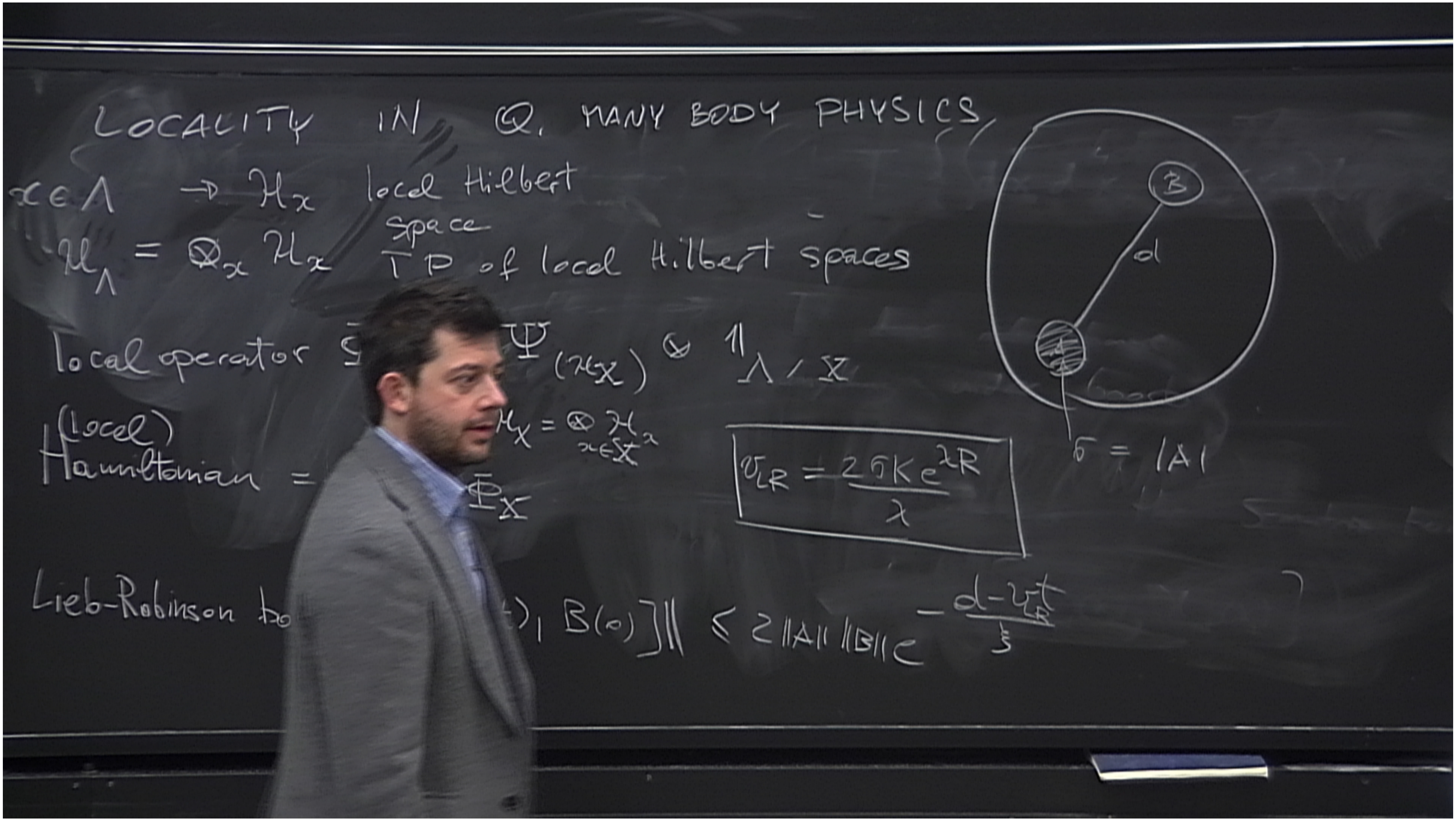
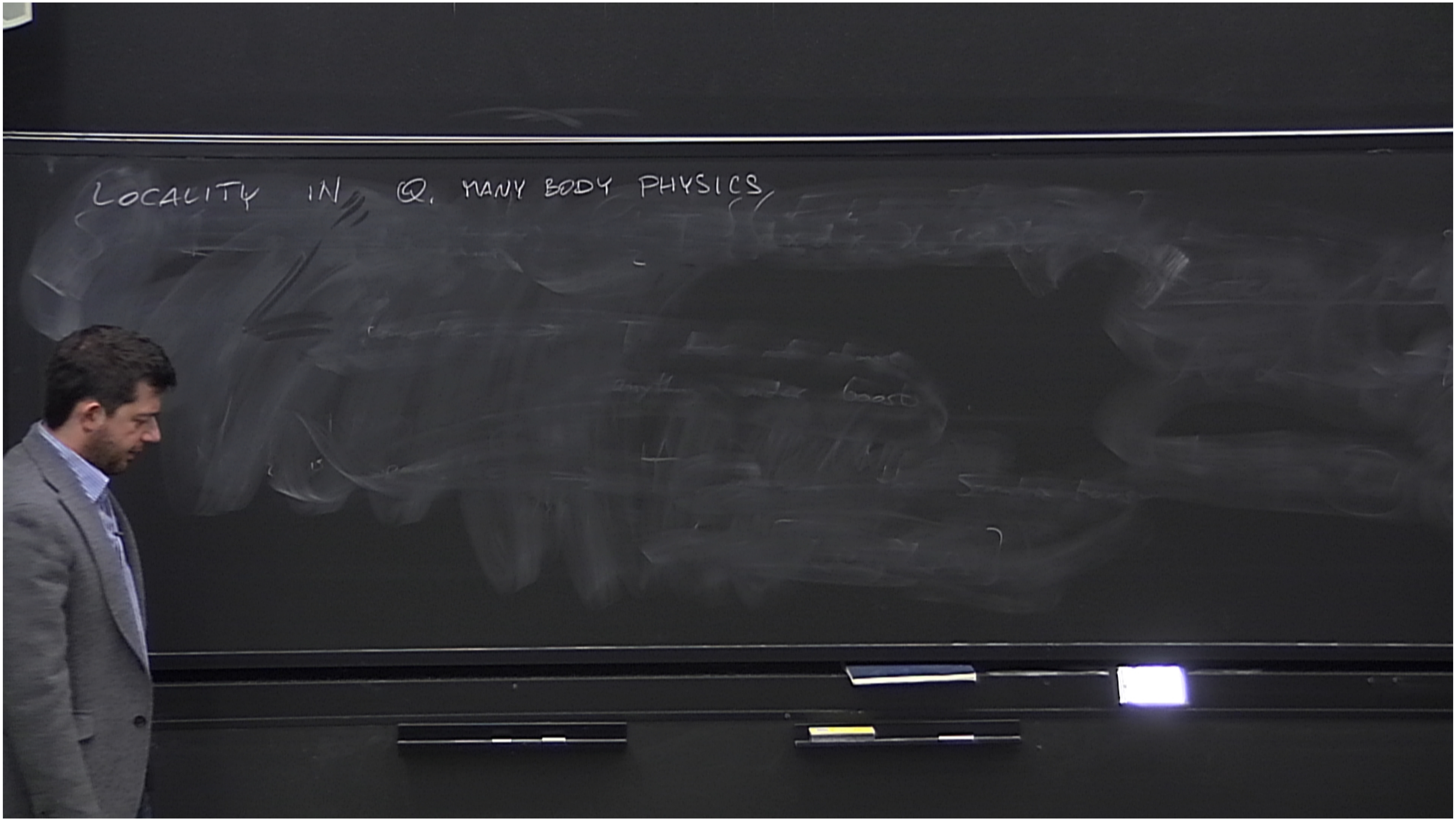
Title: Condensed Matter (Review) - Lecture 6

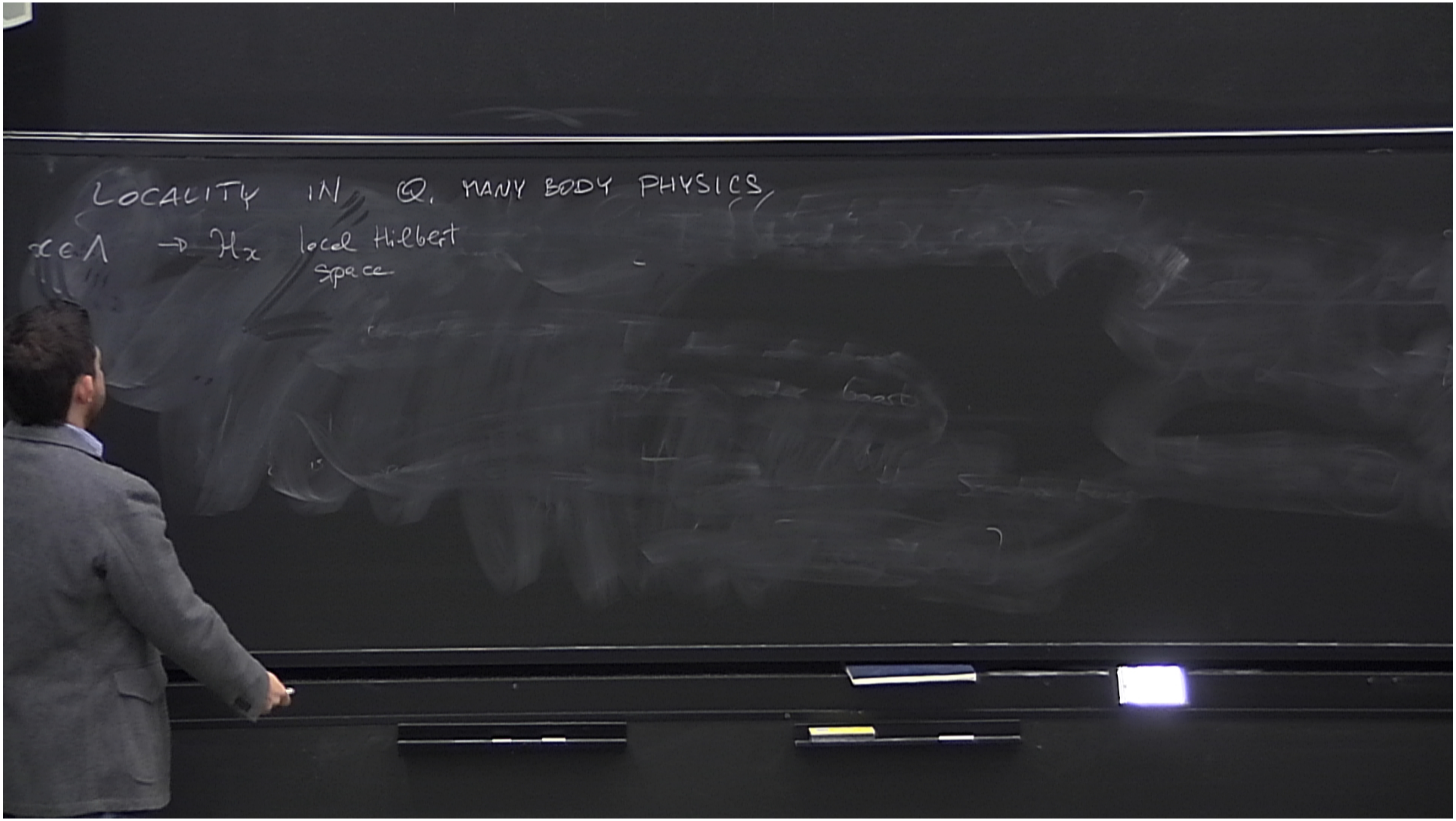
Date: Jan 09, 2012 10:15 AM

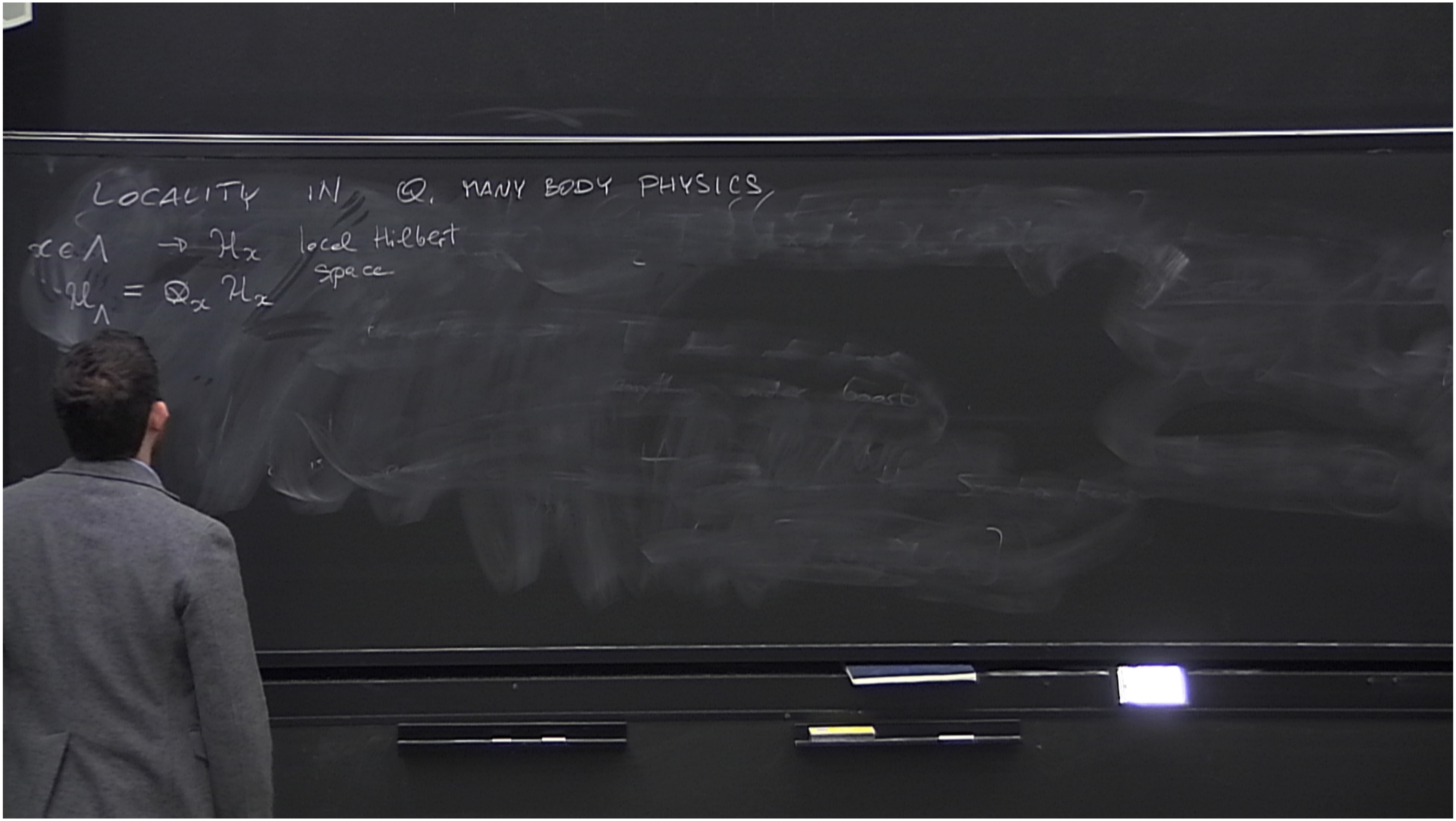
URL: http://pirsa.org/12010090

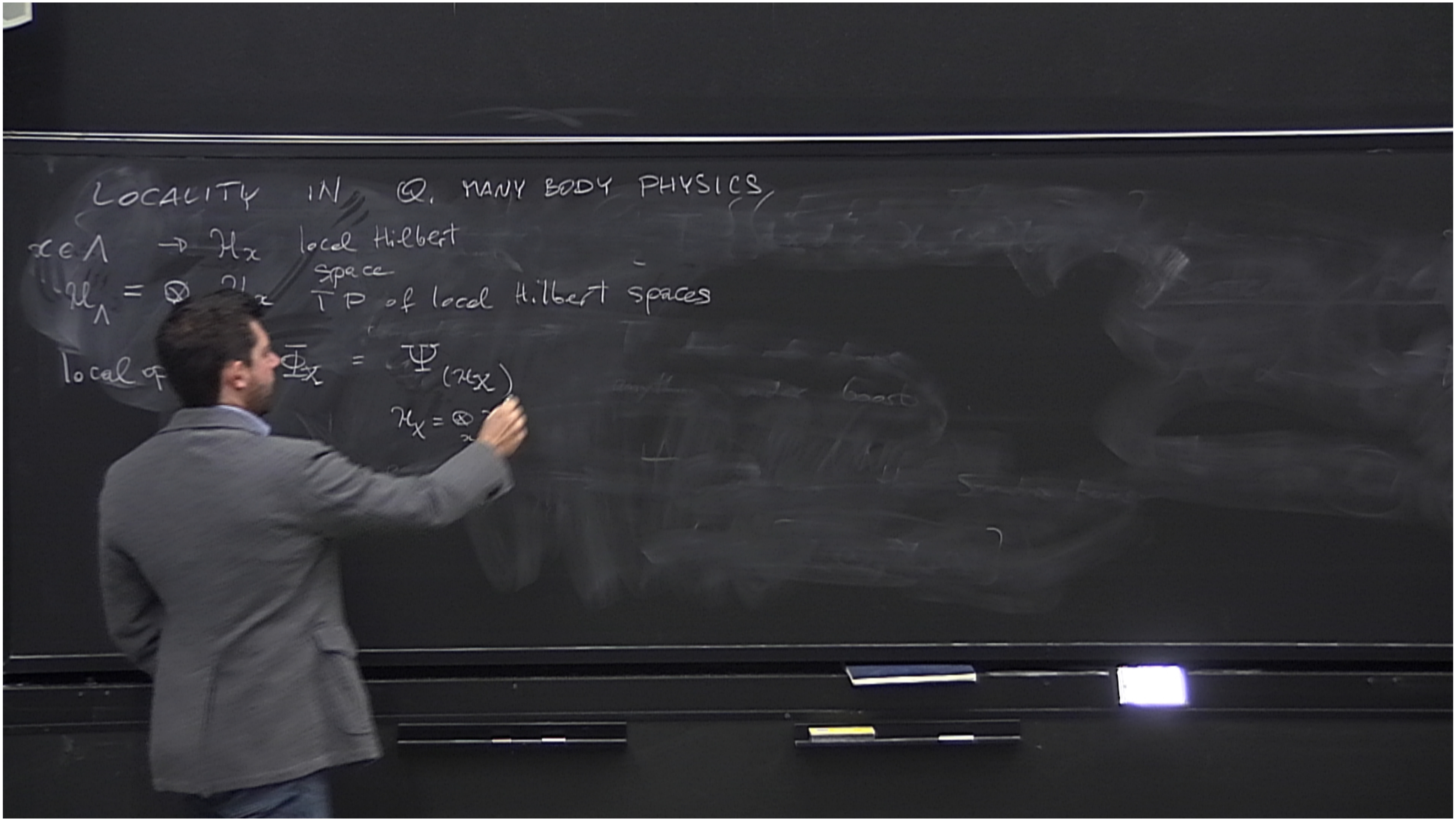
Abstract:

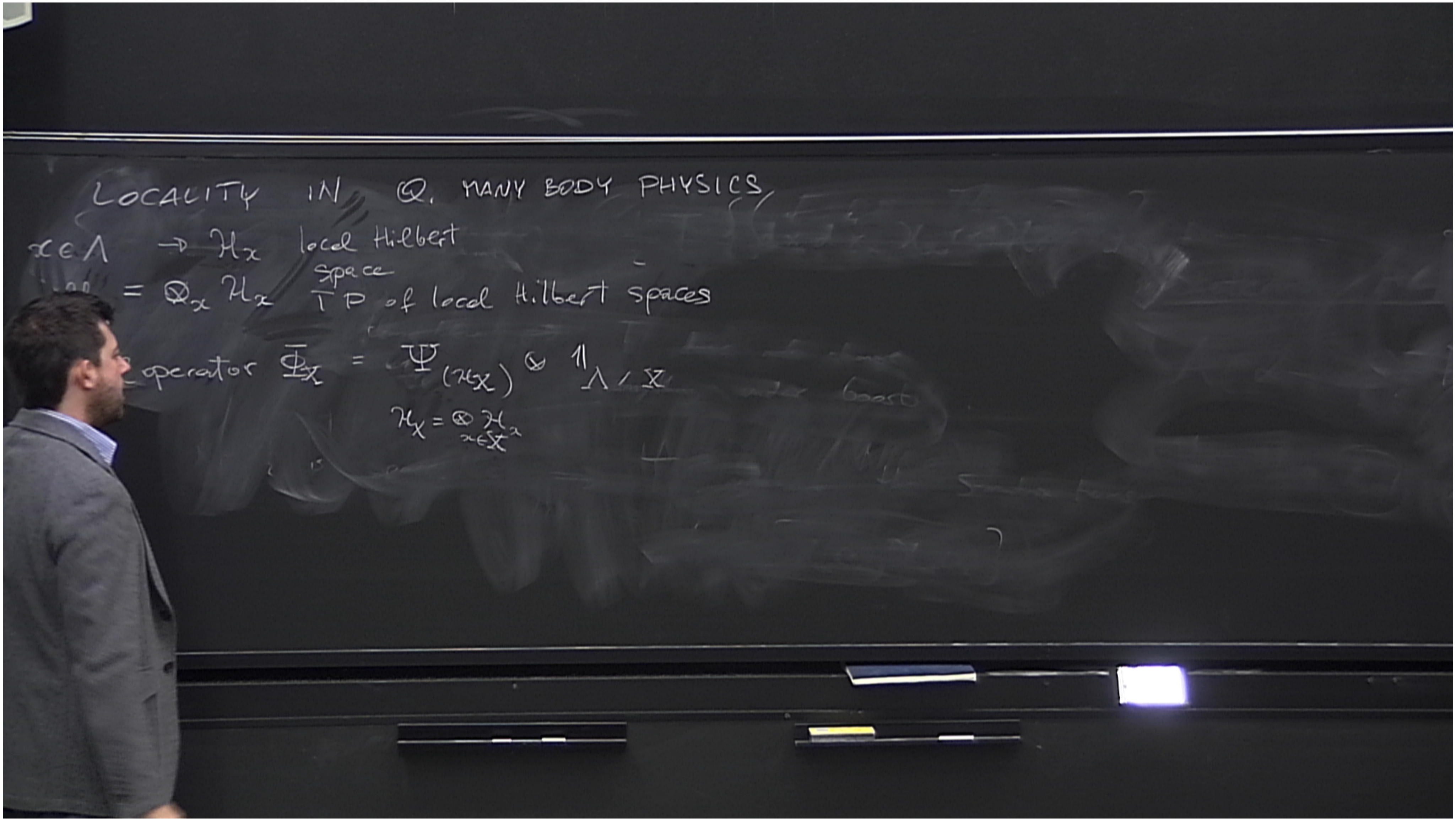

LOCALITY IN Q. MANY BODY PHYSICS
local Hilbert space
TP of local Hilbert spaces
local operator
(local) Hamiltonian
Lieb-Robinson bo
d
B
σ = |A|

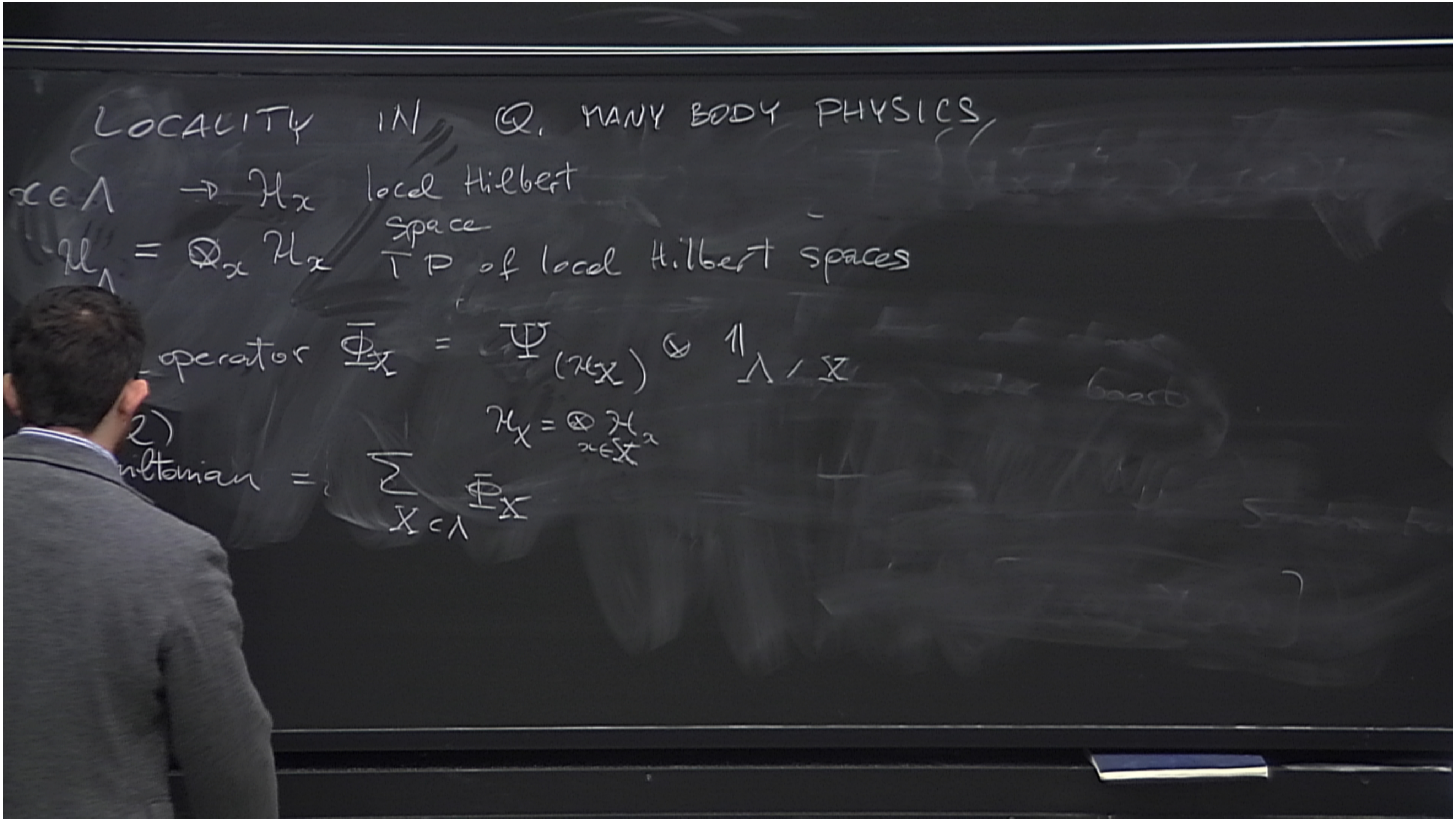

LOCALITY IN Q. MANY BODY PHYSICS

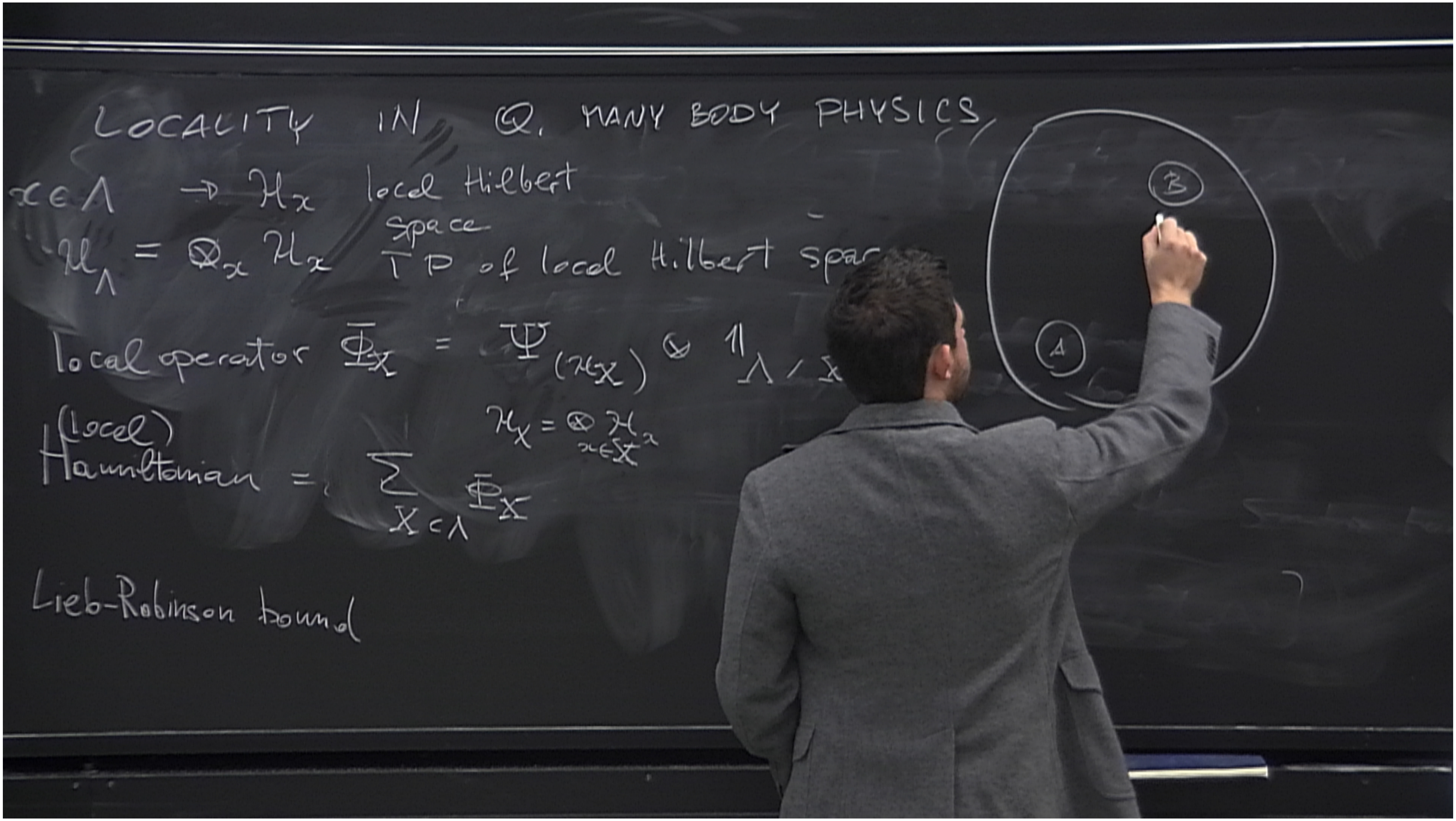

LOCALITY IN Q. MANY BODY PHYSICS
x ∈ Λ → Hx local Hilbert space


LOCALITY IN Q. MANY BODY PHYSICS
$x \in \Lambda \to \mathcal{H}_x$ local Hilbert space
$\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$


LOCALITY IN Q. MANY BODY PHYSICS
local Hilbert space
TP of local Hilbert spaces
local op


LOCALITY IN Q. MANY BODY PHYSICS
$x \in \Lambda \to \mathcal{H}_x$ local Hilbert space
$= \otimes_x \mathcal{H}_x$ TP of local Hilbert spaces
operator $\Phi_X = \Psi_{(\mathcal{H}_X)} \otimes 1_{\Lambda \setminus X}$
$\mathcal{H}_X = \otimes_{x \in X} \mathcal{H}_x$


LOCALITY IN Q. MANY BODY PHYSICS
x ∈ Λ → H_x local Hilbert space
H_Λ = ⊗_x H_x TP of local Hilbert spaces
operator Φ_X = Ψ(H_X) ⊗ 1_{Λ∖X}
H_X = ⊗_{x∈X} H_x
2)
miltonian = Σ_{X⊂Λ} Φ_X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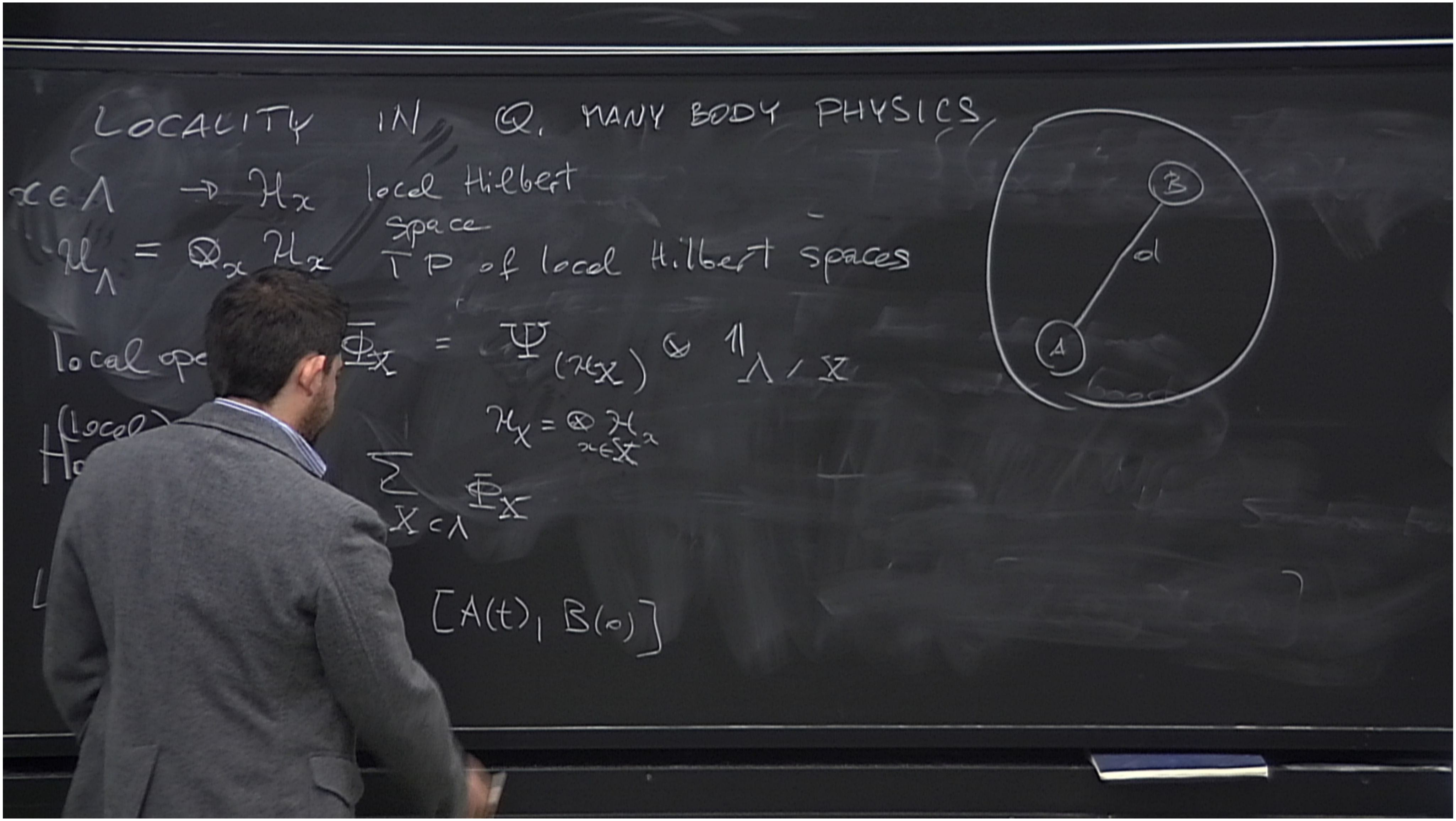

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x ∈ Λ → H_x local Hilbert space
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local ope
Φ_X = Ψ(H_X) ⊗ 1_{Λ/X}
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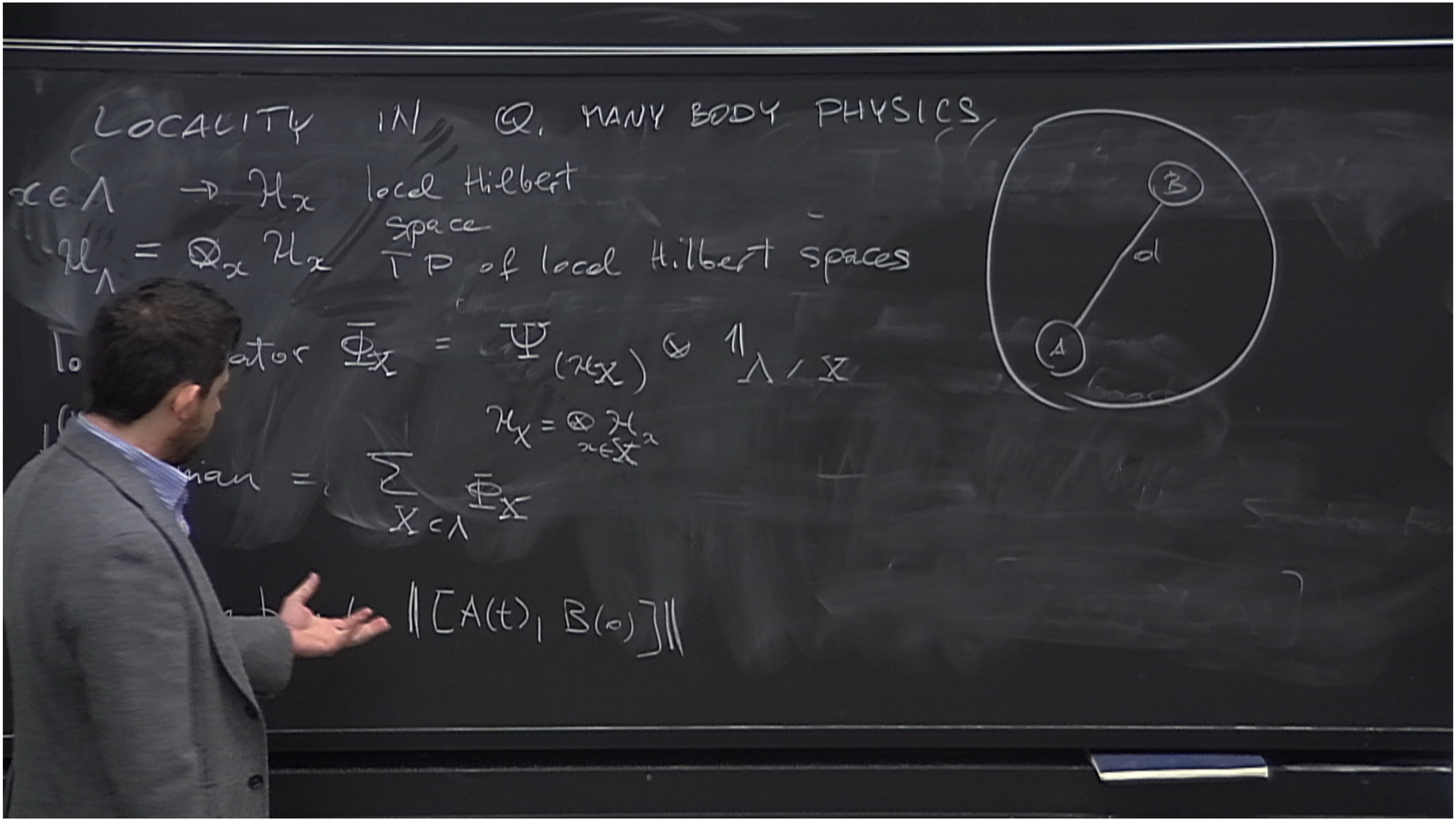
Σ_{X⊂Λ} Φ_X
[A(t), B(0)]
B
A
d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B
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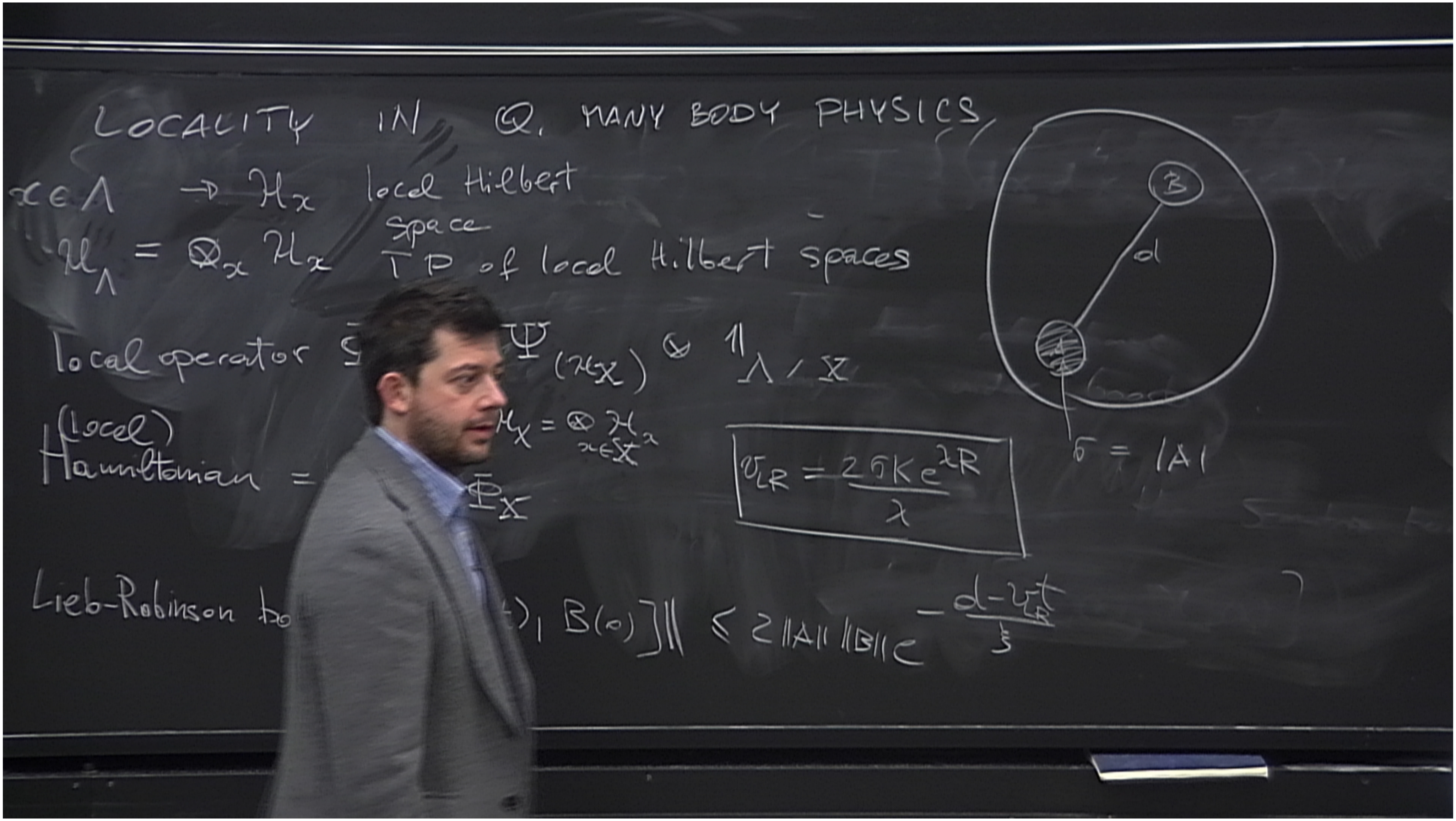
A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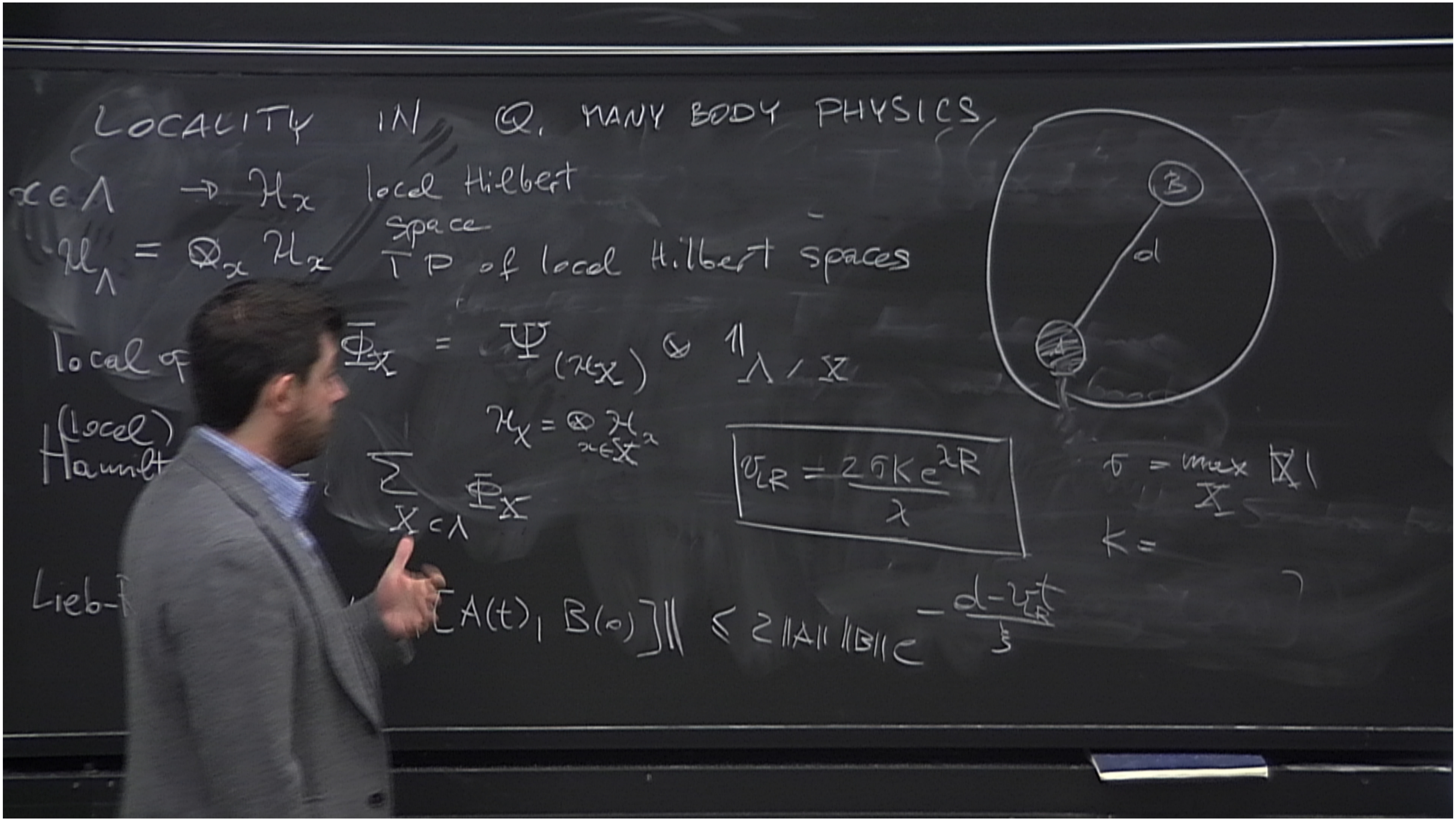
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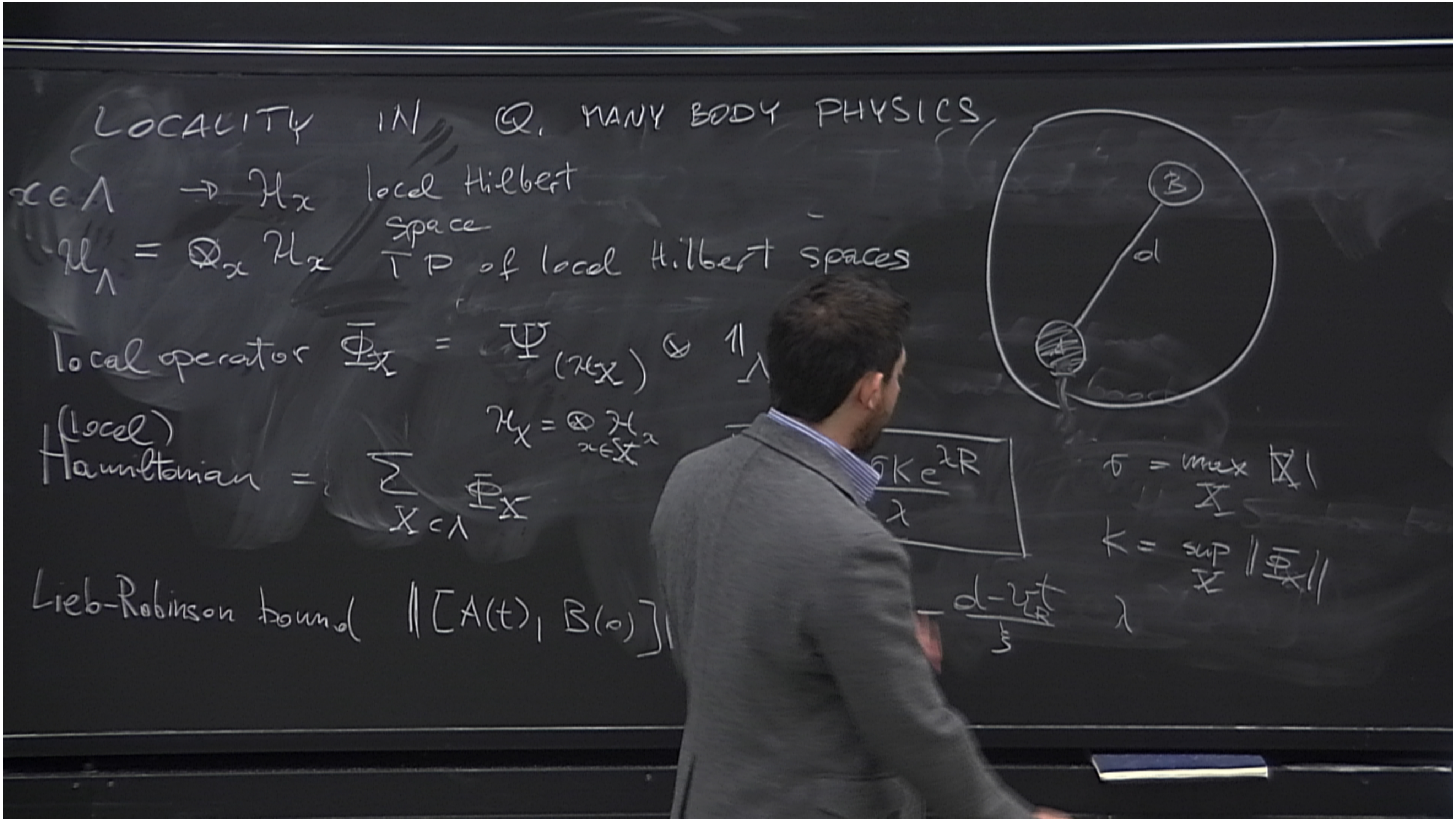
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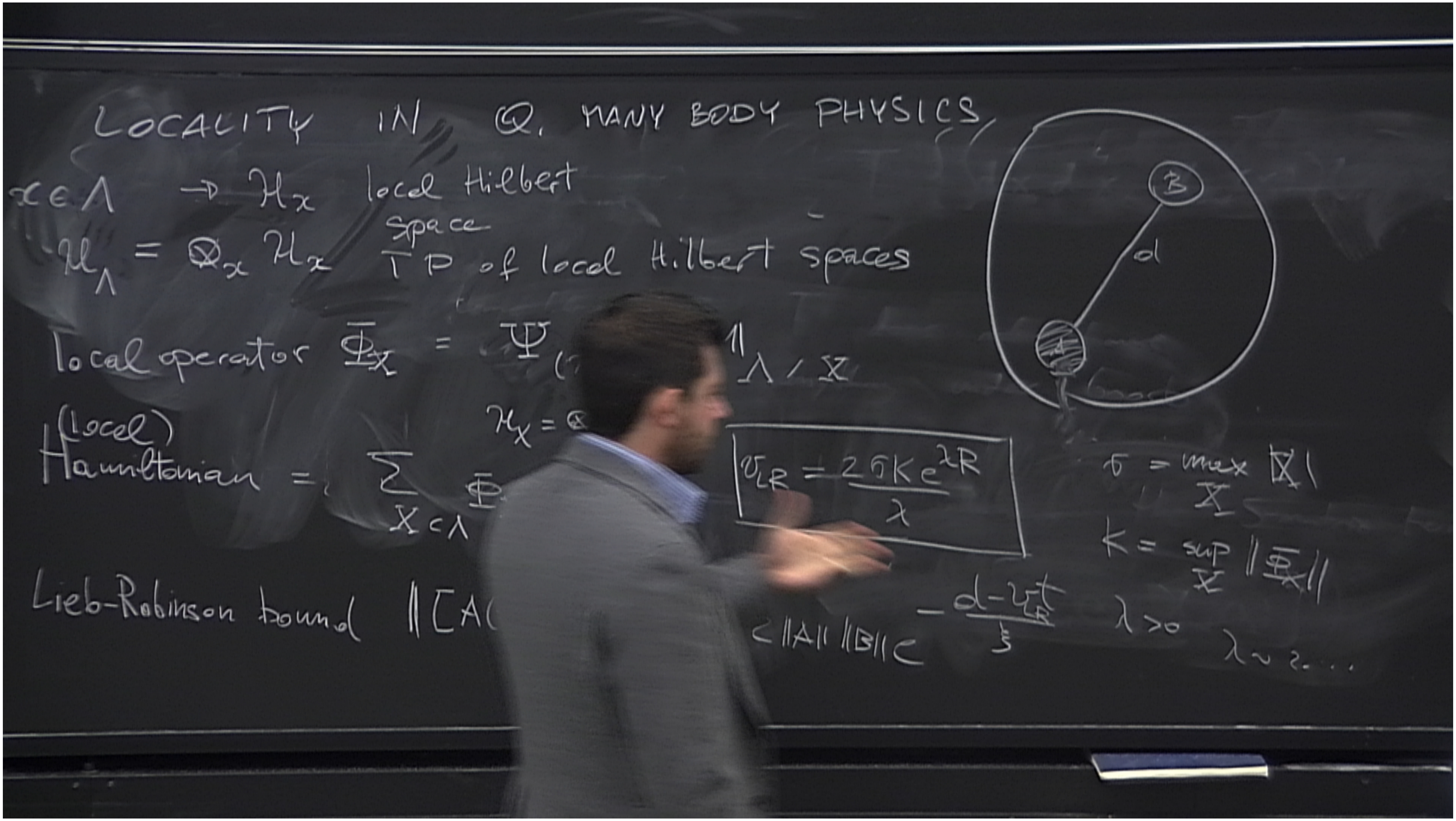
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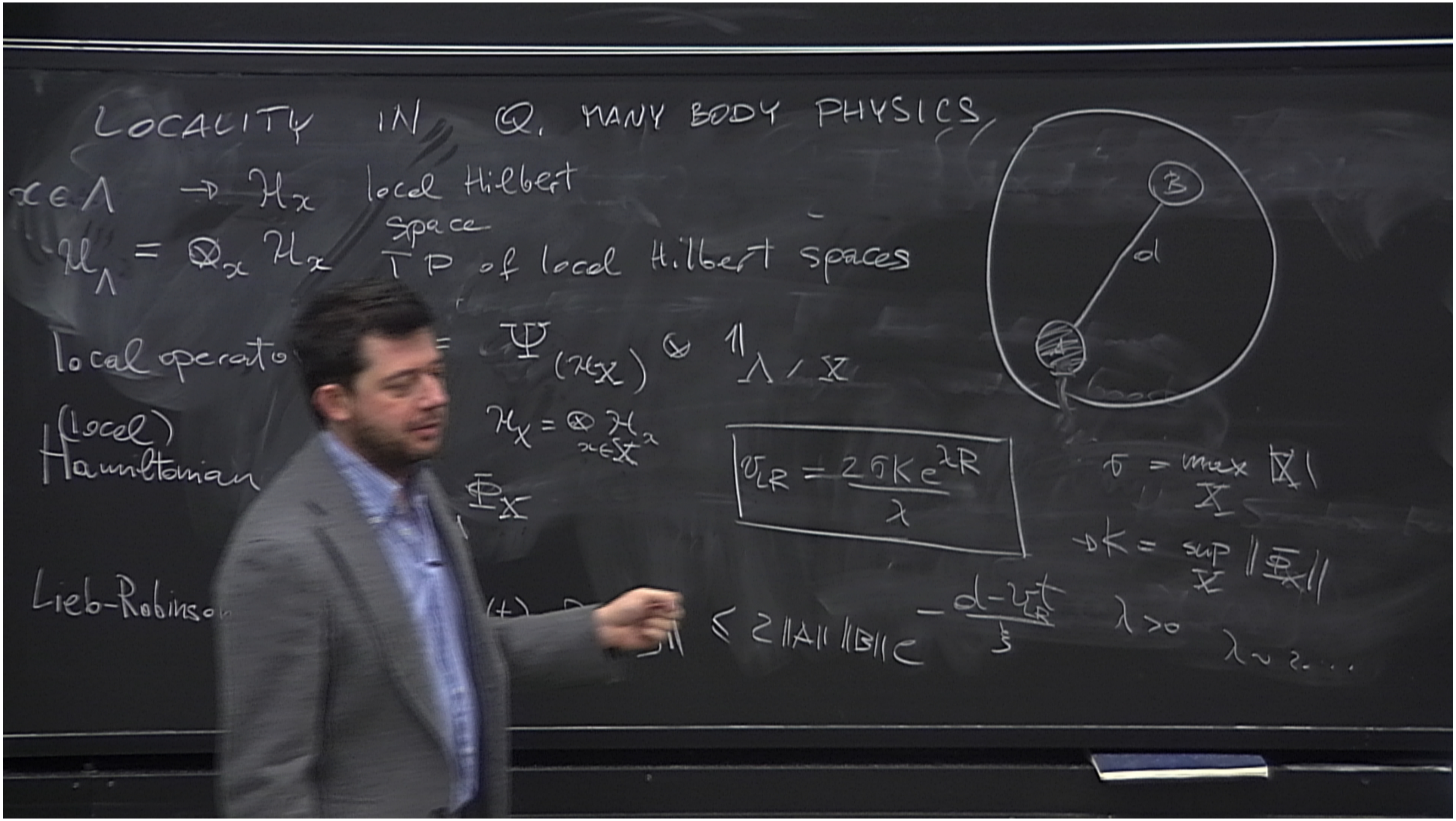
LOCALITY IN Q. MANY BODY PHYSICS
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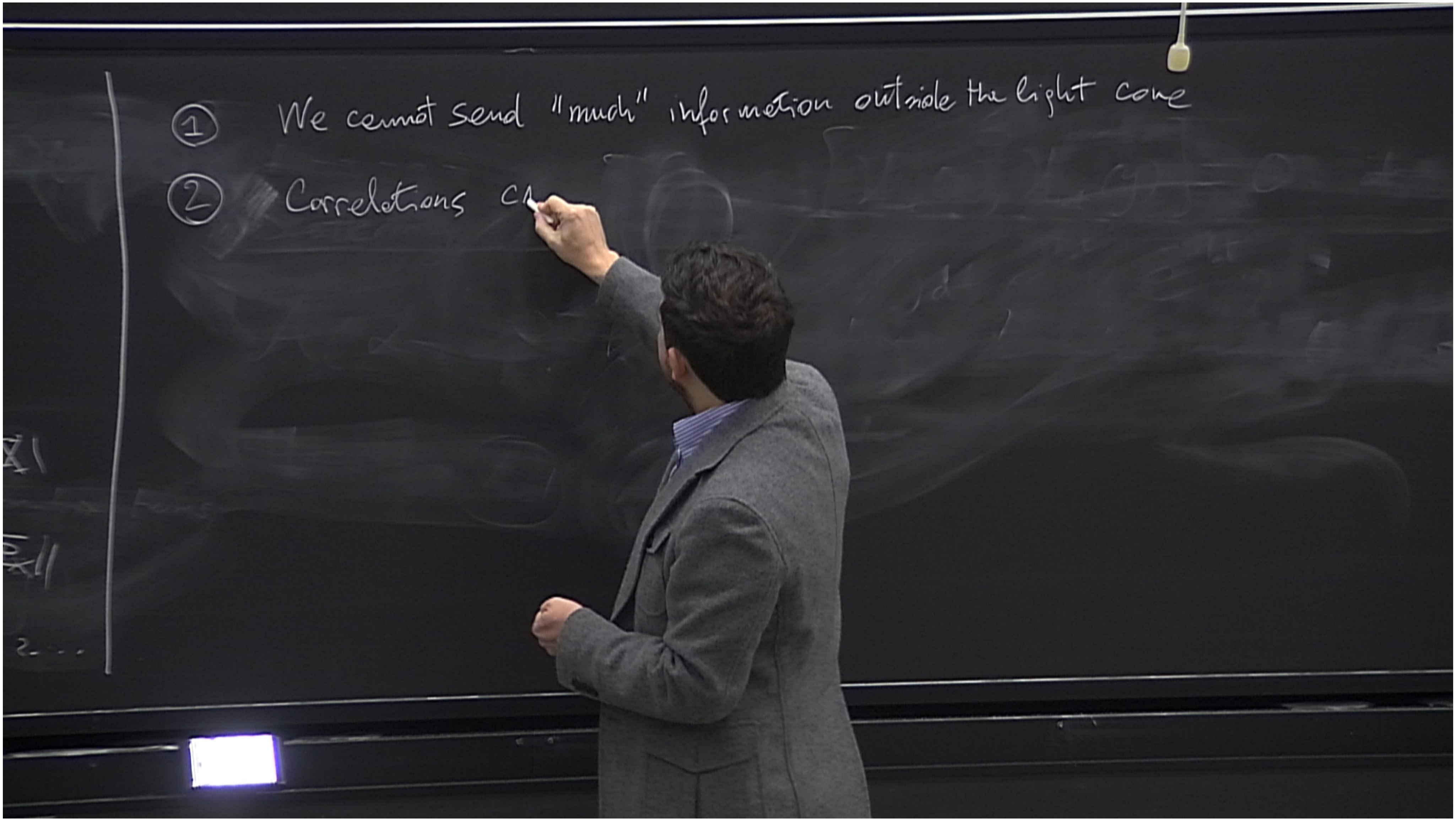
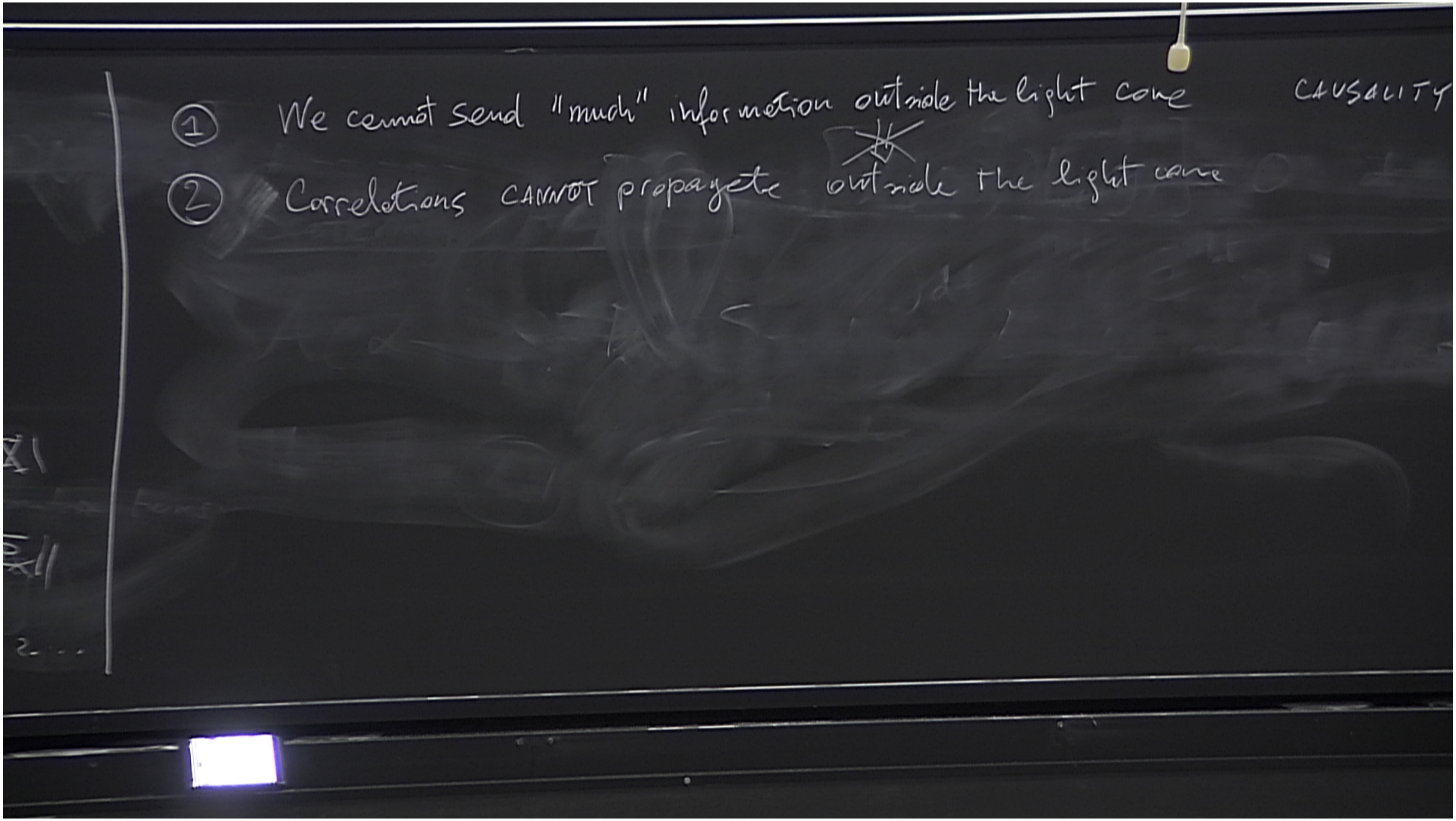
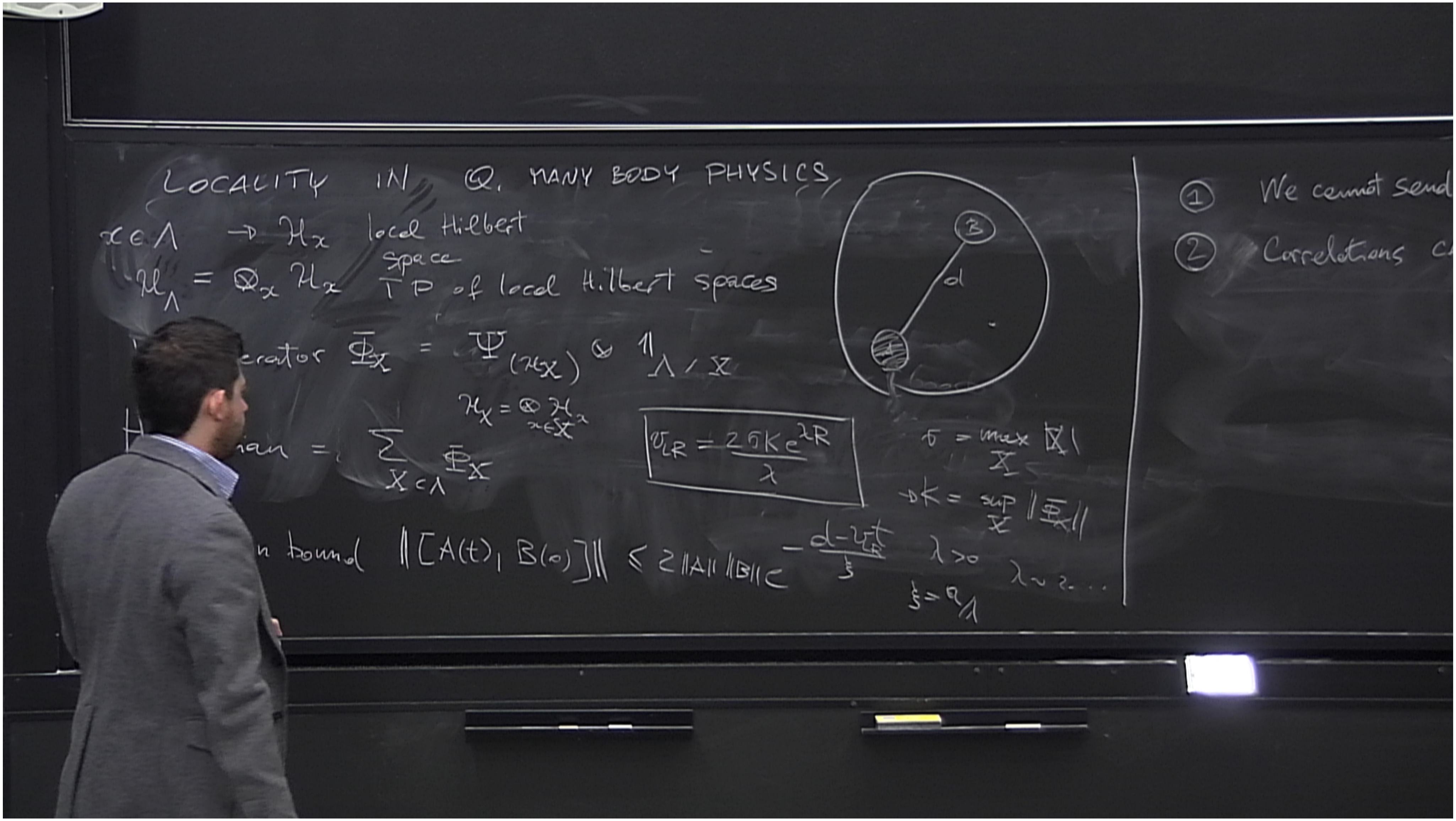
(local) Hamiltonian
Lieb-Robinson
λ > 0


① We cannot send "much" information outside the light cone
② Correlations c


CAUSALITY
① We cannot send "much" information outside the light cone
② Correlations CANNOT propagate outside the light cone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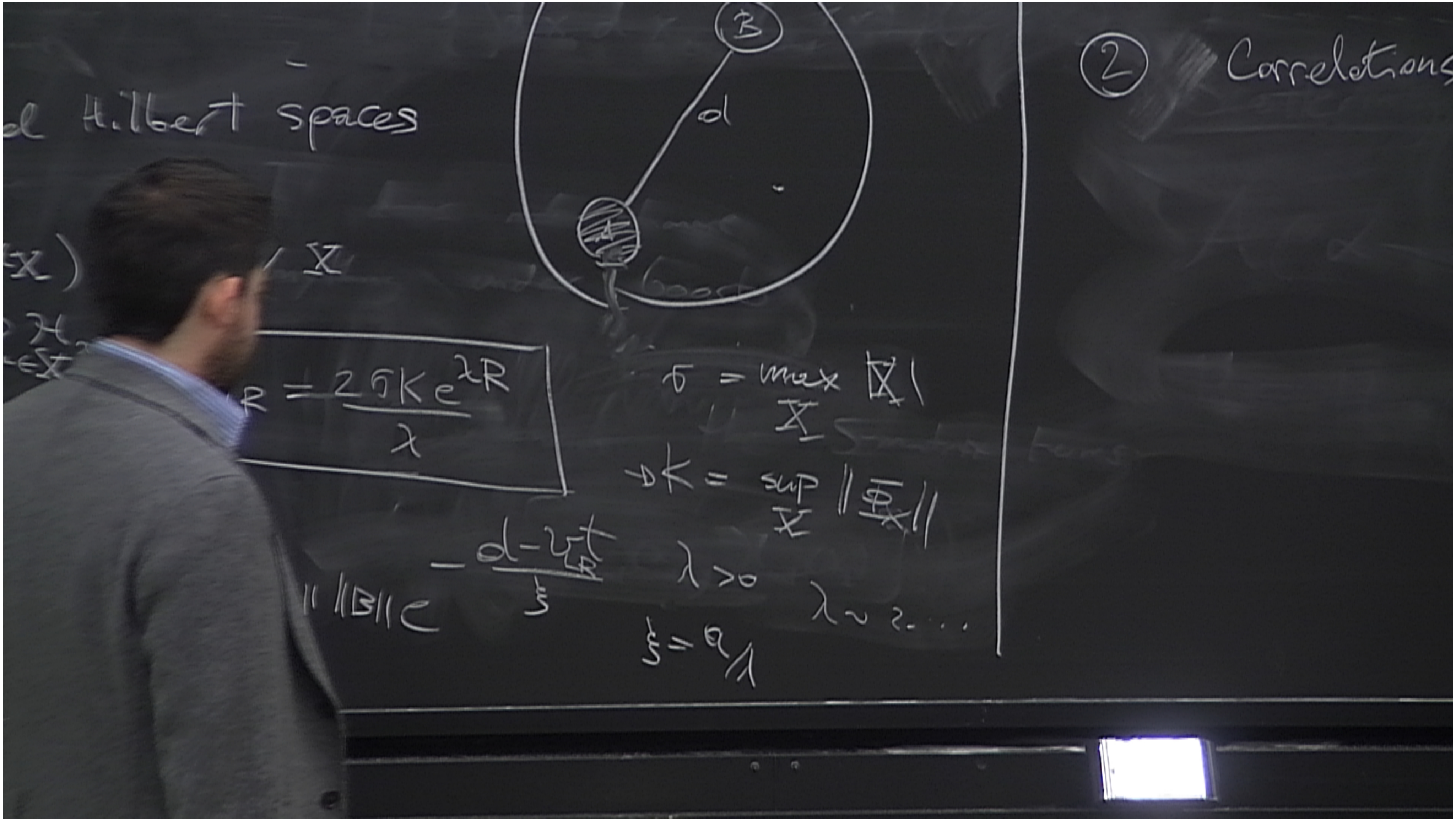
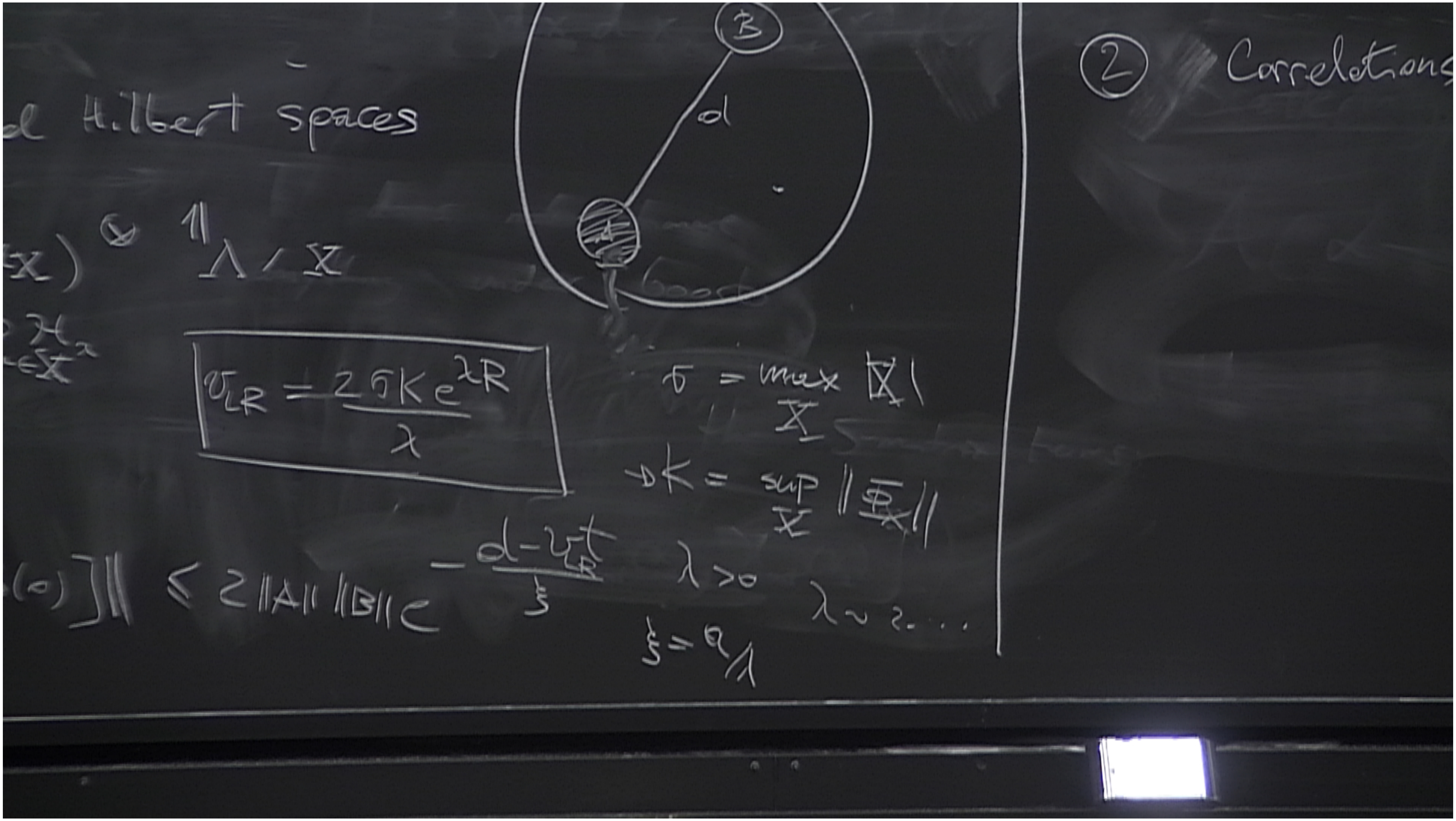
erator $\Phi_X = \Psi(\mathcal{H}_X) \otimes 1_{\Lambda / X}$
$\mathcal{H}_X = \otimes_{x \in X} \mathcal{H}_x$
nian $= \sum_{X \subset \Lambda} \Phi_X$
n bound $\|[A(t), B(0)]\| \leq 2\|A\|\|B\| e^{-\frac{d - v_{LR} t}{\xi}}$
$v_{LR} = \frac{2 \sigma K e^{\lambda R}}{\lambda}$
$\sigma = \max_X |X|$
$K = \sup_X \|\Phi_X\|$
$\lambda > 0$
$\xi = a/\lambda$
d
① We cannot send
② Correlations

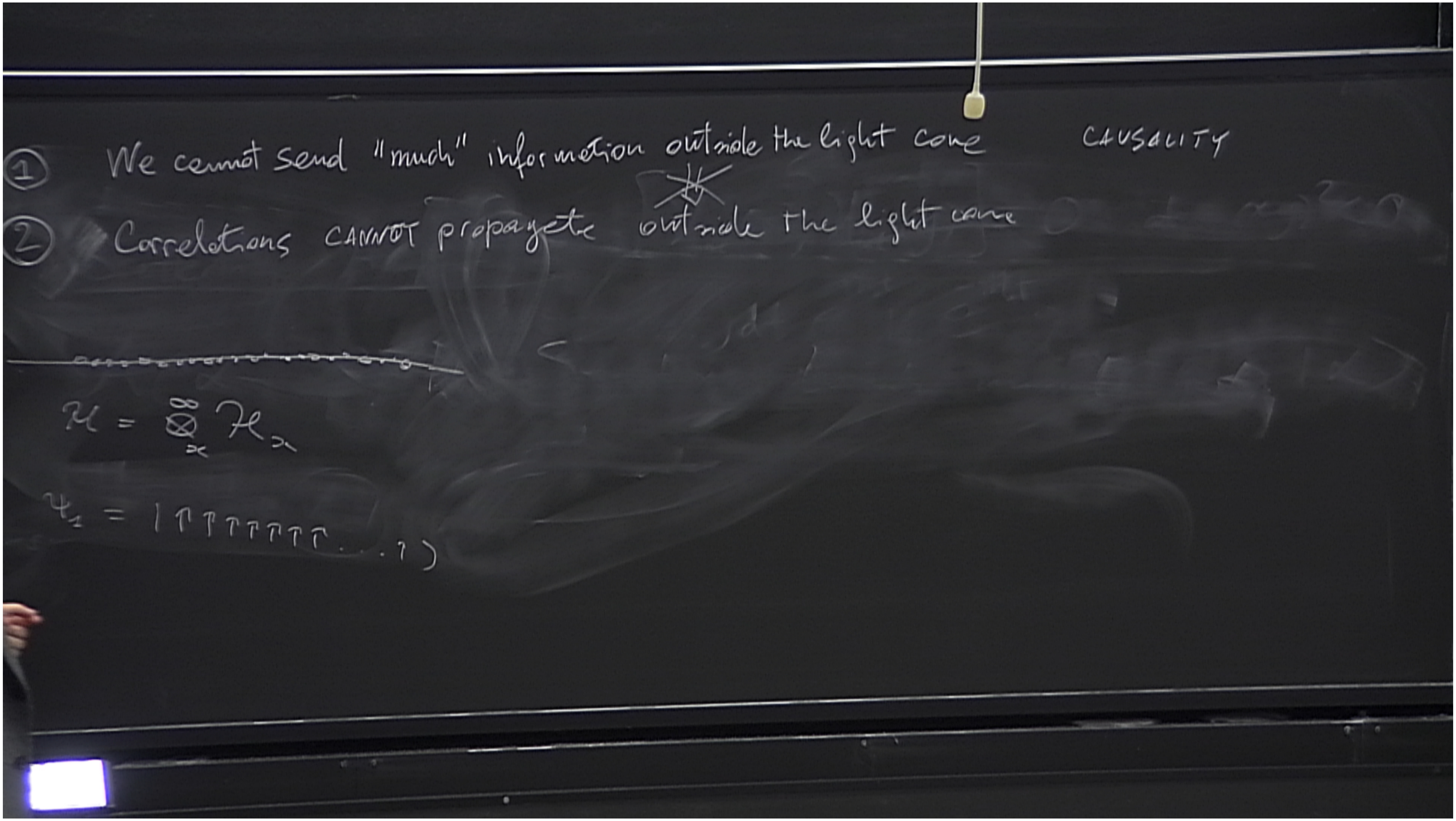

Hilbert spaces
Correlations
d
σ = max |X|
λ > 0

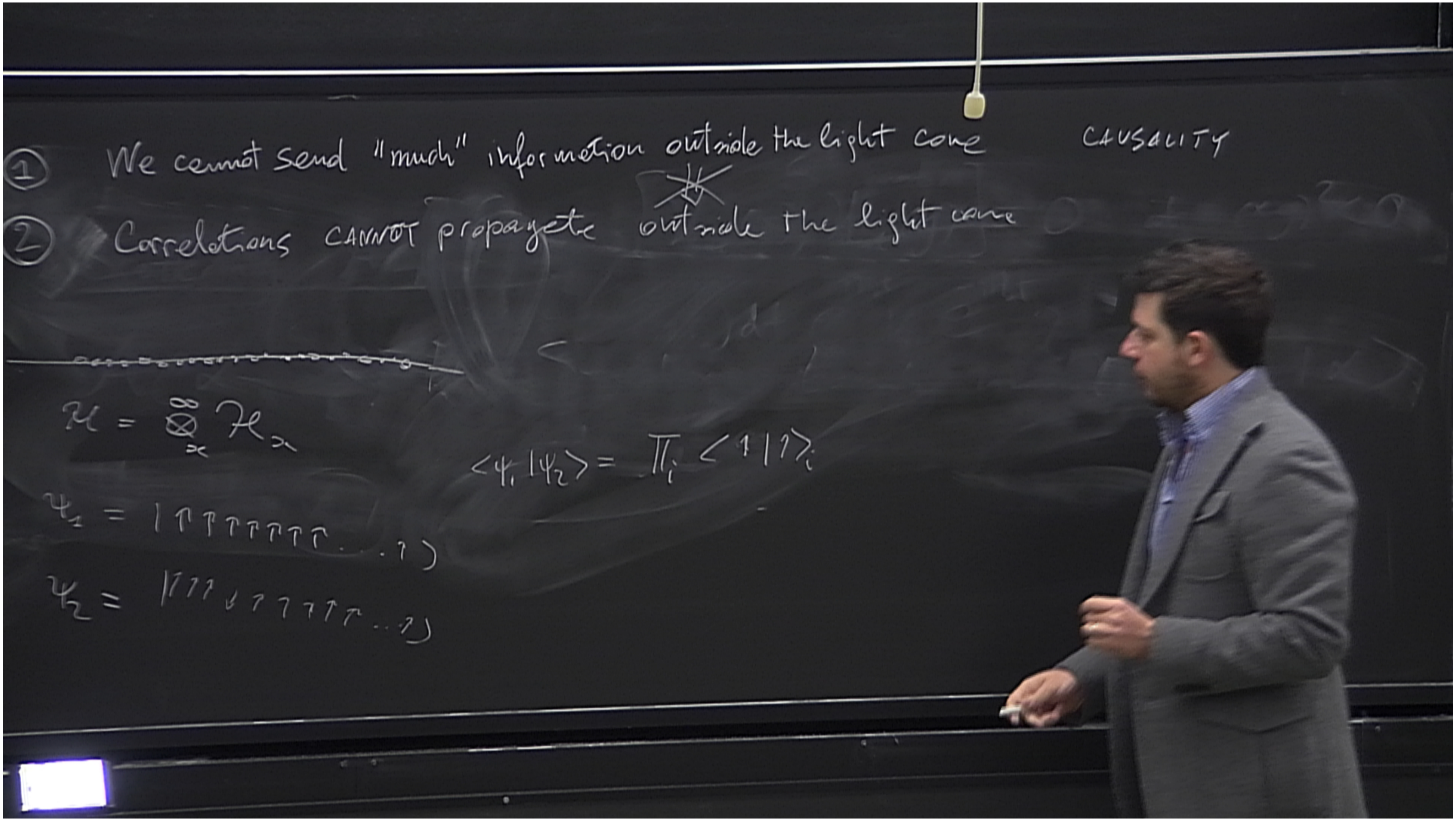

Hilbert spaces
② Correlations
$v_{LR} = \frac{2\sigma K e^{\lambda R}}{\lambda}$
$\xi = \frac{2}{\lambda}$
λ > 0

① We cannot send "much" information outside the light cone CAUSALITY

② Correlations CANNOT propagate outside the light cone

$$\mathcal{H} = \bigotimes_{x}^{\infty} \mathcal{H}_x$$


① We cannot send "much" information outside the light cone
CAUSALITY
② Correlations CANNOT propagate outside the light cone
$\mathcal{H} = \bigotimes_x^\infty \mathcal{H}_x$
$\Psi_1 = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle$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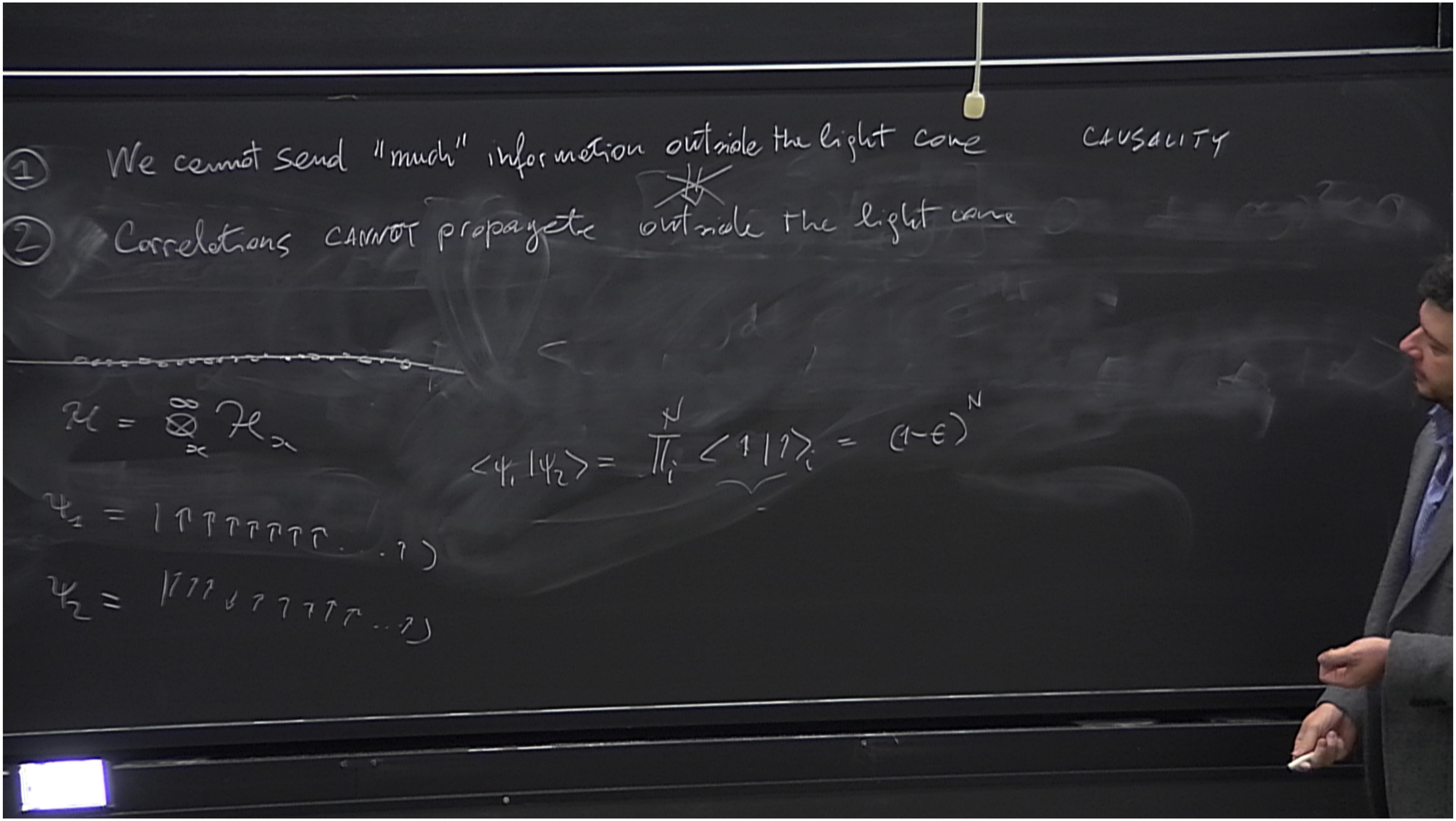
$\langle\psi_1|\psi_2\rangle = \prod_i \langle\uparrow|\uparrow\rangle_i$
$\psi_1 = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle$
$\psi_2 = |\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle$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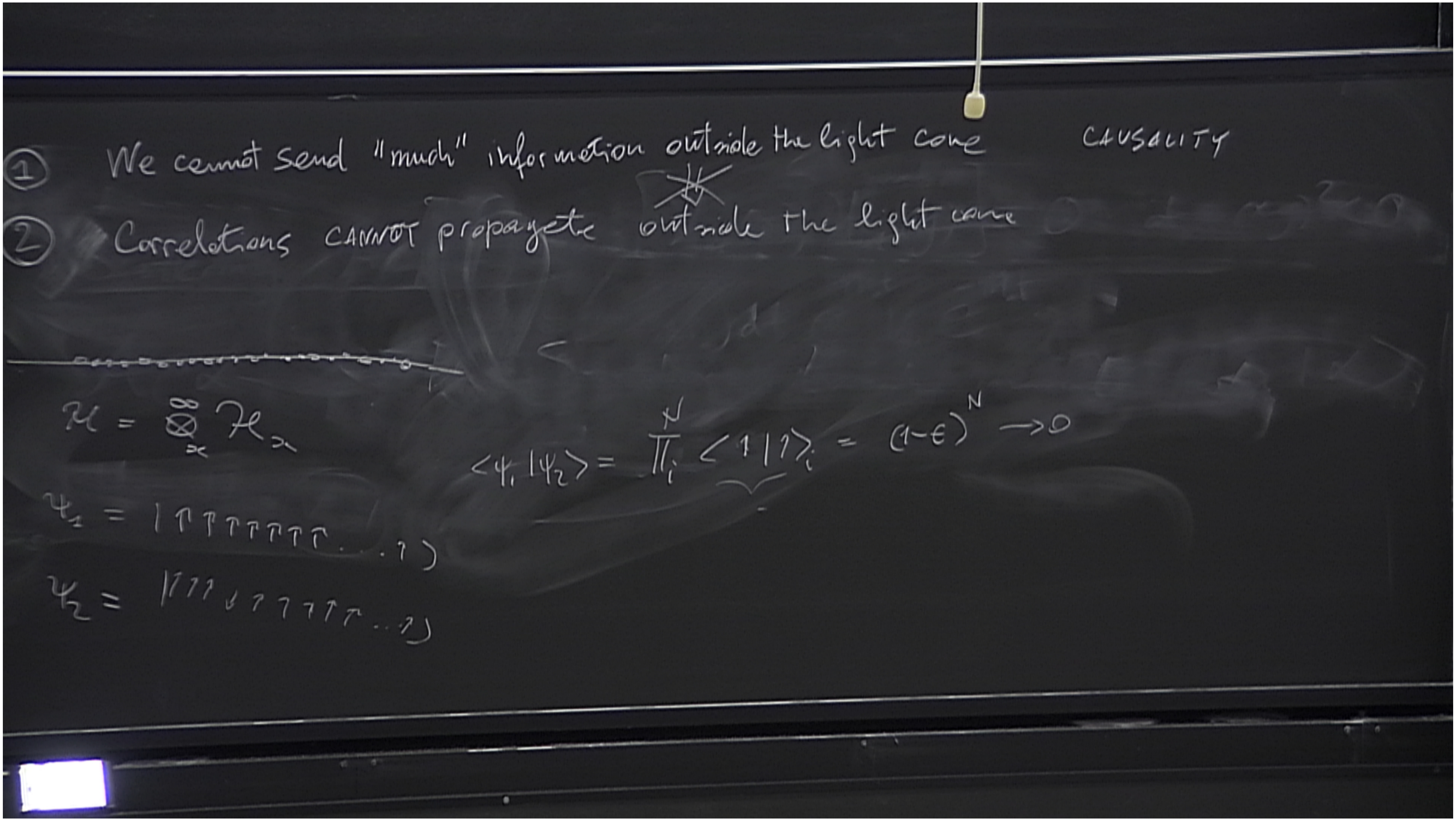
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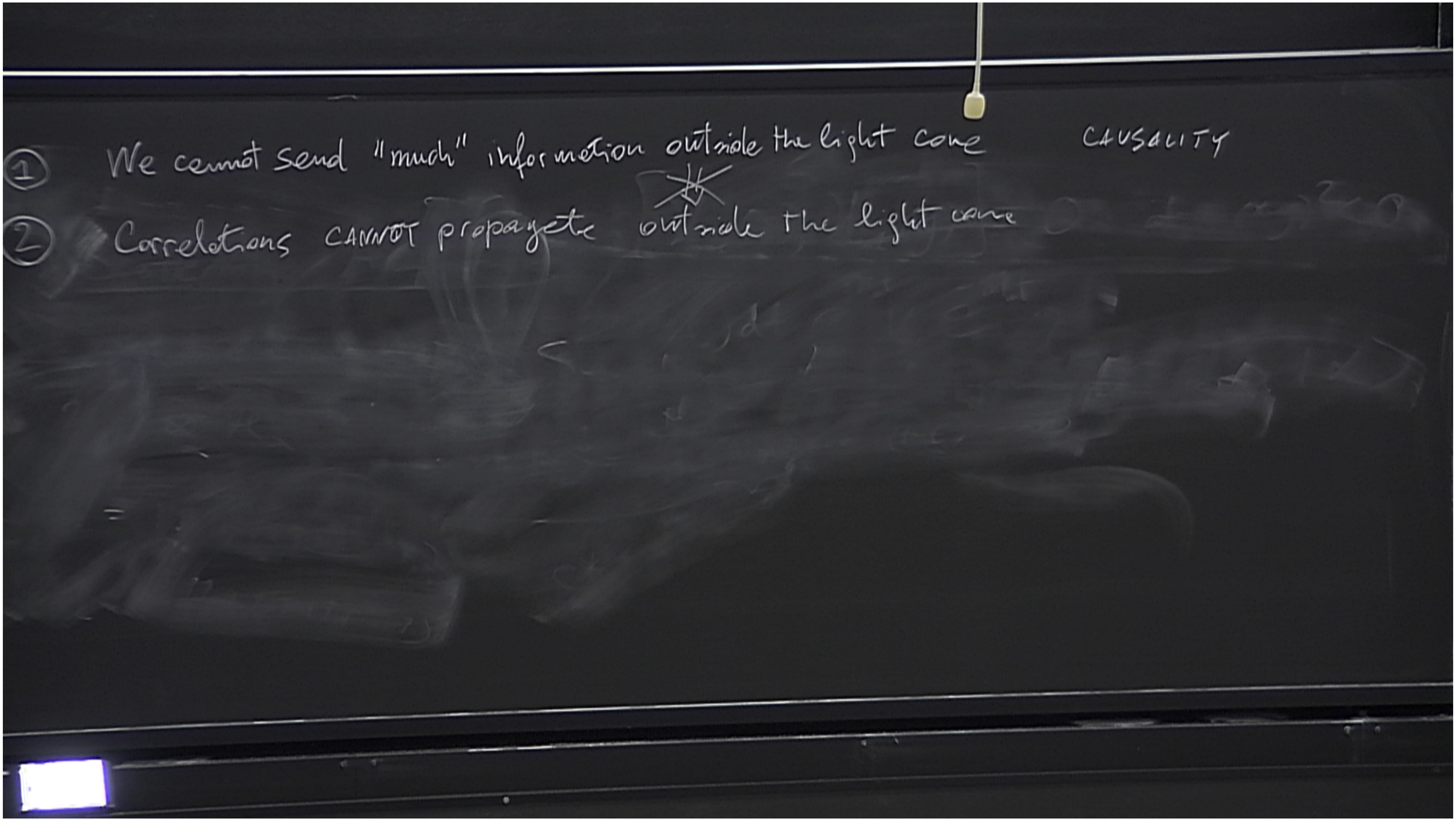
$\mathcal{H} = \bigotimes_x^\infty \mathcal{H}_x$
$\psi_1 = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle$
$\psi_2 = |\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle$
$\langle\psi_1|\psi_2\rangle = \prod_i^N \langle\uparrow|\uparrow\rangle_i = (1-\epsilon)^N \to 0$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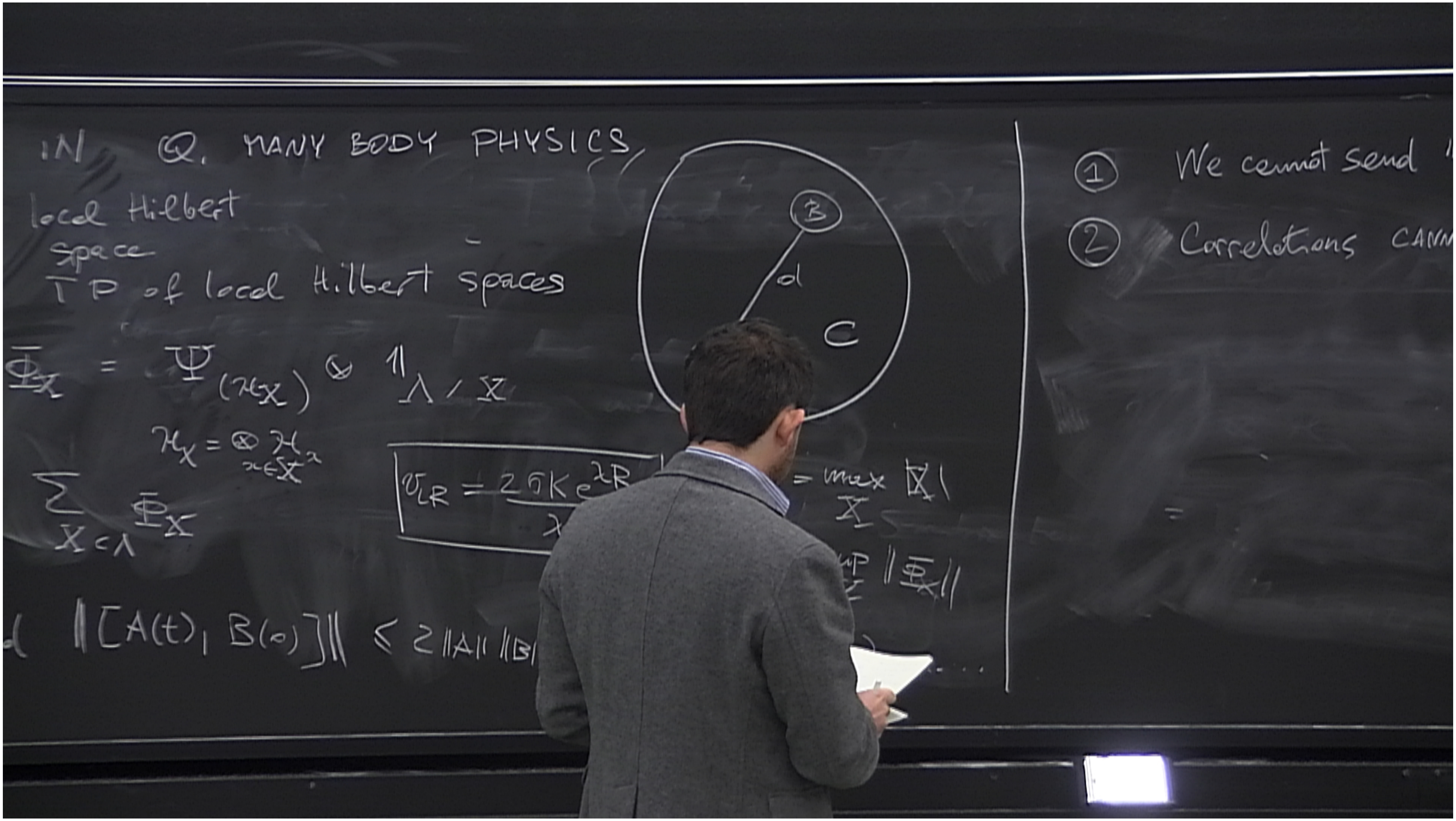
IN Q. MANY BODY PHYSICS
local Hilbert space
TP of local Hilbert spaces
C
d
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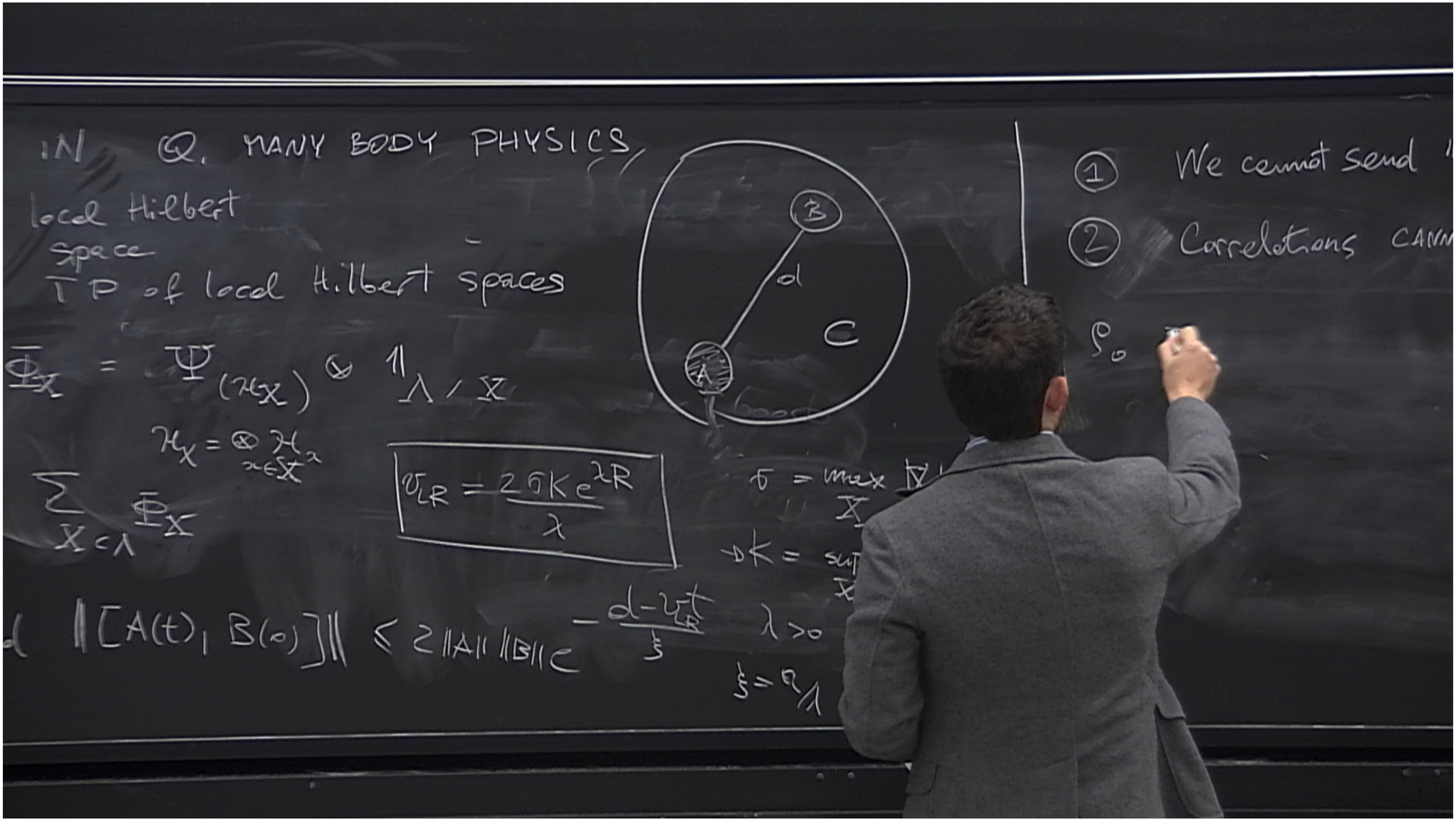
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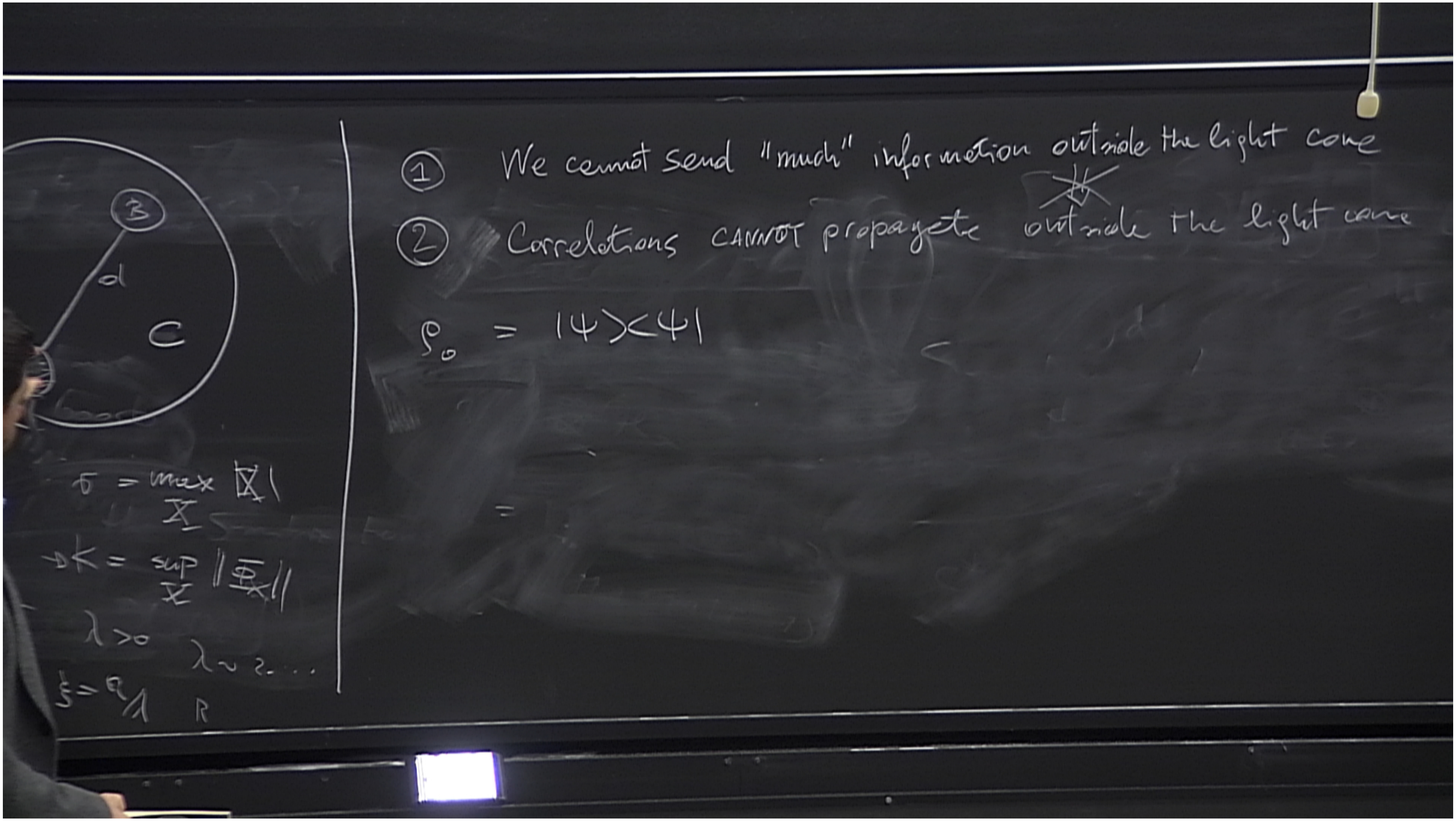
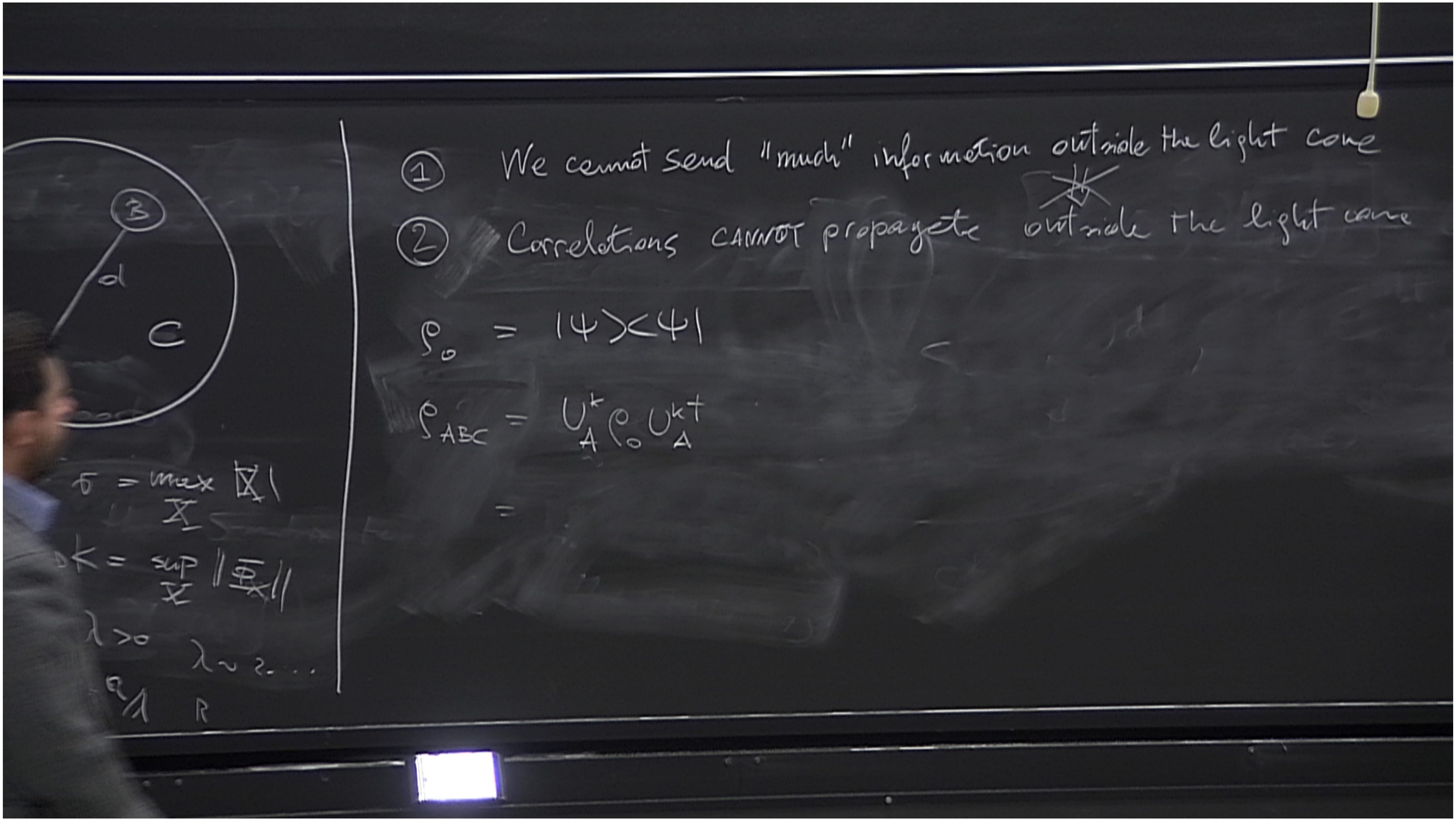
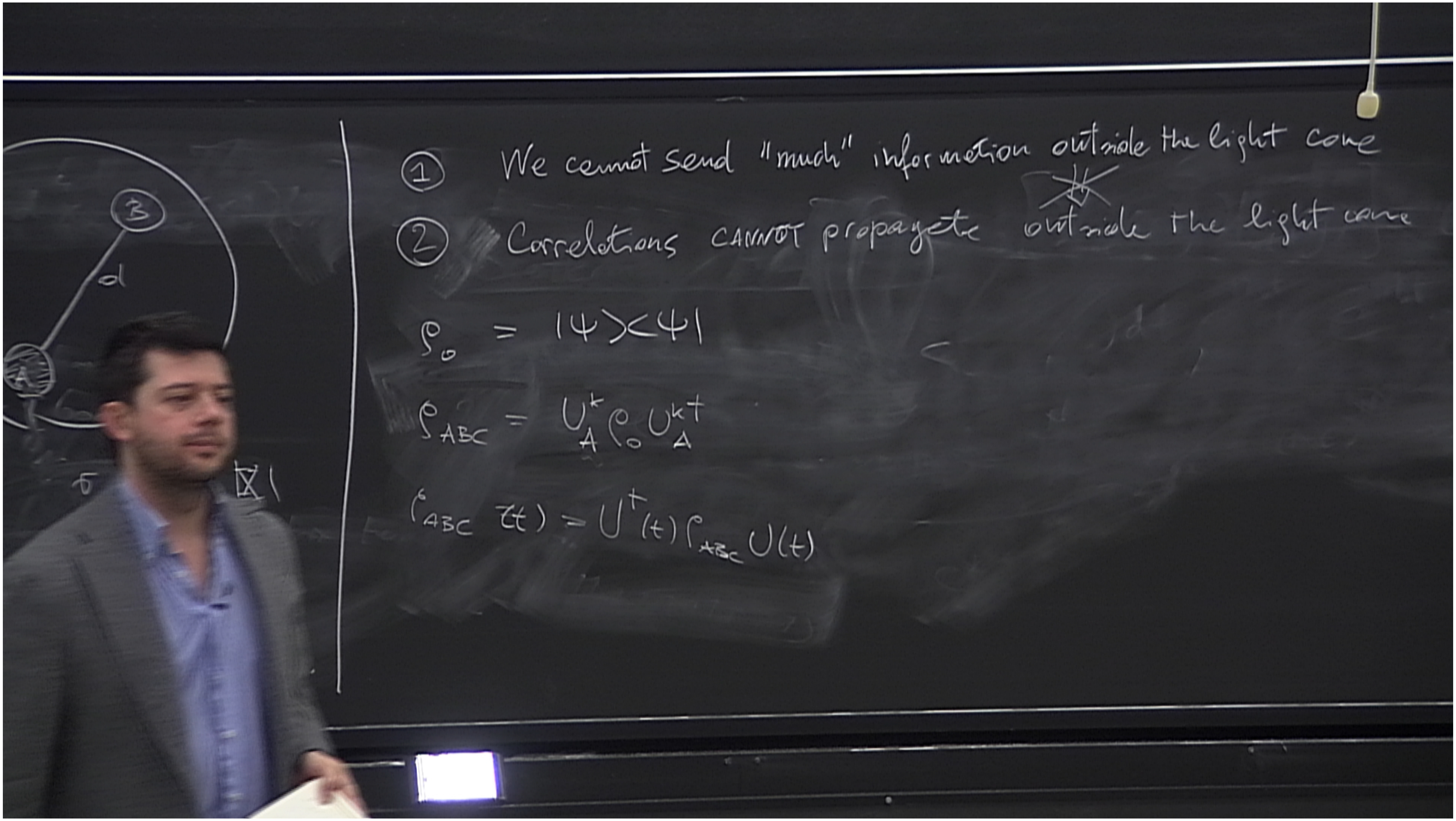
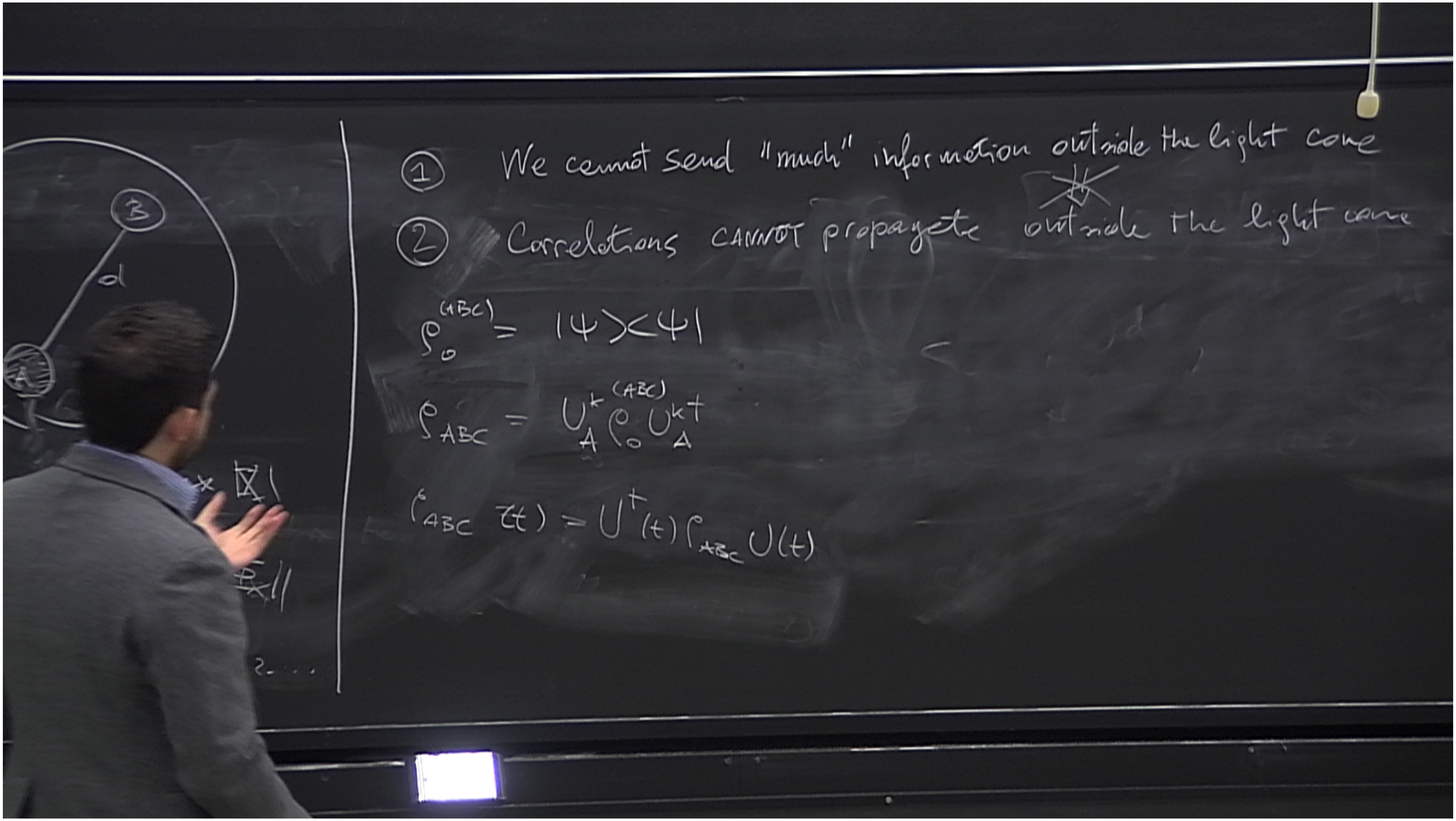
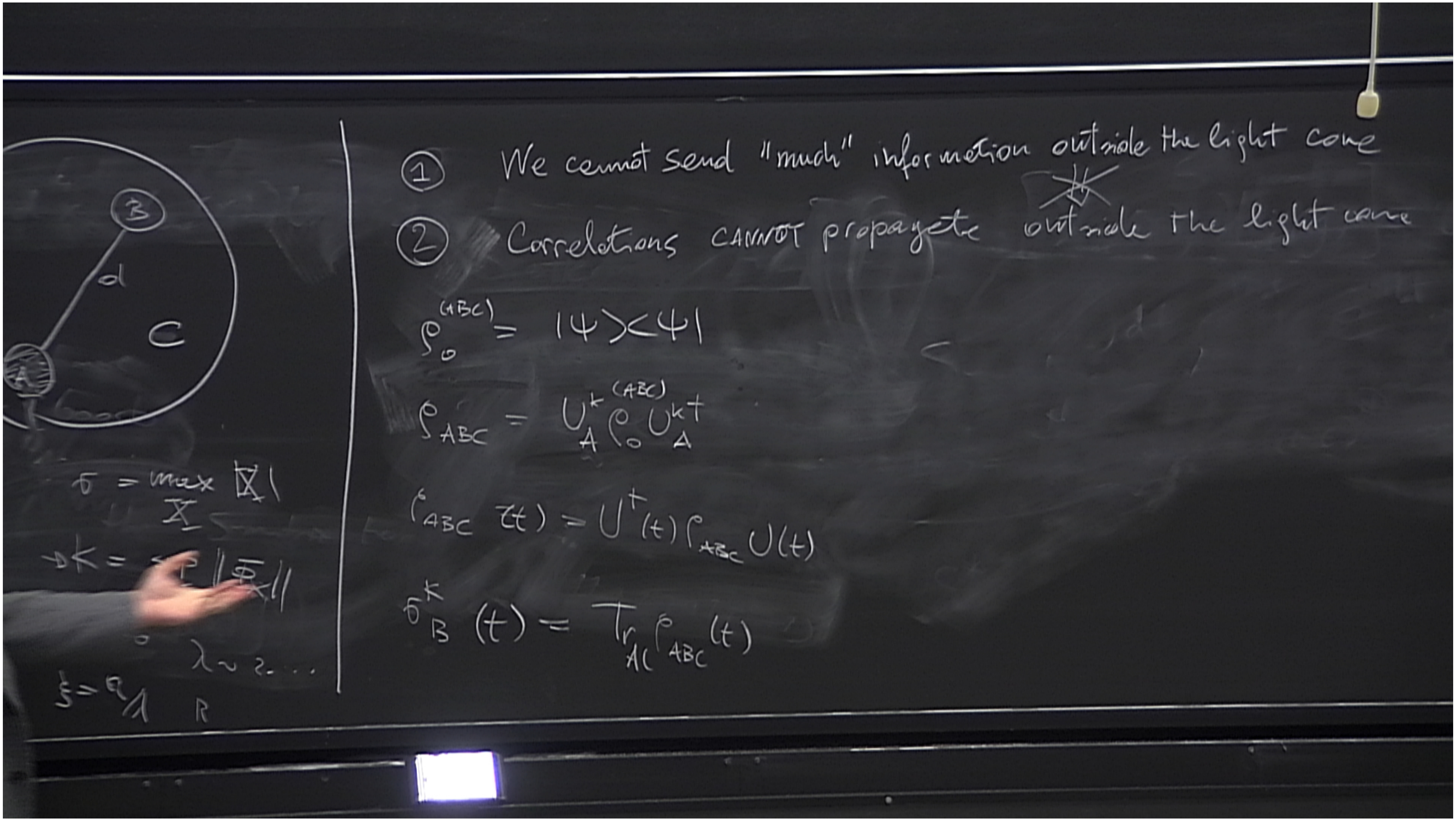

① We cannot send "much" information outside the light cone
② Correlations CANNOT propagate outside the light cone
ρ0 = |ψ><ψ|
B
d
C
λ > 0
λ ~ 2 . . .


① We cannot send "much" information outside the light cone
② Correlations CANNOT propagate outside the light cone
ρ₀ = |ψ⟩⟨ψ|
ρ_ABC = U^k_A ρ₀ U^k†_A
B
d
C
λ>0



B
d
A
① We cannot send "much" information outside the light cone
② Correlations CANNOT propagate outside the light cone
ρ0 = |ψ><ψ|
ρABC = U^k_A ρ0 U^k†_A
ρABC (t) = U†(t) ρABC U(t)


① We cannot send "much" information outside the light cone
② Correlations CANNOT propagate outside the light cone
$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$
$\rho_{ABC} = U_A^k \rho_0^{(ABC)} U_A^{k\dagger}$
$\rho_{ABC}(t) = U^\dagger(t)\rho_{ABC}U(t)$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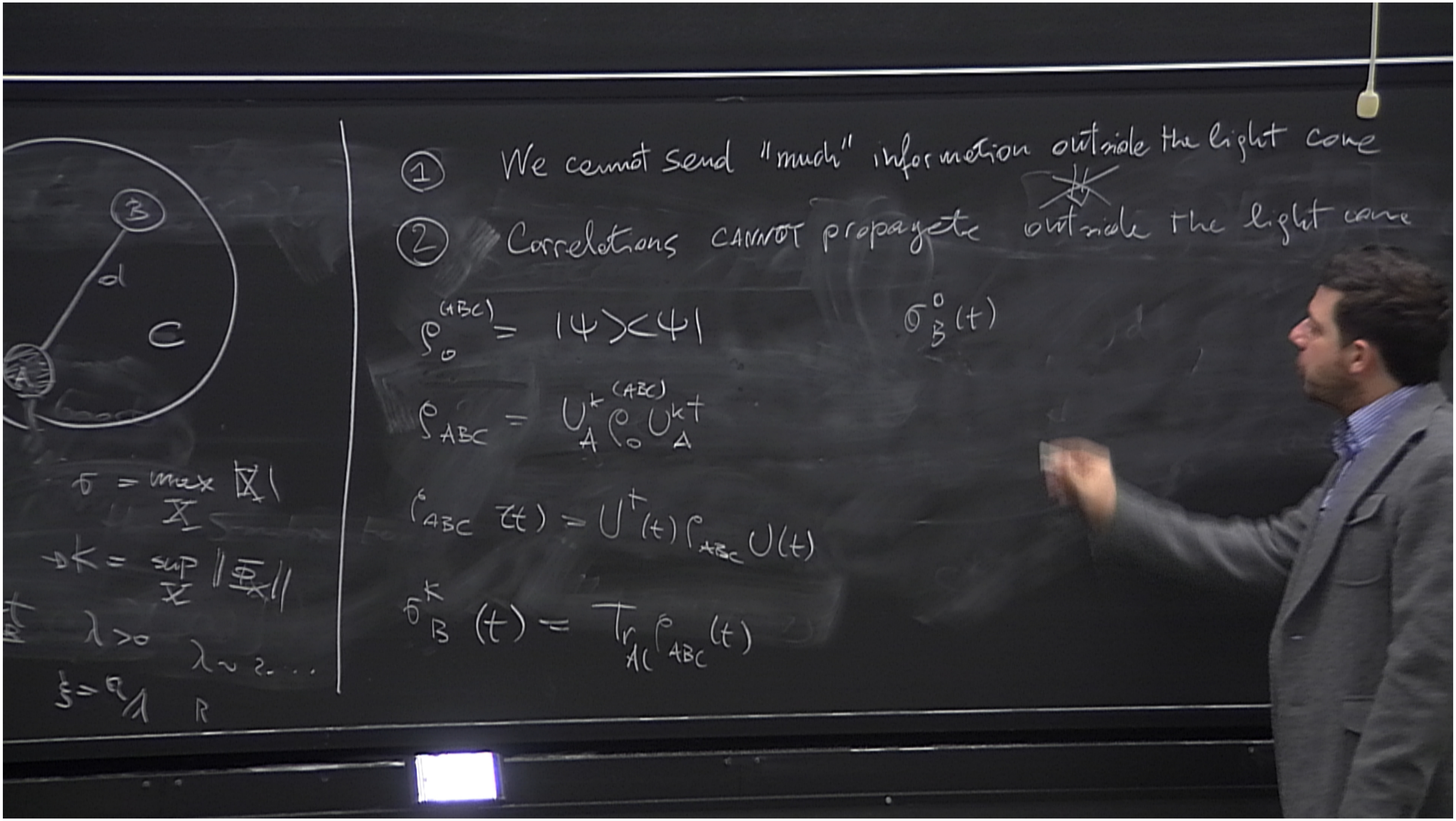
$\rho_0^{(ABC)} = |\Psi\rangle\langle\Psi|$
$\rho_{ABC} = U_A^k \rho_0^{(ABC)} U_A^{k\dagger}$
$\rho_{ABC}(t) = U^\dagger(t)\rho_{ABC}U(t)$
$\rho_B^k(t) = Tr_{AC}\rho_{ABC}(t)$
B
d
C
A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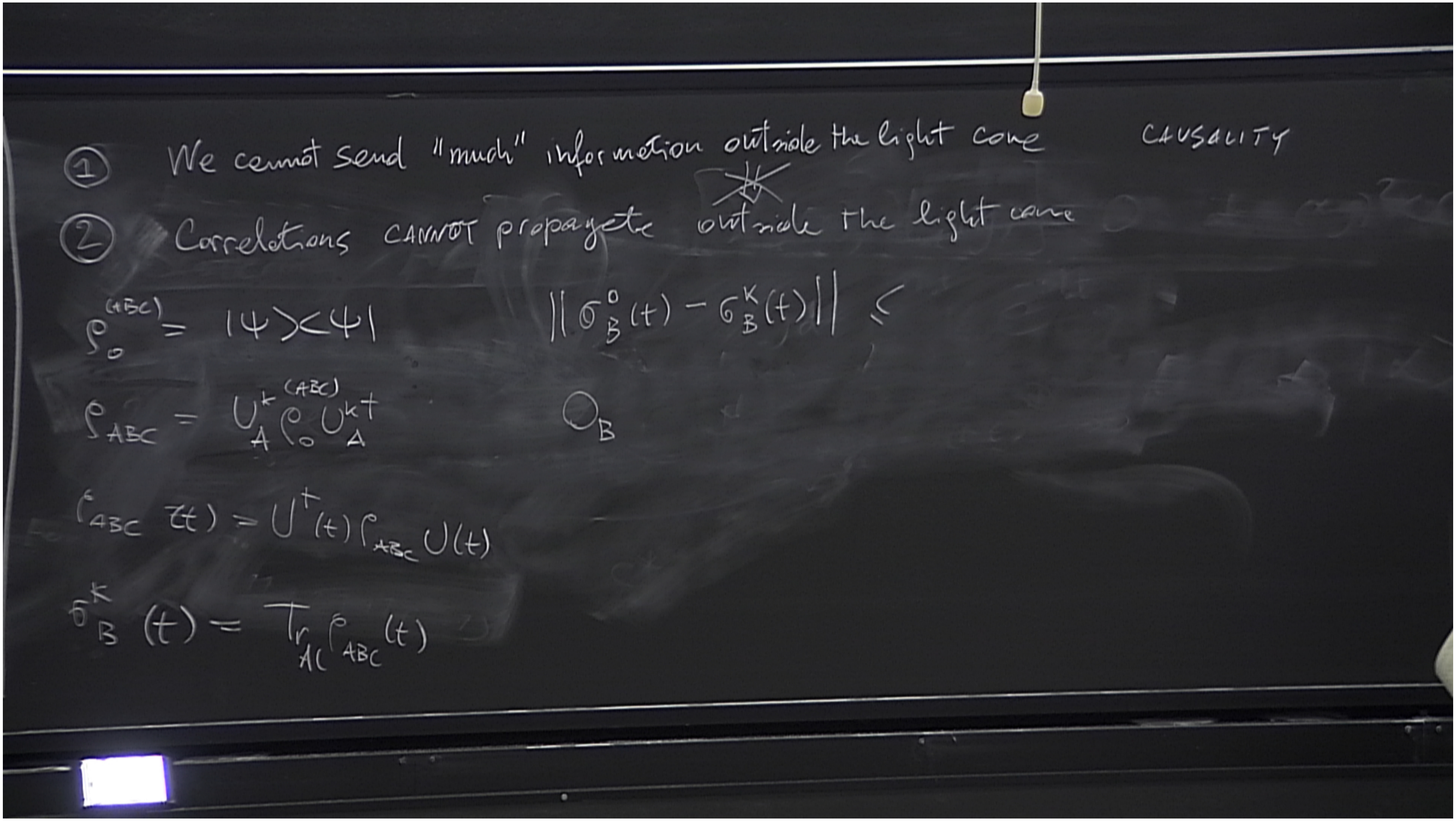
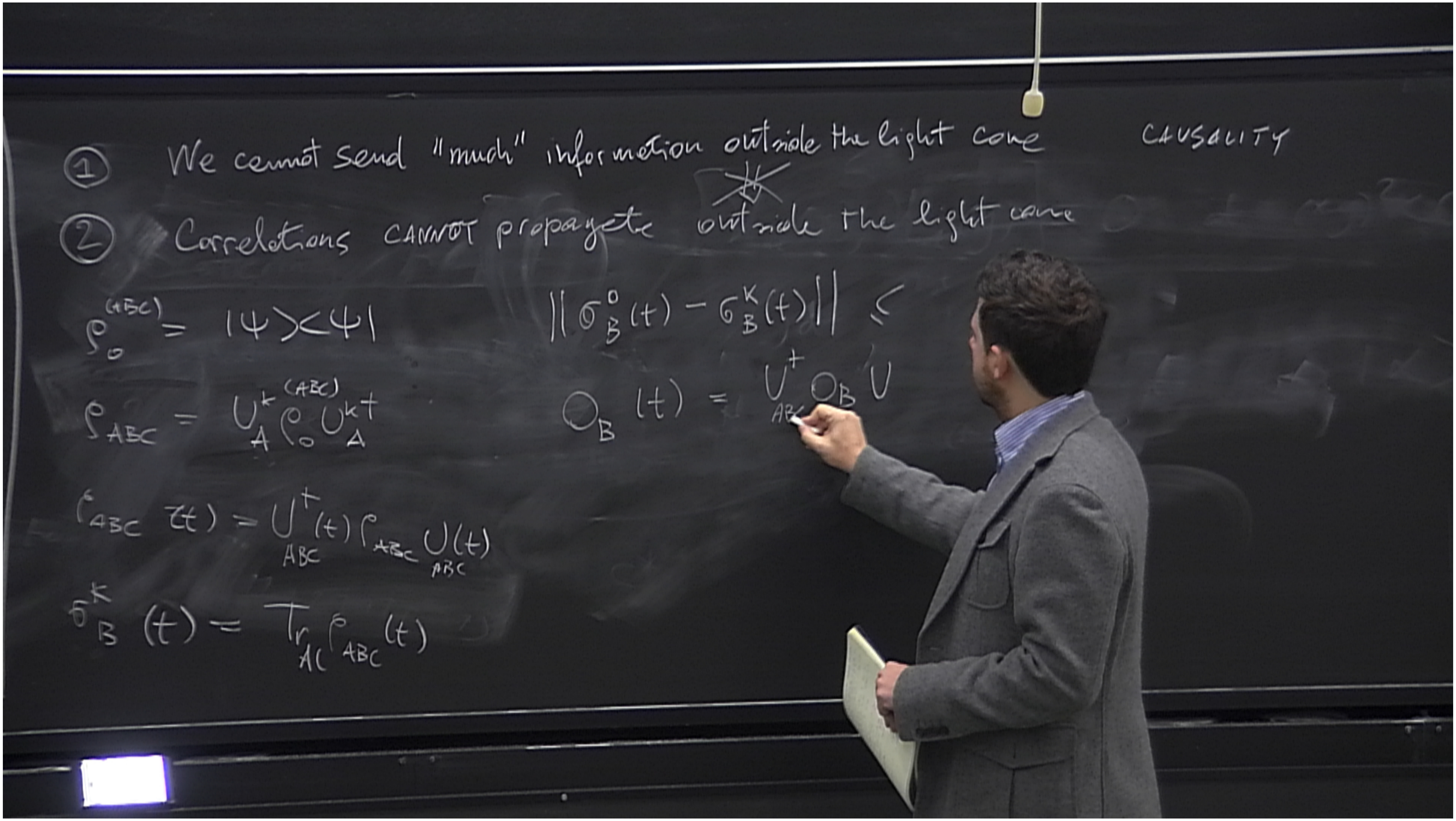
$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$
$\rho_{ABC} = U_A^k \rho_0^{(ABC)} U_A^{k\dagger}$
$\rho_{ABC}(t) = U^\dagger(t)\rho_{ABC}U(t)$
$\sigma_B^k(t) = Tr_{AC}\,\rho_{ABC}(t)$
$\sigma_B^0(t)$
B
d
C
A
$\sigma = \max_X |X|$
$K = \sup_X \|\Phi_X\|$
$\lambda > 0$


① We cannot send "much" information outside the light cone
CAUSALITY
② Correlations CANNOT propagate outside the light cone
$\rho_0^{(ABC)} = |\Psi\rangle\langle\Psi|$
$\| \sigma_B^0(t) - \sigma_B^k(t) \| \leq$
$\rho_{ABC} = U_A^k \rho_0^{(ABC)} U_A^{k\dagger}$
O_B
$\rho_{ABC}(t) = U^\dagger(t) \rho_{ABC} U(t)$
$\sigma_B^k(t) = Tr_{AC} \rho_{ABC}(t)$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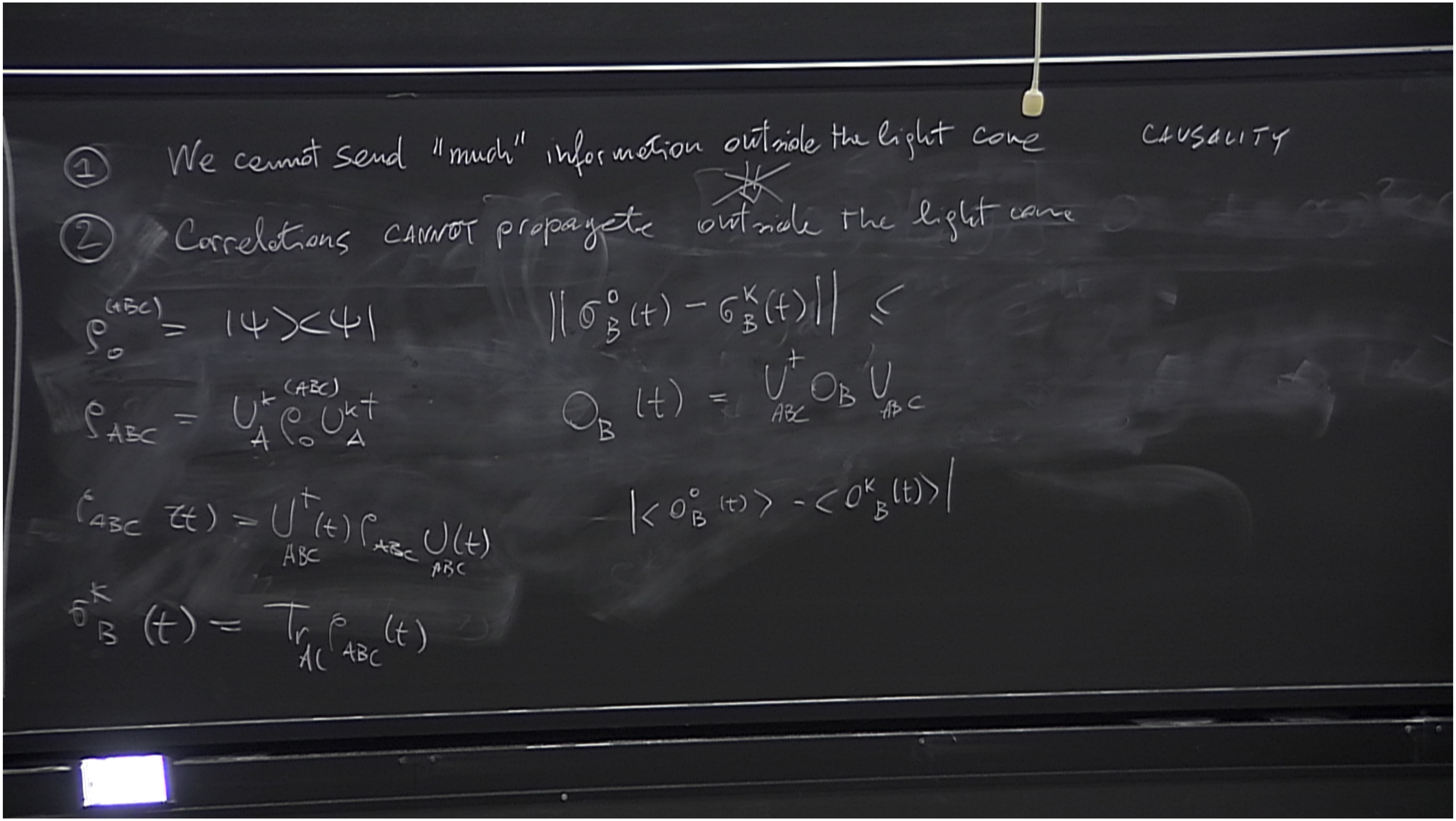
ρ_0^(ABC) = |ψ⟩⟨ψ|
ρ_ABC = U_A^k ρ_0^(ABC) U_A^k†
ρ_ABC (t) = U_ABC^†(t) ρ_ABC U_ABC(t)
σ_B^k (t) = Tr_AC ρ_ABC (t)
|| σ_B^0 (t) − σ_B^k (t) || ≤
O_B (t) = U_ABC^† O_B U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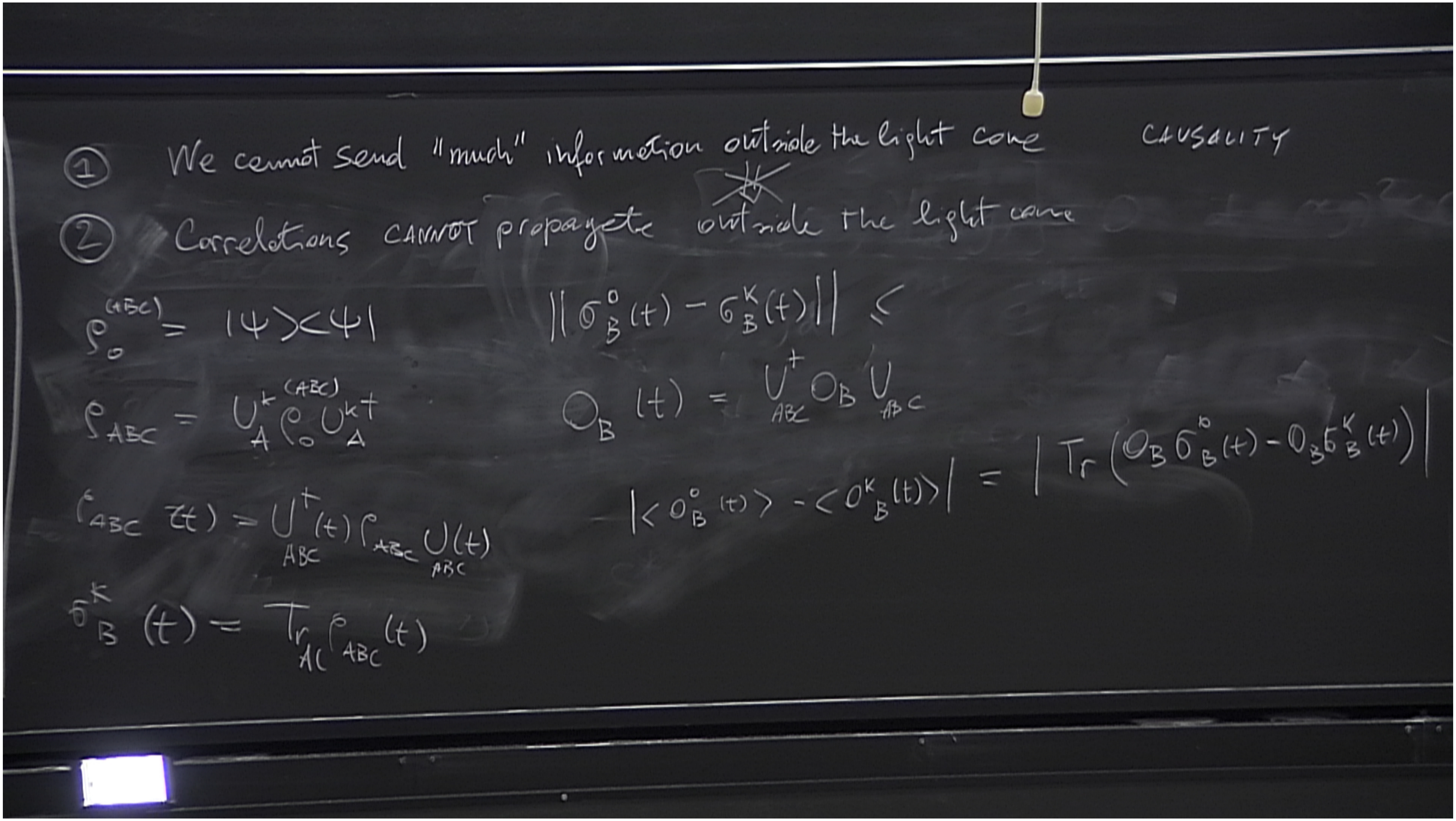
$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$
$\rho_{ABC} = U_A^k \rho_0^{(ABC)} U_A^{k\dagger}$
$\rho_{ABC}(t) = U_{ABC}^\dagger(t)\, \rho_{ABC}\, U_{ABC}(t)$
$\sigma_B^k(t) = \mathrm{Tr}_{AC}\, \rho_{ABC}(t)$
$\| \sigma_B^0(t) - \sigma_B^k(t) \| \leq$
$O_B(t) = U_{ABC}^\dagger O_B U_{ABC}$
$|\langle O_B^0(t)\rangle - \langle O_B^k(t)\rangle|$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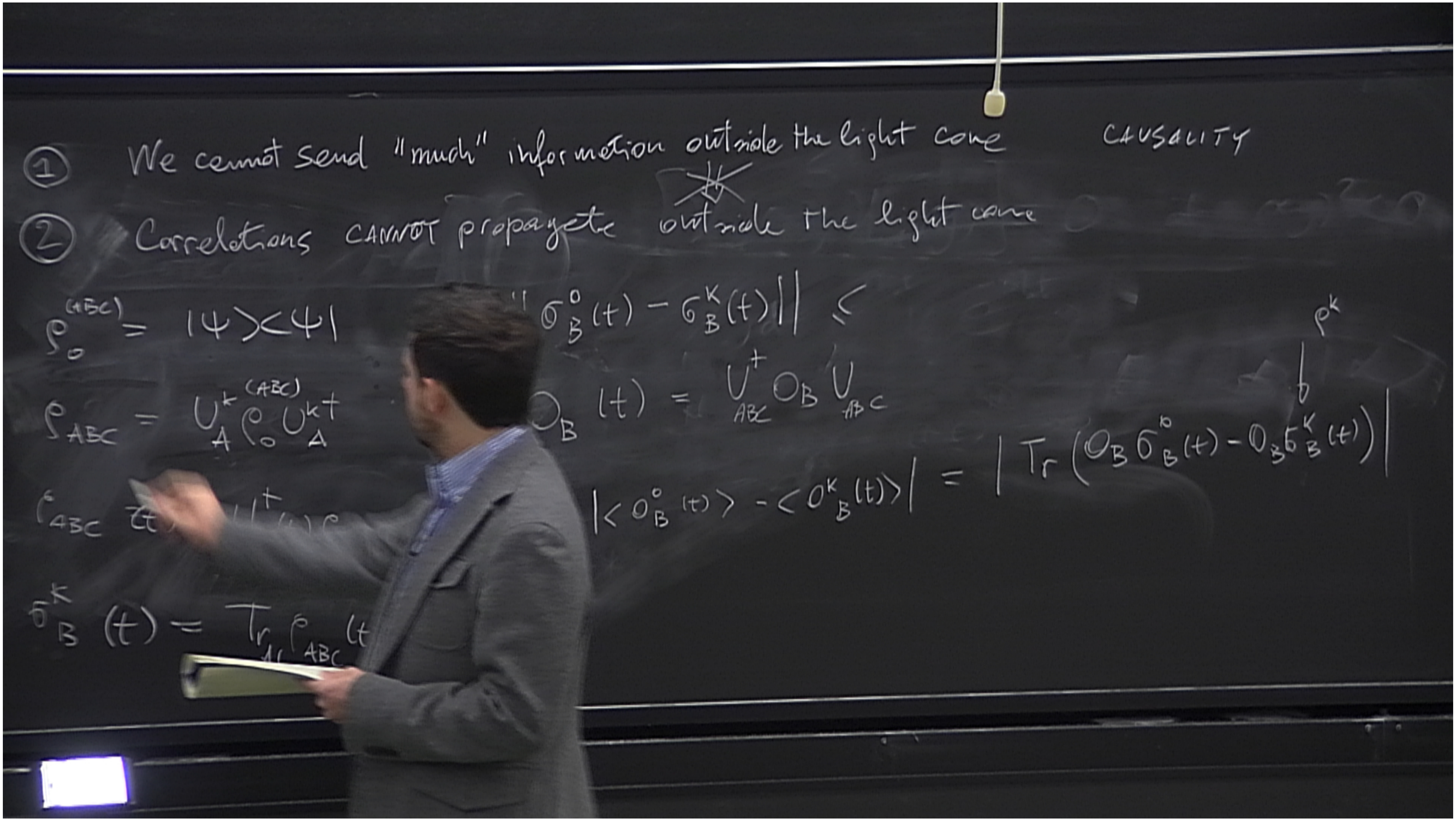
$\rho_{ABC}(t) = U^\dagger_{ABC}(t)\, \rho_{ABC}\, U_{ABC}(t)$
$\sigma_B^k(t) = Tr_{AC}\, \rho_{ABC}(t)$
$\| \sigma_B^0(t) - \sigma_B^k(t) \| \leq$
$O_B(t) = U^\dagger_{ABC} O_B U_{ABC}$
$|\langle O_B^0(t)\rangle - \langle O_B^k(t)\rangle| = |Tr(O_B \sigma_B^0(t) - O_B \sigma_B^k(t))|$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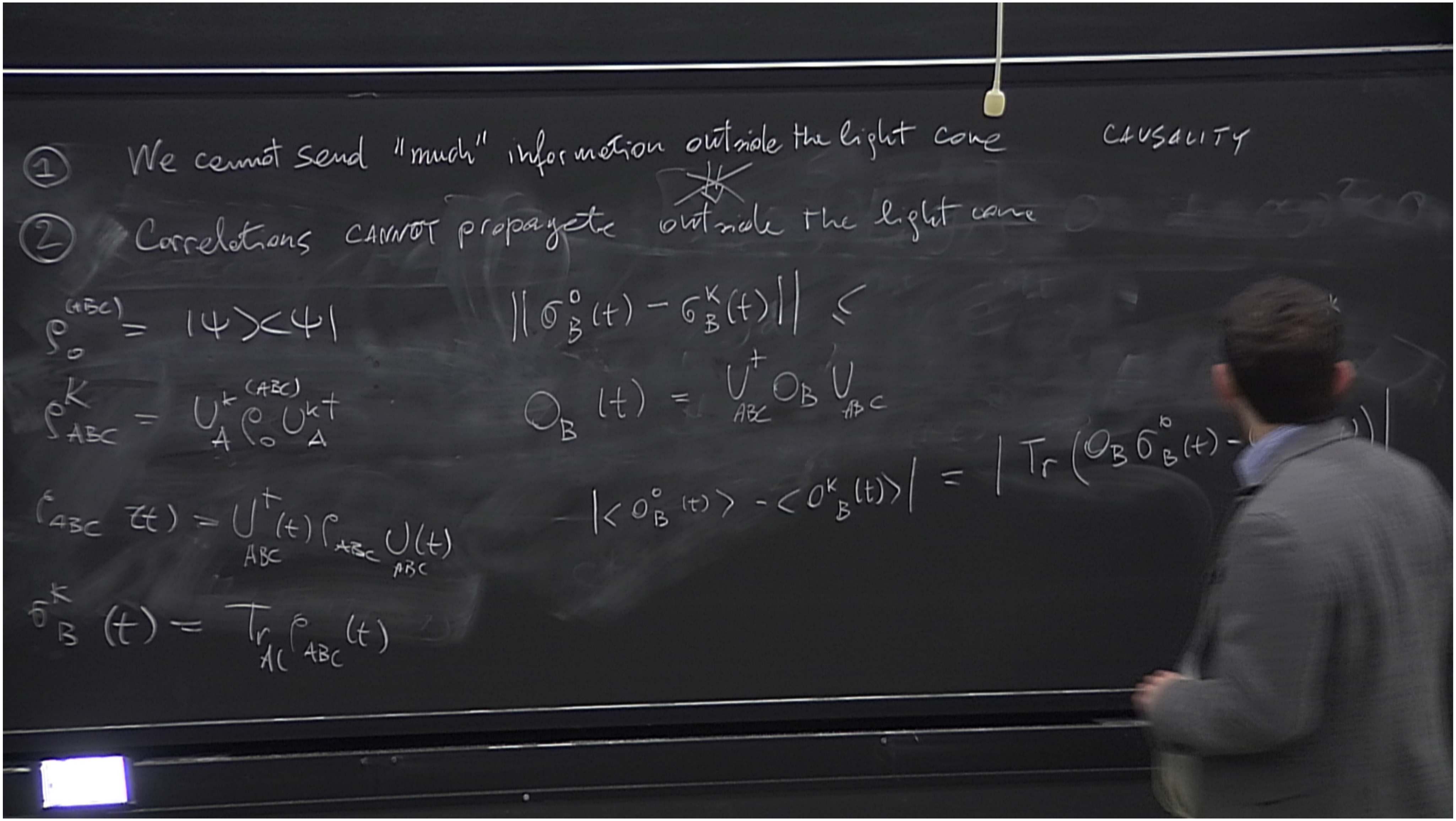
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CAUSALITY
② Correlations CANNOT propagate outside the light cone
$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$
$\rho^k_{ABC} = U^k_A \rho_0^{(ABC)} U^{k\dagger}_A$
$\rho_{ABC}(t) = U^\dagger_{ABC}(t)\,\rho_{ABC}\,U_{ABC}(t)$
$\sigma^k_B(t) = Tr_{AC}\,\rho_{ABC}(t)$
$\| \sigma^0_B(t) - \sigma^k_B(t) \| \leq$
$O_B(t) = U^\dagger_{ABC} O_B U_{ABC}$
$|\langle O^0_B(t)\rangle - \langle O^k_B(t)\rangle| = |Tr(O_B \sigma^0_B(t) -$

① We cannot send "much" information outside the light cone — CAUSALITY

② Correlations CANNOT propagate outside the light cone

$$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$$

$$\rho^K_{ABC} = U^k_A \, \rho_0^{(ABC)} \, U_A^{k\dagger}$$

$$\rho_{ABC}(t) = U^\dagger_{ABC}(t) \, \rho_{ABC} \, U_{ABC}(t)$$

$$\sigma^K_B(t) = \mathrm{Tr}_{AC} \, \rho_{ABC}(t)$$

$$\| \sigma^0_B(t) - \sigma^K_B(t) \| \leq$$

$$O_B(t) = U^\dagger_{ABC} \, O_B \, U_{ABC}$$

$$U^\dagger_{ABC} \, \rho^K \, U_{ABC}$$

$$|\langle O^0_B(t)\rangle - \langle O^K_B(t)\rangle| = |\mathrm{Tr}(O_B \sigma^0_B(t) - O_B \sigma^K_B(t))|$$

① We cannot send "much" information outside the light cone CAUSALITY

② Correlations CANNOT propagate outside the light cone

$$\rho_0^{(ABC)} = |\psi\rangle\langle\psi|$$

$$\rho^K_{ABC} = U^k_A \, \rho_0^{(ABC)} \, U^{k\dagger}_A$$

$$\rho_{ABC}(t) = U^\dagger_{ABC}(t)\, \rho_{ABC}\, U_{ABC}(t)$$

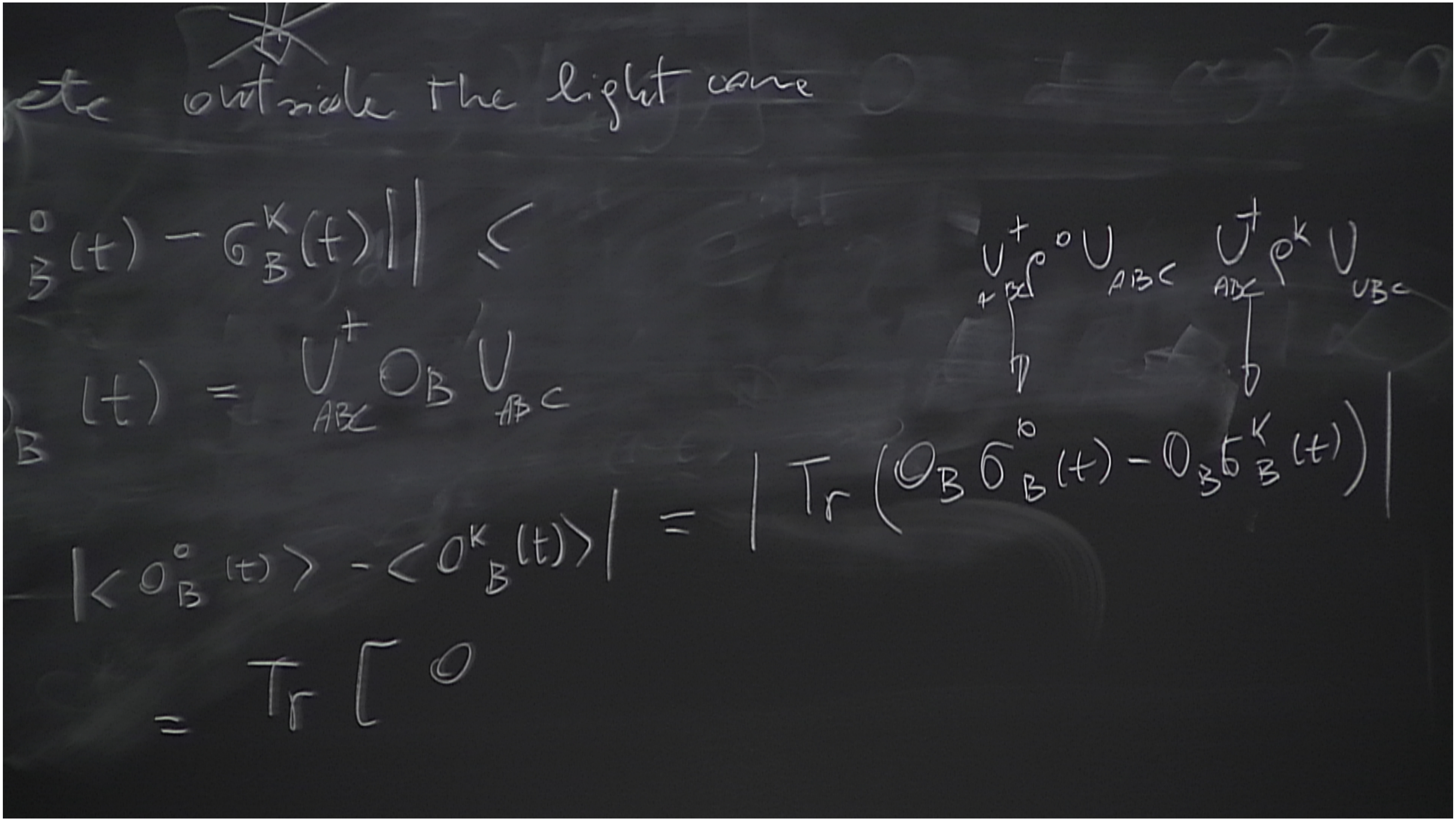
$$\sigma^K_B(t) = Tr_{AC}\, \rho_{ABC}(t)$$

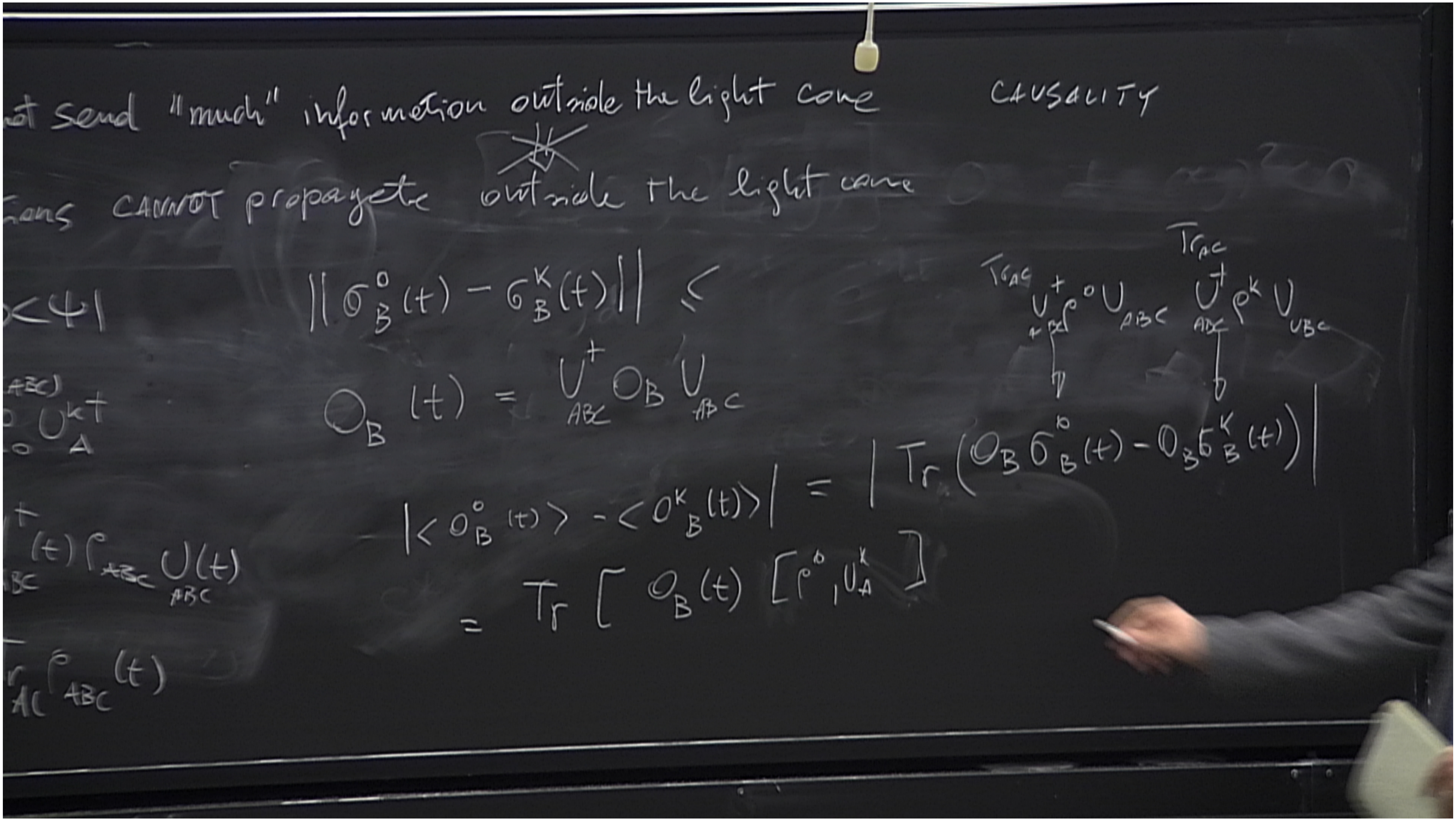
$$\| \sigma^0_B(t) - \sigma^K_B(t) \| \leq$$

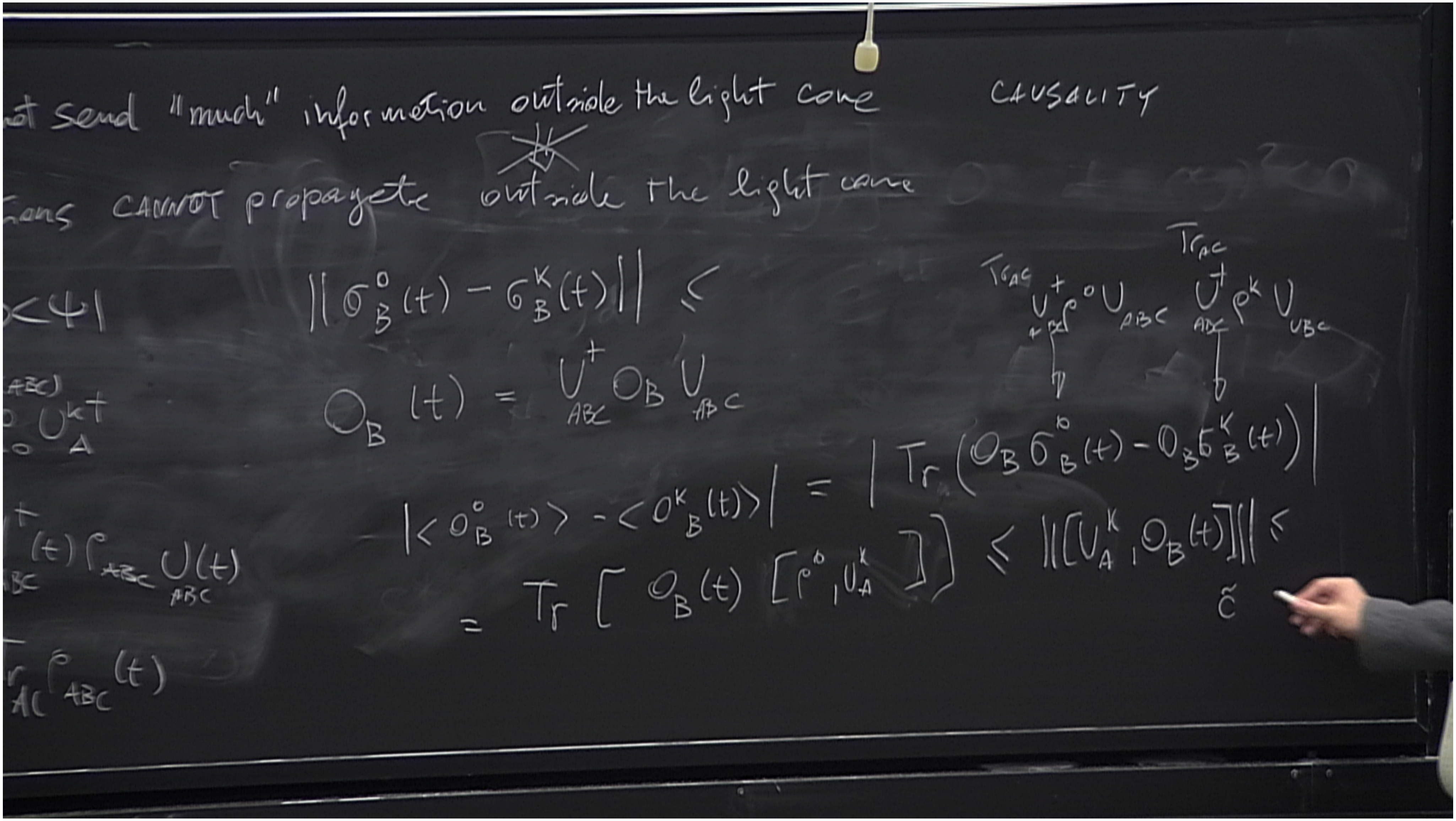
$$O_B(t) = U^\dagger_{ABC}\, O_B\, U_{ABC}$$

$$|\langle O^0_B(t)\rangle - \langle O^K_B(t)\rangle| = |Tr(O_B\sigma^0_B(t) - O_B\sigma^K_B(t))|$$

$U^\dagger_{ABC}\rho^0 U_{ABC}$ ↓ $\qquad U^\dagger_{ABC}\rho^k U_{ABC}$ ↓


etc outside the light cone
$\sigma^{0}_{B}(t) - \sigma^{K}_{B}(t) \| \le$
$U^{\dagger}_{ABC}\,\rho^{0}\,U_{ABC} \quad U^{\dagger}_{ABC}\,\rho^{K}\,U_{ABC}$
$O_{B}(t) = U^{\dagger}_{ABC}\,O_{B}\,U_{ABC}$
$|\langle O^{0}_{B}(t)\rangle - \langle O^{K}_{B}(t)\rangle| = |\mathrm{Tr}(O_{B}\,\sigma^{0}_{B}(t) - O_{B}\,\sigma^{K}_{B}(t))|$
$= \mathrm{Tr}[\,O$


not send "much" information outside the light cone
CAUSALITY
...ions CANNOT propagate outside the light cone


...ot send "much" information outside the light cone
CAUSALITY
...ions CANNOT propagate outside the light cone
$\|\sigma^0_B(t) - \sigma^K_B(t)\| \leq$
$O_B(t) = U^\dagger_{ABC} O_B U_{ABC}$
$|\langle O^0_B(t)\rangle - \langle O^K_B(t)\rangle| = |\mathrm{Tr}(O_B \sigma^0_B(t) - O_B \sigma^K_B(t))|$
$= \mathrm{Tr}[O_B(t)[\rho^0, U^K_A]] \leq \|[U^K_A, O_B(t)]\| \leq$
$\tilde{C}$

...ot send "much" information outside the light cone CAUSALITY

...ions CANNOT propagate outside the light cone

$$\| \sigma^0_B(t) - \sigma^K_B(t) \| \leq$$

$$O_B(t) = U^\dagger_{ABC}\, O_B\, U_{ABC}$$

$$\mathrm{Tr}_{AC}\, U^\dagger_{ABC} \rho^0 U_{ABC} \qquad \mathrm{Tr}_{AC}\, U^\dagger_{ABC} \rho^K U_{ABC}$$

$$|\langle O^0_B(t)\rangle - \langle O^K_B(t)\rangle| = |\mathrm{Tr}(O_B \sigma^0_B(t) - O_B \sigma^K_B(t))|$$

$$= \mathrm{Tr}[\, O_B(t)\, [\rho^0, U^K_A]\,]) \leq \|[U^K_A, O_B(t)]\| \leq \tilde{c}\, e^{-\frac{d - vt}{\xi}}$$

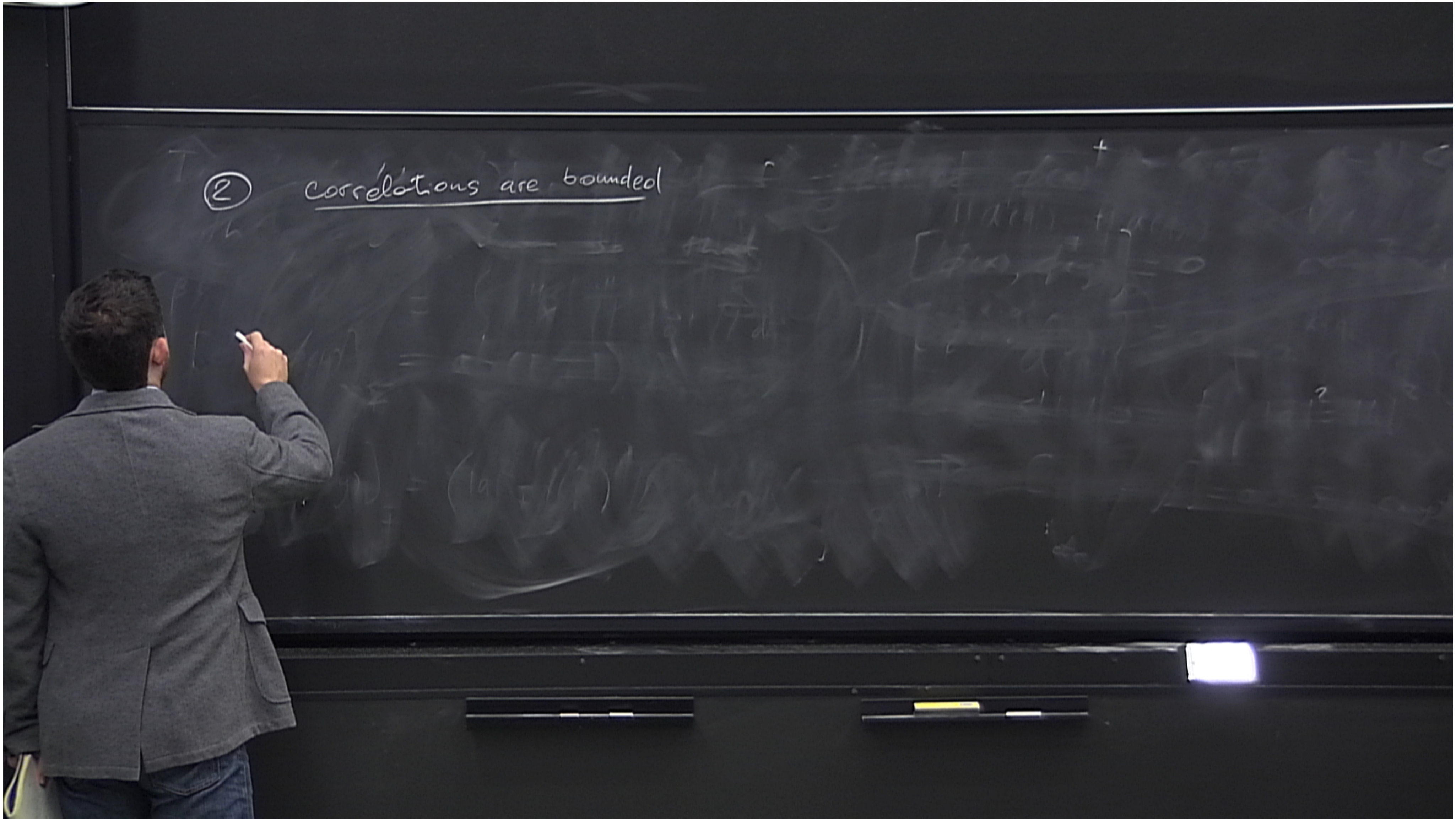
$\langle \Psi |$

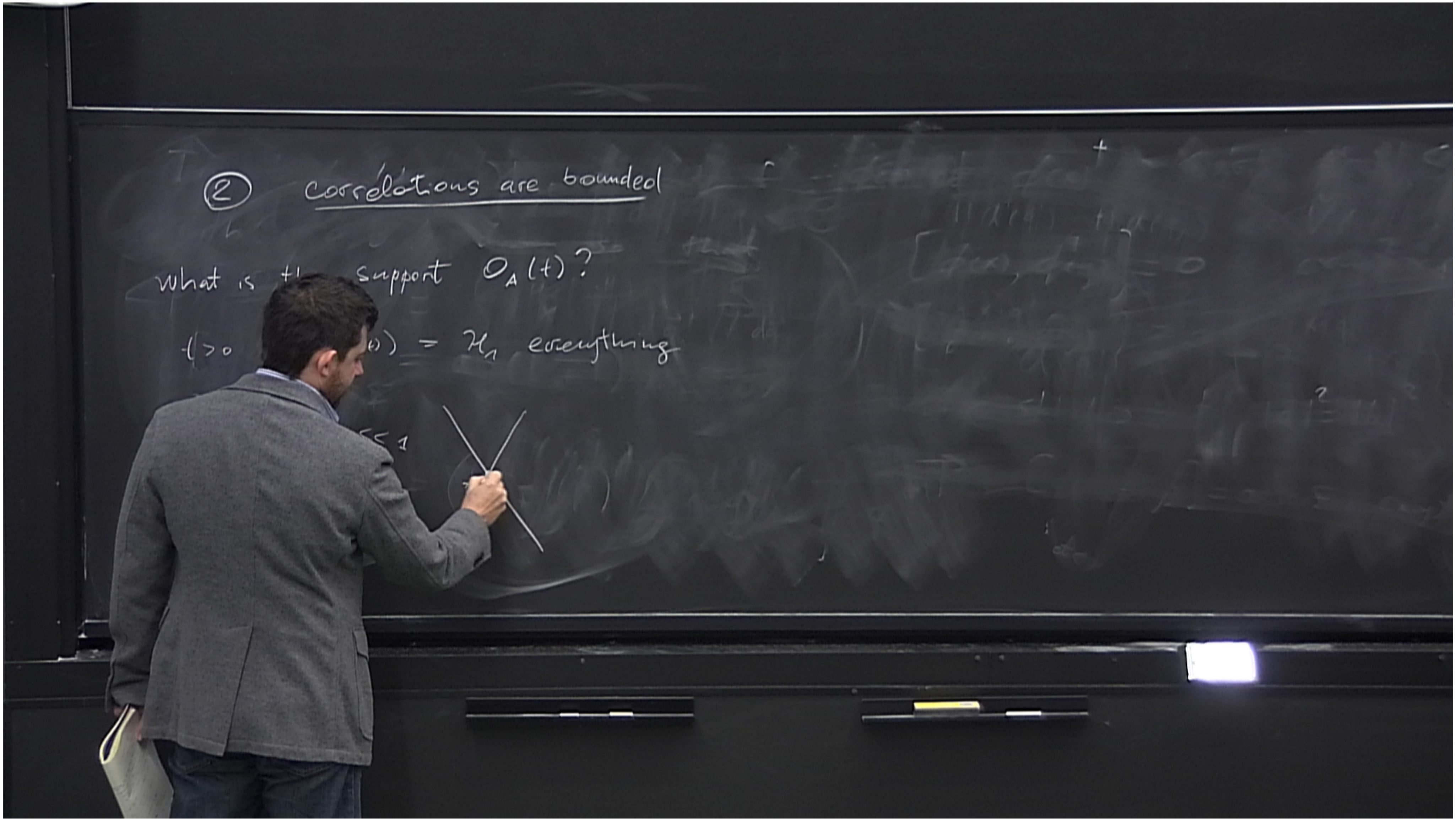
(ABC)

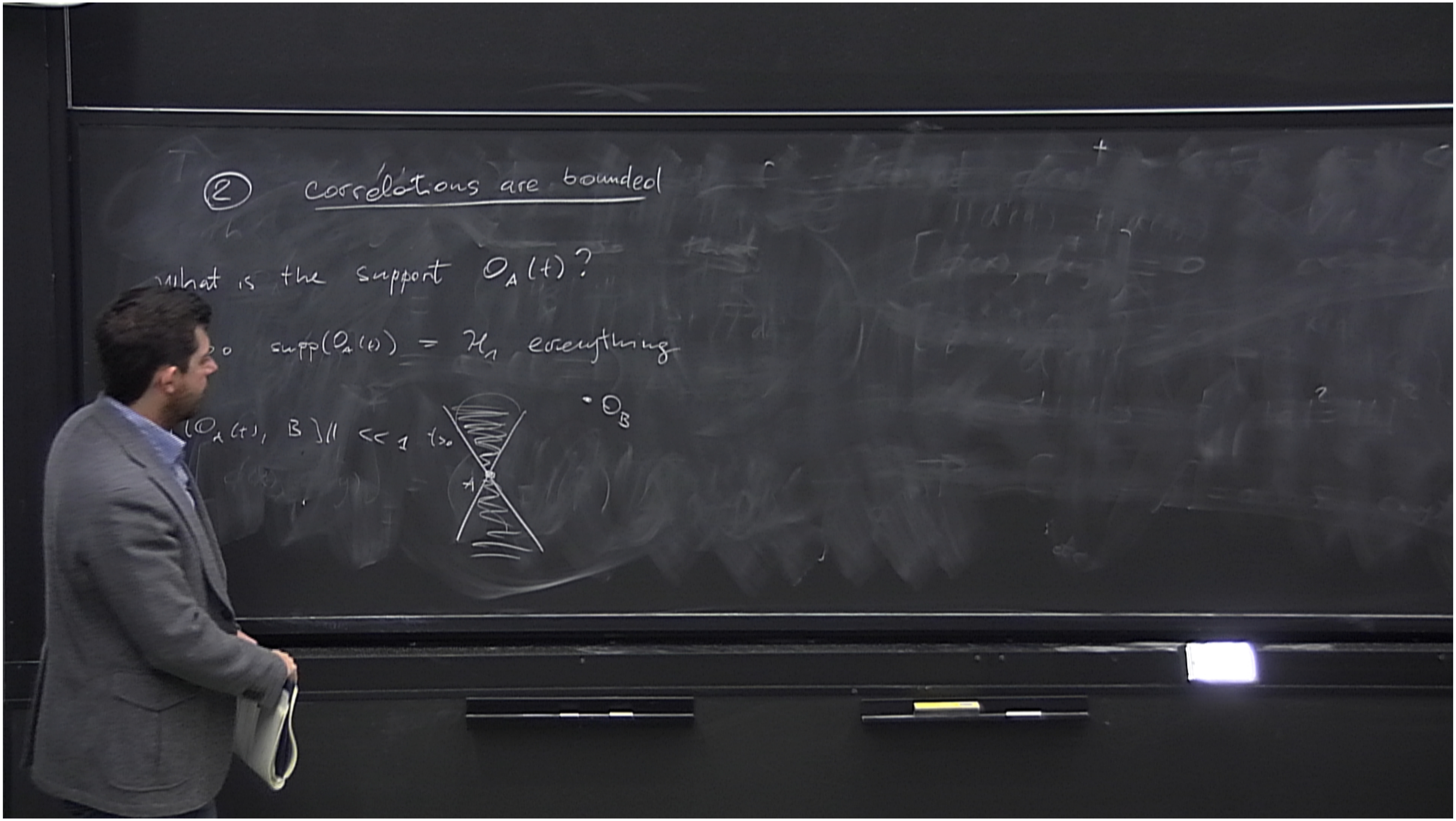
$U^{K\dagger}_A$

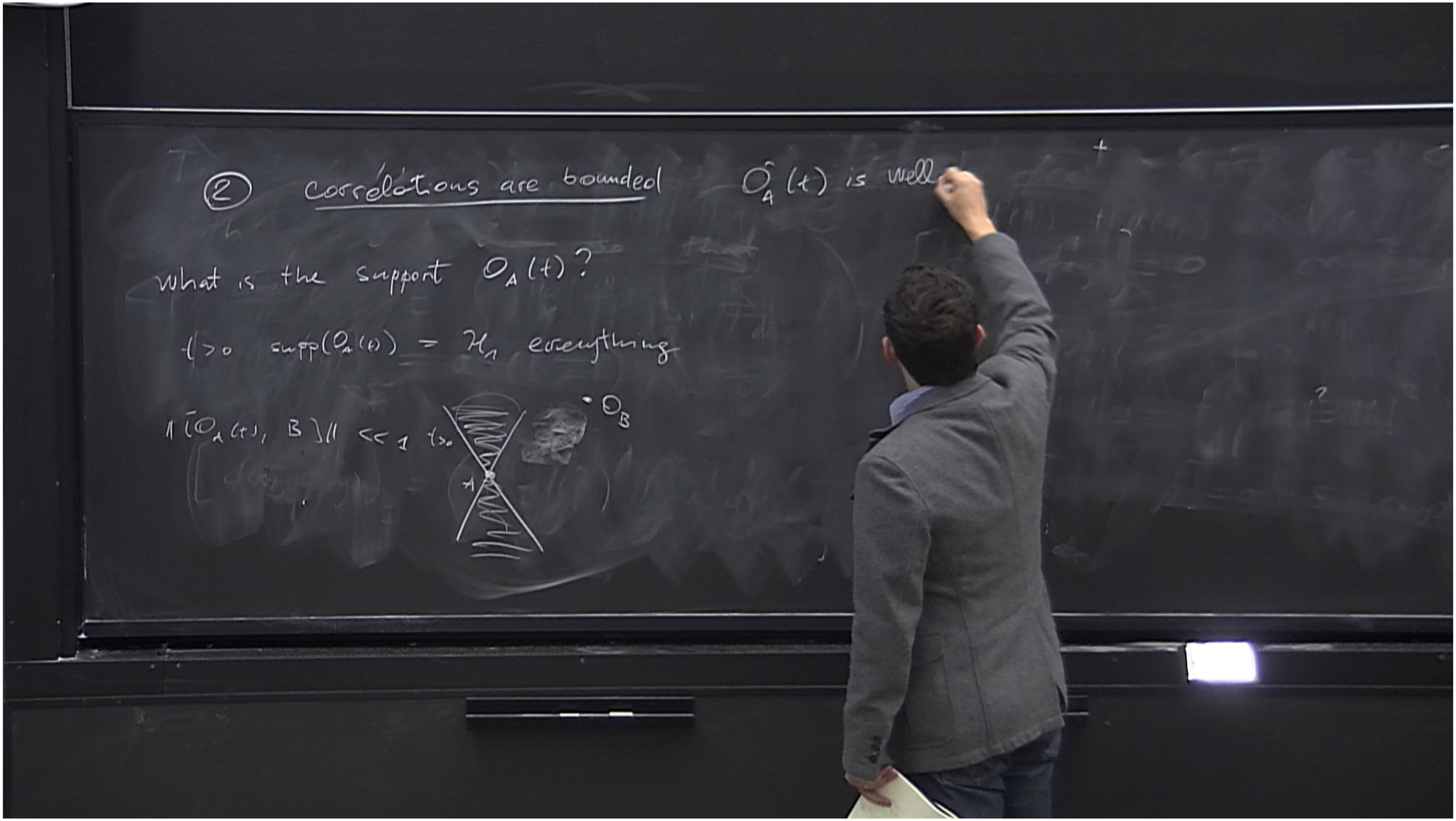
$U^\dagger_{ABC}(t)\, \rho_{ABC}\, U_{ABC}(t)$

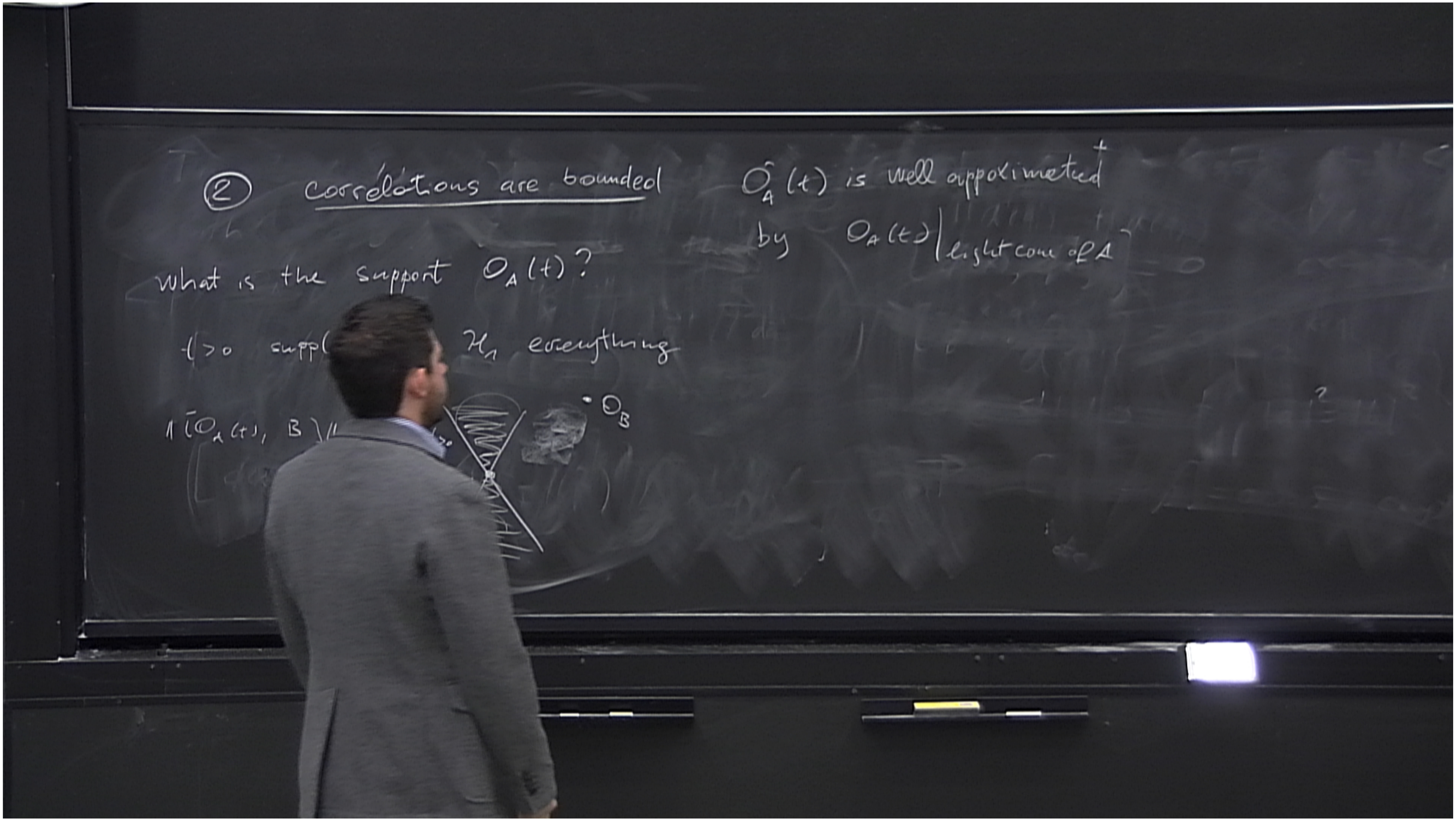
$\mathrm{Tr}_{AC}\, \rho_{ABC}(t)$

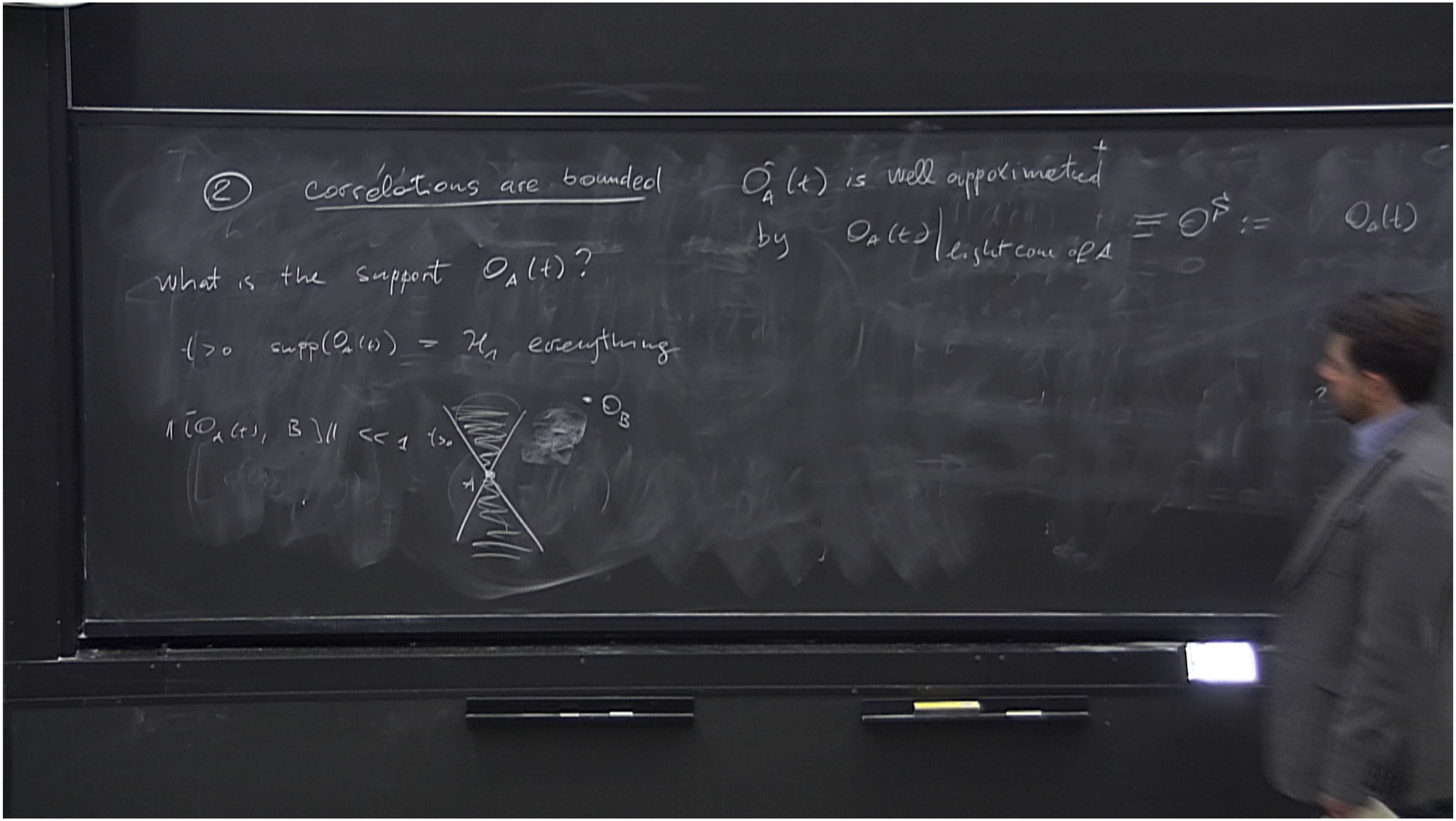

② correlations are bounded


② correlations are bounded
support $O_A(t)$?
everything


② correlations are bounded
What is the support O_A(t)?
supp(O_A(t)) = H_1 everything
O_B


② correlations are bounded
O_A(t) is well
what is the support O_A(t) ?
t>0 supp(O_A(t)) = H_1 everything
‖[O_A(t), B]‖ << 1 t>0
O_B


② correlations are bounded
what is the support O_A(t) ?
H_1 everything
O_B
O_A(t) is well approximated
by O_A(t) | light cone of A

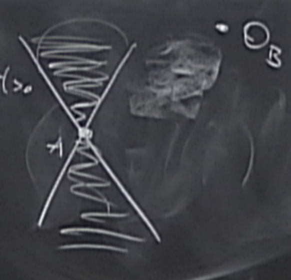

② correlations are bounded
What is the support $O_A(t)$?
$t>0$ supp$(O_A(t)) = \mathcal{H}_1$ everything
$\| [O_A(t), B] \| << 1$ $t>0$
A
O_B
$\hat{O}_A(t)$ is well approximated
by $O_A(t)|_{\text{light cone of } A} \equiv O^S :=$ $O_A(t)$

② correlations are bounded

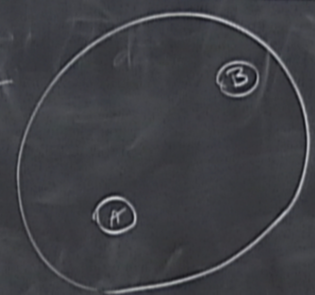
What is the support $O_A(t)$?

$t>0$ $\quad \mathrm{supp}(O_A(t)) = \mathcal{H}_1$ everything

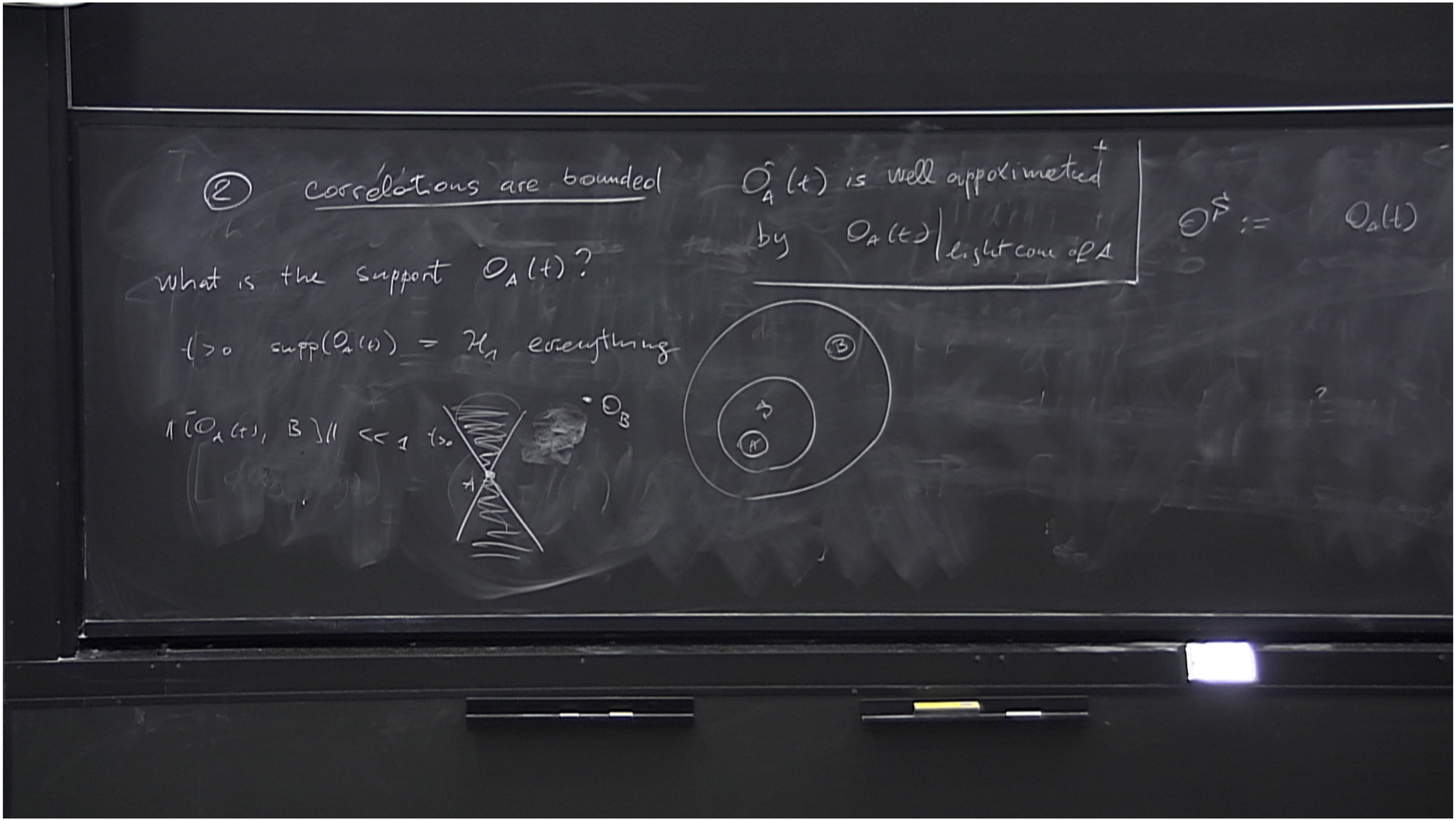
$\| [O_A(t), B] \| \ll 1 \quad t>0$

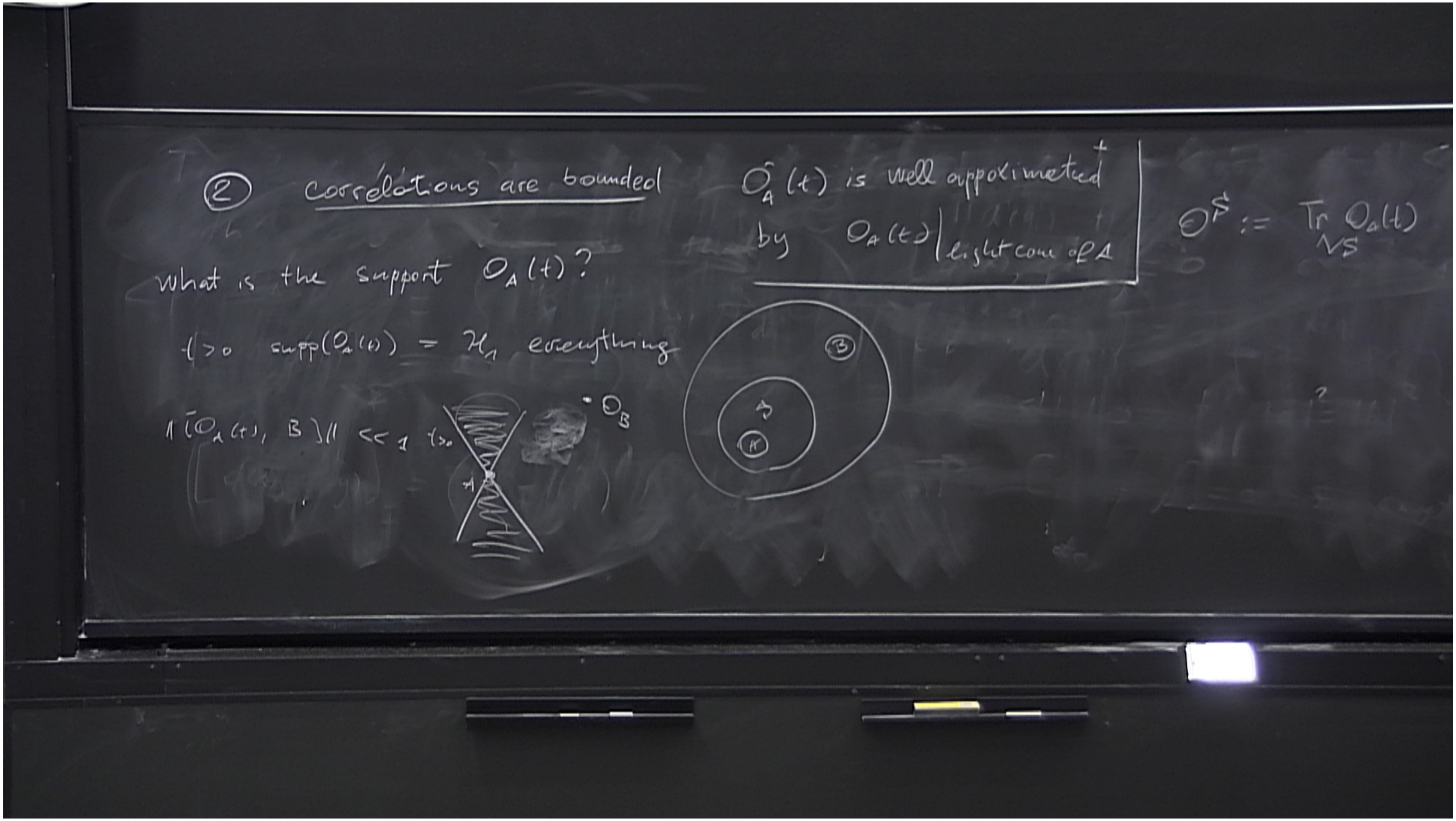


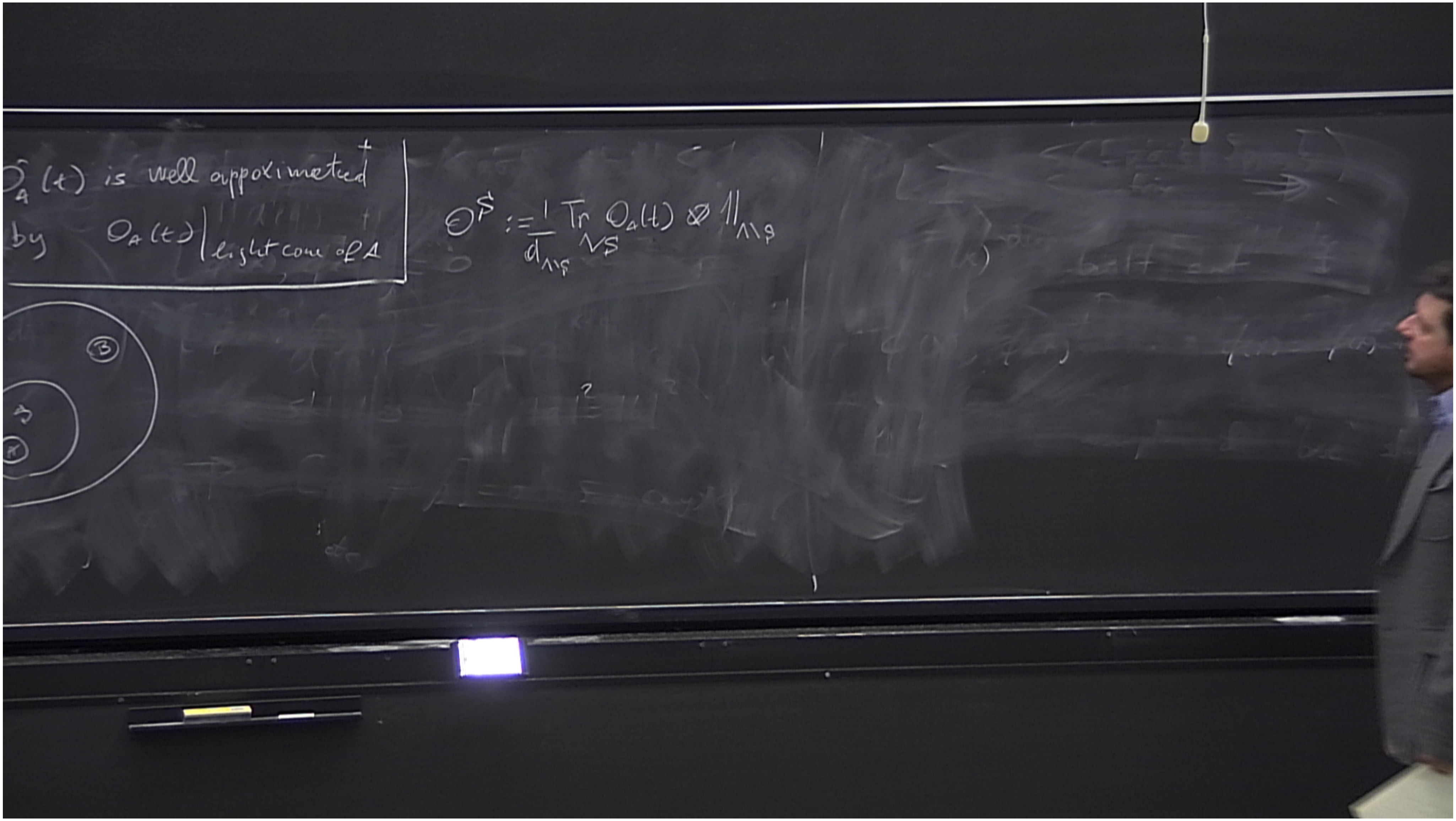
$\hat{O}_A(t)$ is well approximated by $O_A(t)\big|_{\text{light cone of } A}$

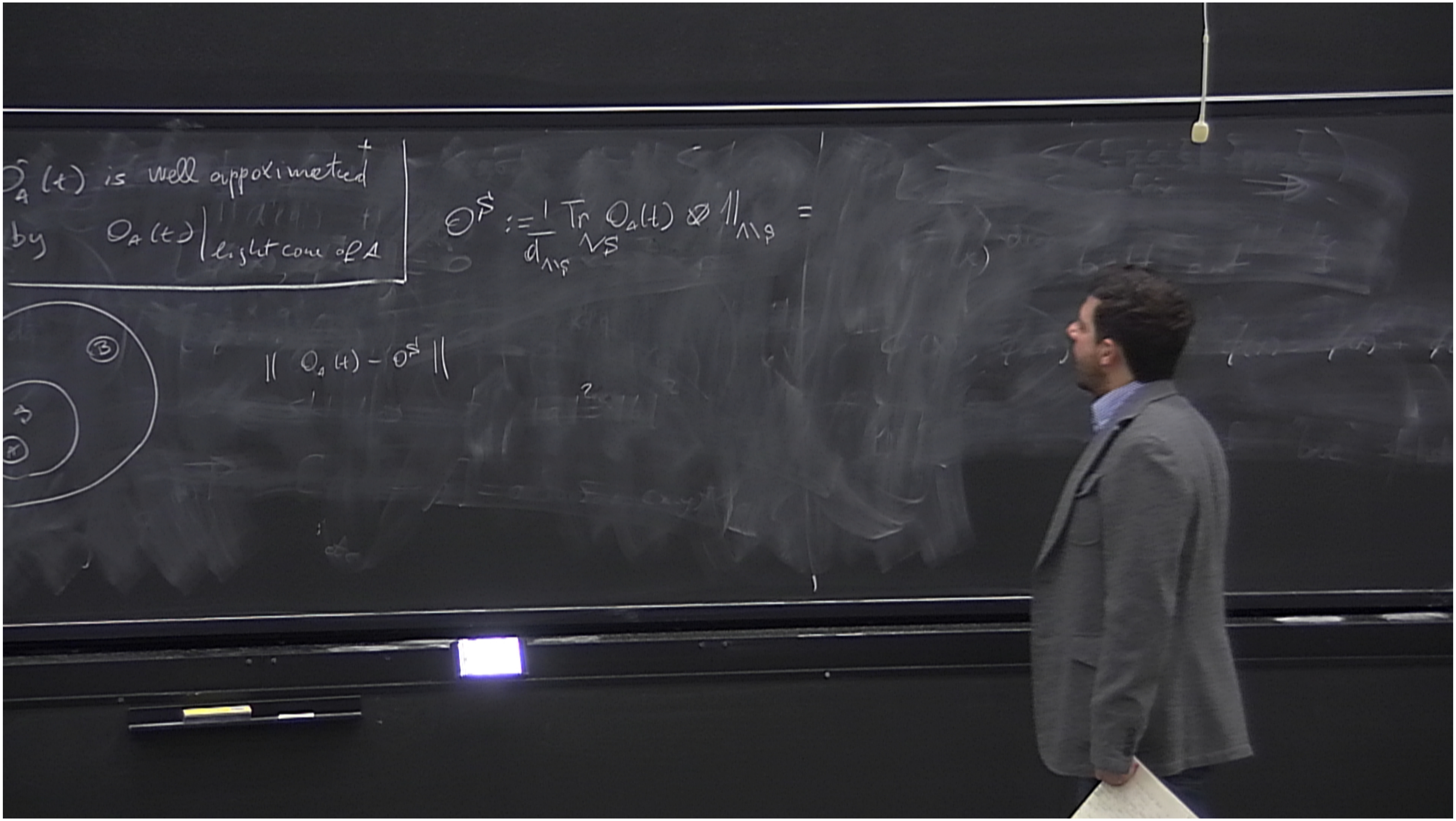


$O^S := \quad O_A(t)$

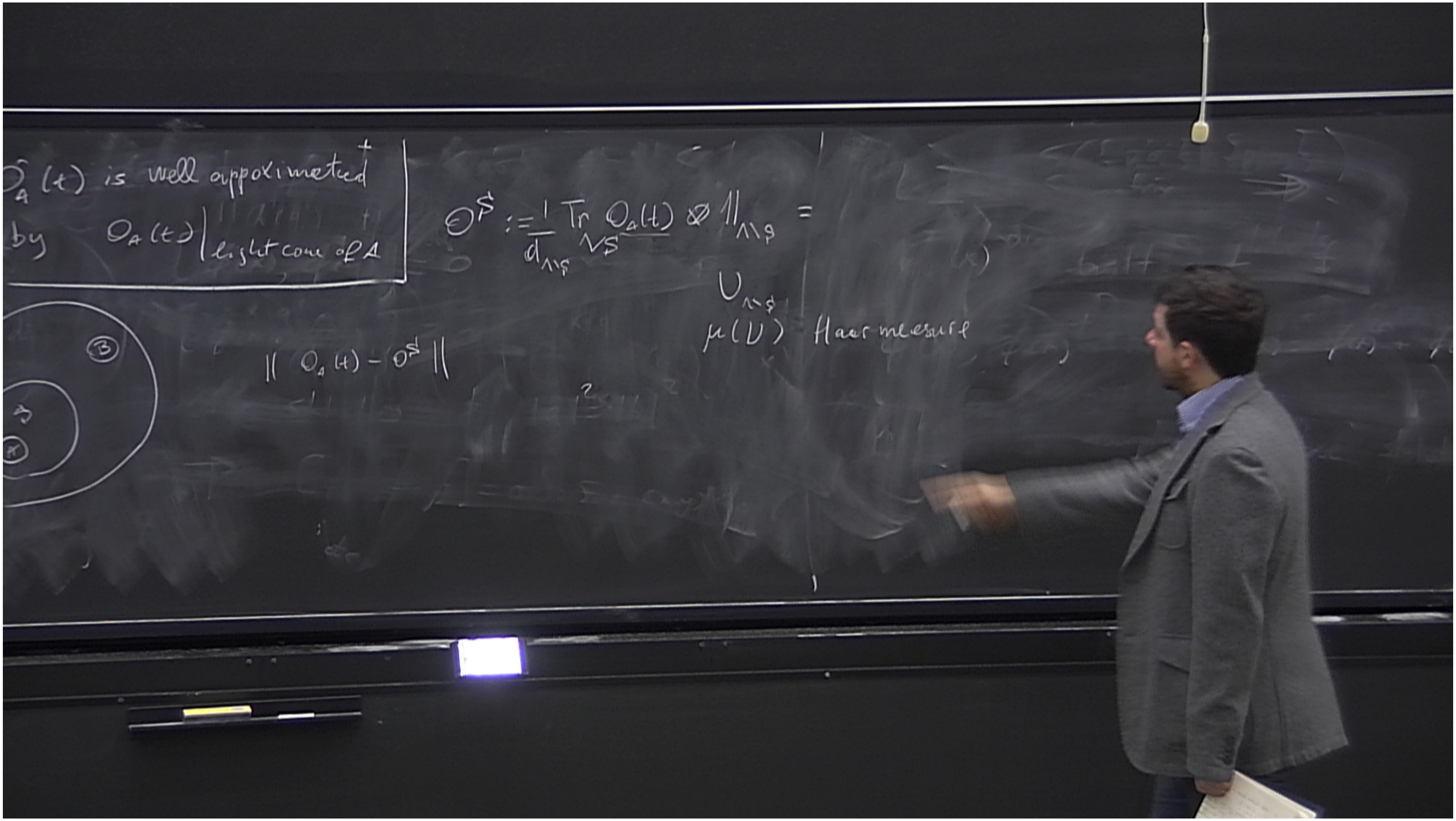

② correlations are bounded
What is the support O_A(t)?
t>0 supp(O_A(t)) = H_1 everything
‖[O_A(t), B]‖ << 1 t>0
O_B
Ô_A(t) is well approximated
by O_A(t)| light cone of A
O^S := O_A(t)

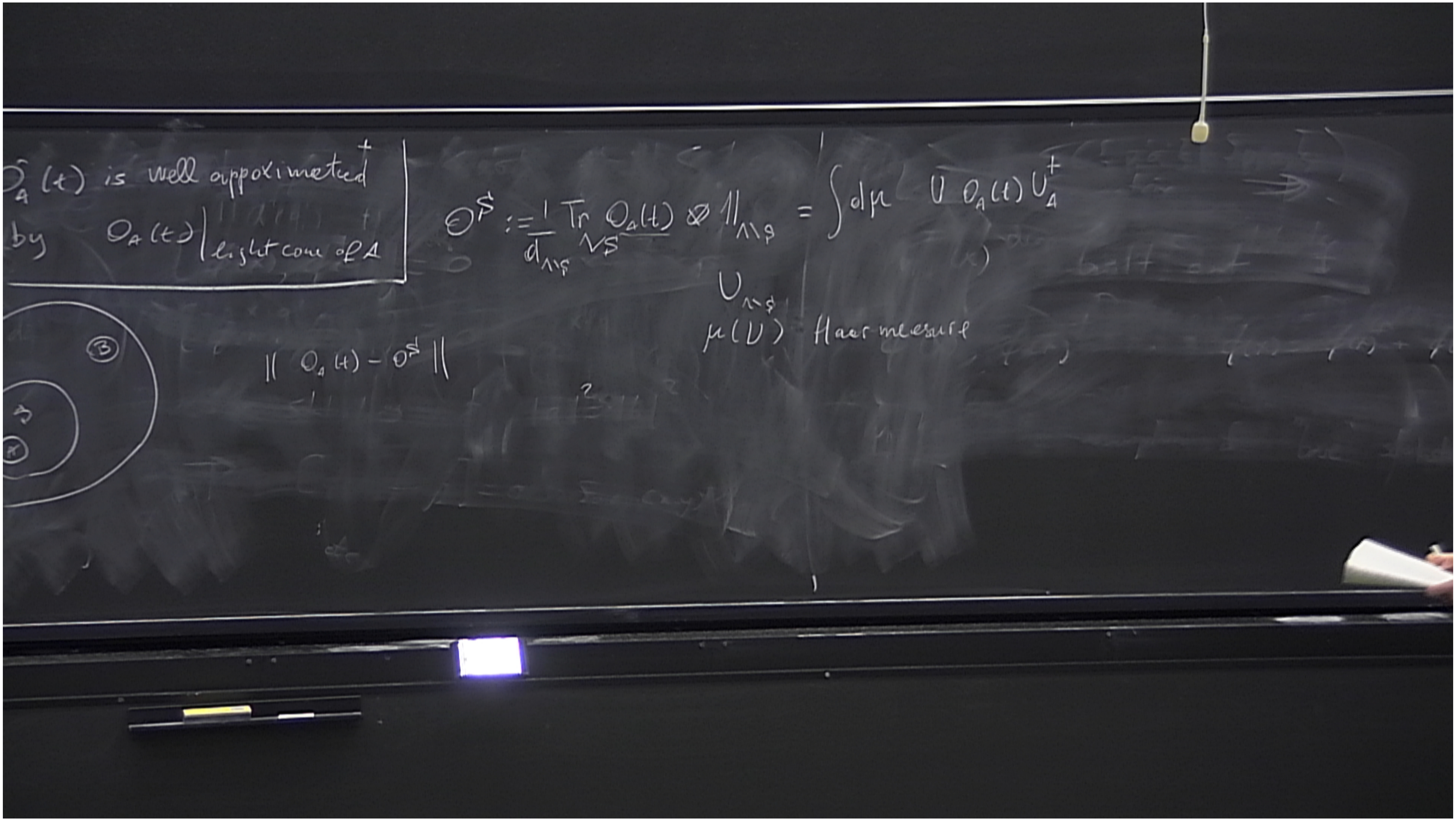

② correlations are bounded
What is the support O_A(t)?
t>0 supp(O_A(t)) = H_1 everything
||[O_A(t), B]|| << 1 t>0
O_B
A
Õ_A(t) is well approximated
by O_A(t)| light cone of A
B
A
O^S := Tr O_A(t)

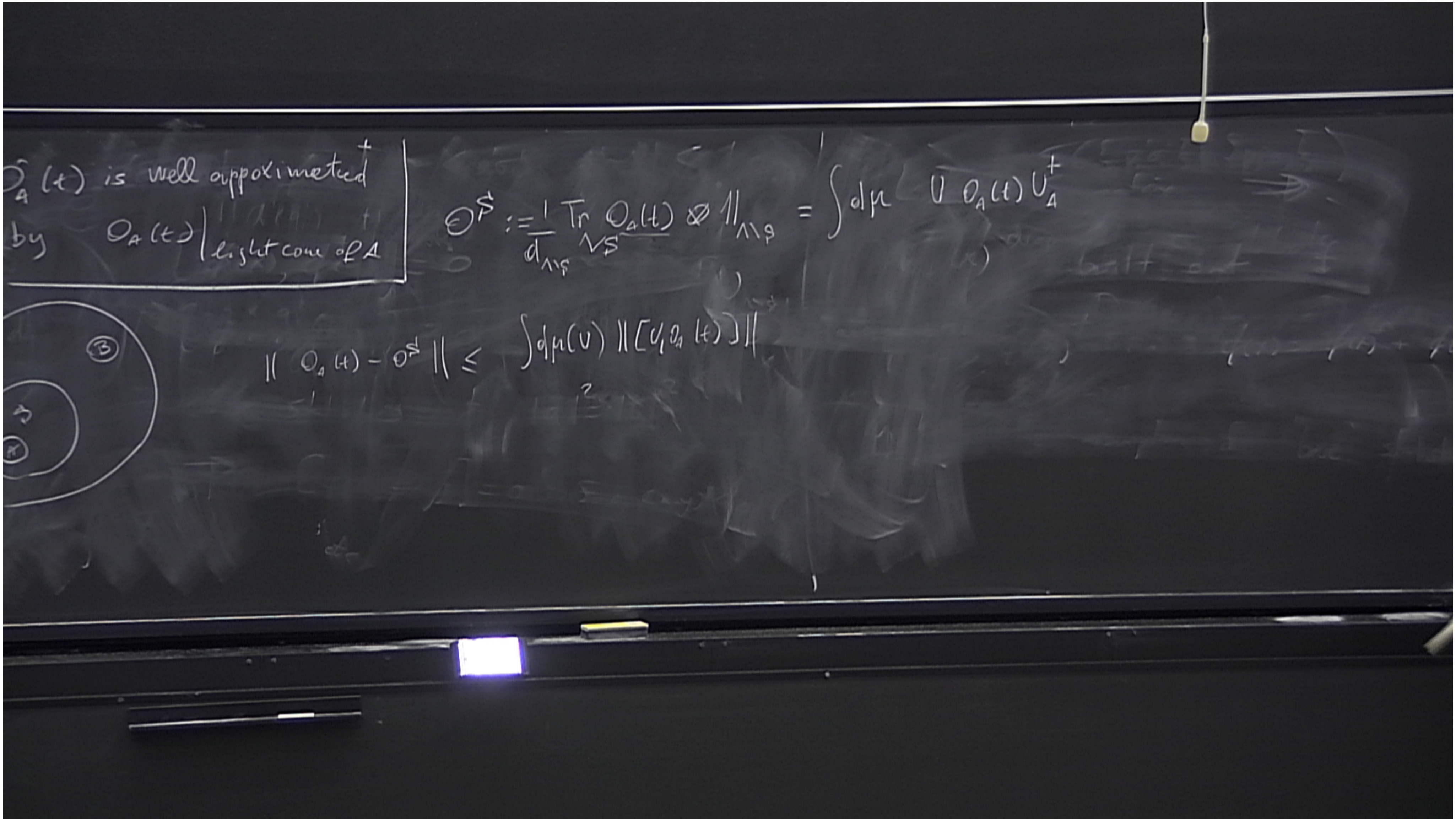

O_A^†(t) is well approximated
by O_A(t)| light cone of A
O^S := 1/d_∧\S Tr_∧\S O_A(t) ⊗ 1l_∧\S



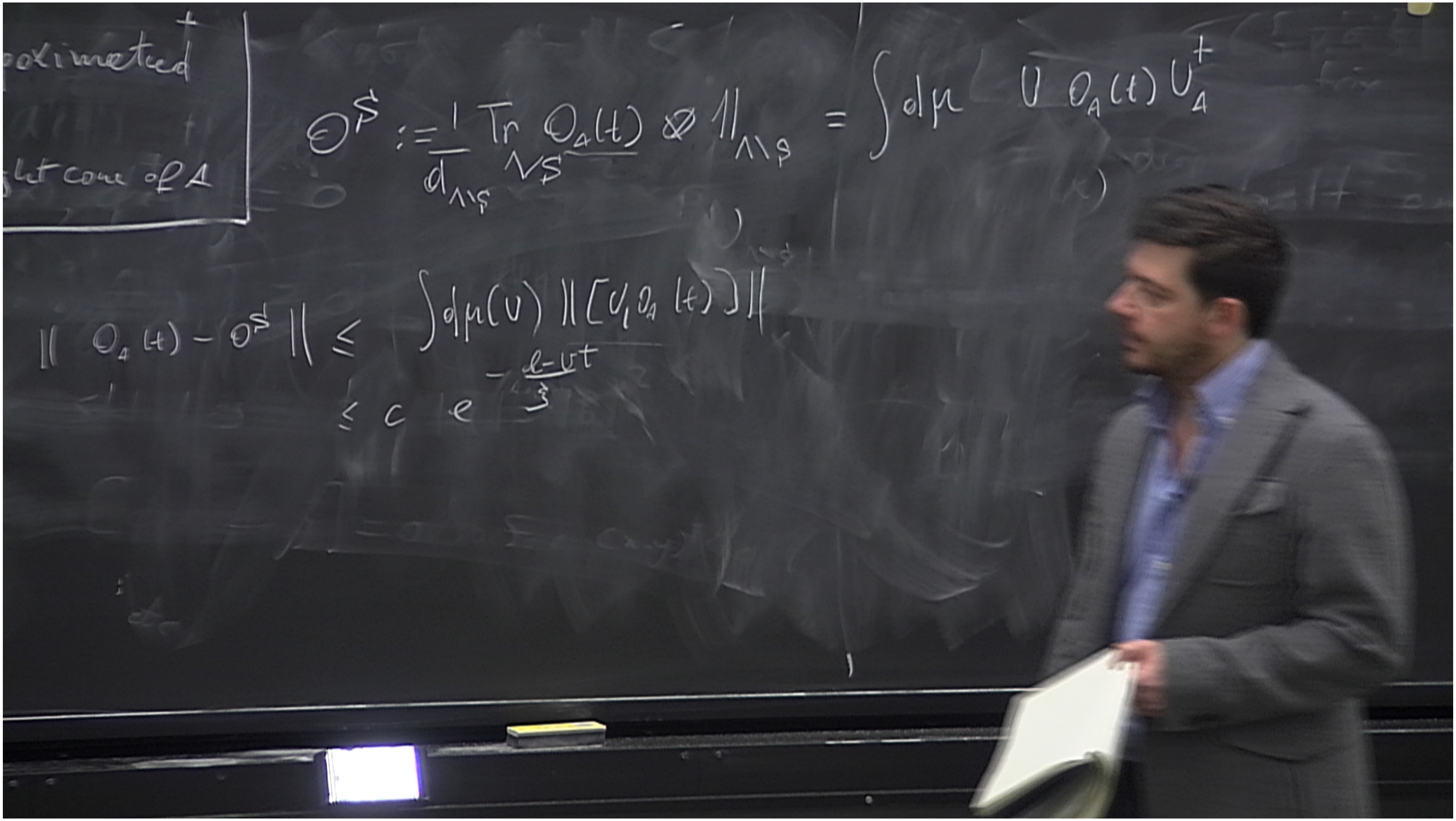
O_A(t) is well approximated
by O_A(t)| light cone of A
O^S := 1/d_Λ\S Tr_Λ\S O_A(t) ⊗ 1_Λ\S =
|| O_A(t) − O^S ||

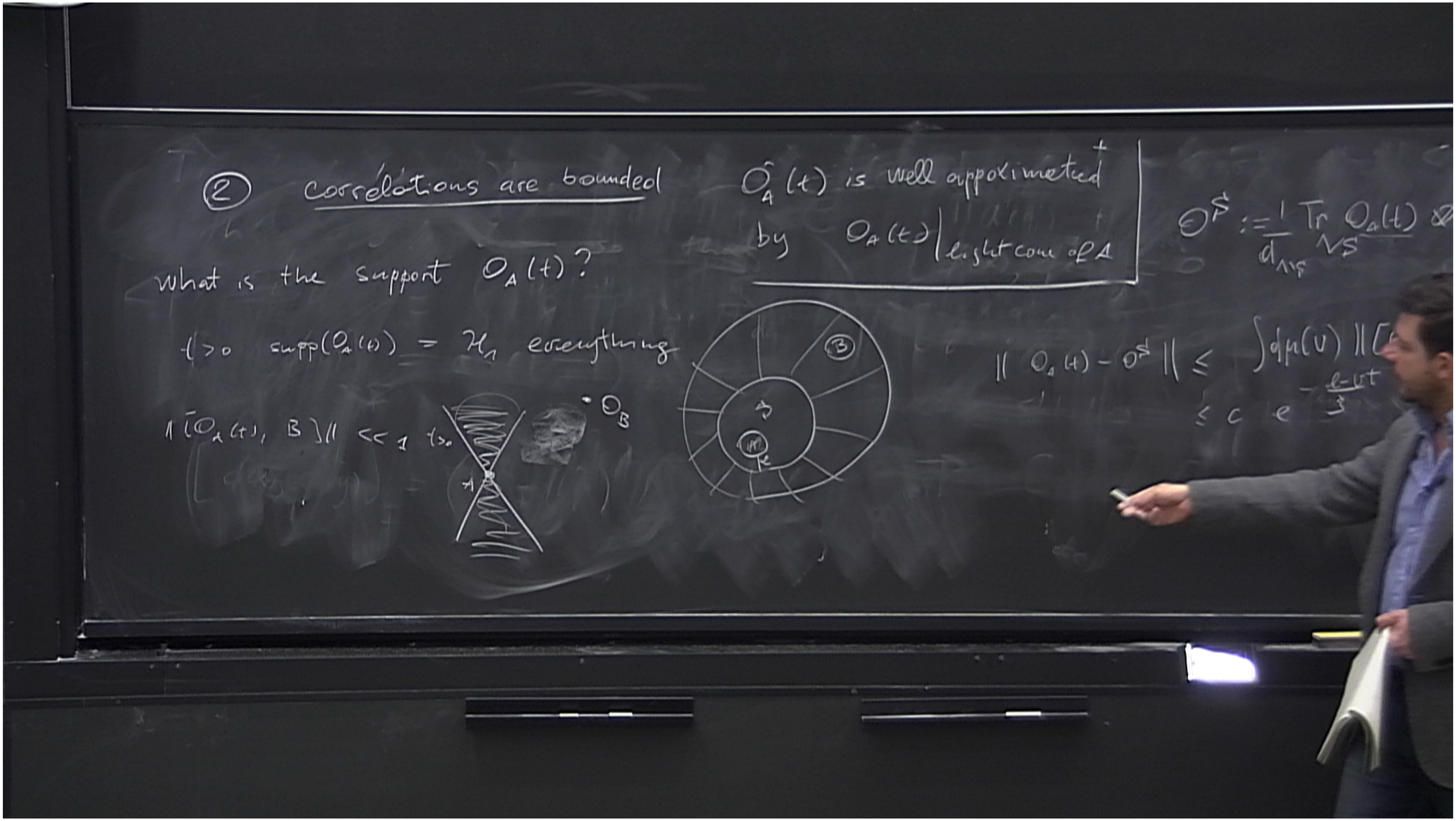

O_A(t) is well approximated
by O_A(t)| light cone of A
O^S := 1/d_{Λ\S} Tr_{Λ\S} O_A(t) ⊗ 1_{Λ\S} =
U_{Λ\S}
μ(U) Haar measure
|| O_A(t) − O^S ||

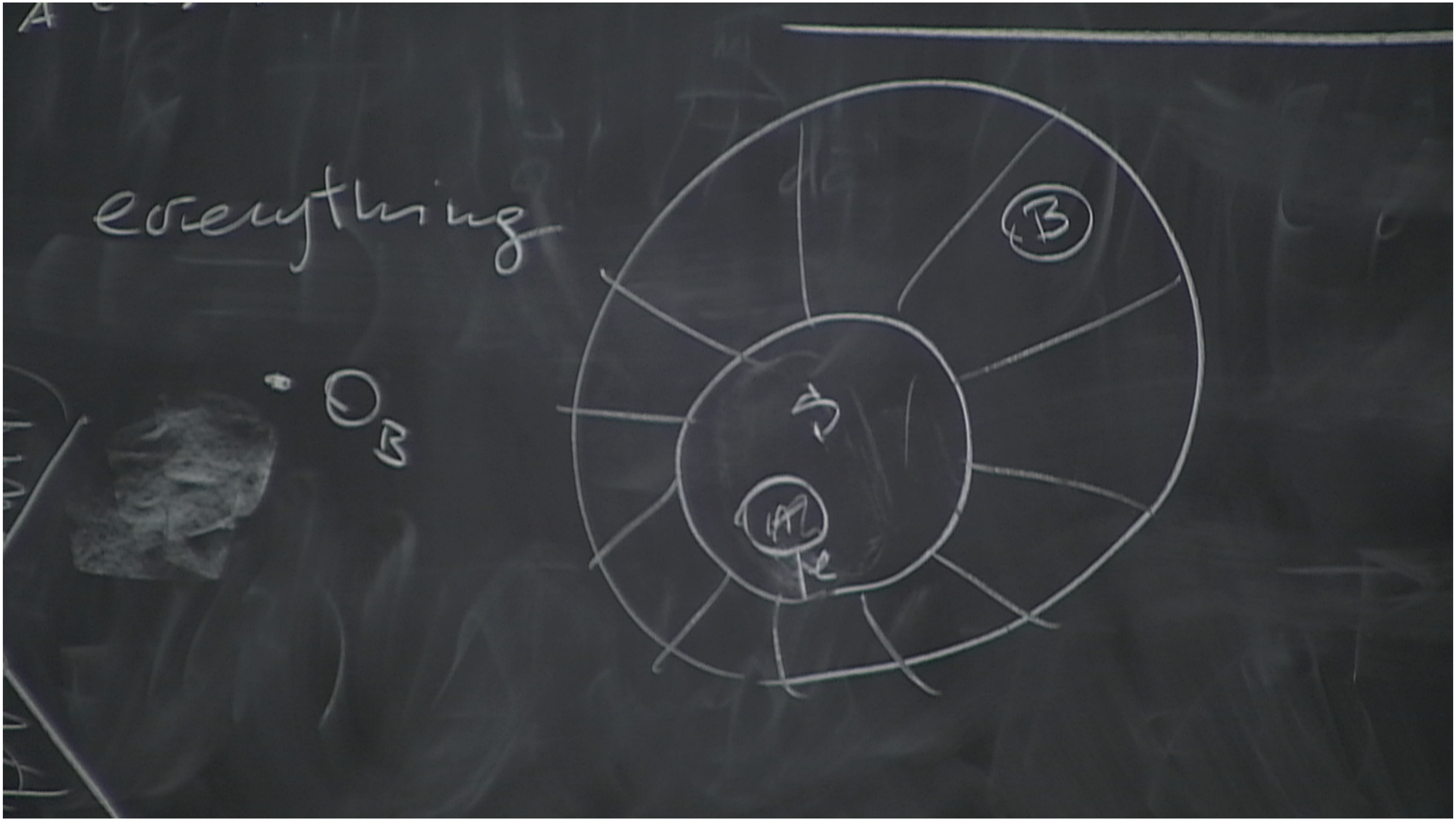

O_A(t) is well approximated by O_A(t)| light cone of A
O^S := 1/d_{Λ\S} Tr_{Λ\S} O_A(t) ⊗ 1_{Λ\S} = ∫dμ U O_A(t) U^†
U_{Λ\S}
μ(U) Haar measure
|| O_A(t) − O^S ||



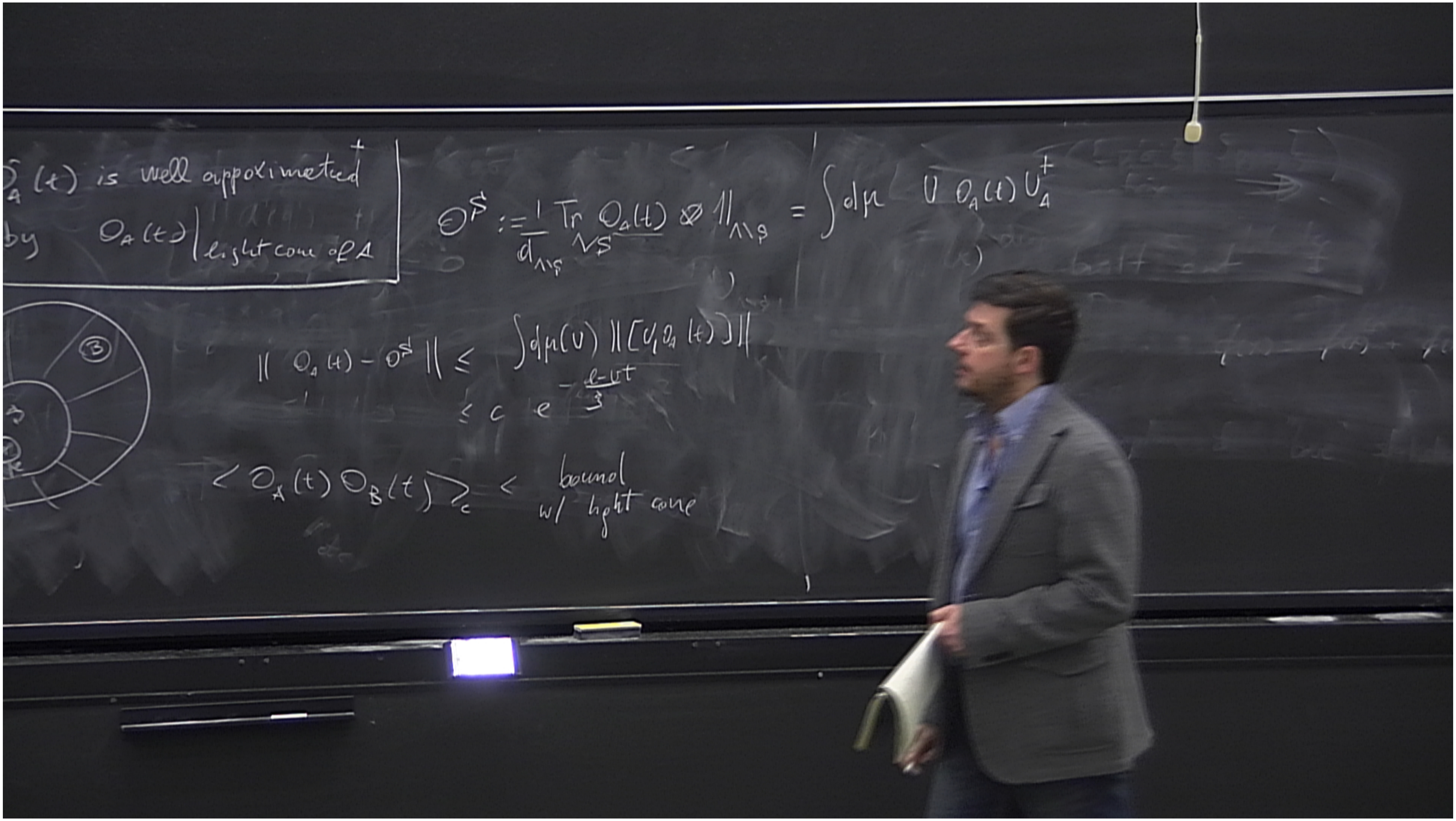
$O_A^\dagger(t)$ is well approximated by $O_A(t)|_{\text{light cone of } A}$
$O^S := \frac{1}{d_{\Lambda\setminus S}} \mathrm{Tr}_{\Lambda\setminus S}\, O_A(t) \otimes \mathbb{1}_{\Lambda\setminus S} = \int d\mu \; U\, O_A(t)\, U_A^\dagger$
$\| O_A(t) - O^S \| \leq \int d\mu(U) \, \| [U, O_A(t)] \|$

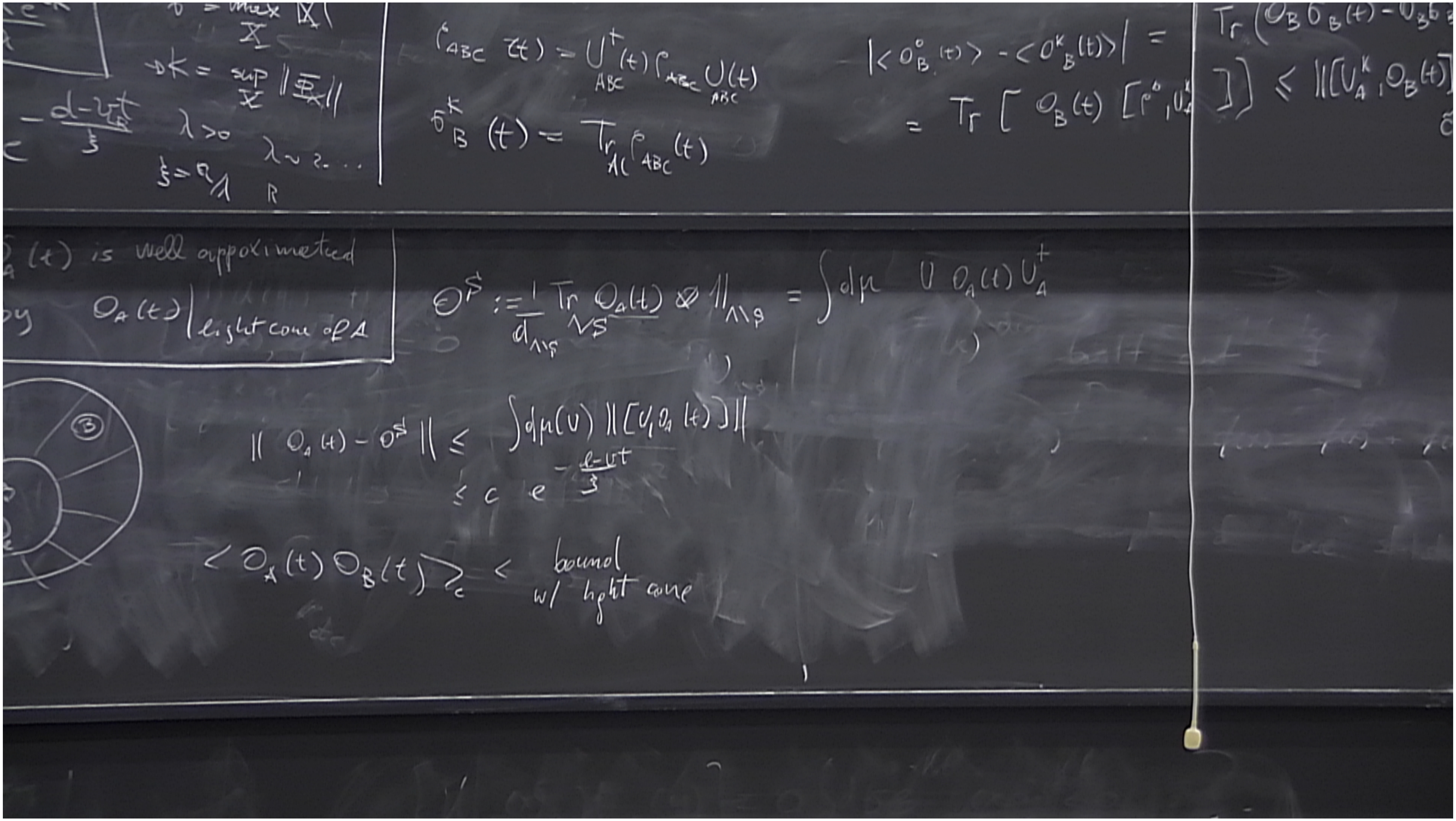

oximated
ght cone of A
O^S := 1/d_{Λ\S} Tr_{Λ\S} O_A(t) ⊗ 1l_{Λ\S} = ∫ dμ U O_A(t) U_A^†
|| O_A(t) − O^S || ≤ ∫ dμ(U) || [U, O_A(t)] ||
≤ C e^{−(l−vt)/ξ}

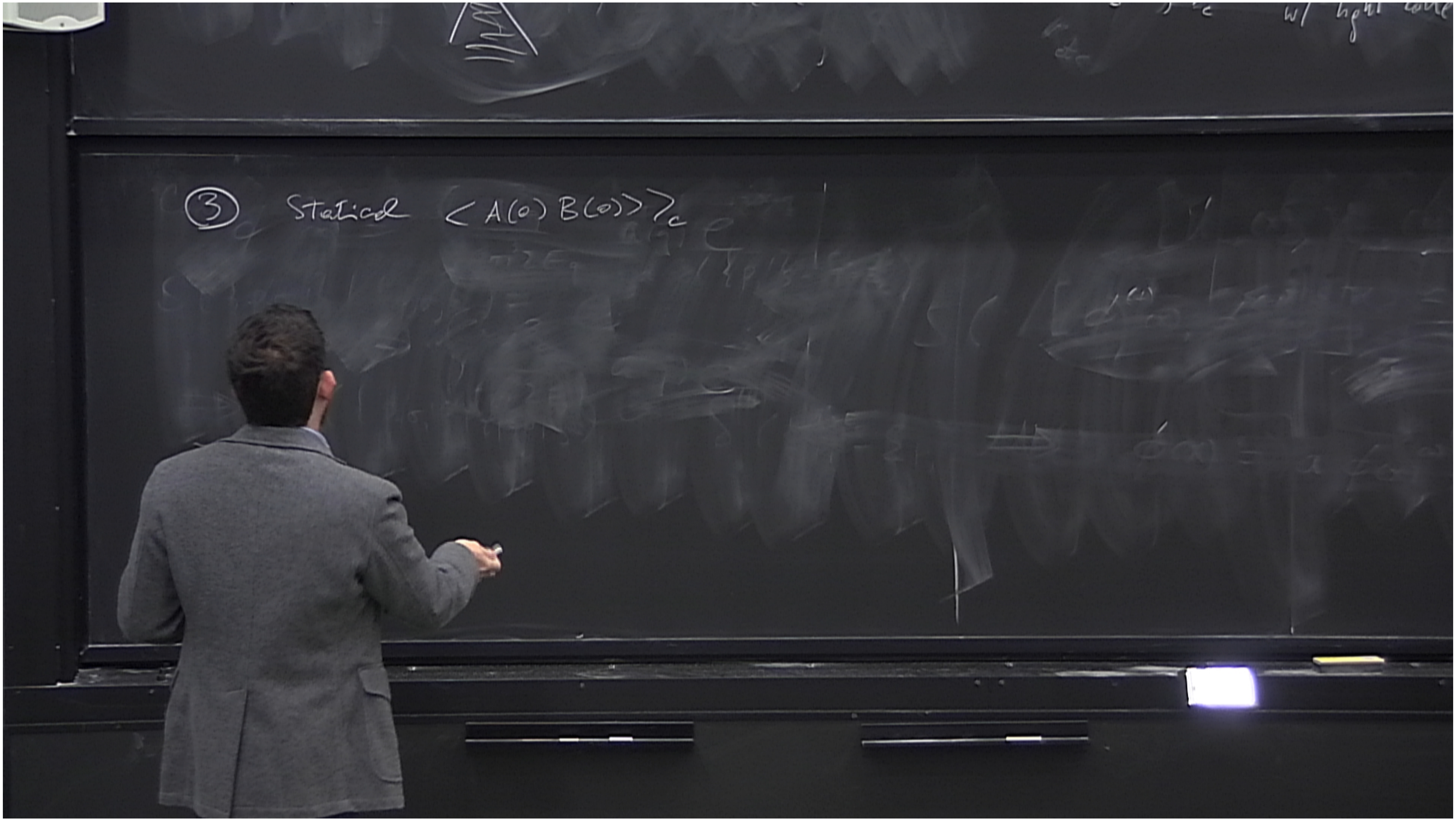

② correlations are bounded
what is the support O_A(t)?
t>0 supp(O_A(t)) = H_Λ everything
‖[O_A(t), B]‖ << 1 t>0
O_B
Ô_A(t) is well approximated
by O_A(t)| light cone of A
B

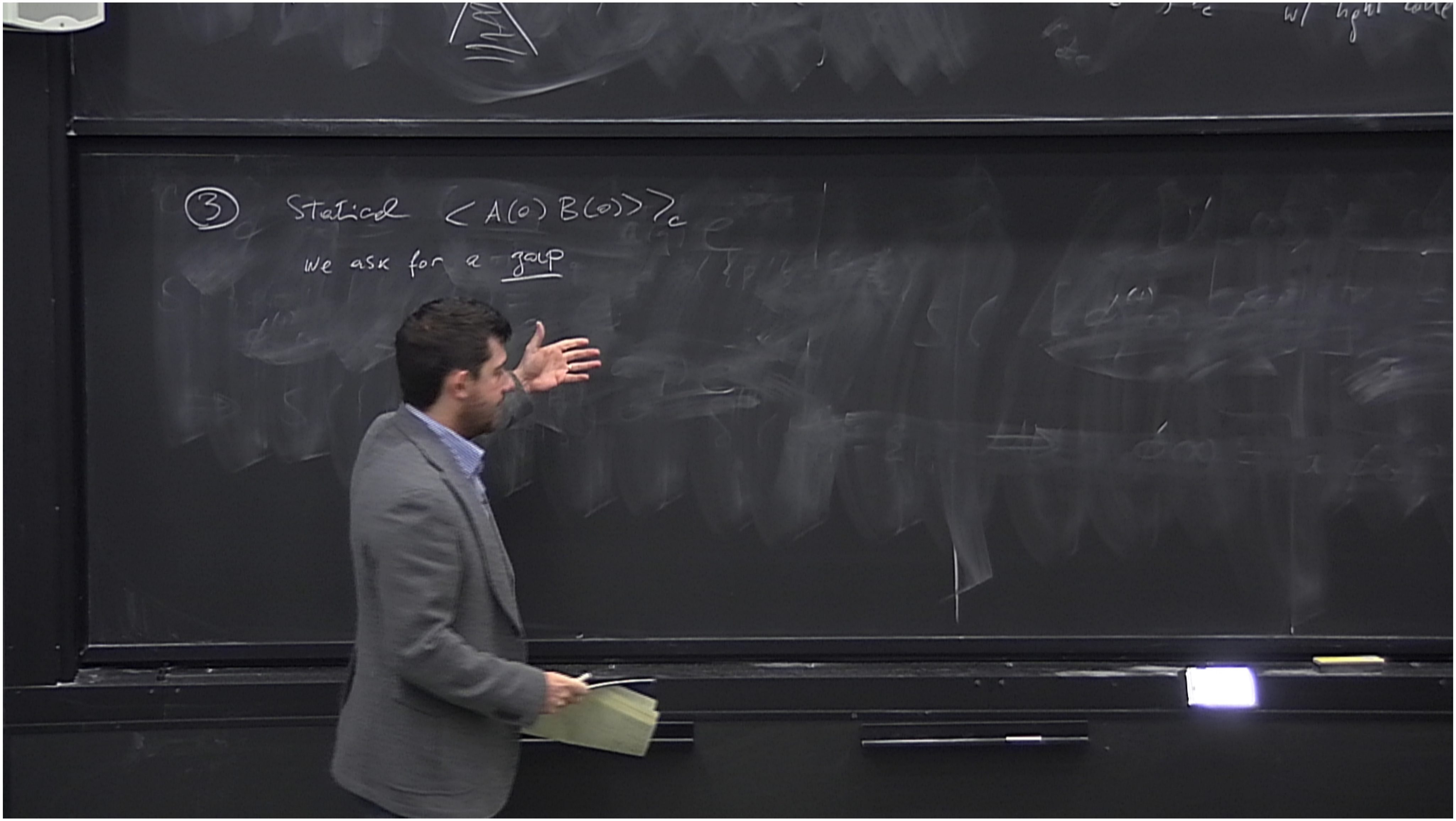


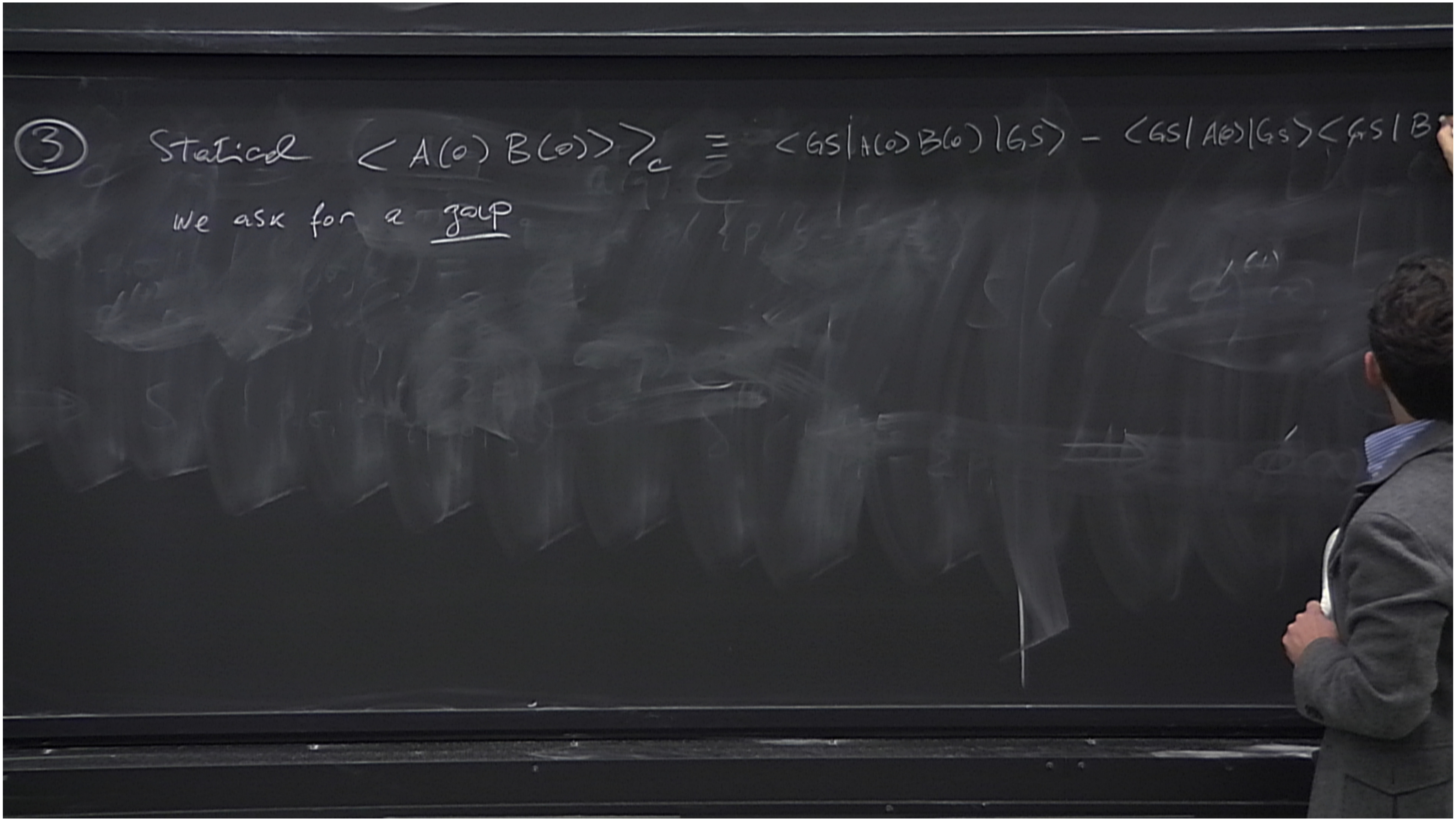
everything
B

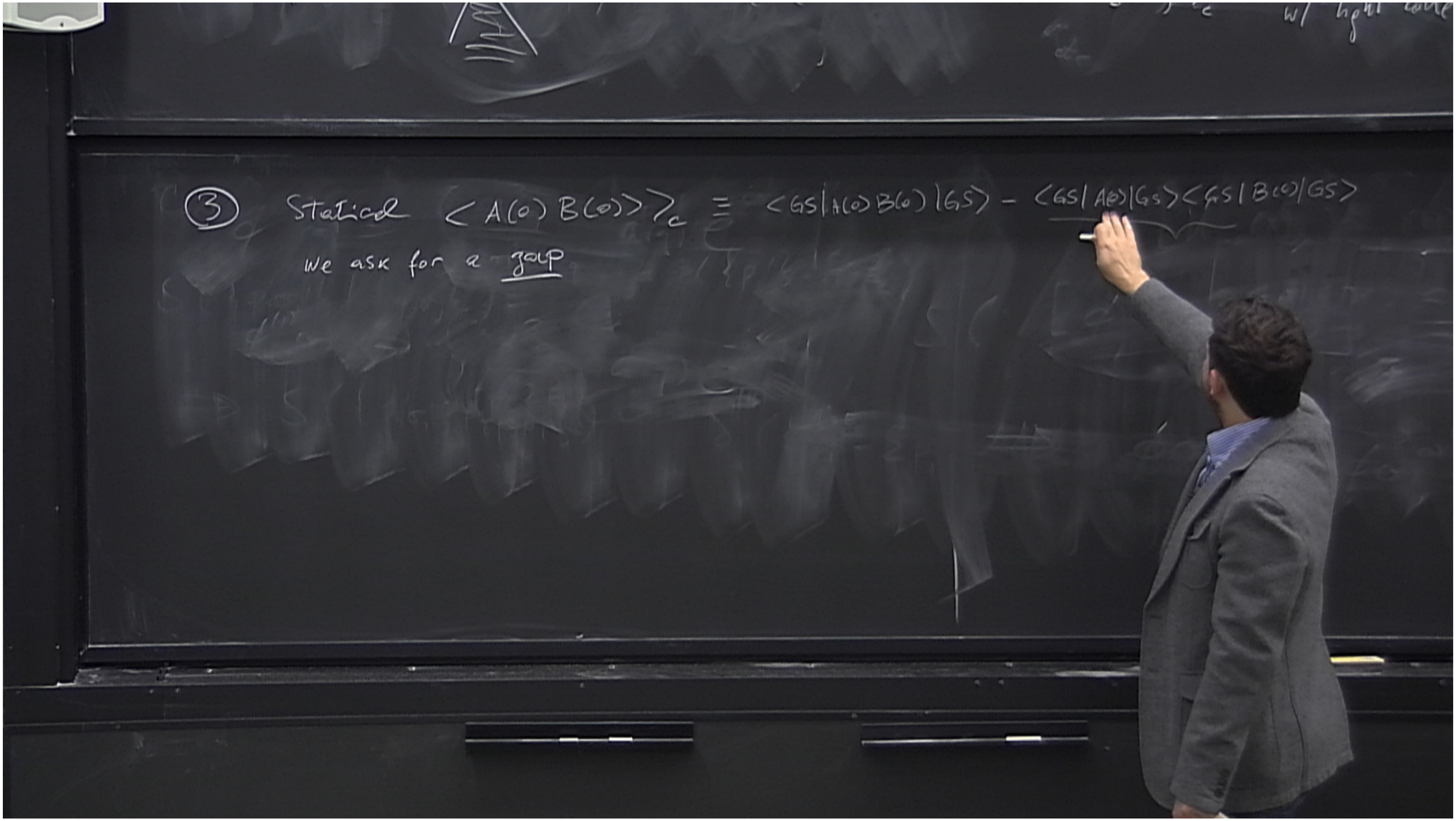

is well approximated
by
light cone of A
bound
w/ light cone

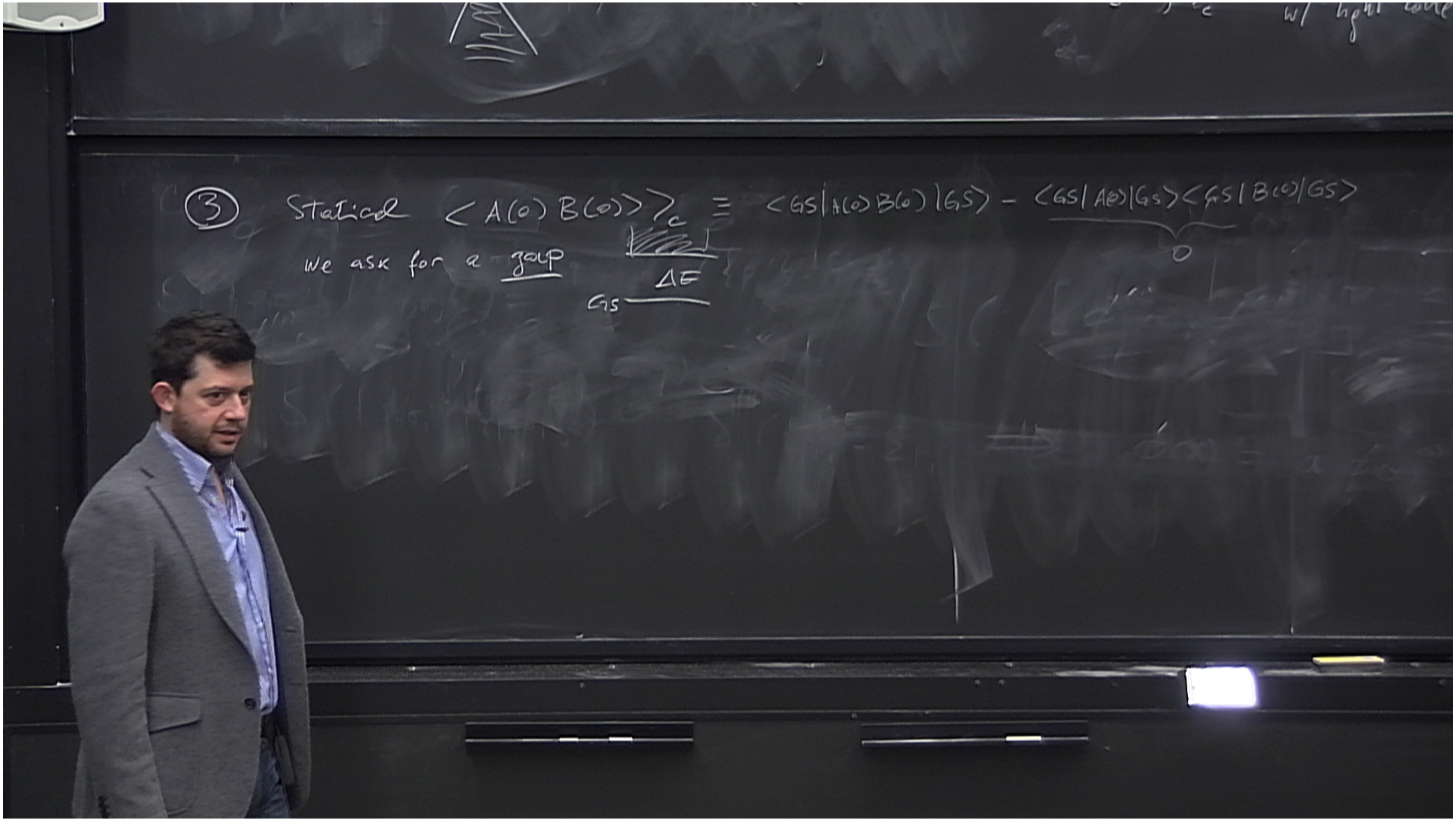

(t) is well approximated
by
light cone of A
bound
w/ light cone


③ Statical < A(0) B(0) >c


③ Statical < A(0) B(0) >$_c$
We ask for a gap


③ Static <A(0) B(0)>_c ≡ <GS|A(0) B(0)|GS> − <GS|A(0)|GS><GS|B
We ask for a gap

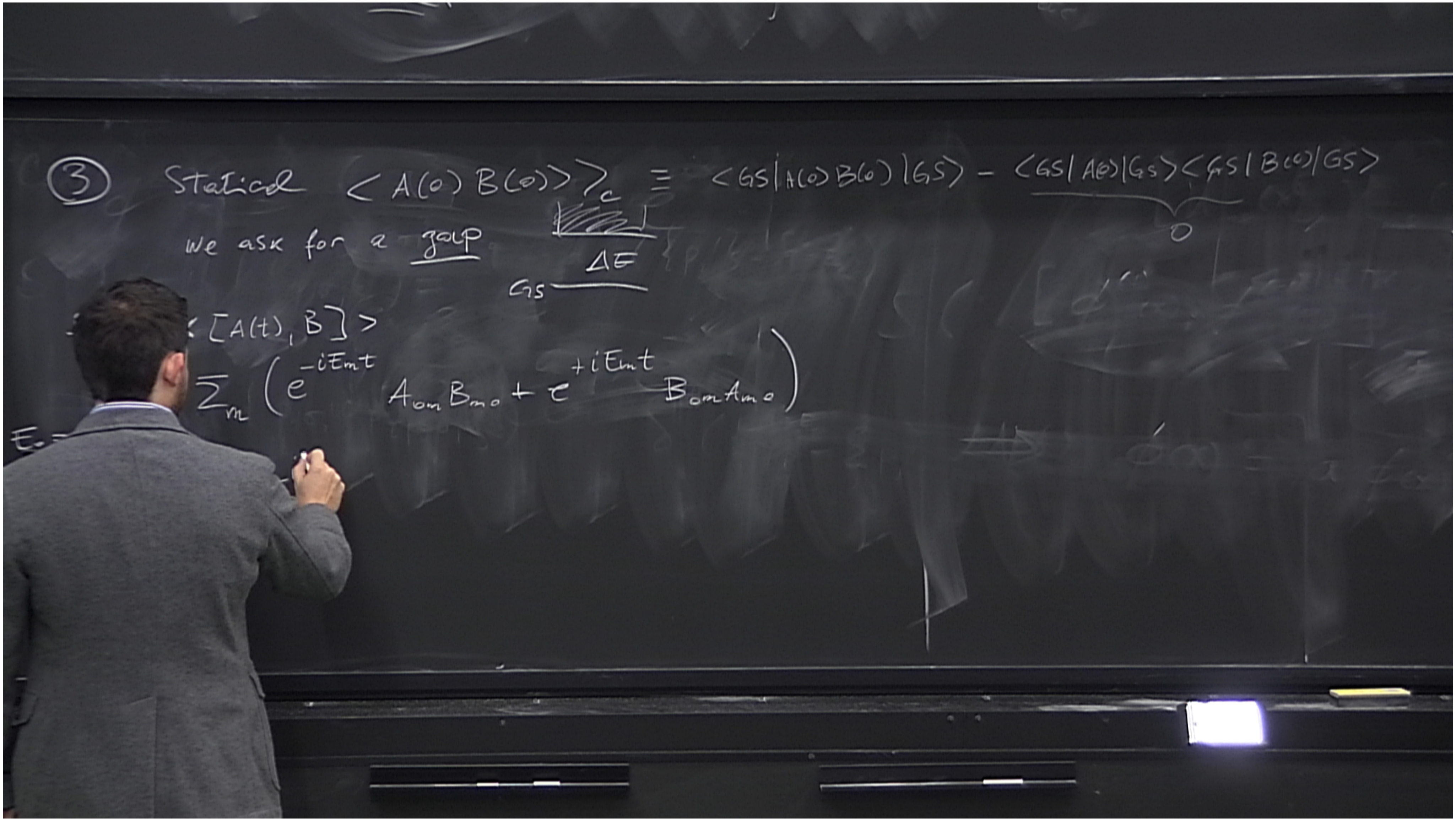

③ Statical < A(0) B(0) >_c ≡ <GS|A(0) B(0)|GS> - <GS|A(0)|GS><GS|B(0)|GS>
We ask for a gap

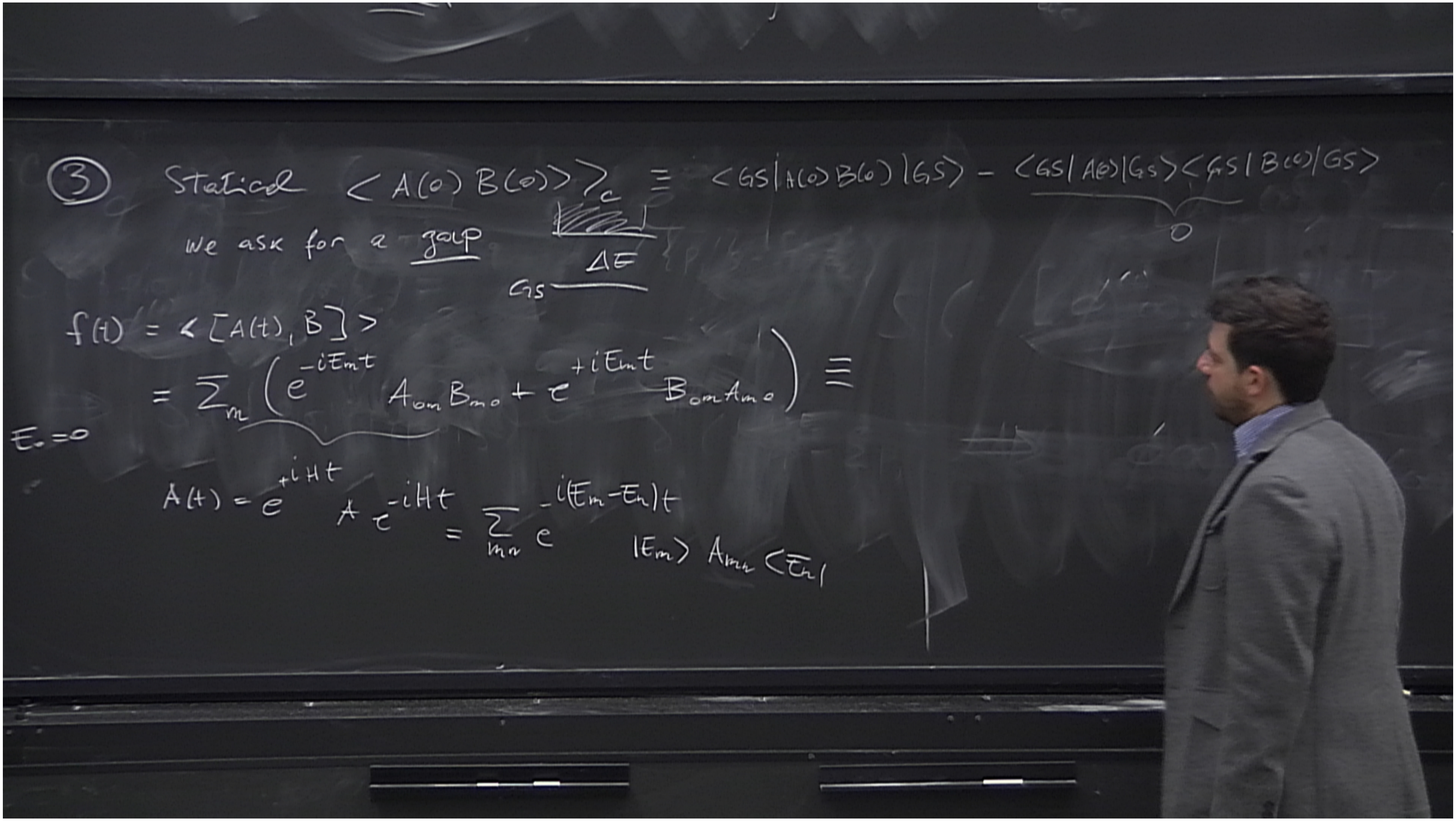

③ Static ⟨A(0) B(0)⟩_c ≡ ⟨GS|A(0)B(0)|GS⟩ − ⟨GS|A(0)|GS⟩⟨GS|B(0)|GS⟩
0
We ask for a gap
ΔE
GS

③ Statical $\langle A(0)\,B(0)\rangle\rangle_c \equiv \langle GS|A(0)B(0)|GS\rangle - \underbrace{\langle GS|A(0)|GS\rangle\langle GS|B(0)|GS\rangle}_{0}$

we ask for a <u>gap</u> ΔE (GS)

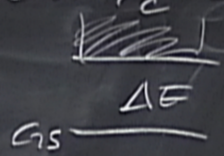
$f(t) = \langle [A(t), B] \rangle$


③ Static < A(0) B(0) >_c ≡ < GS | A(0) B(0) | GS > − < GS | A(0) | GS > < GS | B(0) | GS >
0
We ask for a gap
ΔE
GS
< [A(t), B] >
Σ_m (e^{−iE_m t} A_{om} B_{mo} + e^{+iE_m t} B_{om} A_{mo})


③ Statical <A(0) B(0)>_c ≡ <GS|A(0) B(0)|GS> − <GS|A(0)|GS><GS|B(0)|GS>
0
we ask for a gap
ΔE
GS
f(t) = <[A(t), B]>
= Σ_m (e^{−iE_m t} A_{0m} B_{m0} + e^{+iE_m t} B_{0m} A_{m0}) ≡
E_0 = 0
A(t) = e^{+iHt} A e^{−iHt}
= Σ_{mn} e^{−i(E_m − E_n)t} |E_m> A_{mn} <E_n|

③ Static $\langle A(0)\, B(0) \rangle_c \equiv \langle GS | A(0) B(0) | GS \rangle - \underbrace{\langle GS | A(0) | GS \rangle \langle GS | B(0) | GS \rangle}_{0}$

We ask for a gap

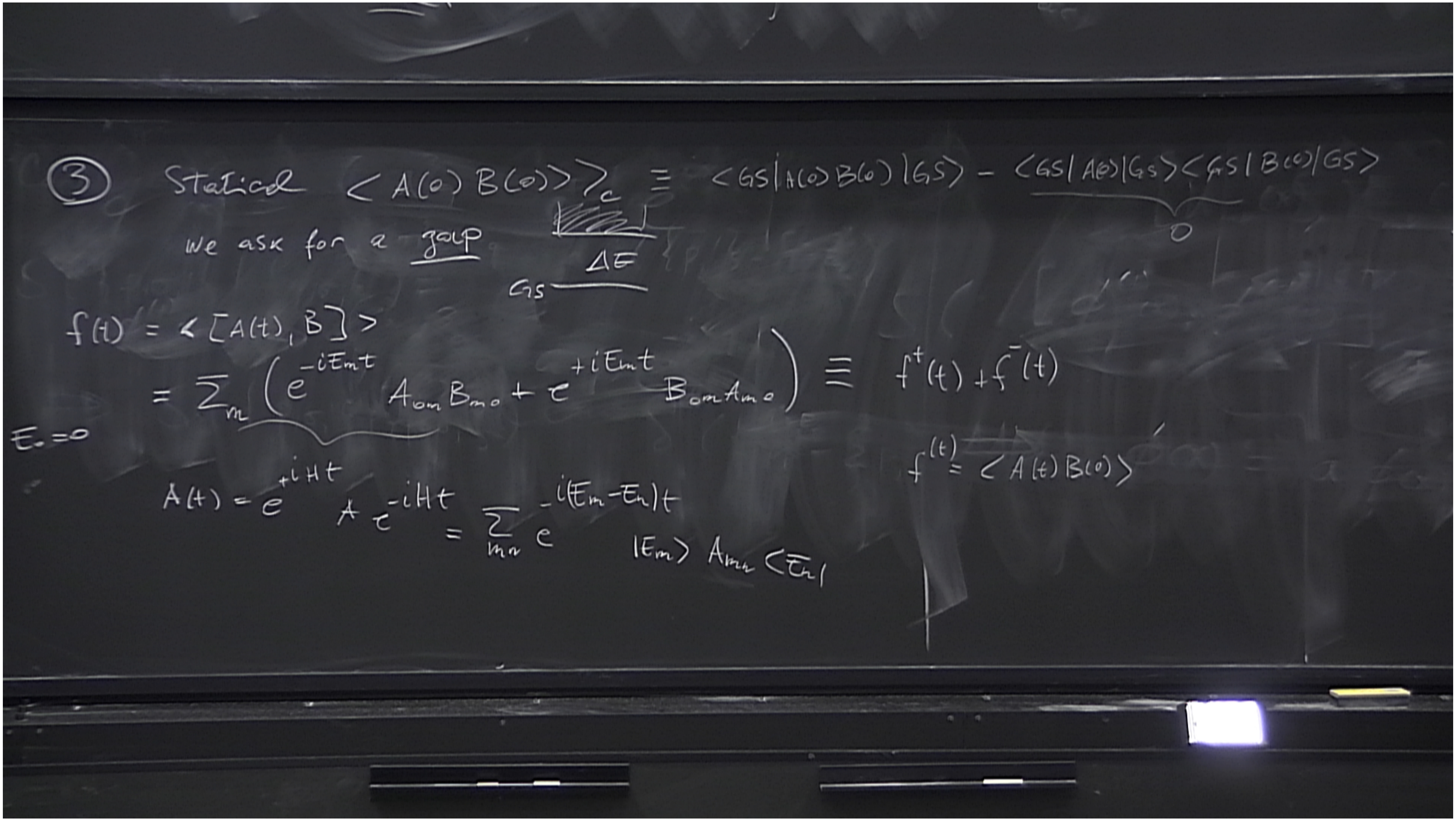


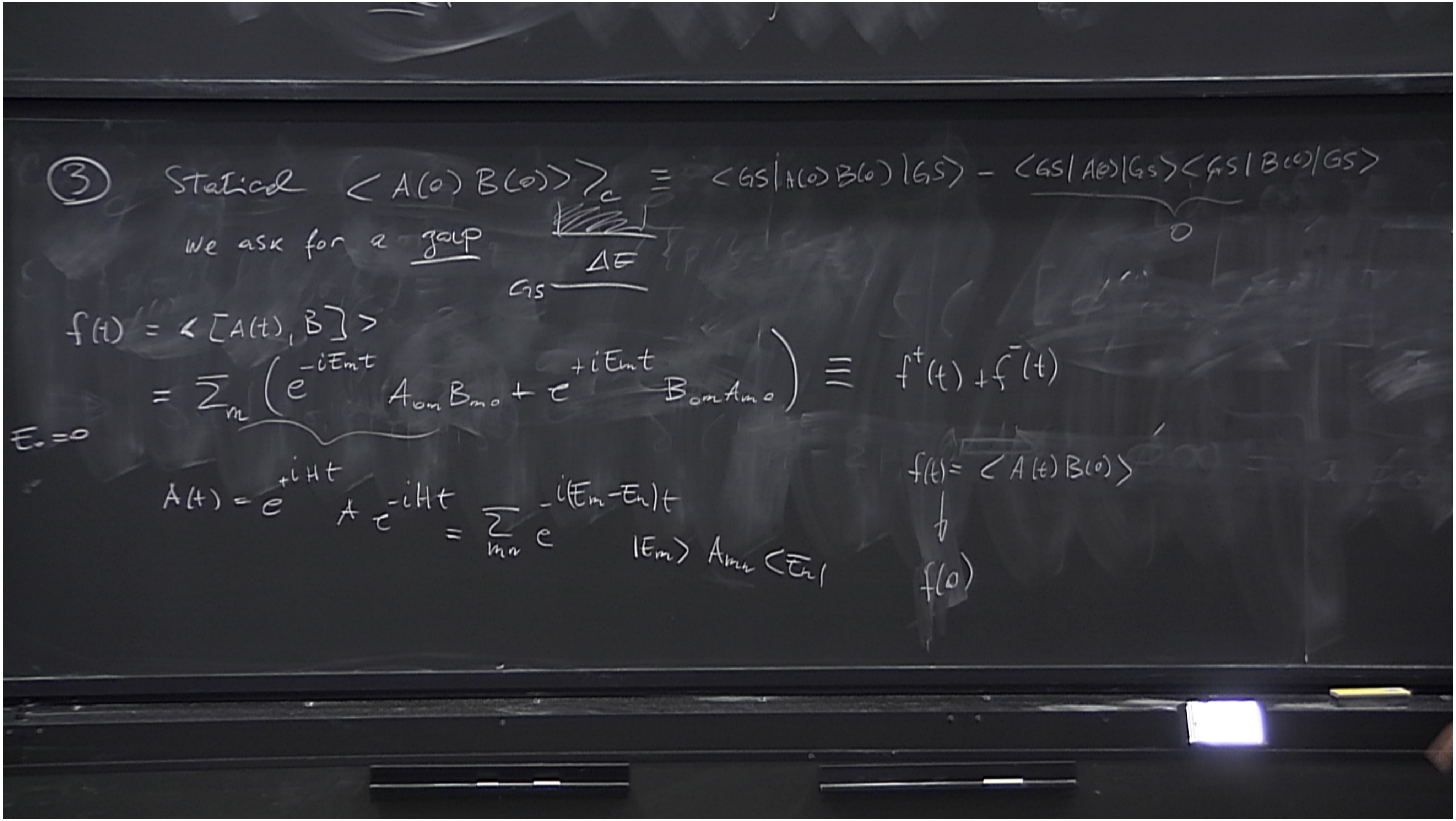
$$f(t) = \langle [A(t), B] \rangle$$

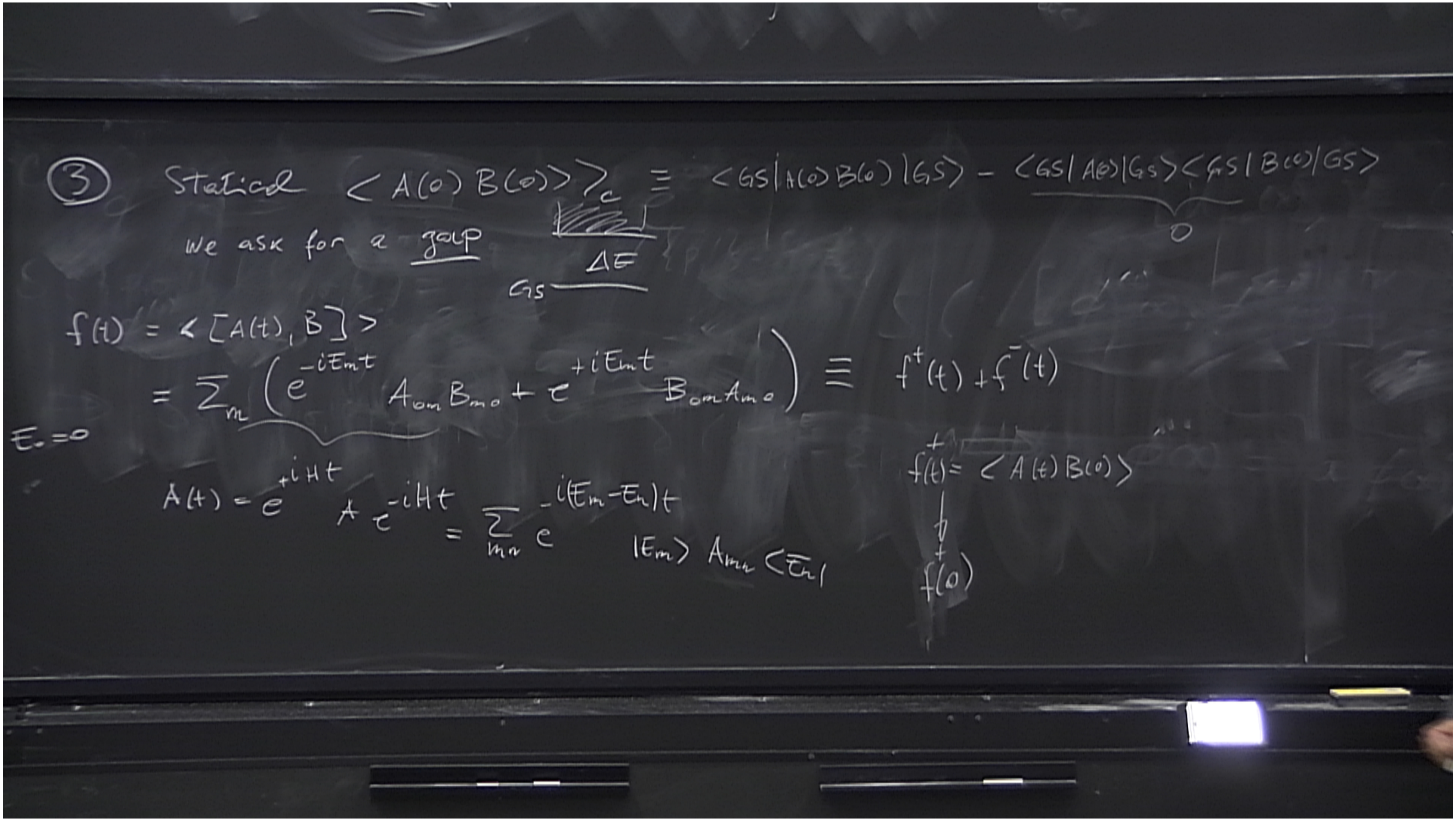
$$= \sum_m \left(e^{-iE_m t} A_{om} B_{mo} + e^{+iE_m t} B_{om} A_{mo} \right) \equiv f^+(t) + f^-(t)$$

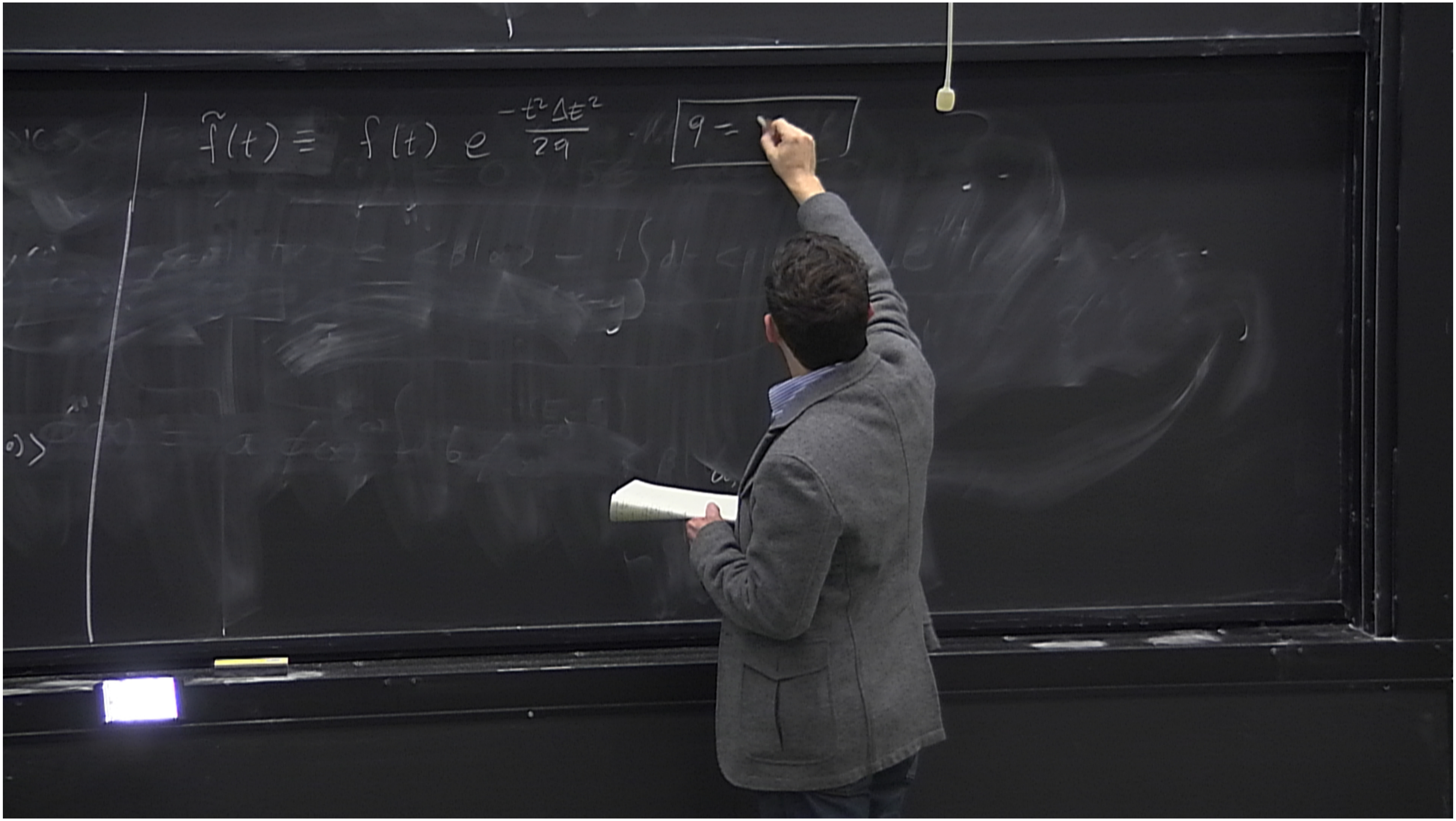
$E_0 = 0$

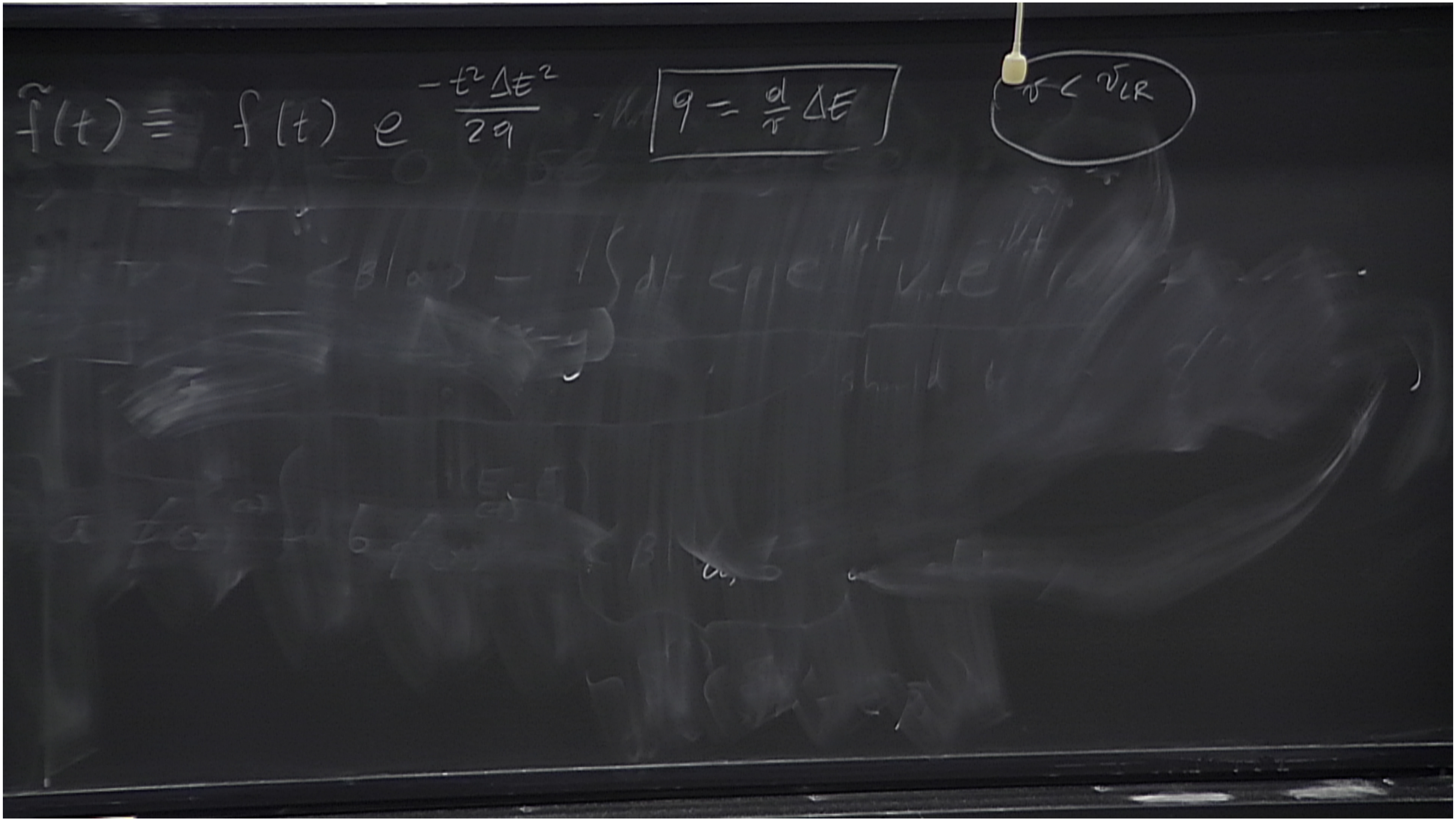
$$A(t) = e^{+iHt} A\, e^{-iHt} = \sum_{mn} e^{-i(E_m - E_n)t} |E_m\rangle A_{mn} \langle E_n|$$


③ Statical < A(0) B(0) >_c ≡ <GS| A(0) B(0) |GS> − <GS| A(0) |GS><GS| B(0) |GS>
0
We ask for a gap
ΔE
GS
f(t) = < [A(t), B] >
= Σ_m (e^{−iE_m t} A_{0m} B_{m0} + e^{+iE_m t} B_{0m} A_{m0}) ≡ f^+(t) + f^−(t)
E_0 = 0
f^(t) = < A(t) B(0) >
A(t) = e^{+iHt} A e^{−iHt} = Σ_{mn} e^{−i(E_m − E_n)t} |E_m> A_{mn} <E_n|


③ Static <A(0) B(0)>_c ≡ <GS|A(0) B(0)|GS> − <GS|A(0)|GS><GS|B(0)|GS>
0
we ask for a gap
ΔE
GS
f(t) = <[A(t), B]>
= Σ_m (e^{−iE_m t} A_{0m} B_{m0} + e^{+iE_m t} B_{0m} A_{m0}) ≡ f^+(t) + f^−(t)
E_0 = 0
f(t) = <A(t) B(0)>
A(t) = e^{+iHt} A e^{−iHt} = Σ_{mn} e^{−i(E_m − E_n)t} |E_m> A_{mn} <E_n|
f(0)


③ Static $\langle A(0) B(0) \rangle_c \equiv \langle GS | A(0) B(0) | GS \rangle - \langle GS | A(0) | GS \rangle \langle GS | B(0) | GS \rangle$
0
We ask for a gap
ΔE
GS
$f(t) = \langle [A(t), B] \rangle$
$= \sum_m \left(e^{-iE_m t} A_{0m} B_{m0} + e^{+iE_m t} B_{0m} A_{m0} \right) \equiv f^+(t) + f^-(t)$
$E_0 = 0$
$f^+(t) = \langle A(t) B(0) \rangle$
$f^+(0)$
$A(t) = e^{+iHt} A e^{-iHt} = \sum_{mn} e^{-i(E_m - E_n)t} |E_m\rangle A_{mn} \langle E_n|$


$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta t^2}{2q}}$
$q =$


$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$
$q = \frac{d}{v} \Delta E$
$v < v_{LR}$

$$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$$

$$q = \frac{d}{v} \Delta E$$

$$v < v_{LR}$$

① $\|\tilde{f}(t)\| \leq \|f(t)\|$

$$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$$

$$\boxed{q = \frac{d}{v} \Delta E}$$

$$v < v_{LR}$$

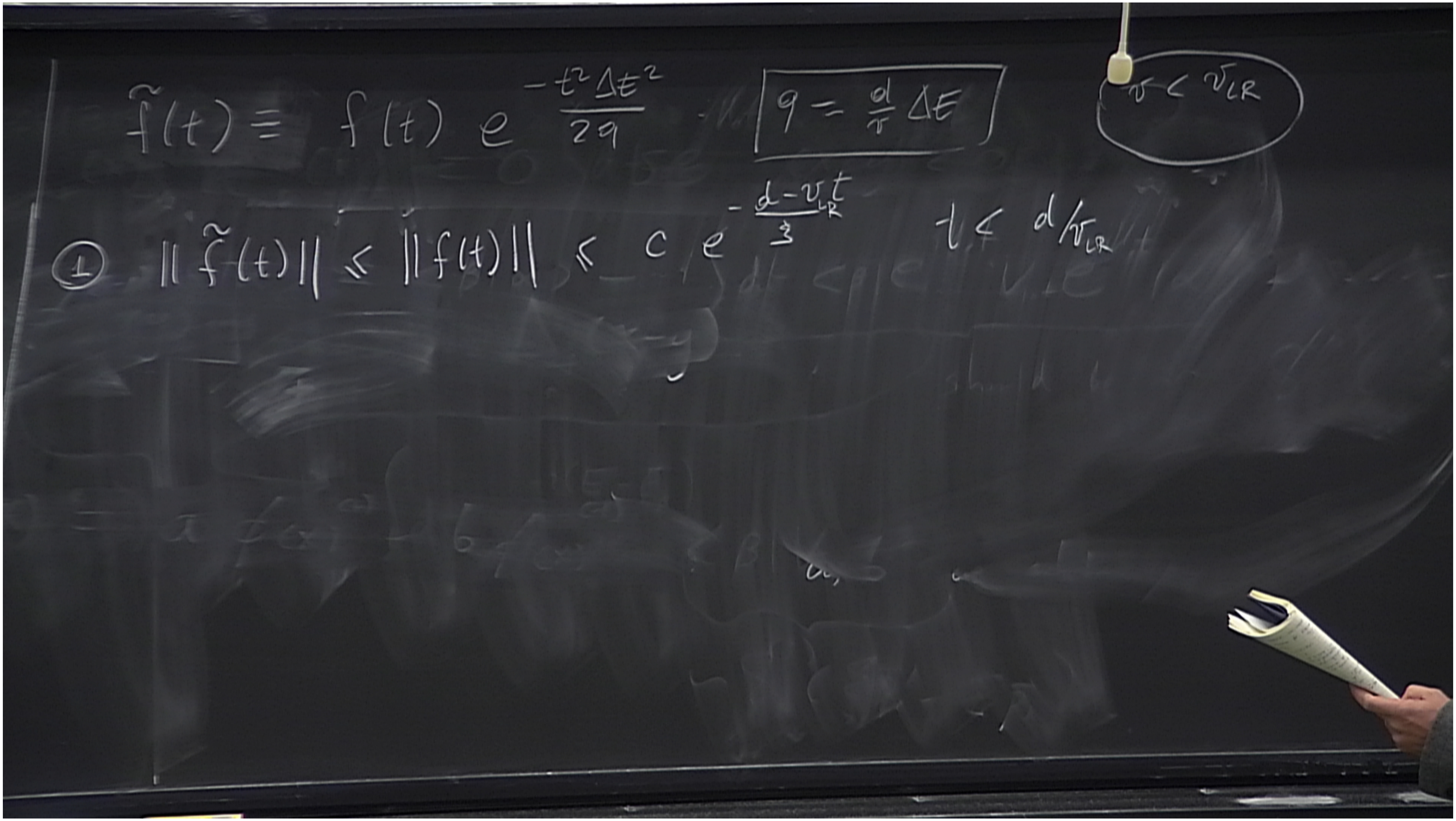
① $$\|\tilde{f}(t)\| \le \|f(t)\| \le c\, e^{-\frac{d - vt}{\xi}}$$

$$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$$

$$q = \frac{d}{v} \Delta E$$

$$v < v_{LR}$$

$$① \quad \|\tilde{f}(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - vt}{\xi}} \qquad t < d/v$$


$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$
$q = \frac{d}{v} \Delta E$
$v < v_{LR}$
① $\|\tilde{f}(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}}$
$t < d/v_{LR}$

$$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$$

$$\boxed{q = \frac{d}{v}\Delta E}$$

$v < v_{LR}$

① $\|\tilde{f}(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}}$ $\quad t < d/v_{LR}$

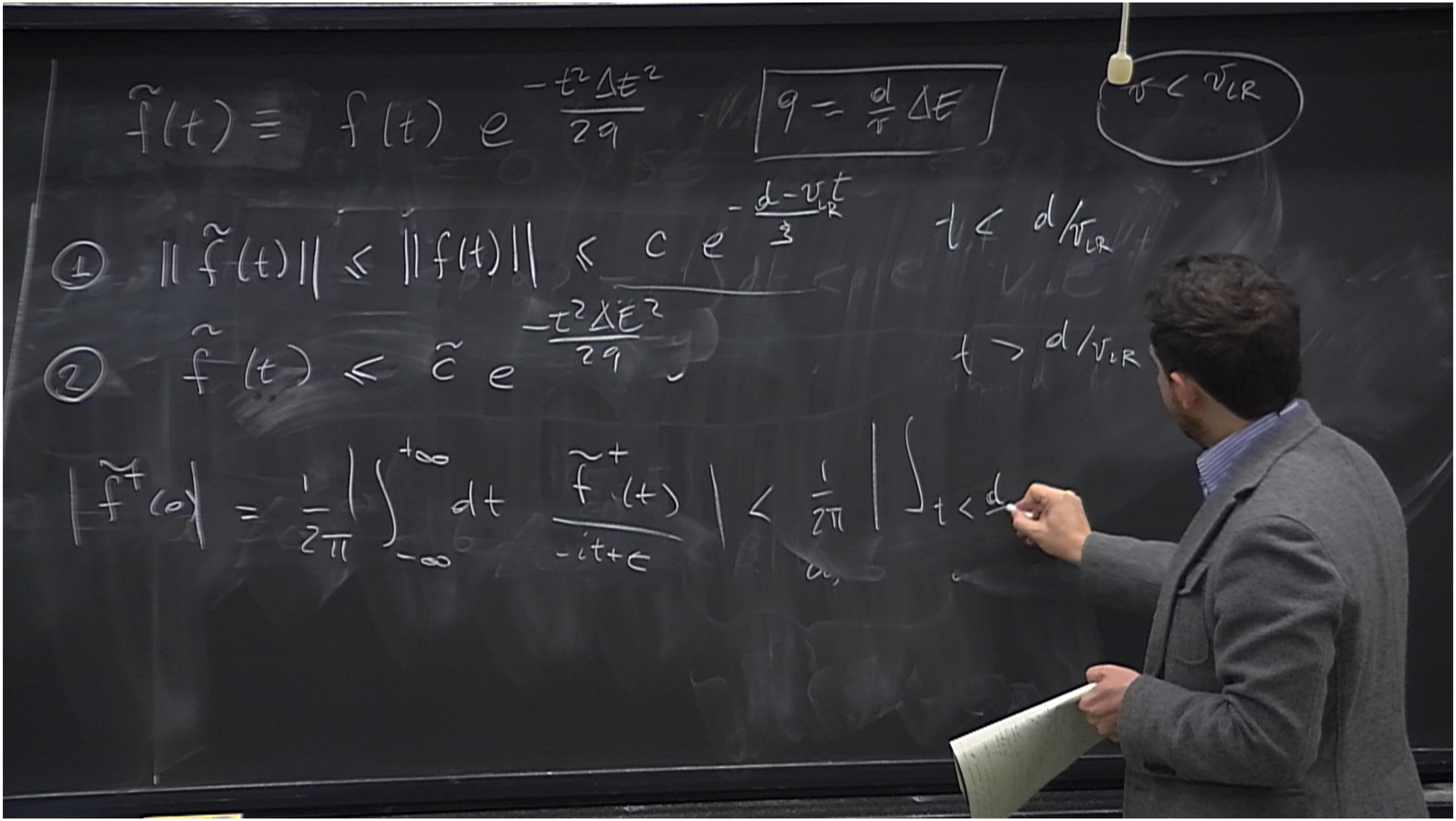
② $\tilde{f}(t) \sim \tilde{c}\, e^{-\frac{t^2 \Delta E^2}{2q}}$ $\quad t > d/v_{LR}$

$$\tilde f(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad \boxed{q = \frac{d}{v}\Delta E} \qquad v < v_{LR}$$

① $$\|\tilde f(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}} \qquad t < d/v_{LR}$$

② $$\tilde f(t) \leq \tilde c\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad t > d/v_{LR}$$

$$\tilde f^{+}(0) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt\, \frac{\tilde f^{+}(t)}{-it+\epsilon}$$


f̃(t) ≡ f(t) e^(−t²ΔE²/2q)
q = (d/v) ΔE
v < v_LR
① ||f̃(t)|| ≤ ||f(t)|| ≤ c e^(−(d − v_LR t)/ξ)
t < d/v_LR
② f̃(t) ≤ c̃ e^(−t²ΔE²/2q)
t > d/v_LR
|f̃⁺(0)| = (1/2π) | ∫_{−∞}^{+∞} dt f̃⁺(t)/(−it+ε) | < (1/2π) | ∫_{t<d}

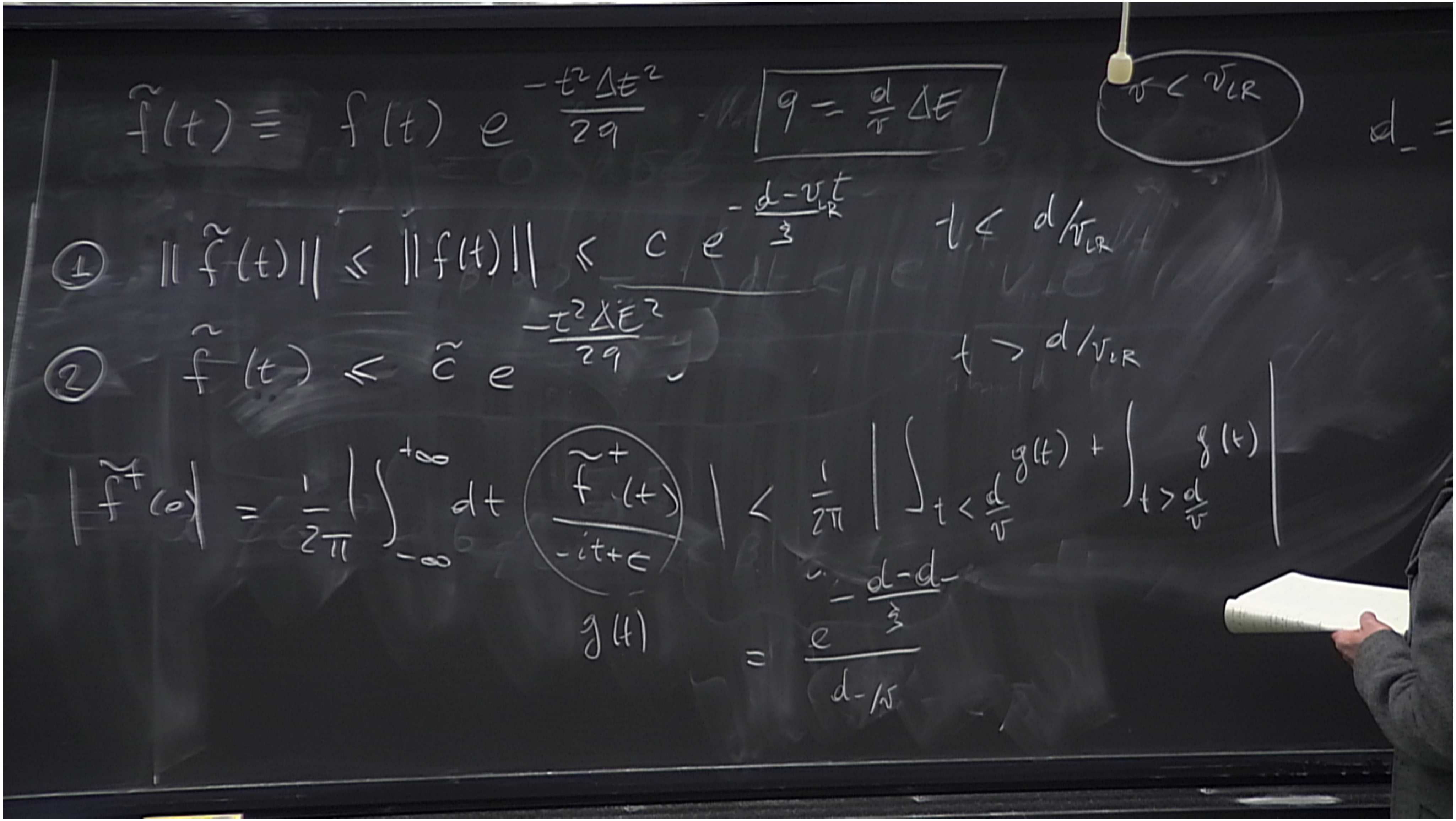
$$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad \boxed{q = \frac{d}{v}\Delta E} \qquad v < v_{LR}$$

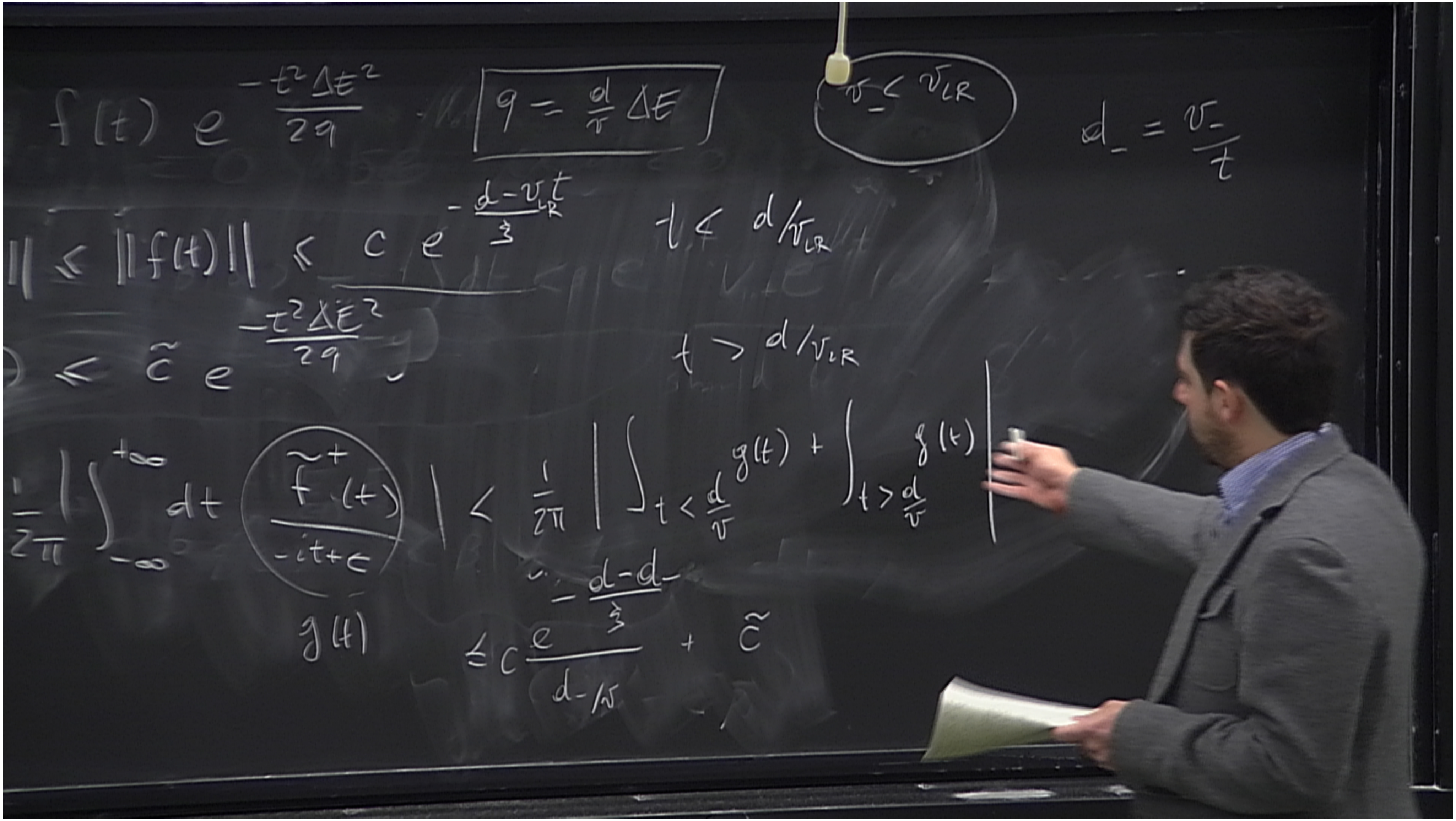
$$① \quad \|\tilde{f}(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}} \qquad t < d/v_{LR}$$

$$② \quad \tilde{f}(t) \leq \tilde{c}\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad t > d/v_{LR}$$

$$|\tilde{f}^{+}(0)| = \frac{1}{2\pi}\left|\int_{-\infty}^{+\infty} dt\, \frac{\tilde{f}^{+}(t)}{-it+\epsilon}\right| < \frac{1}{2\pi}\left|\int_{t<\frac{d}{v}} g(t) + \int_{t>\frac{d}{v}} g(t)\right|$$

$g(t)$


$\tilde{f}(t) \equiv f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}}$
$q = \frac{d}{v} \Delta E$
$v < v_{LR}$
$d_- =$
① $\|\tilde{f}(t)\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}}$ $t < d/v_{LR}$
② $\tilde{f}(t) \leq \tilde{c}\, e^{-\frac{t^2 \Delta E^2}{2q}}$ $t > d/v_{LR}$
$|\tilde{f}^+(0)| = \frac{1}{2\pi} \left| \int_{-\infty}^{+\infty} dt \frac{\tilde{f}^+(t)}{-it+\epsilon} \right| < \frac{1}{2\pi} \left| \int_{t<\frac{d}{v}} g(t) + \int_{t>\frac{d}{v}} g(t) \right|$
$g(t)$
$= \frac{e^{-\frac{d-d_-}{\xi}}}{d_-/v}$


f(t) e^{-t²ΔE²/2q}
q = d/v ΔE
v₋ < v_LR
d₋ = v₋/t
t < d/v_LR
t > d/v_LR
g(t)

$$f(t)\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad \boxed{q = \frac{d}{v} \Delta E}$$

$$v_- < v_{LR}$$

$$d_- = \frac{v_-}{t}$$

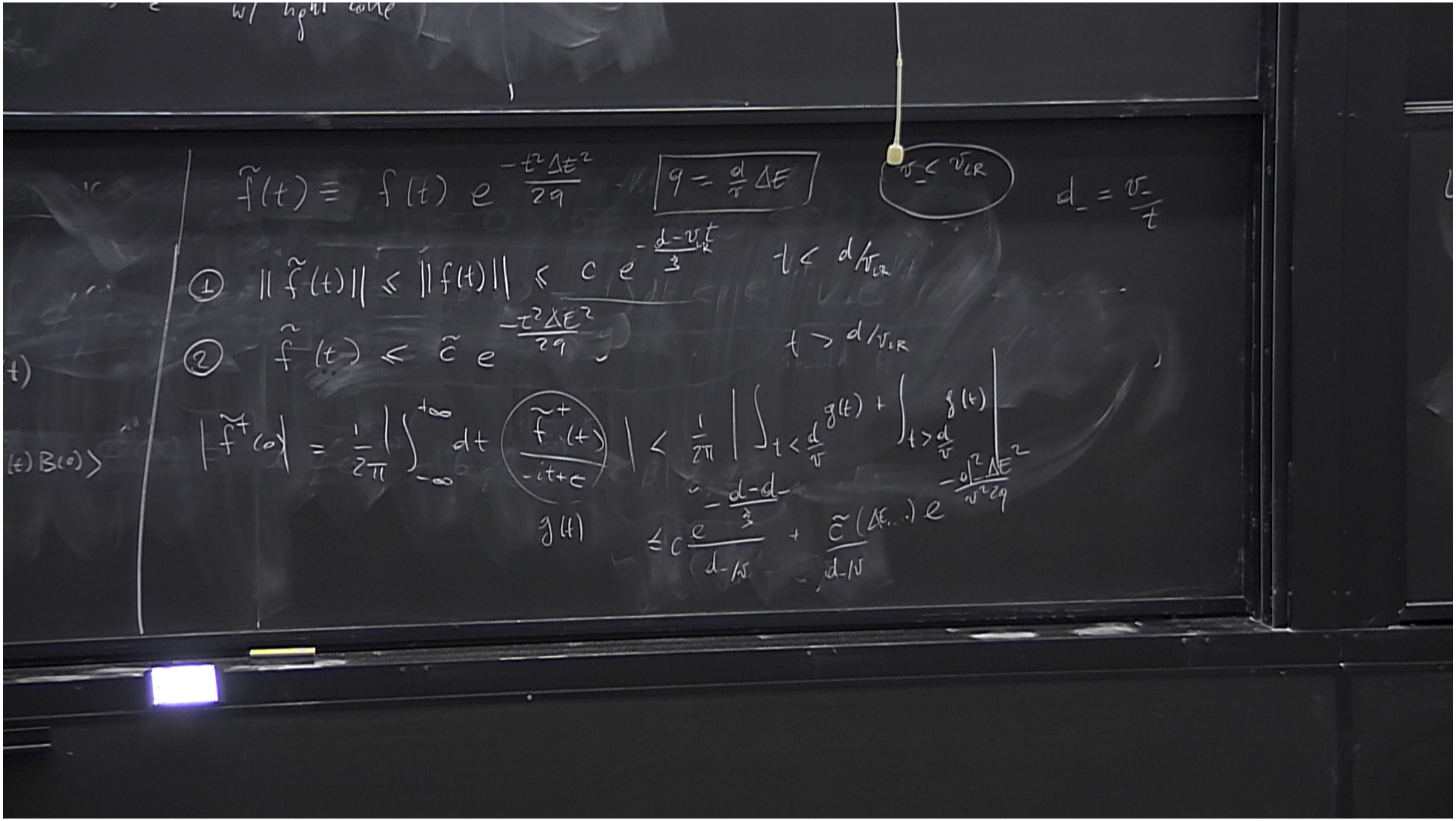
$$\| \leq \|f(t)\| \leq c\, e^{-\frac{d - v_{LR} t}{\xi}} \qquad t \leq d/v_{LR}$$

$$\leq \tilde{c}\, e^{-\frac{t^2 \Delta E^2}{2q}} \qquad t > d/v_{LR}$$

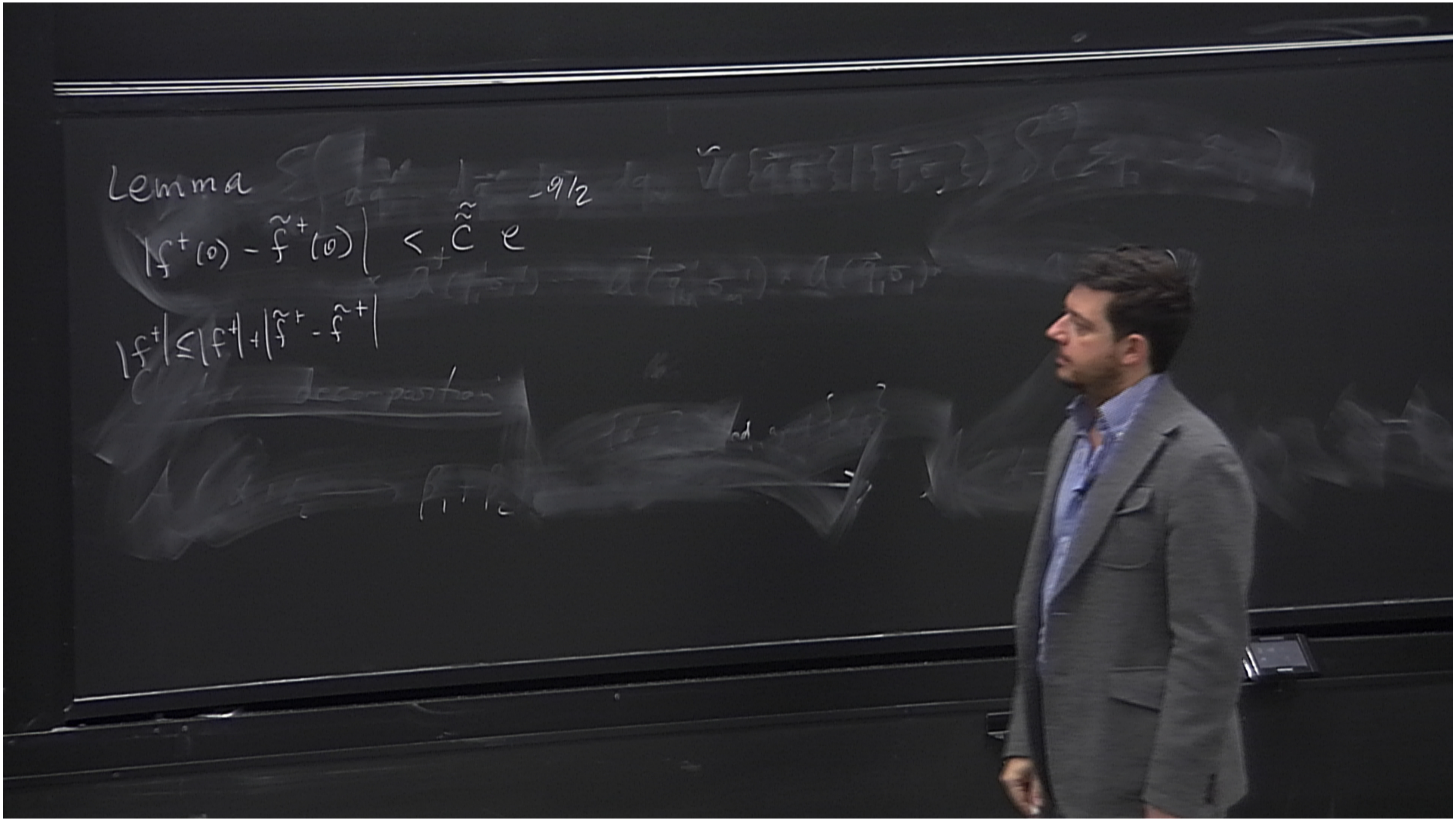
$$\frac{1}{2\pi} \left| \int_{-\infty}^{+\infty} dt\, \frac{\tilde{f}^{+}(t)}{-it+\epsilon} \right| < \frac{1}{2\pi} \left| \int_{t < \frac{d}{v}} g(t) + \int_{t > \frac{d}{v}} g(t) \right|$$

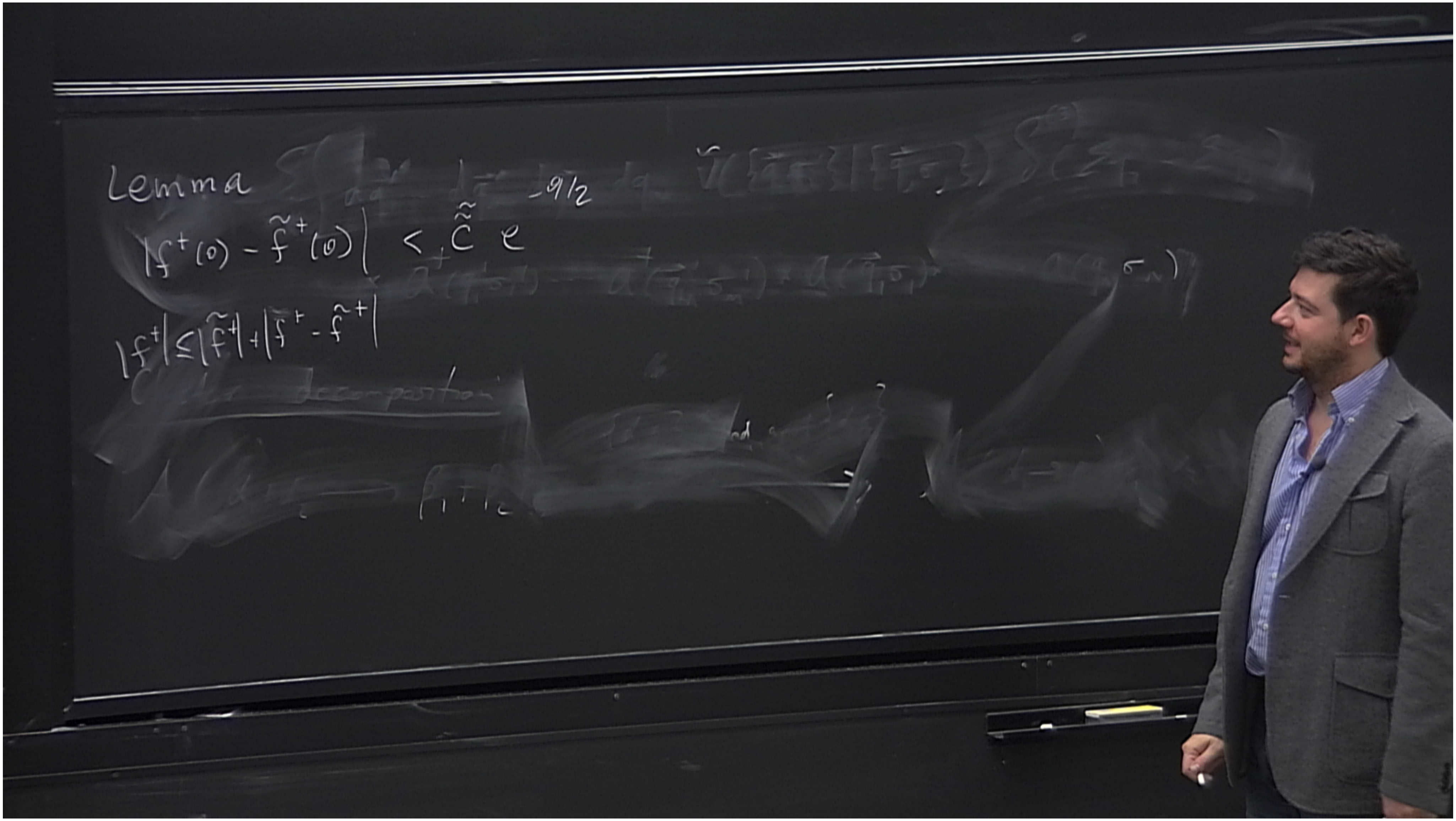
$$g(t)$$

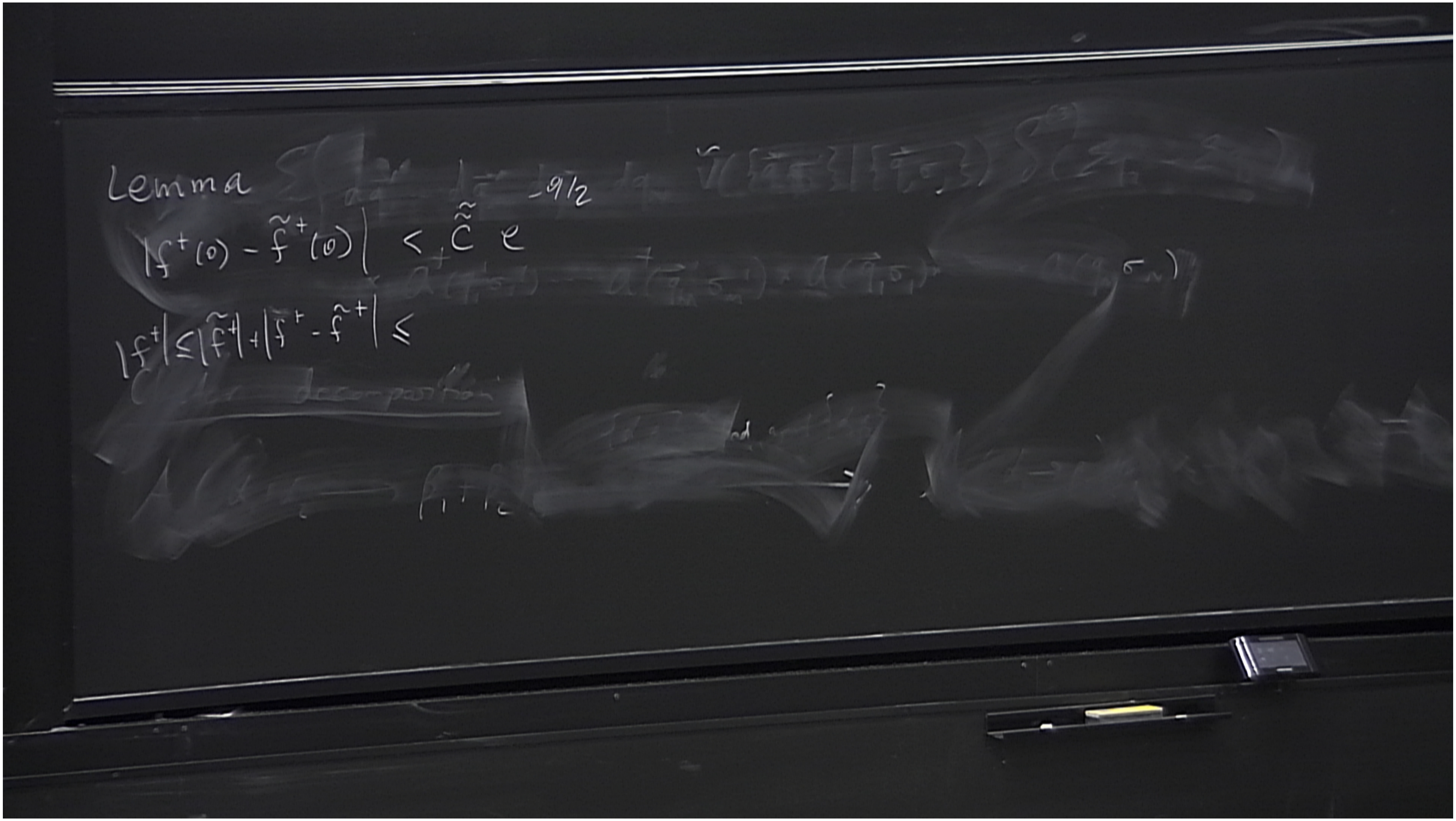
$$\leq c\, \frac{e^{-\frac{d-d_-}{\xi}}}{d_-/v} + \frac{\tilde{c}(\Delta E, \ldots)}{d_-/v}\, e^{-\frac{d_-^2 \Delta E^2}{v^2\, 2q}}$$

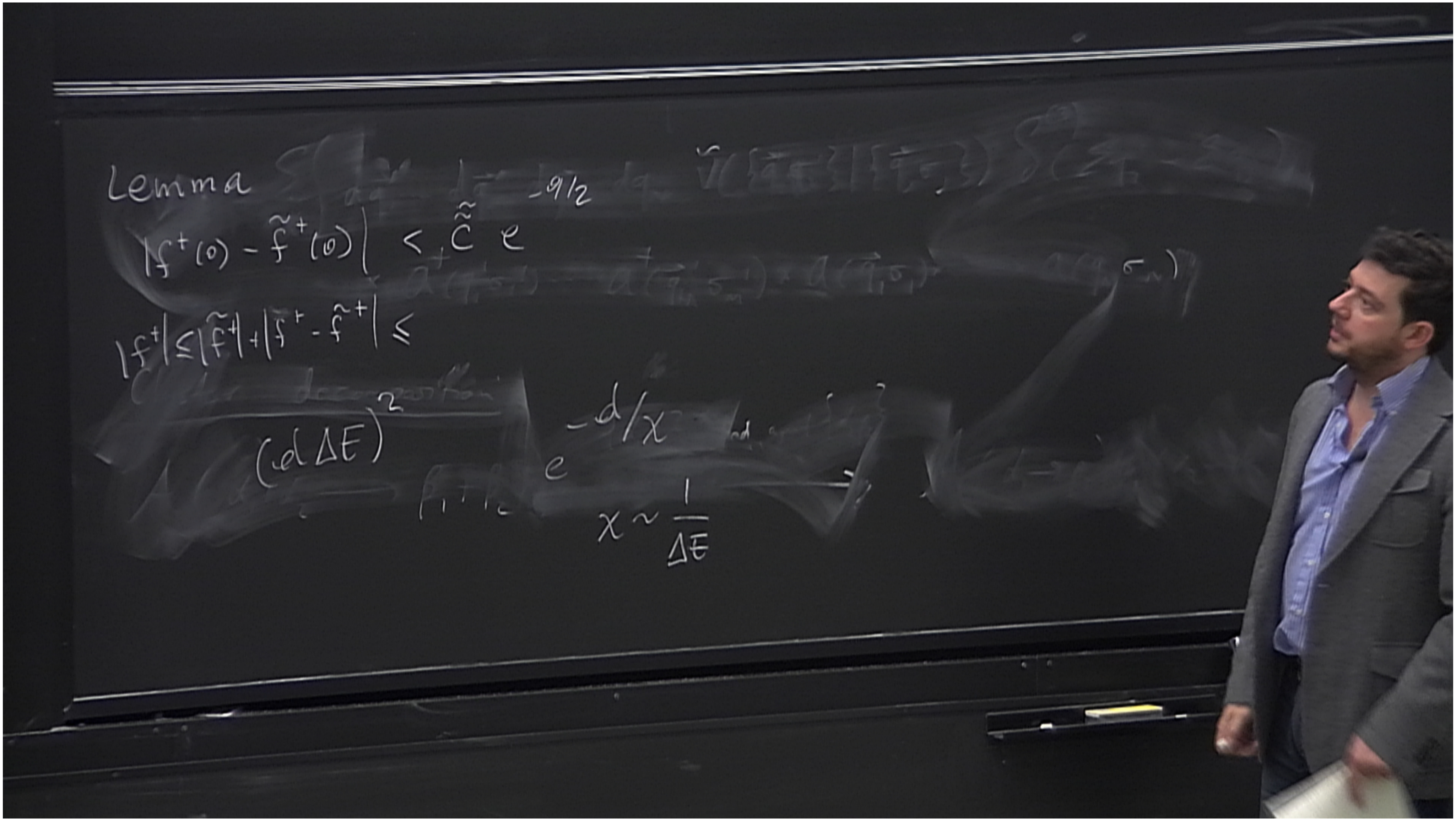


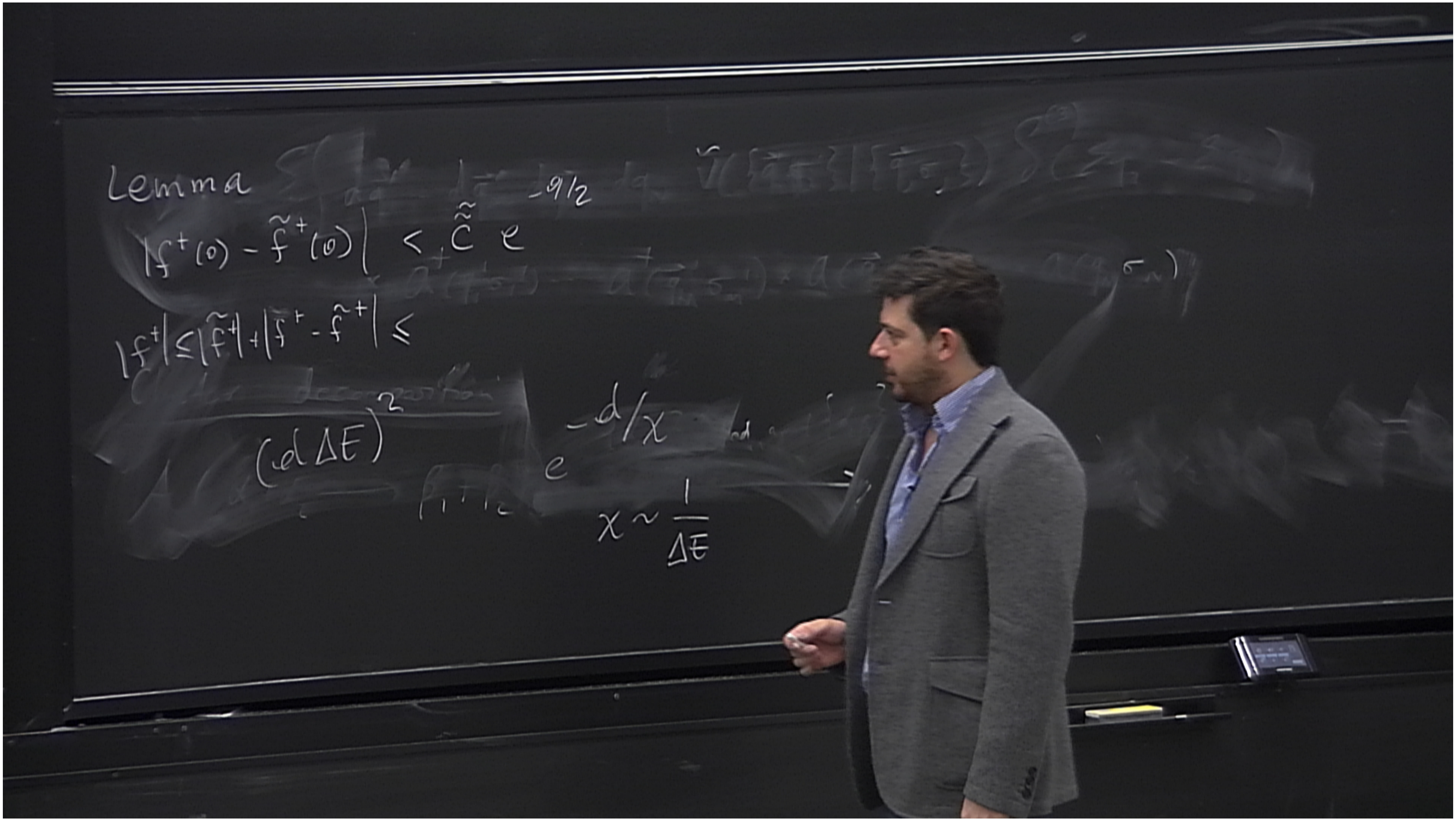
f̃(t) = f(t) e^(−t²ΔE²/2q)
q = (d/v) ΔE
v₋ < v_LR
d₋ = v₋/t
① ‖f̃(t)‖ ≤ ‖f(t)‖ ≤ c e^(−(d − v_LR t)/ξ)   t < d/v_LR
② f̃(t) ≤ c̃ e^(−t²ΔE²/2q)   t > d/v_LR
|f̃⁺(0)| = (1/2π) |∫_{−∞}^{+∞} dt f̃⁺(t)/(−it+ε)| < (1/2π) |∫_{t<d/v} g(t) + ∫_{t>d/v} g(t)|
g(t)
≤ c e^(−(d−d₋)/ξ)/(d₋/v) + c̃(ΔE, ..) e^(−d₋²ΔE²/(v²2q))/(d₋/v)

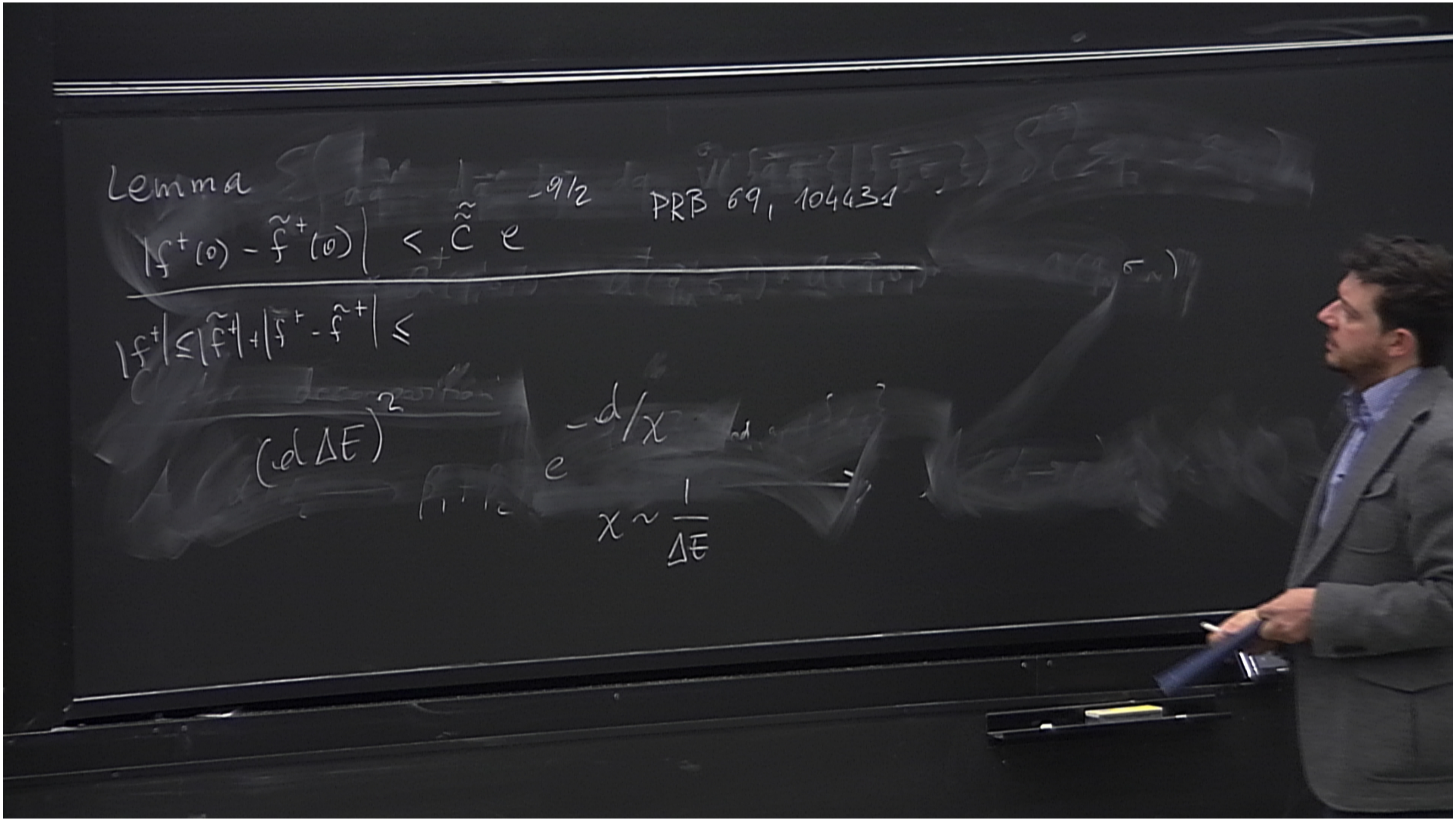

Lemma

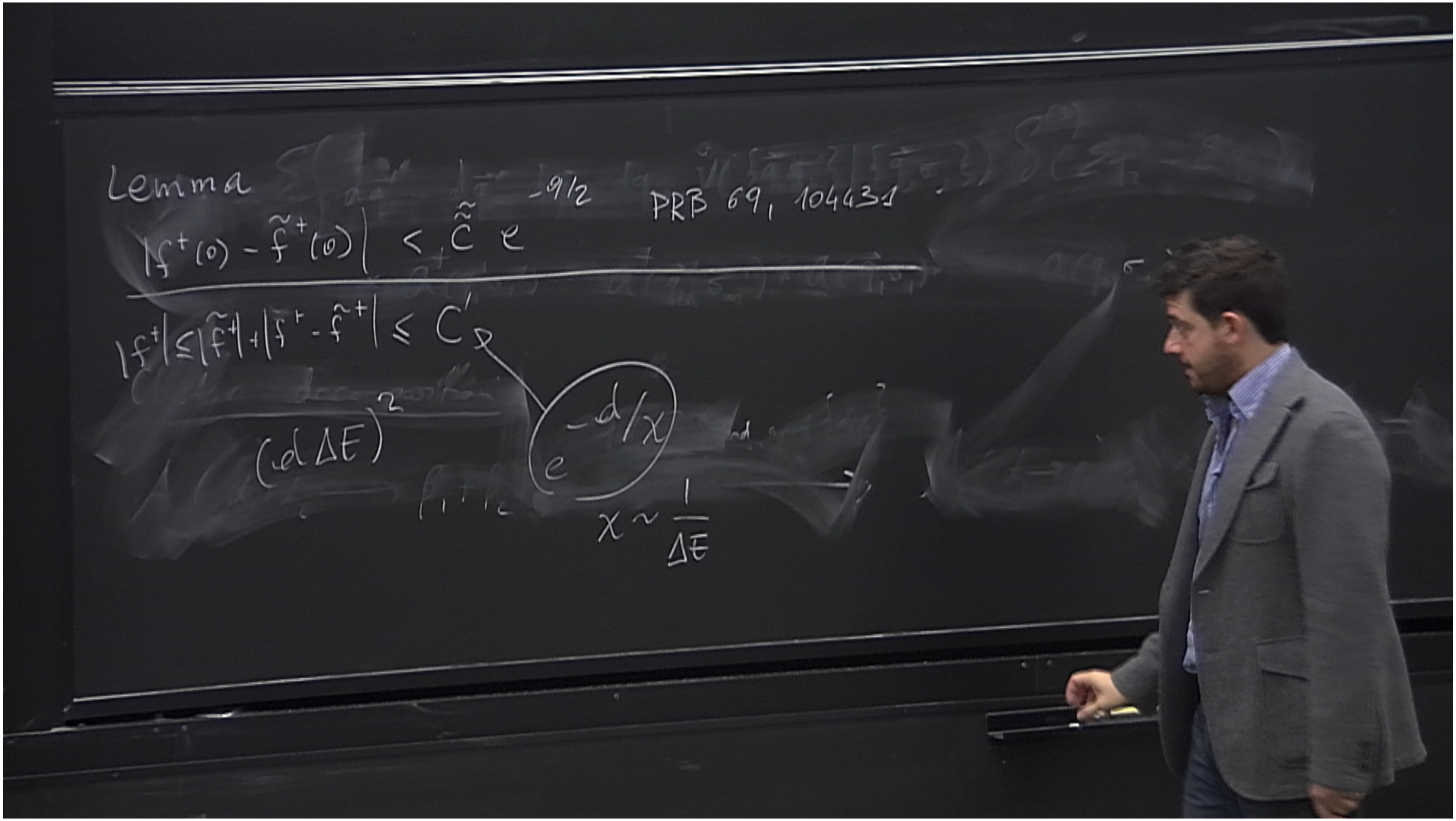

Lemma
$|f^+(0) - \tilde{f}^+(0)| < \tilde{\tilde{C}} e^{-q/2}$
$|f^+| \leq |\tilde{f}^+| + |f^+ - \tilde{f}^+|$


Lemma
|f⁺(0) − f̃⁺(0)| < C̃̃ e^{-g/2}
|f⁺| ≤ |f̃⁺| + |f⁺ − f̃⁺| ≤


Lemma
|f+(0) - f̃+(0)| < C̃̃ e -q/2
|f+| ≤ |f̃+| + |f+ - f̃+| ≤
(d ΔE)2
e -d/χ
χ ~ 1/ΔE


Lemma


Lemma
PRB 69, 104431
$|f^+(0) - \tilde{f}^+(0)| < \tilde{\tilde{C}} e^{-9/2}$
$|f^+| \leq |\tilde{f}^+| + |f^+ - \tilde{f}^+| \leq$
$(d \Delta E)^2$
$e^{-d/\chi}$
$\chi \sim \frac{1}{\Delta E}$


Lemma
PRB 69, 104431
|f⁺(0) − f̃⁺(0)| < C̃̃ e^−9/2
|f⁺| ≤ |f̃⁺| + |f⁺ − f̃⁺| ≤ C′
(d ΔE)²
e^−d/χ
χ ~ 1/ΔE