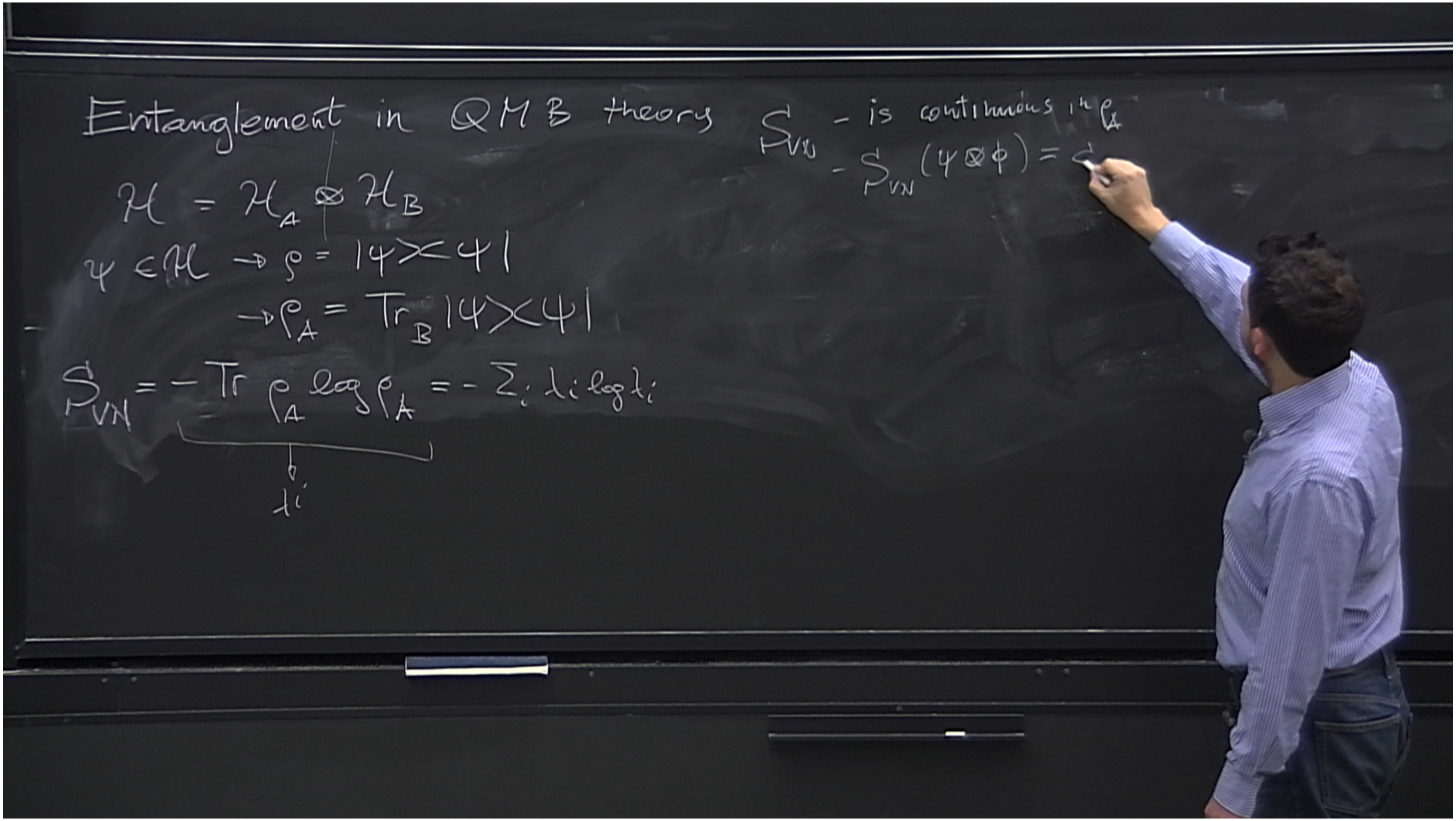
Title: Condensed Matter (Review) - Lecture 9

Date: Jan 12, 2012 10:15 AM

URL: http://pirsa.org/12010093

Abstract:



Entanglement in QMB theory

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

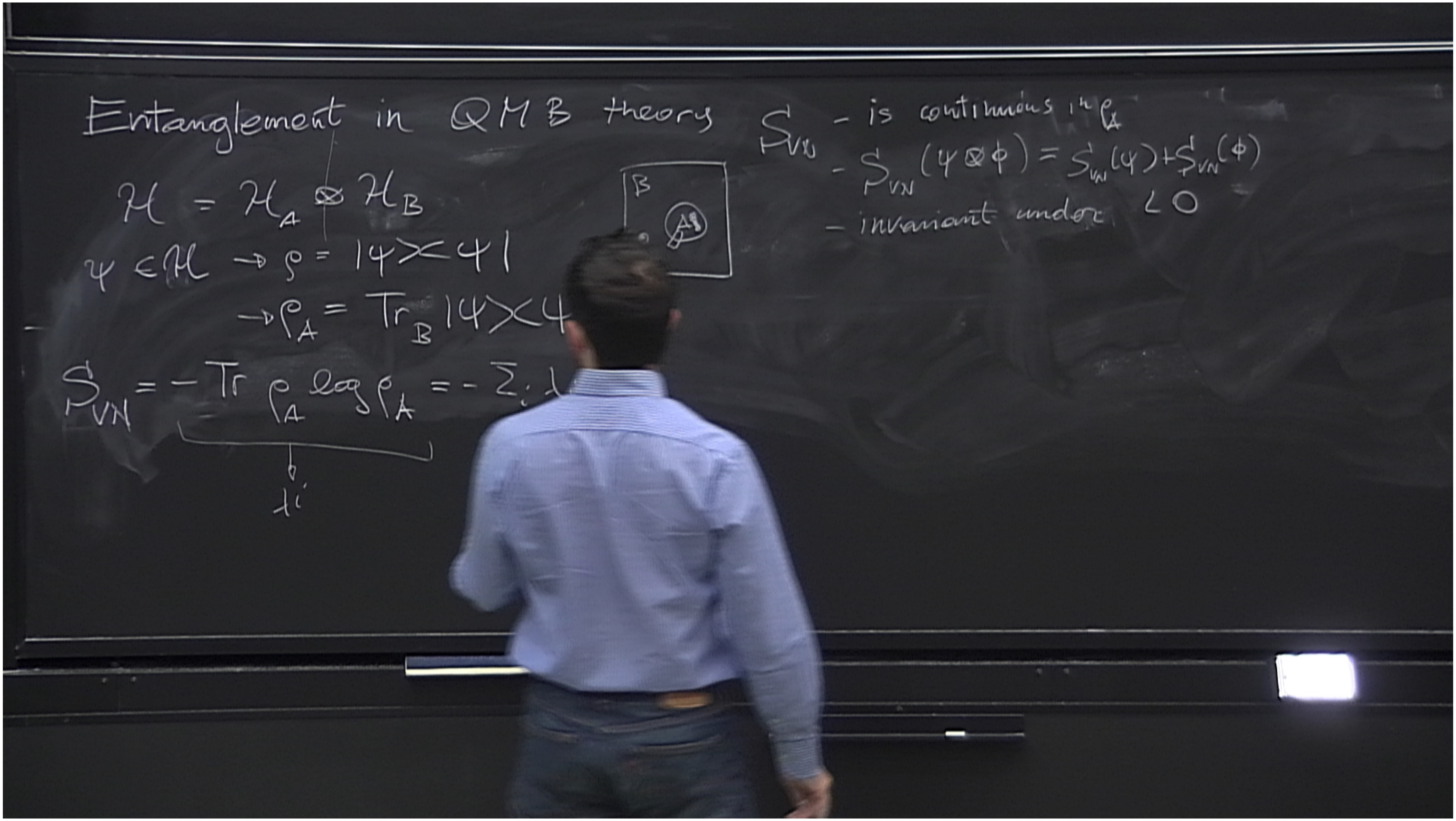
$\psi \in \mathcal{H} \to \rho = |\psi\rangle\langle\psi|$

$\to \rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$

$S_{VN} = -\underbrace{\mathrm{Tr}\, \rho_A \log \rho_A}_{\lambda_i} = -\sum_i \lambda_i \log \lambda_i$

S_{VN} - is continuous in ρ_A

- $S_{VN}(\psi \otimes \phi) = S$



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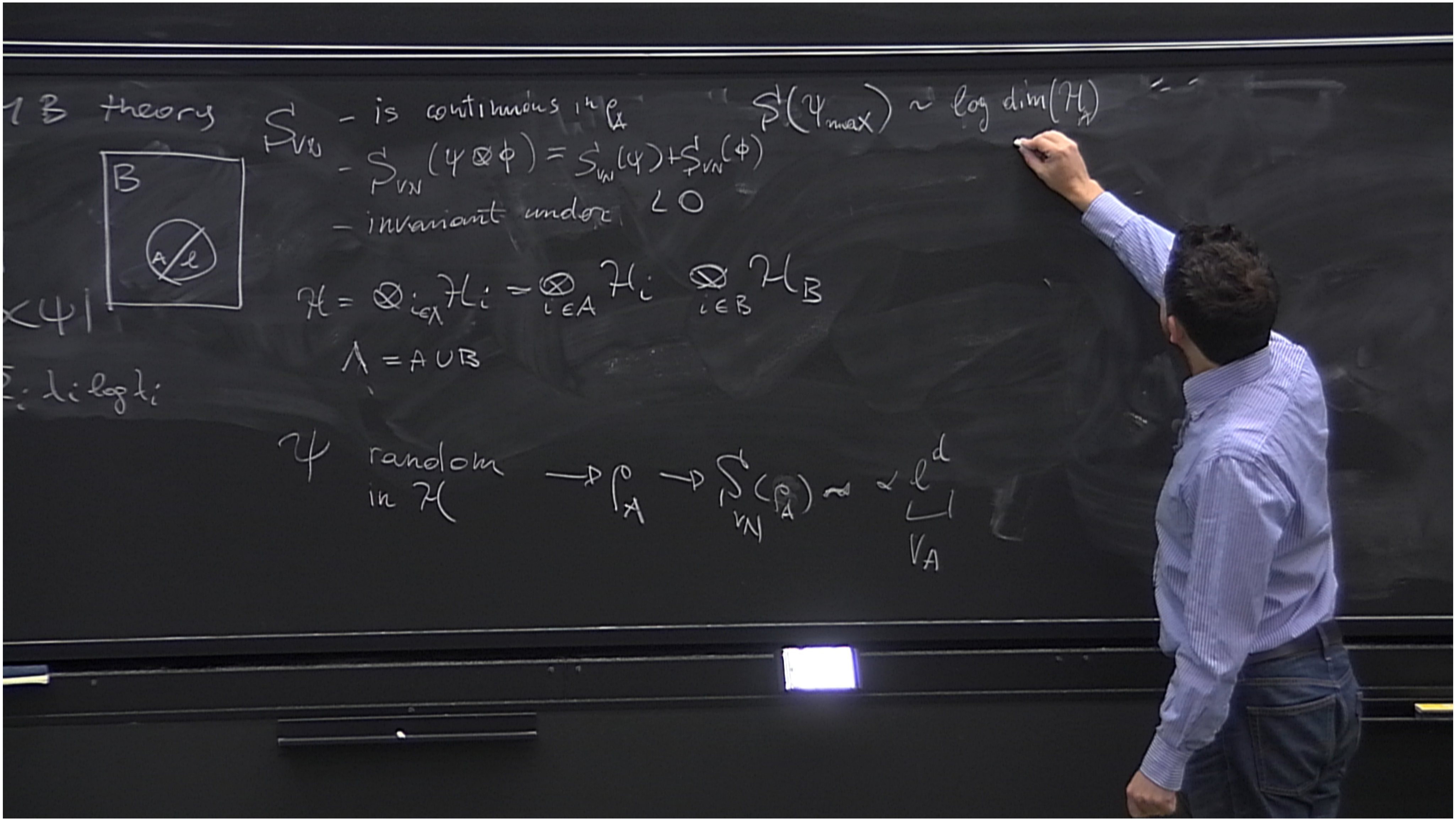
$S_{VN} = \underbrace{-\mathrm{Tr}\, \rho_A \log \rho_A}_{\lambda_i} = -\sum_i \lambda$

S_{VN} - is continuous in ρ_A

- $S_{VN}(\psi \otimes \phi) = S_{VN}(\psi) + S_{VN}(\phi)$

- invariant under LO



1 B theory

$\langle\Psi|$

$\sum_i \lambda_i \log \lambda_i$

S_{vN} – is continuous in ρ_A

– $S_{vN}(\psi\otimes\phi) = S_{vN}(\psi) + S_{vN}(\phi)$

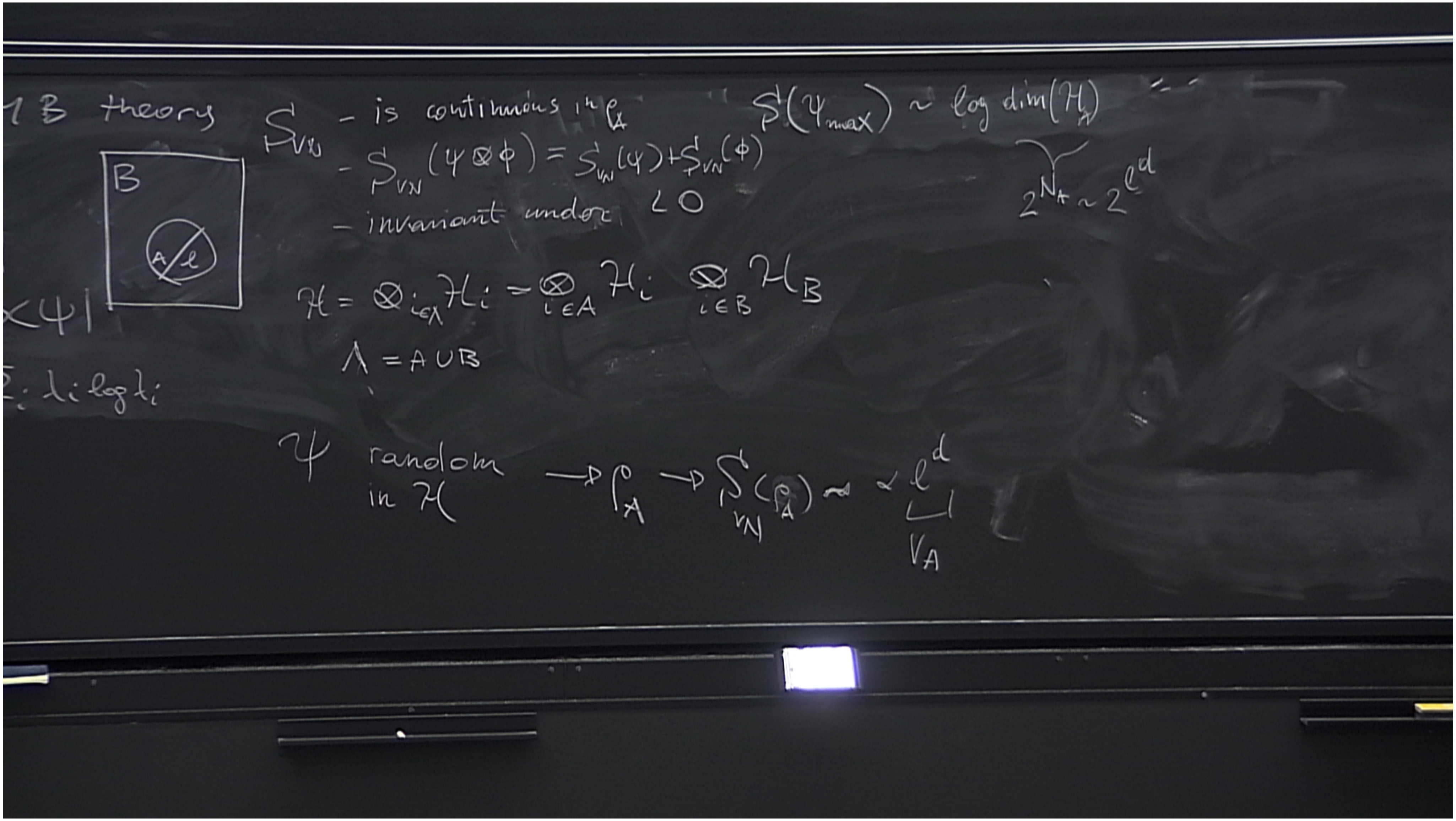
– invariant under LO

$S(\psi_{max}) \sim \log \dim(\mathcal{H}_A)$

$\mathcal{H} = \otimes_{i\in\Lambda}\mathcal{H}_i = \bigotimes_{i\in A}\mathcal{H}_i \bigotimes_{i\in B}\mathcal{H}_B$

$\Lambda = A\cup B$

ψ random in $\mathcal{H}$ $\longrightarrow \rho_A \longrightarrow S_{vN}(\rho_A) \sim \propto \underbrace{\ell^d}_{V_A}$



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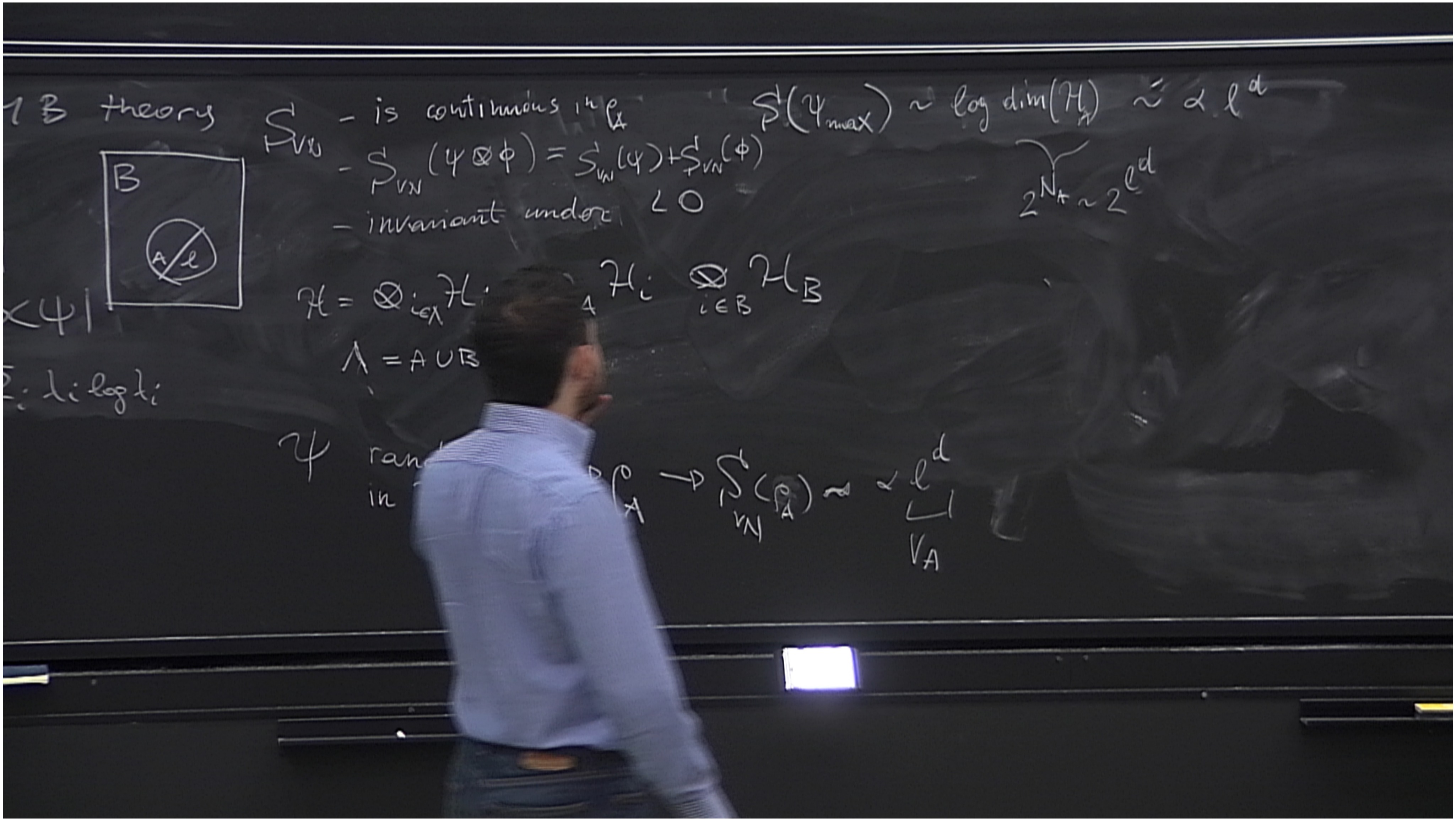

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S_{VN} – is continuous in ρ_A

– $S_{VN}(\psi\otimes\phi) = S_{VN}(\psi) + S_{VN}(\phi)$

– invariant under LO

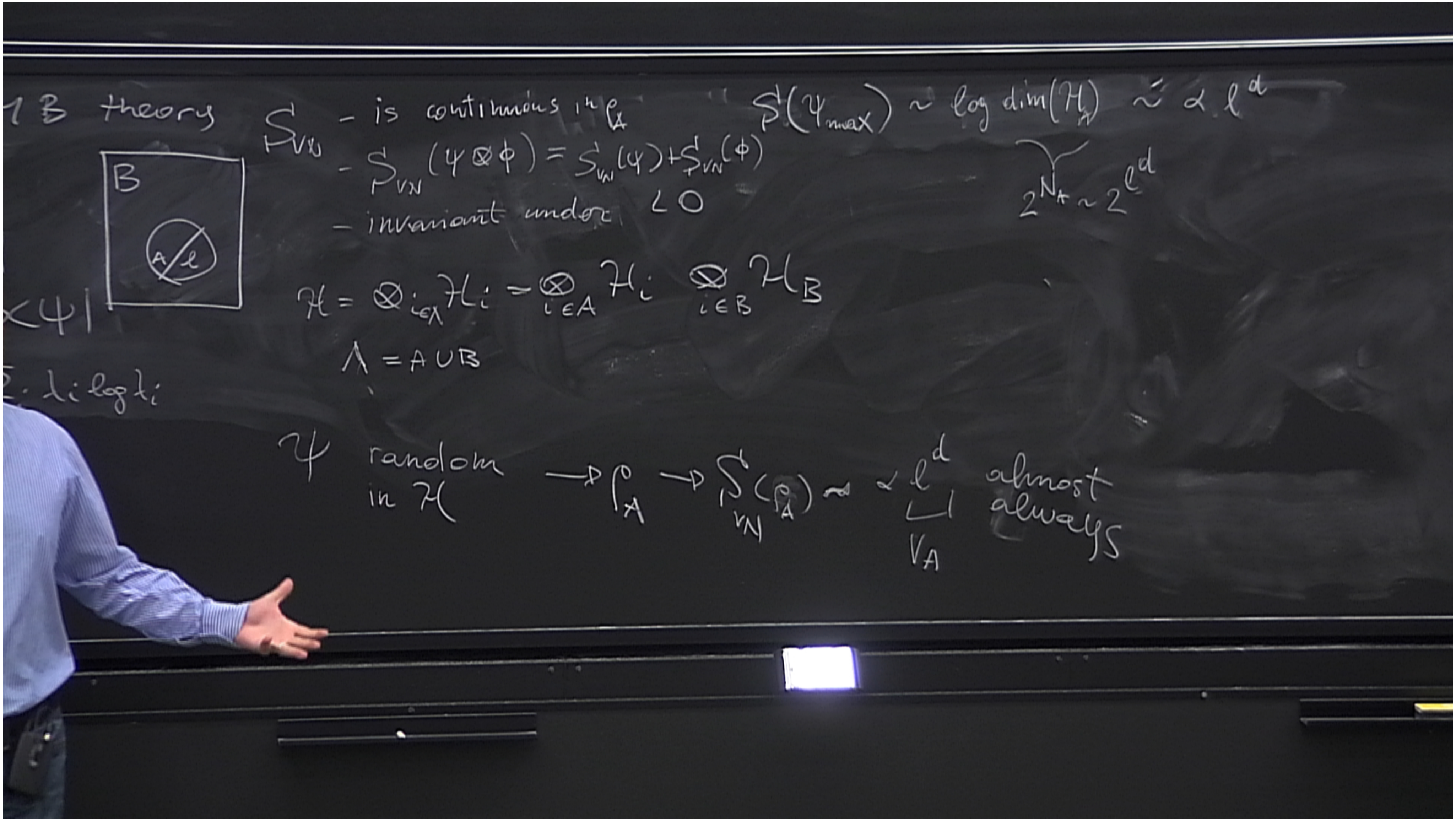
$S(\psi_{max}) \sim \underbrace{\log \dim(\mathcal{H}_A)}_{2^{N_A} \sim 2^{\ell^d}} \simeq \alpha\, \ell^d$

$\mathcal{H} = \otimes_{i\in\Lambda}\mathcal{H}_i \ \ \ _A \mathcal{H}_i \ \otimes_{i\in B} \mathcal{H}_B$

$\Lambda = A \cup B$

ψ ran
in

$\rho_A \to S_{VN}(\rho_A) \sim \alpha \underbrace{\ell^d}_{V_A}$



1 B theory

S_{VN} - is continuous in ρ_A

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$$S(\psi_{max}) \sim \underbrace{\log \dim(\mathcal{H}_A)}_{2^{N_A} \sim 2^{\ell^d}} \tilde{\sim} \alpha\, \ell^d$$

$\langle\psi|$

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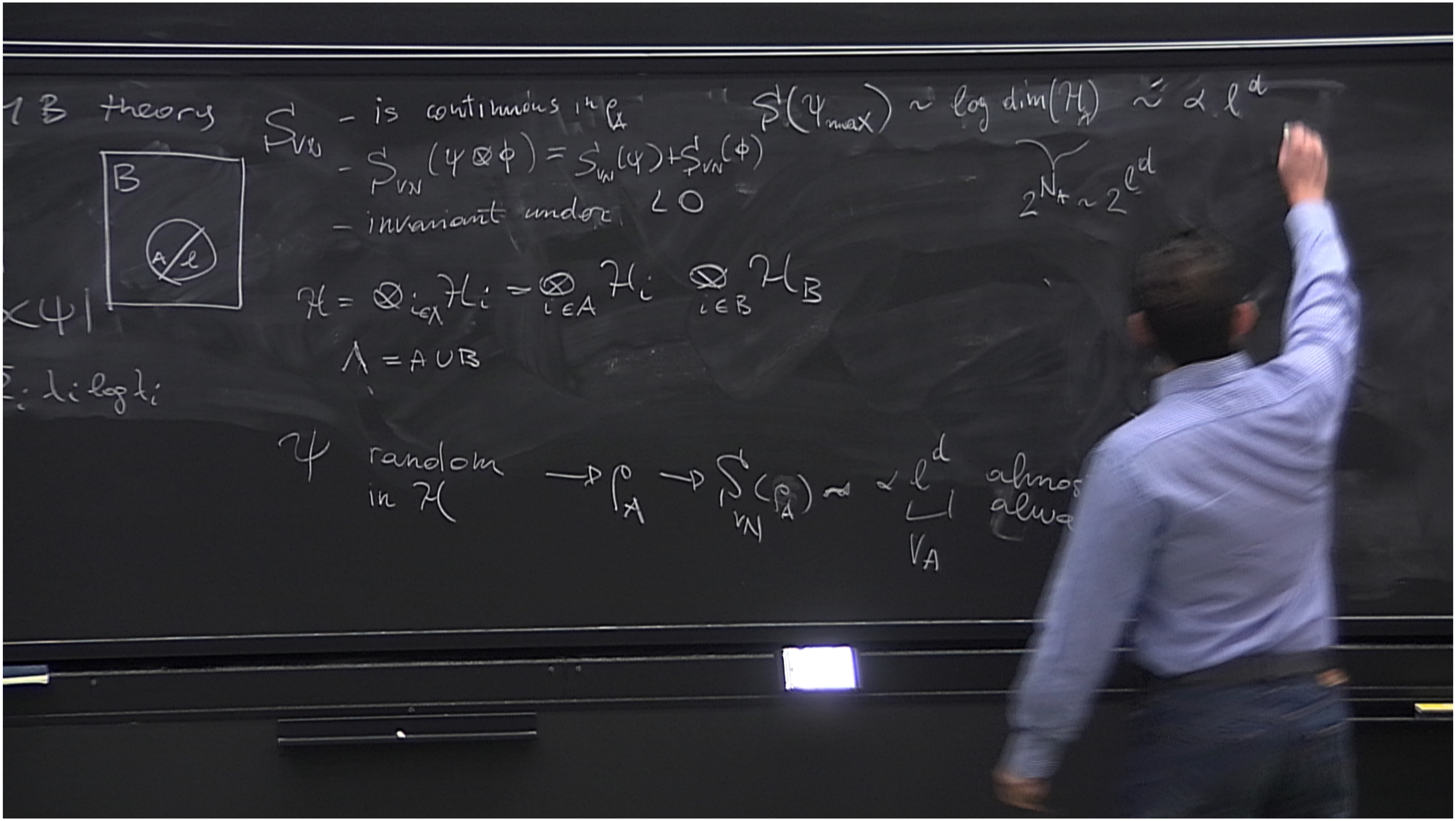
$\Lambda = A \cup B$

$\Sigma_i \lambda_i \log \lambda_i$

$$\psi \text{ random in } \mathcal{H} \longrightarrow \rho_A \longrightarrow S_{VN}(\rho_A) \sim \alpha \underbrace{\ell^d}_{V_A} \text{ almost always}$$



1 B theory

S_{VN} - is continuous in ρ_A

- $S_{VN}(\psi\otimes\phi) = S_{VN}(\psi)+S_{VN}(\phi)$

- invariant under LO

$S(\psi_{max}) \sim \log\dim(\mathcal{H}_A) \simeq \alpha\, l^d$

$$\underbrace{\log\dim(\mathcal{H}_A)}_{2^{N_A}\sim 2^{l^d}}$$

$\langle\Psi|$

$\mathcal{H} = \otimes_{i\in\Lambda}\mathcal{H}_i = \bigotimes_{i\in A}\mathcal{H}_i \bigotimes_{i\in B}\mathcal{H}_B$

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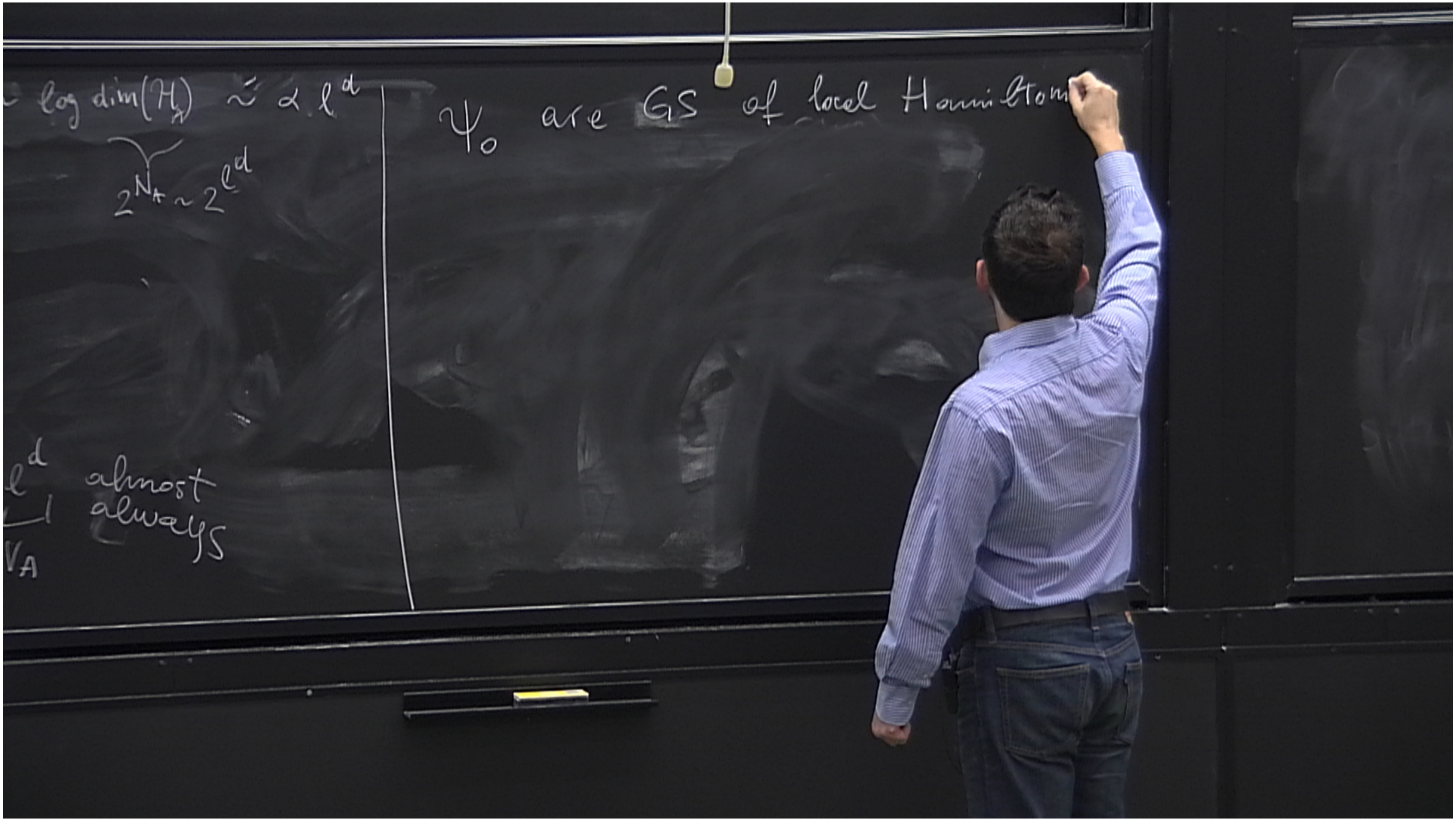
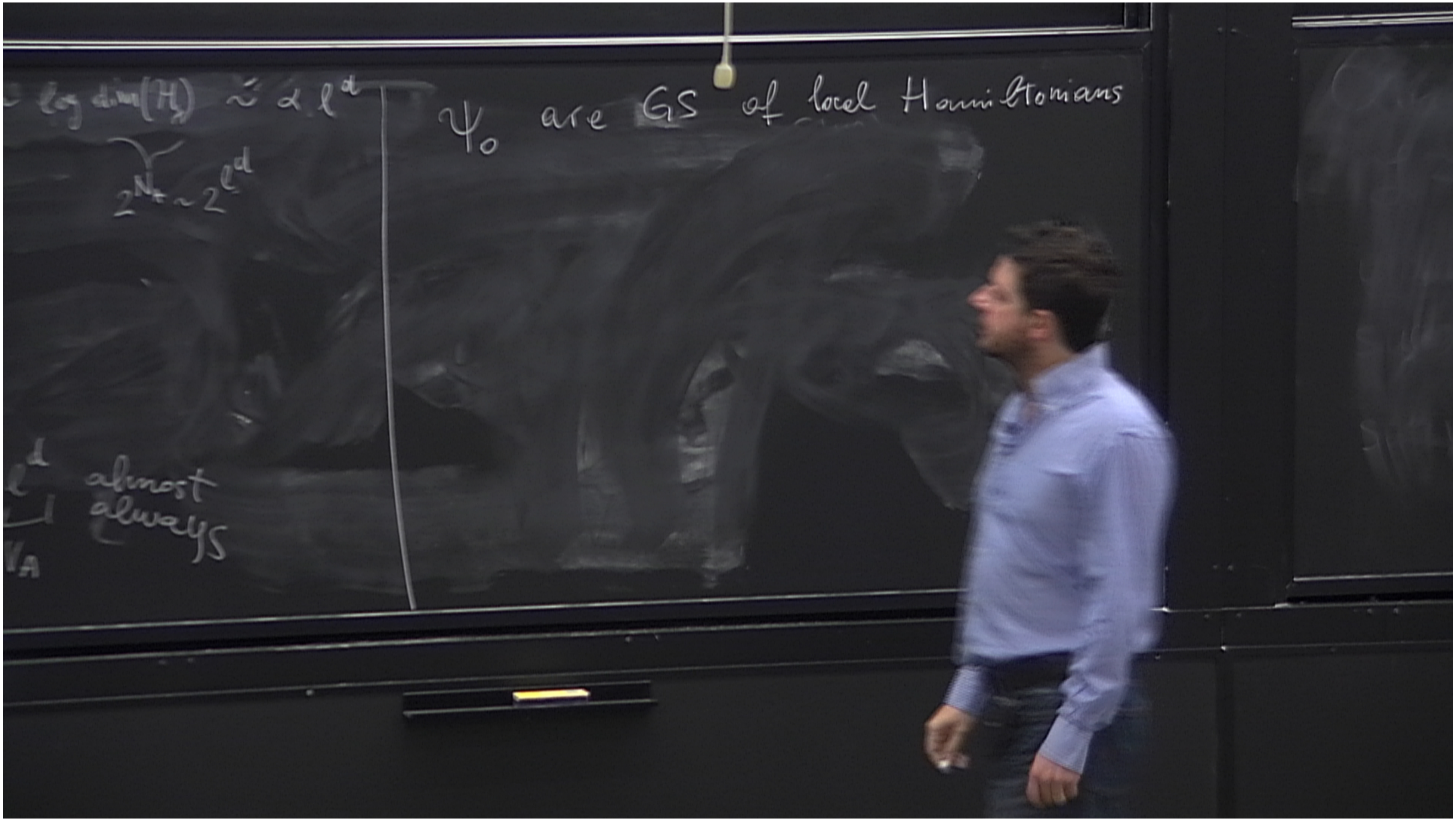
ψ random in $\mathcal{H}$ $\longrightarrow \rho_A \longrightarrow S_{VN}(\rho_A) \sim \alpha\, \underbrace{l^d}_{V_A}$ almos alwa


log dim($\mathcal{H}_A$) $\simeq$ α l^d
$2^{N_A} \sim 2^{l^d}$
l^d almost
1 always
V_A
Ψ_0 are GS of local Hamilton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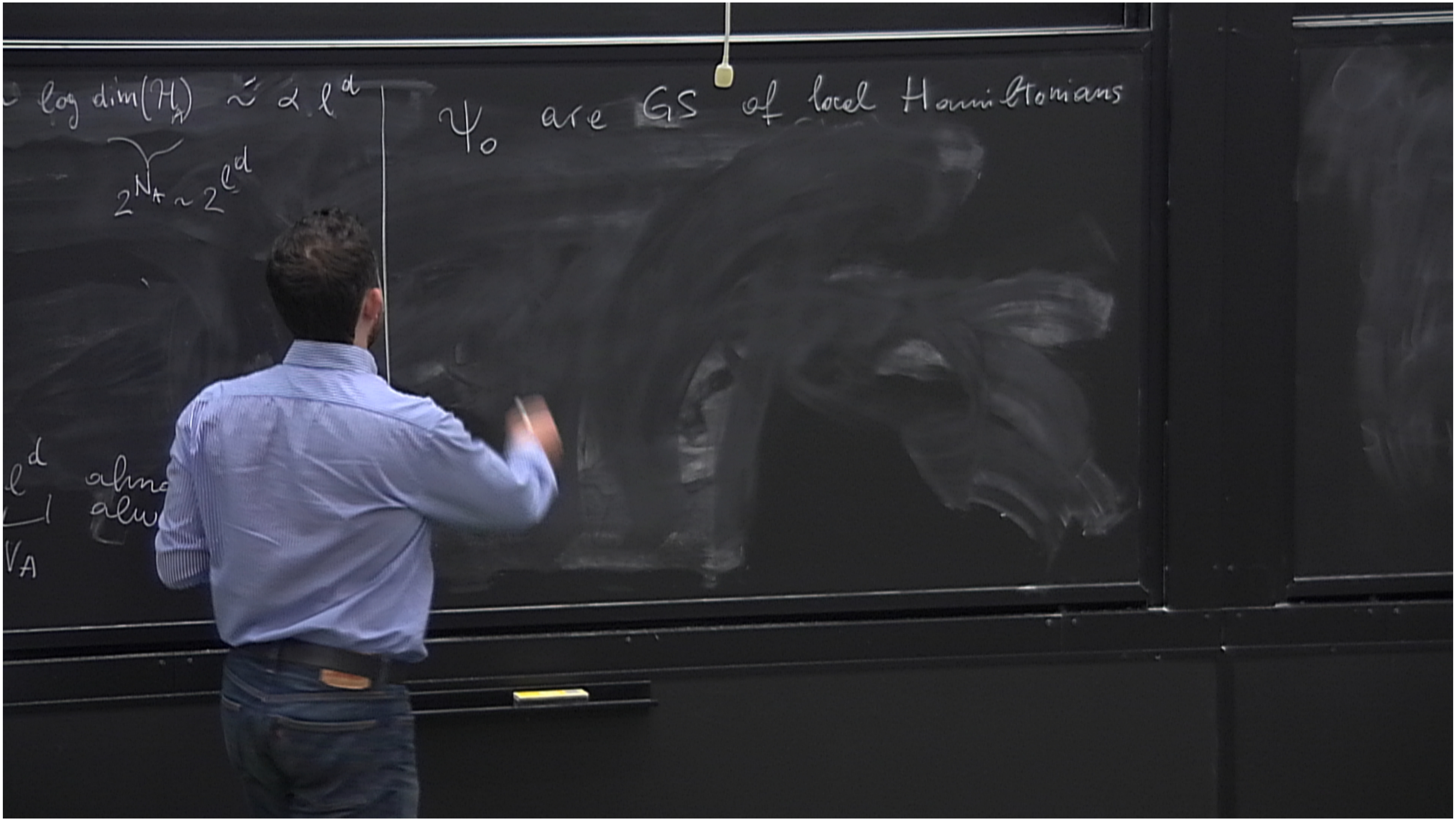


$\log \dim(\mathcal{H}_A) \simeq \alpha\, \ell^d$

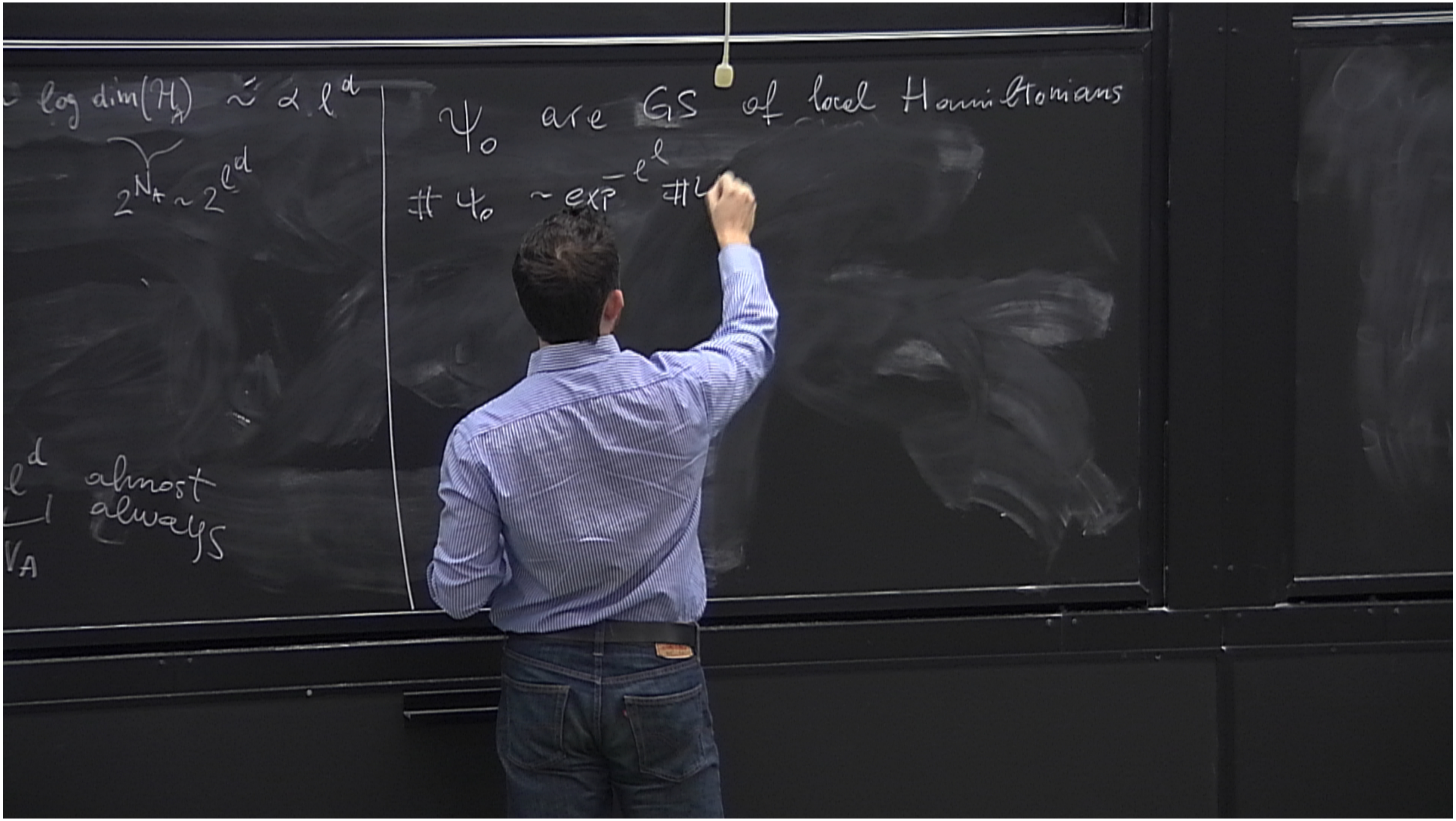
$2^{N_A} \sim 2^{\ell^d}$

Ψ_0 are GS of local Hamiltonians

ℓ^d almo

l alw

V_A



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ℓ^d almost

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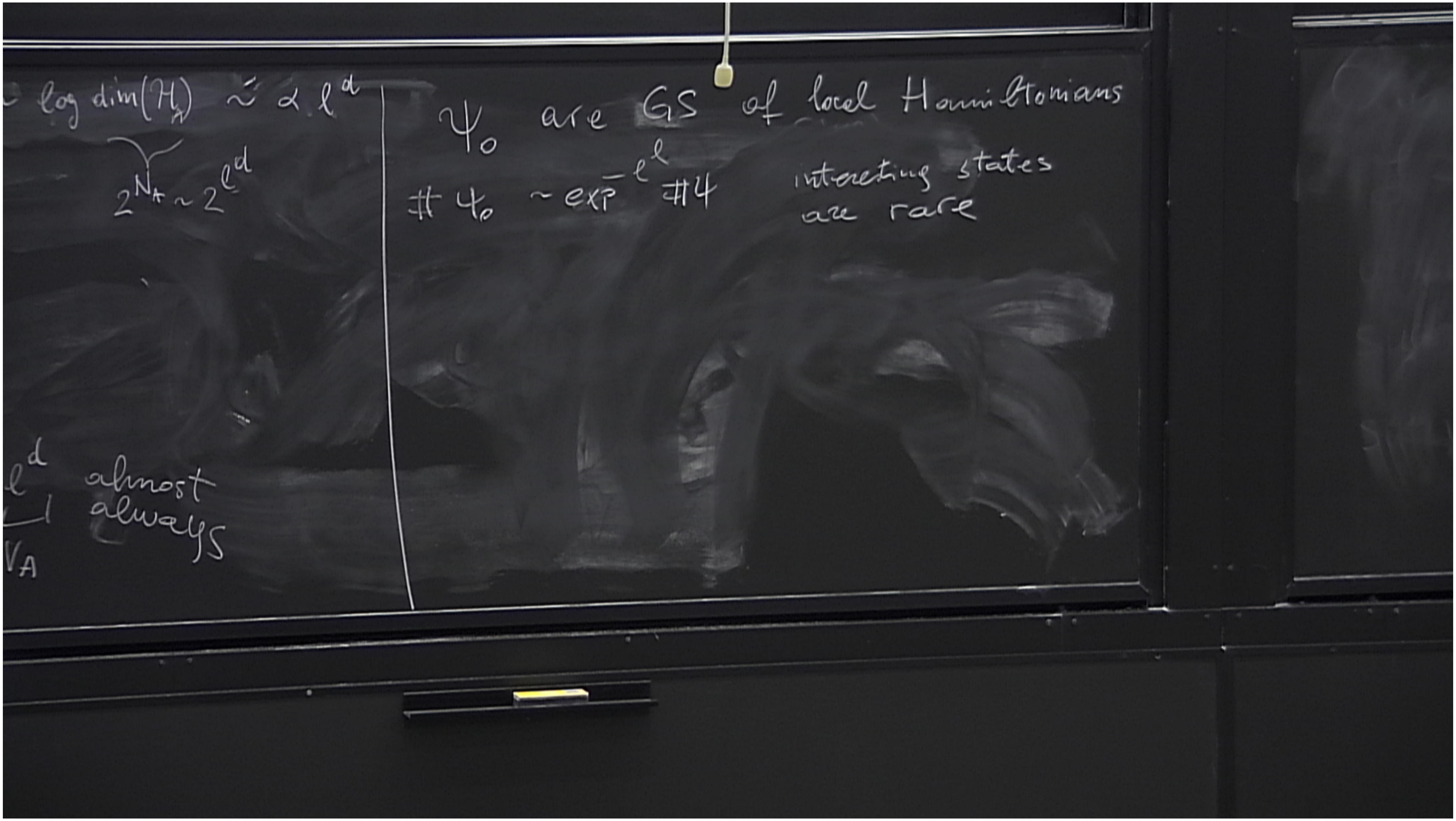
N_A

Ψ_0 are GS of local Hamiltonians

$\# \Psi_0 \sim \exp^{-\ell^{\ell}} \# \Psi$



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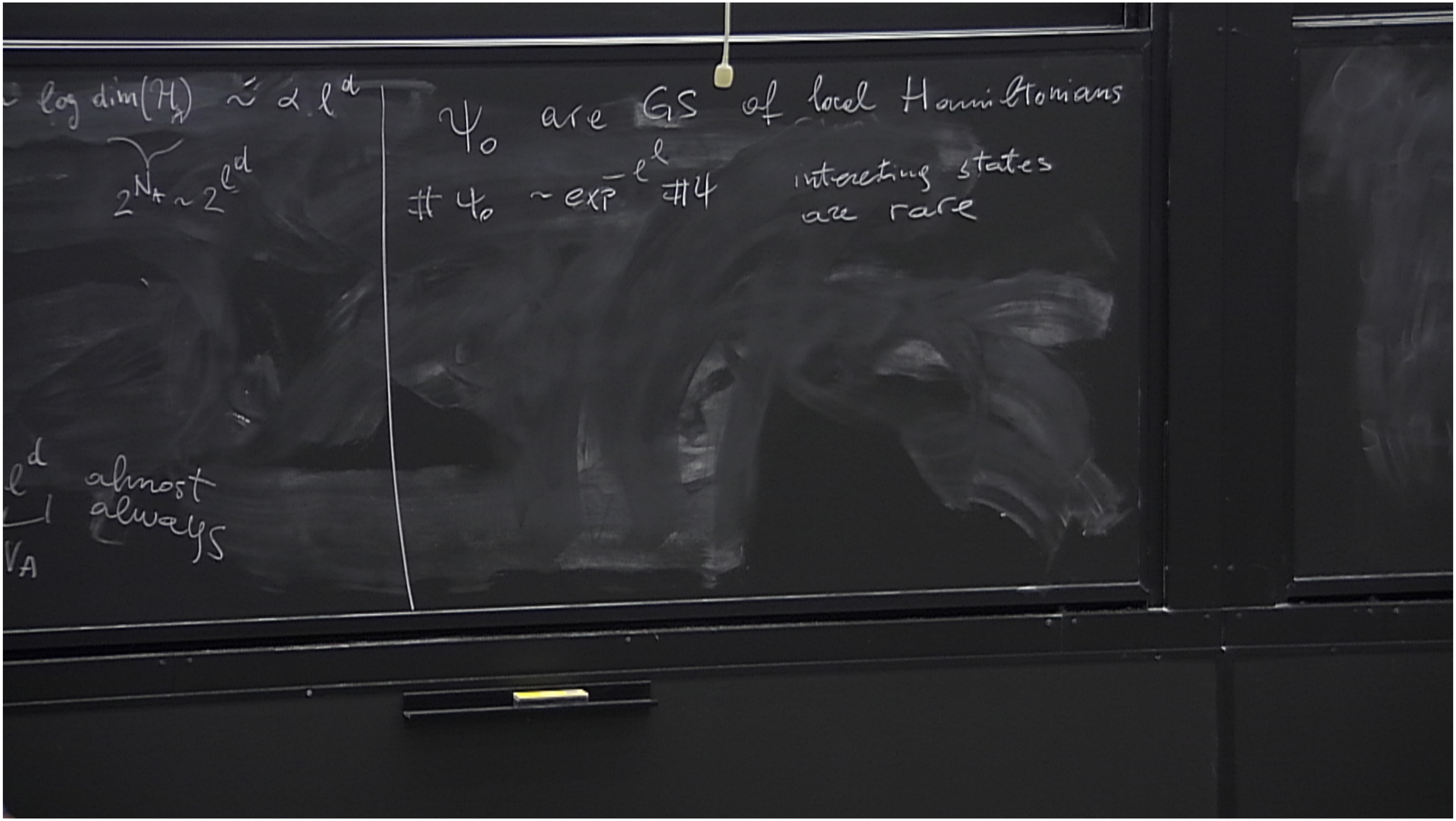
$\# \Psi_0 \sim \exp^{-\ell^\ell} \# \Psi$

interesting states are rare



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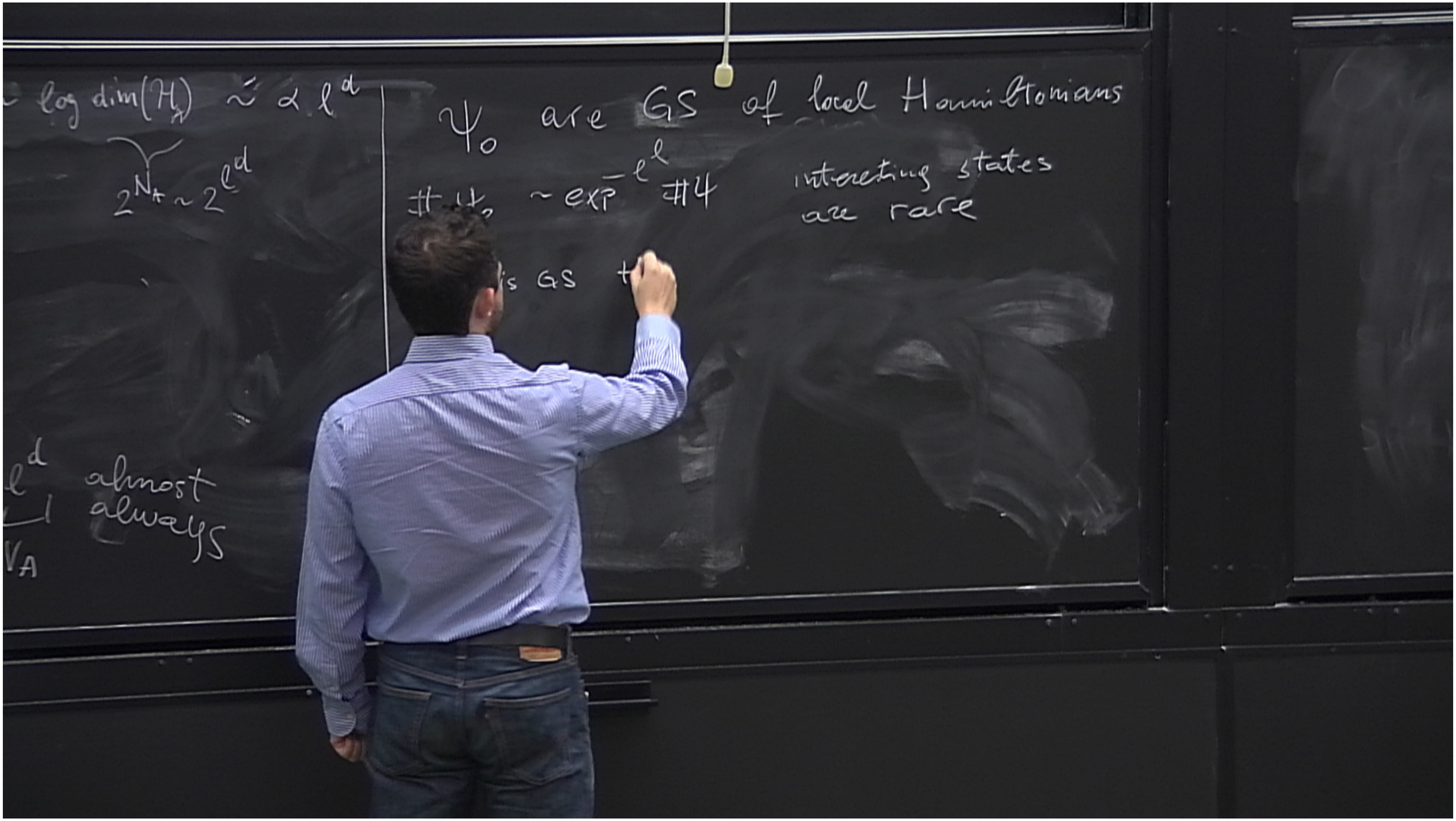

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$2^{N_A} \sim 2^{\ell^d}$

ℓ^d almost always

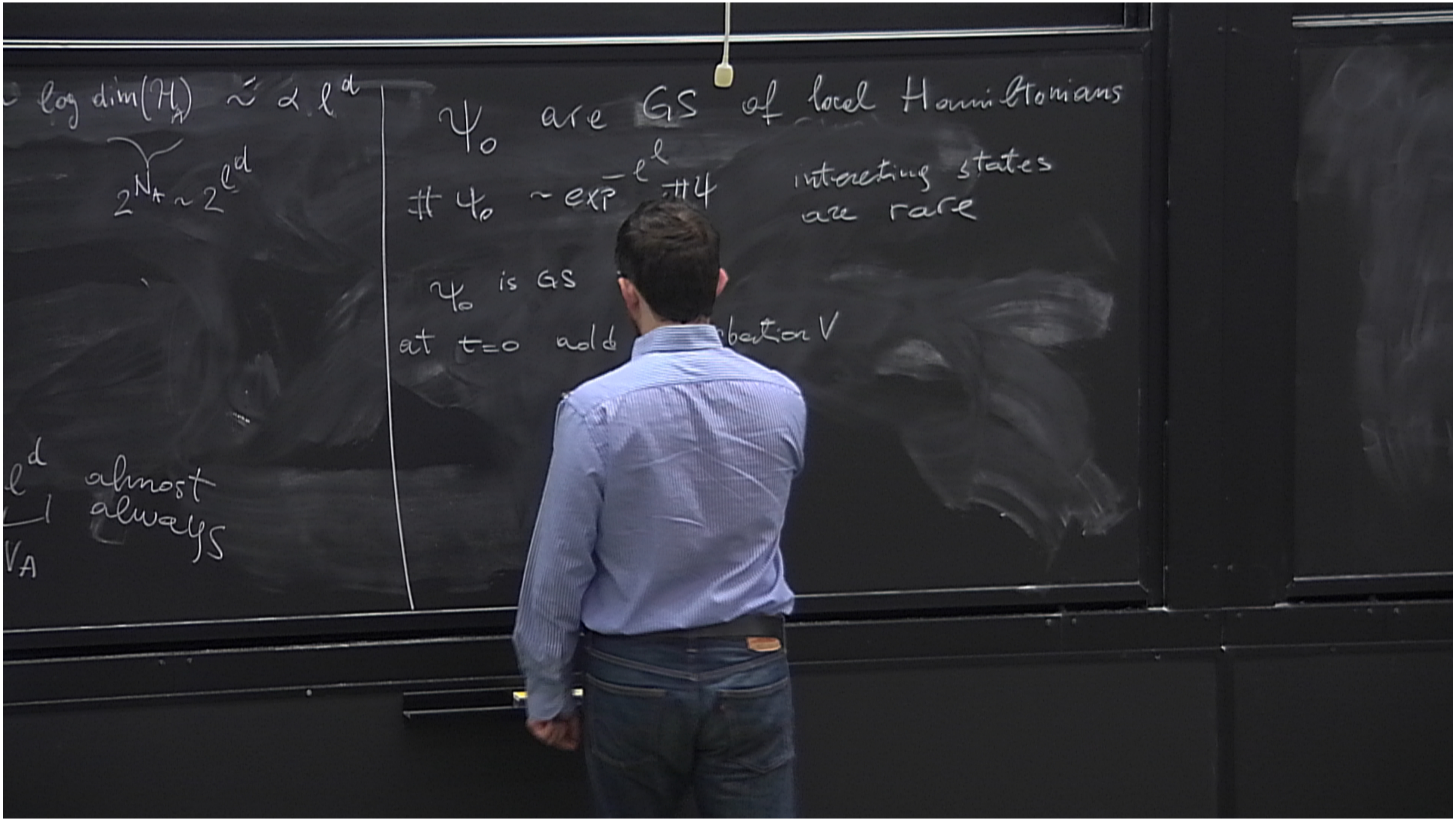
N_A

Ψ_0 are GS of local Hamiltonians

$\# \Psi_0 \sim \exp^{-\ell^{\ell}}$ #4

interesting states are rare

is GS t



$\log \dim(\mathcal{H}_A) \simeq \alpha\, \ell^d$

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ℓ^d almost

$\lfloor 1$ always

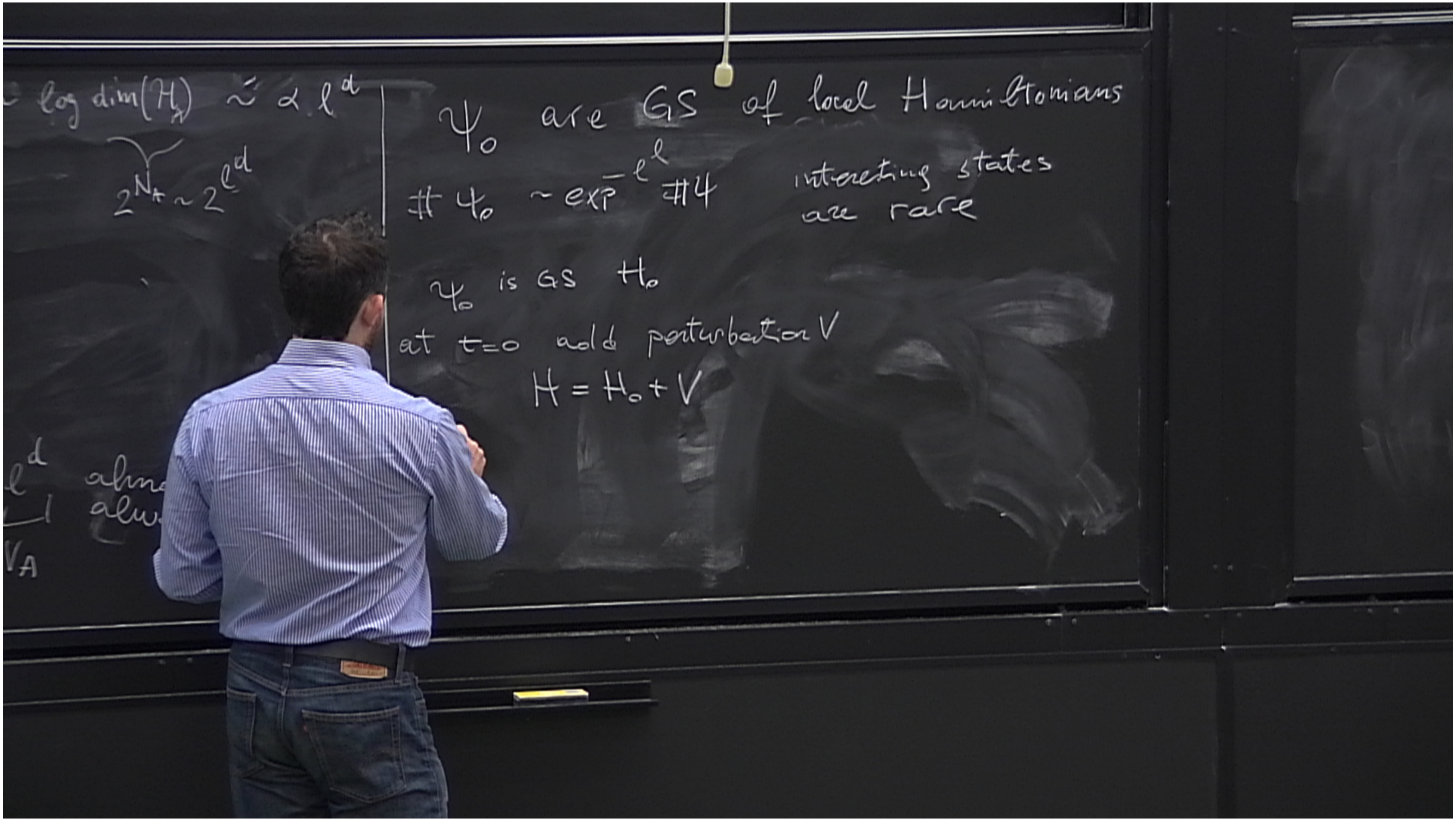
V_A

ψ_0 are GS of local Hamiltonians

$\# \psi_0 \sim \exp^{-\ell^{\ell}} \# \psi$ interesting states are rare

ψ_0 is GS

at $t=0$ add bation V



$\log \dim(\mathcal{H}_A) \simeq \alpha\, \ell^d$

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ℓ^d almo

1 alw

V_A

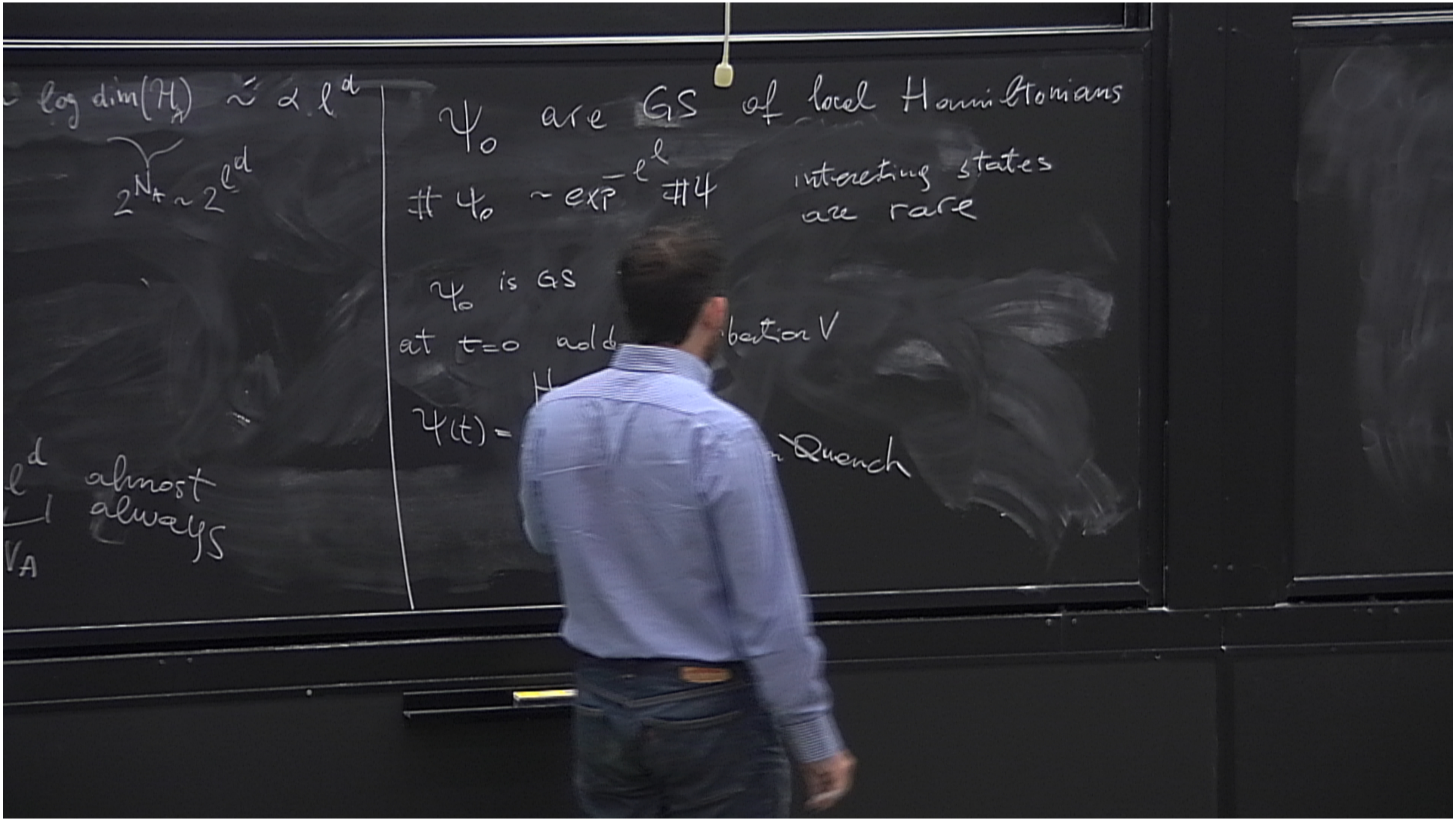
Ψ_0 are GS of local Hamiltonians

$\# \Psi_0 \sim \exp^{-\ell^{\ell}}$ #4 interesting states are rare

Ψ_0 is GS H_0

at $t=0$ add perturbation V

$$H = H_0 + V$$



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$\# \Psi_0 \sim \exp^{-\ell^{\ell}} \# \Psi$ interesting states are rare

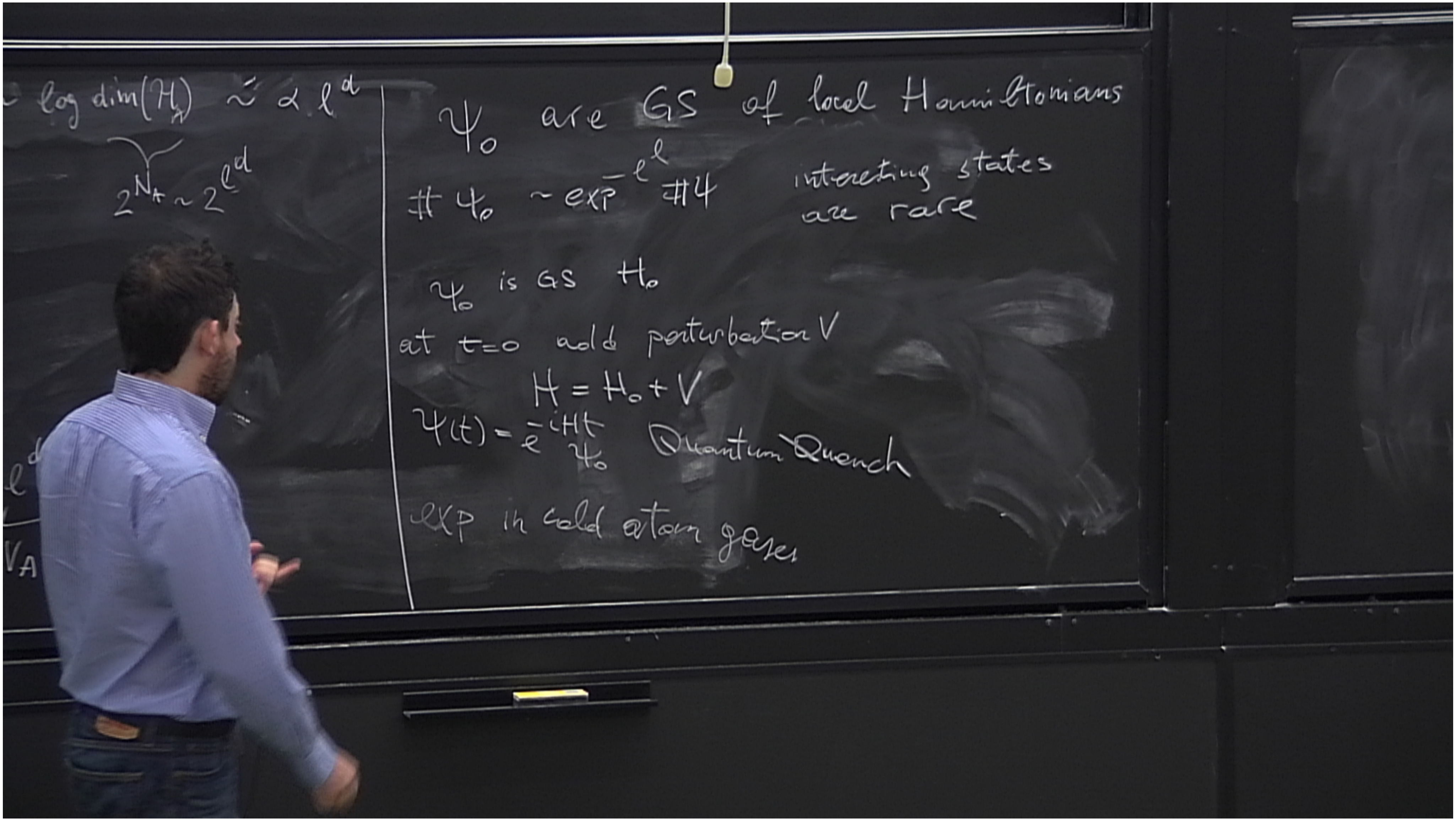
Ψ_0 is GS

at $t=0$ add … bation V

H

$\Psi(t) =$

Quench



$$\underbrace{\log \dim(\mathcal{H}_A)}_{2^{N_A} \sim 2^{\ell^d}} \sim \alpha \, \ell^d$$

ℓ^d

V_A

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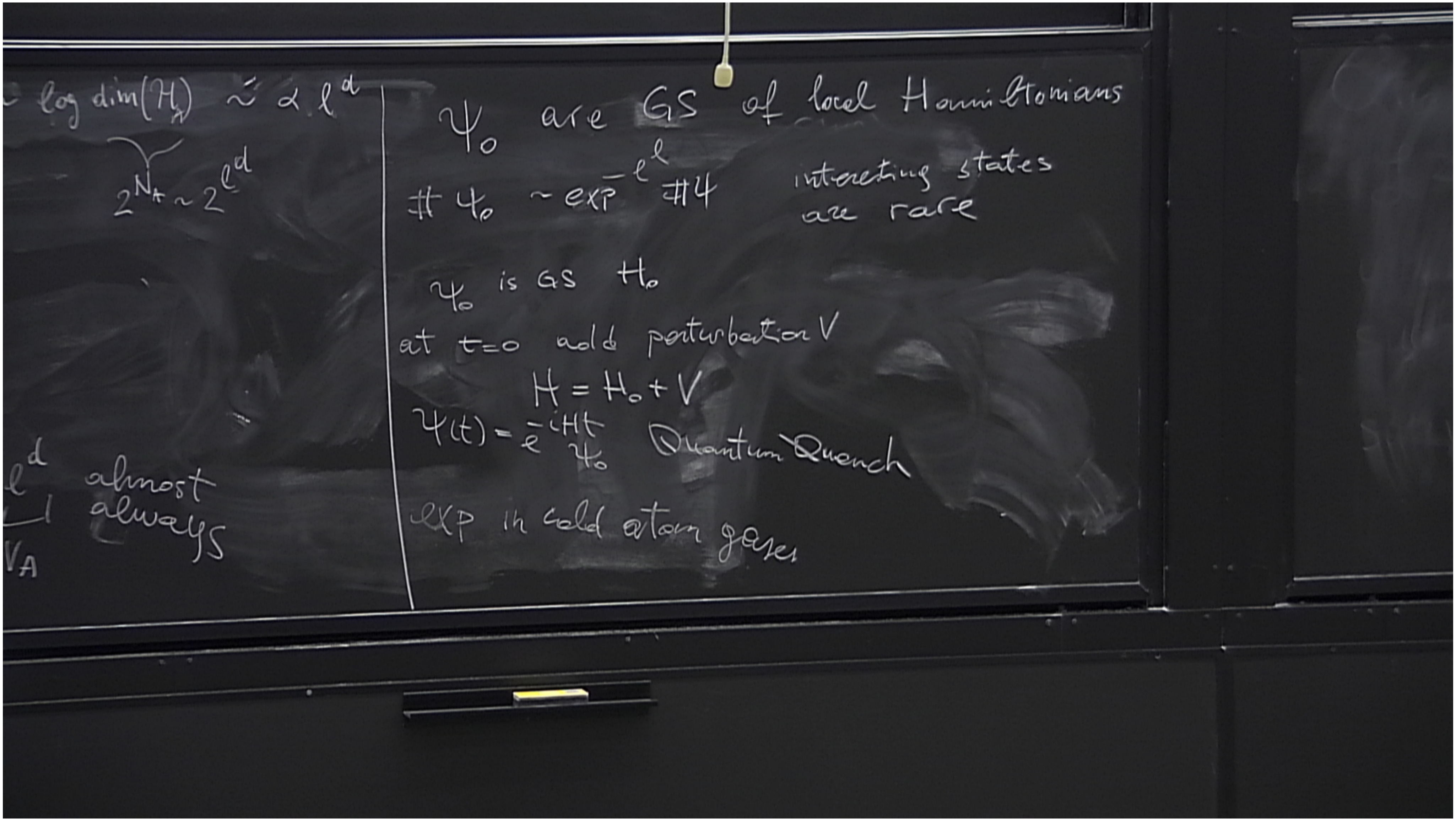
at $t=0$ add perturbation V

$$H = H_0 + V$$

$\Psi(t) = e^{-iHt} \Psi_0$ Quantum Quench

exp in cold atom gases



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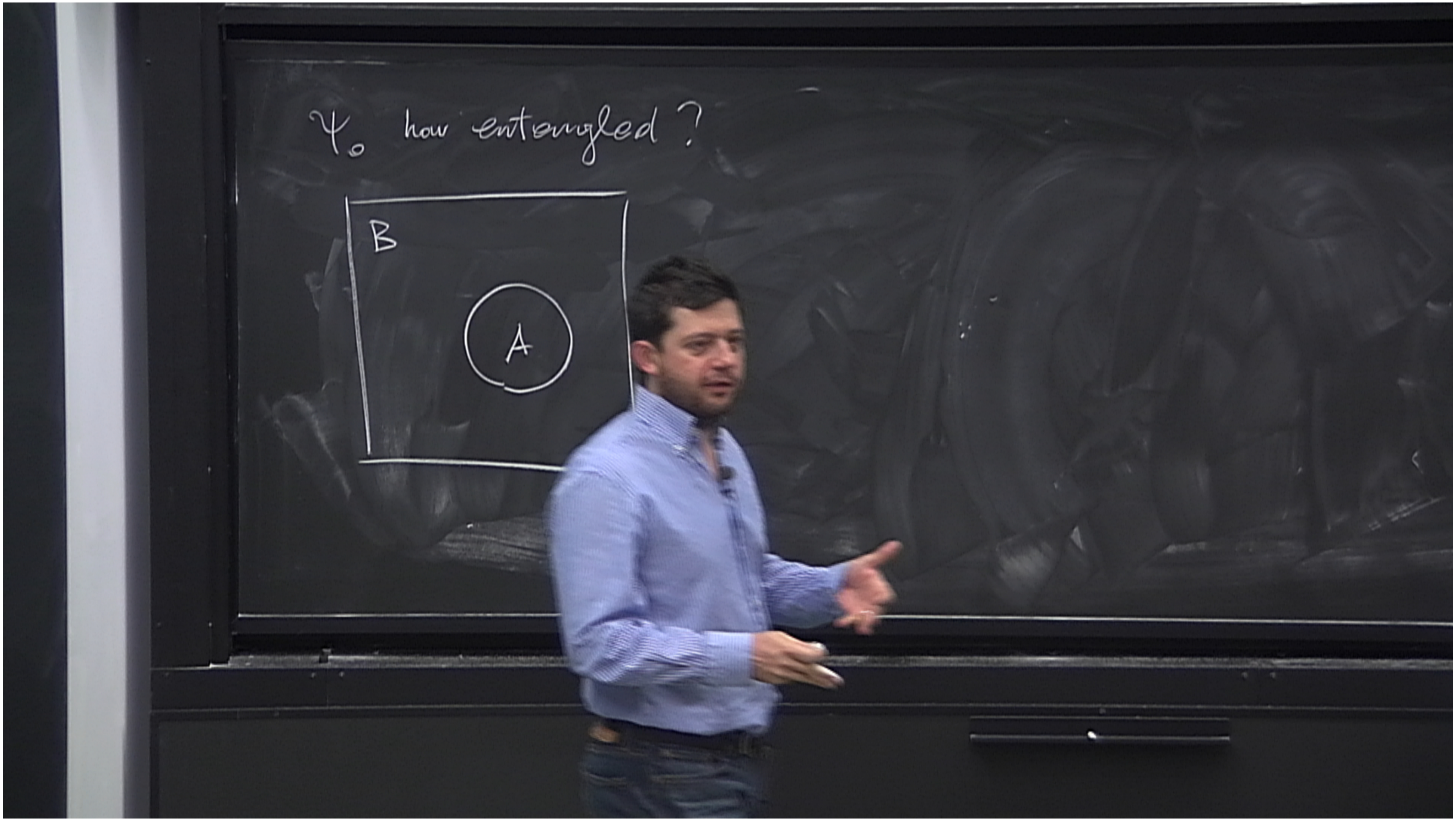
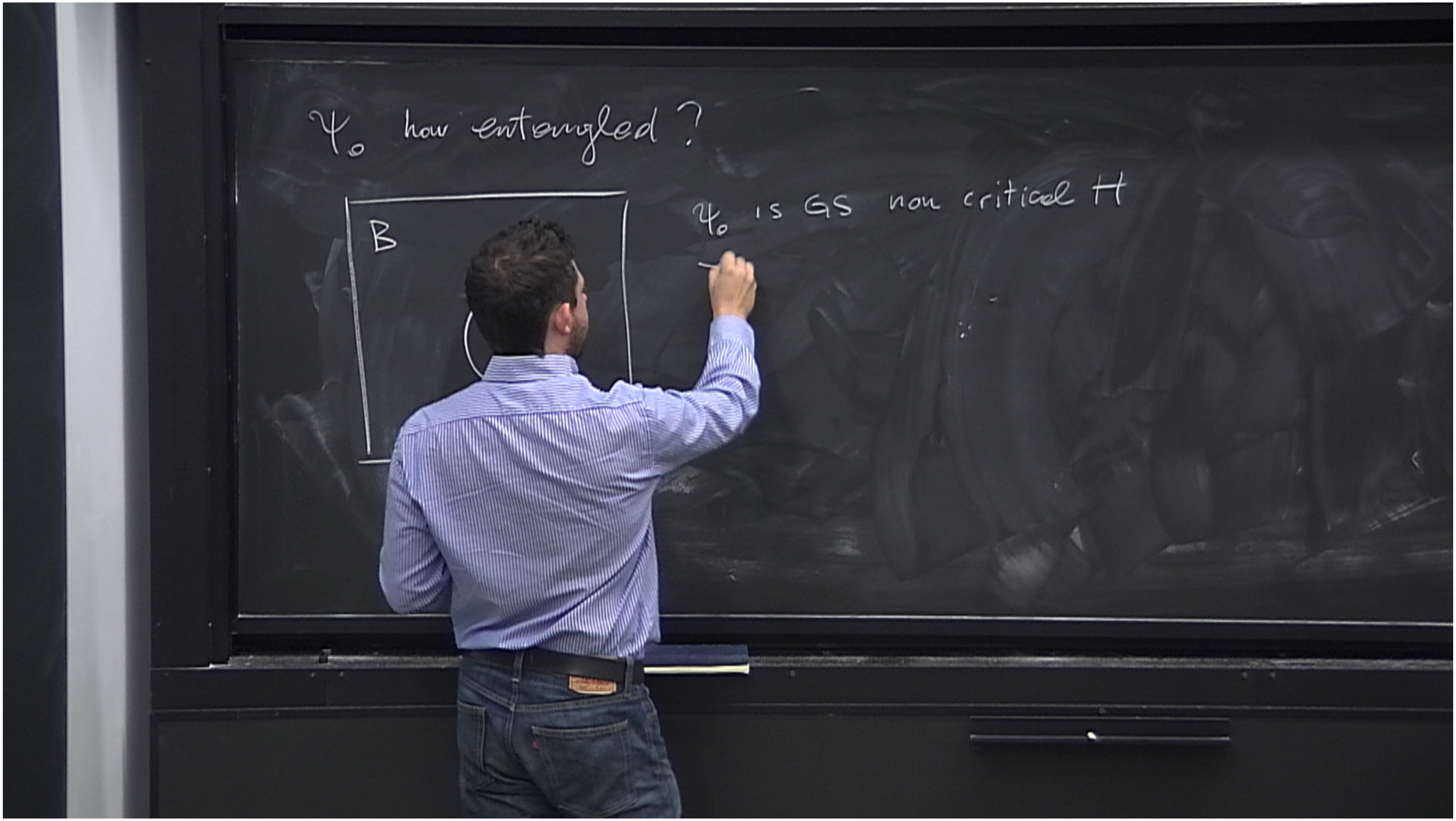
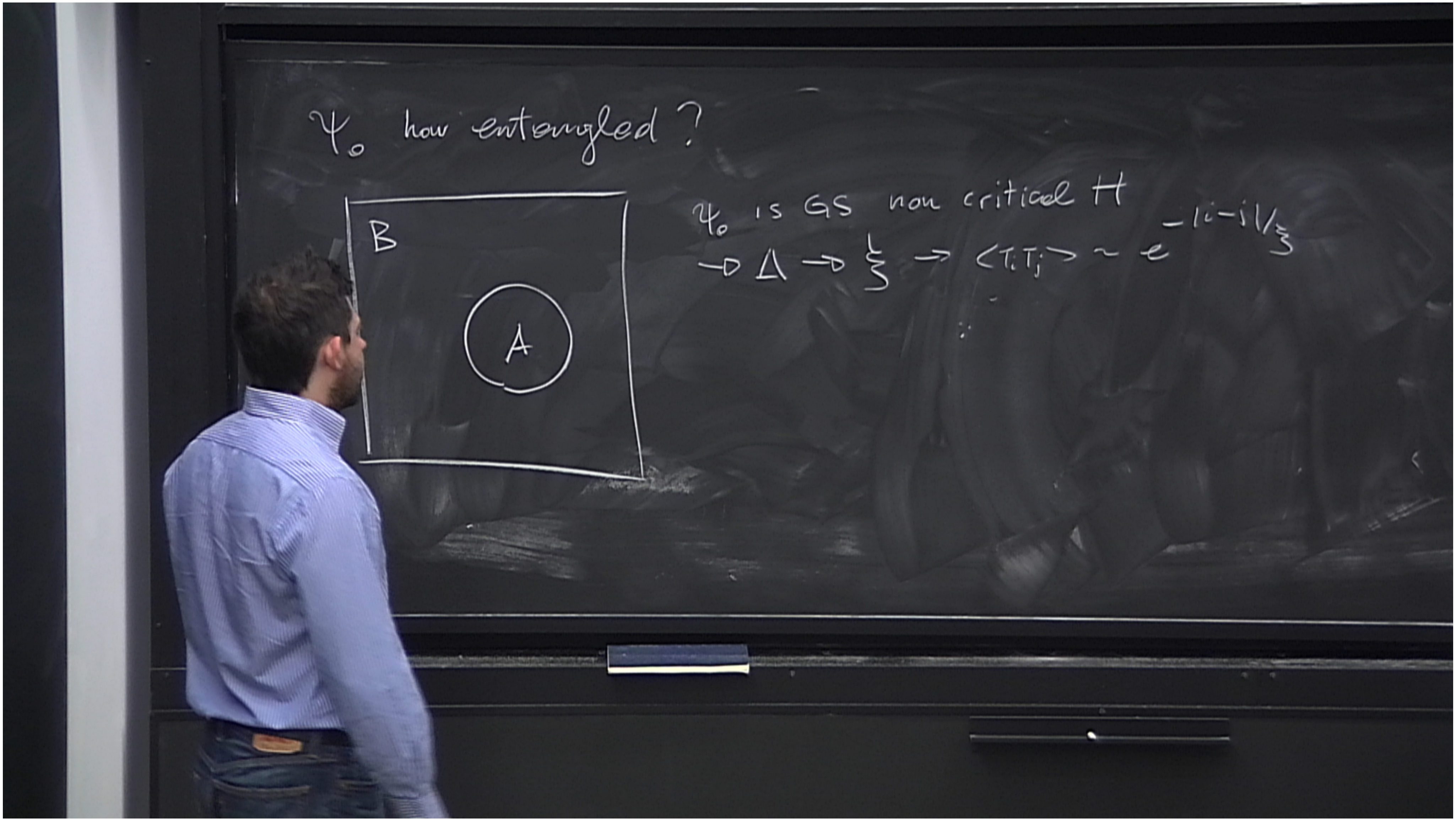

Ψ₀ how entangled?
B
A



Ψ_0 how entangled?

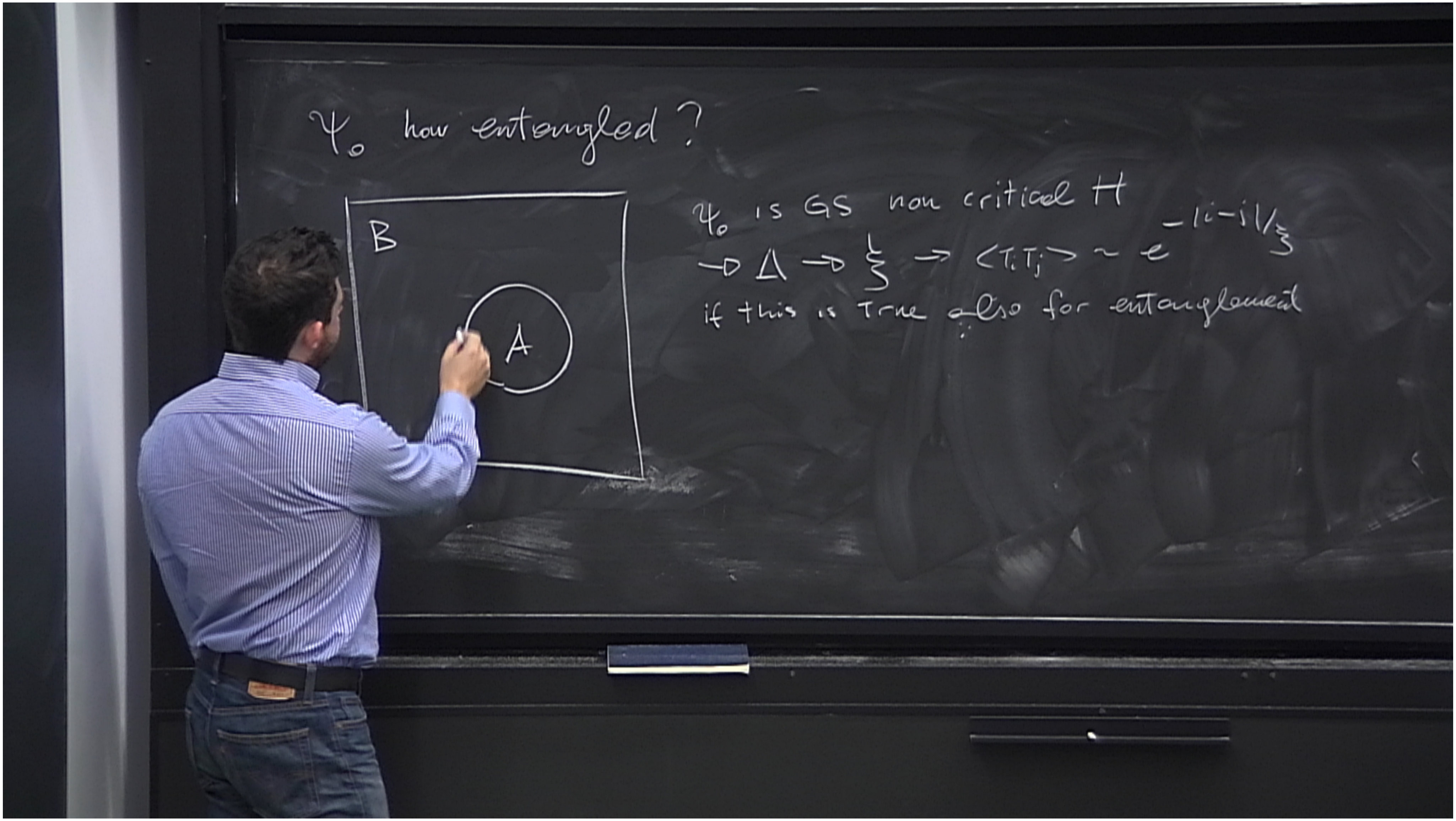
Ψ_0 is GS non critical H



Ψ_0 how entangled ?

Ψ_0 is GS non critical H

$$\to \Delta \to \xi \to \langle T_i T_j \rangle \sim e^{-|i-j|/\xi}$$

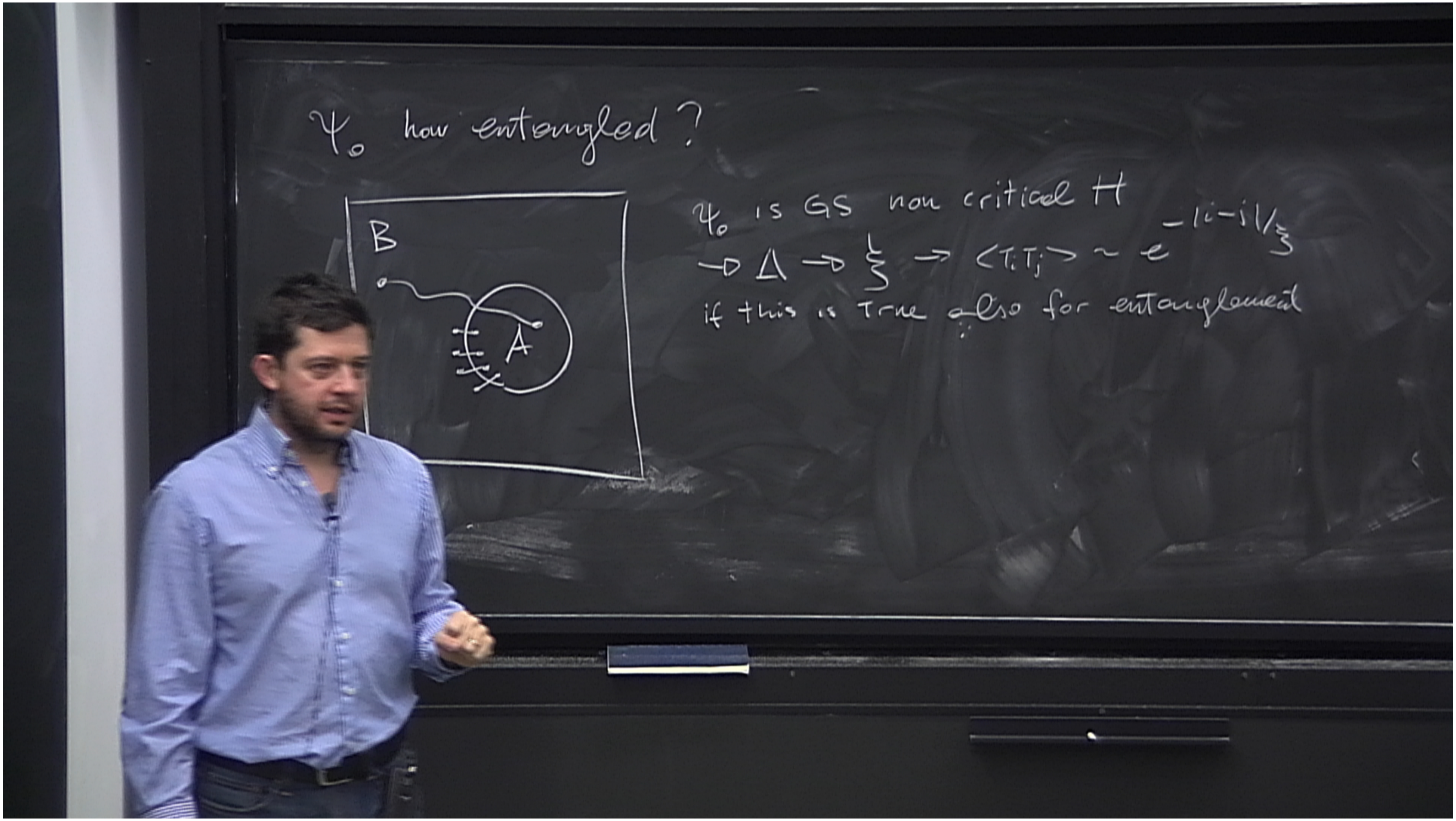


Ψ_0 how entangled?

Ψ_0 is GS non critical H

$\rightarrow \Delta \rightarrow \xi \rightarrow \langle T_i T_j \rangle \sim e^{-|i-j|/\xi}$

if this is True also for entanglement



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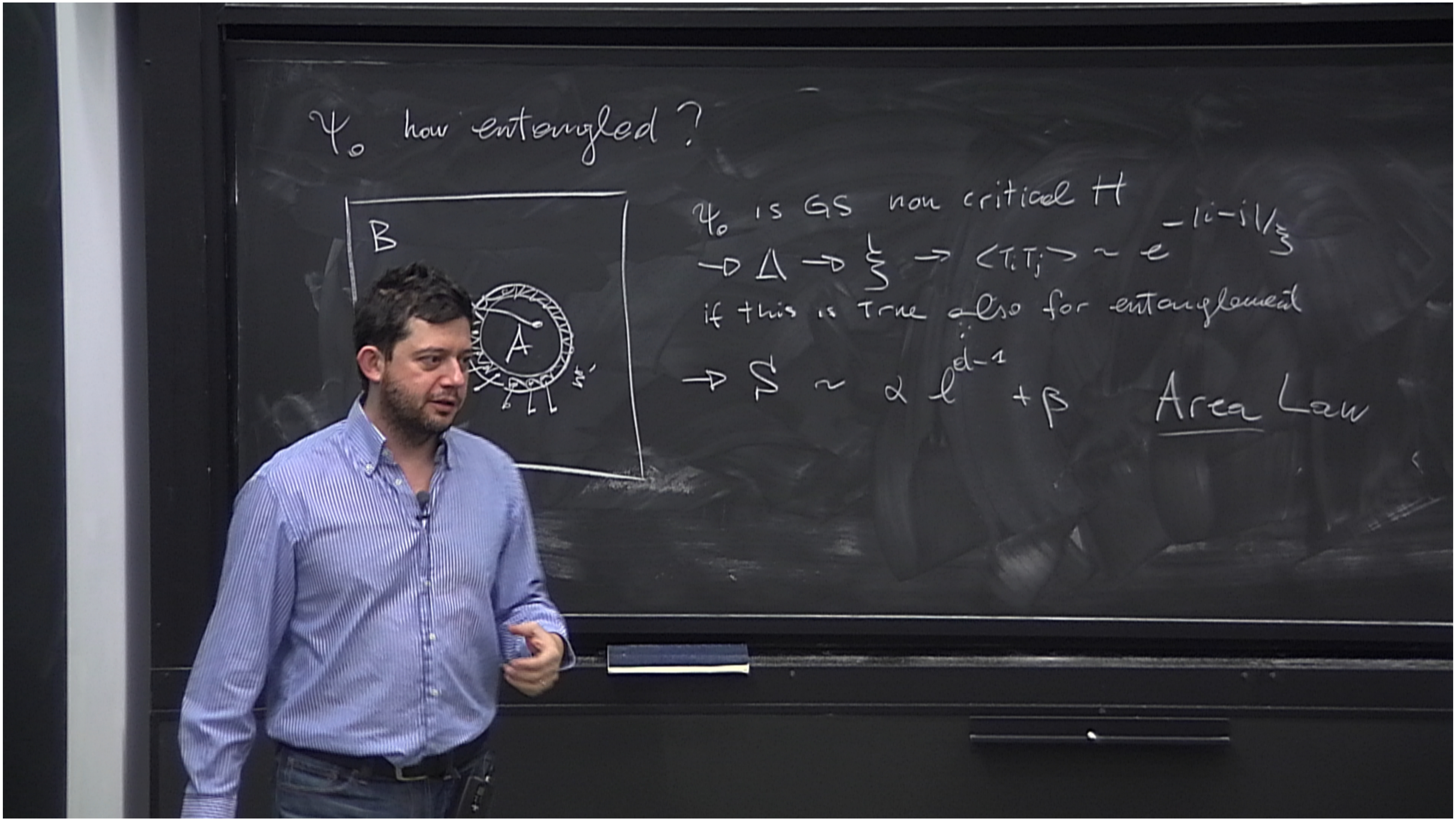

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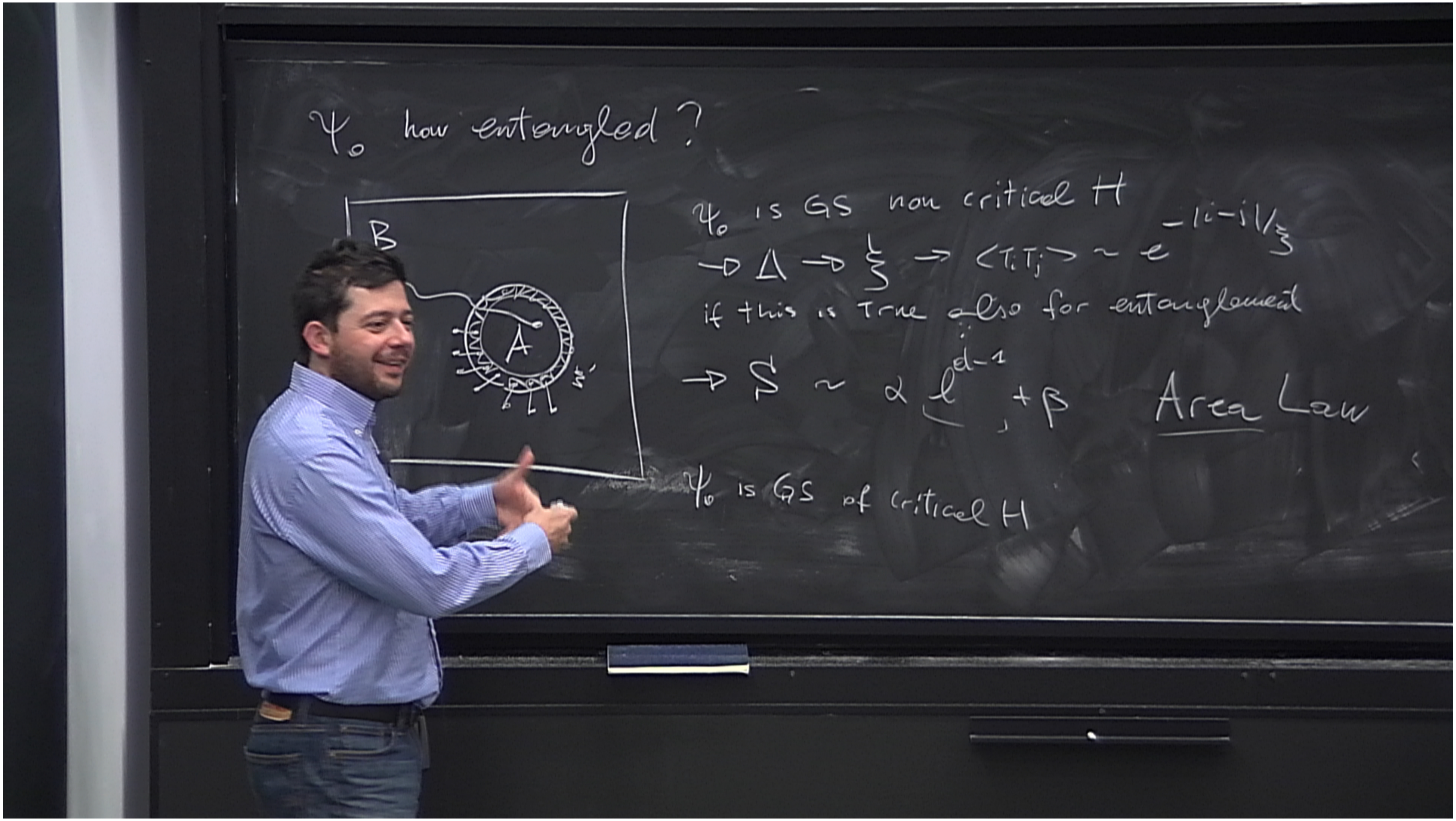
$\to \Delta \to \xi \to \langle T_i T_j \rangle \sim e^{-|i-j|/\xi}$

if this is true also for entanglement

$\to S \sim \alpha \ell^{d-1} + \beta$ Area Law



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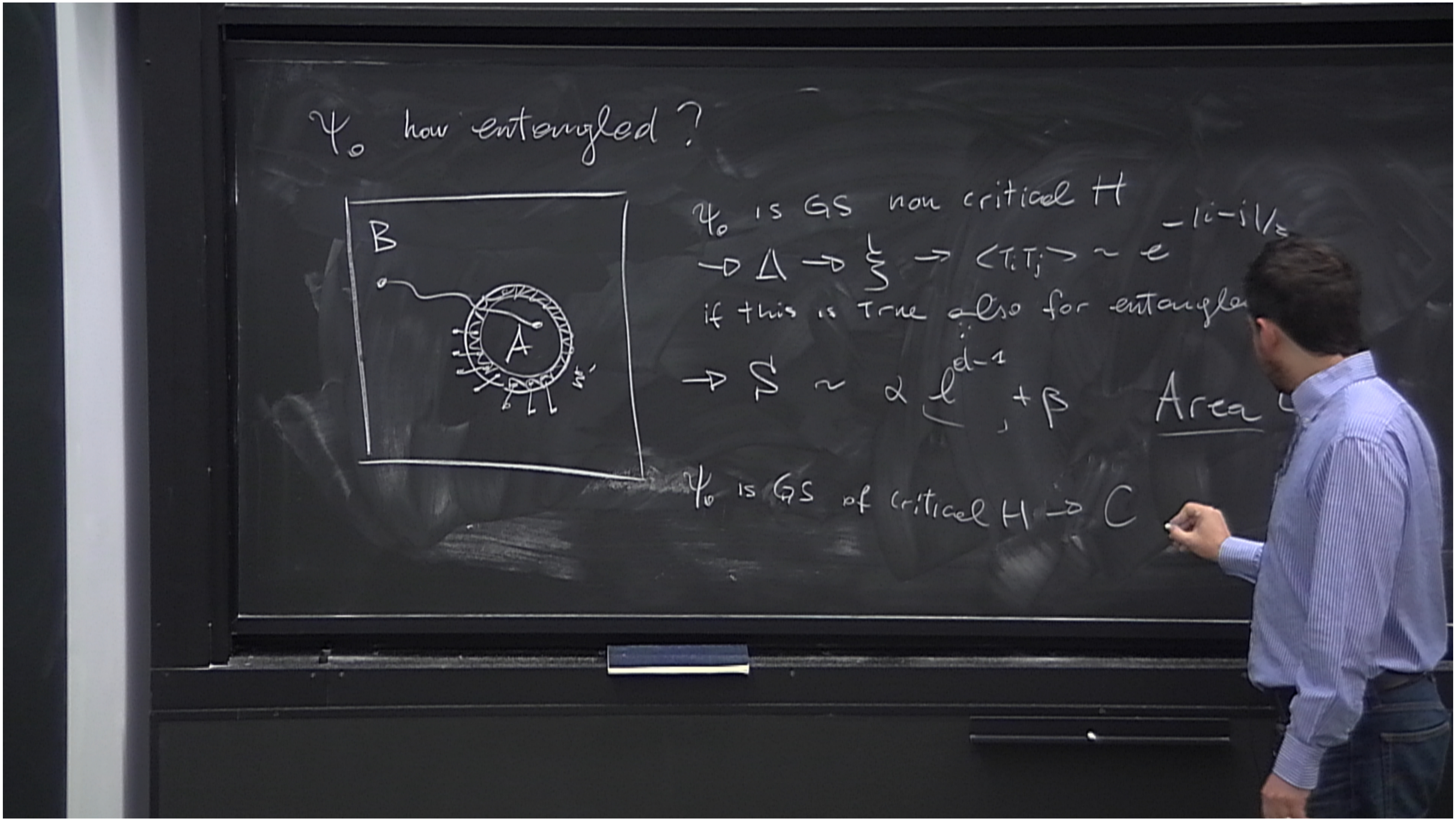
Ψ_0 is GS of critical H



Ψ_0 how entangled?

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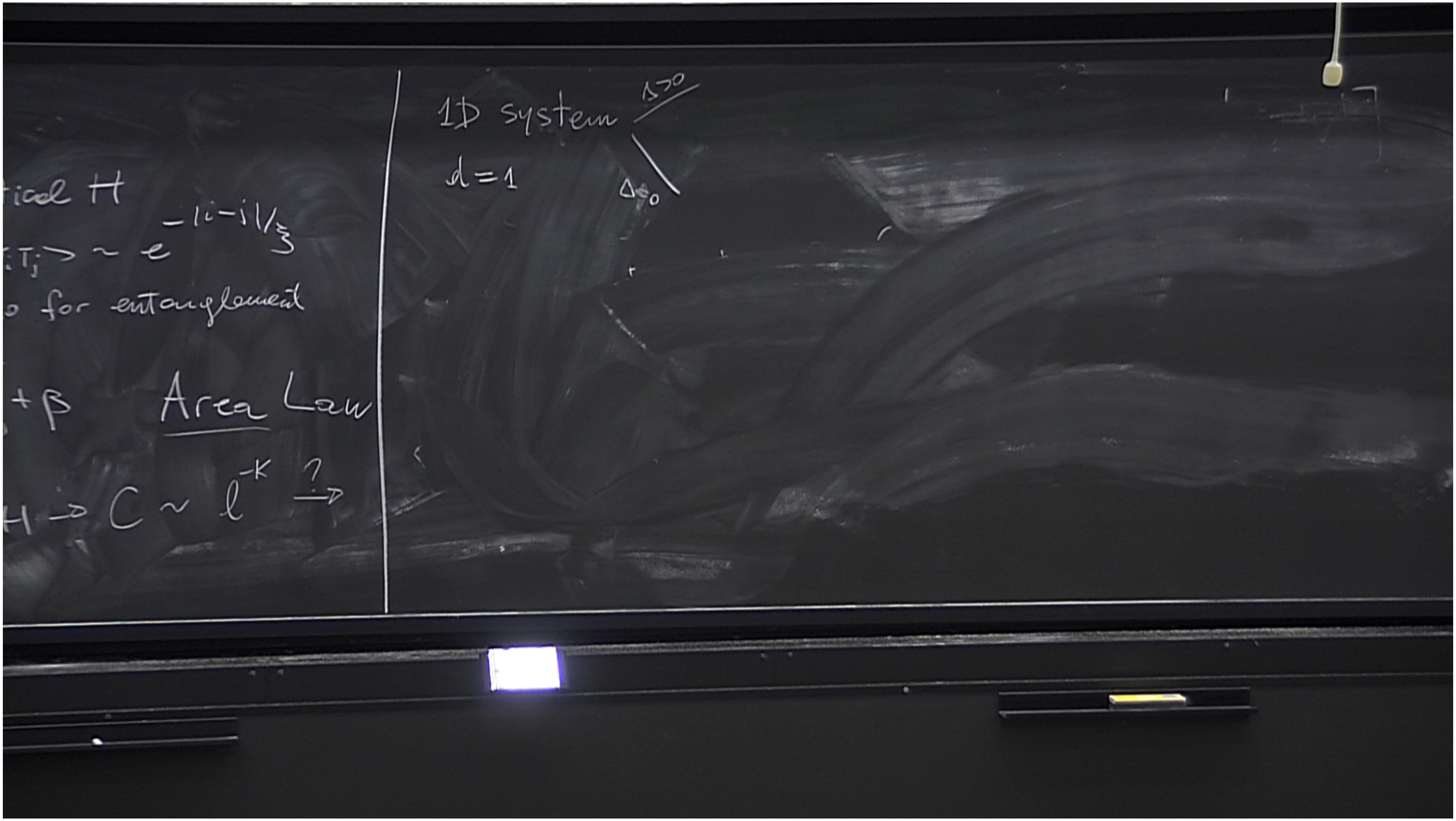
if this is True also for entangle

$\rightarrow S \sim \alpha \, \ell^{d-1} + \beta$ Area

Ψ_0 is GS of critical $H \rightarrow C$



tical H

$T_i T_j\rangle \sim e^{-|i-j|/\xi}$

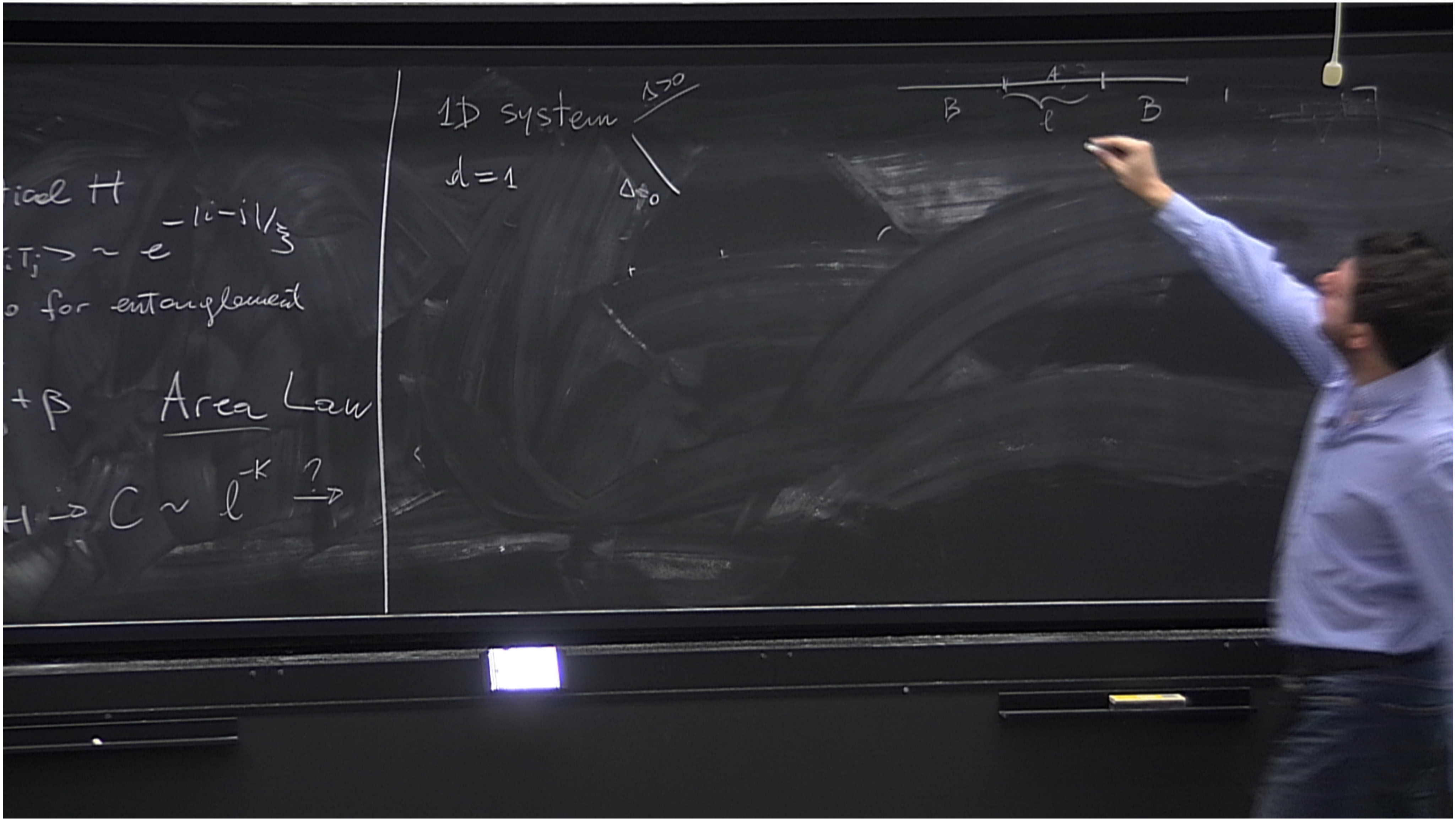
o for entanglement

$+\beta$ Area Law

$H \to C \sim \ell^{-K} \xrightarrow{?}$

1D system $\Delta>0$

$d=1$ $\Delta=0$



...tical H

$\langle ...T_i T_j \rangle \sim e^{-|i-j|/\xi}$

...o for entanglement

...$+\beta$ Area Law

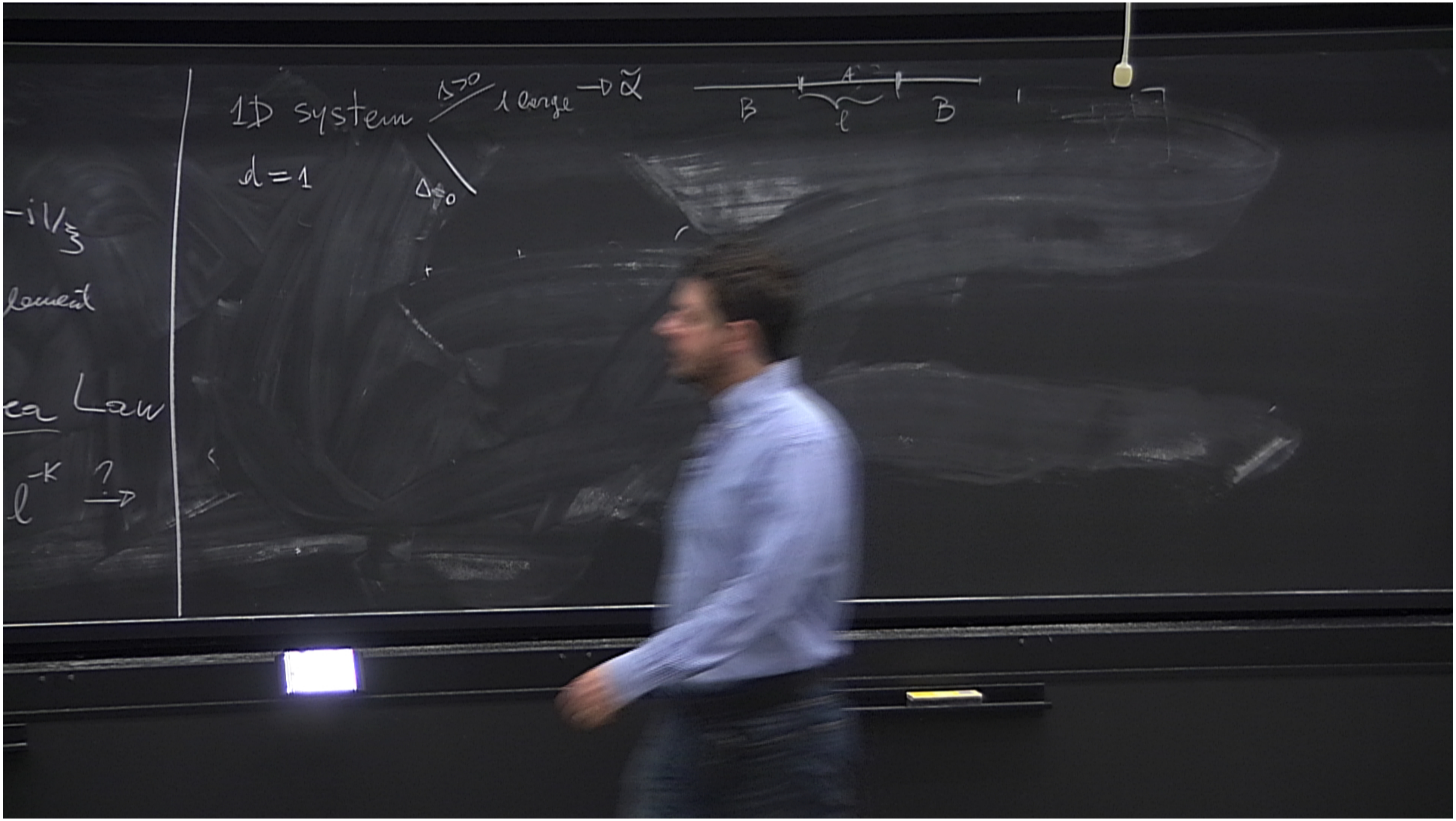
...H → $C \sim \ell^{-\kappa} \xrightarrow{?}$

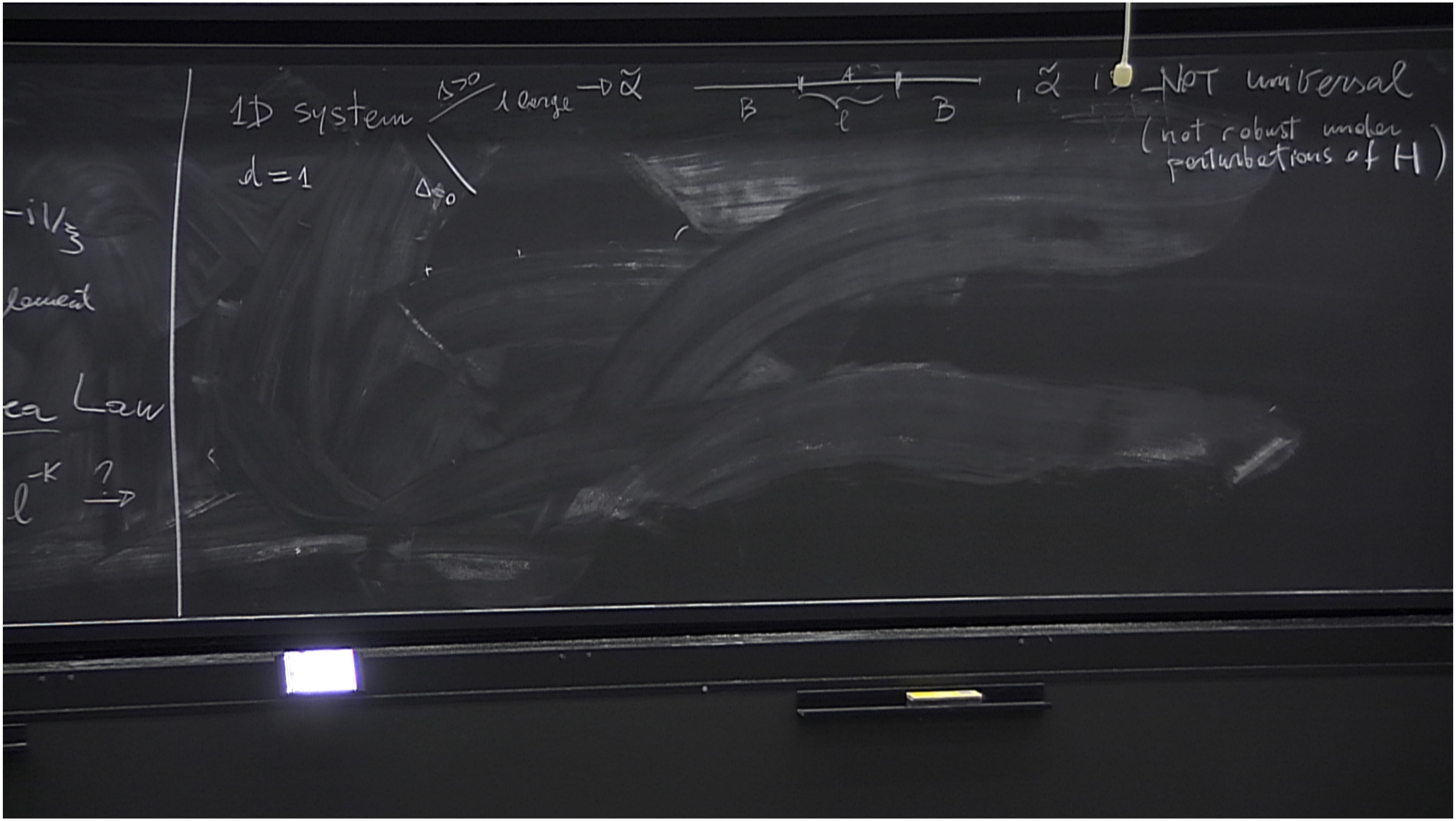
1D system

$d=1$

$\Delta > 0$

$\Delta = 0$


1D system
Δ>0
l large → α̃
d = 1
Δ=0
A
B
l
B
-il/ξ
lement
ea Law
l^{-κ} ?→



1D system $\xrightarrow{\Delta>0}$ l large $\rightarrow \tilde{\alpha}$

$d = 1$ $\searrow$ $\Delta = 0$

, $\tilde{\alpha}$ is NOT universal

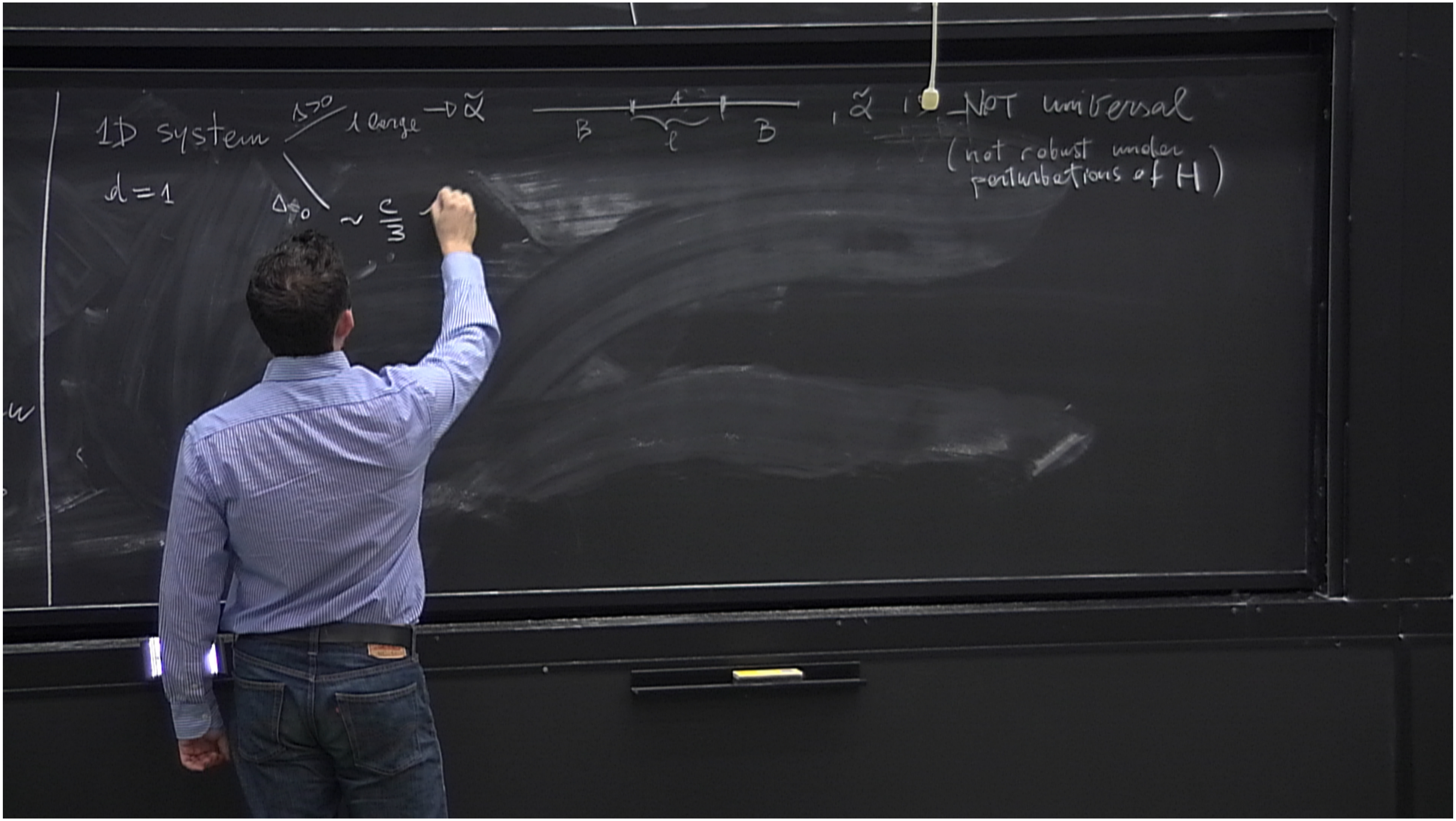
(not robust under perturbations of H)

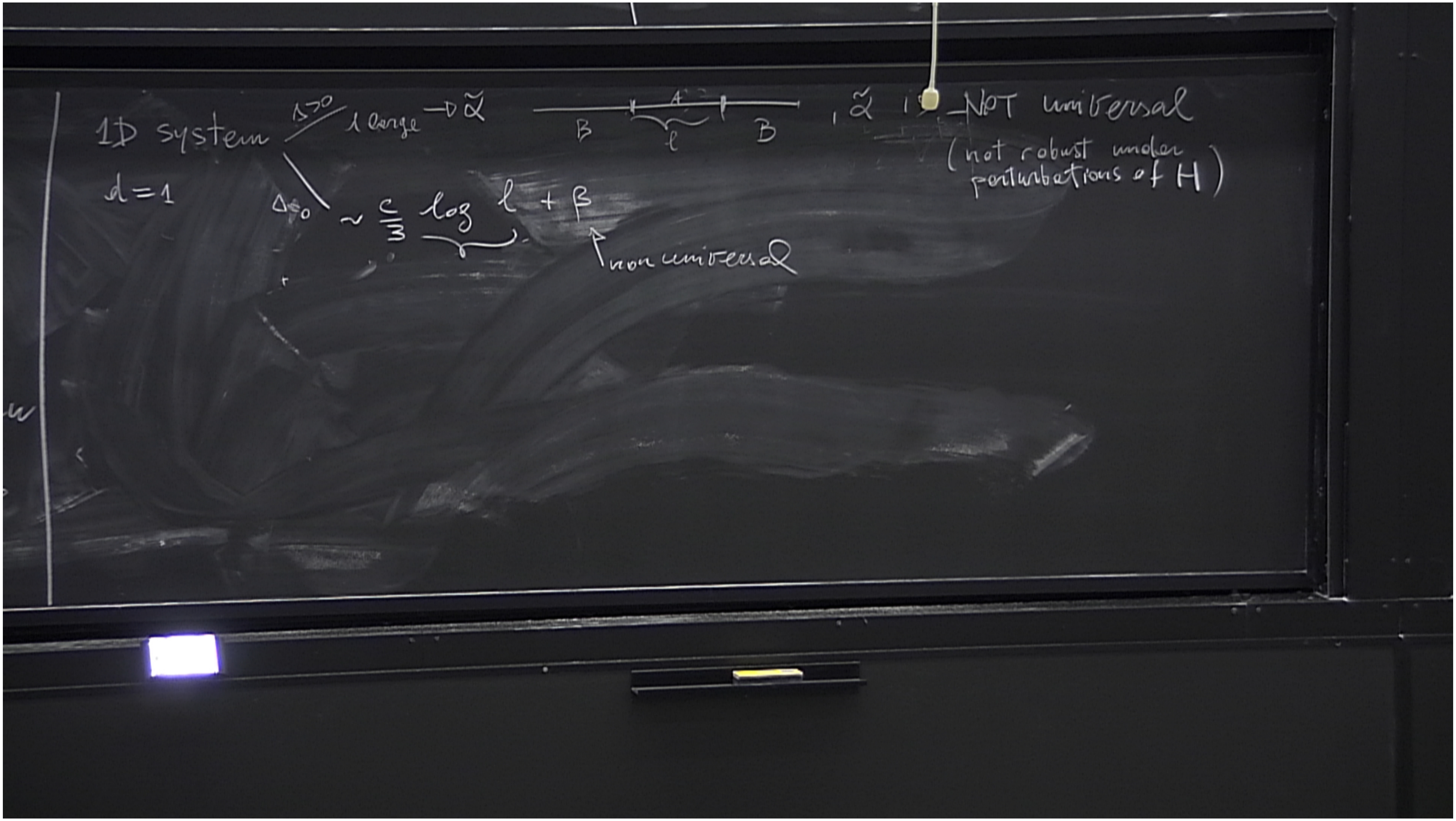
$-i l/\xi$

lement

ea Law

$l^{-k} \overset{?}{\longrightarrow}$



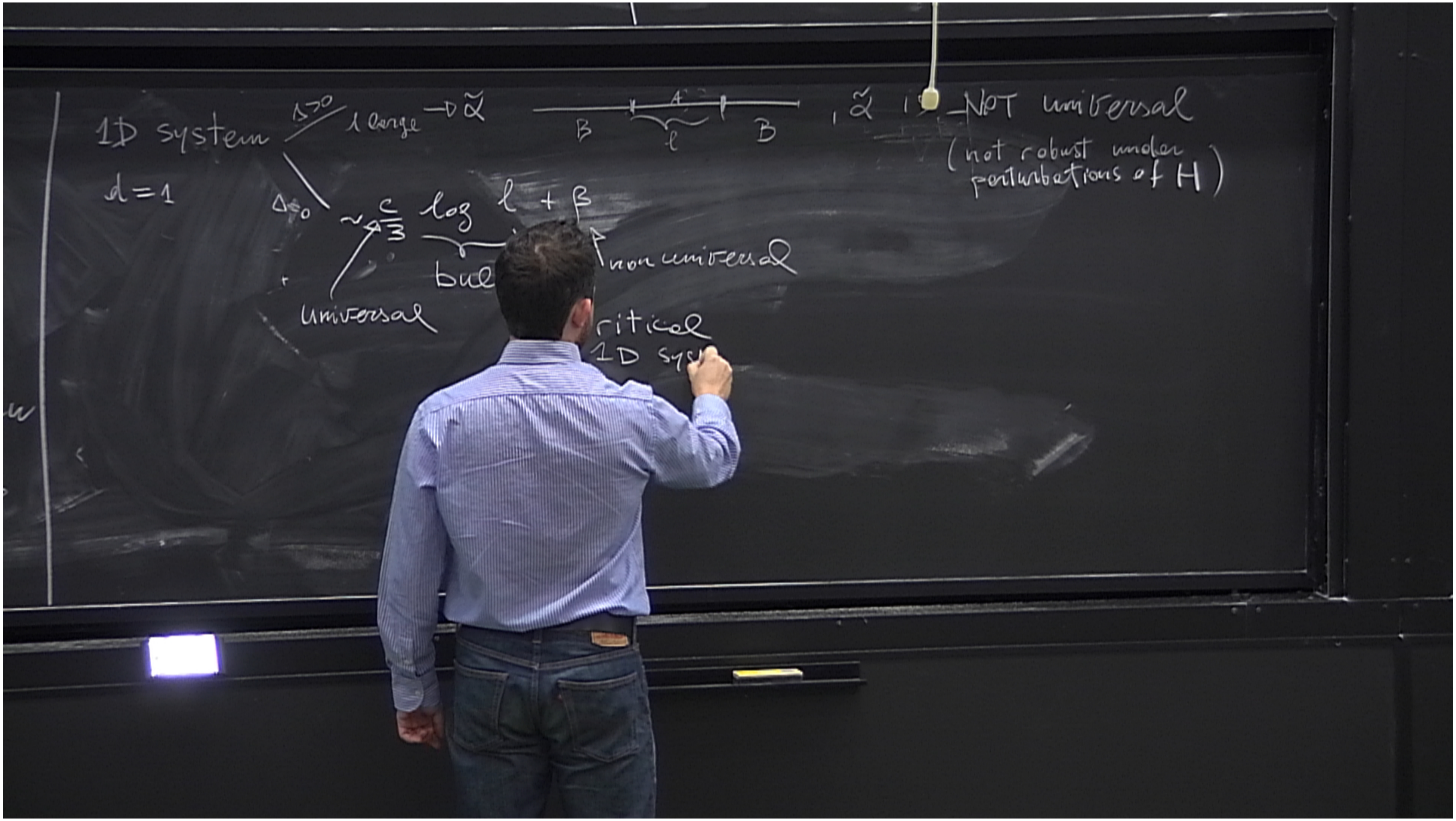


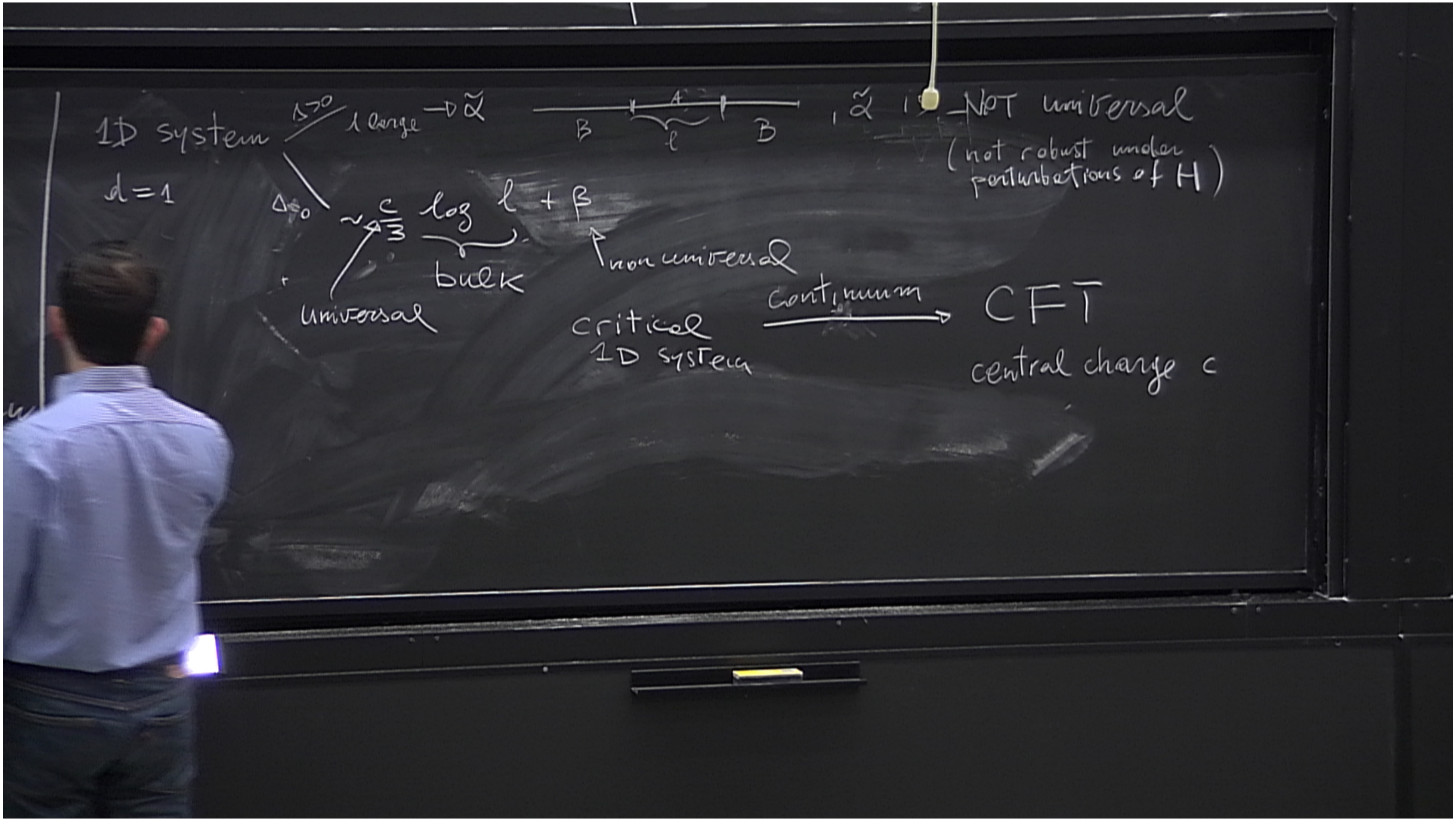
1D system $\xrightarrow{\Delta>0}$ l large $\to \tilde{\alpha}$, $\tilde{\alpha}$ is NOT universal (not robust under perturbations of H)

$d=1$

$\Delta=0$ $\sim \underbrace{\frac{c}{3}\log \ell} + \beta$

non universal


1D system
Δ>0 l large → α̃
d = 1
Δ=0
~ c/3 log l + β
bulk
non universal
universal
critical
1D syst
B
A
B
l
, α̃ is NOT universal
(not robust under perturbations of H)



1D system $\xrightarrow{\Delta>0}$ ℓ large $\to \tilde{\alpha}$, $\tilde{\alpha} \sim 1$ NOT universal (not robust under perturbations of H)

$d = 1$

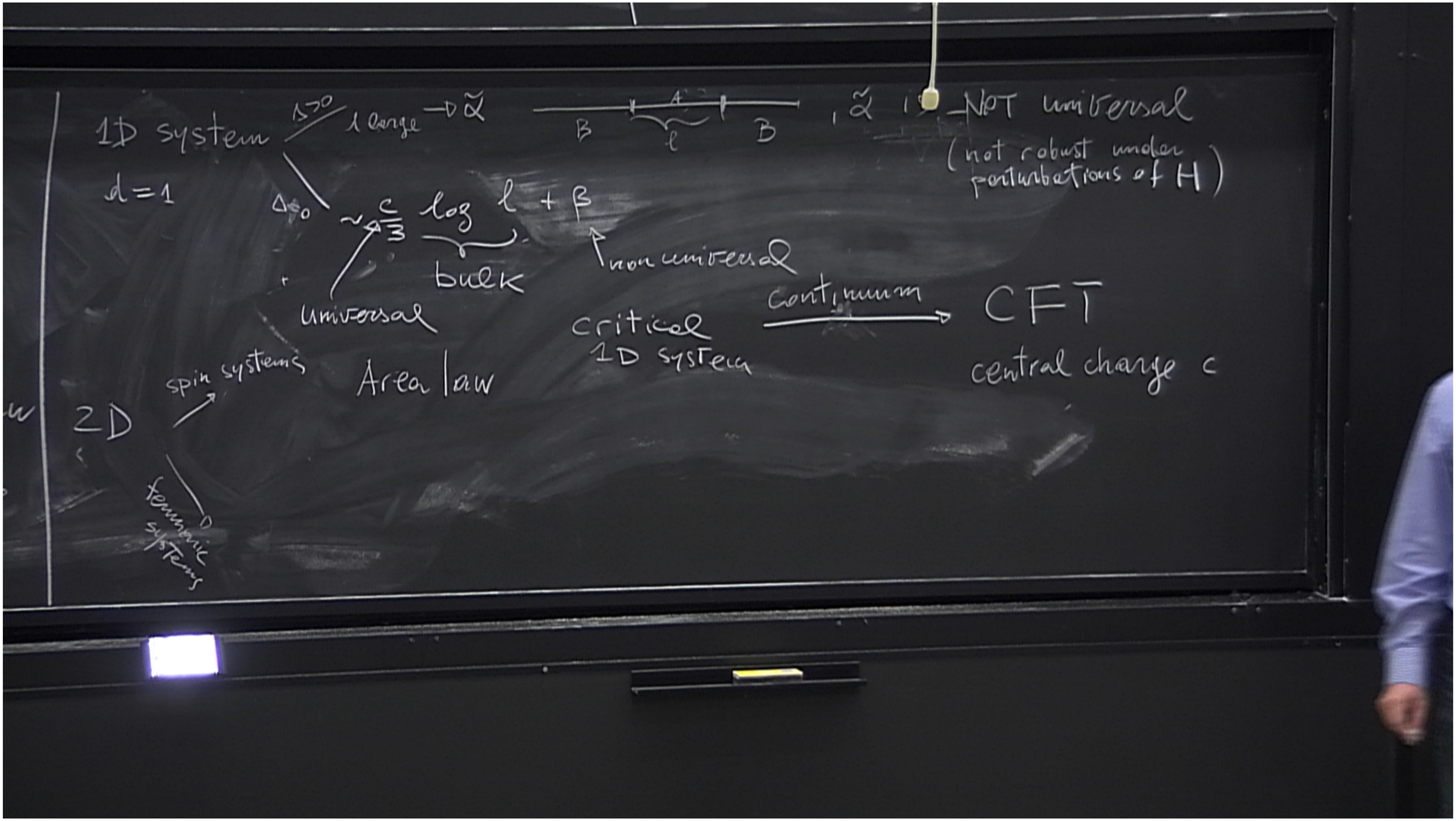
$\xrightarrow{\Delta=0}$ $\sim \underbrace{\frac{c}{3}\log \ell}_{\text{bulk}} + \beta$

universal

non universal

critical 1D system $\xrightarrow{\text{continuum}}$ CFT

central charge c



1D system

$d=1$

$\Delta>0$ l large $\rightarrow \tilde{\alpha}$

$B \;|\; \overset{A}{\underbrace{\quad}_{l}} \;|\; B$

, $\tilde{\alpha}$ is NOT universal (not robust under perturbations of H)

$\Delta=0$ $\sim \underbrace{\frac{c}{3}\log l}_{\text{bulk}} + \beta$

universal

non universal

critical 1D system $\xrightarrow{\text{continuum}}$ CFT

central charge c

Area law

2D

spin systems

fermionic systems

1D system

$d=1$

$\Delta > 0$, ℓ large $\to \tilde{\alpha}$

$B \quad A \quad B$, ℓ

, $\tilde{\alpha}$ is NOT universal (not robust under perturbations of H)

$\Delta = 0$ $\sim \frac{c}{3}\log \ell + \beta$

bulk

β non universal

universal

critical system $\xrightarrow{\text{continuum}}$ CFT

central charge c

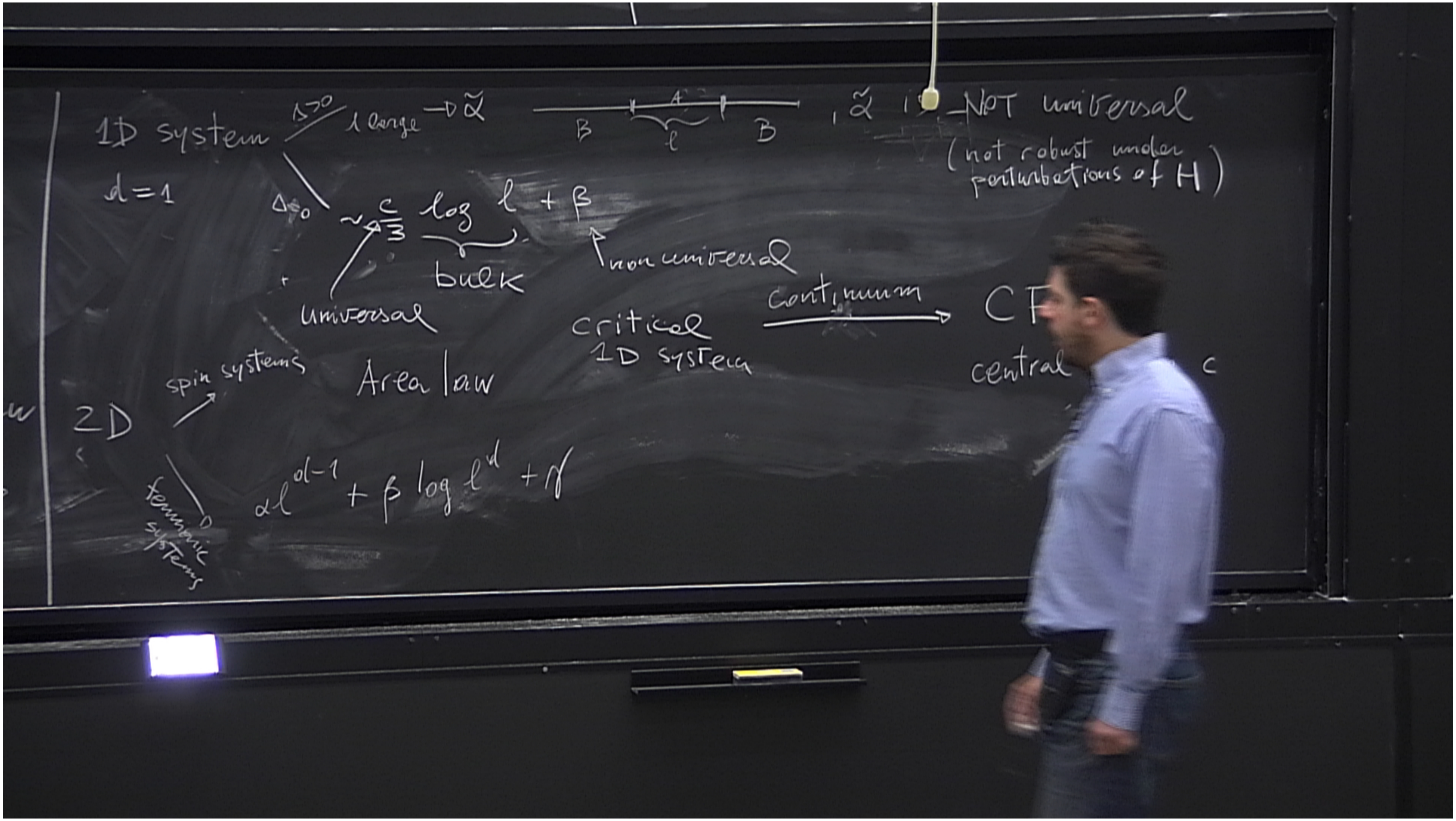
Area law

2D

spin systems

fermionic systems

$\alpha \ell^{d-1} + \beta \log$



1D system $\xrightarrow{\Delta > 0}$ ℓ large $\to \tilde{\alpha}$, $\tilde{\alpha}$ 1 NOT universal

(not robust under perturbations of H)

$d = 1$

$\Delta = 0$ $\sim \underbrace{\frac{c}{3} \log \ell}_{\text{bulk}} + \beta$

universal

non universal

critical 1D system $\xrightarrow{\text{continuum}}$ CF

central c

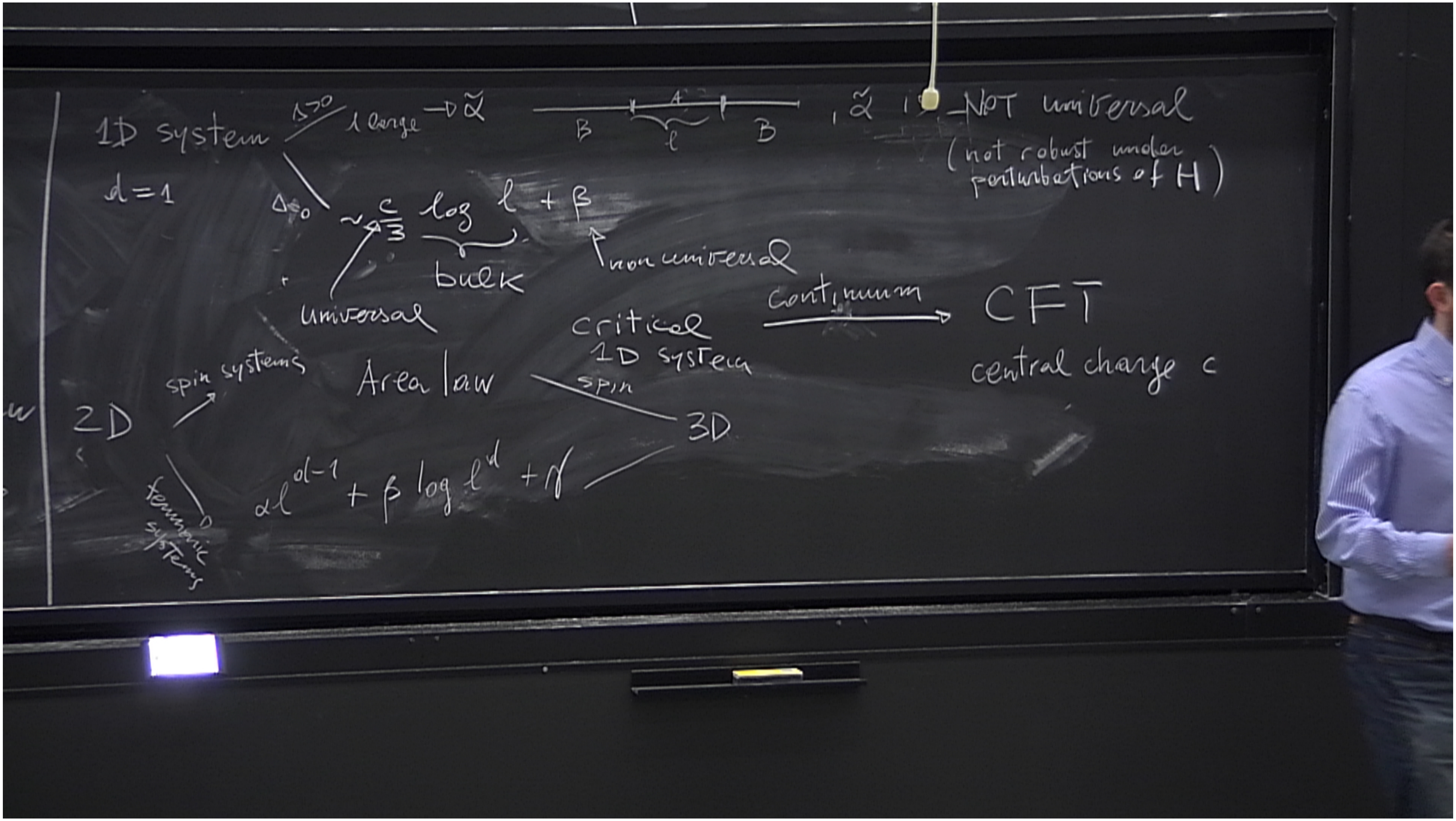
2D

spin systems

fermionic systems

Area law

$\alpha \ell^{d-1} + \beta \log \ell^{d} + \gamma$



1D system $\xrightarrow{\Delta>0}$ l large $\to \tilde{\alpha}$, $\tilde{\alpha}$! NOT universal (not robust under perturbations of H)

$d=1$

$\xrightarrow{\Delta=0}$ $\sim \underbrace{\frac{c}{3} \log \ell}_{\text{bulk}} + \beta$

universal

non universal

critical 1D system $\xrightarrow{\text{continuum}}$ CFT

central charge c

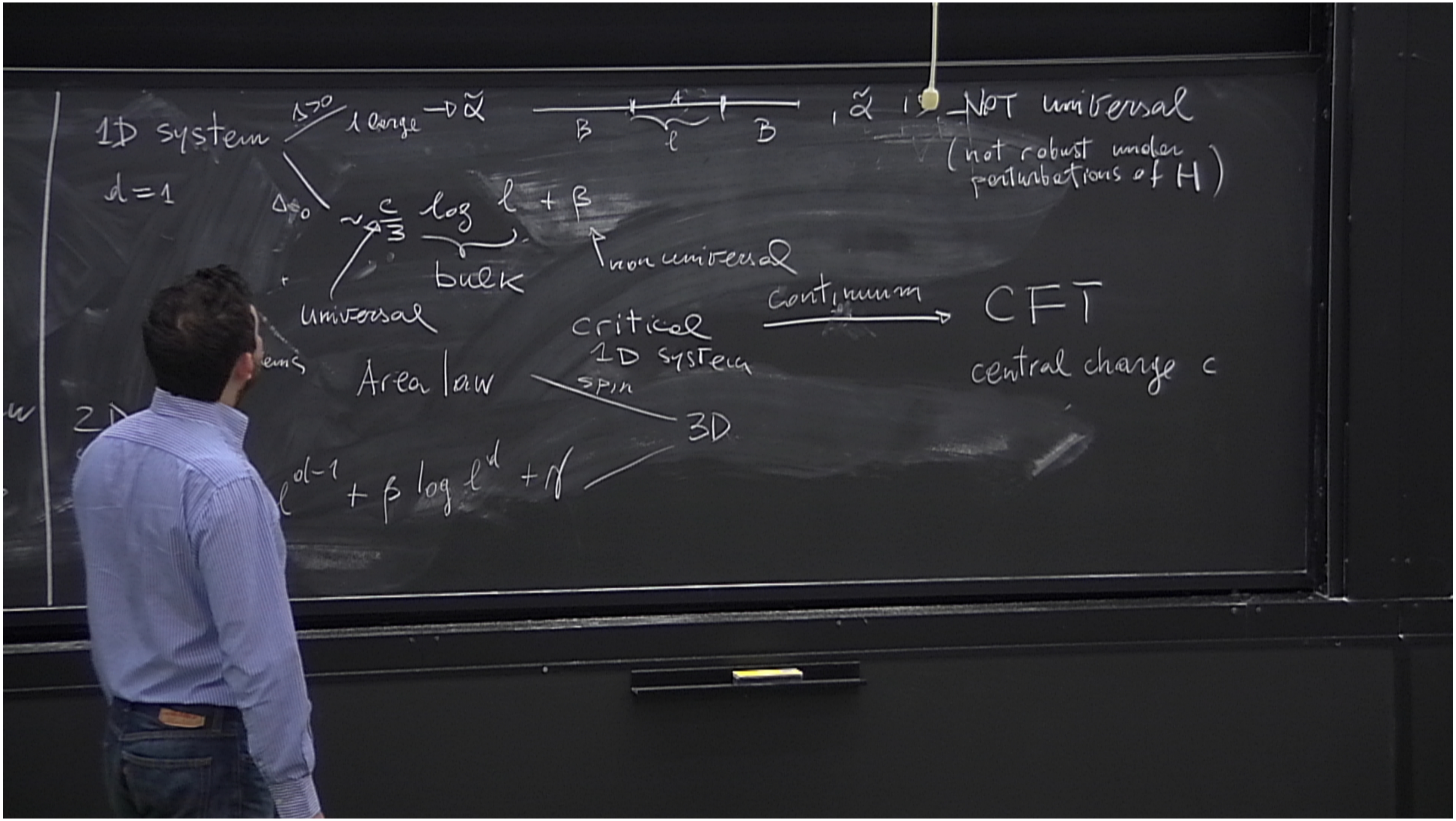
Area law

2D → spin systems

2D → fermionic systems

spin 3D

$\alpha \ell^{d-1} + \beta \log \ell^{d} + \gamma$



1D system

$d=1$

$\Delta > 0$, ℓ large $\to \tilde{\alpha}$

$$\underset{B}{\rule{1cm}{0.4pt}}\underset{\ell}{\underbrace{\overset{A}{\rule{1cm}{0.4pt}}}}\underset{B}{\rule{1cm}{0.4pt}}$$

, $\tilde{\alpha}$ NOT universal (not robust under perturbations of H)

$\Delta = 0$ $\sim \underbrace{\frac{c}{3} \log \ell}_{\text{bulk}} + \beta$

universal

non universal

critical 1D system $\xrightarrow{\text{continuum}}$ CFT

central charge c

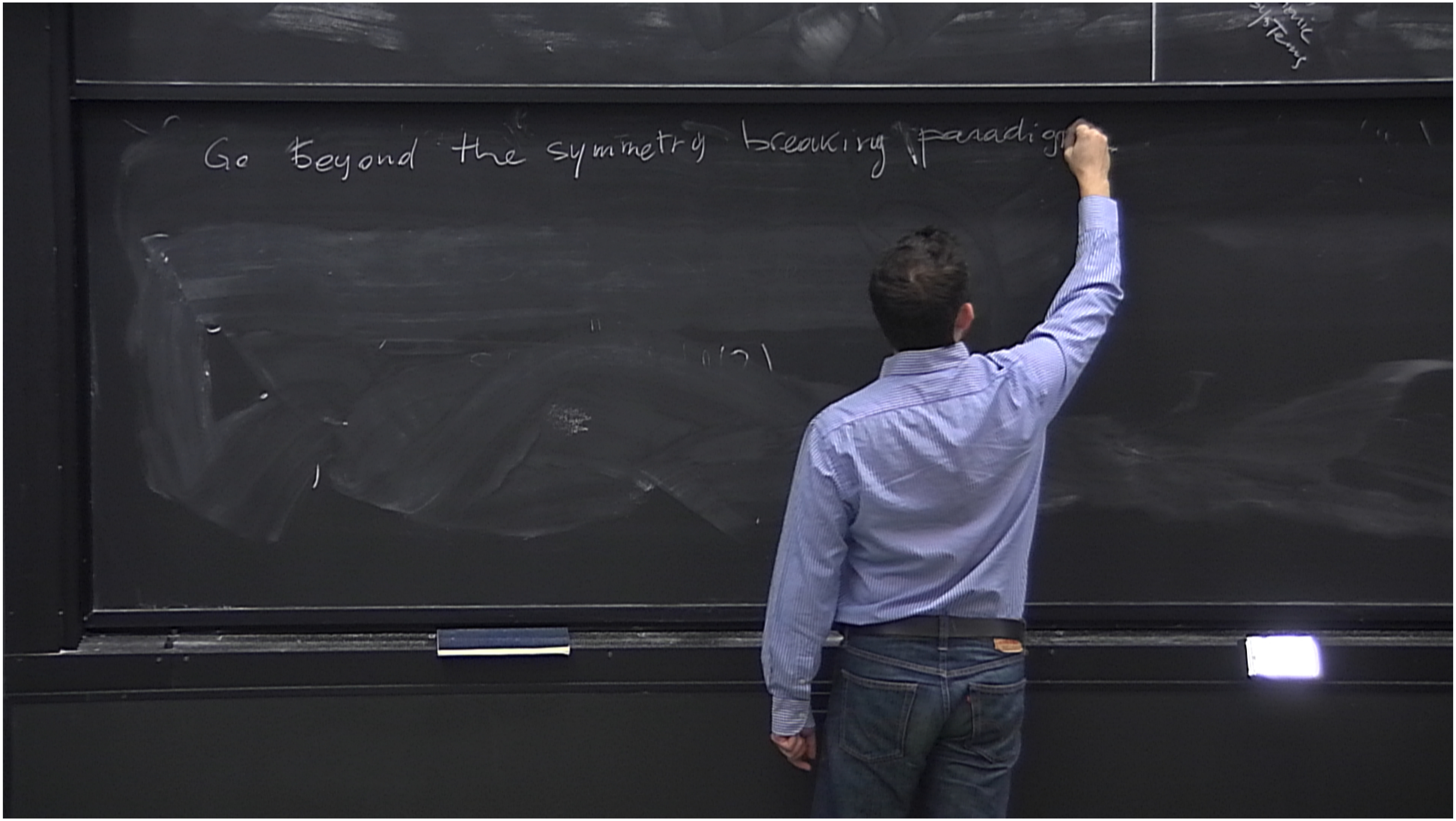
Area law

spin

3D

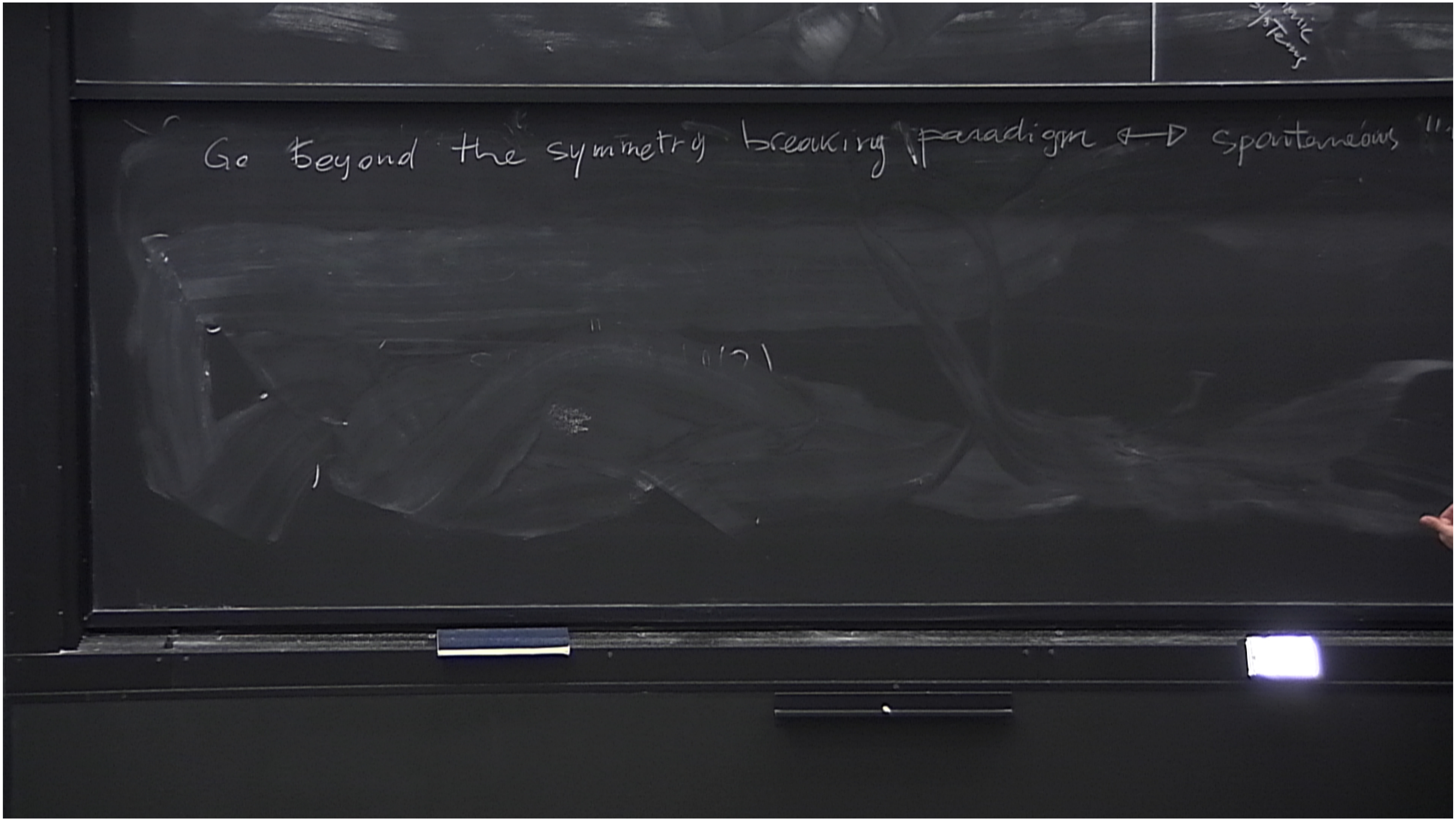
2D

$\ell^{d-1} + \beta \log \ell^{d} + \gamma$



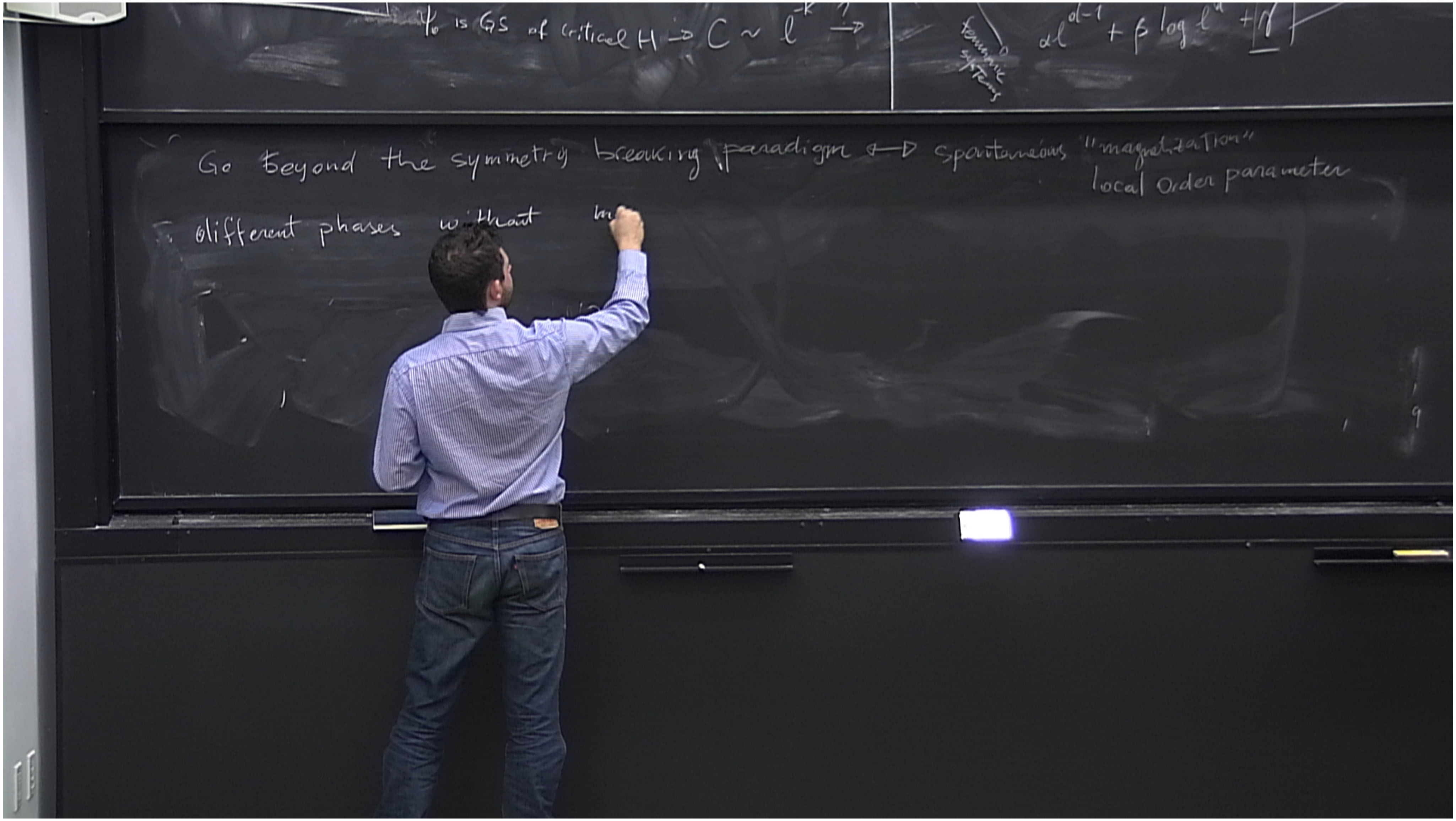
Go beyond the symmetry breaking paradigm

systems



systems

Go beyond the symmetry breaking paradigm ↔ spontaneous "



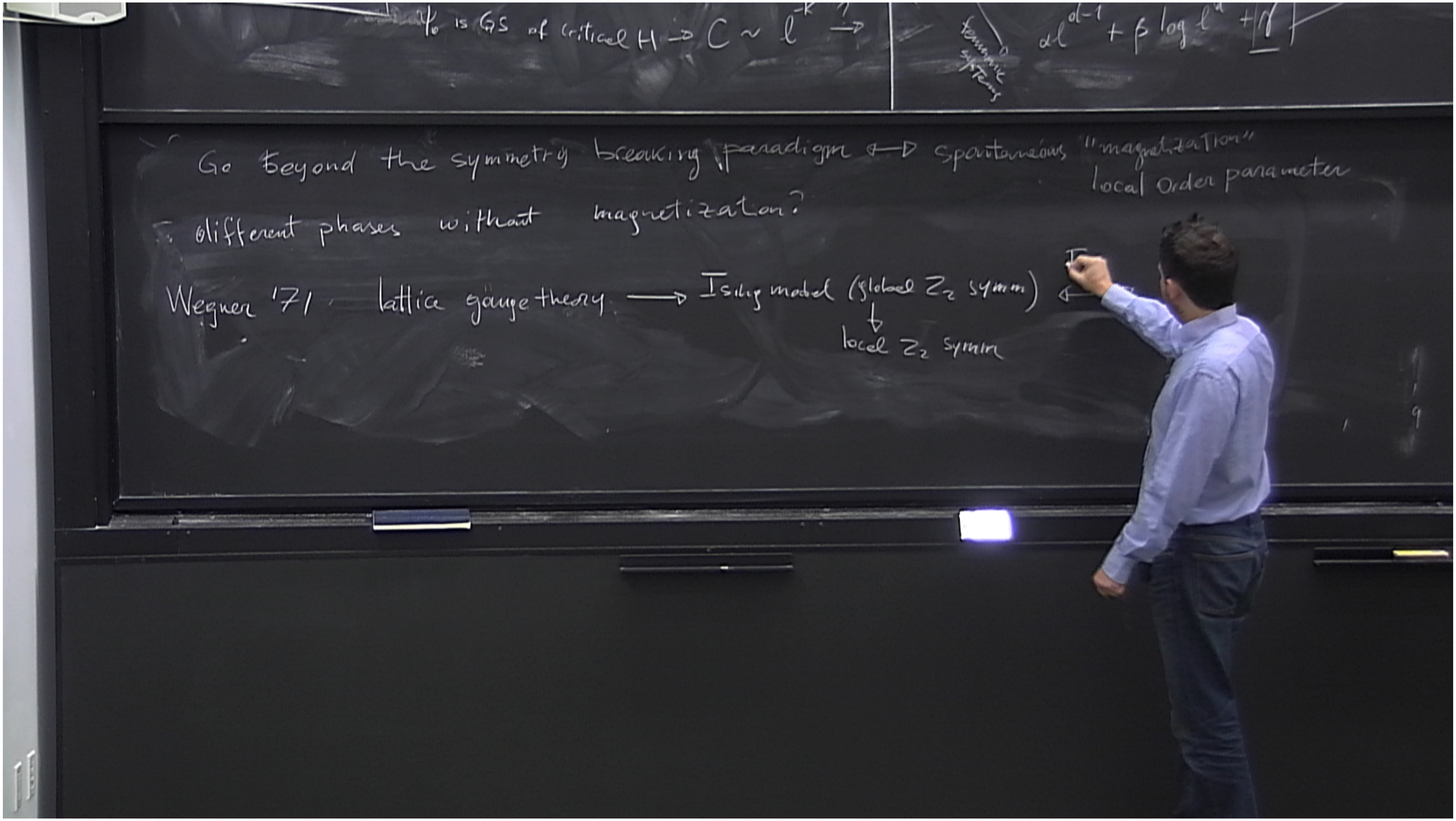
ψ_0 is GS of critical H $\rightarrow C \sim \ell^{-k} \xrightarrow{?}$

fermionic systems

$\alpha \ell^{d-1} + \beta \log \ell^{d} + \boxed{\gamma}$

Go beyond the symmetry breaking paradigm ⟷ spontaneous "magnetization"
local order parameter

different phases without br



Ψ_0 is GS of critical H $\rightarrow C \sim \ell^{-k} \overset{?}{\rightarrow}$

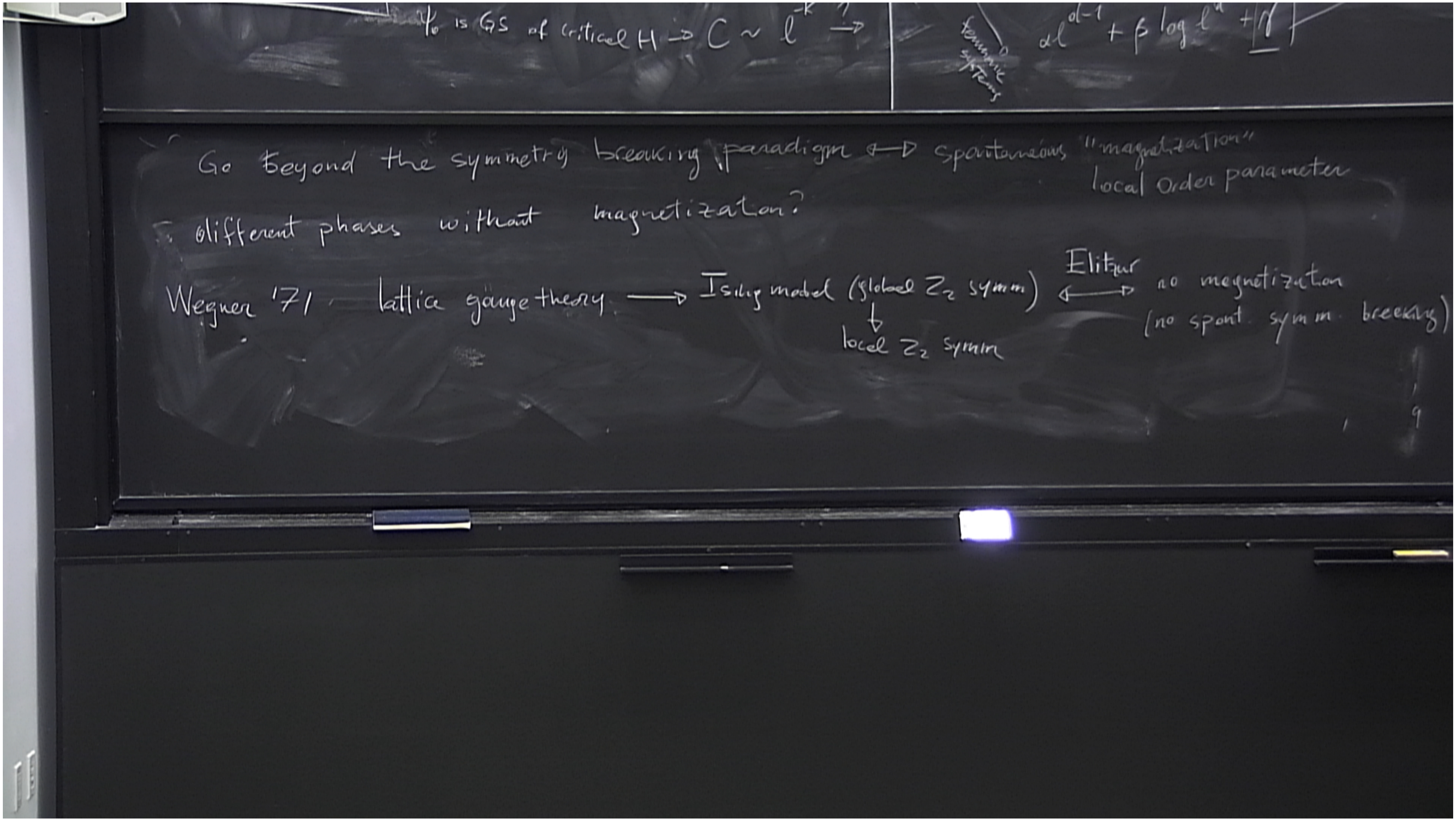
fermionic systems

$\alpha \ell^{d-1} + \beta \log \ell^{d} + \underline{\gamma}$

Go beyond the symmetry breaking paradigm $\leftrightarrow$ spontaneous "magnetization"
local order parameter

different phases without magnetization?

Wegner '71 lattice gauge theory $\longrightarrow$ Ising model (global Z_2 symm) $\leftarrow$ I
$\downarrow$
local Z_2 symm



ψ_0 is GS of critical $H \to C \sim \ell^{-k} \xrightarrow{?}$

fermionic systems

$\alpha \ell^{d-1} + \beta \log \ell^{d} + \boxed{\gamma}$

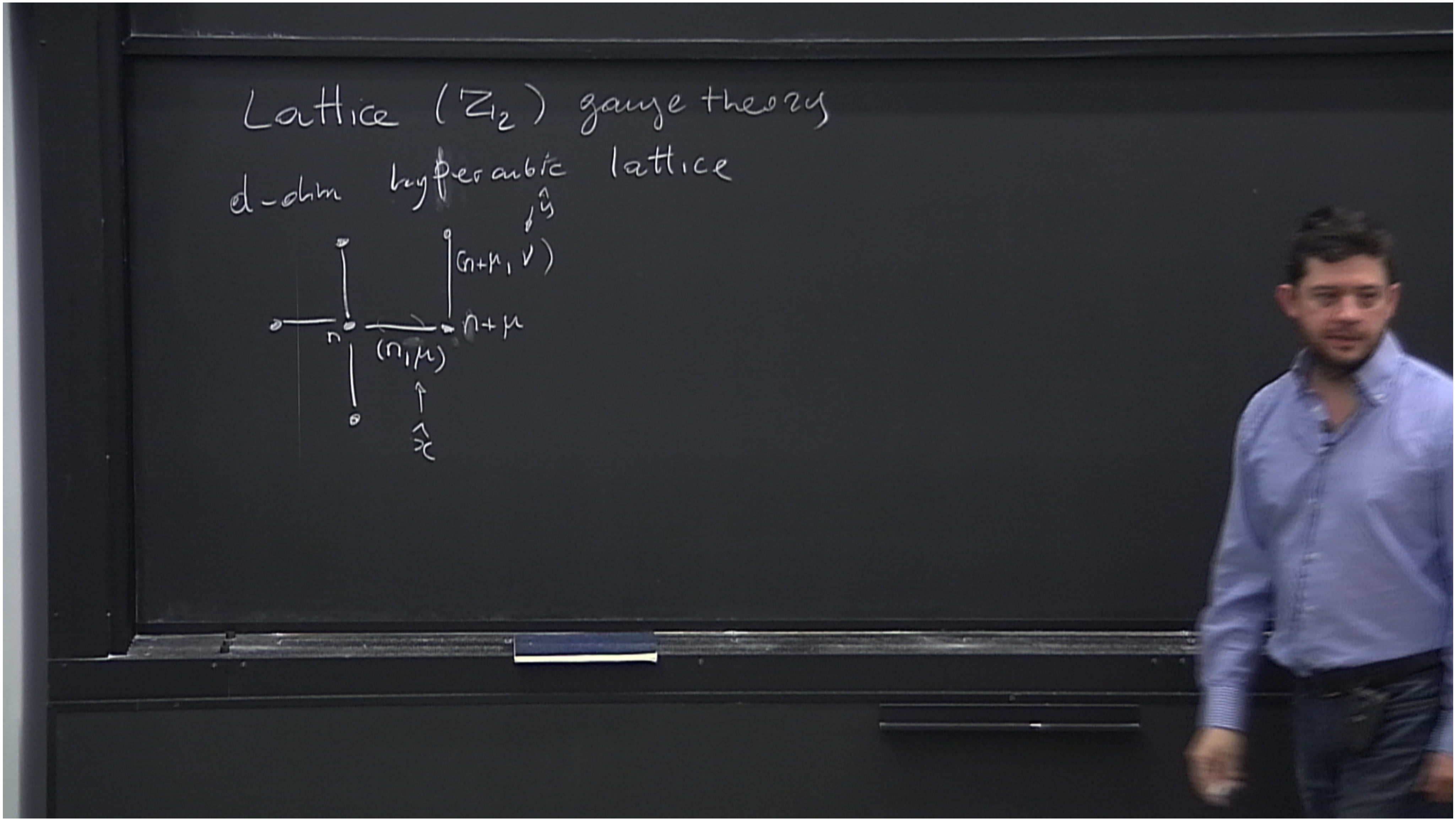
Go beyond the symmetry breaking paradigm $\leftrightarrow$ spontaneous "magnetization"
local order parameter

different phases without magnetization?

Wegner '71 lattice gauge theory $\longrightarrow$ Ising model (global Z_2 symm) $\overset{\text{Elitzur}}{\longleftrightarrow}$ no magnetization
(no spont. symm. breaking)

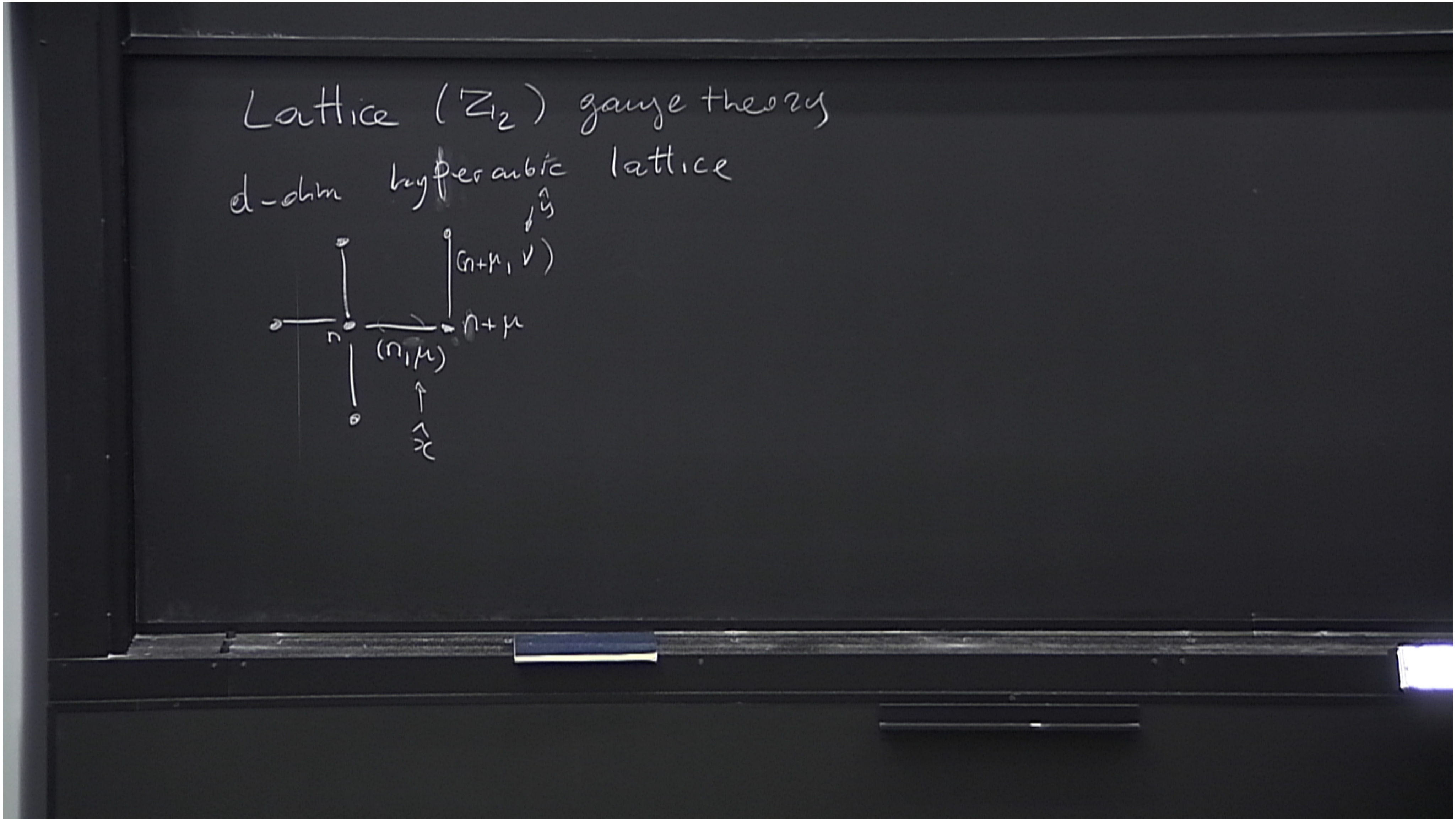
$\downarrow$

local Z_2 symm



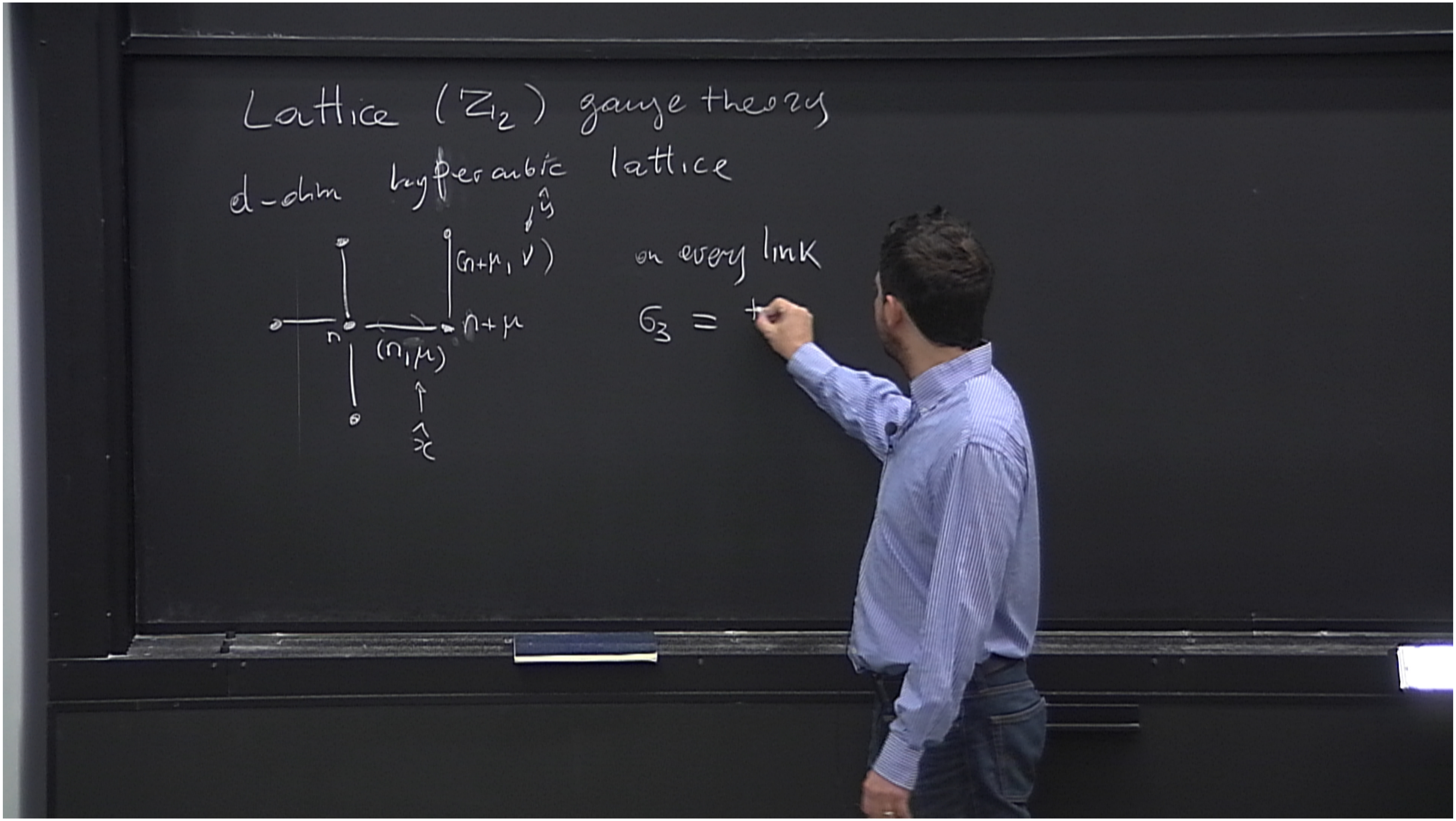
Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice



Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice

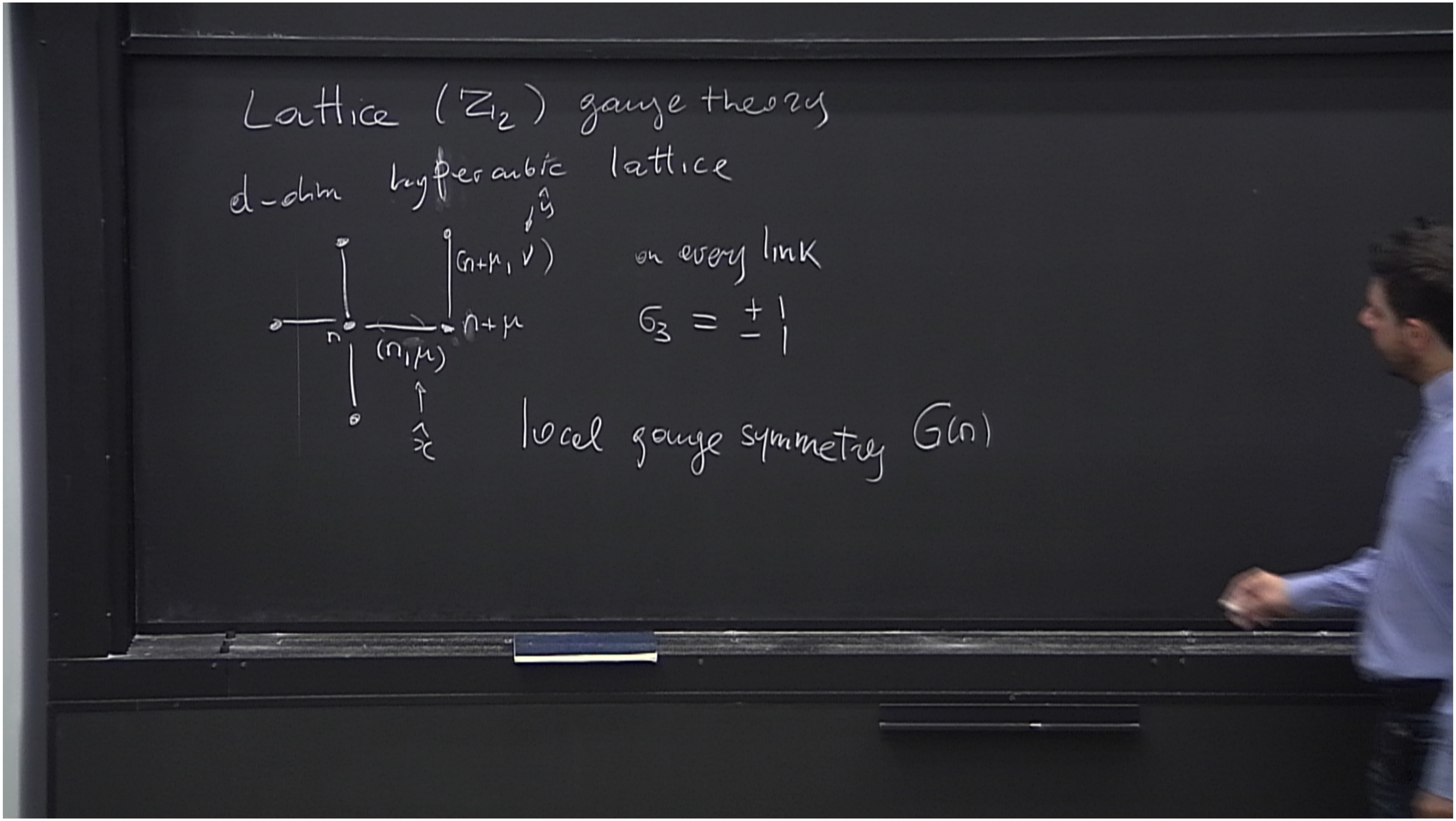


Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice

on every link

$\sigma_3 = +$



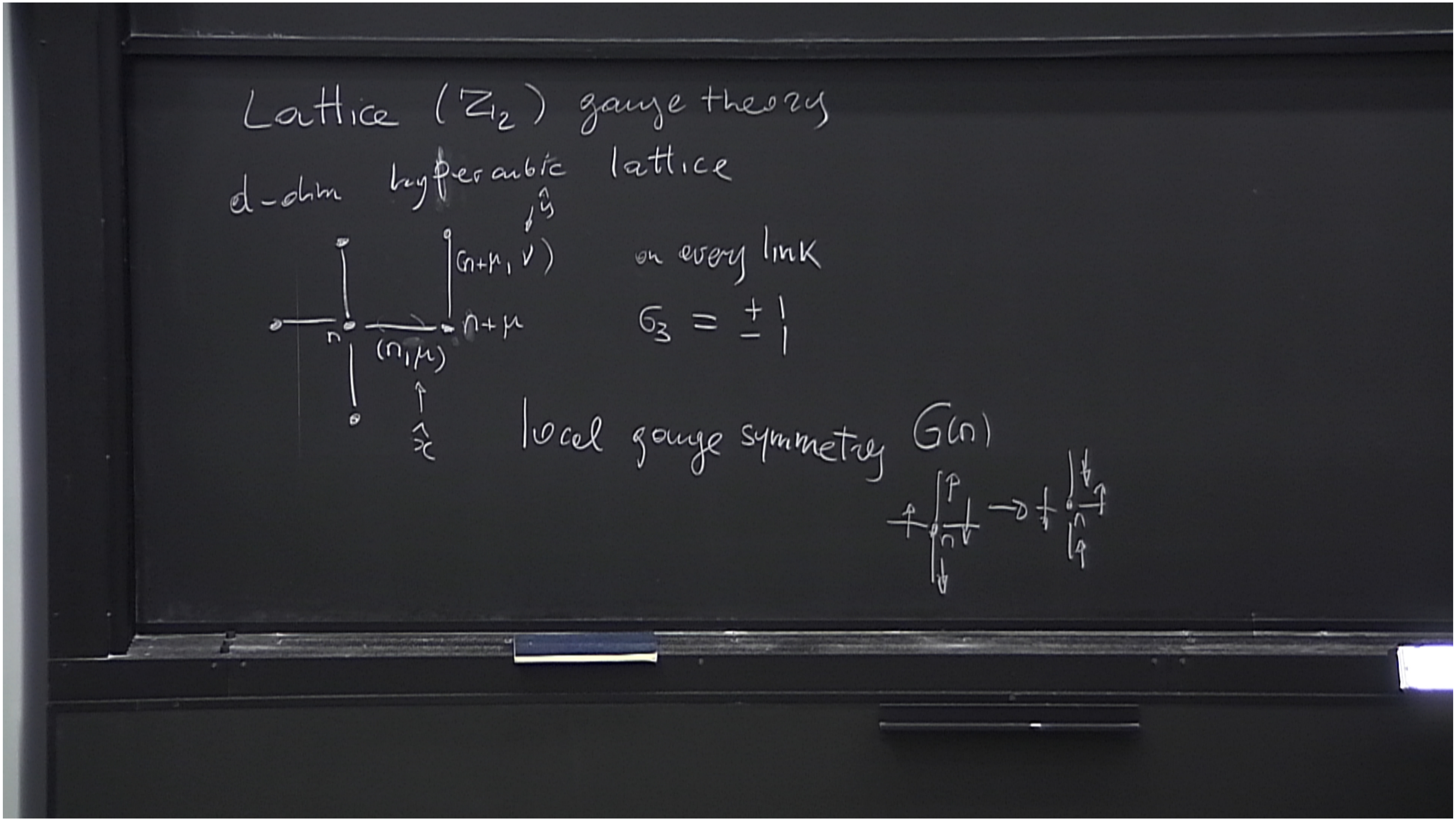
Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice

on every link

$\sigma_3 = \pm 1$

local gauge symmetry $G(n)$



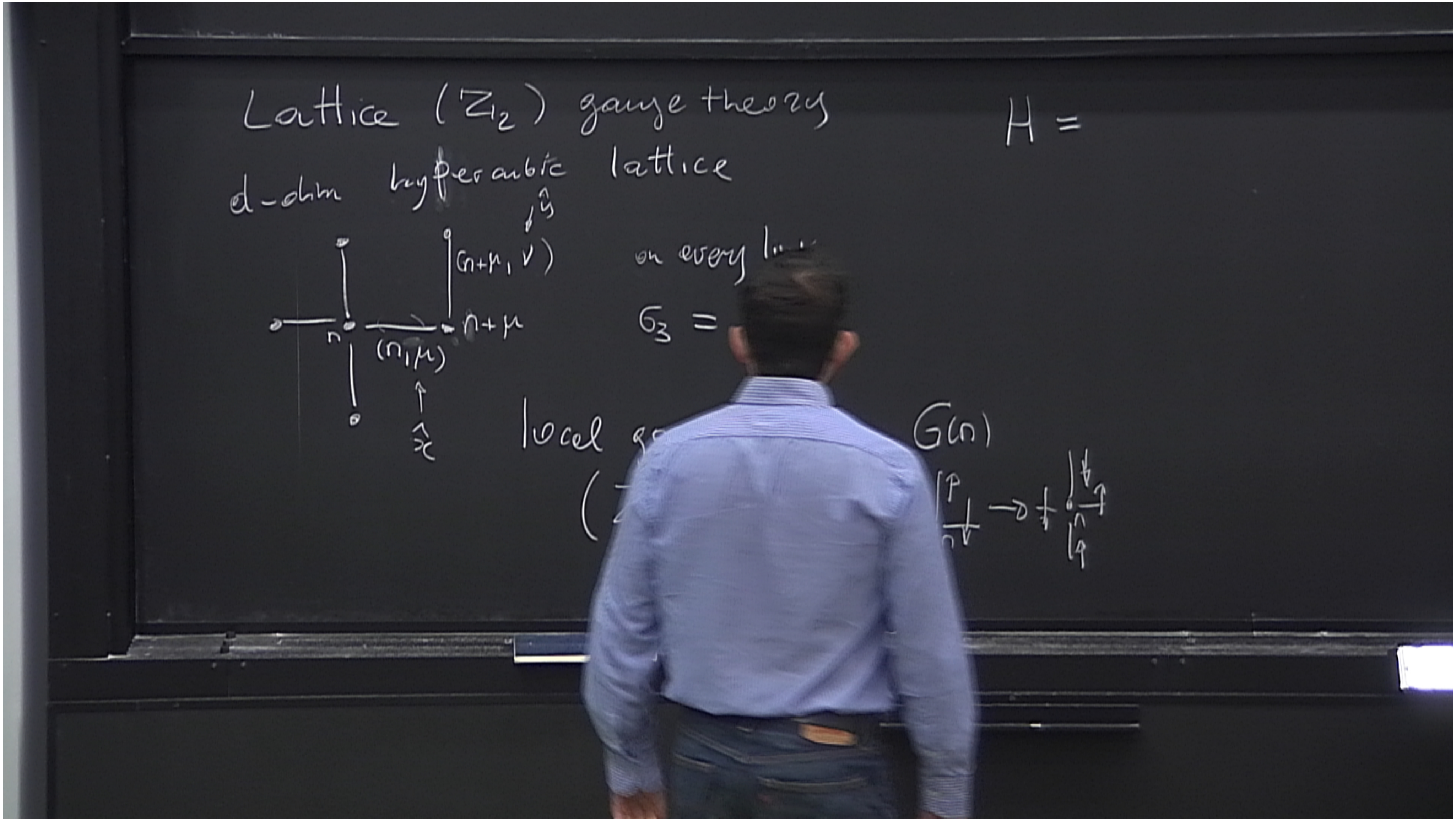
Lattice (Z_2) gauge theory

d-dim hypercubic lattice

on every link

$\sigma_3 = \pm 1$

local gauge symmetry $G(n)$



Lattice ($\mathbb{Z}_2$) gauge theory

$H =$

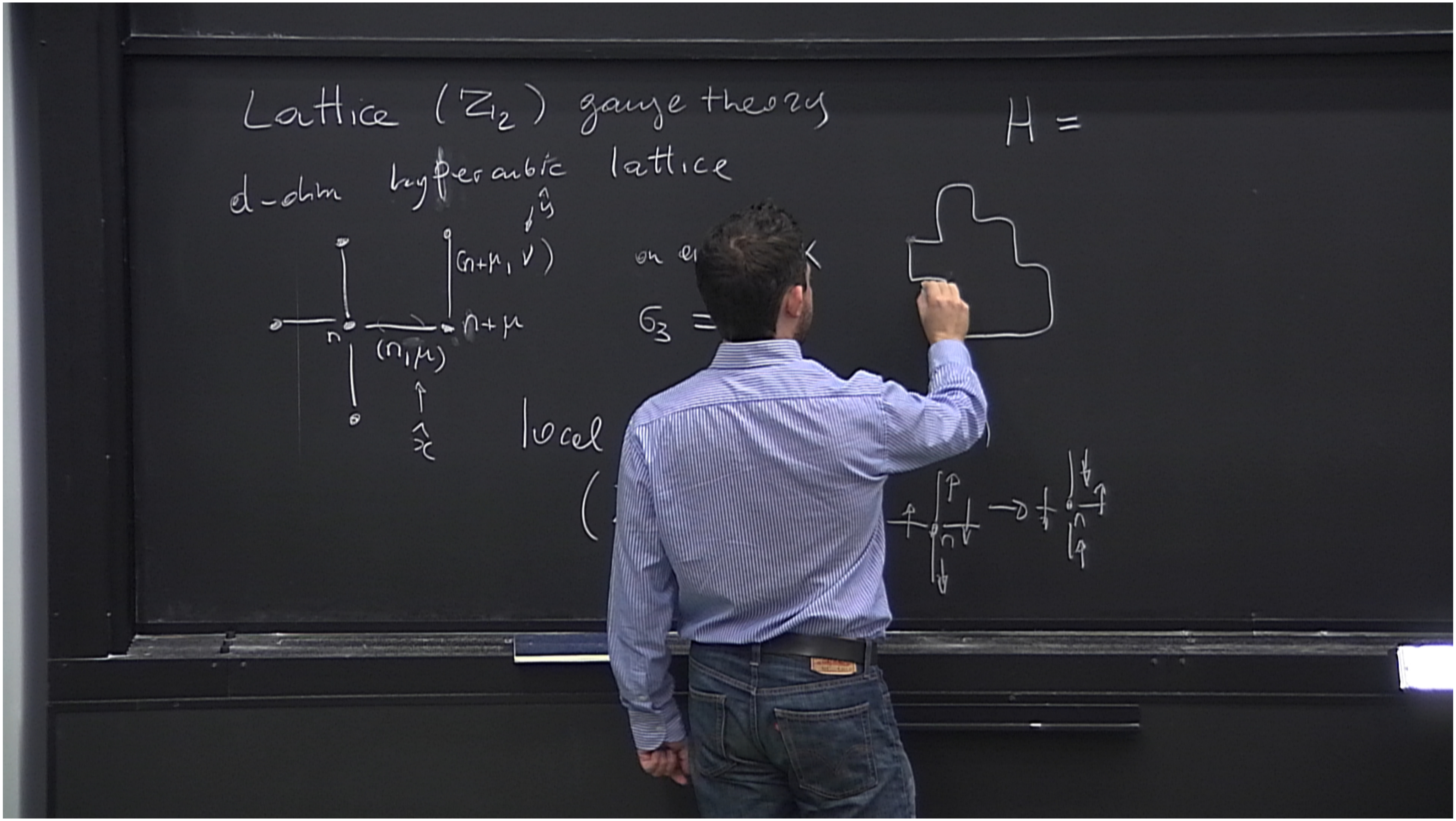
d-dim hypercubic lattice

on every l

$\sigma_3 =$

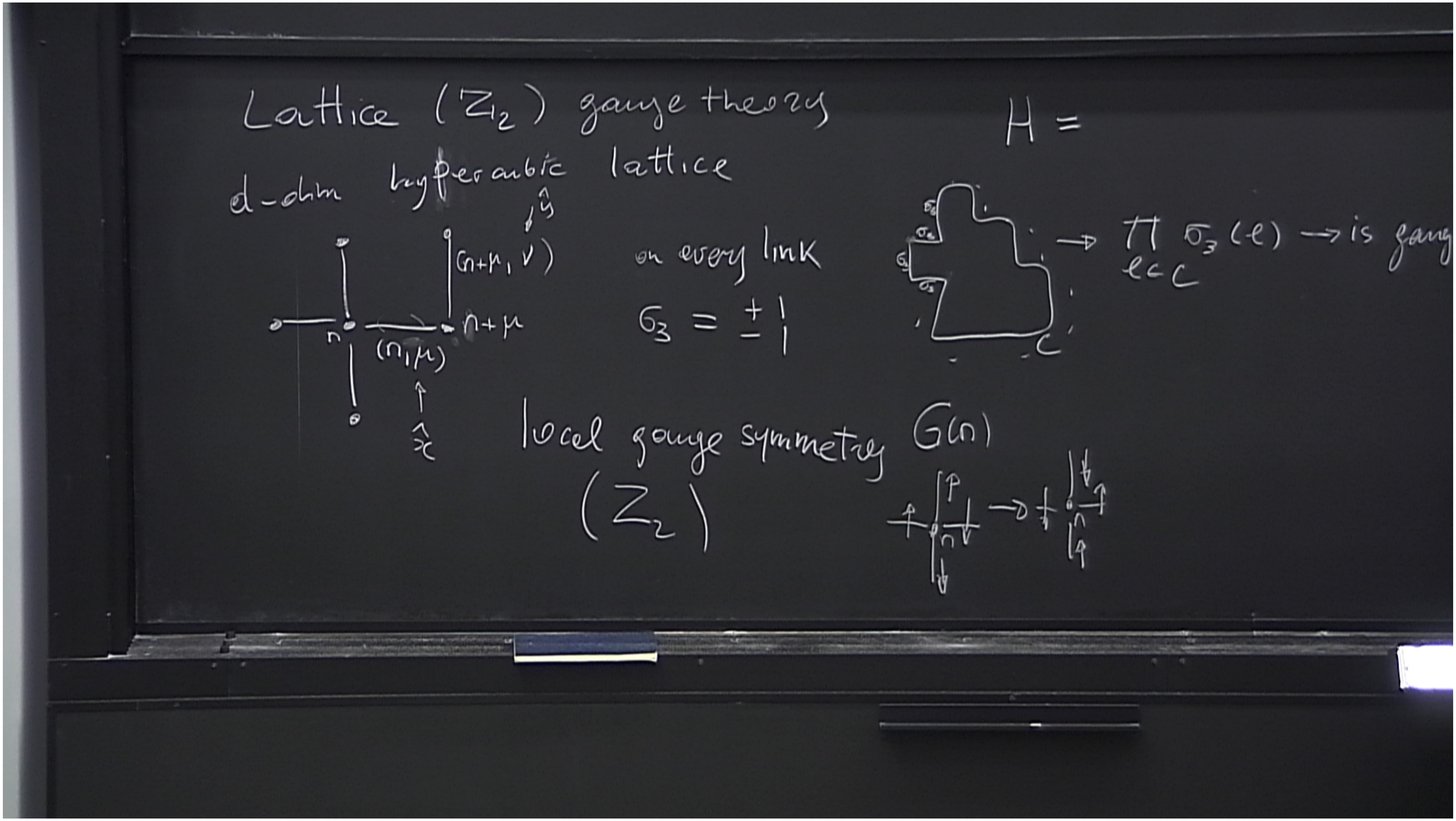
local g

$G(n)$



Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice



Lattice ($\mathbb{Z}_2$) gauge theory

d-dim hypercubic lattice

on every link

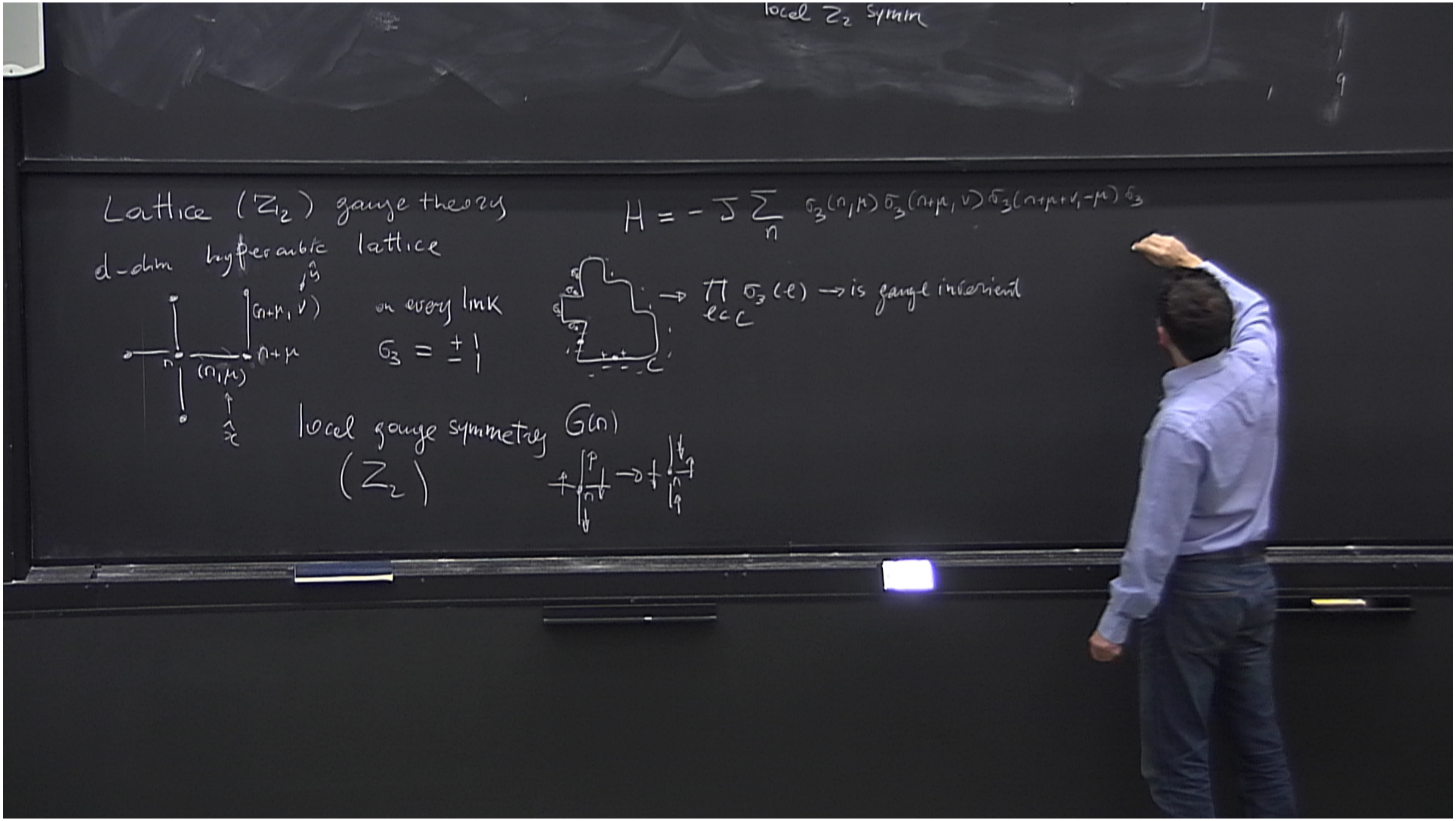
$\sigma_3 = \pm 1$

local gauge symmetry $G(n)$

($\mathbb{Z}_2$)

$H =$

$\xrightarrow{} \prod_{\ell \in C} \sigma_3(\ell) \longrightarrow$ is gaug



local Z_2 symm

Lattice (Z_2) gauge theory

d-dim hypercubic lattice

$(n+\mu, \nu)$ $\hat{y}$

n (n, μ) $n+\mu$

$\hat{x}$

on every link

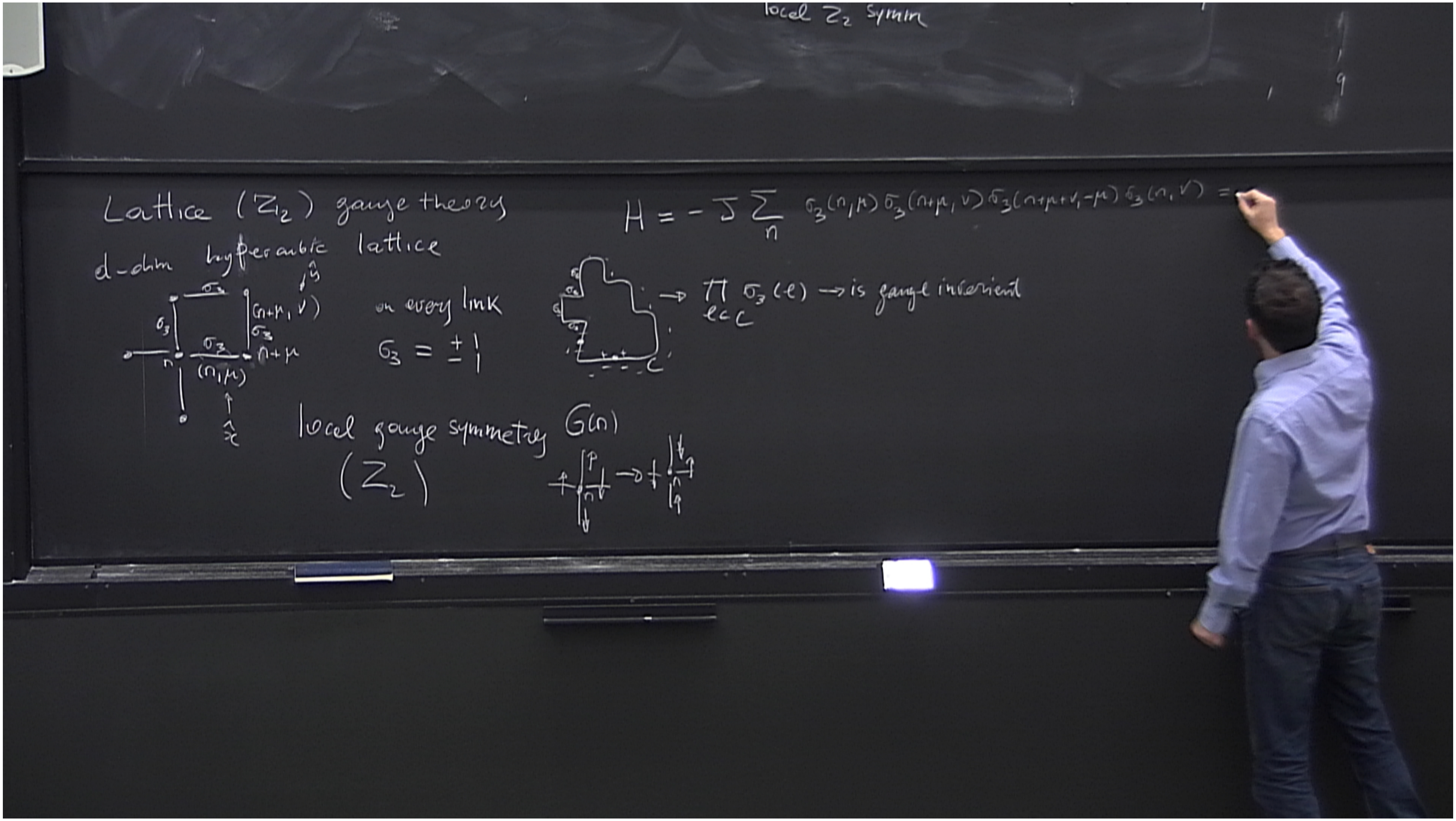
$\sigma_3 = \pm 1$

local gauge symmetry $G(n)$

(Z_2)

$$H = -J \sum_{n} \sigma_3(n,\mu)\, \sigma_3(n+\mu,\nu)\, \sigma_3(n+\mu+\nu,-\mu)\, \sigma_3$$

$\rightarrow \prod_{\ell \in C} \sigma_3(\ell) \rightarrow$ is gauge invariant



local Z_2 symm

Lattice (Z_2) gauge theory

d-dim hypercubic lattice

on every link

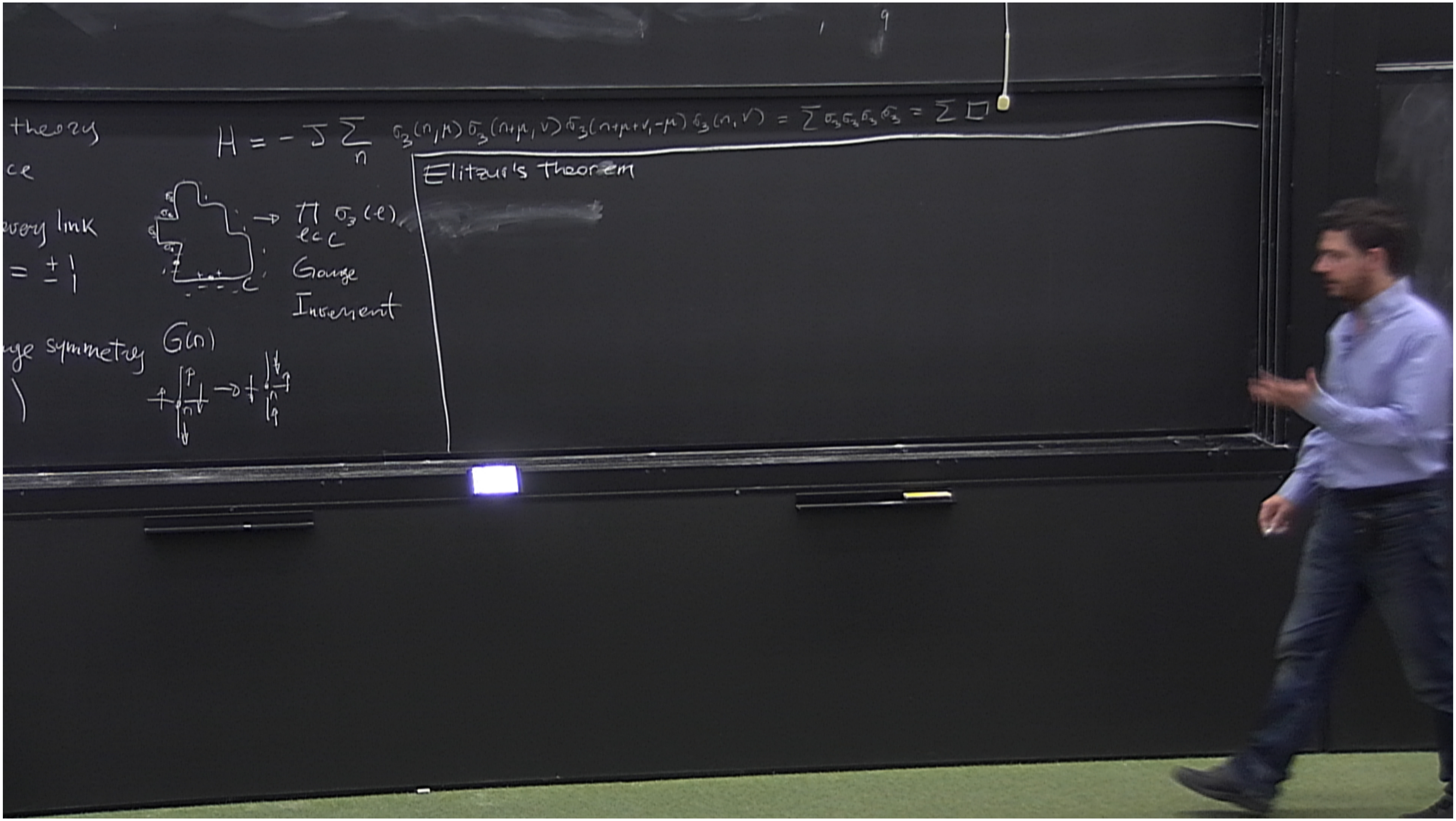
$\sigma_3 = \pm 1$

local gauge symmetry $G(n)$

(Z_2)

$$H = -J \sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) =$$

$\rightarrow \prod_{\ell \in C} \sigma_3(\ell) \rightarrow$ is gauge invariant



theory

ce

every link

$= \pm 1$

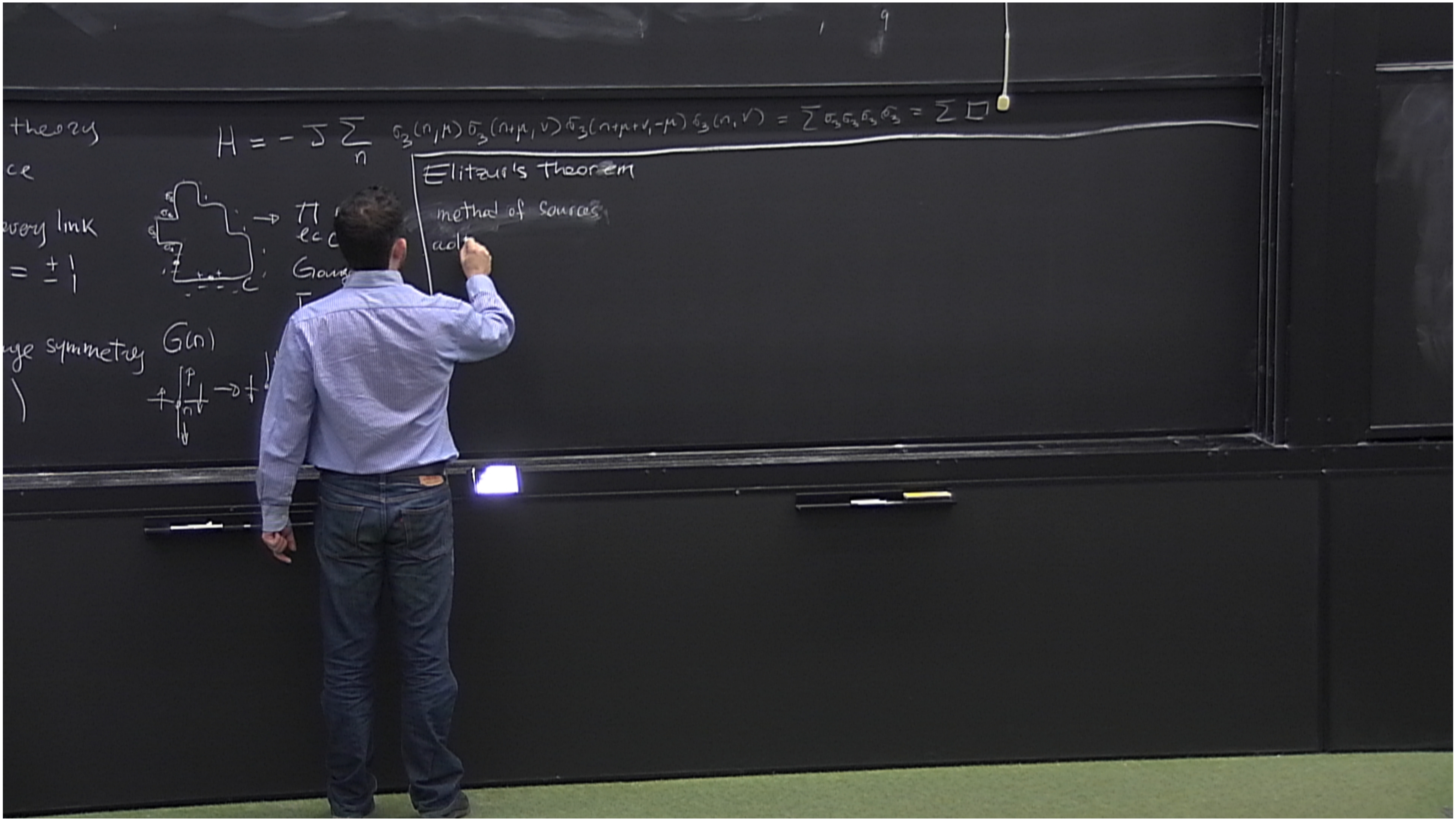
ge symmetry $G(n)$

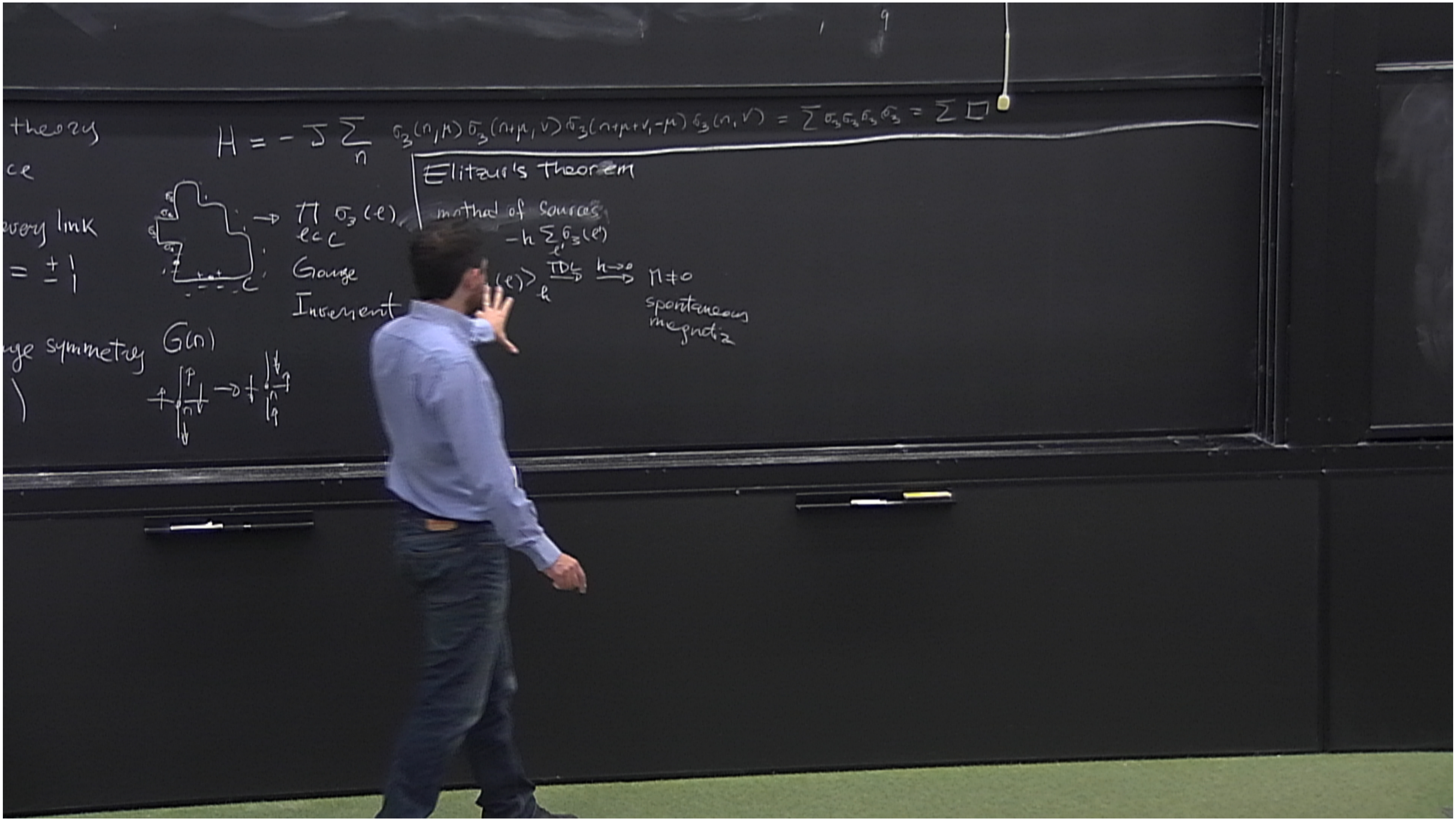
$$H = -J \sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \square$$

$\rightarrow \prod_{l \in C} \sigma_3(l)$

Gauge Invariant

Elitzur's Theorem


theory
ce
every link
= ±1
ge symmetry
G(n)
$H = -J \sum_n \sigma_3(n,\mu)\sigma_3(n+\mu,\nu)\sigma_3(n+\mu+\nu,-\mu)\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \square$
Elitzur's Theorem
method of sources
add



theory

ce

every link

$= \pm 1$

ge symmetry $G(n)$

$$H = -J\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \Box$$

$\rightarrow \prod_{\ell\in C} \sigma_3(\ell)$

Gauge Invariant

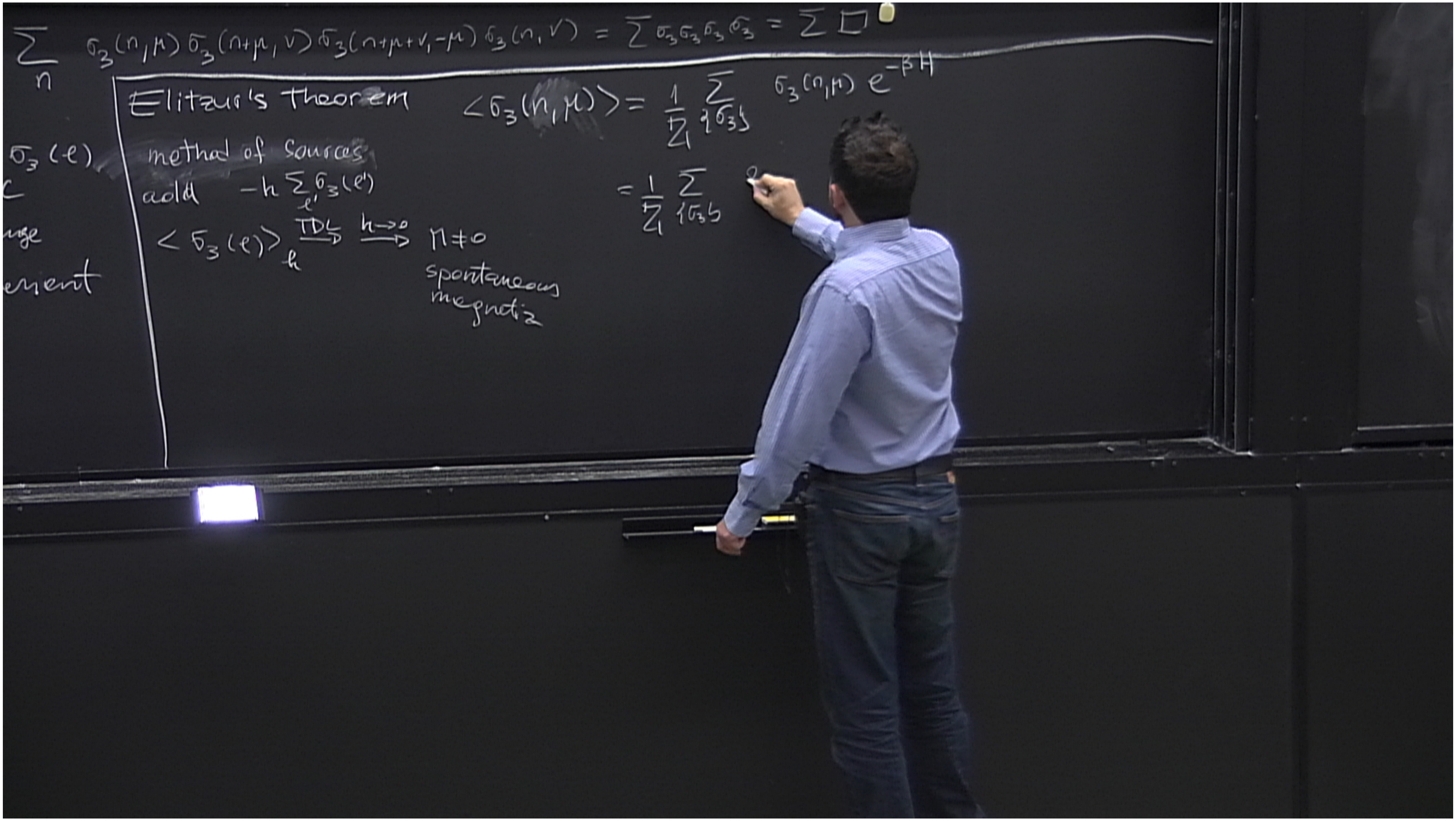
Elitzur's Theorem

method of sources

$-h\sum_{\ell'}\sigma_3(\ell')$

$(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M \neq 0$

spontaneous magnetiz



$$\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \Box$$

$\sigma_3(\ell)$

c

ge

ement

Elitzur's theorem $\langle \sigma_3(n,\mu)\rangle = \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{-\beta H}$

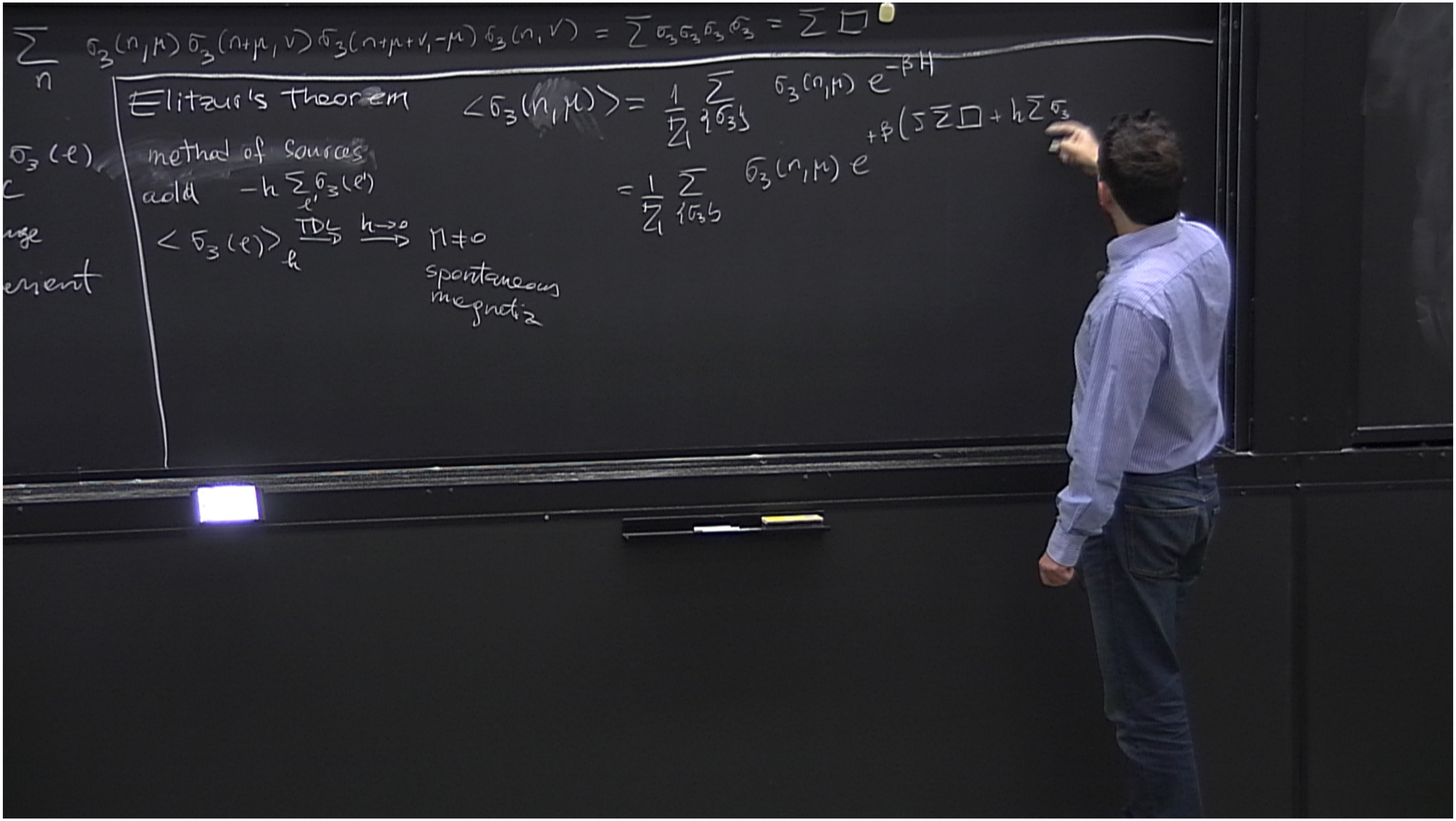
$= \frac{1}{Z_1}\sum_{\{\sigma_3\}}$

method of sources

add $-h\sum_{\ell'}\sigma_3(\ell')$

$\langle \sigma_3(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M \neq 0$

spontaneous
magnetiz



$$\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \Box$$

$\sigma_3(\ell)$

c

uge

ement

Elitzur's theorem

$$\langle\sigma_3(n,\mu)\rangle = \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{-\beta H}$$

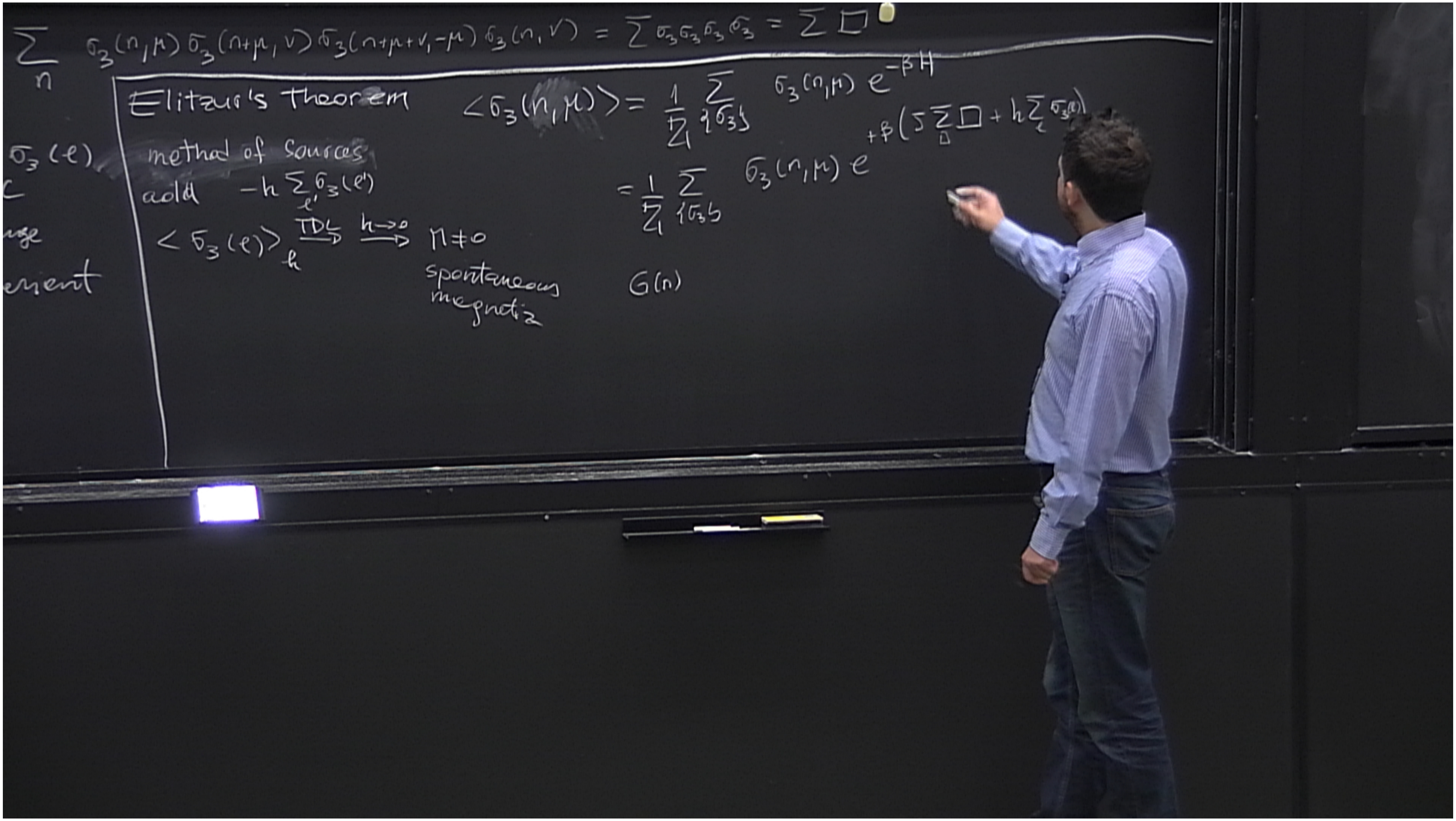
$$= \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{+\beta(J\sum\Box + h\sum\sigma_3}$$

method of sources

add $-h\sum_{\ell'}\sigma_3(\ell')$

$\langle\sigma_3(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M\neq 0$

spontaneous magnetiz



$$\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \square$$

$\sigma_3(\ell)$

C

nge

ement

Elitzur's theorem

$$\langle \sigma_3(n,\mu)\rangle = \frac{1}{Z}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{-\beta H}$$

$$= \frac{1}{Z}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{+\beta\left(J\sum_{\square}\square + h\sum_{\ell}\sigma_3(\ell)\right)}$$

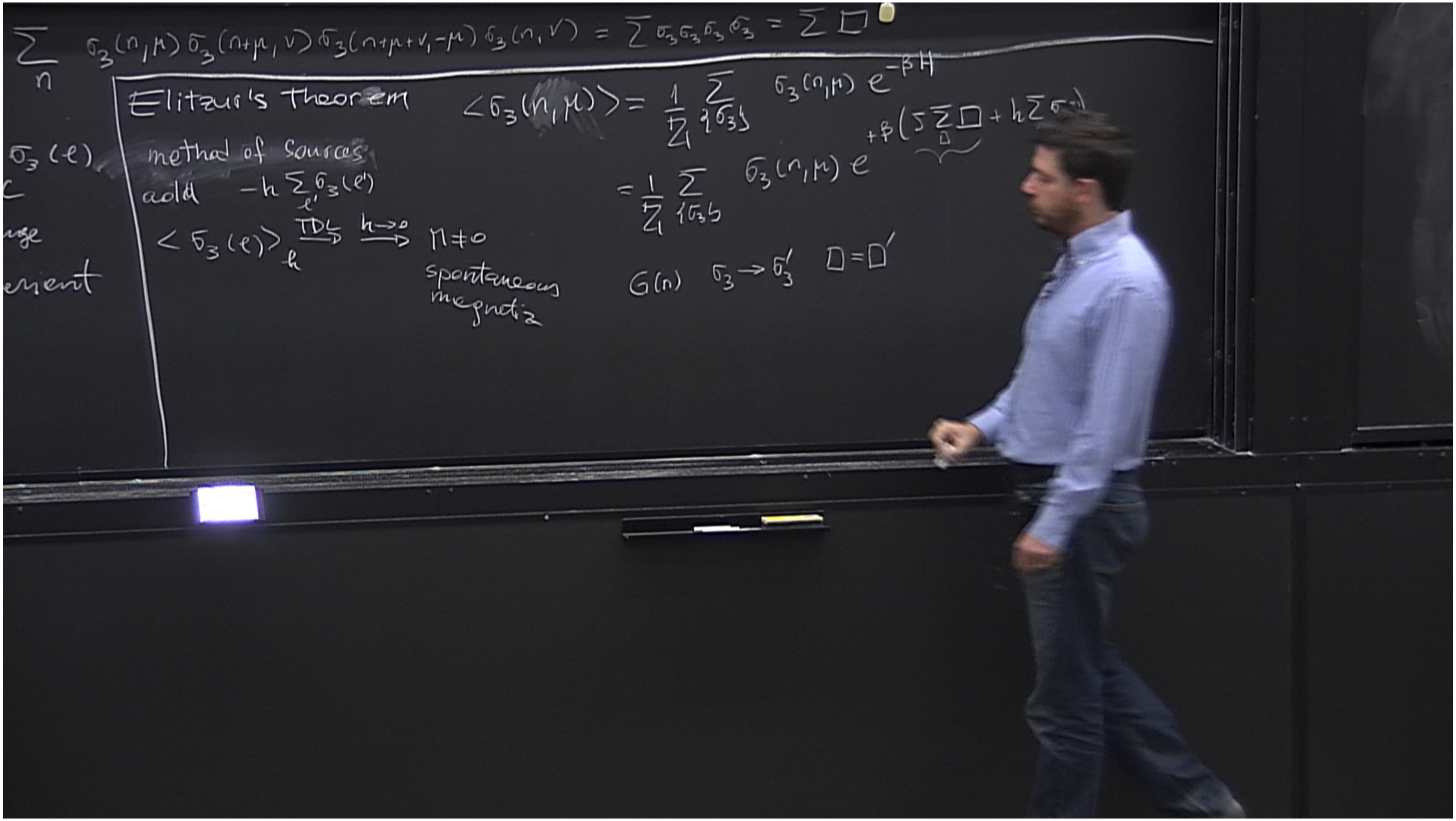
method of sources

add $-h\sum_{\ell'}\sigma_3(\ell')$

$\langle\sigma_3(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M \neq 0$

spontaneous magnetiz

$G(n)$



$$\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \Box$$

$\sigma_3(-e)$

c

nge

ement

Elitzur's theorem

method of sources

add $-h\sum_{\ell'}\sigma_3(\ell')$

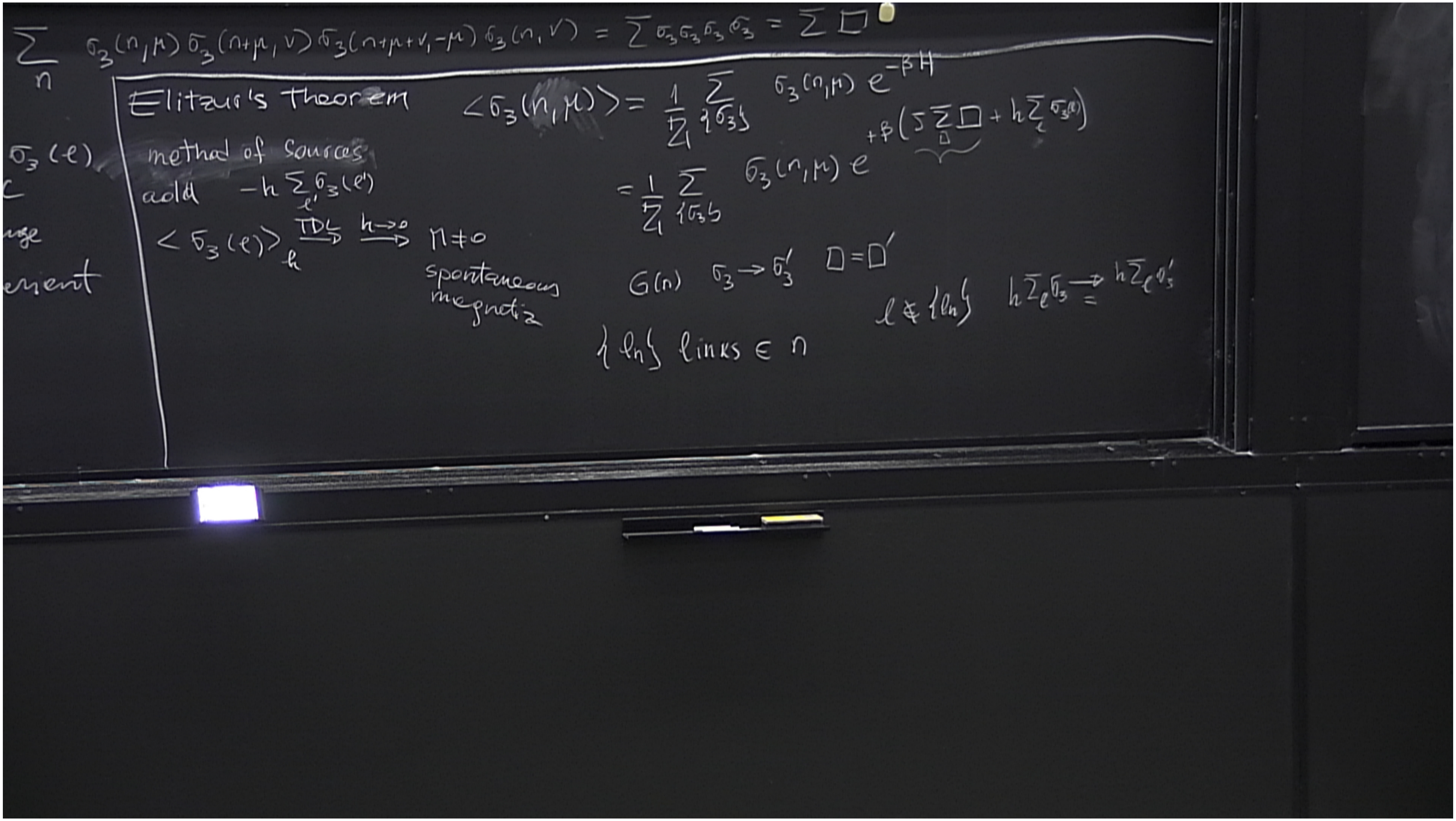
$\langle\sigma_3(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M \neq 0$

spontaneous magnetiz

$$\langle\sigma_3(n,\mu)\rangle = \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{-\beta H}$$

$$= \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{+\beta(J\sum_{\Box}\Box + h\sum\sigma_3)}$$

$G(n) \quad \sigma_3 \to \sigma_3' \quad \Box = \Box'$



$$\sum_n \sigma_3(n,\mu)\,\sigma_3(n+\mu,\nu)\,\sigma_3(n+\mu+\nu,-\mu)\,\sigma_3(n,\nu) = \sum \sigma_3\sigma_3\sigma_3\sigma_3 = \sum \Box$$

$\sigma_3(\ell)$

c

nge

ement

Elitzur's theorem

$$\langle \sigma_3(n,\mu)\rangle = \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{-\beta H}$$

$$= \frac{1}{Z_1}\sum_{\{\sigma_3\}} \sigma_3(n,\mu)\, e^{+\beta\left(\underbrace{J\sum_{\Box}\Box}\, + \,h\sum_{\ell}\sigma_3(\ell)\right)}$$

method of sources

add $-h\sum_{\ell'}\sigma_3(\ell')$

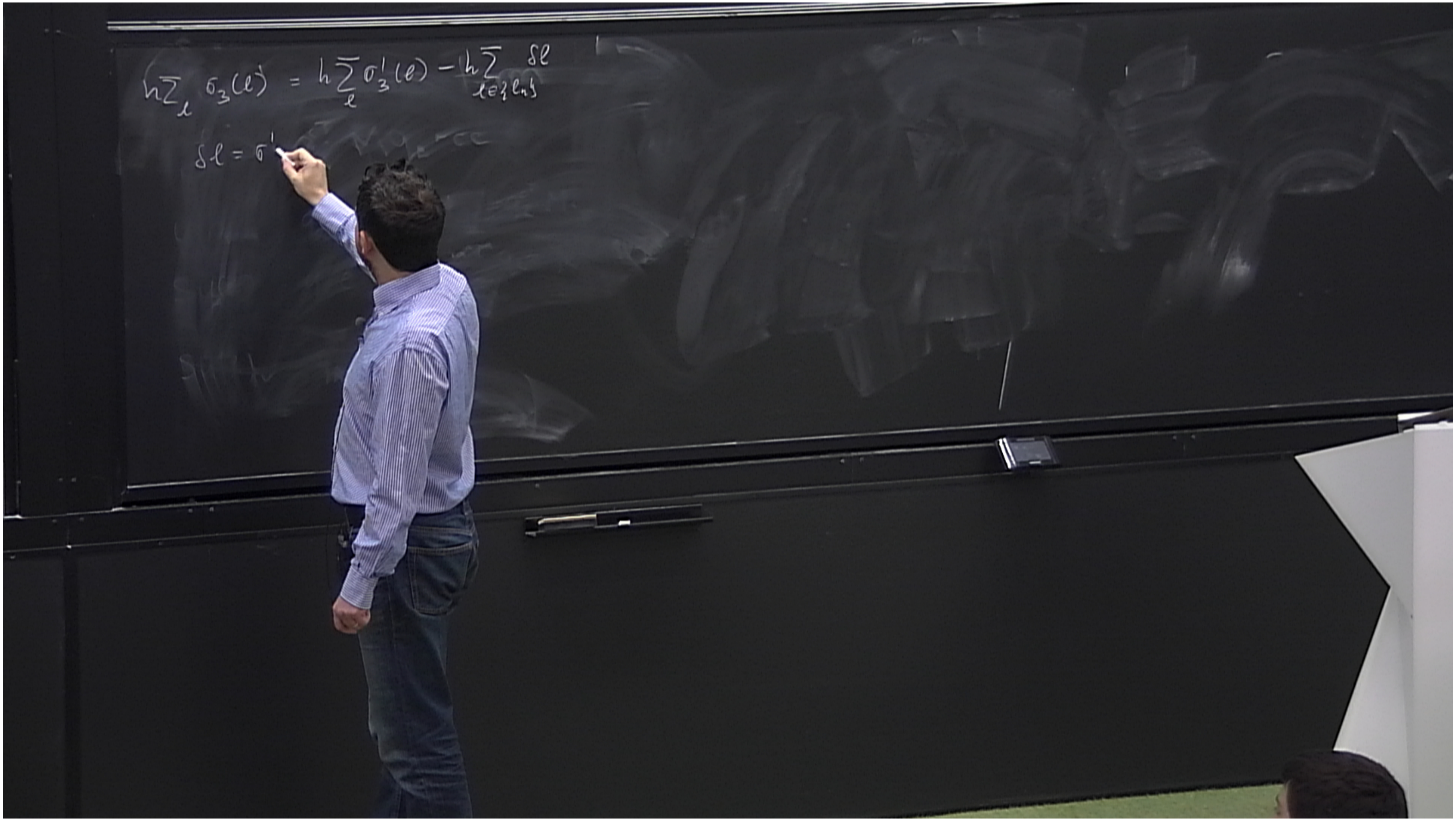
$\langle \sigma_3(\ell)\rangle_h \xrightarrow{TDL} \xrightarrow{h\to 0} M \neq 0$

spontaneous magnetiz

$G(n)$ $\sigma_3 \to \sigma_3'$ $\Box = \Box'$

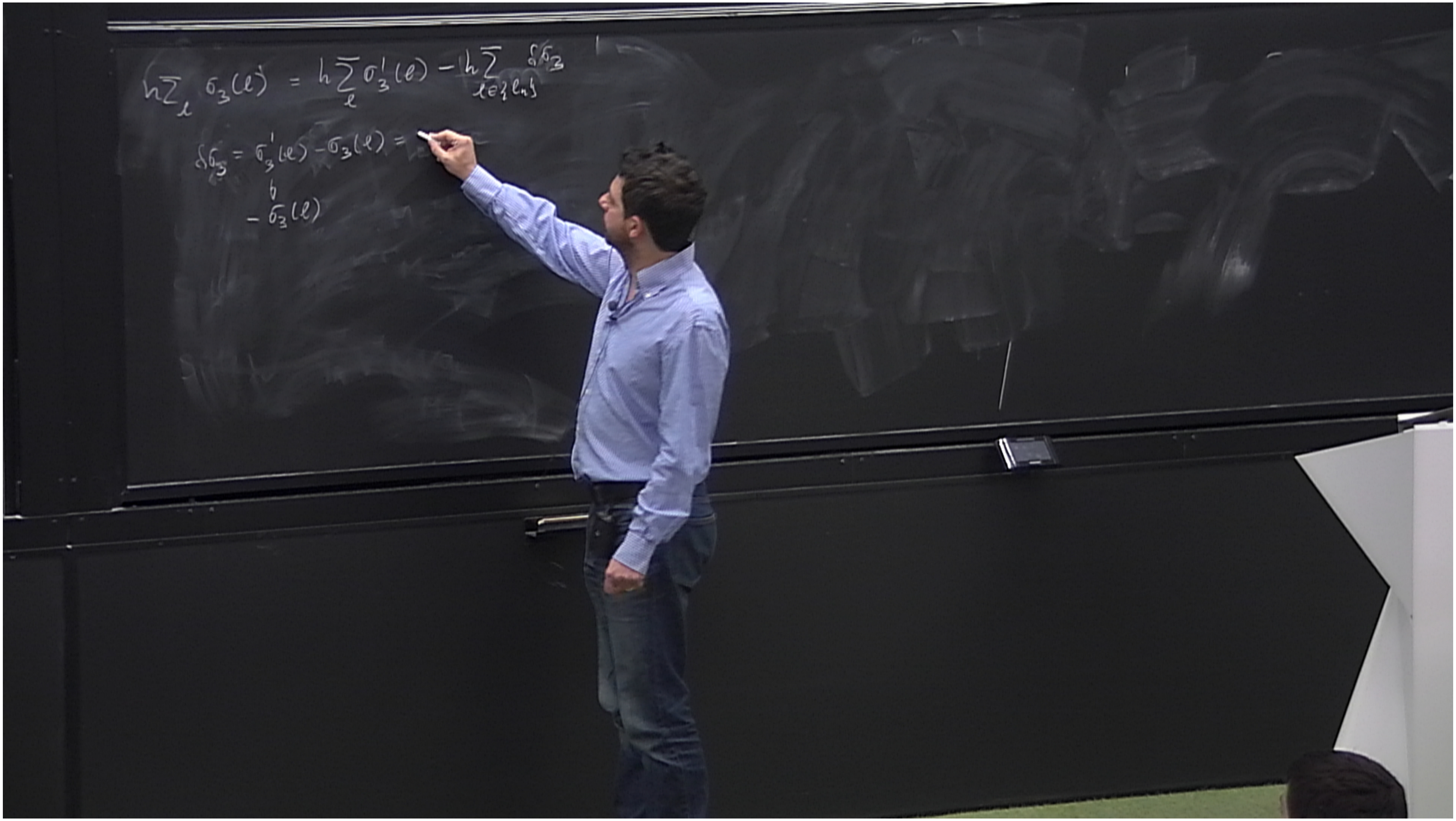
$\ell \notin \{\ell_n\}$ $h\sum_\ell \sigma_3 \xrightarrow{=} h\sum_\ell \sigma_3'$

$\{\ell_n\}$ links $\in n$



$$h\sum_{\ell}\sigma_3(\ell) = h\sum_{\ell}\sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}}\delta\ell$$

$$\delta\ell = \sigma'$$

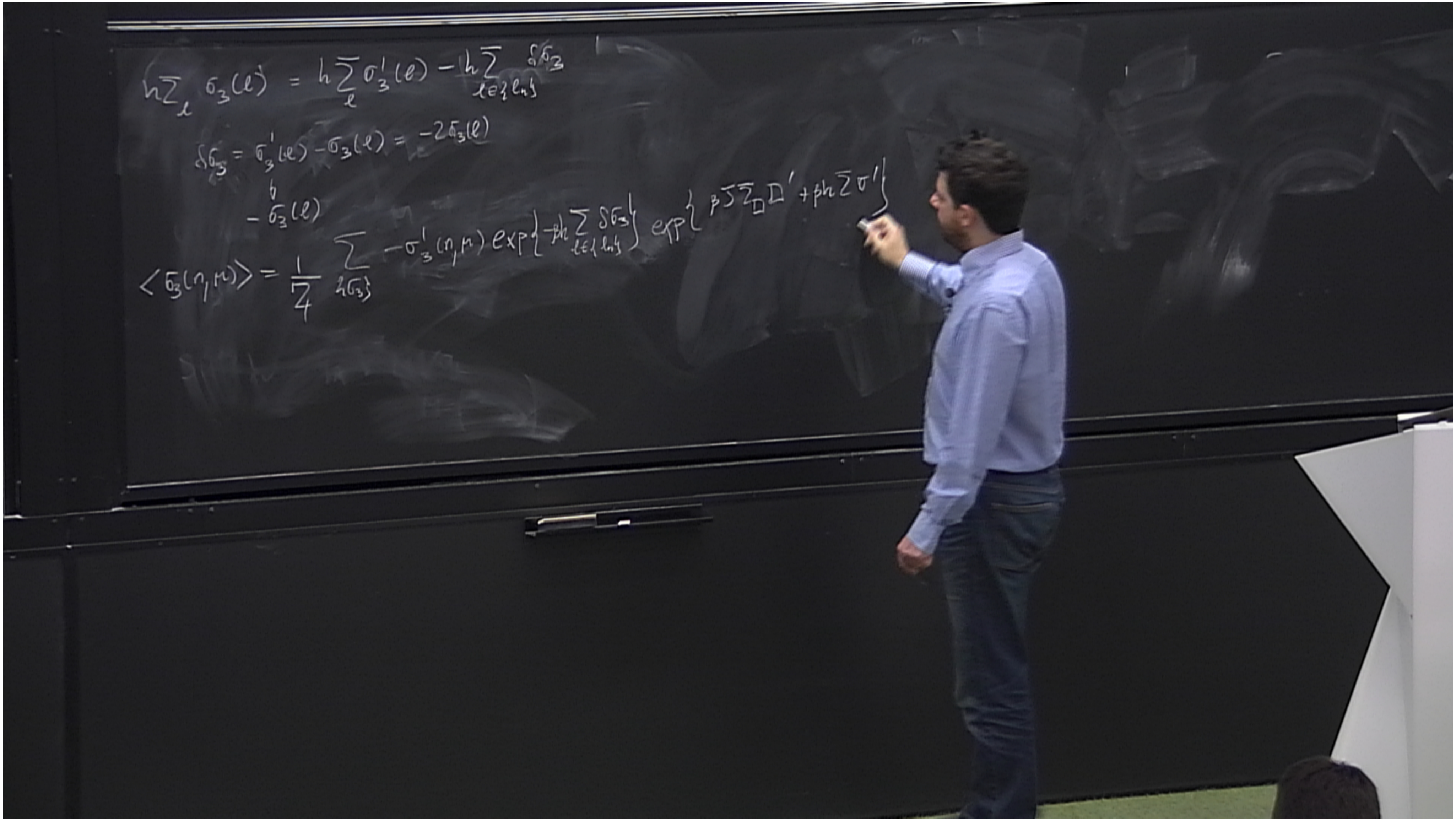


$$h\sum_{\ell}\sigma_3(\ell)' = h\sum_{\ell}\sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3$$

$$\delta\sigma_3 = \sigma_3'(\ell) - \sigma_3(\ell) =$$

$$b$$

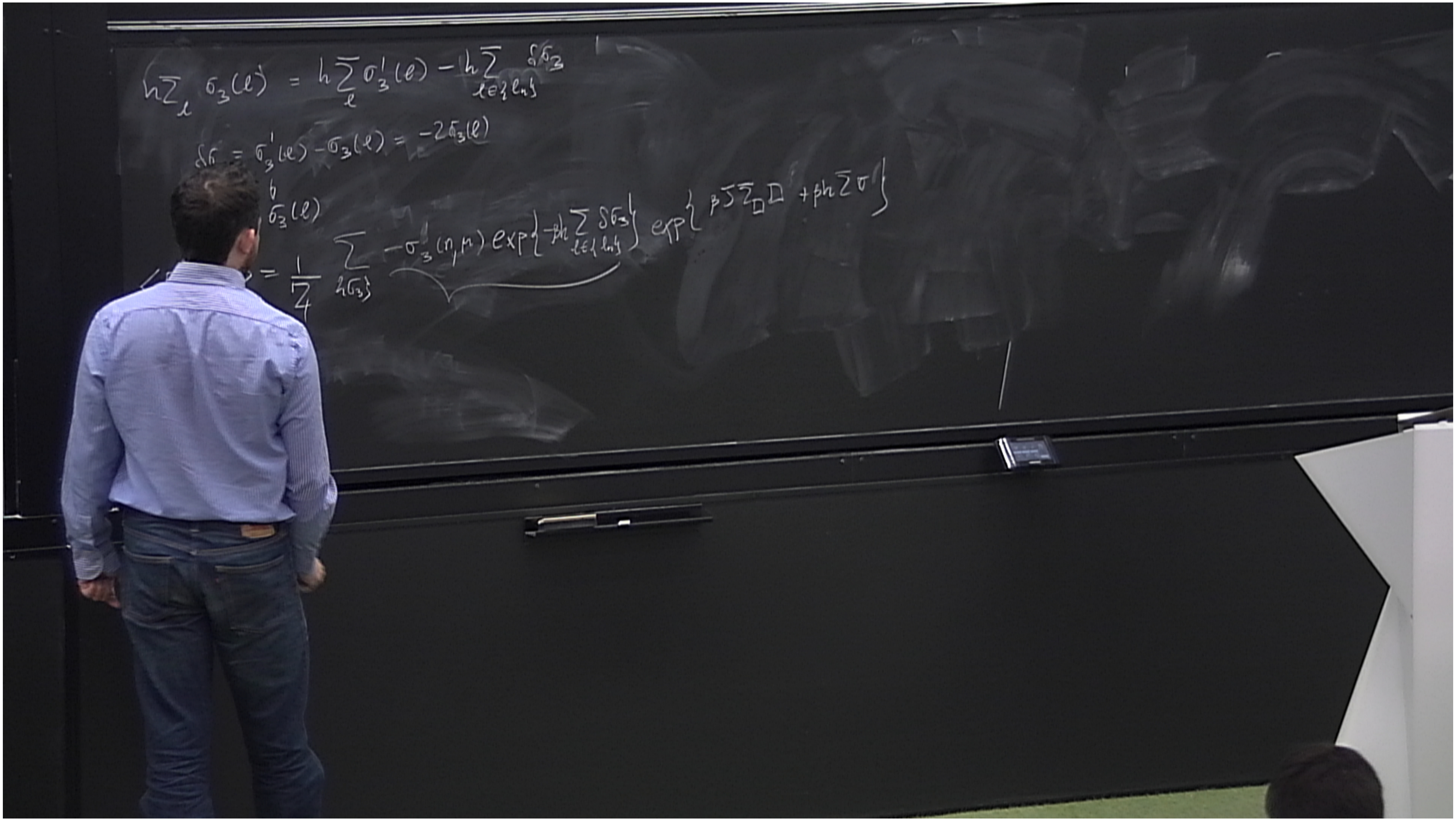
$$-\sigma_3(\ell)$$



$$h\sum_\ell \sigma_3(\ell) = h\sum_\ell \sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}} \delta\sigma_3$$

$$\delta\sigma_3 = \underset{\substack{\downarrow \\ -\sigma_3(\ell)}}{\sigma_3'(\ell)} - \sigma_3(\ell) = -2\sigma_3(\ell)$$

$$\langle\sigma_3(n,m)\rangle = \frac{1}{Z}\sum_{\{\sigma_3\}} -\sigma_3'(n,m)\exp\Big\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3\Big\}\exp\Big\{\beta J\sum_\square \square' + \beta h\sum\sigma'\Big\}$$



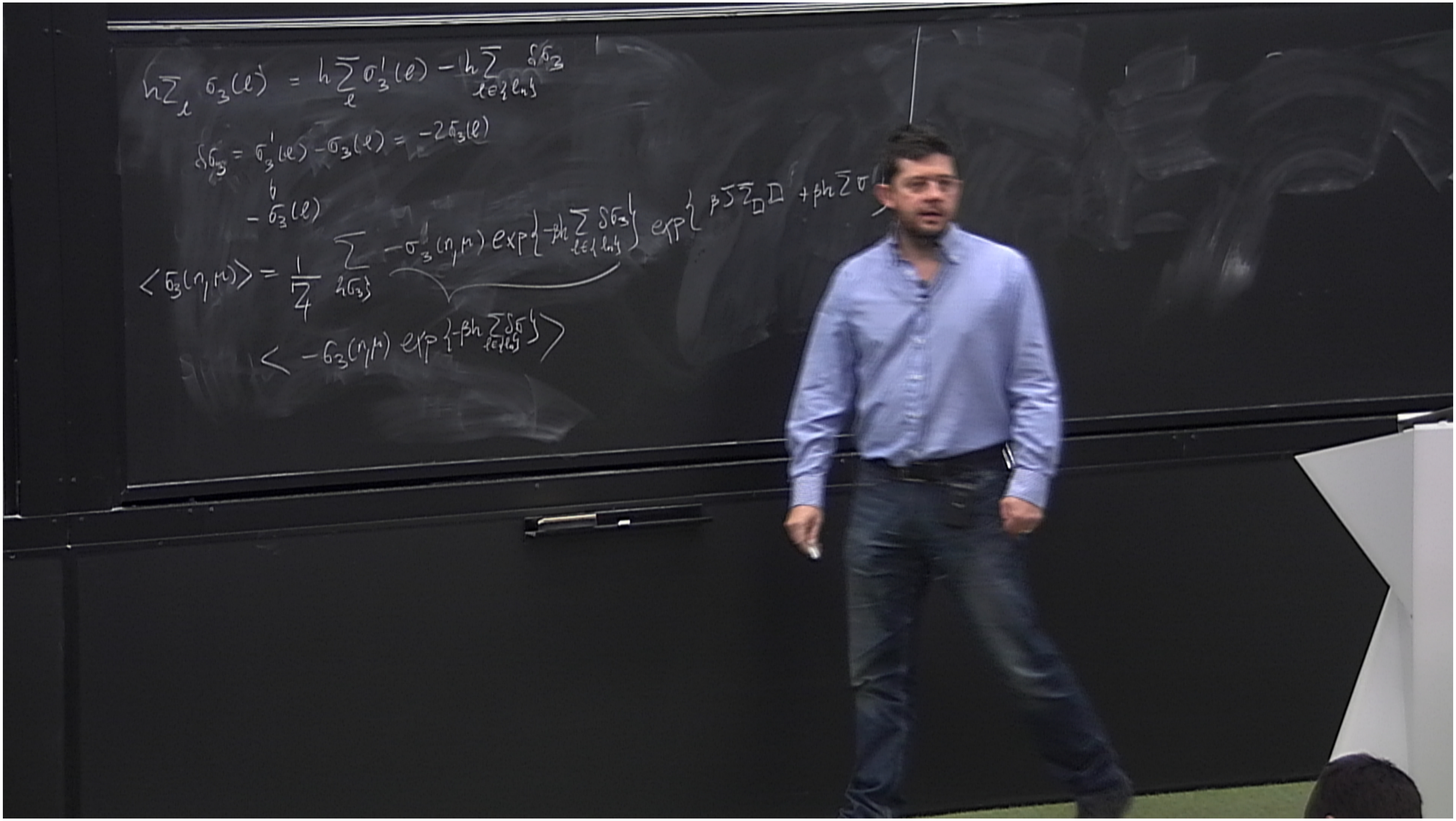
$$h\sum_l \sigma_3(l) = h\sum_l \sigma'_3(l) - h\sum_{l\in\{l_n\}} \delta\sigma_3$$

$$\delta\sigma = \sigma'_3(l) - \sigma_3(l) = -2\sigma_3(l)$$

$$b$$

$$\sigma_3(l)$$

$$\langle \quad \rangle = \frac{1}{Z}\sum_{\{\sigma_3\}} \underbrace{-\sigma'_3(n,\mu)\exp\Big\{-\beta h\sum_{l\in\{l_n\}}\delta\sigma_3\Big\}} \exp\Big\{\beta J\sum_\Box \Box + \beta h\sum\sigma\Big\}$$



$$h\sum_\ell \sigma_3(\ell) = h\sum_\ell \sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}} \delta\sigma_3$$

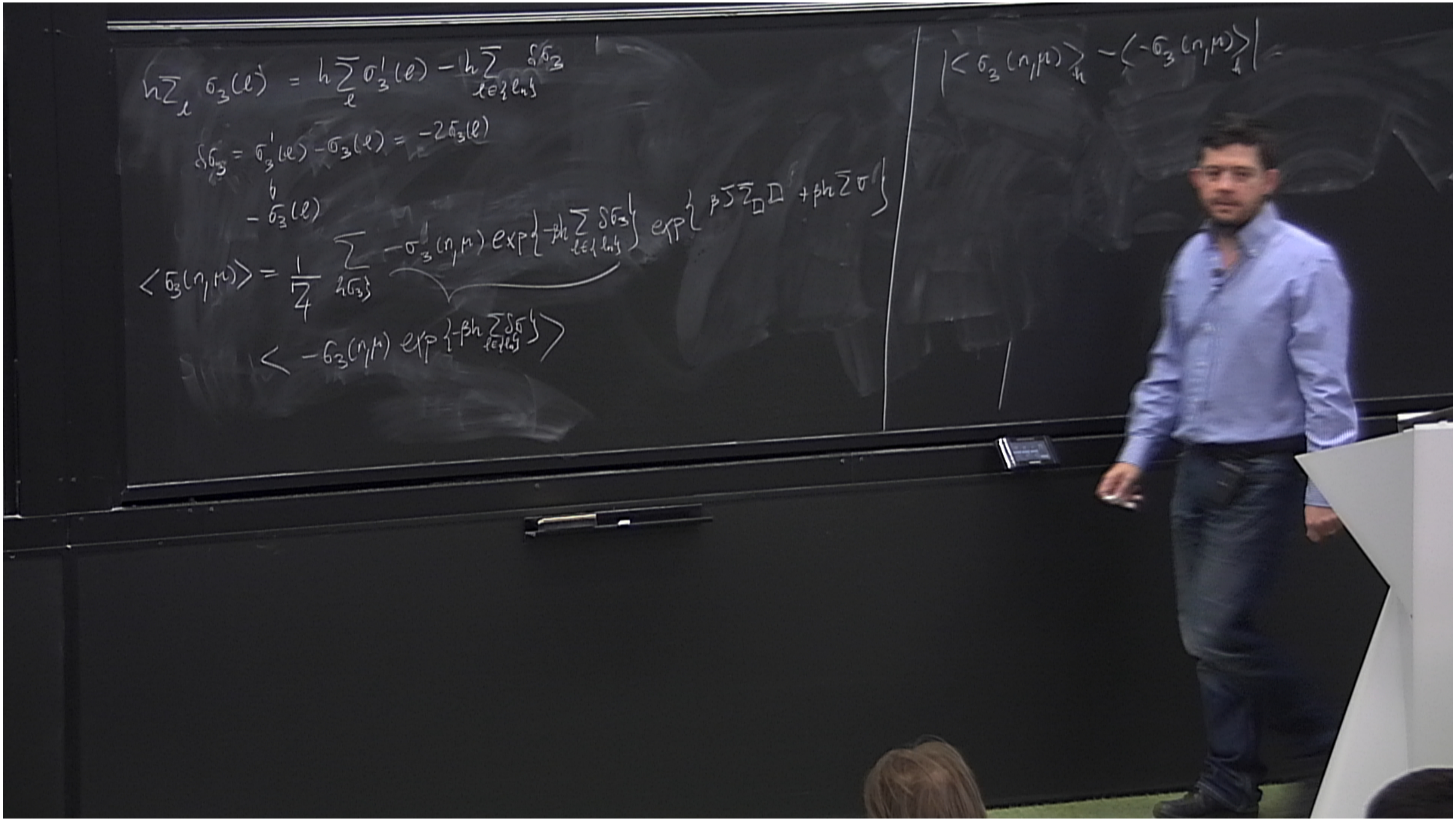
$$\delta\sigma_3 = \sigma_3'(\ell) - \sigma_3(\ell) = -2\sigma_3(\ell)$$

$$\downarrow$$

$$-\sigma_3(\ell)$$

$$\langle\sigma_3(n,\mu)\rangle = \frac{1}{Z}\sum_{\{\sigma_3\}} \underbrace{-\sigma_3'(n,\mu)\exp\Big\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3\Big\}} \exp\Big\{\beta J\sum_\Box \Box + \beta h\sum\sigma$$

$$\Big\langle -\sigma_3(n,\mu)\exp\Big\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma\Big\}\Big\rangle$$



$$h\sum_\ell \sigma_3(\ell) = h\sum_\ell \sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}} \delta\sigma_3$$

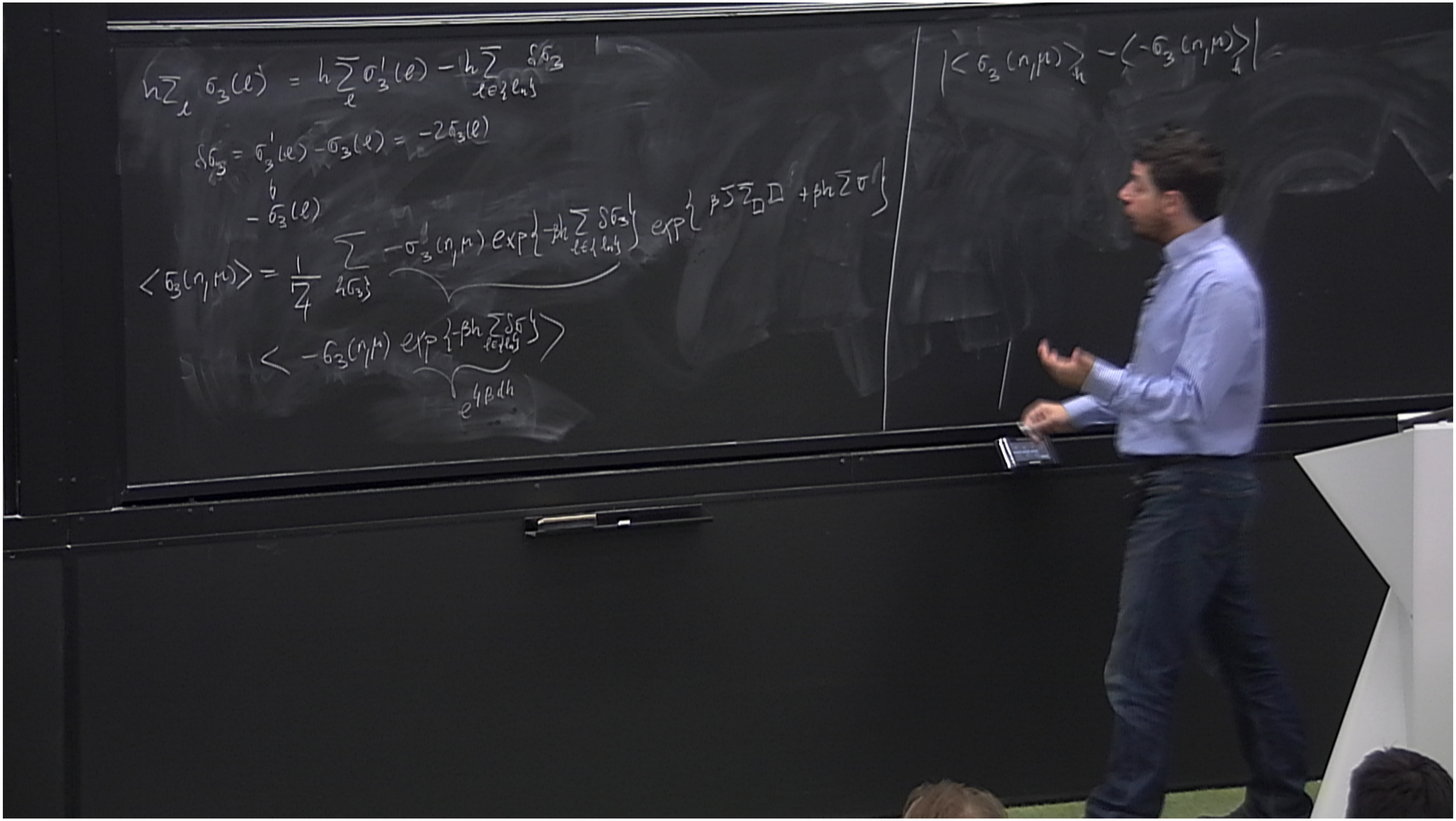
$$\delta\sigma_3 = \sigma_3'(\ell) - \sigma_3(\ell) = -2\sigma_3(\ell)$$

$$\overset{\Downarrow}{-\sigma_3(\ell)}$$

$$\langle\sigma_3(n,\mu)\rangle = \frac{1}{Z}\sum_{\{\sigma_3\}} \underbrace{-\sigma_3'(n,\mu)\exp\left\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3\right\}}\exp\left\{\beta J\sum_\Box \Box + \beta h\sum\sigma'\right\}$$

$$\left\langle -\sigma_3(n,\mu)\exp\left\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma\right\}\right\rangle$$

$$\left|\langle\sigma_3(n,\mu)\rangle_h - \langle-\sigma_3(n,\mu)\rangle_h\right| =$$



$$h\sum_{\ell}\sigma_3(\ell) = h\sum_{\ell}\sigma_3'(\ell) - h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3$$

$$\delta\sigma_3 = \sigma_3'(\ell) - \sigma_3(\ell) = -2\sigma_3(\ell)$$

$$\overset{\downarrow}{-\sigma_3(\ell)}$$

$$\langle\sigma_3(n,\mu)\rangle = \frac{1}{Z}\sum_{\{\sigma_3\}}\underbrace{-\sigma_3'(n,\mu)\exp\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma_3\}}\exp\{\beta J\sum_{\Box}\Box + \beta h\sum\sigma\}$$

$$\langle -\sigma_3(n,\mu)\underbrace{\exp\{-\beta h\sum_{\ell\in\{\ell_n\}}\delta\sigma\}}_{e^{4\beta dh}}\rangle$$

$$\left|\langle\sigma_3(n,\mu)\rangle_h - \langle-\sigma_3(n,\mu)\rangle_h\right|$$