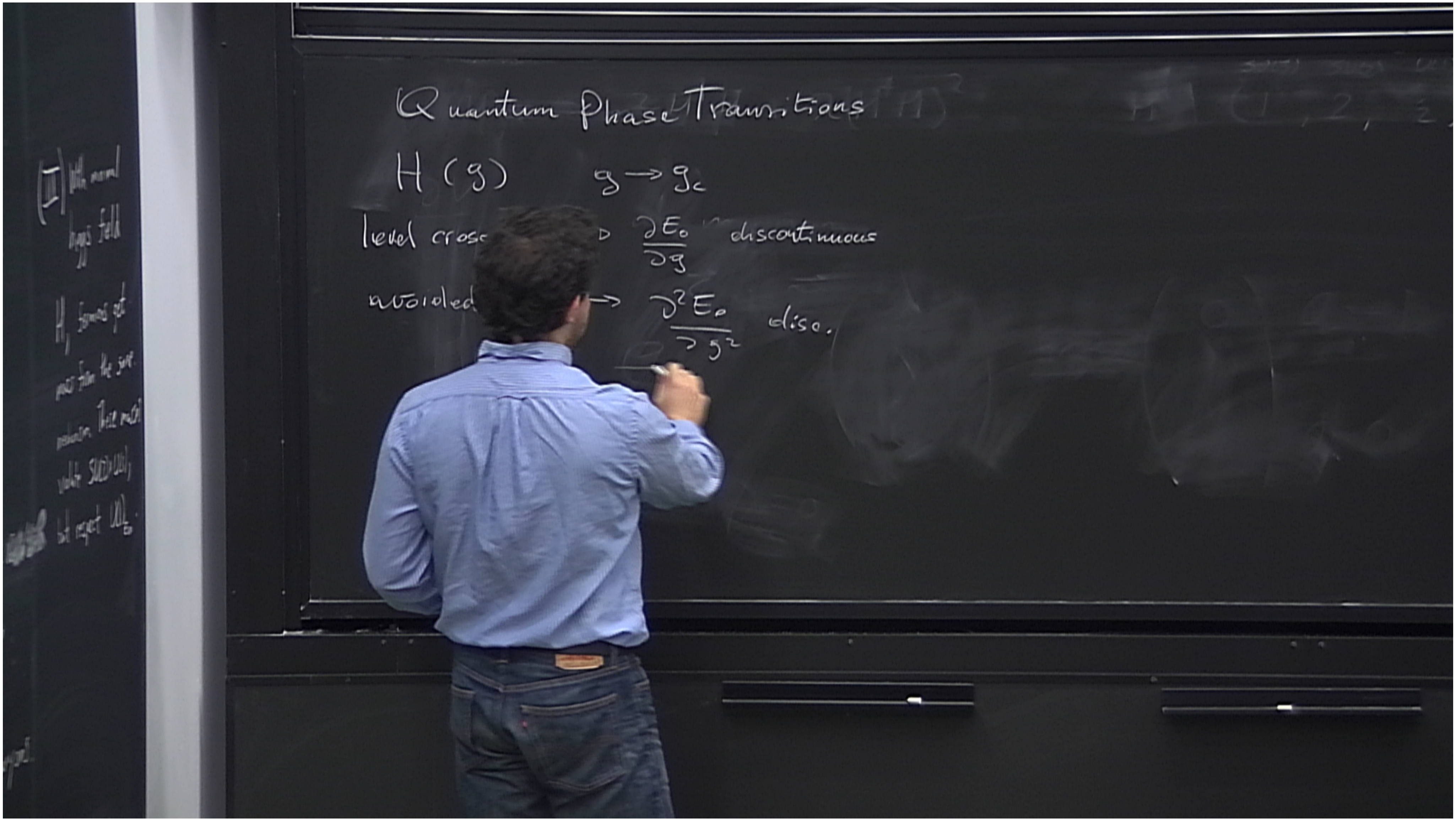
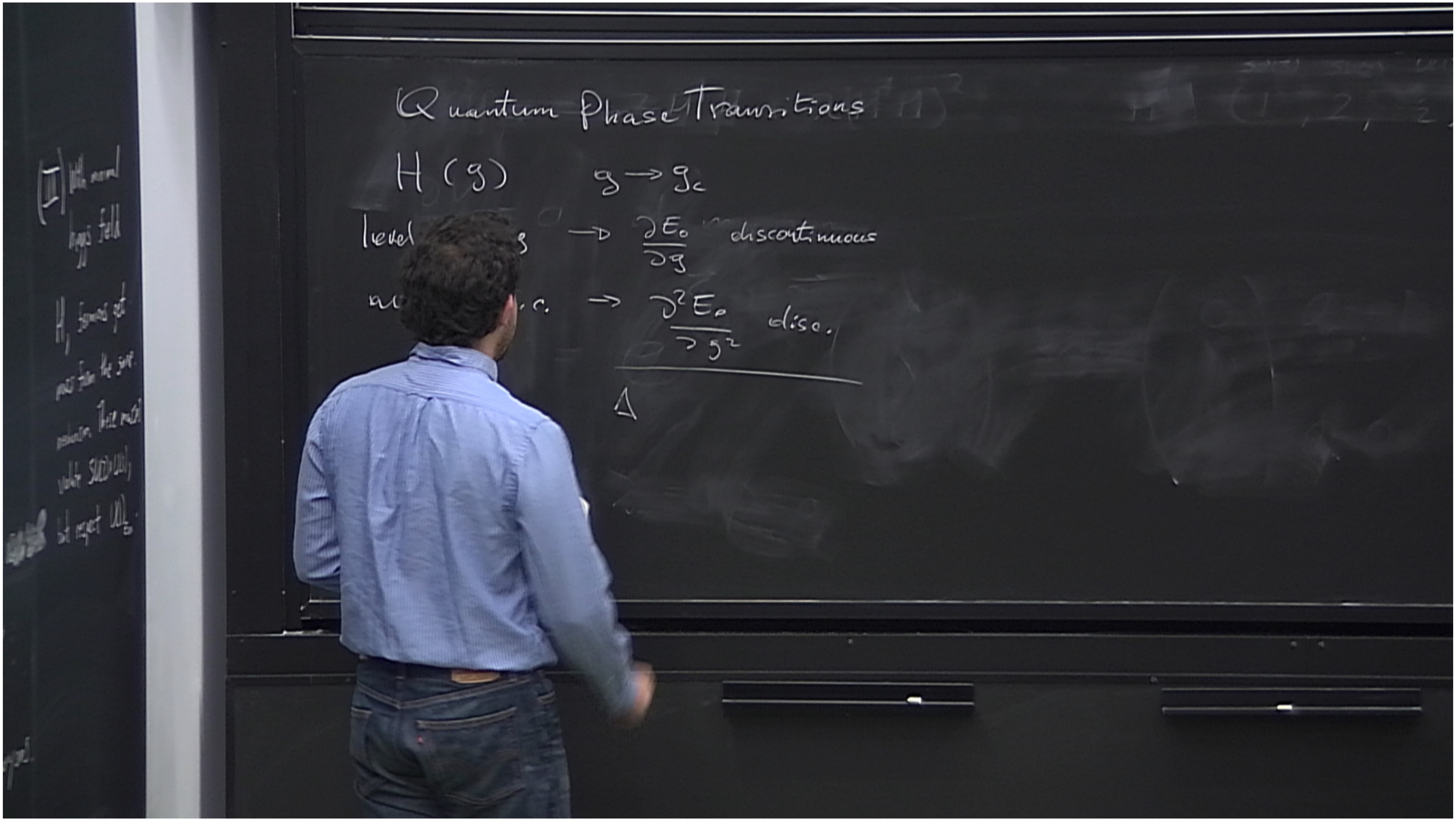
Title: Condensed Matter (Review) - Lecture 4

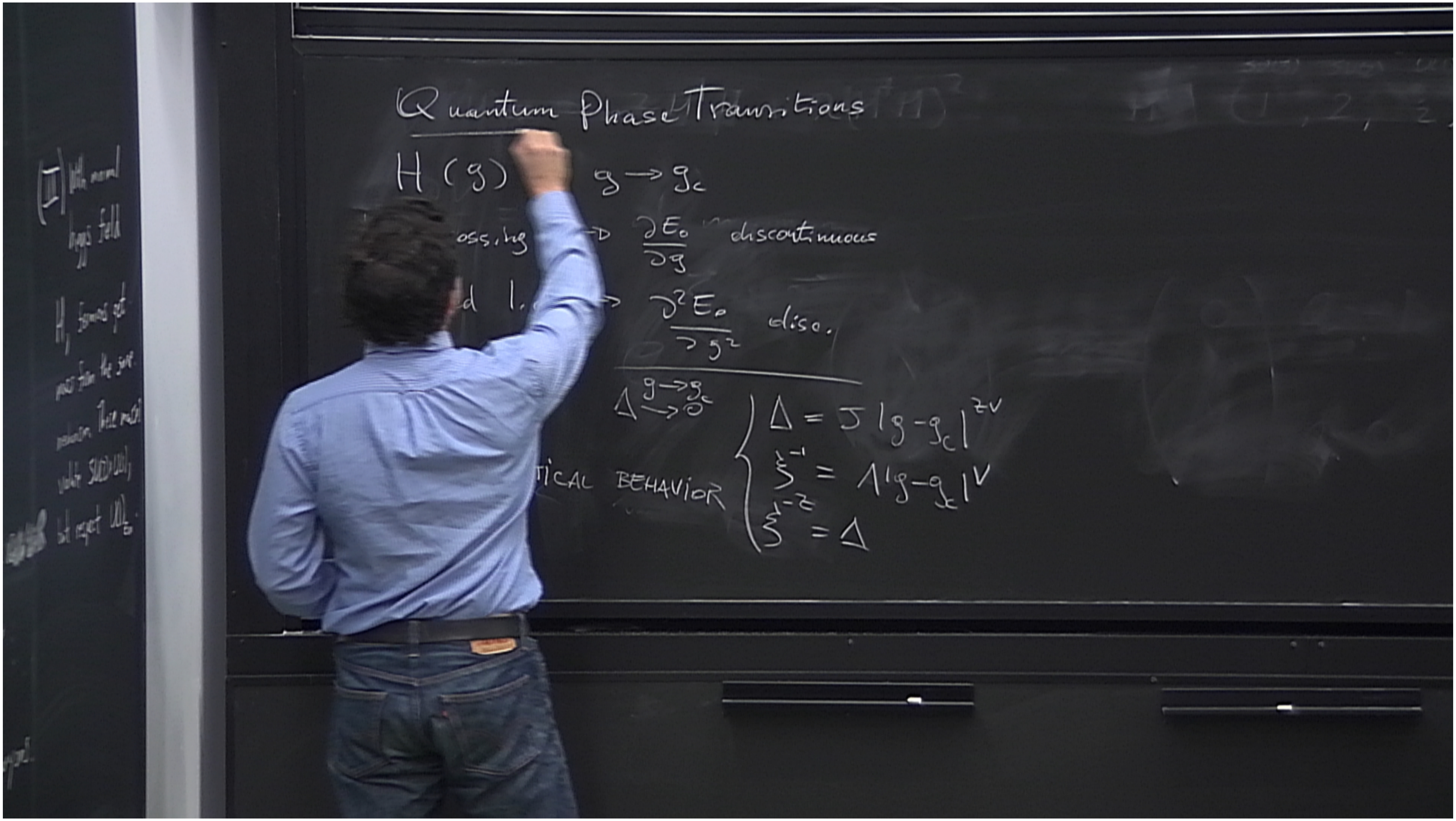
Date: Jan 05, 2012 10:15 AM

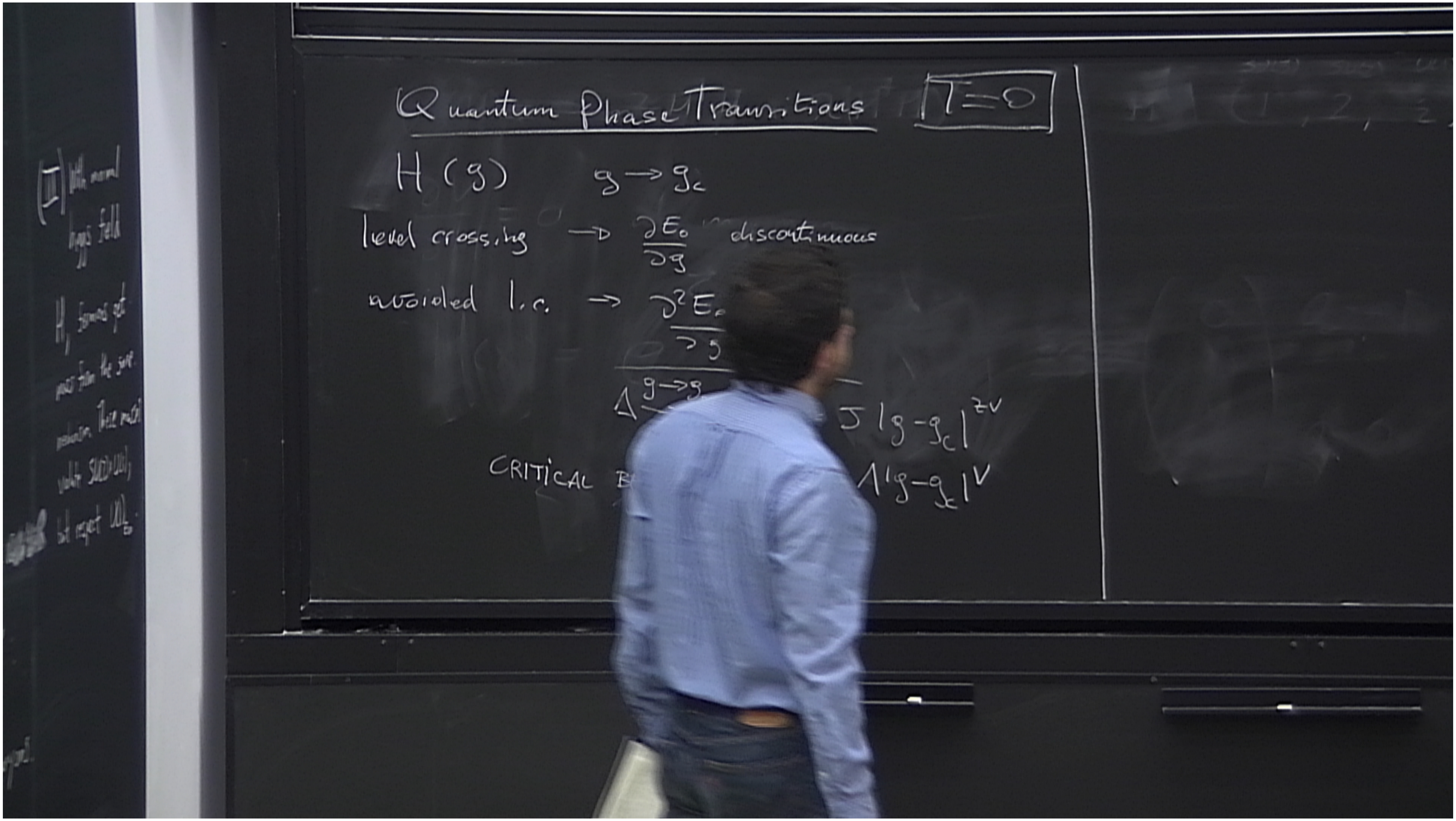
URL: http://pirsa.org/12010086

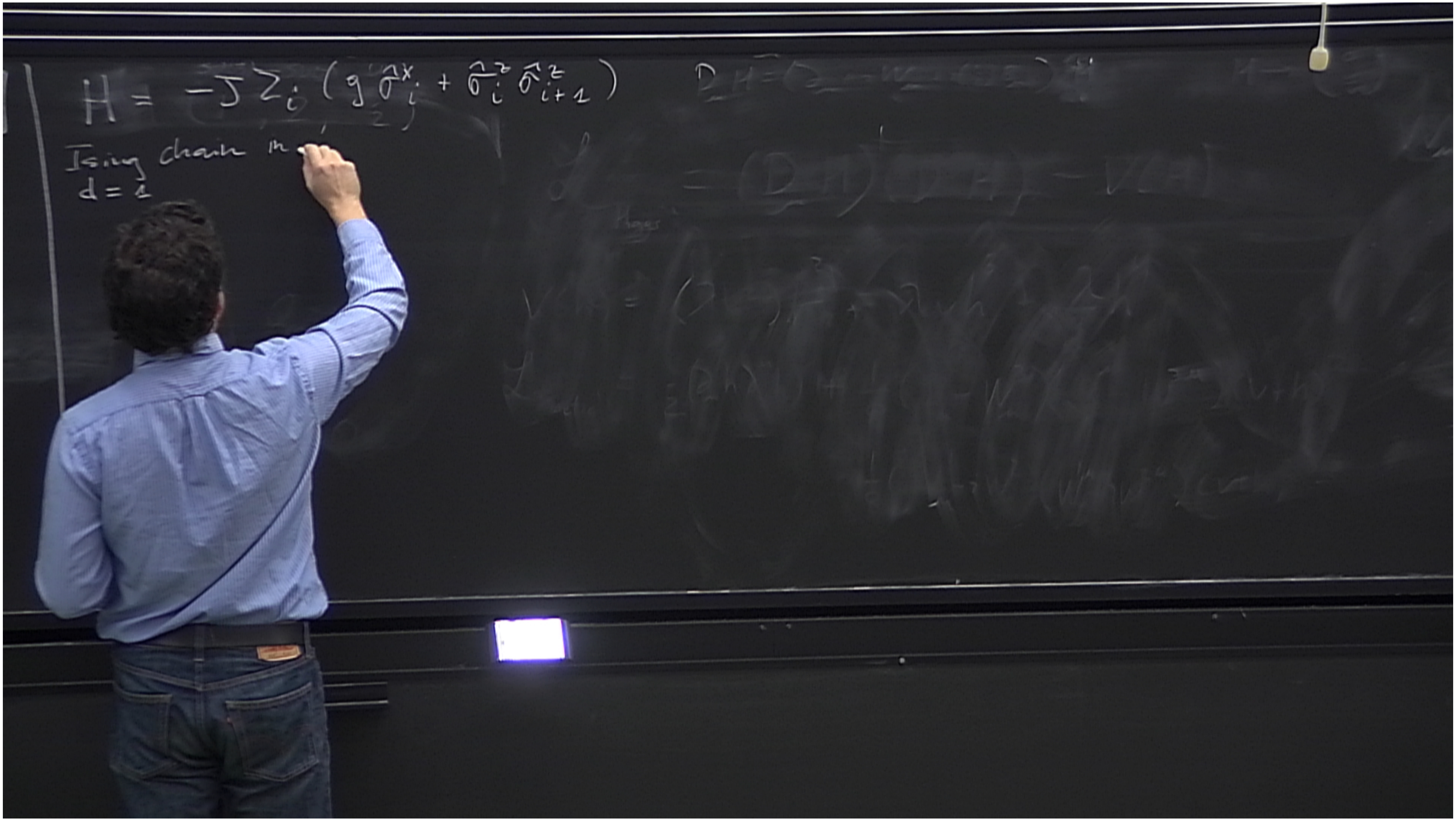
Abstract:

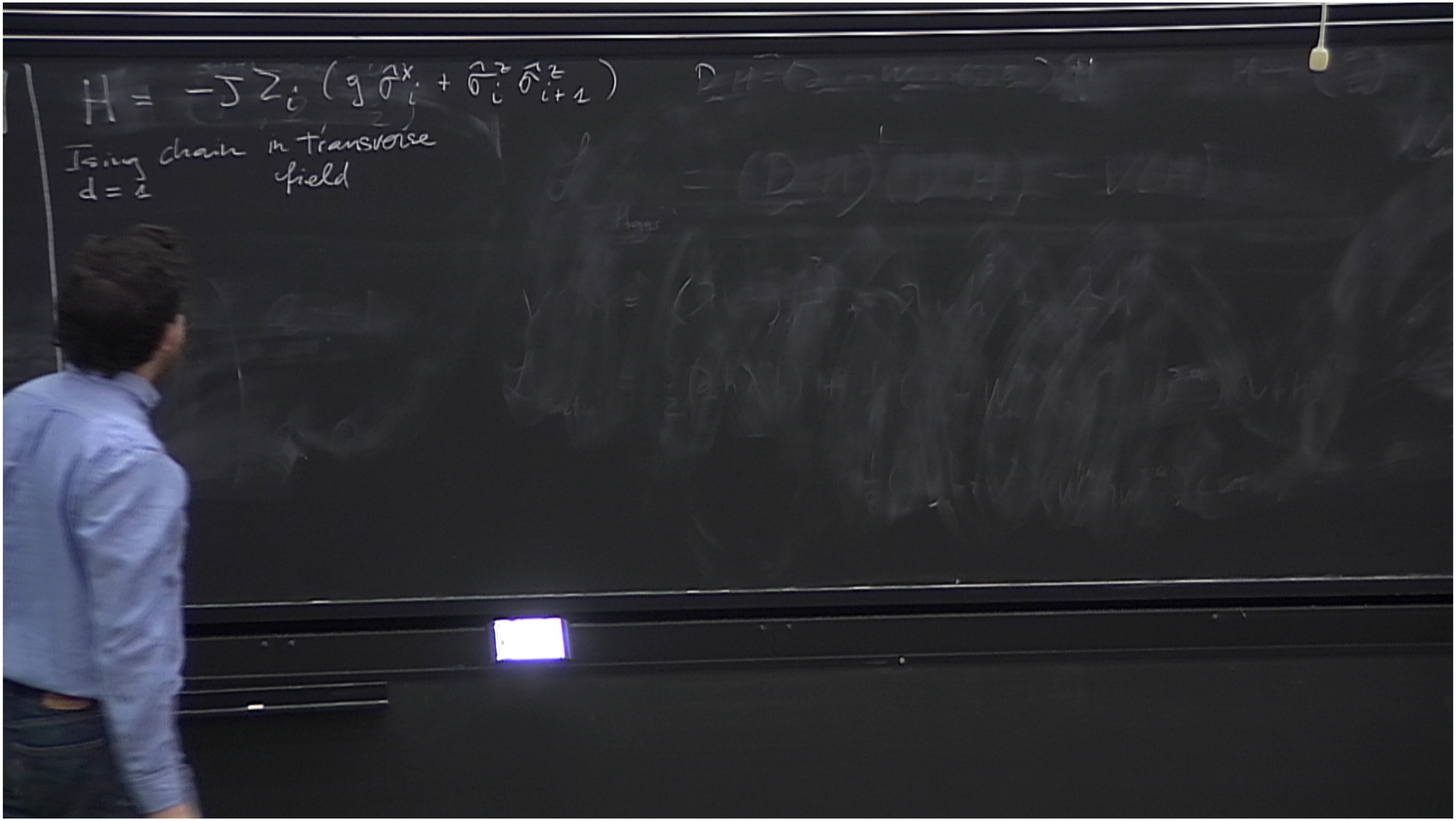

Quantum Phase Transitions
H (g)
g → g_c
level cros
∂E_0/∂g
discontinuous
avoided
∂²E_0/∂g²
disc.

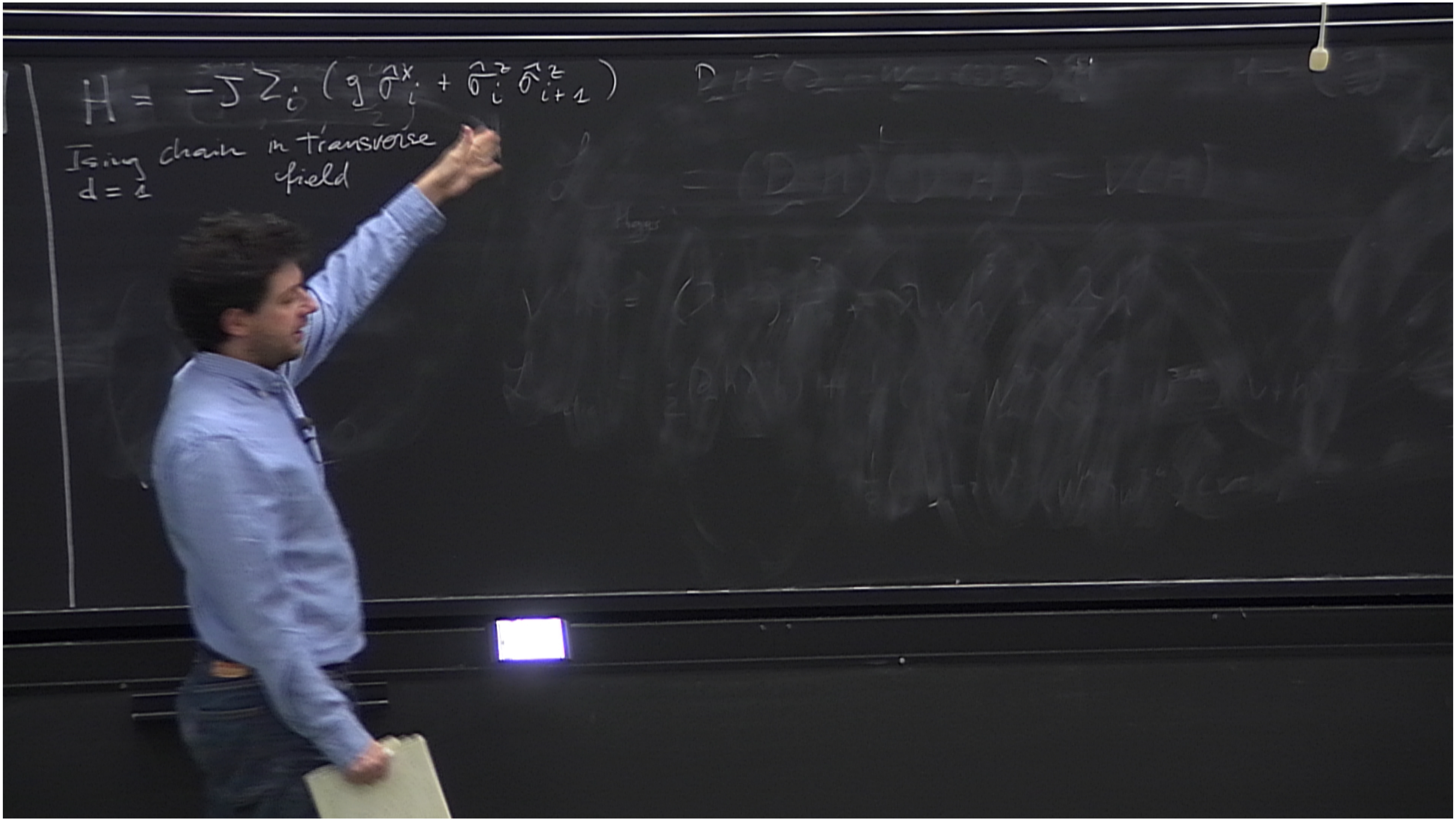

Quantum Phase Transitions
H (g)   g → g_c
level
→ ∂E_0/∂g discontinuous
→ ∂²E_0/∂g² disc.
Δ

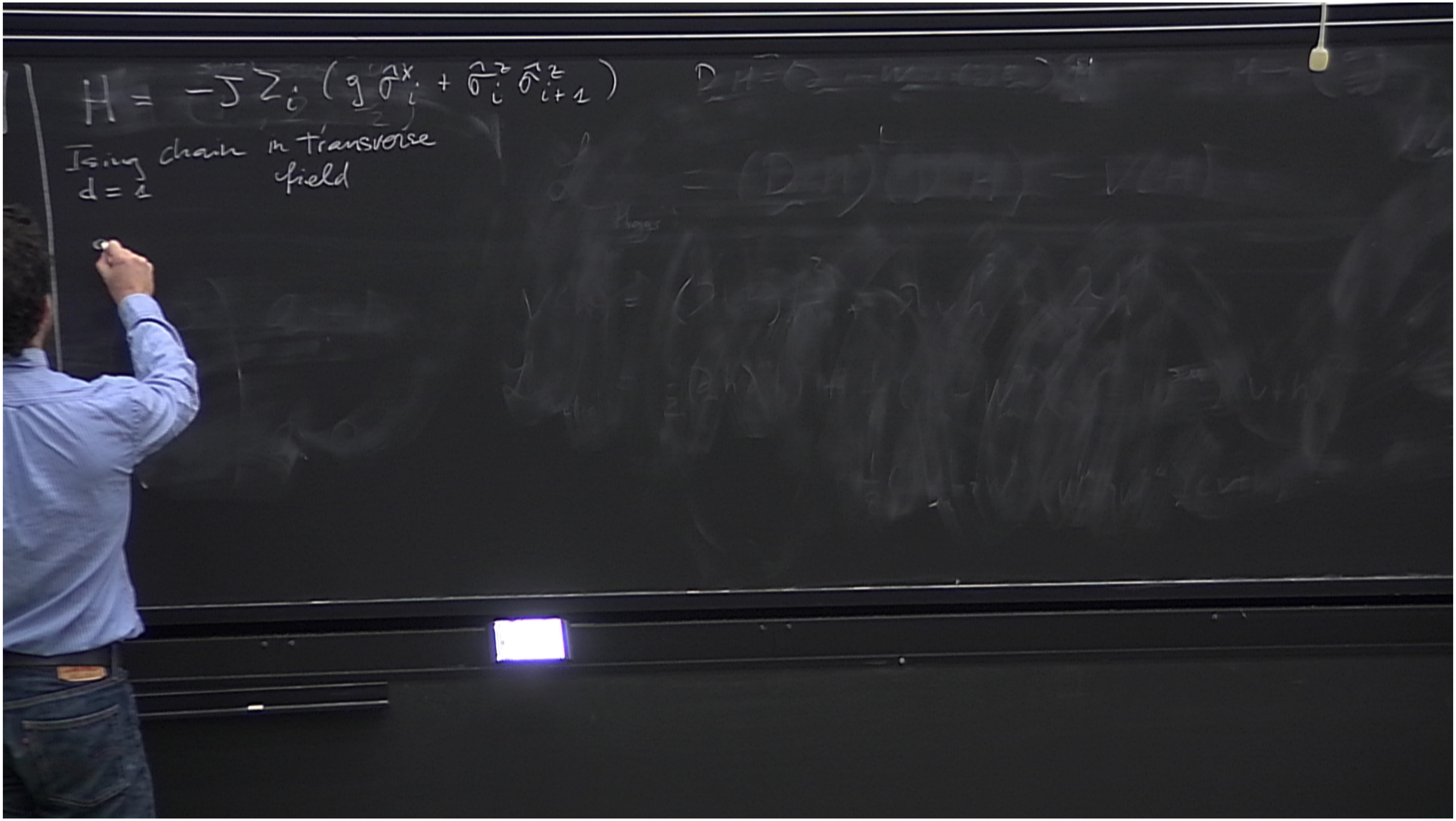

Quantum Phase Transitions
$H(g)$ $g \to g_c$
ossing → $\frac{\partial E_0}{\partial g}$ discontinuous
d 1. → $\frac{\partial^2 E_0}{\partial g^2}$ disc.
$\Delta \xrightarrow{g \to g_c} 0$
TICAL BEHAVIOR
$\Delta = J|g-g_c|^{z\nu}$
$\xi^{-1} = \Lambda|g-g_c|^{\nu}$
$\xi^{-z} = \Delta$

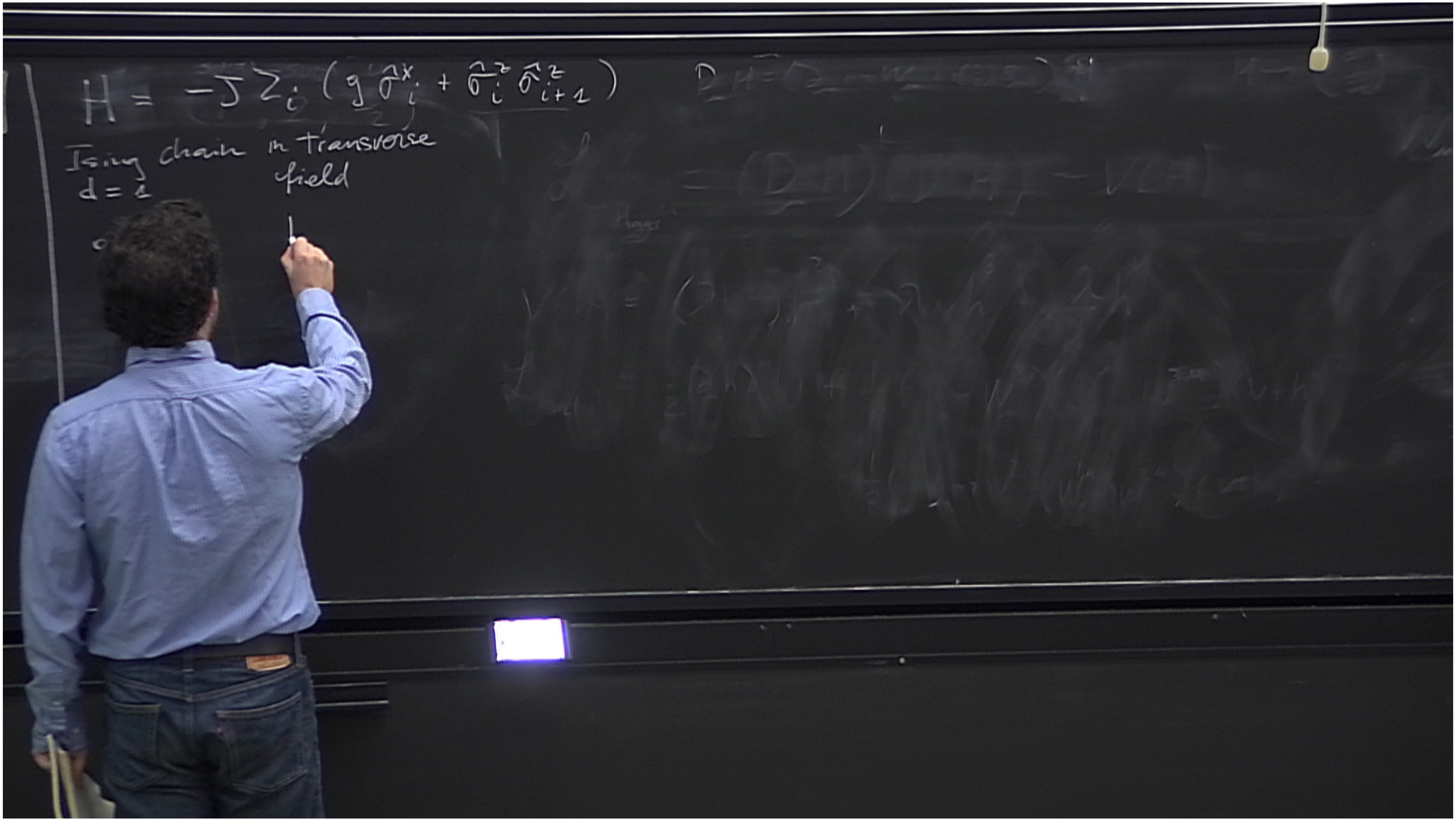

Quantum Phase Transitions
T=0
H (g)   g → g_c
level crossing → ∂E_0/∂g discontinuous
avoided l.c. →
J |g - g_c|^{zν}
CRITICAL
Λ|g - g_c|^ν


H = −J Σi (g σ̂xi + σ̂zi σ̂zi+1)
Ising chain in
d = 1


$H = -J \sum_i \left(g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$
Ising chain in transverse field
$d = 1$


H = −J Σ_i (g σ̂^x_i + σ̂^z_i σ̂^z_{i+1})
Ising chain in transverse field
d = 1


$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
$d = 1$

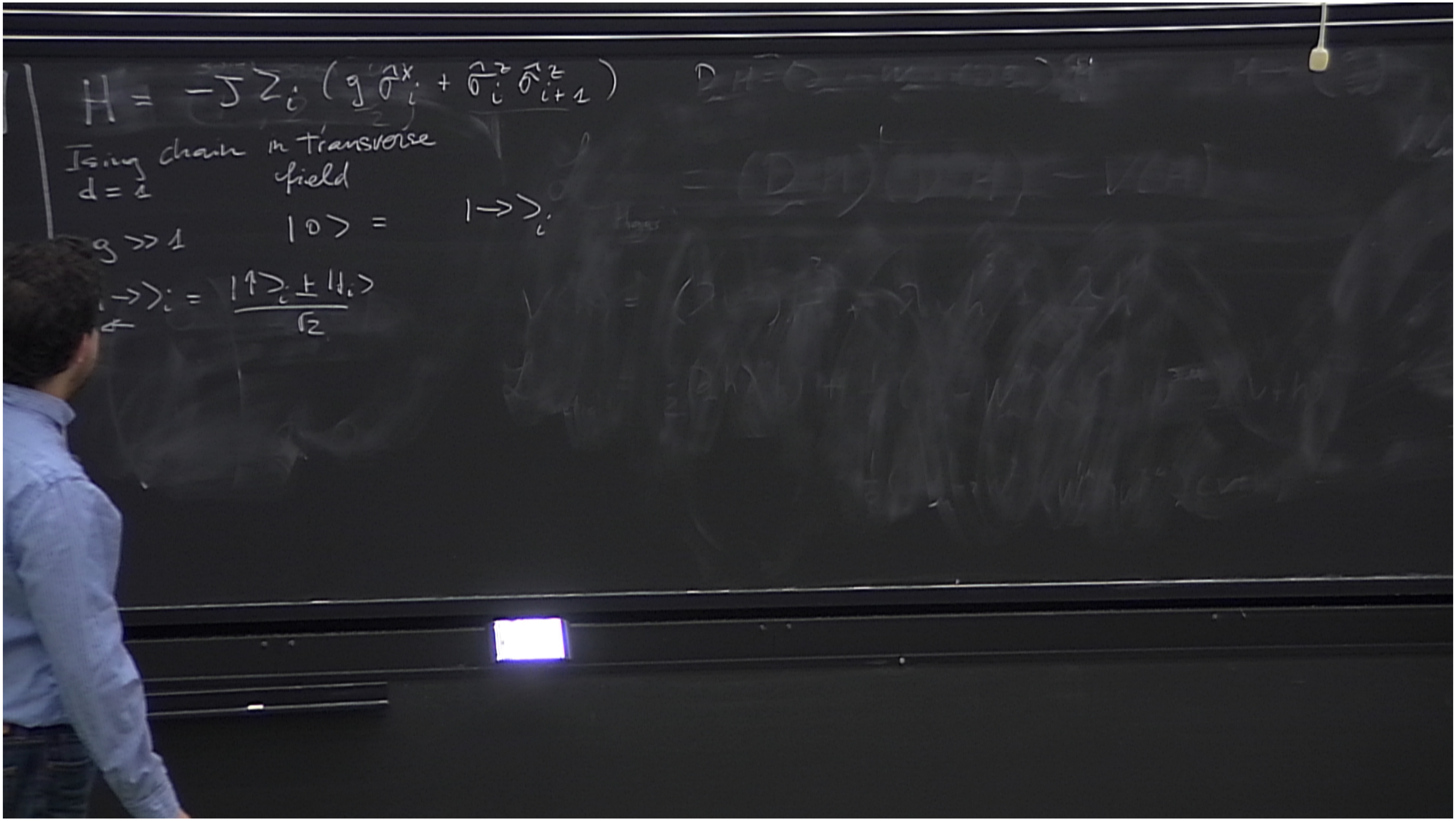

$H = -J \sum_i \left(g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$
Ising chain in transverse field
d = 1

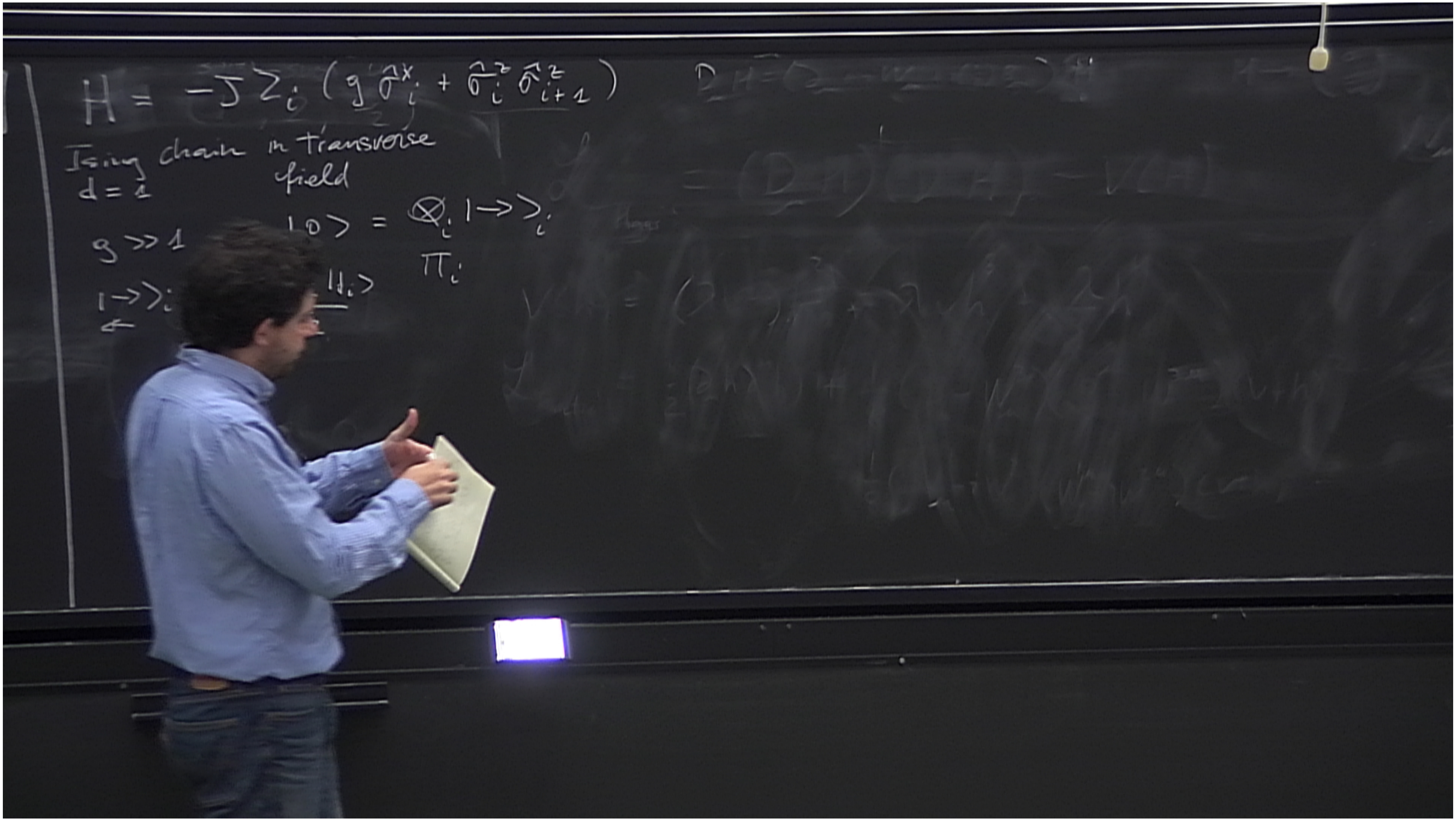
$$H = -J \sum_i \left(g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$$

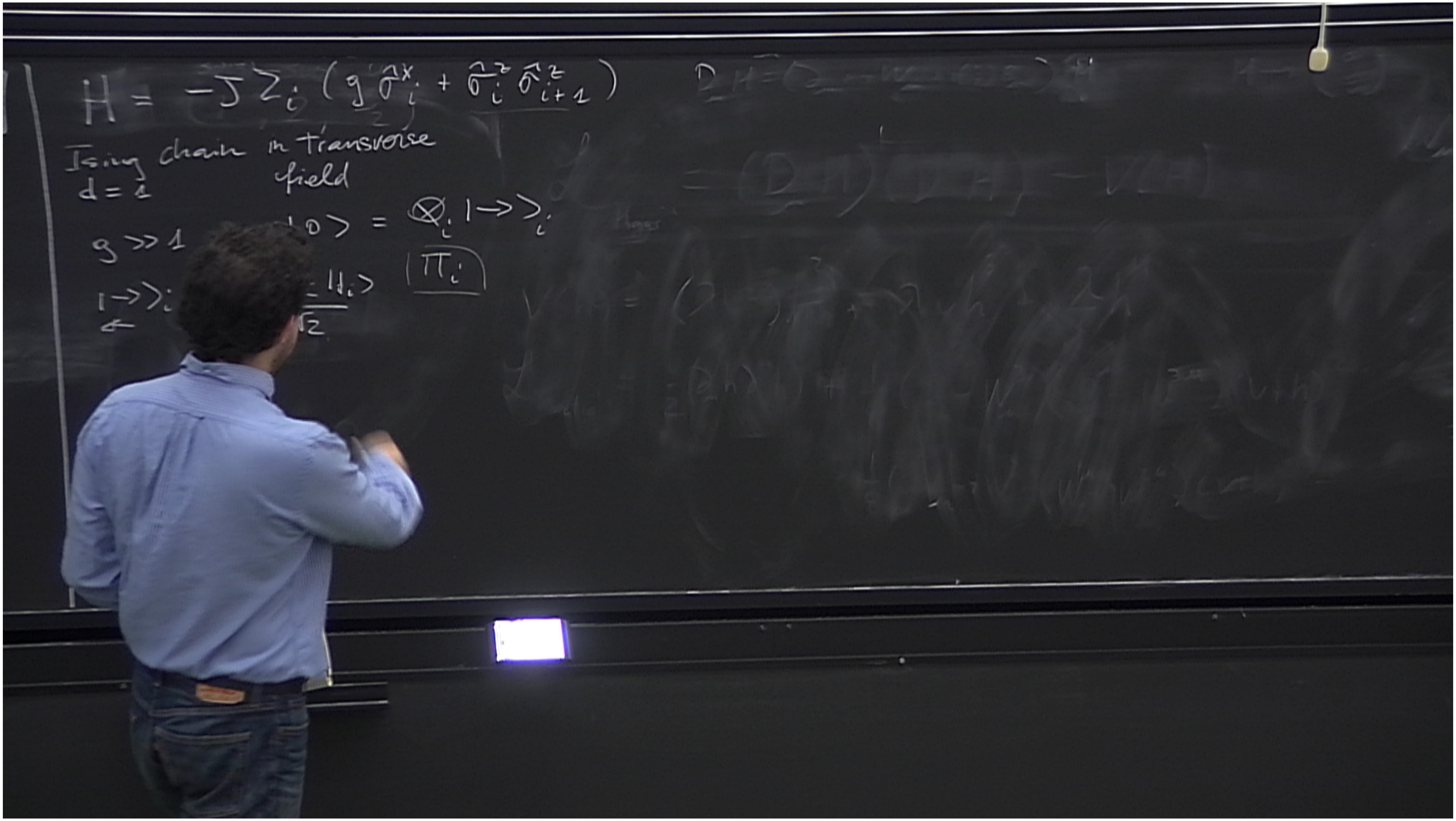
Ising chain in transverse field

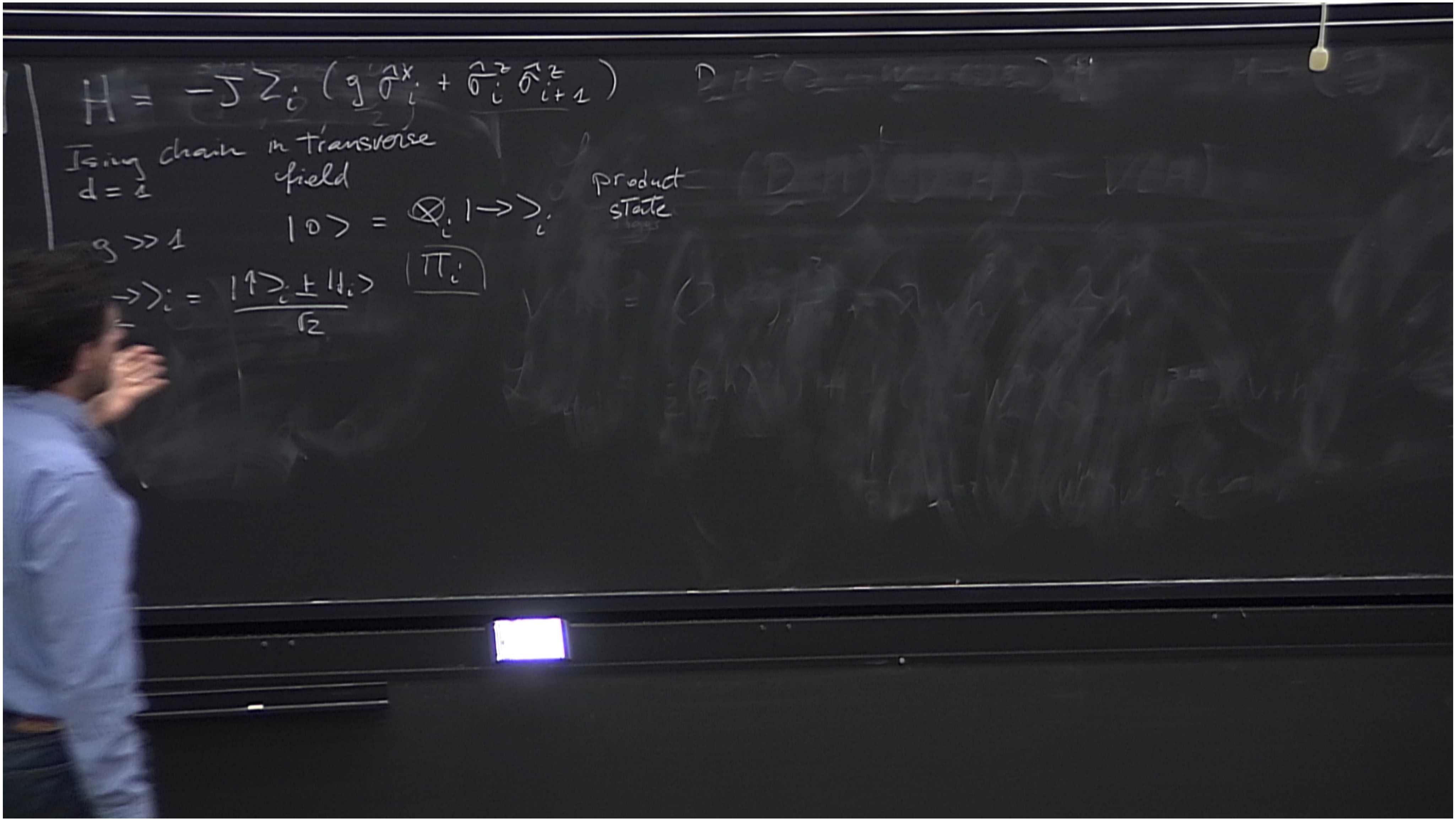
$d = 1$

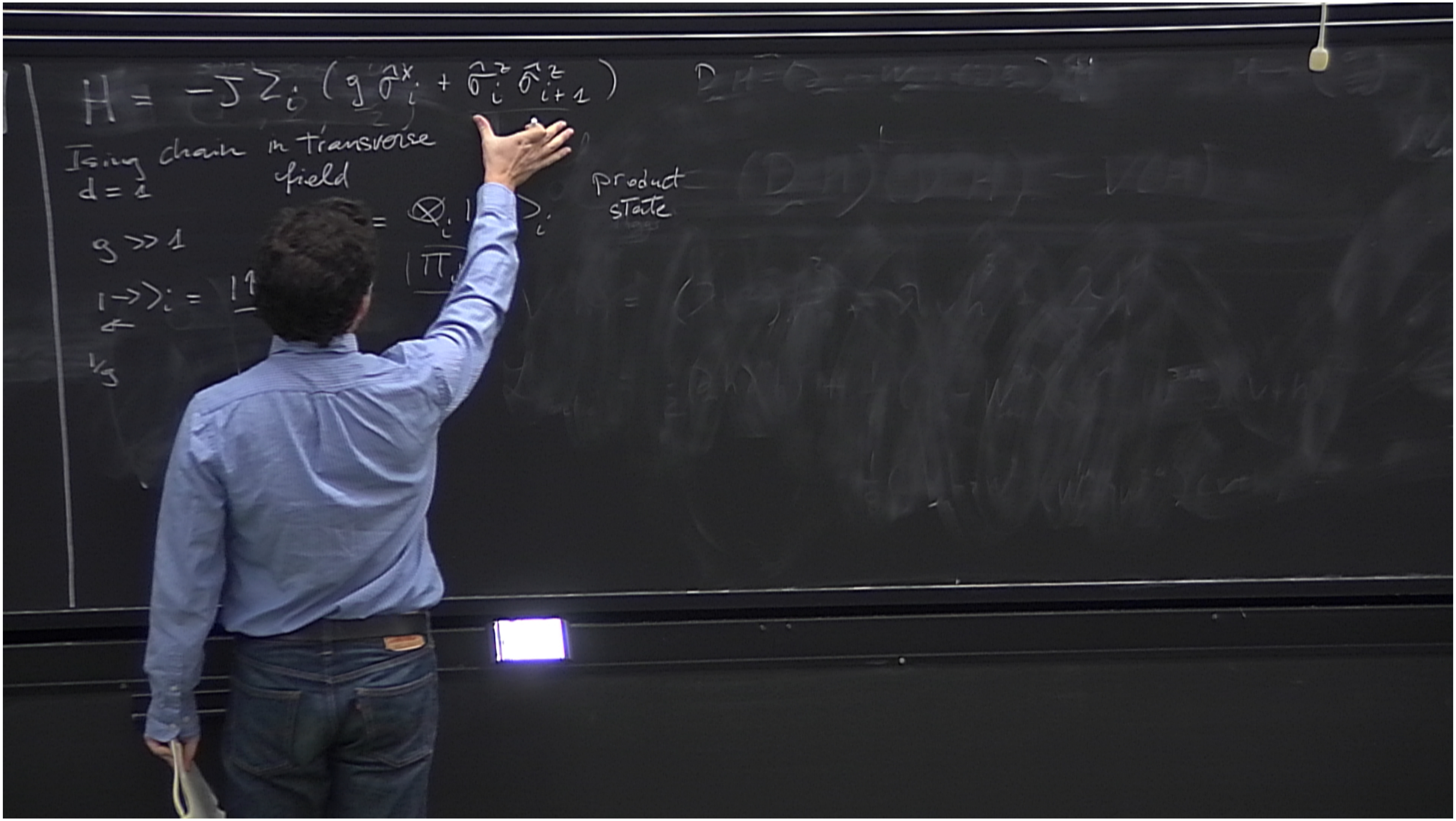
$g \gg 1$ $\qquad |0\rangle =$


$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
$d = 1$
$g \gg 1$
$|0\rangle = \quad |\rightarrow\rangle_i$
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$


H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
Π_i


H = −J Σi (g σ̂xi + σ̂zi σ̂zi+1)
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗i |→>i
Πi


$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
d = 1
g >> 1
$|0\rangle = \otimes_i |\rightarrow\rangle_i$
product state
Π_i
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$



$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
d = 1
g >> 1
product state

$$H = -J \sum_i \left(g\, \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$$

Ising chain in transverse field

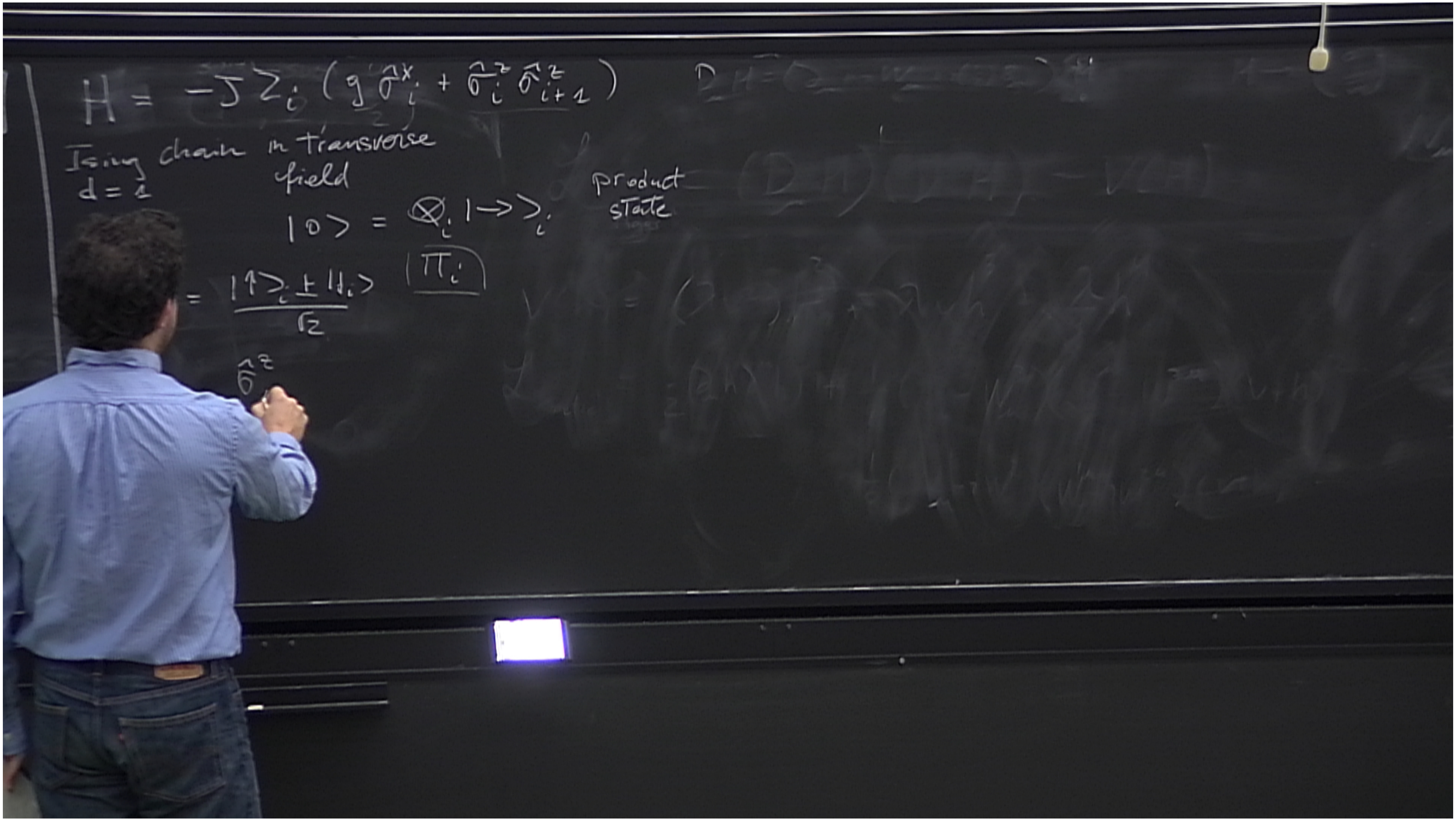
$d = 1$

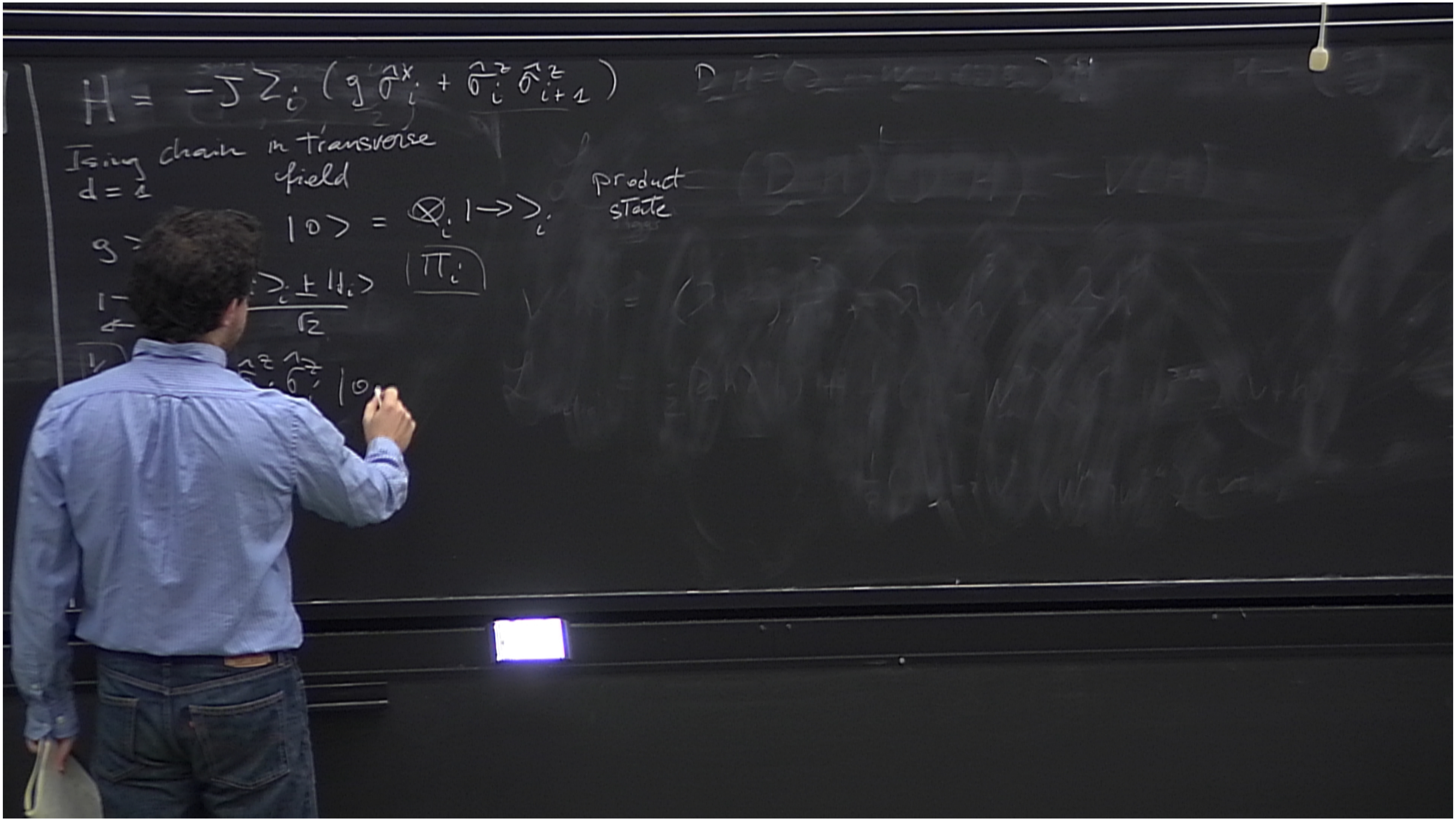
$g \gg 1$ $\qquad |0\rangle = \bigotimes_i |\rightarrow\rangle_i$ product state

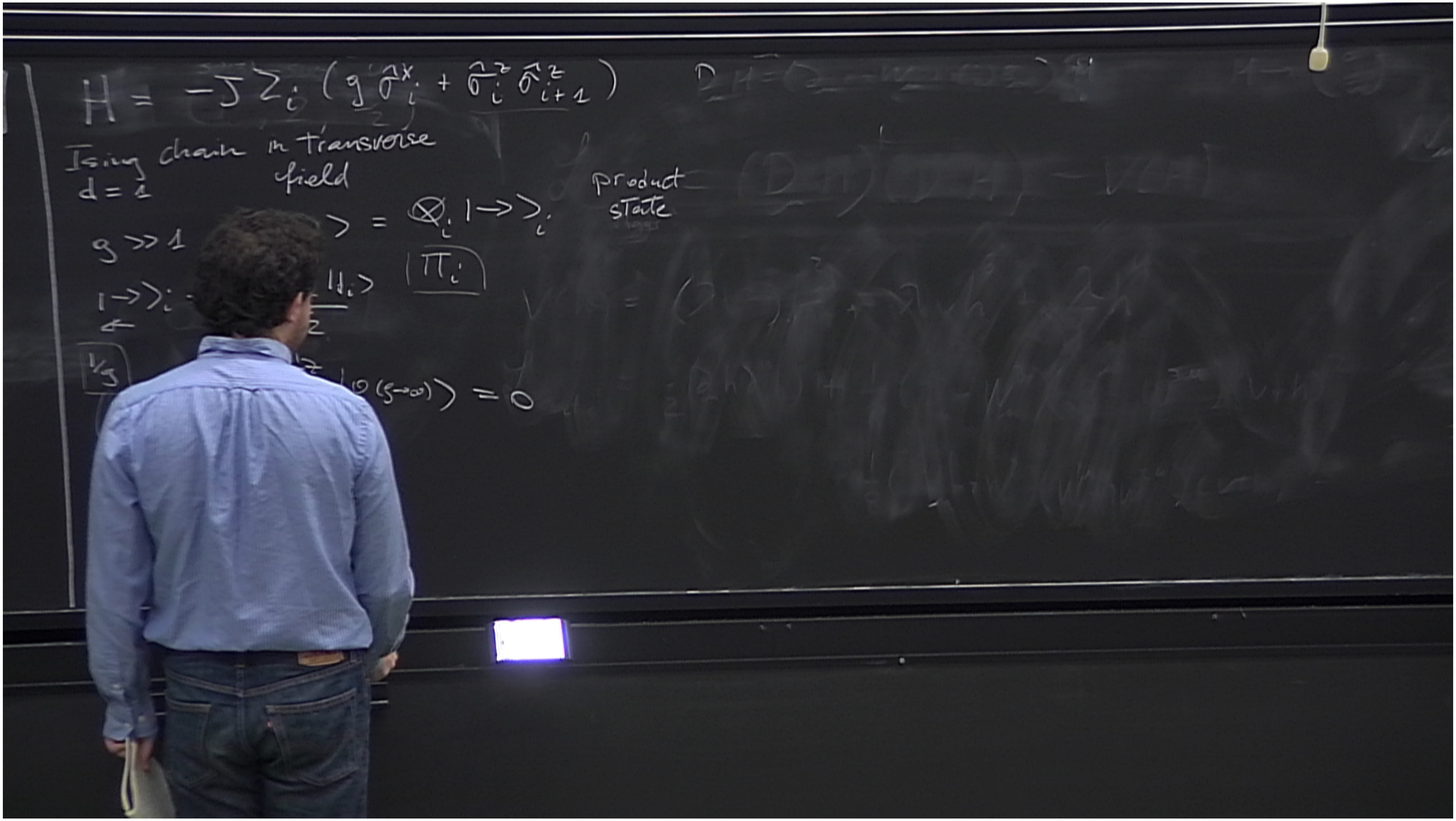
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow\rangle_i}{\sqrt{2}}$ $\quad (\Pi_i)$

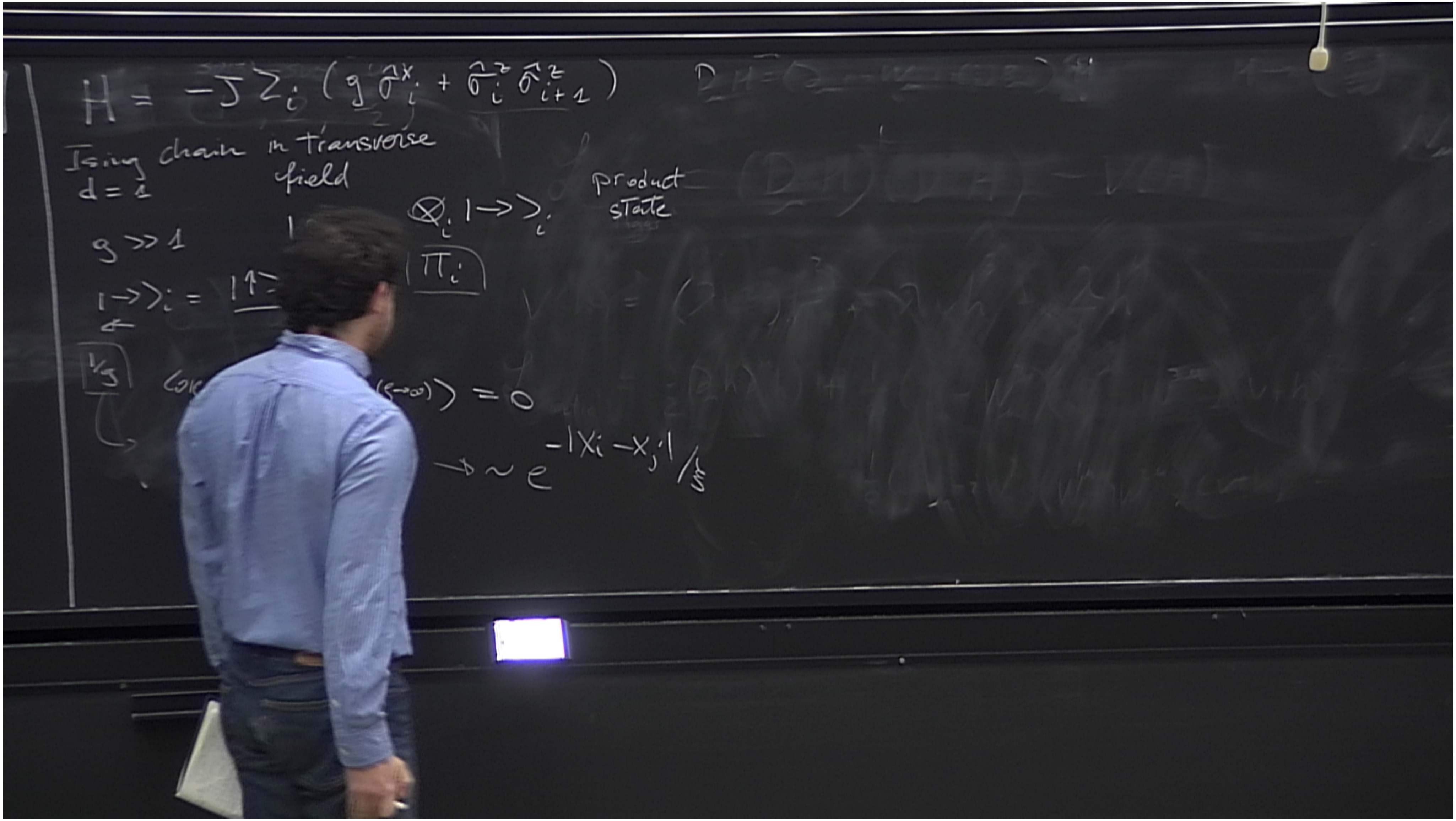
$\leftarrow$

$1/g$


H = −J Σᵢ (g σ̂ˣᵢ + σ̂ᶻᵢ σ̂ᶻᵢ₊₁)
Ising chain in transverse field
d = 1
|0> = ⊗ᵢ |→>ᵢ
product state
= (|↑>ᵢ ± |↓ᵢ>)/√2
Πᵢ
σ̂ᶻ


H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
|0> = ⊗_i |→>_i
product state
Π_i


H = -J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
product state



$H = -J \sum_i (g\, \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
d = 1
g >> 1
$\otimes_i |\rightarrow\rangle_i$
product state
Π_i
$|\rightarrow\rangle_i =$
1/g
= 0
$\rightarrow \sim e^{-|x_i - x_j|/\xi}$

$$H = -J \sum_i \left(g\, \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$$

Ising chain in transverse field

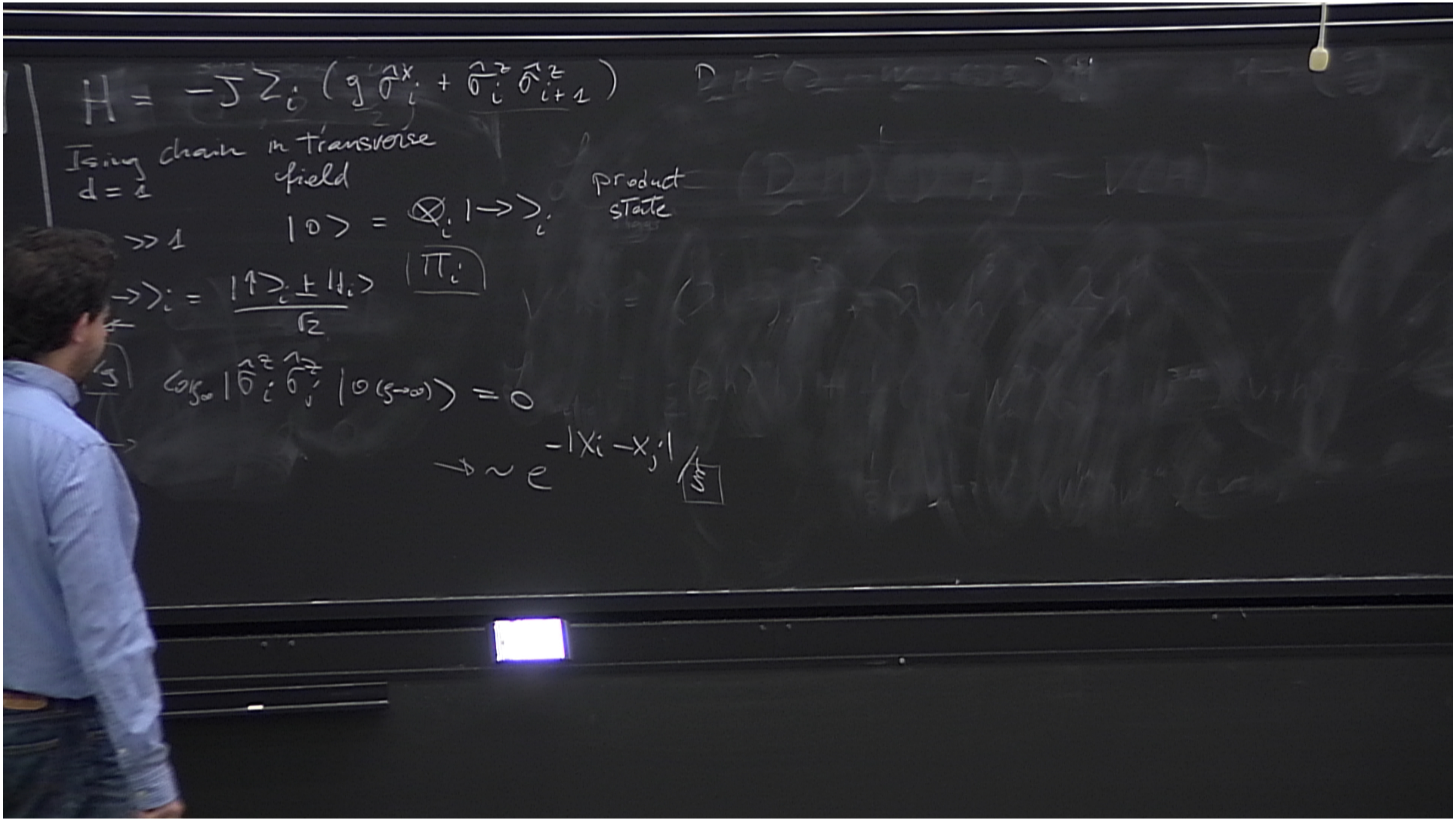
$d = 1$

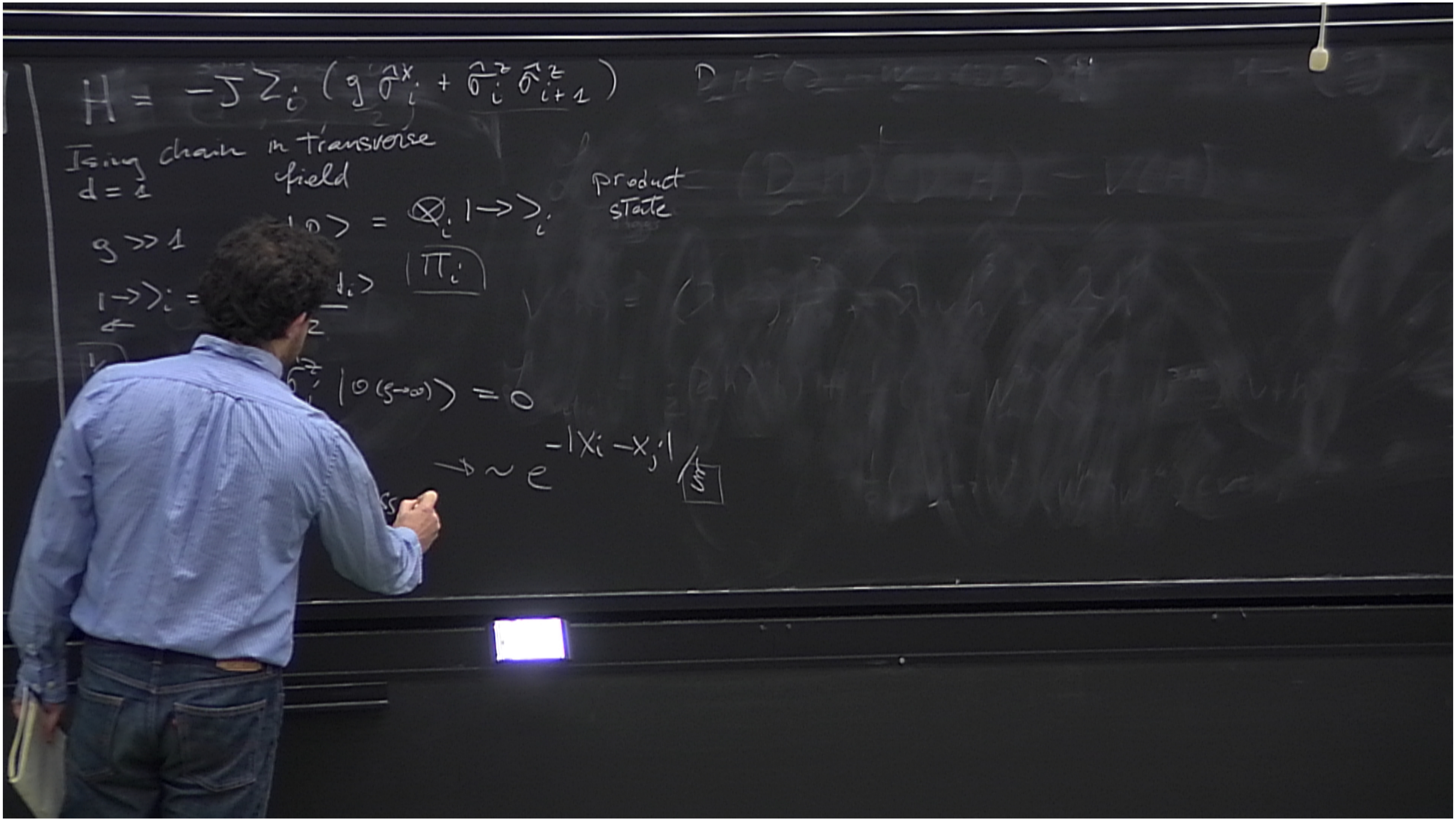
$g \gg 1$ $\qquad |0\rangle = \bigotimes_i |\rightarrow\rangle_i$ $\qquad$ product state

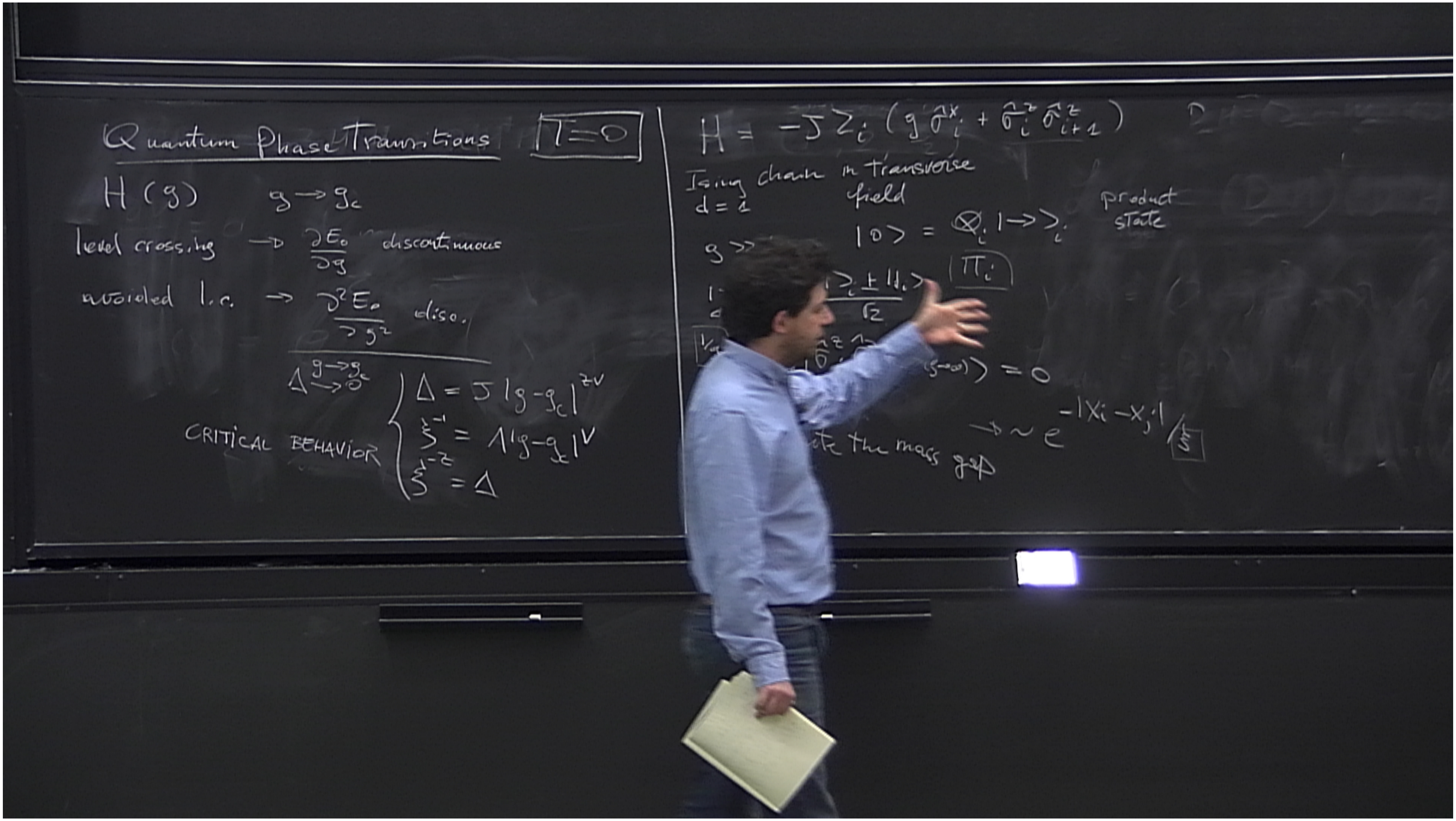
$|\rightarrow\rangle_i = \dfrac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$ $\qquad \Pi_i$

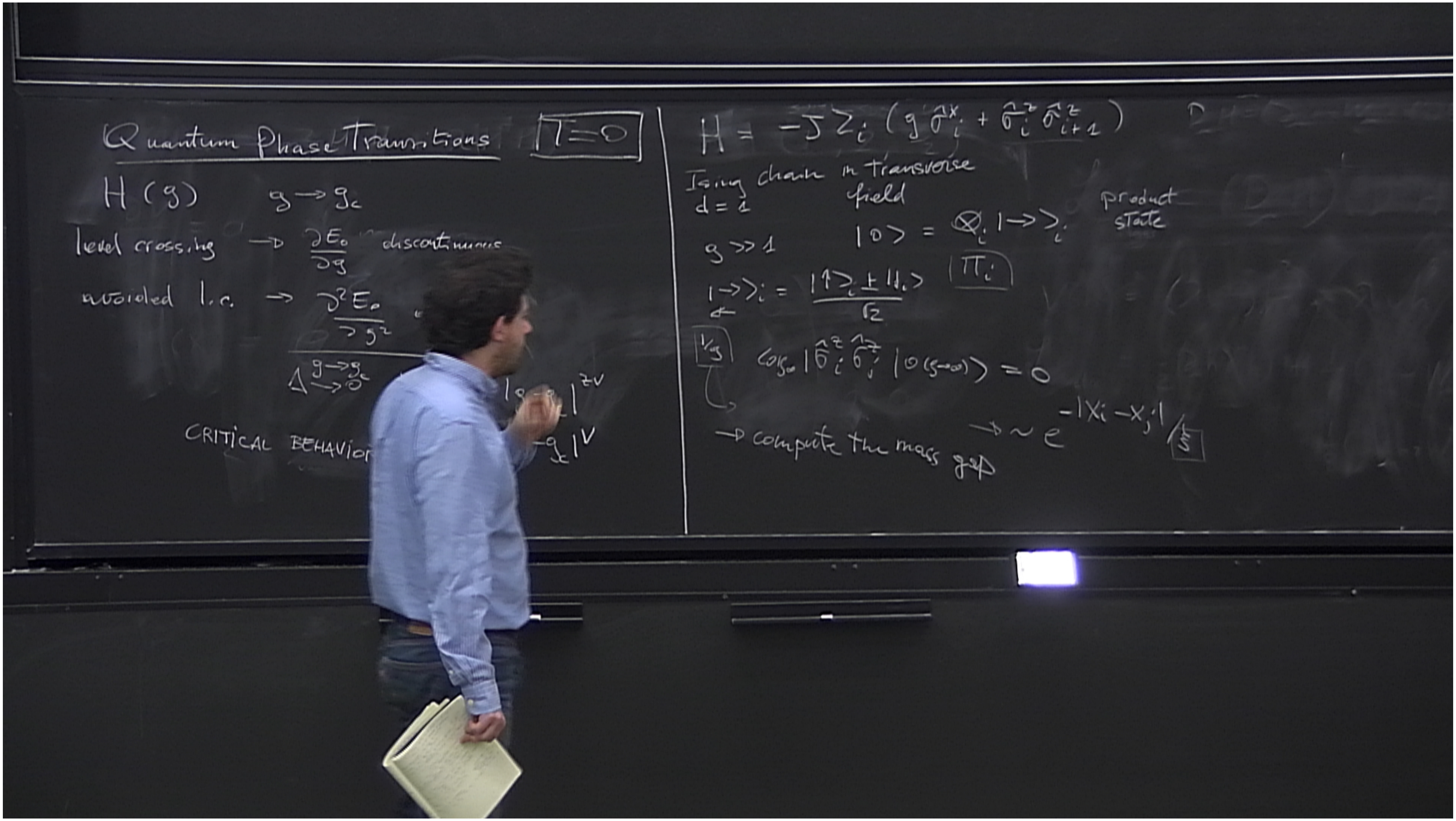
$\boxed{1/g}$ $\qquad \langle 0_{g\to\infty} | \hat{\sigma}^z_i \hat{\sigma}^z_j | 0(g\to\infty)\rangle = 0$

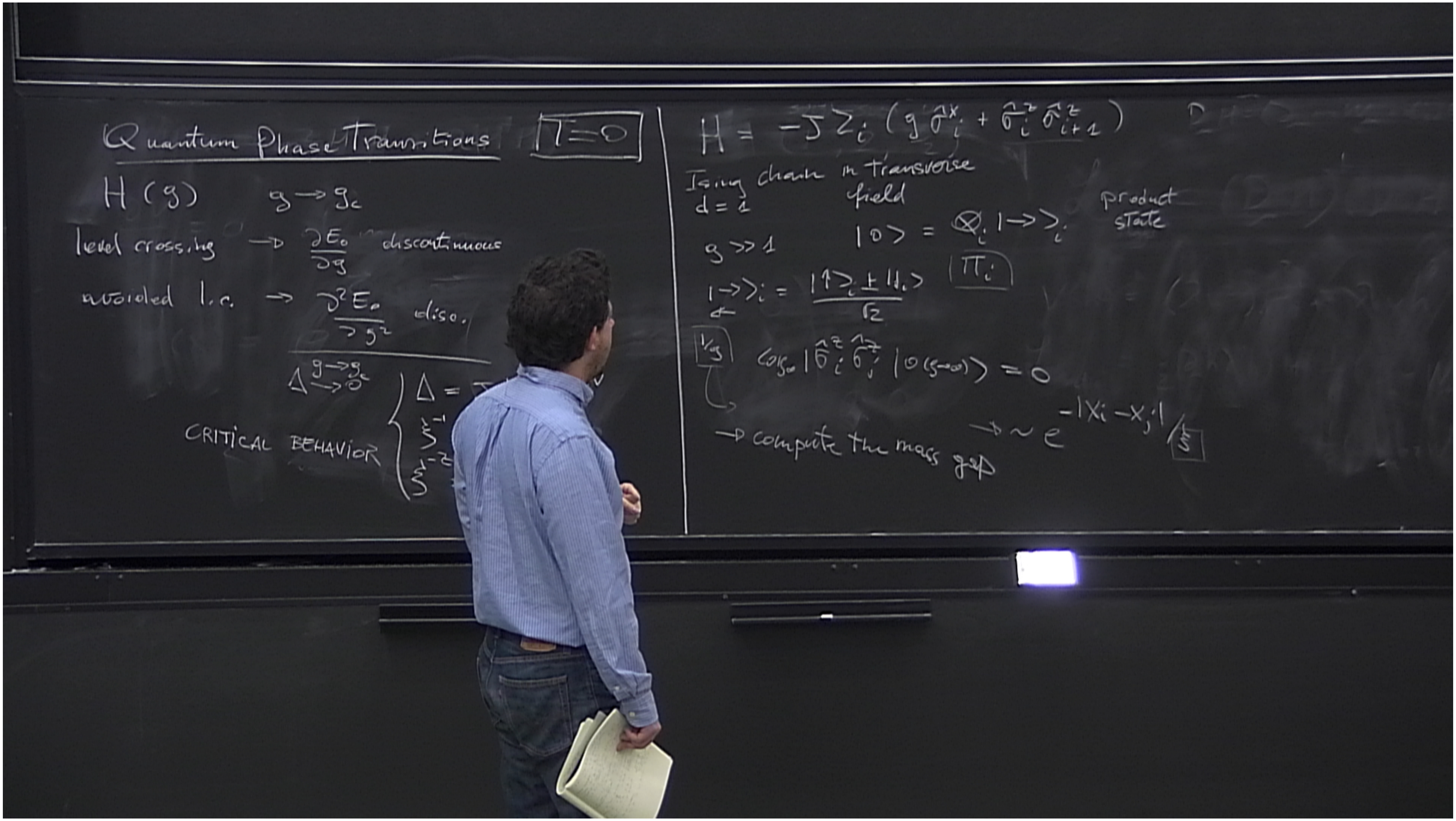
$\rightarrow \sim e^{-|x_i - x_j|/\boxed{\xi}}$

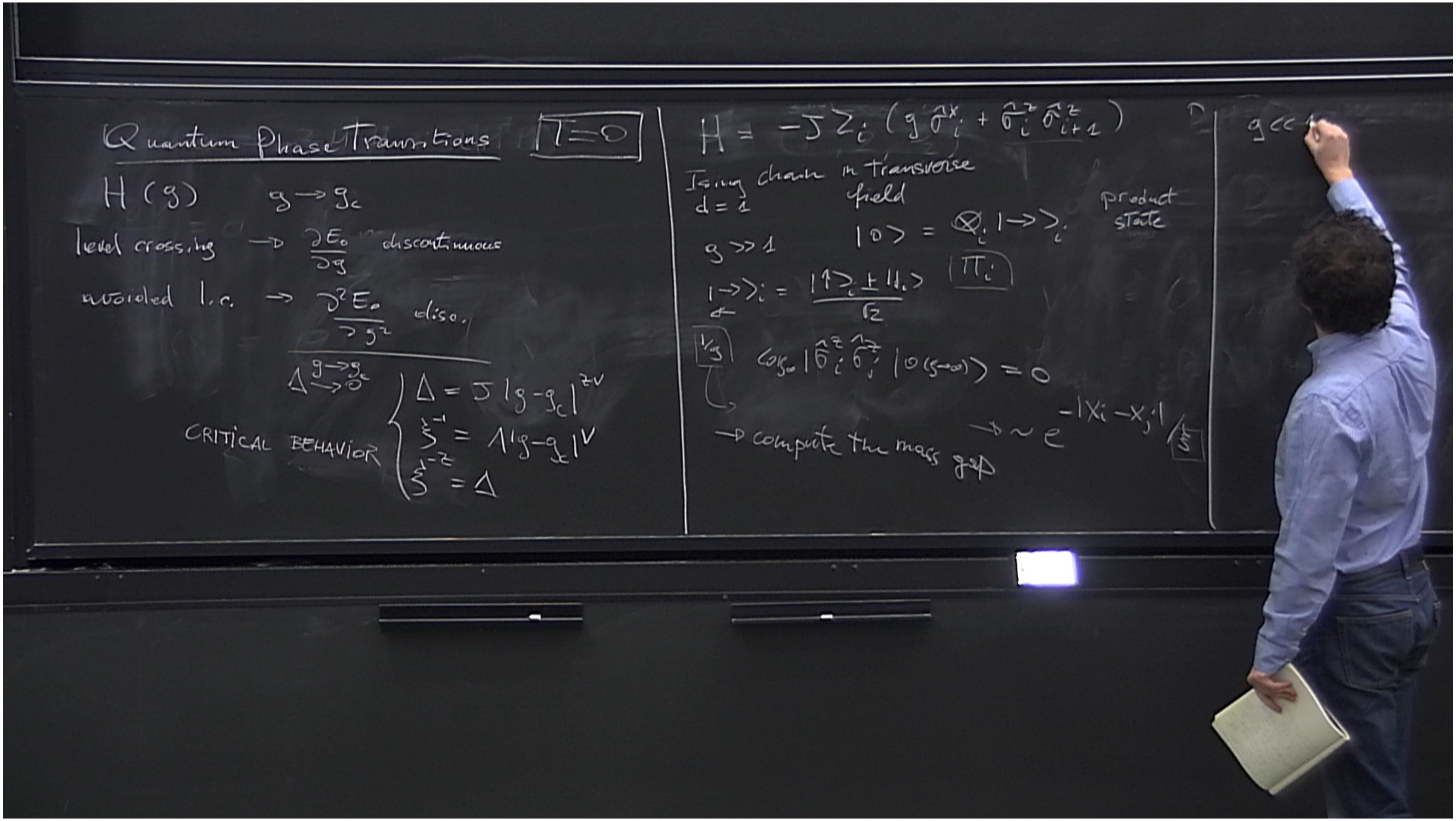

$H = -J \sum_i \left(g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$
Ising chain in transverse field
$d = 1$
$g \gg 1$
$|0\rangle = \bigotimes_i |\rightarrow\rangle_i$
product state
Π_i
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow\rangle_i}{\sqrt{2}}$
$\langle 0_{g\to\infty} | \hat{\sigma}^z_i \hat{\sigma}^z_j | 0(g\to\infty)\rangle = 0$
$\rightarrow \sim e^{-|x_i - x_j| / \xi}$

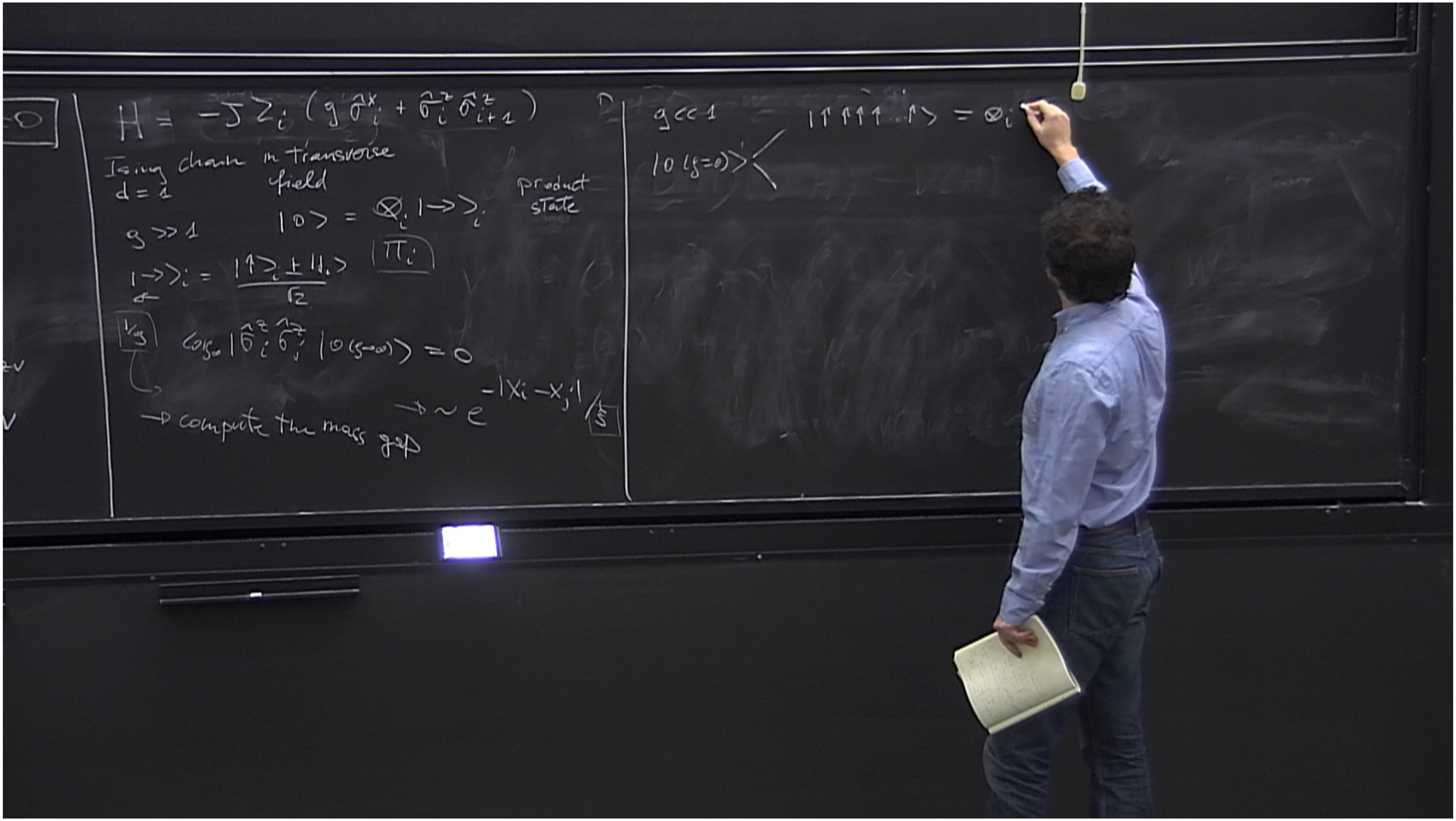

H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
Π_i
|0(g→∞)> = 0
→ ~ e^{−|x_i − x_j| / ξ}

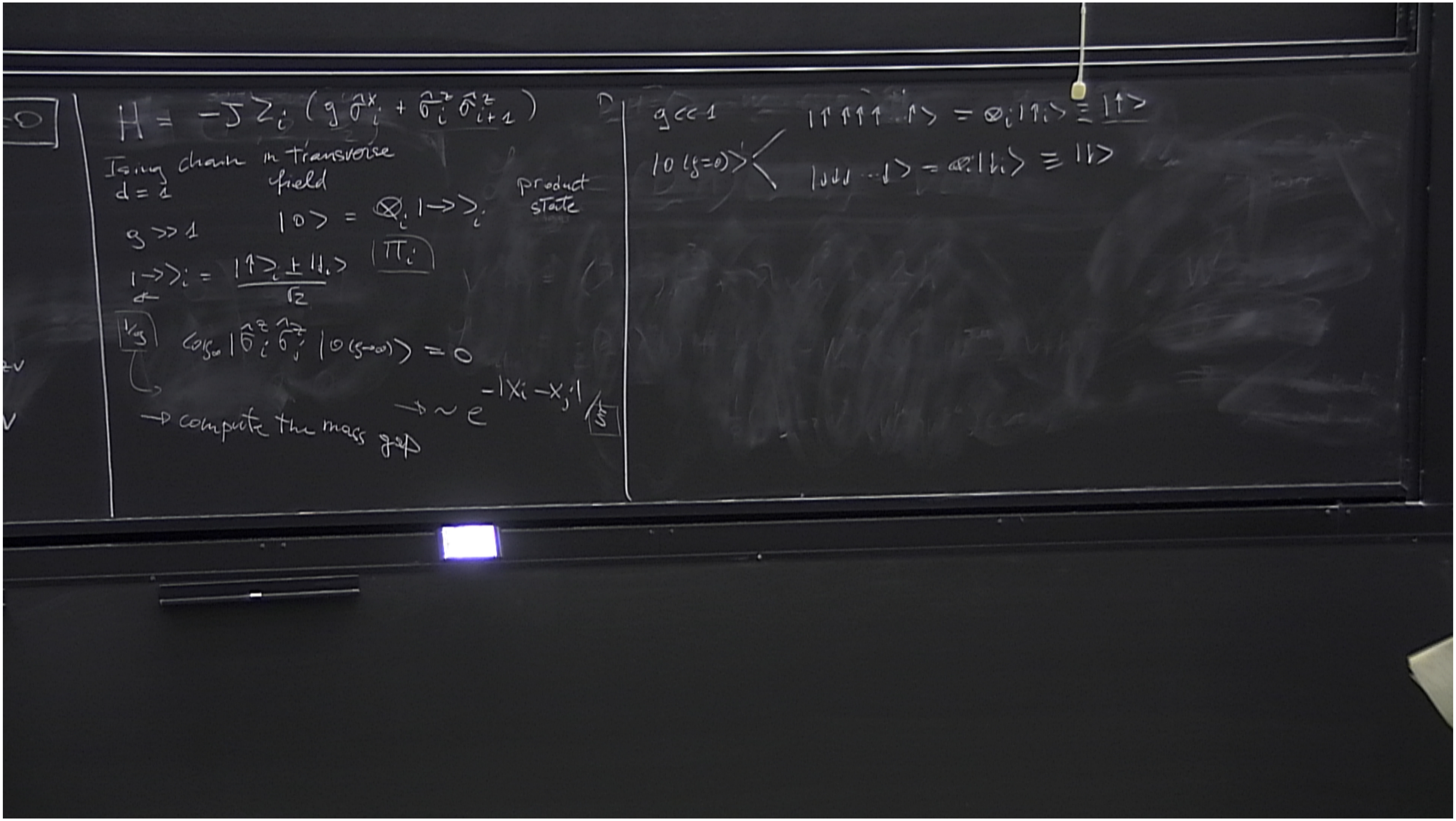

Quantum Phase Transitions
T=0
H (g)
g → g_c
level crossing → ∂E_0/∂g discontinuous
avoided l.c. → ∂²E_0/∂g² disc.
CRITICAL BEHAVIOR
Δ = J |g-g_c|^{zν}
ξ^{-1} = Λ|g-g_c|^ν
ξ^{-z} = Δ
H = -J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
product state
the mass gap

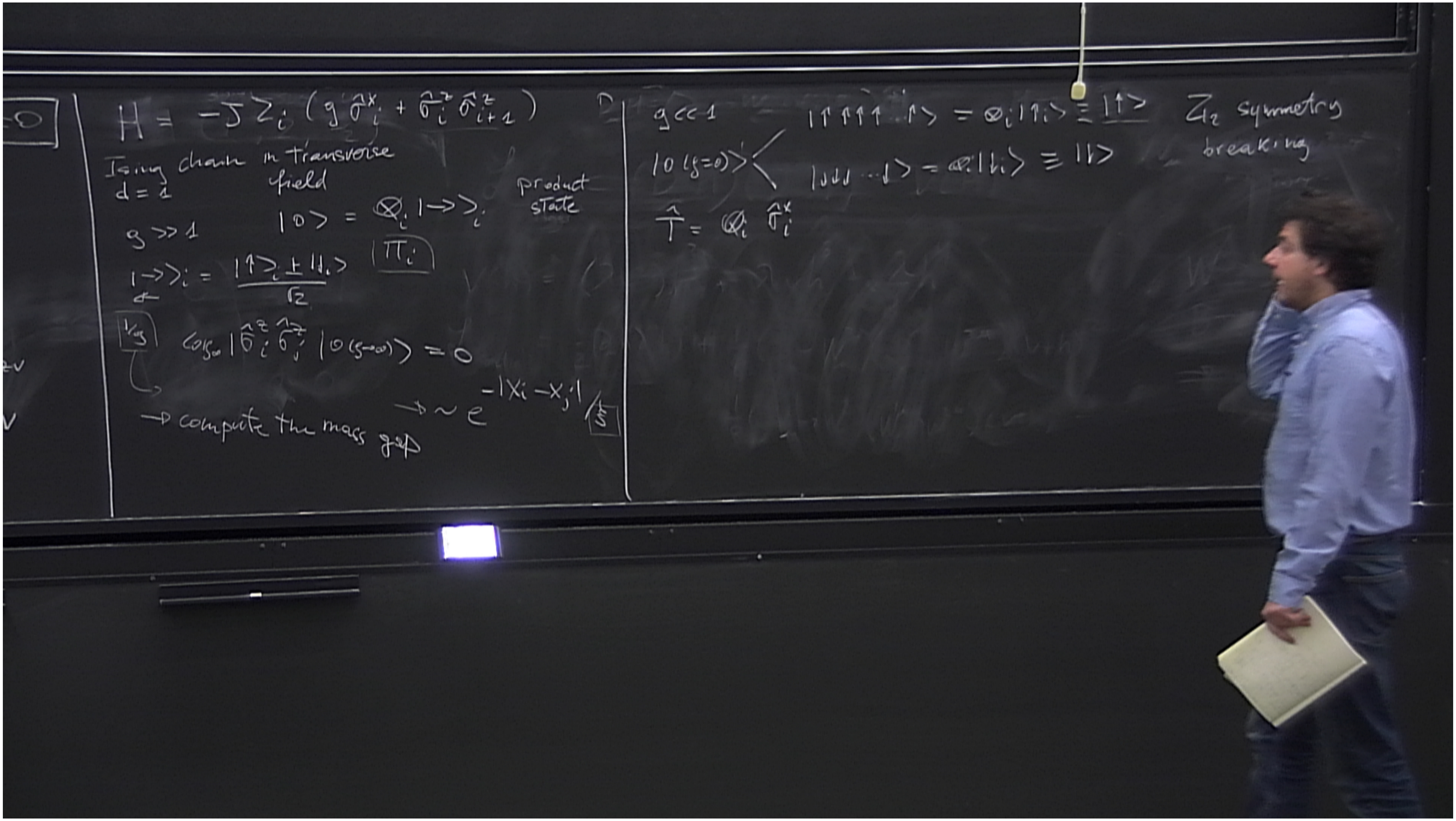

Quantum Phase Transitions
T=0
H(g) g → g_c
level crossing → ∂E_0/∂g discontinuous
avoided l.c. → ∂²E_0/∂g²
CRITICAL BEHAVIOR
H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i ± |↓_i>)/√2
Π_i
1/g
→ compute the mass gap
→ ~ e^{−|x_i − x_j|/ξ}


Quantum Phase Transitions
T=0
H(g) g → g_c
level crossing → ∂E₀/∂g discontinuous
avoided l.c. → ∂²E₀/∂g² disc.
CRITICAL BEHAVIOR
Ising chain in transverse field
d = 1
g >> 1
product state
compute the mass gap

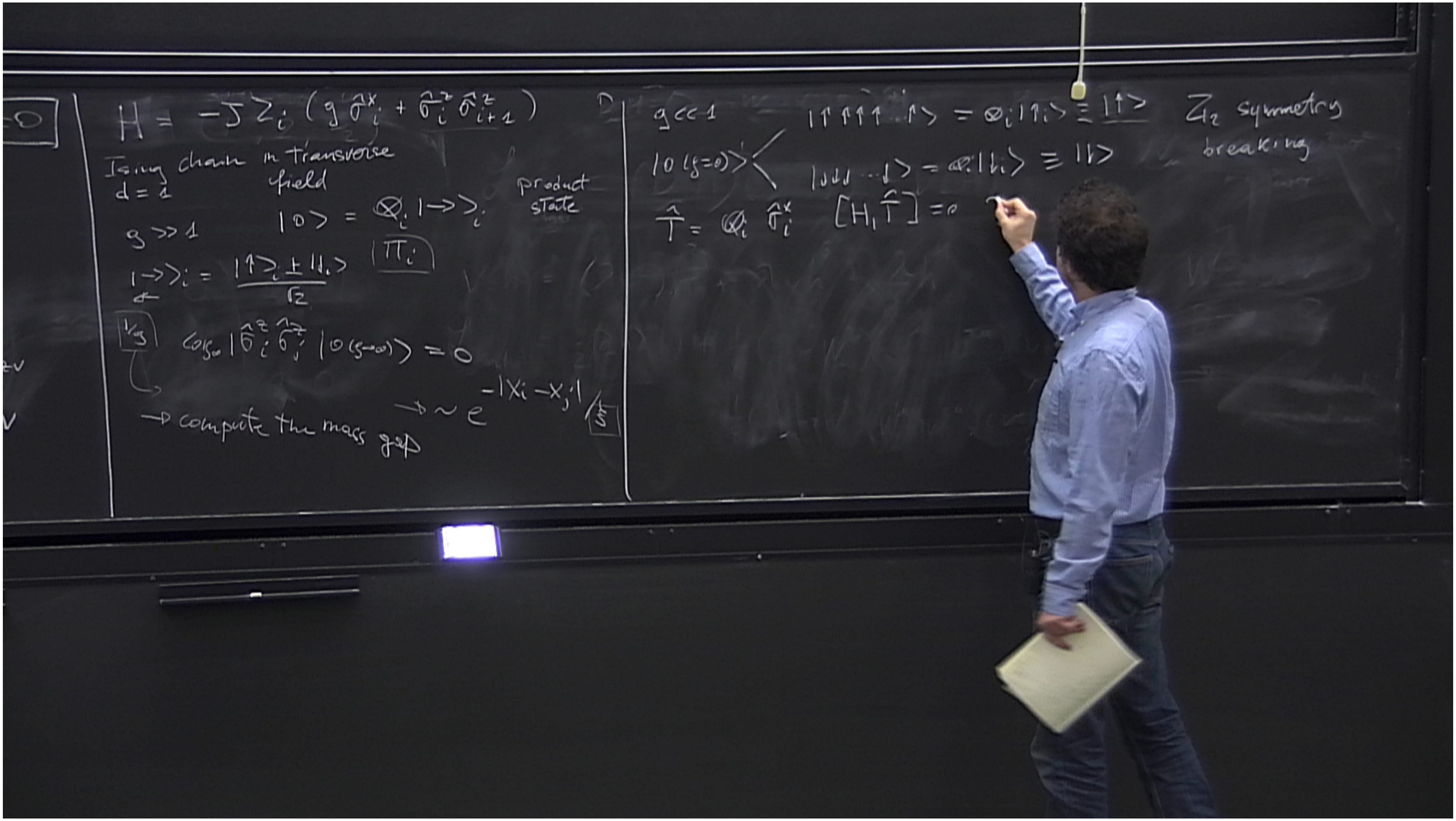

Quantum Phase Transitions
T = 0
H(g)   g → g_c
level crossing → ∂E_0/∂g discontinuous
avoided l.c. → ∂²E_0/∂g² disc.
Δ → 0 as g → g_c
CRITICAL BEHAVIOR
Δ = J |g − g_c|^{zν}
ξ^{-1} = Λ |g − g_c|^ν
ξ^{-z} = Δ
H = −J Σ_i (g σ̂^x_i + σ̂^z_i σ̂^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i + |↓>_i)/√2
Π_i
1/g
<0_{g→∞}| σ̂^z_i σ̂^z_j |0(g→∞)> = 0
→ compute the mass gap
→ ~ e^{−|x_i − x_j|/ξ}
g << 1

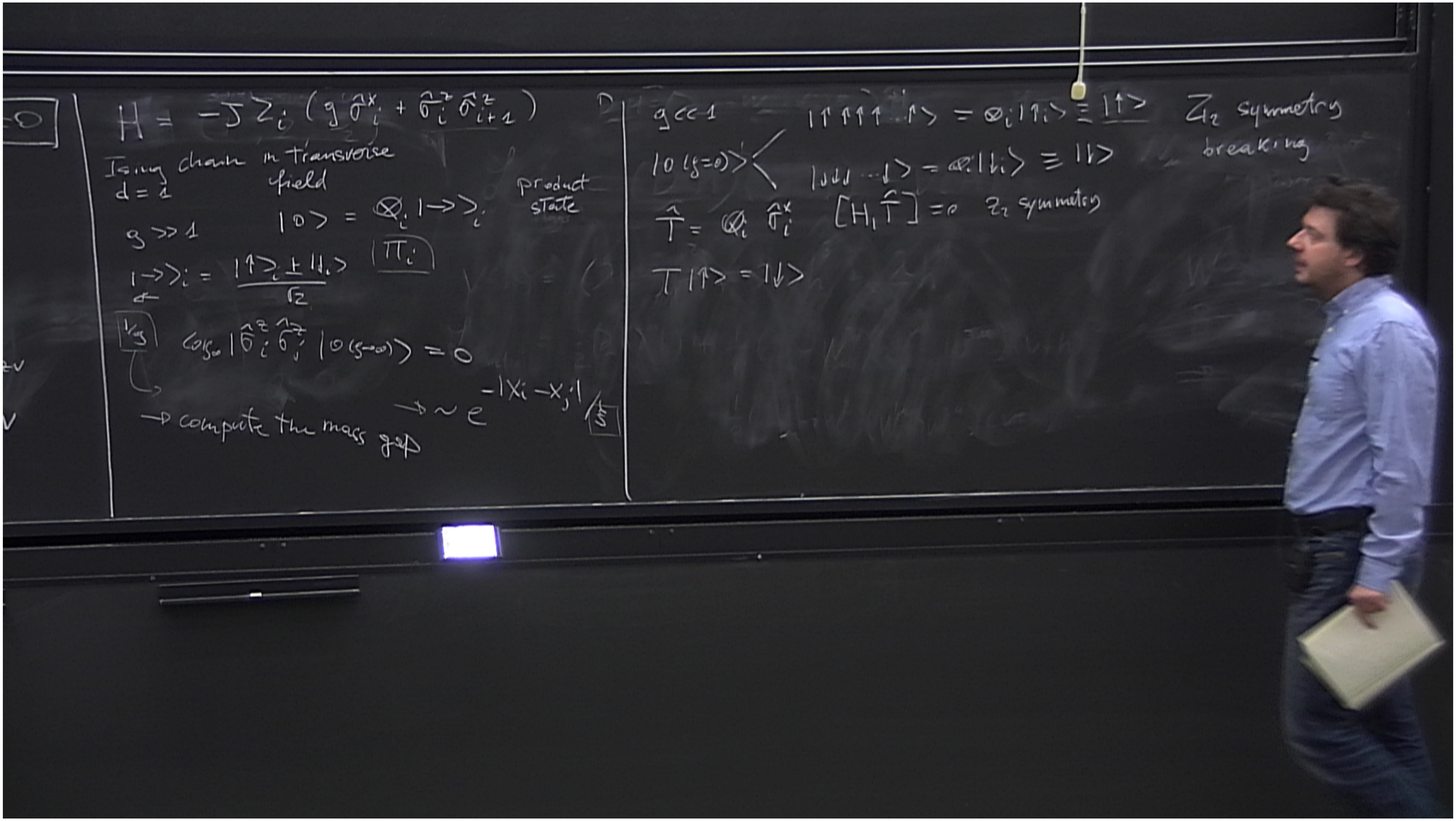

H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i ± |↓>_i)/√2
Π_i
1/g
<0(g→∞)| σ^z_i σ^z_j |0(g→∞)> = 0
→ ~ e^{−|x_i − x_j|/ξ}
→ compute the mass gap
g << 1
|0(g=0)>
|↑↑↑↑ ... ↑> = ⊗_i

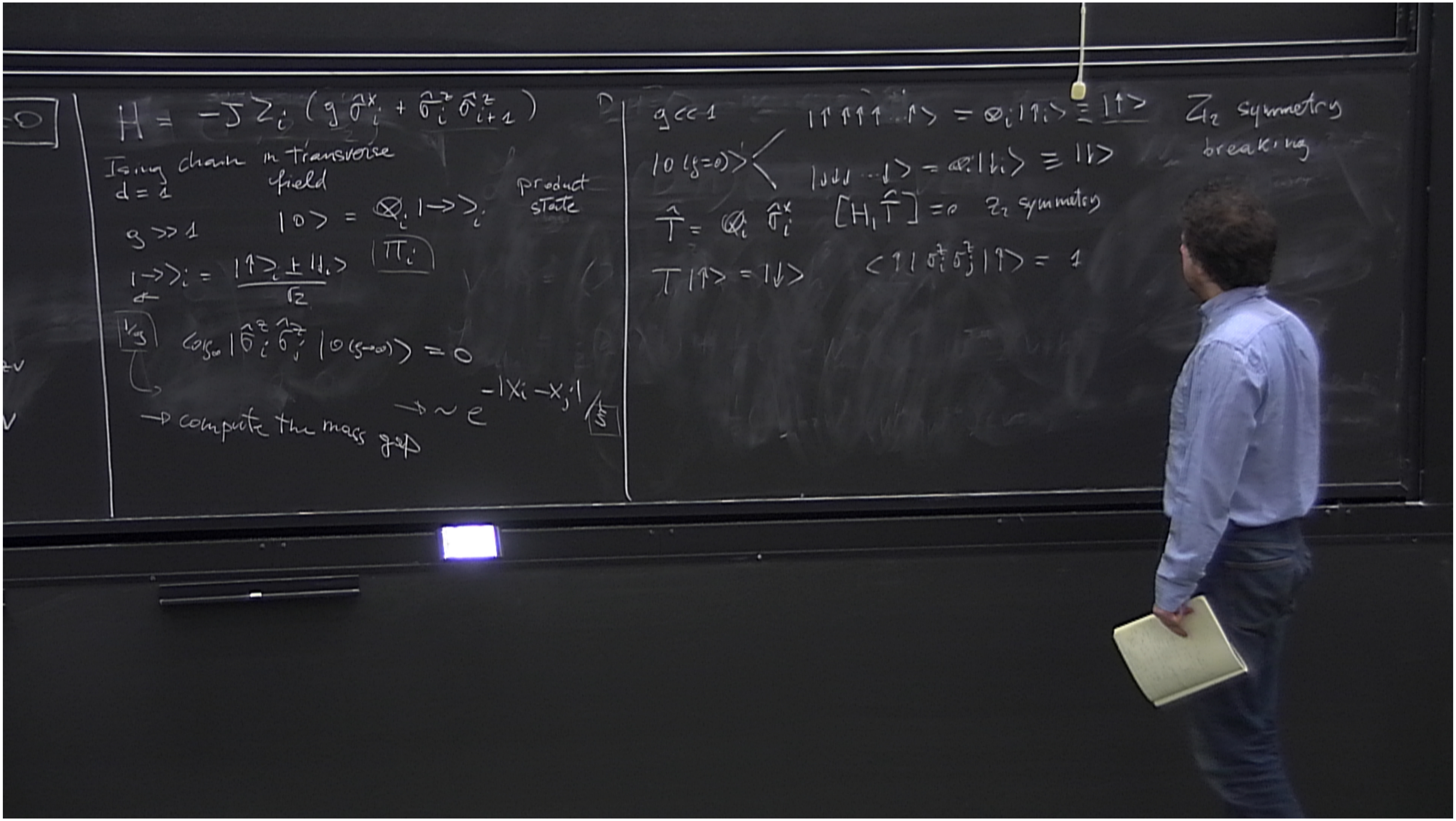

$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
$d = 1$
$g \gg 1$
$|0\rangle = \otimes_i |\rightarrow\rangle_i$
product state
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$
Π_i
$1/g$
$\langle 0|_{g\to\infty} \hat{\sigma}^z_i \hat{\sigma}^z_j |0(g\to\infty)\rangle = 0$
$\to \sim e^{-|x_i - x_j|/\xi}$
→ compute the mass gap
$g \ll 1$
$|0(g=0)\rangle$
$|\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle = \otimes_i |\uparrow_i\rangle \equiv |\Uparrow\rangle$
$|\downarrow\downarrow\downarrow \dots \downarrow\rangle = \otimes_i |\downarrow_i\rangle \equiv |\Downarrow\rangle$

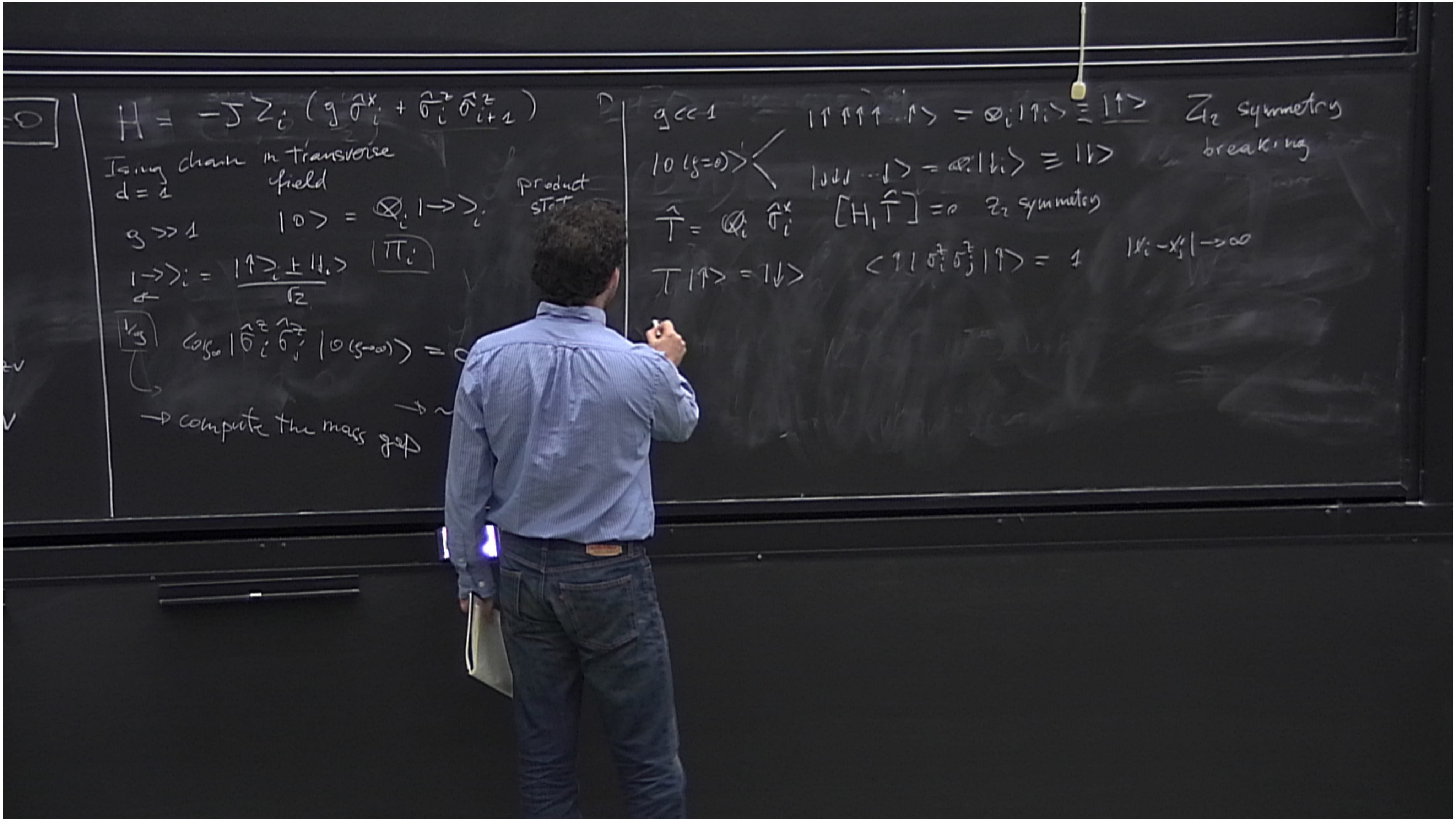


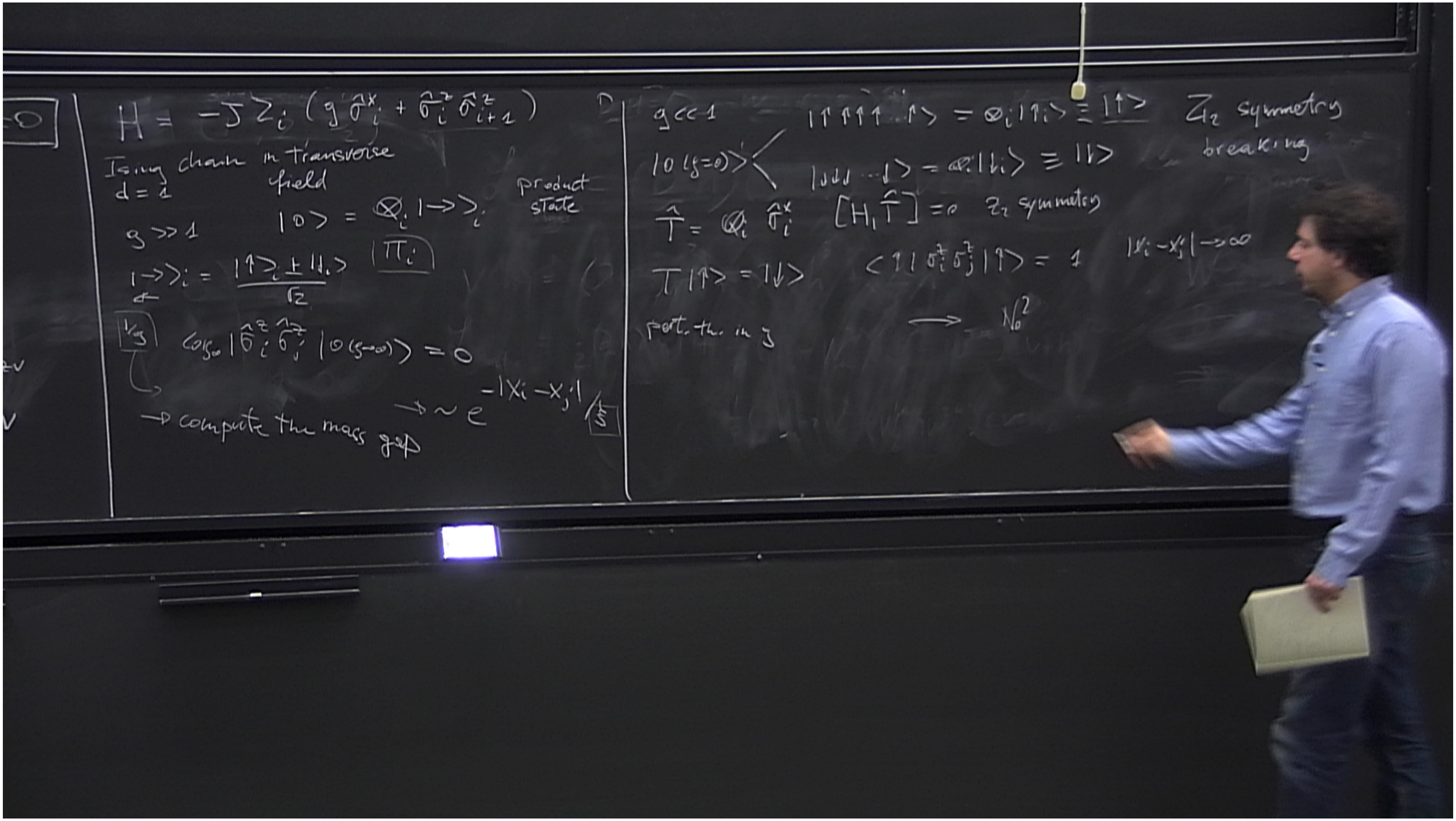
H = -J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
product state
g >> 1
|0> = ⊗_i |->_i
|->_i = (|↑>_i ± |↓_i>)/√2
Π_i
⟨0_{g→∞}| σ^z_i σ^z_j |0(g→∞)⟩ = 0
→ ~ e^{-|x_i - x_j|/ξ}
→ compute the mass gap
g << 1
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |↑>
|0 (g=0)>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |↓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i

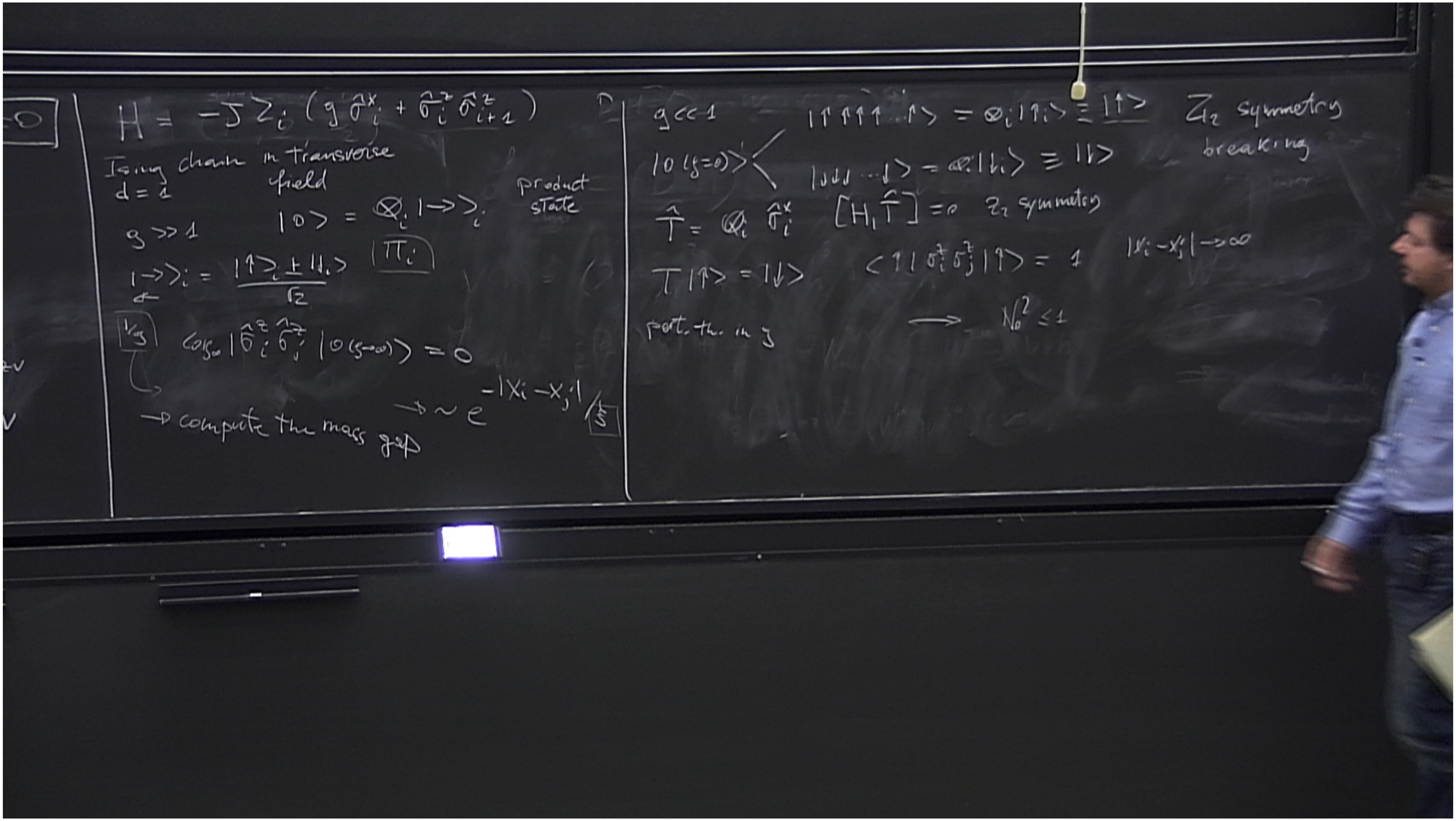

H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
product state
g >> 1
|0> = ⊗_i |→>_i
|→>_i = (|↑>_i ± |↓_i>)/√2
Π_i
1/g
<0_{g→∞}| σ^z_i σ^z_j |0(g→∞)> = 0
→ ~ e^{−|x_i − x_j|/ξ}
→ compute the mass gap
g << 1
|0(g=0)>
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |⇑>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |⇓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i
[H, T] = 0

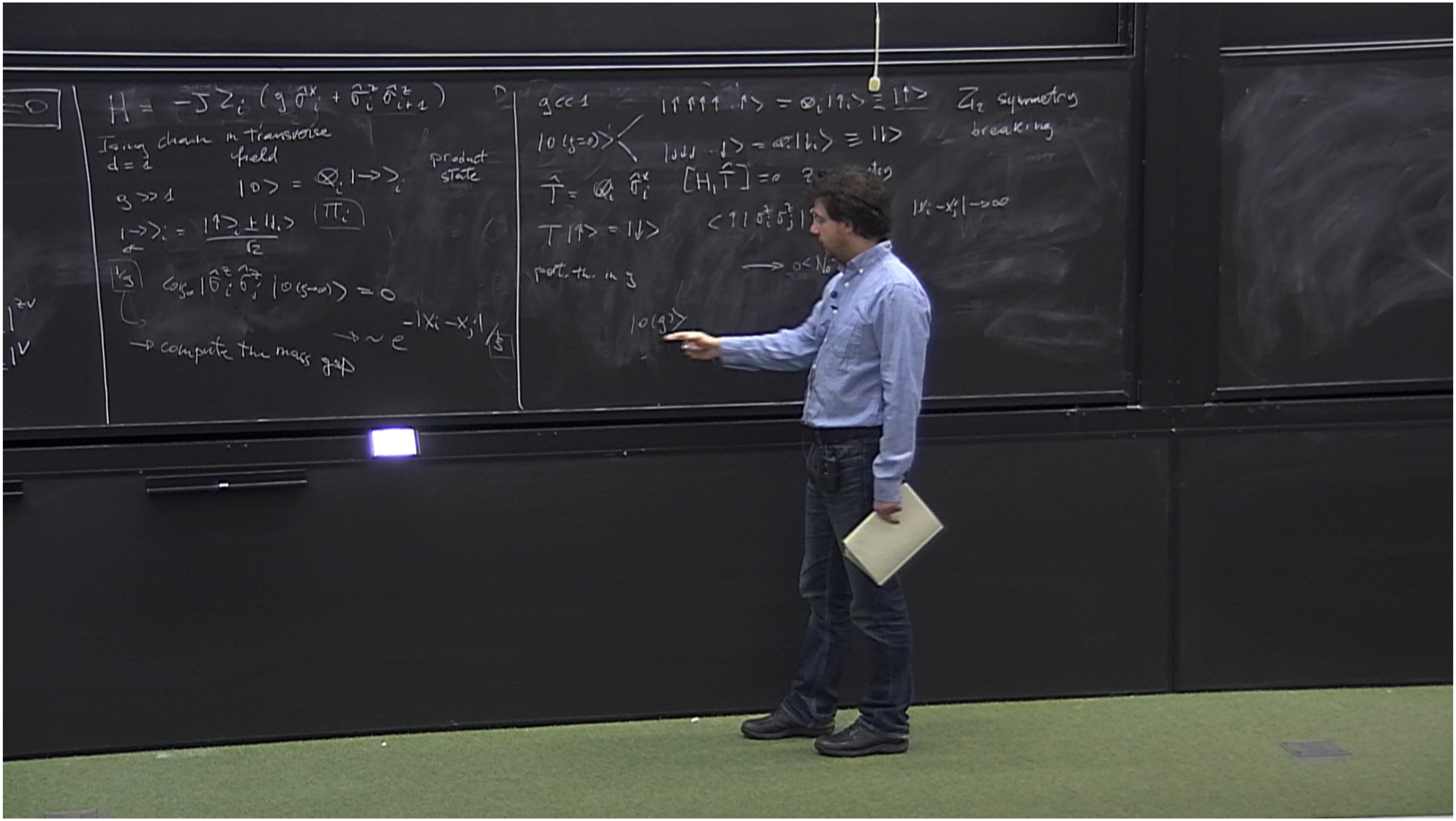

$H = -J \sum_i (g \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1})$
Ising chain in transverse field
d = 1
product state
$g \gg 1$ $|0\rangle = \otimes_i |\rightarrow\rangle_i$
$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$
Π_i
$1/g$
$\langle 0|_{g\to\infty} | \hat{\sigma}^z_i \hat{\sigma}^z_j | 0(g\to\infty)\rangle = 0$
$\to \sim e^{-|x_i - x_j|/\xi}$
→ compute the mass gap
$g \ll 1$
$|0(g=0)\rangle$
$|\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle = \otimes_i |\uparrow_i\rangle \equiv |\Uparrow\rangle$
$|\downarrow\downarrow\downarrow \dots \downarrow\rangle = \otimes_i |\downarrow_i\rangle \equiv |\Downarrow\rangle$
Z_2 symmetry breaking
$\hat{T} = \otimes_i \hat{\sigma}^x_i$ $[H, \hat{T}] = 0$ Z_2 symmetry
$T|\Uparrow\rangle = |\Downarrow\rangle$


H = -J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |->_i
product state
|->_i = (|↑>_i ± |↓_i>)/√2
<0 g→∞| σ^z_i σ^z_j |0 (g→∞)> = 0
→ compute the mass gap
→ ~ e^{-|x_i - x_j| / ξ}
g << 1
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |⇑>
|0 (g=0)>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |⇓>
T = ⊗_i σ^x_i
[H, T] = 0
Z_2 symmetry
T |⇑> = |⇓>
<⇑| σ^z_i σ^z_j |⇑> = 1
Z_2 symmetry breaking


H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i ± |↓_i>)/√2
Π_i
1/g
<0|g→∞| σ^z_i σ^z_j |0 (g→∞)> = 0
→ compute the mass gap
g << 1
|0 (g=0)>
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |⇑>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |⇓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i
[H, T] = 0
Z_2 symmetry
T |⇑> = |⇓>
<⇑| σ^z_i σ^z_j |⇑> = 1
|x_i − x_j| → ∞


H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i ± |↓_i>)/√2
Π_i
1/g
<0(g→∞)| σ^z_i σ^z_j |0(g→∞)> = 0
→ ~ e^{−|x_i − x_j|/ξ}
→ compute the mass gap
g << 1
|↑↑↑↑ … ↑> = ⊗_i |↑_i> ≡ |⇑>
|0(g=0)>
|↓↓↓ … ↓> = ⊗_i |↓_i> ≡ |⇓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i
[H, T] = 0
Z_2 symmetry
T|⇑> = |⇓>
<⇑| σ^z_i σ^z_j |⇑> = 1
|x_i − x_j| → ∞
→ N_0^2
pert. th. in g


H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗_i |→>_i
product state
|→>_i = (|↑>_i ± |↓_i>)/√2
Π_i
1/g
<0(g→∞)| σ^z_i σ^z_j |0(g→∞)> = 0
→ compute the mass gap
→ ~ e^{−|x_i − x_j|/ξ}
g << 1
|0(g=0)>
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |⇑>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |⇓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i
[H, T] = 0 Z_2 symmetry
T|⇑> = |⇓>
<⇑| σ^z_i σ^z_j |⇑> = 1
|x_i − x_j| → ∞
→ N_0^2 ≤ 1
pert. th. in g


Ising chain in transverse field
d = 1
product state
g >> 1
compute the mass gap
g << 1
Z2 symmetry breaking
pert. th. in g

$$H = -J \sum_i \left(g\, \hat{\sigma}^x_i + \hat{\sigma}^z_i \hat{\sigma}^z_{i+1} \right)$$

Ising chain in transverse field

$d = 1$

$g \gg 1$ $\quad |0\rangle = \bigotimes_i |\rightarrow\rangle_i$ product state

$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i \pm |\downarrow_i\rangle}{\sqrt{2}}$ $\quad \prod_i$

$\boxed{1/g}$ $\quad \langle 0_{g\to\infty} | \hat{\sigma}^z_i \hat{\sigma}^z_j | 0(g\to\infty)\rangle = 0$

$\to \sim e^{-|x_i - x_j| / \xi}$

→ compute the mass gap

$g \ll 1$ $\quad |\uparrow\uparrow\uparrow\uparrow \ldots \uparrow\rangle = \otimes_i |\uparrow_i\rangle \equiv |\Uparrow\rangle$ $\quad Z_2$ symmetry breaking

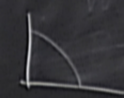
$|0(g=0)\rangle$

$|\downarrow\downarrow\downarrow \ldots \downarrow\rangle = \otimes_i |\downarrow_i\rangle \equiv |\Downarrow\rangle$

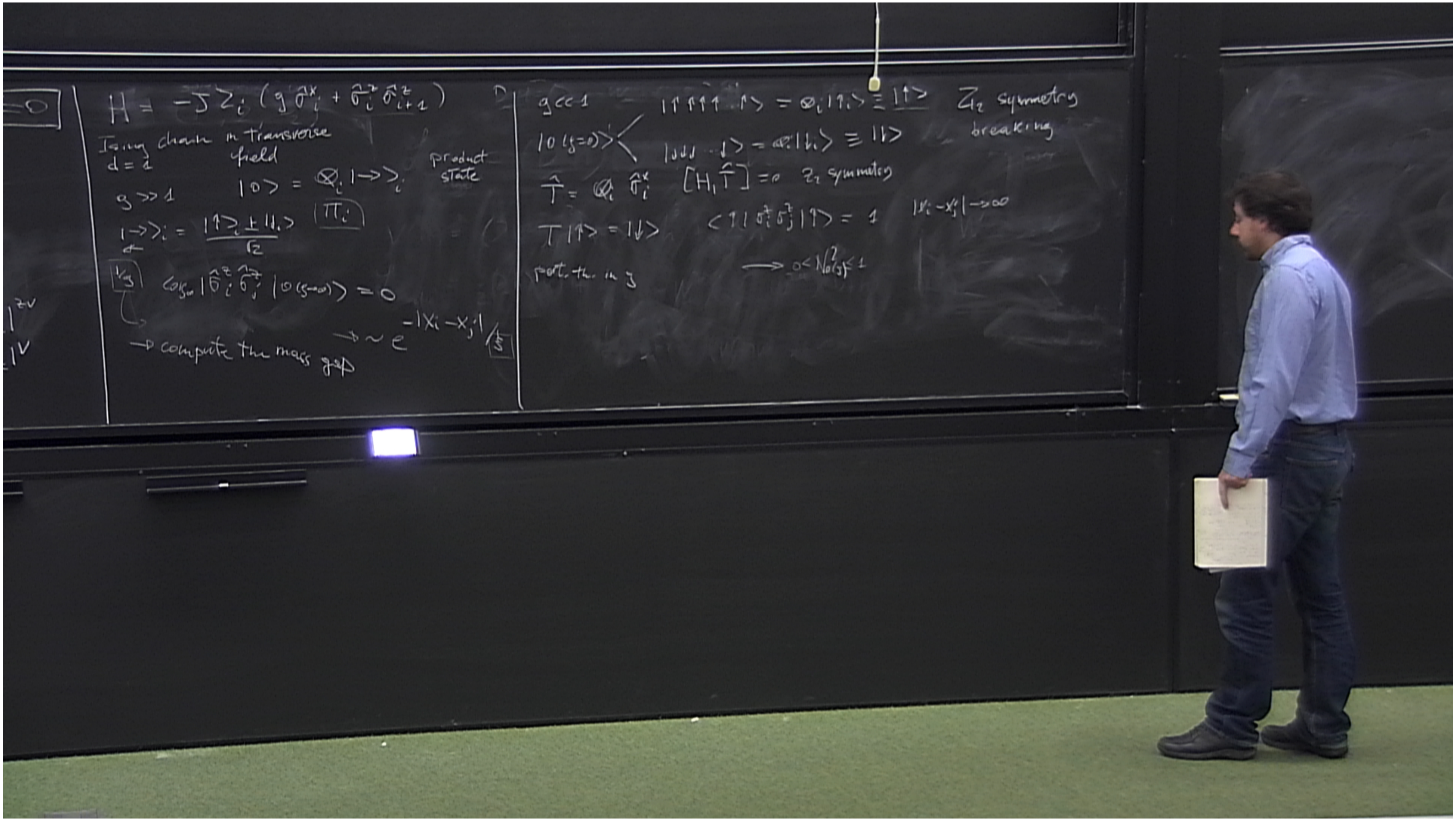
$\hat{T} = \bigotimes_i \hat{\sigma}^x_i$ $\quad [H, \hat{T}] = 0$ $\quad Z_2$ symmetry

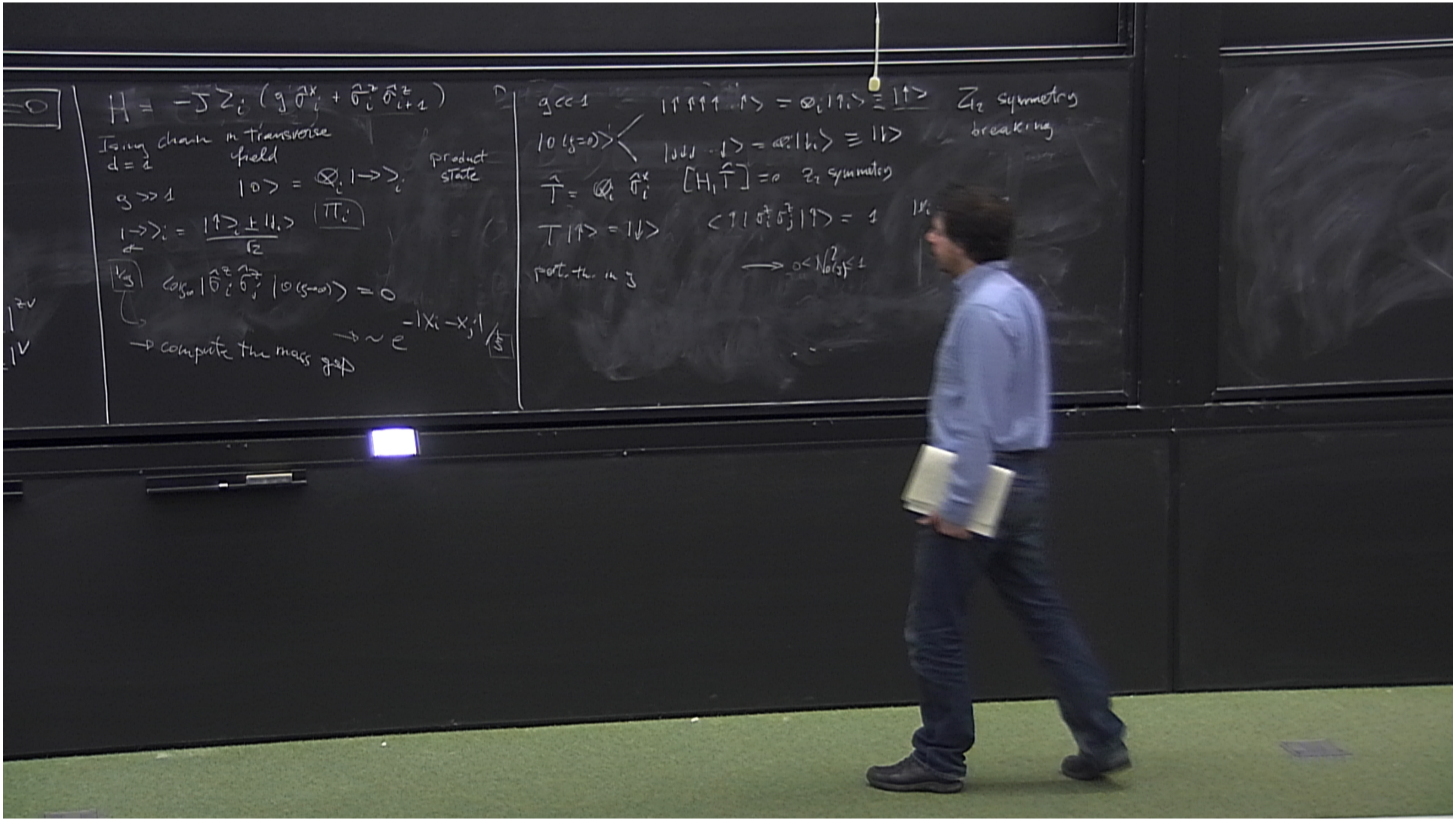
$T|\Uparrow\rangle = |\Downarrow\rangle$ $\quad \langle \Uparrow | \sigma^z_i \sigma^z_j | \Uparrow \rangle = 1$ $\quad |x_i - x_j| \to \infty$

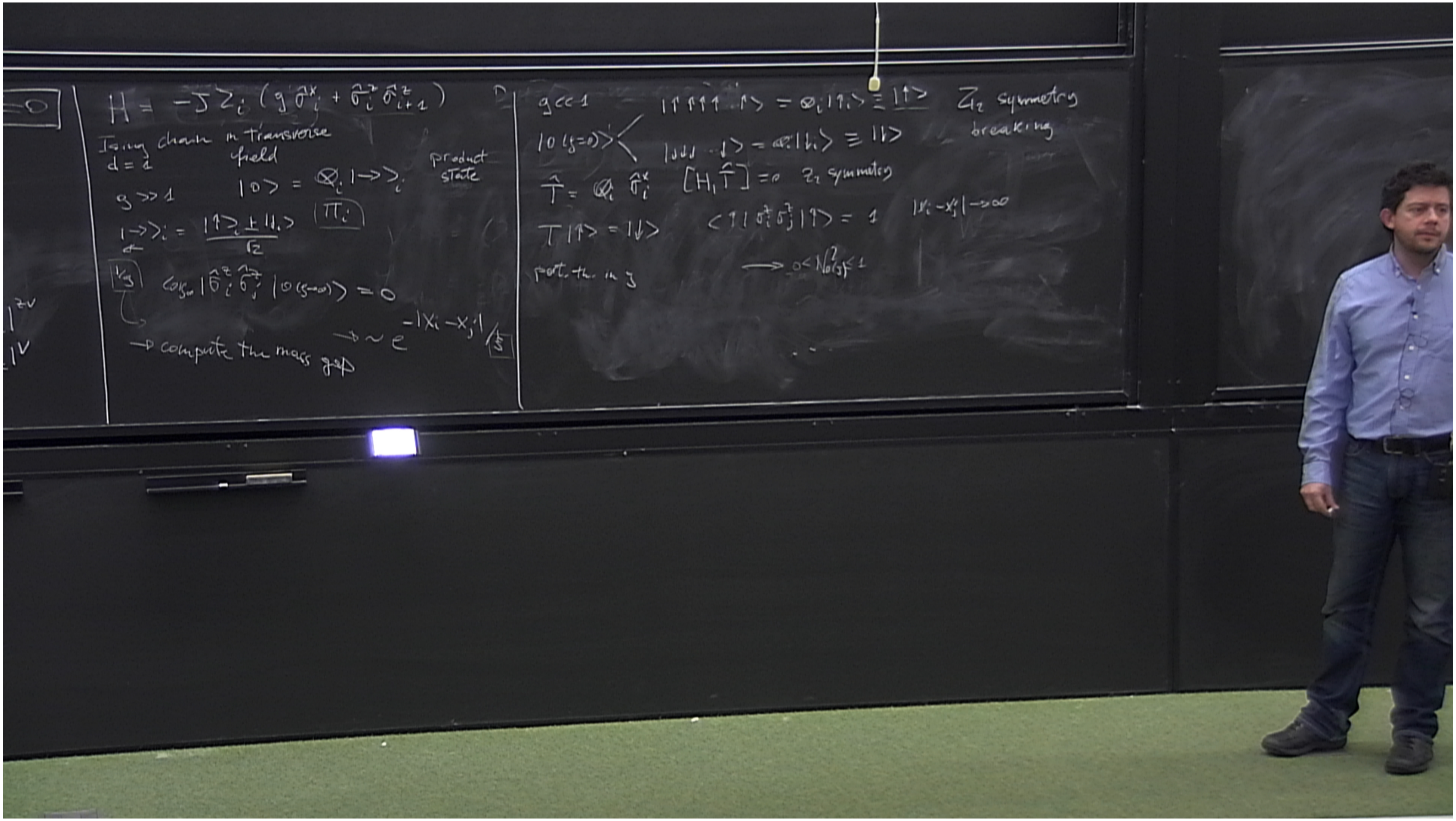
pert. th. in g $\quad \longrightarrow 0 < N_0^2(g) \leq 1$

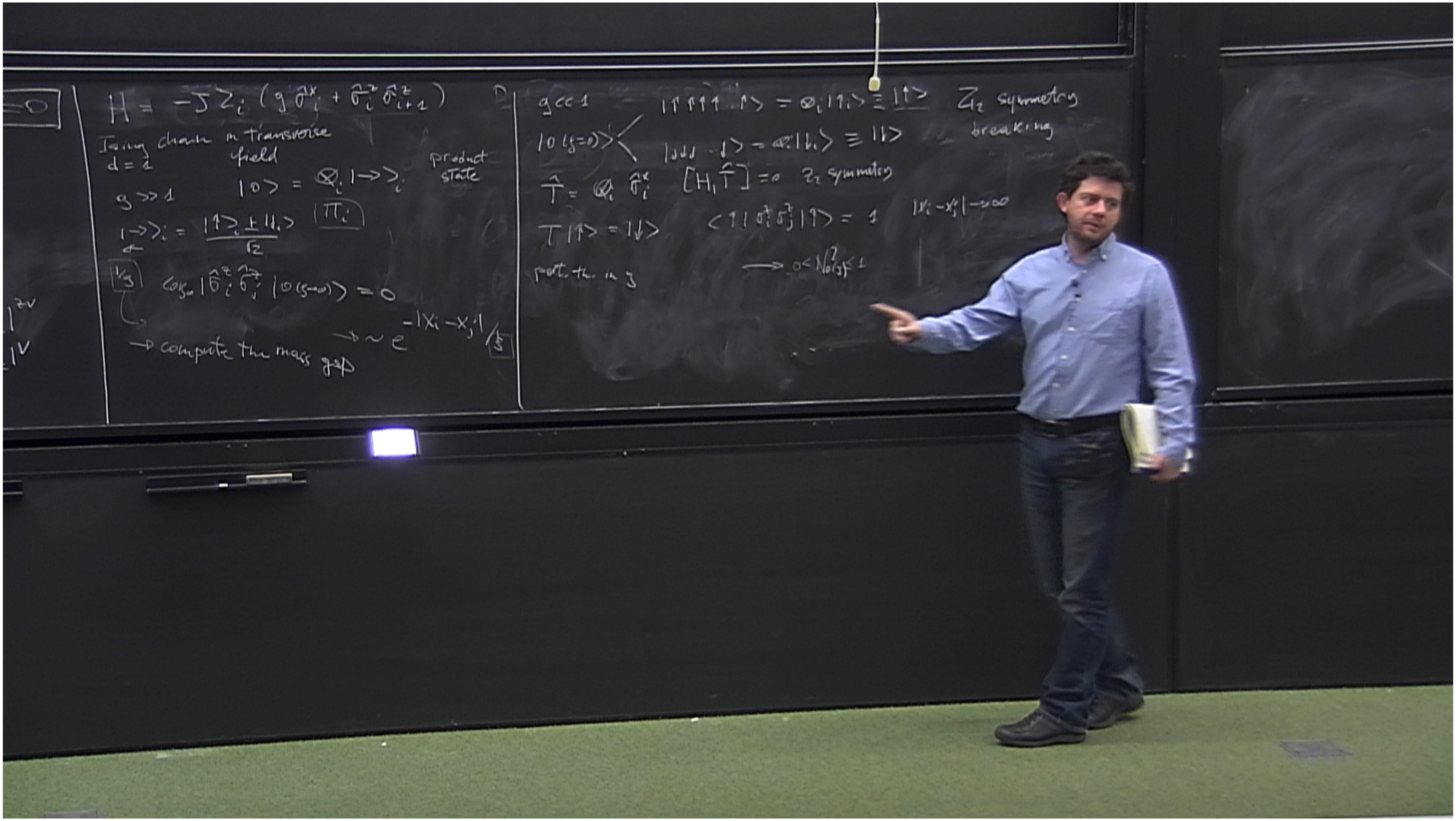


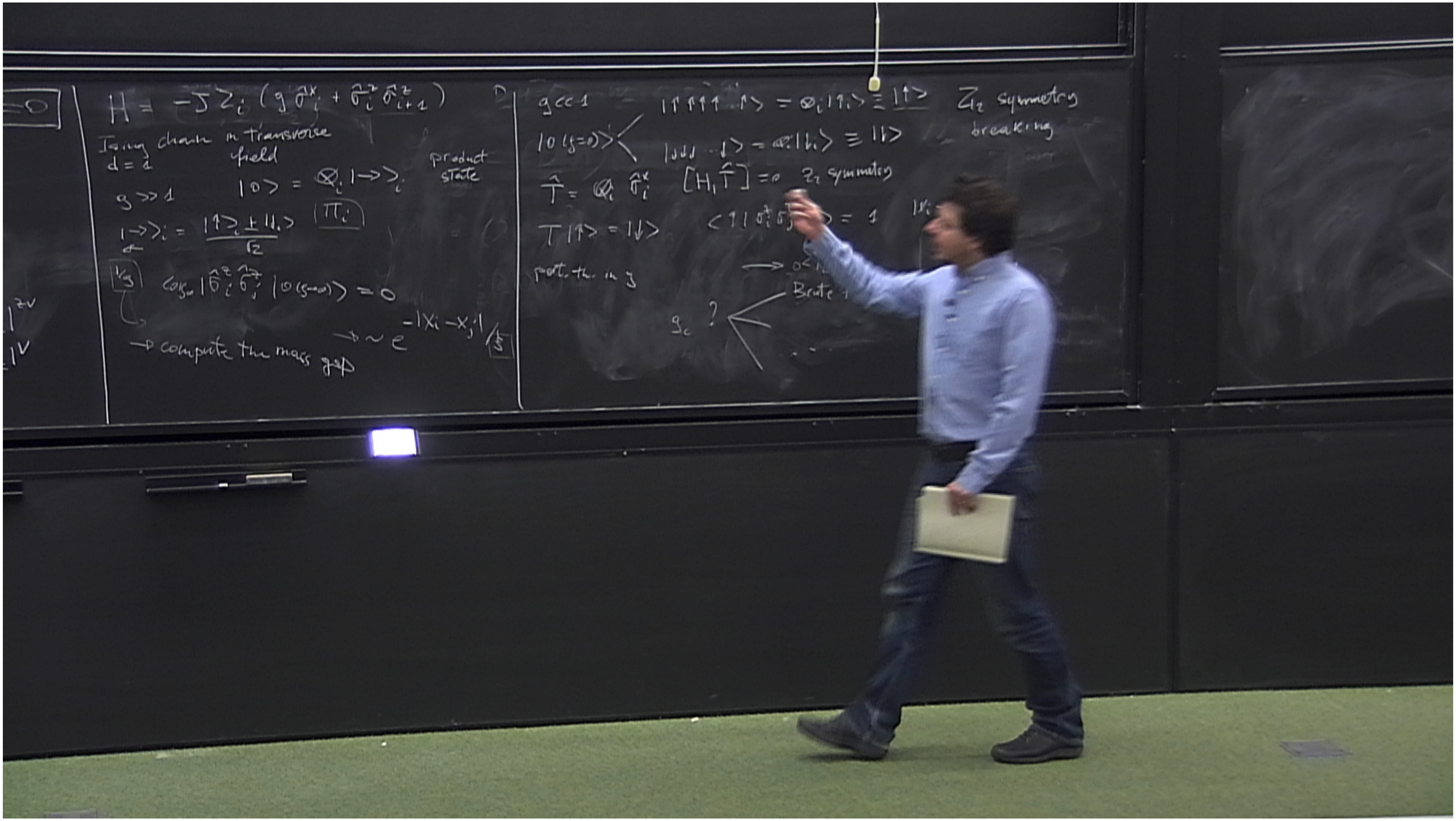
$|0(g)\rangle$

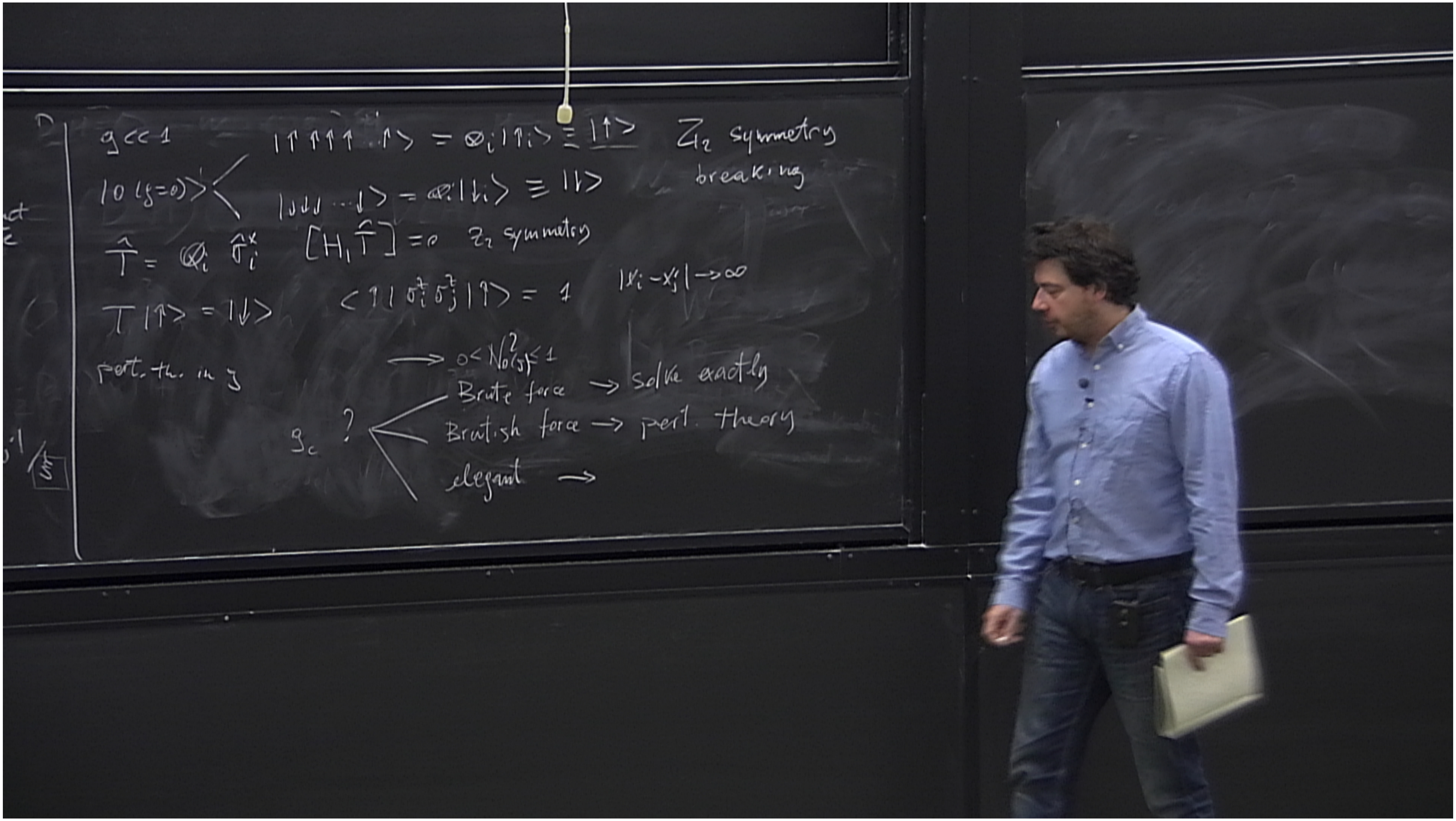

Ising chain in transverse field
d = 1
product state
g >> 1
compute the mass gap
g << 1
Z2 symmetry breaking
Z2 symmetry
pert. th. in g


H = −J Σi (g σxi + σzi σzi+1)
Ising chain in transverse field
d = 1
g >> 1
|0> = ⊗i |→>i
product state
|→>i = |↑>i ± |↓i> / √2
→ compute the mass gap
g << 1
Z2 symmetry breaking
[H, T] = 0 Z2 symmetry
T |↑> = |↓>
pert. th. in g

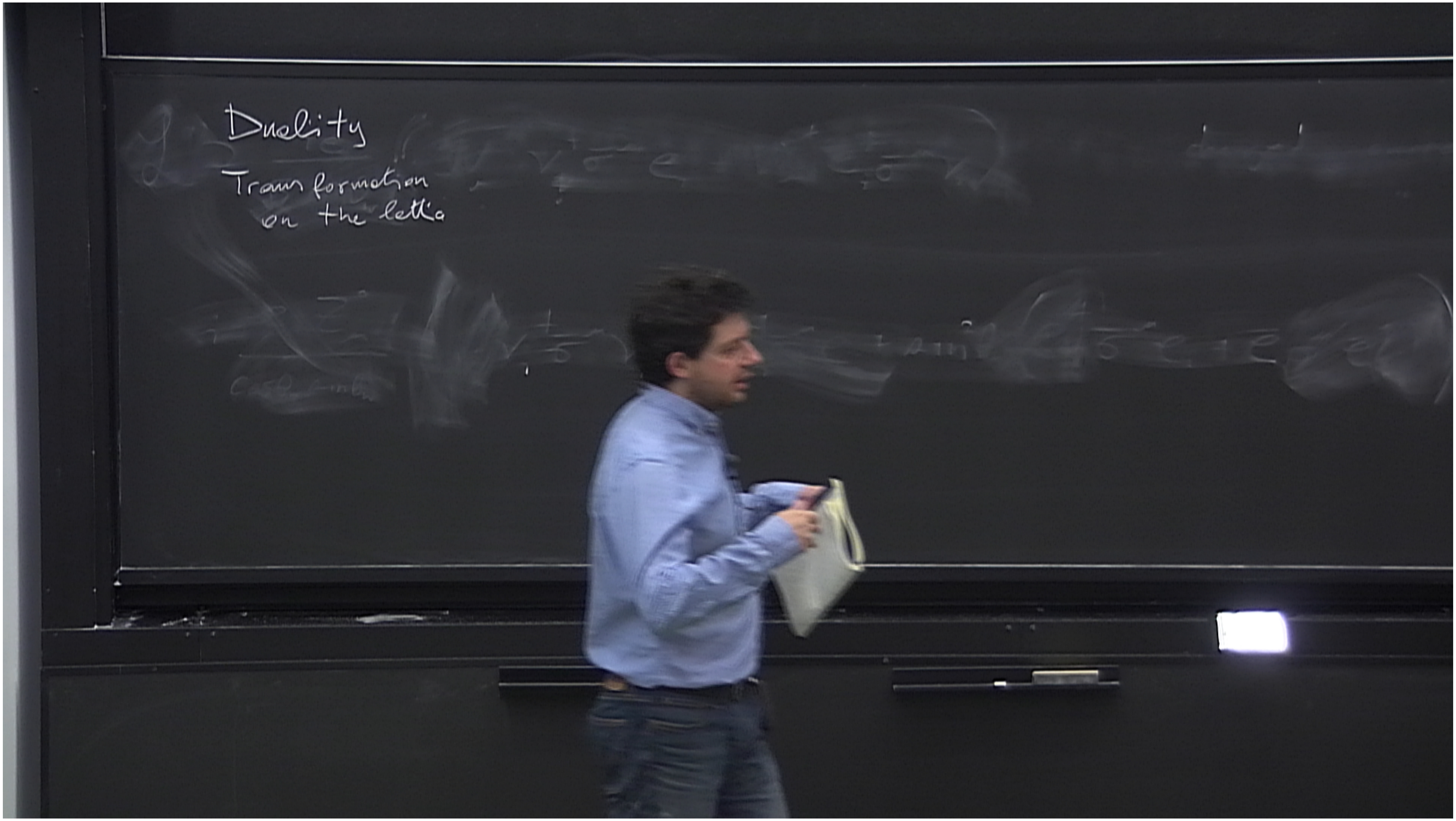

H = −J Σ_i (g σ^x_i + σ^z_i σ^z_{i+1})
Ising chain in transverse field
d = 1
product state
g >> 1
|0> = ⊗_i |→>_i
|→>_i = (|↑>_i ± |↓_i>)/√2
compute the mass gap
g << 1
|0 (g=0)>
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |↑>
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |↓>
Z_2 symmetry breaking
T = ⊗_i σ^x_i
[H, T] = 0 Z_2 symmetry
T |↑> = |↓>
pert. th. in g

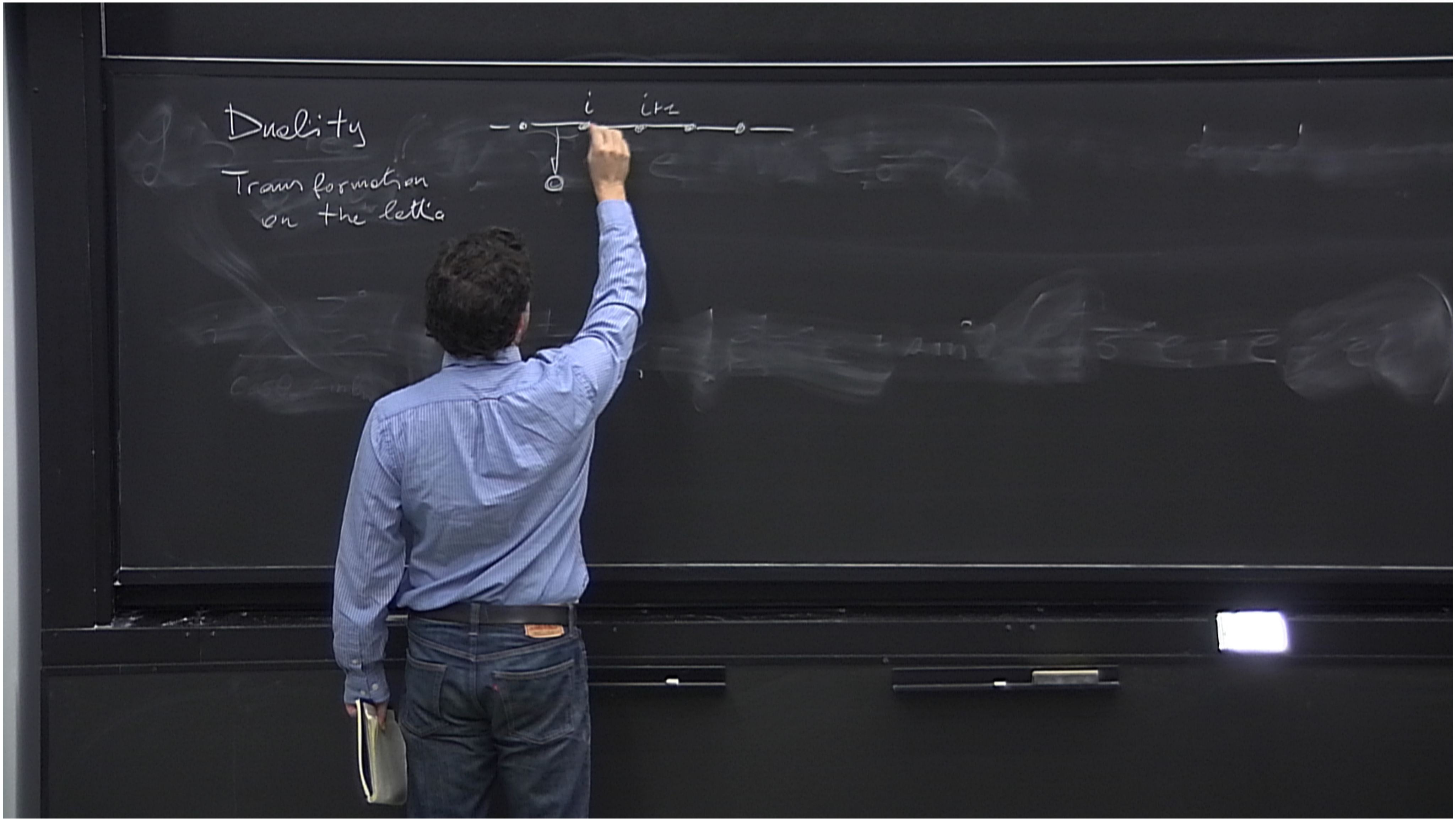

Ising chain in transverse field
product state
g >> 1
g << 1
Z2 symmetry breaking
compute the mass gap

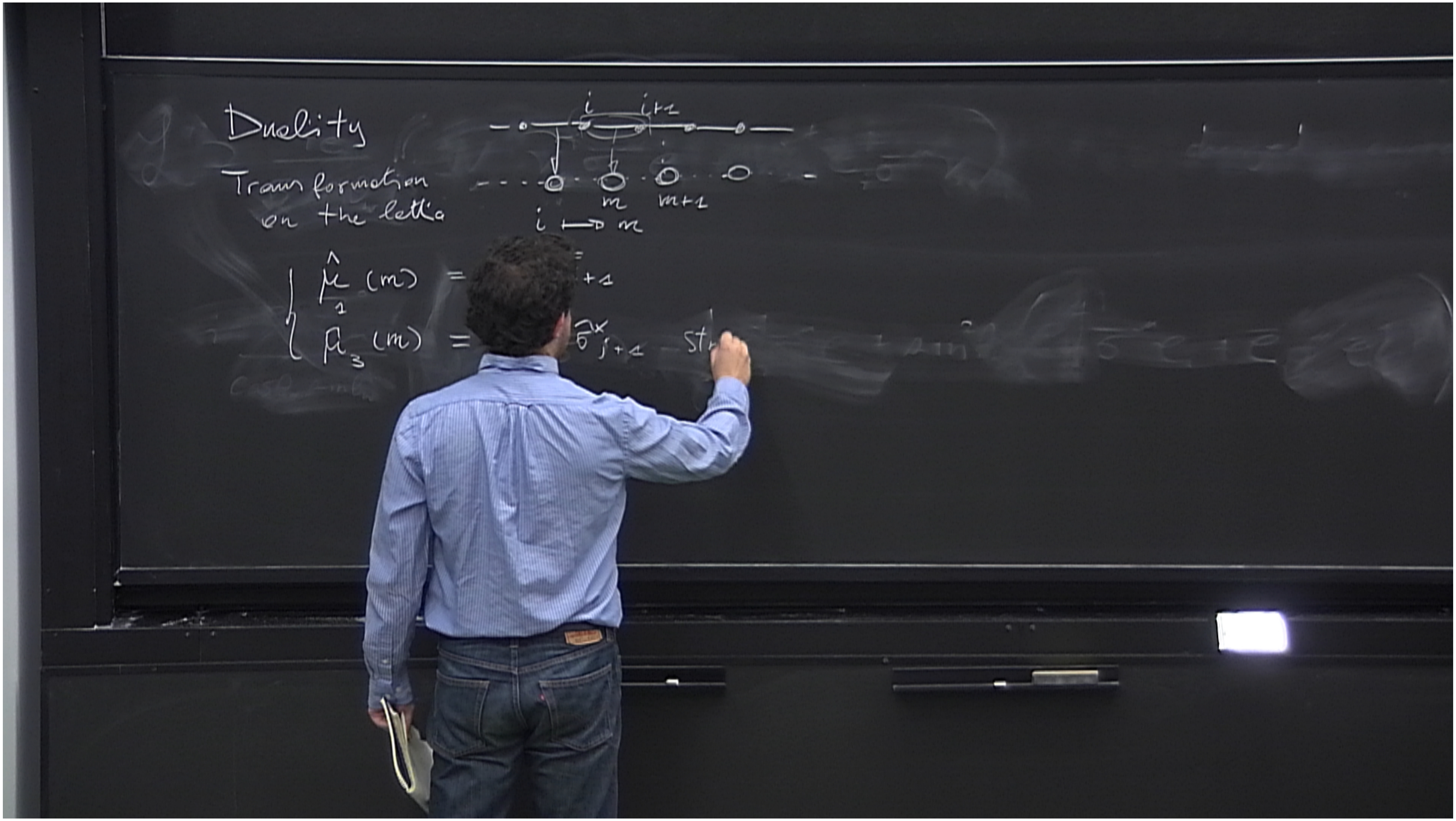

Ising chain in transverse field
d = 1
product state
compute the mass gap
g << 1
Z2 symmetry breaking
Z2 symmetry
Brute

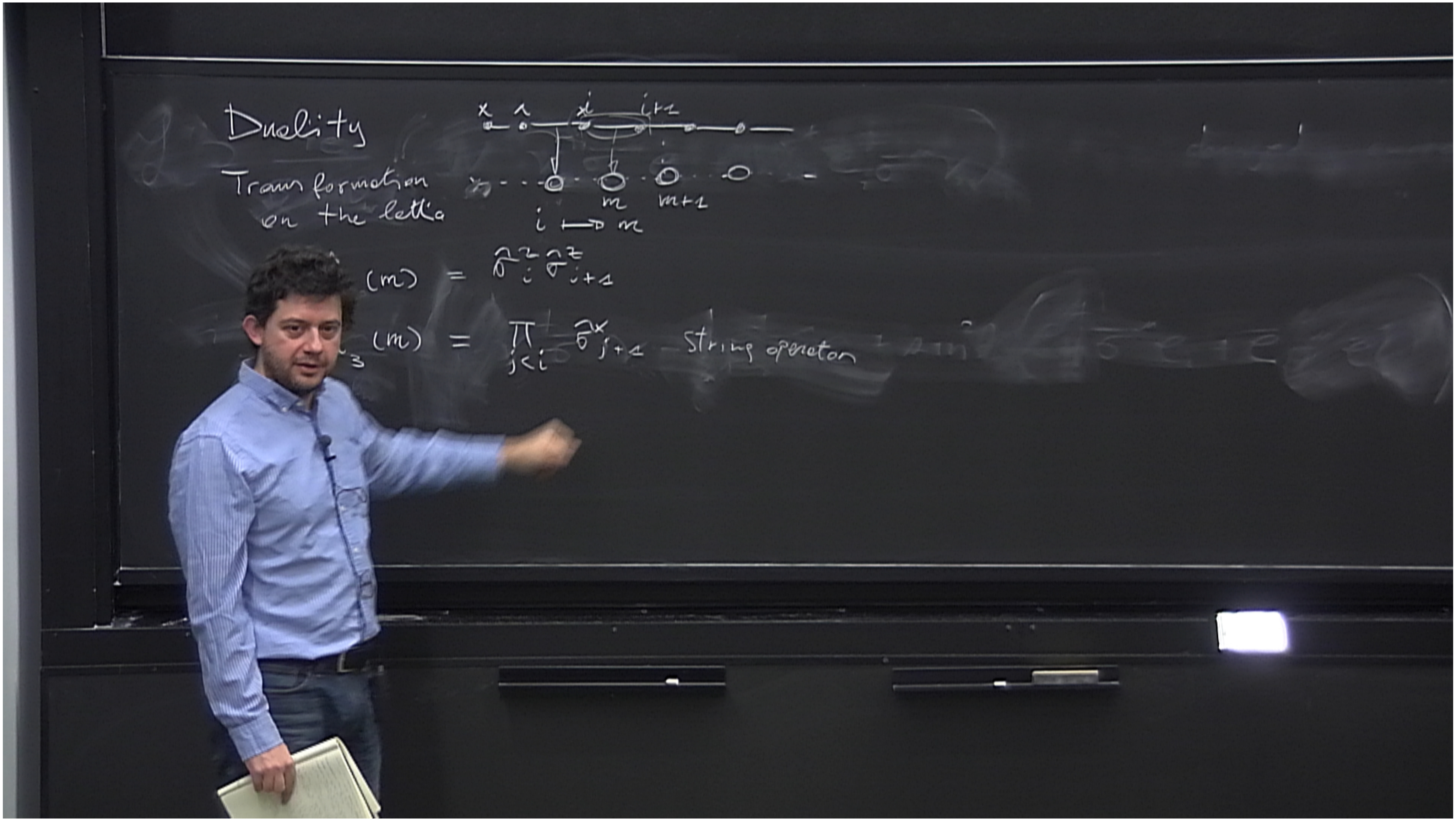


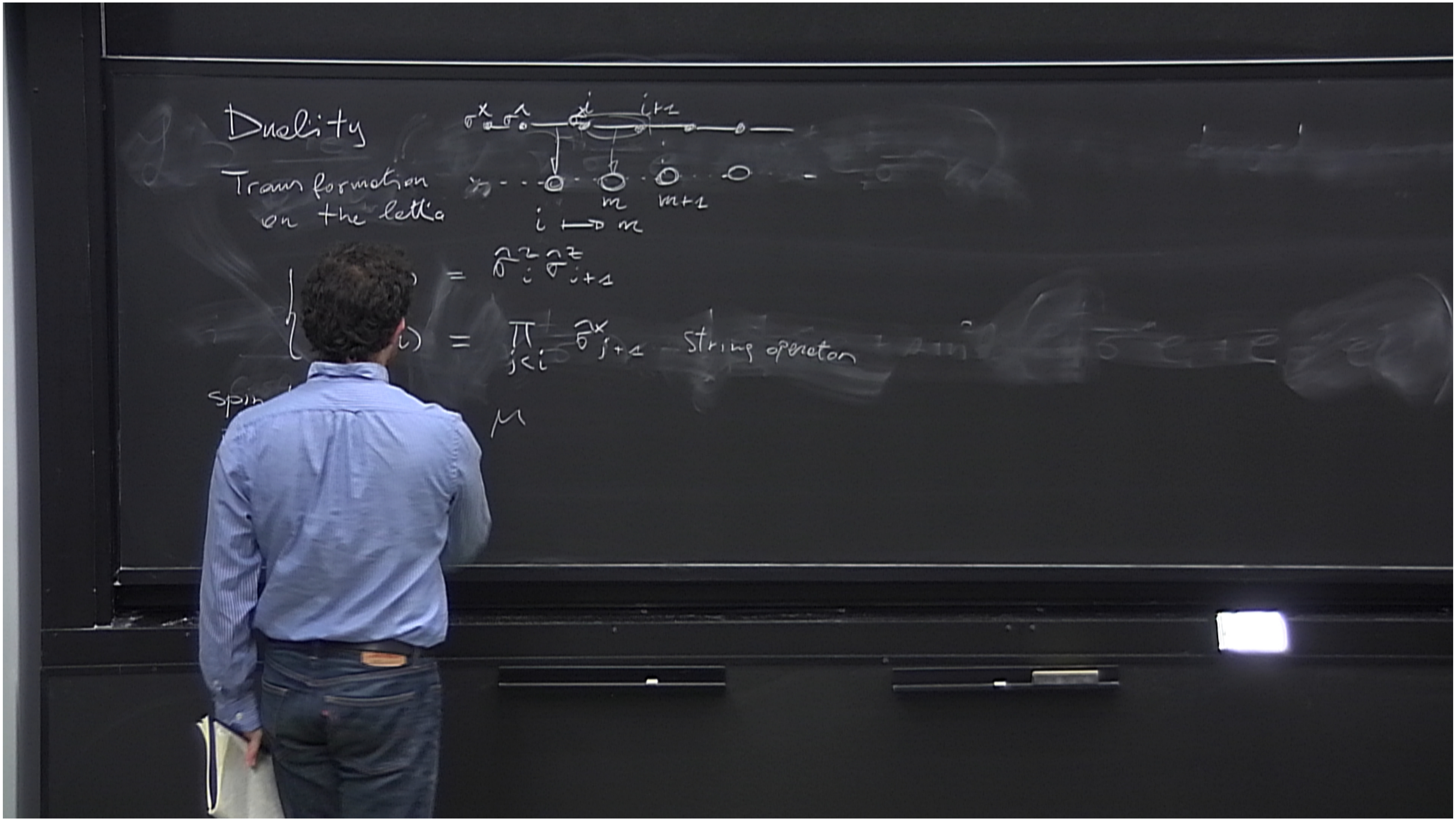
g << 1
|0 (g=0)>
T̂ = ⊗_i σ̂^x_i
T |⇑> = |⇓>
pert. th. in g
|↑↑↑↑ ... ↑> = ⊗_i |↑_i> ≡ |⇑>
Z_2 symmetry breaking
|↓↓↓ ... ↓> = ⊗_i |↓_i> ≡ |⇓>
[H, T̂] = 0 Z_2 symmetry
< ⇑ | σ^z_i σ^z_j | ⇑ > = 1
|x_i − x_j| → ∞
→ 0 < N_0^2(g) ≤ 1
Brute force → solve exactly
g_c ?
Brutish force → pert. theory
elegant →

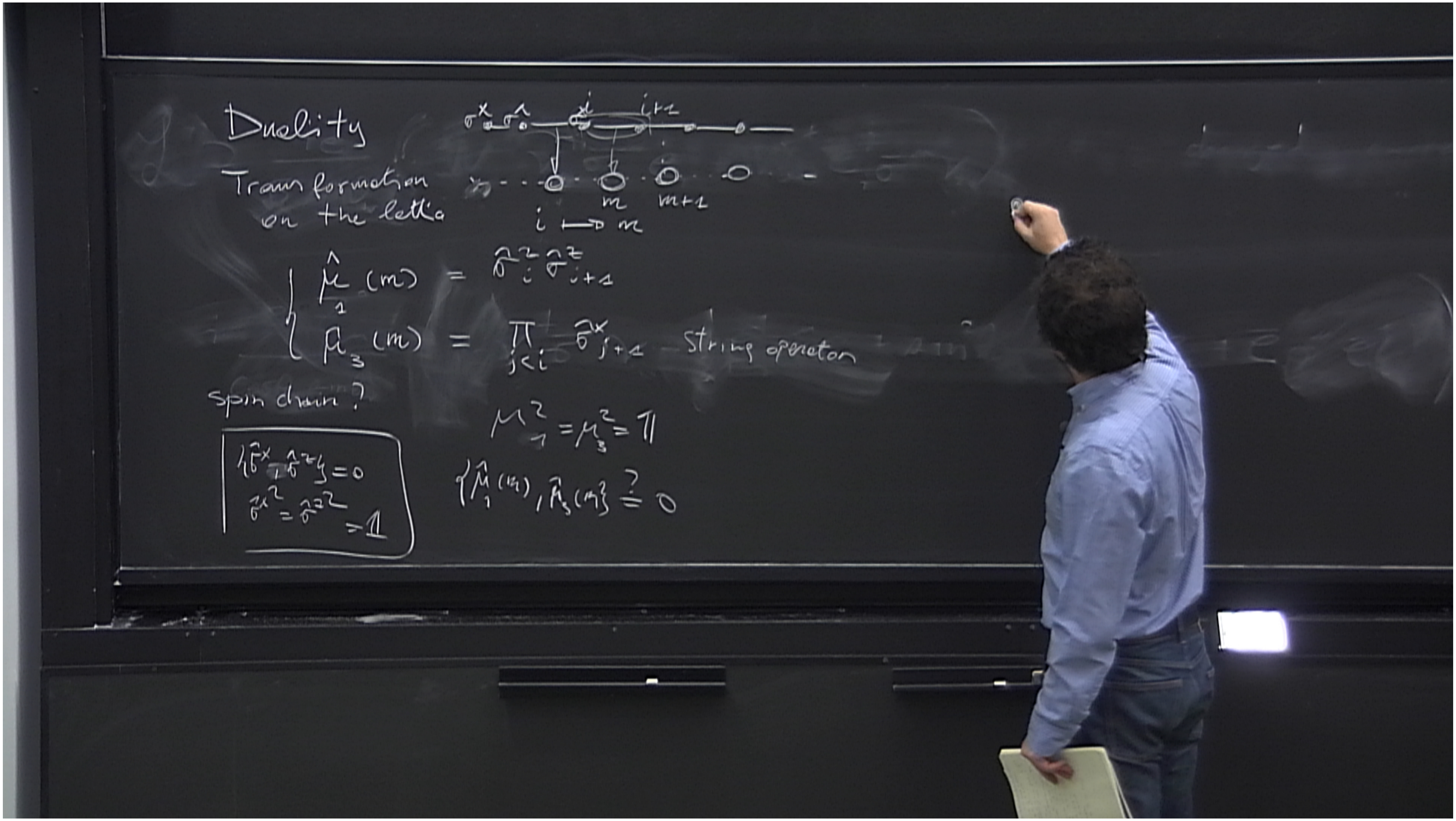

Duality
Transformation
on the lattice

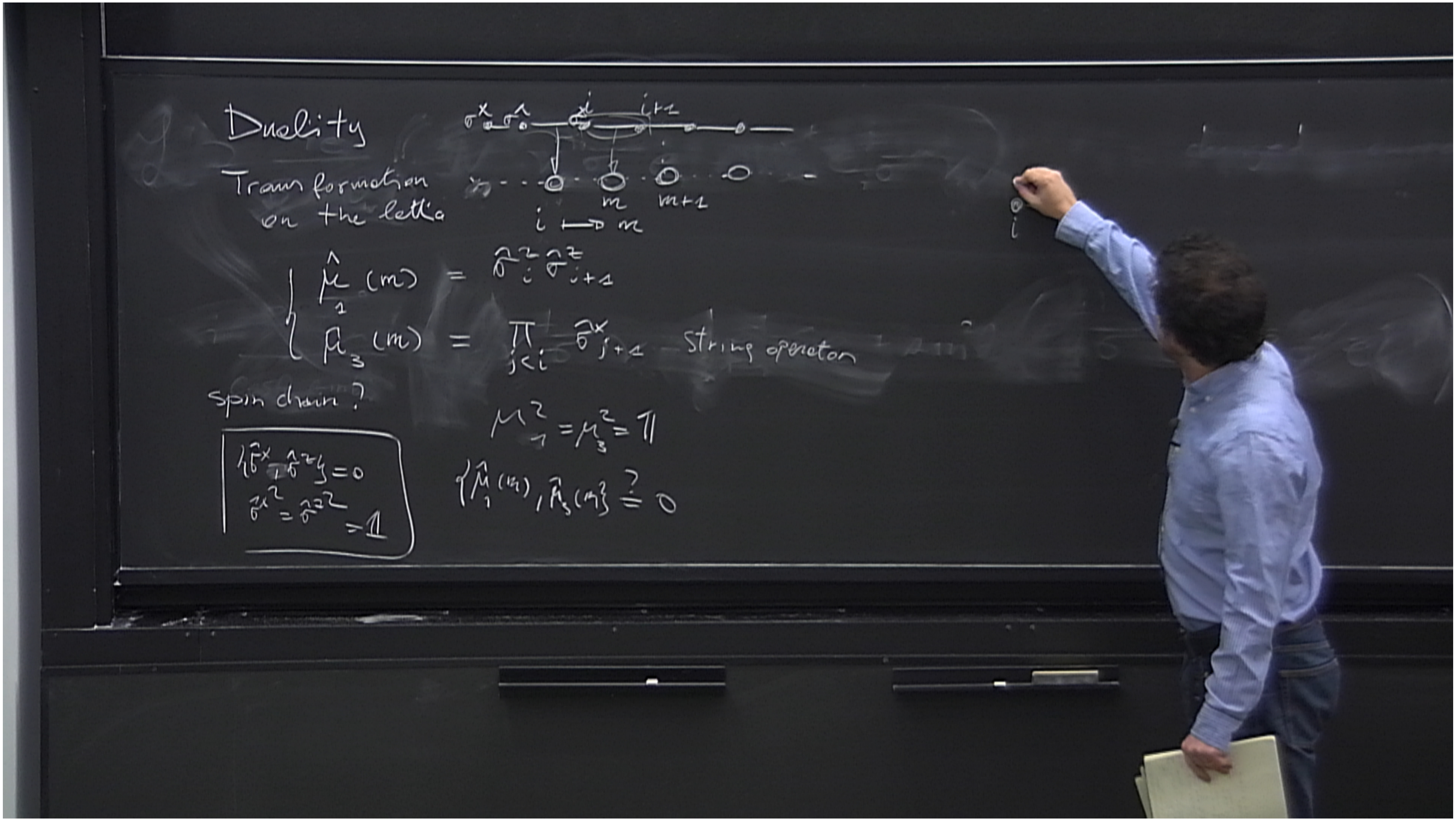

Duality
Transformation on the lattice
i
i+1

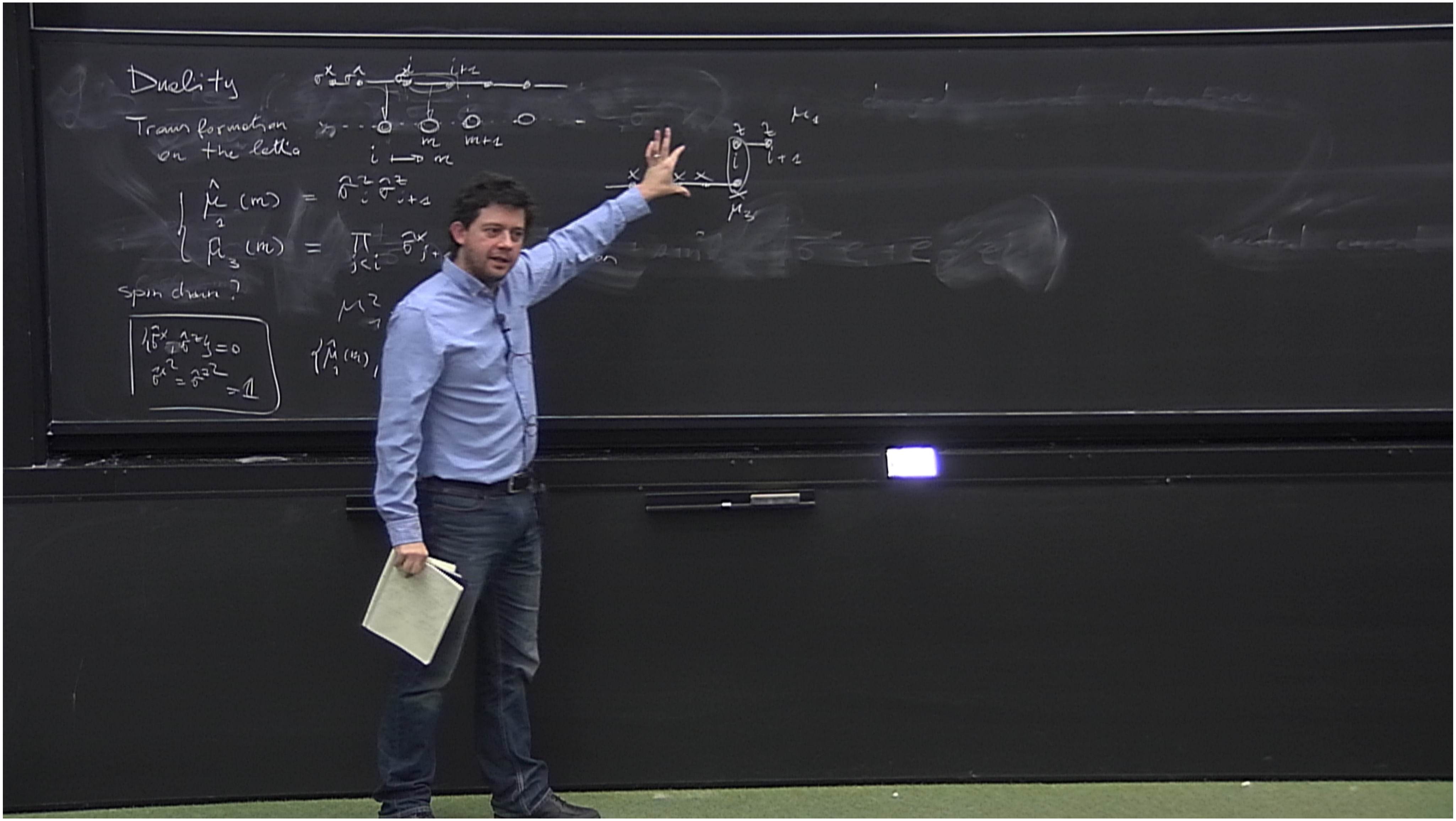

Duality
Transformation
on the lattice
i
i+1
m
m+1
i ⟼ m
$\hat{\mu}_1(m) =$
$\hat{\mu}_3(m) =$

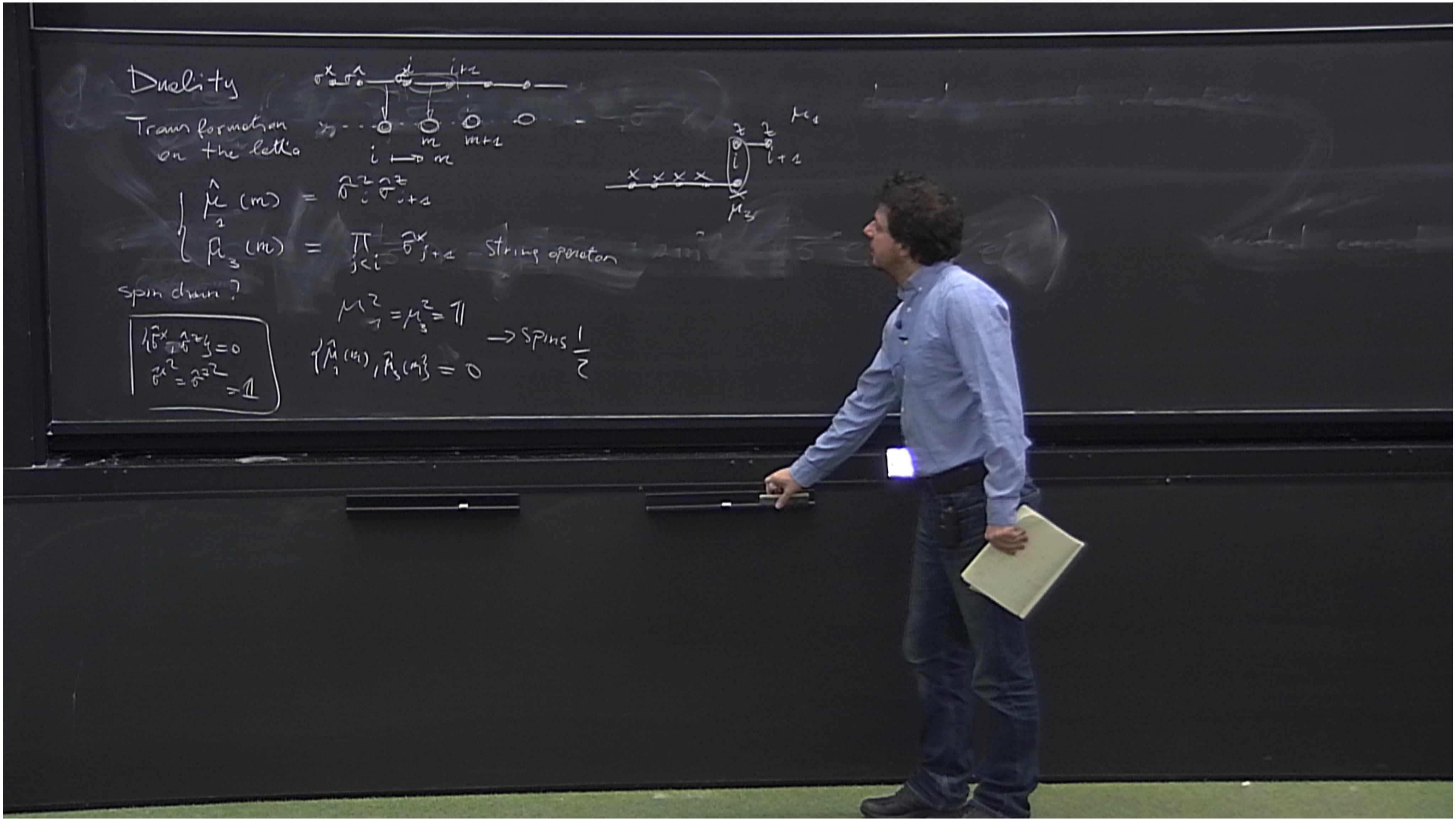

Duality
Transformation
on the lattice
i ⟼ m
(m) = σ^z_i σ^z_{i+1}
(m) = Π_{j<i} σ^x_{j+1}
String operator

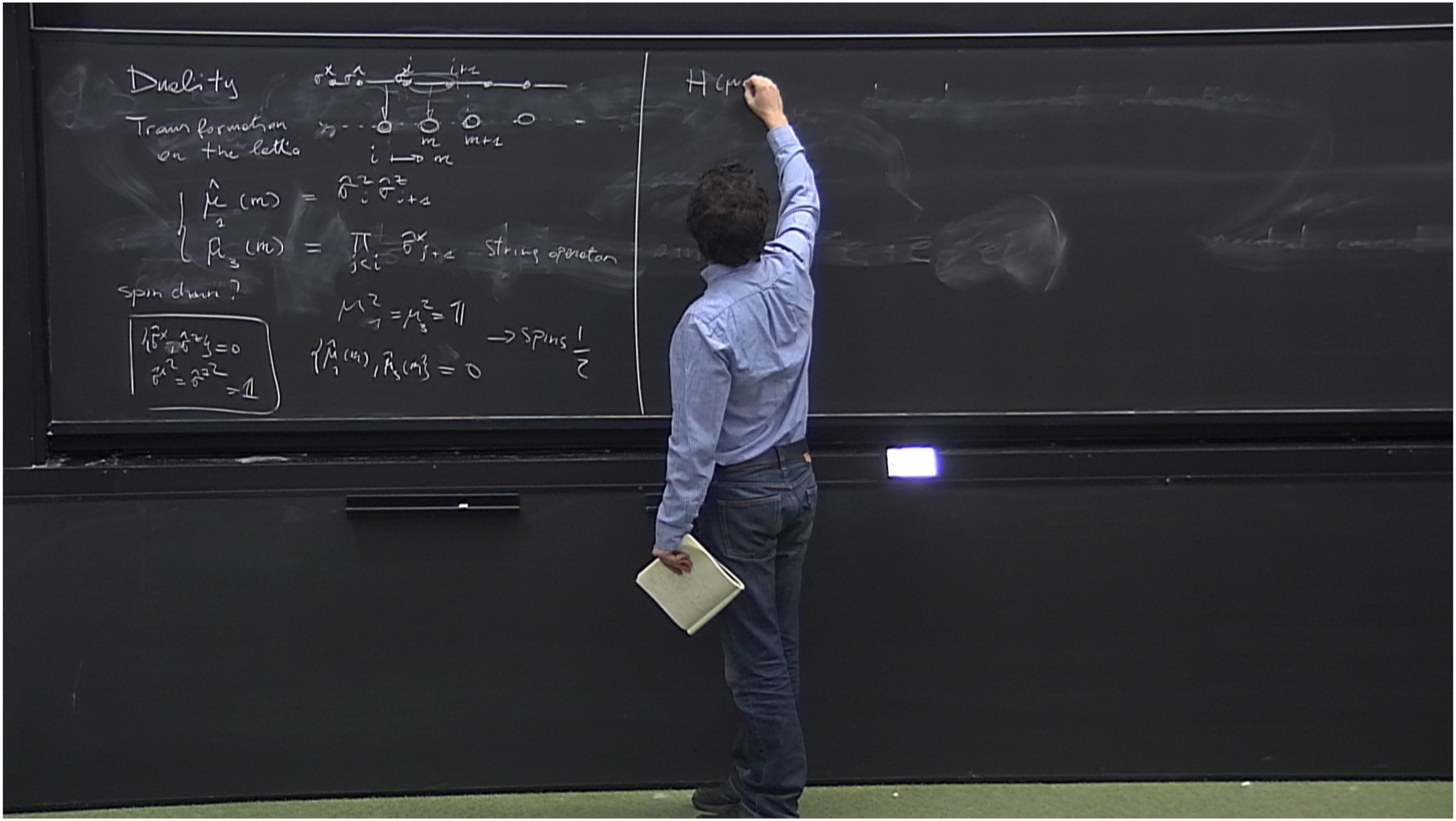

Duality
Transformation on the lattice
i+1
m
m+1
i ⟼ m
String operator

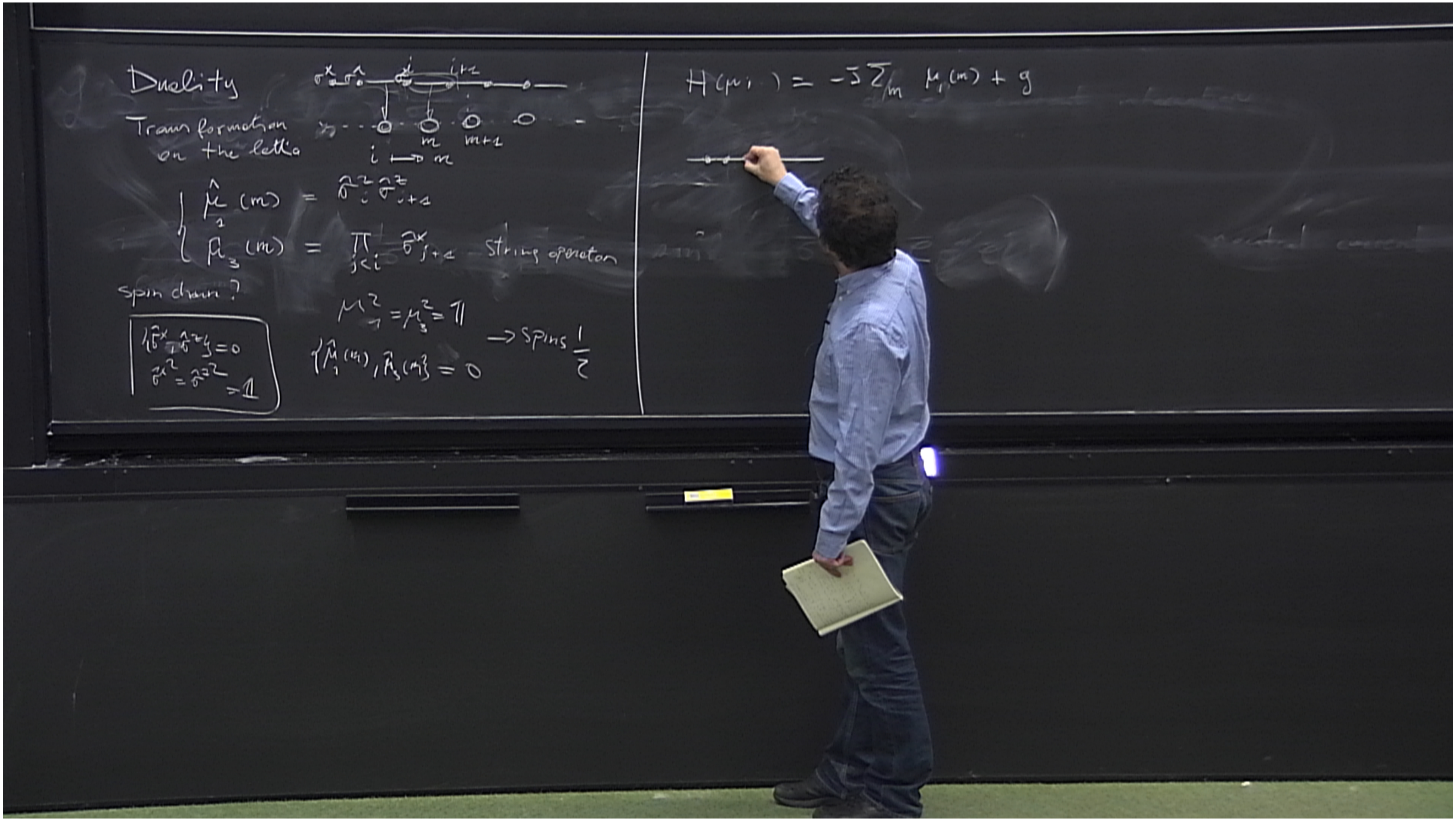

Duality
Transformation on the lattice
i ⟼ m
m   m+1
$\hat{\mu}_1(m) = \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$
$\hat{\mu}_3(m) = \prod_{j<i} \hat{\sigma}^x_{j+1}$
String operator
spin chain?
$\{\hat{\sigma}^x, \hat{\sigma}^z\} = 0$
$\hat{\sigma}^{x2} = \hat{\sigma}^{z2} = \mathbb{1}$
$\mu_1^2 = \mu_3^2 = \mathbb{1}$
$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} \stackrel{?}{=} 0$

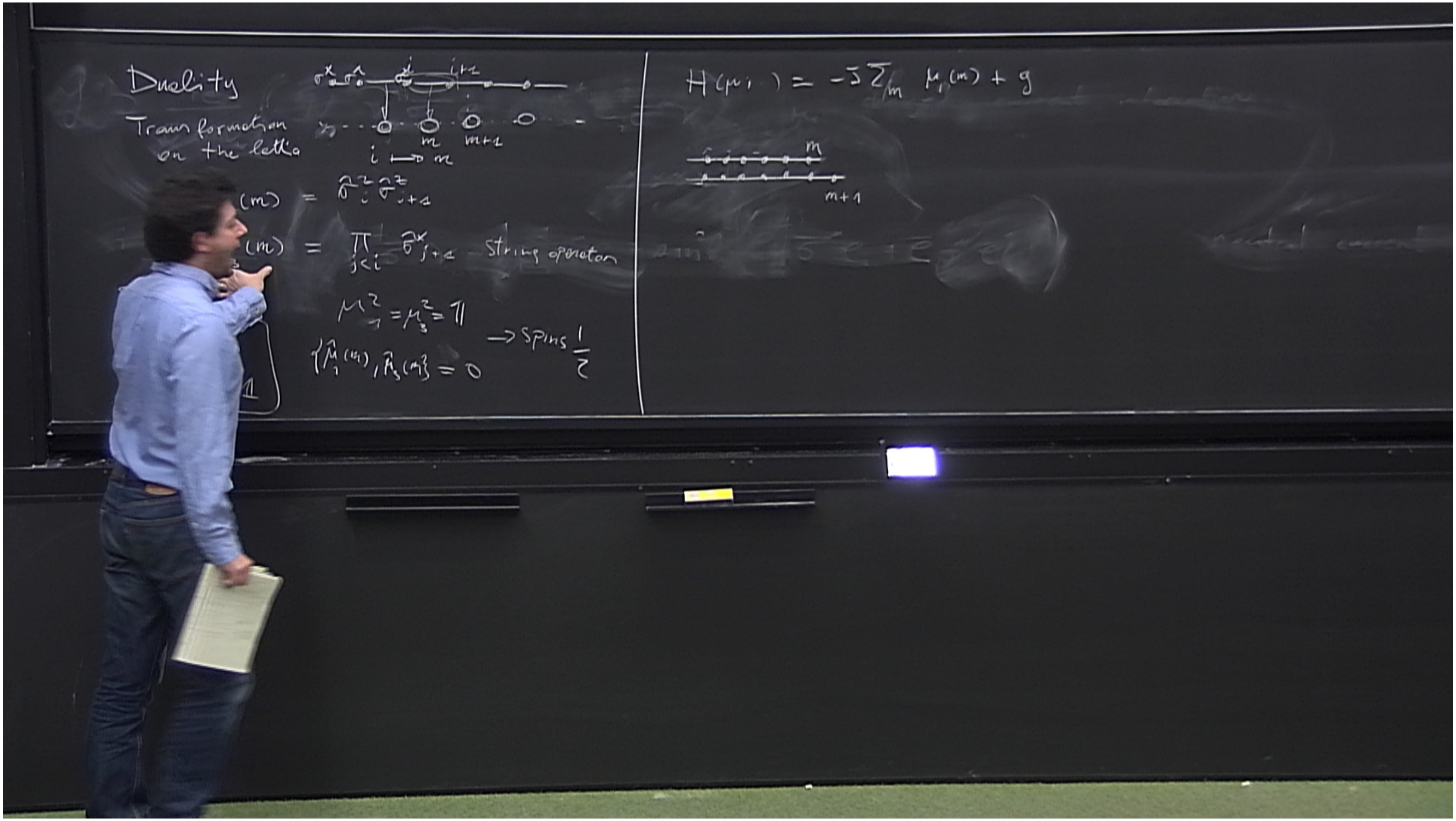

Duality
Transformation on the lattice
i ⟼ m
m
m+1
i+1
$\hat{\mu}_1(m) = \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$
$\hat{\mu}_3(m) = \prod_{j<i} \hat{\sigma}^x_{j+1}$
String operator
spin chain?
$\{\hat{\sigma}^x, \hat{\sigma}^z\} = 0$
$\hat{\sigma}^{x2} = \hat{\sigma}^{z2} = \mathbb{1}$
$\mu_1^2 = \mu_3^2 = \mathbb{1}$
$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} \overset{?}{=} 0$

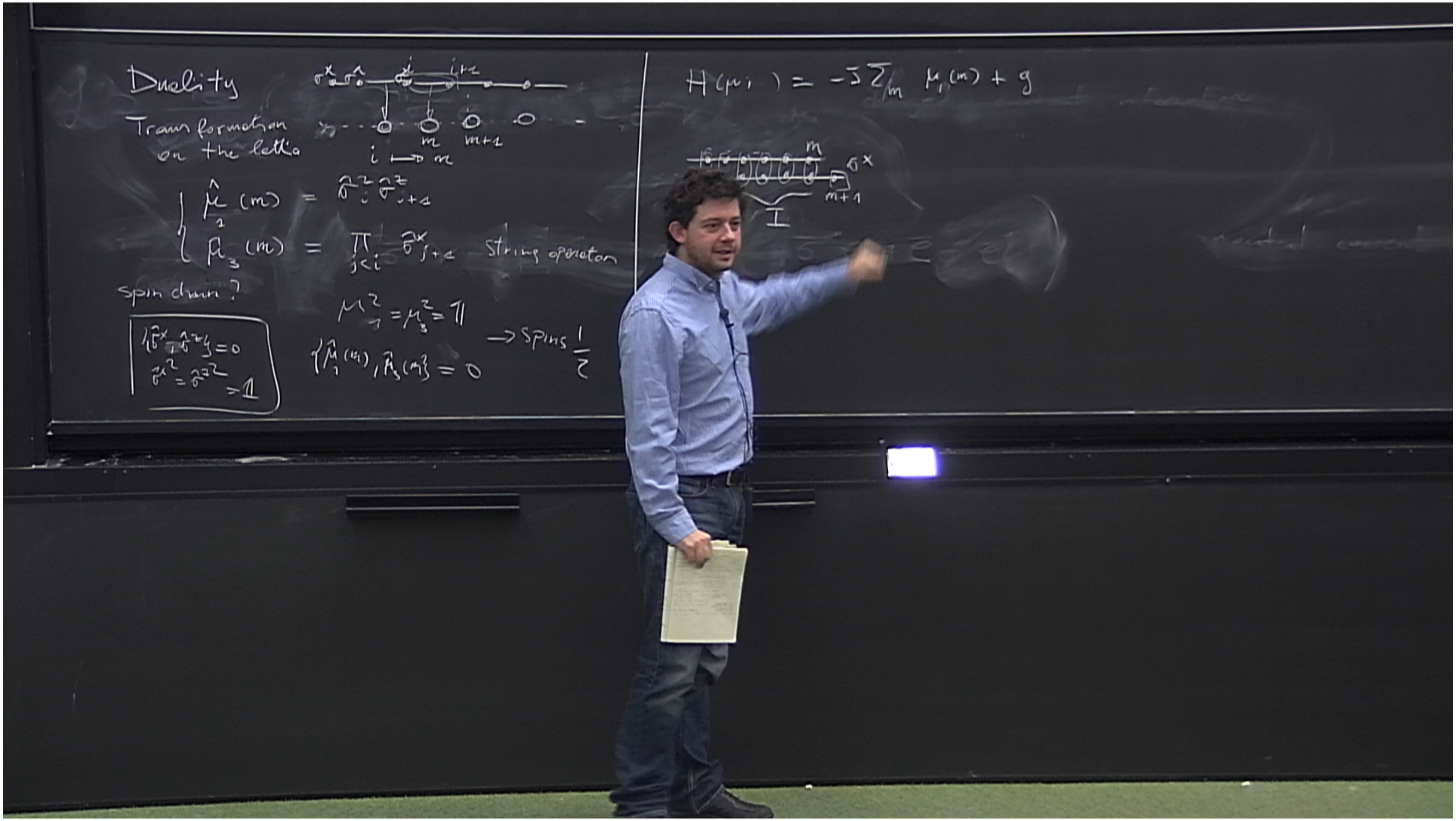

Duality
Transformation on the lattice
i ⟼ m
m
m+1
i+1
spin chain?

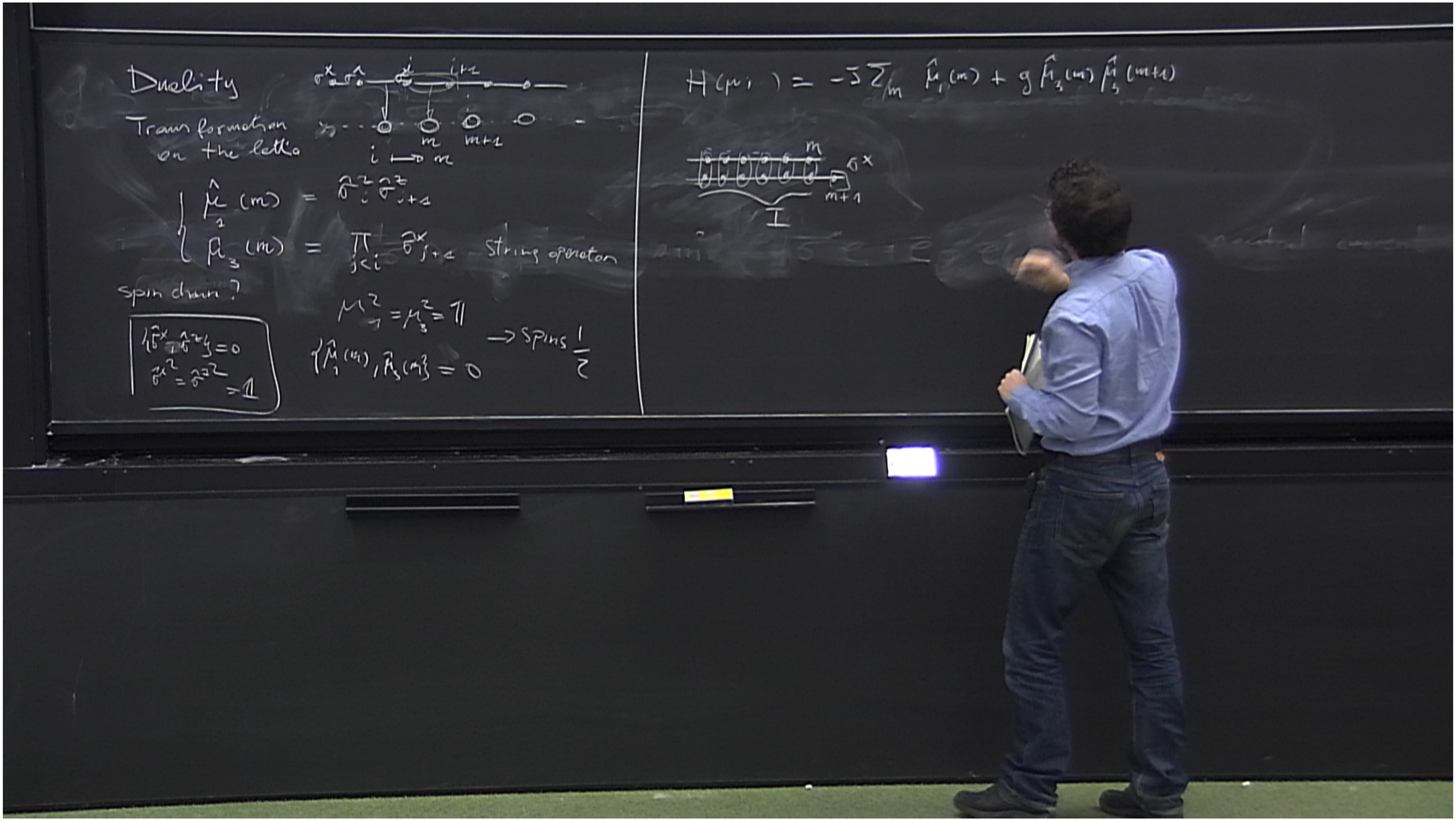

Duality
Transformation on the lattice
$i \mapsto m$
$\hat{\mu}_1(m) = \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$
$\hat{\mu}_3(m) = \prod_{j<i} \hat{\sigma}^x_{j+1}$ string operator
spin chain?
$\{\hat{\sigma}^x, \hat{\sigma}^z\} = 0$
$\hat{\sigma}^{x2} = \hat{\sigma}^{z2} = \mathbb{1}$
$\mu_1^2 = \mu_3^2 = \mathbb{1}$
$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} = 0$ → spins $\frac{1}{2}$

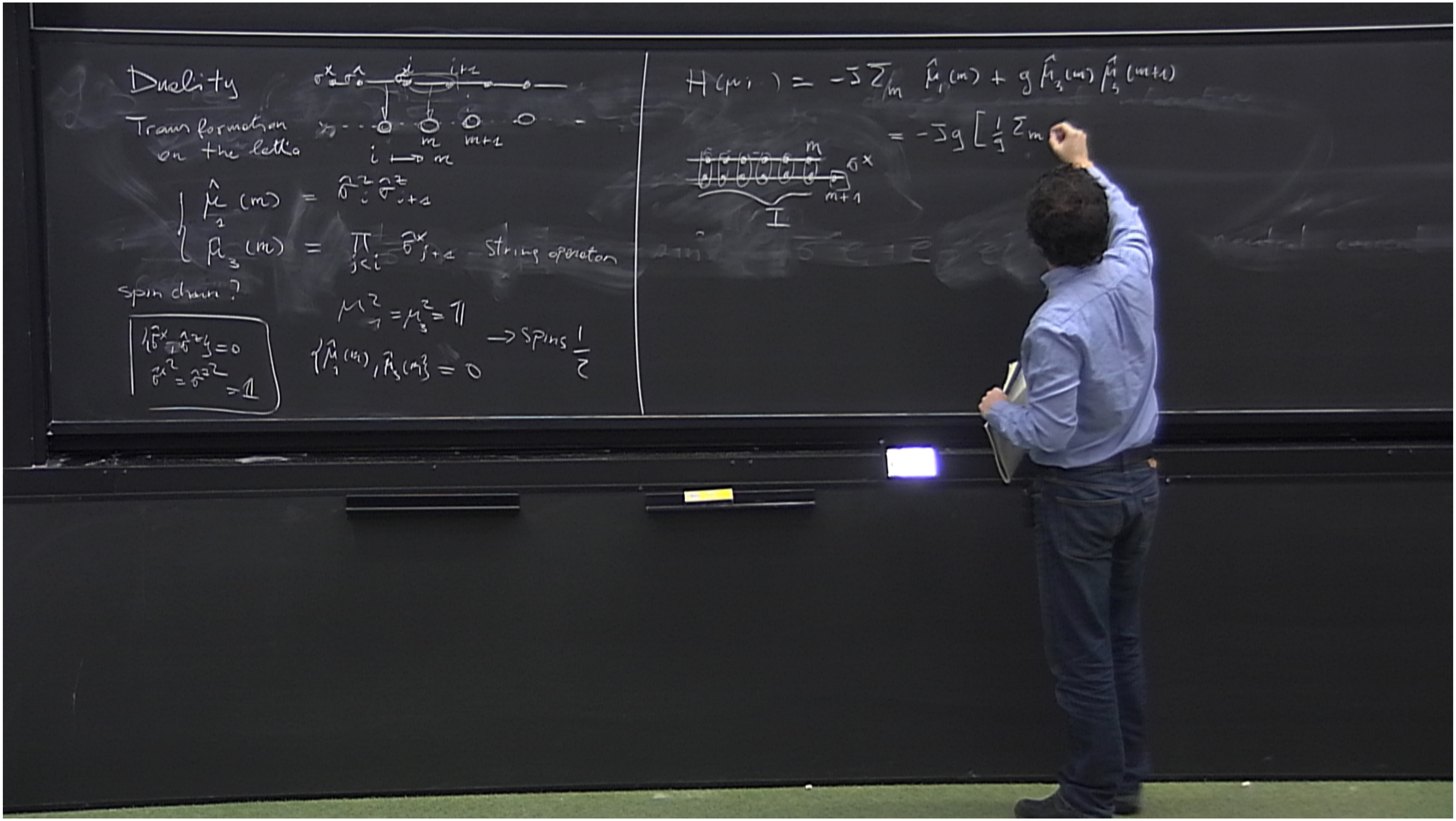

Duality
Transformation
on the lattice
string operator
spin chain?
→ Spins 1/2
H(μ

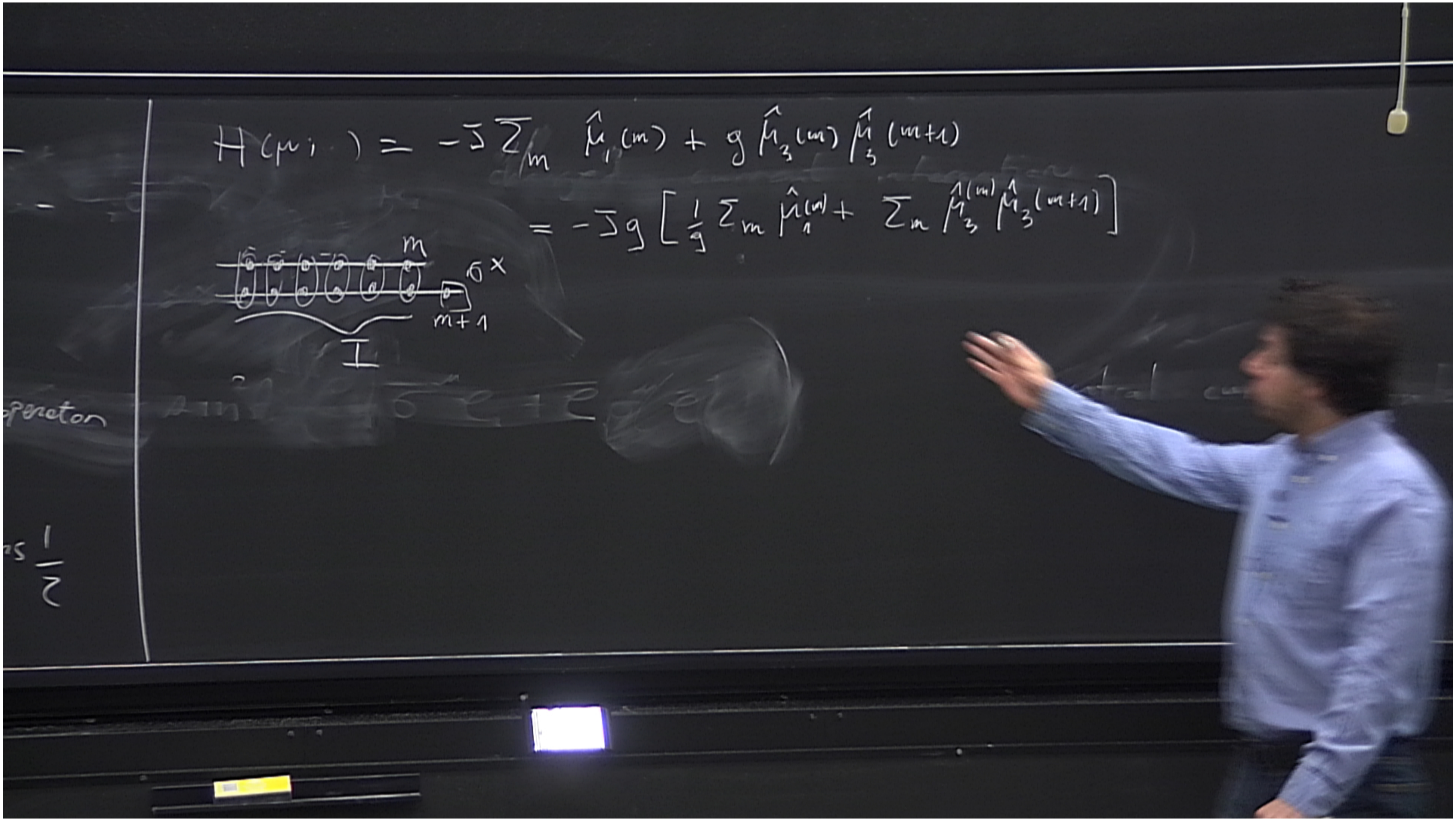

Duality
Transformation
on the lattice
String operator
spin chain?
→ spins 1/2


Duality
Transformation
on the lattice
i ⟶ m
string operator
→ spins 1/2


Duality
Transformation
on the lattice
String operator
spin chain?
spins

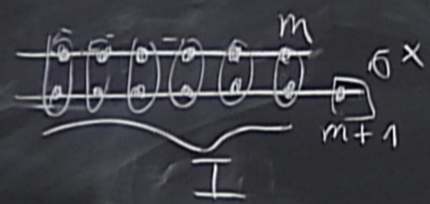

Duality
Transformation on the lattice
i ⟶ m
m m+1
$\hat{\mu}_1(m) = \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$
$\hat{\mu}_3(m) = \prod_{j<i} \hat{\sigma}^x_{j+1}$ string operator
spin chain?
$\{\hat{\sigma}^x_i, \hat{\sigma}^z_i\} = 0$
$\hat{\sigma}^{x2} = \hat{\sigma}^{z2} = \mathbb{1}$
$\mu_1^2 = \mu_3^2 = \mathbb{1}$
$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} = 0$ → spins $\frac{1}{2}$
$H(\mu) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$
σ^x
m+1


Duality
Transformation on the lattice
$i \longmapsto m$
$\hat{\mu}_1(m) = \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$
$\hat{\mu}_3(m) = \prod_{j<i} \hat{\sigma}^x_{j+1}$ String operator
spin chain?
$\{\hat{\sigma}^x, \hat{\sigma}^z\} = 0$
$\hat{\sigma}^{x2} = \hat{\sigma}^{z2} = 1$
$\mu_1^2 = \mu_3^2 = 1$
$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} = 0$
→ Spins $\frac{1}{2}$
$H(\mu) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \right.$
m
m+1
σ^x
I


$H(\mu;\cdot) = -J\sum_m \hat{\mu}_1(m) + g\,\hat{\mu}_3(m)\,\hat{\mu}_3(m+1)$
$= -Jg\left[\frac{1}{g}\sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m)\,\hat{\mu}_3(m+1)\right]$
m
σ^x
m+1
I
operator

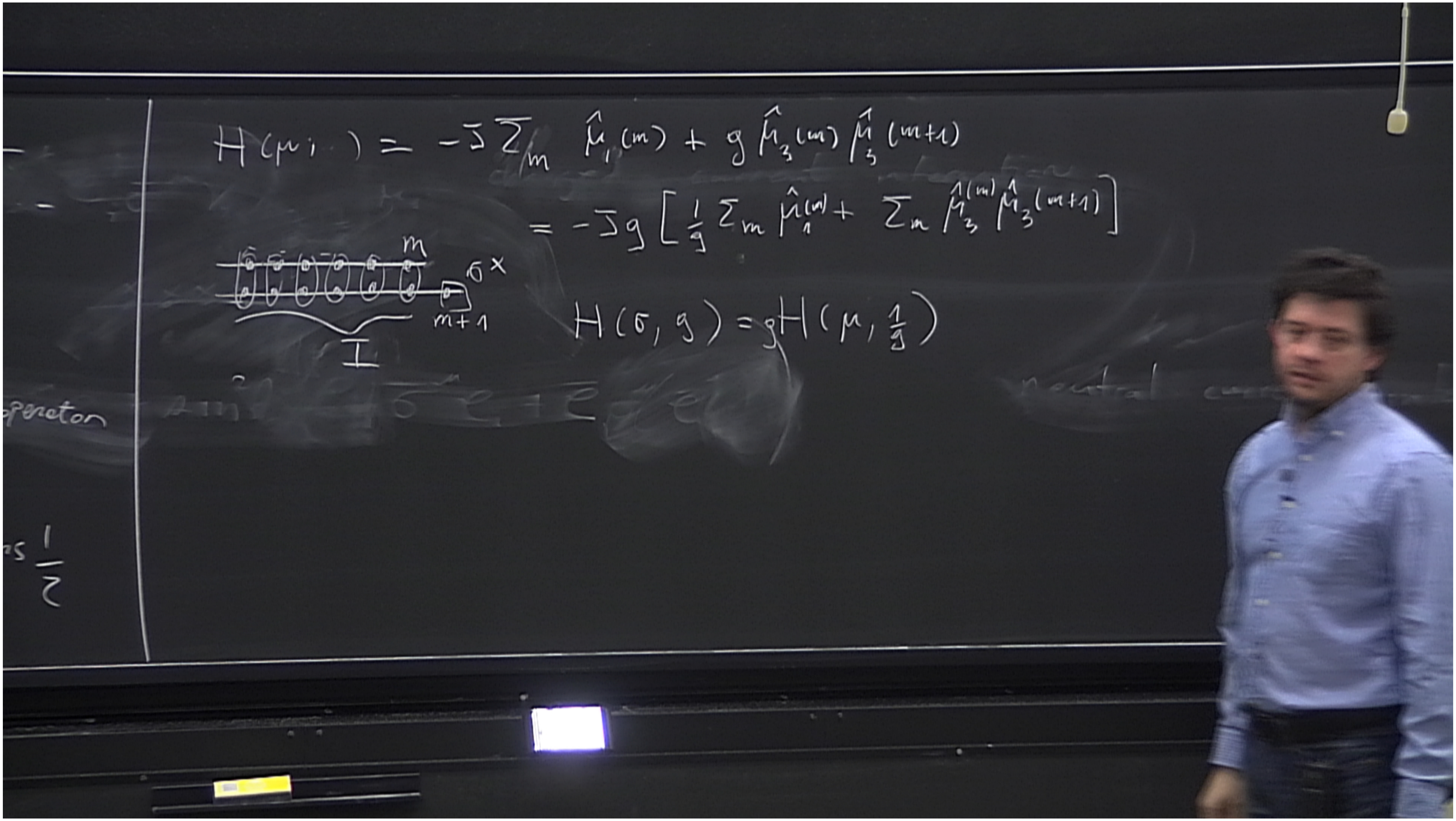
$$H(\mu_i) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$$

$$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$

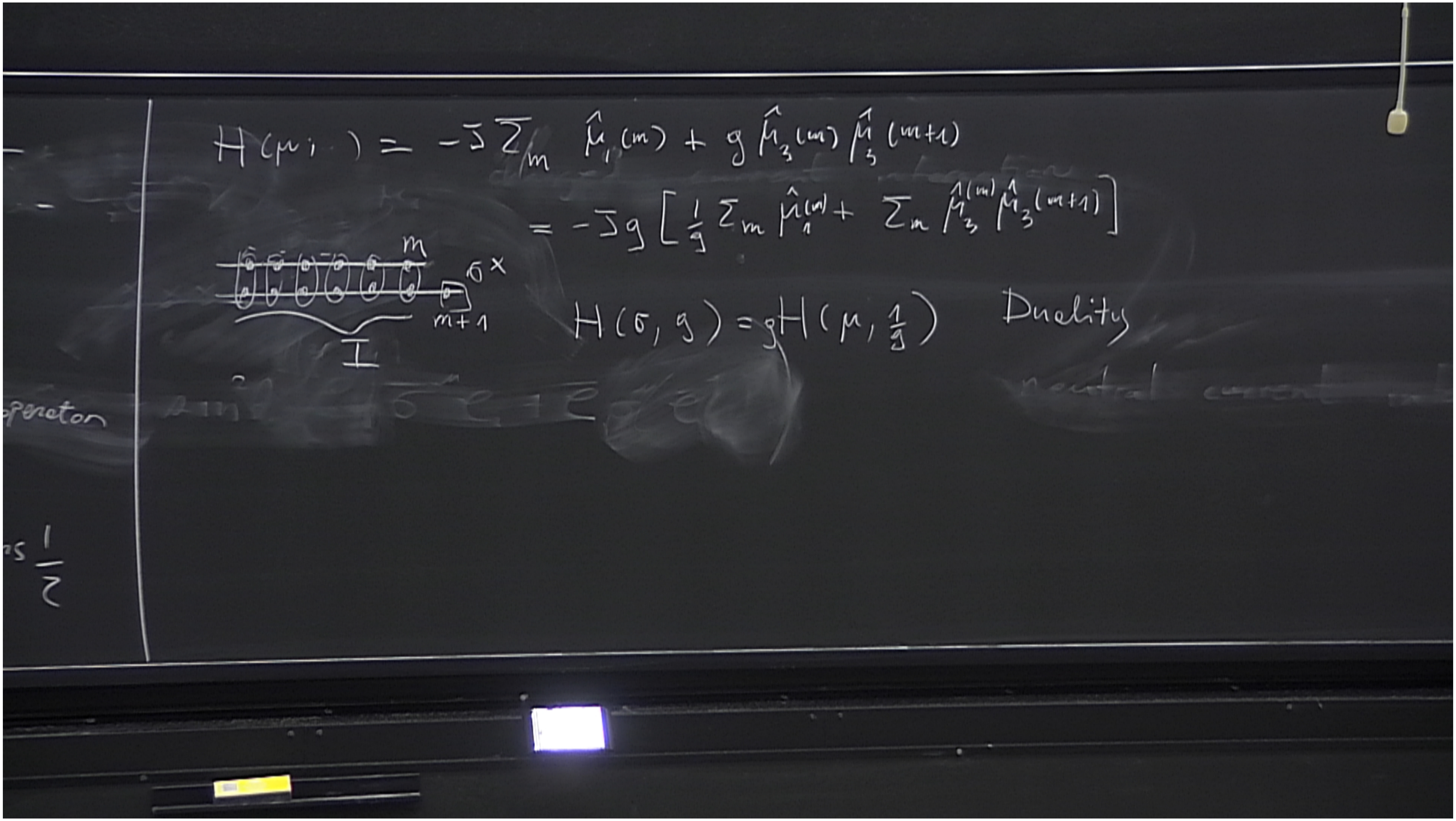


operator

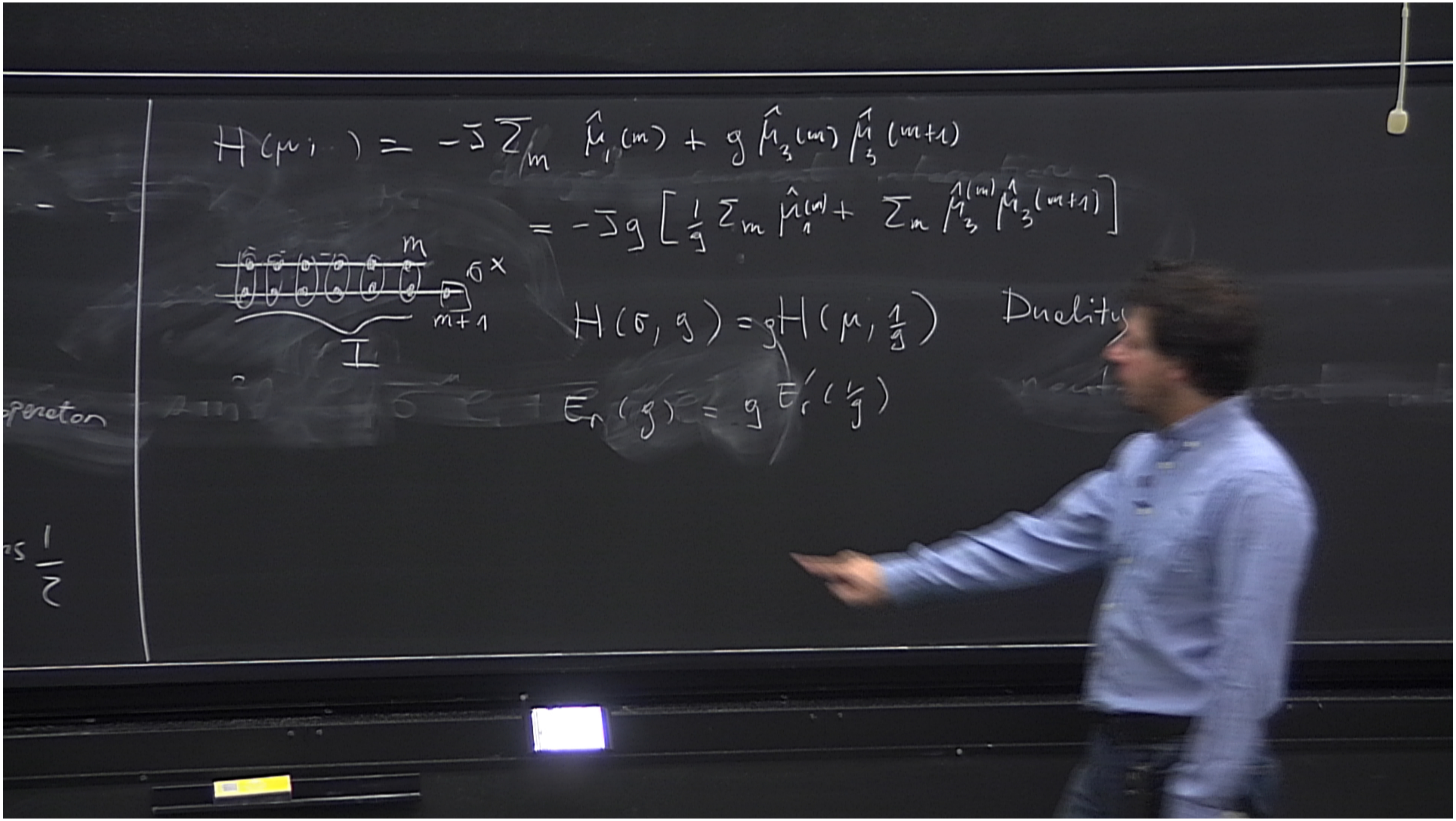
$\frac{1}{2}$



$H(\mu;\ ) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m)\, \hat{\mu}_3(m+1) \right]$
m
σ^x
$m+1$
I
$H(\sigma, g) = g H(\mu, \frac{1}{g})$
operator

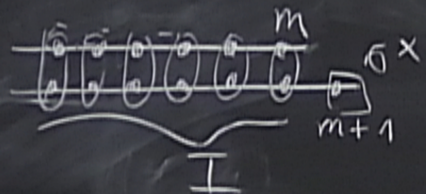


$H(\mu; \cdot) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m)\, \hat{\mu}_3(m+1) \right]$
m
σ^x
$m+1$
I
$H(\sigma, g) = g H(\mu, \frac{1}{g})$
Duality
operator


H(μ, .) = −J Σ_m μ̂_1(m) + g μ̂_3(m) μ̂_3(m+1)
= −Jg [1/g Σ_m μ̂_1(m) + Σ_m μ̂_3(m) μ̂_3(m+1)]
m
σ^x
m+1
I
H(σ, g) = g H(μ, 1/g)
Duality
E_r(g) = g E′_r(1/g)
operator

$$H(\mu; \,) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$$

$$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m)\, \hat{\mu}_3(m+1) \right]$$

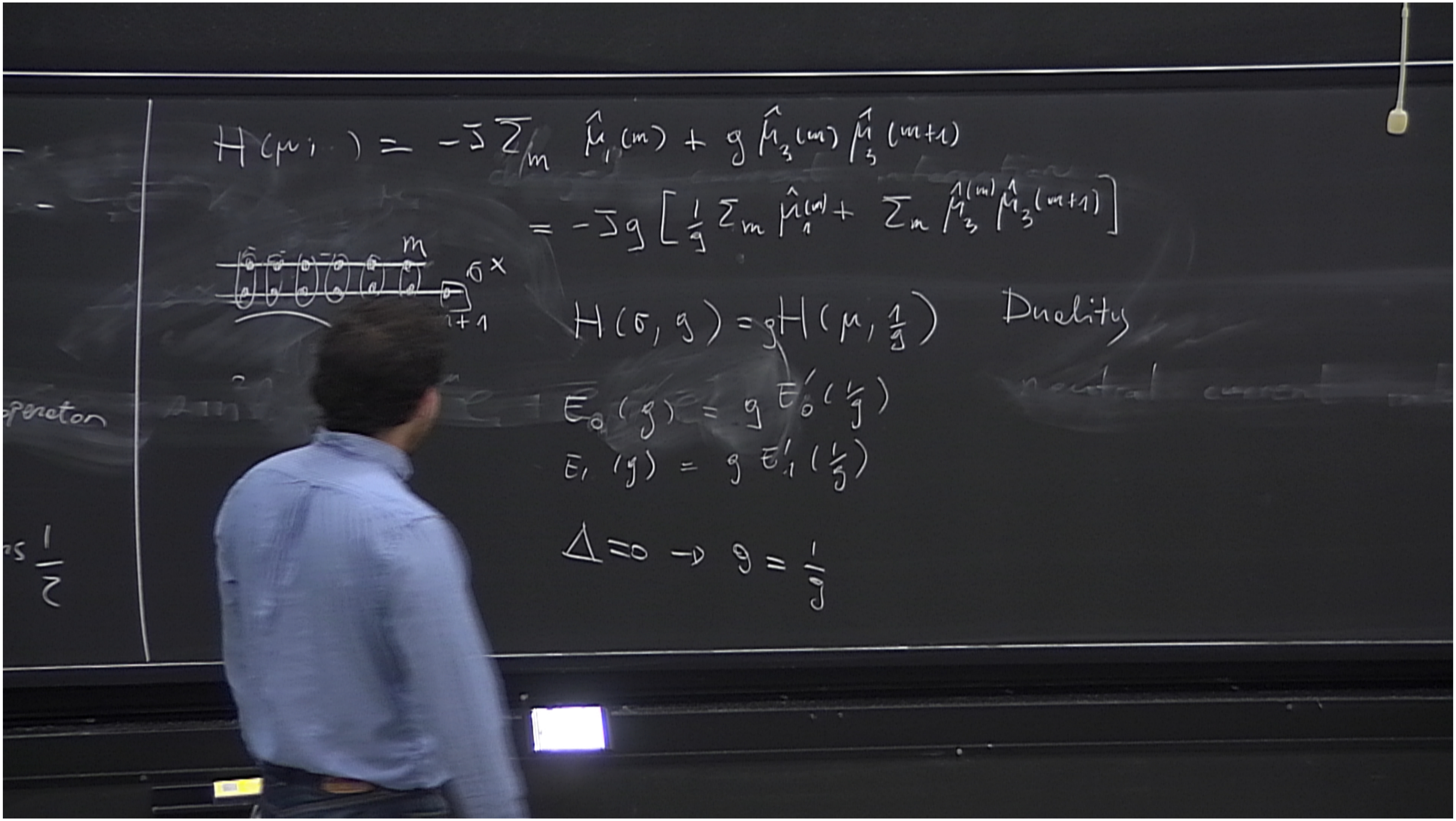


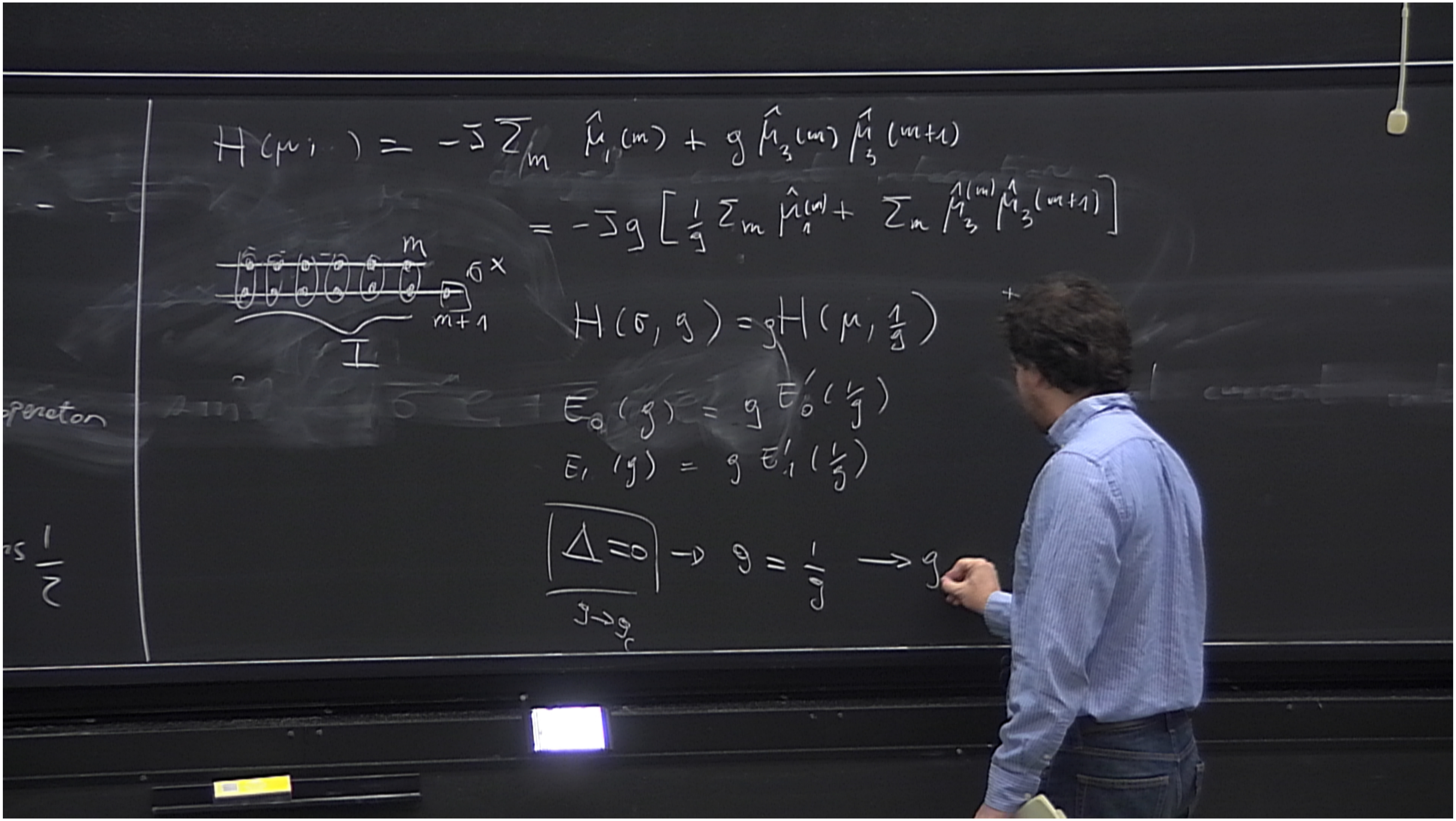
$$H(\sigma, g) = g H(\mu, \tfrac{1}{g})$$ Duality

$$E_0(g) = g\, E_0'(\tfrac{1}{g})$$

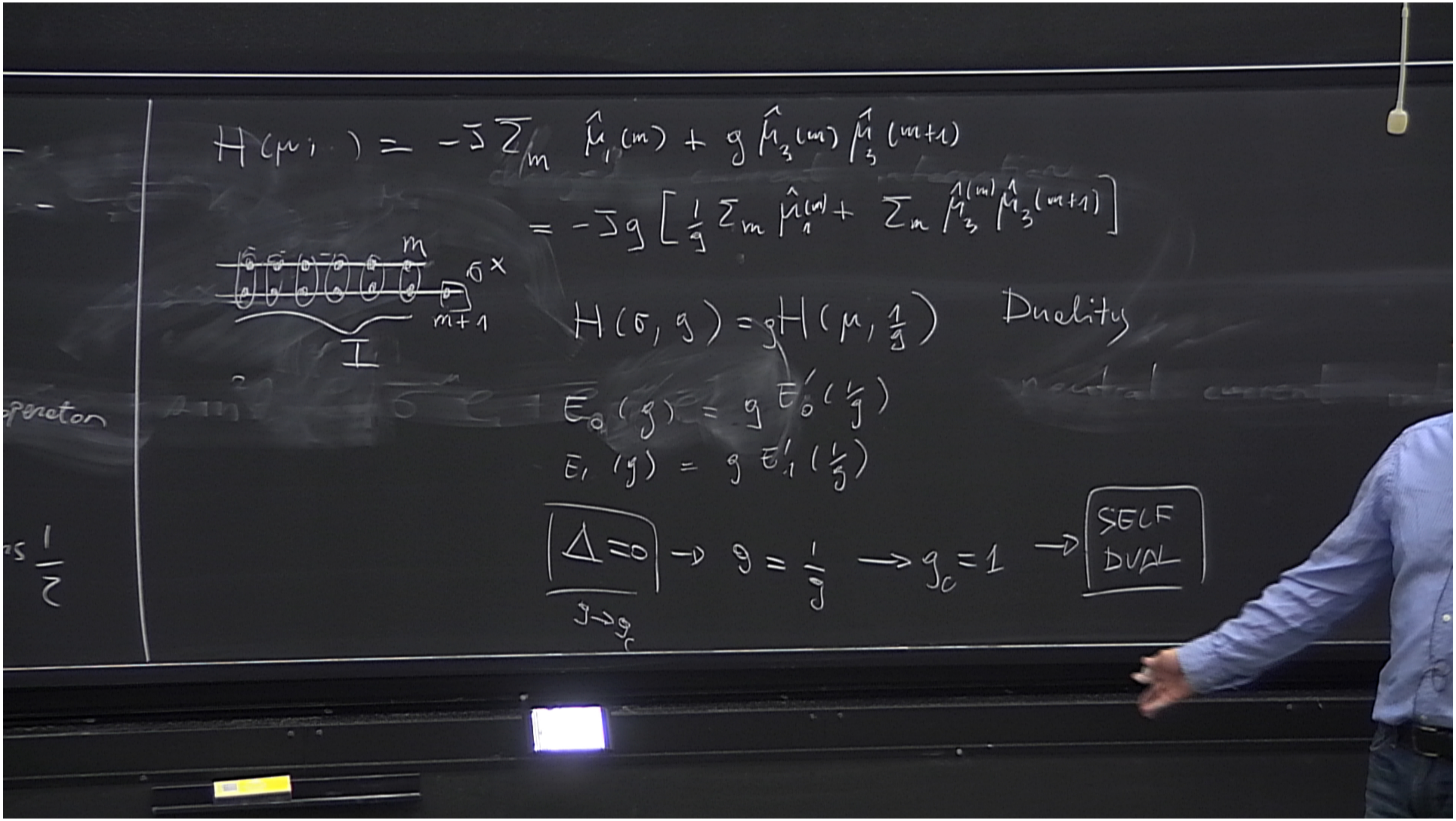
operator

$\frac{1}{2}$

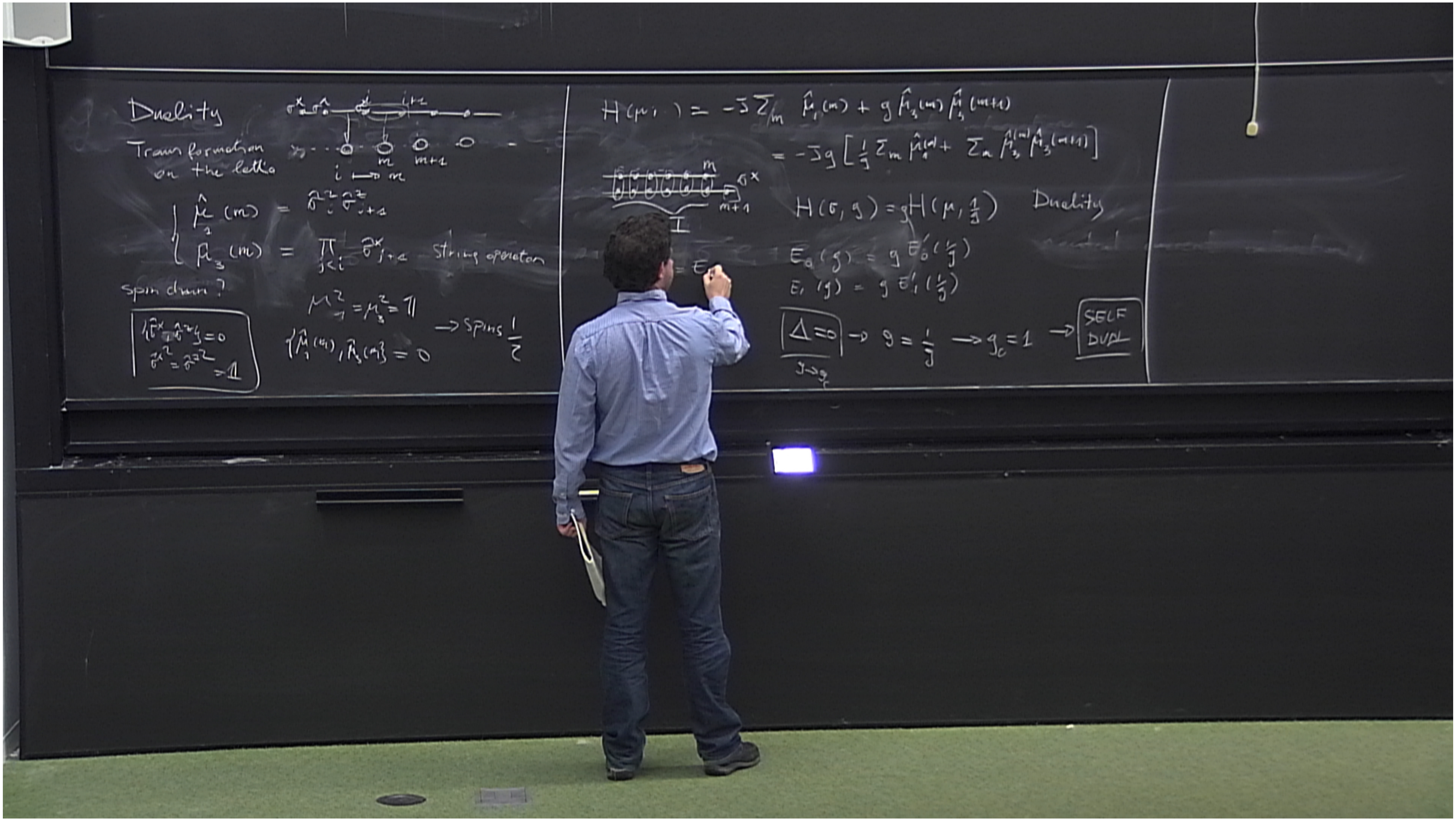

$H(\mu; \cdot) = -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_3(m) \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$
m
σ^x
$H(\sigma, g) = g H(\mu, \frac{1}{g})$
Duality
$E_0(g) = g E_0'(\frac{1}{g})$
$E_1(g) = g E_1'(\frac{1}{g})$
$\Delta = 0 \to g = \frac{1}{g}$
operator

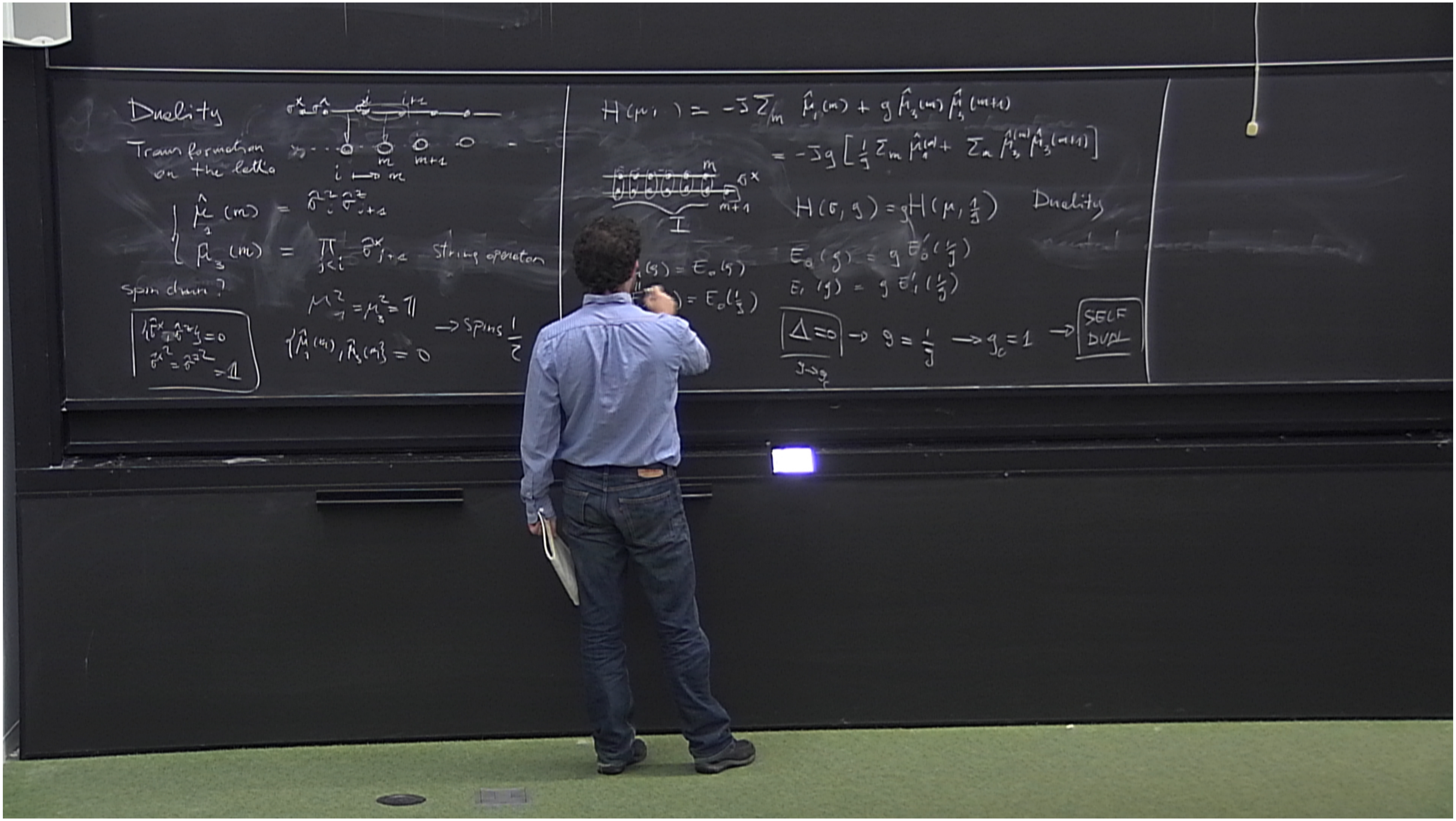


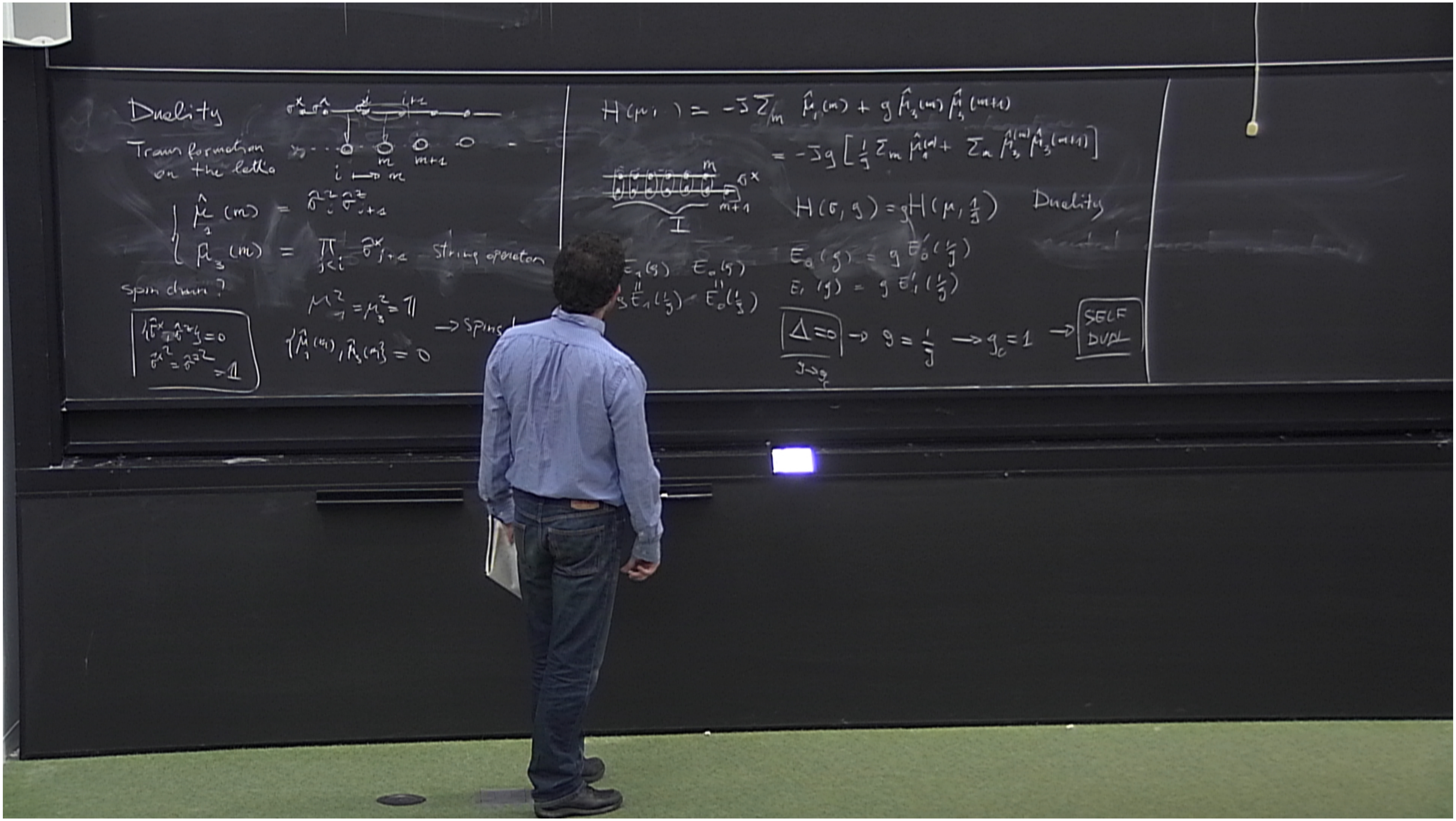
$H(\mu, \,) = -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_3(m) \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m) \hat{\mu}_3(m+1) \right]$
m
σ^x
$m+1$
I
$H(\sigma, g) = g H(\mu, \frac{1}{g})$
$E_0(g) = g E_0'(\frac{1}{g})$
$E_1(g) = g E_1'(\frac{1}{g})$
$\boxed{\Delta = 0} \to g = \frac{1}{g} \to g$
$g \to g_c$
$\frac{1}{2}$

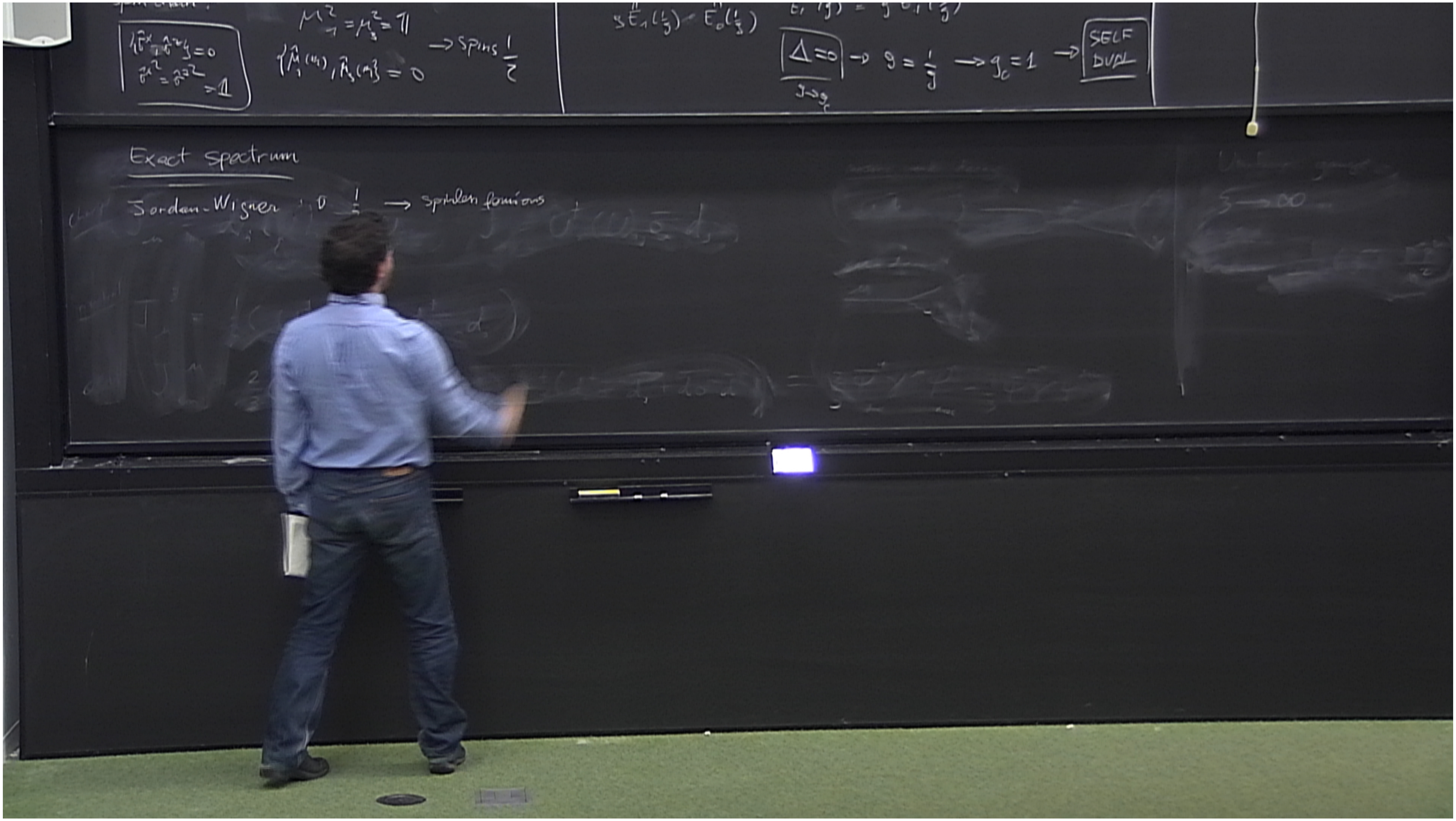


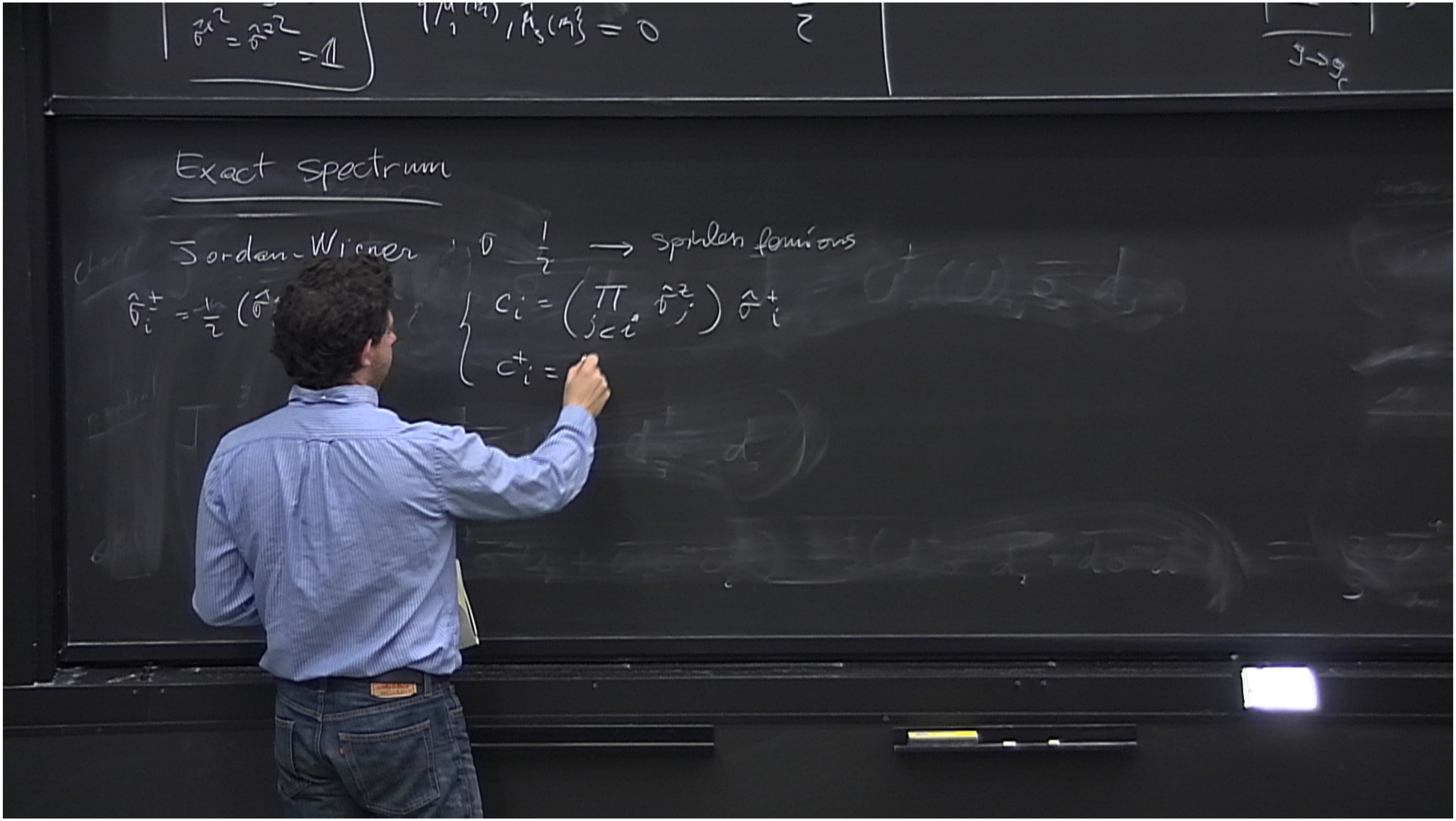
$H(\mu, \cdot) = -J \sum_m \hat{\mu}_1(m) + g\, \hat{\mu}_3(m)\, \hat{\mu}_3(m+1)$
$= -Jg \left[\frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_3(m)\, \hat{\mu}_3(m+1) \right]$
m
σ^x
$m+1$
I
$H(\sigma, g) = g H(\mu, \frac{1}{g})$
Duality
$E_0(g) = g E_0'(\frac{1}{g})$
$E_1(g) = g E_1'(\frac{1}{g})$
$\Delta = 0 \to g = \frac{1}{g} \to g_c = 1 \to$
SELF DUAL
$g \to g_c$
operator

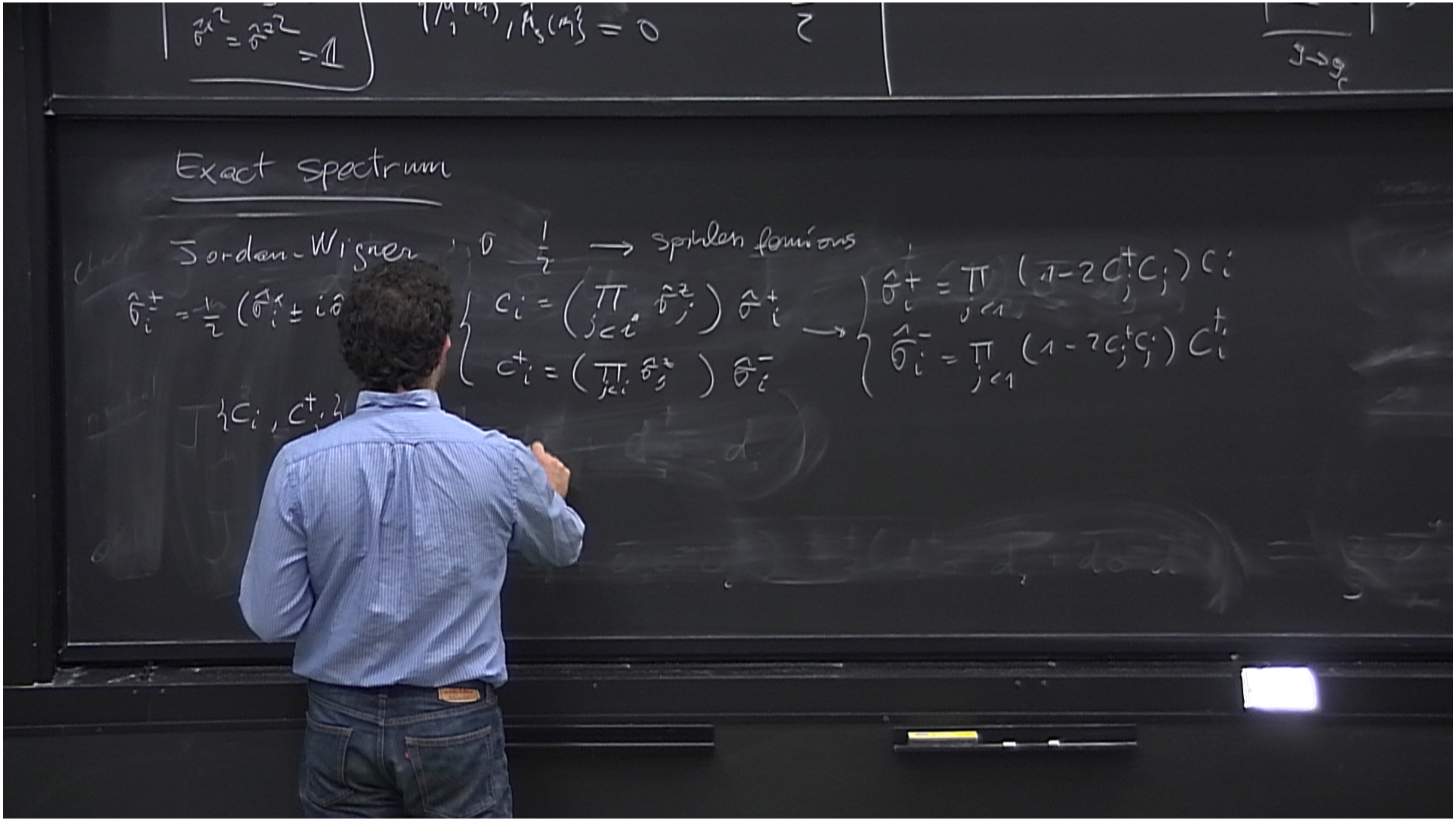

Duality
Transformation on the lattice
String operator
spin chain?
Duality
SELF DUAL


Duality
Transformation on the lattice
String operator
spin chain?
Spins 1/2
Duality
SELF DUAL


Duality
Transformation on the lattice
String operator
spin chain?
Duality
SELF DUAL


Exact spectrum
Jordan-Wigner
spinless fermions
SELF DUAL
spins


$\hat\sigma^{x2} = \hat\sigma^{z2} = \mathbb{1}$
$= 0$
$g \to g_c$
Exact spectrum
Jordan-Wigner
$\longrightarrow$ spinless fermions
$c_i = \left(\prod_{j<i} \hat\sigma^z_j\right) \hat\sigma^+_i$
$c^+_i =$


Exact spectrum
Jordan-Wigner
spinless fermions

Exact spectrum

Jordan-Wigner : σ $\frac{1}{2}$ $\longrightarrow$ spinless fermions

$\hat{\sigma}_i^{\pm}$ ... $\hat{\sigma}_i^x \pm i\hat{\sigma}_i^y)$

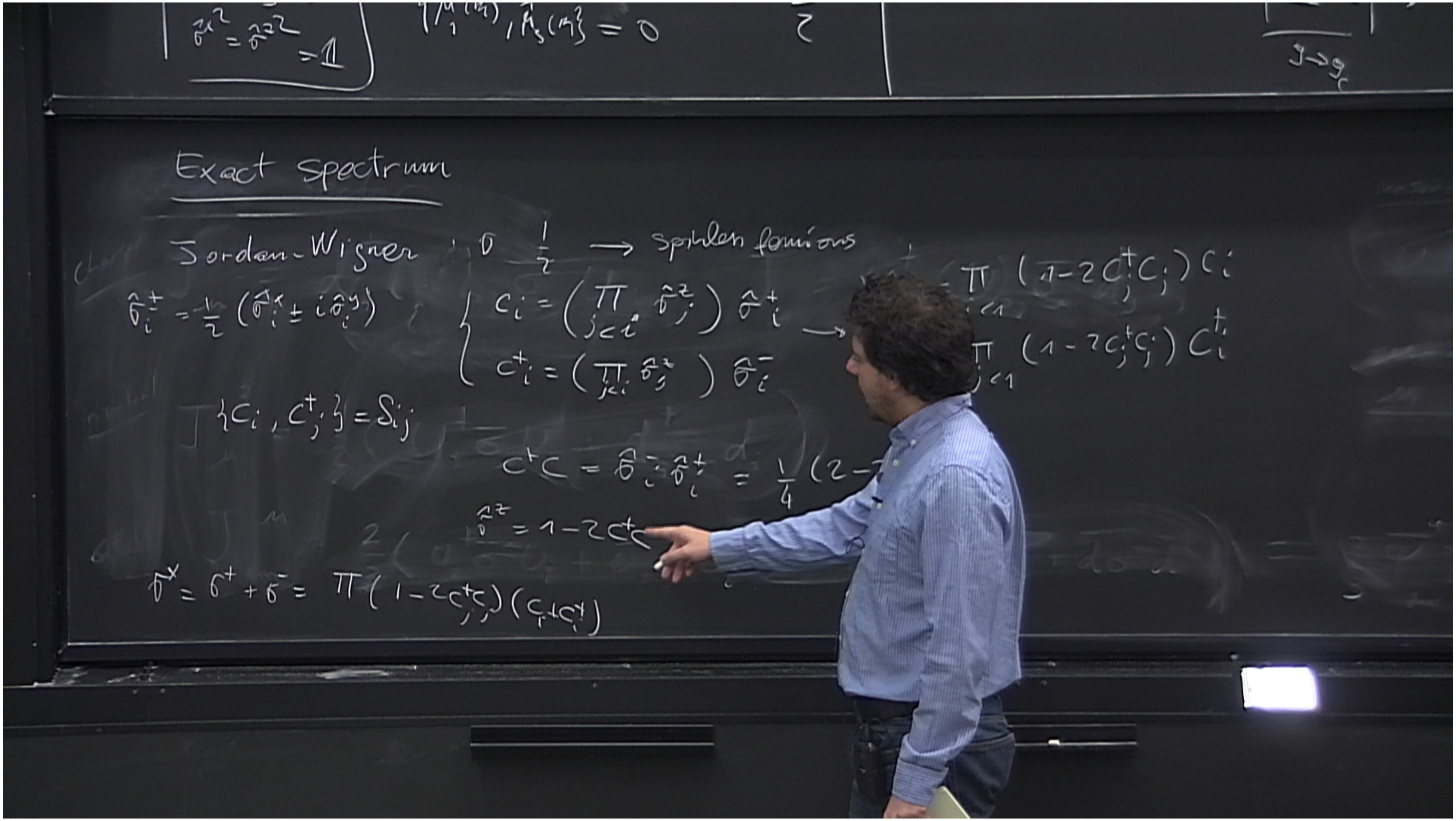
$$\begin{cases} c_i = \left(\prod_{j<i} \hat{\sigma}_j^z\right) \hat{\sigma}_i^+ \\ c_i^\dagger = \left(\prod_{j<i} \hat{\sigma}_j^z\right) \hat{\sigma}_i^- \end{cases} \longrightarrow \begin{cases} \hat{\sigma}_i^+ = \prod_{j<i} (1-2c_j^\dagger c_j)\, c_i \\ \hat{\sigma}_i^- = \prod_{j<i} (1-2c_j^\dagger c_j)\, c_i^\dagger \end{cases}$$

$\{c_i, c_j^\dagger\} = \delta_{ij}$

$$c^\dagger c = \hat{\sigma}_i^- \hat{\sigma}_i^+ = \frac{1}{4}(2-2\sigma_i^z)$$

$$\hat{\sigma}^z = 1-2c^\dagger c$$

$\prod$


Exact spectrum
Jordan-Wigner
spinless fermions

$\hat\sigma^{x\,2} = \hat\sigma^{z\,2} = \mathbb{1}$

$\{\mu_i(n), \mu_j(n)\} = 0$

$g \to g_c$

Exact spectrum

Jordan-Wigner : σ $\frac{1}{2}$ $\longrightarrow$ spinless fermions

$\hat\sigma_i^{\pm} = \frac{1}{2}(\hat\sigma_i^x \pm i\hat\sigma_i^y)$

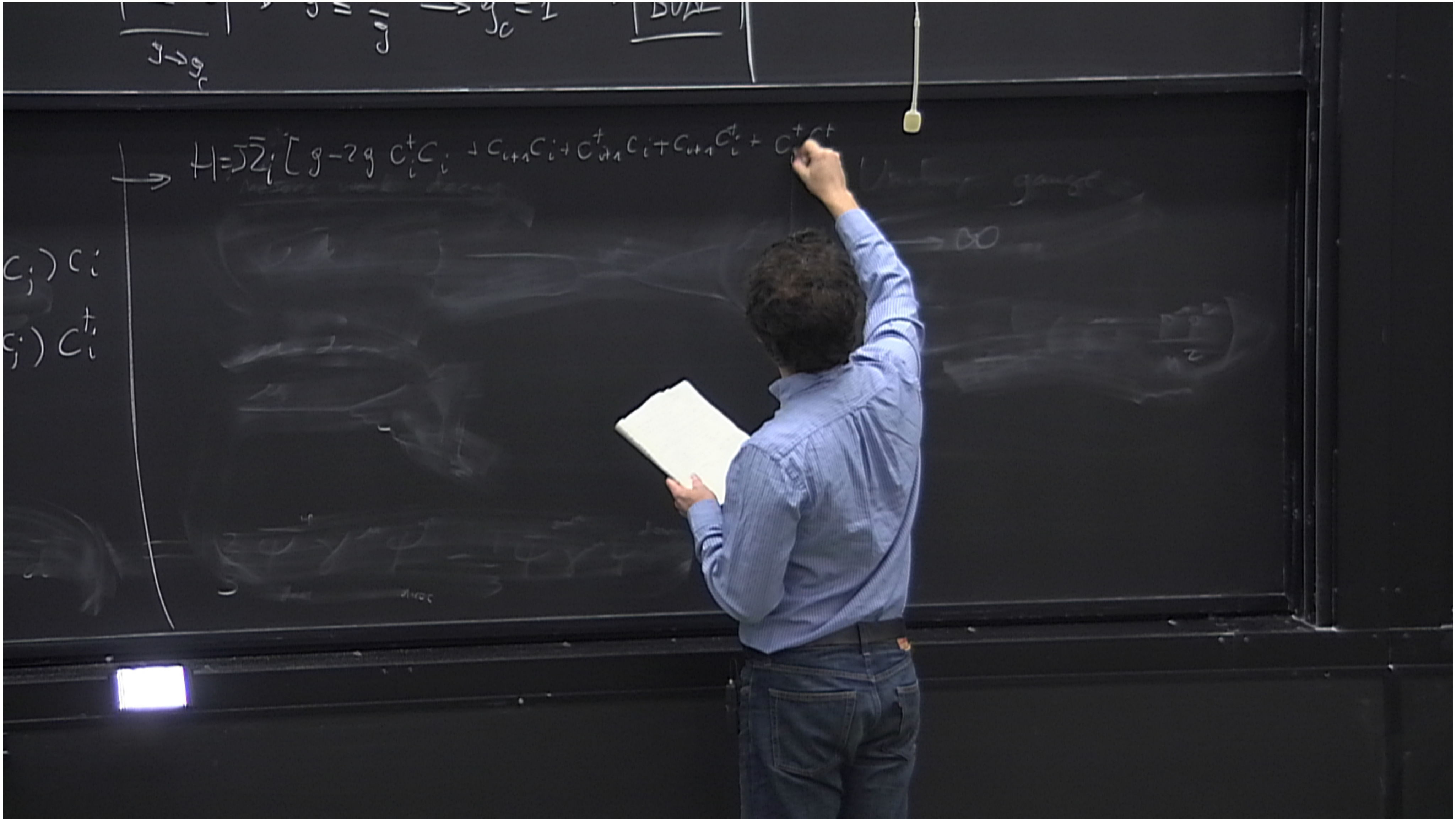
$$\begin{cases} c_i = \left(\prod_{j<i} \hat\sigma_j^z\right)\hat\sigma_i^+ \\ c_i^+ = \left(\prod_{j<i} \hat\sigma_j^z\right)\hat\sigma_i^- \end{cases} \longrightarrow \begin{cases} \hat\sigma_i^+ = \prod_{j<i}(1 - 2c_j^+c_j)\, c_i \\ \hat\sigma_i^- = \prod_{j<i}(1 - 2c_j^+c_j)\, c_i^+ \end{cases}$$

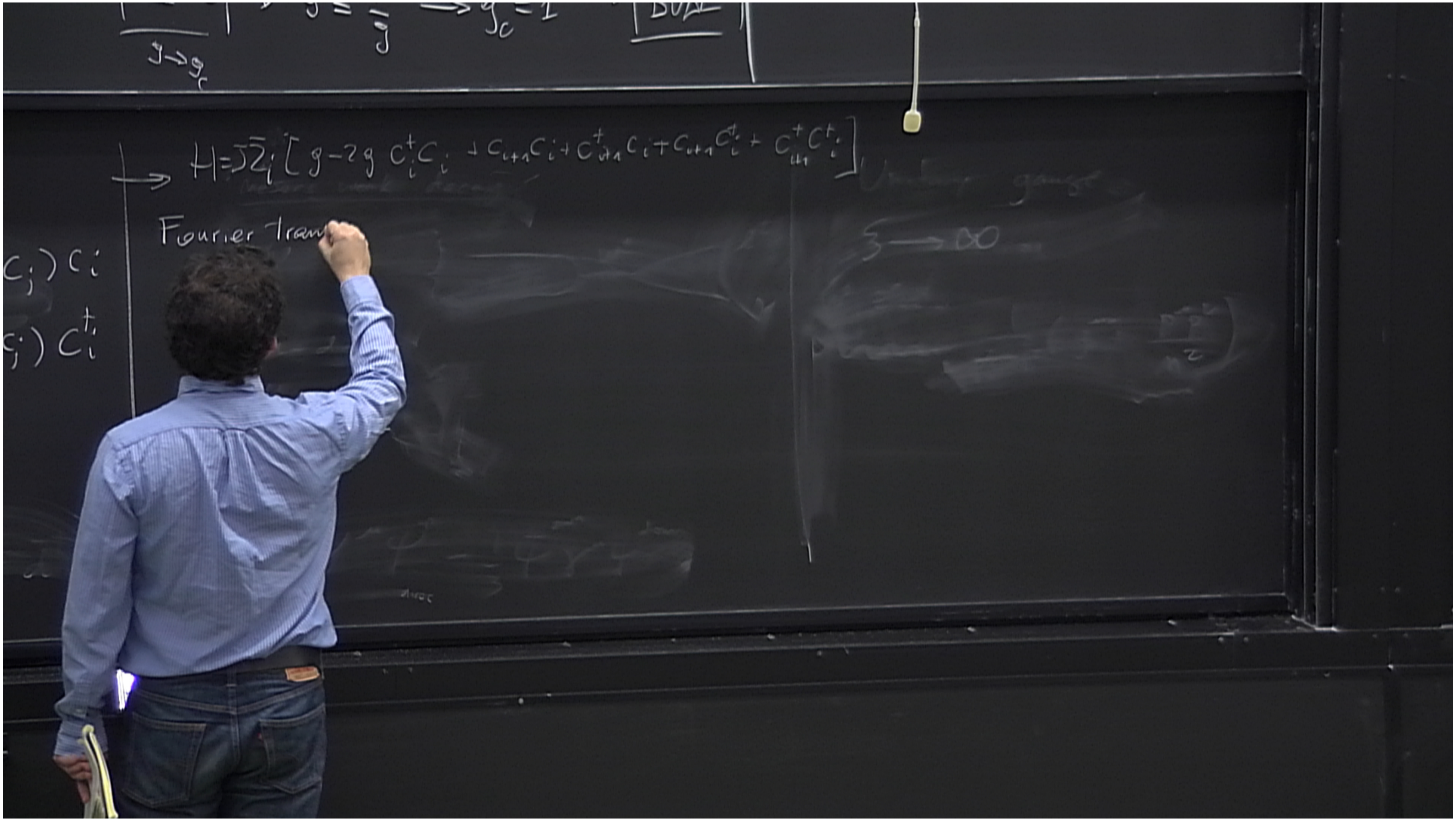
$\{c_i, c_j^+\} = \delta_{ij}$

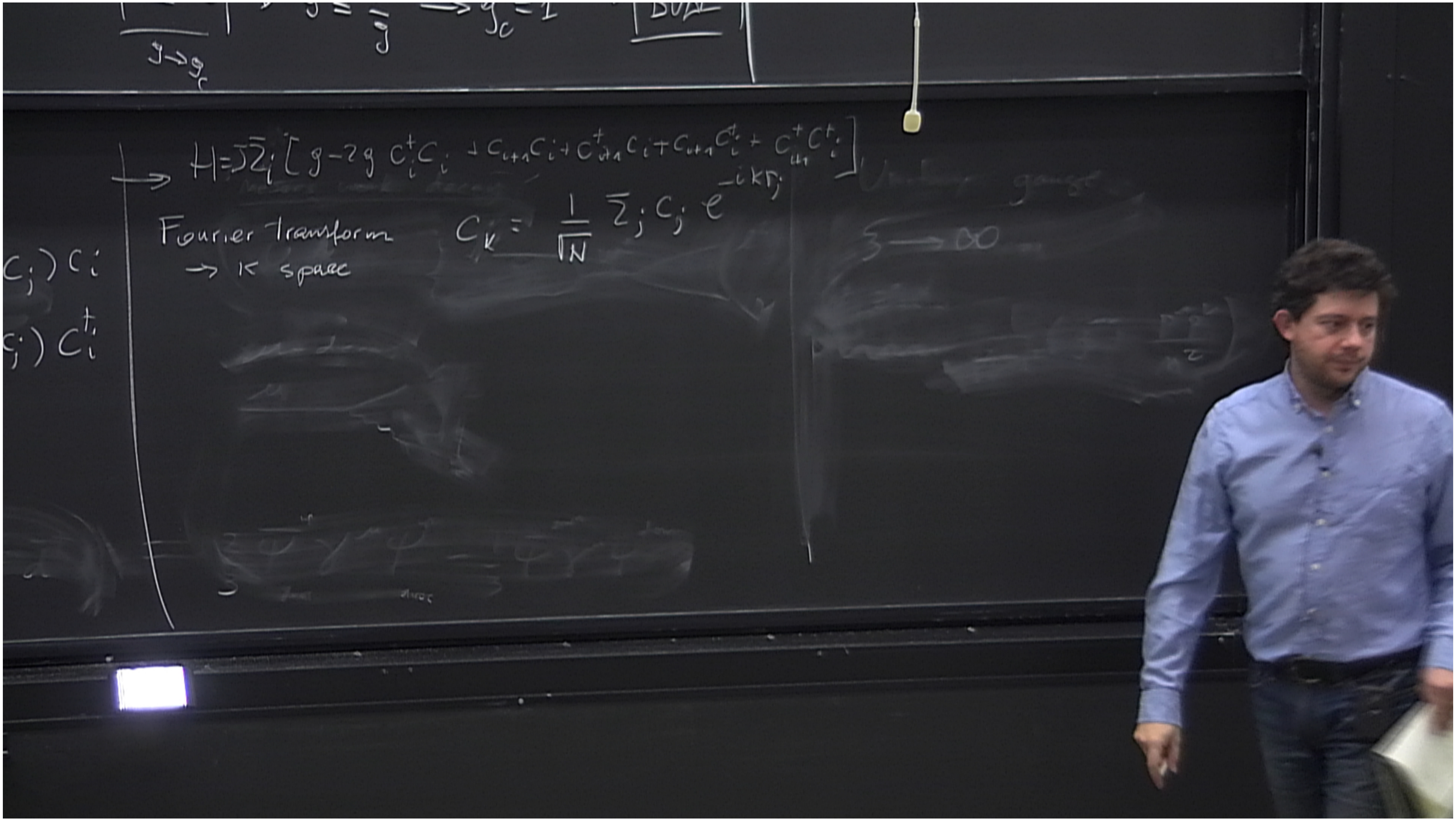
$$c^+c = \hat\sigma_i^-\hat\sigma_i^+ = \frac{1}{4}(2 - 2\sigma_i^z)$$

$$\hat\sigma^z = 1 - 2c^+c \longrightarrow \sigma^x$$

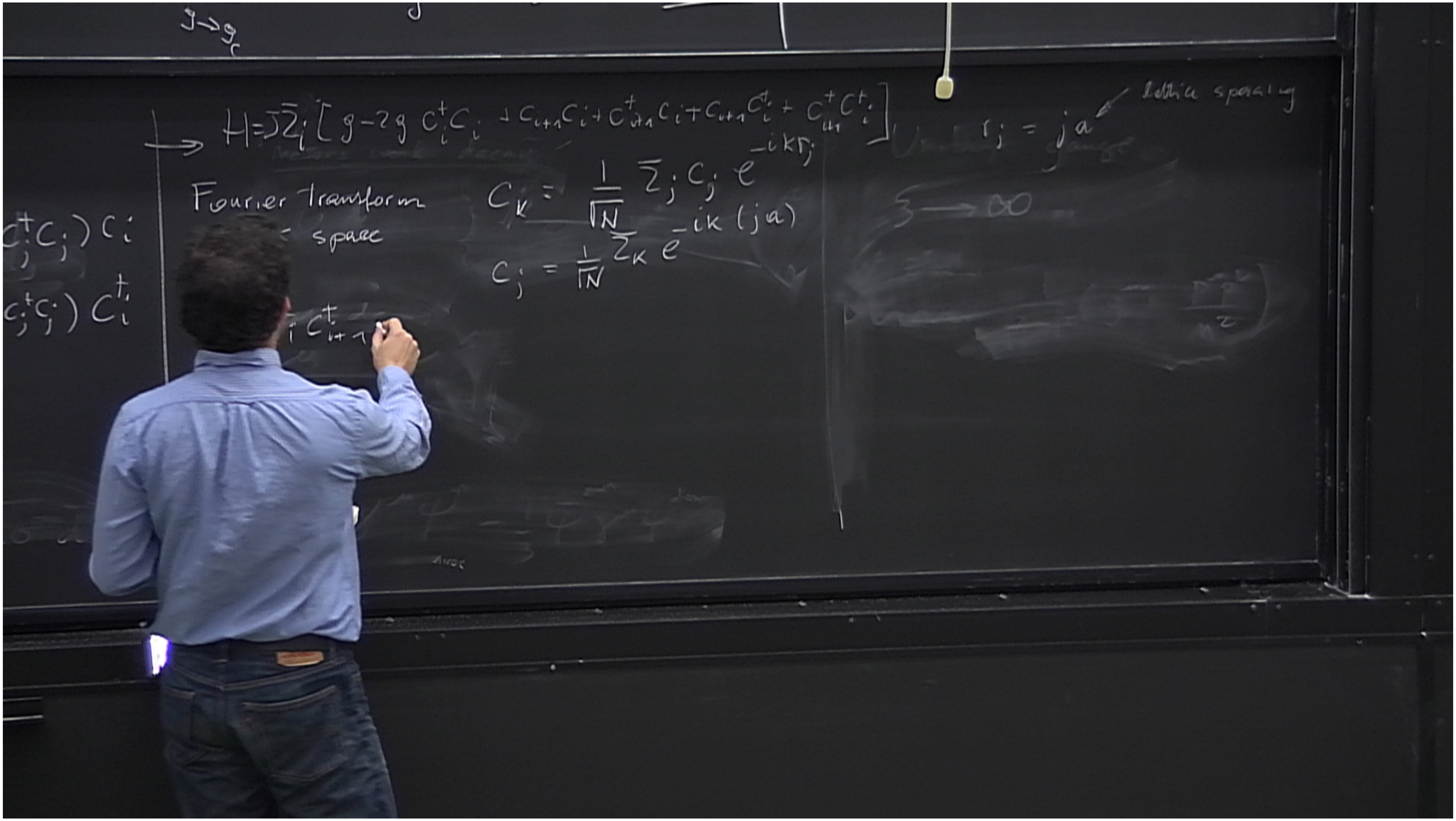
$$\sigma^x = \sigma^+ + \sigma^- = \prod(1 - 2c_j^+c_j)(c_i + c_i^+) \longrightarrow -\sigma^z$$

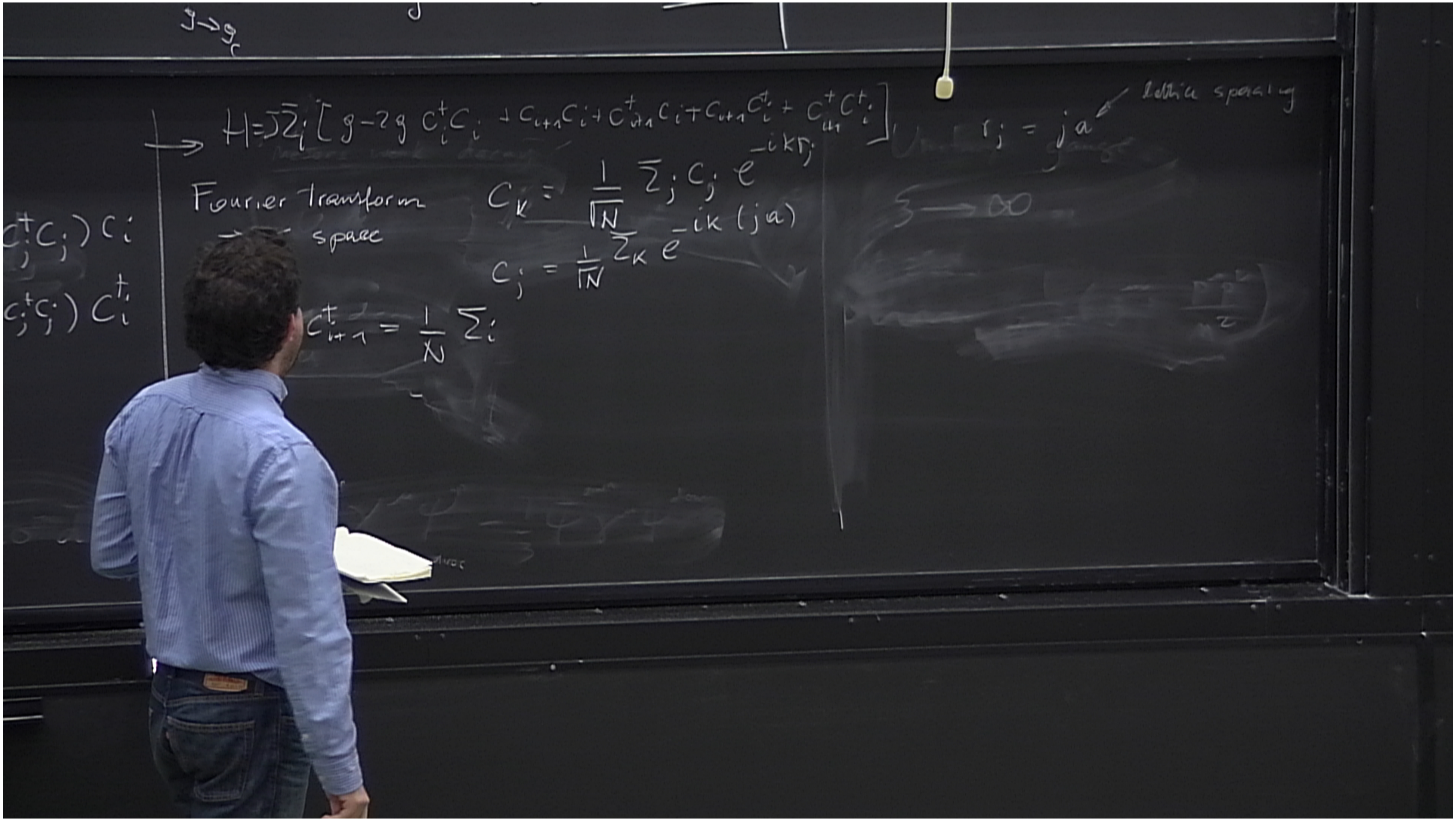

H=JΣi[g−2g ci†ci + ci+1ci + c†i+1ci + ci+1ci† + c†c†

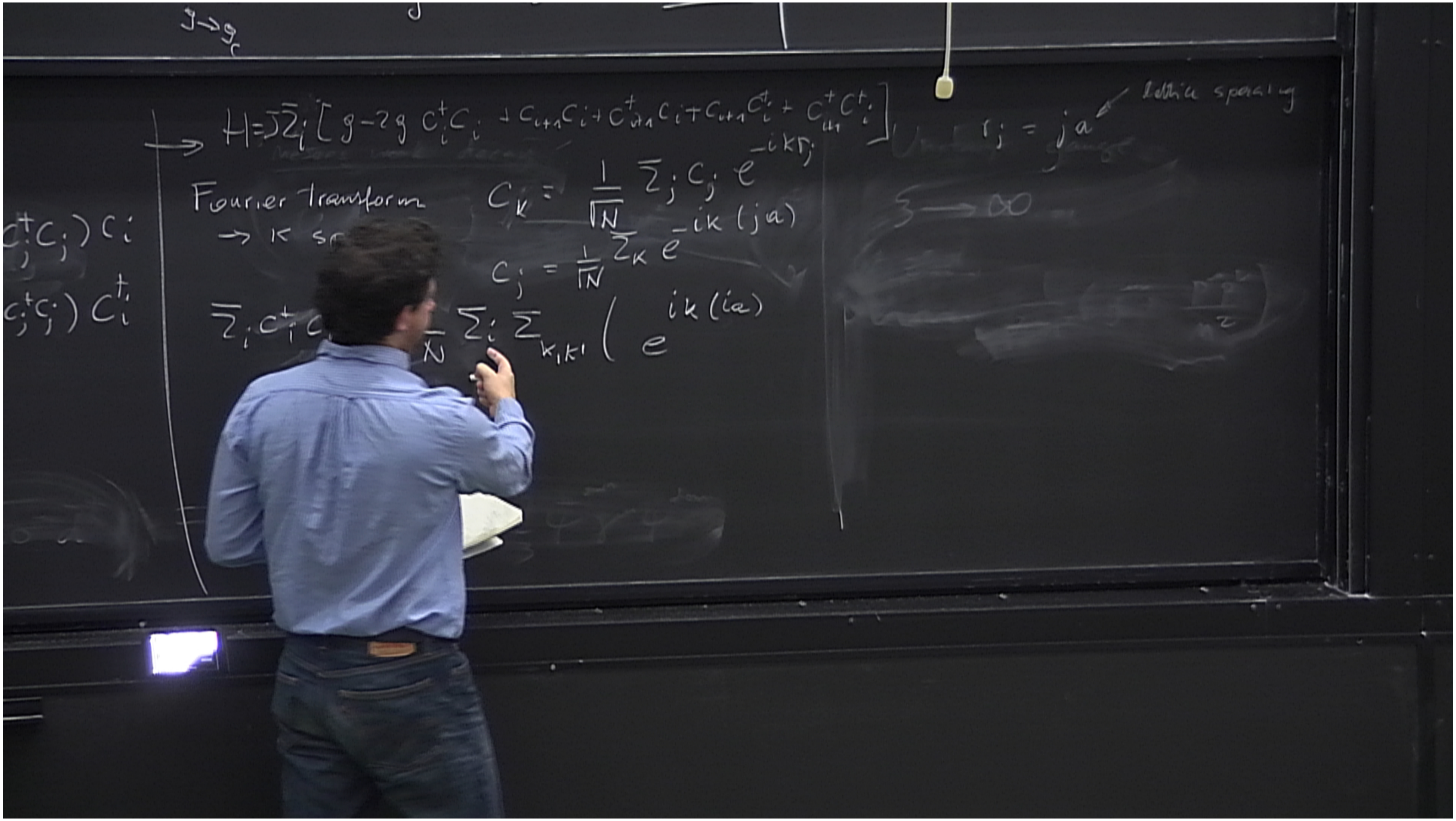

$H = J\sum_i [\, g - 2g\, c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_{i+1}^\dagger c_i^\dagger \,]$
Fourier Tran
$\xi \to \infty$

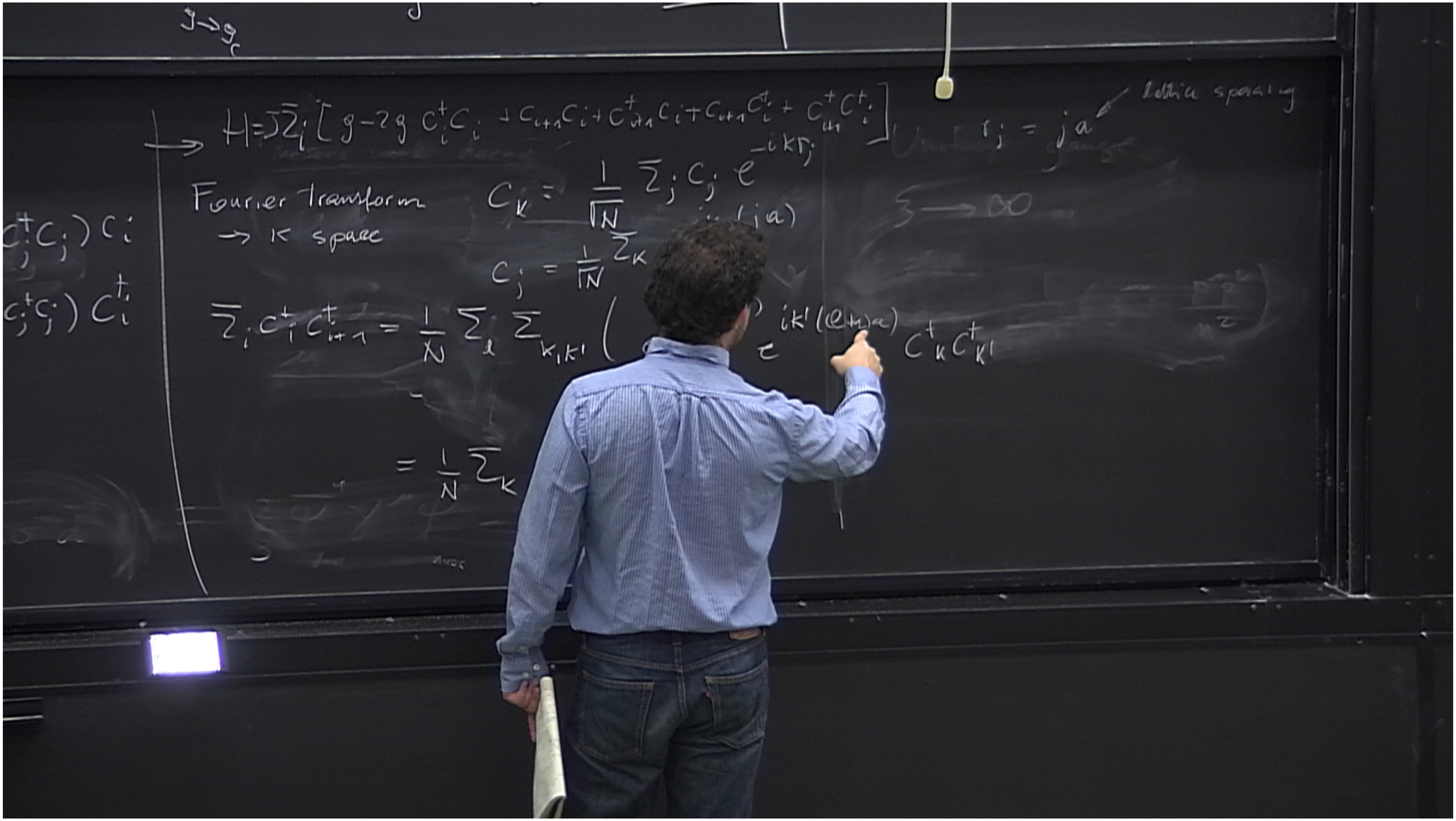


H = J Σ_i [g − 2g c_i^† c_i + c_{i+1} c_i + c_{i+1}^† c_i + c_{i+1} c_i^† + c_{i+1}^† c_i^†]
Fourier Transform
→ k space
c_k = 1/√N Σ_j c_j e^{−ikr_j}
ξ → ∞


Fourier Transform
space
lattice spacing
r_j = ja
C_k = 1/√N Σ_j C_j e^{-ikr_j}
e^{-ik(ja)}
C_j = 1/√N Σ_k e


g → g_c
H = J Σ_i [g − 2g c_i^† c_i + c_{i+1} c_i + c_{i+1}^† c_i + c_{i+1} c_i^† + c_{i+1}^† c_i^†]
lattice spacing
r_j = j a
Fourier Transform
space
C_k = 1/√N Σ_j c_j e^{−ik r_j}
c_j = 1/√N Σ_k e^{−ik (ja)}
c_{i+1}^† = 1/N Σ_i
ξ → ∞


lattice spacing
Fourier Transform
→ k space
r_j = ja


$H = J\sum_i [g - 2g\, c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_{i+1}^\dagger c_i^\dagger]$
$r_j = ja$
lattice spacing
Fourier Transform
→ k space
$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ikr_j}$
$c_j = \frac{1}{\sqrt{N}} \sum_k$
$\sum_i c_i^\dagger c_{i+1}^\dagger = \frac{1}{N} \sum_l \sum_{k,k'}$
$c_k^\dagger c_{k'}^\dagger$
$= \frac{1}{N} \sum_k$

$g \to g_c$

$\rightarrow H = J \sum_i [g - 2g\, c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_{i+1}^\dagger c_i^\dagger]$ — lattice spacing — $r_j = ja$

Fourier Transform $\to$ k space

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ikr_j}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{-ik(ja)}$$

$\xi \to \infty$

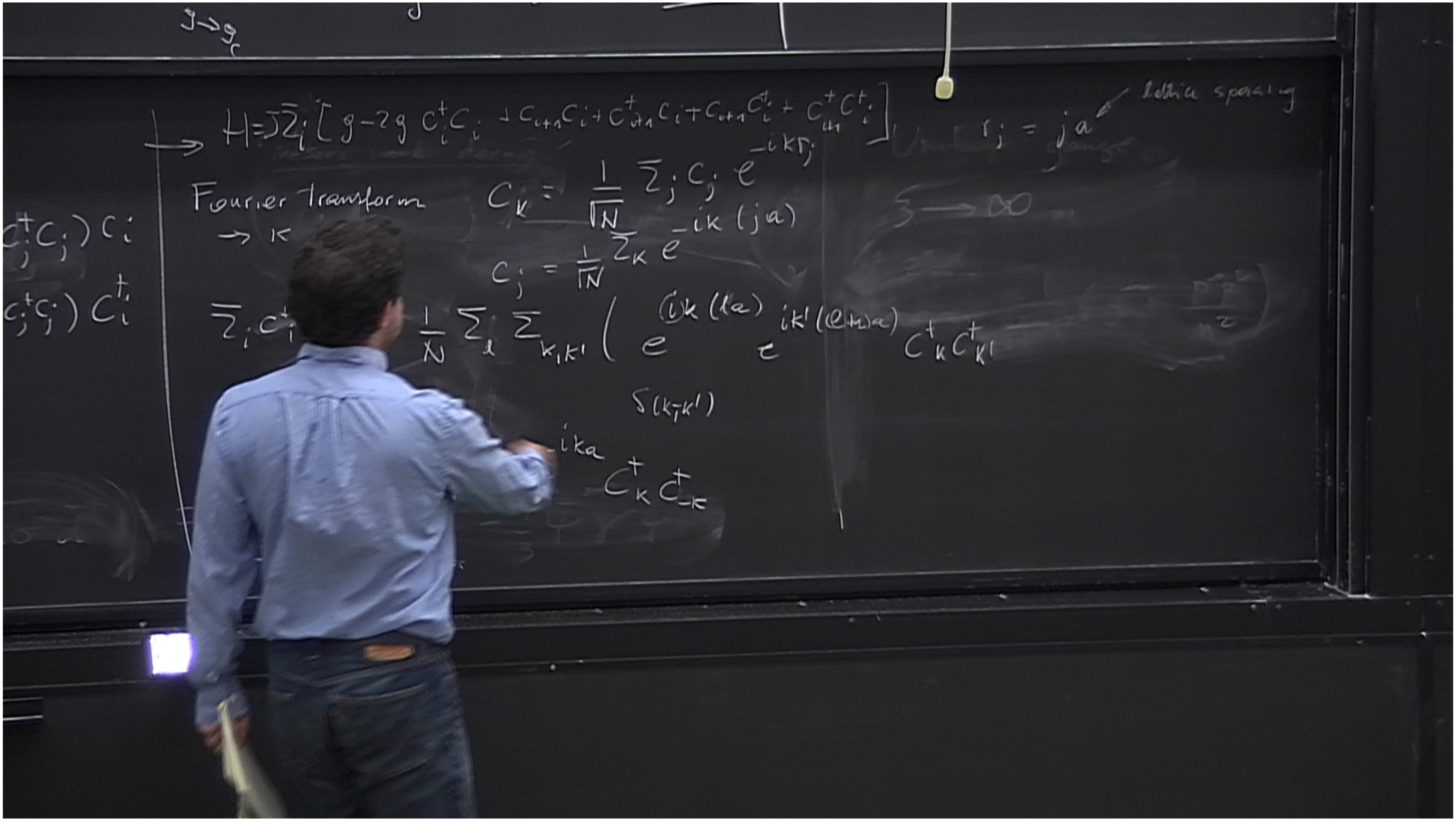
$(c_j^\dagger c_j) c_i$

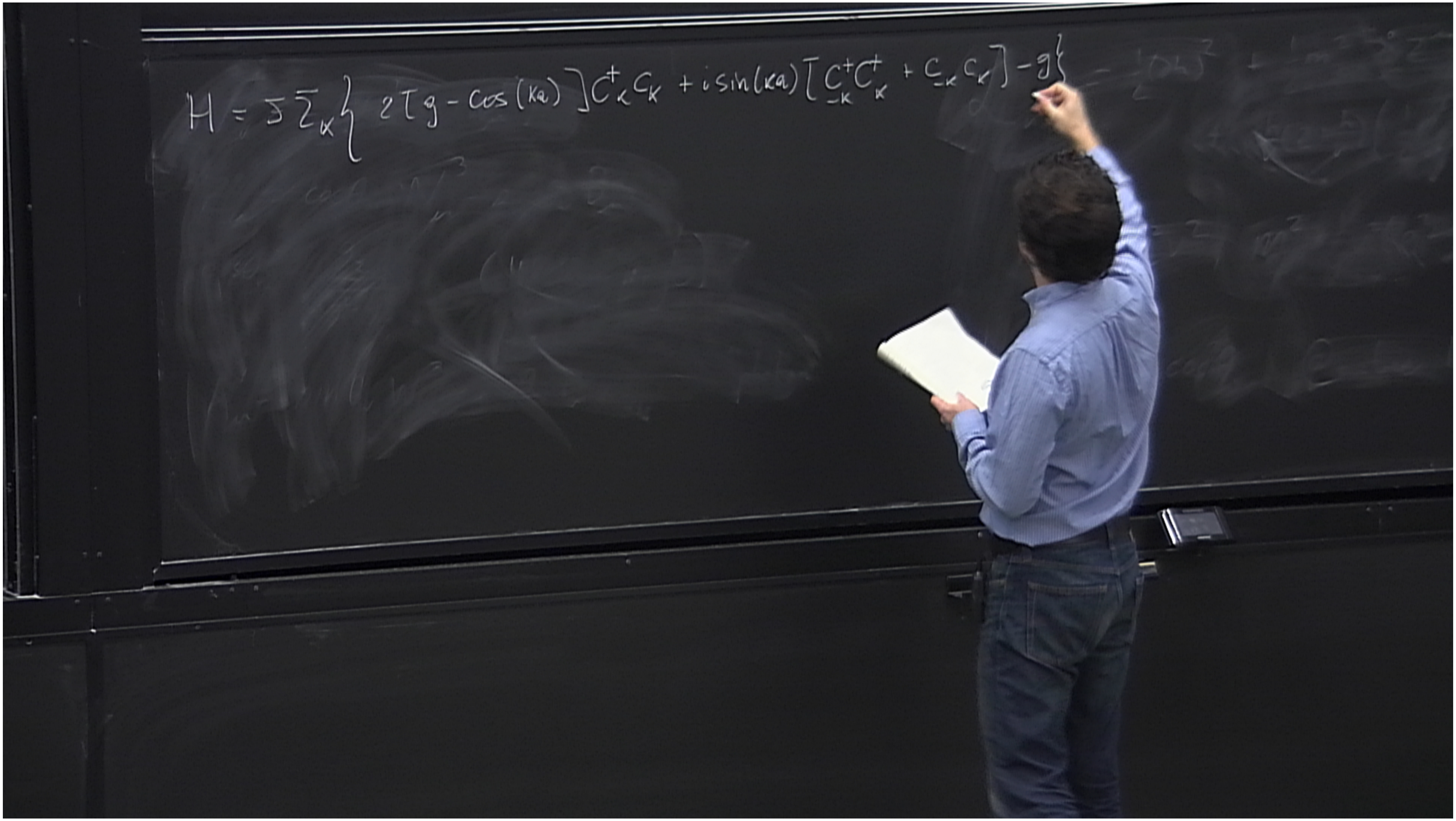
$(c_j^\dagger c_j) c_i^\dagger$

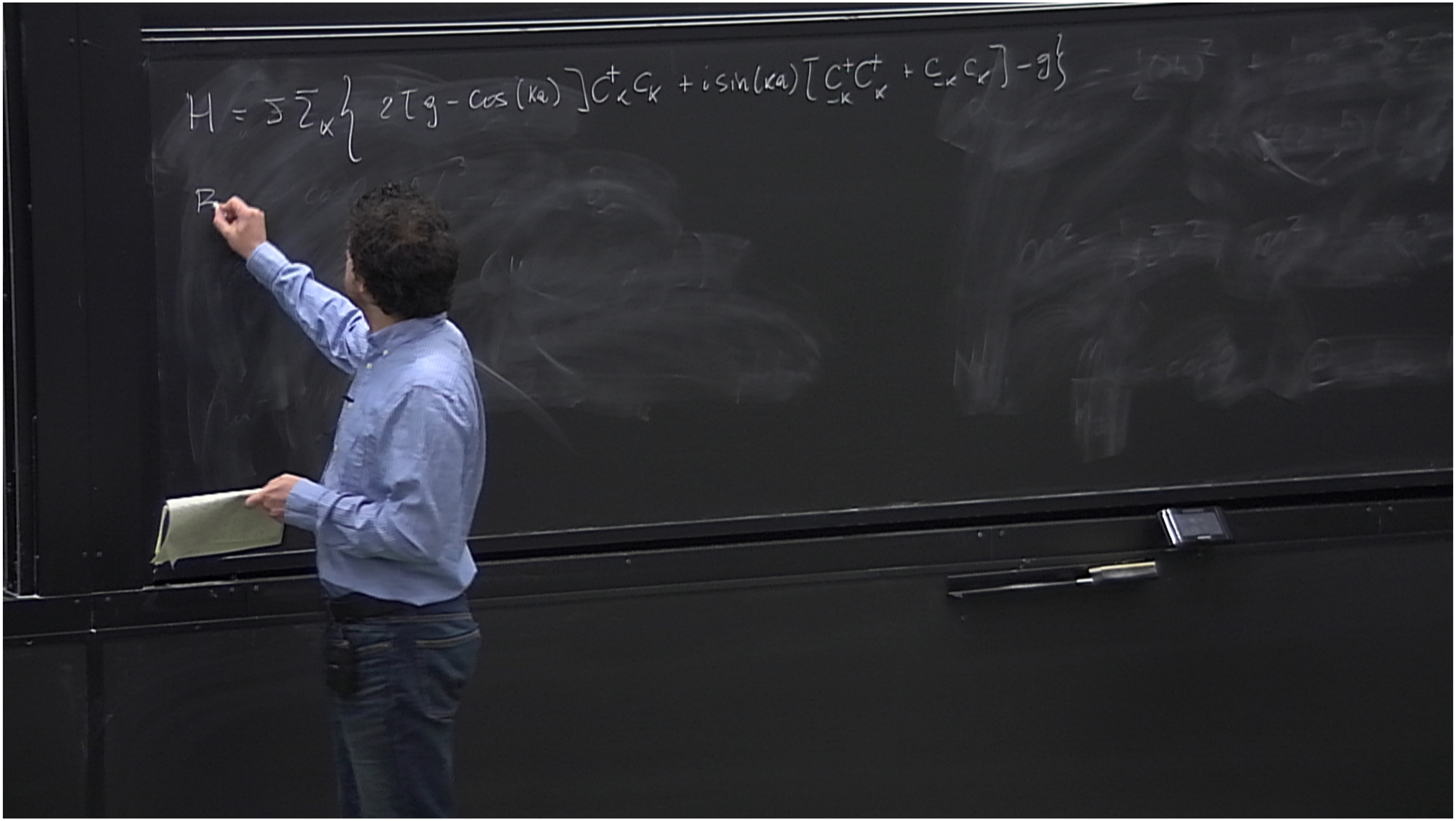
$$\sum_i c_i^\dagger c_{i+1}^\dagger = \frac{1}{N} \sum_l \sum_{k,k'} \left(e^{ik(la)} e^{ik'((l+1)a)} c_k^\dagger c_{k'}^\dagger \right.$$

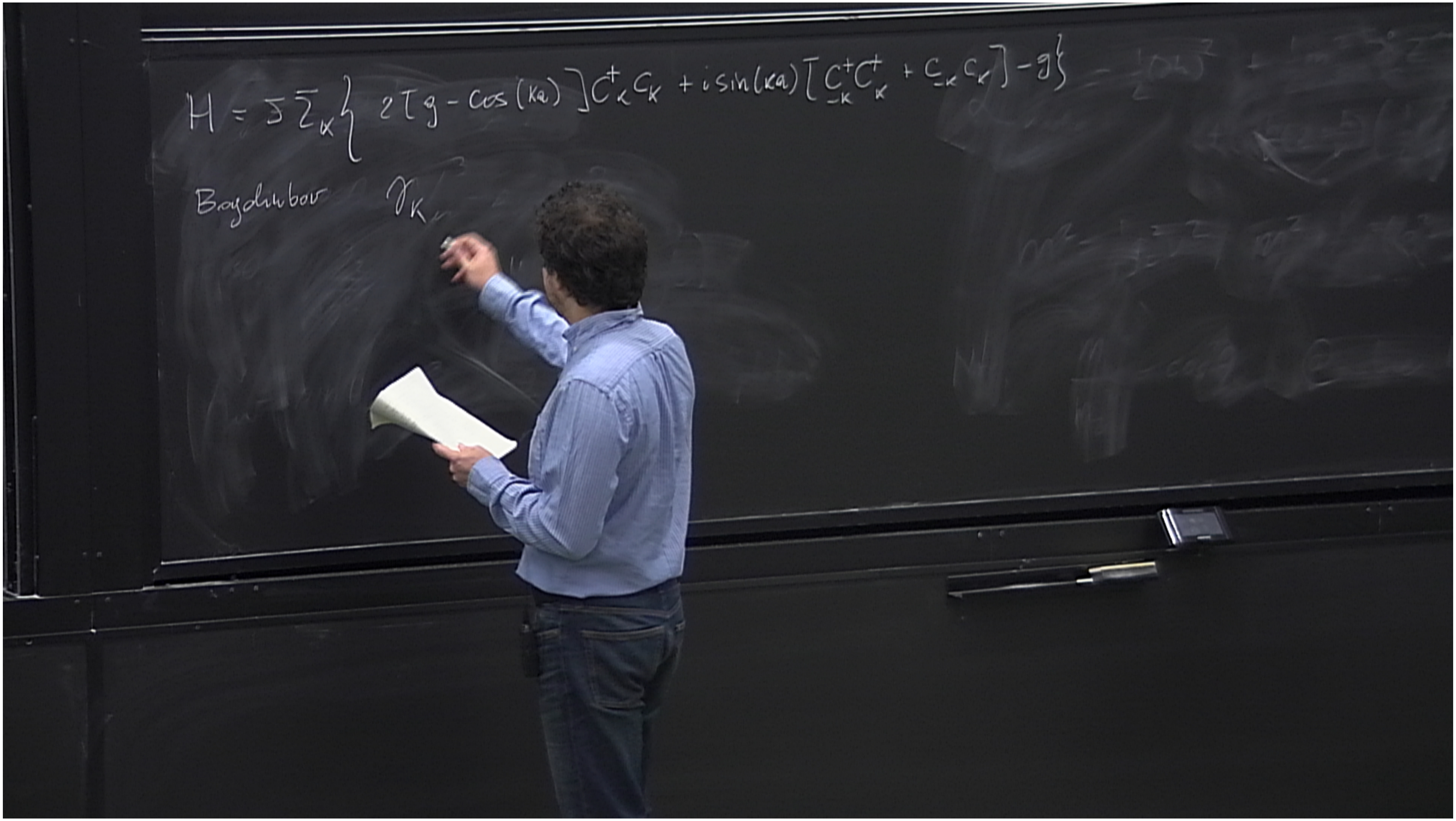
$\delta(k,k')$

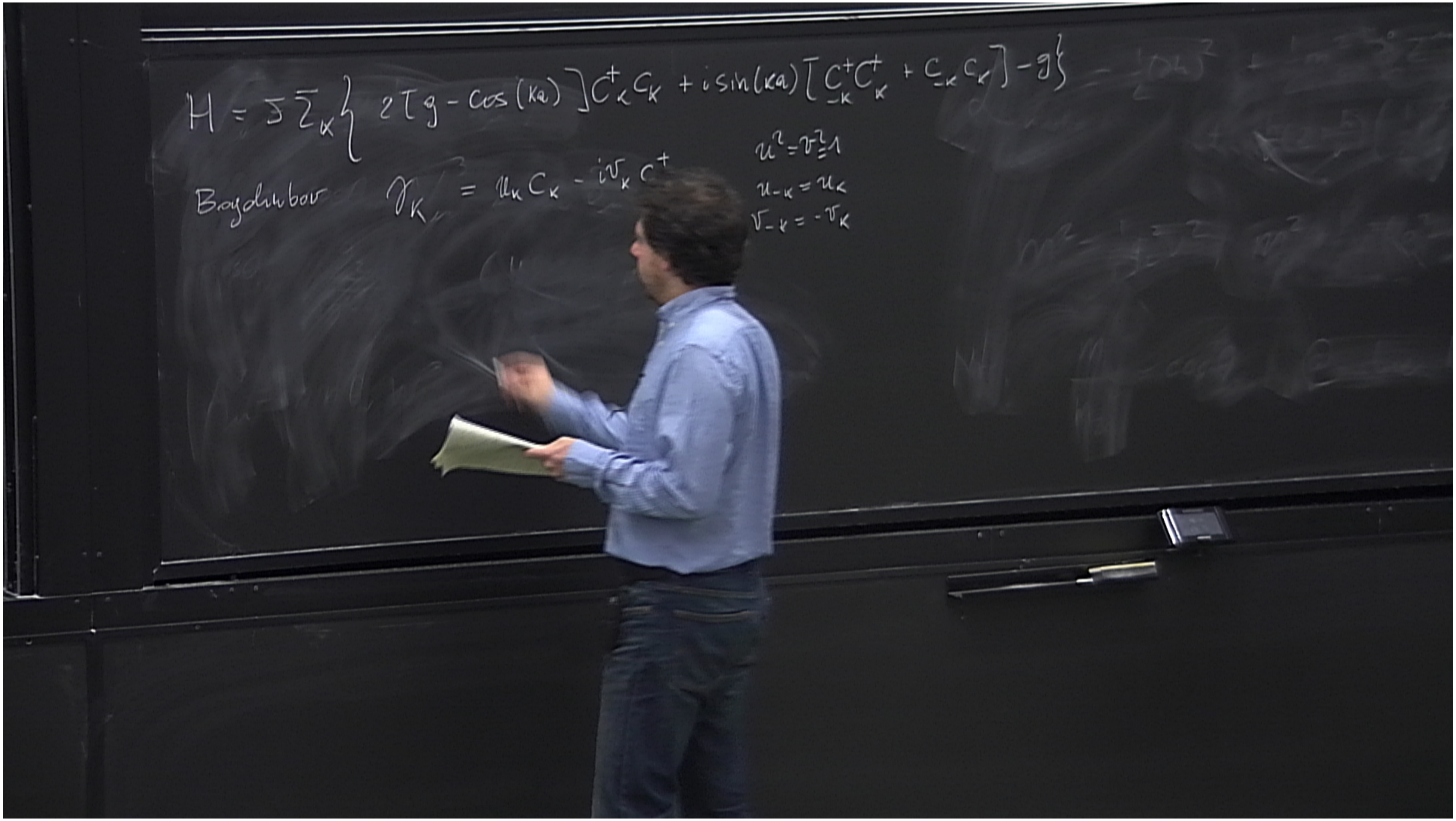
$$= \frac{1}{N} \sum_k e^{-ika} c_k^\dagger c_{-k}^\dagger$$

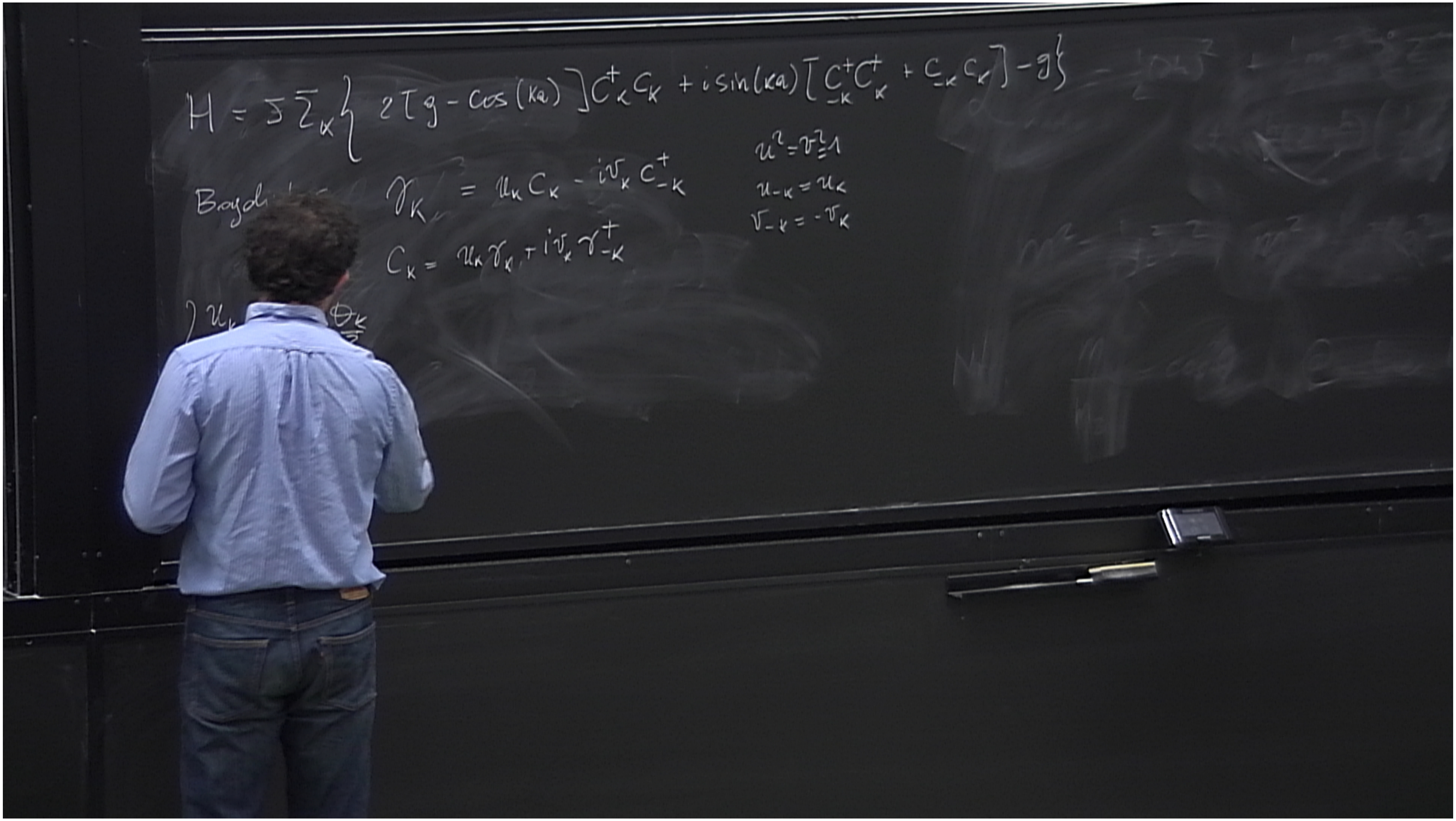

H = J Σ_i [g − 2g c_i^† c_i + c_{i+1} c_i + c_{i+1}^† c_i + c_{i+1} c_i^† + c_{i+1}^† c_i^†]
lattice spacing
r_j = j a
Fourier Transform
→ k
c_k = 1/√N Σ_j c_j e^{−ik r_j}
e^{−ik (ja)}
c_j = 1/√N Σ_k
Σ_i c_i^† … = 1/N Σ_l Σ_{k,k'} (e^{ik (la)} e^{ik' (l+1)a} c_k^† c_{k'}^†
δ(k,−k')
ika
c_k^† c_{−k}^†

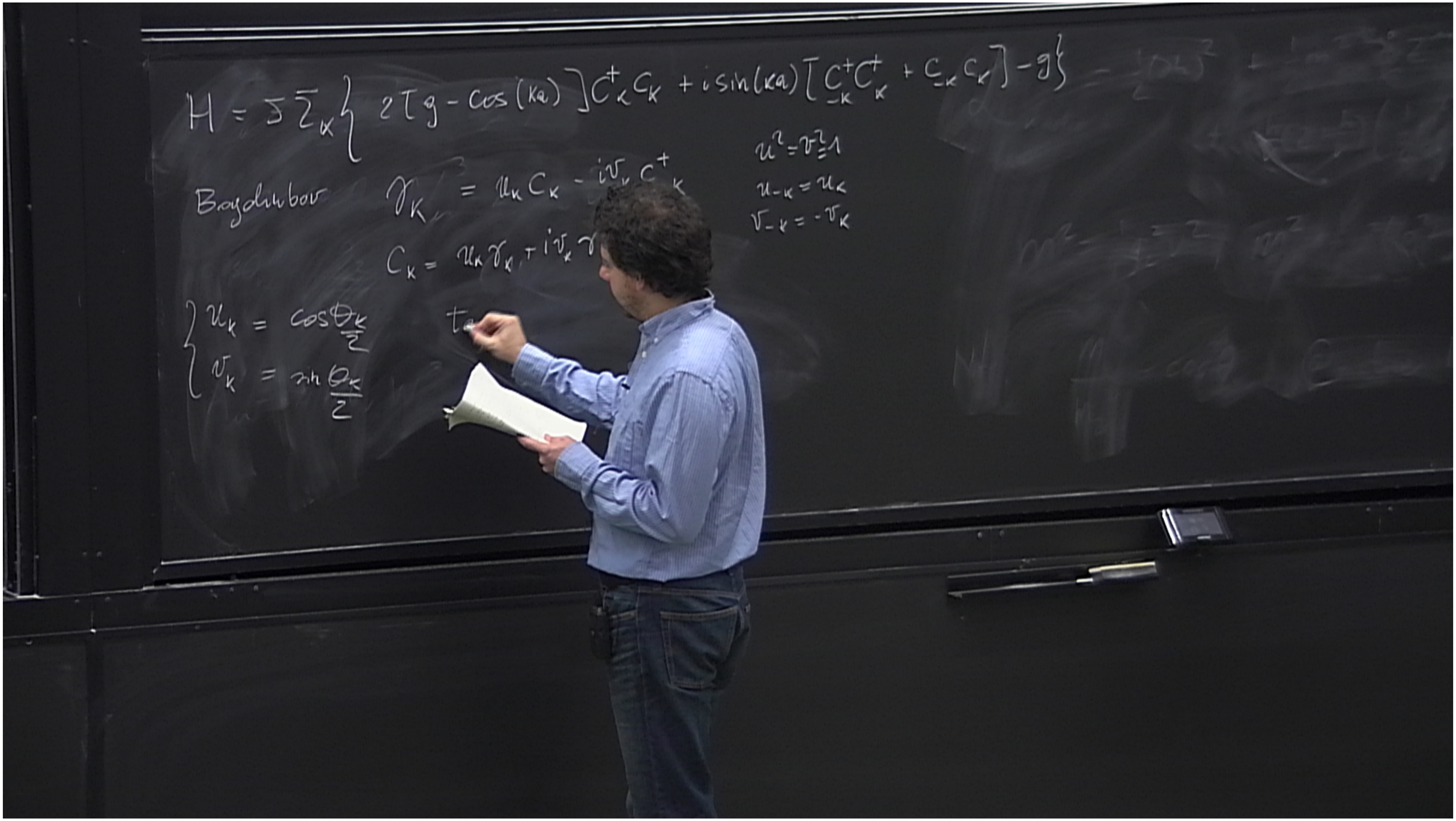

$H = J \sum_k \{ 2[g - \cos(ka)] c_k^{+} c_k + i\sin(ka) [c_{-k}^{+} c_k^{+} + c_{-k} c_k] - g \}$


$H = J \sum_k \{ 2[g - \cos(ka)] C^\dagger_k C_k + i\sin(ka) [C^\dagger_{-k} C^\dagger_k + C_{-k} C_k] - g \}$

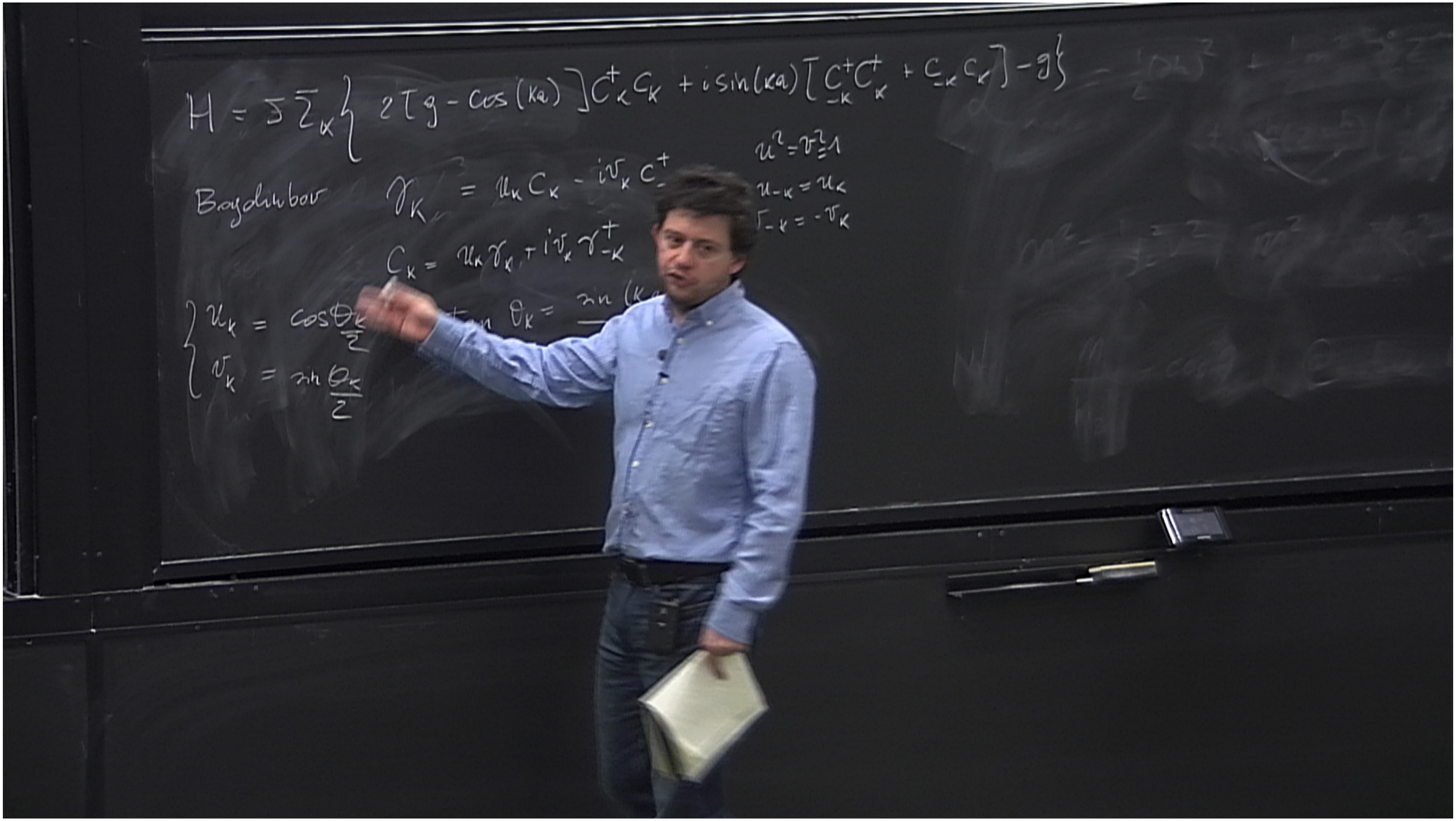

H = J Σ_k { 2[g - cos(ka)] C^+_k C_k + i sin(ka) [C^+_{-k} C^+_k + C_{-k} C_k] - g }
Bogoliubov γ_k


H = J Σ_k { 2[g - cos(ka)] C⁺_k C_k + i sin(ka) [C⁺_{-k} C⁺_k + C_{-k} C_k] - g }
Bogolubov γ_k = u_k C_k - i v_k C⁺
u² + v² = 1
u_{-k} = u_k
v_{-k} = -v_k

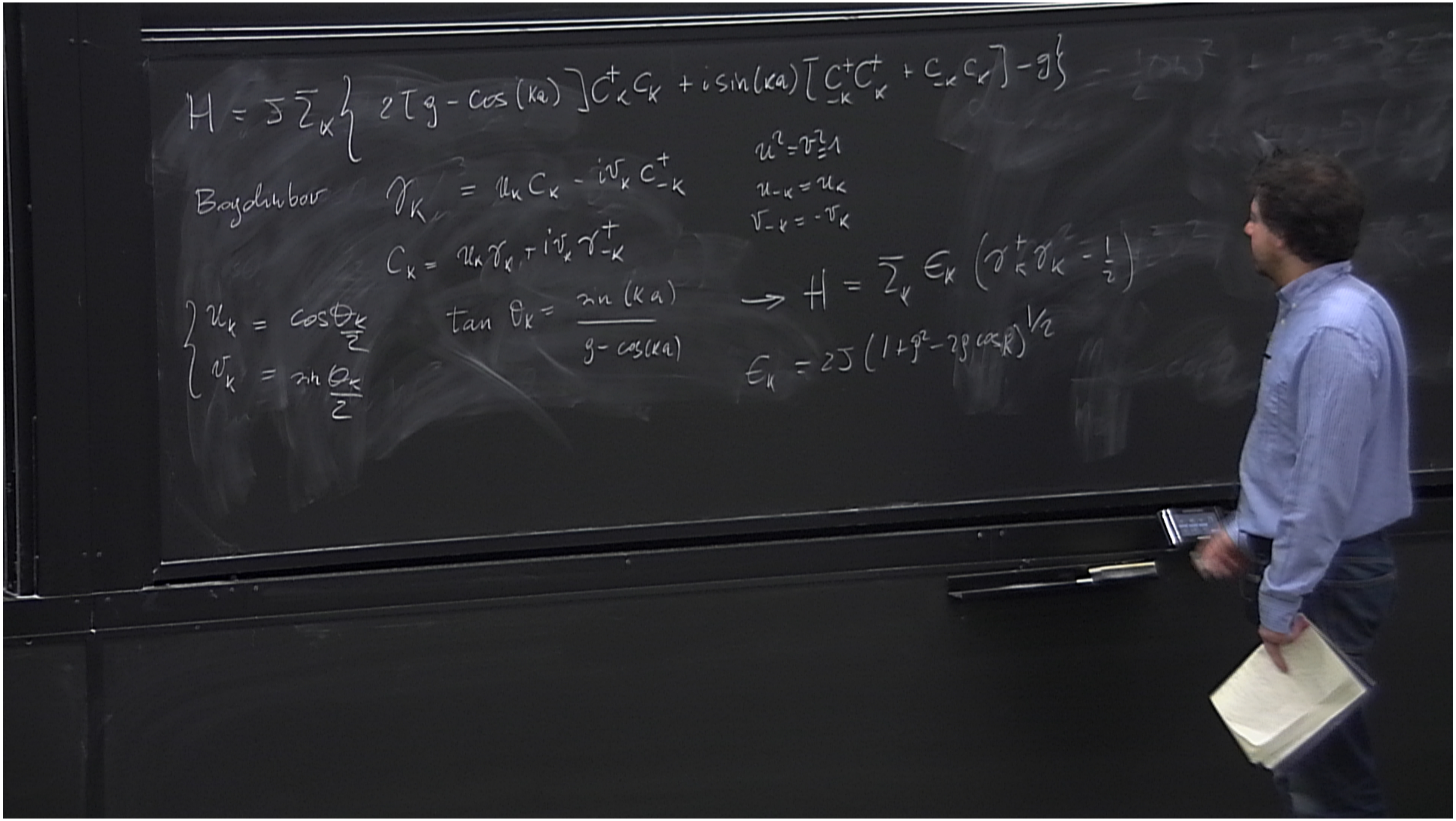

$H = J \sum_k \{ 2[g - \cos(ka)] C^+_k C_k + i \sin(ka) [C^+_{-k} C^+_k + C_{-k} C_k] - g \}$
$u^2 = v^2 = 1$
$\gamma_k = u_k C_k - i v_k C^+_{-k}$
$u_{-k} = u_k$
$v_{-k} = -v_k$
$C_k = u_k \gamma_k + i v_k \gamma^+_{-k}$



$H = J \sum_k \{ 2[g - \cos(ka)] C^+_k C_k + i \sin(ka) [C^+_{-k} C^+_k + C_{-k} C_k] - g \}$
$u^2 + v^2 = 1$
$u_{-k} = u_k$
$v_{-k} = -v_k$
Bogoliubov
$\gamma_k = u_k C_k - i v_k C^+_{-k}$
$C_k = u_k \gamma_k + i v_k \gamma$
$u_k = \cos \frac{\theta_k}{2}$
$v_k = \sin \frac{\theta_k}{2}$



$H = J \sum_k \{ 2[g - \cos(ka)] C^\dagger_k C_k + i \sin(ka) [C^\dagger_{-k} C^\dagger_k + C_{-k} C_k] - g \}$
Bogoliubov
$\gamma_k = u_k C_k - i v_k C^\dagger_{-k}$
$C_k = u_k \gamma_k + i v_k \gamma^\dagger_{-k}$
$u^2 + v^2 = 1$
$u_{-k} = u_k$
$v_{-k} = -v_k$
$u_k = \cos\frac{\theta_k}{2}$
$v_k = \sin\frac{\theta_k}{2}$
$\tan\theta_k =$


$H = J\sum_k \{ 2[g - \cos(ka)] C_k^+ C_k + i\sin(ka)[C_{-k}^+ C_k^+ + C_{-k} C_k] - g \}$
Bogoliubov
$\gamma_k = u_k C_k - i v_k C_{-k}^+$
$C_k = u_k \gamma_k + i v_k \gamma_{-k}^+$
$u^2 + v^2 = 1$
$u_{-k} = u_k$
$v_{-k} = -v_k$
$u_k = \cos\frac{\theta_k}{2}$
$v_k = \sin\frac{\theta_k}{2}$
$\tan\theta_k = \frac{\sin(ka)}{g - \cos(ka)}$
$\rightarrow H = \sum_k \epsilon_k (\gamma_k^+ \gamma_k - \frac{1}{2})$
$\epsilon_k = 2J(1 + g^2 - 2g\cos k)^{1/2}$

$$H = J \sum_k \left\{ 2[g - \cos(ka)] C^+_k C_k + i \sin(ka) [C^+_{-k} C^+_k + C_{-k} C_k] - g \right\}$$

Bogoliubov

$$\gamma_k = u_k C_k - i v_k C^+_{-k}$$

$$C_k = u_k \gamma_k + i v_k \gamma^+_{-k}$$

$$u^2 + v^2 = 1$$

$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

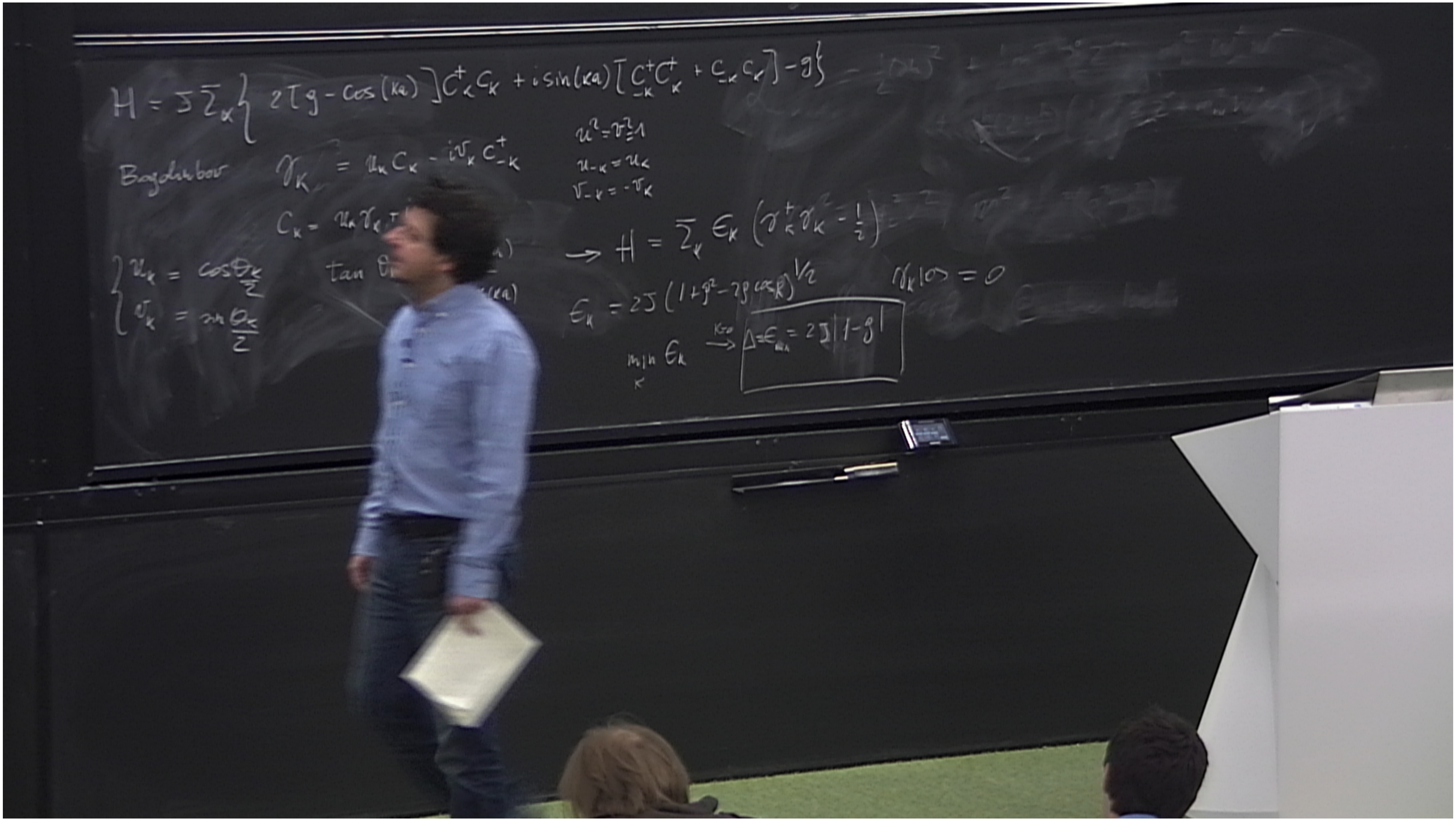
$$\left\{ \begin{array}{l} u_k = \cos \frac{\theta_k}{2} \\ v_k = \sin \frac{\theta_k}{2} \end{array} \right.$$

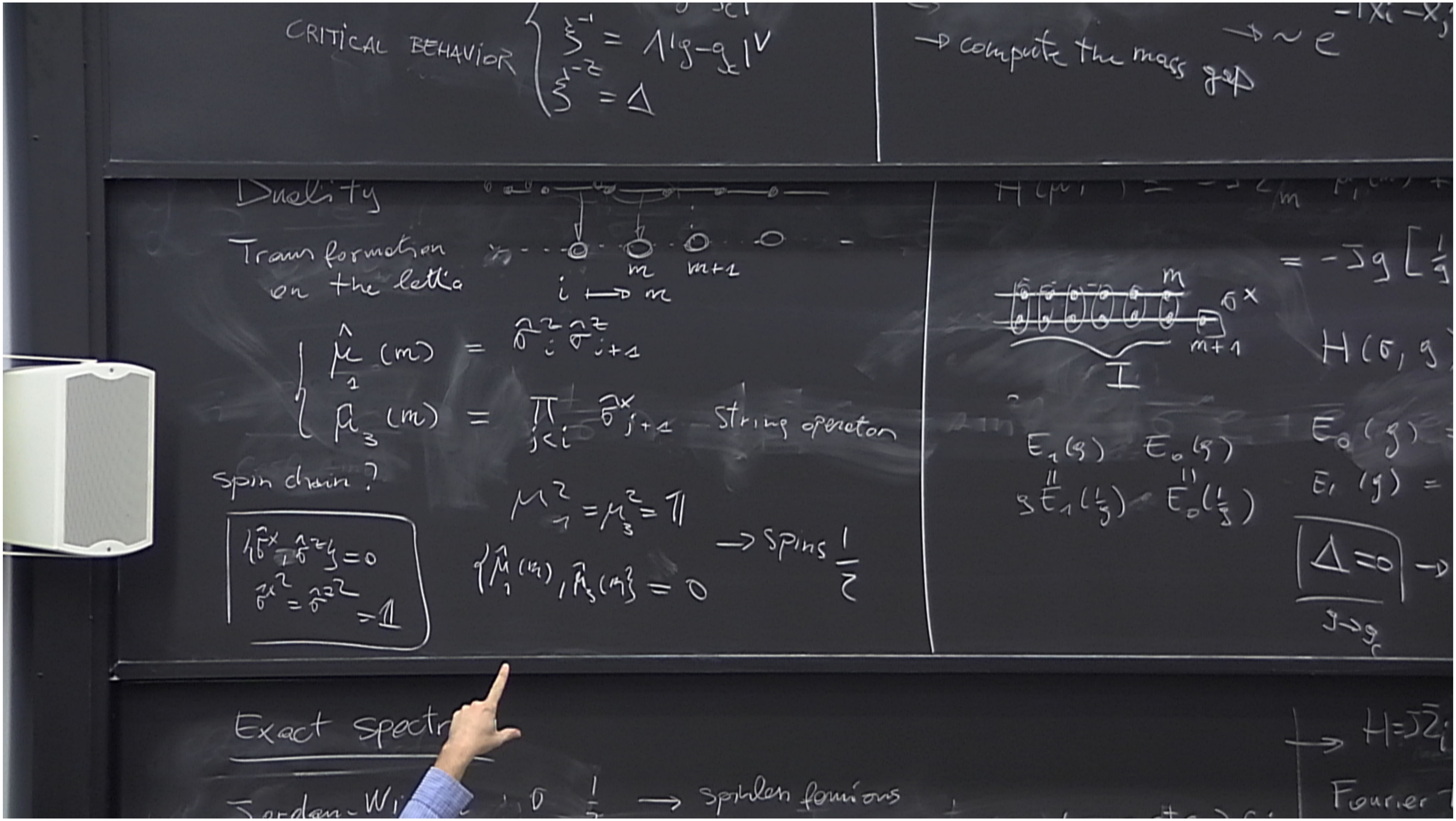
$$\tan \theta_k = \frac{\sin(ka)}{g - \cos(ka)}$$

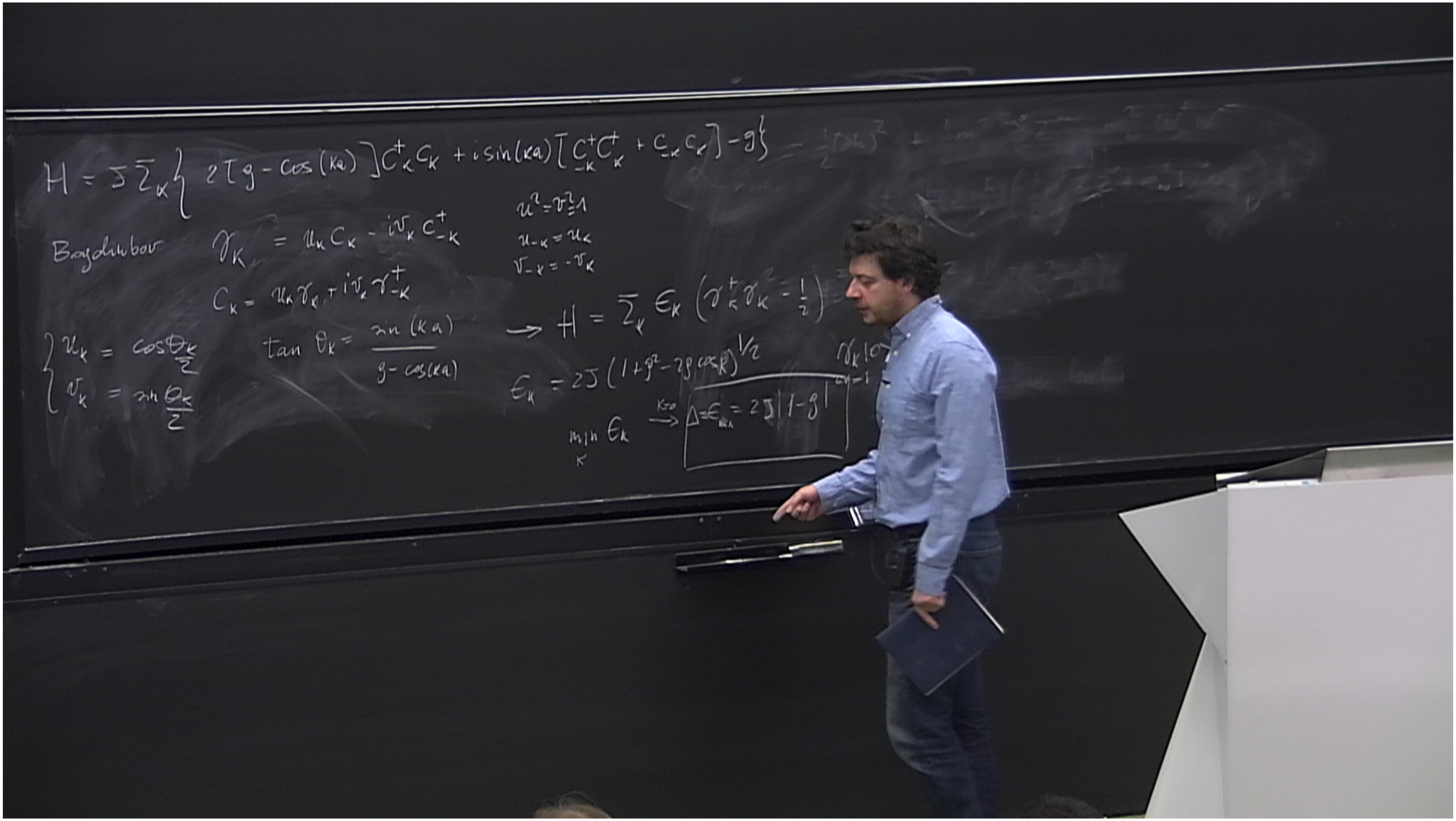
$$\rightarrow H = \sum_k \epsilon_k \left(\gamma^+_k \gamma_k - \frac{1}{2} \right)$$

$$\epsilon_k = 2J (1 + g^2 - 2g \cos k)^{1/2}$$

$$\gamma_k |0\rangle = 0$$


H = J Σ_k { 2[g - cos(ka)] C+_k C_k + i sin(ka) [C+_-k C+_k + C_-k C_k] - g }
→ H = Σ_k ε_k (γ+_k γ_k - 1/2)
γ_k |0> = 0


CRITICAL BEHAVIOR
compute the mass gap
Duality
Transformation on the lattice
String operator
spin chain?
spins 1/2
Exact spectr
spinless fermions
Fourier


H = J Σ_k { 2[g - cos(ka)] C⁺_k C_k + i sin(ka) [C⁺_{-k} C⁺_k + C_{-k} C_k] - g }
Bogoliubov
γ_k = u_k C_k - i v_k C⁺_{-k}
C_k = u_k γ_k + i v_k γ⁺_{-k}
u² + v² = 1
u_{-k} = u_k
v_{-k} = -v_k
u_k = cos θ_k/2
v_k = sin θ_k/2
tan θ_k = sin(ka) / (g - cos(ka))
→ H = Σ_k ε_k (γ⁺_k γ_k - 1/2)
ε_k = 2J (1 + g² - 2g cos k)^{1/2}
min_k ε_k → Δ = ε_min = 2J |1 - g|