

Title: Condensed Matter (Review) - Lecture 4

Date: Jan 05, 2012 10:15 AM

URL: <http://pirsa.org/12010086>

Abstract:

# Quantum Phase Transitions

$$H(g) \quad g \rightarrow g_c$$

level cross  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$  disco.

(III) With normal  
higgs field  
 $H_1$  fermions get  
mass from the same  
mechanism. These masses  
vary smoothly,  
but respect  $U(1)_B$ .

# Quantum Phase Transitions

$$H(g) \quad g \rightarrow g_c$$

$$\text{level } g \rightarrow \frac{\partial E_0}{\partial g} \text{ discontinuous}$$

$$\text{w.c. } \rightarrow \frac{\partial^2 E_0}{\partial g^2} \text{ disco.}$$

$\Delta$

(III) With normal  
higgs field  
 $H_1$  forms get  
mass from the joint  
masses. These masses  
vanish smoothly,  
but respect  $U(1)_S$ .

# Quantum Phase Transitions

$$H(g) \quad g \rightarrow g_c$$

$$\text{1st order} \rightarrow \frac{\partial E_0}{\partial g} \text{ discontinuous}$$

$$\text{2nd order} \rightarrow \frac{\partial^2 E_0}{\partial g^2} \text{ diso.}$$

$$\Delta \begin{matrix} \xrightarrow{g \rightarrow g_c} \\ \xrightarrow{g \rightarrow 0} \end{matrix} \left\{ \begin{array}{l} \Delta = \gamma |g - g_c|^{2\nu} \\ \xi^{-1} = \lambda |g - g_c|^\nu \\ \xi^{-1-2\nu} \\ \xi = \Delta \end{array} \right.$$

CRITICAL BEHAVIOR

(III) With normal  
 Higgs field  
 Higgs bosons get  
 mass from the same  
 mechanism. Their mass  
 is like  $\sqrt{2}v\phi$ ,  
 but respect  $U(1)_Y$ .

# Quantum Phase Transitions

$$T=0$$

$$H(g) \quad g \rightarrow g_c$$

level crossing  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided l.c.  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$

$$\Delta \xrightarrow{g \rightarrow g_c}$$

$$\propto |g - g_c|^{2\nu}$$

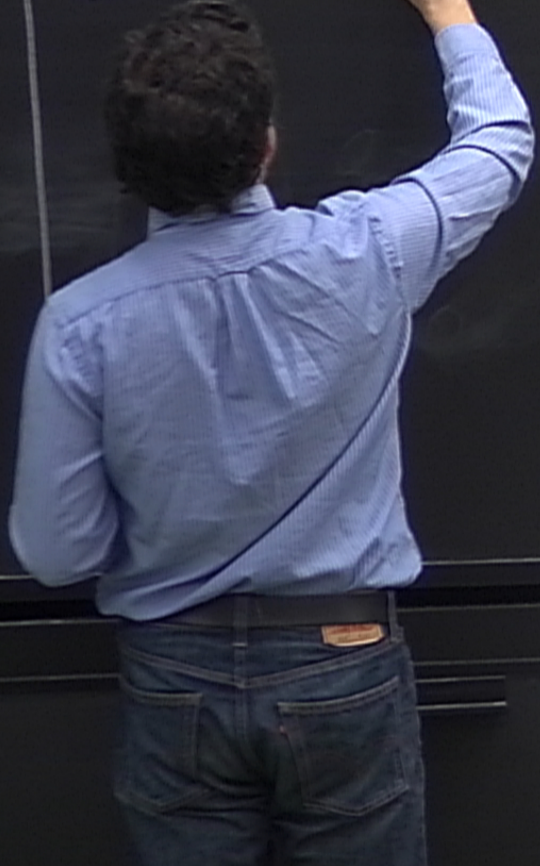
$$\propto |g - g_c|^\nu$$

CRITICAL EXPONENT

(III) With normal  
magnetic field  
 $H$ , fermions get  
mass from the joint  
masses. These masses  
vanish smoothly,  
but respect  $U(1)$ .

$$H = -J \sum_i (\sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in  
 $d=1$

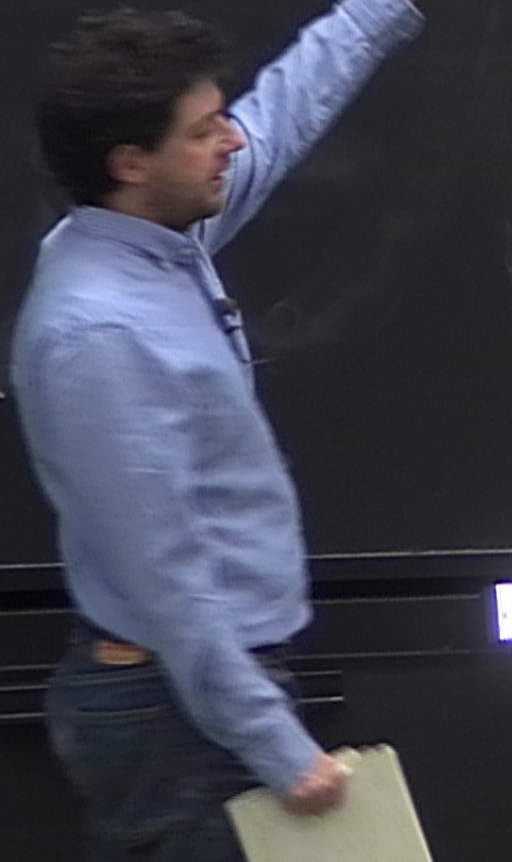


$$H = -J \sum_i (\sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$H = -J \sum_i (\sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$





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Ising chain in transverse field  
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Ising chain in transverse field  
 $d=1$



$$H = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse  
 $d=1$  field

$$g \gg 1 \quad |0\rangle =$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$|0\rangle =$$

$$| \rightarrow \rangle_i$$

$$| \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg J$$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

$$|1\rangle_i \quad \pi_i$$

$$| \rightarrow \rangle_i$$



$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1 \quad |0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i) \quad \boxed{\pi_i}$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg J$$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i) \quad \boxed{\pi_i}$$

$$H = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i)$$

$$1/2$$

$$= \bigotimes_i | \uparrow \rangle_i$$

product state



$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i)$$

$\frac{1}{\sqrt{2}}$

$$H = -J \sum_i (g \sigma_i^x + \frac{\gamma}{2} \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
d=1

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

$$= \frac{| \uparrow \rangle_i + | \downarrow \rangle_i}{\sqrt{2}} \quad (\pi_i)$$

product state

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \rightarrow \rangle_i \pm | \leftarrow \rangle_i \quad \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} (\sigma_i^z \pm 1) |0\rangle$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$| \rightarrow \rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \leftarrow \rangle_i$$

$$| \downarrow \rangle_i \quad \boxed{\pi_i}$$

$$\frac{1}{\sqrt{2}}$$

$$| \psi(g \rightarrow \infty) \rangle = 0$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i)$$

$$| \otimes_i | \rightarrow \rangle_i$$

product state

$$| \pi_i \rangle$$

$$\frac{1}{\sqrt{2}}$$

color

$$| \epsilon \rightarrow 0 \rangle = 0$$

$$\rightarrow \sim e^{-|x_i - x_j| / \xi}$$

$$H = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$|0\rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i) \quad \boxed{\pi_i}$$

$$\frac{1}{\sqrt{2}} \cos \theta \sigma_i^z \sigma_j^z |0(g \rightarrow \infty)\rangle = 0$$

$$\rightarrow \sim e^{-|x_i - x_j|} \quad \boxed{\frac{1}{\sqrt{2}}}$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$\rightarrow 1 \quad |0\rangle = \bigotimes_i | \rightarrow \rangle_i \quad \text{product state}$$

$$\rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}} \quad \boxed{\pi_i}$$

$$\langle \psi | \cos \theta \sigma_i^z \sigma_j^z | \psi \rangle = 0$$

$$\rightarrow \sim e^{-|x_i - x_j|} \quad \boxed{\frac{1}{|x_i - x_j|}}$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=1$

$$g \gg 1$$

$$| \rightarrow \rangle_i = \frac{1}{\sqrt{2}} (| \uparrow \rangle_i + | \downarrow \rangle_i)$$

$$| 0 \rangle = \bigotimes_i | \rightarrow \rangle_i$$

product state

$$| \pi_i \rangle$$

$$| 0 (g \rightarrow \infty) \rangle = 0$$

$$\rightarrow \sim e^{-|x_i - x_j|} \frac{1}{\sqrt{v}}$$



# Quantum Phase Transitions

$$\boxed{T=0}$$

$$H(g) \quad g \rightarrow g_c$$

level crossing  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided l.c.  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$  diso.

$$\Delta \xrightarrow{g \rightarrow g_c} 0 \quad \left\{ \begin{array}{l} \Delta = J |g - g_c|^{2\nu} \\ \sum_{i=1}^{-1} = |g - g_c|^\nu \\ \sum_{i=1}^{-2} = \Delta \end{array} \right.$$

CRITICAL BEHAVIOR

$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$

Ising chain in transverse field  
d=1

product state

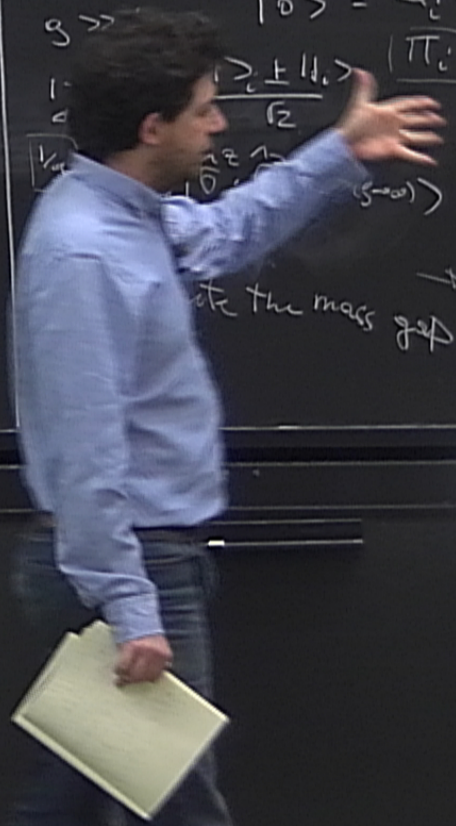
$$|0\rangle = \bigotimes_i |1\rangle_i$$

$$|g_c \pm 1\rangle = \frac{|\pi_i\rangle}{\sqrt{2}}$$

$$E_{\pm}(\omega) = 0$$

$$\rightarrow \sim e^{-|x_i - x_j|} \left( \frac{1}{S} \right)$$

the mass gap



# Quantum Phase Transitions

$$\boxed{T=0}$$

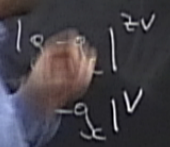
$$H(g) \quad g \rightarrow g_c$$

level crossing  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided l.c.  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$

$$\frac{g \rightarrow g_c}{\Delta \rightarrow 0}$$

CRITICAL BEHAVIOR



$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field

$$g \gg 1$$

$$|0\rangle = \bigotimes_i |1\rangle_i$$

product state

$$|1\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}} \quad (\pi_i)$$

$$\langle \sigma_i^z \sigma_{i+1}^z \rangle = 0$$

$\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

# Quantum Phase Transitions

$$\boxed{T=0}$$

$$H(g) \quad g \rightarrow g_c$$

level crossing  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided l.c.  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$  diso.

$$\frac{g \rightarrow g_c}{\Delta \rightarrow 0} \quad \Delta = \dots$$

CRITICAL BEHAVIOR

$$\left\{ \begin{array}{l} \sum_{i=1}^{\dots} \\ \sum_{i=2}^{\dots} \end{array} \right.$$

$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$

Ising chain in transverse field

$$g \gg 1 \quad |0\rangle = \bigotimes_i |1\rangle_i \quad \text{product state}$$

$$|1\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}} \quad (\pi_i)$$

$$\langle \psi | \cos \theta \hat{\sigma}_i^z \hat{\sigma}_j^z | \psi \rangle = 0$$

$\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

# Quantum Phase Transitions

$$T=0$$

$$H(g) \quad g \rightarrow g_c$$

level crossing  $\rightarrow \frac{\partial E_0}{\partial g}$  discontinuous

avoided l.c.  $\rightarrow \frac{\partial^2 E_0}{\partial g^2}$  diso.

$$\text{CRITICAL BEHAVIOR} \left\{ \begin{array}{l} \Delta \xrightarrow{g \rightarrow g_c} 0 \\ \Delta = J |g - g_c|^{2\nu} \\ \xi^{-1} = A |g - g_c|^\nu \\ \xi^{-1} = \Delta \end{array} \right.$$

$$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field

$$g \gg 1 \quad |0\rangle = \bigotimes_i |1\rangle_i$$

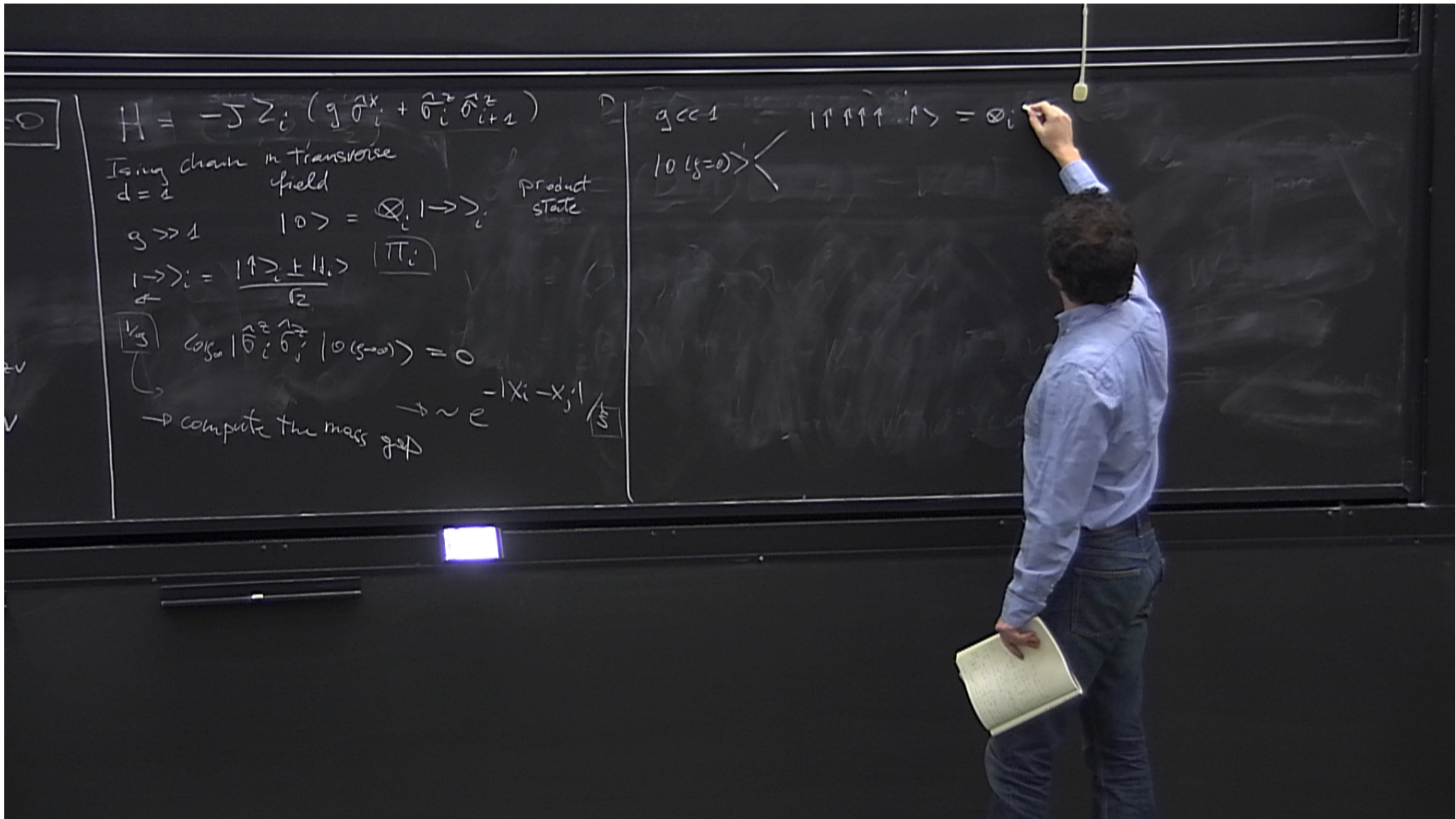
product state

$$|1\rangle_i = \frac{1}{\sqrt{2}} (|\uparrow\rangle_i + |\downarrow\rangle_i) \quad (\pi_i)$$

$$\langle \sigma_i^z \rangle = \langle \sigma_i^x \rangle = 0$$

$\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

$$g \ll 1$$



$$H = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=2$

$g \gg 1$        $|0\rangle = \bigotimes_i | \rightarrow \rangle_i$       product state

$| \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$        $(\pi_i)$

$\frac{1}{\sqrt{2}} \cos \theta \left( \frac{\sigma_i^z \sigma_j^z}{\sigma_i^z \sigma_j^z} \right) |0(g \rightarrow \infty)\rangle = 0$

→ compute the mass gap       $\sim e^{-|x_i - x_j| / \xi}$

$g \ll 1$        $|\uparrow\uparrow\uparrow\uparrow \dots \uparrow\rangle = \bigotimes_i | \rightarrow \rangle_i$

$|0(g=0)\rangle$

0

$$H = -J \sum_i (g \sigma_i^x + \sigma_i^z \sigma_{i+1}^z)$$

Ising chain in transverse field  
 $d=2$

$$g \gg 1$$

$$|0\rangle = \bigotimes_i |\rightarrow\rangle_i$$

product state

$$|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}} \quad (\pi_i)$$

$$\left[ \frac{1}{g} \right] \cos_{\theta} \left[ \frac{\sigma_i^z \sigma_j^z}{g} \right] |0(g \rightarrow \infty)\rangle = 0$$

→ compute the mass gap →  $\sim e^{-|x_i - x_j| / \xi}$

$$g \ll 1$$

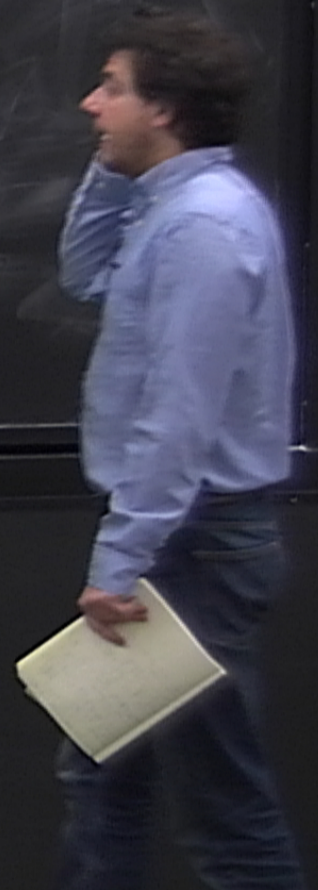
$$|\uparrow\uparrow\uparrow\uparrow\rangle = \bigotimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$$

$$|0(g=0)\rangle$$

$$|\downarrow\downarrow\downarrow\downarrow\rangle = \bigotimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$$

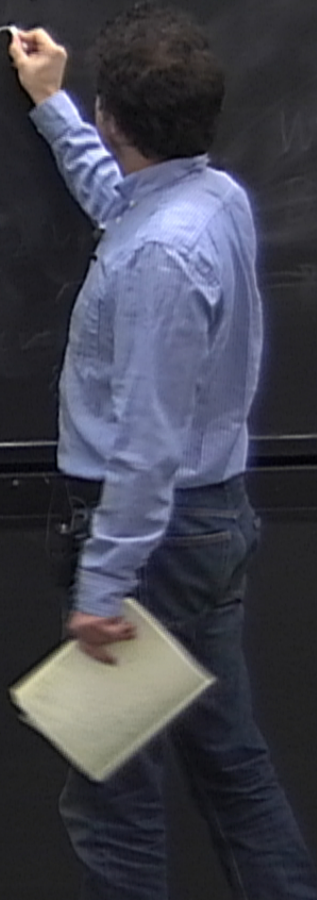
$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \rightarrow \rangle_i$   
 $| \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   $(\pi_i)$   
 $\cos \theta \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j| / \xi}$

$g \ll 1$   
 $|0(g=0)\rangle = \bigotimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|0(g \rightarrow \infty)\rangle = \bigotimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   
 $Z_2$  symmetry breaking



$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \rightarrow \rangle_i$   
 $| \rightarrow \rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   $(\pi_i)$   
 $\cos \theta \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow \sim e^{-|x_i - x_j|} / |x_i - x_j|$   
 $\rightarrow$  compute the mass gap

$g \ll 1$   
 $|0(g=0)\rangle = \bigotimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   $Z_2$  symmetry breaking  
 $|0(g \rightarrow \infty)\rangle = \bigotimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$





$$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$$
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \uparrow \rangle_i$   
 $| \rightarrow \rangle_i = \frac{| \uparrow \rangle_i + | \downarrow \rangle_i}{\sqrt{2}}$   $(\pi_i)$   
 $\cos \theta \hat{\sigma}_i^z \hat{\sigma}_i^x \hat{\sigma}_i^z |0\rangle = 0$   
 $\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

$g \ll 1$   
 $|0\rangle (g=0) = \bigotimes_i | \uparrow \rangle_i \equiv | \uparrow \rangle$   $Z_2$  symmetry  
 $| \downarrow \downarrow \dots \downarrow \rangle = \bigotimes_i | \downarrow \rangle_i \equiv | \downarrow \rangle$  breaking  
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T} | \uparrow \rangle = | \downarrow \rangle$



$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \uparrow \rangle_i$   
 $|\rightarrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   $(\pi_i)$   
 $\cos \theta \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

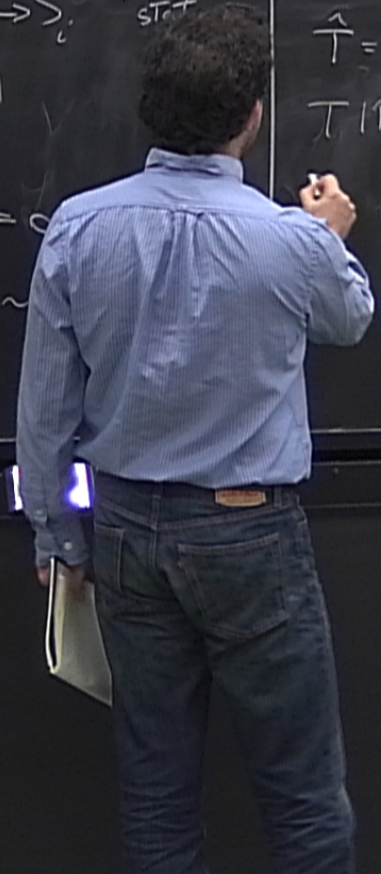
$g \ll 1$   $|\uparrow\uparrow\uparrow\uparrow\rangle = \bigotimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   $Z_2$  symmetry breaking  
 $|0(g=0)\rangle$   $|\downarrow\downarrow\downarrow\downarrow\rangle = \bigotimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$



$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \theta \left| \hat{\sigma}_i^z \hat{\sigma}_j^z \right| 0(g \rightarrow \infty) \rangle = 0$   
 → compute the mass gap

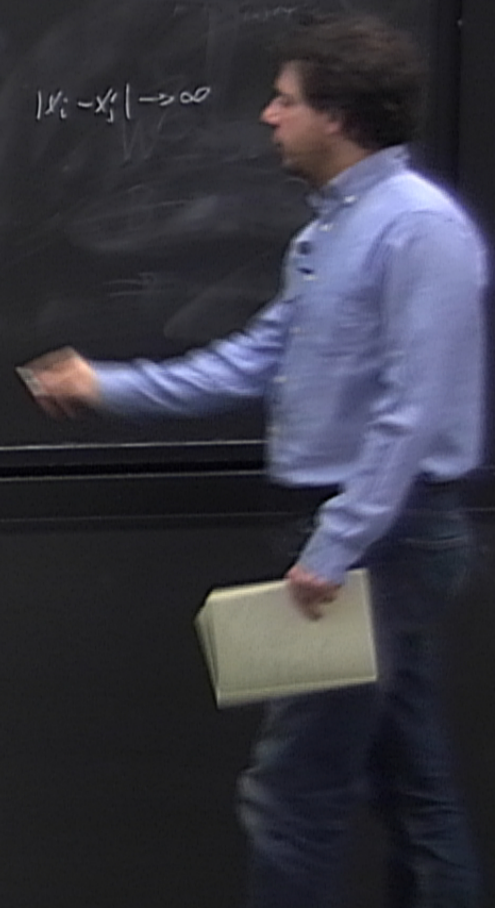
product state

$g \ll 1$   
 $|\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $[\hat{H}, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $\hat{T} |\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 $Z_2$  symmetry breaking



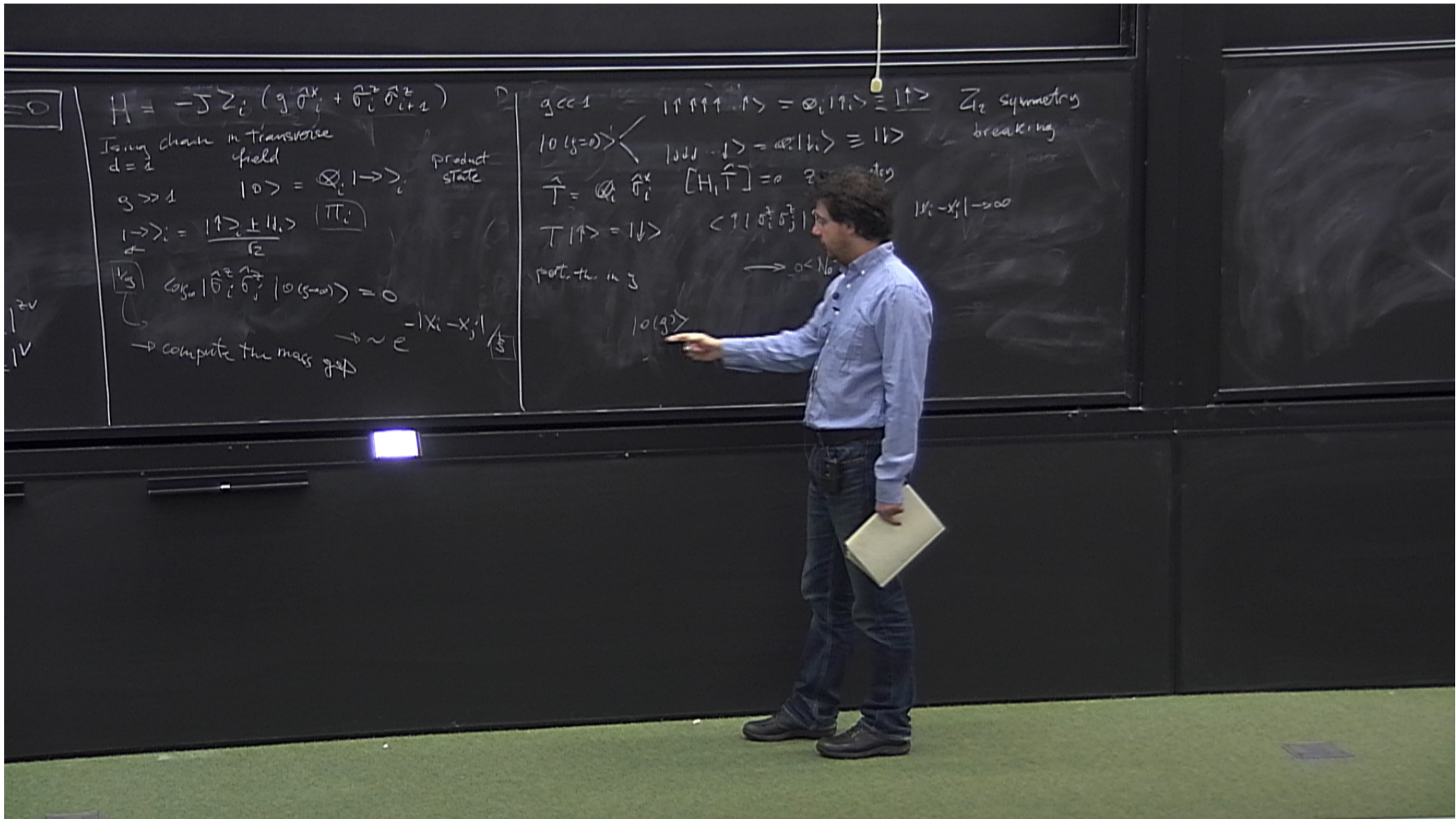
$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \uparrow \rangle_i$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   $(\pi_i)$   
 $\cos \theta \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow \sim e^{-|x_i - x_j| / \xi}$   
 $\rightarrow$  compute the mass gap

$g \ll 1$   $|\uparrow\uparrow\uparrow\uparrow\rangle = \bigotimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   $Z_2$  symmetry  
 $|0(g=0)\rangle$   $|\downarrow\downarrow\downarrow\downarrow\rangle = \bigotimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$  breaking  
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T} |\uparrow\rangle = |\downarrow\rangle$   $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 pert. th. in  $J$   $\rightarrow N_0^2$



$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \bigotimes_i | \uparrow \rangle_i$   
 $| \uparrow \rangle_i = \frac{| \uparrow \rangle_i + | \downarrow_i \rangle}{\sqrt{2}}$   $(\pi_i)$   
 $\cos_{\theta} \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j| / \xi}$

$g \ll 1$   $| \uparrow \uparrow \uparrow \uparrow \dots \uparrow \rangle = \bigotimes_i | \uparrow_i \rangle \equiv | \uparrow \rangle$   $Z_2$  symmetry  
 $|0(g=0)\rangle = \bigotimes_i | \downarrow_i \rangle \equiv | \downarrow \rangle$  breaking  
 $\hat{T} = \bigotimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T} | \uparrow \rangle = | \downarrow \rangle$   $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 pert. th. in  $J \rightarrow N_0^2 \leq 1$



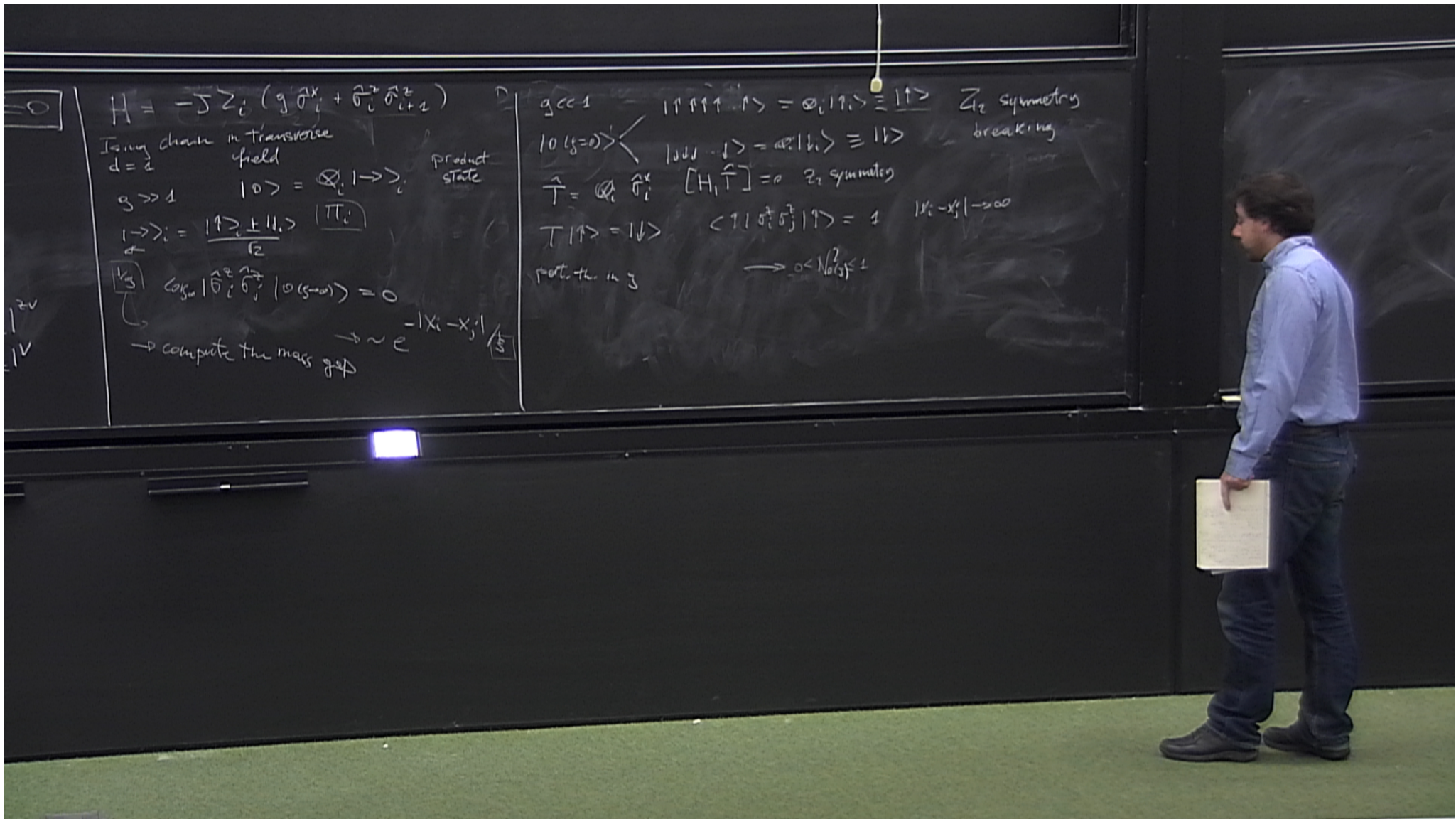
$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \theta_i \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow \sim e^{-|x_i - x_j|/\xi}$   
 $\rightarrow$  compute the mass gap

$g \ll 1$   
 $|\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $[\hat{H}, \hat{T}] = 0$   
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle$   
 $\rightarrow 0 < N \xi$   
 $Z_2$  symmetry breaking  
 $|x_i - x_j| \rightarrow \infty$   
 $|0(g)\rangle$

$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$  product state  
 $|0\rangle = \otimes_i |1\rangle_i$   
 $|1\rangle_i = \frac{|1\rangle_i + |1\rangle_i}{\sqrt{2}}$  ( $\pi_i$ )  
 $\cos \frac{\pi z}{2} \hat{\sigma}_i^z \hat{\sigma}_j^z |0(g \rightarrow \infty)\rangle = 0$   
 $\rightarrow \sim e^{-|x_i - x_j|/\xi}$   
 $\rightarrow$  compute the mass gap

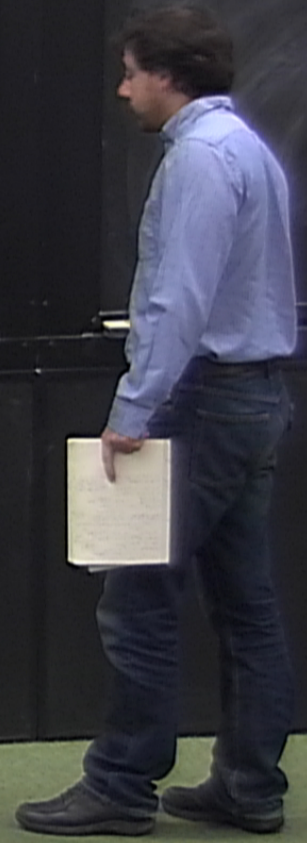
$g \ll 1$   $|1\rangle \langle 1\rangle \langle 1\rangle \langle 1\rangle = \otimes_i |1\rangle_i \equiv |1\rangle$   $Z_2$  symmetry breaking  
 $|0(g=0)\rangle = \otimes_i |1\rangle_i \equiv |1\rangle$   
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   $[H, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T}|1\rangle = |1\rangle$   $\langle 1 | \hat{\sigma}_i^z \hat{\sigma}_j^z | 1 \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 pot. in  $g$   $\rightarrow 0 < \xi < 1$   
 $|0(g)\rangle$



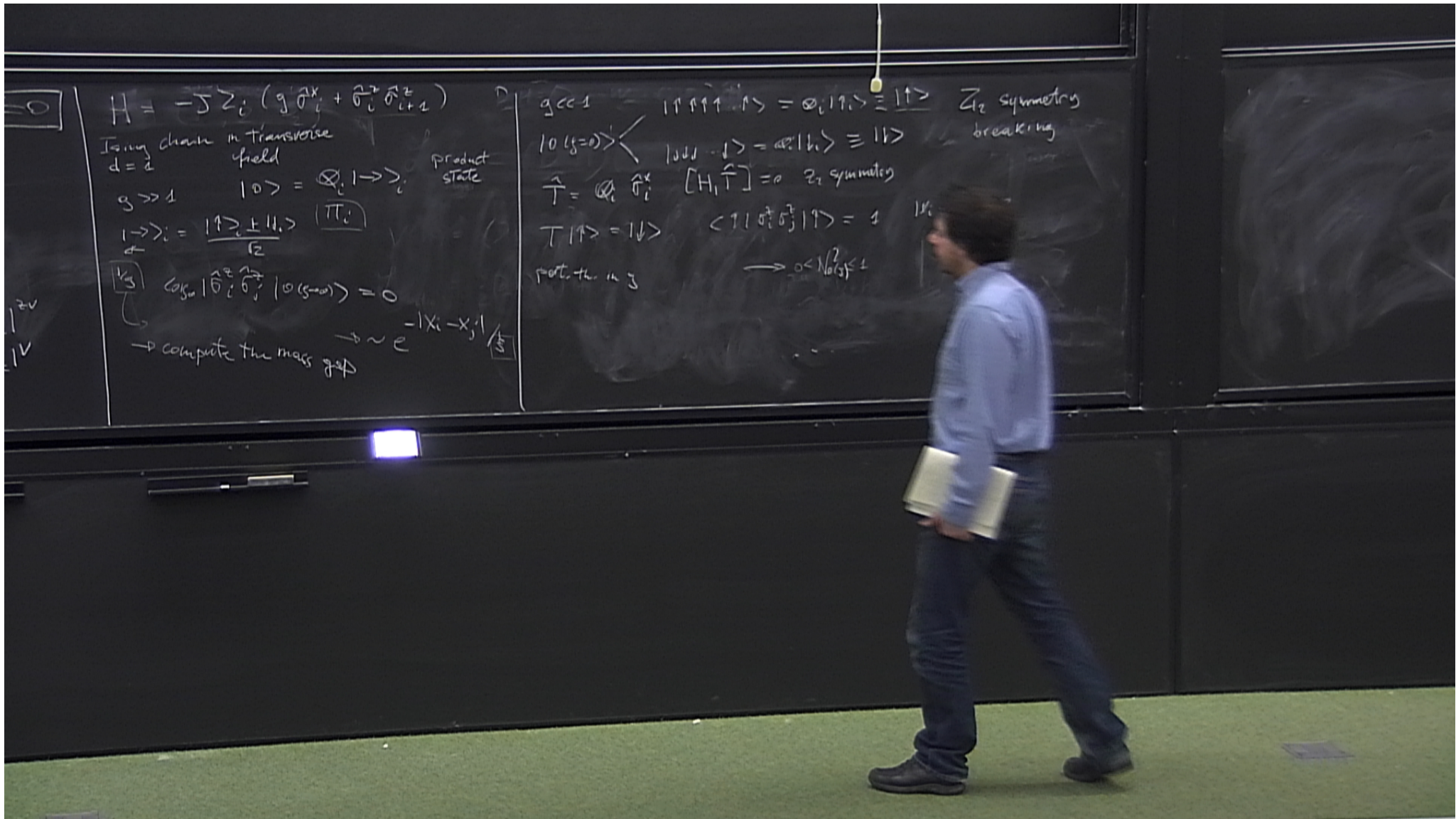


$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \theta \frac{\hat{\sigma}_i^z \hat{\sigma}_j^z}{\sqrt{2}} |\uparrow\rangle_i |\downarrow\rangle_j = 0$   
 $\rightarrow$  compute the mass gap  
 $\sim e^{-|x_i - x_j|/\xi}$

$g \ll 1$   
 $|\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $Z_2$  symmetry breaking  
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $[\hat{H}, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 $\rightarrow 0 < \frac{1}{N} \sum \hat{\sigma}_i^z \hat{\sigma}_j^z$   
 pot. th. in 3

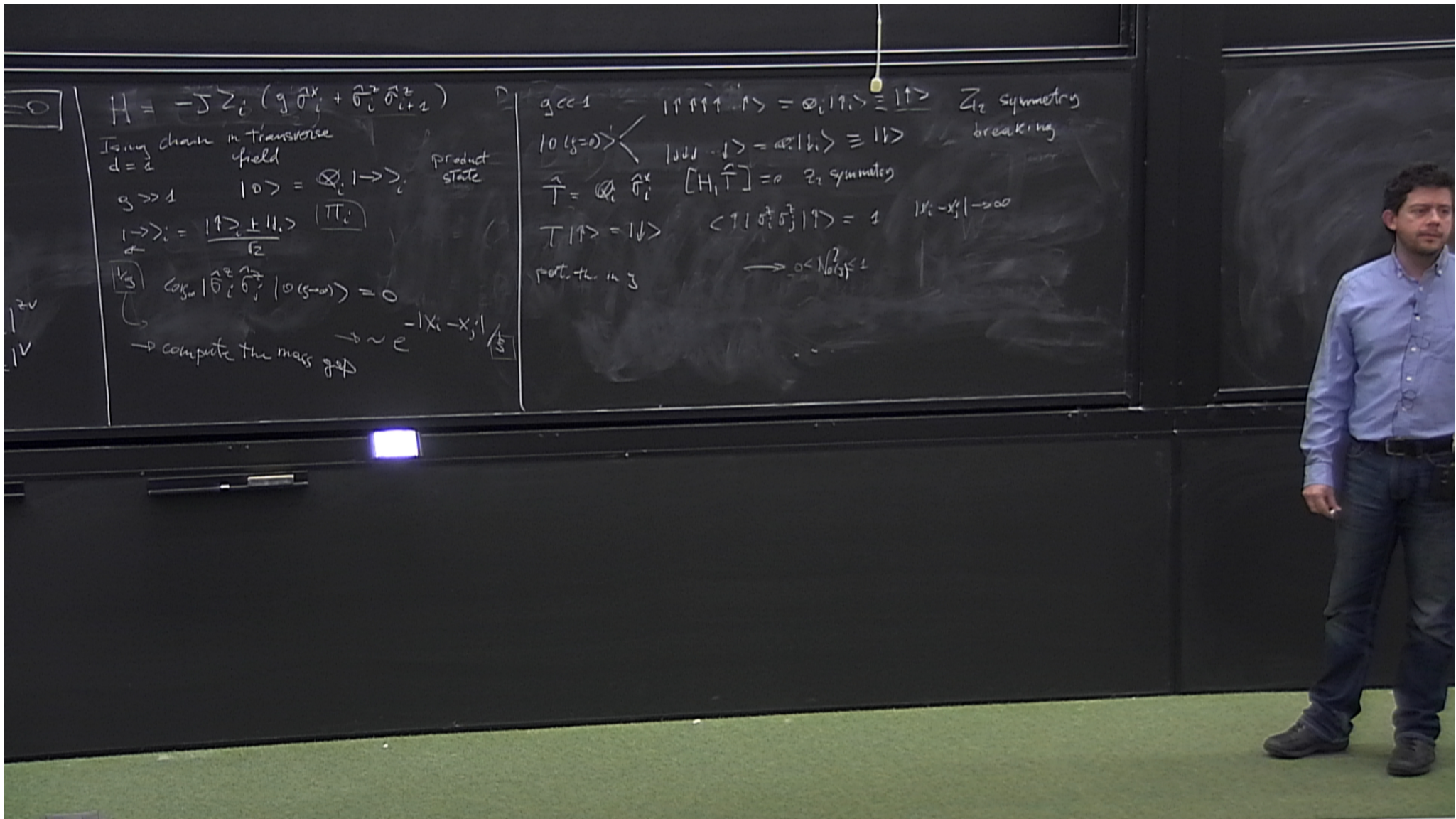






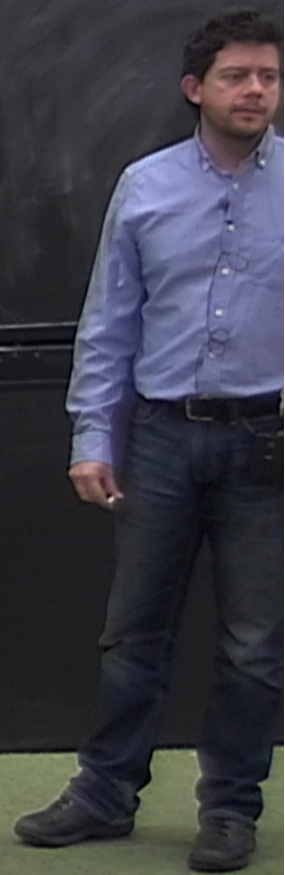
$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \theta \left| \frac{\uparrow}{\sqrt{2}}; \frac{\uparrow}{\sqrt{2}} \right\rangle = 0$   
 $\rightarrow$  compute the mass gap  
 $\sim e^{-|x_i - x_j|/\xi}$

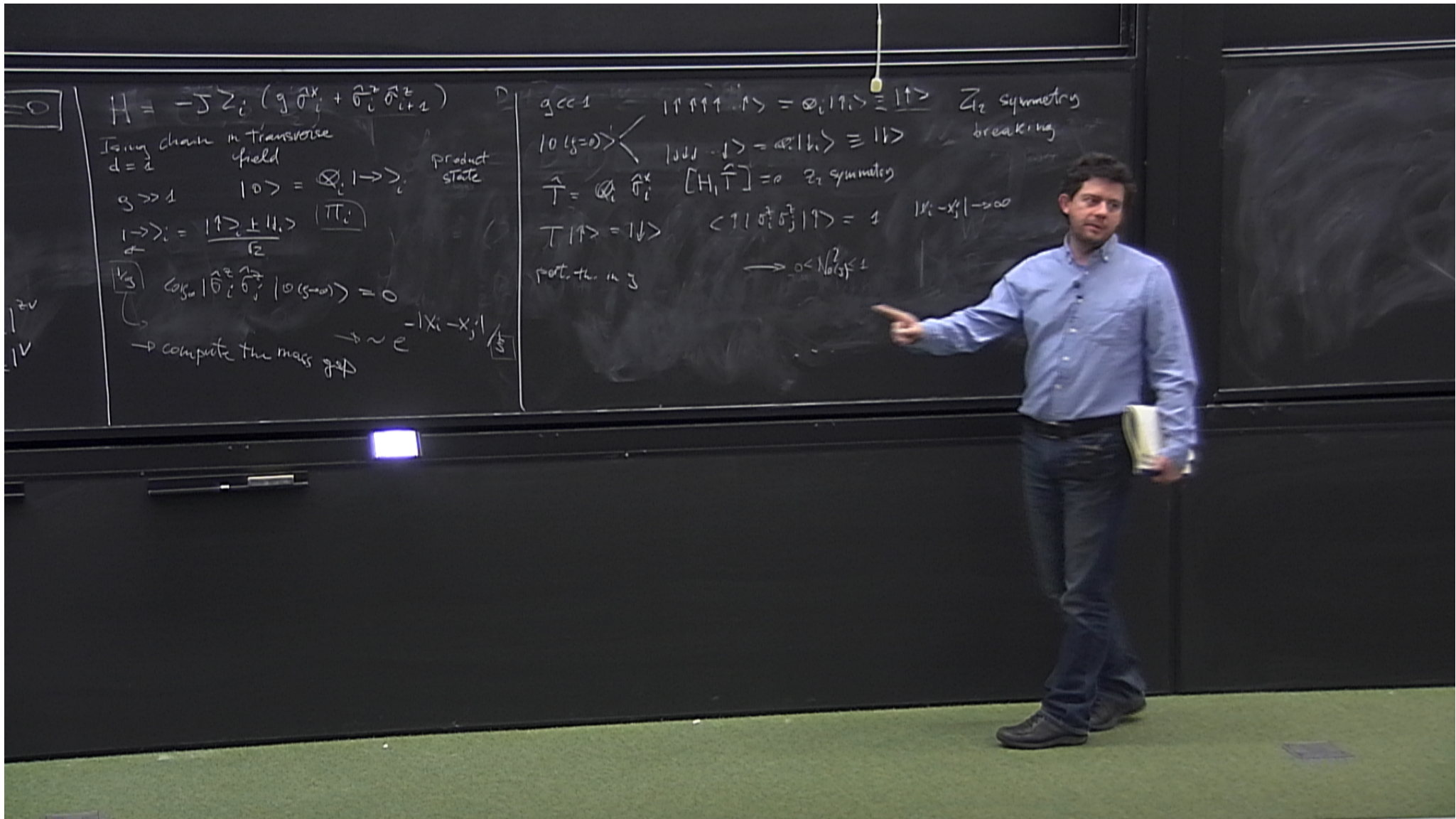
$g \ll 1$   
 $|\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $[\hat{H}, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \hat{\sigma}_i^z | \uparrow \rangle = 1$   
 $\rightarrow \langle \hat{N}_0 | \hat{\sigma}_i^z | \uparrow \rangle = 1$   
 $Z_2$  symmetry breaking  
 product state  
 pot. in 3



$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \frac{\pi z}{s} \frac{\hat{\sigma}_i^z \hat{\sigma}_j^z}{s} |\uparrow\rangle_i |\downarrow\rangle_j = 0$   
 $\rightarrow$  compute the mass gap  
 $\sim e^{-|x_i - x_j|/s}$

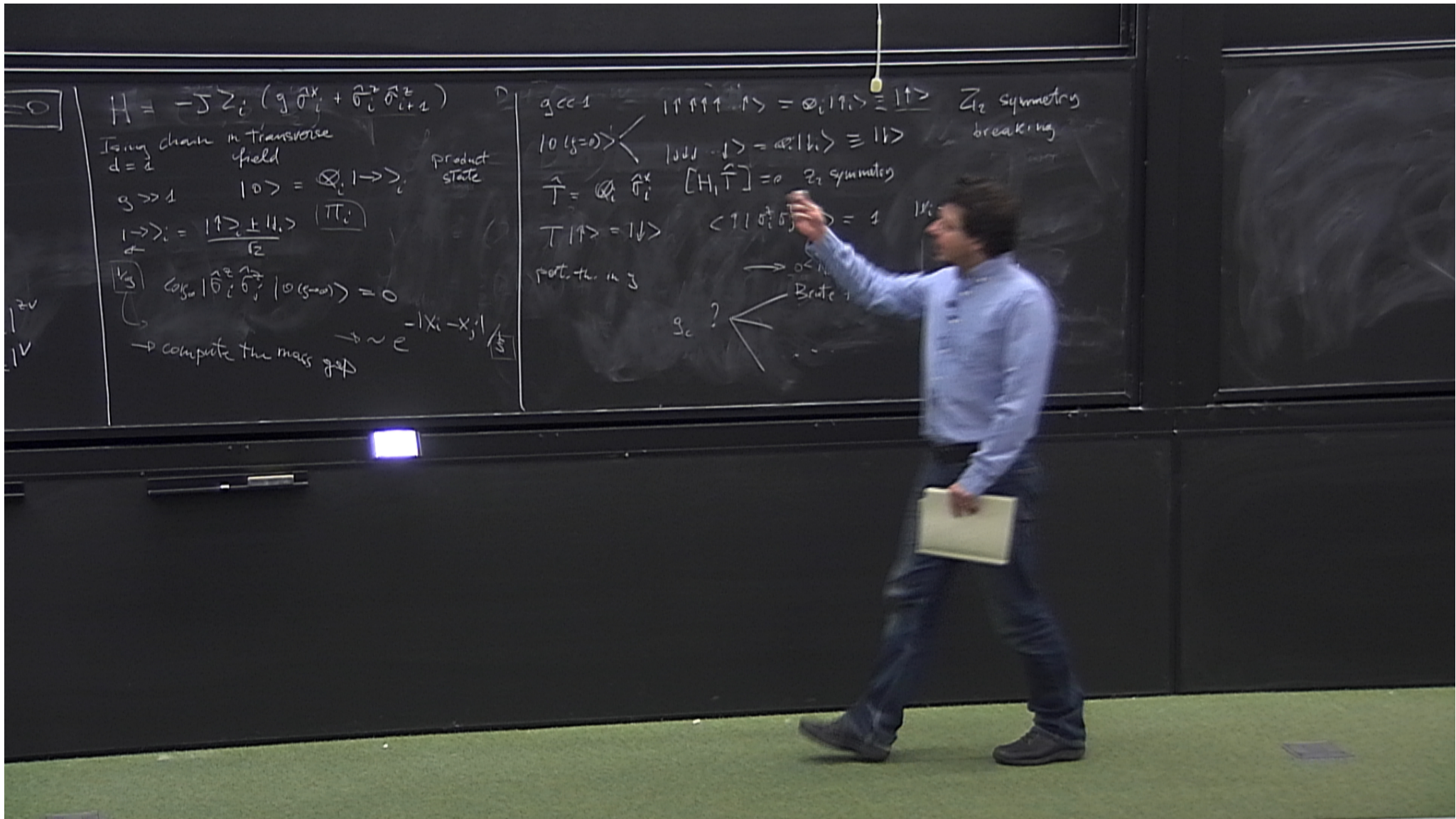
$g \ll 1$   
 $|\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow\rangle_i \equiv |\uparrow\rangle$   
 $|\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow\rangle_i \equiv |\downarrow\rangle$   
 $Z_2$  symmetry breaking  
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $[\hat{H}, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 pot. in 3  $\rightarrow 0 < N \hat{\sigma}_i^z \hat{\sigma}_j^z < 1$





$H = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$   
 $|\uparrow\rangle_i = \frac{|\uparrow\rangle_i + |\downarrow\rangle_i}{\sqrt{2}}$   
 $\cos \theta_i \frac{\sigma_i^z \sigma_{i+1}^z}{2} |\uparrow\rangle_i |\downarrow\rangle_{i+1} = 0$   
 $\rightarrow$  compute the mass gap  $\sim e^{-|x_i - x_j|/\xi}$

$g \ll 1$   
 $|\uparrow\rangle_i |\uparrow\rangle_{i+1} = \frac{1}{\sqrt{2}} (|\uparrow\rangle_i + |\downarrow\rangle_i) \otimes \frac{1}{\sqrt{2}} (|\uparrow\rangle_{i+1} + |\downarrow\rangle_{i+1})$   
 $|\downarrow\rangle_i |\downarrow\rangle_{i+1} = \frac{1}{\sqrt{2}} (|\uparrow\rangle_i - |\downarrow\rangle_i) \otimes \frac{1}{\sqrt{2}} (|\uparrow\rangle_{i+1} - |\downarrow\rangle_{i+1})$   
 $\hat{T} = \sigma_i^x \sigma_{i+1}^x$   
 $[\hat{H}, \hat{T}] = 0$   $Z_2$  symmetry  
 $\hat{T} |\uparrow\rangle = |\downarrow\rangle$   
 $\langle \uparrow | \sigma_i^z \sigma_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 $\rightarrow 0 < \frac{1}{N} \sum \sigma_i^z \sigma_j^z < 1$   
 $Z_2$  symmetry breaking



$H = -J \sum_i (g \hat{\sigma}_i^x + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z)$   
 Ising chain in transverse field  
 $d=2$   
 $g \gg 1$      $|0\rangle = \otimes_i |1\rangle_i$     product state  
 $|1\rangle_i = \frac{|1\rangle_i + |1\rangle_i}{\sqrt{2}}$      $(\pi_i)$   
 $\cos \theta_i \frac{\hat{\sigma}_i^z \hat{\sigma}_i^z}{\sqrt{2}} |0(g=0)\rangle = 0$   
 $\rightarrow \sim e^{-|x_i - x_j|/\xi}$   
 $\rightarrow$  compute the mass gap

$g \ll 1$      $|1\rangle \langle 1\rangle \langle 1\rangle = \otimes_i |1\rangle_i \equiv |1\rangle$      $Z_2$  symmetry breaking  
 $|0(g=0)\rangle$      $|0\rangle \langle 0\rangle = \otimes_i |0\rangle_i \equiv |0\rangle$   
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$      $[H, \hat{T}] = 0$      $Z_2$  symmetry  
 $\hat{T}|1\rangle = |0\rangle$      $\langle 1 | \hat{\sigma}_i^z | 0 \rangle = 1$      $1\%$   
 pot. th. in 3  
 $g_c ?$      $\rightarrow$  Brite

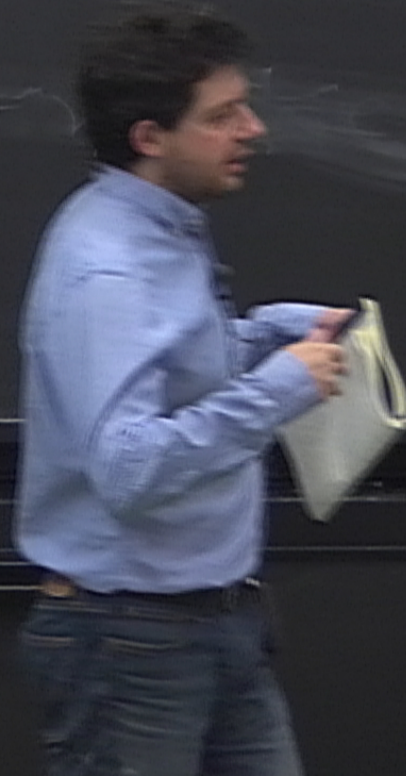
$g \ll 1$   $|\uparrow\uparrow\uparrow\uparrow\uparrow\rangle = \otimes_i |\uparrow_i\rangle \equiv |\uparrow\rangle$   $Z_2$  symmetry  
 $|0(g=0)\rangle \left\langle \begin{array}{l} |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle = \otimes_i |\downarrow_i\rangle \equiv |\downarrow\rangle \\ [H, \hat{T}] = 0 \end{array} \right.$   $Z_2$  symmetry breaking  
 $\hat{T} = \otimes_i \hat{\sigma}_i^x$   
 $\hat{T}|\uparrow\rangle = |\downarrow\rangle$   $\langle \uparrow | \hat{\sigma}_i^z \hat{\sigma}_j^z | \uparrow \rangle = 1$   $|x_i - x_j| \rightarrow \infty$   
 Pert. th. in 3  
 $g_c ?$

- $0 < N_0(g) \leq 1$
- Brute force  $\rightarrow$  solve exactly
- British force  $\rightarrow$  pert. theory
- elegant  $\rightarrow$



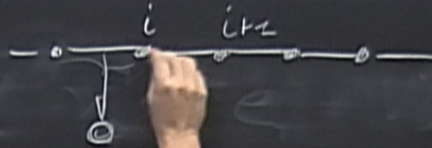
Duality

Transformation  
on the left



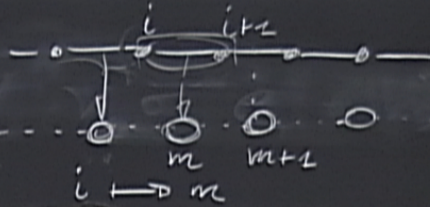
# Duality

Transformation  
on the lattice



# Duality

Transformation  
on the lattice

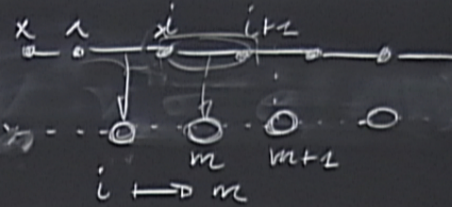


$$\left\{ \begin{array}{l} \hat{\mu}_2(m) = \dots + 1 \\ \hat{\mu}_3(m) = \dots \sigma_{j+1}^x \dots \text{str} \end{array} \right.$$



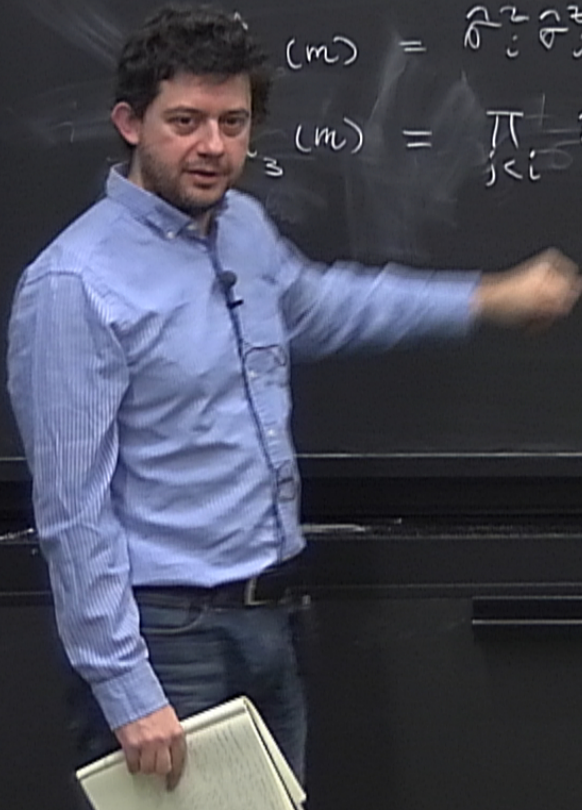
# Duality

Transformation  
on the lattice



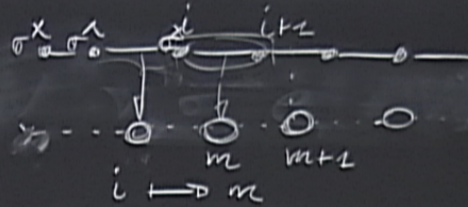
$$(m) = \frac{\sigma_i^z \sigma_{i+1}^z}{\sigma_i^z \sigma_{i+1}^z}$$

$$(m) = \prod_{j < i} \frac{\sigma_j^x}{\sigma_{j+1}^x} \text{ String operator}$$



# Duality

Transformation  
on the lattice



$$= \frac{\sigma_i^z \sigma_{i+1}^z}{\sigma_i^z \sigma_{i+1}^z}$$

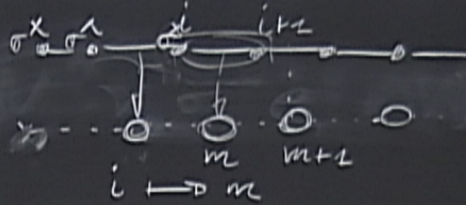
$$= \prod_{j < i} \sigma_{j+1}^x \text{ String operator}$$

$\mathcal{M}$

Spin

# Duality

Transformation  
on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \sigma_i^z \sigma_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_{j+1}^x \text{ -- String operator} \end{cases}$$

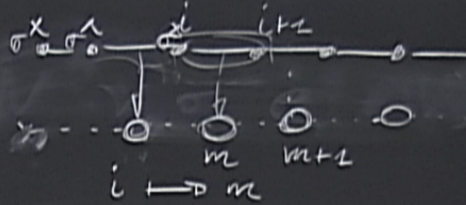
Spin chain?

$$\begin{cases} \hat{\sigma}_i^x \hat{\sigma}_i^z = 0 \\ \hat{\sigma}_i^z = \hat{\sigma}_{i+1}^z = \mathbb{1} \end{cases}$$

$$\begin{aligned} M_1^z &= M_3^z = \mathbb{1} \\ \{\hat{M}_1(m), \hat{M}_3(m)\} &= 0 \end{aligned}$$

# Duality

Transformation  
on the lattice



$$\left\{ \begin{array}{l} \hat{\mu}_2(m) = \sigma_i^z \sigma_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_{j+1}^x \end{array} \right. \text{String operator}$$

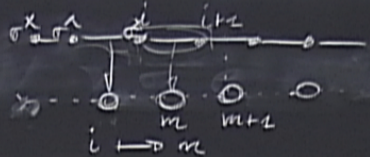
Spin chain?

$$\left\{ \begin{array}{l} \hat{\sigma}_i^x \hat{\sigma}_i^z = 0 \\ \hat{\sigma}_i^z = \hat{\sigma}_{i+1}^z = \mathbb{1} \end{array} \right.$$

$$\begin{aligned} \mu_1^2 = \mu_3^2 = \mathbb{1} \\ \{\hat{\mu}_1(m), \hat{\mu}_3(m)\} = 0 \end{aligned}$$

# Duality

Transformation on the lattice

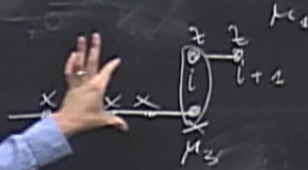


$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma_i^z \sigma_{i+1}^z}{\sigma_i^z \sigma_{i+1}^z} \\ \hat{\mu}_3(m) = \prod_{j < i} \frac{\sigma_j^x}{\sigma_j^x} \end{cases}$$

spin down?

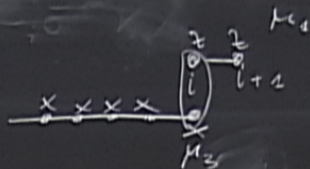
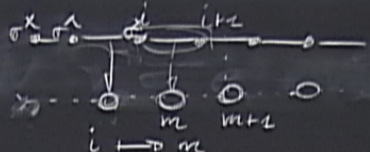
$$\begin{cases} \sigma_i^x \sigma_i^y = 0 \\ \sigma_i^z = \sigma_i^z = 1 \end{cases}$$

$$\hat{\mu}_1(m)$$



# Duality

Transformation on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma_x^z \sigma_x^z}{\sigma^z \sigma^z} \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_x^z \sigma_x^z \text{ - String operator} \end{cases}$$

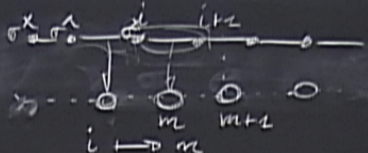
Spin down?

$$\begin{cases} \hat{\mu}_2^x = \hat{\mu}_2^y = 0 \\ \hat{\mu}_2^z = \hat{\mu}_2^z = 1 \end{cases}$$

$$\begin{cases} \mu_2^z = \mu_3^z = 1 \\ \langle \hat{\mu}_1^x, \hat{\mu}_2^x \rangle = 0 \end{cases} \rightarrow \text{Spins } \frac{1}{2}$$

# Duality

Transformation on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma_i^z \sigma_{i+1}^z}{\sigma_i^z \sigma_{i+1}^z} \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_j^x \text{ String operator} \end{cases}$$

Spin down?

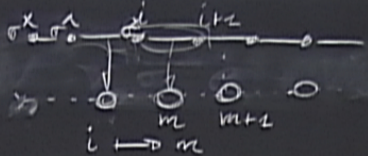
$$\begin{cases} \sigma_i^x \sigma_{i+1}^x = 0 \\ \sigma_i^z = \sigma_{i+1}^z = 1 \end{cases}$$

$$\begin{aligned} \mu_1^2 = \mu_3^2 = 1 \\ \langle \hat{\mu}_1(m), \hat{\mu}_3(m) \rangle = 0 \end{aligned} \rightarrow \text{Spins } \frac{1}{2}$$

H(m)

# Duality

Transformation on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma^z_i \sigma^z_{i+1}}{\sigma^z_i \sigma^z_{i+1}} \\ \hat{\mu}_3(m) = \prod_{j < i} \frac{\sigma^x_{j+1}}{\sigma^x_{j+1}} \text{ String operator} \end{cases}$$

Spin down?

$$\begin{cases} \sigma^x_i \sigma^x_{i+1} = 0 \\ \sigma^z_i = \sigma^z_{i+1} = 1 \end{cases}$$

$$\begin{aligned} \mu_2^2 = \mu_3^2 = 1 \\ \{\hat{\mu}_1(m), \hat{\mu}_2(m)\} = 0 \end{aligned} \rightarrow \text{Spins } \frac{1}{2}$$

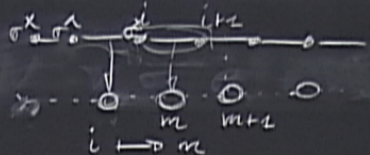
$$H(\mu_i) = -J \sum_m \mu_i(m) + g$$





# Duality

Transformation on the lattice



$$\hat{\mu}_i(m) = \frac{\hat{\mu}_i^z \hat{\mu}_{i+1}^z}{\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z}$$

$$\hat{\mu}_i(m) = \prod_{j < i} \hat{\sigma}_j^x \text{ String operator}$$

$$\mu_1^2 = \mu_3^2 = \mathbb{1}$$

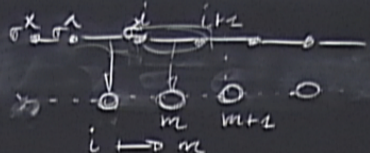
$$\{\hat{\mu}_1(m), \hat{\mu}_3(m)\} = 0 \rightarrow \text{spins } \frac{1}{2}$$

$$H(\mu_i) = -J \sum_m \mu_i(m) + g$$



# Duality

Transformation on the lattice



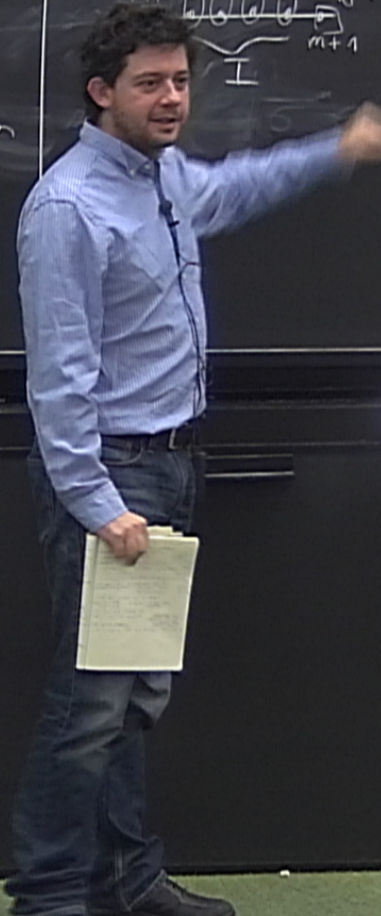
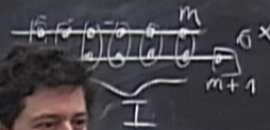
$$\begin{cases} \hat{\mu}_2(m) = \sigma_i^z \sigma_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_j^x \text{ String operator} \end{cases}$$

Spin down?

$$\begin{cases} \sigma_i^x \sigma_{i+1}^x = 0 \\ \sigma_i^z = \sigma_{i+1}^z = 1 \end{cases}$$

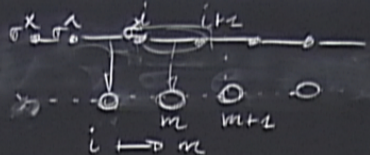
$$\begin{aligned} \mu_2^z = \mu_3^z = 1 &\rightarrow \text{Spins } \frac{1}{2} \\ \langle \hat{\mu}_1(m), \hat{\mu}_2(m) \rangle = 0 \end{aligned}$$

$$H(\mu_i) = -J \sum_m \mu_i(m) + g$$



# Duality

Transformation on the lattice



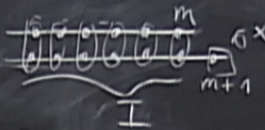
$$\begin{cases} \hat{\mu}_2(m) = \sigma_i^z \sigma_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_j^x \text{ String operator} \end{cases}$$

Spin down?

$$\begin{cases} \sigma_i^x \sigma_{i+1}^x = 0 \\ \sigma_i^z = \sigma_{i+1}^z = 1 \end{cases}$$

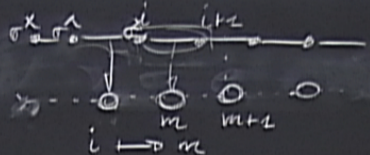
$$\begin{aligned} \mu_2^2 = \mu_3^2 = 1 &\rightarrow \text{Spins } \frac{1}{2} \\ \langle \hat{\mu}_1(m), \hat{\mu}_2(m) \rangle = 0 \end{aligned}$$

$$H(\mu_i) = -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_2(m) \hat{\mu}_3(m+1)$$



# Duality

Transformation on the lattice



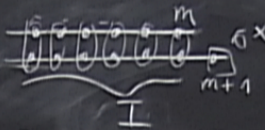
$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma_i^z \sigma_{i+1}^z}{\sigma_i^z \sigma_{i+1}^z} \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_j^x \text{ String operator} \end{cases}$$

spin down?

$$\begin{cases} \sigma_i^x \sigma_{i+1}^x = 0 \\ \sigma_i^z = \sigma_{i+1}^z = 1 \end{cases}$$

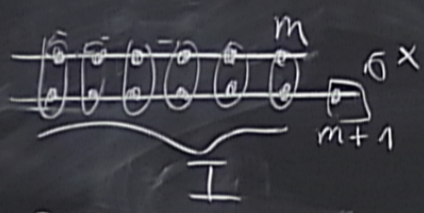
$$\begin{aligned} \mu_2^z = \mu_3^z = 1 &\rightarrow \text{spins } \frac{1}{2} \\ \hat{\mu}_1(m), \hat{\mu}_3(m) = 0 \end{aligned}$$

$$\begin{aligned} H(\mu_i) &= -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_3(m) \hat{\mu}_3(m+1) \\ &= -Jg \left[ \frac{1}{J} \sum_m \right] \end{aligned}$$



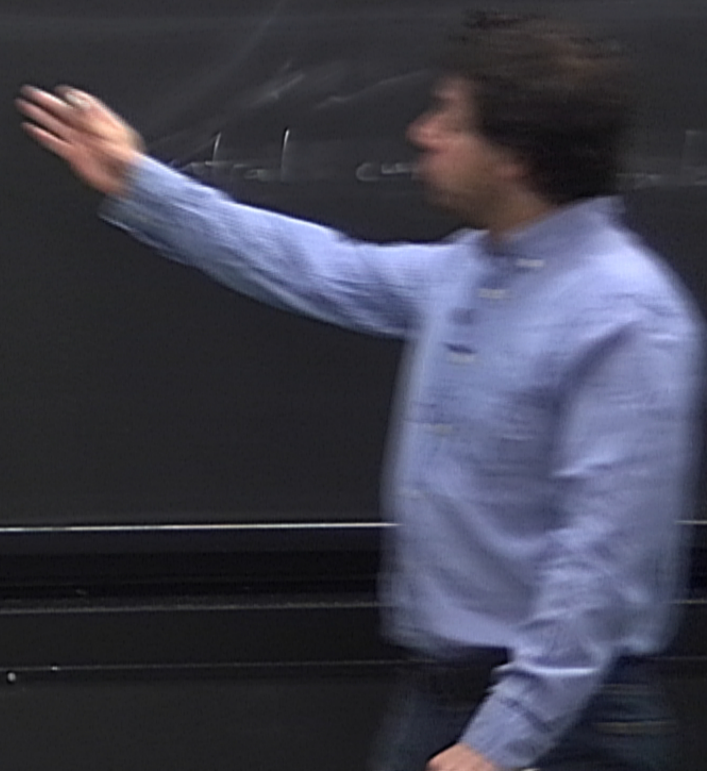
$$H(\mu_i) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



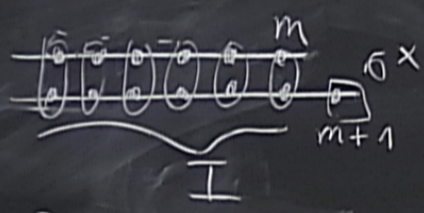
operator

as  $\frac{1}{2}$



$$H(\mu_i) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



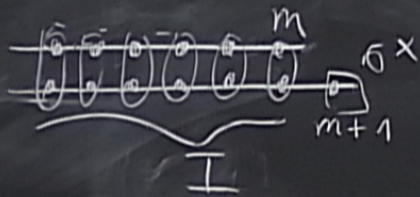
operator

as  $\frac{1}{2}$

neutral current

$$H(\mu, g) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$

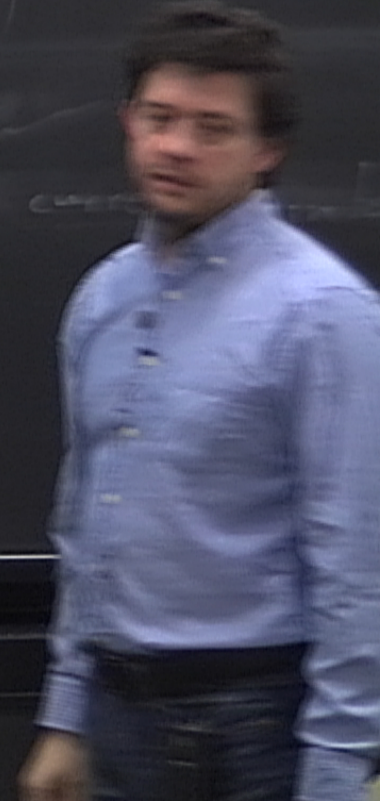


$$H(\sigma, g) = gH(\mu, \frac{1}{g})$$

operator

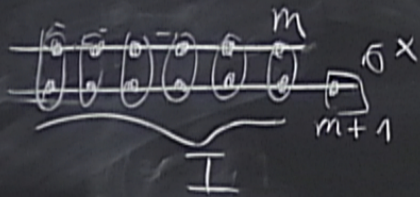
as  $\frac{1}{g}$

neutral current



$$H(\mu, g) = -\bar{J} \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -\bar{J} g \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g})$$

Duality

neutral current

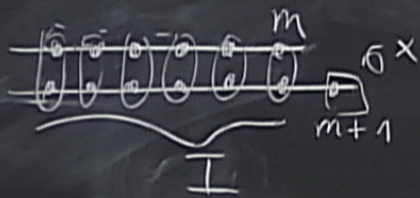
operator

as  $\frac{1}{2}$



$$H(\mu, g) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$

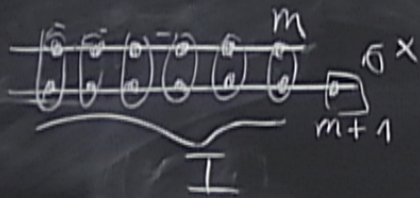


$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \quad \text{Duality}$$

$$E_r(g) = g E_r'(\frac{1}{g})$$

$$H(\mu, g) = -\bar{J} \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -\bar{J} g \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g})$$

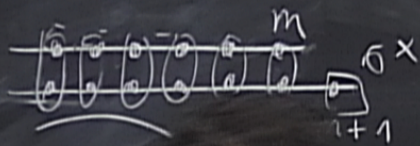
Duality

$$E_0(g) = g E_0(\frac{1}{g})$$

neutral current

$$H(\mu, g) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \quad \text{Duality}$$

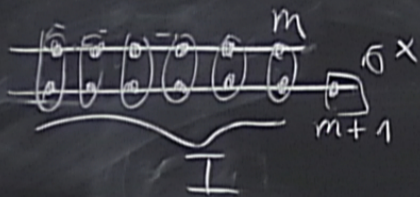
$$E_0(g) = g E'_0(\frac{1}{g})$$

$$E_1(g) = g E'_1(\frac{1}{g})$$

$$\Delta = 0 \rightarrow g = \frac{1}{g}$$

$$H(\mu, \sigma) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g})$$

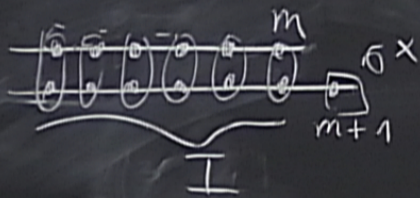
$$E_0(g) = g E'_0(\frac{1}{g})$$

$$E_1(g) = g E'_1(\frac{1}{g})$$

$$\boxed{\Delta = 0} \xrightarrow{g \rightarrow g_c} g = \frac{1}{g} \rightarrow g$$

$$H(\mu, g) = -J \sum_m \hat{\mu}_1^{(m)} + g \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)}$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1^{(m)} + \sum_m \hat{\mu}_3^{(m)} \hat{\mu}_3^{(m+1)} \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \quad \text{Duality}$$

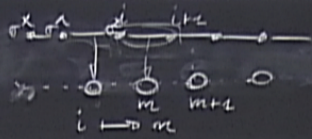
$$E_0(g) = g E'_0(\frac{1}{g})$$

$$E_1(g) = g E'_1(\frac{1}{g})$$

$$\boxed{\Delta = 0} \xrightarrow{g \rightarrow g_c} g = \frac{1}{g} \rightarrow g_c = 1 \rightarrow \boxed{\text{SELF DUAL}}$$

# Duality

Transformation on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \hat{\sigma}_j^x \end{cases} \text{String operator}$$

Spin down?

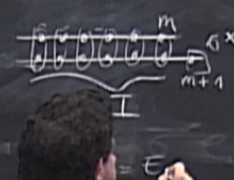
$$\begin{cases} \hat{\mu}_2^x = \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z = 0 \\ \hat{\sigma}_i^z = \hat{\sigma}_{i+1}^z = 1 \end{cases}$$

$$\mu_2^z = \mu_3^z = 1 \rightarrow \text{Spins } \frac{1}{2}$$

$$\{\hat{\mu}_2(m), \hat{\mu}_3(m)\} = 0$$

$$H(\mu, g) = -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_2(m) \hat{\mu}_3(m+1)$$

$$= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_2(m) \hat{\mu}_3(m+1) \right]$$



$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \quad \text{Duality}$$

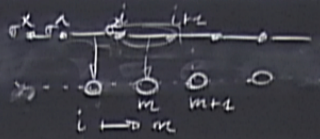
$$\begin{cases} E_0(g) = g E'_0(\frac{1}{g}) \\ E_1(g) = g E'_1(\frac{1}{g}) \end{cases}$$

$$\boxed{\Delta = 0} \xrightarrow{g \rightarrow g_c} g = \frac{1}{g} \rightarrow g_c = 1 \rightarrow \boxed{\text{SELF DUPL}}$$



# Duality

Transformation on the lattice



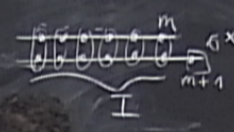
$$\begin{cases} \hat{\mu}_2(m) = \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \hat{\sigma}_j^x \text{ String operator} \end{cases}$$

Spin down?

$$\begin{cases} \hat{\mu}_2^x = \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x = 0 \\ \hat{\mu}_2^z = \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z = 1 \end{cases}$$

$$\begin{cases} \mu_2^z = \mu_3^z = 1 \\ \{\hat{\mu}_2(m), \hat{\mu}_3(m)\} = 0 \end{cases} \rightarrow \text{Spins } \frac{1}{2}$$

$$\begin{aligned} H(\mu, g) &= -J \sum_m \hat{\mu}_2(m) + g \hat{\mu}_2(m) \hat{\mu}_3(m+1) \\ &= -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_2(m) + \sum_m \hat{\mu}_2(m) \hat{\mu}_3(m+1) \right] \end{aligned}$$



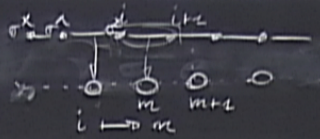
$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \quad \text{Duality}$$

$$\begin{cases} E_0(g) = g E_0(\frac{1}{g}) \\ E_1(g) = g E_1(\frac{1}{g}) \end{cases}$$

$$\boxed{\Delta = 0} \xrightarrow{g \rightarrow \frac{1}{g}} g = \frac{1}{g} \rightarrow g_0 = 1 \rightarrow \boxed{\text{SELF DUPL}}$$

# Duality

Transformation on the lattice



$$\begin{cases} \hat{\mu}_2(m) = \frac{\sigma^x \sigma^z}{\sigma^z \sigma^x} \\ \hat{\mu}_3(m) = \prod_{j < i} \frac{\sigma^x}{\sigma^z} \end{cases} \text{String operator}$$

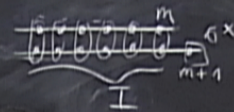
Spin down?

$$\begin{cases} \hat{\mu}_2^x = \hat{\mu}_2^y = 0 \\ \hat{\mu}_2^z = \hat{\mu}_2^x = 1 \end{cases}$$

$$\mu_2^z = \mu_3^z = 1 \rightarrow \text{Spin}$$

$$\{\hat{\mu}_2(m), \hat{\mu}_3(m)\} = 0$$

$$H(\mu, g) = -J \sum_m \hat{\mu}_1(m) + g \hat{\mu}_2(m) \hat{\mu}_3(m+1) = -Jg \left[ \frac{1}{g} \sum_m \hat{\mu}_1(m) + \sum_m \hat{\mu}_2(m) \hat{\mu}_3(m+1) \right]$$

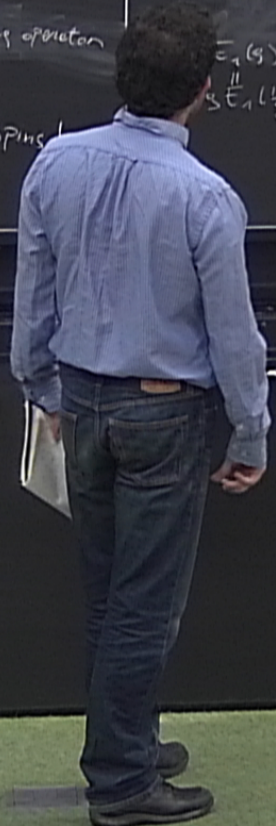


$$\begin{cases} E_1(g) = E_0(g) \\ E_1(g) = g E_0(g) \end{cases}$$

$$H(\sigma, g) = g H(\mu, \frac{1}{g}) \text{ Duality}$$

$$\begin{cases} E_0(g) = g E_0(\frac{1}{g}) \\ E_1(g) = g E_1(\frac{1}{g}) \end{cases}$$

$$\boxed{\Delta = 0} \xrightarrow{g \rightarrow g_c} g = \frac{1}{g} \rightarrow g_c = 1 \rightarrow \boxed{\text{SELF DUPL}}$$

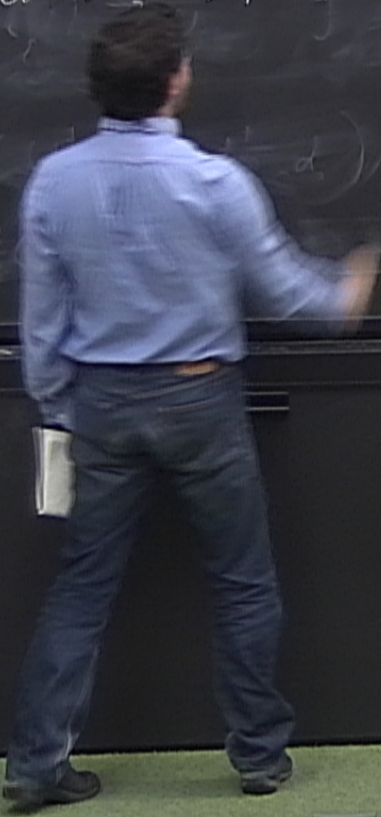




$$\begin{aligned}
 & \left\{ \begin{array}{l} \vec{p}_x \cdot \vec{p}_y = 0 \\ \vec{p}_x^2 = \vec{p}_y^2 = 1 \end{array} \right. \\
 & \mu_1^2 = \mu_2^2 = 1 \rightarrow \text{spins } \frac{1}{2} \\
 & \left\{ \begin{array}{l} \vec{M}_1 \cdot \vec{M}_2 = 0 \\ \vec{M}_1^2 = \vec{M}_2^2 = 1 \end{array} \right. \\
 & \left\{ \begin{array}{l} E_1(L_1) = E_0(L_1) \\ E_1(L_2) = E_0(L_2) \end{array} \right. \\
 & \boxed{\Delta = 0} \rightarrow g = \frac{1}{g} \rightarrow g_c = 1 \rightarrow \boxed{\text{SELF DUAL}}
 \end{aligned}$$

Exact spectrum

Jordan-Wigner :  $0 \rightarrow \frac{1}{2} \rightarrow$  spinless fermions



Exact spectrum

Jordan-Wigner

$0 \quad \frac{1}{2} \rightarrow$  spinen fermions

$$\hat{\sigma}_i^+ = \frac{1}{2} (\hat{\sigma}_i^x + i \hat{\sigma}_i^y)$$

$$\left\{ \begin{aligned} c_i &= \begin{pmatrix} \pi & \hat{\sigma}_i^z \\ j < i \end{pmatrix} \hat{\sigma}_i^+ \\ c_i^+ &= \end{aligned} \right.$$

$$\sigma_x^2 = \sigma_y^2 = 1$$

$$\langle \hat{M}_1 \rangle, \langle \hat{M}_3 \rangle = 0$$

$$j \rightarrow g$$

## Exact spectrum

Jordan-Wigner  $0 \frac{1}{2} \rightarrow$  spinless fermions

$$\hat{\sigma}_i^\pm = \frac{1}{2} (\hat{\sigma}_i^x \pm i \hat{\sigma}_i^y)$$

$$\left\{ \begin{aligned} c_i &= \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^+ \\ c_i^\dagger &= \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^- \end{aligned} \right.$$

$$\left\{ \begin{aligned} \hat{\sigma}_i^+ &= \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i \\ \hat{\sigma}_i^- &= \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i^\dagger \end{aligned} \right.$$

$\{c_i, c_i^\dagger\}$

Exact spectrum

Jordan-Wigner  $\sigma = \frac{1}{2} \rightarrow$  spinless fermions

$\hat{\sigma}_i^{\pm} = \hat{\sigma}_i^z \pm i\hat{\sigma}_i^y$

$$\begin{cases} c_i = \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^+ \\ c_i^\dagger = \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^- \end{cases}$$

$$\begin{cases} \hat{\sigma}_i^+ = \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i \\ \hat{\sigma}_i^- = \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i^\dagger \end{cases}$$

$\{c_i, c_j^\dagger\} = \delta_{ij}$

$$c^\dagger c = \hat{\sigma}_i^- \hat{\sigma}_i^+ = \frac{1}{4} (2 - 2\sigma_i^z)$$

$$\hat{\sigma}_i^z = 1 - 2c^\dagger c$$

$\pi$

Exact spectrum

Jordan-Wigner  $0 \frac{1}{2} \rightarrow$  spinen fermions

$$\hat{\sigma}_i^{\pm} = \frac{1}{2} (\hat{\sigma}_i^x \pm i \hat{\sigma}_i^y)$$

$$\begin{cases} c_i = \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^+ \\ c_i^{\dagger} = \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^- \end{cases} \rightarrow$$

$$\begin{aligned} & \prod_{j < i} (1 - 2c_j^{\dagger} c_j) c_i \\ & \prod_{j < i} (1 - 2c_j c_j^{\dagger}) c_i^{\dagger} \end{aligned}$$

$$\{c_i, c_j^{\dagger}\} = \delta_{ij}$$

$$c^{\dagger} c = \hat{\sigma}_i^- \hat{\sigma}_i^+ = \frac{1}{4} (2 - \hat{\sigma}_i^z)$$

$$\hat{\sigma}_i^z = 1 - 2c^{\dagger} c$$

$$\hat{\sigma}_i^x = \hat{\sigma}_i^+ + \hat{\sigma}_i^- = \prod_{j < i} (1 - 2c_j^{\dagger} c_j) (c_i + c_i^{\dagger})$$

$$\sigma_x^2 = \sigma_y^2 = 1$$

$$\langle \hat{M}_1 \rangle, \langle \hat{M}_3 \rangle = 0$$

$j \rightarrow g$

## Exact spectrum

Jordan-Wigner  $\sigma_x \frac{1}{2} \rightarrow$  spinless fermions

$$\hat{\sigma}_i^{\pm} = \frac{1}{2} (\hat{\sigma}_i^x \pm i \hat{\sigma}_i^y)$$

$$\left\{ \begin{aligned} c_i &= \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^+ \\ c_i^\dagger &= \left( \prod_{j < i} \hat{\sigma}_j^z \right) \hat{\sigma}_i^- \end{aligned} \right. \rightarrow$$

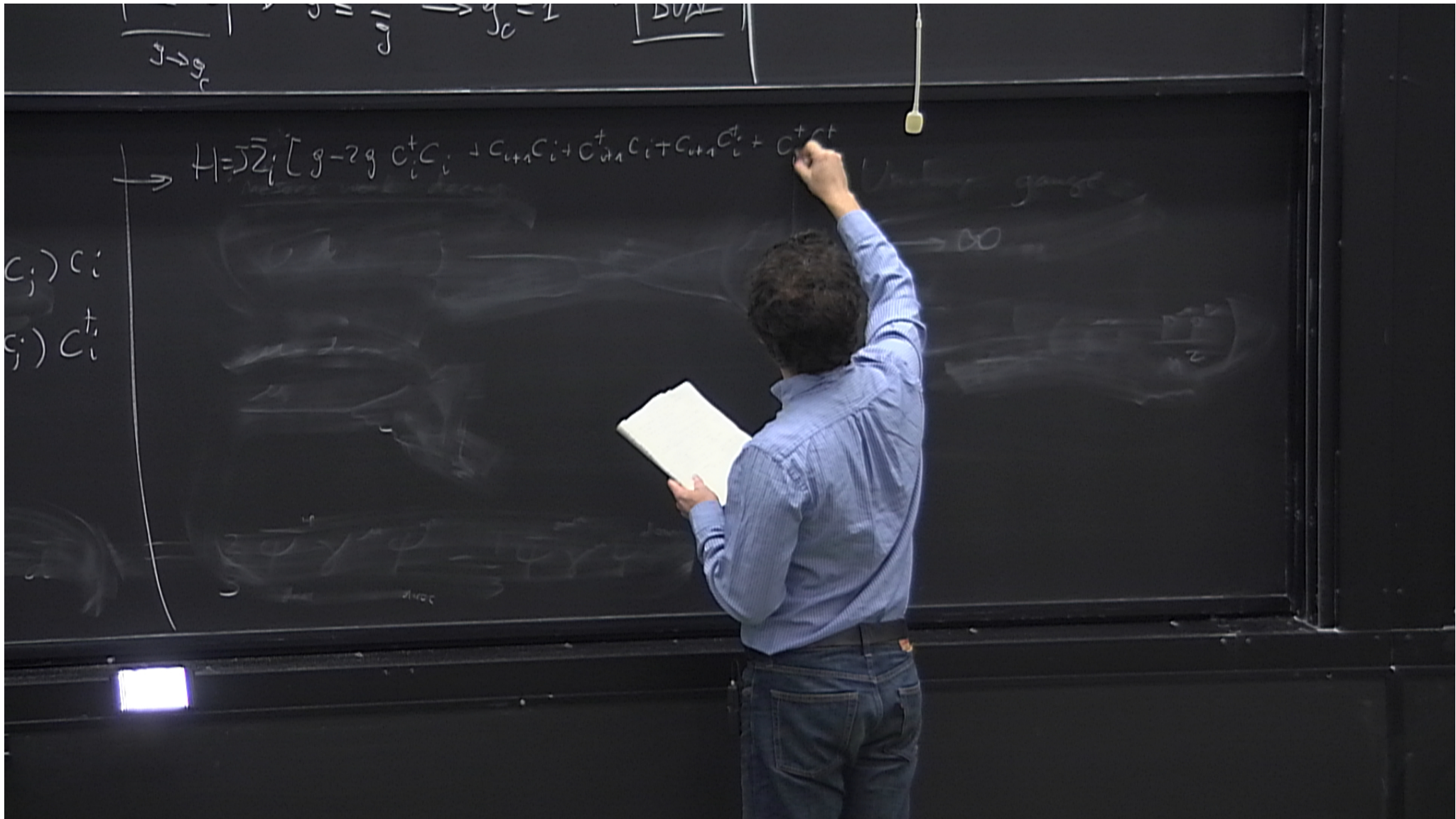
$$\left\{ \begin{aligned} \hat{\sigma}_i^+ &= \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i \\ \hat{\sigma}_i^- &= \prod_{j < i} (1 - 2c_j^\dagger c_j) c_i^\dagger \end{aligned} \right.$$

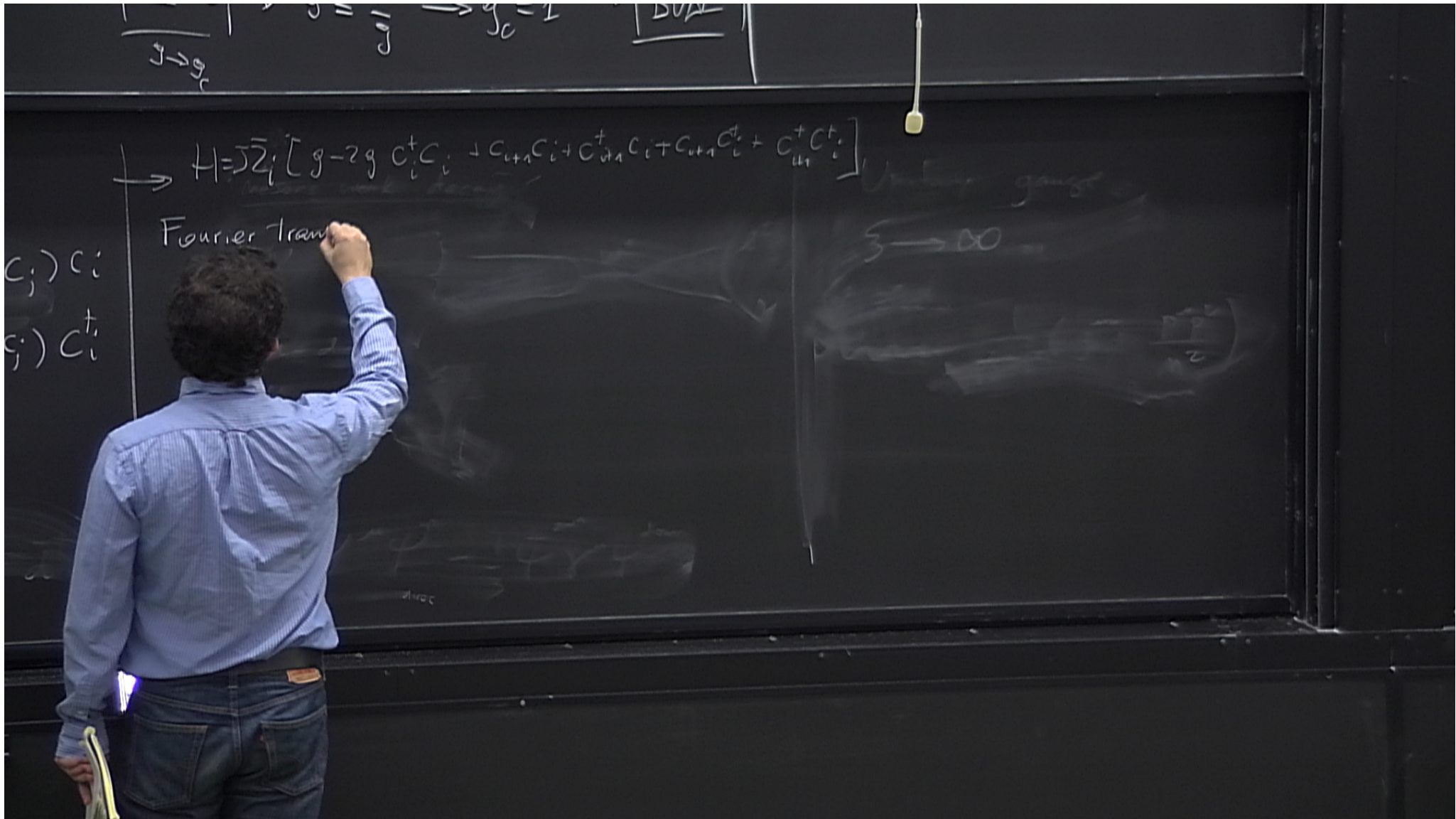
$$\{c_i, c_j^\dagger\} = \delta_{ij}$$

$$c^\dagger c = \hat{\sigma}_i^- \hat{\sigma}_i^+ = \frac{1}{4} (2 - 2\sigma_i^z)$$

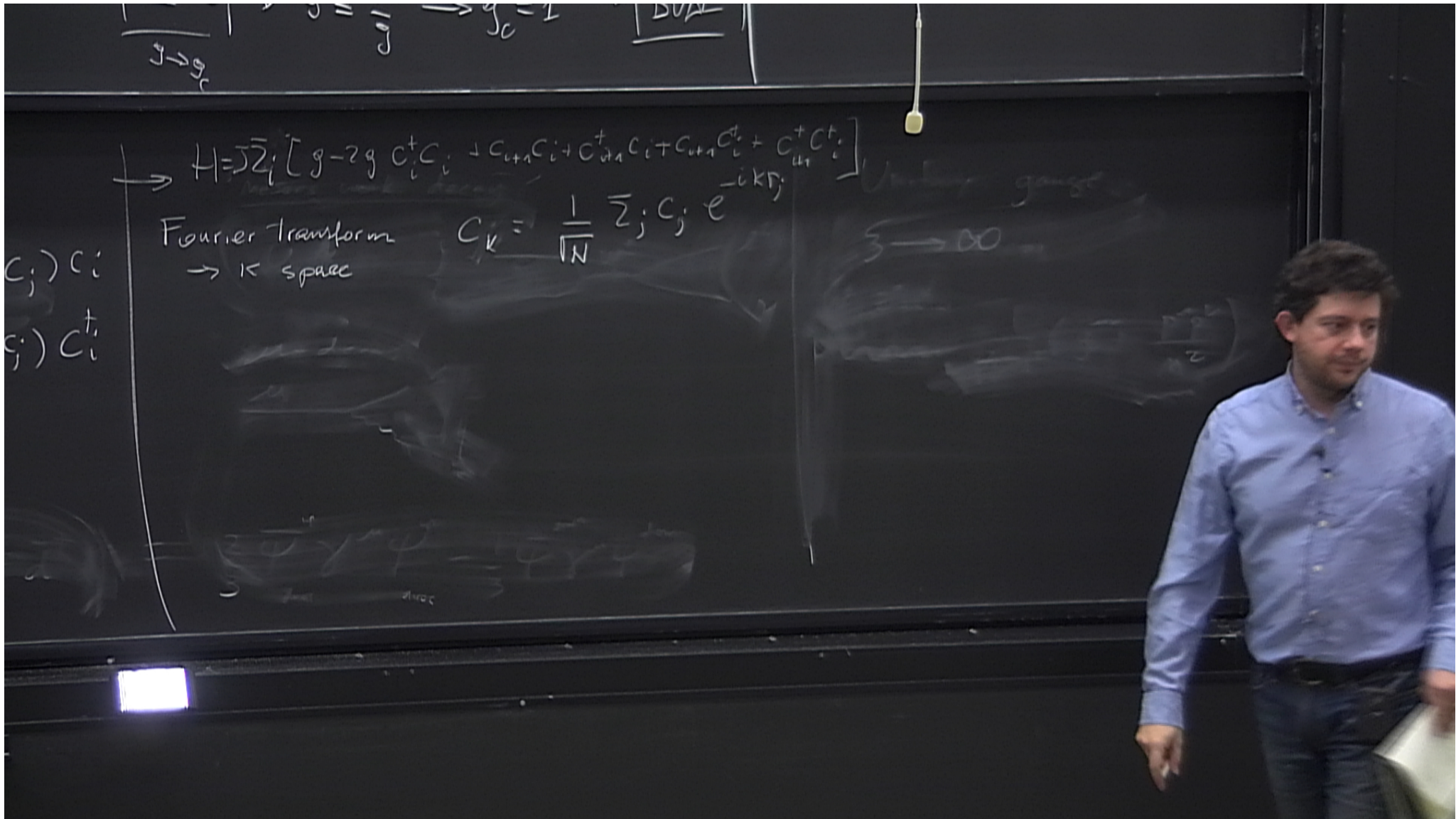
$$\hat{\sigma}_i^z = 1 - 2c^\dagger c \rightarrow \sigma^x$$

$$\hat{\sigma}_i^x = \hat{\sigma}_i^+ + \hat{\sigma}_i^- = \prod_{j < i} (1 - 2c_j^\dagger c_j) (c_i + c_i^\dagger) \rightarrow -\hat{\sigma}_i^z$$









$g \rightarrow g_c$   $g = \frac{1}{g_c} \rightarrow g_c = 1$  DUAL

$c_j \rightarrow c_i$   
 $c_j^+ \rightarrow c_i^+$

$H = J \sum_i [g - 2g c_i^+ c_i + c_{i+1} c_i + c_{i+1}^+ c_i + c_{i+1} c_i^+ + c_{i+1}^+ c_i^+]$

Fourier transform  
 $\rightarrow k$  space

$C_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ikr_j}$

Under gauge

$z \rightarrow \infty$

$g \rightarrow g_c$

$$H = \sum_j \bar{z}_j \left[ g - 2g \sum_i c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_{i+1}^\dagger c_i^\dagger \right] e^{-ikr_j}$$

$r_j = ja$  ← lattice spacing

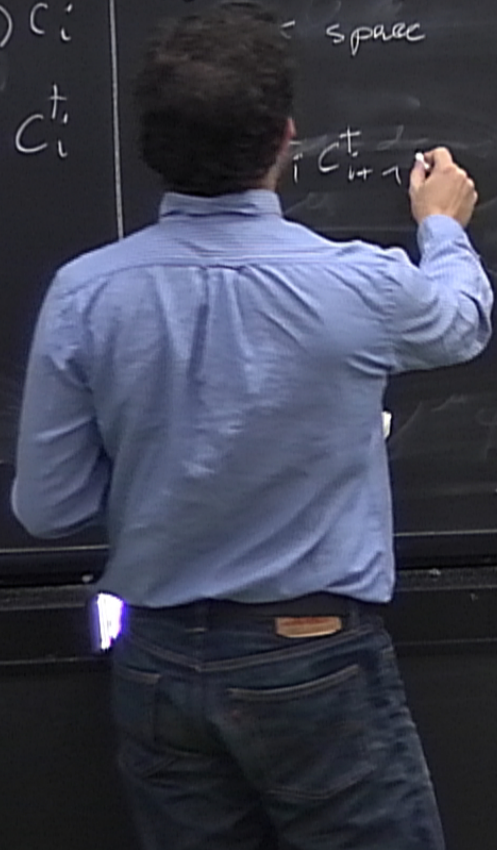
Fourier transform  
= space

$$C_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ik(ja)}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k C_k e^{ik(ja)}$$

$c_j^\dagger c_j) c_i$   
 $c_j^\dagger c_j) c_i^\dagger$

$i c_{i+1}^\dagger$



$g \rightarrow g_c$

$$H = J \sum_i \left[ g - 2g c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i^\dagger + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_i^\dagger c_{i+1}^\dagger + c_i c_{i+1} \right] e^{-ikr_j}$$

$r_j = ja$  lattice spacing

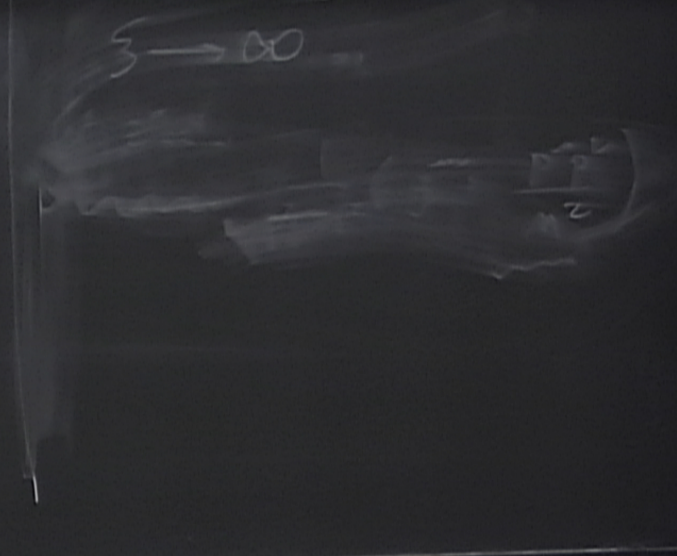
Fourier transform  
→ space

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ik(ja)}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k c_k e^{ik(ja)}$$

$$c_{i+1}^\dagger = \frac{1}{\sqrt{N}} \sum_k c_k^\dagger e^{ik(ja)}$$

$c_j^\dagger c_j$   
 $c_j^\dagger c_j$



$g \rightarrow g_c$

$$H = J \sum_i \left[ g - 2g \left( c_i^\dagger c_i + c_{i+a} c_i + c_{i+a}^\dagger c_i + c_{i+a} c_i^\dagger + c_{i-a}^\dagger c_i + c_{i-a} c_i^\dagger \right) \right]$$

$r_j = ja$  ← lattice spacing  
gauge

Fourier transform

$\rightarrow k$  space

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ik(ja)}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k c_k e^{ik(ja)}$$

$$\sum_i c_i^\dagger c_i = \frac{1}{N} \sum_i \sum_{k, k'} \left( e^{ik(ja)} \right)$$

$c_j^\dagger c_j$   
 $c_j^\dagger c_j$   
 $c_i^\dagger c_i$

$g \rightarrow g_c$

$$H = \sum_j \bar{z}_j \left[ g - 2g \sum_i c_i^\dagger c_i + c_{i+1} c_i + c_{i+1}^\dagger c_i + c_{i+1} c_i^\dagger + c_{i+1}^\dagger c_i^\dagger \right] \quad r_j = ja \quad \text{lattice spacing}$$

Fourier transform  
 $\rightarrow k$  space

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ikr_j}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k c_k e^{ikr_j}$$

$$\sum_i \bar{z}_i c_i^\dagger c_{i+1} = \frac{1}{N} \sum_{\ell} \sum_{k, k'} e^{i\ell k' (e+1)a} c_k^\dagger c_{k'}$$

$$= \frac{1}{N} \sum_k$$

$c_j^\dagger c_j$   
 $c_j^\dagger c_j$   
 $c_i^\dagger c_i$

$j \rightarrow g$

$$H = J \sum_j \left[ g - 2g c_j^\dagger c_j + c_{j+1} c_j + c_{j+1}^\dagger c_j + c_{j+1}^\dagger c_j^\dagger + c_j^\dagger c_j^\dagger \right]$$

$r_j = ja$  lattice spacing

Fourier transform  
 $\rightarrow k$  space

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ik(ja)}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k e^{ik(ja)} c_k$$

$$\sum_j c_j^\dagger c_{j+1} = \frac{1}{N} \sum_l \sum_{k, k'} S(k, k') e^{ik(la)} e^{ik'(l+1)a} c_k^\dagger c_{k'}$$

$$= \frac{1}{N} \sum_k e^{-ika} c_k^\dagger c_{-k}$$

$c_j^\dagger c_j$   
 $c_j^\dagger c_j$

$g \rightarrow g_c$

$$H = J \sum_j \left[ g - 2g \sum_i c_i^\dagger c_i + c_{i+a} c_i + c_{i+a}^\dagger c_i + c_{i-a} c_i^\dagger + c_{i-a}^\dagger c_i^\dagger \right]$$

$r_j = ja$  lattice spacing  
gauge

Fourier transform  
 $\rightarrow k$

$$c_k = \frac{1}{\sqrt{N}} \sum_j c_j e^{-ik(ja)}$$

$$c_j = \frac{1}{\sqrt{N}} \sum_k c_k e^{ik(ja)}$$

$$\sum_i c_i^\dagger$$

$$\frac{1}{N} \sum_l \sum_{k, k'} e^{i(k-k')la} e^{ik'(l+ja)} c_k^\dagger c_{k'}$$

$$\delta(k-k')$$

$$e^{ika} c_k^\dagger c_{-k}$$

$c_j^\dagger c_j$   
 $c_j^\dagger c_j$   
 $c_i^\dagger$   
 $c_i$

$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + i \sin(ka) [c_{-\mathbf{k}}^{\dagger} c_{\mathbf{k}}^{\dagger} + c_{-\mathbf{k}} c_{\mathbf{k}}] - g \right\}$$



$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + i \sin(ka) [c_{-\mathbf{k}}^{\dagger} c_{\mathbf{k}}^{\dagger} + c_{-\mathbf{k}} c_{\mathbf{k}}] - g \right\}$$

R

$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + i \sin(ka) [c_{-\mathbf{k}}^{\dagger} c_{\mathbf{k}}^{\dagger} + c_{-\mathbf{k}} c_{\mathbf{k}}] - g \right\}$$

Baydlinbar

$\gamma_{\mathbf{k}}$

$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + i \sin(ka) [c_{-\mathbf{k}}^{\dagger} c_{\mathbf{k}}^{\dagger} + c_{-\mathbf{k}} c_{\mathbf{k}}] - g \right\}$$

Boydinbar

$$\gamma_{\mathbf{k}} = u_{\mathbf{k}} c_{\mathbf{k}} - i v_{\mathbf{k}} c_{\mathbf{k}}^{\dagger}$$

$$u^2 = v^2 = 1$$

$$u_{-\mathbf{k}} = u_{\mathbf{k}}$$

$$v_{-\mathbf{k}} = -v_{\mathbf{k}}$$

$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] C_{\mathbf{k}}^{\dagger} C_{\mathbf{k}} + i \sin(ka) [C_{-\mathbf{k}}^{\dagger} C_{\mathbf{k}}^{\dagger} + C_{-\mathbf{k}} C_{\mathbf{k}}] - g \right\}$$

Boyd:

$$\gamma_{\mathbf{k}} = u_{\mathbf{k}} C_{\mathbf{k}} - i v_{\mathbf{k}} C_{-\mathbf{k}}^{\dagger}$$

$$C_{\mathbf{k}} = u_{\mathbf{k}} \gamma_{\mathbf{k}} + i v_{\mathbf{k}} \gamma_{-\mathbf{k}}^{\dagger}$$

$$u^2 - v^2 = 1$$

$$u_{-\mathbf{k}} = u_{\mathbf{k}}$$

$$v_{-\mathbf{k}} = -v_{\mathbf{k}}$$

$u_{\mathbf{k}}$   $\frac{\partial \mathbf{k}}$

$$H = \sum_k \left\{ 2[g - \cos(ka)] C_k^+ C_k + i \sin(ka) [C_{-k}^+ C_k^+ + C_{-k} C_k] - g \right\}$$

Boydinbar  $\gamma_k = u_k C_k - i v_k C_k^+$

$$C_k = u_k \gamma_k + i v_k \gamma_k^+$$

$$u^2 = v^2 = 1$$

$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

$$\begin{cases} u_k = \cos \frac{\theta_k}{2} \\ v_k = \sin \frac{\theta_k}{2} \end{cases}$$

tan



$$H = \sum_k \left\{ 2[g - \cos(ka)] C_k^+ C_k + i \sin(ka) [C_{-k}^+ C_k^+ + C_{-k} C_k] - g \right\}$$

Boydinbar

$$\gamma_k = u_k C_k - i v_k C_{-k}^+$$

$$u^2 = v^2 = 1$$

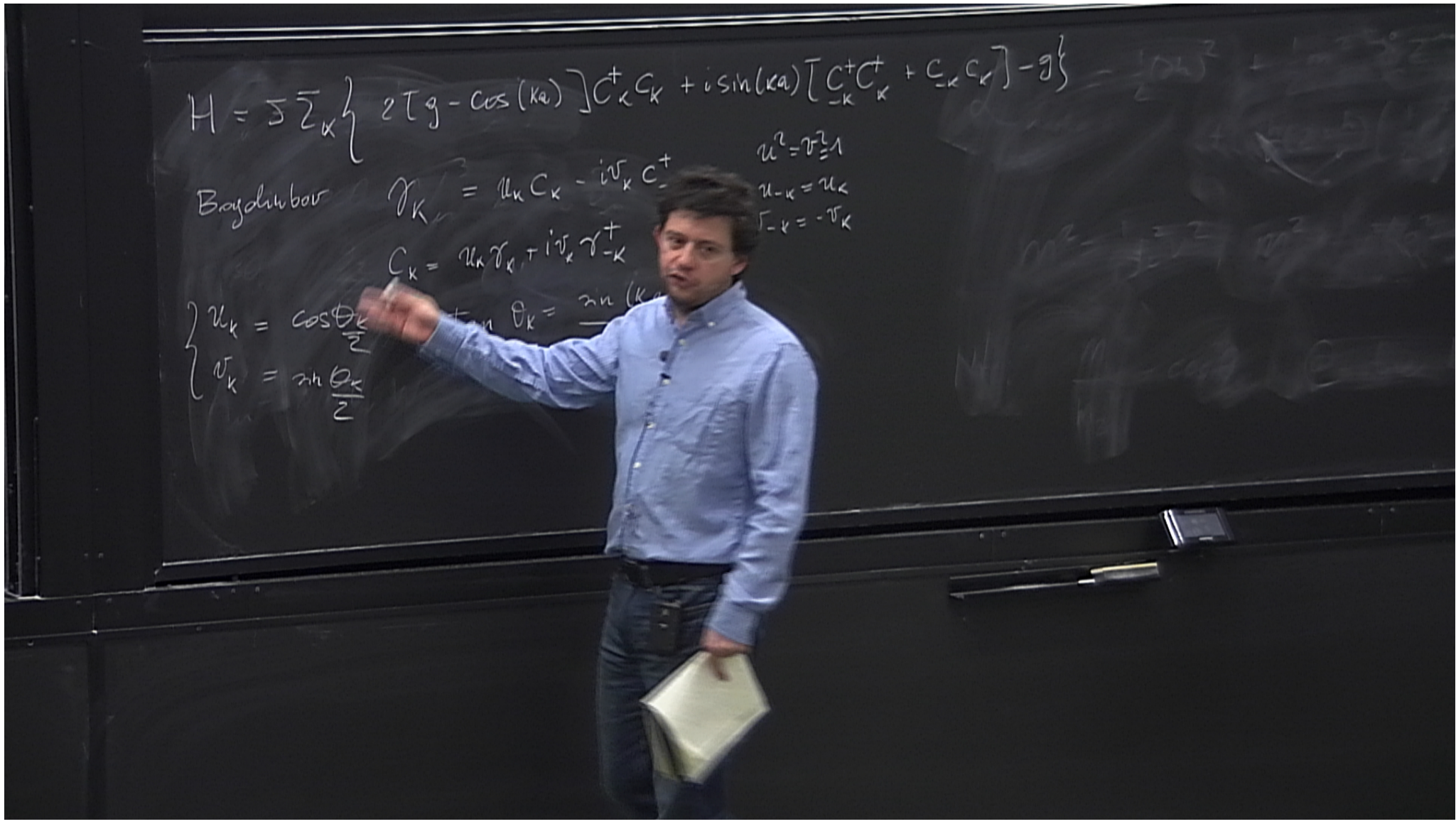
$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

$$C_k = u_k \gamma_k + i v_k \gamma_{-k}^+$$

$$\begin{cases} u_k = \cos \theta_k \\ v_k = \sin \theta_k \end{cases}$$

$$\tan \theta_k = \frac{v_k}{u_k} = \frac{v_k}{\cos \theta_k}$$



$$H = \sum_k \left\{ 2[g - \cos(ka)] C_k^+ C_k + i \sin(ka) [C_{-k}^+ C_k^+ + C_{-k} C_k] - g \right\}$$

Boydinbar  $\gamma_k = u_k C_k - i v_k C_{-k}^+$

$$C_k = u_k \gamma_k + i v_k \gamma_{-k}^+$$

$$\begin{cases} u_k = \cos \frac{\theta_k}{2} \\ v_k = \sin \frac{\theta_k}{2} \end{cases}$$

$$\tan \theta_k = \frac{\sin(ka)}{g - \cos(ka)}$$

$$u^2 - v^2 = 1$$

$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

$$\rightarrow H = \sum_k \epsilon_k \left( \gamma_k^+ \gamma_k - \frac{1}{2} \right)$$

$$\epsilon_k = 2g \left( 1 + g^2 - 2g \cos(ka) \right)^{1/2}$$

$$H = \sum_k \left\{ 2[g - \cos(ka)] C_k^+ C_k + i \sin(ka) [C_{-k}^+ C_k^+ + C_{-k} C_k] - g \right\}$$

Boydinbar

$$\gamma_k = u_k C_k - i v_k C_{-k}^+$$

$$u^2 - v^2 = 1$$

$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

$$C_k = u_k \gamma_k + i v_k \gamma_{-k}^+$$

$$\begin{cases} u_k = \cos \theta_k \\ v_k = \sin \theta_k \end{cases}$$

$$\tan \theta_k = \frac{\sin(ka)}{g - \cos(ka)}$$

$$\rightarrow H = \sum_k \epsilon_k \left( \gamma_k^+ \gamma_k - \frac{1}{2} \right)$$

$$\epsilon_k = 2g \left( 1 + g^2 - 2g \cos(ka) \right)^{1/2}$$

$$v_{k=0} = 0$$



$$H = \sum_{\mathbf{k}} \left\{ 2[g - \cos(ka)] C_{\mathbf{k}}^{\dagger} C_{\mathbf{k}} + i \sin(ka) [C_{-\mathbf{k}}^{\dagger} C_{\mathbf{k}}^{\dagger} + C_{-\mathbf{k}} C_{\mathbf{k}}] - g \right\}$$

Bogoliubov  $\gamma_{\mathbf{k}} = u_{\mathbf{k}} C_{\mathbf{k}} - i v_{\mathbf{k}} C_{-\mathbf{k}}^{\dagger}$

$$u^2 - v^2 = 1$$

$$u_{-\mathbf{k}} = u_{\mathbf{k}}$$

$$v_{-\mathbf{k}} = -v_{\mathbf{k}}$$

$$C_{\mathbf{k}} = u_{\mathbf{k}} \gamma_{\mathbf{k}} + i v_{\mathbf{k}} \gamma_{-\mathbf{k}}^{\dagger}$$

$$\begin{cases} u_{\mathbf{k}} = \cos \frac{\theta_{\mathbf{k}}}{2} \\ v_{\mathbf{k}} = \sin \frac{\theta_{\mathbf{k}}}{2} \end{cases}$$

$$\tan \theta_{\mathbf{k}} = \frac{2g \sin(ka)}{2g - \cos(ka)}$$

$$\rightarrow H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} (\gamma_{\mathbf{k}}^{\dagger} \gamma_{\mathbf{k}} - \frac{1}{2})$$

$$\epsilon_{\mathbf{k}} = 2J (1 + g^2 - 2g \cos(ka))^{1/2} \quad |v_{\mathbf{k}}| \cos \theta_{\mathbf{k}} = 0$$

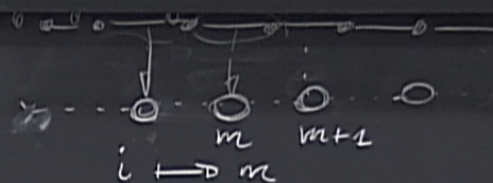
$$\min_{\mathbf{k}} \epsilon_{\mathbf{k}} \xrightarrow{k=0} \Delta = \epsilon_{\mathbf{k}} = 2J |1 - g|$$

CRITICAL BEHAVIOR  $\left\{ \begin{array}{l} \sum_{i=1}^N \sigma_i^z = N(g - g_c) \\ \sum_{i=1}^N \sigma_i^x = \Delta \end{array} \right.$

$\rightarrow$  compute the mass gap  $\rightarrow \sim e^{-\Delta}$

Duality

Transformation on the lattice

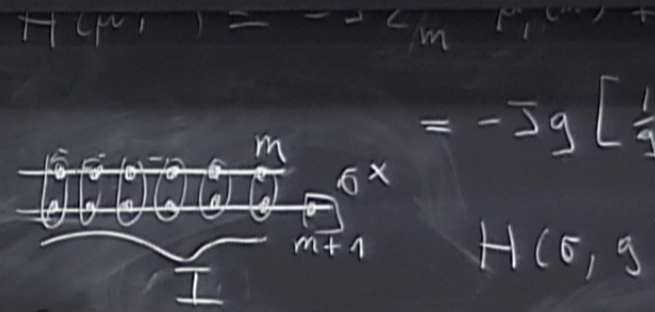


$$\begin{cases} \hat{\mu}_1(m) = \sigma_i^z \sigma_{i+1}^z \\ \hat{\mu}_3(m) = \prod_{j < i} \sigma_j^x \end{cases} \text{String operator}$$

Spin chain?

$$\begin{cases} \hat{\mu}_1^x = \hat{\mu}_3^z = 0 \\ \hat{\mu}_1^z = \hat{\mu}_3^x = 1 \end{cases}$$

$$\begin{cases} \mu_1^z = \mu_3^z = 1 \\ \hat{\mu}_1^x = \hat{\mu}_3^x = 0 \end{cases} \rightarrow \text{Spins } \frac{1}{2}$$



$$\begin{aligned} H(g) &= -J \sum \sigma_i^x \sigma_{i+1}^x \\ &= -Jg \left[ \frac{1}{g} \right] \\ H(\sigma, g) &= \\ E_1(g) &= E_0(g) \\ \frac{1}{g} E_1(\frac{1}{g}) &= E_0(\frac{1}{g}) \\ E_0(g) &= \\ E_1(g) &= \\ \left[ \Delta = 0 \right] &\rightarrow \\ g \rightarrow g_c & \end{aligned}$$

Exact Spectra

Jordan-Wigner  $\rightarrow$  spinless fermions

$H = J \sum \sigma_i^z$   
Fourier

$$H = \sum_k \left\{ 2[g - \cos(ka)] C_k^+ C_k + i \sin(ka) [C_{-k}^+ C_k^+ + C_{-k} C_k] - g k^2 \right\}$$

Boydinbar

$$\psi_k = u_k C_k - i v_k C_{-k}^+$$

$$C_k = u_k \psi_k + i v_k \psi_{-k}^+$$

$$u^2 = v^2 = 1$$

$$u_{-k} = u_k$$

$$v_{-k} = -v_k$$

$$\begin{cases} u_k = \cos \theta_k \\ v_k = \sin \theta_k \end{cases}$$

$$\tan \theta_k = \frac{v u(ka)}{g - \cos(ka)}$$

$$\rightarrow H = \sum_k \epsilon_k \left( \psi_k^+ \psi_k - \frac{1}{2} \right)$$

$$\epsilon_k = 2J \left( 1 + g^2 - 2g \cos(ka) \right)^{1/2}$$

$$\min_k \epsilon_k \xrightarrow{k \rightarrow 0} \Delta \epsilon_{ka} = 2J |1 - g|$$