

Title: Introduction to Massive Gravity

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URL: <http://pirsa.org/12010123>

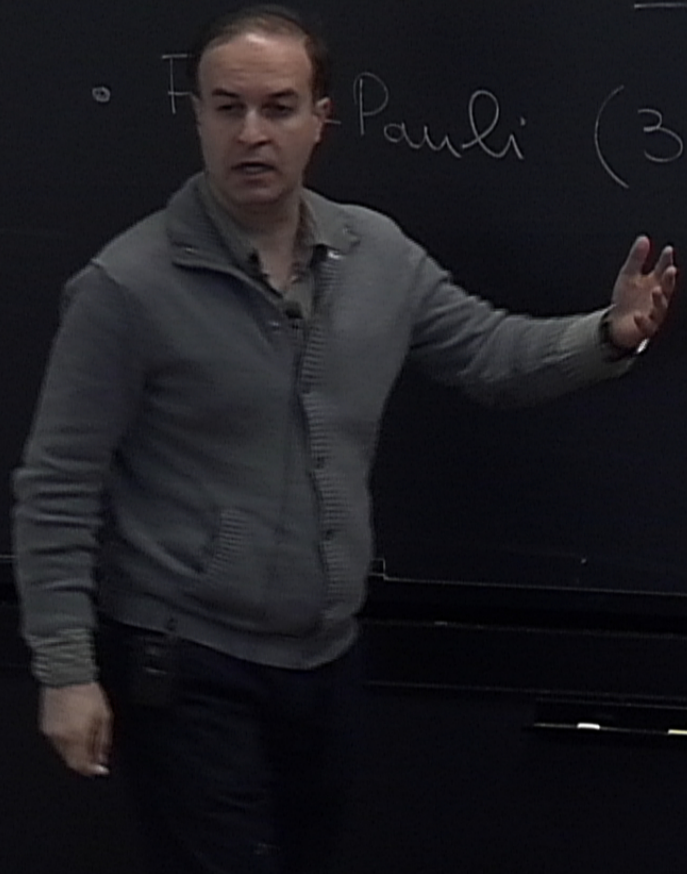
Abstract: For a long time it was believed that a classical nonlinear theory of a single massive spin-2 state does not exist.

Spin-2 $m \neq 0$

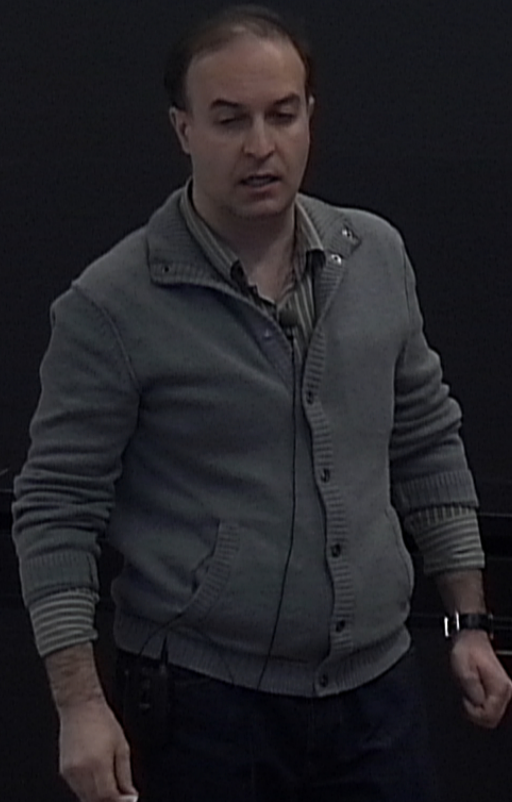
5 dof

• Fierz-Pauli (39)

Theory	}	"Observ"
linear theory		vDVZ
		Vainshtein



Spin-2 $m \neq 0$



Spin-2 $m \neq 0$

5 dof



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5 dof

• Fierz-Pauli (39)

Theory
linear theory

“Observed”

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5 dof

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VDVZ

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vDVZ

Boulware-Deser (71)

Vainshtein



Spin-2 $m \neq 0$

5 dof

• Fierz-Pauli

(39)

(BD)

Theory

linear theory

Boulware-Deser (71)

"Observ"

vDVZ

Vainshtein

Spin-2 $m \neq 0$

5 dof

• Pauli

(39)

Theory

linear theory

“Observ”

vDVZ

(BD)

Boulware-Deser (71)

Vainshtein

branes.

→

decoupling lim.
(Arkani-Hamed, Georgi, Schwartz (AGS))
(Duff and Rubakov, Nicolis et al.)

spin-2 $m \neq 0$

5 dof

• Fierz-Pauli (39)

Theory
linear theory

“Observ”

VDVZ

(BD)

Boulware-Deser (71)

Vainshtein

• 2000 planes →

decoupling lim.

(Arkani-Hamed, Georgi, Schwartz (AGS)
Deffayet, Pombarts, Nicolis et al.

de Rham, GG

de Rham, GG, Tolley

(Arkani-Hamed, Georgi, Schwartz (AGS)
(Deffayet, Kombers, Niessl et al.

de Rham, GG, Tolley

Massive vector field

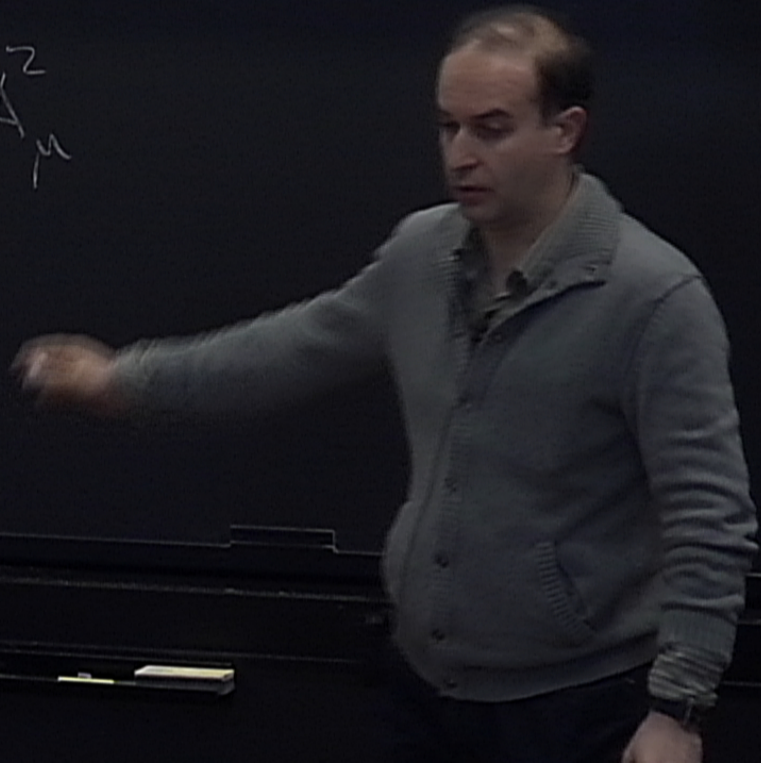
$$\alpha = -\frac{1}{\Lambda^2}$$

(Arkani-Hamed, Georgi, Schwarz (AGS)
Deffayet, Kombers, Nielsen et al.

de Rham, Ge, Tolley

Massive vector field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2$$



(Arkan-Hamed, Georgi, Schwartz (AGS)
Deffayet Kombeants, Nielsen et al.

de Rham, GG, Tolley

Massive vector field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2$$

Stückelberg
method

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 \left(A_\mu - \frac{\partial_\mu \pi}{m} \right)^2$$

(Arkani-Hamed, Georgi, Schwarz (AGS)
Deffayet Kombeants, Niels et al.

de Rham, GG, Tolley

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(Arkani-Hamed, Georgi, Schwarz (AGS)
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de Rham, Ge, Tolley

Massive vector field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2$$

$\delta A_\mu = \partial_\mu \alpha$

Stückelberg
method

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 \left(A_\mu - \frac{\partial_\mu \pi}{m} \right)^2$$

(Arkani-Hamed, Georgi, Schwarz (AGS)
Deffayet, Kombers, Nielsen et al.

de Rham, Ge, Tolley

Massive vector field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 A_\mu^2$$

$$\delta A_\mu = \partial_\mu \alpha \quad \delta \pi = m \alpha$$

Stückelberg
method

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} m^2 \left(A_\mu - \frac{\partial_\mu \pi}{m} \right)^2$$

$m \rightarrow 0$

$$\mathcal{L}_{\text{dec}} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (\partial_\mu \pi)^2$$

\uparrow
 $2g$

\uparrow
 $1g$

$m \rightarrow 0$

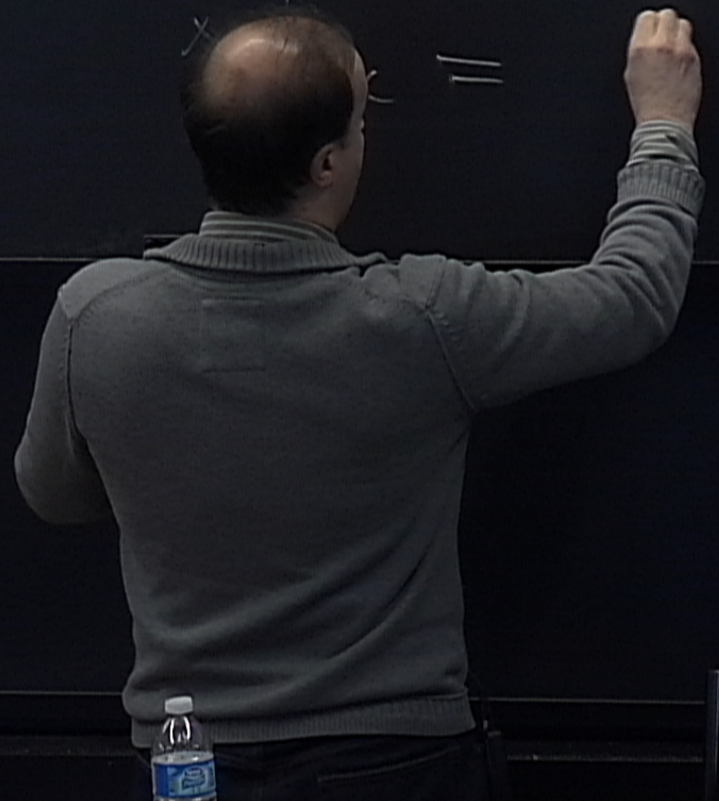
$$\mathcal{L}_{dec} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} (\partial_\mu \pi)^2$$

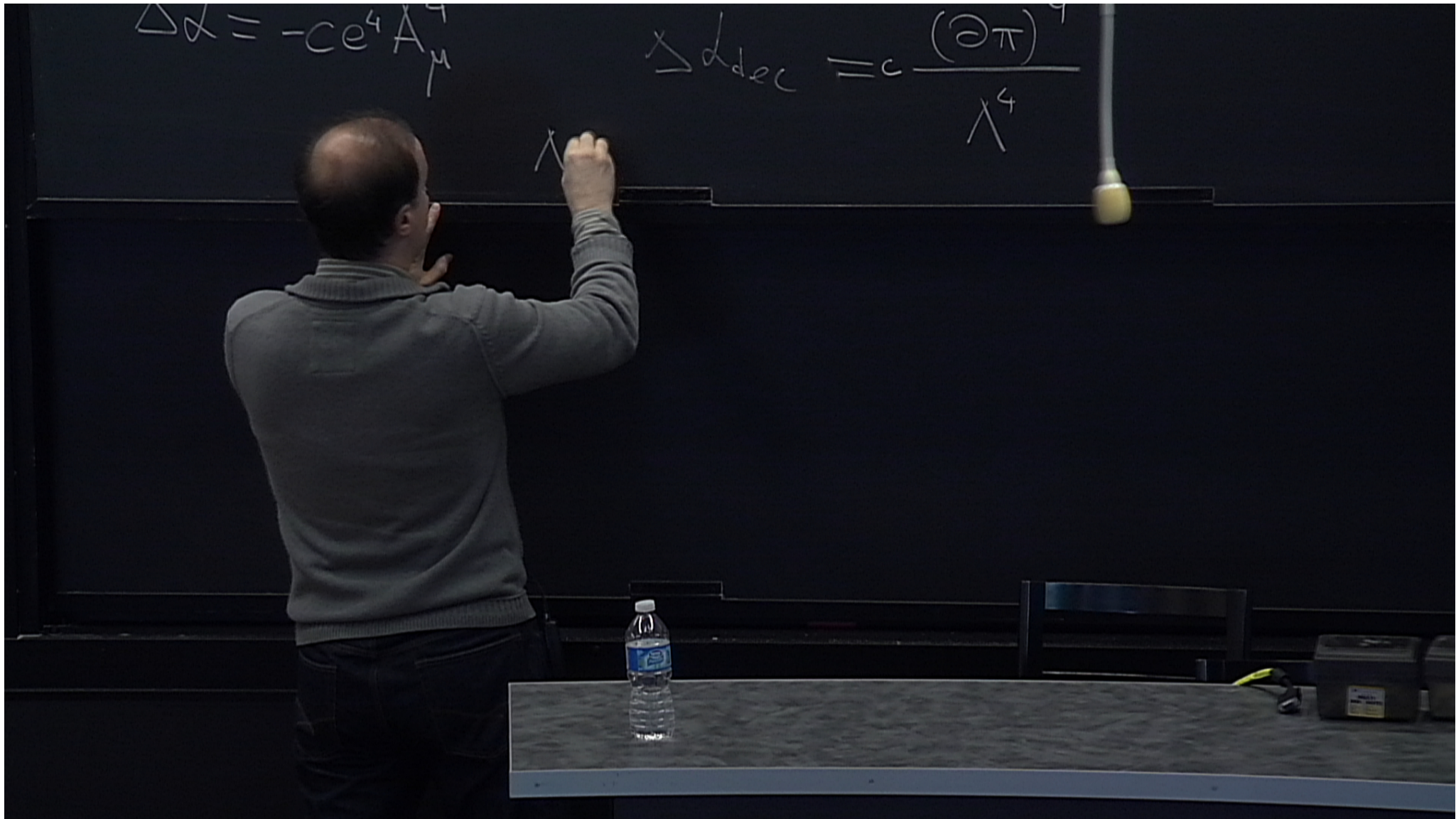
\uparrow
 $2g$

\uparrow
 g

$$\Delta \mathcal{L} = -ce^4 A_\mu^4$$

$$\Delta d = -ce^4 A_M^4$$



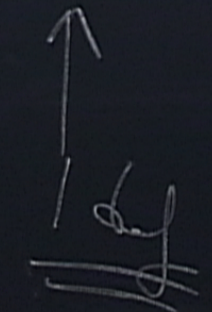
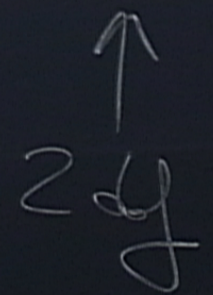


$$\Delta\alpha = -ce^4 A_\mu^4$$

$$\Delta\alpha_{dec} = c \frac{(\partial\pi)^4}{\Lambda^4}$$

$$\Lambda \equiv m/e$$

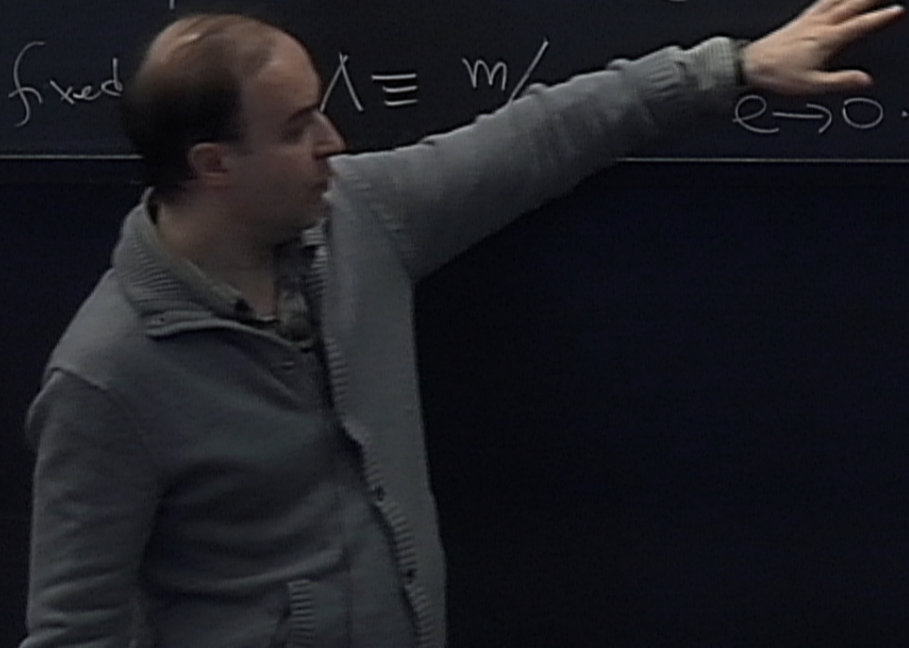
$$\Delta \mathcal{L} = -c e^4 A_\mu^4$$



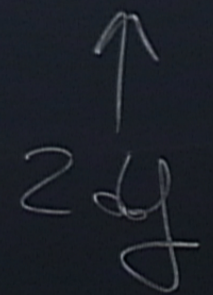
$$\Delta \mathcal{L}_{dec} = c \frac{(\partial \pi)^4}{\Lambda^4}$$

$e \rightarrow 0.$

fixed $\Lambda \equiv m/$

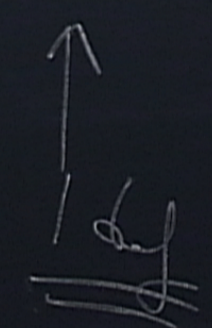


$$\Delta \mathcal{L} = -c e^4 A_\mu^4$$



$$\lambda_{dec}$$

$$= c \frac{(\partial \pi)^4}{\lambda^4}$$



$$\text{fixed} = \lambda \equiv m/e$$

$$e \rightarrow 0.$$

$$\Delta\alpha = -ce^4 A_{\mu}^4$$

$$\Delta\alpha_{dec} = c \frac{(\partial\pi)^4}{\Lambda^4}$$

$$\text{fixed} = \Lambda \equiv m/e \quad e \rightarrow 0.$$

FP: Lin. th.

$$\Delta \alpha = -c e^4 A_{\mu}^4 \quad \Delta d_{dec} = c \frac{(\partial \pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e$ $e \rightarrow 0.$

FP: Lin. th. $h_{\mu\nu}$

$$\alpha = -\frac{M_p^2}{2} h_{\mu\nu} \left(\alpha_{\beta} h^{\alpha\beta} - \frac{M_p^2 m^2}{4} (h_{\mu\nu}^2 - h^2) \right)$$

$$\Delta \alpha = -c e^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial \pi)^4}{\Lambda^4}$$

fixed = $\Lambda \equiv m/e$ $e \rightarrow 0.$

FP: Lin. th. $h_{\mu\nu}$

$$\alpha = -\frac{M_p^2}{2} h_{\mu\nu} F^{\mu\nu} h^{\alpha\beta} - \frac{M_p^2 m^2}{4} (h_{\mu\nu}^2 - h^2)$$

$$(E_h)_{\mu\nu} = -\frac{1}{2} \left(D h_{\mu\nu} - \partial_\mu \partial_\alpha h_\nu^\alpha - \partial_\nu \partial_\alpha h_\mu^\alpha + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h + \eta_{\mu\nu} \right)$$

$$\delta \mathcal{L} = -ce^4 A_\mu^4$$

$$\Delta \mathcal{L}_{loc} = c \frac{(\partial \pi)^4}{\Lambda^4}$$

$$f_{\text{red}} = \lambda \equiv \frac{m}{e} \quad e \rightarrow 0. \quad \Lambda^4$$

EP Lin th. $h_{\mu\nu}$

$$\mathcal{L} = -\frac{M_{\text{Pl}}^2}{2} h_{\mu\nu} E^{\mu\nu} - \frac{M_{\text{Pl}}^2 m^2}{4} (h_{\mu\nu}^2 - h^2)$$

$$(E^h)_{\mu\nu} = -\frac{1}{2} \left(D h_{\mu\nu} - \partial_\mu \partial_\alpha h_\nu^\alpha - \partial_\nu \partial_\alpha h_\mu^\alpha + \partial_\mu \partial_\nu h - \eta_{\mu\nu} D h + \eta_{\mu\nu} \partial^\alpha \partial_\alpha h \right)$$

$$\Delta \alpha = -c e^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial \pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e \quad e \rightarrow 0.$

FP: Lin. th. $h_{\mu\nu}$

$$\alpha = - \underbrace{\frac{M_p^2}{2} h_{\mu\nu} F_{\alpha\beta}^{\mu\nu} h^{\alpha\beta}}_{\alpha_2} - \frac{M_p^2 m^2}{4} (h_{\mu\nu}^2 - h^2)$$

$$(E_h)_{\mu\nu} = -\frac{1}{2} \left(D h_{\mu\nu} - \partial_\mu \partial_\alpha h_\nu^\alpha - \partial_\nu \partial_\alpha h_\mu^\alpha + \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h + \eta_{\mu\nu} \right)$$

(Arkanj-Hamed, Georgi, Schwarz (AGS)
(Deffayet-Rombouts, Nilles et al.

de Rham, GG, Tolby

$$\mathcal{L} = \mathcal{L}_2 - \frac{M_{\text{pl}}^2}{4} \left(h_{\mu\nu} - \frac{2x + 2v A_{\mu}}{2} \right)$$

(Arkanj Hamed, Georg, Schartz (AGS)
Deffner Kombants, Nils et al.

de Ruum, GG, Tolby

$$L = L_2 - \frac{M^2 \mu^2}{4} \left(\left(h_{\mu} - \frac{\partial_{\mu} A_{\nu} + \partial_{\nu} A_{\mu}}{m} - \frac{2 \partial_{\mu} \partial_{\nu} \pi}{m^2} \right)^2 - \left(h - \frac{2 \partial A}{m} - \frac{2 \partial \pi}{m^2} \right)^2 \right)$$

$$\Delta \alpha = -ce^2 A_\mu^2$$

$$\Delta \alpha_{loc} = c \frac{(\partial \pi)^2}{\Lambda^4}$$

$$f_{\text{ind}} = \lambda \equiv m/e \quad e \rightarrow 0$$

EP: $\int \mathcal{L}_m + \mathcal{L}_h$ $h_{\mu\nu}$

$$\mathcal{L} = -\frac{M_{\text{Pl}}^2}{2} h_{\mu\nu} \underbrace{E^{\mu\nu}}_{\mathcal{L}_2} - \frac{M_{\text{Pl}}^2}{4} \left(h_{\mu\nu}^2 - h^2 \right)$$



$$h_{\mu\nu} - \partial_\mu \partial_\nu h + \partial_\nu \partial_\mu h - \partial_\mu \partial_\nu h - \eta_{\mu\nu} \square h + \eta_{\mu\nu} \partial_\alpha \partial^\alpha h$$

(Arkan-Hamed, Georgi, Schwartz (AGS)
 (Deffayet Kombeants, Nilles et al.

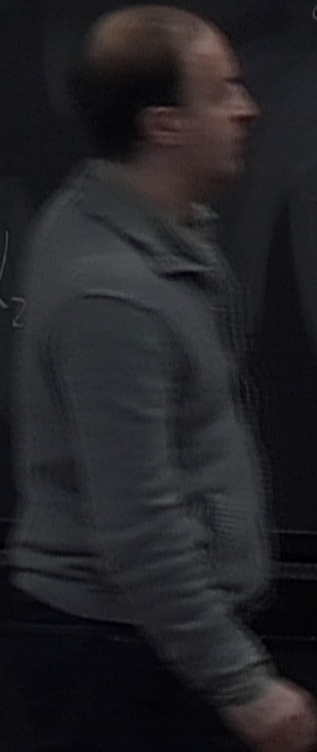
de Rham, Ge, Tolley

$$\begin{aligned}
 \mathcal{L} = \mathcal{L}_2 - \frac{f^2 m^2}{4} & \left(\left(h_{\mu\nu} - \frac{\partial_\mu A_\nu + \partial_\nu A_\mu}{m M_p} - \frac{2 \partial_\mu \partial_\nu \pi}{m^2 M_p} \right)^2 \right. \\
 & \left. - \left(h - \frac{2 \partial A}{m M_p} - \frac{2 \partial \pi}{m^2 M_p} \right)^2 \right)
 \end{aligned}$$

(Arkanj Hamed, Georg, Schachtz (AGS)
 Deffner Kombarts, Nils et al.

de Rham, GG, Tolley

$$\begin{aligned}
 \alpha &= \alpha_2 - \frac{\hbar^2 m^2}{4} \left(\left(h_{\mu\nu} - \frac{2\eta_{\mu\nu} A + 2\nu A_{\mu\nu}}{m} - \frac{2\partial_{\mu}\partial_{\nu}\pi}{m^2} \right)^2 \right. \\
 &= \alpha_2 \left. - \left(h - \frac{2\partial A}{m} - \frac{2\partial\pi}{m^2} \right)^2 \right)
 \end{aligned}$$

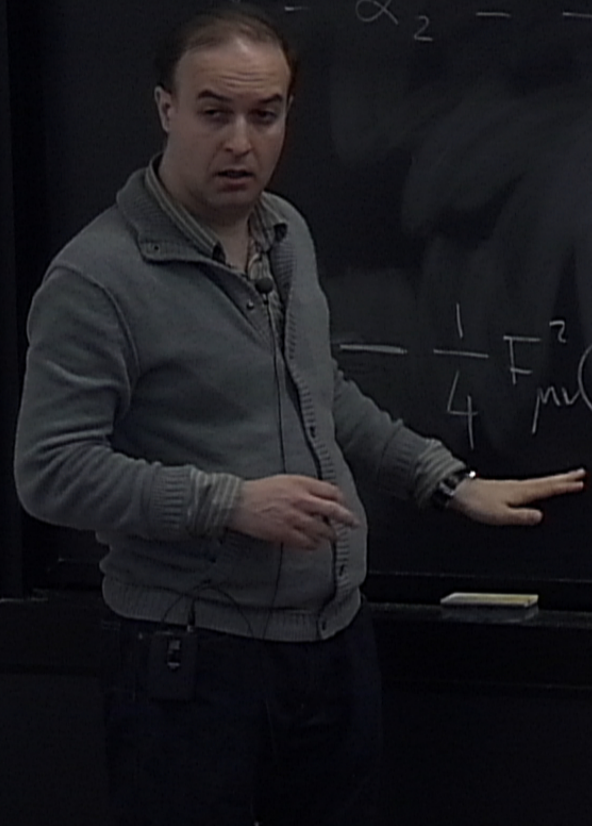


(Arkan-Hamed, Georgi, Schwartz (AGS)
 Deffayet Kombers, Nicolis et al.

de Rham, Ge, Tolley

$$L_2 = \frac{F^2 m^2}{4} \left(\left(h_{\mu\nu} - \frac{2\partial_\mu \partial_\nu A}{m} - \frac{2\partial_\mu \partial_\nu \pi}{m^2} \right)^2 - \left(h - \frac{2\partial A}{m} - \frac{2\partial \pi}{m^2} \right)^2 \right)$$

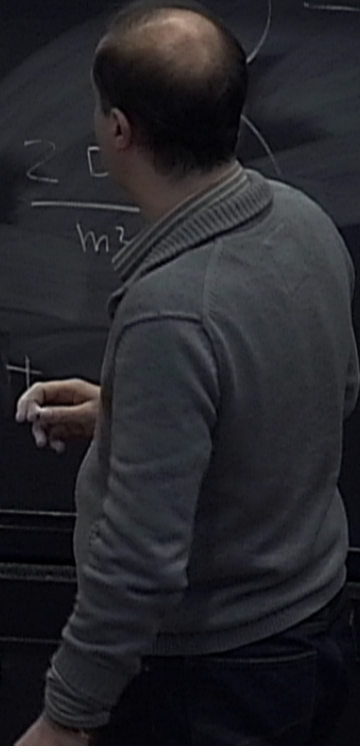
$$= \frac{1}{4} F^2 m^2 (A)$$



(Arkanj-Hamed, Georgi, Schwartz (AGS)
 Deffayet Kombearts, Nilles et al.

de Rham, Ge, Tolley

$$\begin{aligned}
 \mathcal{L} &= \mathcal{L}_2 - \frac{F^2 m^2}{4} \left(h_{\mu\nu} - \frac{\partial_\mu A \partial_\nu A}{m^2} - \frac{2 \partial_\mu \partial_\nu \pi}{m^2} \right)^2 \\
 &= \mathcal{L}_2 - \frac{1}{4} F^2_{\mu\nu}(A) - h_{\mu\nu} (\partial_\mu \partial_\nu \pi - \eta_{\mu\nu} \square \pi) + \dots
 \end{aligned}$$



(Arkan-Hamed, Georgi, Schwartz (AGS)
 (Deffayet Kombeants, Nicolis et al.

de Rham, Ge, Tolley

$$\begin{aligned}
 \mathcal{L} &= \mathcal{L}_2 - \frac{F^2 m^2}{4} \left(\left(h_{\mu\nu} - \frac{\partial_\mu A \partial_\nu A}{m^2} - \frac{2 \partial_\mu \partial_\nu \pi}{m^2} \right)^2 - \left(h - \frac{2 \partial A}{m} - \frac{2 \partial \pi}{m^2} \right)^2 \right) + h_{\mu\nu} T_{\mu\nu} \\
 &= \mathcal{L}_2 - \frac{1}{4} F_{\mu\nu}^2 - h_{\mu\nu} (\partial_\mu \partial_\nu \pi - \eta_{\mu\nu} \partial^2 \pi) + \frac{(\partial^2 \pi)^2 - (\partial \pi)^2}{m^2} + \mathcal{O}(m) + h_{\mu\nu} T_{\mu\nu}
 \end{aligned}$$

(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Komarsnik, Nicolis et al.

de Rham, Ge, Tolley

$$\begin{aligned}
 \mathcal{L} &= \mathcal{L}_2 - \frac{m^2}{2} \left(h_{\mu\nu} - \frac{\partial_\mu A_\nu + \partial_\nu A_\mu}{m} - \frac{2 \partial_\mu \partial_\nu \pi}{m^2} \right)^2 \\
 &= \mathcal{L}_2 - \frac{1}{2} \left(h_{\mu\nu} - \frac{2 \partial A}{m} - \frac{2 \partial \pi}{m^2} \right)^2 + h_{\mu\nu} T_{\mu\nu} \\
 &= \mathcal{L}_2 - \frac{1}{2} \left(h_{\mu\nu} (\partial_\mu \partial_\nu \pi - h_{\mu\nu} \partial \pi) + \frac{(\partial \pi)^2 - (\partial \pi)^2}{m^2} \right) + O(m) + h_{\mu\nu} T_{\mu\nu}
 \end{aligned}$$

$$\Delta\alpha = -ce^4 A_{\mu}^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

$$\text{dec. limit.} \quad m \rightarrow 0 \quad M_p \rightarrow \infty$$

$$m^{\#} M_p = \text{fixed}$$

$$\Delta\alpha = -ce^4 A_\mu^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

dec. limit. $m \rightarrow 0$ $M_p \rightarrow \infty$

$m^\#$ $M_p = \text{fixed}$
 $T \rightarrow \infty$

$$\mathcal{L} = \mathcal{L}_2(h) - \frac{1}{4} F_{\mu\nu}^2 - h_{\mu\nu} (\partial_\mu \partial_\nu \pi - v_{\mu\nu} \partial \pi) + h_{\mu\nu} \overline{T}^{\mu\nu}$$

$$\Delta\alpha = -ce^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e \quad e \rightarrow 0.$

dec. limit. $m \rightarrow 0 \quad M_p \rightarrow \infty$

$m^\# \quad M_p = \text{fixed}$
 $T \rightarrow \infty$

$$\mathcal{L} = \underbrace{\alpha_2(h)}_{\text{lin. dif.}} - \underbrace{\frac{1}{4} F_{\mu\nu}^2}_{O(1)} - \left(\partial_\mu \partial_\nu \pi - \nu_{\mu\nu} \partial \pi \right) + h_{\mu\nu} \bar{T}^{\mu\nu}$$

$h_{\mu\nu} + h$

$$\Delta\alpha = -ce^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e$ $e \rightarrow 0.$

dec. limit. $m \rightarrow 0$ $M_p \rightarrow \infty$

$m^\# M_p = \text{fixed}$
 $T \rightarrow \infty$

$$\mathcal{L} = \underbrace{\alpha_2(h)}_{\text{lin. dif.}} - \underbrace{\frac{1}{4} F_{\mu\nu}^2}_{\text{0(1)-2 dof.}} - h_{\mu\nu} (\partial_\mu \partial_\nu \pi - v_{\mu\nu} \partial \pi) + h_{\mu\nu} \overline{T}^{\mu\nu}$$

$h_{\mu\nu} \rightarrow h_{\mu\nu} + h_{\mu\nu} \pi$

$$\Delta\alpha = -ce^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e$ $e \rightarrow 0.$

dec. limit. $m \rightarrow 0$ $M_p \rightarrow \infty$

$m^\#$ $M_p = \text{fixed}$
 $T \rightarrow \infty$

$$\alpha = \underbrace{\alpha_2(h)}_{\text{lin. dif.}} - \underbrace{\frac{1}{4} F_{\mu\nu}^2}_{\text{0(1)-2dof}} - \frac{3}{2} (\partial\pi)^2 + \pi \bar{T} + h_{\mu\nu} \bar{T}_{\mu\nu}$$

$h_{\mu\nu} \rightarrow h_{\mu\nu} + h_{\mu\nu} \pi$

$$\Delta\alpha = -ce^4 A_\mu^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

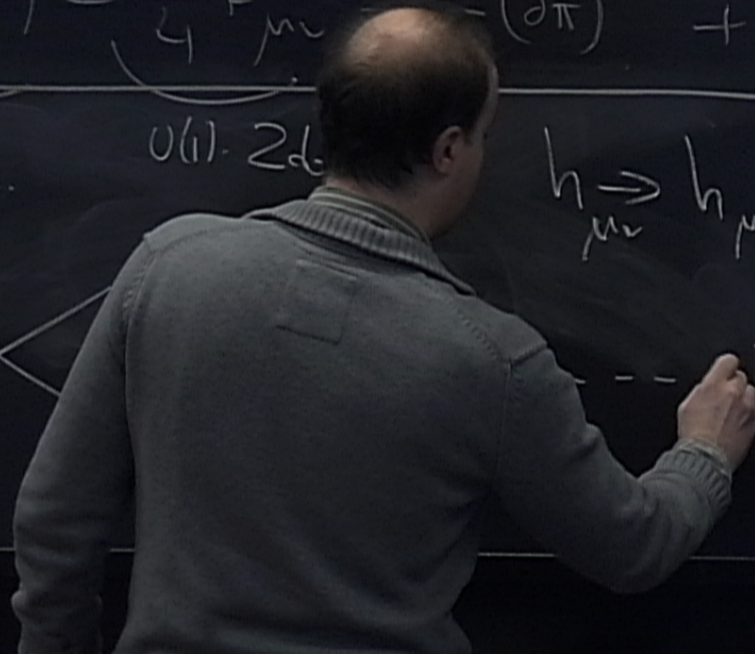
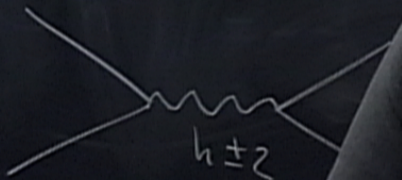
$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

$$\mathcal{L} = \mathcal{L}_2(h) - \frac{1}{4} F_\mu^2 - \frac{3}{2} (\partial\pi)^2 + \pi \bar{T} + h_{\mu\nu} \bar{T}^{\mu\nu}$$

lin. dif.

U(1) - Z₂

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + h_{\mu\nu} \pi$$



$$\Delta\alpha = -ce^4 A_\mu^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

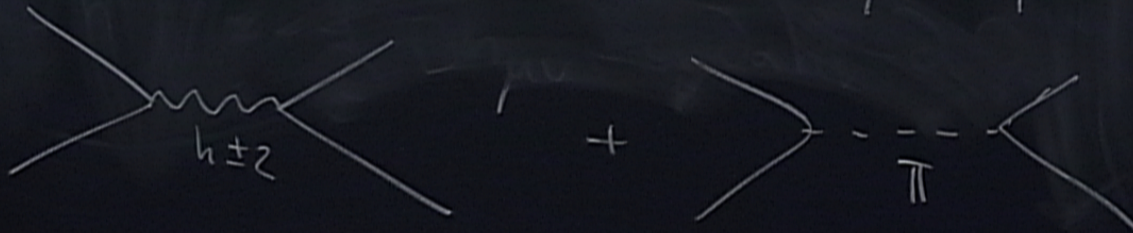
$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

$$\mathcal{L} = \mathcal{L}_2(h) - \frac{1}{4} F_\mu^2 - \frac{3}{2} (\partial\pi)^2 + \pi \bar{T} + h_{\mu\nu} \bar{T}^{\mu\nu}$$

lin. dif.

U(1)-Zdf

$$h_\mu \rightarrow h_\mu + b_\mu \pi$$



$$\Delta\alpha = -ce^4 A_\mu^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

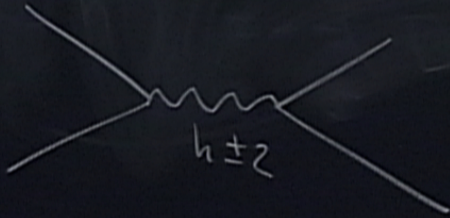
$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

$$\mathcal{L} = \mathcal{L}_2(h) - \frac{1}{4} F_\mu^2 - \frac{3}{2} (\partial\pi)^2 + \pi \bar{T} + h_{\mu\nu} \bar{T}^{\mu\nu}$$

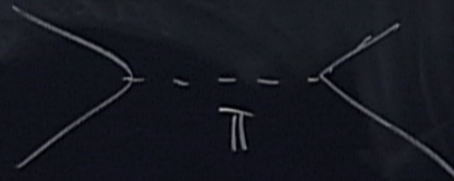
lin. dif.

U(1)-Zdf

$$h_\mu \rightarrow h_\mu + b_\mu \pi$$



+

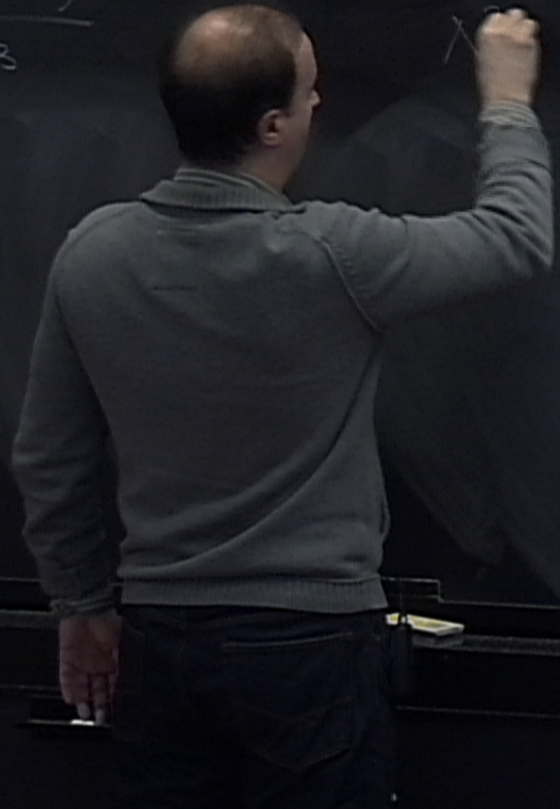


$\nu D\nu Z$

(Arhancini, Flaminio, Georgi, Schwartz (AGS)
(Deffayet, Kombersht, Nilles et al.

de Rham, GG, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\wedge^3}$$



(Arhoni-Horned, Georgi, Schwarz (AGS)
(Deffayet, Kombers, Nilles et al.)

de Rham, GG, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$\Lambda^3 = m^2 M_p$$

(Arkani-Hamed, Georgi, Schwartz (AGS)
Deffayet, Komarsnik, Nicolis et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$\Lambda^3 = m^2 M_p$$

$$\square \pi - \frac{(\partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \frac{1}{1}$$

(Arhany, Hamed, Georg, Schachtz (AGS)
Deffayet, Komars, Nilles et al.

de Rham, GG, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$\Lambda^3 = m^2 M$$

$$\square \pi - \frac{(\partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \frac{\square \pi}{\Lambda^3}$$

(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Komarsnik, Nicolis et al.

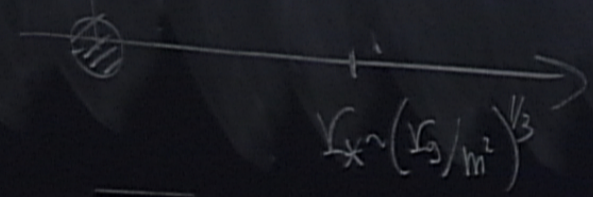
de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$= \square \pi - \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \frac{\square \pi}{\Lambda^3}$$

$$\Lambda^3 = m^2 M_p$$

Vainshtein,



(Arkani-Hamed, Georgi, Shiu (AGS)
 Deffayet, Komarsnik, Niessl et al.

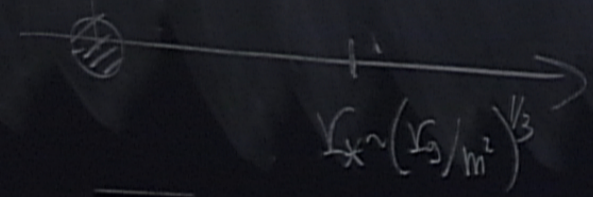
de Rham, Ge, Tolley

$$\Delta \mathcal{L}_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$\Lambda^3 = m^2 M_p$$

$$\square \pi - \frac{(\partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \frac{\square \pi}{\Lambda^3}$$

Vainshtein,



(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet Kombearts, Nicolis et al.

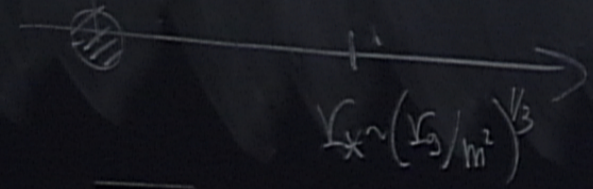
de Rham, GG, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

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$$\square \pi - \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \dots$$

Vainshtein,



(Arkani-Hamed, Georgi, Strassler (AGS)
 Deffayet Kombearts, Nilles et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

$$\Lambda^3 = m^2 M_p$$

$$\square \pi = \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \frac{\dots}{\Lambda^3}$$

Vainshtein,

$$k_p \sim k_p \sim \frac{H}{M_p} \sim \frac{r_s}{r}$$

$$k_* \sim (r_s/m^2)^{1/3}$$



(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Komarskii, Nicolis et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3}$$

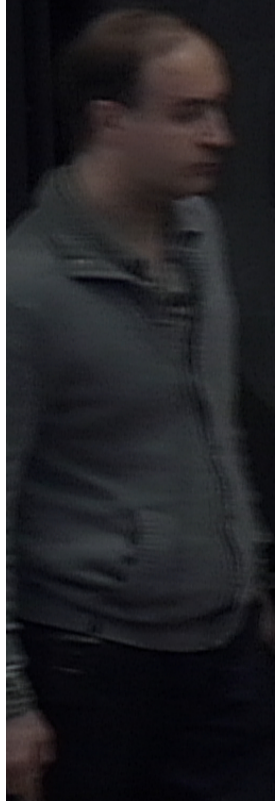
$$\Lambda^3 = m^2 M_p$$

$$\square \pi - \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \dots$$

$\frac{\pi}{M_p} \sim \frac{v_0}{k} \sim \frac{v_0}{k} \left(\frac{k}{\Lambda}\right)^{3/2}$
 $k \sim \left(\frac{v_0}{m}\right)^{1/3}$

Vainshtein,

$$\frac{\pi}{M_p} \sim \frac{v_0}{k}$$



(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet Komarskii, Nicolis et al.

de Rham, GG, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3} \quad \frac{(\partial^2 \pi)^3}{\Lambda^5}$$

Vainshtein,

$$\frac{\hbar}{M_p} \sim \frac{\hbar^2}{M^2} \left(\frac{k}{k_*} \right)^{3/2}$$

$$\frac{\hbar}{M_p} \sim \frac{k_0}{k}$$

$$k_* \sim \left(\frac{k_0}{m^2} \right)^{1/3}$$

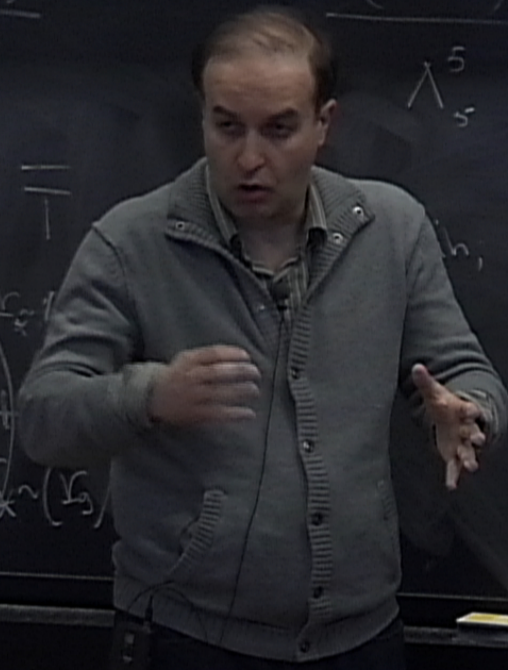
Arkani-Hamed, Georgi, Strassler (AGS)
 Deffayet, Komarsnik, Nilles et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3} \sim \frac{(\partial^2 \pi)^3}{\Lambda^5} \sim \frac{(\partial^2 \pi)_\alpha (\partial^2 \pi)^2}{\Lambda^5}$$

$$\square \pi - \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \dots$$

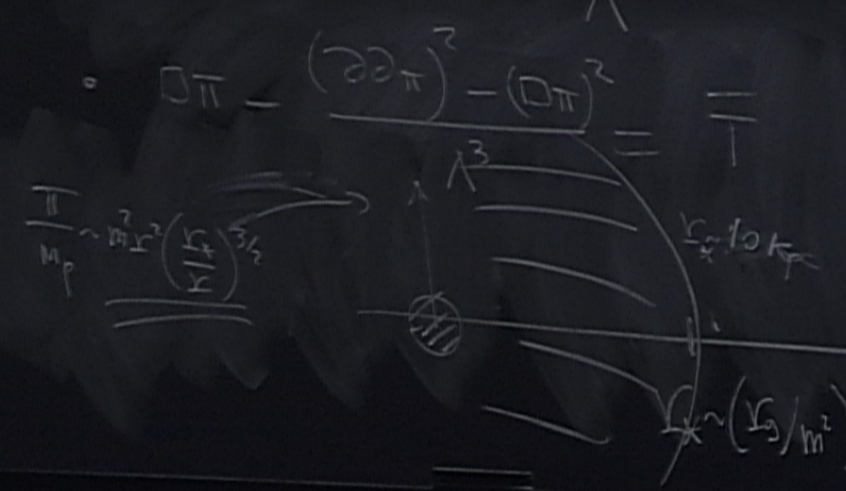
$\frac{\pi}{M_p} \sim \frac{1}{M_p^2} \left(\frac{1}{\Lambda}\right)^{3/2}$
 $\Lambda \sim \left(\frac{M_p}{\sqrt{2}}\right)^{2/3}$
 $\Lambda \sim (M_p)^{2/3}$



(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Komberski, Nicolis et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3} \sim \frac{(\partial^2 \pi)^3}{\Lambda^5} \sim \frac{(\partial^2 \pi)_\alpha}{\Lambda^5} (\partial^2 \delta \pi)^2$$



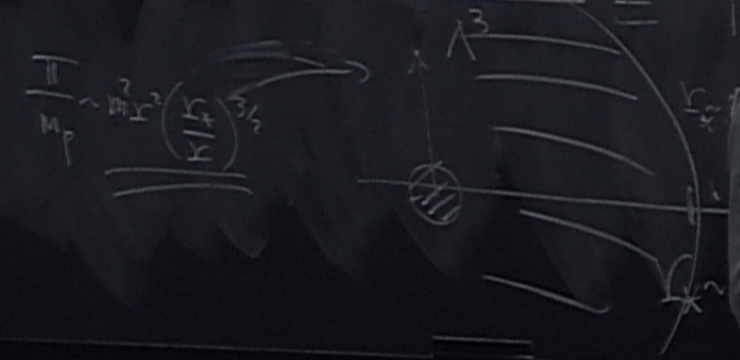
Vainshtein,

Arkani-Hamed, Georgi, Schwarz (AGS)
 Deffayet, Komarsnik, Nilles et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3} \quad \frac{(\partial^2 \pi)^3}{\Lambda^5} \sim \frac{(\partial^2 \pi)_\alpha}{\Lambda^5} (\partial^2 \delta \pi)^2$$

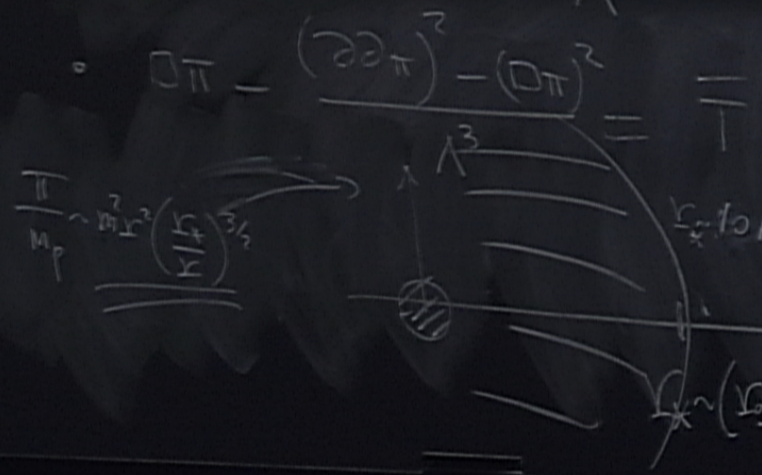
$$\square \pi - \frac{(\partial \partial \pi)^2 - (\square \pi)^2}{\Lambda^3} = \dots$$



(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet Kombearts, Nilles et al.

de Rham, Ge, Tolley

$$\Delta \alpha_{NL} = \frac{\square \pi (\partial_\mu \pi)^2}{\Lambda^3} \sim \frac{(\partial^2 \pi)^3}{\Lambda^5} \sim \frac{(\partial^2 \pi \alpha)}{\Lambda^5} (\partial^2 \delta \pi)^2$$



Vainshtein,

$$\Delta\alpha = -ce^4 A_{\mu}^4$$

$$\Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e$ $e \rightarrow 0$.

$$\mathcal{L} = \frac{M_p^2}{2} \sqrt{g} R - m^2 \left(h_{\mu\nu}^2 - h^2 + \bar{c}_1 h_{\mu\nu}^3 + \bar{c}_2 h_{\mu\nu}^2 h + \bar{c}_3 h^3 + \dots \right)$$

$$\Delta\alpha = -ce^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Delta \alpha = -c e^4 A_\mu^4$$

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$$= h_{\mu\nu} + h_{\mu\nu}$$

$$= \partial_\mu \varphi^a \partial_\nu \varphi^b \eta_{ab} + H_{\mu\nu}$$

$$\varphi^a \quad a=0,1,2,3$$

$$\Delta \alpha = -c e^4 A_\mu^4$$

$$\Delta d_{dec} = c \frac{(\partial \pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e$ $e \rightarrow 0$.

$$\mathcal{L} = \frac{M_p^2}{2} \sqrt{g} R - m^2 \left(h_{\mu\nu}^2 - h^2 + \bar{c}_1 h_{\mu\nu}^3 + \bar{c}_2 \partial_{\mu\nu}^2 h + \bar{c}_3 h^3 + \dots \right)$$

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$$g_{\mu\nu} = \partial_\mu \varphi^a \partial_\nu \varphi^b \eta_{ab} + H_{\mu\nu} \quad \varphi^a \quad a=0,1,2,3$$

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$$\text{fixed} = \lambda \equiv m/e \quad e \rightarrow 0.$$

$$\mathcal{L} = \frac{M_{Pl}^2}{2} \sqrt{g} R - m^2 \left(h_{\mu\nu}^2 - h^2 + \bar{c}_1 h_{\mu\nu}^3 + \bar{c}_2 \partial_\mu h^2 + \bar{c}_3 h^3 + \dots \right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$g_{\mu\nu} = \partial_\mu y^a \partial_\nu y^b \eta_{ab} + H_{\mu\nu} \quad y^a \quad a=0,1,2,3$$

Unit-gauge $y^a = x^a$

$$\Delta \alpha = -c e^4 A_\mu^4 \quad \Delta d_{dec} = c \frac{(\partial \pi)^4}{\lambda^4}$$

fixed = $\lambda \equiv m/e \quad e \rightarrow 0.$

$$\mathcal{L} = \frac{M_p^2}{2} \sqrt{g} R - m^2 \left(h_{\mu\nu}^2 - h^2 + \bar{c}_1 h_{\mu\nu}^3 + \bar{c}_2 \partial_{\mu\nu}^2 h + \bar{c}_3 h^3 + \dots \right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$g_{\mu\nu} = \partial_\mu y^a \partial_\nu y^b \eta_{ab} + H_{\mu\nu} \quad y^a \quad a=0,1,2,3$$

Unit gauge $y^a = x^a$

$$\Delta\alpha = -ce^4 A_M^4 \quad \Delta d_{dec} = c \frac{(\partial\pi)^4}{\lambda^4}$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$g_{\mu\nu} = \partial_\mu y^a \partial_\nu y^b \eta_{ab} + H_{\mu\nu}$$

$$y^a \quad a=0,1,2,3$$

Unit. gauge

$$y^a = x^a$$

$$y^a = x^a - \frac{mA_M^a}{m^2 M_P} - \frac{\partial^a \pi}{m^2 M_P}$$

(Arkan-Hamed, Georgi, Schwartz (AGS)
Duff and Rubakov, Nilles et al.

de Rham, Ge, Tolley

$$\chi = \frac{M_{\text{pl}}^2}{2} \sqrt{g} R - \frac{M_{\text{pl}}^2 m^2}{4} \sqrt{g} \left(H_{\text{pl}}^2 - H^2 + c_1 H_{\text{pl}}^3 + c_2 H_{\text{pl}}^2 H + c_3 H^3 + \dots \right)$$

(Arkani-Hamed, Georgi, Schwarz (AGS)
 Deffayet, Kombers, Niess et al.

de Rham, Ge, Tolley

$$-\frac{M_{\text{pl}}^2}{2} \sqrt{g} R - \frac{M_{\text{pl}}^2 m^2}{4} \sqrt{g} \left(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H_{\mu\nu}^2 H + c_3 H^3 + \dots \right)$$

$$c_1, c_2, c_3 \rightarrow \left\{ c_3 \right\}$$

No BD ghost app in de-f.

(Arkani-Hamed, Georgi, Schwarz (AGS)
 Duff and Romants, Nilles et al.

de Rham, Ge, Tolley

$$\alpha = \frac{M_p^2}{4} \sqrt{g} R - \frac{M_p^2 m^2}{4} \sqrt{g} \left(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H_{\mu\nu}^2 H + c_3 H^3 + d_1 \dots \right)$$

$c_1, c_2, c_3 \rightarrow \{c_3\}$
 $\dots, d_5 \rightarrow \{d_5\}$

No BD ghost app in de-f.
 No BD.

(Arkani-Hamed, Georgi, Schwarz (AGS)
 Deffayet, Kombers, Niels et al.

de Rham, Ge, Tolley

$$\alpha = \frac{M_{pl}^2}{2} \sqrt{g} R - \frac{M_{pl}^2 m^2}{4} \left(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H_{\mu\nu}^2 H + c_3 H^3 + d_1 \dots \right)$$

$\{c_1, c_2, c_3\}$
 $\{d_1, \dots, d_5\}$
 functions

No BD ghost app in de R.

No BD.

No BD.

(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Komarsnik, Nicolis et al.

de Rham, Ge, Tolley

$$\alpha = \frac{M_{\text{pl}}^2}{4} \sqrt{g} R - \frac{M_{\text{pl}}^2 m^2}{4} \sqrt{g} \left(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H_{\mu\nu}^2 H + c_3 H^3 + d_1 \dots \right)$$

$$c_2, c_3 \rightarrow \{c_3\}$$

$$\dots, d_5 \rightarrow \{d_5\}$$

$$\dots \rightarrow \{0\}$$

No BD ghost app in de-f.

No BD.

No BD.

(Arkani-Hamed, Georgi, Schwartz (AGS)
 Deffayet, Kombers, Niiles et al.

de Rham, Ge, Tolley

$$\alpha = \frac{M_p^2}{2} \sqrt{g} R - \frac{M_p^2 m^2}{4} \sqrt{g} \left(H_{\mu\nu}^2 - H^2 + C_1 H_{\mu\nu}^3 + C_2 H_{\mu\nu}^2 H + C_3 H^3 + d_1 \dots \right)$$

- \exists $C_1, C_2, C_3 \rightarrow \{C_3\}$
- $d_1, \dots, d_5 \rightarrow \{d_5\}$
- $f_{\mu\nu} \rightarrow \{0\}$

No BD ghost app in de-f.
 No BD.
 No BD.