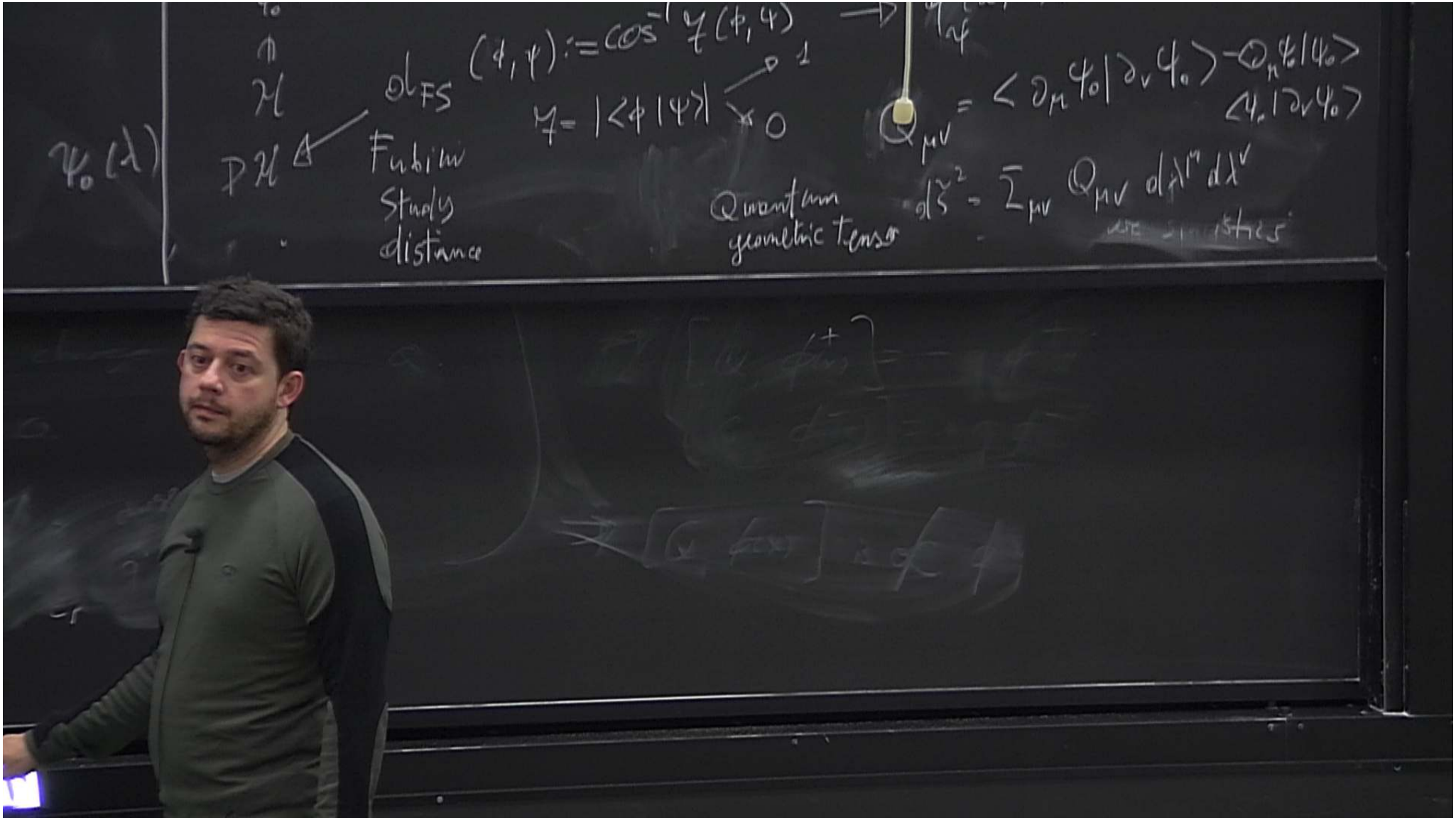
Title: Condensed Matter (Review) - Lecture 7

Date: Jan 10, 2012 10:15 AM

URL: http://pirsa.org/12010091

Abstract:


Fubini
Study
distance
Quantum
geometric tensor

QPT ⟷ Geometry ⟷ Information theoretic
of $\mathcal{H}$ "fidelity"

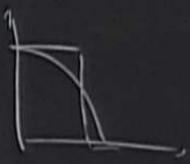
↕

Berry phases

How do we know we have a QPT ?

→ "phenomenological"

$\hat{O}$ order parameter (local)



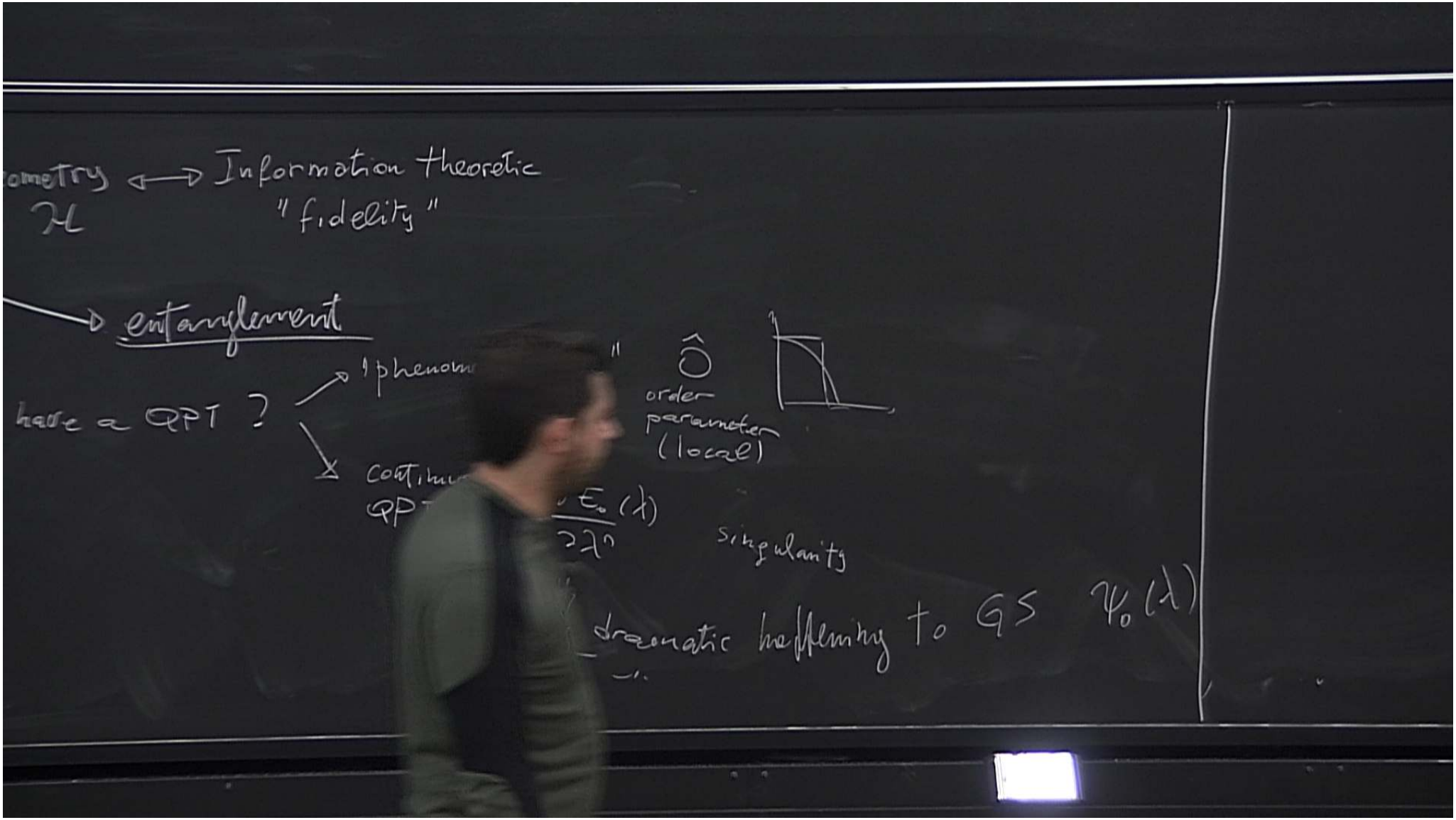
→ continuous QPT

$\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$ singularity

$H = H(\lambda)$

⇓

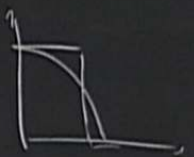
something dramatic happening to GS


ometry ⟷ Information theoretic
"fidelity"
entanglement
have a QPT ?
order parameter (local)
singularity
dramatic
to GS

ometry ⟷ Information theoretic
$\mathcal{H}$ "fidelity"

→ entanglement

have a QPT ? → "phenomenological" $\hat{O}$ order parameter (local)



→ continuous QPT $\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$ singularity

⇓

something dramatic happening to GS $\psi_0(\lambda)$

ometry ⟷ Information theoretic
$\mathcal{H}$ "fidelity"

AdS / CFT

→ entanglement

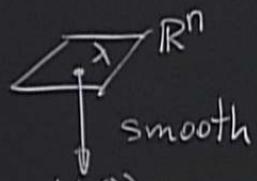
have a QPT ? → "phenomenological" $\hat{O}$ order parameter (local)

→ continuous QPT $\frac{\partial^n E_0(\lambda)}{\partial \lambda^n}$ singularity

⇓

something dramatic happening to GS $\psi_0(\lambda)$

$\lambda = (\lambda^1, \ldots, \lambda^n) \in \mathbb{R}^n$



unique GS $\psi_0(\lambda)$ smooth?

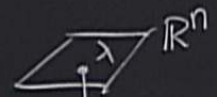
$\psi_0(\lambda)$ $\uparrow$ $\mathcal{H}$ $\mathbb{P}\mathcal{H}$

$d_{FS}(\phi, \psi) := \cos^{-1} \gamma(\phi, \psi)$

$\gamma = |\langle \phi | \psi \rangle|$

Fubini Study distance

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$



smooth

$H(\lambda)$

unique GS $\psi_0(\lambda)$ smooth?

$\Uparrow$

$\mathcal{H}$

$\psi_0(\lambda)$ $\quad \mathcal{P}\mathcal{H}$

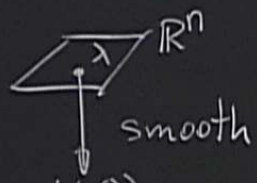
d_{FS} Fubini Study distance

$(\phi, \psi) := \cos^{-1} \mathcal{F}(\phi, \psi)$

$\mathcal{F} = |\langle \phi | \psi \rangle| \to 1$

use statistics

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$



unique GS $\psi_0(\lambda)$ smooth?

$\psi_0(\lambda)$

$\mathcal{H}$

$P\mathcal{H}$

d_{FS} Fubini Study distance

$(\phi, \psi) := \cos^{-1} \gamma(\phi, \psi)$

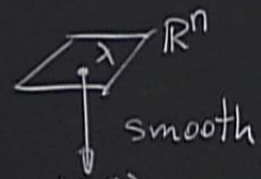
$\gamma = |\langle \phi | \psi \rangle| \to 1$

$\gamma = |\langle \phi | \psi \rangle| \to 0$

metric tensor

$\to g(u, v) = \langle u | 1 - |\psi\rangle\langle\psi| | v \rangle$

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$



unique GS $\psi_0(\lambda)$ smooth?

$\Uparrow$

$\mathcal{H}$

$\psi_0(\lambda)$ $\quad \mathcal{P}\mathcal{H}$ $\quad d_{FS}$ Fubini Study distance

$(\phi, \psi) := \cos^{-1} \mathcal{F}(\phi, \psi)$

$\mathcal{F} = |\langle \phi | \psi \rangle| \to 1, \; 0$

metric tensor $\longrightarrow$ $q_\psi(u, v) \equiv \langle u | \, 1 - |\psi\rangle\langle\psi| \, | v \rangle$

$Q_{\mu\nu} = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle$

use statistics

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$

smooth

$H(\lambda)$

unique GS $\psi_0(\lambda)$ smooth?

$\psi_0(\lambda)$

$\mathcal{H}$

$\mathcal{P}\mathcal{H}$

d_{FS}

Fubini Study distance

$(\phi, \psi) := \cos^{-1} \mathcal{F}(\phi, \psi)$

$\mathcal{F} = |\langle \phi | \psi \rangle|$ $\to 1$, $\to 0$

metric tensor $\longrightarrow$ $q_\psi(u, v) \equiv \langle u | 1 - |\psi\rangle\langle\psi| \, |v\rangle$

$Q_{\mu\nu} = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle$

$d\tilde{s} = \sum_{\mu\nu} Q_{\mu\nu} \, d\lambda^\mu d\lambda^\nu$

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$

$$\boxed{Q_{\mu\nu} = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle}$$

smooth

$H(\lambda)$

$\downarrow$

unique GS $\psi_0(\lambda)$ smooth?

$\Uparrow$

$\mathcal{H}$

$\psi_0(\lambda)$ $\quad \mathcal{P}\mathcal{H}$

$d_{FS}(\phi, \psi) := \cos^{-1} \mathcal{F}(\phi, \psi)$

$\mathcal{F} = |\langle \phi | \psi \rangle| \to 1, \; \to 0$

FUBINI Study distance

$$\partial_\mu = \frac{\partial}{\partial \lambda^\mu}$$

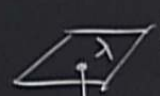
metric tensor $\longrightarrow q_\psi(u, v) \equiv \langle u | 1 - |\psi\rangle\langle\psi| \, |v\rangle$

Quantum geometric tensor $\quad d\tilde{s}^2 = \sum_{\mu\nu} Q_{\mu\nu} \, d\lambda^\mu d\lambda^\nu$

use statistics

$\lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$

$$Q_{\mu\nu} = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle$$



smooth

$H(\lambda)$

$\downarrow$

unique GS $\psi_0(\lambda)$ smooth?

$\Uparrow$

$\mathcal{H}$

$\psi_0(\lambda)$ $\quad \mathcal{P}\mathcal{H}$

$d_{FS}(\phi, \psi) := \cos^{-1} \mathcal{F}(\phi, \psi)$

$\mathcal{F} = |\langle \phi | \psi \rangle| \to 1, \; \to 0$

FUBINI Study distance

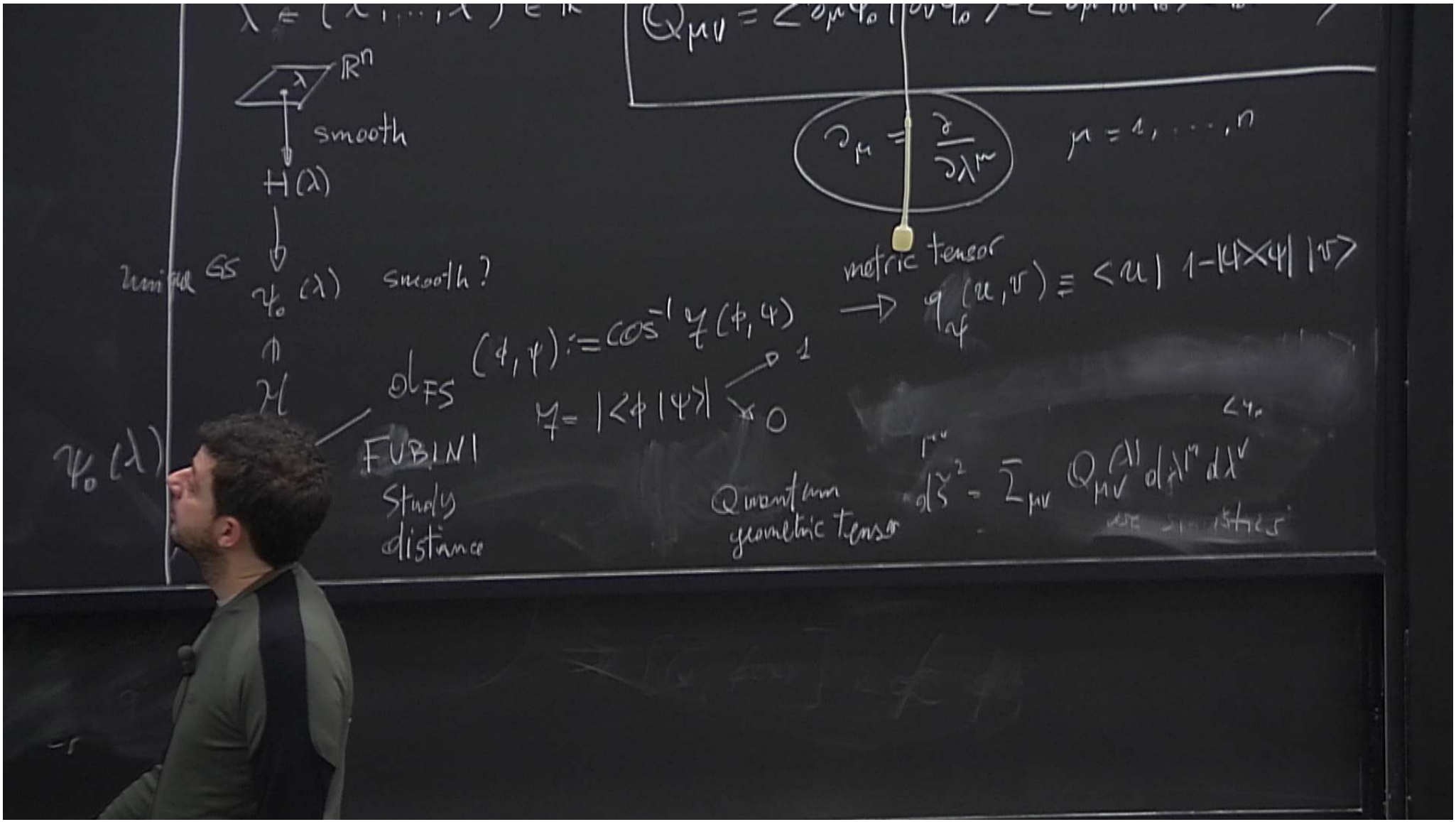
$$\partial_\mu = \frac{\partial}{\partial \lambda^\mu} \qquad \mu = 1, \dots, n$$

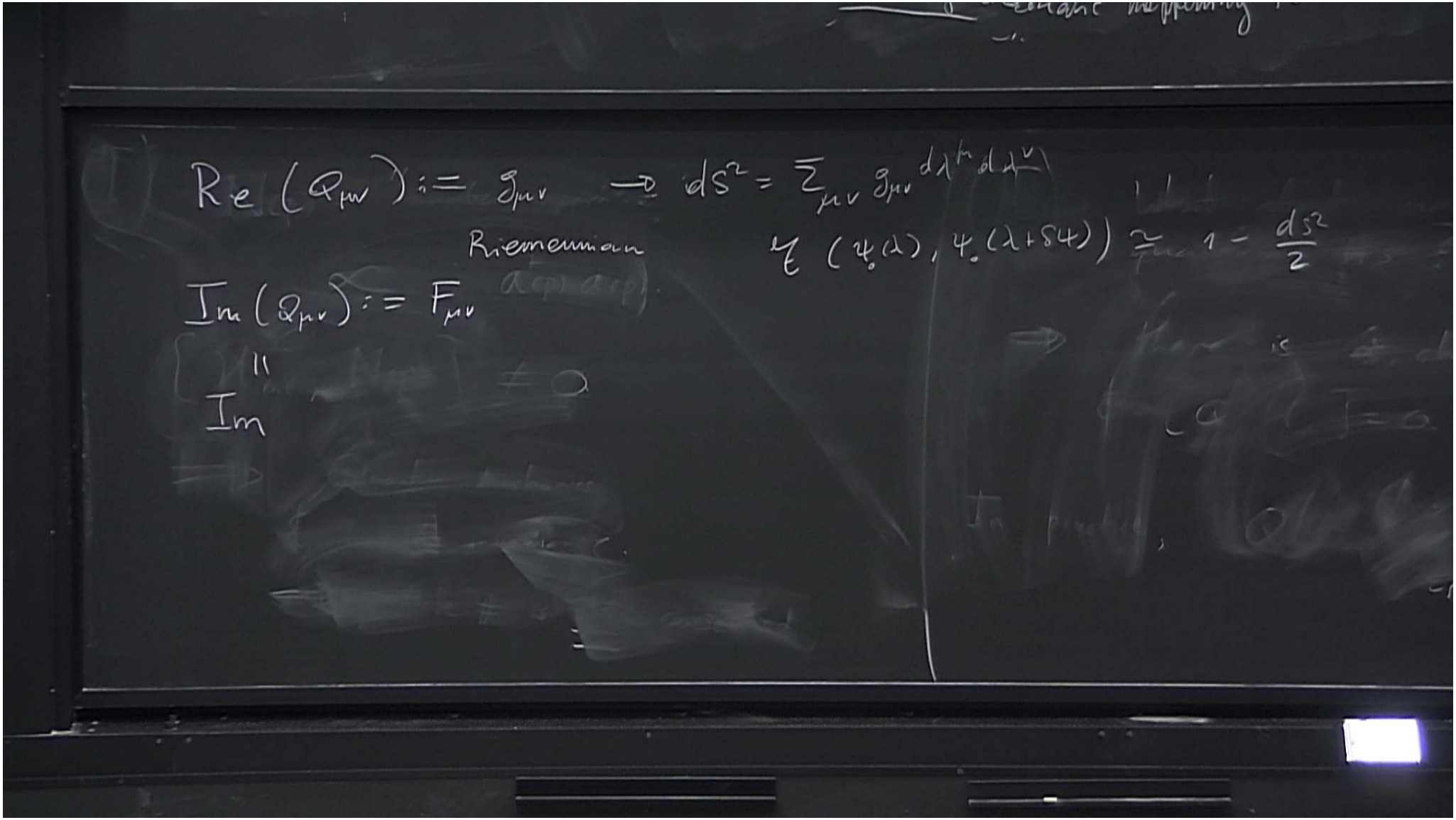
metric tensor $\longrightarrow q_\psi(u, v) \equiv \langle u | \, 1 - |\psi\rangle\langle\psi| \, | v \rangle$

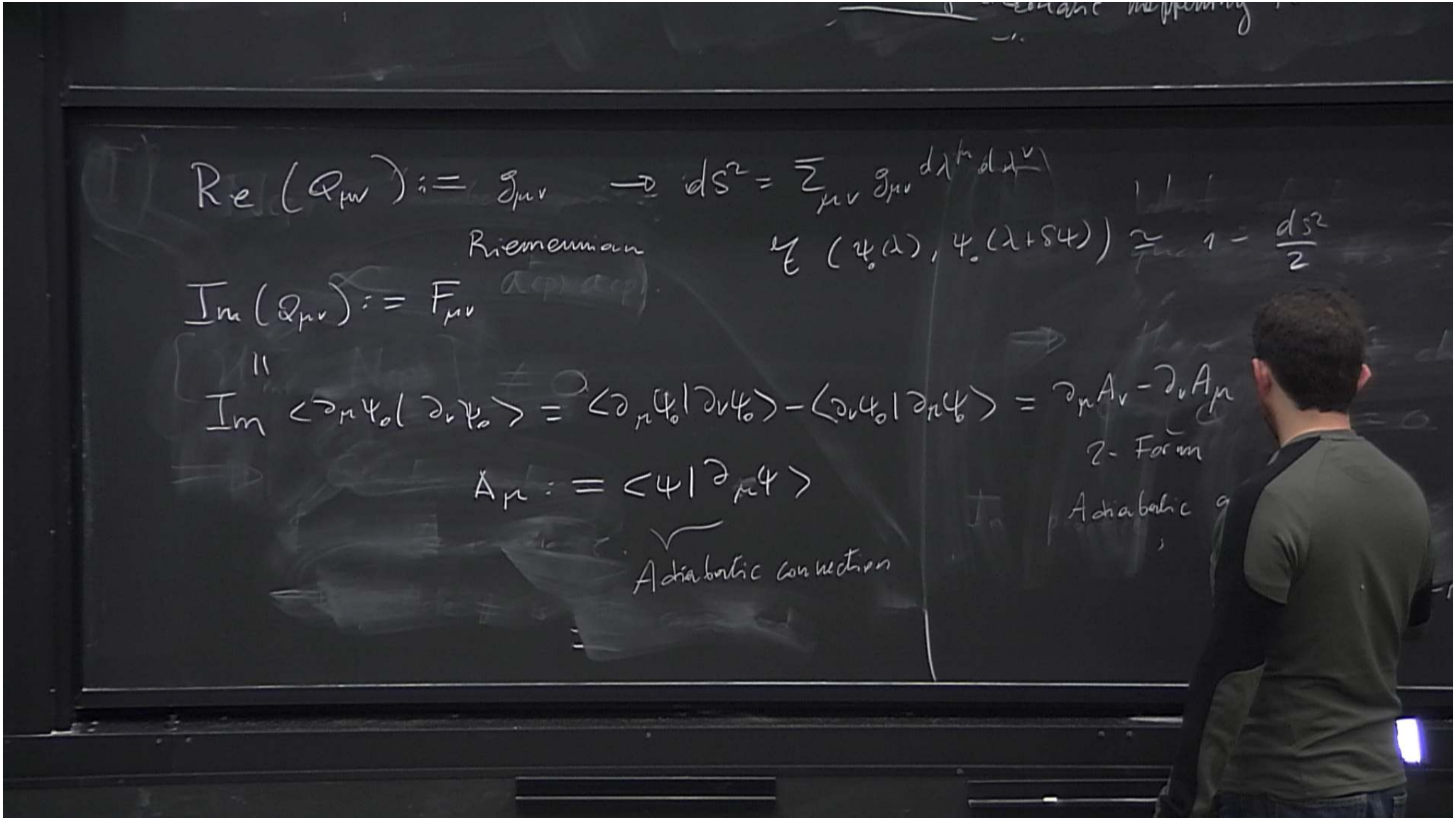
Quantum geometric tensor

$$d\tilde{s}^2 = \sum_{\mu\nu} Q_{\mu\nu} \, d\lambda^\mu d\lambda^\nu$$

use statistics


$\mathbb{R}^n$
smooth
$H(\lambda)$
$\psi_0(\lambda)$
smooth?
$\mathcal{H}$
$d_{FS}(\phi,\psi) := \cos^{-1} |\langle\phi|\psi\rangle|$
$|\langle\phi|\psi\rangle| \to 1$
$\to 0$
FUBINI
Study
distance
$Q_{\mu\nu} = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\mu \psi_0 | \psi_0 \rangle \langle \psi_0 | \partial_\nu \psi_0 \rangle$
$\partial_\mu = \frac{\partial}{\partial \lambda^\mu}$
$\mu = 1, \dots, n$
metric tensor
$g_{\mu\nu}(u,v) \equiv \langle u | 1 - |\psi\rangle\langle\psi| | v \rangle$
Quantum
geometric tensor
$ds^2 = \sum_{\mu\nu} Q_{\mu\nu} d\lambda^\mu d\lambda^\nu$


$Re(g_{\mu\nu}) := g_{\mu\nu} \rightarrow ds^2 = \Sigma_{\mu\nu} g_{\mu\nu} d\lambda^\mu d\lambda^\nu$
Riemannian
$Im(g_{\mu\nu}) := F_{\mu\nu}$
Im


$Re(Q_{\mu\nu}) := g_{\mu\nu} \rightarrow ds^2 = \sum_{\mu\nu} g_{\mu\nu} d\lambda^\mu d\lambda^\nu$
Riemannian
$\mathcal{F}(\psi_0(\lambda), \psi_0(\lambda+\delta\lambda)) \simeq 1 - \frac{ds^2}{2}$
$Im(Q_{\mu\nu}) := F_{\mu\nu}$
$Im \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\nu \psi_0 | \partial_\mu \psi_0 \rangle = \partial_\mu A_\nu - \partial_\nu A_\mu$
2- Form
$A_\mu := \langle \psi | \partial_\mu \psi \rangle$
Adiabatic connection
Adiabatic

$Re(Q_{\mu\nu}) := g_{\mu\nu} \rightarrow ds^2 = \Sigma_{\mu\nu} g_{\mu\nu} d\lambda^\mu d\lambda^\nu$

Riemannian

$\mathcal{F}(\psi_0(\lambda), \psi_0(\lambda+\delta\lambda)) \simeq 1 - \frac{ds^2}{2}$

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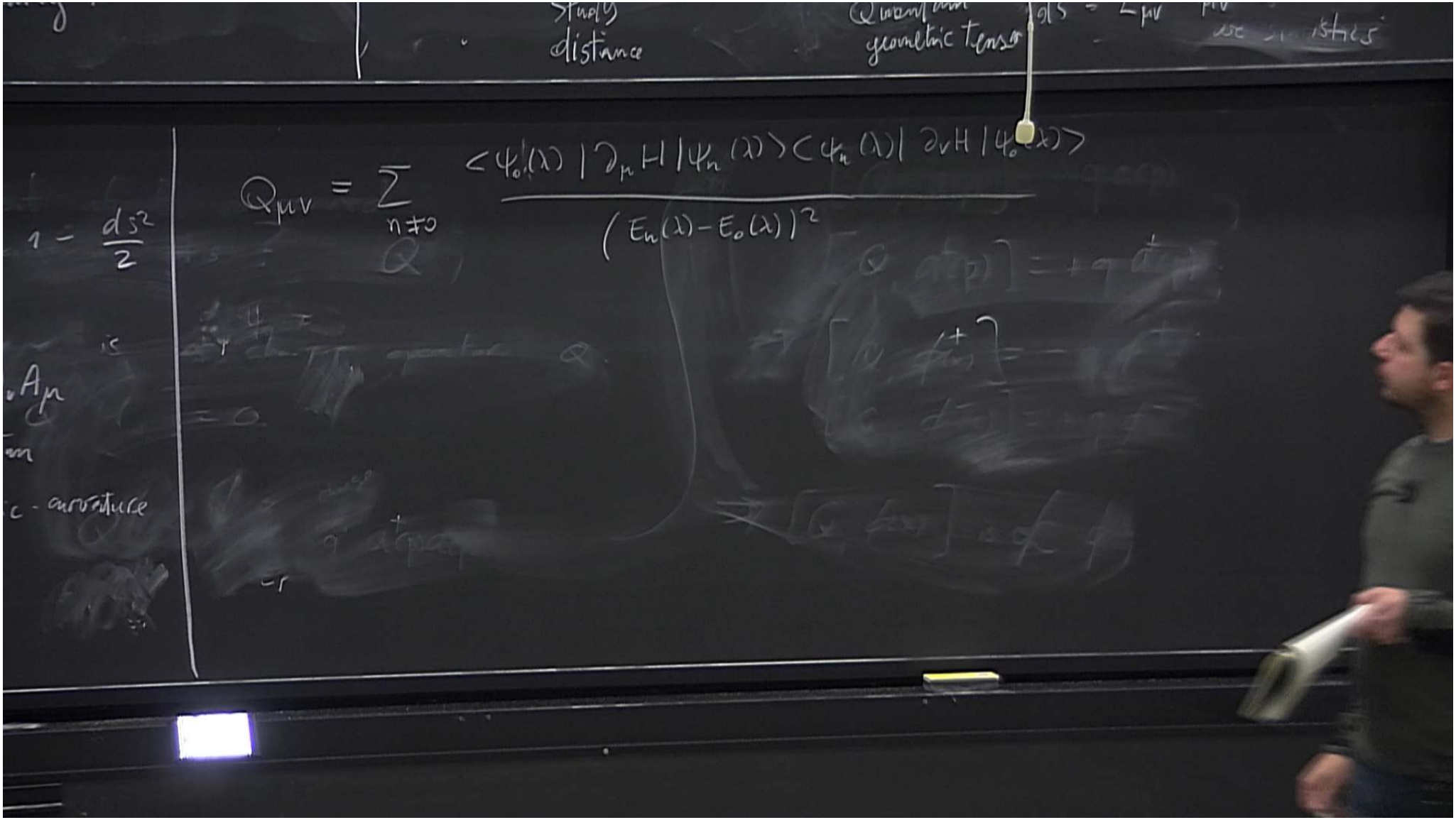
$Im \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle = \langle \partial_\mu \psi_0 | \partial_\nu \psi_0 \rangle - \langle \partial_\nu \psi_0 | \partial_\mu \psi_0 \rangle = \partial_\mu A_\nu - \partial_\nu A_\mu$

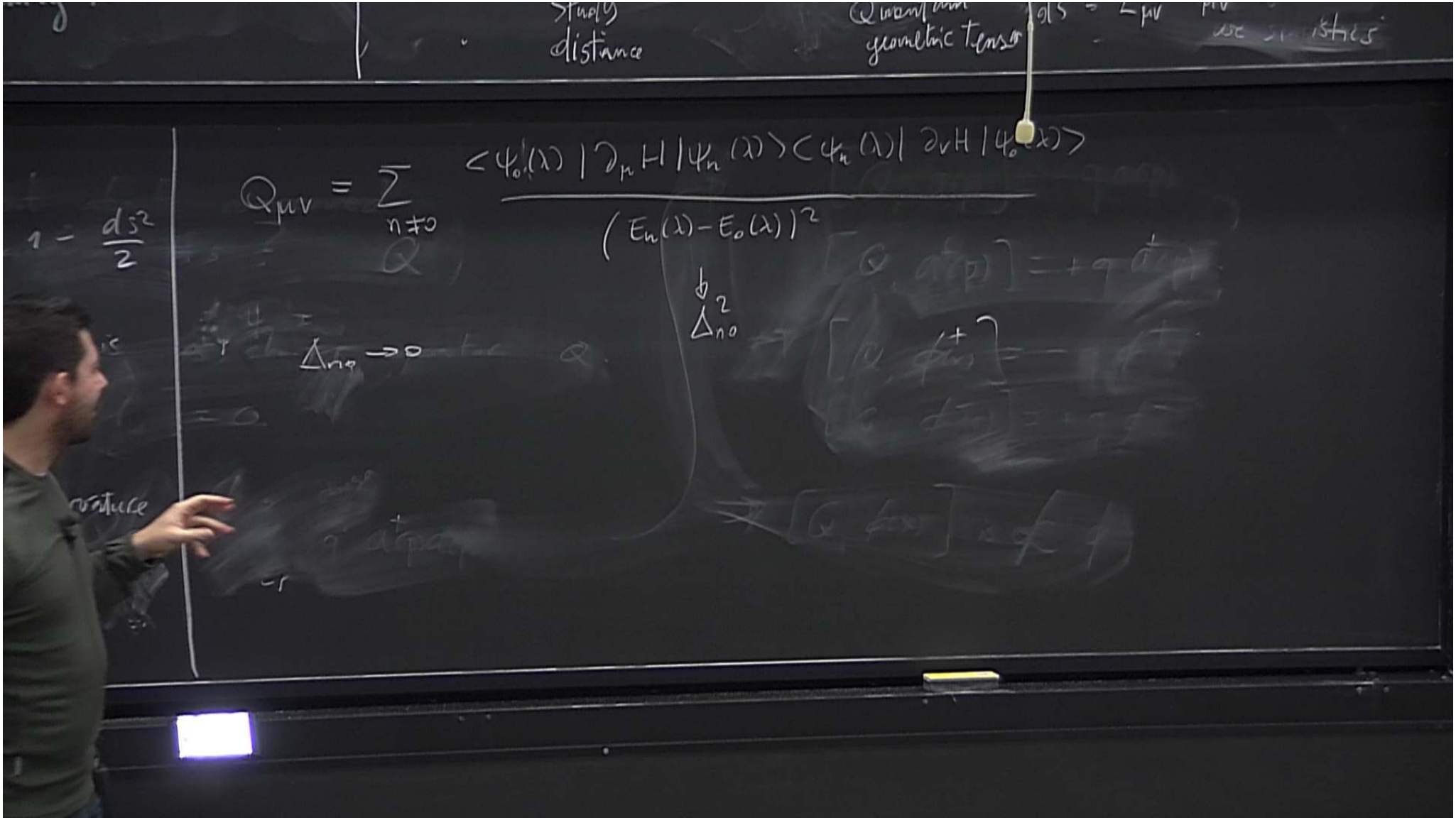
2- Form

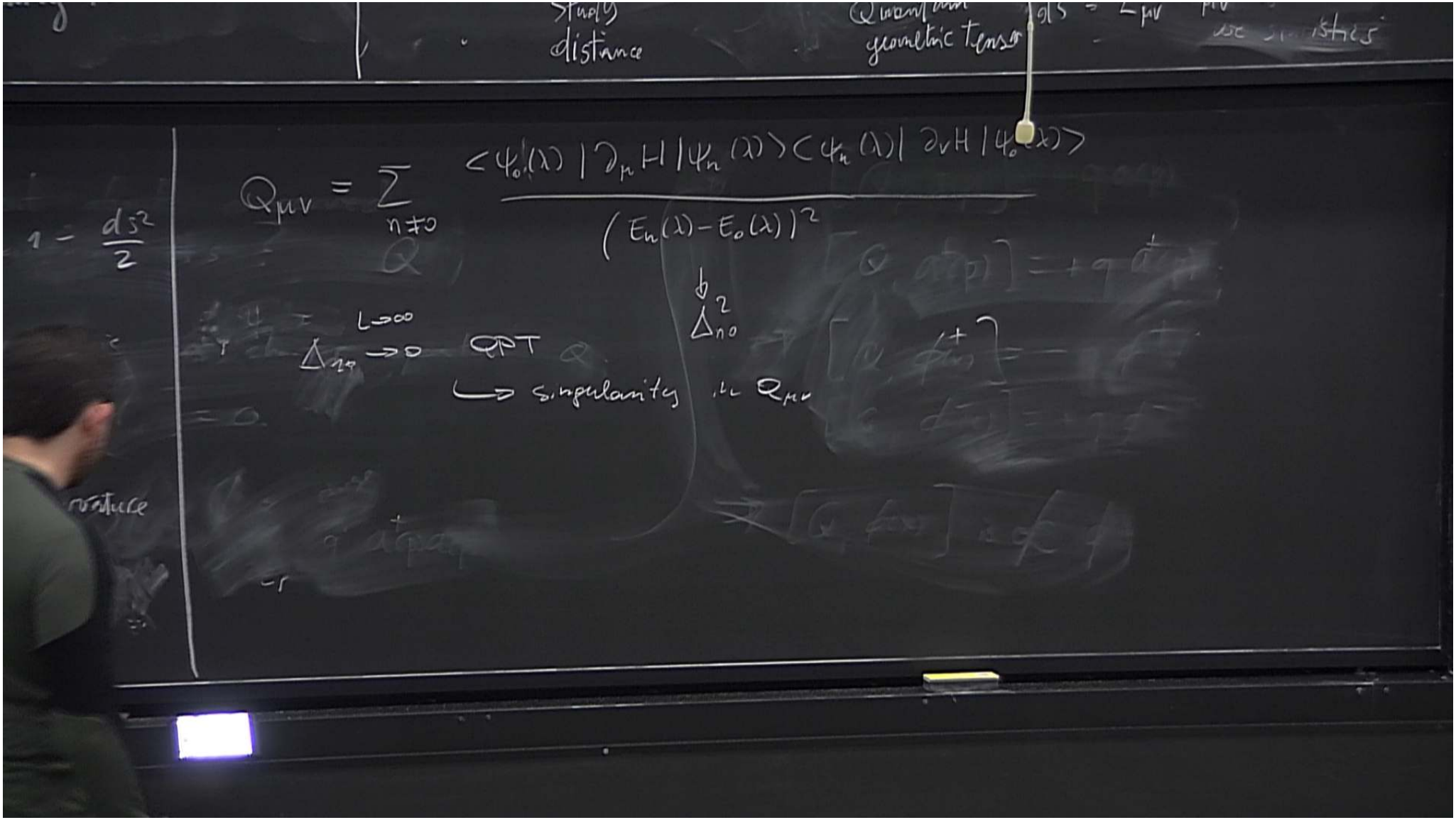
$A_\mu := \langle \psi | \partial_\mu \psi \rangle$

Adiabatic connection

Adiabatic curvature


distance
geometric tensor
$Q_{\mu\nu} = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$
$1 - \frac{ds^2}{2}$


distance
geometric tensor
$Q_{\mu\nu} = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$
$1 - \frac{ds^2}{2}$
$\Delta_{n0} \to 0$
Δ_{n0}^2

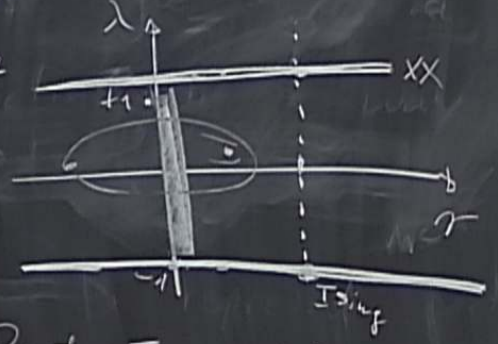

distance
geometric tensor
$Q_{\mu\nu} = \sum_{n \neq 0} \frac{\langle \psi_0(\lambda) | \partial_\mu H | \psi_n(\lambda) \rangle \langle \psi_n(\lambda) | \partial_\nu H | \psi_0(\lambda) \rangle}{(E_n(\lambda) - E_0(\lambda))^2}$
$1 - \frac{ds^2}{2}$
$L \to \infty$
$\Delta_{2n} \to 0$
QPT
$\hookrightarrow$ singularity in $Q_{\mu\nu}$
Δ_{n0}^2

Example XY spin chain

$$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \frac{1+\gamma}{2}\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \lambda\hat{\sigma}_i^z$$

σ^x, σ^z

$L = 2M+1$



$$H(\lambda,\gamma,\phi) = g(\phi)\, H(\lambda,\gamma)\, g^{\dagger}(\phi)$$

$$g(\phi) = \prod_{i=-M}^{M} e^{i\hat{\sigma}_i^z \phi/2}$$

JW, FT, Bog →

$$H = \sum_{k=-M}^{M} \Lambda_k\, b_k^{\dagger} b_k$$

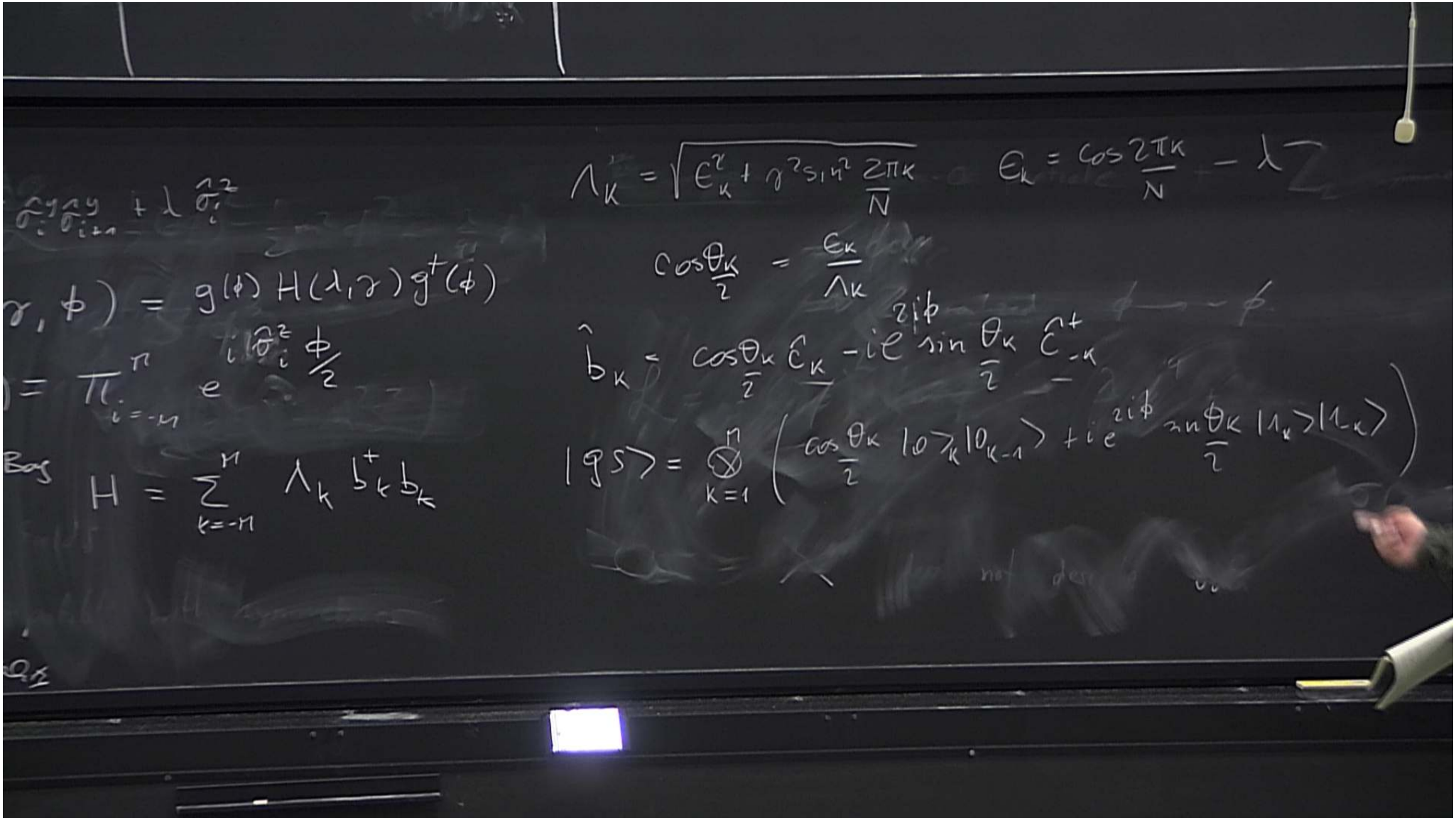
Λ_k

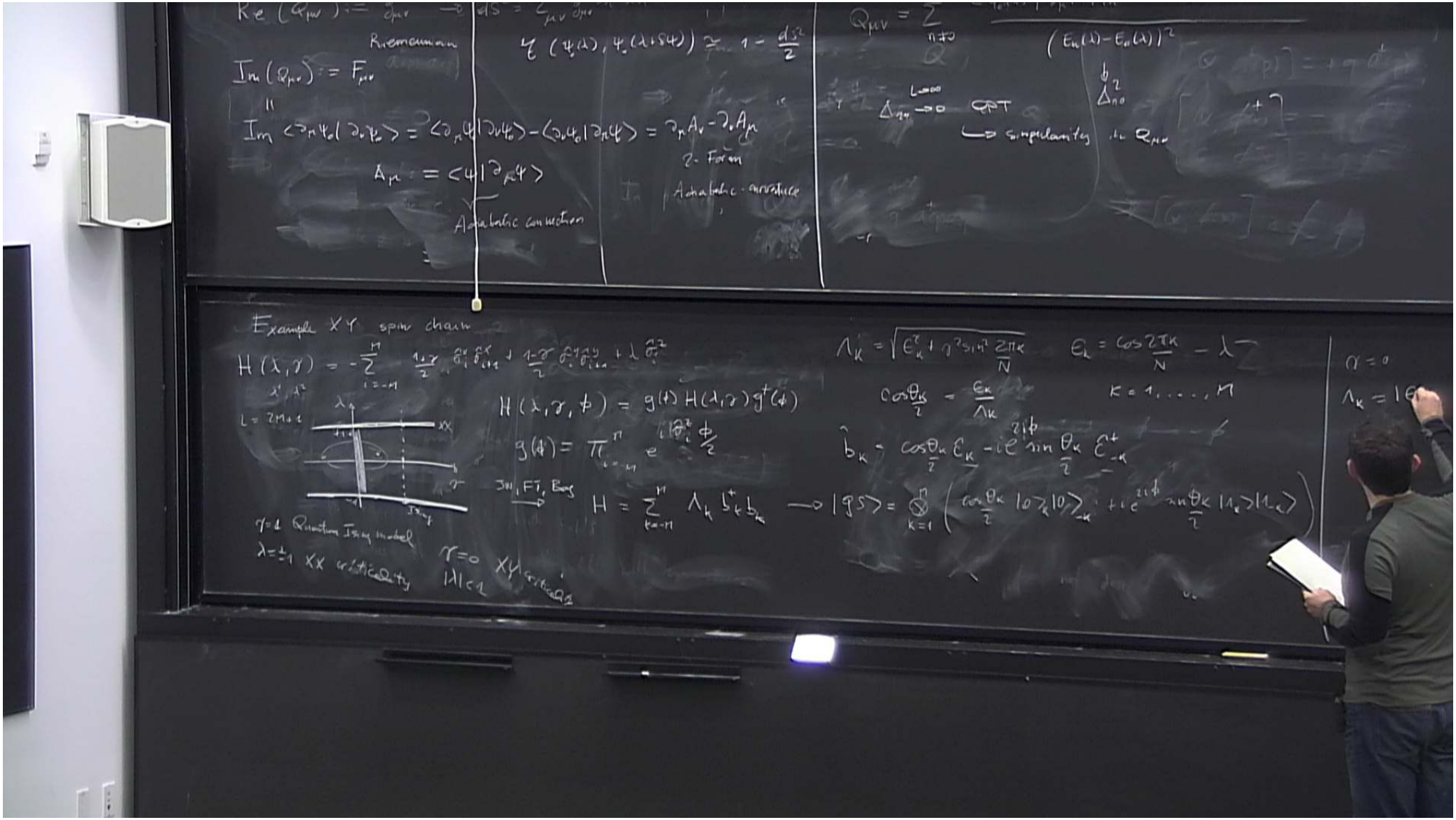
$\gamma = 1$ Quantum Ising model

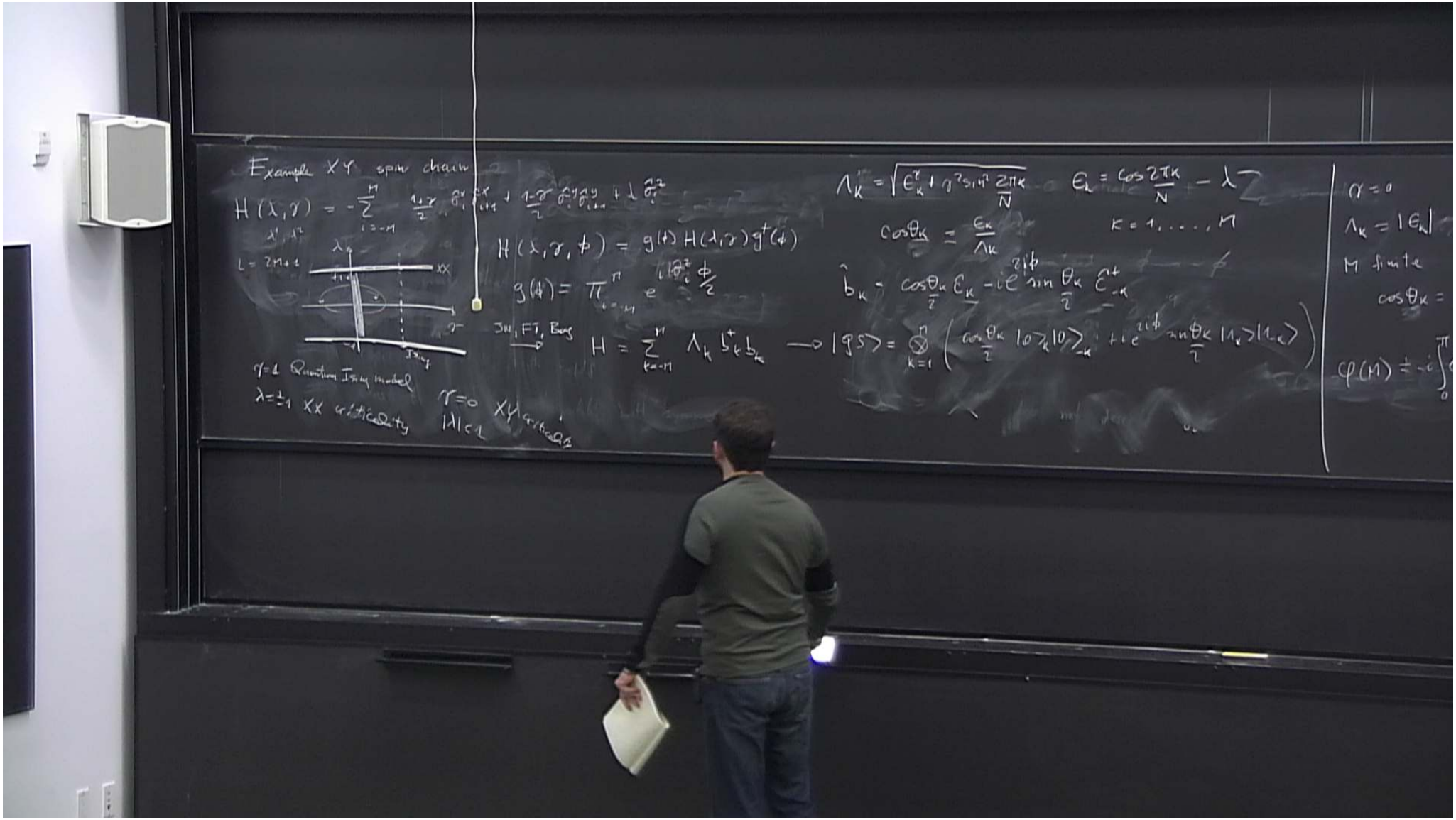
$\lambda = \pm 1$ XX criticality

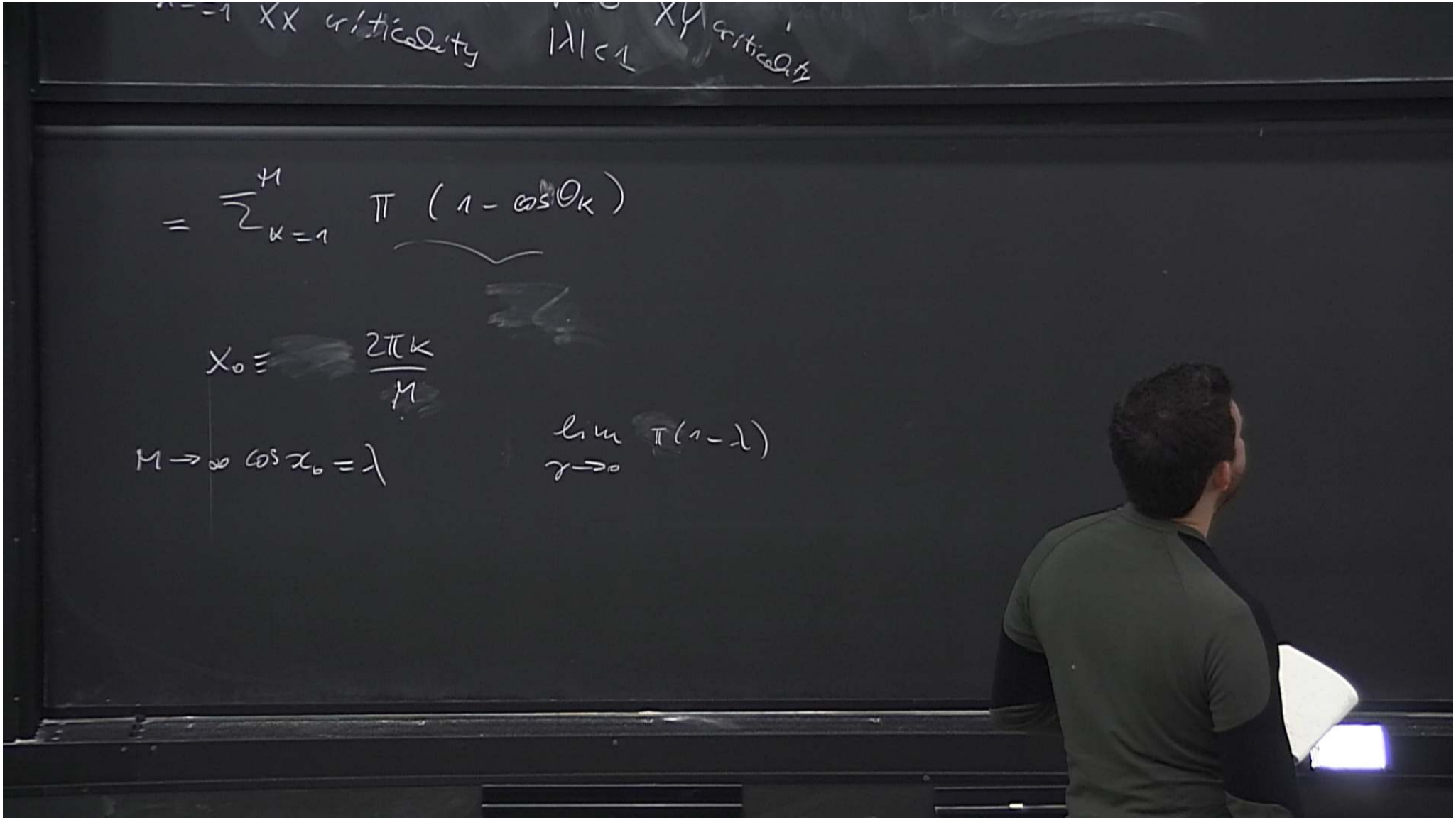
$\gamma = 0$ XY criticality

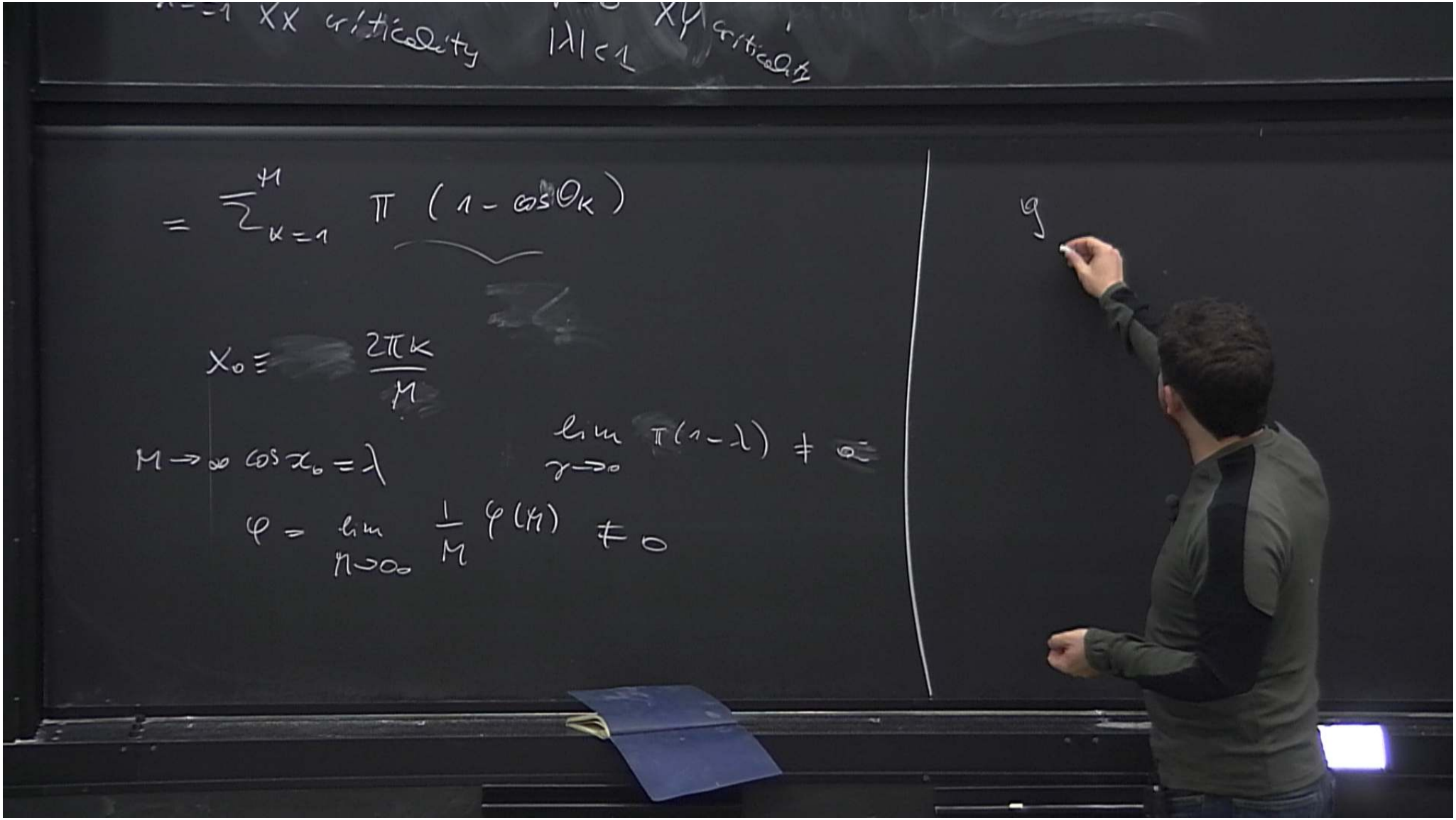
$|\lambda| < 1$

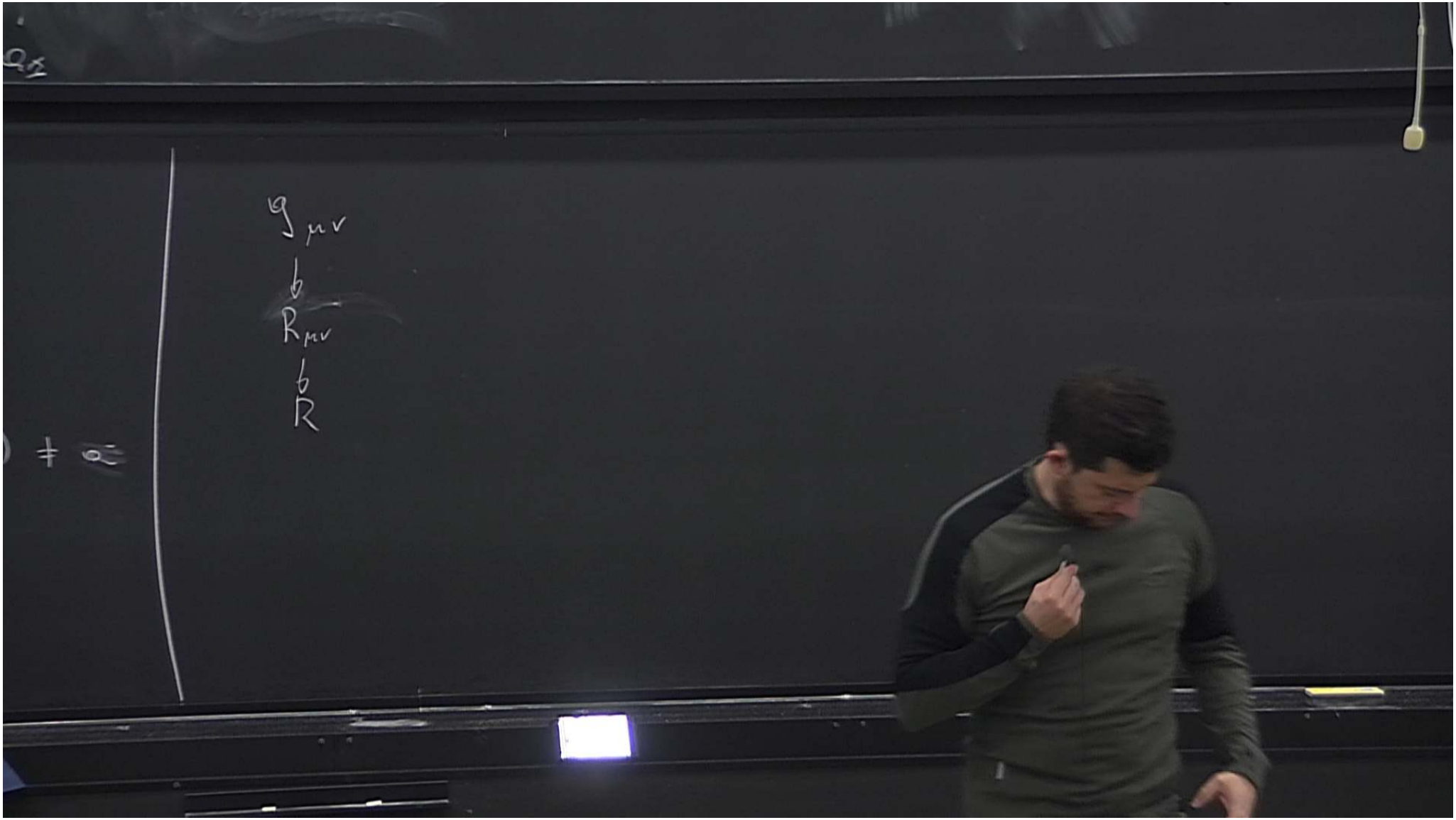

$\hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$
$g, \phi) = g(\phi)\, H(\lambda, g)\, g^\dagger(\phi)$
$= \Pi_{i=-M}^{M} e^{i \hat{\sigma}_i^z \phi/2}$
Bog
$H = \sum_{k=-M}^{M} \Lambda_k b_k^\dagger b_k$
$\Lambda_k = \sqrt{\epsilon_k^2 + g^2 \sin^2 \frac{2\pi k}{N}}$
$\epsilon_k = \cos \frac{2\pi k}{N} - \lambda$
$\cos \frac{\theta_k}{2} = \frac{\epsilon_k}{\Lambda_k}$
$\hat{b}_k = \cos \frac{\theta_k}{2} \hat{c}_k - i e^{2i\phi} \sin \frac{\theta_k}{2} \hat{c}_{-k}^\dagger$
$|gs\rangle = \bigotimes_{k=1}^{M} \left(\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} + i e^{2i\phi} \sin \frac{\theta_k}{2} |1_k\rangle |1_{-k}\rangle \right)$


Riemannian
Adiabatic connection
2- Form
Adiabatic curvature
QPT
Example XY spin chain
Quantum Ising model
XX criticality
XY critical


Example XY spin chain
$H(\lambda,\gamma) = -\sum_{i=-M}^{M} \frac{1+\gamma}{2}\sigma_i^x\sigma_{i+1}^x + \frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y + \lambda\sigma_i^z$
$H(\lambda,\gamma,\phi) = g(\phi)\,H(\lambda,\gamma)\,g^\dagger(\phi)$
$g(\phi) = \prod_{i=-M}^{M} e^{i\frac{\phi}{2}\sigma_i^z}$
JW, FT, Ising
$H = \sum_{k=-M}^{M} \Lambda_k\, b_k^\dagger b_k$
$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2\sin^2\frac{2\pi k}{N}}$
$\epsilon_k = \cos\frac{2\pi k}{N} - \lambda$
$\cos\theta_k = \frac{\epsilon_k}{\Lambda_k}$
$k = 1, \dots, M$
$\hat{b}_k = \cos\frac{\theta_k}{2}\,\mathcal{E}_k - i e^{2i\phi}\sin\frac{\theta_k}{2}\,\mathcal{E}_{-k}^\dagger$
$|gs\rangle = \bigotimes_{k=1}^{n}\left(\cos\frac{\theta_k}{2}|0\rangle_k|0\rangle_{-k} + i e^{2i\phi}\sin\frac{\theta_k}{2}|1_k\rangle|1_{-k}\rangle\right)$
$\gamma = 0$
$\Lambda_k = |\epsilon_k|$
M finite
$\cos\theta_k =$
$\varphi(M) = -i\int_0^{\pi}$
$\gamma = 1$ Quantum Ising model
$\lambda = \pm 1$ XX criticality
$\gamma = 0$ XY critical
$|\lambda| < 1$


XX criticality
$|\lambda| < 1$
XY criticality
$= \sum_{k=1}^{M} \pi (1 - \cos\theta_k)$
$x_0 \equiv \frac{2\pi k}{M}$
$M \to \infty \cos x_0 = \lambda$
$\lim_{\gamma \to 0} \pi(1-\lambda)$


XX criticality
$|\lambda| < 1$
XY criticality
$= \sum_{k=1}^{M} \pi (1 - \cos\theta_k)$
$x_0 \equiv \frac{2\pi k}{M}$
$M \to \infty \cos x_0 = \lambda$
$\lim_{\gamma \to 0} \pi(1-\lambda) \neq 0$
$\varphi = \lim_{M \to \infty} \frac{1}{M} \varphi(M) \neq 0$
g


$g_{\mu\nu}$
$R_{\mu\nu}$
R