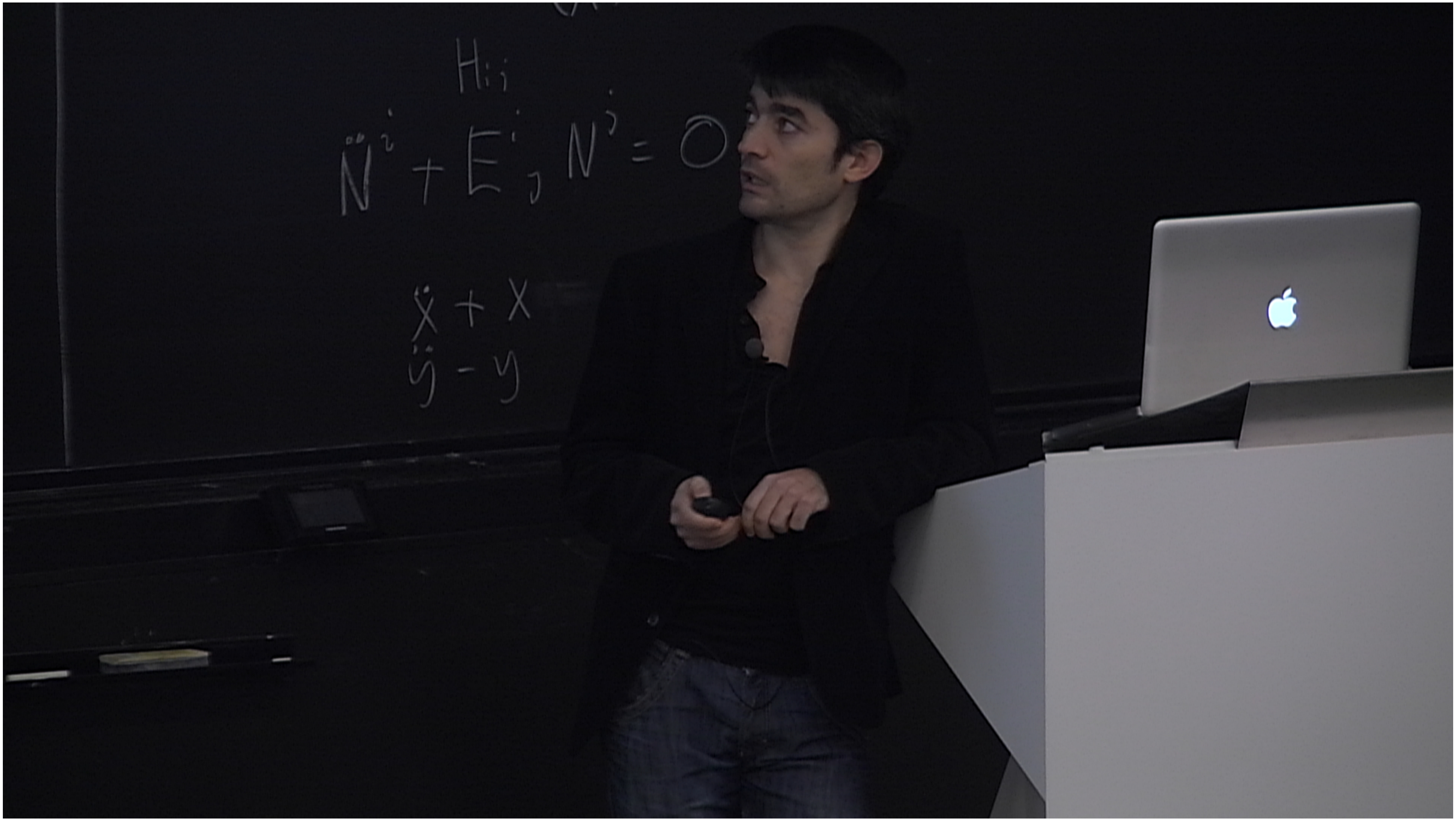
Title: What is Space Made Of?

Date: Jan 25, 2012 02:00 PM

URL: http://pirsa.org/12010129

Abstract: Quantum gravity is about finding out what is the more fundamental nature of spacetime, as a physical system. Several approaches to quantum gravity, suggest that the very description of spacetime as a continuum fails at shorter distances and higher energies, and should be replaced by one in terms of discrete, pre-geometric degrees of freedom, possibly of combinatorial and algebraic nature.


H_{ij}
$\ddot{N}^i + E^i_j N^j = 0$
$\ddot{x} + x$
$\ddot{y} - y$

Plan

- intro: quantum gravity, atoms of space and geometrogenesis
- a bit of history, ingredients from and relation with other approaches
- matrix models
- tensor models
- group field theories - a simple model
- overview of some recent results

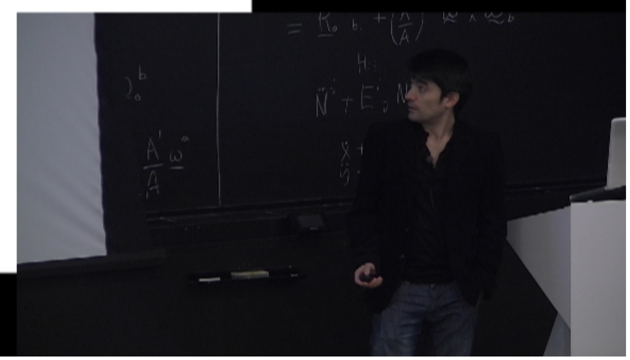
QG, atoms of space and geometrogenesis

The general picture

macroscopic continuum spacetime (and geometry)
from
microscopic, discrete, quantum ("pre-geometric") d.o.f.s

The case for discreteness:

- role of Planck length (a fundamental cut-off? the breakdown scale of continuum?)
- results on black hole entropy (finiteness, proportionality to area, ...)
- infinities in QFT
- singularities in GR



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"The alternative continuum-discontinuum seems to me to be a real alternative; i.e., there is no compromise . . . In a [discontinuum] theory space and time cannot occur . . . It will be especially difficult to derive something like a spatio-temporal quasi-order from such a schema . . . But I hold it entirely possible that the development will lead there . . ." A. Einstein (quoted from R. Sorkin)

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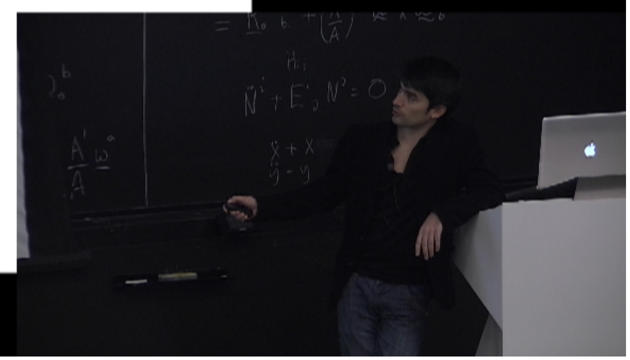
“Atoms of (quantum) space”



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Continuum spacetime (and geometry) result from *collective behaviour* of "atoms of quantum space"

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The case for a phase transition:

- phase transition and bound states are most generic
- necessary if:
  1) renormalizability still key notion for fundamental theory
  2) one wants to explain non-renormalizability of effective GR

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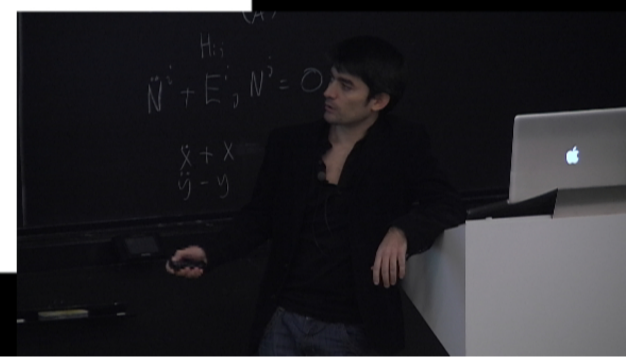

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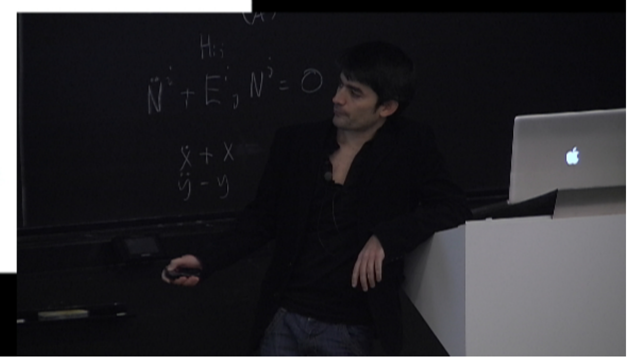
Phase Transition

Geometrogenesis

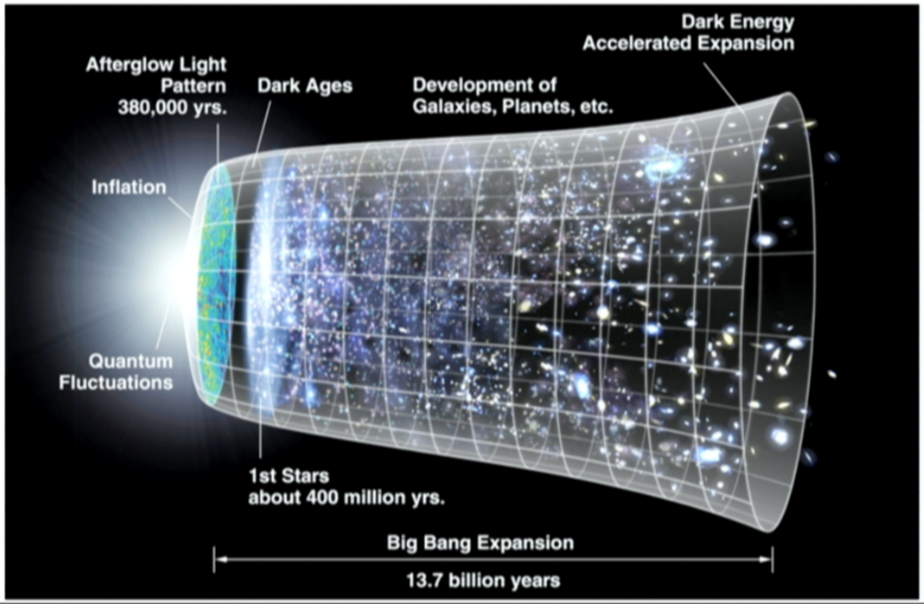
from "non-geometric" or "pre-geometric" phase to a geometric phase



QG, atoms of space and geometrogenesis

Geometrogenesis

cosmological interpretation:



QG, atoms of space and geometrogenesis

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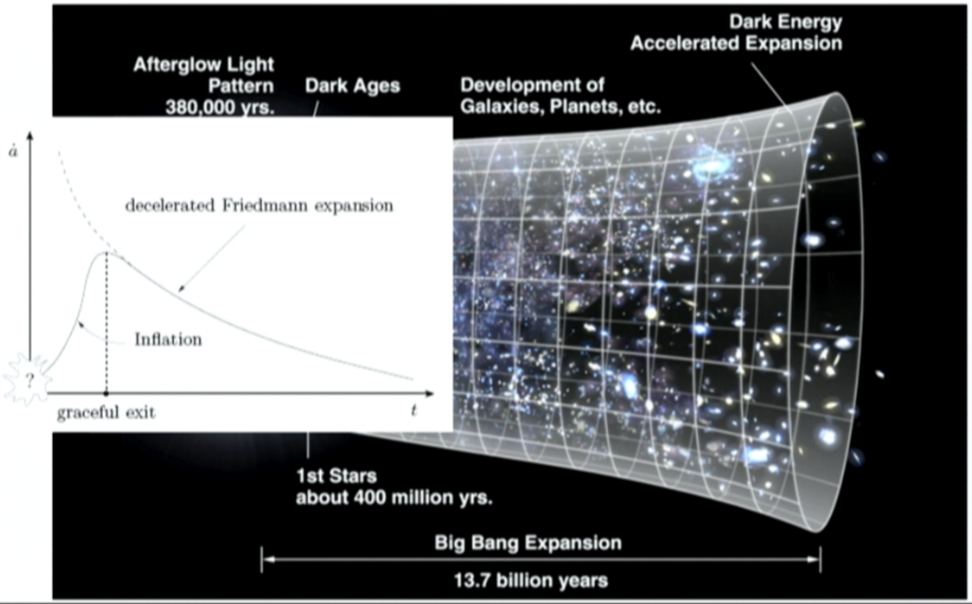

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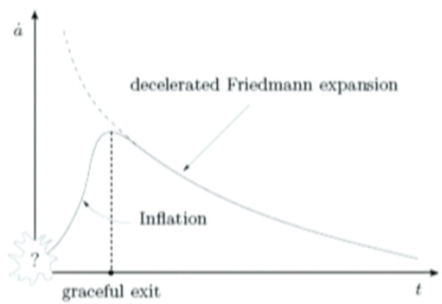
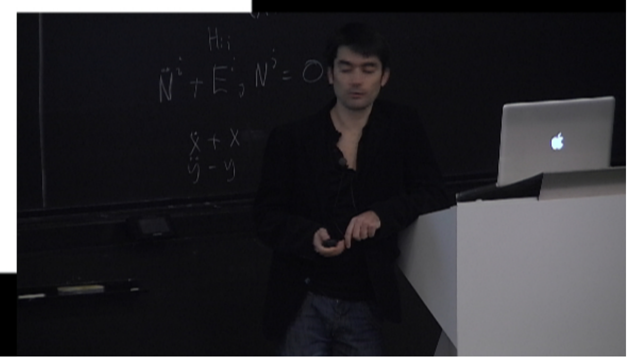


We (QG people) are those of the question mark....



QG, atoms of space and geometrogenesis

key questions of Quantum Gravity (of any QG approach):

- what are the “atoms of space” ?
- how to describe their fundamental dynamics ?

(what is the correct, background independent formalism?)

- what is their fundamental dynamics (eqns, symmetries,..) ?
- how does a continuum spacetime emerge ?
- how to identify/extract effective continuum physics ?

first part: task is not to “quantize GR”, but much more than that

second part: typical problem (thus tools) of condensed matter physics: “how do things organize”?

these are all fundamental, difficult issues.....

... on which many clever people are working since decades:

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... on which many clever people are working since decades:

many failing to find jobs or decent salaries

in solitary, deep thinking

or in large groups and animated discussions

these are all fundamental, difficult issues.....

... on which many clever people are working since decades:

always struggling between
mathematical abstraction and physical reality

Group Field Theory and tensor models

recent general introductions and reviews:

L. Freidel, arXiv: hep-th/0505016

D. Oriti, arXiv: gr-qc/0512103

D. Oriti, arXiv: gr-qc/0607032

D. Oriti, arXiv: 0912.2441 [hep-th]

V. Rivasseau, arXiv:1103.1900 [gr-qc]

R. Gurau, J. Ryan, arXiv: 1109.4812 [hep-th]

D. Oriti, arXiv: 1111.5606 [hep-th]

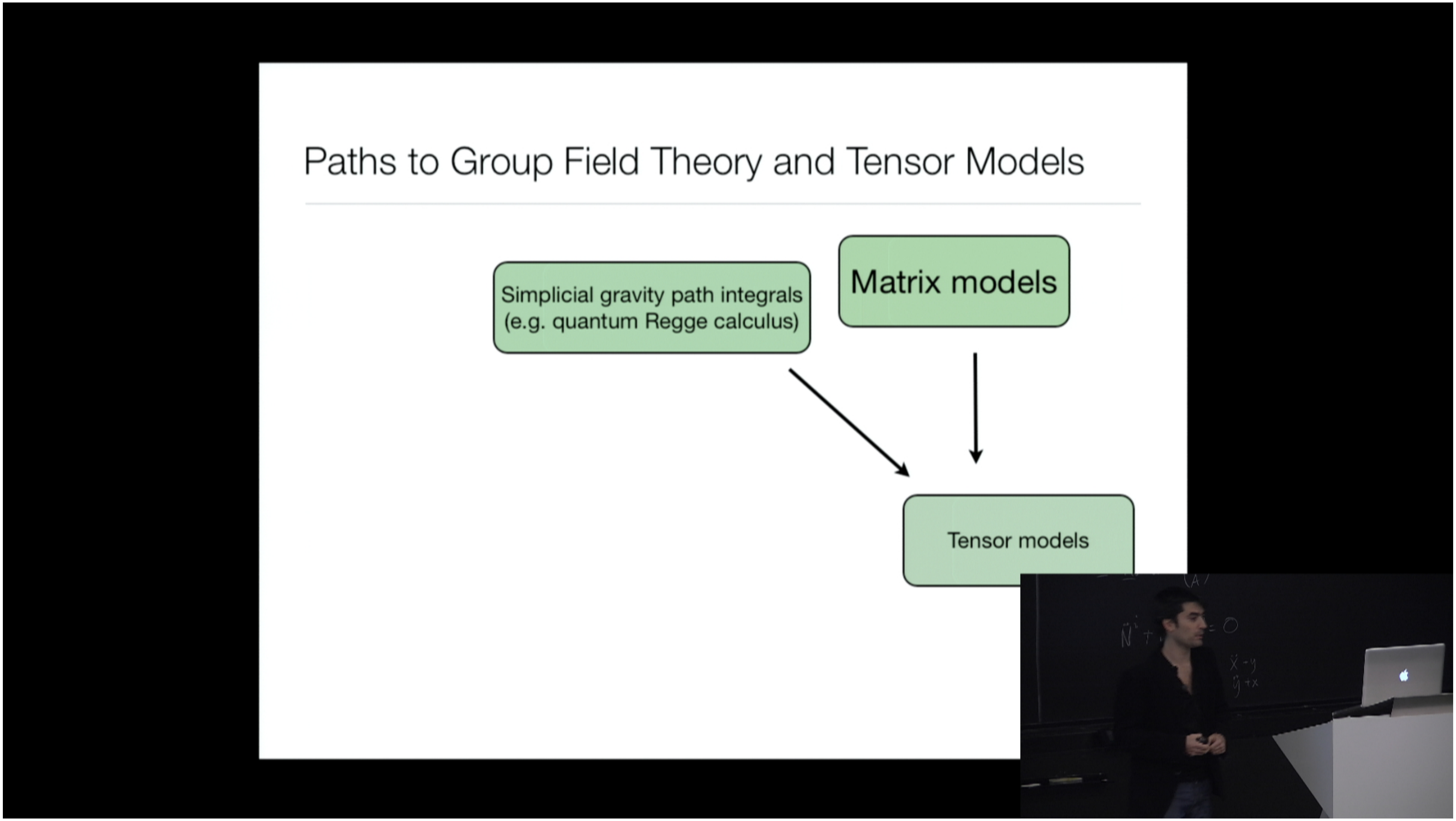
V. Rivasseau, arXiv:1112.5104 [hep-th]

A. Baratin, D. Oriti, to appear

work by:

Baratin, Ben Geloun, Bonzom, Boulatov, Carrozza, De Pietri, Fairbairn, Freidel, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Ooguri, Oriti, Perez, Raasakka, Reisenberger, Rivasseau, Rovelli, Ryan, Sindoni, Smerlak, Tanasa, Vitale, ...

..... fastly growing area of research ....


Paths to Group Field Theory and Tensor Models
Simplicial gravity path integrals
(e.g. quantum Regge calculus)
Matrix models
Tensor models

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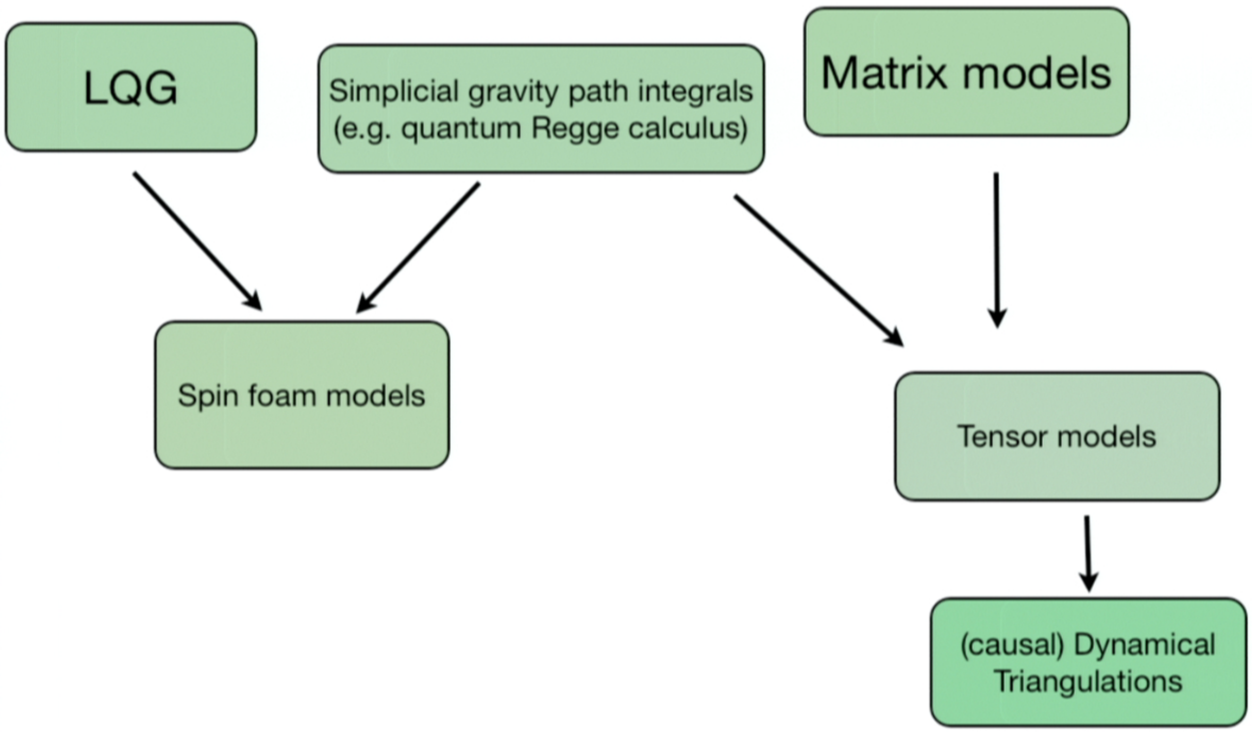

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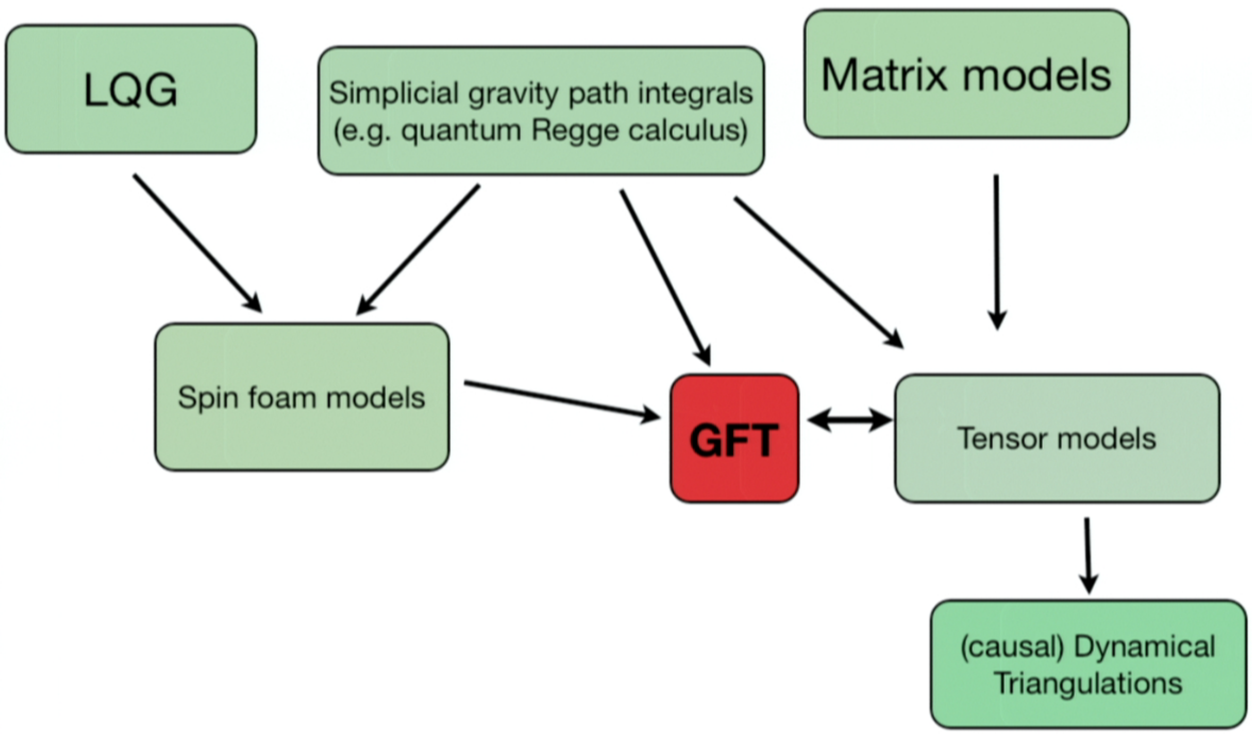

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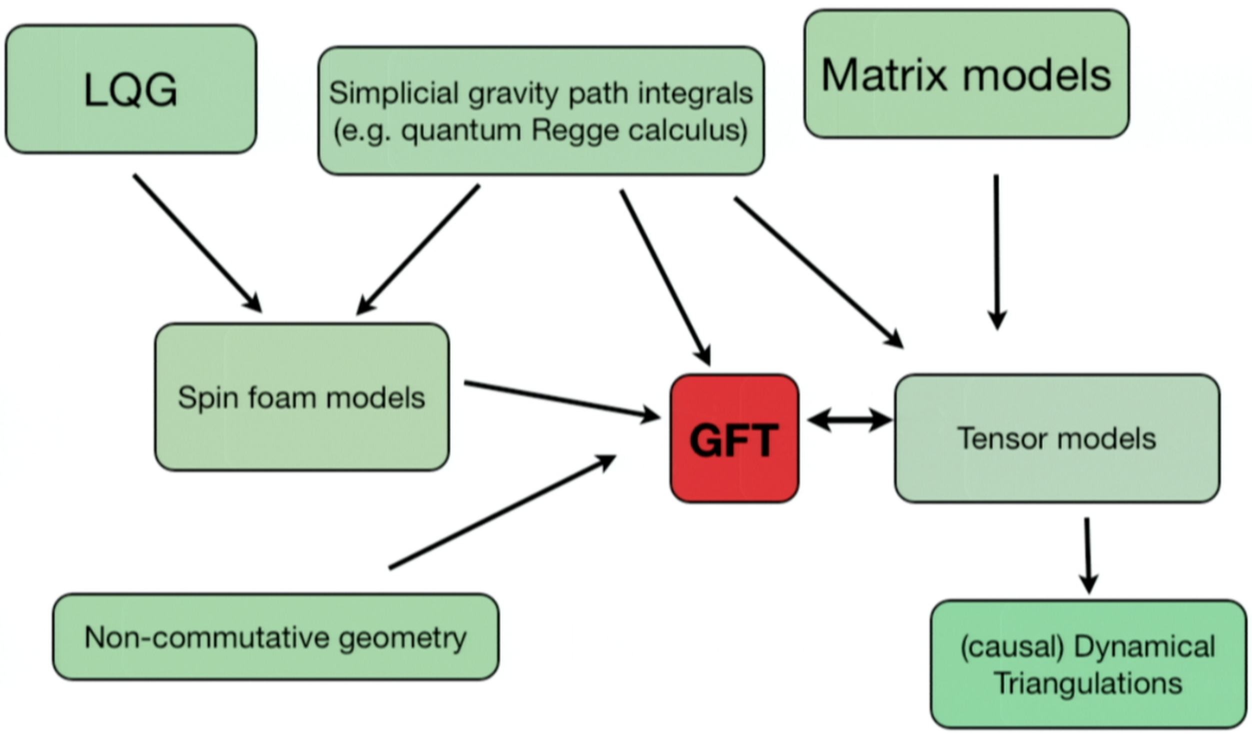

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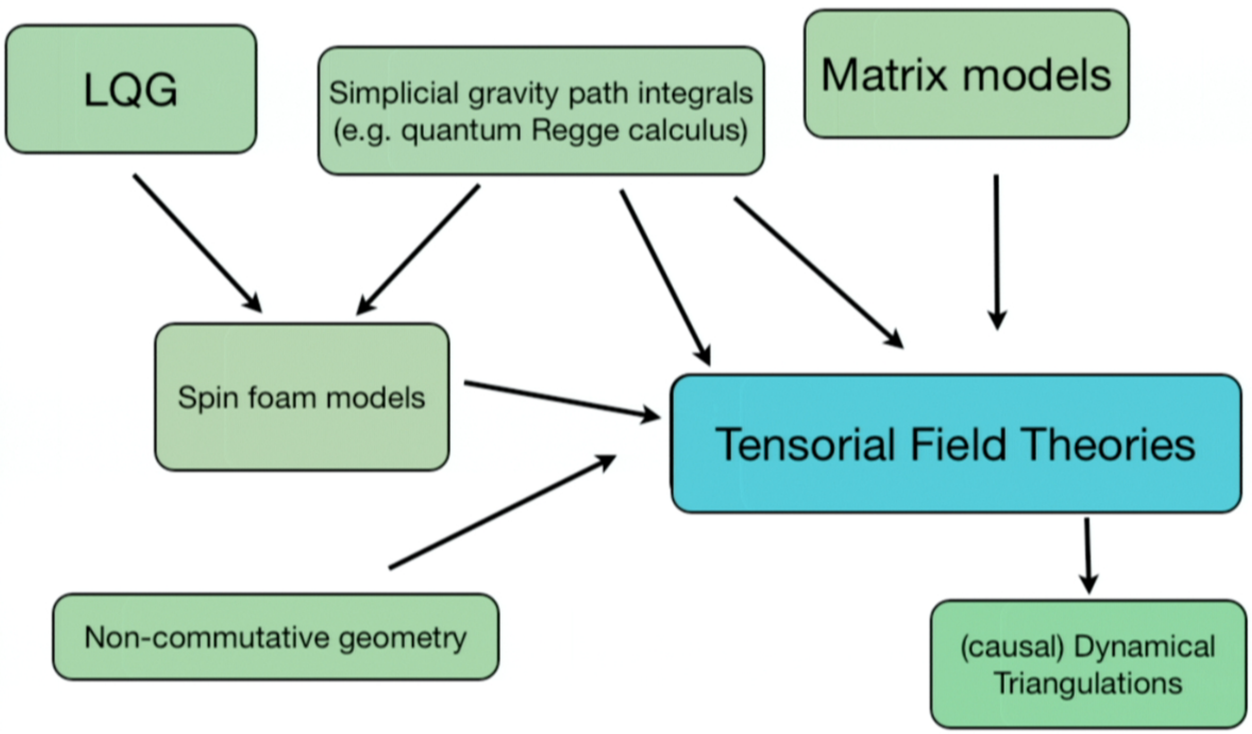

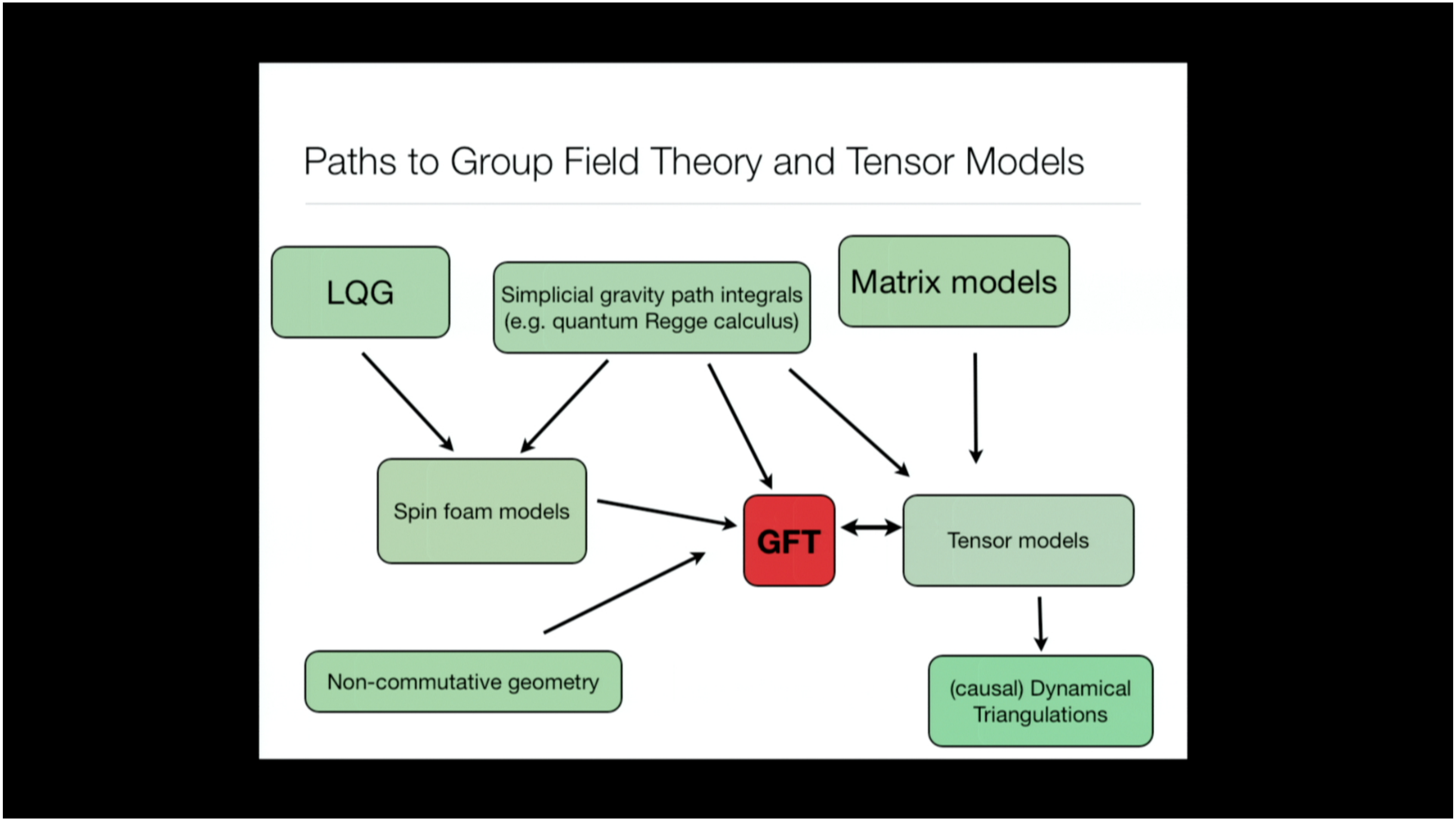
Paths to Group Field Theory and Tensor Models
LQG
Simplicial gravity path integrals (e.g. quantum Regge calculus)
Matrix models
Spin foam models
GFT
Tensor models
Non-commutative geometry
(causal) Dynamical Triangulations

Paths to Group Field Theory and Tensor Models

"Spacetime as a superposition of cellular complexes...."

".... generated by a field theory"

"Spacetime as a statistical system"

Matrix models

Matrix models (Migdal, Kazakov, David, Ambjorn, Kawai, Di Francesco, Zuber, Brezin, Parisi, ...)

Quantum 2d spacetime as (statistical) superposition of discrete surfaces

Fundamental building block of (quantum) space: (hermitian) matrix

$$M^i{}_j \quad i, j = 1, ..., N$$

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Microscopic dynamics:

$$S(M) = \frac{1}{2} tr M^2 - \frac{g}{\sqrt{N}} tr M^3 = \frac{1}{2} M^i{}_j K^{jl}{}_{ki} M^k{}_l - \frac{g}{\sqrt{N}} M^i{}_j M^m{}_n M^k{}_l V^{jnl}{}_{mki}$$

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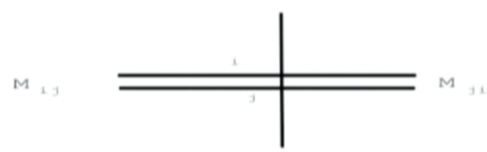
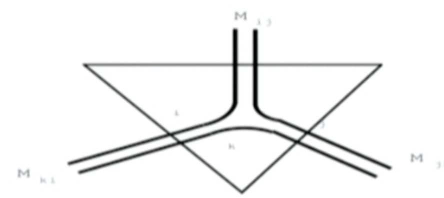
$$K^{jl}{}_{ki} = \delta^j{}_k \, \delta^l{}_i \qquad V^{jnl}{}_{mki} = \delta^j{}_m \, \delta^n{}_k \, \delta^l{}_i$$





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$$Z = \int \mathcal{D}M_{ij}\, e^{-S(M,g)} = \sum_{\Gamma} \left(\frac{g}{\sqrt{N}}\right)^{\frac{1}{2}} Z_{\Gamma} = \sum_{\Gamma} g^{V_\Gamma}\, N^{\chi_\Gamma}$$

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Relation to discrete gravity?

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discretize 2d GR via 2d equilateral triangulation

$$S_{GR} = \int_S d^2x\, \sqrt{g}\,(-R(g) + \Lambda) = -4\pi\,\chi + \Lambda\, A_S \longrightarrow S_{\Delta}(a, G, \Lambda) = -\frac{4\pi}{G}\,\chi(\Delta) + \frac{\Lambda a}{G}\, t_{\Delta}$$

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From matrix model:

$$g = e^{-\frac{\Lambda a}{G}} \qquad N = e^{+\frac{4\pi}{G}}$$

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sum over geometries on each simplicial complex + sum over complexes of all topologies

Matrix models

- control over sum over complexes/topologies?

Yes! sum governed by topological parameters

$$Z = \sum_{\Delta} g^{t_\Delta} N^{\chi(\Delta)} = \sum_{\Delta} g^{t_\Delta} N^{2-2h} = \sum_{h} N^{2-2h} Z_h(g) = N^2 Z_0(g) + Z_1(g) + N^{-2} Z_2(g) + ...$$

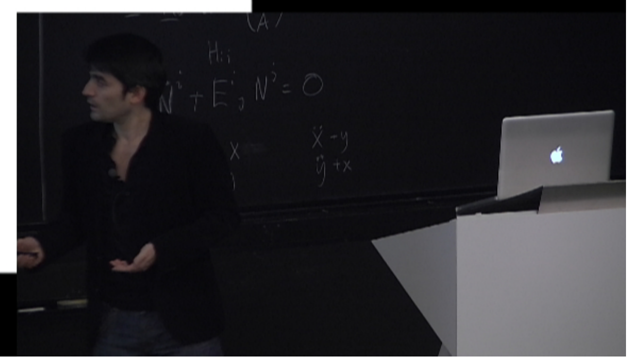
in large-N limit, planar diagrams dominate

- continuum limit (critical behaviour)?
  - Feynman series can be re-summed in large-N limit

$$Z_0(g) \simeq \sum_{V} V^{\gamma-3} \left(\frac{g}{g_c}\right)^V \simeq_{V\to\infty} (g-g_c)^{2-\gamma} \qquad \gamma = -\frac{1}{2}$$

  - free energy and average number of simplices diverge

$$\langle t_\Delta \rangle = \langle V_\Gamma \rangle \propto \frac{\partial}{\partial g} \ln Z_0(g) \simeq \frac{1}{g-g_c}$$



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phase transition (condensation) to theory of large continuum surfaces

Matrix models

- which continuum theory does it correspond to?

2d quantum Liouville gravity (plus matter)

- SD equations on n-point functions ~ 2d WdW equations
- quantum symmetry algebra ~ Virasoro algebra
- critical exponents ......

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many more results and applications: QCD and confinement, statistical mechanics and condensed matter, RNA folding, ...

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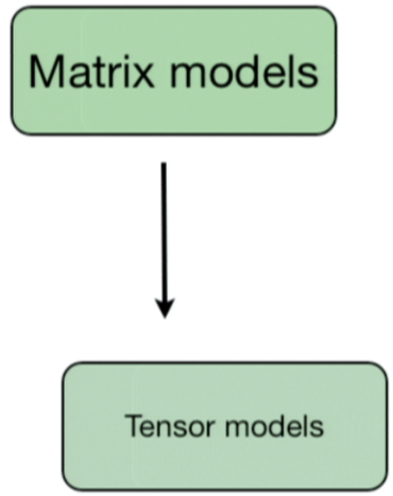
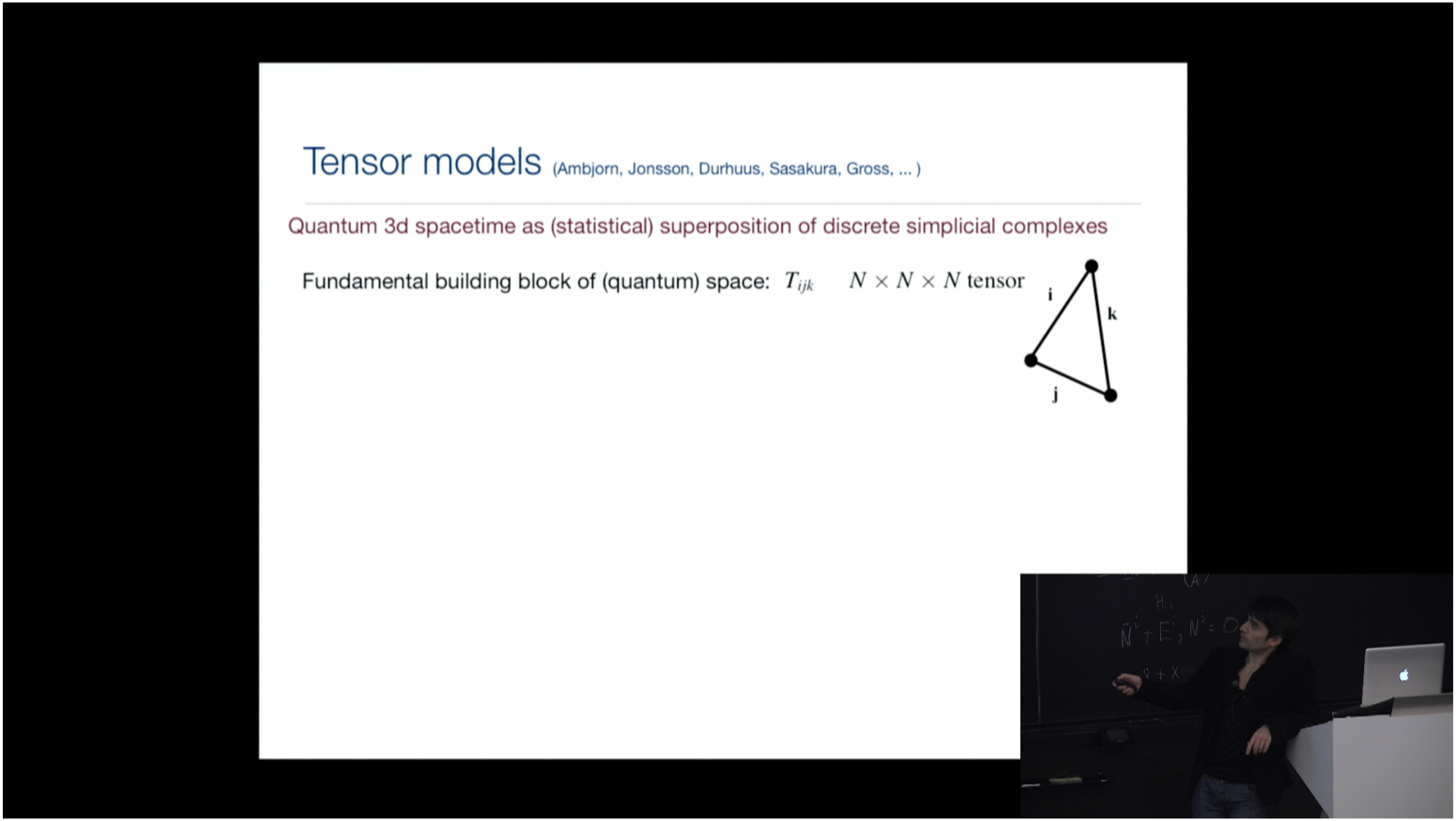
a simple example of Geometrogenesis




Tensor models (Ambjorn, Jonsson, Durhuus, Sasakura, Gross, ...)
Quantum 3d spacetime as (statistical) superposition of discrete simplicial complexes
Fundamental building block of (quantum) space: T_{ijk} $N \times N \times N$ tensor
i
k
j

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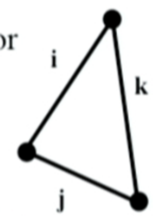

Microscopic dynamics:

$$S(T) = \frac{1}{2} trT^2 - \lambda\, trT^4 = \frac{1}{2} \sum_{i,j,k} T_{ijk} T_{kji} - \lambda \sum_{ijklmn} T_{ijk} T_{klm} T_{mjn} T_{nli}$$

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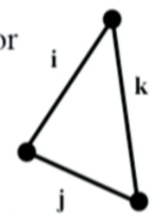

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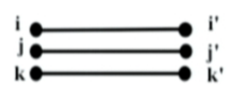

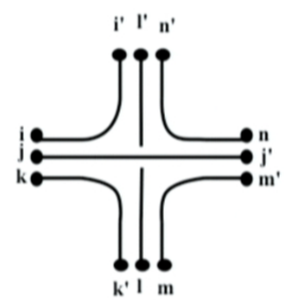

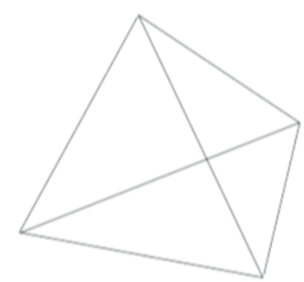

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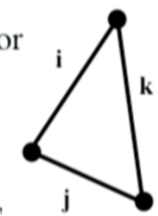

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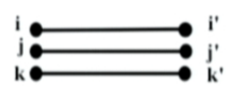
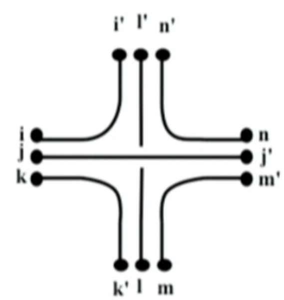
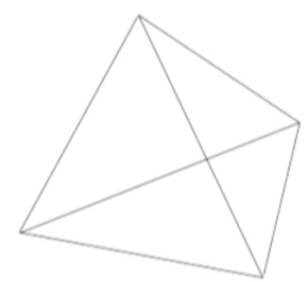






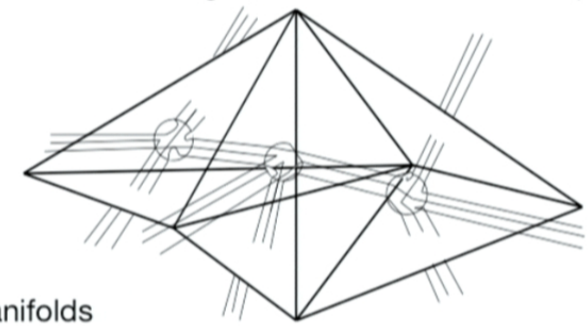
Notice: no GR input,
pure 3d combinatorics

Tensor models

Quantum dynamics:

$$Z = \int \mathcal{D}T \, e^{-S(T,\lambda)} = \sum_\Gamma \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} Z_\Gamma = \sum_\Gamma \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} N^{F_\Gamma - \frac{3}{2}V_\Gamma}$$

Feynman diagrams are stranded graphs dual to 3d simplicial complexes
(nodes dual to tetrahedra, lines dual to triangles, faces dual to edges, 3-cells dual to vertices)



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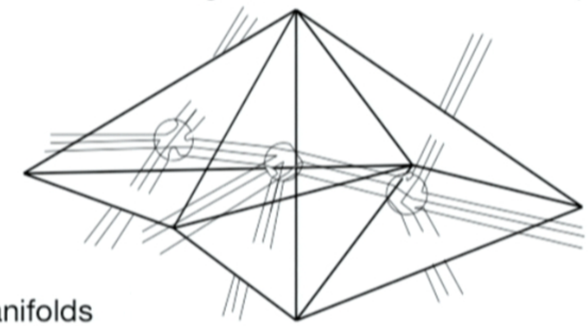


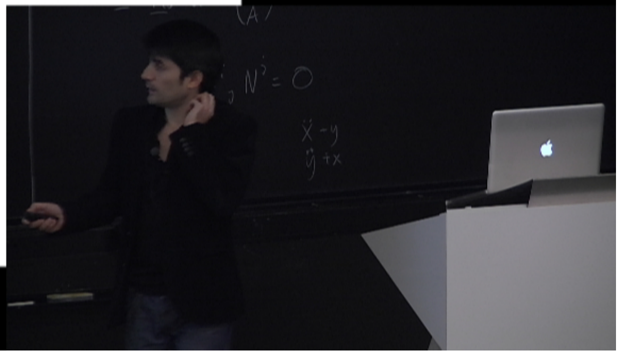
All topological manifolds as well as pseudo-manifolds
included in perturbative sum

Construction can be straightforwardly generalized to arbitrary spacetime dimension
(d-tensors, d-complexes, ...)

Tensor models

Key questions:

- relation to discrete (classical and quantum) gravity?
    - (quantum) simplicial geometry (and gravity) much richer in d>2
    - need more structures/data in boundary states and amplitudes
- control over perturbative sum (topological expansion)
    - complexity of d-simplicial topology and combinatorics
    - scaling of amplitudes and power counting - analogue of large-N limit
    - tensor model/GFT renormalization



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difficult! (no real progress until recently)

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alternative: (causal) dynamical triangulations
(Ambjorn, Loll, Jurkiewicz, ...) ← difficult! (no real progress until recently)

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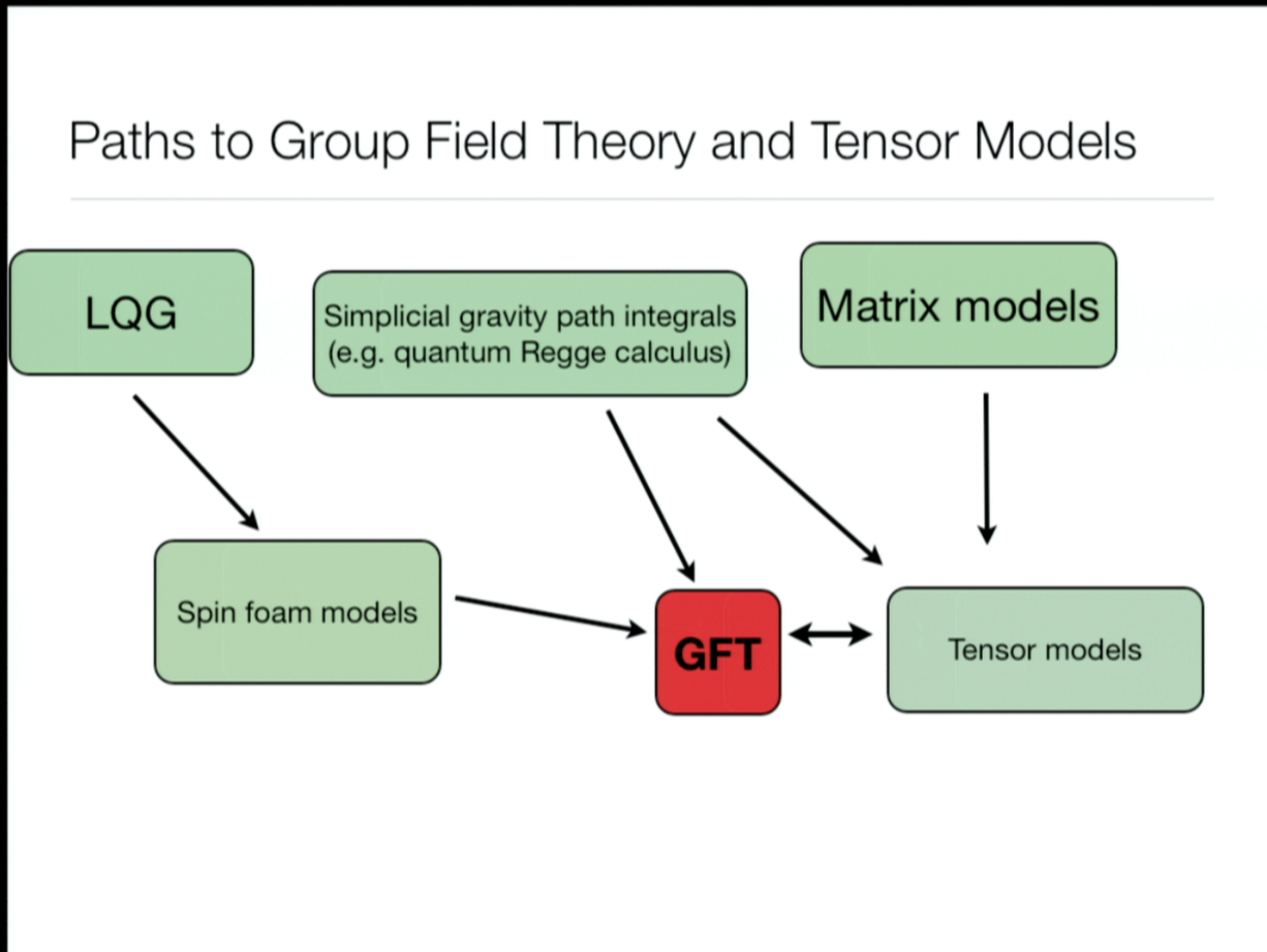
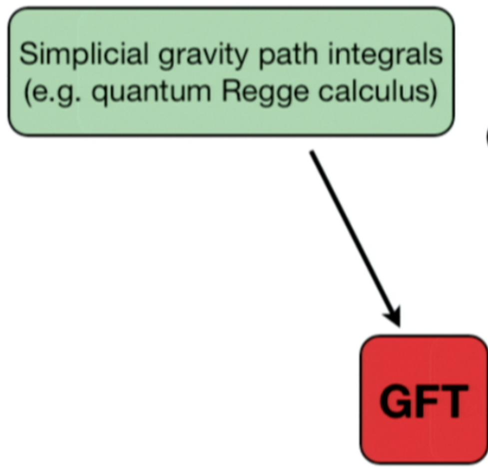
↓

- 3d: what is a quantum triangle (2-simplex)? how to make quantum triangles "interact" to form a 3d simplicial complex?
- 4d: what is a quantum tetrahedron (3-simplex)? how to make quantum tetrahedra "interact" to form a 4d simplicial complex?


Paths to Group Field Theory and Tensor Models
LQG
Simplicial gravity path integrals
(e.g. quantum Regge calculus)
Matrix models
Spin foam models
GFT
Tensor models

Paths to Group Field Theory and Tensor Models



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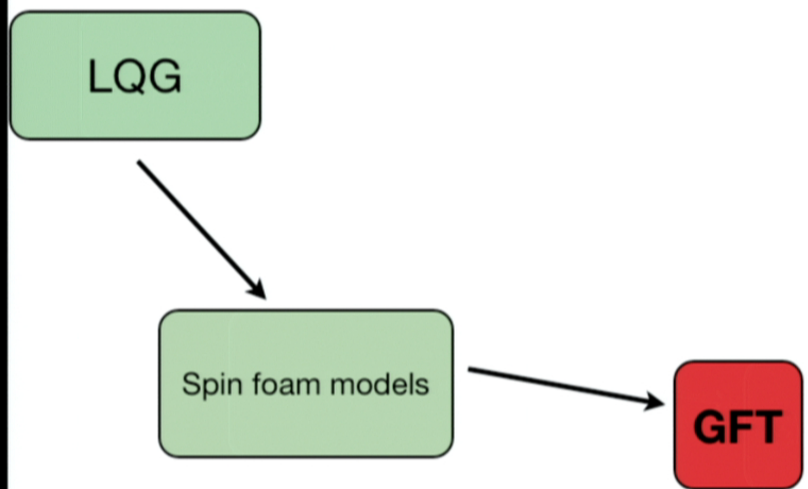
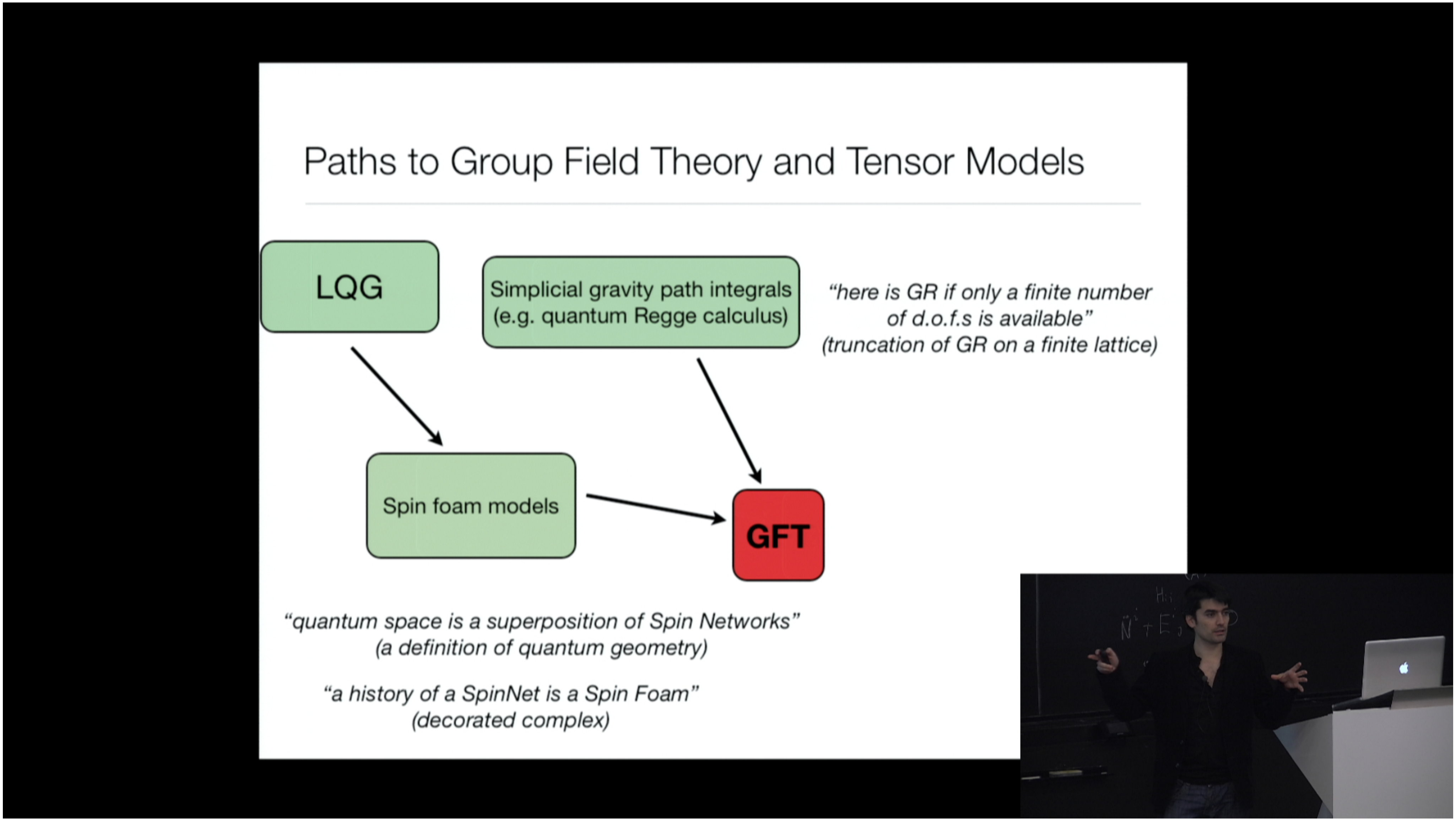


"quantum space is a superposition of Spin Networks"
(a definition of quantum geometry)

"a history of a SpinNet is a Spin Foam"
(decorated complex)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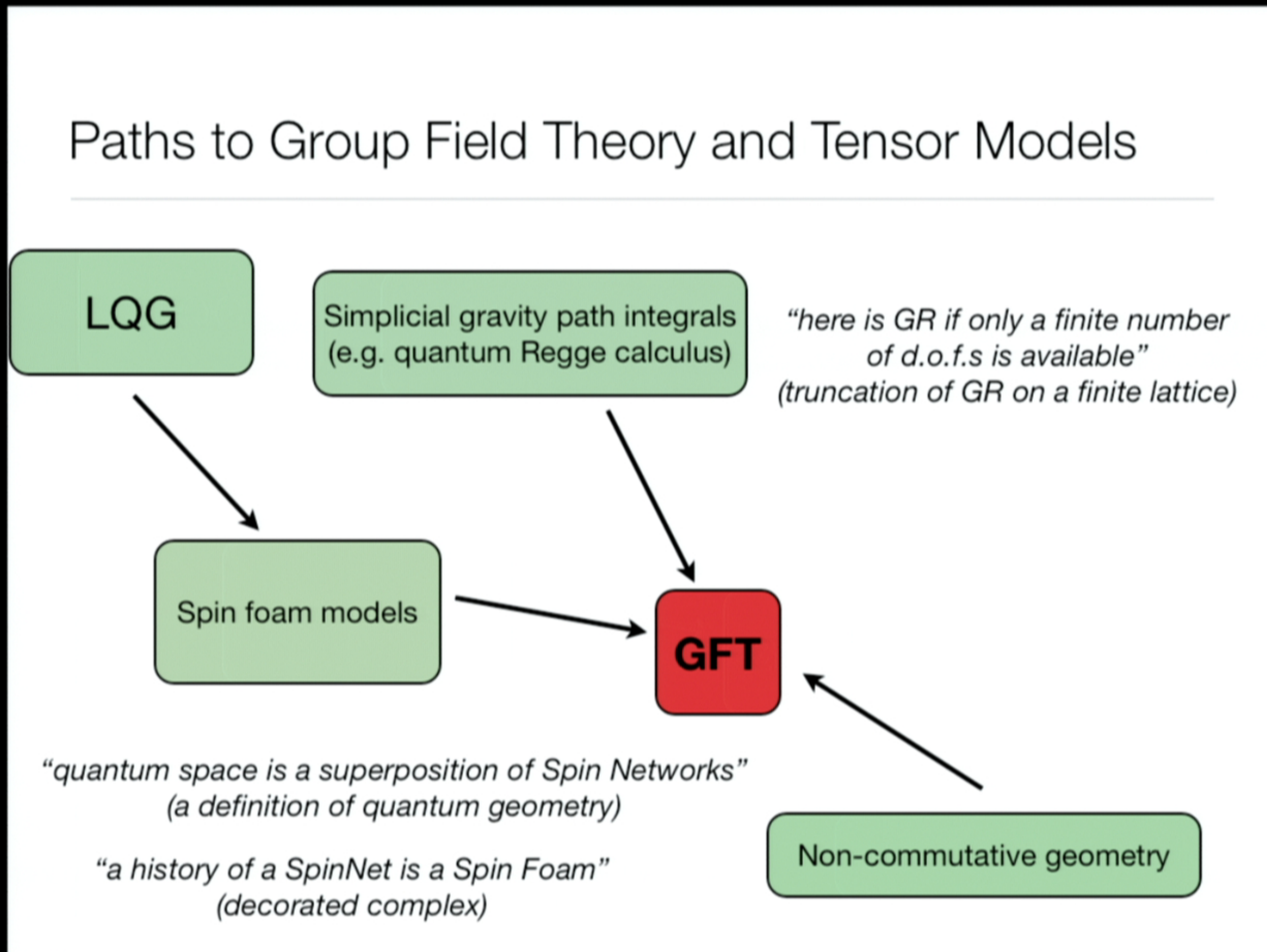
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Non-commutative geometry

Interlude: non-commutative group Fourier transform

(Freidel-Livine, Freidel-Majid, Joung-Mourad-Noui, Livine)

unitary map : $C_\star\left(\mathfrak{su}(2) \simeq \mathbb{R}^3\right) \leftrightarrow C\left(SU(2)\right)$ (here: SO(3) version)

$$f(x) \;=\; \int_{SO(3)} f(g)\, e_g(x) \;\in \mathcal{C}_*(\mathfrak{so}(3))$$

$e_g(x) \;=\; e^{ix\cdot P_g} : \mathfrak{so}(3) \times SO(3) \to \mathbb{C}$ $\qquad P^i_g = tr(|g|\sigma^i)$ $\qquad |g| \equiv sign(trg)g$ $\qquad (e_{g_1} \star e_{g_2}) \equiv e_{g_1 g_2}(x)$

plane waves $\qquad$ coordinates on group $\qquad$ NC star product

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plane waves $\qquad$ coordinates on group $\qquad$ NC star product

- note: $C_*(\mathbb{R}^3)$ is space of functions on $\mathbb{R}^3$ with finite resolution, whose usual Fourier transform has bounded momenta
- one has delta functions: $\delta(g) = \int dx\, e_g(x)$ $\qquad \delta_x(y) = e_{g^{-1}}(x)\, e_g(y)$
- important property: $\int dy\, (\delta_x * \phi)(y) = \int dy\, (\phi * \delta_x)(y) = \phi(x)$
- $\phi(g^{-1}) \to \phi(-x)$

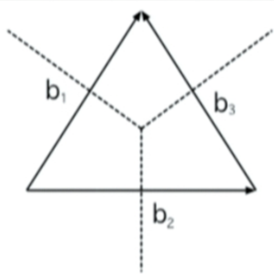
- recently used also to obtain a NC flux representation in LQG (Baratin-Dittrich-Oriti-Tambornino)

Quantum 3d simplicial geometry (euclidean)

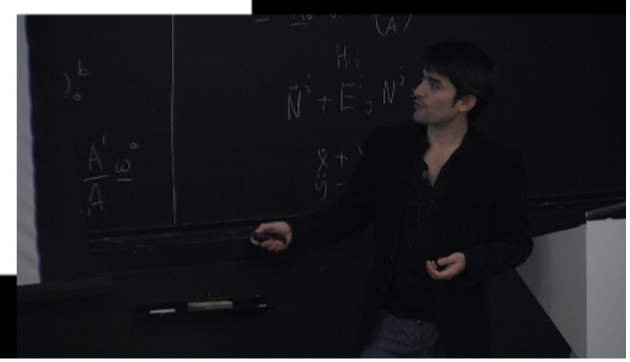
(Barbieri, Baez, Barrett, Freidel, Krasnov, Rovelli, Livine, Speziale, Baratin, Oriti, ...)

classical triangle in $\mathbb{R}^3$

3 edge vectors (or 3 normal vectors) that close $\quad b_i \in \mathbb{R}^3 \quad s.t. \quad \sum_i b_i = 0$



unique intrinsic geometry (up to rotations)



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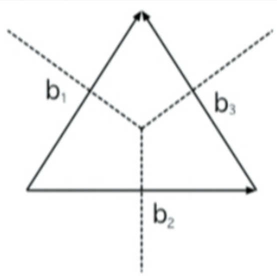

unique intrinsic geometry (up to rotations)

would like to see it as intrinsic geometry of triangle in 2d slice in embedding 3d space → (b_1, b_2, b_3) part of classical phase space

3d gravity in 1st order form - 3d BF theory

classical 3d action : $S(e,\omega) = \int Tr(e \wedge F(\omega))$

$connection: \quad \omega \in \mathfrak{so}(3) \qquad curvature \quad F(\omega) = d_\omega \omega \qquad co-triad \quad e \in \mathfrak{so}(3)$

can discretize on arbitrary 3d simplicial complex:

$$\int_e E = b_e = x_e \in \mathbb{R}^3 \simeq \mathfrak{su}(2) \qquad \mathcal{P}e^{\int_l \omega} = h_l \in SO(3)$$

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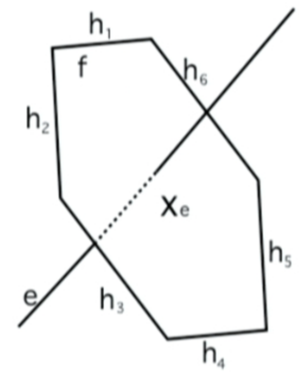

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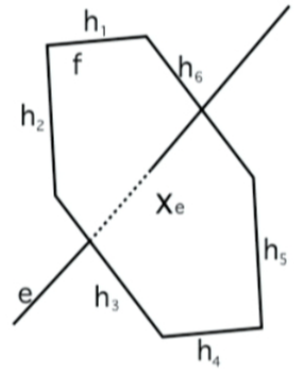

Phase space for triangle in discrete 3d gravity: $[\mathcal{T}^* SU(2)]^{\times 3}$

$$\{h_e\,, h_{e'}\} = 0 \qquad \{x_e^j\,, h_{e'}\} \propto \delta_{e,e'} \tau^j h_e \qquad \{x_e^j\,, x_{e'}^k\} = i\epsilon_{ijk}\, x_e^i\, \delta_{e,e'}$$

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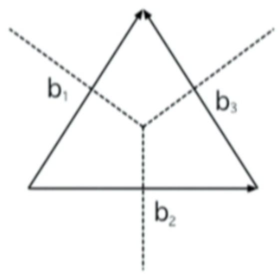

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Phase space for triangle in discrete 3d gravity: $\quad [\mathcal{T}^* SU(2)]^{\times 3}$

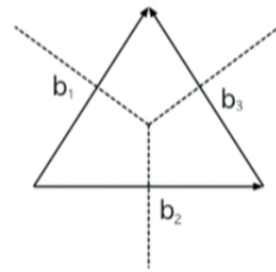
classical tetrahedron in $\mathbb{R}^3$ $\quad$ 4 triangles glued (by identification of edge vectors) across common edges

Quantum 3d simplicial geometry (euclidean)

quantum triangle:

- non-commutative metric (edge vector) representation:

$$\mathcal{H}_{triangle} = Inv\left(\otimes_i \mathcal{H}_i^{SU(2)}\right) \ni \psi(x_1, x_2, x_3) \star \delta\left(x_1 + x_2 + x_3\right)$$



GFT for 3d Riemannian gravity (Boulatov, '92)

colored version (Gurau) : 4 fields $\varphi_\ell \,:\, SO(3)^3/SO(3) \,\rightarrow\, \mathbb{R}$

$$\forall h \in \mathrm{SO}(3), \qquad \varphi_\ell(hg_1, hg_2, hg_3) \,=\, \varphi_\ell(g_1, g_2, g_3)$$

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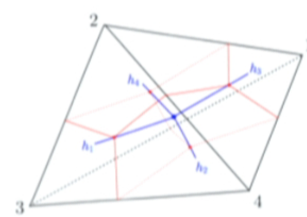
action: $S(\varphi_\ell) \,=\, S_{kin}(\varphi_\ell) \,+\, S_{int}(\varphi_\ell)$

$$S_{kin}[\varphi_\ell] = \int [\mathrm{d}g_i]^3 \sum_{\ell=1}^{4} \varphi_\ell(g_1, g_2, g_3)\overline{\varphi_\ell}(g_1, g_2.g_3),$$

$$S_{int}[\varphi_\ell] \quad = \quad \lambda \int [\mathrm{d}g_i]^6\, \varphi_1(g_1, g_2, g_3)\varphi_2(g_3, g_4, g_5)\varphi_3(g_5, g_2, g_6)\varphi_4(g_6, g_4, g_1)$$
$$+\lambda \int [\mathrm{d}g_i]^6\, \overline{\varphi_4}(g_1, g_4, g_6)\overline{\varphi_3}(g_6, g_2, g_5)\overline{\varphi_2}(g_5, g_4, g_3)\overline{\varphi_1}(g_3, g_2, g_1)$$

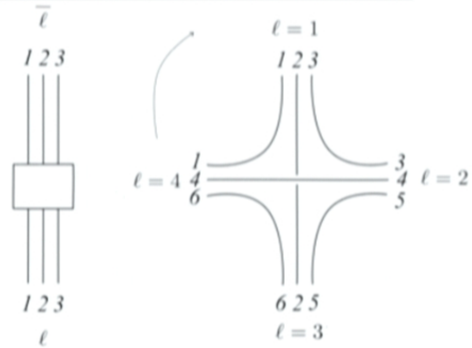
$$\varphi(g_1, g_2, g_3) \;\leftrightarrow\; \varphi(x_1, x_2, x_3)$$



GFT for 3d Riemannian quantum gravity

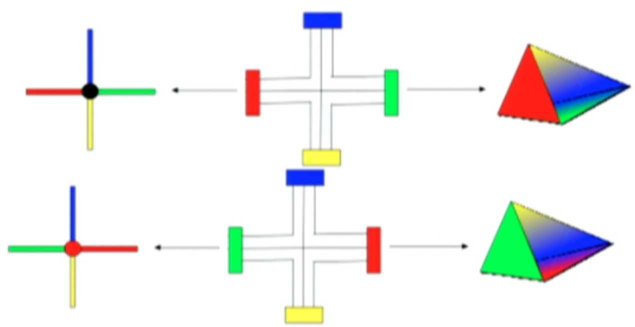
Feynman perturbative expansion around trivial vacuum:

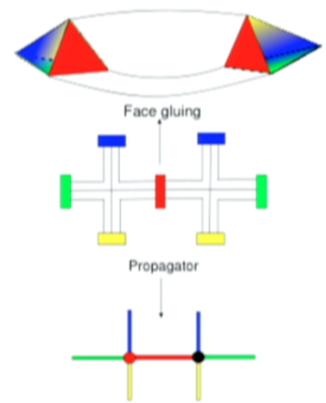
extract propagator and vertex function from action (need to keep track of combinatorics of field arguments)



Feynman diagrams dual to 3d simplicial complexes of arbitrary topology (including pseudomanifolds)

coloring: restrictions on gluing + topological information





GFT for 3d Riemannian quantum gravity

Feynman amplitudes can be computed in different representations (group, Lie algebra, spin)

$$\mathcal{A}_\Gamma = \int \prod_l \mathrm{d}h_l \prod_f \delta\left(H_f(h_l)\right) = \int \prod_l \mathrm{d}h_l \prod_f \delta\left(\overrightarrow{\prod}_{l\in\partial f} h_l\right) =$$

$$= \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \left\{ \begin{array}{ccc} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{array} \right\} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e]\, e^{i \sum_e \mathrm{Tr}\, x_e H_e}$$

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exact duality: simplicial gravity path integral $\leftrightarrow$ spin foam model (Baratin-Oriti)

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complements/adds to earlier result:

any spin foam model can be obtained as GFT Feynman amplitude (Reisenberger-Rovelli)

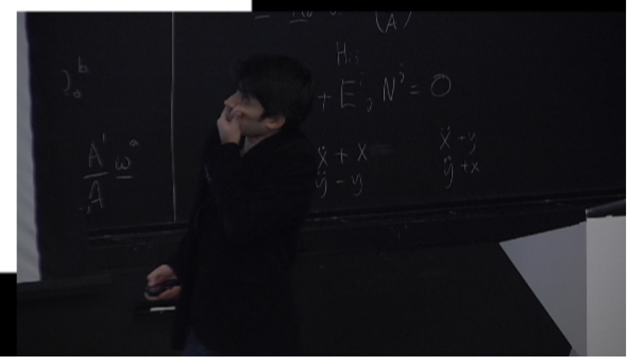
(importance of GFT for LQG via spin foam models)

GFT for 3d Riemannian quantum gravity

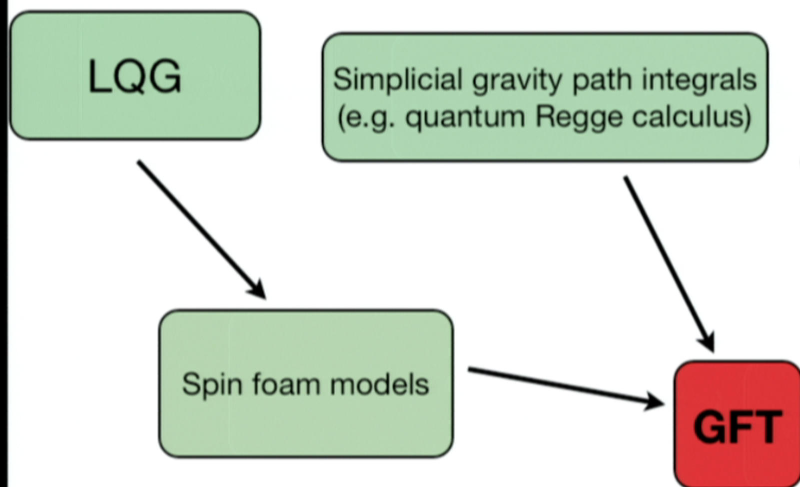
because of structure and representations of GFT field,
boundary states are expressed as glued geometric simplices
or spin networks

the GFT model defines their transition amplitudes

these are expressed as sums over complexes, which
associated amplitude given by a simplicial gravity path integral
or a spin foam model (history of spin networks in LQG)



Paths to Group Field Theory and Tensor Models



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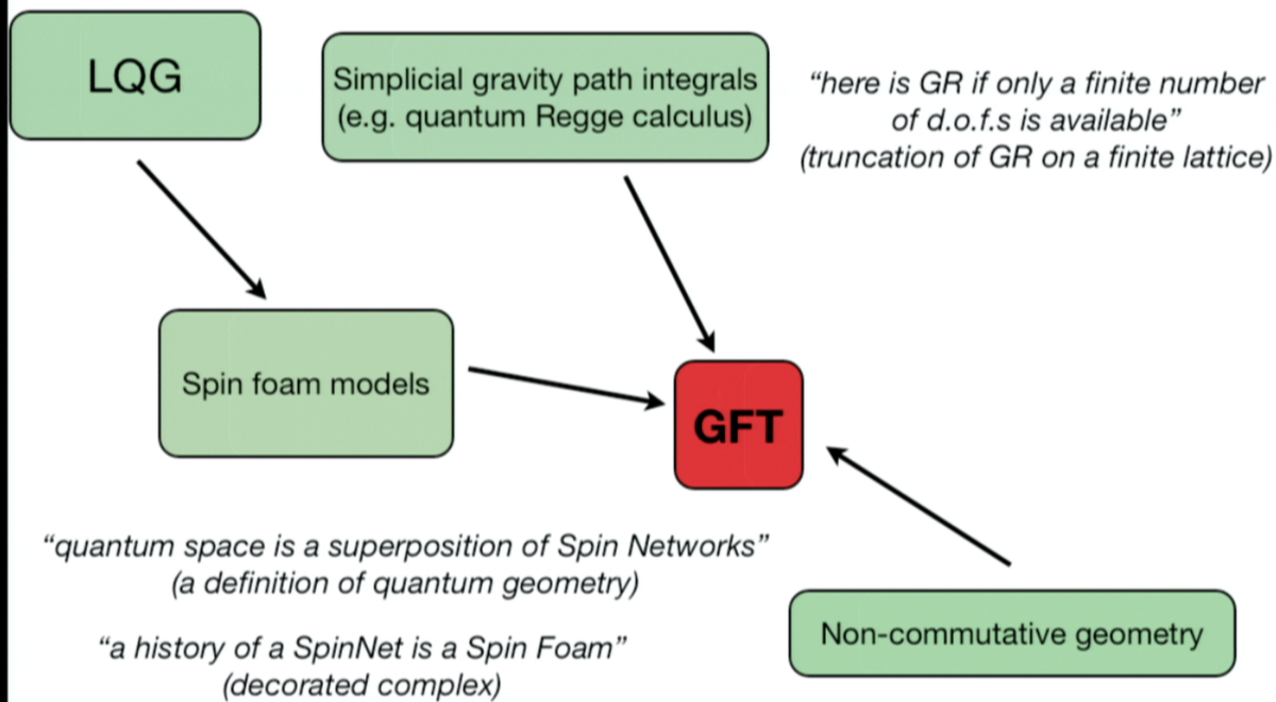


Group Field Theory - summary

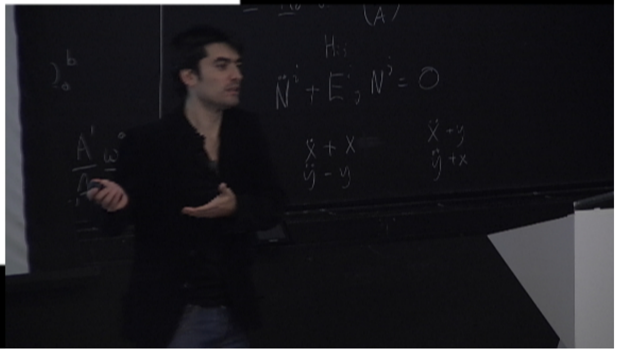
- GFTs = new class of (Tensorial) Field Theories, defined on groups or Lie algebras
- GFTs = field theories of spin network vertices or quantum simplices
- tentative definition of the microscopic structure and dynamics of quantum space (both geometry and topology)
- d.o.f. of LQG and simplicial gravity into framework of matrix and tensor models
- new challenges for renormalization theory, applications to statistical mechanics (Bonzom)

- interaction process (Feynman diagram): 2-complex ~ triangulation
- Feynman amplitude: spin foam model ~ simplicial path integral
- dynamics: GFT n-point functions, Ward identities, SD eqns, ...
- microscopic dynamics from elementary simplicial geometry --> far from GR
- framework for continuum limit and emergent continuum dynamics
- tools from QFT: renormalization, mean field theory, symmetries, ...

GFT - key questions

key questions of Quantum Gravity (of any QG approach):

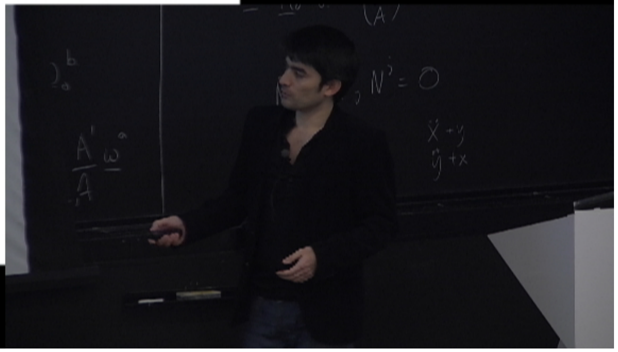
- what are the “atoms of space” ?
- how to describe their fundamental dynamics ?



GFT - key questions

key questions of Quantum Gravity (of any QG approach):

- what are the “atoms of space” ?
- how to describe their fundamental dynamics ?
(what is the correct, background independent formalism?)
- what is their fundamental dynamics (eqns, symmetries,..) ?



GFT - key questions

key questions of Quantum Gravity (of any QG approach):

- what are the “atoms of space” ?
- how to describe their fundamental dynamics ?
(what is the correct, background independent formalism?)
- what is their fundamental dynamics (eqns, symmetries,..) ?
- how does a continuum spacetime emerge ?
- how to identify/extract effective continuum physics ?

Geometrogenesis (or the problem of the continuum)

a phase transition from a pre-geometric/discrete to
a geometric/continuous phase of quantum space

quantum Regge calculus and spin foams
(on given lattice)
(methods from lattice gauge theory)
Dittrich et al, '09,'10,'11

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(causal) dynamical triangulations
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Quantum Graphity
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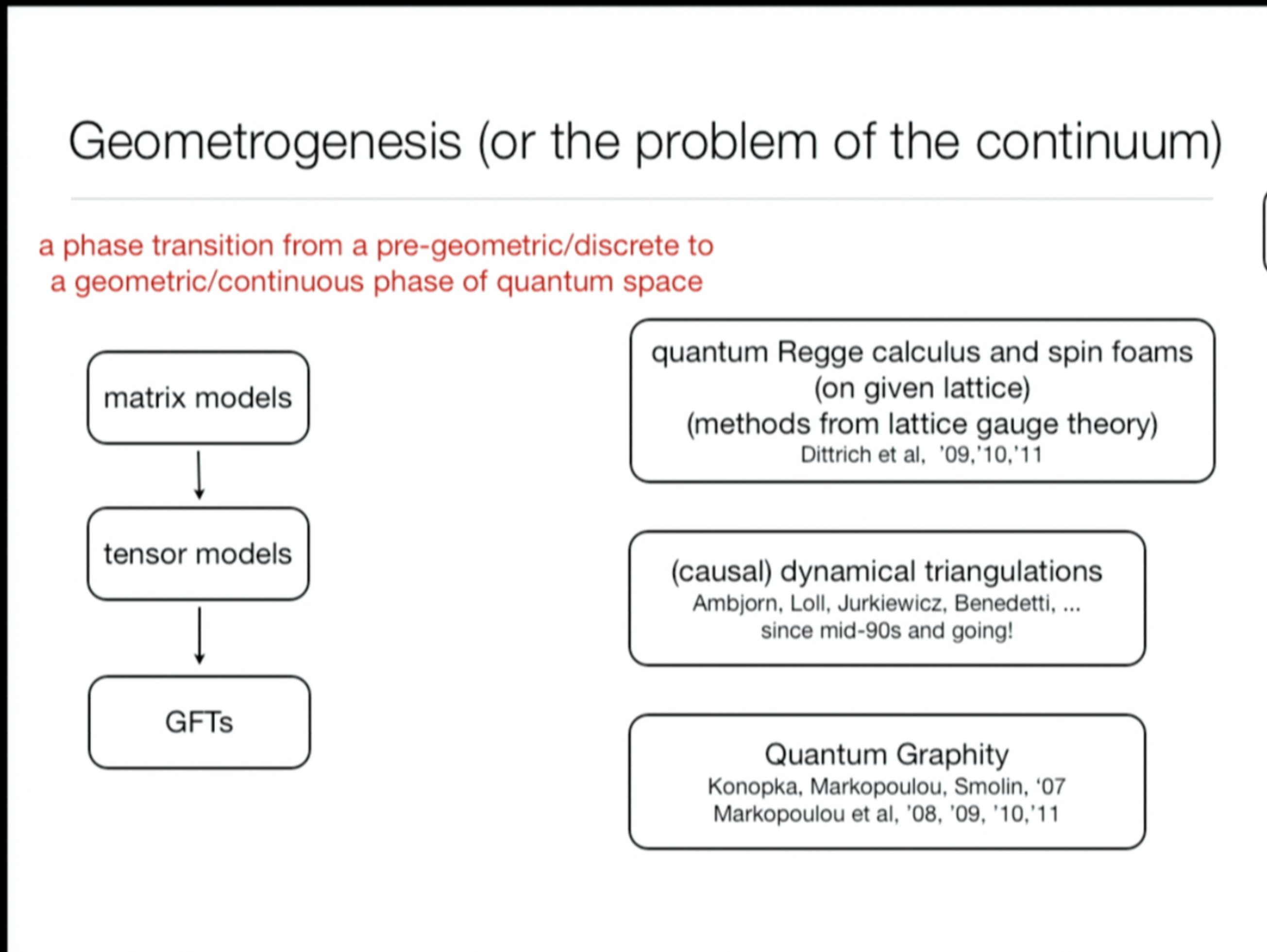
Geometrogenesis (or the problem of the continuum)
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matrix models
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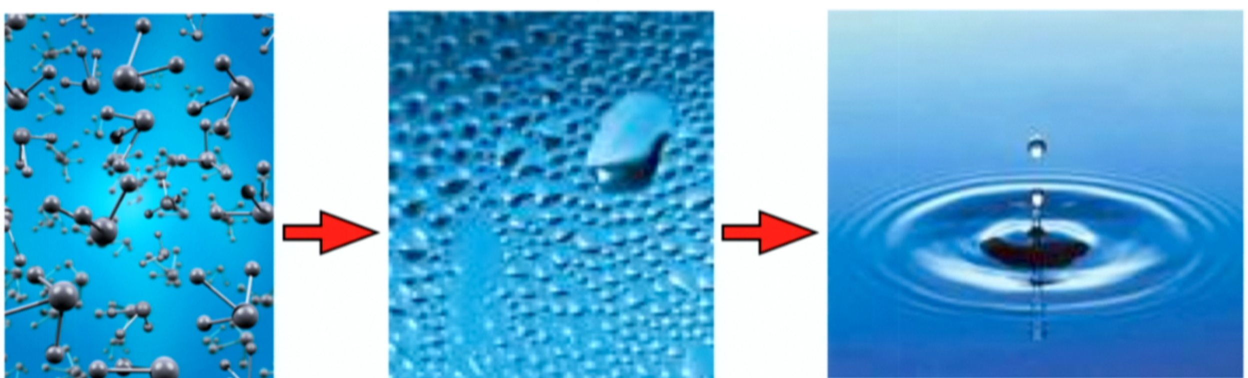
Emergence of continuum spacetime in GFT?

- expectation: physics of continuum spacetime to emerge from non-perturbative GFT in ‘thermodynamic limit’ of large number of microscopic GFT ‘building blocks’ in one phase
- important advantage: tools from statistical field theory and condensed matter physics
  - calculational tools (e.g. RG), approximation methods (e.g. mean field approx.)
- what is the physical interpretation? a *condensate* of GFT atoms?
  quantum space as a condensed matter system? geometrogenesis is GFT condensation?

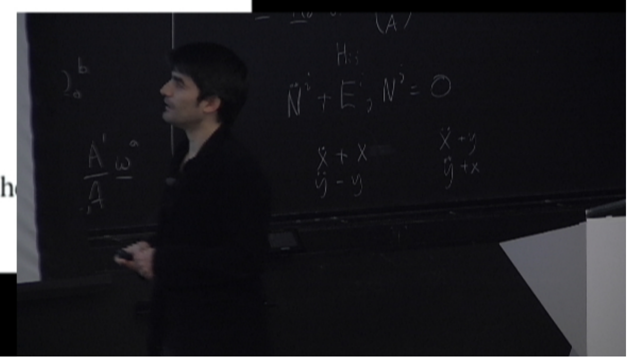


An analogy: Bose Condensates

a Bose condensate below the critical temperature is a (super)fluid, described in hydrodynamic approximation:

- hydrodynamic variables: density of fluid $\rho(x)$, velocity of fluid $\mathbf{v}(x)$
  $\partial_t \rho = \{H, \rho\} = -\nabla \cdot (\rho \mathbf{v})$, $\qquad \partial_t \mathbf{v} = \{H, \mathbf{v}\} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \nabla \frac{d\epsilon}{d\rho}$.
  with energy density $\epsilon(\rho)$.
- for vortex-free flows, derived from extended Lagrangian (with gradient terms):
  $L_{\text{hydro}}(\rho, \theta) = \int d^4x \, \left(\frac{1}{2}\rho v^2 + \epsilon(\rho) - \rho \partial_t \theta + \frac{1}{2} K (\nabla \rho)^2\right)$, $\mathbf{v} = \nabla \theta$.

- What if we quantize this theory?
- note: it is non-renormalizable (quadratic divergences, just like GR)
- $\mathbf{v}(v) \to \hat{\mathbf{v}}(x)$, $\rho(x) \to \hat{\rho}(x)$,
- resulting quantum theory is, first of all, not so physically relevant (before quantum fluctuations of $\hat{\rho}$ or $\hat{\mathbf{v}}$ become relevant, whole hydrodynamic approximation breaks down
- second, classical limit of such *quantum hydrodynamic* does not give back th correct GP Lagrangian, i.e. one obtains different energy functional $\epsilon'(\rho)$



An analogy: Bose Condensates

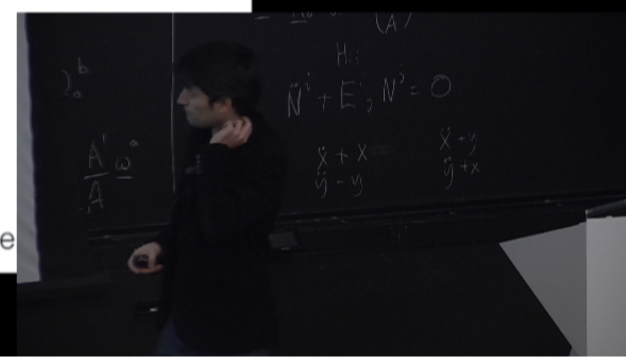
Here we know what the “fundamental” theory is (the theory of everything):
a (non-relativistic) renormalizable scalar quantum field theory with quartic interaction

$$L_{\mathrm{micro}} = \int d^4x\, \Psi^* \partial_t \Psi - \Psi \partial_t \Psi^* \;+\; \Psi^\dagger(\mathbf{x}) \left(-\frac{\bar{\kappa}^2}{2} \nabla^2 - \mu \right) \Psi(\mathbf{x})$$
$$+ \frac{1}{2} \int dt d^3x \int d^3y\; \Psi^\dagger(\mathbf{x}) \Psi^\dagger(\mathbf{y}) U(\mathbf{x}-\mathbf{y}) \Psi(\mathbf{y}) \Psi(\mathbf{x}),$$

to find the effective dynamics of the fluid from the microscopic theory, we need:

- some approximation of microscopic theory
- prove or argue for phase transition to condensed phase
- identify new condensate vacuum
- extract effective dynamics around it

...and all this is quite



An analogy: Bose Condensates

- start by approximating microscopic dynamics as:

$$\hat{H} = \int \hat{\Psi}^*(x) \left(-\frac{\hbar^2 \nabla^2}{2m} + \frac{\kappa}{2} \hat{\Psi}^*(x) \hat{\Psi}(x) \right) \hat{\Psi}(x) \, d^3x,$$

- move away from Fock vacuum $\hat{\Psi} \mid F.S.\rangle = 0$ to a non-trivial vacuum state such that $\hat{\Psi}(x)|G.S.\rangle \approx \psi(x)|G.S.\rangle$
- one assumes that the system is in a macroscopic configuration close to $\mid G.S.\rangle$:

$$\hat{\Psi}(x) \approx \psi(x)\mathbb{I} + \hat{\chi}(x),$$

  with $\psi(x)$ the condensate wavefunction, and $\hat{\chi}$ deviations from the mean field ψ.
- the microscopic equations for $\hat{\Psi}$ give then the effective equations for the order parameter ψ, which are the GP hydrodynamic equations

$$\Psi = \sqrt{\rho} e^{i\theta}$$

collective variables

An analogy: Bose Condensates

maybe:

- quantum space is like a quantum fluid (condensate)
- (some) GFT is like the microscopic field theory for its atoms
- the continuum (hydrodynamic) approximation is appropriate only after a phase transition
- GR is like the effective hydrodynamics of the fluid
- to go from one to the other we should use ideas and methods from statistical field theory and condensed matter theory

GFT - overview of recent results (last 5 years)

- encoding of quantum geometry in GFT - construction of models for 4d quantum gravity

issue: can we construct models that encode correctly simplicial geometry on each complex (GFT Feynman diagram)?

status:

- can build on results in LQG and spin foams (Rovelli, Engle, Pereira, Speziale, Livine, Freidel, ...)
- non-commutative geometry tools allow better encoding/understanding of geometry (Freidel, Livine, Majid, Noui, Baratin, Oriti, ...)
- have several interesting candidate models in 4d, both riemannian and lorentzian (Engle-Pereira-Rovelli-Livine, Freidel-Krasnov, Baratin-Oriti)

GFT - overview of recent results (last 5 years)

- symmetries - diffeomorphisms?

issues: what are the symmetries of your GFT/tensor model? where are the diffeomorphisms?

status:

- the field theory formalism allows to define and study GFT symmetries
- in 3d model, we know the GFT field symmetry corresponding to simplicial diffeos (vertex translations) in the simplicial path integral (GFT Feynman amplitude)
(Baratin-Girelli-Oriti)

GFT - overview of recent results (last 5 years)

- control over perturbative expansion - scaling and divergences, sum over topologies

issues: can control sum over complexes? do manifolds dominate over singular configurations? which manifold topologies dominate the sum? if the Feynman amplitudes are divergent, how do they diverge?

status:

- many results obtained for "colored models" (Gurau)
- good control over combinatorial/topological properties of diagrams (Gurau, Ben Geloun-Krajewski-Magnen-Rivasseau, Tanasa, Ryan, Caravelli, Bonzom-Smerlak)
- several power counting theorems (for topological models) (Freidel-Gurau-Oriti, Ben Geloun-Magnen-Rivasseau, Bonzom-Smerlak, Gurau-Rivasseau,...)
- pseudo-manifolds are generically suppressed

GFT - overview of recent results (last 5 years)

- perturbative GFT renormalization

issue: can GFT models be renormalized as standard QFTs?
is your GFT model renormalizable?

status:

- standard and rigorous QFT renormalization techniques can be adapted and applied to GFTs (and tensor models)
- some calculations of basic divergences and counterterms have been performed for topological models
(Ben Geloun-Bonzom)
- some simplified models have been proven to be renormalizable at all orders
(Ben Geloun-Rivasseau, Ben Geloun-Samary)

GFT - overview of recent results (last 5 years)

- summability and critical behaviour - continuum limit

issues: is the perturbative series (Borel) summable?
is there a phase transition to a continuum space?

status:

- some models have been shown to be Borel summable
(Freidel-Louapre, Magnen-Noui-Rivasseau-Smerlak)

- the critical behaviour of some simple tensor models has been studied (analytically) and a phase transition identified, in the large cut-off limit; the nature of the transition depends on the model
(Bonzom-Gurau-Riello-Rivasseau, Benedetti-Gurau)

GFT - overview of recent results (last 5 years)

- effective (non-perturbative) continuum physics?

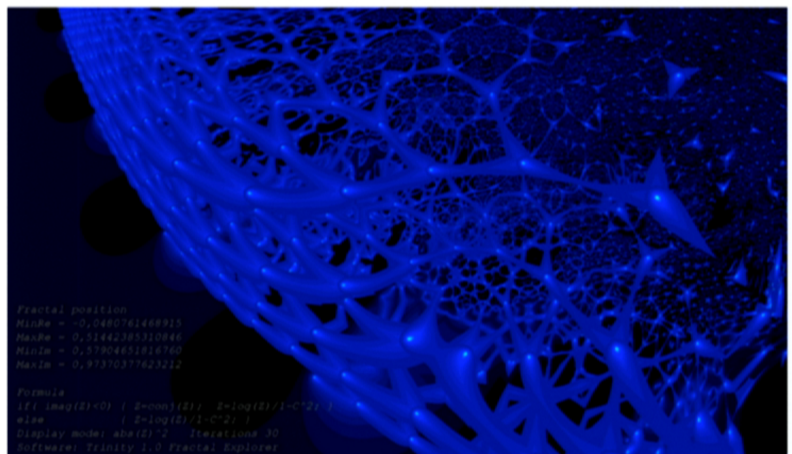
issues: what is the continuum theory? can extract interesting effective physics?

status:

- limited understanding of continuum theory for simple tensor models
- no exact results for more involved models
- algebra of symmetries for the n-point functions (generalizing the Virasoro algebra of matrix models) identified for simple tensor models
  (Gurau, Caravelli-Carrozza-Oriti)
- (limited results) mean field theory approximation resulting in: effective classical equations for geometry, effective Hamiltonian constraint operator for spin networks, effective non-commutative matter field theories
  (Oriti-Sindoni, Livine-Oriti-Ryan, Livine-Fairbairn, Girelli-Livine-Oriti)
- construction of simplified GFT-like models for (quantum) cosmology

Conclusions

...... a bit early to draw conclusions....



Conclusions

...in particular, we should be able to do both (platonic) beautiful mathematics and (aristotelian) real physics!