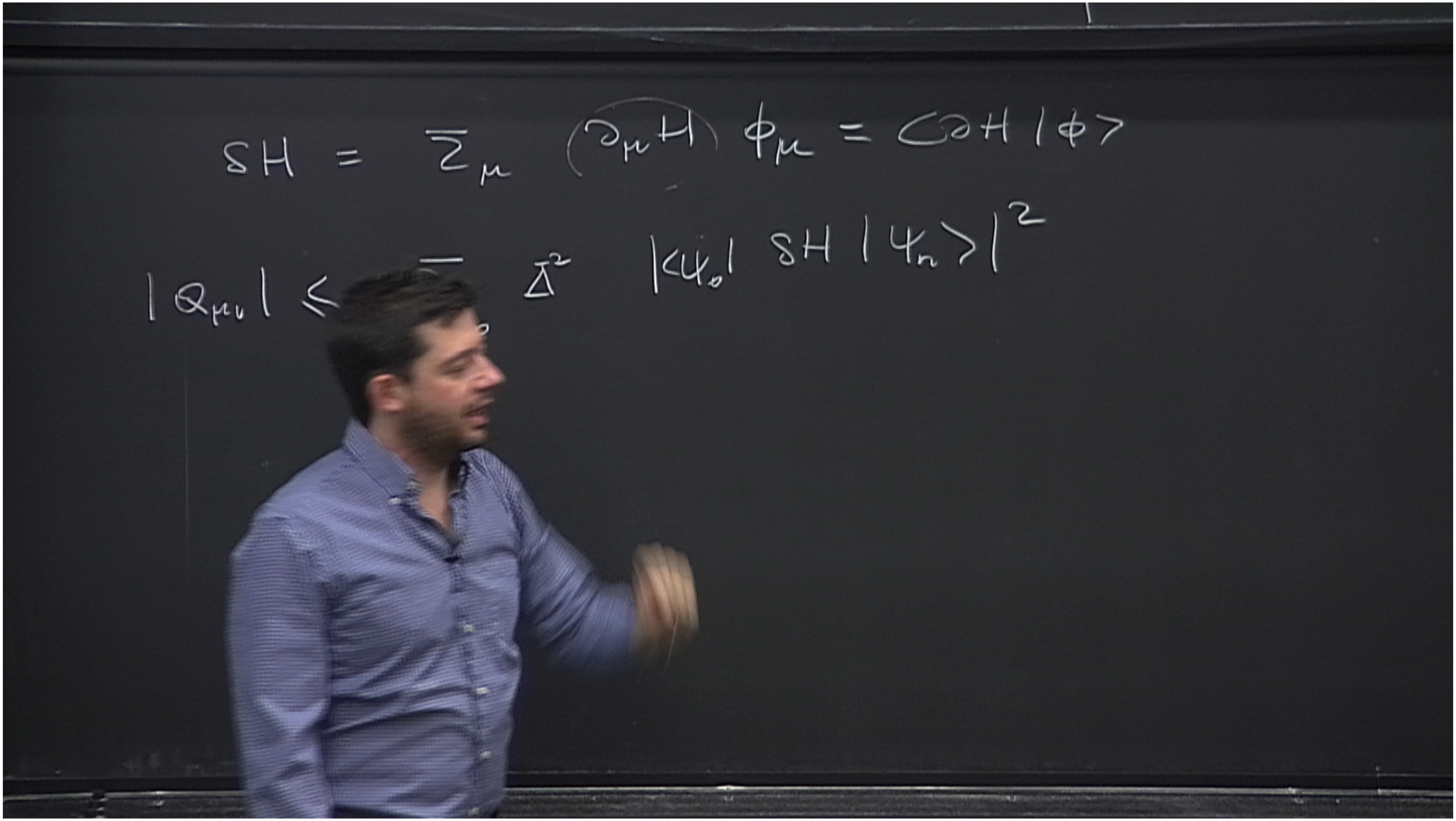
Title: Condensed Matter (Review) - Lecture 8

Date: Jan 11, 2012 10:15 AM

URL: http://pirsa.org/12010092

Abstract:


$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$
$|Q_{\mu\nu}| \leq \ldots \Delta^{2} \, |\langle \psi_0 | \, \delta H \, | \psi_n \rangle|^2$

$H(\lambda)$

$\downarrow$ GS $\quad \lambda \in \mathcal{M}$

$\psi_0(\lambda)$

ground-state manifold

$$Q_{\mu\nu} \equiv \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle = \sum$$

Quantum geometric tensor

$$\psi_0 | \partial_\nu \psi_0 \rangle = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(\underbrace{E_n(\lambda) - E_0(\lambda)}_{\Delta^2})^2}$$

1st order pert. theory

$H(\lambda)$

$\downarrow$ GS $\quad \lambda \in \mathcal{M}$

$\psi_0(\lambda)$

ground-state manifold

$$Q_{\mu\nu} \equiv \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle =$$

Quantum geometric tensor

$\mathrm{Im}\, Q_{\mu\nu} \equiv F_{\mu\nu}$ adiabatic curvature

$\mathrm{Re}\, Q_{\mu\nu}$

1st. or

pert.

$$\partial_\nu \psi_0 \rangle = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(\underbrace{E_n(\lambda) - E_0(\lambda)}_{\Delta^2})^2}$$

1st order pert. theory

Q critical point $\rightarrow \Delta^2 \rightarrow 0 \xrightarrow{?} Q_{\mu\nu}$ might blow-up

$H(\lambda)$ $\qquad Q_{\mu\nu} \equiv \langle \partial_\mu \Psi_0 | \partial_\nu \Psi_0 \rangle - \langle \partial_\mu \Psi_0 | \Psi_0 \rangle \langle \Psi_0 | \partial_\nu \Psi_0 \rangle$

$\downarrow$ GS $\quad \lambda \in \mathcal{M}$ $\qquad$ Quantum geometric tensor

1st s

$\Psi_0(\lambda)$ $\qquad \mathrm{Im}\, Q_{\mu\nu} \equiv F_{\mu\nu}$ adiabatic curvature

pert

ground-state manifold

$\mathrm{Re}\, Q_{\mu\nu} \equiv g_{\mu\nu}$ this would blow up at a QCP $(\Delta \to 0)$

$g_{\mu\nu}$ to blow up ?

$$\mathcal{O}(\lambda) = \sum_n |\psi_n(\lambda+\delta\lambda)\rangle\langle\psi_n(\lambda)|$$

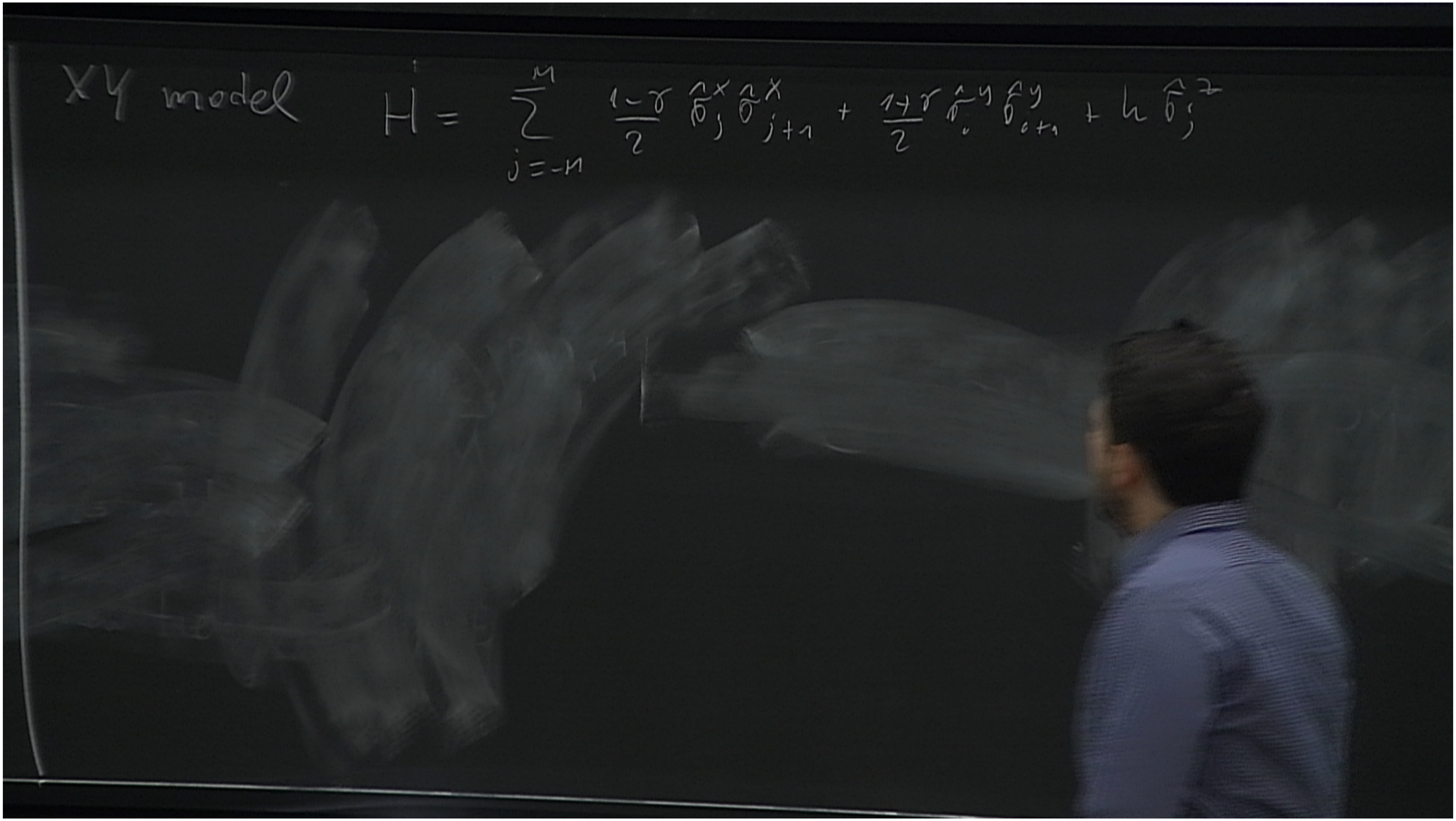
$$X_\mu := i(\partial_\mu \mathcal{O})\mathcal{O}^\dagger$$

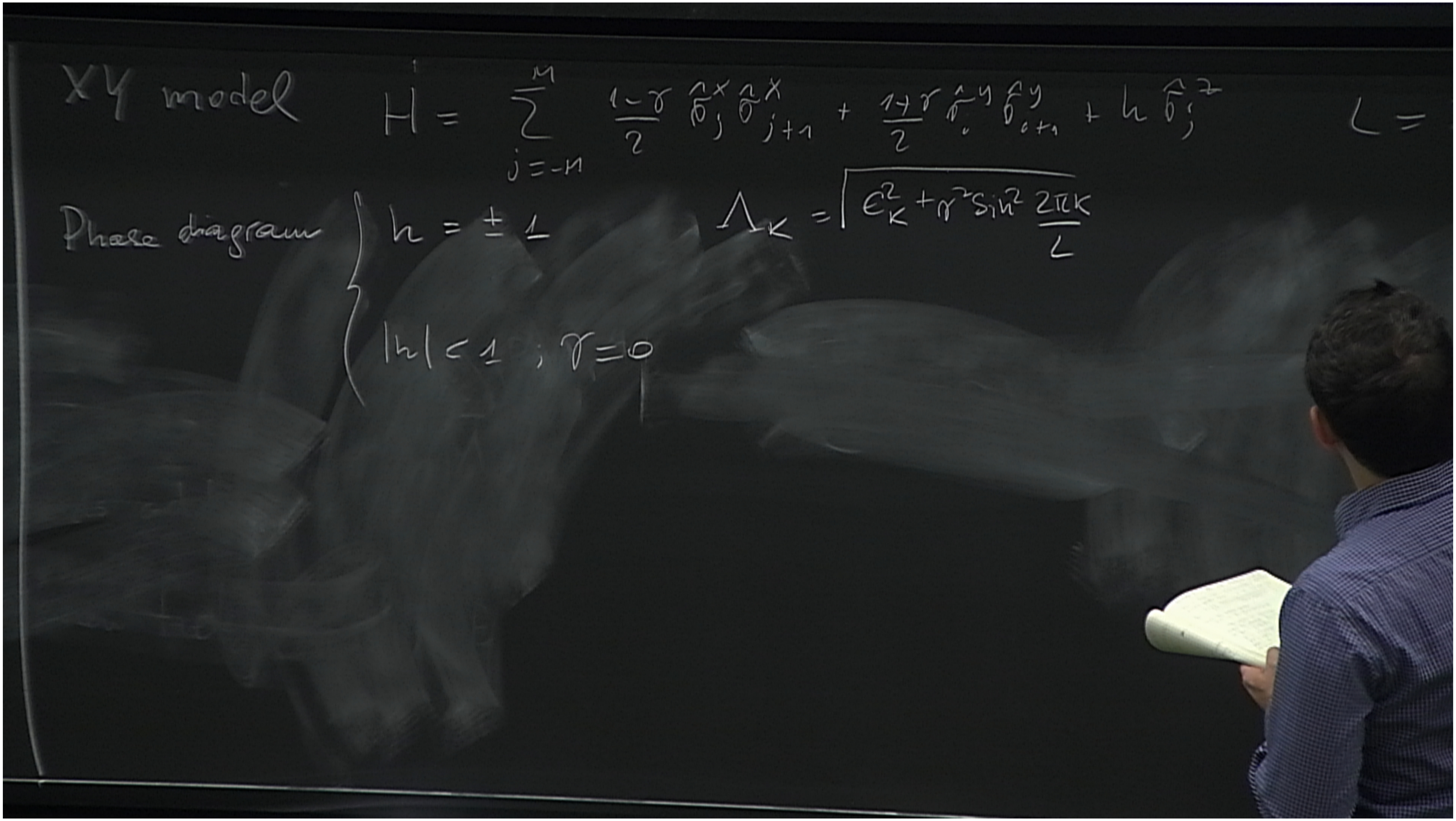
$$\bar{X}_\mu := X_\mu - \langle X_\mu \rangle \qquad X := i\, d\mathcal{O}\, \mathcal{O}^\dagger$$

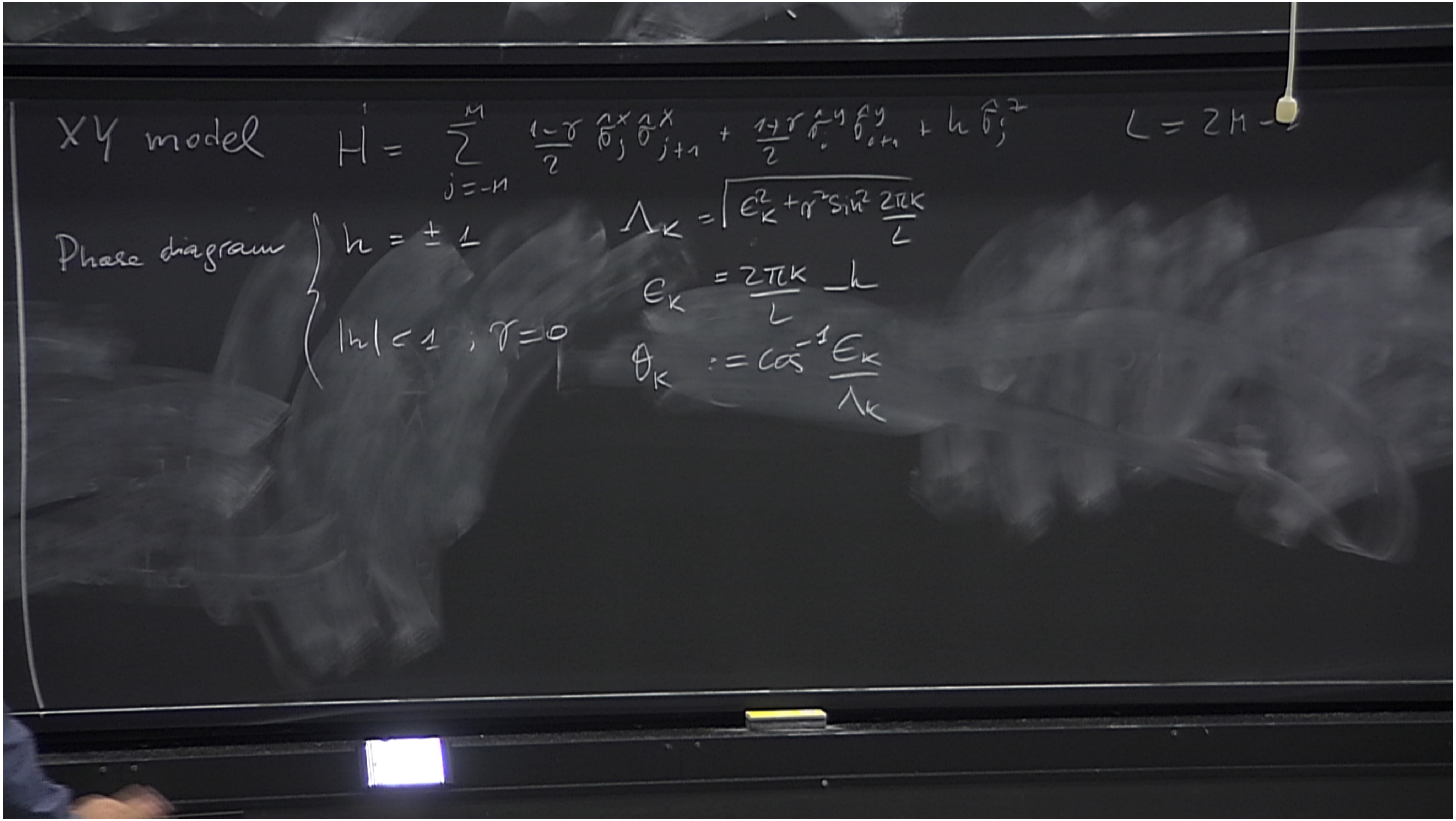
$$d\mathcal{O} = \frac{\partial \mathcal{O}}{\partial \lambda^\mu} d\lambda^\mu$$

$$\Rightarrow \begin{cases} g_{\mu\nu} = \frac{1}{2}\{\bar{X}_\mu, \bar{X}_\nu\} \\ ds^2 = \langle \bar{X}^2 \rangle \end{cases}$$

$ds^2 \to \infty \Rightarrow \mathrm{Var}(\bar{X})$ blows up


XY model
$H = \sum_{j=-M}^{M} \frac{1-\gamma}{2} \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \frac{1+\gamma}{2} \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + h \hat{\sigma}_j^z$


XY model
$H = \sum_{j=-M}^{M} \frac{1-\gamma}{2} \hat{\sigma}^x_j \hat{\sigma}^x_{j+1} + \frac{1+\gamma}{2} \hat{\sigma}^y_j \hat{\sigma}^y_{j+1} + h \hat{\sigma}^z_j$
$L =$
Phase diagram
$h = \pm 1$
$|h| < 1 \; ; \; \gamma = 0$
$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{L}}$


XY model
$\hat{H} = \sum_{j=-M}^{M} \frac{1-\gamma}{2} \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \frac{1+\gamma}{2} \hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + h \hat{\sigma}_j^z$
$L = 2M -$
Phase diagram
$h = \pm 1$
$|h| < 1 \; ; \gamma = 0$
$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{L}}$
$\epsilon_k = \frac{2\pi k}{L} - h$
$\theta_k := \cos^{-1} \frac{\epsilon_k}{\Lambda_k}$

XY model $H = \sum_{j=-M}^{M} \frac{1-\gamma}{2}\hat\sigma_j^x\hat\sigma_{j+1}^x + \frac{1+\gamma}{2}\hat\sigma_j^y\hat\sigma_{j+1}^y + h\hat\sigma_j^z$ $L = 2M$

Phase diagram $\begin{cases} h = \pm 1 \\ |h| < 1 \,;\ \gamma = 0 \end{cases}$

$$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2\frac{2\pi k}{L}}$$

$$\epsilon_k = \frac{2\pi k}{L} - h$$

$$\theta_k := \cos^{-1}\frac{\epsilon_k}{\Lambda_k}$$

Compute the Riemannian tensor $g_{\mu\nu} = \frac{1}{2}\sum_{k=1}^{M}\frac{\partial\theta_k}{\partial\lambda^\mu}\frac{\partial\theta_k}{\partial\lambda^\nu}$

straight-forward

$$\ldots \sigma^x_{j+1} + \frac{1+r}{2}\hat{\sigma}^y_i \hat{\sigma}^y_{i+1} + h\,\hat{\sigma}^z_j$$

$$L = 2M - 1$$

$$\lambda^\mu = \gamma, h \qquad \mu = 1, 2$$

$$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{L}}$$

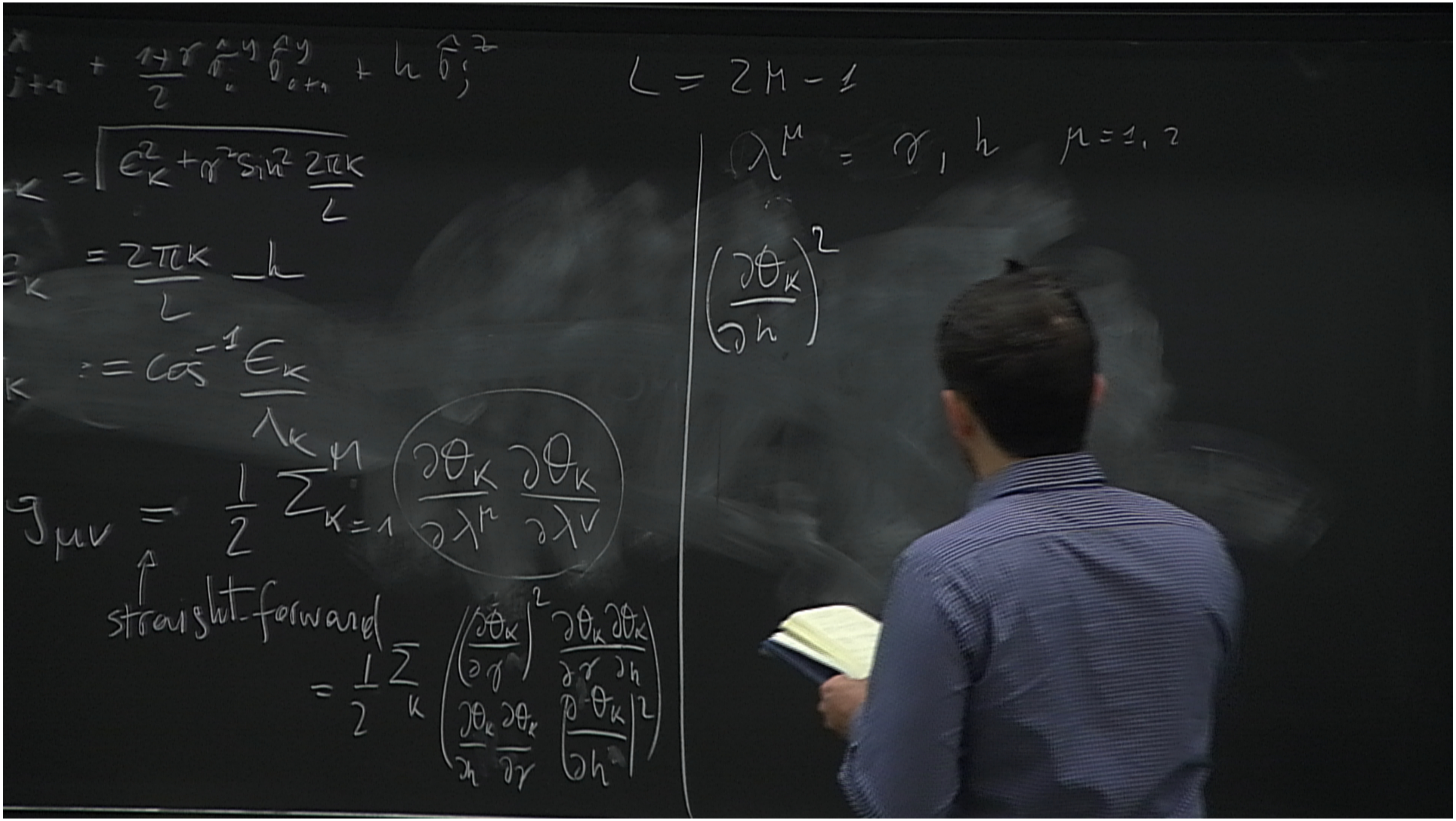
$$\epsilon_k = \frac{2\pi k}{L} - h$$

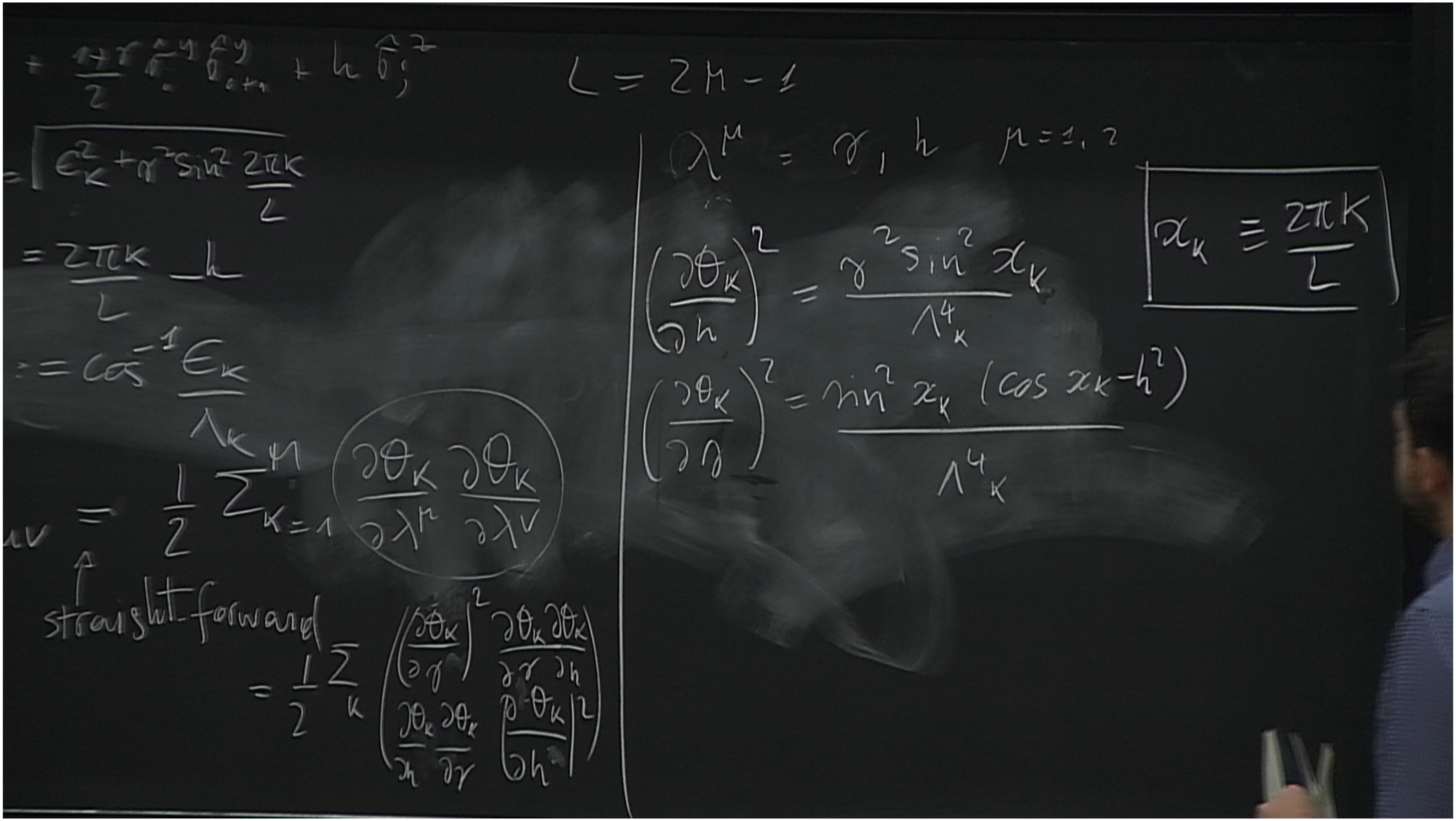
$$\theta_k := \cos^{-1} \frac{\epsilon_k}{\Lambda_k}$$

$$g_{\mu\nu} = \frac{1}{2} \sum_{k=1}^{M} \frac{\partial\theta_k}{\partial\lambda^\mu} \frac{\partial\theta_k}{\partial\lambda^\nu}$$

straight-forward

$$= \frac{1}{2} \sum_k \begin{pmatrix} \frac{\partial^2\theta_k}{\partial\gamma^2} & \frac{\partial\theta_k \partial\theta_k}{\partial\gamma\,\partial h} \\ \frac{\partial\theta_k}{\partial h}\frac{\partial\theta_k}{\partial\gamma} & \frac{\partial^2\theta_k}{\partial h^2} \end{pmatrix}$$


L = 2M − 1
straight-forward



$L = 2M - 1$
$\lambda^\mu = \gamma, h \quad \mu = 1, 2$
$x_k \equiv \frac{2\pi k}{L}$
$\left(\frac{\partial\theta_k}{\partial h}\right)^2 = \frac{\gamma^2 \sin^2 x_k}{\Lambda_k^4}$
$\left(\frac{\partial\theta_k}{\partial \gamma}\right)^2 = \frac{\sin^2 x_k (\cos x_k - h^2)}{\Lambda_k^4}$
$= \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{L}}$
$= \frac{2\pi k}{L} - h$
$= \cos^{-1} \frac{\epsilon_k}{\Lambda_k}$
$= \frac{1}{2} \sum_{k=1}^{M} \frac{\partial\theta_k}{\partial\lambda^\mu} \frac{\partial\theta_k}{\partial\lambda^\nu}$
straight-forward

$$+\frac{1+\gamma}{2}\hat{\sigma}^y_i\hat{\sigma}^y_{i+1}+h\hat{\sigma}^z_i$$

$$L=2M-1$$

$$=\sqrt{\epsilon_k^2+\gamma^2\sin^2\frac{2\pi k}{L}}$$

$$=\frac{2\pi k}{L}-h$$

$$=\cos^{-1}\frac{\epsilon_k}{\Lambda_k}$$

$$\lambda^\mu=\gamma,h \quad \mu=1,2$$

$$\boxed{x_k\equiv\frac{2\pi k}{L}}$$

$$\left(\frac{\partial\theta_k}{\partial h}\right)^2=\frac{\gamma^2\sin^2 x_k}{\Lambda_k^4}$$

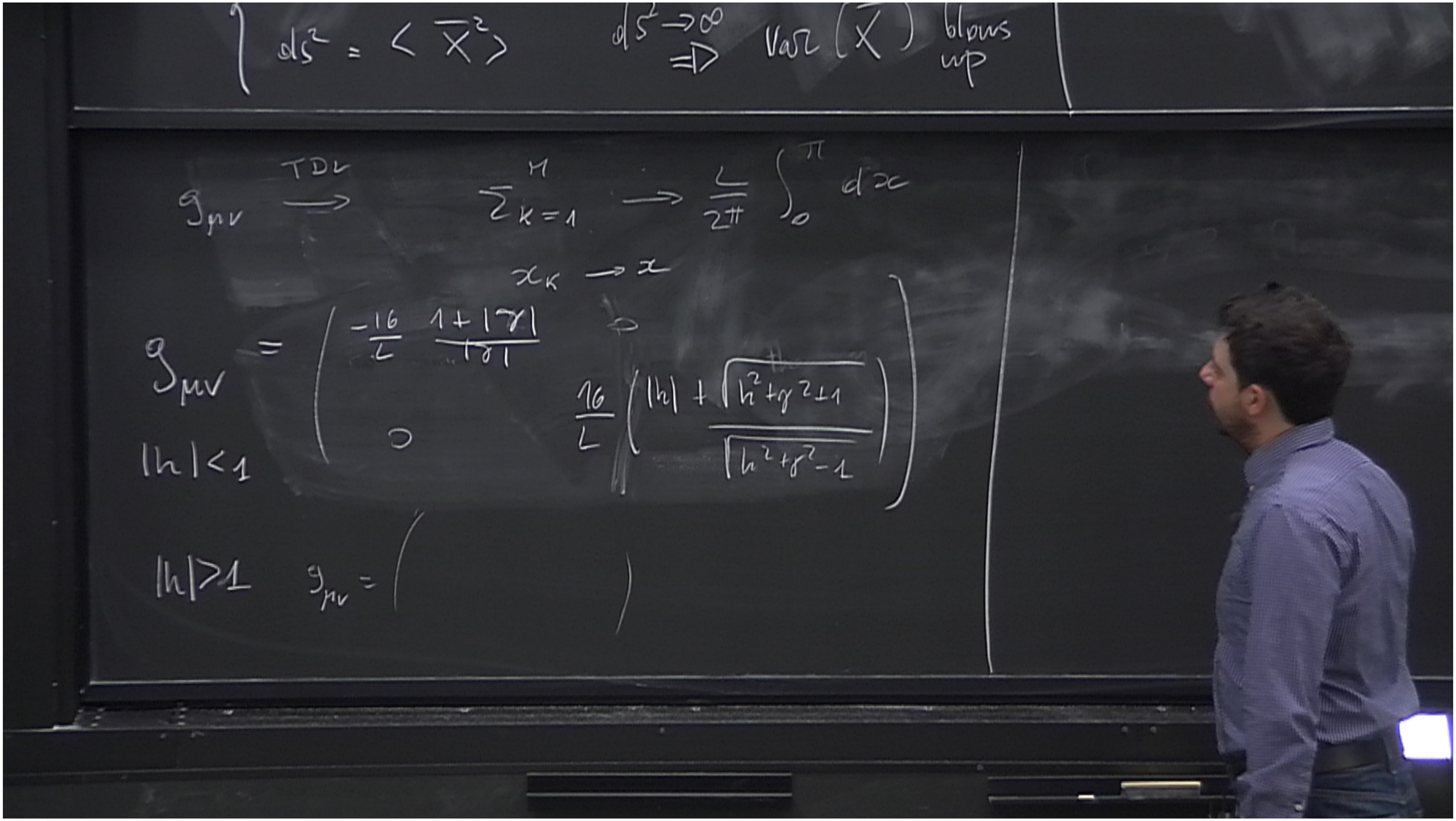
$$\left(\frac{\partial\theta_k}{\partial\gamma}\right)^2=\frac{\sin^2 x_k(\cos x_k-h^2)}{\Lambda_k^4}$$

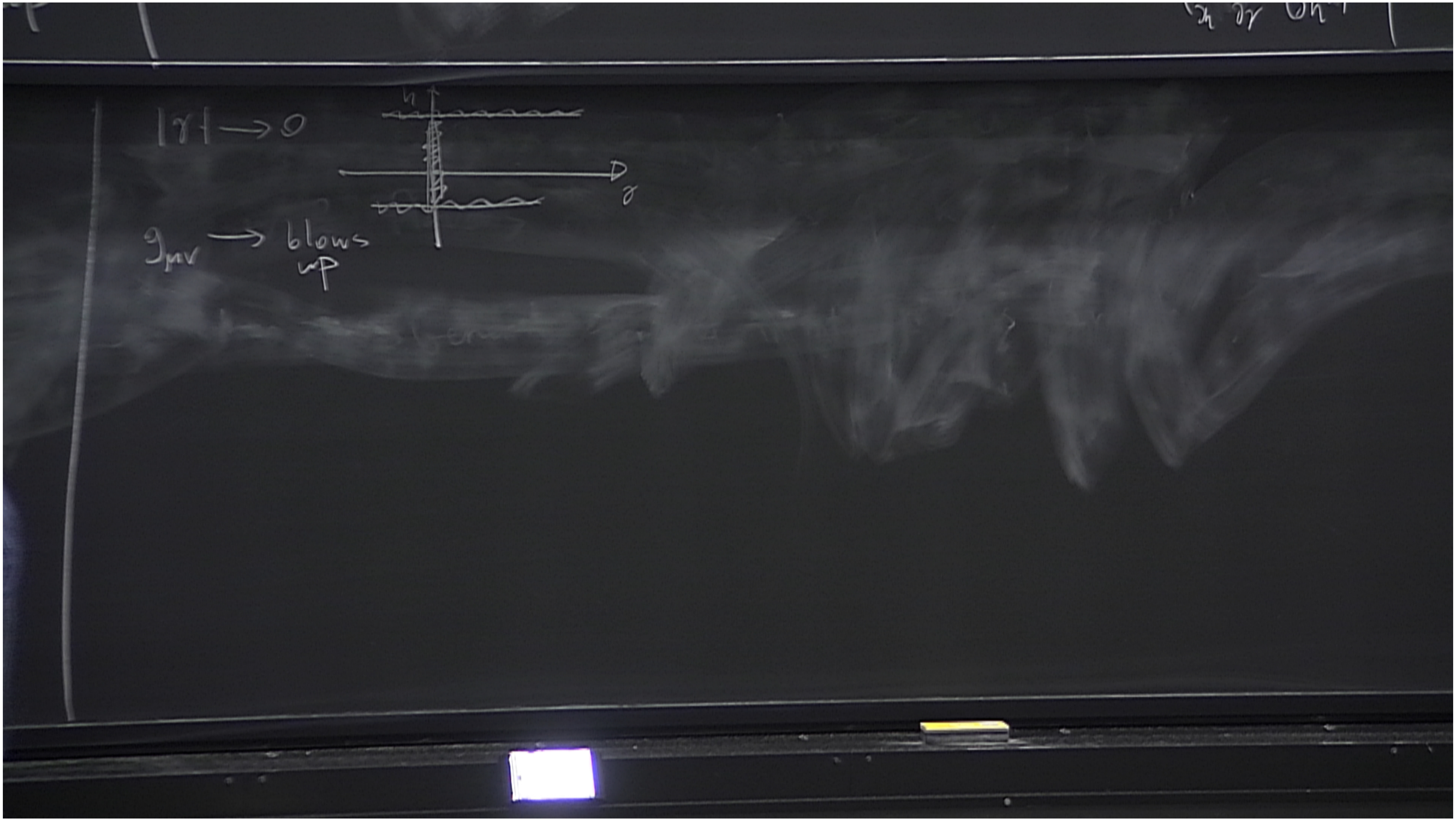
$$\frac{\partial\theta_k}{\partial\gamma}\frac{\partial\theta_k}{\partial h}=\frac{\gamma\sin^2 x_k(\cos x_k-h)}{\Lambda_k^4}$$

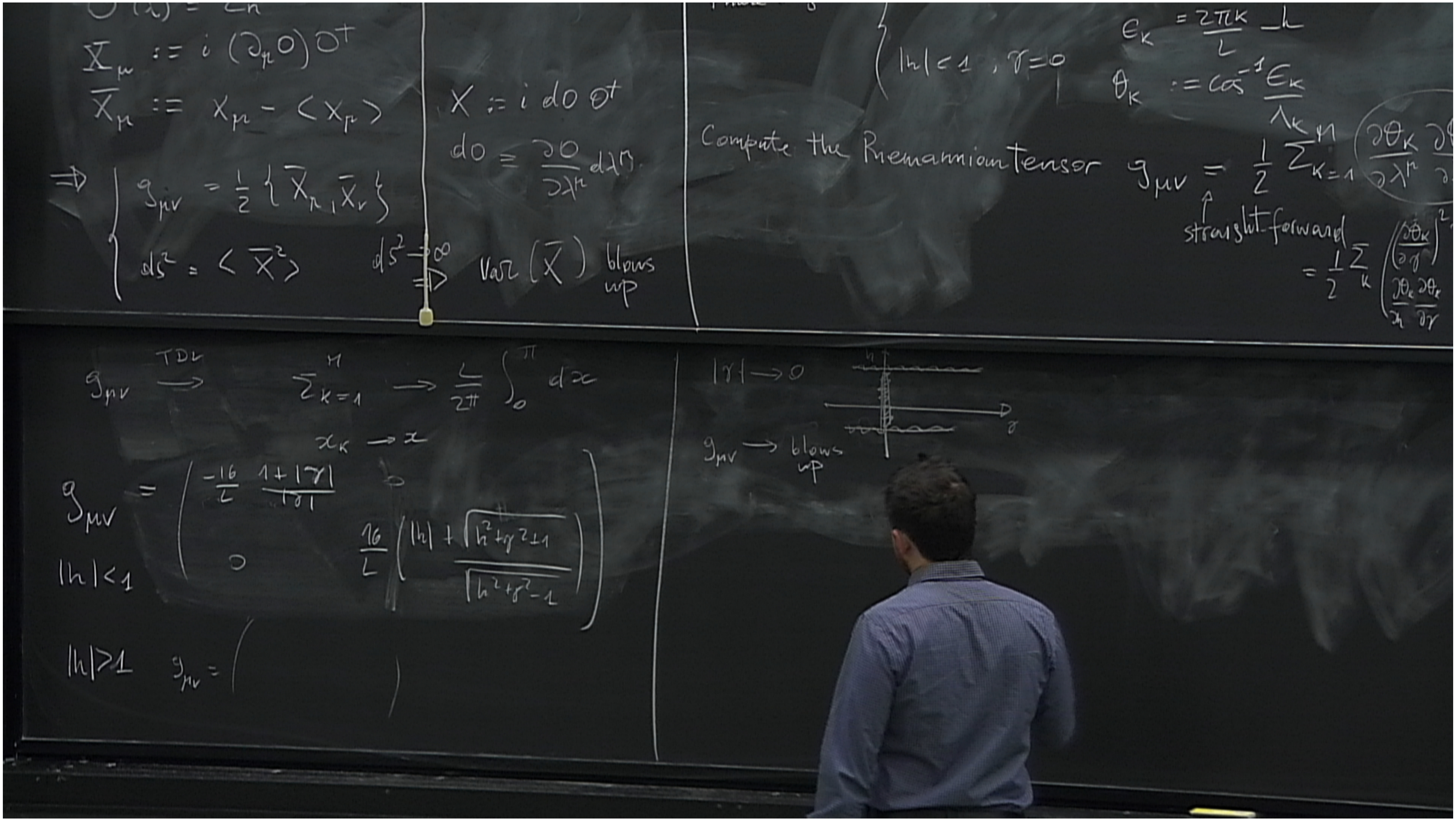
$$_{\mu\nu}=\frac{1}{2}\sum_{k=1}^{M}\frac{\partial\theta_k}{\partial\lambda^\mu}\frac{\partial\theta_k}{\partial\lambda^\nu}$$

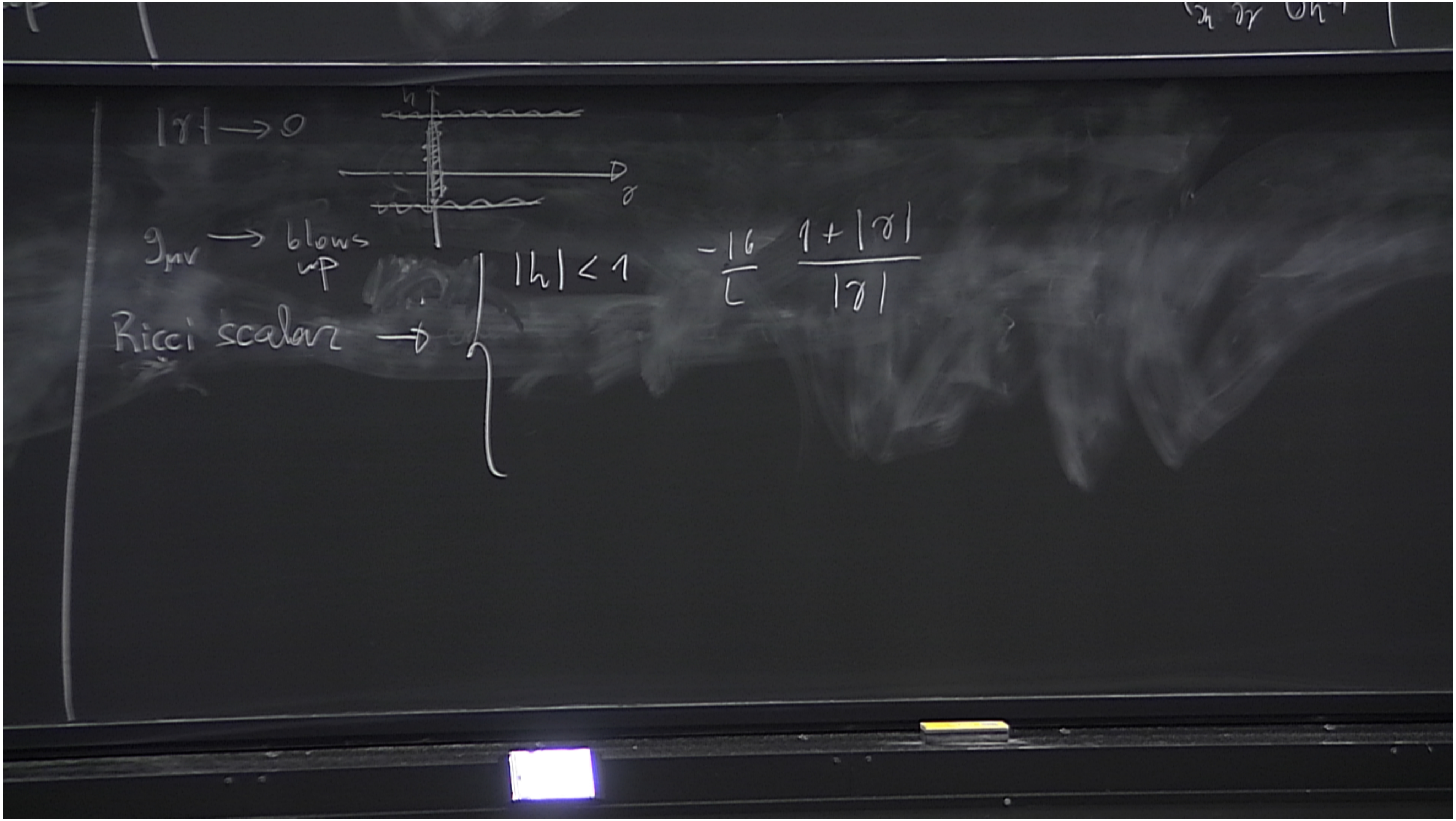
straightforward

$$=\frac{1}{2}\sum_k\begin{pmatrix}\left(\frac{\partial\theta_k}{\partial\gamma}\right)^2 & \frac{\partial\theta_k}{\partial\gamma}\frac{\partial\theta_k}{\partial h}\\ \frac{\partial\theta_k}{\partial h}\frac{\partial\theta_k}{\partial\gamma} & \left(\frac{\partial\theta_k}{\partial h}\right)^2\end{pmatrix}$$

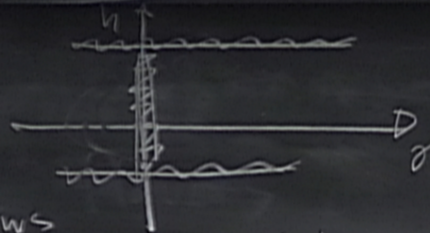

$ds^2 = \langle \bar{X}^2 \rangle$
$ds^2 \to \infty \Rightarrow \mathrm{Var}(\bar{X})$ blows up
$g_{\mu\nu} \xrightarrow{\mathrm{TDL}} \sum_{k=1}^{M} \to \frac{L}{2\pi}\int_0^\pi dx$
$x_k \to x$
$g_{\mu\nu} = \begin{pmatrix} -\frac{16}{L}\frac{1+|\gamma|}{|\gamma|} & 0 \\ 0 & \frac{16}{L}\left(|h| + \frac{\sqrt{h^2+\gamma^2+1}}{\sqrt{h^2+\gamma^2-1}}\right) \end{pmatrix}$
$|h|<1$
$|h|>1 \quad g_{\mu\nu} = \left(\quad \right)$


|γ| → 0
g_{μν} → blows up


X_μ := i (∂_μ O) O†
X̄_μ := X_μ − ⟨X_μ⟩
g_μν = 1/2 {X̄_μ, X̄_ν}
ds² = ⟨X̄²⟩
X := i dO O†
dO = ∂O/∂λ^μ dλ^μ
ds² → ∞ ⟹ Var(X̄) blows up
|h| < 1 ; γ = 0
ε_k = 2πk/L − h
θ_k := cos⁻¹ ε_k/Λ_k
Compute the Riemannian tensor
straight forward
g_μν → TDL
Σ_{k=1}^{M} → L/2π ∫_0^π dx
x_k → x
|h| < 1
|h| > 1
|γ| → 0
g_μν → blows up


$|\gamma| \to 0$
h
γ
$g_{\mu\nu} \to$ blows up
Ricci scalar $\to$
$|h| < 1$
$-\frac{16}{L} \frac{1+|\gamma|}{|\gamma|}$

$|\gamma| \to 0$



$g_{\mu\nu} \longrightarrow$ blows up

$$\text{Ricci scalar} \longrightarrow \begin{cases} |h|<1 & -\dfrac{16}{L}\,\dfrac{1+|\gamma|}{|\gamma|} \\[2ex] |h|>1 & \left(|h| + \dfrac{\sqrt{h^2+\gamma^2+1}}{\sqrt{h^2+\gamma^2-1}}\right) \end{cases}$$

$$g_{\mu\nu} \xrightarrow{TDL} \qquad \sum_{k=1}^{M} \longrightarrow \frac{L}{2\pi}\int_0^{\pi} dx$$

$$x_k \to x$$

$$g_{\mu\nu} = \begin{pmatrix} \frac{1}{1-h^2} & 0 \\ 0 & \frac{1}{1+|\gamma|^2} \end{pmatrix}$$

$|h|<1$

$|h|>1 \qquad g_{\mu\nu} = \left(\quad \right)$

R

$$\frac{16}{L}\,\frac{1+|\gamma|}{|\gamma|}$$

$$|h| + \frac{\sqrt{h^2+\gamma^2+1}}{\sqrt{h^2+\gamma^2-1}}\Bigg)^{\frac{16}{L}}$$

General argument

scaling behavior

$|Q_{\mu\nu}|$

$$\frac{16}{L}\,\frac{1+|\gamma|}{|\gamma|}$$

$$|h| + \frac{\sqrt{h^2+\gamma^2+1}}{\sqrt{h^2+\gamma^2-1}}\Big)\frac{16}{L}$$

General argument

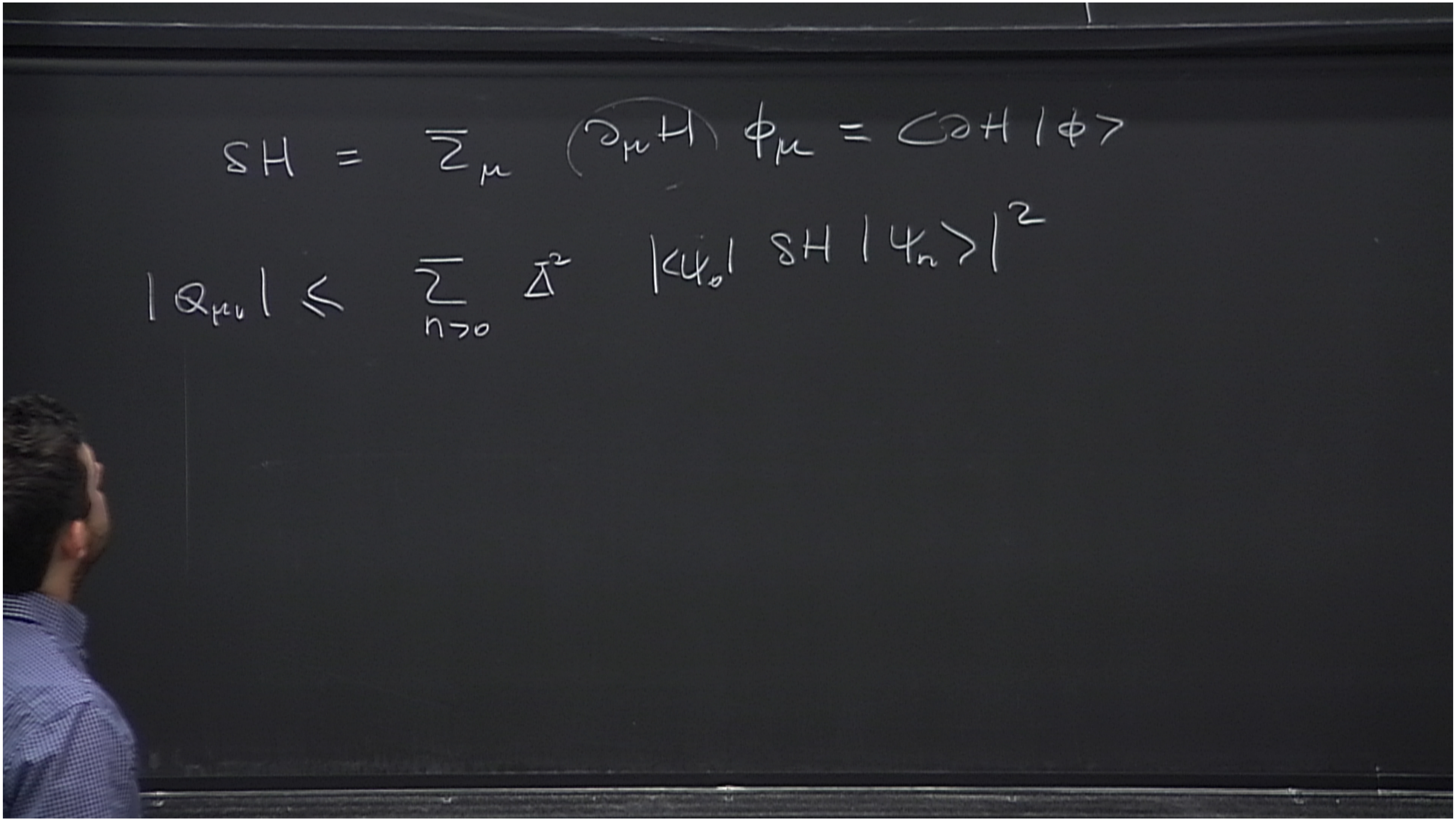
scaling behavior

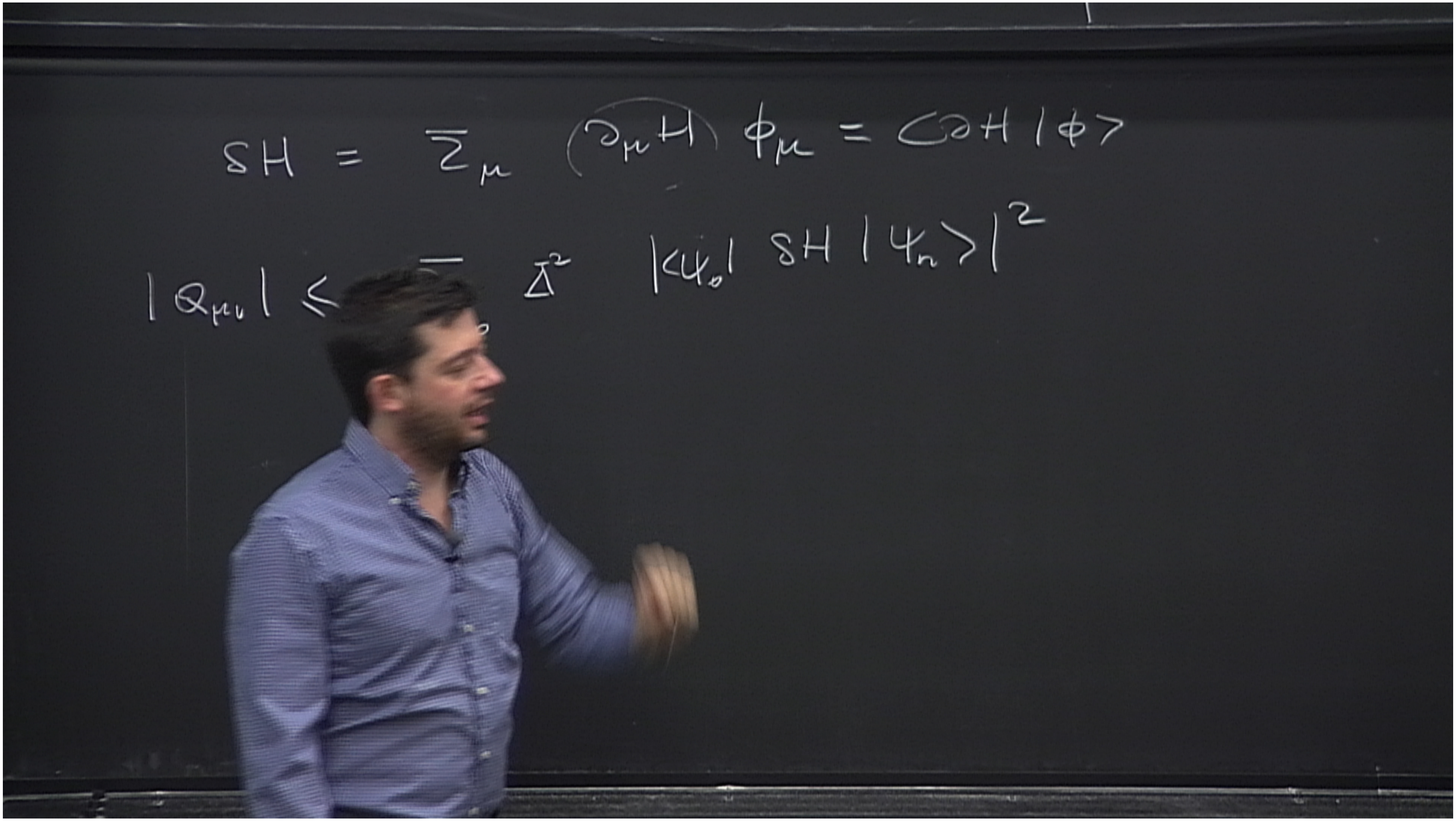
$$|Q_{\mu\nu}| \leqslant \|Q_{\mu\nu}\| = \langle\phi|Q|\phi\rangle$$

$|\phi\rangle$ is the eigenvector corr. to the max eigenvalue

$$|\phi\rangle = (\phi^1, \ldots, \phi^n)$$

$$n = \dim |M|$$


$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$
$|Q_{\mu\nu}| \leq \sum_{n>0} \Delta^{-2} \, |\langle \psi_0 | \delta H | \psi_n \rangle|^2$


$\delta H = \sum_\mu (\partial_\mu H) \phi_\mu = \langle \partial H | \phi \rangle$
$|Q_{\mu\nu}| \leq$ Δ^{-2} $|\langle \psi_0 | \delta H | \psi_n \rangle|^2$

$$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$$

$$|Q_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \, \delta H \, | \psi_n \rangle|^2$$

$$\leq \Delta_{10}^{-2} \Big\{ \sum_n |\langle \psi_0 | \, \delta H \, | \psi_n \rangle|^2 - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$$

$$\Delta_{10} \leq \Delta_{n0}$$

$$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$$

$$|Q_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2}\, |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

$$\leq \Delta_{10}^{-2} \left\{ \sum_n |\langle \psi_0 | \delta H | \psi_n \rangle|^2 - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right\}$$

$$\Delta_{10} \leq \Delta_{n0}$$

$$\downarrow$$

$$\sum_n \langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H^\dagger | \psi_0 \rangle$$

$$\|$$

$$\sum_n |\psi_n\rangle\langle\psi_n| = \mathbb{1} \qquad \langle \psi_0 | \delta H \delta H^\dagger | \psi_0 \rangle$$

$1 \quad g_{\mu\nu} = \left(\quad \right)$

$$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$$

$$|Q_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

$$\leq \Delta_{10}^{-2} \left\{ \sum_n |\langle \psi_0 | \delta H | \psi_n \rangle|^2 - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right\} = \Delta_{10}^{-2} \left\{ \langle \delta H \delta H^\dagger \rangle - \langle \delta H \rangle^2 \right\}$$

$\Delta_{10} \leq \Delta_{n0}$

$$\sum_n \langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H^\dagger | \psi_0 \rangle$$

$$\sum_n |\psi_n\rangle\langle\psi_n| = \mathbb{1} \qquad \parallel \qquad \langle \psi_0 | \delta H \delta H^\dagger | \psi_0 \rangle$$

$g_{\mu\nu} = \left(\quad \right)$

$$\delta H = \sum_\mu (\partial_\mu H)\, \phi_\mu = \langle \partial H | \phi \rangle$$

$$|Q_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \, \delta H \, | \psi_n \rangle|^2$$

$$\leq \Delta_{10}^{-2} \left\{ \sum_n |\langle \psi_0 | \, \delta H \, | \psi_n \rangle|^2 - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right\} = \Delta_{10}^{-2} \left\{ \langle \delta H \delta H^\dagger \rangle - \langle \delta H \rangle^2 \right\}$$

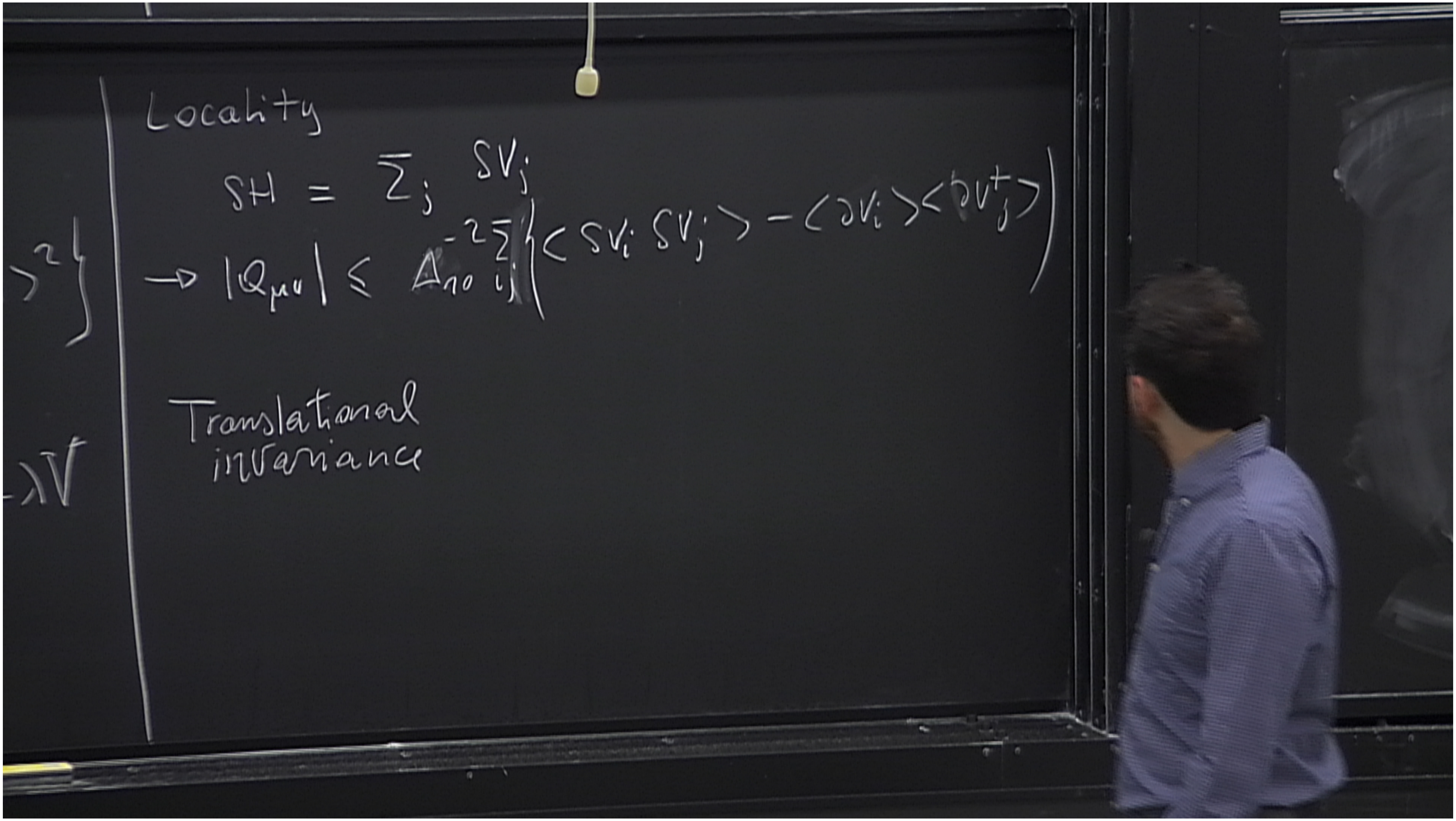
$\Delta_{10} \leq \Delta_{n0}$

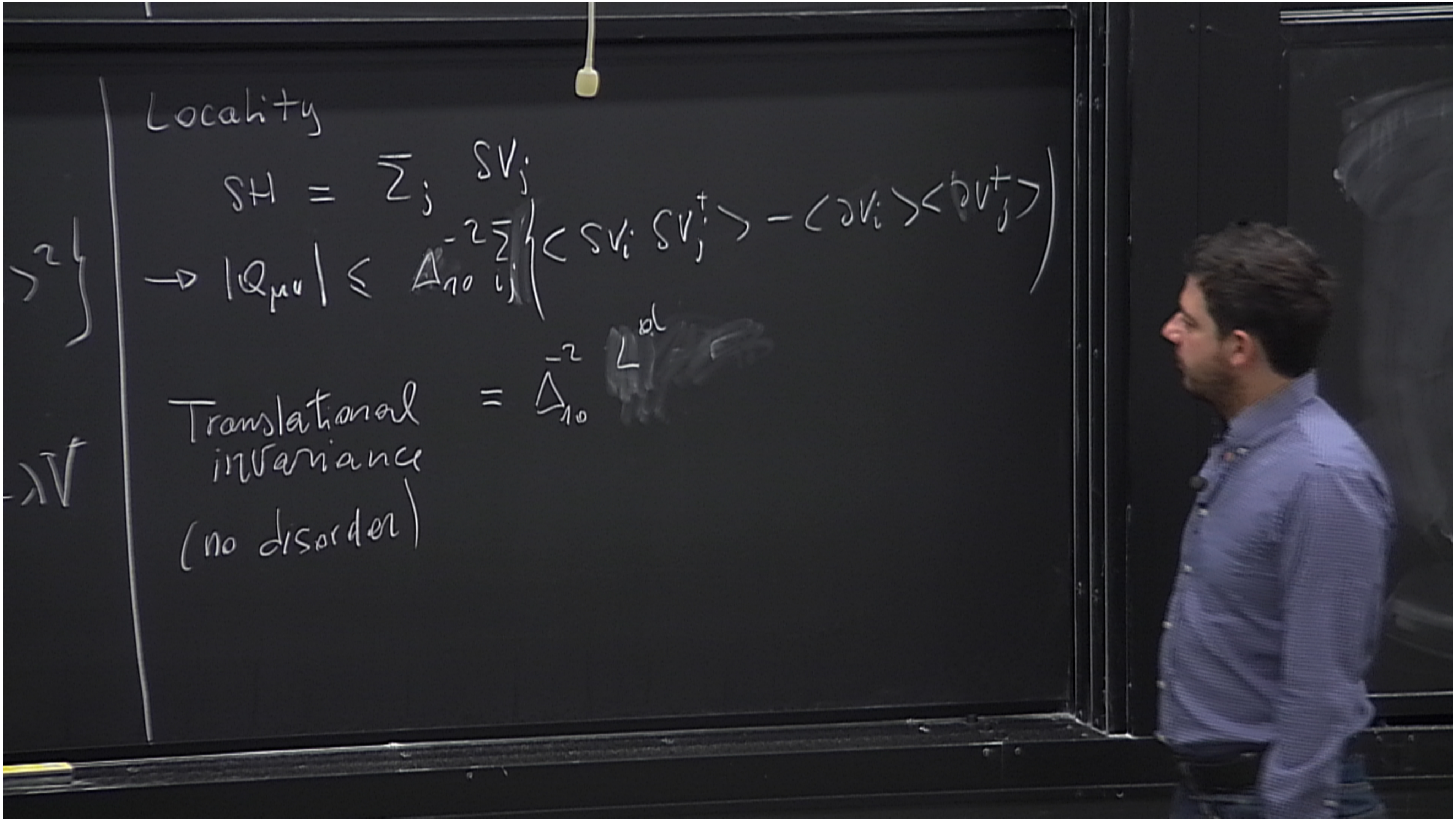
$$\sum_n \langle \psi_0 | \delta H | \psi_n \rangle \langle \psi_n | \delta H^\dagger | \psi_0 \rangle$$

$$\| \qquad \sum_n |\psi_n\rangle\langle\psi_n| = \mathbb{1}$$

$$\langle \psi_0 | \, \delta H \delta H^\dagger | \psi_0 \rangle$$

$$H(\lambda) = H(0) + \lambda V$$


Locality
δH = Σ_j δV_j
→ |Q_μν| ≤ Δ_10^-2 Σ_ij (<δV_i δV_j> − <δV_i><δV_j^†>)
Translational invariance


Locality
$\delta H = \sum_j \delta V_j$
$\rightarrow |Q_{\mu\nu}| \leq \Delta_{10}^{-2} \sum_{i,j} \left(\langle \delta V_i \, \delta V_j^\dagger \rangle - \langle \delta V_i \rangle \langle \delta V_j^\dagger \rangle \right)$
$= \Delta_{10}^{-2} L^{d}$
Translational invariance
(no disorder)

Locality

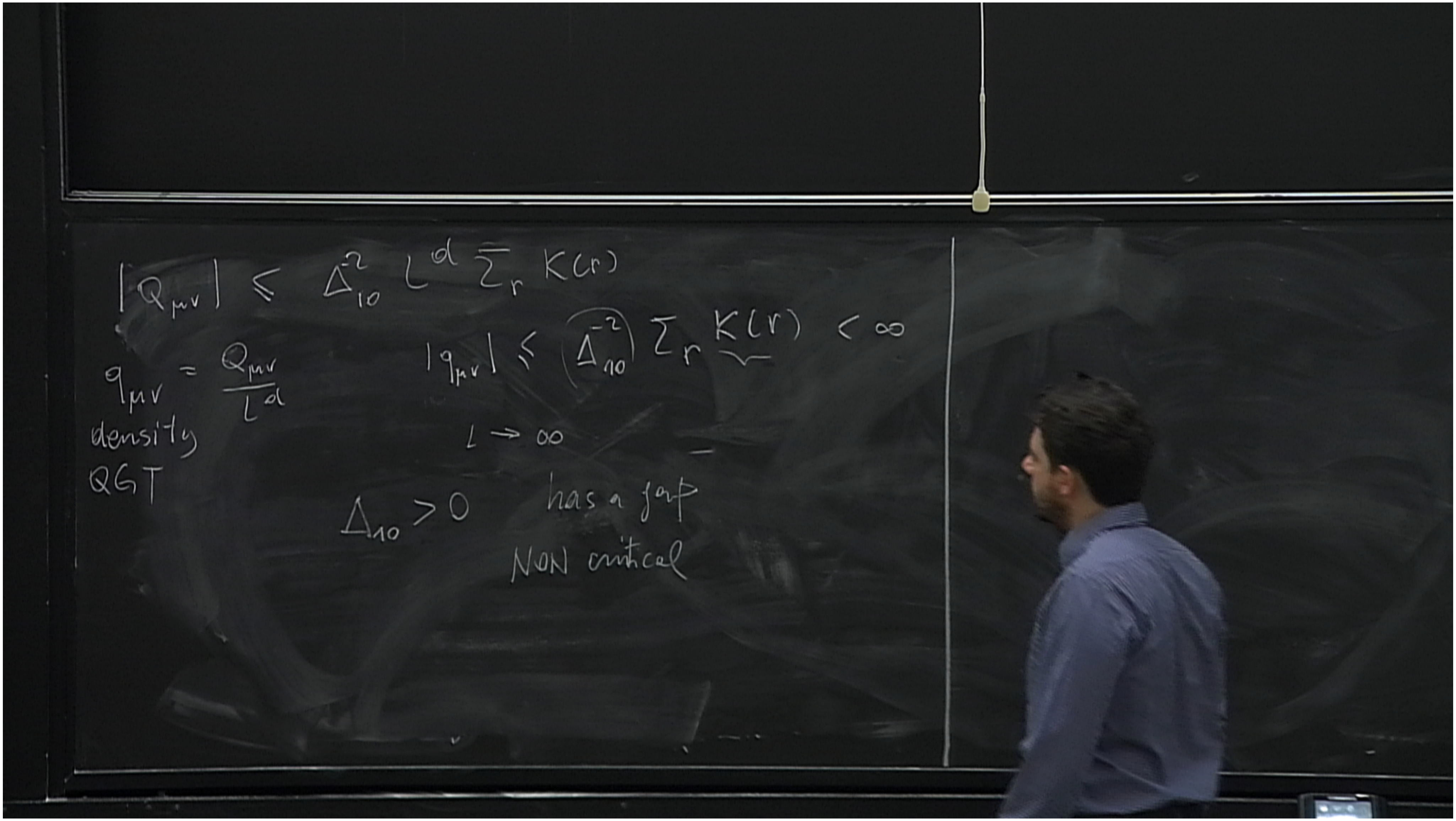
$$\delta H = \sum_j \delta V_j$$

$$\rightarrow |Q_{\mu\nu}| \leq \Delta_{10}^{-2} \sum_{ij} \left(\langle \delta V_i \, \delta V_j^\dagger \rangle - \langle \delta V_i \rangle \langle \delta V_j^\dagger \rangle \right)$$

$$= \Delta_{10}^{-2} L^d \sum_r K(r)$$

Translational invariance (no disorder)

$$K(r) = \langle \delta V(r) \, \delta V^\dagger(0) \rangle - \langle \delta V(r) \rangle \langle \delta V^\dagger(0) \rangle$$


|Q_μν| ≤ Δ_10^-2 L^d Σ_r K(r)
|q_μν| ≤ Δ_10^-2 Σ_r K(r) < ∞
q_μν = Q_μν / L^d
density
QGT
L → ∞
Δ_10 > 0
has a gap
NON critical

$$|Q_{\mu\nu}| \leq \Delta_{10}^{-2} L^d \sum_r K(r)$$

$q_{\mu\nu} = \dfrac{Q_{\mu\nu}}{L^d}$

$q_{\mu\nu}$ density

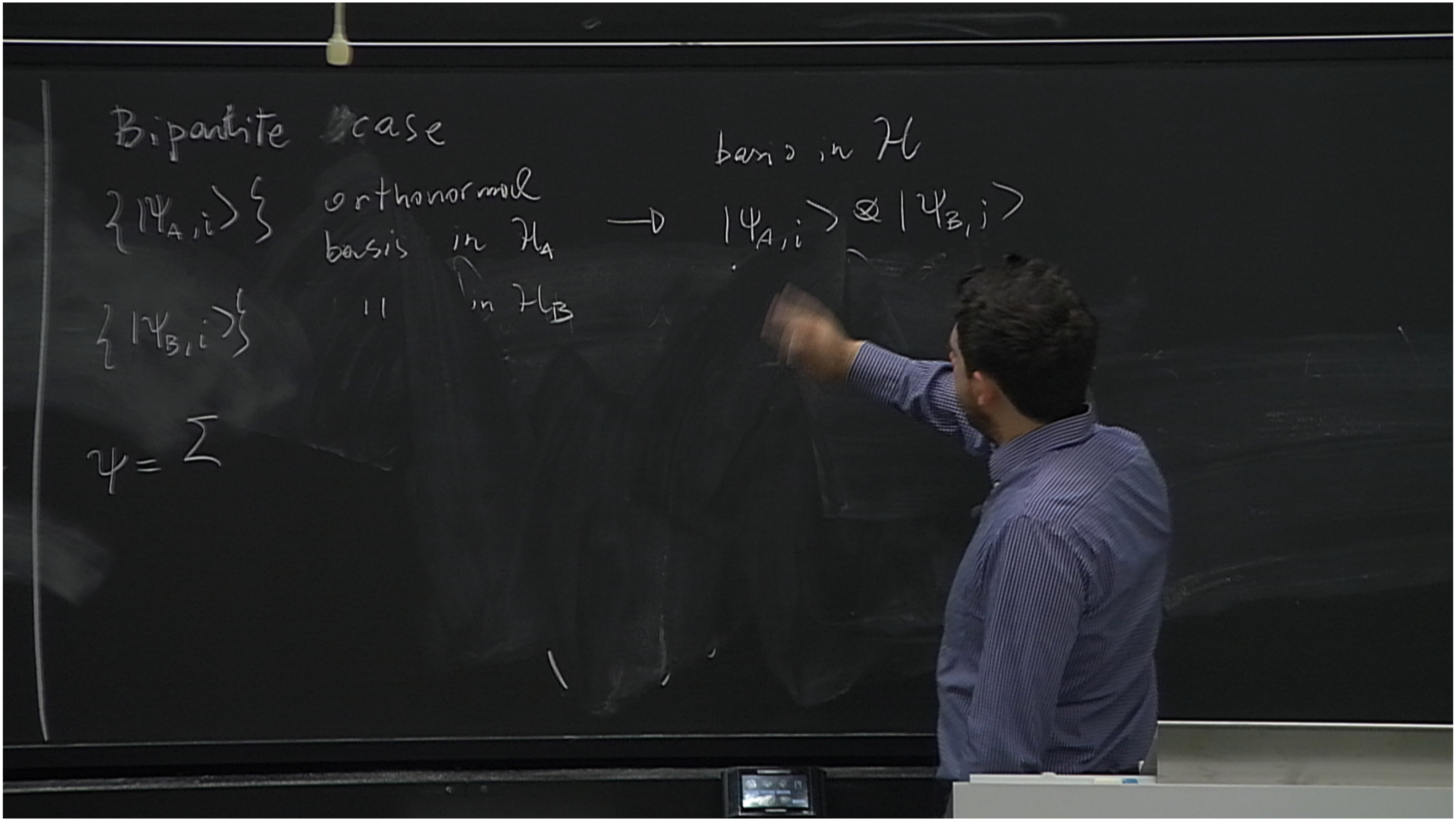
QGT

$$|q_{\mu\nu}| \leq \Delta_{10}^{-2} \sum_r \underbrace{K(r)} < \infty$$

$L \to \infty$

$\Delta_{10} > 0$ has a gap

NON critical


Bipartite case
{|ψ_{A,i}>} orthonormal basis in H_A
{|ψ_{B,i}>} " in H_B
basis in H
→ |ψ_{A,i}> ⊗ |ψ_{B,i}>
ψ = Σ

Bipartite case

$\{|\psi_{A,i}\rangle\}$ orthonormal basis in $\mathcal{H}_A$ $\longrightarrow$ $|\psi_{A,i}\rangle \otimes |\psi_{B,i}\rangle$ basis in $\mathcal{H}$

$\{|\psi_{B,i}\rangle\}$ " in $\mathcal{H}_B$

$$\psi = \sum_i \alpha_i \, |\psi_{Ai}\rangle |\psi_{Bi}\rangle$$

Schmidt decomposition

$$\rho_A = Tr_B \left(|\psi\rangle\langle\psi| \right) = \sum_i \alpha_i^2 \, |\psi_{Ai}\rangle\langle\psi_{Ai}|$$

Bipartite case

$\{|\psi_{A,i}\rangle\}$ orthonormal basis in $\mathcal{H}_A$ $\longrightarrow$ $|\psi_{A,i}\rangle \otimes |\psi_{B,i}\rangle$ basis in $\mathcal{H}$

$\{|\psi_{B,i}\rangle\}$ " in $\mathcal{H}_B$

$$\psi = \sum_i \alpha_i \, |\psi_{Ai}\rangle |\psi_{Bi}\rangle$$

Schmidt decomposition

$$\rho_A = \mathrm{Tr}_B\left(|\psi\rangle\langle\psi|\right) = \sum_i \alpha_i^2 \, |\psi_{Ai}\rangle\langle\psi_{Ai}|$$

$$\mathrm{Tr}\,\rho_A = 1 \rightarrow \sum_i \alpha_i^2 = 1$$

$\{\alpha_i^2\}$ prob. distribution

Bipartite case

$\{|\psi_{A,i}\rangle\}$ orthonormal basis in $\mathcal{H}_A$ $\rightarrow$ basis in $\mathcal{H}$: $|\psi_{A,i}\rangle \otimes |\psi_{B,i}\rangle$

$\{|\psi_{B,i}\rangle\}$ " in $\mathcal{H}_B$

$$\psi = \sum_i \alpha_i \, |\psi_{Ai}\rangle |\psi_{Bi}\rangle$$ Schmidt decomposition

$$\rho_A = Tr_B \left(|\psi\rangle\langle\psi| \right) = \sum_i \alpha_i^2 \, |\psi_{Ai}\rangle\langle\psi_{Ai}|$$

$Tr \rho_A = 1 \rightarrow \sum_i \alpha_i^2 = 1$

$\{\alpha_i^2\}$ prob. distribution

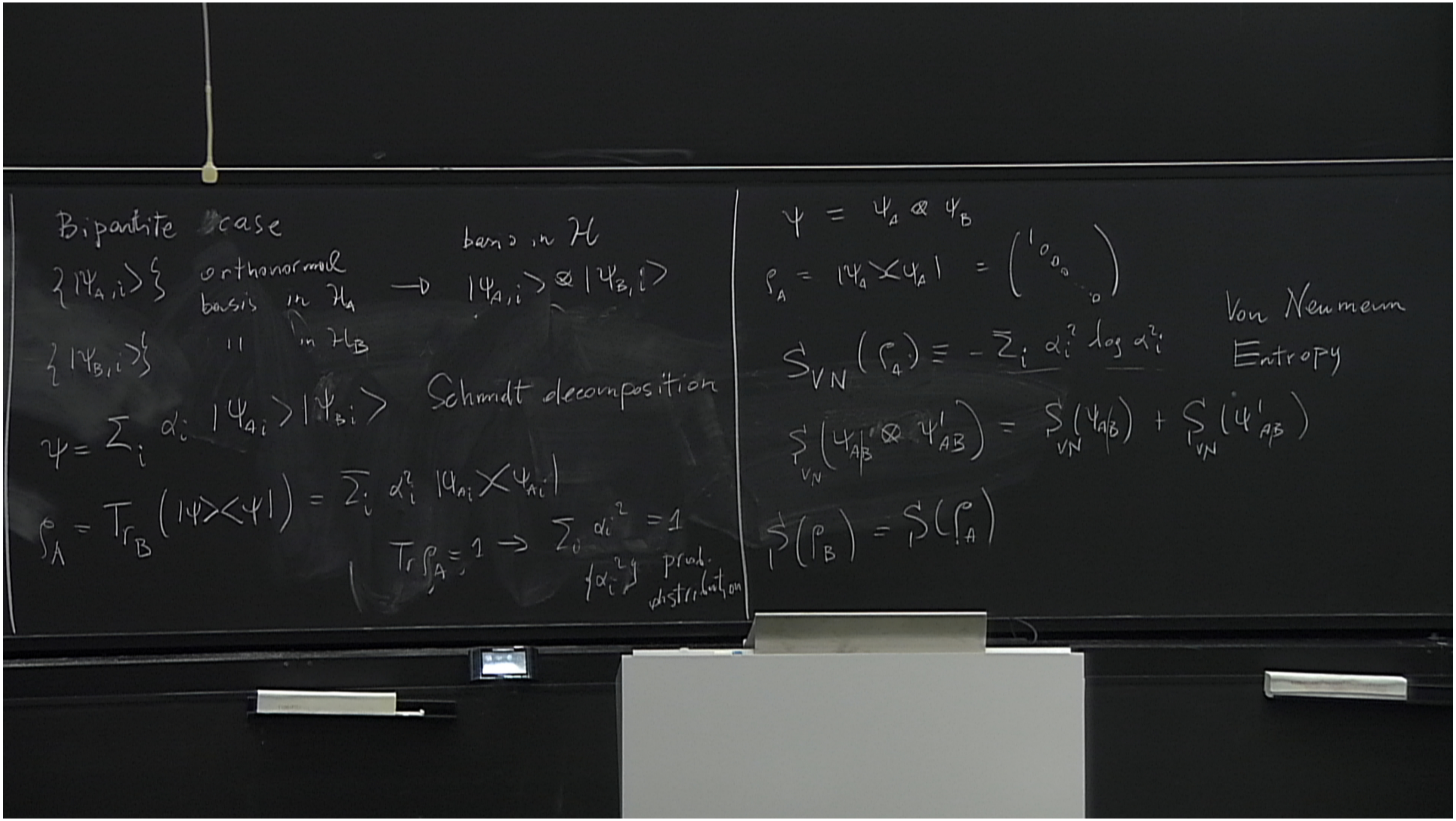
$$\psi = \psi_A \otimes \psi_B$$

$$\rho_A = |\psi_A\rangle\langle\psi_A| = \begin{pmatrix} 1 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}$$

Von Neumen Entropy

$$S_{VN}(\rho_A) \equiv -\sum_i \alpha_i^2 \log \alpha_i^2$$

$$S_{VN}\left(\psi_{A|B} \otimes \psi'_{AB}\right) = S_{VN}(\psi_{A|B}) + S_{VN}(\psi'_{AB})$$


Bipartite case
$\{|\psi_{A,i}\rangle\}$ orthonormal basis in $\mathcal{H}_A$
$\{|\psi_{B,i}\rangle\}$ " in $\mathcal{H}_B$
basis in $\mathcal{H}$
$\longrightarrow |\psi_{A,i}\rangle \otimes |\psi_{B,i}\rangle$
$\psi = \sum_i \alpha_i |\psi_{Ai}\rangle |\psi_{Bi}\rangle$ Schmidt decomposition
$\rho_A = Tr_B(|\psi\rangle\langle\psi|) = \sum_i \alpha_i^2 |\psi_{Ai}\rangle\langle\psi_{Ai}|$
$Tr\,\rho_A = 1 \rightarrow \sum_i \alpha_i^2 = 1$
$\{\alpha_i^2\}$ prob. distribution
$\psi = \psi_A \otimes \psi_B$
$\rho_A = |\psi_A\rangle\langle\psi_A| = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$
$S_{VN}(\rho_A) \equiv -\sum_i \alpha_i^2 \log \alpha_i^2$
Von Neumann Entropy
$S_{VN}(\psi_{AB} \otimes \psi'_{AB}) = S_{VN}(\psi_{AB}) + S_{VN}(\psi'_{AB})$
$S(\rho_B) = S(\rho_A)$