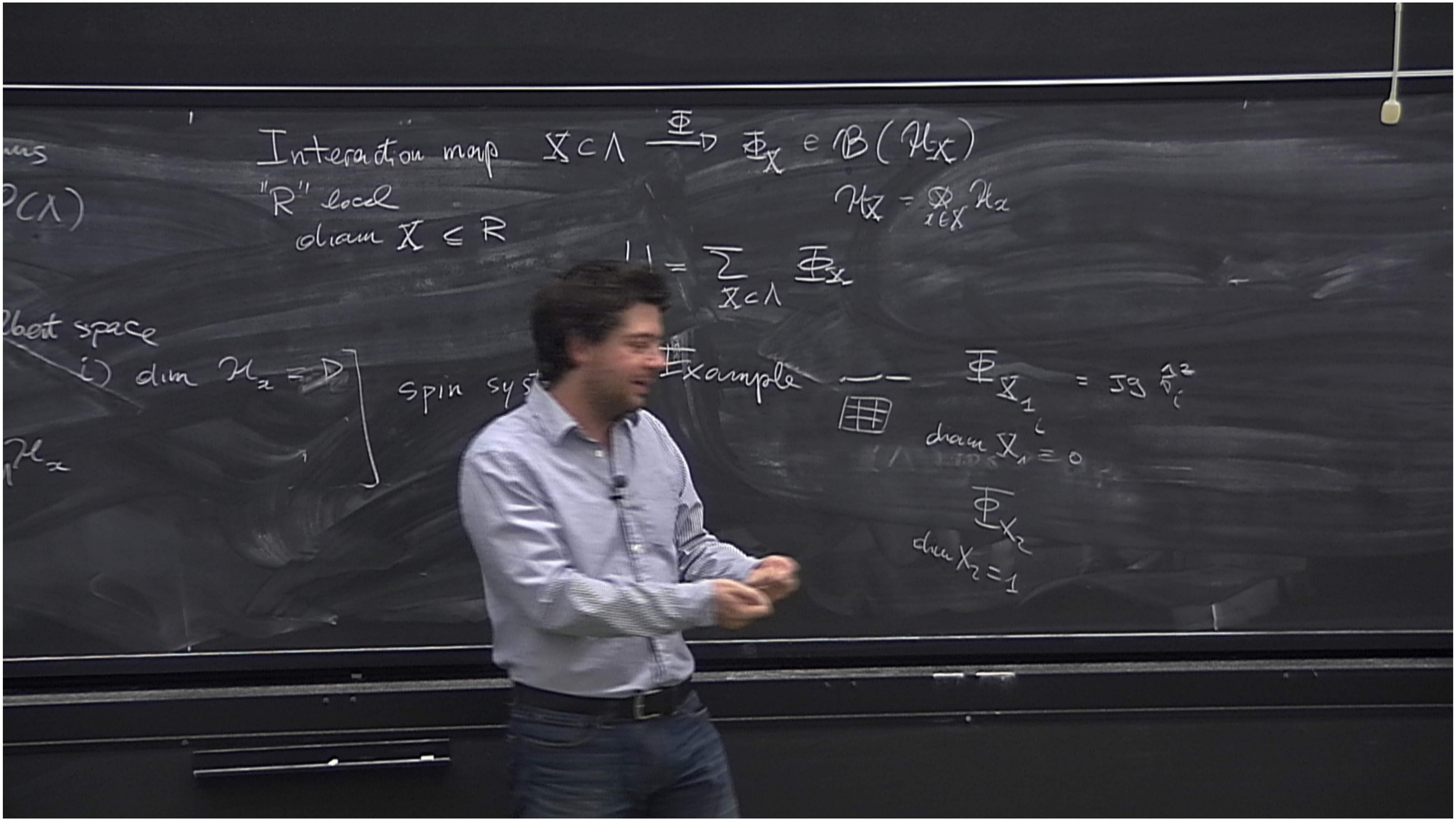
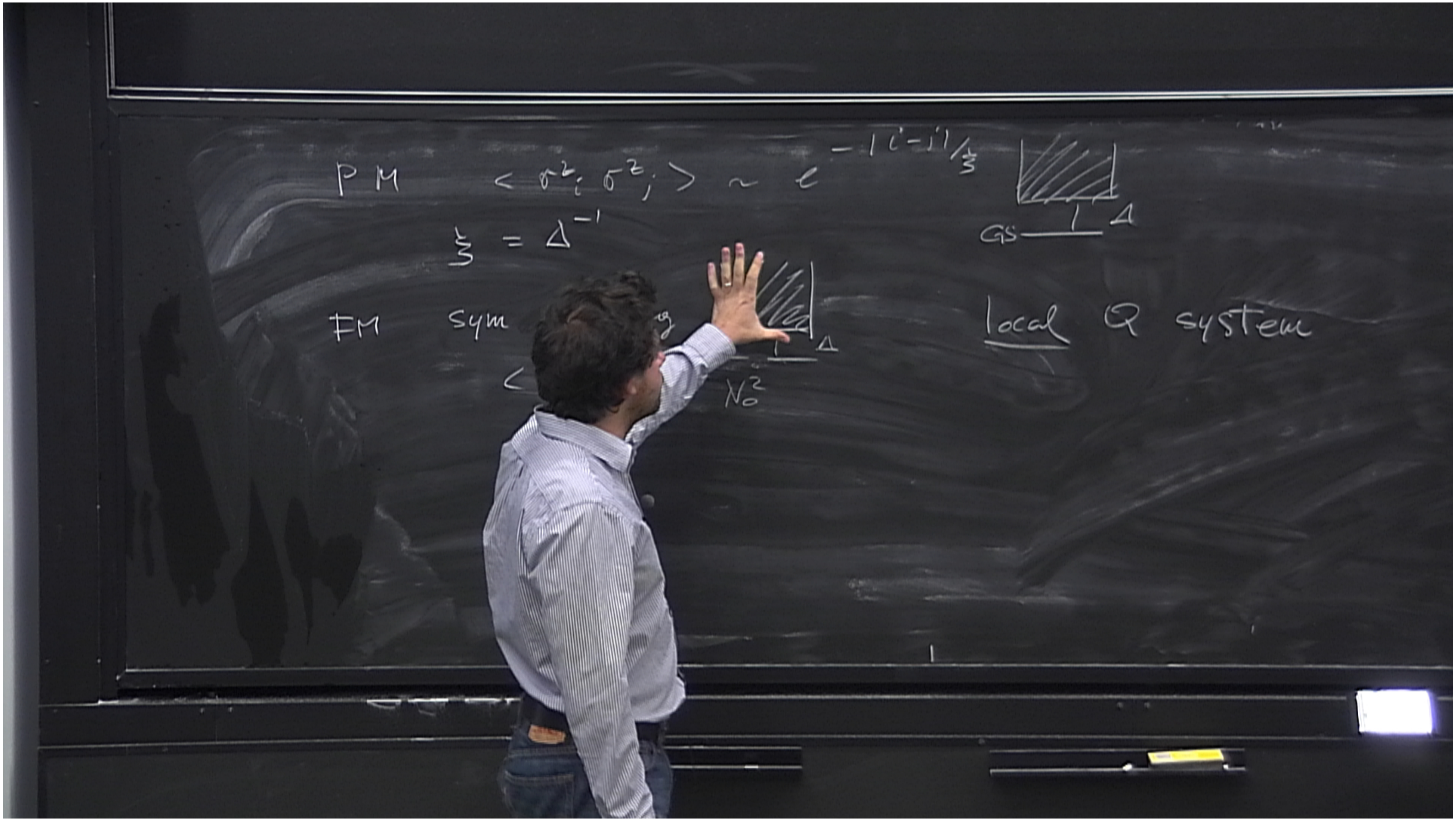
Title: Condensed Matter (Review) - Lecture 5

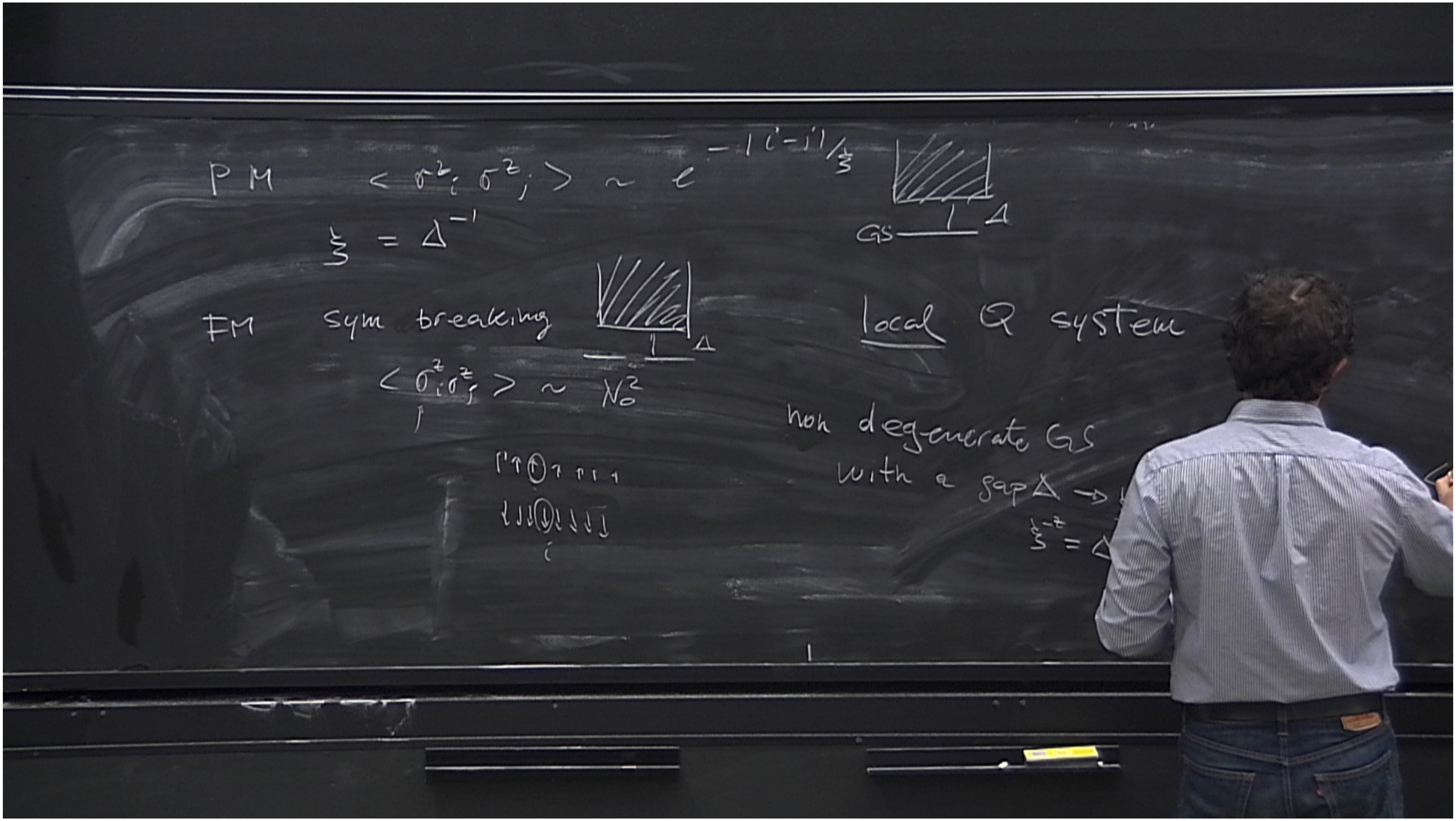
Date: Jan 06, 2012 10:15 AM

URL: http://pirsa.org/12010087

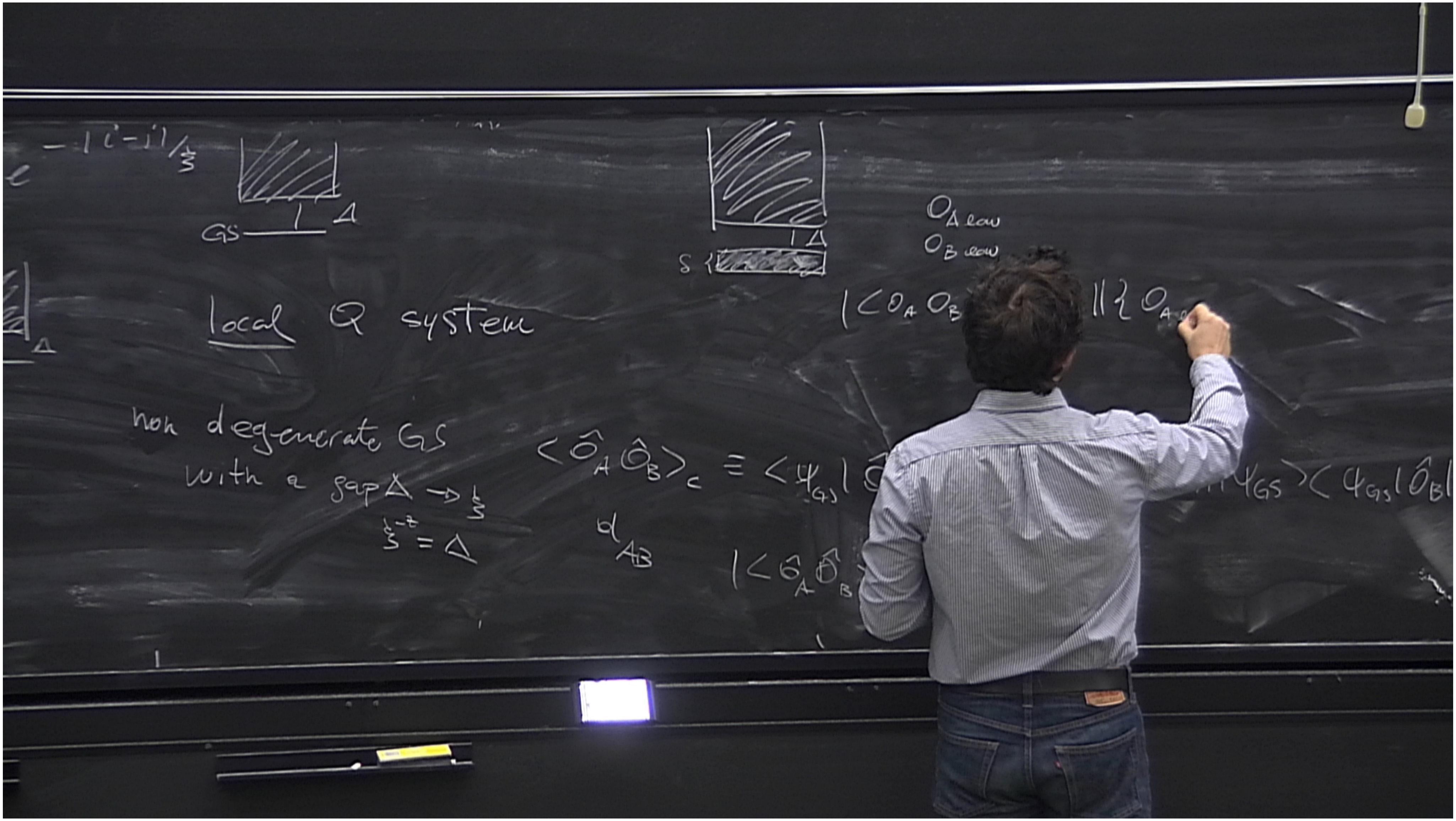
Abstract:

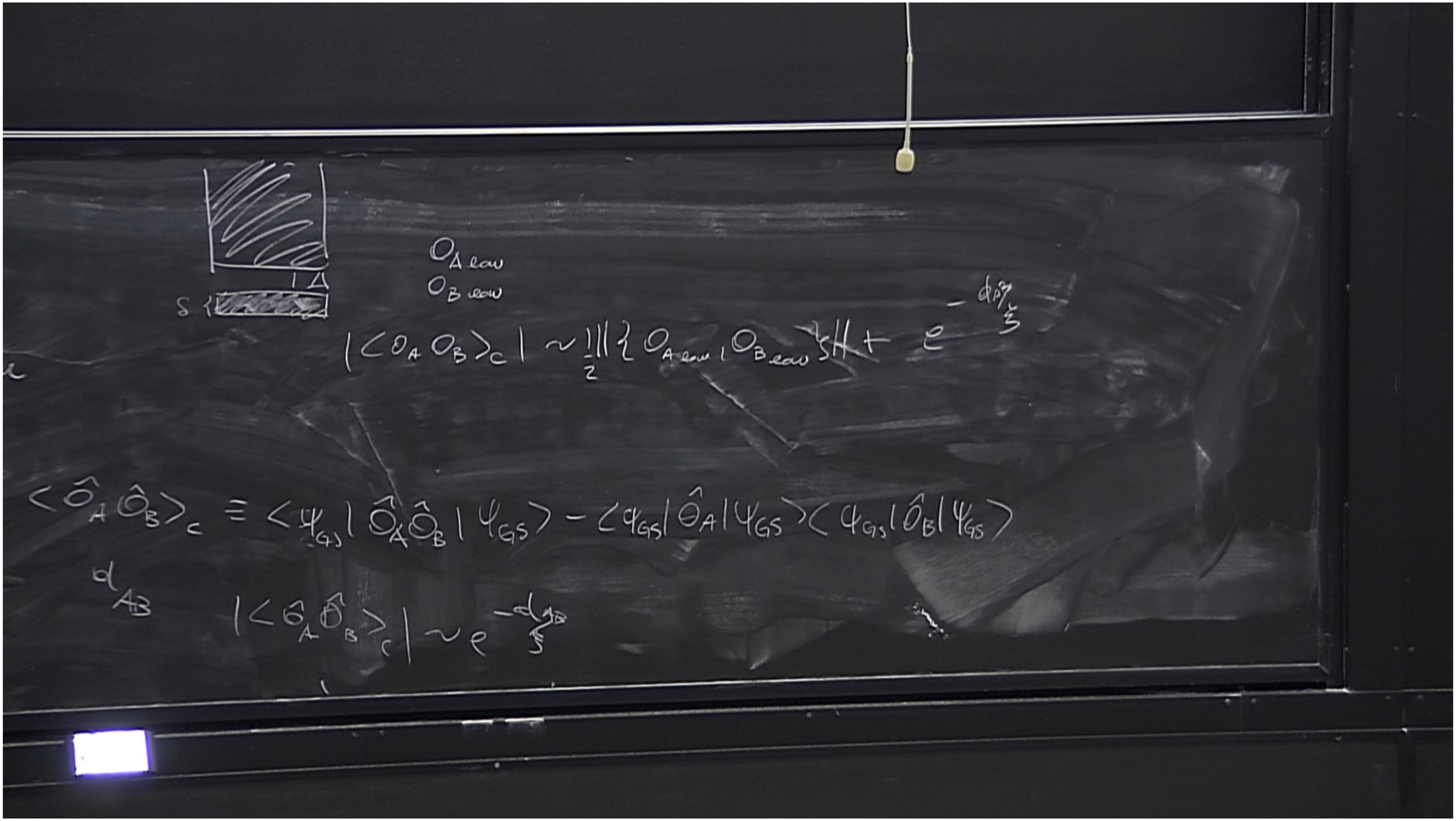

Interaction map
"R" local
diam X ≤ R
spin sys
Example
dim X₂ = 1

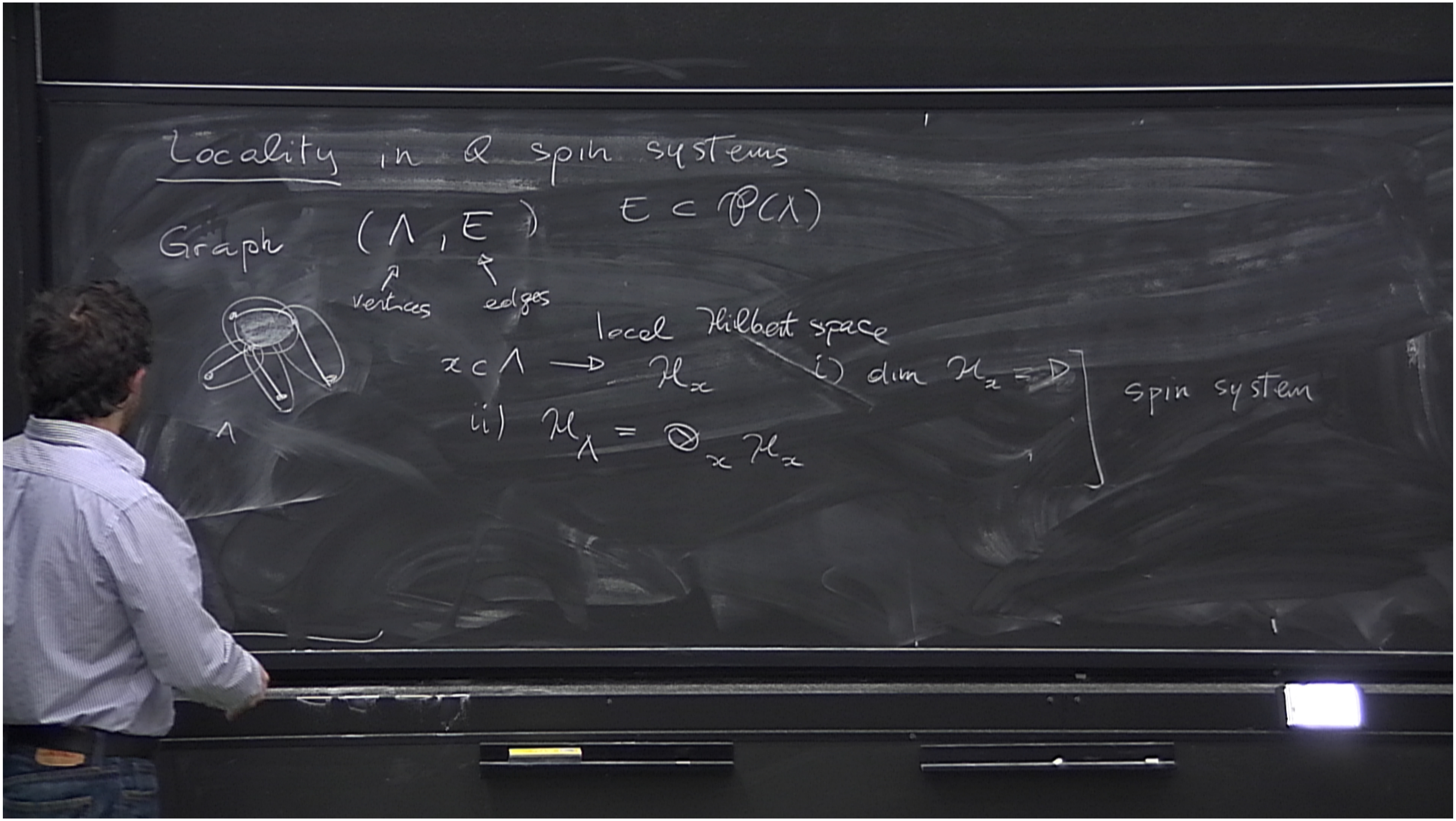

PM $\langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$
$\xi = \Delta^{-1}$
GS
Δ
FM sym
N_0^2
local Q system



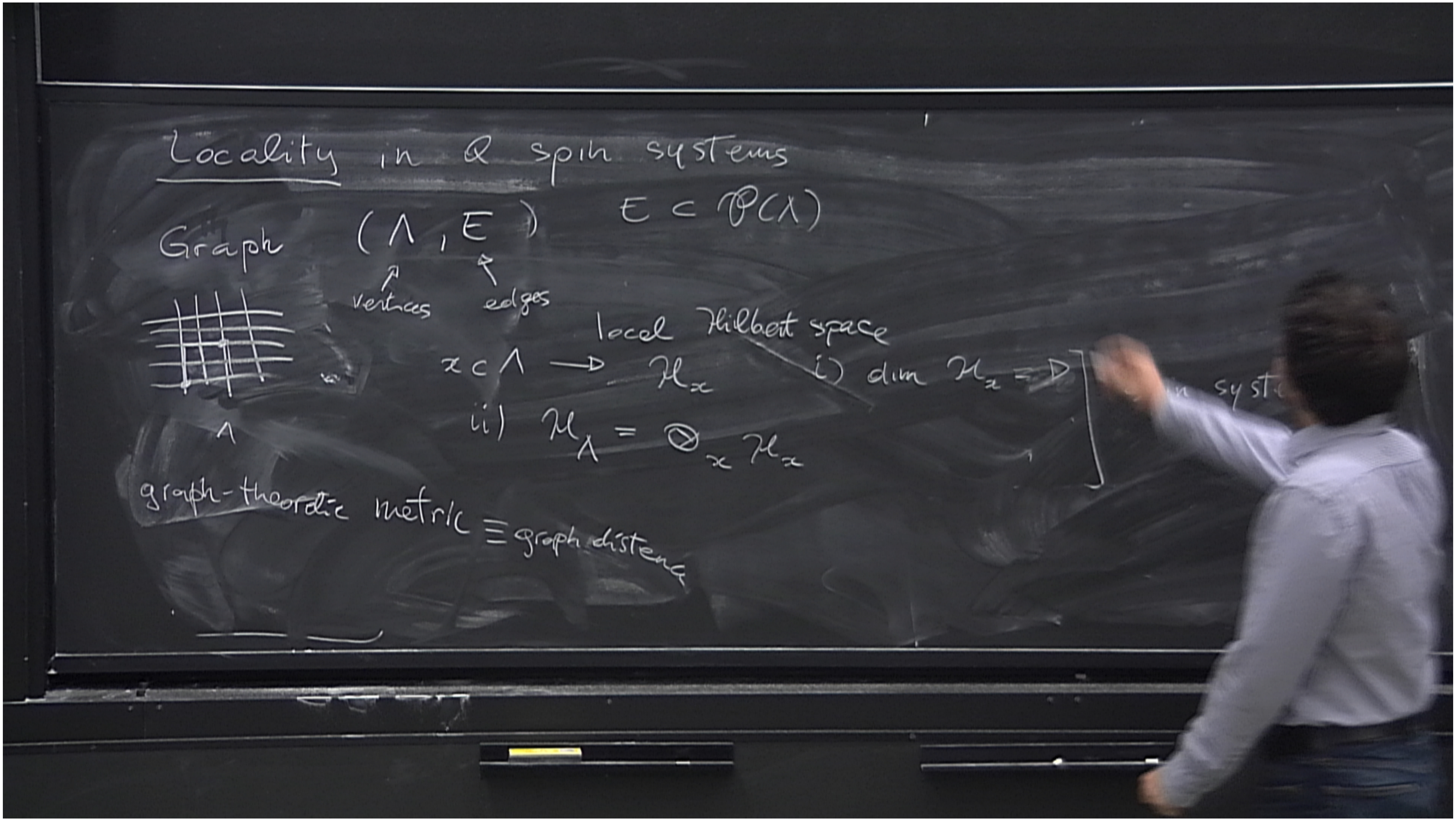
PM $\langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$
$\xi = \Delta^{-1}$
GS
Δ
FM sym breaking
Δ
$\langle \sigma^z_i \sigma^z_j \rangle \sim N_0^2$
i
Local Q system
non degenerate GS
with a gap $\Delta \rightarrow$
$\xi^{-z} = \Delta$

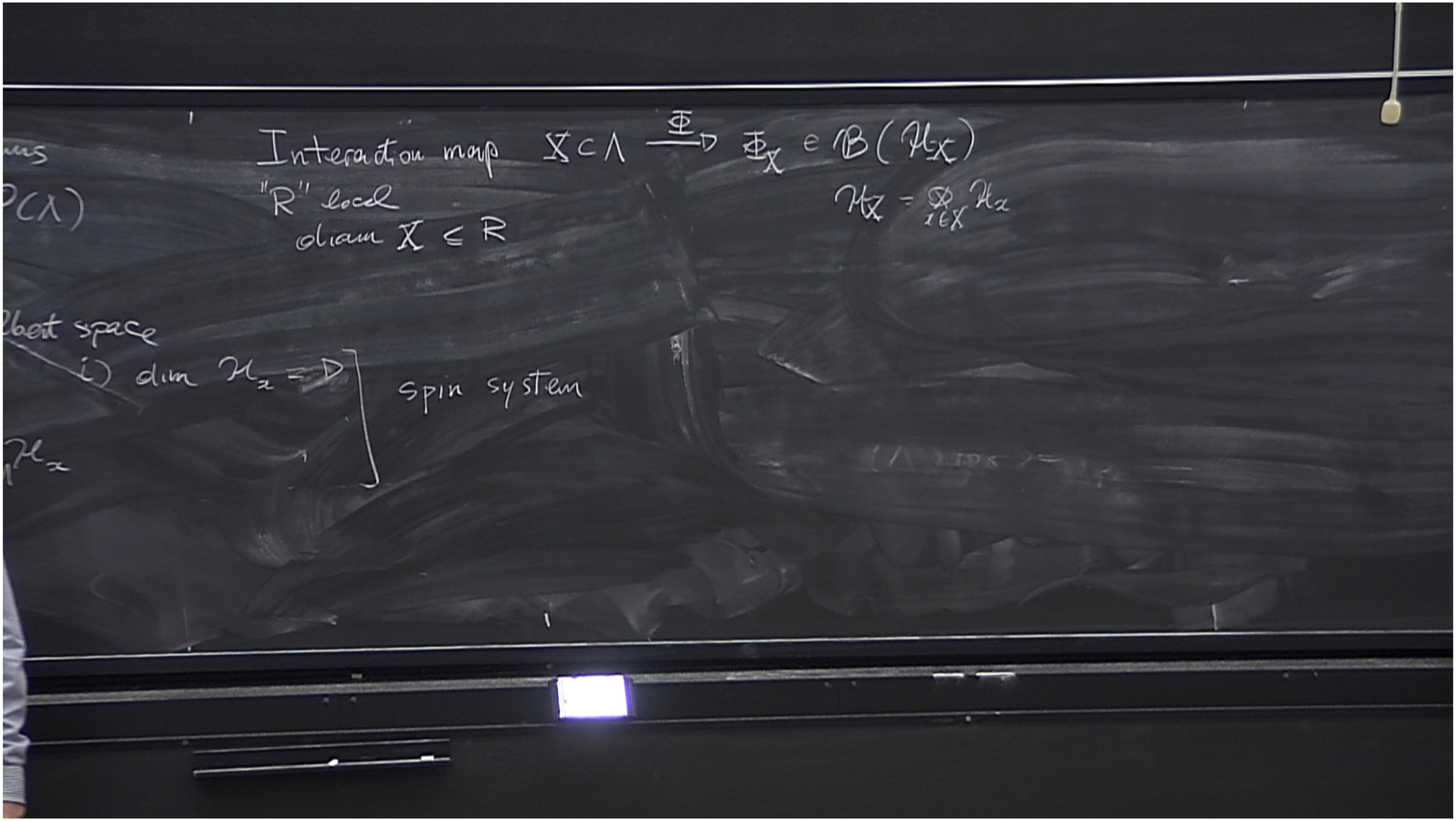

GS
local Q system
non degenerate GS
with a gap Δ


Δ
δ
$O_{A\,low}$
$O_{B\,low}$
$|\langle O_A O_B\rangle_c| \sim \frac{1}{2}\|\{O_{A\,low}, O_{B\,low}\}\| + e^{-d_{AB}/\xi}$
$\langle \hat{O}_A \hat{O}_B\rangle_c \equiv \langle \Psi_{GS}|\hat{O}_A \hat{O}_B|\Psi_{GS}\rangle - \langle \Psi_{GS}|\hat{O}_A|\Psi_{GS}\rangle\langle \Psi_{GS}|\hat{O}_B|\Psi_{GS}\rangle$
d_{AB}
$|\langle \hat{O}_A \hat{O}_B\rangle_c| \sim e^{-d_{AB}/\xi}$

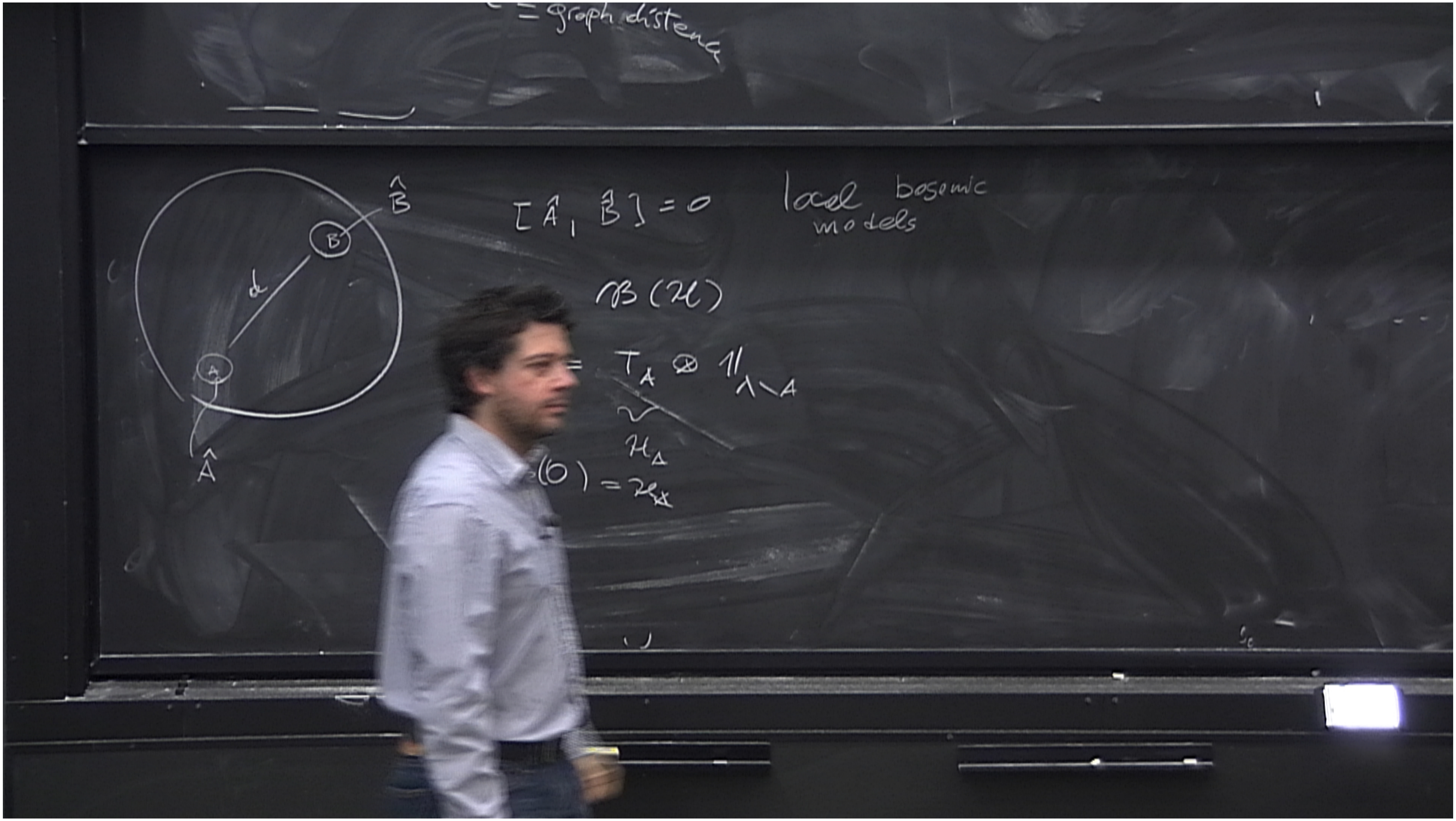


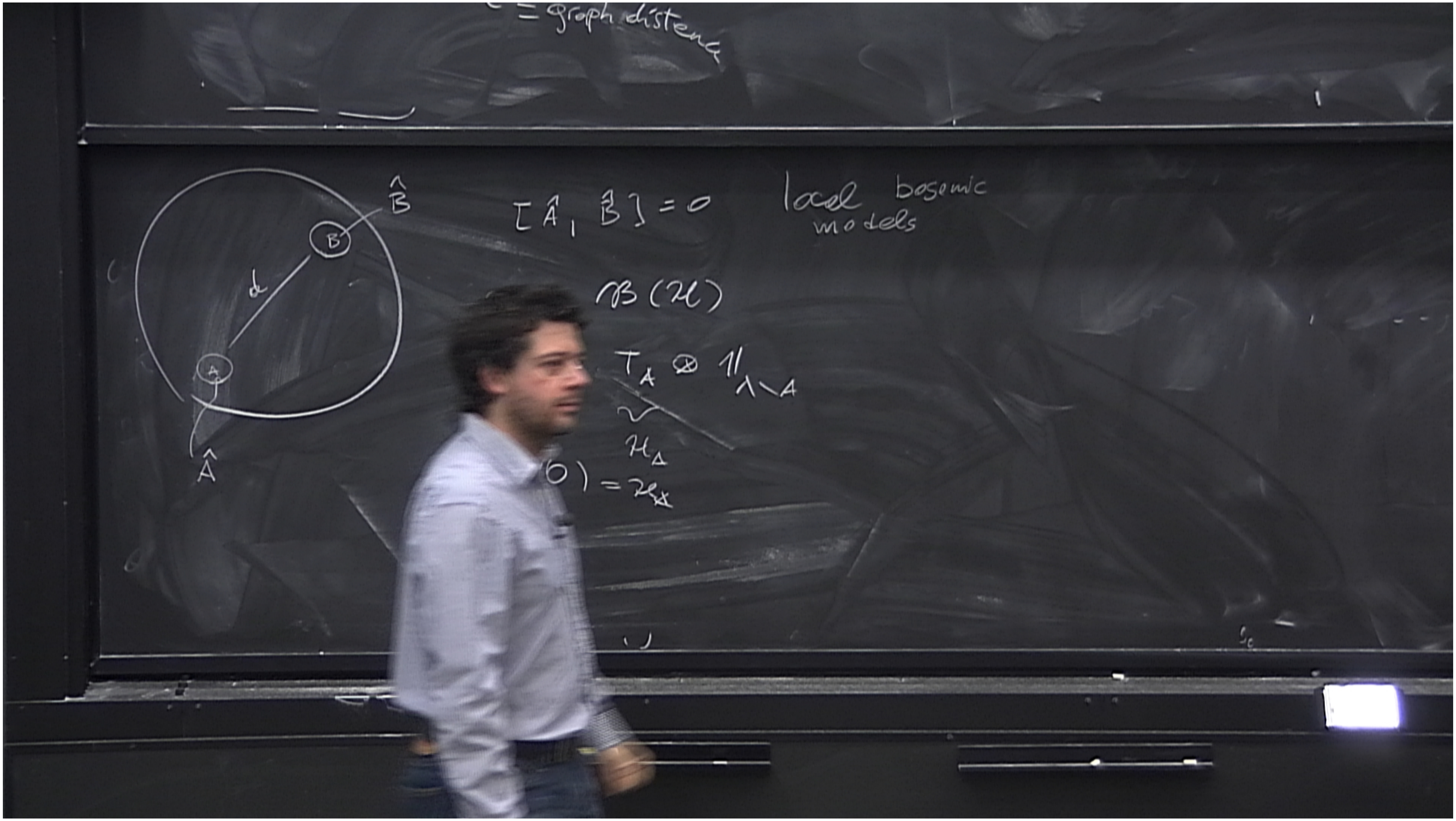
Locality in Q spin systems
Graph (Λ, E)
$E \subset \mathcal{P}(\Lambda)$
vertices
edges
Λ
local Hilbert space
$x \in \Lambda \longrightarrow \mathcal{H}_x$
i) $\dim \mathcal{H}_x = D$
ii) $\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$
spin system

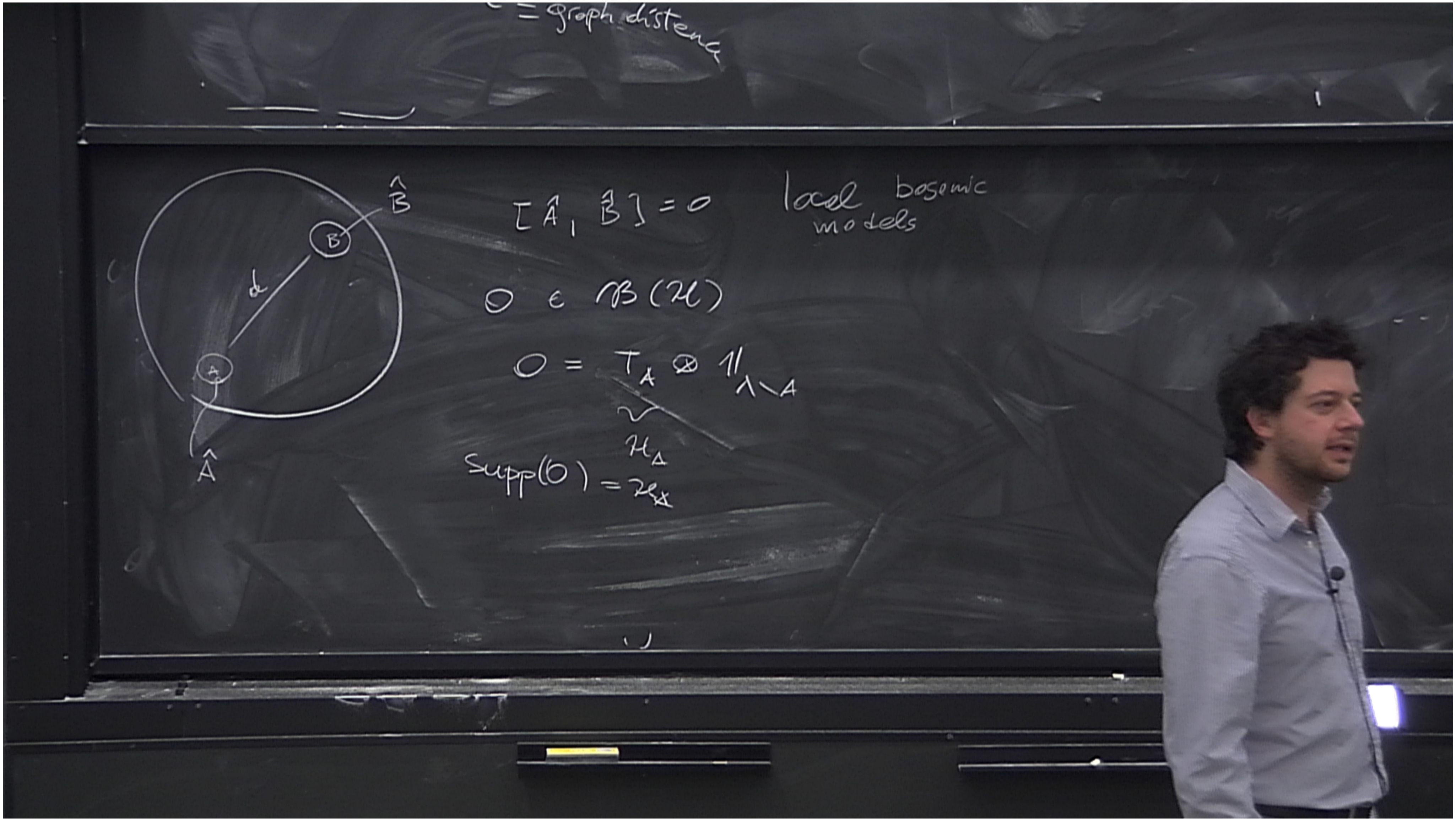

Locality in Q spin systems
Graph (Λ, E)
$E \subset \mathcal{P}(\Lambda)$
vertices
edges
Λ
local Hilbert space
$x \in \Lambda \to \mathcal{H}_x$
i) $\dim \mathcal{H}_x = D$
ii) $\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$
graph-theoretic metric $\equiv$ graph distance

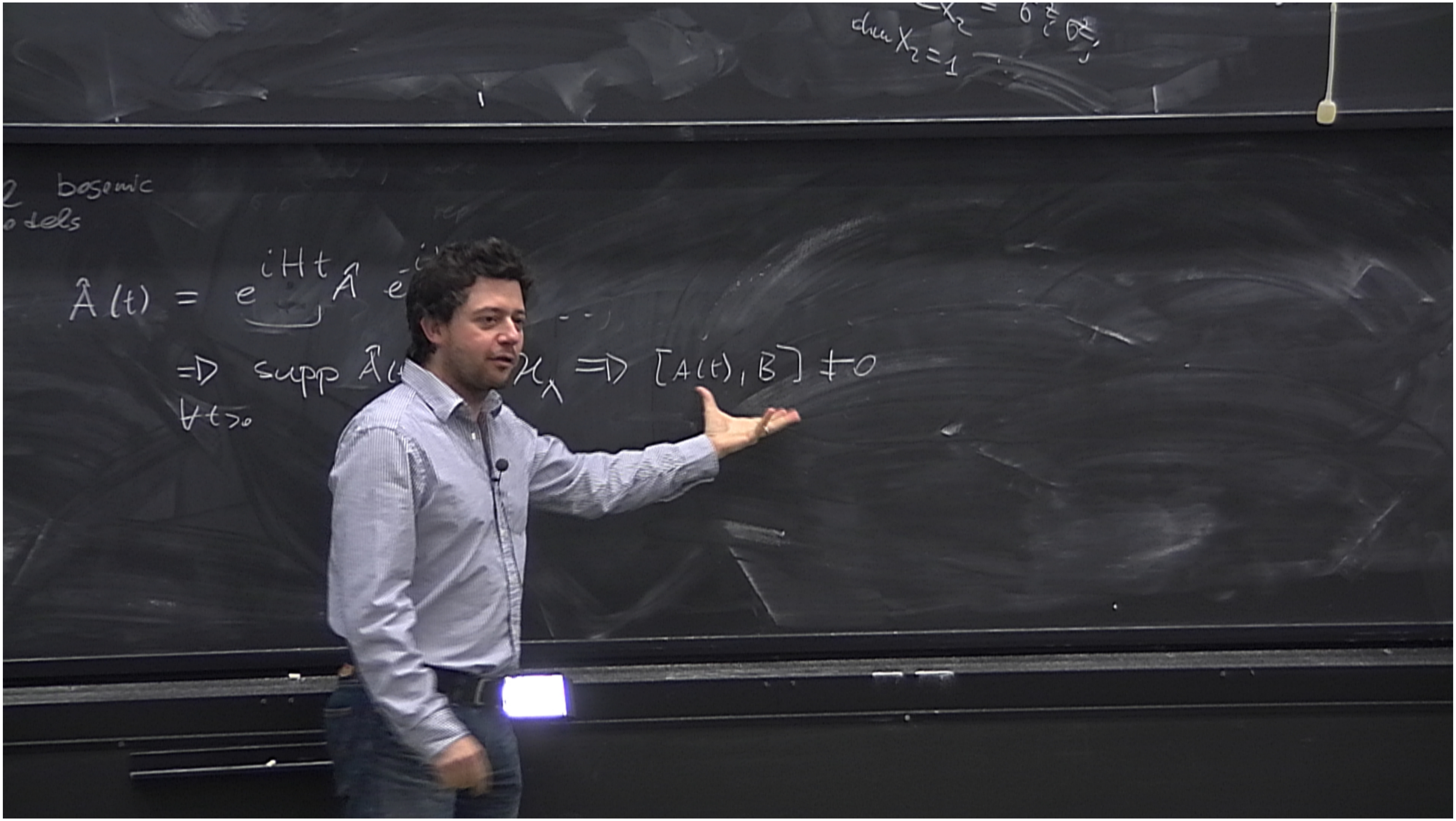


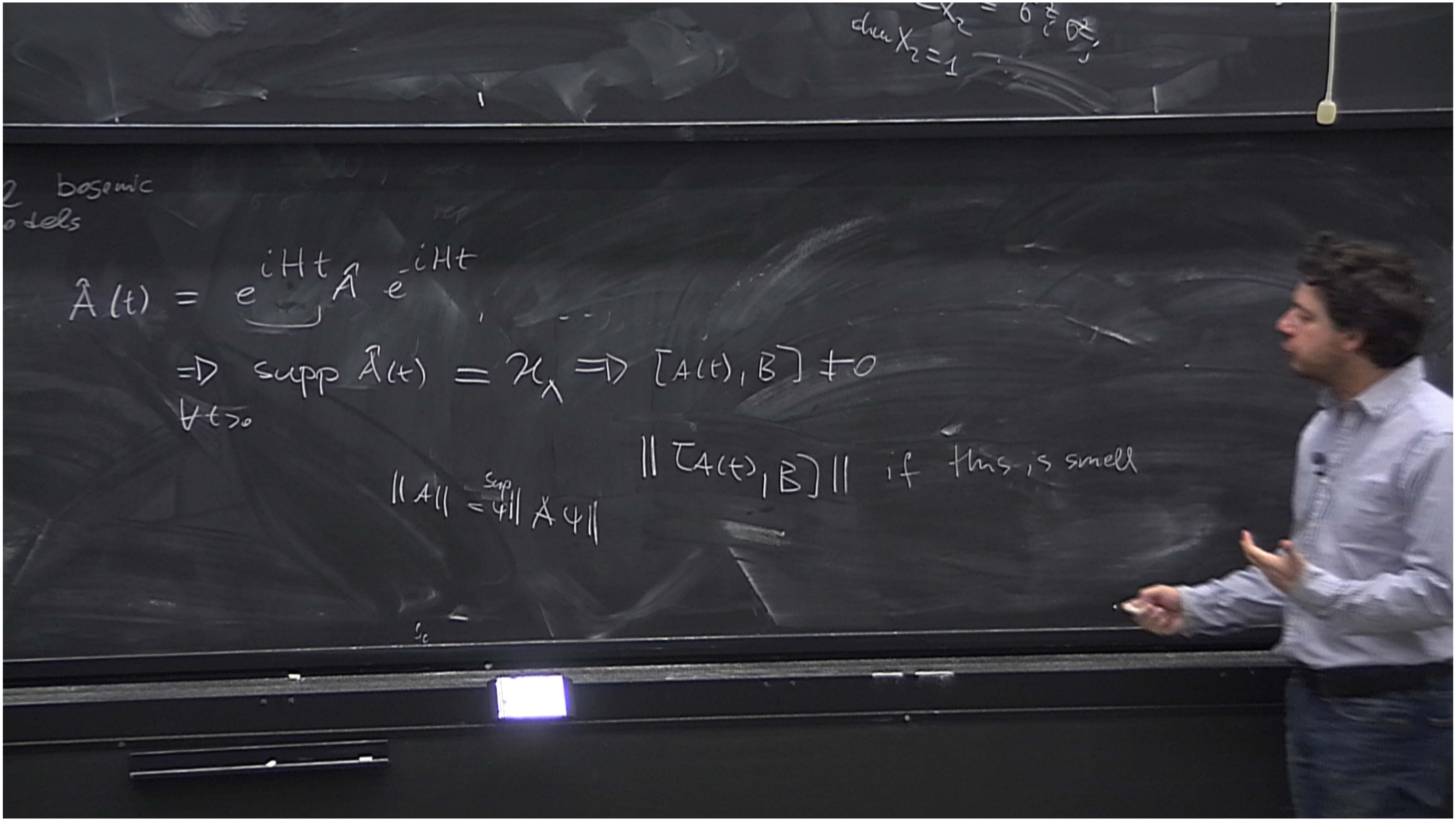
ns
P(Λ)
Interaction map $X \subset \Lambda \xrightarrow{\Phi} \Phi_X \in \mathcal{B}(\mathcal{H}_X)$
"R" local
diam $X \leq R$
$\mathcal{H}_X = \bigotimes_{x \in X} \mathcal{H}_x$
lbert space
i) dim $\mathcal{H}_x = D$
spin system
$\mathcal{H}_x$

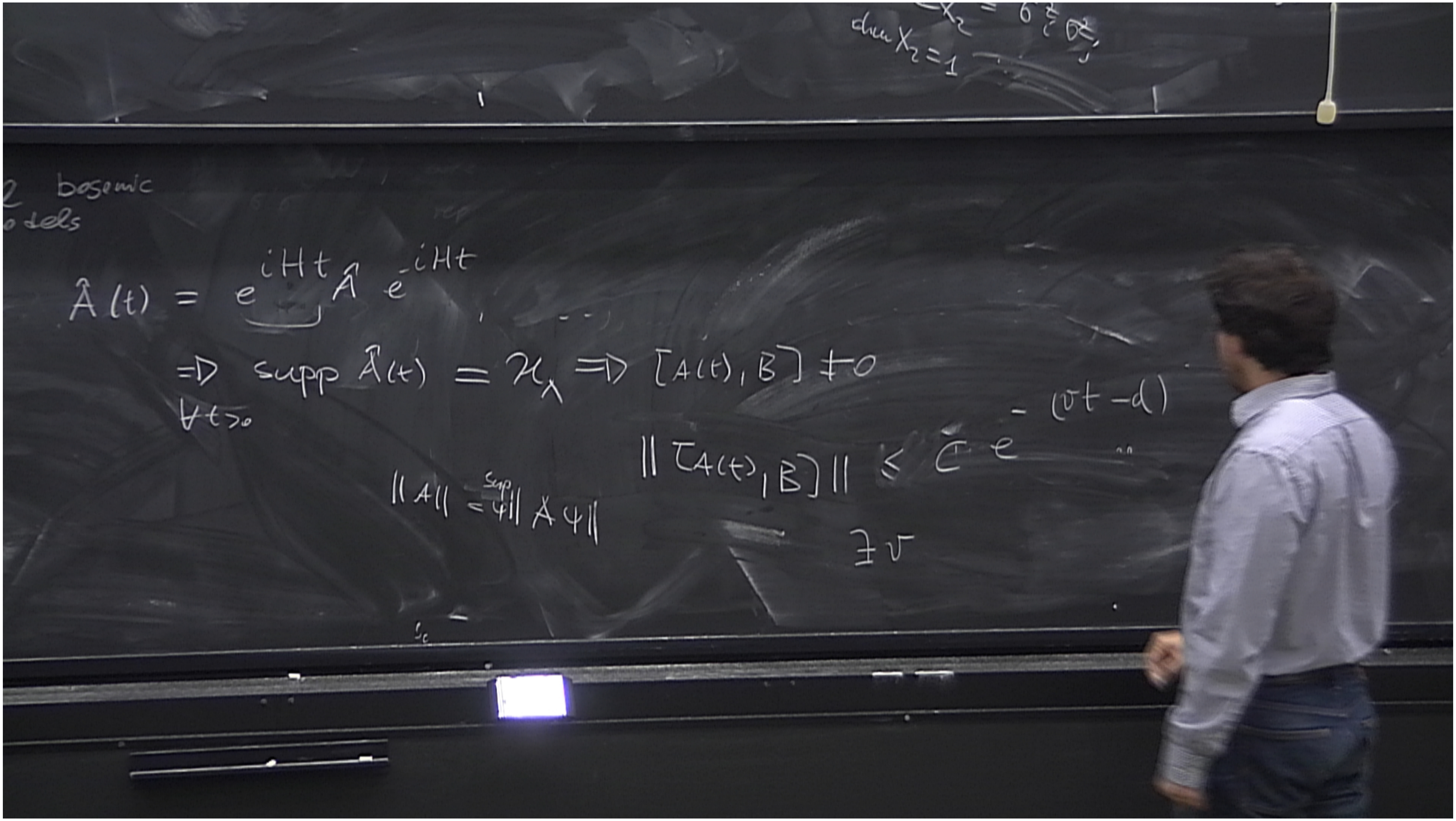

[Â, B̂] = 0
local bosonic models
B(H)
T_A ⊗ 1_{Λ∖A}
H_A
B̂
Â

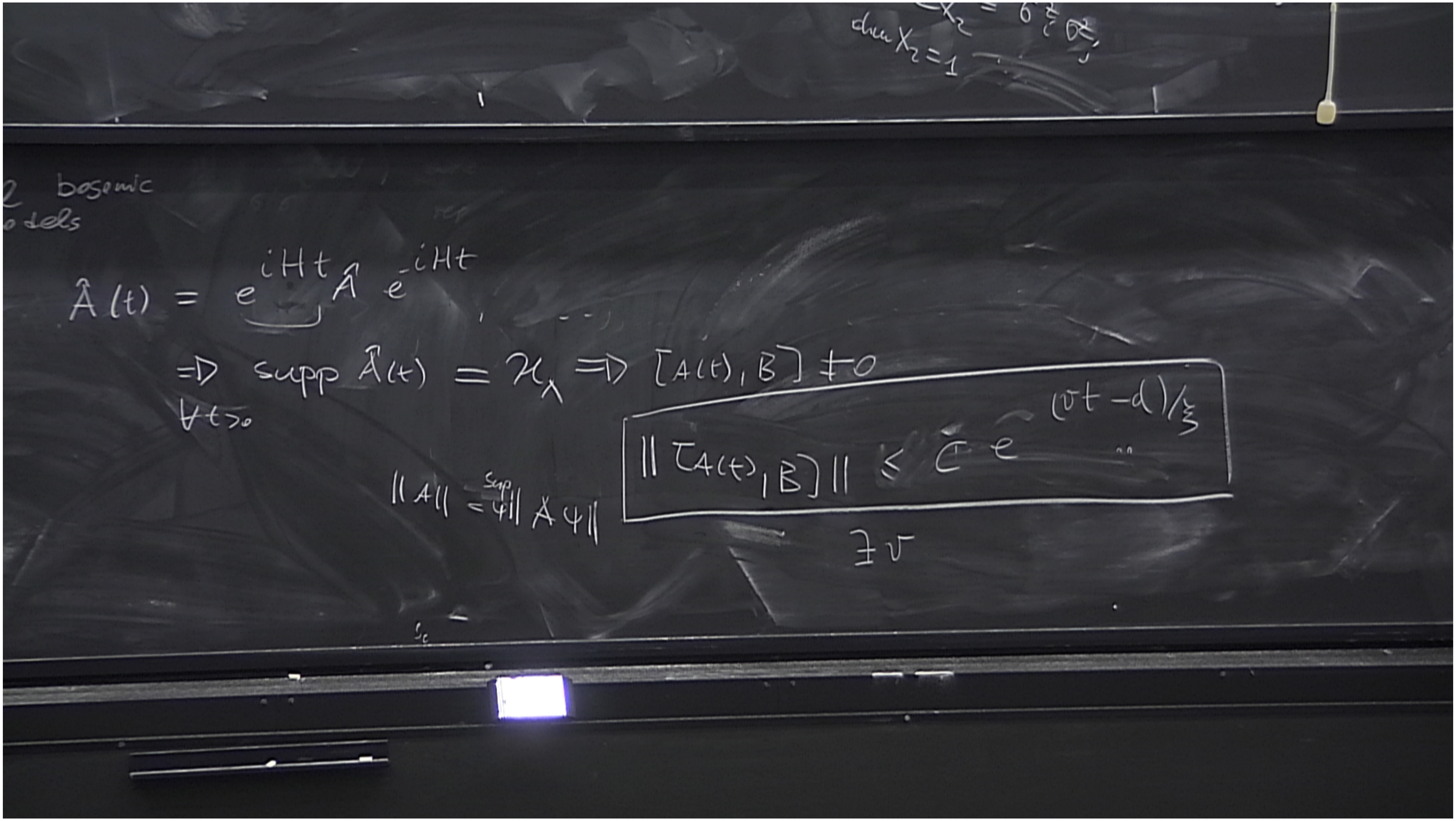

= graph distance
[Â, B̂] = 0
local bosonic models
d
Â
B̂


= graph distance
[Â, B̂] = 0
local bosonic models
O ∈ B(H)
O = T_A ⊗ 1_{Λ\A}
H_A
Supp(O) = H_A
B̂
Â


bosonic
odels
$\hat{A}(t) = e^{iHt}\hat{A}\,e^{-i}$
$\Rightarrow$ supp $\hat{A}$
$\Rightarrow [A(t), B] \neq 0$
$\forall t>0$
$\dim X_2 = 1$


bosonic
models
Â(t) = e^{iHt} Â e^{-iHt}
=D supp Â(t) = H_Λ =D [A(t), B] ≠ 0
∀t>0
||[A(t), B]|| if this is small
||A|| = sup_ψ ||Aψ||
dim X_2 = 1


bosonic
models
$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$\Rightarrow \text{supp } \hat{A}(t) = \mathcal{H}_\Lambda \Rightarrow [A(t), B] \neq 0$
$\forall t > 0$
$\| [A(t), B] \| \leq C e^{-(vt-d)}$
$\|A\| = \sup_\psi \| A \psi \|$
$\exists v$


$\dim X_2 = 1$
bosonic
odels
$\hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$\Rightarrow \operatorname{supp} \hat{A}(t) = \mathcal{H}_\Lambda \Rightarrow [A(t), B] \neq 0$
$\forall t > 0$
$\|A\| = \sup_\psi \|\hat{A}\psi\|$
$\|[A(t), B]\| \leq C e^{(vt-d)/\xi}$
$\exists v$

$$f(t) = [A(t), B]$$

$$f' = -i\,[[A(t), H], B] = -i\,[U_t [A, H] U_t^{\dagger}, B]$$

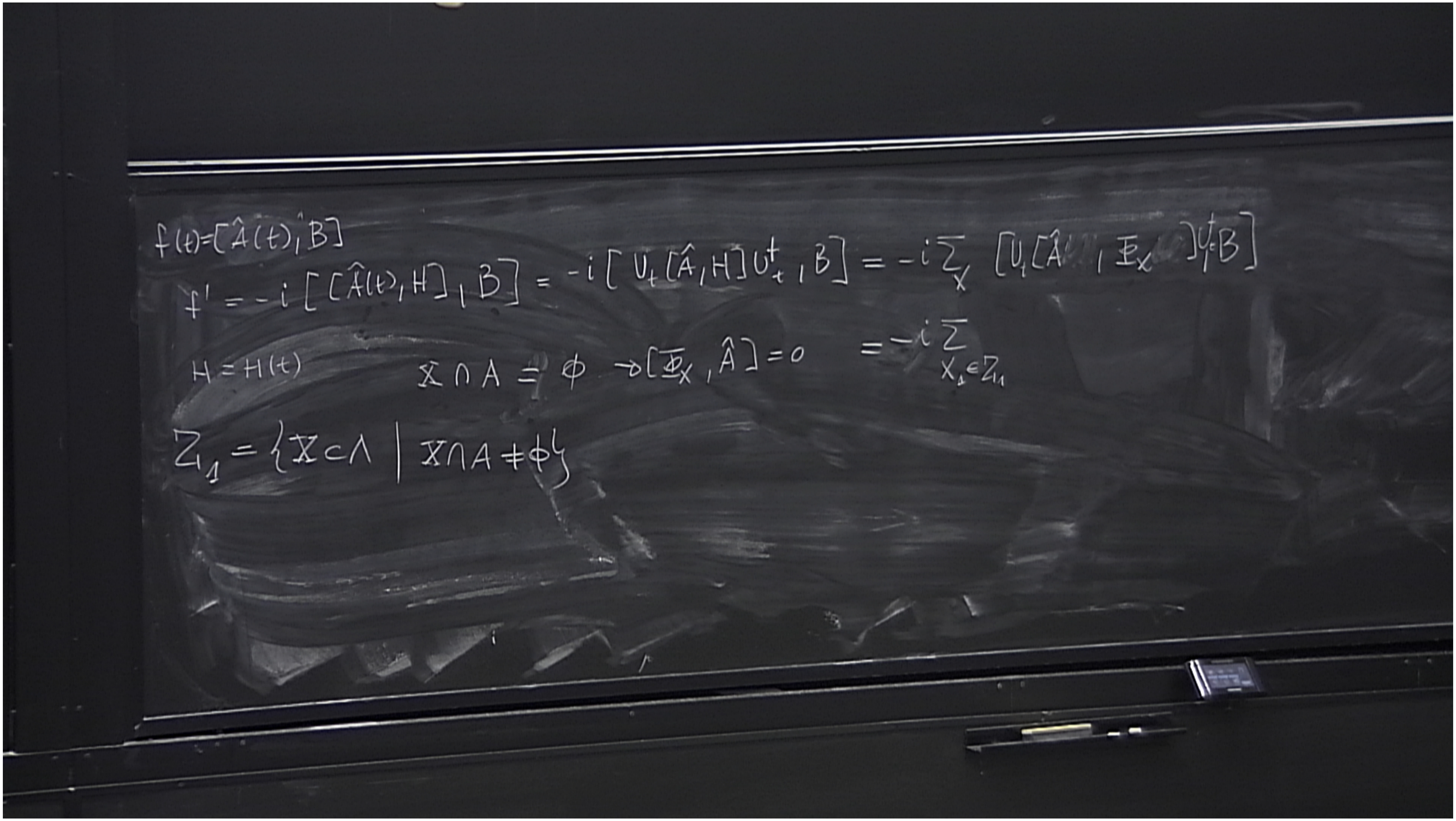
$$H = H(t)$$

$f(t) = [A(t), B]$

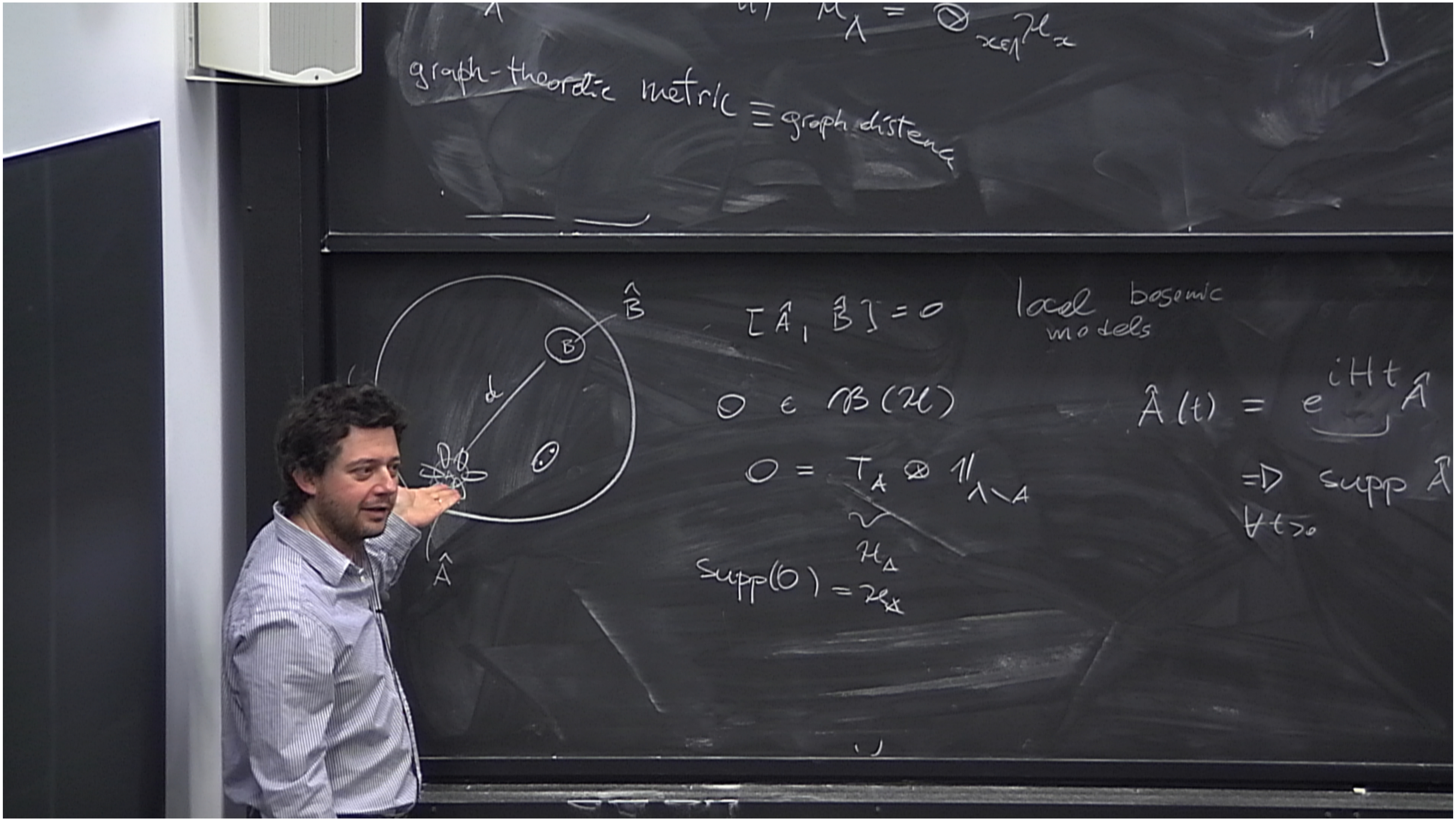
$f' = -i\,[[A(t), H], B] = -i\,[U_t [A, H] U_t^\dagger, B] = -i \sum_x [U_t [A, \Phi_x] U_t^\dagger, B]$

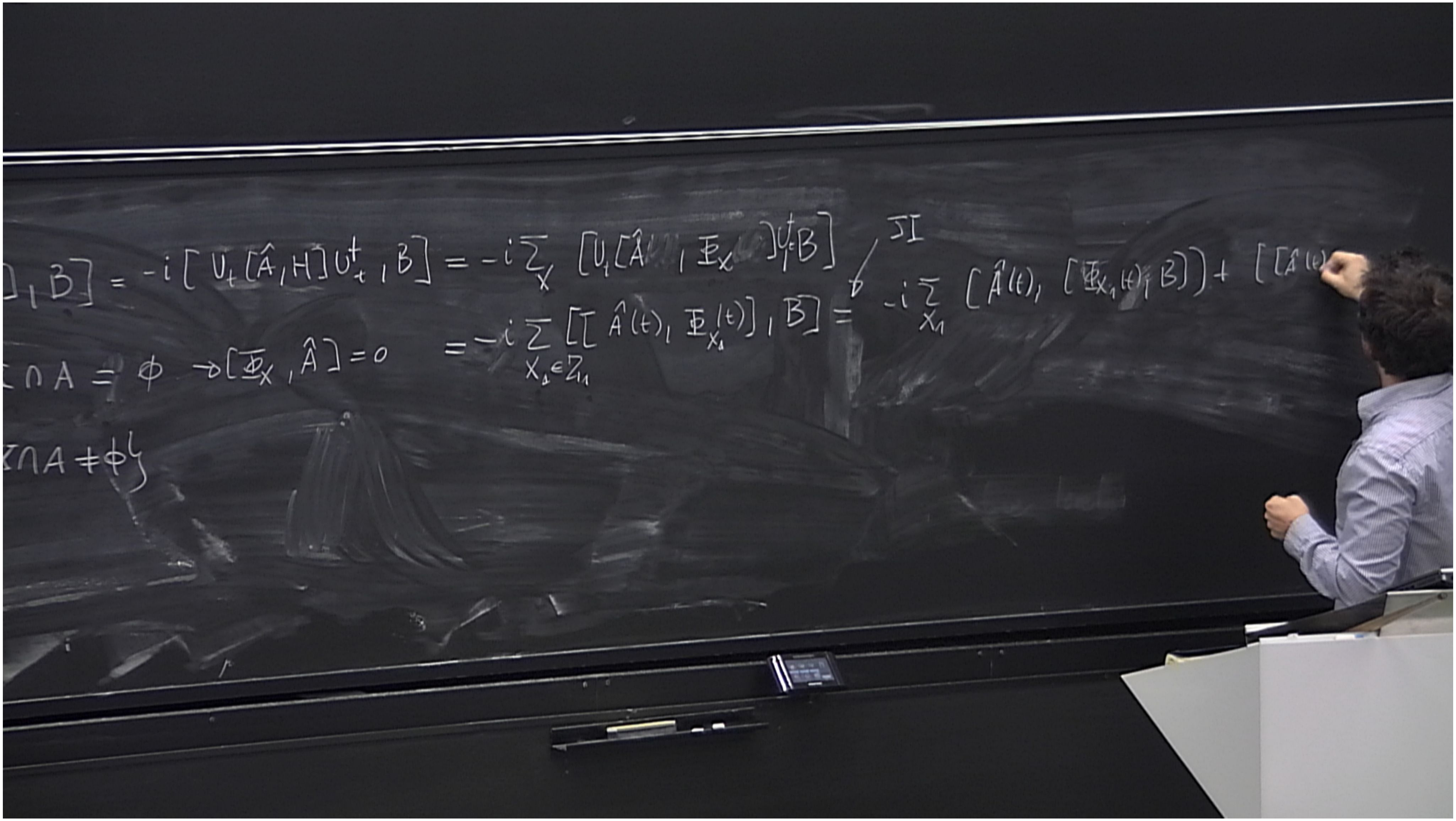
$H = H(t)$

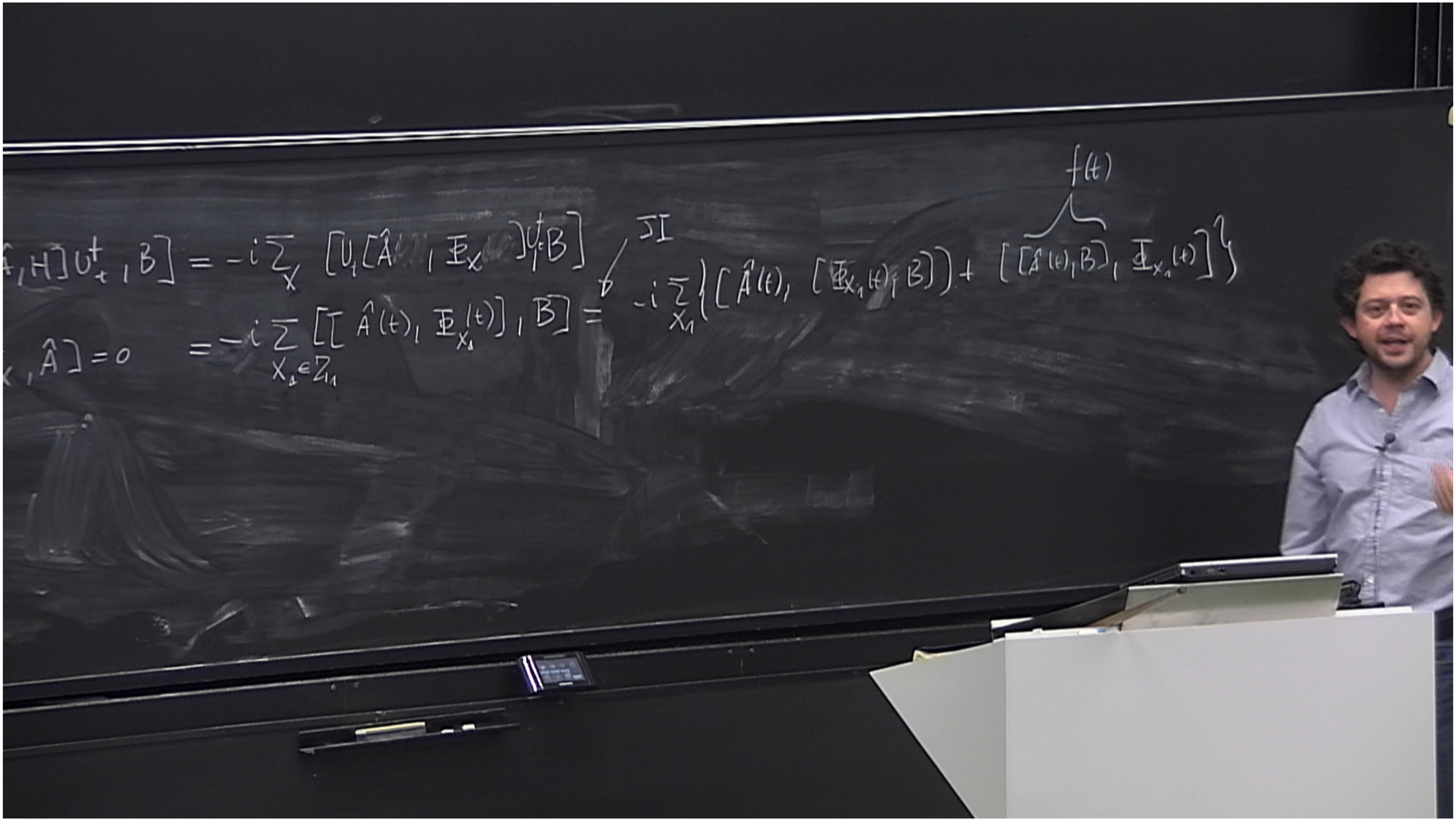
X



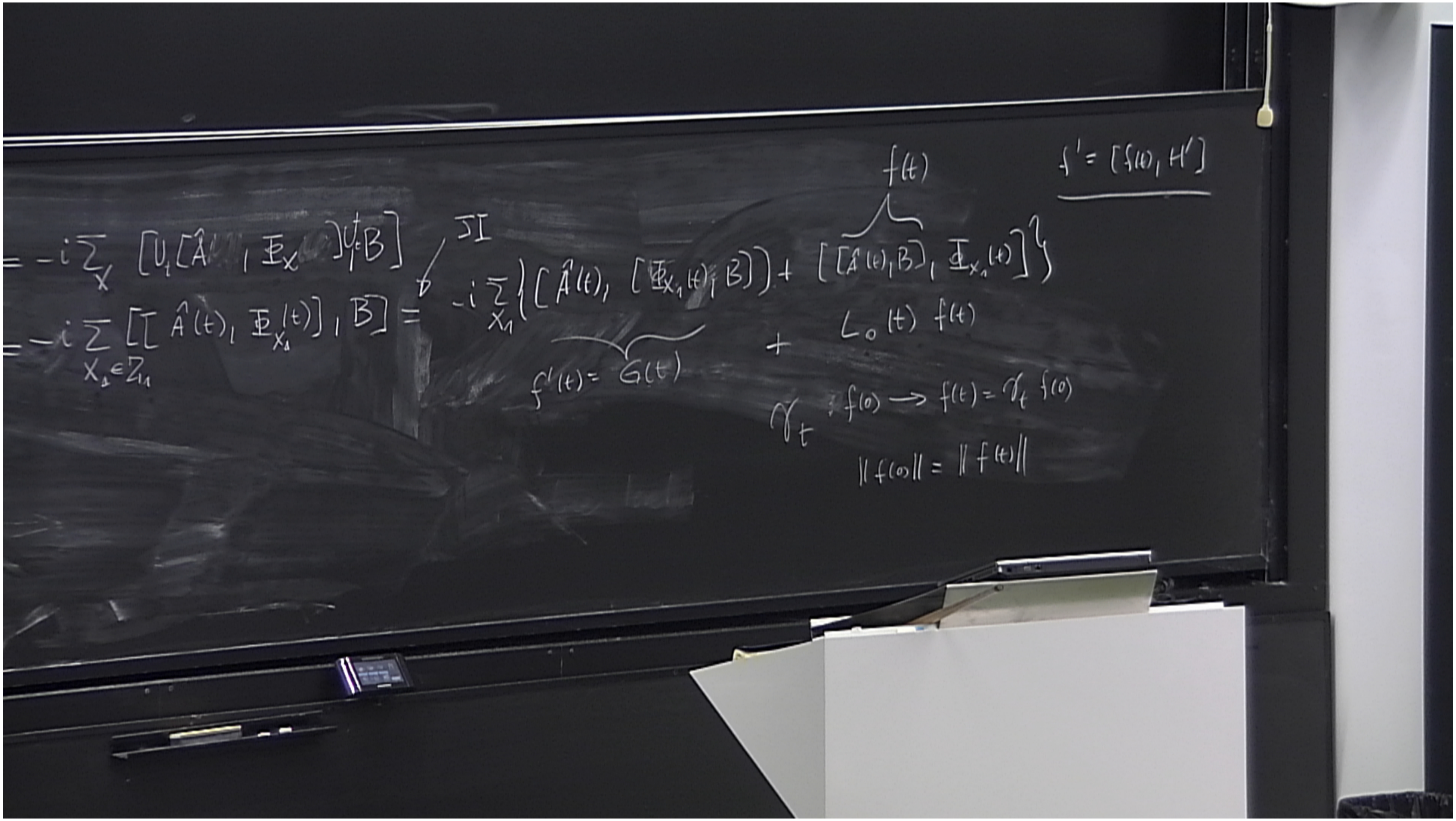
$f(t) = [\hat{A}(t), B]$
$f' = -i[[\hat{A}(t), H], B] = -i[U_t[\hat{A}, H]U_t^\dagger, B] = -i\sum_X [U_t[\hat{A}, \Phi_X]U_t^\dagger B]$
$H = H(t)$
$X \cap A = \phi \to [\Phi_X, \hat{A}] = 0$
$= -i \sum_{X_1 \in Z_1}$
$Z_1 = \{X \subset \Lambda \mid X \cap A \neq \phi\}$


graph-theoretic metric ≡ graph distance
[Â, B̂] = 0
local bosonic models
O ∈ B(H)
O = T_A ⊗ 1_{Λ∖A}
H_A
supp(O) = H_A
Â(t) = e^{iHt} Â
⇒ supp Â
∀ t > 0


], B] = -i [U_t [Â, H] U_t^†, B] = -i Σ_X [U_t [Â, Φ_X] U_t^† B]
JI
-i Σ_{X_1} [Â(t), [Φ_{X_1}(t), B]] + [[Â(t)
= -i Σ_{X_1 ∈ Z_1} [[Â(t), Φ_{X_1}(t)], B] =
X ∩ A = φ → [Φ_X, Â] = 0
X ∩ A ≠ φ}



$f(t)$
$\Sigma\mathrm{I}$
$\hat{A}, H] U_t^\dagger, B] = -i \sum_X [U_t [\hat{A}, \Phi_X] U_t^\dagger, B]$
$= -i \sum_{X_1} \{ [\hat{A}(t), [\Phi_{X_1}(t), B]] + [[\hat{A}(t), B], \Phi_{X_1}(t)] \}$
$= -i \sum [[\hat{A}(t), \Phi_{X_1}(t)], B] =$
$, \hat{A}] = 0$
$= -i \sum_{X_1 \in Z_{1,1}}$


f' = [f(t), H']
f(t)
= -i Σ_X [U_t [Â , Φ_X] U_t* B]
JI
= -i Σ_{X_1 ∈ Z_1} [[Â(t), Φ_{X_1}(t)], B] = -i Σ_{X_1} {[Â(t), [Φ_{X_1}(t), B]] + [[Â(t), B], Φ_{X_1}(t)]}
f'(t) = G(t)
+ L_0(t) f(t)
γ_t : f(0) → f(t) = γ_t f(0)
||f(0)|| = ||f(t)||

$f(t) = [\hat{A}(t), B]$

$f' = -i[[\hat{A}(t), H], B] = -i[U_t[\hat{A}, H]U_t^\dagger, B] = -i\sum_X [U_t[\hat{A}, \Phi_X]U_t^\dagger, B] = -i\sum_{X_1}\{[\hat{A}(t)$

$H = H(t)$

$X \cap A = \phi \to [\Phi_X, \hat{A}] = 0$

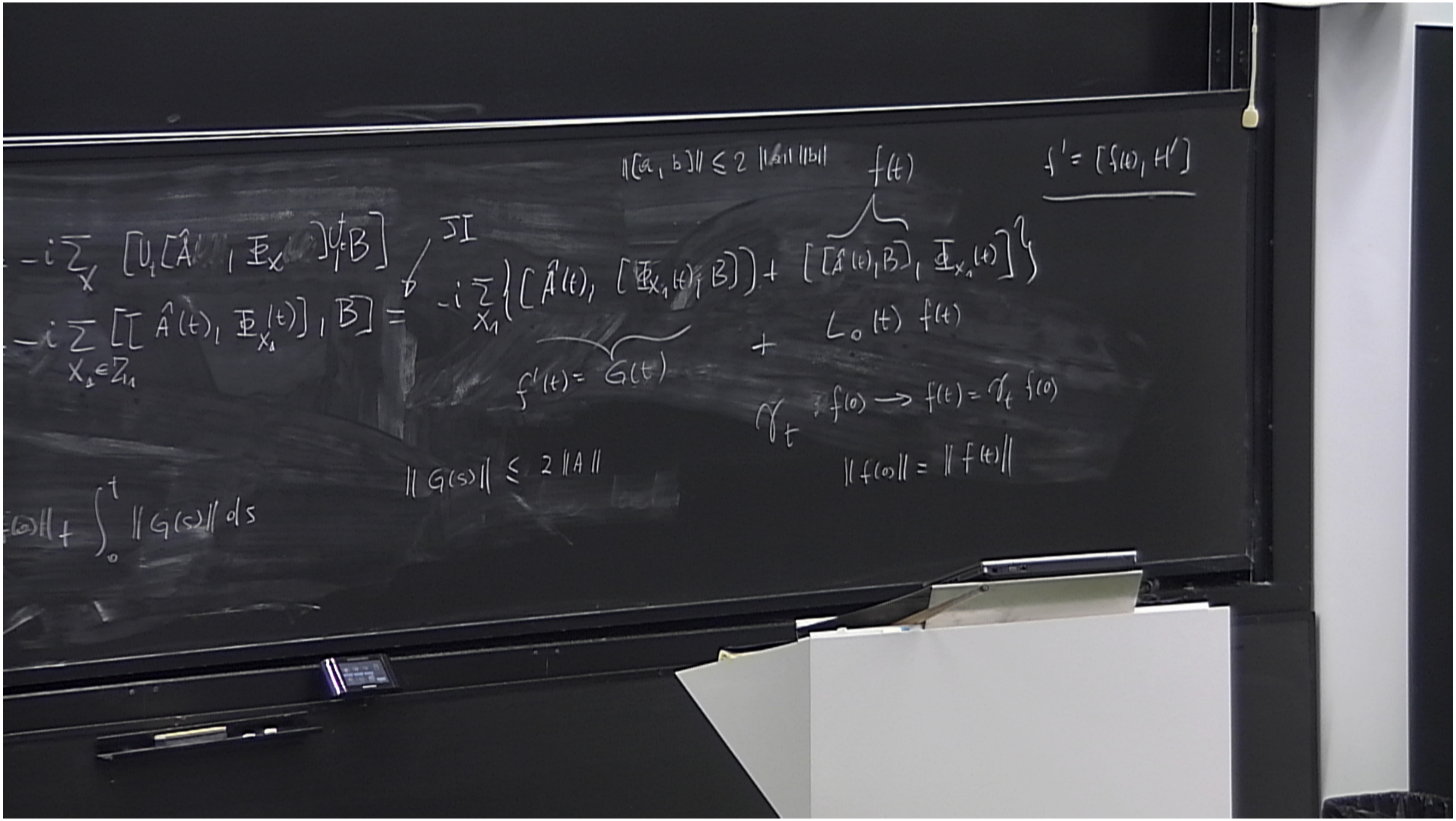
$= -i\sum_{X_1 \in Z_1} [[\hat{A}(t), \Phi_{X_1}(t)], B] =$

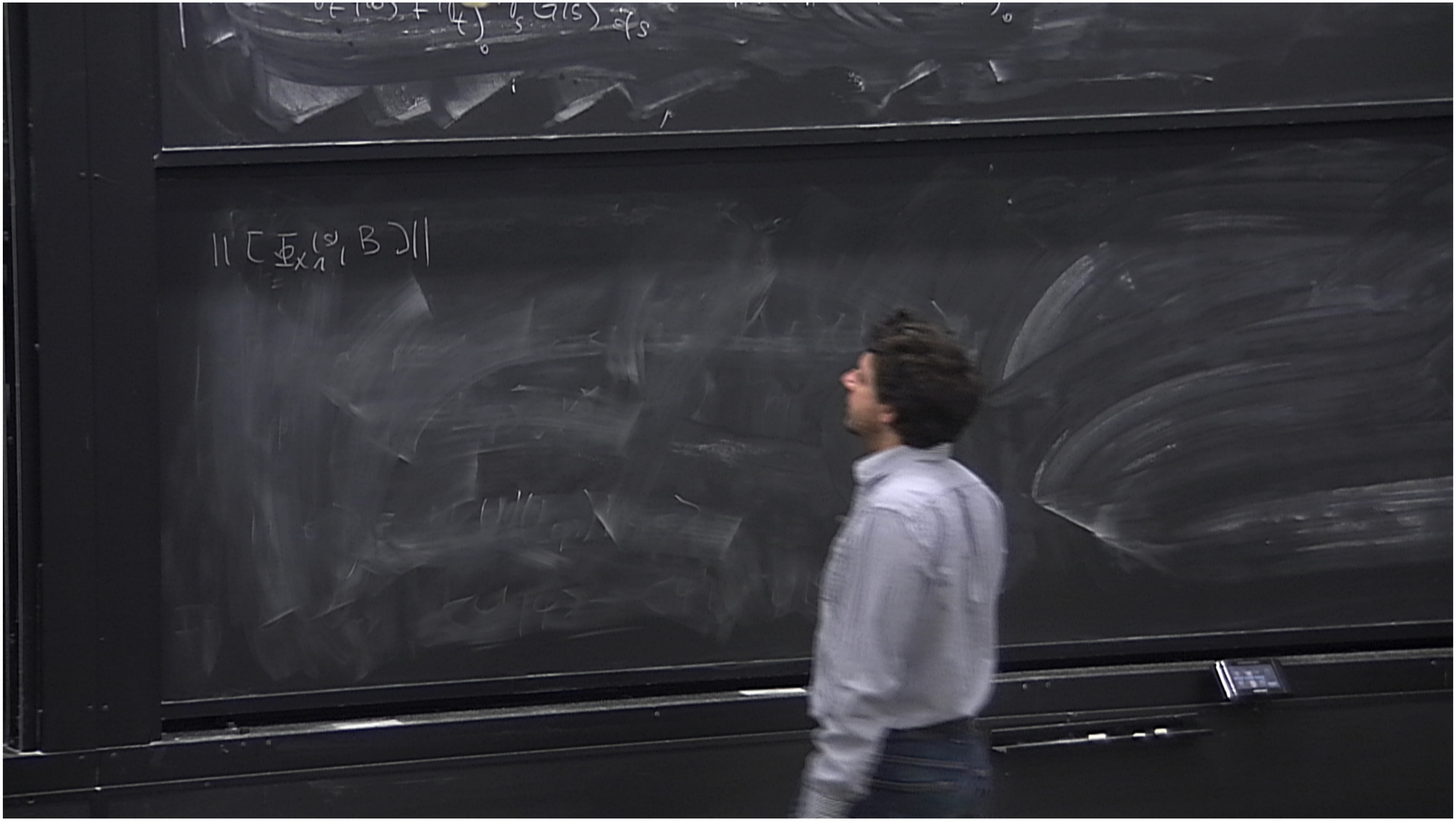
$f'(t) =$

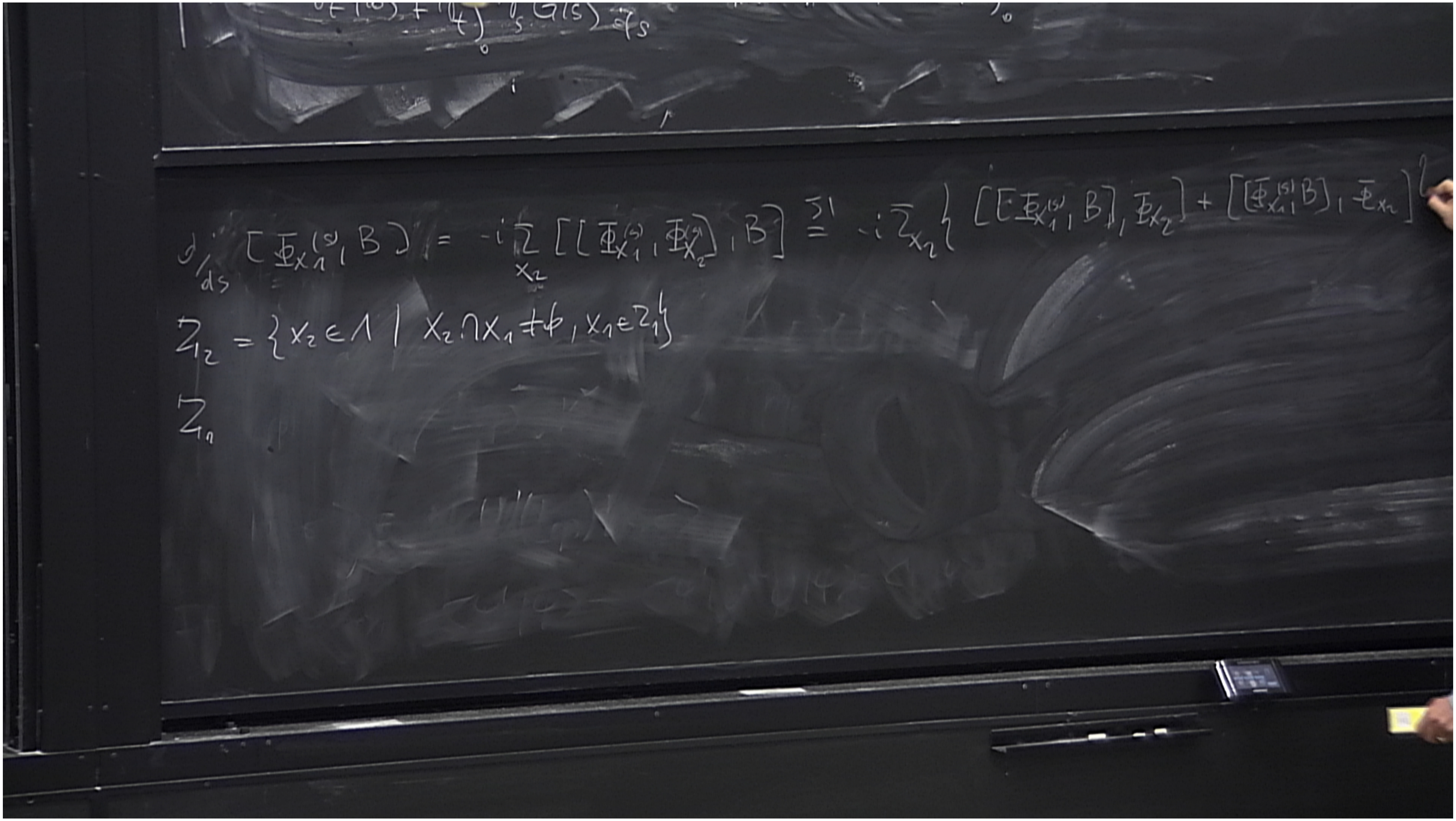
$Z_1 = \{X \subset \Lambda \mid X \cap A \neq \phi\}$

$f(t) = \gamma_t f(0) + \gamma_t \int_0^t \gamma_s^{-1} G(s)\, ds$

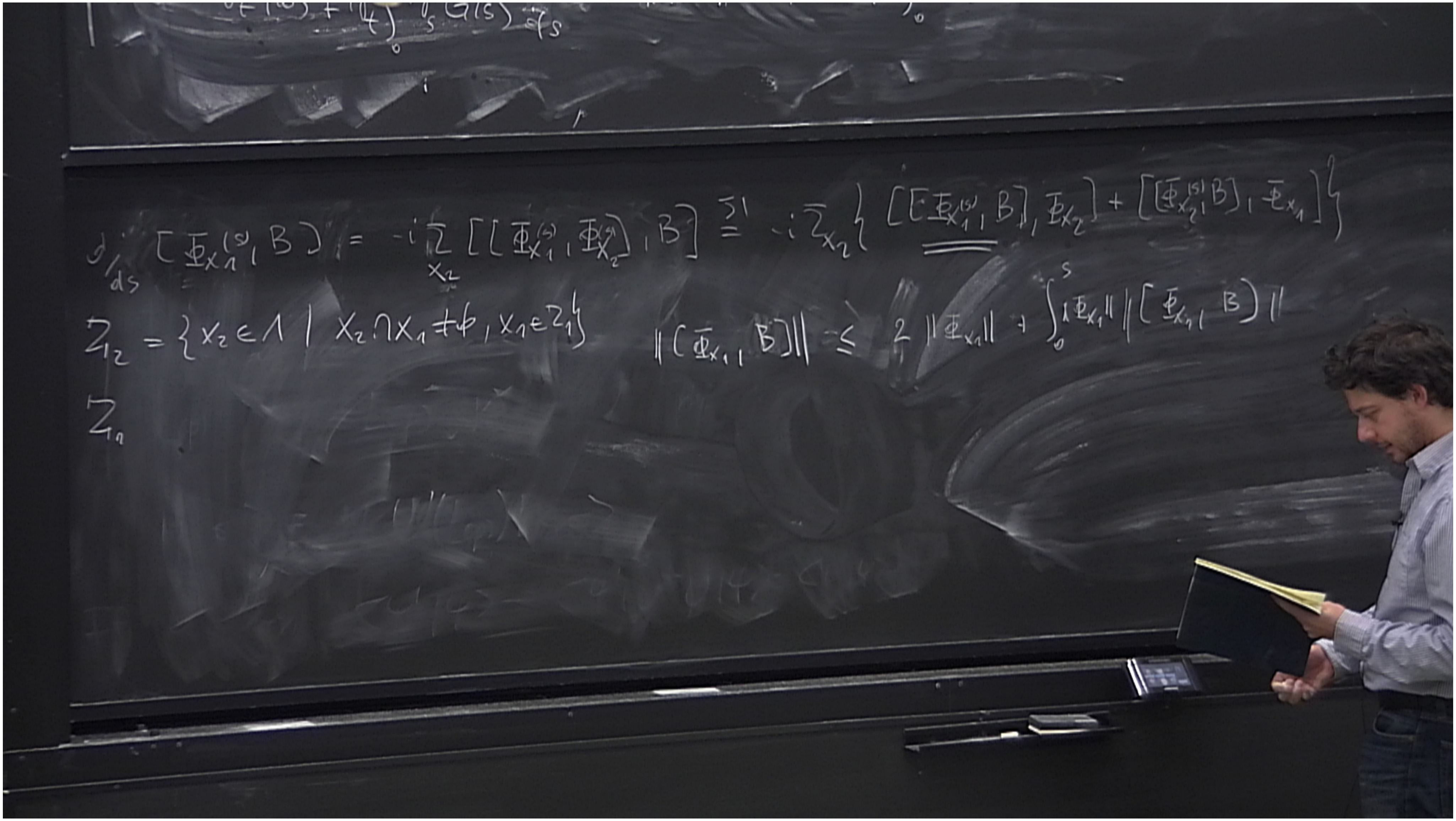
$\|f(t)\| \leq \|f(0)\| + \int_0^t \|G(s)\|\, ds$


‖[a, b]‖ ≤ 2 ‖a‖ ‖b‖
f(t)
f' = [f(t), H']
II
L₀(t) f(t)
f'(t) = G(t)
‖G(s)‖ ≤ 2 ‖A‖
‖f(0)‖ = ‖f(t)‖





$\frac{d}{ds}[\Phi_{X_1}(s), B] = -i\sum_{X_2}[[\Phi_{X_1}(s), \Phi_{X_2}(s)], B] \stackrel{JI}{=} -i\sum_{X_2}\{[[\Phi_{X_1}(s), B], \Phi_{X_2}] + [[\Phi_{X_1}(s) B], \Phi_{X_2}]\}$
$Z_{1_2} = \{X_2 \in \Lambda \mid X_2 \cap X_1 \neq \emptyset, X_1 \in Z_1\}$
Z_{1_1}

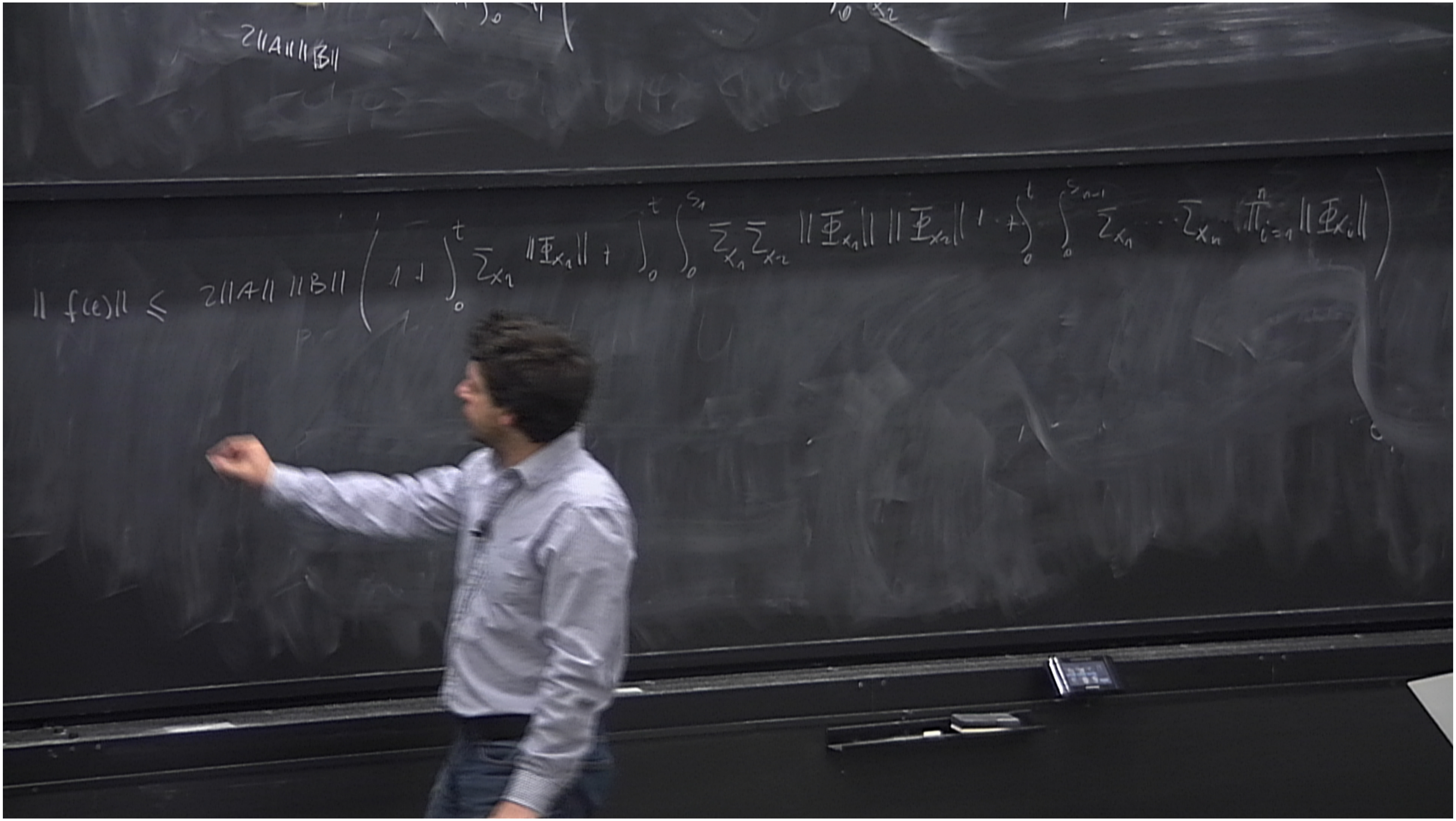

d/ds [Φ_{X_1}^{(s)}, B] = -i Σ_{X_2} [[Φ_{X_1}^{(s)}, Φ_{X_2}^{(s)}], B] = -i Σ_{X_2} { [[Φ_{X_1}^{(s)}, B], Φ_{X_2}] + [[Φ_{X_2}^{(s)}, B], Φ_{X_1}] }
Z_{12} = { X_2 ∈ Λ | X_2 ∩ X_1 ≠ φ, X_1 ∈ Z_1 }
||[Φ_{X_1}, B]|| ≤ 2 ||Φ_{X_1}|| + ∫_0^s ||Φ_{X_1}|| ||[Φ_{X_1}, B]||
Z_{1n}

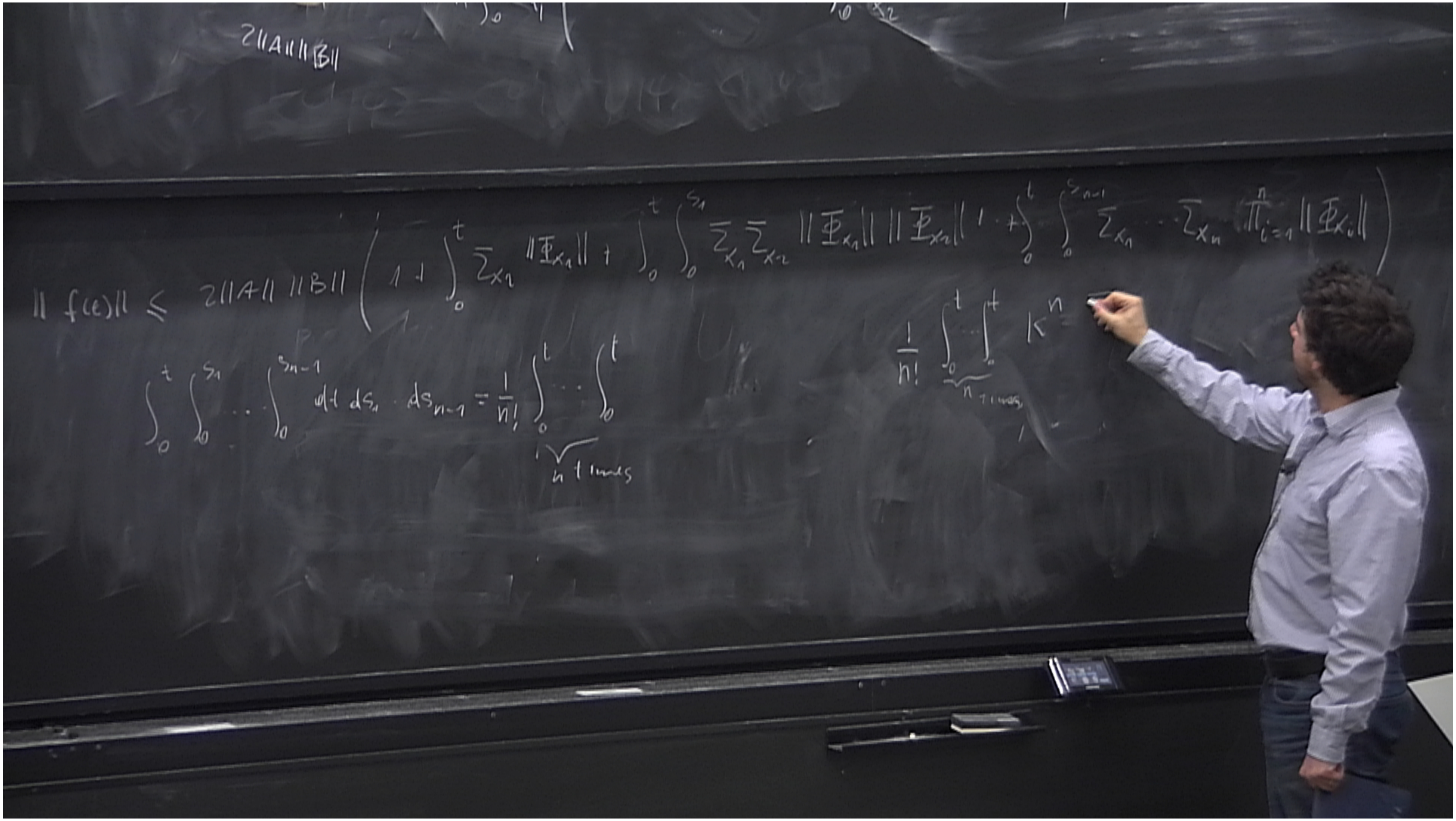
$$\frac{d}{ds}[\Phi_{X_1}(s), B] = -i \sum_{X_2} [[\Phi_{X_1}(s), \Phi_{X_2}(s)], B] \stackrel{JI}{=} -i \sum_{X_2} \{ [[\Phi_{X_1}(s), B], \Phi_{X_2}] + [[\Phi_{X_2}(s), B], \Phi_{X_1}] \}$$

$$Z_{12} = \{ X_2 \in \Lambda \mid X_2 \cap X_1 \neq \phi, X_1 \in Z_1 \}$$

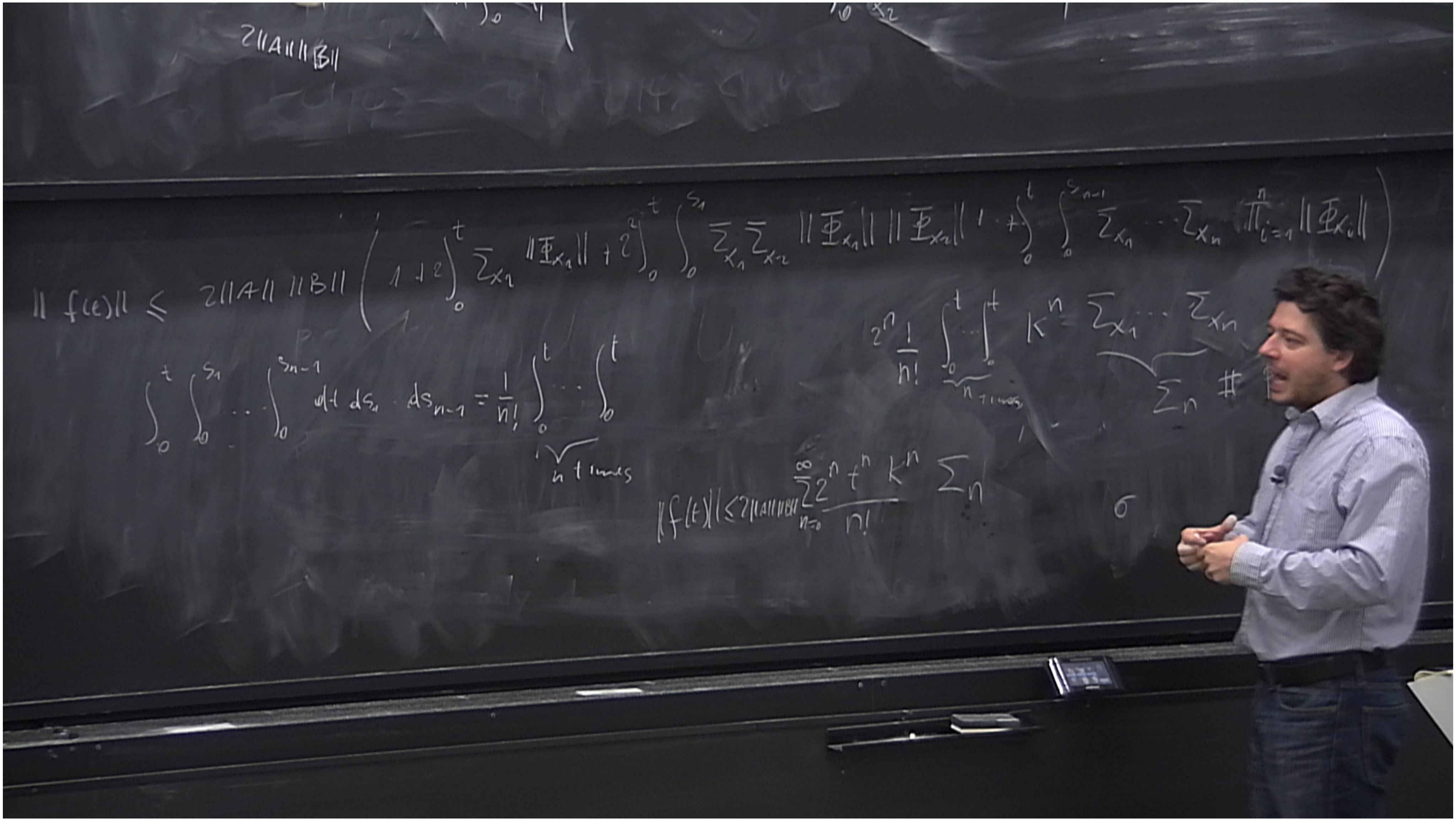
$$\|[\Phi_{X_1}, B]\| \leq \|[\Phi_{X_1}, B]\| + 2\|\Phi_{X_1}\| \int_0^s \sum_{X_2} \|[\Phi_{X_2}(s), B$$

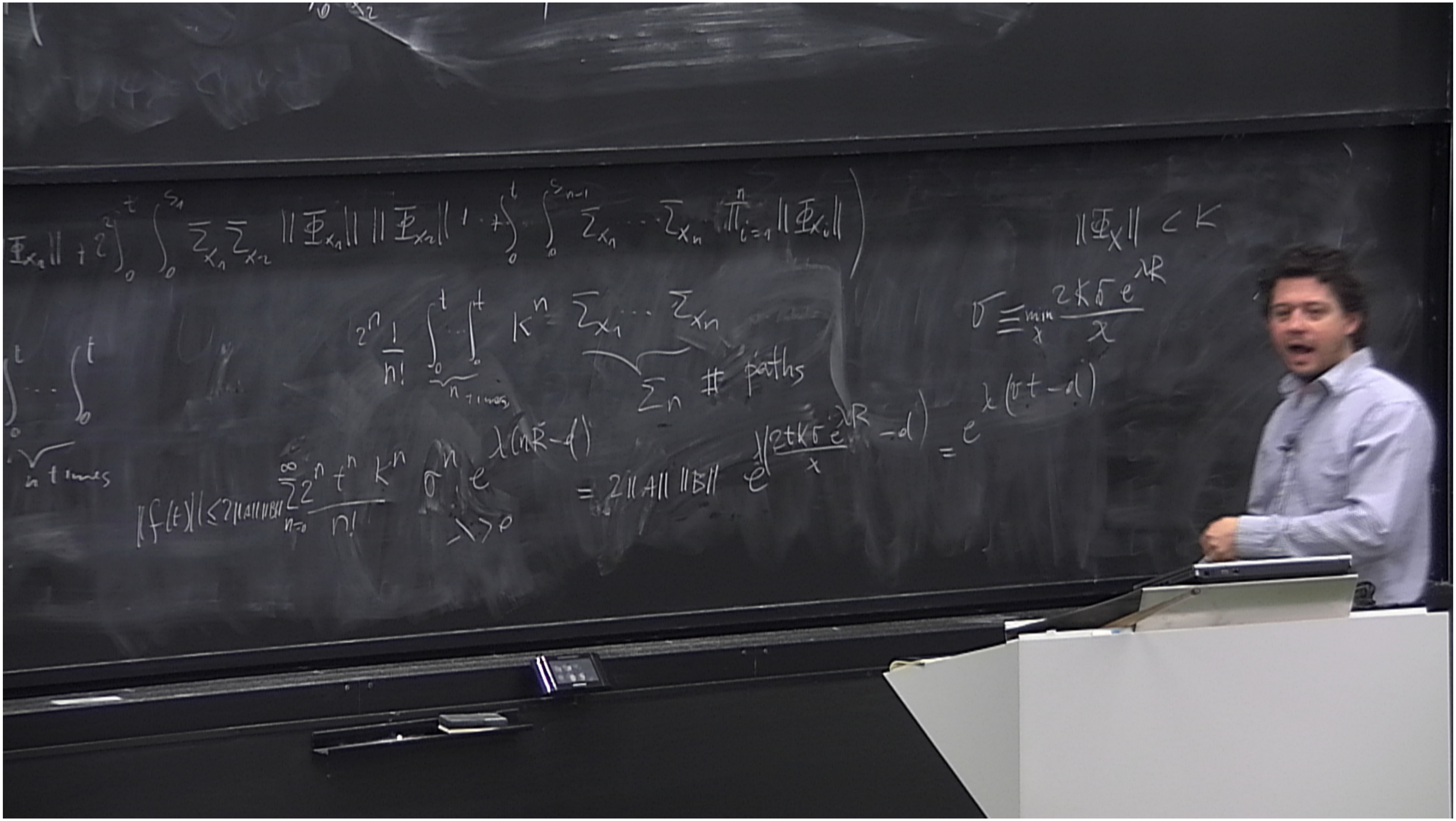
$$Z_{1n}$$

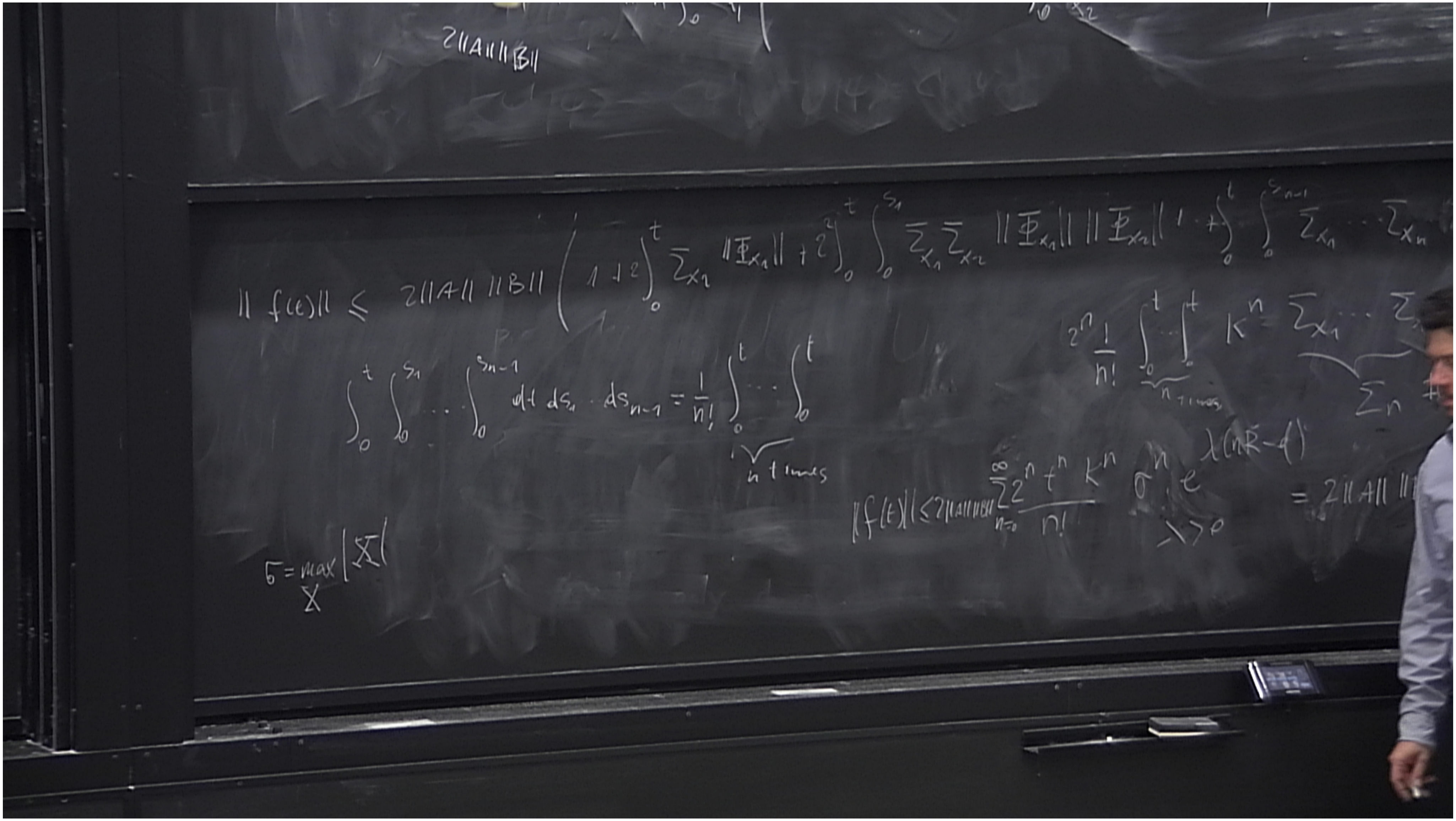

$2\|A\|\|B\|$
$\|f(t)\| \leq 2\|A\|\,\|B\|\left(1 + \int_0^t \sum_{x_1} \|\Phi_{x_1}\| + \int_0^t \int_0^{s_1} \sum_{x_1}\sum_{x_2} \|\Phi_{x_1}\|\,\|\Phi_{x_2}\| + \dots + \int_0^t \int_0^{s_{n-1}} \sum_{x_1} \dots \sum_{x_n} \prod_{i=1}^{n} \|\Phi_{x_i}\|\right)$



$2\|A\|\|B\|$
$\|f(t)\| \leq 2\|A\|\,\|B\| \left(1 + \int_0^t \sum_{x_1} \|\Phi_{x_1}\| + \int_0^t \int_0^{s_1} \sum_{x_1} \sum_{x_2} \|\Phi_{x_1}\|\,\|\Phi_{x_2}\| + \dots + \int_0^t \int_0^{s_{n-1}} \sum_{x_1} \dots \sum_{x_n} \prod_{i=1}^n \|\Phi_{x_i}\| \right)$
$\int_0^t \int_0^{s_1} \dots \int_0^{s_{n-1}} dt\, ds_1 \dots ds_{n-1} = \frac{1}{n!} \underbrace{\int_0^t \dots \int_0^t}_{n \text{ times}}$
$\frac{1}{n!} \underbrace{\int_0^t \dots \int_0^t}_{n \text{ times}} K^n$


2||A|| ||B||
|| f(t)|| ≤ 2||A|| ||B|| (1 + 2 ∫ Σx1 ||Φx1|| + 2² ∫∫ Σx1 Σx2 ||Φx1|| ||Φx2|| + … + ∫ … ∫ Σx1 … Σxn Π ||Φxi||)
∫ ∫ … ∫ dt ds1 … dsn-1 = 1/n! ∫ … ∫
n times
2ⁿ 1/n! ∫ … ∫ Kⁿ Σx1 … Σxn
n times
Σn
||f(t)|| ≤ 2||A|| ||B|| Σ 2ⁿ tⁿ Kⁿ Σn / n!
σ


$\|\Phi_X\| < K$
$\sigma \equiv \min_\lambda \frac{2K\sigma e^{\lambda R}}{\lambda}$
$\sum_n$ # paths
n times
$\|f(t)\| \le 2\|A\|\|B\| \sum_{n=0}^{\infty} \frac{2^n t^n K^n}{n!} \sigma^n e^{\lambda(nR-d)}$
$\lambda > 0$
$= 2\|A\|\|B\| e^{\lambda\left(\frac{2tK\sigma e^{\lambda R}}{\lambda} - d\right)} = e^{\lambda(vt-d)}$


2||A|| ||B||
|| f(t)|| ≤ 2||A|| ||B|| (1 + 2 ∫_0^t Σ_{x_1} ||Φ_{x_1}|| + 2^2 ∫_0^t ∫_0^{s_1} Σ_{x_1} Σ_{x_2} ||Φ_{x_1}|| ||Φ_{x_2}|| + ... + ∫_0^t ∫_0^{s_{n-1}} Σ_{x_1} ... Σ_{x_n}
∫_0^t ∫_0^{s_1} ... ∫_0^{s_{n-1}} dt ds_1 ... ds_{n-1} = 1/n! ∫_0^t ... ∫_0^t
n times
2^n 1/n! ∫_0^t ... ∫_0^t K^n Σ_{x_1} ... Σ
n times
Σ_n
||f(t)|| ≤ 2||A|| ||B|| Σ_{n=0}^∞ 2^n t^n K^n / n!
σ^n e^{λ(nR - d)}
λ > 0
= 2||A|| ||
σ = max |X|
X