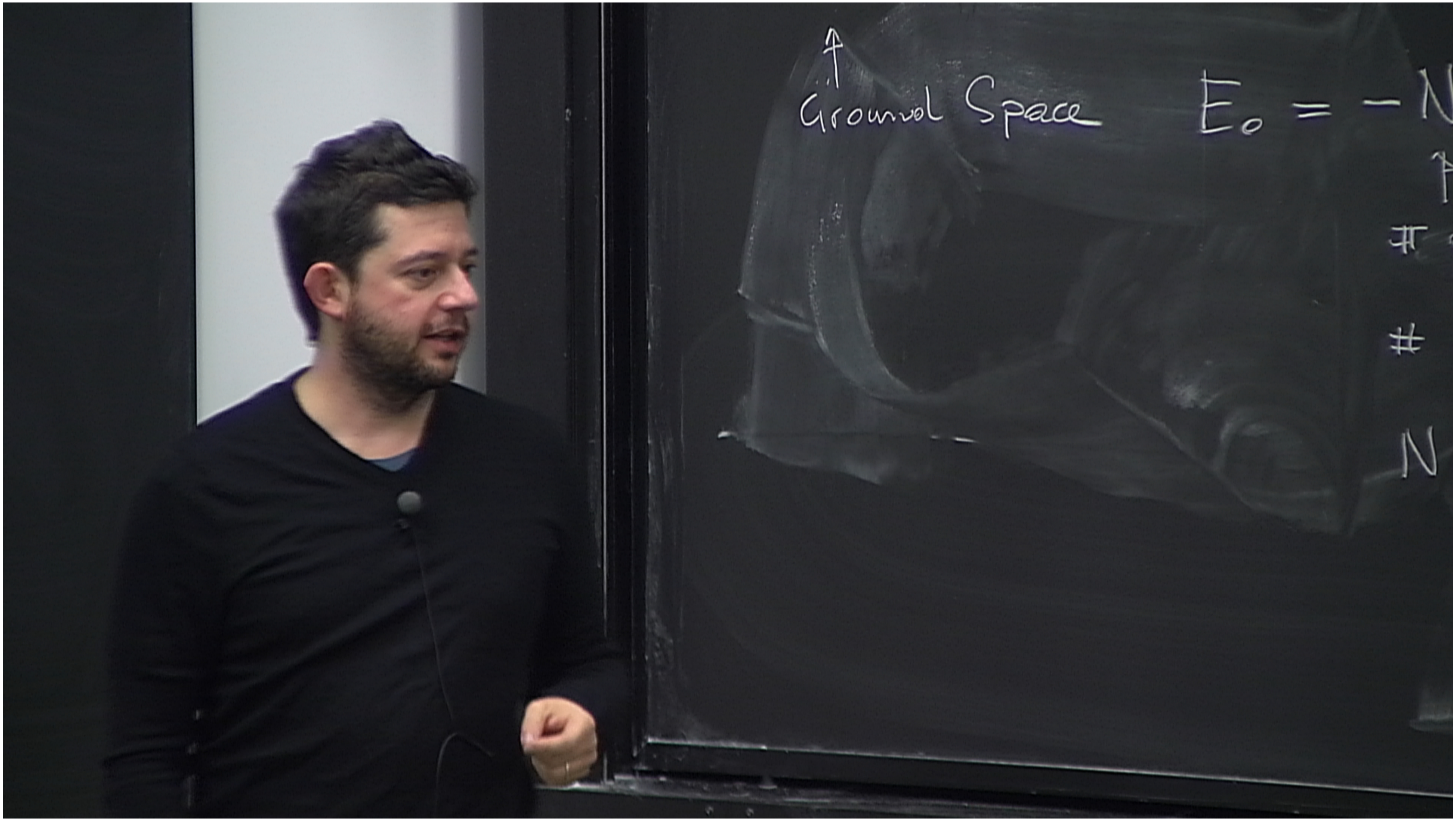
Title: Condensed Matter (Review) - Lecture 12

Date: Jan 17, 2012 10:15 AM

URL: http://pirsa.org/12010098

Abstract:

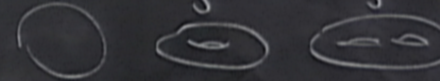


Ground Space
$E_0 = -N$

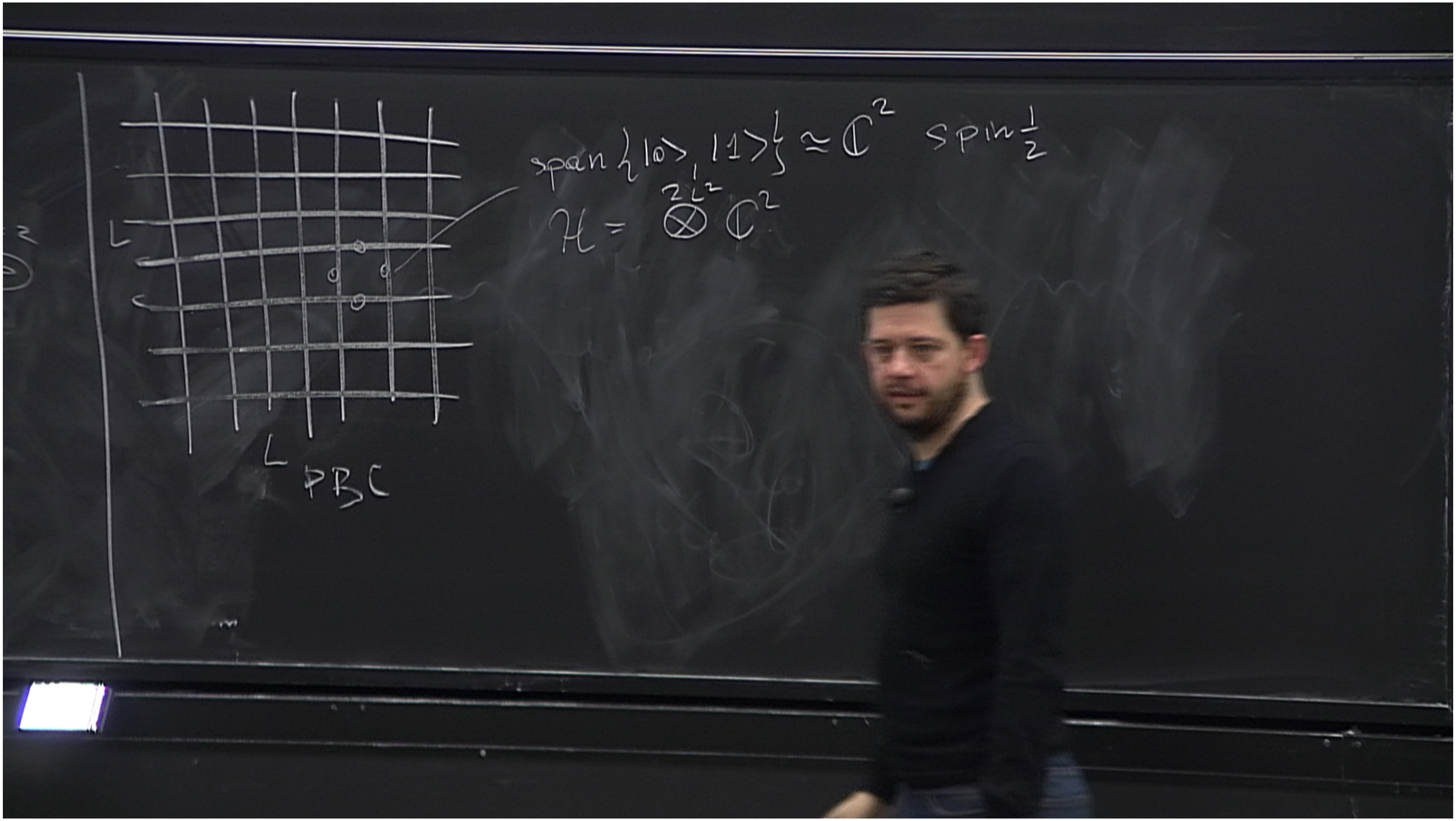
Topological Quantum order

1. Degenerate GS with a gap Δ
2. degeneracy depends on topology q^g

$g=0$ $g=1$ $g=2$

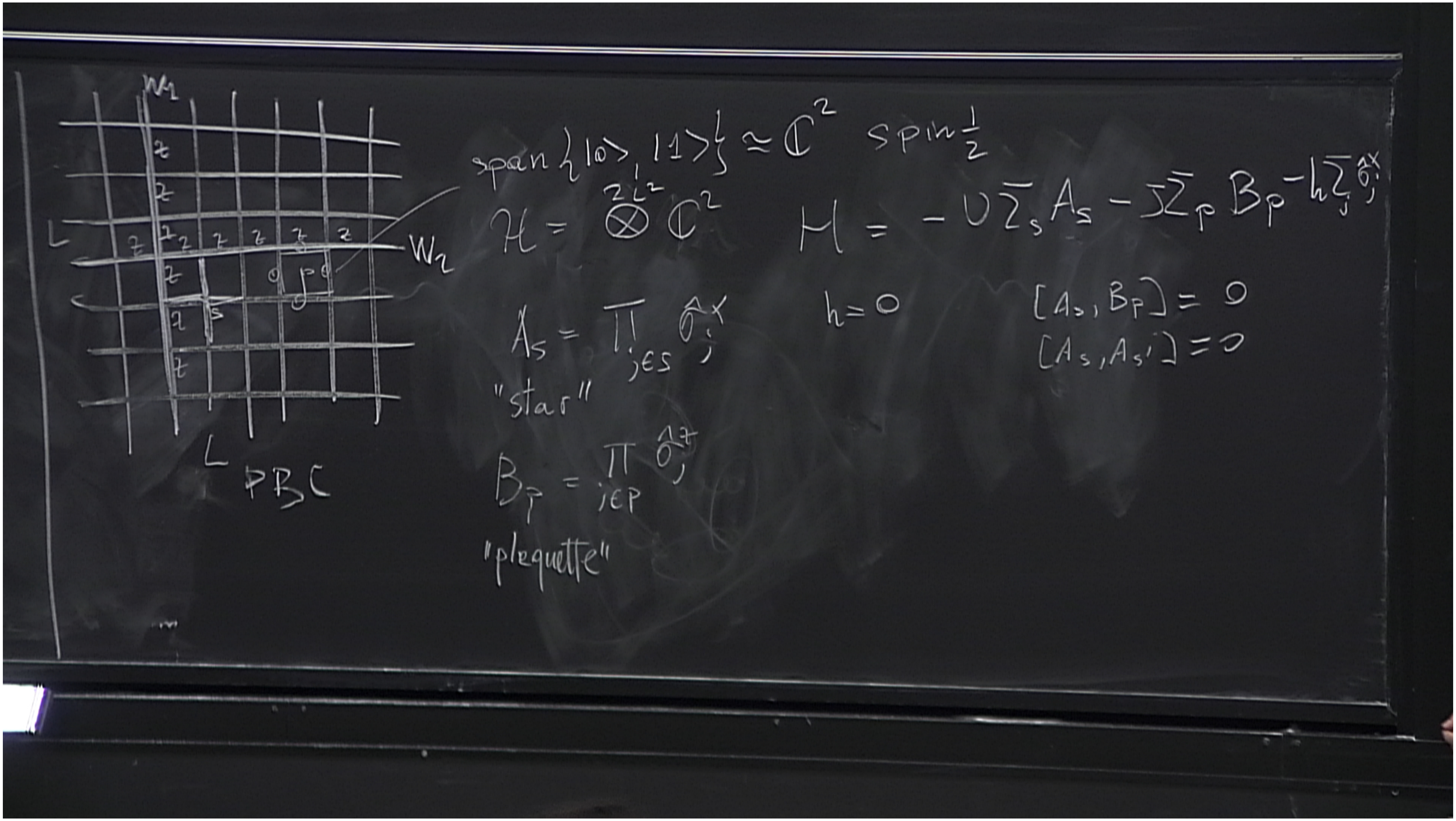


3. different GS are locally indistinguishable

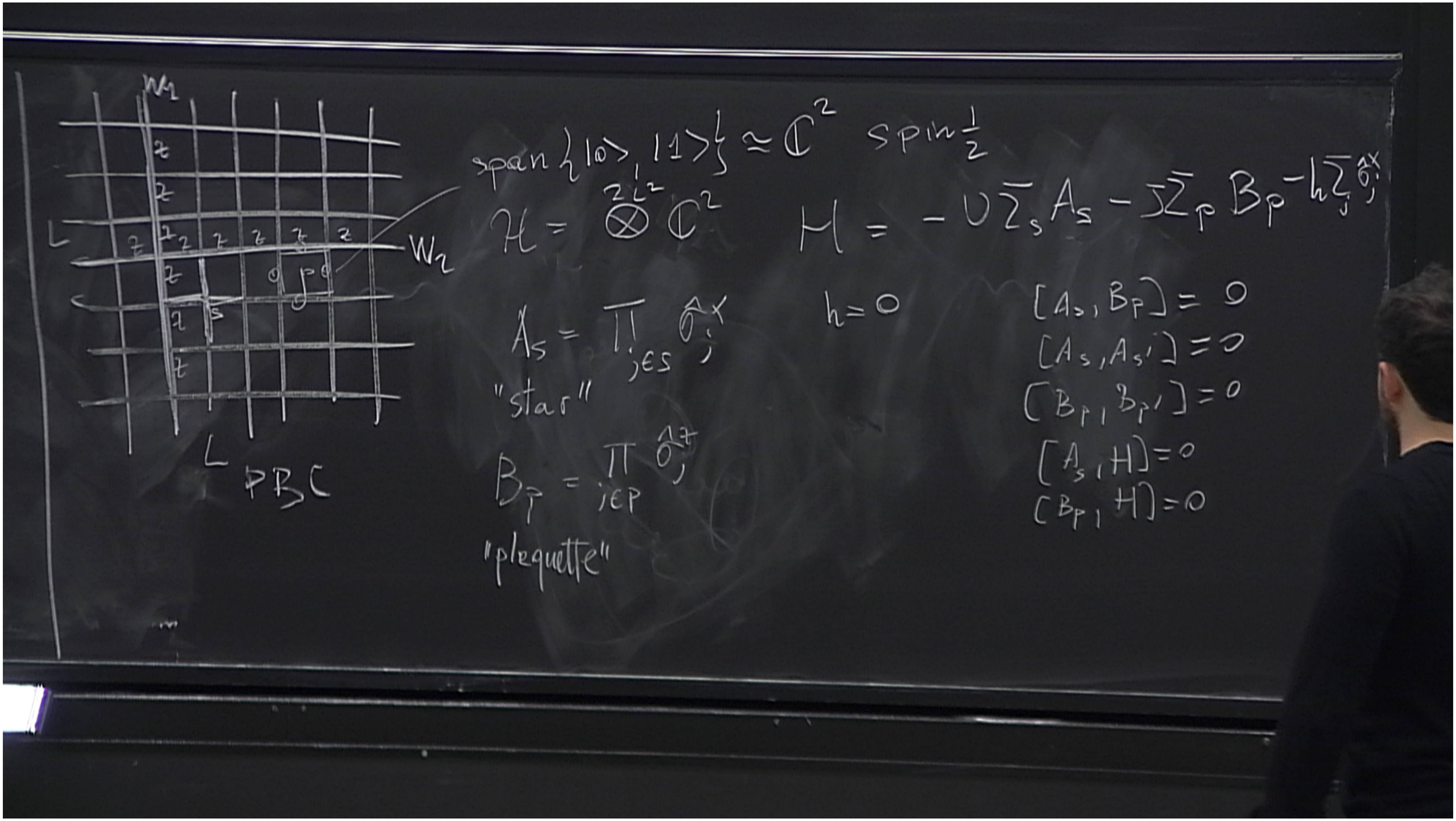

span{|0>, |1>} ≃ ℂ2 spin 1/2
H = ⊗ 2L2 ℂ2
L
L
PBC

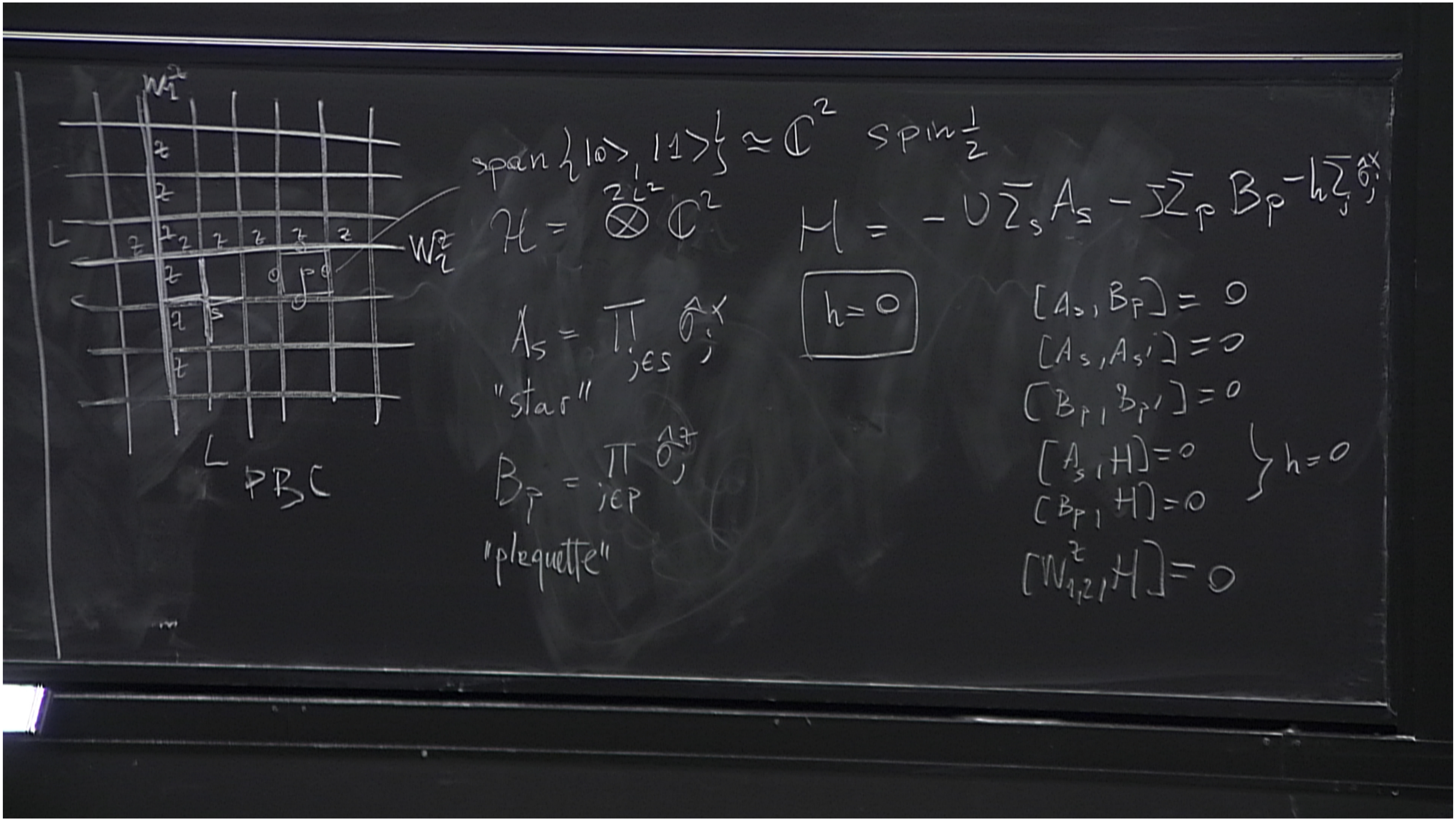
$$\text{span}\{|0\rangle, |1\rangle\} \simeq \mathbb{C}^2 \quad \text{spin } \tfrac{1}{2}$$

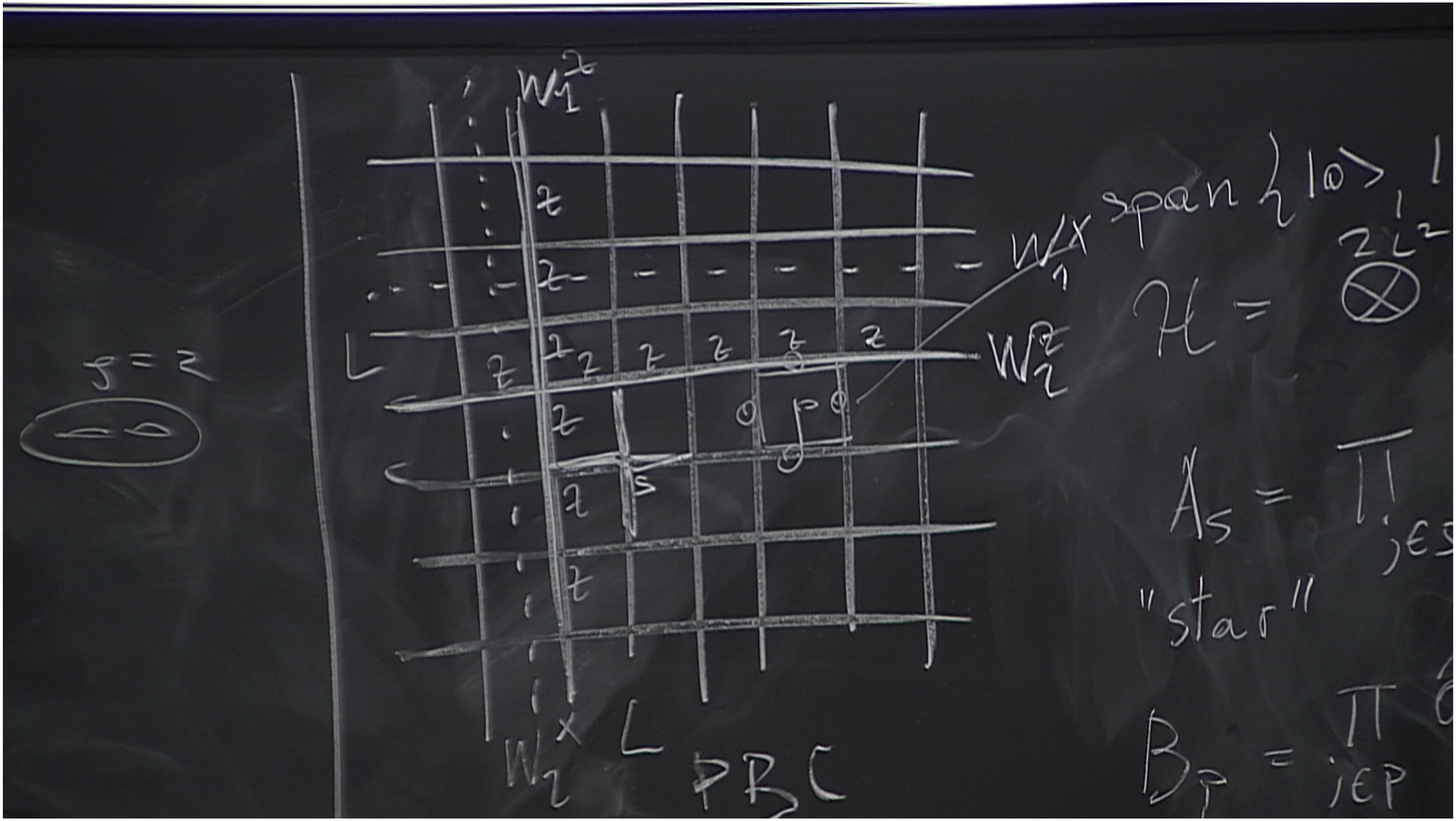
$$\mathcal{H} = \bigotimes^{2L^2} \mathbb{C}^2$$

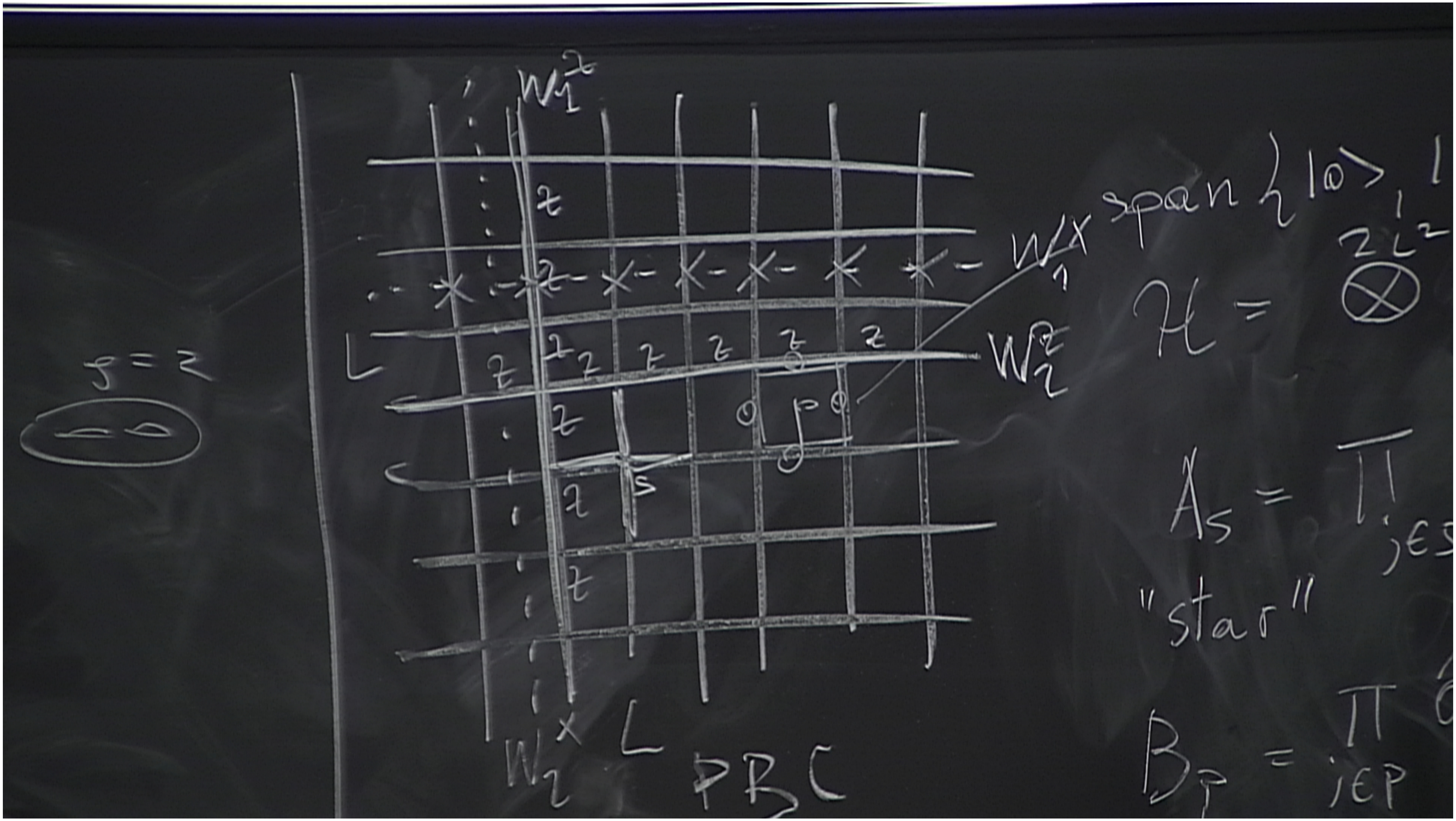


W1
W2
L
L
PBC
$\mathrm{span}\{|0\rangle, |1\rangle\} \simeq \mathbb{C}^2$ spin $\frac{1}{2}$
$\mathcal{H} = \bigotimes^{2L^2} \mathbb{C}^2$
$H = -U\sum_s A_s - J\sum_p B_p - h\sum_j \hat{\sigma}_j^x$
$A_s = \prod_{j\in s} \hat{\sigma}_j^x$
"star"
$B_p = \prod_{j\in p} \hat{\sigma}_j^z$
"plequette"
$h=0$
$[A_s, B_p] = 0$
$[A_s, A_{s'}] = 0$


W1
W2
L
L
PBC
span{|0>, |1>} ≃ $\mathbb{C}^2$ spin $\frac{1}{2}$
$\mathcal{H} = \bigotimes^{2L^2} \mathbb{C}^2$
$H = -U\sum_s A_s - J\sum_p B_p - h\sum_j \hat{\sigma}_j^x$
$A_s = \prod_{j\in s} \hat{\sigma}_j^x$
"star"
$B_p = \prod_{j\in p} \hat{\sigma}_j^z$
"plequette"
$h=0$
$[A_s, B_p] = 0$
$[A_s, A_{s'}] = 0$
$[B_p, B_{p'}] = 0$
$[A_s, H] = 0$
$[B_p, H] = 0$


span{|0>, |1>} ≃ C^2 spin 1/2
H = ⊗^{2L^2} C^2
H = −U Σ_s A_s − J Σ_p B_p − h Σ_j σ_j^x
h = 0
A_s = Π_{j∈s} σ_j^x
"star"
B_p = Π_{j∈p} σ_j^z
"plequette"
[A_s, B_p] = 0
[A_s, A_s'] = 0
[B_p, B_p'] = 0
[A_s, H] = 0
[B_p, H] = 0
h = 0
[W^z_{1,2}, H] = 0
W_1^z
W_2^z
L
PBC


g = 2
L
W_1^z
W_1^x
W_2^z
W_2^x
L
PBC
span{|0>, |1>}
$\mathcal{H} = \bigotimes$
$A_s = \prod_{j \in s}$
"star"
$B_p = \prod_{j \in p}$


g = 2
"star"
PBC

$j \in S$

tar"

$= \prod_{j \in P} \hat{\sigma}_j^z$

quette"

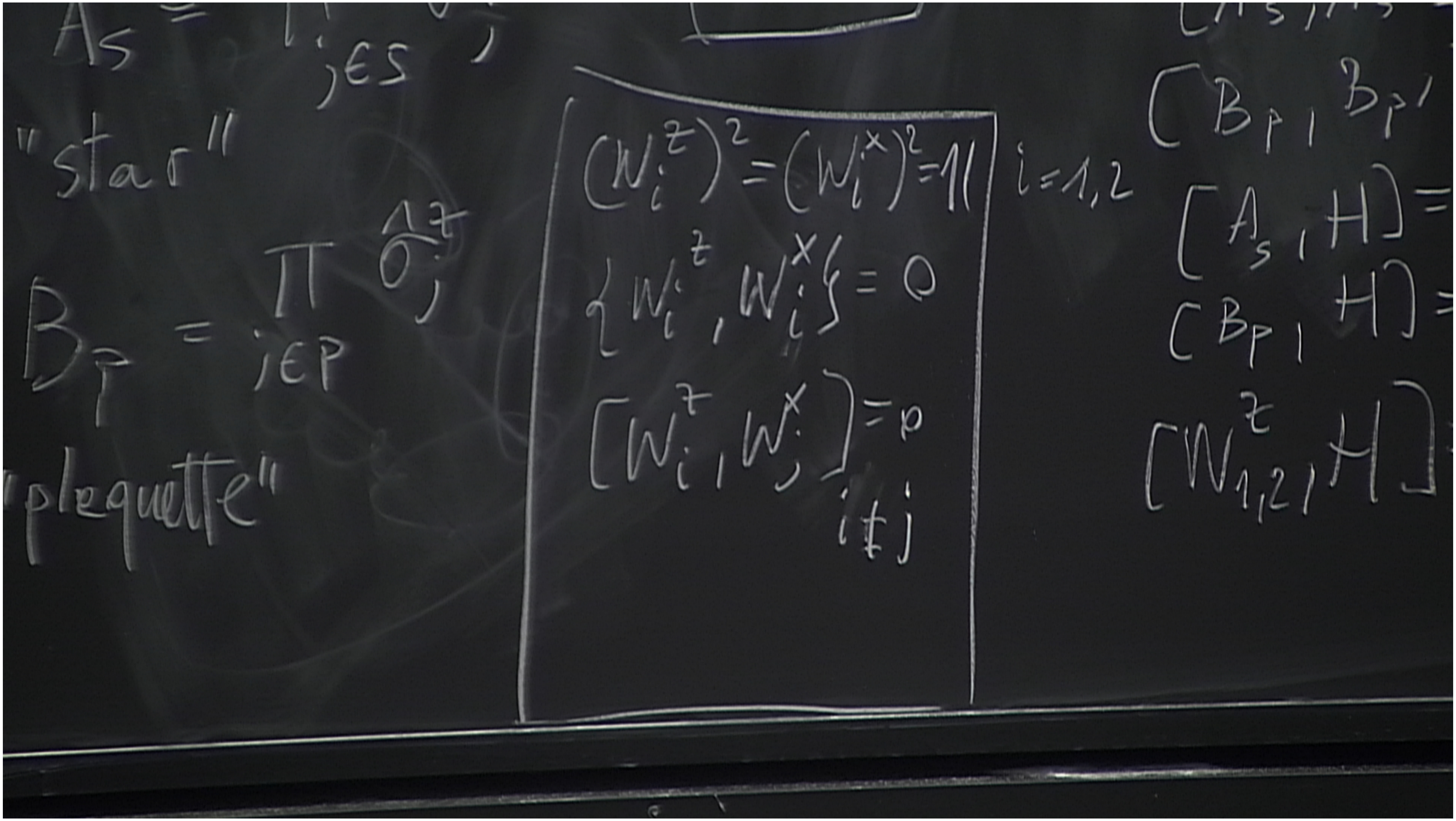
$$(W_i^z)^2 = (W_i^x)^2 = 1\!\!1 \qquad i = 1,2$$

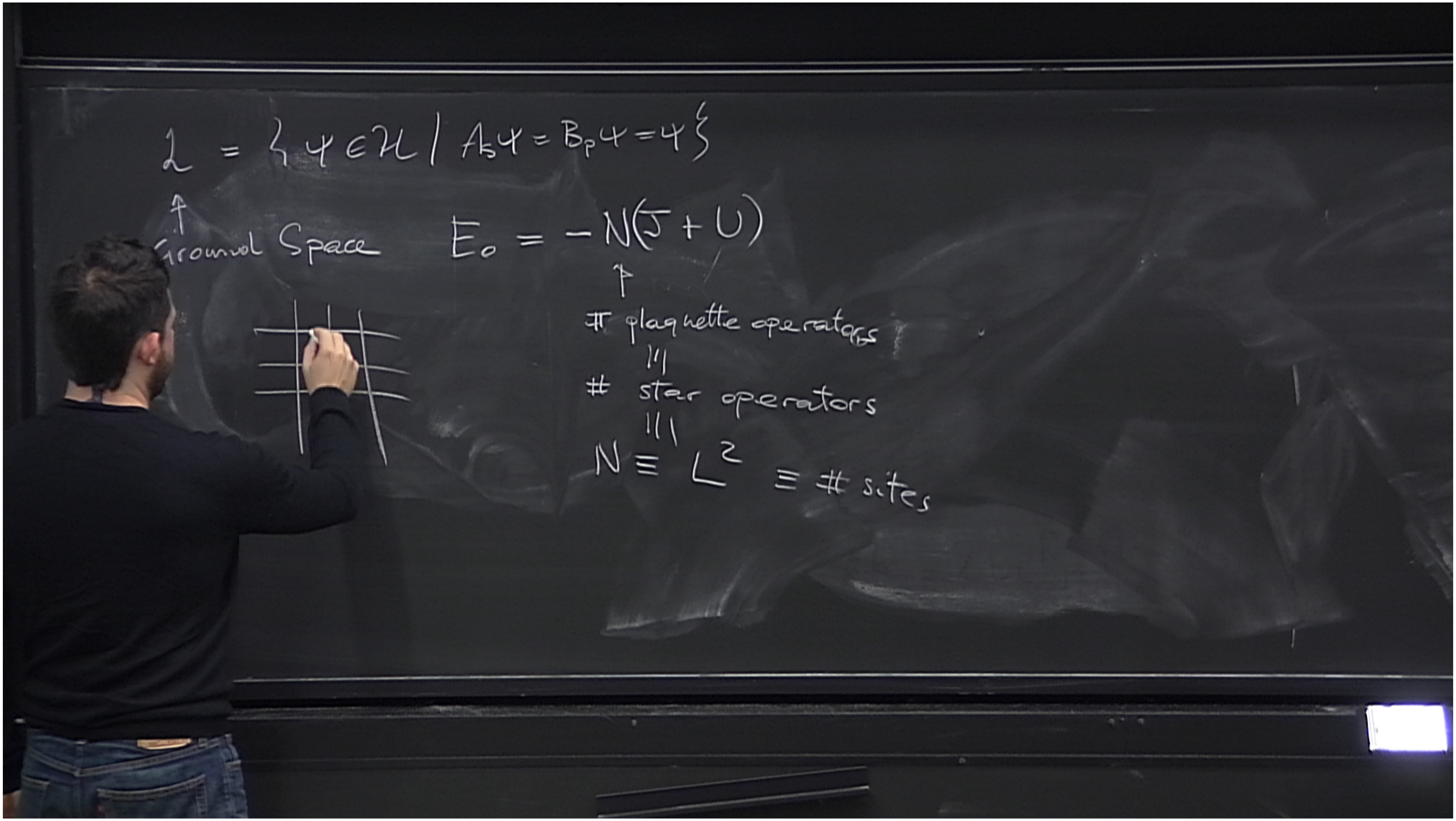
$[B_p, B$

$[A_s, H$

$[B_p, H$

$[W^z_{1,2}, H$


$A_s = \prod_{j \in S}$
"star"
$B_p = \prod_{j \in P} \sigma_j^z$
"plaquette"
$(W_i^z)^2 = (W_i^x)^2 = 1 \quad i = 1,2$
$\{W_i^z, W_i^x\} = 0$
$[W_i^z, W_j^x] = 0 \quad i \neq j$
$[B_p, B_{p'}$
$[A_s, H] =$
$[B_p, H] =$
$[W^z_{1,2}, H] =$


$\mathcal{L} = \{ \psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi \}$
Ground Space
$E_0 = -N(J + U)$
plaquette operators
star operators
$N \equiv L^2 \equiv$ # sites

$$\mathcal{L} = \{\psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi\}$$

$\uparrow$

Ground Space

$$E_0 = -N(J+U)$$

$\uparrow$

plaquette operators

$\parallel$

star operators

$\parallel$

$N \equiv L^2 \equiv$ # sites

$$\phi_0 \in \mathcal{L}$$

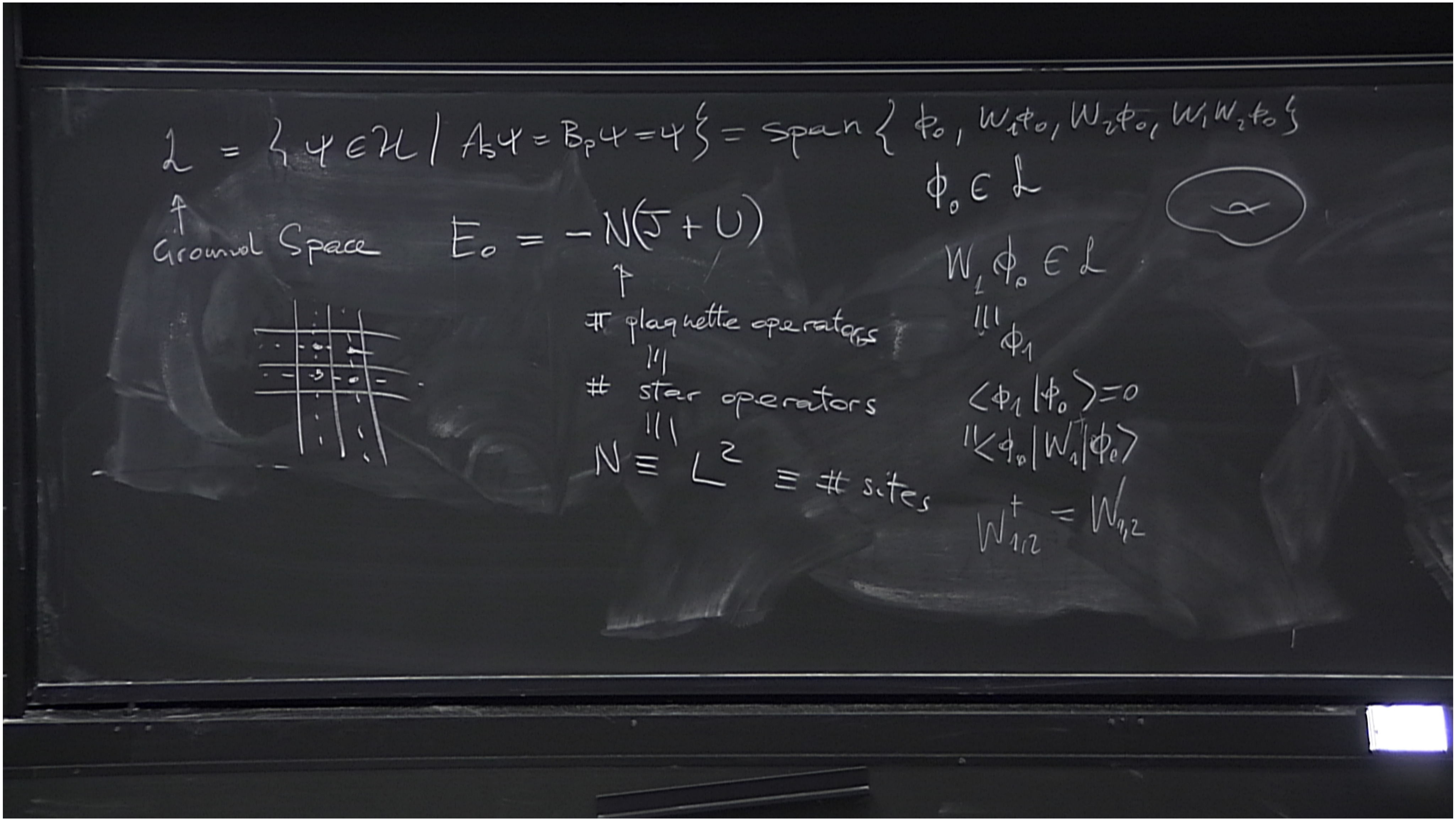
$$W_1\phi_0 \in \mathcal{L}$$

$$\parallel \; \phi_1$$

$$\langle\phi_1|\phi_0\rangle = 0$$

$$= \langle\phi_0|W_1^\dagger|\phi_0\rangle$$

$$W_{1,2}^\dagger = W_{1,2}$$



$\mathcal{L} = \{ \psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi \} = \text{span}\{ \phi_0, W_1\phi_0, W_2\phi_0, W_1W_2\phi_0 \}$
Ground Space
$E_0 = -N(J+U)$
plaquette operators
star operators
$N \equiv L^2 \equiv$ # sites
$\phi_0 \in \mathcal{L}$
$W_1\phi_0 \in \mathcal{L}$
ϕ_1
$\langle \phi_1 | \phi_0 \rangle = 0$
$\langle \phi_0 | W_1^\dagger | \phi_0 \rangle$
$W_{1,2}^\dagger = W_{1,2}$

$$\mathcal{L} = \{\psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi\} = \text{span}\{\phi_0,\ W_1^x\phi_0,\ W_2^x\phi_0,\ W_1^x W_2^x\phi_0\}$$

Ground Space

$$E_0 = -N(J+U)$$

\# plaquette operators

$\parallel$

\# star operators

$\parallel$

$N \equiv L^2 \equiv$ # sites

$\phi_0 \in \mathcal{L}$

$W_1^x \phi_0 \in \mathcal{L}$

$\parallel$

ϕ_1

$\langle \phi_1 | \phi_0 \rangle = 0$

$\parallel \langle \phi_0 | W_1^x | \phi_0 \rangle$

$$W_{1,2}^{x\dagger} = W_{1,2}^x$$

$$\mathcal{L} = \{ \psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi \} = \text{span}\{ \phi_0 , W_1^x\phi_0 , W_2^x\phi_0 , W_1^xW_2^x\phi_0 \}$$

$\uparrow$

Ground Space

$$E_0 = -N(J+U)$$

$\uparrow$

\# plaquette operators

$\equiv$

\# star operators

$\equiv$

$N \equiv L^2 \equiv$ \# sites

$\phi_0 \in \mathcal{L}$

$W_1^x\phi_0 \in \mathcal{L}$

$\equiv$

ϕ_1

$\langle \phi_1 | \phi_0 \rangle = 0$

$\| \langle \phi_0 | W_1^x | \phi_0 \rangle$

$W_{1,2}^{x\dagger} = W_{1,2}^x$

$$\mathcal{L} = \{\psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi\} = \text{span}\{\phi_0, W_1^x\phi_0, W_2^x\phi_0, W_1^xW_2^x\phi_0\}$$

Ground Space

$$E_0 = -N(J+U)$$

plaquette operators

$|||$

star operators

$|||$

$N \equiv L^2 \equiv$ # sites

$\phi_0 \in \mathcal{L}$

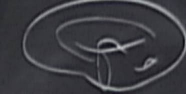
$W_1^x\phi_0 \in \mathcal{L}$

$||| \; \phi_1$

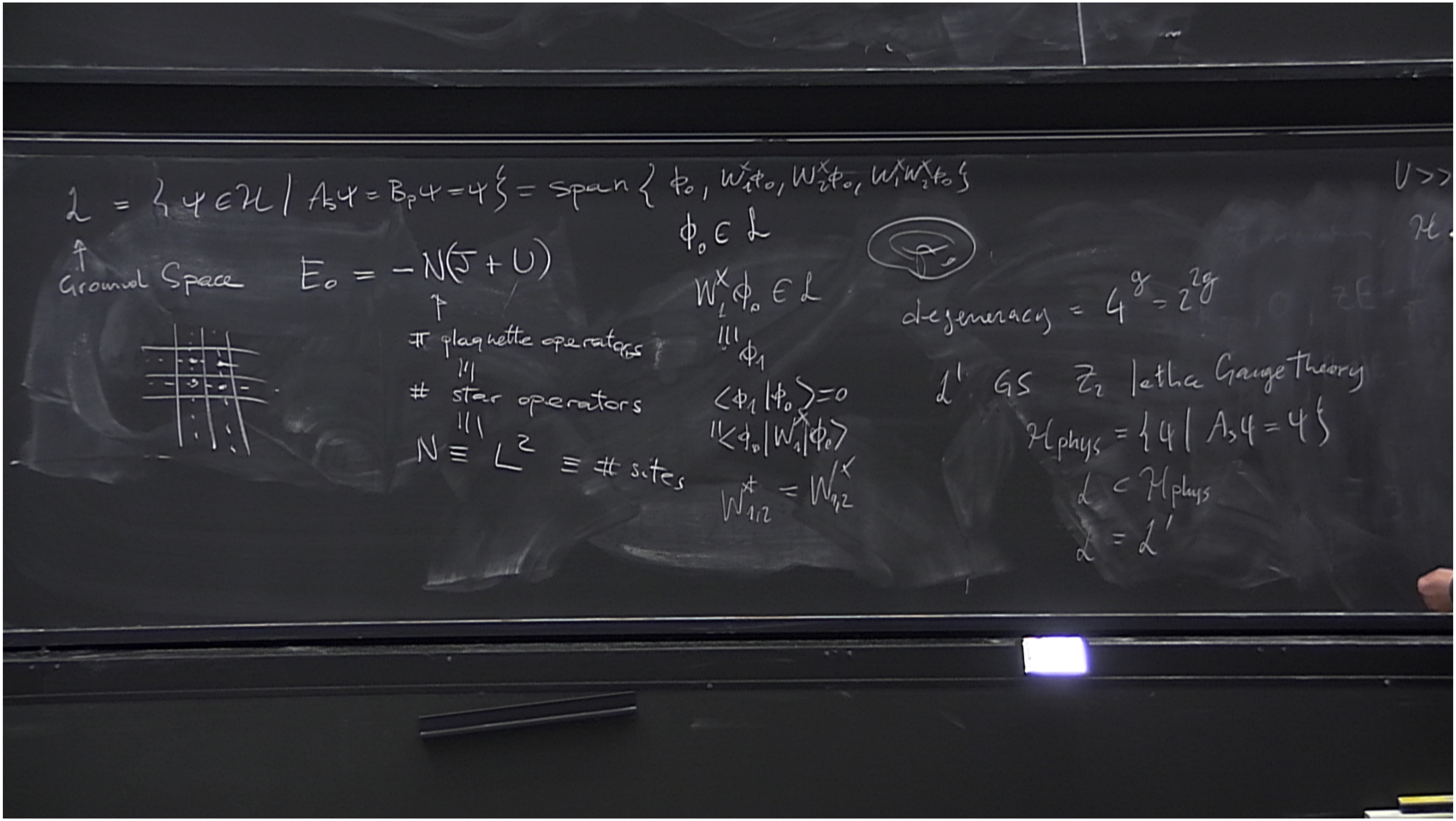
$\langle\phi_1|\phi_0\rangle = 0$

$|| \; \langle\phi_0|W_1^x|\phi_0\rangle$

$W_{1,2}^{x\dagger} = W_{1,2}^x$



degeneracy $= 4^g = 2^{2g}$


$\mathcal{L} = \{\psi \in \mathcal{H} \mid A_s\psi = B_p\psi = \psi\} = \text{span}\{\phi_0, W_1^x\phi_0, W_2^x\phi_0, W_1^xW_2^x\phi_0\}$
Ground Space
$E_0 = -N(J + U)$
plaquette operators
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$N \equiv L^2 \equiv$ # sites
$\phi_0 \in \mathcal{L}$
$W_1^x\phi_0 \in \mathcal{L}$
ϕ_1
$\langle\phi_1|\phi_0\rangle = 0$
$\langle\phi_0|W_1^x|\phi_0\rangle$
$W_{1,2}^{x\dagger} = W_{1,2}^x$
degeneracy $= 4^g = 2^{2g}$
$\mathcal{L}'$: GS $\mathbb{Z}_2$ lattice Gauge Theory
$\mathcal{H}_{phys} = \{\psi \mid A_s\psi = \psi\}$
$\mathcal{L} \subset \mathcal{H}_{phys}$
$\mathcal{L} = \mathcal{L}'$
$U \gg$

$, W_1^x W_2^x \psi_0 \}$

degeneracy $= 4^g = 2^{2g}$

$\mathcal{L}'$: GS $\mathbb{Z}_2$ lattice Gauge Theory

$$\mathcal{H}_{phys} = \{\psi \mid A_s \psi = \psi\}$$

$$\mathcal{L} \subset \mathcal{H}_{phys}$$

$$\mathcal{L} = \mathcal{L}'$$

$U >> J$

$$\mathcal{H}_{low} \subset \mathcal{H}$$

$$\mathcal{H}_{low} = \{\psi \in \mathcal{H} \mid \langle\psi|H|\psi\rangle < U\}$$

$$\mathcal{H}_{low} = \mathcal{H}_{phys}$$

$\{ W_1^x W_2^x \psi_0 \}$

degeneracy $= 4^g = 2^{2g}$

$\mathcal{L}'$ GS Z_2 lattice Gauge Theory

$$\mathcal{H}_{phys} = \{ \psi \mid A_s \psi = \psi \}$$

$$\mathcal{L} \subset \mathcal{H}_{phys}$$

$$\mathcal{L} = \mathcal{L}'$$

$U >> J$

$$\mathcal{H}_{low} \subset \mathcal{H}$$

$$\{ \psi \in \mathcal{H} \mid \langle \psi | H | \psi \rangle < U \}$$

$$\mathcal{H}_{low} = \mathcal{H}_{phys}$$

ϕ_0 in terms of spins

$\phi_0 \in \mathcal{L}$

reference state $|\tilde{0}\rangle$

$= |\uparrow\uparrow\uparrow \ldots \uparrow\rangle$

z - basis

$W_1^x W_2^x \phi_0 \}$

degeneracy $= 4^g = 2^{2g}$

$\mathcal{L}'$ GS Z_2 lattice Gauge Theory

$\mathcal{H}_{phys} = \{\psi \mid A_s \psi = \psi\}$

$\mathcal{L} \subset \mathcal{H}_{phys}$

$\mathcal{L} = \mathcal{L}'$

$U >> J$

$\mathcal{H}_{low} \subset \mathcal{H}$

$\{\psi \in \mathcal{H} \mid \langle\psi|H|\psi\rangle < U\}$

$\mathcal{H}_{low} = \mathcal{H}_{phys}$

ϕ_0 in terms of spins

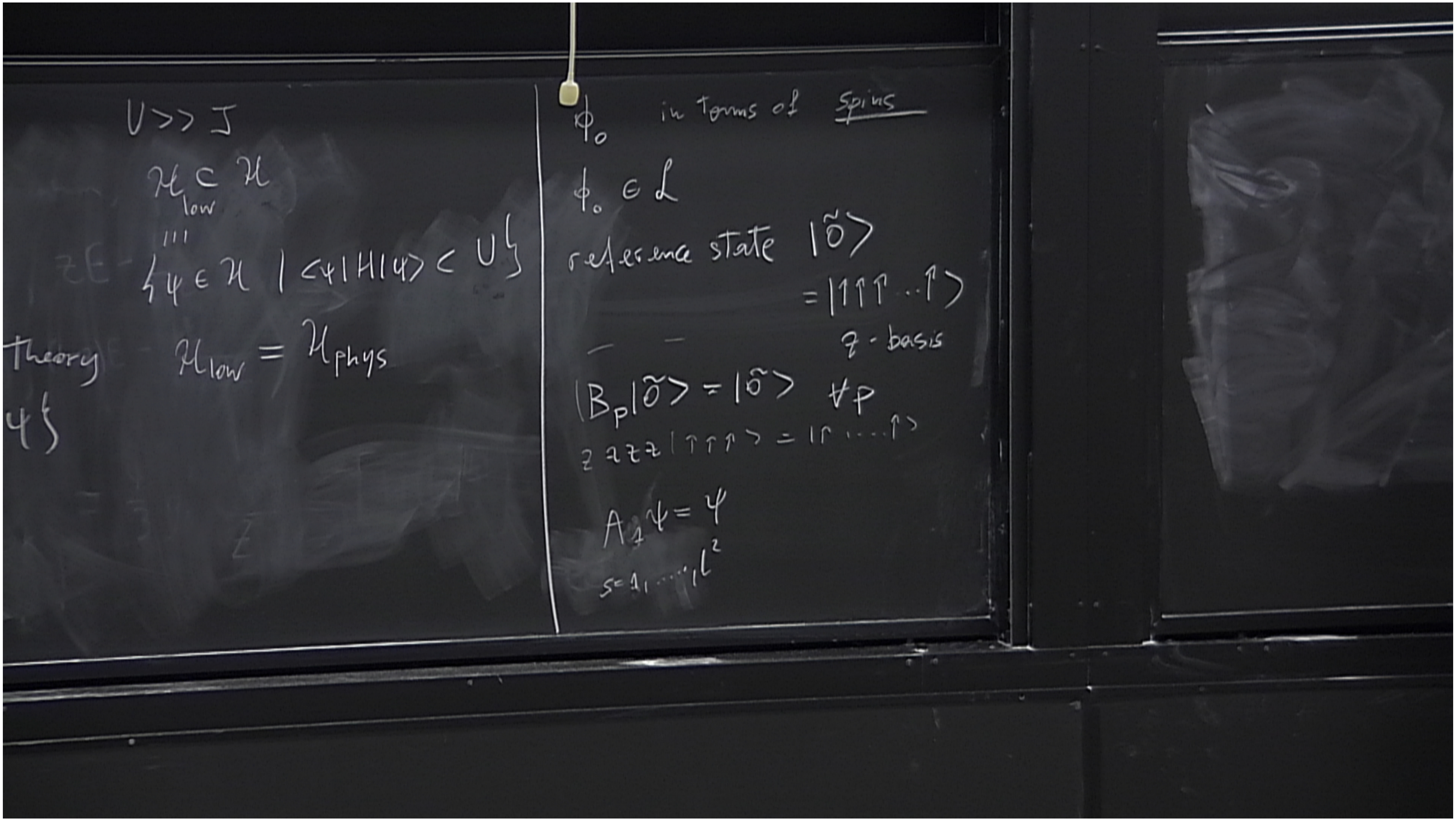
$\phi_0 \in \mathcal{L}$

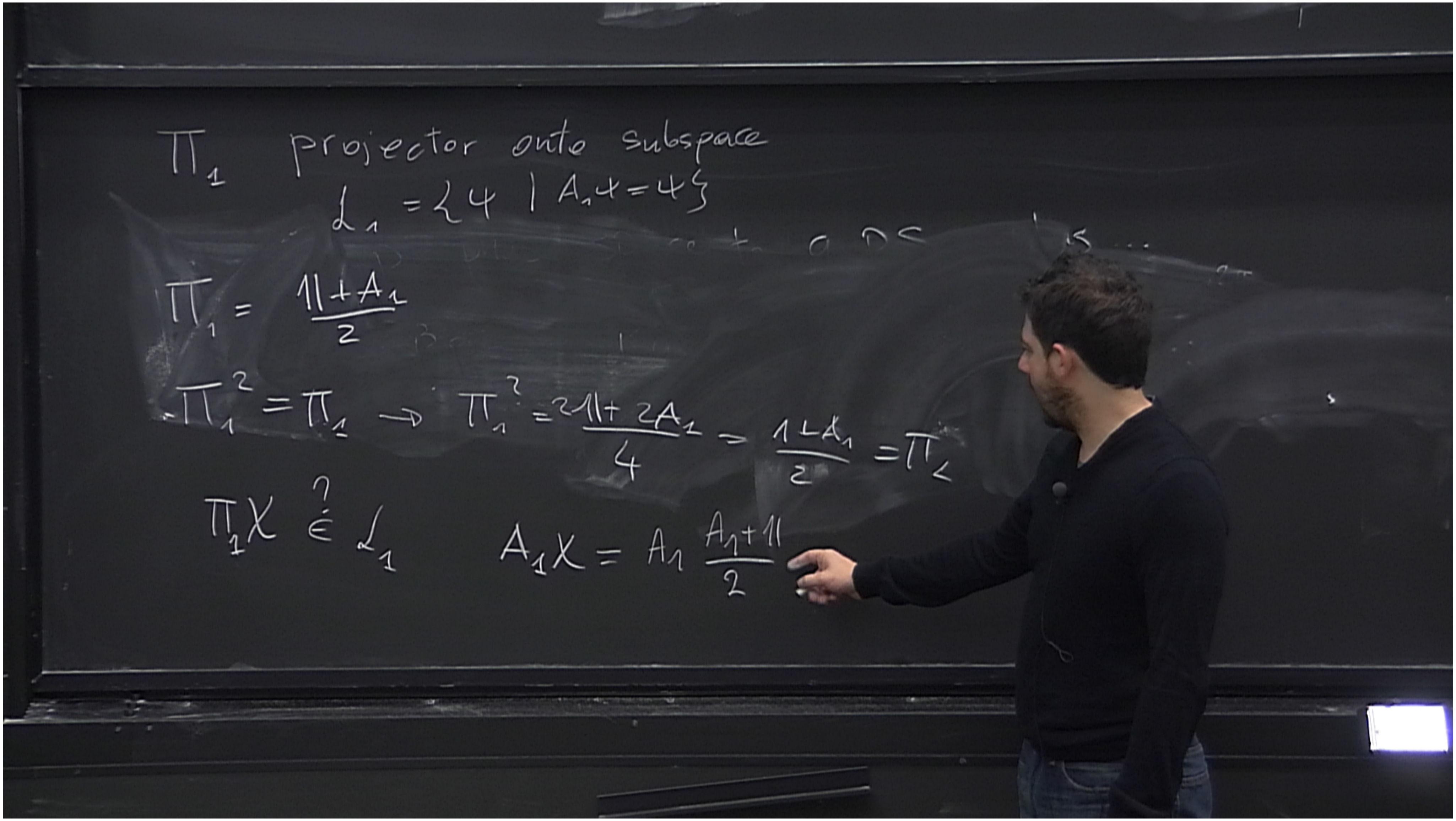
reference state $|\tilde{0}\rangle = |\uparrow\uparrow\uparrow\ldots\uparrow\rangle$

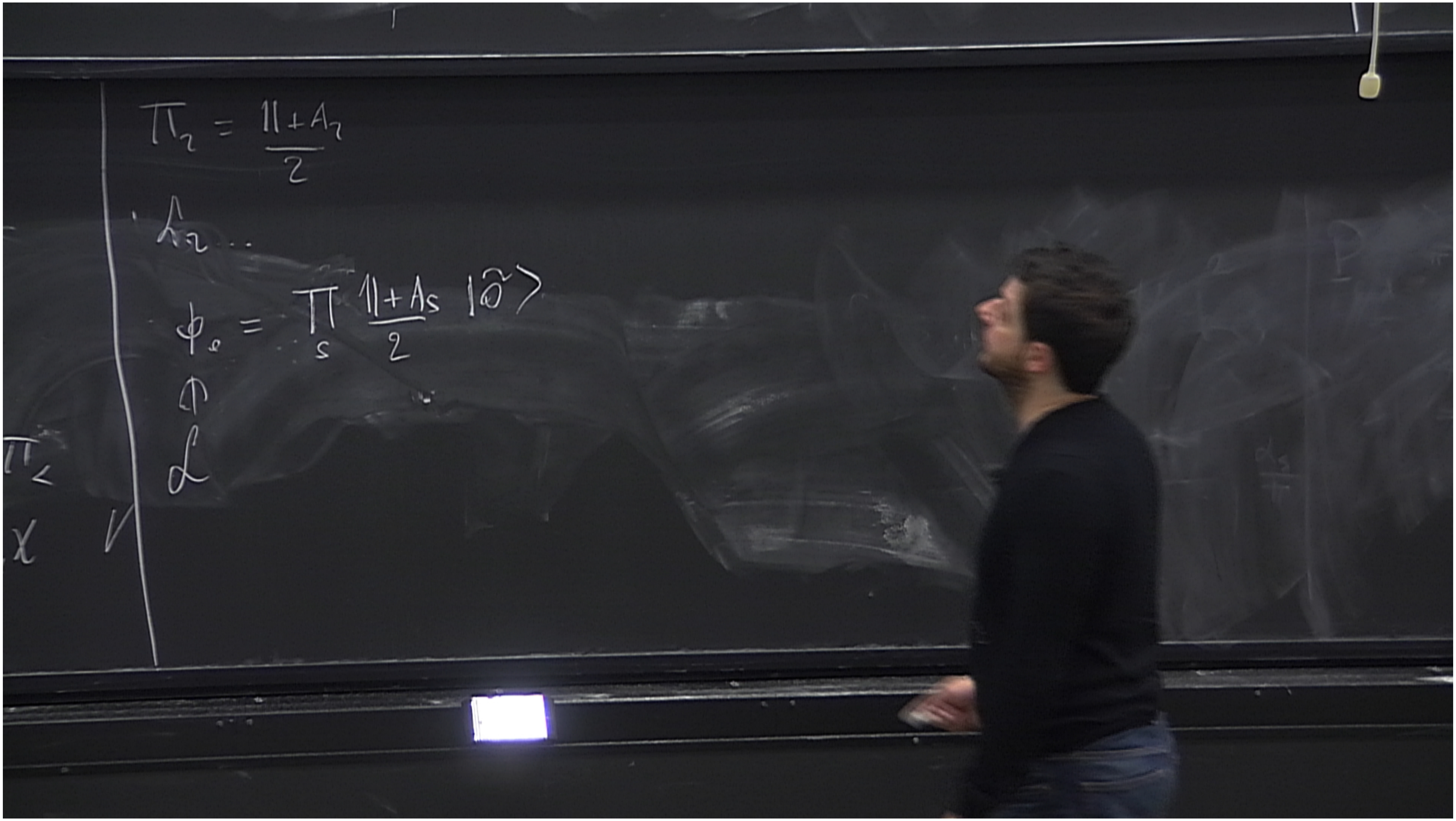
z - basis

$B_p|\tilde{0}\rangle = |\tilde{0}\rangle \quad \forall p$

$zzzz|\uparrow\uparrow\uparrow\rangle = |\uparrow\ldots\uparrow\rangle$


$U >> J$
$\mathcal{H}_{low} \subset \mathcal{H}$
$\{\psi \in \mathcal{H} \mid \langle\psi|H|\psi\rangle < U\}$
Theory
$\mathcal{H}_{low} = \mathcal{H}_{phys}$
ϕ_0 in terms of spins
$\phi_0 \in \mathcal{L}$
reference state $|\tilde{0}\rangle$
$= |\uparrow\uparrow\uparrow \cdots \uparrow\rangle$
z - basis
$B_p|\tilde{0}\rangle = |\tilde{0}\rangle \quad \forall p$
$zzzz|\uparrow\uparrow\uparrow\rangle = |\uparrow \cdots \uparrow\rangle$
$A_s\psi = \psi$
$s = 1, \ldots, L^2$


Π1 projector onto subspace
L1 = {ψ | A1ψ = ψ}
Π1 = (1 + A1)/2
Π1² = Π1 → Π1² = (21 + 2A1)/4 = (1 + A1)/2 = Π1
Π1χ ∈? L1
A1χ = A1 (A1 + 1)/2


$\Pi_z = \frac{1\!\!1 + A_z}{2}$
$A_z \ldots$
$\phi_o = \prod_s \frac{1\!\!1 + A_s}{2} |\tilde{0}\rangle$
$\mathcal{L}$

$= \Pi_2$

$\Pi_1 \chi$

$$\Pi_2 = \frac{\mathbb{1} + A_2}{2}$$

$A_2 \ldots$

$i^2 = 1$

$$\phi_0 = N \prod_{s=1} \frac{\mathbb{1} + A_s}{2} |\tilde{0}\rangle = N' \left(\mathbb{1} + \sum_s A_s + \sum_{s,s'} A_s A_{s'} + \sum_{s,s',s''} A_s A_{s'} A_{s''} + \ldots \right)$$

Φ

$\mathcal{L}$

$$\prod_s A_s = \mathbb{1}$$

$$\Pi_2 = \frac{\mathbb{1}+A_2}{2}$$

$A_2 \dots$

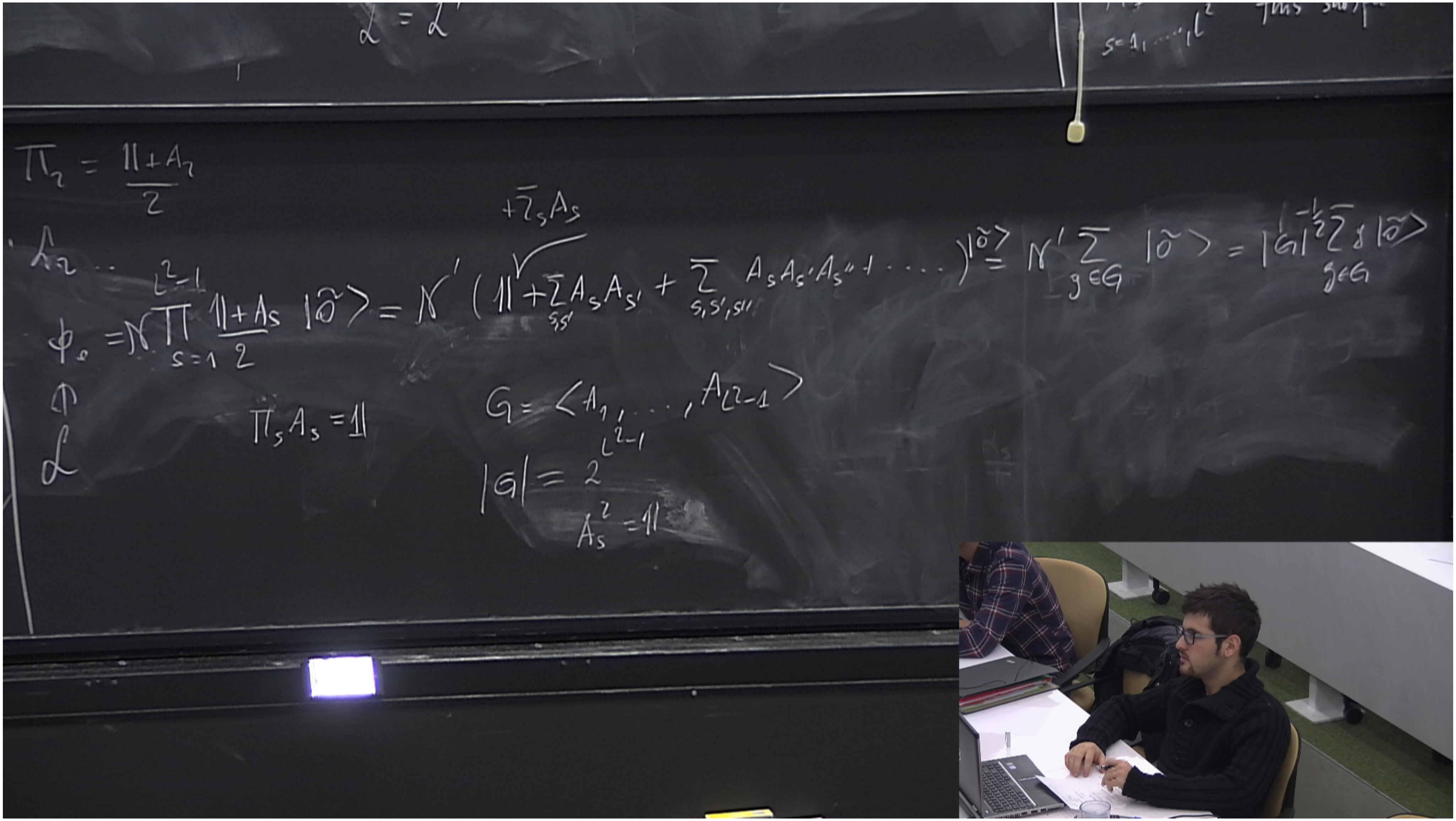
$$\phi_0 = \mathcal{N} \prod_{s=1}^{L^2-1} \frac{\mathbb{1}+A_s}{2}\, |\tilde{0}\rangle = \mathcal{N}' \Big(\mathbb{1} + \sum_s A_s + \sum_{s,s'} A_s A_{s'} + \sum_{s,s',s''} A_s A_{s'} A_{s''} + \dots\Big)|\tilde{0}\rangle = \mathcal{N}' \sum_{g\in G} |\tilde{0}\rangle$$

Φ

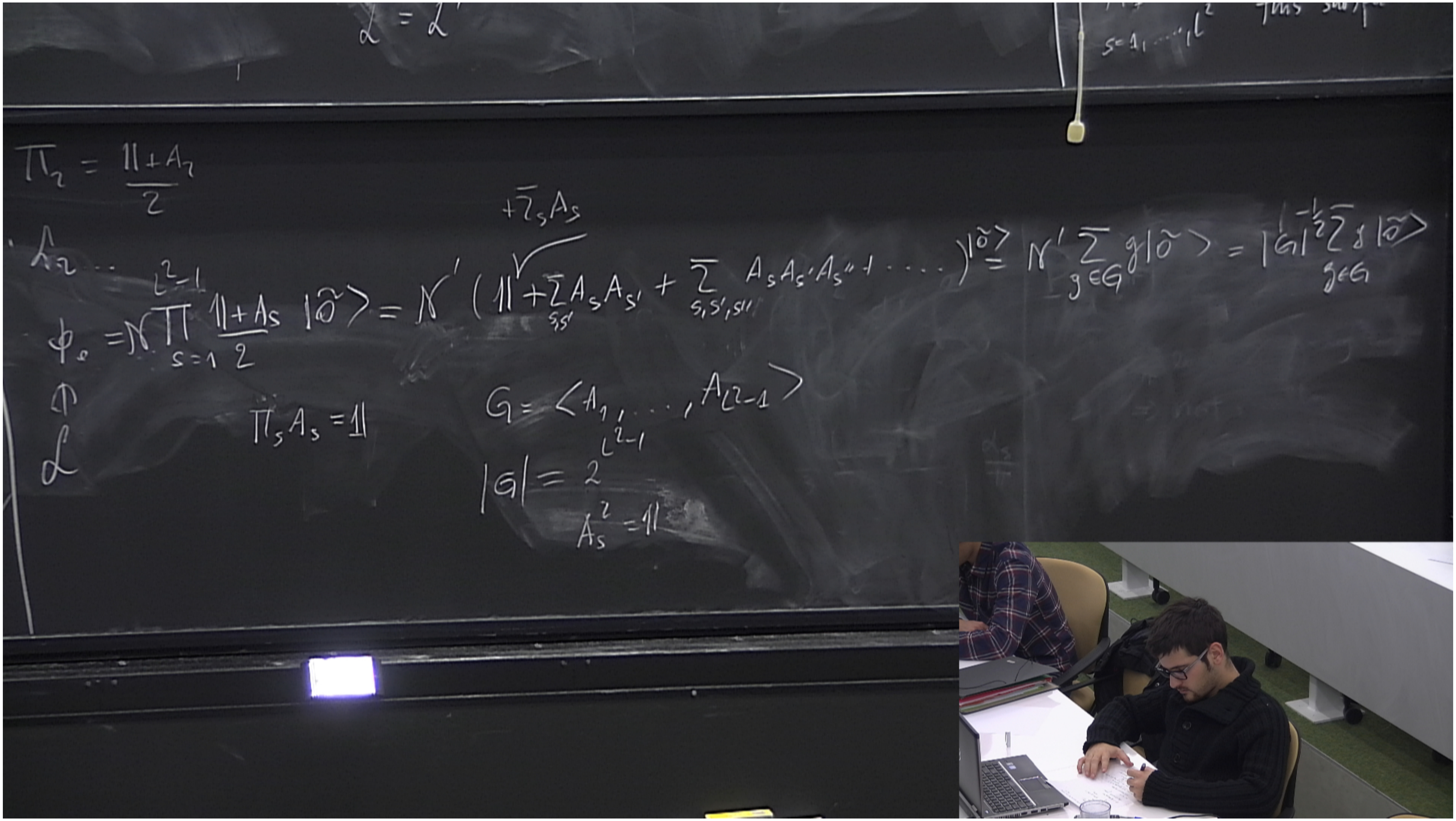
$\mathcal{L}$

$$\prod_s A_s = \mathbb{1}$$

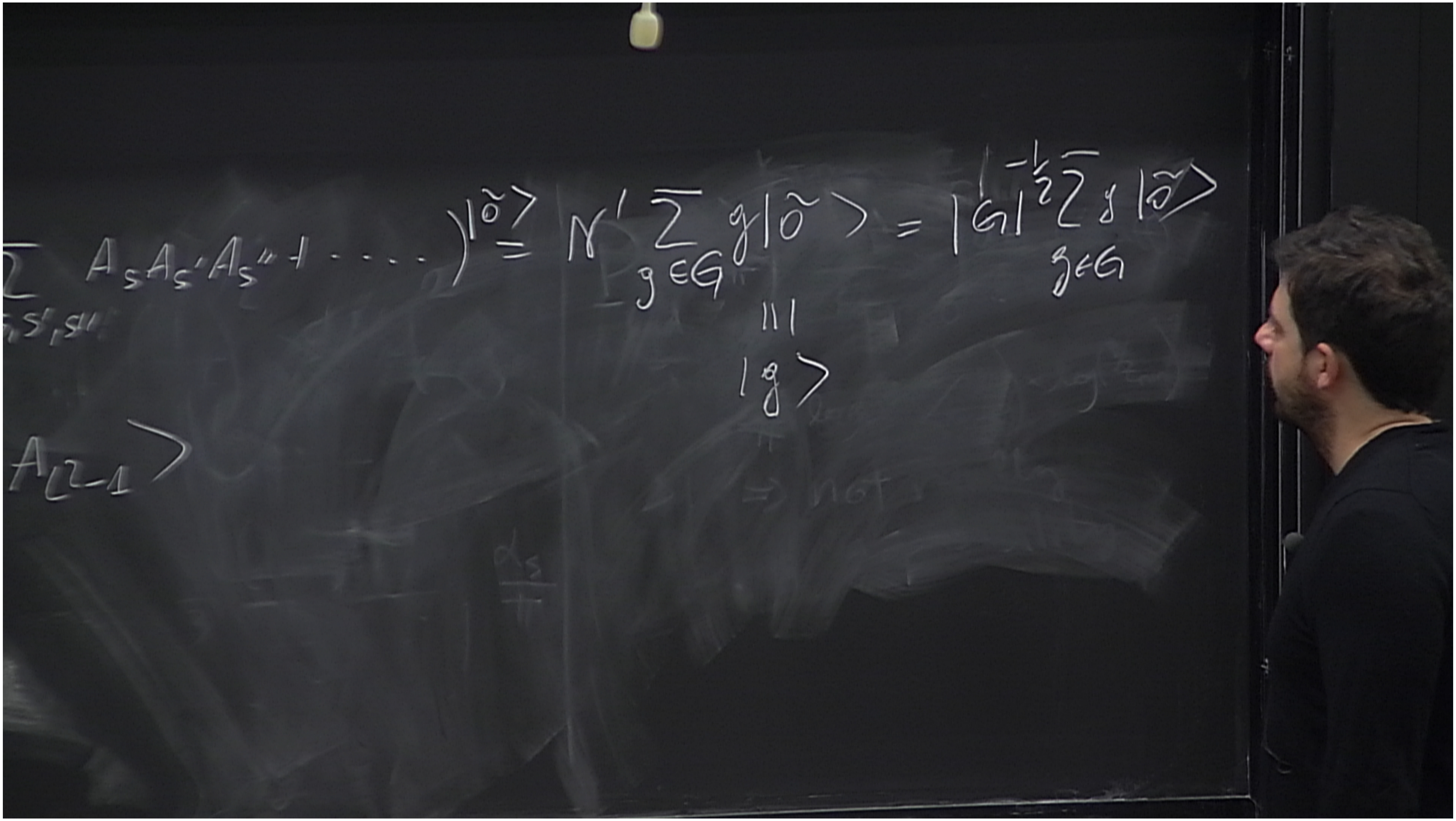
$$G = \langle A_1, \dots, A_{L^2-1} \rangle$$



Π_2 = (1 + A_2)/2
+Σ_s A_s
φ_0 = N Π_{s=1}^{L²−1} (1 + A_s)/2 |0̃> = N'(1 + Σ_{s,s'} A_s A_{s'} + Σ_{s,s',s''} A_s A_{s'} A_{s''} + ...)|0̃> = N' Σ_{g∈G} g|0̃> = |G|^{-1/2} Σ_{g∈G} g|0̃>
Π_s A_s = 1
G = <A_1, ..., A_{L²−1}>
|G| = 2^{L²−1}
A_s² = 1



$\Pi_2 = \frac{1 + A_2}{2}$
$+\sum_s A_s$
$\phi_0 = N \prod_{s=1}^{L^2-1} \frac{1+A_s}{2} |\tilde{0}\rangle = N'\left(1 + \sum_{s,s'} A_s A_{s'} + \sum_{s,s',s''} A_s A_{s'} A_{s''} + \dots\right)|\tilde{0}\rangle = N' \sum_{g\in G} g|\tilde{0}\rangle = |G|^{-\frac{1}{2}}\sum_{g\in G} g|\tilde{0}\rangle$
$\prod_s A_s = 1$
$G = \langle A_1, \dots, A_{L^2-1}\rangle$
$|G| = 2^{L^2-1}$
$A_s^2 = 1$

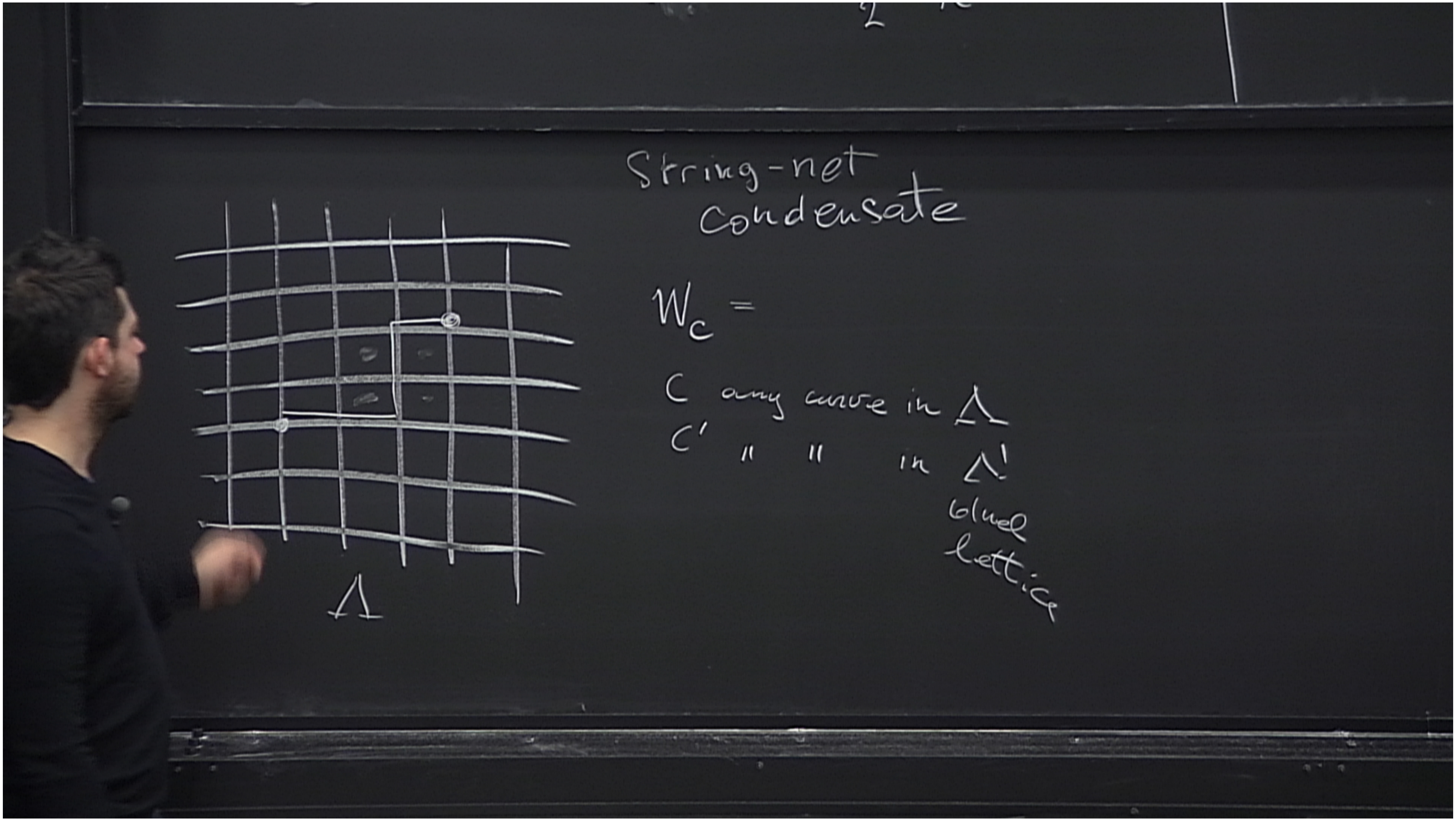

$\sum A_s A_{s'} A_{s''} + \ldots)|\tilde{0}\rangle = N' \sum_{g \in G} g|\tilde{0}\rangle = |G|^{-\frac{1}{2}} \sum_{g \in G} g|\tilde{0}\rangle$
$|||$
$|g\rangle$
$A_{L^2-1}\rangle$

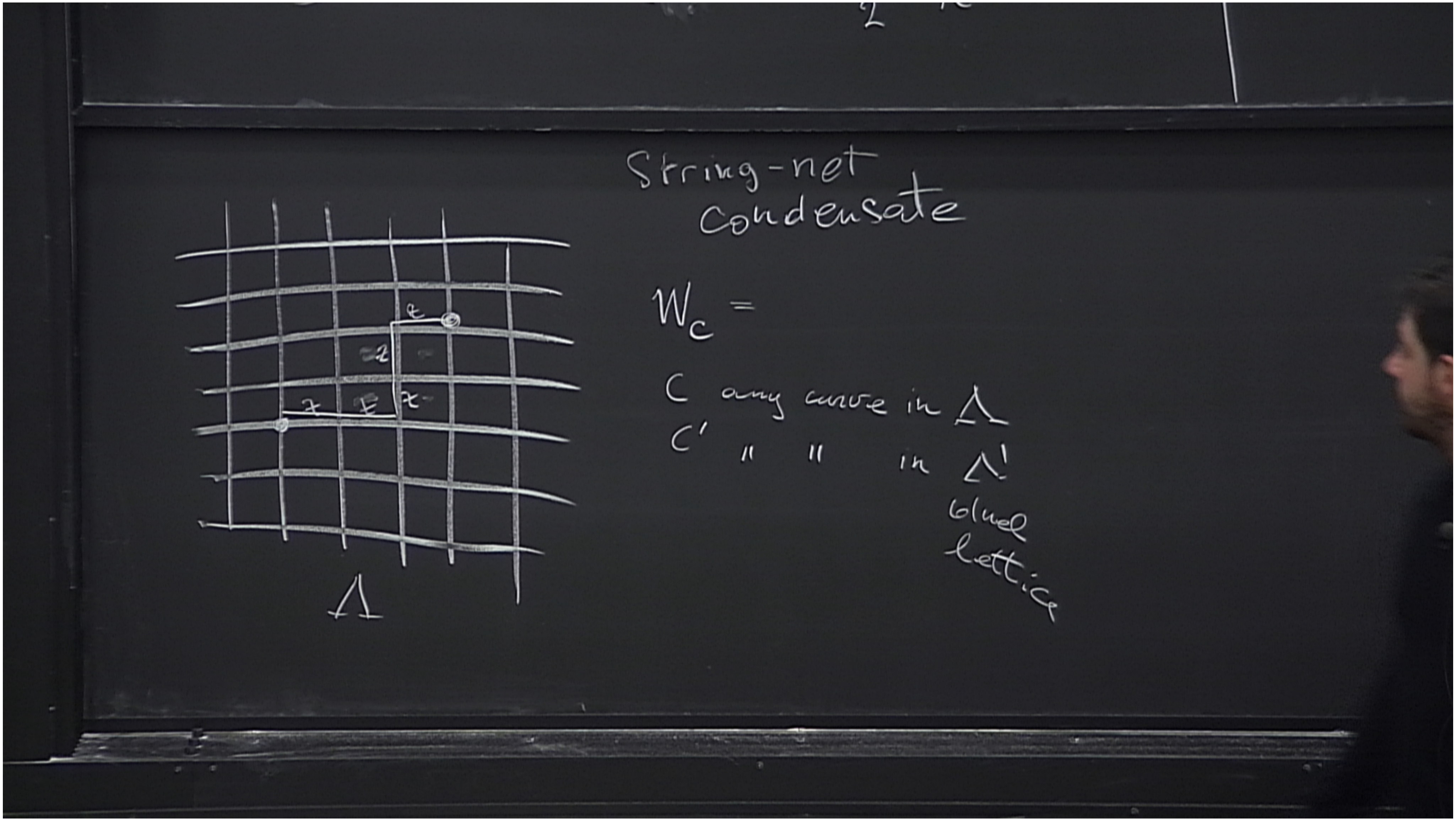
$$\sum_{s,s',s''} A_s A_{s'} A_{s''} + \dots \,)|\tilde{0}\rangle \equiv N' \sum_{g\in G} g|\tilde{0}\rangle = |G|^{-\frac{1}{2}} \sum_{g\in G} g|\tilde{0}\rangle$$

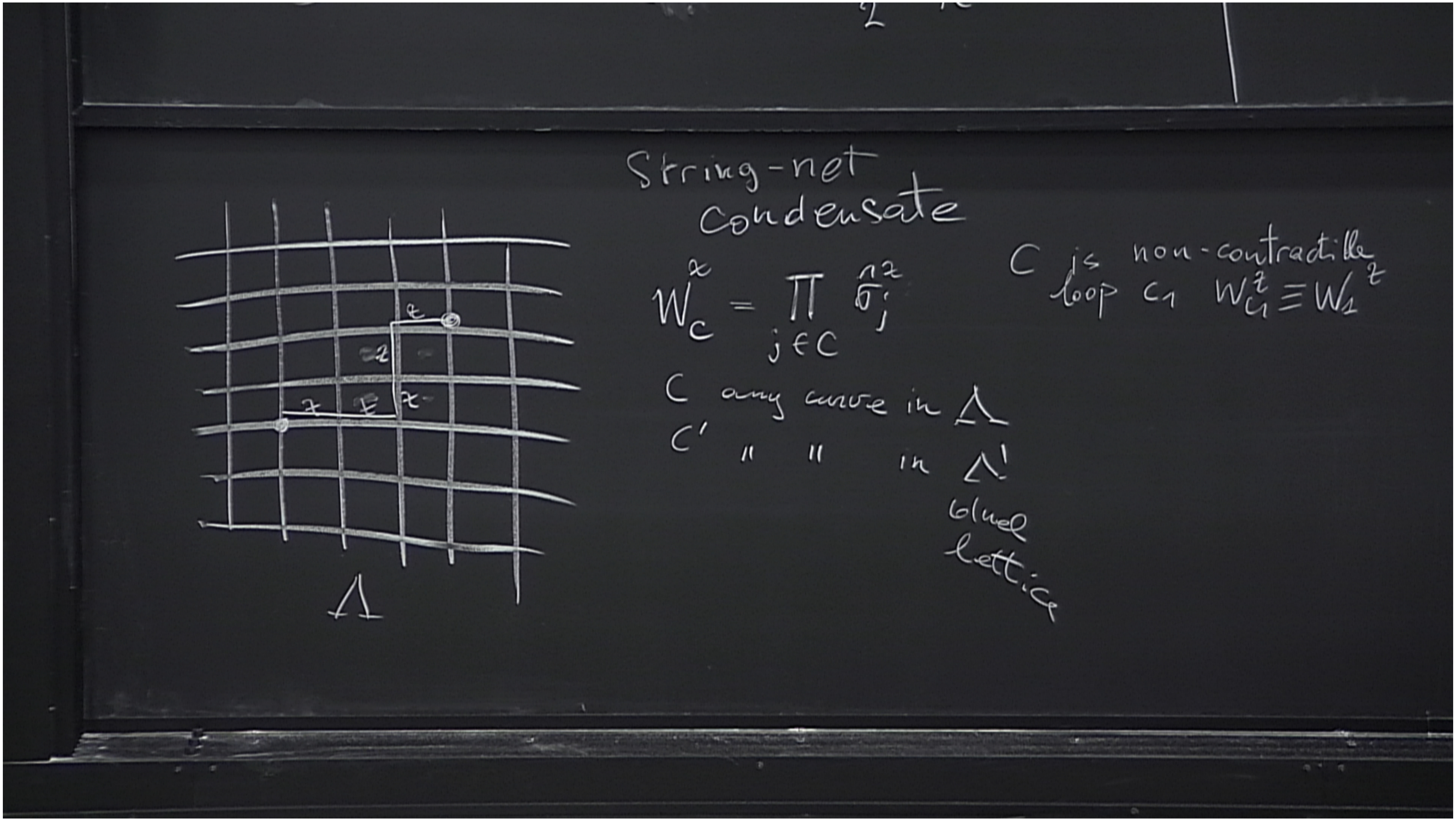
$$g|\tilde{0}\rangle \equiv |g\rangle$$

$$A_{L^2-1}\rangle$$

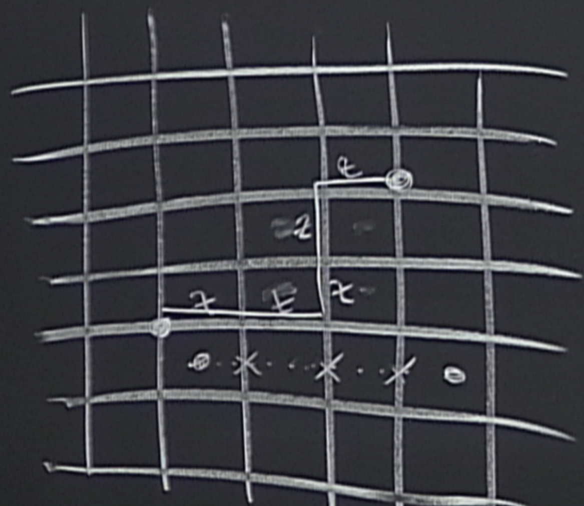
$$|\tilde{0}\rangle \equiv \mathbb{1}|\tilde{0}\rangle \equiv |\tilde{\mathbb{1}}\rangle$$


String-net
condensate
W_C =
C any curve in Λ
C' " " in Λ'
blue lattice
Λ


String-net
condensate
$W_C =$
C any curve in Λ
C' " " in Λ'
dual
lattice
Λ


String-net condensate
$W^x_C = \prod_{j\in C} \hat{\sigma}^x_j$
C any curve in Λ
C' " " in Λ'
dual lattice
C is non-contractible loop C_1 $W^z_{C_1} \equiv W^z_1$
Λ

String-net condensate



Λ

$$W_C^z = \prod_{j \in C} \hat{\sigma}_j^z$$

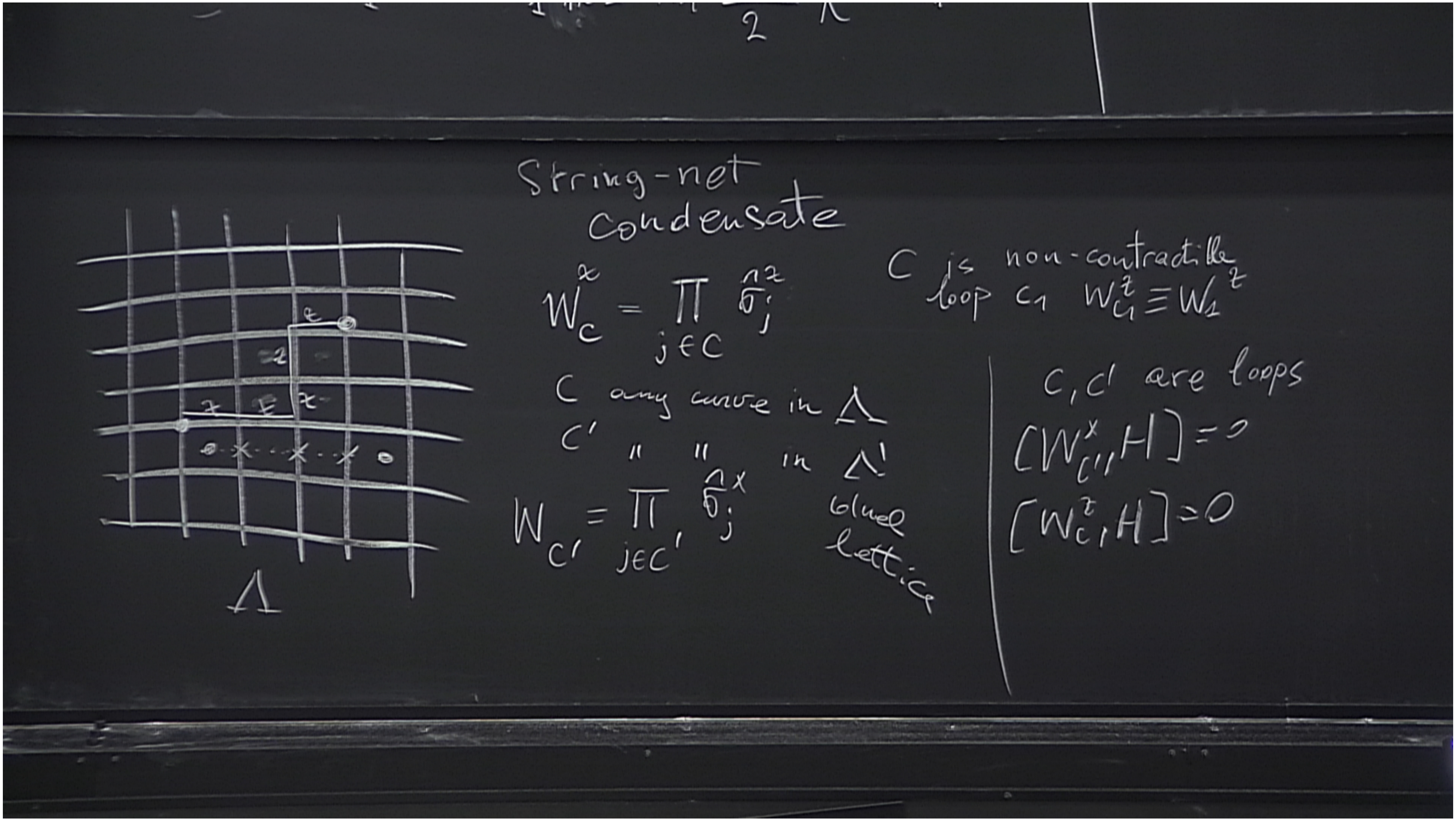
C any curve in Λ

C' " " in Λ'

$$W_{C'} = \prod_{j \in C'} \hat{\sigma}_j^x$$

blue lattice

C is non-contractible loop C_1 $W_{C_1}^z \equiv W_1^z$



String-net condensate
$W_C^x = \prod_{j \in C} \hat{\sigma}_j^z$
C any curve in Λ
C' " " in Λ'
$W_{C'} = \prod_{j \in C'} \hat{\sigma}_j^x$
(blue lattice)
C is non-contractible loop C_1 $W_{C_1}^z \equiv W_1^z$
C, C' are loops
$[W^x_{C'}, H] = 0$
$[W^z_C, H] = 0$
Λ

String-net condensate

$$W_C^x = \prod_{j\in C} \hat{\sigma}_j^z$$

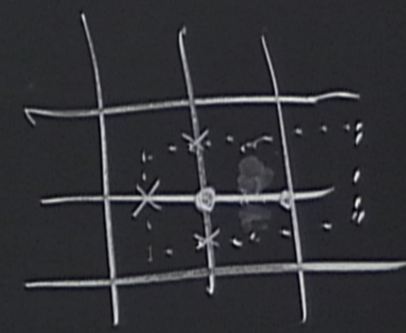
C any curve in Λ

C' " " in Λ'

$$W_{C'} = \prod_{j\in C'} \hat{\sigma}_j^x$$ blue lattice

C is non-contractible loop C_1 $W^z_{C_1} \equiv W_1^z$

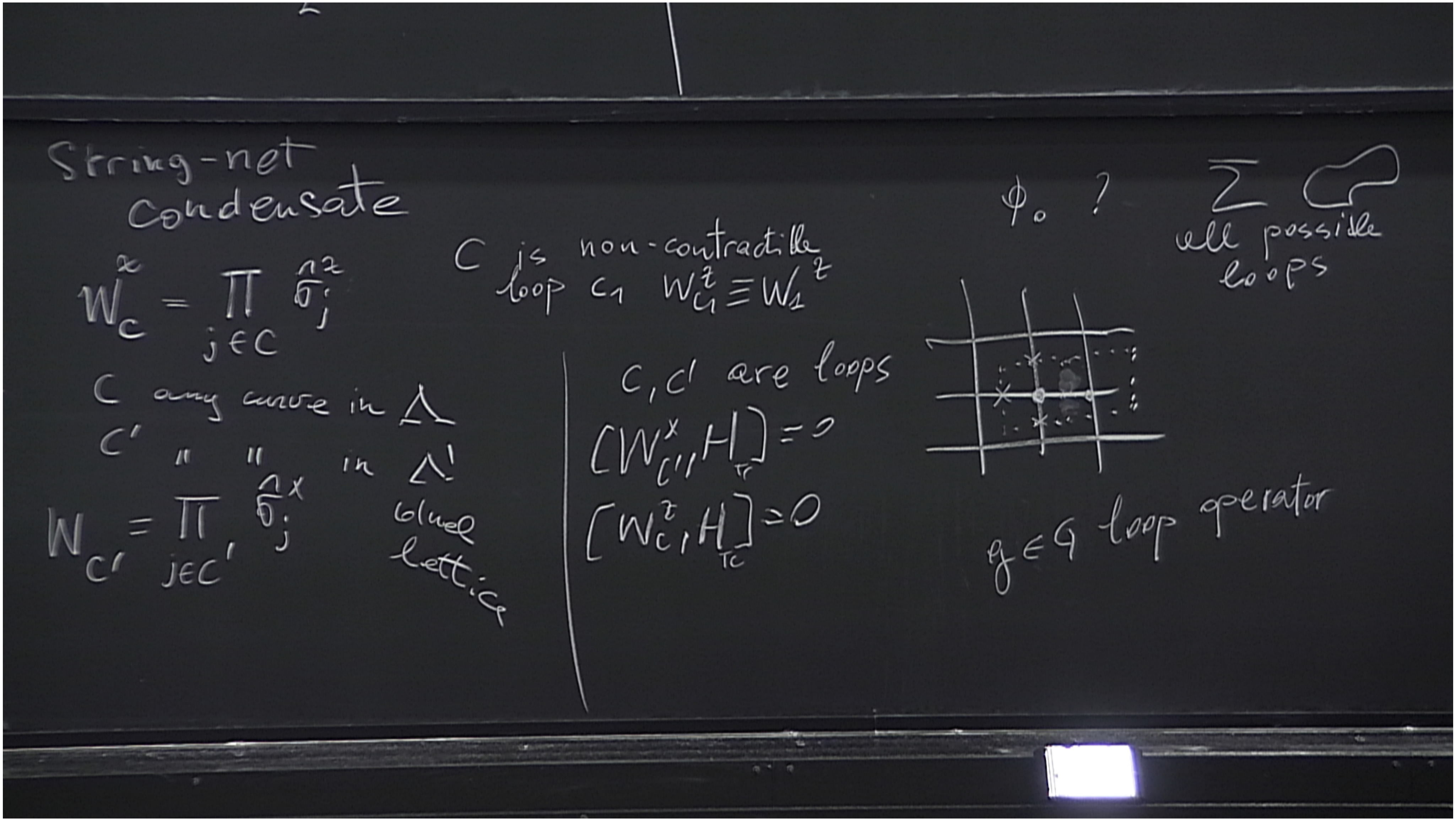
ϕ_0 ?

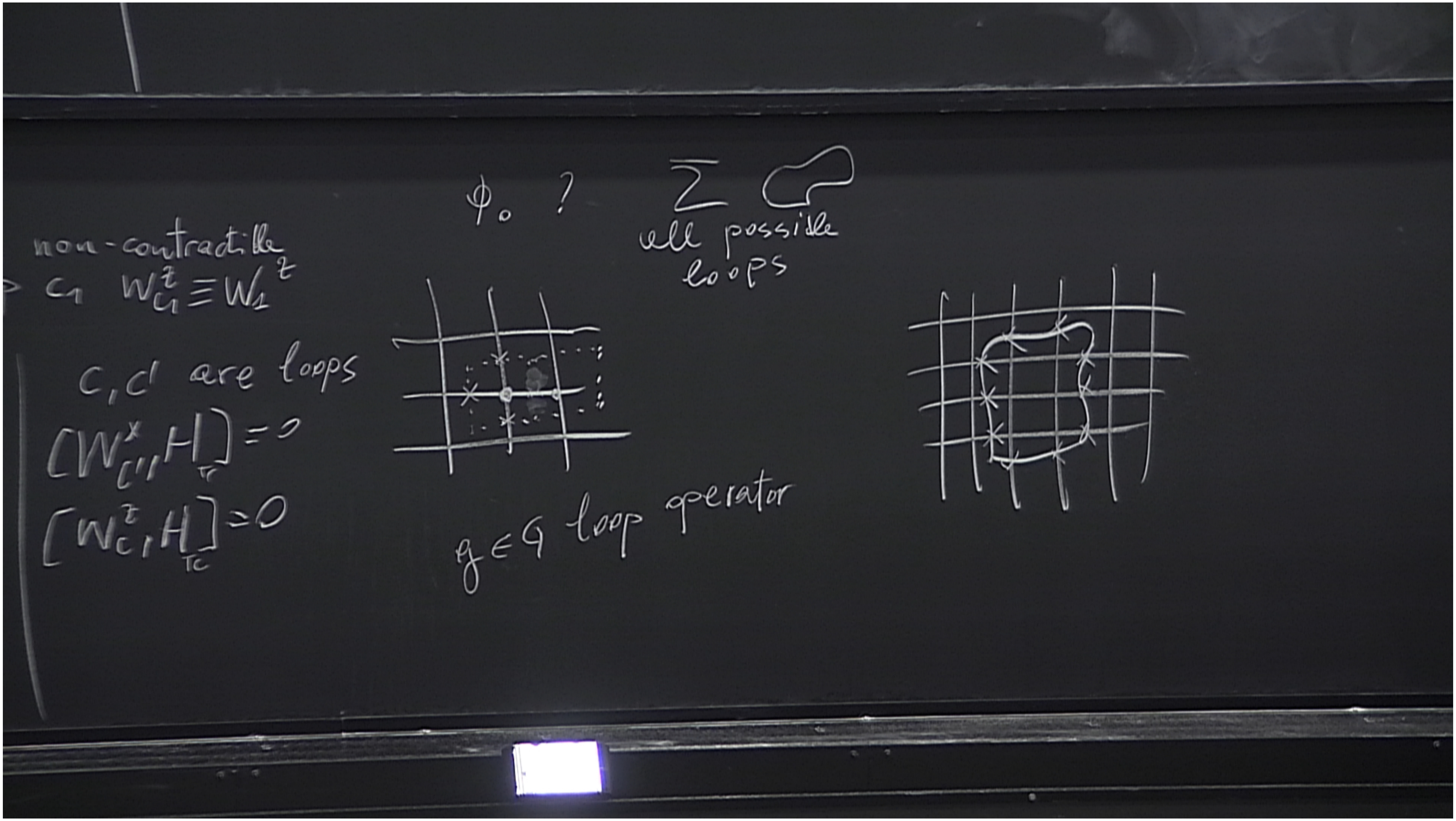


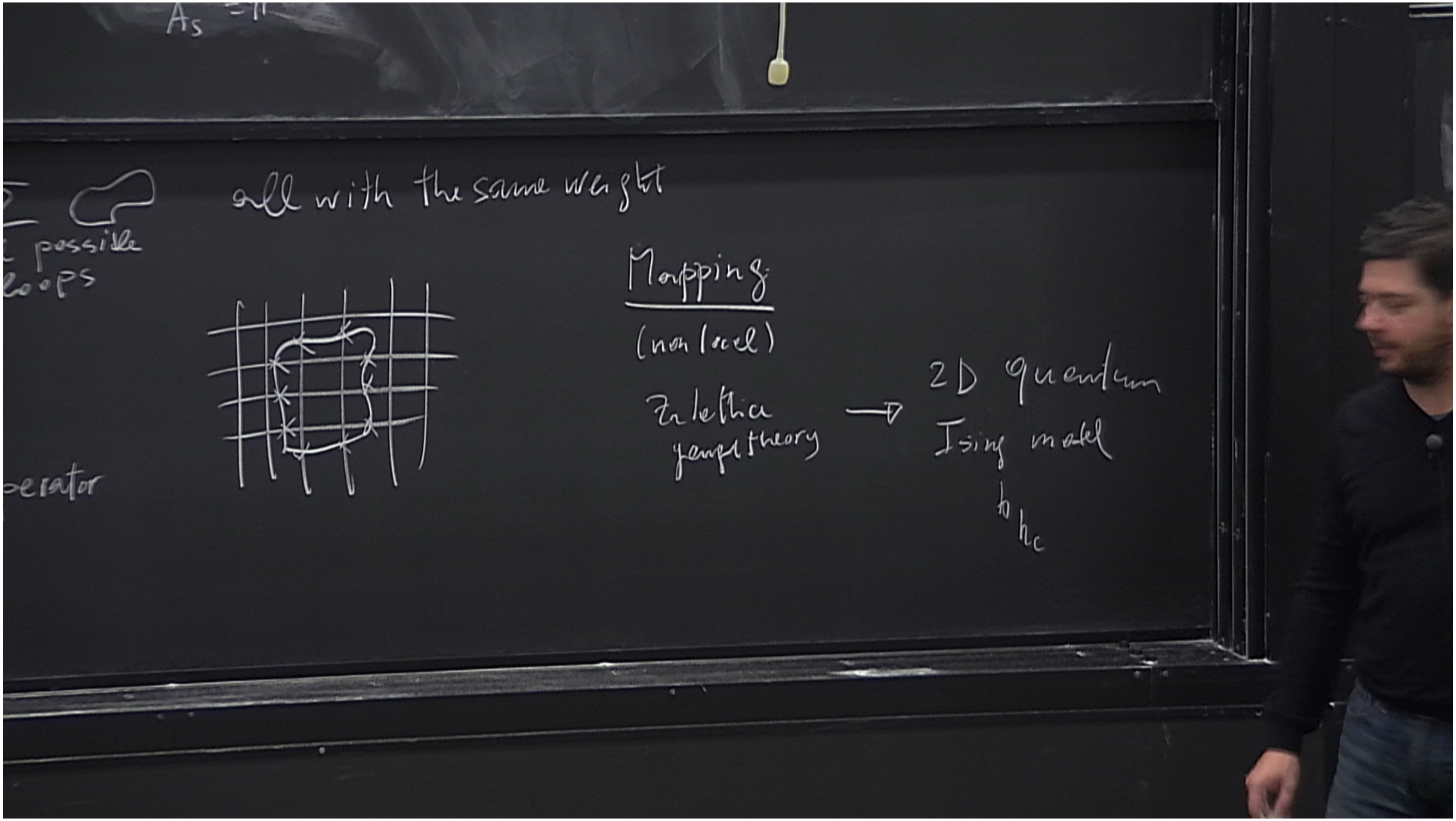
C, C' are loops

$$[W^x_{C'}, H_{TC}] = 0$$

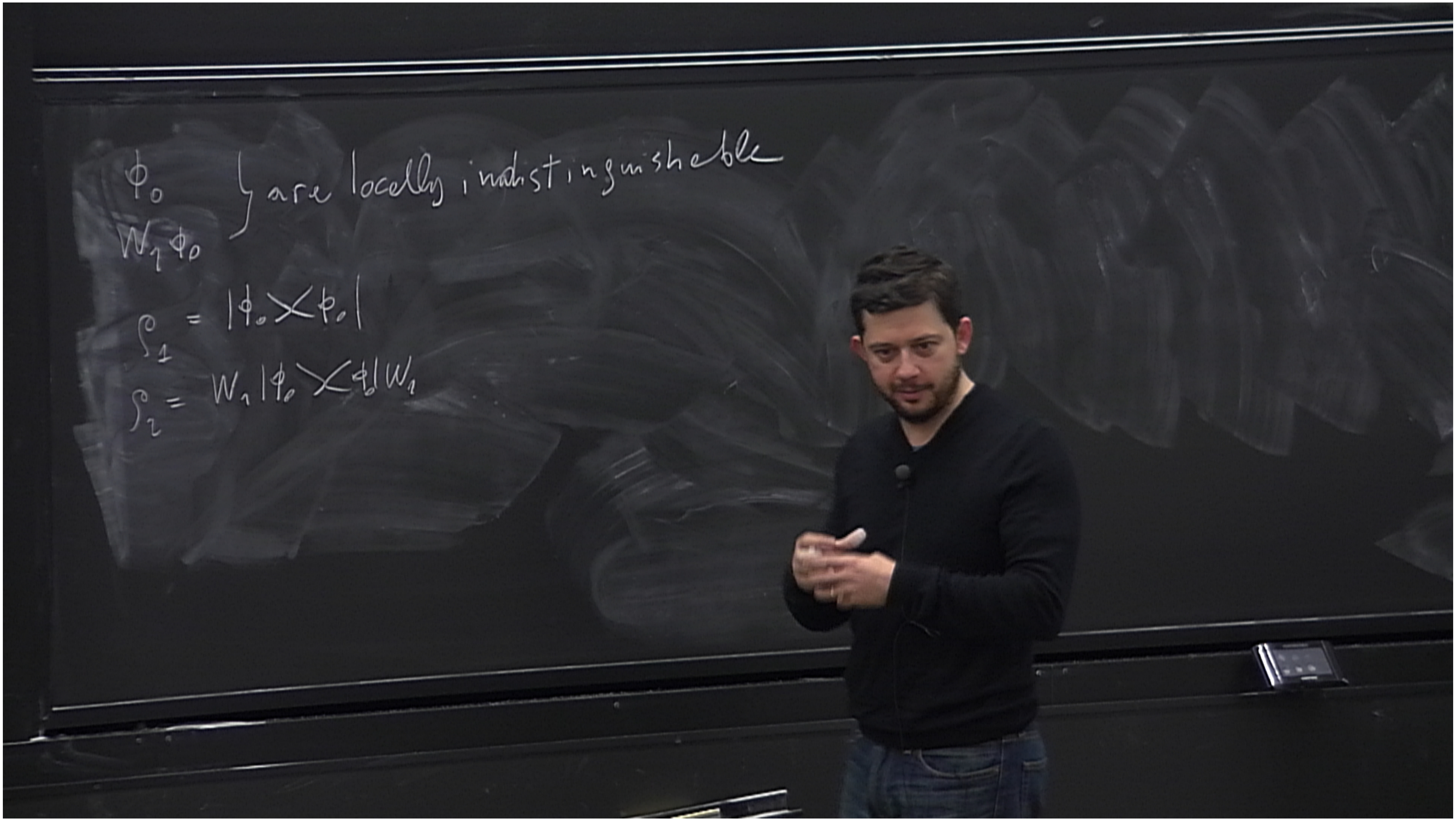
$$[W^z_{C}, H_{TC}] = 0$$

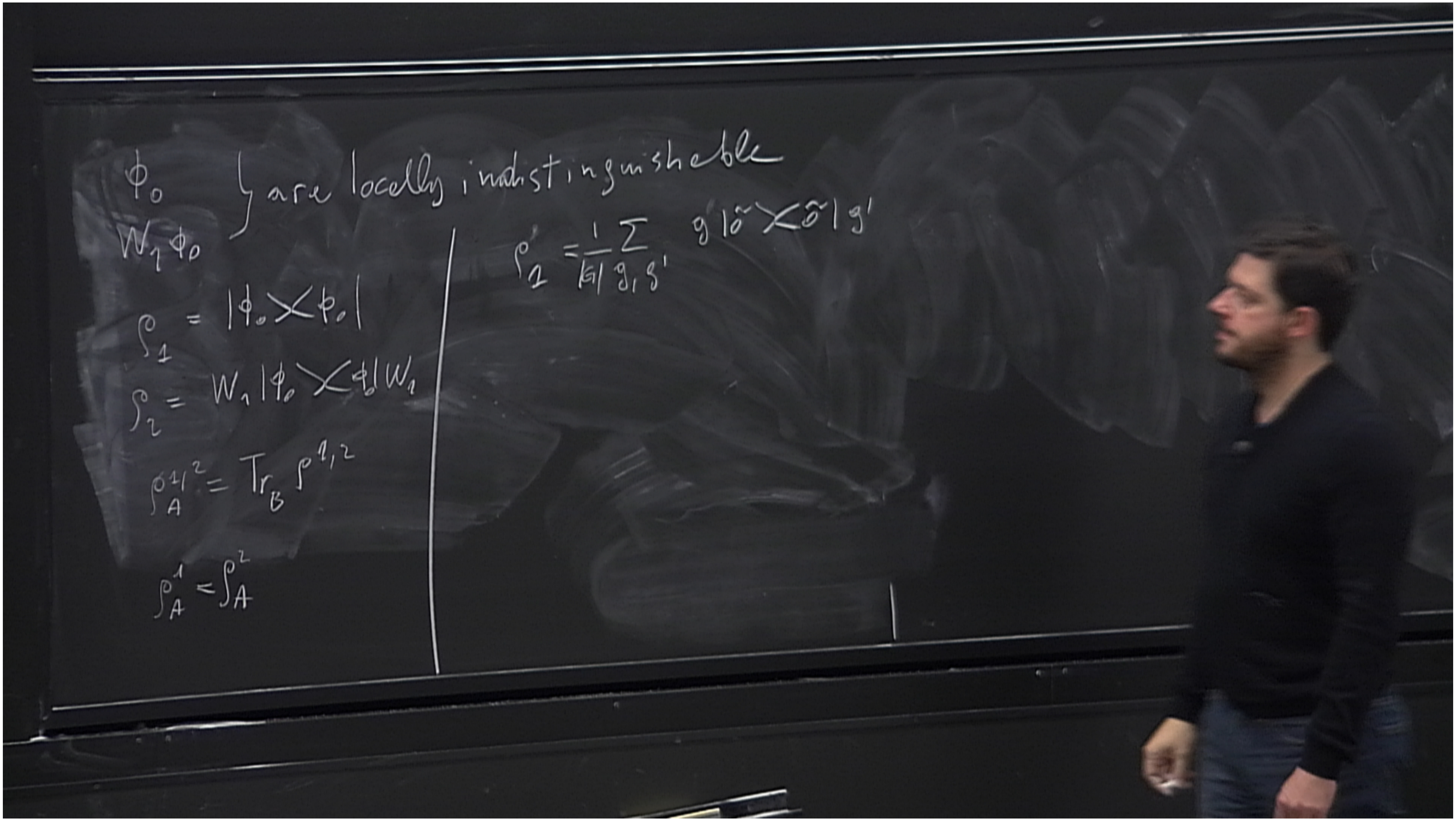

String-net condensate
$W^x_C = \prod_{j \in C} \hat{\sigma}^z_j$
C any curve in Λ
C' " " in Λ'
$W_{C'} = \prod_{j \in C'} \hat{\sigma}^x_j$
blue lattice
C is non-contractible loop c_1 $W^z_{c_1} \equiv W^z_1$
C, C' are loops
$[W^x_{C'}, H_{TC}] = 0$
$[W^z_C, H_{TC}] = 0$
ϕ_0 ?
$\sum$ all possible loops
$g \in G$ loop operator

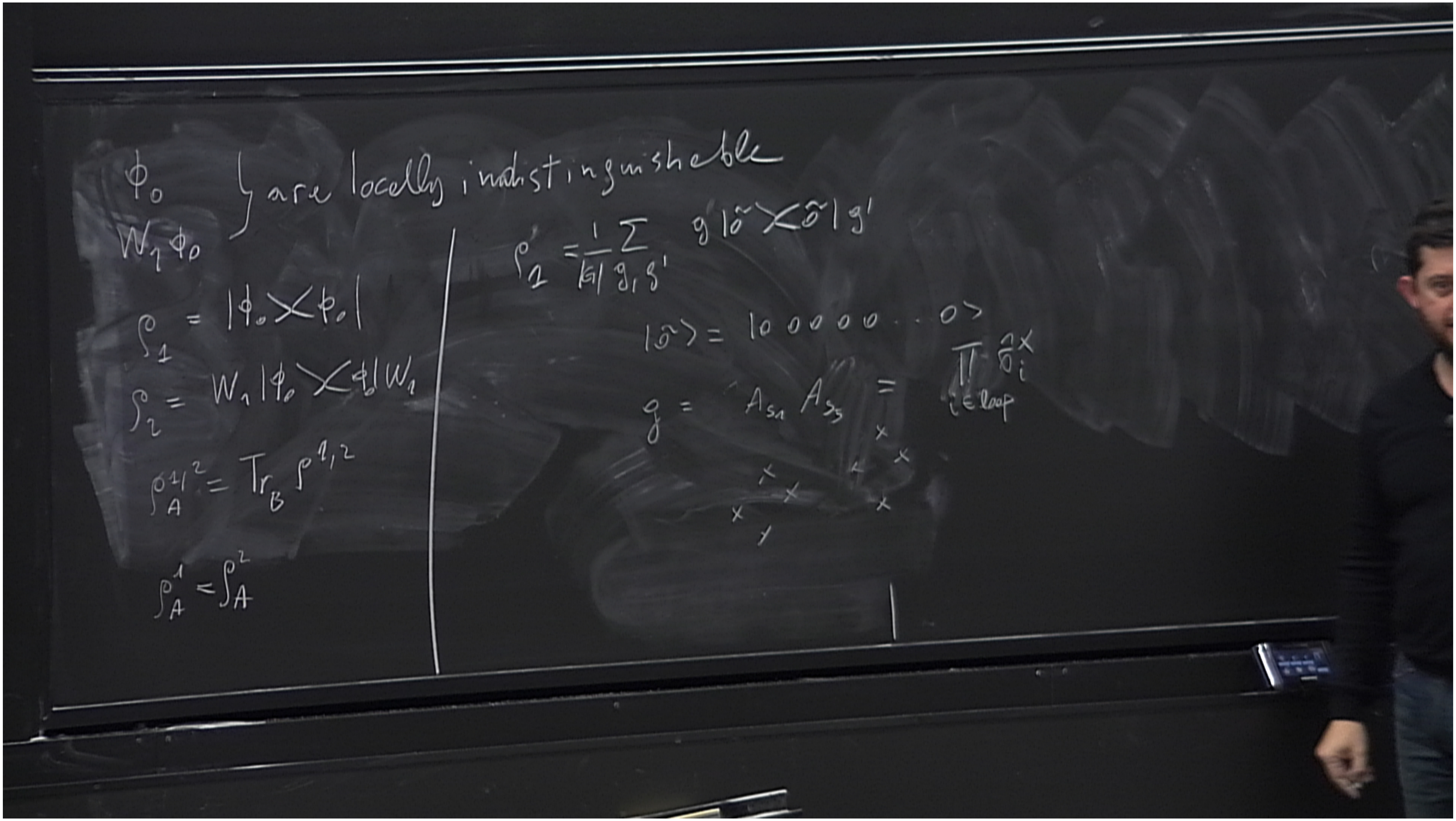

non-contractible
c_1 $W^z_{c_1} \equiv W_1^z$
c, c' are loops
$[W^x_{c'}, H_{Tc}] = 0$
$[W^z_{c}, H_{Tc}] = 0$
ϕ_0 ?
Σ
all possible loops
$g \in G$ loop operator



all with the same weight
possible
loops
perator
Mapping
(non local)
Z2 lattice gauge theory → 2D quantum Ising model
to h_c


ϕ_0 } are locally indistinguishable
$W_1 \phi_0$
$\rho_1 = |\phi_0\rangle\langle\phi_0|$
$\rho_2 = W_1 |\phi_0\rangle\langle\phi_0| W_1$


ϕ_0, $W_1\phi_0$ } are locally indistinguishable
$\rho_1 = |\phi_0\rangle\langle\phi_0|$
$\rho_2 = W_1|\phi_0\rangle\langle\phi_0|W_1$
$\rho_A^{1,2} = Tr_B\, \rho^{1,2}$
$\rho_A^1 = \rho_A^2$
$\rho_2' = \frac{1}{|G|}\sum_{g,g'} g|\tilde{o}\rangle\langle\tilde{o}|g'$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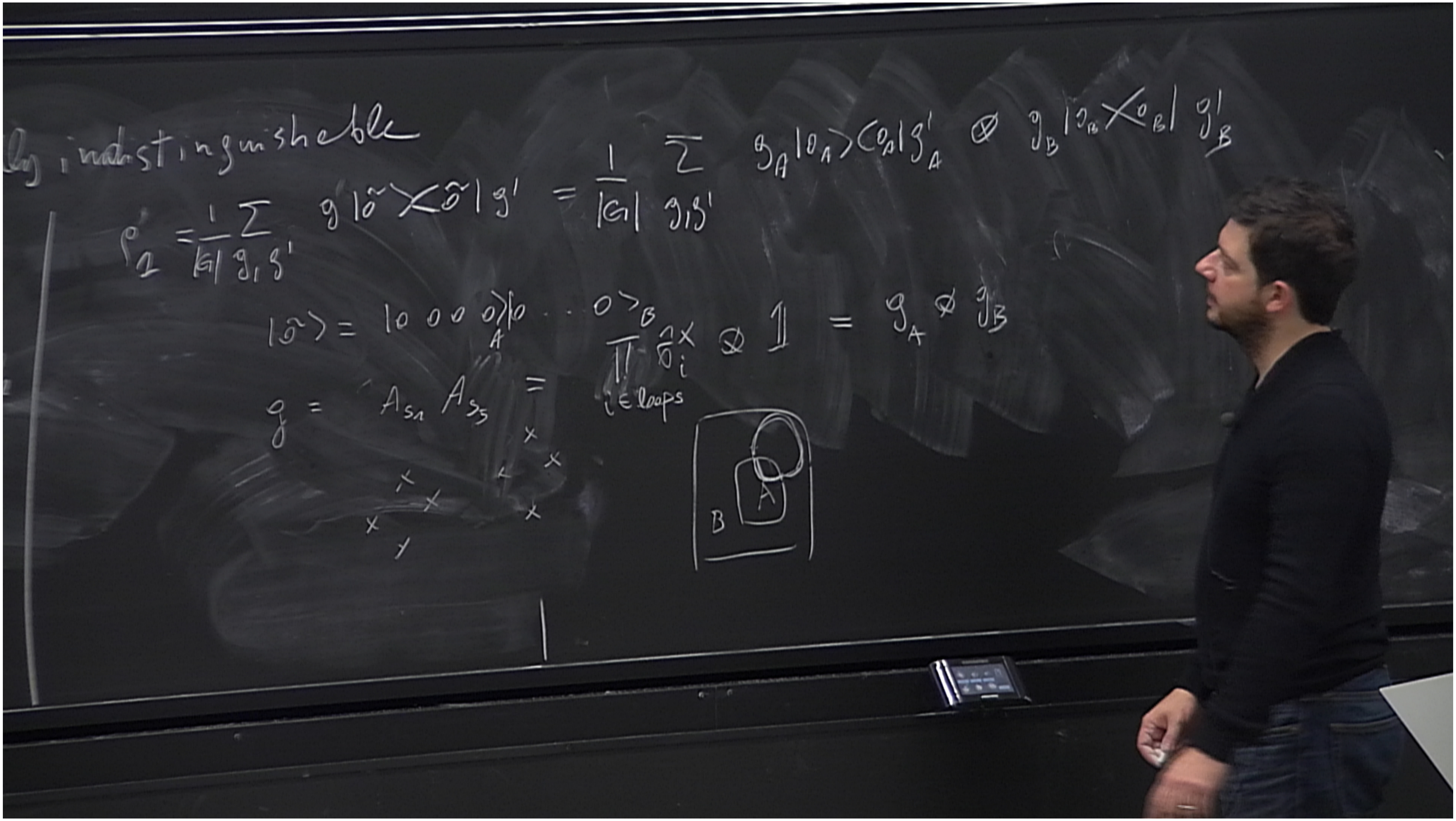
$\rho'_2 = \frac{1}{|G|}\sum_{g,g'} g|\tilde{o}\rangle\langle\tilde{o}|g'$
$|\tilde{o}\rangle = |000000\ldots0\rangle$
$g = A_{s_1} A_{s_5} = \prod_{i\in loop} \hat{\sigma}_i^x$


ly indistinguishable
$\rho_1 = \frac{1}{|G|}\sum_{g,g'} g|\tilde{0}\rangle\langle\tilde{0}|g' = \frac{1}{|G|}\sum_{g,g'} g_A|0_A\rangle\langle 0_A|g'_A \otimes g_B|0_B\rangle\langle 0_B|g'_B$
$|\tilde{0}\rangle = |0000\rangle_A|0\ldots0\rangle_B$
$\prod_{i\in \text{loops}} \hat{\sigma}^x_i \otimes \mathbb{1} = g_A \otimes g_B$
$g = A_{s_1} A_{s_5} =$
A
B

...ly indistinguishable

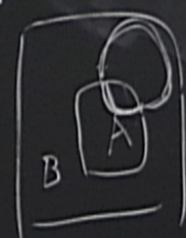
$$\rho_1 = \frac{1}{|G|}\sum_{g,g'} g|\tilde{0}\rangle\langle\tilde{0}|g' = \frac{1}{|G|}\sum_{g,g'} g_A|0_A\rangle\langle 0_A|g'_A \otimes g_B|0_B\rangle\langle 0_B|g'_B$$

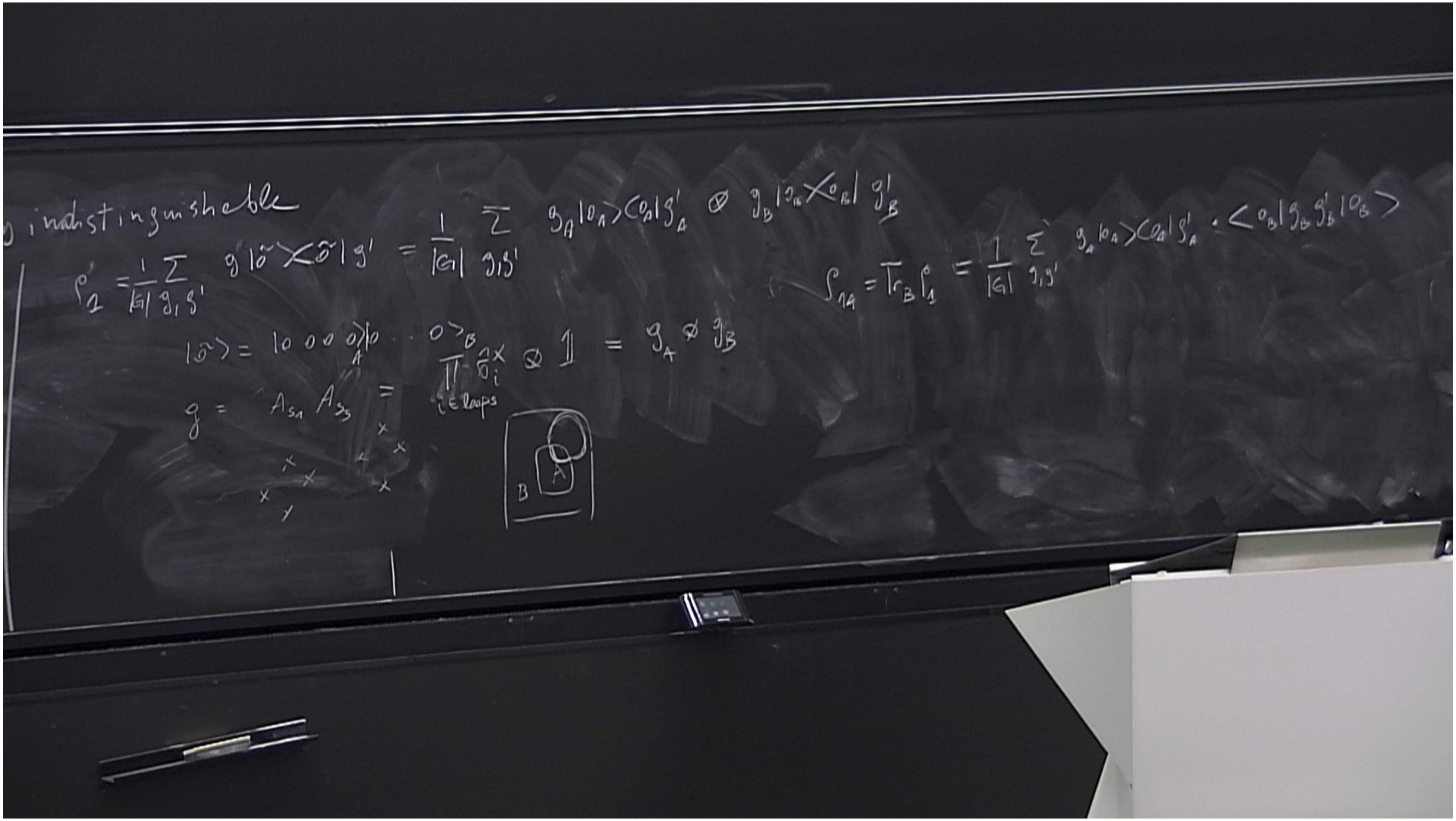
$$\rho_{1A} = \mathrm{Tr}_B\, \rho_1$$

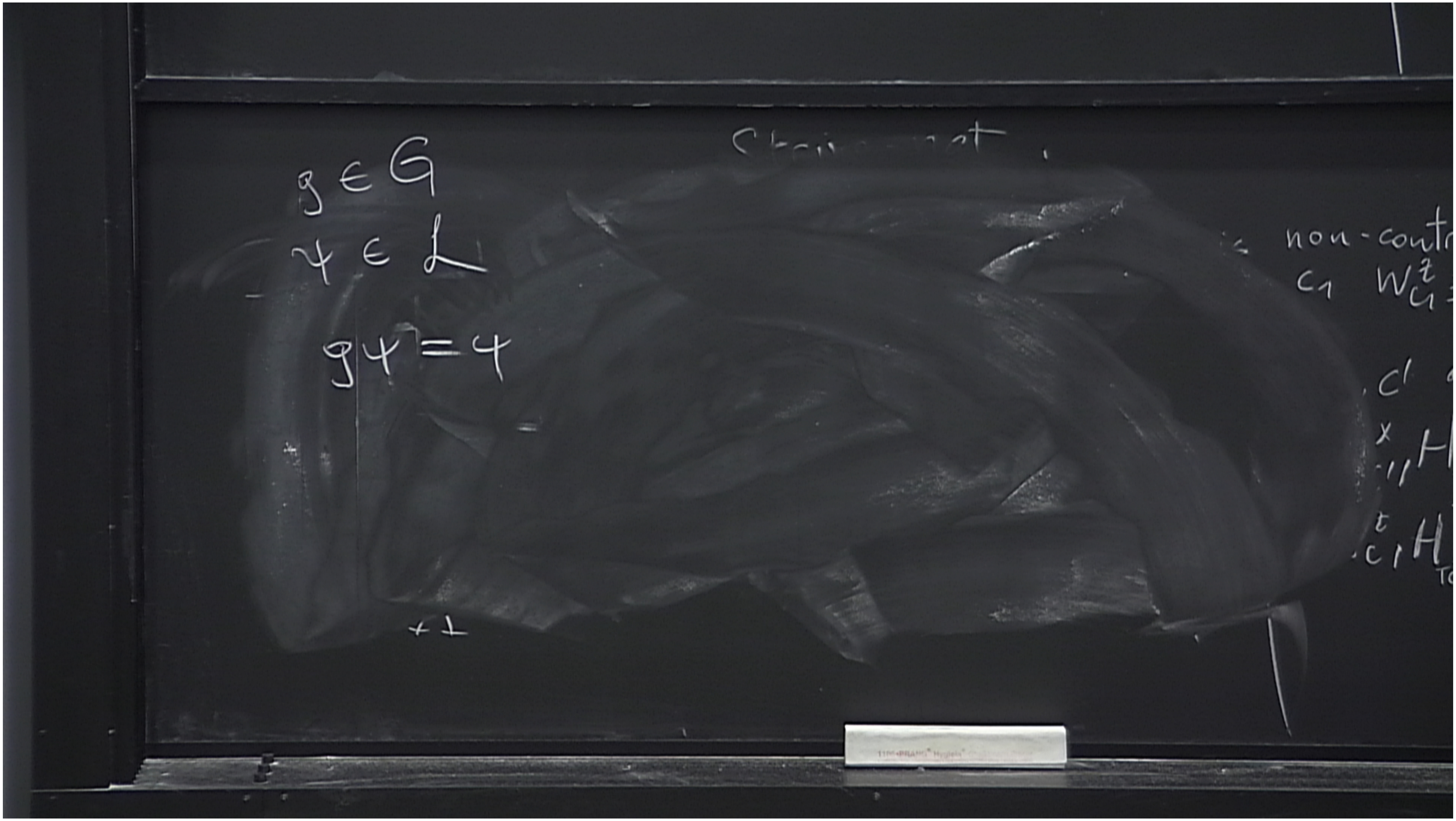
$$|\tilde{0}\rangle = |0000\rangle_A|0\ldots0\rangle_B$$

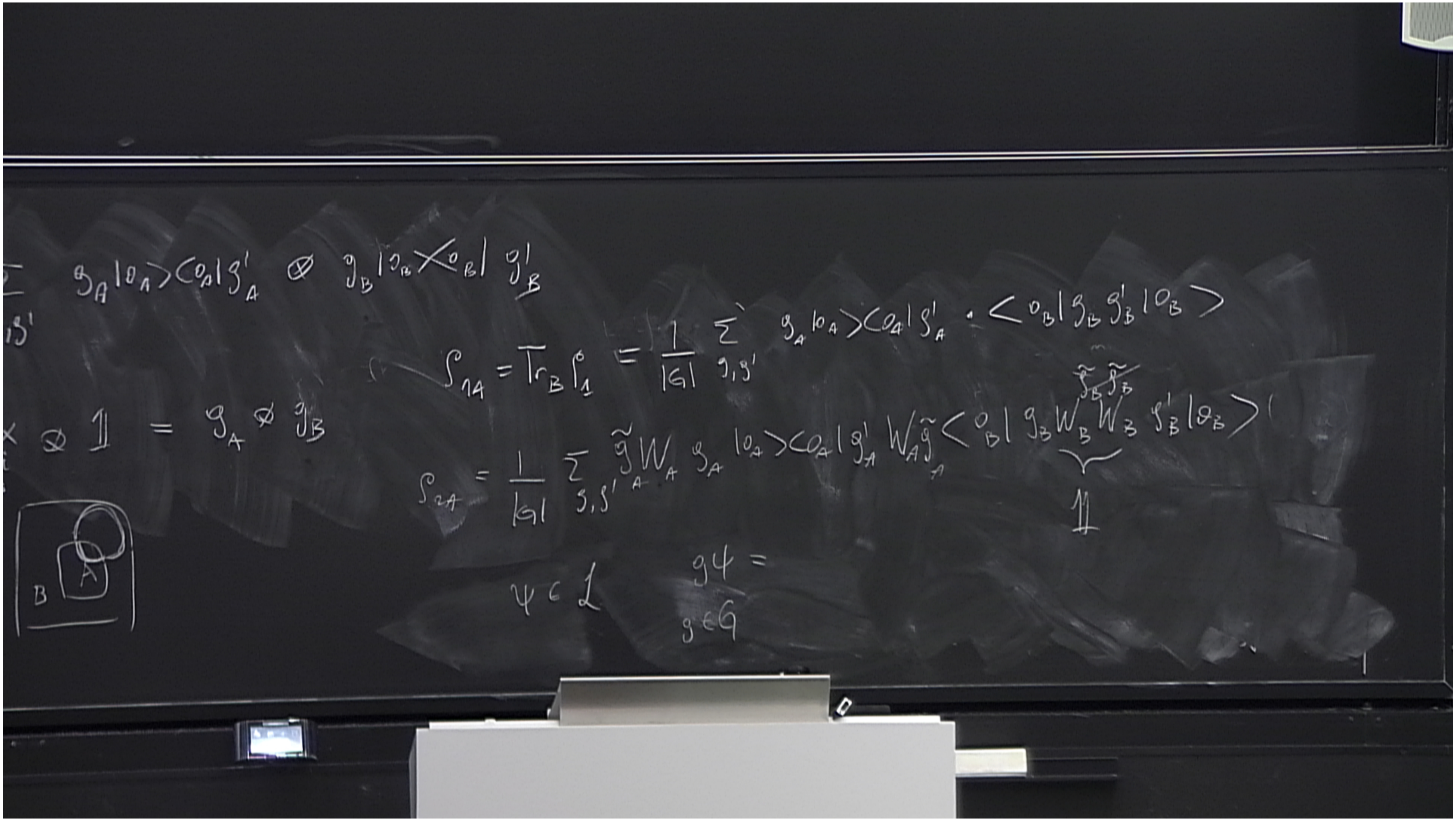
$$\prod_{i\in\text{loops}} \hat{\sigma}_i^x \otimes \mathbb{1} = g_A \otimes g_B$$

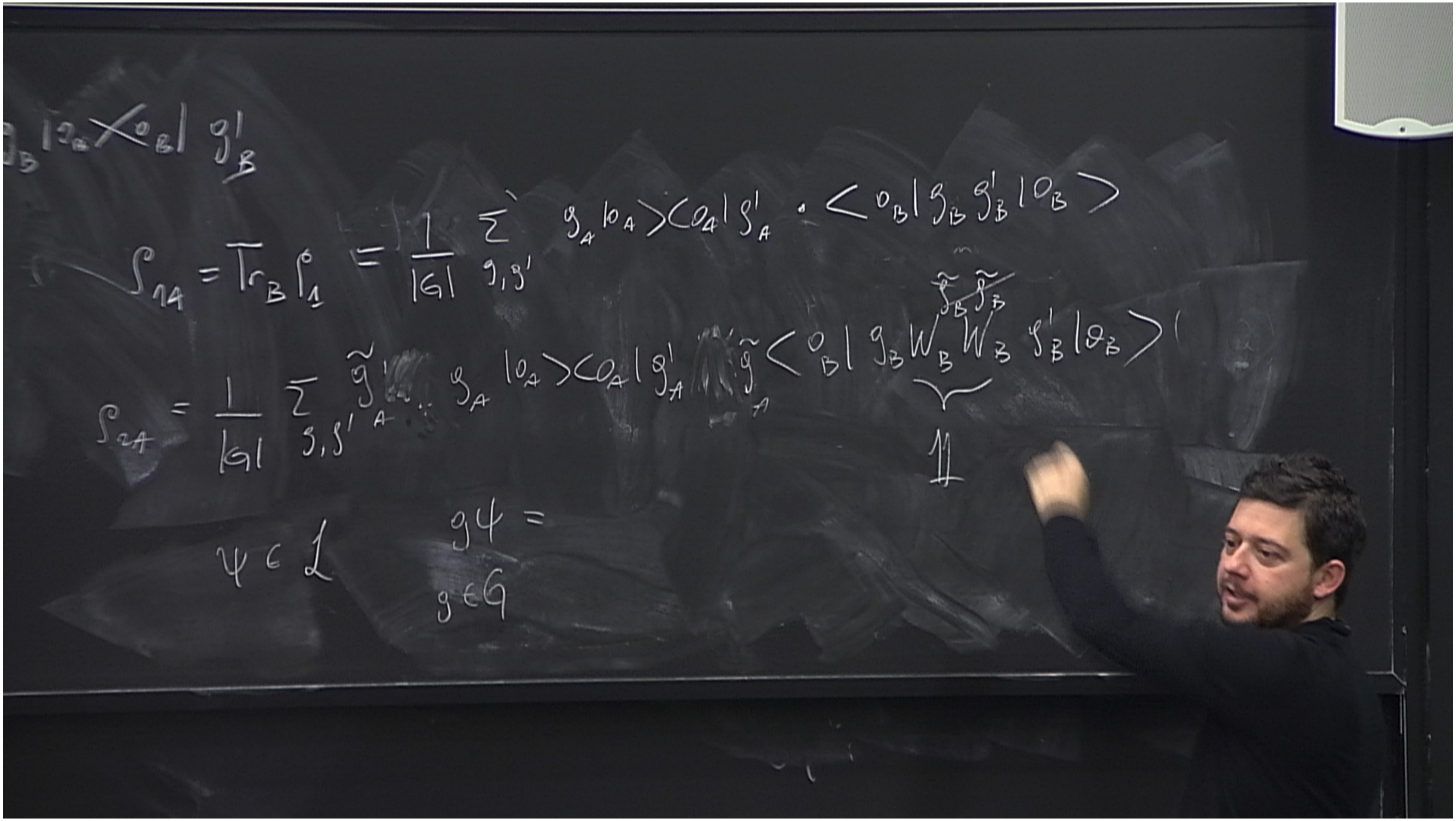
$$g = A_{s_1} A_{s_3} =$$




indistinguishable
$\rho_1' = \frac{1}{|G|}\sum_{g,g'} g|\tilde{o}\rangle\langle\tilde{o}|g' = \frac{1}{|G|}\sum_{g,g'} g_A|0_A\rangle\langle 0_A|g'_A \otimes g_B|0_B\rangle\langle 0_B|g'_B$
$\rho_{1A} = \mathrm{Tr}_B\,\rho_1 = \frac{1}{|G|}\sum_{g,g'} g_A|0_A\rangle\langle 0_A|g'_A \cdot \langle 0_B|g_B g'_B|0_B\rangle$
$|\tilde{o}\rangle = |0\,0\,0\,0\rangle_A|0\ldots 0\rangle_B$
$\prod_{i\in \text{loops}} \hat{\sigma}_i^x \otimes \mathbb{1} = g_A \otimes g_B$
$g = A_{s_1} A_{s_5} =$
B
A


$g \in G$
$\psi \in L$
$g\psi = \psi$
non-contr


$\rho_{1A} = \mathrm{Tr}_B \rho_1 = \frac{1}{|G|} \sum_{g,g'} g_A |0_A\rangle\langle 0_A| g'_A \cdot \langle 0_B | g_B g'_B | 0_B \rangle$
$\rho_{2A} = \frac{1}{|G|} \sum_{g,g'} \tilde{g}_A W_A g_A |0_A\rangle\langle 0_A| g'_A W_A \tilde{g}_A \langle 0_B | g_B W_B W_B g'_B | 0_B \rangle$
$\psi \in \mathcal{L}$
$g\psi =$
$g \in G$


g'_B
$\rho_{1A} = \mathrm{Tr}_B \rho_1 = \frac{1}{|G|}\sum_{g,g'} g_A|0_A\rangle\langle 0_A|g'_A \cdot \langle 0_B|g_B g'_B|0_B\rangle$
$\rho_{2A} = \frac{1}{|G|}\sum_{g,g'} \tilde{g}'_A g_A|0_A\rangle\langle 0_A|g'_A \tilde{g}_A \langle 0_B|g_B \underbrace{W_B W_B}_{1\!\!1} g'_B|0_B\rangle$
$\psi \in \mathcal{L}$
$g\psi =$
$g \in G$