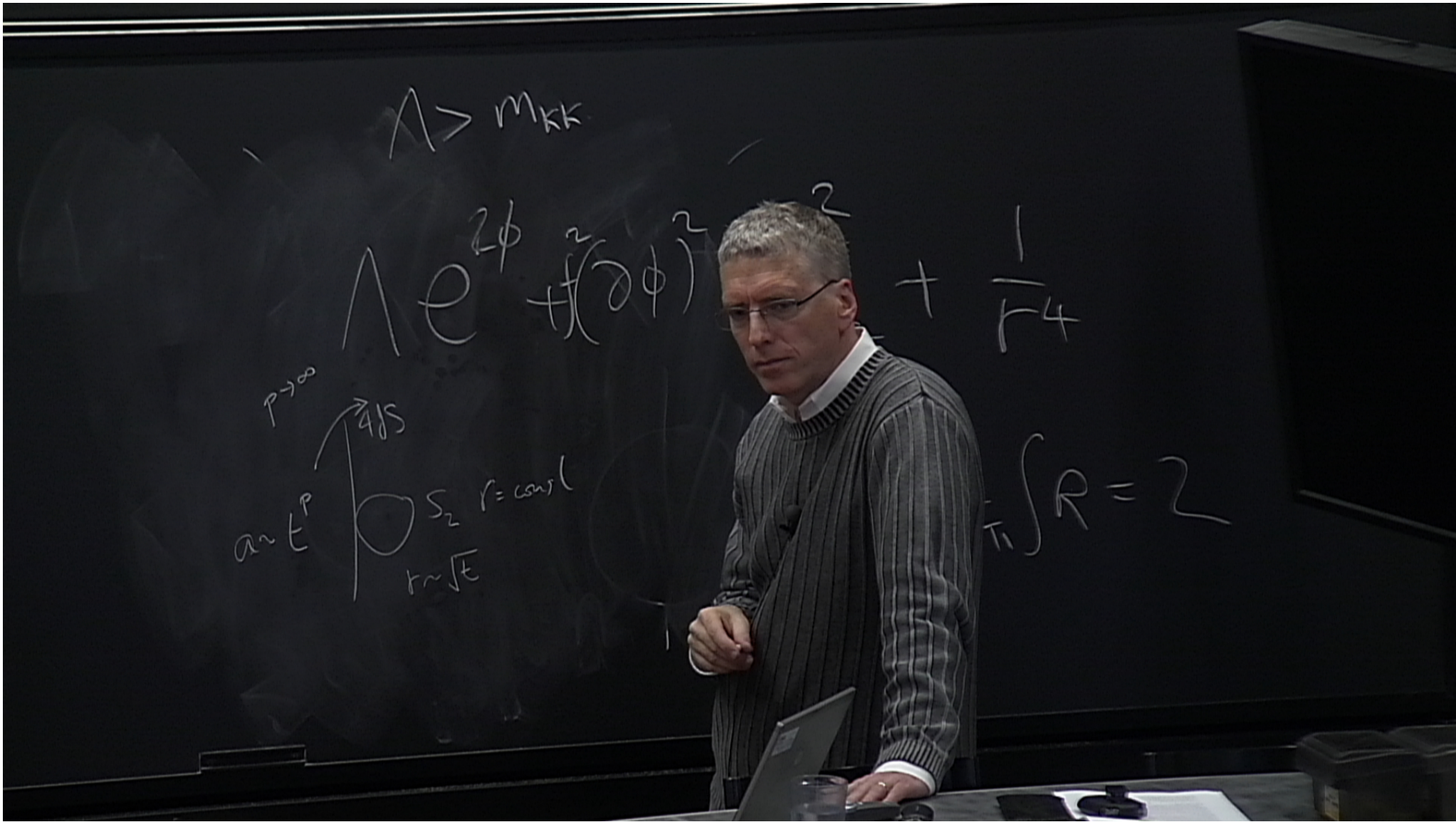


Title: Extra Dimensions, the Cosmological Constant Problem and the LHC

Date: Jan 10, 2012 12:45 PM

URL: <http://pirsa.org/12010130>

Abstract: Two uncertainties define the prevailing attitude toward the LHC: uncertainty about what new physics it may find (if any); together with dissatisfaction with the "technical naturalness" arguments which (when applied to the hierarchy problem) help suggest what it should be looking for. The dissatisfaction arises because of a wide-spread despair about finding a technically natural solution to the cosmological constant problem, despite much effort spent seeking it. In this talk I describe a mechanism within supersymmetric extra-dimensional theories that allows the low-energy effective cosmological constant naturally to be of order the Kaluza-Klein scale. If this is the solution to the cosmological constant problem, then it requires extra dimensions that are both very supersymmetric and large enough to be relevant to the LHC (with the - so far successful - prediction that no MSSM particles will be discovered there, despite the low-energy supersymmetry)





Extra Dimensions, the LHC & the Cosmological Constant Problem



Extra Dimensions, the LHC & the Cosmological Constant Problem

Why extra dimensions must be
large and supersymmetric



w Leo van Nierop
idea: hep-th/0304256, hep-ph/0404135
mechanism: 1012.2638; 1101.0152; 1108.0345
some implications: 1103.4556; 1108.2553

The message:

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)

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“...when you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

A. Conan Doyle

The message:

- The cosmological constant problem is telling us that there must be two micron-sized dimensions (plus possibly more smaller ones)
- These dimensions must be supersymmetric (but need *NOT* require the MSSM)
- *More generally: back-reaction for higher codimension objects is a very promising, but largely unexplored area*

Outline

- Hierarchy problems in nature
 - Cosmological constant: the dog that didn't bark

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- How extra dimensions can help
 - Why they must be big and supersymmetric
 - An explicit realization

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 - Cosmological constant: the dog that didn't bark
- How extra dimensions can help
 - Why they must be big and supersymmetric
 - An explicit realization
- Opportunities and concerns

Hierarchy problems

Hierarchy problems

- The
- Ideas for what lies beyond the Standard Model are largely driven by ‘technical naturalness’.
 - Motivated by belief SM is an effective field theory.

$$L_{SM} = m^2_0 H^* H + \textit{dimensionless}$$

- The

$$m^2 = m^2_0 + \textit{higher order} \sim (126 \text{ GeV})^2$$

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Hierarchy problems

- But the SM has another unnatural parameter
 - Even more unnatural than the EW hierarchy.

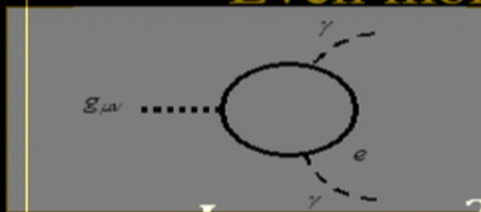
$$L_{SM} = \mu^2_0 + m^2_0 H^* H + \text{dimensionless}$$

$$\mu^2 = \mu^2_0 + \text{higher order} \sim (3 \times 10^{-3} \text{ eV})^4$$

Hierarchy problems

- But the SM has another unnatural parameter

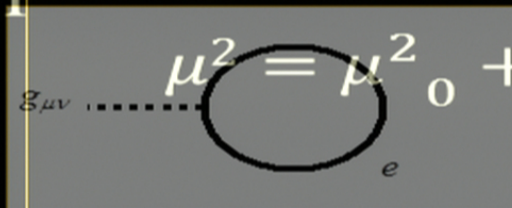
Why this?



$$L_{SM} = \mu^2_0 + m_0^2 \frac{H^\dagger H}{\Lambda^2} + \text{dimensionless}$$

How do you change properties of low-energy particles (like the electron) so that their zero-point energy does not gravitate, even though quantum effects do gravitate in atoms!

But not this?



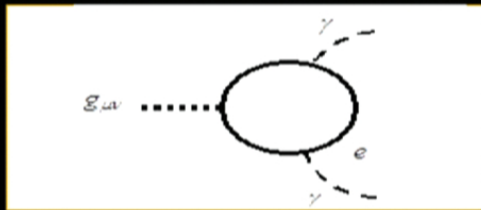
Must change only gravity and not any of their other well-tested properties.

higher order $\sim (3 \times 10^{-3} \text{ eV})^4$

Hierarchy problems

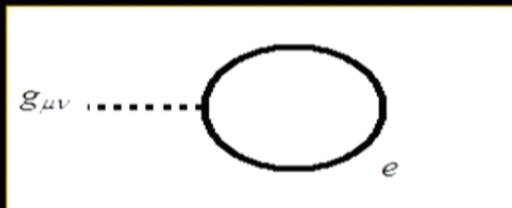
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How do you change properties of *low-energy* particles (like the electron) so that their zero-point energy does not gravitate, *even though quantum effects do gravitate in atoms!*

But not this?



Must change only gravity and not any of their other well-tested properties.

Hierarchy problems

But the SM has another unnatural parameter

- The
- Where does absence of a technically natural cc take us as a field?
 - Abandon naturalness as a criterion (and along with it motivations for supersymmetry, technicolour, etc...)?

- The



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*Extra dimensions
can help*

Helpful extra dimensions

- General arguments
- An explicit realization

Helpful extra dimensions

- Ge
- The Problem:
 - Einstein's equations make a lorentz-invariant vacuum energy (*which is generically large*) an obstruction to a close-to-flat spacetime (*which we see around us*)
- A

$$T_{\mu\nu} = \lambda g_{\mu\nu}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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Helpful extra dimensions

*Arkani-Hamed et al
Kachru et al
Carroll & Guica
Aghababaie et al*

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a cl

*But this need not be true if there are
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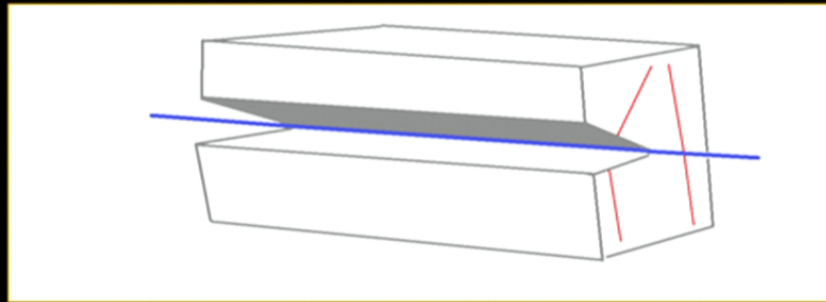
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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Helpful extra dimensions

Vilenkin et al

- Why not?
 - Need not be lorentz invariant in the extra dimensions
 - Vacuum energy might curve extra dimensions, rather than the ones we see (eg gravity field of a cosmic string)



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Helpful extra dimensions

*Carroll & Guica
Aghababaie et al*

- A higher-dimensional analog:
 - Similar (*classical*) examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*



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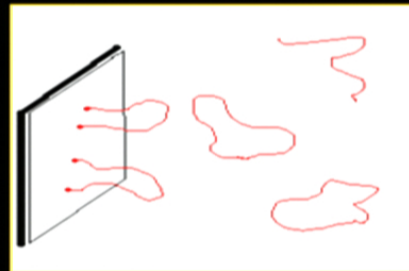
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Helpful extra dimensions

Rubakov & Shaposhnikov
Polchinski

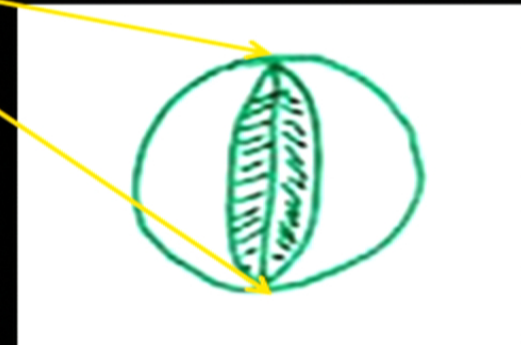
- Particles can be localized on surfaces (branes, or defects) within the extra dimensions

Gravity is not similarly localized



Analogy:

Examples also with a 4D brane in e.g. the rugby ball and related



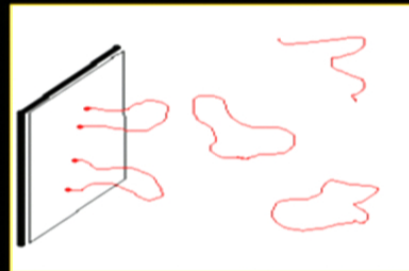
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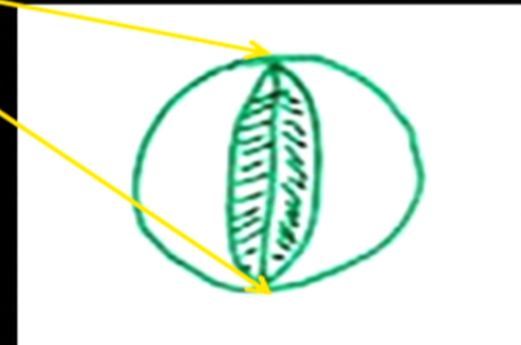
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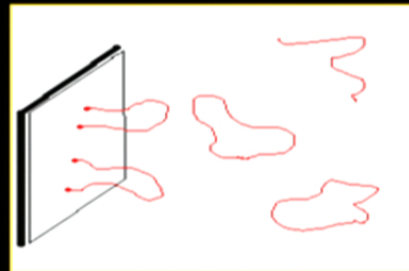


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Helpful extra dimensions

- Particles can be localized on surfaces (branes, or defects) within the extra dimensions

Gravity is not similarly localized



Notice: *this framework manages to modify how things gravitate without strongly modifying other interactions*



Helpful extra dimensions

Chen, Luty & Ponton

- A higher-dimensional analog:
 - Similar (*classical*) examples also with a 4D brane in two extra dimensions: *e.g. the rugby ball and related solutions*

$$R = -2\kappa^2 \sum T_i \delta^2(x_i)$$

$$\begin{aligned} 4\text{D cc} &= \sum T_i + \frac{1}{2\kappa^2} \int d^2x R \\ &= 0 \text{ for all } T_i \end{aligned}$$



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Helpful extra dimensions

Adelberger et al

Remarkably: *this is possible if they are smaller than $45 \mu\text{m}$ and particles stuck on branes*

- A higher-dimensional analog of gravity
 - Similar (*classical*) examples require two extra dimensions: e.g., Kaluza-Klein theory
- Requires:
 - Radius as large as microns



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Helpful extra dimensions

Arkani-Hamed et al

Remarkably: *consistent with EW hierarchy if precisely two dimensions this large since $M_p = M_g^2 r$*



- A higher-dimensional analog
 - Similar (*classical*) examples with two extra dimensions: e.g.
- Requires:
 - Radius as large as microns
 - At most two dimensions

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Helpful extra dimensions

*Golberger & Wise
CB, de Rham,
van Nierop, Tasinato*

- A higher-dimensional action
 - Similar (*classical*) examples with two extra dimensions: e.g.
- *Requires:*
 - *Radius as large as microns*
 - *At most two dimensions*
 - *Back-reaction of the branes*

Otherwise bulk cannot respond to branes.
Technical difficulty: bulk fields diverge at brane positions



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Helpful extra dimensions

- General arguments

- An explicit realization

Helpful extra dimensions

- Must re-ask the cosmological constant problem:
 - Some choices for the branes make the resulting on-brane geometry flat (classically), but other known choices do not: must identify the 'flat' choices.
 - Once flat choices are made in UV, *do they stay made* at the quantum level as successive scales are integrated out?
- G
- A

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Helpful extra dimensions

Nishino, Sezgin

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2} [R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

Two cases (both with flat directions):

6D sugra: choose $a = 1$ and $V = \frac{2g_R^2}{\kappa^2} e^\phi$

6D axion with SUSY: $a = 0$ and $V = \lambda$

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Helpful extra dimensions

Aghababaie et al

- Exact classical result (for SUSY case): *if*

$$ds^2 = e^{2W} \hat{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{2B} d\theta^2$$

- *then*

$$\hat{R} = \frac{1}{\kappa^2} \int d^2x \nabla^2 \phi$$

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Helpful extra dimensions

*Aghababaie et al
Gibbons, Guven & Pope*

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$$ds^2 = e^{2W} \hat{g}_{\mu\nu} dx^\mu dx^\nu$$

then

$$\hat{R} = \frac{1}{\kappa^2} \int d^2x \nabla^2 \phi$$

In particular,

$$\hat{R} = 0 \text{ if } n \cdot \nabla \phi = 0$$

*at the brane positions
(All such solutions
are explicitly known)*

Helpful extra dimensions

*Carroll & Guica
Aghababaie et al*

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L}\right) d\theta^2] e^{-\alpha\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin \left(\frac{r}{L}\right) e^{-\alpha\phi_0} \quad \phi = \phi_0$$



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Magnetic flux required
to stabilize extra
dimensions against
gravitational collapse

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Helpful extra dimensions

Carroll & Guica
Aghababaie et al

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Labels flat direction
(which exists due to
shift symmetry or scale
invariance)

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Helpful extra dimensions

Carroll & Guica
Aghababaie et al

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For later: notice radius is exponential in the flat direction ϕ_0 in the SUSY case

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Helpful extra dimensions

- Simple solution (including back-reaction)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L}\right) d\theta^2] e^{-\alpha\phi_0}$$

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$$1 - \alpha = \frac{\kappa^2 T}{2\pi}$$

Helpful extra dimensions

Carroll & Guica

- Simple solution (non-SUSY case)

$$ds^2 = \hat{g}_{mn} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left(\frac{r}{L} \right) d\theta^2$$

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Field equations

$$\frac{2}{L^2} = \kappa^2 \left(\frac{3Q^2}{2} + \Lambda \right)$$
$$\hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q$$

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$$Q = \frac{n}{2\alpha g L^2} \quad \hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

$$\frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2 \kappa^2} \left[1 \mp \sqrt{1 - \left(\frac{3n^2 \kappa^4 \Lambda}{8\alpha^2 g^2} \right)} \right]$$

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Helpful extra dimensions

- Simple solution (non-SUSY case)

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$$\text{Tune } \Lambda = \frac{Q^2}{2} \quad \text{so } \hat{R} = 0$$

$$\text{If } T \rightarrow T + \delta T \text{ then } \hat{R} \rightarrow - \frac{\kappa^2 \rho}{\pi \alpha L^2} \quad \text{where } \rho = 2 \delta T$$

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Aghababaie et al

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Helpful extra dimensions

Salam & Sezgin

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m dx^n$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right)$$



On-source geometry is always flat.
 Noticed in mid-80s in special case where $n = \alpha = 1$, in which case:

$$L = \sqrt{g} [R + e^{-\phi} F^2 + e^{\phi}]$$

with $R = -1/r^2$ and $F = 1/r^2$

gives $L = r^2 e^{-\phi} [e^{\phi} - 1/r^2]^2$

$$\frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2}$$

$$\kappa^2 Q^2 L^2 = 1$$

$$\hat{R} = 0$$

$$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

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Helpful extra dimensions

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$ Obstructs T to δT

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Helpful extra dimensions

- In SUSY case, how does system respond to changes in brane tension?

Flux quantization: $\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$ Obstructs T to δT

- On other hand, general argument:

$$\rho = \int dV L_{bulk} = -\frac{1}{2\kappa^2} \int dV \partial^2 \phi = \oint dS n \cdot \partial \phi \propto \frac{\partial T}{\partial \phi}$$

Helpful extra dimensions

CB & van Nierop

- Resolution: subdominant effects in the brane action are important for flux quantization

$$\text{if } L_b = T_b(\phi) + \Phi_b(\phi) *F + \dots$$

$$\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b e^\phi$$

- New function Φ has interpretation as brane-localized flux

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Helpful extra dimensions

- Energetics of perturbations: *explore the ansatz*

$$ds^2 = e^{2W} \hat{g}_{mn} dx^m dx^n + dr^2 + e^{2B} d\theta^2$$

$$F_{r\theta} = Q e^{B-4W} \quad \phi = \phi(r)$$

Helpful extra dimensions

- Perturb brane properties

$$T \rightarrow T + \delta T(\phi)$$

- To evade time-dependence add current

$$\Delta L_{bulk} = J\phi \quad \text{or} \quad \Delta L_{bulk} = J$$

- Find general solution to linearized equations

$$\kappa^2 J L^2 \ll 1$$

Helpful extra dimensions

- Sample solutions

$$\delta W = W_0 + W_1 \cos\left(\frac{r}{L}\right)$$

$$\delta\phi = \phi_0 + \phi_1 \ln\left(\frac{1 - \cos(r/L)}{\sin(r/L)}\right) - \kappa^2 J L^2 \ln\left[\sin\left(\frac{r}{L}\right)\right]$$

and so on

Helpful extra dimensions

*CB, Hoover & Tasinato
Bayntun, CB, van Nierop*

- Brane-bulk boundary conditions:

$$(e^B \phi')_b = \frac{\kappa^2}{2\pi} \left(\frac{\partial L_b}{\partial \phi} \right)$$

$$(e^B W')_b = \frac{\kappa^2}{4\pi} \left(\frac{\partial L_b}{\partial g_{\theta\theta}} \right) = U_b$$

$$(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi} \left[\left(\frac{\partial L_b}{\partial \phi} + \frac{3}{2} \frac{\partial L_b}{\partial g_{\theta\theta}} \right) \right]$$

$$\text{Constraint: } 4U_b [2 - 2L_b - 3U_b] - \left(\frac{\partial L_b}{\partial \phi} \right)^2 = 0$$

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- Non-SUSY result:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[\frac{\pi\alpha L^2 \hat{R}(\phi)}{\kappa^2} \right]$$

$$\left[\frac{\partial}{\partial \phi} \sum_b \delta T_b - Q \delta \Phi_b \right]_{\phi_*} = 0$$

$$\rho = \left[\sum_b \delta T_b - 2Q \delta \Phi_b \right]_{\phi_*}$$

This is δL_b
while
this is *not*

Helpful extra dimensions

- SUSY result:

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if δT independent of ϕ and $\delta\Phi = C e^{-\phi}$ then $V(\phi) = A e^{2\phi}$

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Case 2: exponentially large volume:

$\delta T_b = A + B (\phi + \nu)^2$ with $\nu \sim 50$ then $r = L e^{-\phi/2} \gg L$

Helpful extra dimensions

- Three intriguing choices:

Case 3: parametrically small vacuum energy:

If brane action completely independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

$$\frac{n}{g} = \int F + \frac{1}{2\pi} \sum_b \Phi_b e^\phi$$

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As required by Weinberg's no-go theorem

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Helpful extra dimensions

- Upshot: interplay between bulk and brane lifts flat direction and stabilizes size of extra dimensions (*similar to Golberger-Wise mechanism in 5 dimensions*)
 - The extra dimensions are exponential in the dilaton
- G
- A

Helpful extra dimensions

- **What about loops?**
 - **Pure brane loops have no effect on curvature because they cannot generate a dilaton coupling to the brane**
- **Ge**
- **Ar**

Helpful extra dimensions

- **What about loops?**
 - Pure brane loops have no effect on curvature because they cannot generate a dilaton coupling to the brane
 - Each bulk loop comes with a factor of $e^{2\phi}$ (since this is the loop-counting parameter), but flux stabilization relates this to the radius by $e^{2\phi} = 1/r^4$ making the cc equal the KK scale.
- G
- A

Helpful extra dimensions

- Short-wavelength loops in the bulk (*eg* particle of mass M) generate local terms in both the bulk effective action

$$L_B + \delta L_B = \left[\frac{2g_R^2}{\kappa^2} e^\phi + a_1 M^6 e^{3\phi} + \dots \right] \\ + \left[\frac{1}{2\kappa^2} + b_1 M^4 e^{2\phi} + \dots \right] R \\ + [c_1 M^2 e^\phi + \dots] R^2 + \dots$$

and source actions

$$L_b + \delta L_b = T_0 + t_1 M^4 e^{2\phi} + \dots$$

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Bulk contributions from short-wavelength modes *cancel* once summed over 6D supermultiplets.

Brane terms *do not* cancel

$$\left[\frac{2g_R^2}{\kappa^2} e^\phi + a_1 M^6 e^{3\phi} + \dots \right] + \left[\frac{1}{2\kappa^2} + b_1 M^4 e^{2\phi} + \dots \right] R + \left[c_1 M^2 e^\phi + \dots \right] R^2 + \dots$$

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Helpful extra dimensions

Short-
both t

This generates the following potential as a function of the zero mode, $e^\phi = 1/r^2$

$$V(r) = A_{-1}M^6r^2 + A_0M^4 + \frac{A_1M^2}{r^2} + \frac{A_2}{r^4} + \dots$$

with

$$A_{-1} \cong a_1 e^{3\phi} \cong \frac{a_1}{(Mr)^6},$$

$$A_0 \cong b_1 e^{2\phi} \cong \frac{b_1}{(Mr)^4},$$

$$A_1 \cong c_1 e^\phi \cong \frac{c_1}{(Mr)^2}$$

and so on

and so

$$V(r) \cong \frac{k}{r^4} + \dots$$

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Opportunities & Concerns

- Observational opportunities

Opportunities

- Where is the catch? *and concerns*

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Opportunities & Concerns

- Observational opportunities

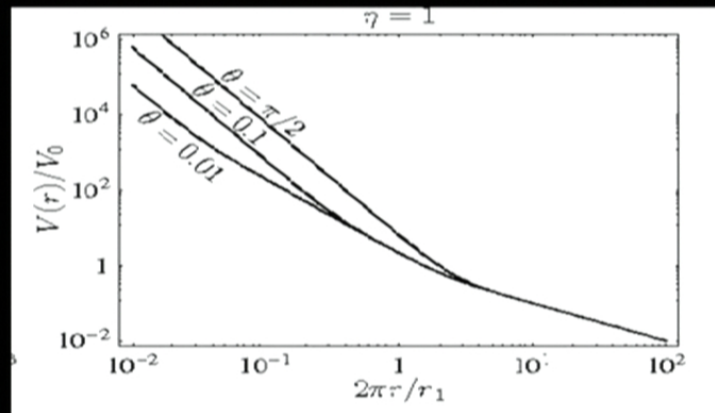
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Opportunities & Concerns

Callin et al

- If true, many striking implications:
 - Deviations from Newton's inverse square law at distances of order 1 – 10 microns



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Opportunities & Concerns

*Hannestad & Raffelt
CB, Matias & Quevedo*

- **O**
- **W**
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Lust et al

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Opportunities & Concerns

Lust et al

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CB, Matias & Quevedo

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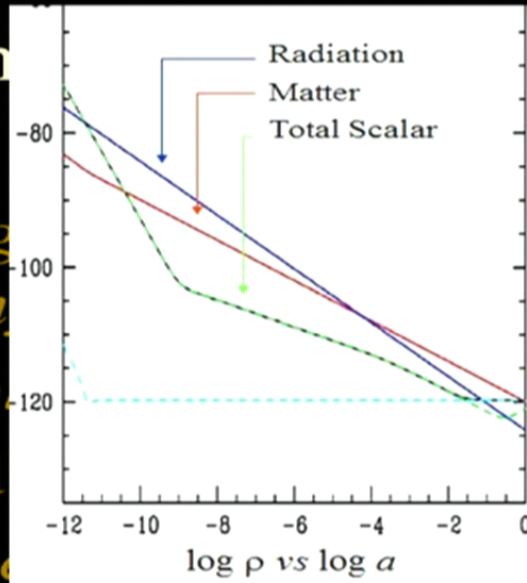
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Opportunities & Concerns

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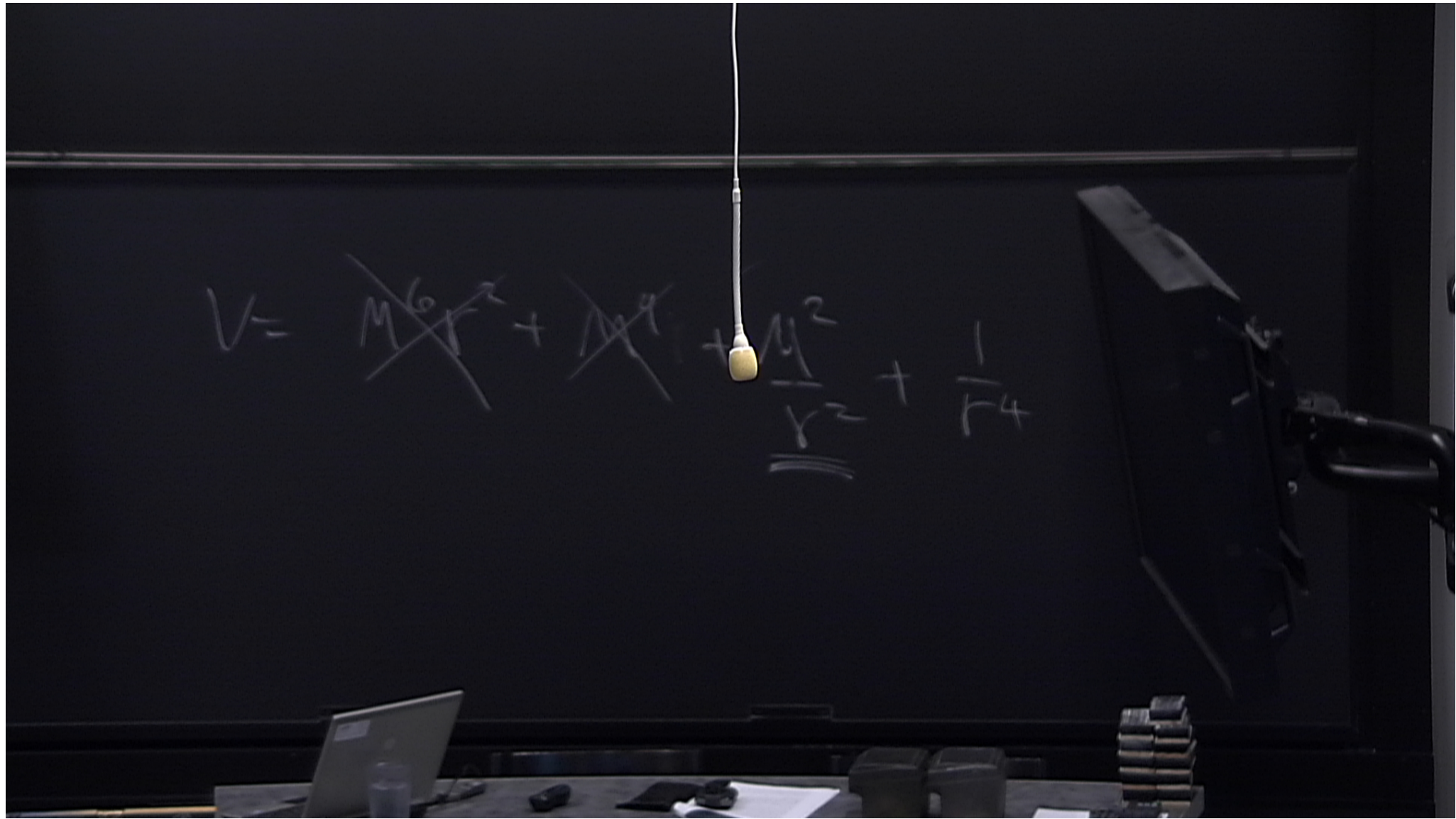
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Implications:

- inverse square law
- HC and in
- $M_g > 10 \text{ TeV}$
- Higgs
- (QG) at the LHC
- out the MSSM

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Opportunities & Concerns

CB & Matias

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 - P
 - E
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 - *Low energy SUSY without the MSSM*
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 - *Sterile neutrinos from the bulk?*
- W

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Opportunities & Concerns

S Weinberg

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- W
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Opportunities & Concerns

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- If you claim to solve the cosmological constant problem, aren't you crazy?
 - Weinberg's no-go theorem?
 - Didn't we see this all before in 5D?
 - What about Nima's argument against x dims
 - What stops proton decay?
 - How is inflation possible?
 - Long range scalars are unnatural/ruled out?
 - Don't constraints already force $(1/r)^4 > cc$?

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Summary

- Brane backreaction is largely unexplored with more than one transverse dimension:
 - Many cool features in 4 dimension (RS models)
 - Requires renormalizing singularities at sources

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Potentially wide-ranging observational implications for Dark Energy cosmology, the LHC and elsewhere...

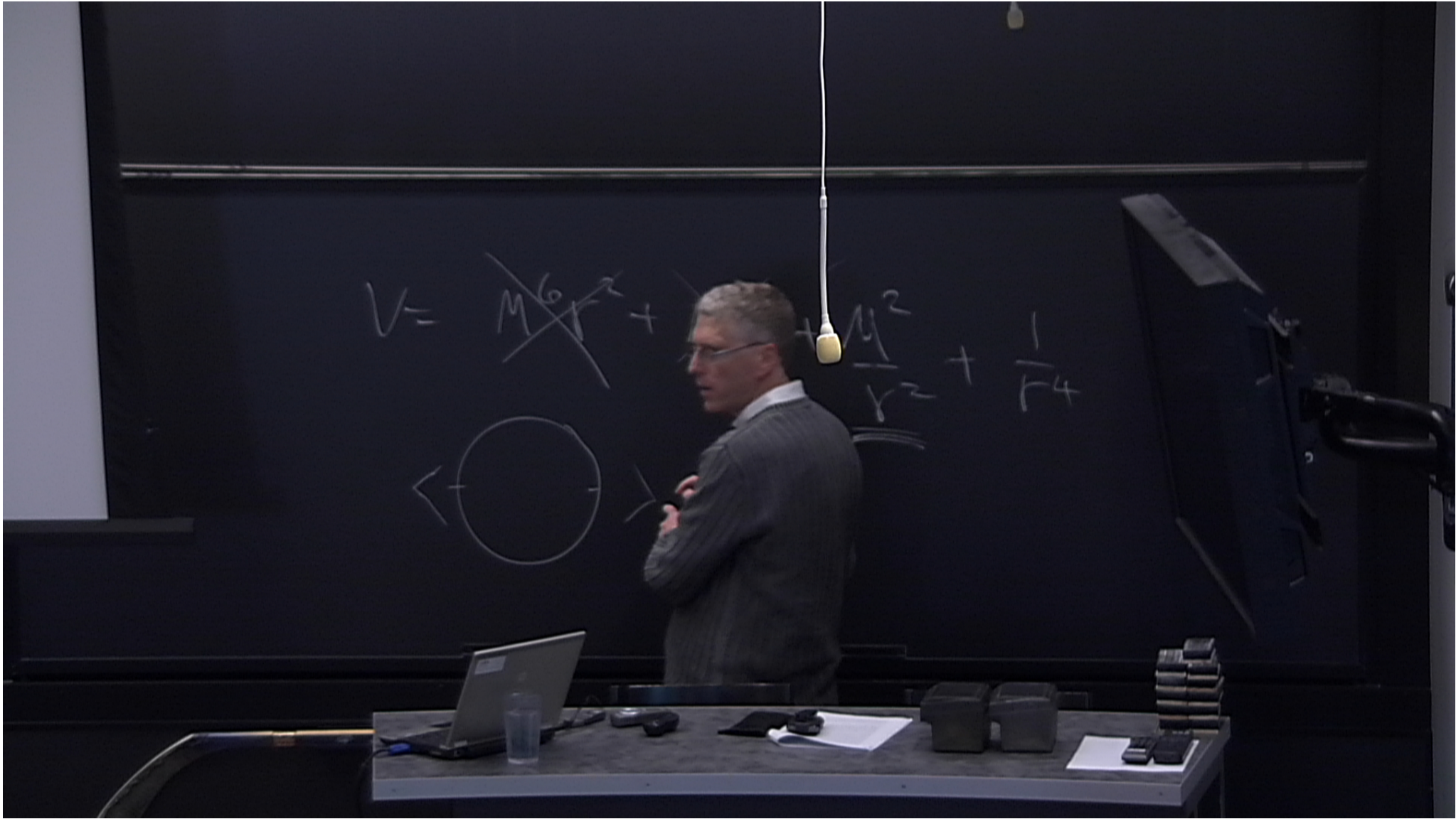


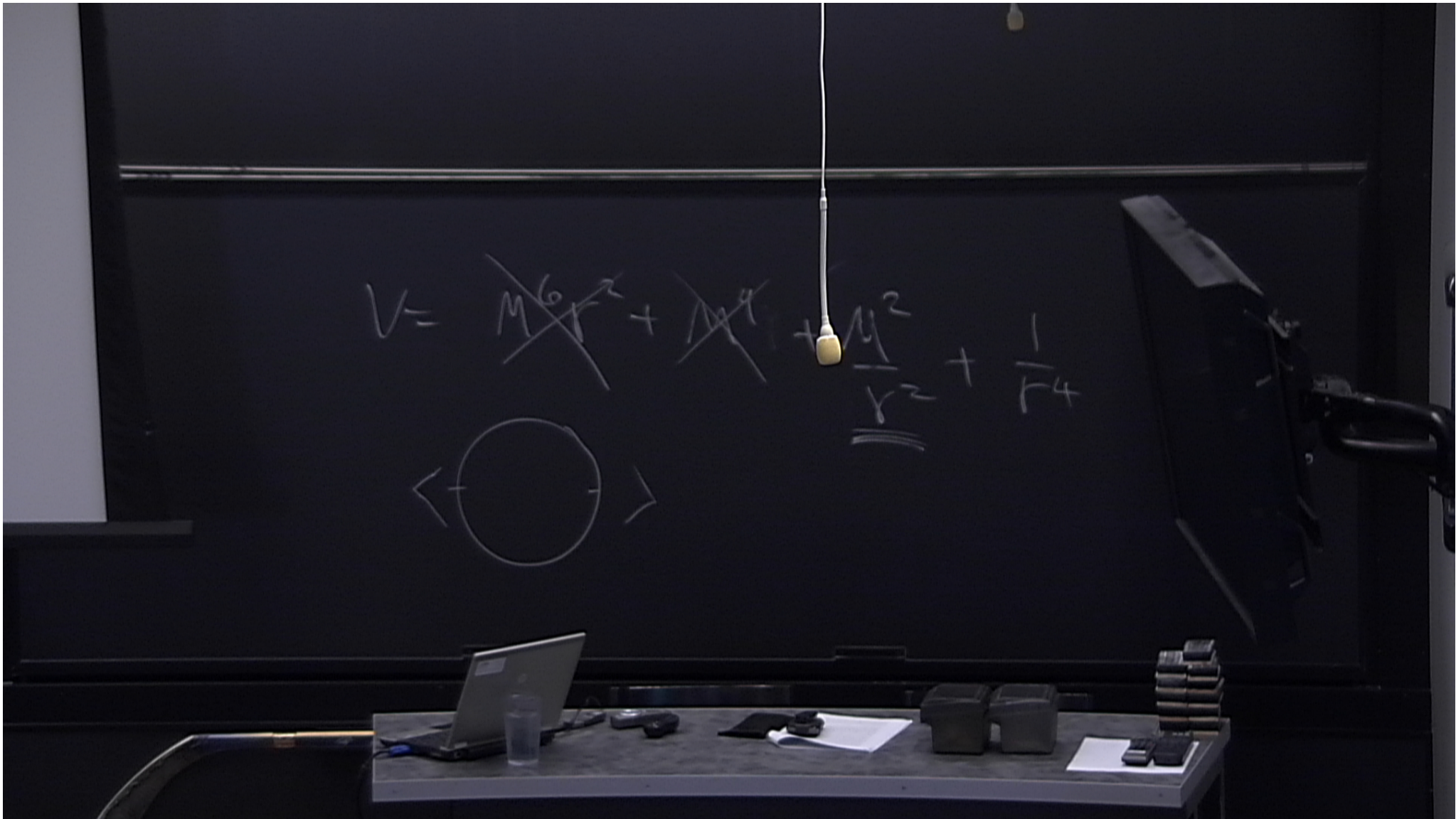
“...when you have eliminated the impossible, whatever remains, however improbable, must be the truth.”

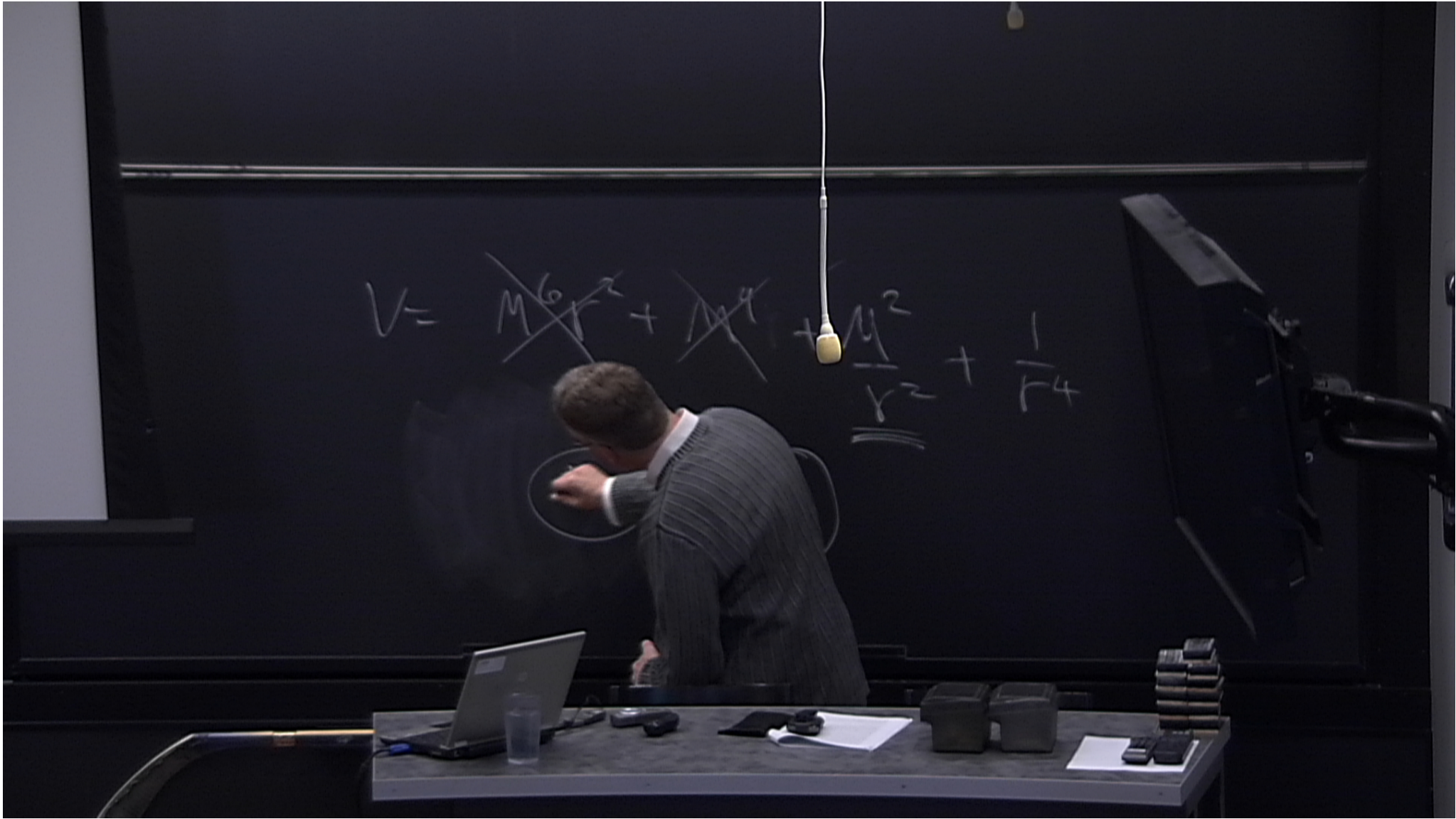
A. Conan Doyle

Opportunities & Concerns

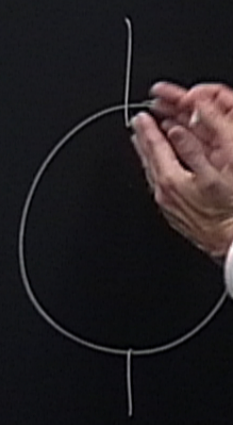
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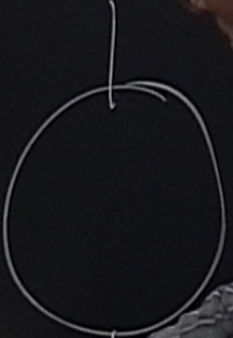
$$\sum G T_i = 0$$

$$V = \cancel{M^6 r^2} + \cancel{M^4} + M^2 + \frac{1}{r^2} + \frac{1}{r^4}$$

$$+ \frac{1}{4\pi} \int R = 2$$

$$\sum_i G T_i + \frac{1}{4\pi}$$


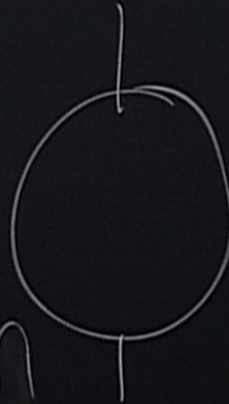
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$$\sum G T_i + \frac{1}{4\pi} \int R = 2$$

$$\sum G T_i + \frac{1}{4\pi} \int R = n$$

