Title: Goldstone Fermion Dark Matter

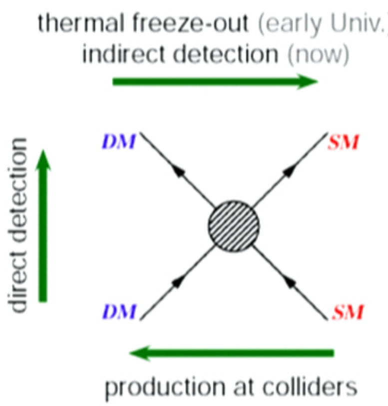
Date: Jan 06, 2012 10:00 AM

URL: http://pirsa.org/12010121

Abstract: We propose that the fermionic superpartner of a weak-scale Goldstone boson can be a nat- ural WIMP candidate. The p-wave annihilation of this `Goldstone fermion' into pairs of Gold- stone bosons automatically generates the correct relic abundance, whereas the XENON100 direct detection bounds are evaded due to suppressed couplings to the Standard Model. Further, it is able to avoid indirect detection constraints because the relevant s-wave annihi- lations are small. The interactions of the Goldstone supermultiplet can induce non-standard Higgs decays and novel collider phenomenology.

SUSY LSP as WIMP

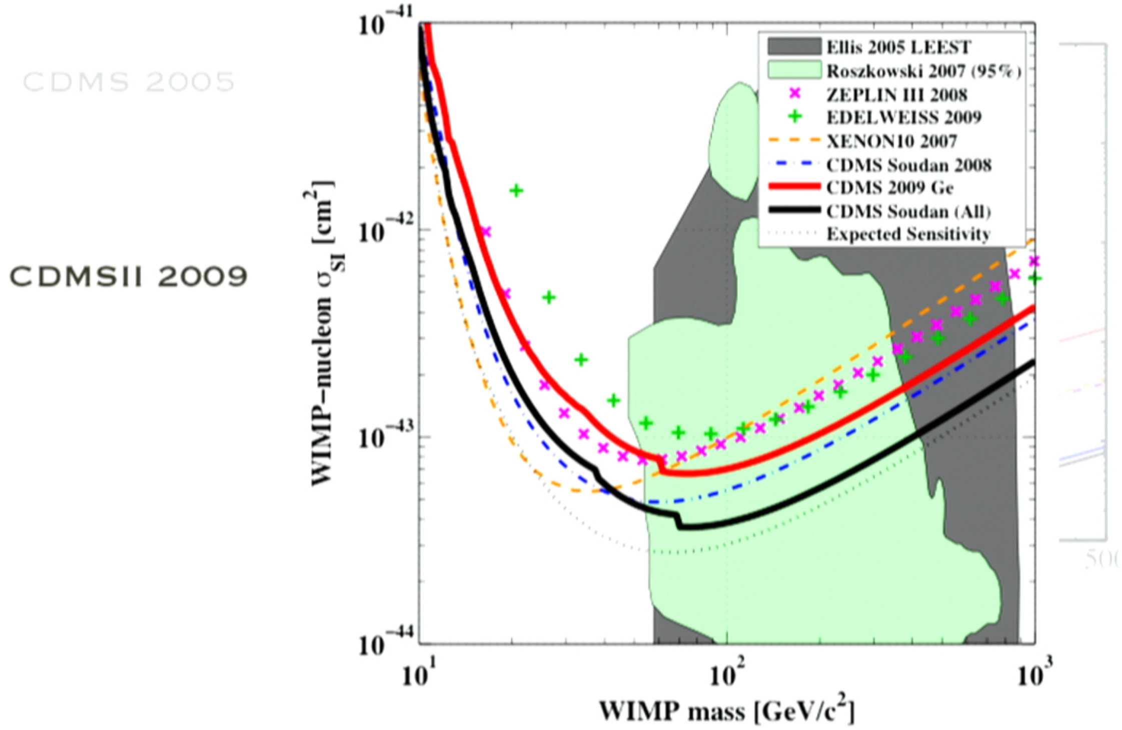
- R-Parity -> Stability
- Weak Scale -> Naturalness



Natural WIMP

Miracle within orders of magnitude!

Increased Tension w/ Direct Detection Experiment



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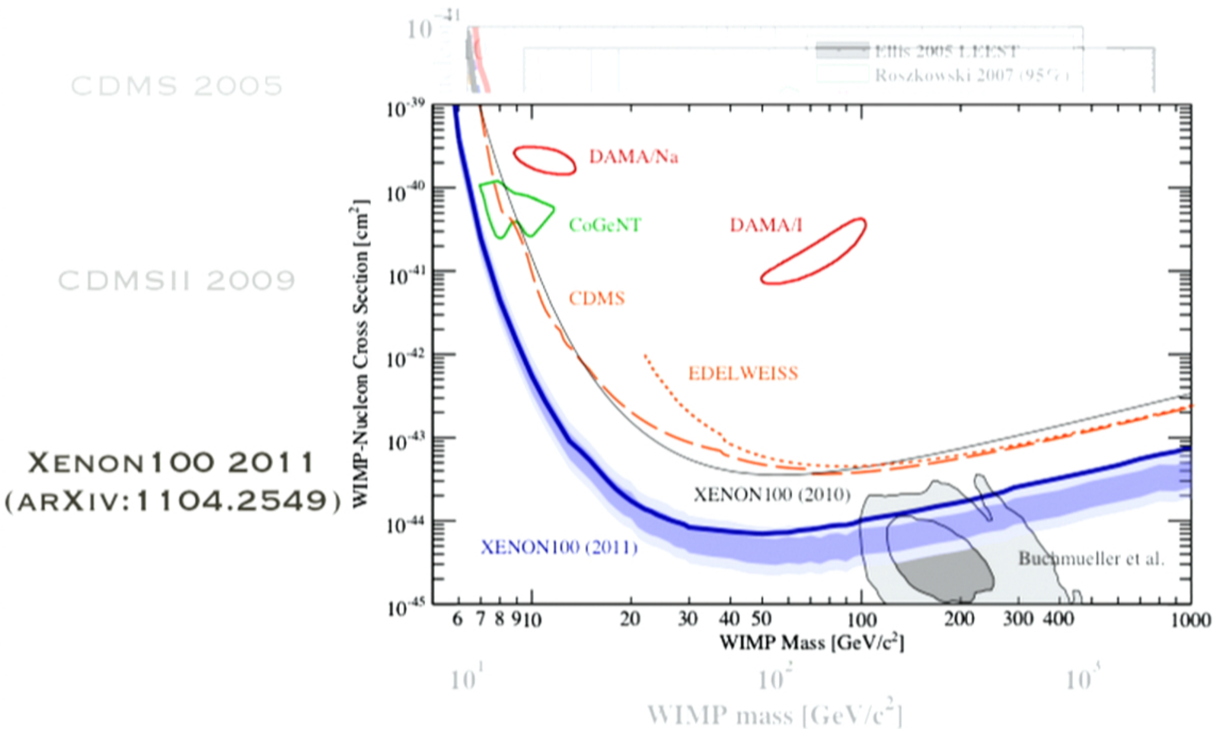
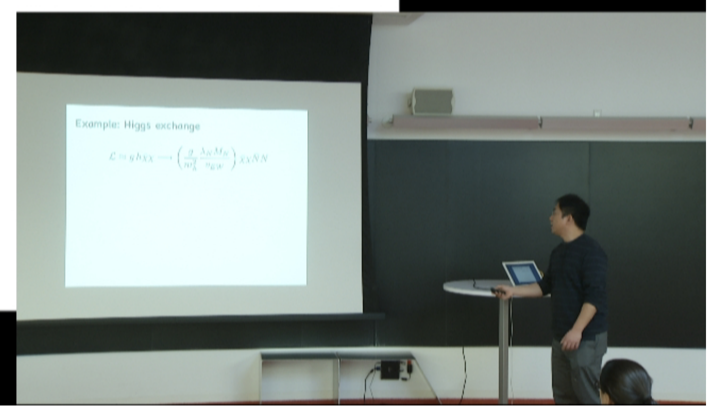


Example: Higgs exchange

$$\mathcal{L} \approx g\, h \bar{\chi}\chi \longrightarrow \left(\frac{g}{m_h^2} \frac{\lambda_N M_N}{v_{EW}} \right) \bar{\chi}\chi \bar{N} N$$



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$$\sigma_{SI} \sim \frac{4g^2}{\pi} \lambda_N^2 \frac{\mu^2 M_N^2}{m_h^4 v_{EW}^2} \approx \boxed{10^{-42} - 10^{-41}\ \text{cm}^2}$$

Way above Xenon100 bound

$$\sigma_{SI} \sim 7.0 \times 10^{-45}\ \text{cm}^2$$

Additional Motivation

Interesting extension of MSSM that lead to "buried higgs" type models.

Bellazzini, Csaba, Falkowski, Weiler

- additional global symmetry broken at

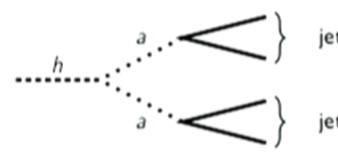
f ~ 500GeV

- PGB ~O(10)GeV, couple derivatively to Higgs $\frac{1}{f^2} h^2 \partial_\mu a \partial^\mu a$

- Light Higgs dominantly decay to jets, and

buried in QCD background; Higgs mass

possibly below LEP bound



Could the fermionic partner of PGB be the LSP?

Goldstone multiplet in SUSY

- Add a global U(1) broken in the SUSY limit
- simplest example: $W = S\left(N\bar{N} - f^2\right)$
- Goldstone Boson (~axion)
  - Goldstone Fermion (~axino)
  - S-Goldstone (~saxion)

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\theta\chi + \theta^2 F$$

Low energy effective theory

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\theta\chi + \theta^2 F$$

S-GOLDSTONE — GOLDSTONE BOSON — GOLDSTONE FERMION

Low energy effective theory

$$A = \frac{1}{\sqrt{2}}(s + ia) + \sqrt{2}\theta\chi + \theta^2 F$$

S-GOLDSTONE GOLDSTONE BOSON GOLDSTONE FERMION

$$a + const$$

$$A \longrightarrow A + i\, const$$

U(1)-SYMMETRY

INVARIANT: $(A + A^\dagger)$

KAHLER POTENTIAL

$$K = K(A + A^\dagger)$$

NO SUPERPOTENTIAL

$$W = 0$$

Simplest Example

$$W = \underset{0}{S}(\underset{-1}{\bar{N}}\underset{+1}{N} - f^2)$$

$$K = \bar{N}^\dagger \bar{N} + N^\dagger N$$

THEORY-UV

$$N \sim f e^{A/f}$$

$$\bar{N} \sim f e^{-A/f}$$

FIELD CONTENT-IR

integrate out heavy modes with mass ~ f

$$K = \cosh(A + A^\dagger)$$

$$W = 0$$

THEORY-IR

MORE GENERALLY

$$\Phi_i = f_i e^{q_i A/f}$$

$$f^2 = \sum_i q_i^2 f_i^2$$

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Supersymmetric Non-linear sigma model

B. Zumino, Phys. Lett. B **87** (1979) 203.

$$\mathcal{L} = K''(s)\left(\tfrac{1}{2}\partial^\mu s\partial_\mu s + \tfrac{1}{2}\partial^\mu a\partial_\mu a + \tfrac{i}{2}\chi^\dagger\bar\sigma^\mu\partial_\mu\chi - \tfrac{i}{2}\partial_\mu\chi^\dagger\bar\sigma^\mu\chi\right)$$
$$-\tfrac{1}{\sqrt{2}}K'''(s)\left(\chi^\dagger\bar\sigma^\mu\chi\partial_\mu a\right) + \tfrac{1}{4}\left(K''''(s) - \frac{K'''^{\,2}(s)}{K''(s)}\right)(\chi\chi)(\chi^\dagger\chi^\dagger)$$

$$\frac{\partial^2 K}{\partial A\partial A^\dagger} = 1 + b_1\frac{q}{f}(A + A^\dagger) + \cdots \qquad b_1 = \frac{1}{qf^2}\sum_i q_i^3 f_i^2$$

Goldstone-current interaction — **4-fermion interaction**

$$\frac{1}{2\sqrt{2}}\left(b_1\frac{1}{f} + o(s/f^2)\right)(\bar\chi\gamma^\mu\gamma^5\chi)\,\partial_\mu a \qquad -\frac{1}{16f^2}\left(b_1^2 + \cdots\right)\left[(\bar\chi\chi)^2 - (\bar\chi\gamma^5\chi)^2\right]$$

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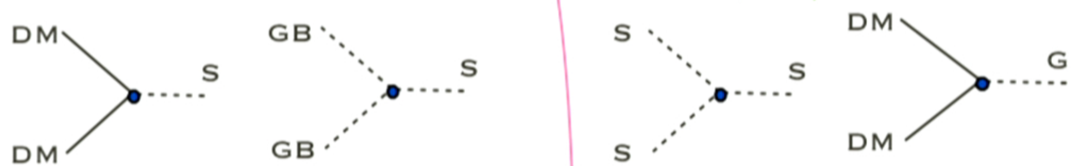

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4-fermion interaction

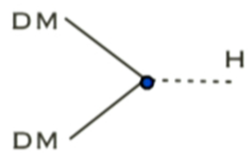
$$-\frac{1}{16 f^2}\left(b_1^2 + \cdots\right)\left[(\bar{\chi}\chi)^2 - (\bar{\chi}\gamma^5\chi)^2\right]$$

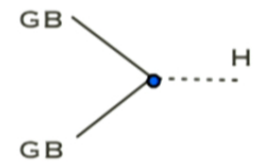
Scalar Mixing with Higgs

$$K = K(A + A^\dagger, \Phi_{SM})$$

$$K \subset \frac{1}{f}(A + A^\dagger) H_u H_d + \ldots \rightarrow \frac{v}{f}(\partial s \partial h)$$

$$\left(1 + b_1 \frac{\sqrt{2}}{f} s + c_h \frac{v}{f^2} h\right) \left[\frac{1}{2}\partial^\mu s \partial_\mu s + \frac{1}{2}\partial^\mu a \partial_\mu a + \frac{i}{2}\bar{\chi}\gamma^\mu \partial_\mu \chi\right]$$





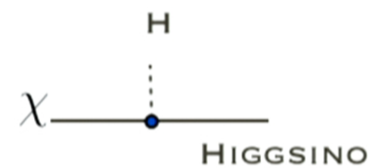
Fermion kinetic mixing

$$\mathcal{L}_{\rm KM} = \frac{i}{2f}\left[\left(\chi^\dagger\bar\sigma^\mu\partial_\mu\tilde H_u - \partial_\mu\chi^\dagger\bar\sigma^\mu\tilde H_u\right)(c_1 H_d + \ldots) + {\rm h.c.}\right.$$

$$\rightarrow \quad i\epsilon_u\chi^\dagger\bar\sigma^\mu\partial_\mu\tilde H_u^0 + i\epsilon_d\chi^\dagger\bar\sigma^\mu\partial_\mu\tilde H_d^0 + {\rm h.c.}$$

$$\epsilon_{u,d} \sim v_{\rm EW}/f$$

canonical normalizing kinetic term
leads to a mixing of GF and Higgsino



small Higgsino component when mu large

$$\epsilon_{u,d}\, m_\chi/\mu \sim v_{\rm EW}\, m_\chi/f\mu.$$

In our scenario, we restrict us to
this case, and ignore this mixing

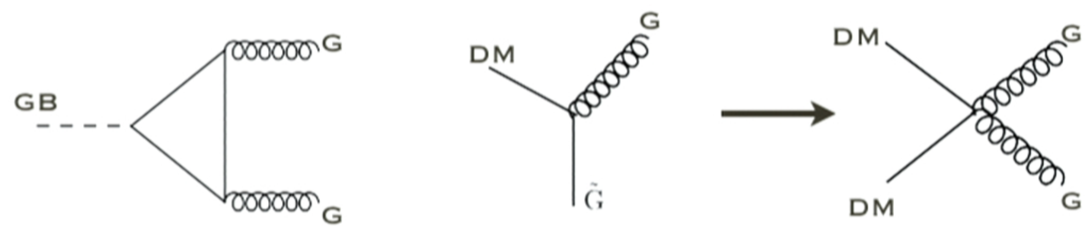
Anomaly induced interactions

$$V_{an} = \frac{c_{an}}{\sqrt{2}f} a G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} \qquad W_{an} = -\frac{c_{an}}{f} A \mathcal{W}^a \mathcal{W}^a$$

$$\mathcal{L}_{anomaly} \subset \frac{c_{an}}{f\sqrt{2}} \left(a G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu} + i \bar{\chi} G^a_{\mu\nu} \sigma^{\mu\nu} \gamma^5 \lambda^a \right)$$

$$\mathcal{L}_y = ia \sum_{i=1}^{N_\Psi} y_i \bar{\Psi}_i \gamma^5 \Psi_i \, .$$

If all fermions masses equal $\quad c_{\rm an} = \frac{\alpha}{8\pi} q_\Psi N_\Psi$



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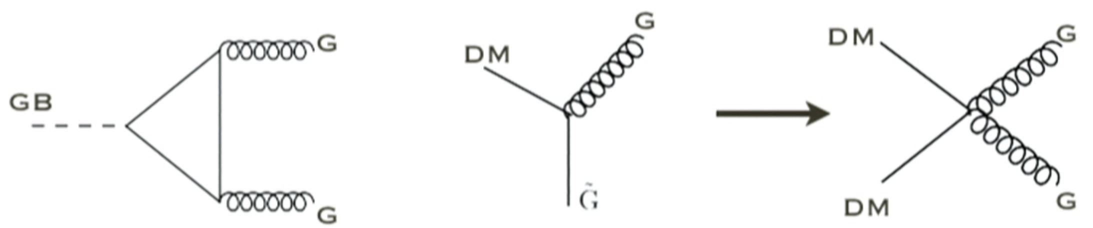


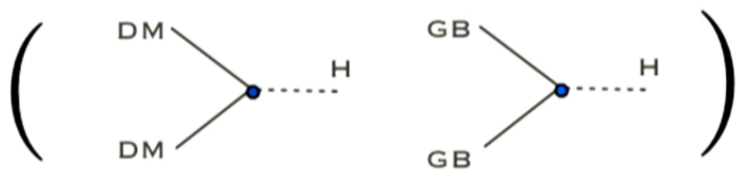
The Story So far …

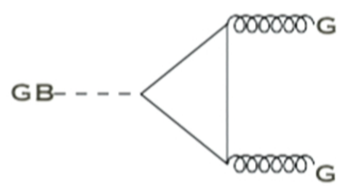
U(1)-broken spont. $W = S(\bar{N}N - f^2)$

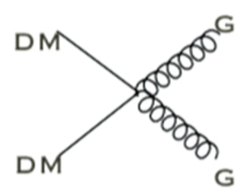
Summary of interactions

$$W = S(\bar{N}N - f^2) + SH_uH_d + N\Phi_1\Phi_2$$

$\frac{b_1}{f} \times$ 

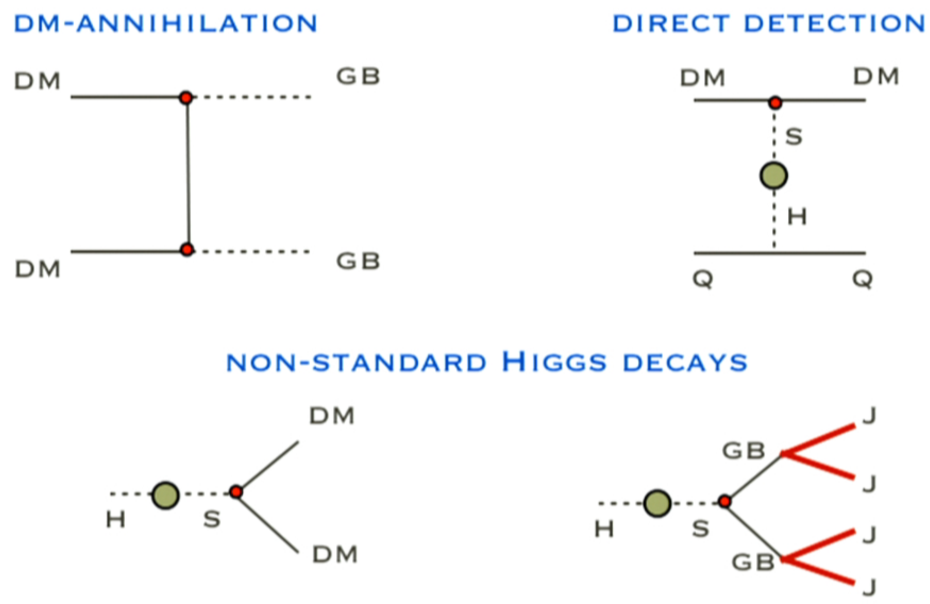
$\frac{v}{f^2} \times$ 

$\frac{\alpha_s N}{8\pi f} \times$ 



$\left(\frac{\alpha_s N}{8\pi f}\right)^2 \times \frac{1}{M_\lambda}$

General Feature



SUSY Breaking

- Until now entire A massless, but SUSY breaking can give mass to s and χ
- Add soft SUSY breaking, assume all globally symmetric
- Goldstone Fermion $\chi = \sum_i \frac{q_i f_i}{f} \psi_i$

U(1) invariance of W $\sum_j W_{ij} q_j f_j = -W_i q_i = -F_i q_i$

→ $m_\chi \approx q_i \langle F_i \rangle / f.$

Tamvakis-Wyler: If F_i vanish for all U(1) charged fields, X remain massless

SUSY Breaking

- however SUSY breaking globally symmetric does NOT imply $q_i F_i = 0$
- reason: mixing among F-terms $F_X^\dagger F_i$
- assume spurion $\langle X \rangle = x + F_X \theta^2$
- For example $K \sim \frac{X^\dagger X}{M^2} \phi_i^\dagger \phi_i \longrightarrow \frac{x f_i}{M^2} F_X^* F_i$

$$F_i \to F_i^0 + \frac{x f_i}{M^2} F_X \longrightarrow m_\chi = F_i q_i / f \sim \frac{x}{M} m_i$$

($F_i^0 \to 0$)

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(F_i^0 → 0)

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Mass Spectrum

GOLDSTONE BOSON $m_a = 0$ UP TO U(1)-BREAKING TERMS

GOLDSTONE FERMION

$m_\chi \ll m_i$ CAN BE ACHIEVED -> LSP
DEPEND ON DETAILS OF SUSY BREAKING

S-GOLDSTONE $m_s \approx m_i$

$$m_a < m_\chi \ll m_s$$

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Parameters

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	$[0, 2]$
$c_{\rm an}$	Anomaly coefficient	0.06
c_h	Higgs coupling	$[-1, 1]$
δ	Explicit breaking $ia\overline{\chi}\gamma^5\chi$ coupling	$3/2$

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\overline{\chi}\partial\!\!\!/\chi\right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f}\left(\overline{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a + \frac{c_{\rm an}}{f\sqrt{2}} a G\widetilde{G} + i\delta a\overline{\chi}\gamma^5\chi$$

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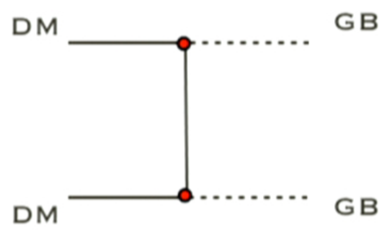
DM annihilation

DM-ANNIHILATION



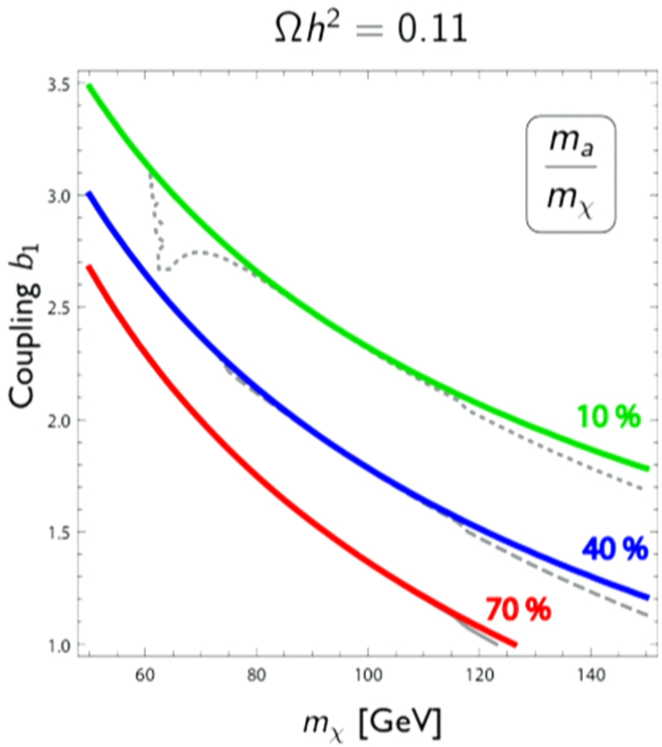
$$\left(b_1 \frac{1}{2\sqrt{2}f}\right) \left(\bar{\chi}\gamma^\mu\gamma^5\chi\right) \partial_\mu a = b_1 \frac{m_\chi}{f} (\bar{\chi}\gamma^5\chi) a$$

LEADING

$$\langle \sigma v \rangle = a + b\langle v^2 \rangle = \left(\frac{1}{8\pi} b_1^4 \frac{m_\chi^2}{f^4} \right) \frac{T_f}{m_\chi} \approx \text{ pb}$$

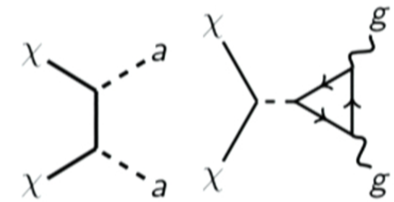
P-WAVE

DM annihilation



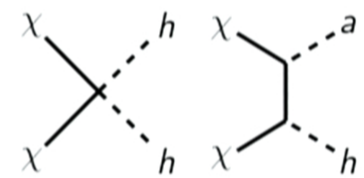
Dominant contribution

Kähler, anomaly, ~~U(1)~~



Subleading

Mixing with Higgs



Negligible

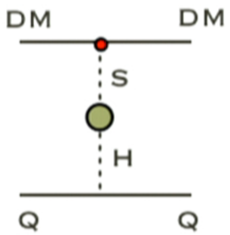
$\chi\chi \to s \to aa$, $\chi\chi \overset{t,u}{\to} hh$

Direct Detection

operator	coefficient	origin
$\bar{\chi}\chi G^a_{\mu\nu} G^a_{\mu\nu}$	$\frac{1}{f^2 M_\lambda}$	anomaly
$\bar{\chi}\chi\bar{q}q$	$\frac{y_q}{m_h^2} \times \left(\frac{m_\chi v_{EW}}{f^2}\right)$	mixing

Higgs exchange

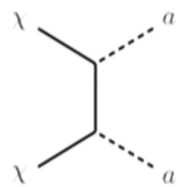
DIRECT DETECTION



$$\sigma_{SI} \approx \sigma_{SI}^{MSSM} \times \left(\frac{m_\chi v_{EW}}{f^2}\right)^2$$

10^{-46} $\quad 10^{-42} \quad \times \quad 10^{-4}$

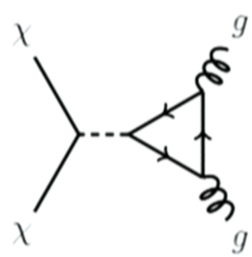
Indirect Detection: FERMI



LEADING P-WAVE

$$\langle v\sigma \rangle_{\mathrm{Th}} = 3 \times 10^{-26} \mathrm{cm}^3/\mathrm{s}$$

TODAY SUPPRESSED BY VELOCITY $v \simeq 10^{-3}$



$g\ \gamma$ SUBLEADING S-WAVE

$$0.2 - 0.3 \times \langle v\sigma \rangle_{\mathrm{Th}} \times 10^{-3}$$

$g\ \gamma$ $\dfrac{\Gamma(\gamma\gamma)}{\Gamma(gg)}$

$\gamma -$ ray LINES (30-200 GeV)

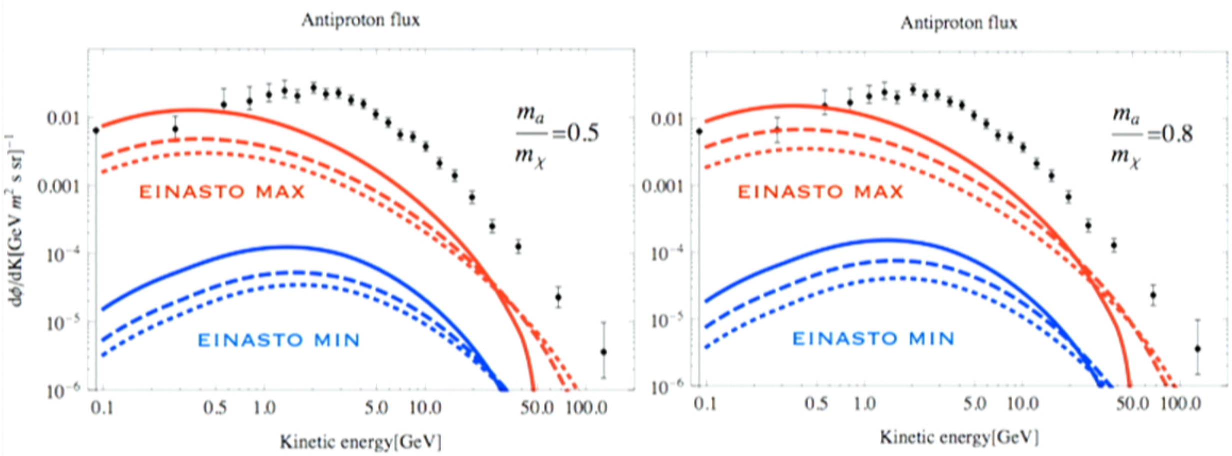
$\mathcal{O}(10) \times \langle \sigma v \rangle_{\mathrm{Th}}$

$\gamma -$ ray DIFFUSE (20-100 GeV)

$\chi\chi \to \pi^0, \ldots$

$\mathcal{O}(10)$ smaller

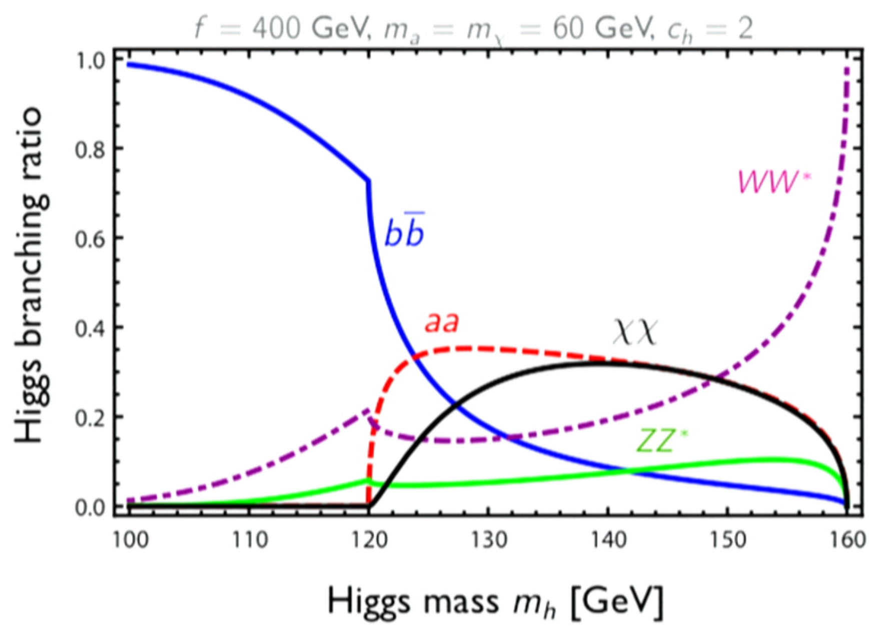
Indirect Detection: PAMELA



WE USED THE **PPPC 4 DM ID:** POOR PARTICLE PHYSICIST COOKBOOK FOR DARK MATTER INDIRECT DETECTION"
DEVELOPED BY M. CIRELLI ET AL. ARXIV 1012.4515

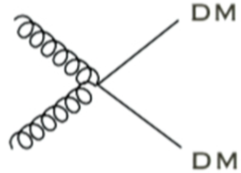
Non-standard Higgs Decay

partial buried & invisible: suppressed SM channels, MET, Width < 1



Goldstone Fermion @ LHC

Direct production through gluons -> ISR Monojets



operator	coefficient	origin	overall scale
$\bar{\chi}\chi G^a_{\mu\nu} G^a_{\mu\nu}$	$\frac{\alpha_s^2}{64\pi^2 f^2 M_\lambda}$	anomaly	$\frac{\alpha_s}{8M_*^3}$
$\bar{\chi}\chi\bar{q}q$	$\frac{y_q}{m_h^2} \times \left(\frac{m_\chi v_{EW}}{f^2}\right)$	mixing	$\frac{m_q}{2M_*^3}$

$gg \to a^* \to \chi\chi$ may be within 5σ reach with 100 fb^{-1}

1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196

Cascade decays, LOSP $\to \chi$: $\overline{\chi}G\lambda$ anomaly and χ–$\widetilde{H}$ kinetic mixing

Decays typically prompt, *a* reconstruction is difficult for light masses.
Heavy fermions Ψ in anomaly may appear as "fourth generation" quarks

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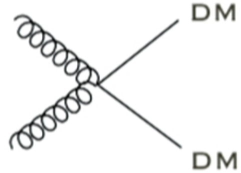

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Conclusion

- minimal SUSY DM squeezed by direct detection bounds
- simple extension: add a U(1) and break it spontaneously in susy
    - like an axino at the TeV scale
    - GF could be the LSP and ordinary WIMP
    - annihilation rate dominated by
    - direct detection naturally below Xenon100 but not too far via Higgs exchange

Scalar Mixing with Higgs

$$K = K(A + A^{\dagger}, \Phi_{SM})$$

$$K \subset \frac{1}{f}(A + A^{\dagger})H_u H_d + \ldots \rightarrow \frac{v}{f}(\partial s \partial h)$$

$$\left(1 + b_1 \frac{\sqrt{2}}{f} s + c_h \frac{v}{f^2} h\right) \left[\frac{1}{2}\partial^{\mu} s \partial_{\mu} s + \frac{1}{2}\partial^{\mu} a \partial_{\mu} a + \frac{i}{2}\bar{\chi}\gamma^{\mu}\partial_{\mu}\chi\right]$$