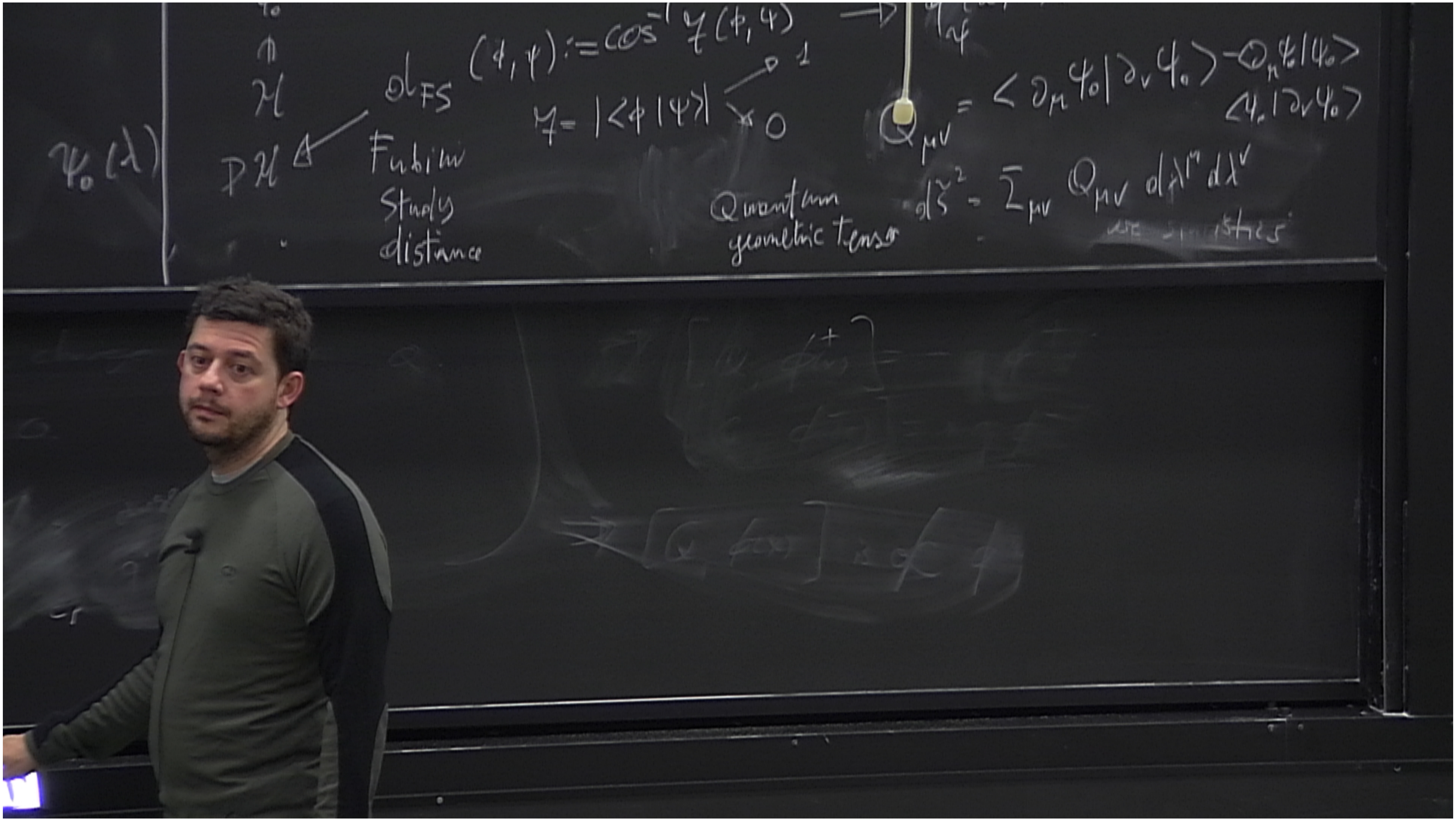
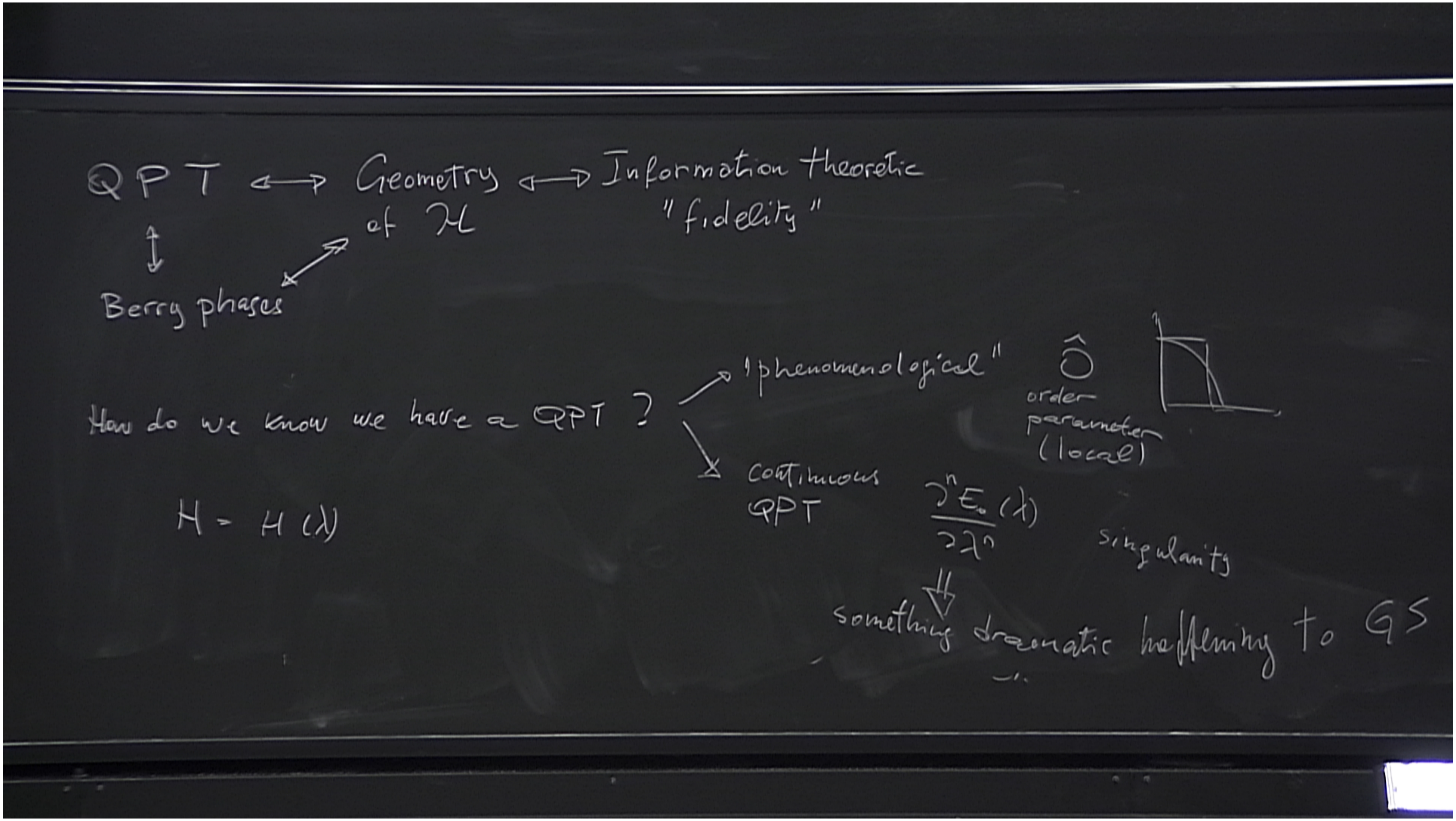
Title: Condensed Matter (Review) - Lecture 7

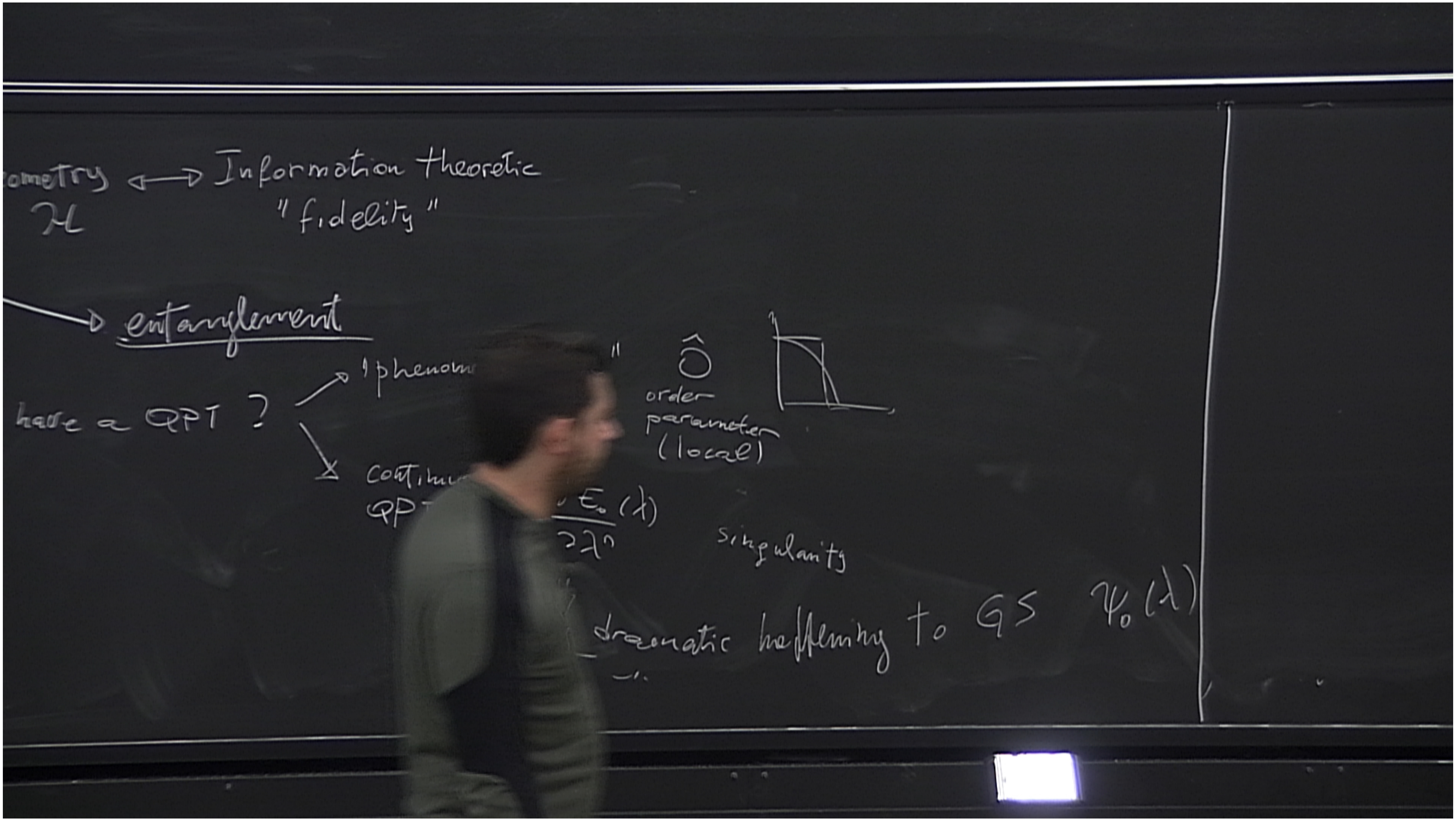
Date: Jan 10, 2012 10:15 AM

URL: http://pirsa.org/12010091

Abstract:


Fubini
Study
distance
Quantum
geometric tensor


QPT ⟷ Geometry of $\mathcal{H}$ ⟷ Information theoretic "fidelity"
Berry phases
How do we know we have a QPT ?
'phenomenological"
$\hat{O}$
order parameter (local)
continuous QPT
$H = H(\lambda)$
$\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$
singularity
something dramatic happening to GS


Information theoretic
"fidelity"
entanglement
have a QPT ?
order parameter
(local)
singularity
dramatic happening to GS

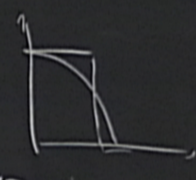
ometry ⟷ Information theoretic

$\mathcal{H}$ "fidelity"

→ entanglement

have a QPT ?

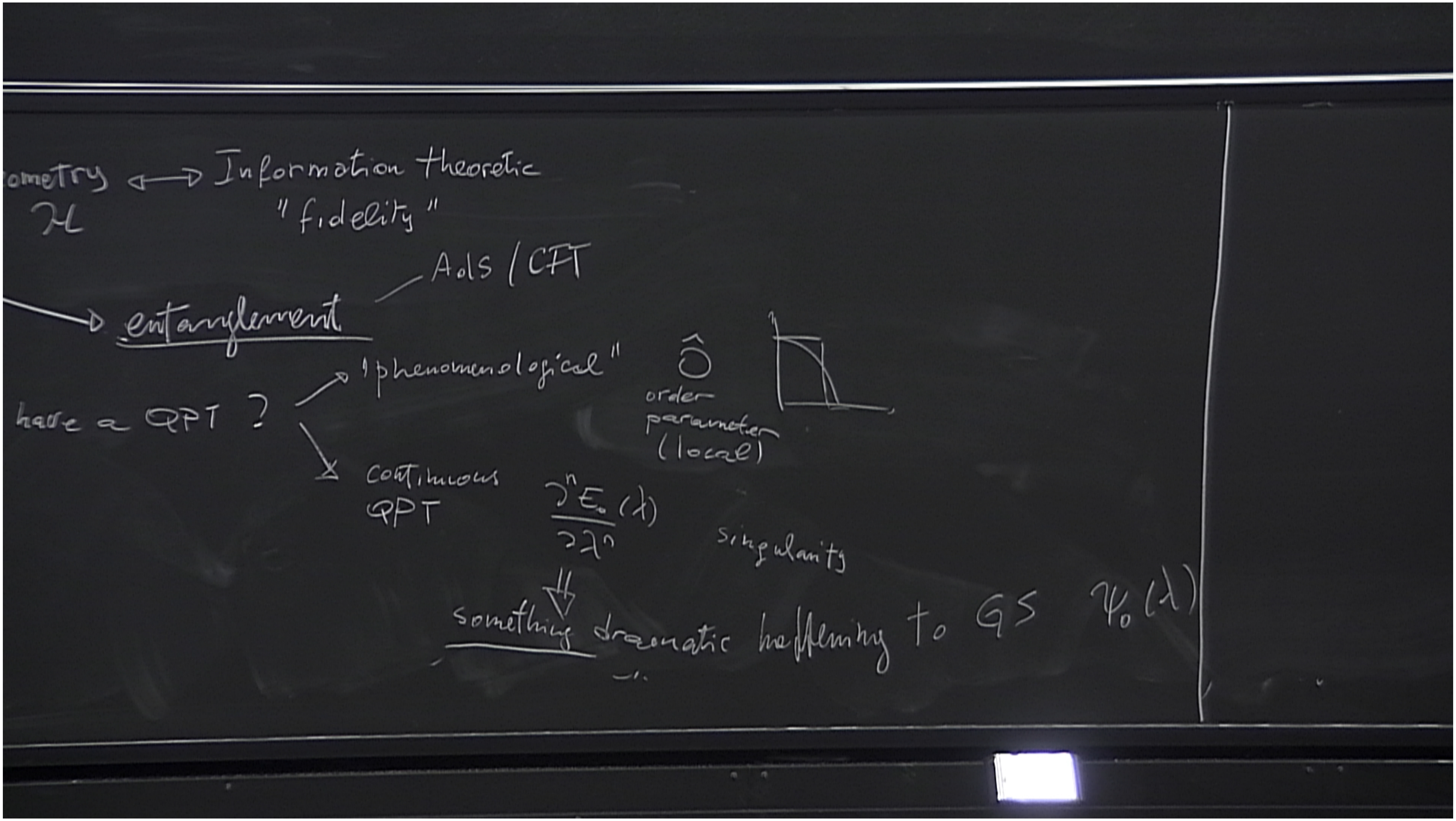
→ "phenomenological" $\hat{O}$ order parameter (local)

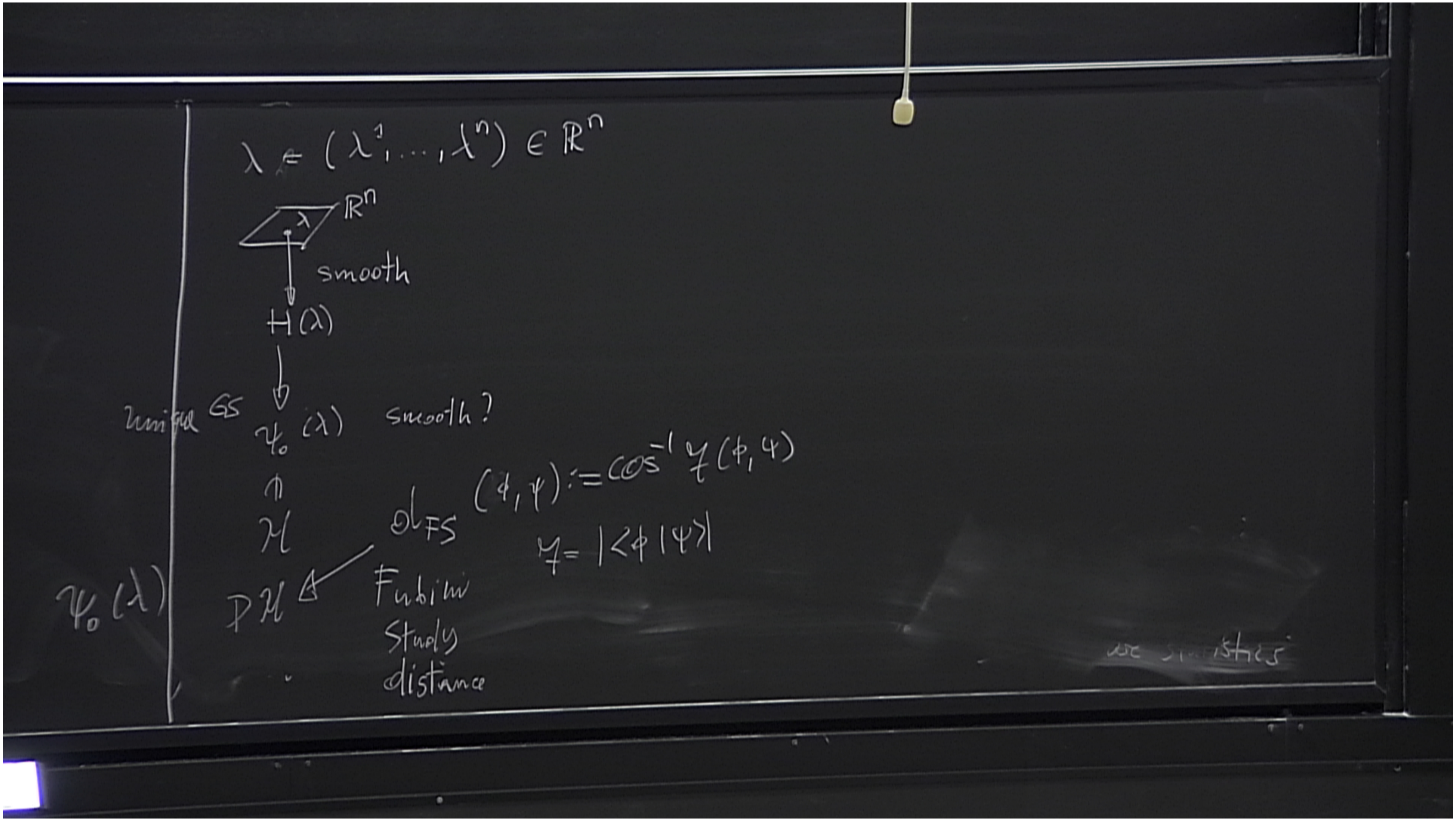


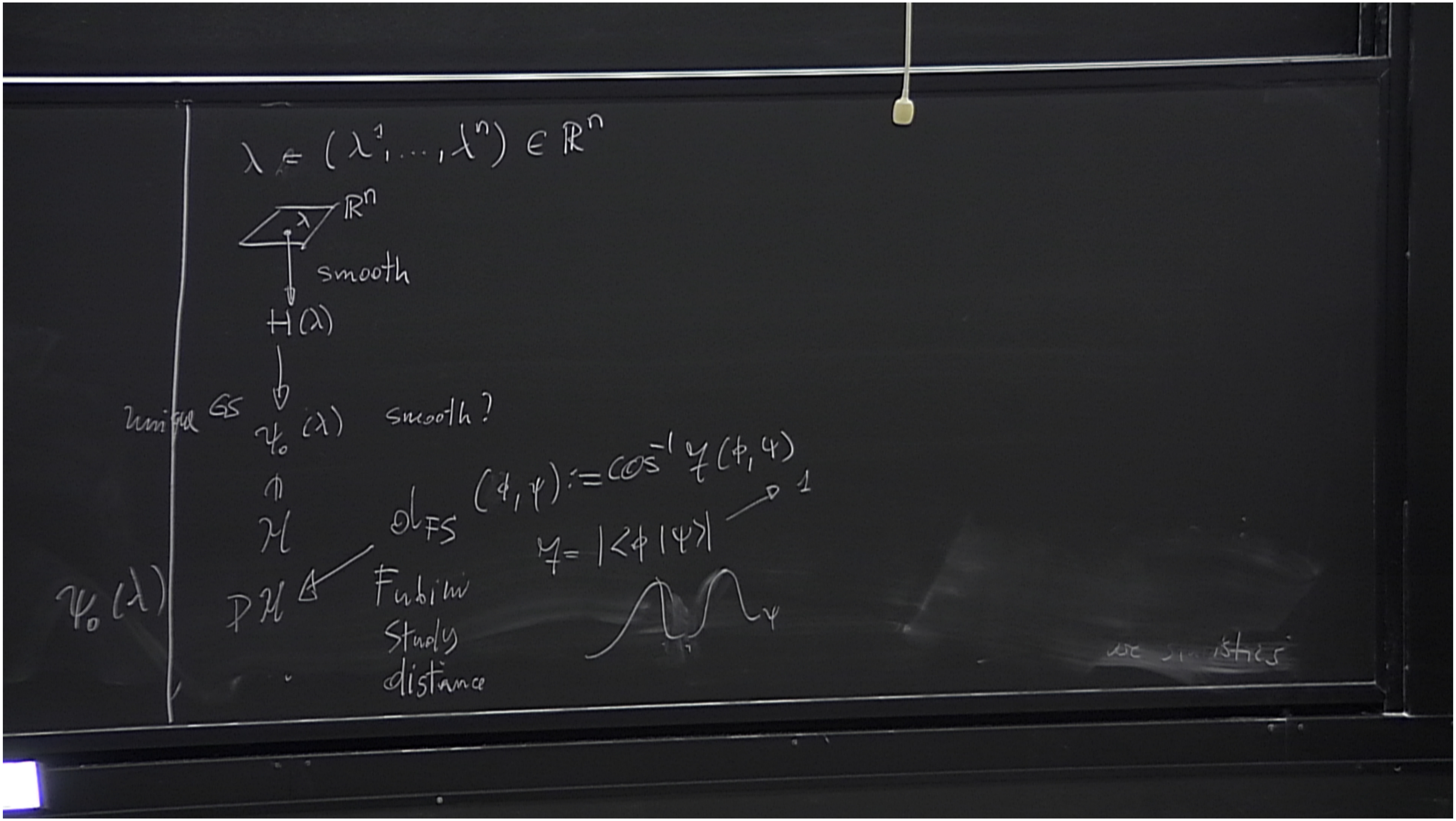
→ continuous QPT $\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$ singularity

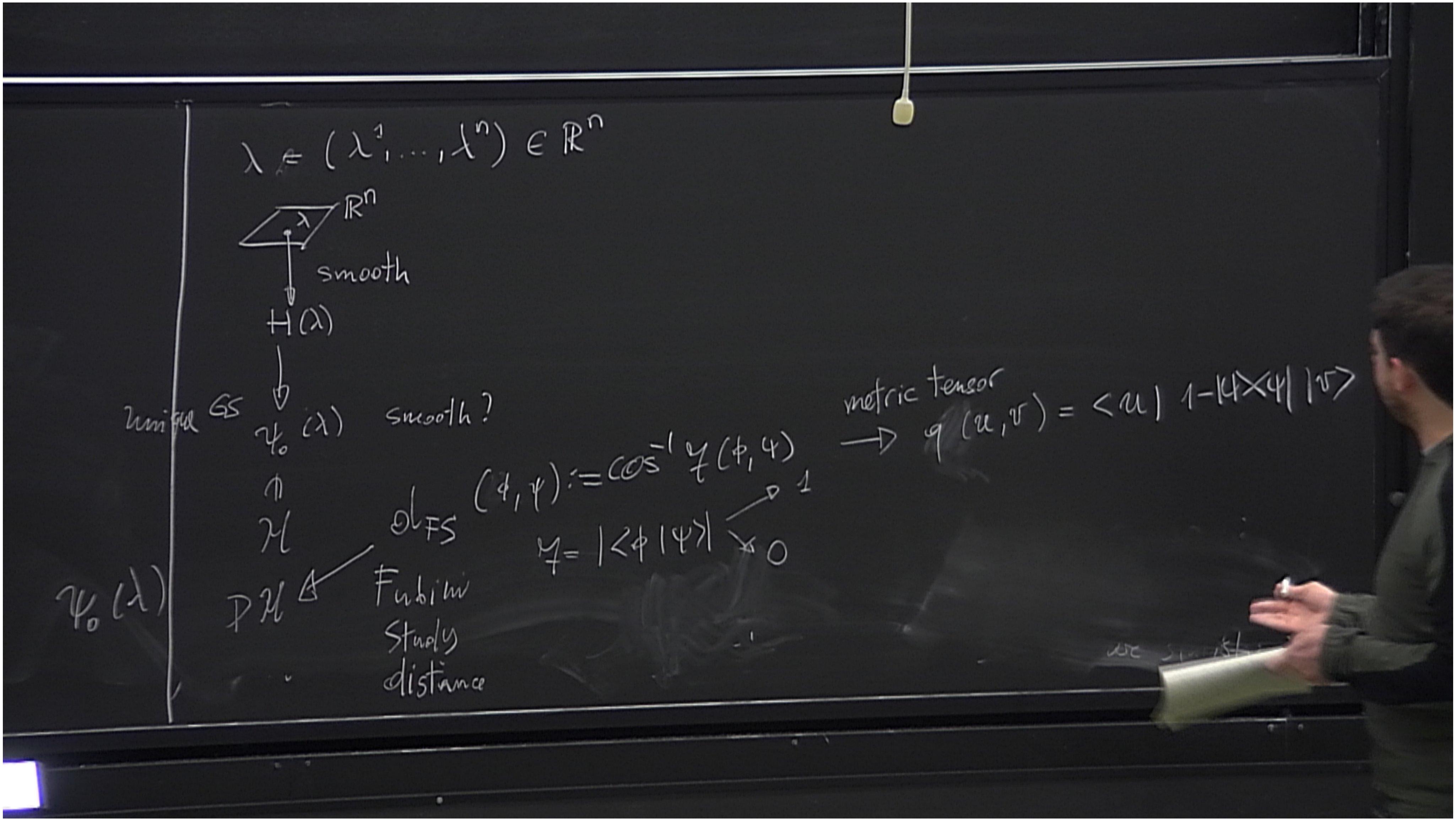
⇓

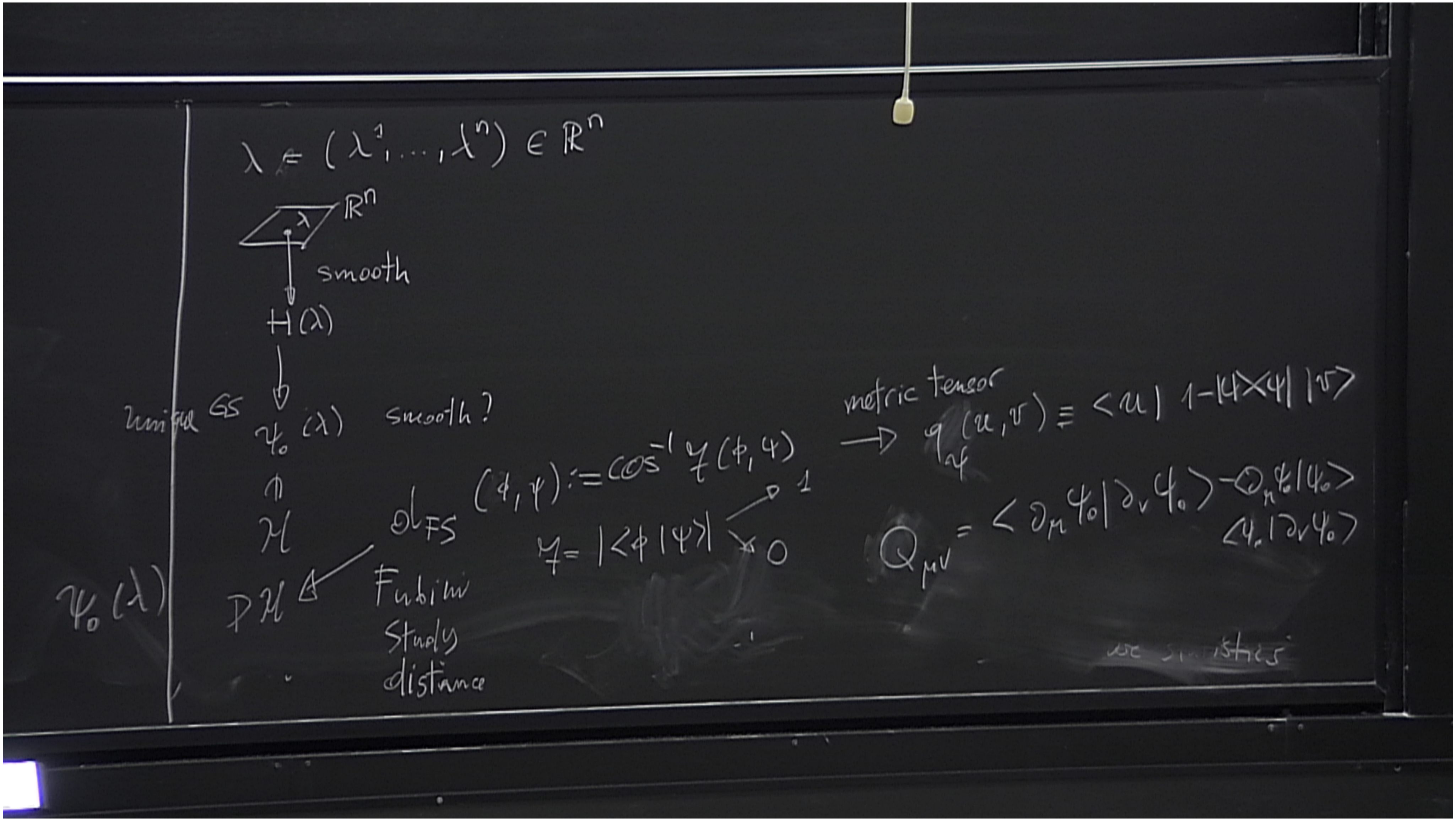
something dramatic happening to GS $\Psi_0(\lambda)$

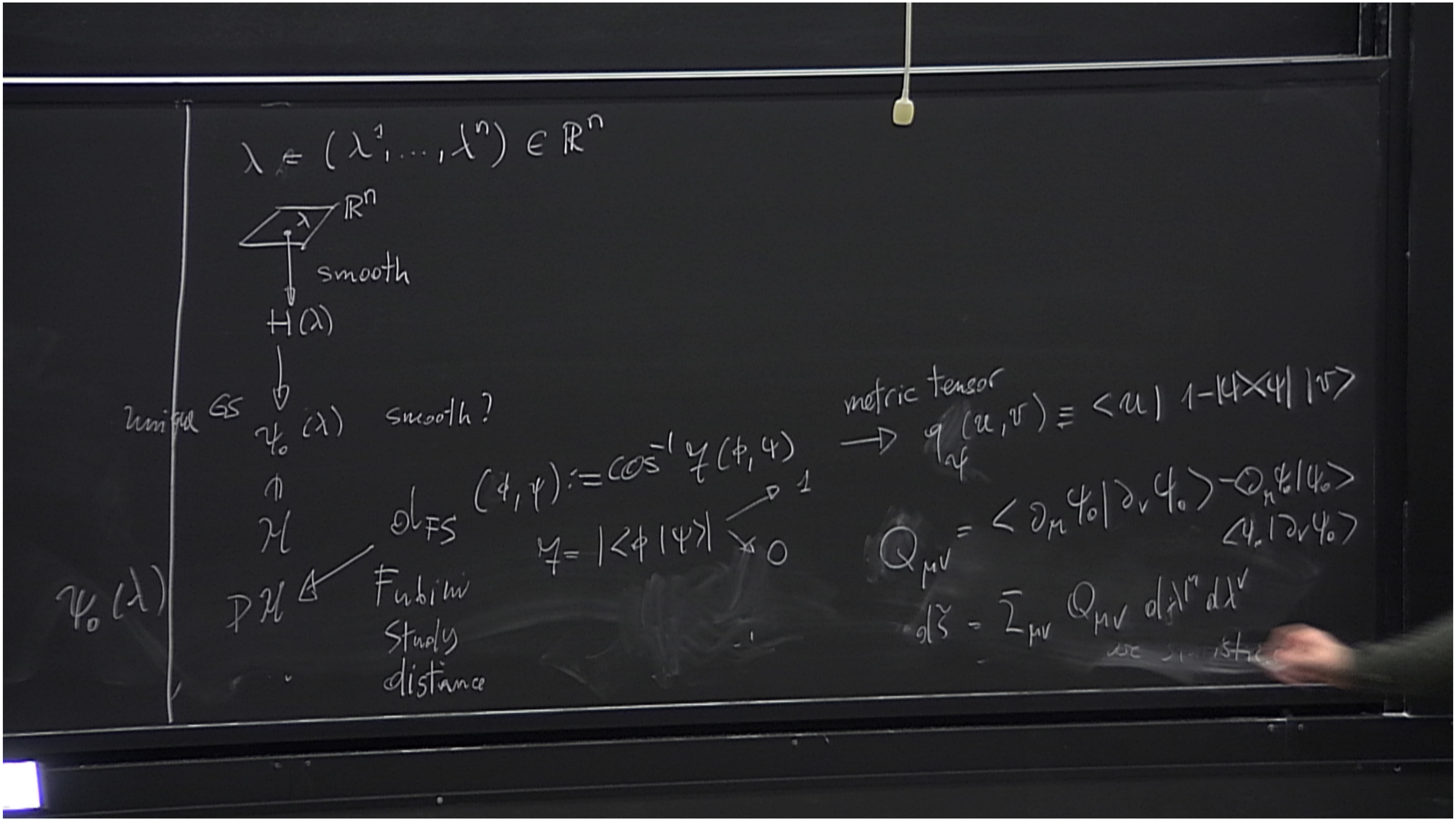

ometry ⟷ Information theoretic
$\mathcal{H}$
"fidelity"
AdS / CFT
entanglement
"phenomenological"
$\hat{O}$
order parameter (local)
have a QPT ?
continuous QPT
$\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$
singularity
something dramatic happening to GS $\Psi_0(\lambda)$

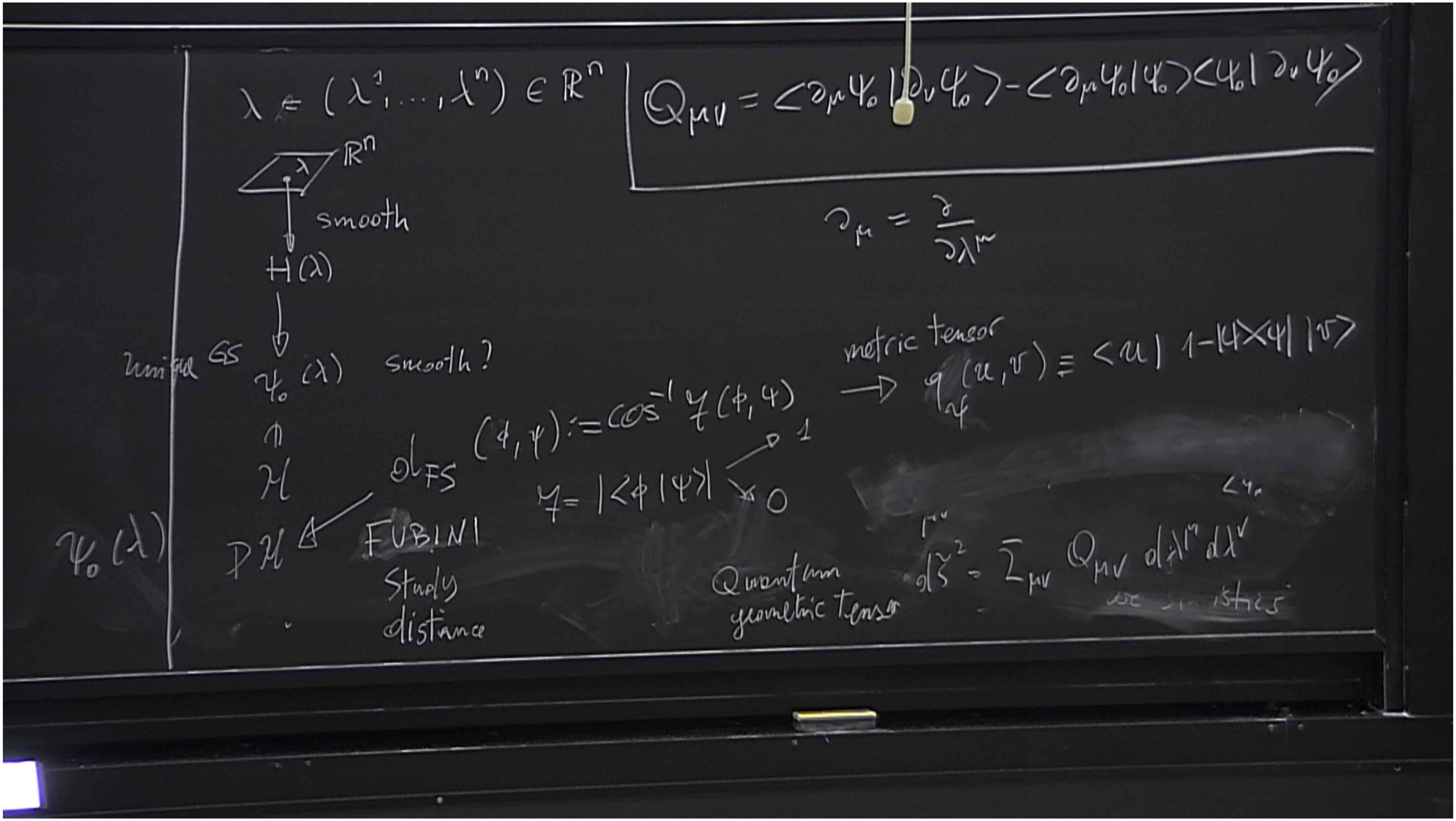

λ = (λ^1, …, λ^n) ∈ ℝ^n
ℝ^n
λ
smooth
H(λ)
unique GS
ψ_0 (λ) smooth?
ℋ
Pℋ
d_FS (φ, ψ) := cos^{-1} γ(φ, ψ)
γ = |⟨φ|ψ⟩|
Fubini
Study
distance
ψ_0 (λ)

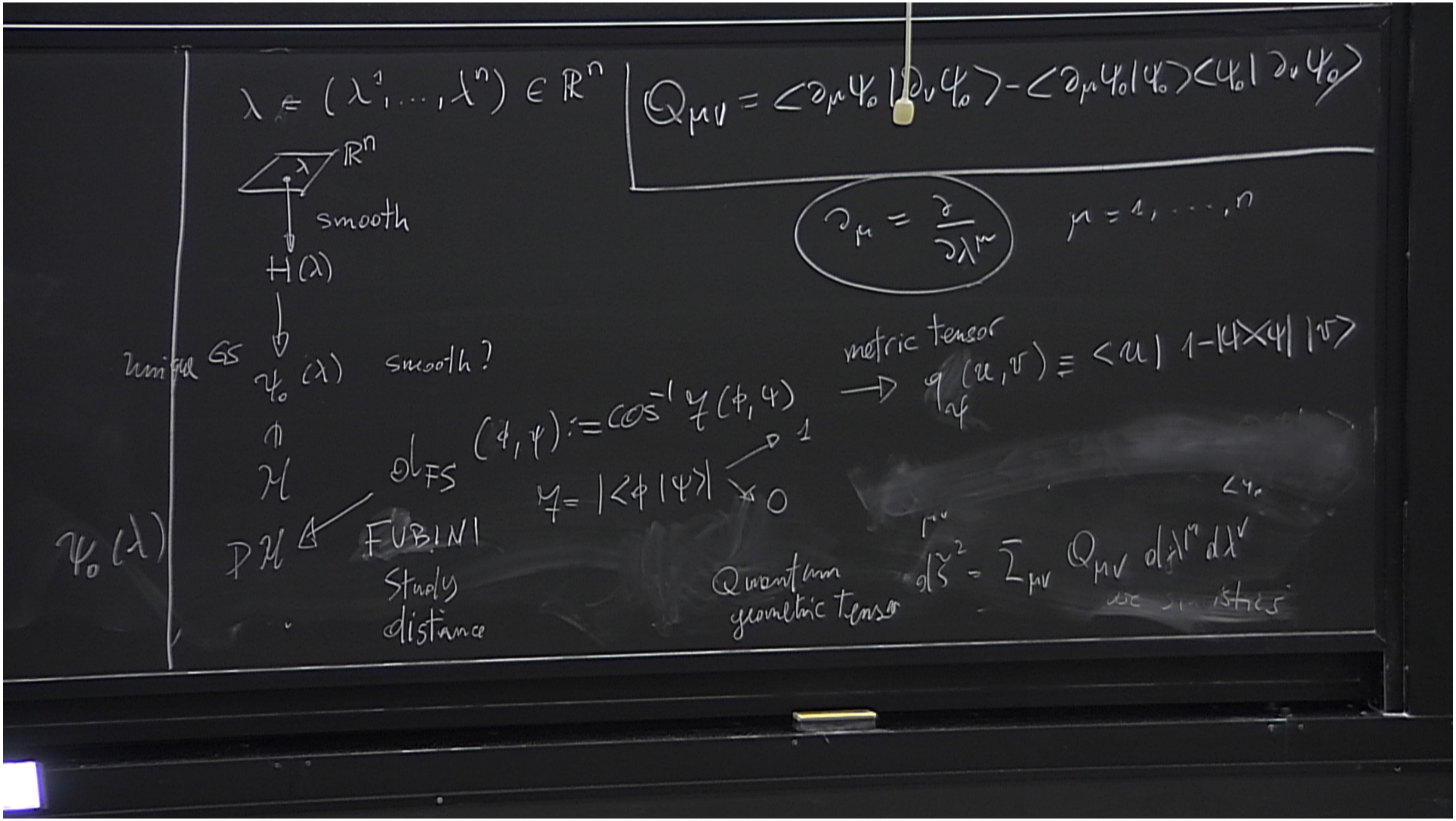

smooth
H(λ)
unique GS
smooth?
Fubini
Study
distance

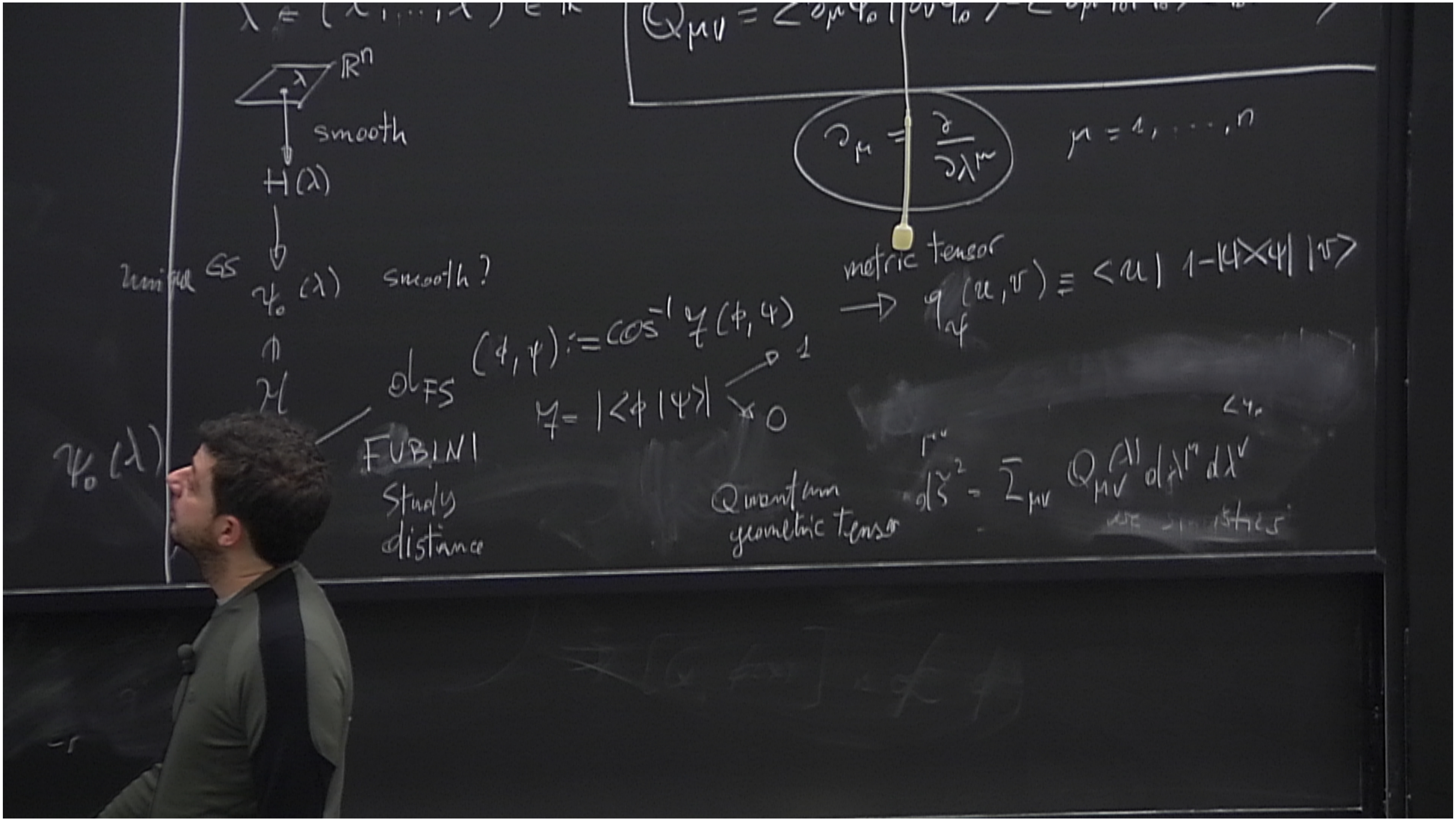

λ = (λ^1, …, λ^n) ∈ ℝ^n
ℝ^n
smooth
H(λ)
unique GS
ψ0(λ) smooth?
ℋ
Pℋ
ψ0(λ)
dFS (ϕ, ψ) := cos^-1 γ(ϕ, ψ)
γ = |<ϕ|ψ>|
1
0
Fubini Study distance
metric tensor
g(u, v) = <u| 1-|ψ><ψ| |v>

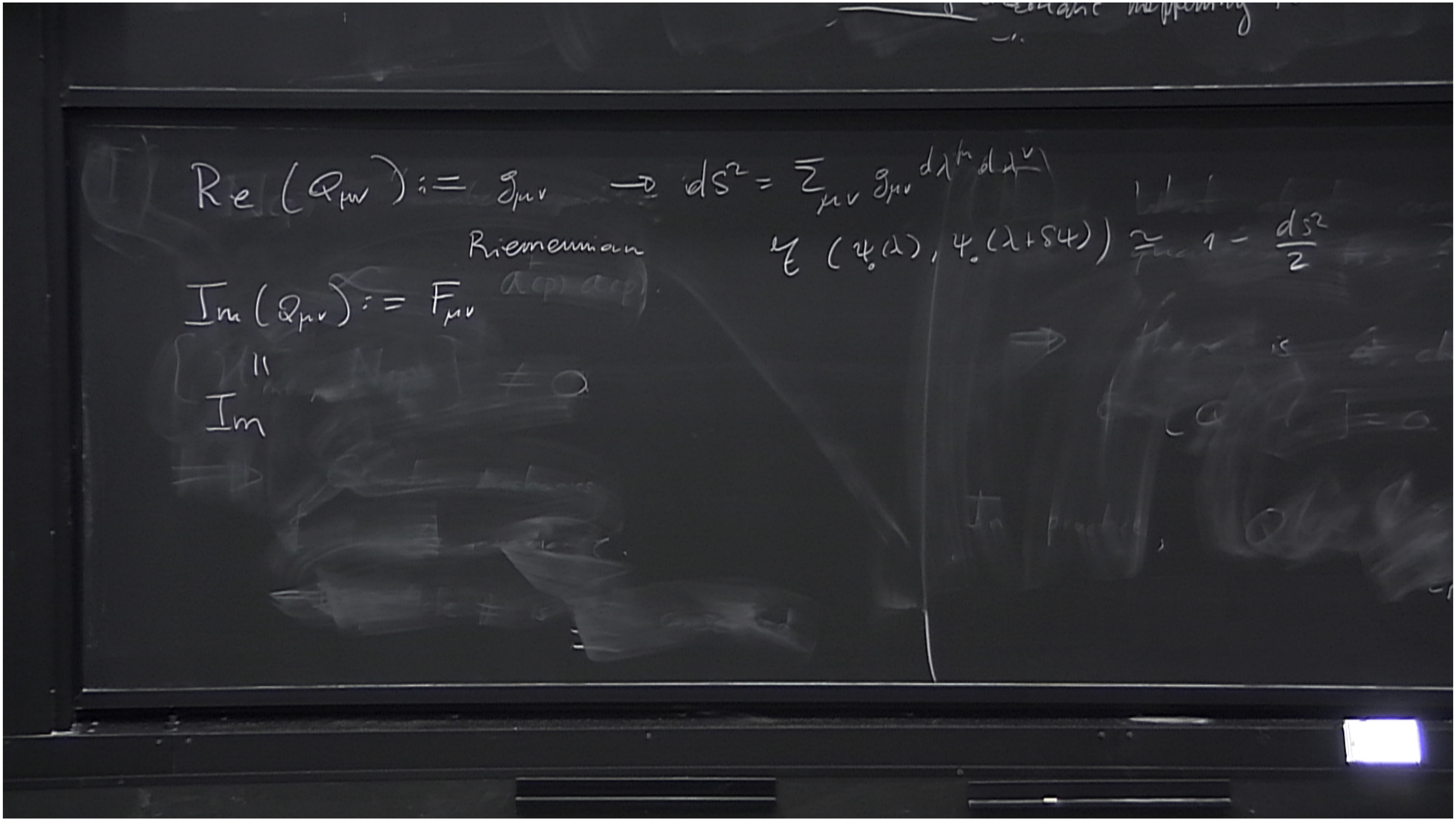

λ = (λ^1, …, λ^n) ∈ ℝ^n
ℝ^n
smooth
H(λ)
Ψ_0(λ) smooth?
ℋ
Pℋ
d_FS
Fubini
Study
distance
(φ, ψ) := cos^{-1} γ(φ, ψ)
γ = |⟨φ|ψ⟩|
metric tensor
g_ψ(u, v) ≡ ⟨u| 1 − |ψ⟩⟨ψ| |v⟩
Q_{μν} = ⟨∂_μ Ψ_0|∂_ν Ψ_0⟩ − ⟨∂_μ Ψ_0|Ψ_0⟩⟨Ψ_0|∂_ν Ψ_0⟩
Ψ_0(λ)

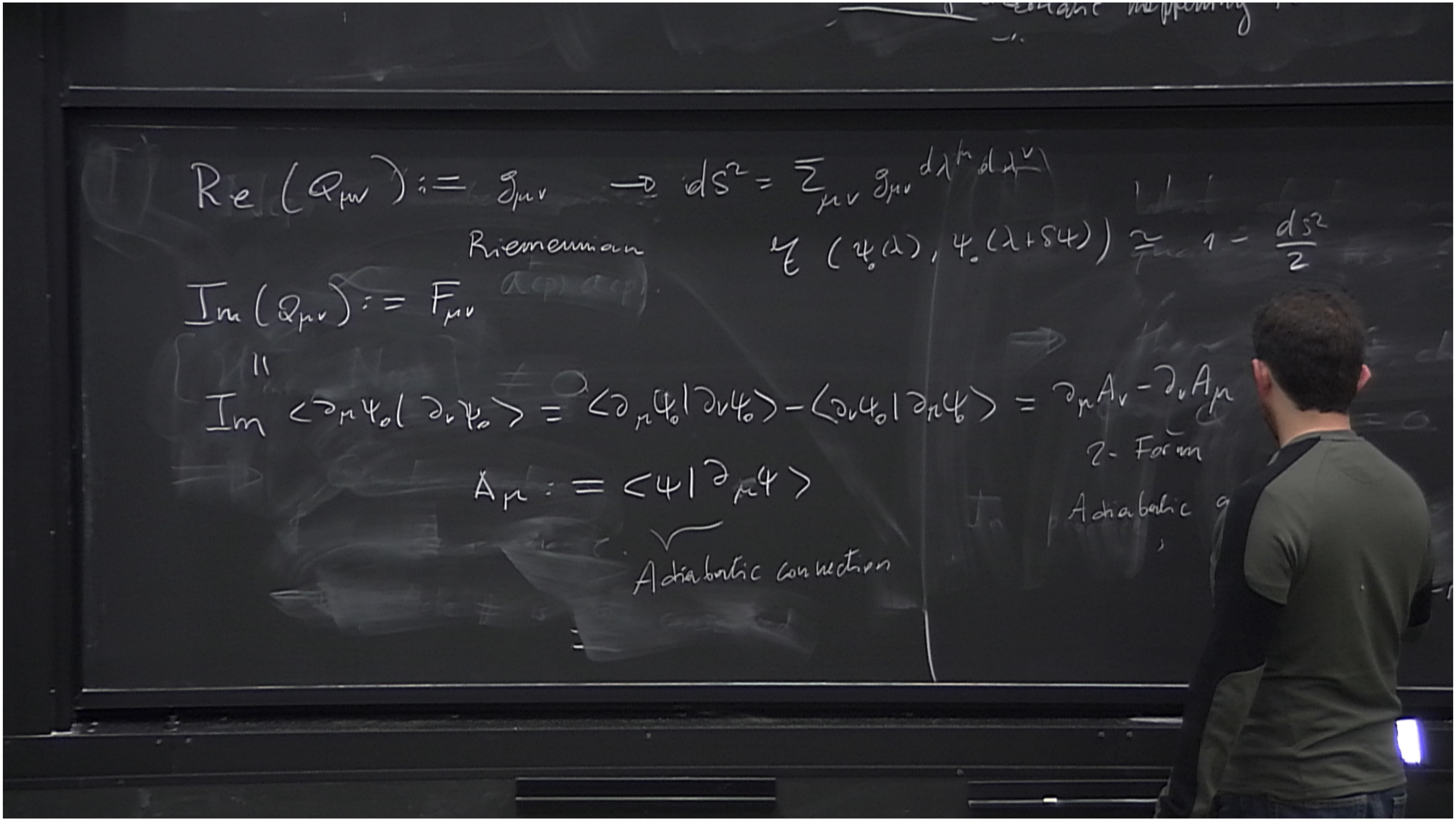

λ = (λ¹, …, λⁿ) ∈ ℝⁿ
ℝⁿ
λ
smooth
H(λ)
unique GS
ψ₀(λ)
smooth?
ℋ
Pℋ
ψ₀(λ)
d_FS
Fubini
Study
distance
(φ, ψ) := cos⁻¹ η(φ, ψ)
η = |⟨φ|ψ⟩|
1
0
metric tensor
g_ψ(u, v) ≡ ⟨u| 1−|ψ⟩⟨ψ| |v⟩
Q_μν = ⟨∂_μ ψ₀|∂_ν ψ₀⟩ − ⟨∂_μ ψ₀|ψ₀⟩⟨ψ₀|∂_ν ψ₀⟩
dζ = Σ_μν Q_μν dλ^μ dλ^ν

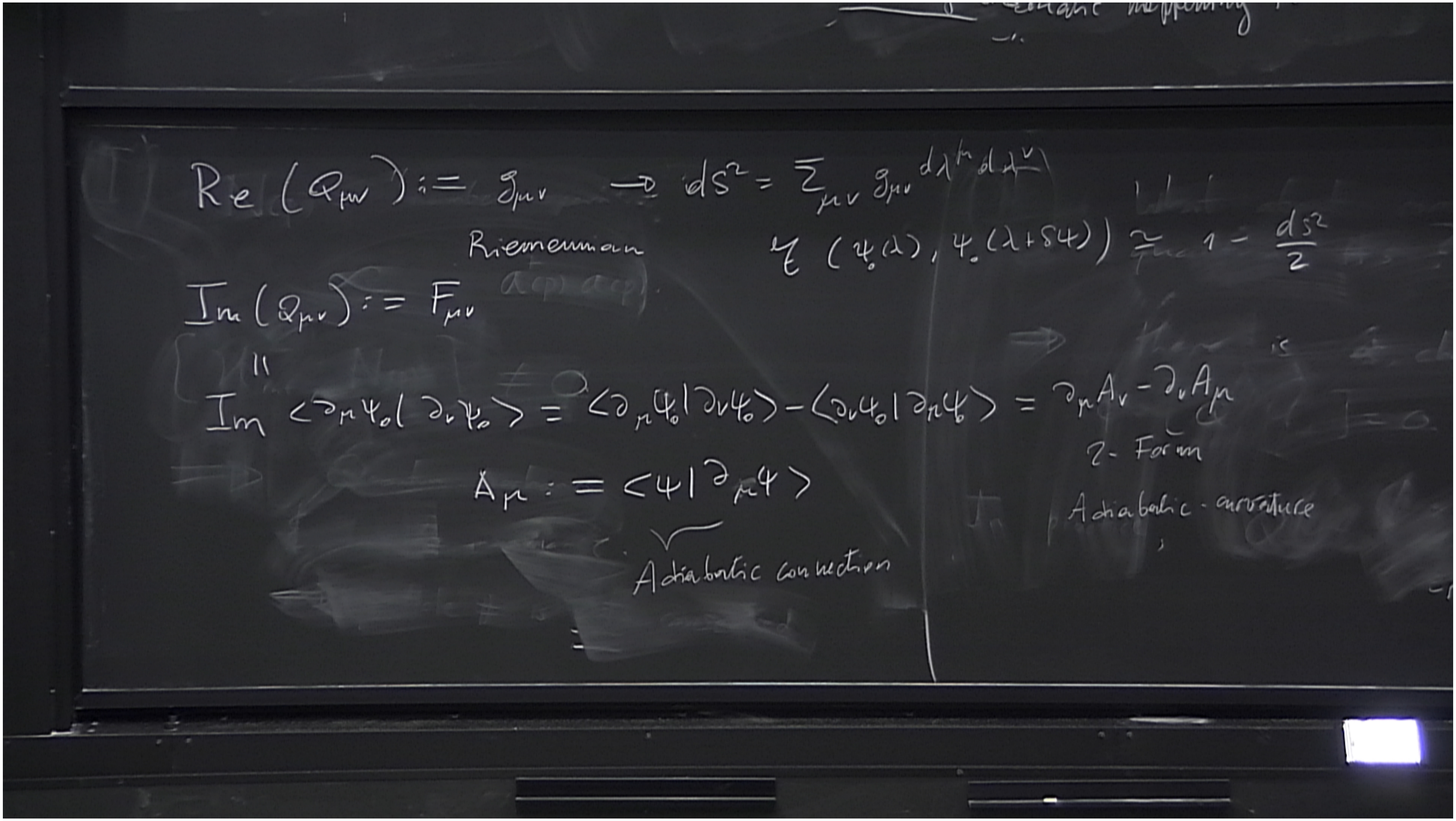

λ = (λ¹, …, λⁿ) ∈ ℝⁿ
Q_μν = <∂_μ Ψ₀ | ∂_ν Ψ₀> − <∂_μ Ψ₀ | Ψ₀><Ψ₀ | ∂_ν Ψ₀>
ℝⁿ
smooth
H(λ)
∂_μ = ∂/∂λ^μ
unique GS
Ψ₀(λ) smooth?
metric tensor
g_ψ(u, v) ≡ <u| 1−|ψ><ψ| |v>
d_FS(φ, ψ) := cos⁻¹ γ(φ, ψ)
γ = |<φ|ψ>|
1
0
ℋ
Pℋ
FUBINI
Study
distance
Ψ₀(λ)
Quantum
geometric tensor
dζ² = Σ_μν Q_μν dλ^μ dλ^ν

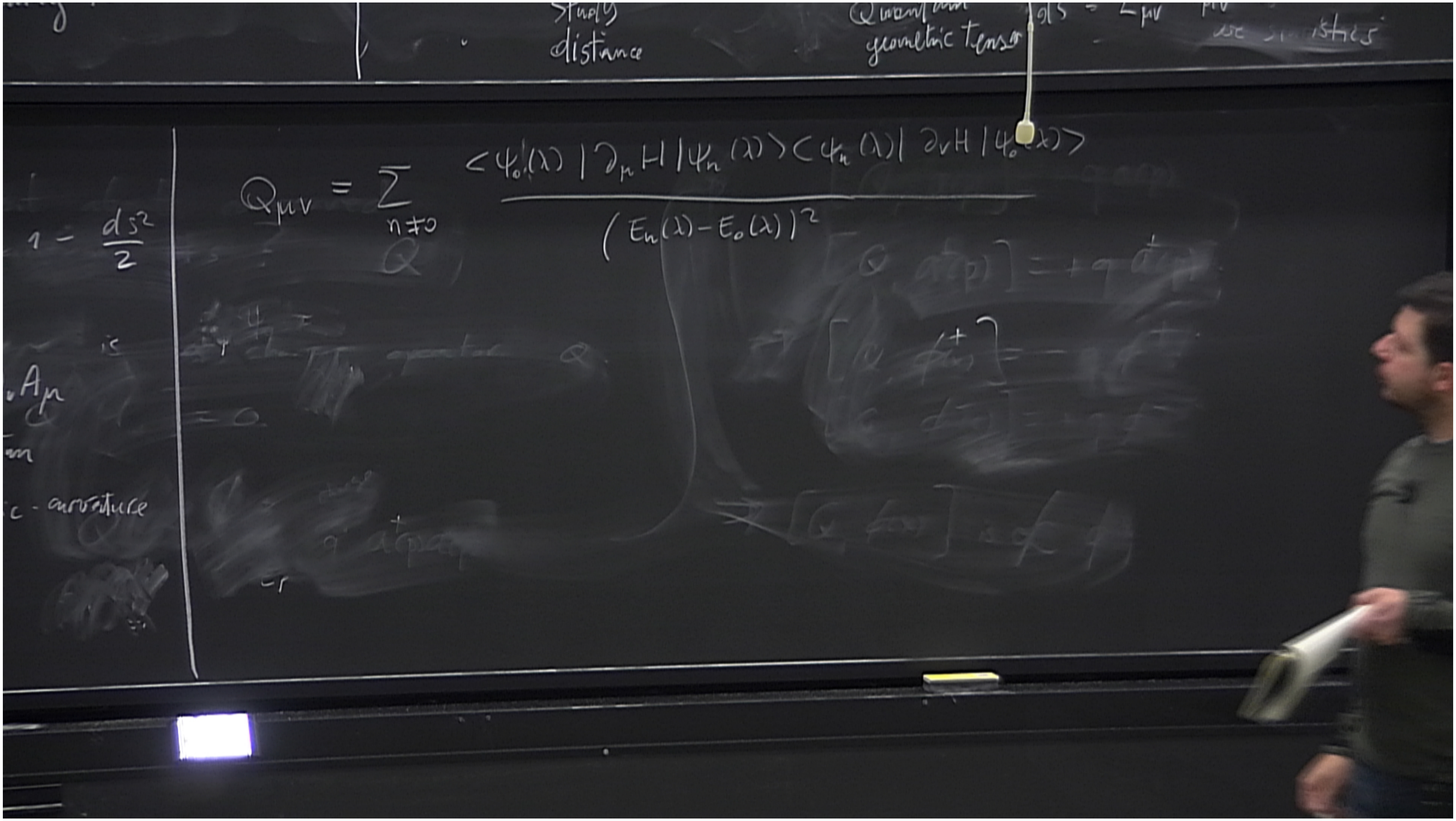

λ = (λ^1, …, λ^n) ∈ ℝ^n
Q_μν = ⟨∂_μ Ψ_0 | ∂_ν Ψ_0⟩ − ⟨∂_μ Ψ_0 | Ψ_0⟩⟨Ψ_0 | ∂_ν Ψ_0⟩
ℝ^n
smooth
H(λ)
∂_μ = ∂/∂λ^μ
μ = 1, …, n
unique GS
Ψ_0(λ) smooth?
metric tensor
g_Ψ(u, v) ≡ ⟨u| 1 − |Ψ⟩⟨Ψ| |v⟩
d_FS (φ, ψ) := cos^{-1} γ(φ, ψ)
γ = |⟨φ|ψ⟩|
1
0
ℋ
Pℋ
Ψ_0(λ)
FUBINI
Study
distance
Quantum
geometric tensor
ds^2 = Σ_μν Q_μν dλ^μ dλ^ν

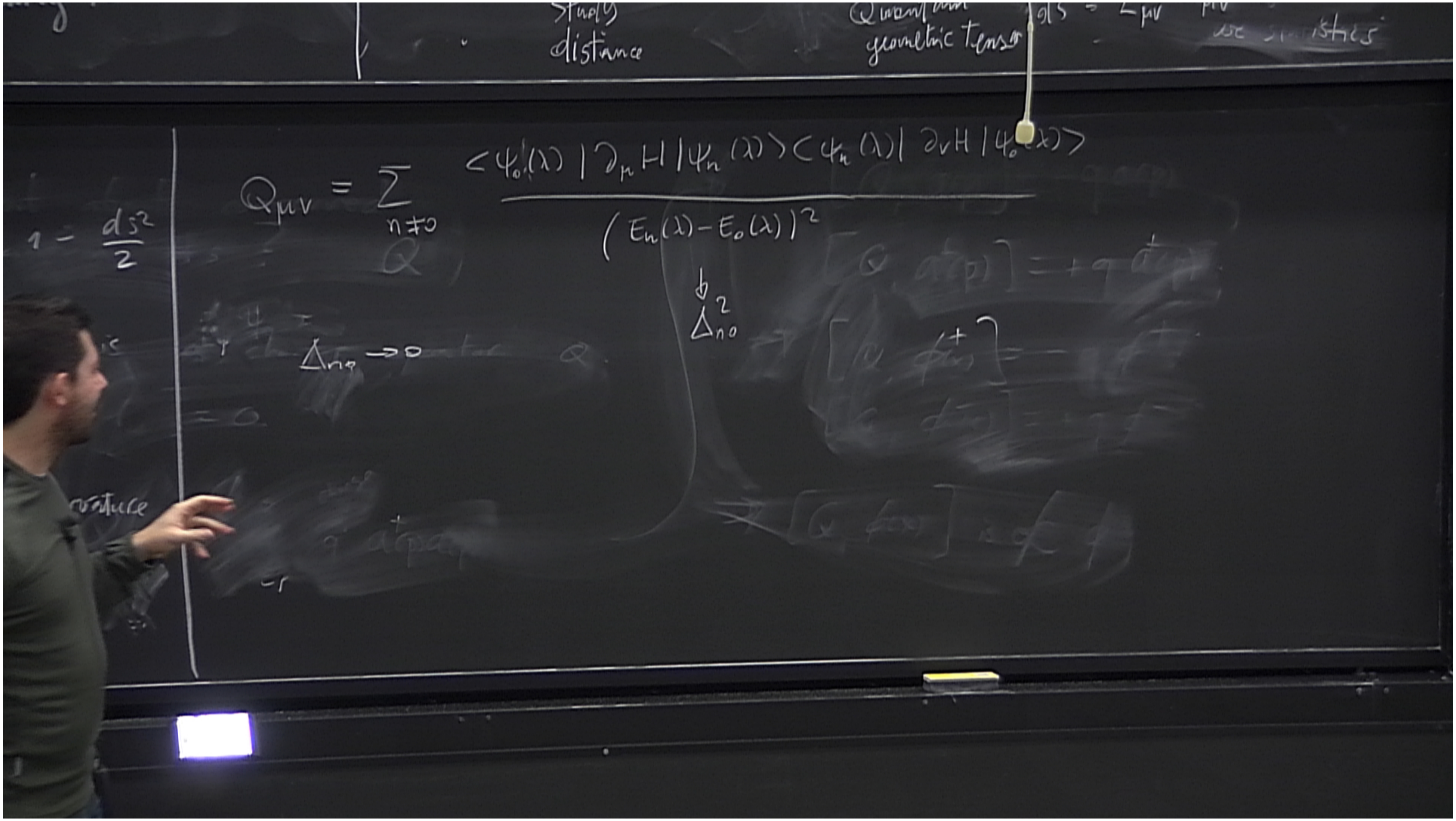

smooth
H(λ)
unique GS
smooth?
metric tensor
FUBINI
Study
distance
Quantum
geometric tensor

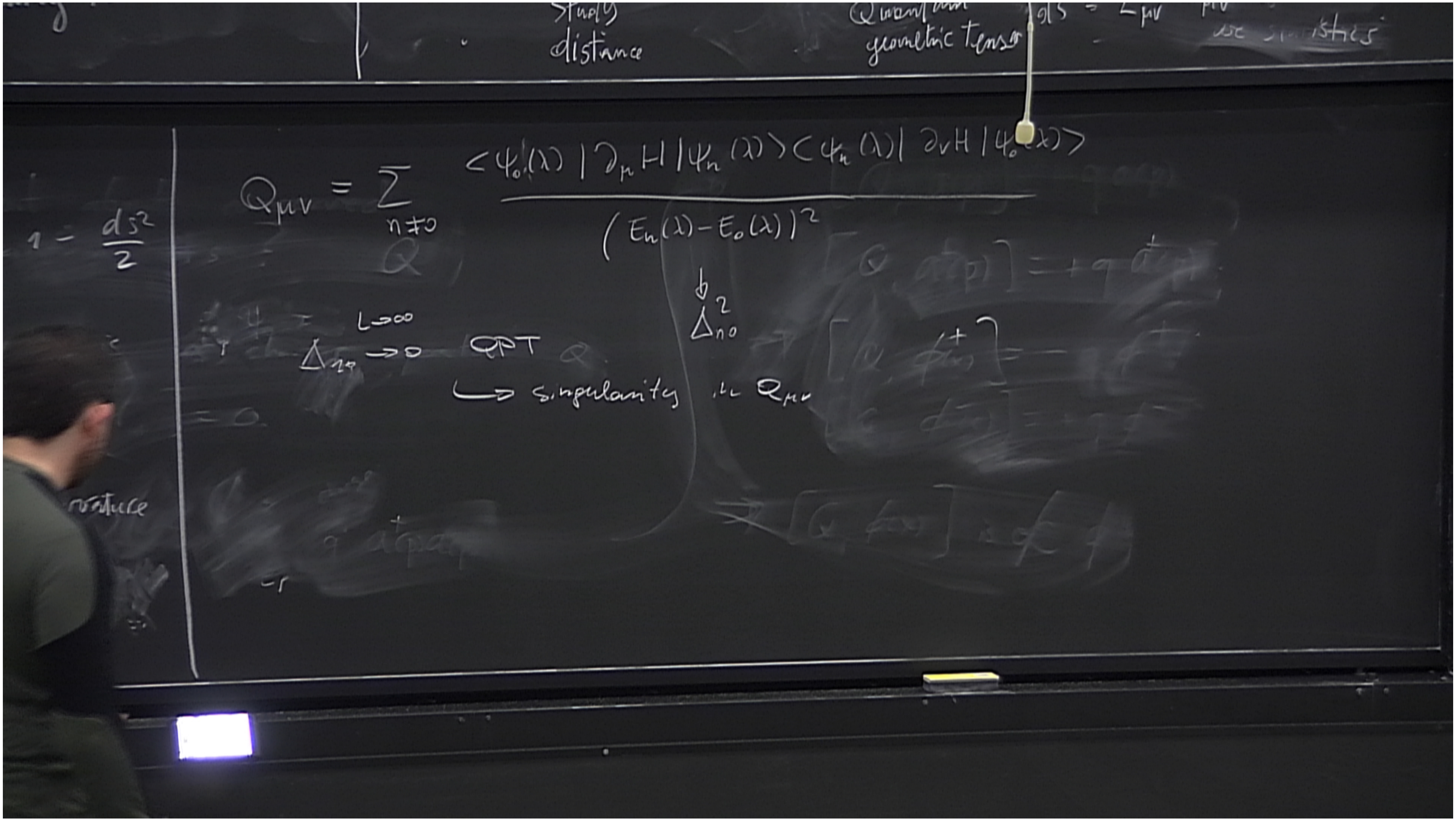

$Re(Q_{\mu\nu}) := g_{\mu\nu} \rightarrow ds^2 = \sum_{\mu\nu} g_{\mu\nu} d\lambda^\mu d\lambda^\nu$
Riemannian
$\zeta(\psi_0(\lambda), \psi_0(\lambda+\delta\lambda)) \simeq 1 - \frac{ds^2}{2}$
$Im(Q_{\mu\nu}) := F_{\mu\nu}$
Im

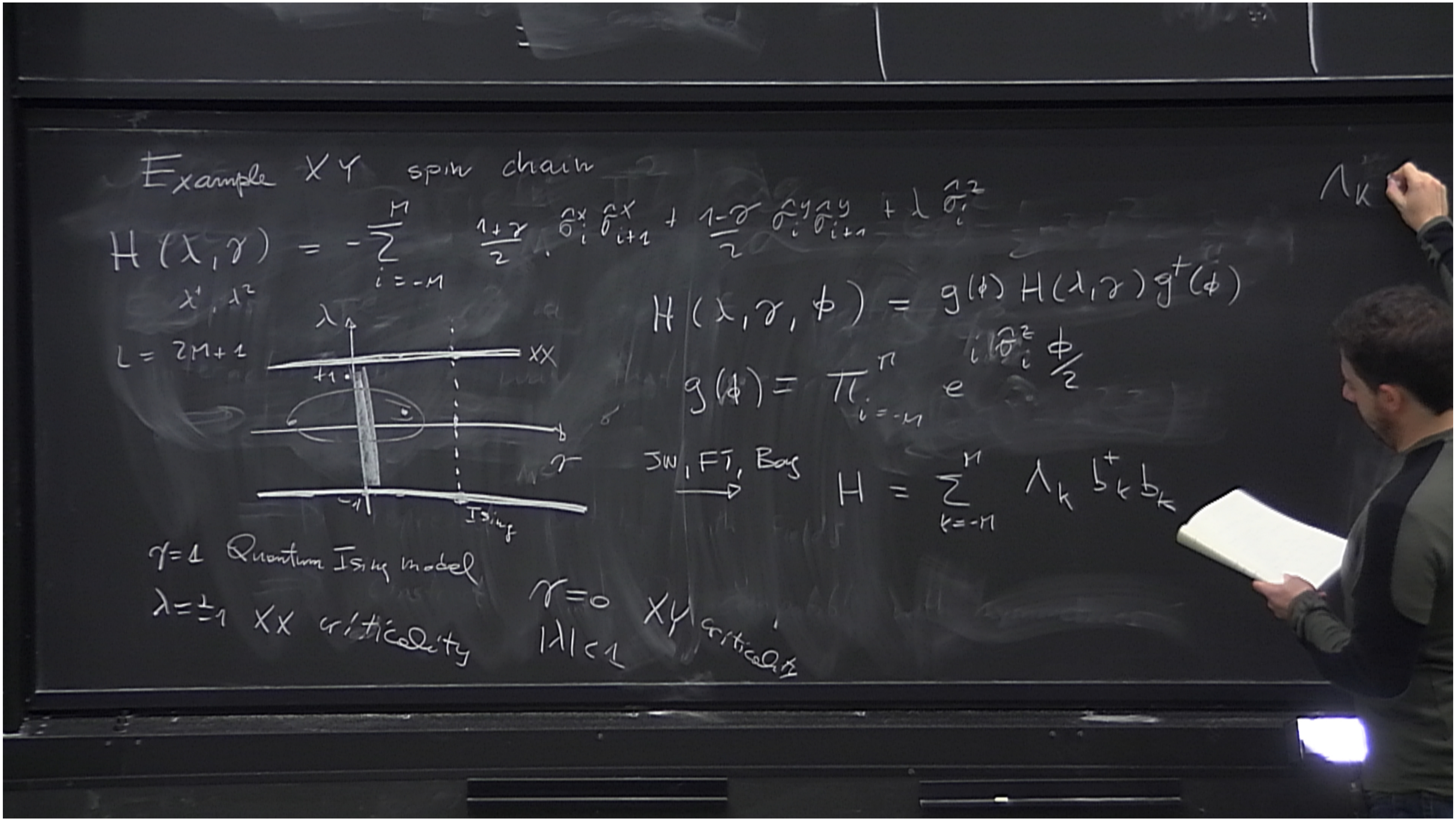

Re (Q_{μν}) := g_{μν} → ds² = Σ_{μν} g_{μν} dλ^μ dλ^ν
Riemannian
Im (Q_{μν}) := F_{μν}
Im ⟨∂_μ ψ_0 | ∂_ν ψ_0⟩ = ⟨∂_μ ψ_0 | ∂_ν ψ_0⟩ − ⟨∂_ν ψ_0 | ∂_μ ψ_0⟩ = ∂_μ A_ν − ∂_ν A_μ
2- Form
A_μ := ⟨ψ | ∂_μ ψ⟩
Adiabatic connection
Adiabatic

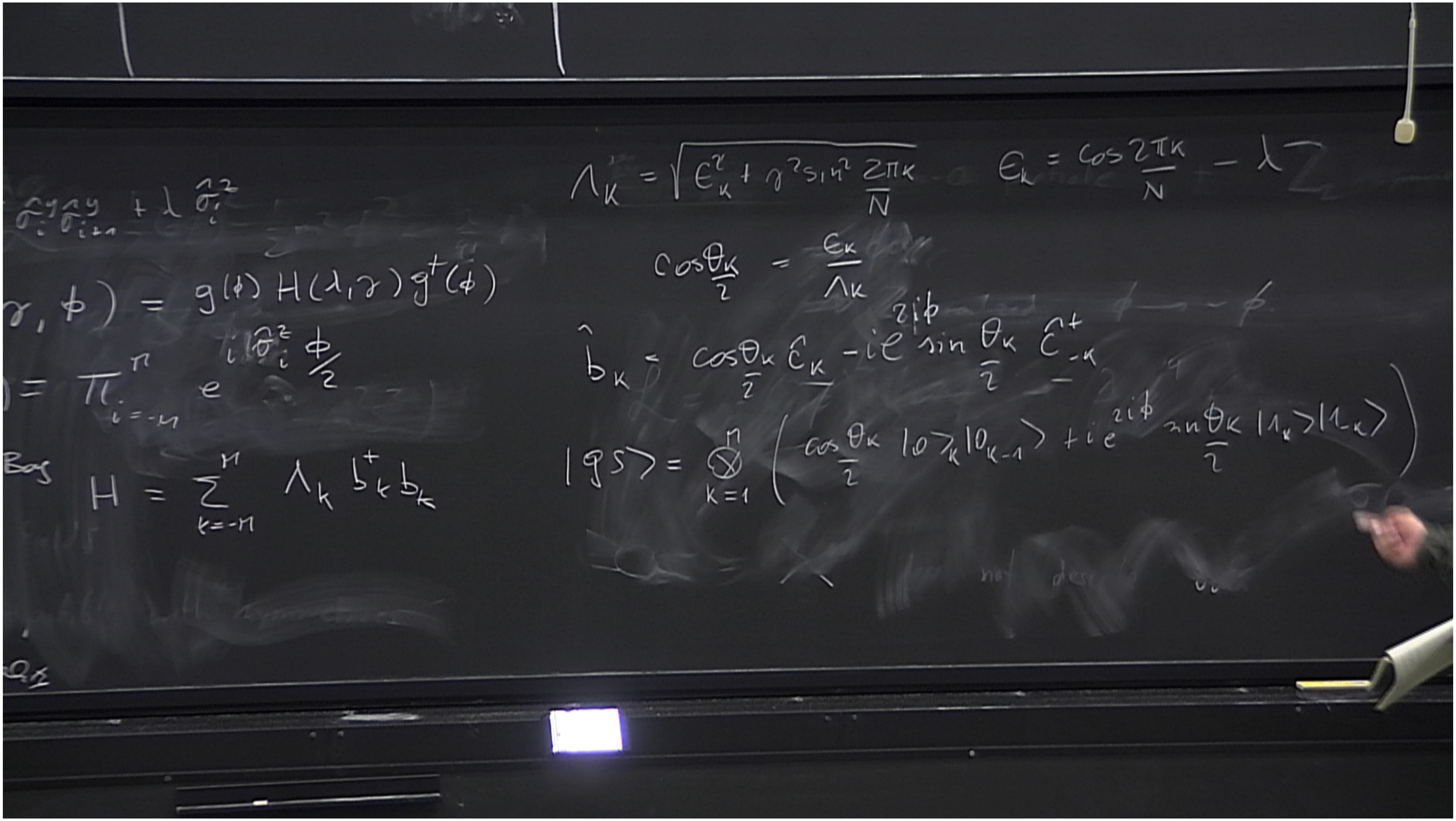

$\mathrm{Re}(Q_{\mu\nu}) := g_{\mu\nu} \rightarrow ds^2 = \sum_{\mu\nu} g_{\mu\nu} d\lambda^\mu d\lambda^\nu$
Riemannian
$\mathcal{F}(\psi_0(\lambda), \psi_0(\lambda+\delta\lambda)) \simeq 1 - \frac{ds^2}{2}$
$\mathrm{Im}(Q_{\mu\nu}) := F_{\mu\nu}$
$\mathrm{Im}\langle\partial_\mu\psi_0|\partial_\nu\psi_0\rangle = \langle\partial_\mu\psi_0|\partial_\nu\psi_0\rangle - \langle\partial_\nu\psi_0|\partial_\mu\psi_0\rangle = \partial_\mu A_\nu - \partial_\nu A_\mu$
2-Form
$A_\mu := \langle\psi|\partial_\mu\psi\rangle$
Adiabatic curvature
Adiabatic connection

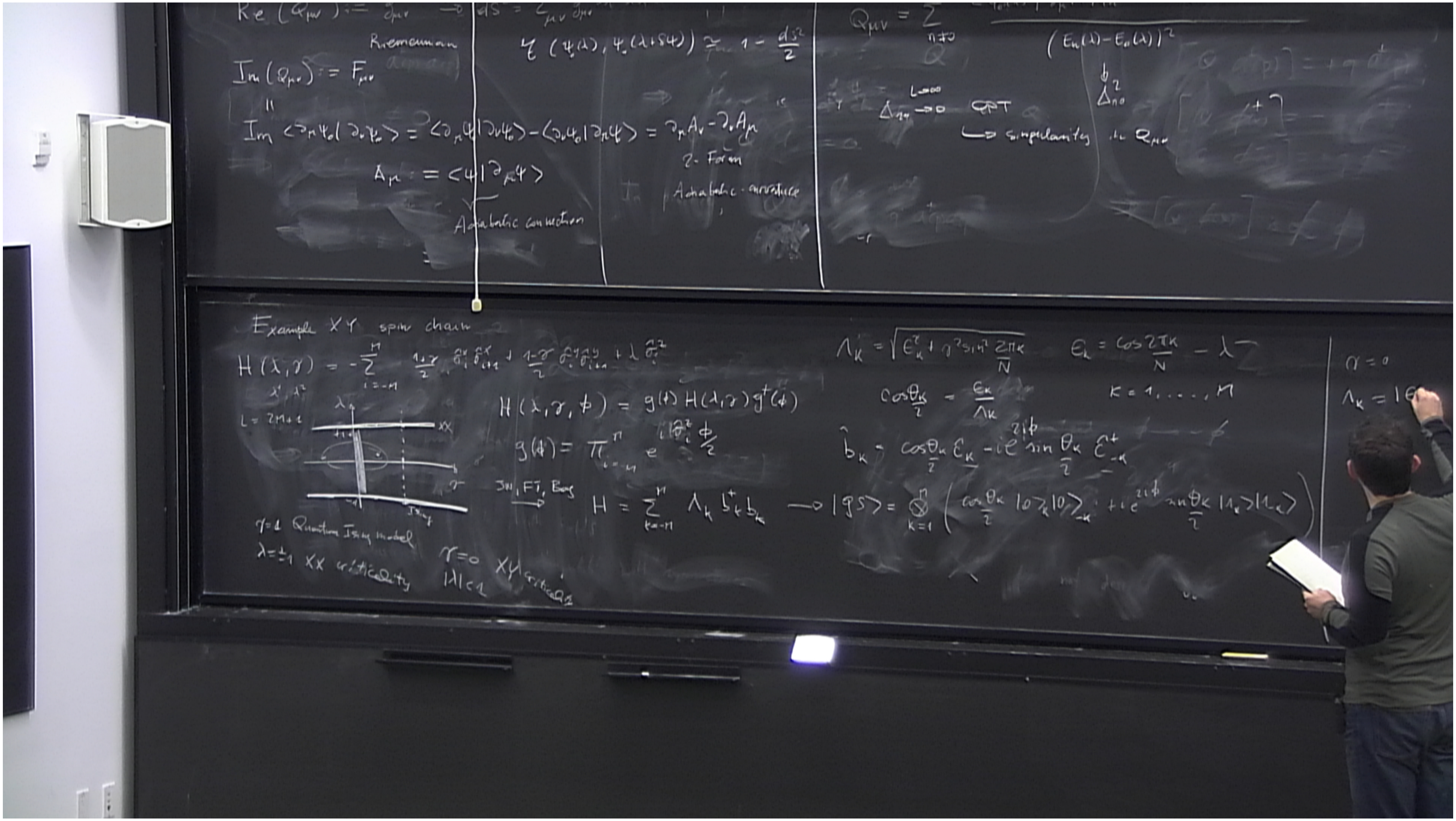

distance
geometric tensor
$$Q_{\mu\nu} = \sum_{n\neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$$
$1 - \frac{ds^2}{2}$
c - curvature

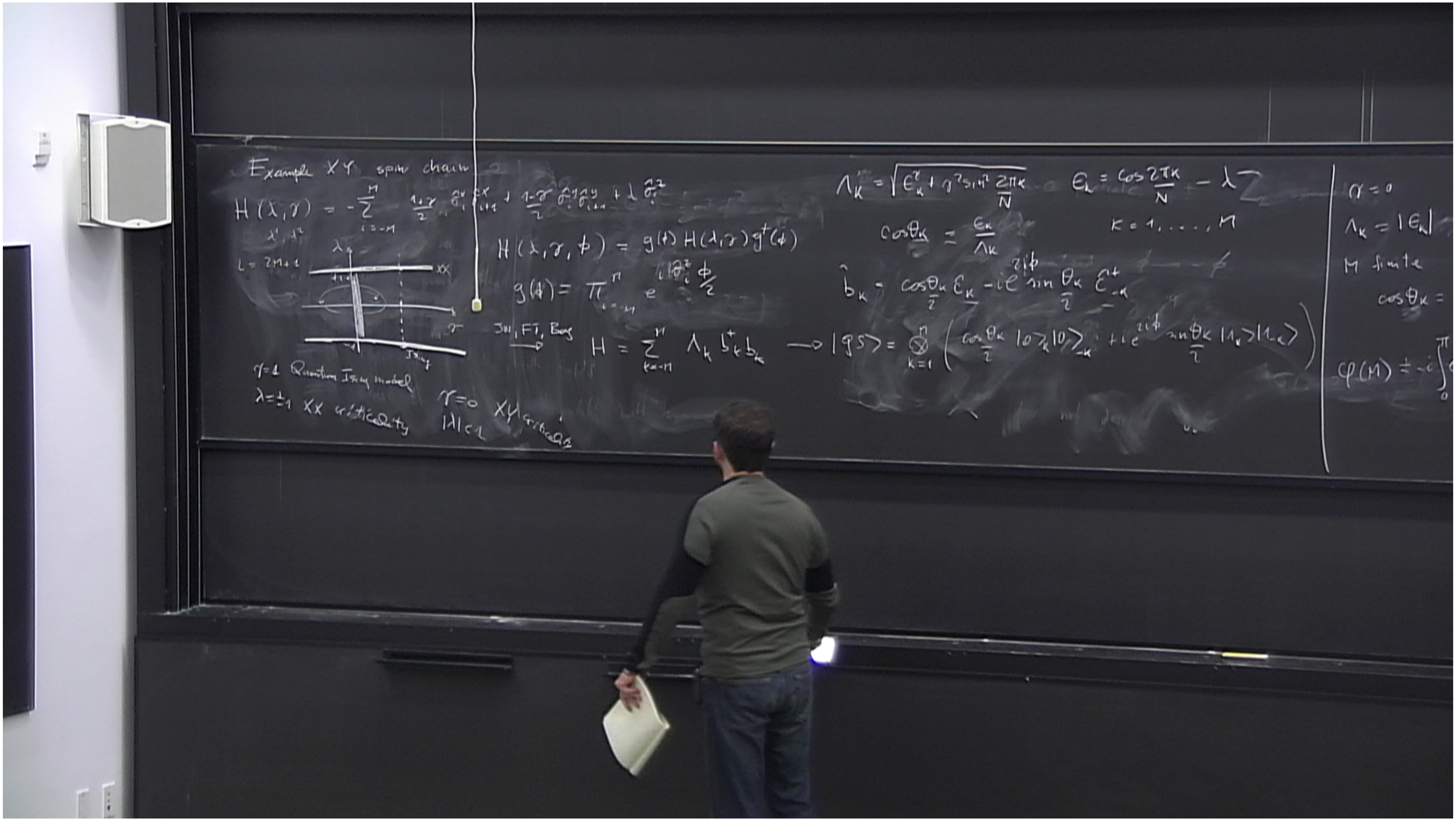

distance
geometric tensor
$Q_{\mu\nu} = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$
Δ_{n0}^2
$\Delta_{n0} \to 0$
$1 - \frac{ds^2}{2}$

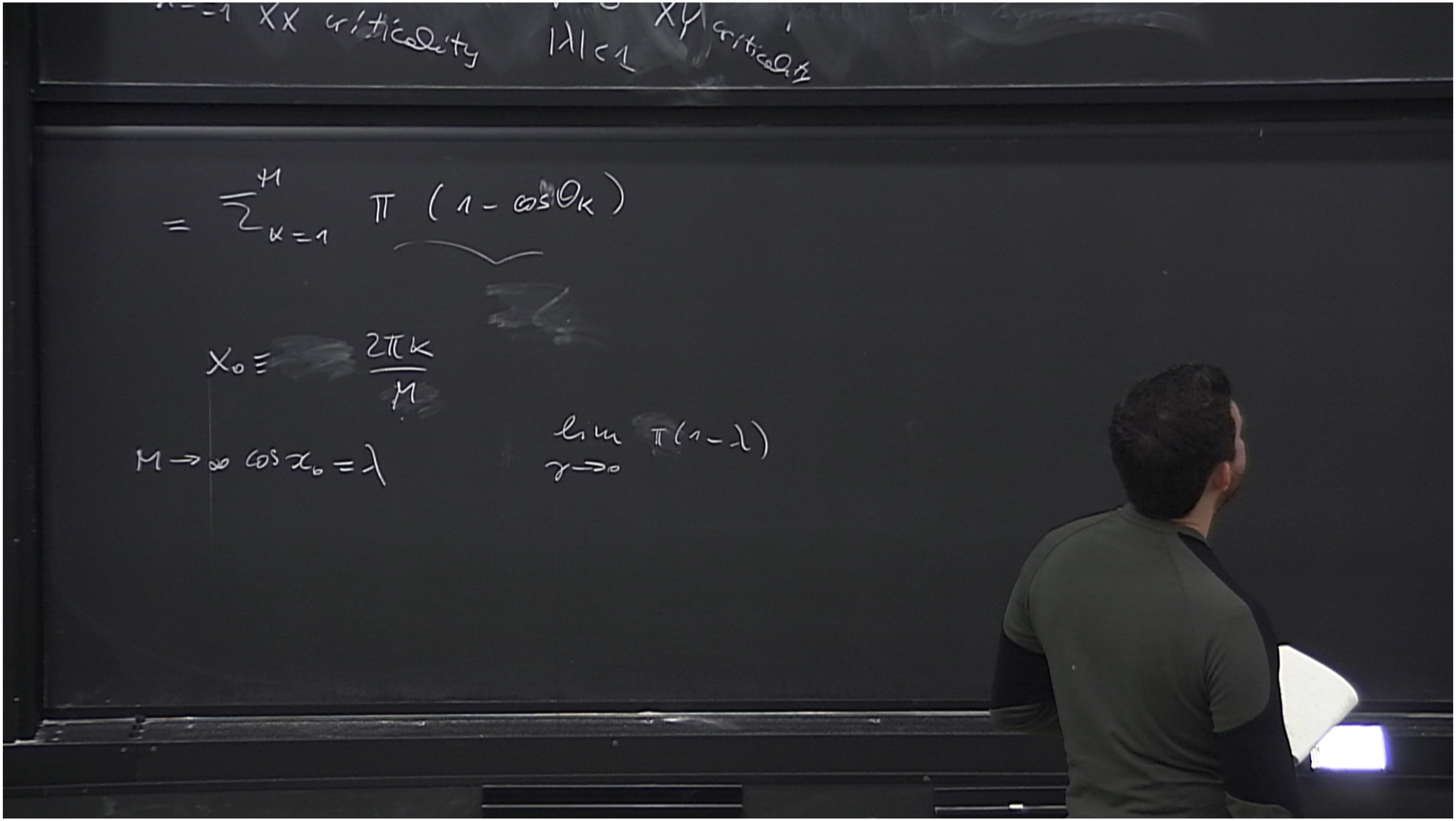

distance
geometric tensor
$Q_{\mu\nu} = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$
Δ_{n0}^2
$L \to \infty$
$\Delta_{10} \to 0$ QPT
singularity in $Q_{\mu\nu}$
$1 - \frac{ds^2}{2}$

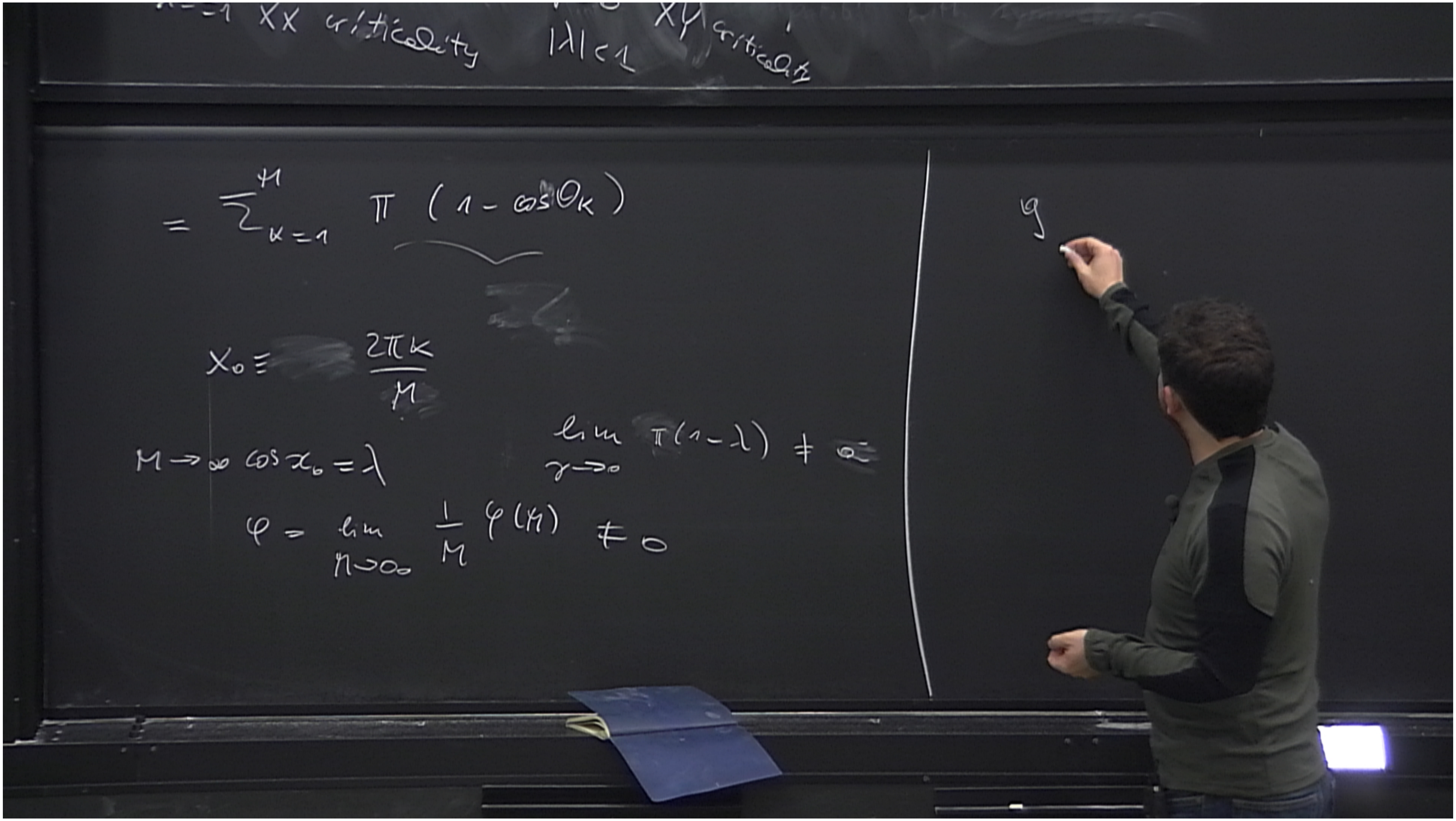

Example XY spin chain
$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \frac{1+\gamma}{2} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$
λ^1, λ^2
$L = 2M+1$
λ
XX
+1
γ
-1
Ising
$H(\lambda,\gamma,\phi) = g(\phi) H(\lambda,\gamma) g^+(\phi)$
$g(\phi) = \prod_{i=-M}^{M} e^{i \hat{\sigma}_i^z \frac{\phi}{2}}$
JW, FT, Bog
$H = \sum_{k=-M}^{M} \Lambda_k b_k^+ b_k$
Λ_k
$\gamma = 1$ Quantum Ising model
$\lambda = \pm 1$ XX criticality
$\gamma = 0$ XY criticality
$|\lambda| < 1$

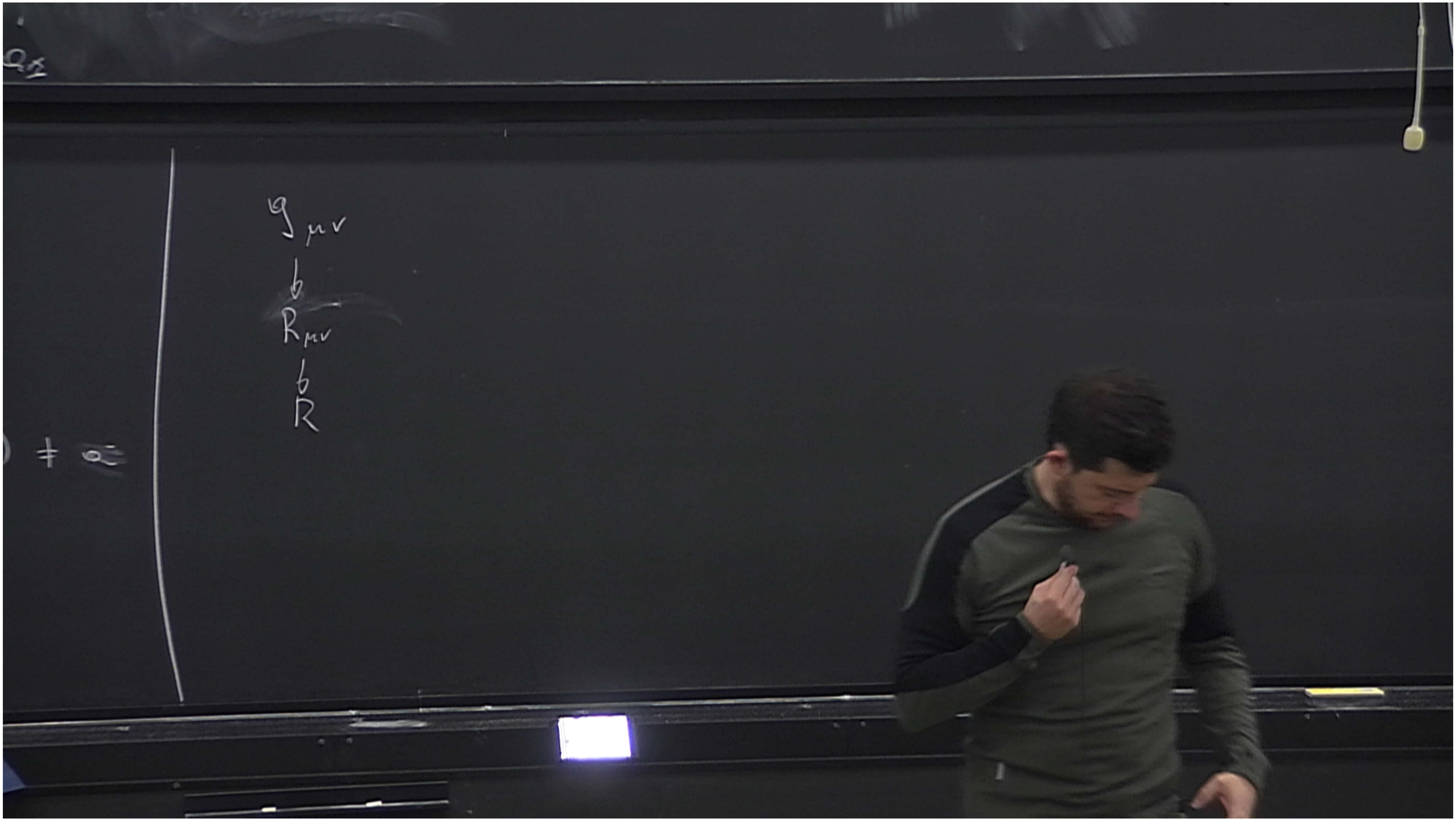

Λk = √(εk² + γ² sin² 2πk/N)
εk = cos 2πk/N − λ
cos θk/2 = εk/Λk
b̂k = cos θk/2 ĉk − i e^{2iφ} sin θk/2 ĉ†−k
|gs⟩ = ⊗_{k=1}^{n} (cos θk/2 |0⟩k|0−k⟩ + i e^{2iφ} sin θk/2 |1k⟩|1−k⟩)
H = Σ_{k=−n}^{n} Λk b†k bk


Riemannian
2-Form
Adiabatic curvature
Adiabatic connection
QPT
singularity in $Q_{\mu\nu}$
Example XY spin chain
$\gamma=1$ Quantum Ising model
$\lambda=\pm1$ XX criticality
$\gamma=0$ XY criticality
$|\lambda|<1$
JW, FT, Bog
$k=1,\ldots,M$


Example XY spin chain
Quantum Ising model
XX criticality
M finite


XX criticality
|λ|<1
XY criticality
= Σ k=1 M Π (1 - cos θk)
X0 ≡ 2πk / M
M → ∞ cos x0 = λ
lim γ→0 π(1-λ)


XX criticality
|λ|<1
XY criticality
= Σ_{k=1}^{M} π (1 - cos θ_k)
X_0 ≡ 2πk / M
M → ∞ cos x_0 = λ
lim π(1-λ) ≠ 0
φ = lim_{M→∞} (1/M) φ(M) ≠ 0


$g_{\mu\nu}$
$\downarrow$
$R_{\mu\nu}$
$\downarrow$
R