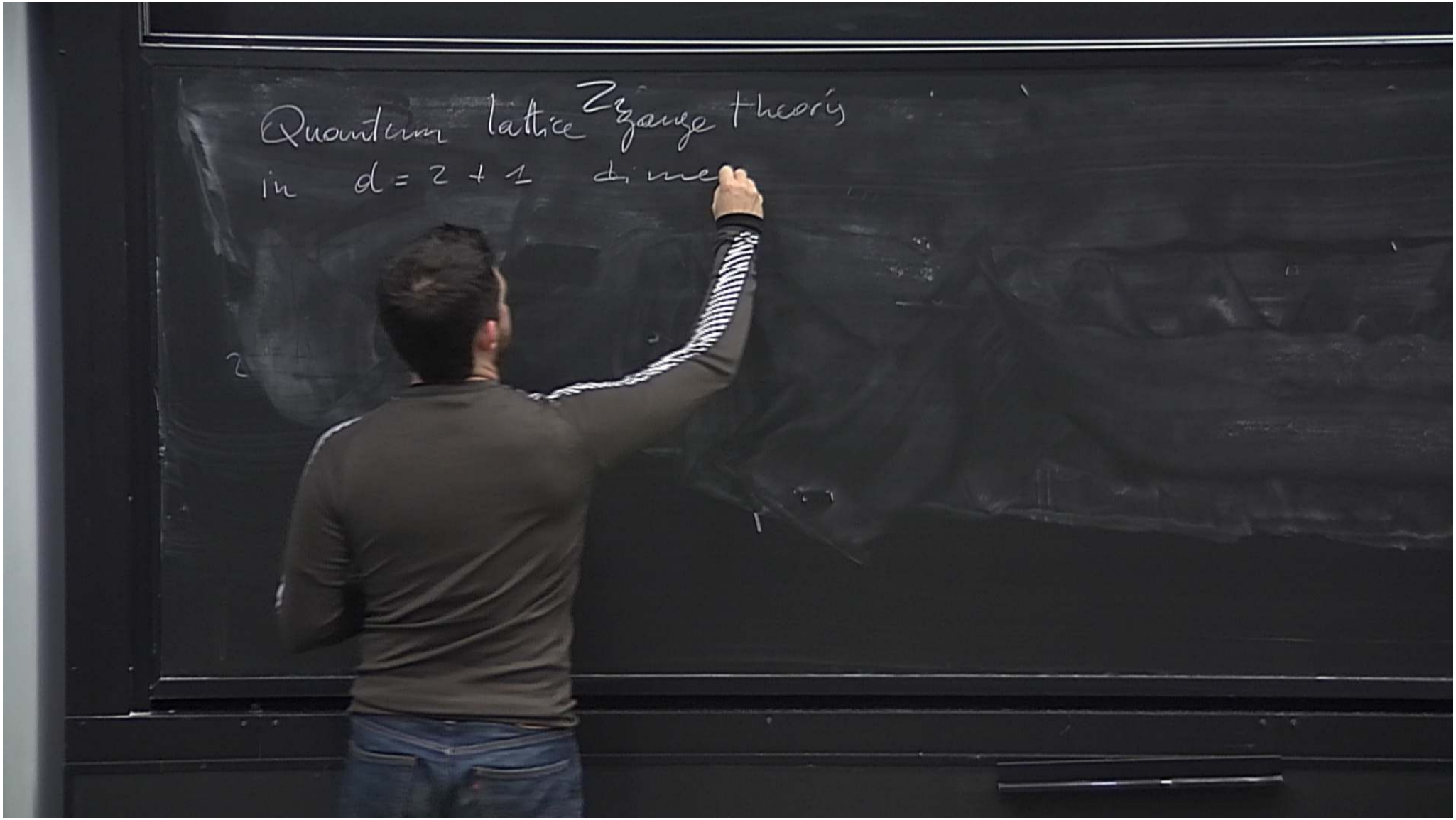
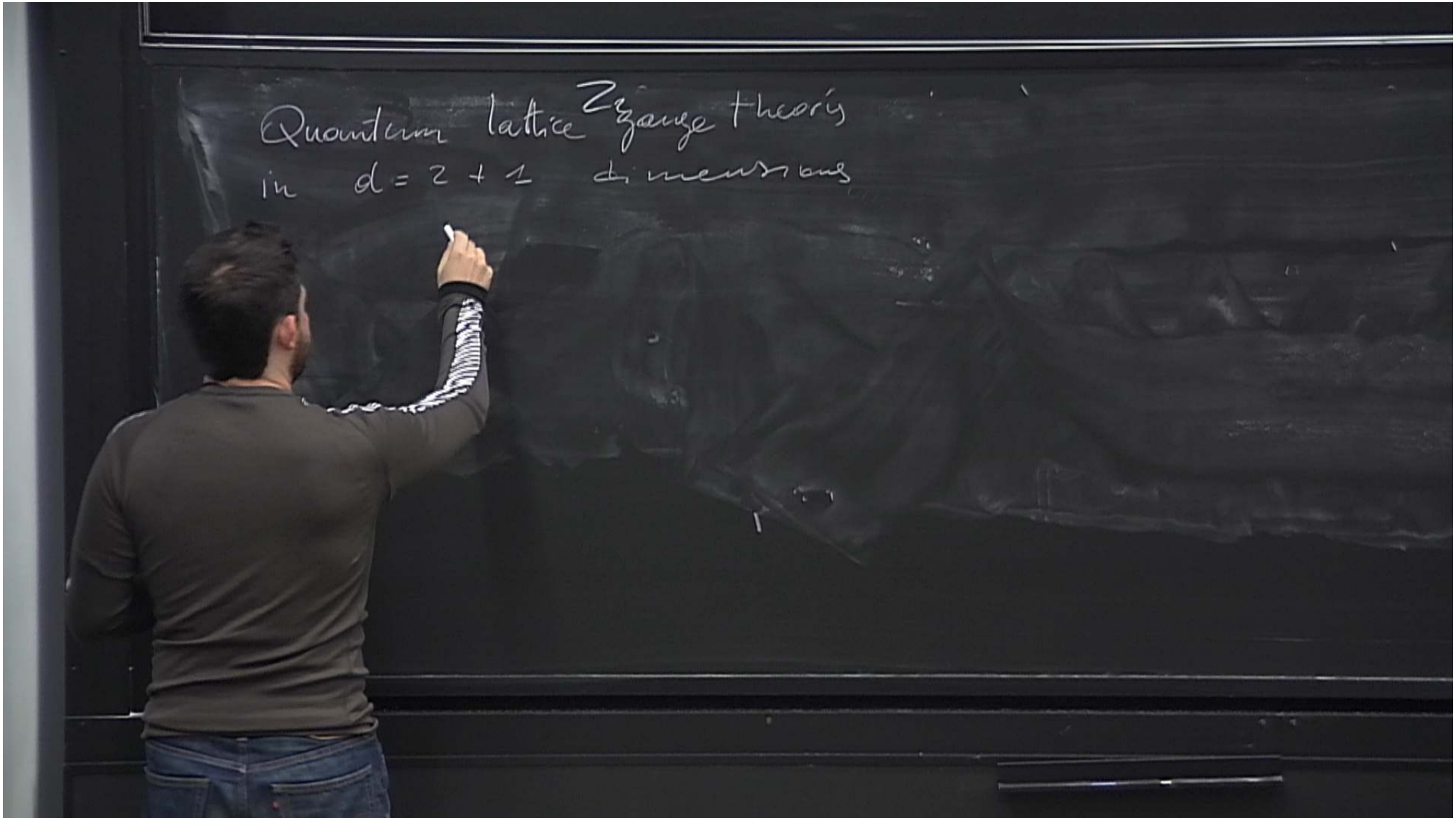
Title: Condensed Matter (Review) - Lecture 11

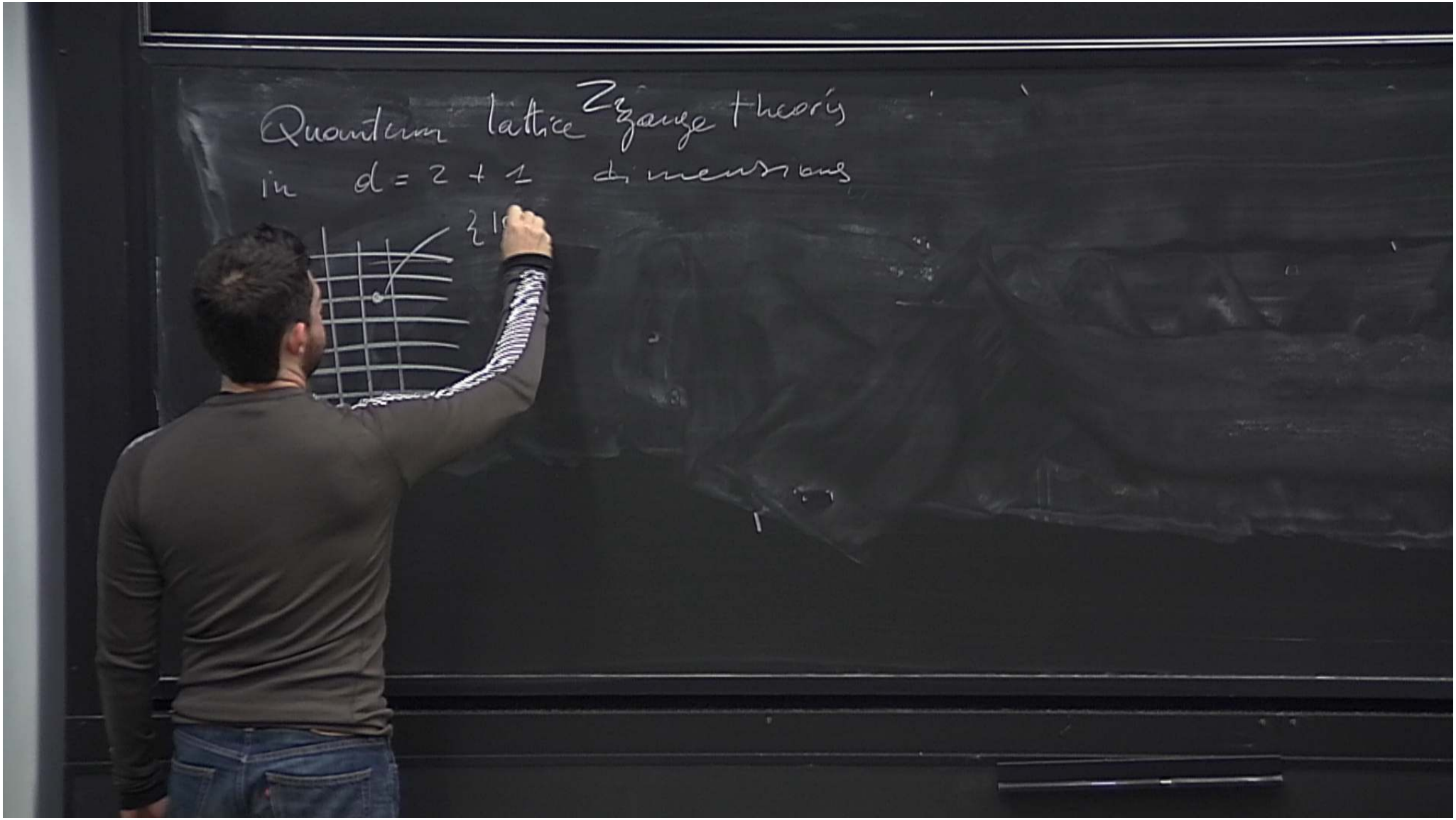
Date: Jan 16, 2012 10:15 AM

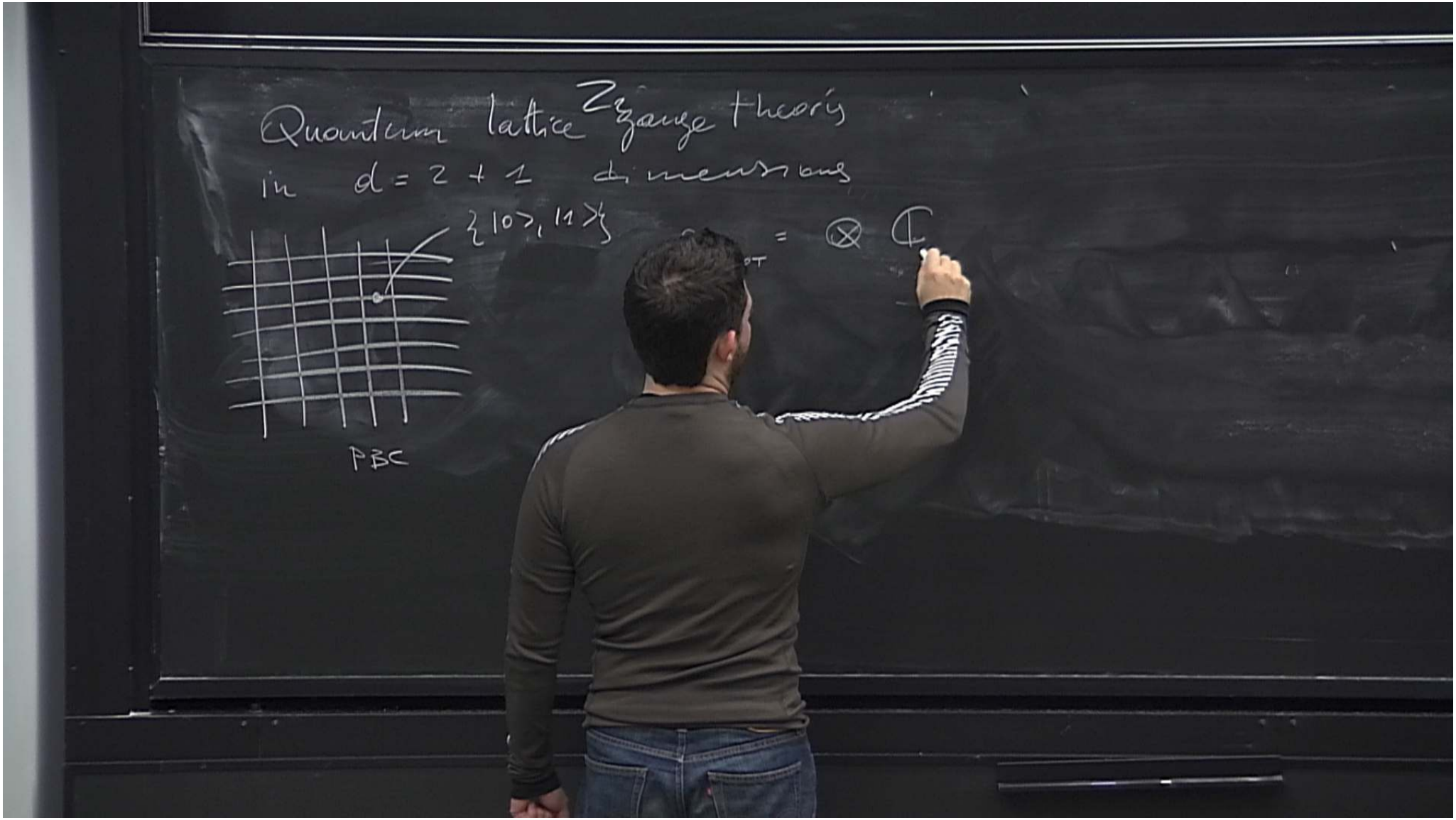
URL: http://pirsa.org/12010097

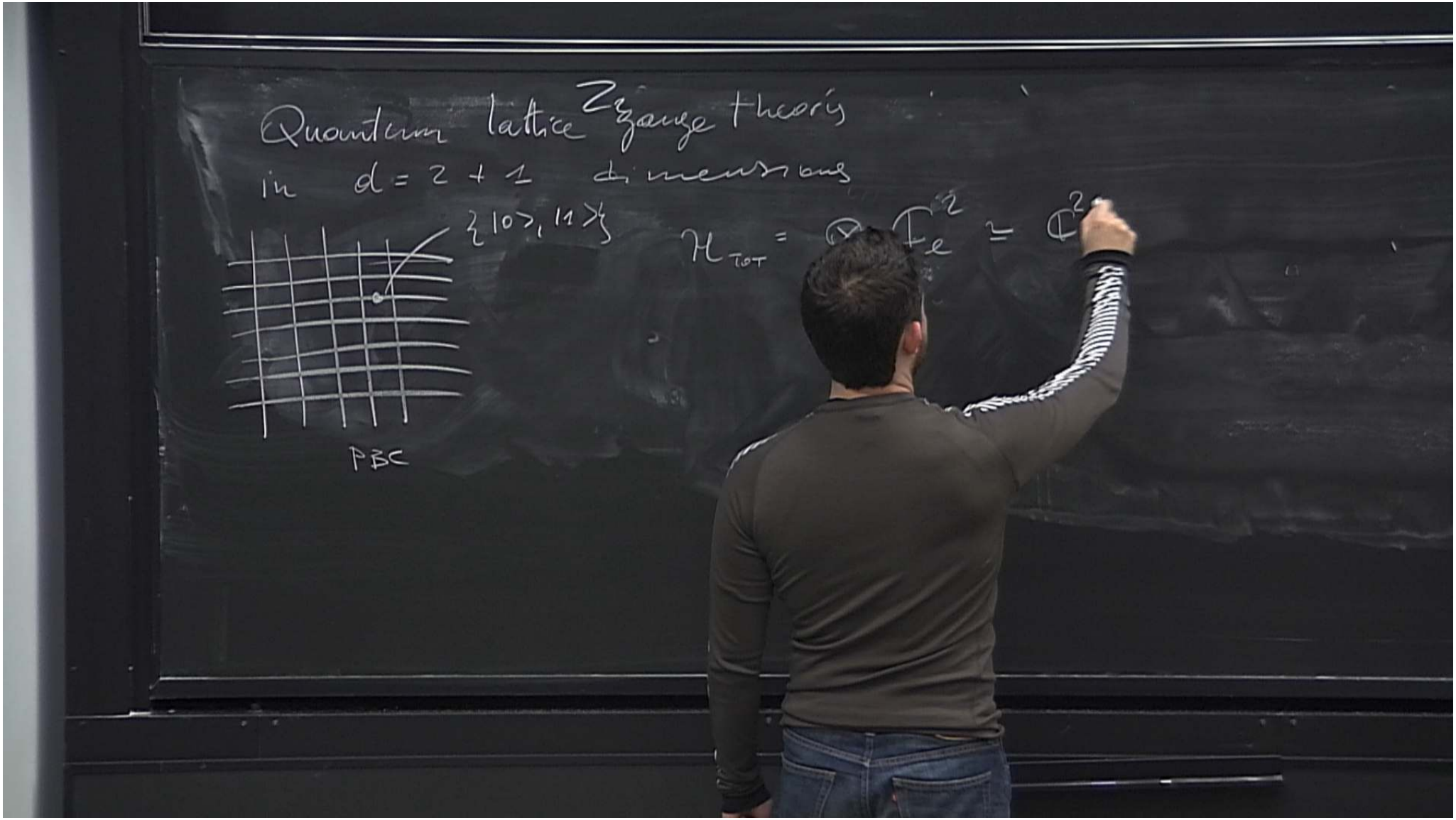
Abstract:

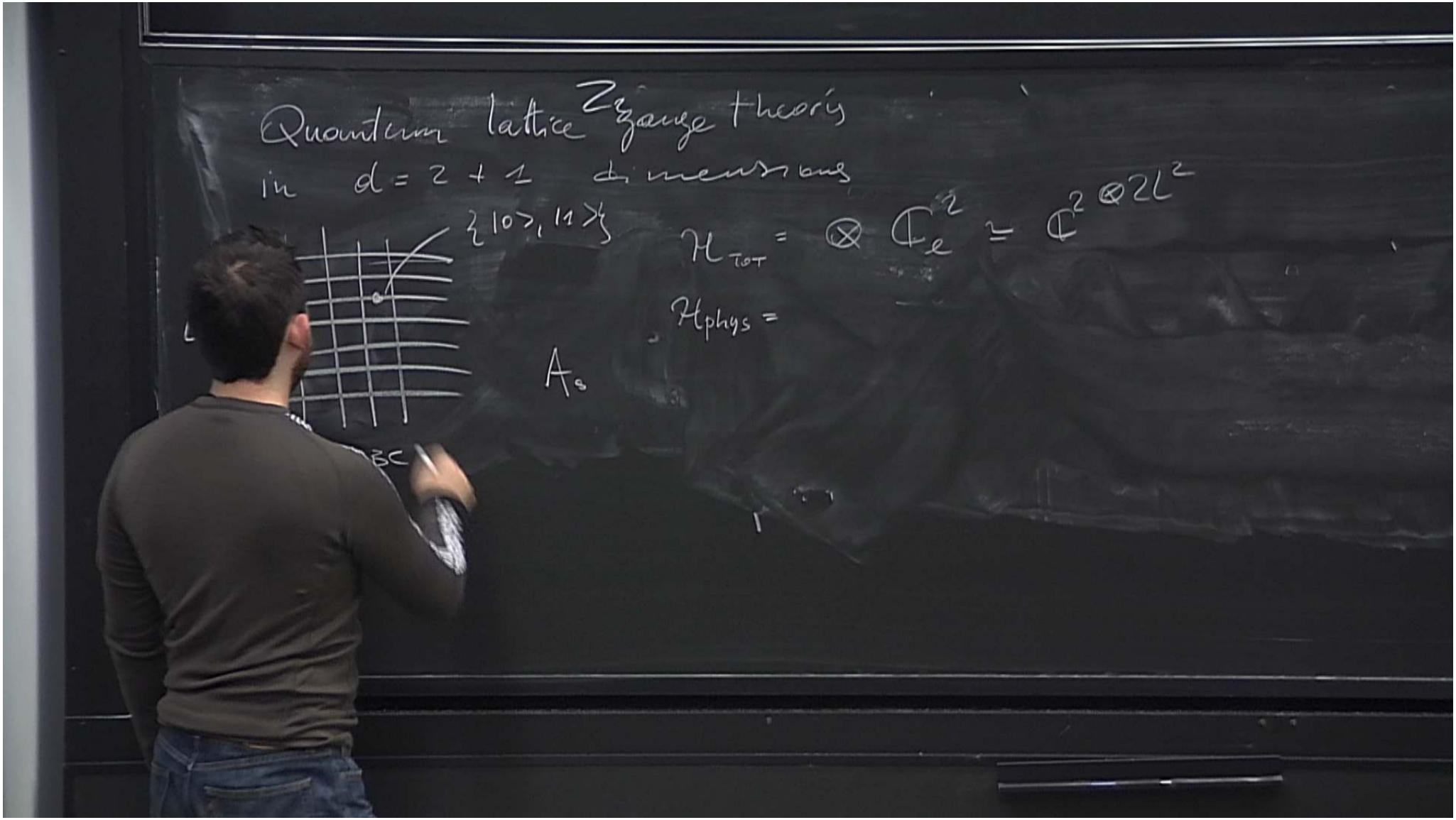

Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dime

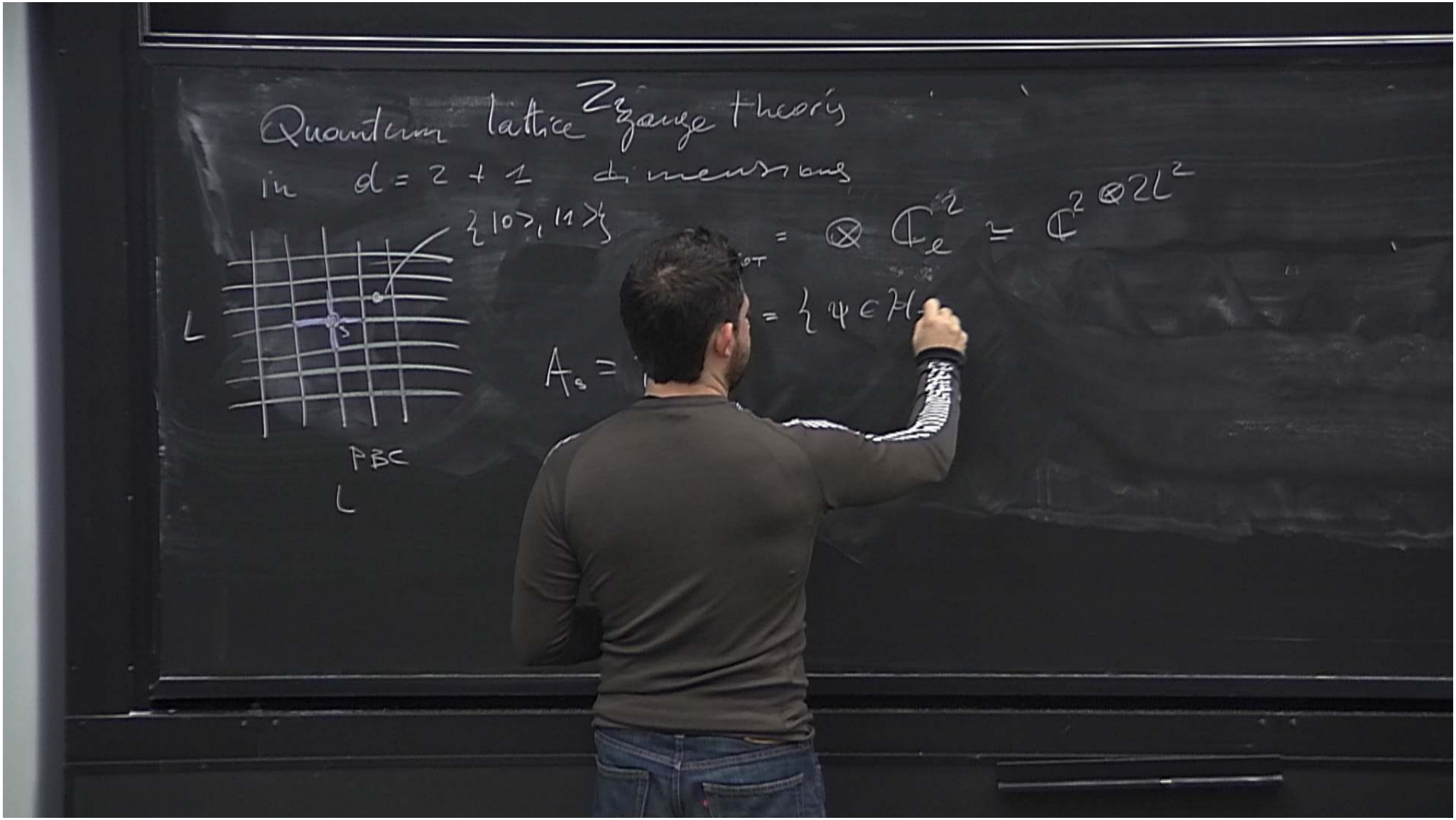

Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions

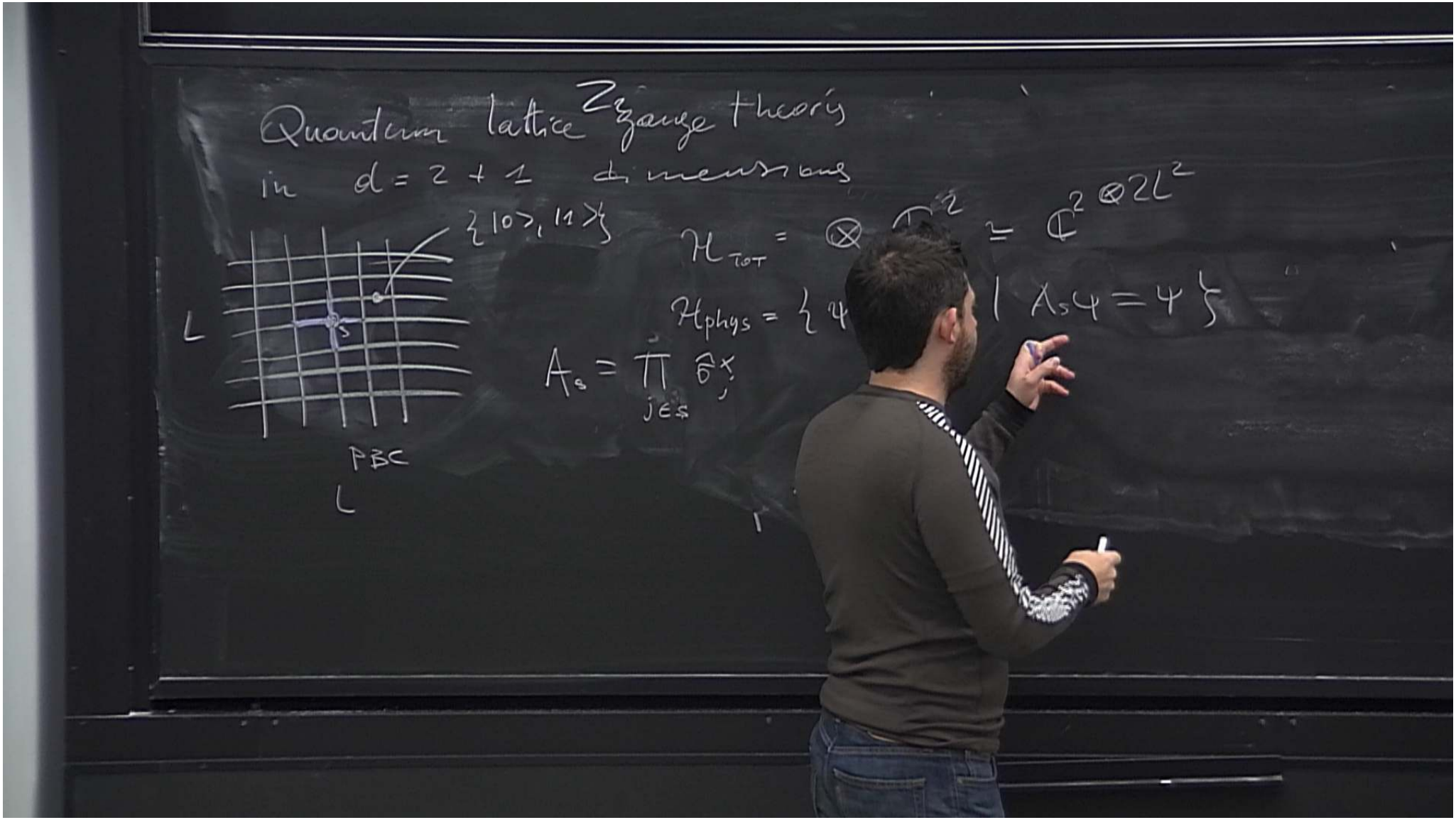

Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions


Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
PBC

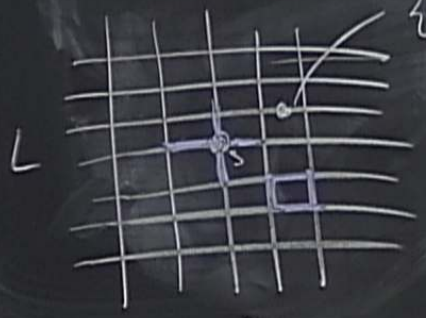

Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
$\mathcal{H}_{TOT} = \Phi \Gamma_e^2 = \Phi^2$
PBC


Quantum lattice $\mathbb{Z}_2$ gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}^2_e \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} =$
A_s
BC


Quantum lattice $\mathbb{Z}_2$ gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
$= \bigotimes \mathbb{C}^2_e \simeq \mathbb{C}^{2 \otimes 2L^2}$
$= \{\psi \in \mathcal{H}$
$A_s =$
L
s
PBC
L


Quantum lattice Z_2 gauge theory
in $d = 2 + 1$ dimensions
$\{|0\rangle, |1\rangle\}$
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{ \psi$
$| A_s \psi = \psi \}$
$A_s = \prod_{j \in s} \hat{\sigma}^x_j$
L
s
PBC
L

Quantum lattice $\mathbb{Z}_2$ gauge theory
in $d = 2 + 1$ dimensions



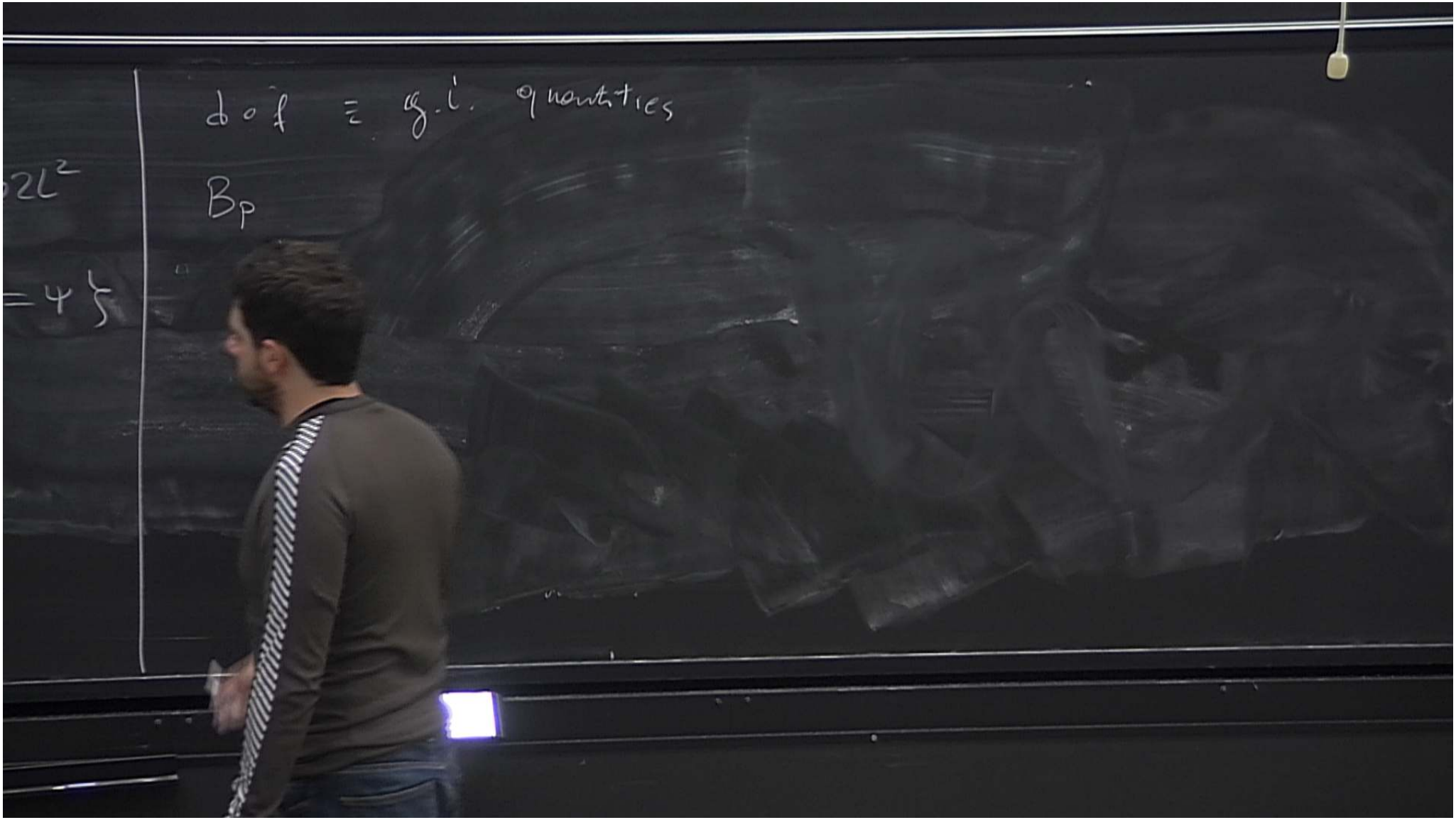
$$\mathcal{H}_{TOT} = \bigotimes_e \mathbb{C}_e^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$$

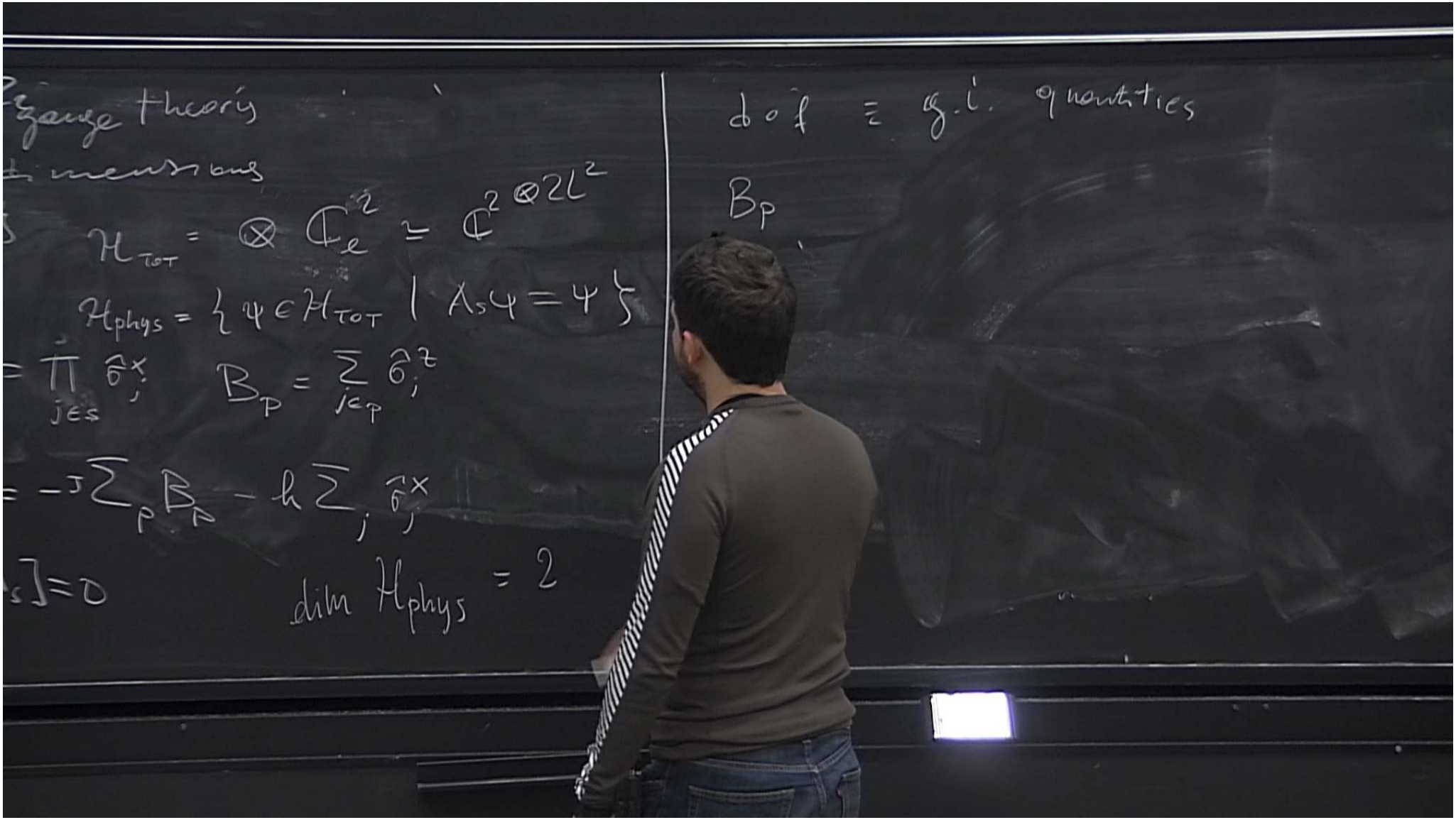
$$\mathcal{H}_{phys} = \{\psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi\}$$

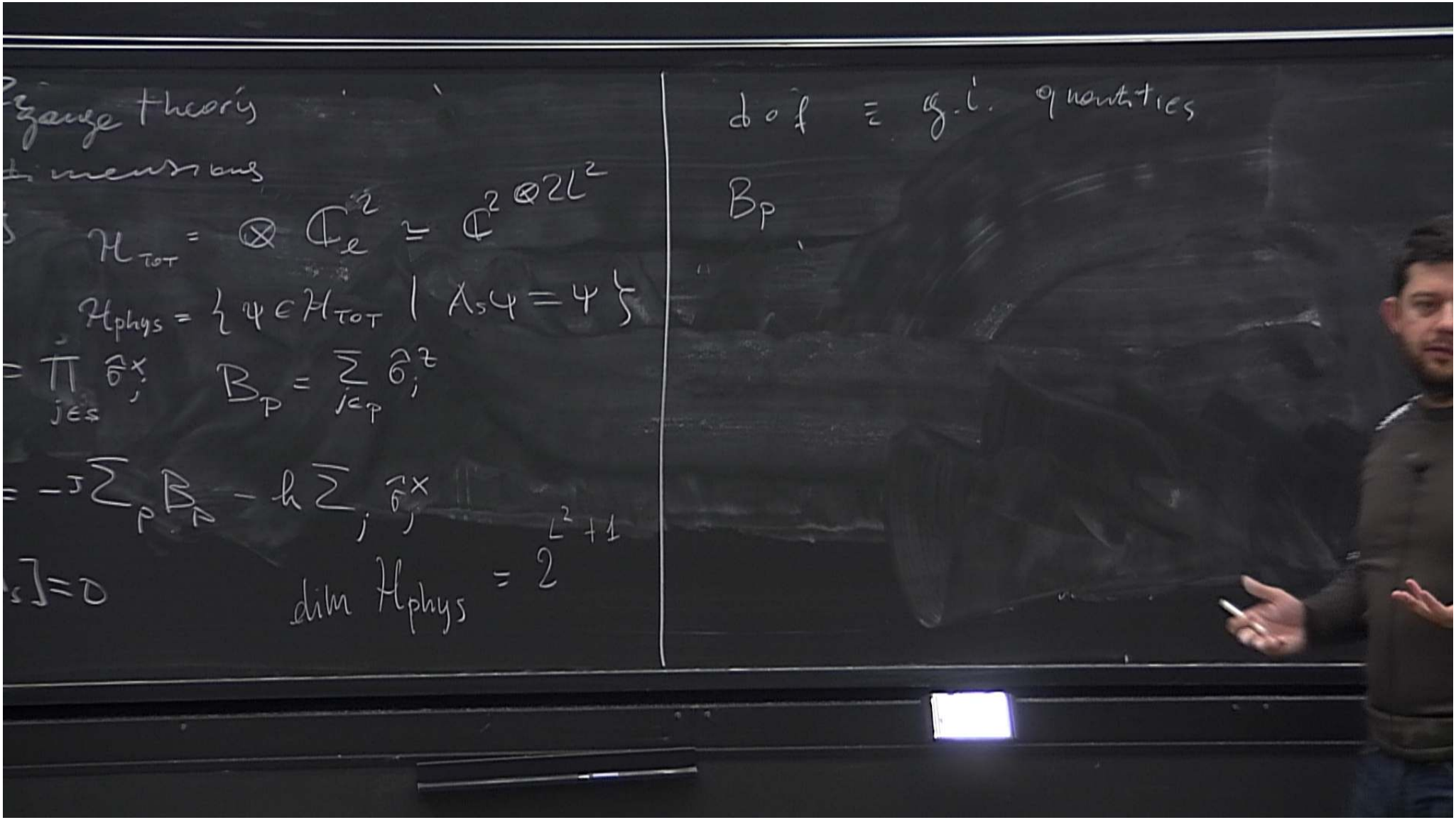
$$A_s = \prod_{j \in s} \hat{\sigma}_j^x \qquad B_p = \sum_{j \in p} \hat{\sigma}_j^z$$

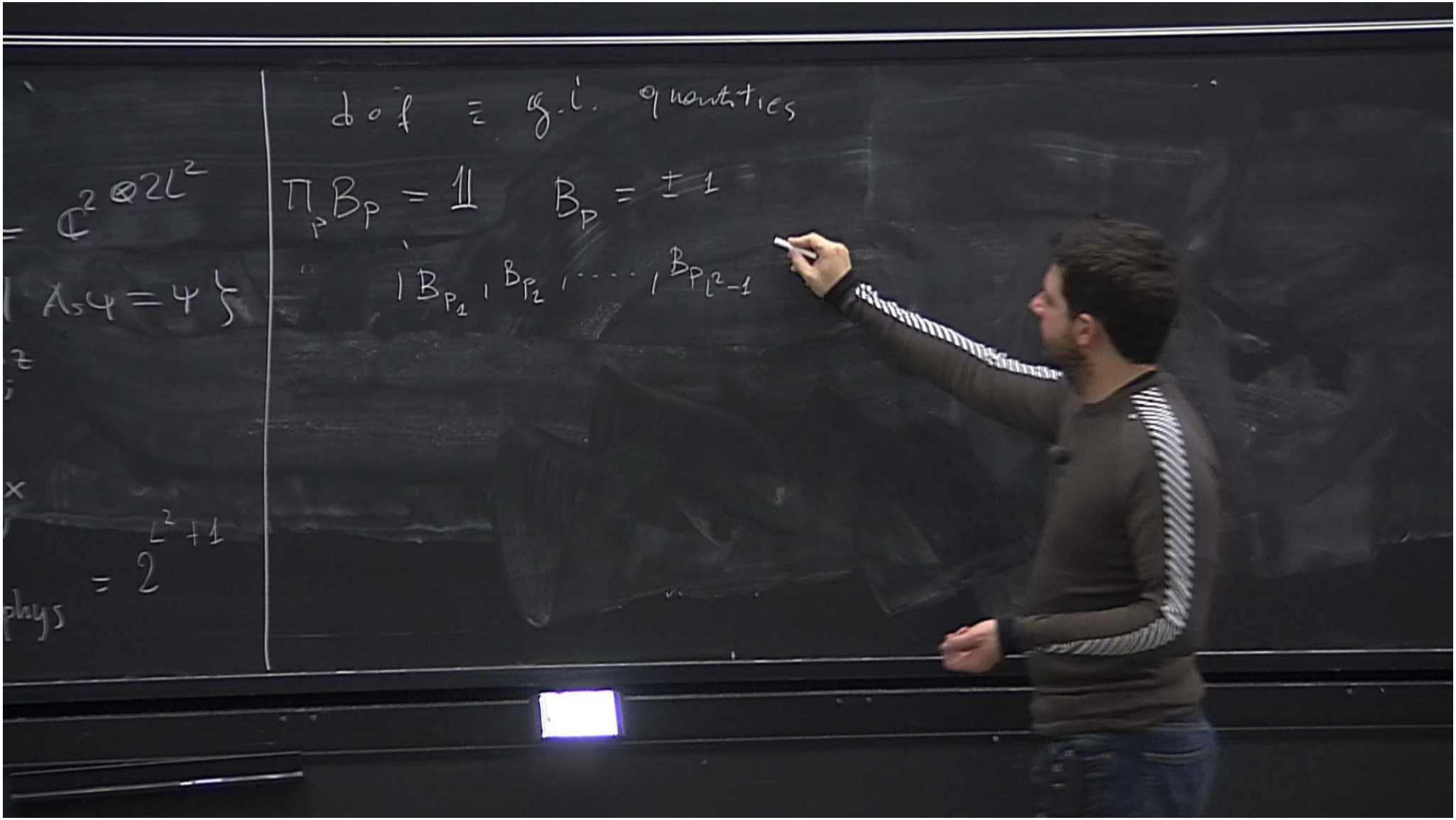
$$H = -J \sum_p B_p - h \sum_j \hat{\sigma}_j^x$$

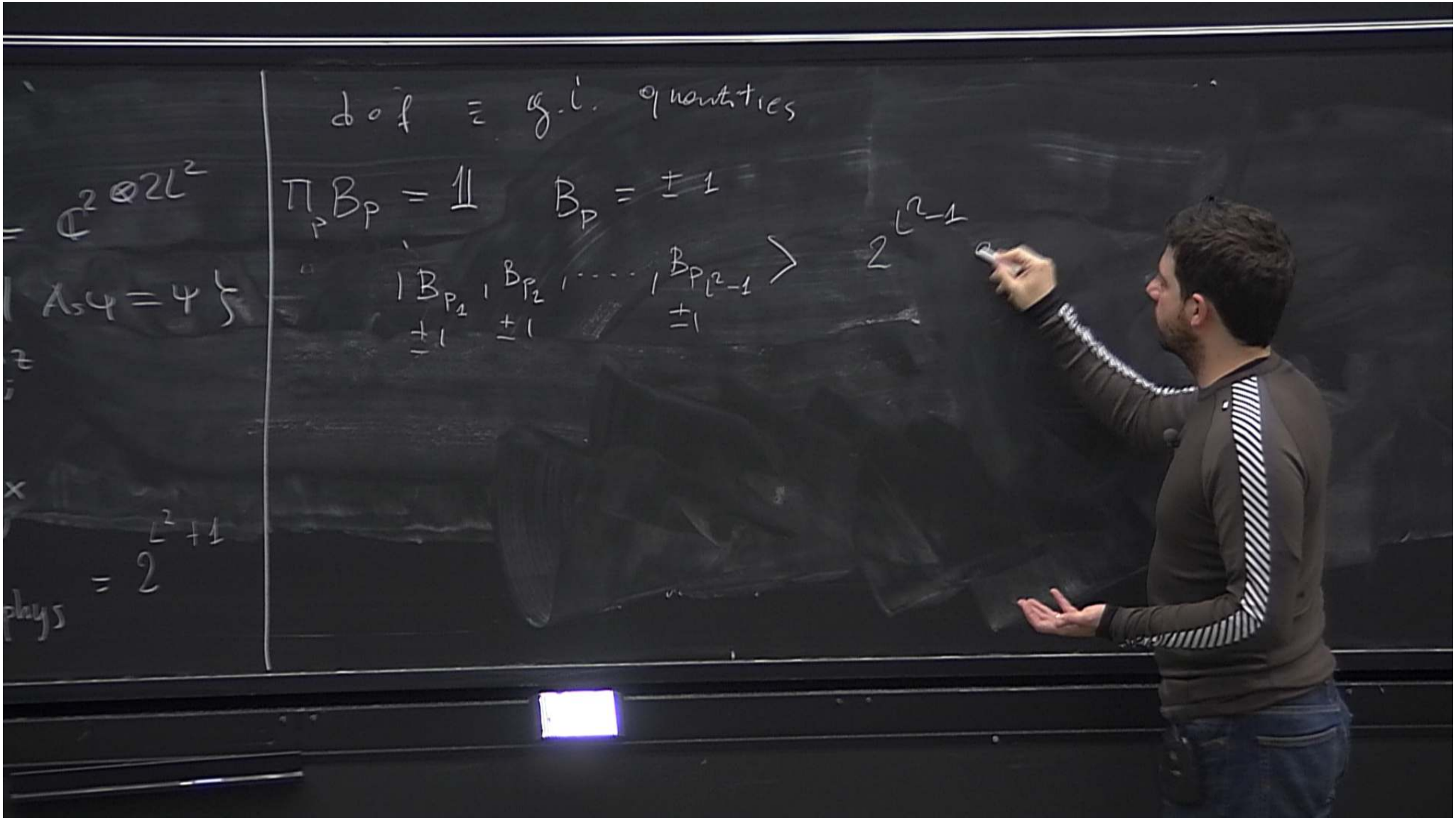
$$[H, A_s] = 0$$

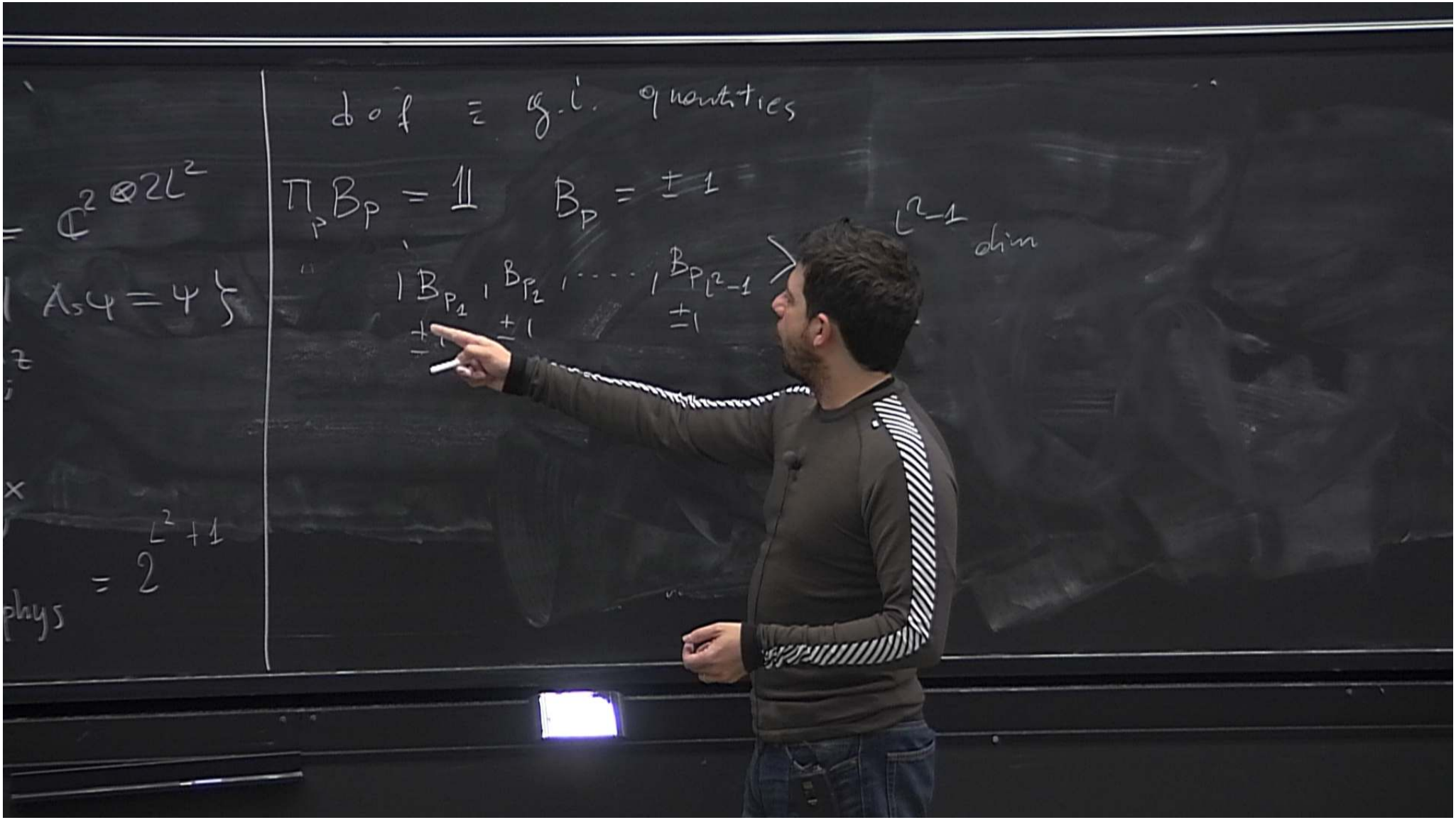

dof ≡ g.i. quantities
B_p

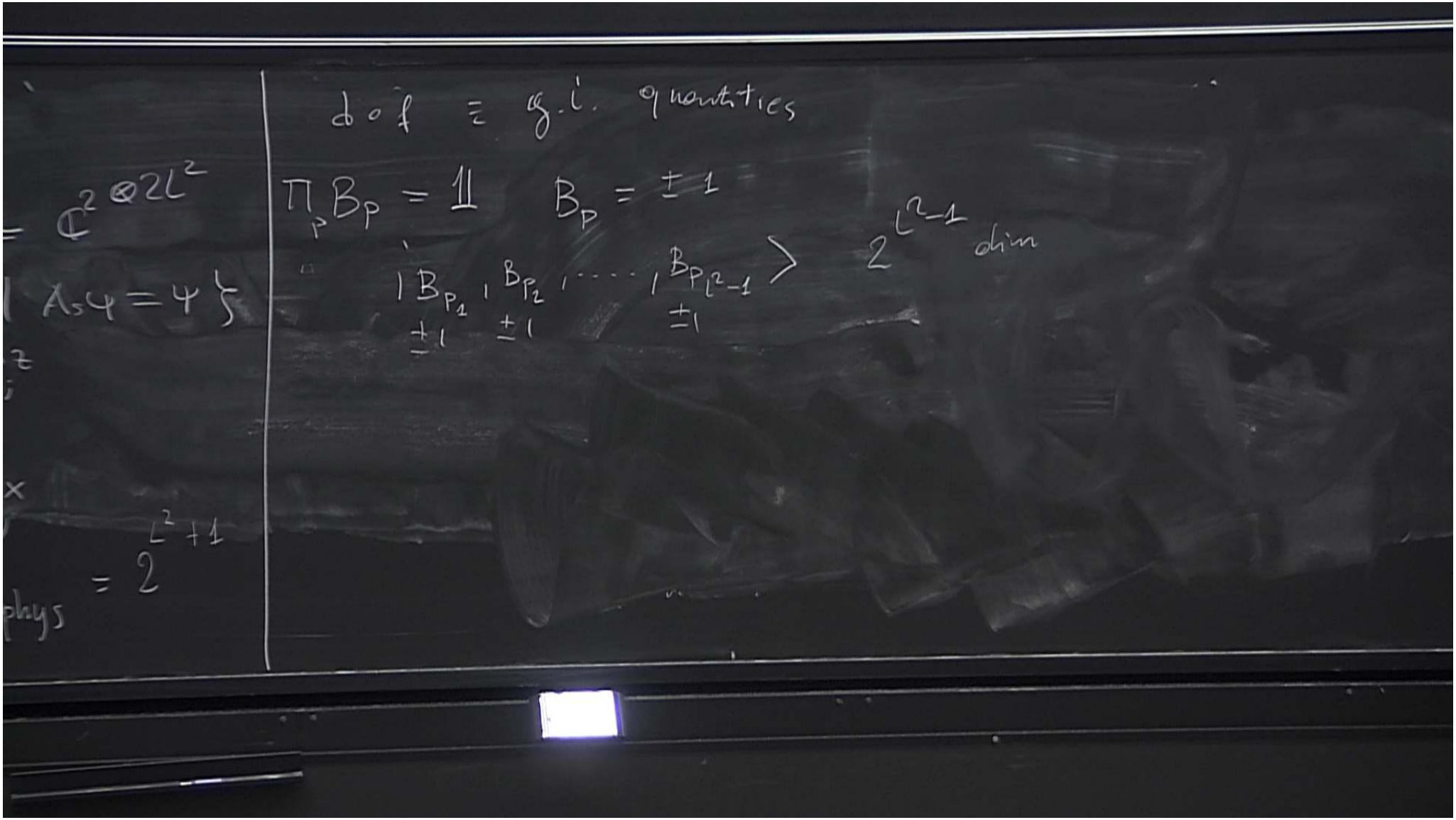

gauge theory
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$
$\prod_{j \in s} \hat{\sigma}_j^x$
$B_p = \sum_{j \in p} \hat{\sigma}_j^z$
$\sum_p B_p$
$\sum_j \hat{\sigma}_j^x$
$\dim \mathcal{H}_{phys} = 2$
dof = q.i. quantities
B_p

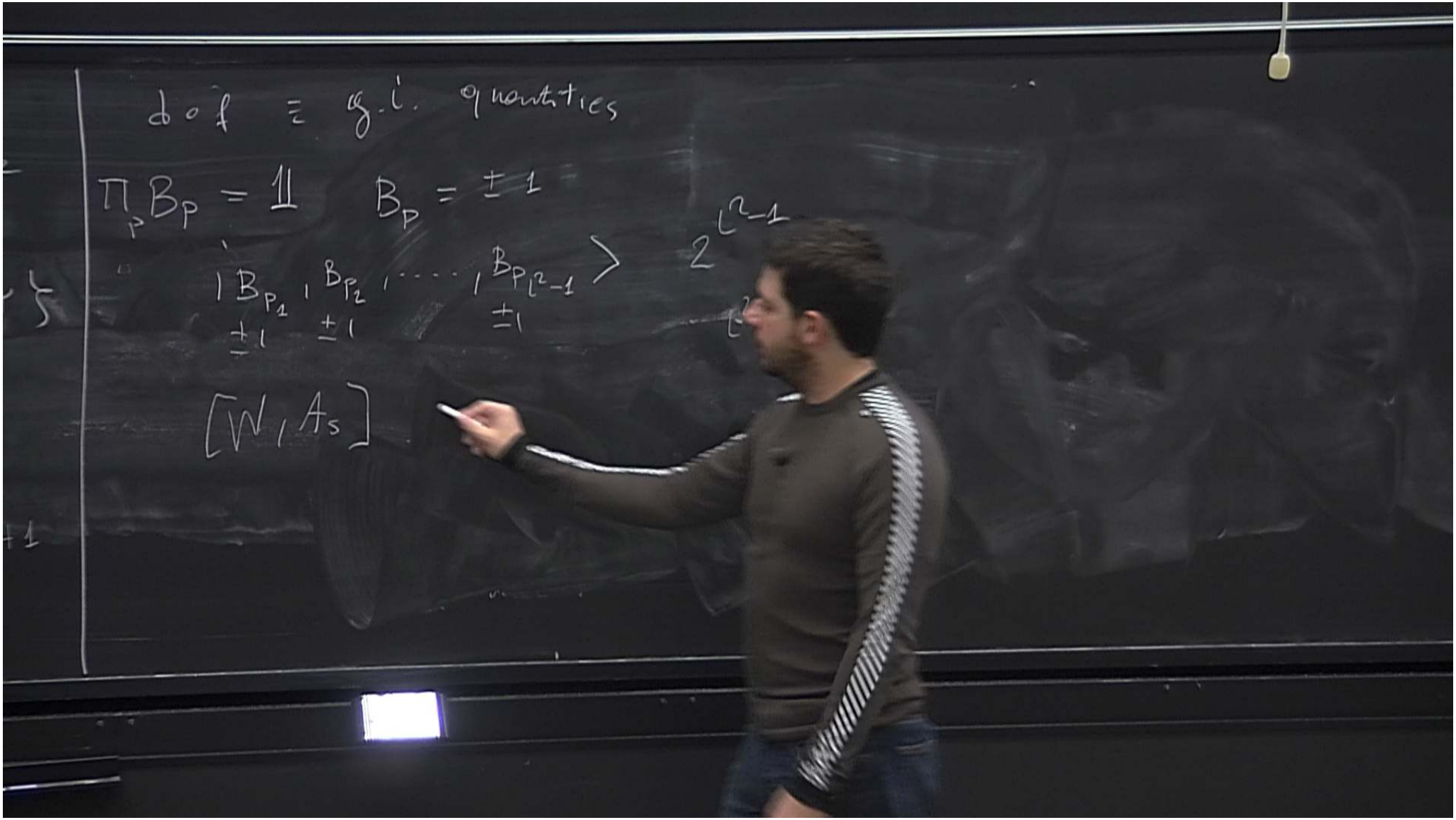

gauge theory
$\mathcal{H}_{TOT} = \bigotimes \mathbb{C}^2 \simeq \mathbb{C}^{2 \otimes 2L^2}$
$\mathcal{H}_{phys} = \{\psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi\}$
$\prod_{j \in s} \hat{\sigma}_j^x \qquad B_p = \sum_{j \in p} \hat{\sigma}_j^z$
$= -J \sum_p B_p - h \sum_j \hat{\sigma}_j^x$
$\dim \mathcal{H}_{phys} = 2^{L^2+1}$
dof ≡ q.i. quantities
B_p

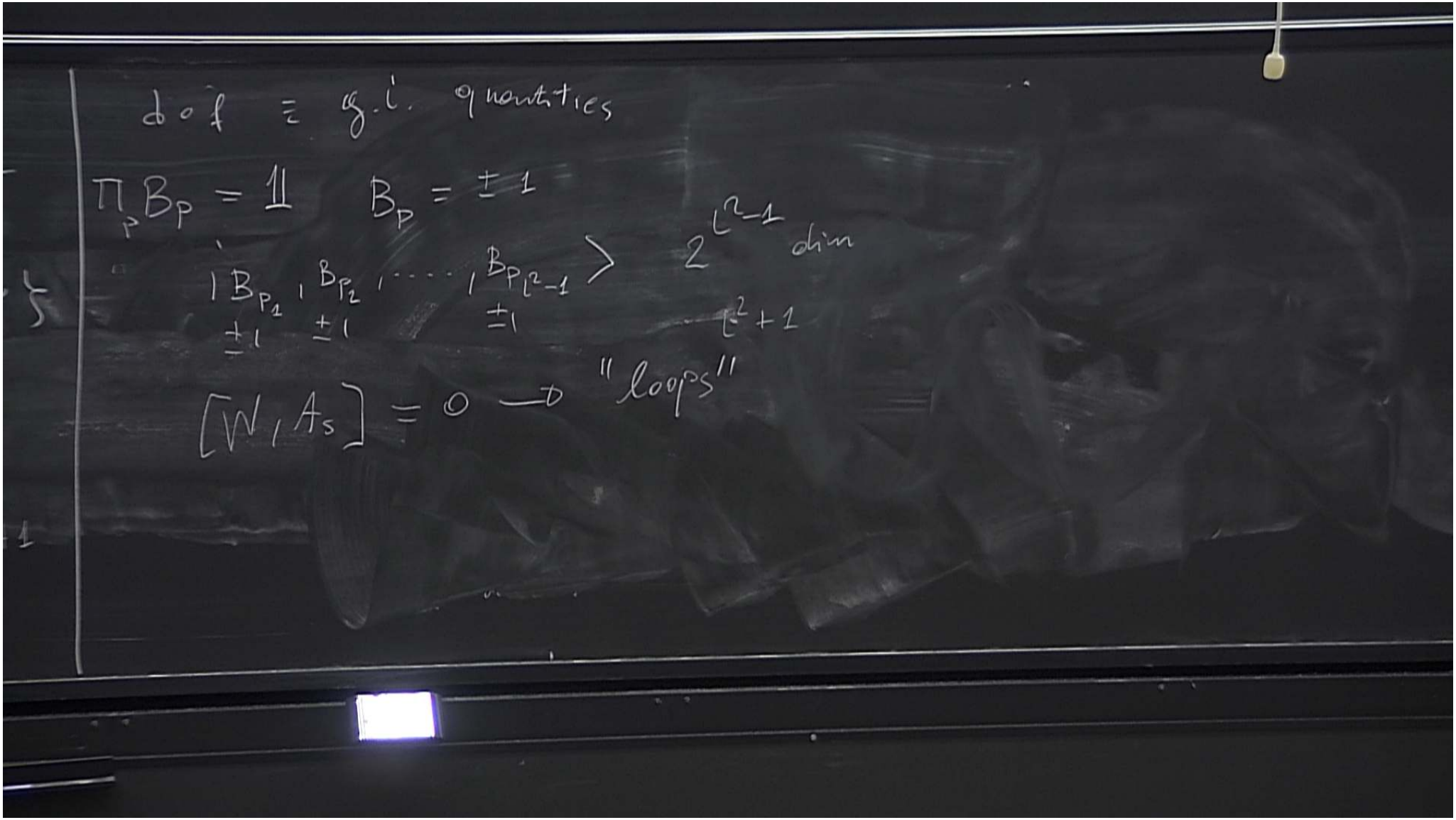

dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \dots, B_{p_{L^2-1}}$
$= \mathbb{C}^2 \otimes 2L^2$
$A_s 4 = 4$
$= 2^{L^2+1}$
phys


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \dots, B_{p_{L^2-1}}\rangle$
2^{L^2-1}
± 1 ± 1 ± 1
$= \mathbb{C}^2 \otimes 2L^2$
$A_s 4 = 4$
$= 2^{L^2+1}$


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
L^2-1
dim
$|B_{p_1}, B_{p_2}, \dots, B_{p_{L^2-1}}\rangle$
± 1
± 1
± 1
$\mathbb{C}^2 \otimes 2L^2$
$A_5 4 = 4$
$= 2^{L^2+1}$
phys


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}\rangle$
2^{L^2-1} dim
± 1 ± 1 ± 1
$\mathbb{C}^2 \otimes 2L^2$
$A_s\psi = \psi\}$
$= 2^{L^2+1}$
phys


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$
$B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}\rangle$
2^{L^2-1}
± 1 ± 1 ± 1
$[W, A_s]$


dof ≡ g.i. quantities
$\prod_p B_p = \mathbb{1}$ $B_p = \pm 1$
$|B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}\rangle$ 2^{L^2-1} dim
± 1 ± 1 ± 1 L^2+1
$[W, A_s] = 0 \rightarrow$ "loops"

dof ≡ g.i. quantities

$\prod_p B_p = \mathbb{1}$ $\quad B_p = \pm 1$

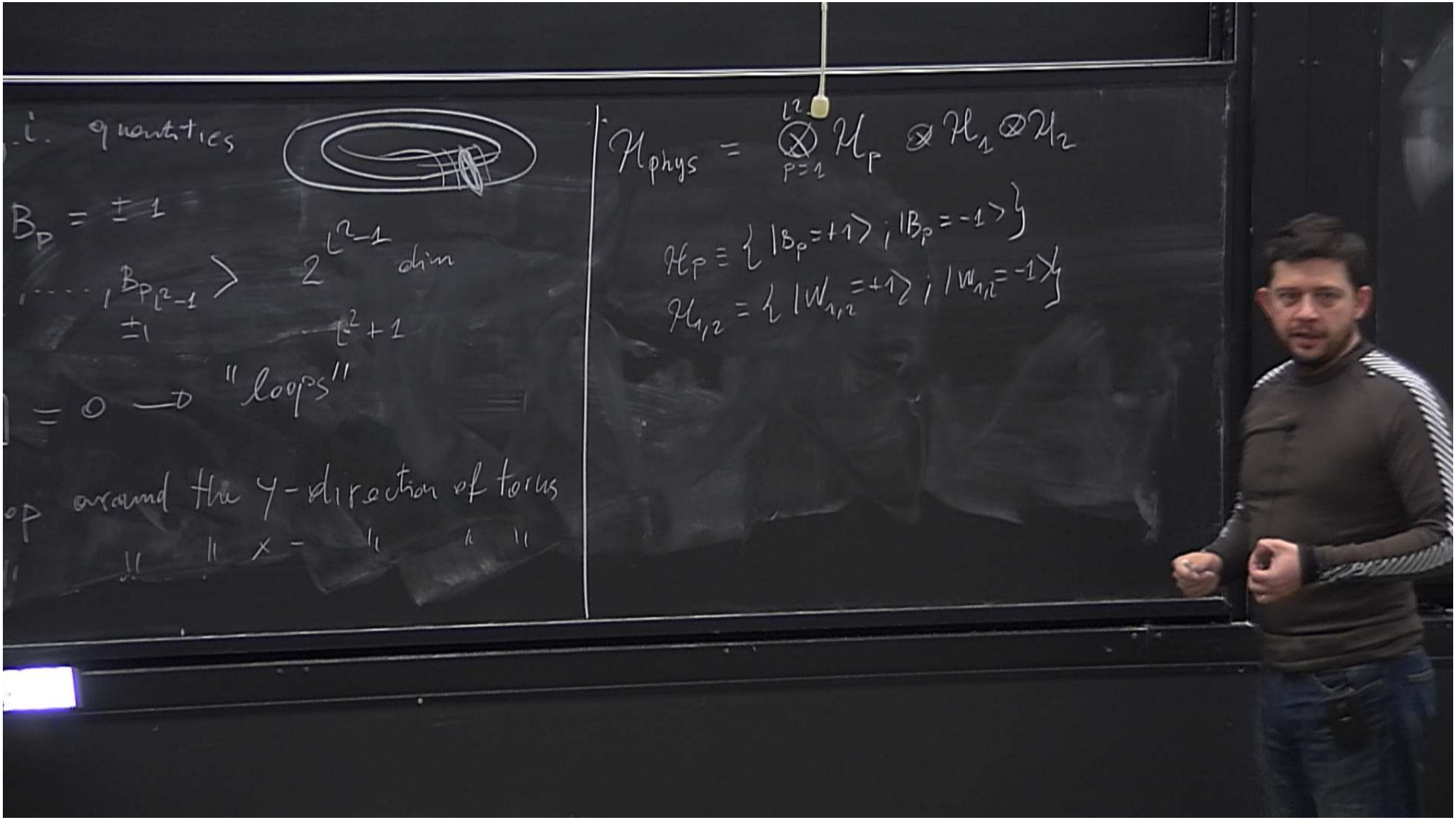
$|B_{p_1}, B_{p_2}, \ldots, B_{p_{L^2-1}}\rangle$ $\quad 2^{L^2-1}$ dim

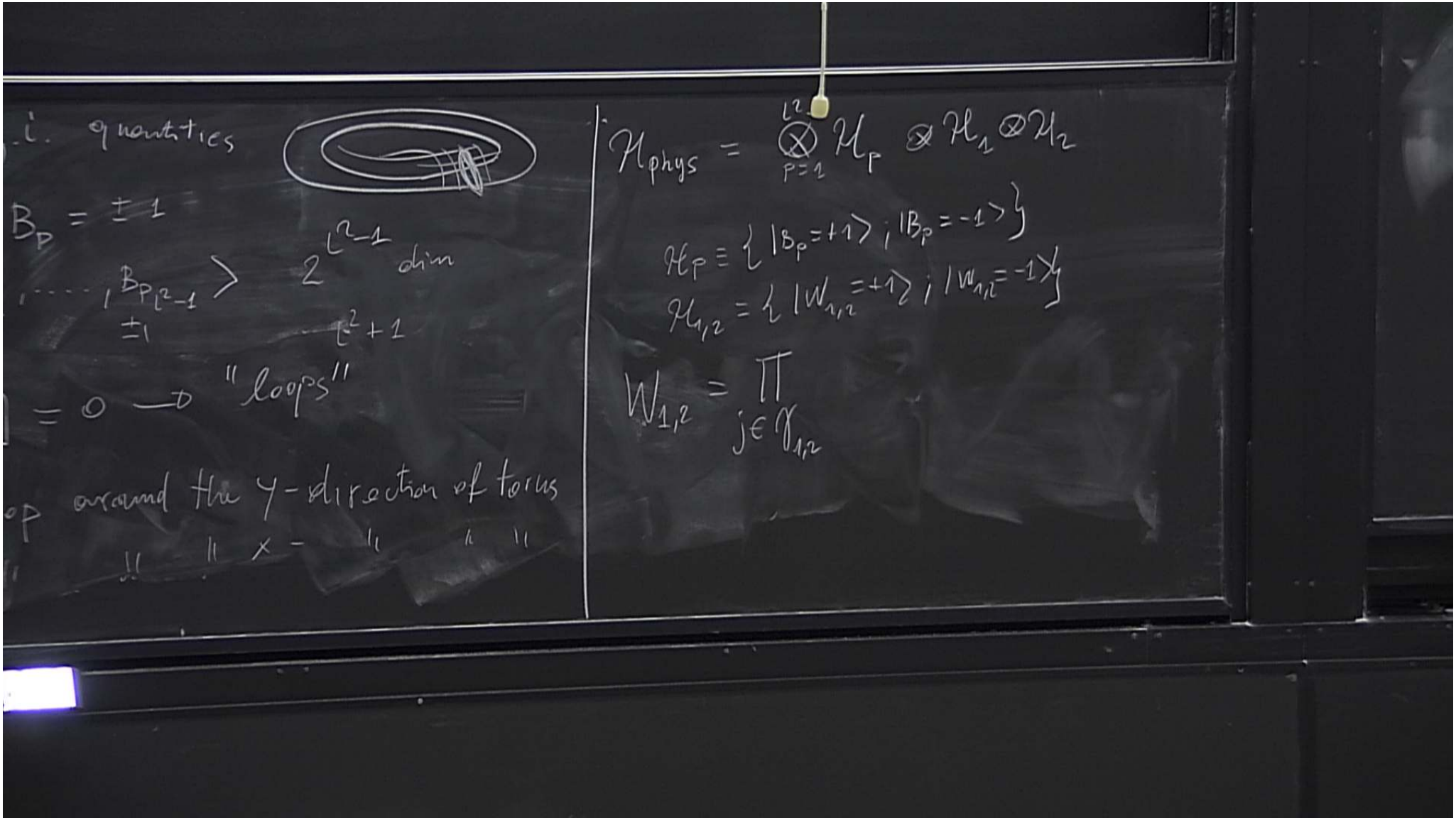
$\pm 1 \quad \pm 1 \quad \pm 1 \quad \ldots \quad L^2+1$

$[W, A_s] = 0 \rightarrow$ "loops"

W_1 loop around the y-direction of torus

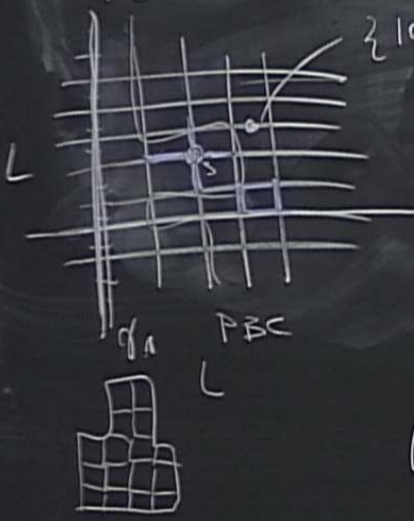
W_2 " " " x- " " "


i. quantics
$B_p = \pm 1$
$|B_{p_1} \ldots B_{p_{L^2-1}}\rangle$
± 1
2^{L^2-1} dim
L^2+1
$= 0 \rightarrow$ "loops"
around the y-direction of torus
$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$
$\mathcal{H}_p \equiv \{|B_p = +1\rangle ; |B_p = -1\rangle\}$
$\mathcal{H}_{1,2} = \{|W_{1,2} = +1\rangle ; |W_{1,2} = -1\rangle\}$


i. quantities
$B_p = \pm 1$
$|B_p \ldots B_{p L^2-1}\rangle$
± 1
2^{L^2-1} dim
L^2+1
$= 0 \rightarrow$ "loops"
op around the y-direction of torus
" " x- " "
$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$
$\mathcal{H}_p \equiv \{|B_p=+1\rangle, |B_p=-1\rangle\}$
$\mathcal{H}_{1,2} = \{|W_{1,2}=+1\rangle, |W_{1,2}=-1\rangle\}$
$W_{1,2} = \prod_{j \in \gamma_{1,2}}$

Quantum lattice $\mathbb{Z}_2$ gauge theory

in $d = 2 + 1$ dimensions



$$\mathcal{H}_{TOT} = \bigotimes_e \mathbb{C}^2_e \simeq \mathbb{C}^{2 \otimes 2L^2}$$

$$\mathcal{H}_{phys} = \{ \psi \in \mathcal{H}_{TOT} \mid A_s \psi = \psi \}$$

$$A_s = \prod_{j \in s} \hat{\sigma}^x_j \qquad B_p = \sum_{j \in p} \hat{\sigma}^z_j$$

$$H = -J \sum_p B_p - h \sum_j \hat{\sigma}^x_j$$

$$[H, A_s] = 0$$

$$\dim \mathcal{H}_{phys} = 2^{L^2+1}$$

dof $\simeq$ g.i. quanti

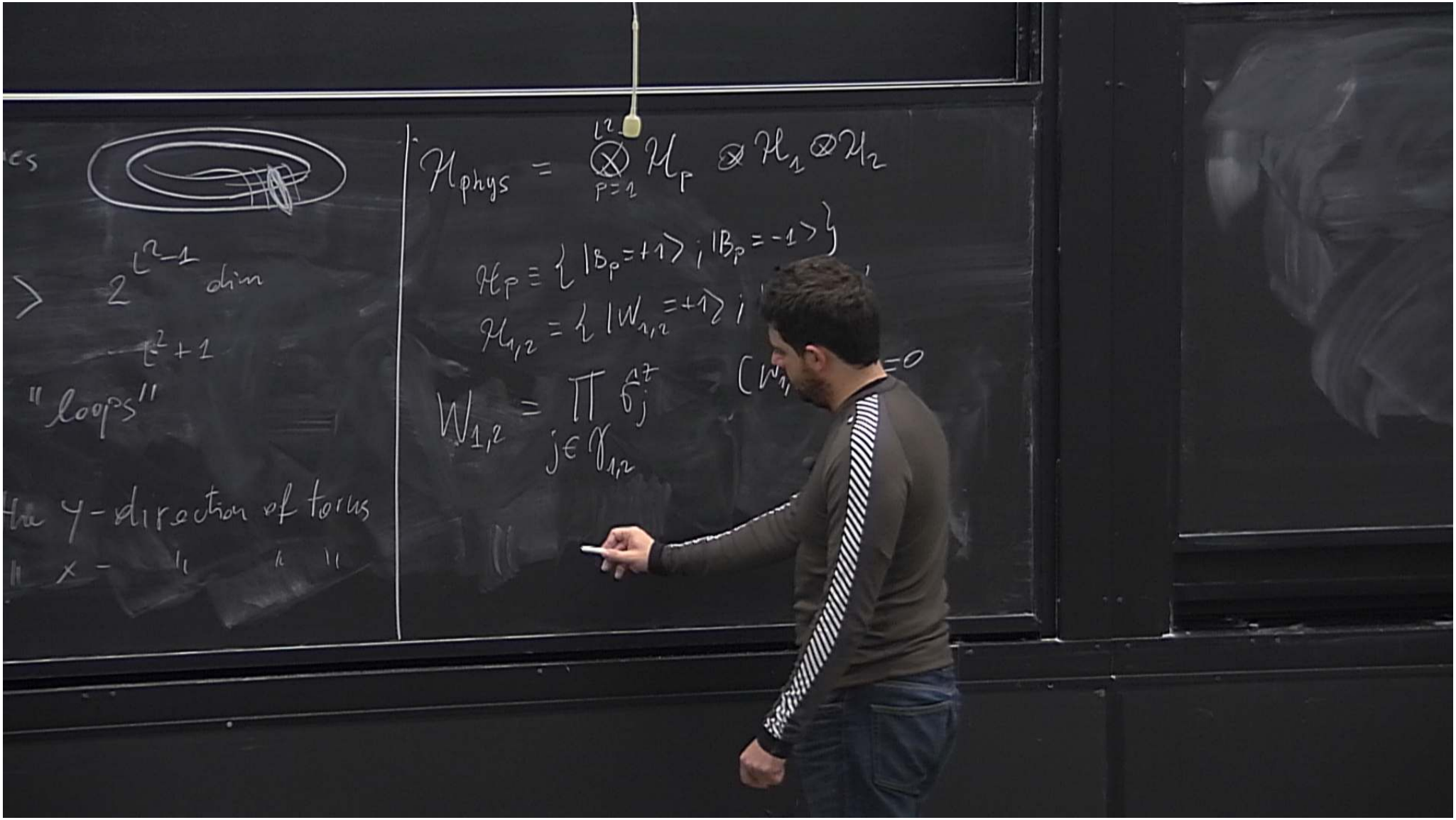
$$\prod_p B_p = \mathbb{1} \qquad B_p = \pm 1$$

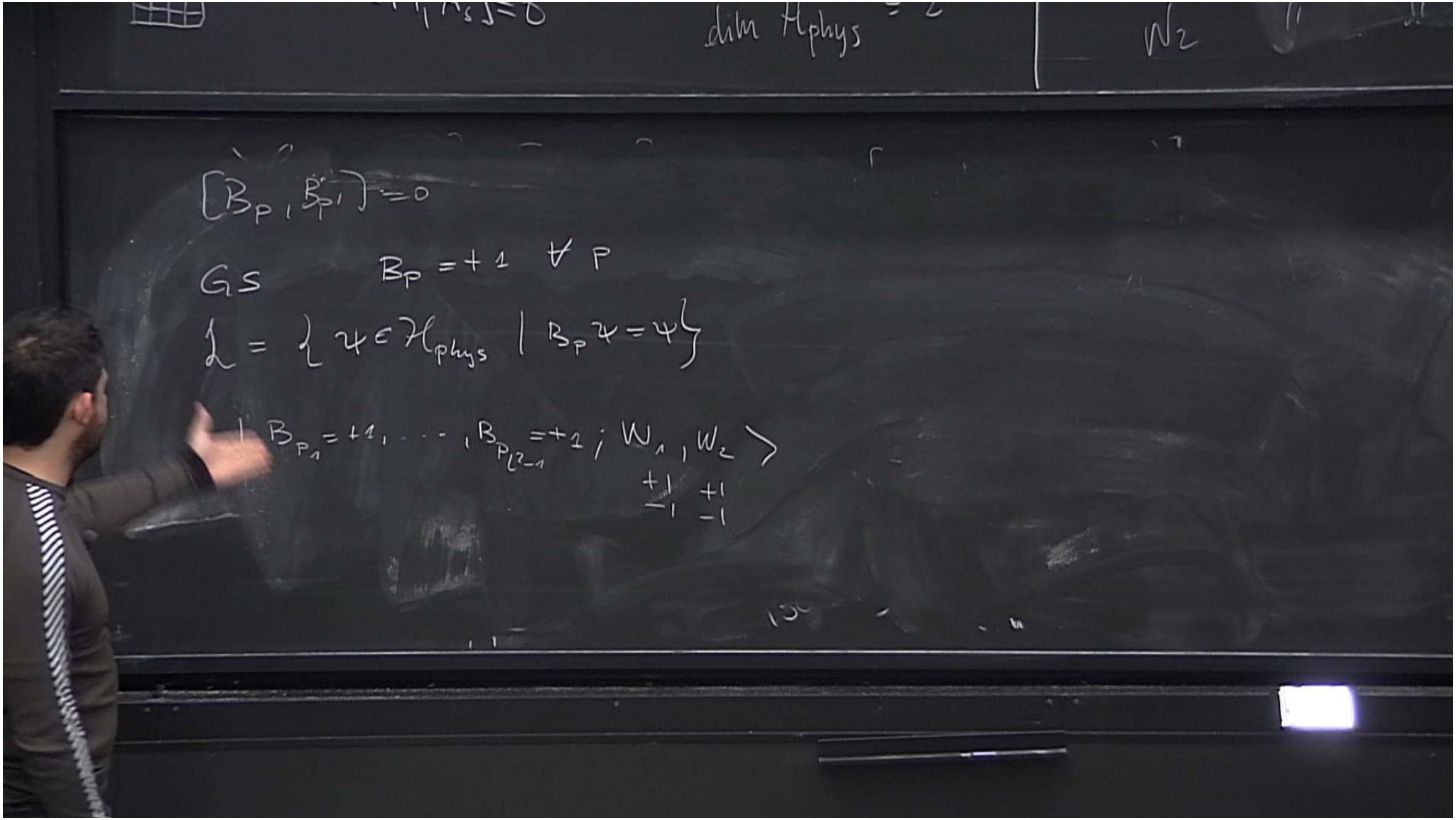
$|B_{p_1}, B_{p_2}, \dots, B_{p_{L^2-1}}$
$\pm 1 \quad \pm 1 \quad \pm 1$

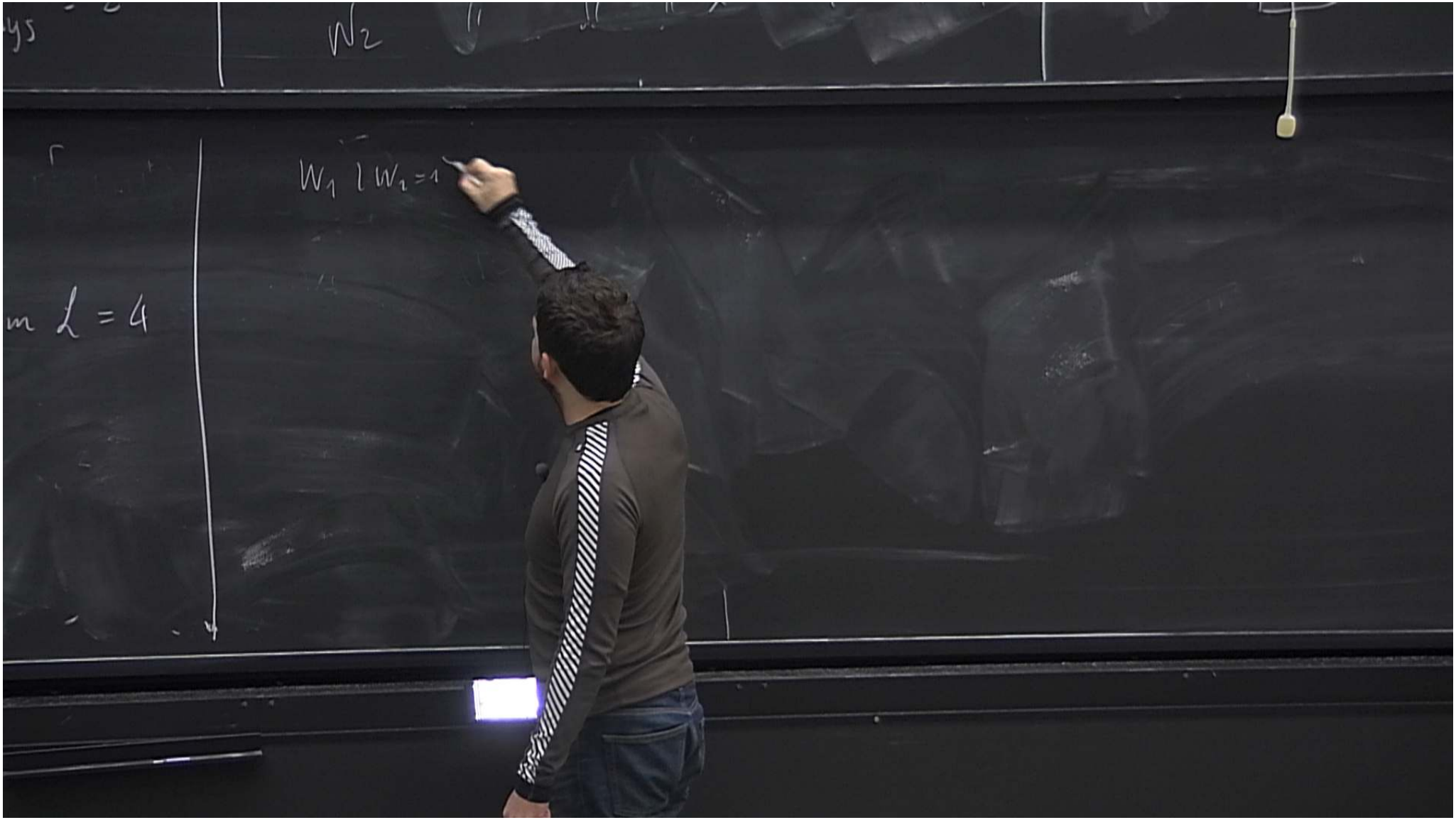
$$[W, A_s] = 0 \longrightarrow$$

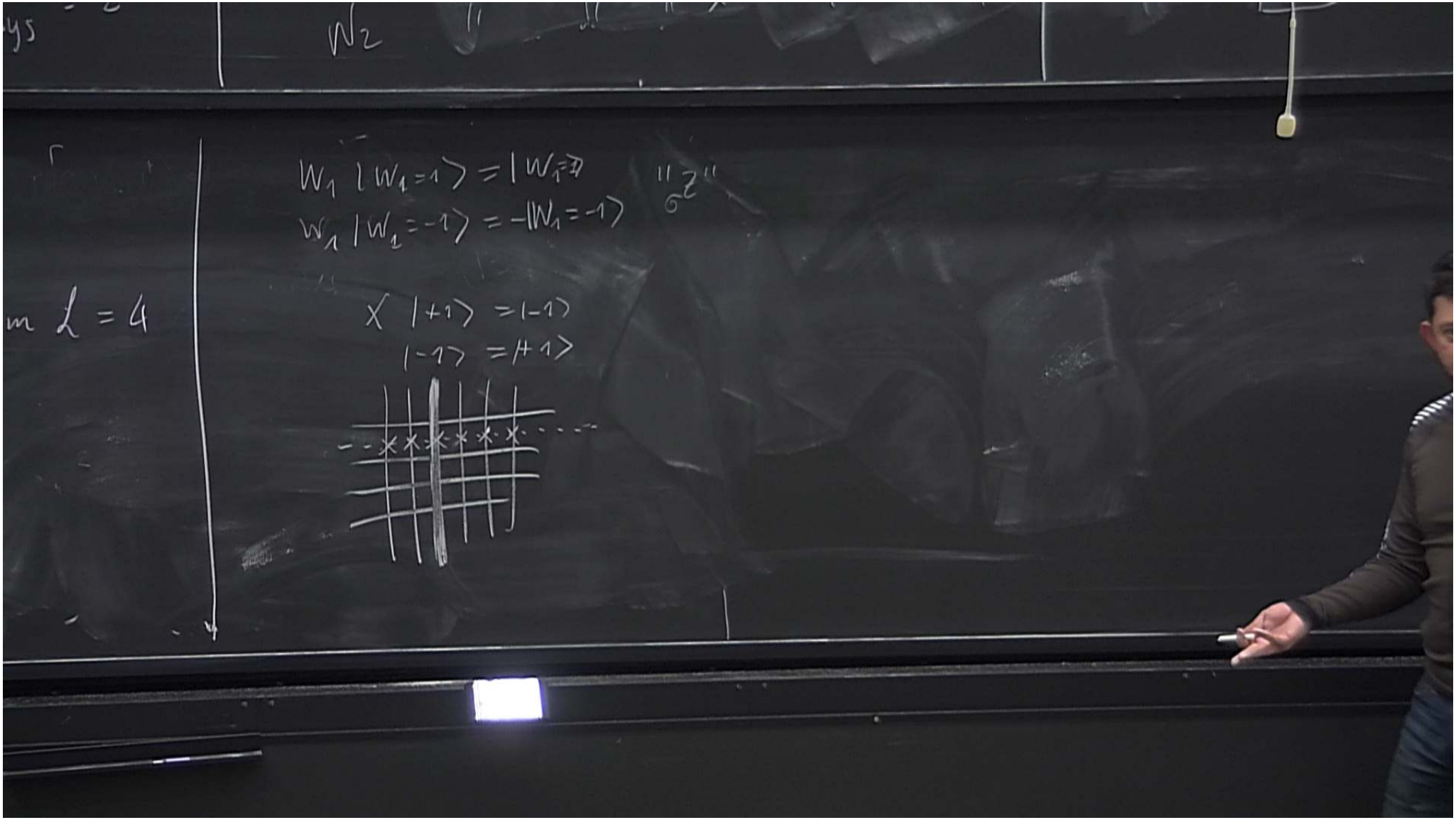
W_1 loop around

W_2 " "


$\mathcal{H}_{phys} = \bigotimes_{p=1}^{L^2} \mathcal{H}_p \otimes \mathcal{H}_1 \otimes \mathcal{H}_2$
$\mathcal{H}_p \equiv \{|B_p=+1\rangle, |B_p=-1\rangle\}$
$\mathcal{H}_{1,2} = \{|W_{1,2}=+1\rangle$
$W_{1,2} = \prod_{j \in \gamma_{1,2}} \sigma_j^z$
2^{L^2-1} dim
L^2+1
"loops"
the y-direction of torus


dim $\mathcal{H}_{phys}$ = 2
W_2
$[B_p, B_{p'}] = 0$
GS $B_p = +1 \quad \forall\, p$
$\mathcal{L} = \{\psi \in \mathcal{H}_{phys} \mid B_p \psi = \psi\}$
$|B_{p_1} = +1, \dots, B_{p_{L^2-1}} = +1; W_1, W_2\rangle$
$+1 \quad +1$
$-1 \quad -1$


W_2
$W_1 \wedge W_2 = 1$
$m\ \ell = 4$


W_2
$W_1 |W_1=1\rangle = |W_1=1\rangle$
$W_1 |W_1=-1\rangle = -|W_1=-1\rangle$
"σ^z"
$m\ l=4$
$X\ |+1\rangle = |-1\rangle$
$|-1\rangle = |+1\rangle$

W_2

$W_1^z |W_1 = 1\rangle = |W_1 = 1\rangle$

$W_1 |W_1 = -1\rangle = -|W_1 = -1\rangle$

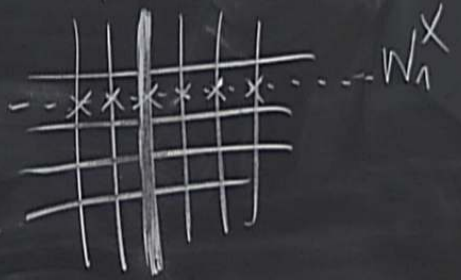
"σ^z"

$\{W_1^x, W_1^z\} = 0$

$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$ } spinska algebra

$X\ |+1\rangle = |-1\rangle$

$|-1\rangle = |+1\rangle$



$m\ l = 4$

W_2

$W_1^z |W_1 = 1\rangle = |W_1 = 1\rangle$

$W_1 |W_1 = -1\rangle = -|W_1 = -1\rangle$

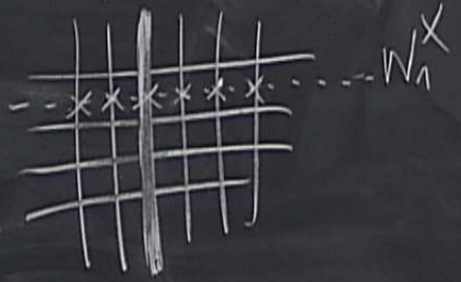
"σ^z"

$\{W_1^x, W_1^z\} = 0$

$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$ } spinski algebra

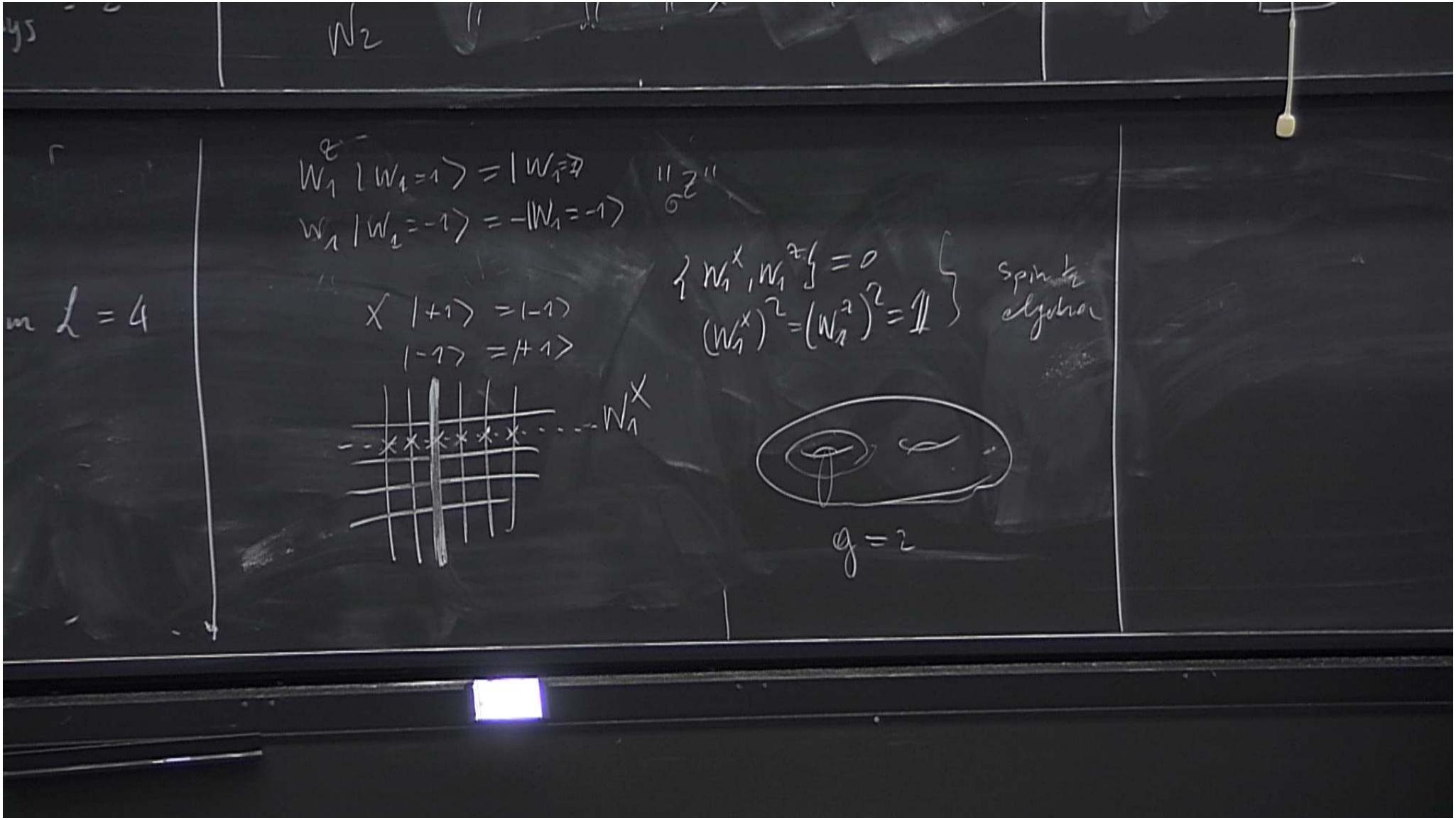
$X\ |+1\rangle = |-1\rangle$

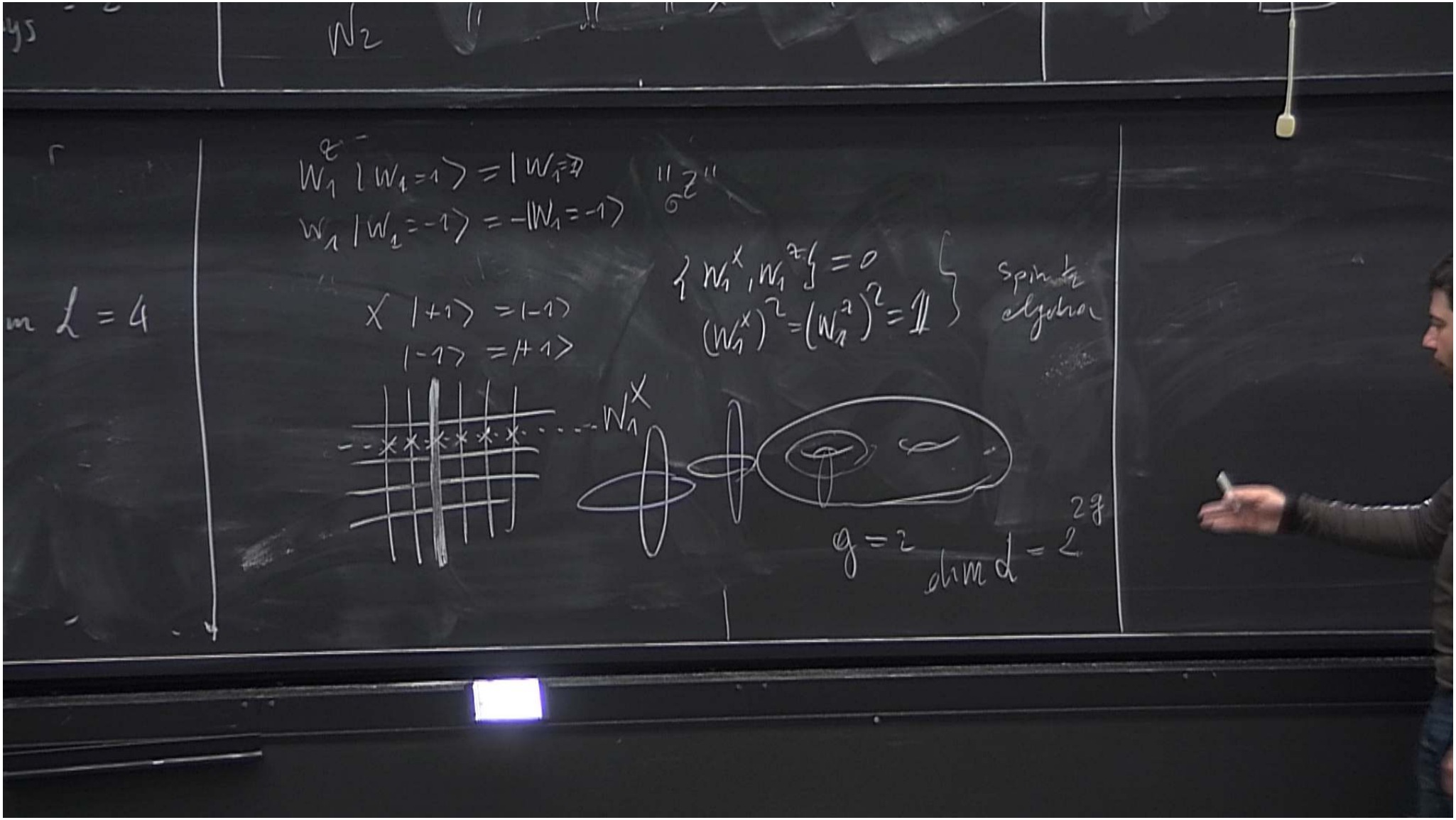
$|-1\rangle = |+1\rangle$

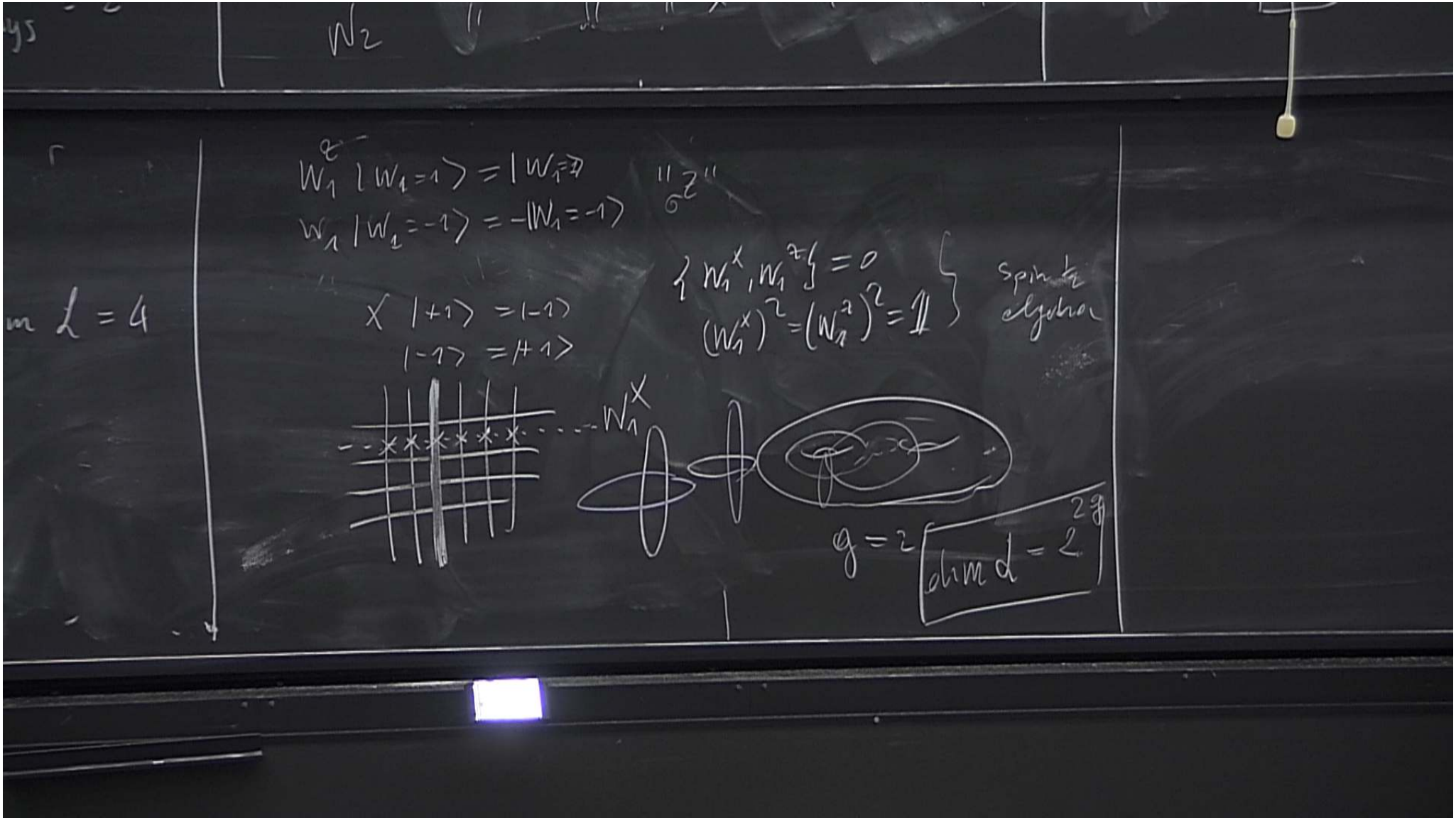


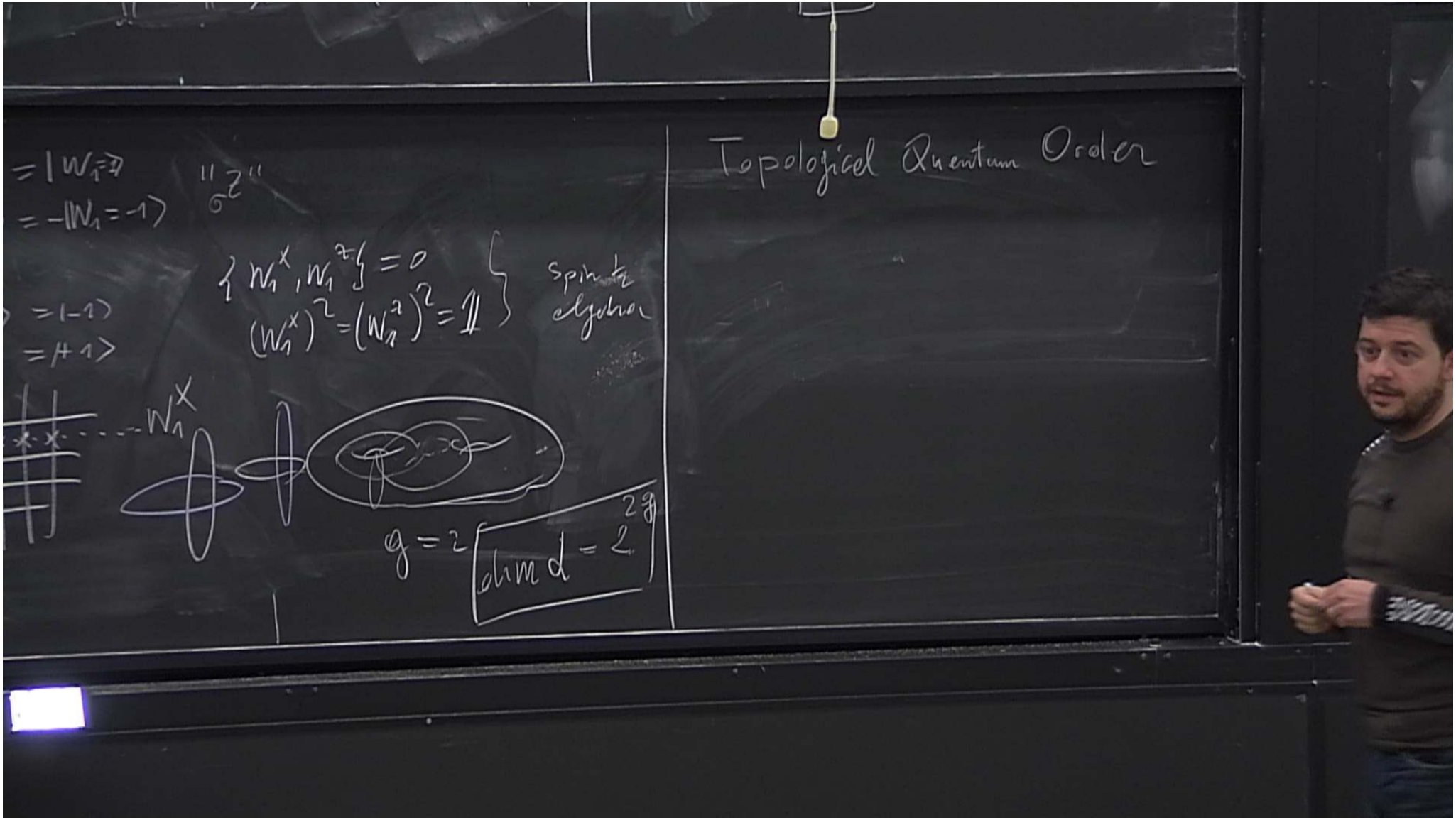
$g = 2$

$L = 4$


W_2
$W_1^x |W_1=1\rangle = |W_1=1\rangle$
$W_1 |W_1=-1\rangle = -|W_1=-1\rangle$
"σ^z"
$X |+1\rangle = |-1\rangle$
$|-1\rangle = |+1\rangle$
W_1^x
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin-1/2 algebra
$g = 2$
$L = 4$


W_2
$W_1^x |W_1=1\rangle = |W_1=1\rangle$
$W_1 |W_1=-1\rangle = -|W_1=-1\rangle$
"$\mathbb{Z}_2$"
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin-1/2 algebra
$X |+1\rangle = |-1\rangle$
$|-1\rangle = |+1\rangle$
W_1^x
$g = 2$
$\dim \mathcal{A} = 2^{2g}$


W_2
$W_1^x |W_1=1\rangle = |W_1=1\rangle$
$W_1 |W_1=-1\rangle = -|W_1=-1\rangle$
"$\mathbb{Z}_2$"
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin $\frac{1}{2}$ algebra
$X |+1\rangle = |-1\rangle$
$|-1\rangle = |+1\rangle$
W_1^x
$g = 2$
$\dim \mathcal{A} = 2^{2g}$
$m\ \mathcal{L} = 4$


$= |W_1 = 1\rangle$
$= -|W_1 = -1\rangle$
"σ^z"
$= |-1\rangle$
$= |+1\rangle$
$\{W_1^x, W_1^z\} = 0$
$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$
spin-½ algebra
W_1^x
$g = 2$
$\dim \mathcal{A} = 2^{2g}$
Topological Quantum Order

$= |W_1 = 1\rangle$ "Z"

$= -|W_1 = -1\rangle$ σ^z

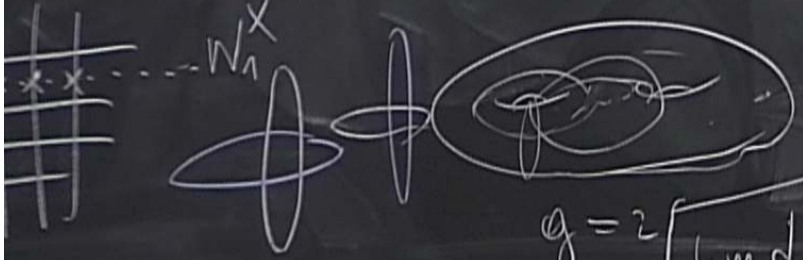
$= |-1\rangle$

$= |+1\rangle$

$\{W_1^x, W_1^z\} = 0$

$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$ } spin-½ algebra

W_1^x



$g = 2$ $\boxed{\dim \mathcal{A} = 2^{2g}}$

Topological Quantum Order

-1 Local Model $\mathcal{H} = \otimes_i \mathcal{H}_i$

$= |W_1 = 1\rangle$ "Z"

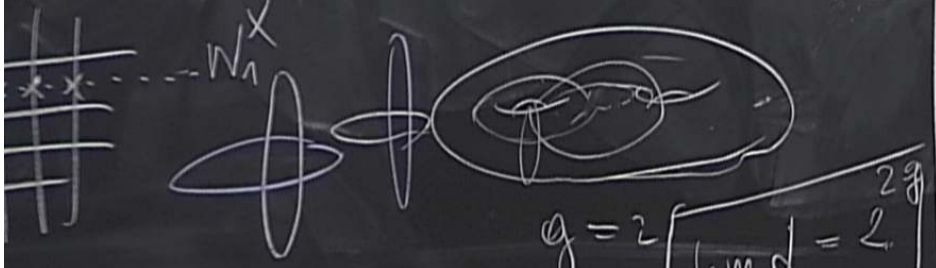
$= -|W_1 = -1\rangle$

$= |-1\rangle$

$= |+1\rangle$

$\{W_1^x, W_1^z\} = 0$

$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$ } spin-$\frac{1}{2}$ algebra



$g = 2$ $\boxed{\dim \mathcal{A} = 2^{2g}}$

Topological Quantum Order

-1 Local Model $\mathcal{H} = \otimes_i \mathcal{H}_i$

Gauge Theory $\equiv$ emerge

$\mathcal{H}_{TOT}$ $H = -U \sum_s A_s - J \sum_p B_p$

$= |W_1 = 1\rangle$ "Z"

$= -|W_1 = -1\rangle$ σ^z

$= |-1\rangle$

$= |+1\rangle$

$\{W_1^x, W_1^z\} = 0$

$(W_1^x)^2 = (W_1^z)^2 = \mathbb{1}$ } spin-$\frac{1}{2}$ algebra

W_1^x

$g = 2$ $\boxed{\dim \mathcal{A} = 2^{2g}}$

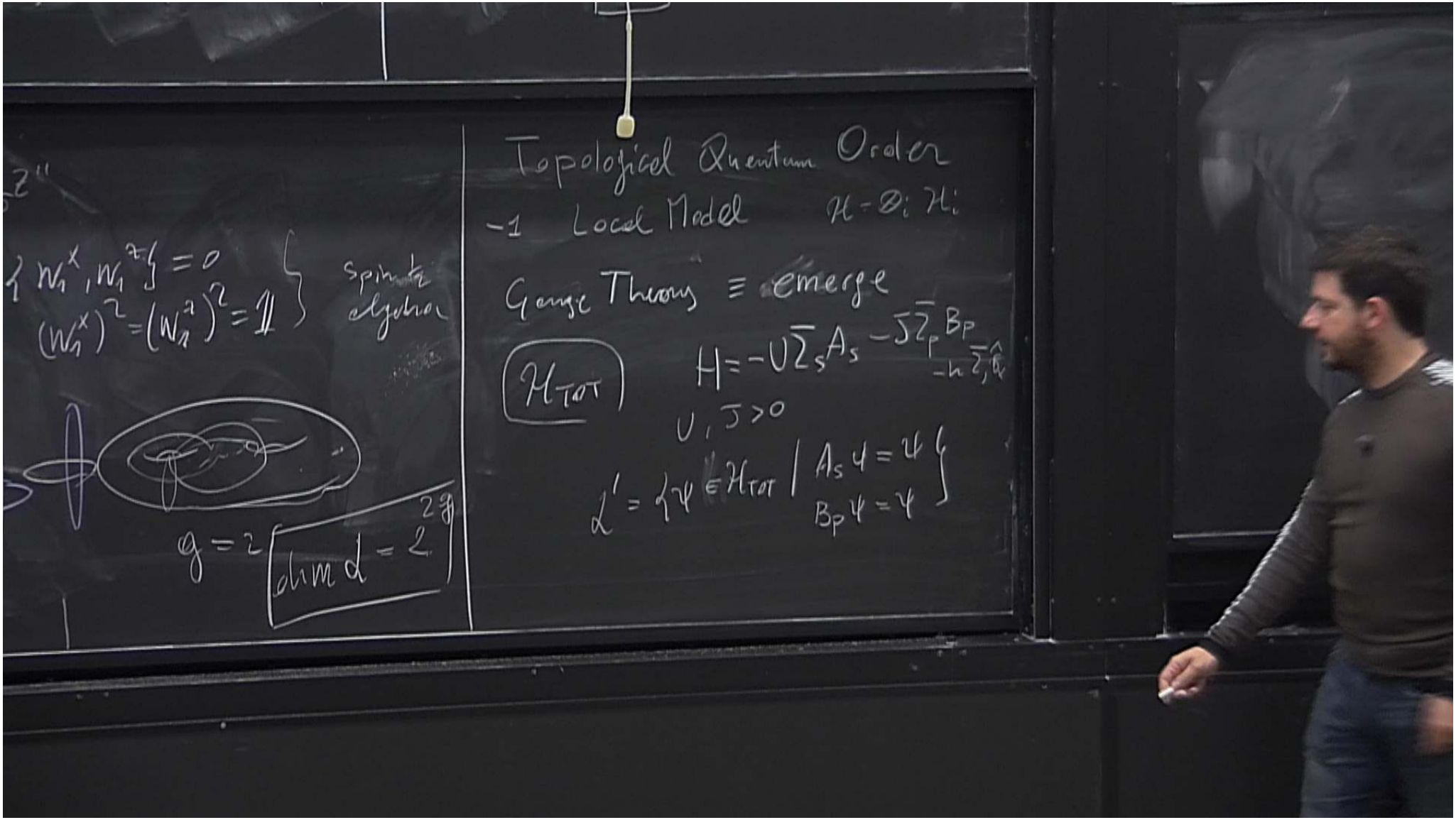
Topological Quantum Order

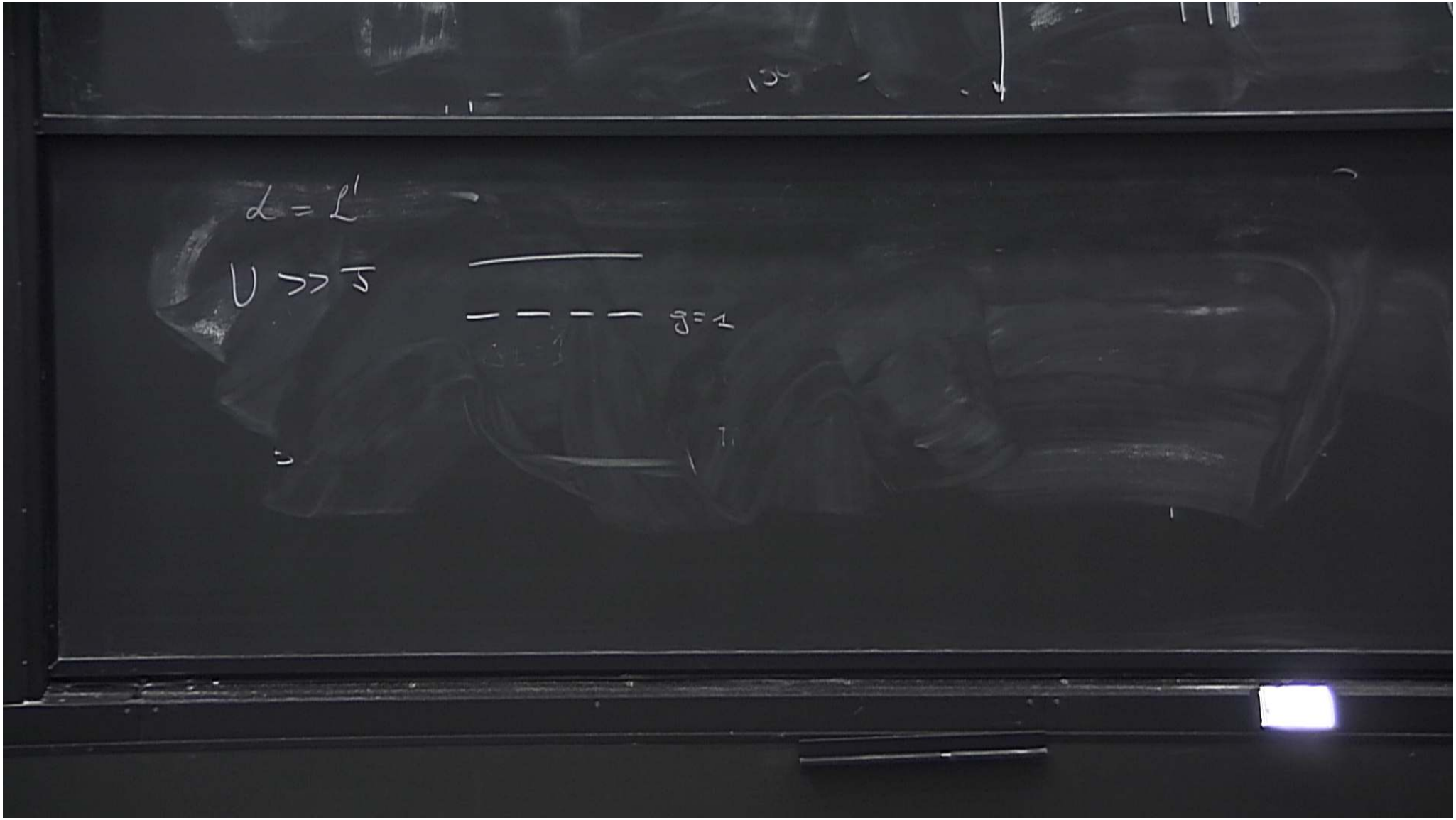
-1 Local Model $\mathcal{H} = \otimes_i \mathcal{H}_i$

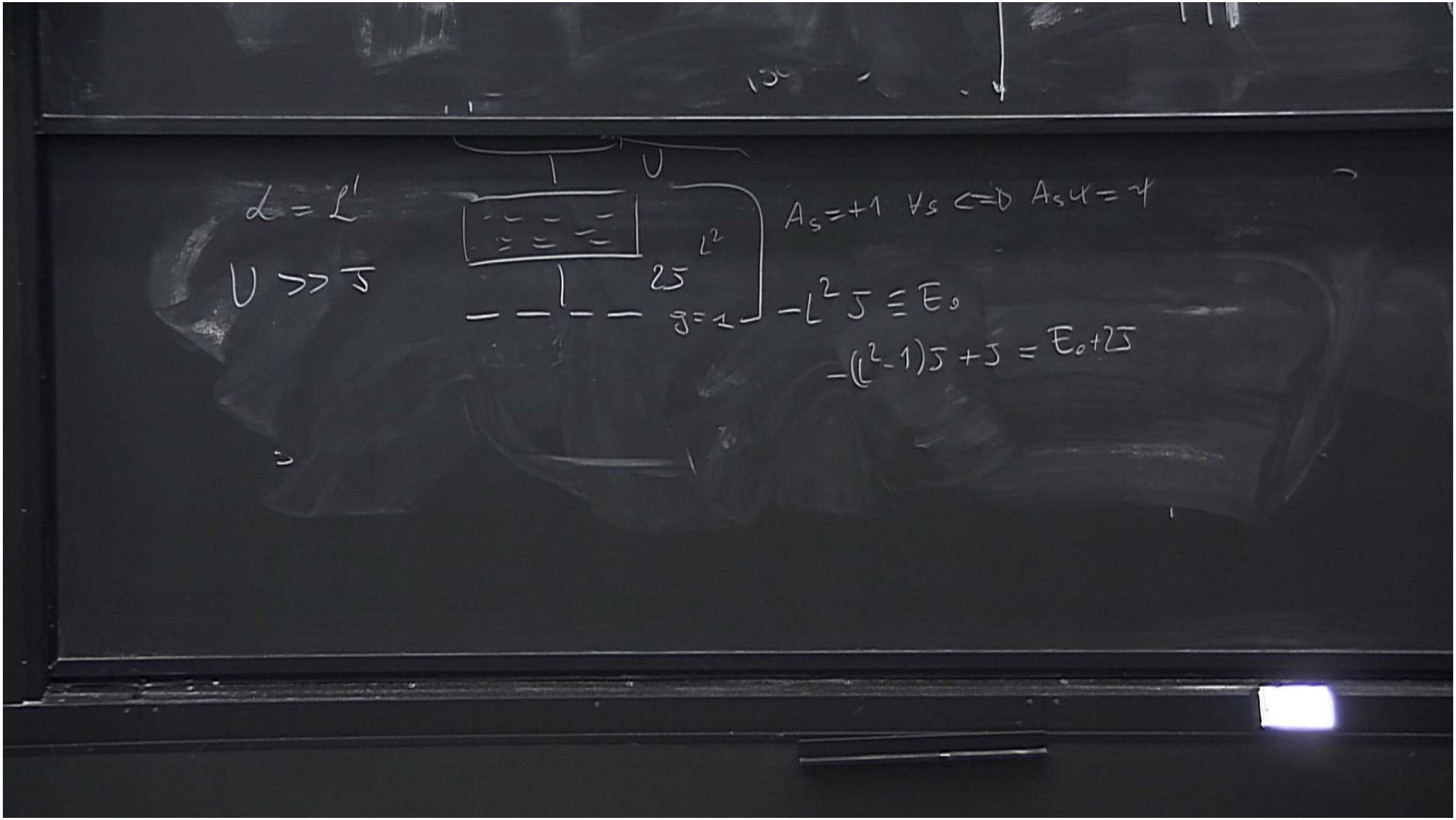
Gauge Theory ≡ emerge

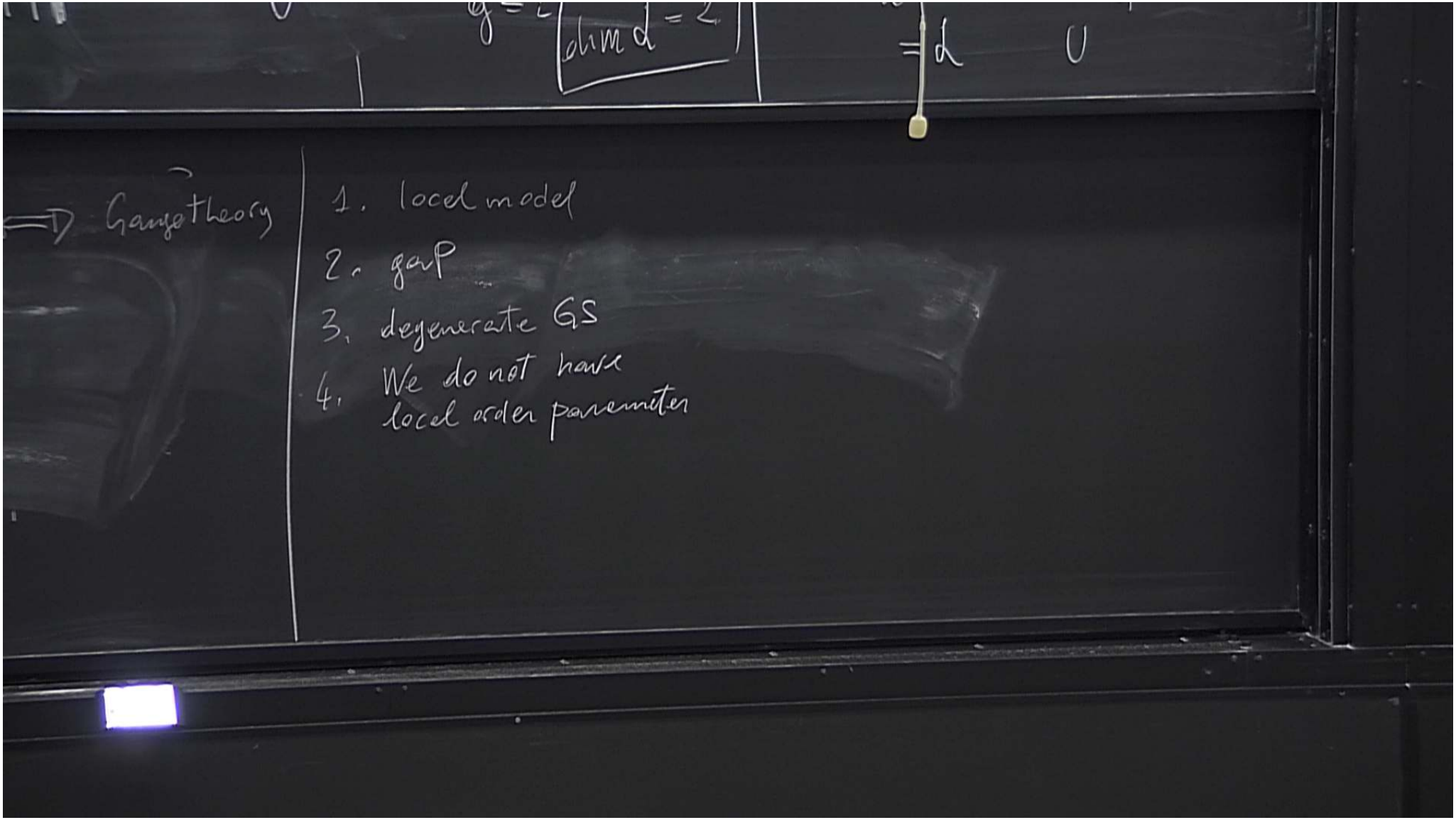
$\mathcal{H}_{TOT}$ $H = -U \sum_s A_s - J \sum_p B_p$

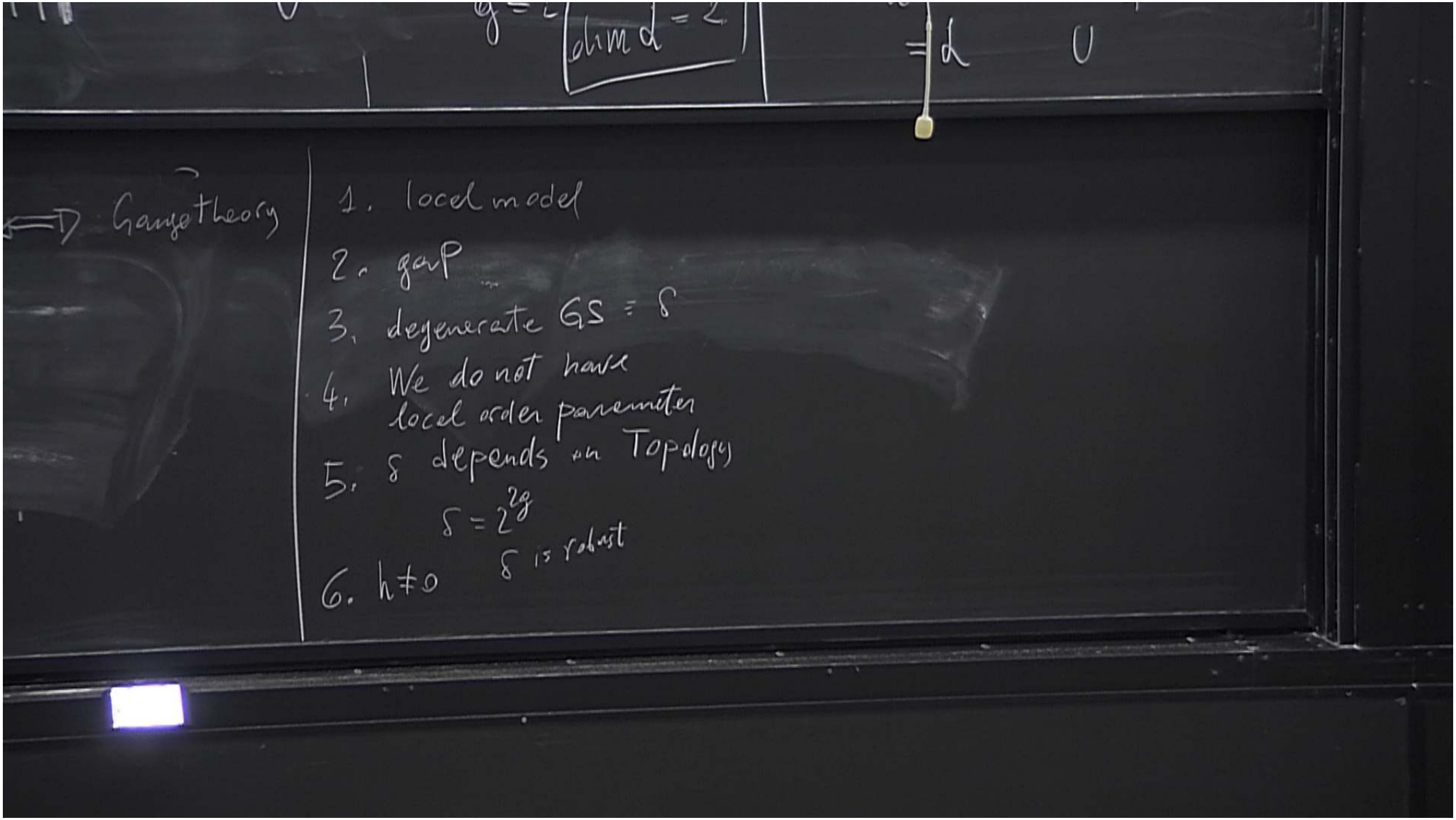
$U, J > 0$

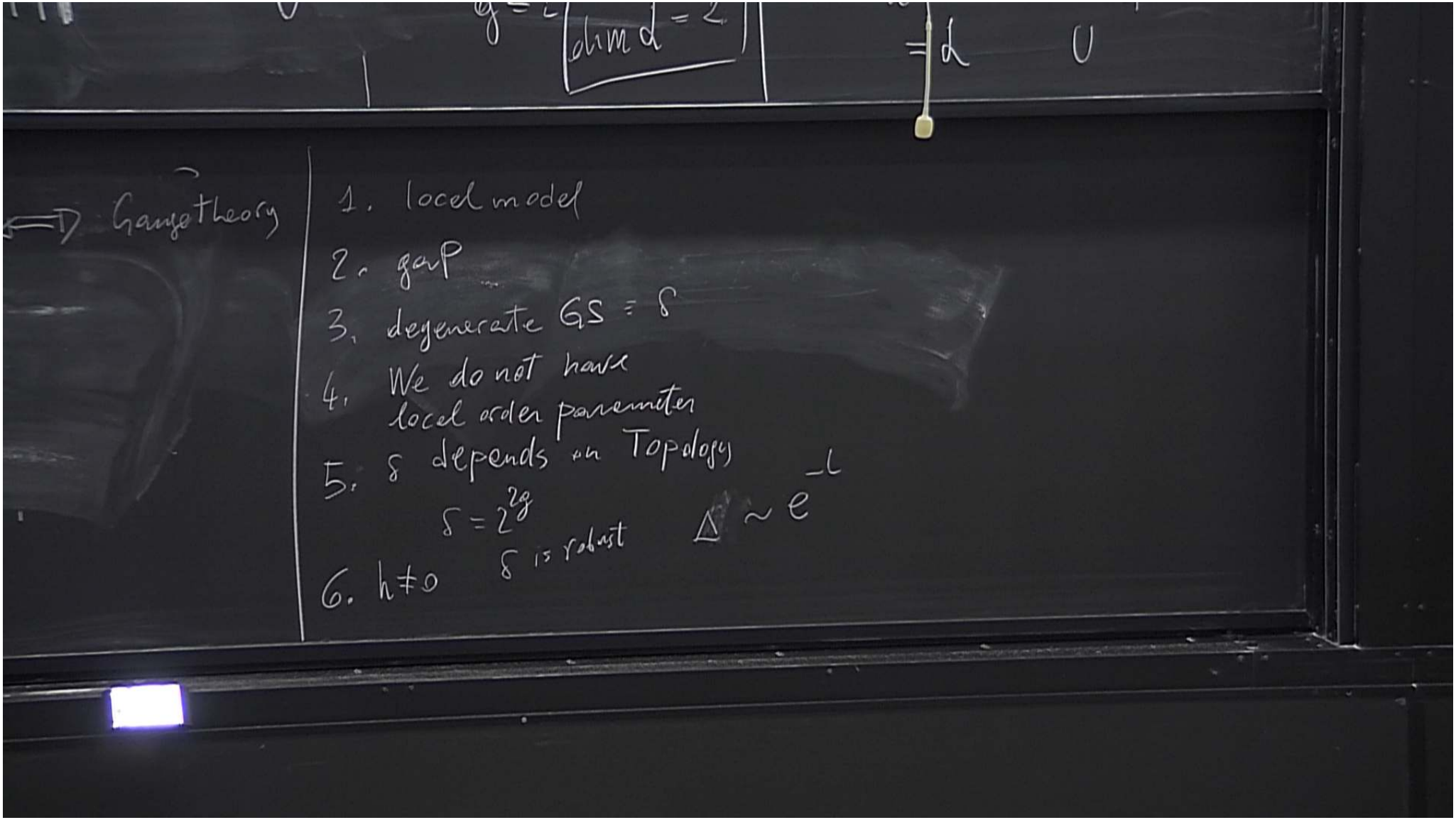

Topological Quantum Order
-1 Local Model
Gauge Theory ≡ emerge
spin-1/2 algebra
g = 2
dim d = 2^{2g}

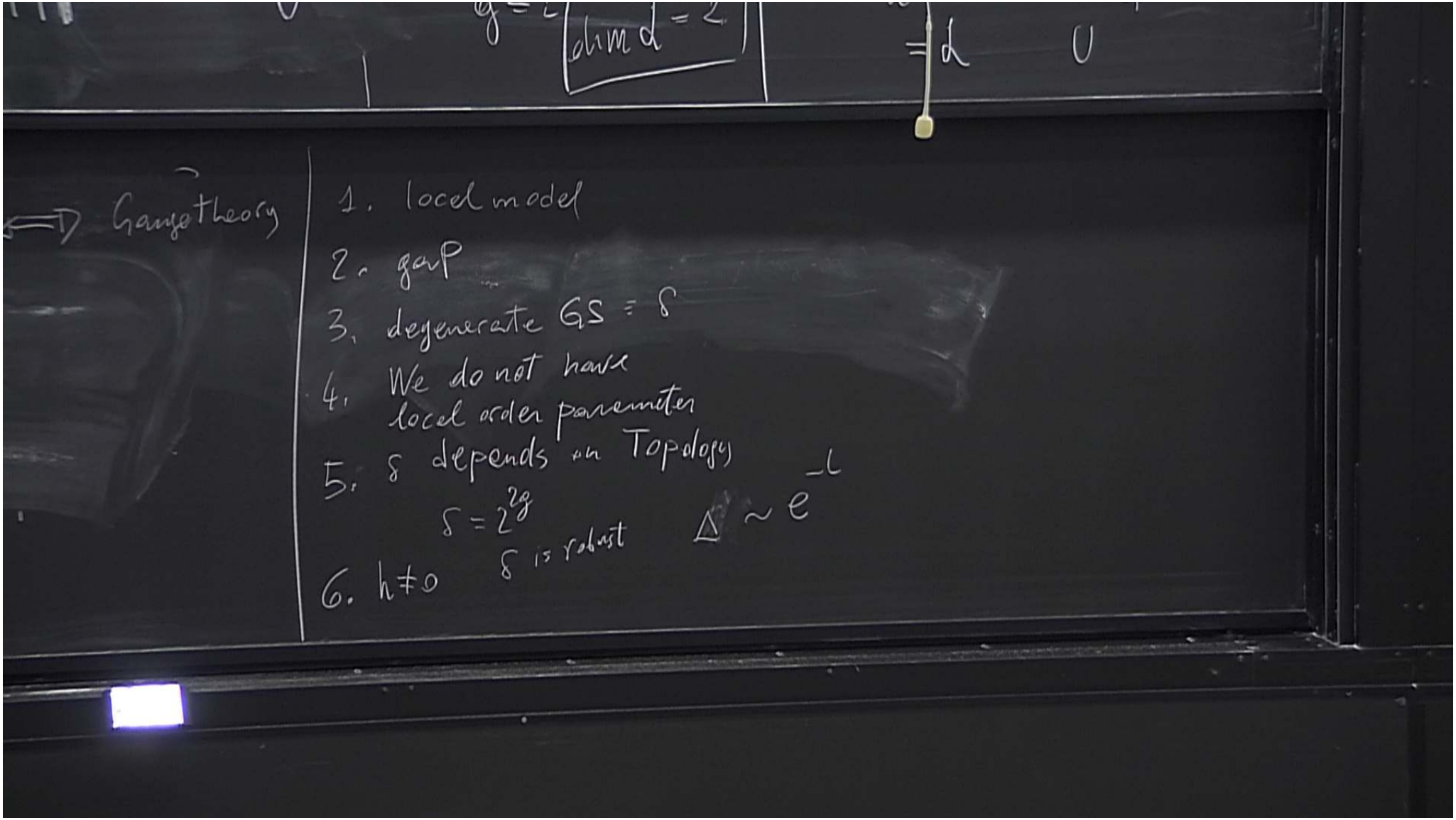

a = L'
U >> J
g = 1

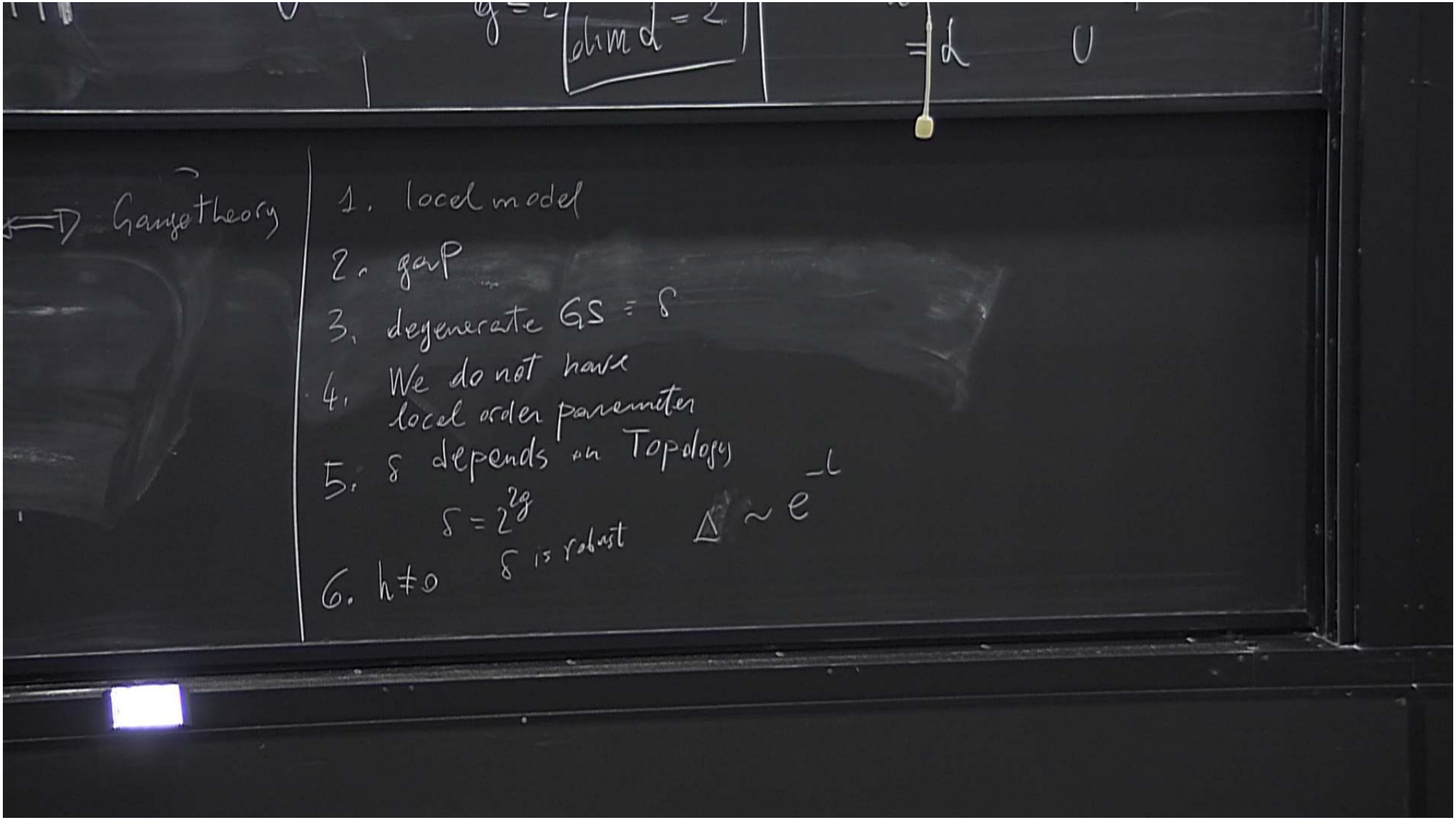

$L = L'$
$U >> J$
L^2
$2J$
$g=1$
$A_s = +1 \quad \forall s \subset \Omega \quad A_s \psi = \psi$
$-L^2 J \equiv E_0$
$-(L^2-1)J + J = E_0 + 2J$

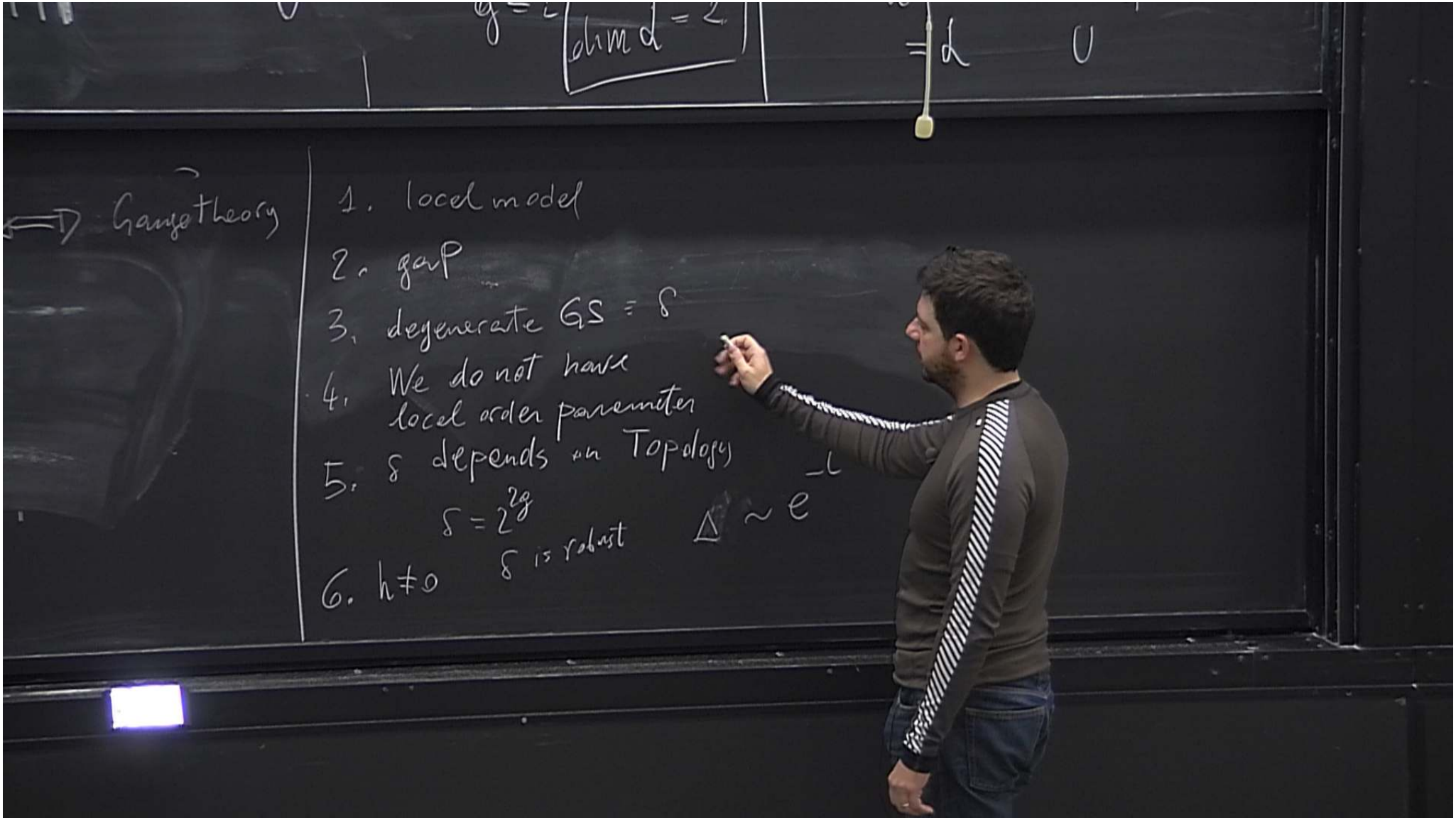

$\dim \mathcal{A} = 2$
$= \mathcal{A}$ U
$\Rightarrow$ Gauge theory
1. local model
2. gap
3. degenerate GS
4. We do not have local order parameter

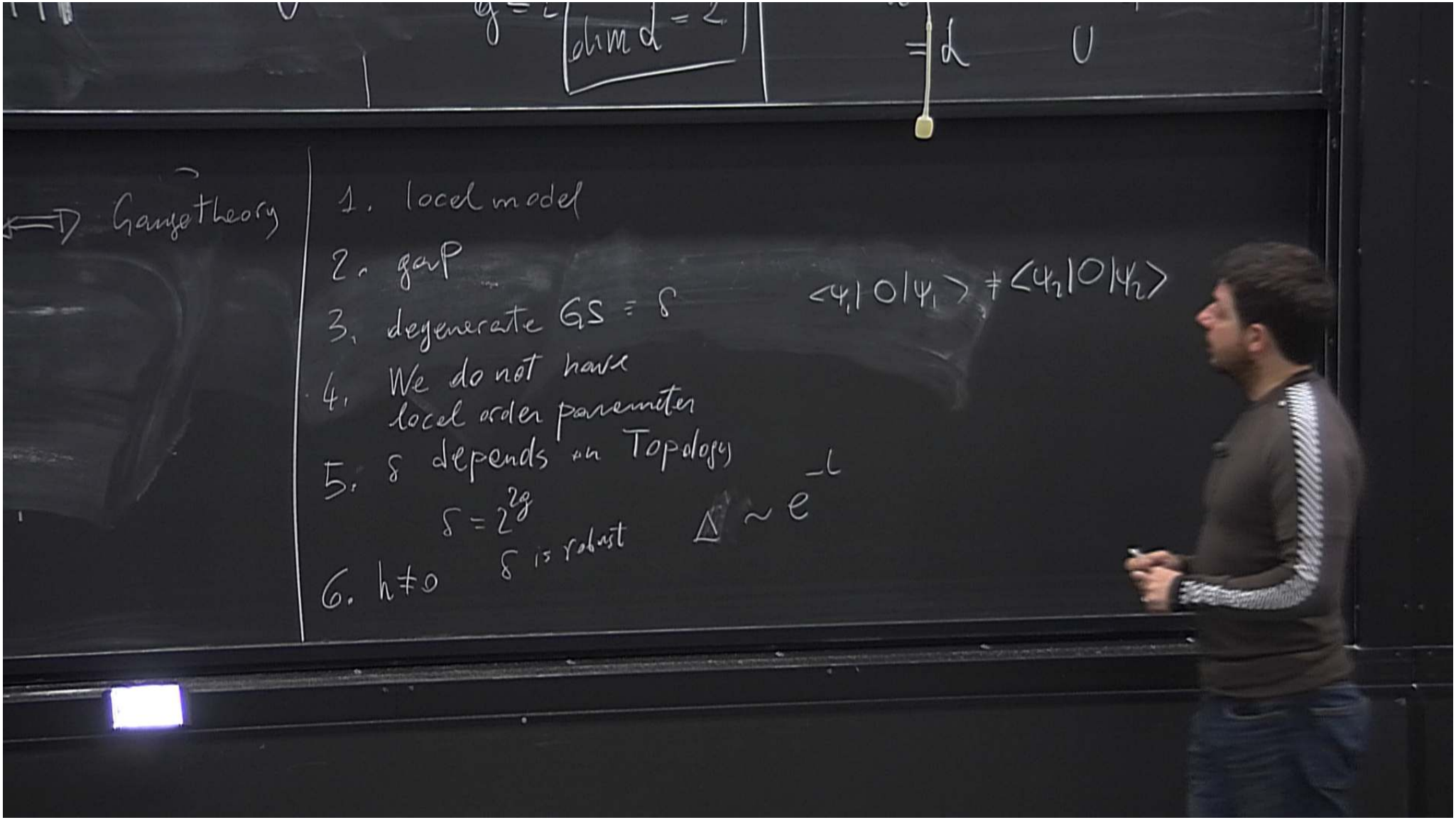

dim d = 2
d
U
Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
6. h≠0 δ is robust

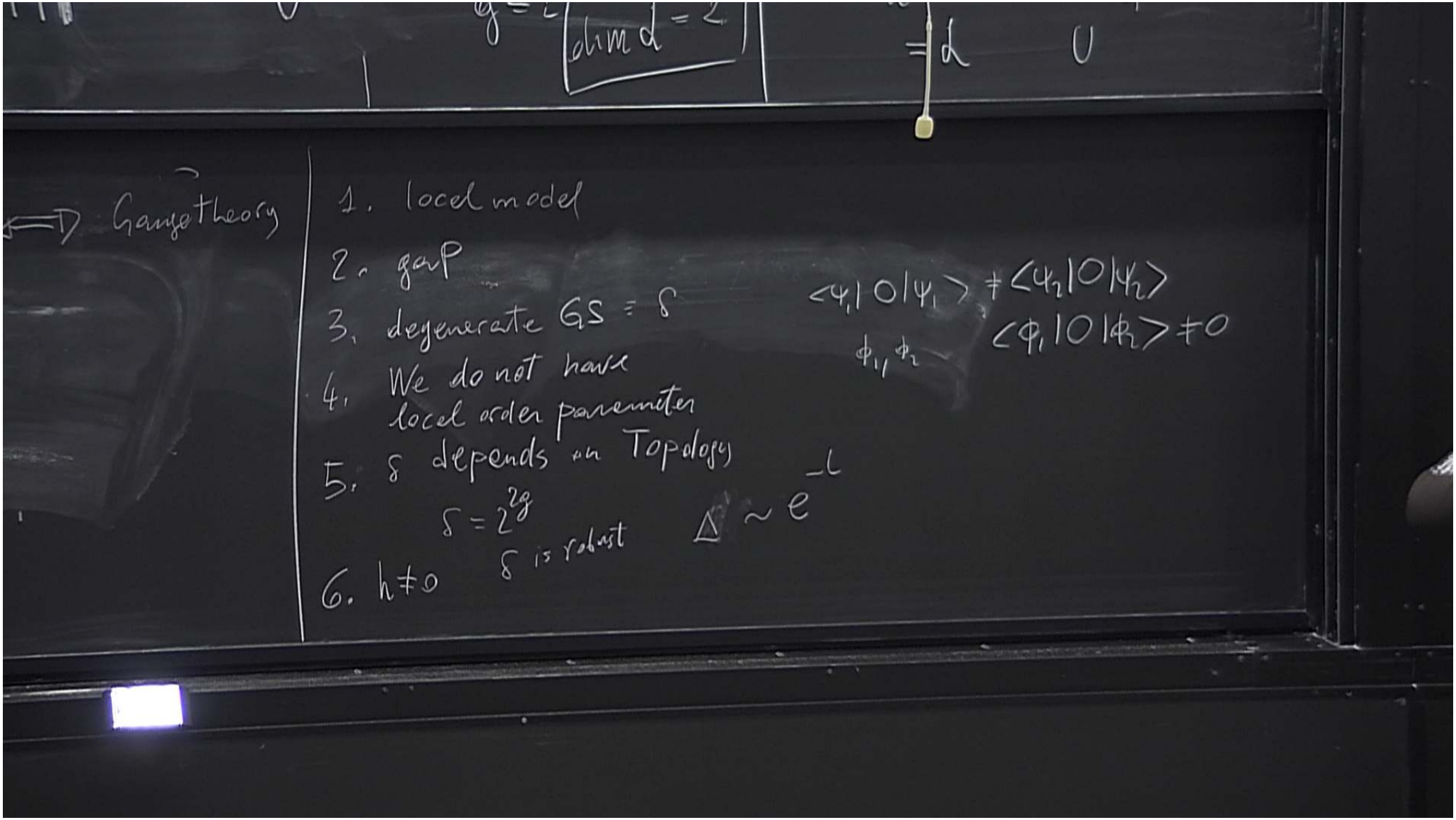

dim d = 2
⟹ Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
$\Delta \sim e^{-L}$
6. h ≠ 0 δ is robust

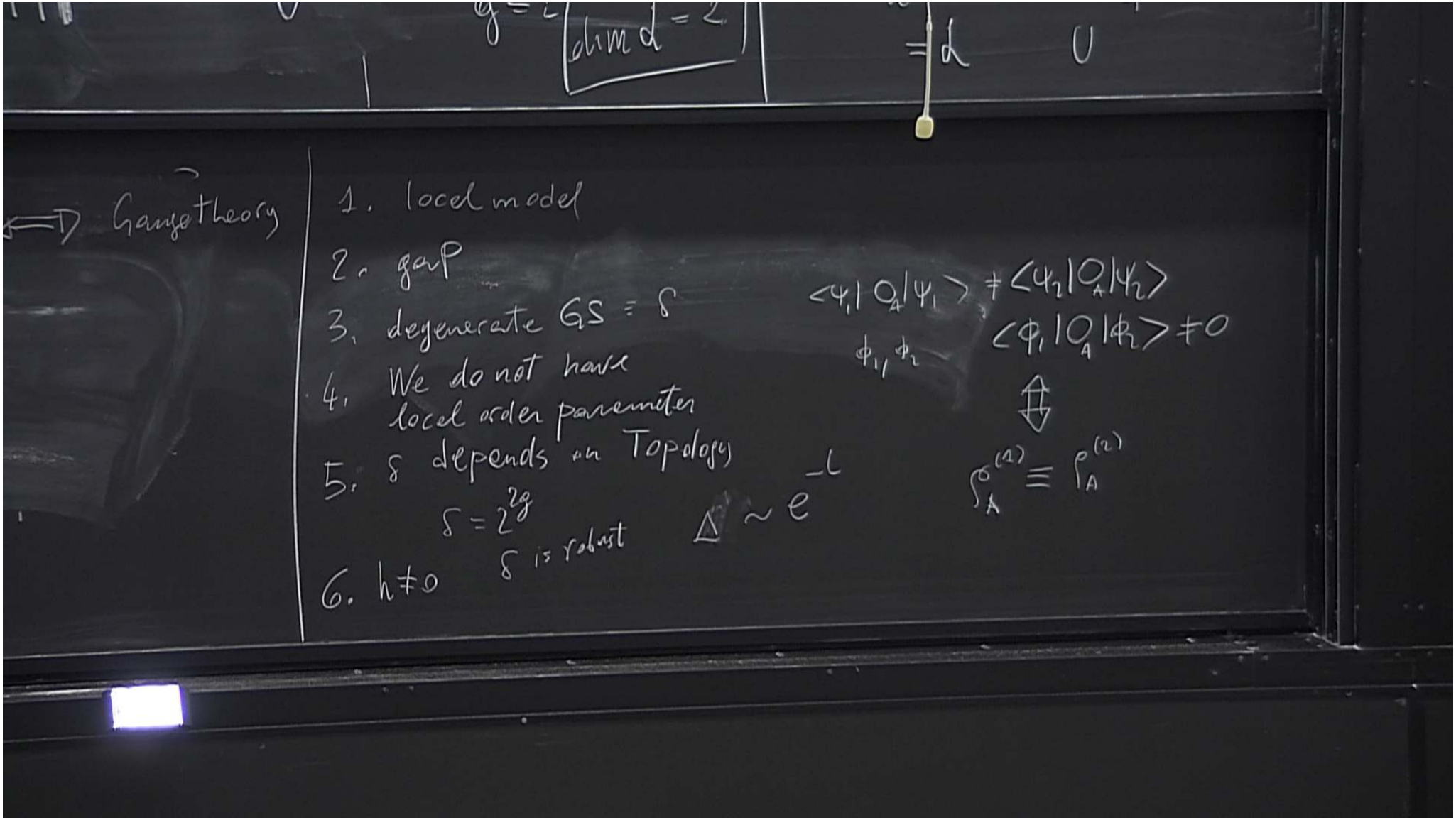

$\dim \mathcal{A} = 2$
$= \mathcal{A}$ U
$\Leftarrow$ Gauge theory
1. local model
2. gap
3. degenerate GS $= \delta$
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
$\Delta \sim e^{-L}$
6. $h \neq 0$ δ is robust


dim d = 2
Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
$\Delta \sim e^{-L}$
6. h ≠ 0 δ is robust

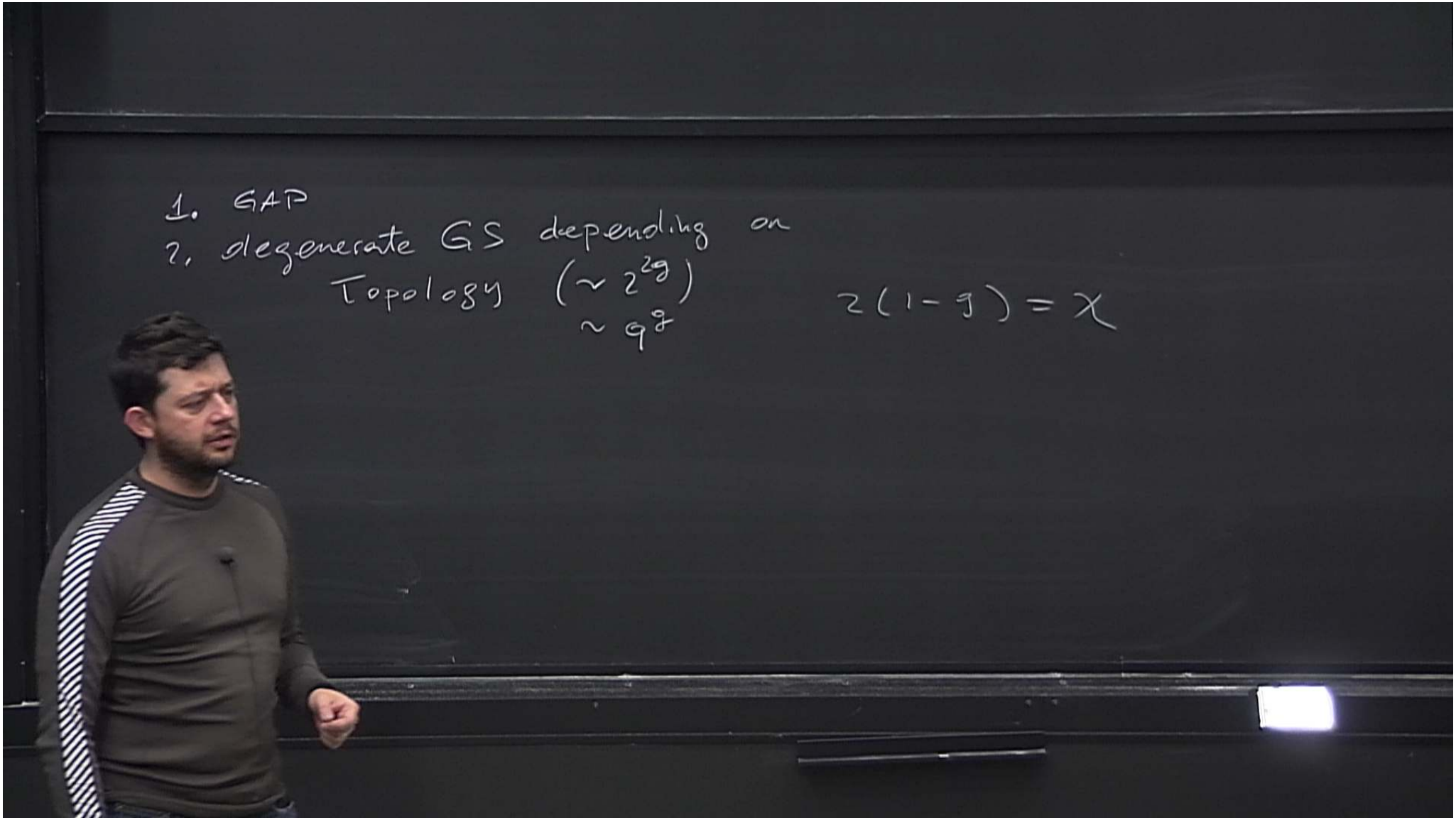

dim d = 2
Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
$\Delta \sim e^{-L}$
6. h≠0 δ is robust


dim d = 2
⟹ Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
$\delta = 2^{2g}$
6. h≠0 δ is robust
$\langle \psi_1 | O | \psi_1 \rangle \neq \langle \psi_2 | O | \psi_2 \rangle$
$\Delta \sim e^{-L}$


dim d = 2
⟹ Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
δ = 2^{2g}
6. h ≠ 0 δ is robust
⟨ψ1|O|ψ1⟩ ≠ ⟨ψ2|O|ψ2⟩
Φ1, Φ2
⟨Φ1|O|Φ2⟩ ≠ 0
Δ ~ e^{-L}


dim d = 2
⟸ Gauge theory
1. local model
2. gap
3. degenerate GS = δ
4. We do not have local order parameter
5. δ depends on Topology
δ = 2^{2g}
6. h ≠ 0 δ is robust
Δ ~ e^{-L}
⟨ψ1|O_A|ψ1⟩ ≠ ⟨ψ2|O_A|ψ2⟩
Φ1, Φ2
⟨Φ1|O_A|Φ2⟩ ≠ 0
ρ_A^(1) ≡ ρ_A^(2)

Topological Quantum Order
Gauge Theory = emerge
Hint
gap
degenerate GS = 1
We do not know
local order parameter
S depends on Topology


1. GAP
2. degenerate GS depending on
Topology ($\sim 2^{2g}$)
$\sim g^g$
$2(1-g) = \chi$

1. GAP
2. degenerate GS depending on Topology $(\sim 2^{2g} \sim g^g$
$2(1-g) = \chi$
3.