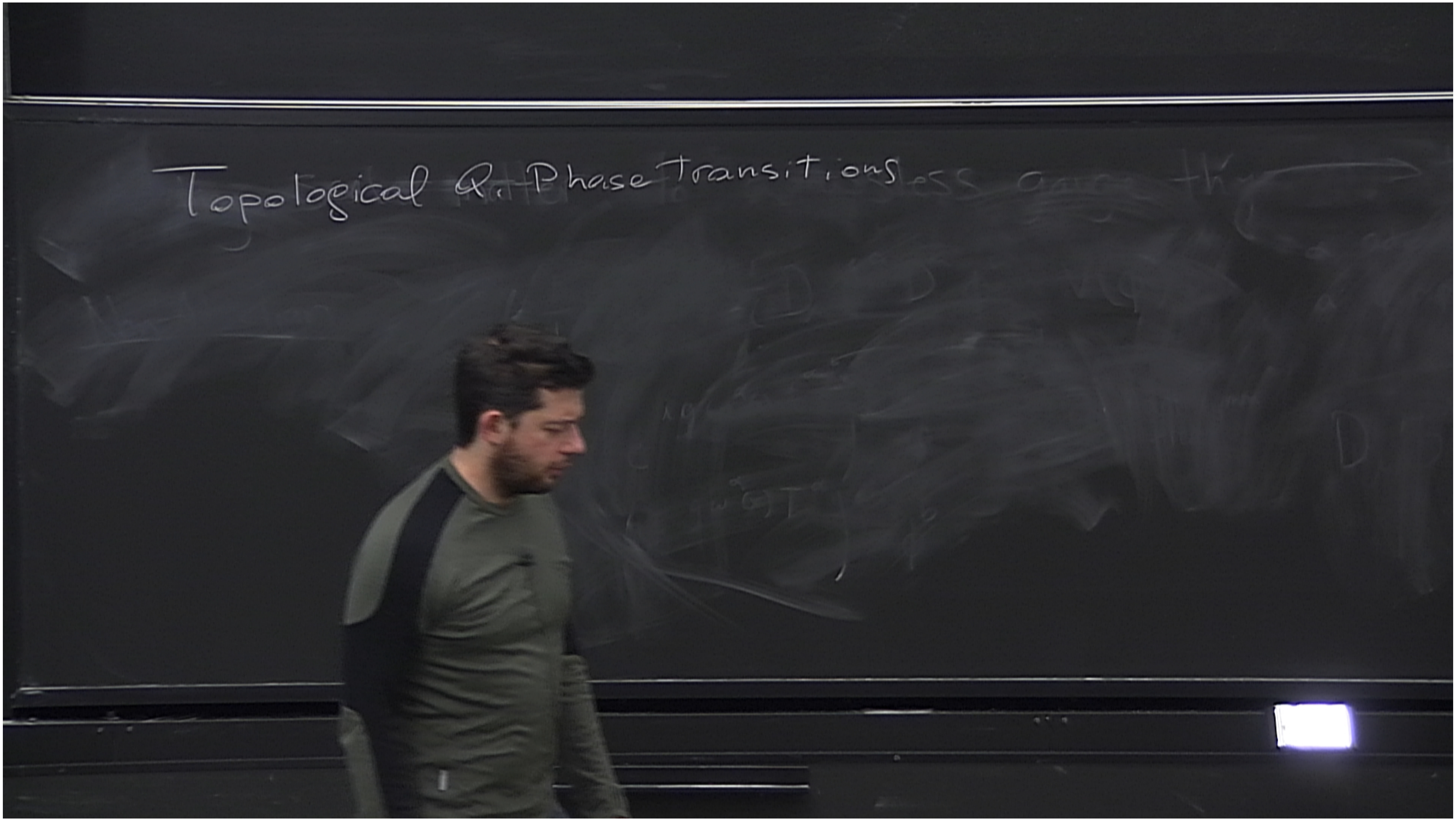
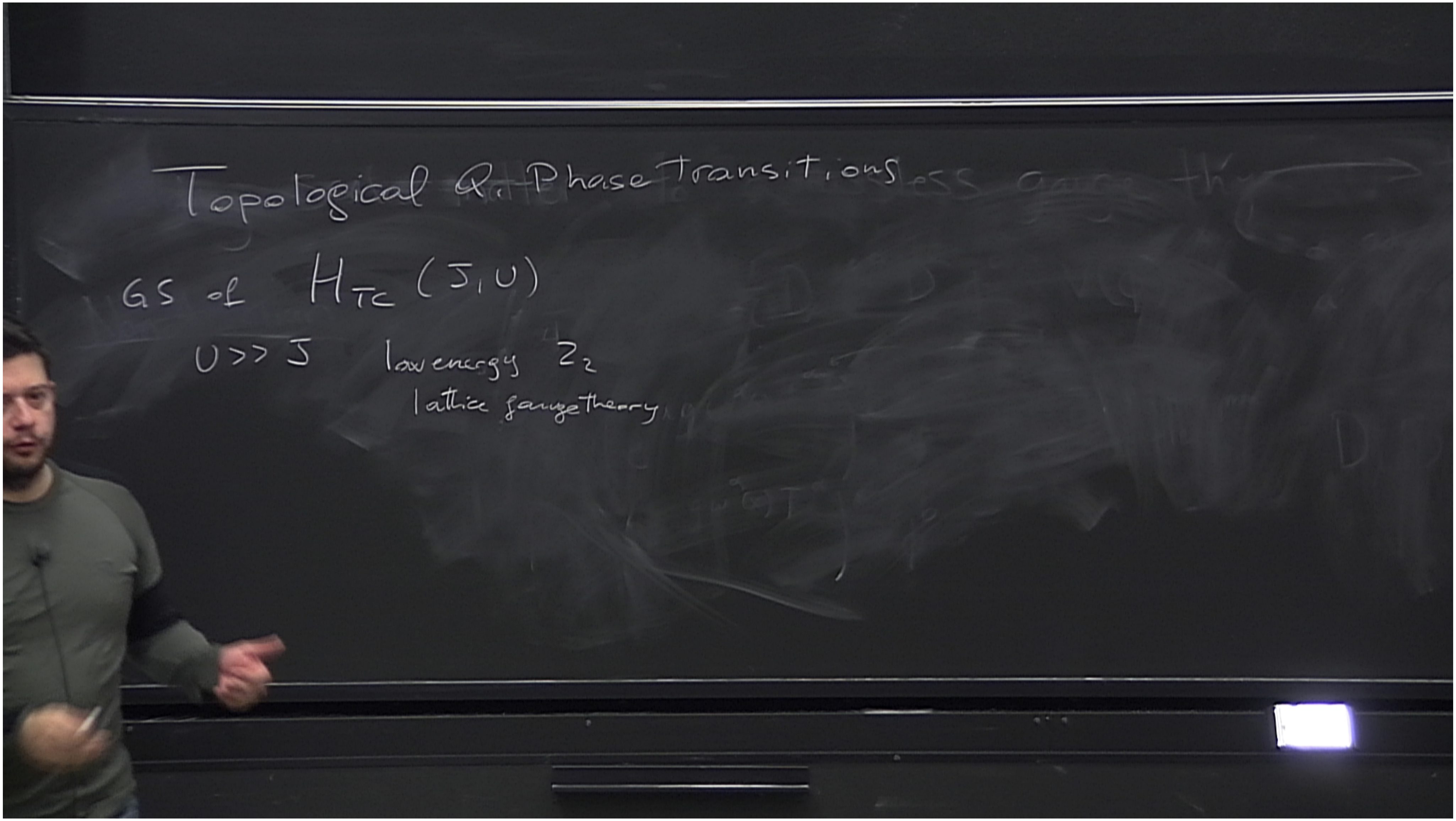
Title: Condensed Matter (Review) - Lecture 14

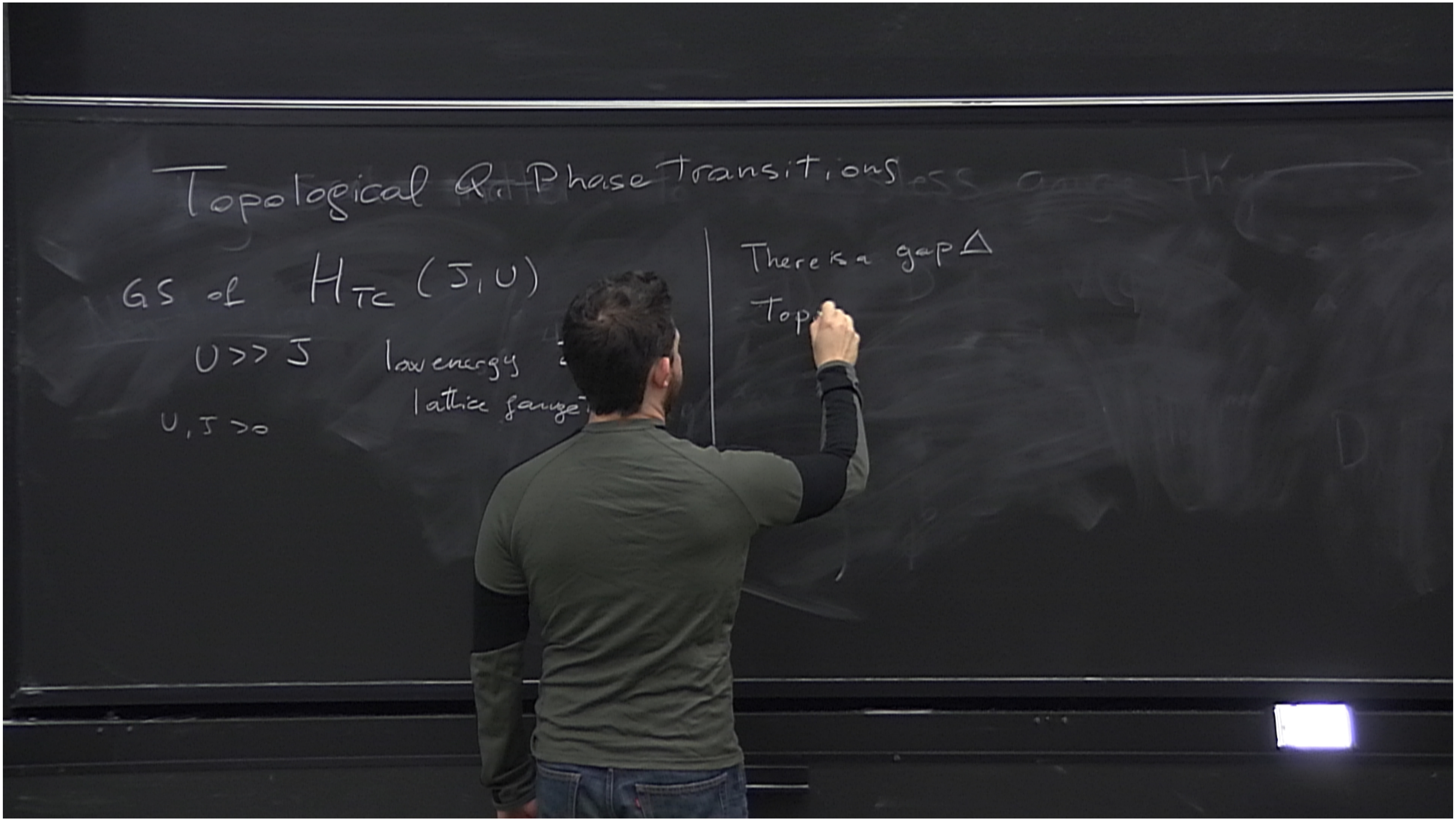
Date: Jan 19, 2012 10:15 AM

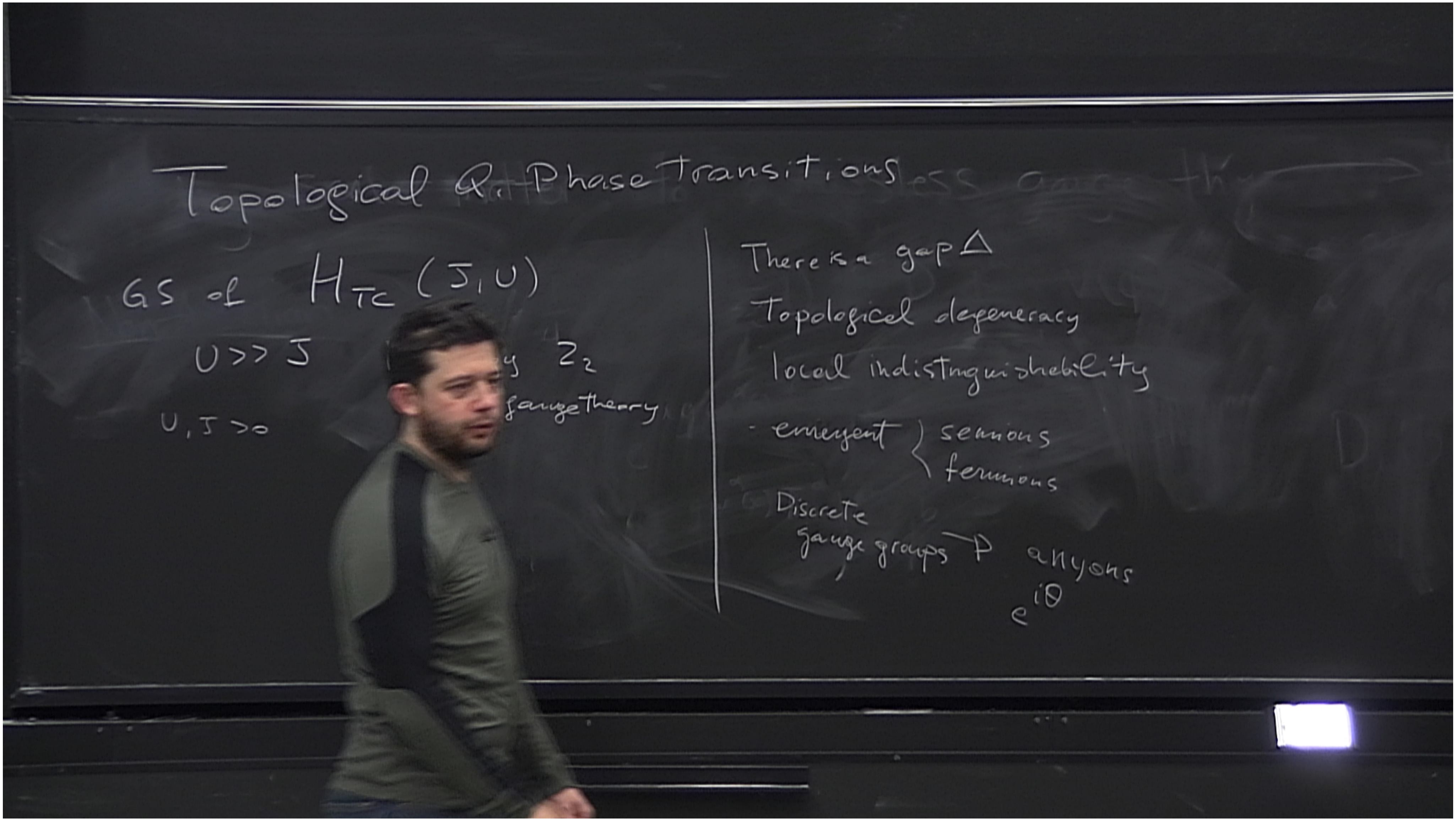
URL: http://pirsa.org/12010100

Abstract:

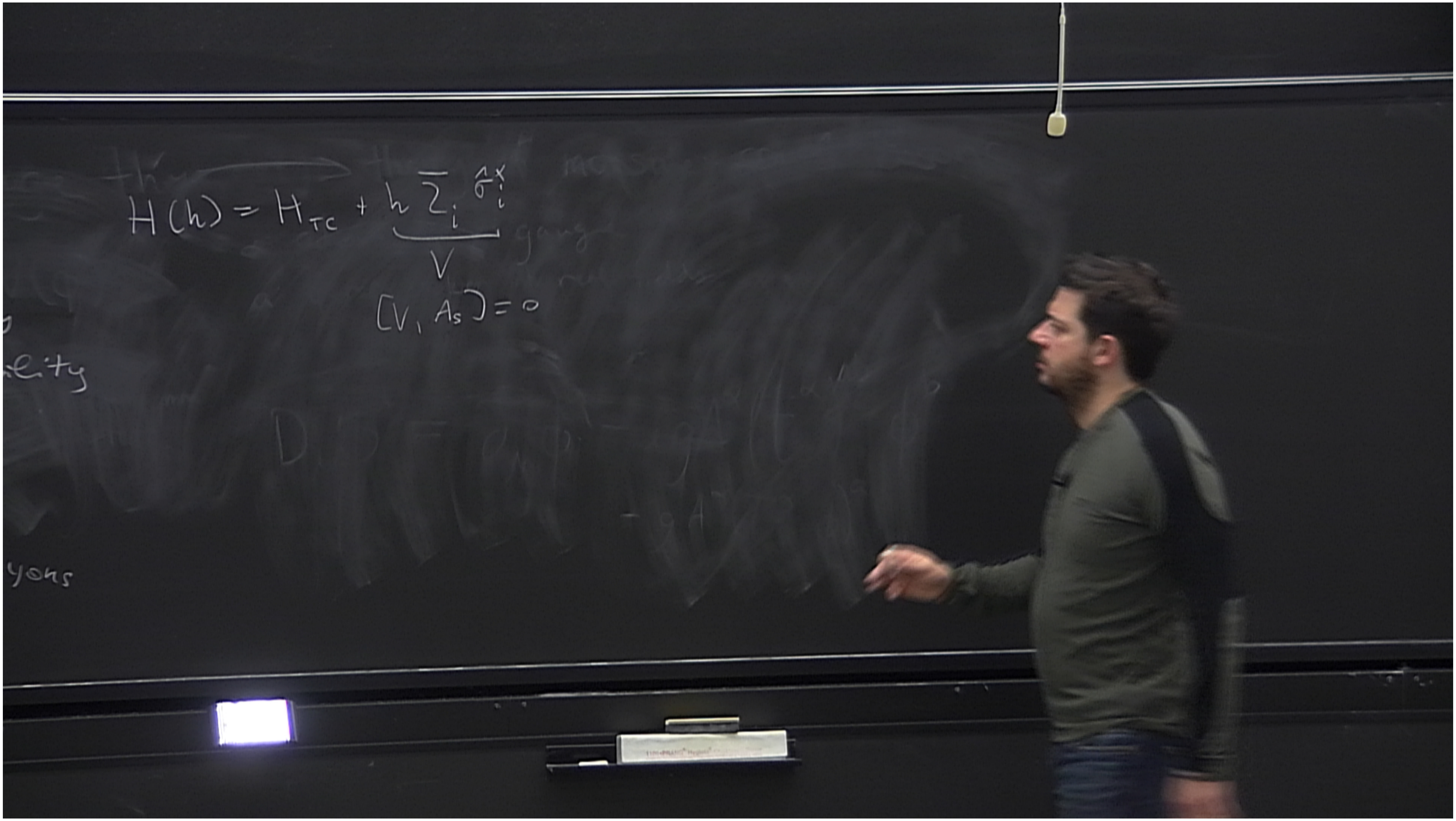

Topological Q. Phase Transitions

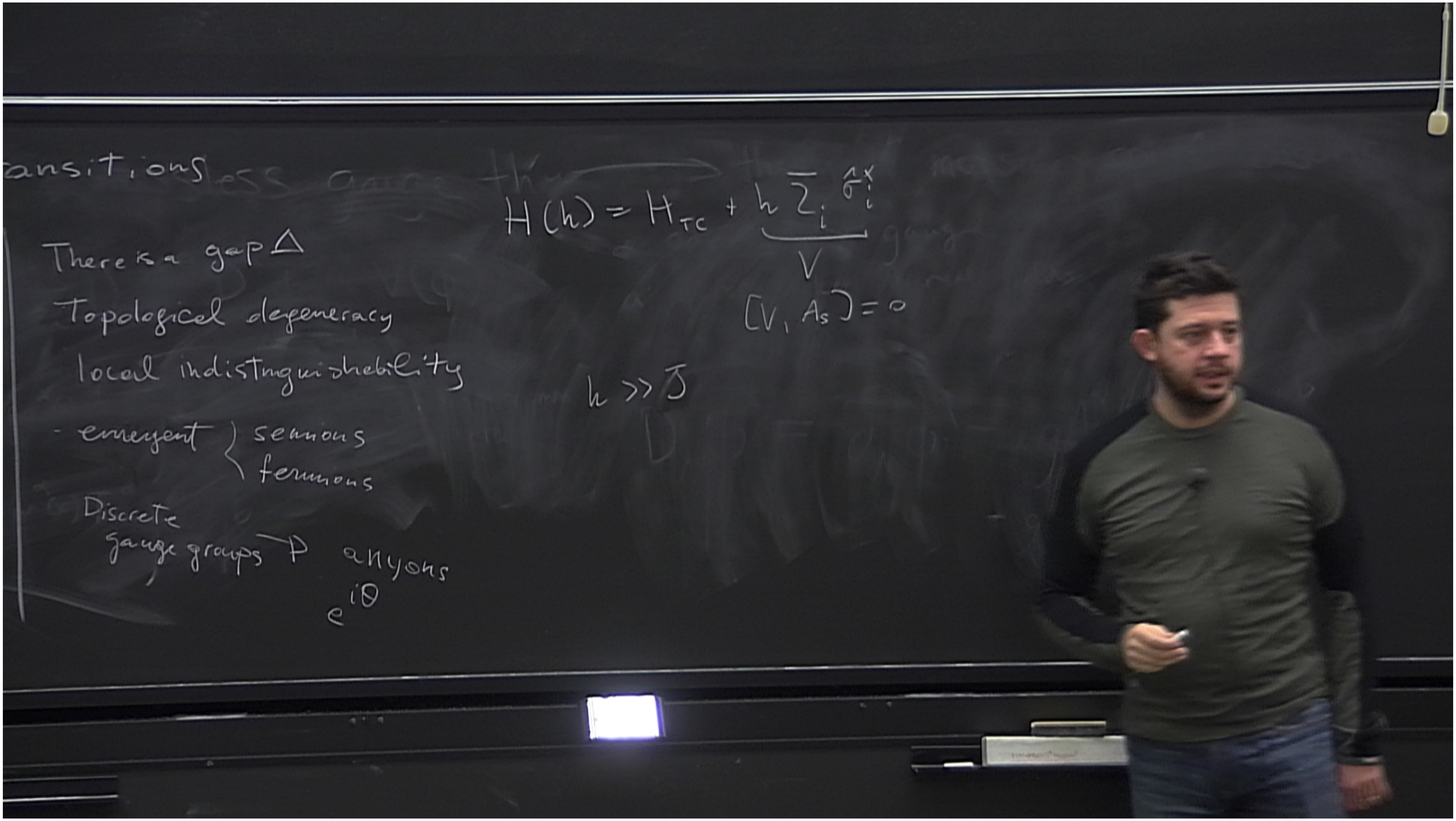

Topological Q. Phase Transitions
GS of $H_{TC}(J,U)$
$U >> J$ low energy Z_2
lattice gauge theory


Topological Q. Phase Transitions
GS of H TC (J, U)
U >> J
low energy
lattice gauge
U, J > 0
There is a gap Δ
Top

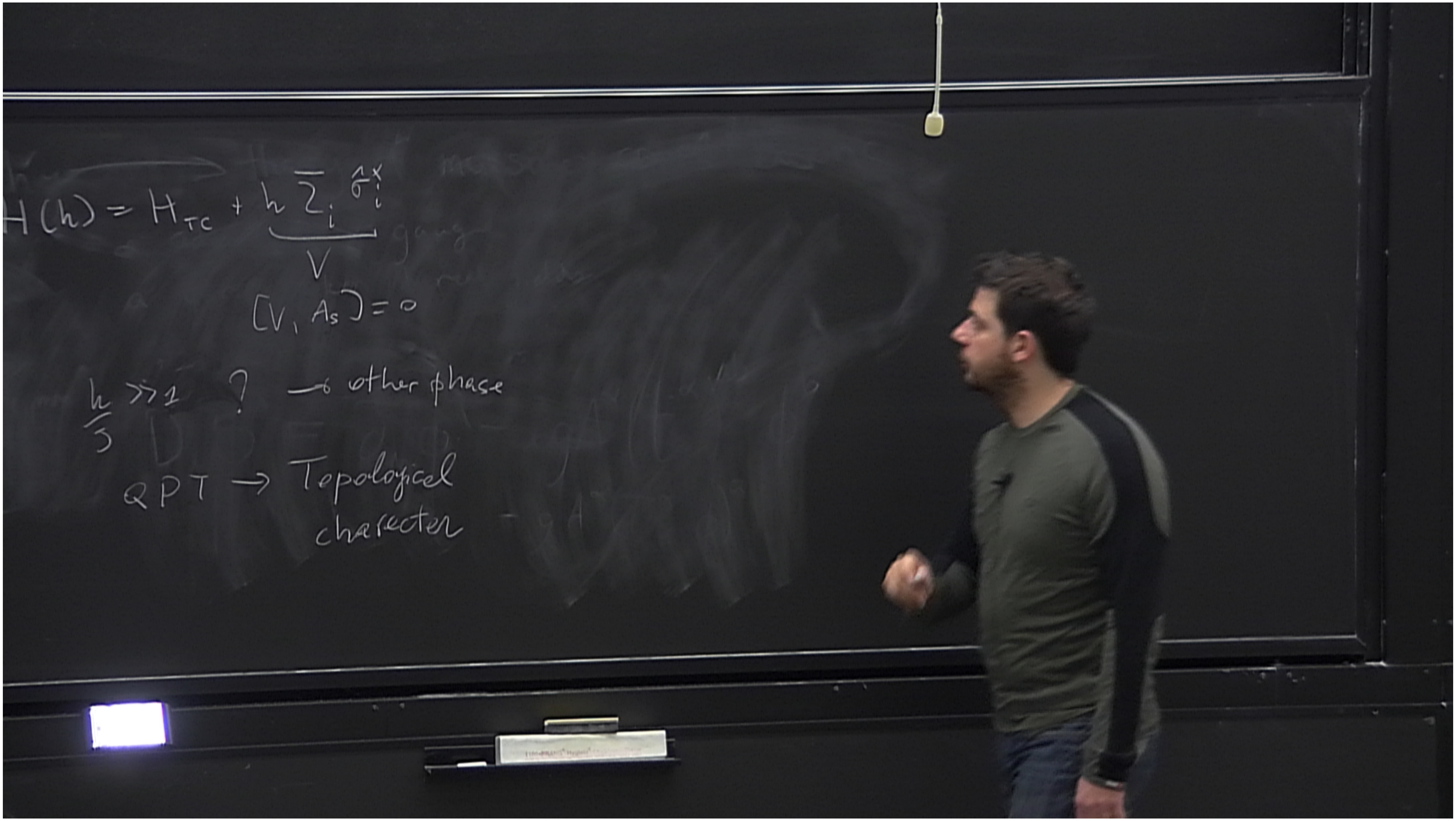

Topological Q. Phase Transitions
GS of H_TC (J, U)
U >> J
Z_2
gauge theory
U, J > 0
There is a gap Δ
Topological degeneracy
local indistinguishability
emergent
semions
fermions
Discrete
gauge groups → anyons
e^iθ

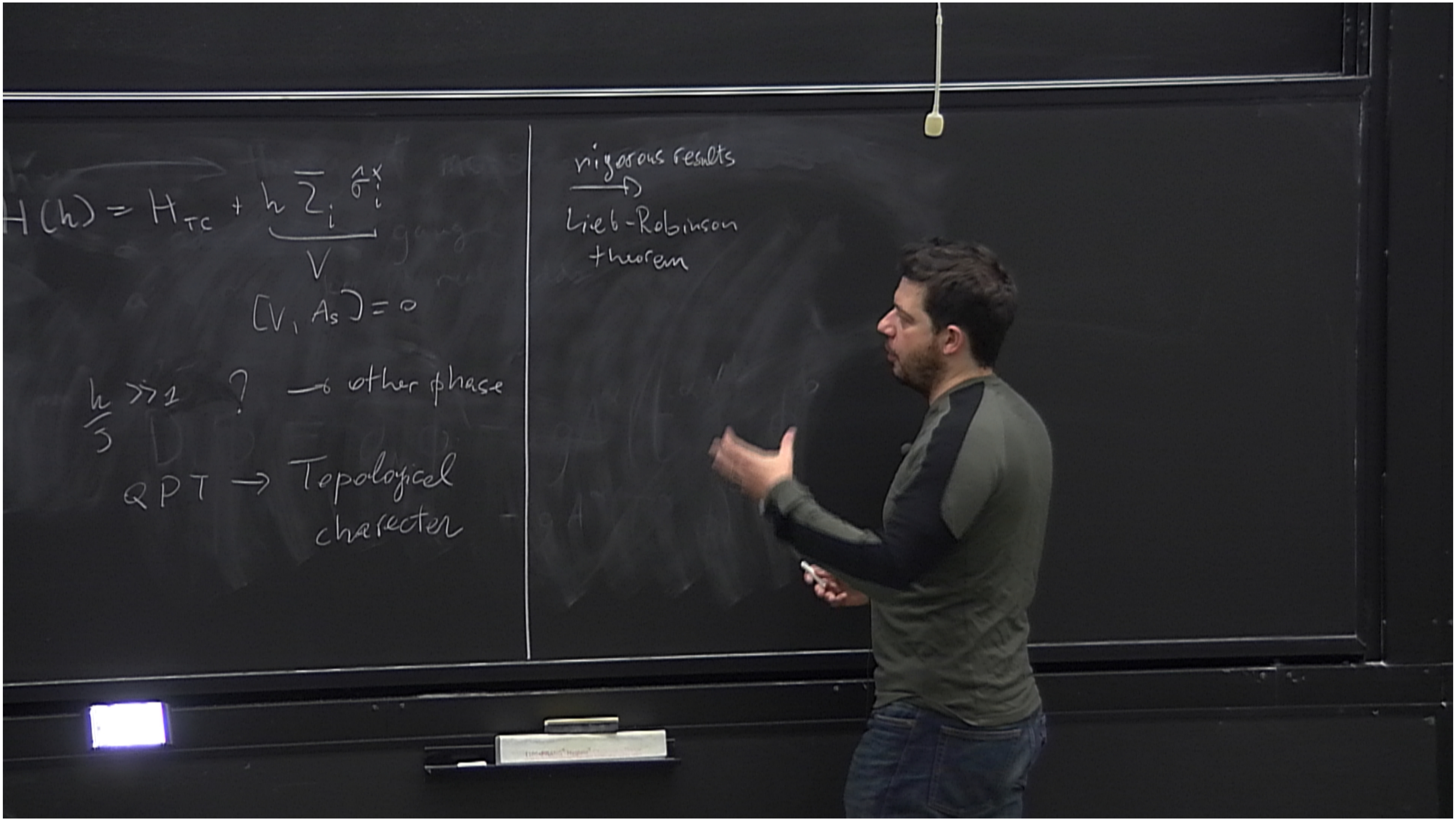

$H(h) = H_{TC} + h \sum_i \hat{\sigma}^x_i$


H(h) = H_TC + h Σ_i σ̂_i^x
V
[V, A_s] = 0
ity
yons


ansitions
There is a gap Δ
Topological degeneracy
local indistinguishability
- emergent { semions
fermions
Discrete gauge groups → anyons
e^iθ
H(h) = H_TC + h Σ_i σ_i^x
V
[V, A_s] = 0
h >> J


H(h) = H_TC + h Σ_i σ^x_i
V
[V, A_s] = 0
h/J >> 1 ?


$H(h) = H_{TC} + h \sum_i \hat{\sigma}_i^x$
V
$[V, A_s] = 0$
$\frac{h}{J} \gg 1$? → other phase
QPT → Topological character


H(h) = H_TC + h Σ_i σ̂_i^x
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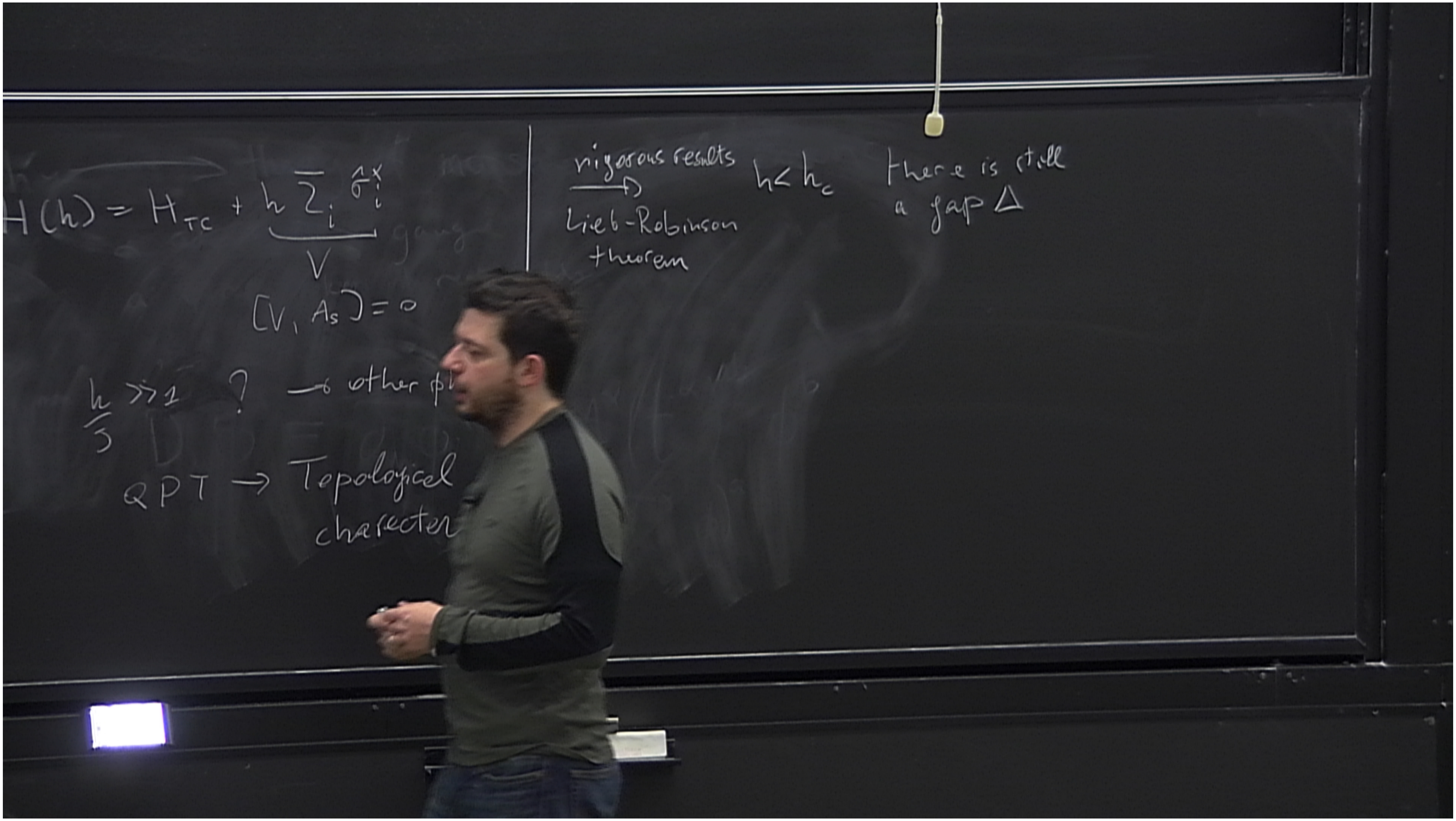
h/J >> 1 ? → other phase
QPT → Topological character
rigorous results
Lieb-Robinson theorem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[V, A_s] = 0
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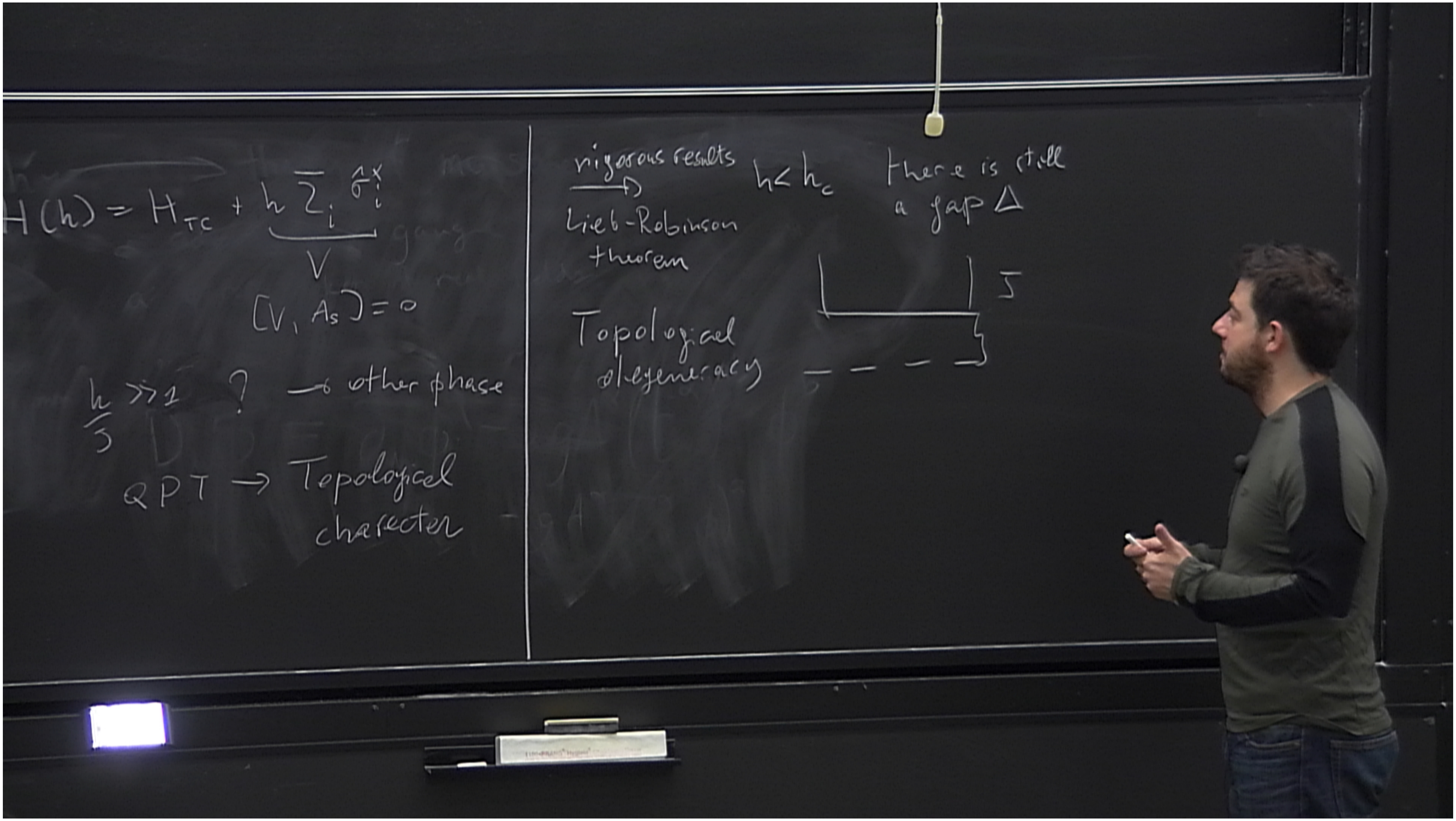
QPT → Topological
character
rigorous results
Lieb-Robinson
theorem
h < h_c
there is still
a gap Δ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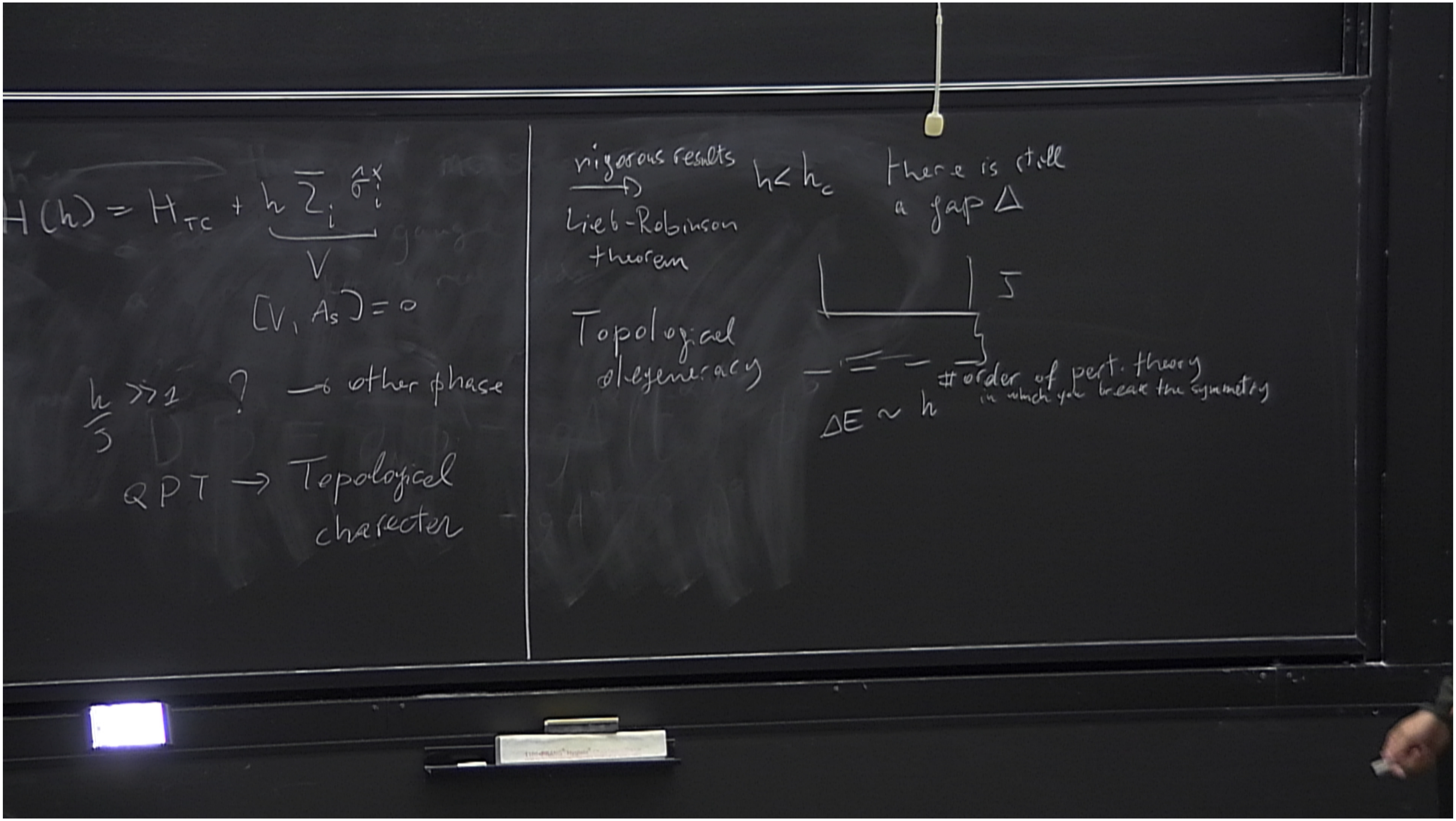

H(h) = H_TC + h Σ_i σ_i^x
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rigorous results
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Topological degeneracy
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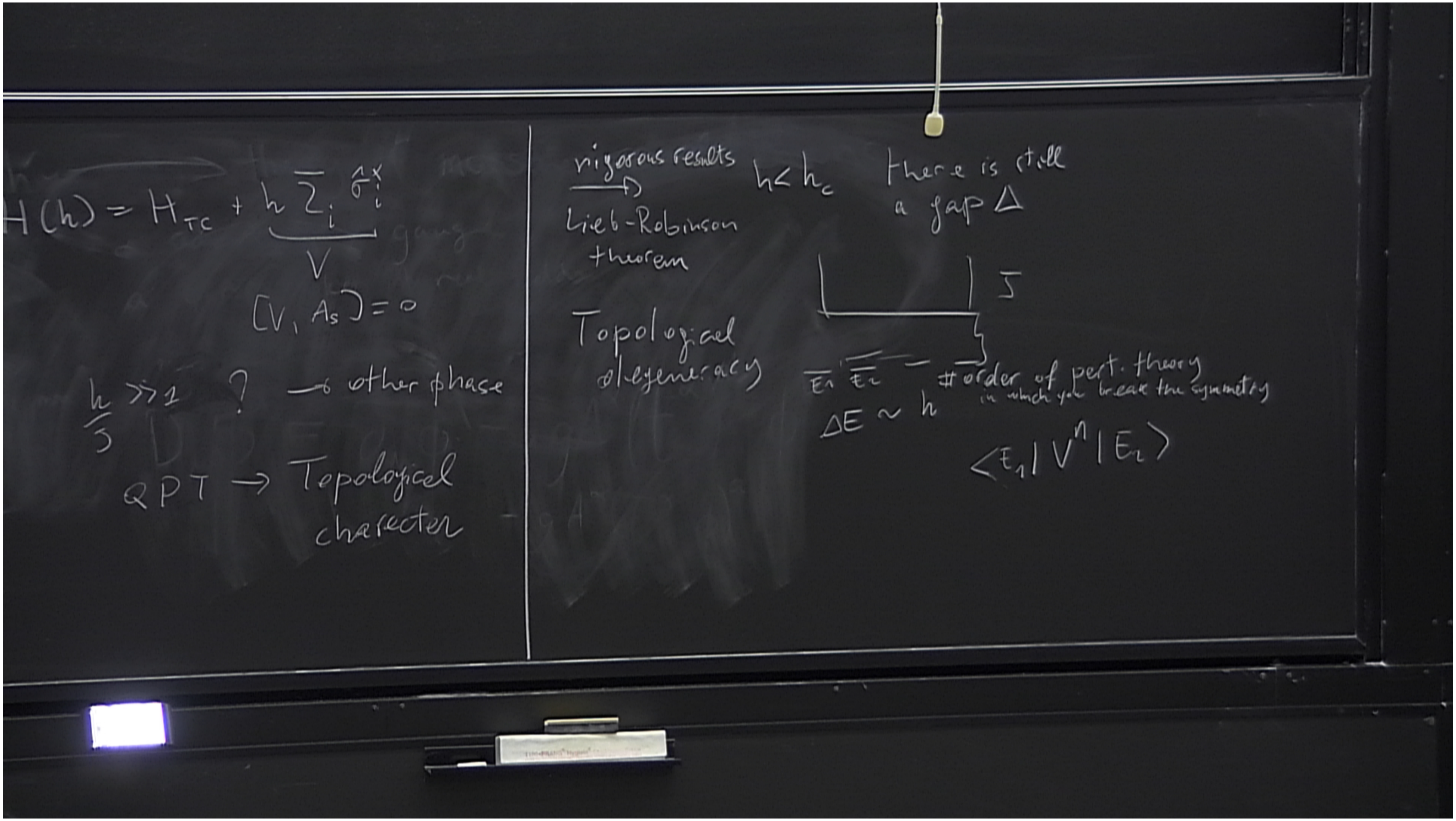
there is still a gap Δ
J
ΔE ~ h
order of pert. theory in which you break the symmetry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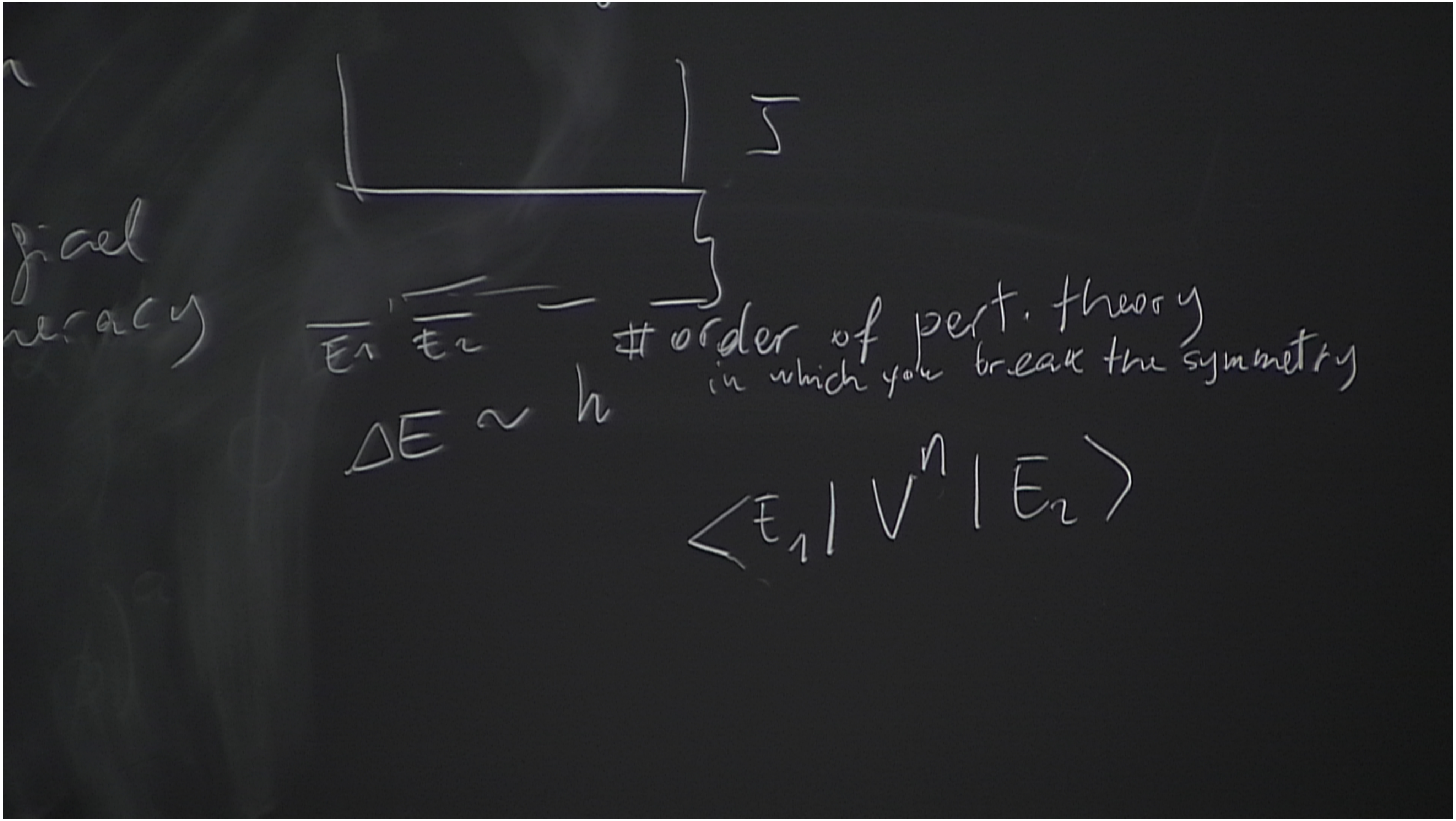
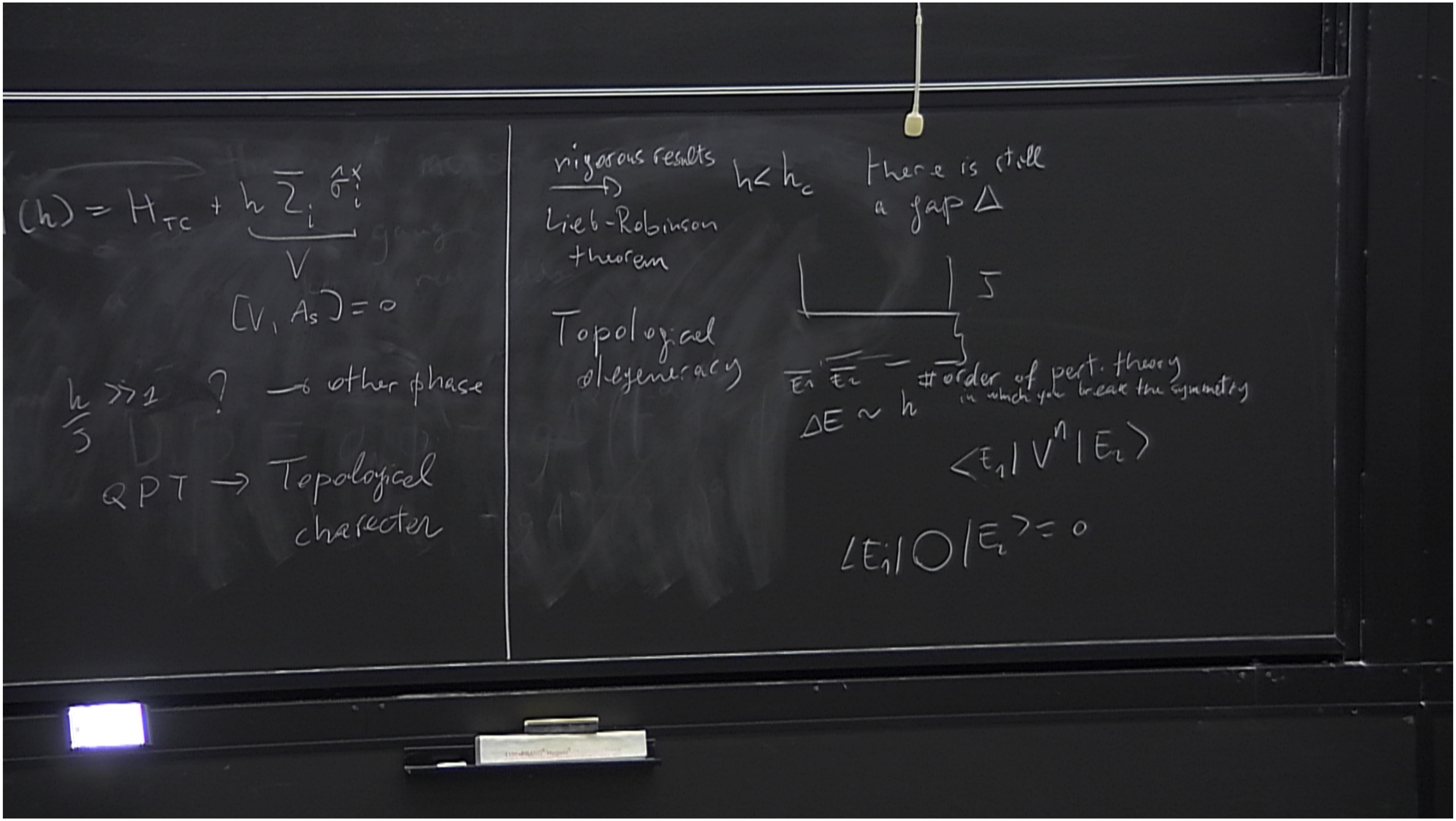
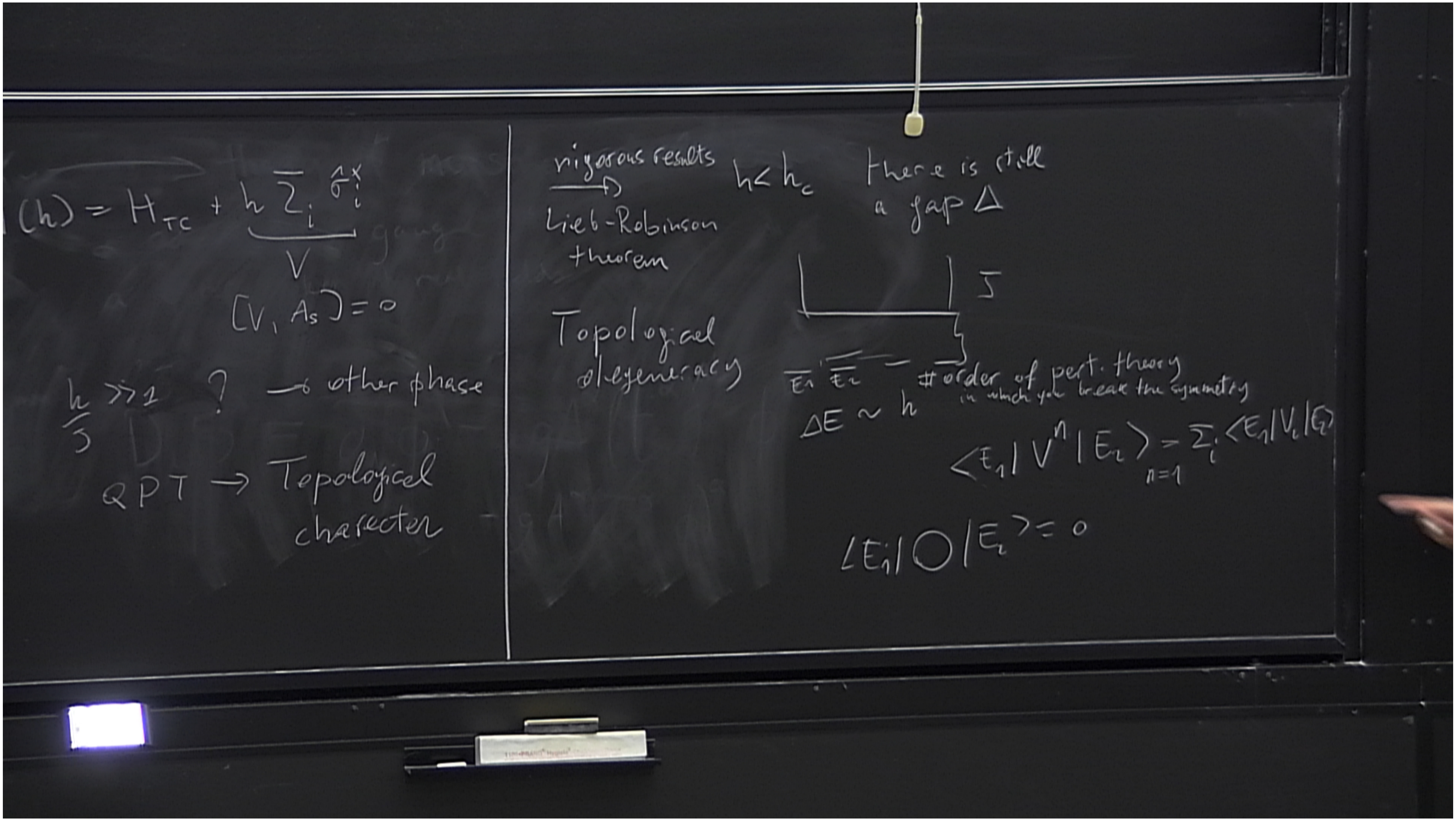
rigorous results
Lieb-Robinson theorem
Topological degeneracy
h < h_c
there is still a gap Δ
J
E_1 E_2
order of pert. theory in which you break the symmetry
ΔE ~ h^n
<E_1 | V^n | E_2>


gical
eracy
J
E1 E2
order of pert. theory in which you break the symmetry
ΔE ~ h
<E1| V^n |E2>


(h) = H_TC + h Σ_i σ̂^x_i
V
[V, A_s] = 0
h/J >> 1 ? → other phase
QPT → Topological character
rigorous results → Lieb-Robinson theorem
Topological degeneracy
h < h_c
there is still a gap Δ
J
E1 E2
ΔE ~ h^# order of pert. theory in which you break the symmetry
<E1| V^n |E2>
<E1| O |E2> = 0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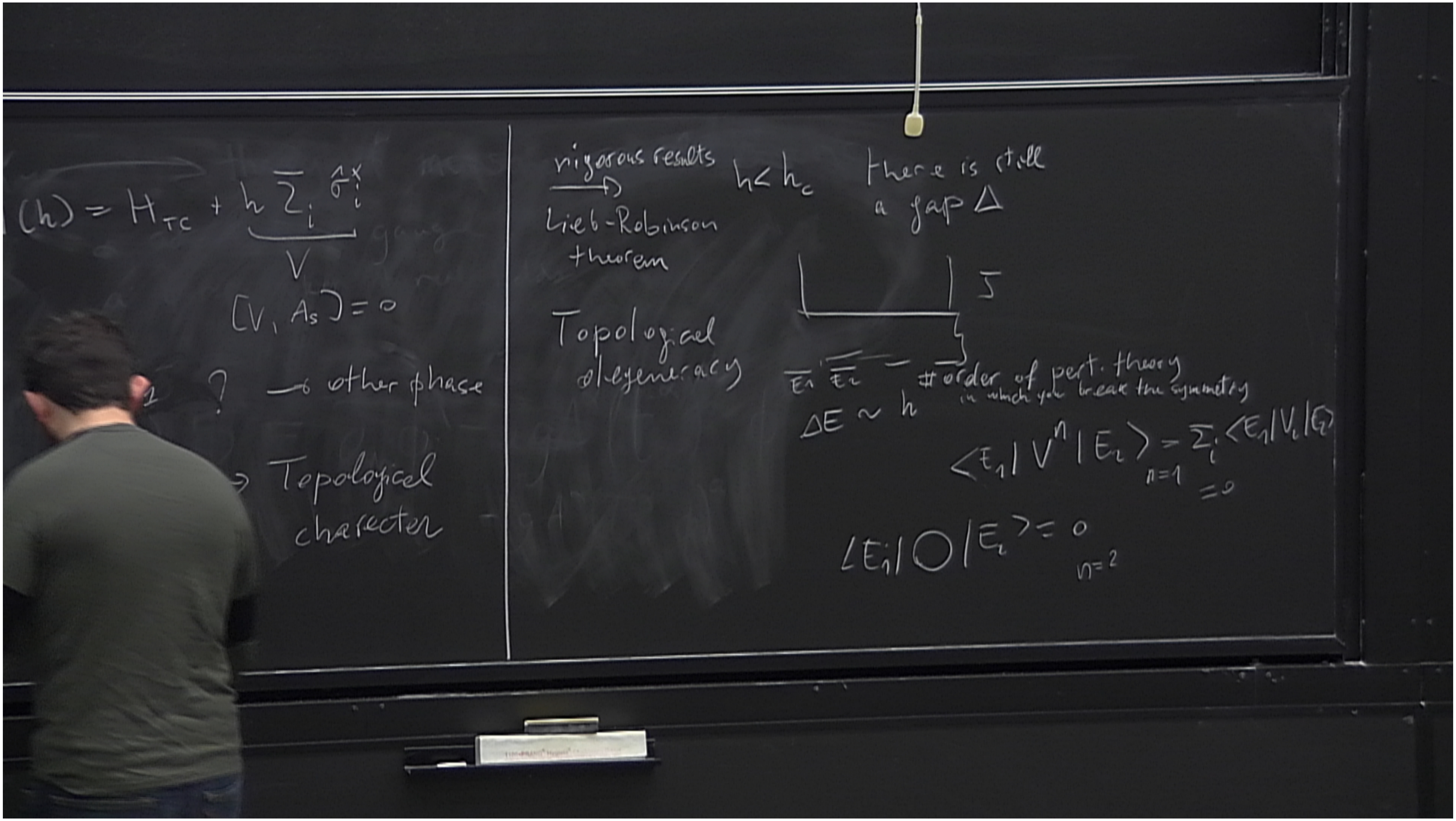
E_1 E_2
order of pert. theory
in which you break the symmetry
ΔE ~ h
<E_1 | V^n | E_2> = Σ_i <E_1|V_i|E_2>
n=1
<E_1| O |E_2> = 0


(h) = H_TC + h Σ_i σ̂_i^x
V
[V, A_s] = 0
? → other phase
→ Topological
character
rigorous results
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theorem
Topological
degeneracy
h < h_c
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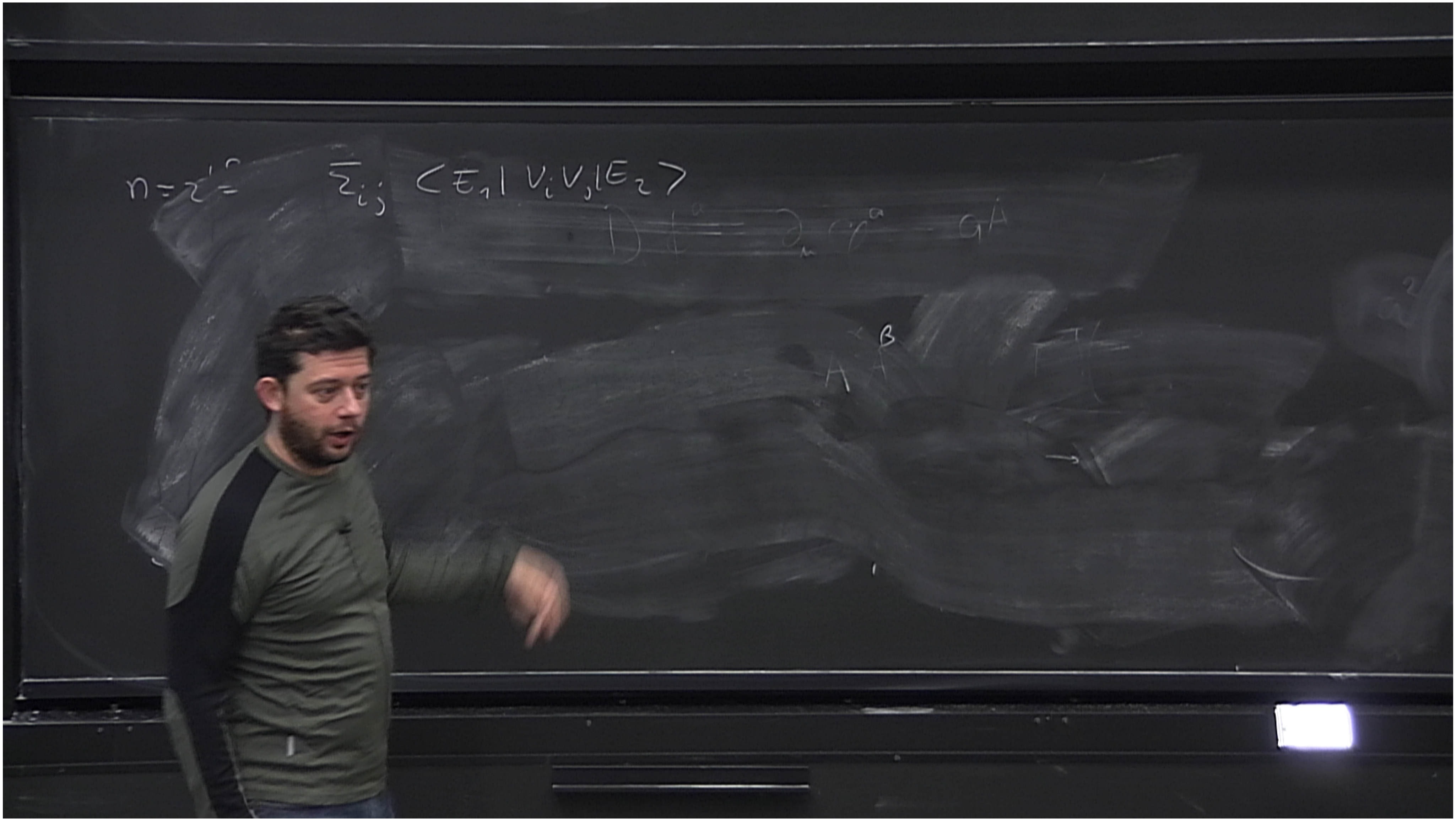
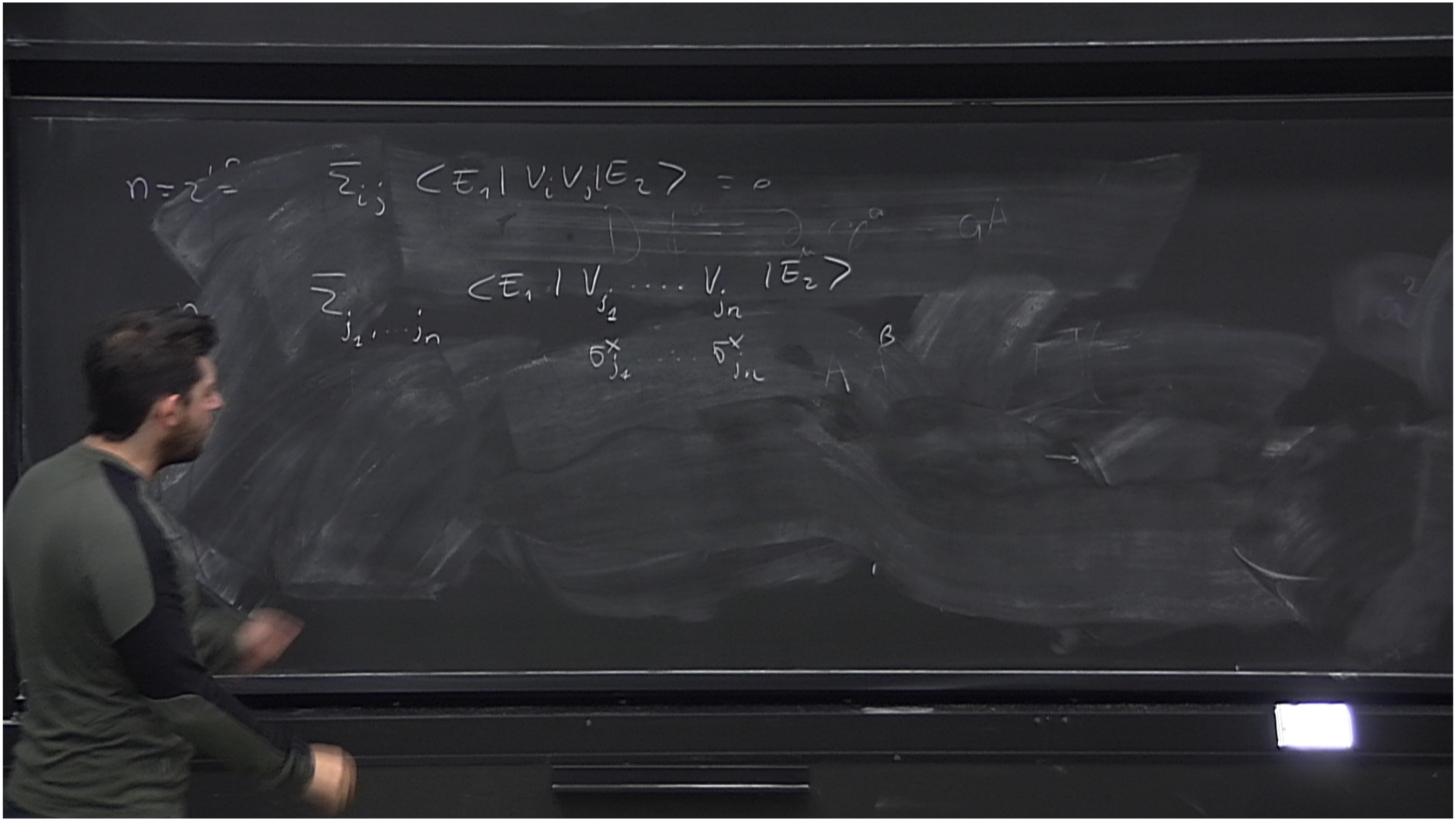
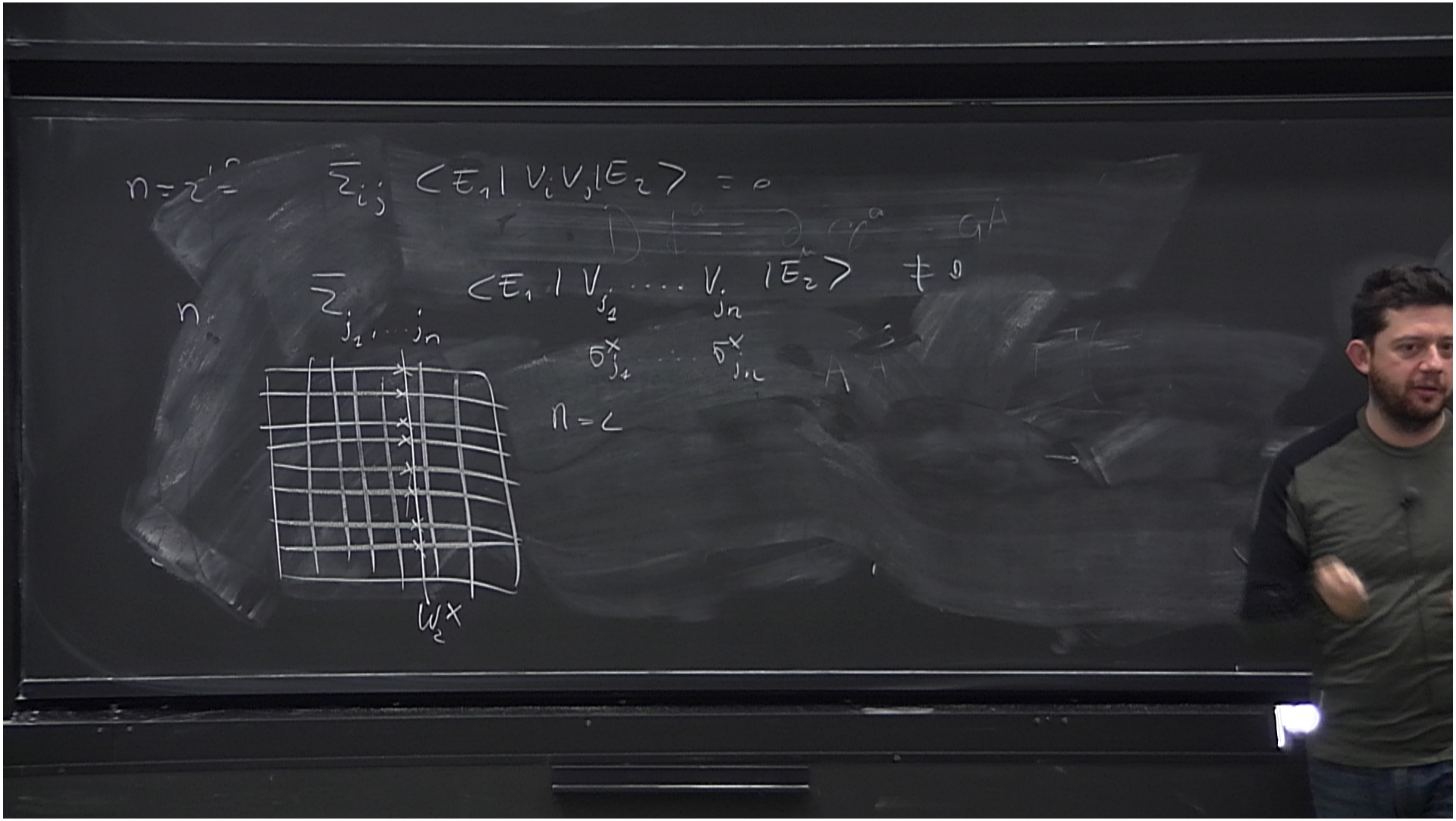
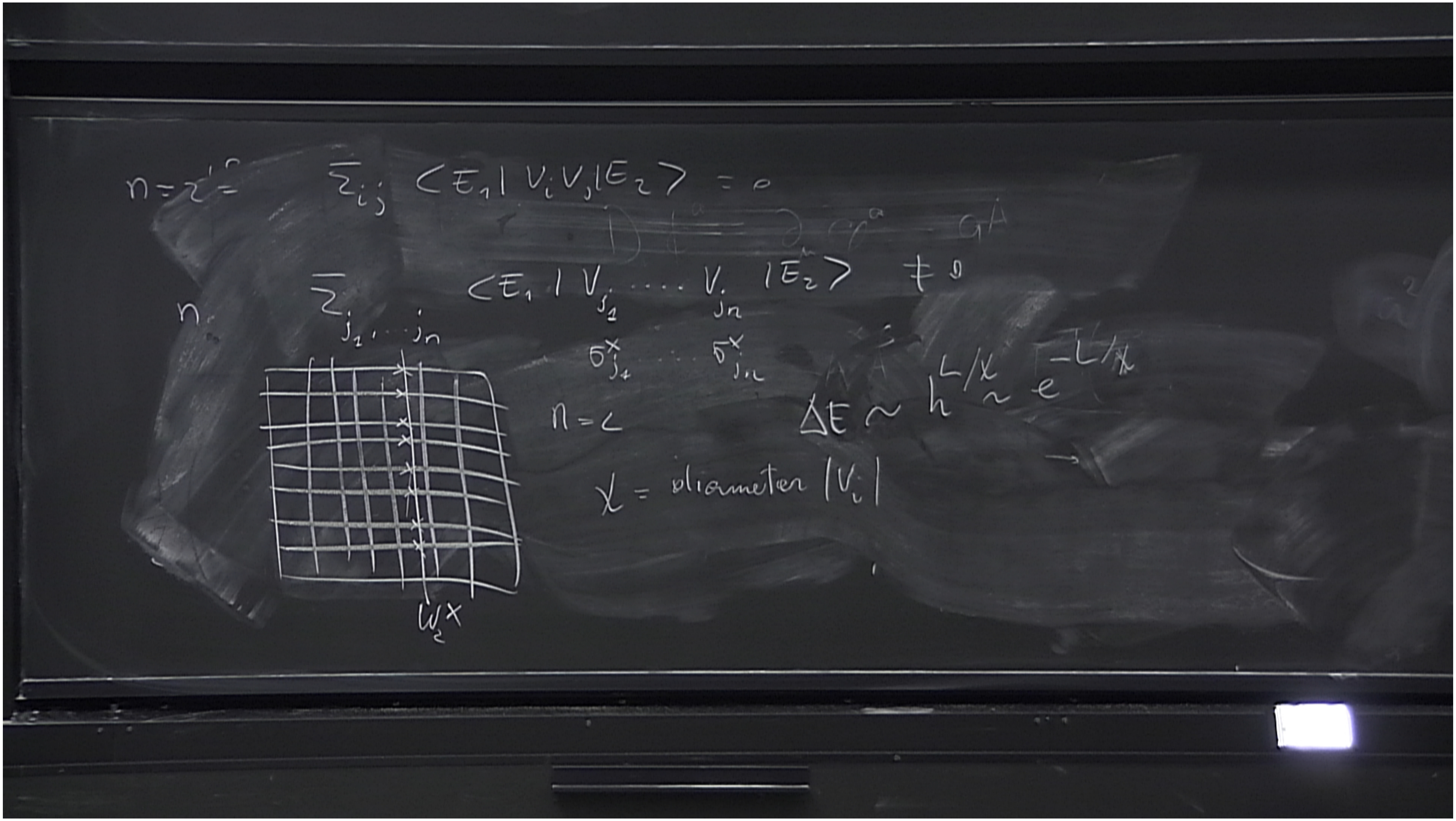
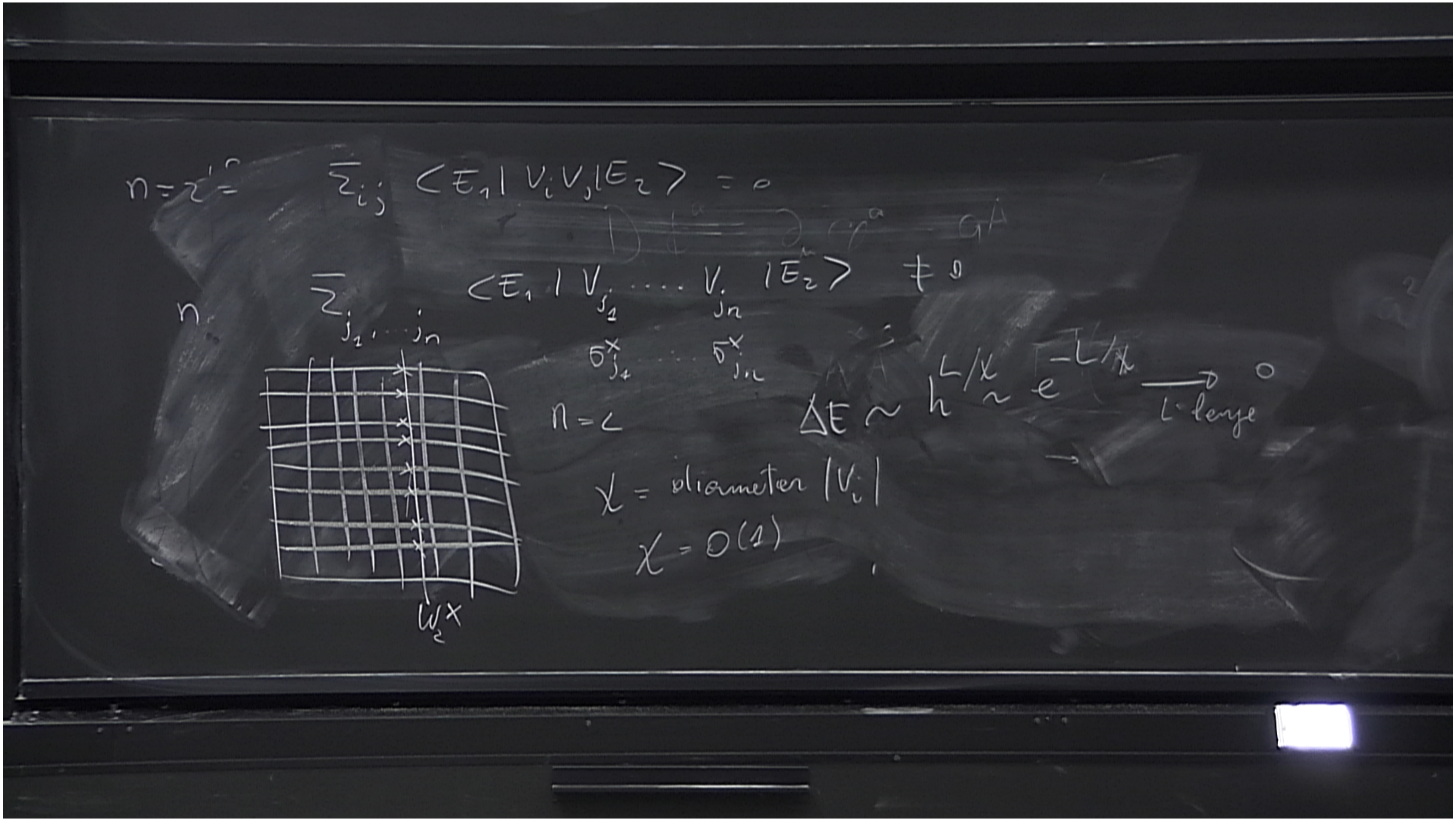
J
E1 E2
ΔE ~ h
order of pert. theory
in which you break the symmetry
<E1| V^n |E2> = Σ_i <E1|V_i|E2>
n=1
= 0
<E1| O |E2> = 0
n=2

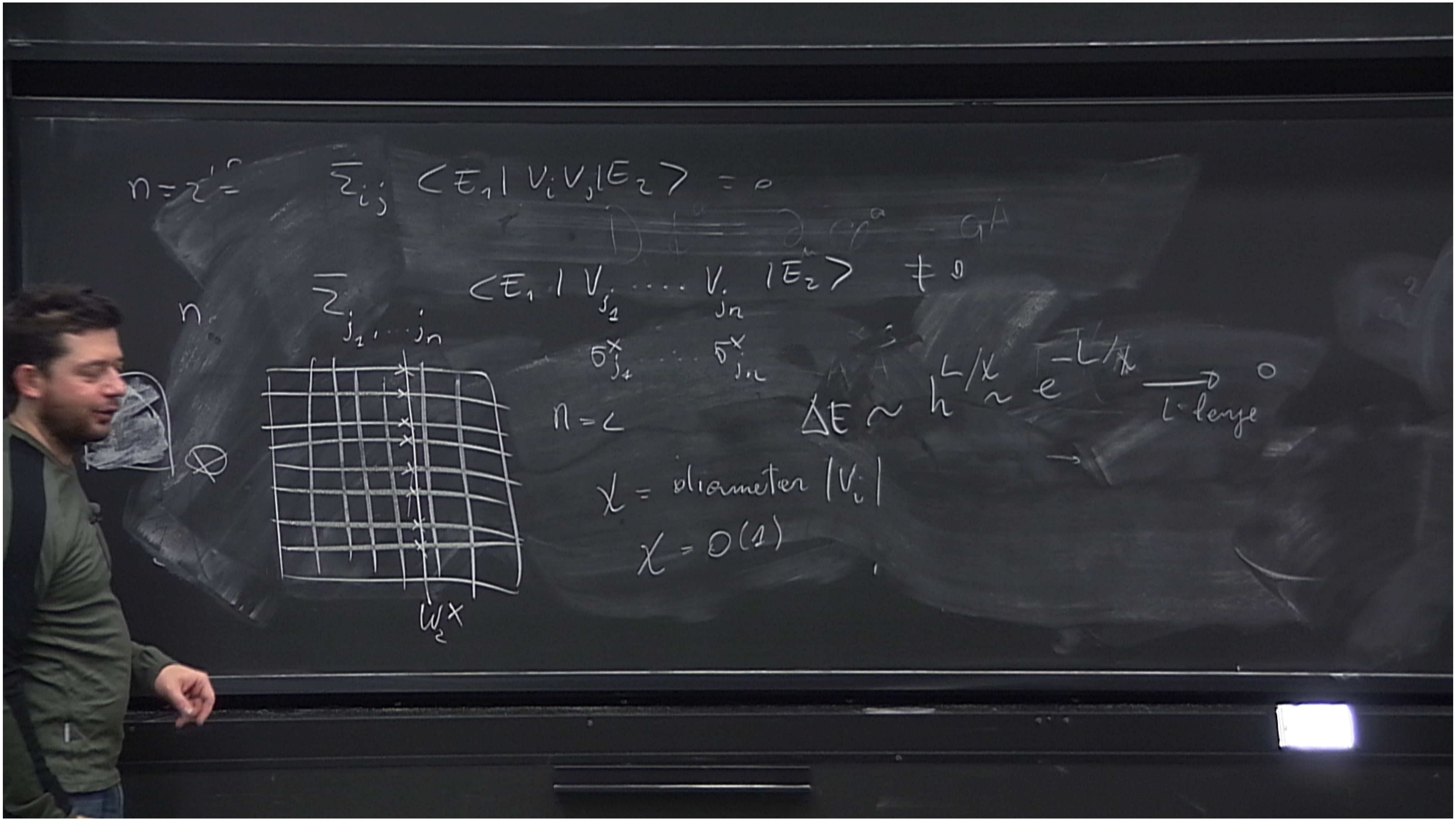

⟨E1| Vi Vj |E2⟩


Σ_{ij} ⟨E_1| V_i V_j |E_2⟩ = 0
Σ_{j_1 ... j_n} ⟨E_1| V_{j_1} .... V_{j_n} |E_2⟩
σ^x_{j_1} ..... σ^x_{j_n}

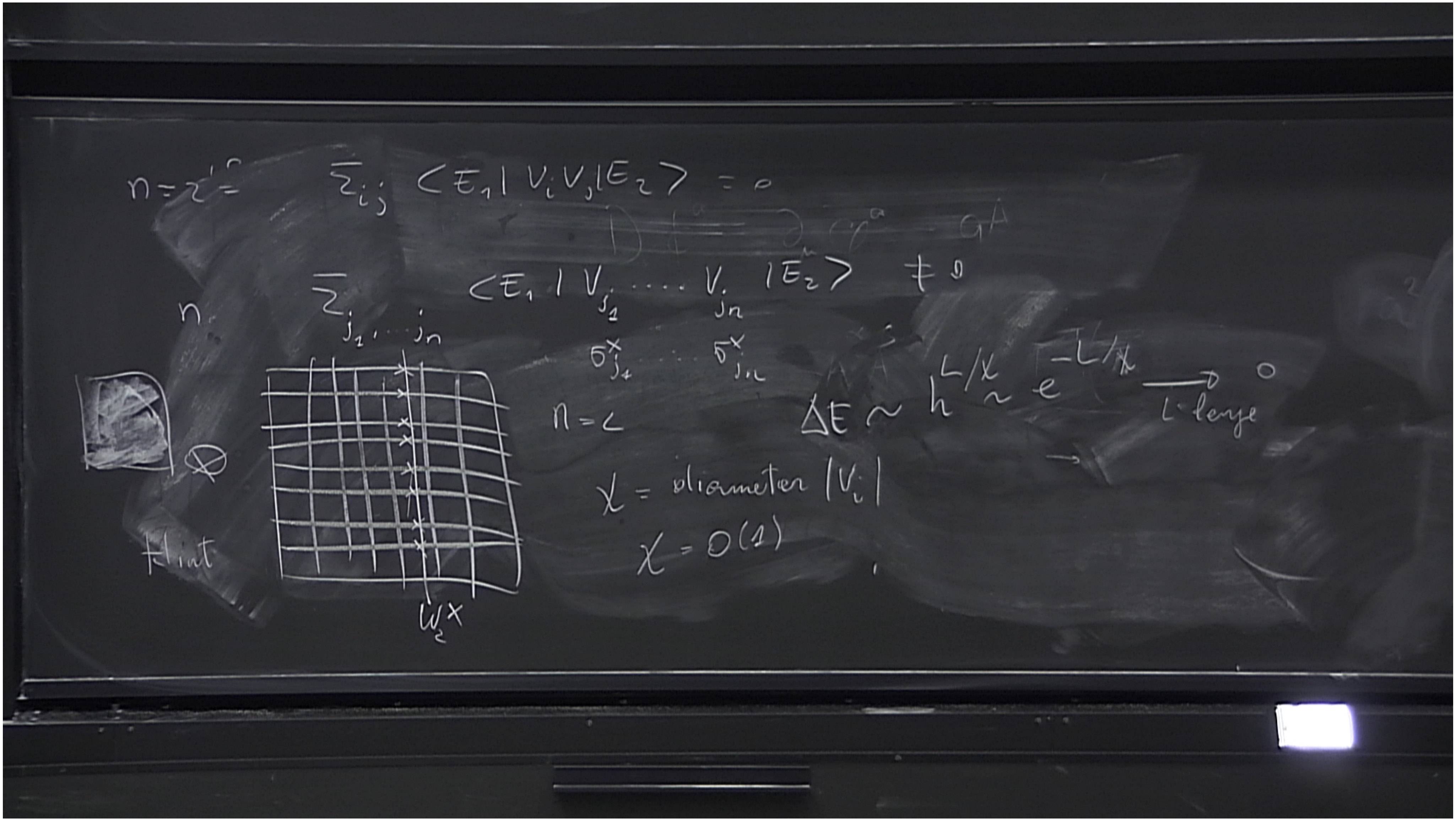


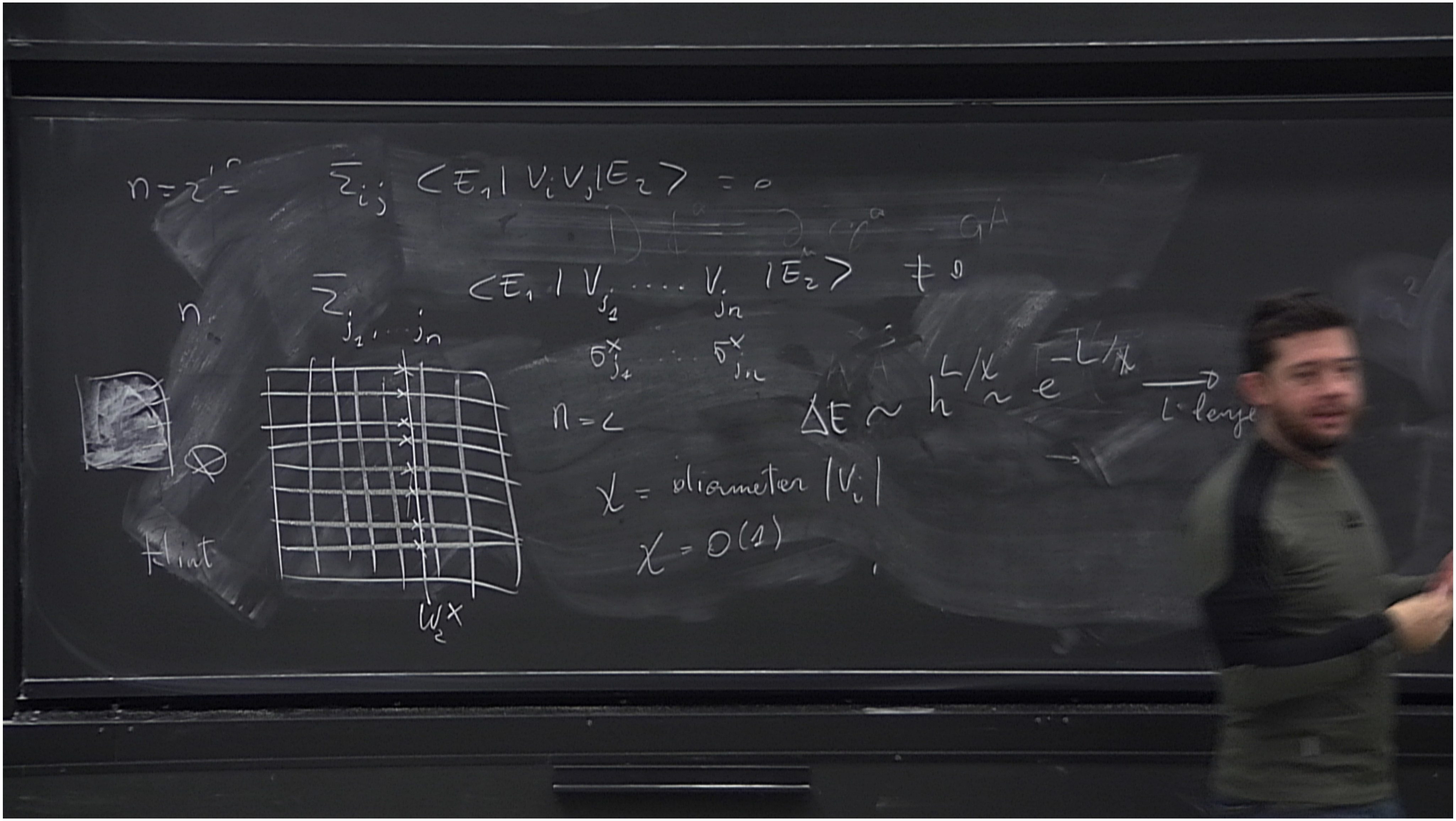

$\sum_{ij} \langle E_1 | V_i V_j | E_2 \rangle = 0$
$\sum_{j_1, \dots, j_n} \langle E_1 | V_{j_1} \dots V_{j_n} | E_2 \rangle \neq 0$
$\sigma^x_{j_1} \dots \sigma^x_{j_n}$
$\Delta E \sim h^{L/\chi} \sim e^{-L/\chi}$
χ = diameter $|V_i|$
W_z^x


Σ_{ij} ⟨E_1| V_i V_j |E_2⟩ = 0
Σ_{j_1,...,j_n} ⟨E_1| V_{j_1} .... V_{j_n} |E_2⟩ ≠ 0
σ^x_{j_1} ..... σ^x_{j_n}
n = L
ΔE ~ h^{L/χ} ~ e^{-L/χ} → 0
L large
χ = diameter |V_i|
χ = O(1)
W_2^x

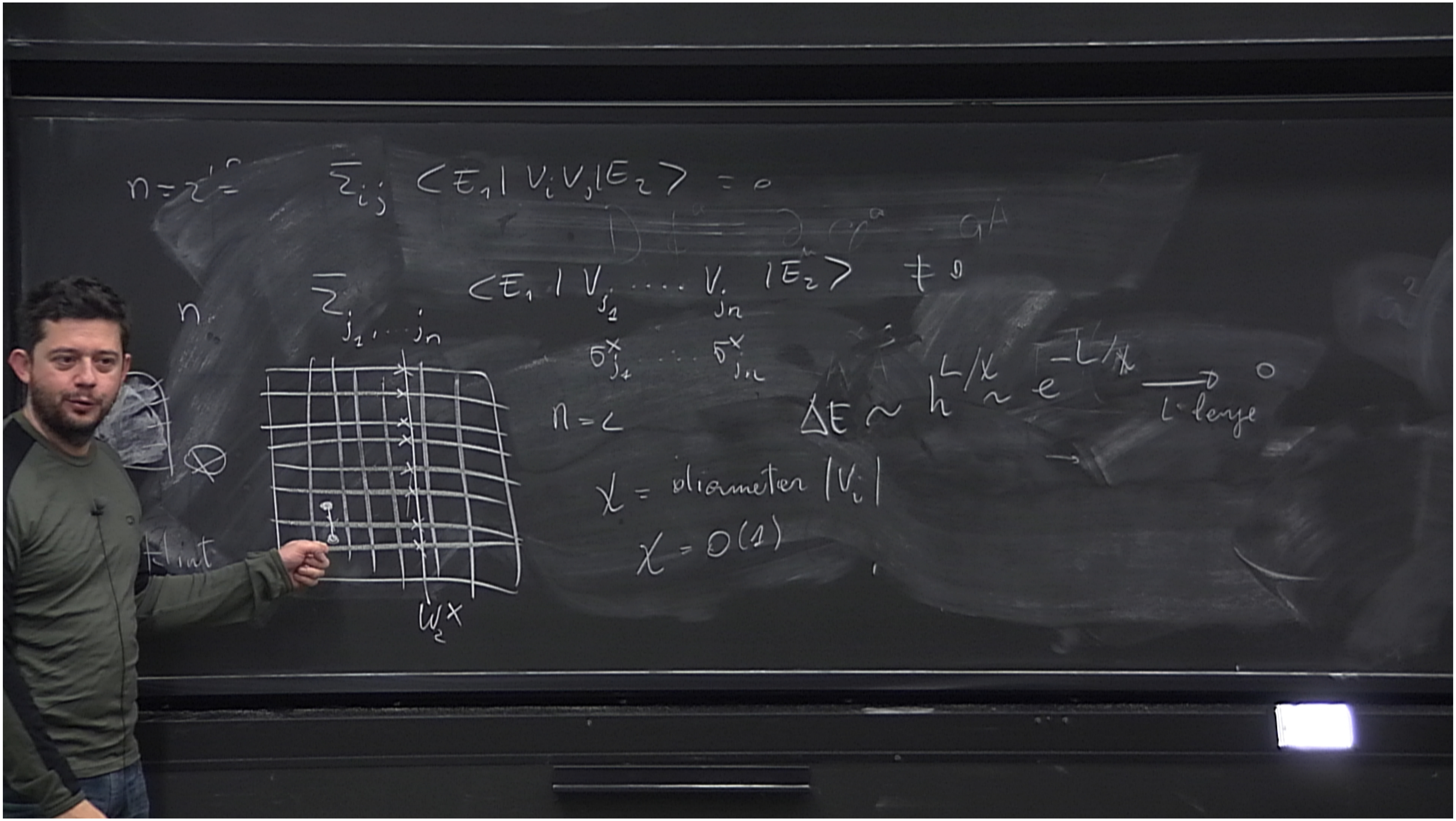


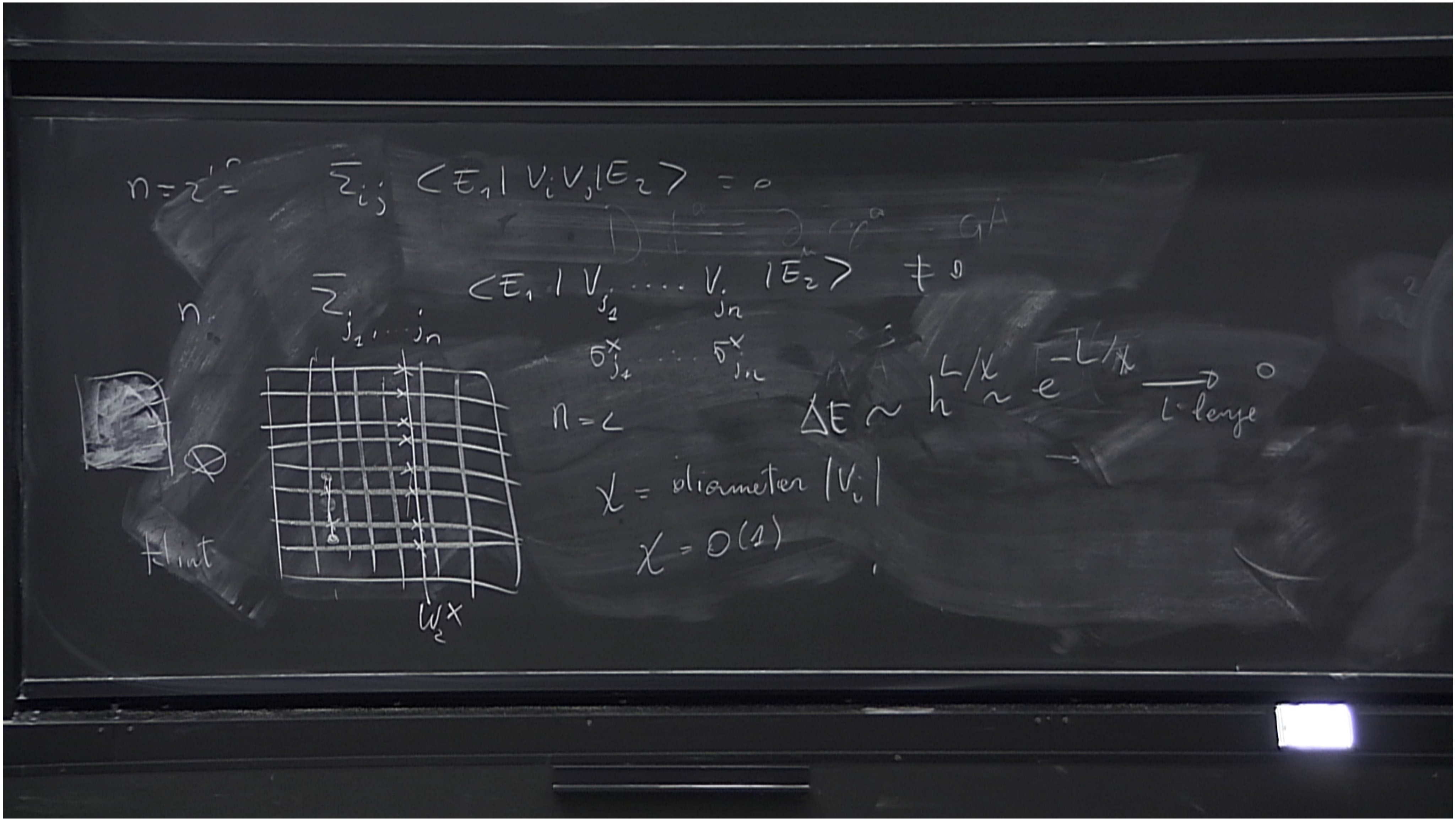
$\sum_{ij} \langle E_1 | V_i V_j | E_2 \rangle = 0$
$\sum_{j_1 \ldots j_n} \langle E_1 | V_{j_1} \ldots V_{j_n} | E_2 \rangle \neq 0$
$\sigma^x_{j_1} \ldots \sigma^x_{j_n}$
$n = L$
$\Delta E \sim h^{L/\chi} \sim e^{-L/\chi} \to 0$
L large
χ = diameter $|V_i|$
$\chi = O(1)$
W_2^x

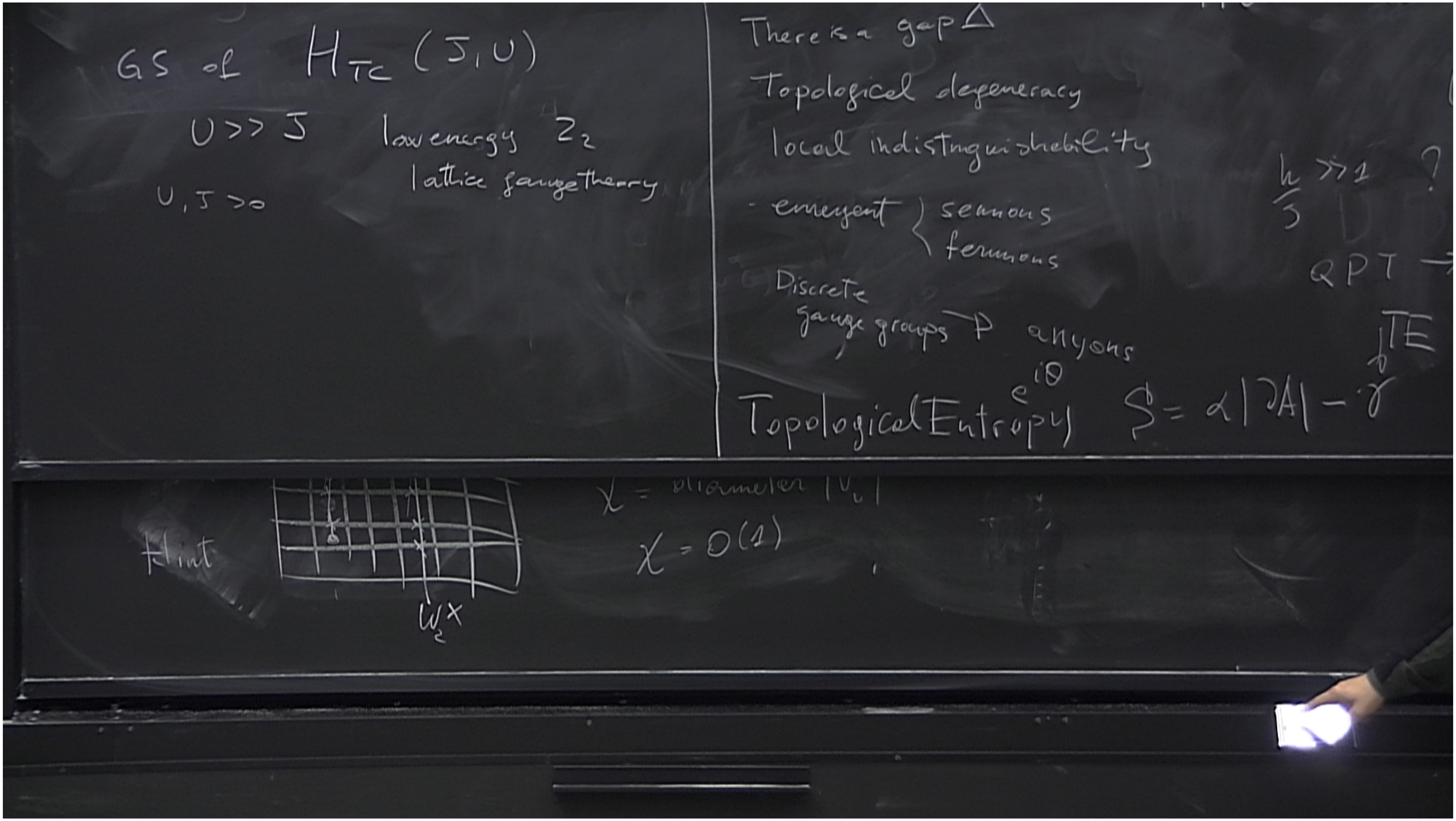

Σ_ij <E_1| V_i V_j |E_2> = 0
Σ_{j_1 … j_n} <E_1| V_{j_1} …. V_{j_n} |E_2> ≠ 0
σ^x_{j_1} …. σ^x_{j_n}
n = L
ΔE ~ h^{L/χ} ~ e^{-L/χ} → 0
L large
χ = diameter |V_i|
χ = O(1)
Hint
W_2^x

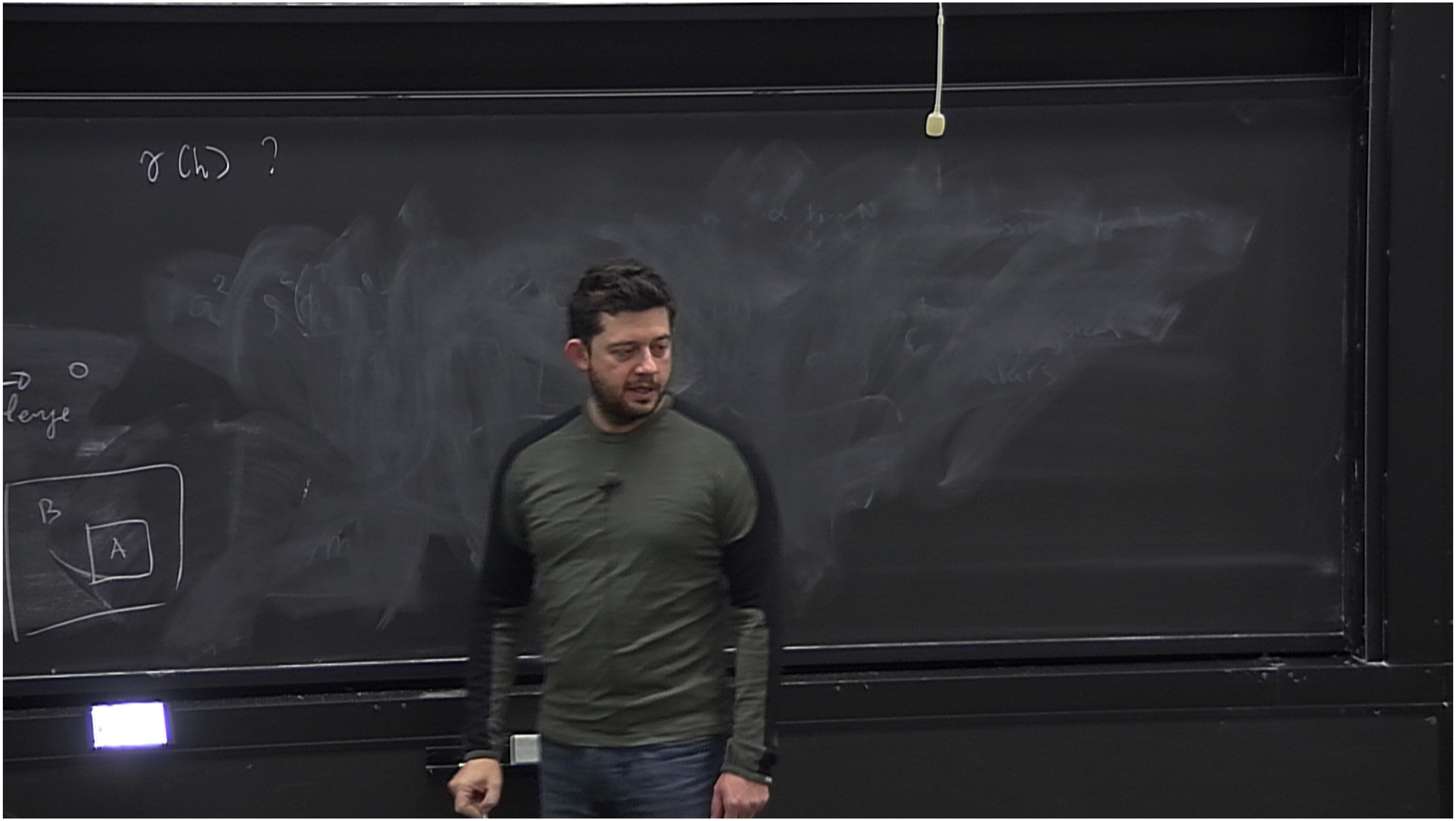


n=2
$\sum_{ij} \langle E_1 | V_i V_j | E_2 \rangle = 0$
n
$\sum_{j_1 \ldots j_n} \langle E_1 | V_{j_1} \ldots V_{j_n} | E_2 \rangle \neq 0$
$\sigma^x_{j_1} \ldots \sigma^x_{j_n}$
n = L
$\Delta E \sim h^{L/\chi} \sim e^{-L/\chi}$
χ = diameter $|V_i|$
$\chi = O(1)$
L · length
W_z^x

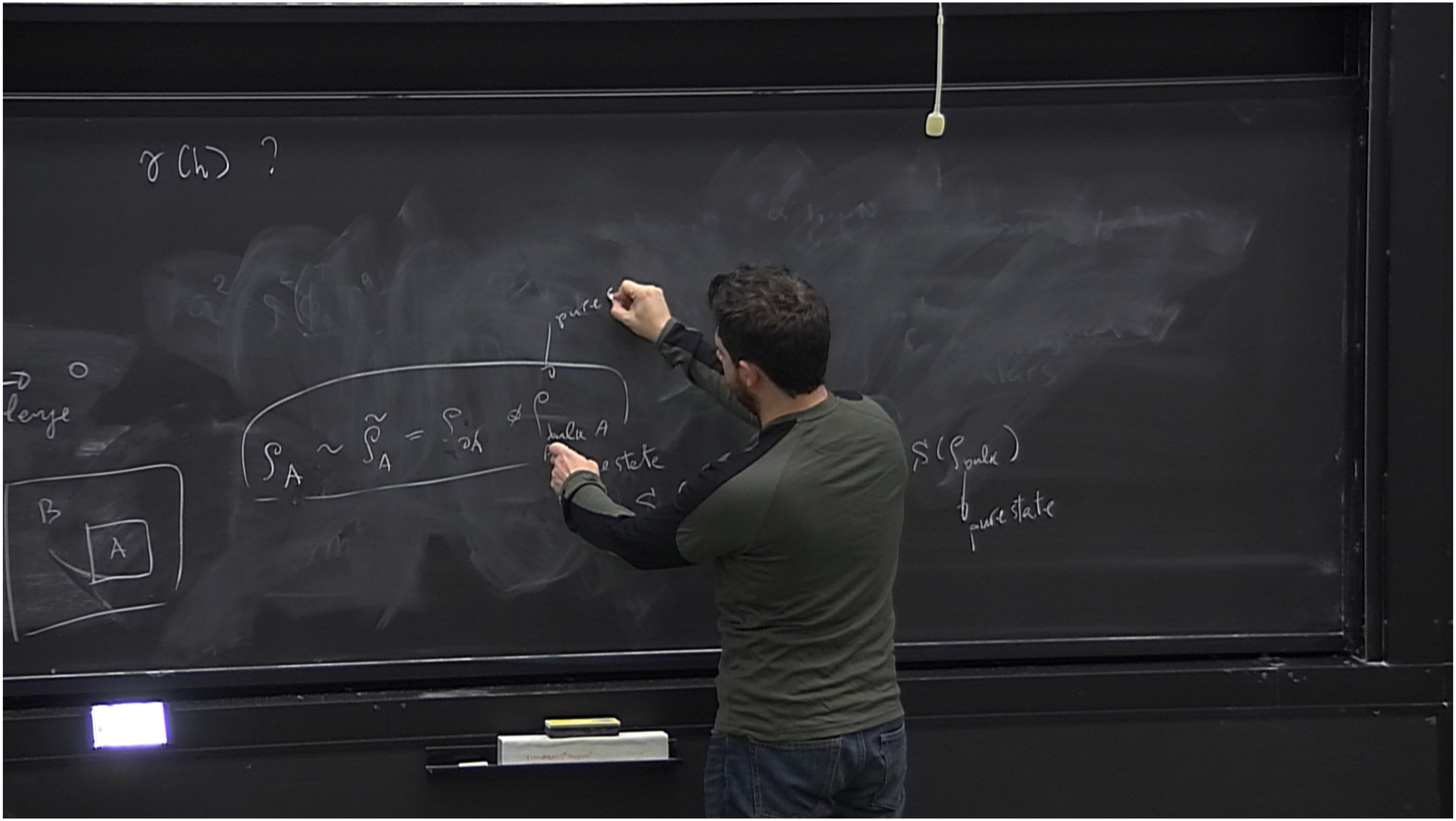

Σ_ij ⟨E_1 | V_i V_j | E_2⟩ = 0
Σ_{j_1 … j_n} ⟨E_1 | V_{j_1} …. V_{j_n} | E_2⟩ ≠ 0
σ^x_{j_1} …. σ^x_{j_n}
n = 2
ΔE ~ h^{L/χ} ~ e^{−L/χ} → 0
L large
χ = diameter |V_i|
χ = O(1)
W_2^x

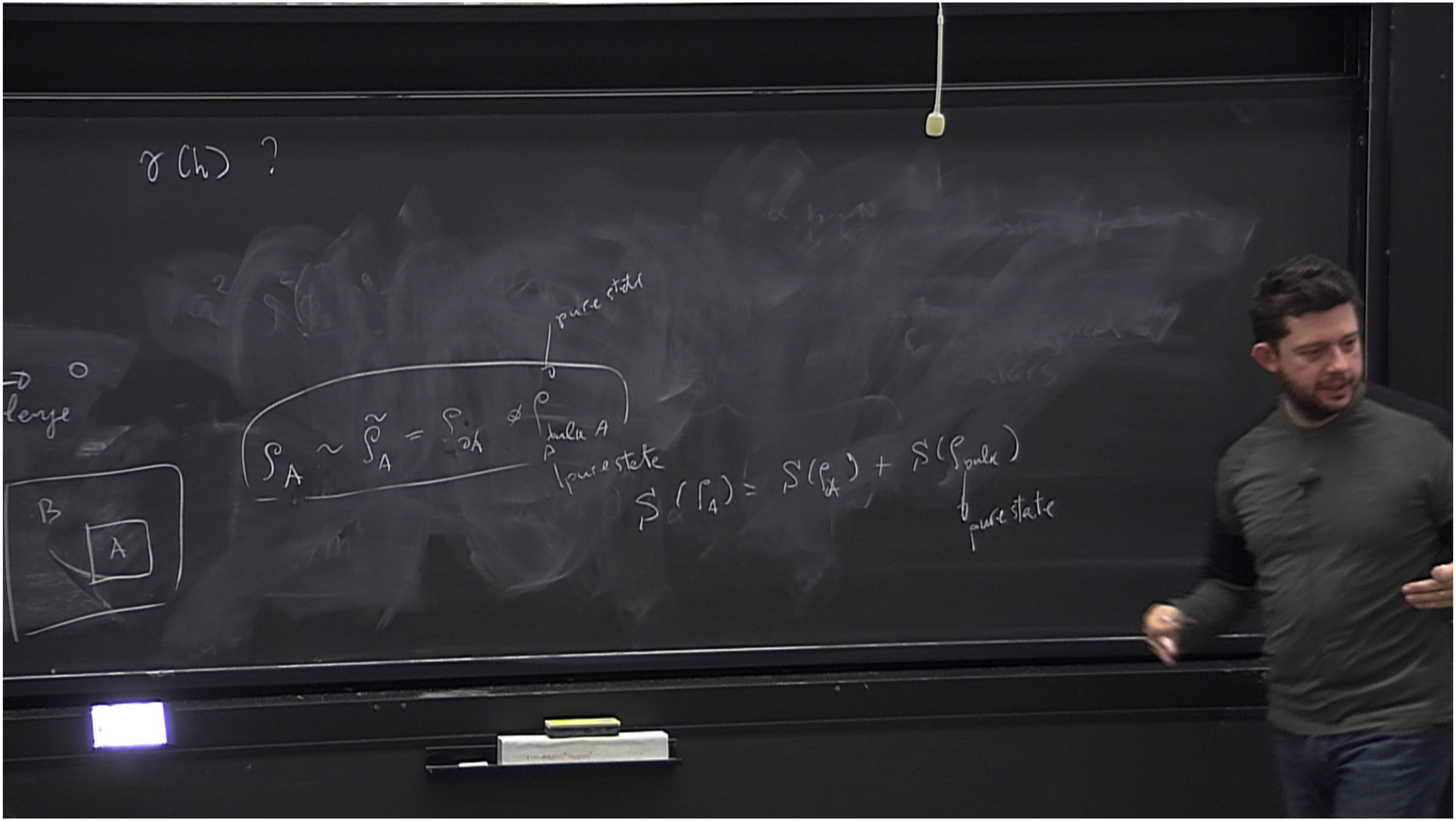

Σij ⟨E1 | Vi Vj | E2⟩ = 0
Σj1,…,jn ⟨E1 | Vj1 …. Vjn | E2⟩ ≠ 0
σxj1 …. σxjn
n = L
ΔE ~ h^(L/χ) ~ e^(−L/χ) → 0
L large
χ = diameter |Vi|
χ = O(1)
Hint
W2x

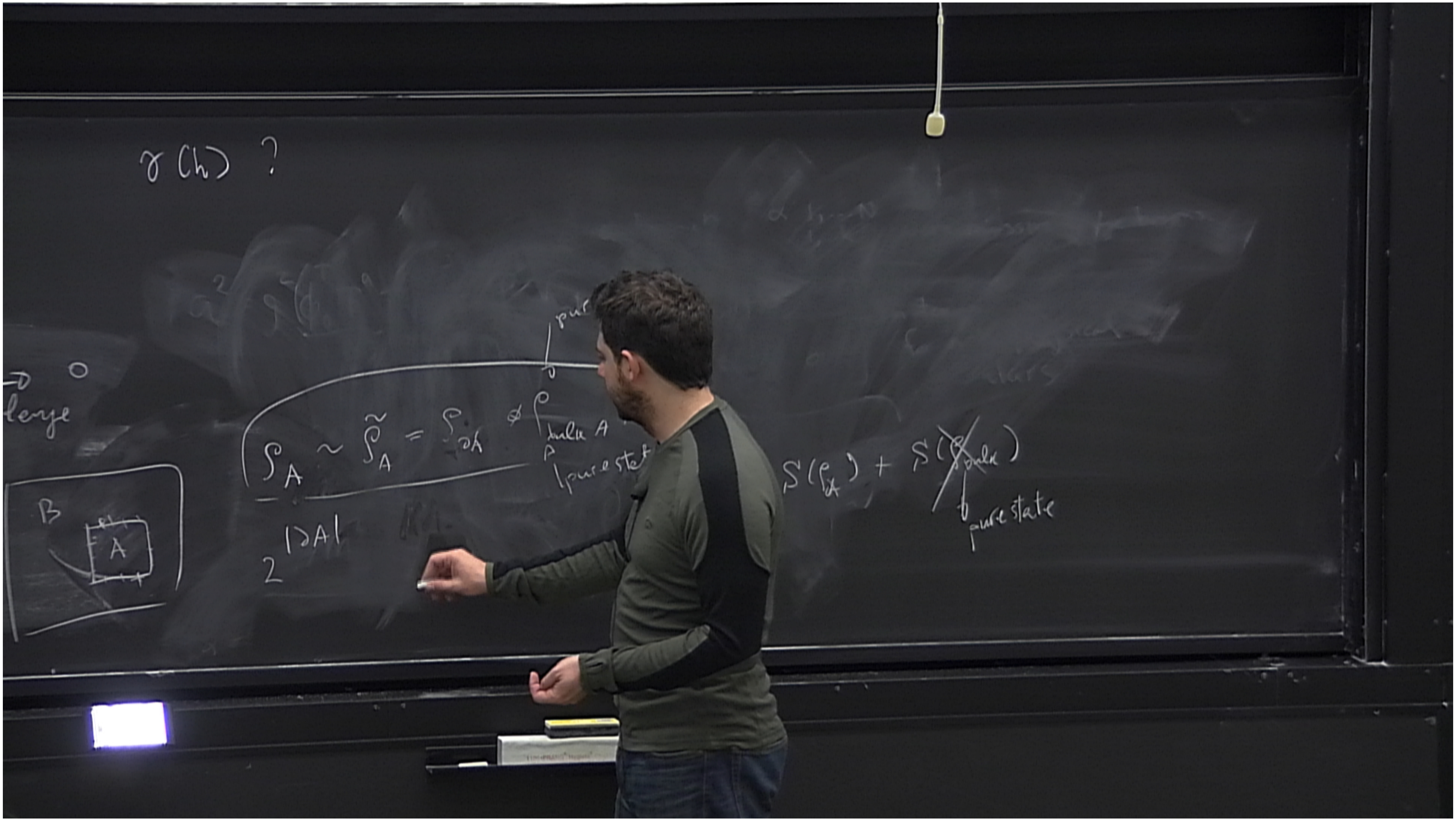

GS of $H_{TC}(J, U)$
$U >> J$ low energy Z_2 lattice gauge theory
$U, J > 0$
There is a gap Δ
Topological degeneracy
local indistinguishability
emergent { semions fermions
Discrete gauge groups → anyons $e^{i\theta}$
Topological Entropy $S = \alpha|\partial A| - \gamma$
$\chi = O(1)$
QPT
TE

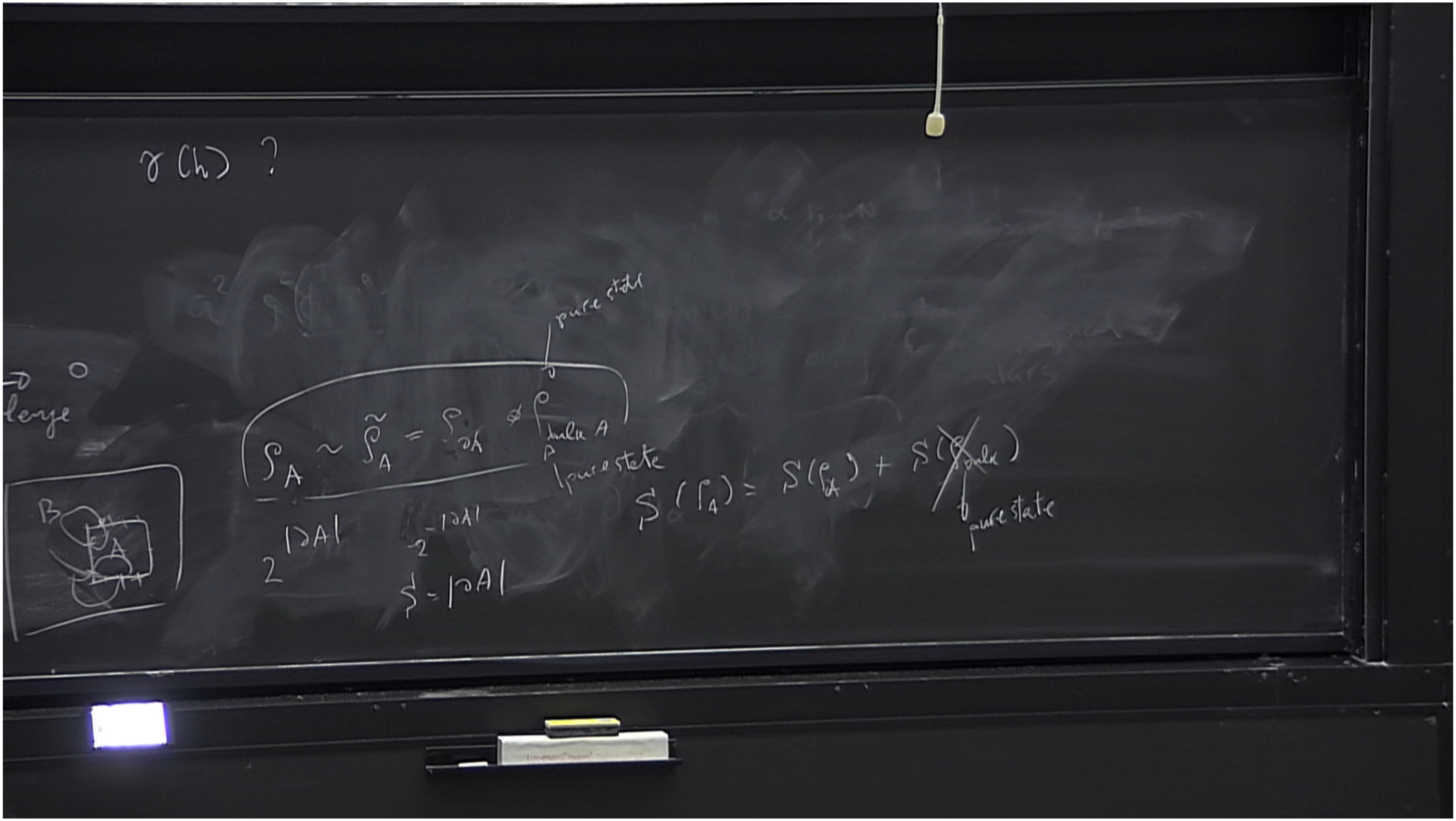


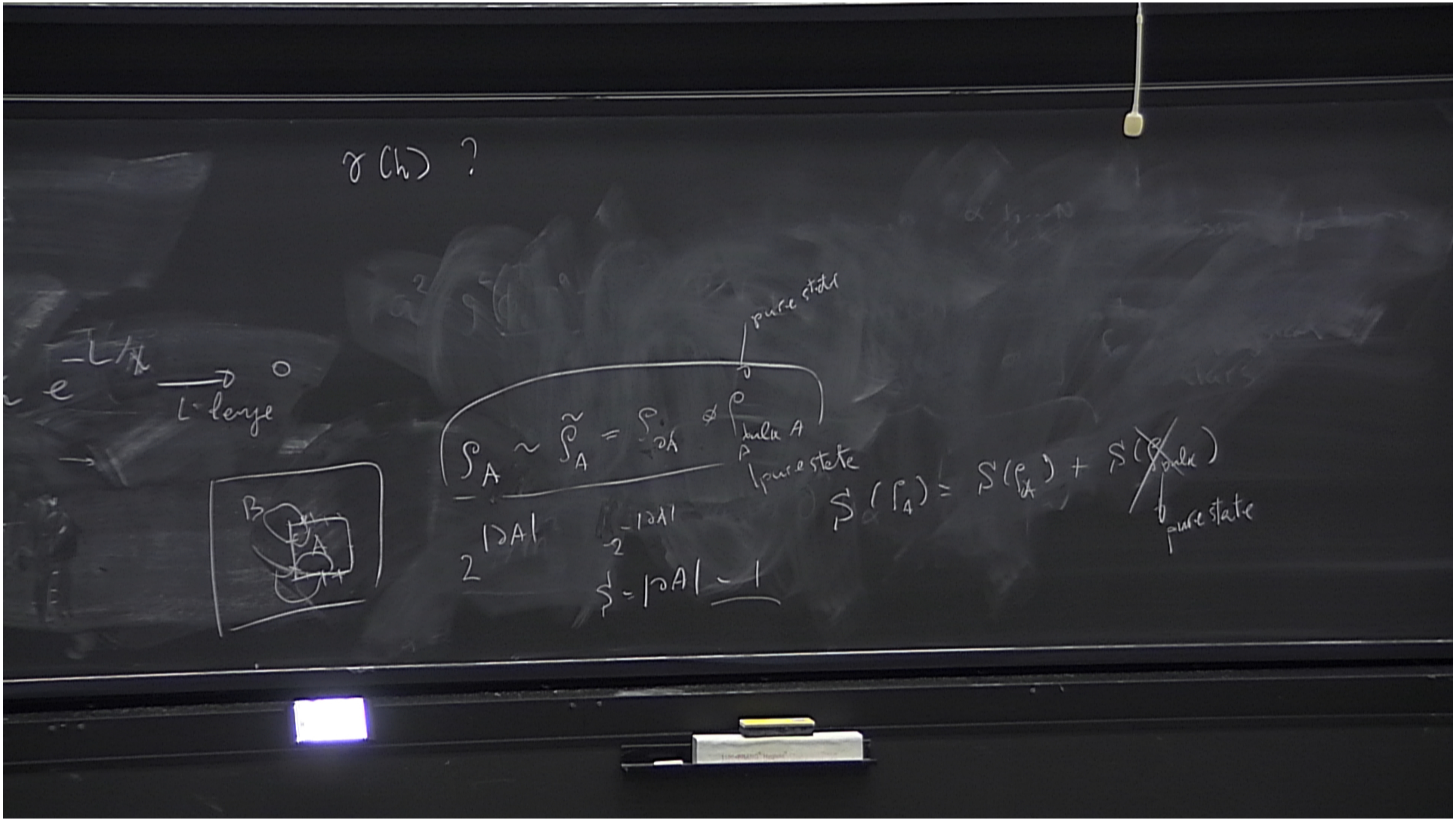
γ(h) ?
B
A

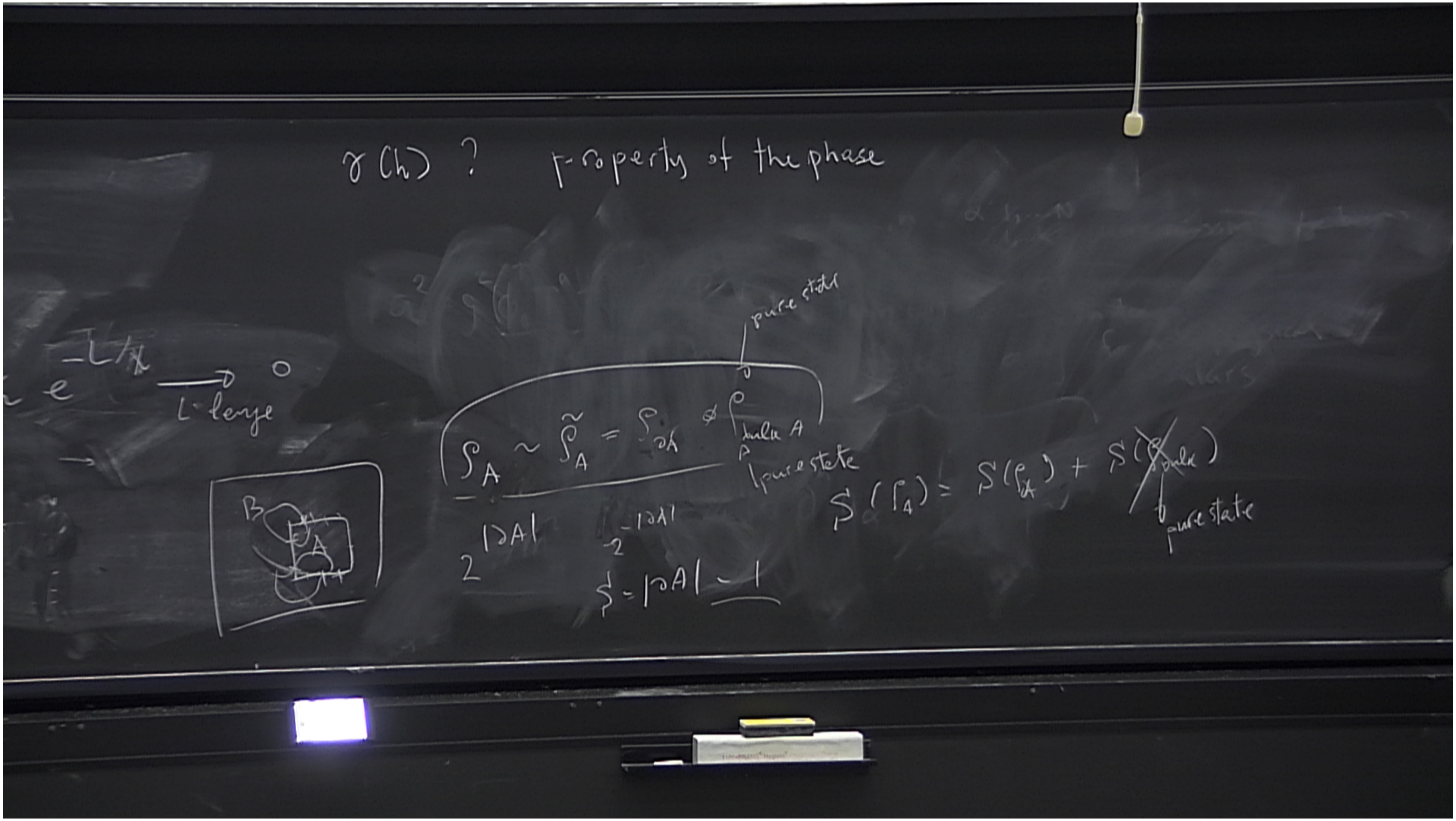

γ(h) ?
pure state
S(ρ_bulk)
pure state
B
A

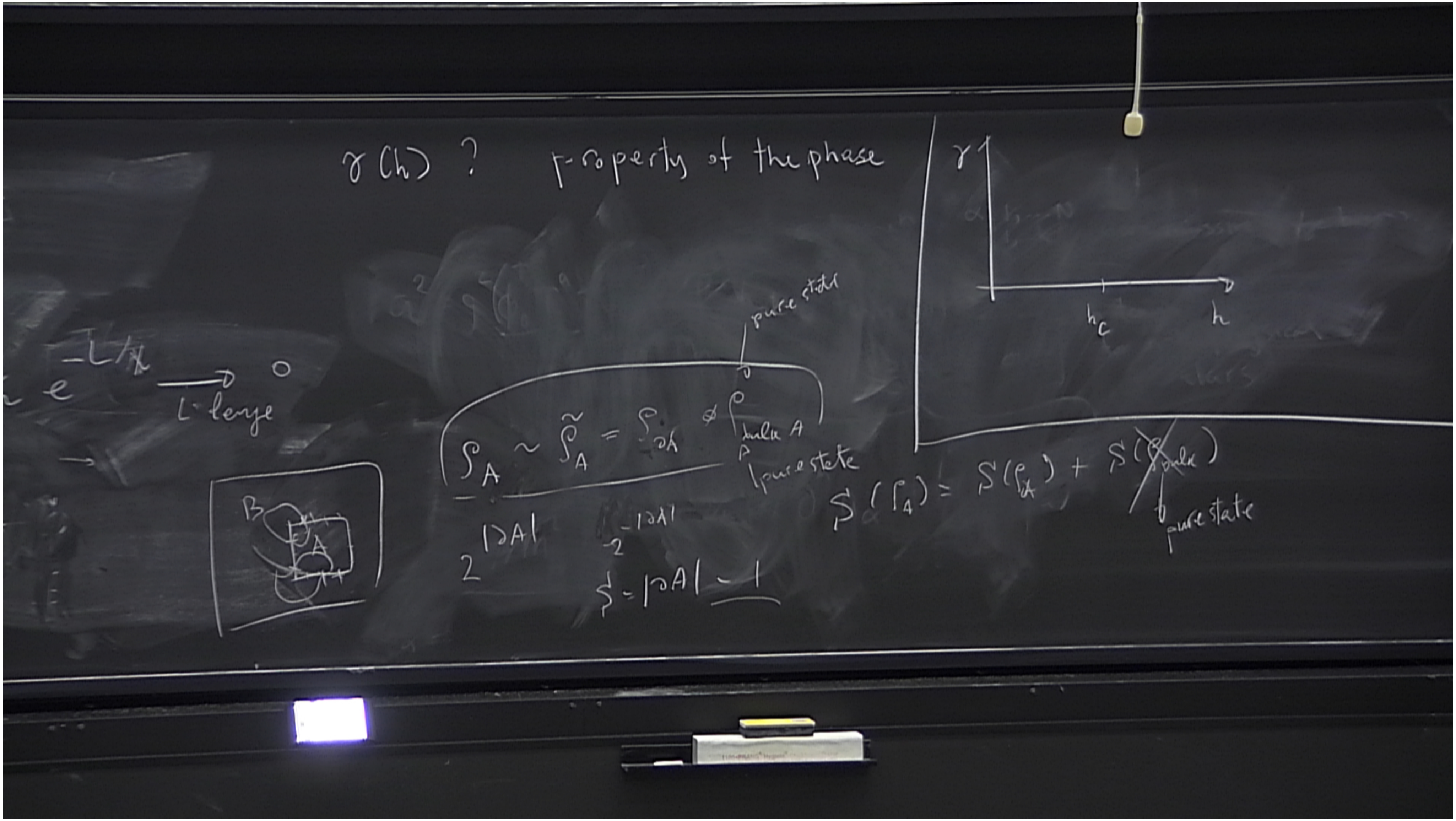

pure state
pure state
pure state
B
A

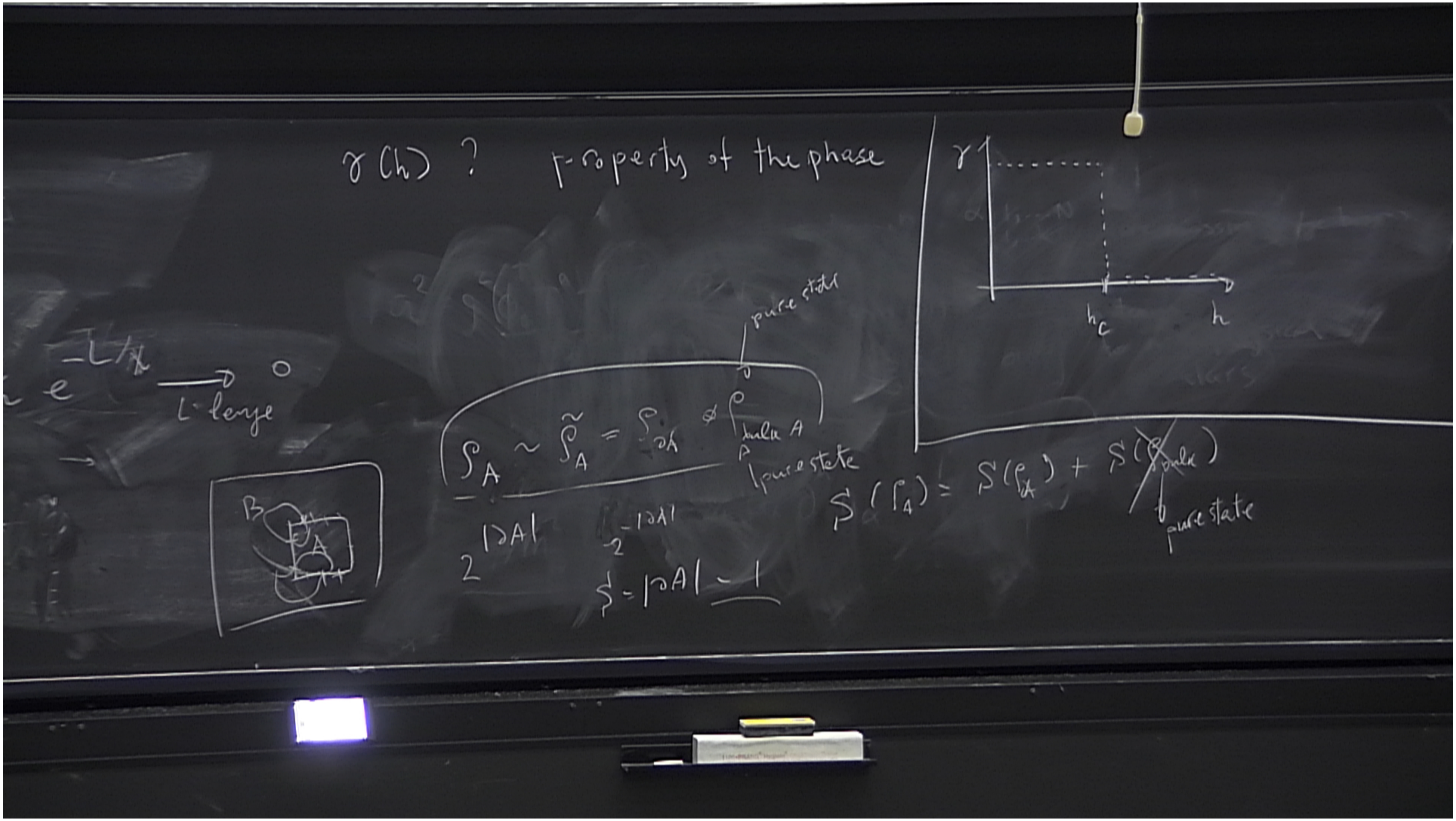

γ(h) ?
pure state
pure state

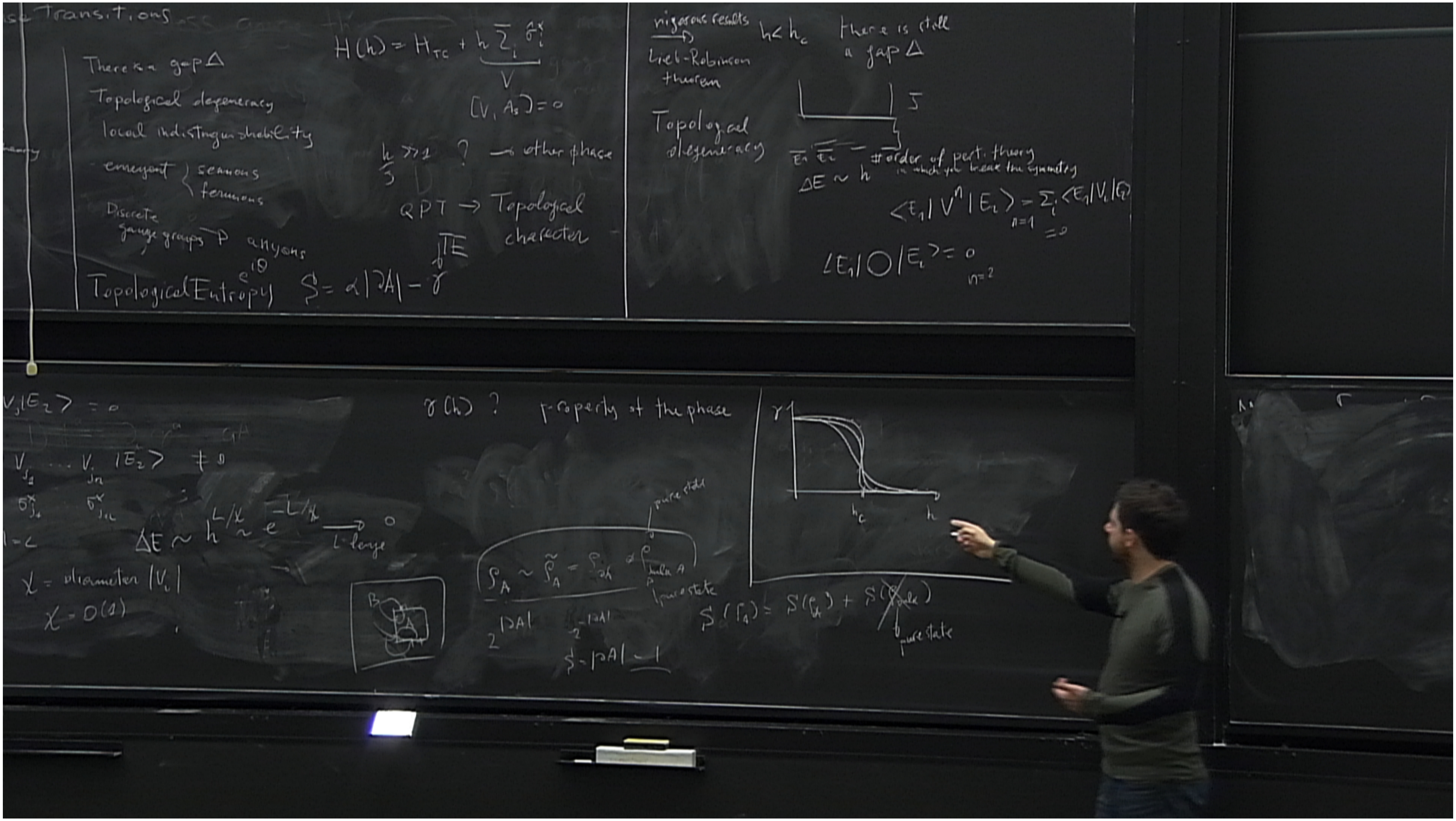

γ(h) ?
pure state
ρ_A ~ ρ̃_A = ρ_∂A ⊗ ρ_bulk A
pure state
S(ρ_A) = S(ρ_∂A) + S(ρ_bulk)
pure state
2^|∂A|
S = |∂A|


γ(h) ?
pure state
L large
bulk A
pure state
pure state


γ(h) ? property of the phase
pure state
L large
bulk A
pure state
pure state


γ(h) ? property of the phase
pure state
L large
pure state
pure state
h_c
h


γ(h) ?
property of the phase
pure state
L large
pure state
pure state
h_c
h
B
A


Transitions
There is a gap Δ
Topological degeneracy
local indistinguishability
emergent { bosons fermions
Discrete gauge groups anyons
Topological Entropy
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[V, A_s] = 0
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rigorous results
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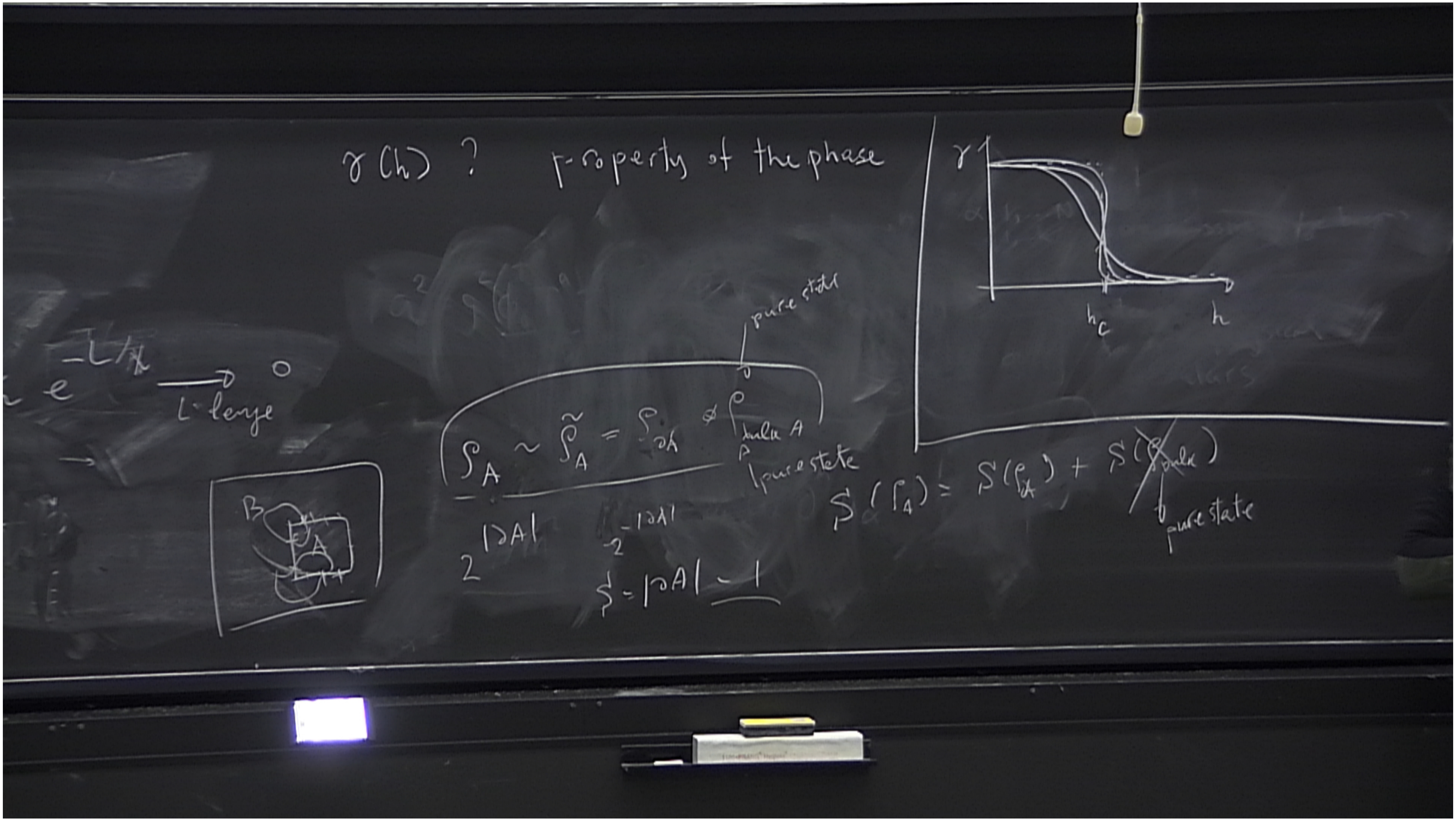
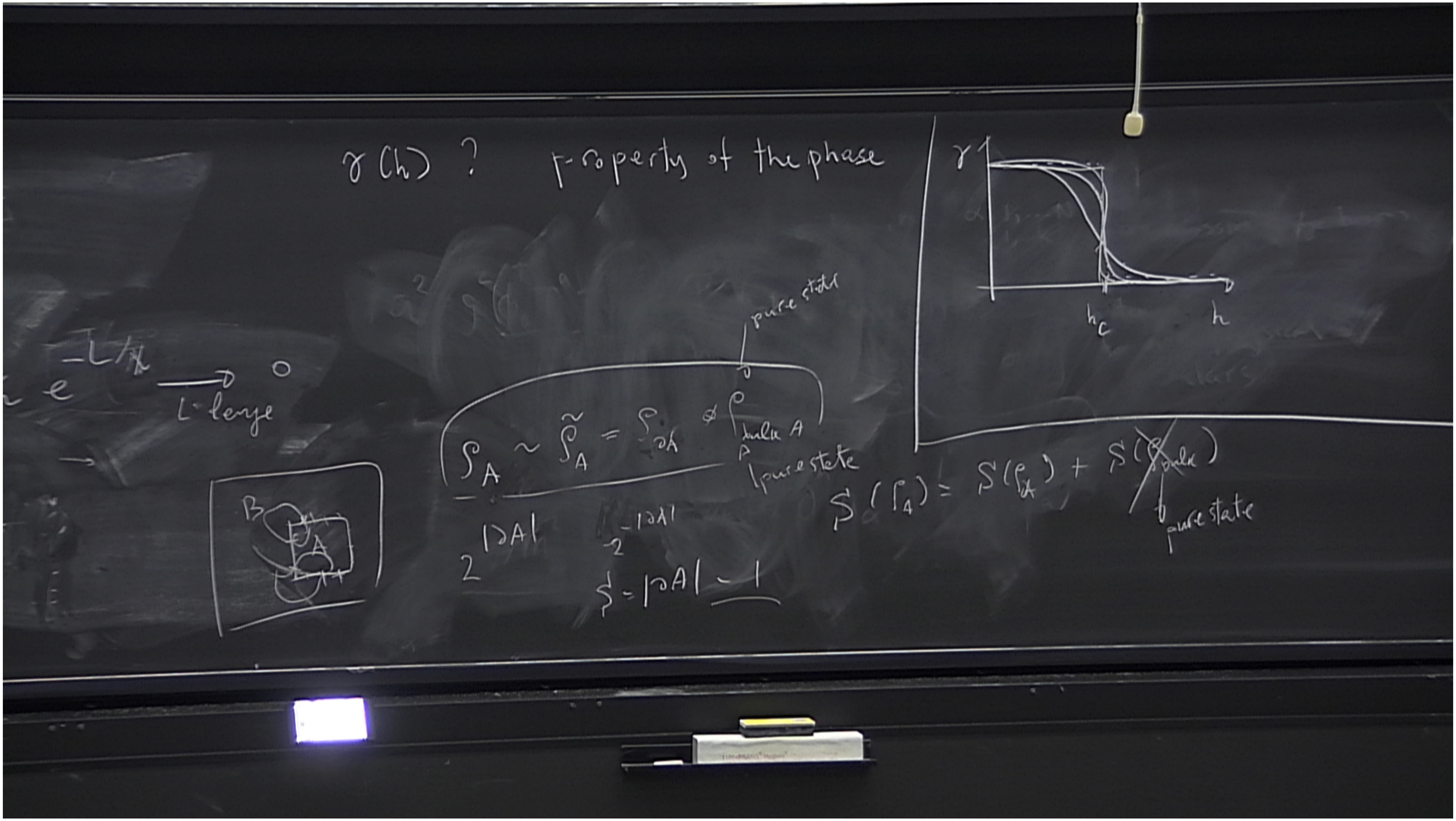
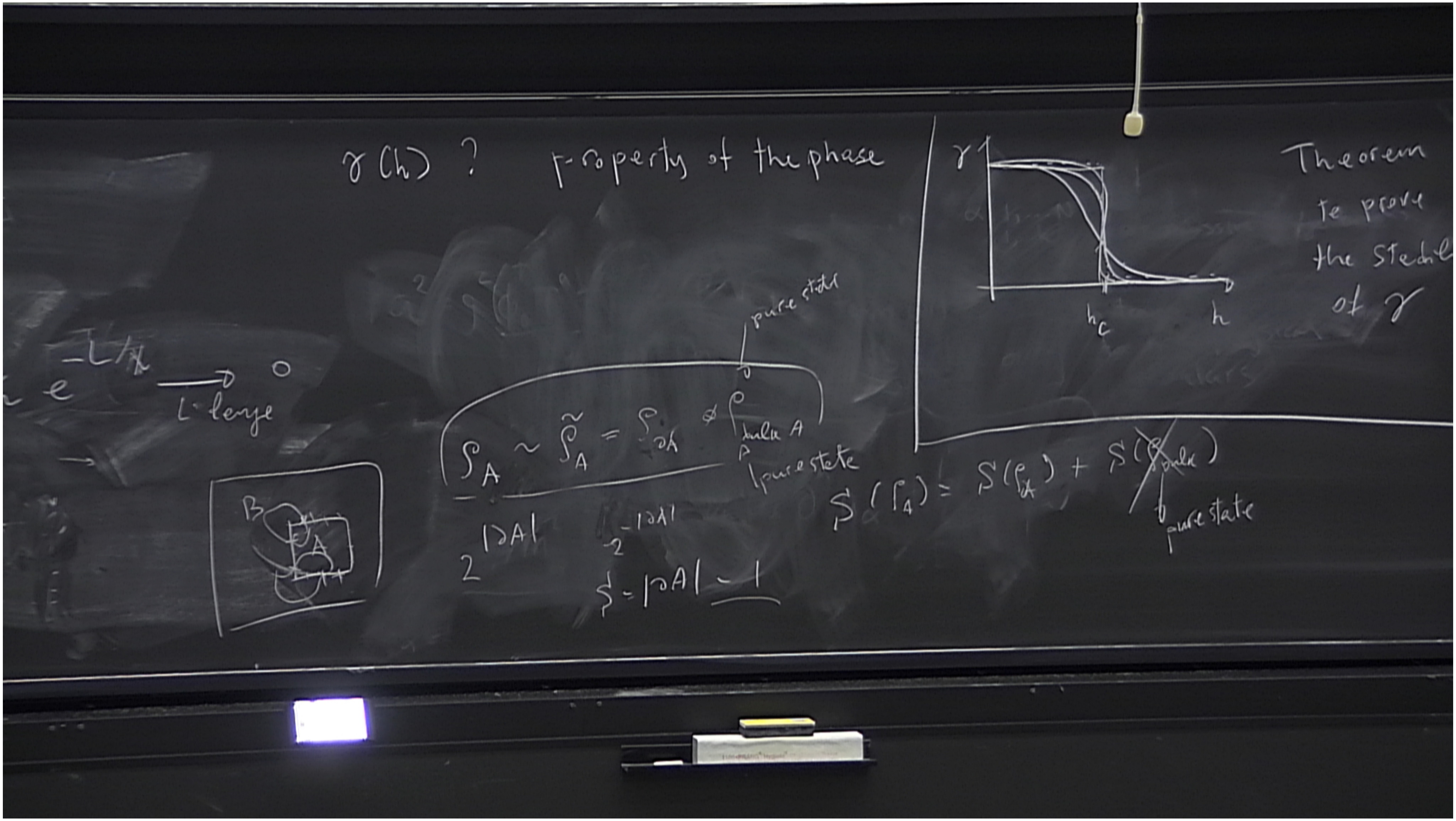
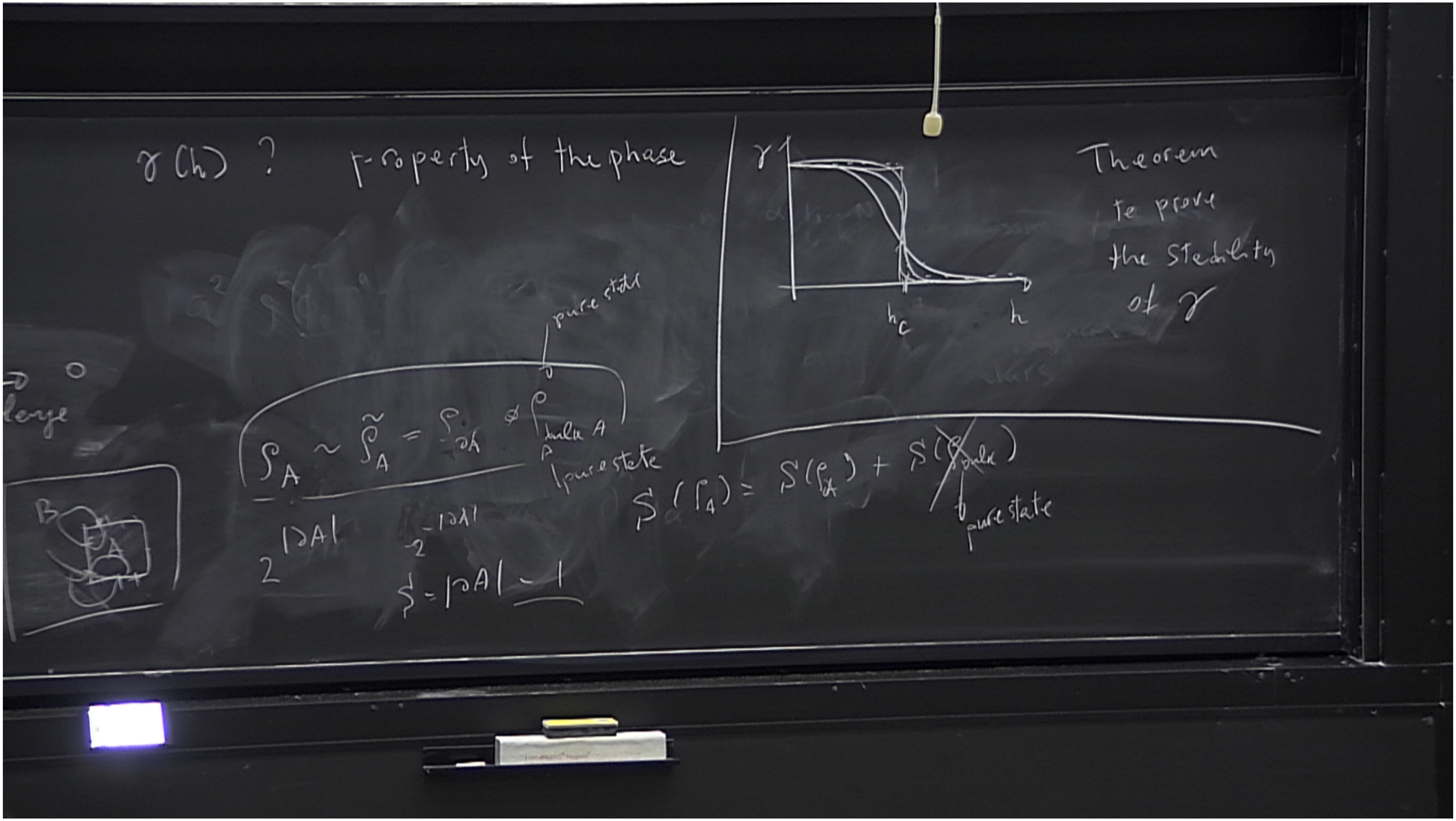
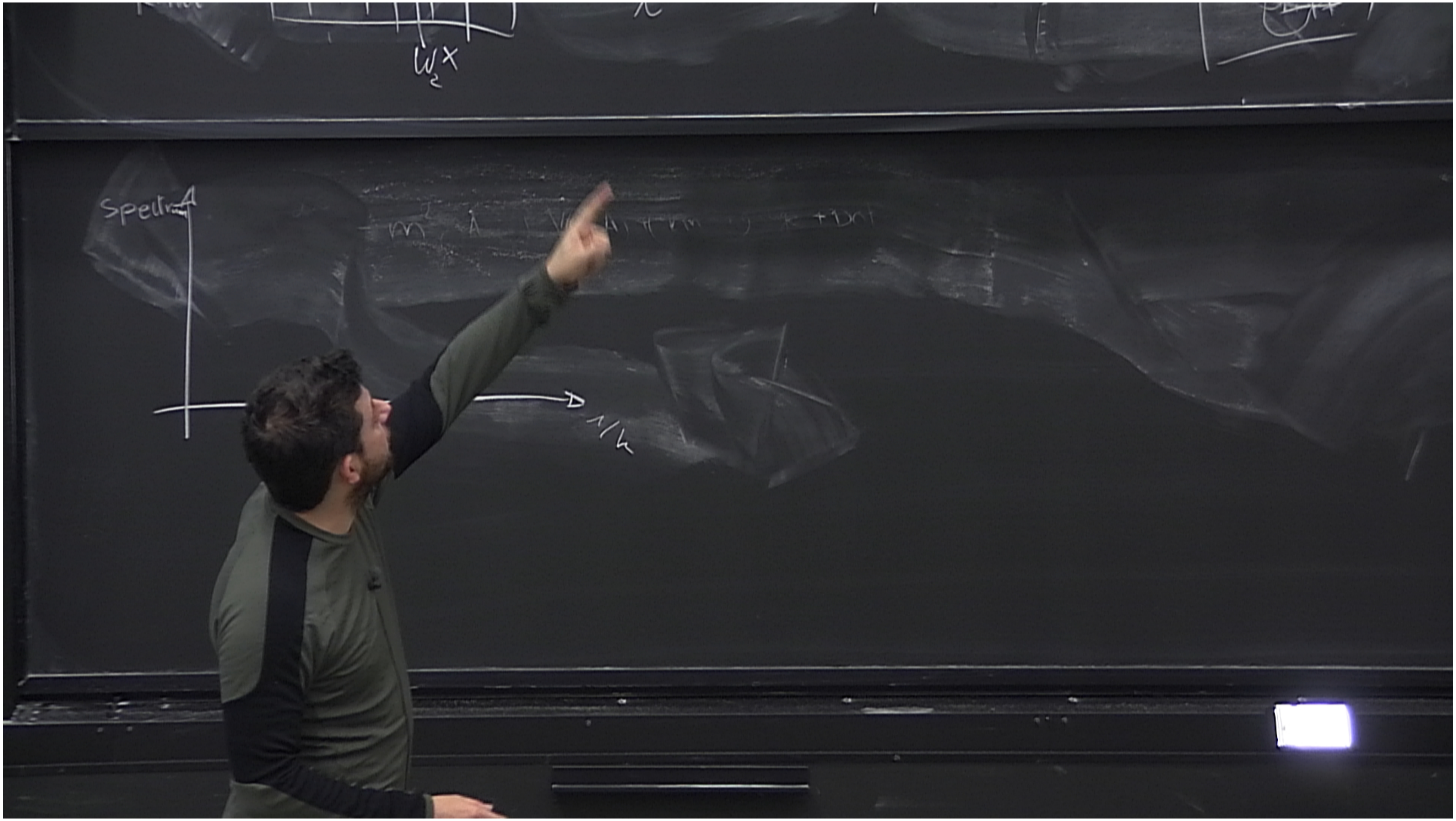
There is still a gap Δ
Topological degeneracy
ΔE ~ h^{# order of pert. theory}
γ(h) ?
property of the phase
pure state
L-large
χ = diameter |V_i|
χ = O(1)
h_c
h

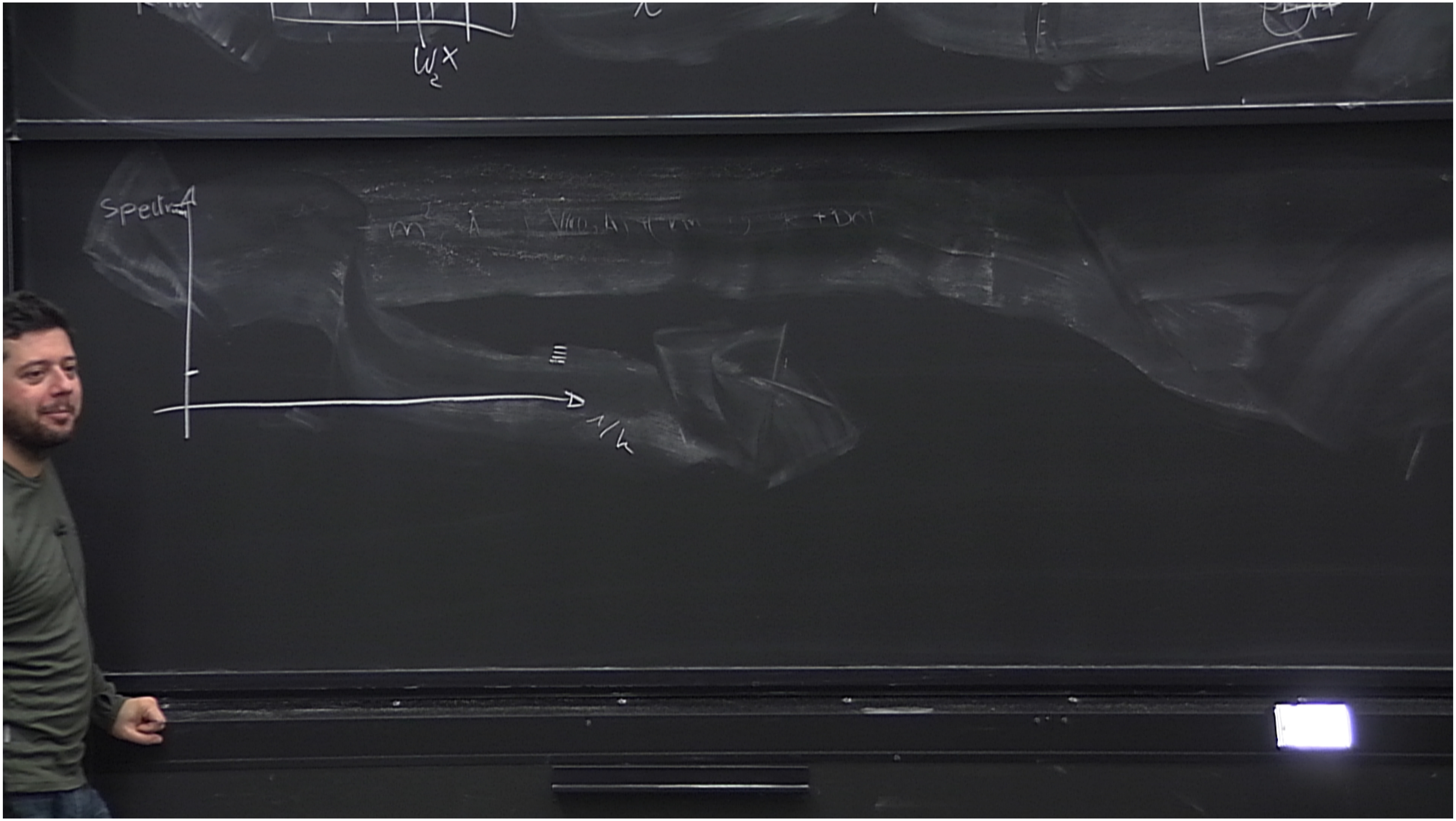

γ(h) ?
property of the phase
L large
pure state
bulk A
pure state
pure state
h

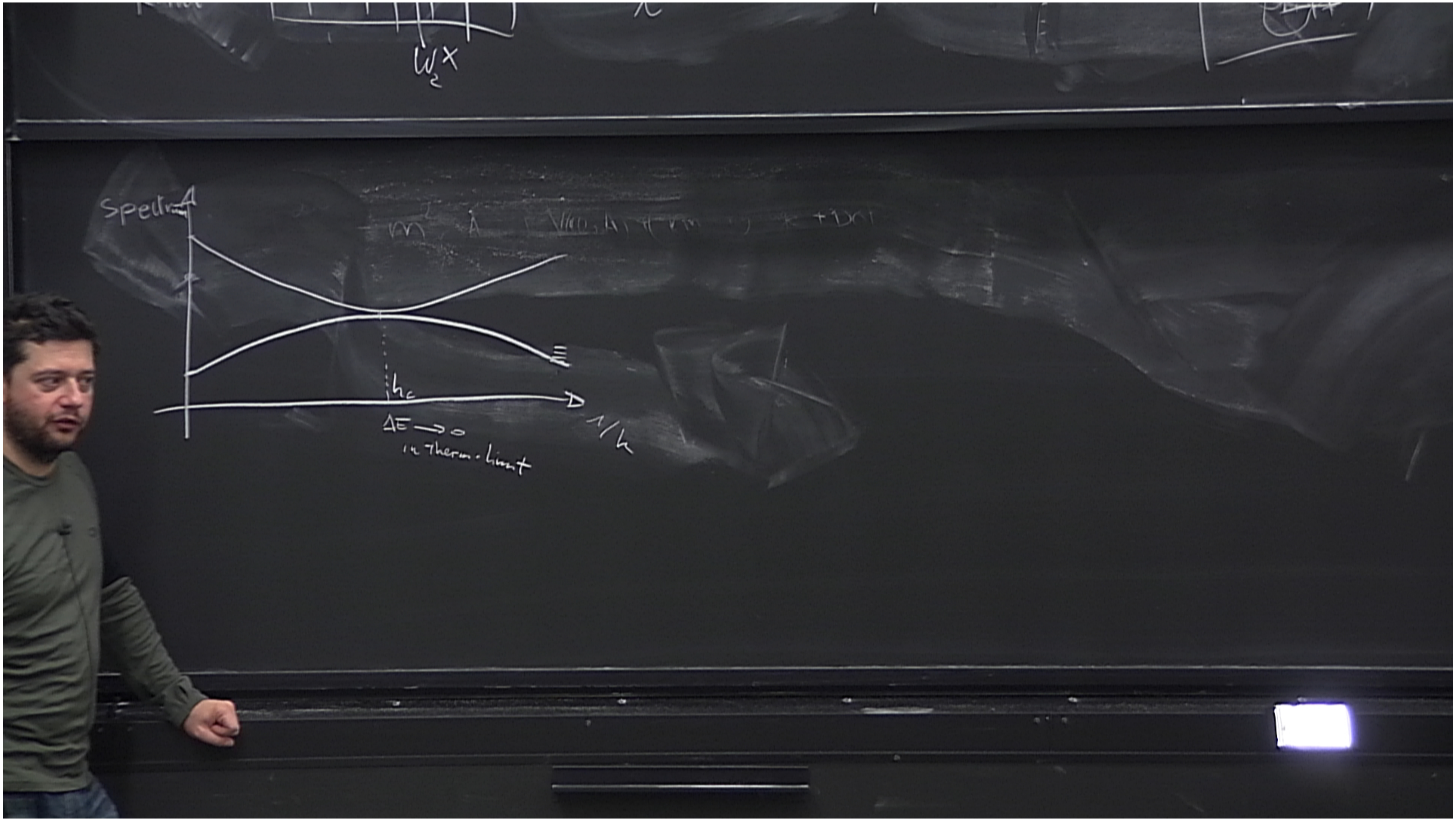

γ(h) ?   property of the phase
L large
pure state
pure state
pure state
h

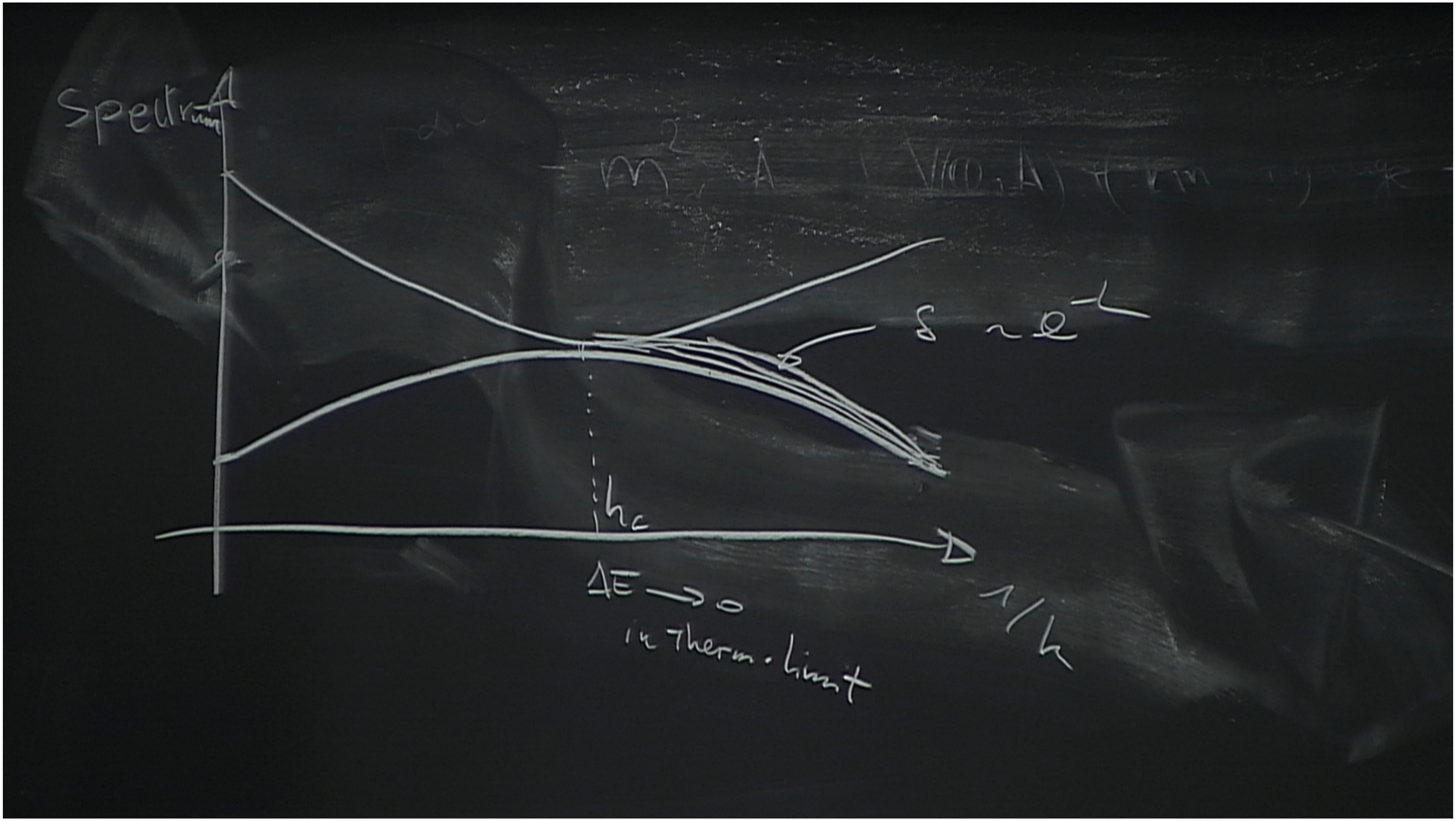

γ(h) ? property of the phase
Theorem
to prove
of γ
L large
pure state
bulk A
pure state
pure state
h


γ(h) ?
property of the phase
Theorem
to prove
the Stability
of γ
pure state
pure state
pure state
h_c
h
S(ρ_A) = S(ρ_∂A) + S(ρ_bulk)

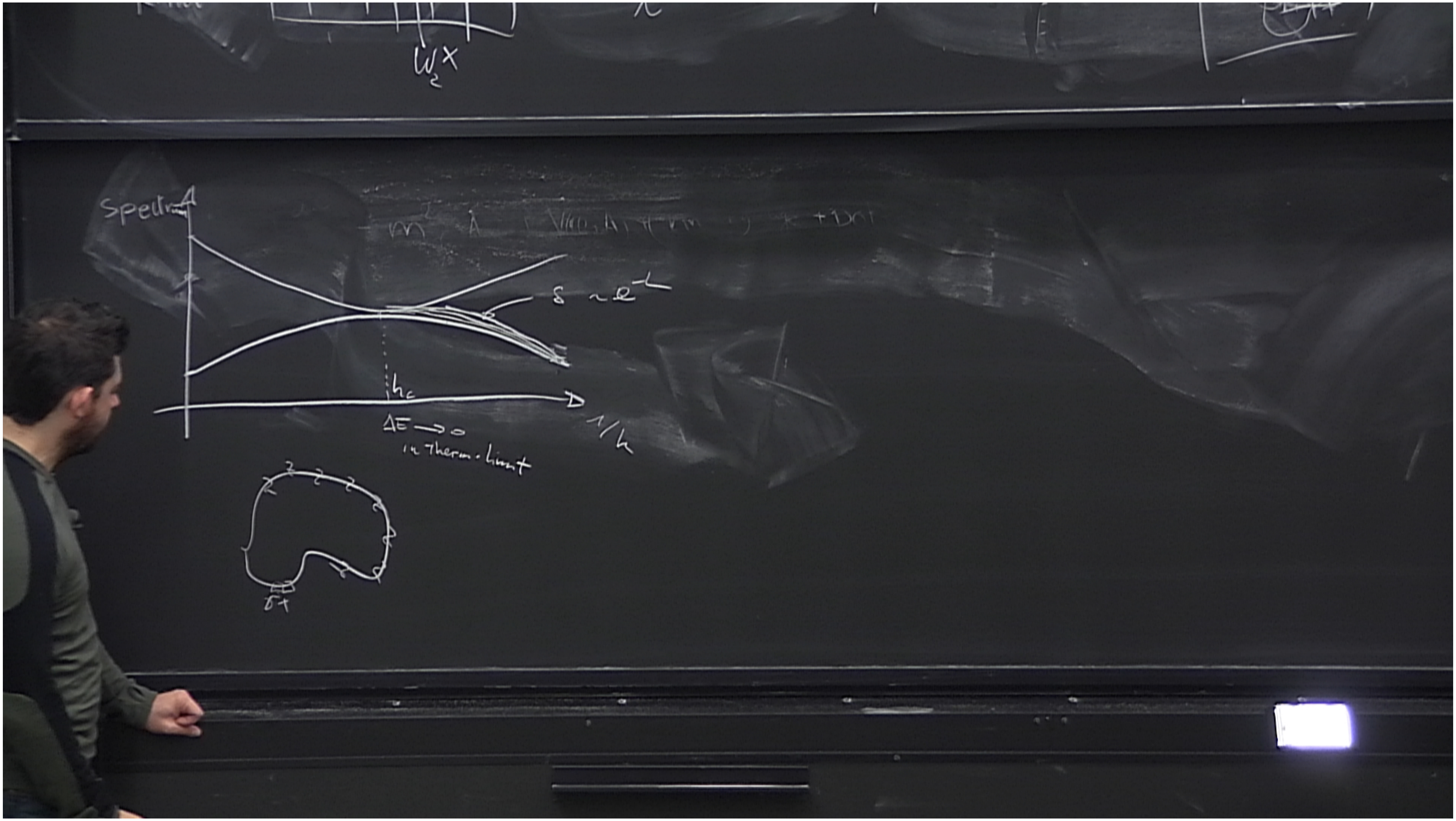

Spectra

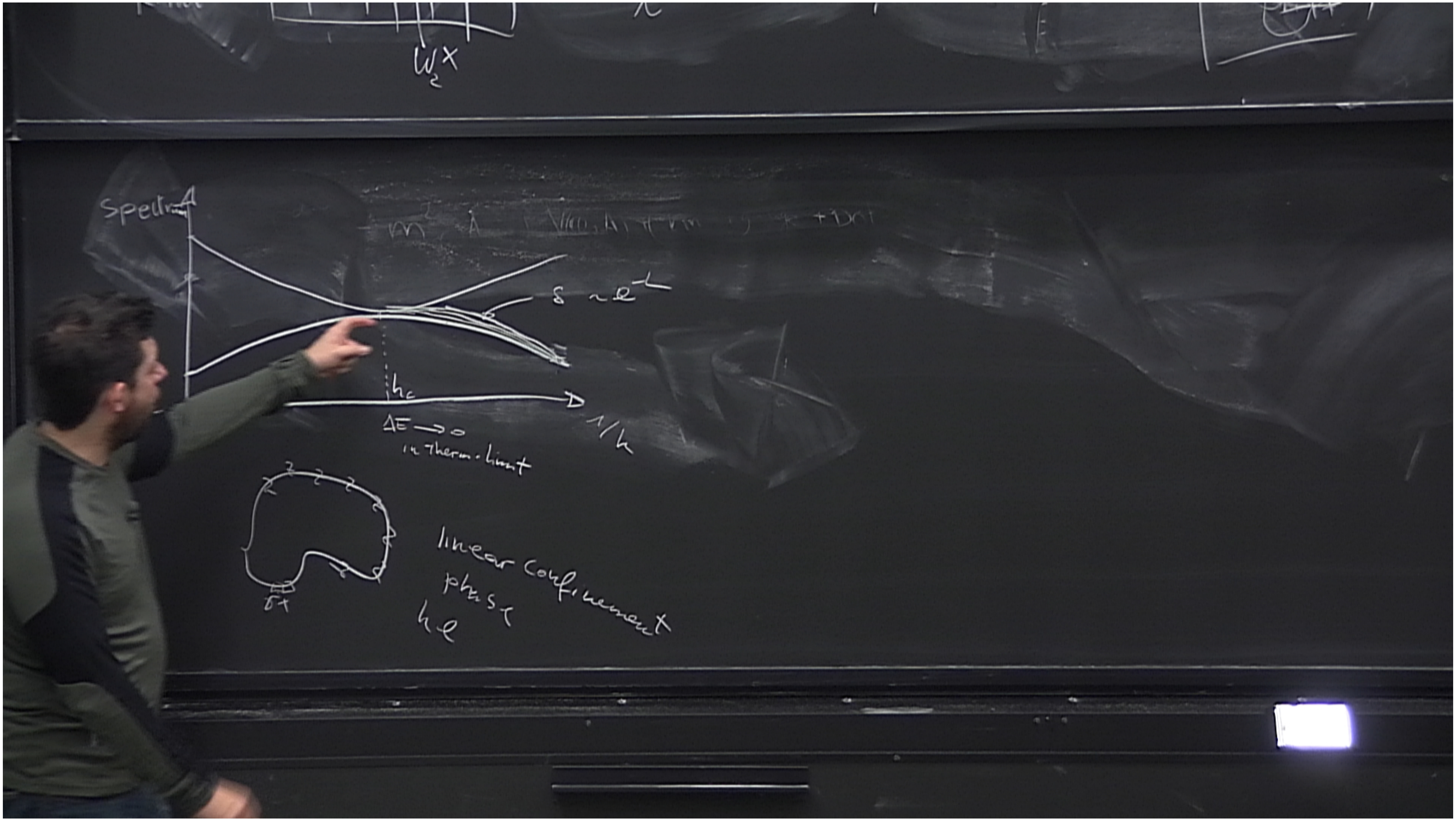

Spectra
1/L

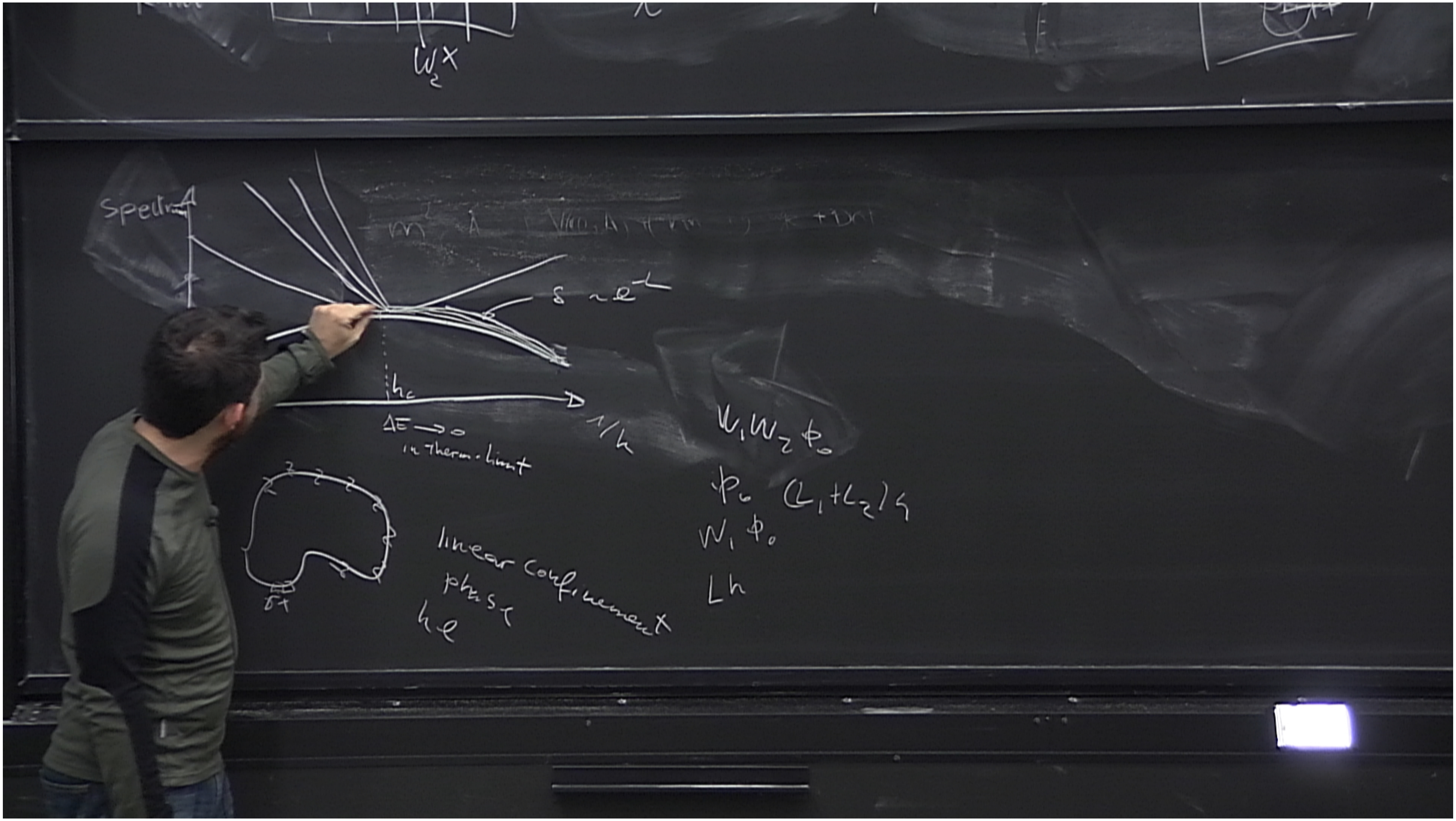

Spectrum
h_c
$\Delta E \to 0$
in Therm. limit
$1/h$

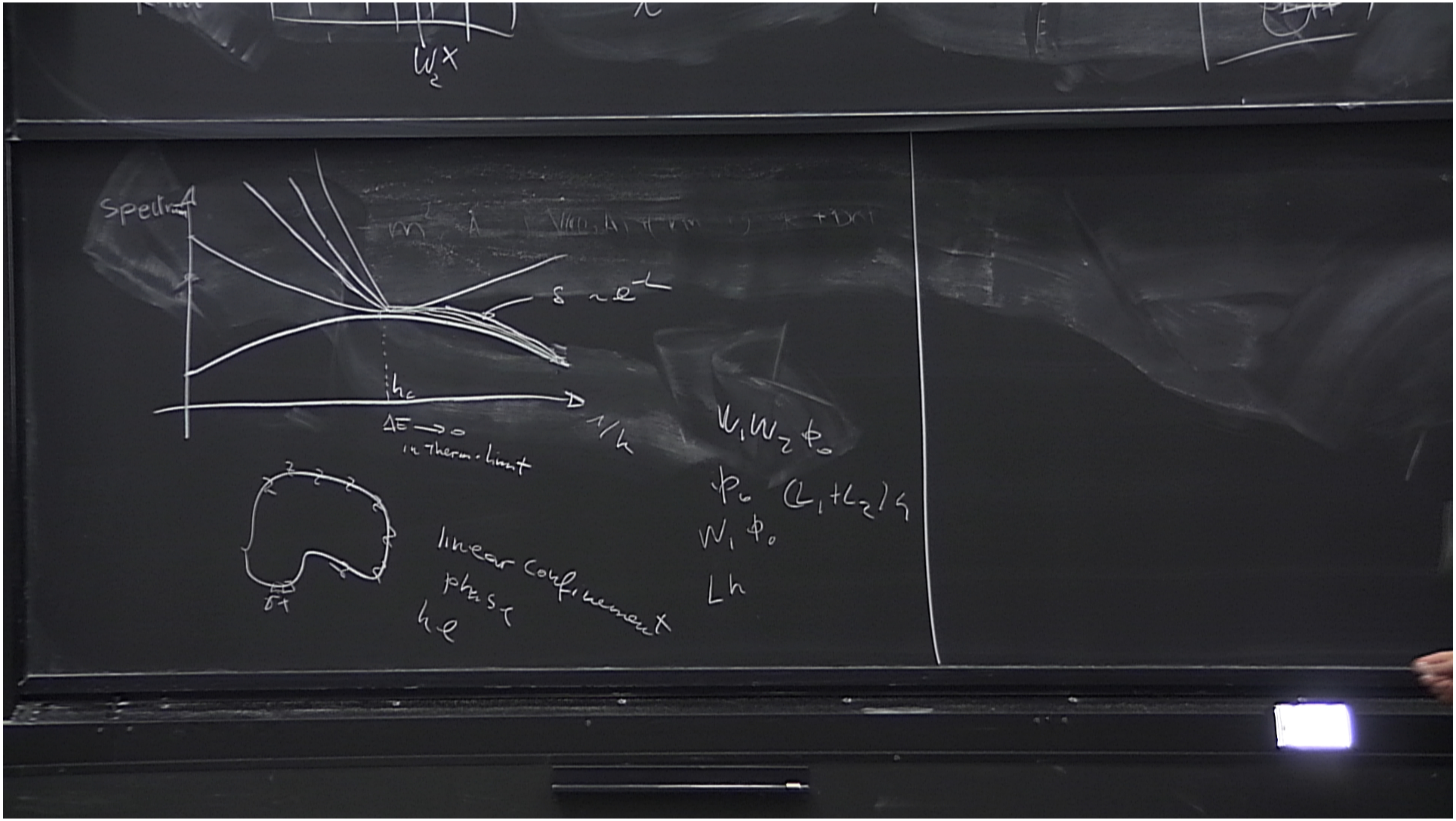


Spectrum
$\delta \sim e^{-L}$
h_c
$\Delta E \to 0$
in therm. limit
$1/h$

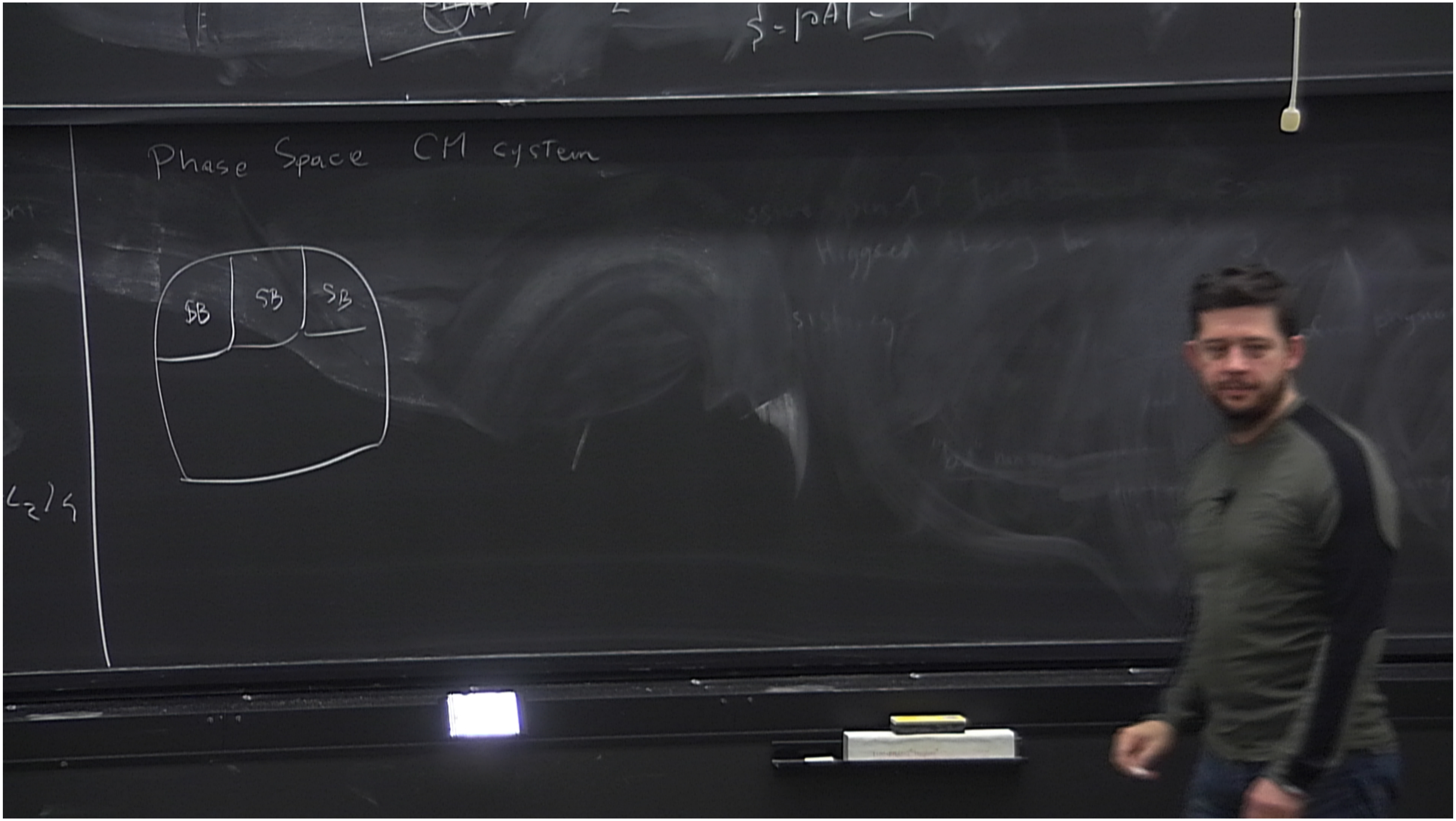

Spectr
$\delta \sim e^{-L}$
h_c
$\Delta E \to 0$
in Therm. limit
$1/h$
δt

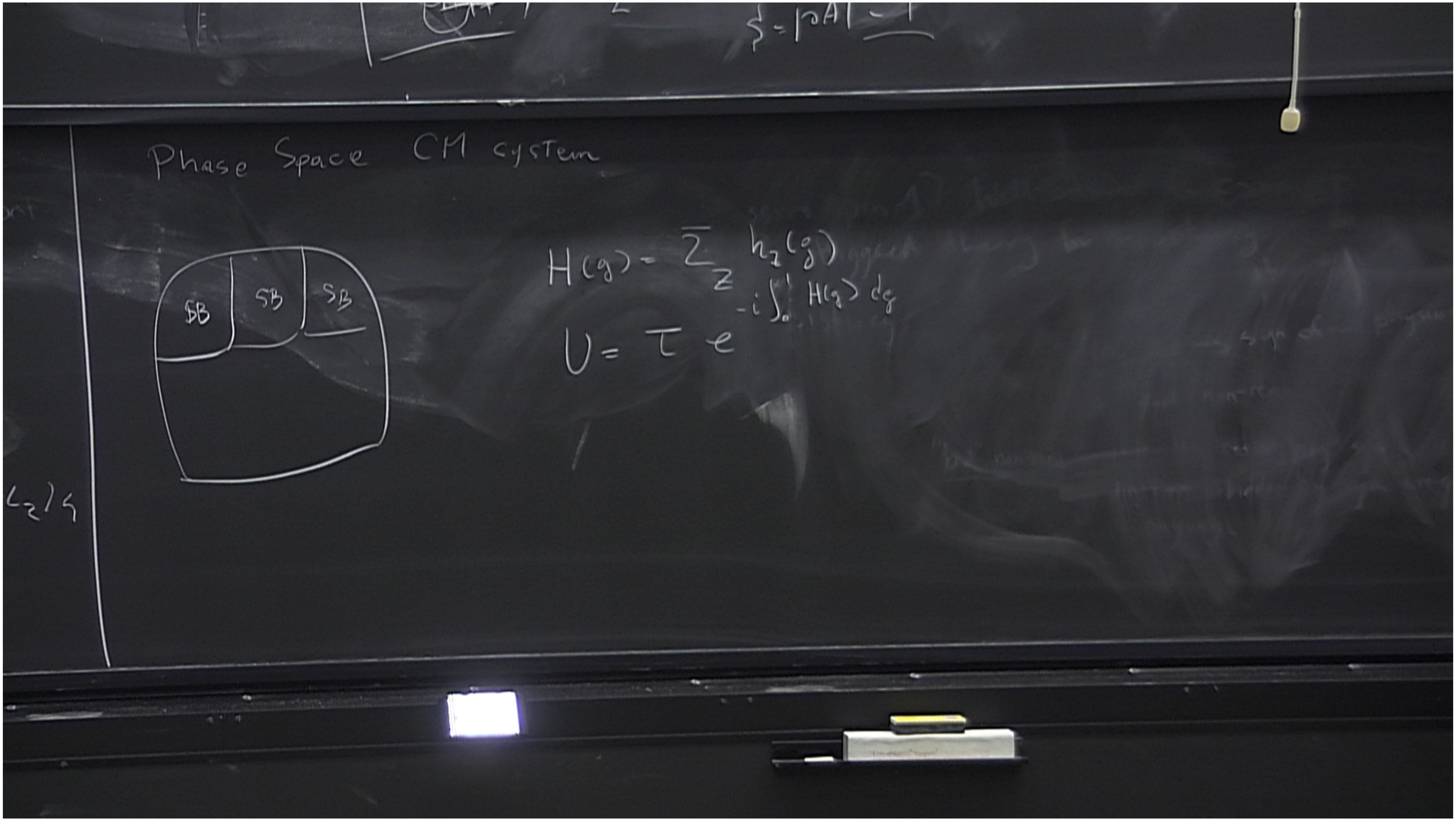

Spectra
$\delta \sim e^{-L}$
h_c
$1/h$
$\Delta E \to 0$
in thermo. limit
linear confinement
phase
h_e

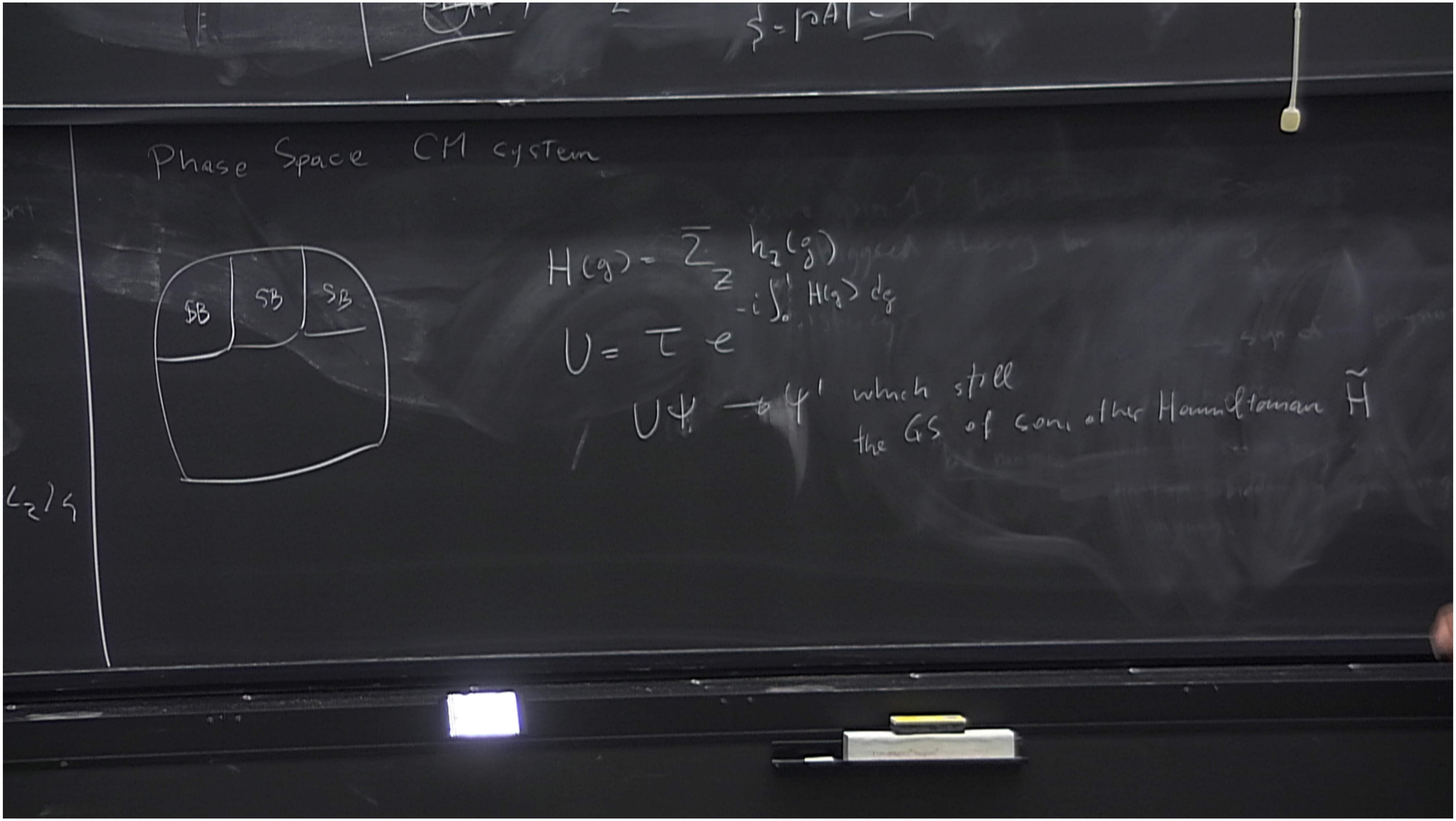

Spectrum
$\delta \sim e^{-L}$
h_c
$\Delta E \to 0$
in thermo. limit
$1/h$
$W_1 W_2 \Phi_0$
$\Phi_0 \; (L_1+L_2)/4$
$W_1 \Phi_0$
Lh
linear confinement
phase
h_e

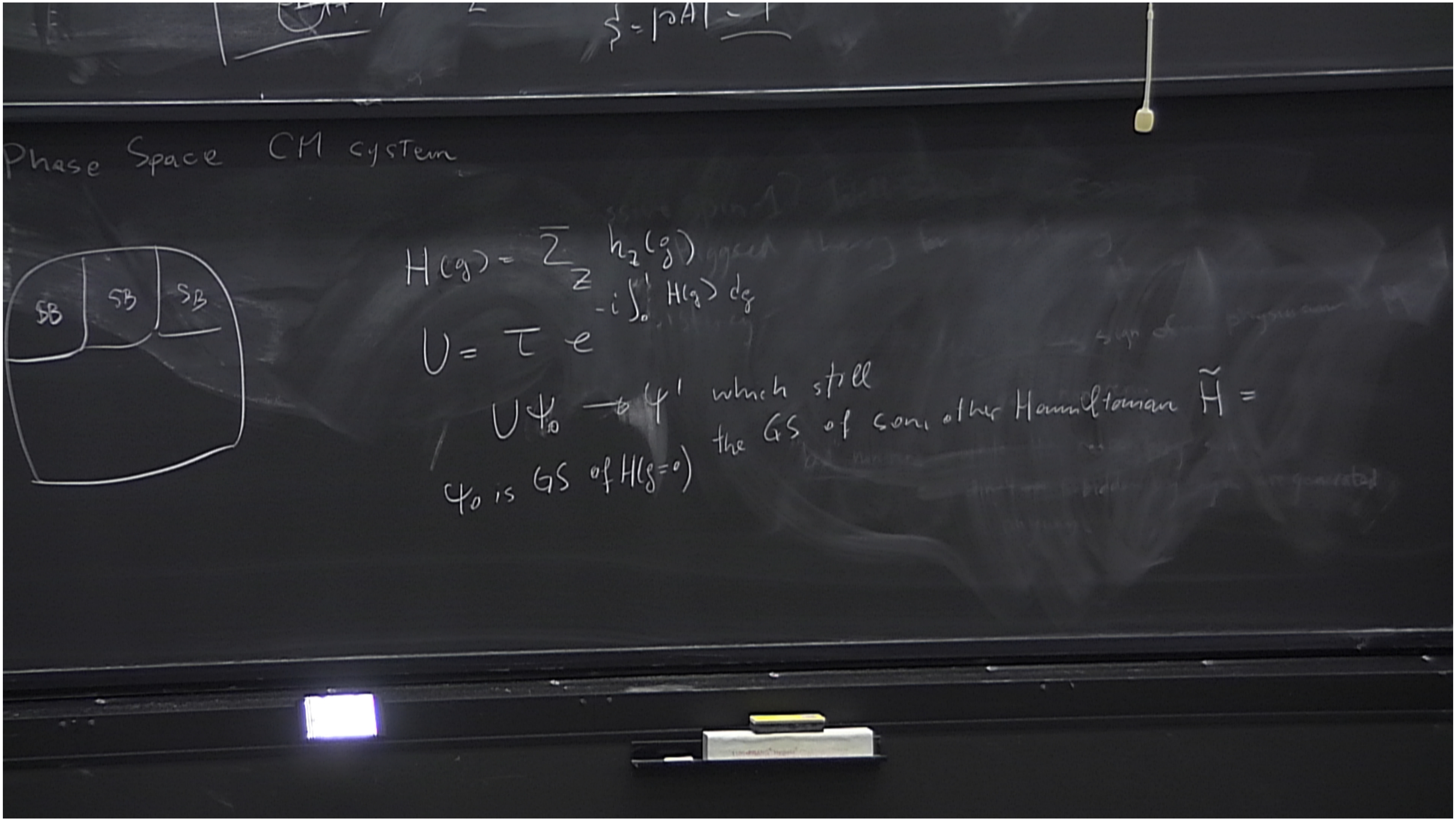

Spectrum
h_c
$\Delta E \to 0$
in Therm. limit
$\delta \sim e^{-L}$
1/h
linear confinement
phase
$W_1 W_2 \phi_0$
$W_1 \phi_0$
Lh

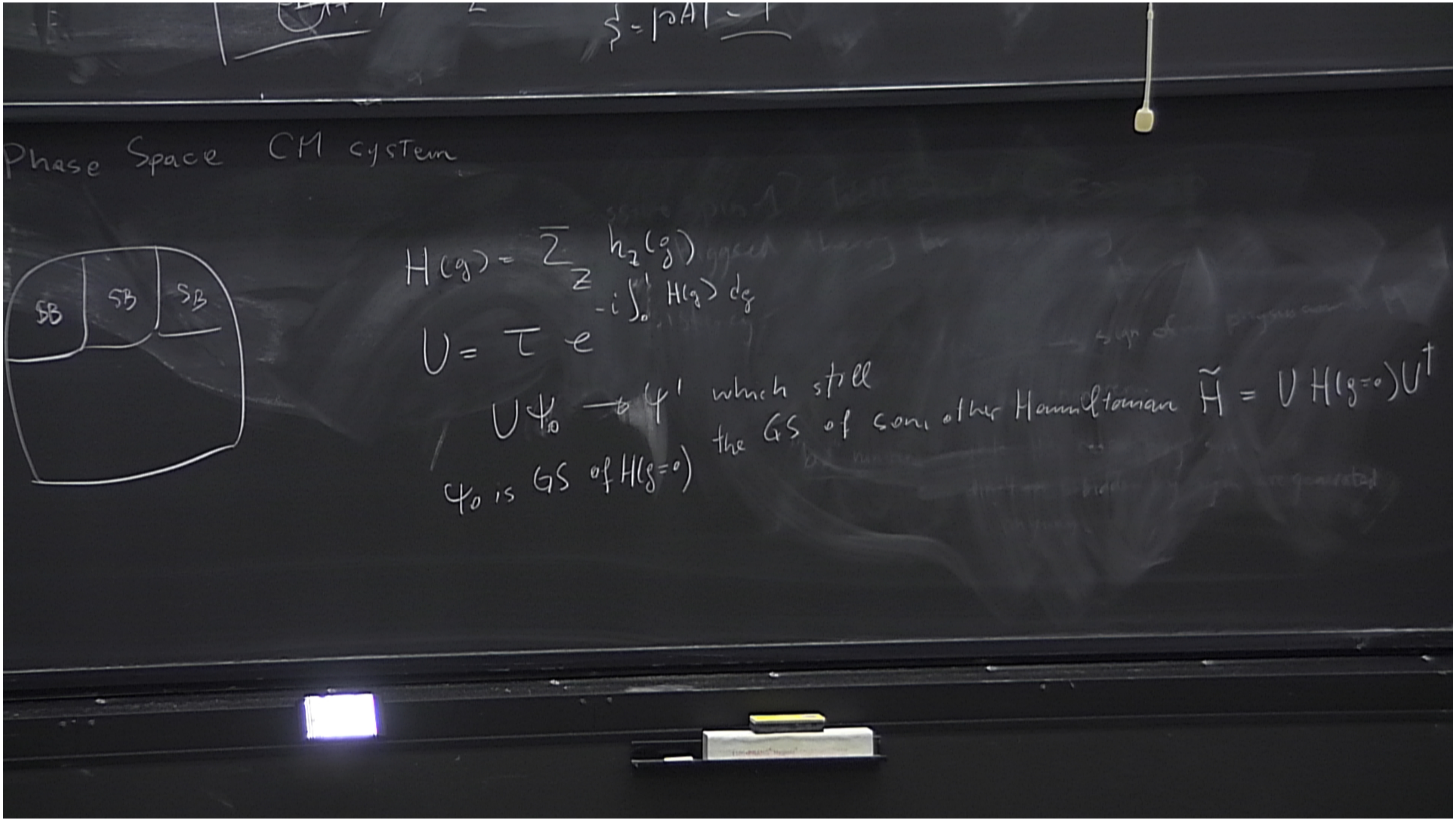

Phase Space CM system

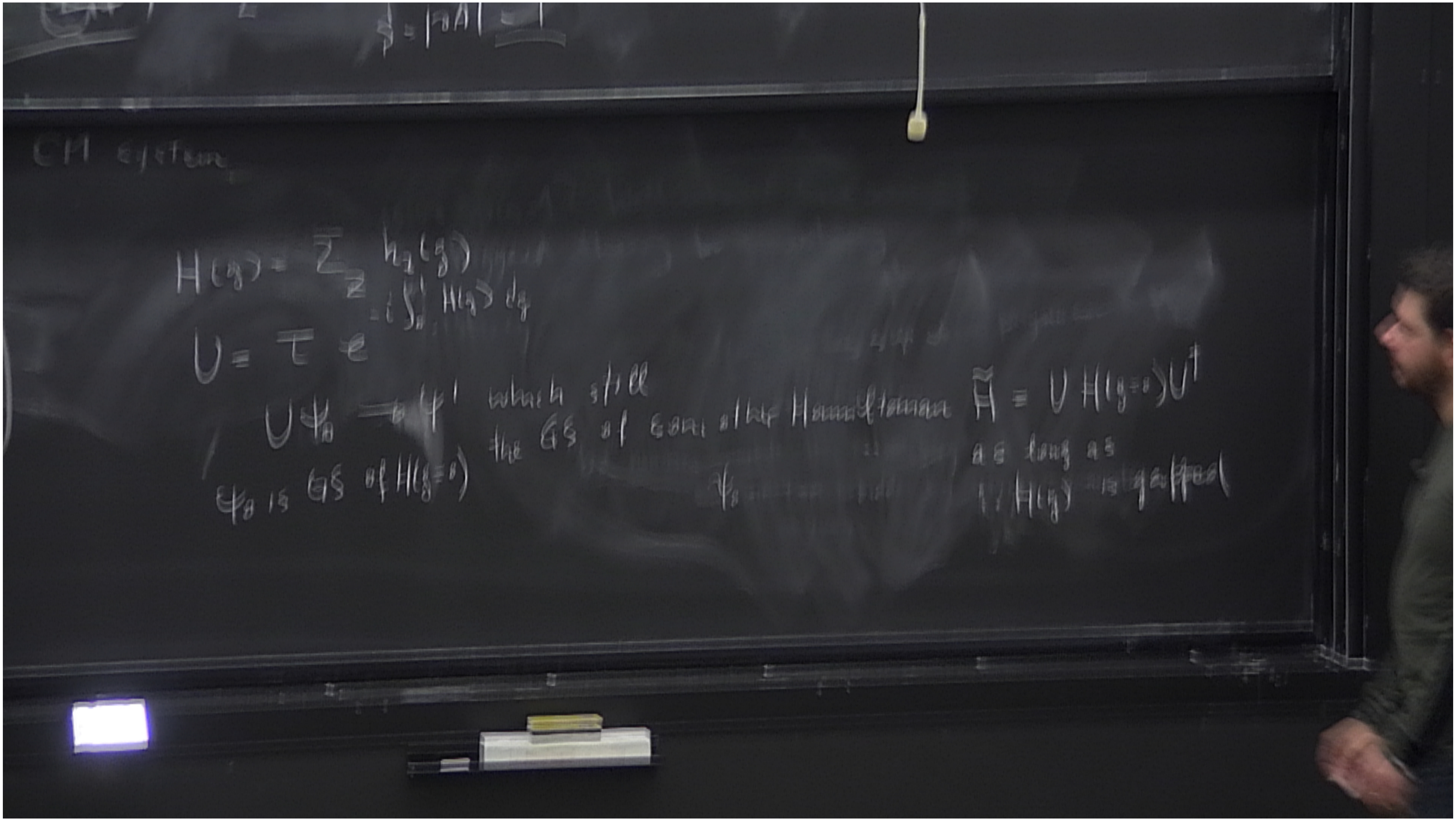

Phase Space CM system
SB
SB
SB

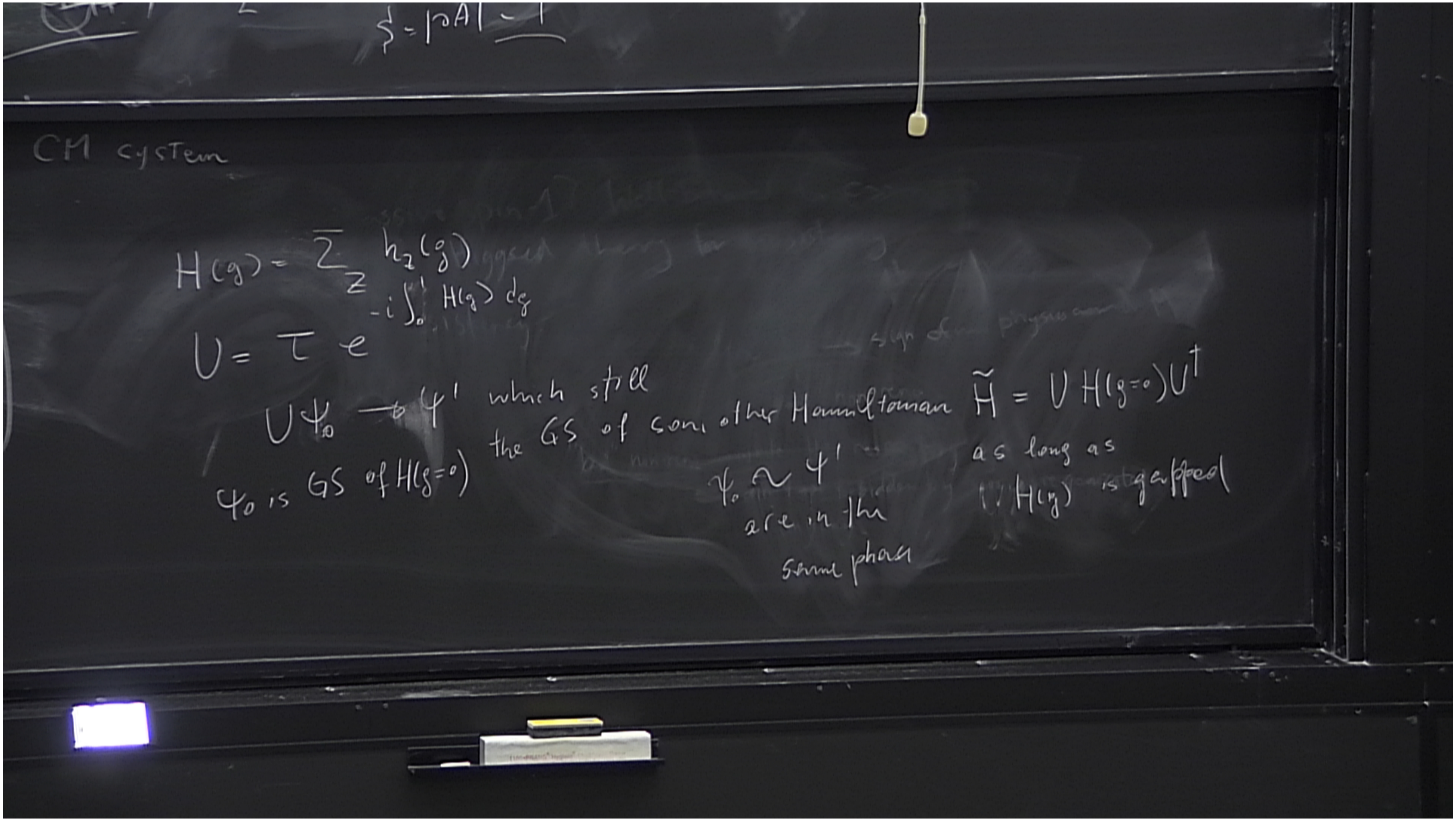

Phase Space CM system
SB
SB
SB
$H(g) = \sum_z h_z(g)$
$U = T e^{-i\int_0^1 H(g)\,dg}$

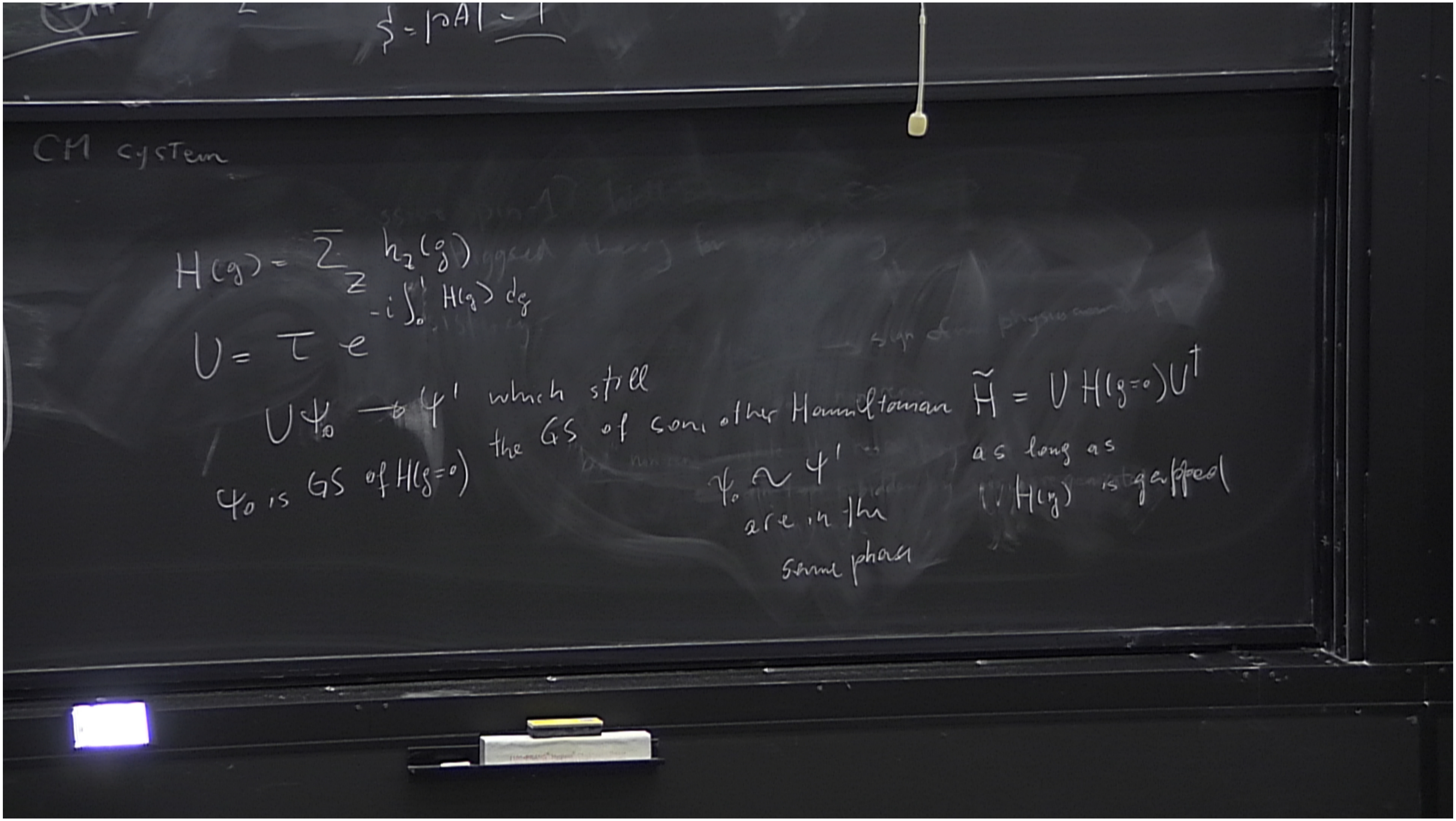

Phase Space CM system
SB
SB
SB
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
UΨ → Ψ' which still
the GS of some other Hamiltonian H̃

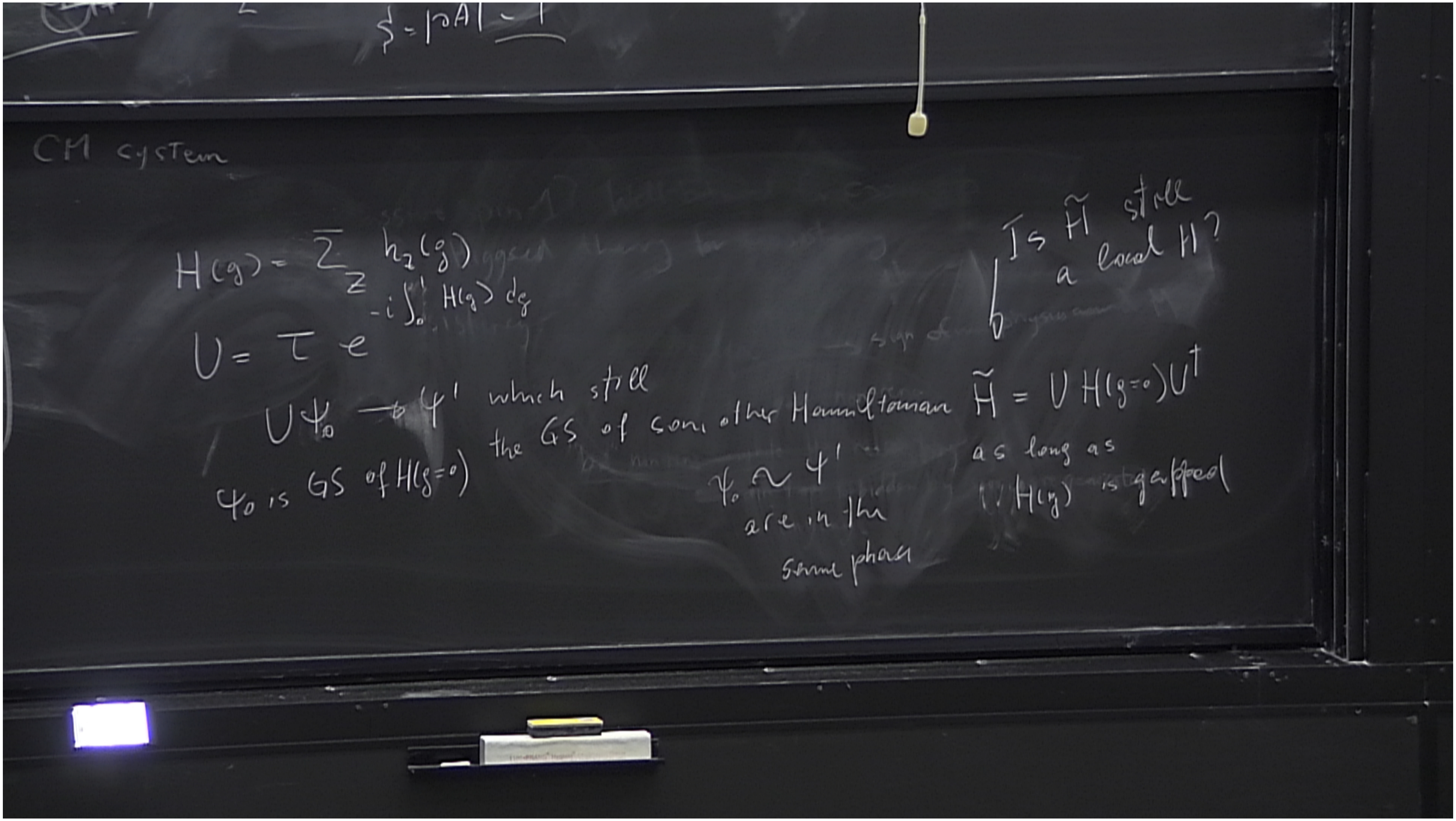

Phase Space CM system
SB
SB
SB
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
UΨ_0 → Ψ' which still the GS of some other Hamiltonian H̃ =
Ψ_0 is GS of H(g=0)

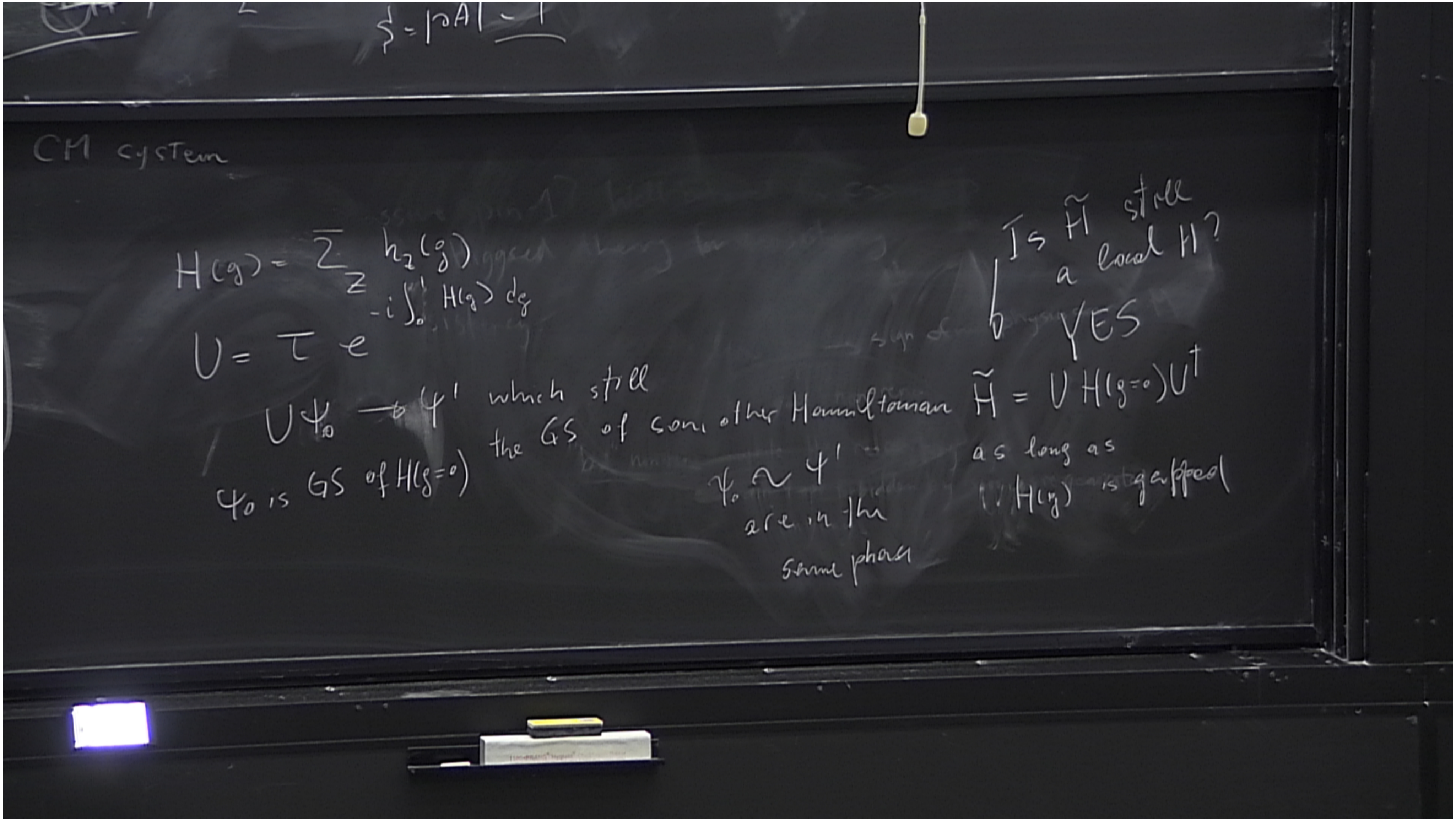

Phase Space CM system
SB
SB
SB
H(g) = Σ_Z h_Z(g)
U = T e^{-i ∫_0^1 H(g) dg}
U Ψ_0 → Ψ' which still the GS of some other Hamiltonian H̃ = U H(g=0) U†
Ψ_0 is GS of H(g=0)

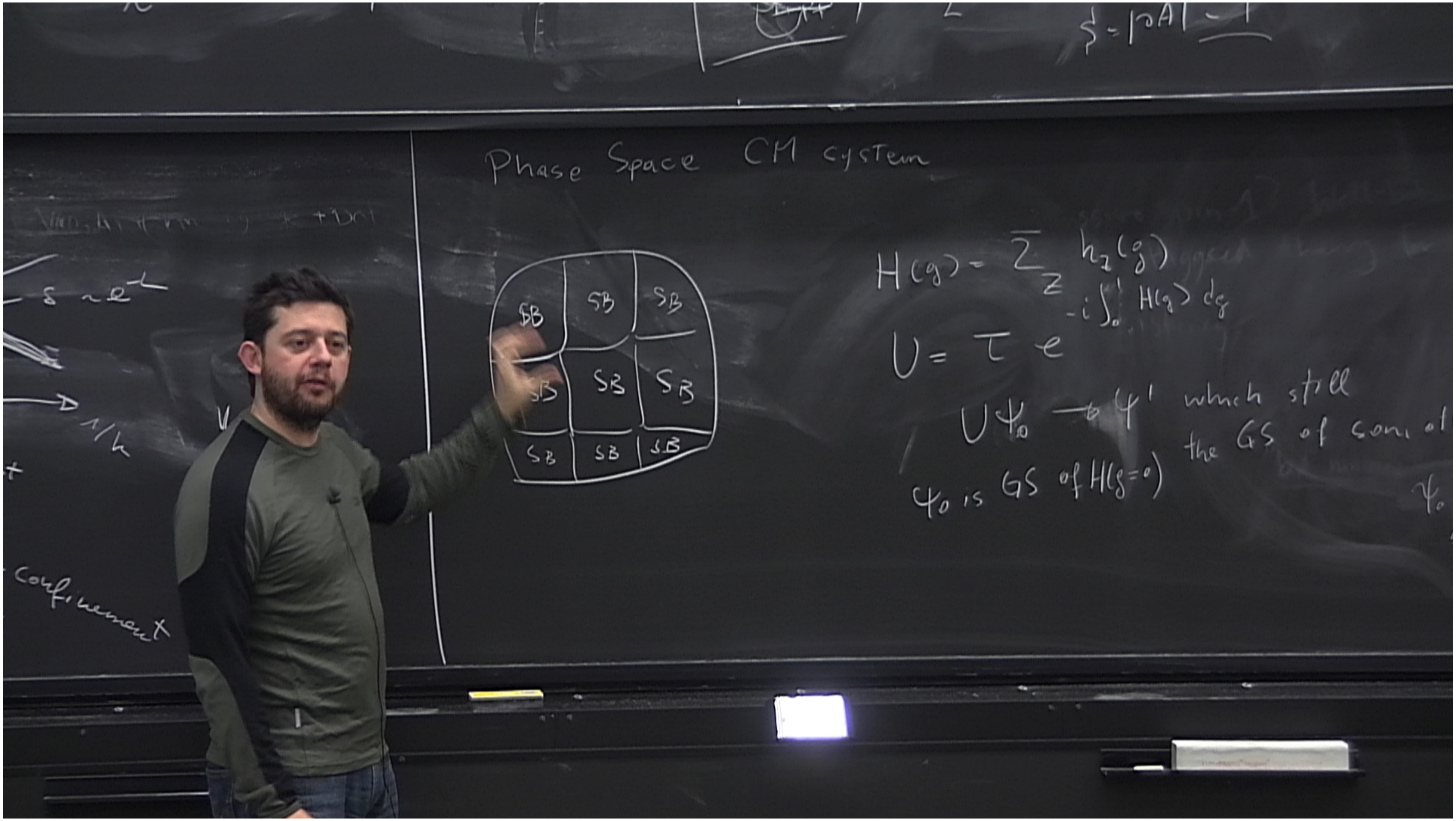

CM system
which still
the GS of some other Hamiltonian
as long as
is gapped
is GS of

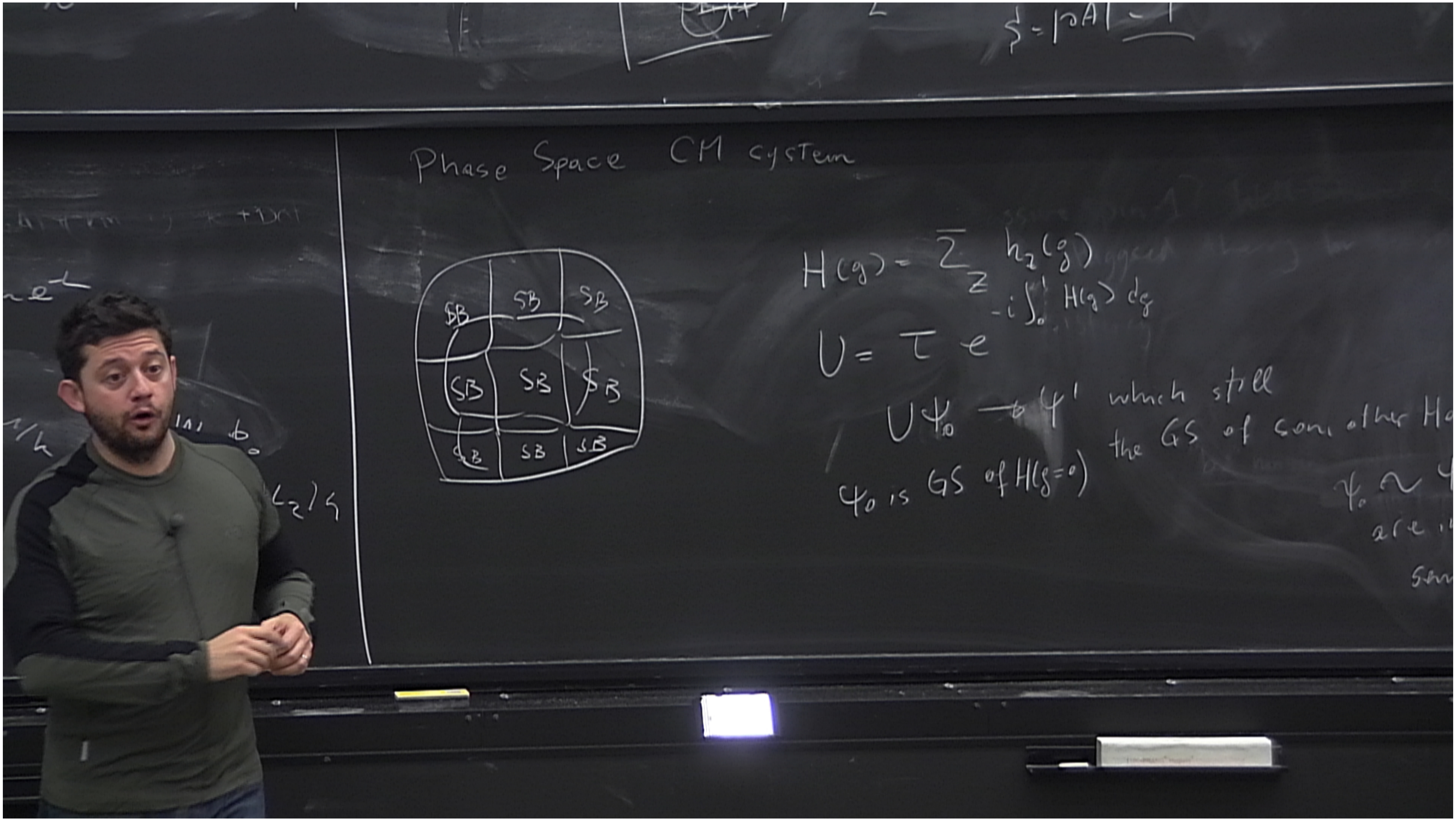

CM system
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
UΨ_0 → Ψ' which still the GS of some other Hamiltonian H̃ = U H(g=0) U†
Ψ_0 is GS of H(g=0)
Ψ_0 ~ Ψ' are in the same phase
as long as H(g) is gapped

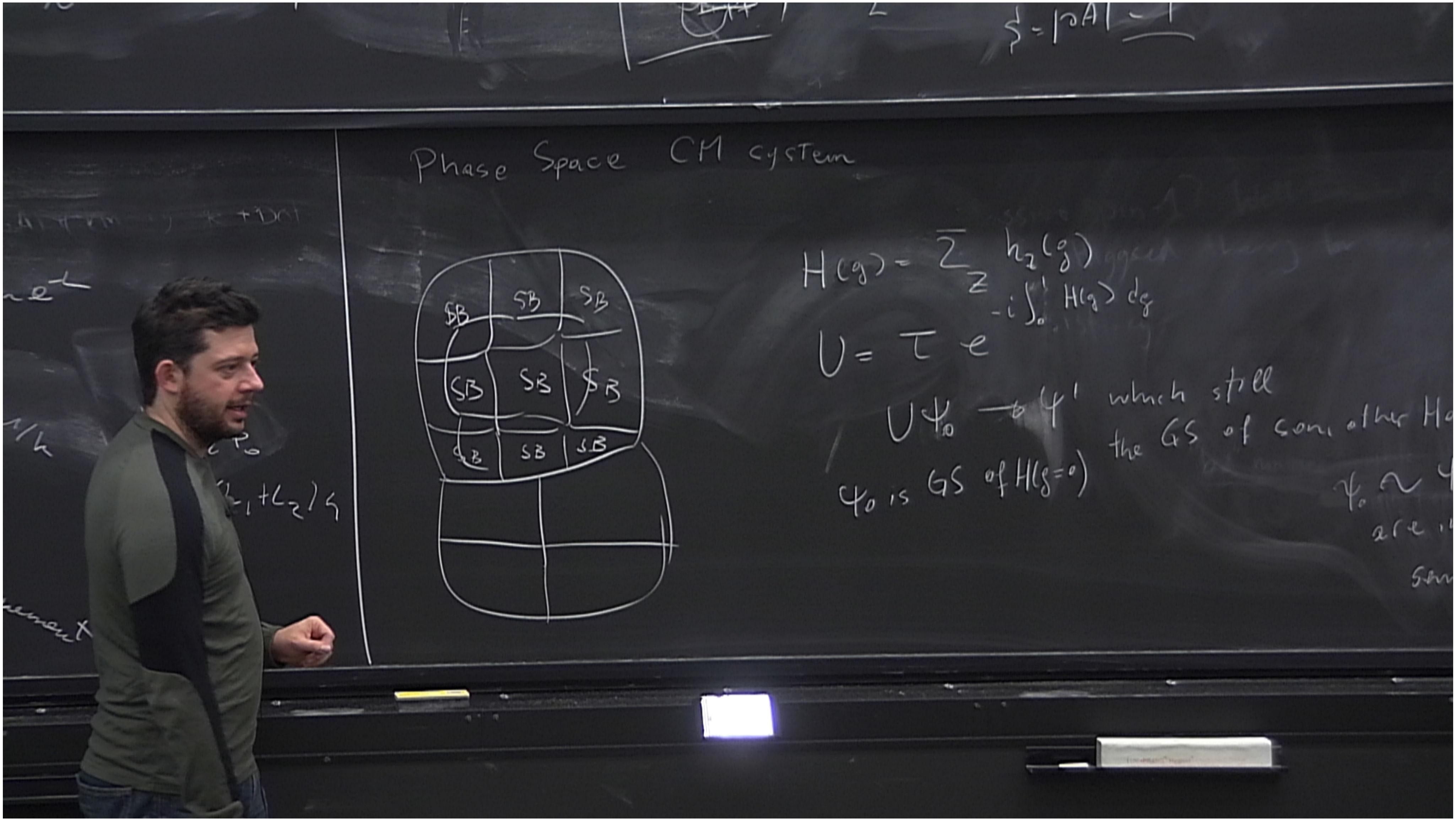

CM system
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
UΨ_0 → Ψ' which still the GS of some other Hamiltonian H̃ = U H(g=0) U†
Ψ_0 is GS of H(g=0)
Ψ_0 ~ Ψ' are in the same phase
as long as H(g) is gapped

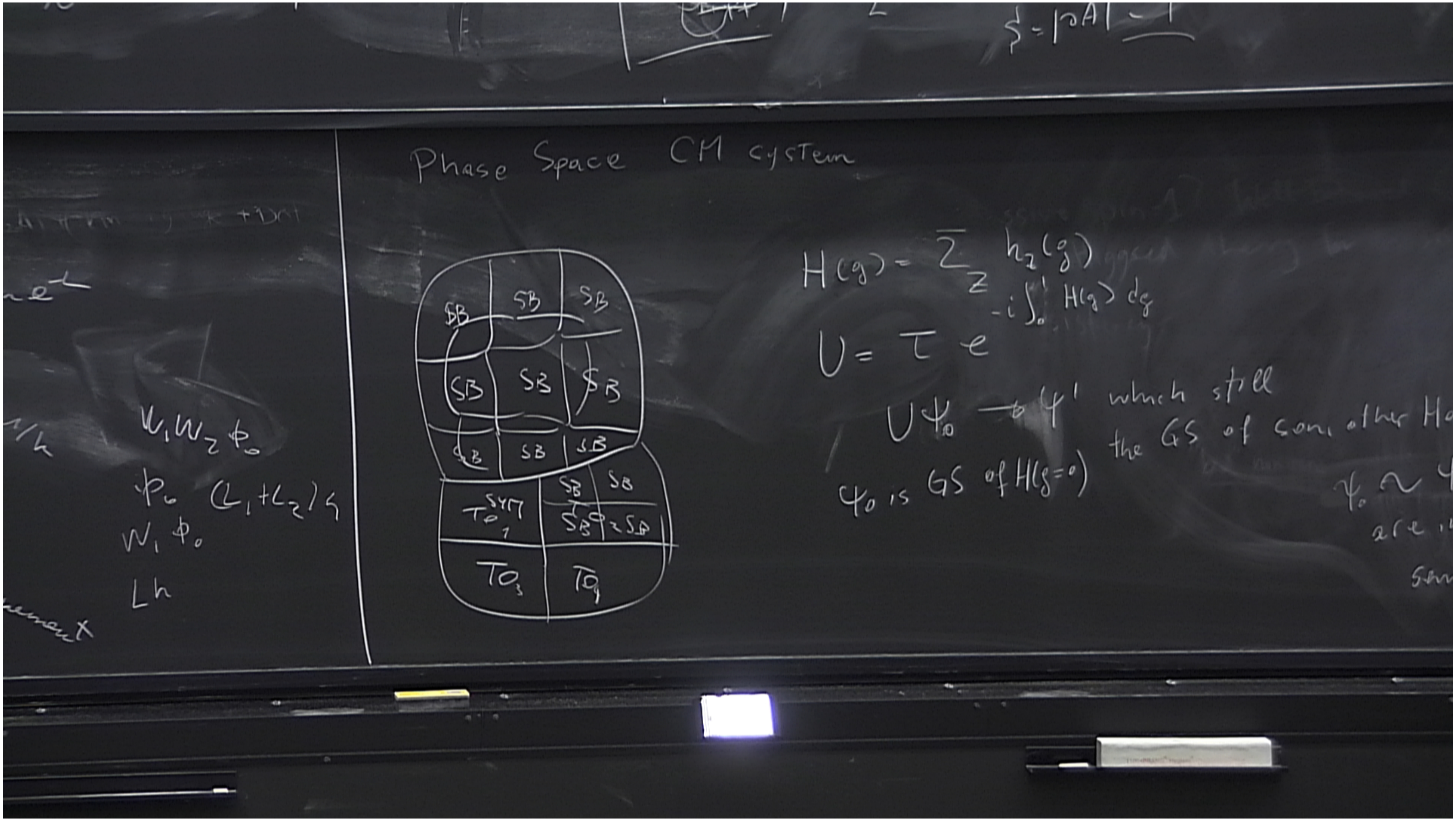

CM system
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
U Ψ0 → Ψ' which still the GS of some other Hamiltonian H̃ = U H(g=0) U†
Ψ0 is GS of H(g=0)
Ψ0 ~ Ψ' are in the same phase
as long as H(g) is gapped
Is H̃ still a local H?

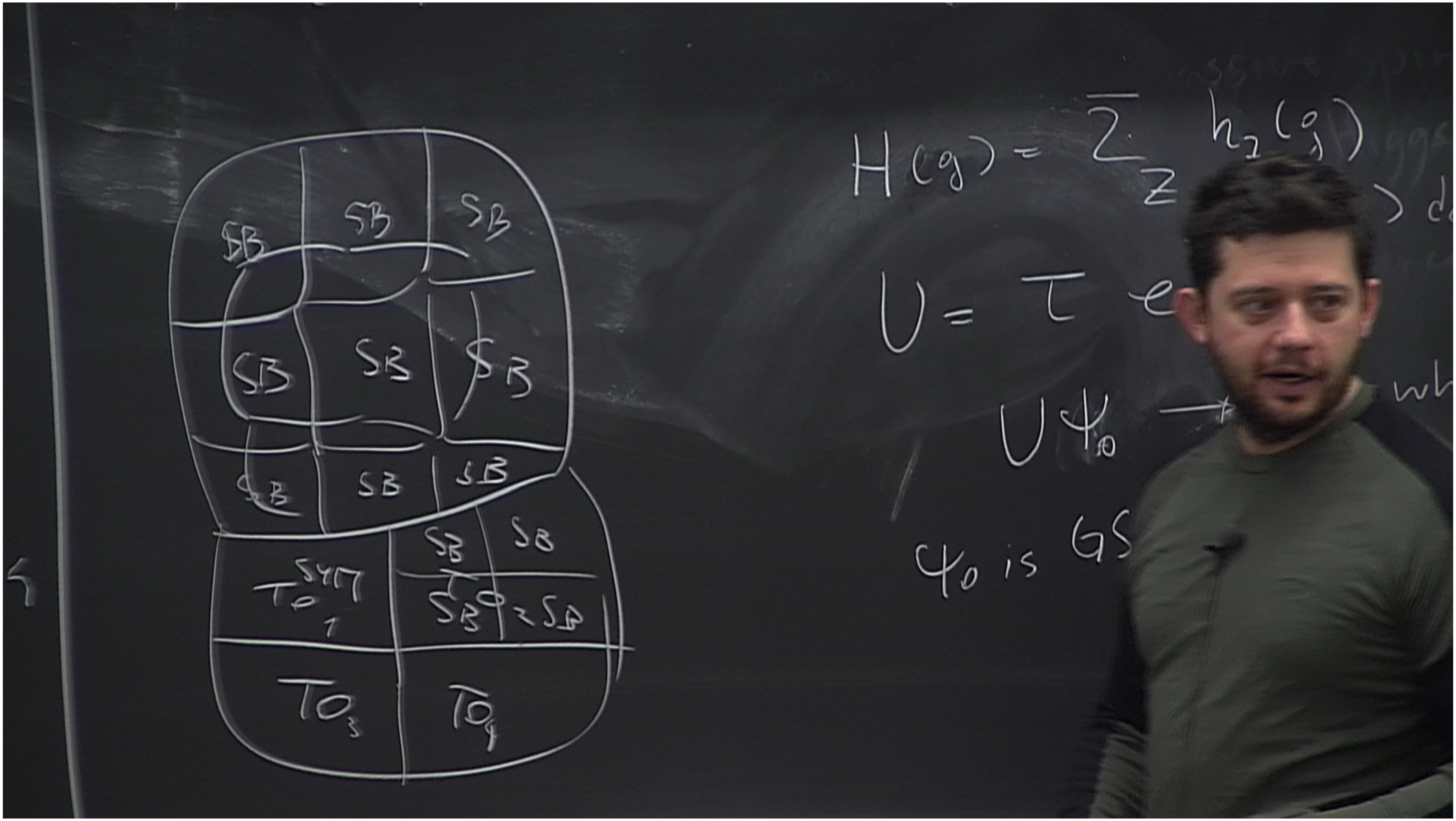

CM system
H(g) = Σ_z h_z(g)
U = T e^{-i ∫_0^1 H(g) dg}
U Ψ_0 → Ψ' which still the GS of some other Hamiltonian H̃ = U H(g=0) U†
Ψ_0 is GS of H(g=0)
Is H̃ still a local H?
YES
Ψ_0 ~ Ψ'
are in the same phase
as long as H(g) is gapped


Phase Space CM system
SB SB SB
SB SB SB
SB SB SB
H(g) = Σ_Z h_Z(g)
U = T e^{-i ∫_0^1 H(g) dg}
U ψ_0 → ψ' which still
the GS of some ...
ψ_0 is GS of H(g=0)
confinement


Phase Space CM system
SB
H(g) = Σ_z h_z(g)
U = T e^{-i∫_0^1 H(g) dg}
UΨ_0 → Ψ' which still
the GS of some other H
Ψ_0 is GS of H(g=0)


Phase Space CM system
SB SB SB
SB SB SB
SB SB SB
H(g) = Σ_Z h_Z(g)
U = T e^{-i ∫_0^1 H(g) dg}
U ψ_0 → ψ' which still
the GS of some other H
ψ_0 is GS of H(g=0)


Phase Space CM system
SB
TO
H(g) = Σ h(g)
U = T e
which still
the GS of some other H
φ₀ is GS of H(g=0)
Lh



SB
SB
SB
SB
SB
SB
SB
SB
SB
SB
SB
TO₃
TO₄
H(g) = Σ_Z h_Z(g_i)
U = τ e
Uψ₀
ψ₀ is GS