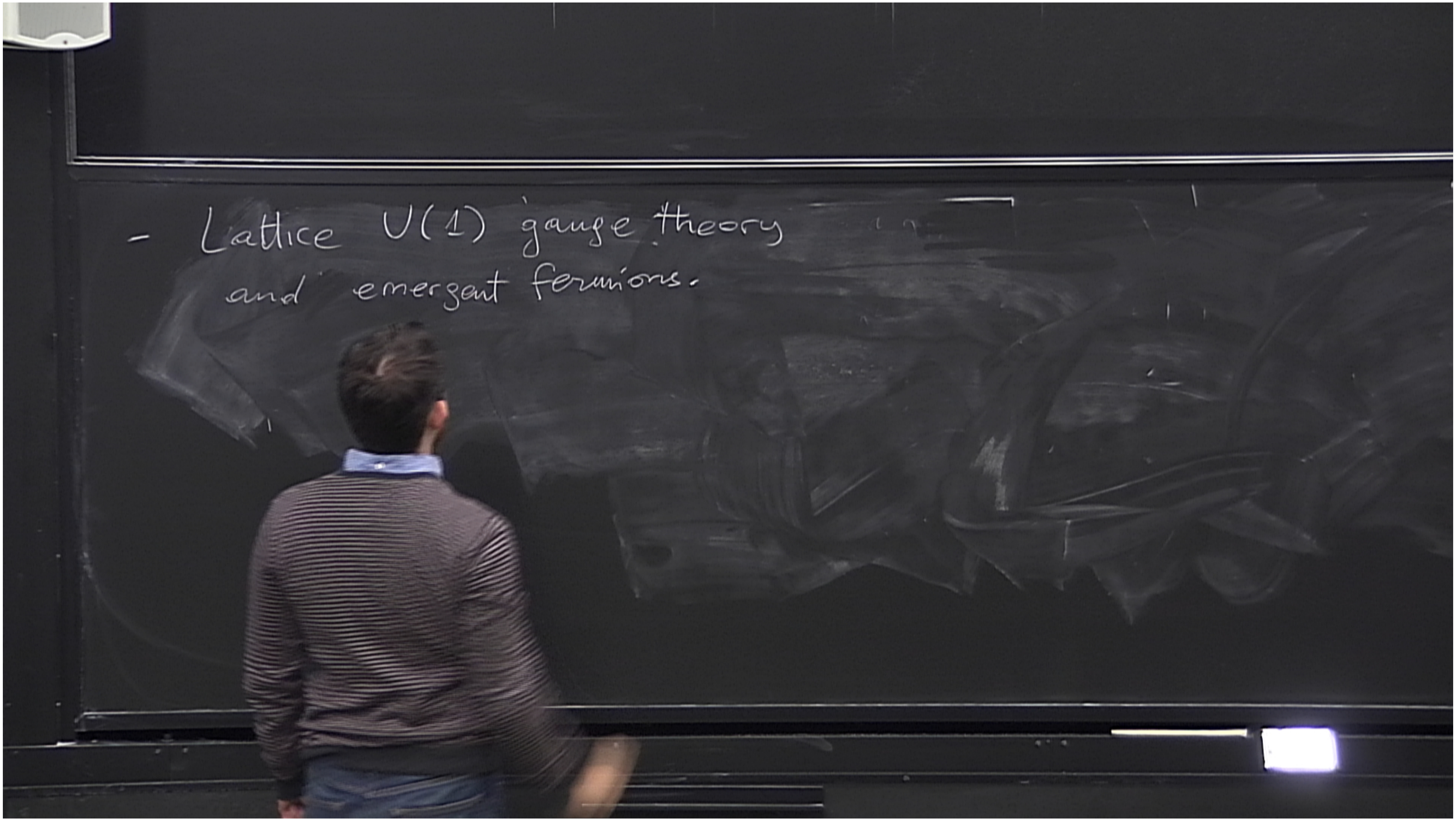
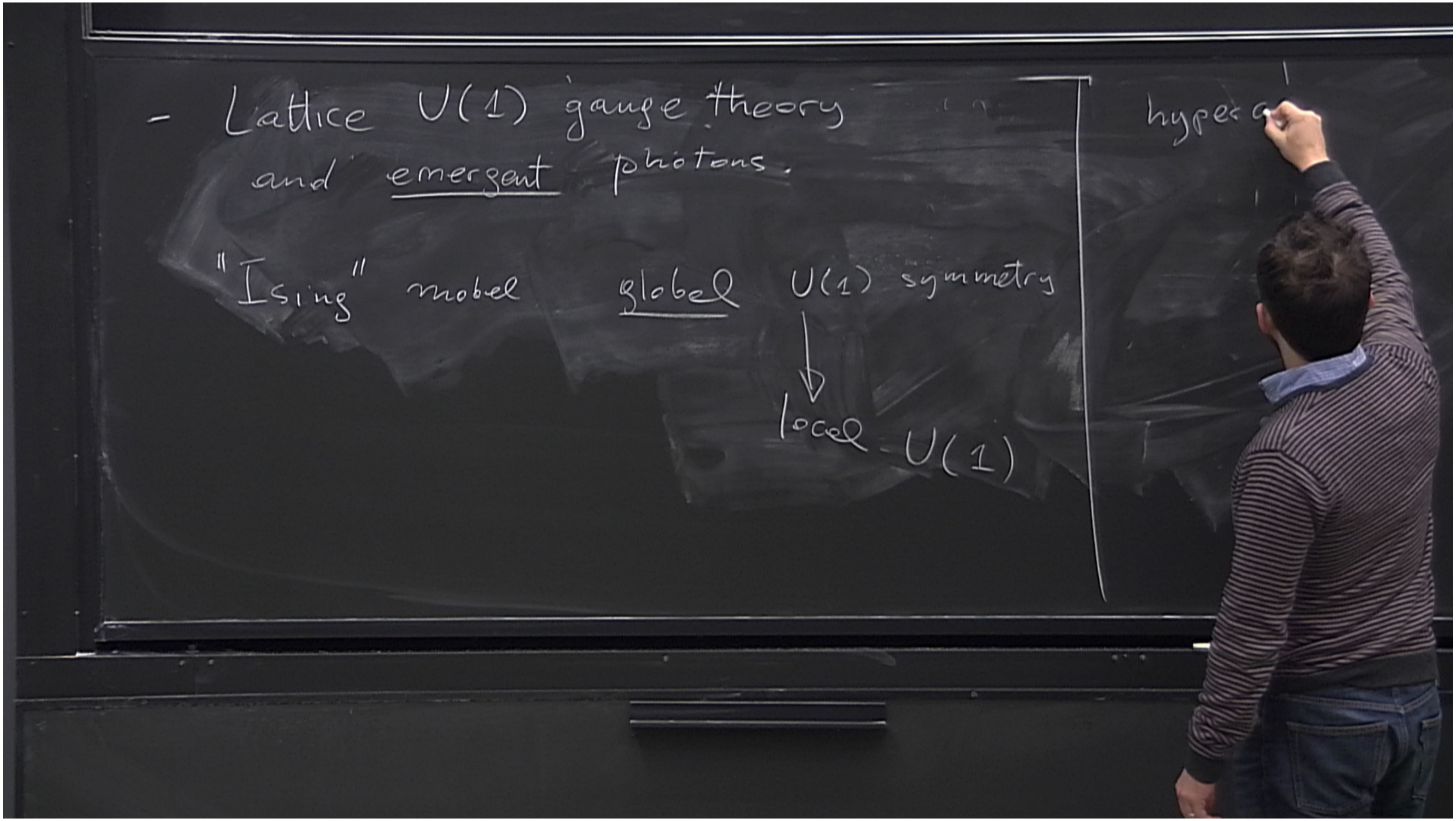
Title: Condensed Matter (Review) - Lecture 15

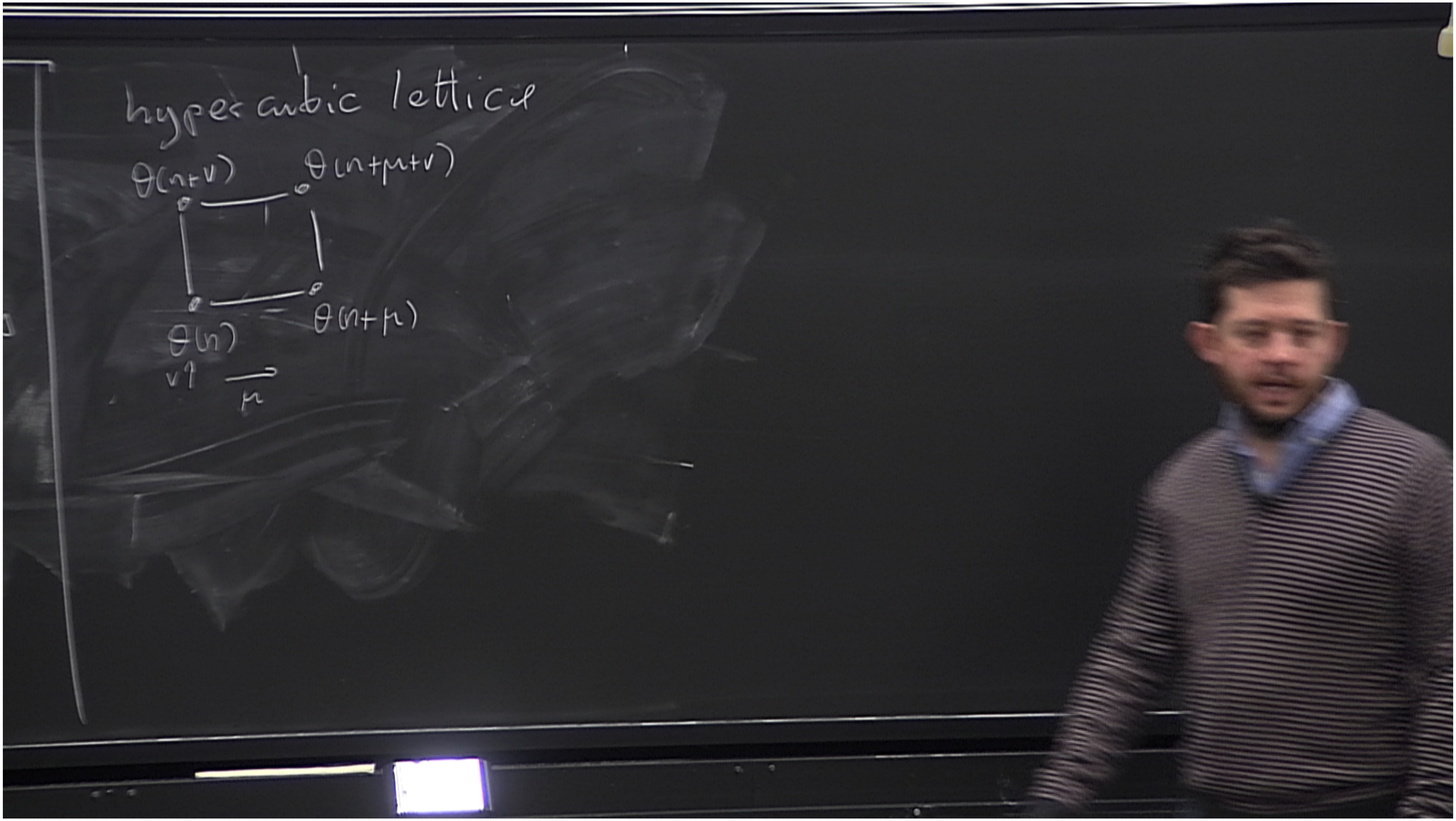
Date: Jan 20, 2012 10:15 AM

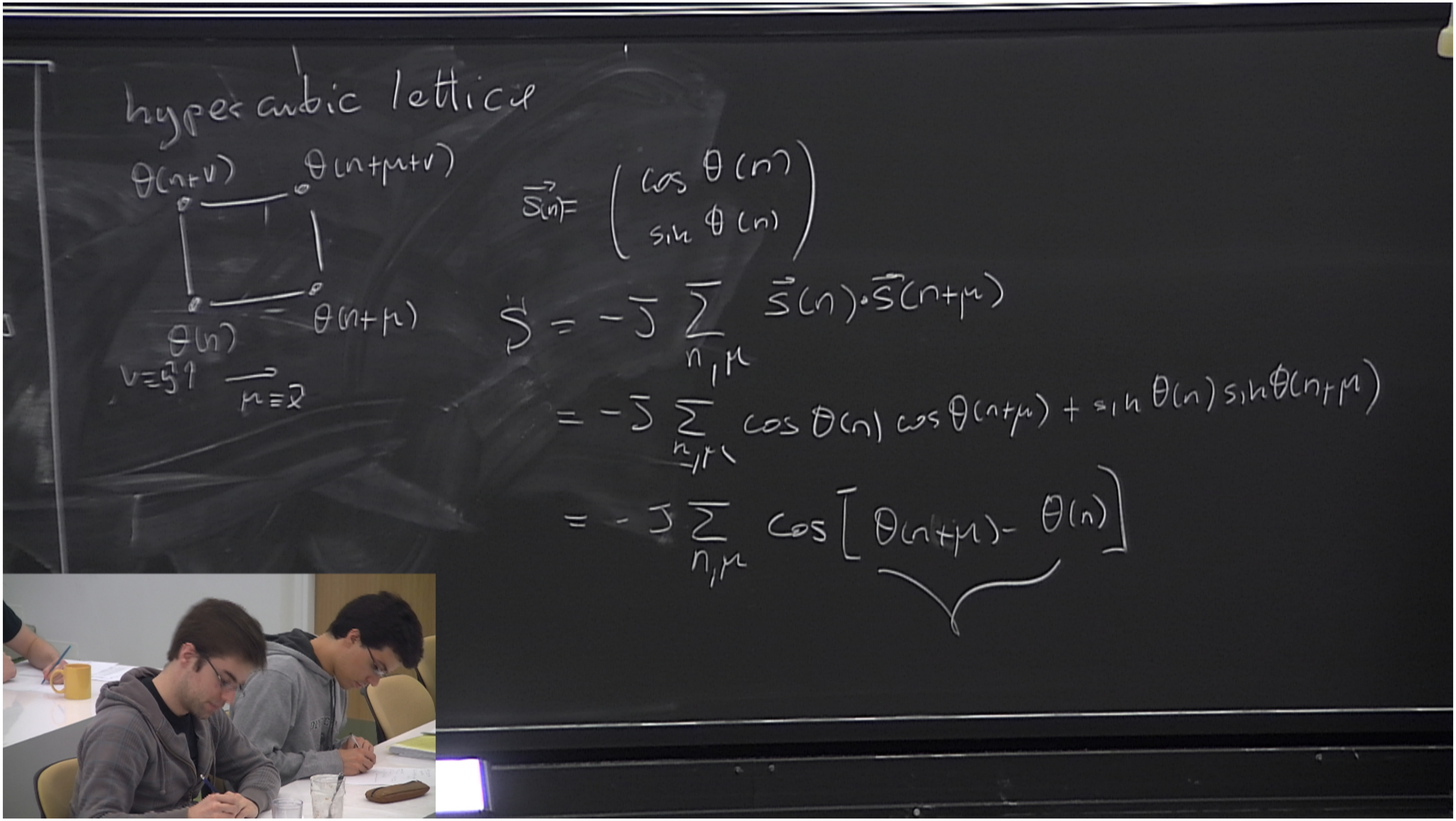
URL: http://pirsa.org/12010101

Abstract:

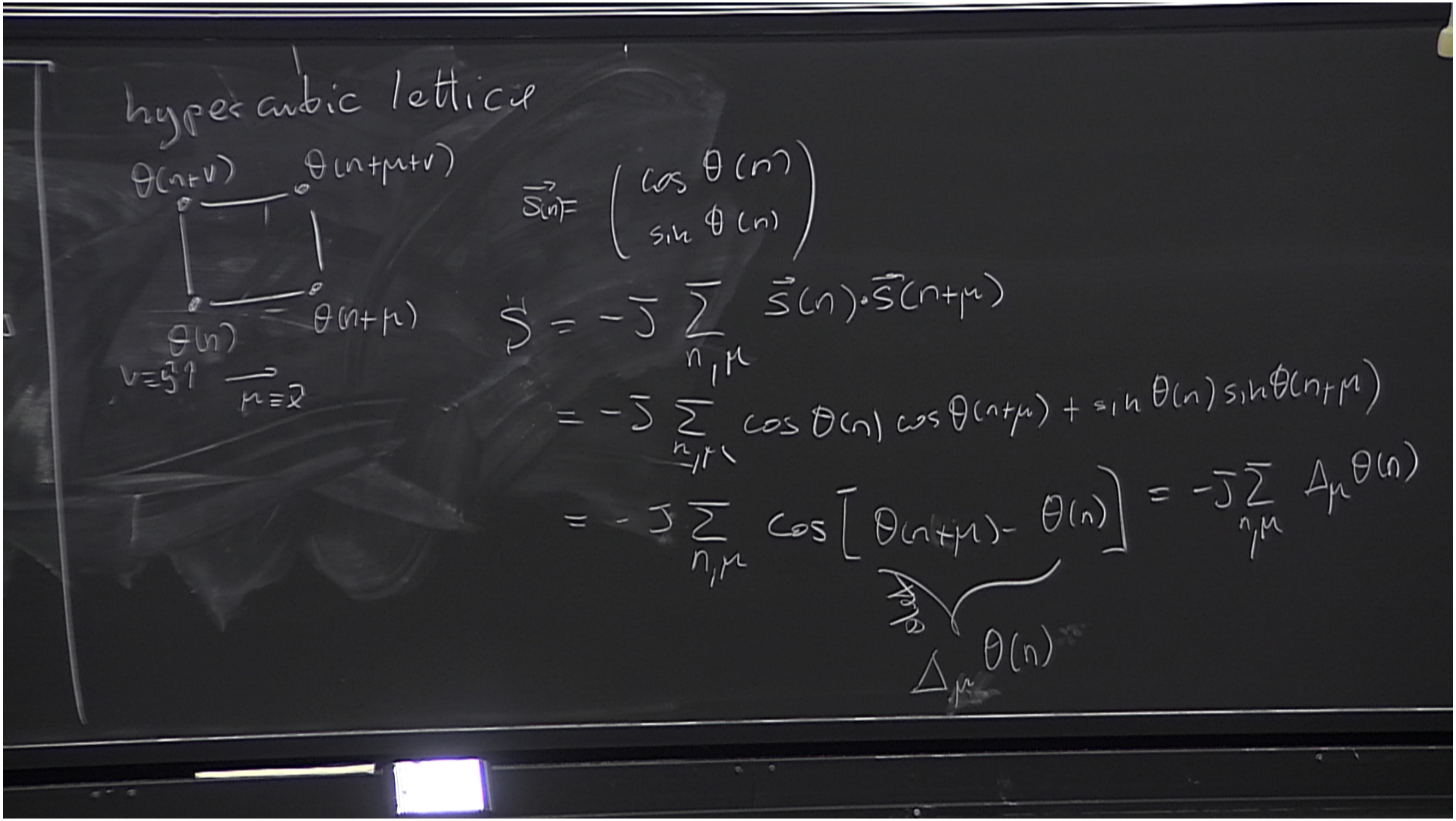

- Lattice U(1) gauge theory
and emergent fermions.

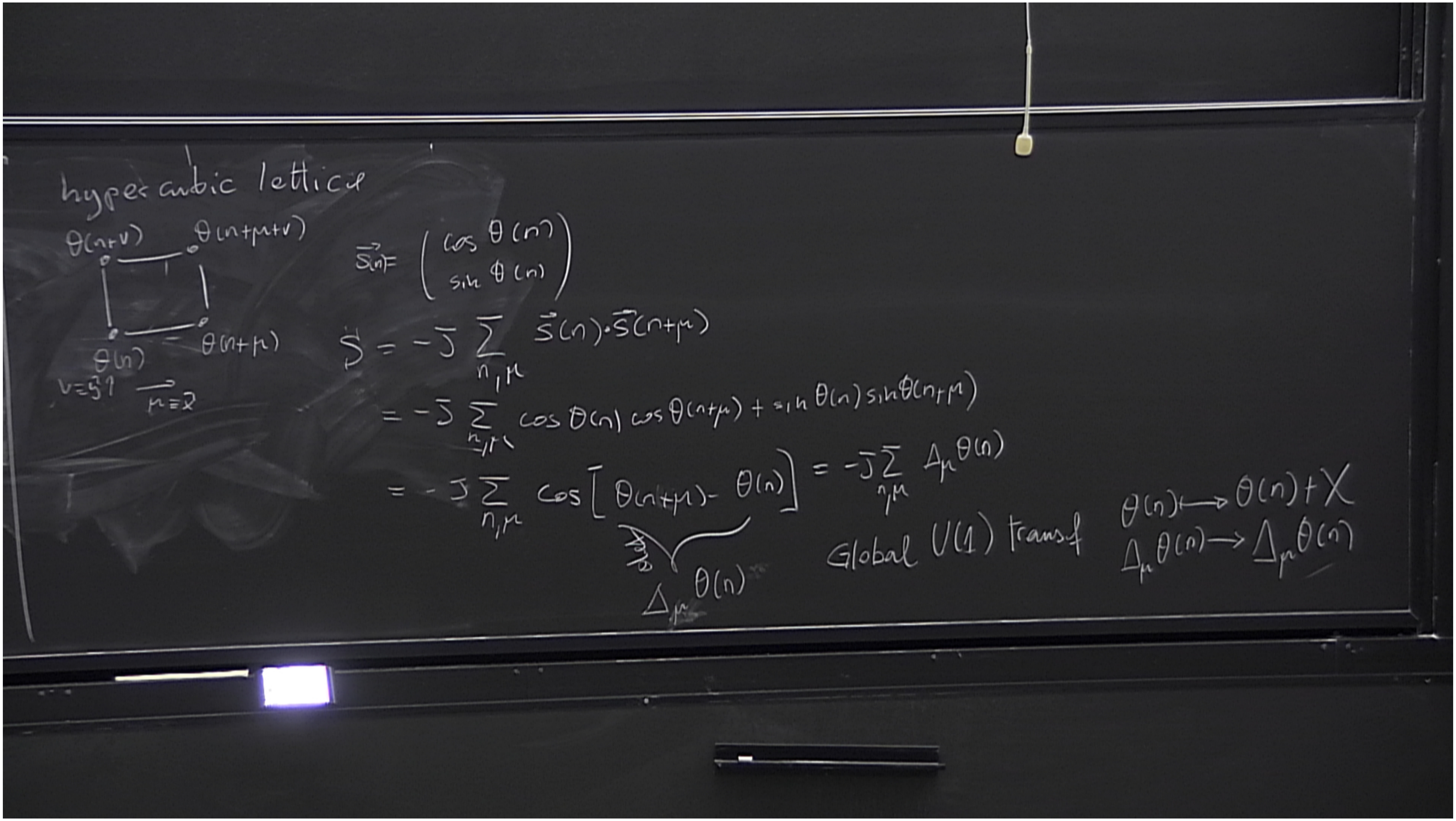

- Lattice U(1) gauge theory
and emergent photons.
"Ising" model
global U(1) symmetry
local U(1)

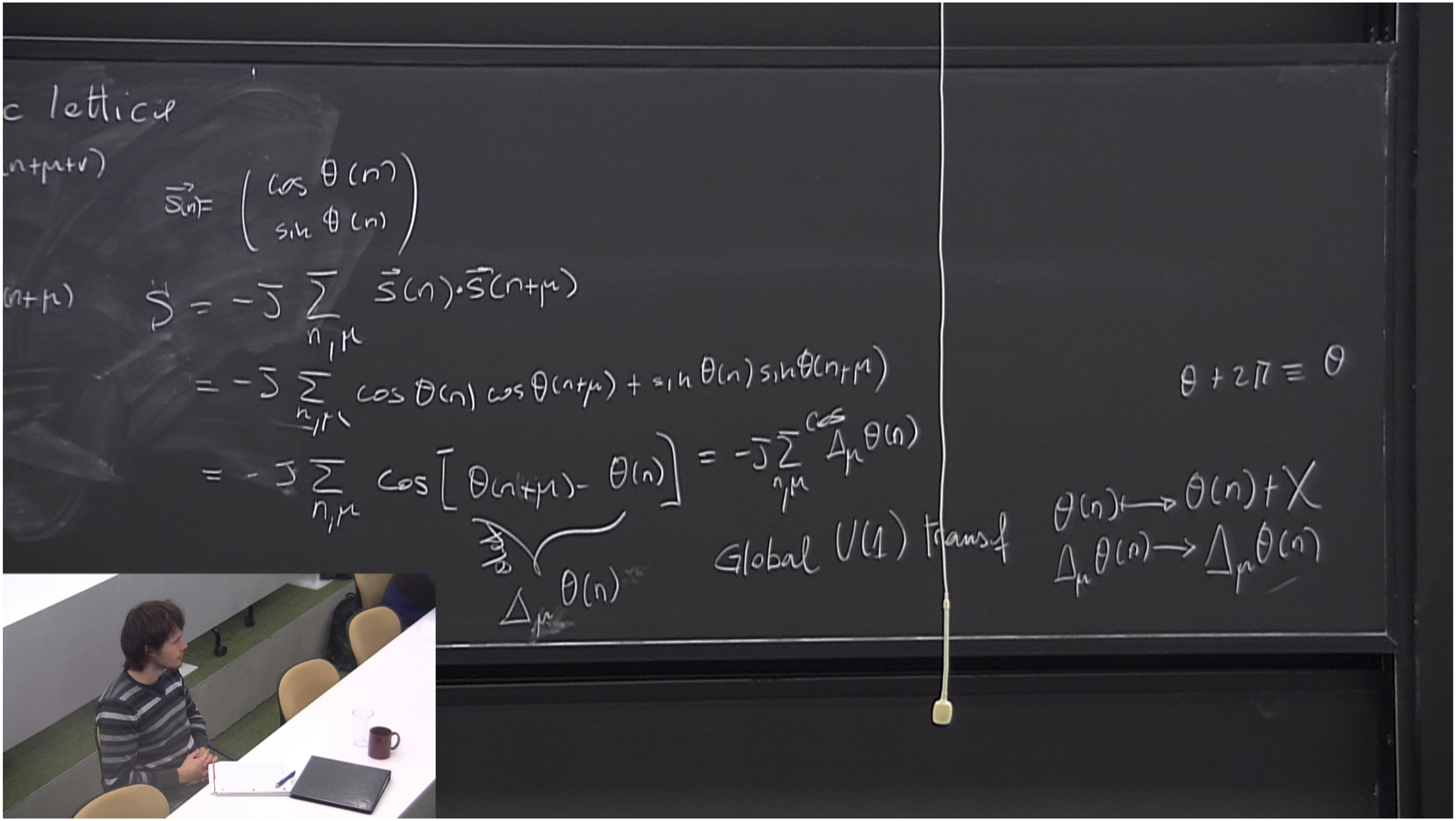

hypercubic lattice
θ(n+ν)
θ(n+μ+ν)
θ(n+μ)
θ(n)
ν
μ

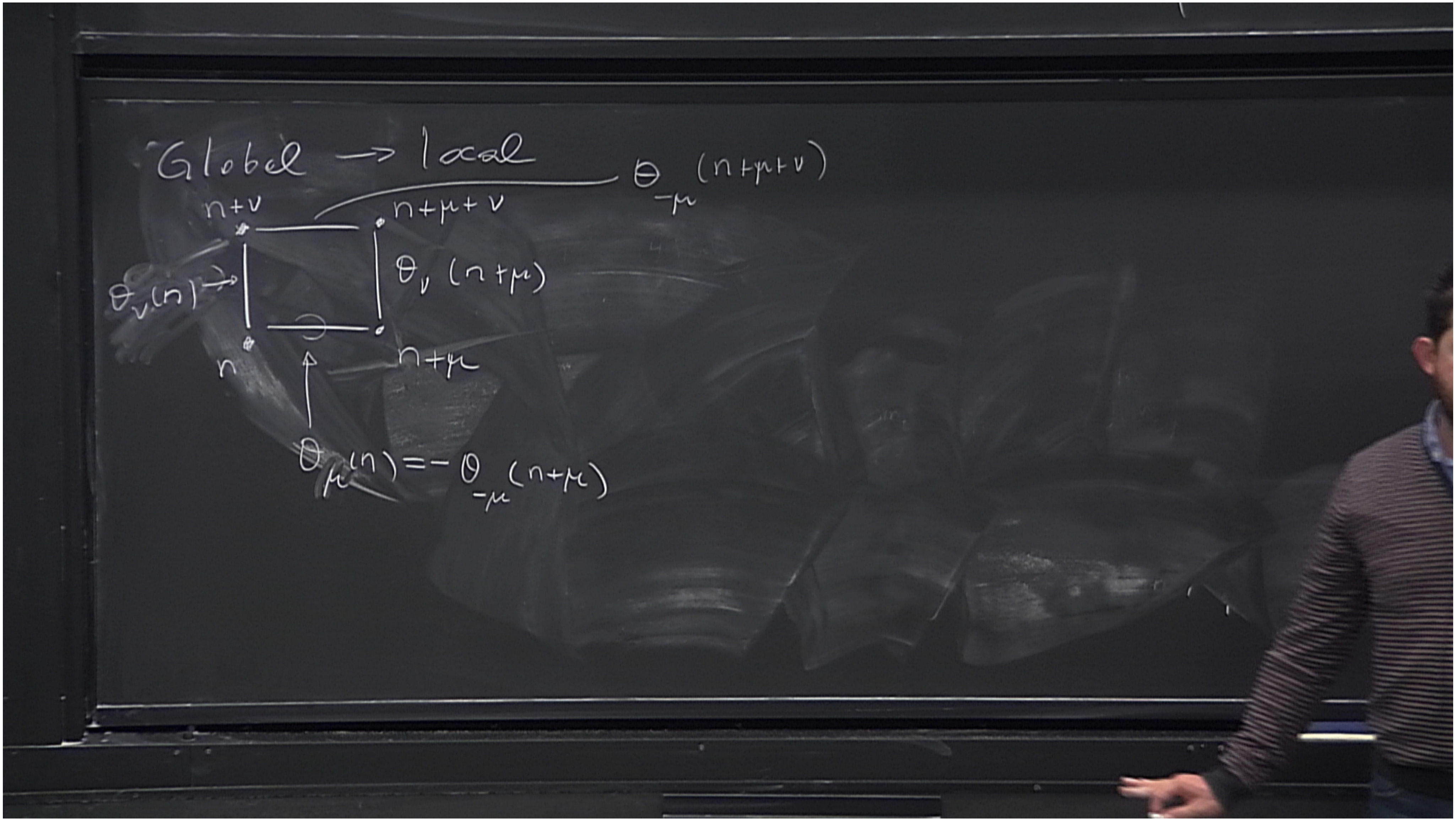


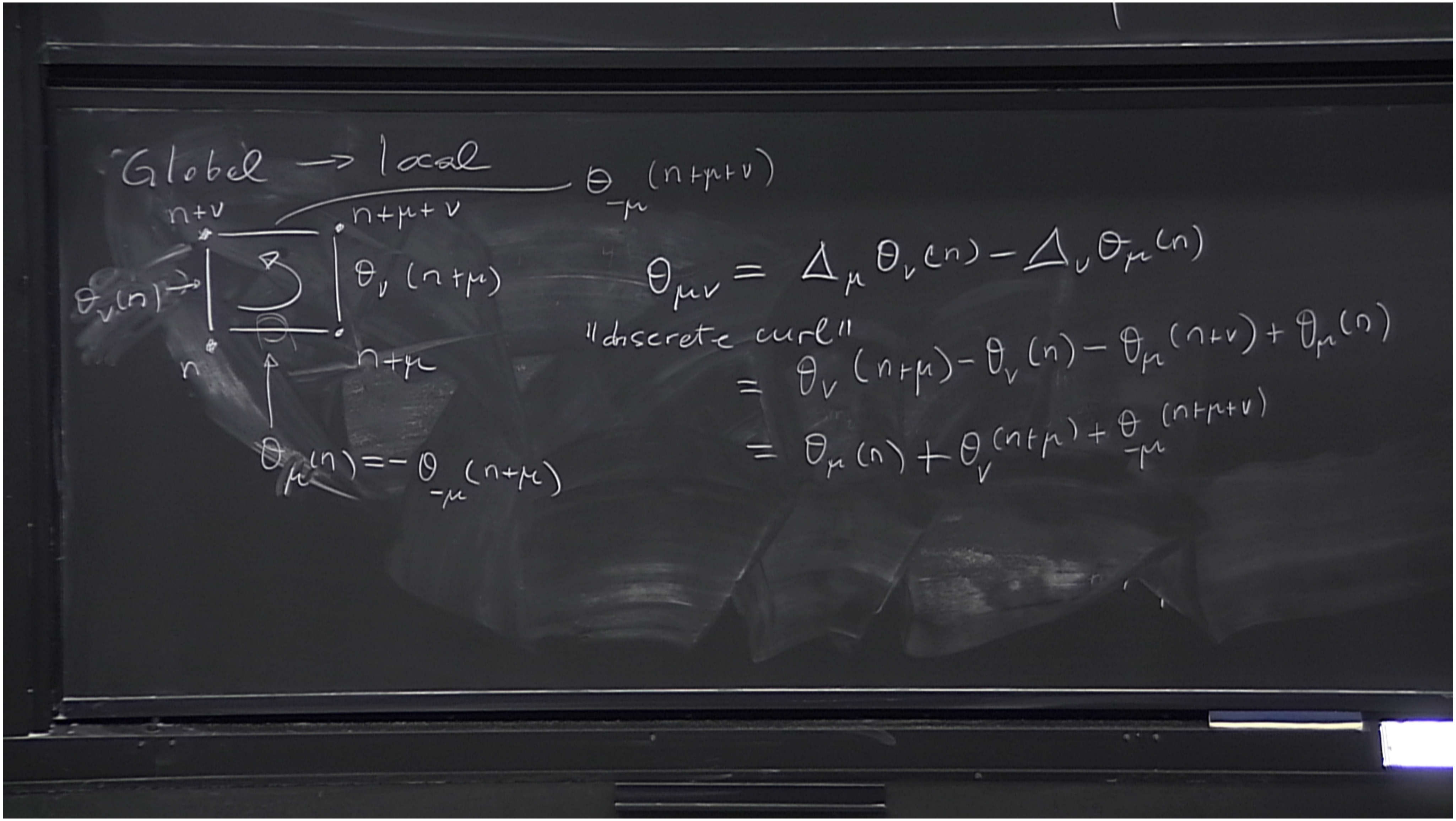
hypercubic lattice
$\theta(n+\nu)$
$\theta(n+\mu+\nu)$
$\theta(n)$
$\theta(n+\mu)$
$\nu \equiv \hat{y}$
$\mu \equiv \hat{x}$
$\vec{S}(n) = \begin{pmatrix} \cos\theta(n) \\ \sin\theta(n) \end{pmatrix}$
$S = -J \sum_{n,\mu} \vec{S}(n)\cdot\vec{S}(n+\mu)$
$= -J \sum_{n,\mu} \cos\theta(n)\cos\theta(n+\mu) + \sin\theta(n)\sin\theta(n+\mu)$
$= -J \sum_{n,\mu} \cos\left[\theta(n+\mu) - \theta(n)\right]$


hypercubic lattice
θ(n+ν)
θ(n+μ+ν)
θ(n)
θ(n+μ)
ν ≡ ŷ
μ ≡ x̂
$\vec{S}(n) = \begin{pmatrix} \cos\theta(n) \\ \sin\theta(n) \end{pmatrix}$
$S = -J \sum_{n,\mu} \vec{S}(n)\cdot\vec{S}(n+\mu)$
$= -J \sum_{n,\mu} \cos\theta(n)\cos\theta(n+\mu) + \sin\theta(n)\sin\theta(n+\mu)$
$= -J \sum_{n,\mu} \cos[\underbrace{\theta(n+\mu) - \theta(n)}_{\Delta_\mu \theta(n)}] = -J \sum_{n,\mu} \Delta_\mu \theta(n)$

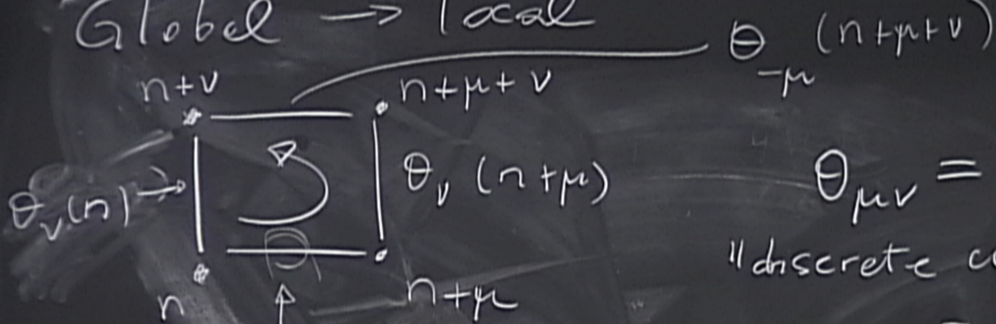

hypercubic lattice
$\theta(n+\nu)$ $\theta(n+\mu+\nu)$
$\theta(n)$ $\theta(n+\mu)$
$\nu = \hat{1}$ $\mu = \hat{2}$
$\vec{S}(n) = \begin{pmatrix} \cos\theta(n) \\ \sin\theta(n) \end{pmatrix}$
$S = -J \sum_{n,\mu} \vec{S}(n)\cdot\vec{S}(n+\mu)$
$= -J \sum_{n,\mu} \cos\theta(n)\cos\theta(n+\mu) + \sin\theta(n)\sin\theta(n+\mu)$
$= -J \sum_{n,\mu} \cos\left[\theta(n+\mu) - \theta(n)\right] = -J \sum_{n,\mu} \Delta_\mu \theta(n)$
$\Delta_\mu \theta(n)$
Global $U(1)$ transf $\theta(n) \longmapsto \theta(n) + X$
$\Delta_\mu\theta(n) \longrightarrow \Delta_\mu\theta(n)$


c lattice
(n+μ+ν)
$\vec{S}(n) = \begin{pmatrix} \cos\theta(n) \\ \sin\theta(n) \end{pmatrix}$
(n+μ)
$S = -J \sum_{n,\mu} \vec{S}(n)\cdot\vec{S}(n+\mu)$
$= -J \sum_{n,\mu} \cos\theta(n)\cos\theta(n+\mu) + \sin\theta(n)\sin\theta(n+\mu)$
$= -J \sum_{n,\mu} \cos\left[\theta(n+\mu) - \theta(n)\right] = -J \sum_{n,\mu} \cos \Delta_\mu \theta(n)$
$\Delta_\mu \theta(n)$
$\theta + 2\pi \equiv \theta$
Global U(1) transf $\theta(n) \longmapsto \theta(n) + X$
$\Delta_\mu\theta(n) \longrightarrow \Delta_\mu\theta(n)$


Global → local
$\Theta_{-\mu}(n+\mu+\nu)$
$n+\nu$
$n+\mu+\nu$
$\Theta_\nu(n)$
$\Theta_\nu(n+\mu)$
n
$n+\mu$
$\Theta_\mu(n) = -\Theta_{-\mu}(n+\mu)$


Global → local
$\theta_{-\mu}(n+\mu+\nu)$
$n+\nu$
$n+\mu+\nu$
$\theta_\nu(n)$
$\theta_\nu(n+\mu)$
n
$n+\mu$
$\theta_\mu(n) = -\theta_{-\mu}(n+\mu)$
$\theta_{\mu\nu} = \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)$
"discrete curl"
$= \theta_\nu(n+\mu) - \theta_\nu(n) - \theta_\mu(n+\nu) + \theta_\mu(n)$
$= \theta_\mu(n) + \theta_\nu(n+\mu) + \theta_{-\mu}(n+\mu+\nu)$

Global → local



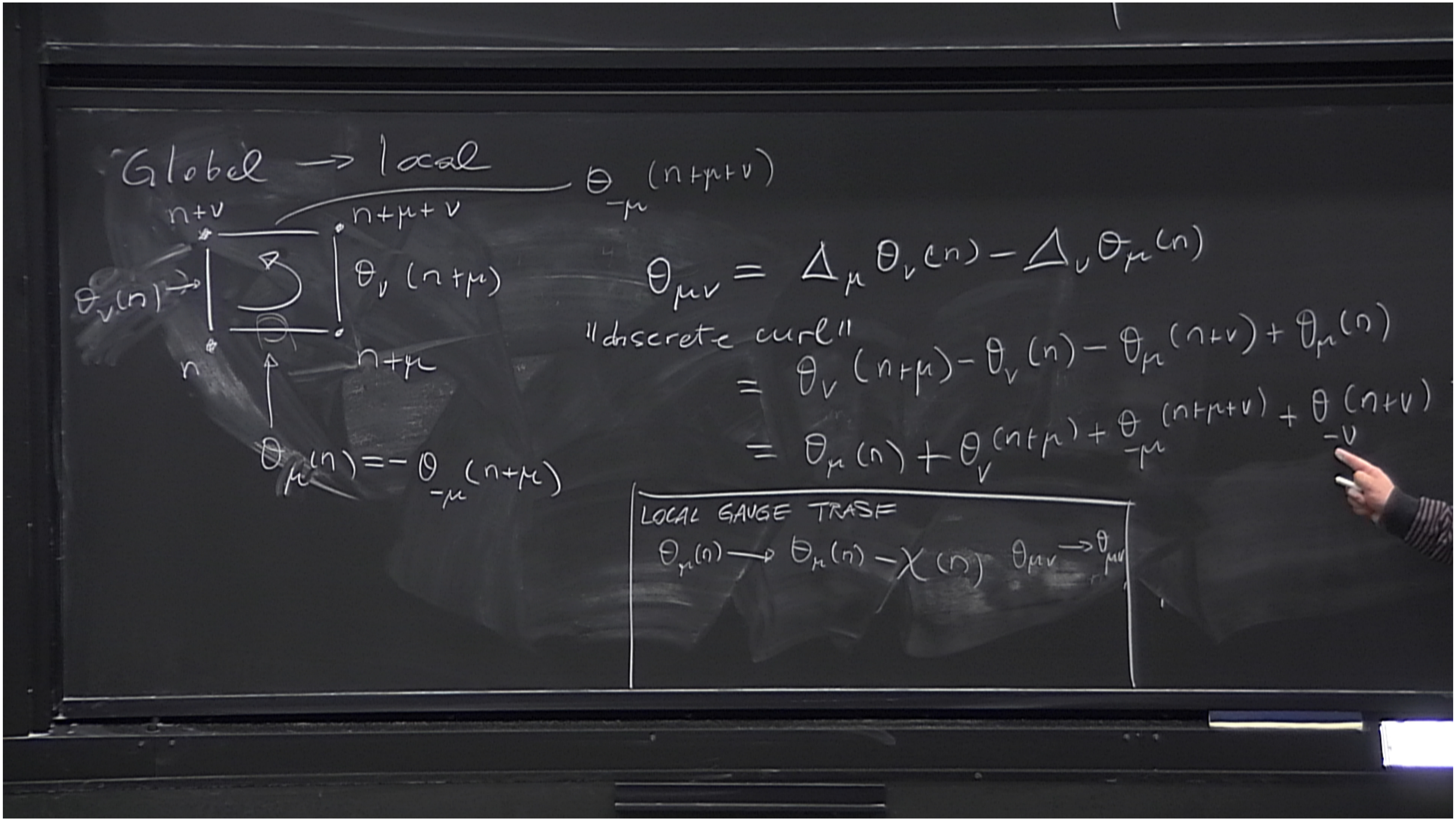
$$\theta_\mu(n) = -\theta_{-\mu}(n+\mu)$$

$$\theta_{\mu\nu} = \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)$$

"discrete curl"

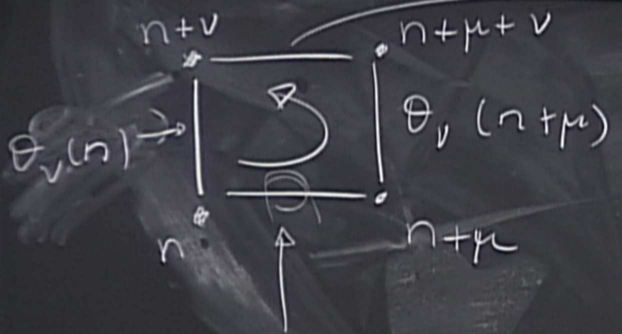
$$= \theta_\nu(n+\mu) - \theta_\nu(n) - \theta_\mu(n+\nu) + \theta_\mu(n)$$

$$= \theta_\mu(n) + \theta_\nu(n+\mu) + \theta_{-\mu}(n+\mu+\nu) + \theta_{-\nu}($$



Global → local
$\theta_{-\mu}(n+\mu+\nu)$
$n+\nu$
$n+\mu+\nu$
$\theta_\nu(n+\mu)$
$\theta_\nu(n)$
n
$n+\mu$
$\theta_\mu(n) = -\theta_{-\mu}(n+\mu)$
$\theta_{\mu\nu} = \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)$
"discrete curl"
$= \theta_\nu(n+\mu) - \theta_\nu(n) - \theta_\mu(n+\nu) + \theta_\mu(n)$
$= \theta_\mu(n) + \theta_\nu(n+\mu) + \theta_{-\mu}(n+\mu+\nu) + \theta_{-\nu}(n+\nu)$
LOCAL GAUGE TRASF
$\theta_\mu(n) \to \theta_\mu(n) - \chi(n)$
$\theta_{\mu\nu} \to \theta_{\mu\nu}$

Global → local



$\theta_{-\mu}(n+\mu+\nu)$

$$\theta_{\mu\nu} = \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)$$

"discrete curl"

$$\theta_\nu(n+\mu) - \theta_\nu(n) - \theta_\mu(n+\nu) + \theta_\mu(n)$$

$$= \theta_\nu(n+\mu) - \theta_\nu(n) + \theta_{-\mu}(n+\mu+\nu) + \theta_{-\nu}(n+\nu)$$

$$= \theta_\mu(n) + \theta_\nu(n+\mu) + \theta_{-\mu}(n+\mu+\nu) + \theta_{-\nu}(n+\nu)$$

$$\theta_\mu(n) = -\theta_{-\mu}(n+\mu)$$

LOCAL GAUGE TRASF

$$\theta_\mu(n) \to \theta_\mu(n) - \chi(n) + \chi(n+\mu)$$

$$\theta_{\mu\nu} \to \theta_{\mu\nu}$$

$$S = \beta \sum_{n,\mu\nu} \left[1 - \cos\theta_{\mu\nu}(n) \right]$$

action for the U(1) gauge theory

$\theta_{-\mu}(n+\mu+\nu)$

$+\mu)$

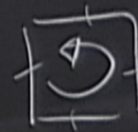
$(n+\mu)$

$$\theta_{\mu\nu} = \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)$$

"discrete curl"

$$= \theta_\nu(n+\mu) - \theta_\nu(n) - \theta_\mu(n+\nu) + \theta_\mu(n)$$

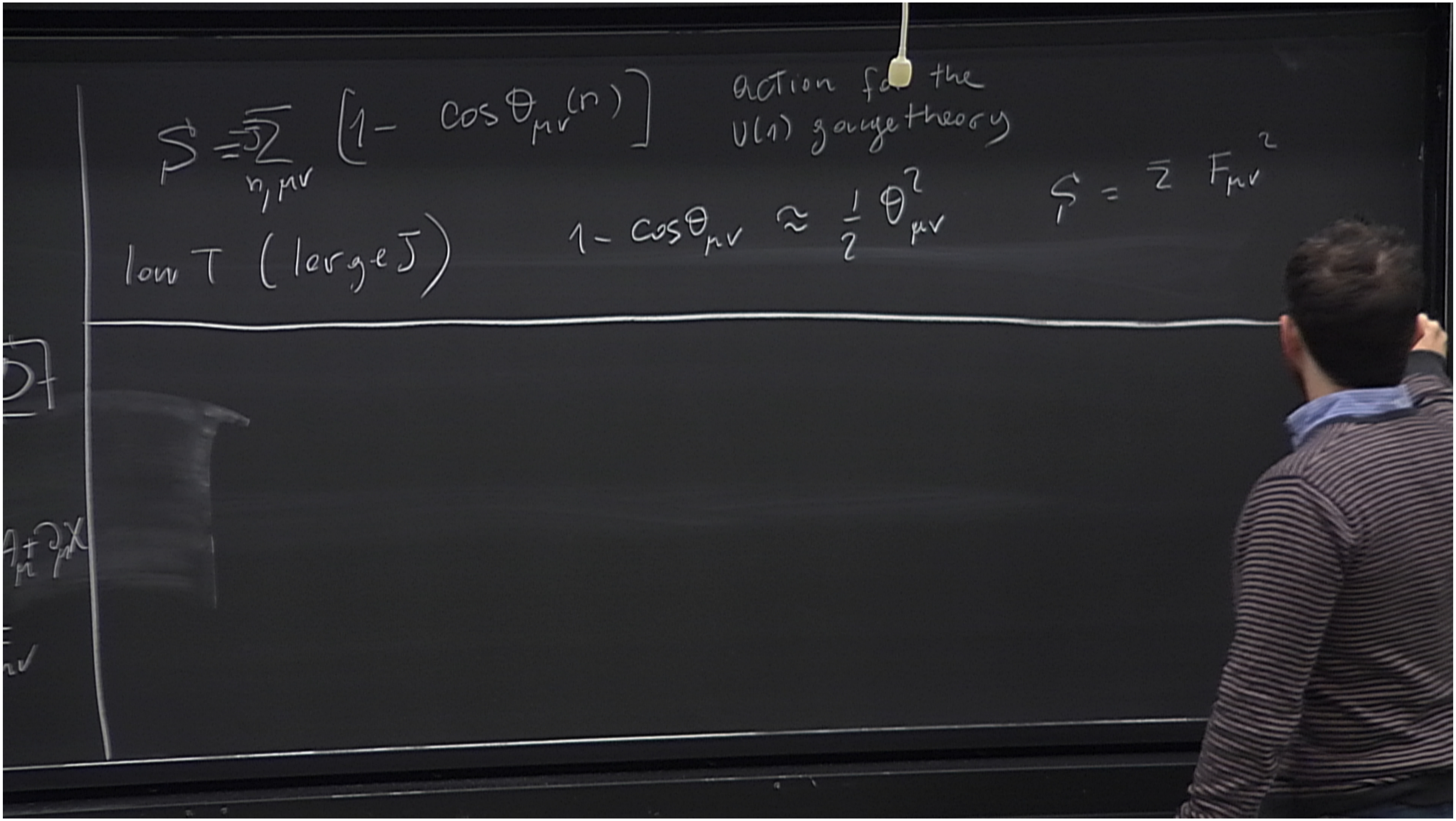
$$= \theta_\mu(n) + \theta_\nu(n+\mu) + \theta_{-\mu}(n+\mu+\nu) + \theta_{-\nu}(n+\nu) \equiv$$

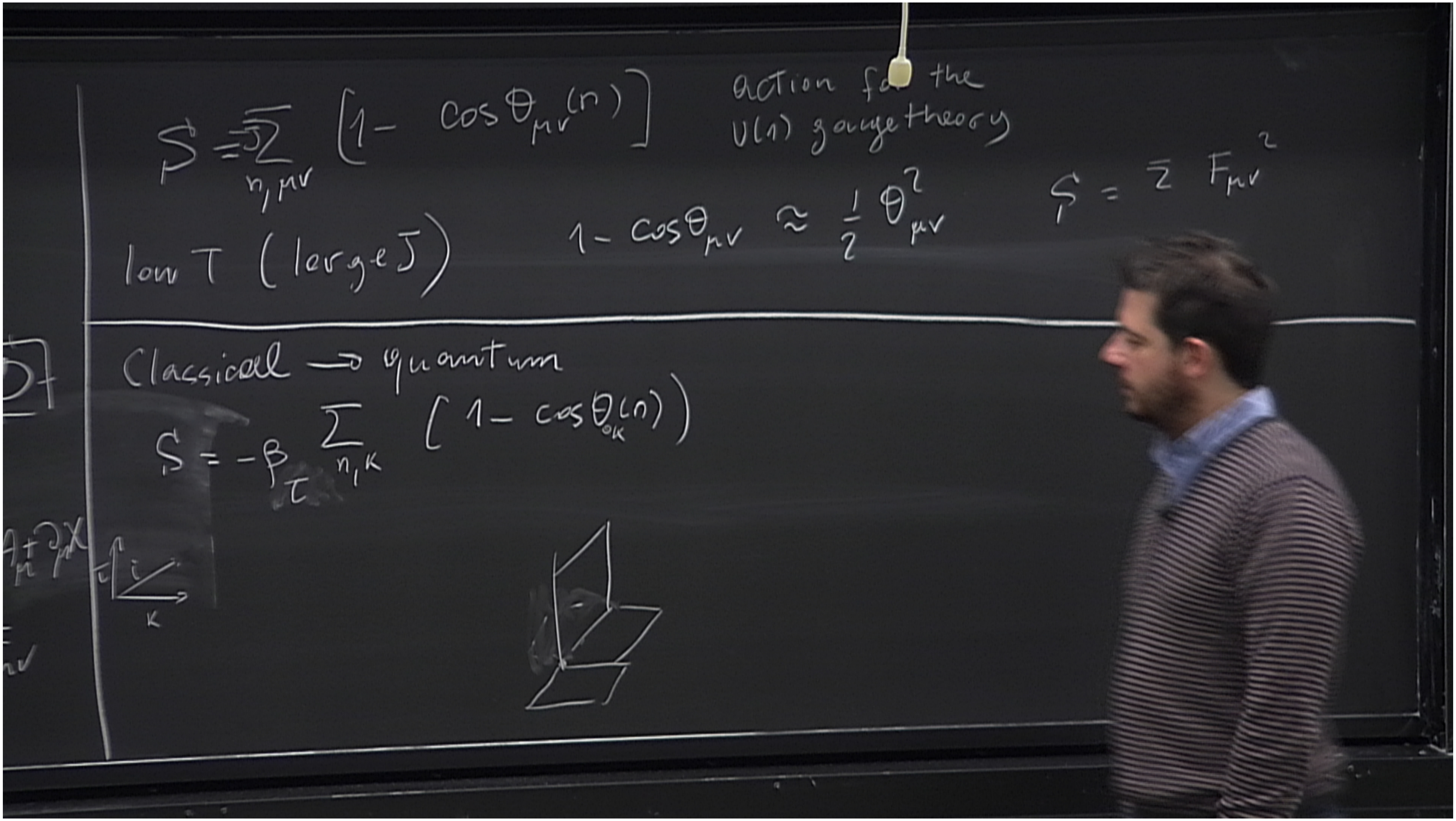


LOCAL GAUGE TRASF

$$\theta_\mu(n) \longrightarrow \theta_\mu(n) - \chi(n) + \chi(n+\mu) = \theta_\mu + \Delta_\mu \chi(n) \rightsquigarrow A_\mu \to A_\mu + \partial_\mu \chi$$

$$\theta_{\mu\nu} \longrightarrow \theta_{\mu\nu} \qquad \rightsquigarrow F_{\mu\nu} \to F_{\mu\nu}$$


$S = -J \sum_{n,\mu\nu} \left[1 - \cos\theta_{\mu\nu}(n)\right]$
action for the U(1) gauge theory
$S = \sum F_{\mu\nu}^2$
low T (large J)
$1 - \cos\theta_{\mu\nu} \approx \frac{1}{2}\theta_{\mu\nu}^2$


S = -J Σ_{n,μν} [1 - cos θ_{μν}(n)]
action for the
U(1) gauge theory
S = Σ F_{μν}^2
low T (large J)
1 - cos θ_{μν} ≈ 1/2 θ_{μν}^2
Classical → quantum
S = -β_τ Σ_{n,k} [1 - cos θ_{0k}(n))

$$S = -J \sum_{n,\mu\nu} \left[1 - \cos\theta_{\mu\nu}(n)\right]$$ action for the U(1) gauge theory

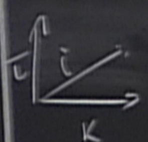
$$S = \sum F_{\mu\nu}^2$$

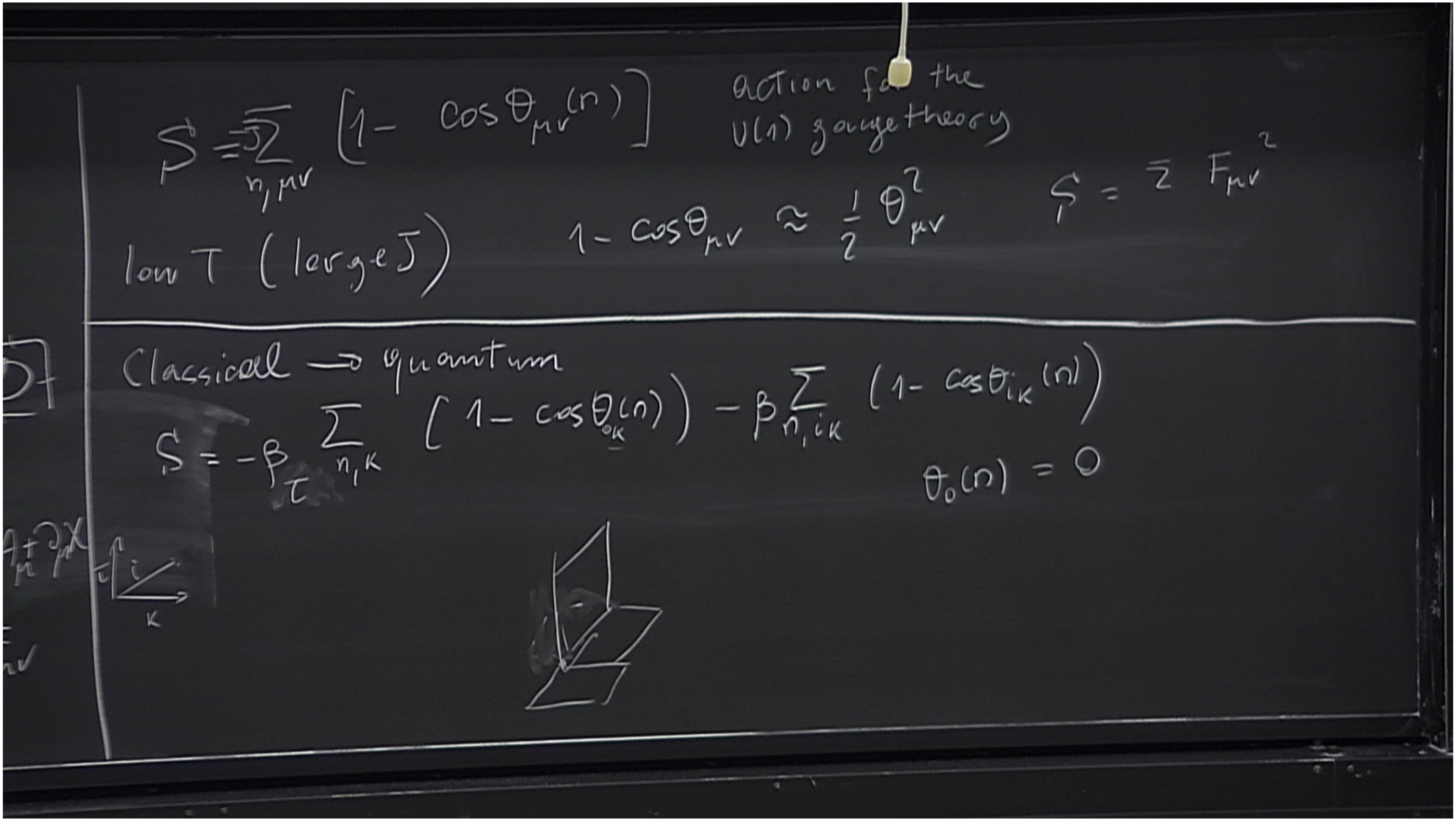
low T (large J) $\quad 1 - \cos\theta_{\mu\nu} \approx \frac{1}{2}\theta_{\mu\nu}^2$

Classical → quantum

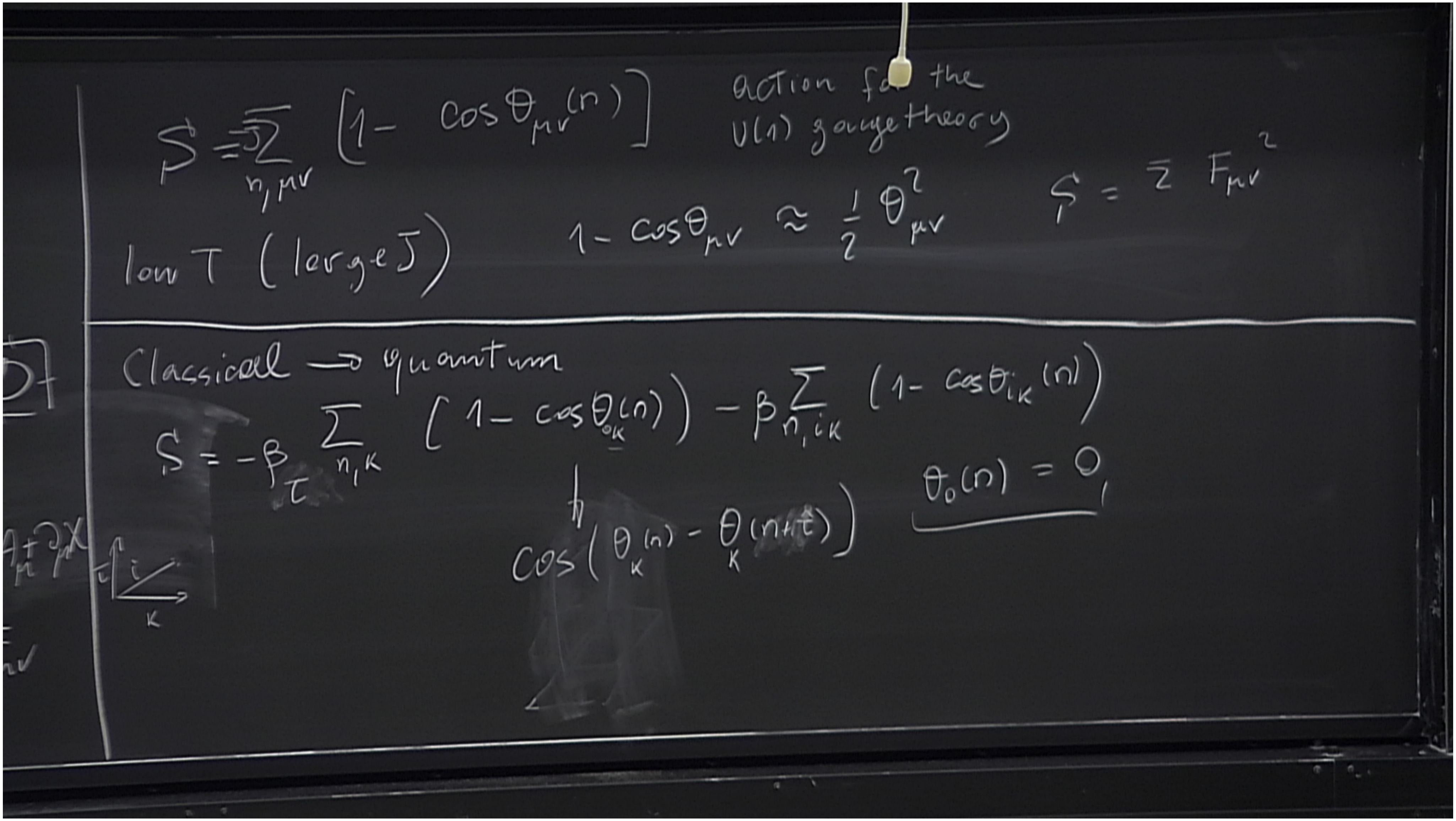
$$S = -\beta_\tau \sum_{n,k} \left(1 - \cos\theta_{0k}(n)\right) - \beta \sum_{n,ik} \left(1 - \cos\theta_{ik}(n)\right)$$

$A_\mu + \partial_\mu \chi$





$S = -J \sum_{n,\mu\nu} \left[1 - \cos\theta_{\mu\nu}(n)\right]$
action for the
U(1) gauge theory
$S = \sum F_{\mu\nu}^2$
low T (large J)
$1 - \cos\theta_{\mu\nu} \approx \frac{1}{2}\theta_{\mu\nu}^2$
Classical → quantum
$S = -\beta_\tau \sum_{n,k} \left(1 - \cos\theta_{0k}(n)\right) - \beta \sum_{n,ik} \left(1 - \cos\theta_{ik}(n)\right)$
$\theta_0(n) = 0$
i
k


$S = -J \sum_{n,\mu\nu} [1 - \cos\theta_{\mu\nu}(n)]$
action for the U(1) gauge theory
$S = \sum F_{\mu\nu}^2$
low T (large J)
$1 - \cos\theta_{\mu\nu} \approx \frac{1}{2}\theta_{\mu\nu}^2$
Classical → quantum
$S = -\beta_\tau \sum_{n,k} [1 - \cos\theta_{0k}(n)) - \beta \sum_{n,ik} (1 - \cos\theta_{ik}(n))$
$\cos(\theta_k(n) - \theta_k(n+\hat{\tau}))$
$\theta_0(n) = 0$
τ
i
k

$$S = -J \sum_{n,\mu\nu} \left[1 - \cos\theta_{\mu\nu}(n)\right]$$

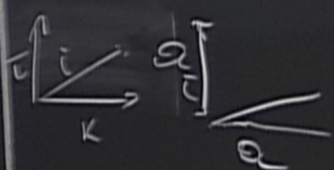
action for the U(1) gauge theory

low T (large J) $\quad 1 - \cos\theta_{\mu\nu} \approx \frac{1}{2}\theta_{\mu\nu}^2 \qquad S = \sum F_{\mu\nu}^2$

Classical $\rightarrow$ quantum

$$S = -\beta_\tau \sum_{n,k} \left(1 - \cos\theta_{0k}(n)\right) - \beta \sum_{n,ik} \left(1 - \cos\theta_{ik}(n)\right)$$

$$\cos\left(\theta_k(n) - \theta_k(n+\hat\tau)\right) \qquad \underline{\theta_0(n) = 0},$$



$a_\tau \to 0 \qquad \frac{\beta_\tau}{a_\tau} = g \qquad \sum_{n,k} \rightarrow \frac{1}{a_\tau}\int d\tau \sum_{n,k}$

$\beta_\tau \to \infty$

everywhere

n only on spatial slices at fixed τ

$$\Theta_{\mu\nu} \longrightarrow \Theta_{\mu\nu}$$

$$\rightsquigarrow F_{\mu\nu}$$

$$S = \int d\tau \left\{ \sum_{n,k} \underbrace{-\frac{\beta_\tau}{a_\tau}}_{g} \frac{a_\tau^2}{2} \dot{\Theta}_{0k}^2(n) \; \underbrace{-\frac{\beta}{a_\tau}}_{J} \sum_{n,ik} \left(1 - \cos\Theta_{ik}(n)\right) \right\}$$

$$1 - \cos \Delta_0(n) \approx \frac{1}{2} \Delta_0^2 \simeq \frac{a_\tau^2}{2} \left(\dot{\Theta}_{0k}^{(n)}\right)^2$$

$$\dot{\Theta}_{0k} = \frac{\partial}{\partial \tau} \Theta_{0k}(n)$$

$$\beta \to 0$$
$$a_\tau \to 0 : \quad \frac{\beta}{a_\tau} = J$$

$$\sum_{n,ik} \underbrace{(1-\cos\theta_{ik}(n))}_{W_p} \Big\}$$

homework

$$H = g\sum_{n,k}\left[S_k^{0z}(n)\right]^2 - J\underbrace{\sum_{n,ik}\left(1-\cos\theta_{ik}(n)\right)}_{\sum_p \hat{W}_p}$$

$$G_\chi = \exp\Big\{-i\sum_{n,j} S_j^z(n)\,\chi(n)\Big\}$$

$$\mathcal{H}_{phys} = \{\psi\in\mathcal{H}_{TOT} \mid G_\chi\psi=\psi\}$$

$$\text{span}\{|\theta_{e_1}\dots\theta_{e_N}\rangle\} = \mathcal{H}_{TOT}$$

$$aH = g\sum_{n,k}\left[S_k^{z}(n)\right]^2 - J\sum_{n,ik}\underbrace{\left(1-\cos\theta_{ik}(n)\right)}_{\sum_p \hat{W}_p}$$

U(1) lattice quantum gauge theory

$$G_\chi = \exp\left\{-i\sum_{n,j} S_j^{z}(n)\chi(n)\right\}$$

$$\mathcal{H}_{phys} = \left\{\psi\in\mathcal{H}_{TOT} \mid G_\chi\psi=\psi\right\}$$

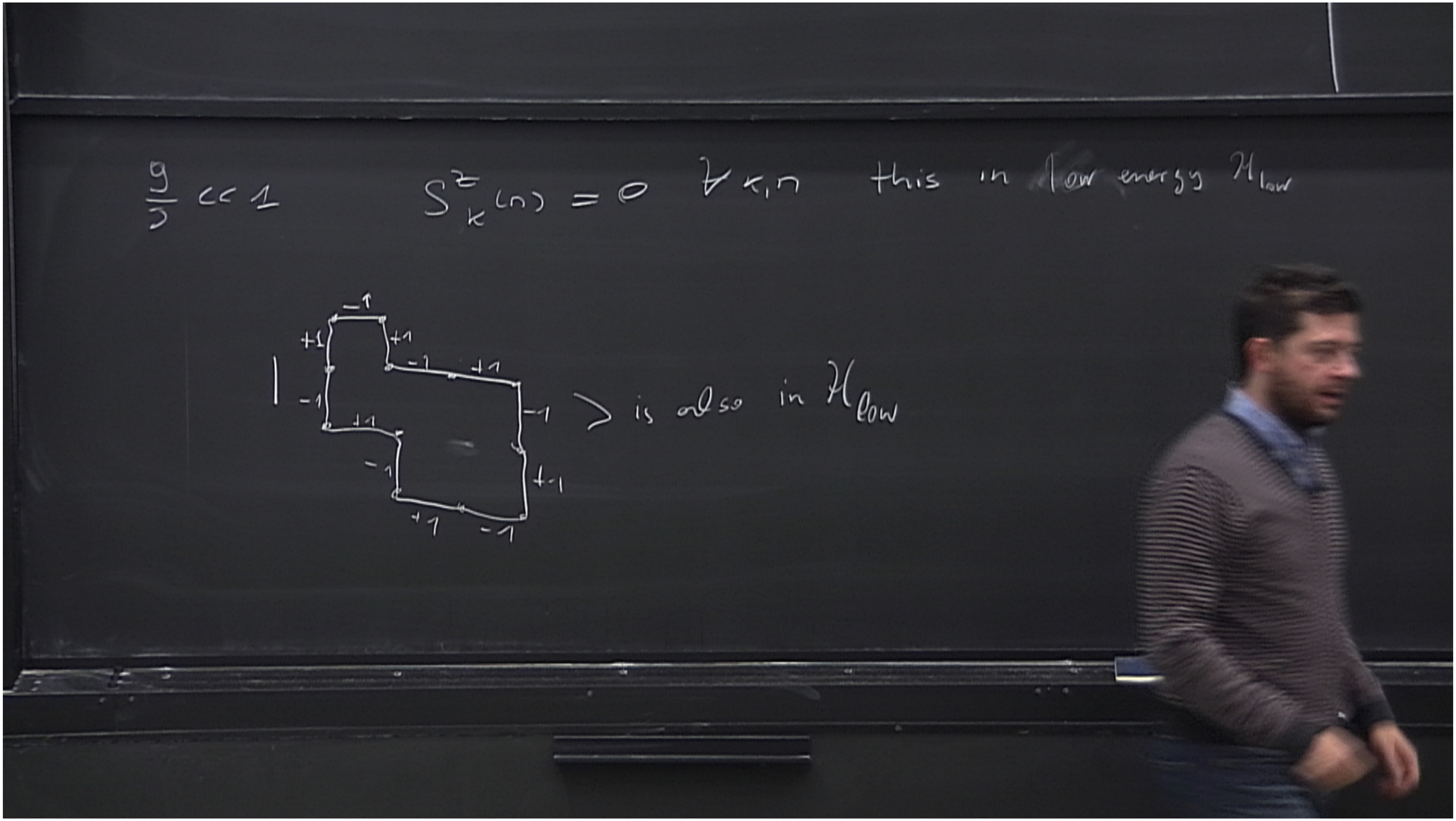
$$\text{span}\left\{|\theta_{e_1}\cdots\theta_{e_N}\rangle\right\} = \mathcal{H}_{TOT}$$

$\frac{g}{J}\gg 1$ $\quad S_k^{z} = 0,\pm 1,\pm 2,\dots$

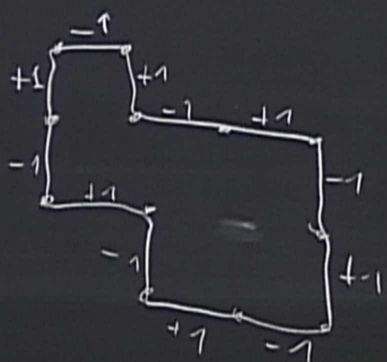
gapped theory

$$\langle A \rangle \sim e^{-A}$$

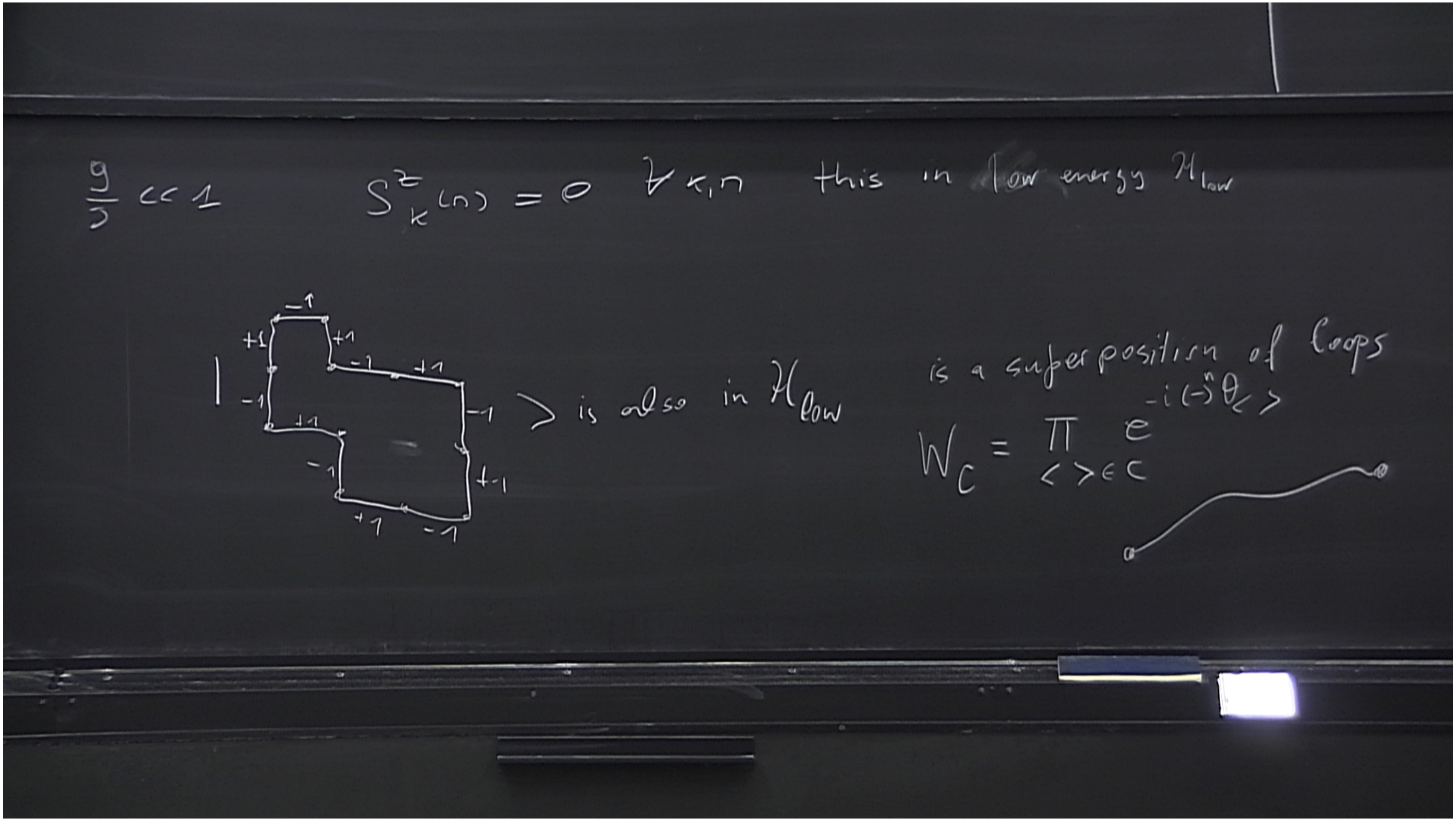
$$\langle \quad \rangle$$


$\frac{g}{J} \ll 1$
$S^z_k(n) = 0 \quad \forall k, n$ this in low energy $\mathcal{H}_{low}$
$|$ $\rangle$ is also in $\mathcal{H}_{low}$
-1
+1
+1
-1
+1
-1
+1
-1
-1
+1
+1
-1

$\frac{g}{J} << 1$ $\quad S^z_k(n) = 0 \quad \forall k,n$ this in low energy $\mathcal{H}_{low}$

$|$ 

$\rangle$ is also in $\mathcal{H}_{low}$ is a superposition of loops


g/J << 1
S^z_k(n) = 0 ∀ k, n this in low energy H_low
| > is also in H_low
+1 −1 +1 −1 +1 −1 +1 −1 +1 −1 +1 −1
is a superposition of loops
W_C = Π_{< > ∈ C} e^{−i(−)^n θ_{< >}}

- emergence of $U(1)$ theory

$$\mathcal{H}_{phys} \equiv \mathcal{H}_{loops}$$

$$H = H_{gauge} + H_{const}$$

$$U \gg g, J$$

$$U \sum_{n,i,k} \left(S^z_{ik}(n) - S^z_{k}(n+i) \right)^2 = H_{const}$$

- emergence of U(1) theory

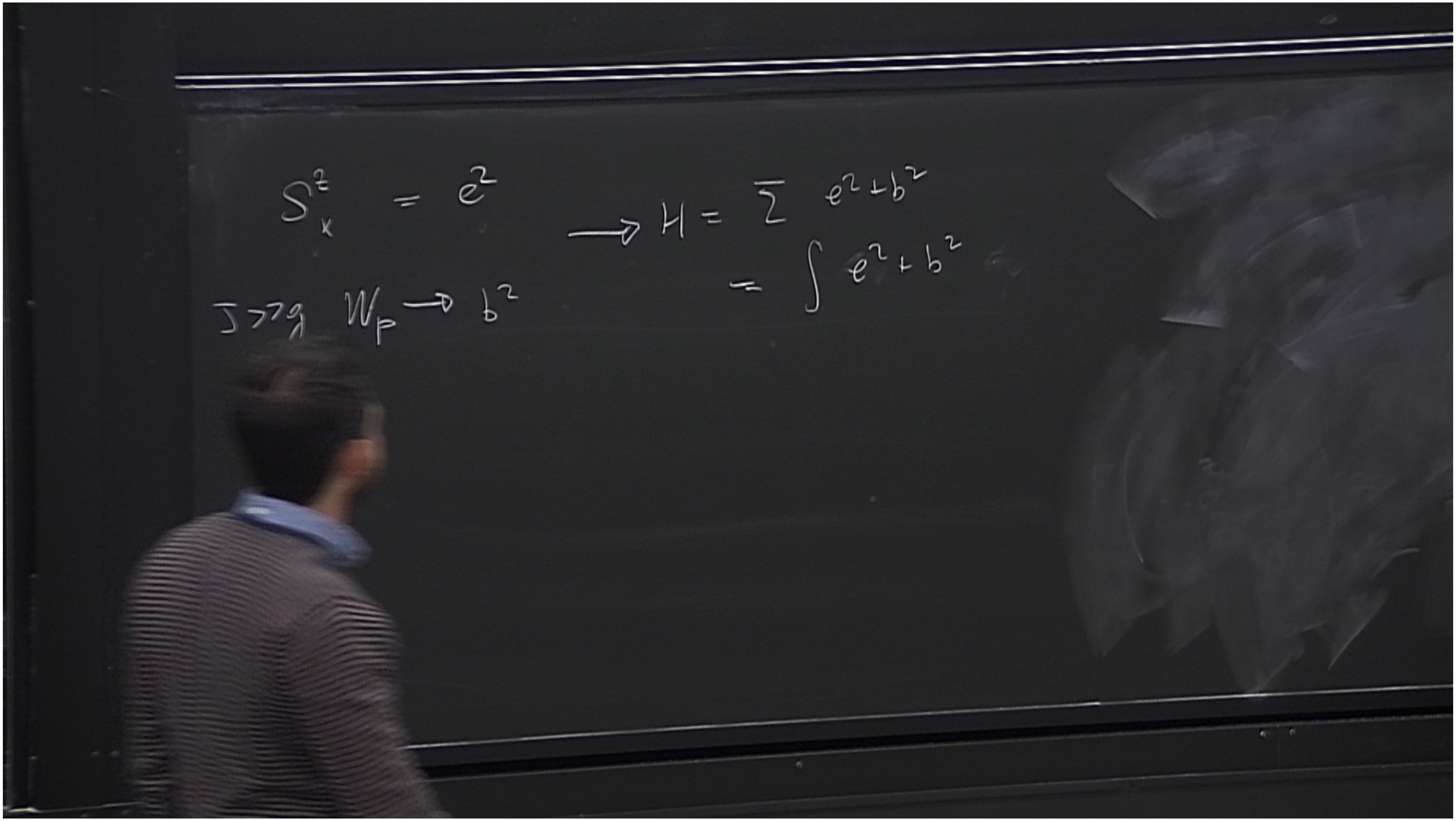
$$\mathcal{H}_{phys} \equiv \mathcal{H}_{loops}$$

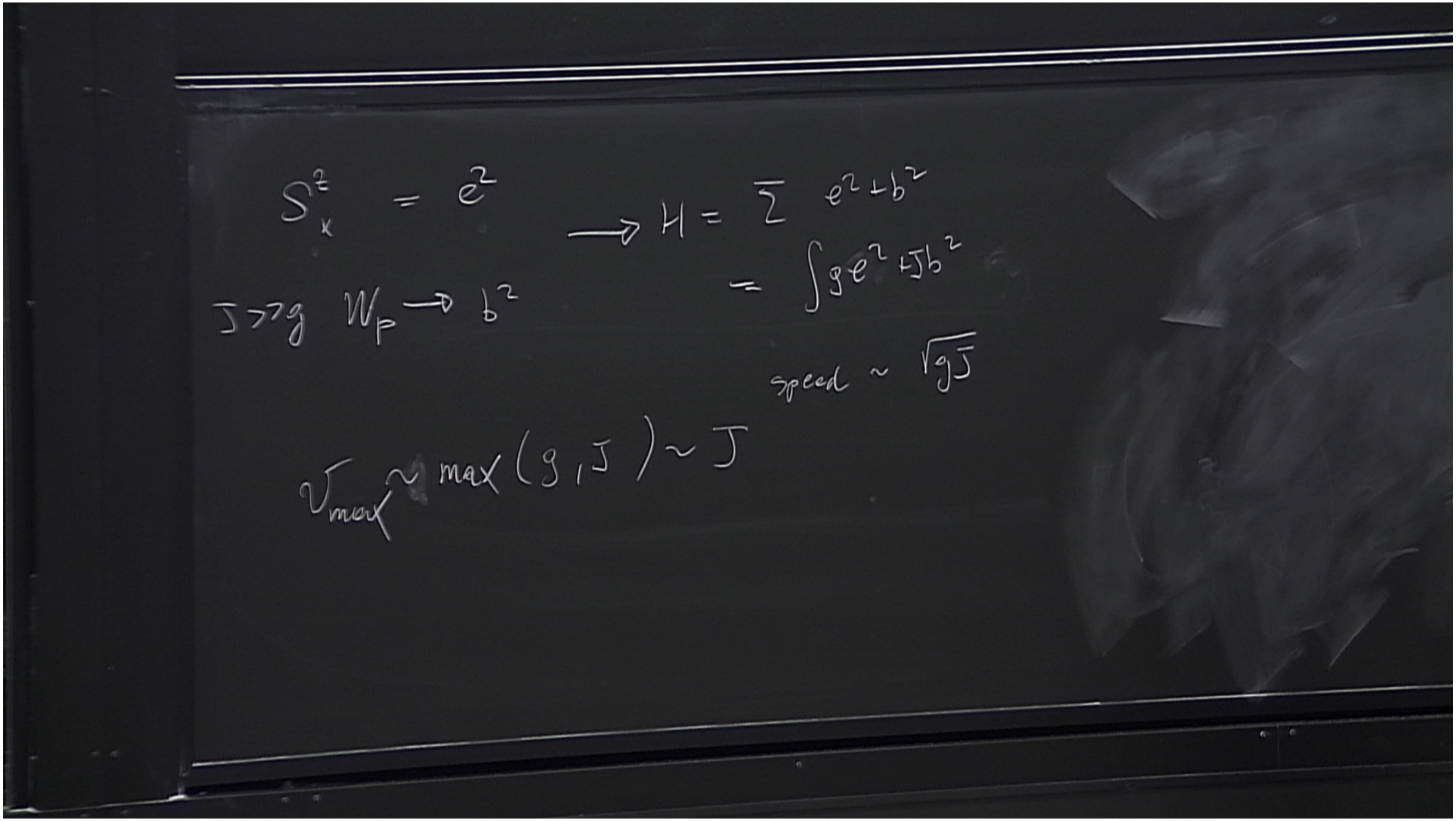
$$H = H_{gauge} + H_{const}$$

$$U \gg g, J$$

$$\mathcal{H}_{low} \equiv \mathcal{H}_{loops} \equiv \mathcal{H}_{phys}$$

$$U \sum_{n,k} \left(S^z_k(n) - S^z_k(n+\hat{i}) \right)^2 = H_{const}$$


$S_x^z = e^2$
$J >> g \quad W_p \rightarrow b^2$
$\rightarrow H = \Sigma \; e^2 + b^2$
$= \int e^2 + b^2$


$S^z_x = e^2$
$\longrightarrow H = \sum e^2 + b^2$
$= \int g e^2 + J b^2$
$J >> g \quad W_p \longrightarrow b^2$
speed $\sim \sqrt{gJ}$
$v_{max} \sim max(g, J) \sim J$

$S_x^z = e^2$ $\longrightarrow H = \sum e^2 + b^2$

$J \gg g \quad W_p \longrightarrow b^2$ $\qquad = \int g e^2 + J b^2$

speed $\sim \sqrt{gJ}$

$v_{max} \sim \max(g, J) \sim J$

with improvements $v_{max} \sim \sqrt{gJ}$