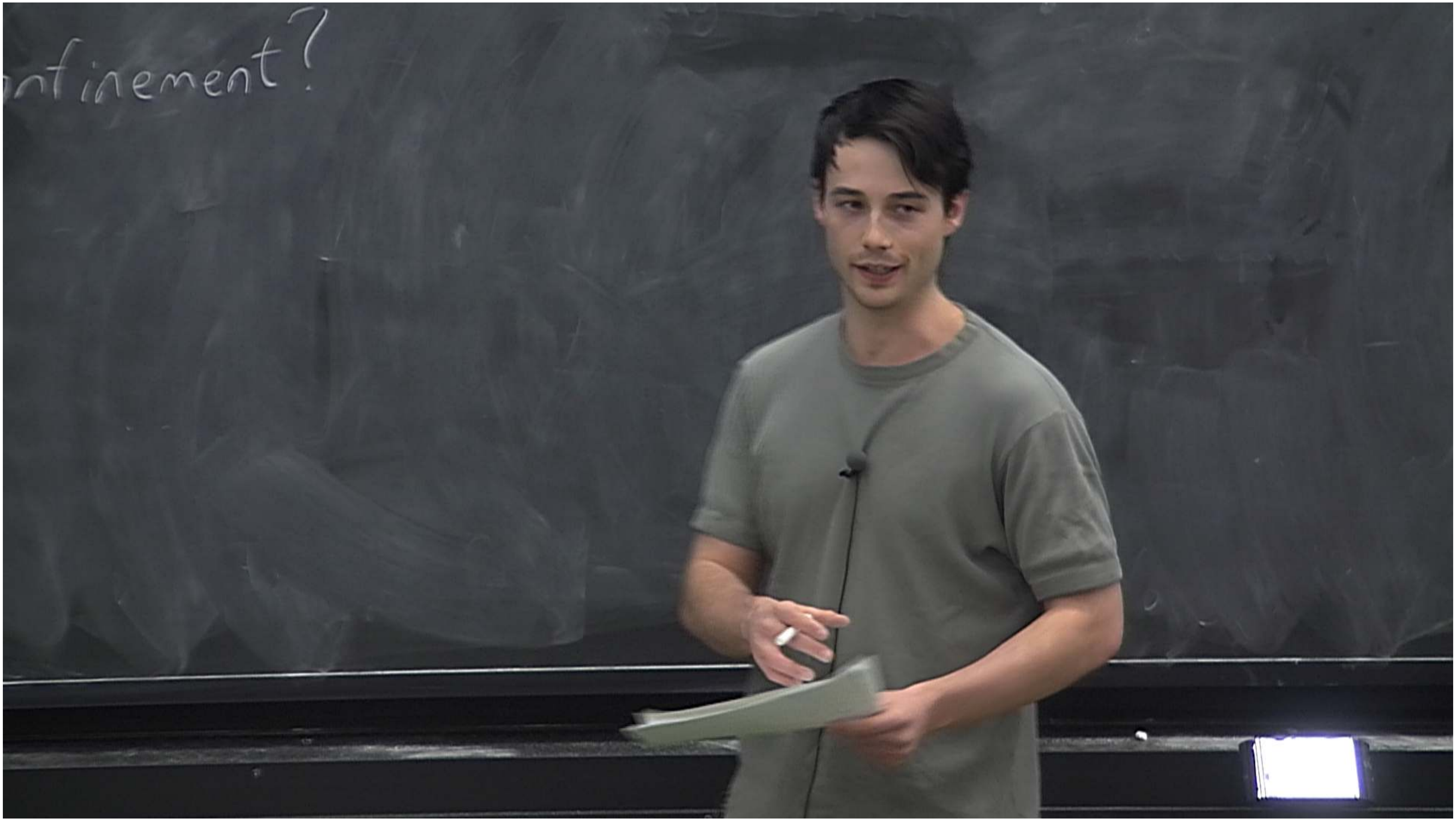
Title: AdS/CFT Correspondence - Lecture 25

Date: Jan 23, 2012 05:15 PM

URL: http://pirsa.org/12010108

Abstract:


nfinement?

Confinement

What is confinement?

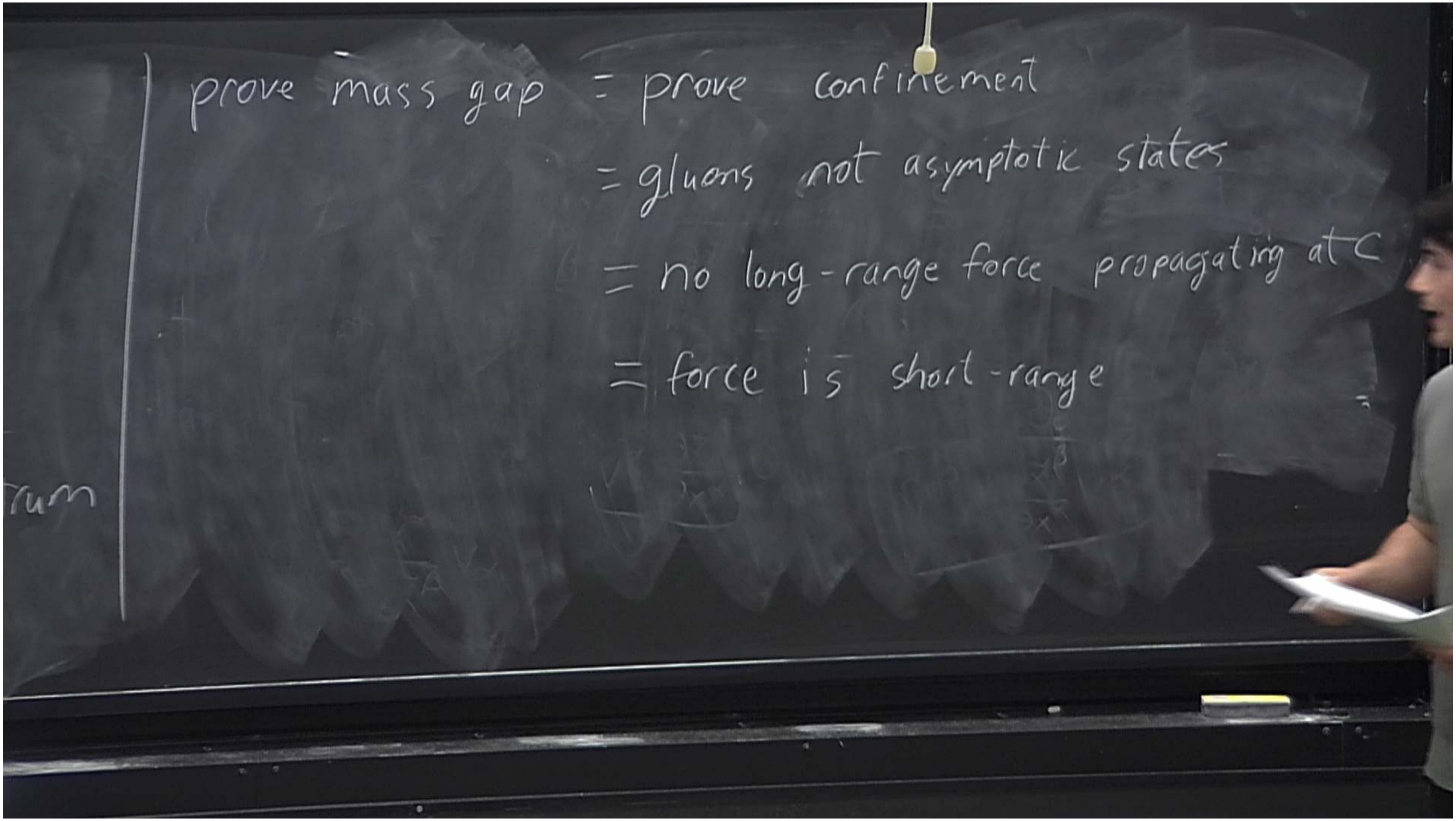
CFT

Gravity

Does it work?

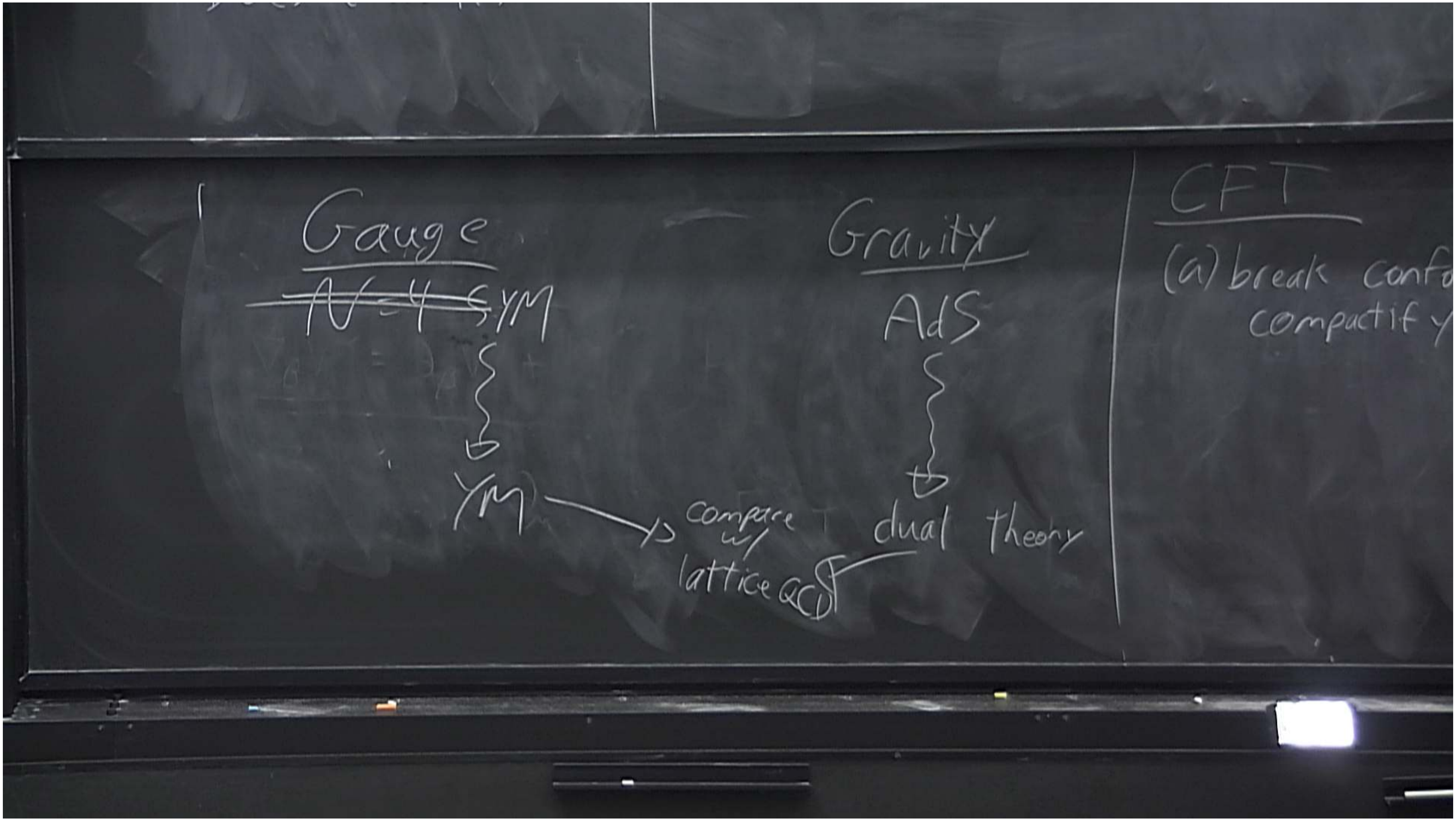
Confinement

- strong coupling
- $V(r) \sim r \Rightarrow$ no free gluons
- 2 properties:
  - mass gap
  - discrete spectrum


rum
prove mass gap = prove confinement
= gluons not asymptotic states
= no long-range force propagating at c
= force is short-range

rum

- prove mass gap = prove confinement
  - = gluons not asymptotic states
  - = no long-range force propagating at c
  - = force is short-range
- lattice QCD
- rigorous proof $\rightarrow$ Millenium prize


Gauge
~~N=4 S~~YM
YM
compare w/ lattice QCD
Gravity
AdS
dual theory
CFT
(a) break

rigorous proof $\longrightarrow$ Millenium prize

ony

CFT

(a) break conformal invariance

compactify 1 dimensions

$$\mathbb{R}^{3,1} \longrightarrow \mathbb{R}^{2,1} \times \underbrace{S^1}_{\text{radius } R}$$

(b) Break SUSY

SUSY-breaking B.C. on S^1

bosons periodic

fermions anti-periodic

(i) Fermions gain mass

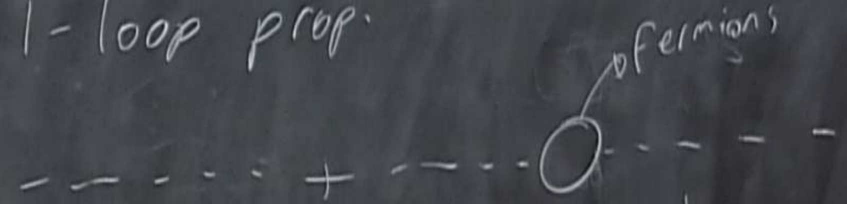
$$\partial_y^2 \phi = 0 \to \phi = a + b \cdot y$$

has no antiperiodic solutions

$$m_F \sim 1/R$$

(ii) Scalars gain mass

1-loop prop.



$$= \frac{1}{p^2} + \frac{1}{p^2}\left(\Sigma\right)\frac{1}{p^2} + \dots$$

$$\simeq \frac{1}{p^2 - \Sigma}$$

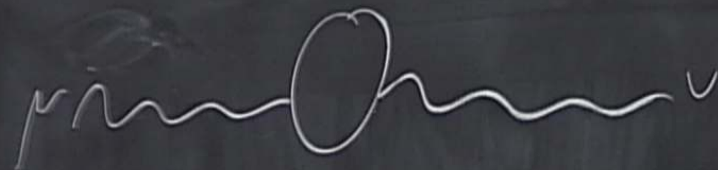
$m^2 \sim$

$$m^2 \sim \Sigma =$$

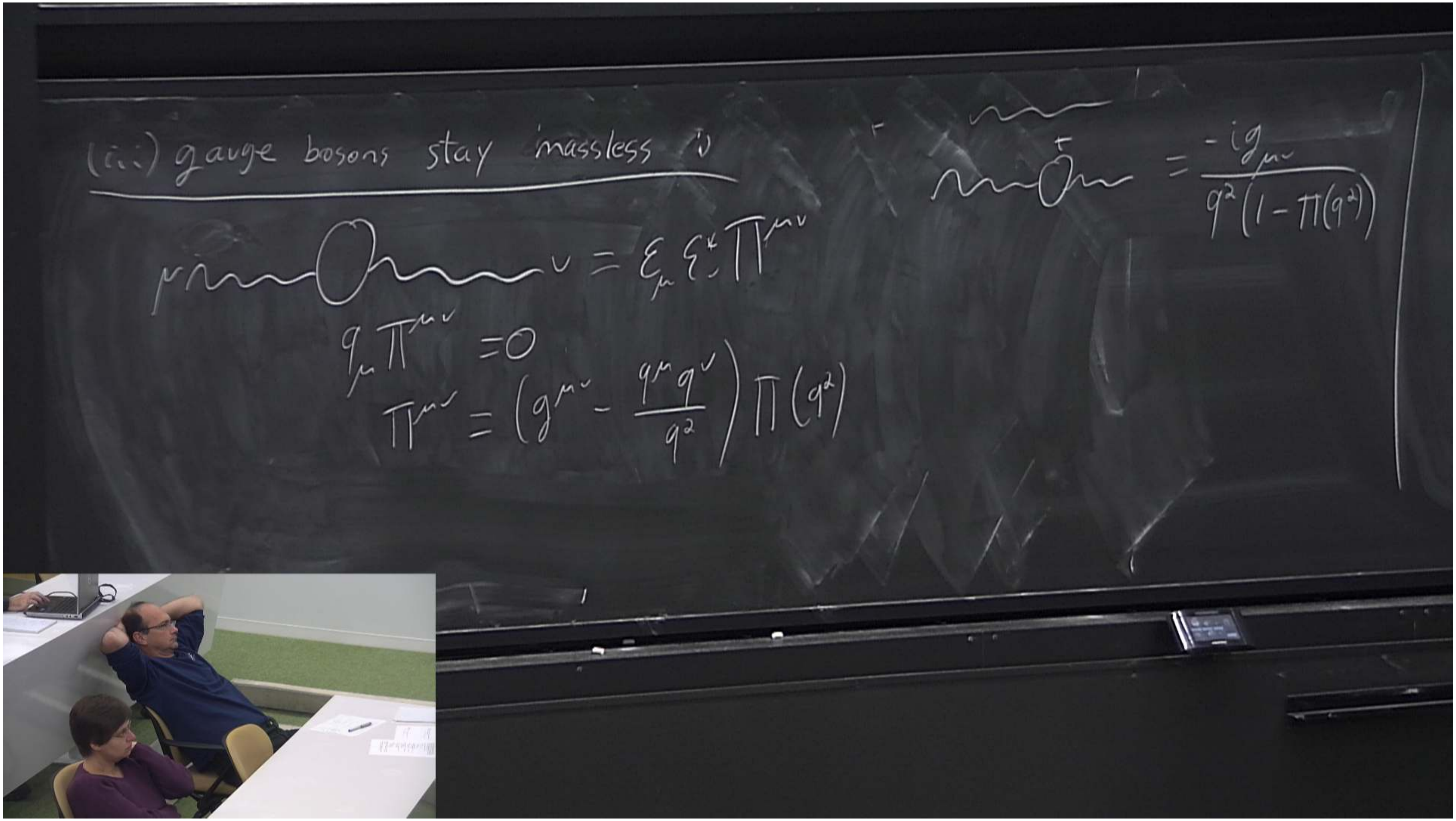
$$m^2_{\text{scalar}} = \frac{g^2 N}{R^2}$$

$$\int \frac{d^4k}{(\not{k}-m)(\not{p}-\not{k}-m)} \sim \text{energy}^2$$

(iii) gauge bosons stay massless

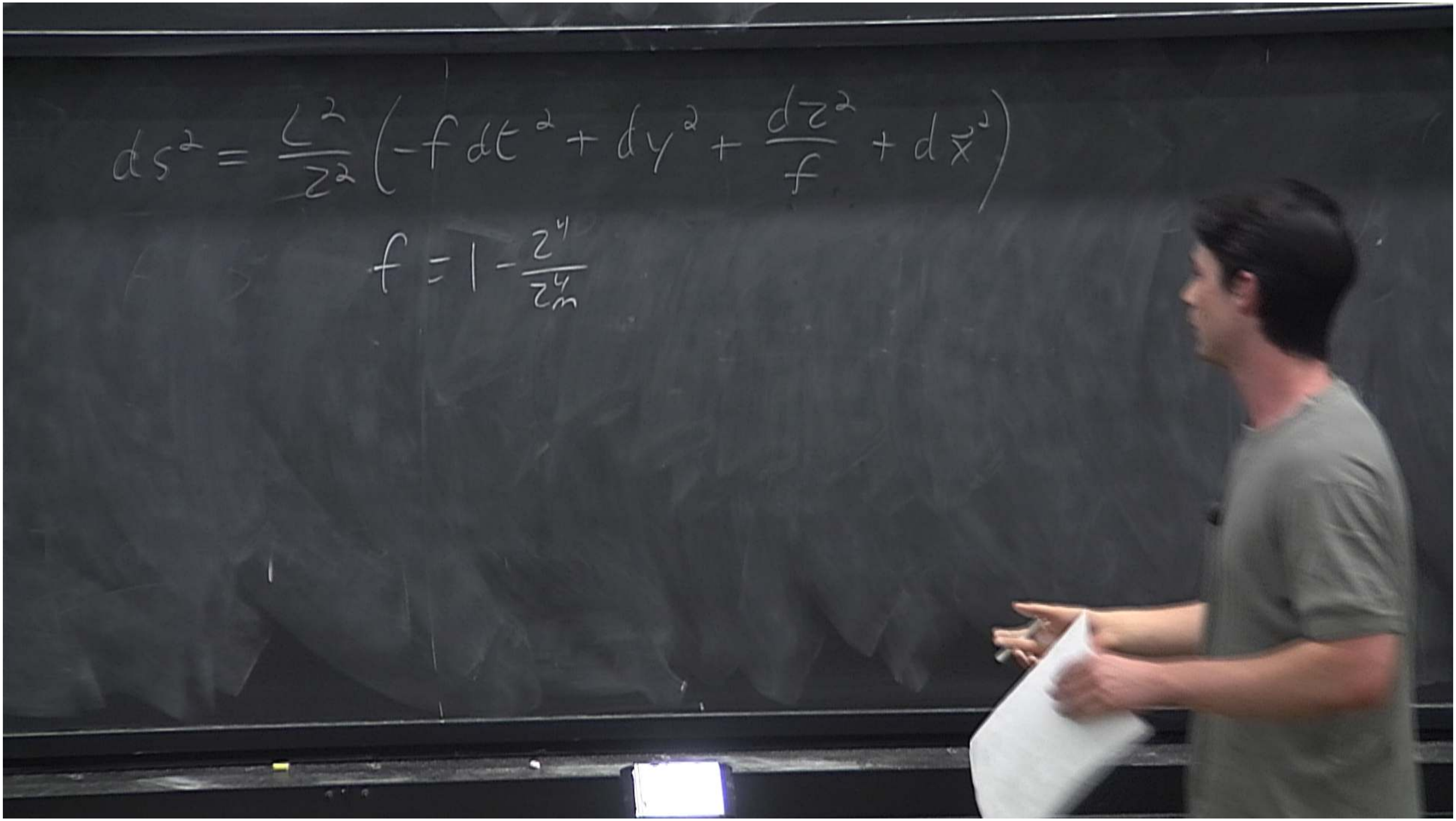


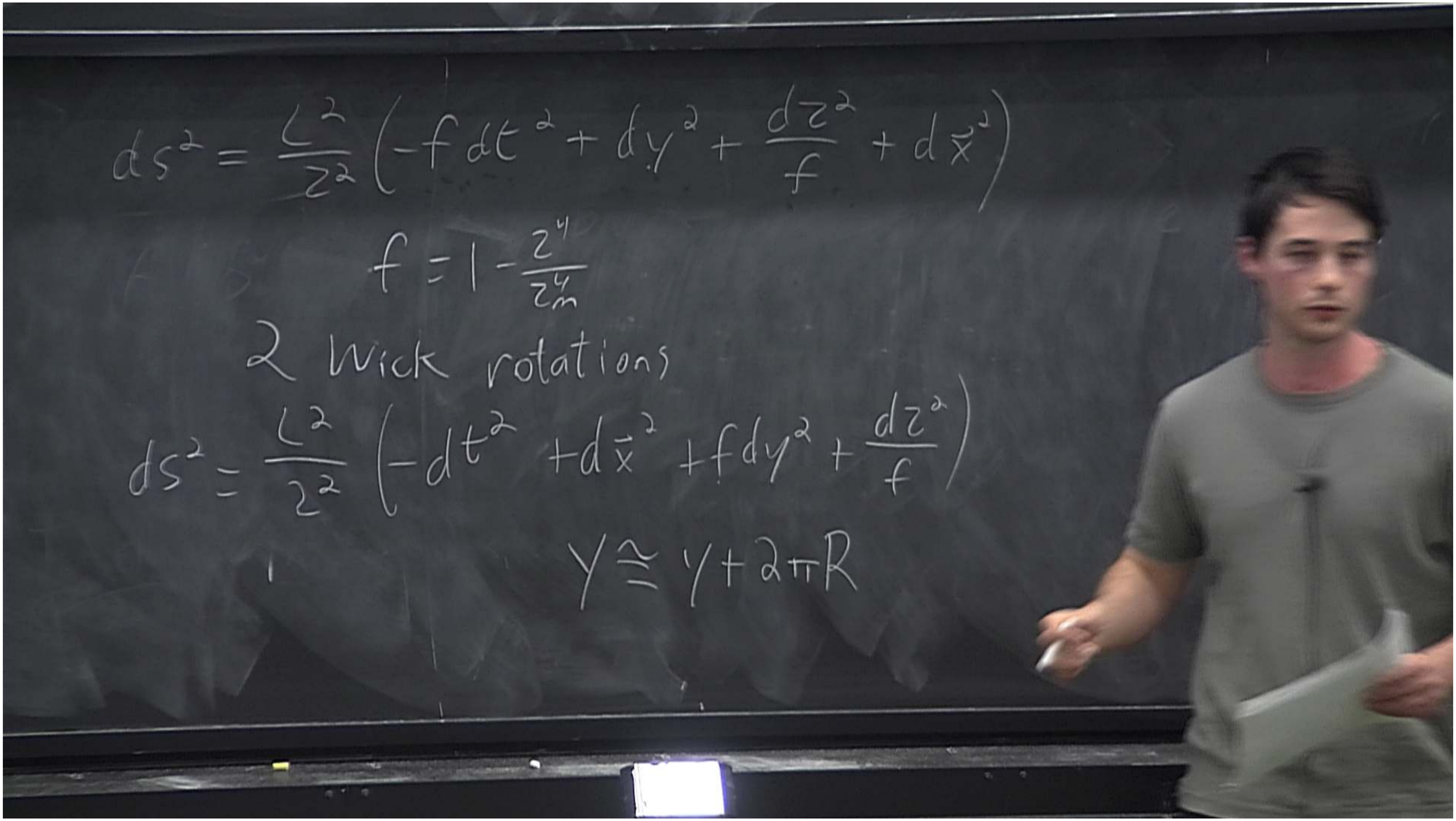
$$= \varepsilon_\mu \varepsilon^*_\nu \Pi^{\mu\nu}$$


(iii) gauge bosons stay massless
$= \epsilon_\mu \epsilon^*_\nu \Pi^{\mu\nu}$
$q_\mu \Pi^{\mu\nu} = 0$
$\Pi^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi(q^2)$
$= \frac{-ig_{\mu\nu}}{q^2(1-\Pi(q^2))}$

Dual AdS geometry

- asymptotically AdS
- boundary = $\mathbb{R}^{2,1} \times S^1$


$ds^2 = \frac{L^2}{z^2}\left(-f\,dt^2 + dy^2 + \frac{dz^2}{f} + d\vec{x}^2\right)$
$f = 1 - \frac{z^4}{z_m^4}$


$ds^2 = \frac{L^2}{z^2}\left(-f\,dt^2 + dy^2 + \frac{dz^2}{f} + d\vec{x}^2\right)$
$f = 1 - \frac{z^4}{z_m^4}$
2 Wick rotations
$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + f\,dy^2 + \frac{dz^2}{f}\right)$
$y \cong y + 2\pi R$

$$ds^2 = \frac{L^2}{z^2}\left(-f\,dt^2 + dy^2 + \frac{dz^2}{f} + d\vec{x}^2\right)$$

$$f = 1 - \frac{z^4}{z_m^4}$$

2 Wick rotations

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\vec{x}^2 + f\,dy^2 + \frac{dz^2}{f}\right)$$

$$y \simeq y + 2\pi R$$

$z = z_m \simeq$ origin in spherical coords

confinement

strong coupling

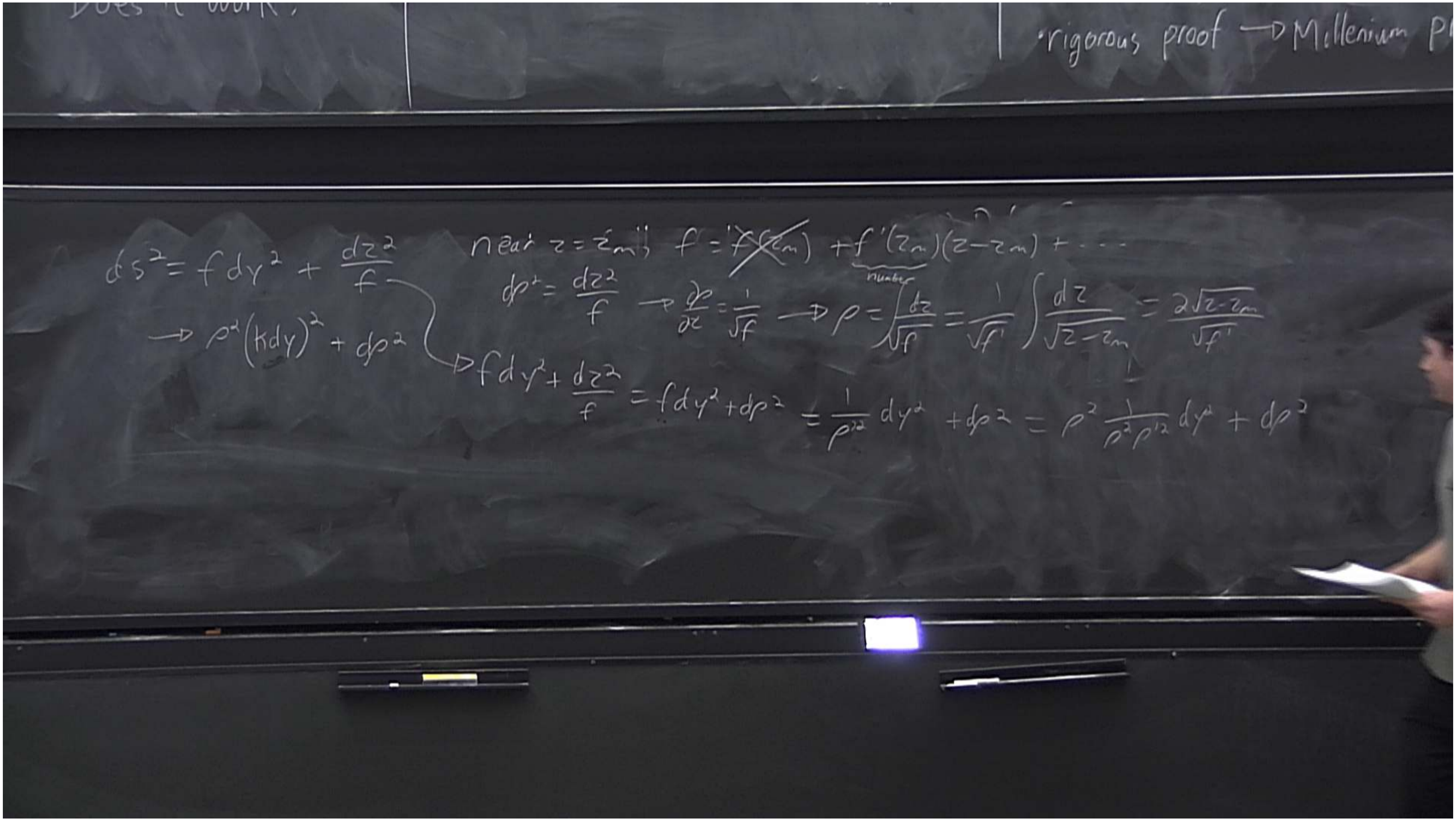
$V(r) \sim r \to$ no free gluons

- prove mass gap = prove confinement

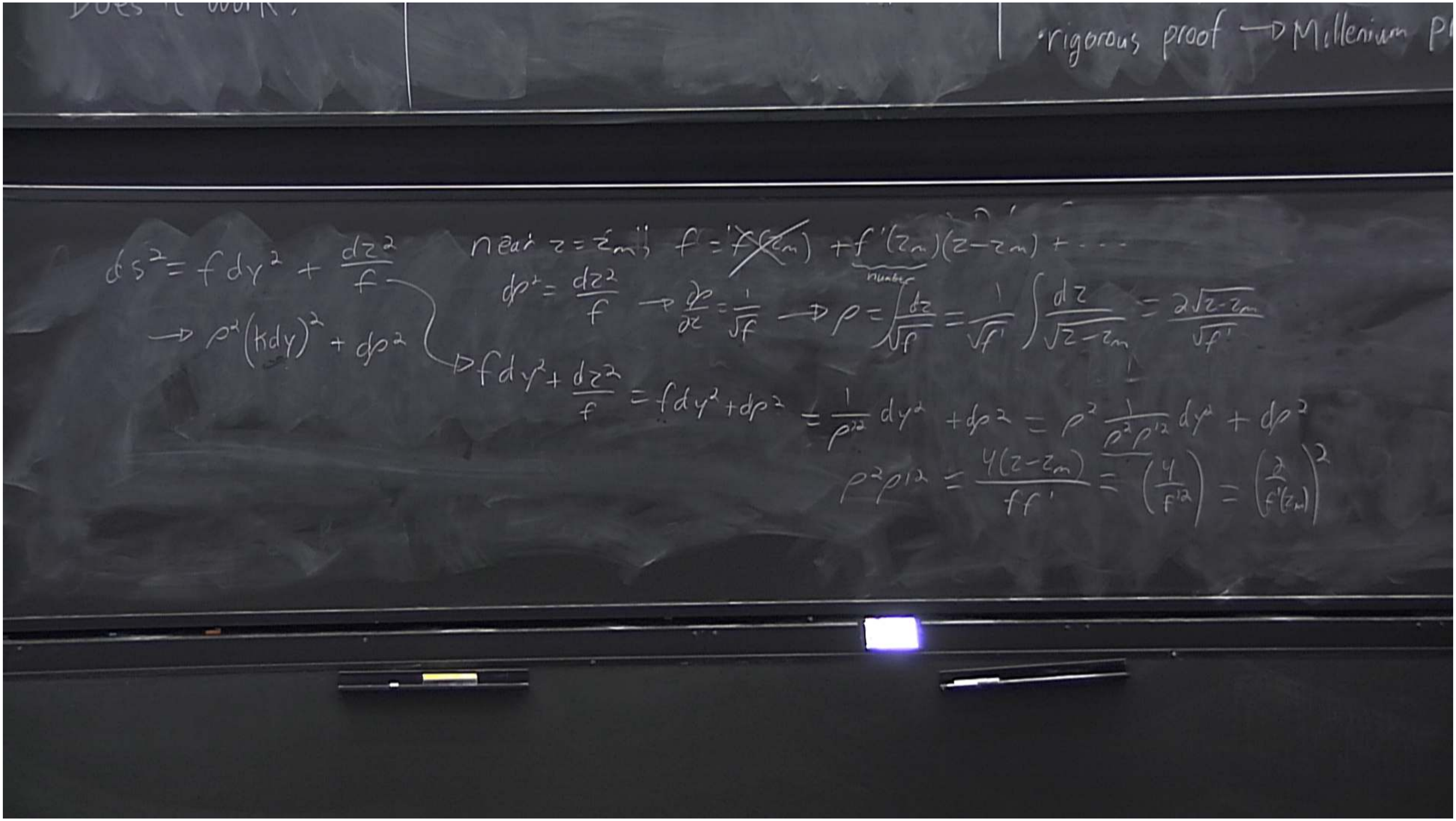
= gluons not asymptotic state

= no long-range force propag

= force is short-range



Does it work?
·rigorous proof → Millenium P
$ds^2 = f\,dy^2 + \frac{dz^2}{f}$
near $z = z_m$: $f = \cancel{f(z_m)} + \underbrace{f'(z_m)}_{\text{number}}(z - z_m) + \dots$
$d\rho^2 = \frac{dz^2}{f} \rightarrow \frac{\partial\rho}{\partial z} = \frac{1}{\sqrt{f}} \rightarrow \rho = \int \frac{dz}{\sqrt{f}} = \frac{1}{\sqrt{f'}} \int \frac{dz}{\sqrt{z - z_m}} = \frac{2\sqrt{z - z_m}}{\sqrt{f'}}$
$\rightarrow \rho^2 (\kappa\,dy)^2 + d\rho^2$
$\rightarrow f\,dy^2 + \frac{dz^2}{f} = f\,dy^2 + d\rho^2 = \frac{1}{\rho'^2} dy^2 + d\rho^2 = \rho^2 \frac{1}{\rho^2 \rho'^2} dy^2 + d\rho^2$


rigorous proof → Millenium P
$ds^2 = f\,dy^2 + \frac{dz^2}{f}$
$\to \rho^2 (k\,dy)^2 + d\rho^2$
near $z = z_m$: $f = \cancel{f(z_m)} + \underbrace{f'(z_m)}_{\text{number}}(z - z_m) + \dots$
$d\rho^2 = \frac{dz^2}{f} \to \frac{\partial\rho}{\partial z} = \frac{1}{\sqrt{f}} \to \rho = \int \frac{dz}{\sqrt{f}} = \frac{1}{\sqrt{f'}} \int \frac{dz}{\sqrt{z - z_m}} = \frac{2\sqrt{z - z_m}}{\sqrt{f'}}$
$\to f\,dy^2 + \frac{dz^2}{f} = f\,dy^2 + d\rho^2 = \frac{1}{\rho'^2} dy^2 + d\rho^2 = \rho^2 \frac{1}{\rho^2 \rho'^2} dy^2 + d\rho^2$
$\rho^2 \rho'^2 = \frac{4(z - z_m)}{f f'} = \left(\frac{4}{f'^2}\right) = \left(\frac{2}{f'(z_m)}\right)^2$

Does it work?

· rigorous proof $\longrightarrow$ Millenium P

$$ds^2 = f\,dy^2 + \frac{dz^2}{f}$$

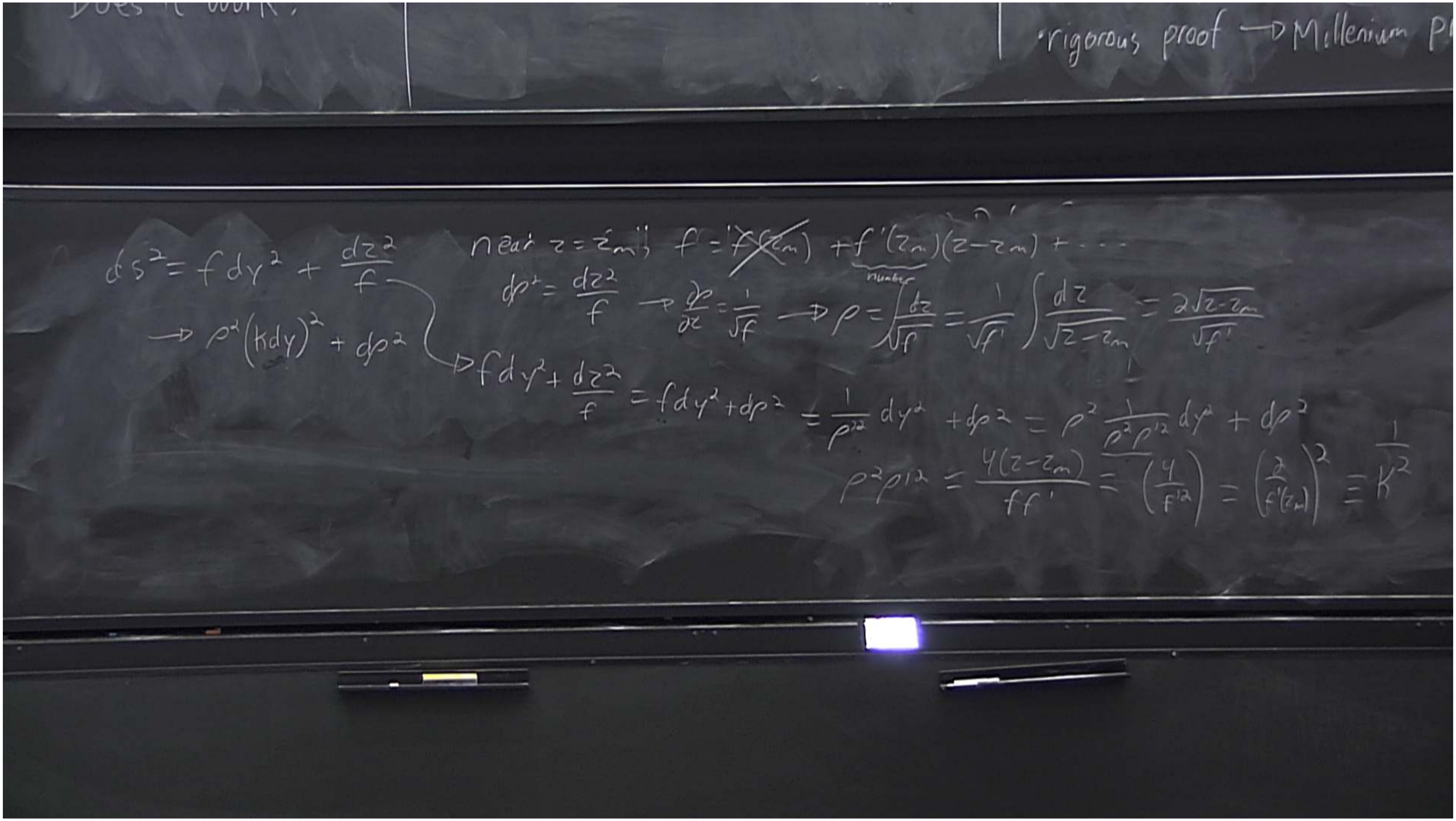
near $z = z_m$: $f = \cancel{f(z_m)} + \underbrace{f'(z_m)}_{\text{number}}(z - z_m) + \cdots$

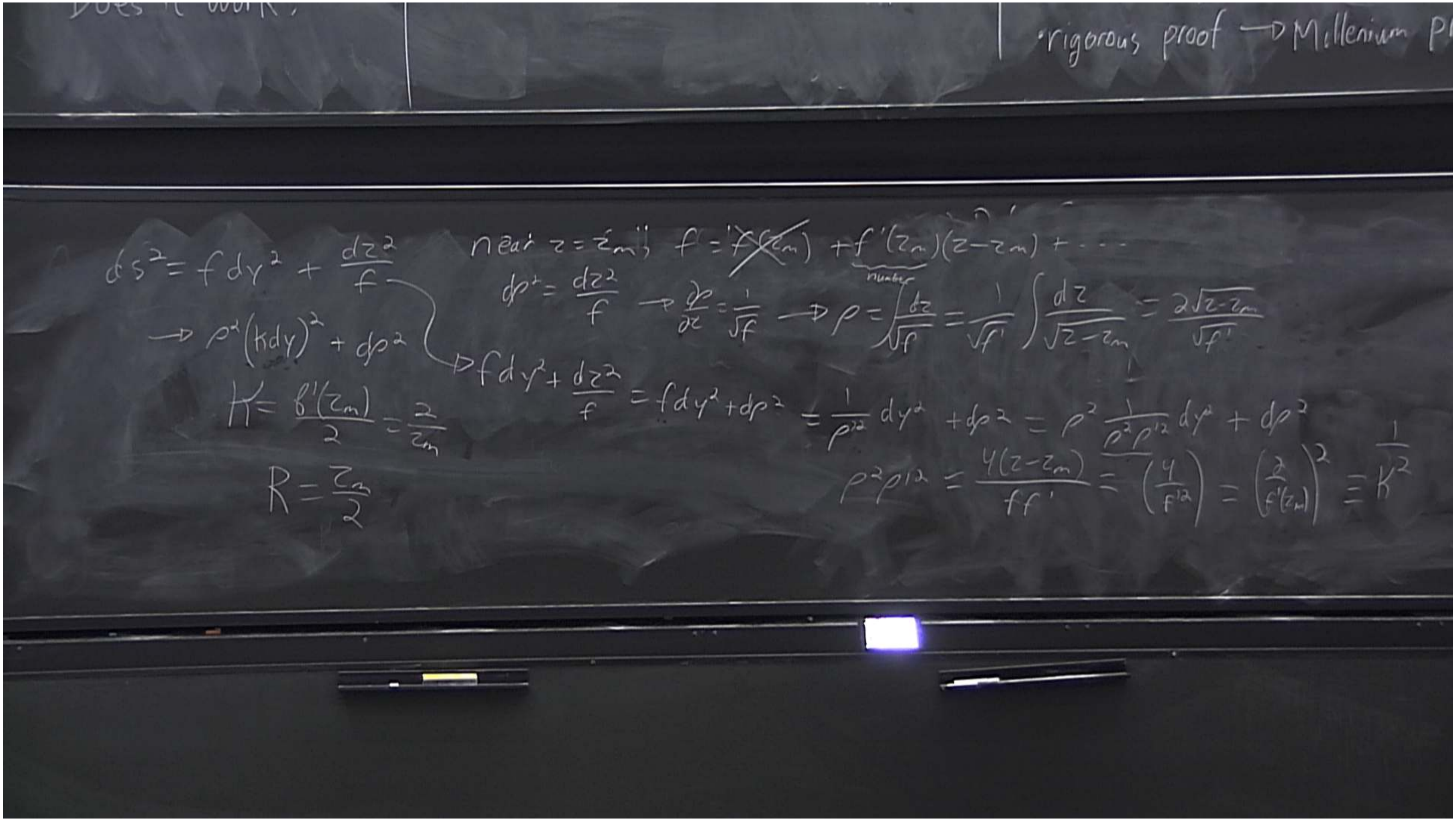
$$d\rho^2 = \frac{dz^2}{f} \rightarrow \frac{d\rho}{dz} = \frac{1}{\sqrt{f}} \rightarrow \rho = \int \frac{dz}{\sqrt{f}} = \frac{1}{\sqrt{f'}} \int \frac{dz}{\sqrt{z - z_m}} = \frac{2\sqrt{z - z_m}}{\sqrt{f'}}$$

$$\rightarrow \rho^2 (K\,dy)^2 + d\rho^2$$

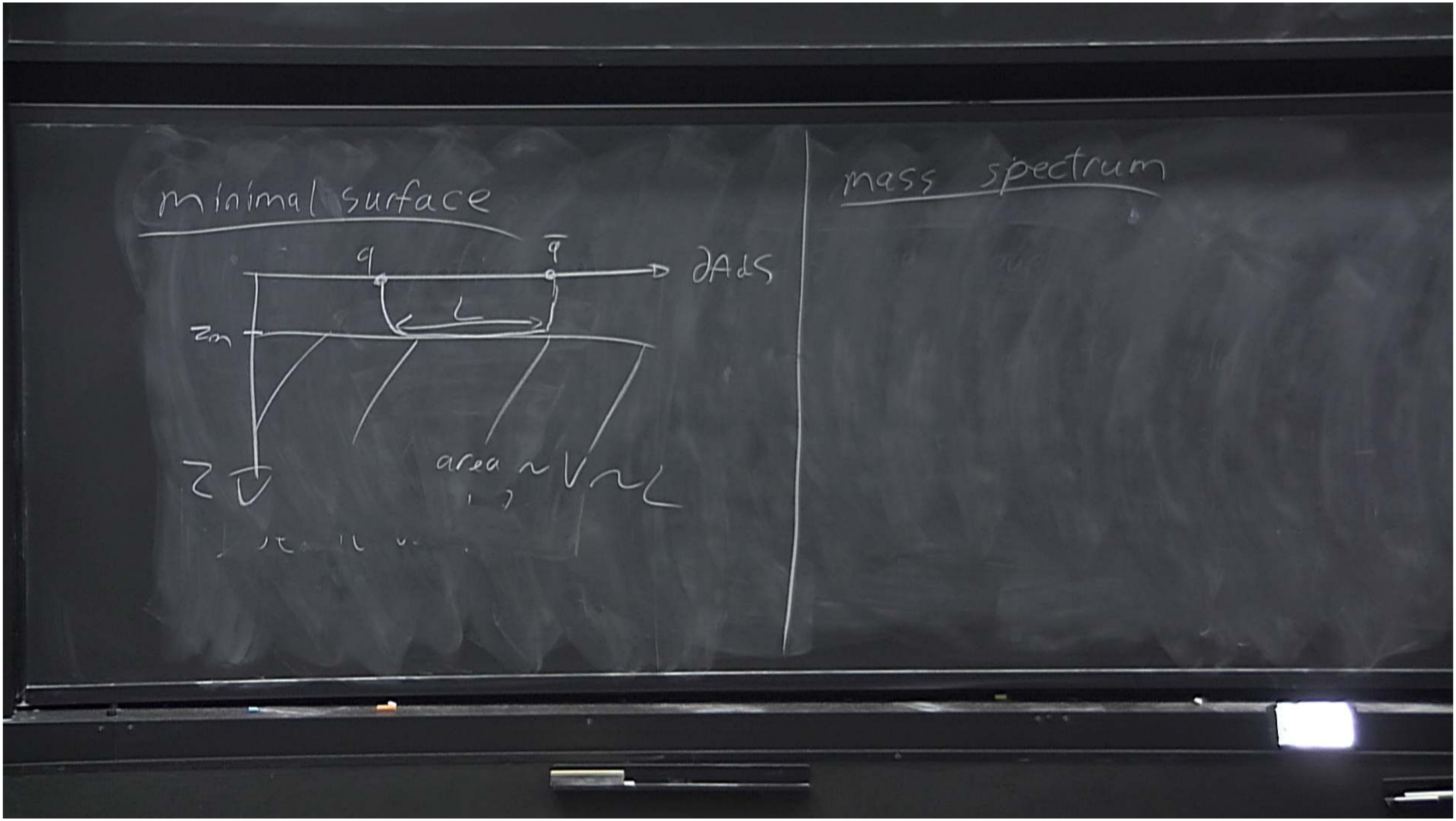
$$\rightarrow f\,dy^2 + \frac{dz^2}{f} = f\,dy^2 + d\rho^2 = \frac{1}{\rho'^2}\,dy^2 + d\rho^2 = \rho^2 \frac{1}{\rho^2 \rho'^2}\,dy^2 + d\rho^2$$

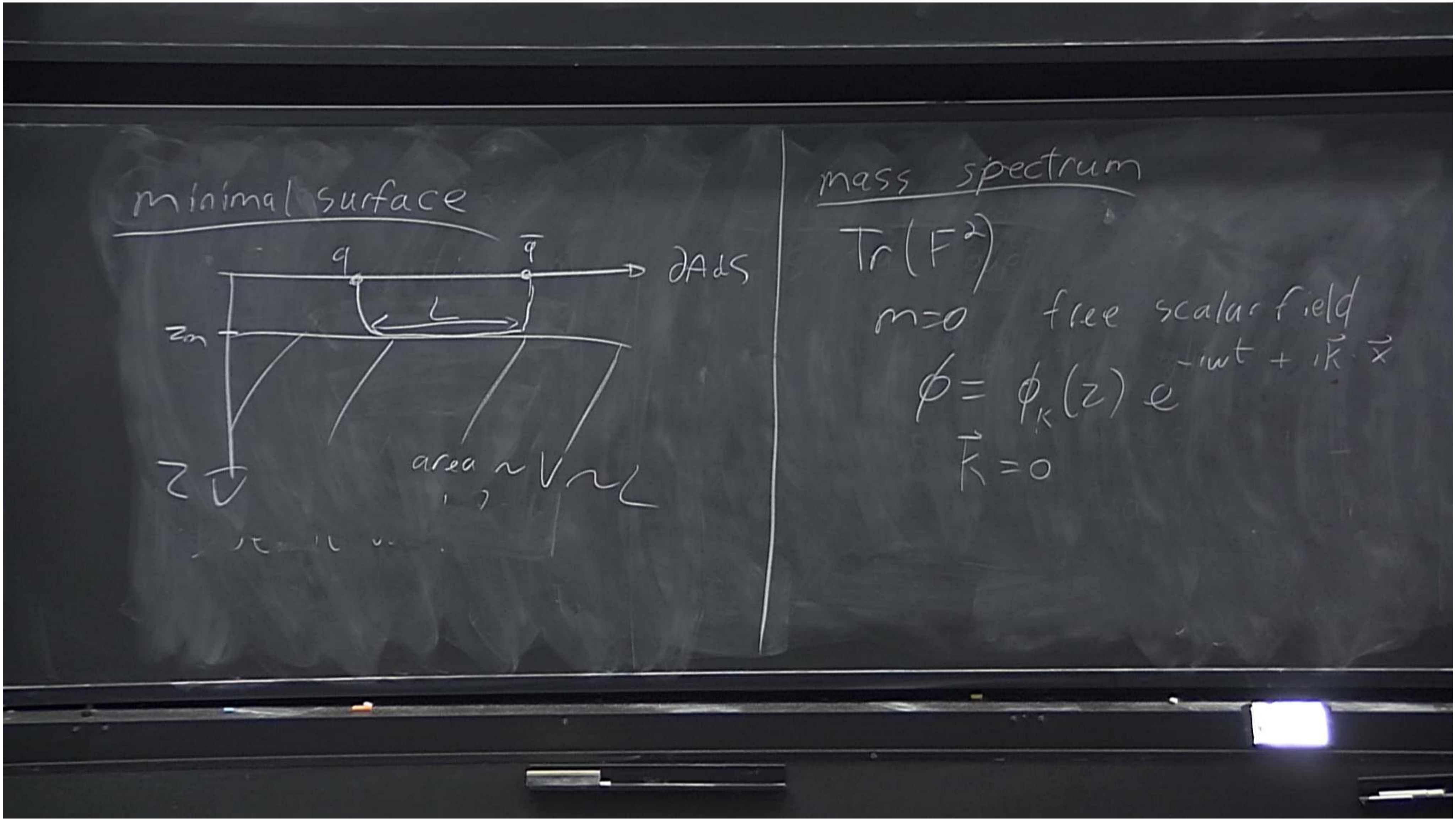
$$\rho^2 \rho'^2 = \frac{4(z - z_m)}{f f'} = \left(\frac{4}{f'^2}\right) = \left(\frac{2}{f'(z_m)}\right)^2 \equiv K^2$$


·rigorous proof → Millenium P
$ds^2 = f\,dy^2 + \frac{dz^2}{f}$
$\rightarrow \rho^2(\kappa\,dy)^2 + d\rho^2$
near $z = z_m$: $f = \cancel{f(z_m)} + \underbrace{f'(z_m)}_{\text{number}}(z-z_m) + \dots$
$d\rho^2 = \frac{dz^2}{f} \rightarrow \frac{d\rho}{dz} = \frac{1}{\sqrt{f}} \rightarrow \rho = \int\frac{dz}{\sqrt{f}} = \frac{1}{\sqrt{f'}}\int\frac{dz}{\sqrt{z-z_m}} = \frac{2\sqrt{z-z_m}}{\sqrt{f'}}$
$\rightarrow f\,dy^2 + \frac{dz^2}{f} = f\,dy^2 + d\rho^2 = \frac{1}{\rho'^2}dy^2 + d\rho^2 = \rho^2\frac{1}{\rho^2\rho'^2}dy^2 + d\rho^2$
$\rho^2\rho'^2 = \frac{4(z-z_m)}{ff'} = \left(\frac{4}{f'^2}\right) = \left(\frac{2}{f'(z_m)}\right)^2 = \frac{1}{\kappa^2}$



rigorous proof → Millenium P
$ds^2 = f\,dy^2 + \frac{dz^2}{f}$
near $z = z_m$; $f = \cancel{f(z_m)} + \underbrace{f'(z_m)}_{\text{number}}(z-z_m) + \dots$
$d\rho^2 = \frac{dz^2}{f} \to \frac{d\rho}{dz} = \frac{1}{\sqrt{f}} \to \rho = \int \frac{dz}{\sqrt{f}} = \frac{1}{\sqrt{f'}}\int \frac{dz}{\sqrt{z-z_m}} = \frac{2\sqrt{z-z_m}}{\sqrt{f'}}$
$\to \rho^2 (K\,dy)^2 + d\rho^2$
$\to f\,dy^2 + \frac{dz^2}{f} = f\,dy^2 + d\rho^2 = \frac{1}{\rho'^2}dy^2 + d\rho^2 = \rho^2 \frac{1}{\rho^2\rho'^2}dy^2 + d\rho^2$
$K = \frac{f'(z_m)}{2} = \frac{2}{z_m}$
$\rho^2\rho'^2 = \frac{4(z-z_m)}{ff'} = \left(\frac{4}{f'^2}\right) = \left(\frac{2}{f'(z_m)}\right)^2 = \frac{1}{K^2}$
$R = \frac{z_m}{2}$


minimal surface
q
q̄
∂AdS
L
z_m
z
area ~ V ~ L
mass spectrum


minimal surface
q
$\bar{q}$
∂AdS
L
z_m
z
area ~ V ~ L
mass spectrum
$\mathrm{Tr}(F^2)$
m=0 free scalar field
$\phi = \phi_k(z)\, e^{-i\omega t + i\vec{k}\cdot\vec{x}}$
$\vec{k} = 0$

field

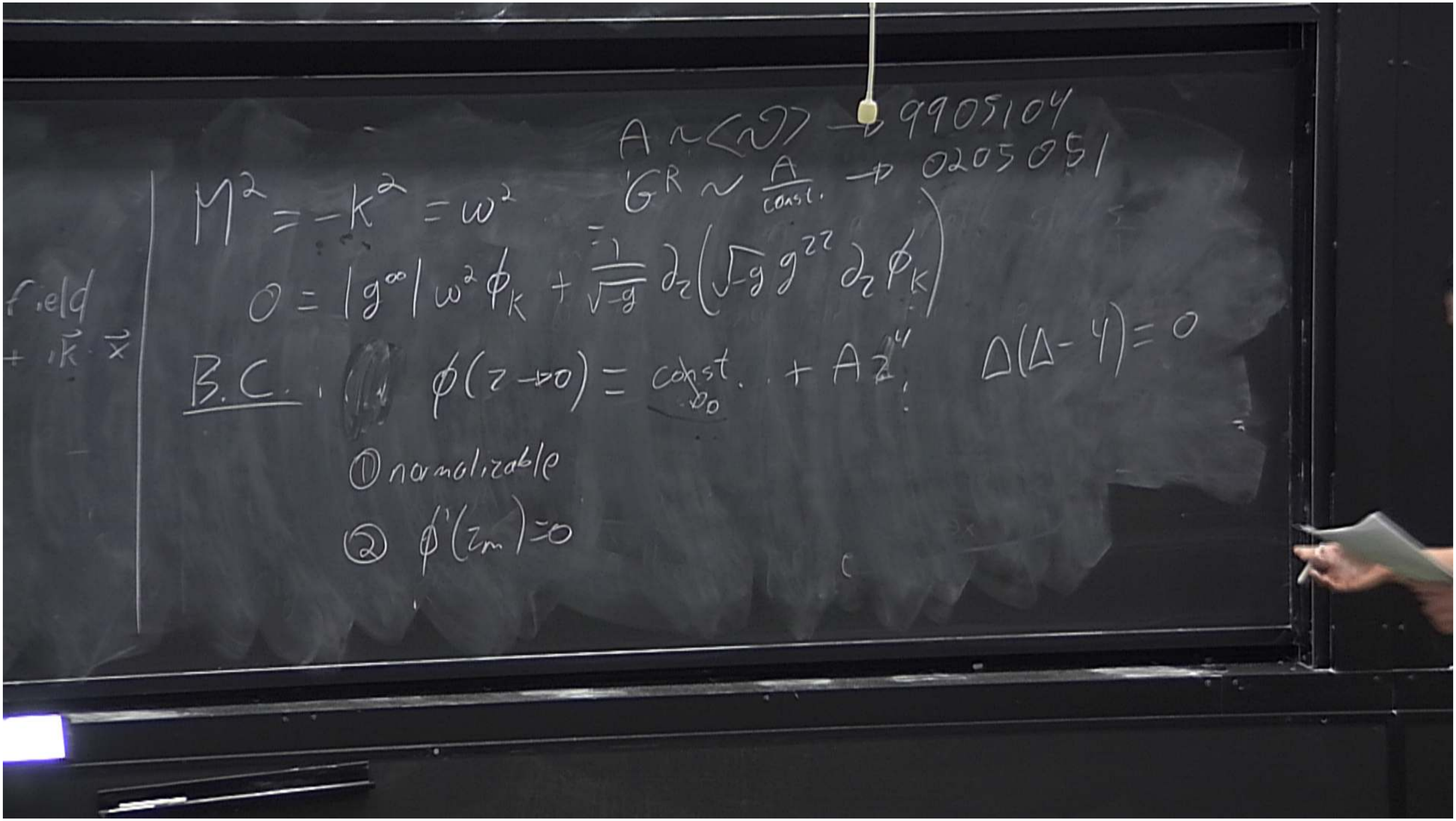
$+ i\vec{k}\cdot\vec{x}$

$$M^2 = -k^2 = \omega^2$$

$$0 = |g^{00}|\,\omega^2 \phi_k + \frac{1}{\sqrt{-g}}\,\partial_z\left(\sqrt{-g}\,g^{zz}\,\partial_z \phi_k\right)$$

B.C. $\phi(z \to 0) = \text{const.}\,\phi_0 + A z^4$ $\Delta(\Delta - 4) = 0$

① normalizable


A ~ ⟨O⟩ → 9905104
G^R ~ A/const. → 0205051
M^2 = −k^2 = ω^2
0 = |g^00| ω^2 φ_k + (−1/√−g) ∂_z(√−g g^zz ∂_z φ_k)
field
+ ik·x
B.C.
φ(z→0) = const. + A z^4
φ_0
Δ(Δ−4) = 0
① normalizable
② φ'(z_m) = 0

(a) $$\omega^2 \int \underbrace{\sqrt{-g}\,|g^{tt}|\,|\phi|^2}_{>0,\ <\infty} = \int \underbrace{\sqrt{g}\, g^{zz} (\partial_z \phi)^2}_{>0,\ <\infty}$$

$\neq 0$ unless $\phi = 0$

$\omega^2 > 0$

(b) Sturm-Liouville problem

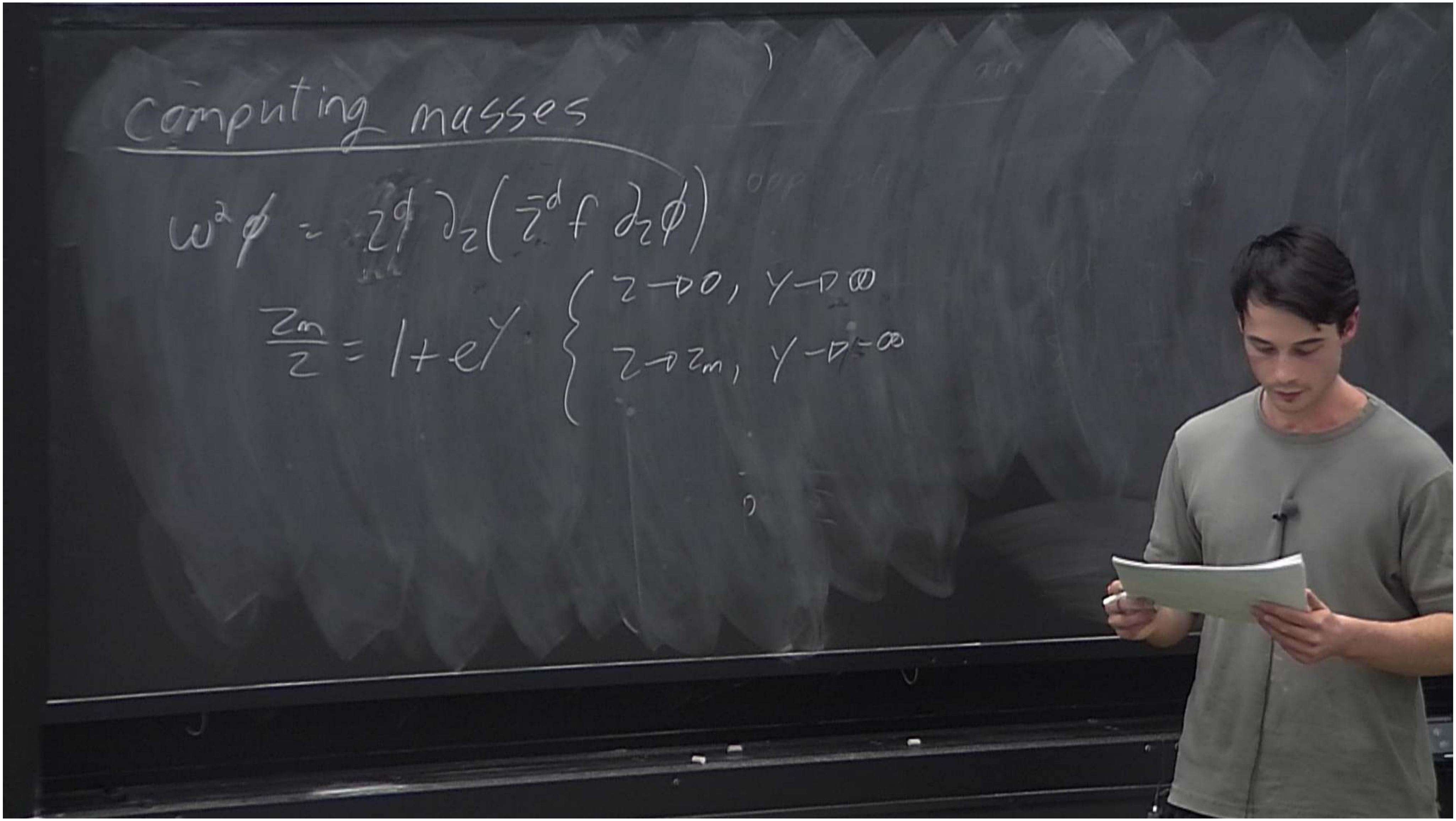
discrete spectrum of eigenvalues ω^2

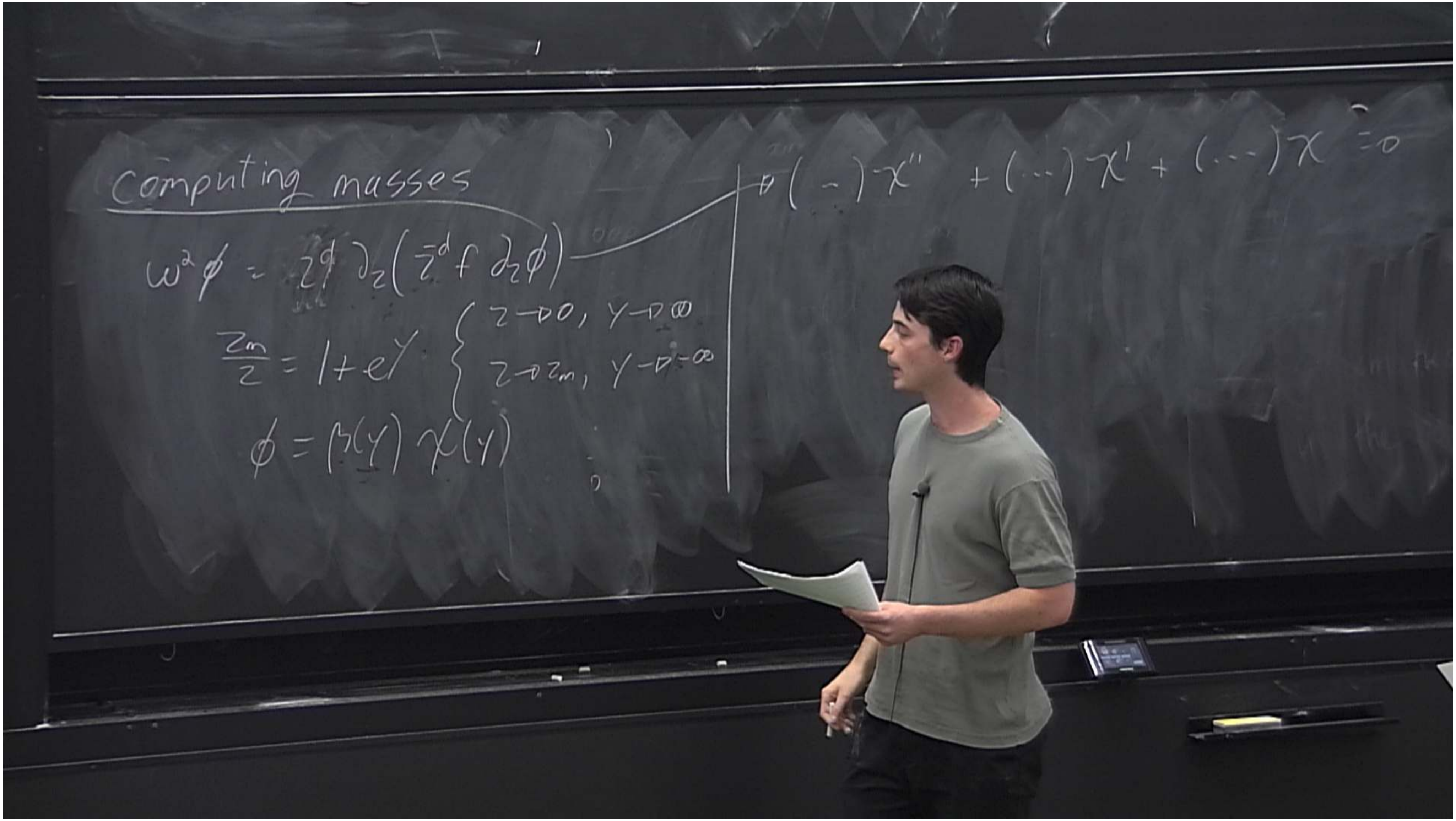
pure AdS?

$\phi \sim \cos(z) \quad z \to \infty$

...rs gain mass

$m^2 \sim \Sigma = \dots$


computing masses
$\omega^2 \phi = z^d \partial_z (z^{-d} f \partial_z \phi)$
$\frac{z_m}{z} = 1 + e^y$
$z \to 0, \; y \to \infty$
$z \to z_m, \; y \to -\infty$



Computing masses
ω²φ = z^d ∂_z(z^{-d} f ∂_z φ)
z_m/z = 1+e^y
z→0, y→∞
z→z_m, y→-∞
φ = β(y) χ(y)
(...)χ'' + (...)χ' + (...)χ = 0

Computing masses

$$\omega^2 \phi = z^d \partial_z \left(z^{-d} f \partial_z \phi \right)$$

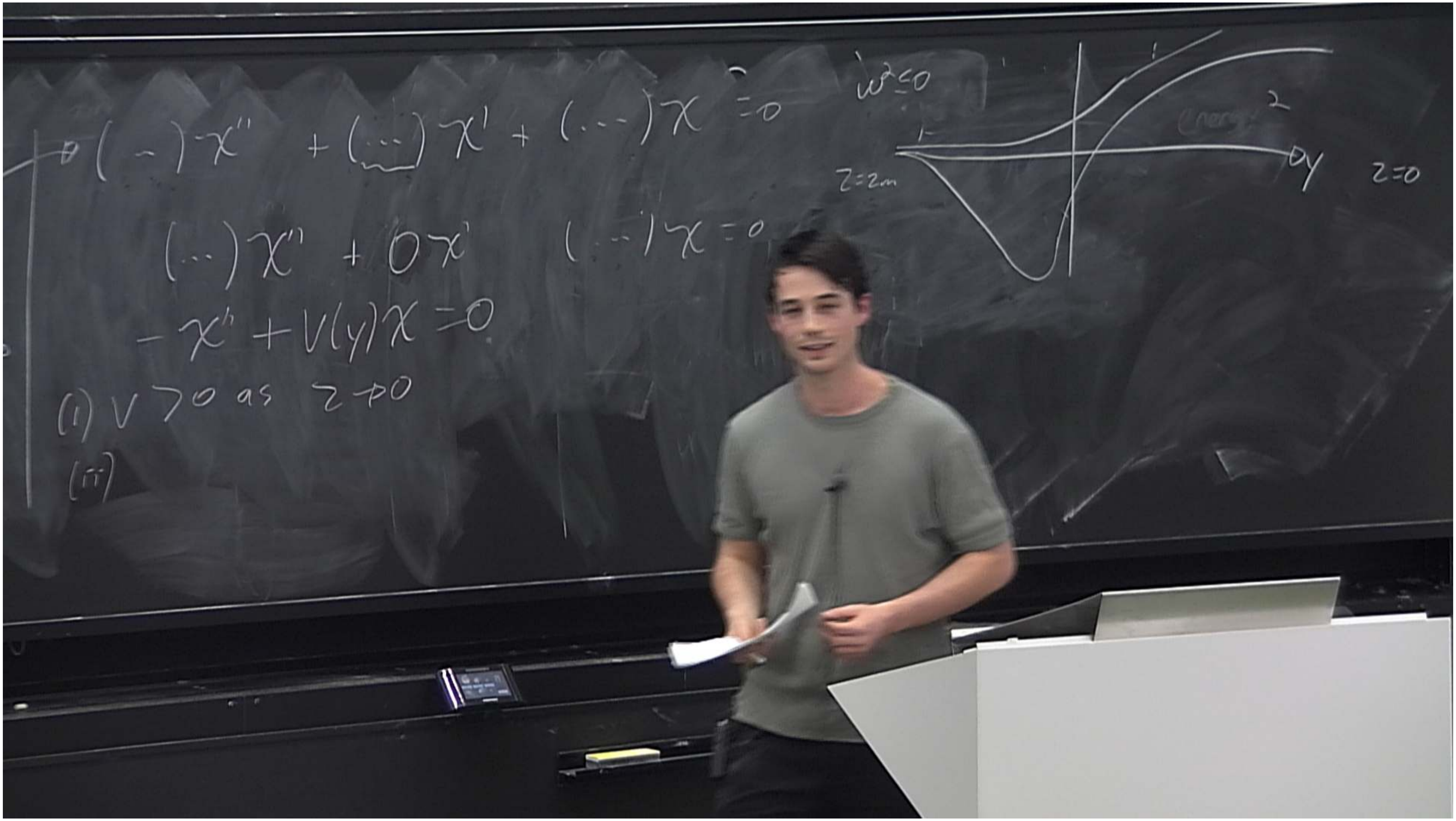
$$\frac{z_m}{z} = 1 + e^y \quad \begin{cases} z \to 0, & y \to \infty \\ z \to z_m, & y \to -\infty \end{cases}$$

$$\phi = \underline{B(y)}\, \chi(y)$$

$$(\ldots)\chi'' + (\ldots)\chi' + (\ldots)\chi = 0$$

$$(\ldots)\chi'' + 0\,\chi' + (\ldots)\chi = 0$$

$$-\chi'' + V(y)\chi = 0$$



ω² ≤ 0
(...) χ'' + (...) χ' + (...) χ = 0
(...) χ'' + 0 χ' (...) χ = 0
− χ'' + V(y) χ = 0
(i) V > 0 as z → 0
(ii)
z=zm
y
z=0

Computing masses

$$\omega^2 \phi = -z^d \partial_z \left(z^{-d} f \partial_z \phi \right)$$

$$\frac{z_m}{z} = 1 + e^y \quad \begin{cases} z \to 0, & y \to \infty \\ z \to z_m, & y \to -\infty \end{cases}$$

$$\phi = \underline{B(y)}\, \chi(y)$$

$$\to (\dots)\chi'' + (\dots)\chi' +$$

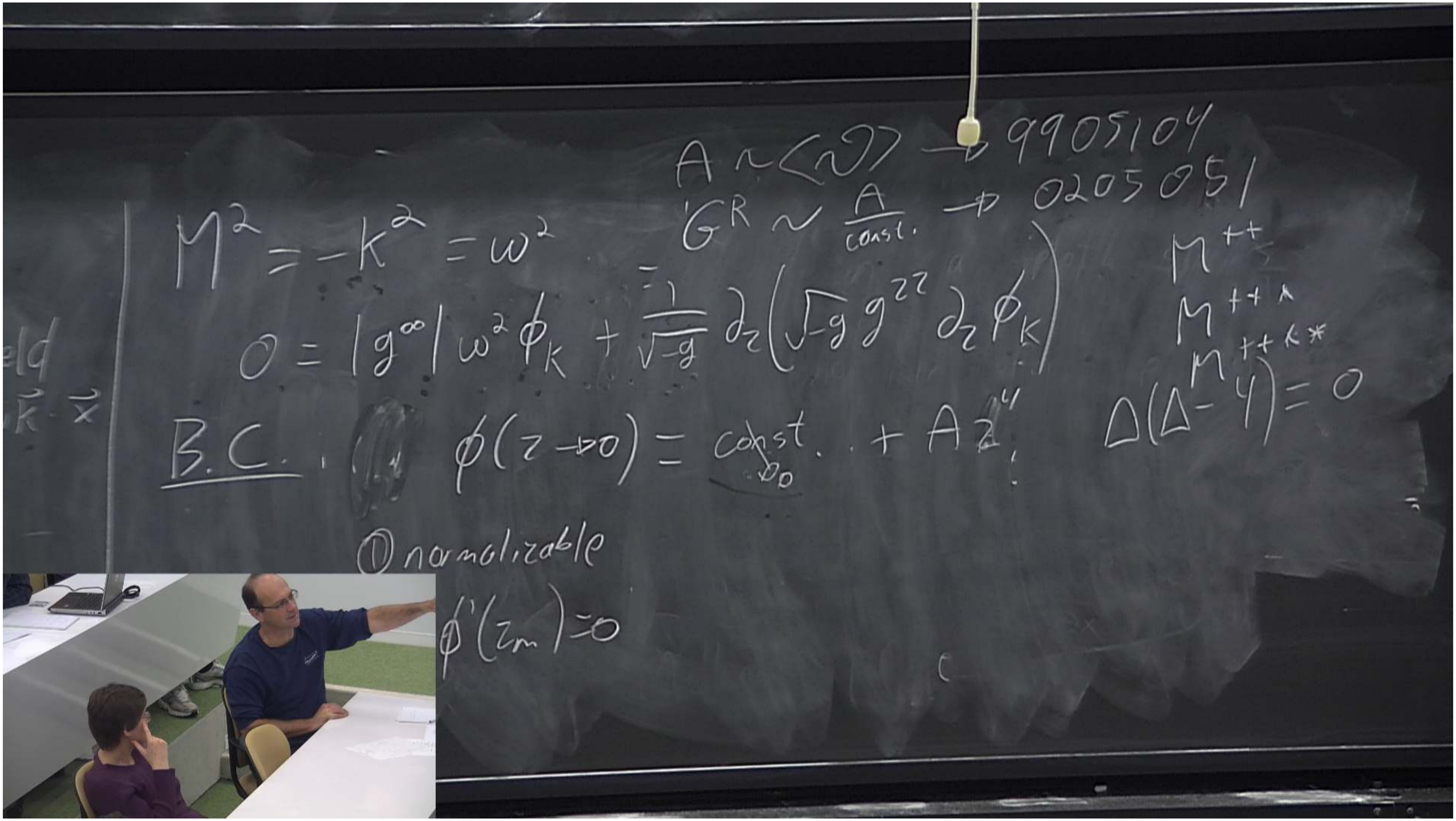
$$(\dots)\chi'' + 0\,\chi'$$

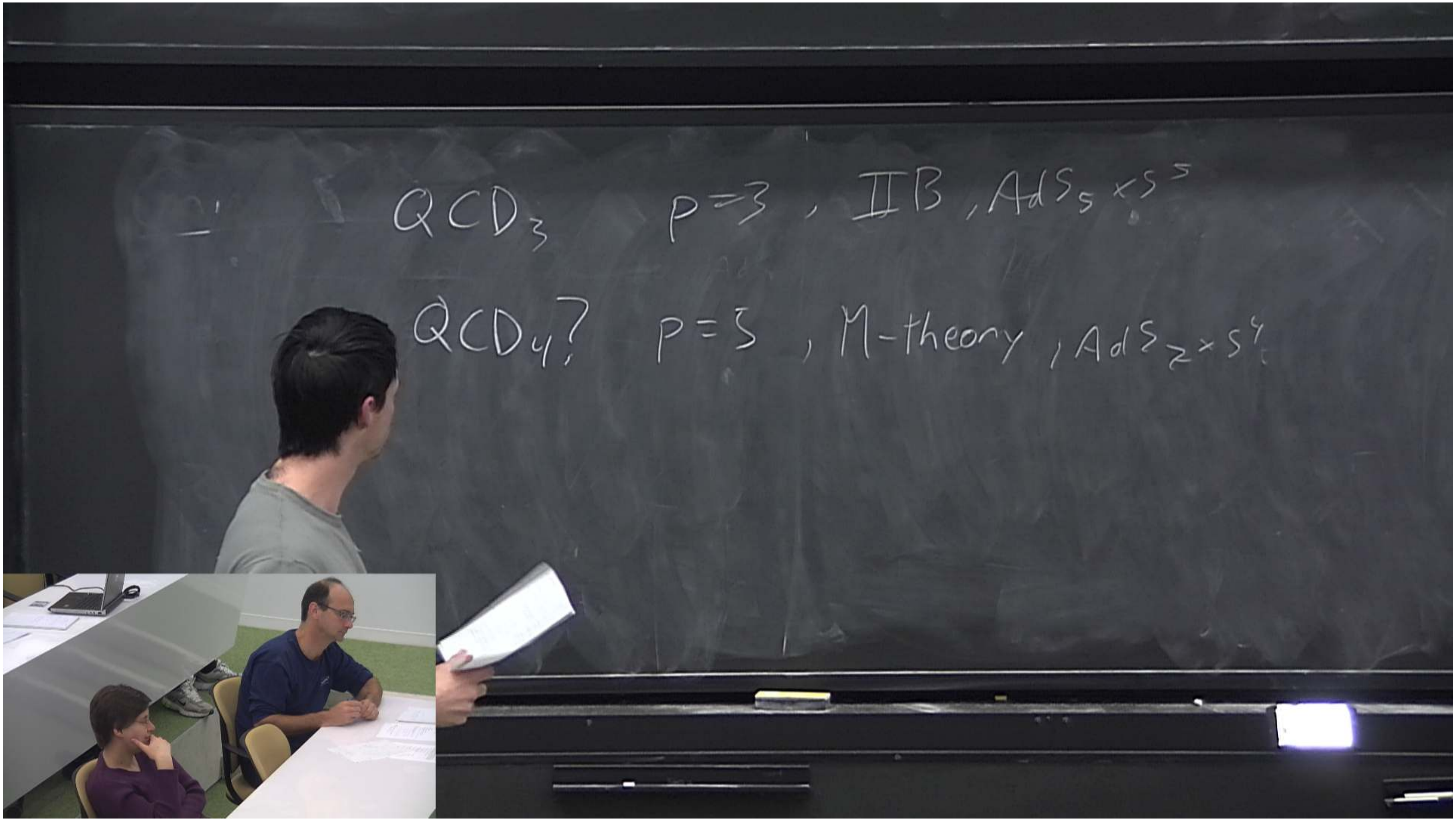
$$-\chi'' + V(y)\chi = 0$$

(i) $V > 0$ as $z \to 0$

(ii)

	lattice, $N=3$	lattice, $N \to \infty$	AdS/CFT
$\frac{M_2}{M_1}$	1.506	1.520	1.725
$\frac{M_3}{M_1}$	1.901	1.966	2.44


A ~ <𝒪> → 9905104
G^R ~ A/const. → 0205051
M² = −k² = ω²
0 = |g⁰⁰| ω² φ_k + (−1/√−g) ∂_z (√−g g^zz ∂_z φ_k)
B.C. φ(z → 0) = const. + A z⁴
φ₀
Δ(Δ − 4) = 0
M⁺⁺
M⁺⁺ᵏ
M⁺⁺ᵏ*
① normalizable
φ′(z_m) = 0


QCD3   p=3 , IIB , AdS5 × S5
QCD4?   p=5 , M-theory , AdS7 × S4