Title: Condensed Matter (Review) - Lecture 13

Date: Jan 18, 2012 10:15 AM

URL: http://pirsa.org/12010099

Abstract:


γ
C2
C1

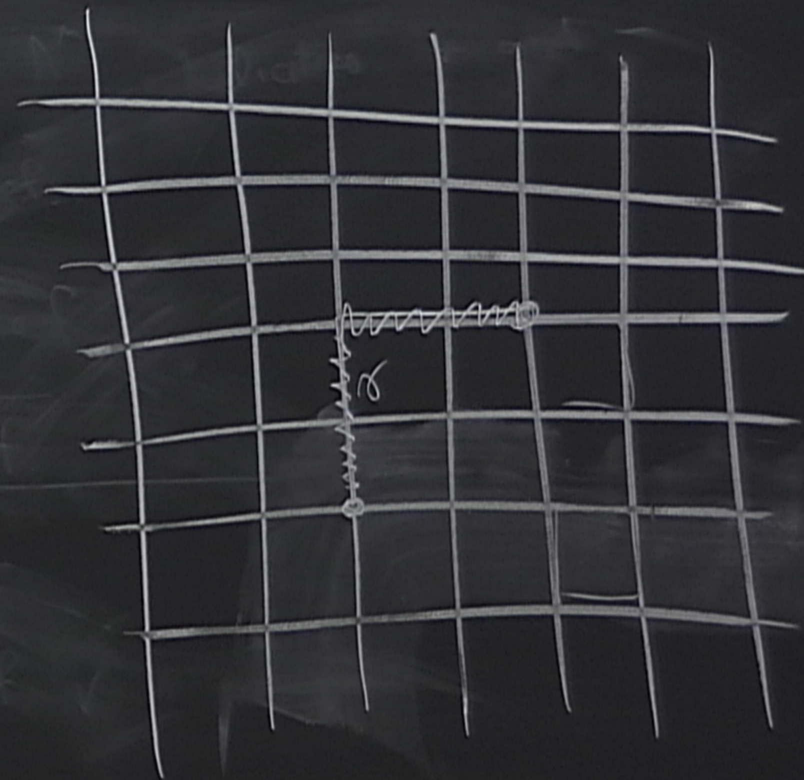
$\mathbb{Z}_2$ Topological Order

$$H_{TC} = -U\sum_s A_s - J\sum_p B_p$$

$$\begin{cases} A_s = \prod_{j\in s} \hat{\sigma}^x_j \\ B_p = \prod_{j\in p} \hat{\sigma}^z_p \end{cases}$$

$$W^z_\gamma = \prod_{j\in\gamma} \hat{\sigma}^z_j$$

B_p is such a string operator



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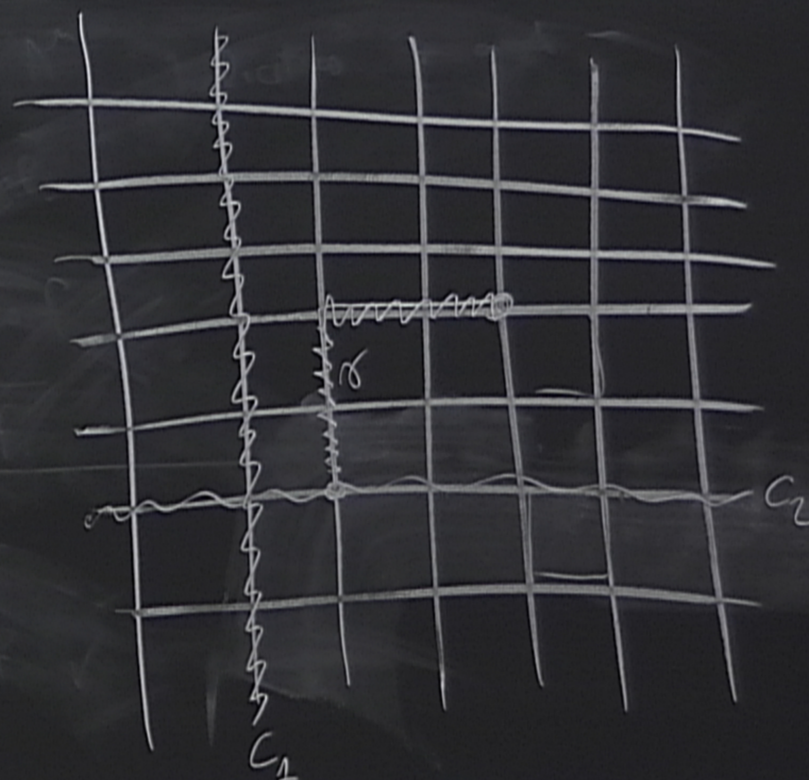

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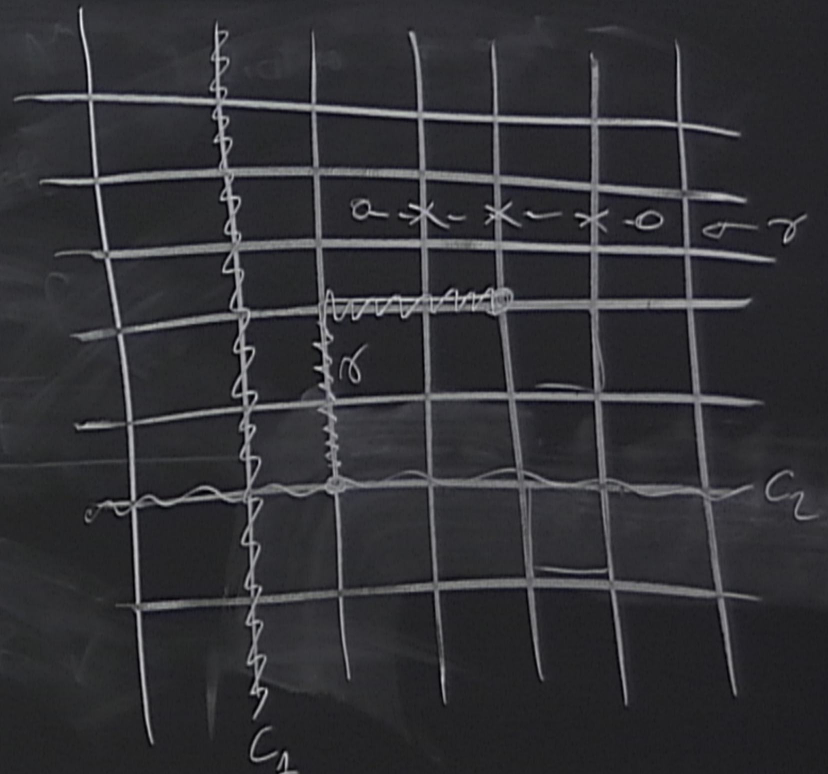

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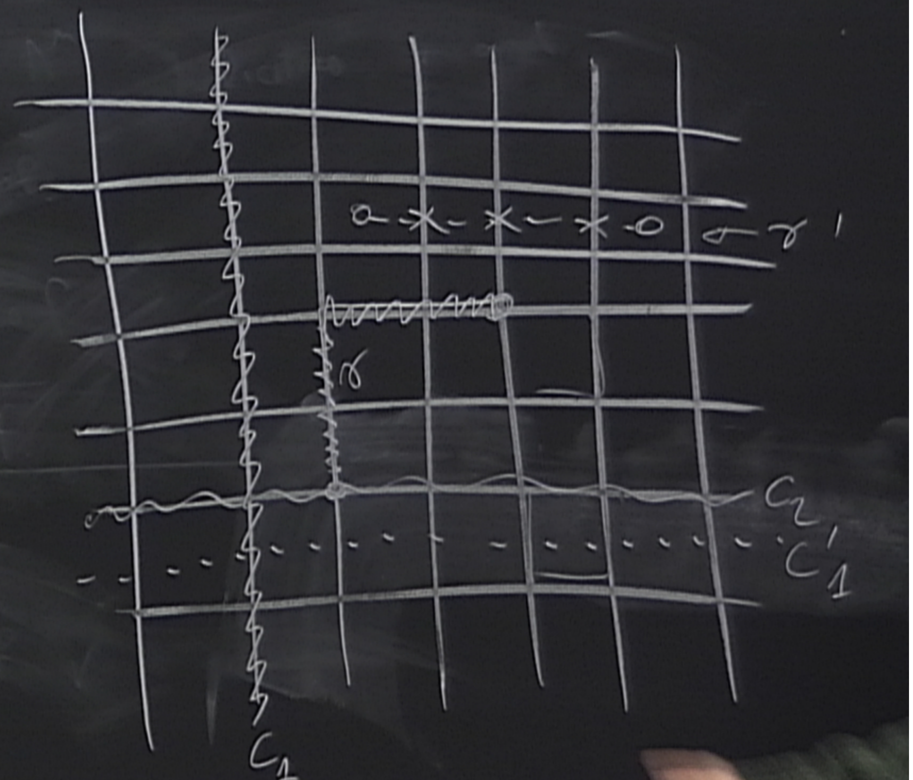

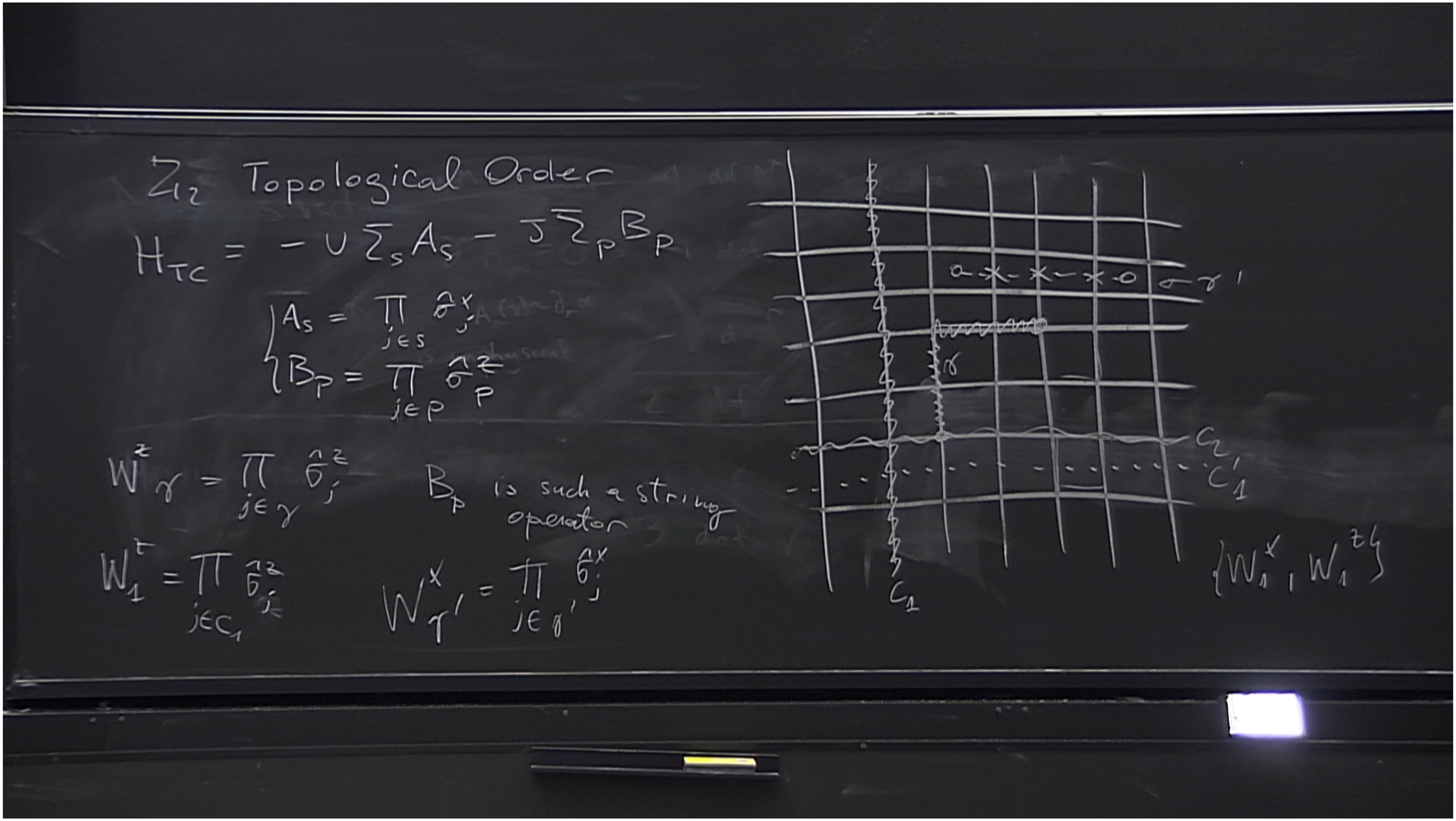
Z_2 Topological Order
$H_{TC} = -U\sum_s A_s - J\sum_p B_p$
$A_s = \prod_{j\in s} \hat\sigma^x_j$
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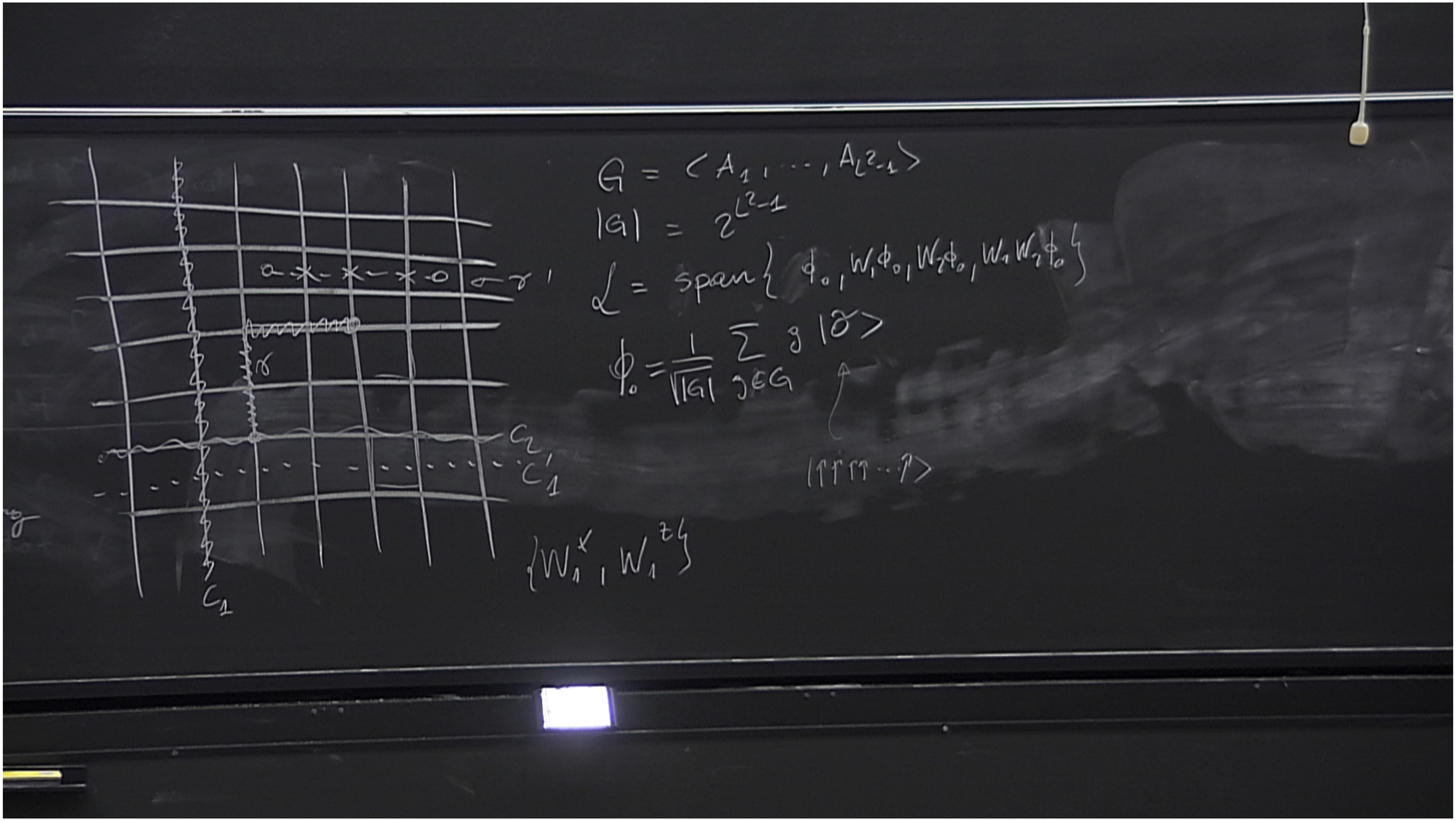
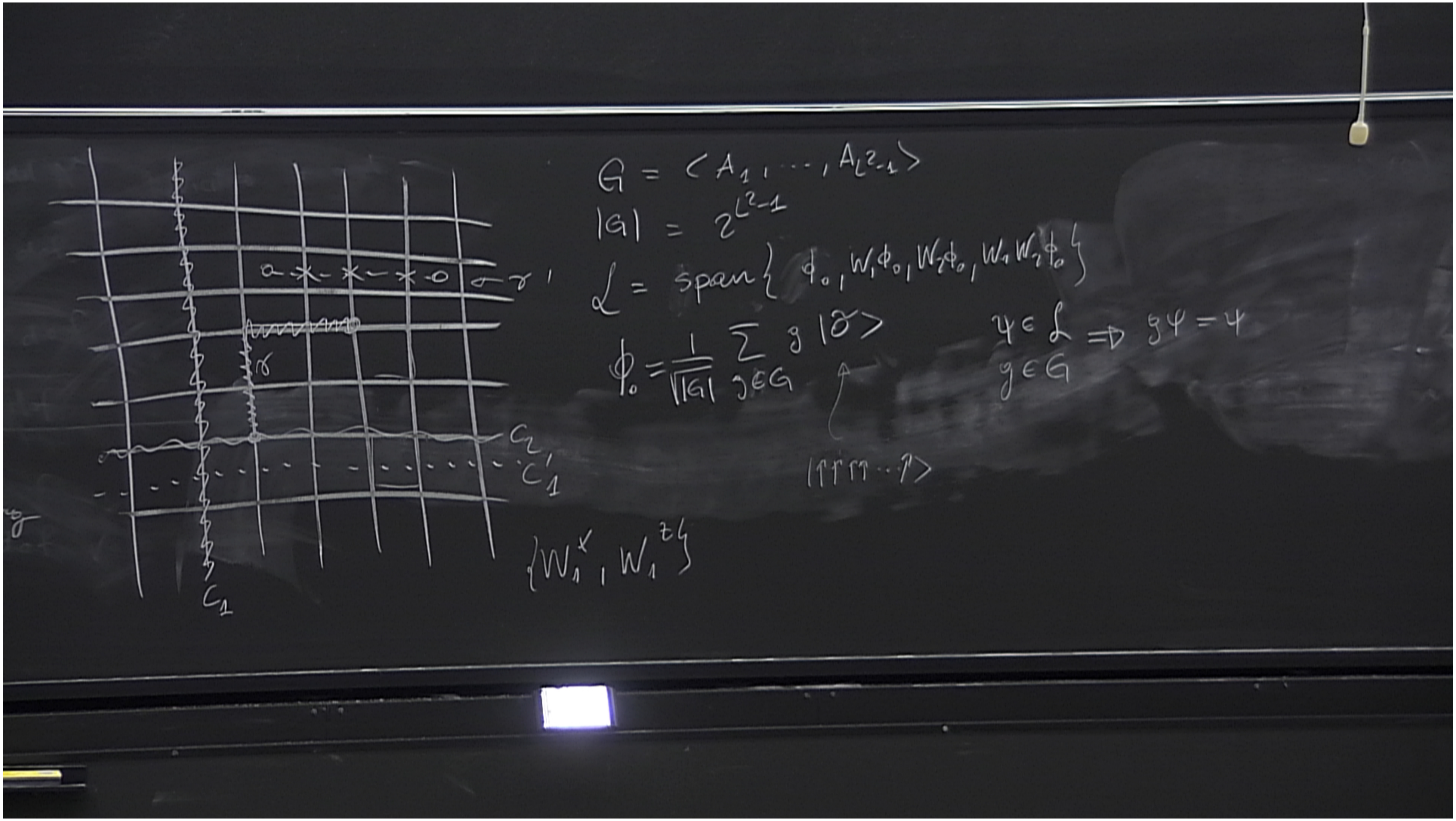
γ'
γ
C_2
C_1'
C_1
$\{W^x_1, W^z_1\}$


$G = \langle A_1, \dots, A_{L^2-1} \rangle$
$|G| = 2^{L^2-1}$
$\mathcal{L} = \mathrm{span}\{ \phi_0, W_1\phi_0, W_2\phi_0, W_1W_2\phi_0 \}$
$\phi_0 = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g|\sigma\rangle$
$|\uparrow\uparrow\uparrow\uparrow \cdots \uparrow\rangle$
$\{W_1^x, W_1^z\}$
γ'
γ
C_2
C_1'
C_1


G = < A_1 , ..., A_{L^2-1} >
|G| = 2^{L^2-1}
L = span{ φ_0 , W_1 φ_0 , W_2 φ_0 , W_1 W_2 φ_0 }
φ_0 = 1/√|G| Σ_{g∈G} g |σ>
ψ ∈ L, g ∈ G ⇒ gψ = ψ
|↑↑↑↑...↑>
{W_1^x , W_1^z}
C_1
C_2
C_1'
γ
γ'

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$\phi_0 = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g |\sigma\rangle$

$|\uparrow\uparrow\uparrow\uparrow \cdots \uparrow\rangle$

$\psi \in \mathcal{L}$, $g \in G$ $\Rightarrow \boxed{g\psi = \psi}$

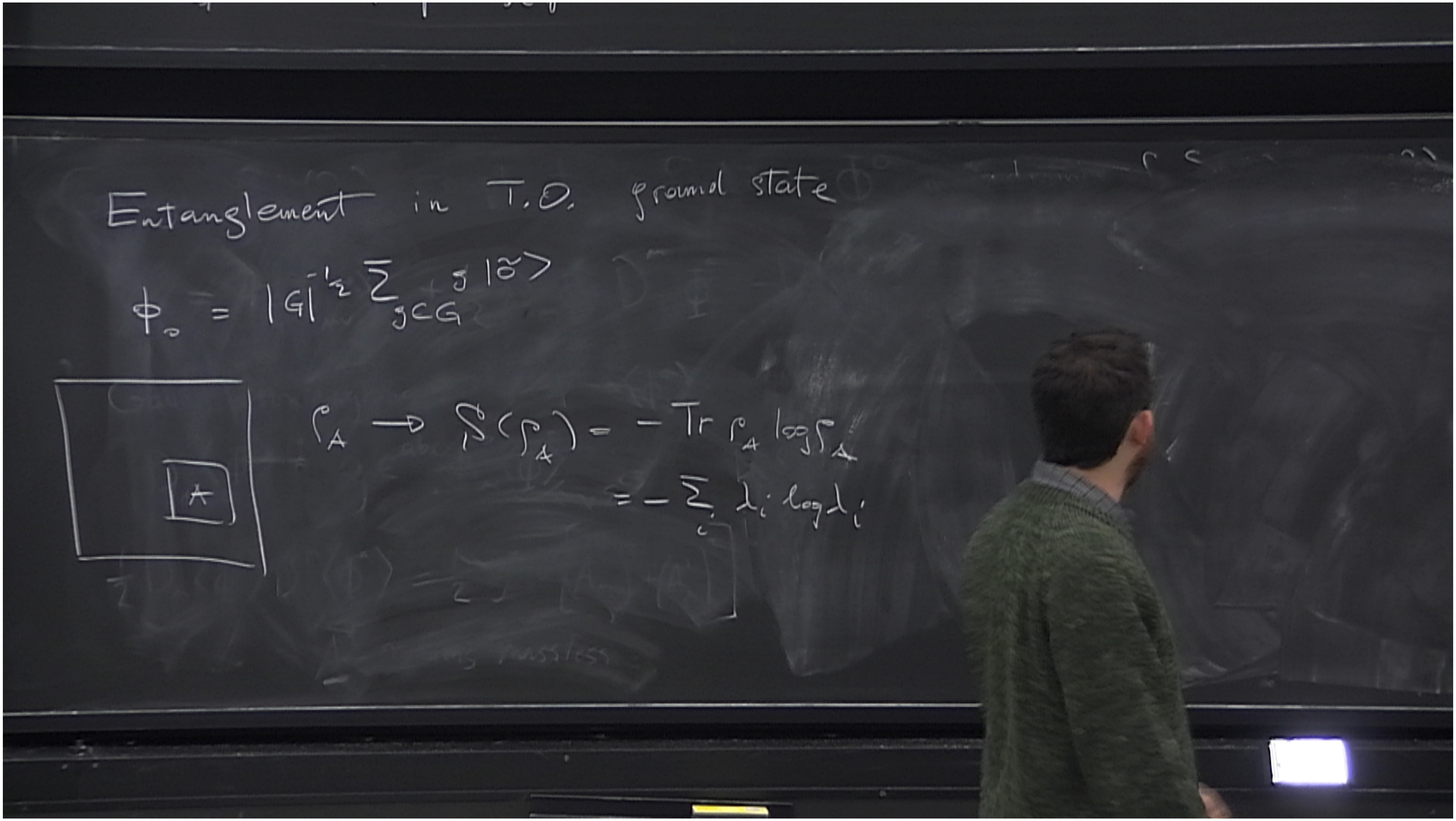
$\Rightarrow$ $\phi_0 \to \rho_A$ $\quad$ A is a subregion

$W_1 \phi_0 \to \rho'_A$

$\rho_A = \rho'_A$

C_2, C_1'

$\{W_1^x, W_1^z\}$


Entanglement in T.O. ground state
$\phi_0 = |G|^{-1/2} \sum_{g \in G} g|\tilde{0}\rangle$
A
$\rho_A \rightarrow S(\rho_A) = -\mathrm{Tr}\, \rho_A \log \rho_A$
$= -\sum_i \lambda_i \log \lambda_i$

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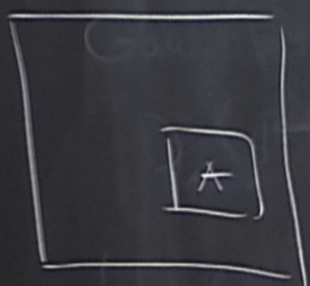
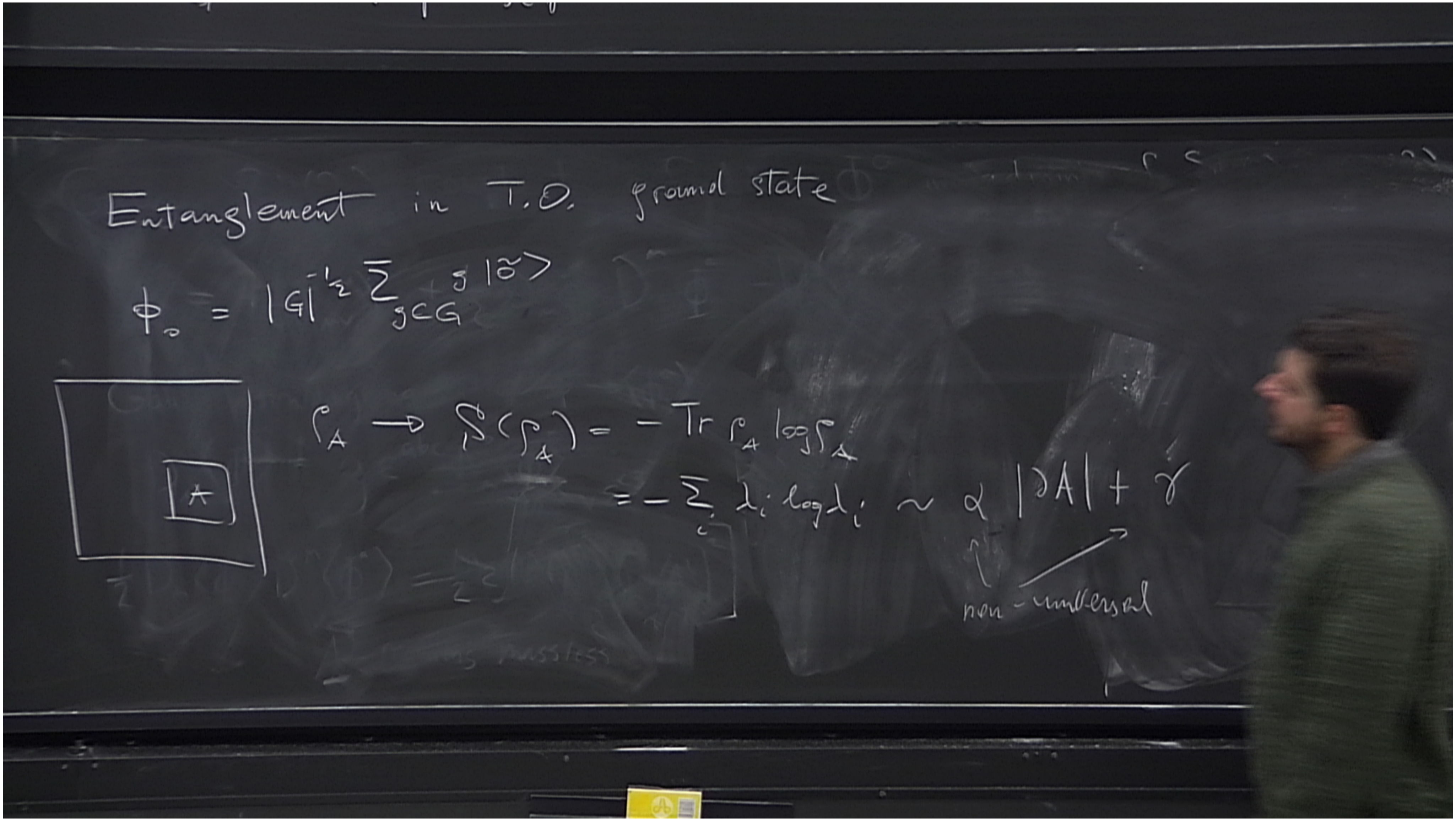


$$\rho_A \longrightarrow S(\rho_A) = -\mathrm{Tr}\,\rho_A \log \rho_A$$

$$= -\sum_i \lambda_i \log \lambda_i \sim \alpha\,|\partial A| + \gamma$$

non-universal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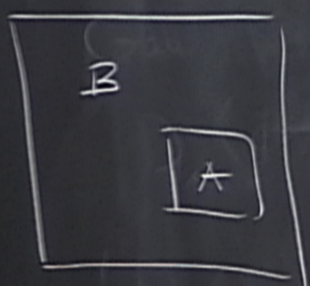
Entanglement in T.O. ground state

$$\phi_0 = |G|^{-1/2} \sum_{g\in G} g|\tilde{0}\rangle$$

$$G_A = \{ g\in G \mid g = g_A \otimes 1\!\!1_B \}$$

$$G_B$$



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S is invariant under local unitaries

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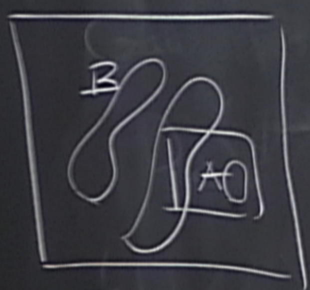

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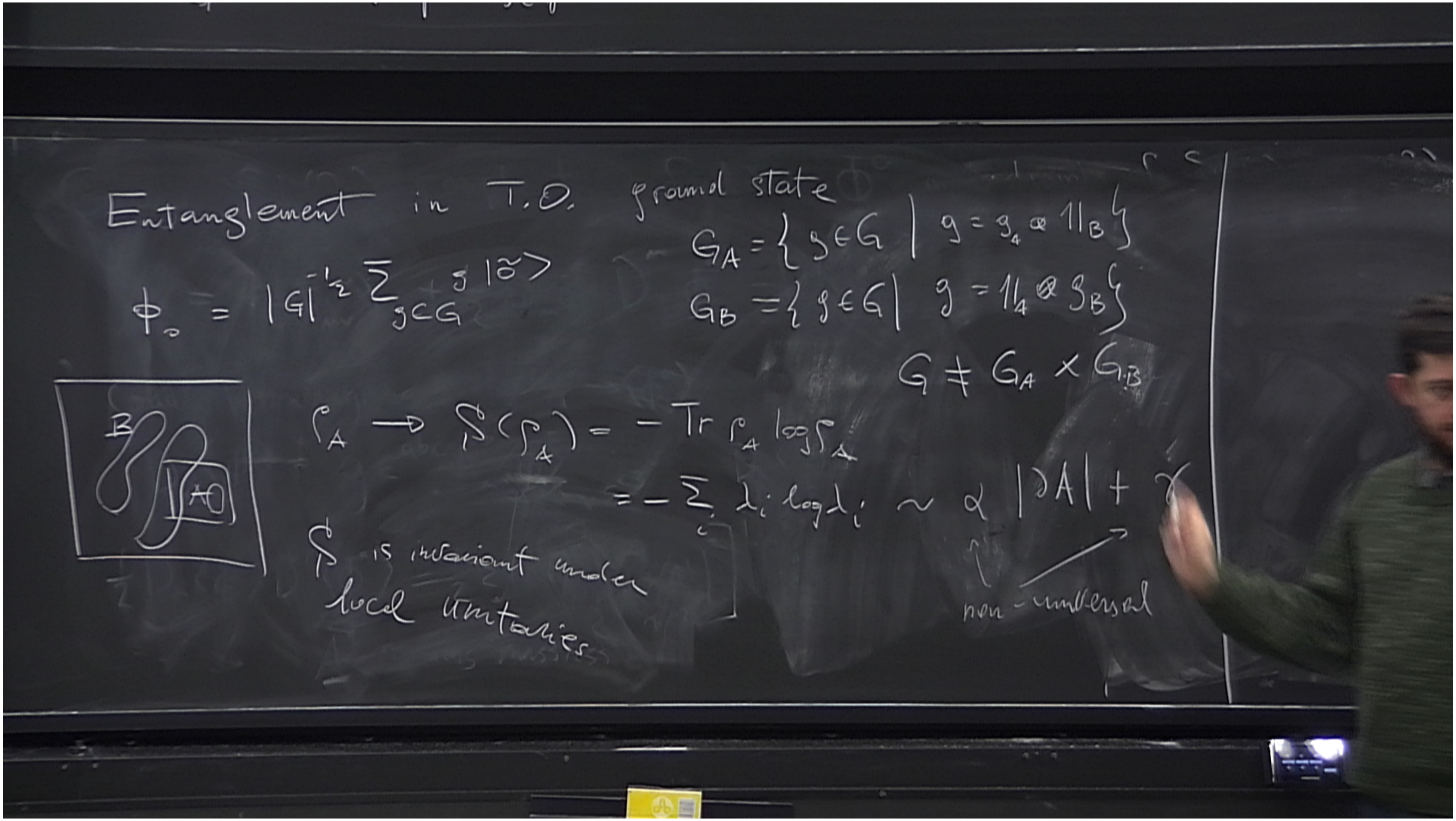

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$G \neq G_A \times G_B$
B
A
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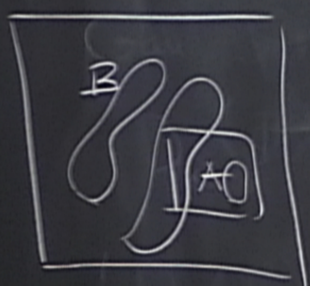

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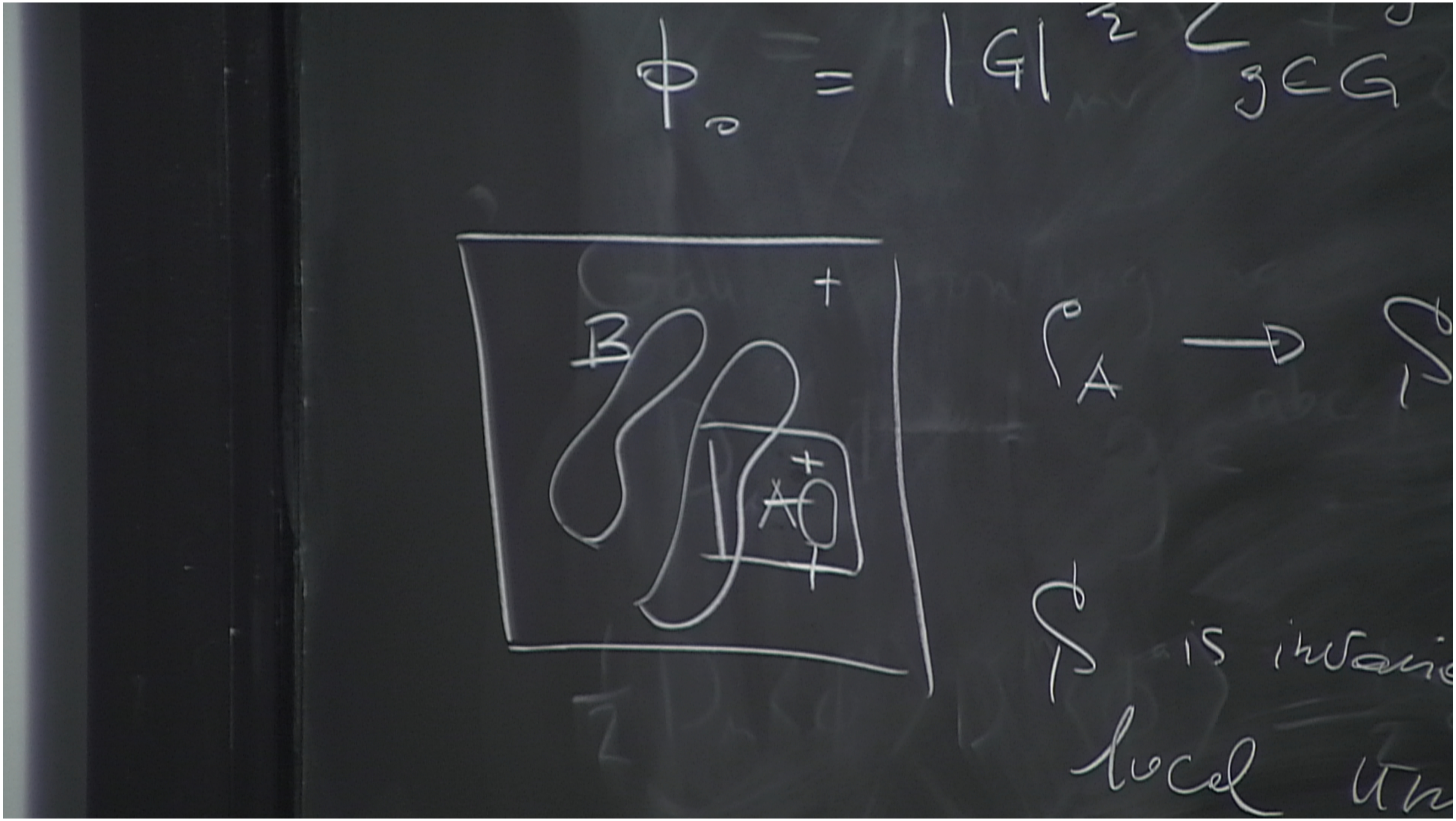

B
A
is
local

$$S = \log \frac{|G|}{|G_A||G_B|} =$$

$\log|G| \sim$ number of A operators

$\log|G_A|, \log|G_B| \sim$ number of A operators that just in A or B

$$G_{AB} = G / G_A \times G_B$$

$\log|G_{AB}| \sim$ the # of A operators near

$$S = \log \frac{|G|}{|G_A|\,|G_B|} =$$

$\log|G| \sim$ number of A operators

$\log|G_A|, \log|G_B| \sim$ number of A operators that just in A or B

$$G_{AB} = G/_{G_A \times G_B}$$

$\log|G_{AB}| \sim$ the # of A operators near the boundary

$$S = \log \frac{|G|}{|G_A||G_B|} = |\partial A| - 1$$

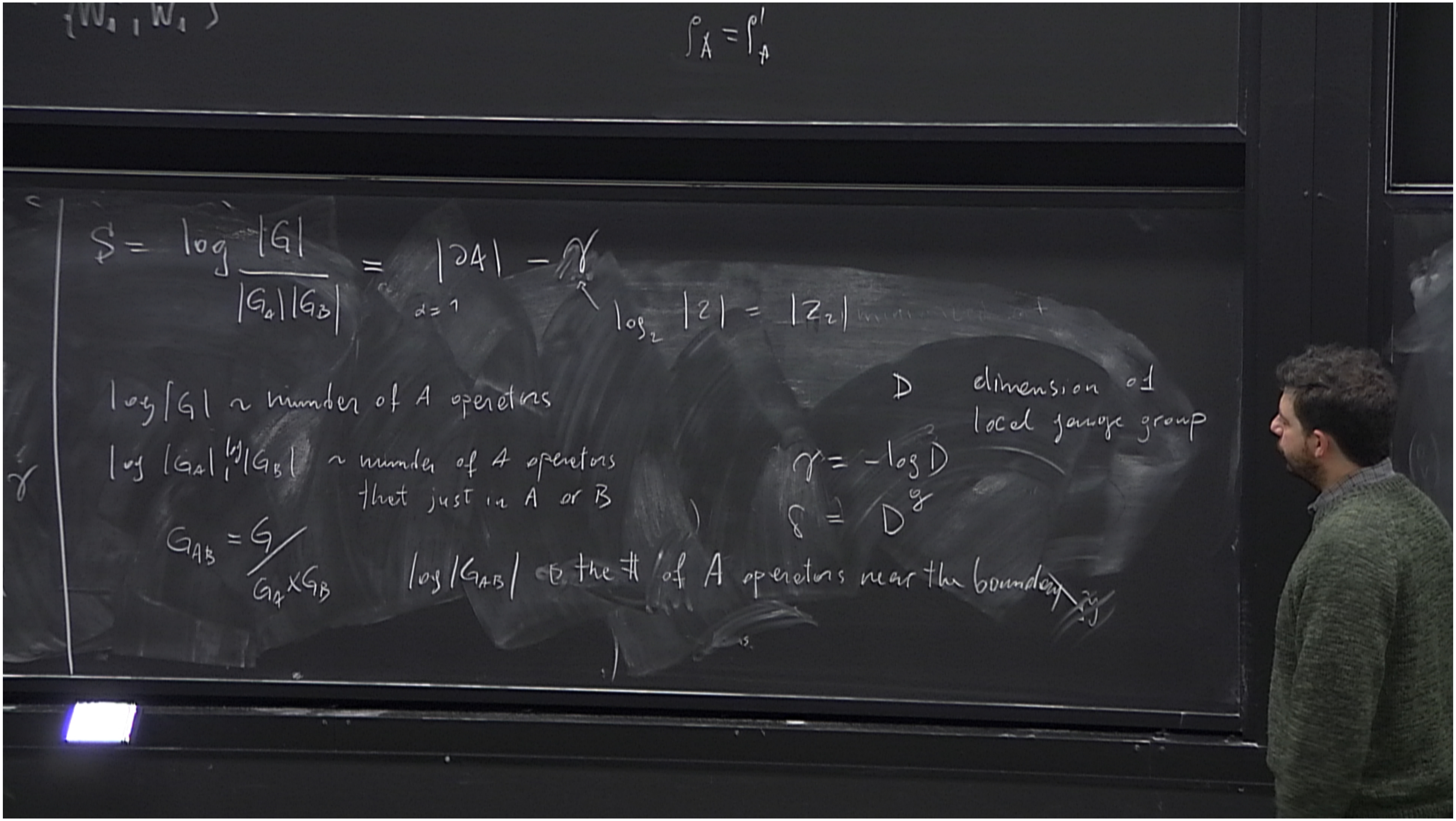
$\alpha = 1$

$\log_2 |Z| = |Z_2|$

$\log |G| \sim$ number of A operators

$\log |G_A|, \log |G_B| \sim$ number of A operators that just in A or B

$G_{AB} = G/$


$\rho_A = \rho'_A$
$S = \log \frac{|G|}{|G_A||G_B|} = |\partial A| - \gamma$
$\alpha = 1$
$\log_2 |Z| = |Z_2|$
$\log|G| \sim$ number of A operators
$\log |G_A|, \log |G_B|$ ~ number of A operators that just in A or B
$G_{AB} = G / G_A \times G_B$
$\log|G_{AB}|$ ~ the # of A operators near the boundary
D dimension of local gauge group
$\gamma = -\log D$
$g = D^{\gamma}$

Z_2 $\quad |0\rangle, |1\rangle$

$e|1\rangle \equiv |0\rangle \equiv |e\rangle$

$\sigma^x|0\rangle = |1\rangle$

$L_g^+ |\tilde{g}\rangle = |g\tilde{g}\rangle$

$L_g^- |\tilde{g}\rangle = |\tilde{g}g^{-1}\rangle$

D dimensional

$|0\rangle \equiv |e\rangle$

$|1\rangle \equiv |g_1\rangle$

$\vdots$

$|D-1\rangle \equiv |g_{D-1}\rangle$

$\mathbb{Z}_2 \qquad |0\rangle , |1\rangle$

$e|1\rangle \equiv |0\rangle \equiv |e\rangle$

$\sigma^x |0\rangle = |1\rangle$

D dimensional

$|0\rangle \equiv |e\rangle$

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$\vdots$

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$L_g^+ |\tilde{g}\rangle = |g\tilde{g}\rangle$

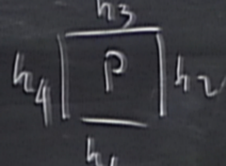
$L_g^- |\tilde{g}\rangle = |\tilde{g}g^{-1}\rangle$

$T_h^+ |g\rangle = \delta_{g,h} |g\rangle$

$T_h^- |g\rangle = \delta_{g,h^{-1}} |g\rangle$

$$A_g(s) = \prod_{j \in s} L^+_g \qquad A(s) = \sum_g A_g(s)$$

$$H = \sum_p (1 - B_p) + \sum_s (1 - A_s$$



$$B_p = \sum_{h_1 h_2 h_3 h_4 = e} T_{h_1} T_{h_2} T_{h_3} T_{h_4}$$

$G_A \times G_B$ $\quad \log |G_{AB}|$ is the # of A operators near the boundary

$A_g(s) = \prod_{j \in s} L^+_g \qquad A(s) = \sum_g A_g(s)$

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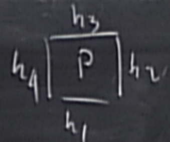
Quantum Double



$$B_p = \sum_{h_1 h_2 h_3 h_4 = e} T_{h_1} T_{h_2} T_{h_3} T_{h_4}$$

$$\gamma = \log D$$

Spectrum

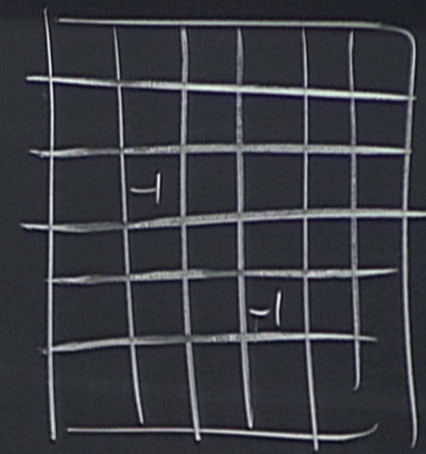
$$|\overbrace{A_1, \dots, A_{L^2-1}}^{A_{L^2}}; B_1, \dots, B_{L^2-1}; W_1 W_2 \rangle$$

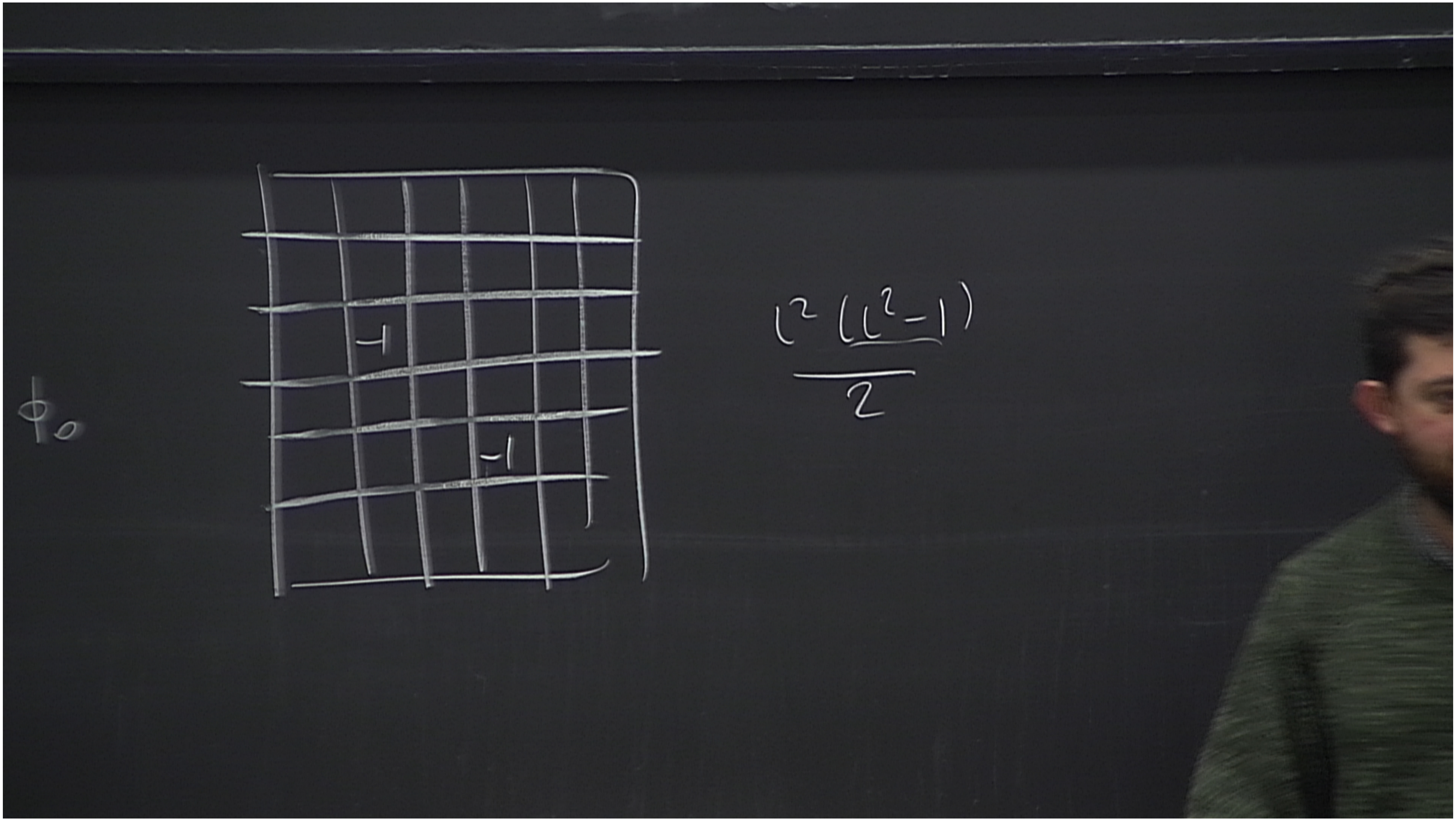
$$\quad +1 \quad\quad +1 \quad\quad +1 \quad\quad +1 \quad\quad +1 \quad +1 \qquad \equiv \phi_0$$

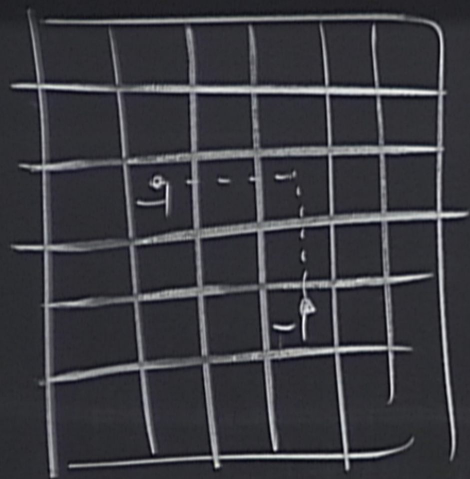
B_{L^2}

L^2 sites $\to 2L^2$ links

$$A_s \psi = \psi$$

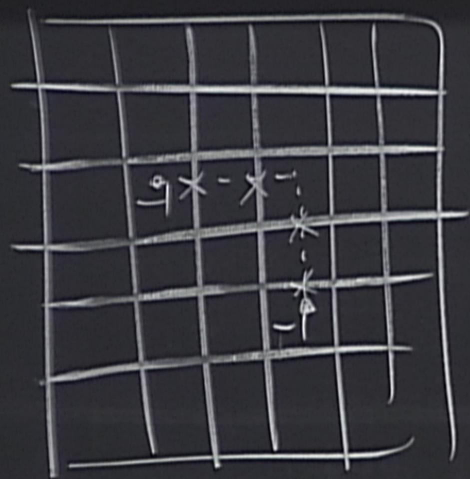



ϕ_0
-1
-1
$\frac{L^2(L^2-1)}{2}$



$$\frac{L^2(L^2-1)}{2}$$ degeneracy of 1st excited state

$$\prod_p B_p = +1$$



$$\frac{L^2(L^2-1)}{2}$$ degeneracy of 1st excited state

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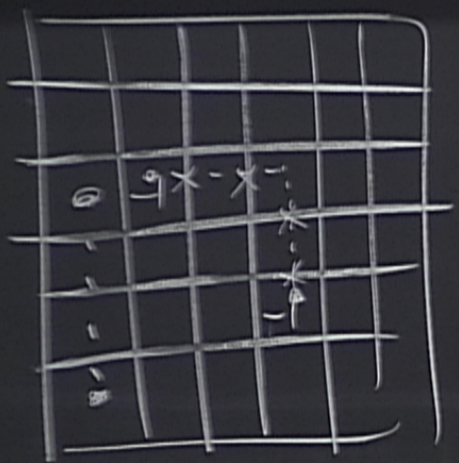
$$W_\gamma = \prod_{j \in \gamma} \sigma^x_j$$

$\underline{W_\gamma |\psi_0\rangle}$ is an eigenstate of H w/ energy

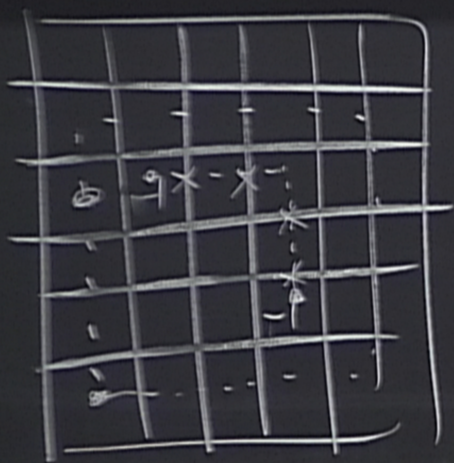


$$4\times\frac{L^2(L^2-1)}{2}$$ degeneracy of 1st excited state

$$\prod_p B_p = +1$$

$$W_\gamma = \prod_{j\in\gamma}\sigma_j^x$$

$\underline{W_\gamma|\phi_0\rangle}$ is an eigenstate of H w/ energy $4J$



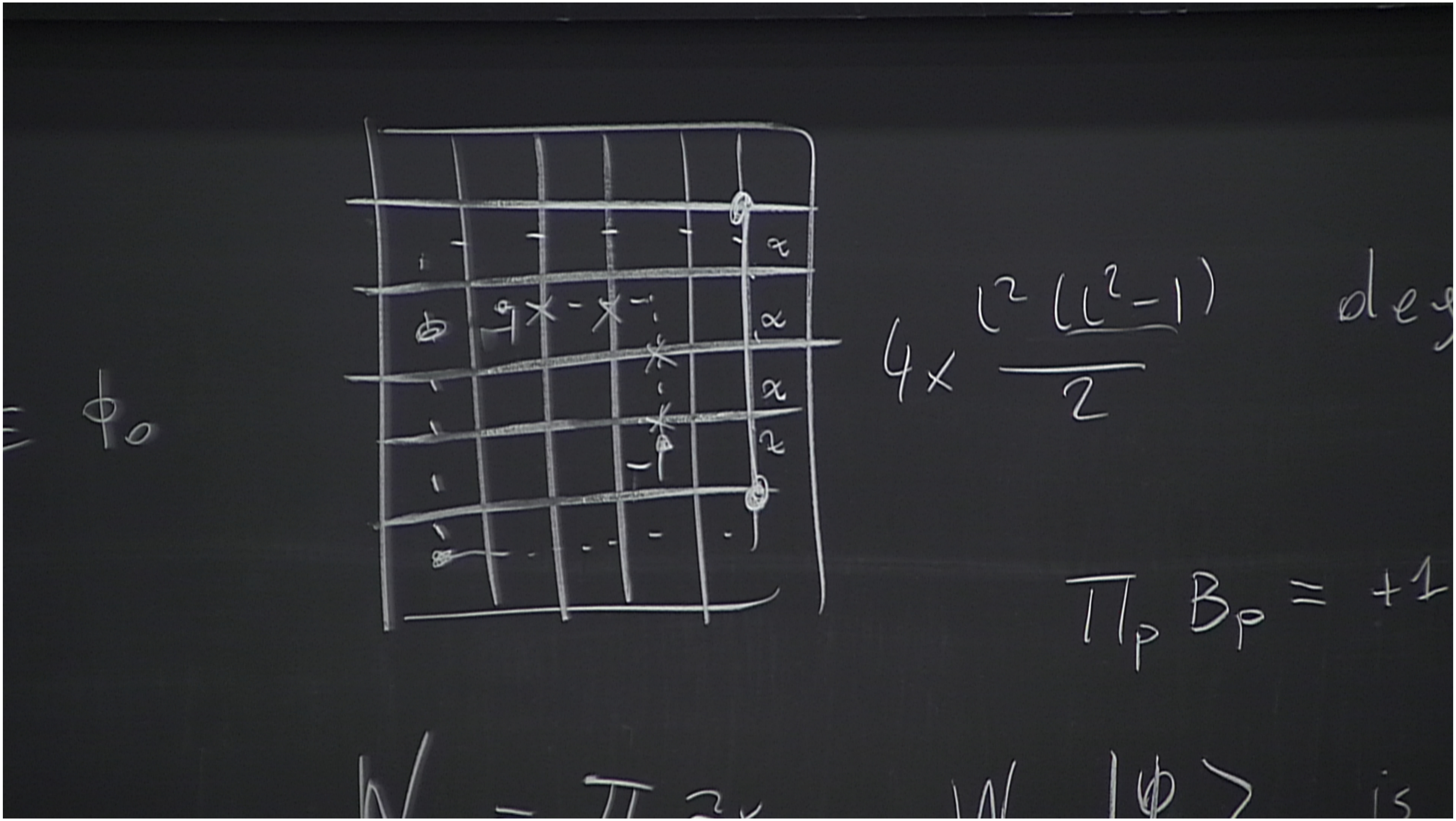
$$4\times\frac{L^2(L^2-1)}{2}$$ degeneracy of 1st excited state

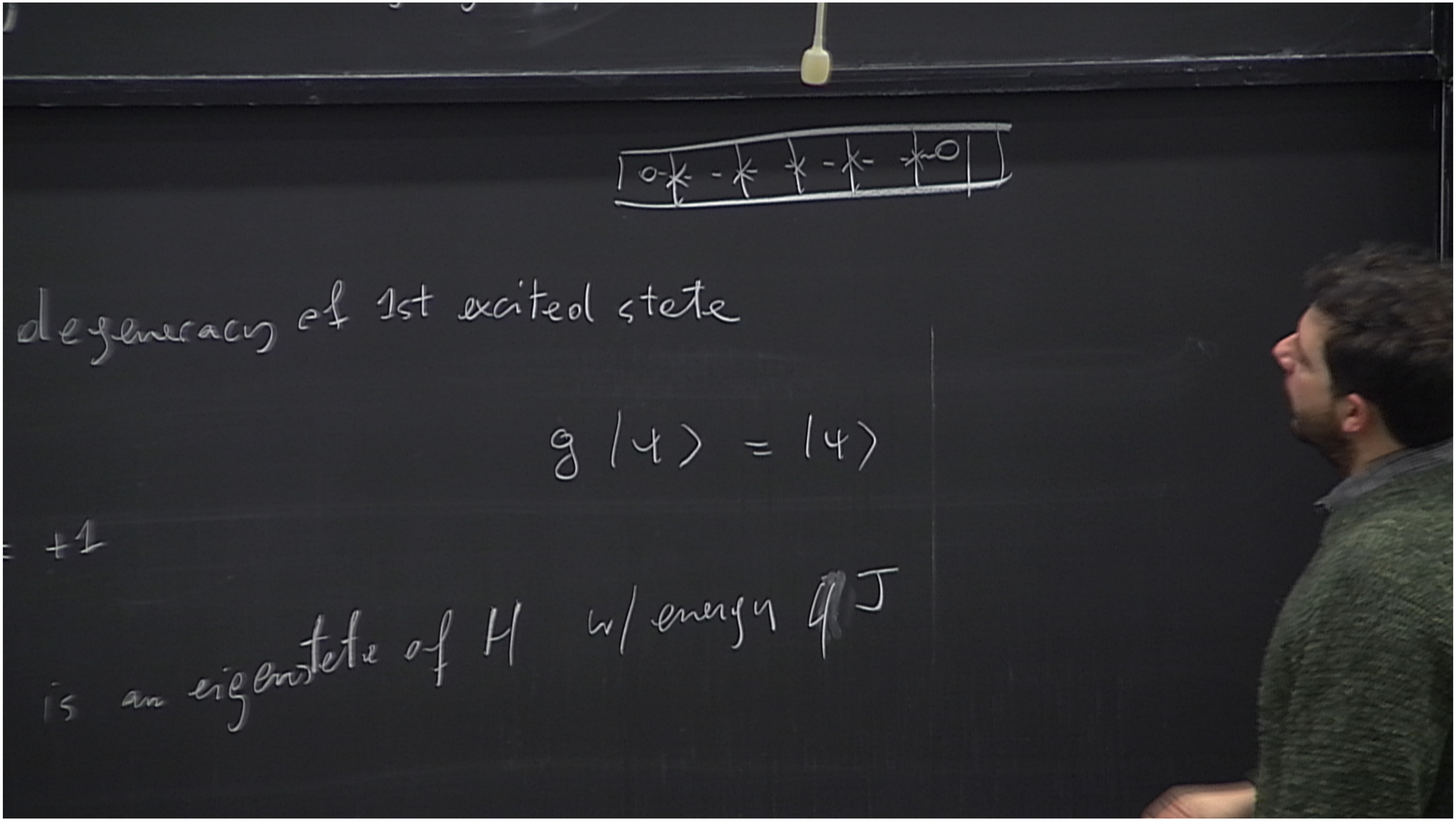
$$g\,|\psi\rangle = |\psi\rangle$$

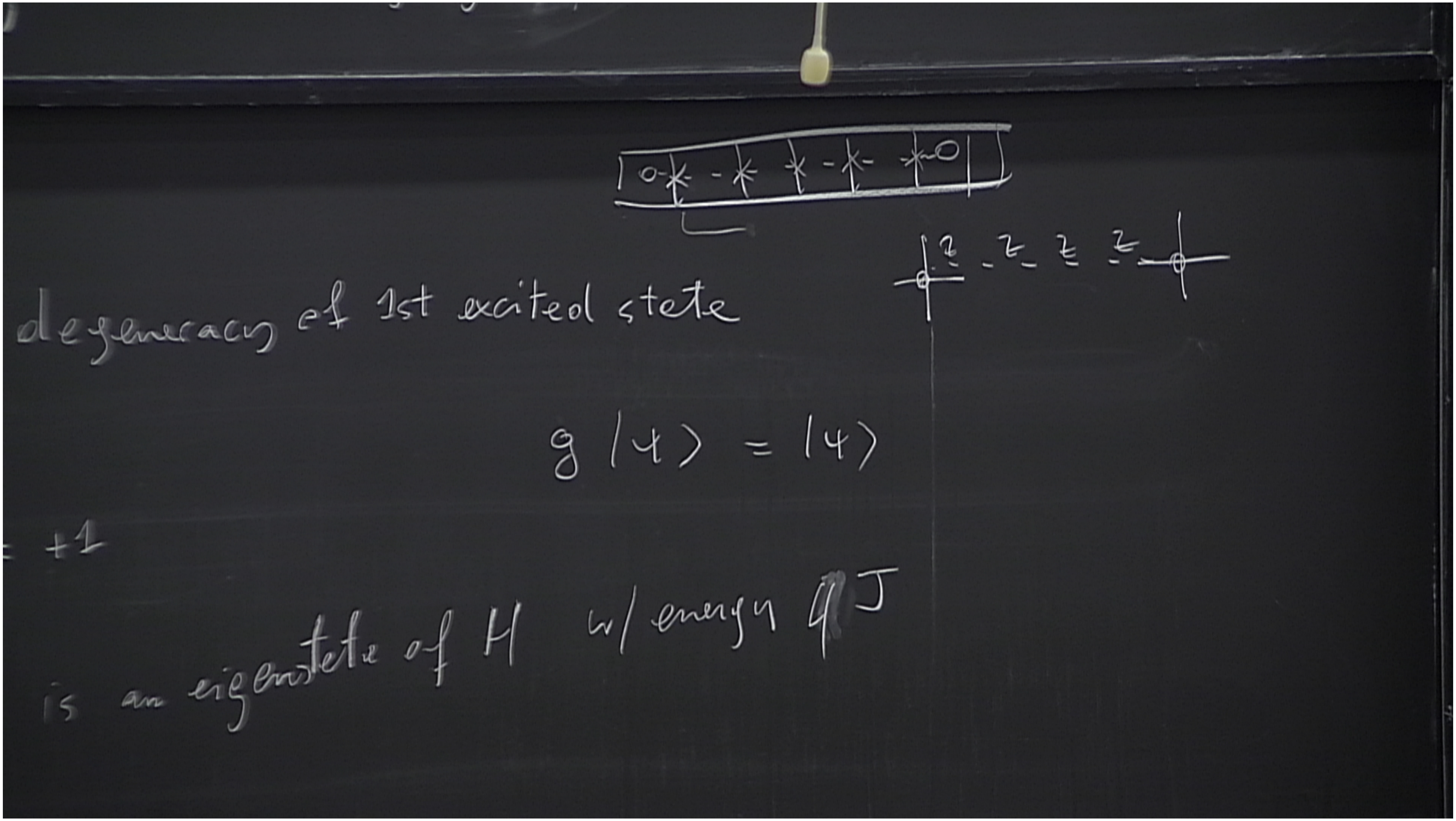
$$\prod_p B_p = +1$$

$$W_\gamma = \prod_{j\in\gamma} \sigma_j^x$$

$g\,\underline{W_\gamma\,|\phi_0\rangle}$ is an eigenstate of H w/ energy $4J$


ϕ_0
$4 \times \frac{L^2(L^2-1)}{2}$
deg
$\Pi_p B_p = +1$
is


degeneracy of 1st excited state
g |ψ> = |ψ>
= +1
is an eigenstate of H
w/ energy 4J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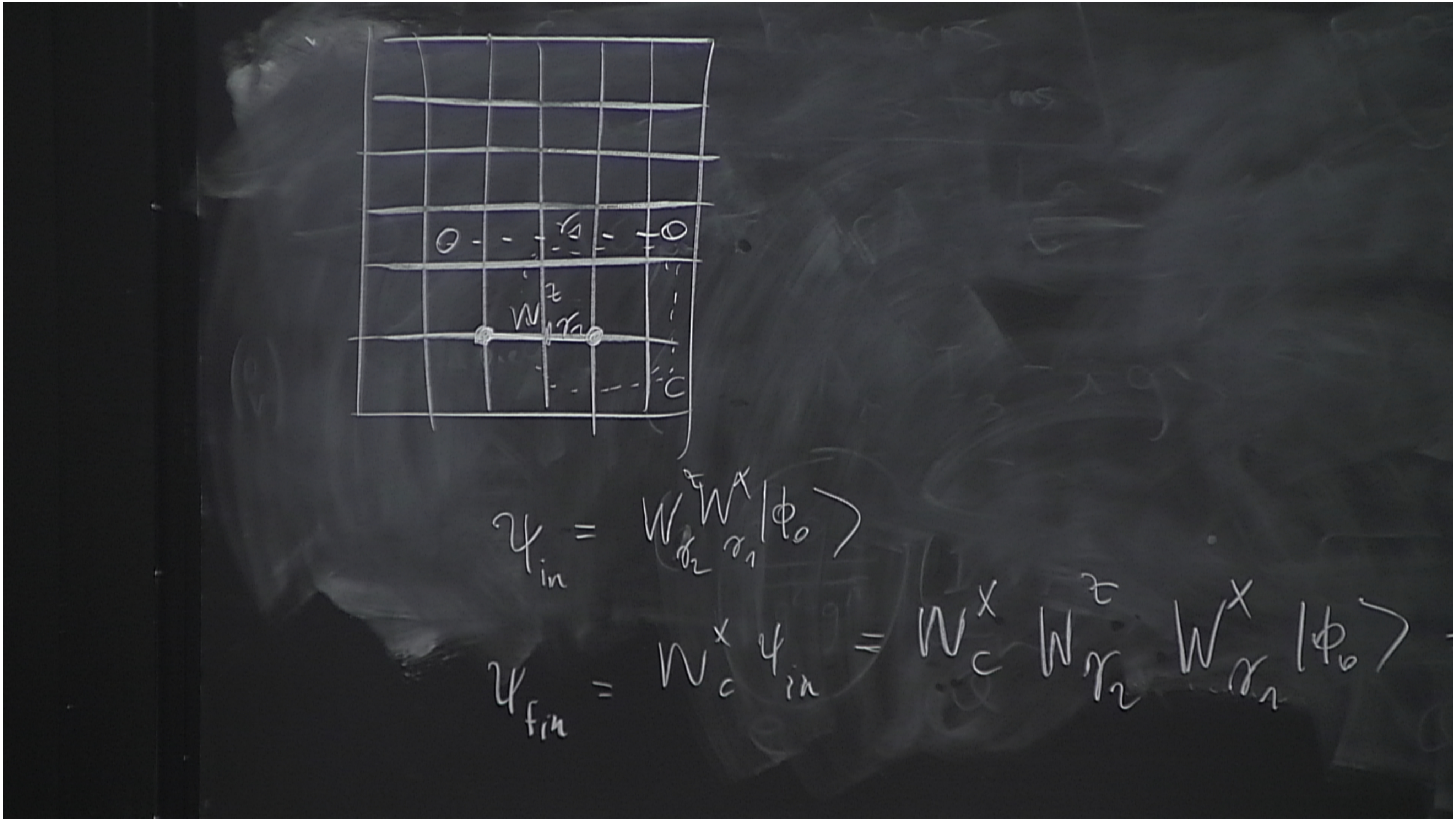
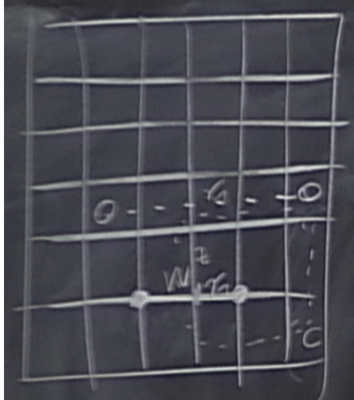
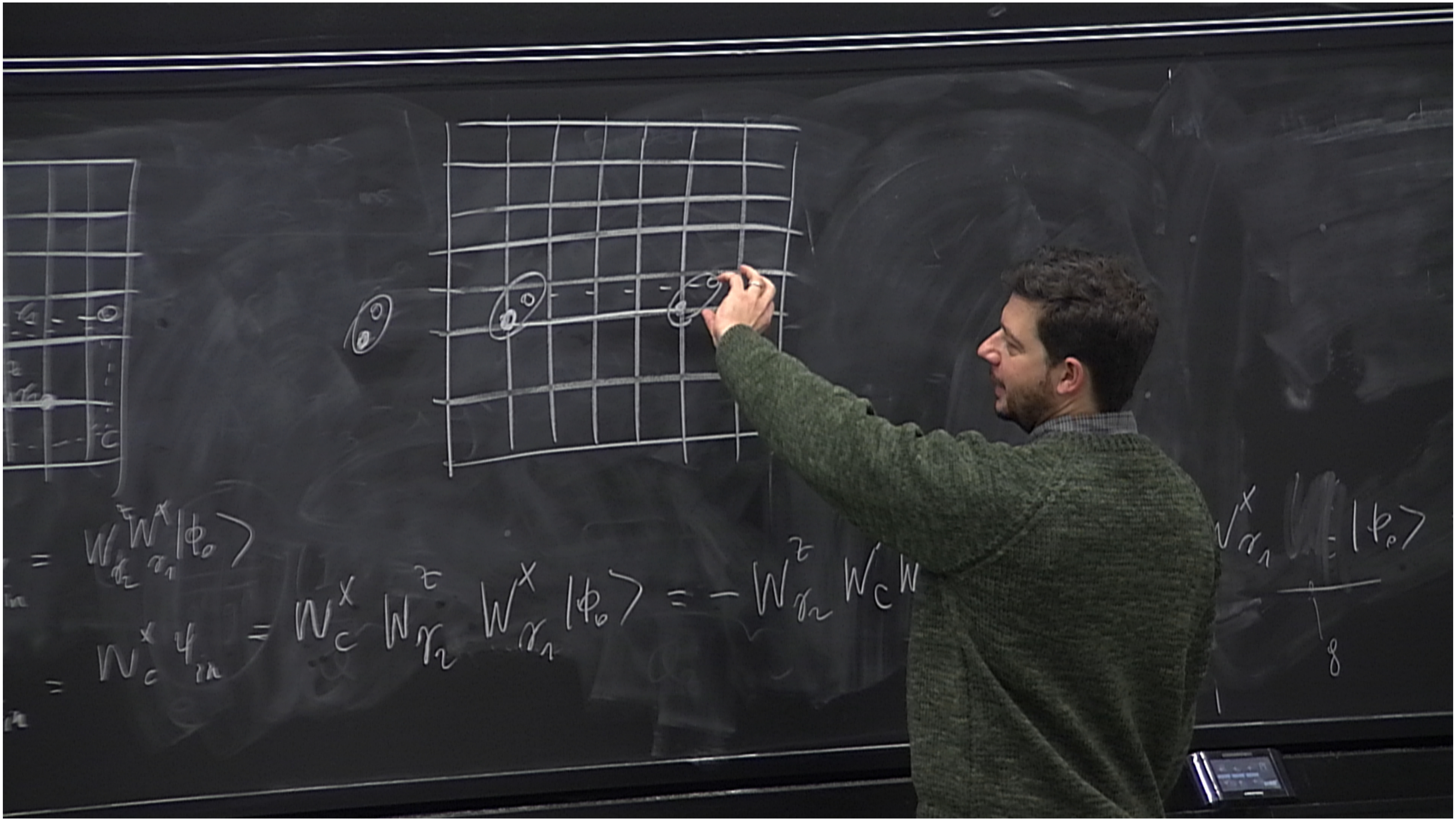
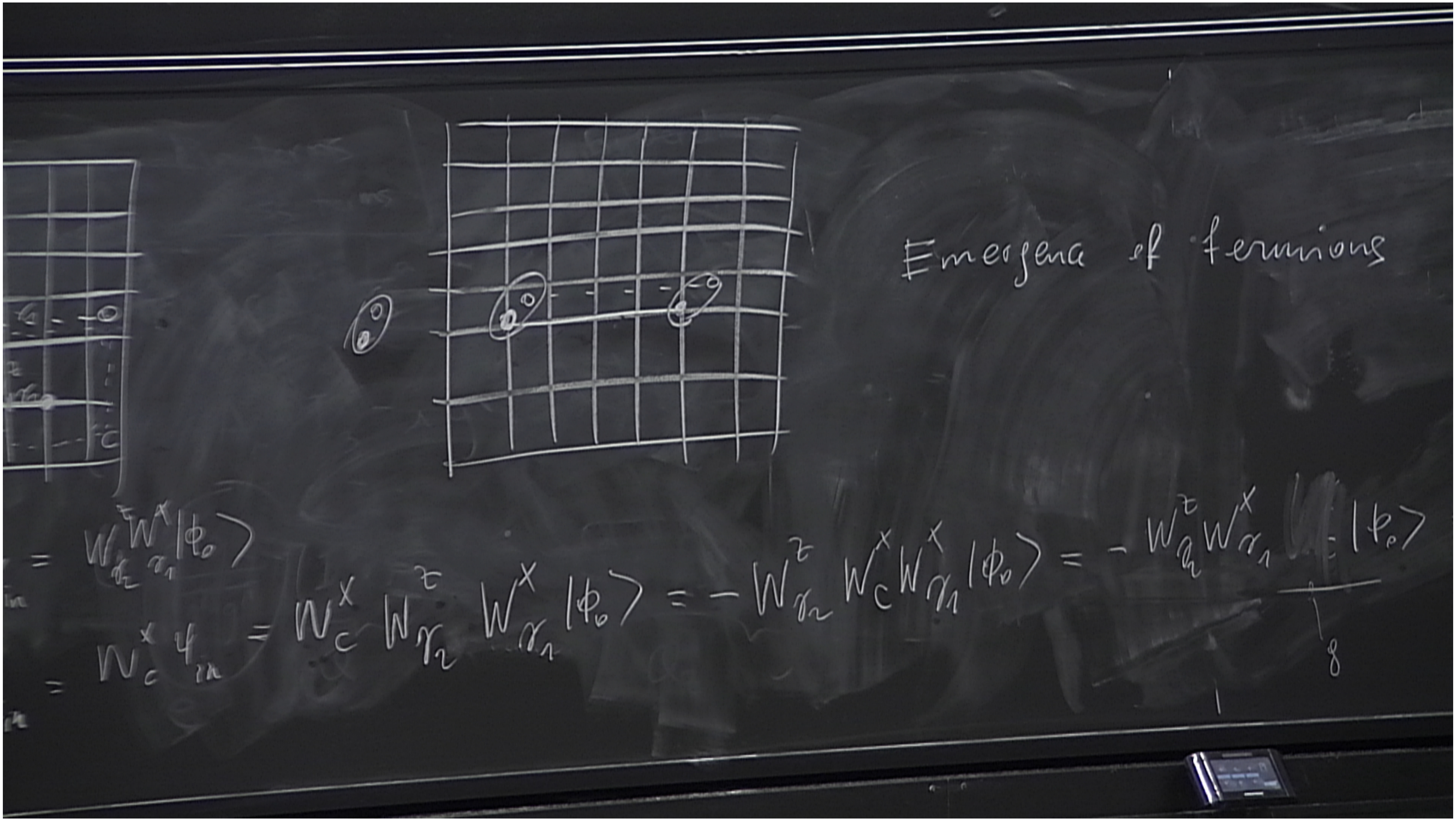

γ_1
$W^z_{\gamma_1}$
C
$\psi_{in} = W^z_{\gamma_2} W^x_{\gamma_1} |\phi_0\rangle$
$\psi_{fin} = W^x_c \psi_{in} = W^x_c W^z_{\gamma_2} W^x_{\gamma_1} |\phi_0\rangle$



$$\Psi_{in} = W^z_{\gamma_2} W^x_{\gamma_1} |\phi_0\rangle$$

$$\Psi_{fin} = W^x_c \Psi_{in} = W^x_c W^z_{\gamma_2} W^x_{\gamma_1} |\phi_0\rangle = -W^z_{\gamma_2} W^x_c W^x_{\gamma_1} |\phi_0\rangle = -W^z_{\gamma_2} W^x_{\gamma_1} |\phi_0\rangle = -\Psi_{in}$$




Emergence of fermions