Title: A Quantum Information Approach to Statistical Physics

Date: Jan 25, 2012 04:00 PM

URL: http://pirsa.org/12010120

Abstract: I will first present a theorem based on the Decoupling Theorem of [1] which gives sufficient and necessary conditions for a quantum channel (CPTPM) being such that it yields the same output for almost all possible inputs. This theorem allows us to reproduce and generalize results oft [2,3], in which cornerstones of statistical physics are derived from first principles of quantum mechanics, in a very natural and easy way. Specifically, we express them in a way which allows to apply results about random 2-qubit interactions [4]. Furthermore, we apply this theorem to provide a criterion for whether different initial states of some subspace of a quantum mechanical system in contact with an environment have at some given time already evolved to the same state or not. As it turns out, this question can be answered by examining a simple entropic inequality evaluated for just one particular state [5]. Applying this criterion to realistic Hamiltonians with local interactions may lead to improved bounds on the thermalization times of quantum mechanical systems.


$= d(A' \omega_0^q) +$
$= A'' dr \wedge \omega_0^q -$
$+$

A Quantum Information Approach to Statistical Physics

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University of Basel

Joint work with Stephanie Wehner
National University of Singapore

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Contents

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Smooth entropies

Key features of "one-shot" or "smooth" entropy measures:

- "Smoothing parameter" $\varepsilon \Rightarrow$ insensitive to small variations of ρ_{AB}
- For $\varepsilon \to 0$ have

$$\mathrm{H}^{\varepsilon}_{\min}(A|B)_\rho \leq \overbrace{H(A|B)_\rho}^{\text{von Neumann entropy}} \leq \mathrm{H}^{\varepsilon}_{\max}(A|B)_\rho$$

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Independence of channel input

When does a quantum channel (CPTPM) $\mathcal{T}_{A\to B}$ yield outputs which are the same for almost all possible inputs ρ_A?
Define the *Choi-Jamiołkowski representation* of the channel

$$\tau_{A'B} := (\mathcal{I}_{A'} \otimes \mathcal{T}_{A\to B})\,\Psi_{A'A}$$

where $\Psi_{A'A}$ is the maximally entangled state.

Formal version

As a corollary of the Decoupling Theorem of Dupuis *et al.* , 2010 we find

Theorem

$$\Pr_{U_A}\left\{\left\|\mathcal{T}_{A\to B}\left(U_A\rho_A U_A^\dagger\right) - \mathcal{T}_{A\to B}\left(\frac{\mathbb{1}_A}{|A|}\right)\right\|_1 \geq 2^{-\zeta/2}+12\varepsilon+\delta\right\}$$
$$\leq 2\exp\left(-|A|\delta^2/16\right)$$

where

$$\begin{aligned}\zeta &= \mathrm{H}_{\min}^{\varepsilon}(A'|B)_\tau \\ &\geq \mathrm{H}_{\min}^{\varepsilon/2}(A'B)_\tau - \mathrm{H}_{\max}^{\varepsilon/2}(B)_\tau - O\left(\log\frac{1}{\varepsilon}\right)\end{aligned}$$

Informal version

The channel output is the same for almost all inputs if

$$\mathrm{H}^{\varepsilon}_{\min}(A'B)_{\tau} \gtrapprox \mathrm{H}^{\varepsilon}_{\max}(B)_{\tau} \ .$$

Converse: We also show that this is not the case if

$$\mathrm{H}^{\varepsilon}_{\max}(A'B)_{\tau} \lessapprox \mathrm{H}^{\varepsilon}_{\min}(B)_{\tau} \ .$$

Equal a priori probability postulate

- *"Given an isolated system in equilibrium, it is found with equal probability in each of its accessible microstates."*
- Let $\mathcal{H}_\Omega \subseteq \mathcal{H}_S \otimes \mathcal{H}_E$ describe the space of "accessible microstates".

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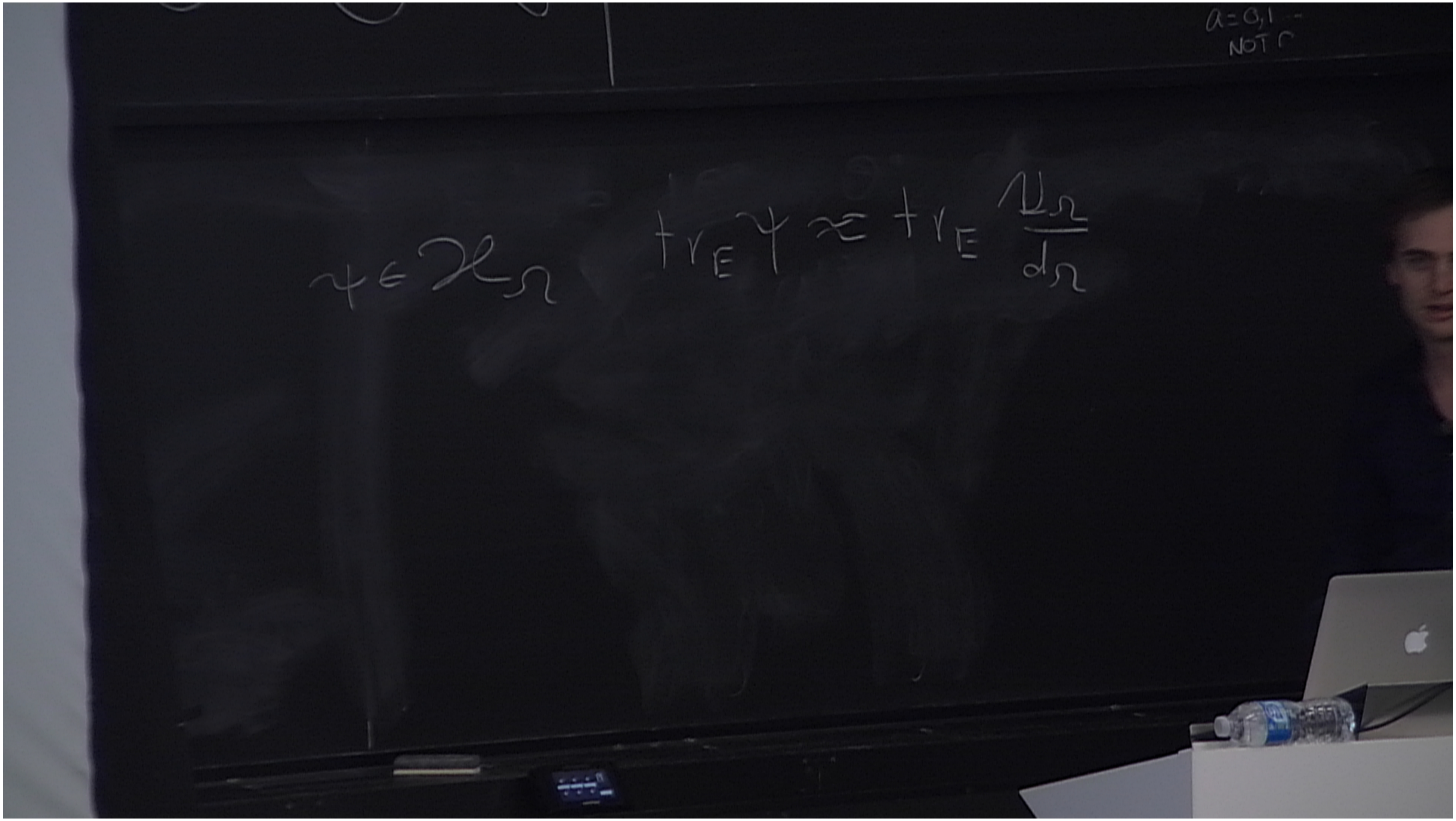
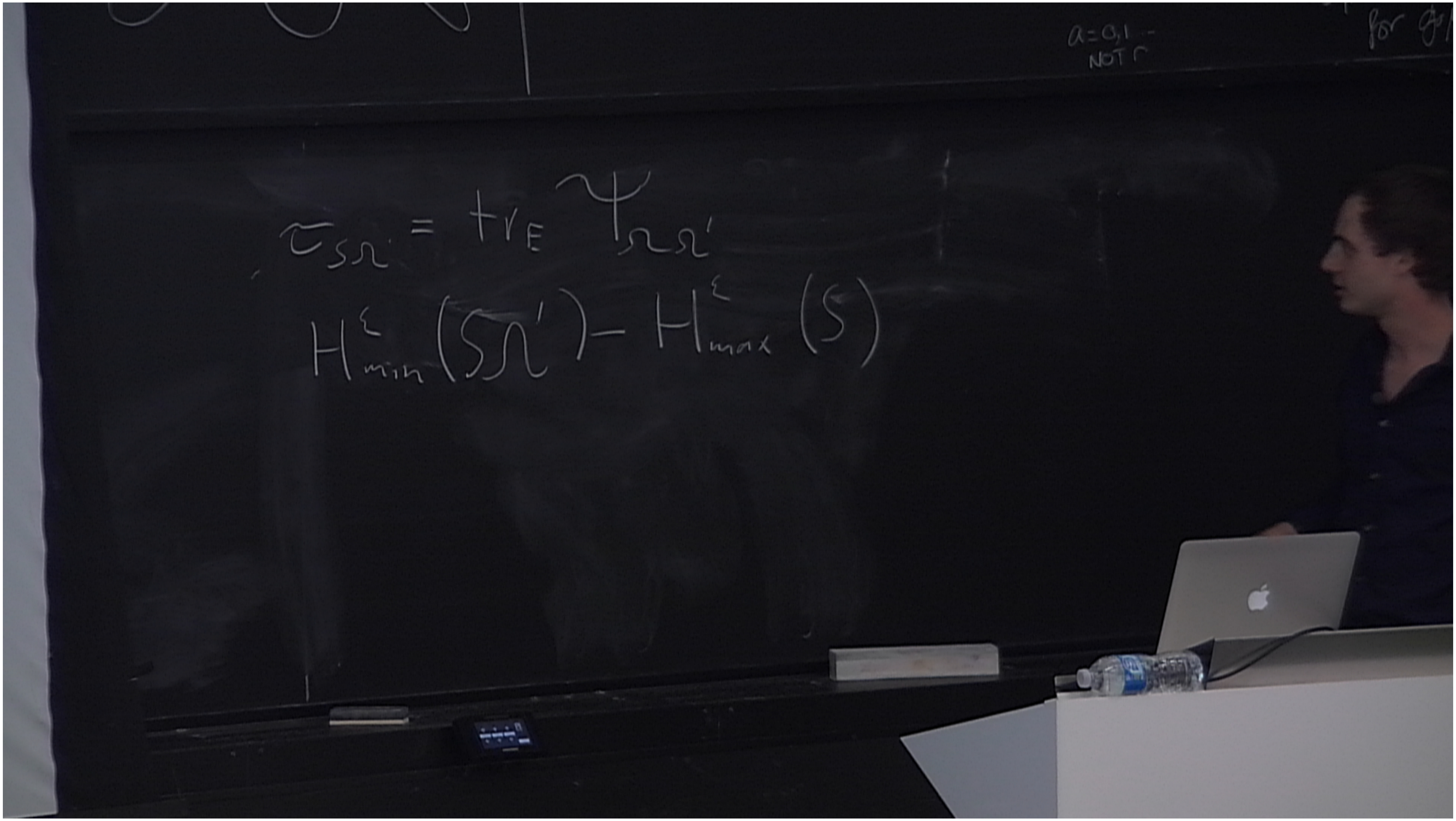

a=0,1 ...
NOT r
ψ ∈ H_Ω
tr_E ψ ≈ tr_E 1_Ω / d_Ω

Applying our theorem

- Let $\mathcal{T}_{\Omega\to S} \equiv \mathrm{Tr}_E$.
- We find: For any state ρ_Ω and almost all unitaries U_Ω we have

$$\mathrm{Tr}_E\left(U_\Omega \rho_\Omega U_\Omega^\dagger\right) \approx \mathrm{Tr}_E \frac{\mathbb{1}_\Omega}{|\Omega|}$$

if $\mathrm{H}^\varepsilon_{\max}(S)_{\frac{\mathbb{1}_\Omega}{|\Omega|}} \lessapprox \mathrm{H}^\varepsilon_{\min}(E)_{\frac{\mathbb{1}_\Omega}{|\Omega|}}$.


$\tau_{S\Omega'} = tr_E \tilde{\Psi}_{\Omega\Omega'}$
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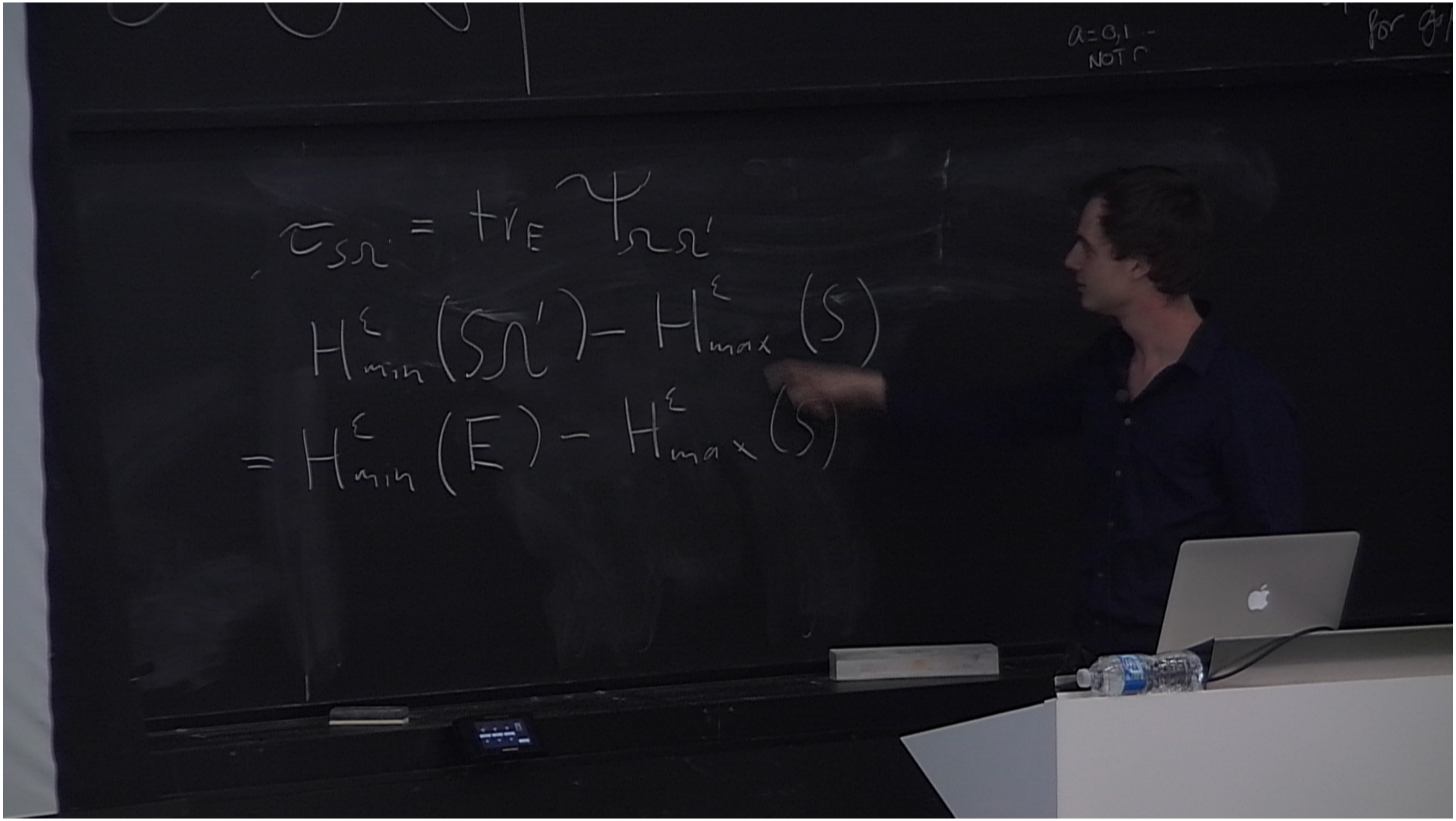
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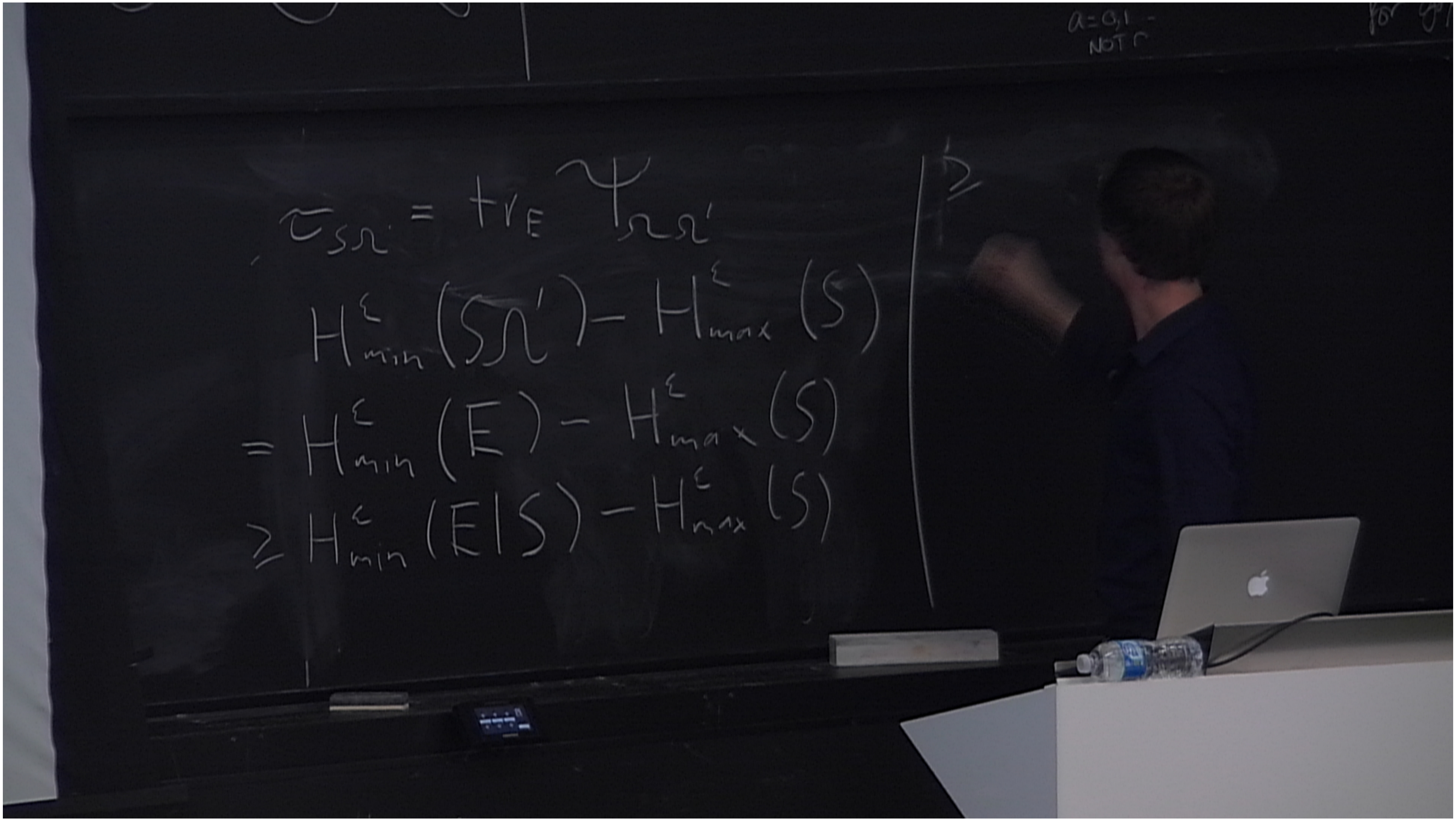
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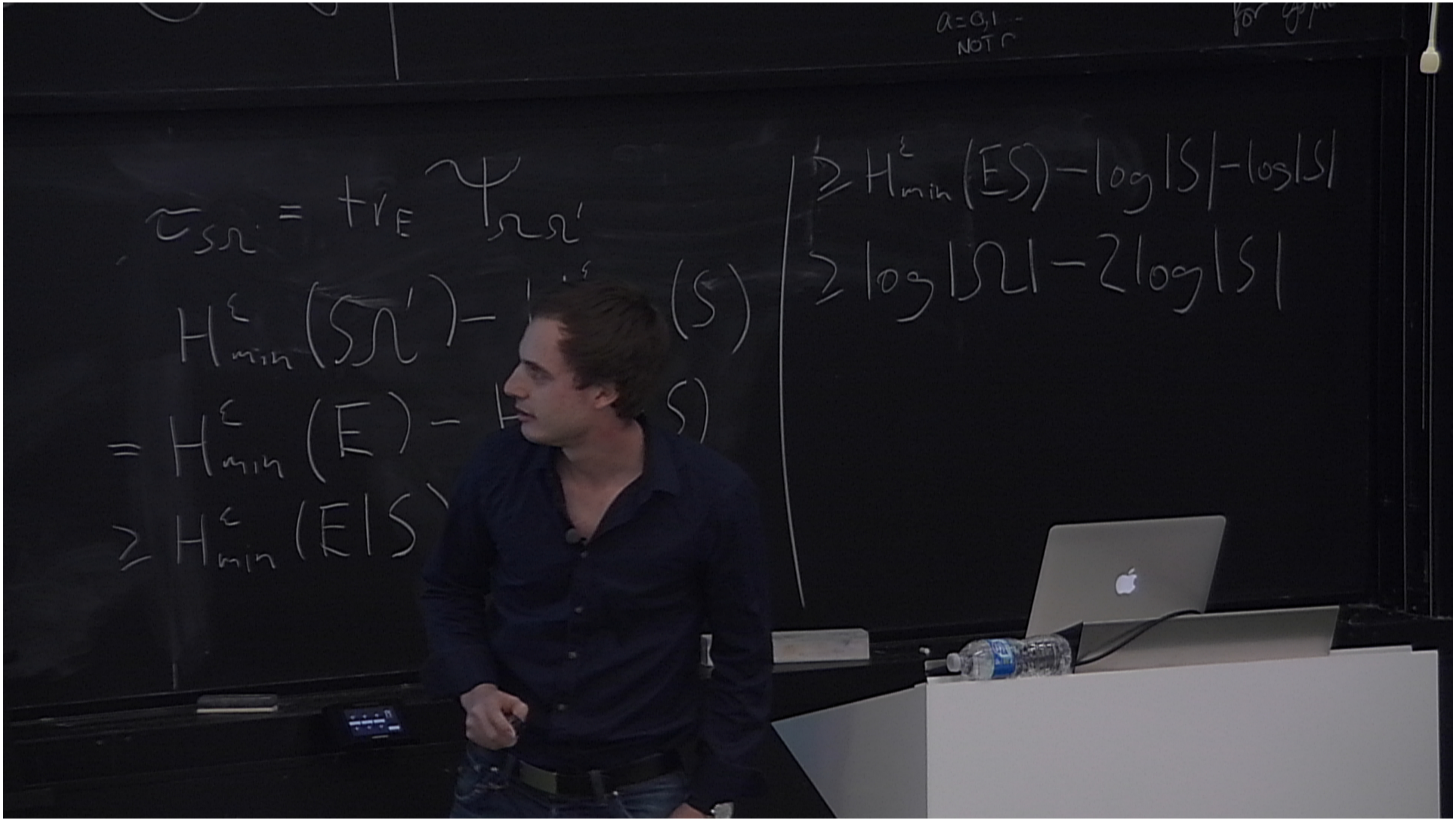
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Approximate 2-designs

- In order to apply our theorem we do not need the full Haar distribution, but only need that the first two moments of the distribution are approximated: *approximate 2-designs* are sufficient.
- A. Harrow and R. Low, 2009: "Random Quantum Circuits are Approximate 2-designs"

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Two aspects of *thermalization* of a system S:

- The equilibrium state should not depend on the precise initial state of the environment E, but only on its macroscopic properties.
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Formal setting

Product initial state $\phi_S \otimes \psi_E$ with $\phi_S \in \mathcal{H}_{\bar{S}} \subseteq \mathcal{H}_S$ and $\psi_E \in \mathcal{H}_{\bar{E}} \subseteq \mathcal{H}_E$, Hamiltonian H_{SE}. We are interested in the following two questions:

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- Given ϕ_S, when does the state of S not depend on the precise initial state $\psi_E \in \mathcal{H}_{\bar{E}}$?

The channel we are interested in

The state of S after some time t is given by

$$\rho_S(t) = \mathrm{Tr}_E \left[e^{-\,\mathrm{i}\, H_{SE} t} \left(\phi_S \otimes \psi_E \right) e^{+\,\mathrm{i}\, H_{SE} t} \right] \; .$$

We can interpret this either as a channel $\bar{S} \to S$ (taking ϕ_S as an input) or as a channel $\bar{E} \to S$ (taking ψ_E as an input).

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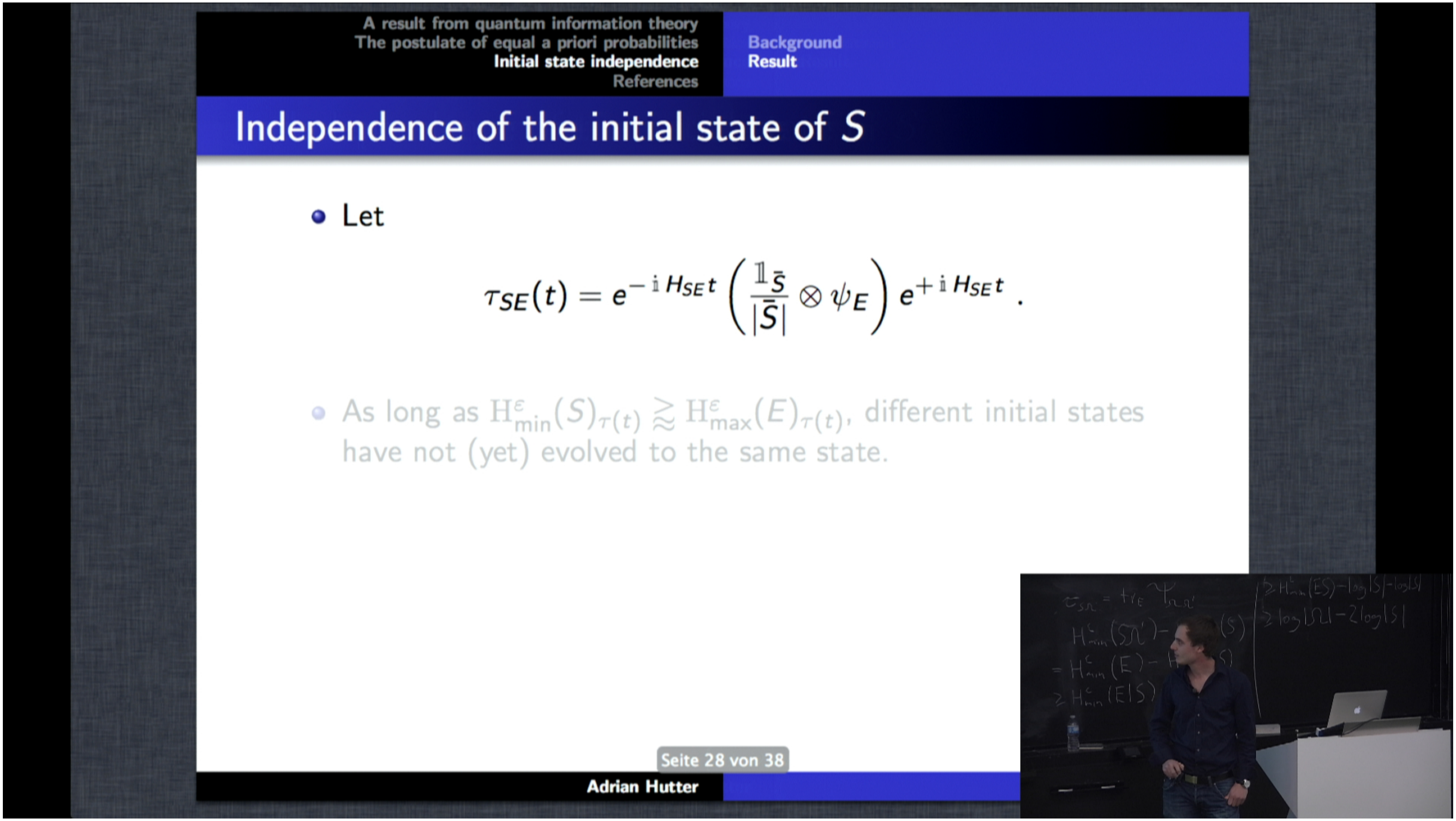
A result from quantum information theory
The postulate of equal a priori probabilities
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Background
Result
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Similarly, we can show:

- If $\log|\bar{E}| > 2\log|S|$, almost all initial states $\psi_E \in \mathcal{H}_{\bar{E}} \subseteq \mathcal{H}_E$ will yield the same time-evolved state $\rho_S(t)$, *at any time* t.
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Timescales

- How long can we guarantee that $\mathrm{H}^{\varepsilon}_{\min}(S)_{\tau(t)} \gtrapprox \mathrm{H}^{\varepsilon}_{\max}(E)_{\tau(t)}$.
- In the most general scenario: for a time

$$O\left(\max\left\{\frac{1}{4\left\|H_{int}\right\|_{\infty}}, \frac{1}{\left\|\left[\frac{\mathbb{1}_S}{|S|}\otimes\psi_E, H_{SE}\right]\right\|_1}\right\}\right).$$

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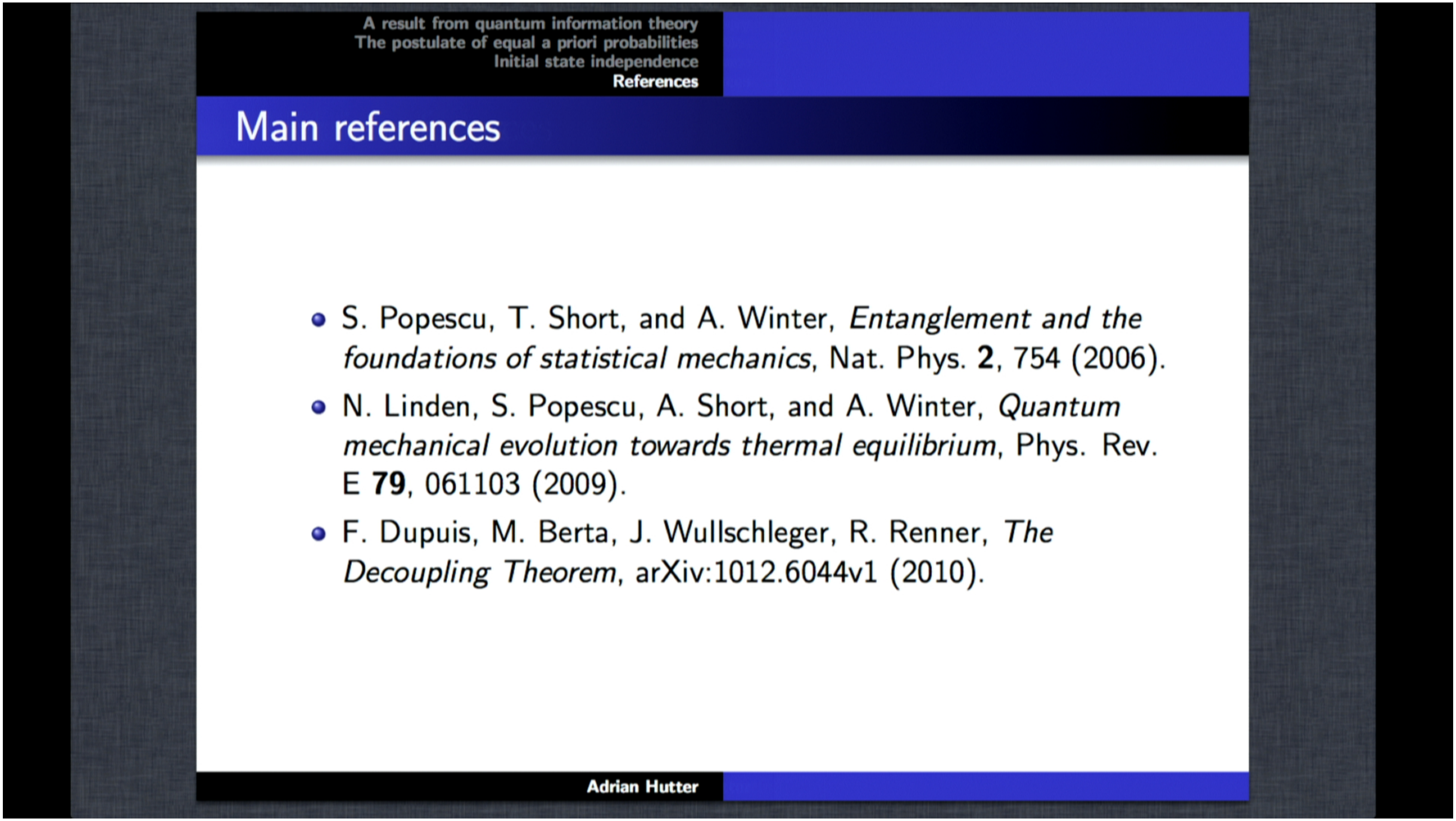
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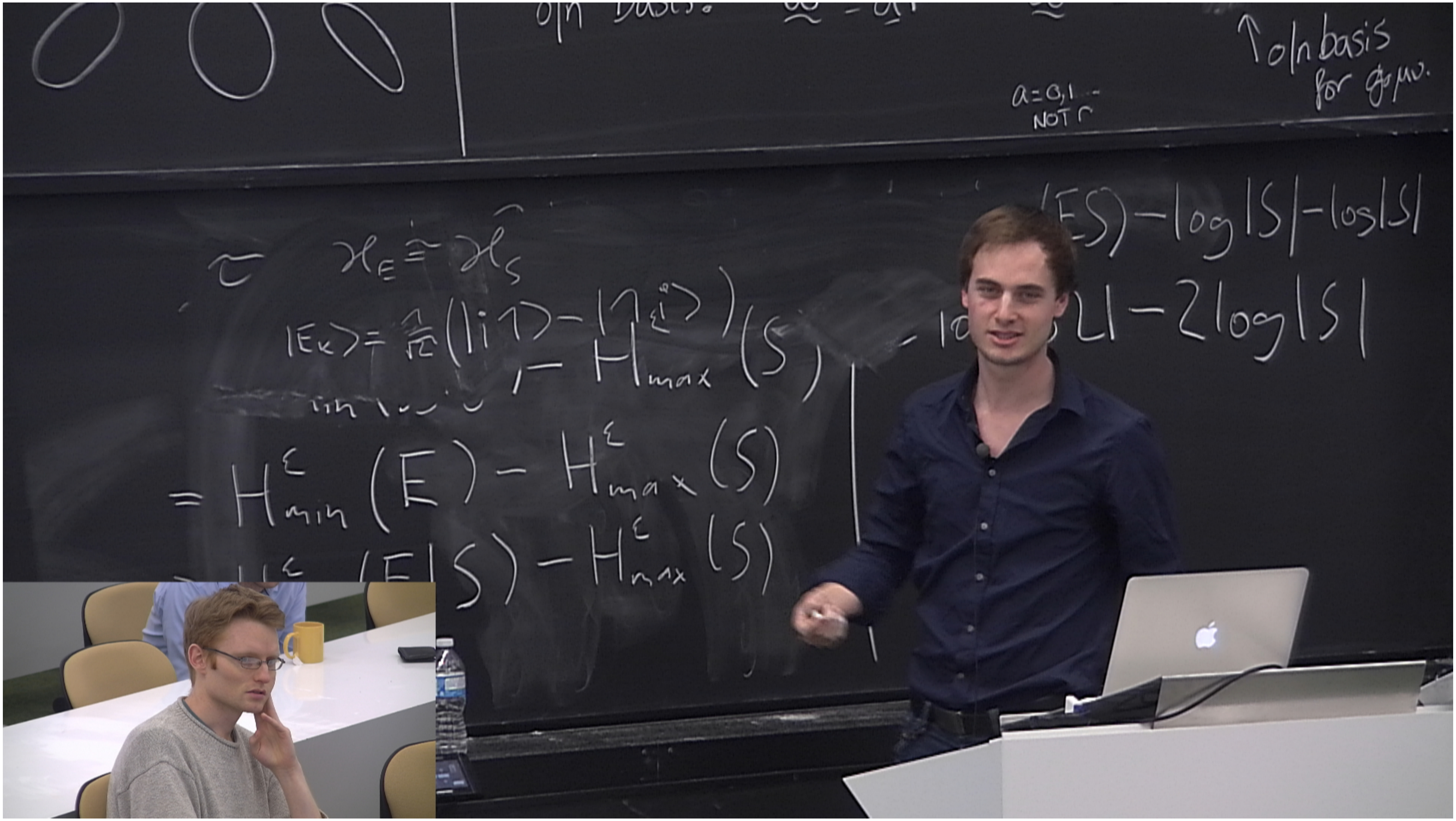
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