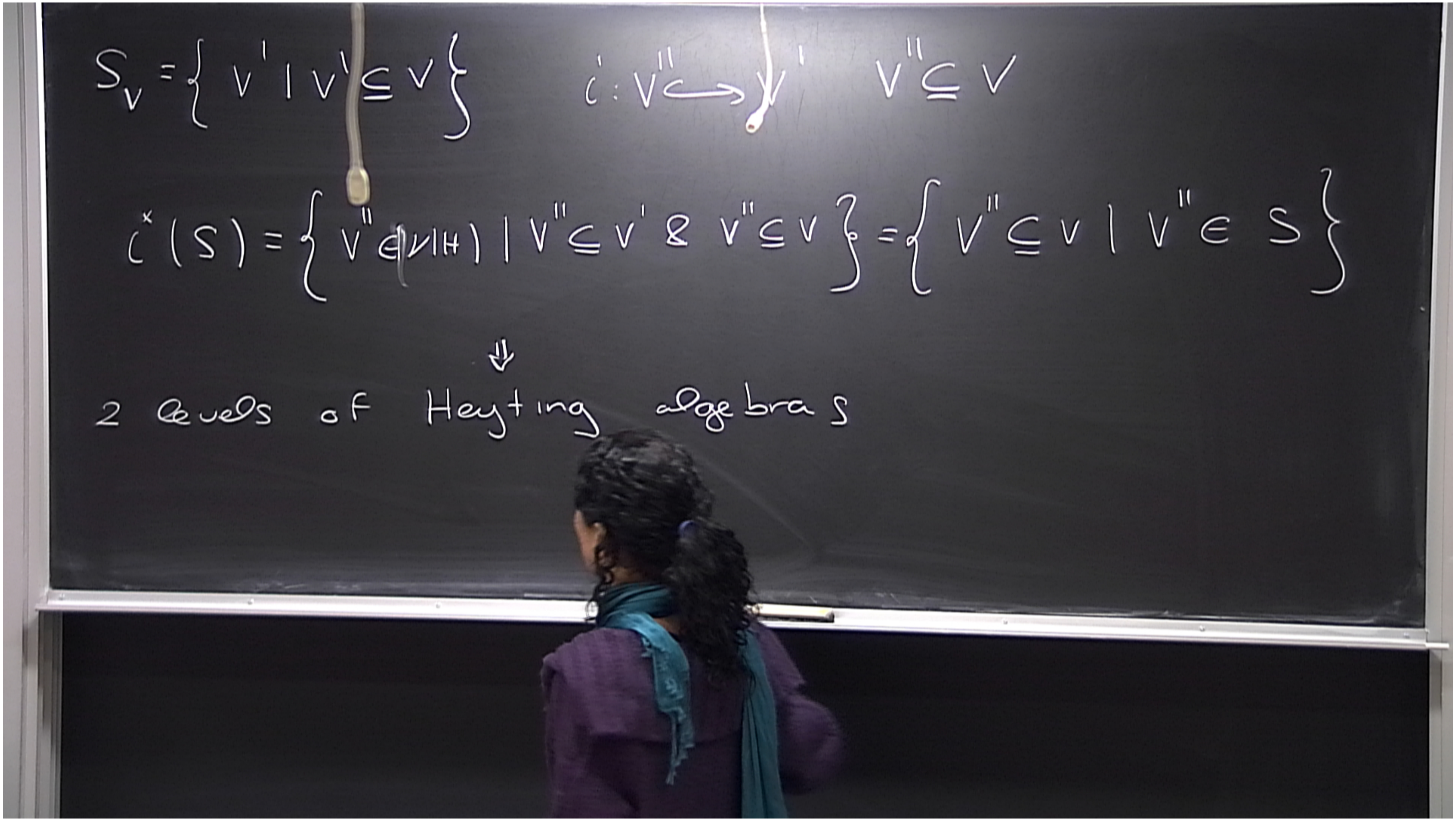
$S_A = \{ f: X \to A \mid cod(A) \} \Rightarrow g: B \to X$ Then $f \circ g \in S$

$V(H) =$ poset

V_1, V_2 — $V_1 \hookrightarrow V_2$ $(V_2 \hookrightarrow V_1)$ / no map $\Rightarrow$ V_1, V_2 $\exists$ at most 1 map between them


$S_V = \{ V' \mid V' \subseteq V \}$
$i : V'' \hookrightarrow V'$
$V'' \subseteq V$


$S_V = \{ V' \mid V' \subseteq V \}$
$i : V'' \hookrightarrow V'$
$V'' \subseteq V$
$i^*(S) = \{ V'' \in \mathcal{V}(\mathcal{H}) \mid V'' \subseteq V' \ \& \ V'' \subseteq V \} = \{ V'' \subseteq V \mid V'' \in S \}$


S_V = { V' | V' ⊆ V }
i : V'' ↪ V'   V'' ⊆ V
i*(S) = { V'' ∈ V(H) | V'' ⊆ V' & V'' ⊆ V } = { V'' ⊆ V | V'' ∈ S }
⇓
2 levels of Heyting algebras

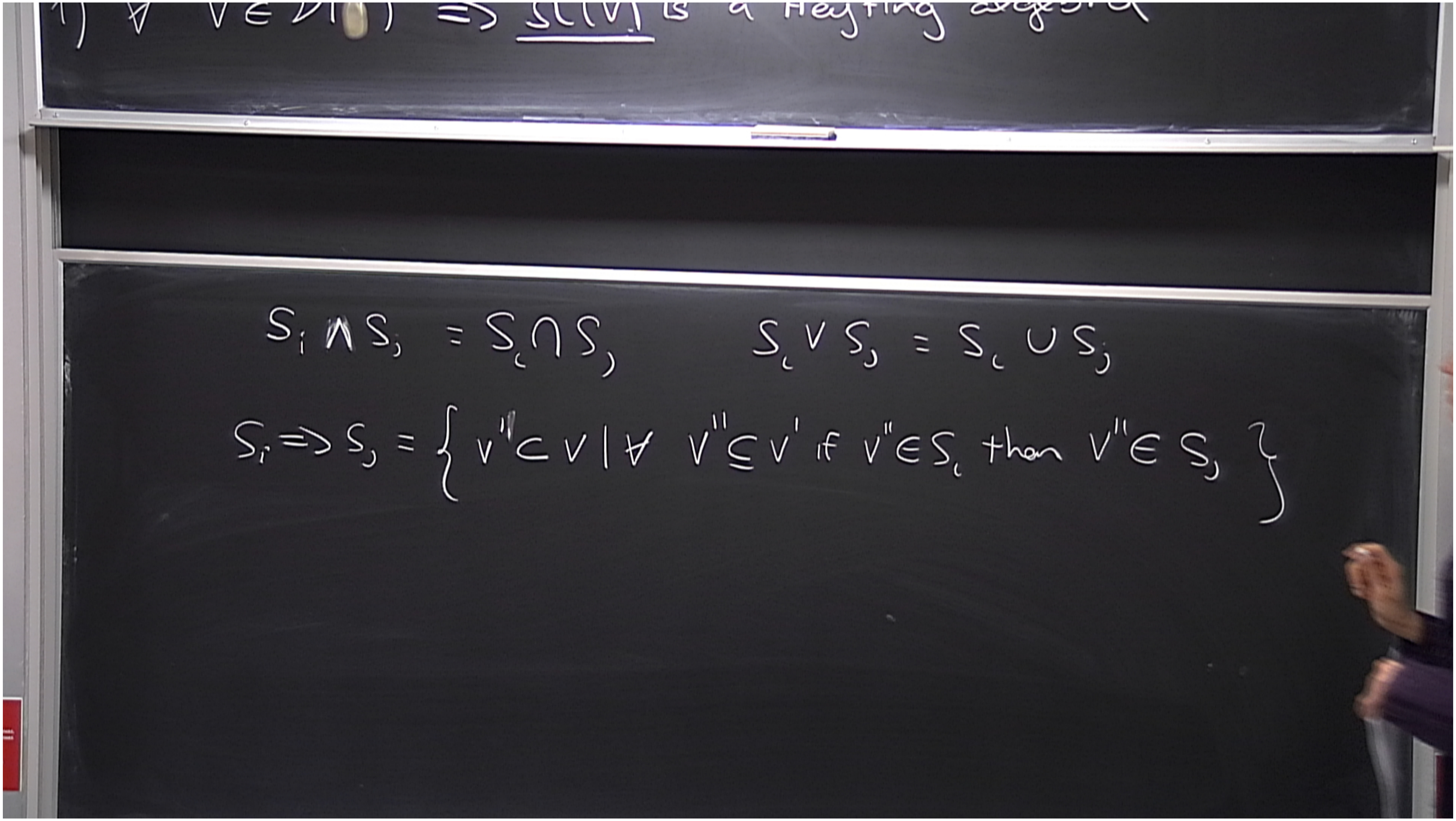
$$S_V = \{ V' \mid V' \subseteq V \} \qquad i : V'' \hookrightarrow V' \qquad V'' \subseteq V$$

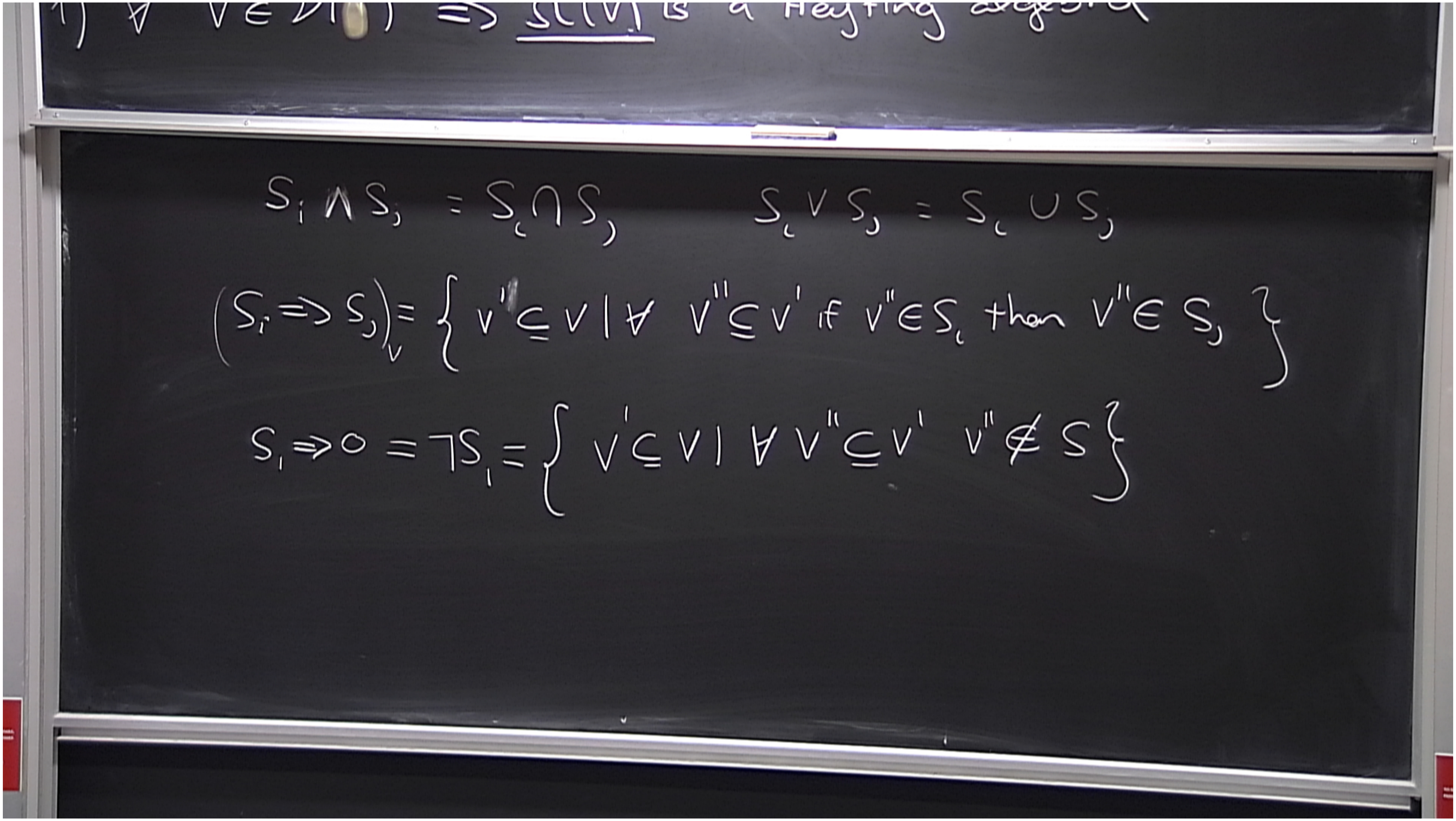
$$i^*(S) = \{ V'' \in \mathcal{V}(\mathcal{H}) \mid V'' \subseteq V' \ \& \ V'' \subseteq V \} = \{ V'' \subseteq V \mid V'' \in S \}$$

$$\Downarrow$$

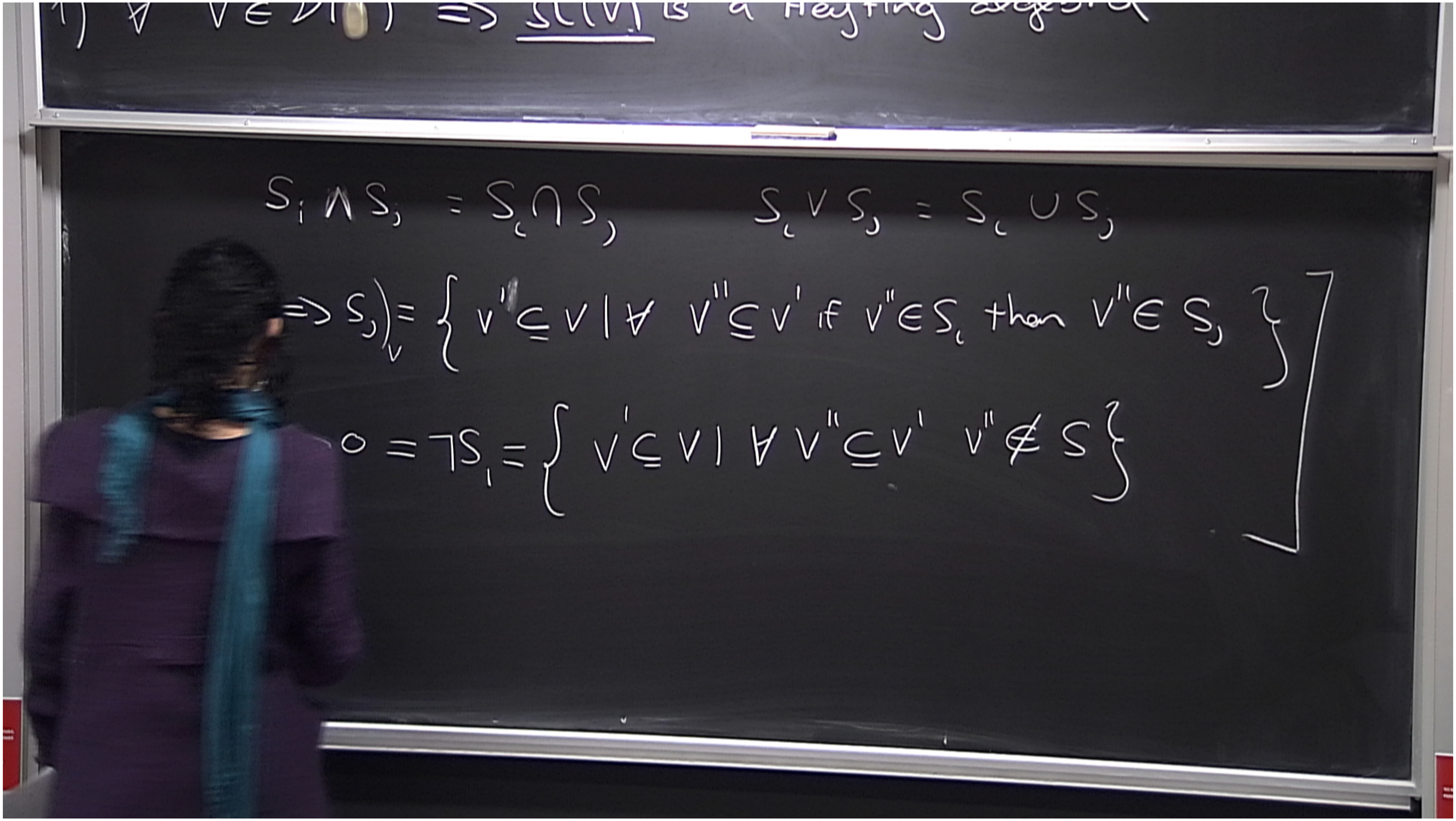
2 levels of Heyting algebras

1) $\forall \; V \in \mathcal{V}(\mathcal{H}) \Rightarrow \underline{\Omega(V)}$ is a Heyting algebra

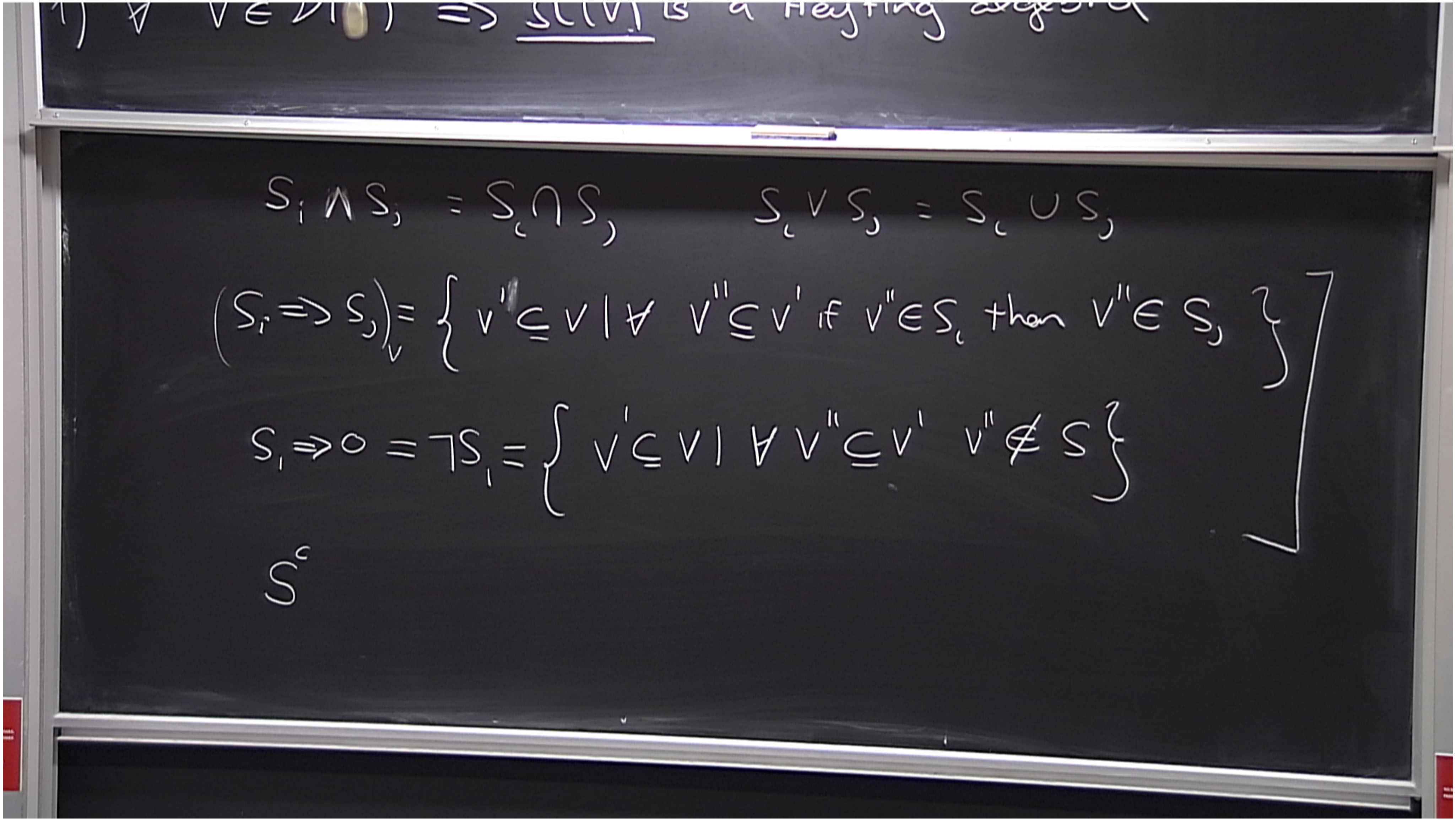

$\Rightarrow$ is a Heyting algebra
$S_i \wedge S_j = S_i \cap S_j$
$S_i \vee S_j = S_i \cup S_j$
$S_i \Rightarrow S_j = \{ V' \subset V \mid \forall V'' \subseteq V' \text{ if } V'' \in S_i \text{ then } V'' \in S_j \}$

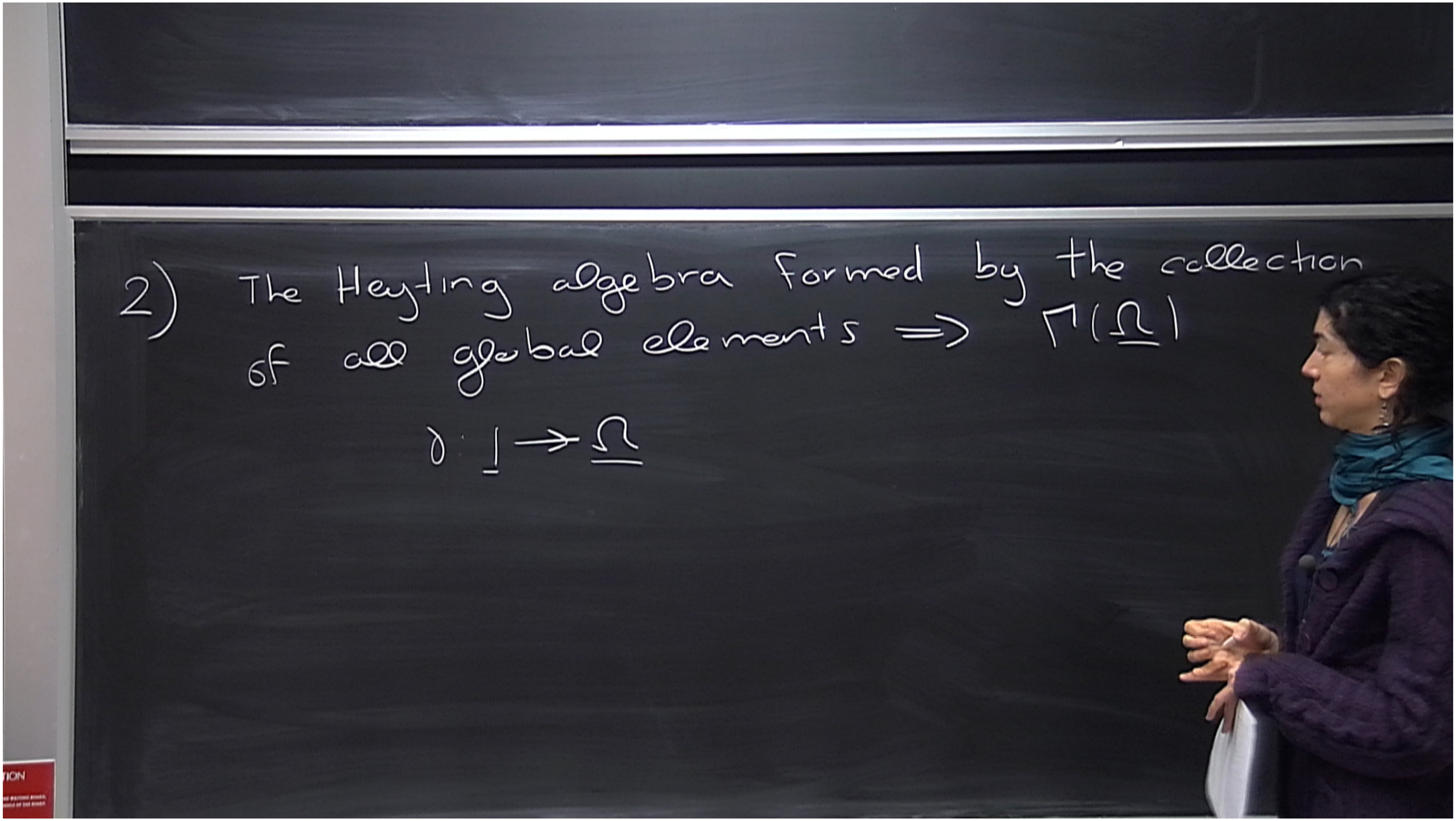


$S_i \wedge S_j = S_i \cap S_j$
$S_i \vee S_j = S_i \cup S_j$
$(S_i \Rightarrow S_j)_V = \{ V' \subseteq V \mid \forall V'' \subseteq V' \text{ if } V'' \in S_i \text{ then } V'' \in S_j \}$
$S_1 \Rightarrow 0 = \neg S_1 = \{ V' \subseteq V \mid \forall V'' \subseteq V' \; V'' \notin S \}$



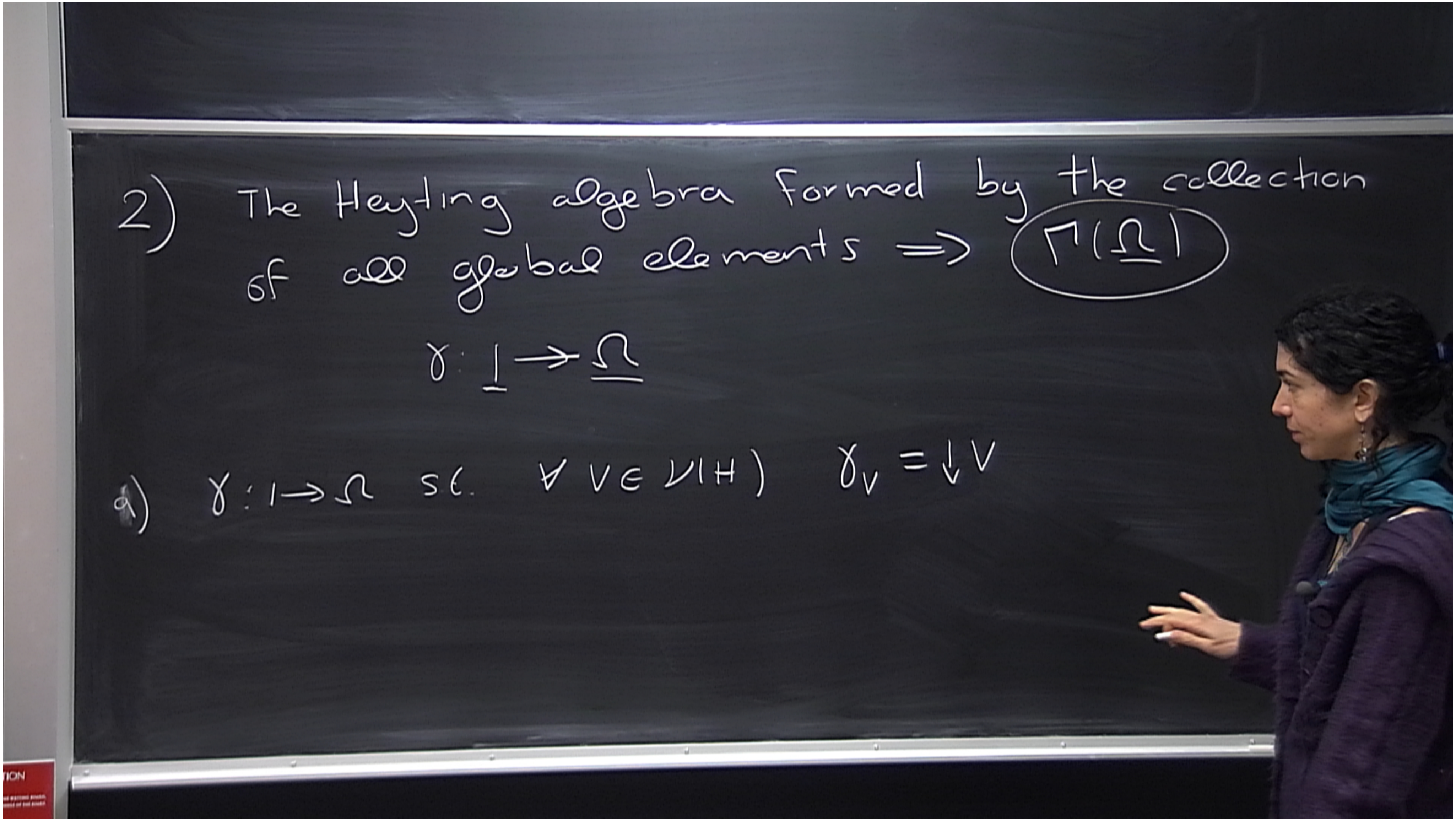
S_i ∧ S_j = S_i ∩ S_j
S_i ∨ S_j = S_i ∪ S_j
⇒ S_j)_V = { V' ⊆ V | ∀ V'' ⊆ V' if V'' ∈ S_i than V'' ∈ S_j }
0 = ¬S_i = { V' ⊆ V | ∀ V'' ⊆ V' V'' ∉ S }

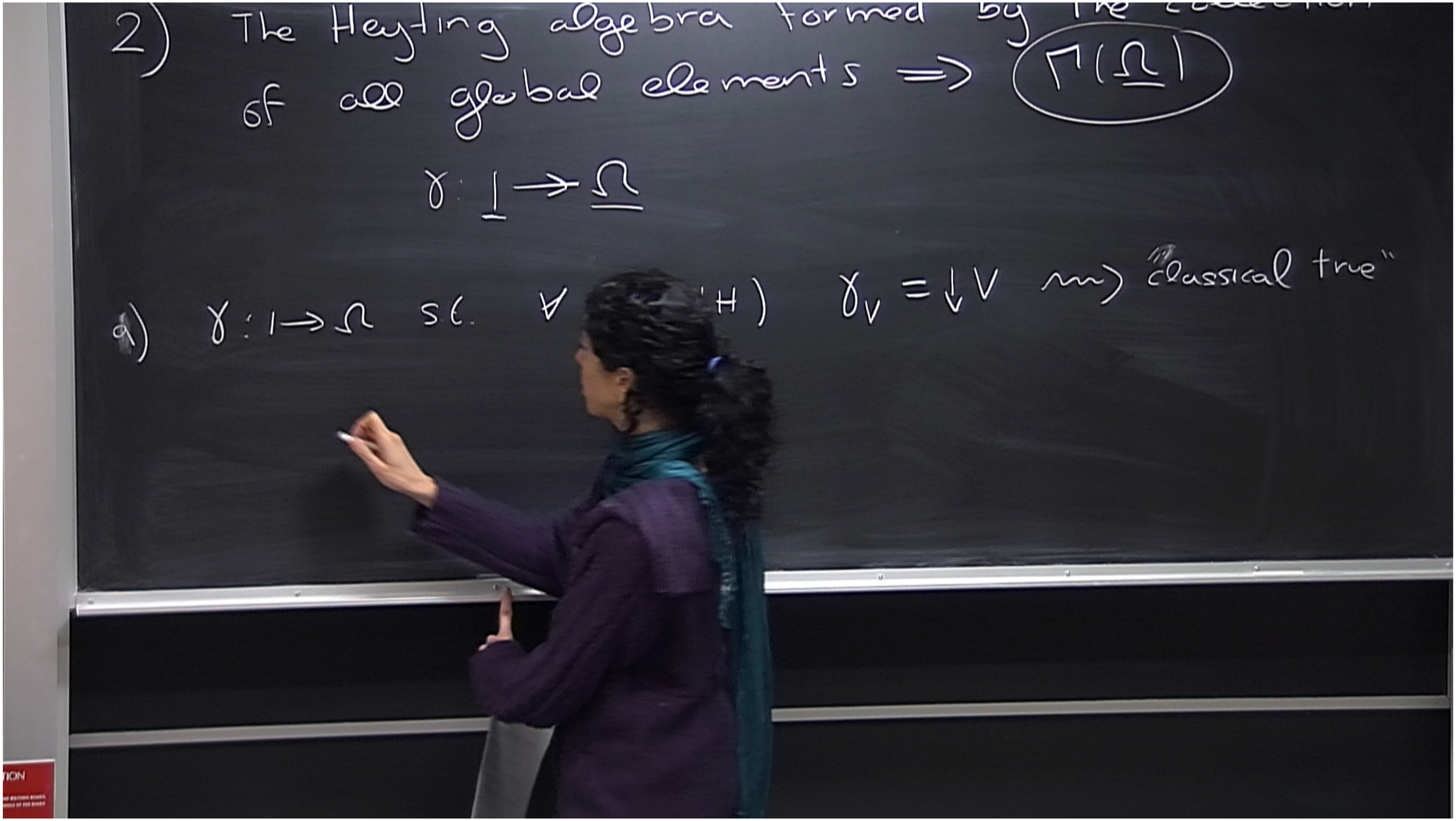

S_i ∧ S_j = S_i ∩ S_j
S_i ∨ S_j = S_i ∪ S_j
(S_i ⇒ S_j)_V = { V' ⊆ V | ∀ V'' ⊆ V' if V'' ∈ S_i than V'' ∈ S_j }
S_1 ⇒ 0 = ¬S_1 = { V' ⊆ V | ∀ V'' ⊆ V' V'' ∉ S }
S^c

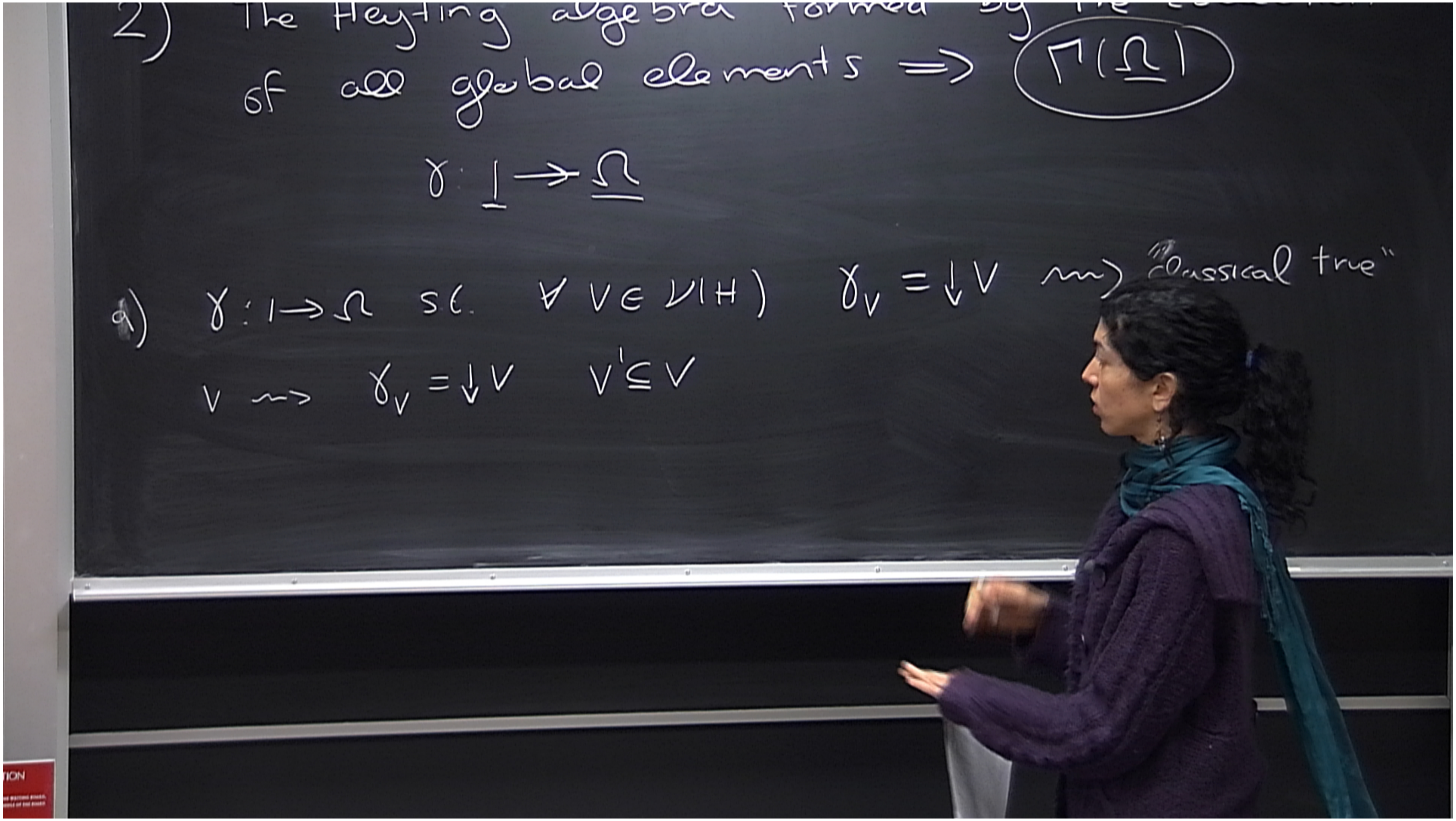

2) The Heyting algebra formed by the collection
of all global elements ⟹ Γ(Ω)
γ : 1 → Ω

2) The Heyting algebra formed by the collection of all global elements $\Longrightarrow$ $\Gamma(\underline{\Omega})$

$$\gamma : \underline{1} \rightarrow \underline{\Omega}$$


2) The Heyting algebra formed by the collection of all global elements $\Rightarrow$ $\Gamma(\underline{\Omega})$
$\gamma : \underline{1} \rightarrow \underline{\Omega}$
a) $\gamma : 1 \rightarrow \Omega$ s.t. $\forall\, V \in \mathcal{V}(\mathcal{H})$ $\gamma_V = \downarrow V$


2) The Heyting algebra formed by the
of all global elements ⟹ Γ(Ω)
γ : 1 → Ω
γ_V = ↓V ⇝ "classical true"
a) γ : 1 → Ω s.t. ∀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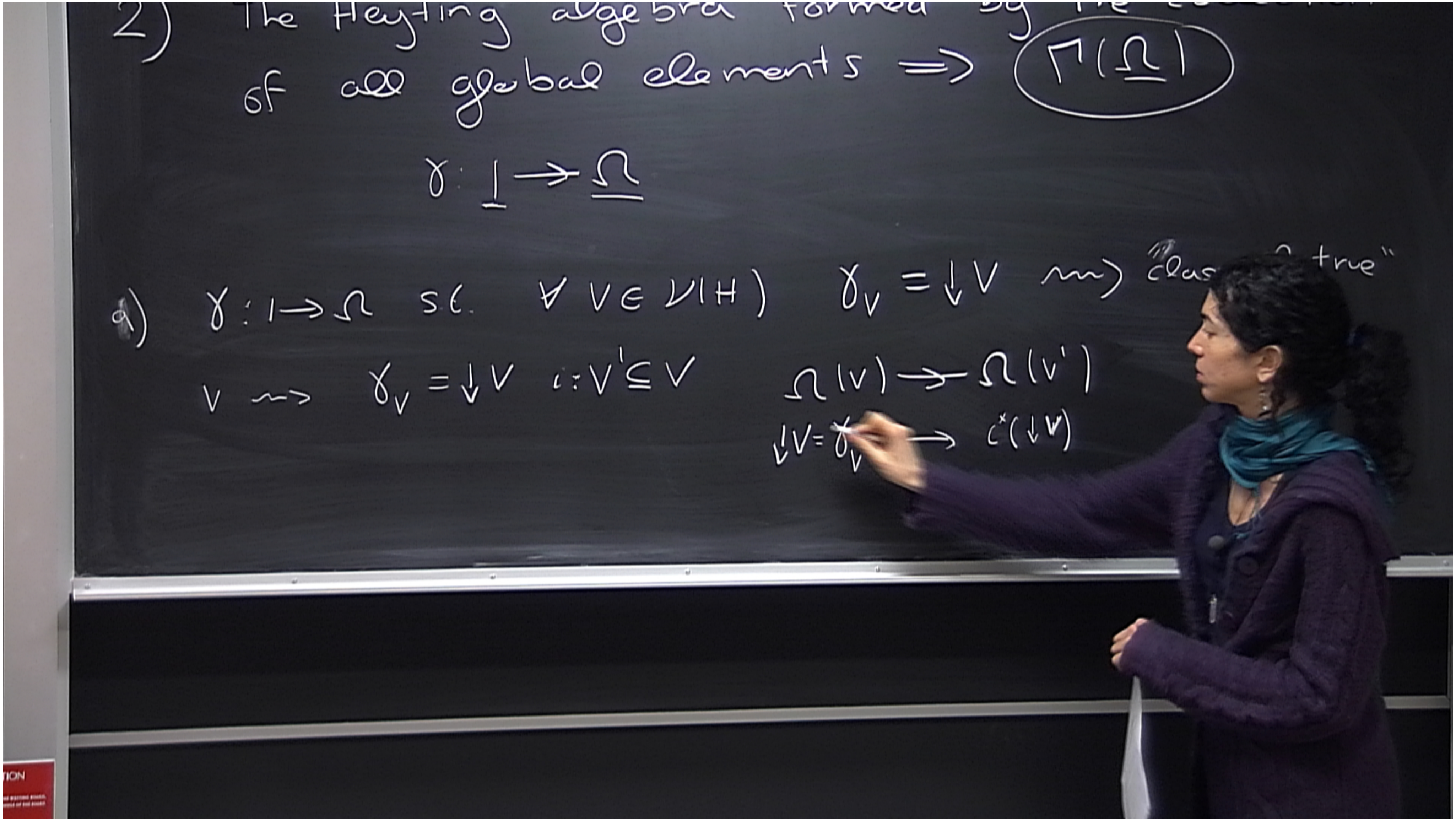
a) γ : 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⤳ "classical true"
V ⤳ γ_V = ↓V V' ⊆ V



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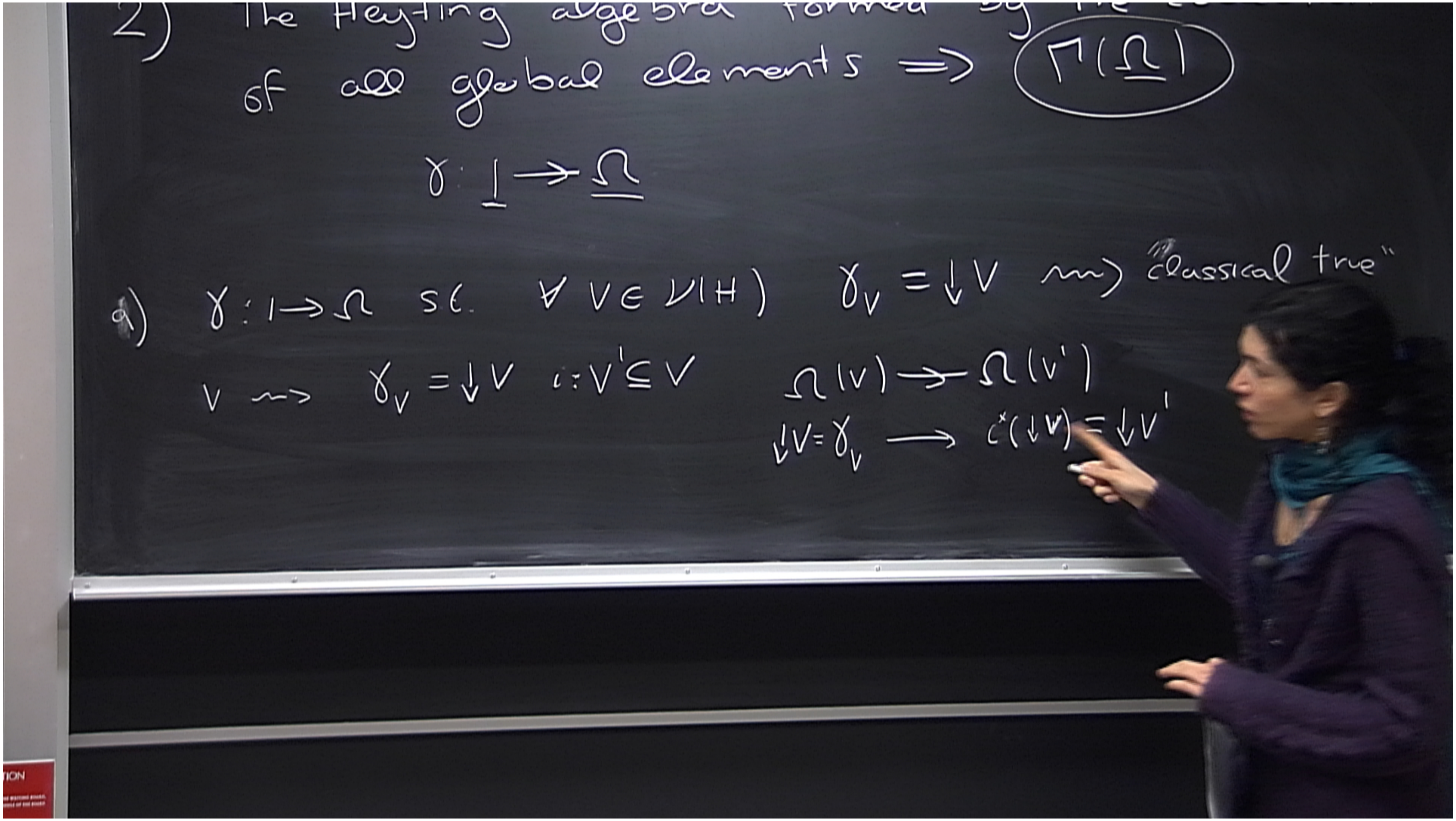
γ : 1 → Ω
a) γ : 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⇝
true"
V ⇝ γ_V = ↓V i : V' ⊆ V
Ω(V) → Ω(V')
↓V = γ_V → i*(↓V)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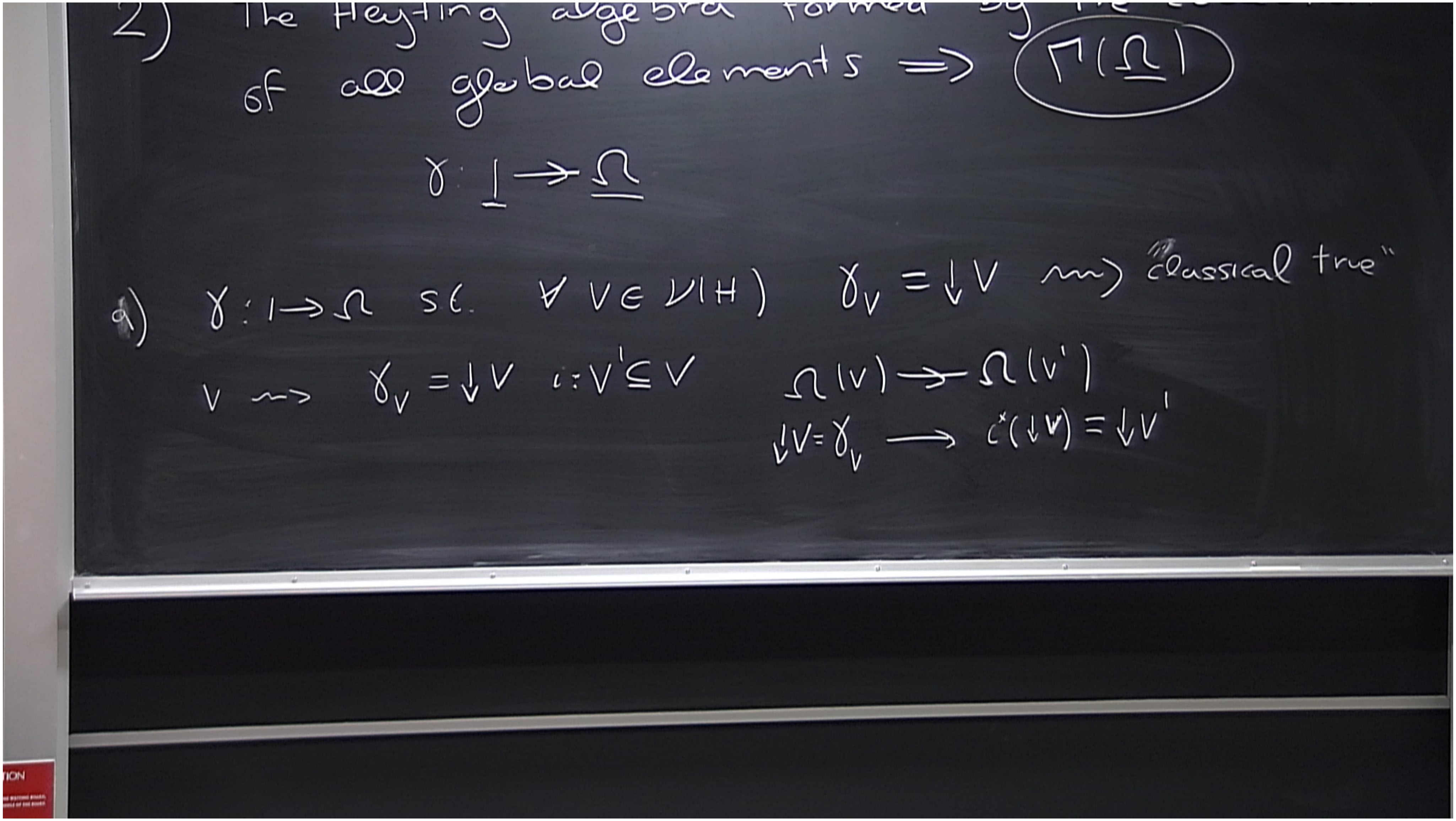
γ: 1 → Ω
a) γ: 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⇝ "classical true"
V ⇝ γ_V = ↓V i: V' ⊆ V
Ω(V) → Ω(V')
↓V = γ_V → i*(↓V) = ↓V'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γ: 1 → Ω
a) γ: 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⇝ "classical true"
V ⇝ γ_V = ↓V i: V′ ⊆ V Ω(V) → Ω(V′)
↓V = γ_V ⟶ i*(↓V) = ↓V′

- obj: $V \in \mathcal{V}(H)$ $\quad \Omega(V) = \{$ collection of sieves on $V \}$

- Morphism $i : V' \hookrightarrow V$ $\quad \Omega(V) \rightarrow \Omega(V')$

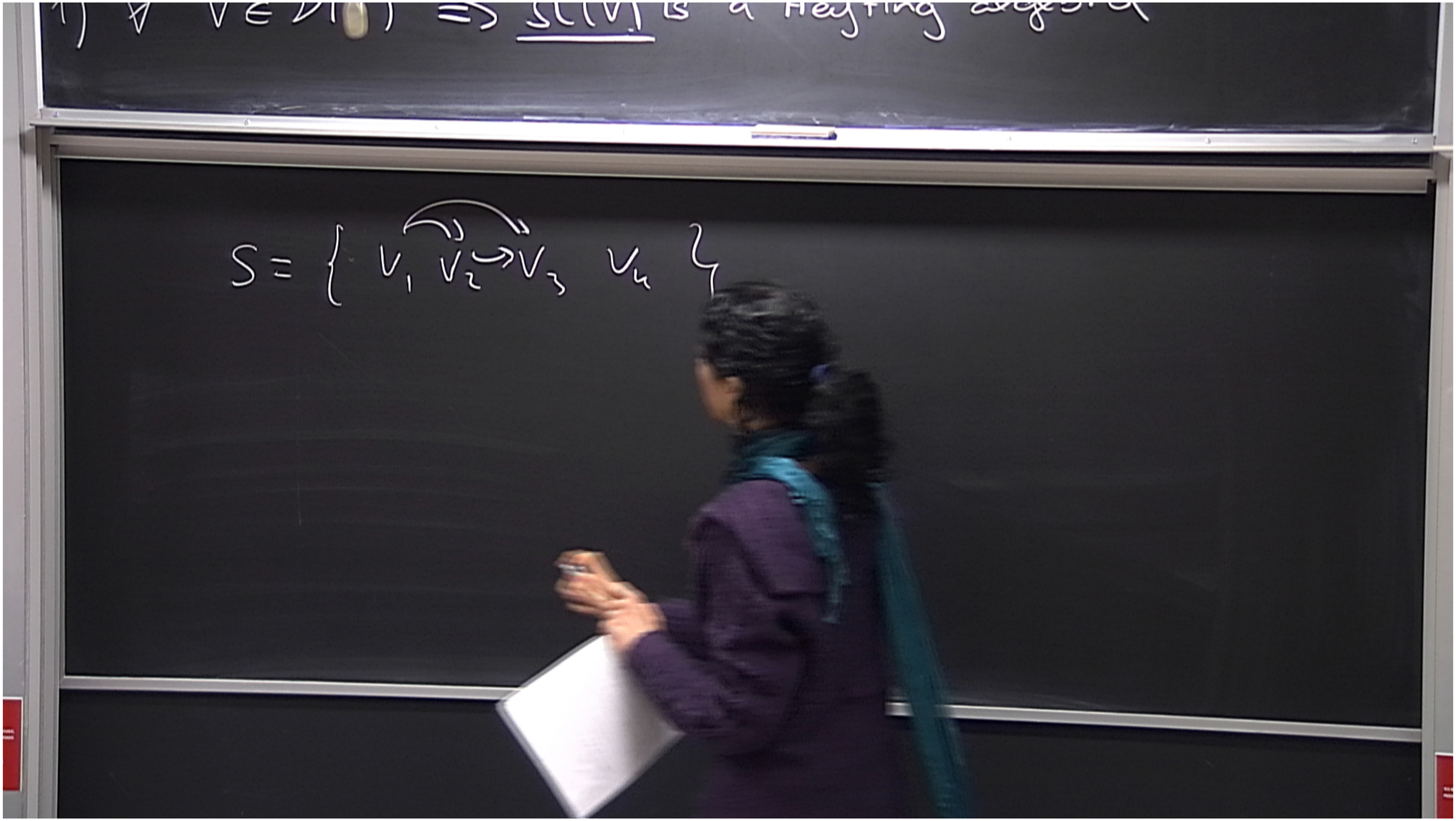
$$S \rightarrow i^{*}(S) = S \cap \downarrow V'$$

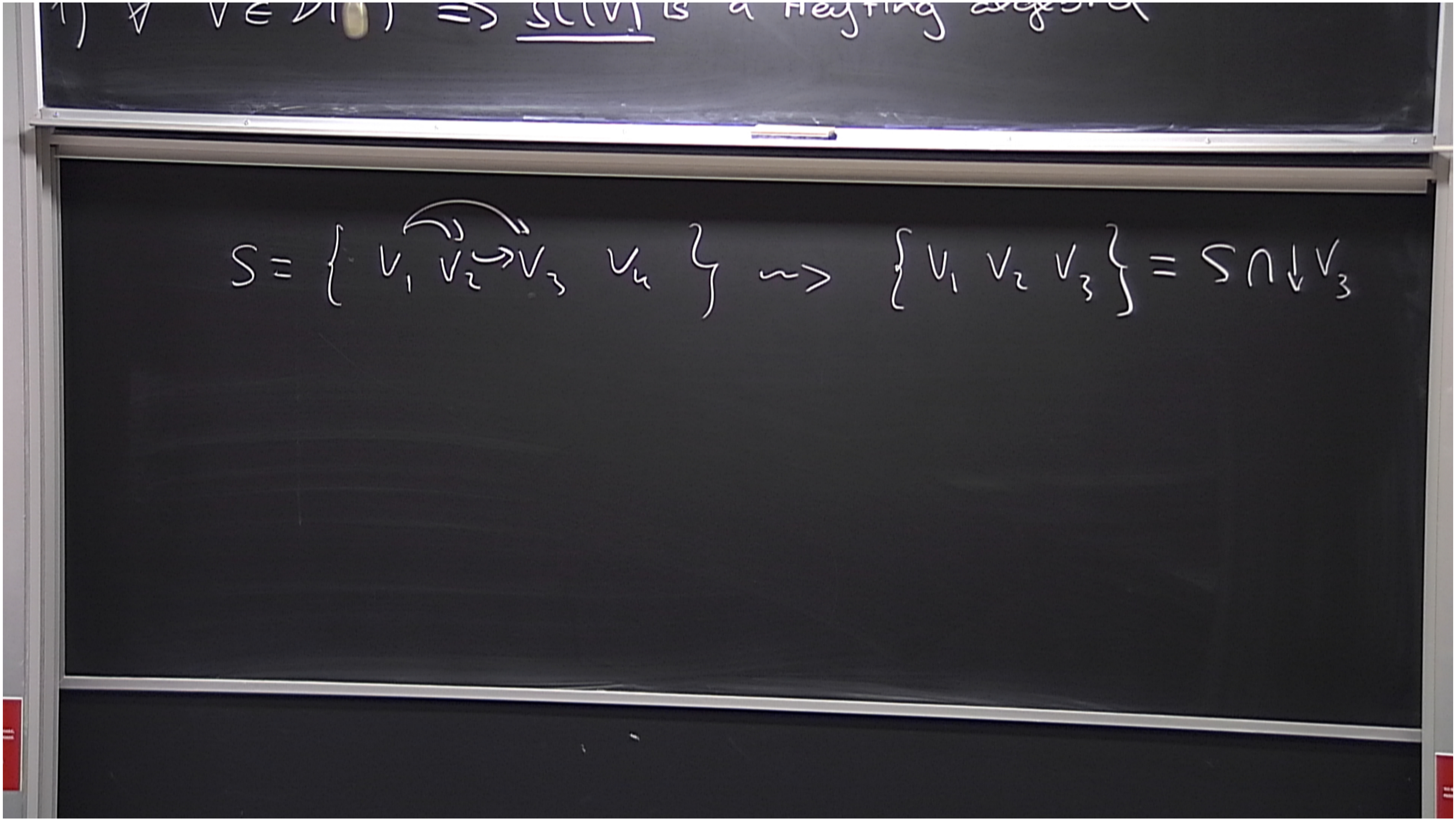
$$S_A := \{ f : X \rightarrow A \mid cod(A) \} \Rightarrow g : B \rightarrow X \text{ Then } f \circ g \in S$$

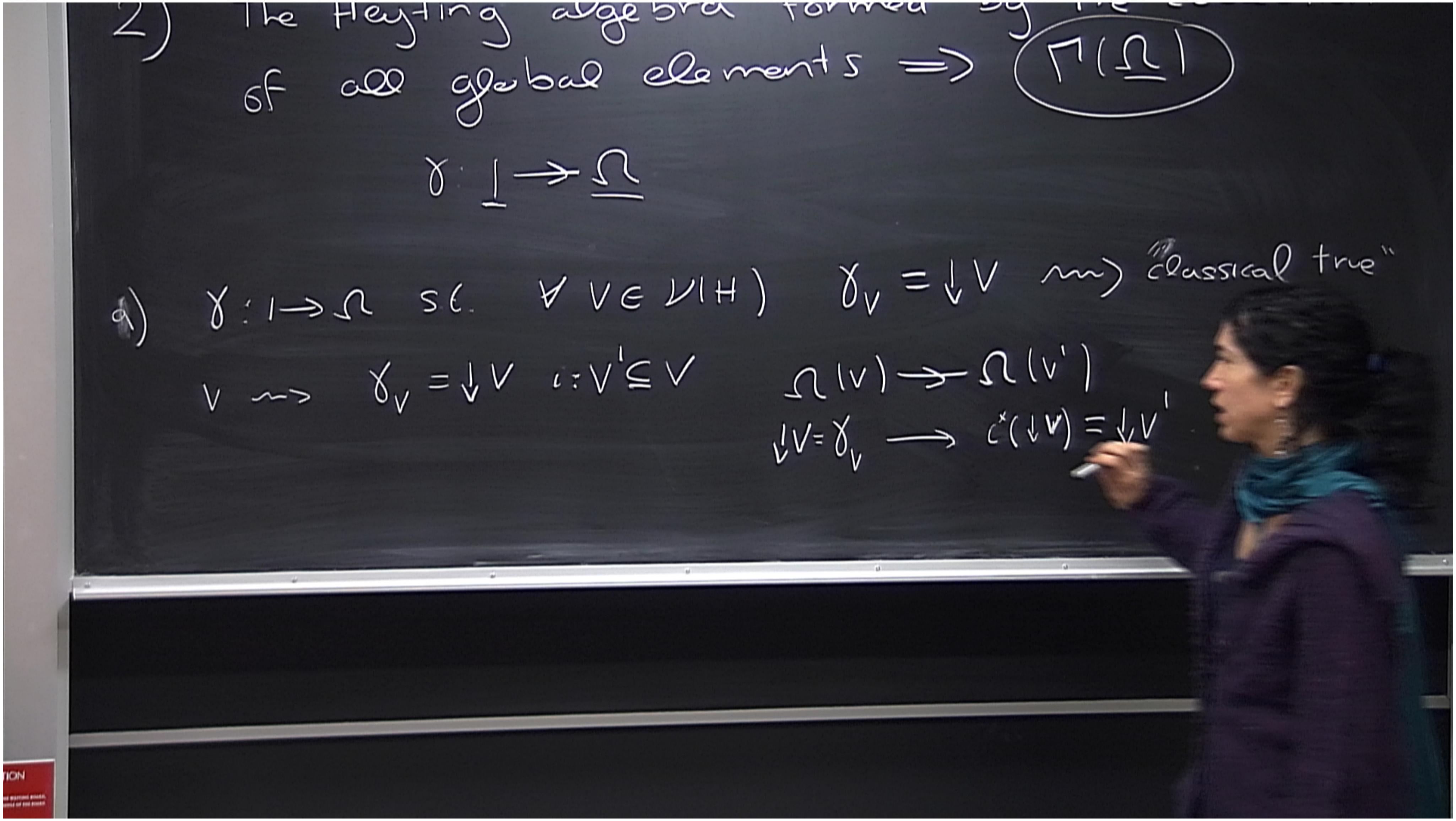
$\mathcal{V}(H)$ = poset

V_1, V_2:
- $V_1 \hookrightarrow V_2$ $\quad (V_2 \hookrightarrow V_1)$
- no map

$\Rightarrow$ V_1, V_2 $\exists$ at most 1 map between them


S = { V1 V2 V3 V4 }


S = { V1 V2 V3 V4 } ~> { V1 V2 V3 } = S ∩ ↓V3


2) The Heyting algebra formed by the
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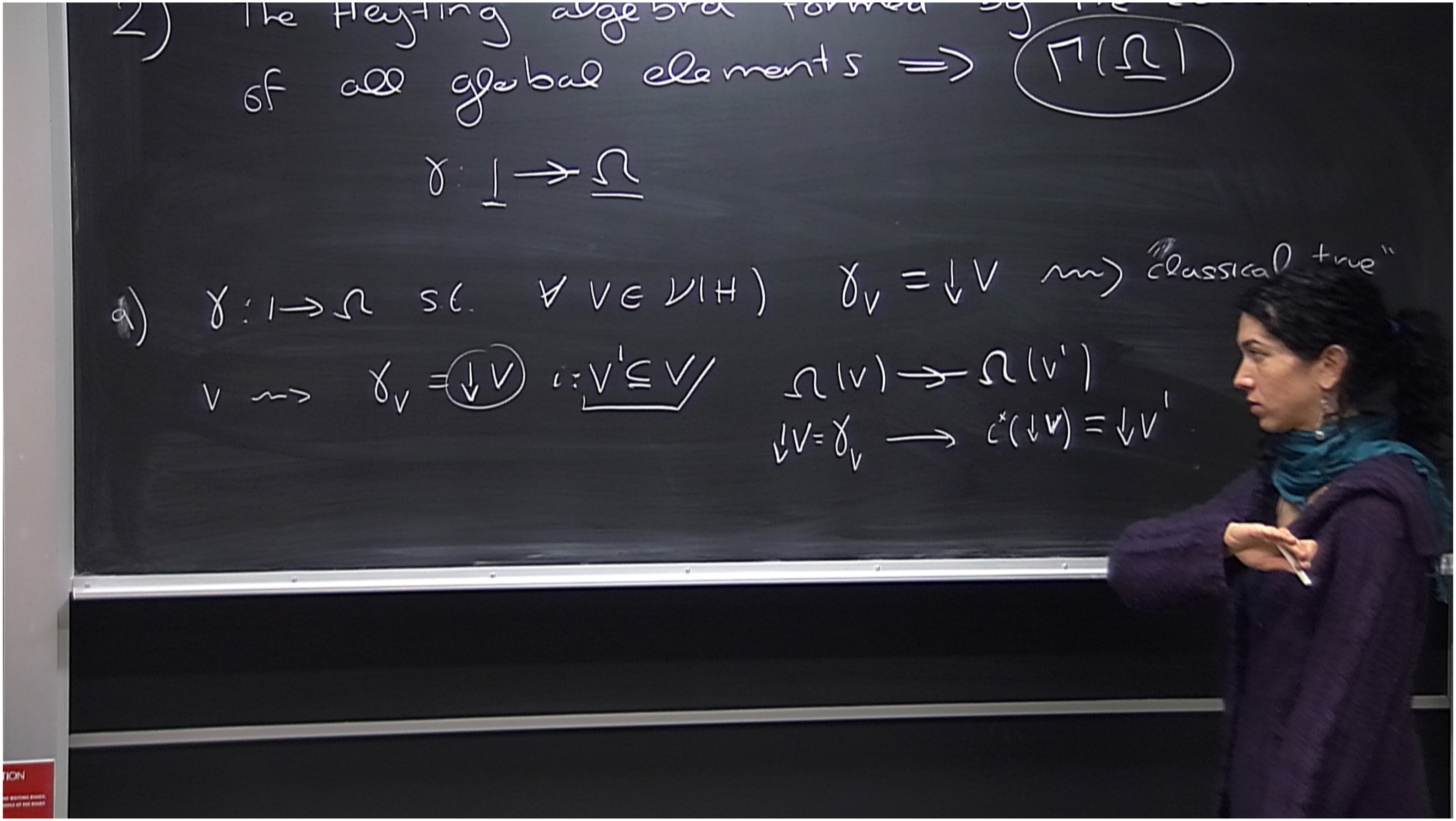
γ : 1 → Ω
a) γ : 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⇝ "classical true"
V ⇝ γ_V = ↓V i : V' ⊆ V
Ω(V) → Ω(V')
↓V = γ_V ⟶ i*(↓V) = ↓V'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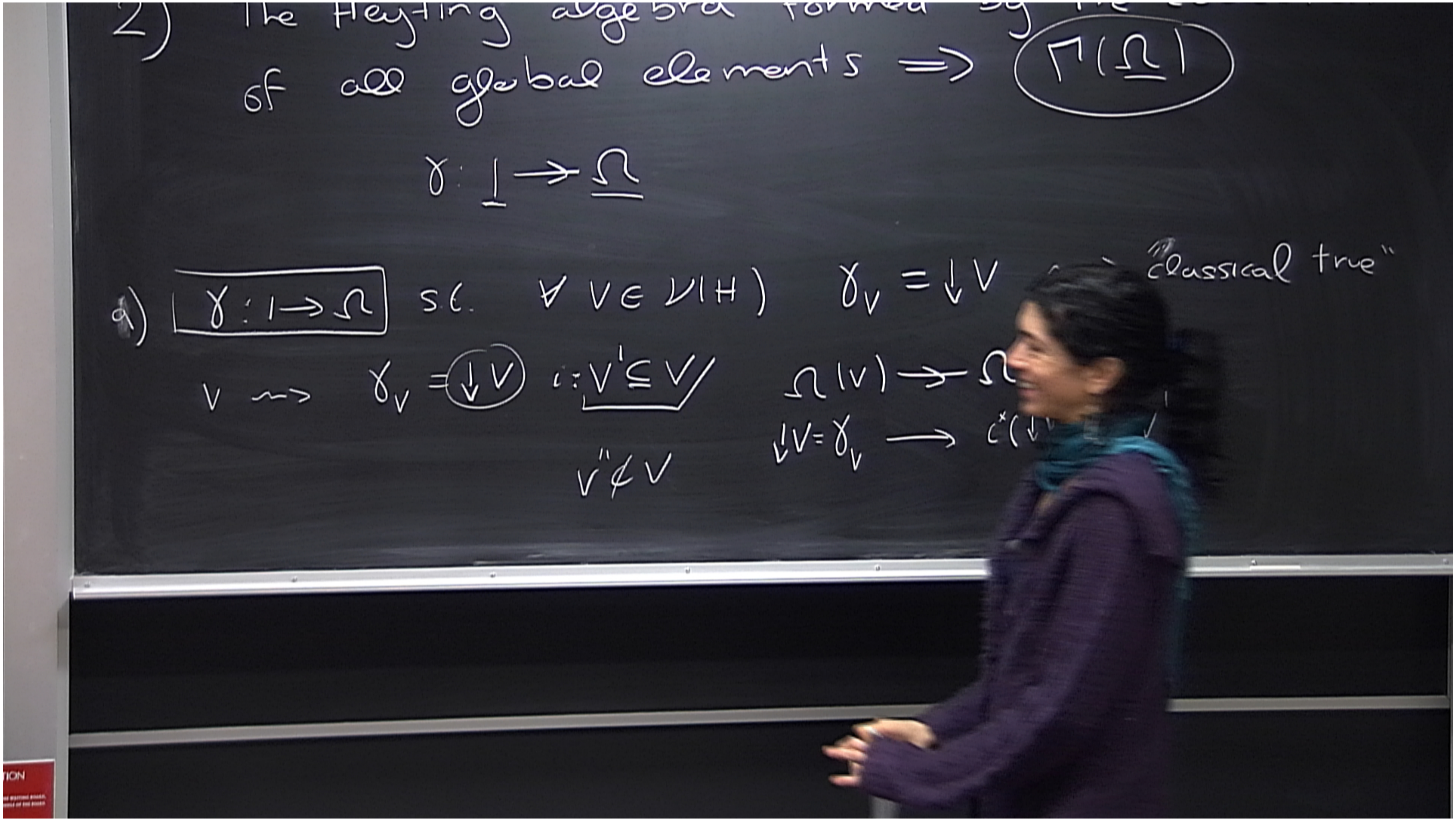
a) γ: 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V ⇝ "classical true"
V ⇝ γ_V = ↓V i: V' ⊆ V
Ω(V) → Ω(V')
↓V = γ_V → i*(↓V) = ↓V'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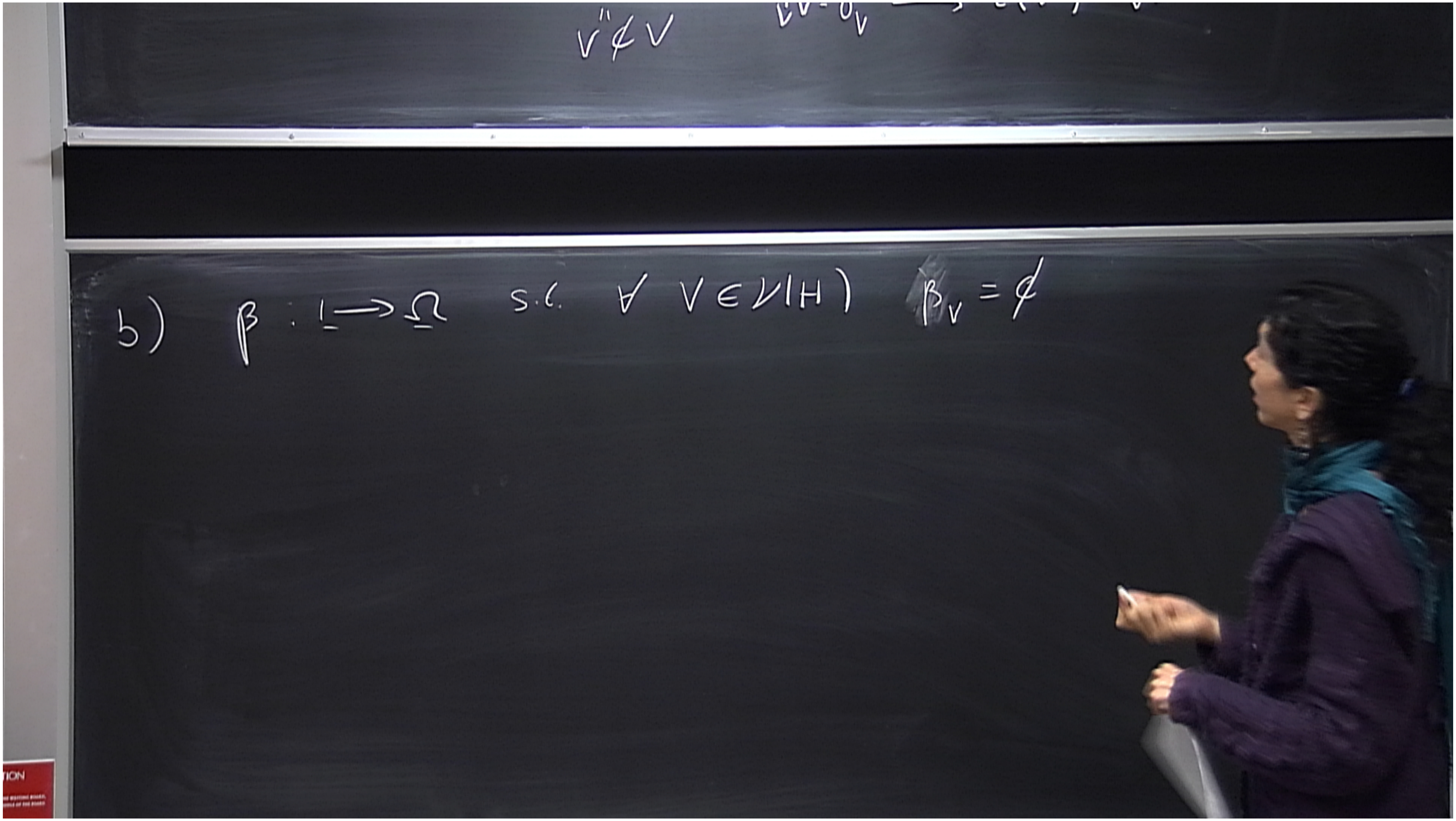
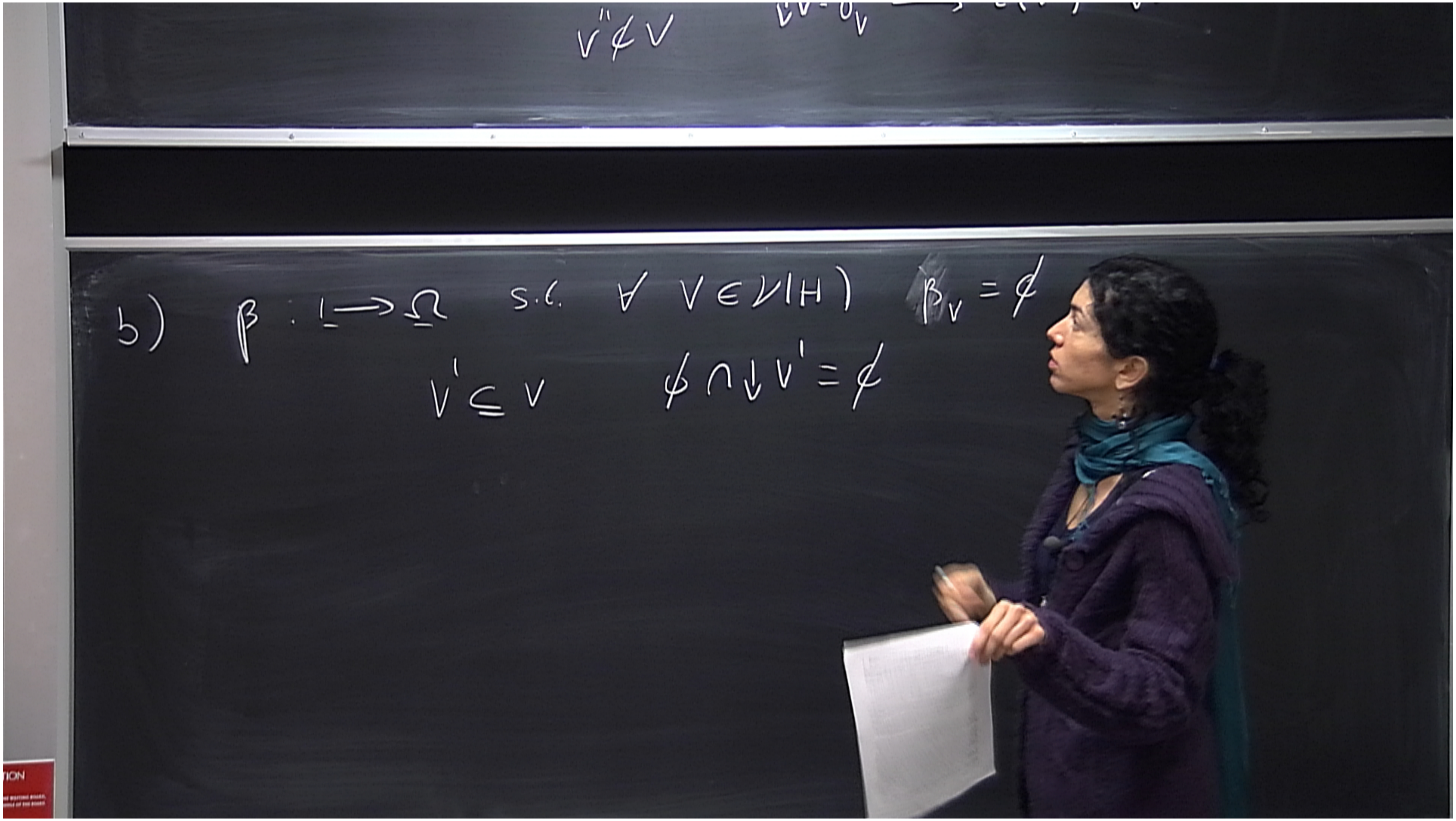
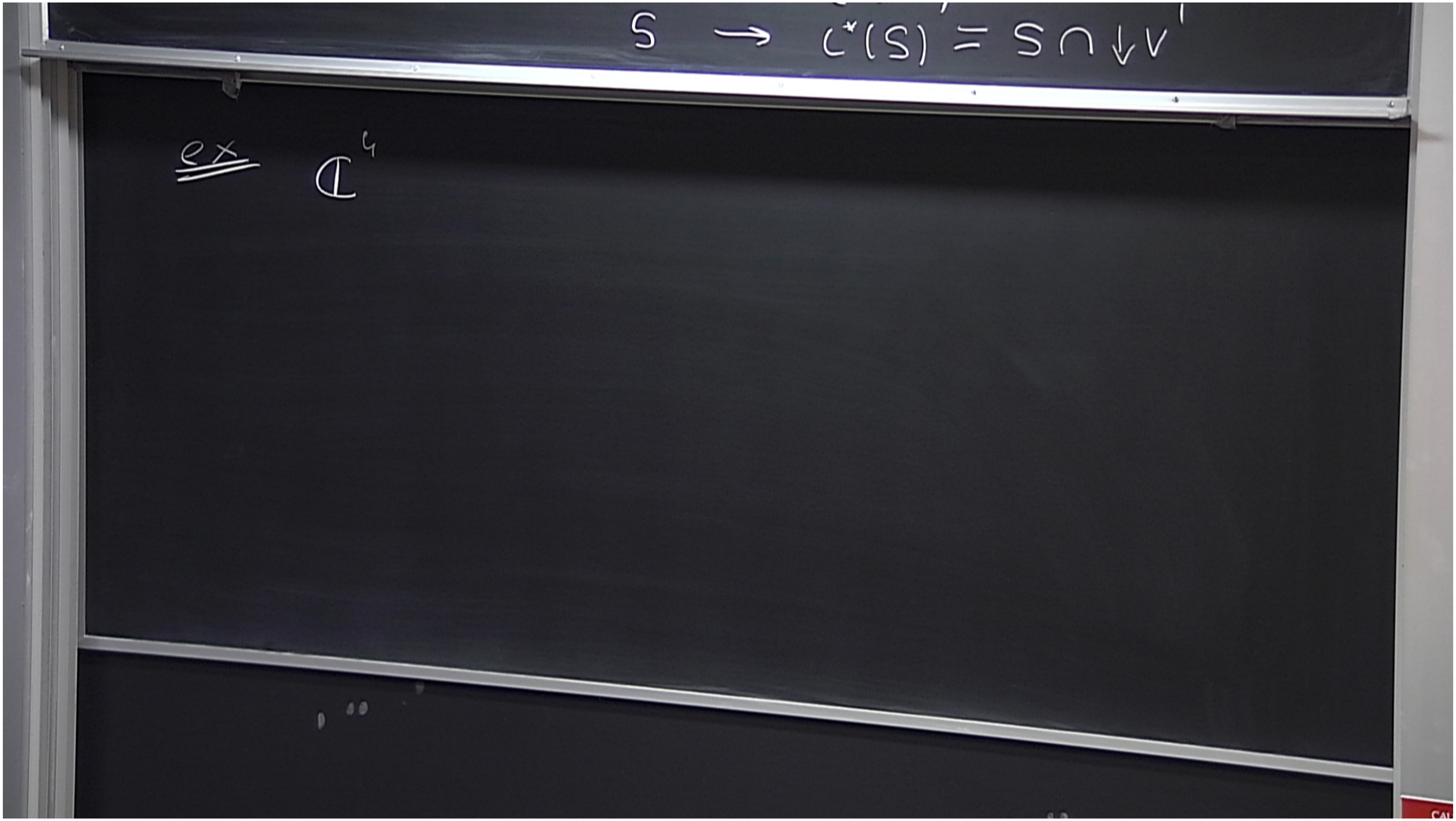
γ : 1 → Ω
a) γ : 1 → Ω s.t. ∀ V ∈ V(H) γ_V = ↓V "classical true"
V ⇝ γ_V = ↓V i : V′ ⊆ V Ω(V) → Ω
V″ ⊄ V ↓V = γ_V → i*(↓V


V'' ⊄ V
b) β : 1 ⟶ Ω s.t. ∀ V ∈ V(H) β_V = ∅


V'' ⊄ V
b) β : 1 → Ω s.t. ∀ V ∈ V(H) β_V = ϕ
V' ⊆ V ϕ ∩ ↓V' = ϕ

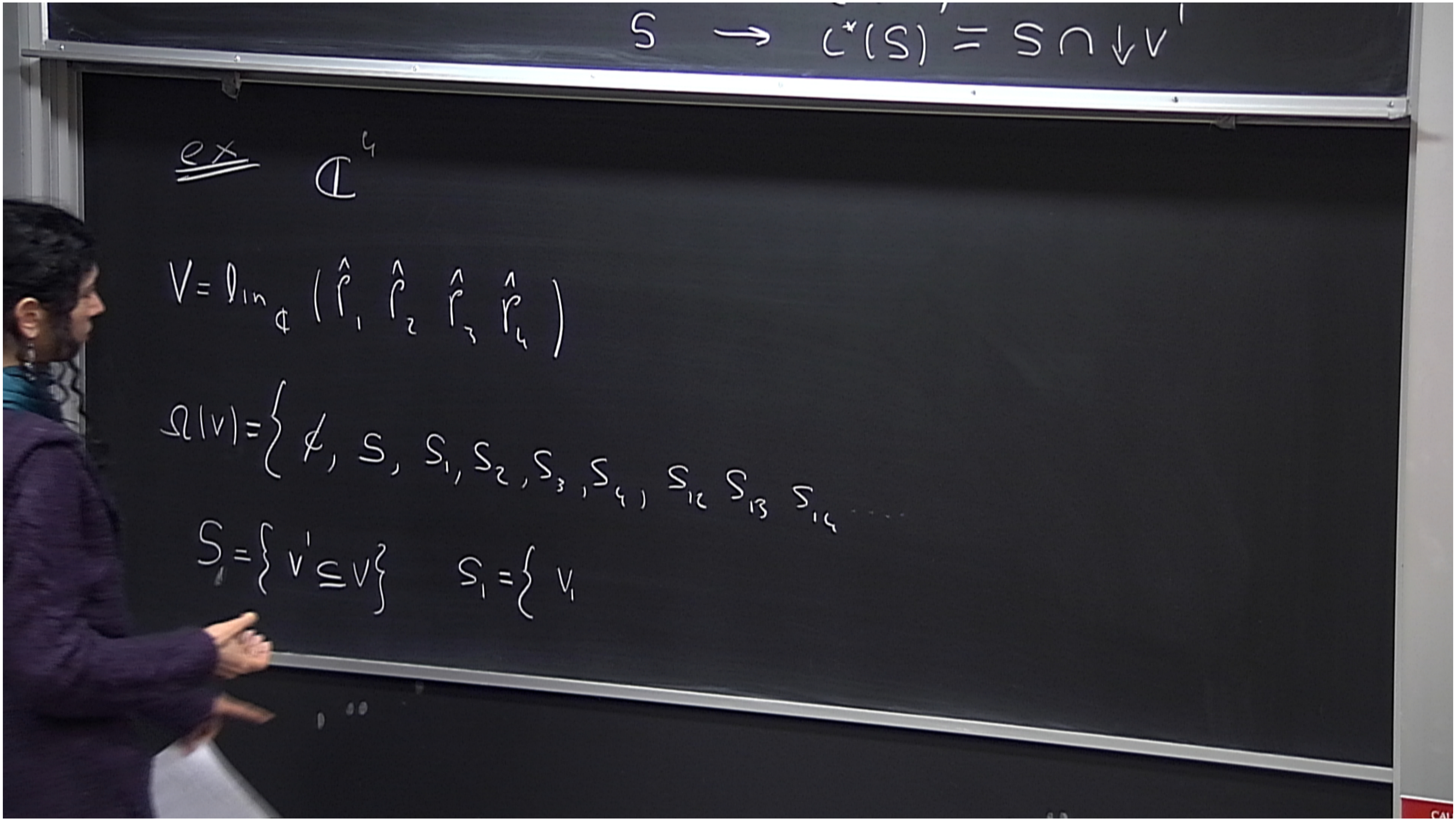

S → ι*(S) = S ∩ ↓V
ex ℂ^4

$$S \rightarrow \iota^*(S) = S \cap {\downarrow} V'$$

ex $\mathbb{C}^4$

$$V = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1\ \hat{P}_2\ \hat{P}_3\ \hat{P}_4)$$

$$\Omega(V) = \{ \emptyset,$$


S → ι*(S) = S ∩ ↓V'
ex ℂ^4
V = lin_ℂ (P̂_1 P̂_2 P̂_3 P̂_4)
Ω(V) = { ∅, S, S_1, S_2, S_3, S_4, S_12 S_13 S_14 ....
S = { V' ⊆ V }
S_1 = { V_1

$$S \to \iota^*(S) = S \cap \downarrow V'$$

ex $\mathbb{C}^4$

$$V = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4) \qquad V_1 = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1, \hat{P}_2 + \hat{P}_3 + \hat{P}_4)$$

$$\Omega(V) = \{ \emptyset, S, S_1, S_2, S_3, S_4, S_{12}\, S_{13}\, S_{14} \dots$$

$$S = \{ V' \subseteq V \} \qquad S_1 = \{ V_1$$

$$S \to \iota^*(S) = S \cap \downarrow V'$$

ex $\mathbb{C}^4$

$$V = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4)$$

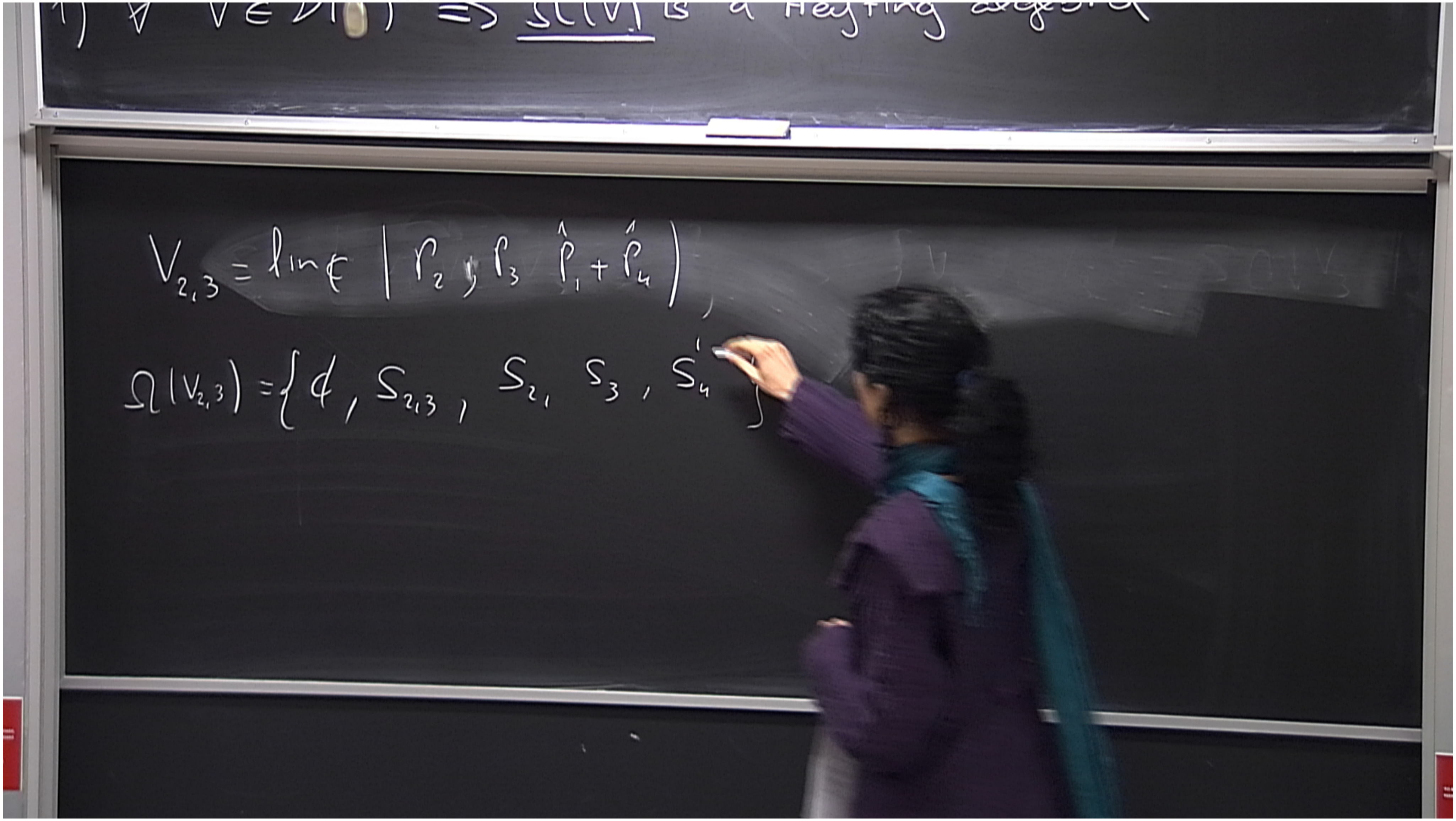
$$V_{1,2} = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1, \hat{P}_2, \hat{P}_3 + \hat{P}_4)$$

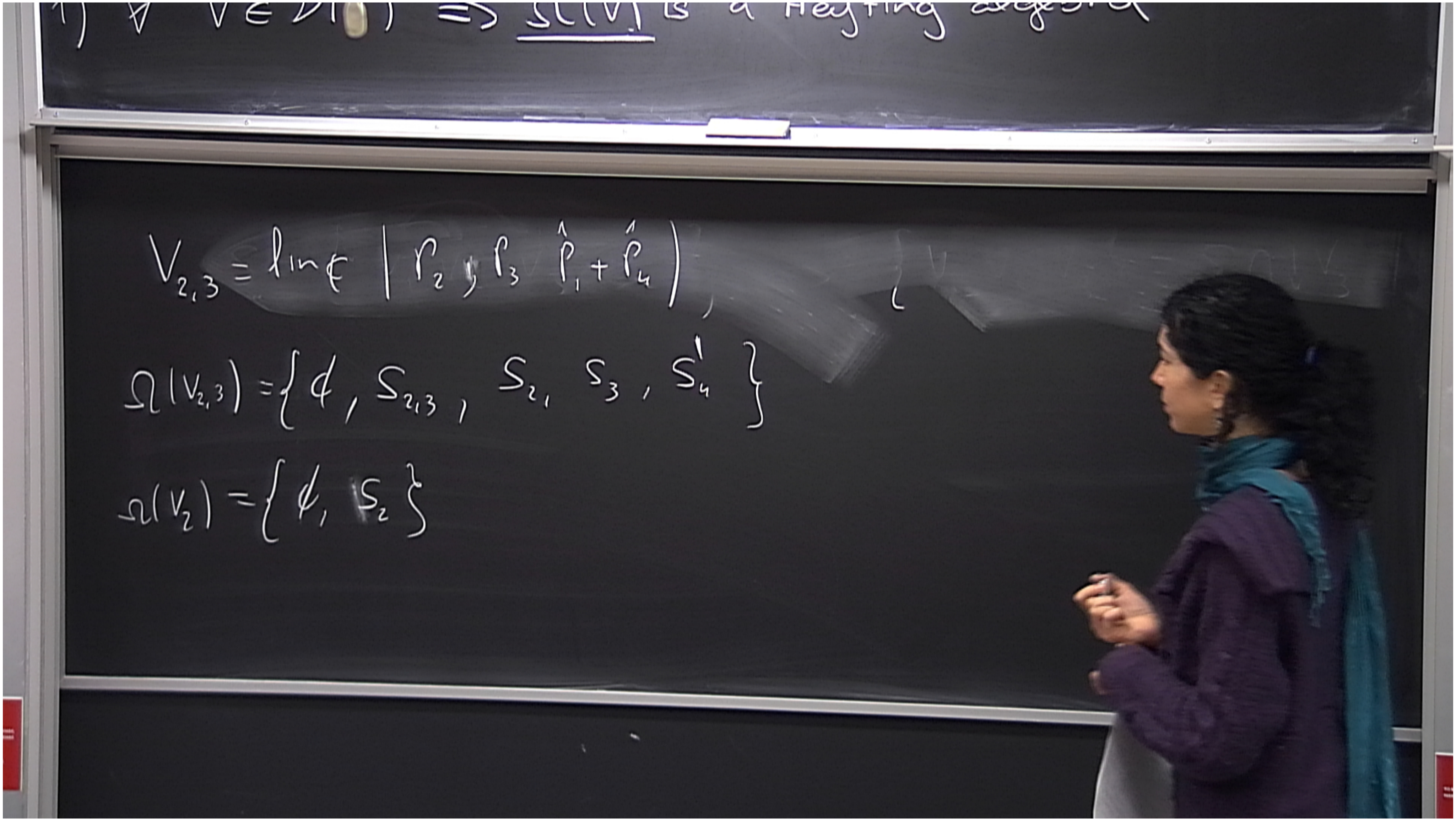
$$V_1 = \mathrm{lin}_{\mathbb{C}}(\hat{P}_1, \hat{P}_2 + \hat{P}_3 + \hat{P}_4)$$

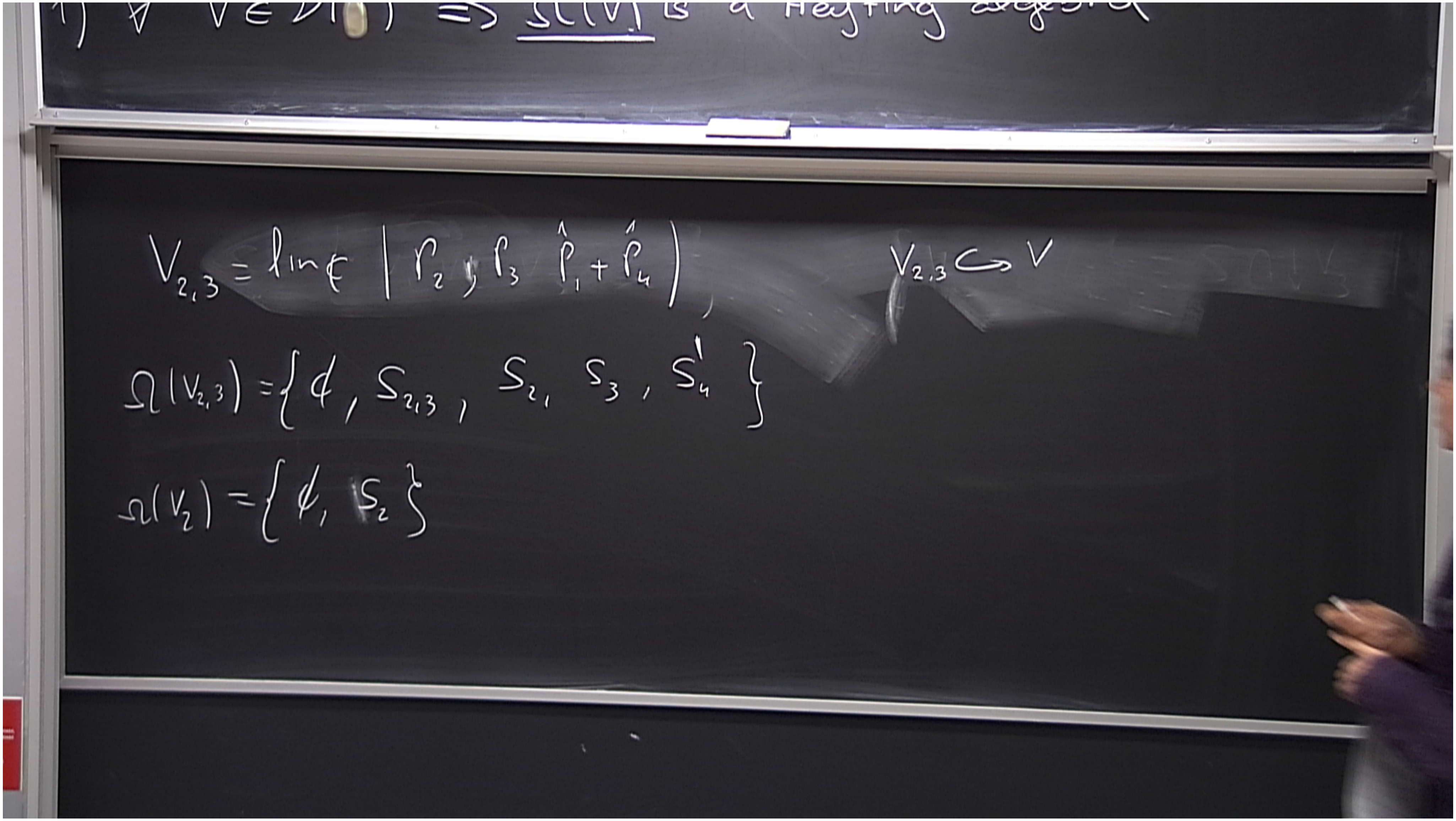
$$V_3 = \mathrm{lin}_{\mathbb{C}}(\hat{P}_3, \hat{P}_1 + \hat{P}_2 + \hat{P}_4)$$

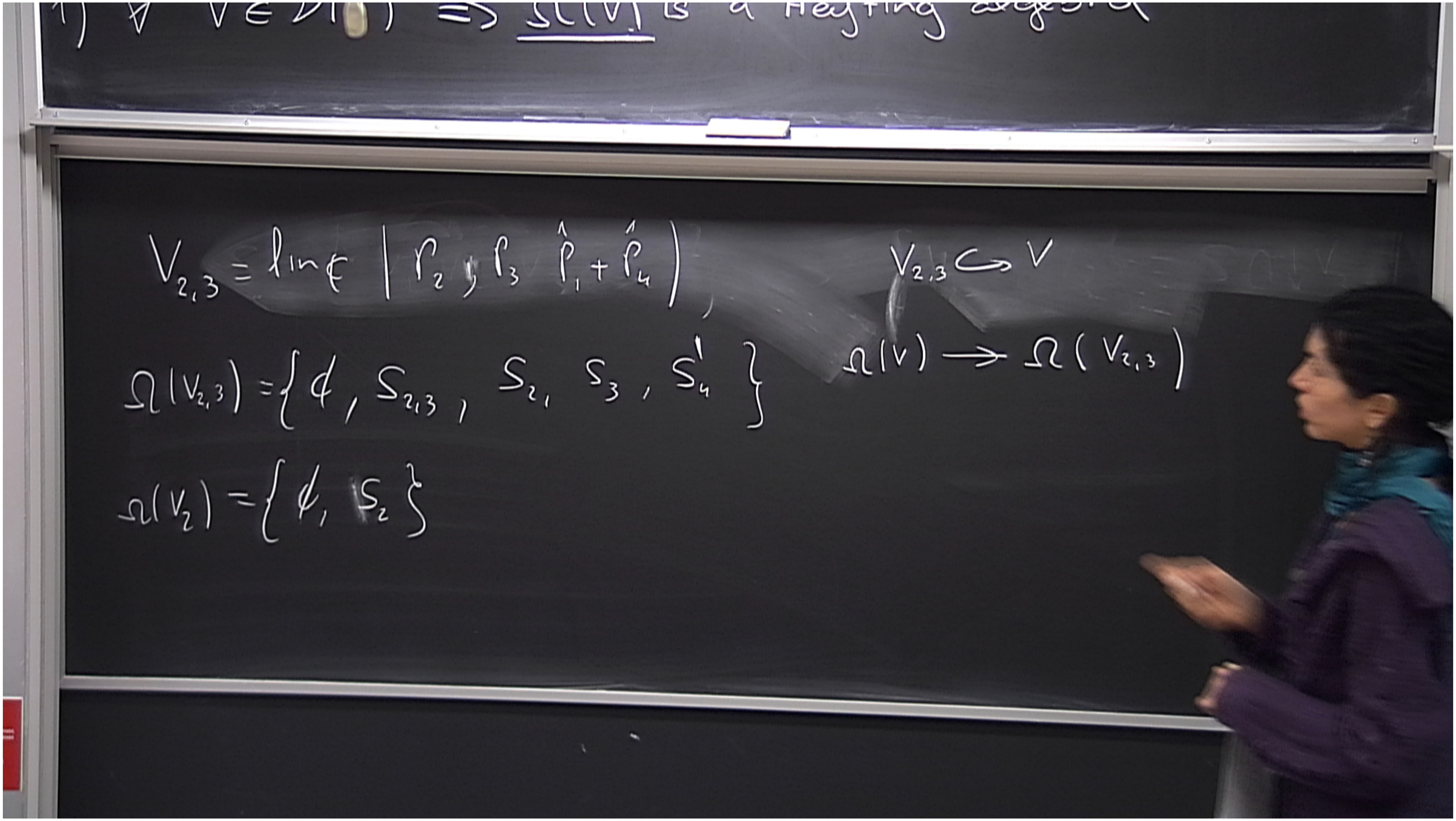
$$\Omega(V) = \{\emptyset, S, S_1, S_2, S_3, S_4, S_{12}\ S_{13}\ S_{14} \ldots$$

$$S = \{V' \subseteq V\} \qquad S_1 = \{V_1\} \qquad S_3 = \{V_3\} \qquad S_{12} = \{V_{12}, V_1, V_2\}$$

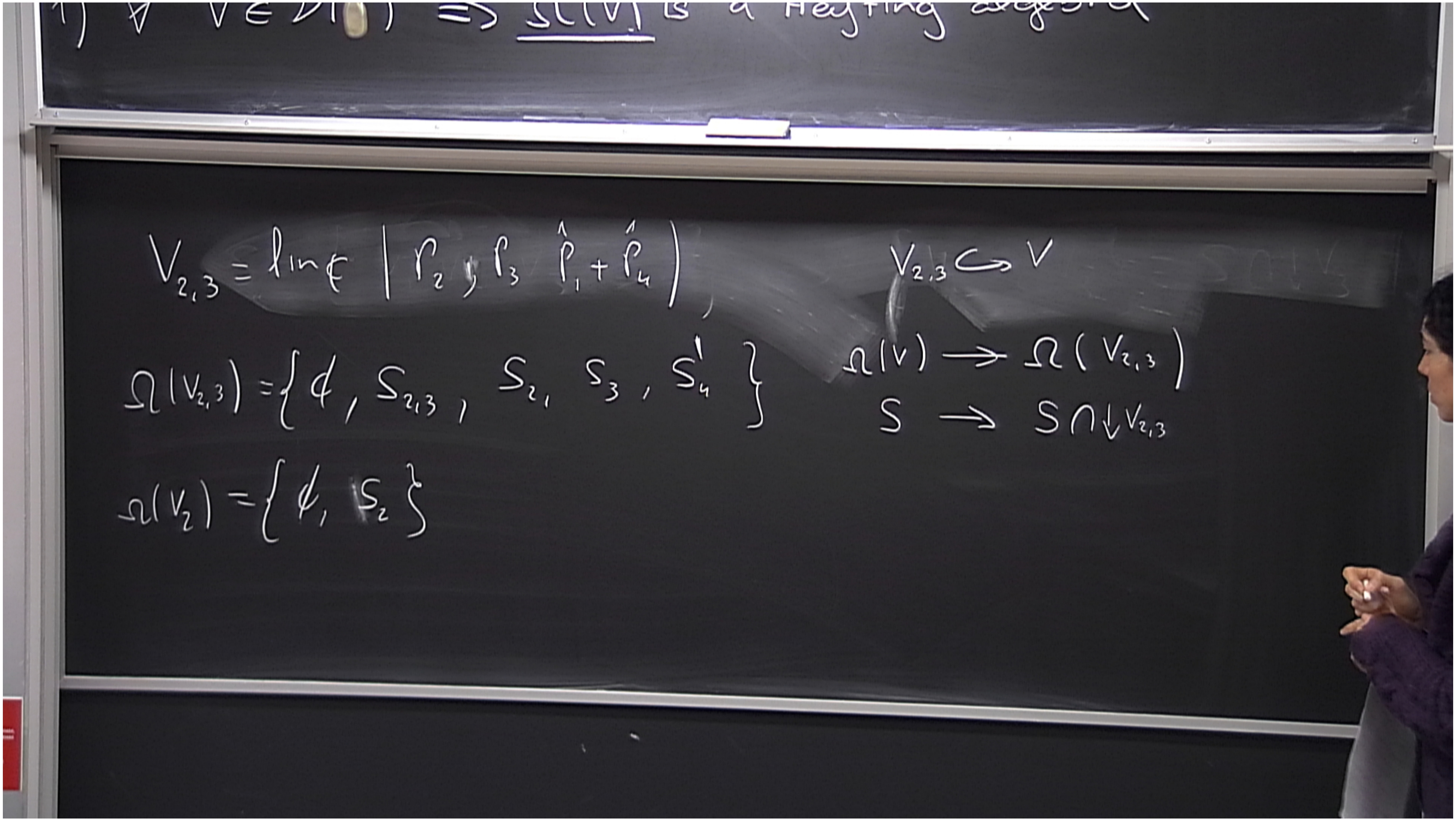

V2,3 = lin ℂ | P2, P3, P̂1 + P̂4)
Ω(V2,3) = { φ, S2,3, S2, S3, S4' }


V2,3 = lin ℂ | P2, P3, P̂1 + P̂4)
Ω(V2,3) = {∅, S2,3, S2, S3, S'4}
Ω(V2) = {∅, S2}

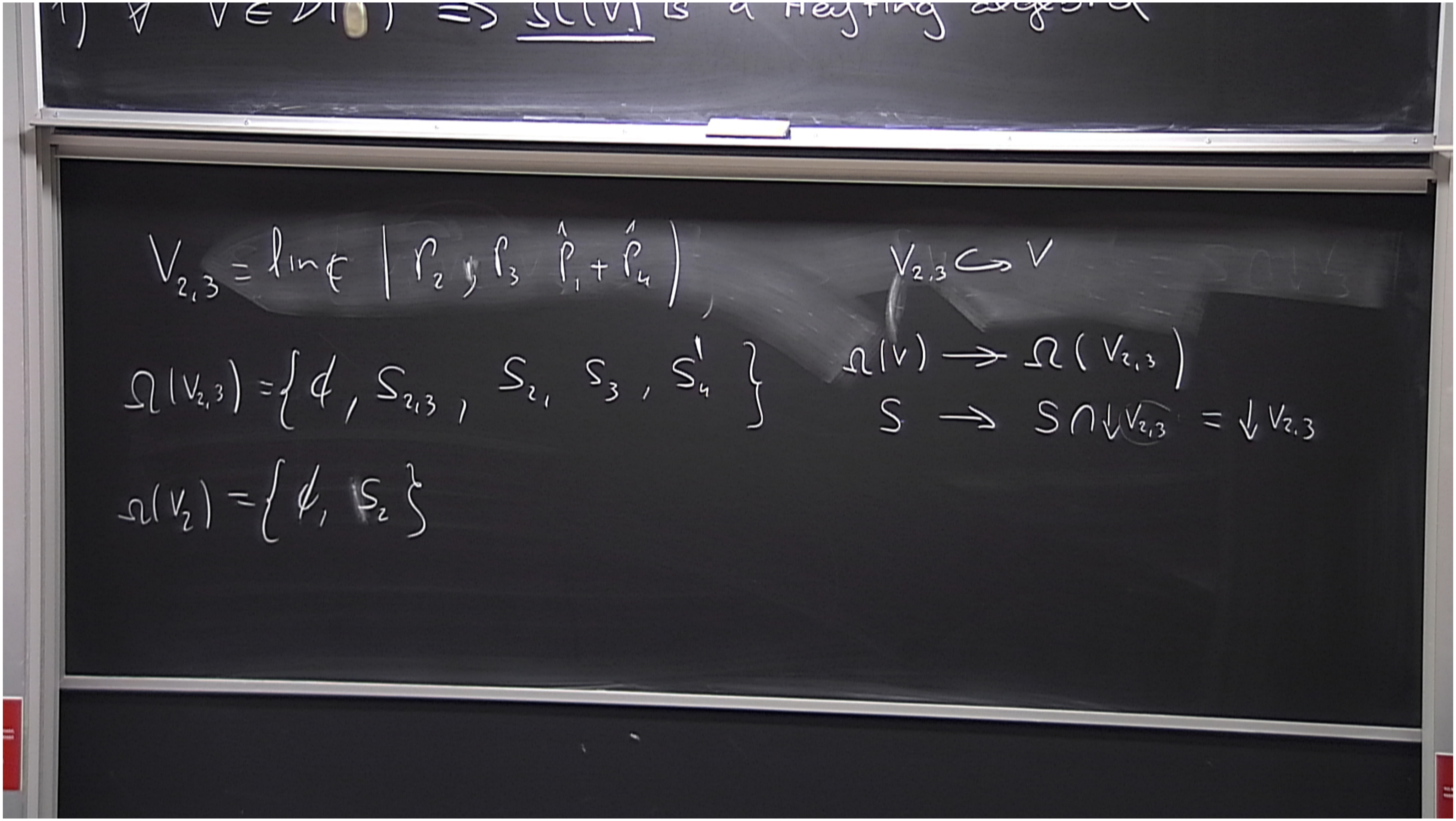

is a Heyting algebra
$V_{2,3} = \text{lin}_{\mathbb{C}} (P_2, P_3, \hat{P}_1 + \hat{P}_4)$
$V_{2,3} \hookrightarrow V$
$\Omega(V_{2,3}) = \{ \emptyset, S_{2,3}, S_2, S_3, S_4' \}$
$\Omega(V_2) = \{ \emptyset, S_2 \}$

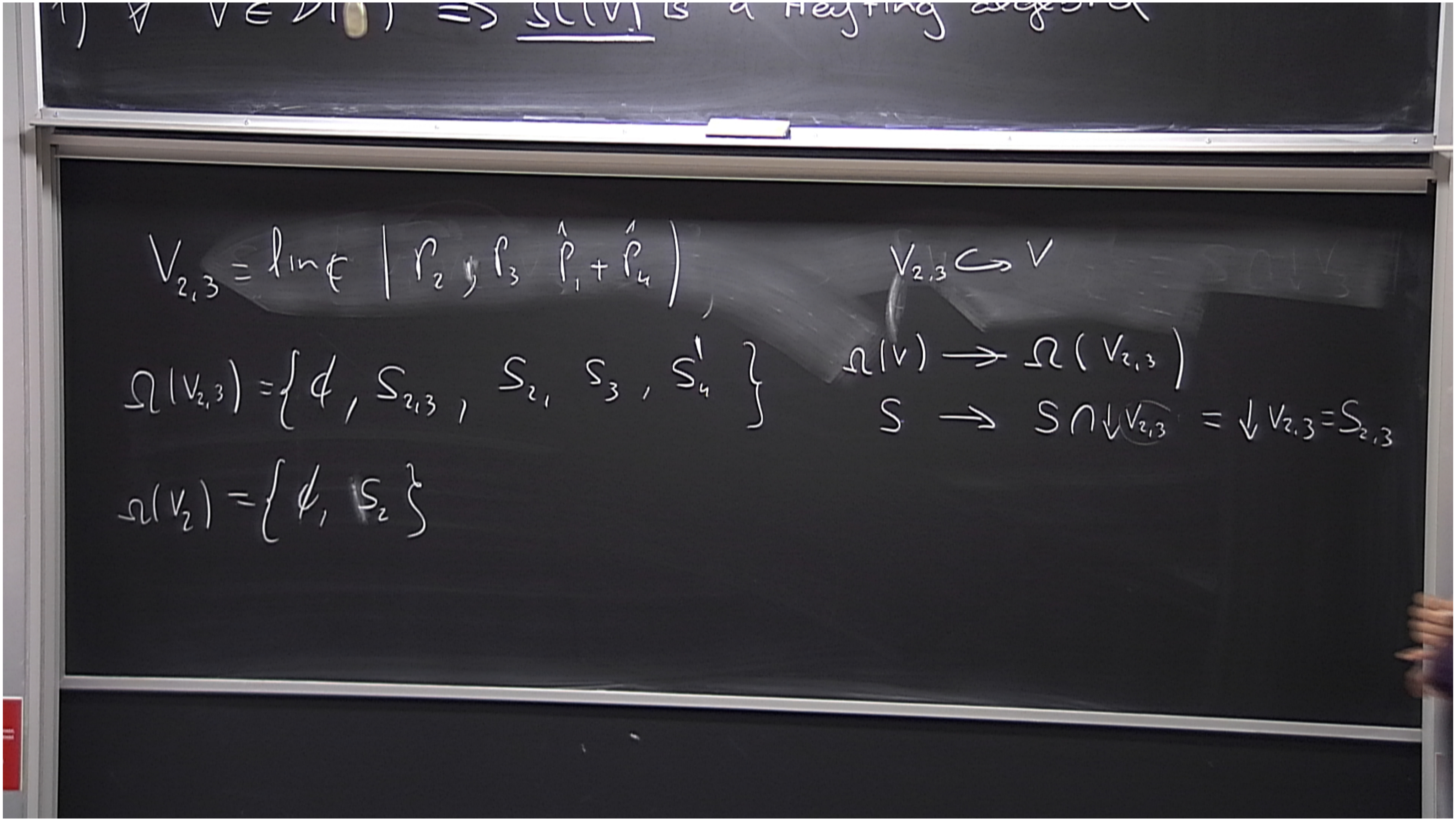


$V_{2,3} = \text{lin}_{\mathbb{C}} | P_2, P_3, \hat{P}_1 + \hat{P}_4)$
$V_{2,3} \hookrightarrow V$
$\Omega(V) \rightarrow \Omega(V_{2,3})$
$\Omega(V_{2,3}) = \{ \emptyset, S_{2,3}, S_2, S_3, S_4' \}$
$\Omega(V_2) = \{ \emptyset, S_2 \}$

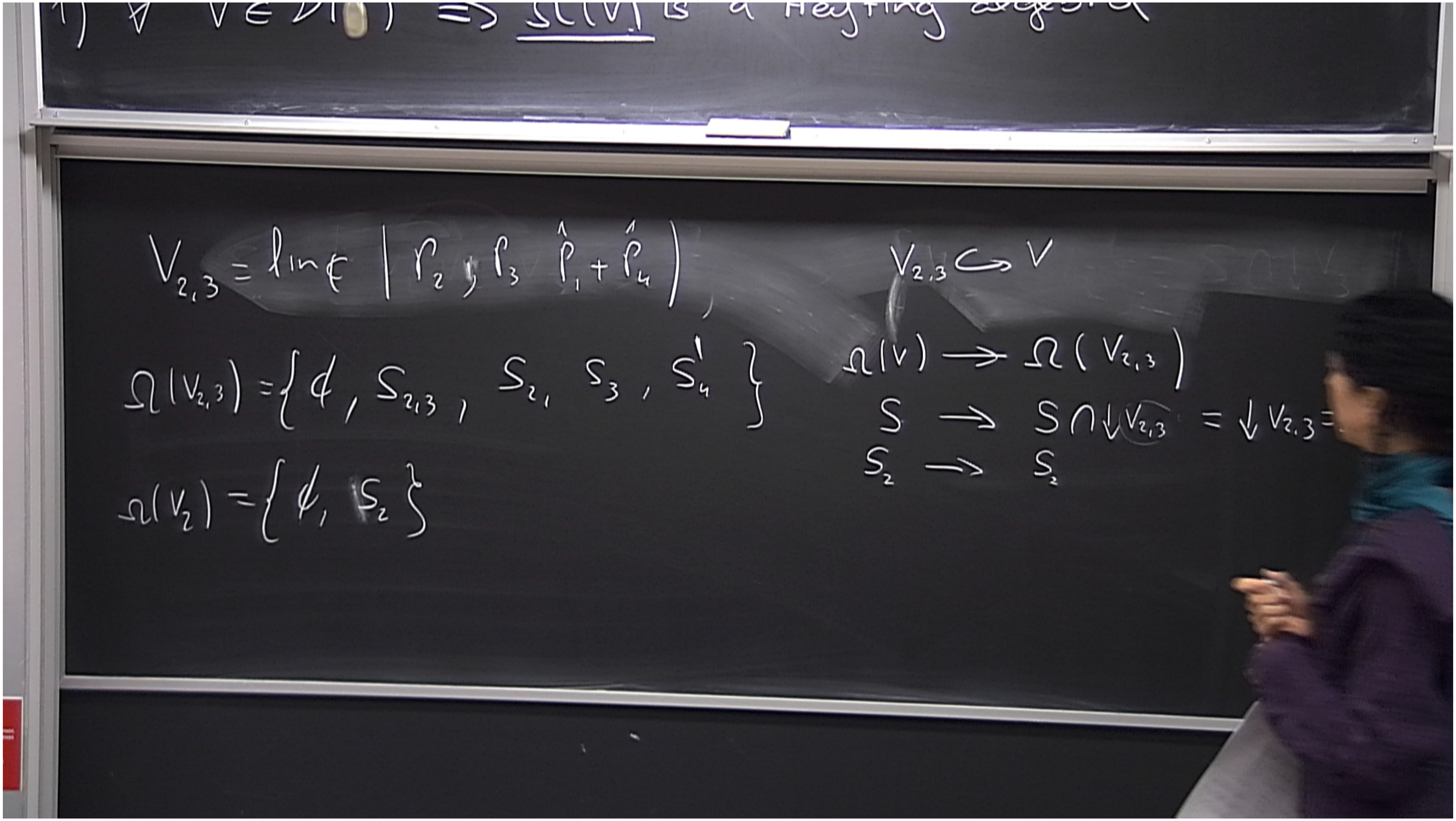


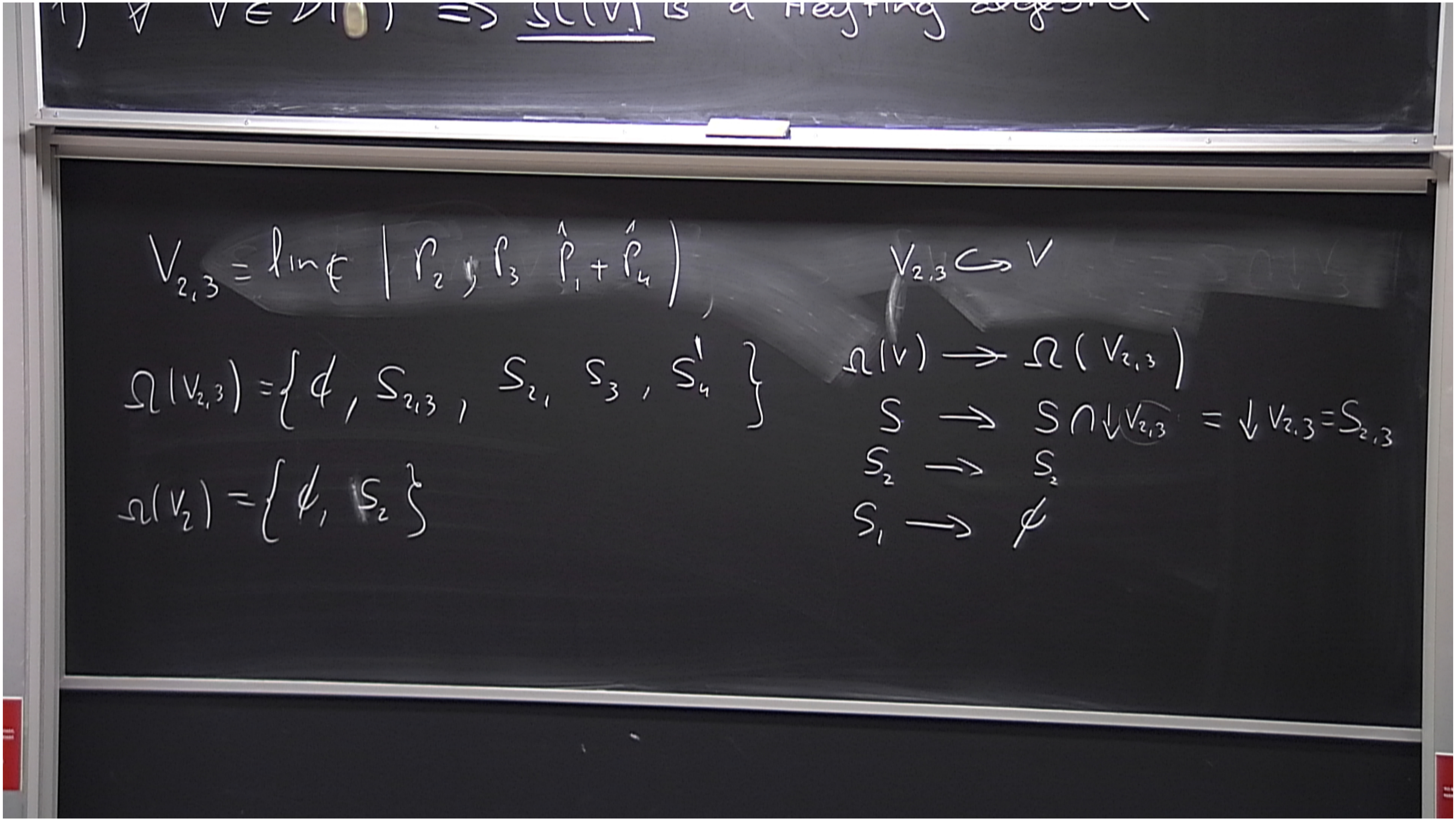
is a Heyting algebra
$V_{2,3} = \mathrm{lin}_{\mathbb{C}} \, | \, \hat{P}_2, \hat{P}_3, \hat{P}_1 + \hat{P}_4)$
$V_{2,3} \hookrightarrow V$
$\Omega(V_{2,3}) = \{ \emptyset, S_{2,3}, S_2, S_3, S_4' \}$
$\Omega(V) \rightarrow \Omega(V_{2,3})$
$S \rightarrow S \cap \downarrow V_{2,3}$
$\Omega(V_2) = \{ \emptyset, S_2 \}$

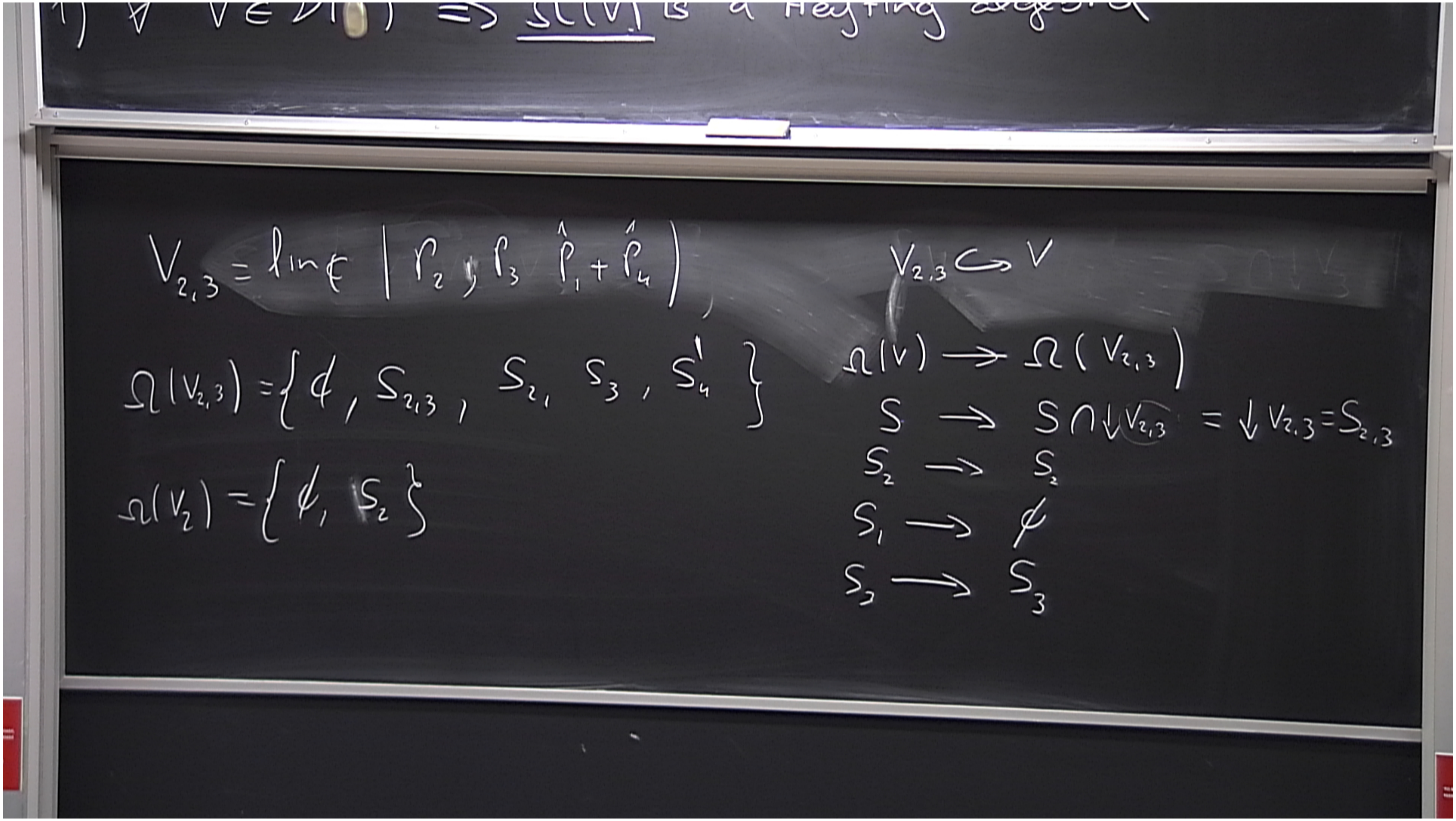

is a Heyting algebra
V2,3 = linC | P2, P3, P̂1 + P̂4)
V2,3 ↪ V
Ω(V) → Ω(V2,3)
S → S ∩ ↓V2,3 = ↓V2,3
Ω(V2,3) = { ∅, S2,3, S2, S3, S'4 }
Ω(V2) = { ∅, S2 }



V2,3 = linC | P2, P3, P̂1 + P̂4)
Ω(V2,3) = {∅, S2,3, S2, S3, S4' }
Ω(V2) = {∅, S2}
V2,3 ↪ V
Ω(V) → Ω(V2,3)
S → S ∩ ↓V2,3 = ↓V2,3 = S2,3


V2,3 = lin C | P2 , P3 , P1 + P4)
Ω(V2,3) = { ∅ , S2,3 , S2 , S3 , S'4 }
Ω(V2) = { ∅ , S2 }
V2,3 ↪ V
Ω(V) → Ω(V2,3)
S → S ∩ ↓V2,3 = ↓V2,3 =
S2 → S2


1) ∀ V ∈ D(...) ⇒ Ω(V) is a Heyting algebra
V_{2,3} = lin_C | P_2, P_3, P̂_1 + P̂_4)
Ω(V_{2,3}) = { ∅, S_{2,3}, S_2, S_3, S_4 }
Ω(V_2) = { ∅, S_2 }
V_{2,3} ↪ V
Ω(V) → Ω(V_{2,3})
S → S ∩ ↓V_{2,3} = ↓V_{2,3} = S_{2,3}
S_2 → S_2
S_1 → ∅


V2,3 = lin C | P2, P3, P1 + P4)
Ω(V2,3) = { ∅, S2,3, S2, S3, S'4 }
Ω(V2) = { ∅, S2 }
V2,3 ↪ V
Ω(V) → Ω(V2,3)
S → S ∩ ↓V2,3 = ↓V2,3 = S2,3
S2 → S2
S1 → ∅
S3 → S3

1) $\forall\, V \in \mathcal{V}(\mathcal{H}) \Rightarrow \underline{\Omega(V)}$ is a Heyting algebra

$$V_{2,3} = \text{lin}_{\mathbb{C}} \left| \hat{P}_2, \hat{P}_3, \hat{P}_1 + \hat{P}_4 \right)$$

$$\Omega(V_{2,3}) = \{ \emptyset, S_{2,3}, S_2, S_3, S'_4 \}$$

$$\Omega(V_2) = \{ \emptyset, S_2 \}$$

$$V_{2,3} \hookrightarrow V$$

$$\Omega(V) \rightarrow \Omega(V_{2,3})$$

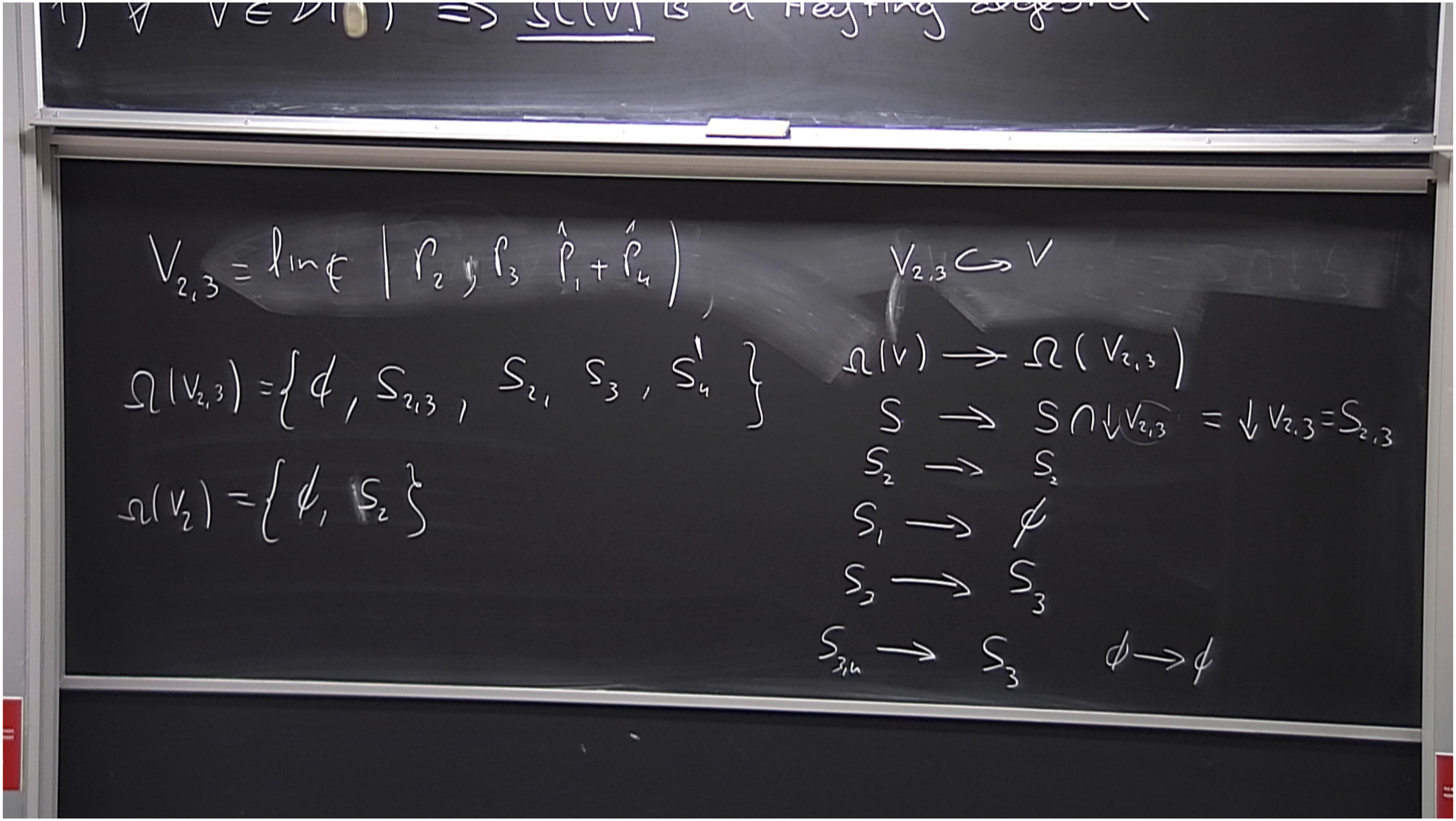
$$S \rightarrow S \cap \downarrow V_{2,3} = \downarrow V_{2,3} = S_{2,3}$$

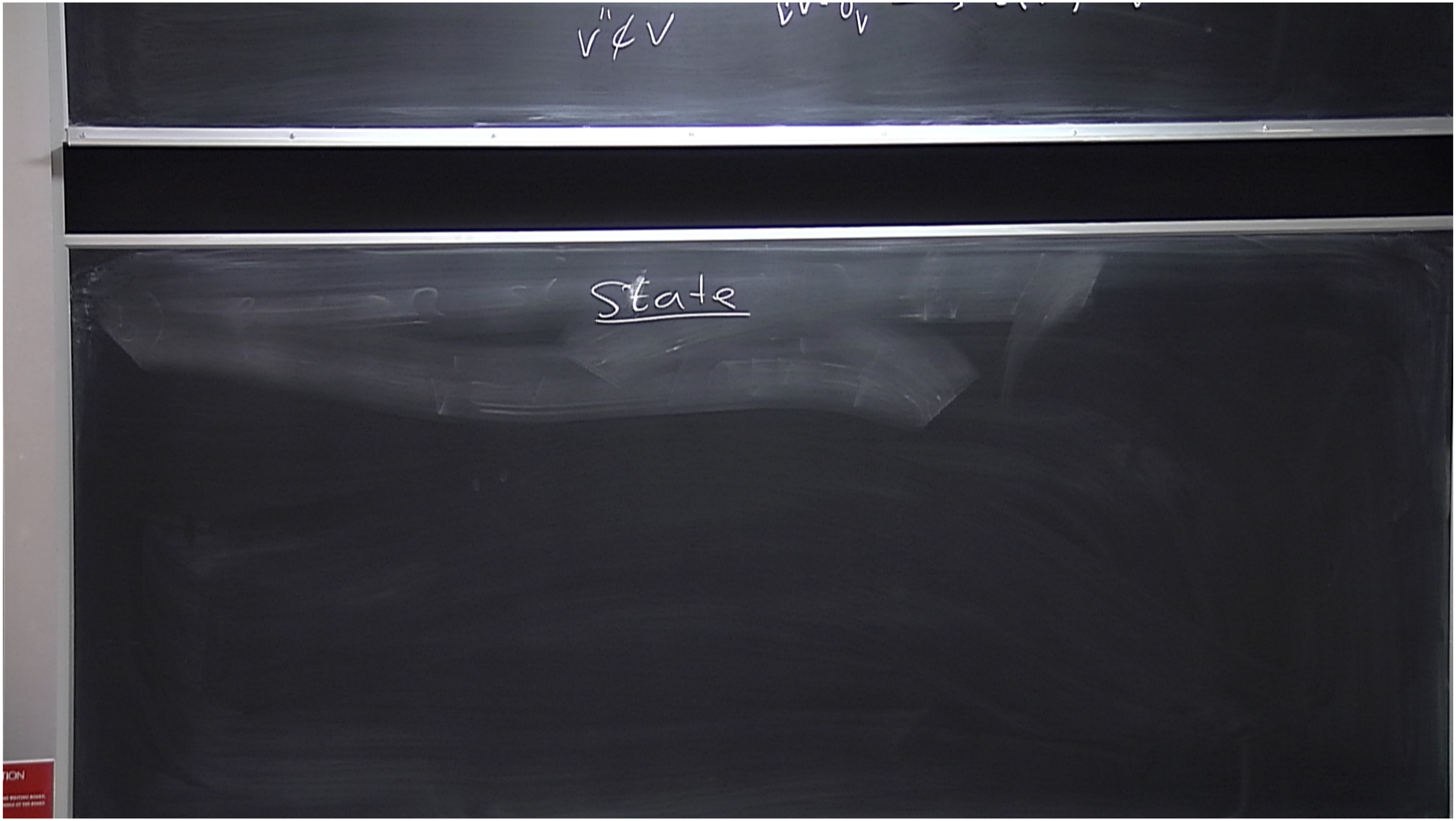
$$S_2 \rightarrow S_2$$

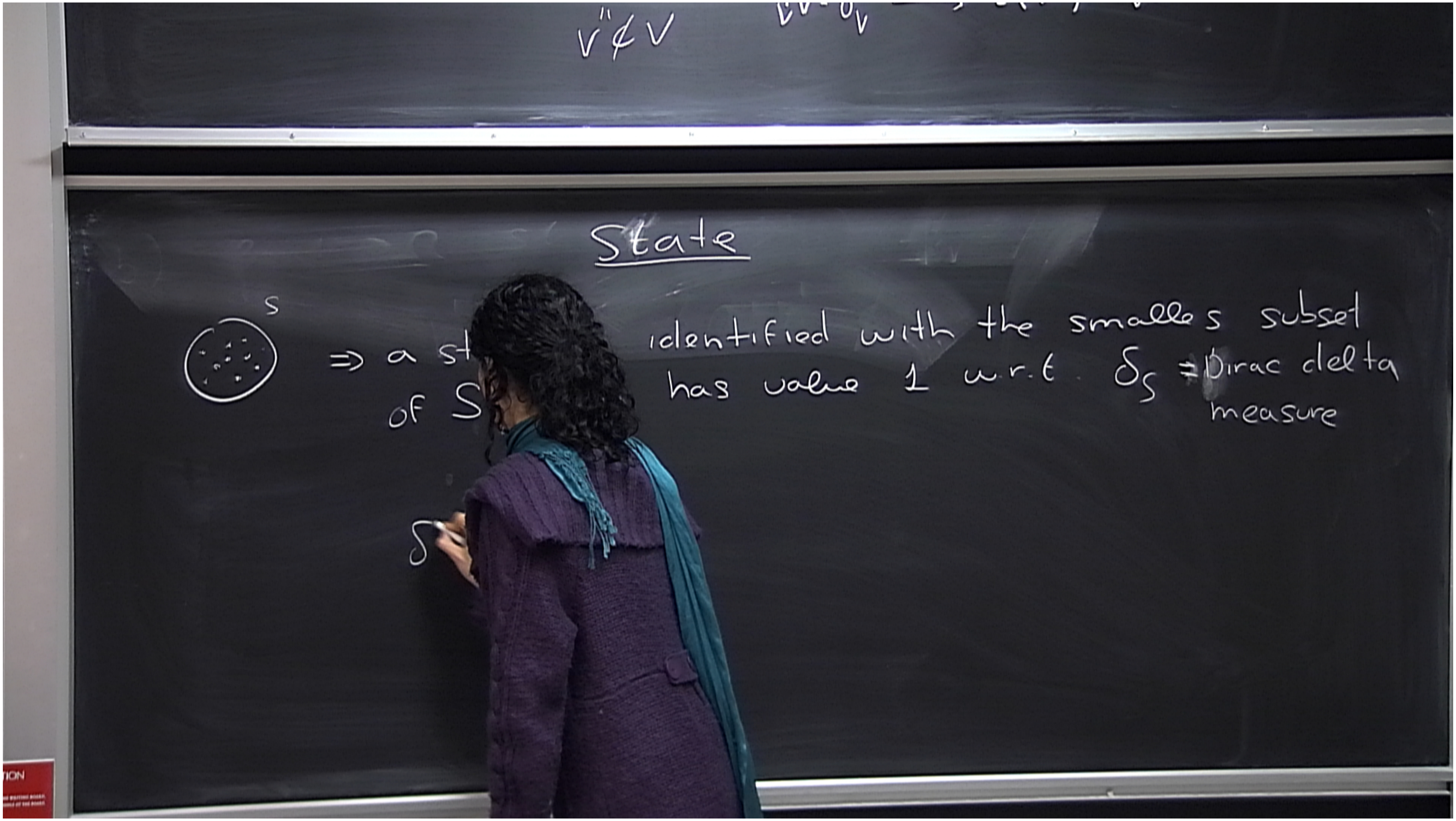
$$S_1 \rightarrow \emptyset$$

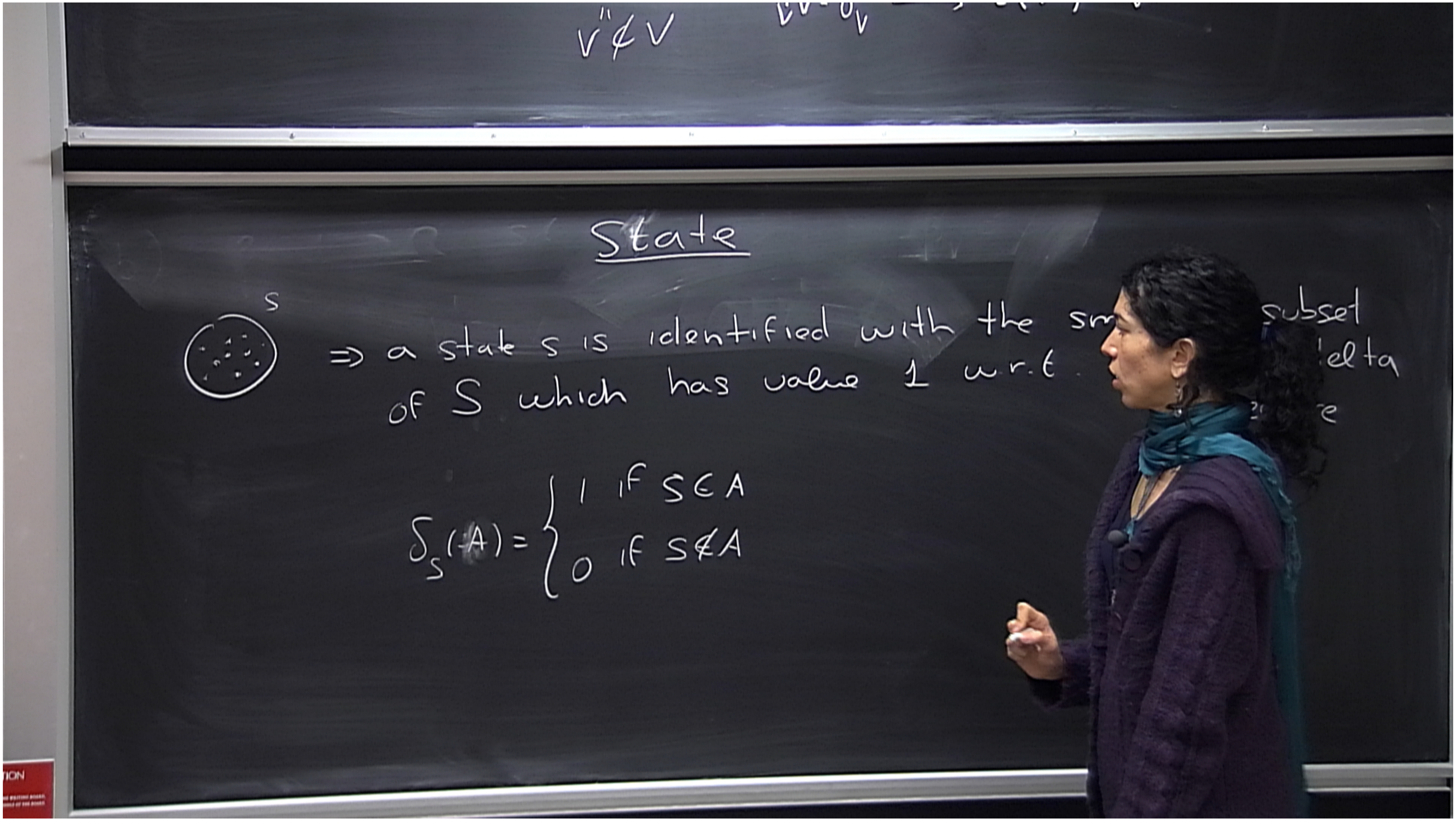
$$S_3 \rightarrow S_3$$

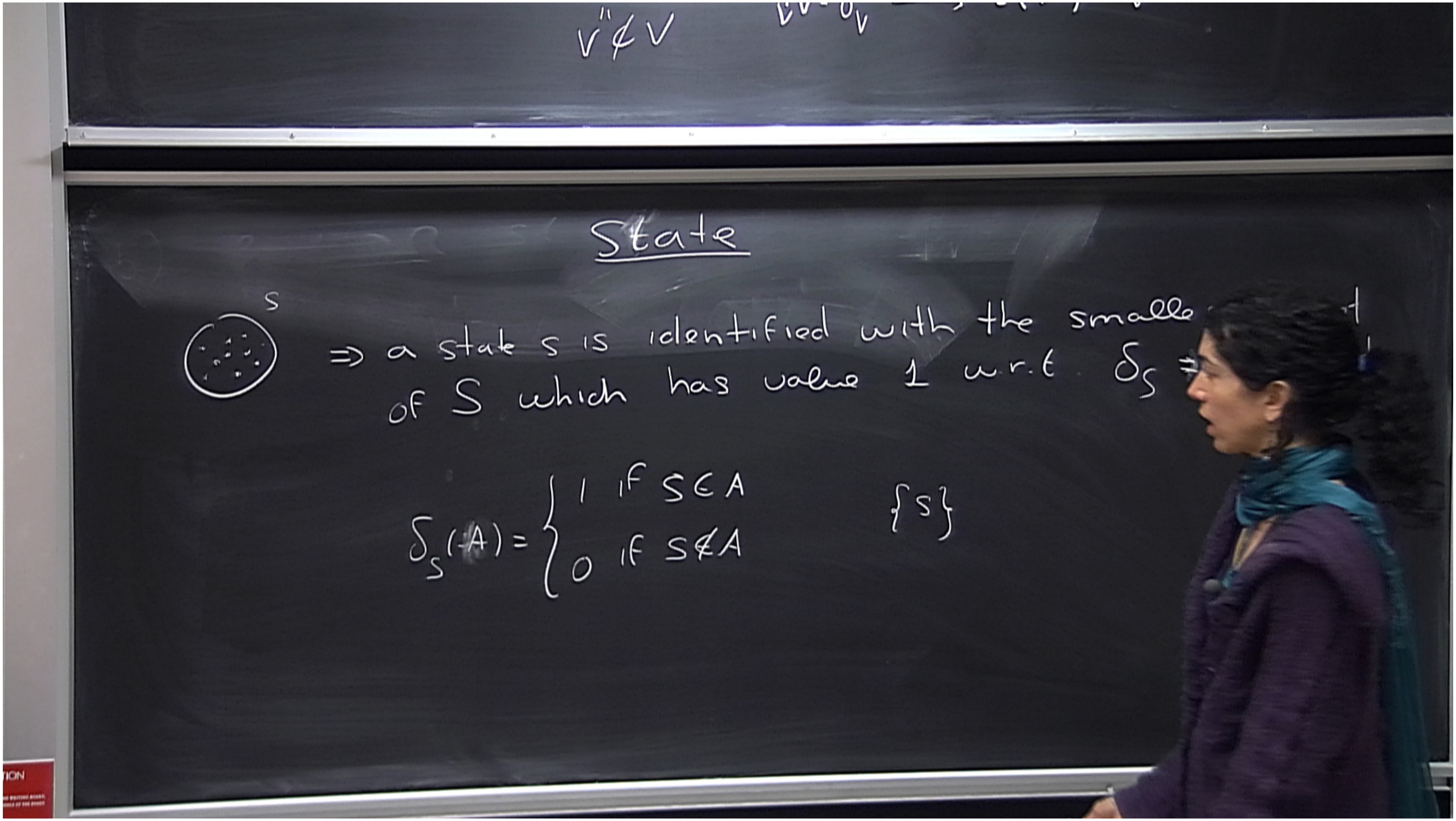
$$S_{3,4} \rightarrow S_3$$


⇒ Ω(V) is a Heyting algebra
V2,3 = linC | P2, P3, P̂1 + P̂4)
Ω(V2,3) = { ∅, S2,3, S2, S3, S'4 }
Ω(V2) = { ∅, S2 }
V2,3 ↪ V
Ω(V) → Ω(V2,3)
S → S ∩ ↓V2,3 = ↓V2,3 = S2,3
S2 → S2
S1 → ∅
S3 → S3
S3,4 → S3
∅ → ∅

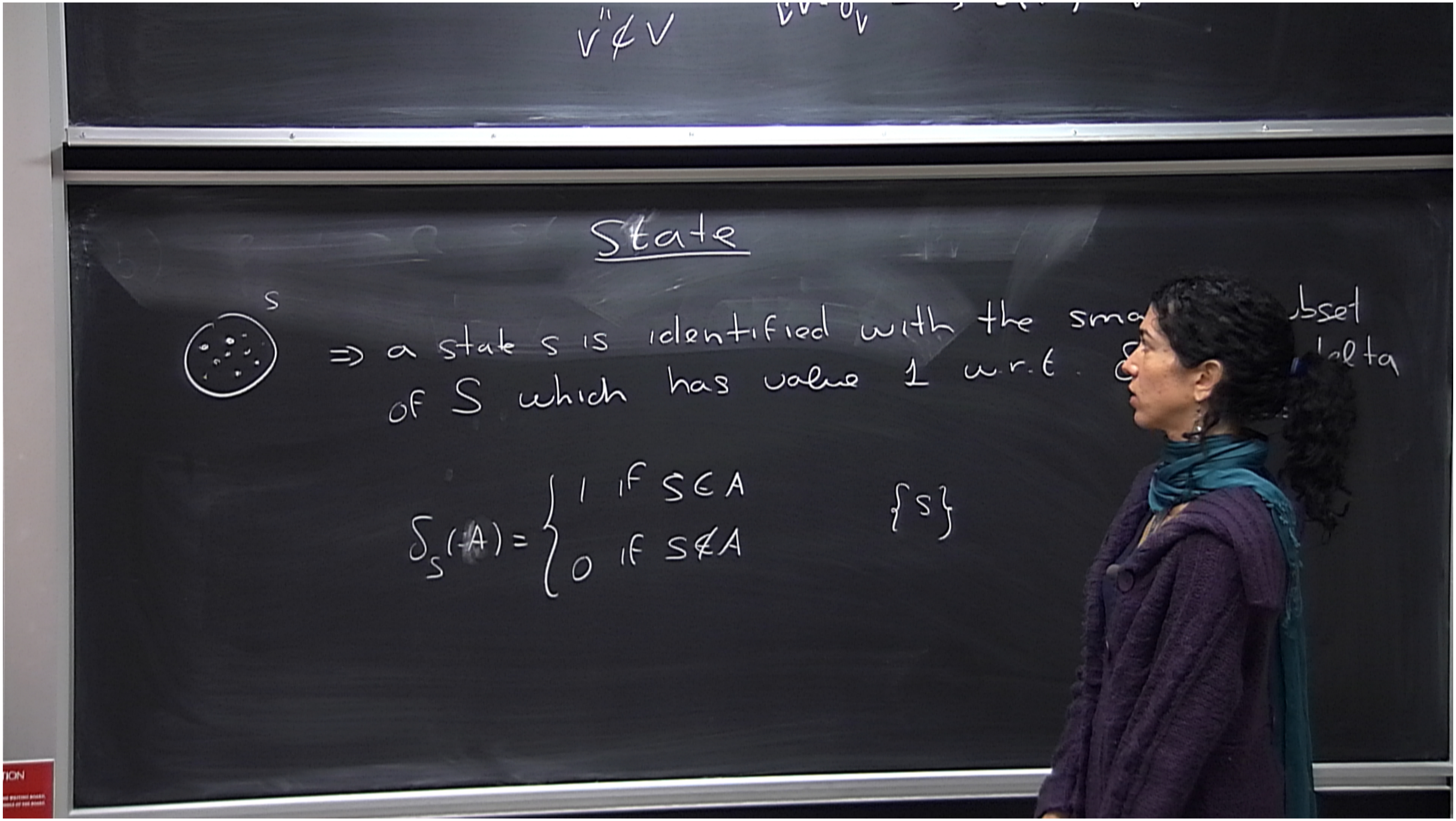

State

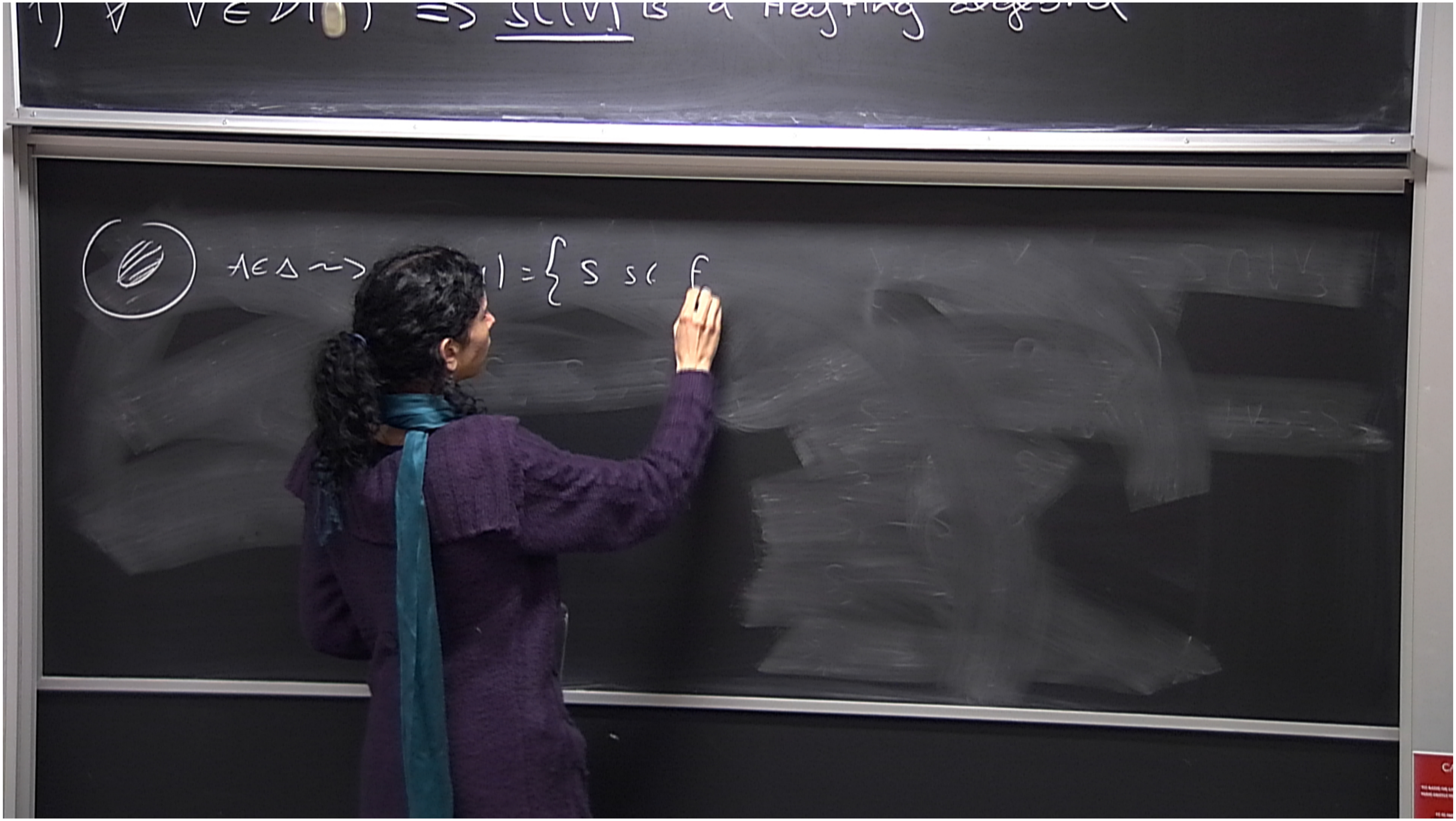

State
S
⇒ a st
identified with the smalles subset
has value 1 w.r.t
δS = Dirac delta
measure
of S

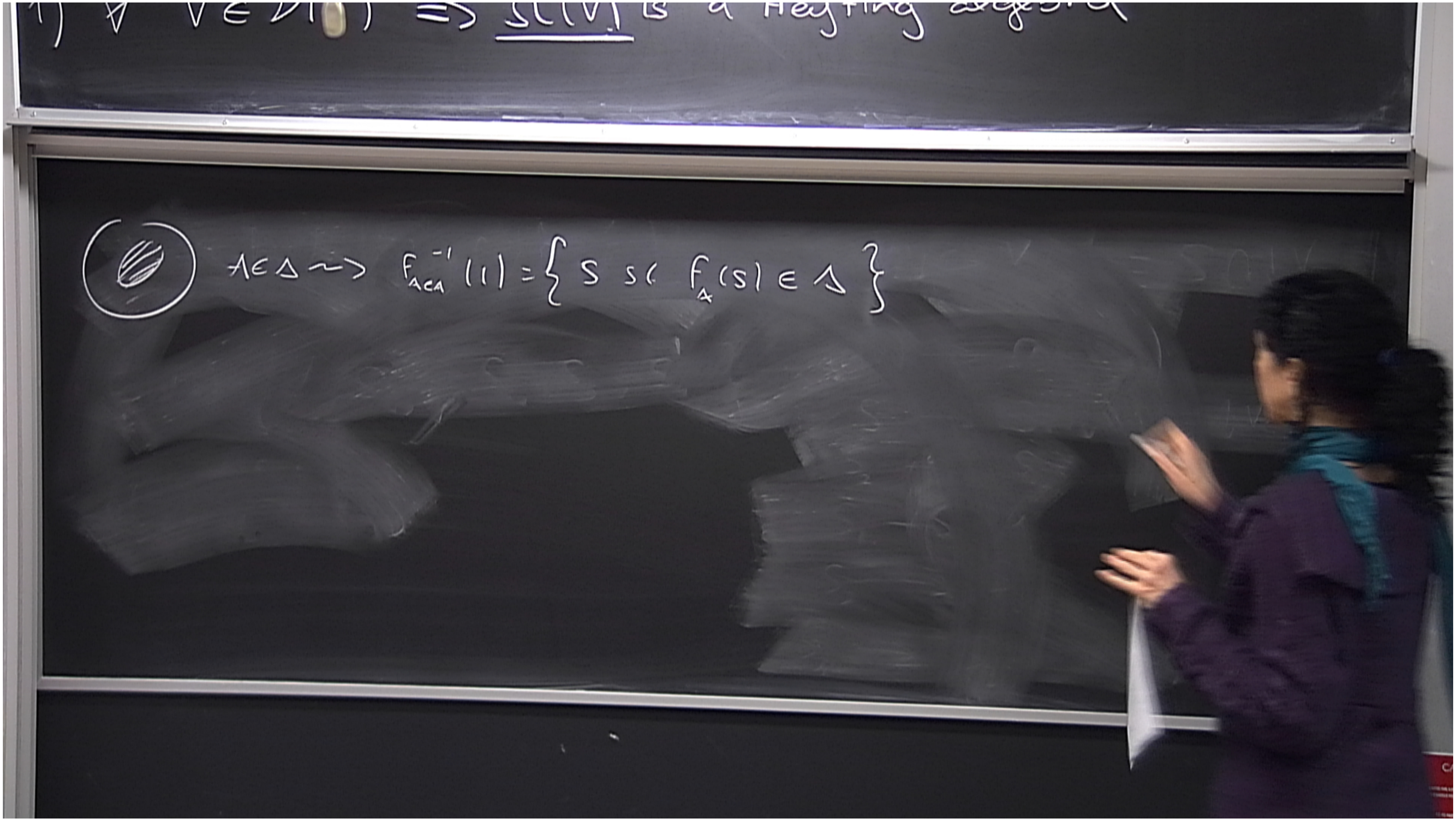

State
S
⇒ a state s is identified with the sm
subset
delta
of S which has value 1 w.r.t.
δs(A) = { 1 if s ∈ A ; 0 if s ∉ A

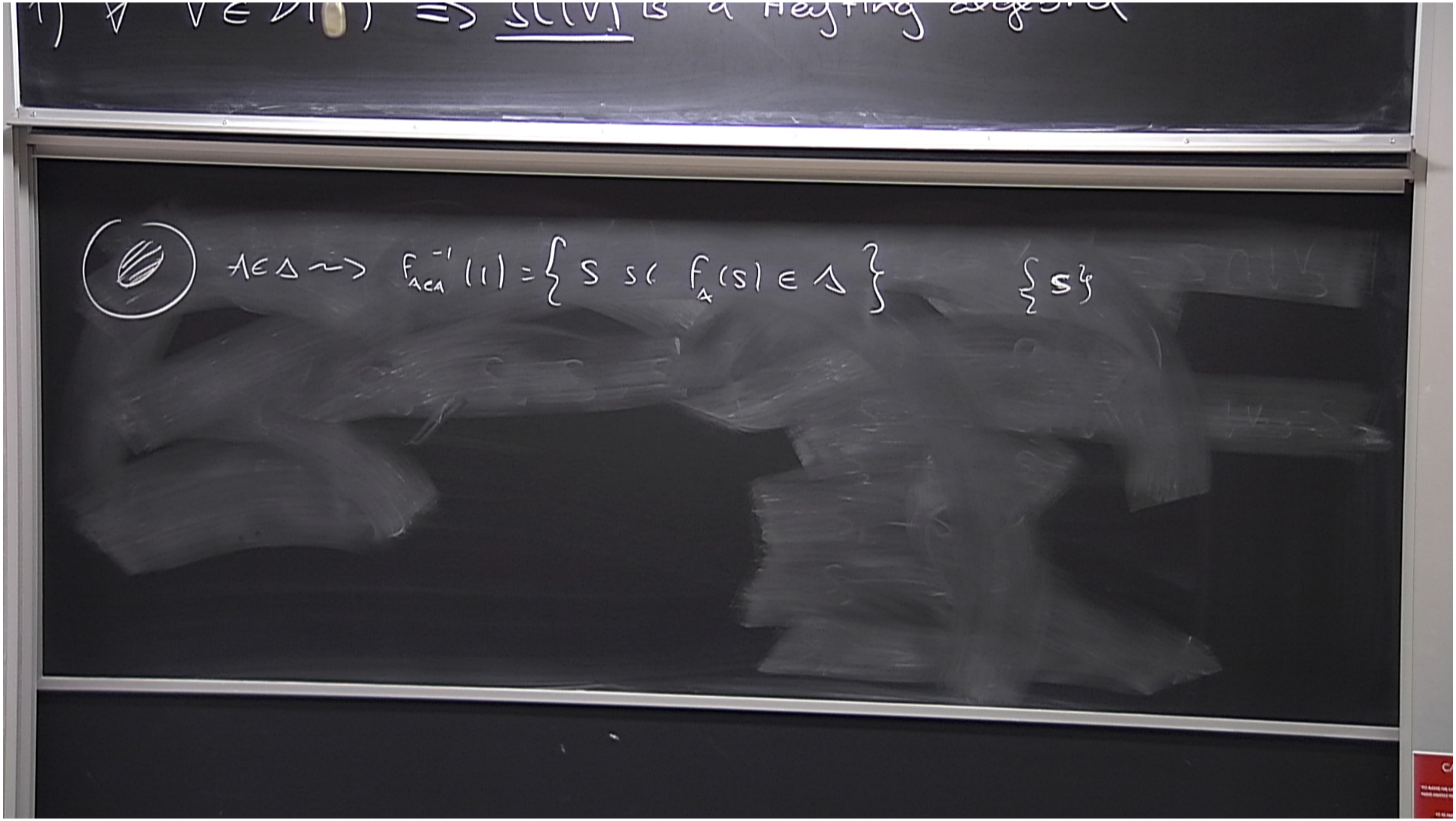


State
S
⇒ a state s is identified with the smalle
of S which has value 1 w.r.t δ_s
$\delta_s(A) = \begin{cases} 1 & \text{if } s \in A \\ 0 & \text{if } s \notin A \end{cases}$
{s}

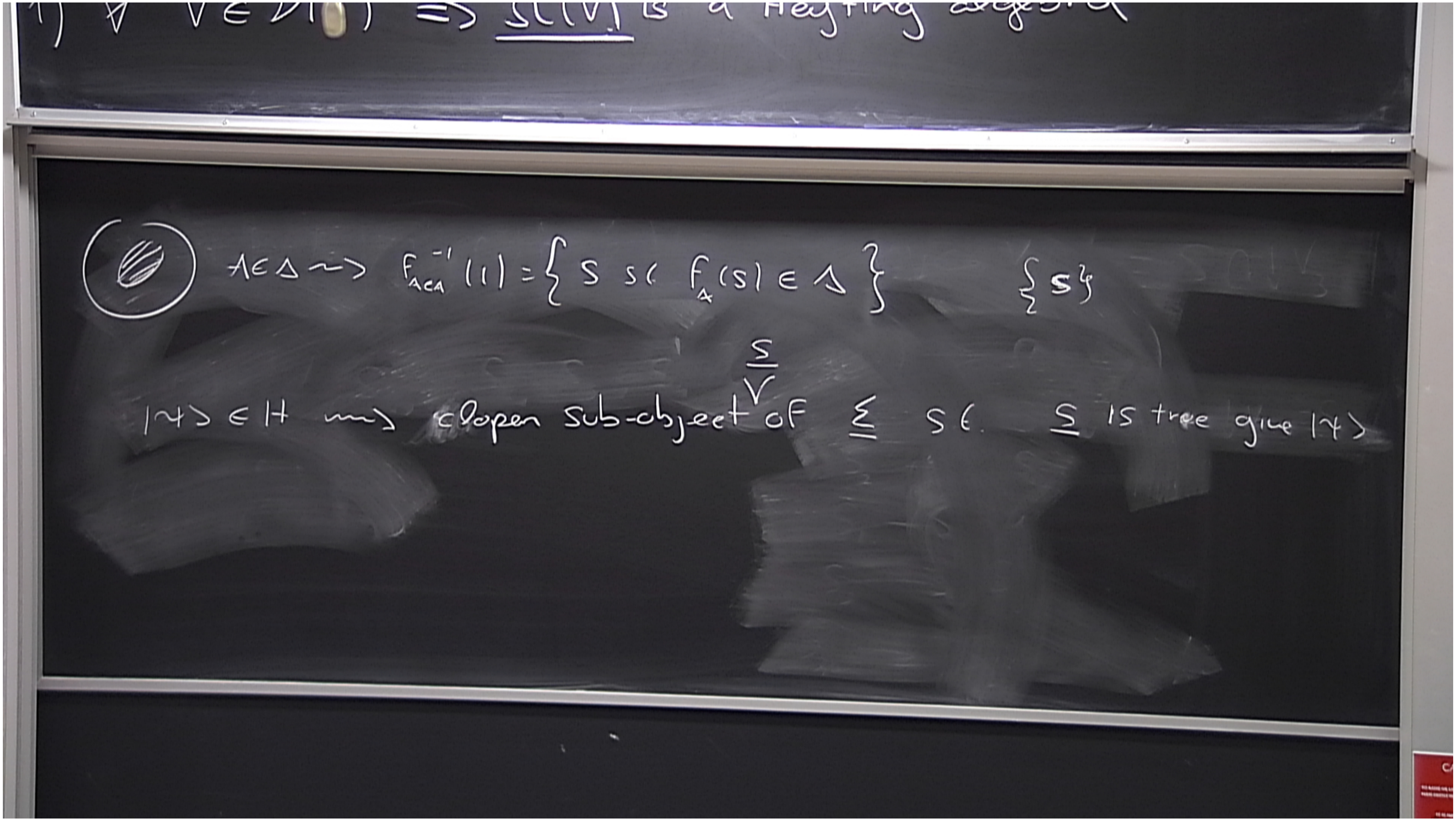

State
S
⇒ a state s is identified with the sma
bset
of S which has value 1 w.r.t.
elta
δs(A) = 1 if s∈A
0 if s∉A
{s}

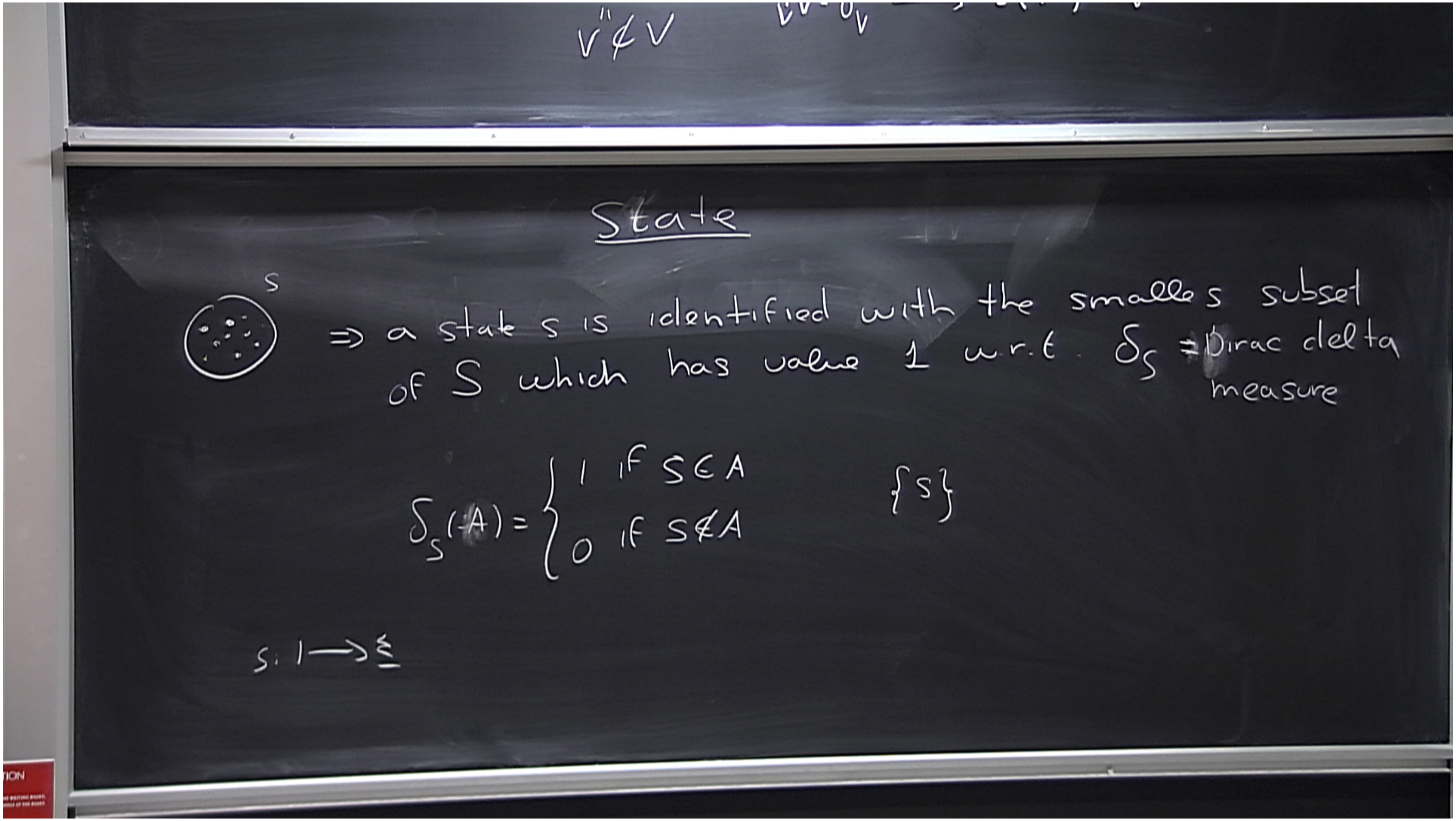


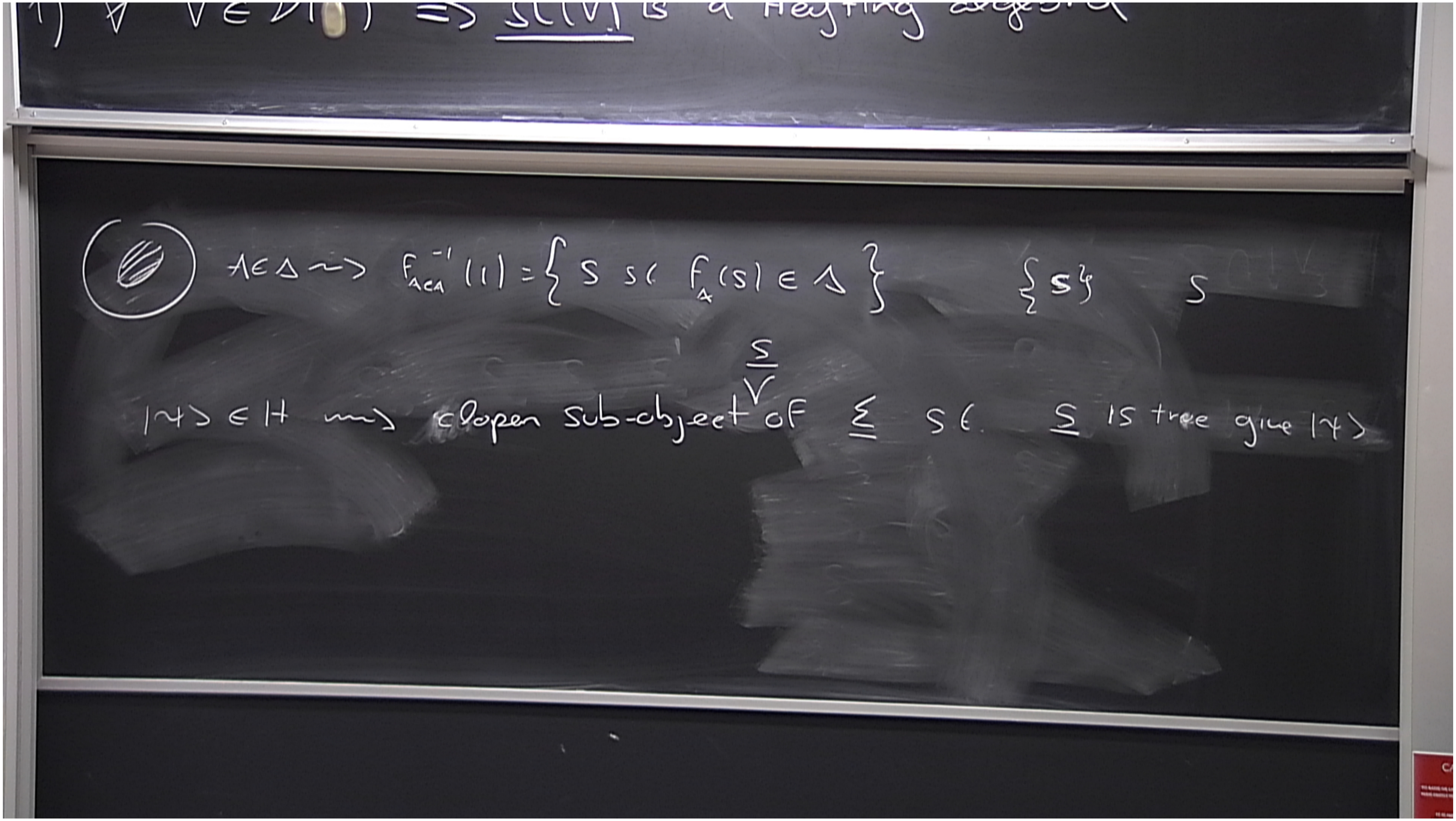

λ∈Δ ⇝ F_λ∈Δ^-1(1) = { s s(F_λ(s) ∈ Δ }



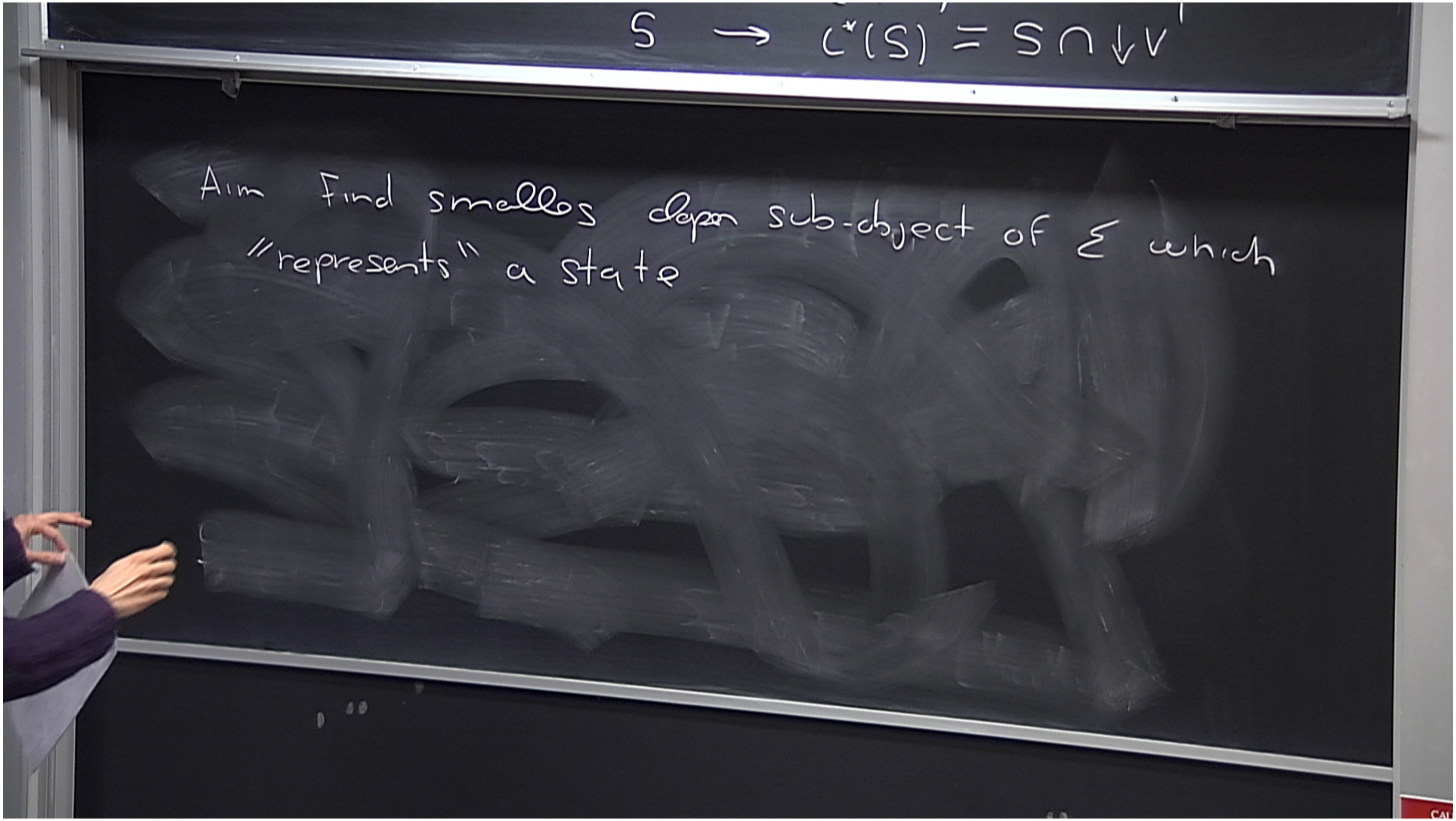
⇒ S(V) is a Heyting algebra
λ ∈ Δ ⇝ f⁻¹(1) = { s s.t. f(s) ∈ Δ }
{s}

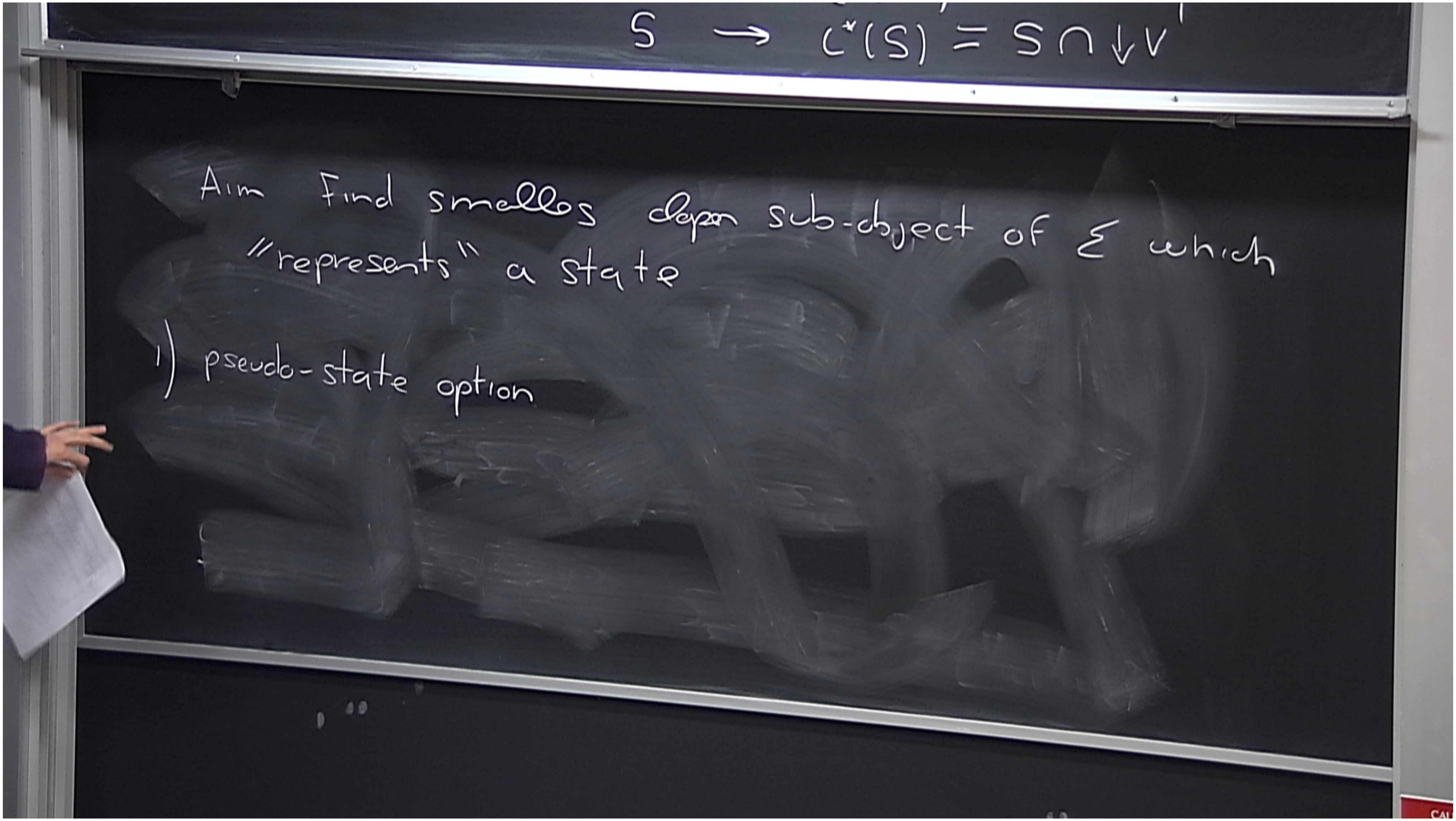

⇒ S(V) is a Heyting algebra
λ∈Δ ~> f⁻¹λ∈Δ (1) = { s s.t. fλ(s) ∈ Δ }
{s}
|ψ> ∈ H ~> clopen sub-object of Σ s.t. Σ is true give |ψ>

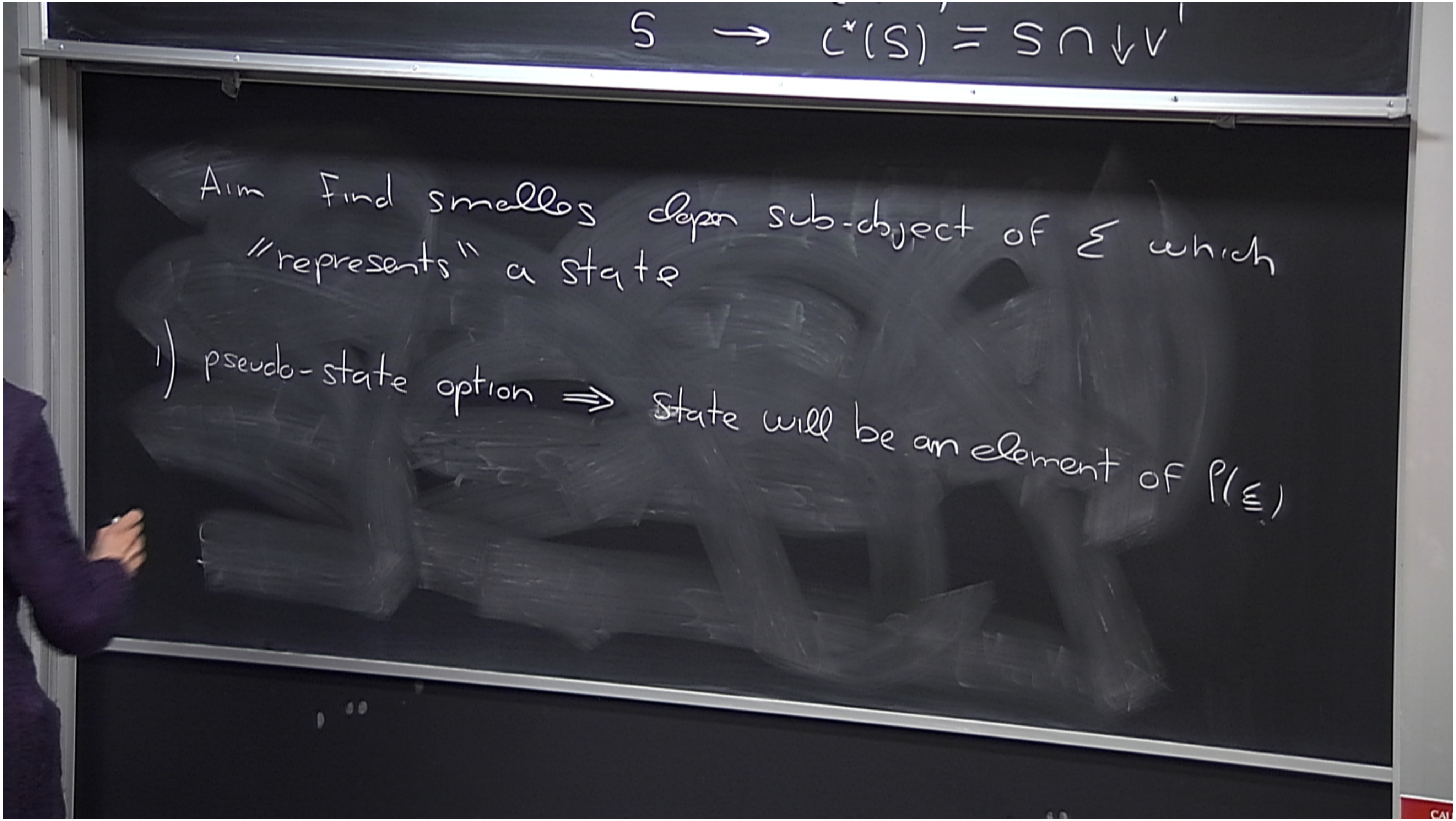

V'' ∉ V
State
S
⇒ a state s is identified with the smalles subset of S which has value 1 w.r.t. δS ≡ Dirac delta measure
δS(A) = 1 if s ∈ A, 0 if s ∉ A
{s}
s: 1 ⟶ S

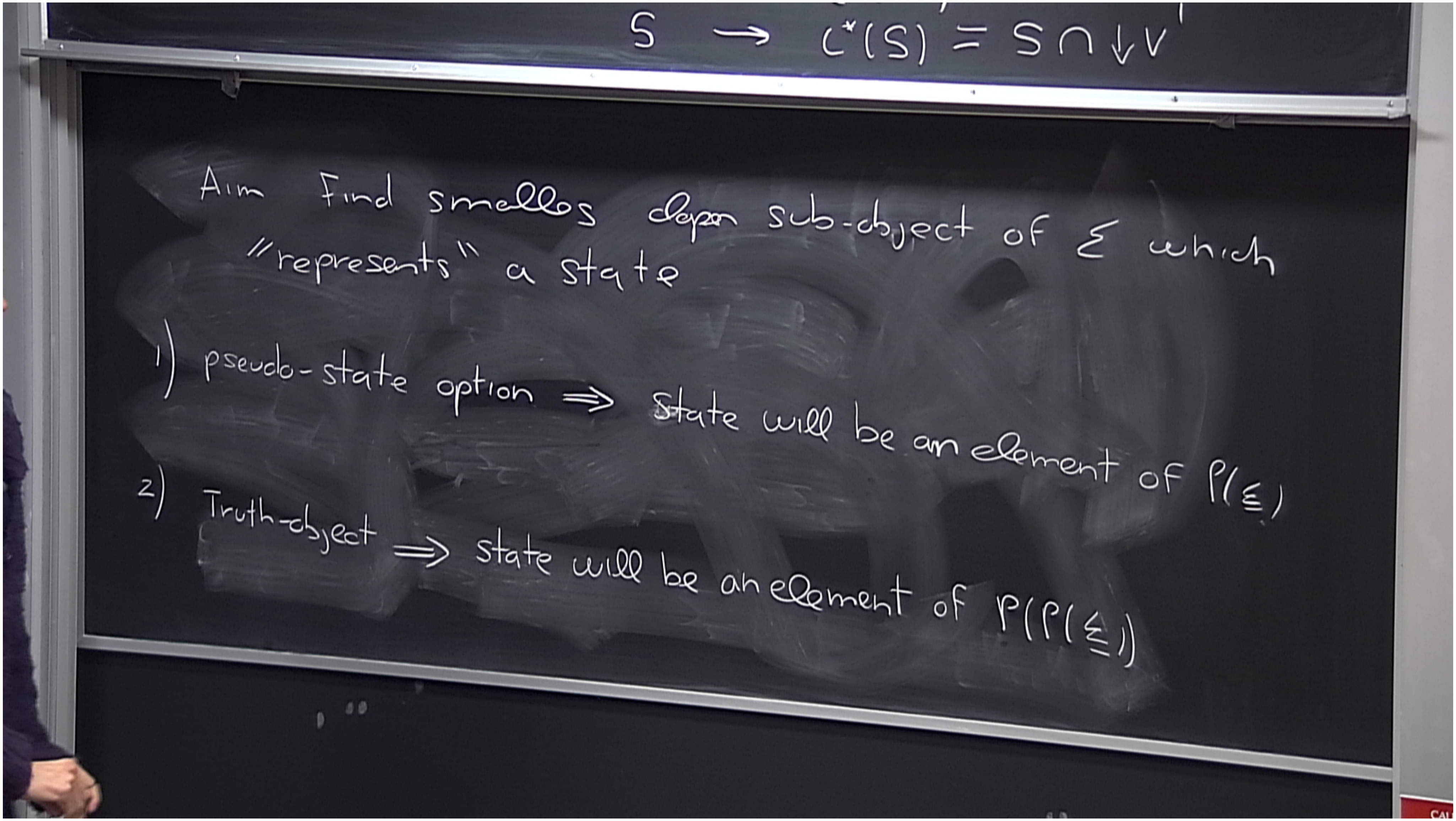


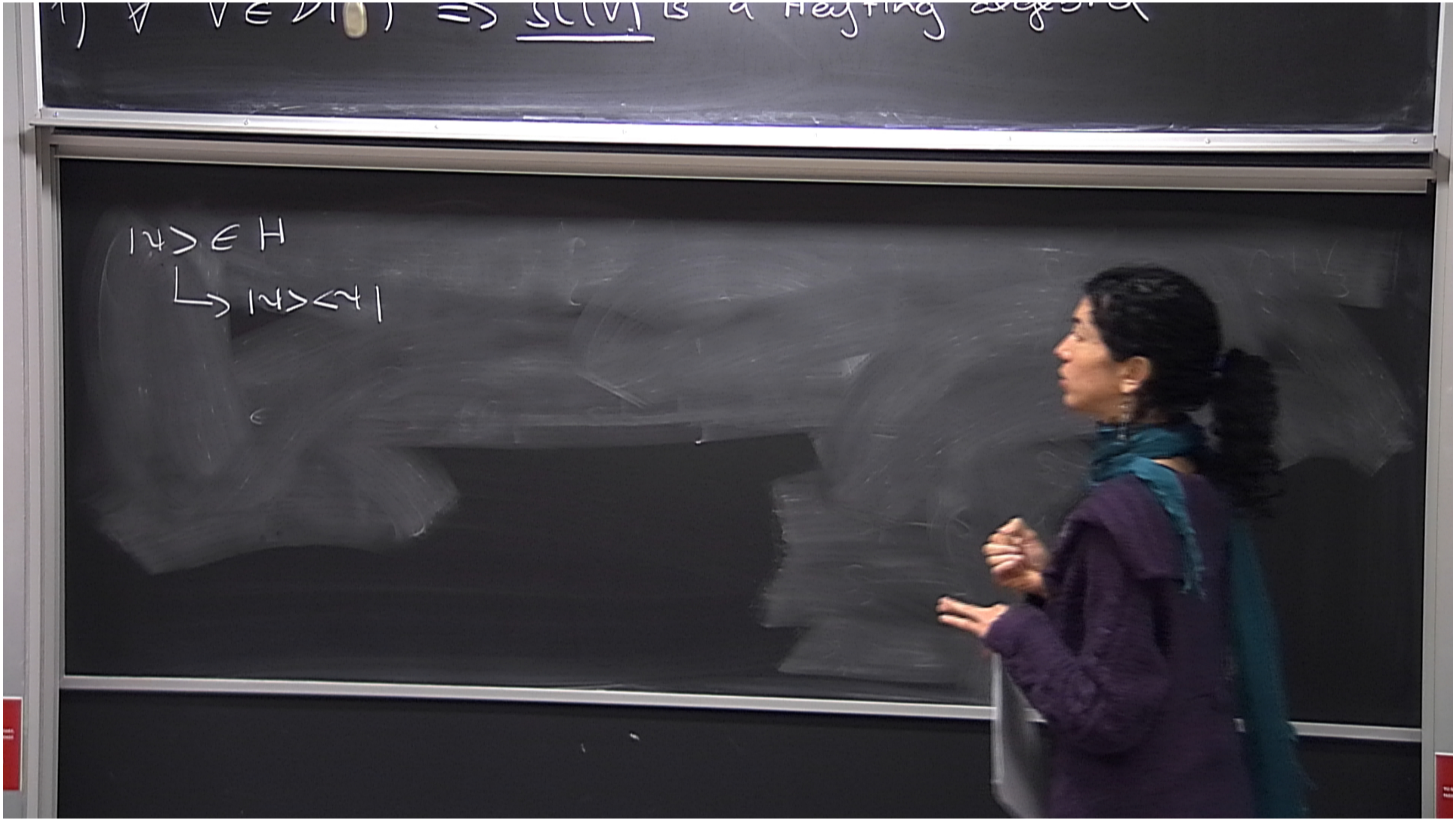
Ω(V) is a Heyting algebra
λ∈Δ ~> f⁻¹λ∈Δ(1) = { s s(fλ(s) ∈ Δ }
{s}
S
|ψ> ∈ H ~> clopen sub-object of Σ s(Σ is true give |ψ>

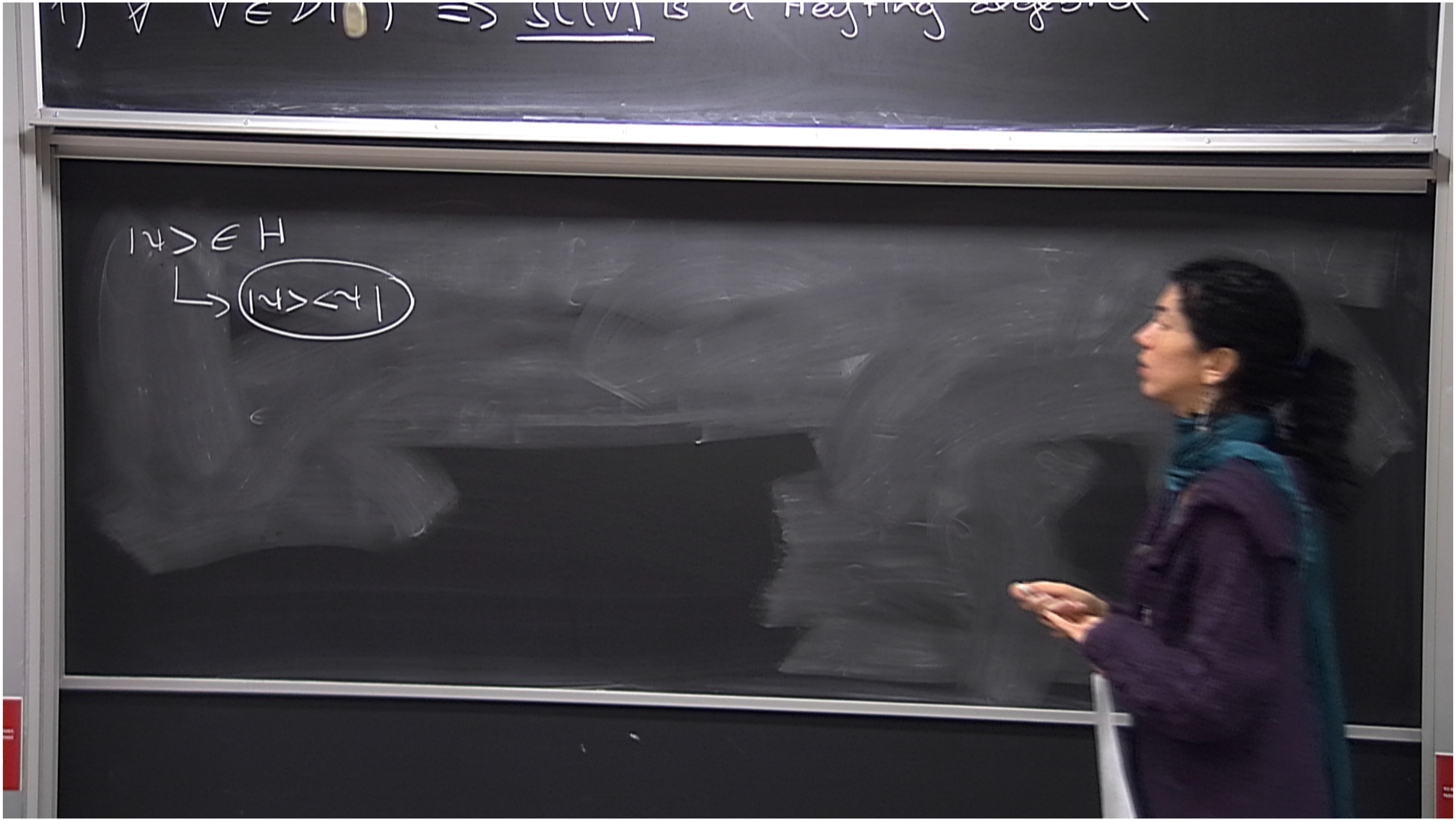

S → ι*(S) = S ∩ ↓V
Aim Find smallest clopen sub-object of Σ which
"represents" a state

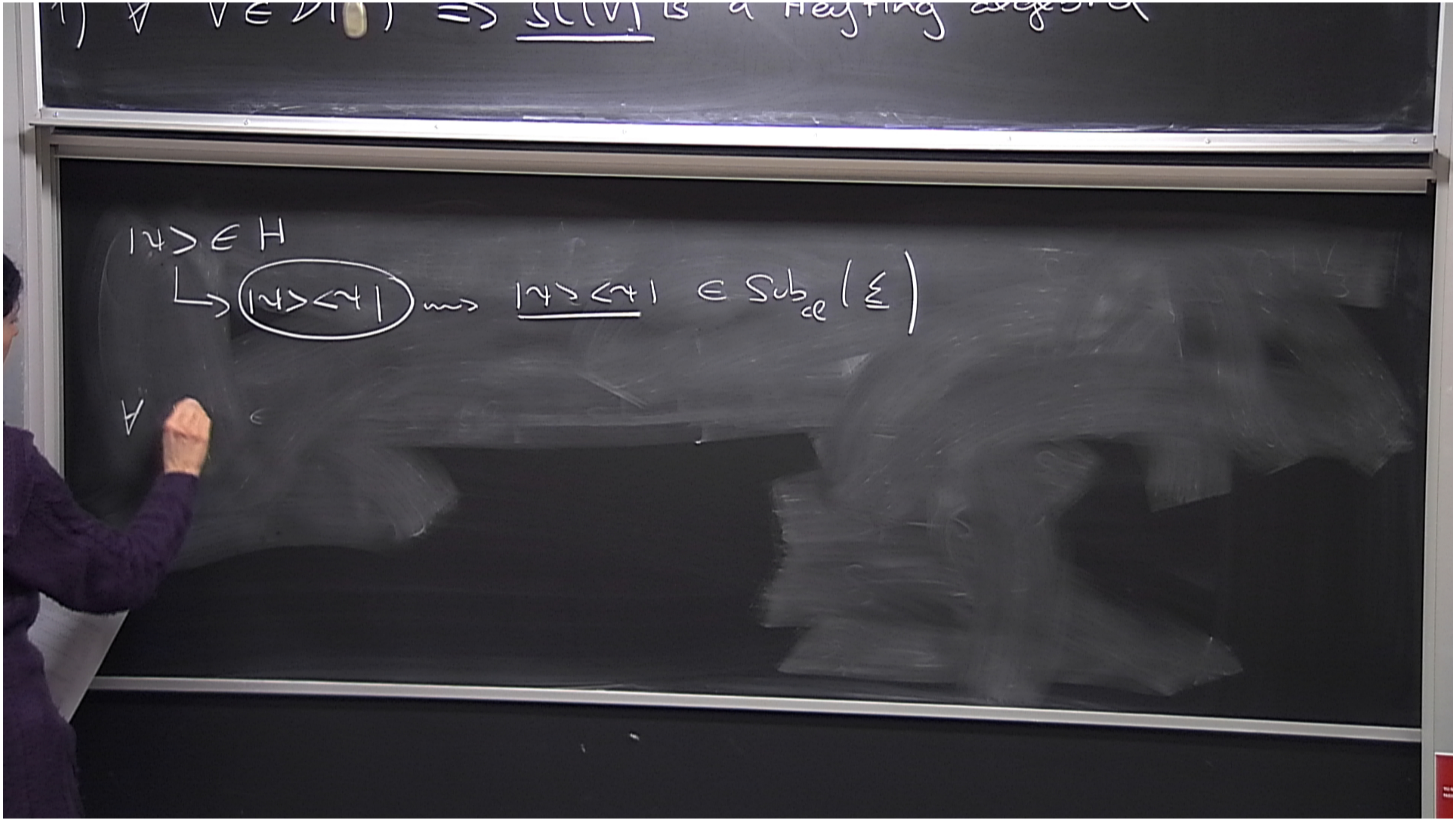

S → ι*(S) = S ∩ ↓V
Aim Find smallest clopen sub-object of Σ which
"represents" a state
1) pseudo-state option

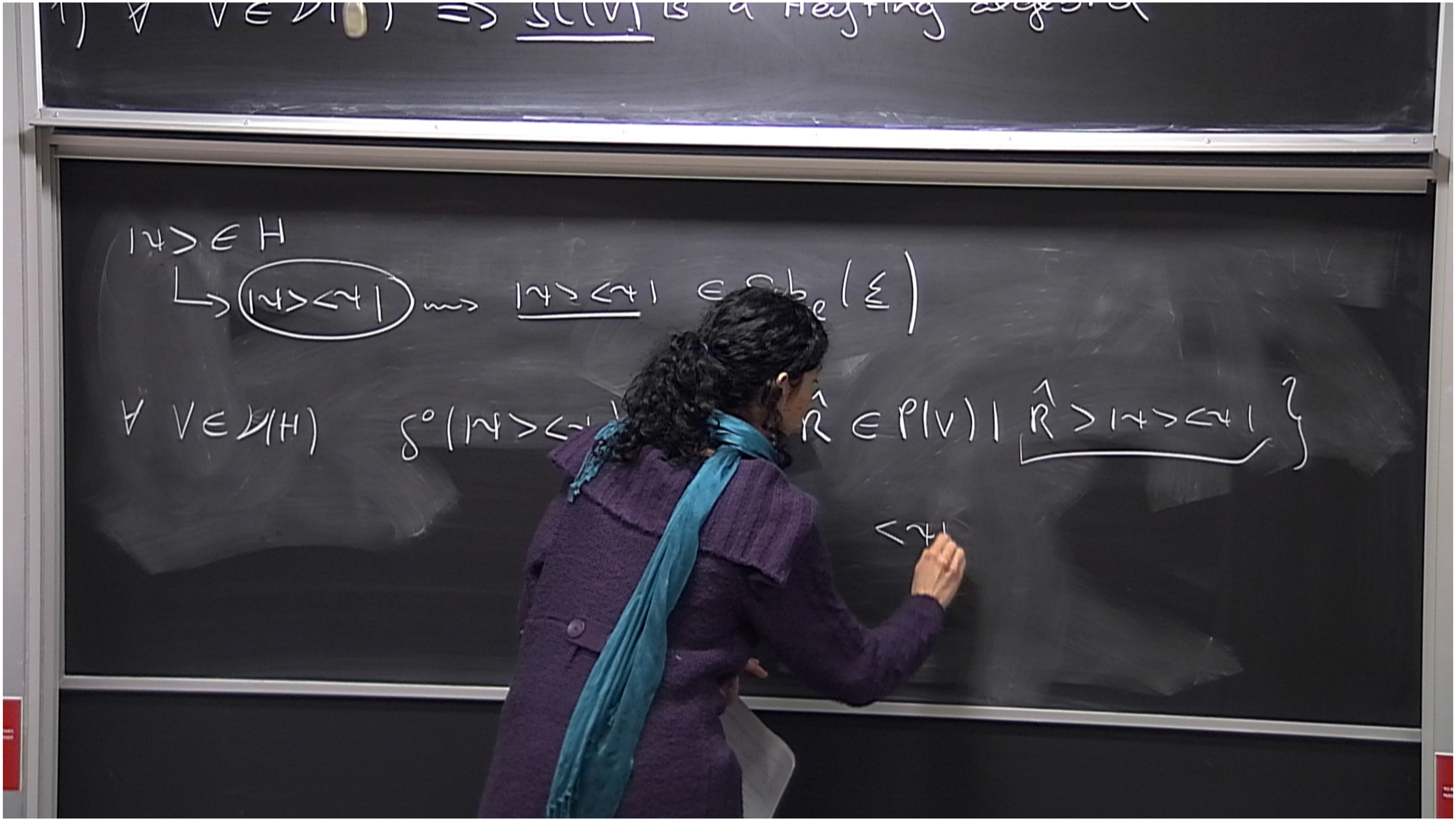

S → c*(S) = S ∩ ↓V
Aim Find smallest clopen sub-object of Σ which "represents" a state
1) pseudo-state option ⇒ state will be an element of P(Σ)

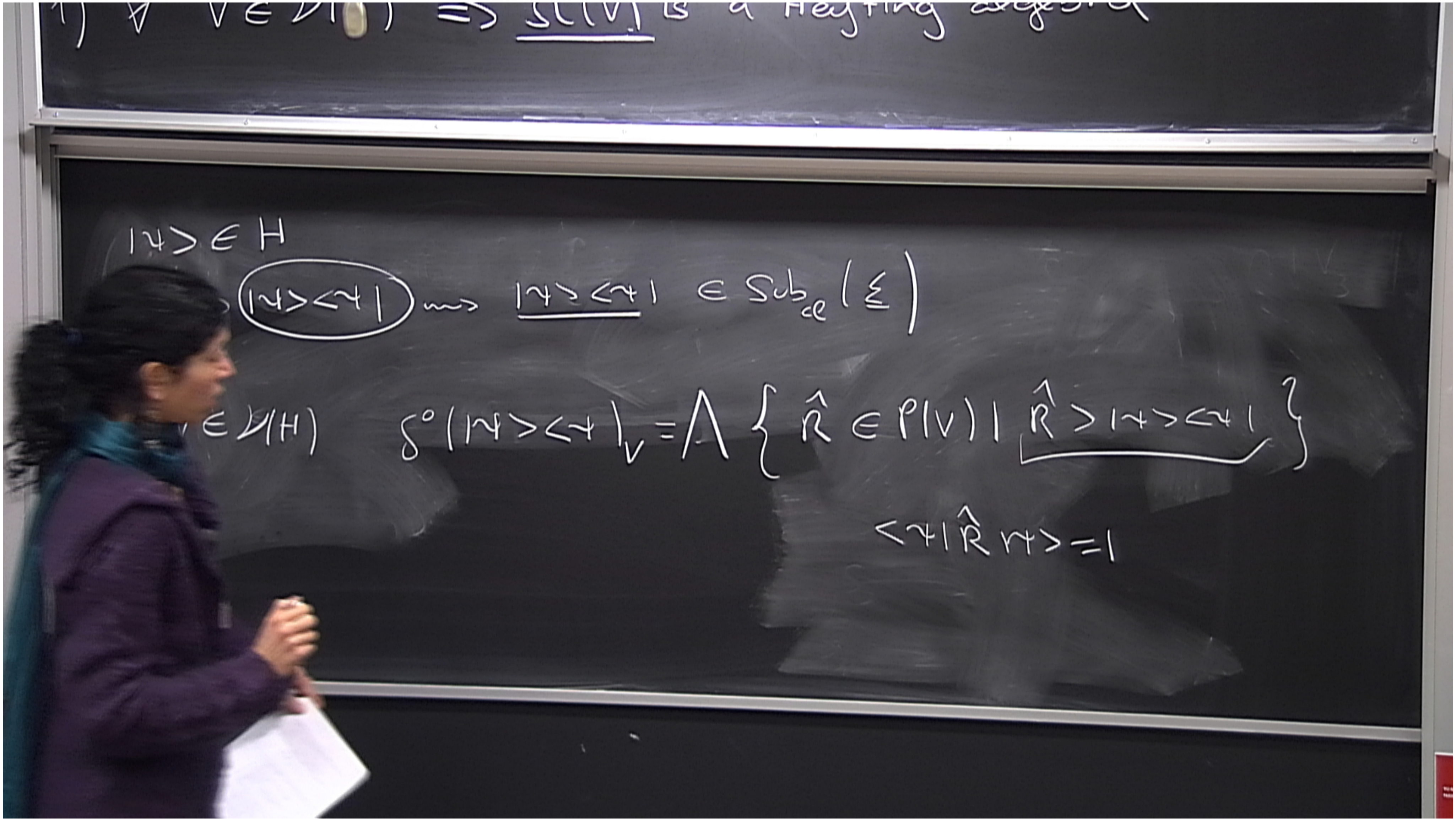

S → c*(S) = S ∩ ↓V
Aim Find smallest clopen sub-object of Σ which "represents" a state
1) pseudo-state option ⇒ State will be an element of P(Σ)
2) Truth-object ⇒ state will be an element of P(P(Σ))

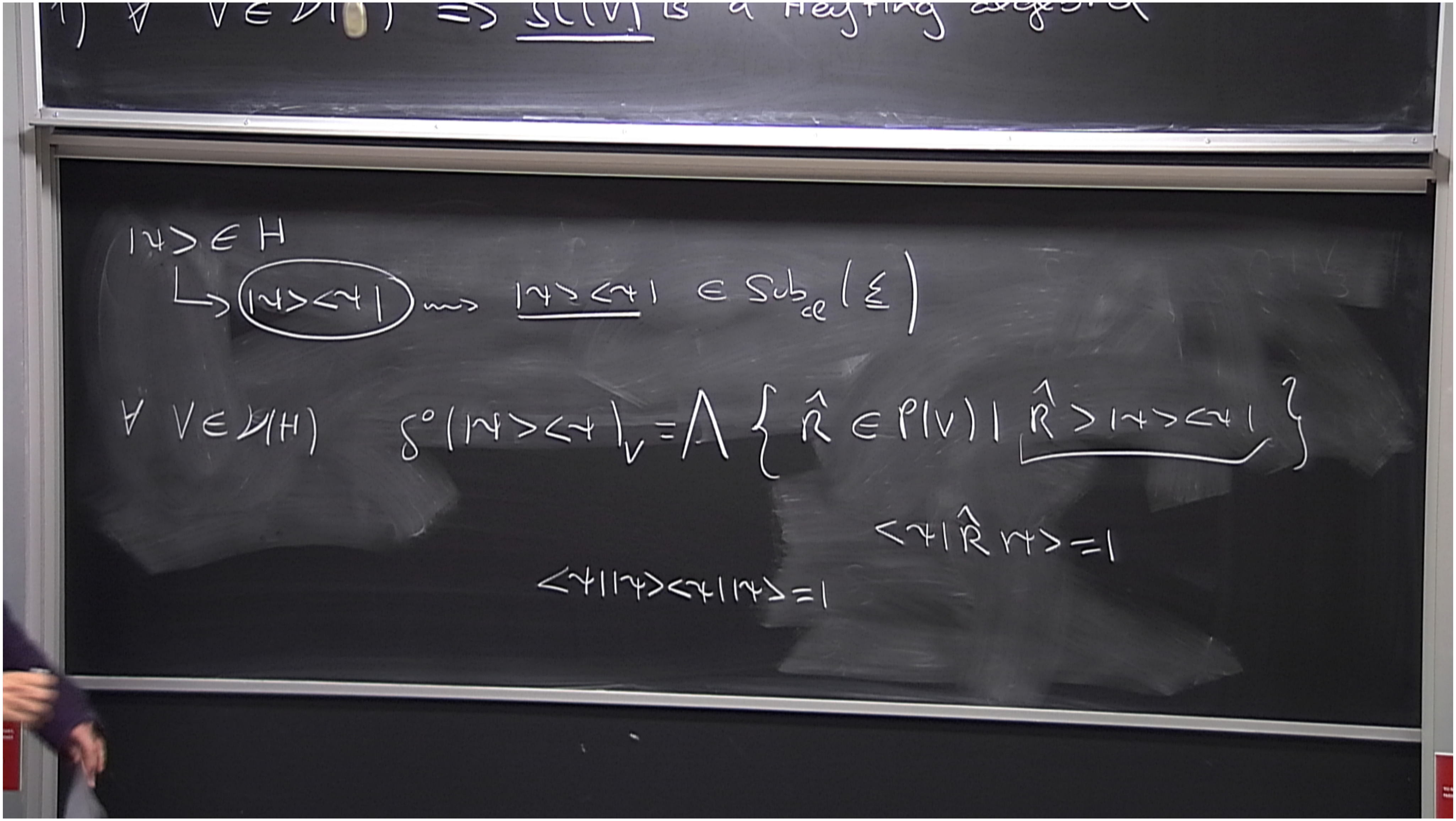

|ψ> ∈ H
↳ |ψ><ψ|

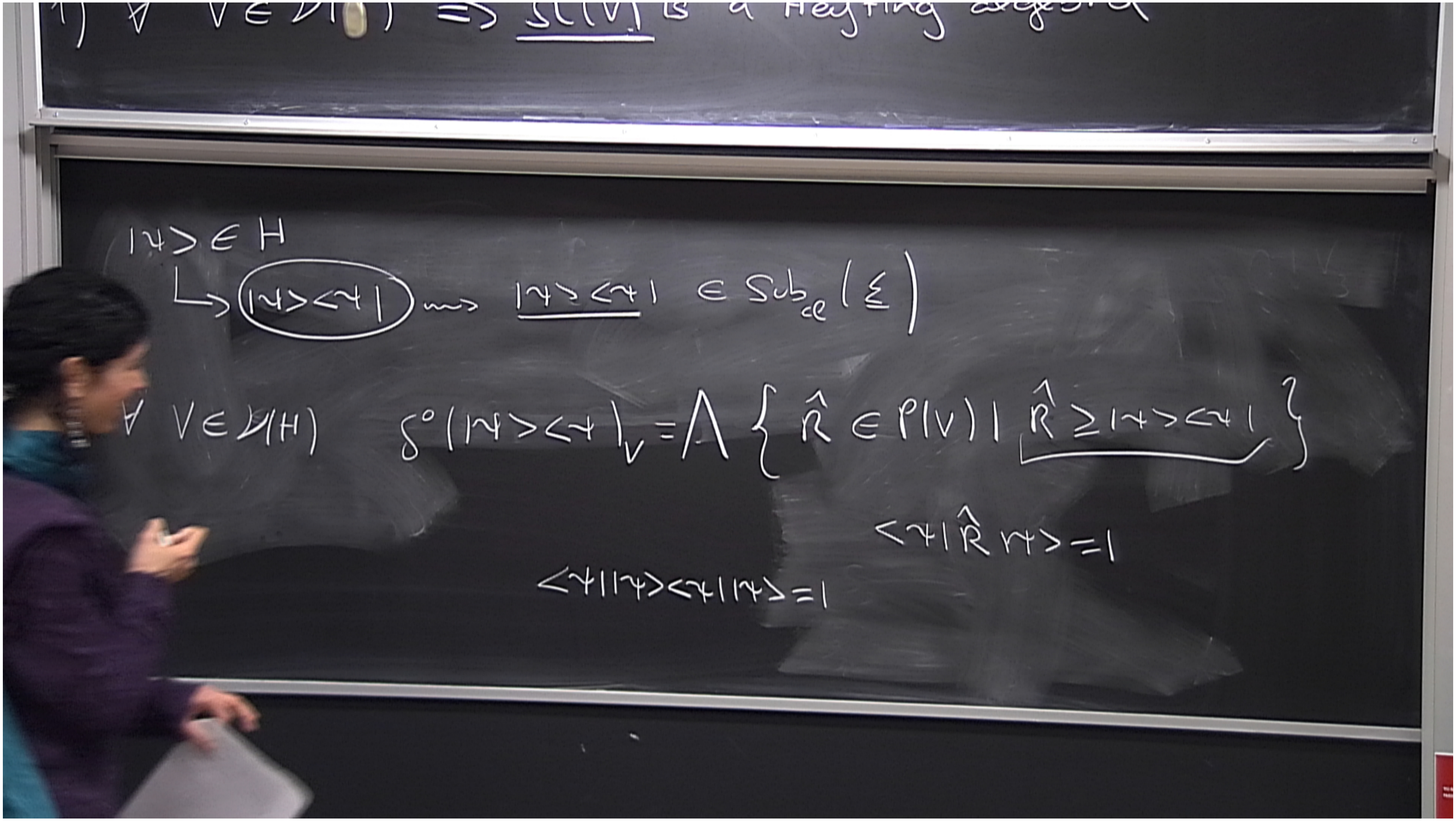

|ψ> ∈ H
↳ |ψ><ψ|

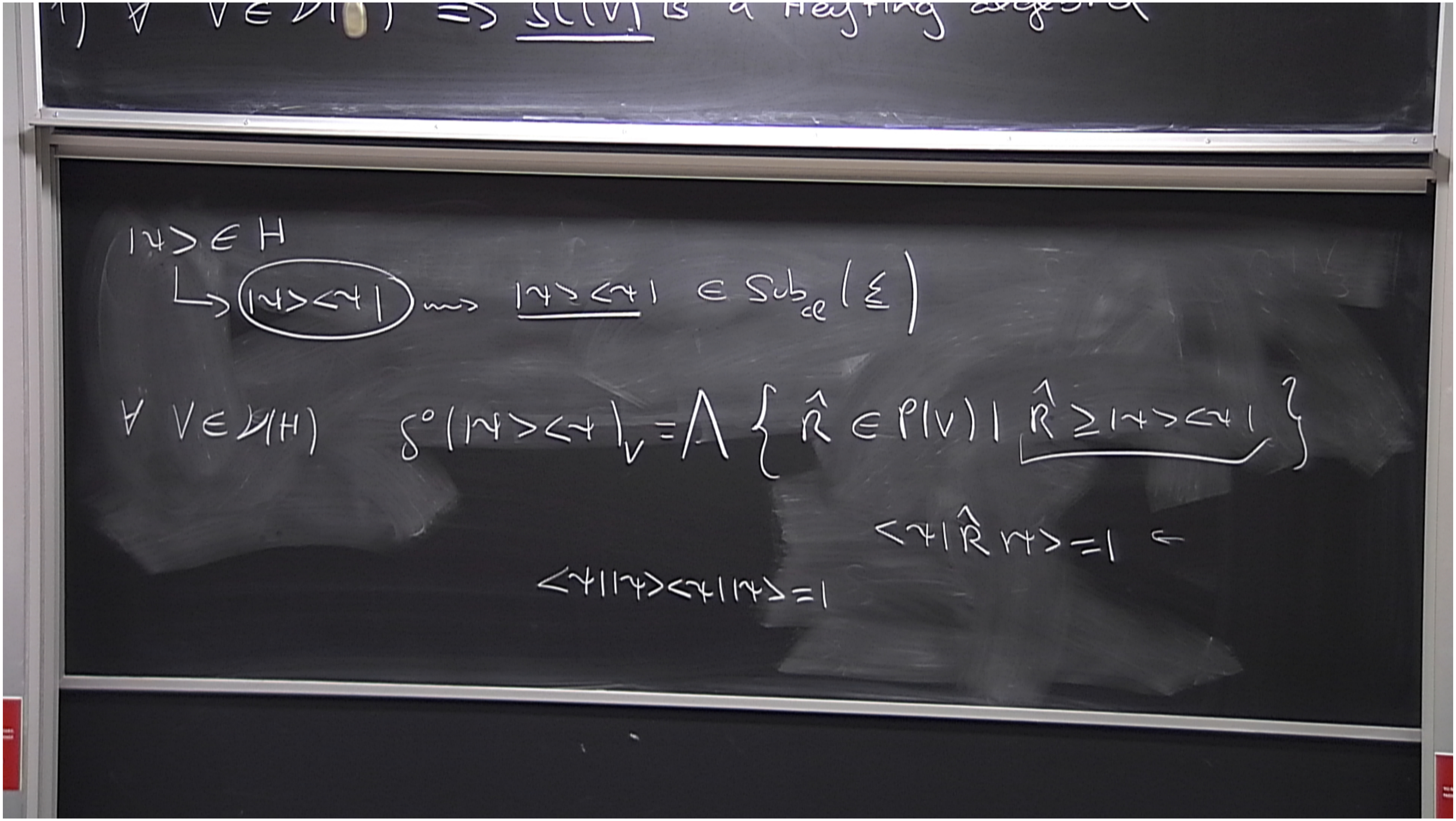

⇒ S(V) is a Heyting algebra
|ψ> ∈ H
↳ |ψ><ψ| ⟿ |ψ><ψ| ∈ Sub_cl(Σ)
∀

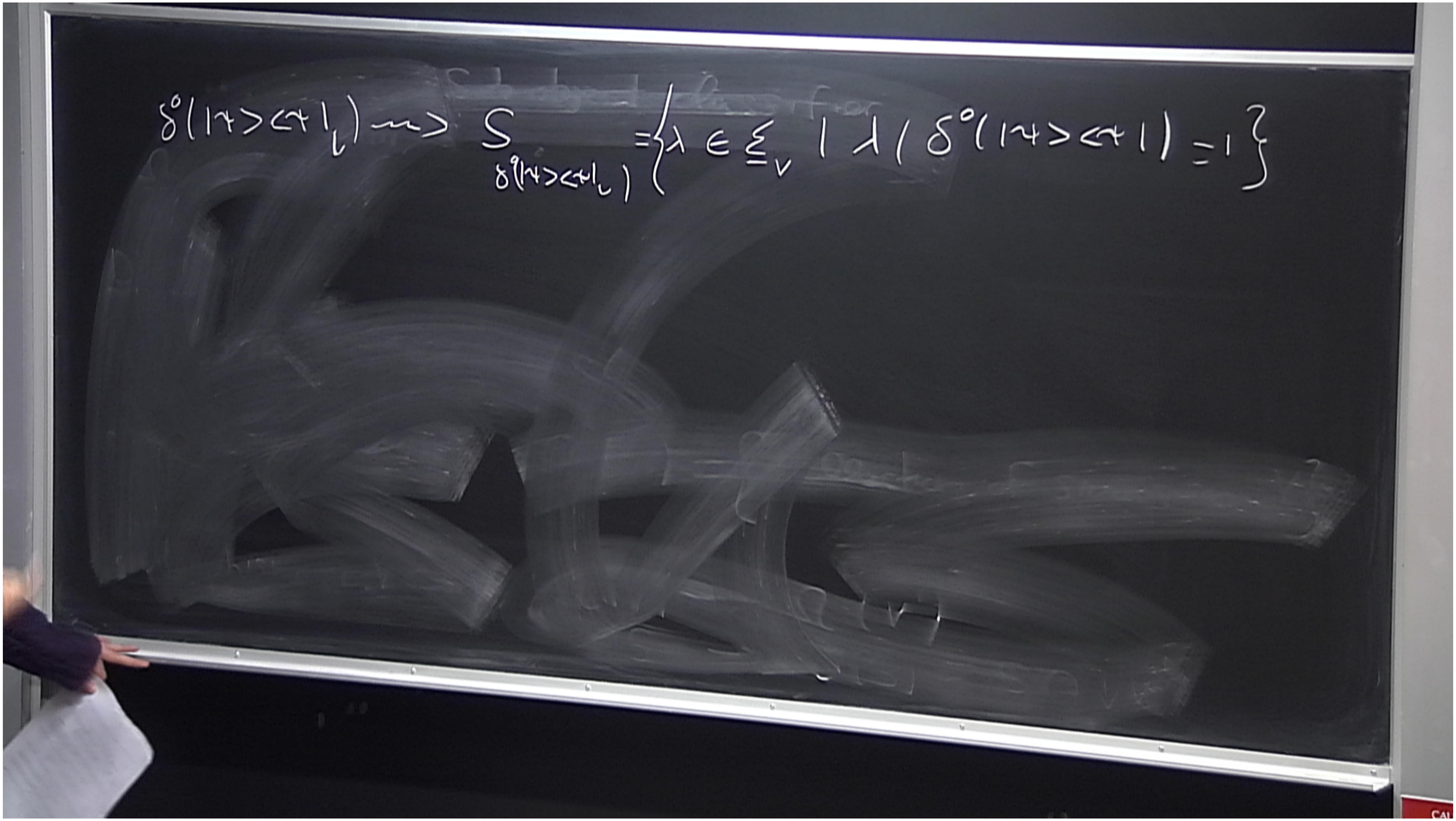

is a Heyting algebra
|ψ> ∈ H
|ψ><ψ| ∈
∀ V ∈ V(H)
R̂ ∈ P(V) | R̂ ≥ |ψ><ψ| }
<ψ|

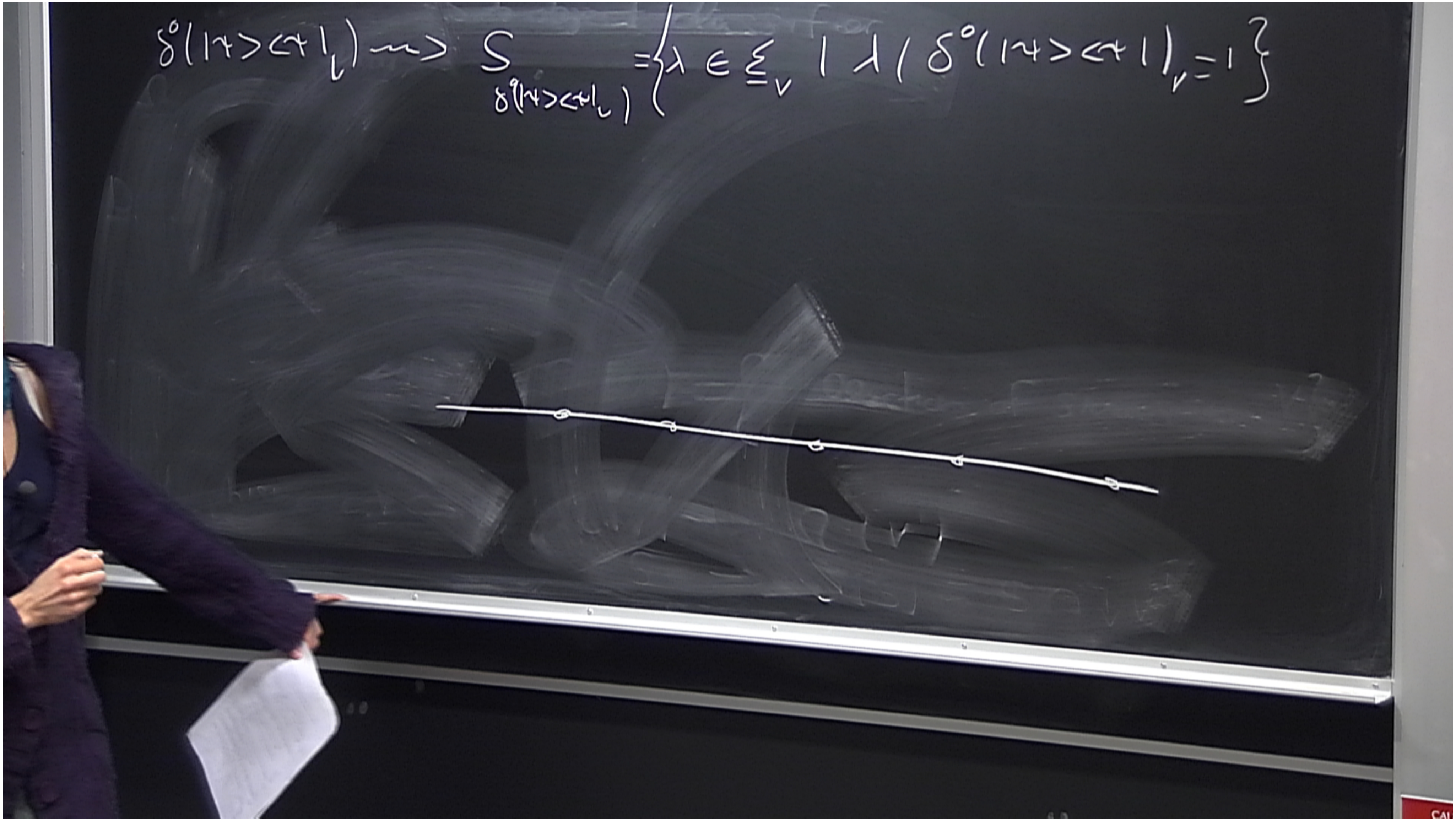

⇒ Sub(V) is a Heyting algebra
|ψ> ∈ H
|ψ><ψ| ⇝ |ψ><ψ| ∈ Sub_cl(Σ)
∈ V(H)
δ°(|ψ><ψ|)_V = ∧{R̂ ∈ P(V) | R̂ ≥ |ψ><ψ|}
<ψ|R̂|ψ> = 1

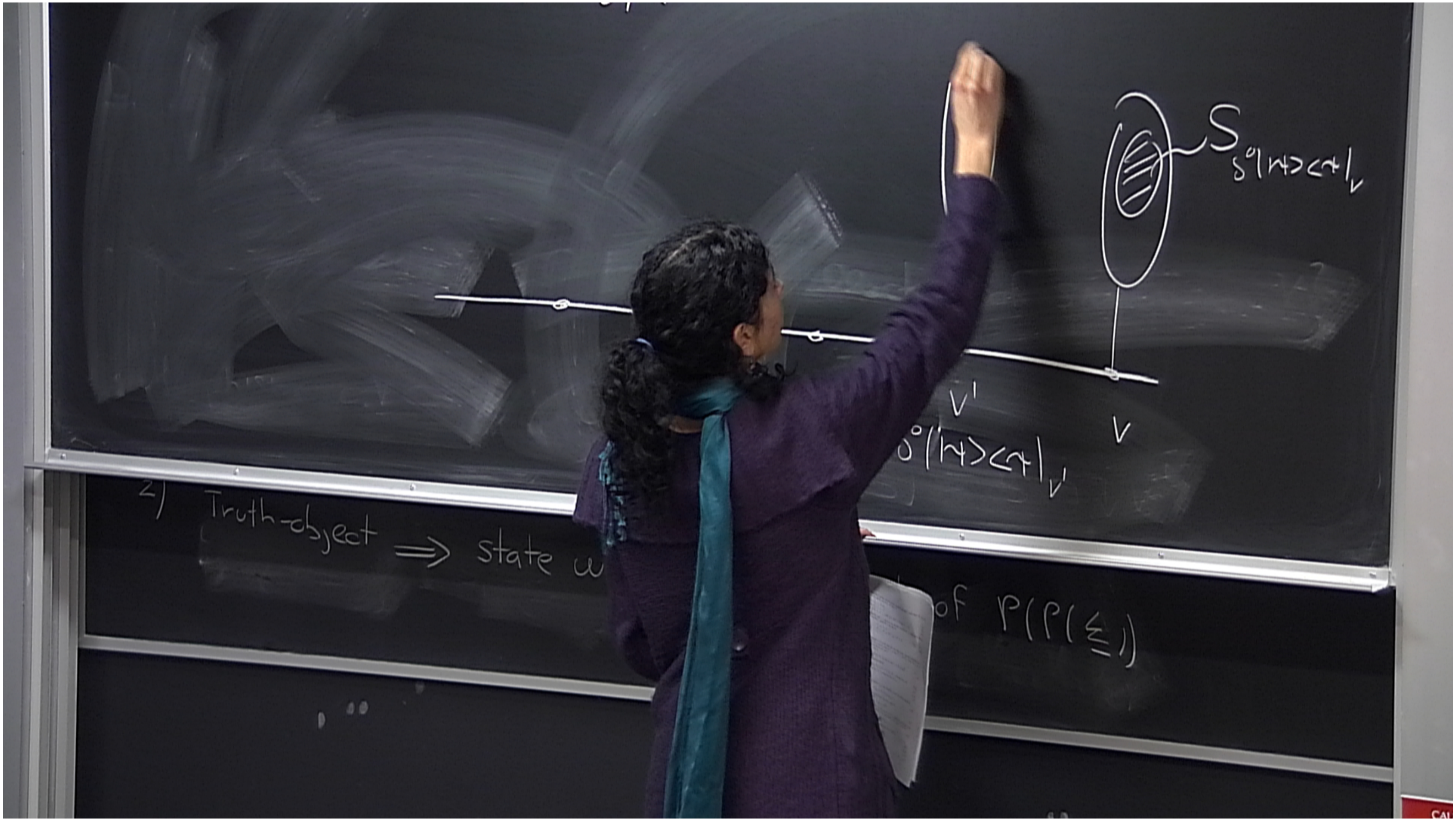

|ψ⟩ ∈ H
↳ |ψ⟩⟨ψ| ⇝ |ψ⟩⟨ψ| ∈ Sub_cl(Σ)
∀ V ∈ V(H) δ°(|ψ⟩⟨ψ|)_V = ∧ { R̂ ∈ P(V) | R̂ ≥ |ψ⟩⟨ψ| }
⟨ψ|R̂|ψ⟩ = 1
⟨ψ|ψ⟩⟨ψ|ψ⟩ = 1

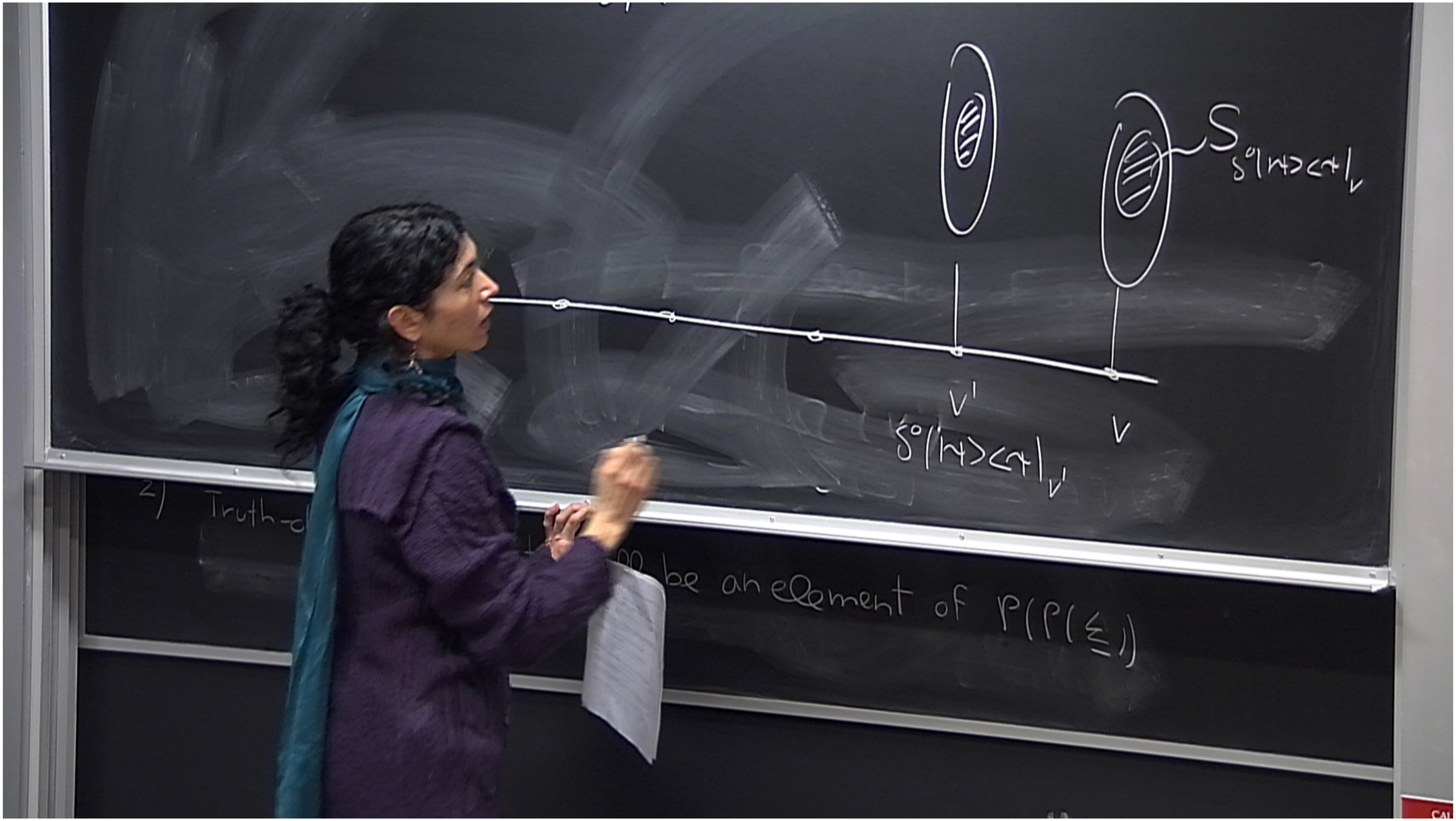

is a Heyting algebra
|ψ> ∈ H
↳ |ψ><ψ| ⇝ |ψ><ψ| ∈ Sub_cl(Σ)
∀ V ∈ V(H) δ°(|ψ><ψ|)_V = ∧{R̂ ∈ P(V) | R̂ ≥ |ψ><ψ|}
<ψ|R̂|ψ> = 1
<ψ||ψ><ψ||ψ> = 1


|ψ> ∈ H
↳ |ψ><ψ| ⇝ |ψ><ψ| ∈ Sub_cl(Σ)
∀ V ∈ V(H)   δ°(|ψ><ψ|)_V = ∧{R̂ ∈ P(V) | R̂ ≥ |ψ><ψ|}
<ψ|R̂|ψ> = 1 ←
<ψ||ψ><ψ||ψ> = 1

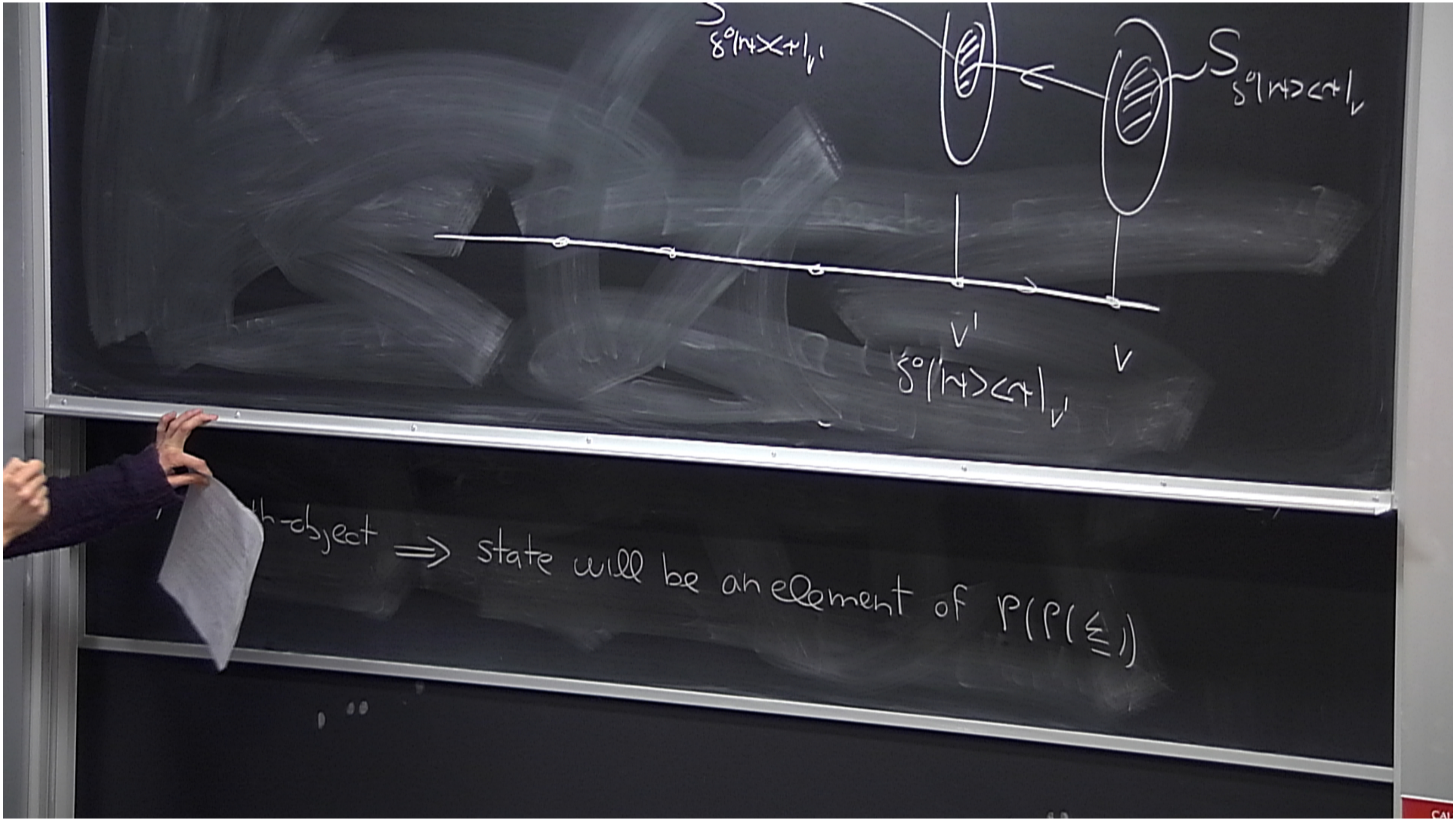

δ°(|ψ><ψ|_i) ⟿ S_{δ°(|ψ><ψ|_i)} = {λ ∈ Ξ_v | λ(δ°(|ψ><ψ|)) = 1}

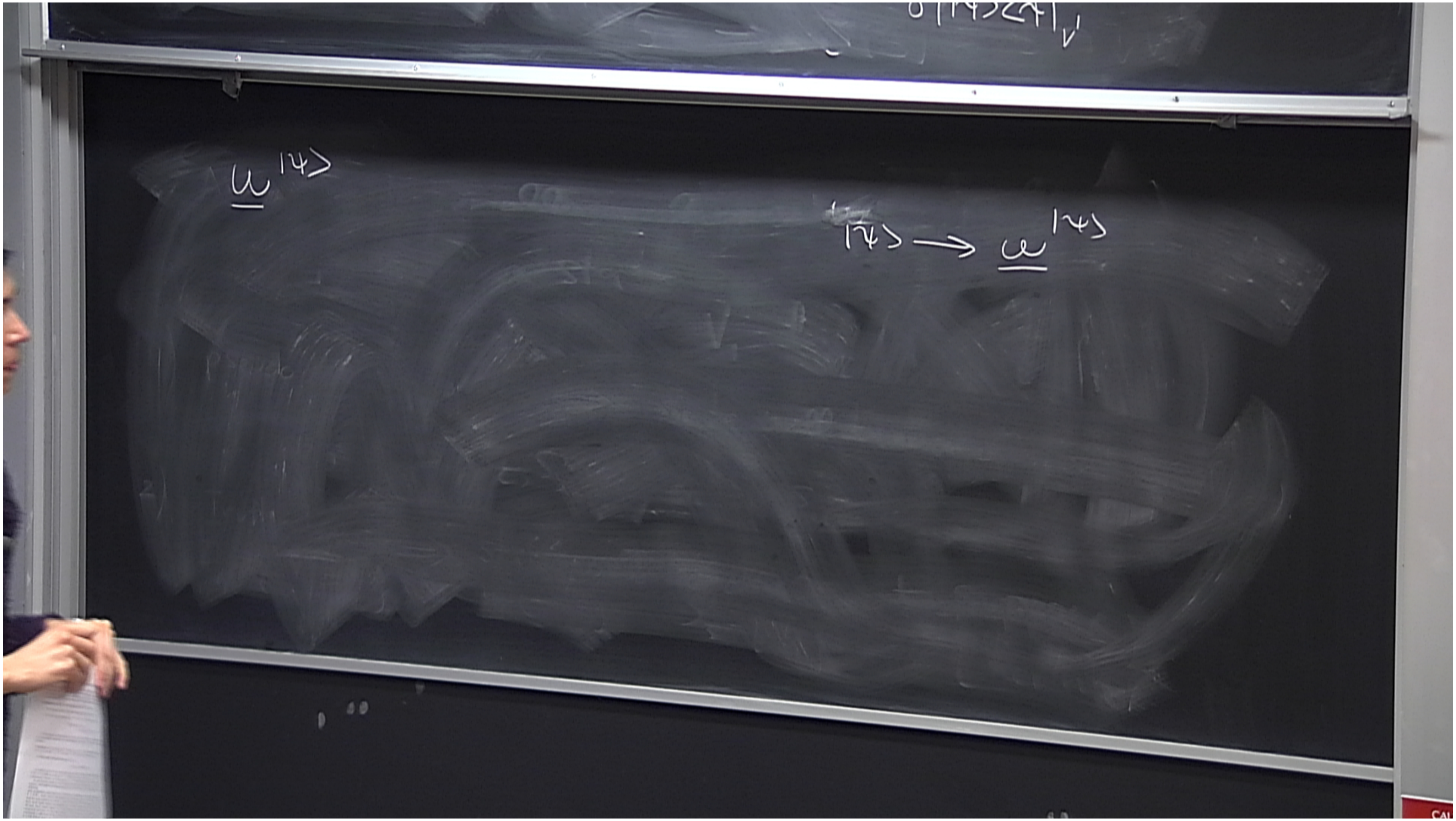

$\delta^o(|\psi\rangle\langle\psi|_L) \rightsquigarrow S_{\delta^o(|\psi\rangle\langle\psi|_L)} = \{\lambda \in \Xi_V \mid \lambda(\delta^o(|\psi\rangle\langle\psi|)_V = 1\}$

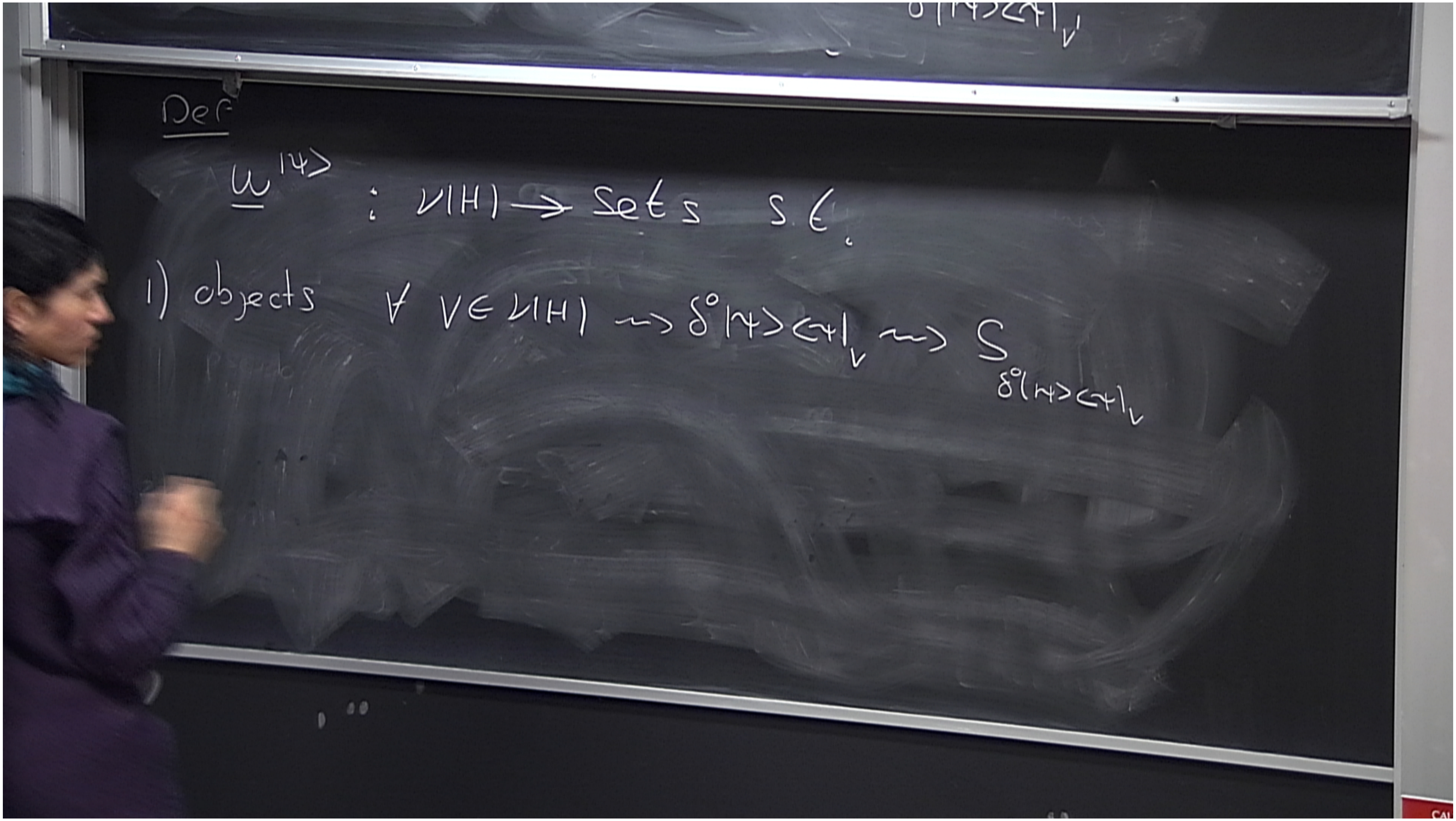

S δ°|ψ><ψ|V
V'
V
δ°(|ψ><ψ|V')
2) Truth-object ⟹ state ω
of P(P(Σ))

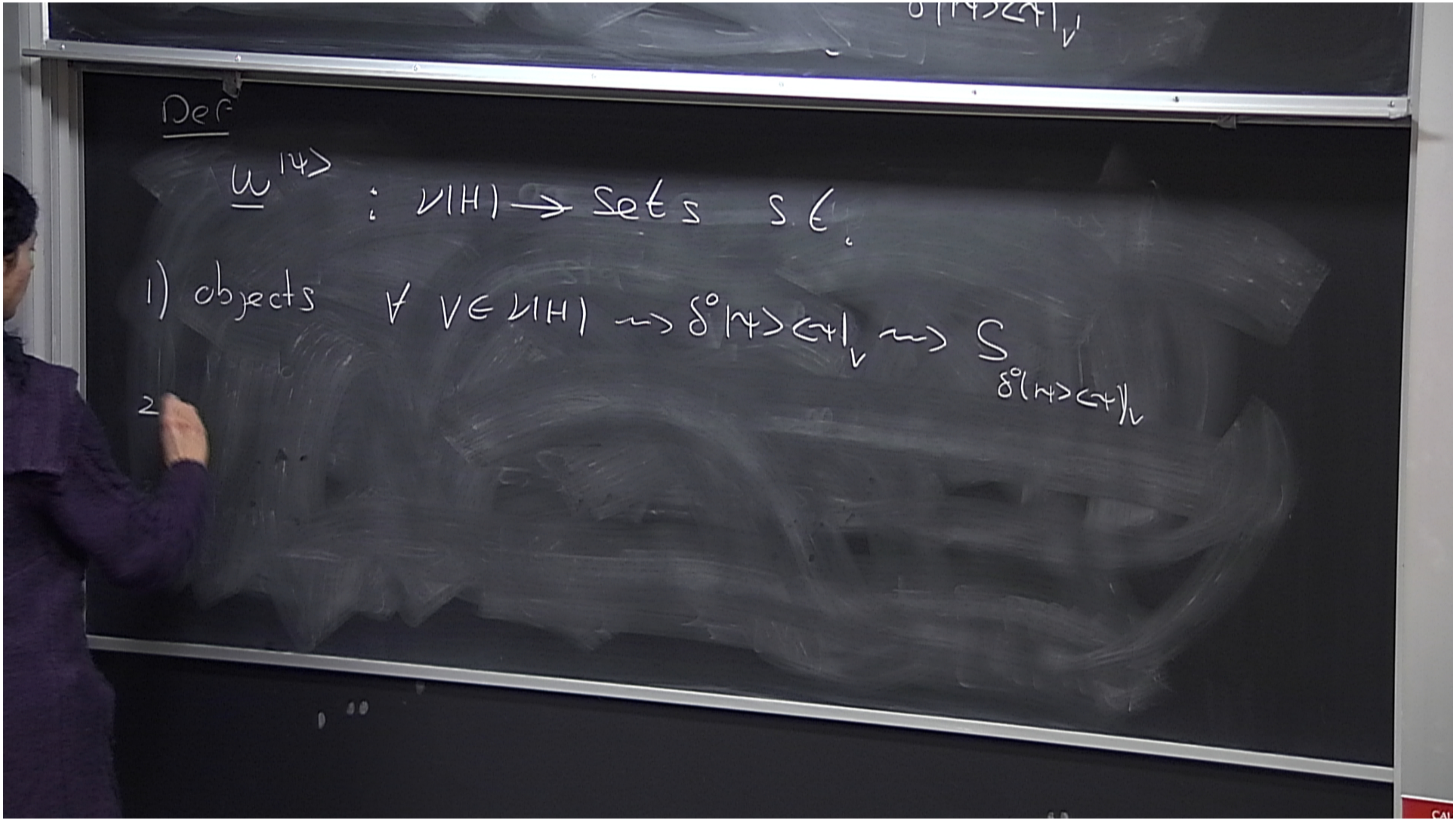


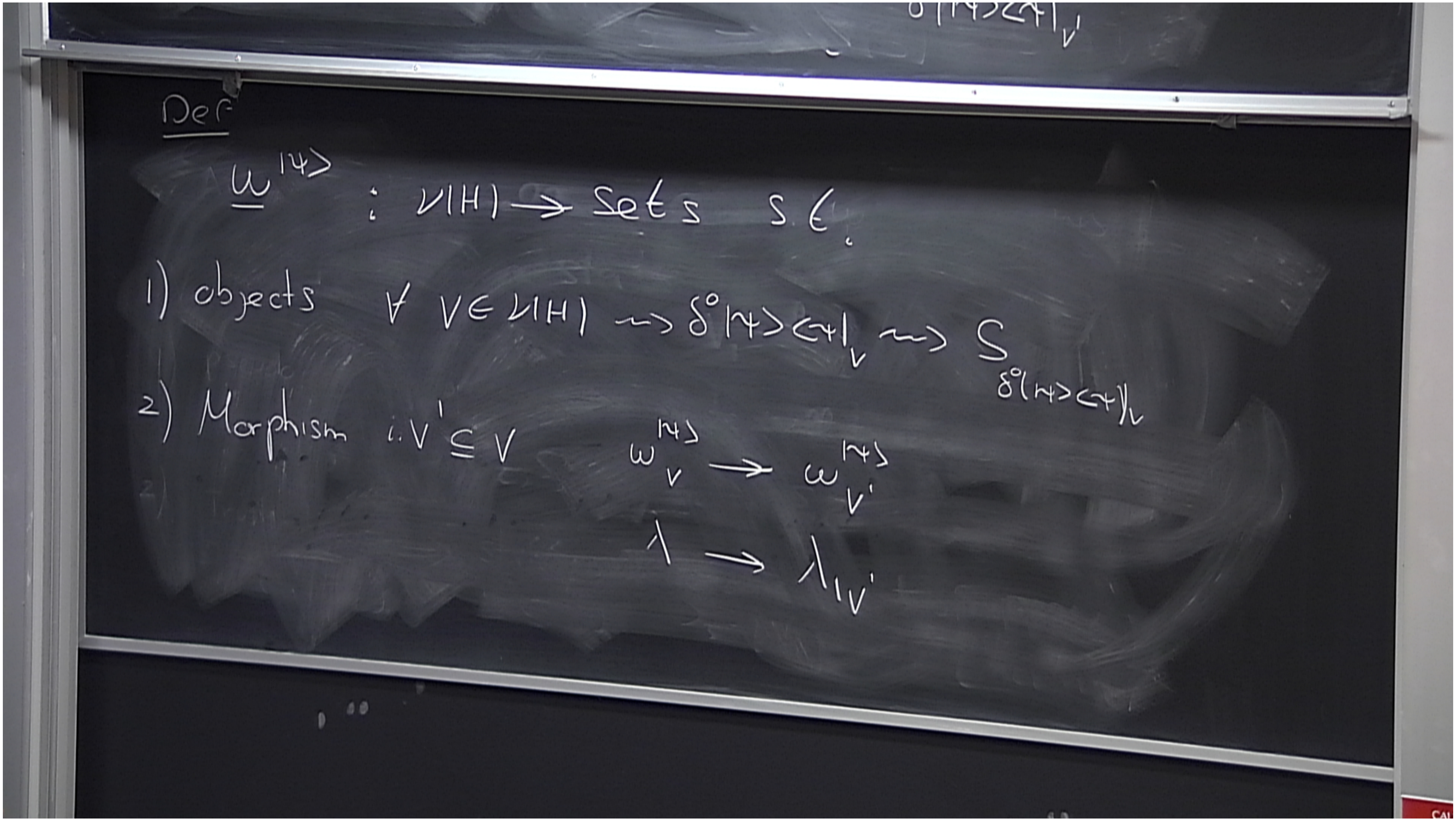
v'
v
2) Truth-
be an element of

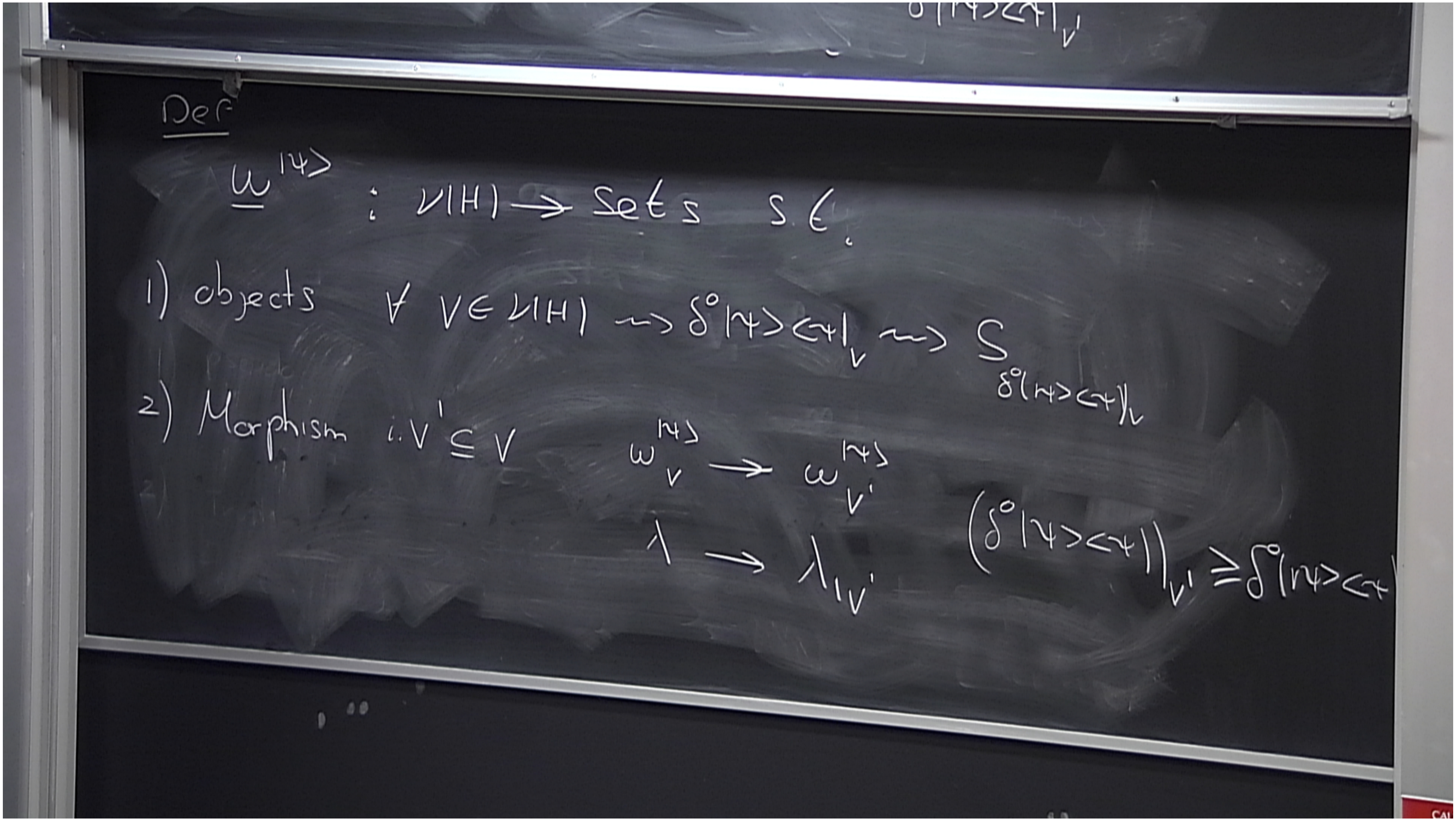

v'
v
-object ⟹ state will be an element of P(P(Σ))

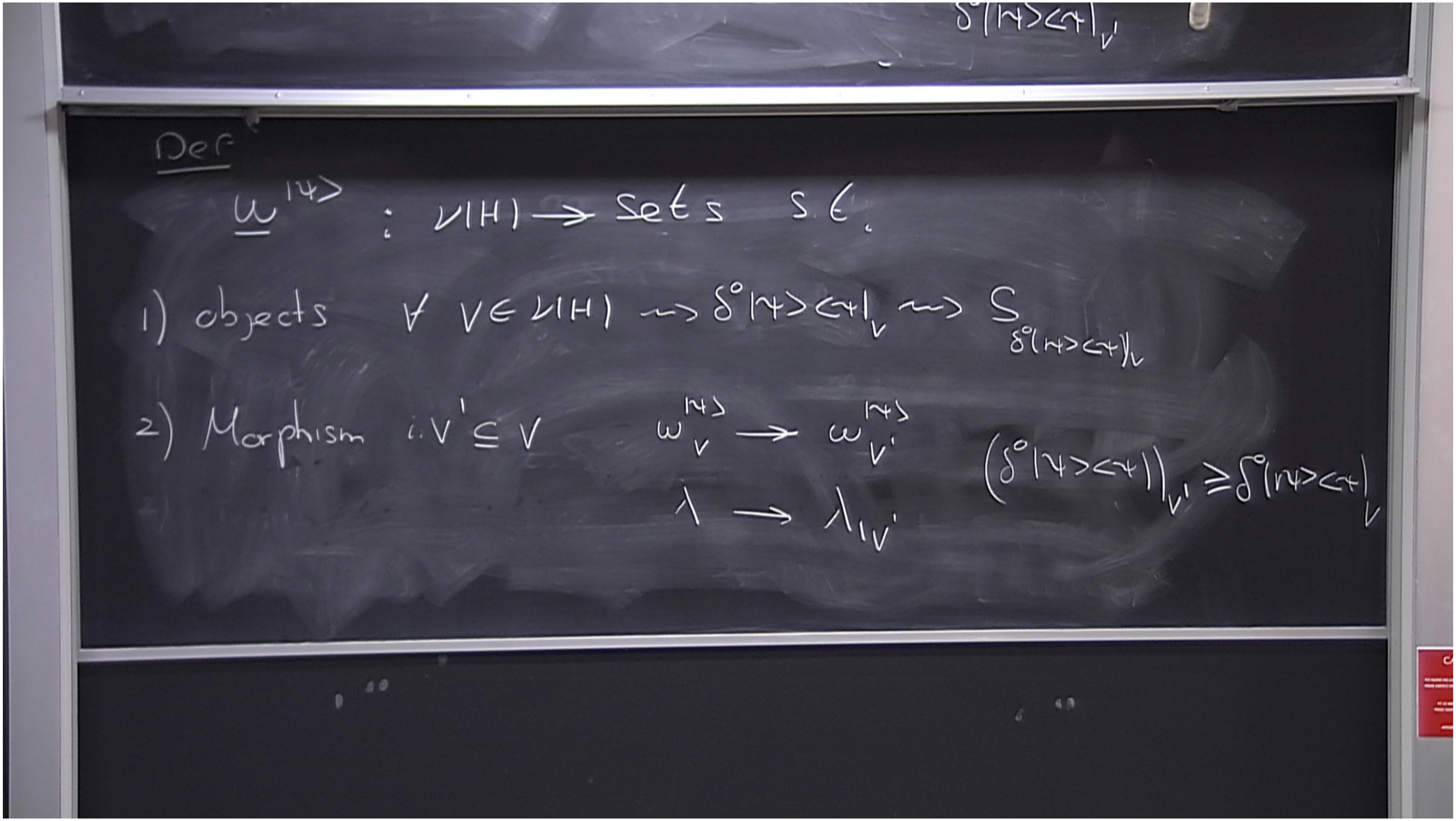

ω^{|ψ⟩}
|ψ⟩ → ω^{|ψ⟩}

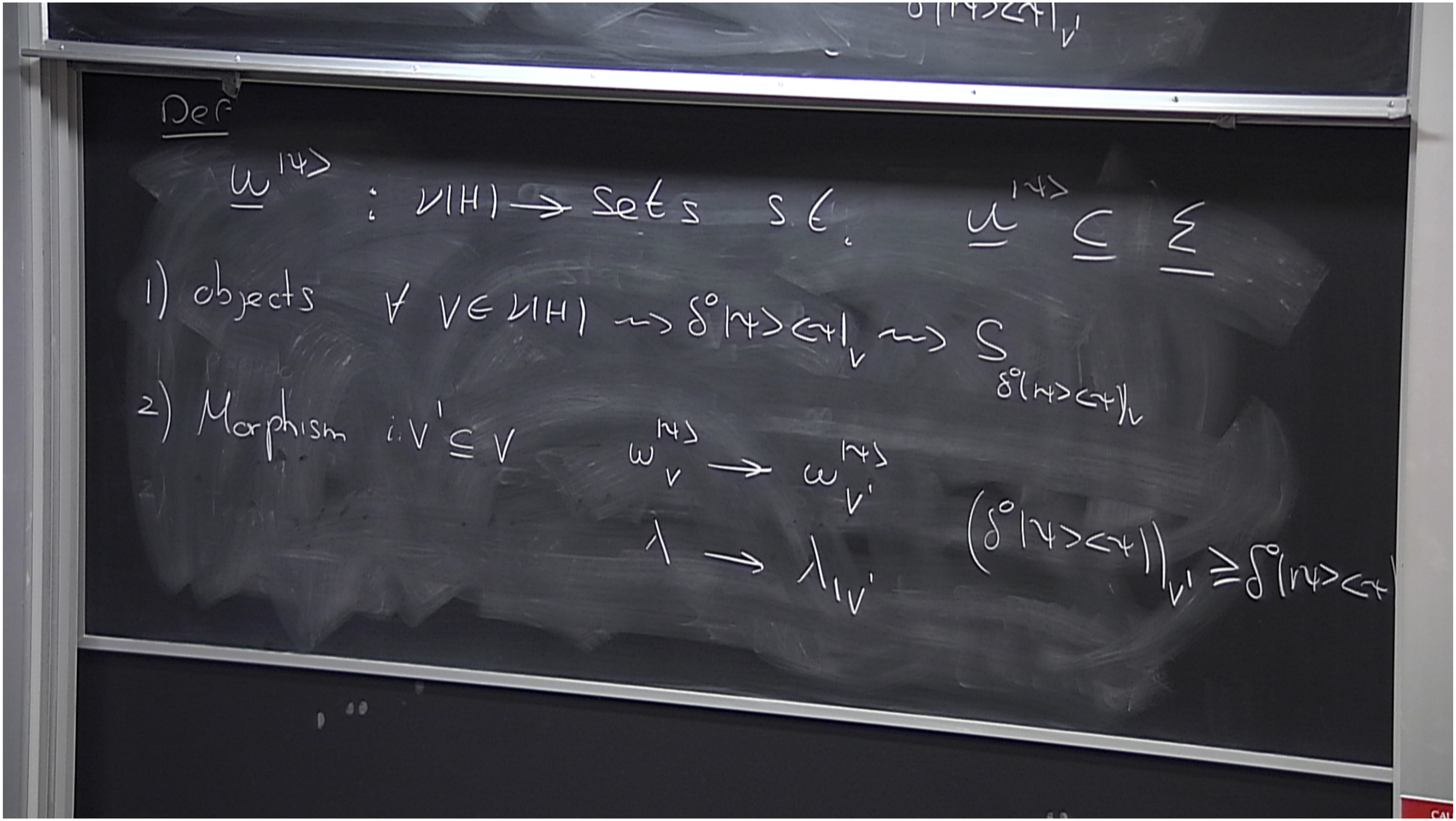

Def
ω^{|ψ>} : V(H) → Sets s.t.
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ><ψ|_V ⇝ S_{δ°|ψ><ψ|_V}

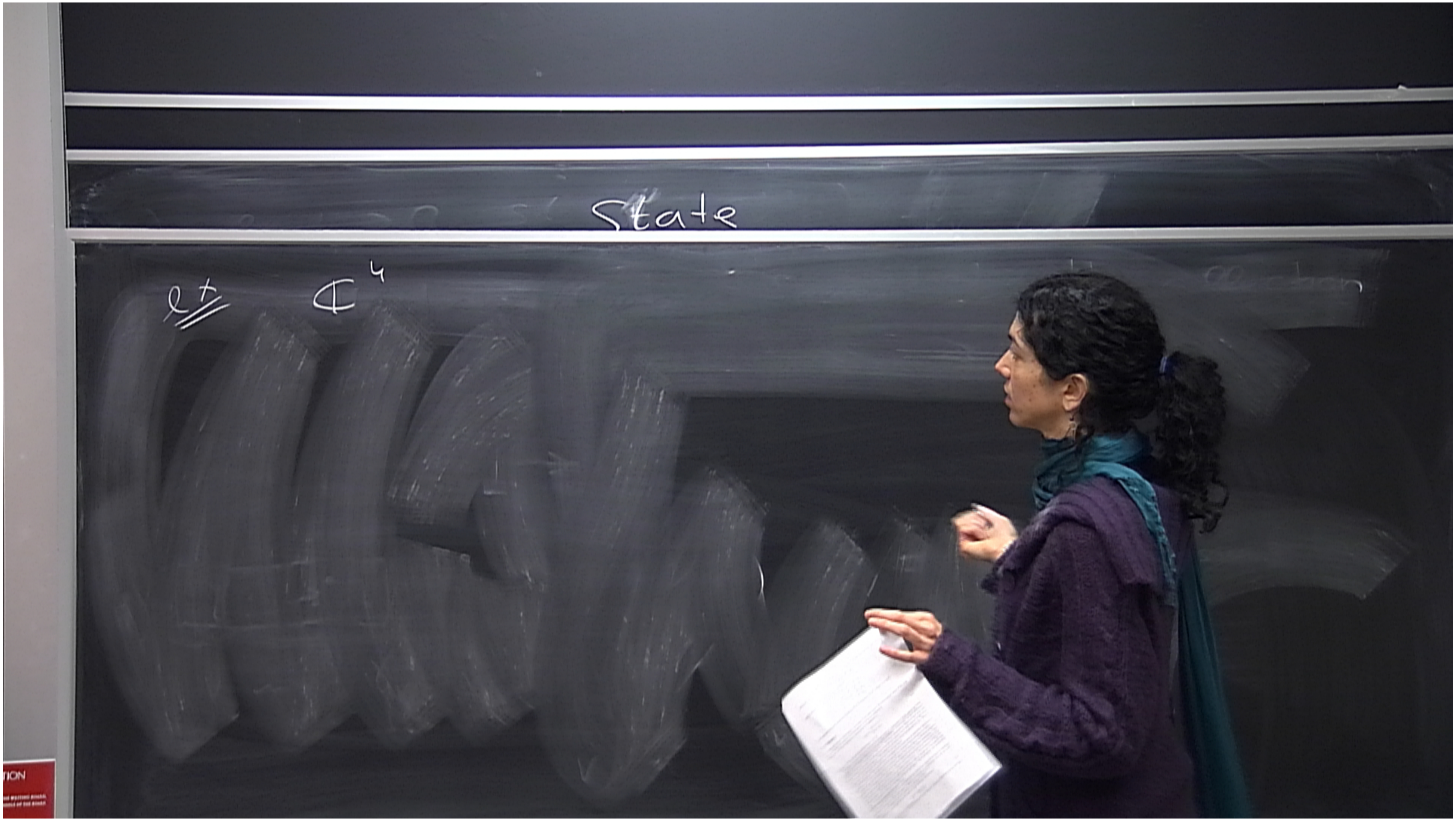

Def
ω^{|ψ>} : V(H) → Sets s.t.
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ><ψ|_V ⇝ S_{δ°(|ψ><ψ|)_V}
2

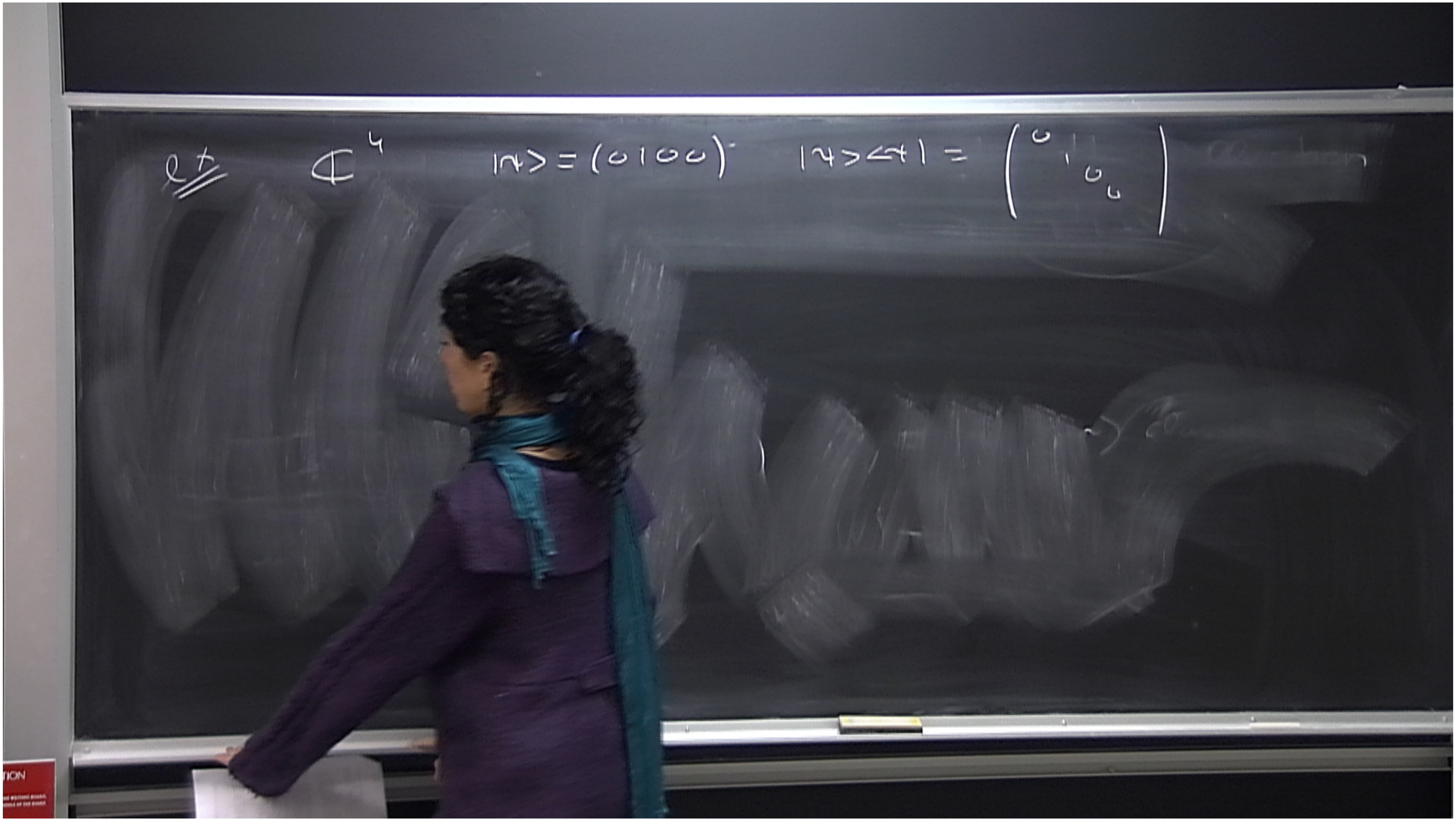

Def
ω^{|ψ>} : V(H) → Sets s.t.
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ><ψ|_V ⇝ S_{δ°(|ψ><ψ|)_V}
2) Morphism i: V' ⊆ V
ω_V^{|ψ>} → ω_{V'}^{|ψ>}
λ → λ|_{V'}


Def
ω^{|ψ⟩} : V(H) → Sets s.t.
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ⟩⟨ψ|_V ⇝ S_{δ°(|ψ⟩⟨ψ|)_V}
2) Morphism i: V' ⊆ V
ω^{|ψ⟩}_V → ω^{|ψ⟩}_{V'}
λ → λ|_{V'}
(δ°|ψ⟩⟨ψ|)_{V'} ≥ δ°|ψ⟩⟨ψ|

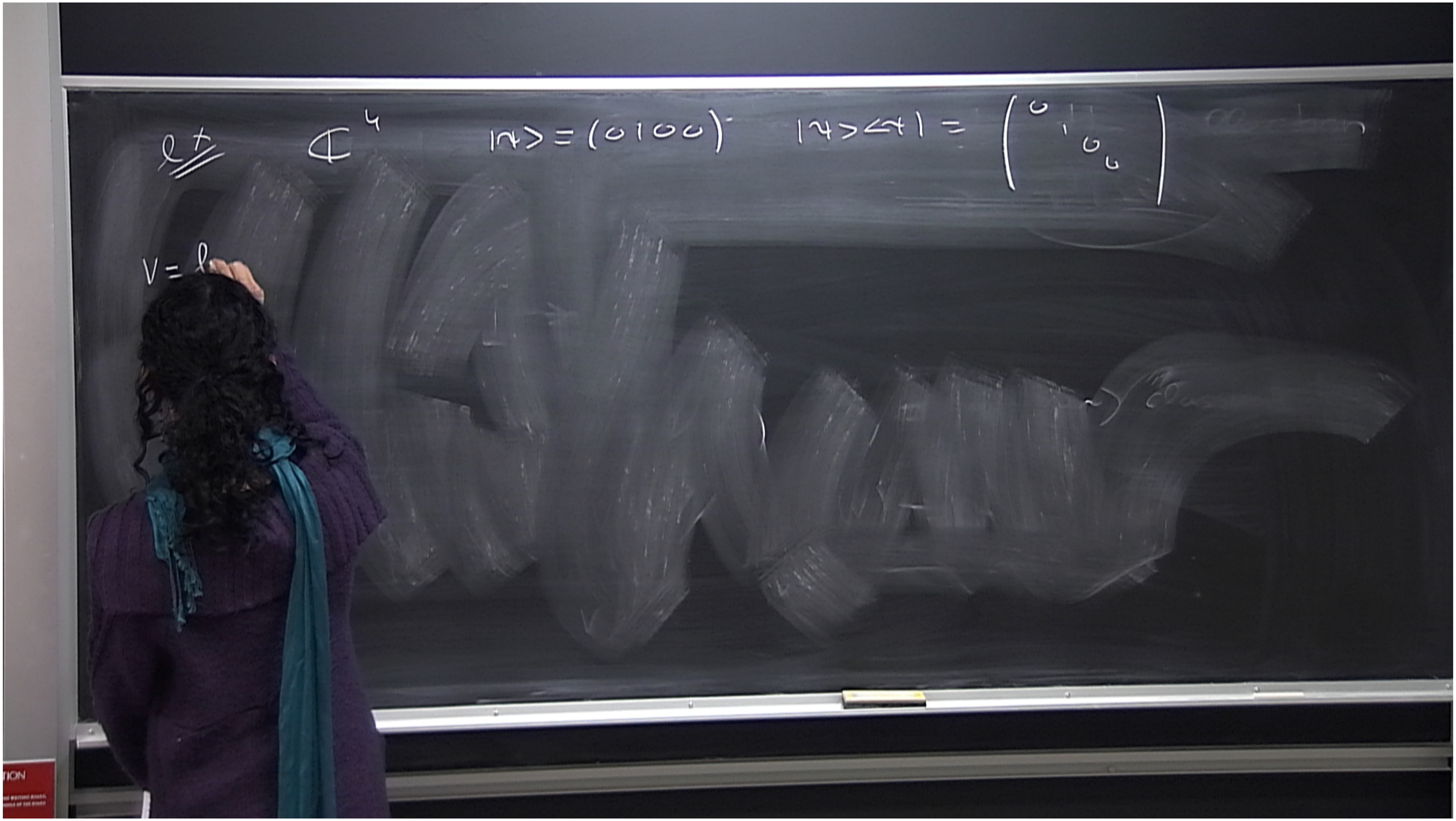

δ°|ψ⟩⟨ψ|_V'
Def
ω^{|ψ⟩} : V(H) → Sets s.t.
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ⟩⟨ψ|_V ⇝ S_{δ°(|ψ⟩⟨ψ|)_V}
2) Morphism i: V' ⊆ V   ω_V^{|ψ⟩} → ω_{V'}^{|ψ⟩}
(δ°|ψ⟩⟨ψ|)_{V'} ≥ δ°(|ψ⟩⟨ψ|)_V
λ → λ|_{V'}


Def
ω^{|ψ⟩} : V(H) → Sets s.t. ω^{|ψ⟩} ⊆ Σ
1) objects ∀ V ∈ V(H) ⇝ δ°|ψ⟩⟨ψ|_V ⇝ S_{δ°|ψ⟩⟨ψ|_V}
2) Morphism i: V' ⊆ V
ω_V^{|ψ⟩} → ω_{V'}^{|ψ⟩}
λ → λ|_{V'}
(δ°|ψ⟩⟨ψ|)|_{V'} ≥ δ°|ψ⟩⟨ψ|

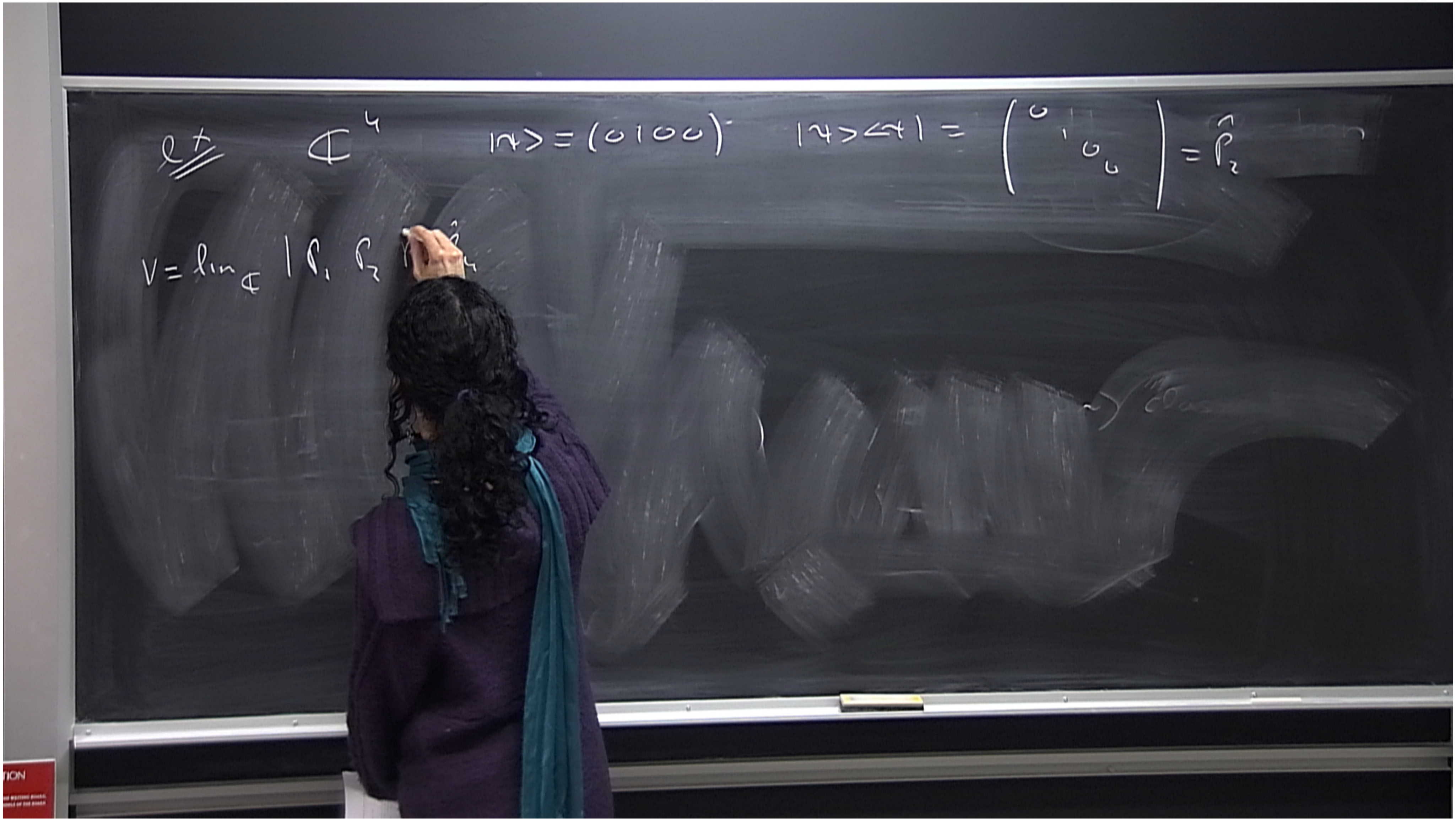

State



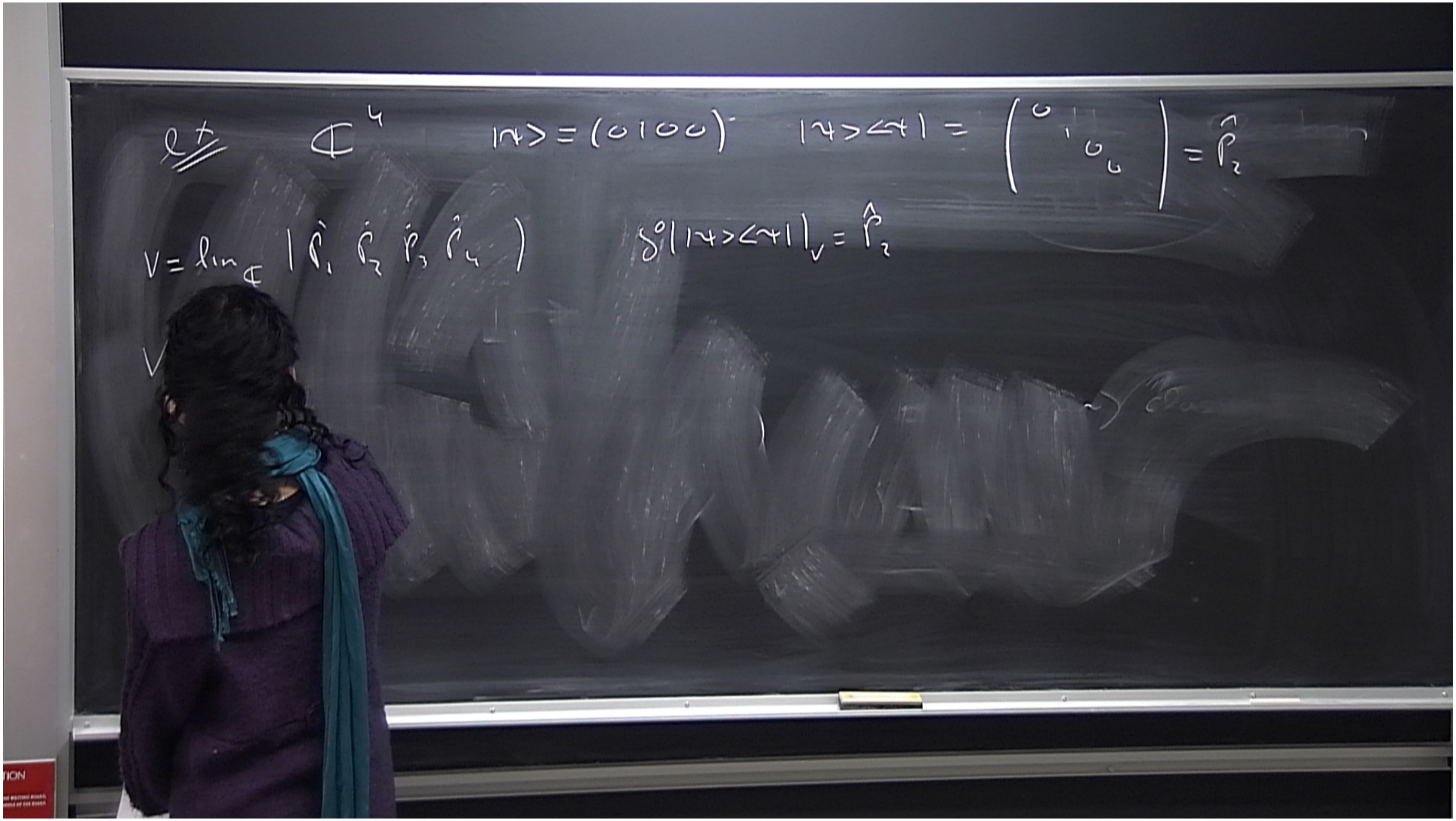
$\mathbb{C}^4$
$|\psi\rangle = (0100)$
$|\psi\rangle\langle\psi| =$

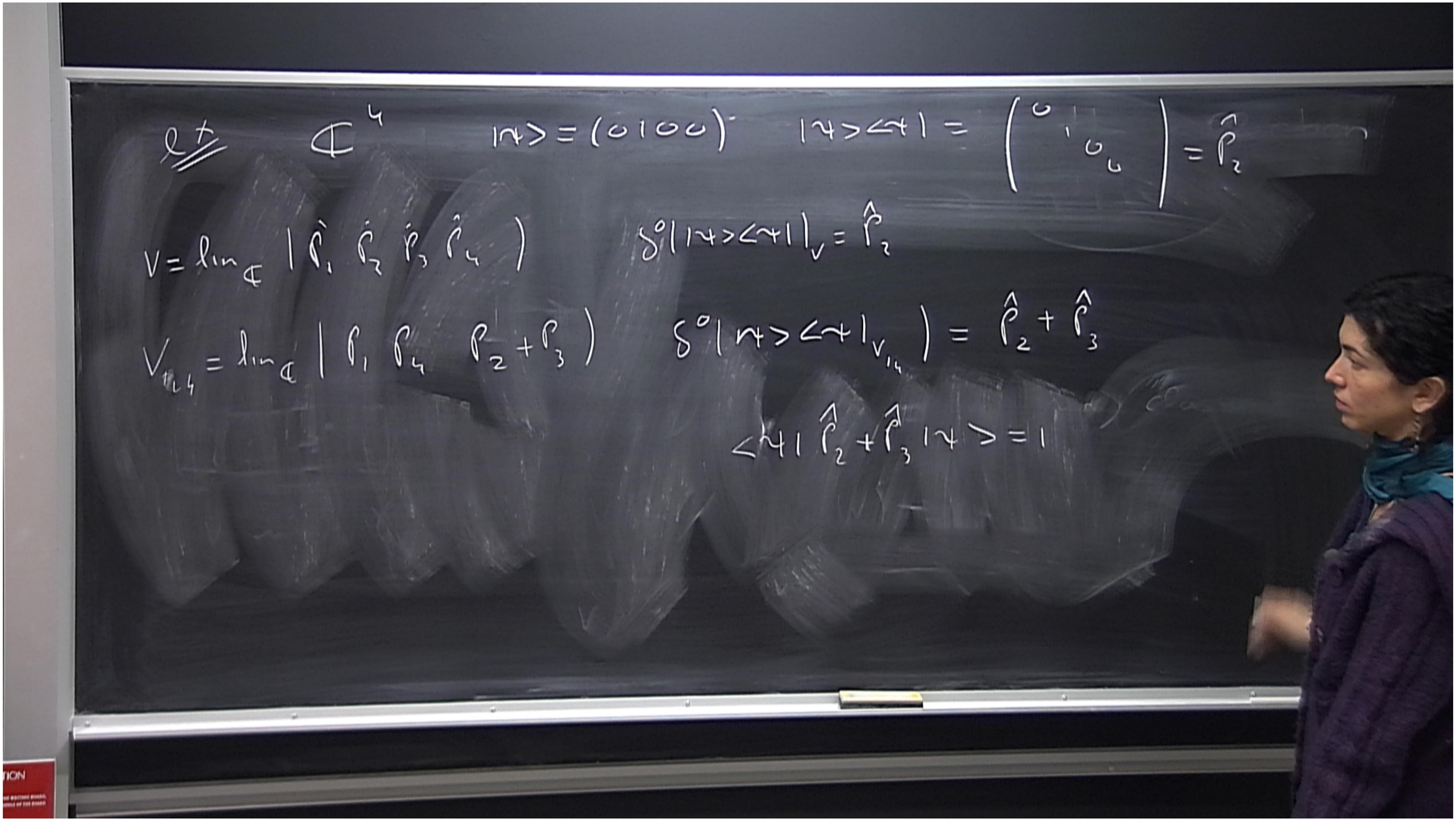


ℂ⁴
|ψ> = (0100)
|ψ><ψ| =
V = e

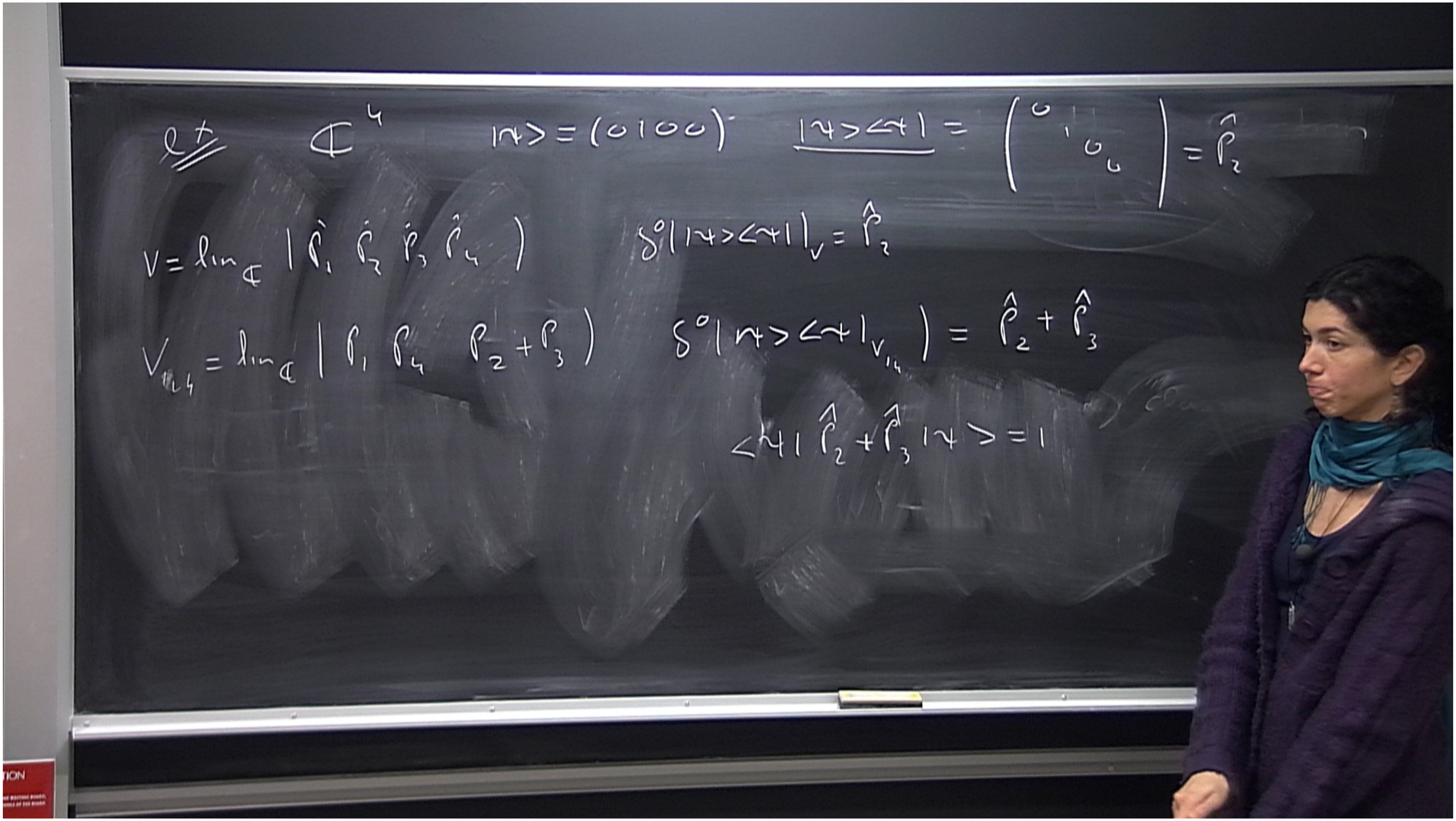


ex
$\mathbb{C}^4$
$|\psi\rangle = (0100)^T$
$|\psi\rangle\langle\psi| = \hat{P}_2$
$V = \text{lin}_{\mathbb{C}} | P_1 \; P_2$

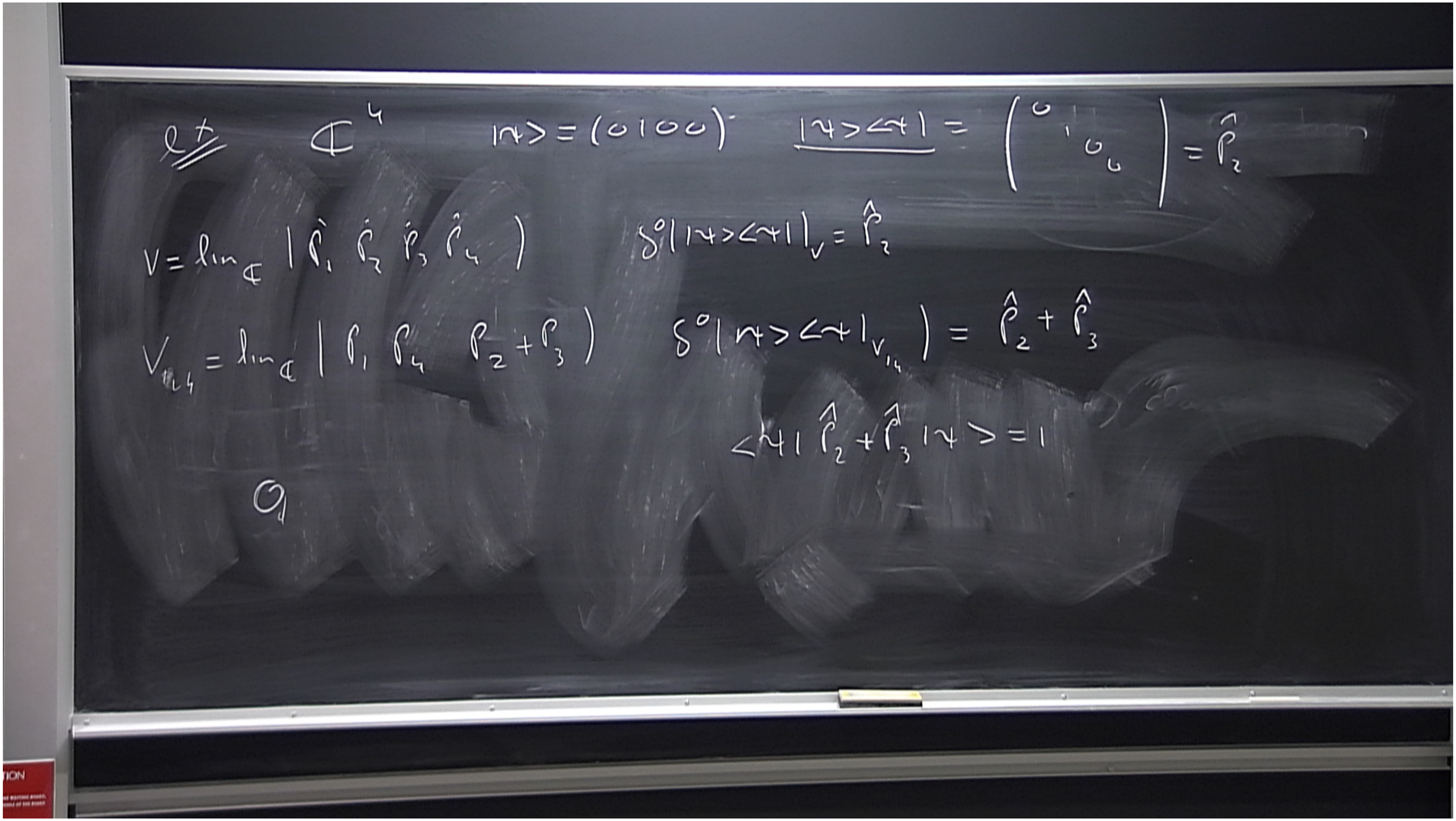

ex
ℂ^4
|ψ> = (0100)^T
|ψ><ψ| =
= P̂_2
V = lin_ℂ | P̂_1 P̂_2 P̂_3 P̂_4)
g°(|ψ><ψ|)_V = P̂_2

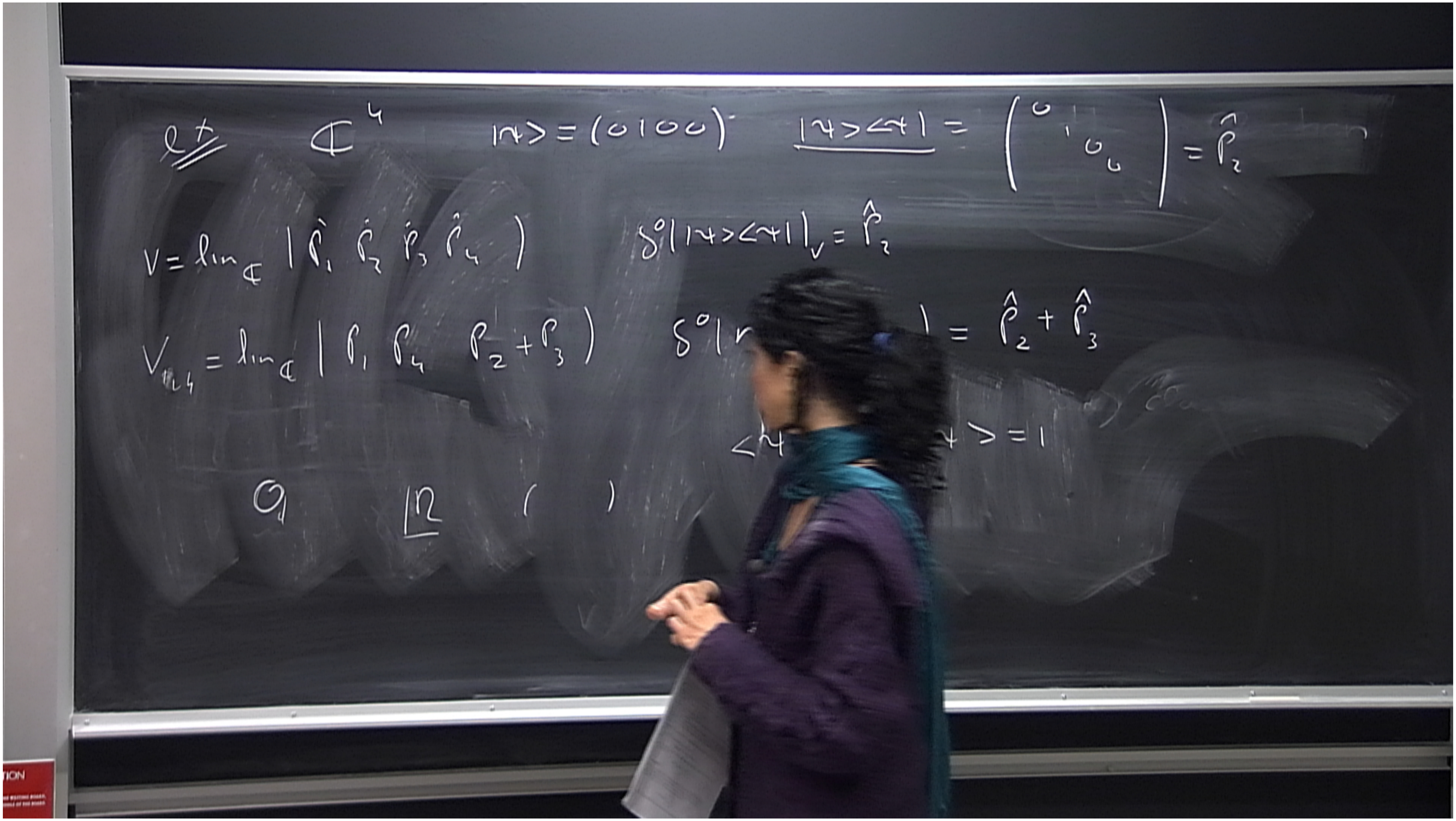


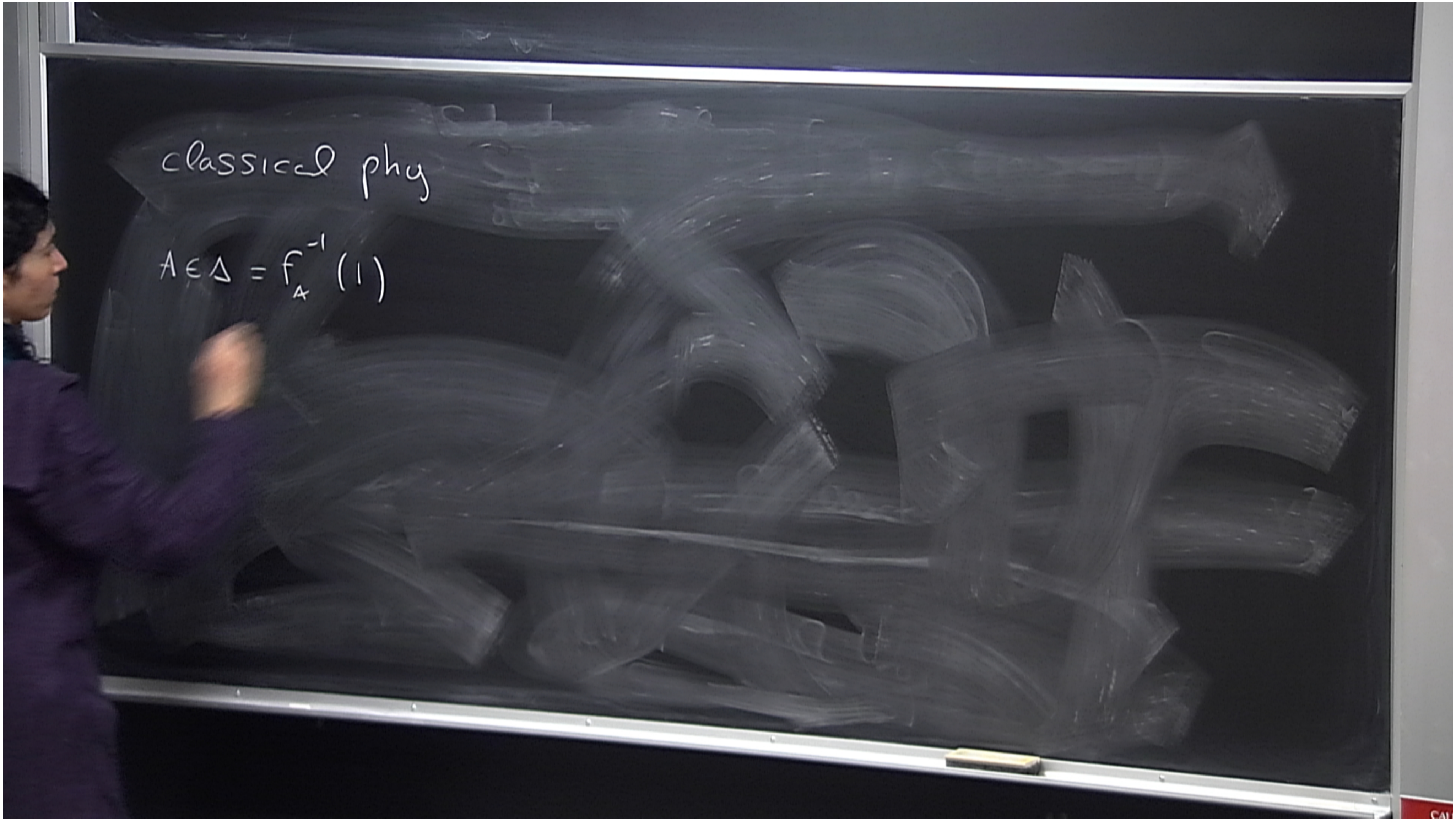
ex   ℂ^4 |ψ> = (0100)^T |ψ><ψ| = ( ) = P̂2
V = lin_ℂ | P̂1 P̂2 P̂3 P̂4)   δ°| |ψ><ψ| |_V = P̂2
V_14 = lin_ℂ | P1 P4 P2+P3)   δ°| |ψ><ψ|_V14) = P̂2 + P̂3
<ψ| P̂2 + P̂3 |ψ> = 1

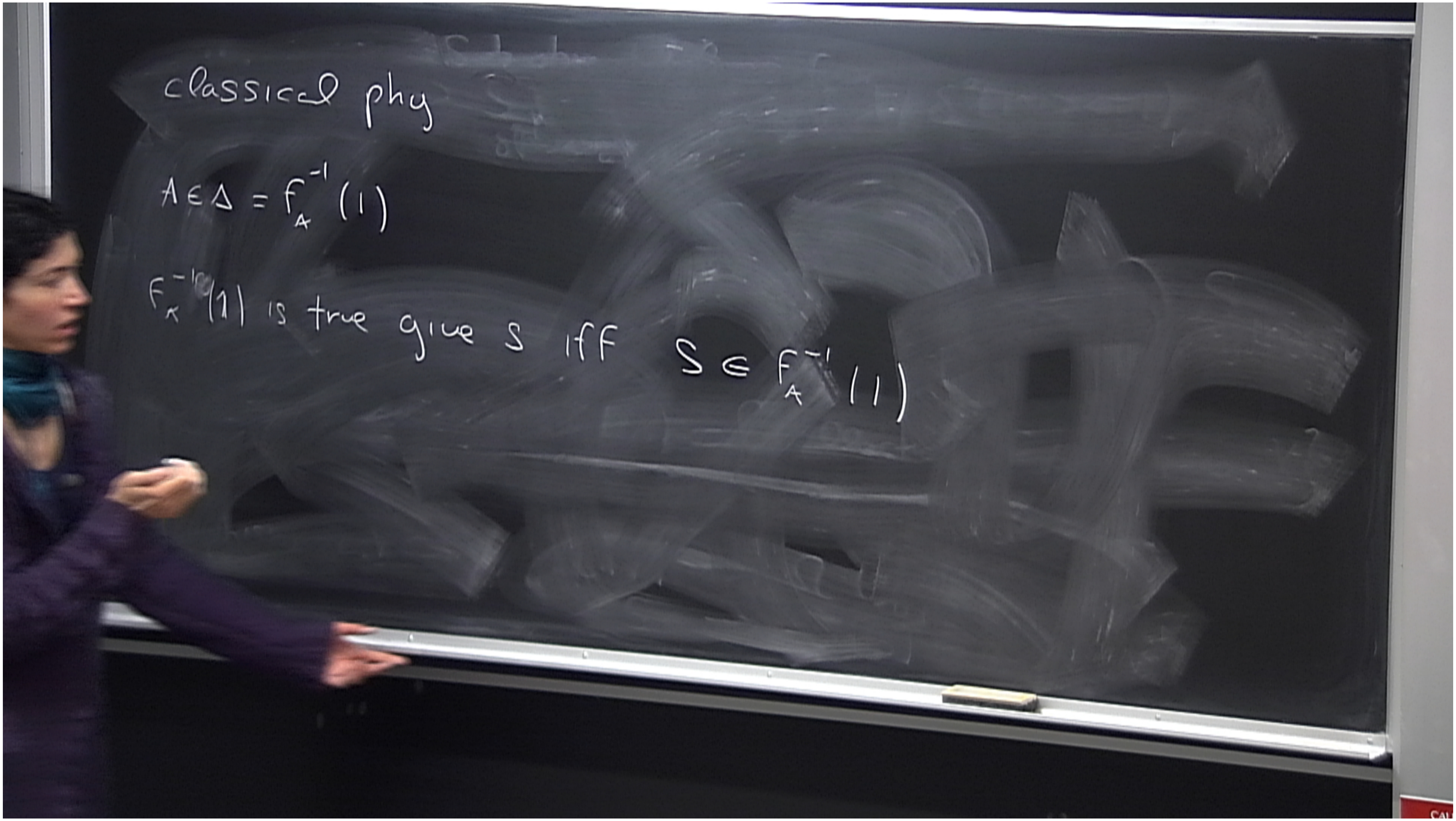


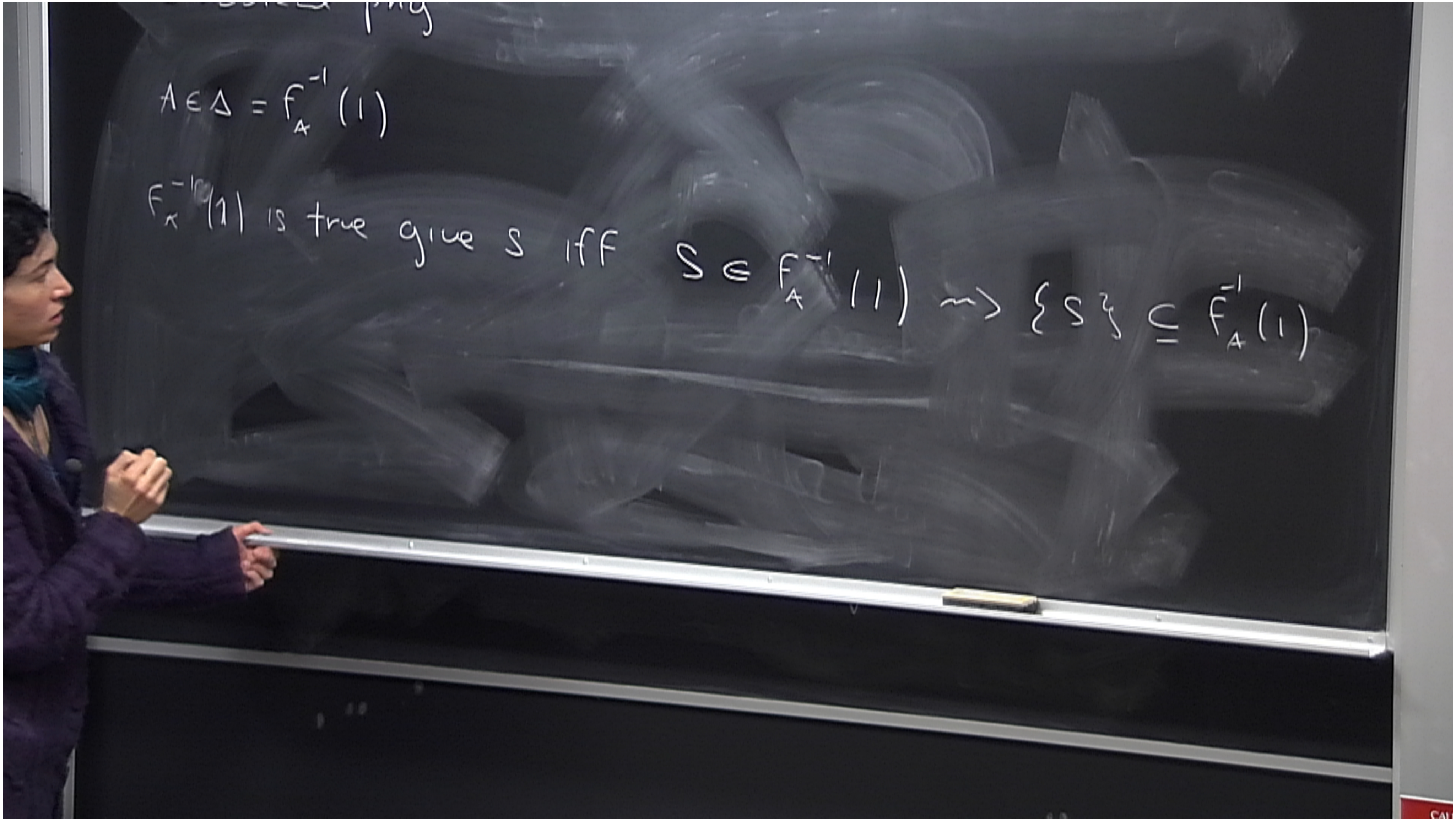
eg
ℂ 4
|ψ> = (0100)
|ψ><ψ| =
= P̂2
V = linℂ | P̂1 P̂2 P̂3 P̂4)
δ°||ψ><ψ||V = P̂2
V14 = linℂ | P1 P4 P2 + P3)
δ°|ψ><ψ|V14) = P̂2 + P̂3
<ψ| P̂2 + P̂3 |ψ> = 1

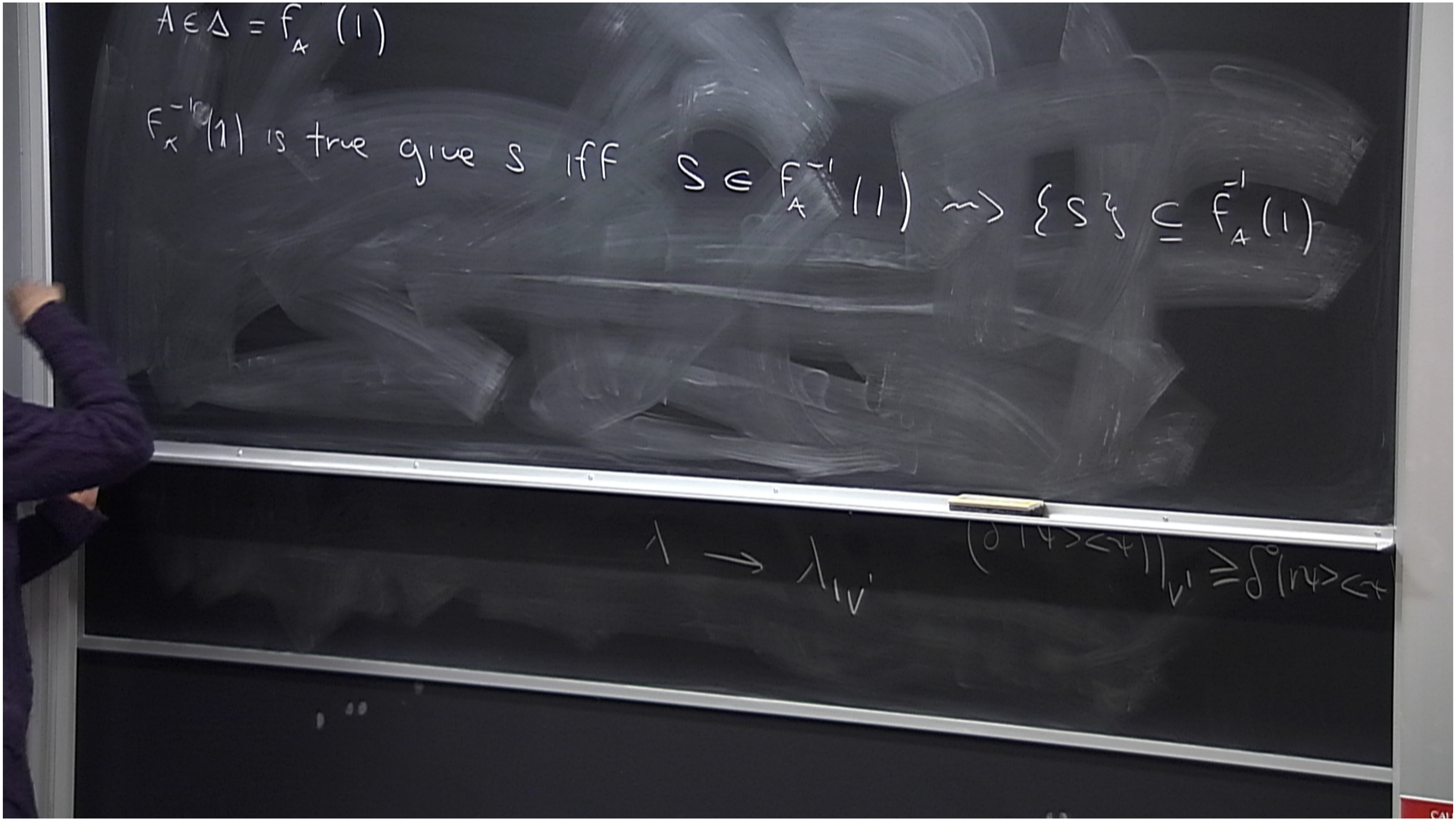

ex
$\mathbb{C}^4$
$|\psi\rangle = (0100)^T$
$|\psi\rangle\langle\psi| = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} = \hat{P}_2$
$V = \mathrm{lin}_{\mathbb{C}} \,|\, \hat{P}_1\ \hat{P}_2\ \hat{P}_3\ \hat{P}_4)$
$\delta^0 |\,|\psi\rangle\langle\psi|\,|_V = \hat{P}_2$
$V_{14} = \mathrm{lin}_{\mathbb{C}} \,|\, P_1\ P_4\ P_2 + P_3)$
$\delta^0 |\,|\psi\rangle\langle\psi|_{V_{14}}) = \hat{P}_2 + \hat{P}_3$
$\langle\psi|\,\hat{P}_2 + \hat{P}_3\,|\psi\rangle = 1$


|ψ> = (0100)
|ψ><ψ| =
V = lin | ρ̂1 ρ̂2 ρ̂3 ρ̂4)
= ρ̂2 + ρ̂3


classical phy
A ∈ Δ = f_A^-1 (1)


classical phy
A ∈ Δ = f_A^-1 (1)
f_A^-1 (1) is true give S iff S ∈ f_A^-1 (1)


A ∈ Δ = f_A^-1 (1)
f_A^-1(1) is true give s iff s ∈ f_A^-1 (1) ~> {s} ⊆ f_A^-1 (1)


A ∈ Δ = f_A (1)
f_A^-1(1) is true give s iff S ∈ f_A^-1(1) ~> {S} ⊆ f_A^-1(1)
λ → λ|_V'

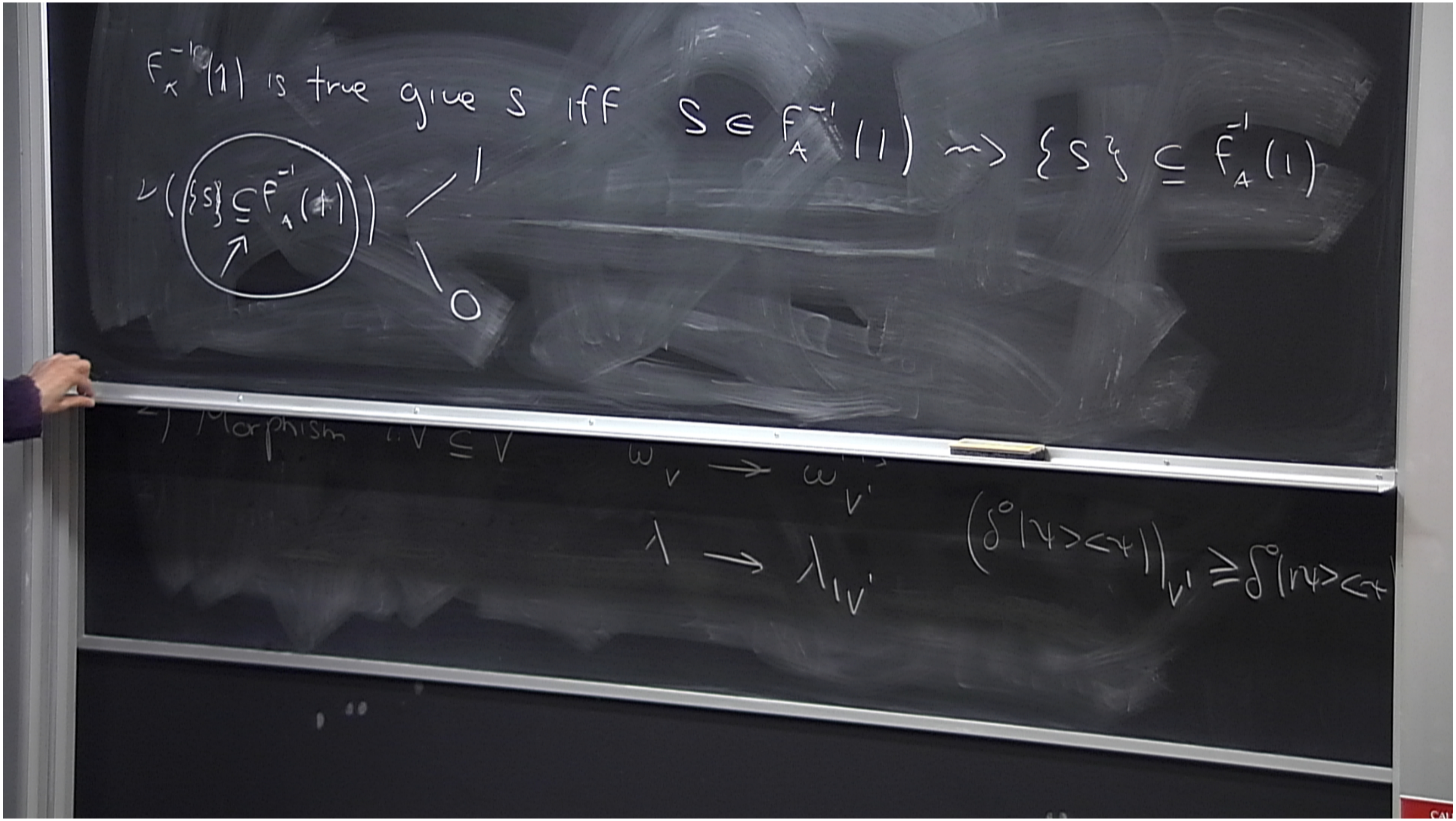
$A \in \Delta = f_A^{-1}(1)$

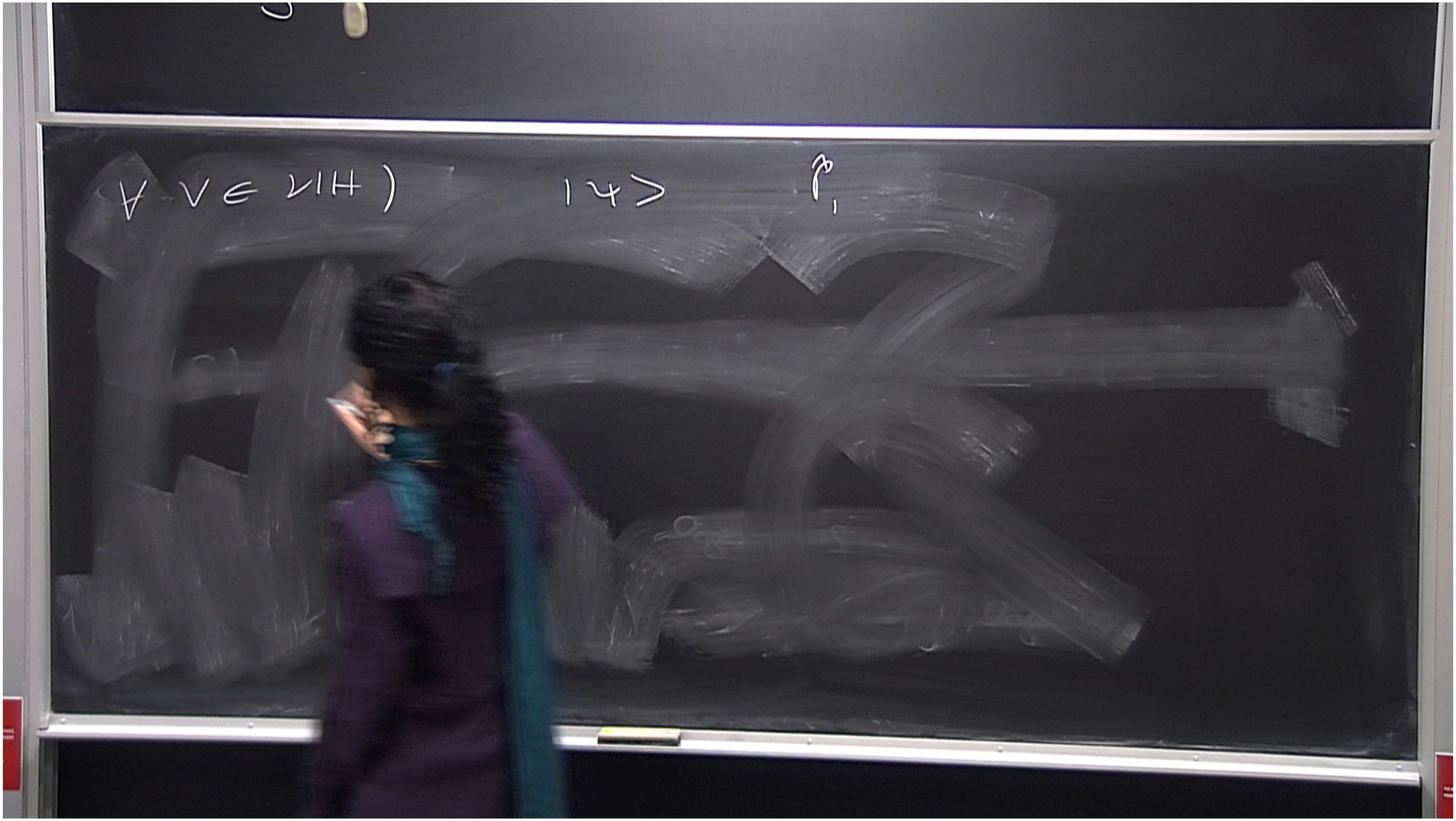
$f_A^{-1}(1)$ is true give s iff $s \in f_A^{-1}(1) \rightsquigarrow \{s\} \subseteq f_A^{-1}(1)$

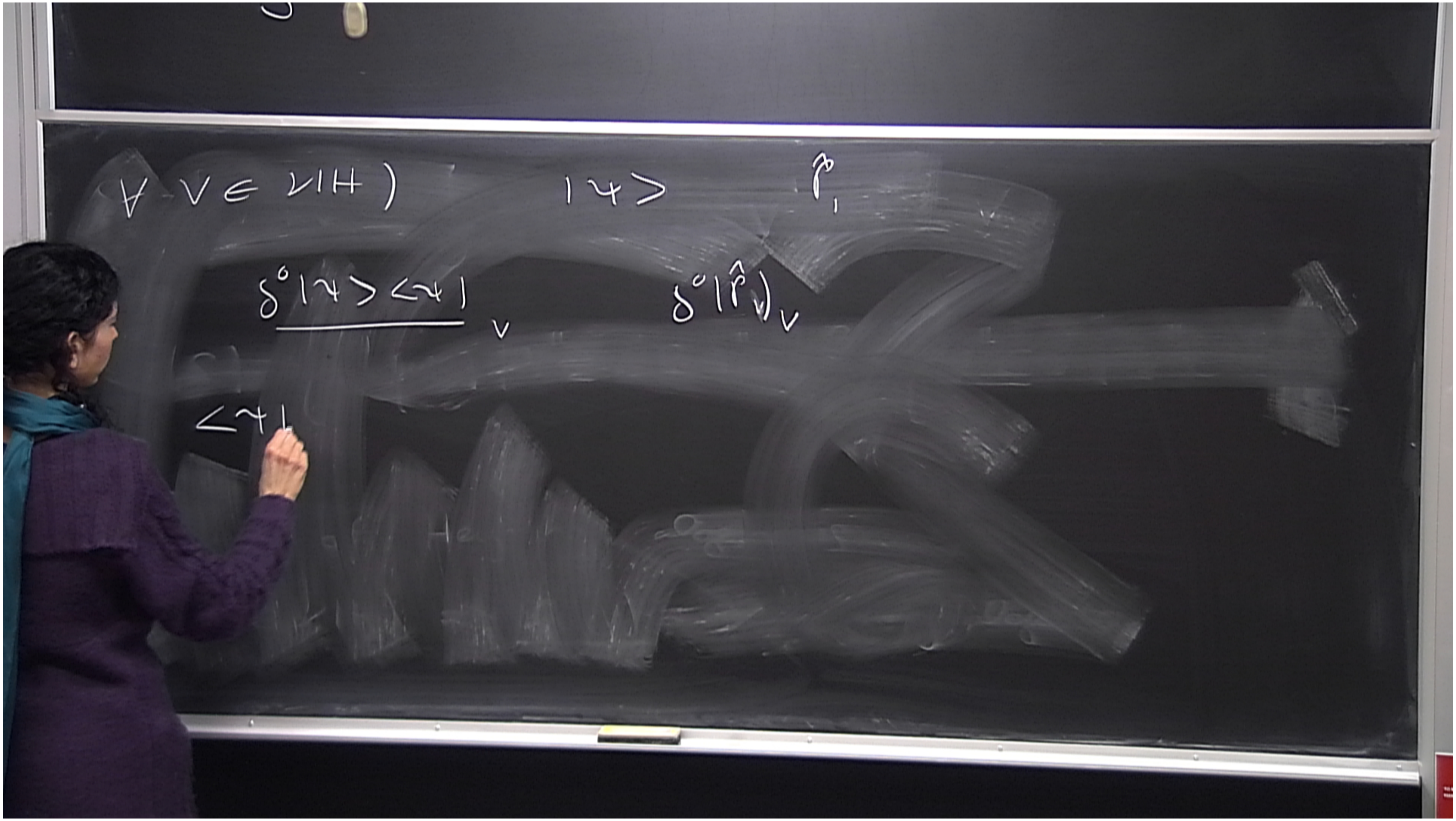
$\nu(\{s\} \subseteq f_A^{-1}(1))$ → 1, 0

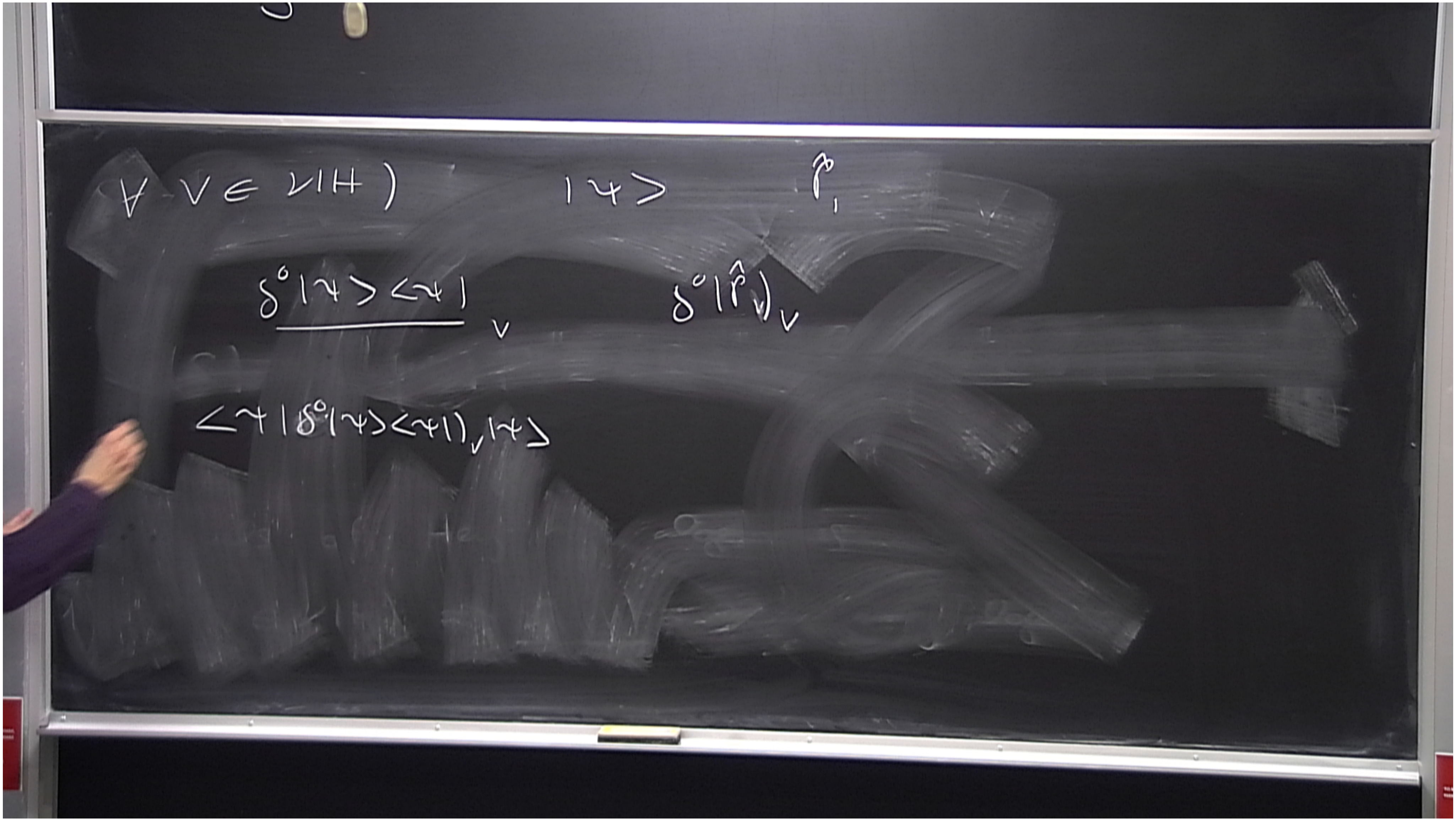
$\lambda \to \lambda_{|V'}$

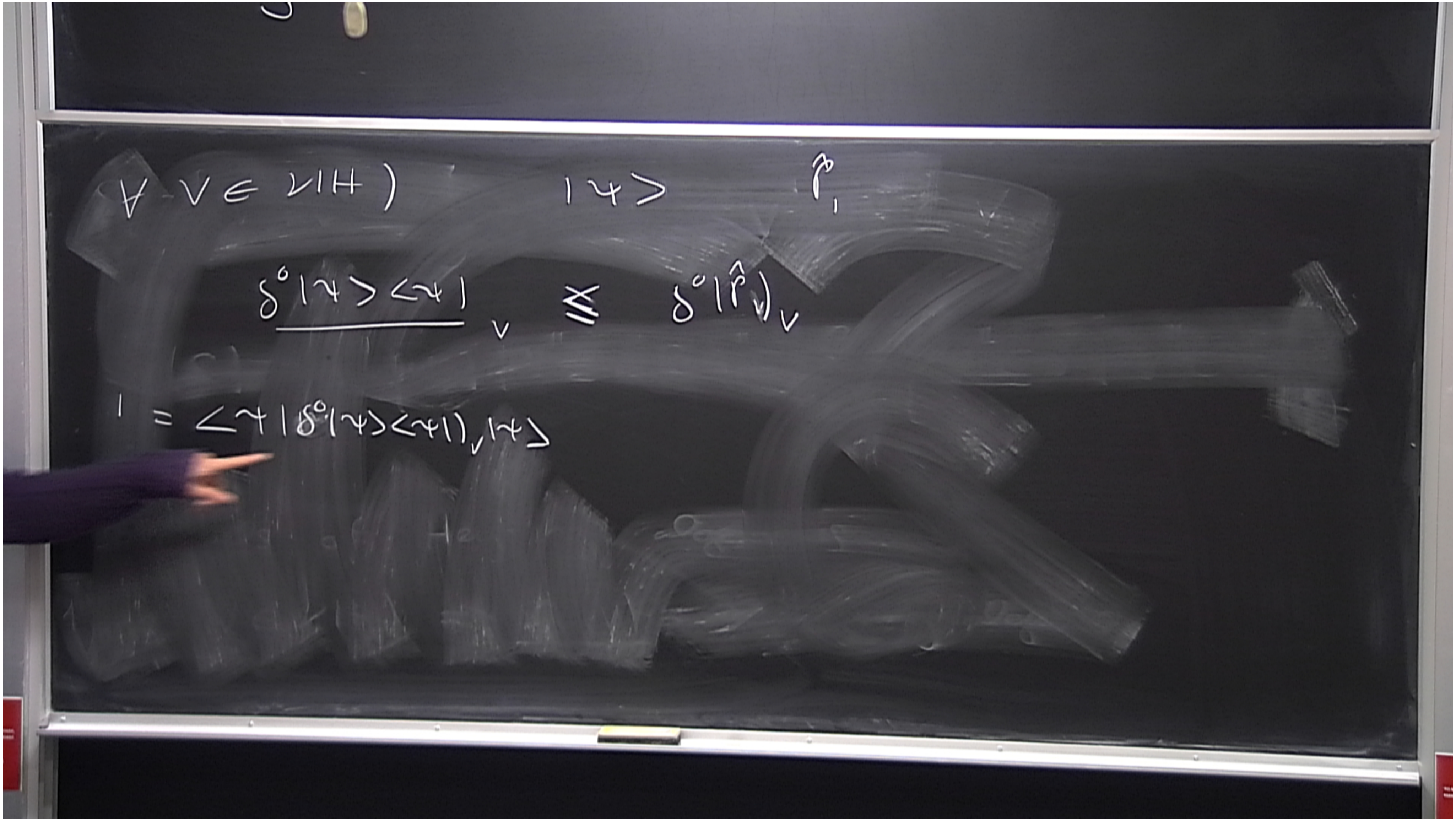
$(\delta |\psi\rangle\langle\psi|)|_{V'} \geq \delta(|\psi\rangle\langle\psi|$

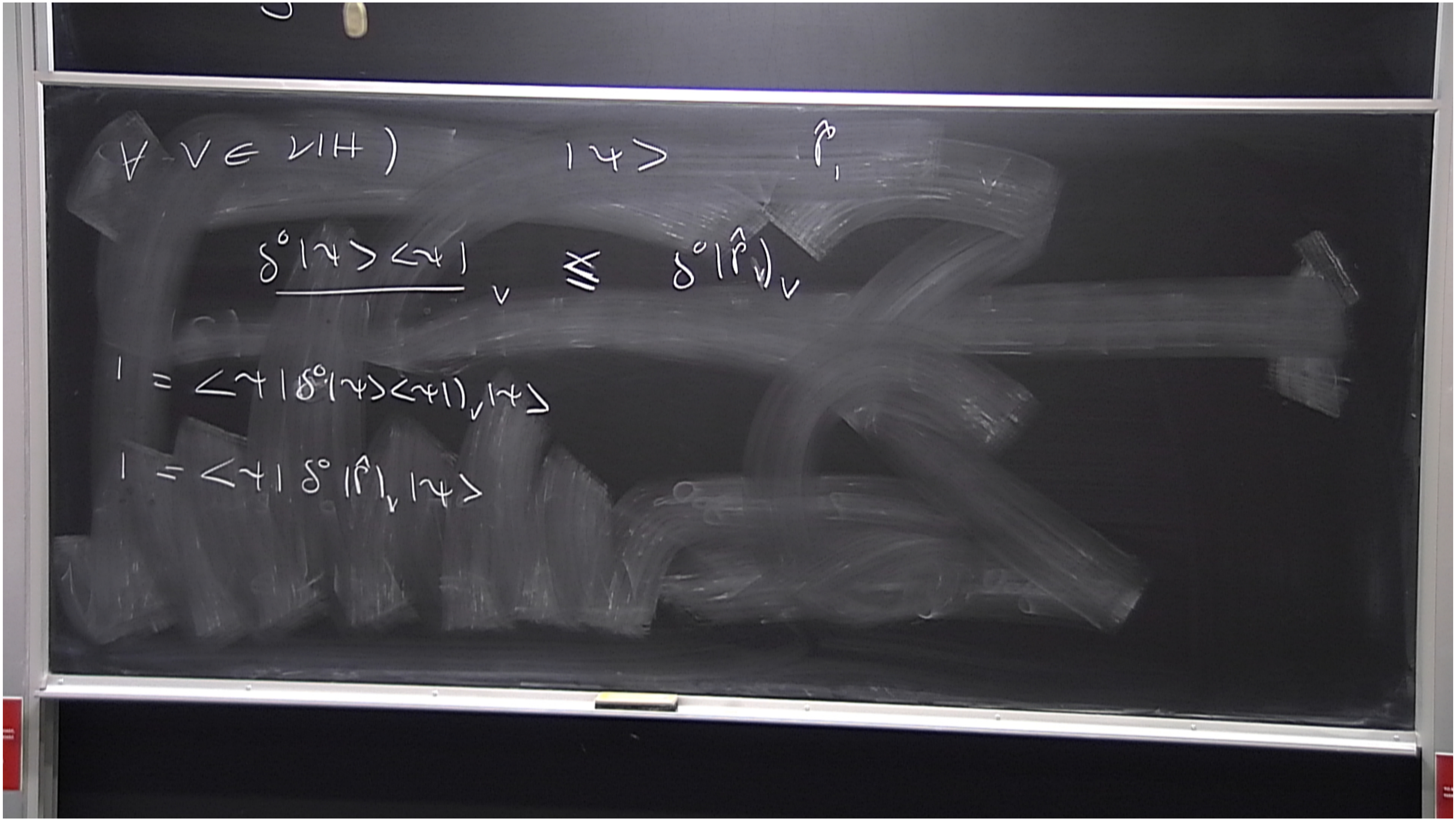

F_A^{-1}(1) is true give S iff S ∈ F_A^{-1}(1) ~> {S} ⊆ F_A^{-1}(1)
ν({S} ⊆ F_A^{-1}(1))
1
0
2) Morphism V' ⊆ V
ω_V → ω_V'
λ → λ_{V'}
(δ°|ψ⟩⟨ψ|)|_{V'} ≥ δ°|ψ⟩⟨ψ|

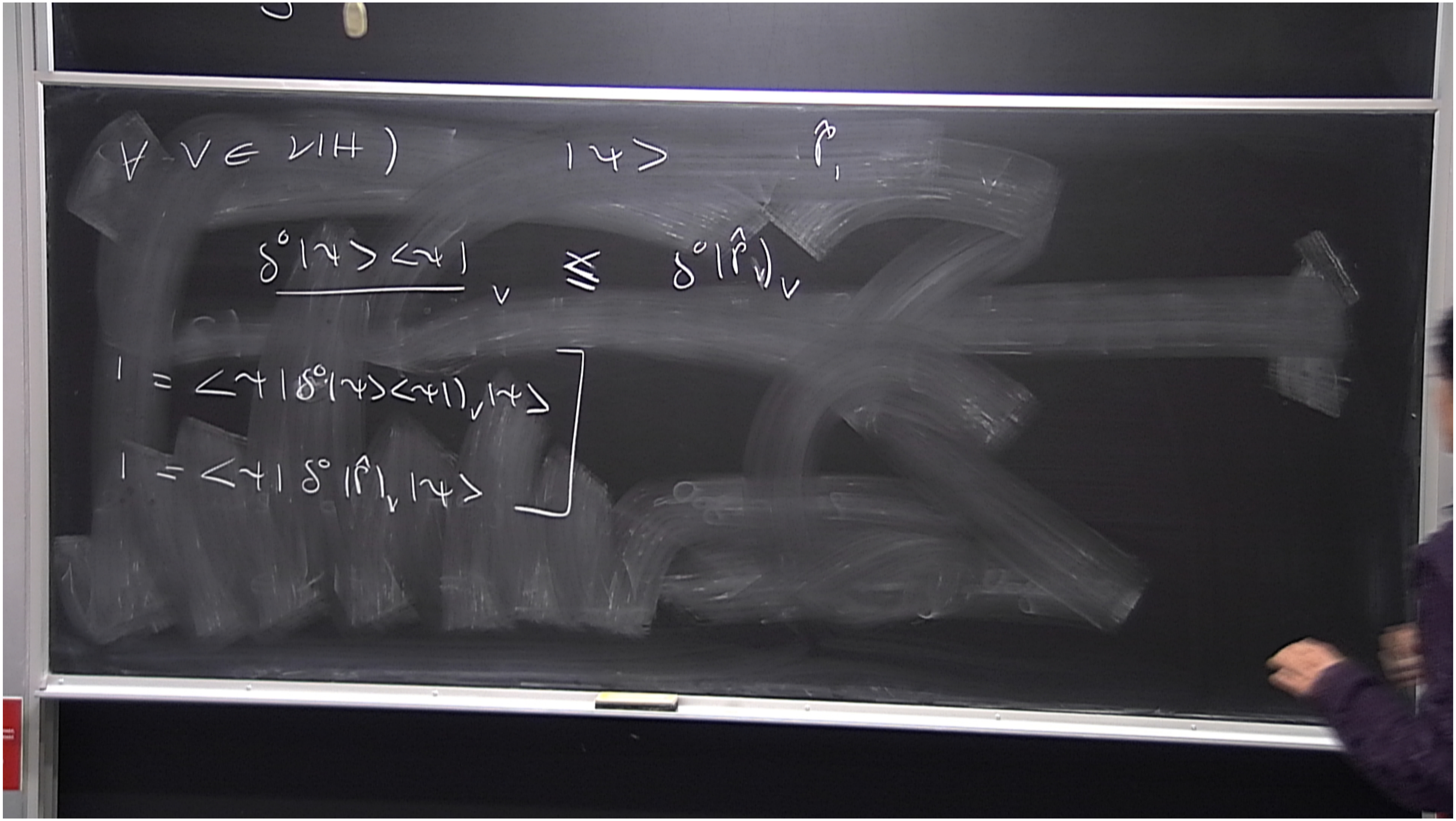

∀ v ∈ V(H))
|ψ>
ρ1

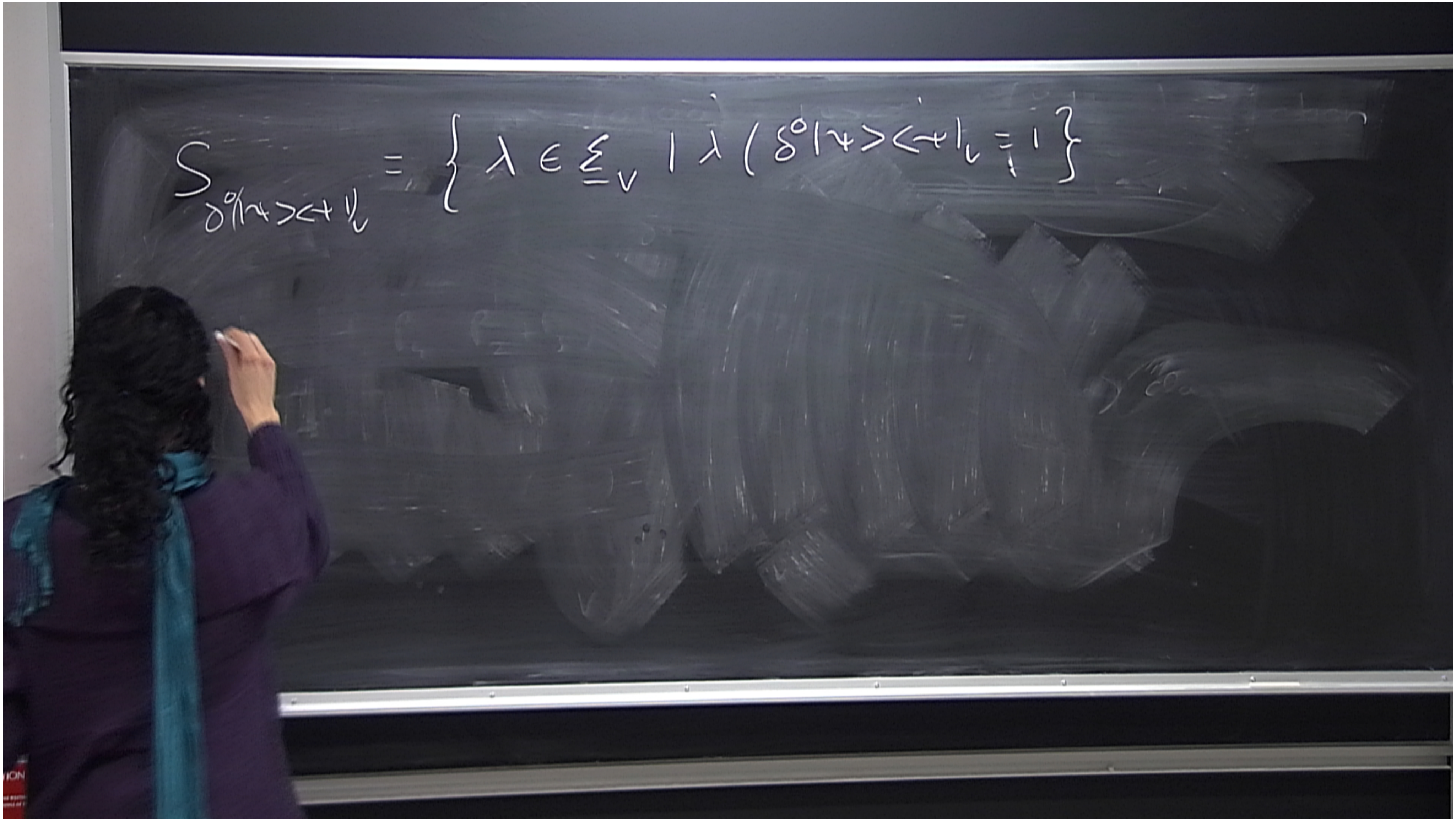


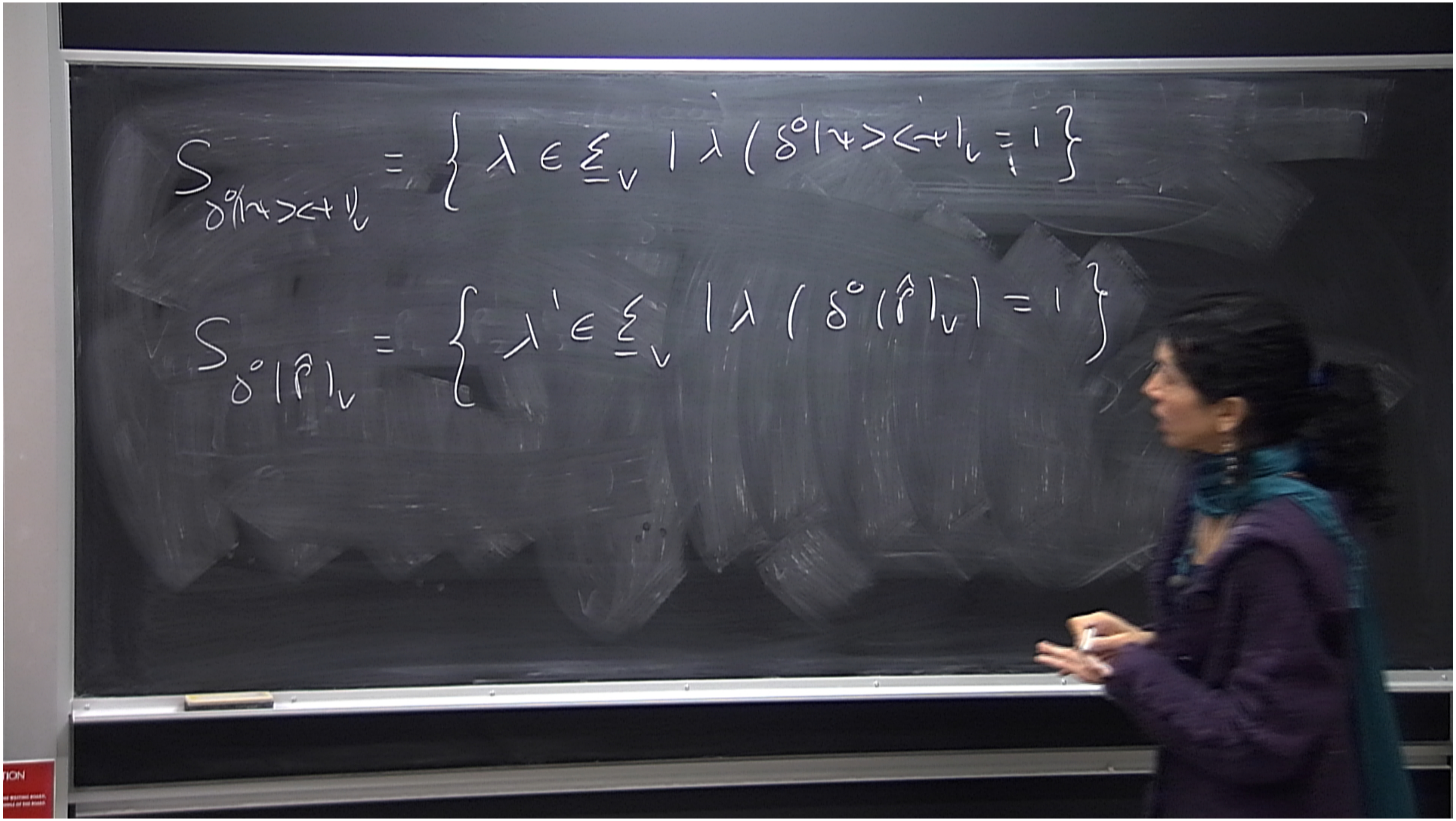

∀ V ∈ L(H)
|ψ>
ρ̂
δ°|ψ><ψ|
V
δ°(ρ̂)V
<ψ|δ°(|ψ><ψ|)V|ψ>

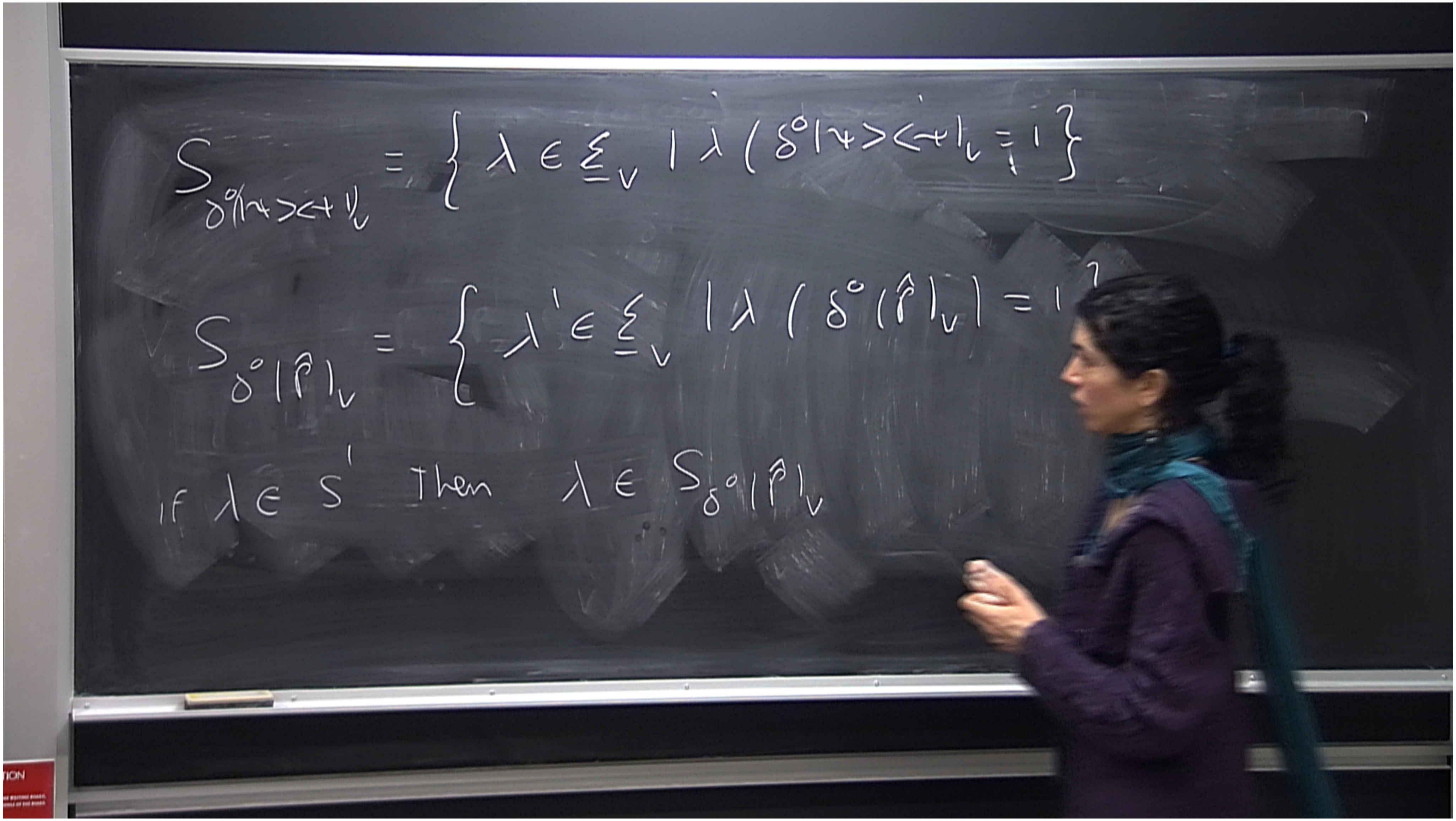

∀ V ∈ V(H)
|ψ>
ρ̂_ψ
δ°|ψ><ψ| _V ≥ δ°(ρ̂_ψ)_V
1 = <ψ|δ°(|ψ><ψ|)_V|ψ>

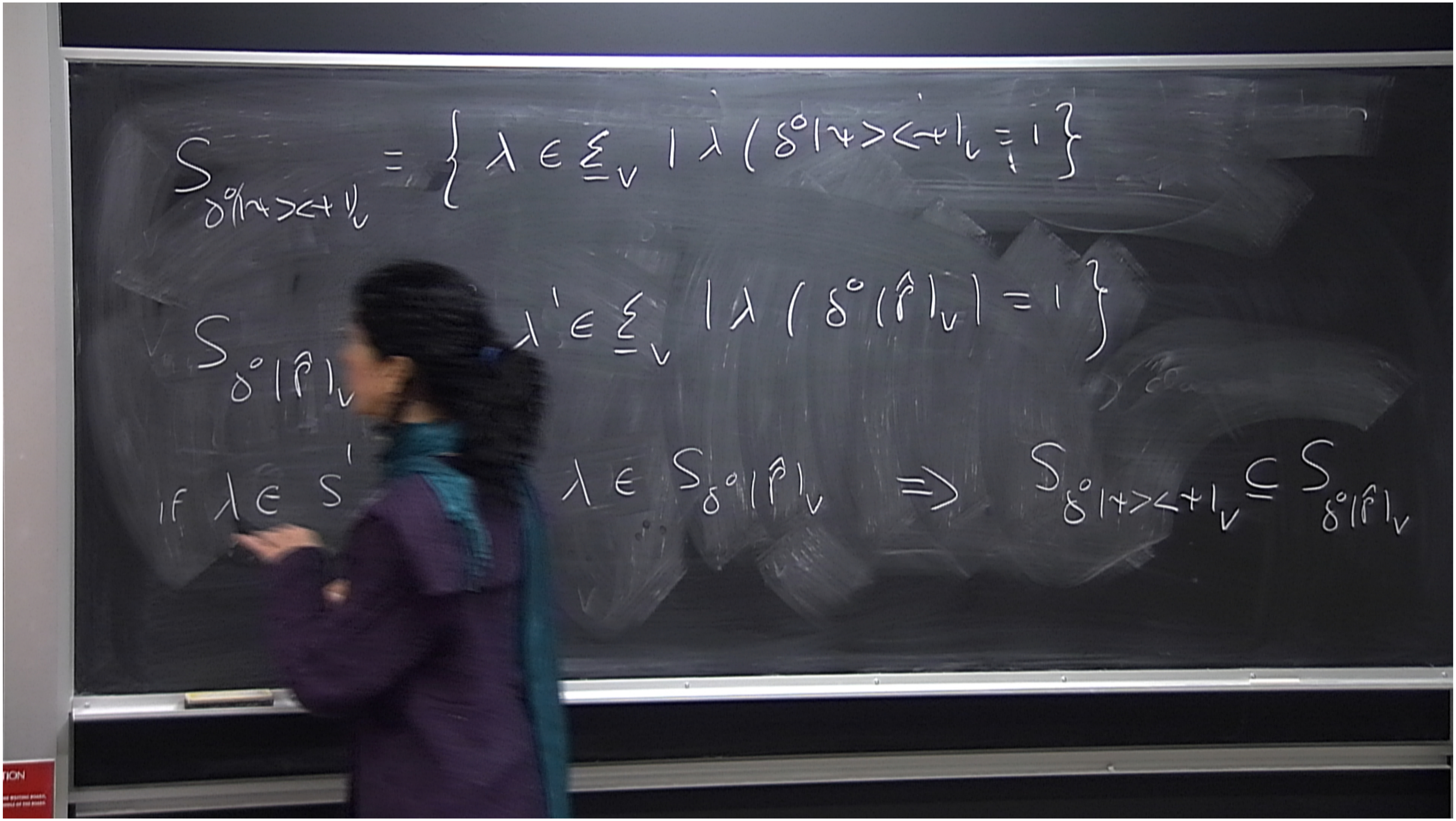

∀ V ∈ V(H) |ψ> ρ̂,
δ°|ψ><ψ|_V ≤ δ°(ρ̂)_V
1 = <ψ|δ°(|ψ><ψ|)_V|ψ>
1 = <ψ|δ°(ρ̂)_V|ψ>

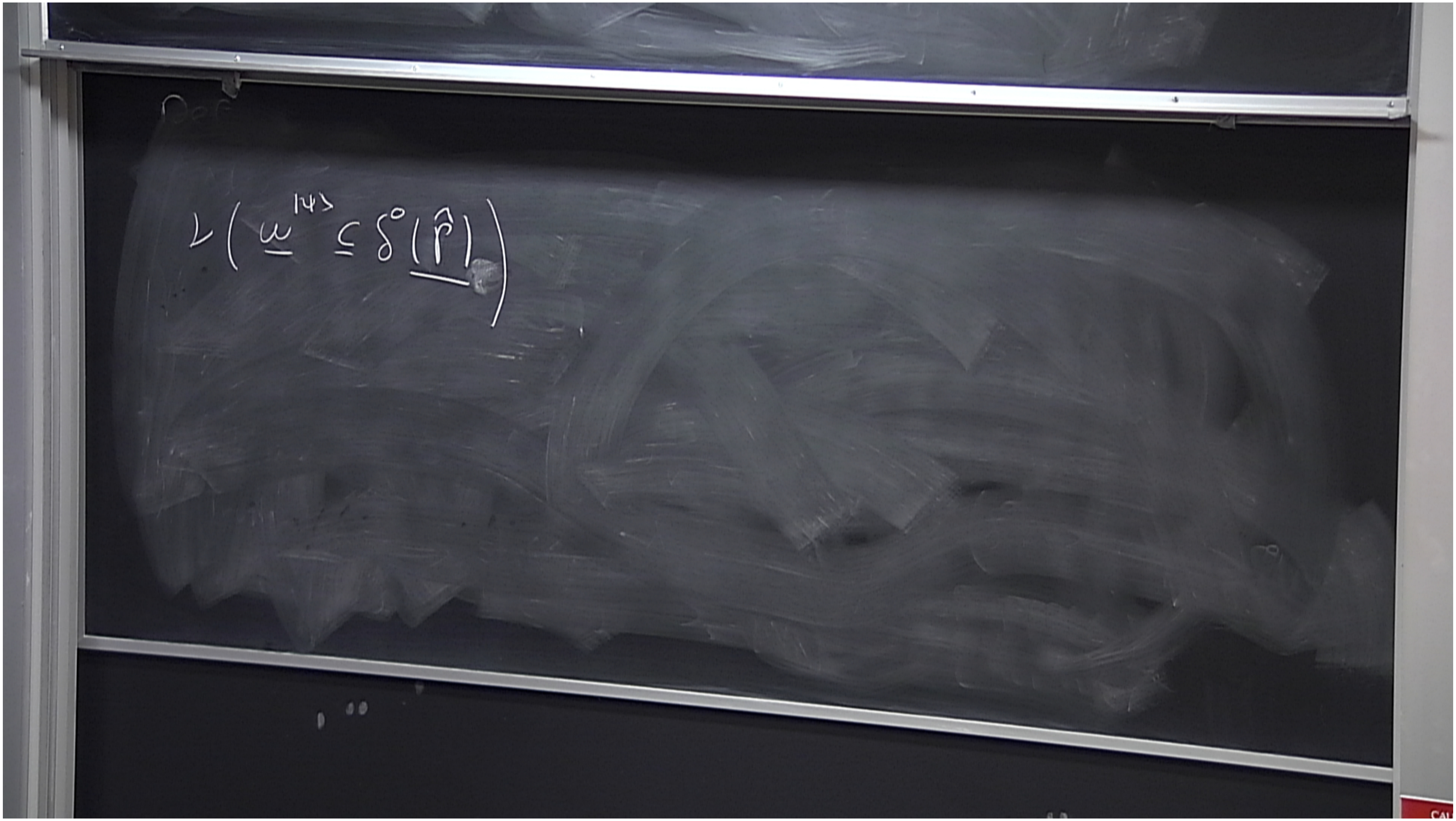

∀ V ∈ V(H) |ψ>   ρ̂
δ°|ψ><ψ|_V ≡ δ°(ρ̂)_V
1 = <ψ|δ°(|ψ><ψ|)_V|ψ>
1 = <ψ|δ°(ρ̂)_V|ψ>

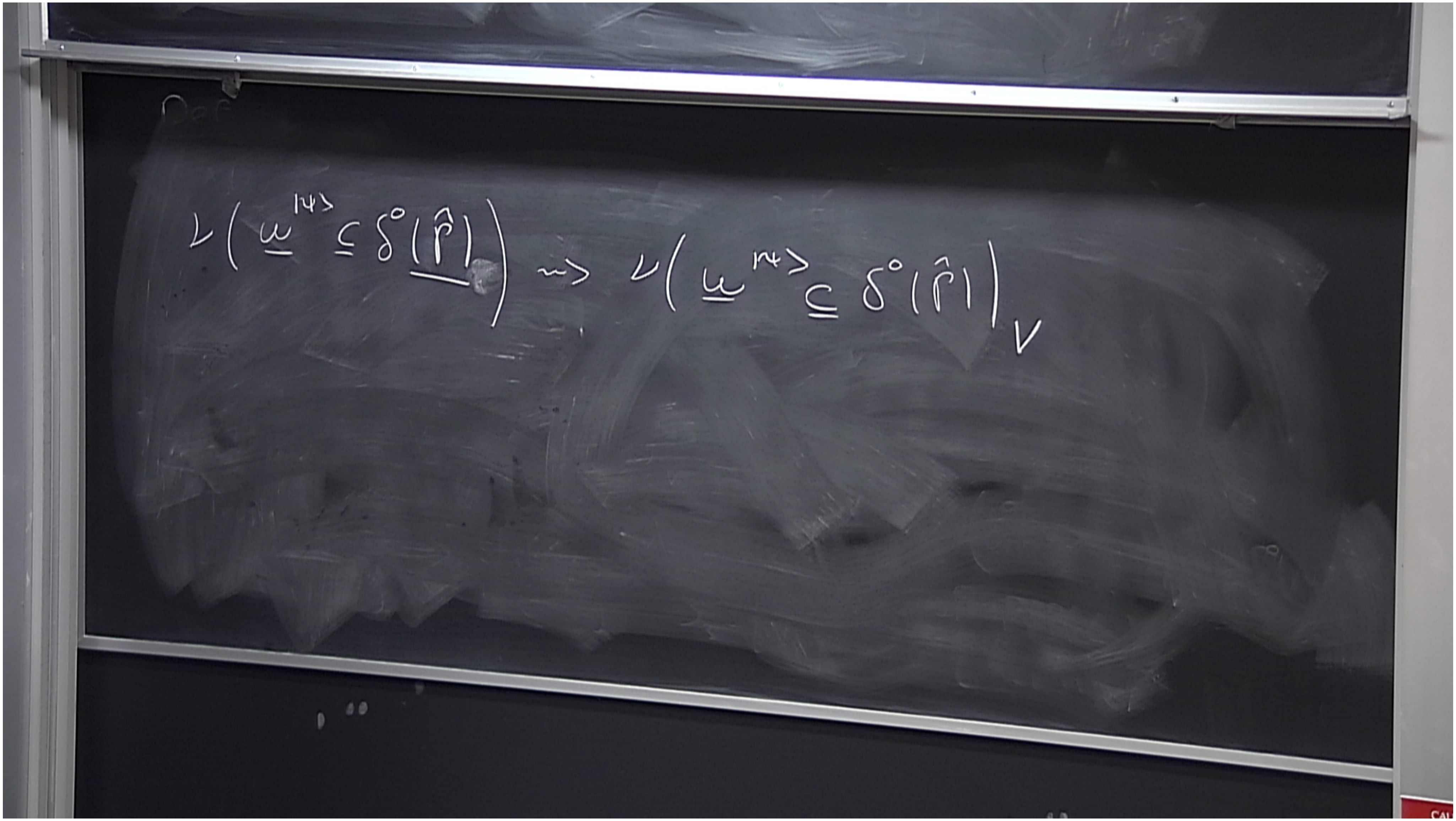

S δ°|ψ><ψ|v = { λ ∈ Σv | λ (δ°|ψ><ψ|v = 1 }

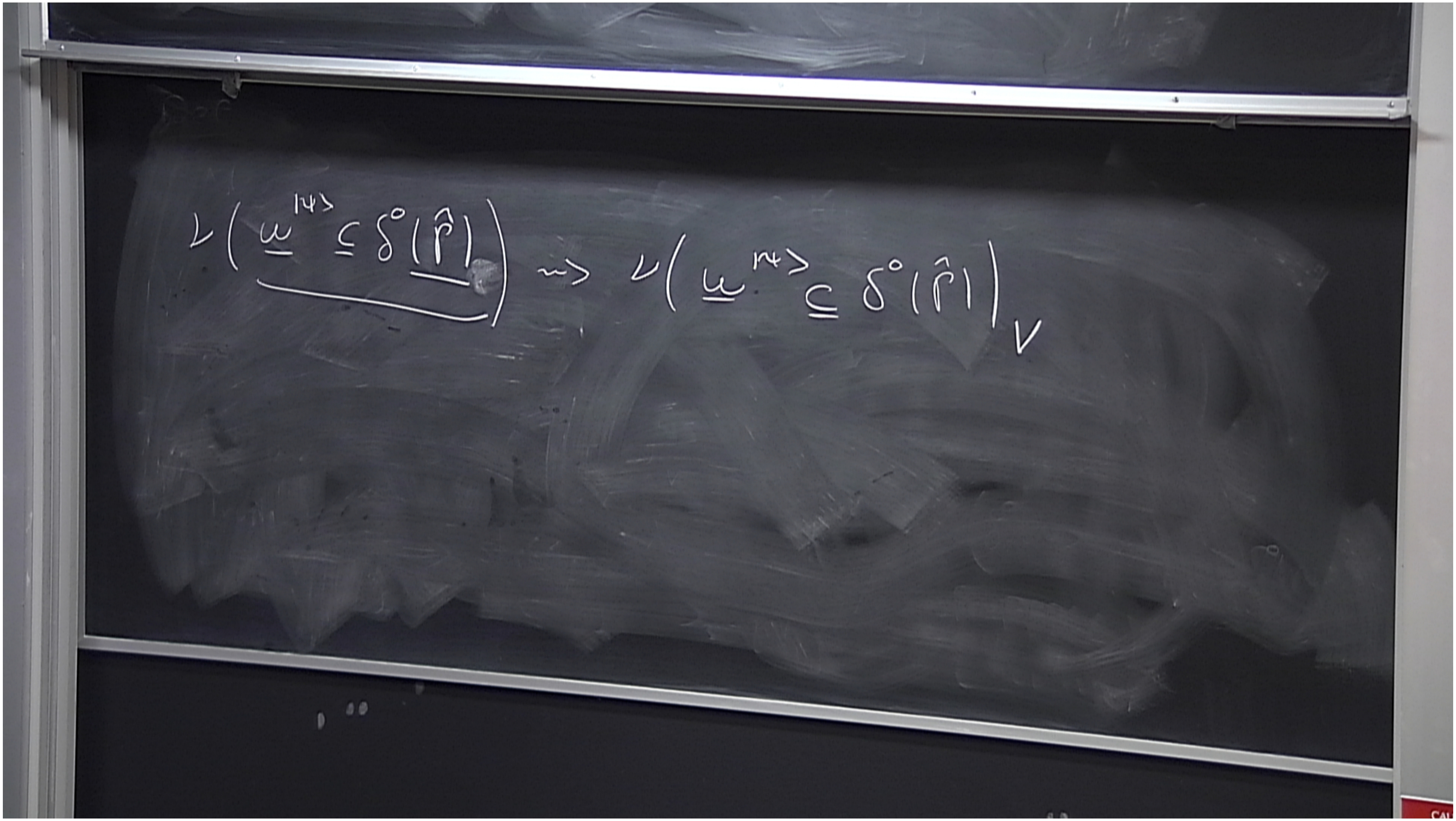

$S_{\delta^o|\psi\rangle\langle\psi|_v} = \{ \lambda \in \underline{\Sigma}_v \mid \lambda(\delta^o|\psi\rangle\langle\psi|_v) = 1 \}$
$S_{\delta^o|\hat{P}|_v} = \{ \lambda' \in \underline{\Sigma}_v \mid \lambda(\delta^o(\hat{P})_v) = 1 \}$

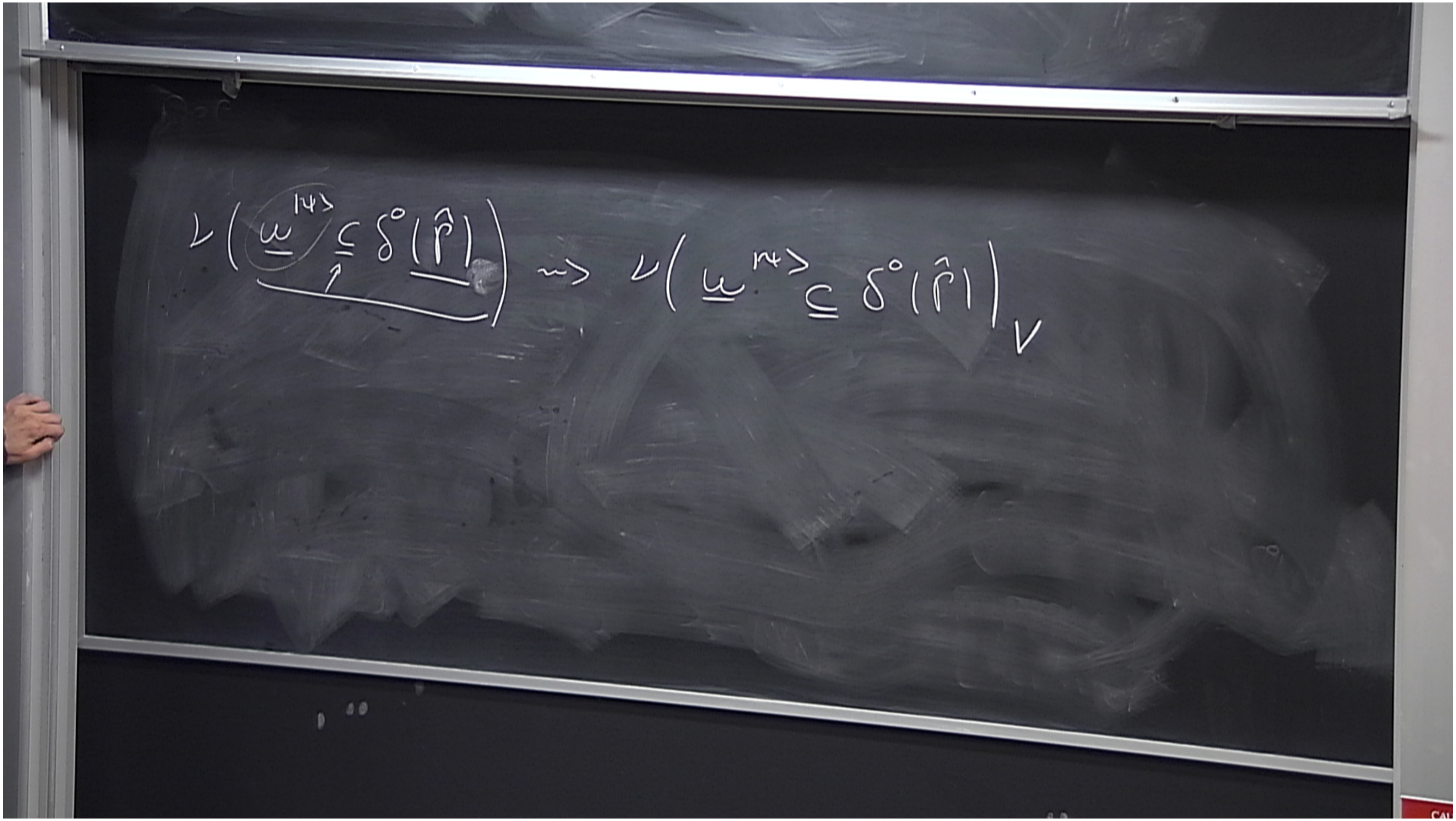

$S_{\delta^o|\psi\rangle\langle\psi|_v} = \{ \lambda \in \underline{\Sigma}_v \mid \lambda(\delta^o|\psi\rangle\langle\psi|_v) = 1 \}$
$S_{\delta^o(\hat{P})_v} = \{ \lambda' \in \underline{\Sigma}_v \mid \lambda(\delta^o(\hat{P})_v) = 1$
if $\lambda \in S'$ then $\lambda \in S_{\delta^o(\hat{P})_v}$

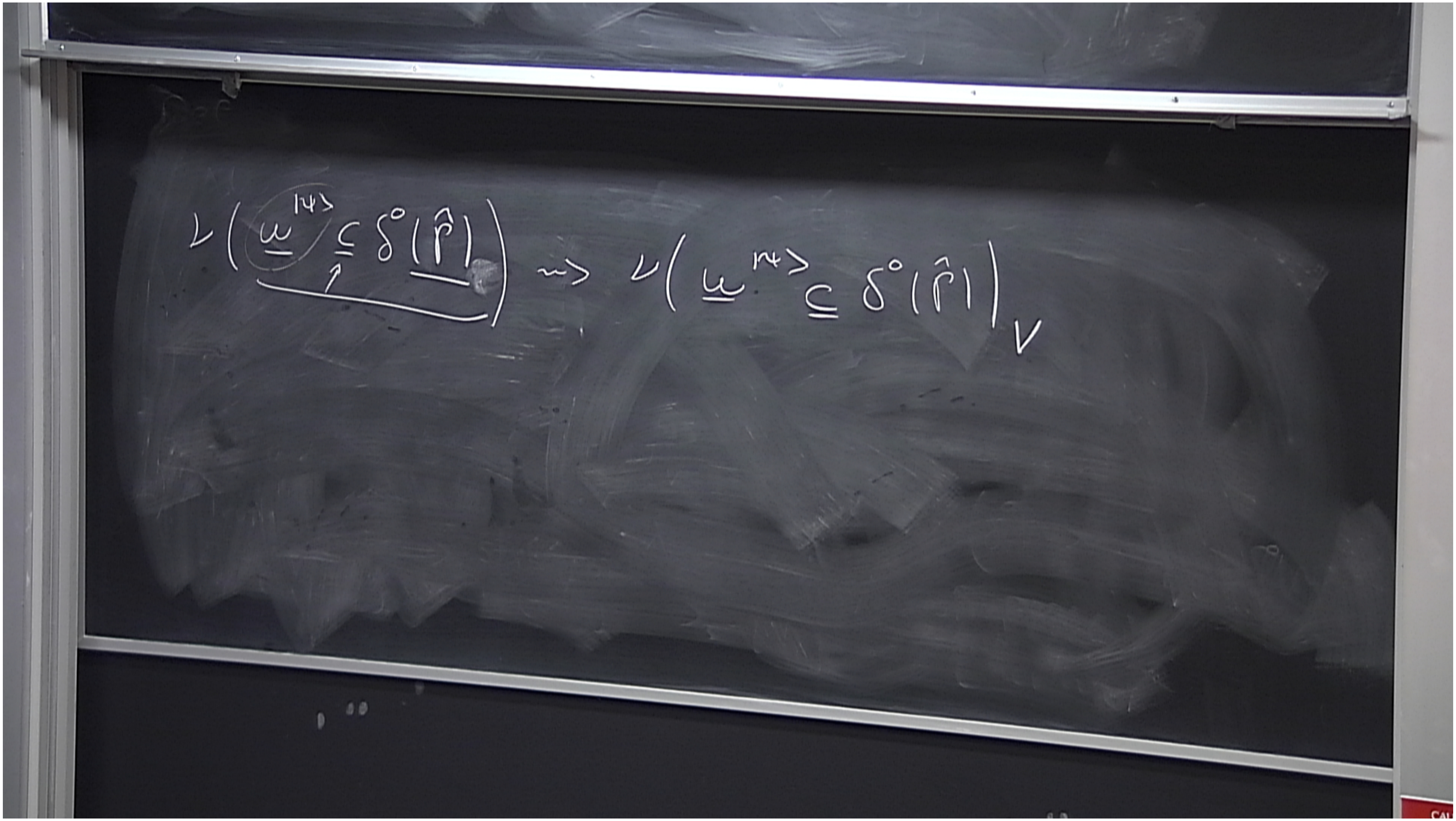

S_{δ°|+><+|_v} = { λ ∈ Σ_v | λ(δ°|+><+|_v = 1 }
S_{δ°|P̂|_v} λ' ∈ Σ_v | λ(δ°(P̂|_v) = 1 }
if λ ∈ S' λ ∈ S_{δ°|P̂|_v} ⇒ S_{δ°|+><+|_v} ⊆ S_{δ°|P̂|_v}




ν(w^{n+} ⊆ δ°(P̂)) ~> ν(w^{n+} ⊆ δ°(P̂))_V


ν(w^{|ψ>} ⊆ δ°(P̂)) ~> ν(w^{|ψ>} ⊆ δ°(P̂))_V


ν(w ⊆ δ°(p̂)) ~> ν(w ⊆ δ°(p̂))V


ν(w^{14>} ⊆ δ°(|p̂|)) ~> ν(w^{14>} ⊆ δ°(|p̂|))_V