Title: Scalar Perturbations in Loop Quantum Cosmology

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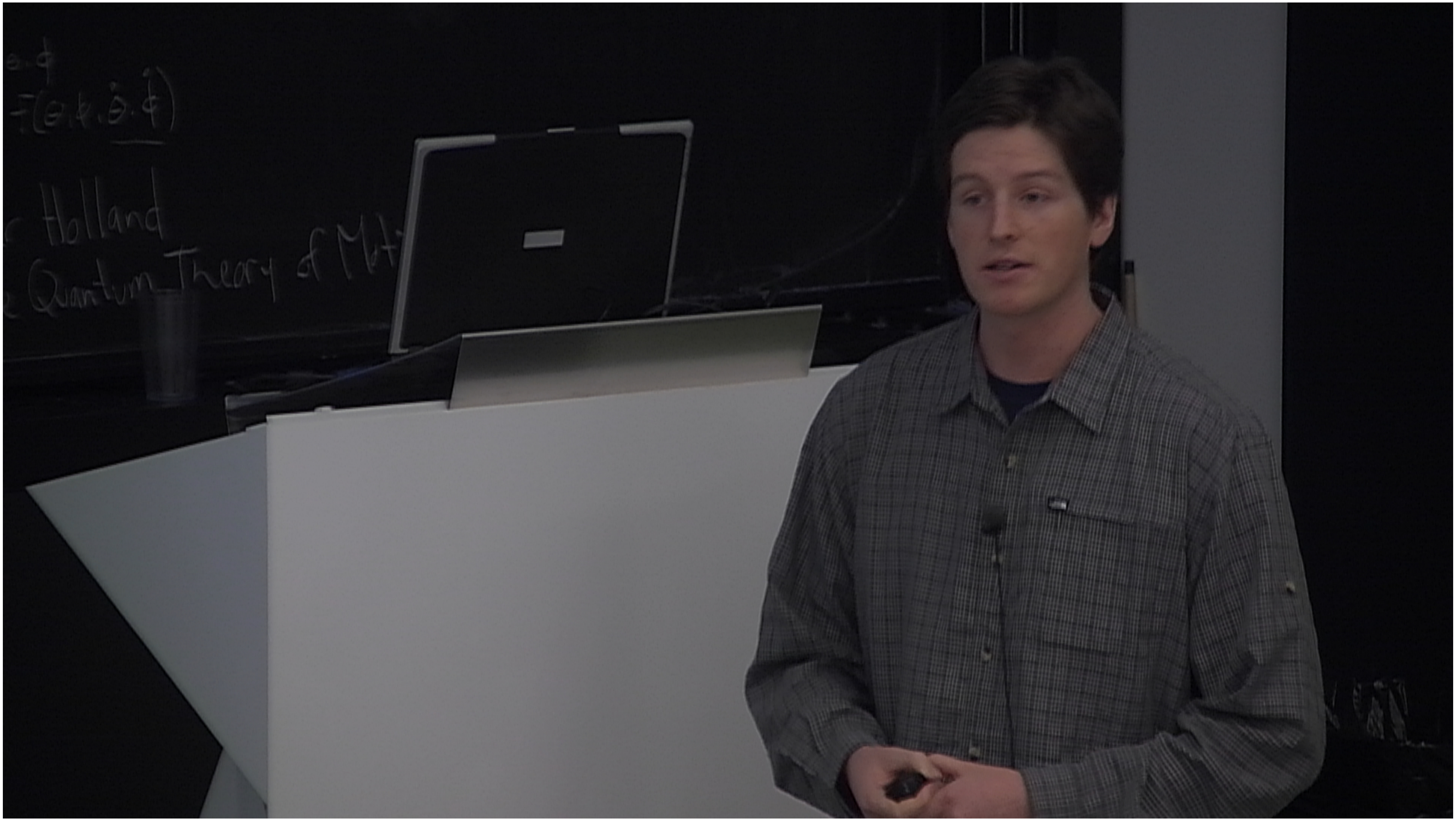
Abstract: We study the dynamics of the scalar modes of linear perturbations around a flat, homogeneous and isotropic background in loop quantum cosmology.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Quantum Theory of

Motivation: Quantum Gravity and Cosmology

- Quantum Gravity: combine the theories of general relativity and quantum mechanics.
- But quantum gravity effects will only become important in extreme situations and therefore any theory of quantum gravity is hard to test.
- Cosmology seems to be the best chance. Compare:
  - Predictions of the theory in a cosmological setting,
  - Observations of the cosmic microwave background and primordial gravitational waves.

Testing LQC

The goal is to test LQC by comparing its predictions to the observations of the cosmic microwave background (CMB). In order to do this, it is necessary to understand how perturbations —especially scalar perturbations— behave in LQC.

What is observed in the CMB is that the spectrum of the scalar perturbations is almost scale invariant, with a slight red shift. This can be explained by inflation and ekpyrotic models among others.

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In order to test LQC, we must determine how it modifies the predictions of these models: calculate subleading effects.

Perhaps LQC alone gives a scale-invariant spectrum with a small red shift?

The Flat FLRW Model

The Friedmann-Lemaître-Robertson-Walker (FLRW) space-time is homogeneous and isotropic. In the spatially flat case,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2).$$

The basic variables in LQG are holonomies of the Ashtekar connection

$$A_a^i = \Gamma_a^i + \gamma K_a^i = \gamma \dot{a}\,(\mathrm{d}x^i)_a = c\,(\mathrm{d}x^i)_a,$$

and fluxes of densitized triads

$$E_i^a = \sqrt{\det q}\, e_i^a = a^2 \left(\frac{\partial}{\partial x^i}\right)^a = p \left(\frac{\partial}{\partial x^i}\right)^a,$$

through surfaces. In the case of the FLRW models, A_a^i and E_i^a can be parametrized by one variable each, c and p.

The Canonical Picture

The Ashtekar connection and the densitized triads are conjugate to one another, it follows that

$$\{c, p\} = \frac{8\pi\gamma G}{3}.$$

With the choice of variables on the previous slide, the Gauss and diffeomorphism constraints are automatically satisfied. Then, the Hamiltonian constraint $\mathcal{C}_H = \int [N\mathcal{H} + N^a\mathcal{H}_a + \Lambda^i\mathcal{G}_i]$ becomes

$$\mathcal{C}_H = \int_{\mathcal{M}} \left[\frac{-NE_i^a E_j^b}{16\pi G\gamma^2\sqrt{|q|}} \epsilon^{ij}{}_k \Big(F_{ab}{}^k - (1+\gamma^2)\Omega_{ab}{}^k \Big) + N\mathcal{H}_{\rm m} \right] = 0,$$

where $F_{ab}{}^k$ and $\Omega_{ab}{}^k$ are the curvatures of A_a^i and Γ_a^i respectively, while $\mathcal{H}_{\rm m}$ is the matter Hamiltonian density. For the flat FLRW model, $\Omega_{ab}{}^k = 0$.

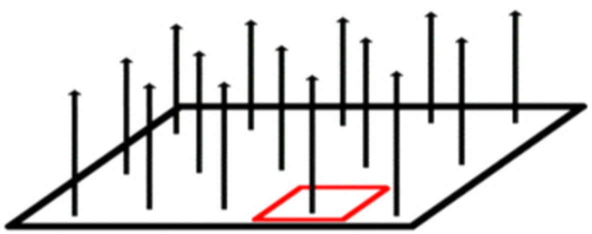
The Main Idea Behind LQC

In loop quantum cosmology, there is no operator $\hat{c} \sim i\partial_p$, just as there is no operator corresponding directly to the connection in LQG.

Therefore, the curvature operator is defined via the relation

$$F(A) \sim \lim_{Ar_\square \to 0} \frac{h_\square(A) - \mathbb{I}}{Ar_\square},$$

where we have expressed the field strength in terms of the holonomy of the Ashtekar connection around a square loop of area $Ar_\square$ in the relevant plane.



The limit $Ar_\square \longrightarrow 0$ does not exist in LQC. Since the area of a surface in LQG is determined by the field lines crossing through it, the minimal nonzero area will be given by a surface which is crossed by only one field line.

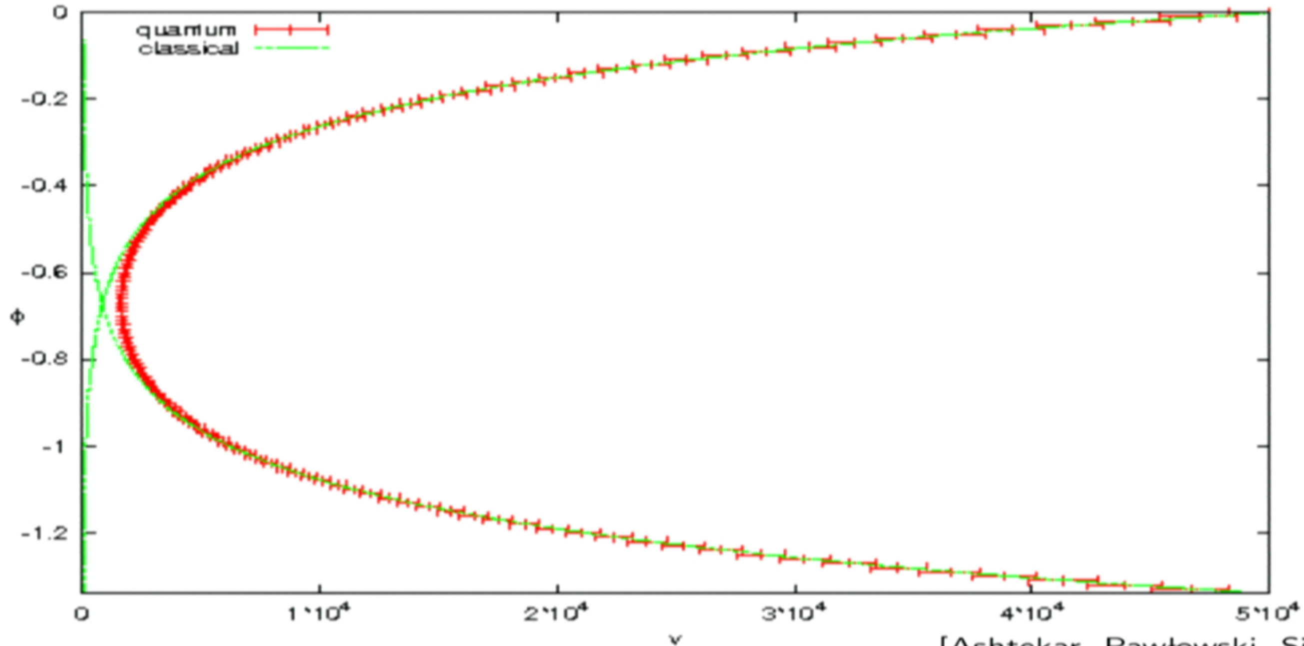
The Hamiltonian Constraint Operator

Assuming a massless scalar field φ, the Hamiltonian constraint operator equation $\widehat{\mathcal{C}_H}\,\psi(\nu,\varphi) = 0$ in LQC implies

$$\partial_\varphi^2\,\psi(\nu,\varphi) = \frac{3\pi G}{4}\Big[(\nu+2)\sqrt{\nu(\nu+4)}\,\psi(\nu+4,\varphi) - 2\nu^2\,\psi(\nu,\varphi) + (\nu-2)\sqrt{\nu(\nu-4)}\,\psi(\nu-4,\varphi)\Big].$$

For simplicity, this equation is written for the lapse $N = a^3$ and in terms of the variable $\nu \propto p^{3/2} = a^3$.

Numerical Study of the Dynamics



[Ashtekar, Pawłowski, Singh]

Effective Dynamics

Note that the wave function remains sharply peaked throughout its evolution, even at the bounce point. Its trajectory can be described by the dynamics due to an effective Hamiltonian constraint,

$$\mathcal{C}_H{}^{(\text{eff})} = -\frac{3\sqrt{p}}{8\pi\gamma^2 G} \frac{p}{\Delta \ell_{\text{Pl}}^2} \sin^2(\bar{\mu}c) + \frac{p_\phi^2}{2p^{3/2}} = 0;$$

which can be obtained from $\mathcal{C}_H$ by the substitution $c \longrightarrow (\sin \bar{\mu}c)/\bar{\mu}$ where $\bar{\mu} = \sqrt{\Delta \ell_{\text{Pl}}^2/p}$, and $\Delta \ell_{\text{Pl}}^2$ is the smallest area eigenvalue in LQG.

By calculating $\dot{p} = \{p, \mathcal{C}_H{}^{(\text{eff})}\}$, one obtains the modified Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),$$

where $H = \dot{a}/a$ is the Hubble rate and the critical density is $\rho_c = (3/8\pi\gamma^2\Delta)\rho_{\text{Pl}} \approx 0.41\rho_{\text{Pl}}$.

Quantum-Corrected Effective Equations

The Friedmann equation is modified by quantum geometry effects,

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right),$$

as is the Raychaudhuri equation,

$$\frac{\ddot{a}}{a} = H^2 - 4\pi G\,(\rho + P)\left(1 - \frac{2\rho}{\rho_c}\right),$$

but the equation governing the conservation of energy remains the same:

$$\dot{\rho} + 3H\,(\rho + P) = 0.$$

Types of Corrections

In LQC, there are typically two types of corrections: holonomy and inverse triad corrections.

In homogeneous LQC models, effects due to inverse triad corrections are negligible if the physical volume of the manifold is large compared to $\ell_{\rm Pl}$ at the bounce point. In the equations on the previous page only effects due to holonomy corrections were shown.

It is possible that this is because inverse triad operators are not implemented correctly in LQC. Unfortunately, it is not clear how to fix this.

For the remainder of this talk, we will continue to only consider holonomy corrections.

What About Perturbations?

The next step is to try and determine how the classical equations for linear perturbations around a flat FLRW background are modified by quantum geometry corrections.

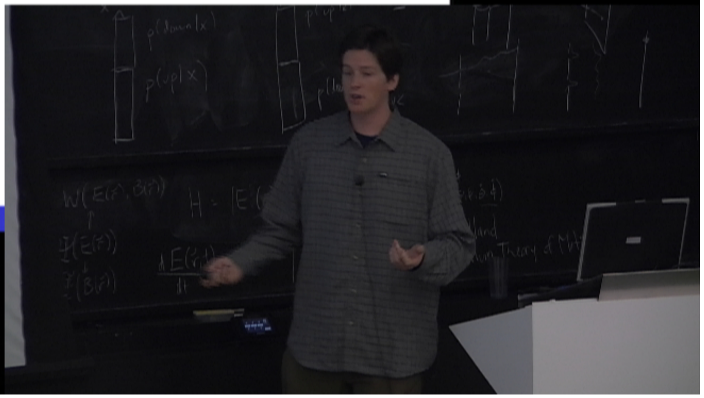
There has been a lot of work studying inverse triad corrections to the dynamics of linear perturbations in an effective setting in LQC as this type of correction is easier to implement at the effective level [Bojowald, Hossain, Kagan, Shankaranarayanan].

It has been considerably more difficult to implement holonomy corrections for perturbations, especially in the case of scalar perturbations.

Two of the Main Difficulties

In LQC we usually work with a minisuperspace: quantum mechanics is sufficient to describe the quantum dynamics of the FLRW space-time in LQC.

Once perturbations are allowed, we move to a quantum field theory setting.



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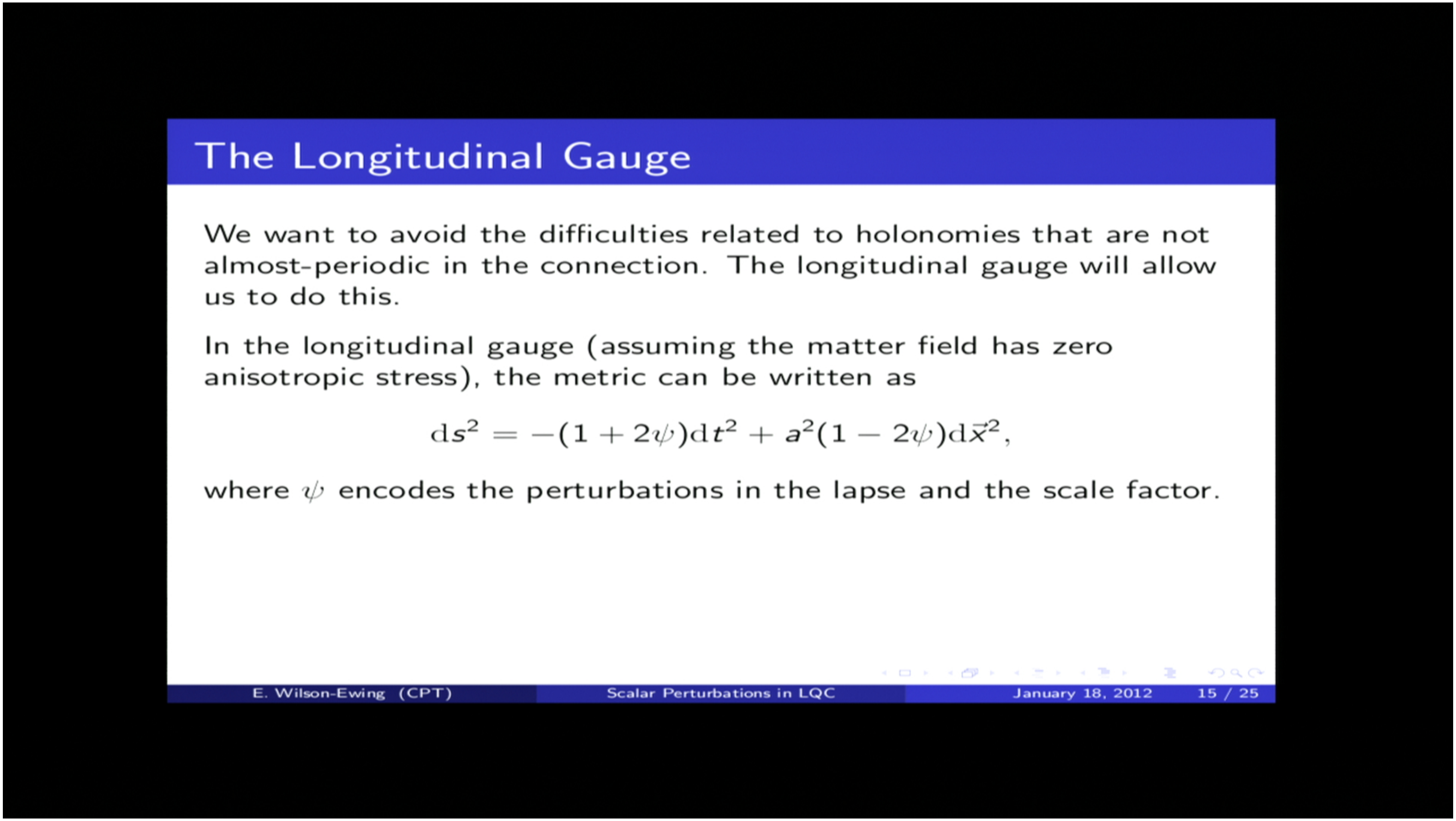
In addition, the degrees of freedom are different: in the homogeneous and isotropic case, it is sufficient to only consider holonomies that are almost-periodic in the connection.

When perturbations are included, this is no longer possible.



The Longitudinal Gauge
We want to avoid the difficulties related to holonomies that are not almost-periodic in the connection. The longitudinal gauge will allow us to do this.
In the longitudinal gauge (assuming the matter field has zero anisotropic stress), the metric can be written as
$$\mathrm{d}s^2 = -(1+2\psi)\mathrm{d}t^2 + a^2(1-2\psi)\mathrm{d}\vec{x}^2,$$
where ψ encodes the perturbations in the lapse and the scale factor.
E. Wilson-Ewing (CPT)
Scalar Perturbations in LQC
January 18, 2012
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where ψ encodes the perturbations in the lapse and the scale factor.

The important point for us is that the spatial metric in this gauge can be viewed as the FLRW metric where the scale factor now depends on position as well as time. This will allow us to incorporate perturbations in LQC by using a new approach.

Lattice LQC

The idea is the following: discretize the manifold (say a cubulation of T^3 for the sake of simplicity) and assume that each cell is homogeneous. Then the metric in each cell is that of a flat FLRW metric, which we know how to quantize in LQC!

The interactions between the neighbouring cells can be turned on in the Hamiltonian. In order to determine precisely how to do this, we must get the Hamiltonian for linear perturbations in the variables that are naturally suited for lattice LQC: $c(\vec{x})$, $p(\vec{x})$.

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We will find that the Hamiltonian can be split into an ultralocal "homogeneous" term and an interaction term. We will perform the standard LQC quantization of the homogeneous term and then turn on the interactions in the simplest consistent manner. In this way, effective equations describing the quantum-geometry-corrected dynamics of scalar perturbations will be obtained.

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Linear Perturbations in Unusual Variables

In the longitudinal gauge, the Gauss constraint is automatically satisfied, $N = 1 + \psi$ and $N^a = 0$. Therefore, the Hamiltonian is given by

$$\mathcal{C}_H = \int N\mathcal{H},$$

where $\mathcal{H}$ is the scalar constraint.

The lapse can be written in terms of the fundamental variable $p = a^2(1 - 2\psi)$ as follows:

$$N = 1 + \psi = 1 + \frac{\bar{p} - p}{2\bar{p}},$$

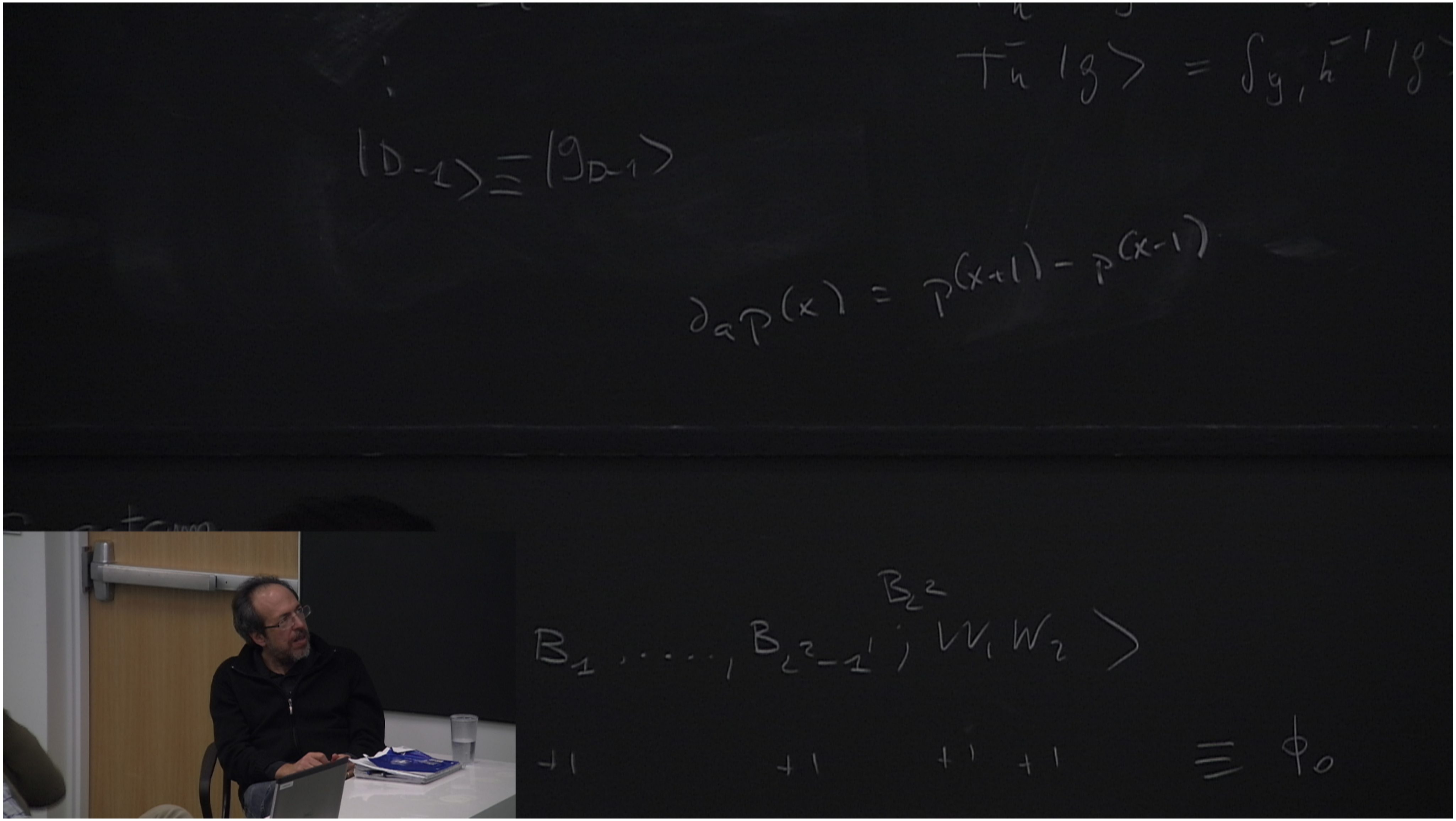
where $\bar{p} = \int p$ up to some normalization by the volume of the 3-torus with respect to the x^i coordinates. Then, $\bar{p} = a^2$.

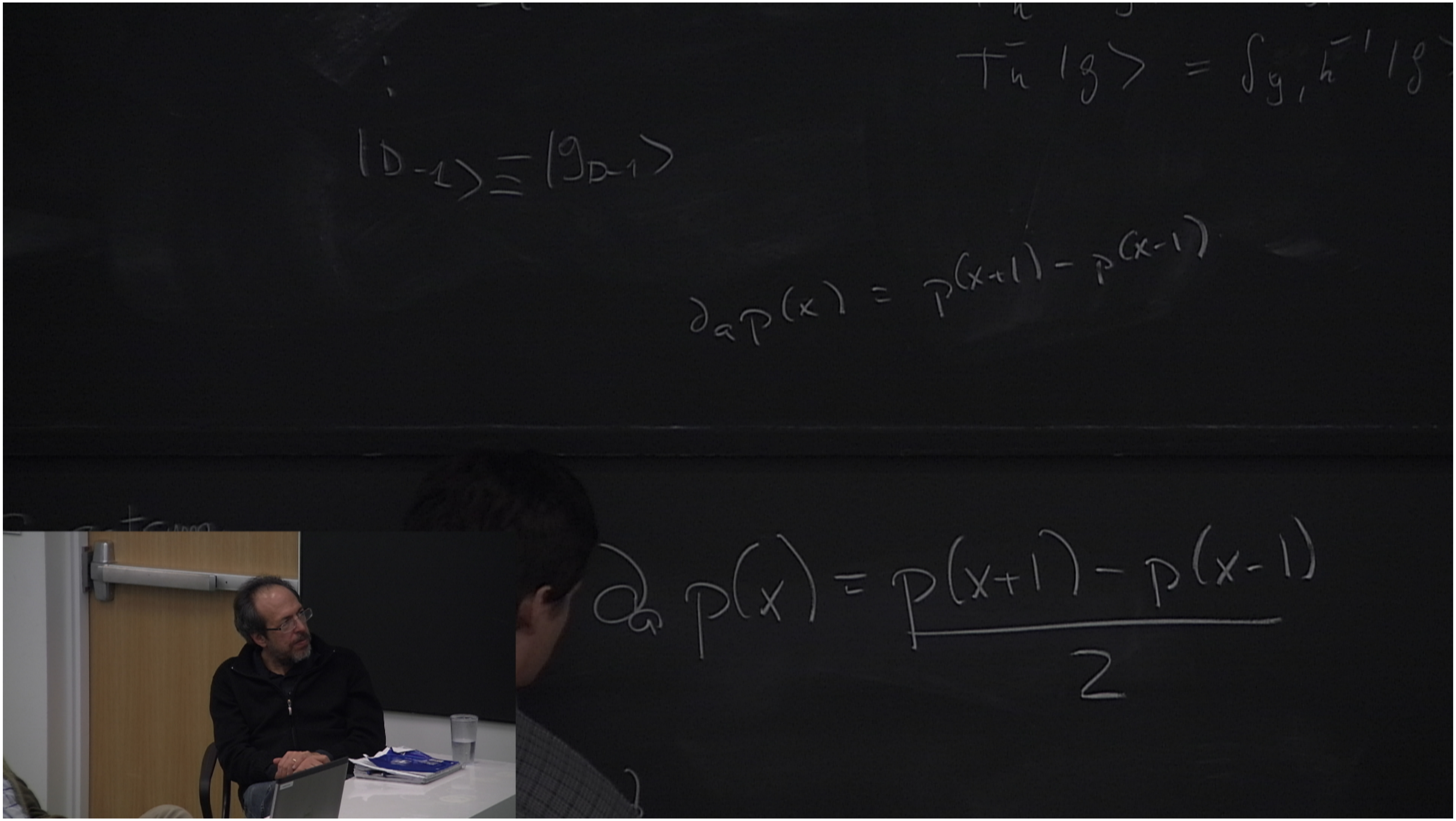
The Constraints

Assuming a massless scalar field φ, the scalar constraint separates into homogeneous and interaction terms and is given by

$$\mathcal{H} = -\frac{3\sqrt{\bar{p}}c^2}{8\pi\gamma^2 G} + \frac{\pi_\varphi^2}{2\bar{p}^{3/2}} - \frac{\sqrt{\bar{p}}}{8\pi G}\left(2\nabla^2\left(\frac{\bar{p}-p}{2\bar{p}}\right) - \left(\vec{\nabla}\frac{\bar{p}-p}{2\bar{p}}\right)^2\right) + \frac{\sqrt{\bar{p}}}{2}\left(\vec{\nabla}\varphi\right)^2 \approx 0,$$

which is constrained to vanish to first order in the perturbations.


|D-1> ≡ |g_{D-1}>
∂_a p(x) = p(x+1) − p(x−1)
≡ φ_0


|D-1> ≡ |g_D-1>
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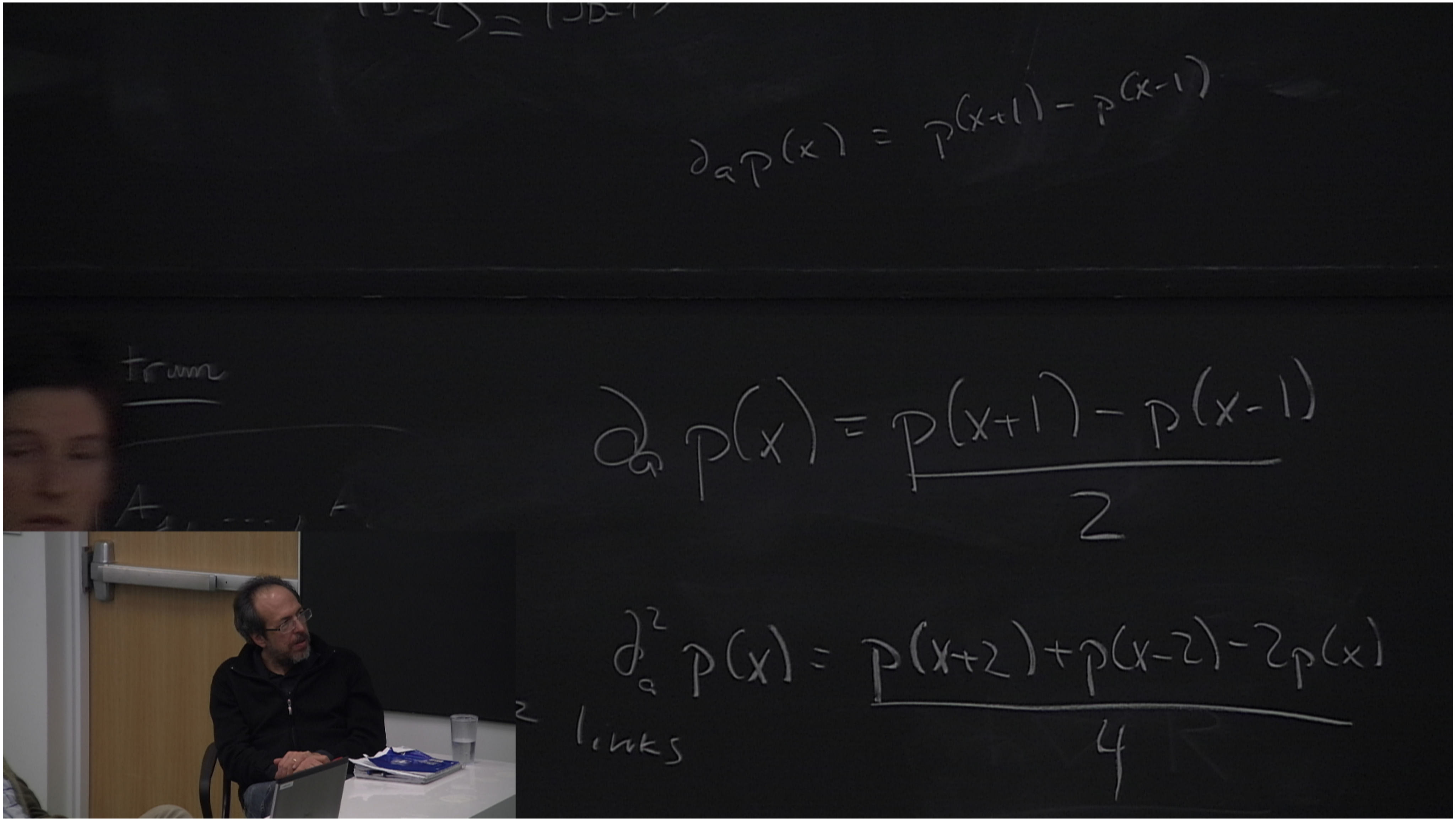
∂a p(x) = p(x+1) - p(x-1) / 2


$\partial_a p(x) = p(x+1) - p(x-1)$
trum
$\partial_a p(x) = \frac{p(x+1) - p(x-1)}{2}$
$\partial_a^2 p(x) = \frac{p(x+2) + p(x-2) - 2p(x)}{4}$
links

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The diffeomorphism constraint is

$$\mathcal{H}_a = \frac{p}{4\pi G\gamma}\left[\partial_a c + c\partial_a\left(\tfrac{\bar{p}-p}{2\bar{p}}\right)\right] + \pi_\varphi\partial_a\varphi \approx 0.$$

Standard Results

The initial conditions must satisfy the constraints. Then the dynamics are the usual local ones for linear scalar perturbations around an FLRW background, despite the unusual variables that were used in the nonlocal Hamiltonian:

$$\dot{\varphi} = (1+\psi)\frac{\pi_\varphi}{p^{3/2}},$$
$$\dot{\pi}_\varphi = a\,\nabla^2\varphi,$$
$$\dot{p} = \frac{2}{\gamma}(1+\psi)\sqrt{p}c,$$
$$\dot{c} = -\frac{1}{2\gamma a}(1+2\psi)c^2 - 2\pi G\gamma a\frac{\pi_\varphi^2}{p^3}.$$

The constraints are automatically preserved by the dynamics.

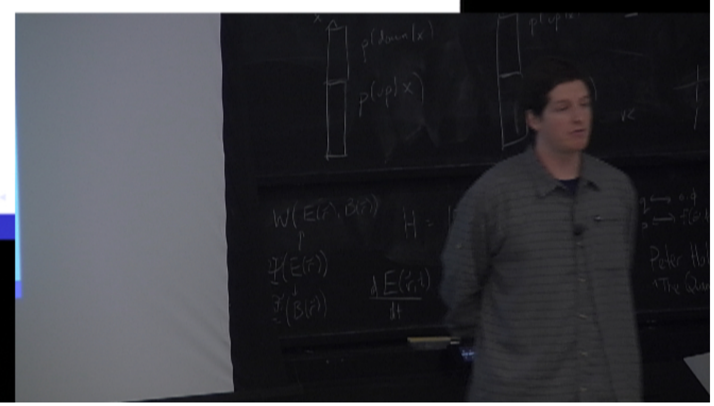
Effective Hamiltonian

We will not modify the diffeomorphism constraint as we wish to preserve spatial diffeomorphisms as a symmetry of our theory.

By using the substitution $c \longrightarrow (\sin \bar{\mu} c)/\bar{\mu}$, the homogeneous part of the scalar constraint becomes

$$\mathcal{H}^{(\mathrm{eff})}_{\mathrm{hom}} = -\frac{3\sqrt{p}}{8\pi\gamma^2 G} \frac{p}{\Delta \ell_{\mathrm{Pl}}^2} \sin^2 \bar{\mu} c + \frac{\pi_\varphi^2}{2p^{3/2}}.$$

E. Wilson-Ewing (CPT) Scalar Perturbations in LQC



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$$\mathcal{H}_{\rm hom}^{\rm (eff)} = -\frac{3\sqrt{p}}{8\pi\gamma^2 G}\frac{p}{\Delta \ell_{\rm Pl}^2}\sin^2 \bar{\mu} c + \frac{\pi_\varphi^2}{2p^{3/2}}.$$

The interaction terms are only modified slightly in order to ensure $\dot{\mathcal{H}}^{\rm (eff)} = \dot{\mathcal{H}}_a^{\rm (eff)} = 0$ so that the constraints are preserved by the dynamics. This gives

$$\mathcal{H}_{\rm int}^{\rm (eff)} = -\frac{\sqrt{p}}{8\pi G}\left[2\nabla^2\left(\tfrac{\bar{p}-p}{2\bar{p}}\right) - \left(\vec{\nabla}\tfrac{\bar{p}-p}{2\bar{p}}\right)^2\right] + \frac{\sqrt{p}}{2}\cos 2\bar{\mu} c\left(\vec{\nabla}\varphi\right)^2.$$

Constraints on the Initial Conditions

The initial conditions must satisfy the quantum-corrected scalar and diffeomorphism constraints everywhere.

However, if the initial conditions are set well away from the Planck regime (i.e., $\bar{\rho} \ll \rho_c$), then it is enough to impose the classical scalar and diffeomorphism constraints.

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The equations of motion generated by $\mathcal{C}_H{}^{(\text{eff})} = \int N[\mathcal{H}_{\text{hom}}^{(\text{eff})} + \mathcal{H}_{\text{int}}^{(\text{eff})}]$ are easily generalized for any perfect fluid.

Quantum-Corrected Effective Equations

The holonomy-corrected equations of motion for scalar perturbations in LQC are:

$$\begin{aligned}
\dot{\varphi} =& (1+\psi)\frac{\pi_\varphi}{p^{3/2}}, \\
\dot{\pi}_\varphi =& a\cos 2\bar{\mu}c\,\nabla^2\varphi, \\
\dot{p} =& \frac{2}{\gamma}(1+\psi)\sqrt{p}\,\frac{\sqrt{p}\sin\bar{\mu}c\cos\bar{\mu}c}{\sqrt{\Delta}\ell_{\rm Pl}}, \\
\dot{c} =& -\frac{3}{2\gamma a}(1+2\psi)\frac{p\sin^2\bar{\mu}c}{\Delta\ell_{\rm Pl}^2} + \frac{1}{\gamma a}(1+2\psi)c\,\frac{\sqrt{p}\sin\bar{\mu}c\cos\bar{\mu}c}{\sqrt{\Delta}\ell_{\rm Pl}} \\
& - 2\pi G\gamma a\frac{\pi_\varphi^2}{p^3}.
\end{aligned}$$

These equations of motion ensure that $\dot{\mathcal{H}} = \dot{\mathcal{H}}_a = 0$.

Effective Equations in a Cosmological Language

For a general perfect fluid, the holonomy-corrected effective equations become:

$$\dot{\delta\rho} + 3H(\delta\rho + \delta P) - 3(\bar{\rho} + \bar{P})\dot{\psi} + \frac{(\bar{\rho} + \bar{P})}{a^2}\left(1 - \frac{2\bar{\rho}}{\rho_c}\right)\nabla^2(\delta u) = 0;$$

$$\partial_t\Big[(\bar{\rho} + \bar{P})\delta u\Big] + \delta P + (\bar{\rho} + \bar{P})\psi + 3H(\bar{\rho} + \bar{P})\delta u = 0;$$

$$\frac{8\pi G}{3}\delta\rho\left(1 - \frac{2\bar{\rho}}{\rho_c}\right) + 2H^2\psi + 2H\dot{\psi} - \frac{2}{3a^2}\left(1 - \frac{2\bar{\rho}}{\rho_c}\right)\nabla^2\psi = 0;$$

$$2\frac{\ddot{a}}{a}\psi - 2H^2\psi + H\dot{\psi} + \ddot{\psi} - 4\pi G\left(1 - \frac{2\bar{\rho}}{\rho_c}\right)(\delta\rho + \delta P) - \frac{1}{a^2}\left(1 + \frac{2\bar{P}}{\rho_c}\right)\nabla^2\psi + \frac{8\pi G(\bar{\rho} + \bar{P})}{\rho_c}\delta\rho = 0.$$

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The holonomy-corrected equations of motion for scalar perturbations in LQC are:

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$$\dot{\pi}_\varphi = a \cos 2\bar{\mu}c \, \nabla^2\varphi,$$

$$\dot{p} = \frac{2}{\gamma}(1+\psi)\sqrt{p}\,\frac{\sqrt{p}\sin\bar{\mu}c\cos\bar{\mu}c}{\sqrt{\Delta}\ell_{\rm Pl}},$$

$$\dot{c} = -\frac{3}{2\gamma a}(1+2\psi)\frac{p\sin^2\bar{\mu}c}{\Delta\ell_{\rm Pl}^2} + \frac{1}{\gamma a}(1+2\psi)c\,\frac{\sqrt{p}\sin\bar{\mu}c\cos\bar{\mu}c}{\sqrt{\Delta}\ell_{\rm Pl}} - 2\pi G\gamma a\frac{\pi_\varphi^2}{p^3}.$$

These equations of motion ensure that $\dot{\mathcal{H}} = \dot{\mathcal{H}}_a = 0$.

Conclusions

- LQC naturally leads to a bouncing universe scenario.
- We now have holonomy-corrected effective equations for scalar perturbations in LQC that will allow us to study the dynamics of the perturbations through the bounce.
- Gravity seems to become repulsive when $\bar{\rho} = \rho_c/2$, in both the background and perturbation equations in LQC.
- These holonomy-corrected equations of motion for scalar perturbations have also been obtained in a recent paper that uses a completely different (and gauge-invariant) approach which gives the same results [Cailleteau, Mielczarek, Barrau, Grain].

Outlook

Directions for further work:

- Gain a better understanding of how the physics changes near ρ_c.
- Develop the quantum theory of lattice LQC.
- Derive observational signatures of LQC:
  - in inflationary models [Bojowald, Calcagni, Tsujikawa; Agullo, Ashtekar, Nelson],
  - in the ekpyrotic scenario,
  - in LQC alone. (Does LQC naturally give an almost scale-invariant spectrum for scalar perturbations with a slight red tilt?)