Title: Tensor Field Theory: Renormalization and One-loop Beta Functions

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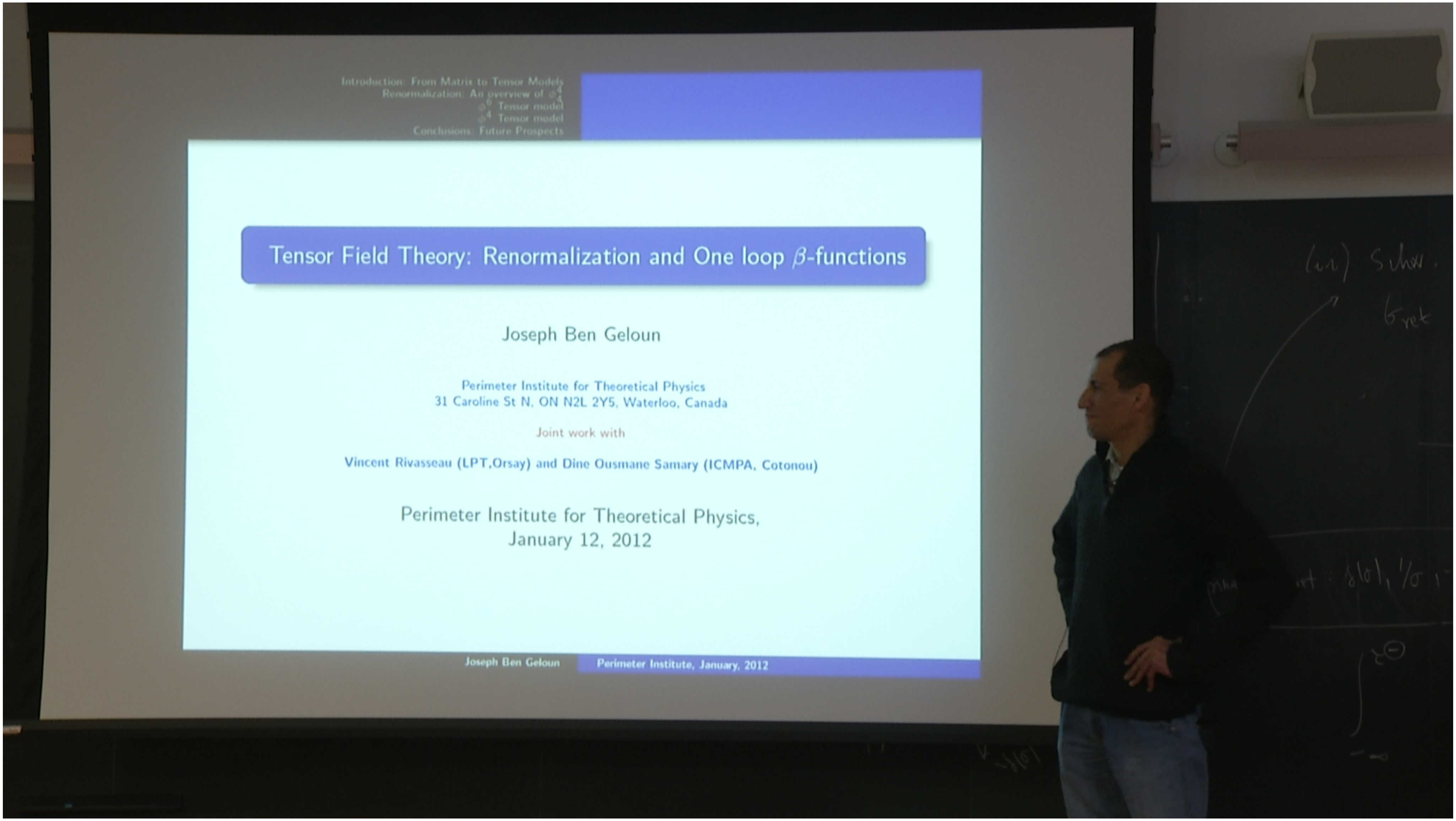
Abstract: Tensor models appear as the higher dimensional extension of the so-called matrix models describing 2D quantum gravity through the sum over triangulations of surfaces. In the light of the recent $1/N$ expansion for these tensor models, we uncover a new class of tensor models for 4D and 3D gravity which are renormalizable at all orders of perturbation theory. An overview of two papers, [arXiv:1111.4997 [hep-th]] and [arXiv:1201.0176 [hep-th]], on the renormalization of these tensor models and their beta function will be given.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Renormalization: An overview of ϕ^4_4
ϕ^6 Tensor model
ϕ^4 Tensor model
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Tensor Field Theory: Renormalization and One loop β-functions
Joseph Ben Geloun
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31 Caroline St N, ON N2L 2Y5, Waterloo, Canada
Joint work with
Vincent Rivasseau (LPT,Orsay) and Dine Ousmane Samary (ICMPA, Cotonou)
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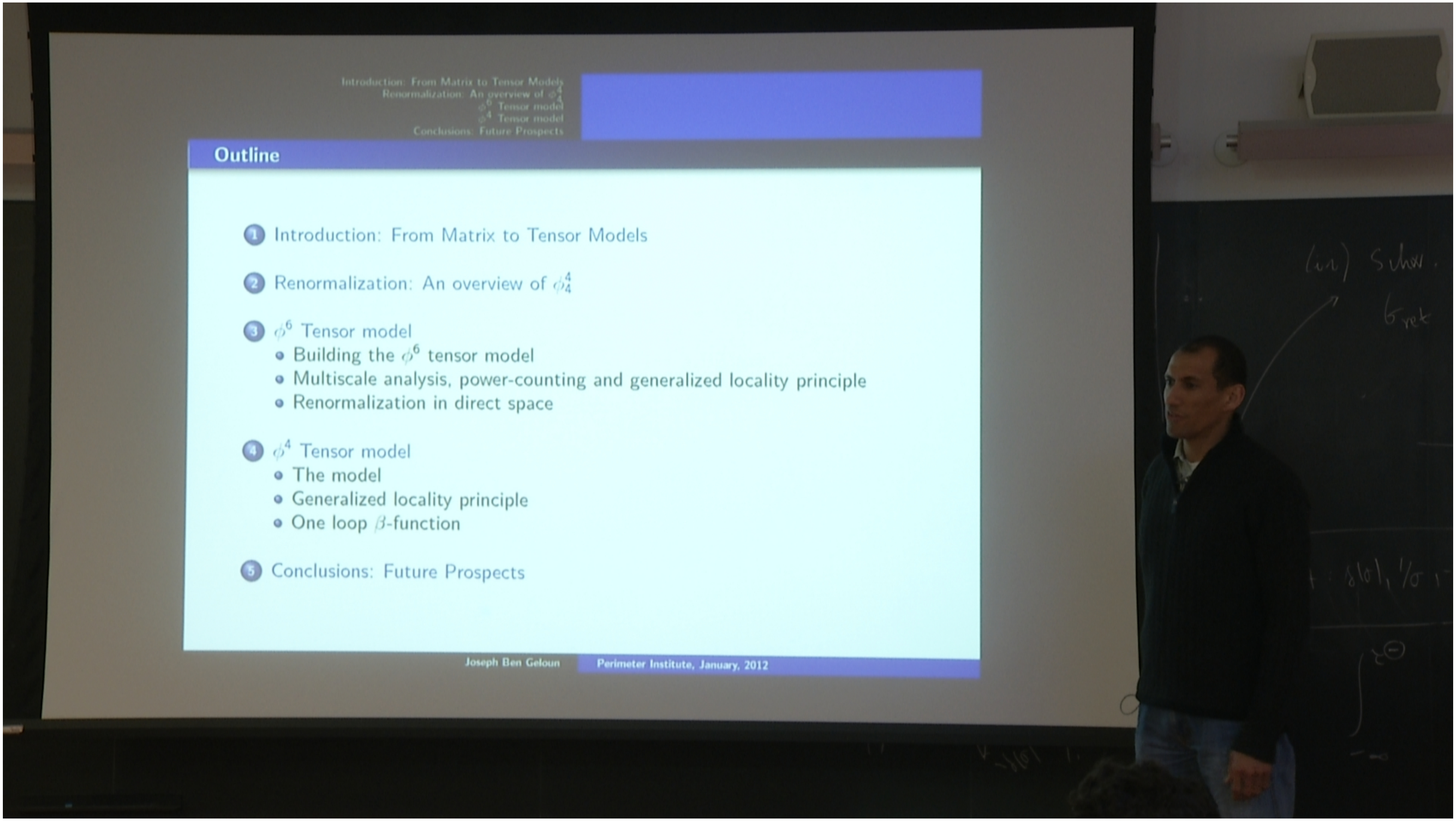
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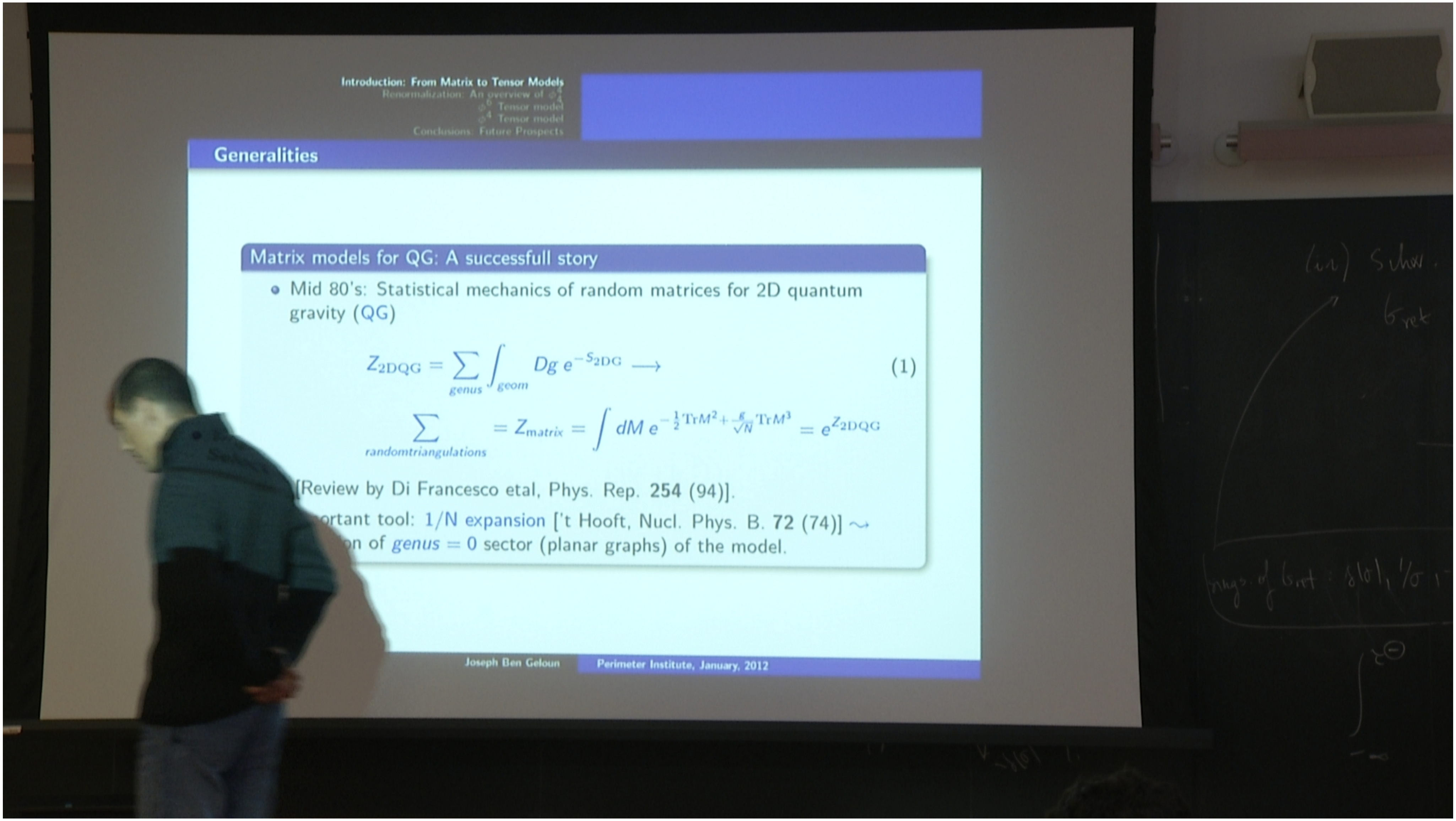

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Matrix models for QG: A successfull story
Mid 80's: Statistical mechanics of random matrices for 2D quantum gravity (QG)
$$Z_{2DQG} = \sum_{genus} \int_{geom} Dg\, e^{-S_{2DG}} \longrightarrow \qquad (1)$$
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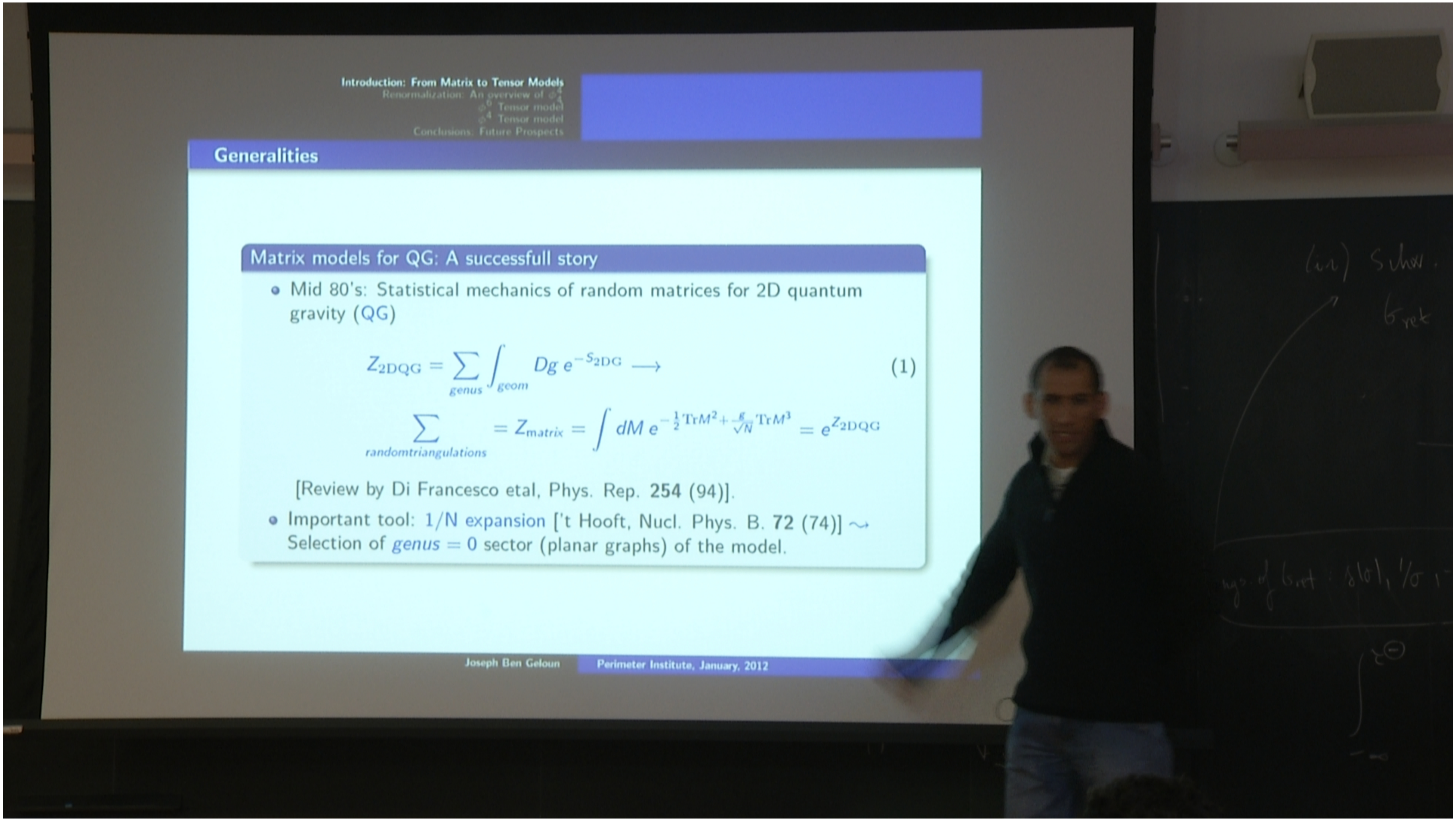
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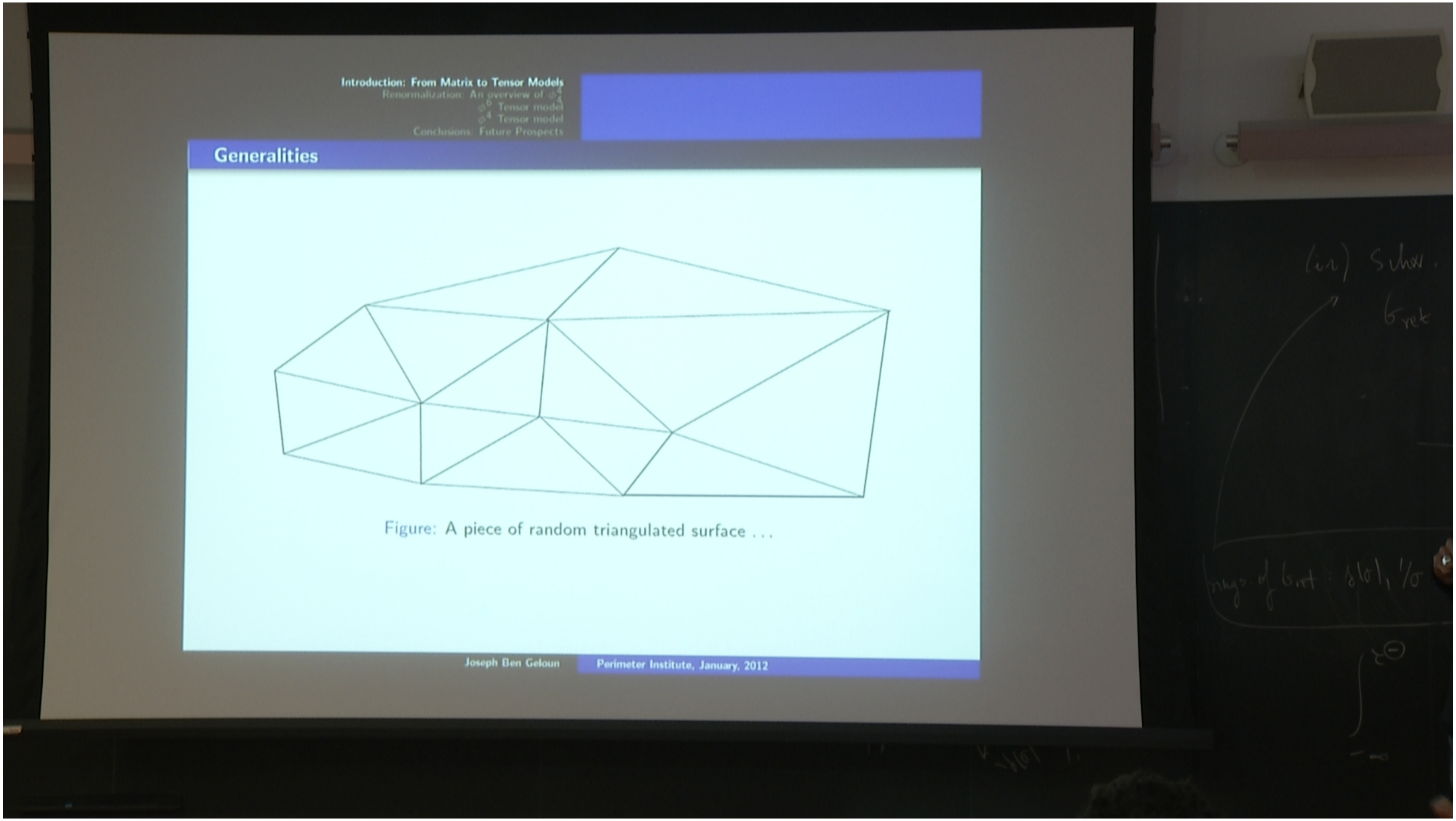
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Figure: A piece of random triangulated surface . . .
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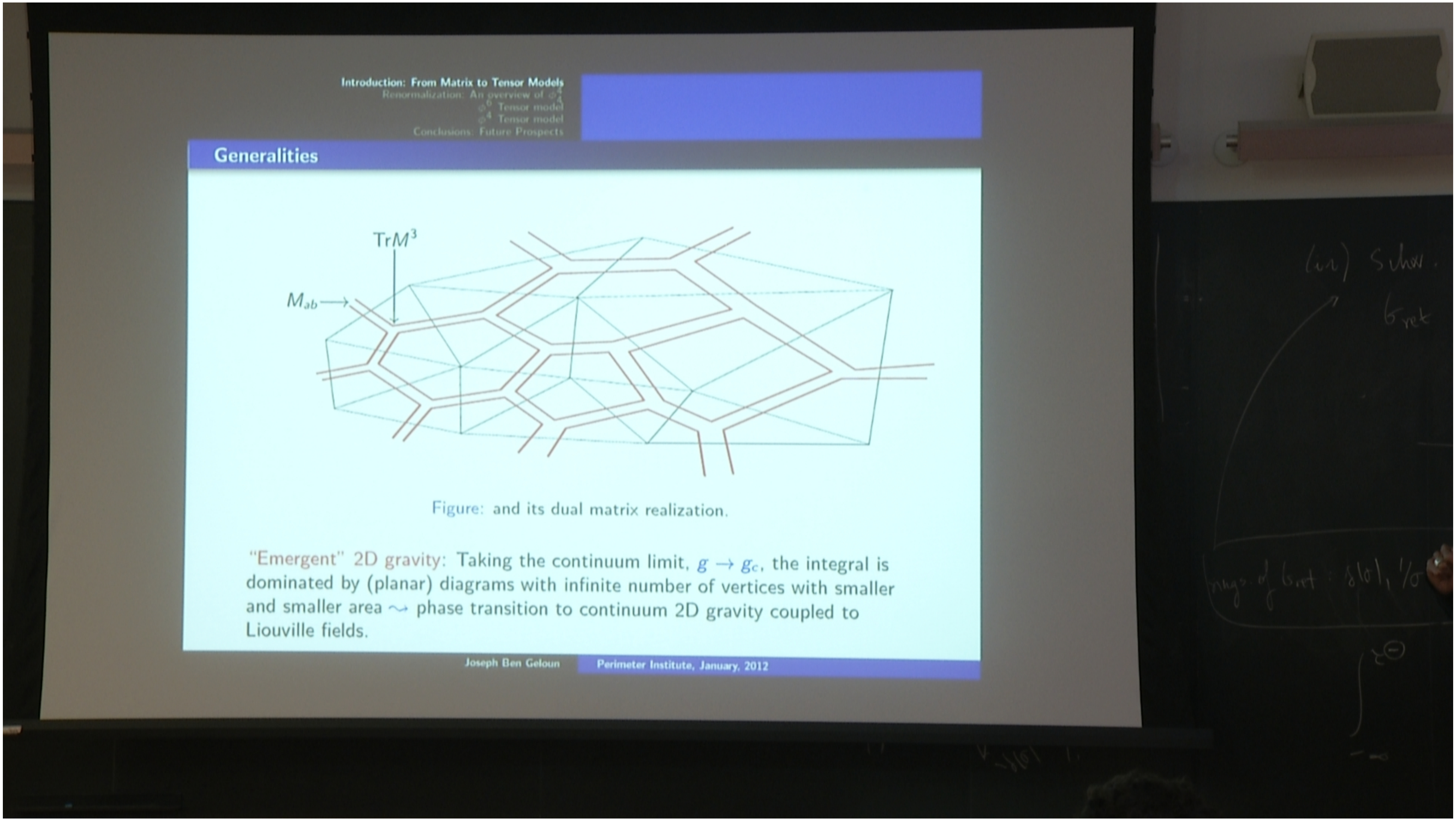
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TrM³
M_ab
Figure: and its dual matrix realization.
"Emergent" 2D gravity: Taking the continuum limit, g → g_c, the integral is dominated by (planar) diagrams with infinite number of vertices with smaller and smaller area ⤳ phase transition to continuum 2D gravity coupled to Liouville fields.
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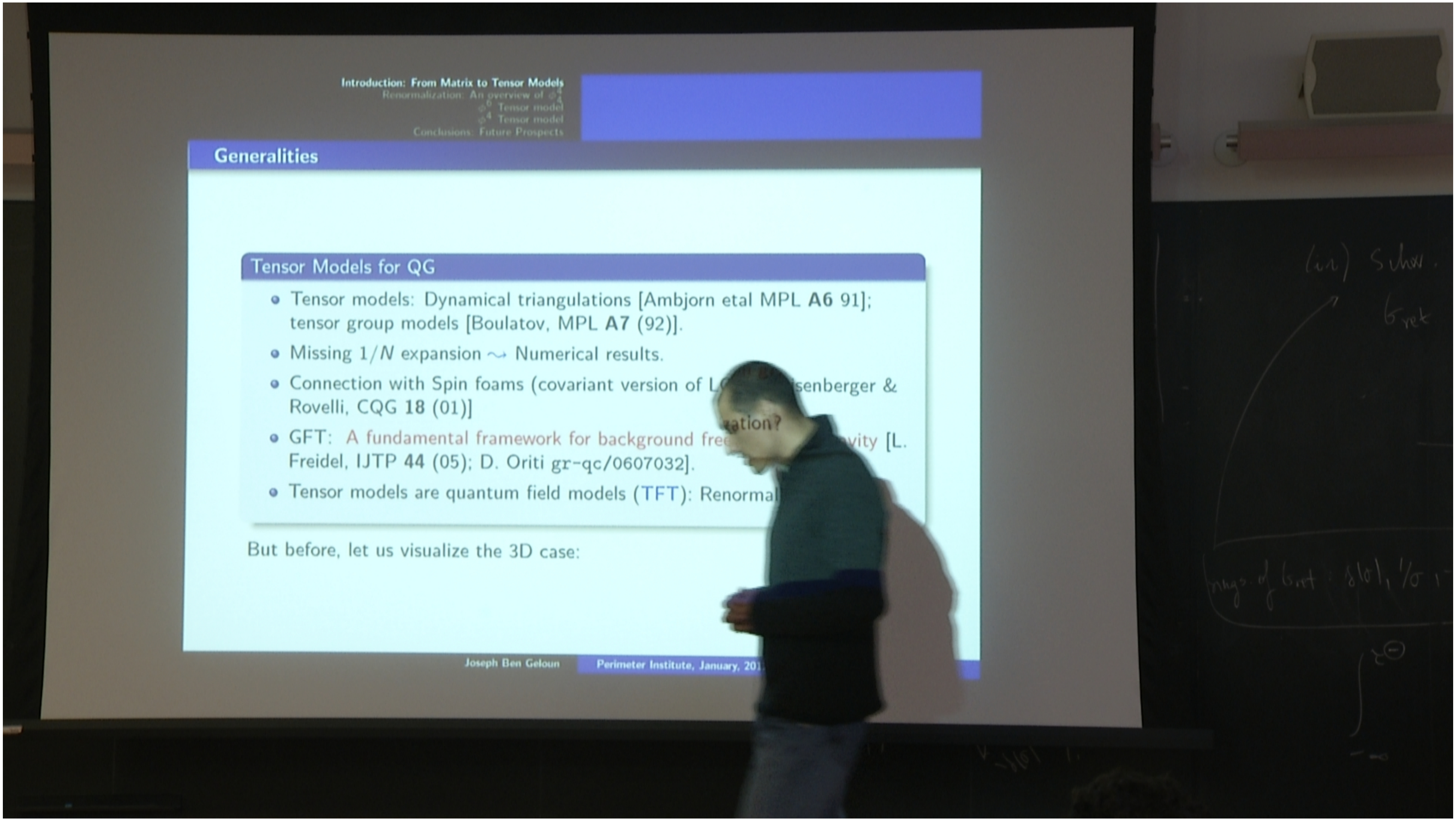
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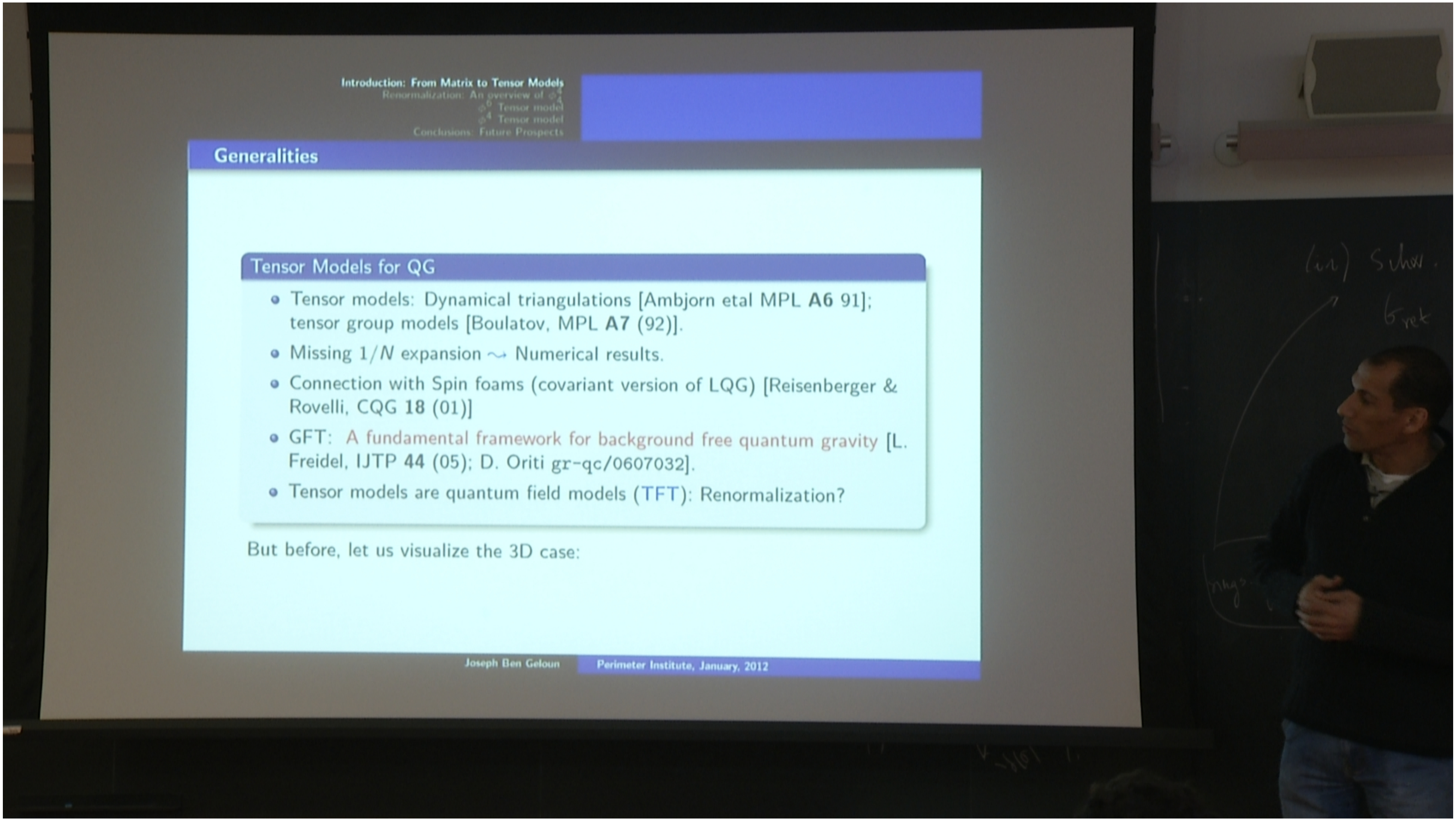
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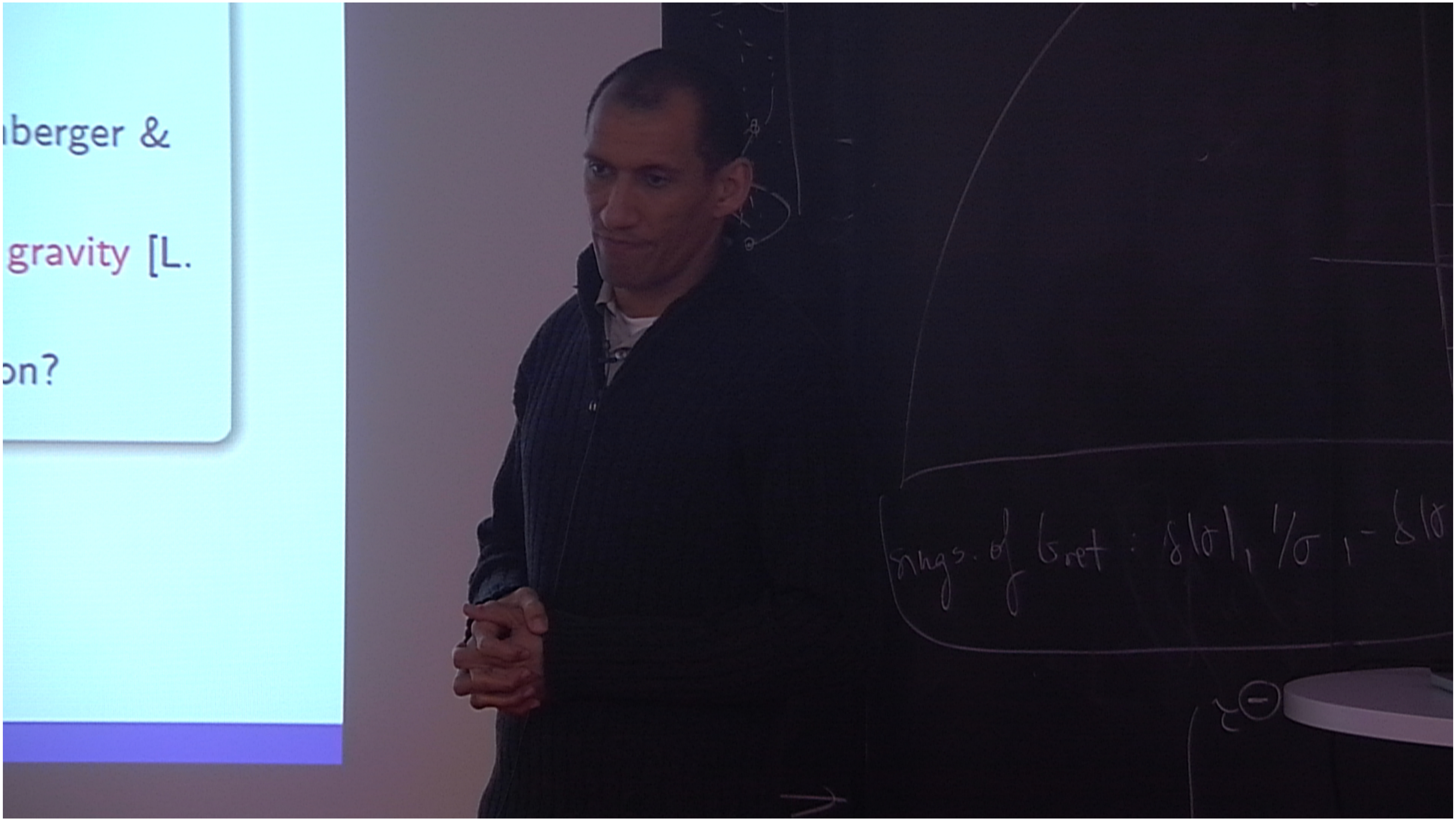
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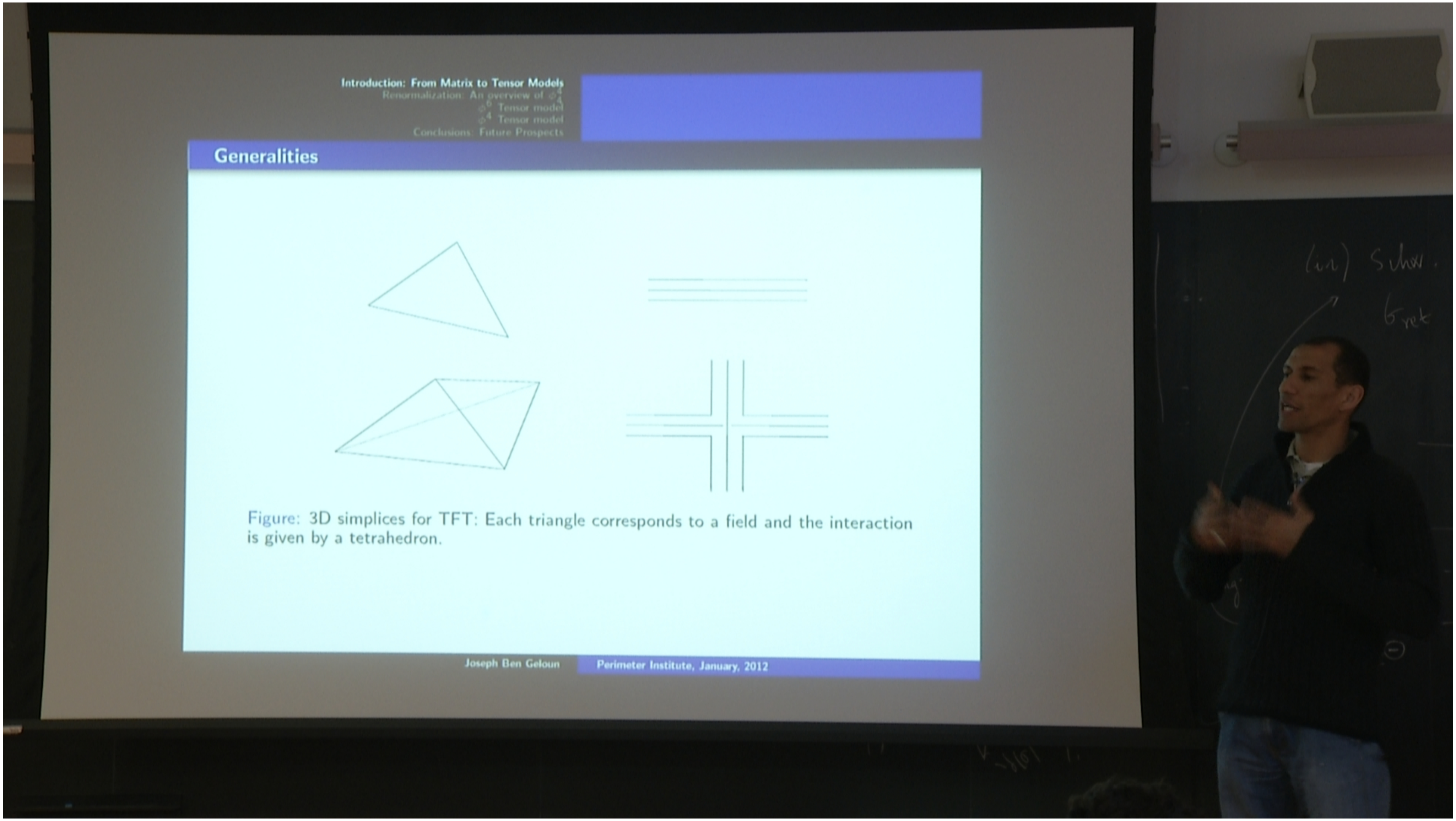
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Figure: 3D simplices for TFT: Each triangle corresponds to a field and the interaction is given by a tetrahedron.
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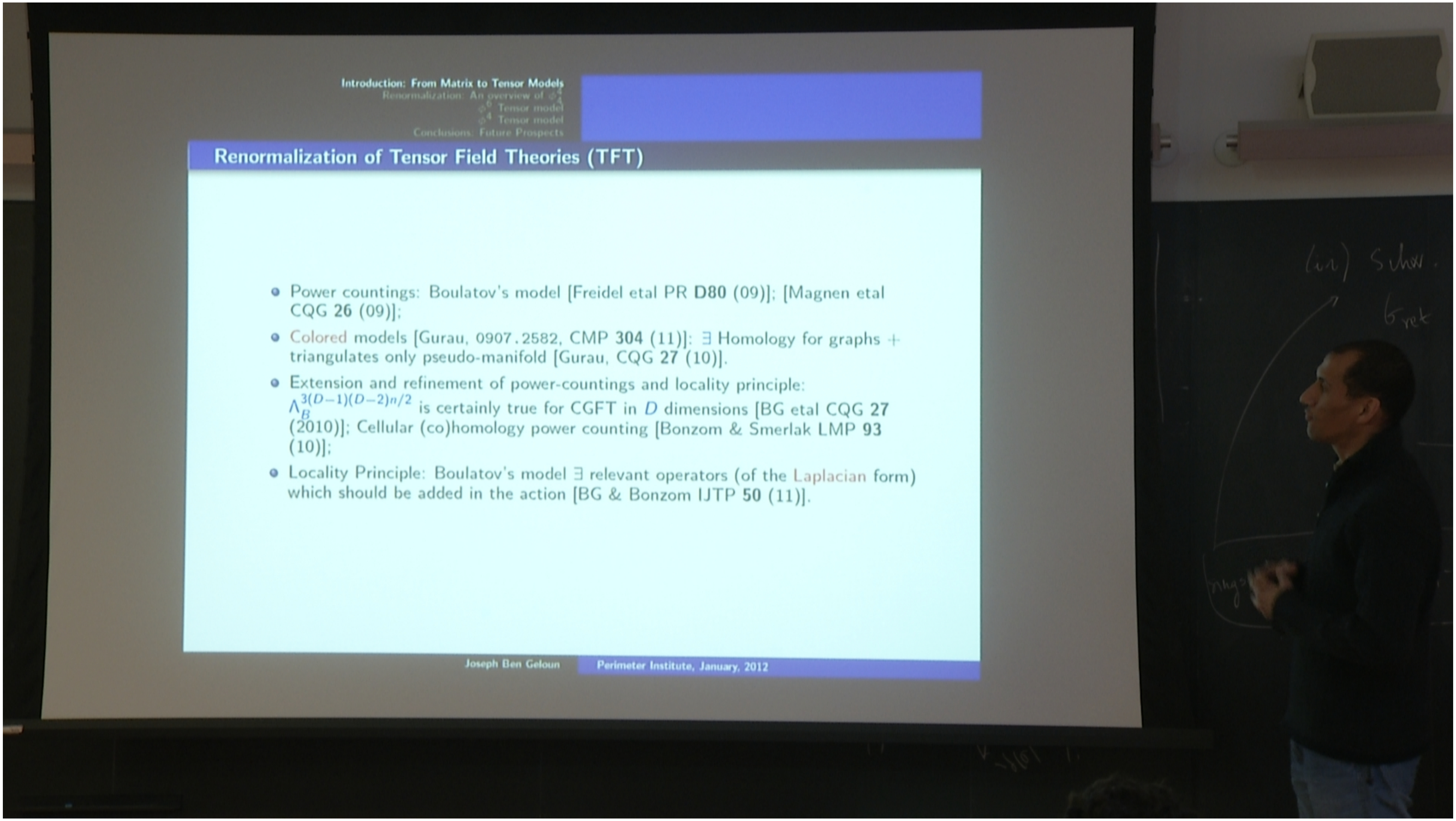
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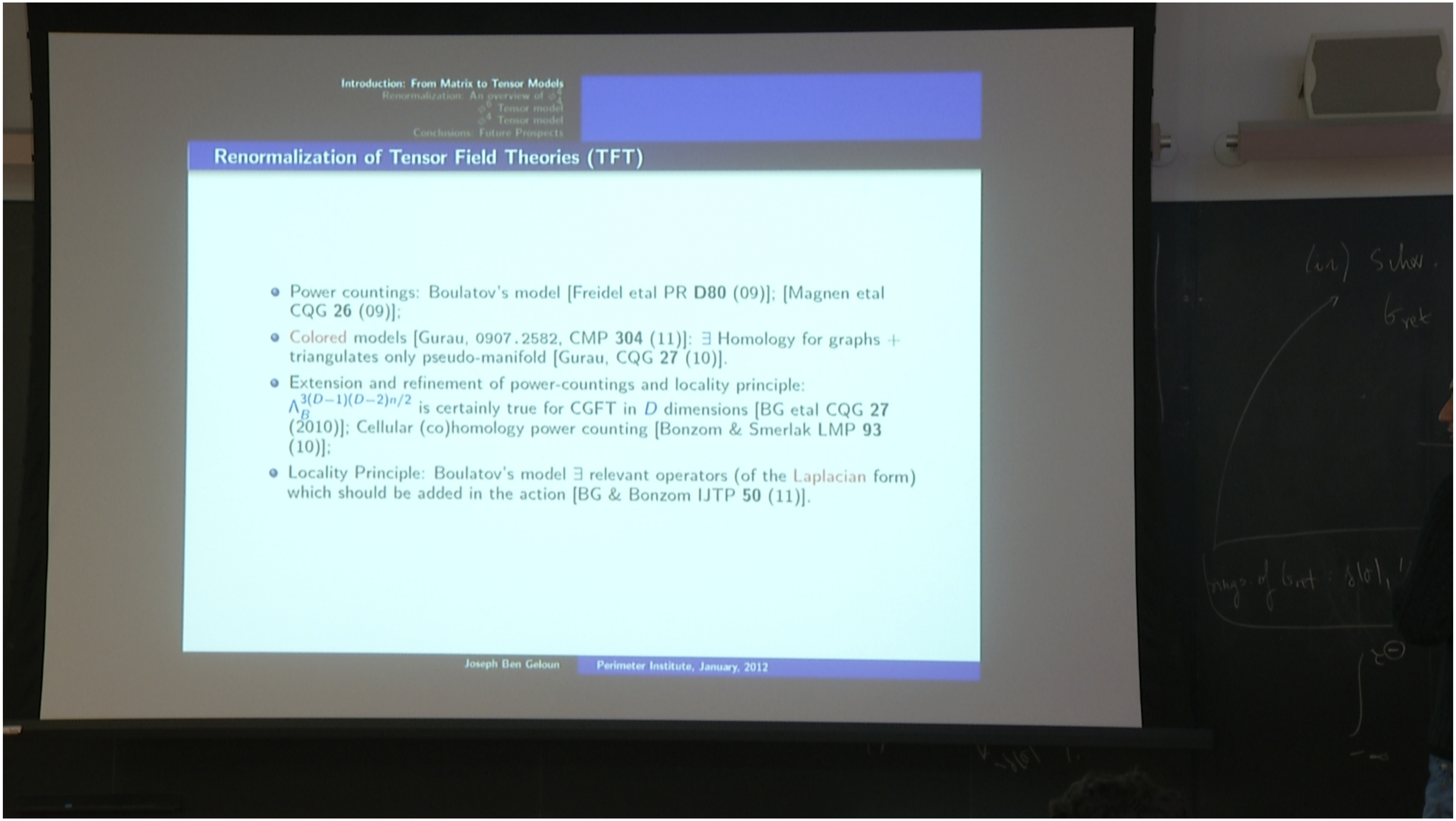
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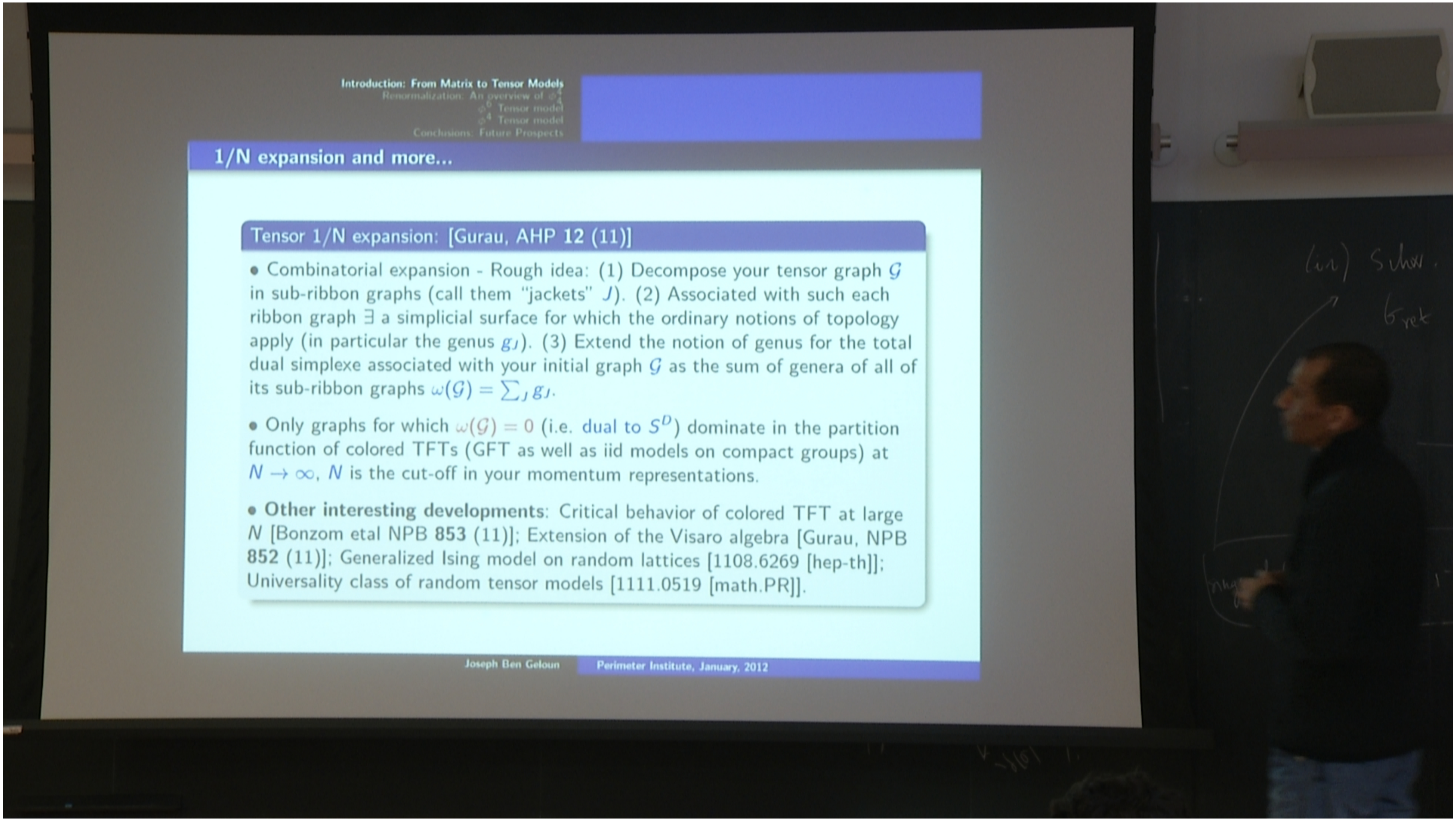
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1/N expansion and more...
Tensor 1/N expansion: [Gurau, AHP 12 (11)]
• Combinatorial expansion - Rough idea: (1) Decompose your tensor graph $\mathcal{G}$ in sub-ribbon graphs (call them "jackets" J). (2) Associated with such each ribbon graph ∃ a simplicial surface for which the ordinary notions of topology apply (in particular the genus g_J). (3) Extend the notion of genus for the total dual simplexe associated with your initial graph $\mathcal{G}$ as the sum of genera of all of its sub-ribbon graphs $\omega(\mathcal{G}) = \sum_J g_J$.
• Only graphs for which $\omega(\mathcal{G}) = 0$ (i.e. dual to S^D) dominate in the partition function of colored TFTs (GFT as well as iid models on compact groups) at $N \to \infty$, N is the cut-off in your momentum representations.
• **Other interesting developments**: Critical behavior of colored TFT at large N [Bonzom etal NPB **853** (11)]; Extension of the Visaro algebra [Gurau, NPB **852** (11)]; Generalized Ising model on random lattices [1108.6269 [hep-th]]; Universality class of random tensor models [1111.0519 [math.PR]].
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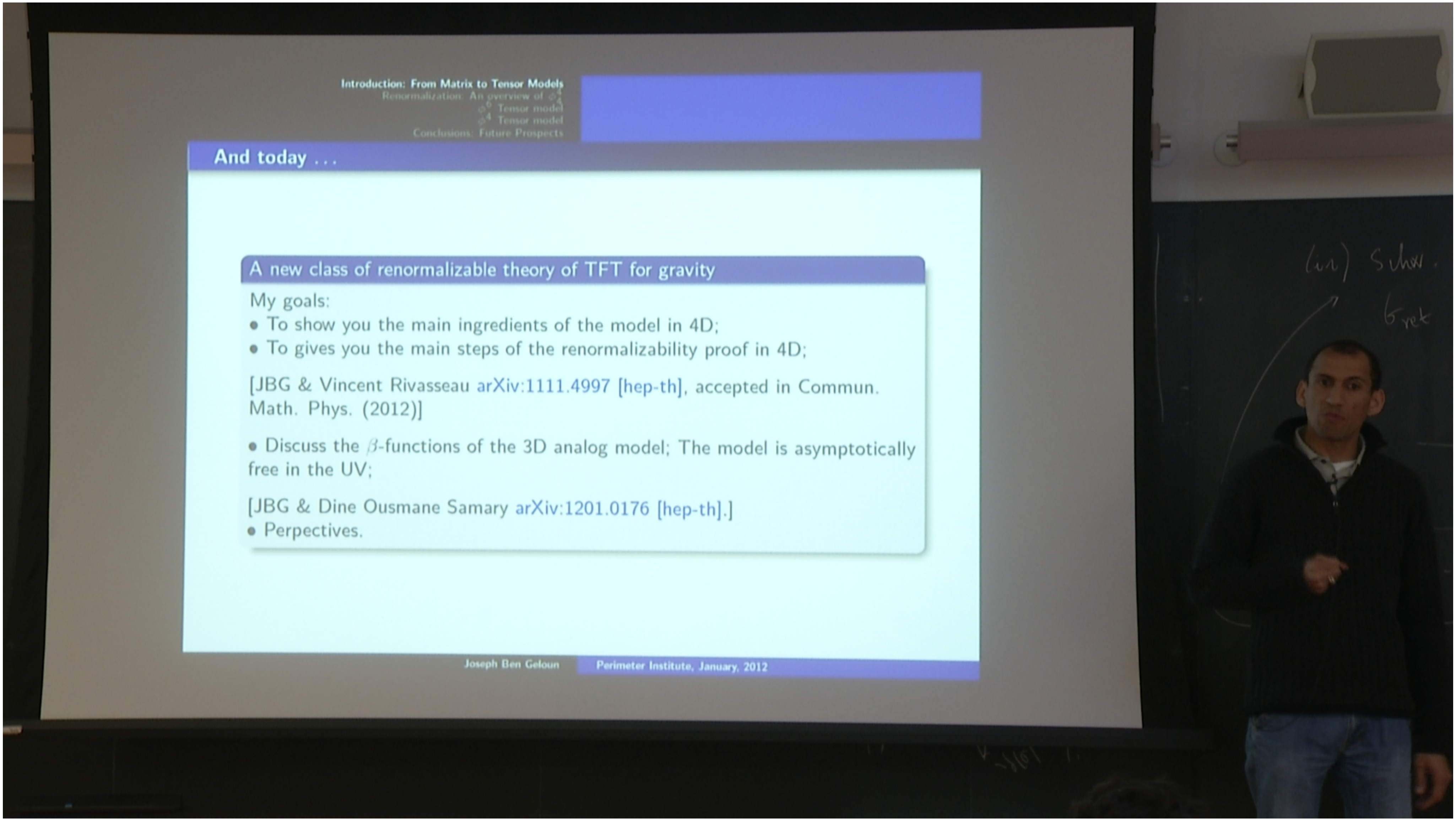
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A new class of renormalizable theory of TFT for gravity
My goals:
• To show you the main ingredients of the model in 4D;
• To gives you the main steps of the renormalizability proof in 4D;
[JBG & Vincent Rivasseau arXiv:1111.4997 [hep-th], accepted in Commun. Math. Phys. (2012)]
• Discuss the β-functions of the 3D analog model; The model is asymptotically free in the UV;
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• Perpectives.
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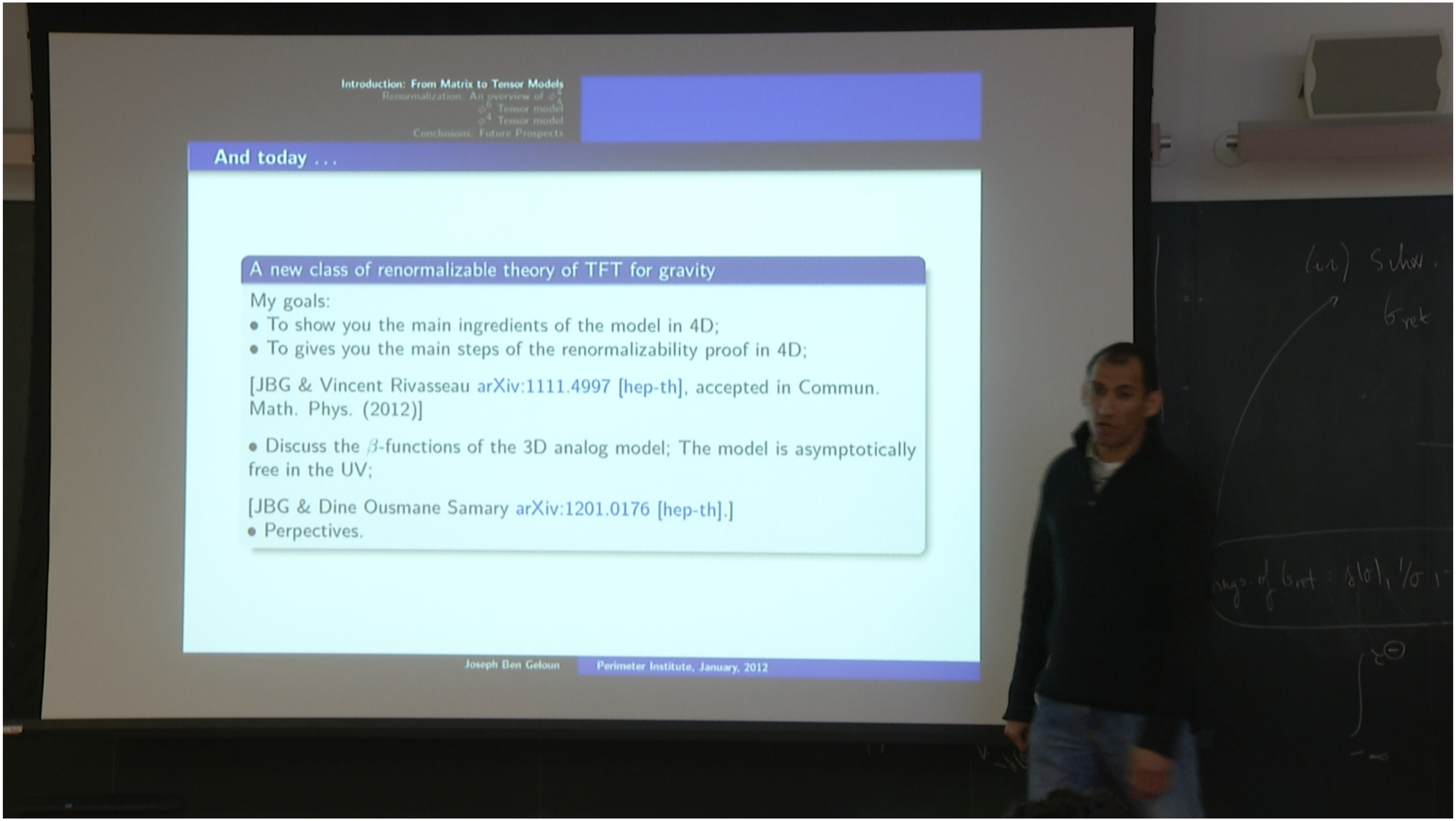
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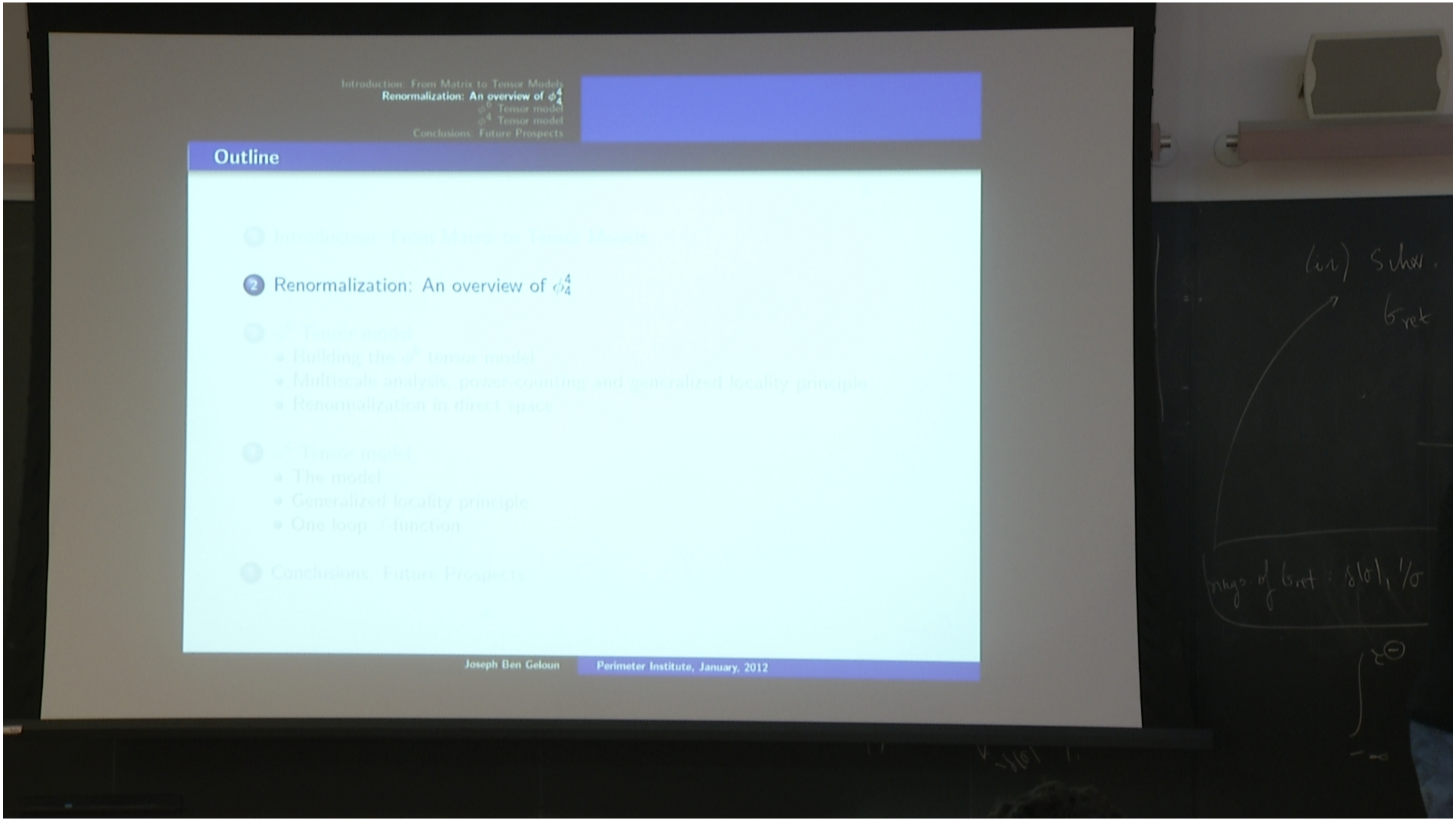
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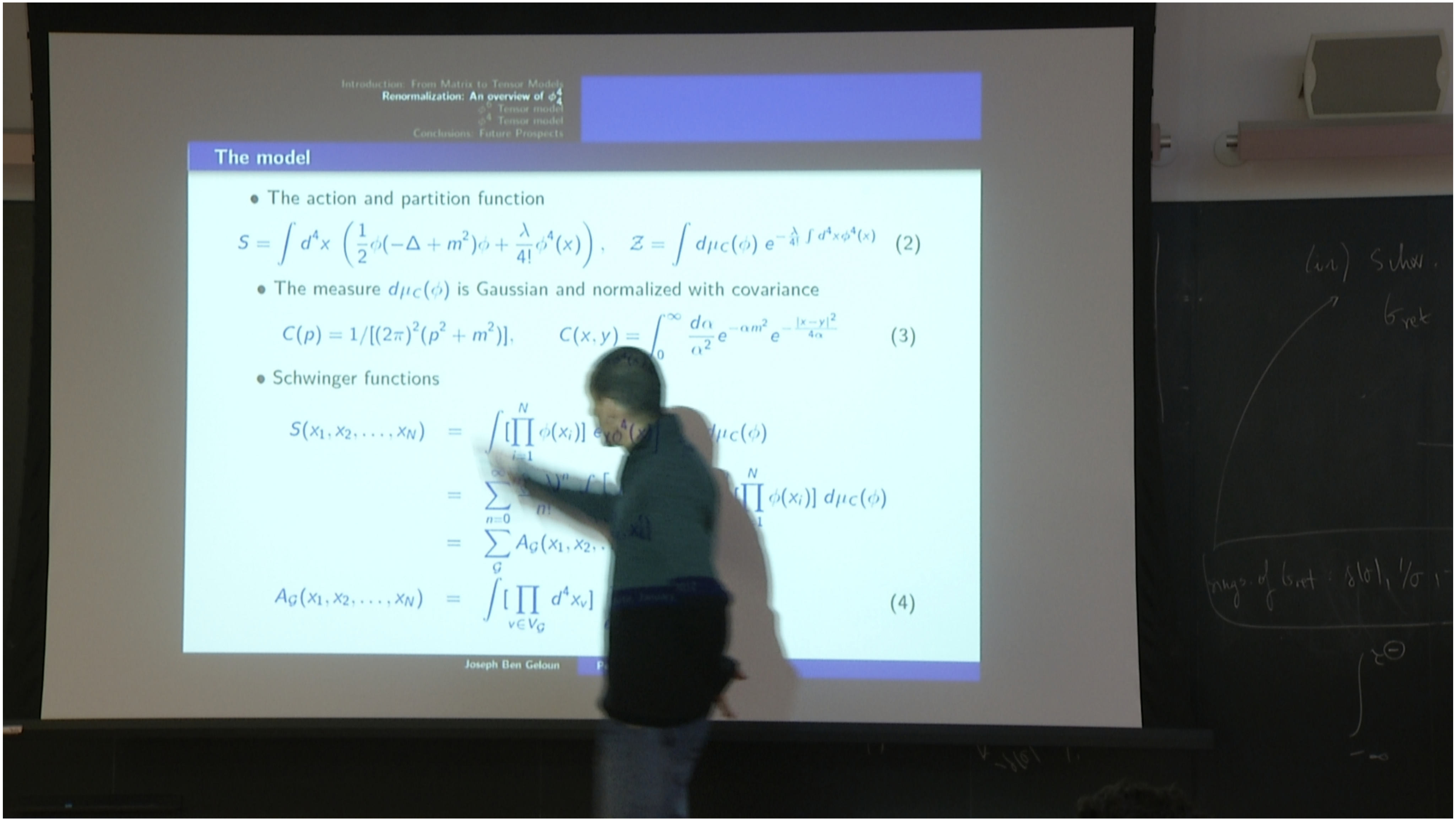
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The model
• The action and partition function
$S = \int d^4x \left(\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{\lambda}{4!}\phi^4(x) \right), \quad \mathcal{Z} = \int d\mu_C(\phi)\, e^{-\frac{\lambda}{4!}\int d^4x \phi^4(x)}$ (2)
• The measure $d\mu_C(\phi)$ is Gaussian and normalized with covariance
$C(p) = 1/[(2\pi)^2(p^2+m^2)], \quad C(x,y) = \int_0^\infty \frac{d\alpha}{\alpha^2} e^{-\alpha m^2} e^{-\frac{|x-y|^2}{4\alpha}}$ (3)
• Schwinger functions
$S(x_1, x_2, \dots, x_N)$
$A_{\mathcal{G}}(x_1, x_2, \dots, x_N) = \int [\prod_{v \in V_{\mathcal{G}}} d^4x_v]$
(4)
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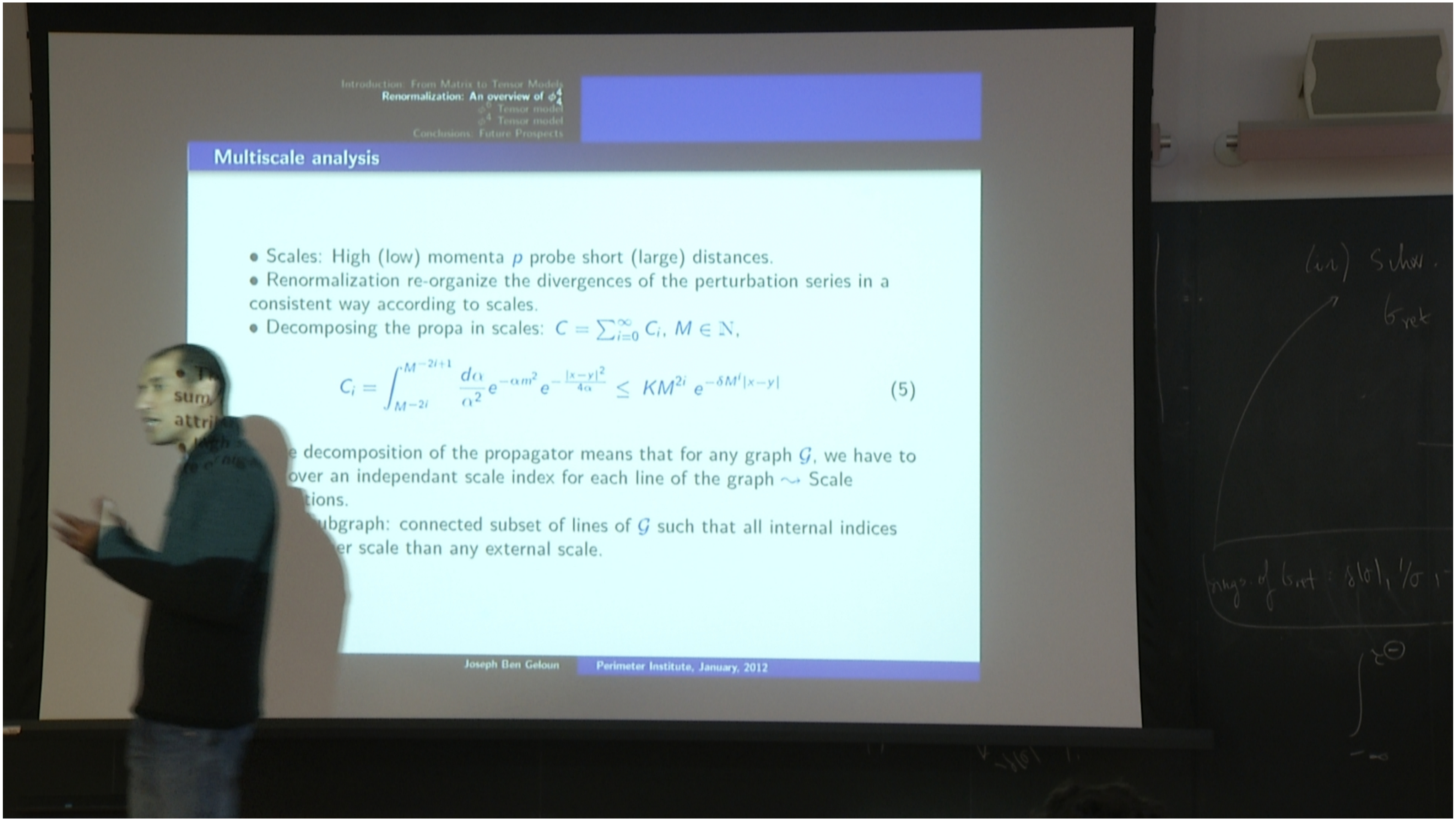
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Multiscale analysis
• Scales: High (low) momenta p probe short (large) distances.
• Renormalization re-organize the divergences of the perturbation series in a consistent way according to scales.
• Decomposing the propa in scales: $C = \sum_{i=0}^{\infty} C_i$, $M \in \mathbb{N}$,
$$C_i = \int_{M^{-2i}}^{M^{-2i+1}} \frac{d\alpha}{\alpha^2} e^{-\alpha m^2} e^{-\frac{|x-y|^2}{4\alpha}} \leq KM^{2i} e^{-\delta M^i |x-y|} \quad (5)$$
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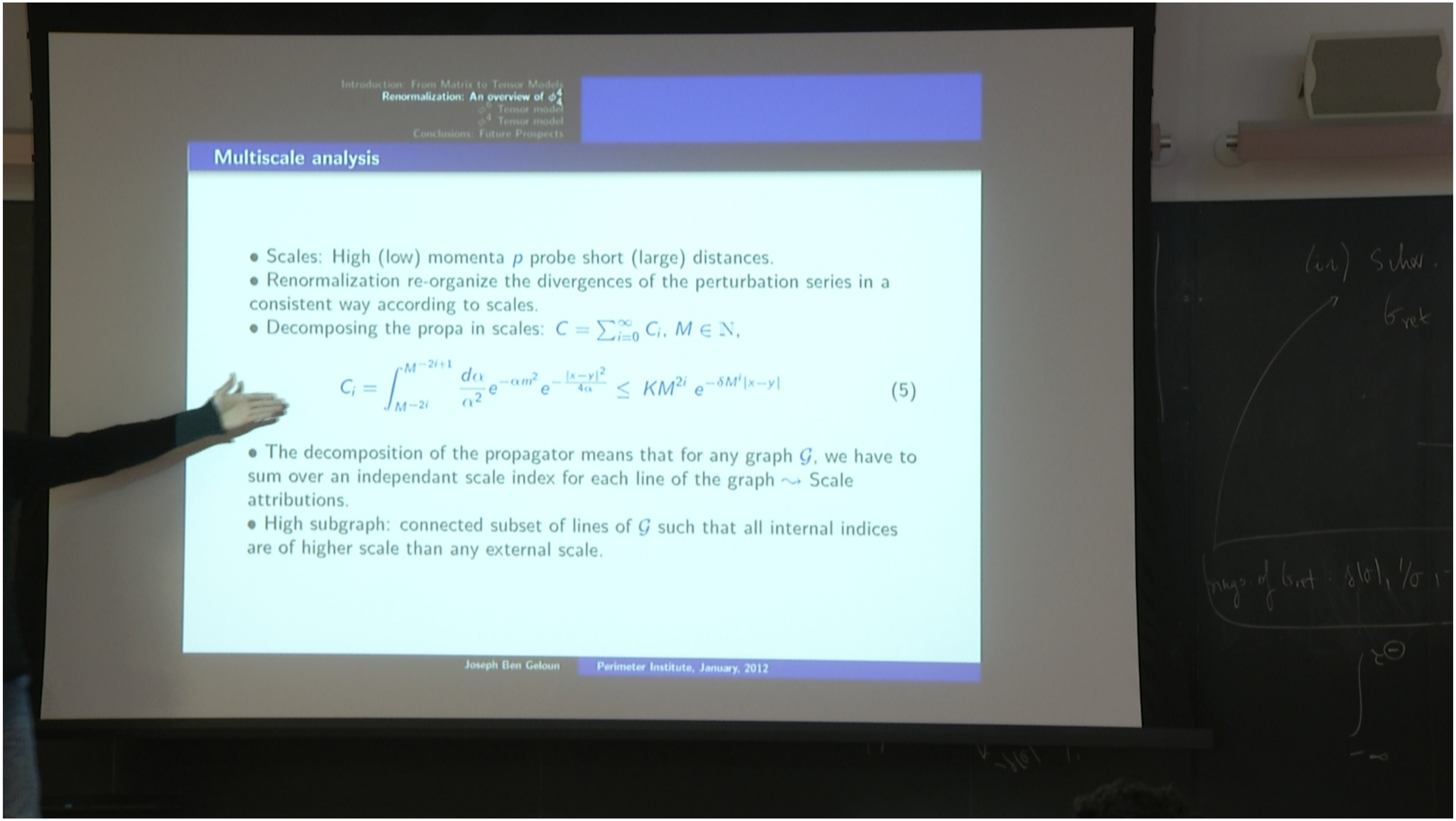
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• The decomposition of the propagator means that for any graph $\mathcal{G}$, we have to sum over an independant scale index for each line of the graph $\rightsquigarrow$ Scale attributions.
• High subgraph: connected subset of lines of $\mathcal{G}$ such that all internal indices are of higher scale than any external scale.
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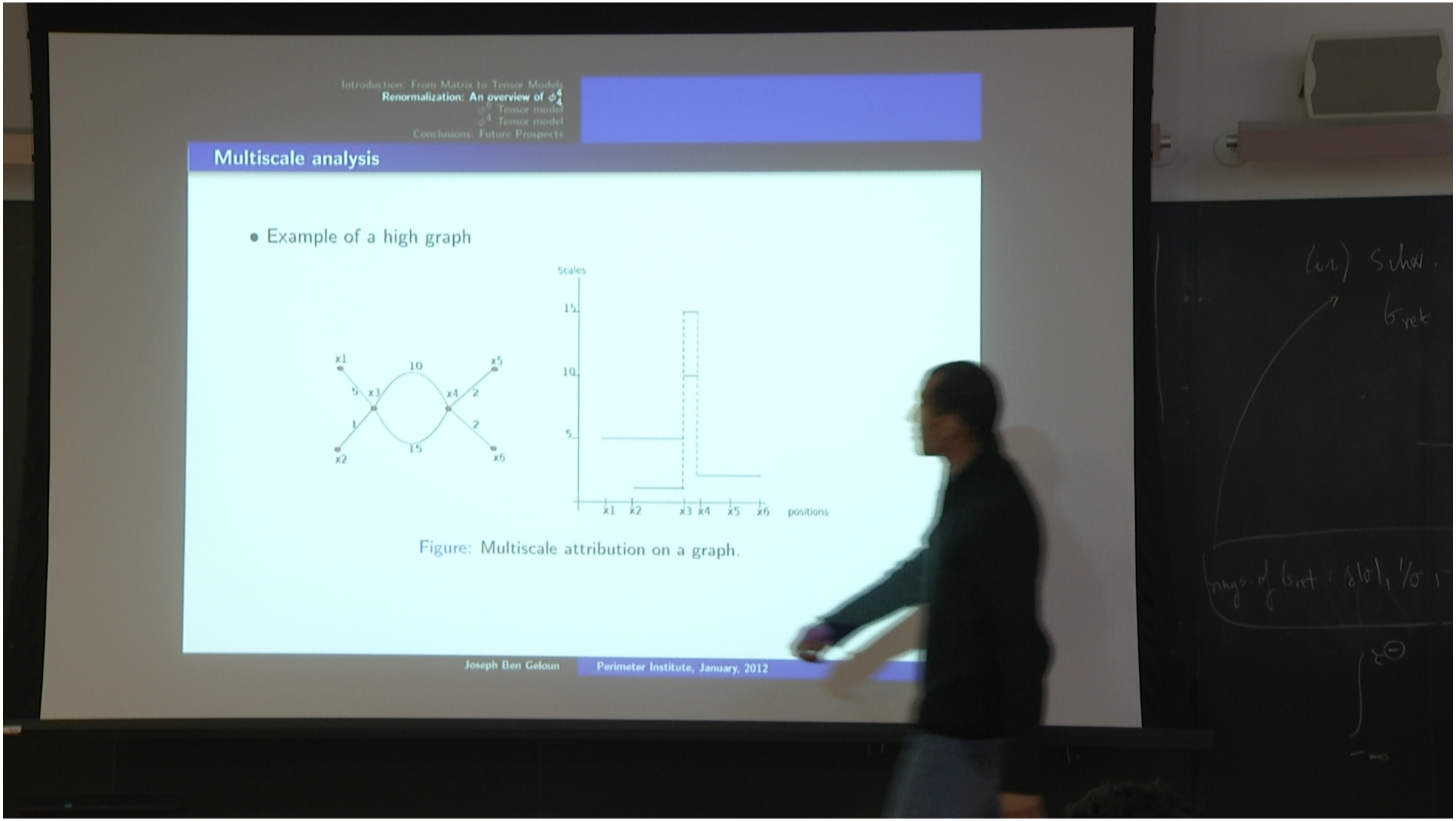
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Figure: Multiscale attribution on a graph.
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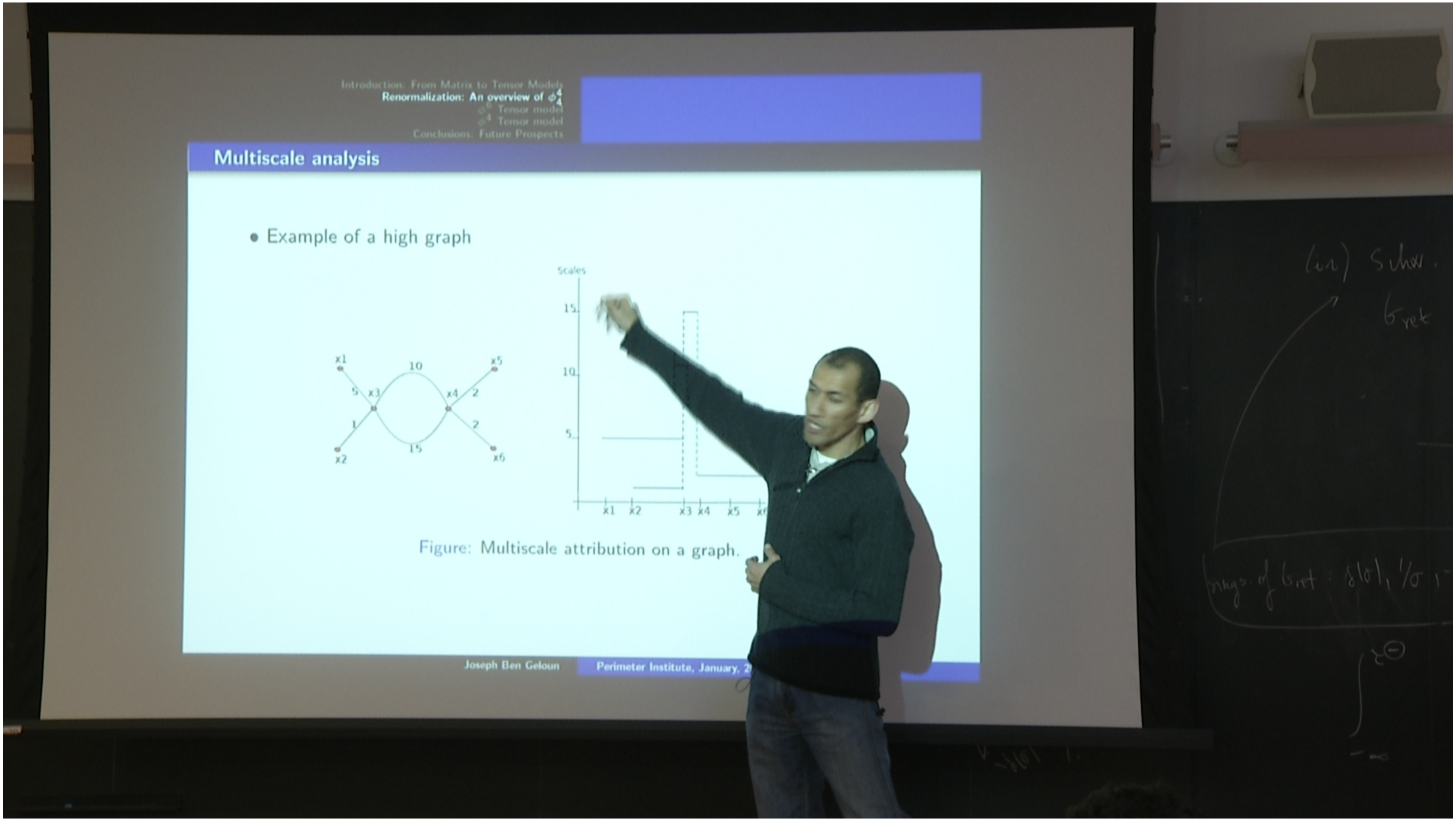
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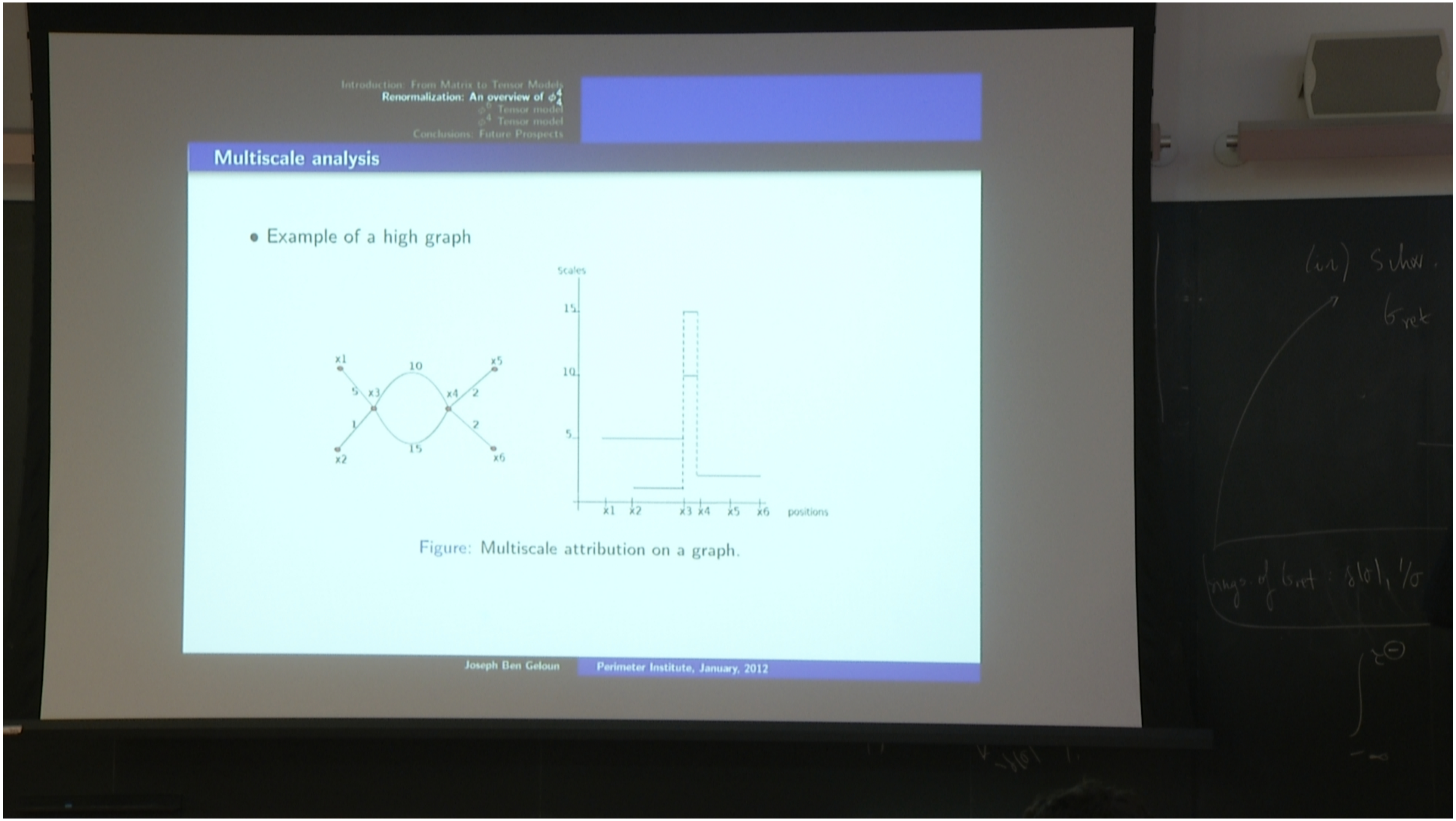
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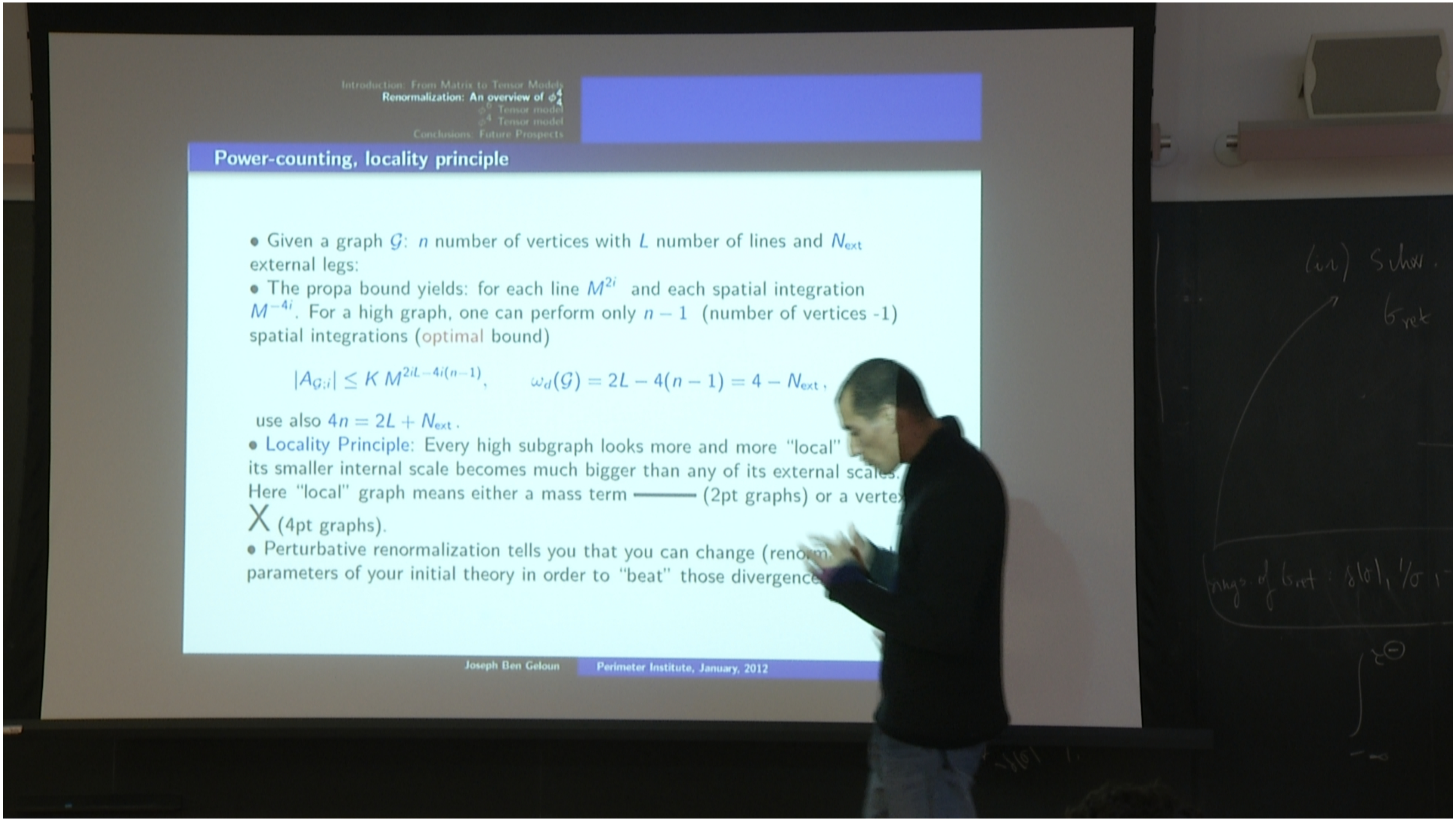
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Power-counting, locality principle
• Given a graph 𝒢: n number of vertices with L number of lines and N_ext external legs:
• The propa bound yields: for each line M^{2i} and each spatial integration M^{−4i}. For a high graph, one can perform only n − 1 (number of vertices -1) spatial integrations (optimal bound)
|A_{𝒢;i}| ≤ K M^{2iL−4i(n−1)},   ω_d(𝒢) = 2L − 4(n − 1) = 4 − N_ext ,
use also 4n = 2L + N_ext .
• Locality Principle: Every high subgraph looks more and more "local"
its smaller internal scale becomes much bigger than any of its external scales.
Here "local" graph means either a mass term (2pt graphs) or a vertex
X (4pt graphs).
• Perturbative renormalization tells you that you can change (renorm.
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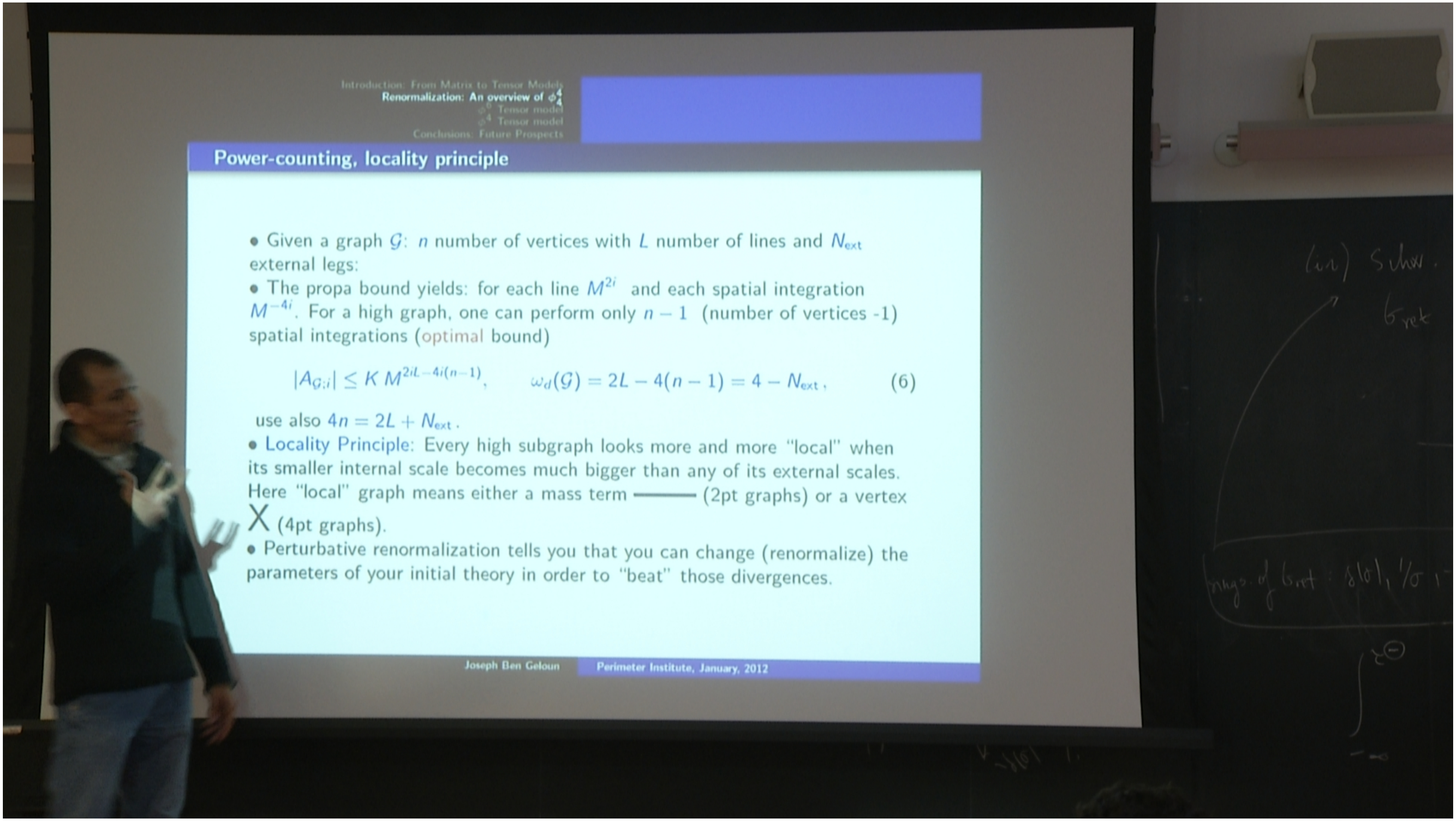
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$|A_{\mathcal{G};i}| \leq K\, M^{2iL-4i(n-1)}, \qquad \omega_d(\mathcal{G}) = 2L - 4(n-1) = 4 - N_{ext}\,, \qquad (6)$
use also $4n = 2L + N_{ext}$.
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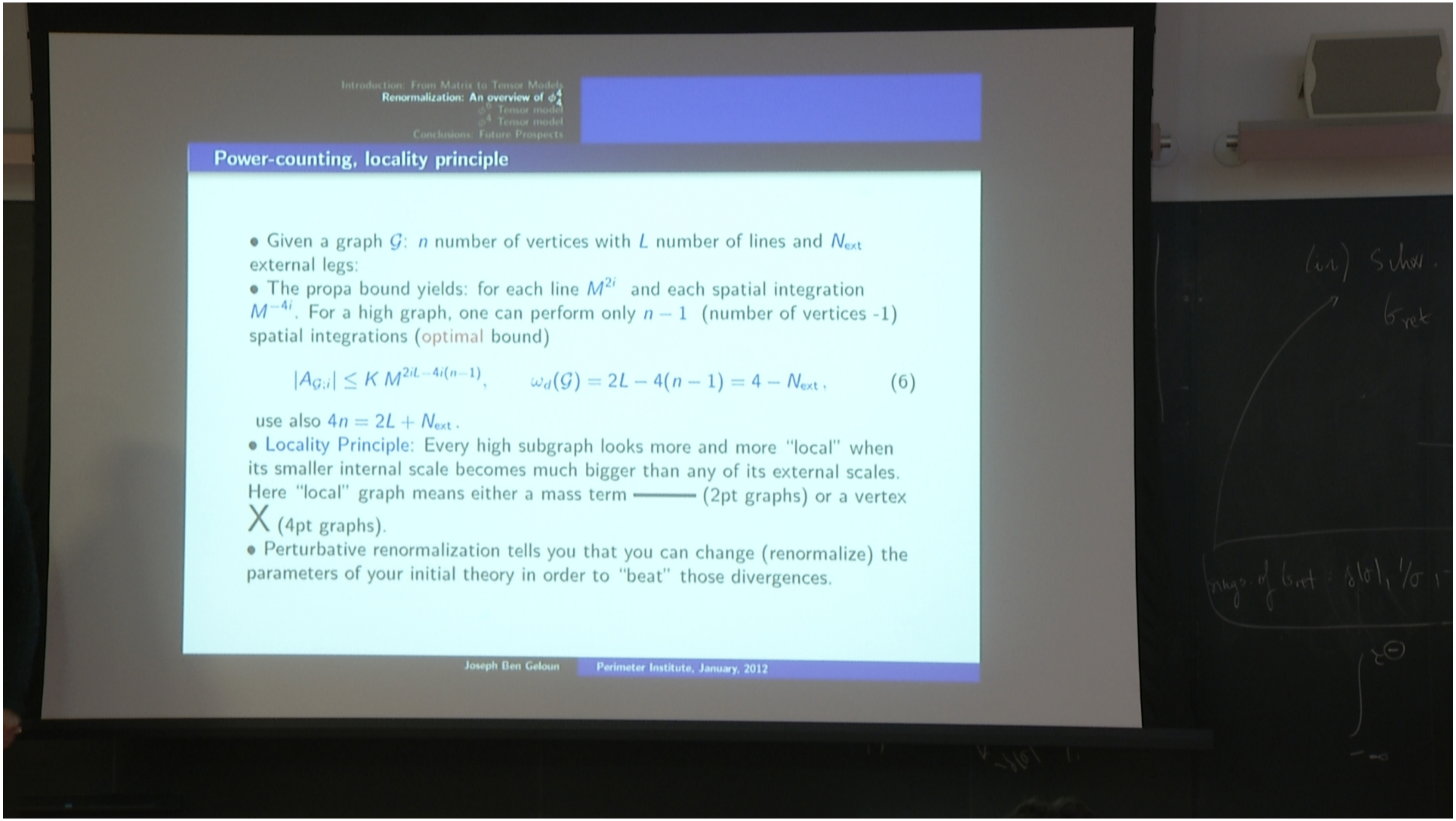
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• The propa bound yields: for each line M^{2i} and each spatial integration M^{-4i}. For a high graph, one can perform only $n-1$ (number of vertices -1) spatial integrations (optimal bound)
$|A_{\mathcal{G};i}| \leq K\, M^{2iL-4i(n-1)}, \qquad \omega_d(\mathcal{G}) = 2L - 4(n-1) = 4 - N_{\text{ext}}\,,$ (6)
use also $4n = 2L + N_{\text{ext}}$.
• Locality Principle: Every high subgraph looks more and more "local" when its smaller internal scale becomes much bigger than any of its external scales. Here "local" graph means either a mass term (2pt graphs) or a vertex X (4pt graphs).
• Perturbative renormalization tells you that you can change (renormalize) the parameters of your initial theory in order to "beat" those divergences.
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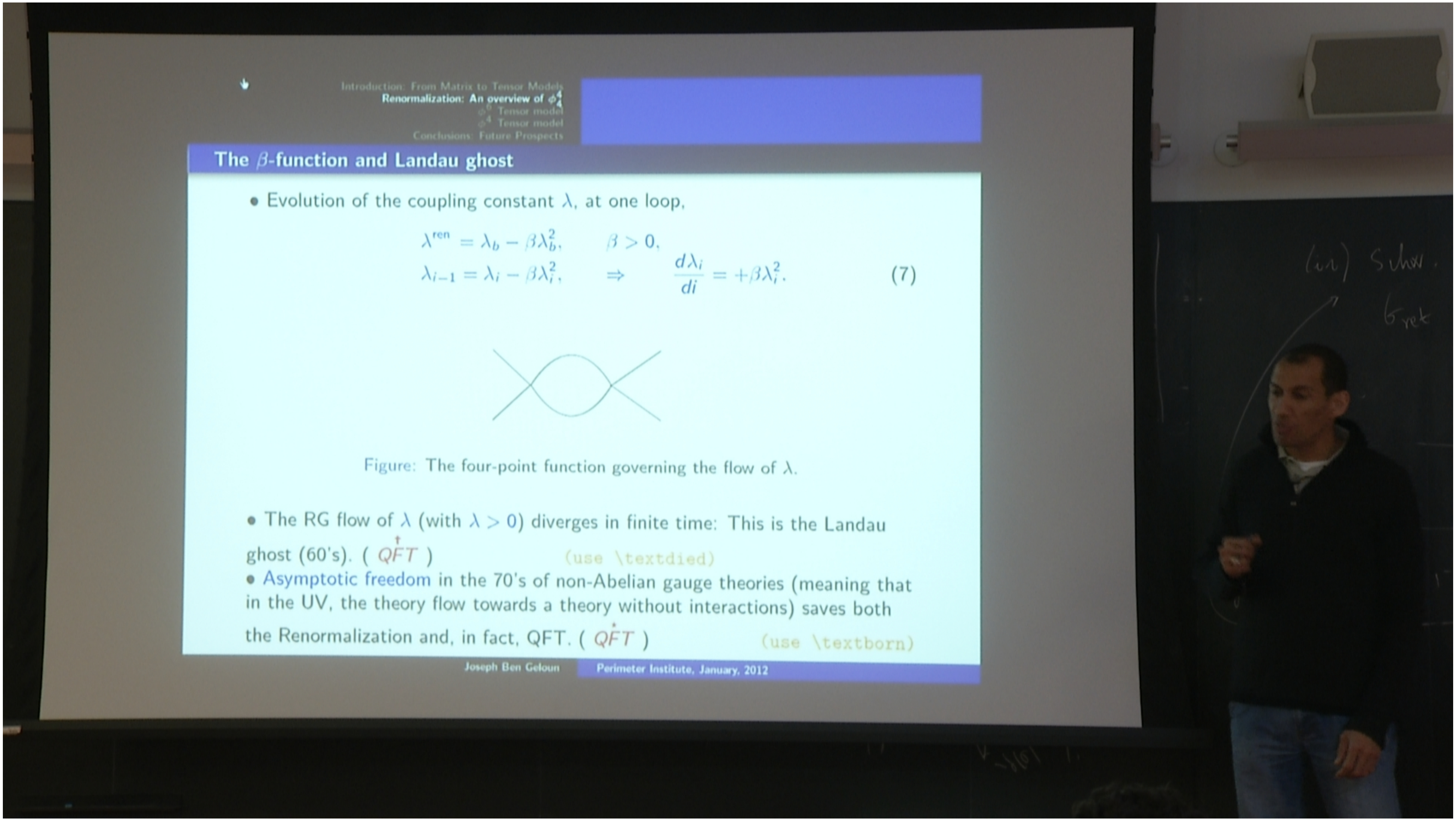
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$\lambda^{\text{ren}} = \lambda_b - \beta\lambda_b^2, \qquad \beta > 0,$
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Figure: The four-point function governing the flow of λ.
• The RG flow of λ (with $\lambda > 0$) diverges in finite time: This is the Landau ghost (60's). (QFT) (use \textdied)
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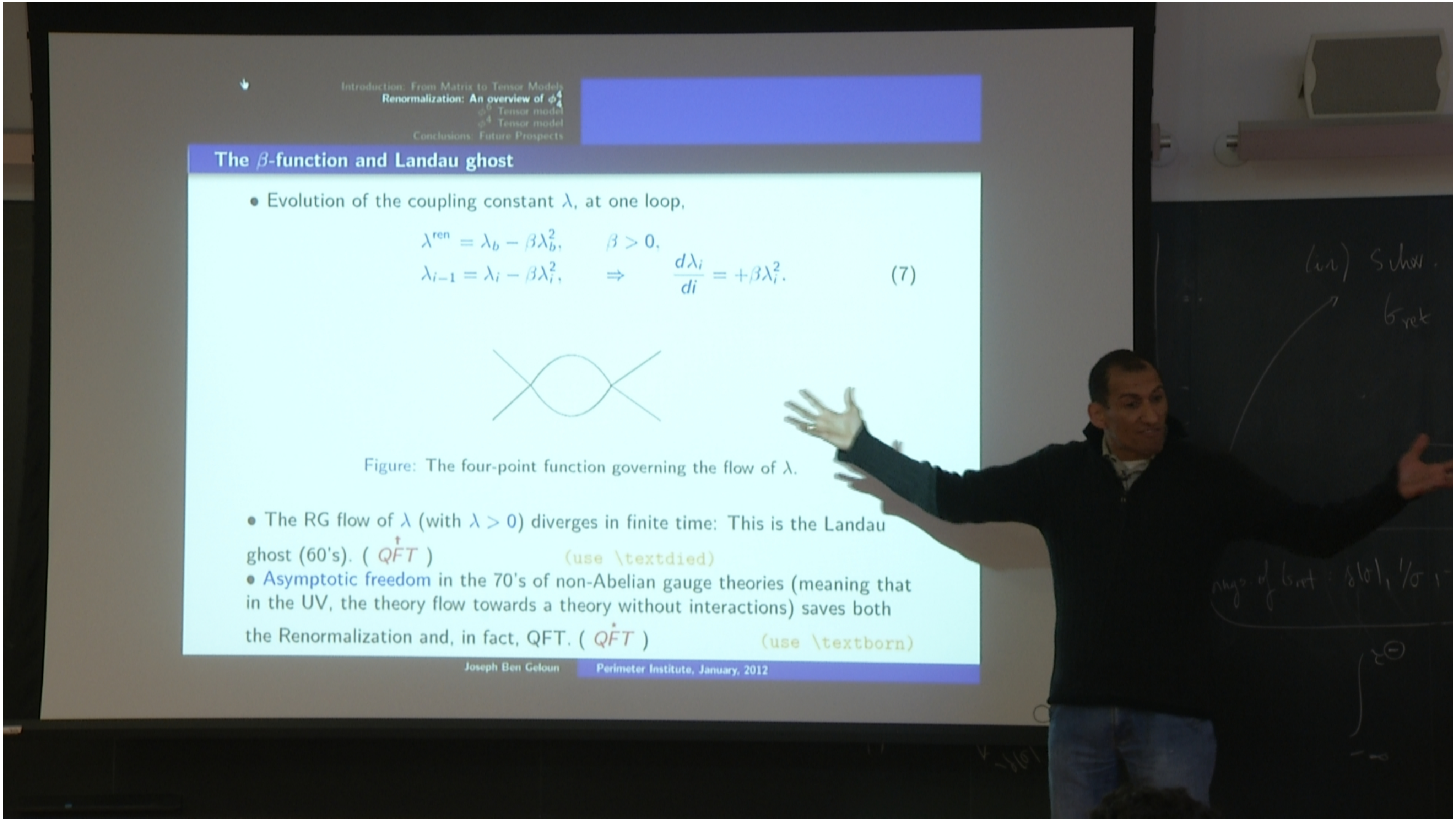
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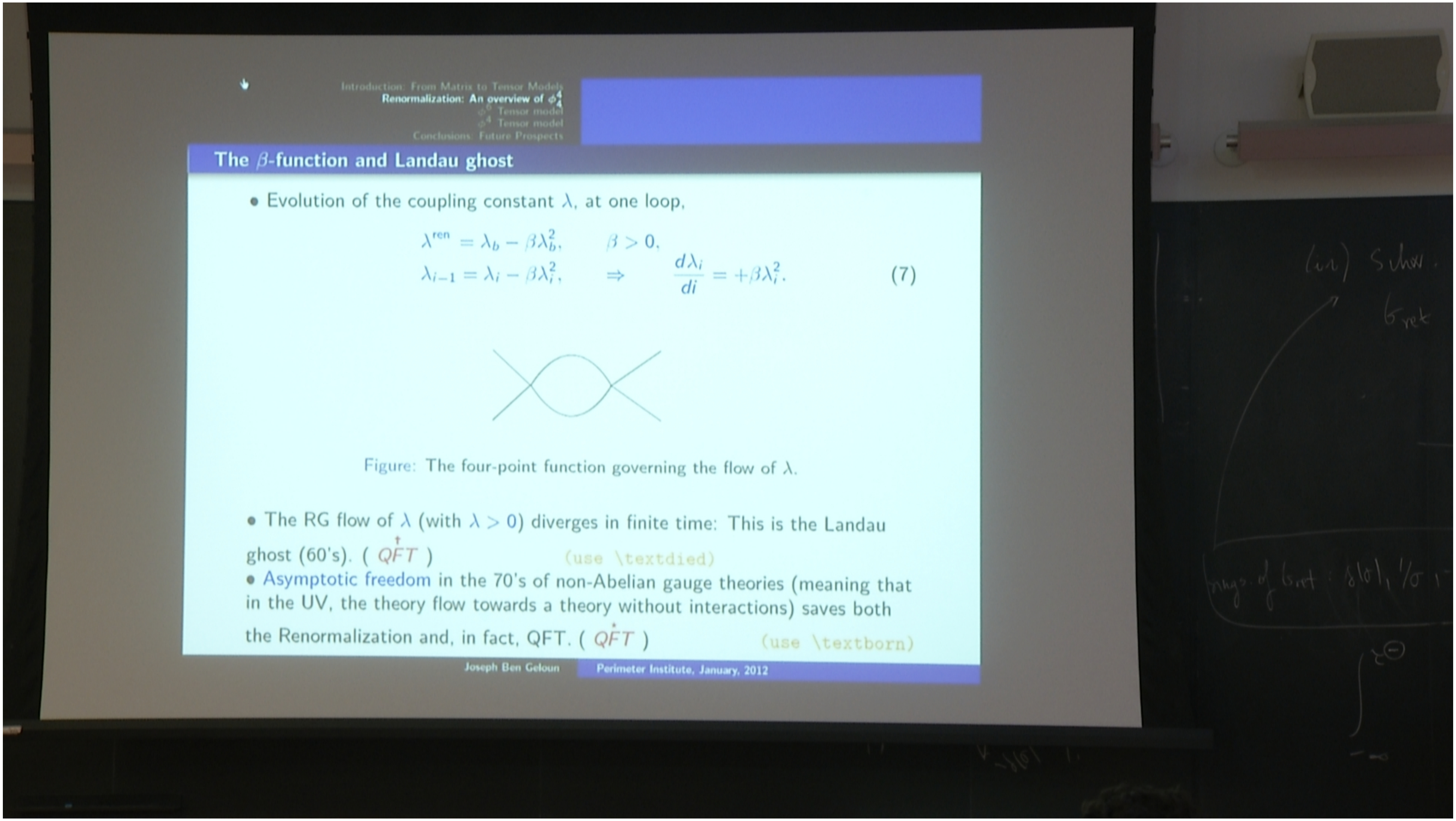
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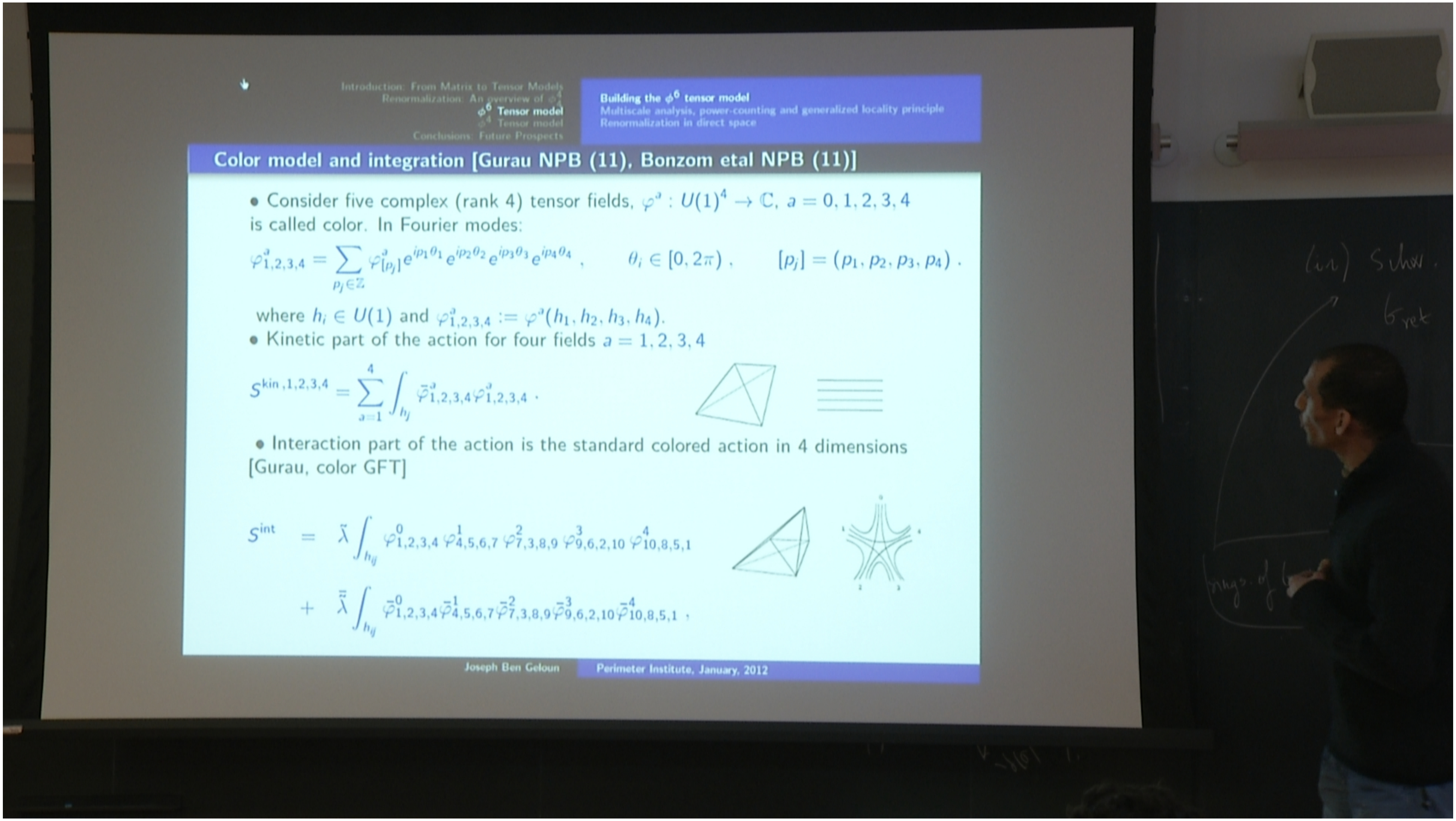
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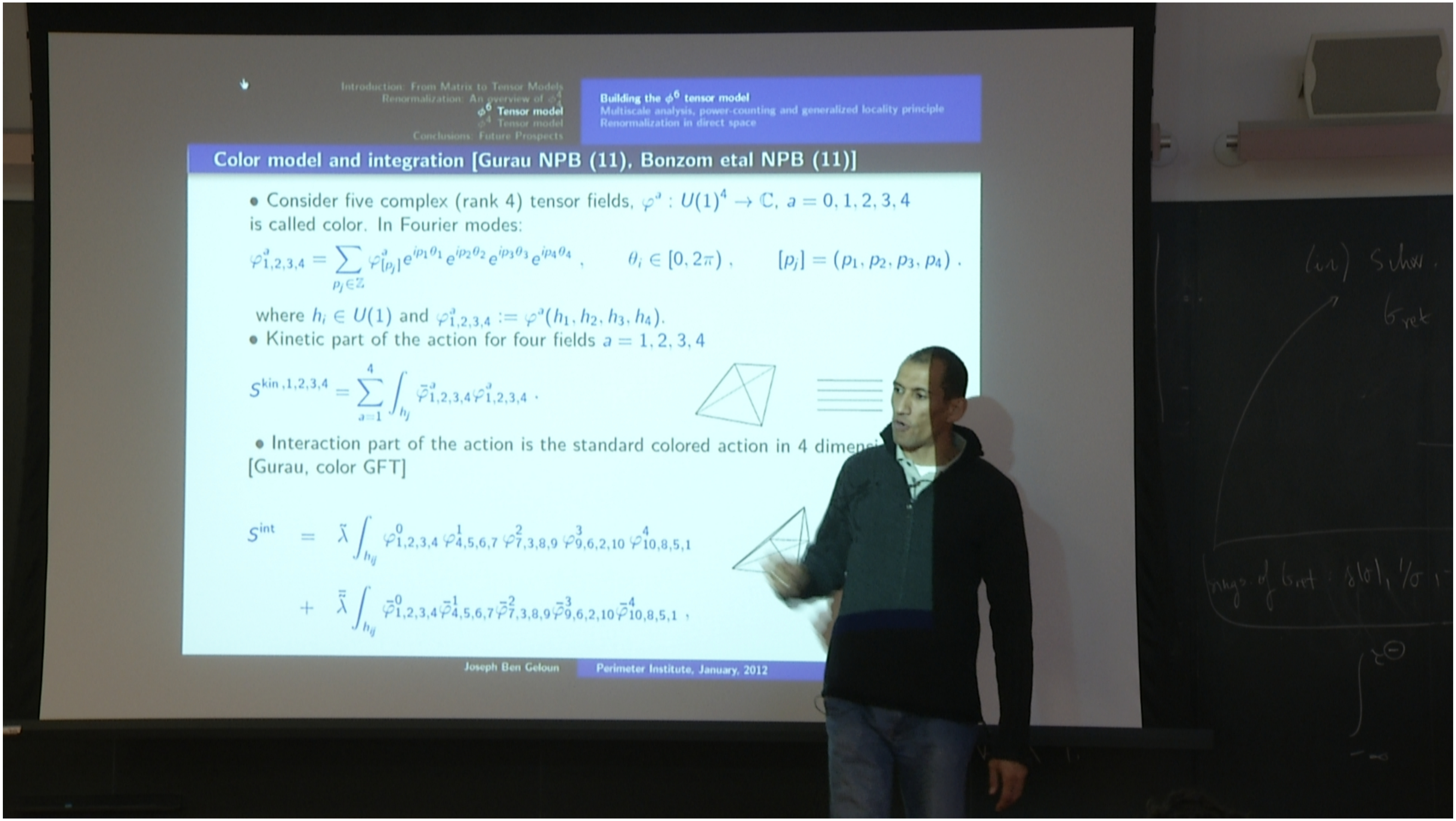

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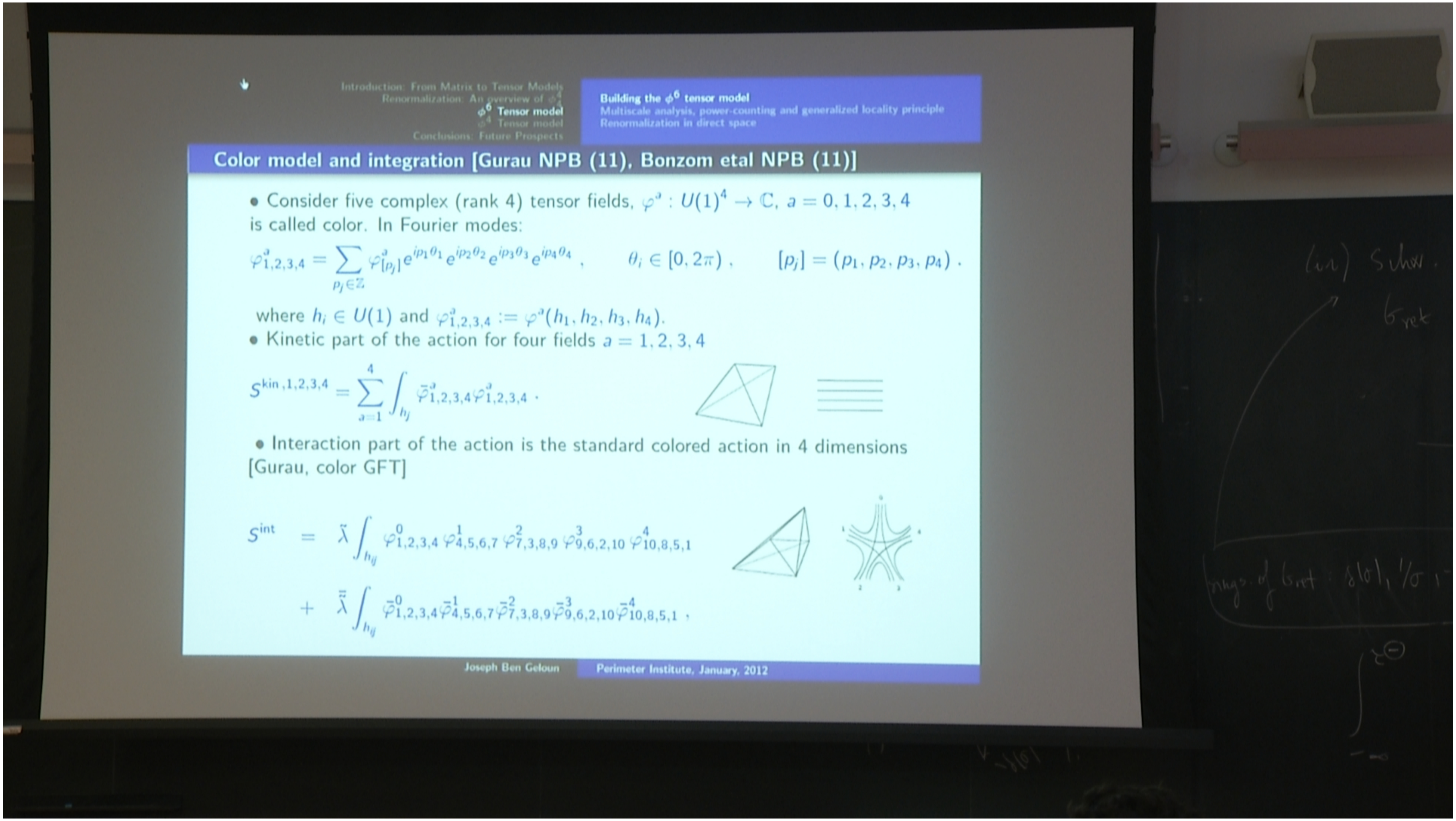
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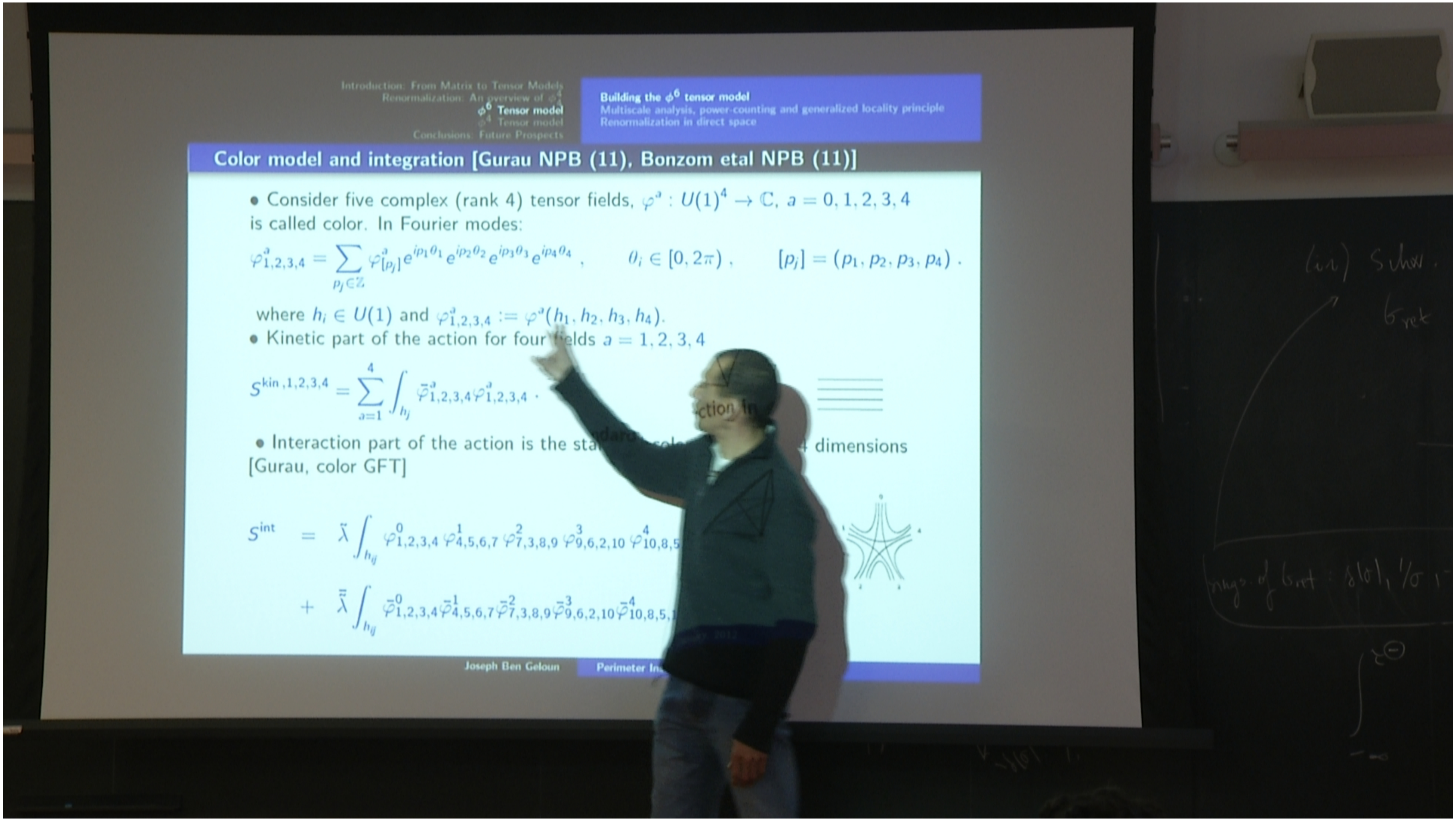
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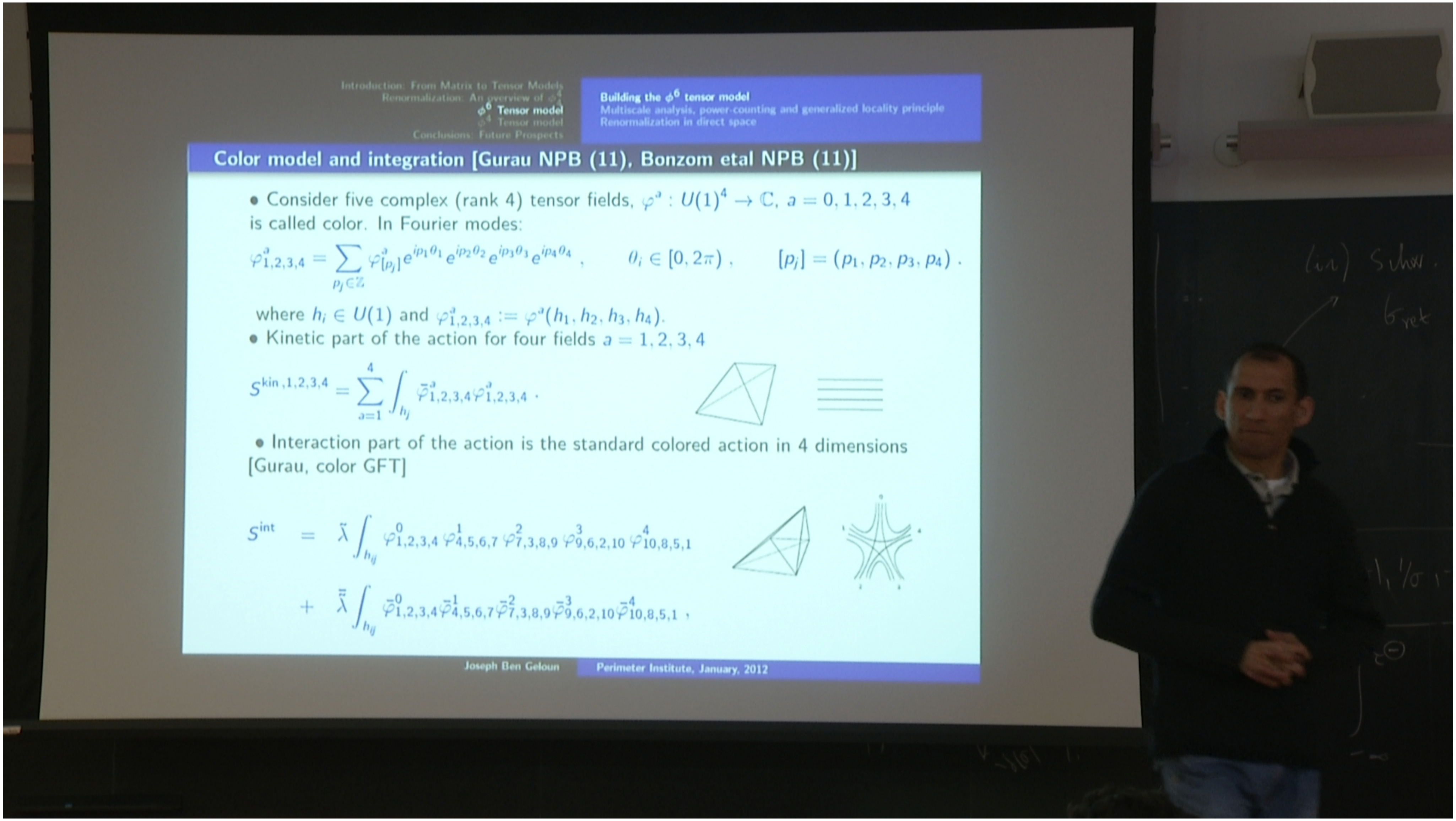
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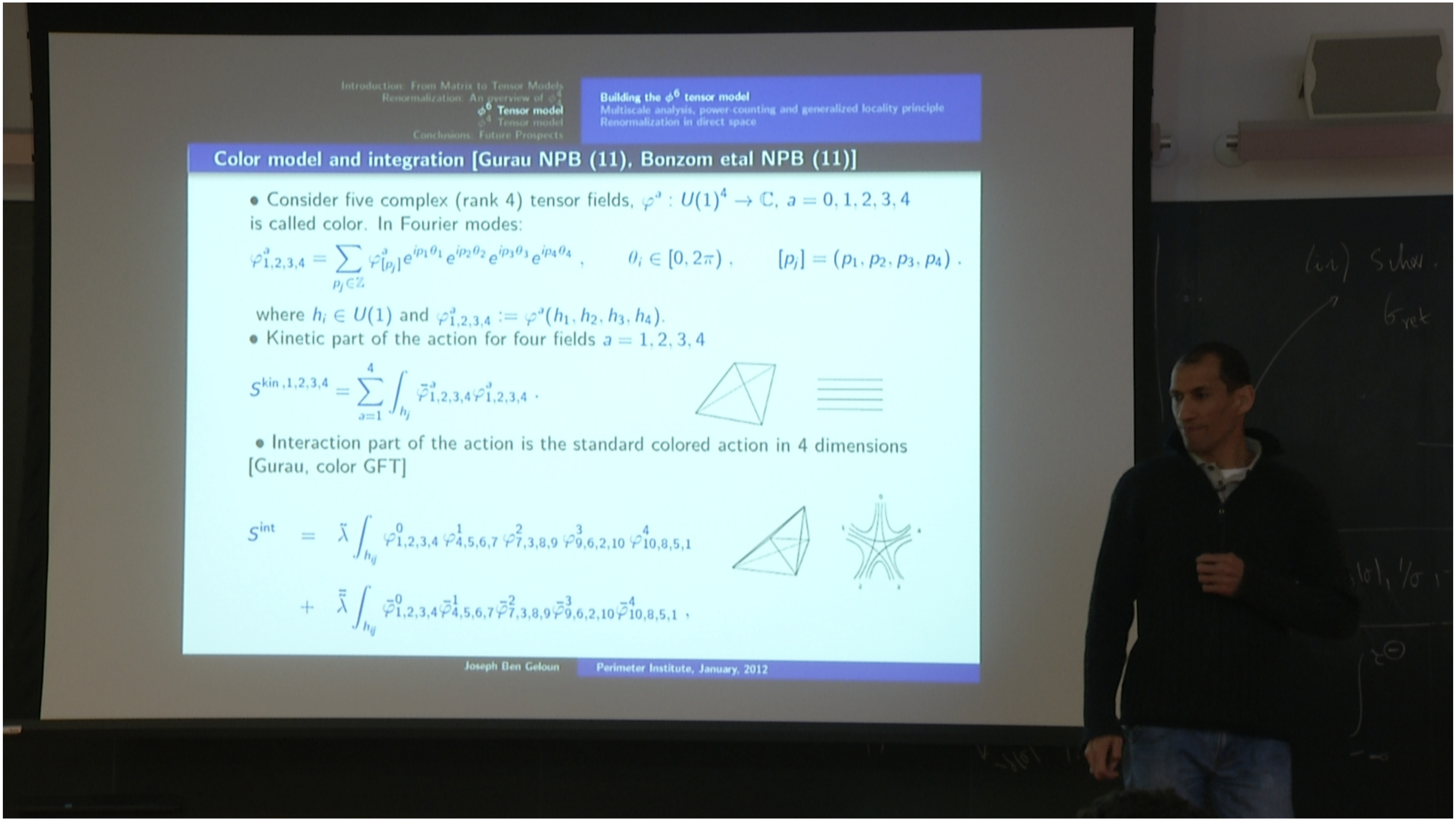

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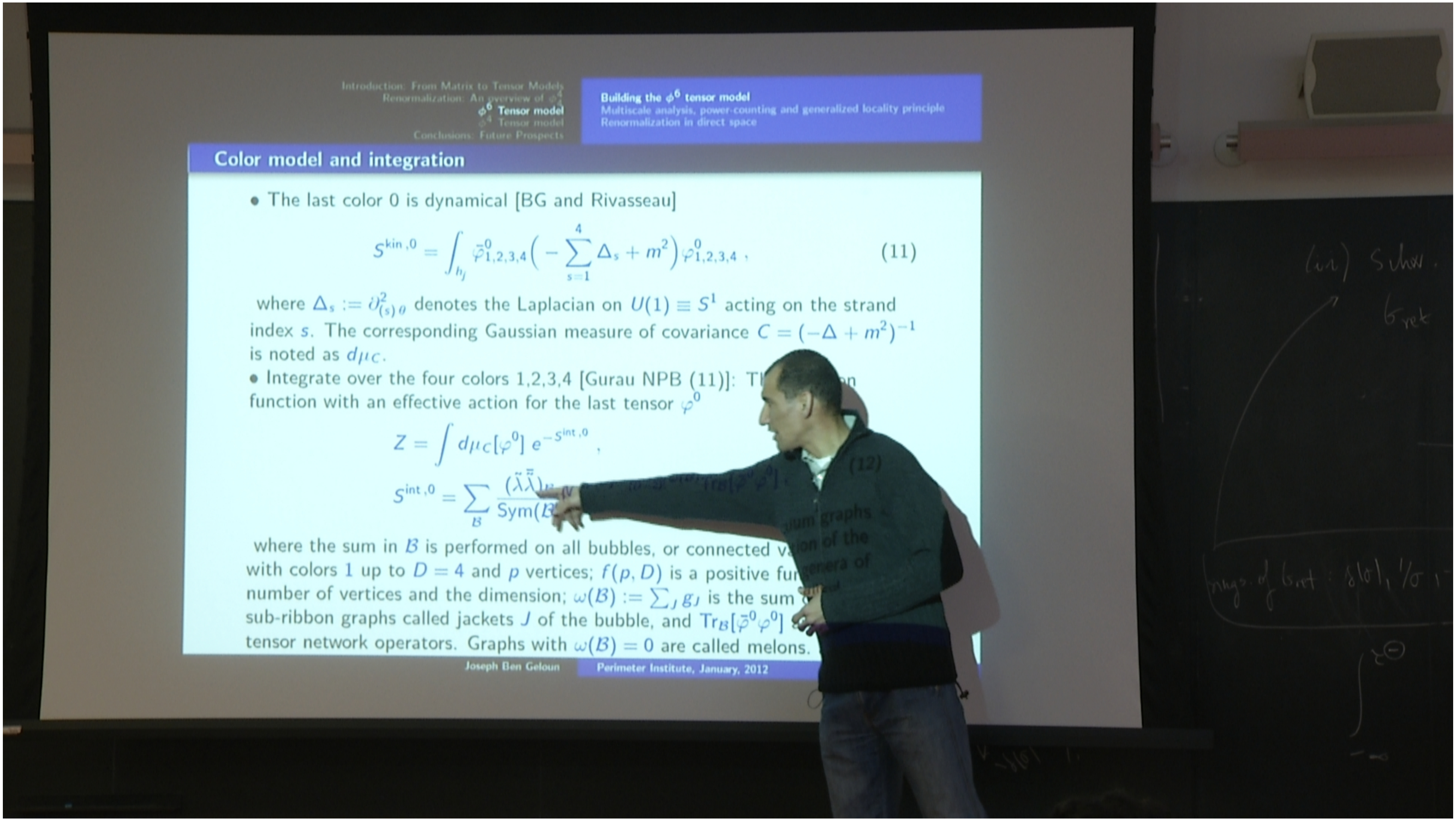
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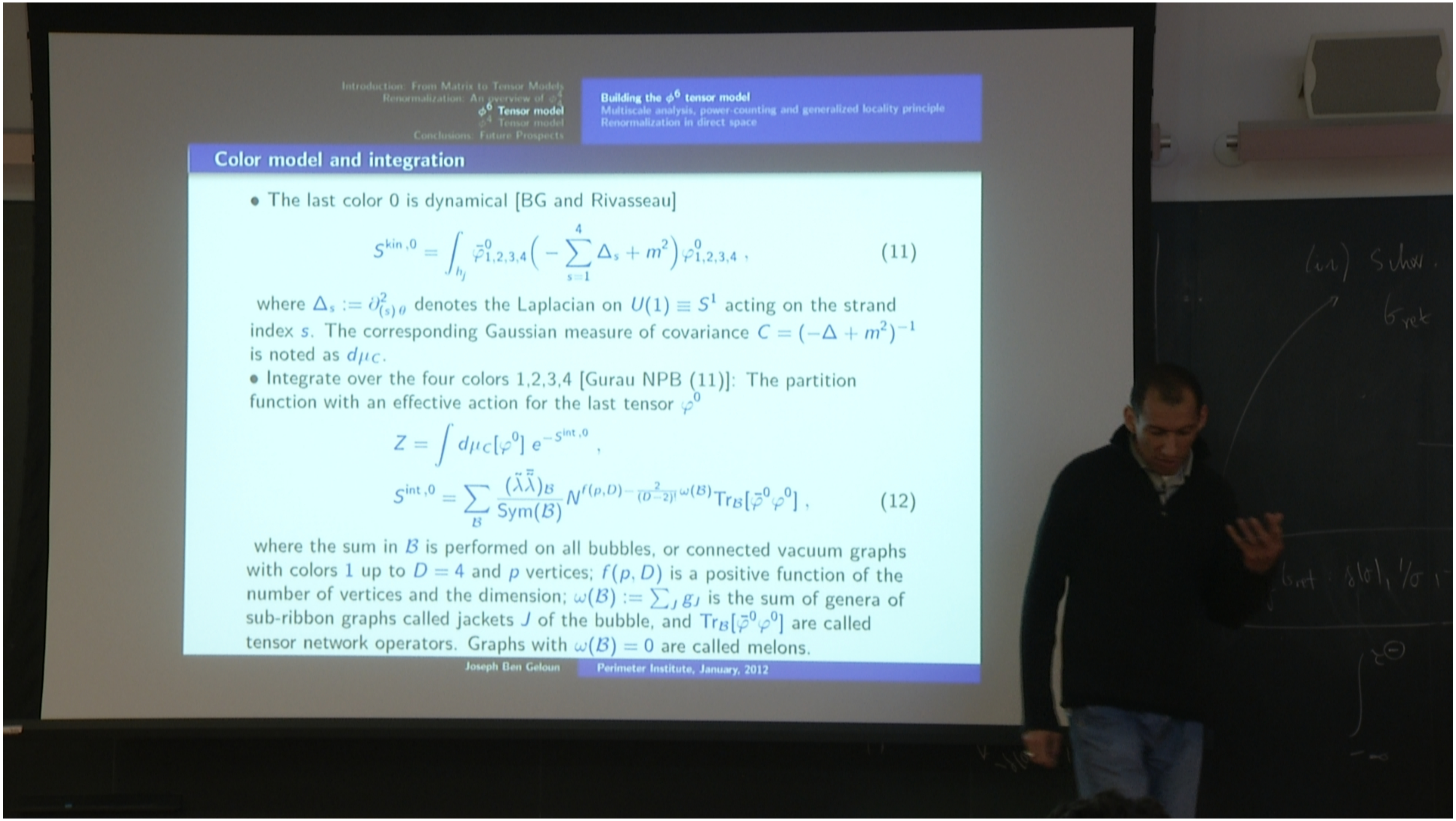
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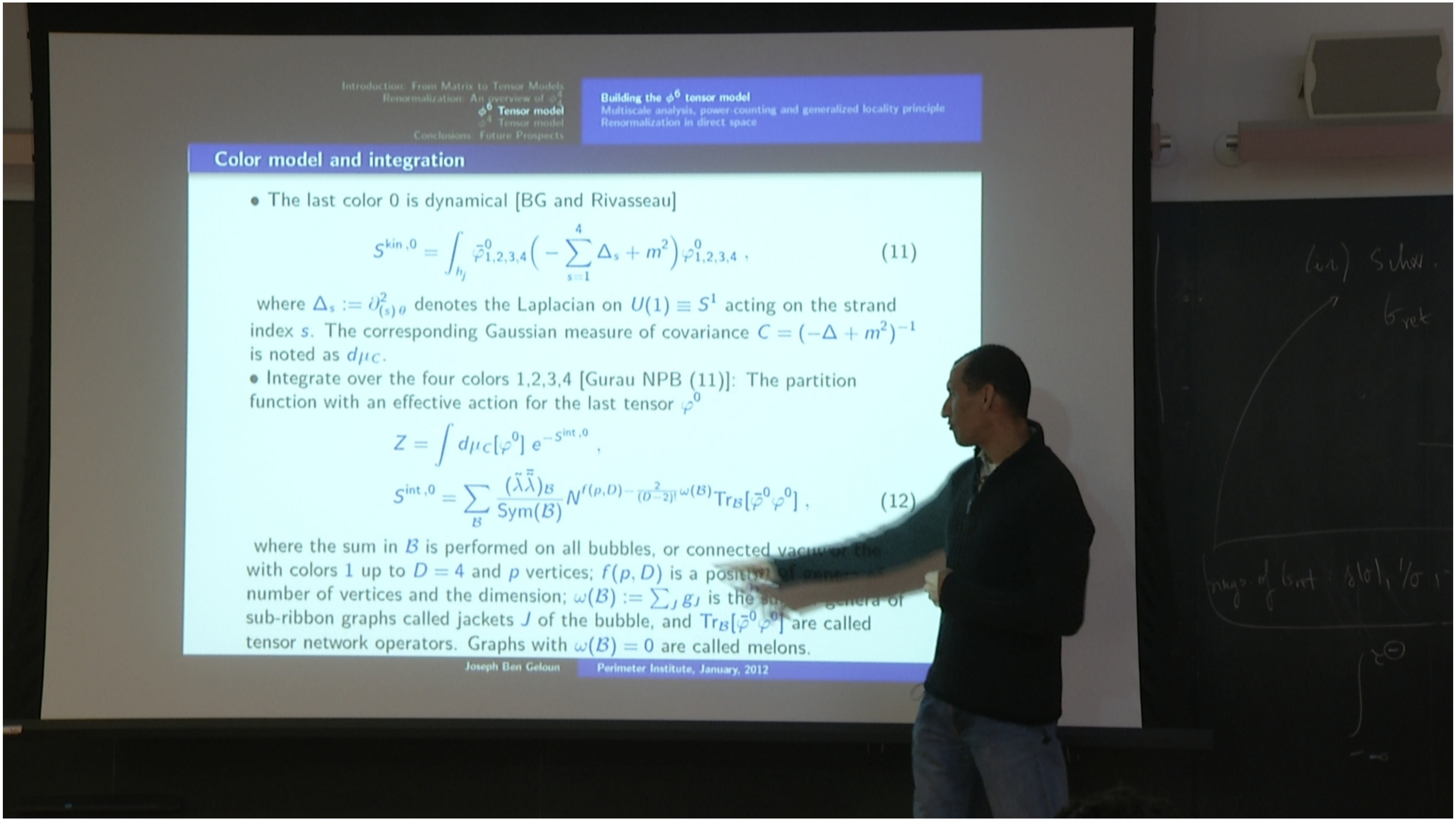

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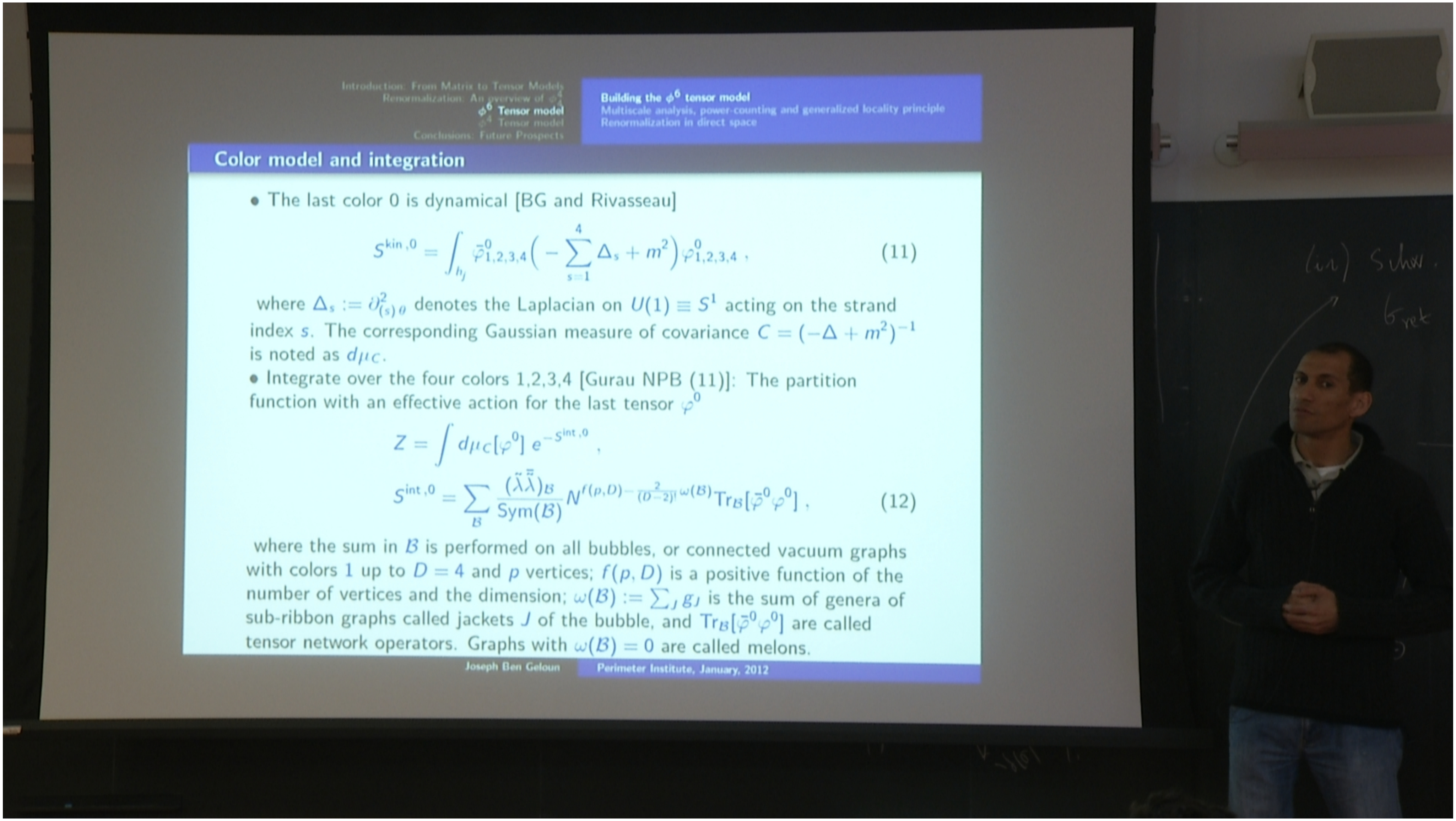
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where the sum in $\mathcal{B}$ is performed on all bubbles, or connected vacuum graphs with colors 1 up to $D = 4$ and p vertices; $f(p,D)$ is a positive function of the number of vertices and the dimension; $\omega(\mathcal{B}) := \sum_J g_J$ is the sum of genera of sub-ribbon graphs called jackets J of the bubble, and $\text{Tr}_{\mathcal{B}}[\bar\varphi^0\varphi^0]$ are called tensor network operators. Graphs with $\omega(\mathcal{B}) = 0$ are called melons.
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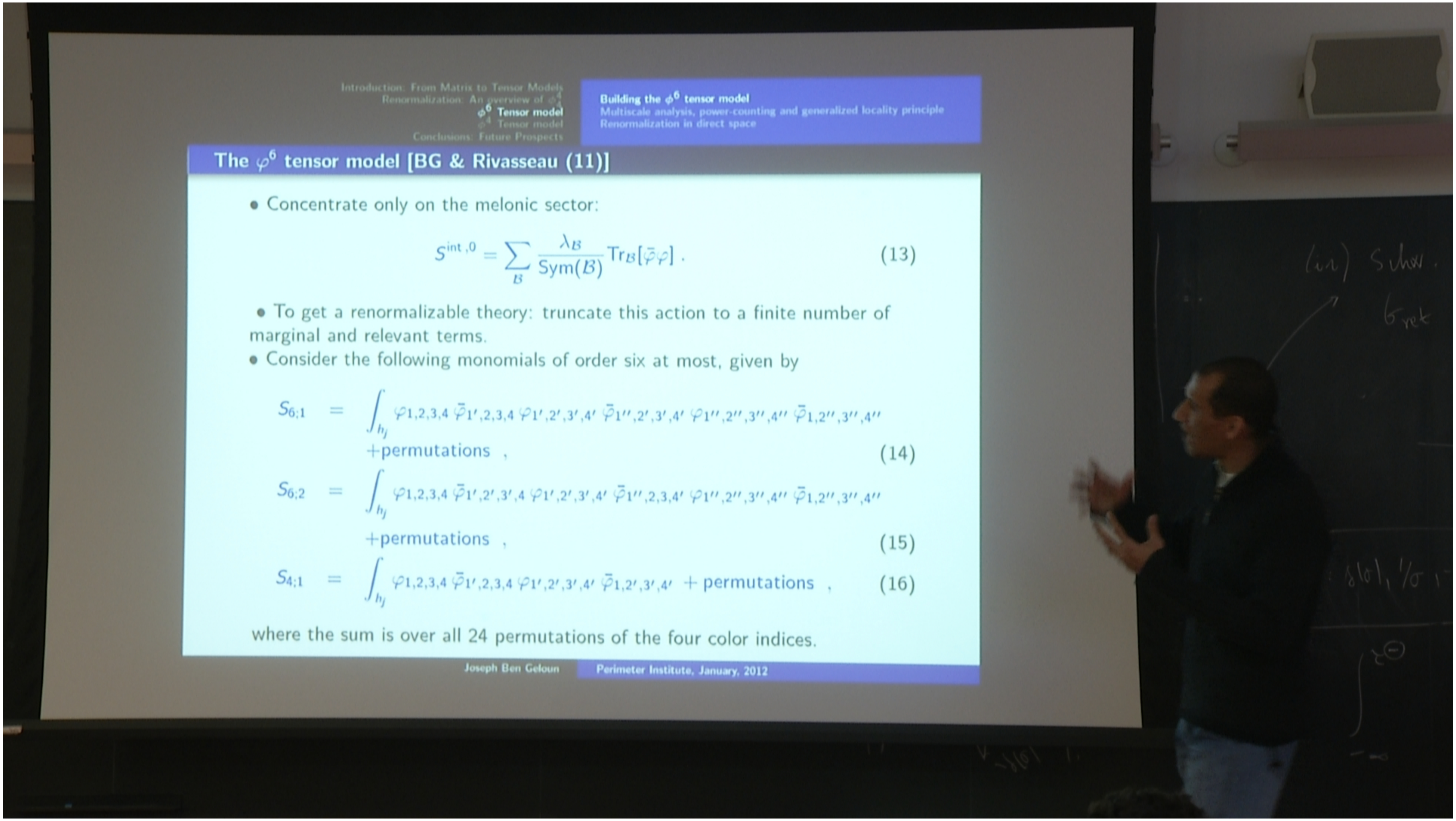
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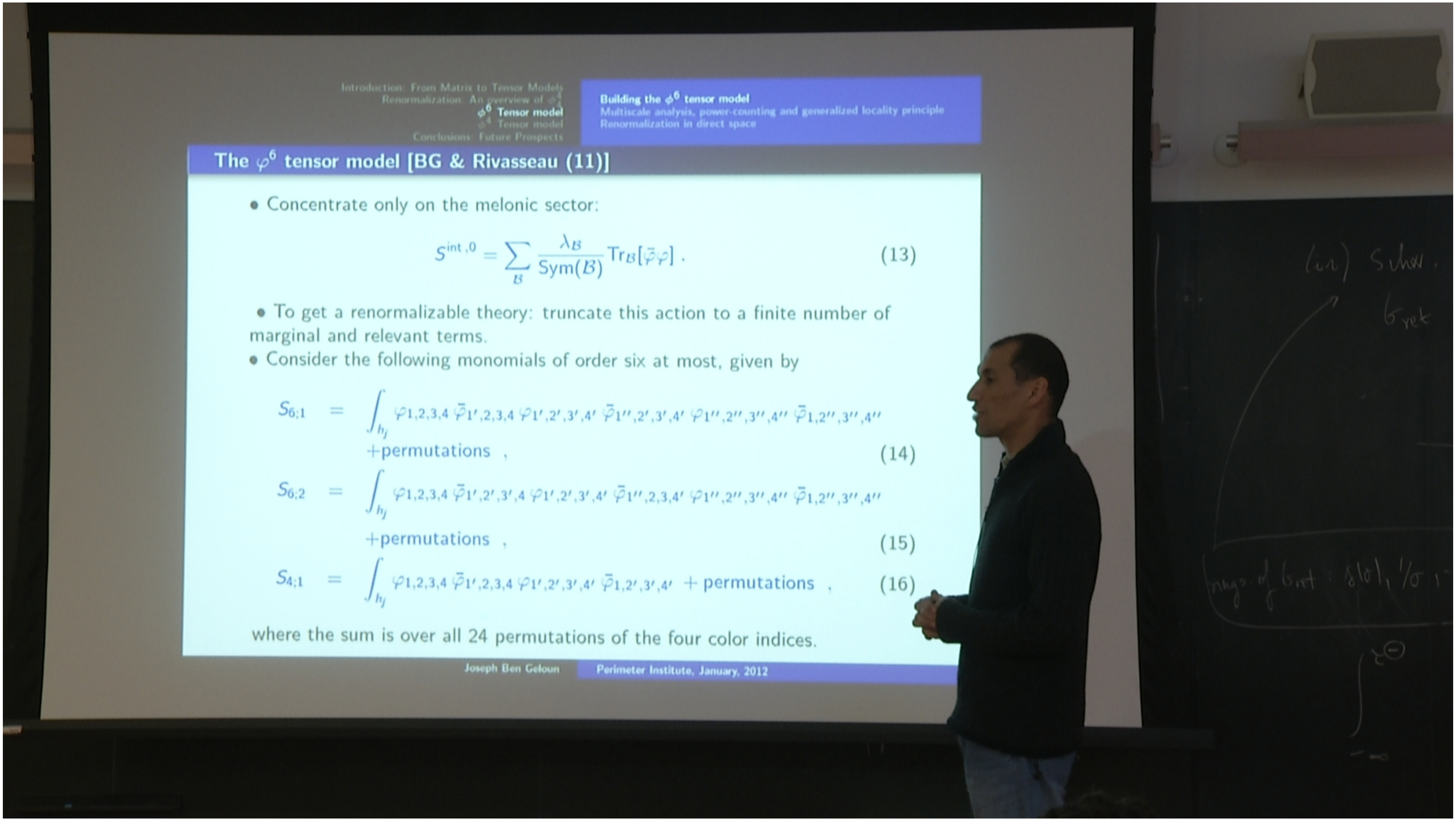
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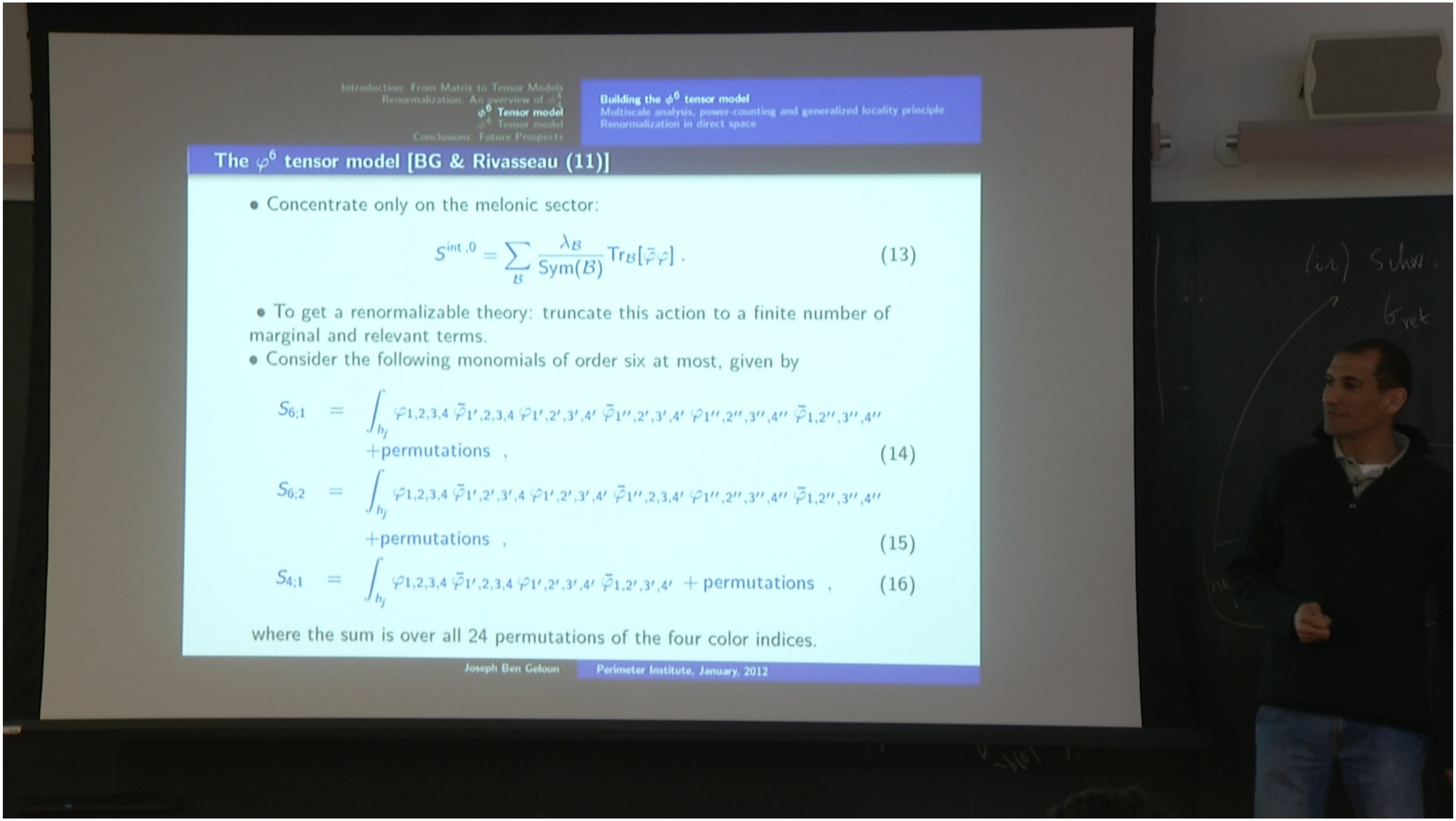
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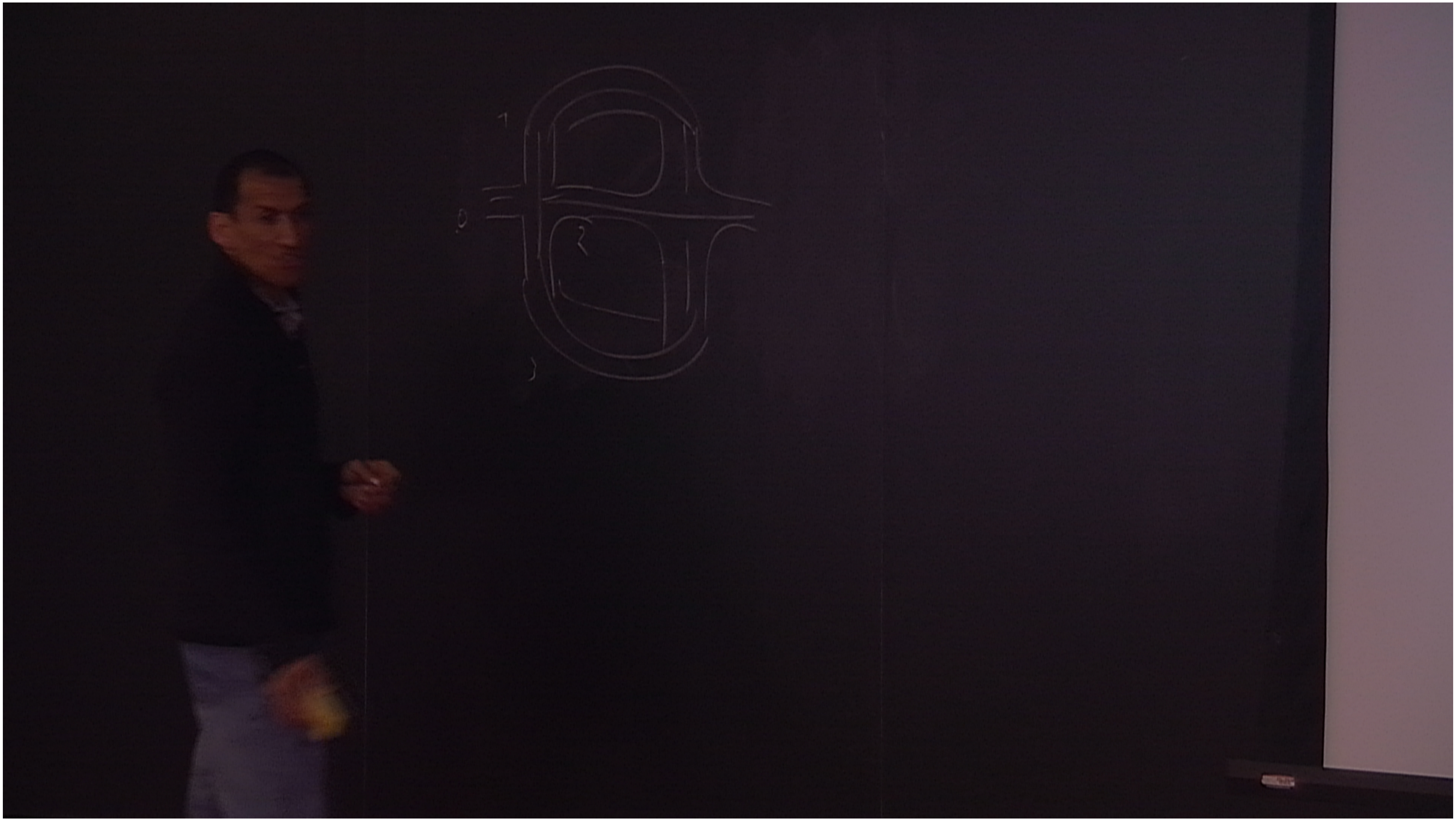

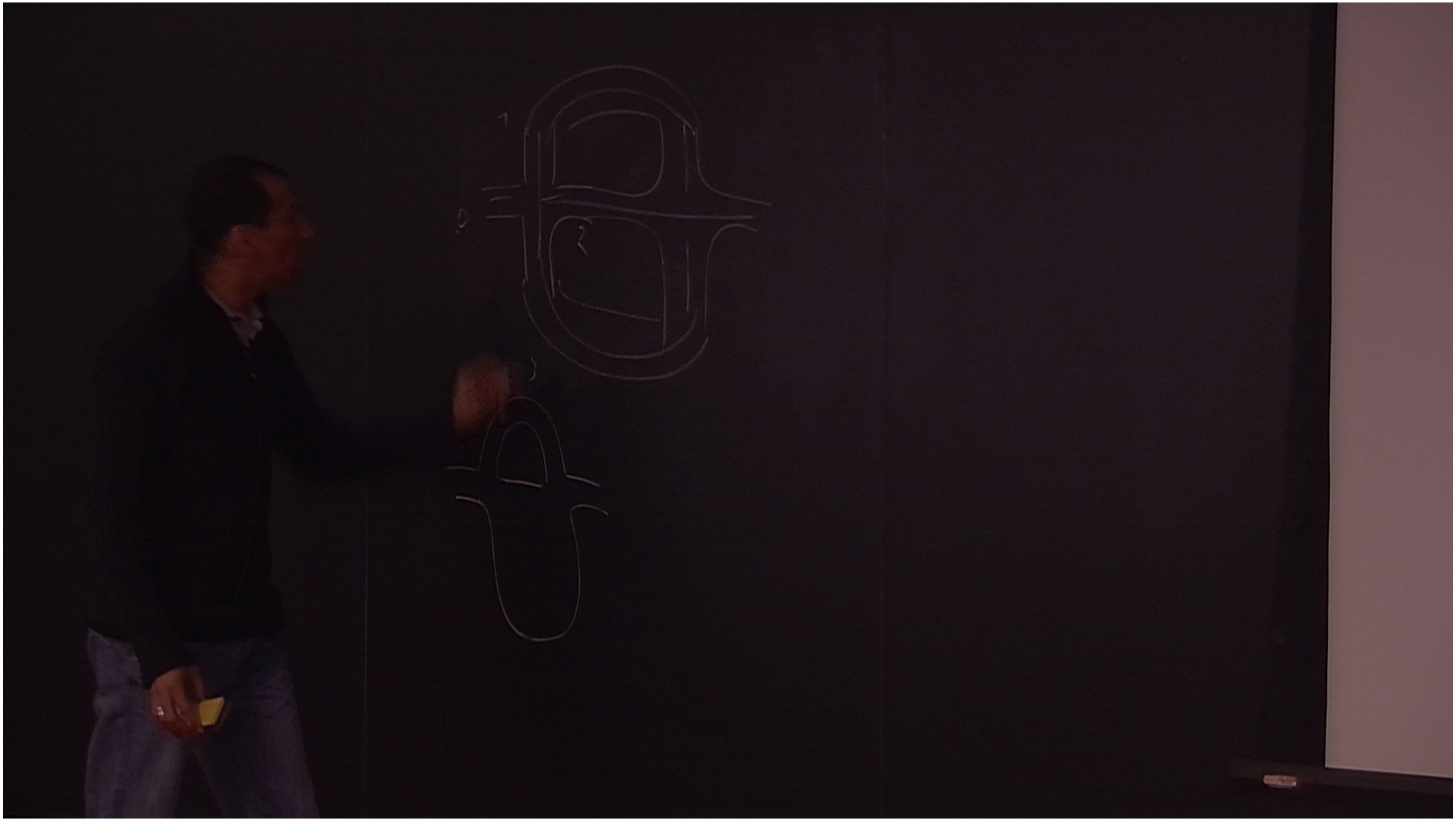

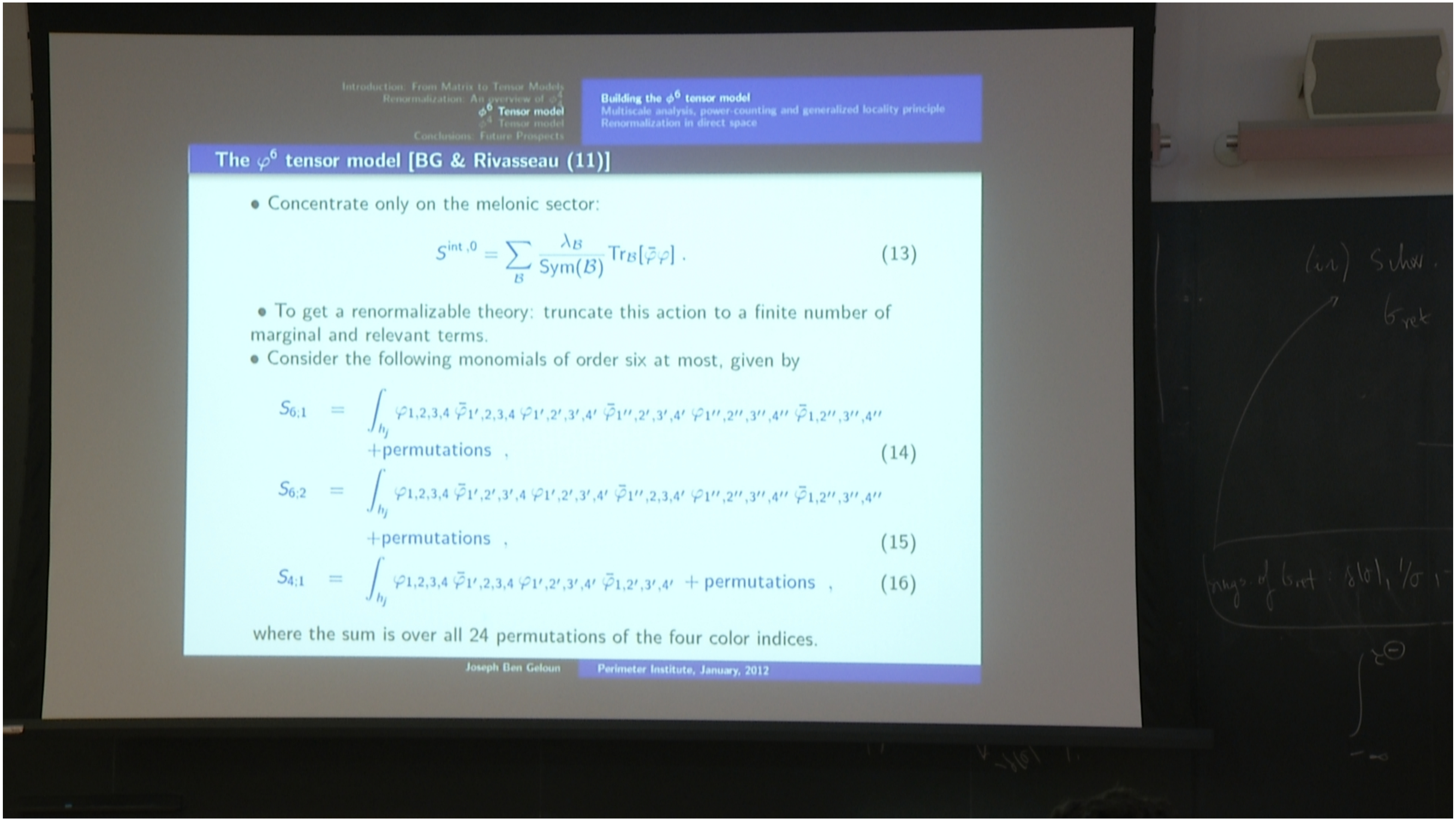
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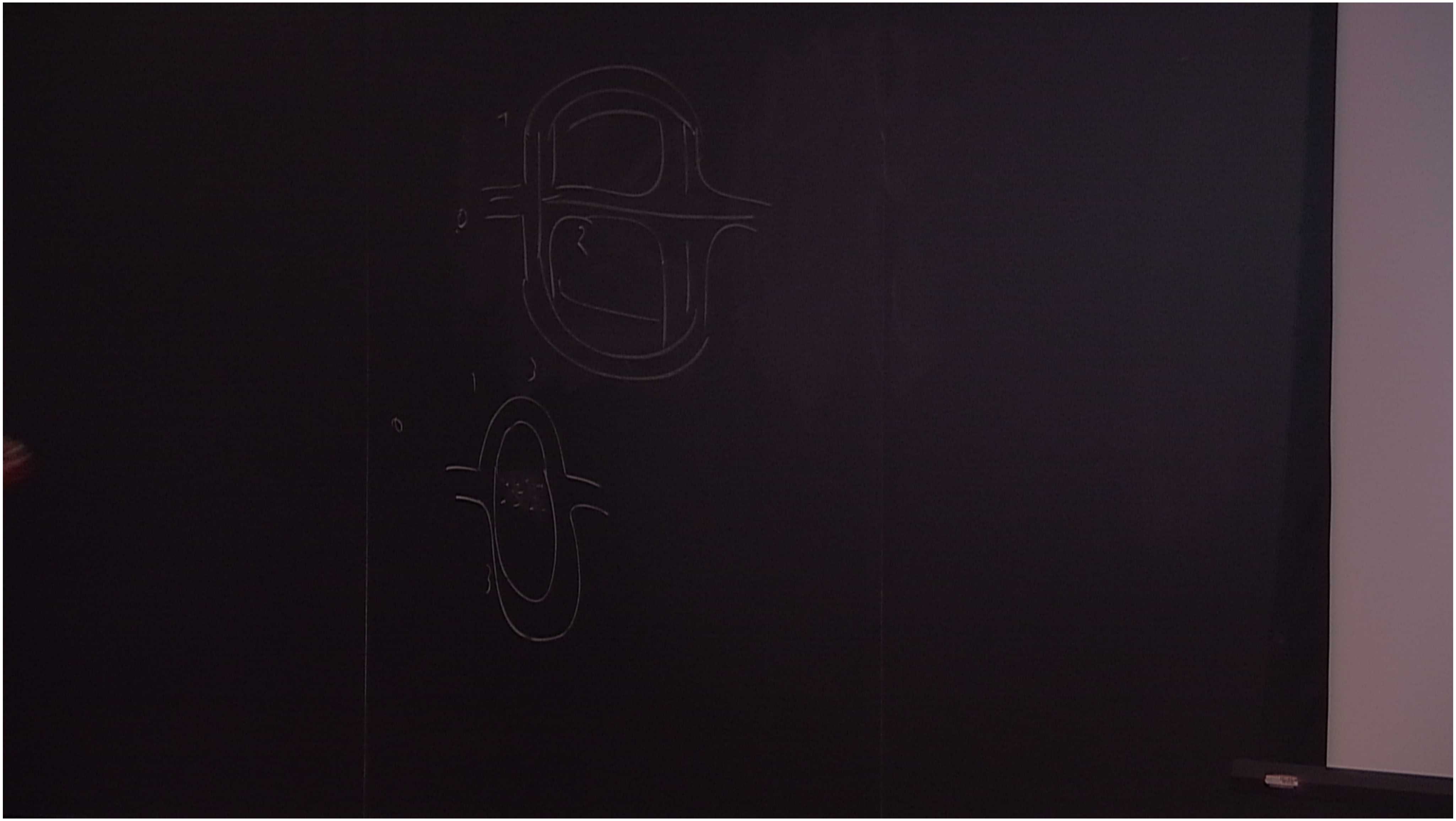

The φ^6 tensor model [BG & Rivasseau (11)]

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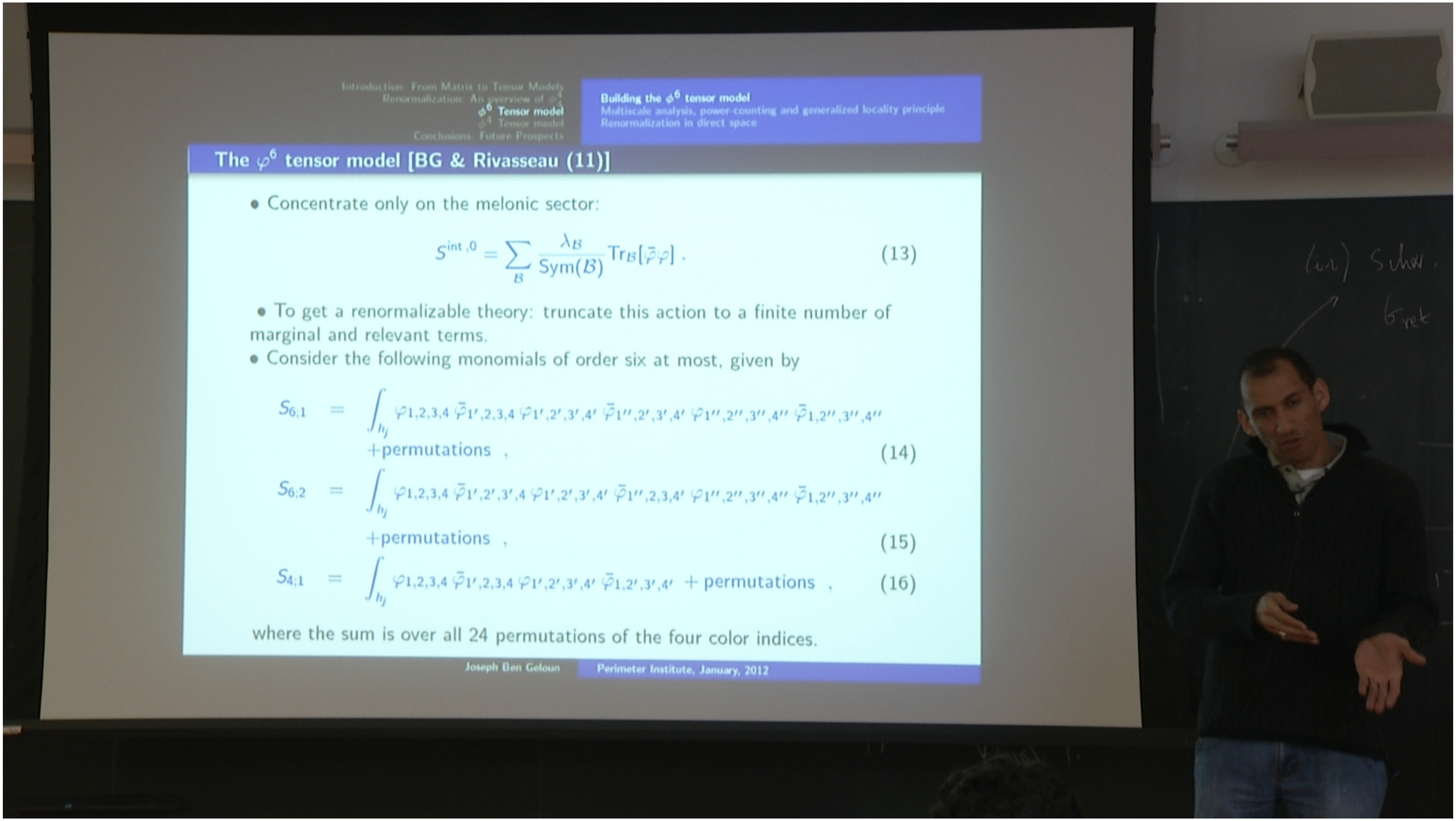
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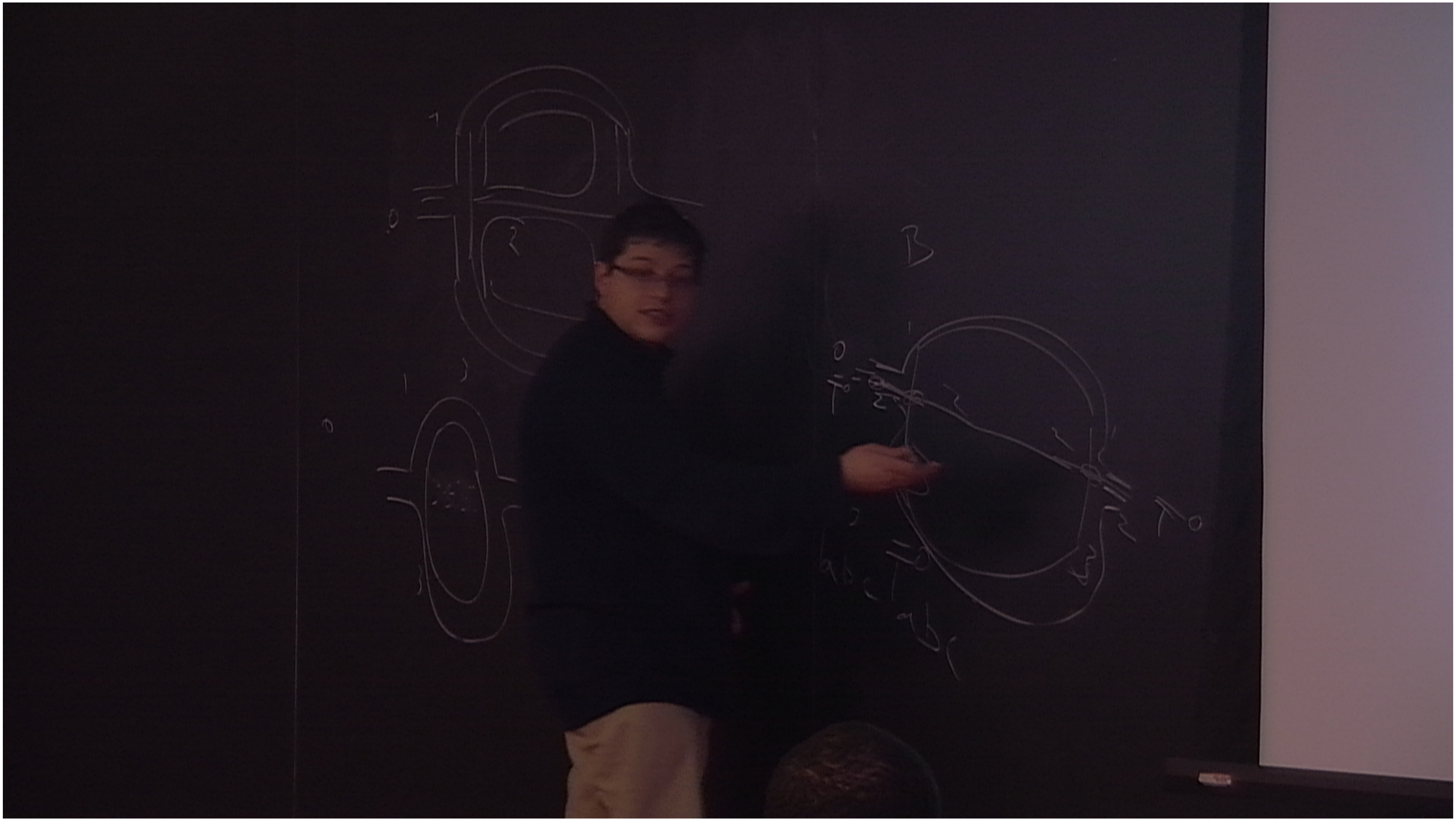

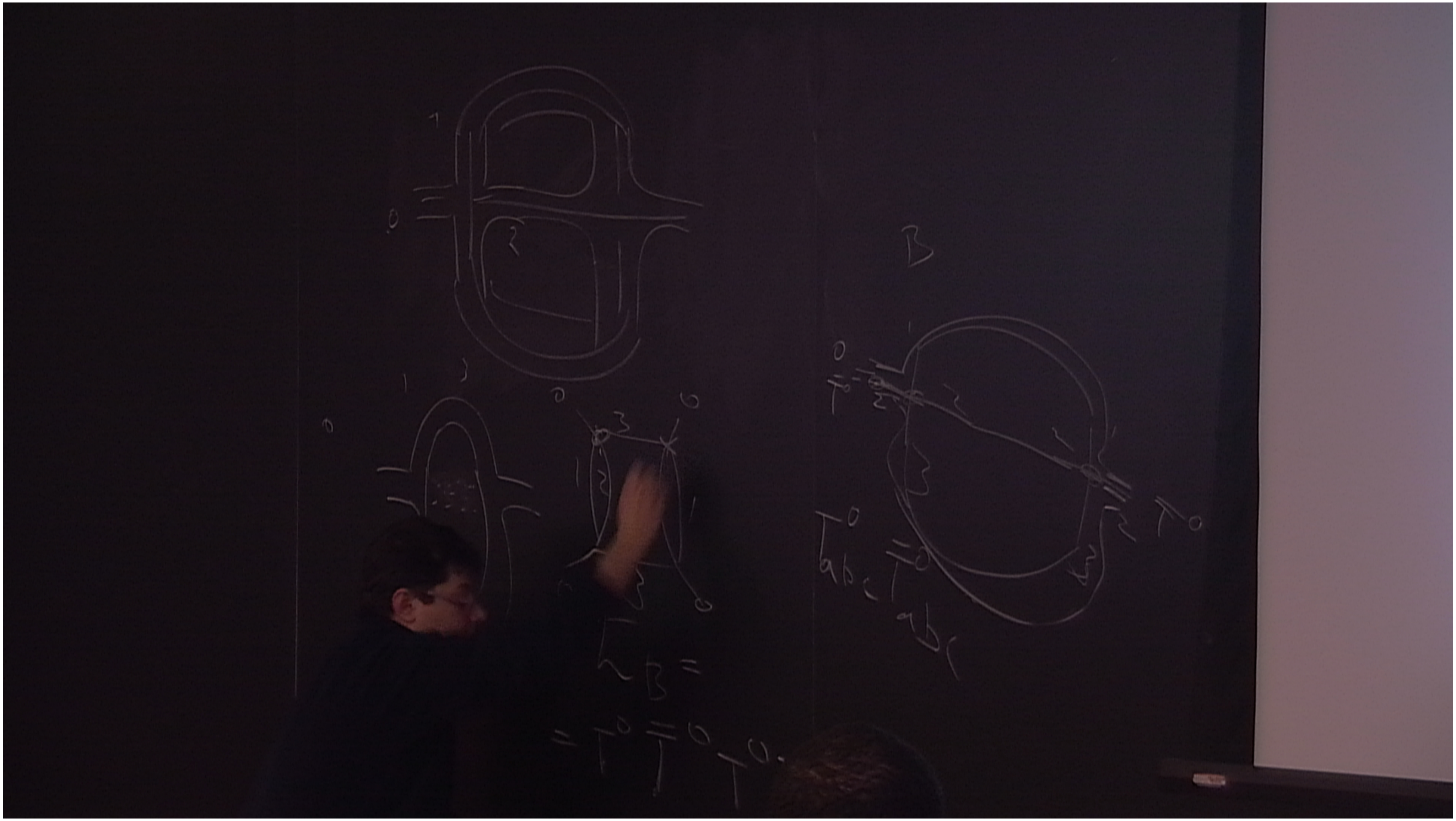

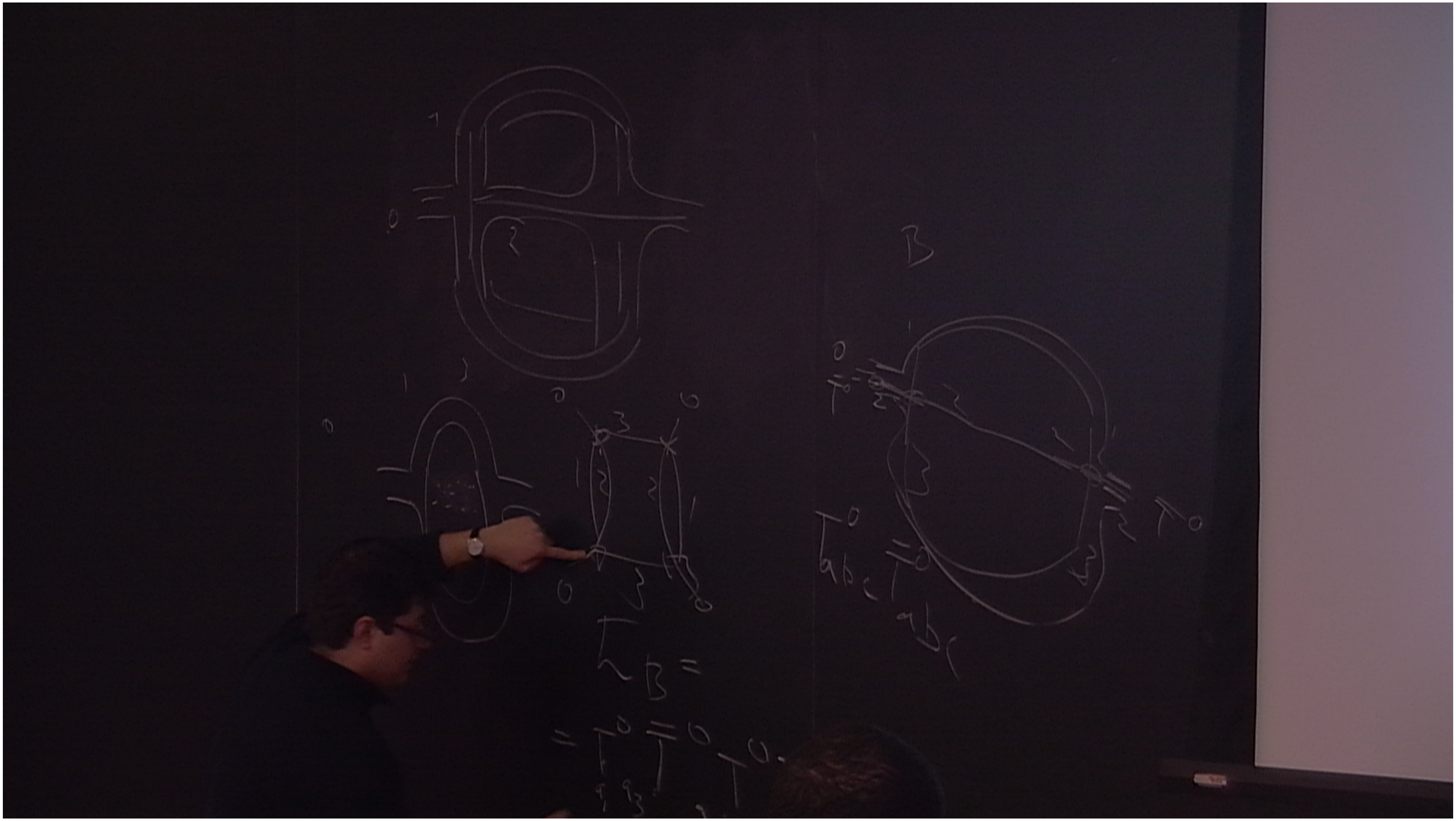

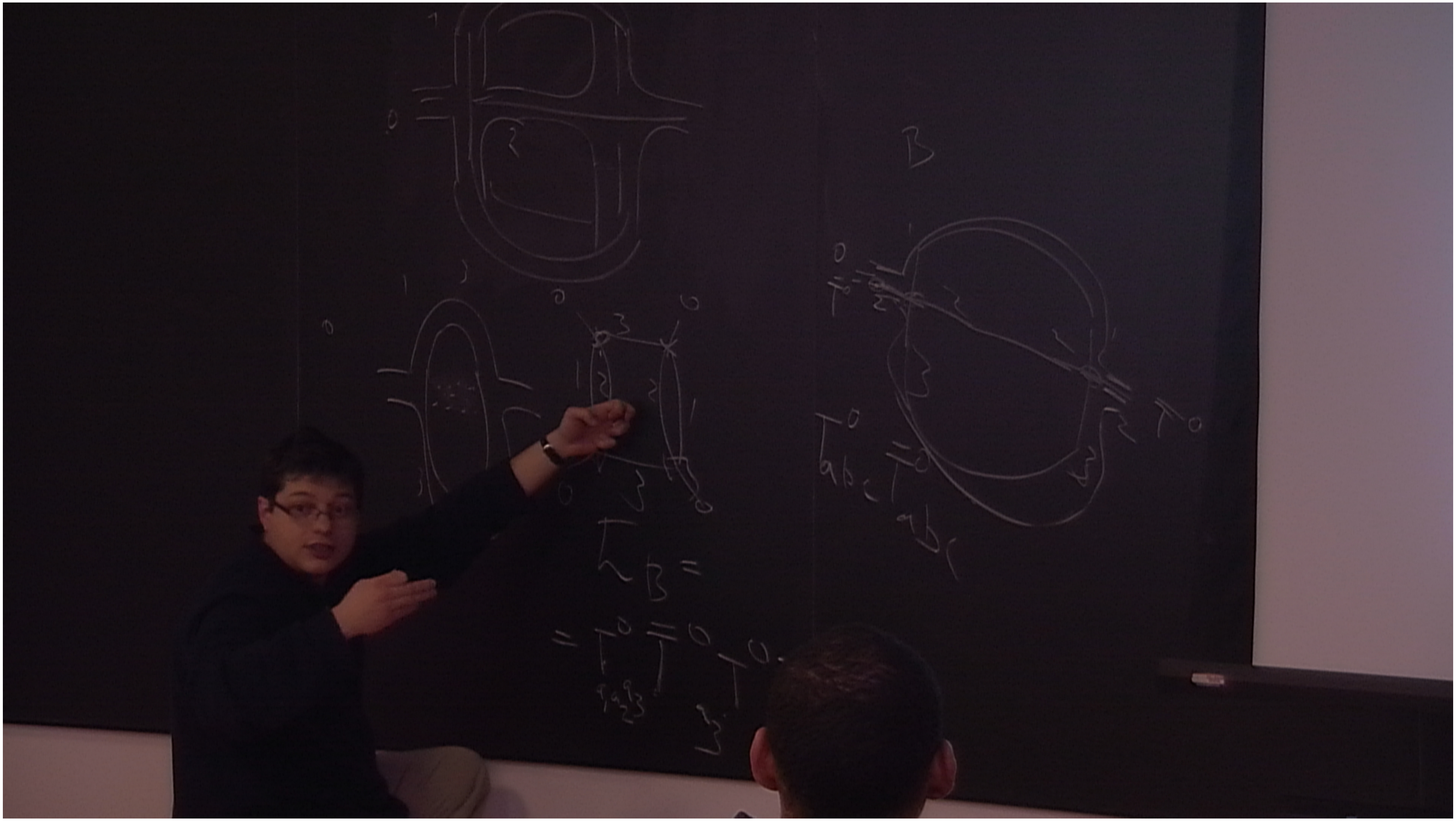

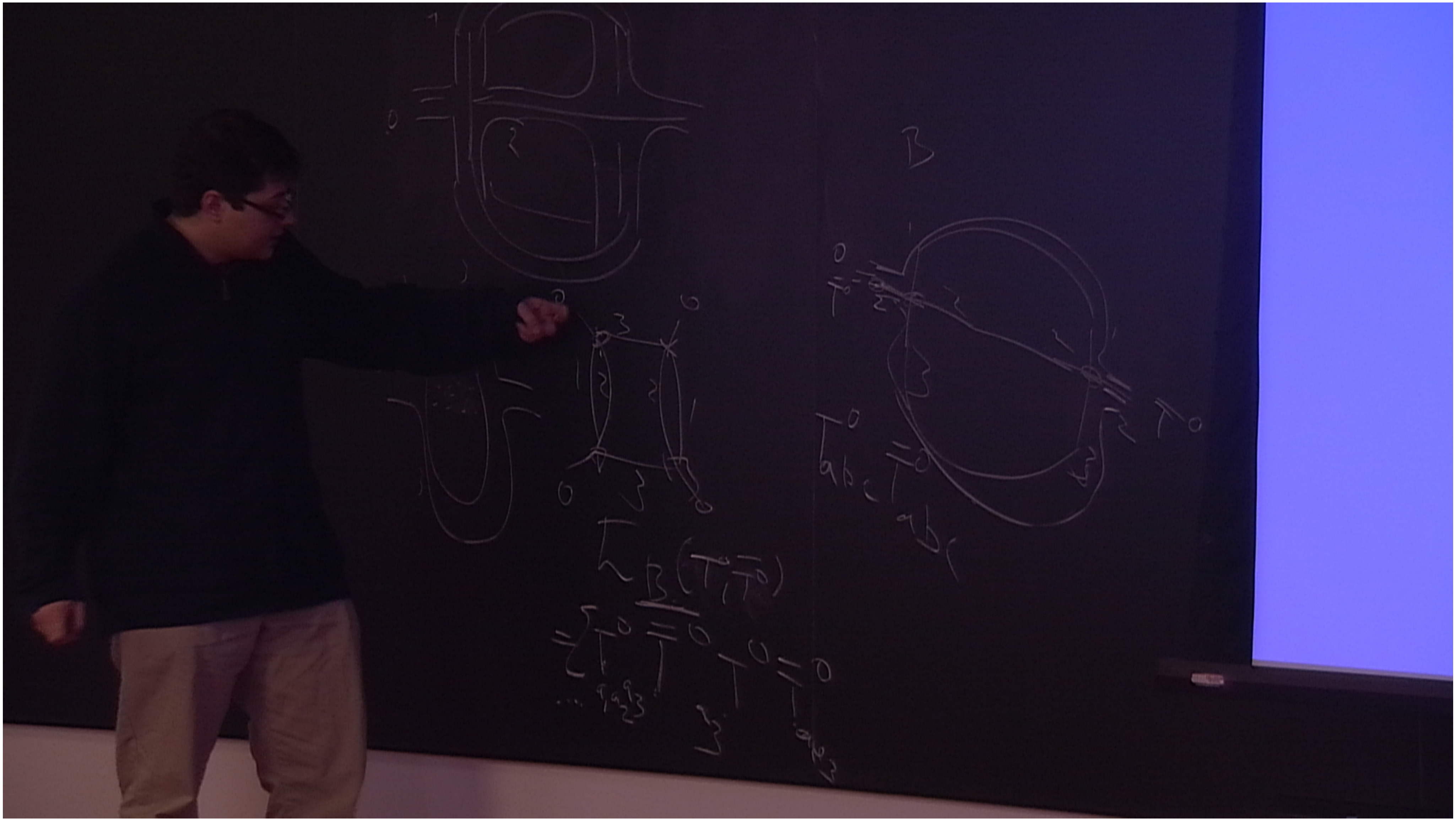

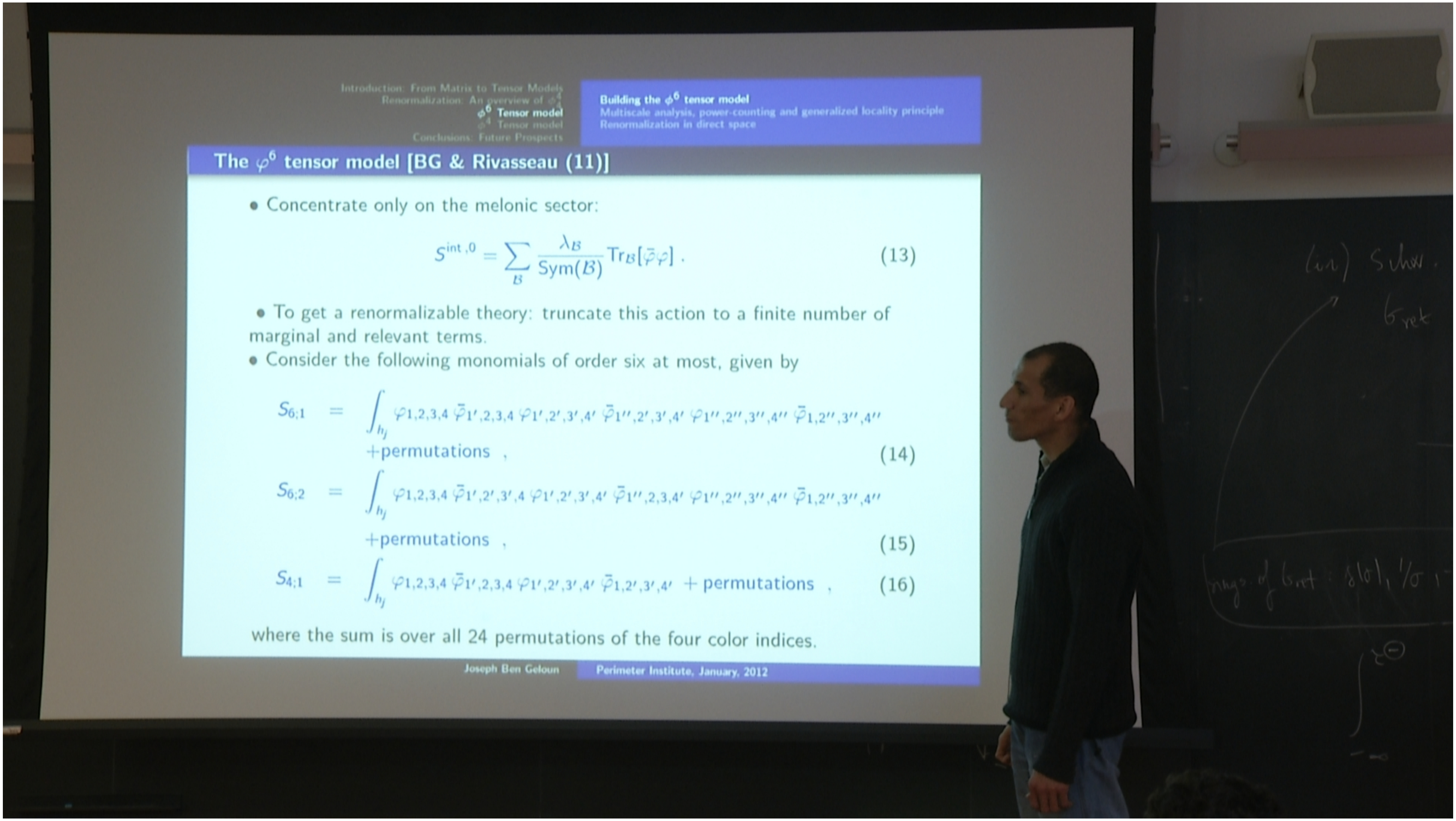
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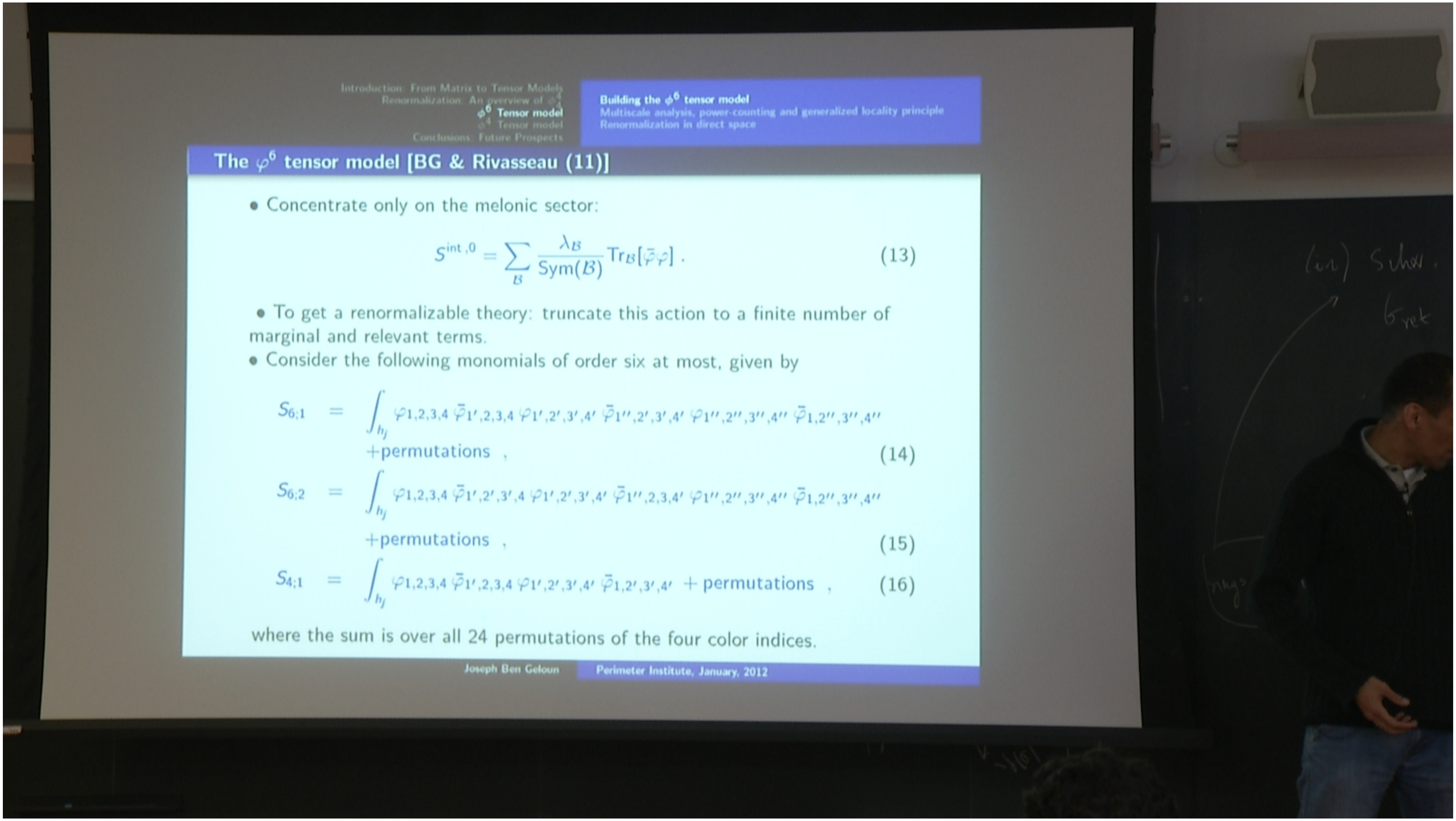
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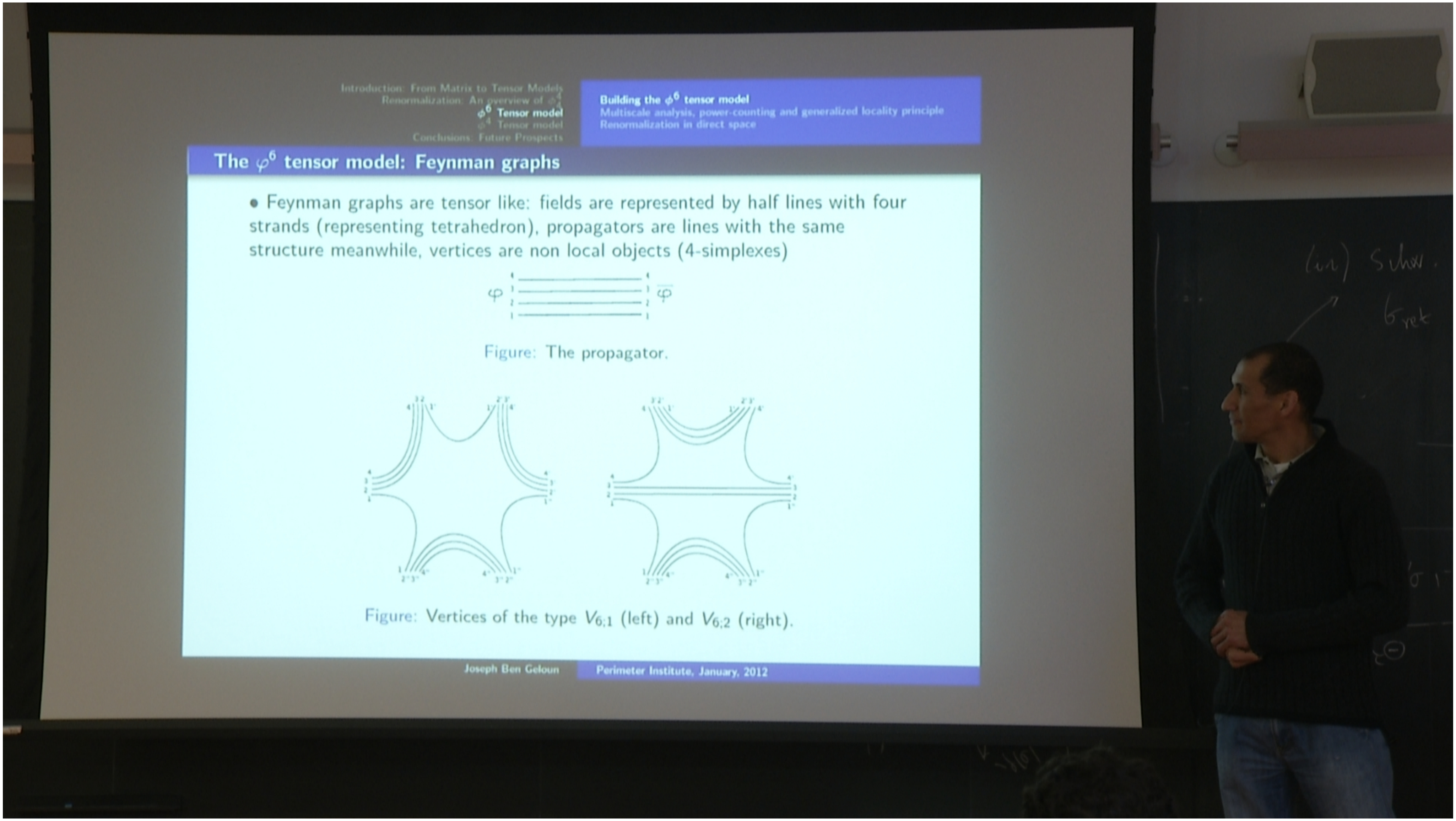
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The φ^6 tensor model: Feynman graphs
• Feynman graphs are tensor like: fields are represented by half lines with four strands (representing tetrahedron), propagators are lines with the same structure meanwhile, vertices are non local objects (4-simplexes)
φ
$\overline{\varphi}$
Figure: The propagator.
Figure: Vertices of the type $V_{6;1}$ (left) and $V_{6;2}$ (right).
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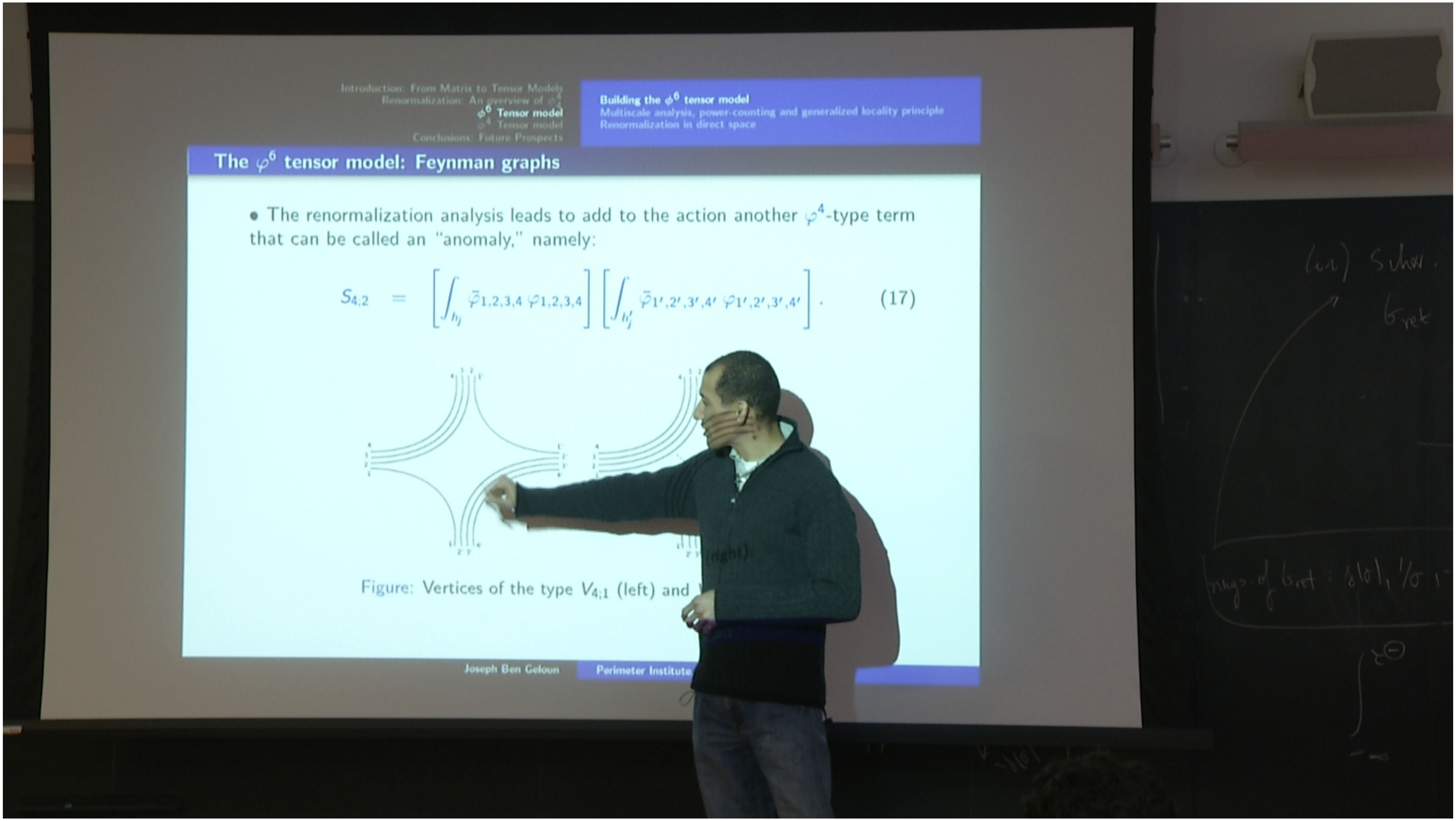

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The φ^6 tensor model: Feynman graphs
• The renormalization analysis leads to add to the action another φ^4-type term that can be called an "anomaly," namely:
$$S_{4;2} = \left[\int_{h_j} \bar{\varphi}_{1,2,3,4}\, \varphi_{1,2,3,4}\right] \left[\int_{h'_j} \bar{\varphi}_{1',2',3',4'}\, \varphi_{1',2',3',4'}\right]. \quad (17)$$
Figure: Vertices of the type $V_{4;1}$ (left) and
(right)
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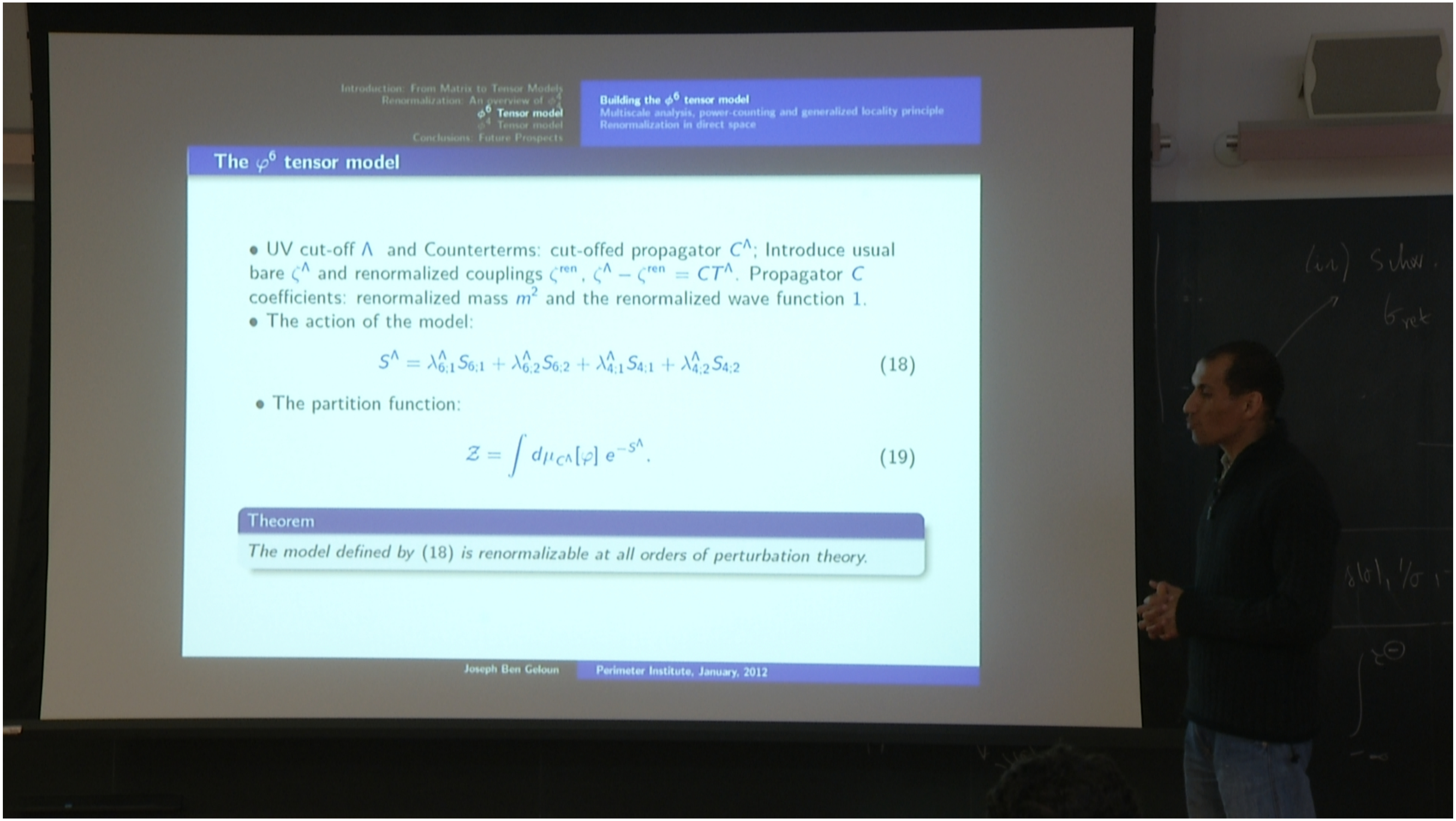
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The φ^6 tensor model
• UV cut-off Λ and Counterterms: cut-offed propagator C^Λ; Introduce usual bare ζ^Λ and renormalized couplings ζ^{ren}, $\zeta^\Lambda - \zeta^{ren} = CT^\Lambda$. Propagator C coefficients: renormalized mass m^2 and the renormalized wave function 1.
• The action of the model:
$$S^\Lambda = \lambda^\Lambda_{6;1} S_{6;1} + \lambda^\Lambda_{6;2} S_{6;2} + \lambda^\Lambda_{4;1} S_{4;1} + \lambda^\Lambda_{4;2} S_{4;2} \quad (18)$$
• The partition function:
$$\mathcal{Z} = \int d\mu_{C^\Lambda}[\varphi]\; e^{-S^\Lambda}. \quad (19)$$
Theorem
The model defined by (18) is renormalizable at all orders of perturbation theory.
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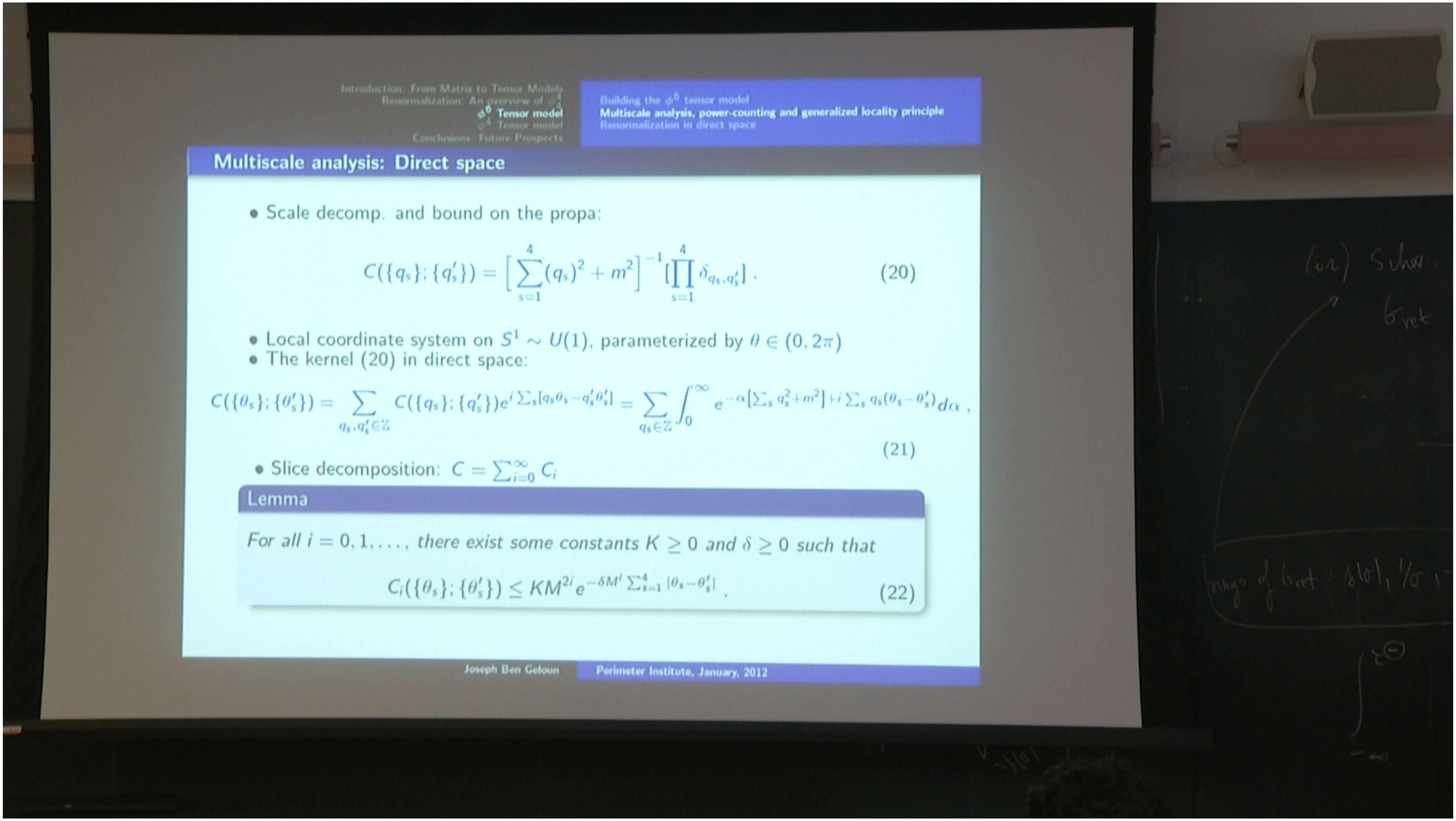
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Multiscale analysis: Direct space
• Scale decomp. and bound on the propa:
$C(\{q_s\};\{q'_s\}) = \Big[\sum_{s=1}^{4}(q_s)^2 + m^2\Big]^{-1}[\prod_{s=1}^{4}\delta_{q_s,q'_s}]\,.$ (20)
• Local coordinate system on $S^1 \sim U(1)$, parameterized by $\theta \in (0, 2\pi)$
• The kernel (20) in direct space:
$C(\{\theta_s\};\{\theta'_s\}) = \sum_{q_s,q'_s\in\mathbb{Z}} C(\{q_s\};\{q'_s\})e^{i\sum_s[q_s\theta_s - q'_s\theta'_s]} = \sum_{q_s\in\mathbb{Z}}\int_0^{\infty} e^{-\alpha[\sum_s q_s^2+m^2]+i\sum_s q_s(\theta_s-\theta'_s)}d\alpha\,,$ (21)
• Slice decomposition: $C = \sum_{i=0}^{\infty} C_i$
Lemma
For all $i = 0, 1, \ldots,$ there exist some constants $K \geq 0$ and $\delta \geq 0$ such that
$C_i(\{\theta_s\};\{\theta'_s\}) \leq KM^{2i}e^{-\delta M^i\sum_{s=1}^{4}|\theta_s-\theta'_s|}\,.$ (22)
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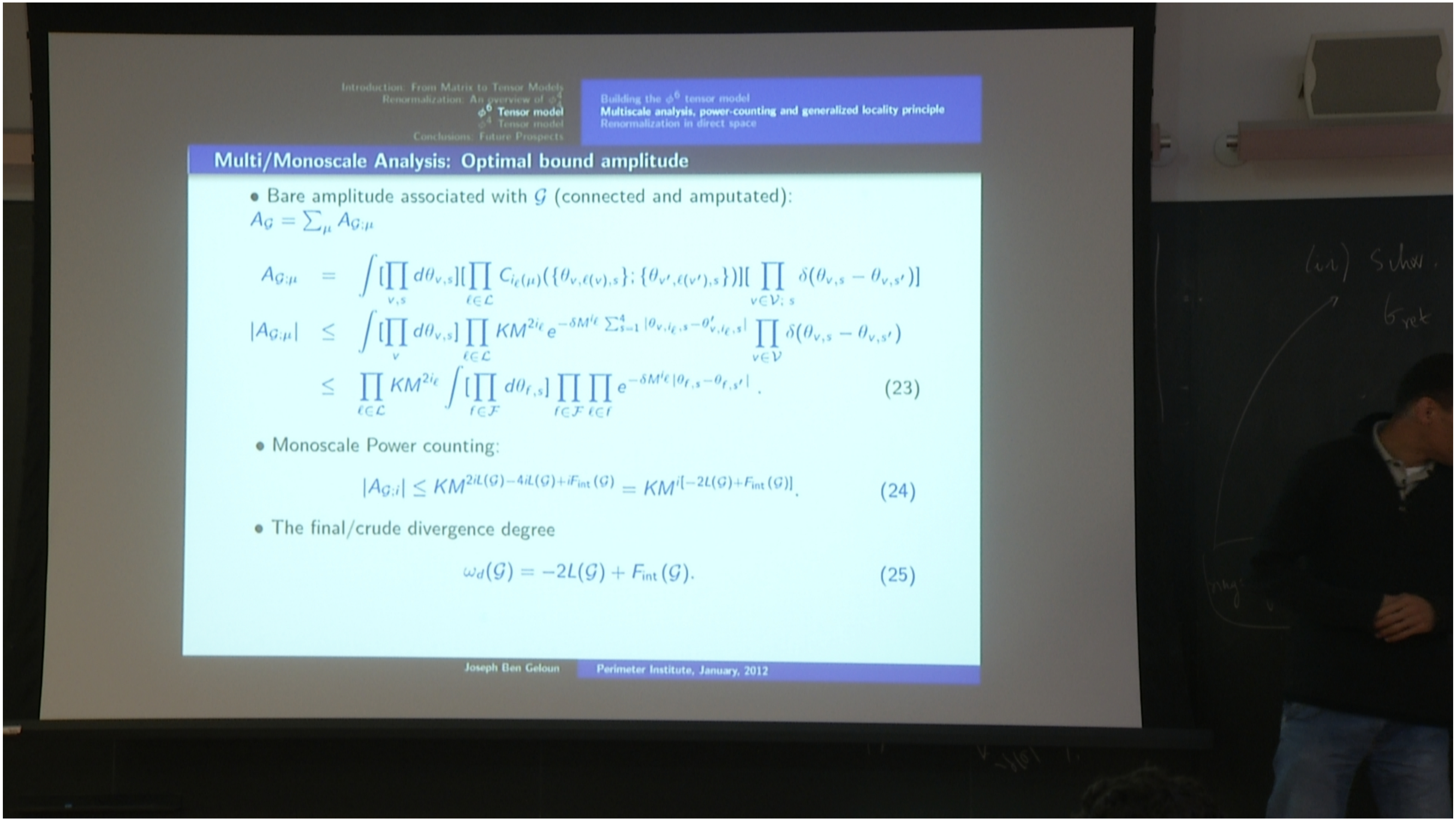
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Multi/Monoscale Analysis: Optimal bound amplitude
• Bare amplitude associated with $\mathcal{G}$ (connected and amputated):
$A_{\mathcal{G}} = \sum_\mu A_{\mathcal{G};\mu}$
$A_{\mathcal{G};\mu} = \int [\prod_{v,s} d\theta_{v,s}][\prod_{\ell \in \mathcal{L}} C_{i_\ell(\mu)}(\{\theta_{v,\ell(v),s}\};\{\theta_{v',\ell(v'),s}\})][\prod_{v \in \mathcal{V};\, s} \delta(\theta_{v,s} - \theta_{v,s'})]$
$|A_{\mathcal{G};\mu}| \leq \int [\prod_{v} d\theta_{v,s}] \prod_{\ell \in \mathcal{L}} K M^{2i_\ell} e^{-\delta M^{i_\ell} \sum_{s=1}^{4} |\theta_{v,i_\ell,s} - \theta'_{v,i_\ell,s}|} \prod_{v \in \mathcal{V}} \delta(\theta_{v,s} - \theta_{v,s'})$
$\leq \prod_{\ell \in \mathcal{L}} K M^{2i_\ell} \int [\prod_{f \in \mathcal{F}} d\theta_{f,s}] \prod_{f \in \mathcal{F}} \prod_{\ell \in f} e^{-\delta M^{i_\ell} |\theta_{f,s} - \theta_{f,s'}|}$. (23)
• Monoscale Power counting:
$|A_{\mathcal{G};i}| \leq K M^{2iL(\mathcal{G}) - 4iL(\mathcal{G}) + iF_{\rm int}(\mathcal{G})} = K M^{i[-2L(\mathcal{G}) + F_{\rm int}(\mathcal{G})]}$. (24)
• The final/crude divergence degree
$\omega_d(\mathcal{G}) = -2L(\mathcal{G}) + F_{\rm int}(\mathcal{G})$. (25)
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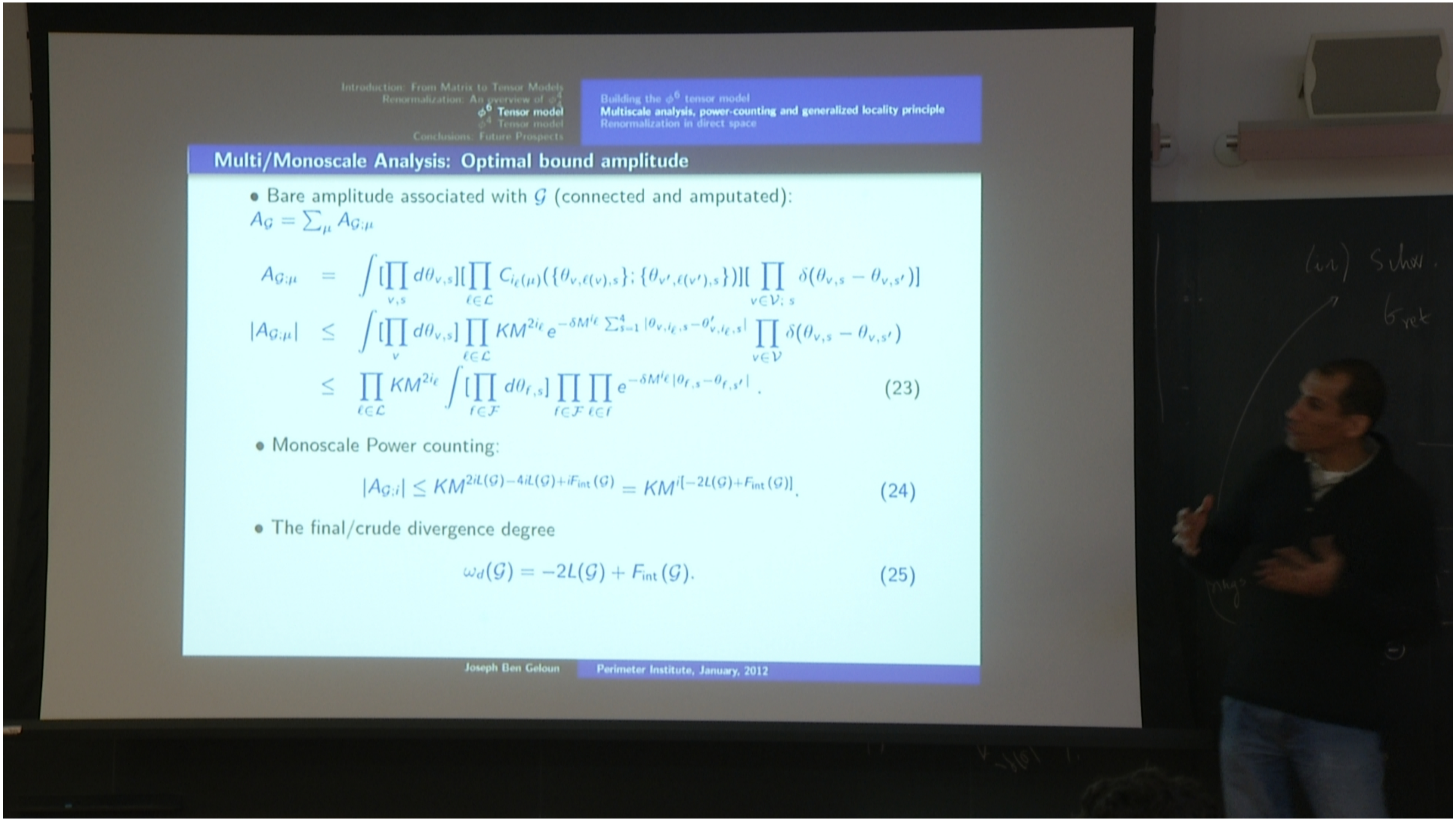
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$A_{\mathcal{G}} = \sum_\mu A_{\mathcal{G};\mu}$
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$|A_{\mathcal{G};\mu}| \leq \int [\prod_{v} d\theta_{v,s}] \prod_{\ell \in \mathcal{L}} KM^{2i_\ell} e^{-\delta M^{i_\ell} \sum_{s=1}^4 |\theta_{v,i_\ell,s} - \theta'_{v,i_\ell,s}|} \prod_{v \in \mathcal{V}} \delta(\theta_{v,s} - \theta_{v,s'})$
$\leq \prod_{\ell \in \mathcal{L}} KM^{2i_\ell} \int [\prod_{f \in \mathcal{F}} d\theta_{f,s}] \prod_{f \in \mathcal{F}} \prod_{\ell \in f} e^{-\delta M^{i_\ell} |\theta_{f,s} - \theta_{f,s'}|}$. (23)
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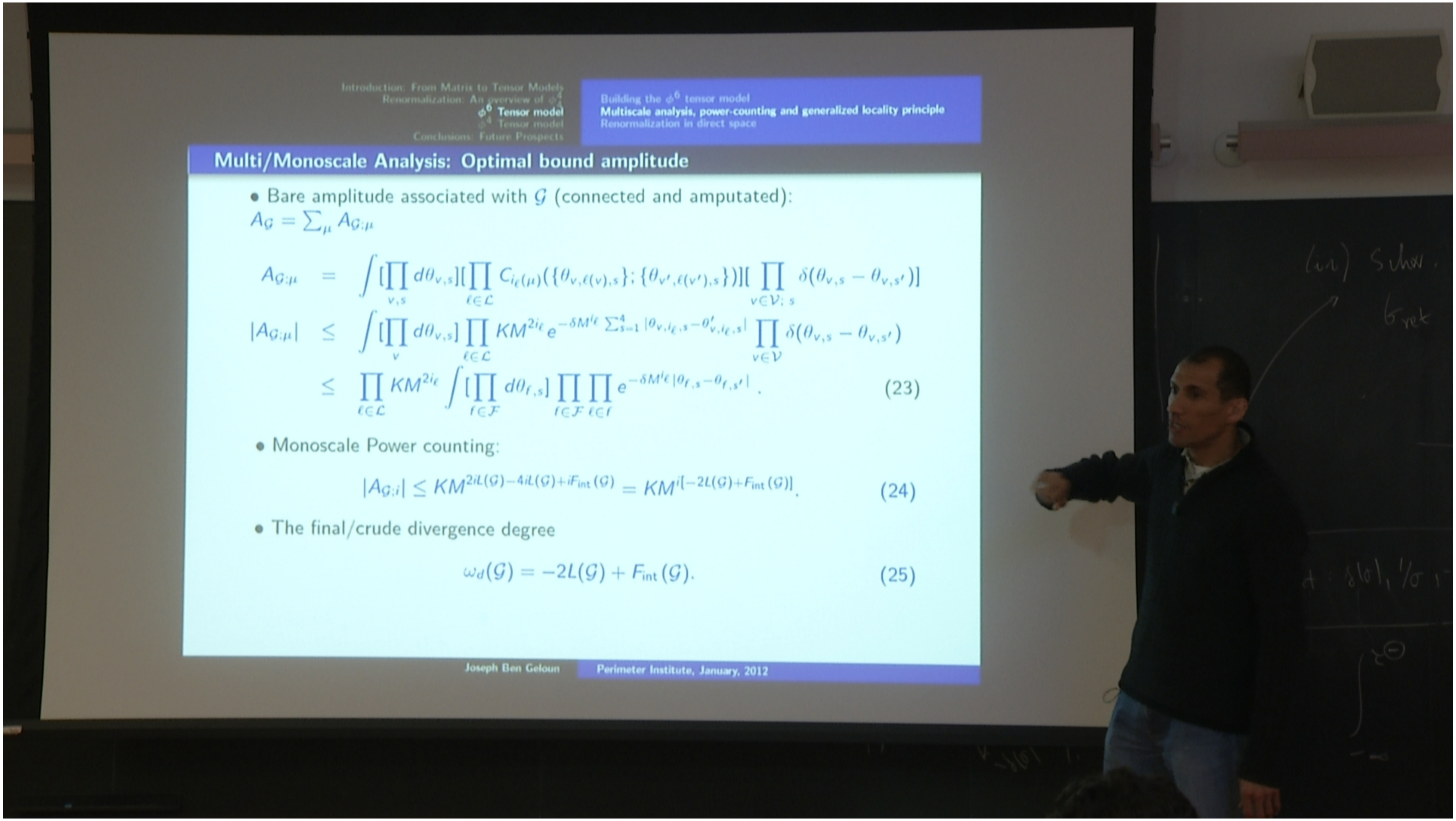
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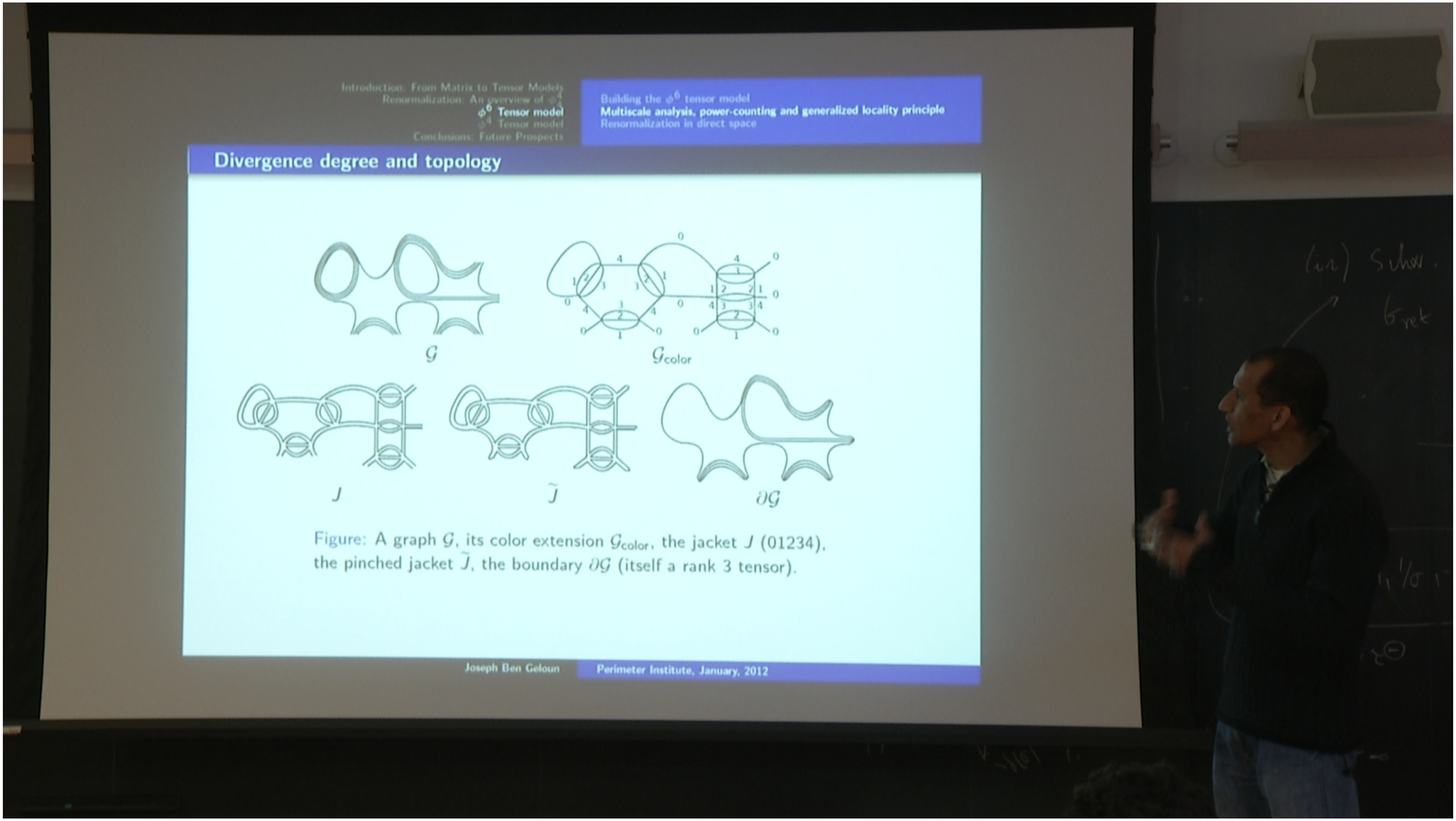
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J
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Figure: A graph $\mathcal{G}$, its color extension $\mathcal{G}_{\text{color}}$, the jacket J (01234), the pinched jacket $\widetilde{J}$, the boundary $\partial\mathcal{G}$ (itself a rank 3 tensor).
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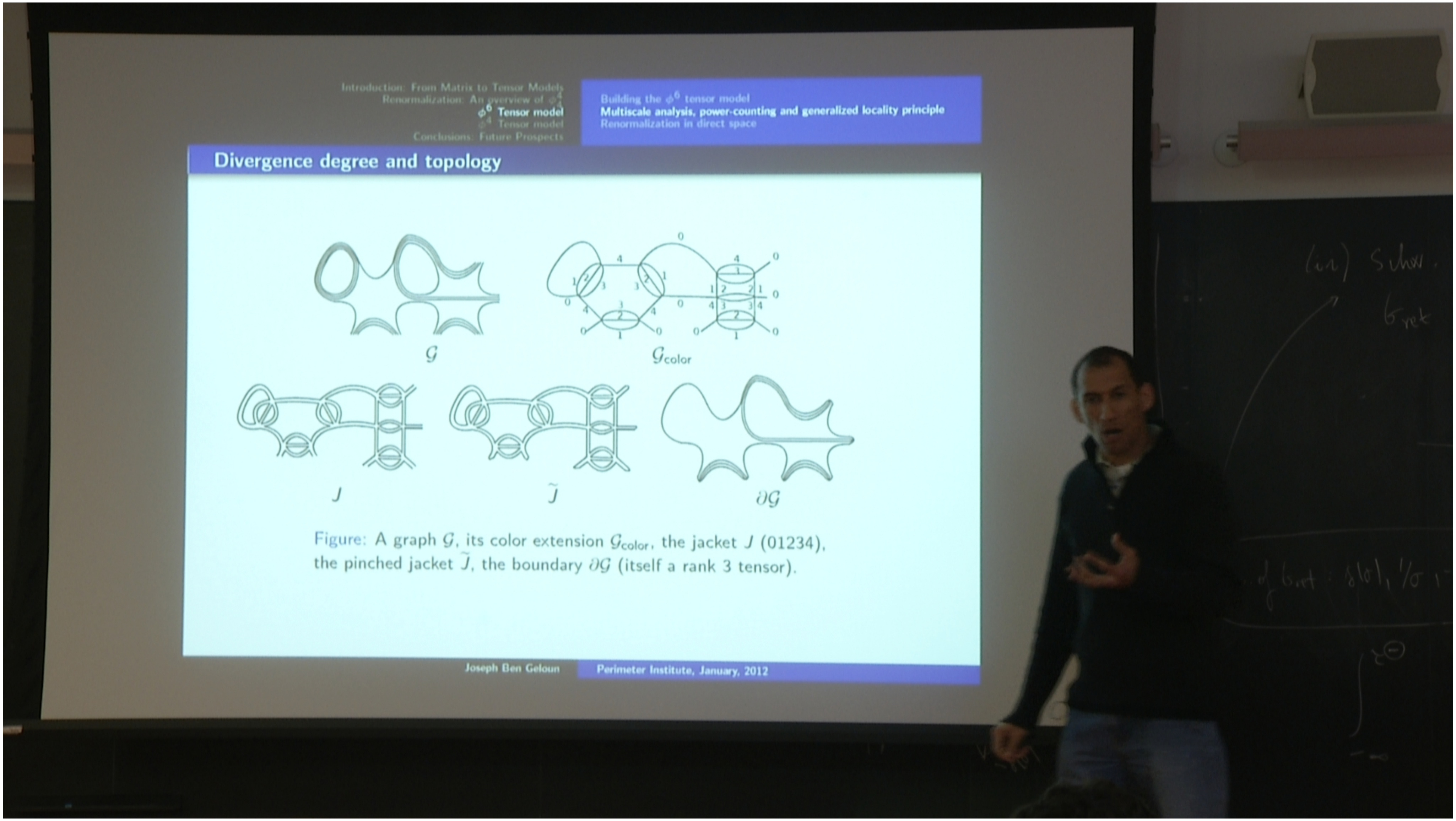

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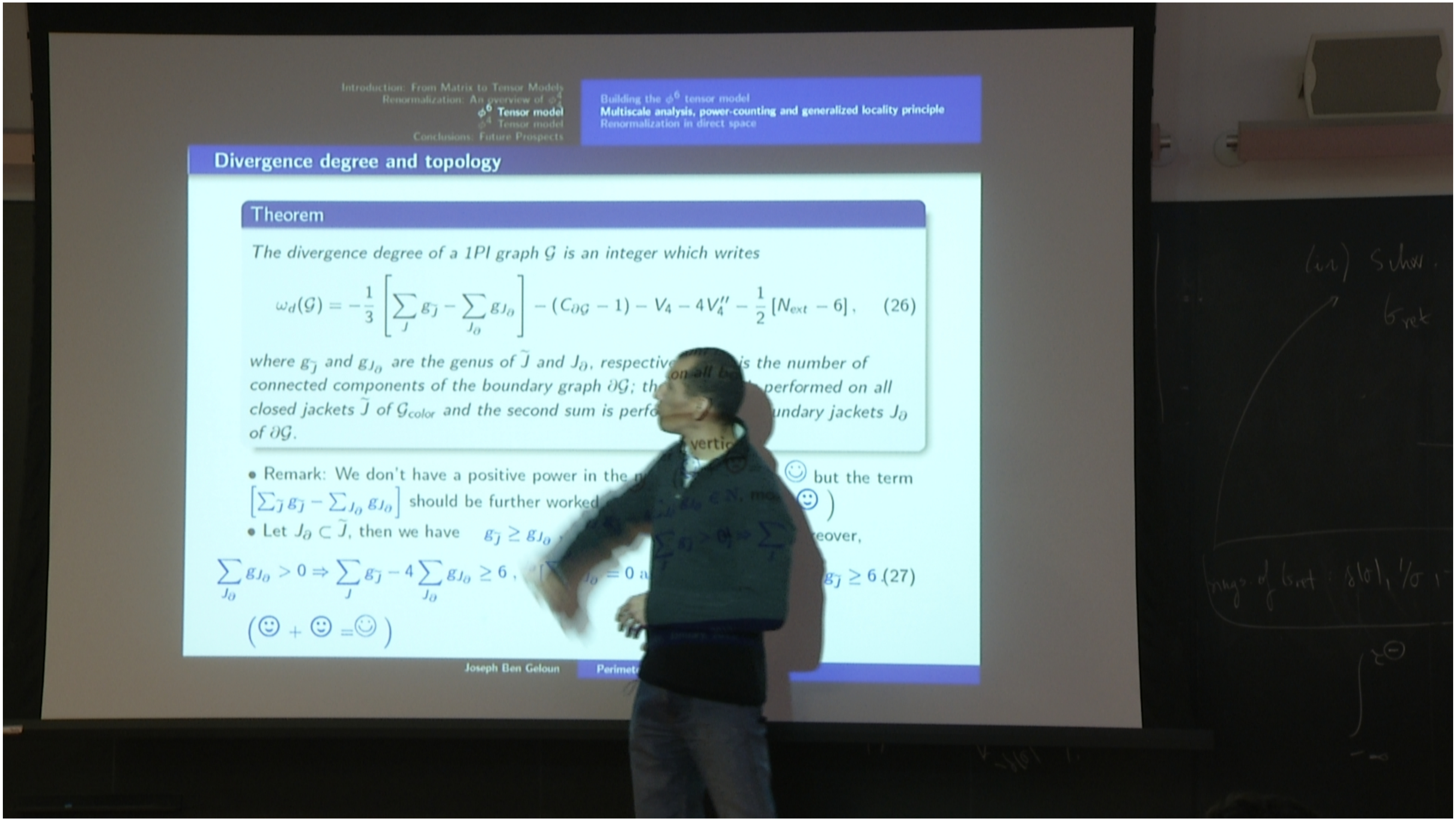

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where $g_{\widetilde{J}}$ and g_{J_∂} are the genus of $\widetilde{J}$ and J_∂, respectiv
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(in) Schw.

Divergence degree and topology

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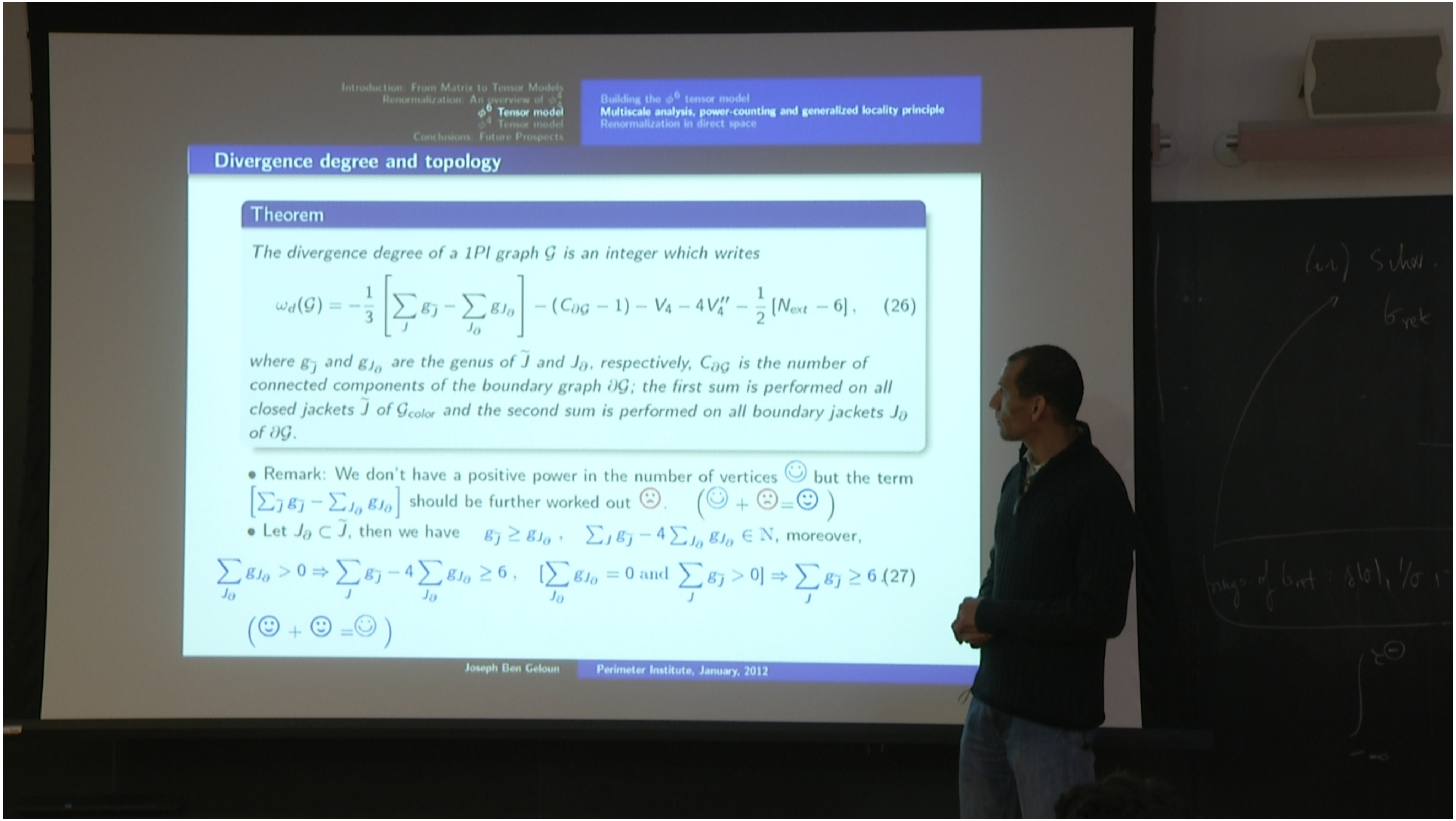
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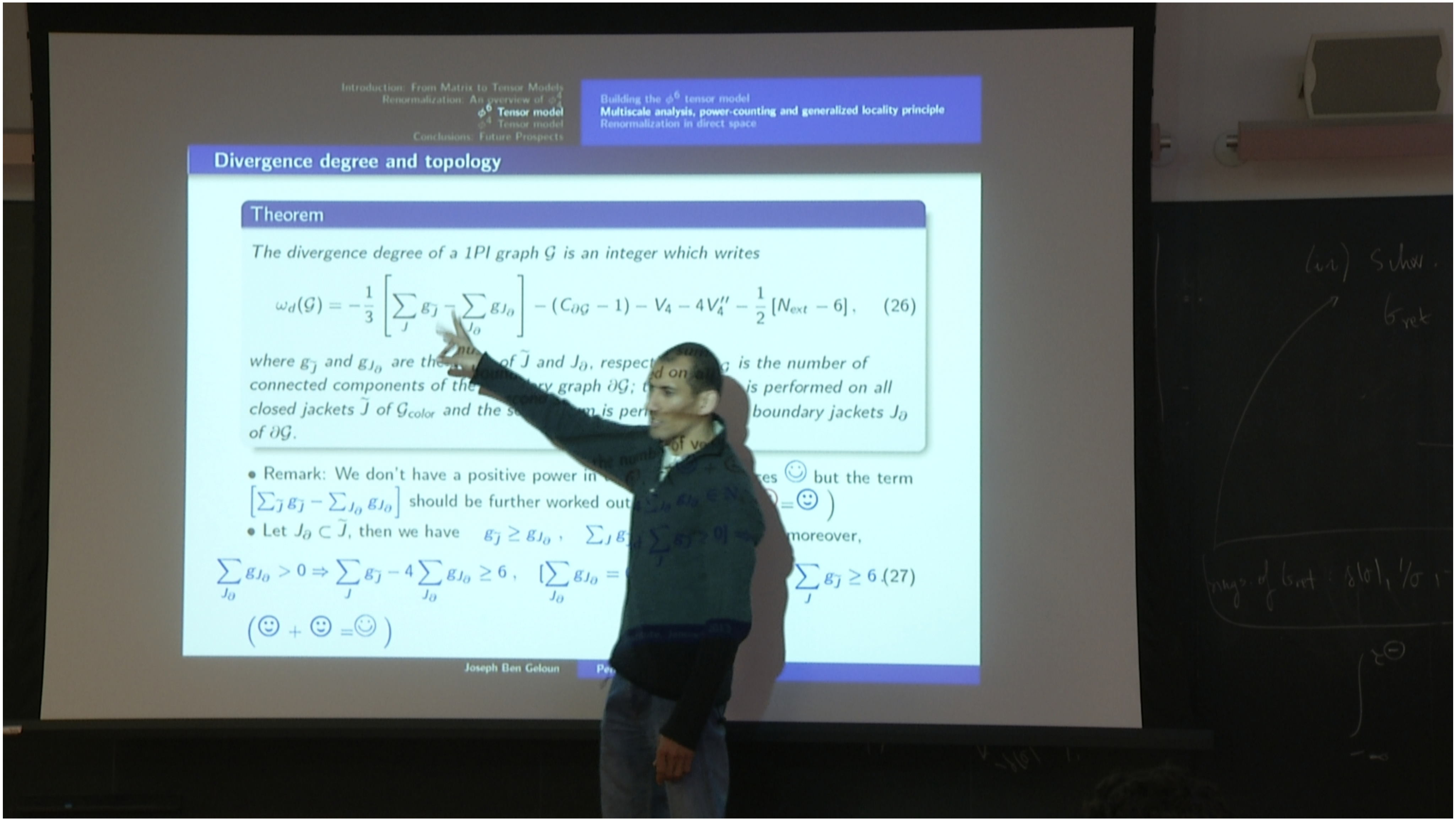
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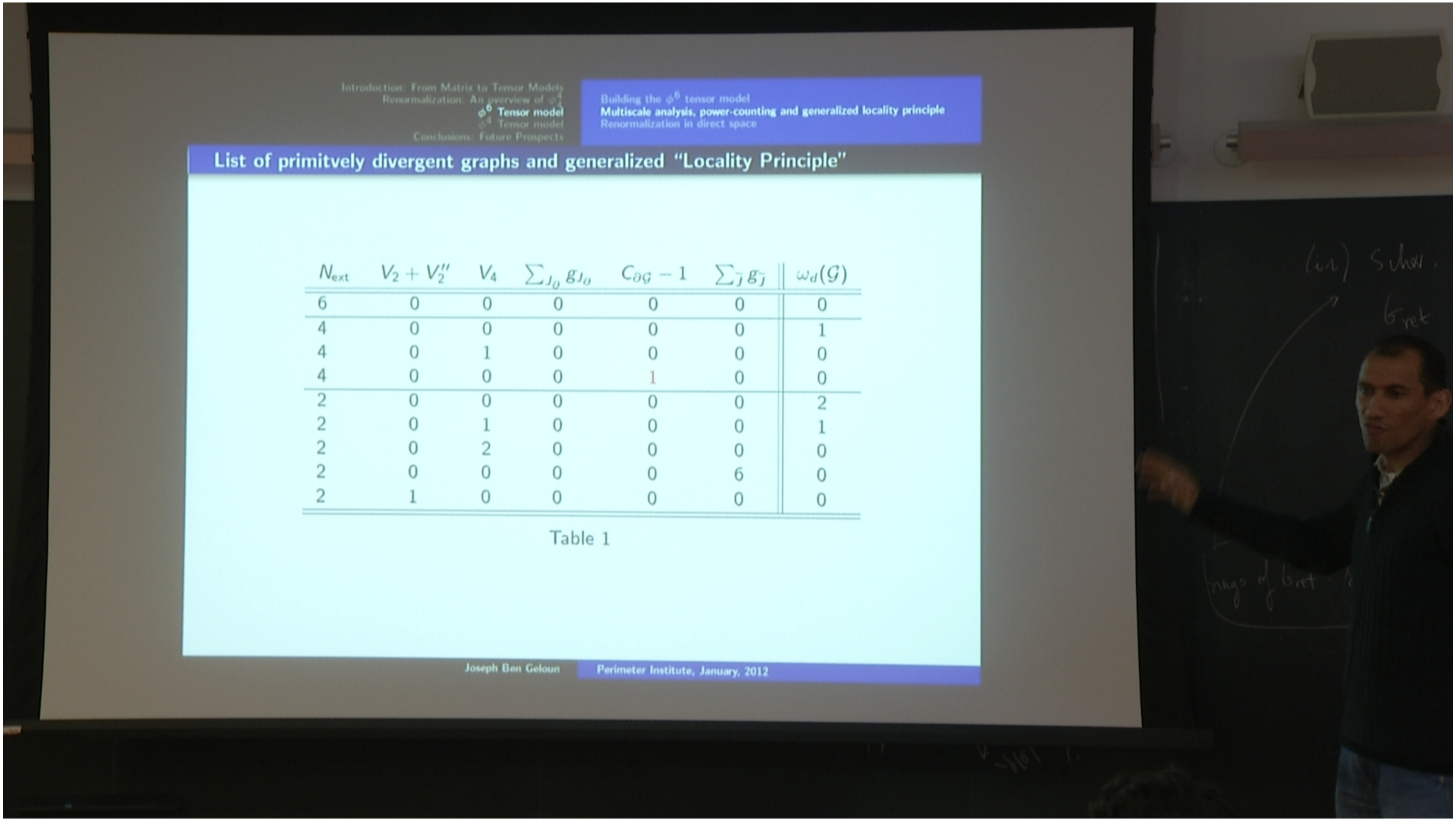
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N_{ext}	$V_2 + V_2''$	V_4	$\sum_{J_\emptyset} g_{J_\emptyset}$	$C_{\partial\mathcal{G}} - 1$	$\sum_{\tilde{J}} g_{\tilde{J}}$	$\omega_d(\mathcal{G})$
6	0	0	0	0	0	0
4	0	0	0	0	0	1
4	0	1	0	0	0	0
4	0	0	0	1	0	0
2	0	0	0	0	0	2
2	0	1	0	0	0	1
2	0	2	0	0	0	0
2	0	0	0	0	6	0
2	1	0	0	0	0	0

Table 1
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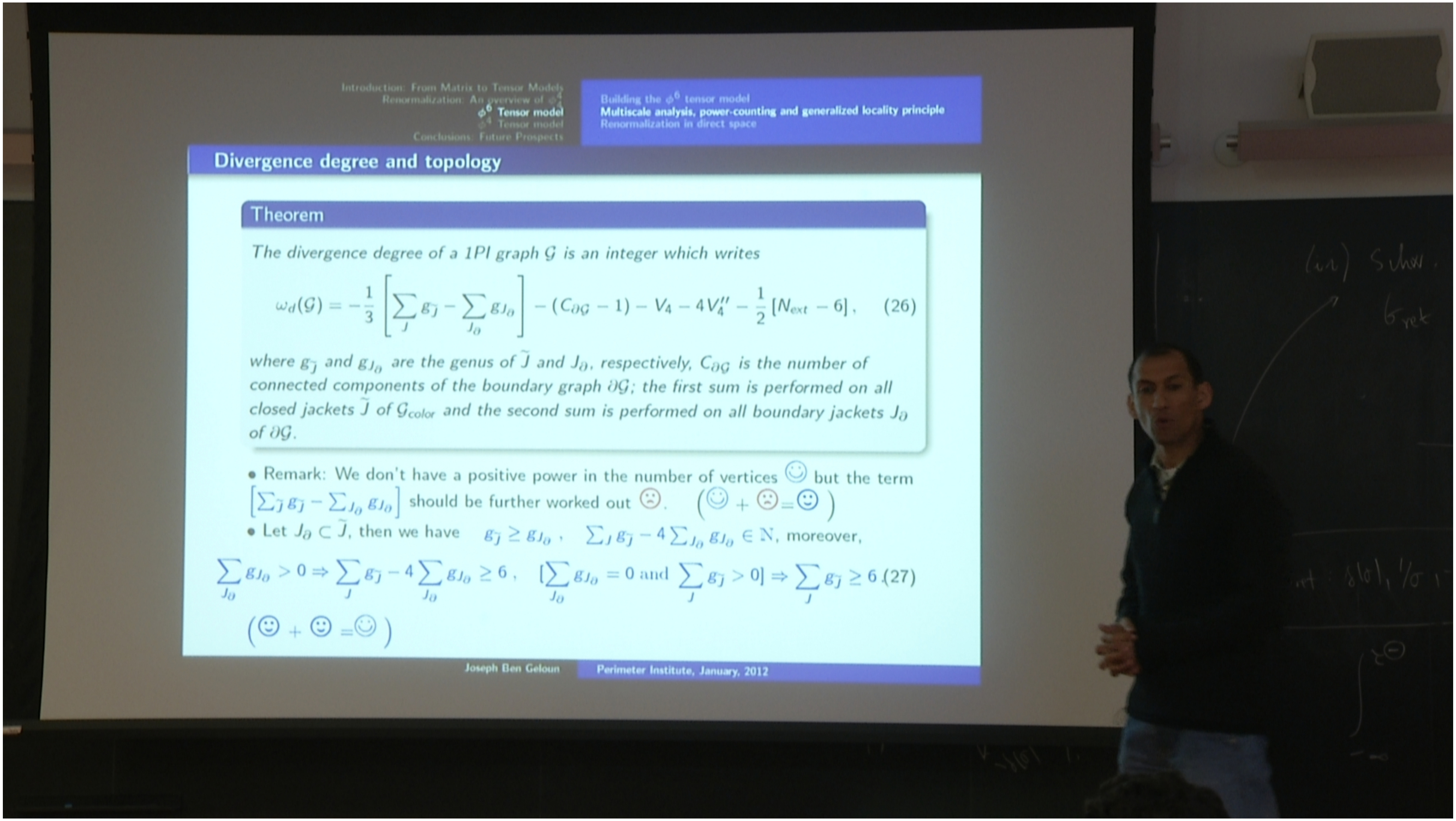
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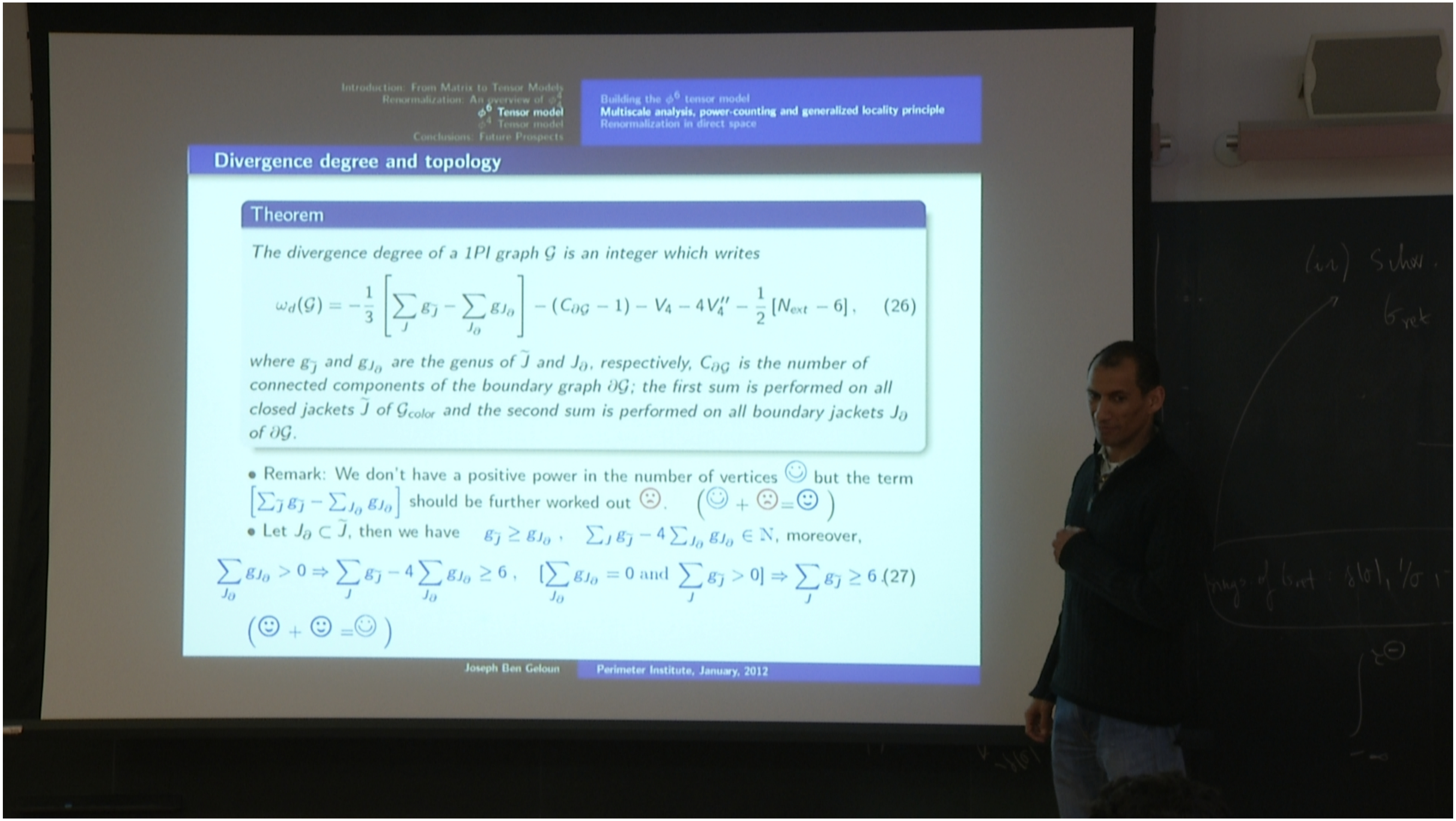
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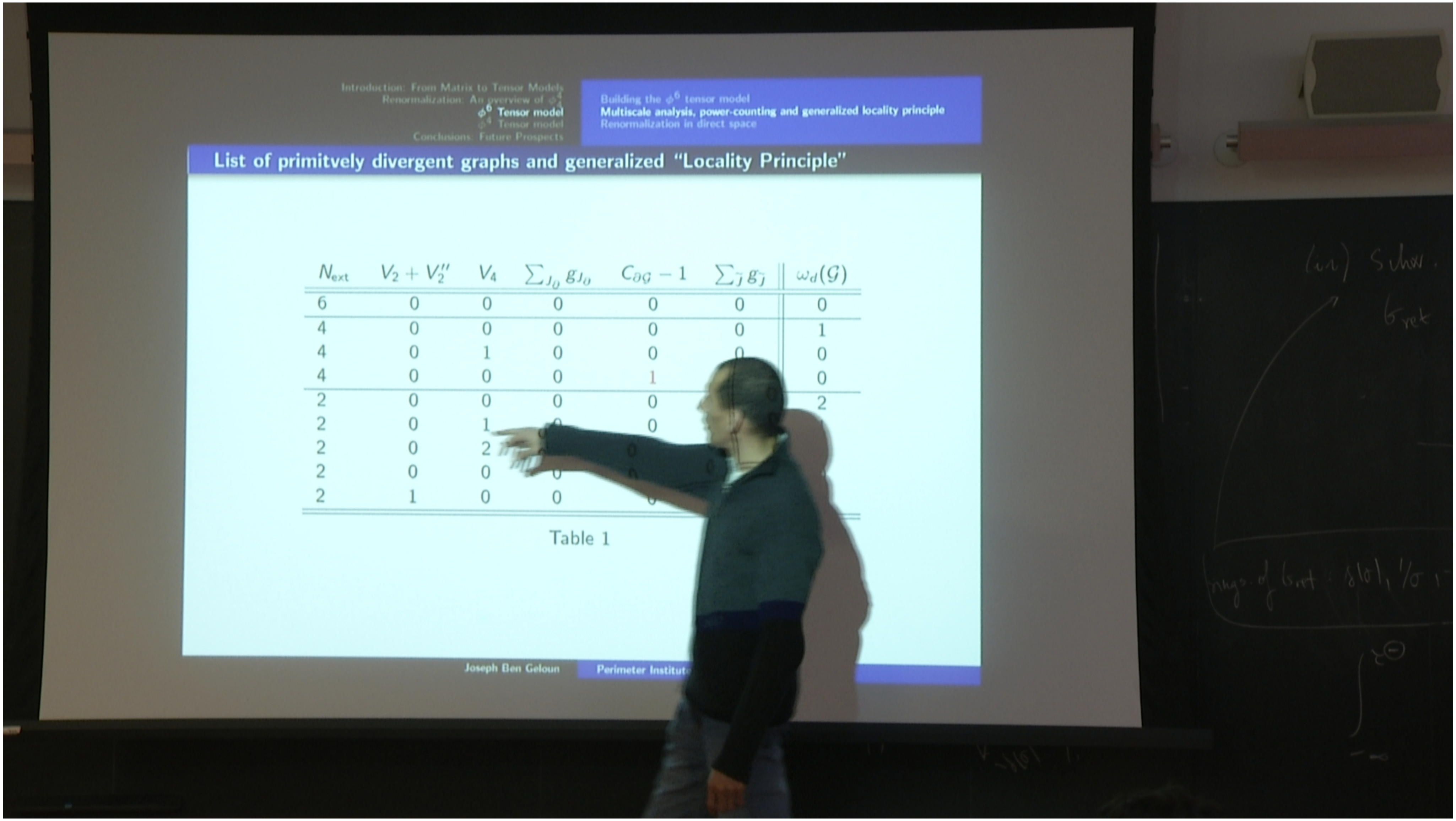
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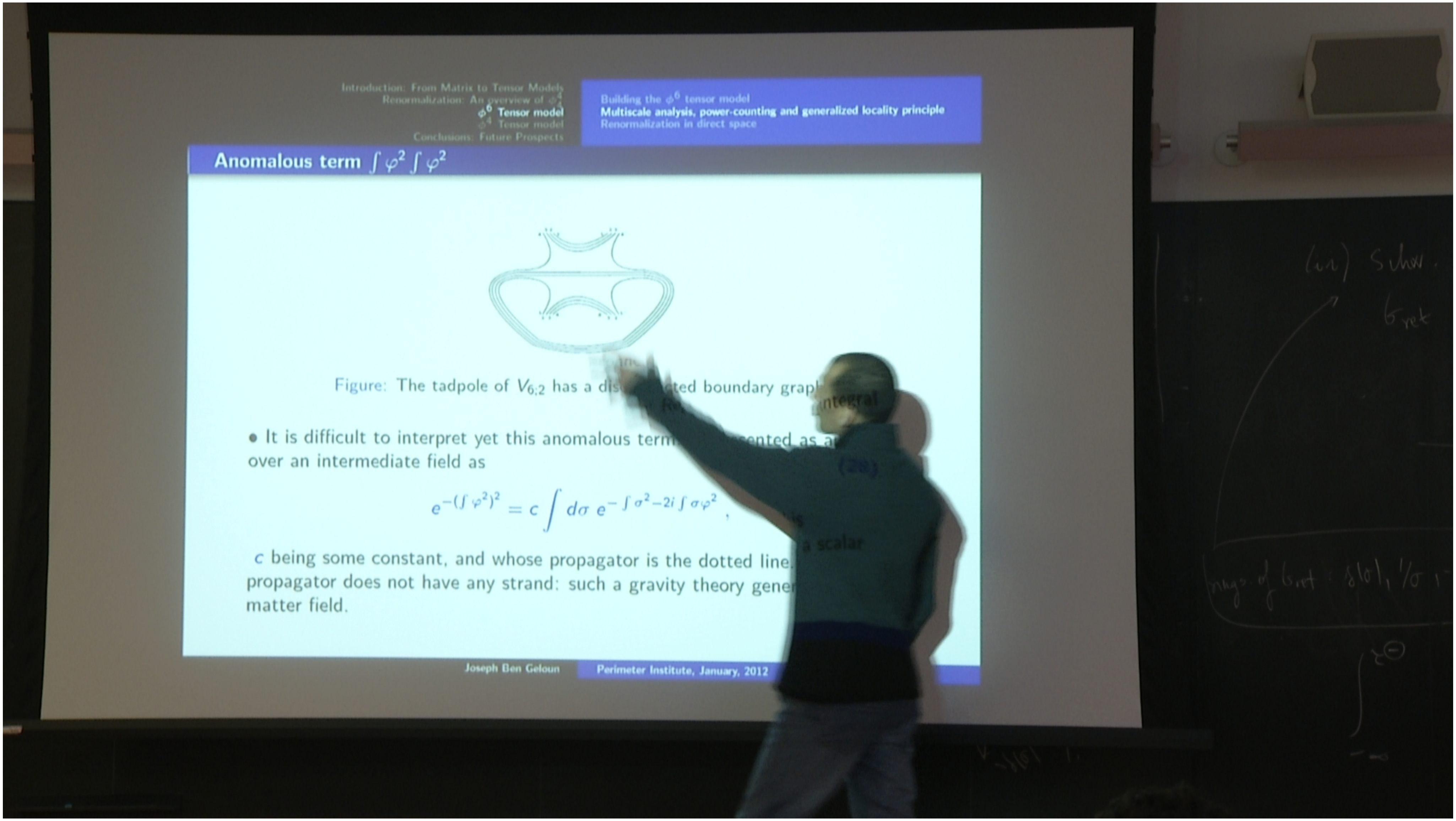
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Anomalous term ∫φ² ∫φ²
Figure: The tadpole of V6;2 has a dis
ted boundary grap
• It is difficult to interpret yet this anomalous term
over an intermediate field as
e^{−(∫φ²)²} = c ∫ dσ e^{−∫σ²−2i∫σφ²},
c being some constant, and whose propagator is the dotted line
propagator does not have any strand: such a gravity theory gene
matter field.
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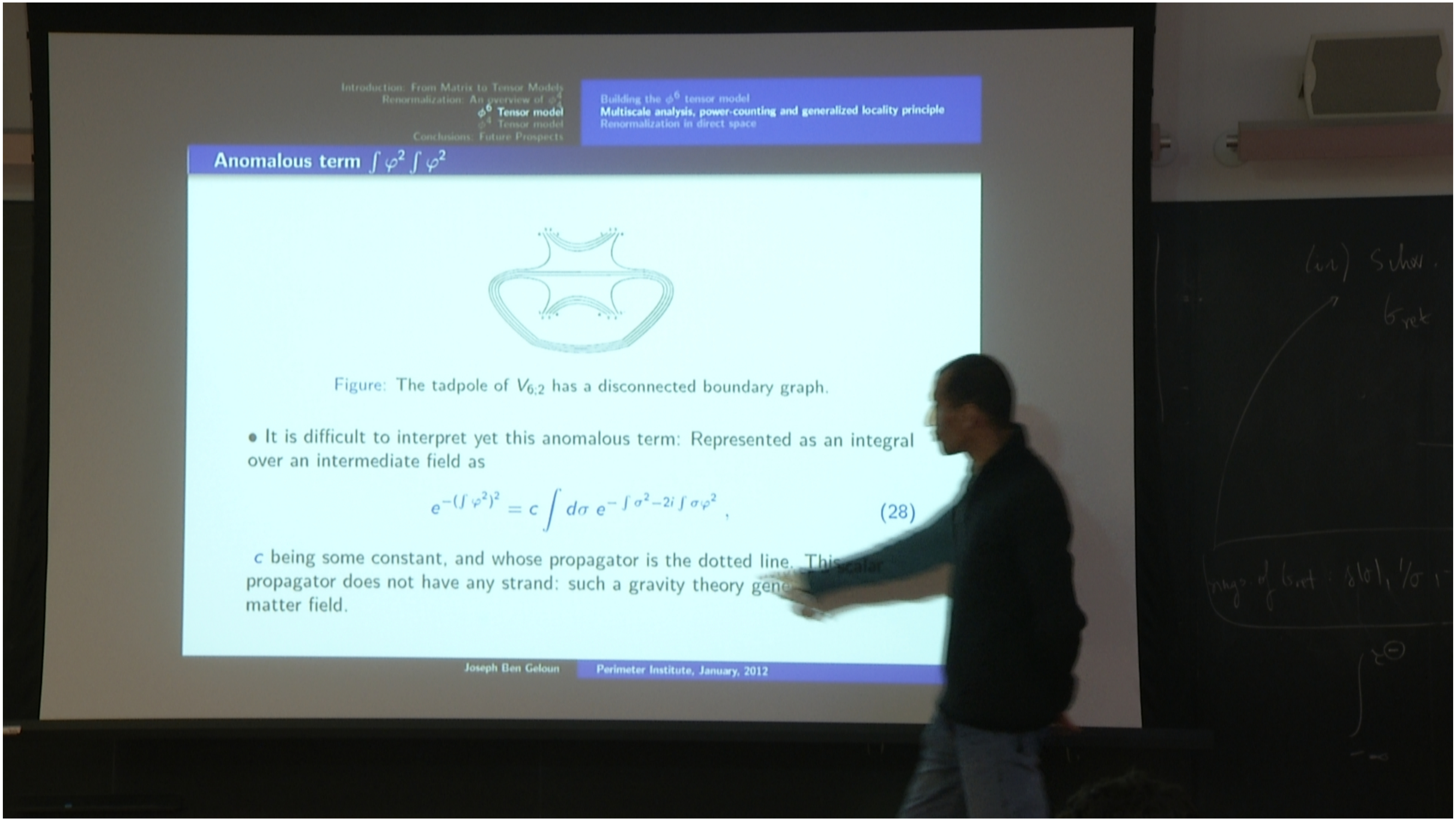
Perimeter Institute, January, 2012



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Anomalous term $\int \varphi^2 \int \varphi^2$
Figure: The tadpole of $V_{6;2}$ has a disconnected boundary graph.
• It is difficult to interpret yet this anomalous term: Represented as an integral over an intermediate field as
$$e^{-(\int \varphi^2)^2} = c \int d\sigma \; e^{-\int \sigma^2 - 2i \int \sigma \varphi^2}, \qquad (28)$$
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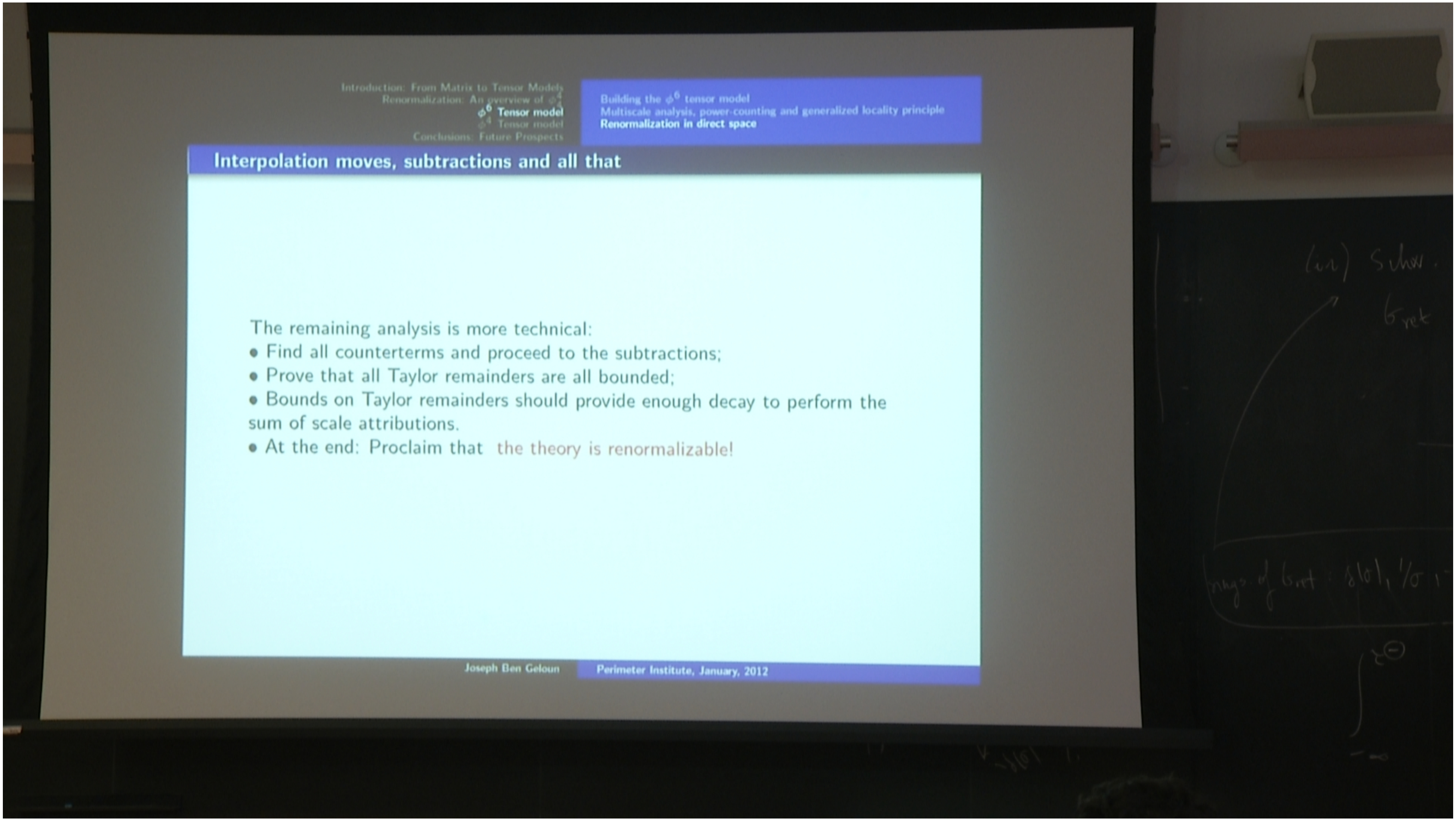
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The remaining analysis is more technical:
• Find all counterterms and proceed to the subtractions;
• Prove that all Taylor remainders are all bounded;
• Bounds on Taylor remainders should provide enough decay to perform the sum of scale attributions.
• At the end: Proclaim that the theory is renormalizable!
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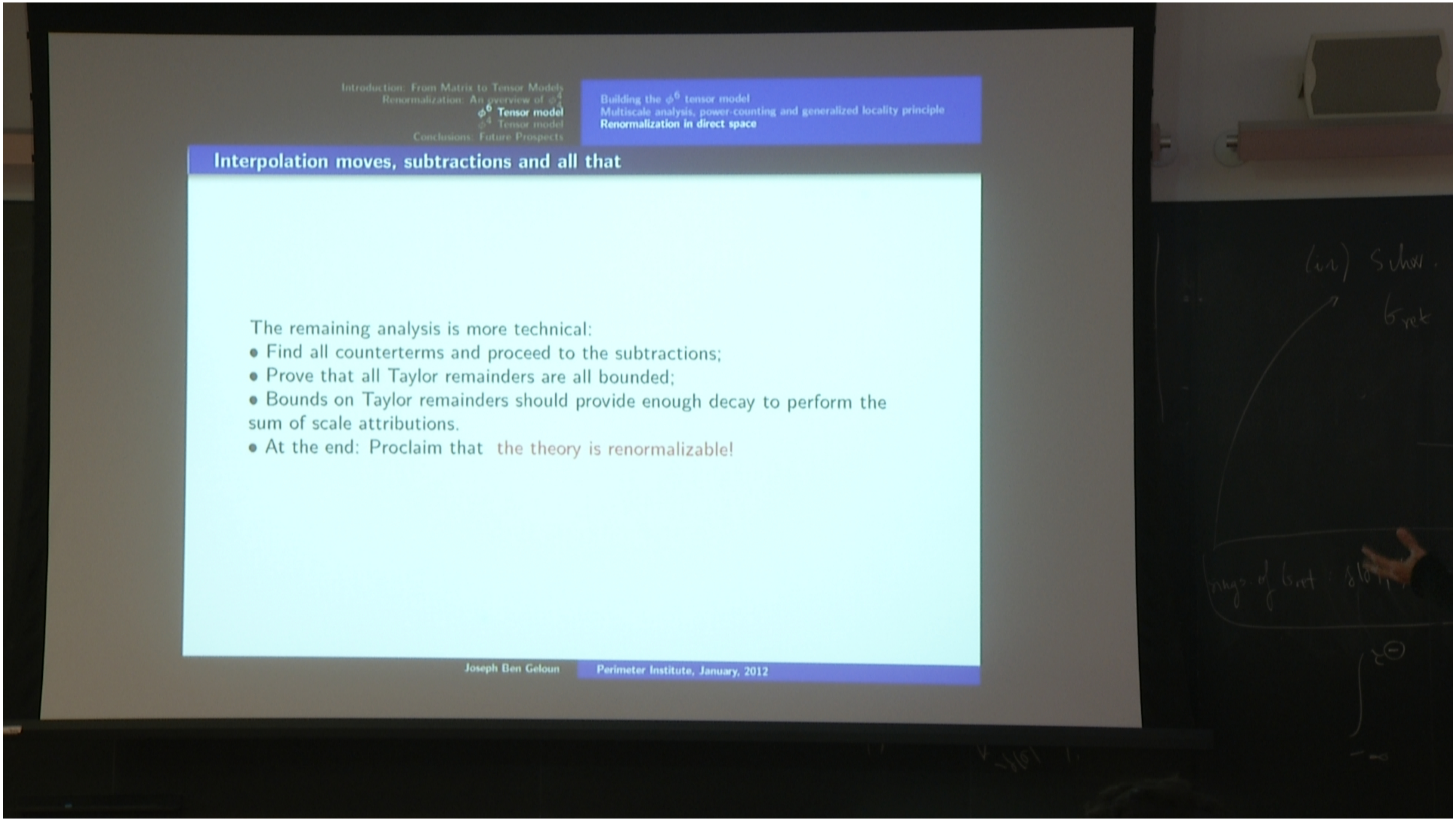
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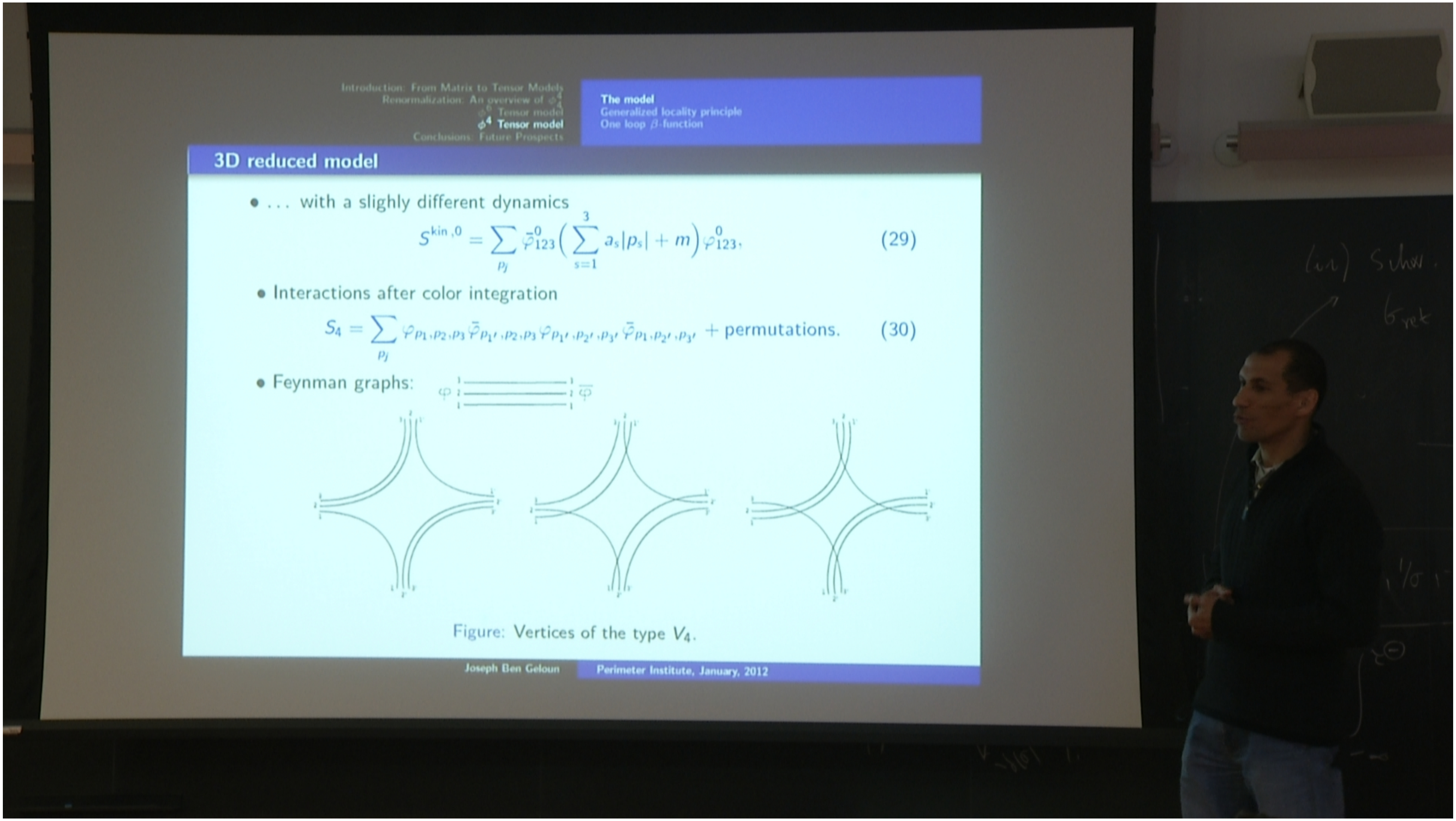

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One loop β-function
3D reduced model
• ... with a slighly different dynamics
$S^{\text{kin},0} = \sum_{p_j} \bar{\varphi}^0_{123} \Big(\sum_{s=1}^{3} a_s |p_s| + m \Big) \varphi^0_{123},$ (29)
• Interactions after color integration
$S_4 = \sum_{p_j} \varphi_{p_1,p_2,p_3} \bar{\varphi}_{p_{1'},p_2,p_3} \varphi_{p_{1'},p_{2'},p_{3'}} \bar{\varphi}_{p_1,p_{2'},p_{3'}}$ + permutations. (30)
• Feynman graphs:
φ $\bar{\varphi}$
Figure: Vertices of the type V_4.
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Theorem and List of divergent graphs

Theorem

The 3D tensor model described above is renormalizable at all orders and, by identifying $a_s = a$, the reduced model is asymptotically free in the UV direction.

Proof. Perform a similar multiscale analysis as previously done and get the divergence degree of connected graph $\mathcal{G}$:

$$\omega_d(\mathcal{G}) = -\frac{1}{2}(N_{\text{ext}} - 4) - \sum_{J} g_{\tilde{J}} + g_{\partial\mathcal{G}} - (C_{\partial\mathcal{G}} - 1). \qquad (31)$$

where $g_{\partial\mathcal{G}}$ is the genus of $\partial\mathcal{G}$ (reducing the dimension the former $\sum_{J_\partial} g_{J_\partial} \to g_{\partial\mathcal{G}}$). The list of primitvely divergent graph

N_{ext}	V_2	$g_{\partial\mathcal{G}}$	$C_{\partial\mathcal{G}} - 1$	$\sum_{\tilde{J}} g_{\tilde{J}}$	$\omega_d(\mathcal{G})$
4	0	0	0	0	0
2	0	0	0	0	1
2	1	0	0	0	0
2	0	0	0	1	0

Table 2

Theorem and List of divergent graphs

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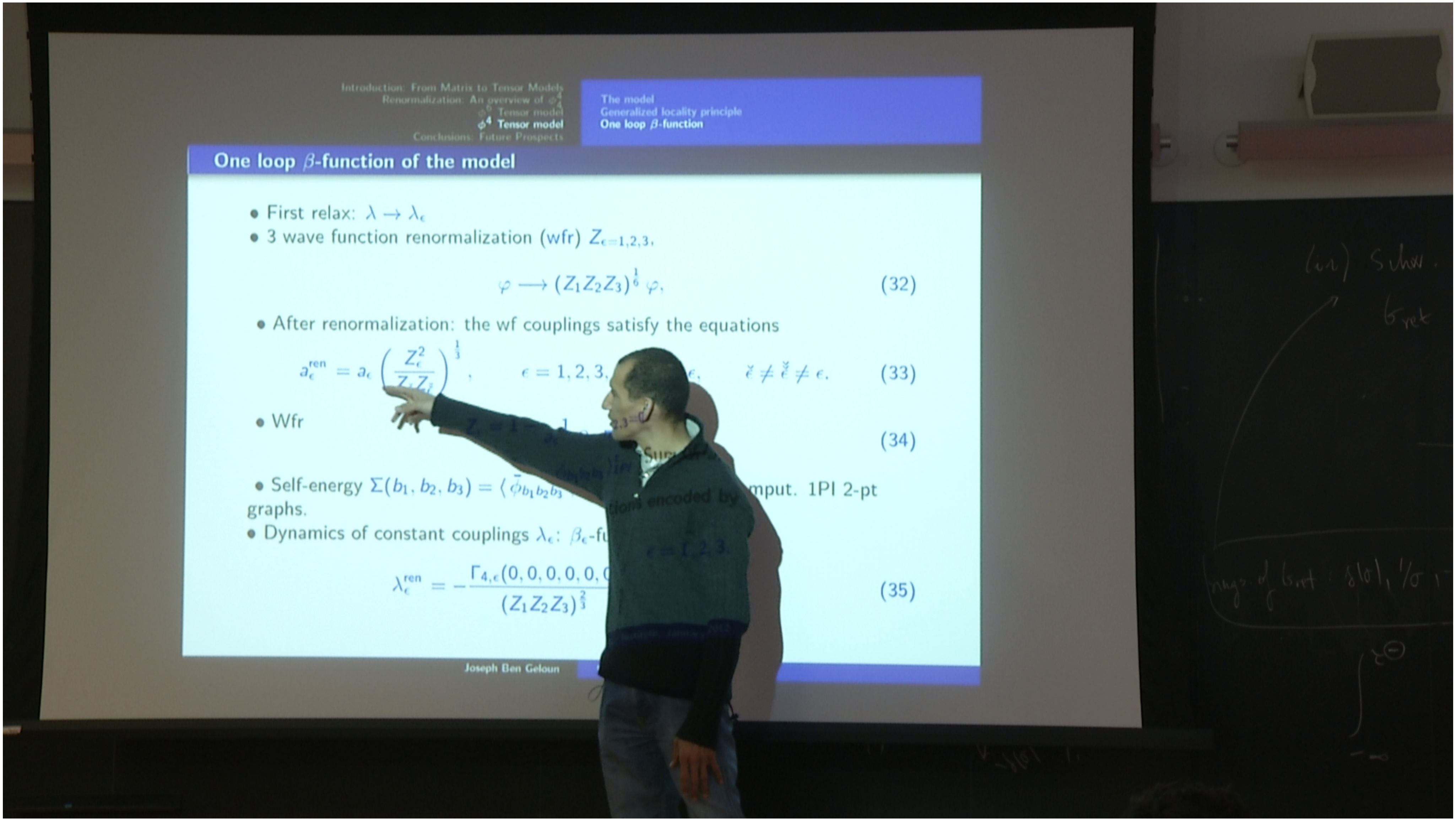
Table 2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The model
Generalized locality principle
One loop β-function
One loop β-function of the model
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• 3 wave function renormalization (wfr) Z_{ϵ=1,2,3},
φ ⟶ (Z₁Z₂Z₃)^{1/6} φ, (32)
• After renormalization: the wf couplings satisfy the equations
ϵ = 1, 2, 3, (33)
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(34)
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mput. 1PI 2-pt
graphs.
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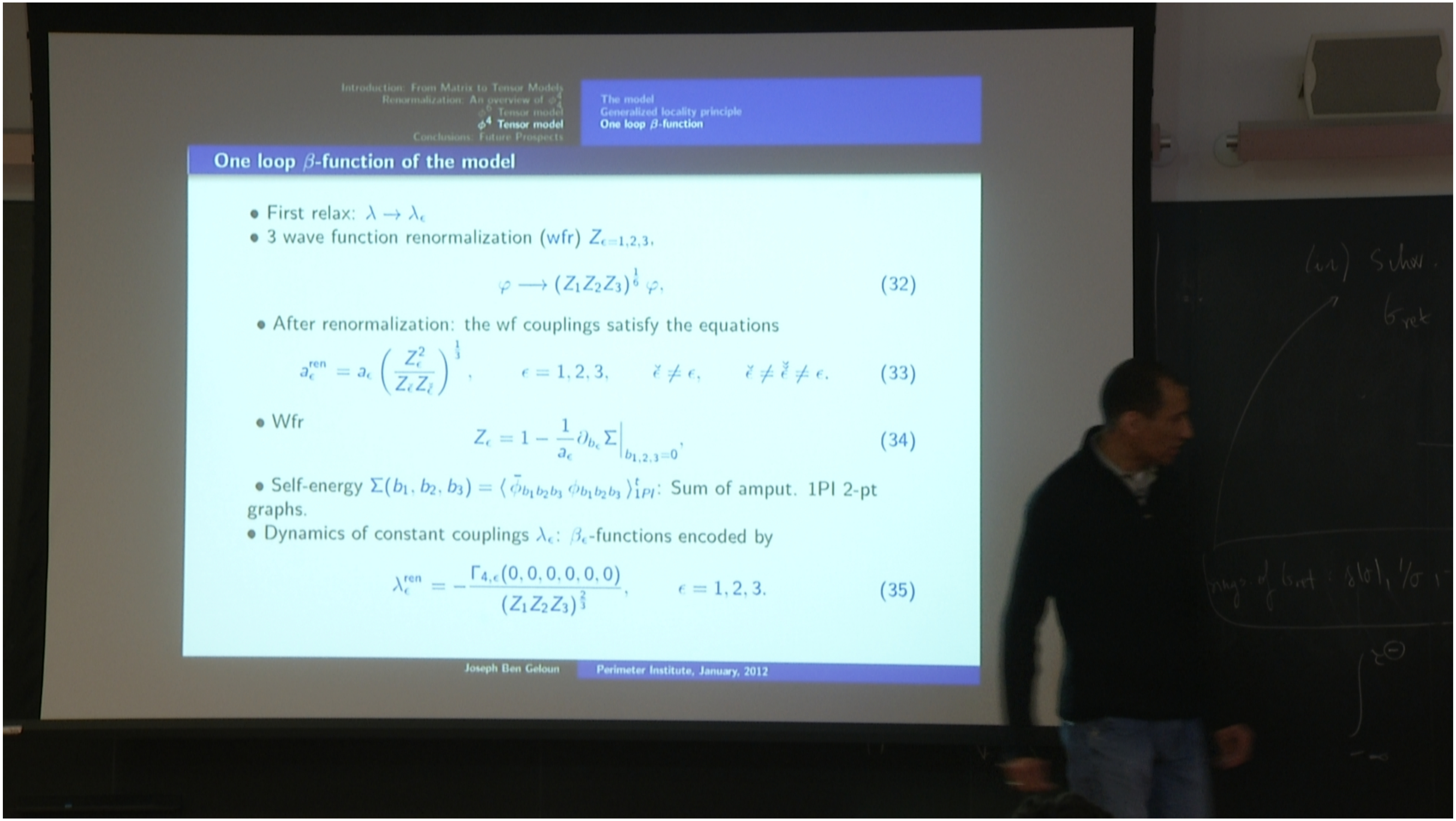
(35)
Joseph Ben Geloun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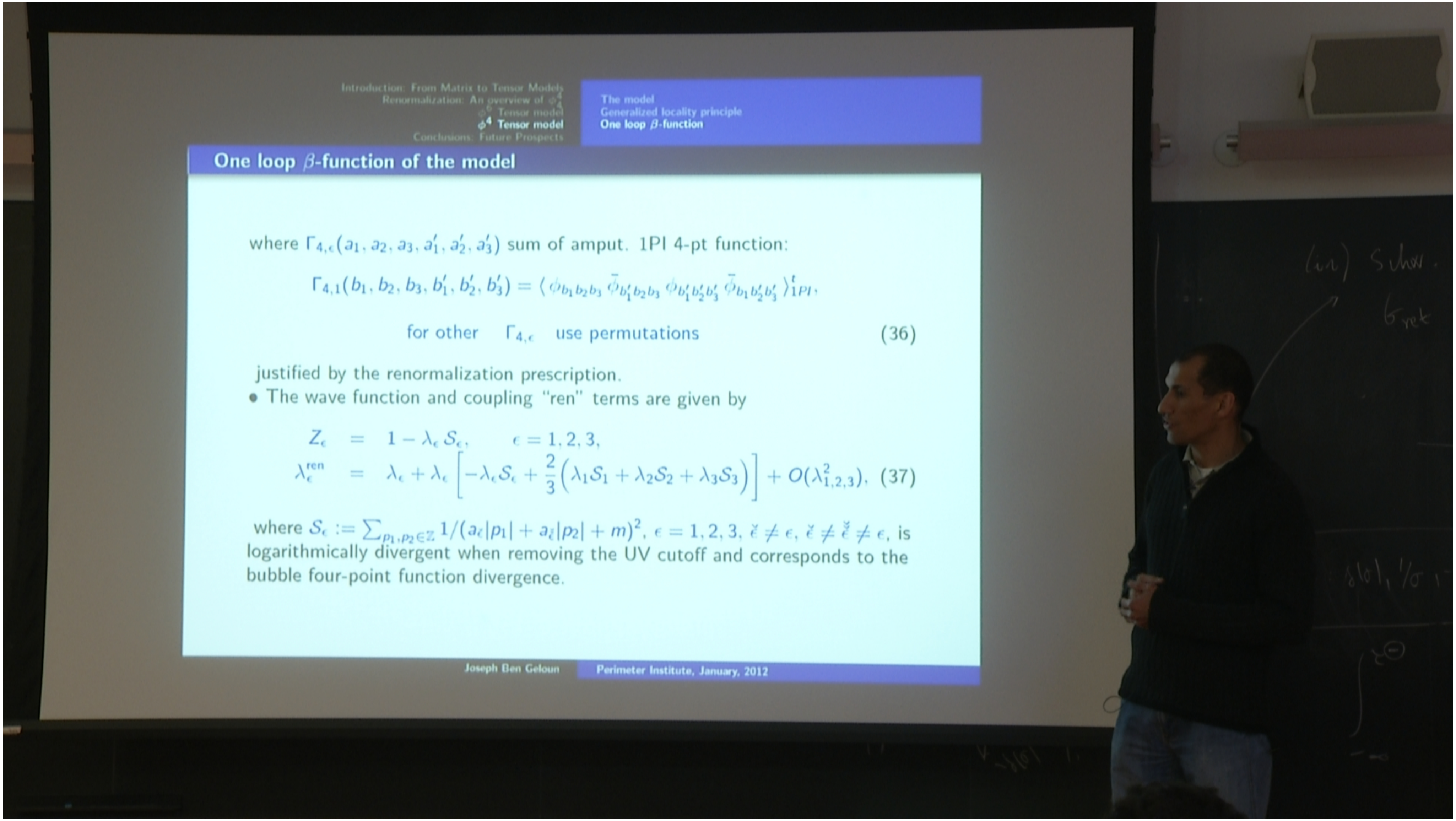
• First relax: $\lambda \to \lambda_\epsilon$
• 3 wave function renormalization (wfr) $Z_{\epsilon=1,2,3}$,
$\varphi \longrightarrow (Z_1 Z_2 Z_3)^{\frac{1}{6}} \varphi$, (32)
• After renormalization: the wf couplings satisfy the equations
$a^{\text{ren}}_\epsilon = a_\epsilon \left(\frac{Z_\epsilon^2}{Z_{\check\epsilon} Z_{\check{\check\epsilon}}} \right)^{\frac{1}{3}}, \quad \epsilon = 1,2,3, \quad \check\epsilon \neq \epsilon, \quad \check\epsilon \neq \check{\check\epsilon} \neq \epsilon.$ (33)
• Wfr
$Z_\epsilon = 1 - \frac{1}{a_\epsilon} \partial_{b_\epsilon} \Sigma \Big|_{b_{1,2,3}=0},$ (34)
• Self-energy $\Sigma(b_1, b_2, b_3) = \langle \bar\phi_{b_1 b_2 b_3} \phi_{b_1 b_2 b_3} \rangle^t_{1PI}$: Sum of amput. 1PI 2-pt graphs.
• Dynamics of constant couplings λ_ϵ: β_ϵ-functions encoded by
$\lambda^{\text{ren}}_\epsilon = -\frac{\Gamma_{4,\epsilon}(0,0,0,0,0,0)}{(Z_1 Z_2 Z_3)^{\frac{2}{3}}}, \quad \epsilon = 1,2,3.$ (35)
Joseph Ben Geloun
Perimeter Institute, January, 2012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where $\Gamma_{4,\epsilon}(a_1, a_2, a_3, a'_1, a'_2, a'_3)$ sum of amput. 1PI 4-pt function:
$\Gamma_{4,1}(b_1, b_2, b_3, b'_1, b'_2, b'_3) = \langle \phi_{b_1 b_2 b_3} \bar\phi_{b'_1 b_2 b_3} \phi_{b'_1 b'_2 b'_3} \bar\phi_{b_1 b'_2 b'_3} \rangle^t_{1PI}$,
for other $\Gamma_{4,\epsilon}$ use permutations (36)
justified by the renormalization prescription.
• The wave function and coupling "ren" terms are given by
$Z_\epsilon = 1 - \lambda_\epsilon \mathcal{S}_\epsilon, \quad \epsilon = 1, 2, 3,$
$\lambda^{\text{ren}}_\epsilon = \lambda_\epsilon + \lambda_\epsilon \Big[-\lambda_\epsilon \mathcal{S}_\epsilon + \frac{2}{3}\Big(\lambda_1 \mathcal{S}_1 + \lambda_2 \mathcal{S}_2 + \lambda_3 \mathcal{S}_3\Big)\Big] + O(\lambda^2_{1,2,3})$. (37)
where $\mathcal{S}_\epsilon := \sum_{p_1,p_2 \in \mathbb{Z}} 1/(a_{\check\epsilon}|p_1| + a_{\check{\check\epsilon}}|p_2| + m)^2$, $\epsilon = 1, 2, 3$, $\check\epsilon \neq \epsilon$, $\check\epsilon \neq \check{\check\epsilon} \neq \epsilon$, is logarithmically divergent when removing the UV cutoff and corresponds to the bubble four-point function divergence.
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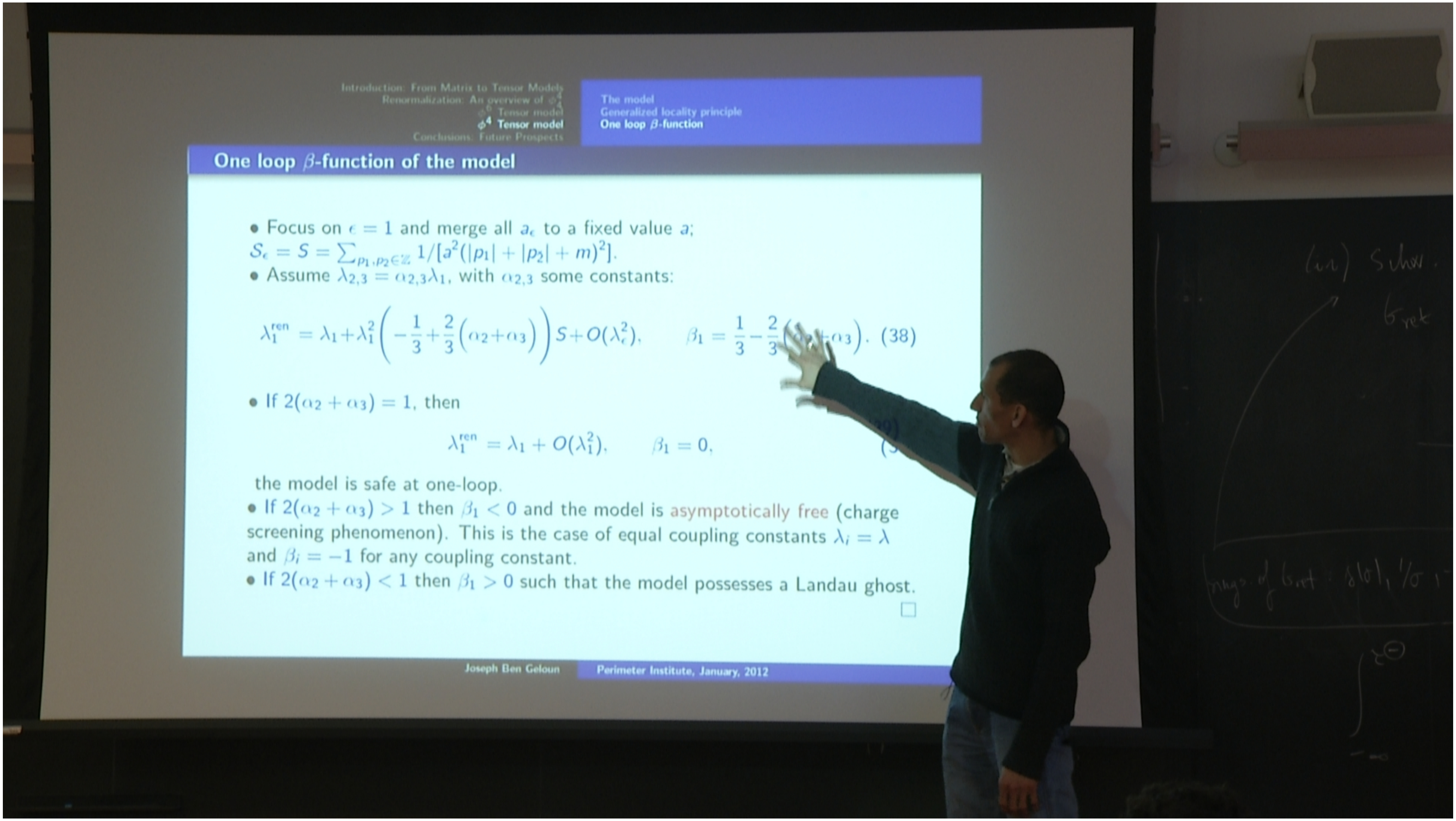
Perimeter Institute, January, 2012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• Focus on $\epsilon = 1$ and merge all a_ϵ to a fixed value a;
$\mathcal{S}_\epsilon = S = \sum_{p_1, p_2 \in \mathbb{Z}} 1/[a^2(|p_1| + |p_2| + m)^2]$.
• Assume $\lambda_{2,3} = \alpha_{2,3}\lambda_1$, with $\alpha_{2,3}$ some constants:
$\lambda_1^{\rm ren} = \lambda_1 + \lambda_1^2\left(-\frac{1}{3} + \frac{2}{3}(\alpha_2 + \alpha_3)\right)S + O(\lambda_\epsilon^2)$, $\beta_1 = \frac{1}{3} - \frac{2}{3}$ $\alpha_3)$. (38)
• If $2(\alpha_2 + \alpha_3) = 1$, then
$\lambda_1^{\rm ren} = \lambda_1 + O(\lambda_1^2)$, $\beta_1 = 0$,
the model is safe at one-loop.
• If $2(\alpha_2 + \alpha_3) > 1$ then $\beta_1 < 0$ and the model is asymptotically free (charge screening phenomenon). This is the case of equal coupling constants $\lambda_i = \lambda$ and $\beta_i = -1$ for any coupling constant.
• If $2(\alpha_2 + \alpha_3) < 1$ then $\beta_1 > 0$ such that the model possesses a Landau ghost.
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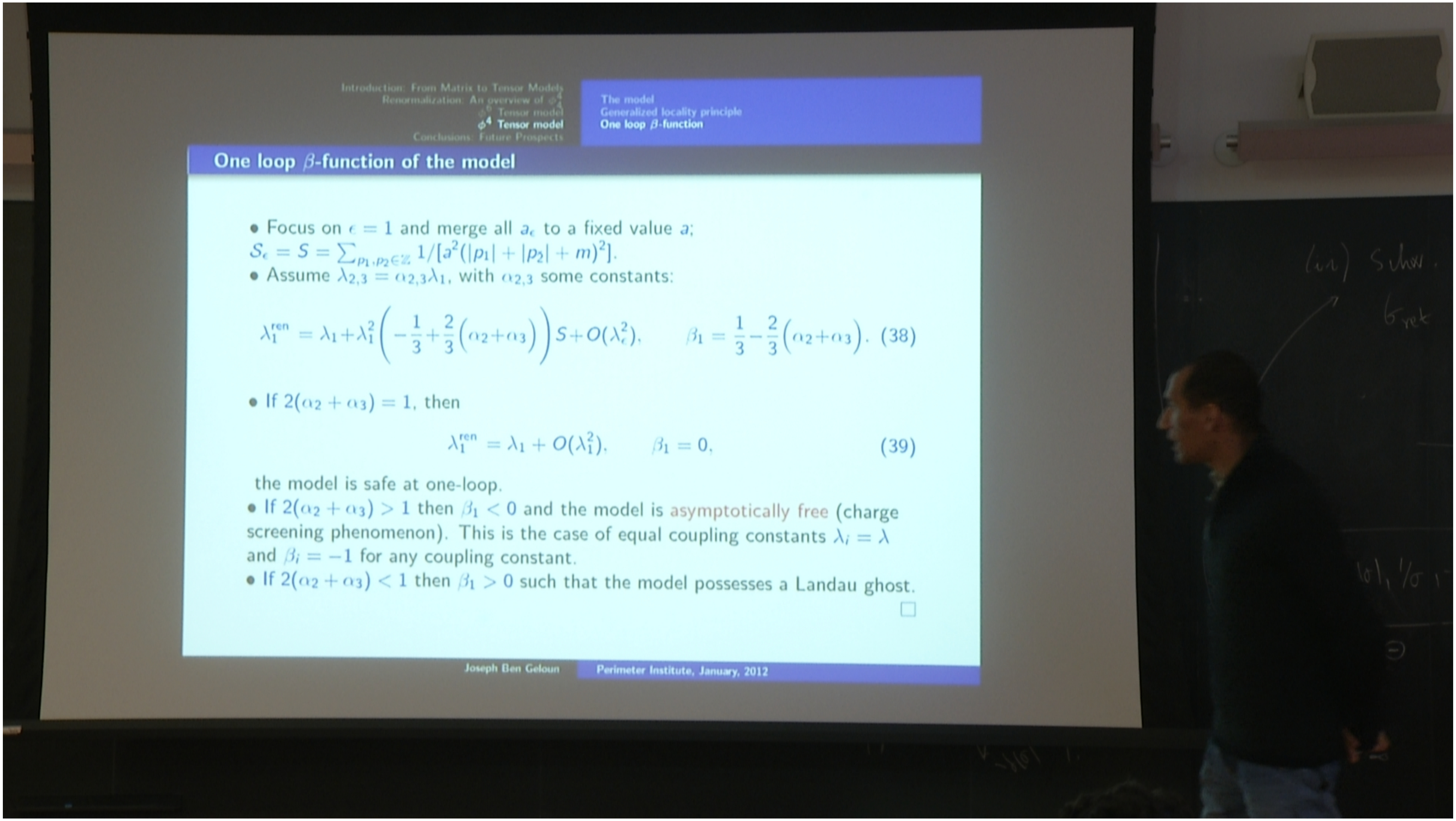
Perimeter Institute, January, 2012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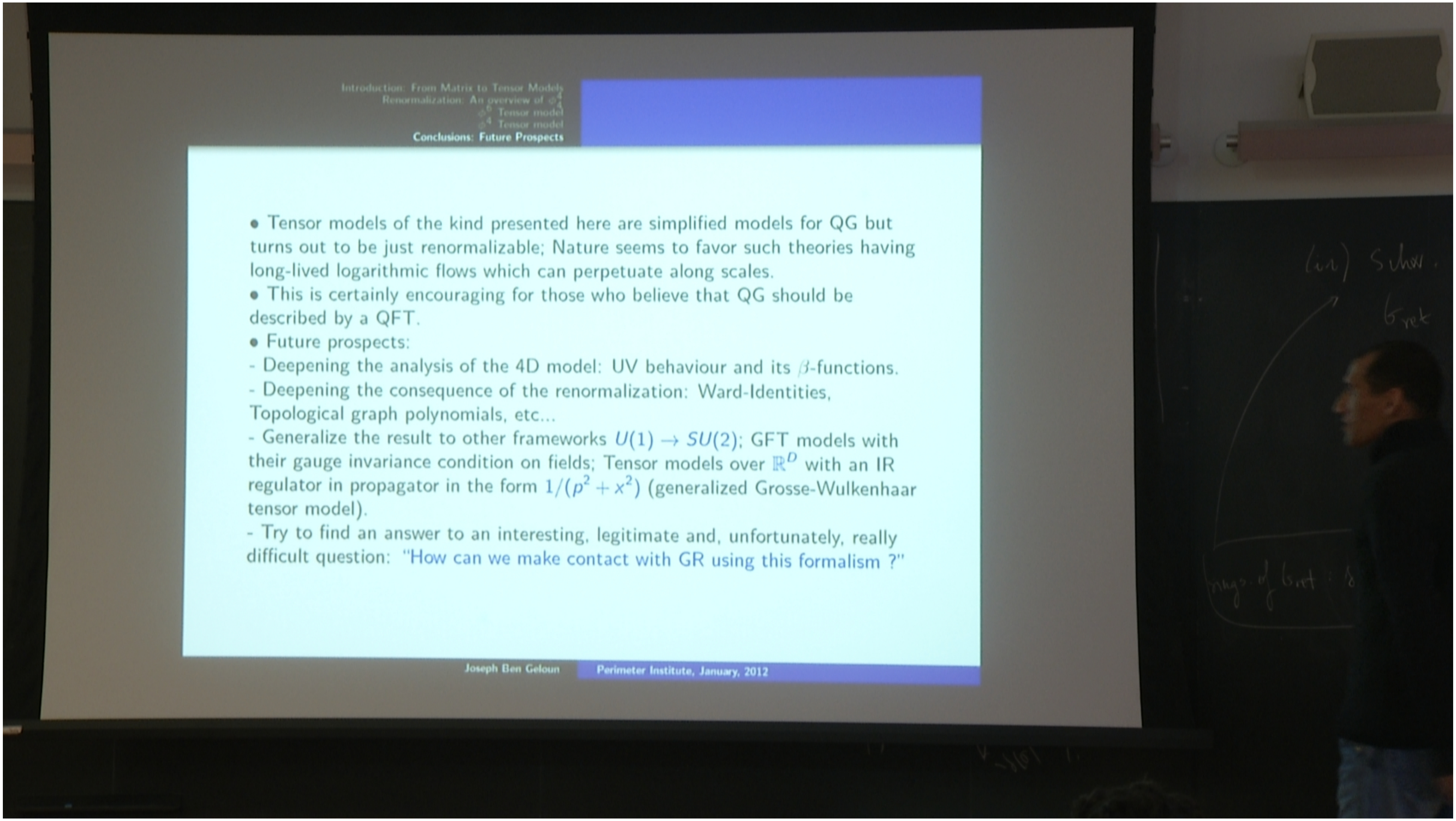
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• This is certainly encouraging for those who believe that QG should be described by a QFT.
• Future prospects:
- Deepening the analysis of the 4D model: UV behaviour and its β-functions.
- Deepening the consequence of the renormalization: Ward-Identities, Topological graph polynomials, etc...
- Generalize the result to other frameworks $U(1) \to SU(2)$; GFT models with their gauge invariance condition on fields; Tensor models over $\mathbb{R}^D$ with an IR regulator in propagator in the form $1/(p^2+x^2)$ (generalized Grosse-Wulkenhaar tensor model).
- Try to find an answer to an interesting, legitimate and, unfortunately, really difficult question: "How can we make contact with GR using this formalism ?"
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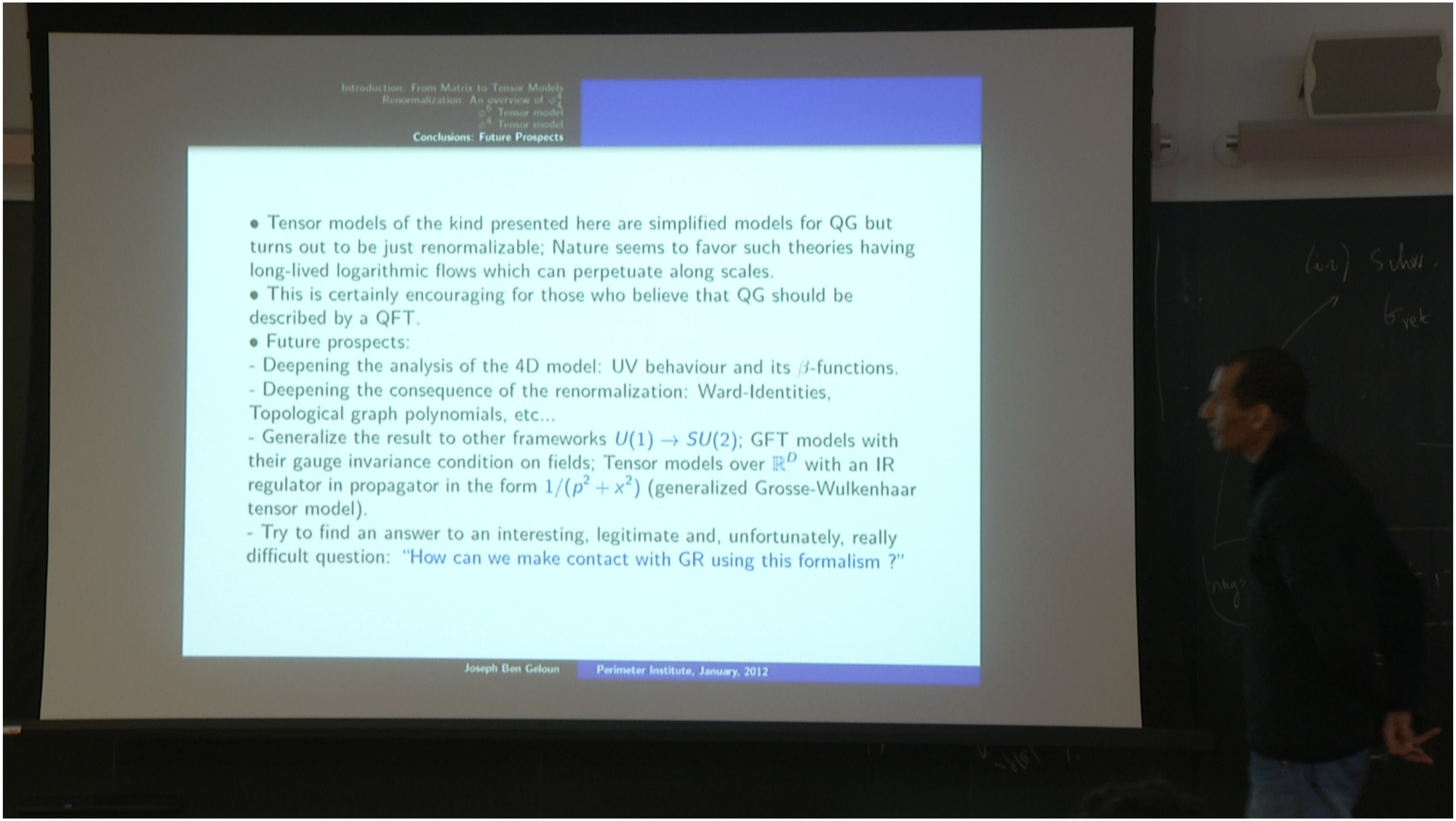
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