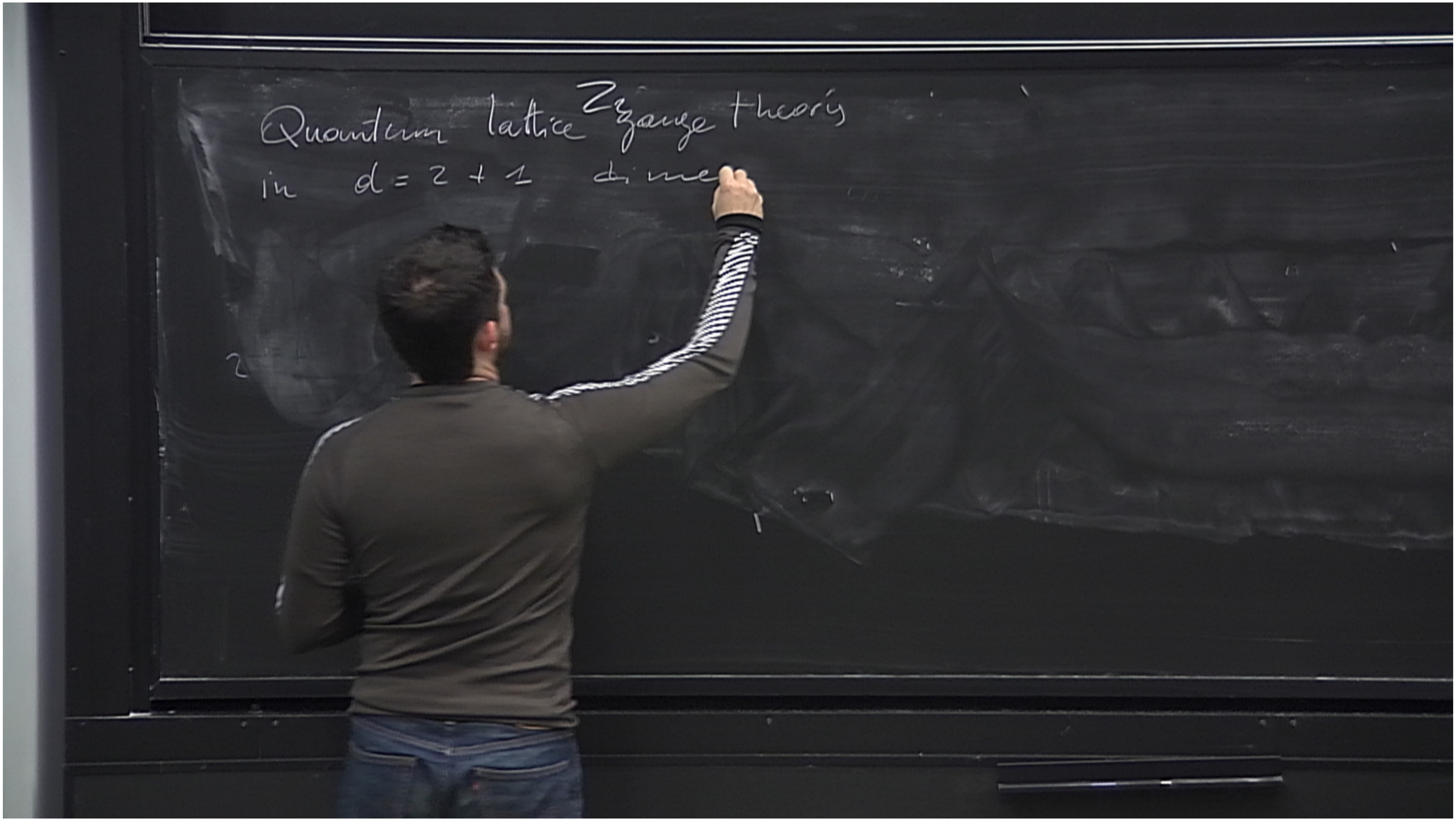
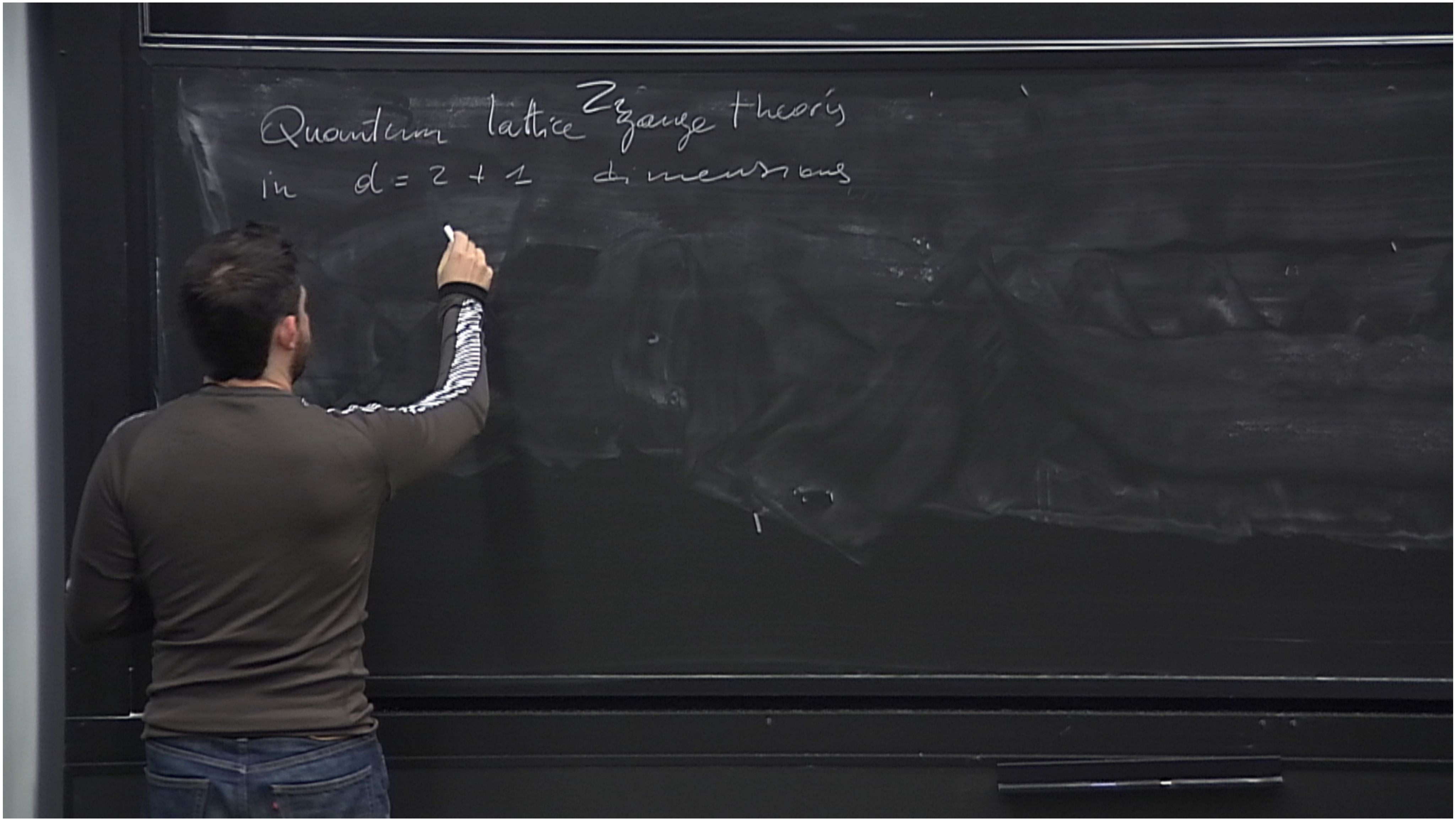
Title: Condensed Matter (Review) - Lecture 11

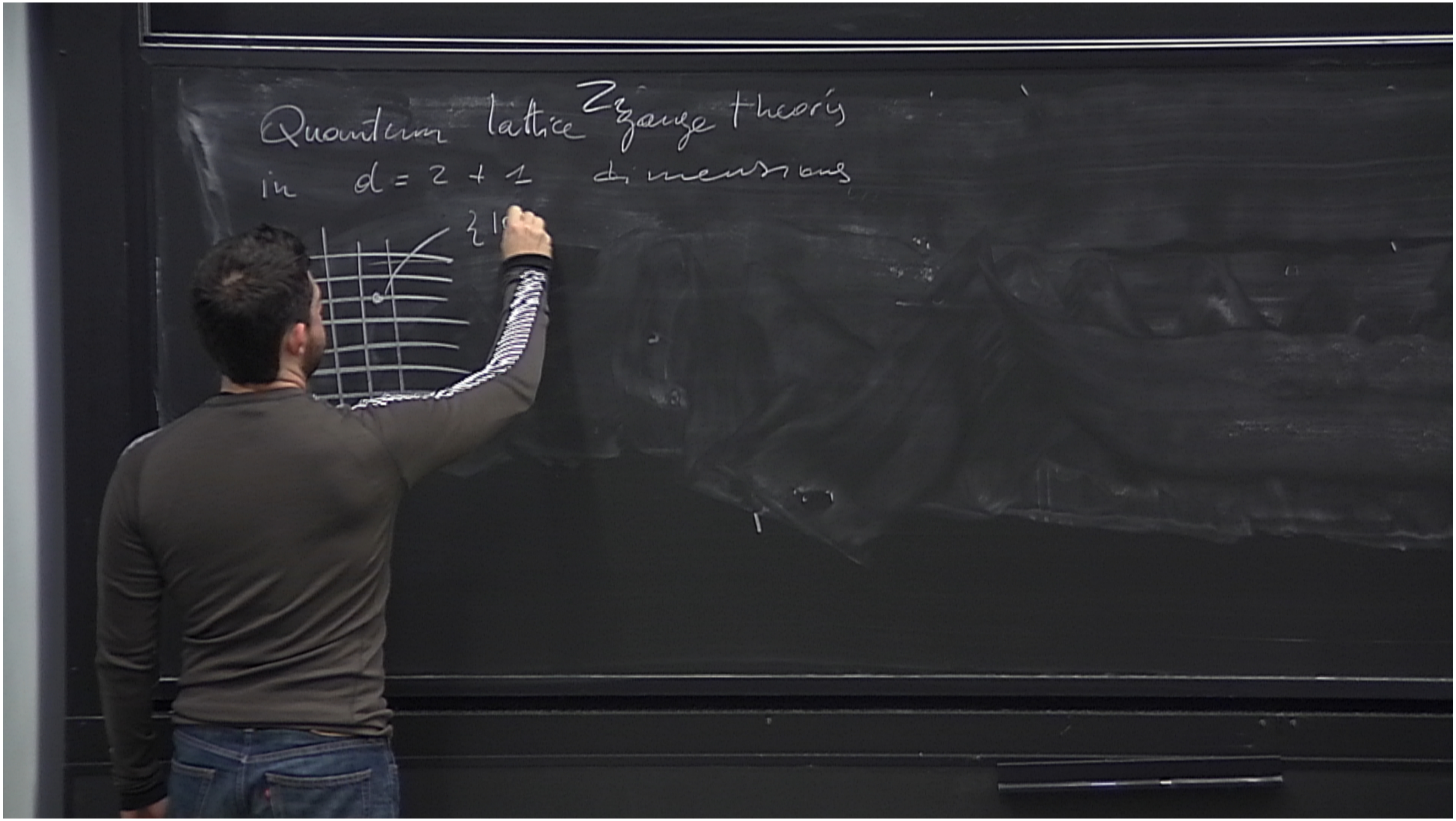
Date: Jan 16, 2012 10:15 AM

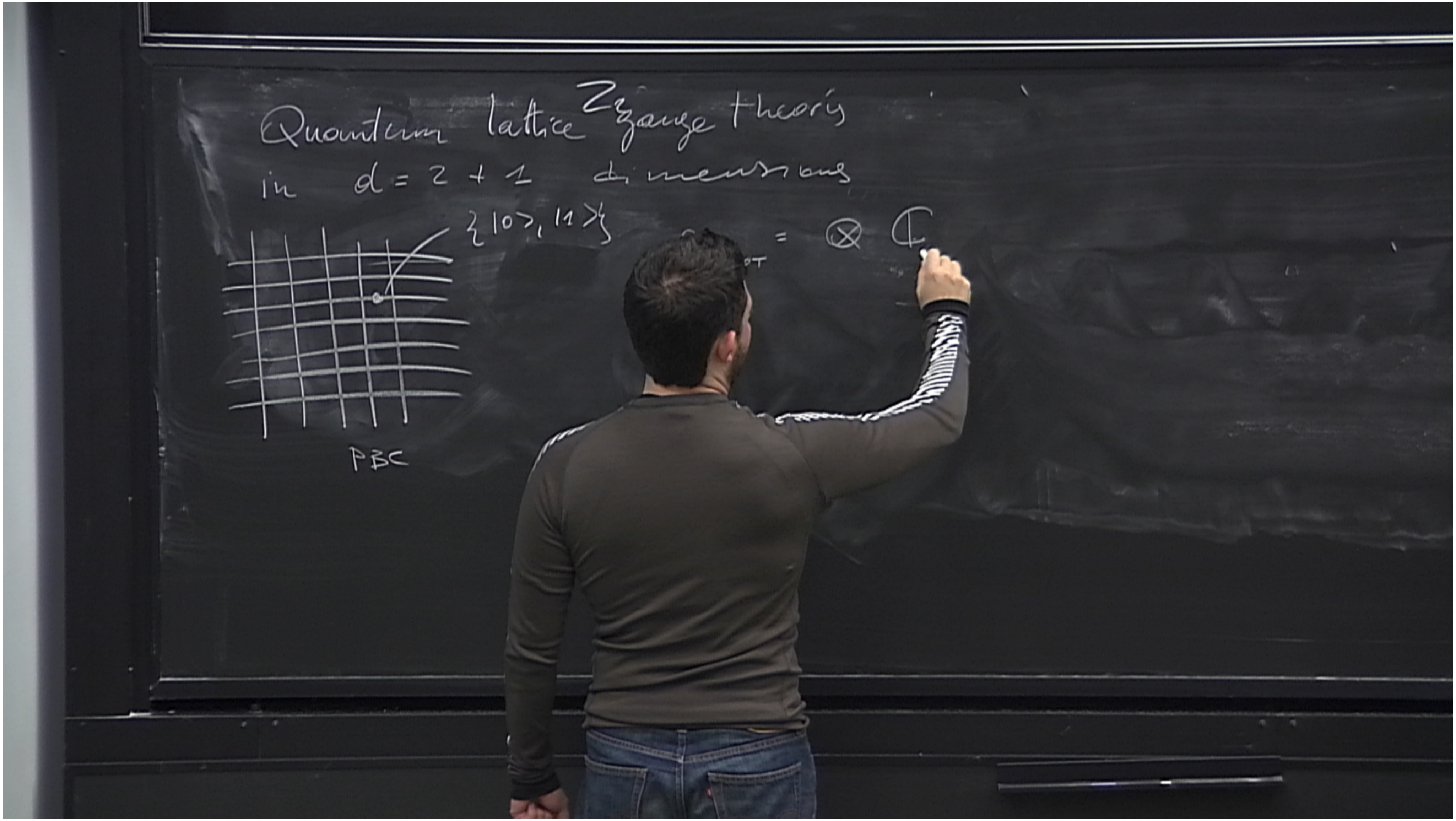
URL: http://pirsa.org/12010097

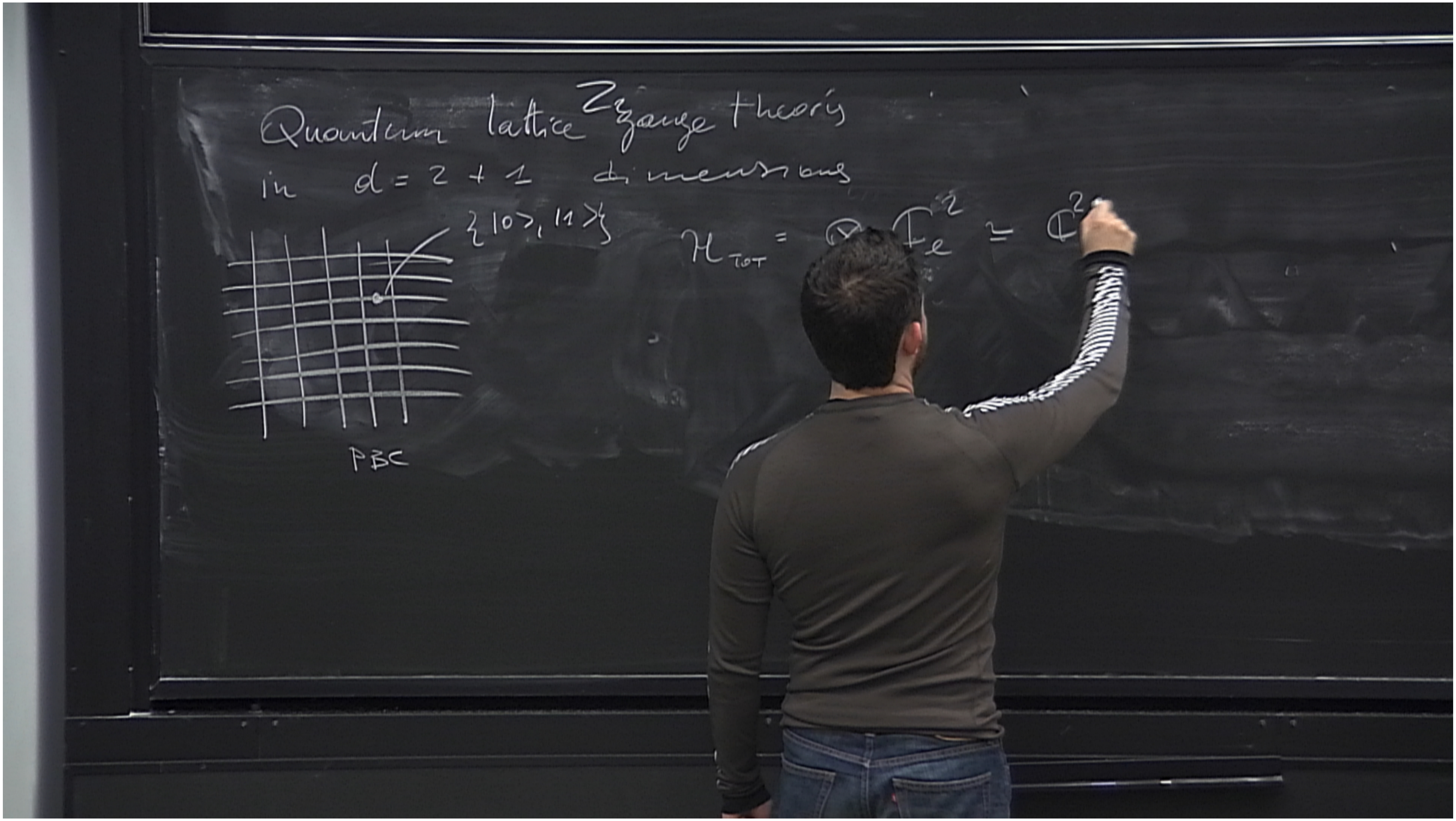
Abstract:

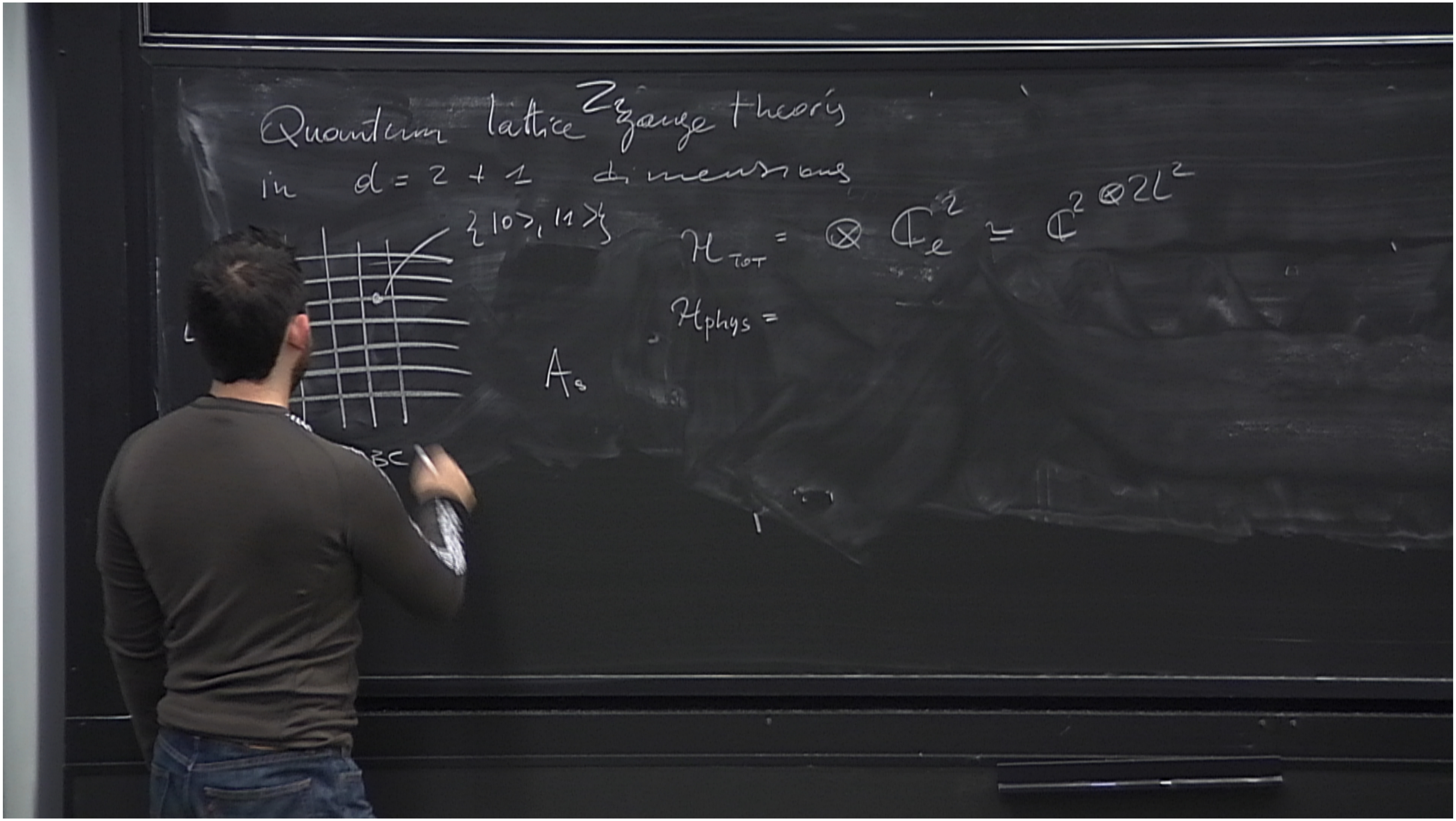

Quantum lattice Z_2 gauge theory
in d = 2 + 1 dime

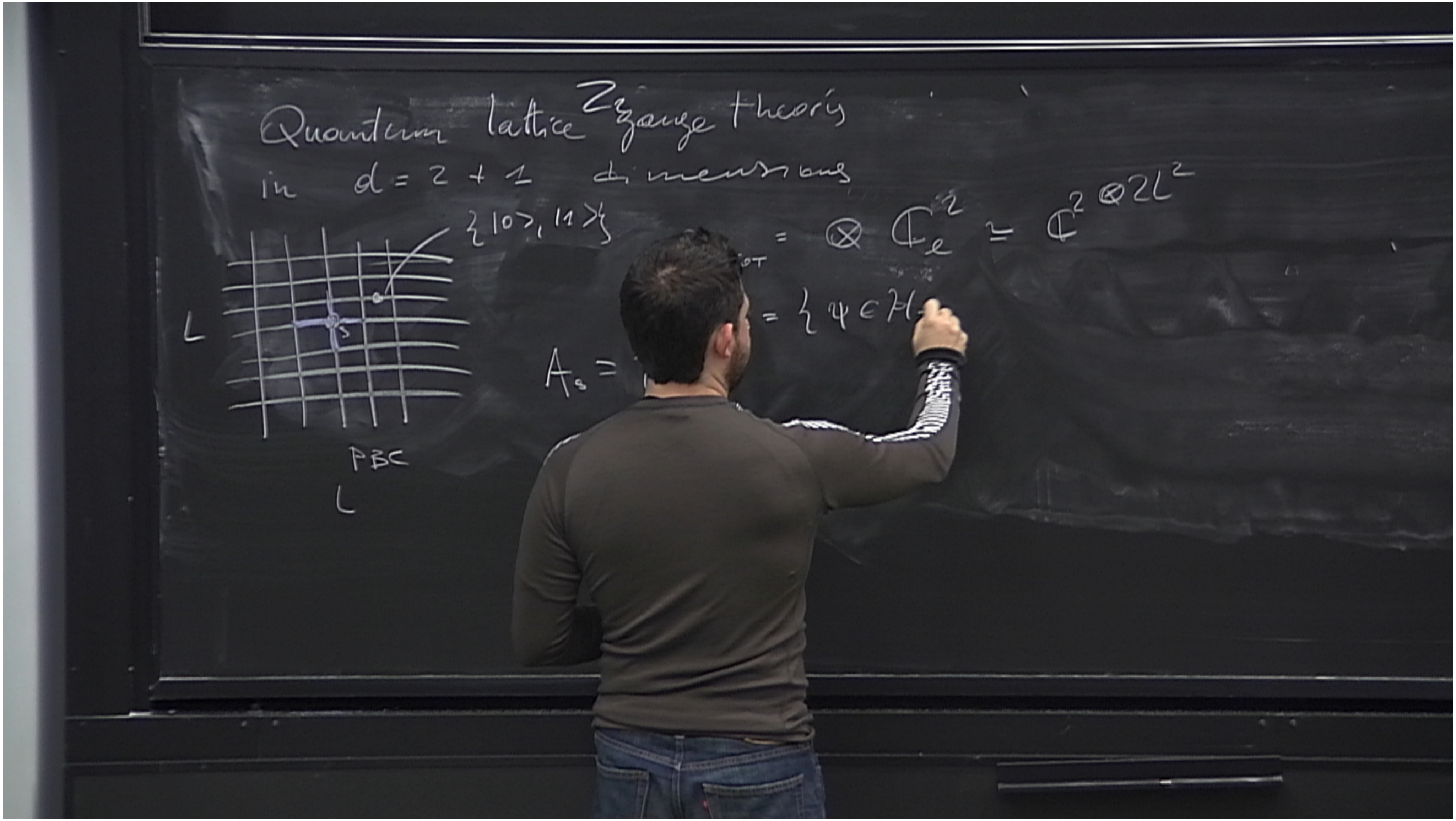

Quantum lattice Z_2 gauge theories
in $d = 2 + 1$ dimensions

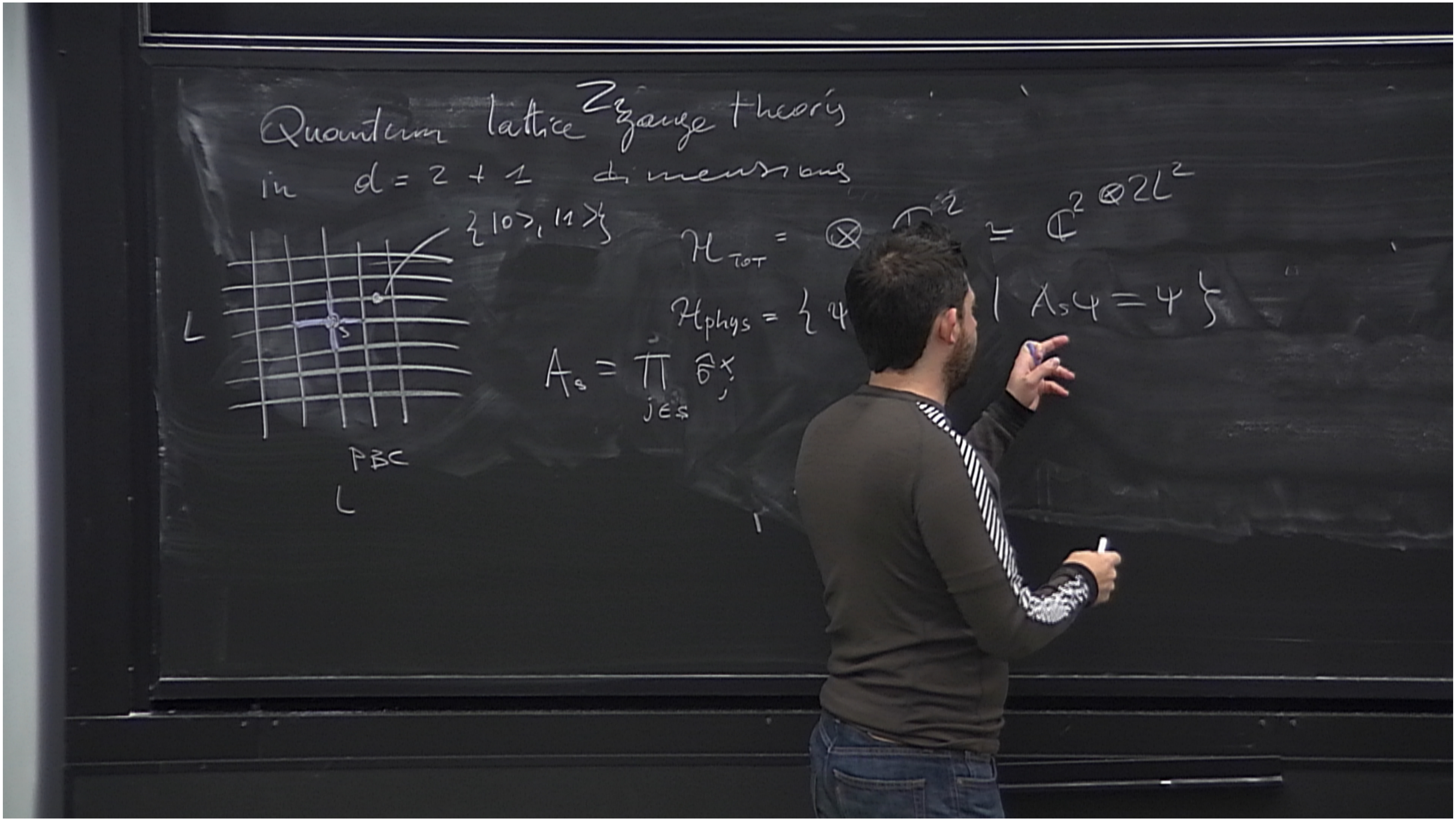

Quantum lattice Z_2 gauge theories
in $d = 2 + 1$ dimensions

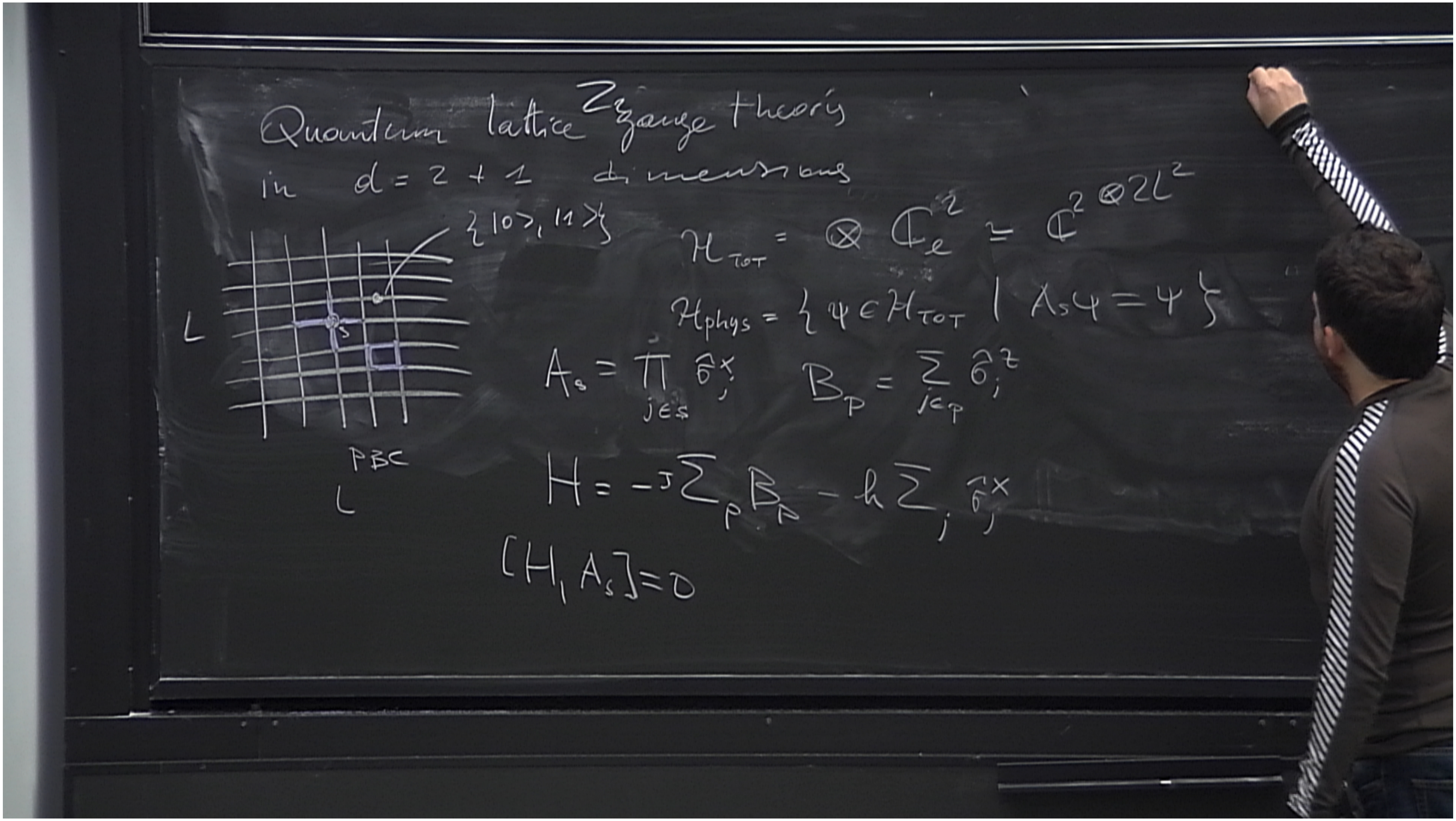

Quantum lattice Z_2 gauge theory
in d = 2 + 1 dimensions
{|0>, |1>}
PBC

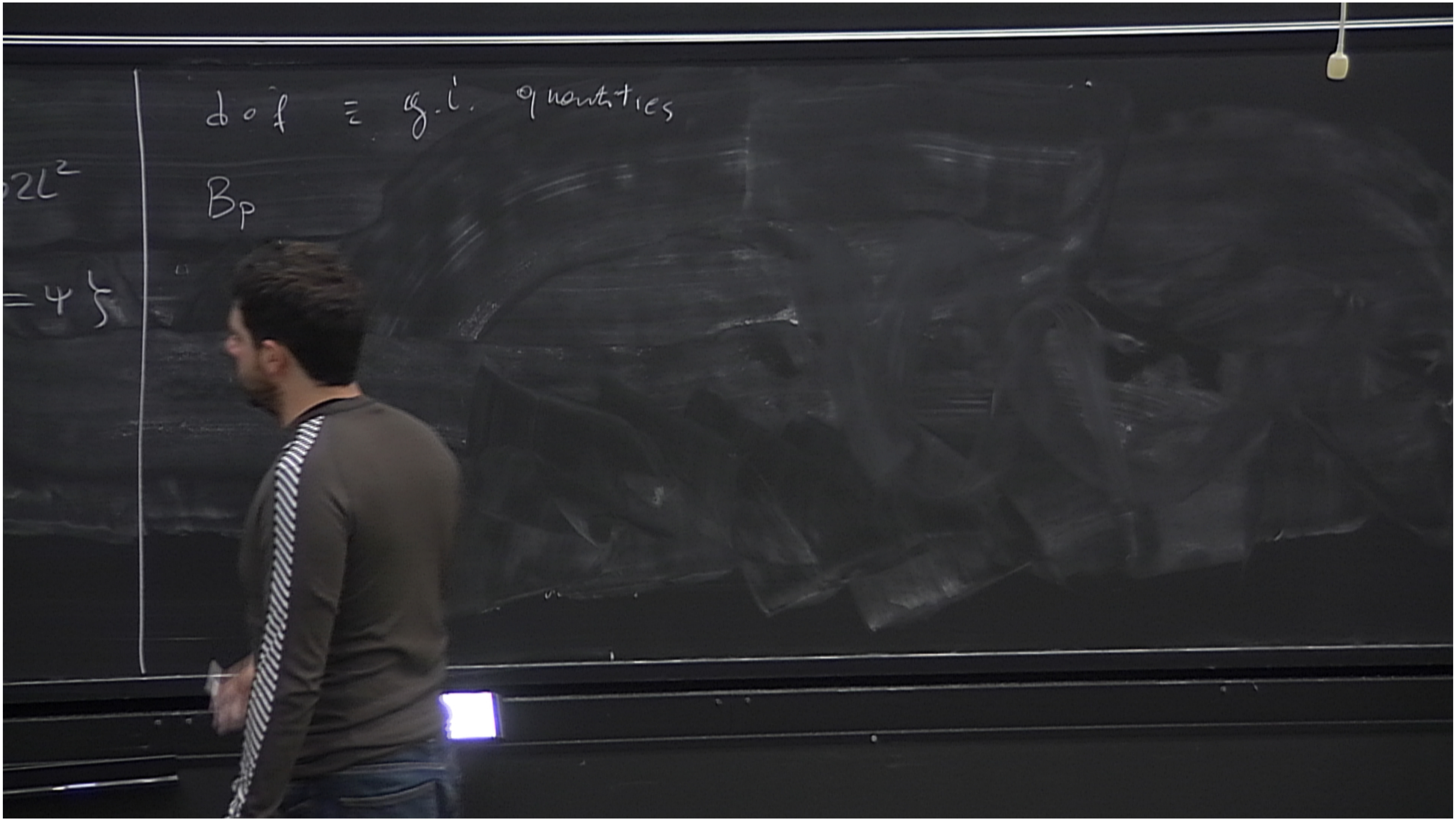

Quantum lattice Z2 gauge theory
in d = 2 + 1 dimensions
{|0>, |1>}
PBC

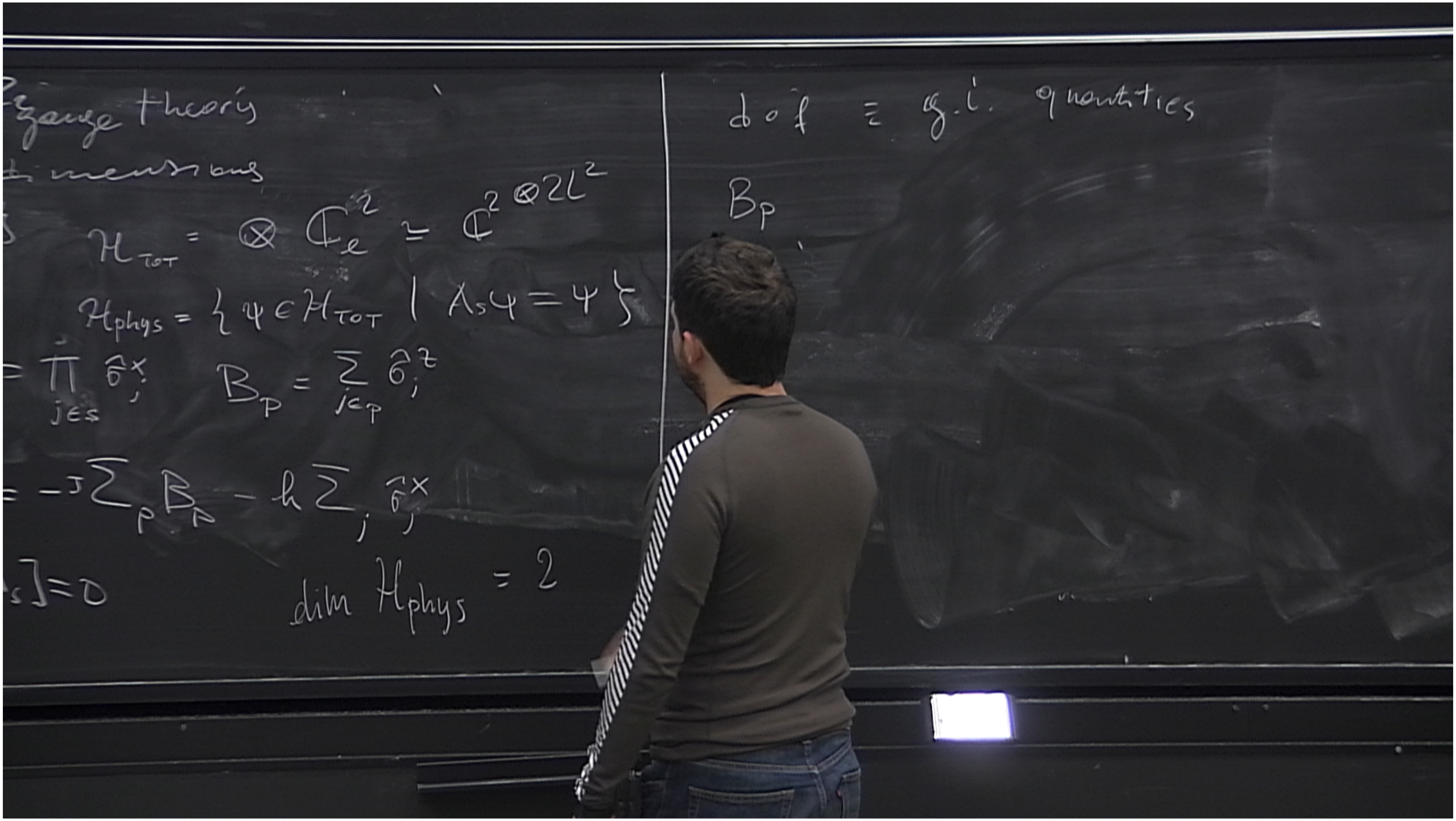

Quantum lattice Z2 gauge theory
in d = 2 + 1 dimensions
{|0>, |1>}
H_TOT = ⊗ C_ℓ^2 ≃ C^2 ⊗ 2L^2
H_phys =
A_s

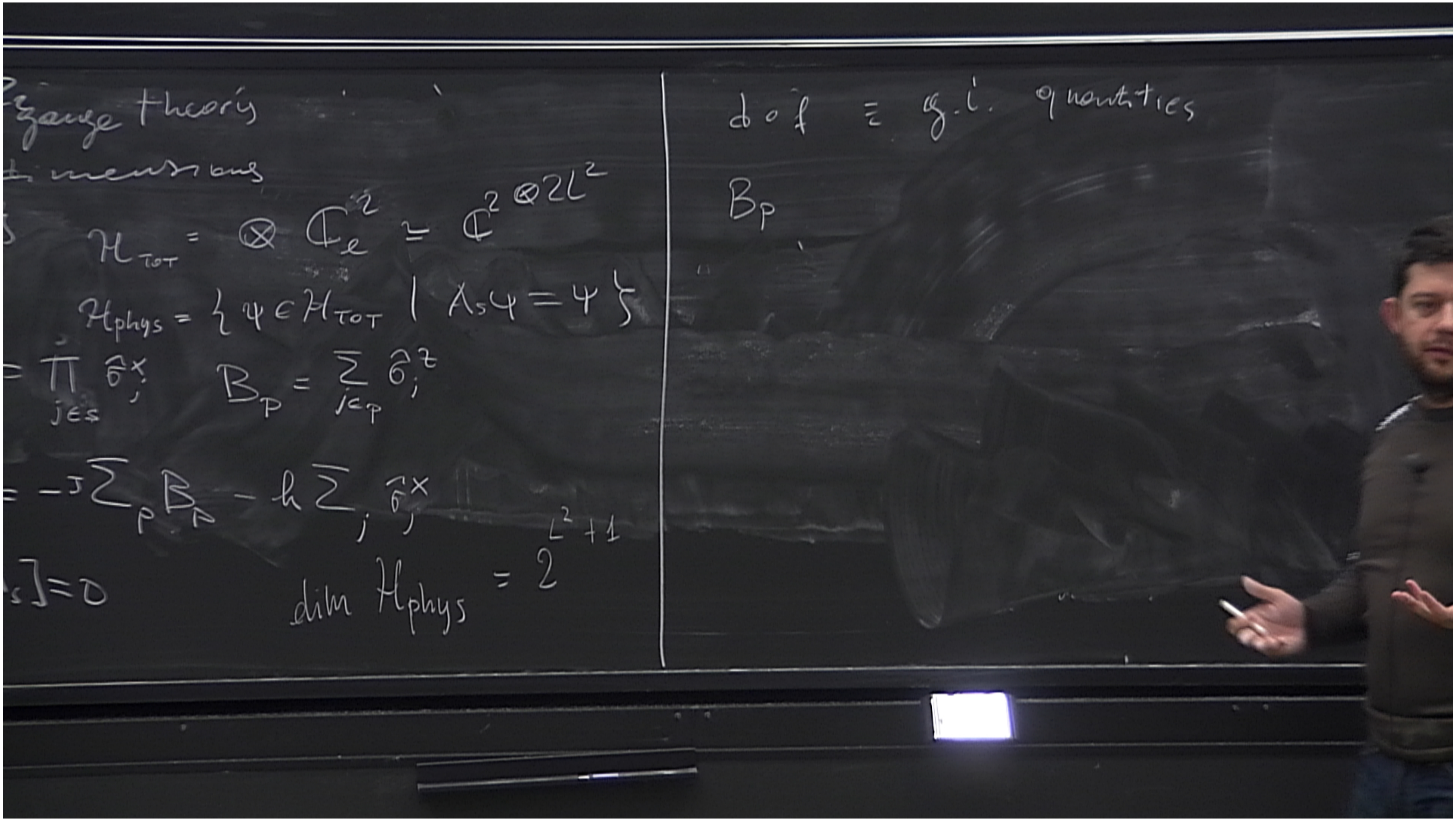

Quantum lattice Z2 gauge theory
in d = 2 + 1 dimensions
{|0>, |1>}
L
s
PBC
L
= ⊗ C_e^2 ≅ C^2 ⊗ 2L^2
= {ψ ∈ H
A_s =

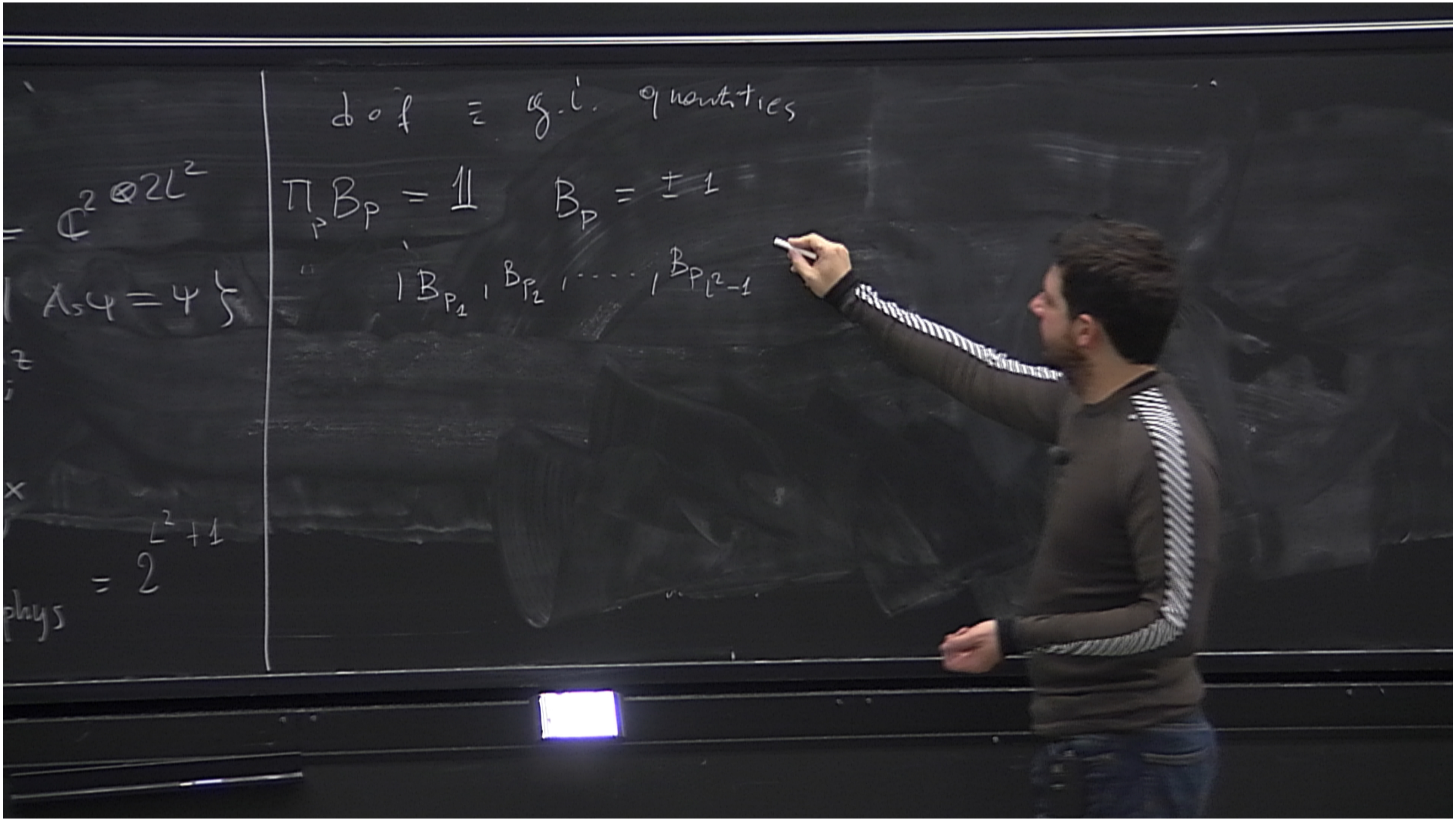

Quantum lattice Z_2 gauge theory
in d = 2 + 1 dimensions
{|0>, |1>}
L
PBC
L
$\mathcal{H}_{TOT} =$
$\mathcal{H}_{phys} =$
$A_s = \prod_{j \in s} \hat{\sigma}^x_j$

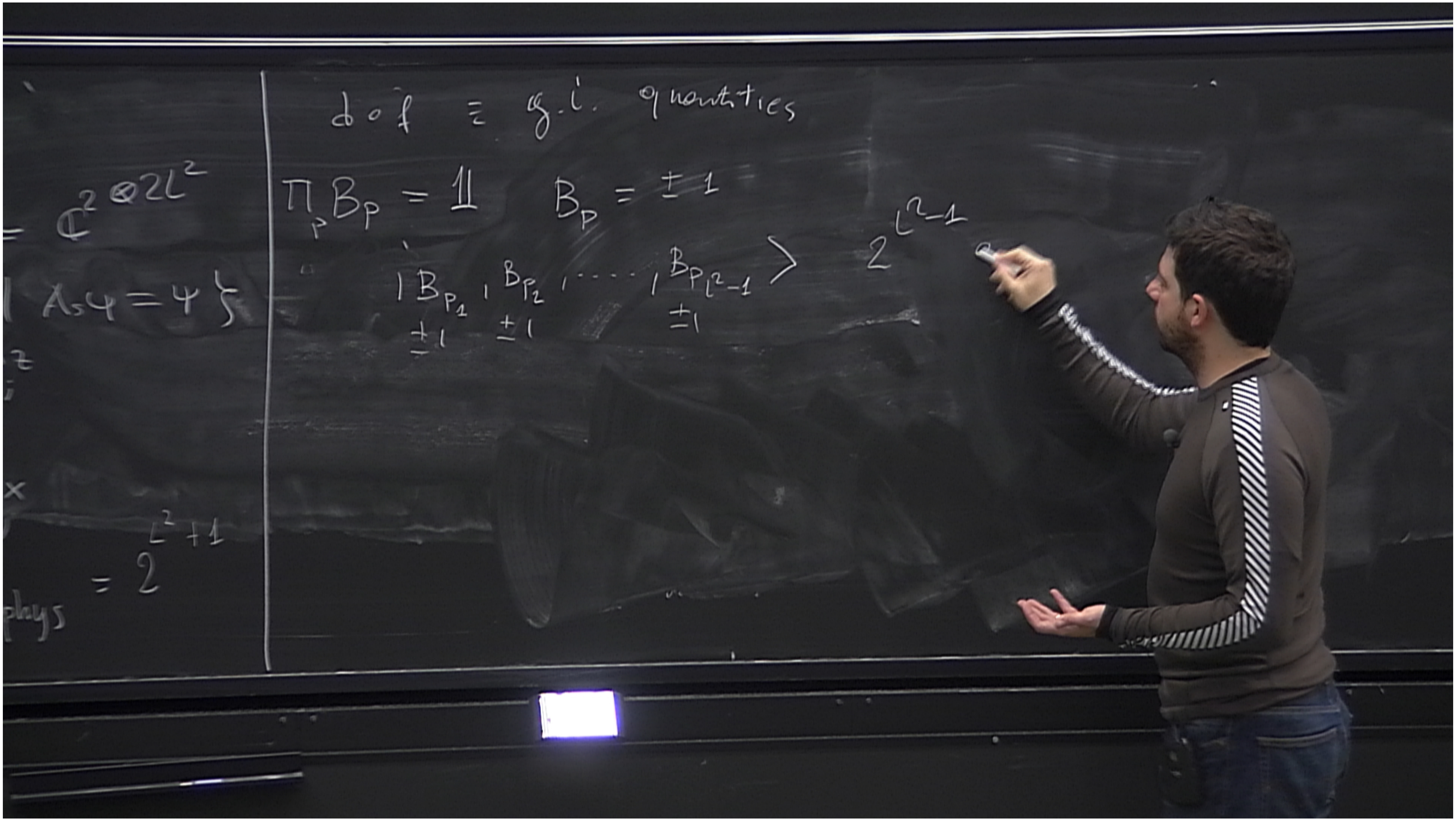

Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
L
s
PBC
L
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}_\ell^2 \cong \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$
$A_s = \prod_{j \in s} \hat{\sigma}_j^x$
$B_p = \sum_{j \in p} \hat{\sigma}_j^z$
$H = -J \sum_p B_p - h \sum_j \hat{\sigma}_j^x$
$[H, A_s] = 0$

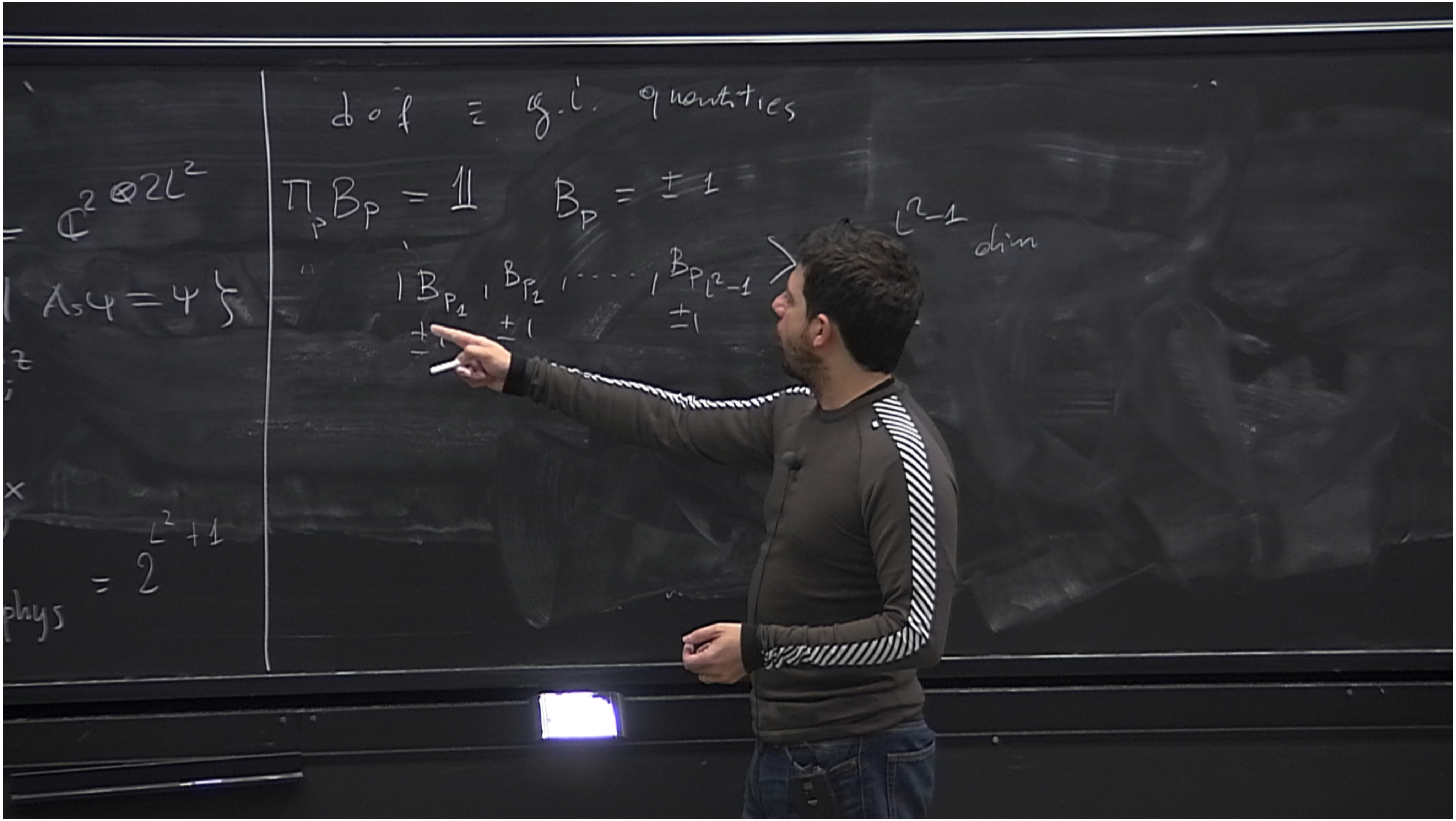

dof ≡ g.i. quantities
B_p
$2L^2$
$=4\}$

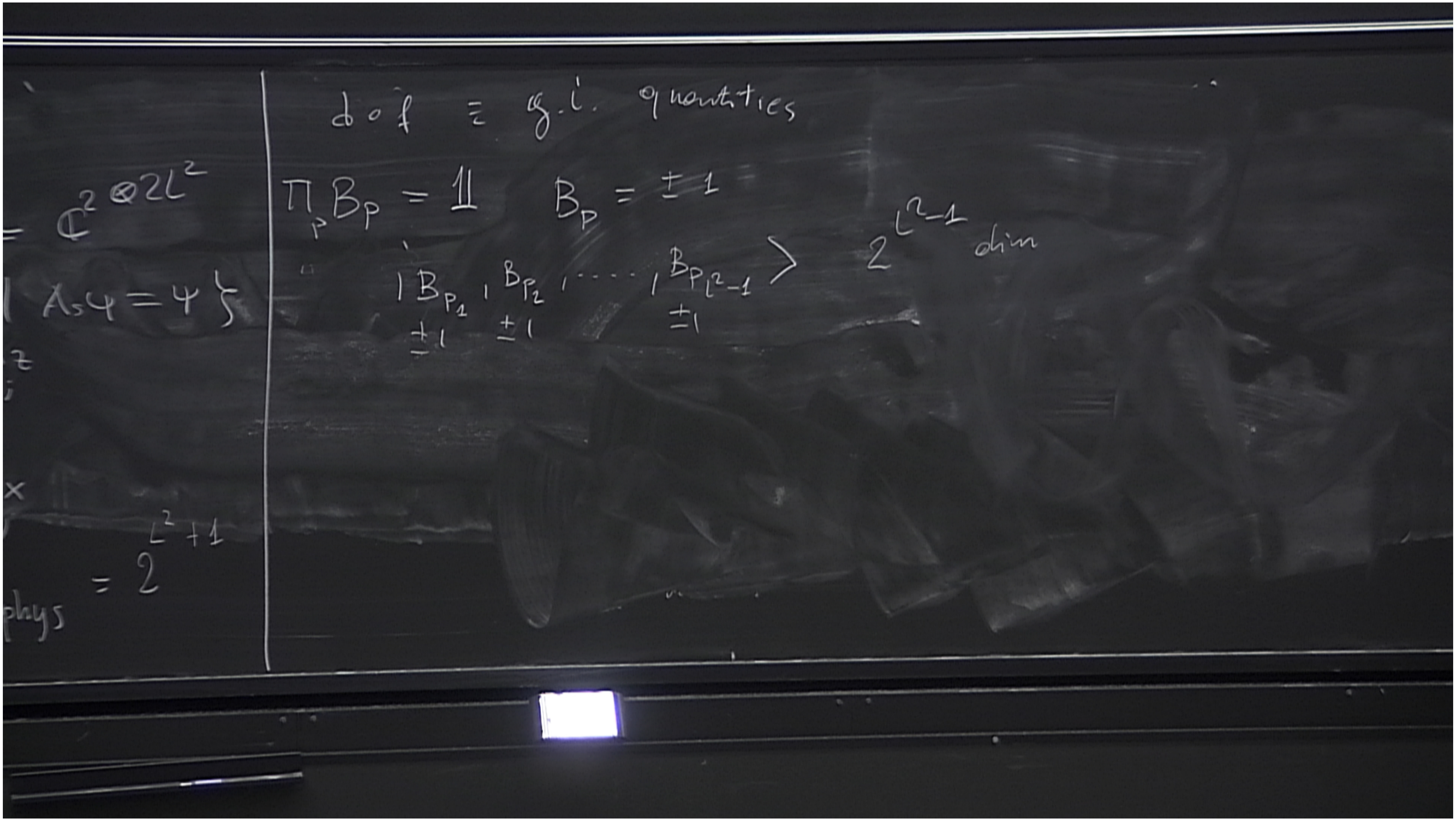

gauge theory
mensions
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}_e^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$
$= \prod_{j \in s} \hat{\sigma}_j^x$
$B_p = \sum_{j \in p} \hat{\sigma}_j^z$
$= -J \sum_p B_p - h \sum_j \hat{\sigma}_j^x$
$]=0$
$\dim \mathcal{H}_{phys} = 2$
dof ≡ g.i. quantities
B_p

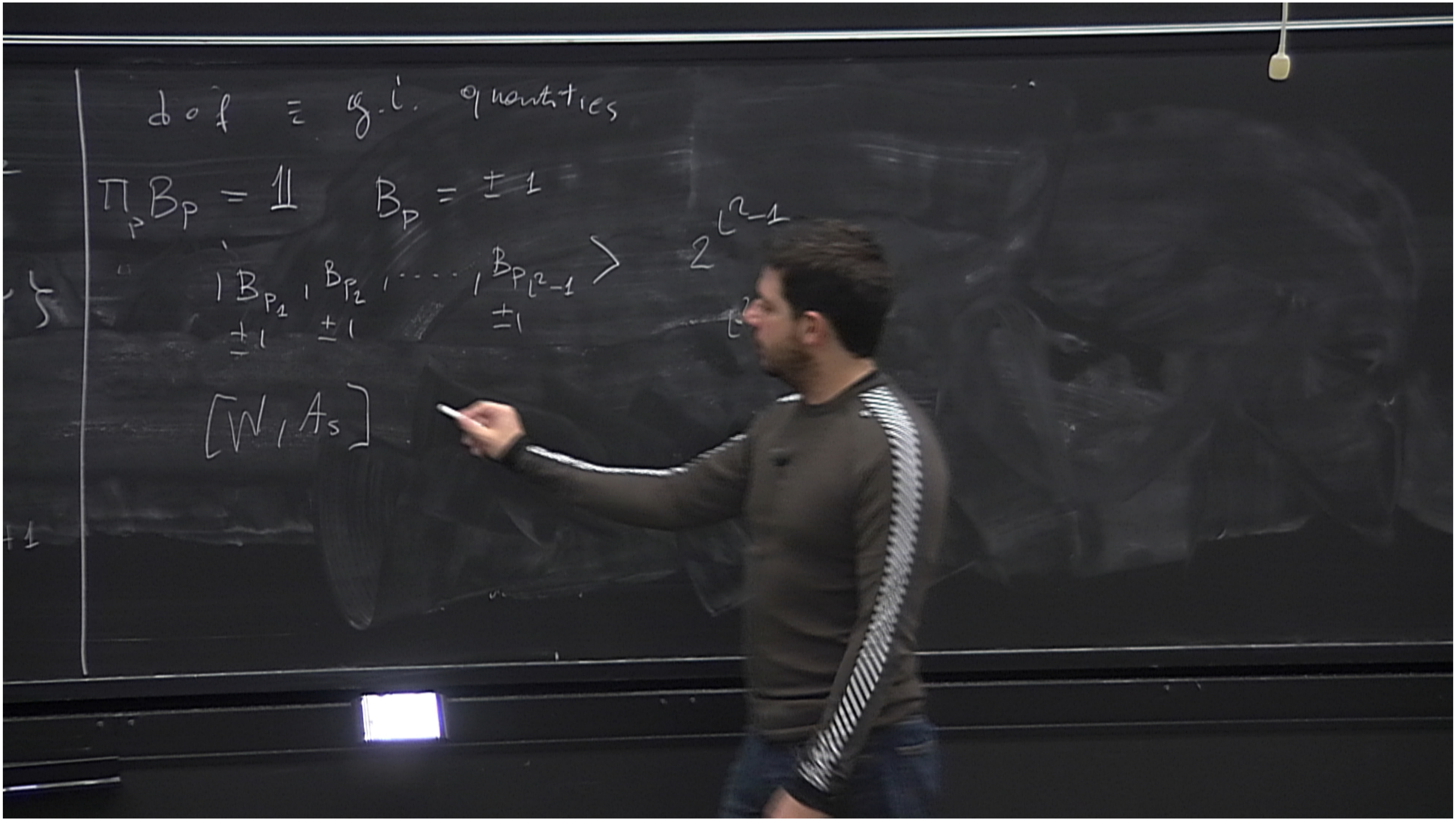

gauge theory
dimensions
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}_e^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$
$= \prod_{j \in s} \hat{\sigma}_j^x \quad B_p = \sum_{j \in p} \hat{\sigma}_j^z$
$= -J \sum_p B_p - h \sum_j \hat{\sigma}_j^x$
$] = 0$
$\dim \mathcal{H}_{phys} = 2^{L^2+1}$
d.o.f. $\equiv$ g.i. quantities
B_p


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}$
$= \mathbb{C}^2 \otimes 2L^2$
$A_s \psi = \psi \}$
phys
$= 2^{L^2+1}$


dof ≡ g.i. quantities
Π_p B_p = 𝟙
B_p = ± 1
| B_p1 , B_p2 , ... , B_pL²-1 >
±1 ±1 ±1
2^(L²-1)
A_s ψ = ψ }
= 2^(L²+1)
phys


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}\rangle$
± 1
± 1
± 1
L^2-1
dim
$= \mathbb{C}^2 \otimes 2L^2$
$A_s \psi = \psi \}$
phys
$= 2^{L^2+1}$


$= \mathbb{C}^2 \otimes 2L^2$
$A_s \psi = \psi \}$
$= 2^{L^2+1}$
phys
dof $\equiv$ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$| B_{p_1}, B_{p_2}, \ldots\ldots, B_{p_{L^2-1}} \rangle$
± 1 ± 1 ± 1
2^{L^2-1} dim


dof ≡ g.i. quantities
Π_p B_p = 1
B_p = ± 1
| B_p1 , B_p2 , ... , B_p(L²−1) ⟩
±1 ±1 ±1
2^(L²−1)
[W , A_s]

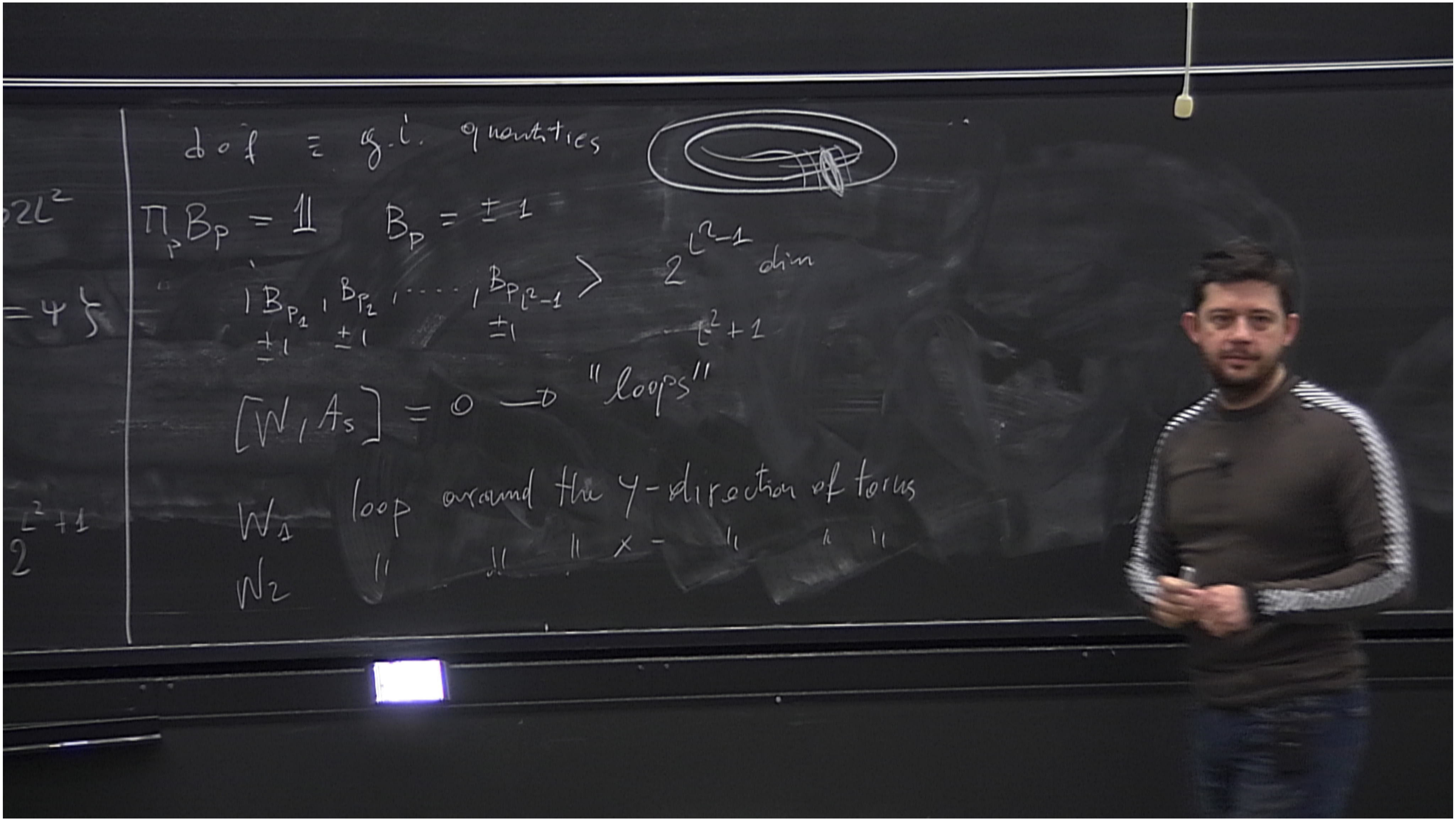
dof $\equiv$ g.i. quantities

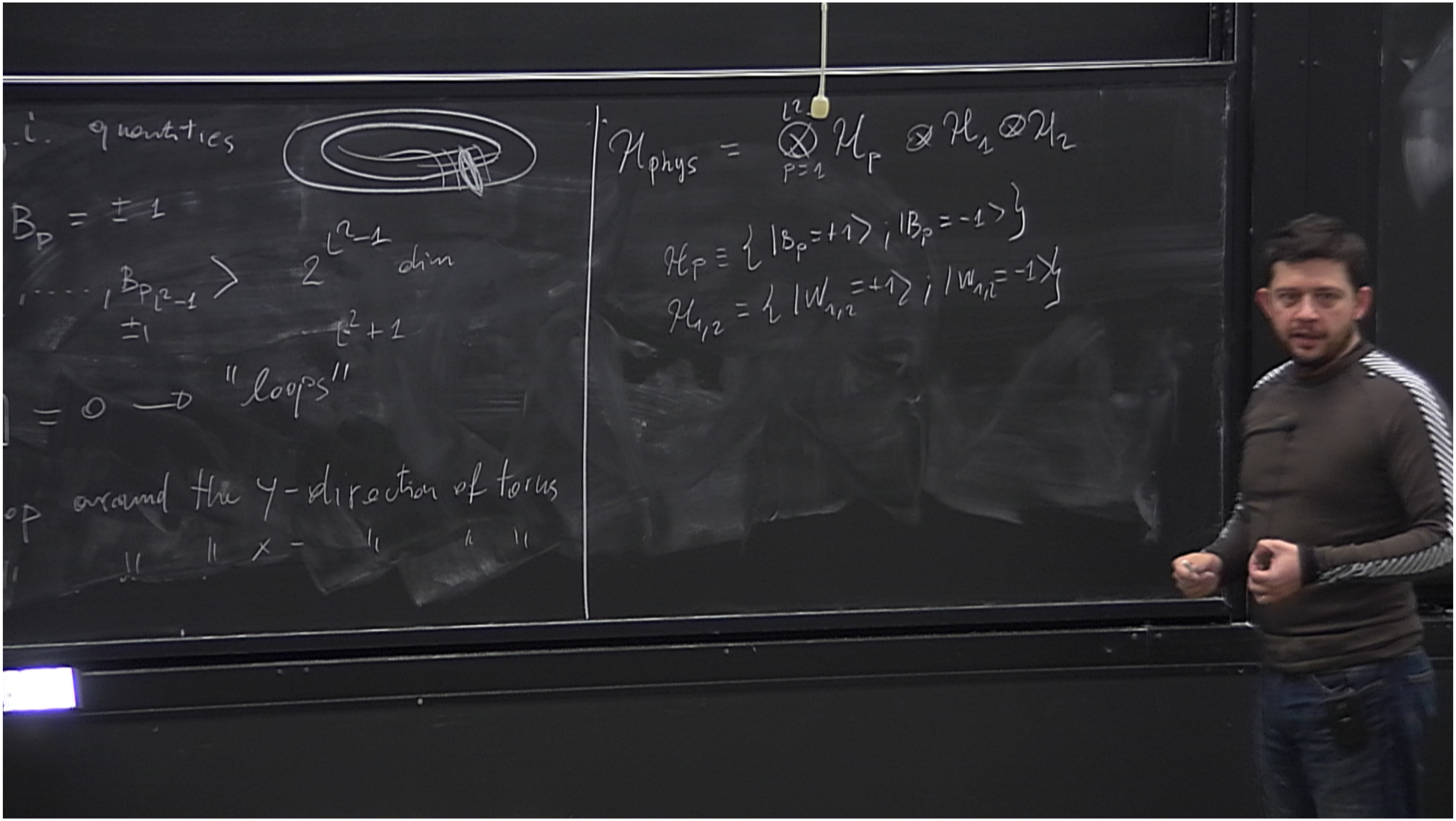
$$\prod_p B_p = \mathbb{1} \qquad B_p = \pm 1$$

$$|B_{p_1}, B_{p_2}, \dots\dots, B_{p_{L^2-1}}\rangle \qquad 2^{L^2-1} \text{ dim}$$

$$\pm 1 \quad \pm 1 \quad \pm 1 \qquad L^2+1$$

$$[W, A_s] = 0 \longrightarrow \text{"loops"}$$


dof ≡ g.i. quantities
Π_p B_p = 1   B_p = ±1
|B_p1 , B_p2 , ... , B_p(L²−1)⟩   2^(L²−1) dim
±1   ±1   ±1
L² + 1
[W , A_s] = 0 → "loops"
W_1   loop around the y-direction of torus
W_2   "   "   "   x - "   "   "


i. quantities
$B_p = \pm 1$
$, \dots, B_{p_{L^2-1}} \rangle$ 2^{L^2-1} dim
± 1
L^2+1
$= 0 \to$ "loops"
op around the y-direction of torus
$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$
$\mathcal{H}_p \equiv \{ |B_p = +1\rangle ; |B_p = -1\rangle \}$
$\mathcal{H}_{1,2} = \{ |W_{1,2} = +1\rangle ; |W_{1,2} = -1\rangle \}$

..i. quantities

$B_p = \pm 1$

$|\ldots, B_{p_{L^2-1}}\rangle \quad 2^{L^2-1}$ dim

$\pm 1 \qquad L^2+1$

$= 0 \longrightarrow$ "loops"

op around the y-direction of torus

" " " x - " " "

$$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$$

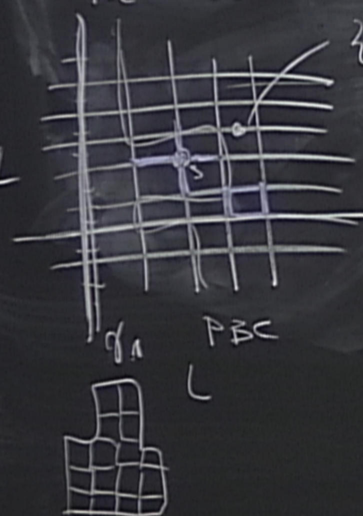
$$\mathcal{H}_p \equiv \{ |B_p=+1\rangle , |B_p=-1\rangle \}$$

$$\mathcal{H}_{1,2} = \{ |W_{1,2}=+1\rangle ; |W_{1,2}=-1\rangle \}$$

$$W_{1,2} = \prod_{j\in\gamma_{1,2}}$$

Quantum lattice $\mathbb{Z}_2$ gauge theory

in $d = 2+1$ dimensions



$$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}^2_e \simeq \mathbb{C}^{2 \otimes 2L^2}$$

$$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$$

$$A_s = \prod_{j \in s} \hat{\sigma}^x_j \qquad B_p = \sum_{j \in p} \hat{\sigma}^z_j$$

$$H = -J \sum_p B_p - h \sum_j \hat{\sigma}^x_j$$

$$[H, A_s] = 0 \qquad \dim \mathcal{H}_{phys} = 2^{L^2+1}$$

d.o.f $\equiv$ g.i. quanti

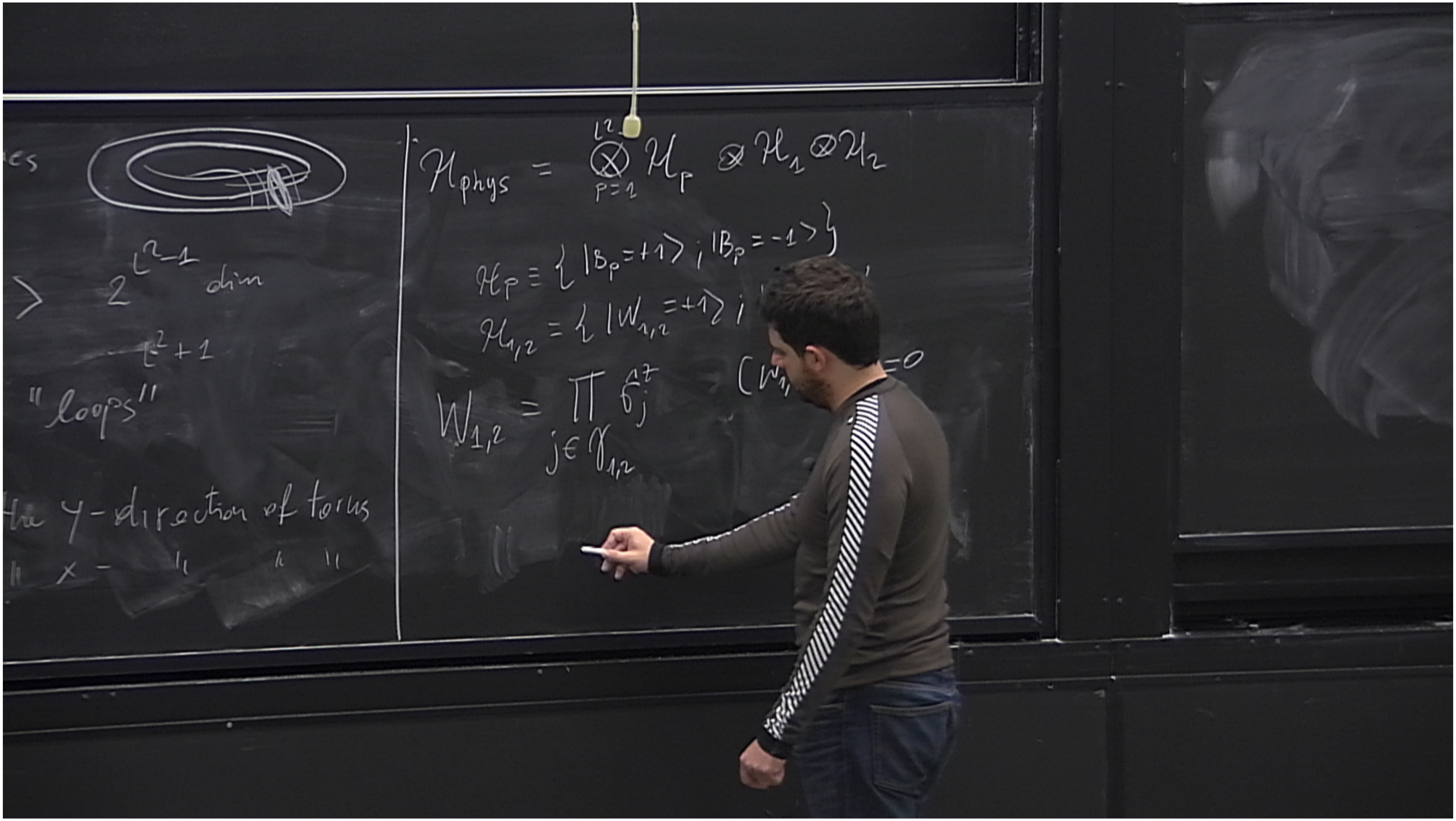
$$\prod_p B_p = \mathbb{1} \qquad B_p = \pm 1$$

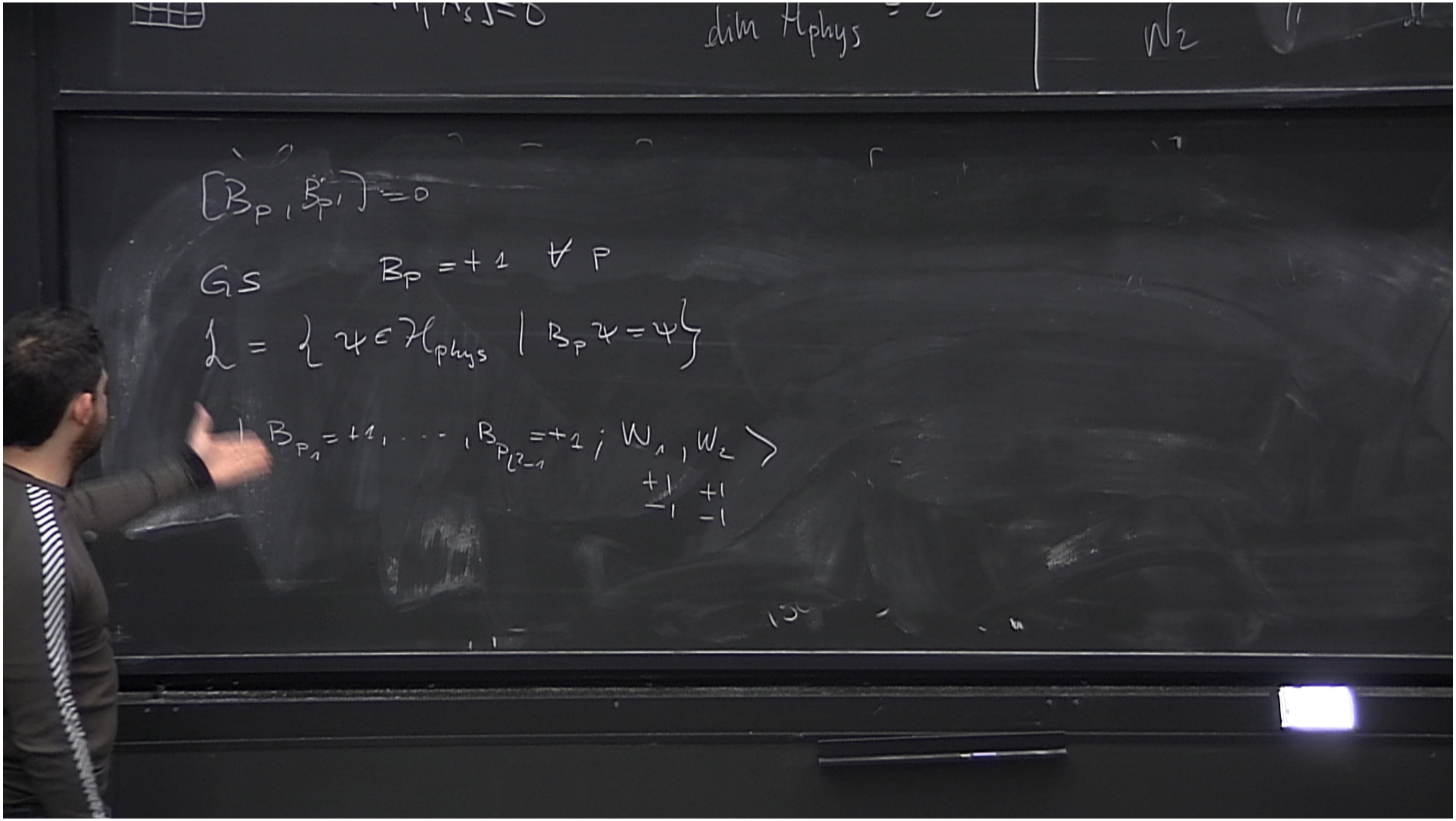
$$\underset{\pm 1}{B_{p_1}}, \underset{\pm 1}{B_{p_2}}, \ldots, \underset{\pm 1}{B_{p_{L^2-1}}}$$

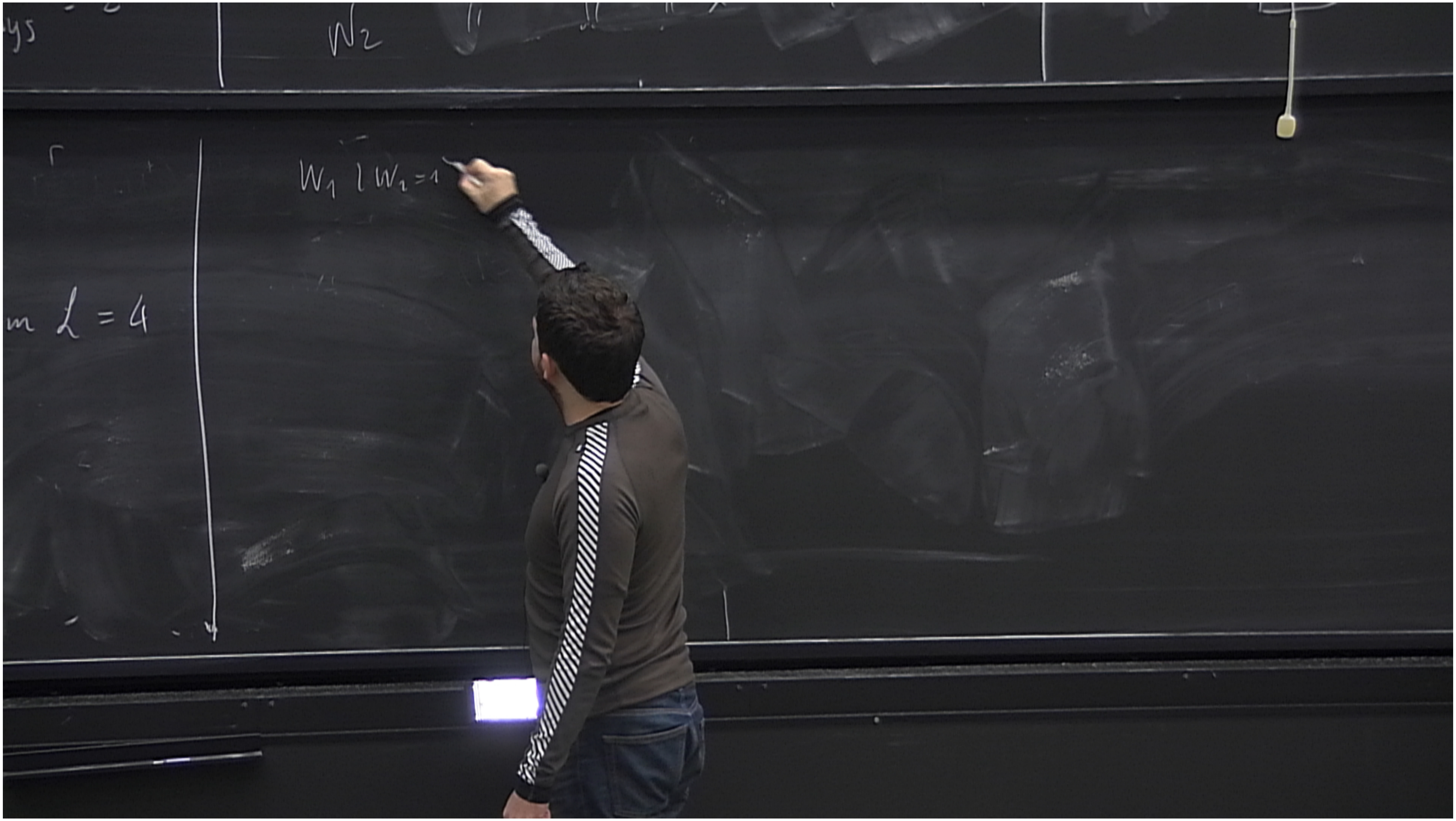
$$[W, A_s] = 0 \longrightarrow$$

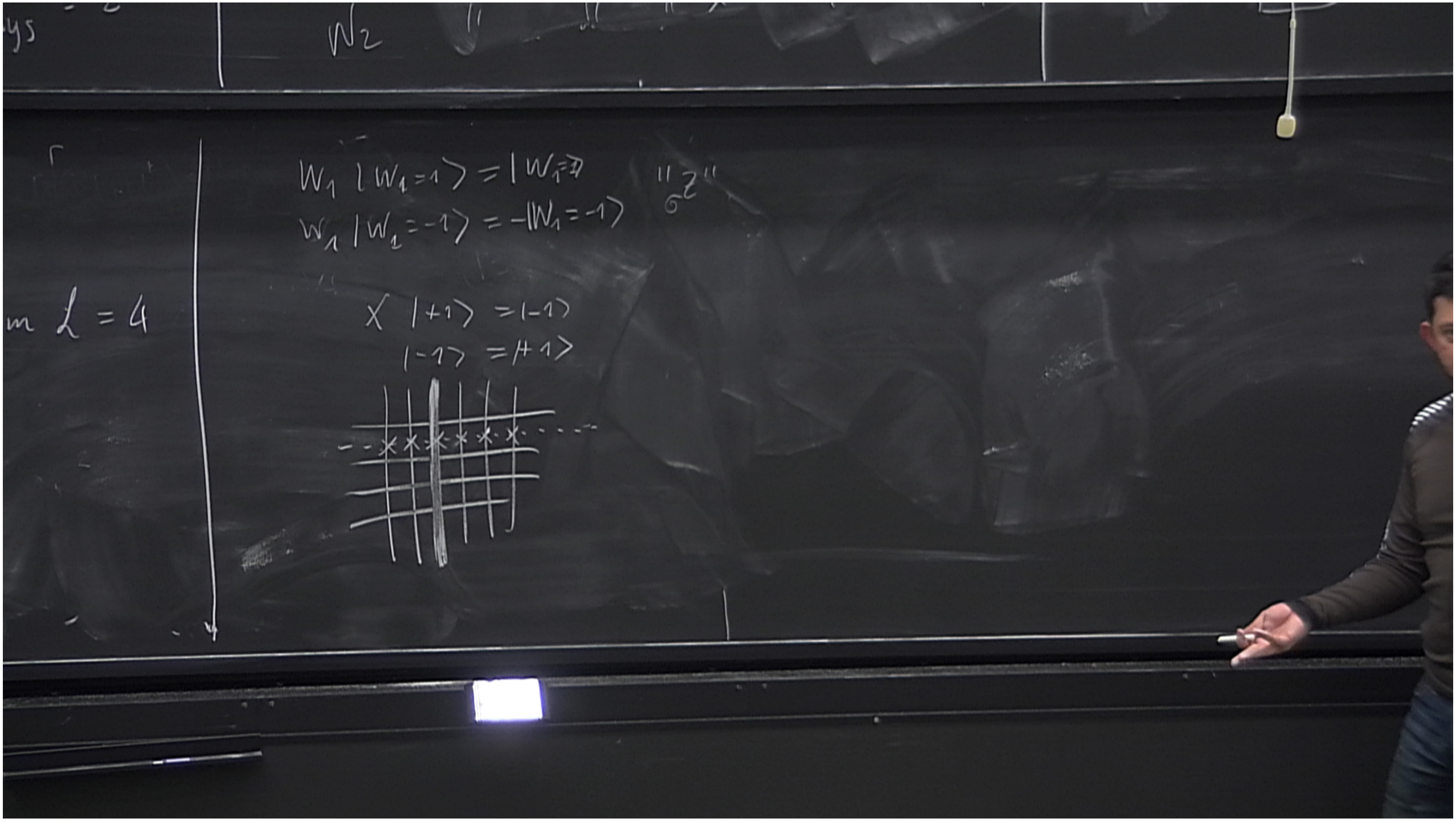
W_1 loop around

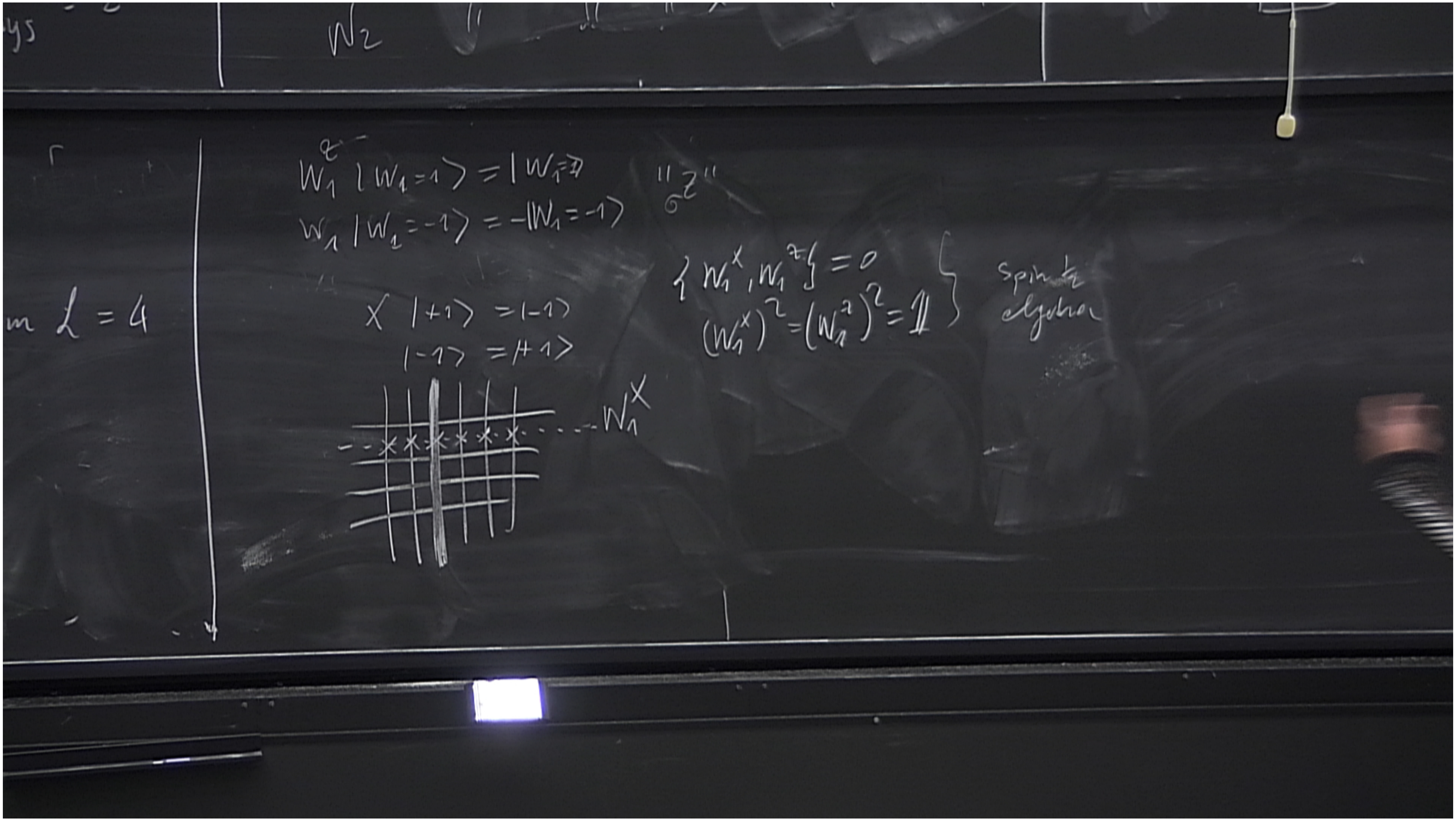
W_2 " "

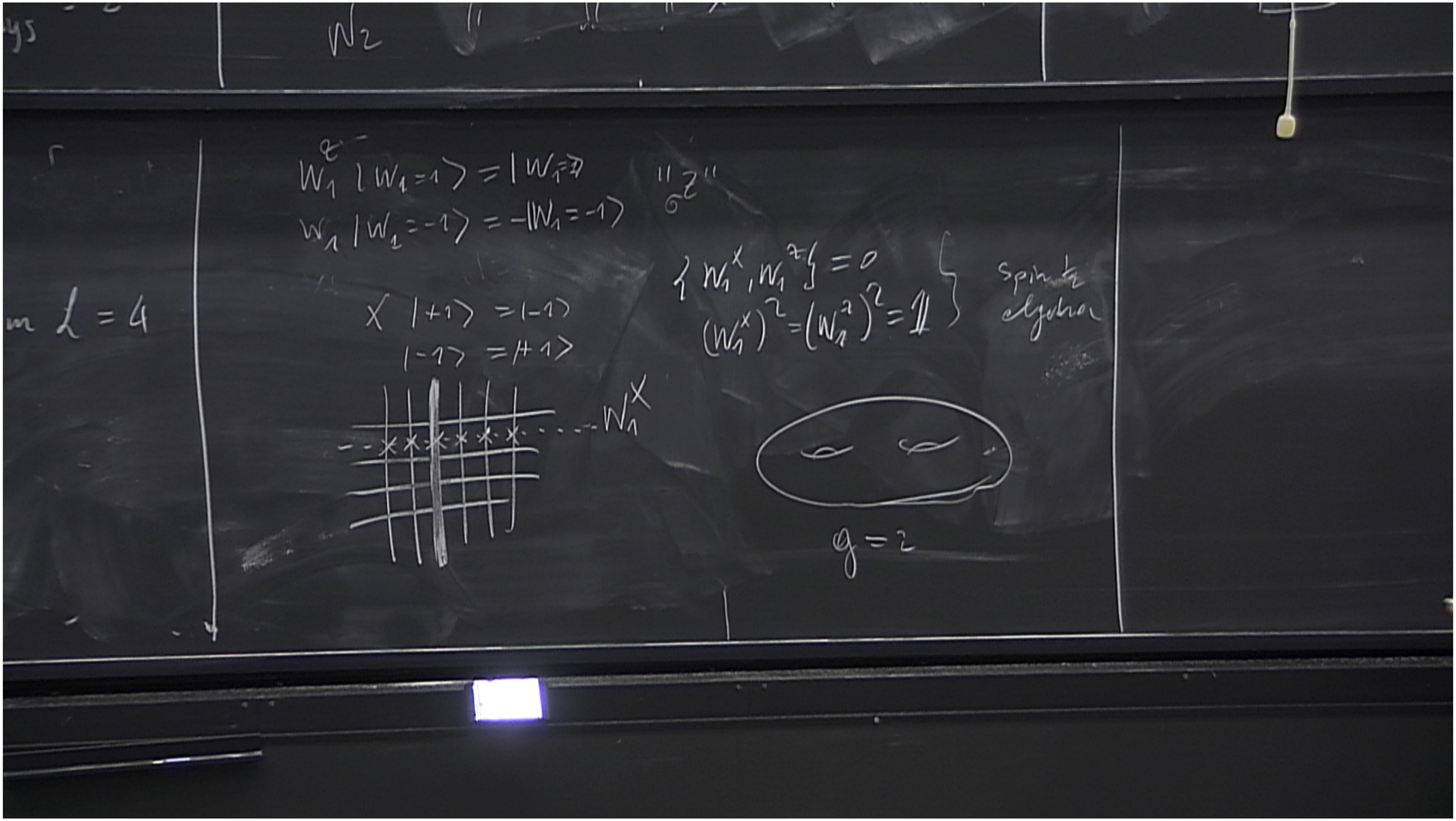

"loops"
the y-direction of torus
2^{L^2-1} dim
L^2+1
$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$
$\mathcal{H}_p = \{ |B_p=+1\rangle ; |B_p=-1\rangle \}$
$\mathcal{H}_{1,2} = \{ |W_{1,2}=+1\rangle ;$
$W_{1,2} = \prod_{j \in \gamma_{1,2}} \sigma_j^z$

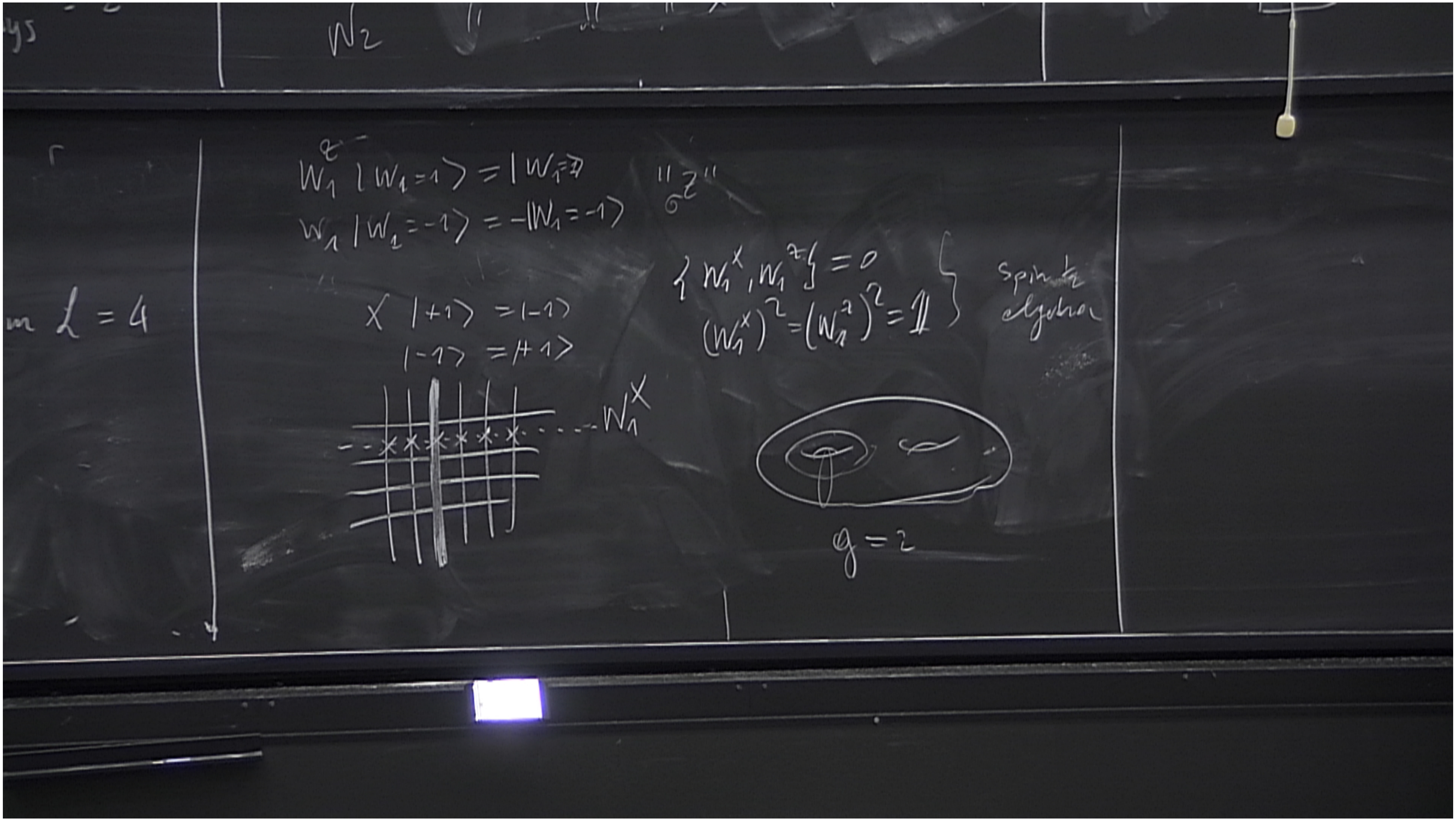

dim $\mathcal{H}_{phys}$
W_2
$[B_p, B_{p'}] = 0$
GS $B_p = +1 \ \forall\ p$
$\mathcal{L} = \{ \Psi \in \mathcal{H}_{phys} \mid B_p \Psi = \Psi \}$
$| B_{p_1} = +1, \ldots, B_{p_{L^2-1}} = +1 ; W_1, W_2 \rangle$
+1 +1
−1 −1


W_2
$W_1 \, l \, W_1 = 1$
$m \, \mathcal{L} = 4$

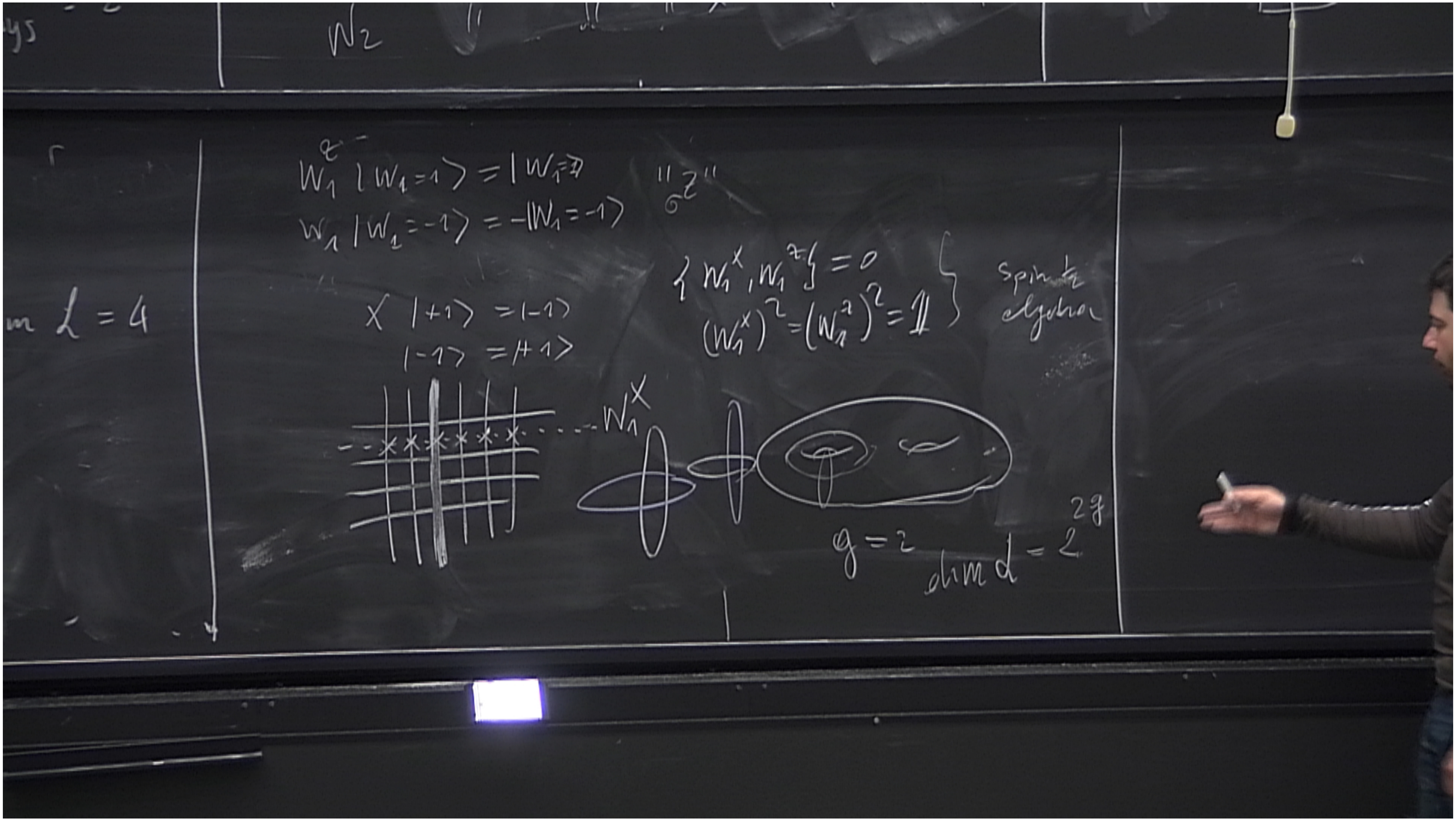

W2
m L = 4
W1 |W1=1> = |W1=1>
"σz"
W1 |W1=-1> = -|W1=-1>
X |+1> = |-1>
|-1> = |+1>

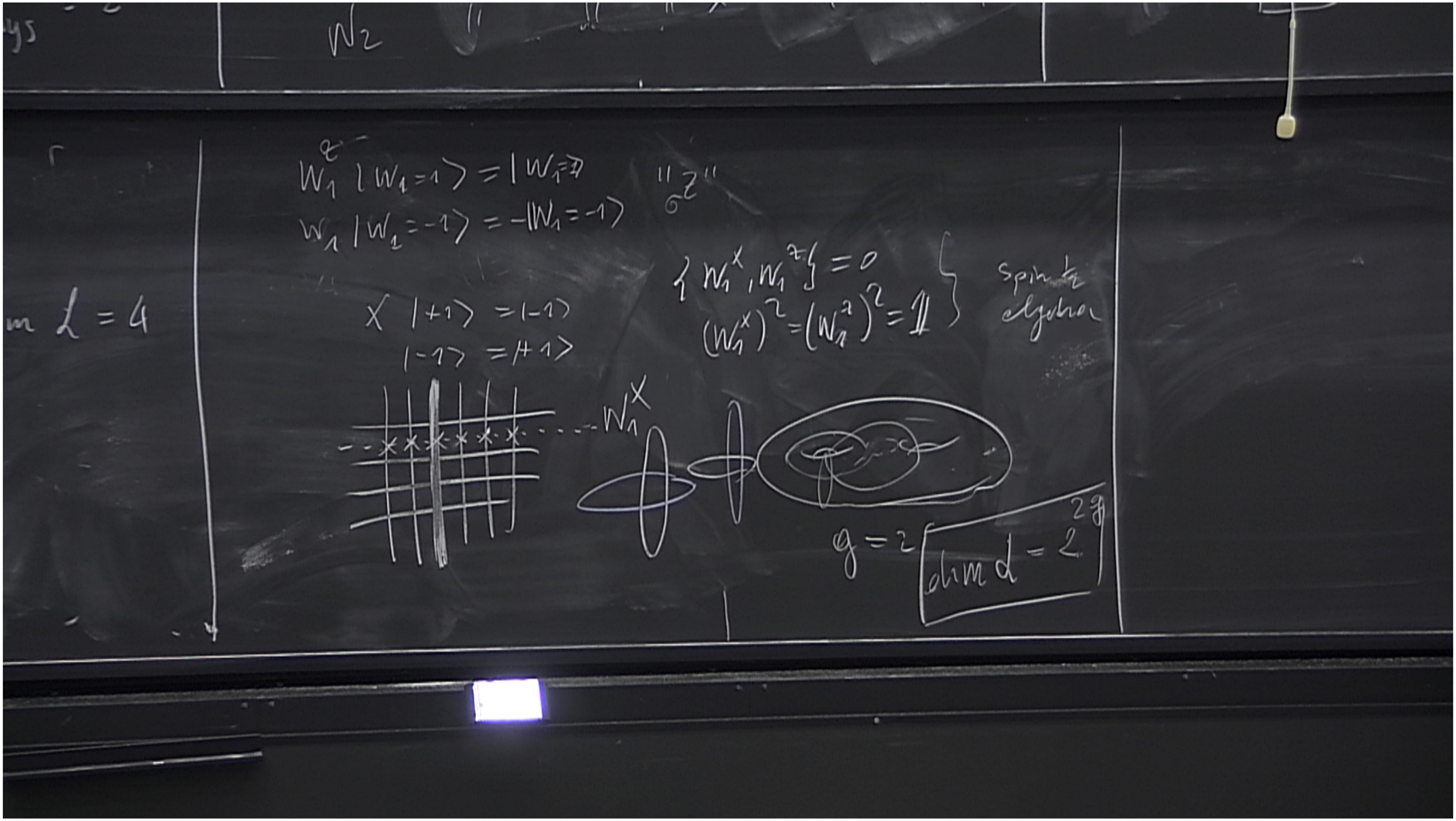

W2
m L = 4
W1z |W1=1> = |W1=1>
W1 |W1=-1> = -|W1=-1>
"σz"
X |+1> = |-1>
|-1> = |+1>
W1x
{W1x, W1z} = 0
(W1x)2 = (W1z)2 = 1
spin 1/2 algebra

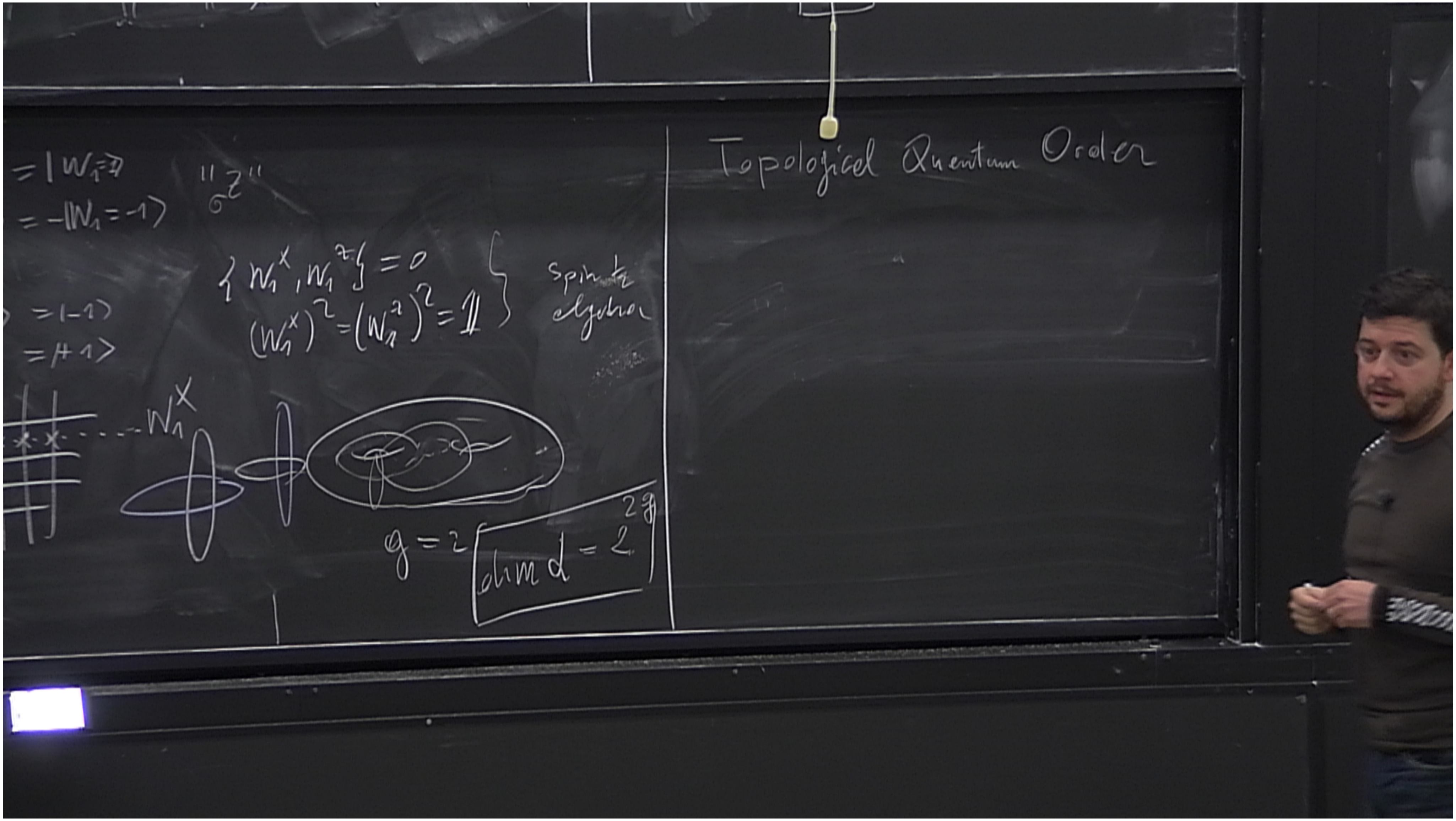

W2
W1 |W1=1> = |W1=1>
"σz"
W1 |W1=-1> = -|W1=-1>
L = 4
X |+1> = |-1>
|-1> = |+1>
W1X
{W1X, W1Z} = 0
(W1X)2 = (W1Z)2 = 1
spin 1/2 algebra
g = 2

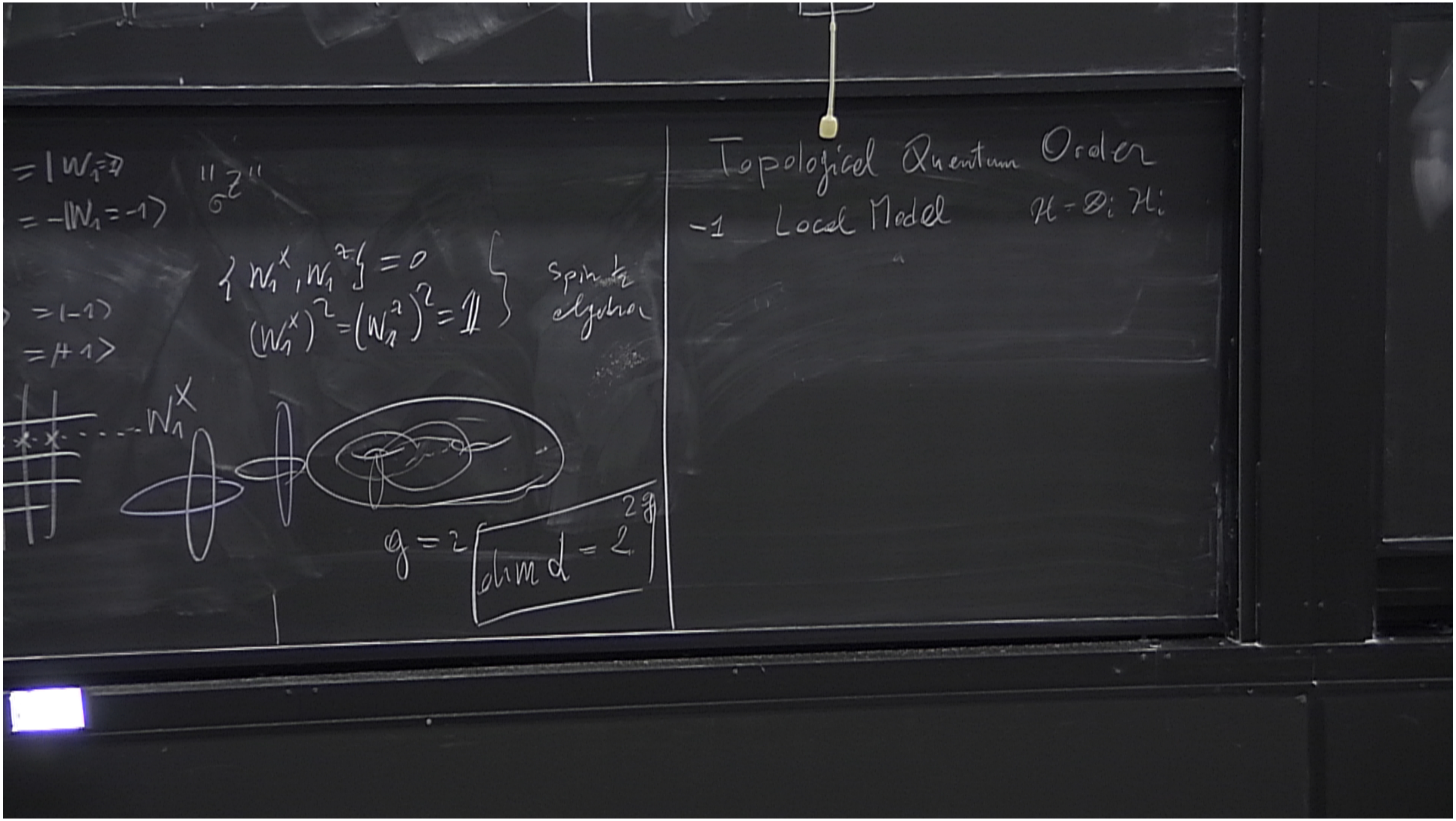


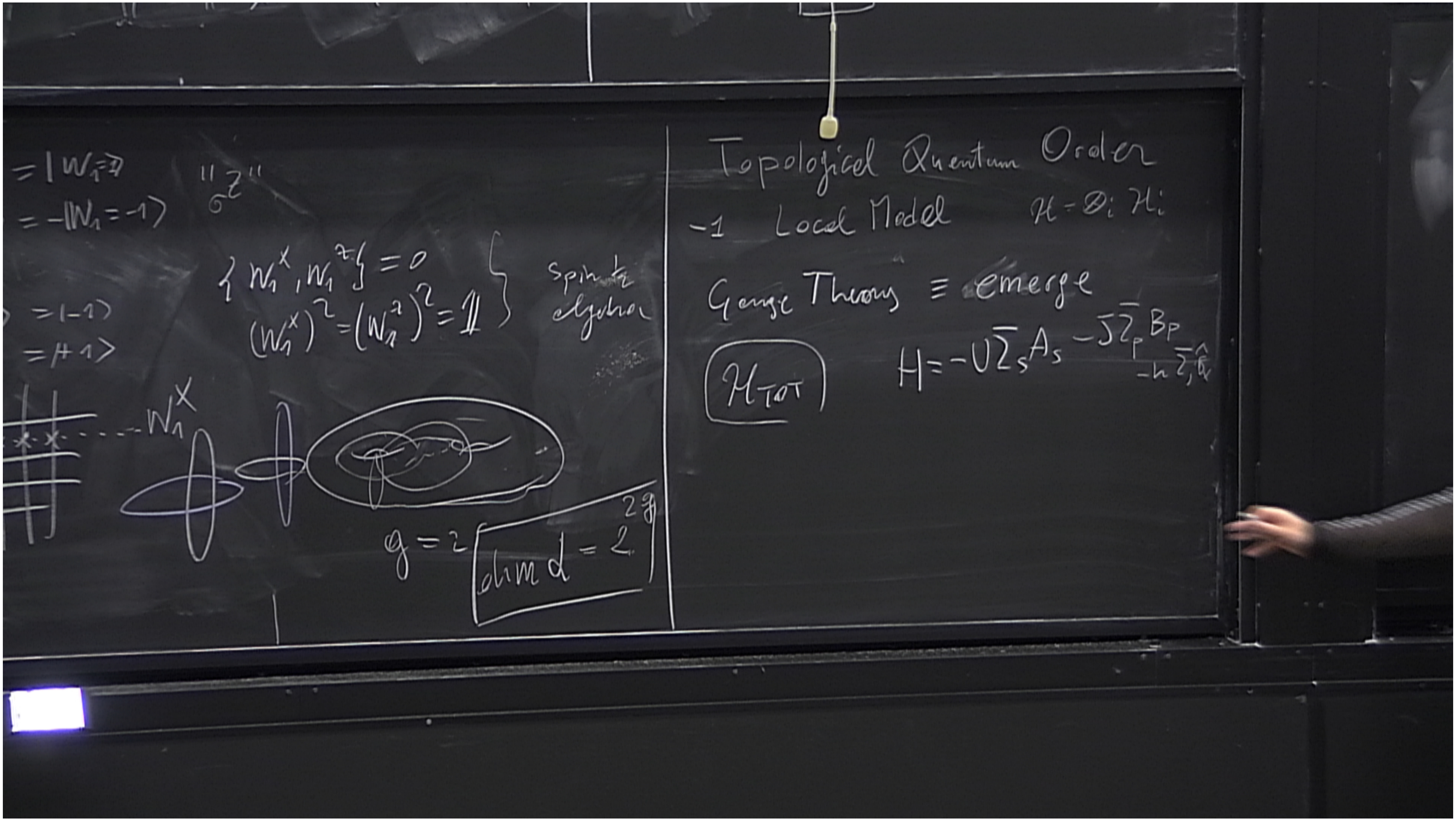
W2
ℓ = 4
W1^z |W1=1> = |W1=1>
"σ^z"
W1 |W1=-1> = -|W1=-1>
{W1^x, W1^z} = 0
(W1^x)^2 = (W1^z)^2 = 1
spin ½ algebra
X |+1> = |-1>
|-1> = |+1>
W1^X
g = 2

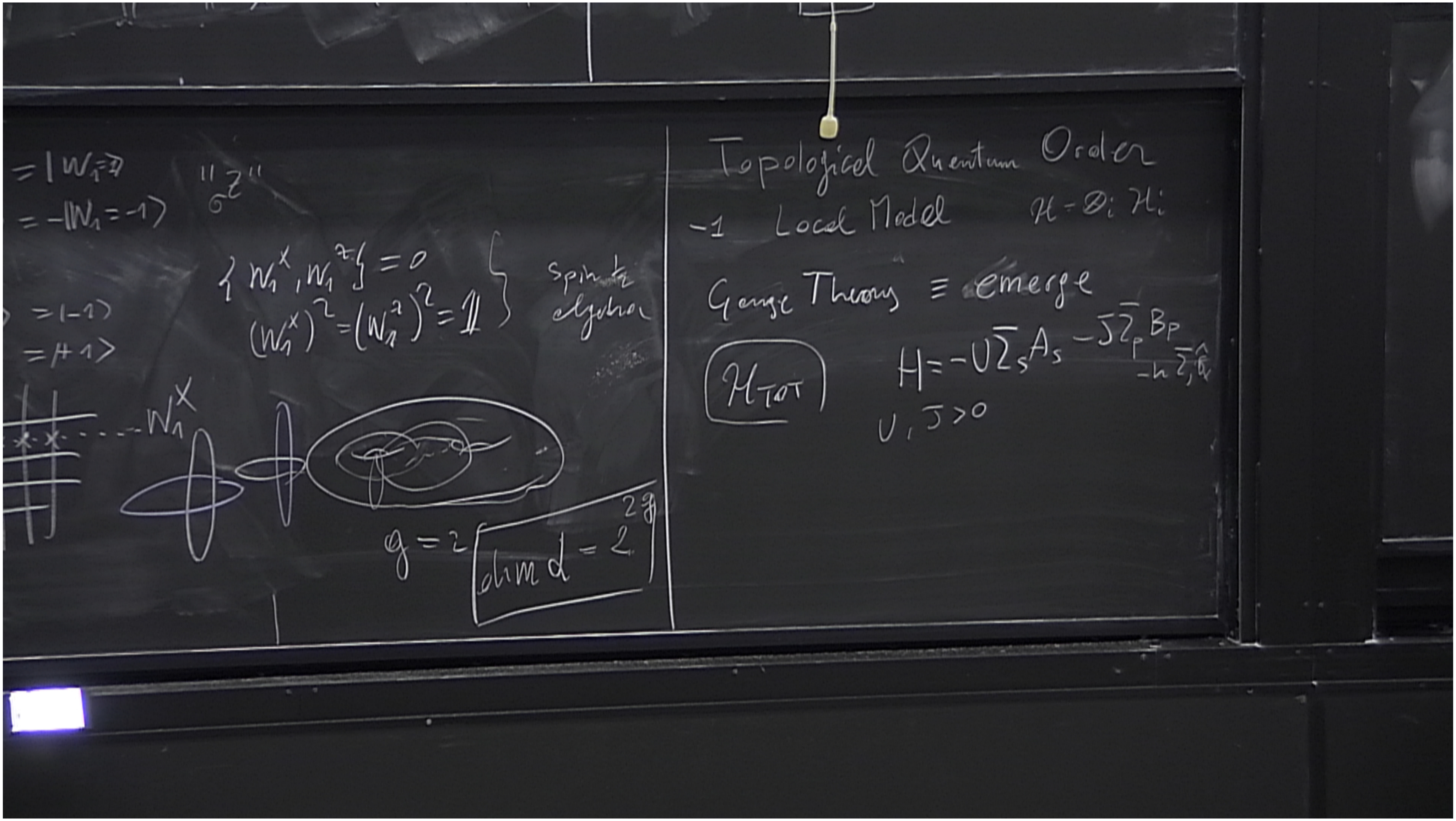

W2
m L = 4
W1^z |W1=1> = |W1=1>
"σ^z"
W1 |W1=-1> = -|W1=-1>
{W1^x, W1^z} = 0
(W1^x)^2 = (W1^z)^2 = 1
spin 1/2 algebra
X |+1> = |-1>
|-1> = |+1>
W1^X
g = 2
dim L = 2^2g


W2
m L = 4
g = 2
dim L = 2^2g
spin 1/2 algebra

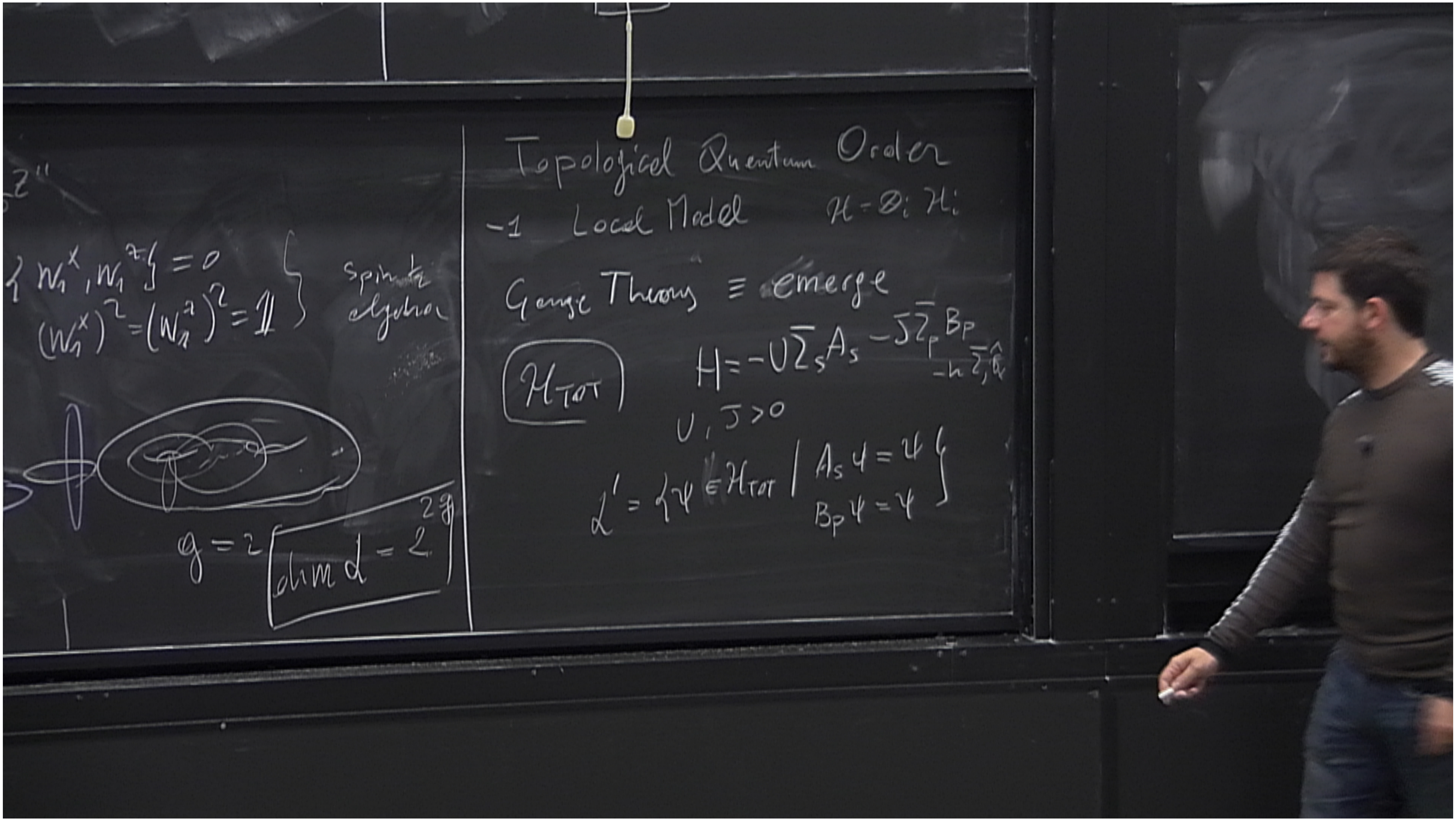

Topological Quantum Order
{W1x, W1z} = 0
(W1x)2 = (W1z)2 = 1
spin 1/2 algebra
g = 2
dim = 22g


Topological Quantum Order
-1 Local Model $\mathcal{H} = \otimes_i \mathcal{H}_i$
$= |W_1 = 1\rangle$
"σ^z"
$= -|W_1 = -1\rangle$
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin ½ algebra
$= |-1\rangle$
$= |+1\rangle$
W_1^x
$g = 2$
$\dim \mathcal{L} = 2^{2g}$


Topological Quantum Order
-1 Local Model
Gauge Theory ≡ emerge
dim = 2
g = 2
spin ½ algebra



Topological Quantum Order
-1 Local Model
Gauge Theory ≡ emerge
H_{TOT}
$H = -U\sum_s A_s - J\sum_p B_p$
$U, J > 0$
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin ½ algebra
$g = 2$
$\dim d = 2^{2g}$



$\{ W_1^x, W_1^z \} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin-1/2 algebra
$g = 2$
$\dim \mathcal{L} = 2^{2g}$
Topological Quantum Order
-1 Local Model $\mathcal{H} = \otimes_i \mathcal{H}_i$
Gauge Theory ≡ emerge
$\mathcal{H}_{TOT}$
$H = -U \sum_s A_s - J \sum_p B_p - h \sum_i \hat{Q}$
$U, J > 0$
$\mathcal{L}' = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi, \; B_p \psi = \psi \}$

$L = L'$

$U \gg J$

$g = 1$

$\alpha = L'$

$U \gg J$

$1 \quad U$

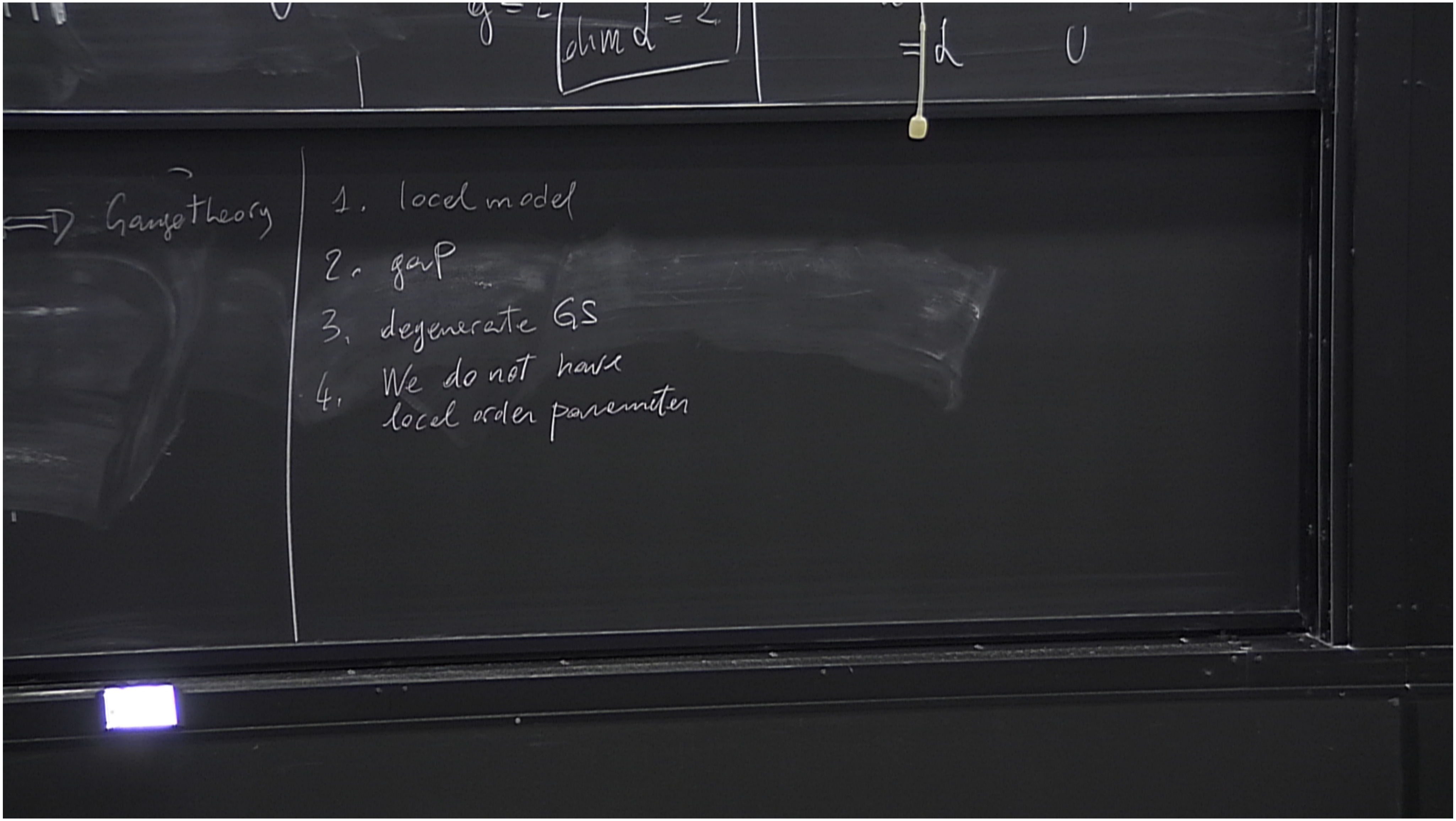
$1 \quad 2J \quad L^2$

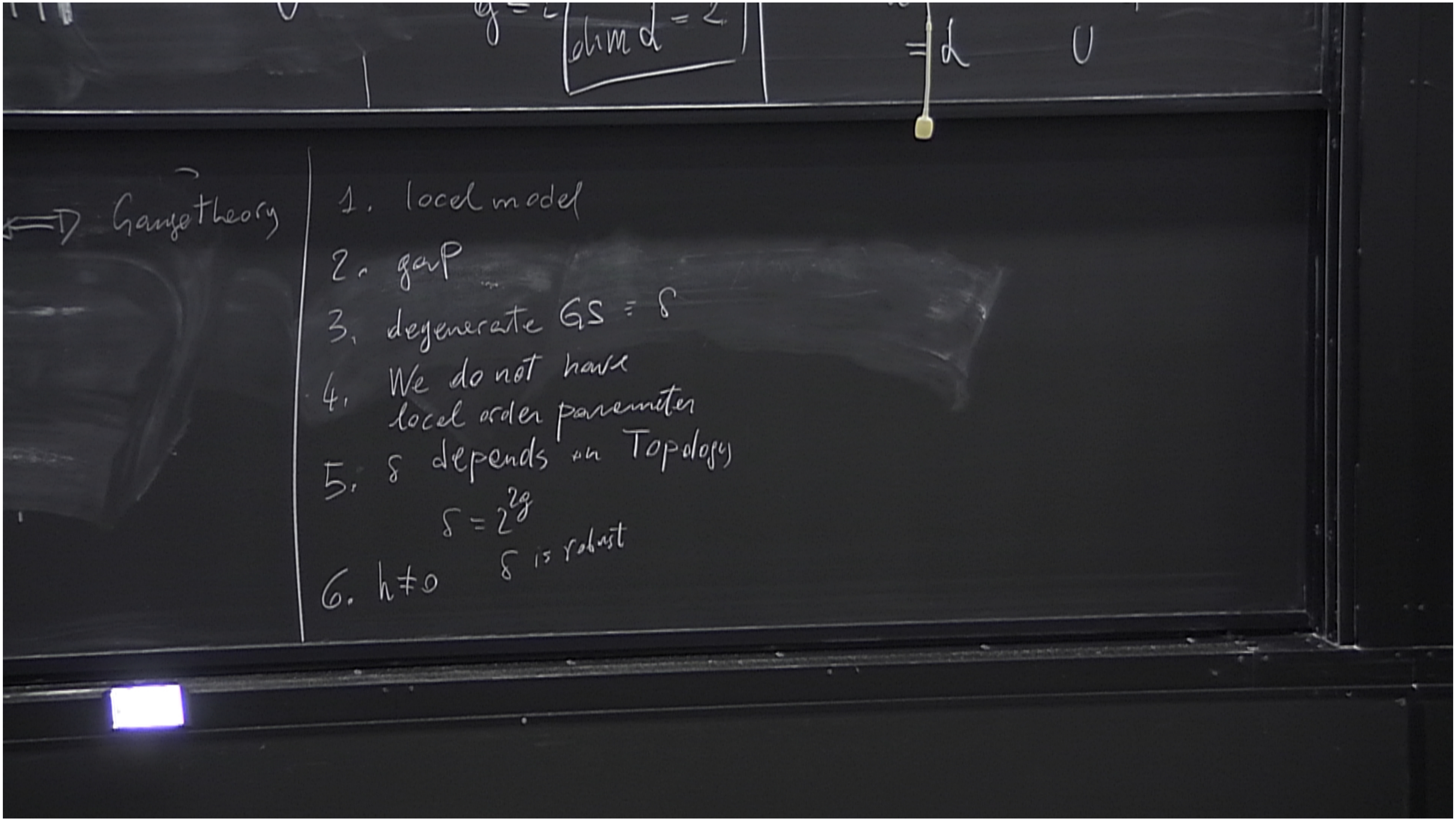
$g = 1$

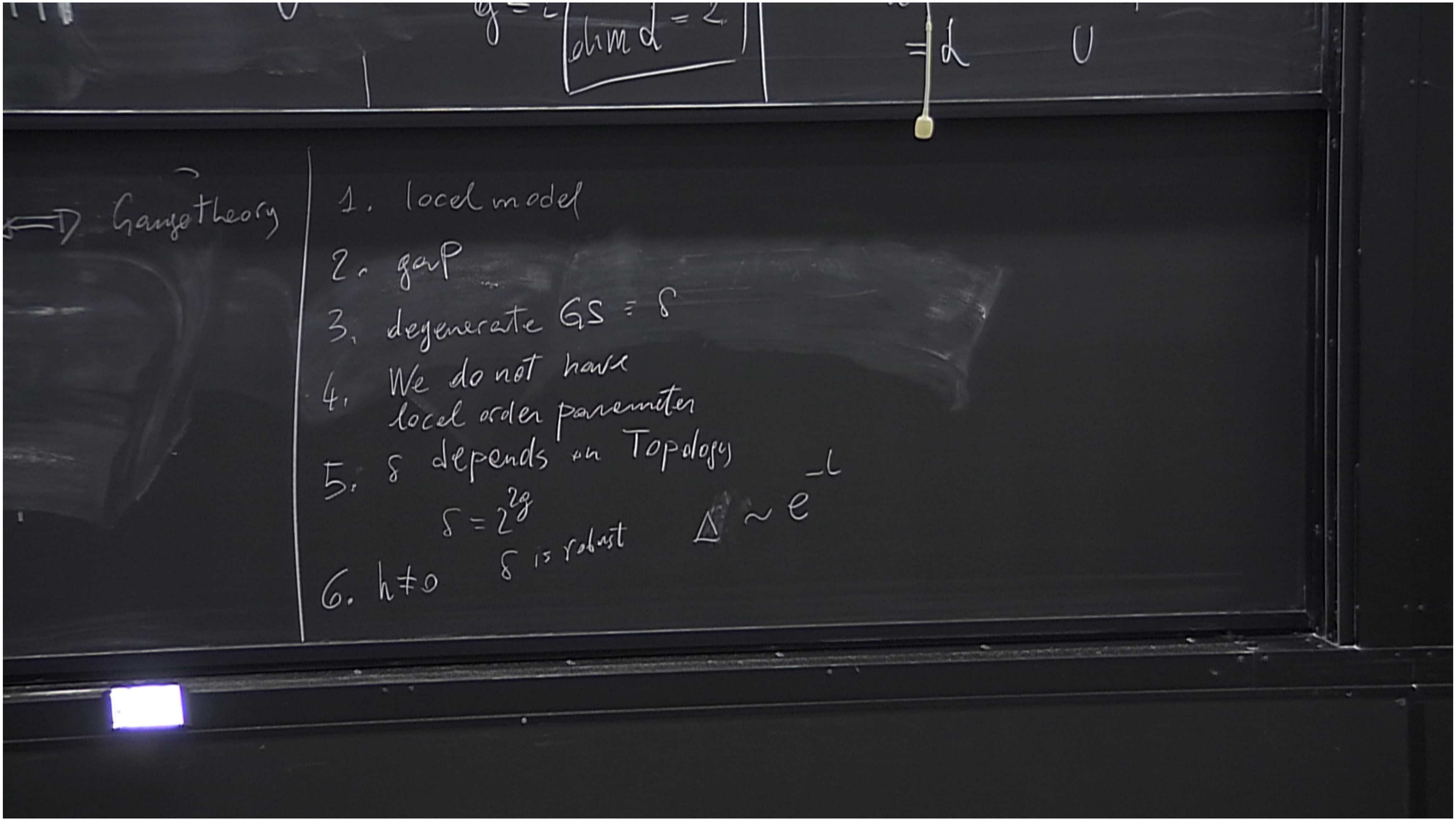
$A_s = +1 \ \forall s \iff A_s \psi = \psi$

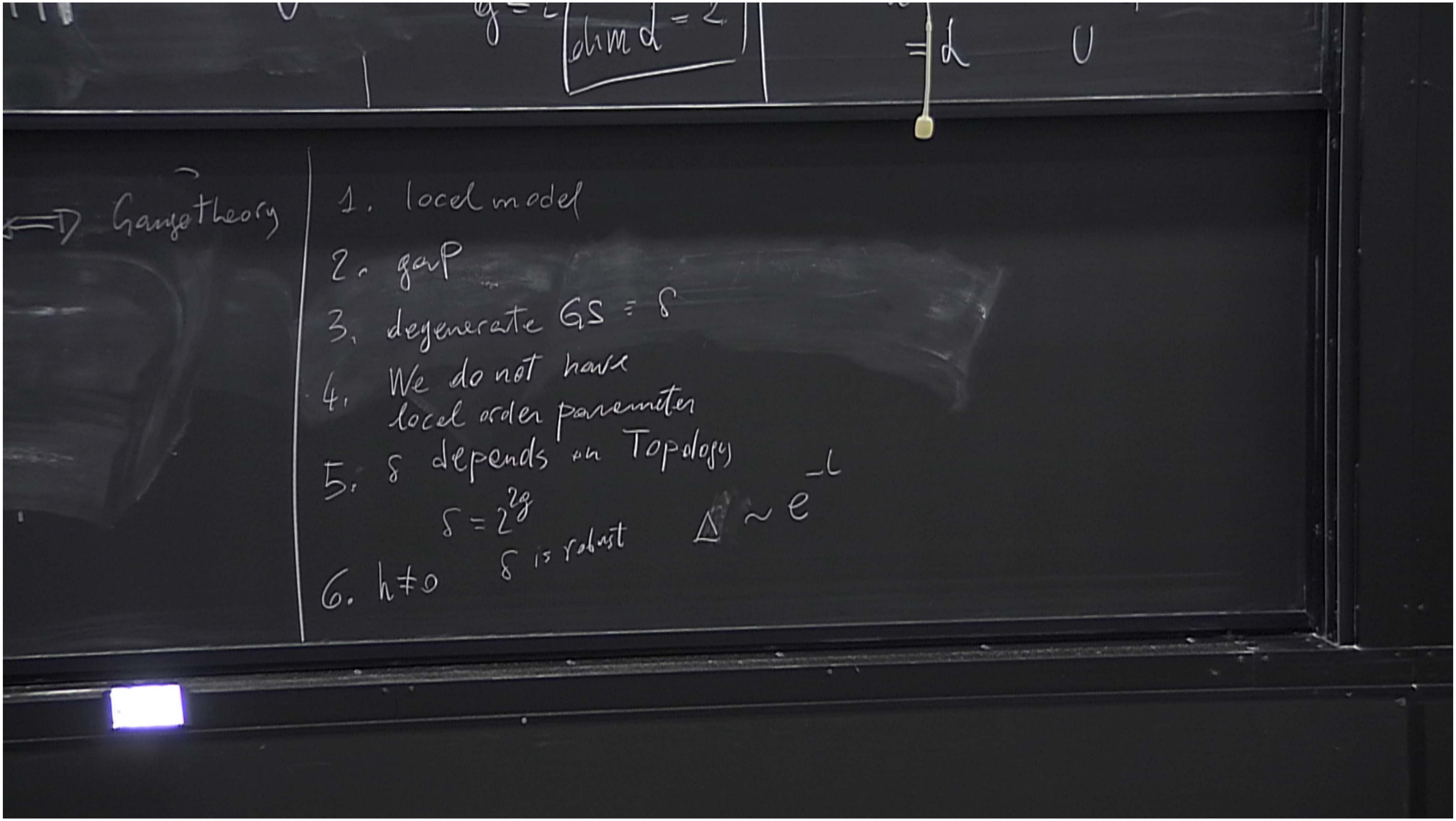
$-L^2 J \equiv E_0$

$-(L^2 - 1)J + J = E_0 + 2J$


dim d = 2
⟹ Gauge theory
1. local model
2. gap
3. degenerate GS
4. We do not have
local order parameter


dim d = 2
Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have
local order parameter
5. δ depends on Topology
δ = 2^2g
6. h≠0 δ is robust


dim d = 2
⟺ Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
δ = 2^2g
Δ ~ e^-L
6. h ≠ 0 δ is robust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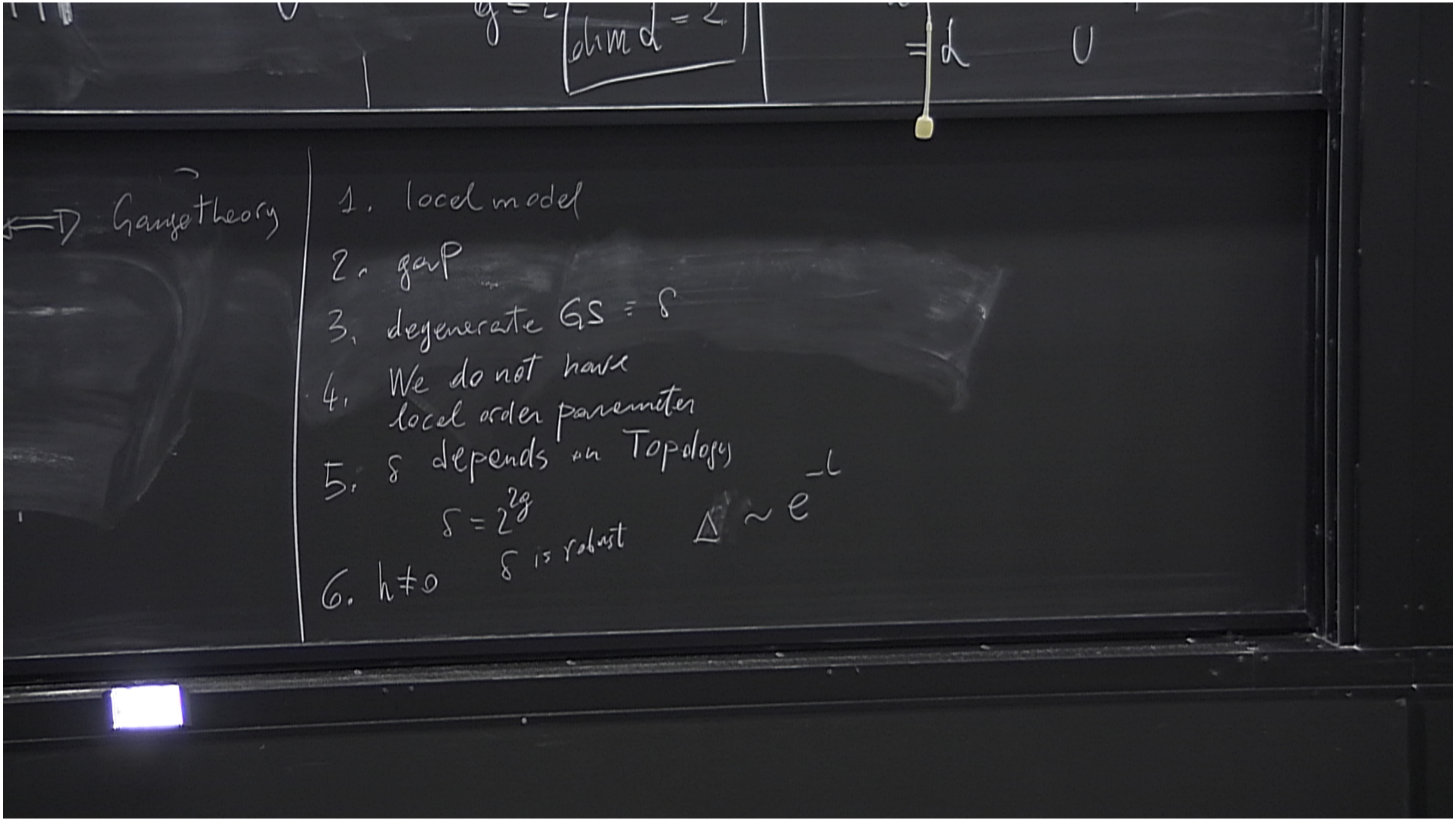
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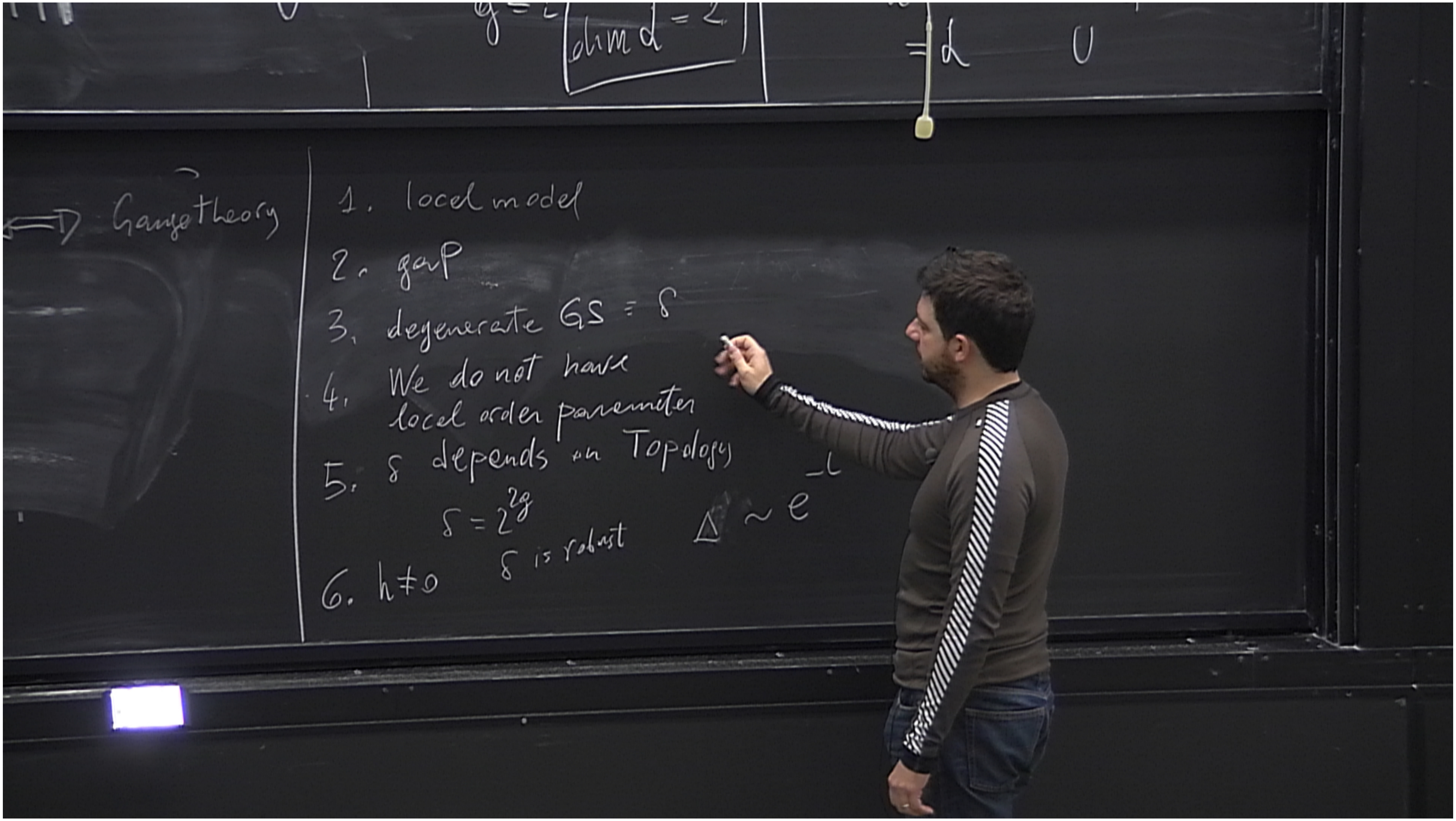
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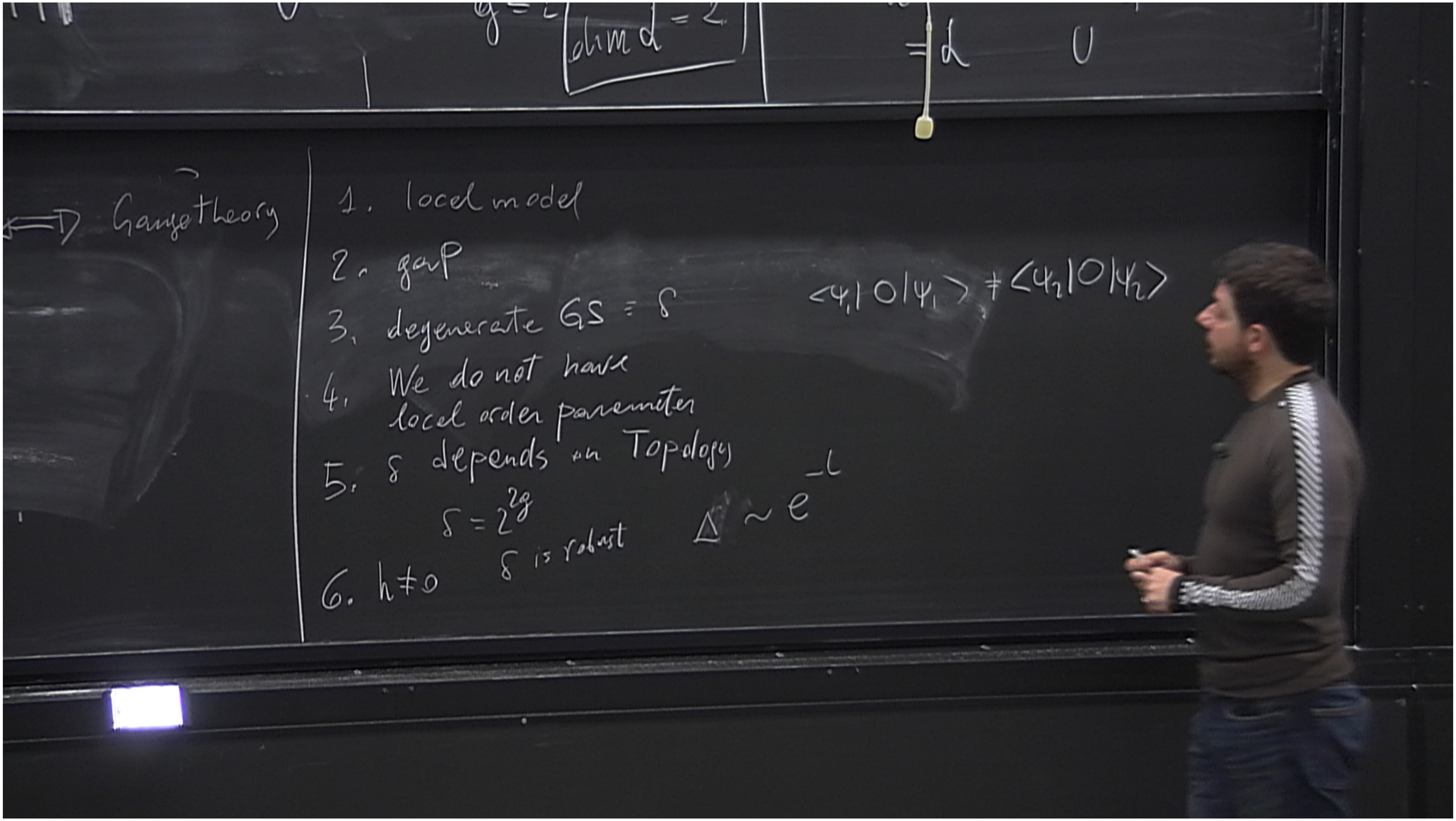
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<ψ1|O|ψ1> ≠ <ψ2|O|ψ2>

$g = 2$ $\boxed{\dim d = 2}$

$= d$ $\quad U$

$\Longleftrightarrow$ Gauge theory

1. local model
2. gap
3. degenerate GS $= \delta$
4. We do not have local order parameter
5. δ depends on Topology

$$\delta = 2^{2g}$$

6. $h \neq 0$ $\quad \delta$ is robust $\quad \Delta \sim e^{-L}$

$$\langle \psi_1 | O | \psi_1 \rangle \neq \langle \psi_2 | O | \psi_2 \rangle$$

ϕ_1, ϕ_2

$$\langle \phi_1 | O | \phi_2 \rangle \neq 0$$

$g = 2$ $\quad \boxed{\dim d = 2}$ $\quad = d \quad U$

$\Longleftrightarrow$ Gauge theory

1. local model
2. gap
3. degenerate GS $= \delta$
4. We do not have local order parameter
5. δ depends on Topology

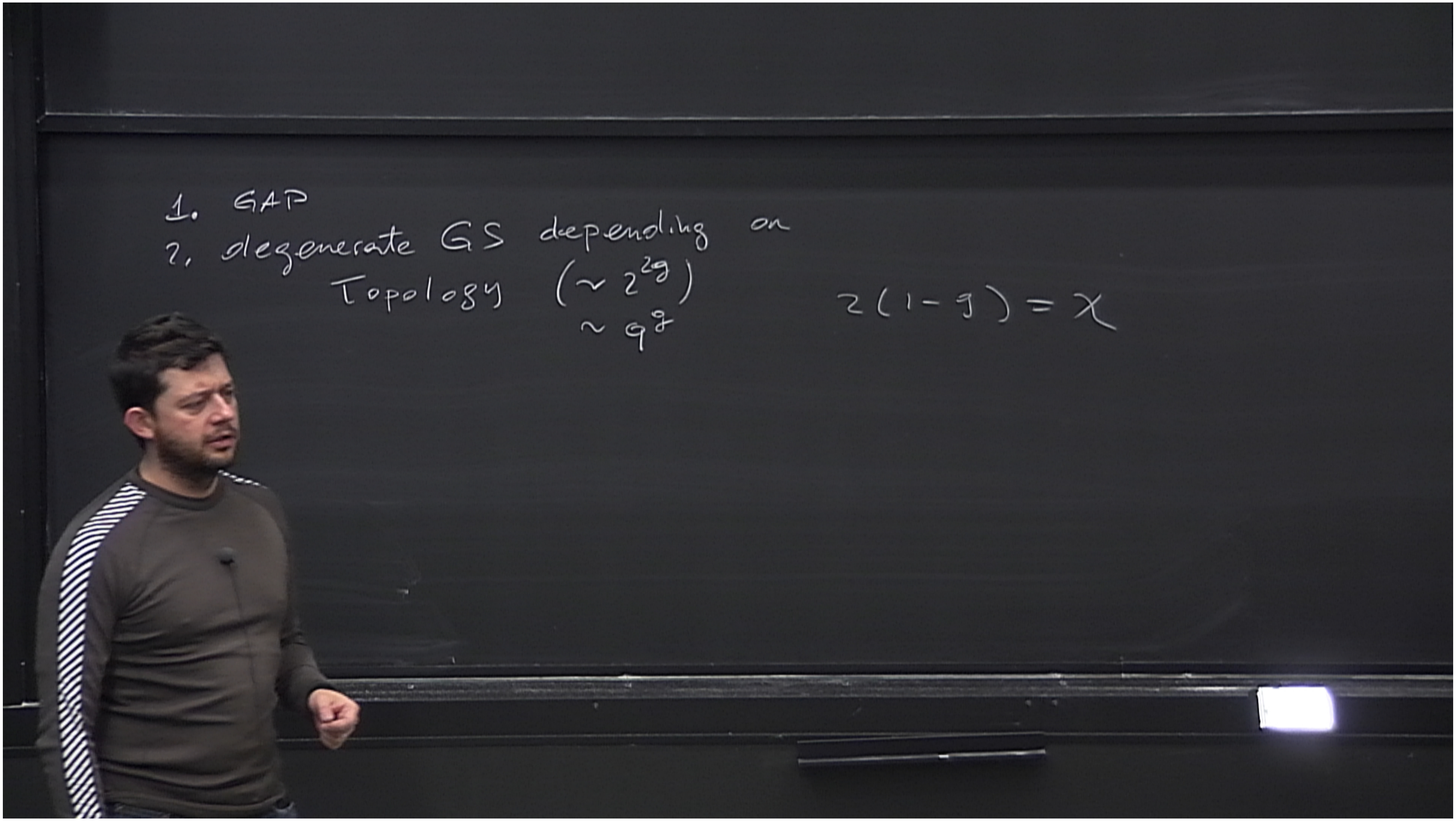
   $\delta = 2^{2g}$
6. $h \neq 0$ $\quad$ δ is robust $\quad$ $\Delta \sim e^{-L}$

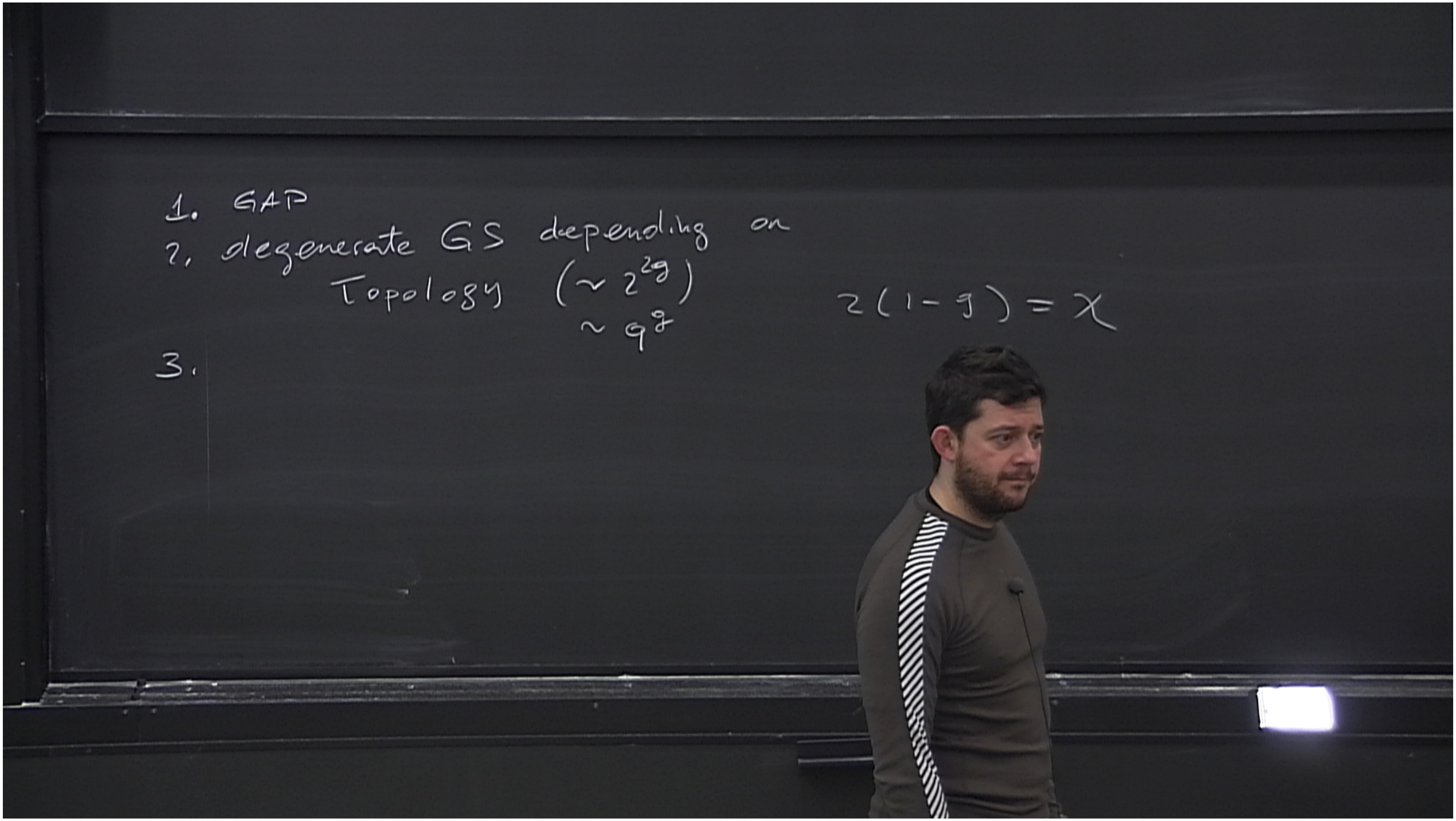
$\langle \psi_1 | O_A | \psi_1 \rangle \neq \langle \psi_2 | O_A | \psi_2 \rangle$

ϕ_1, ϕ_2 $\quad$ $\langle \phi_1 | O_A | \phi_2 \rangle \neq 0$

$\Updownarrow$

$\rho_A^{(1)} \equiv \rho_A^{(2)}$


1. GAP
2. degenerate GS depending on
Topology $(\sim 2^{2g})$
$\sim q^g$
$2(1-g) = \chi$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