

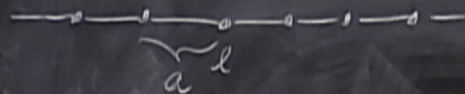
Title: Condensed Matter (Review) - Lecture 3

Date: Jan 04, 2012 10:15 AM

URL: <http://pirsa.org/12010085>

Abstract:

# CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\xi^{-1} \equiv \frac{1}{a} \log \coth(K)$$

$-|\tau|/\xi$

# TRANSFER MATRIX

1d Ising Model  $H = \lambda \sum_i s_i + K \sum_i s_i s_{i+1}$

$$\begin{cases} \lambda = \beta h \\ K = \beta J \end{cases}$$

$$\sum_N e^{-H} = \text{Tr} (T^N) = \epsilon_1^N + \epsilon_2^N$$

$$T = \begin{pmatrix} e^{\lambda+K} & e^{\lambda-K} \\ e^{\lambda-K} & e^{\lambda+K} \end{pmatrix} \sim \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$$

$\epsilon$

$\epsilon$

# TRANSFER MATRIX

1d Ising Model  $H = \lambda \sum_i s_i + K \sum_i s_i s_{i+1}$

$$\begin{cases} \lambda = \beta h \\ K = \beta J \end{cases}$$

$$Z_N = \text{Tr} e^{-H} = \text{Tr} (T^N) = \epsilon_1^N + \epsilon_2^N$$

$$T = \underbrace{\begin{pmatrix} e^k & e^{-k} \\ e^{-k} & e^k \end{pmatrix}}_{T_1} \times \underbrace{\begin{pmatrix} e^\lambda & \\ & e^{-\lambda} \end{pmatrix}}_{T_2} \rightsquigarrow \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$\epsilon_1 > \epsilon_2$   
 $\epsilon_1 \rightarrow 0$   
 $\epsilon_1$  analytic

$k \rightarrow \infty$  1st order PT

i) Compute correlation functions

$$\langle S_i S_{i'} \rangle$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$
$$\rightarrow \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle S_i S_{i'} \rangle$$

$$S_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix}$$

$$H = \lambda \sum_i \sigma_i^z$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\rightarrow \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle S_i S_{i'} \rangle$$

$$S_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix}$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^x$$

$$| \uparrow \rangle \otimes | \uparrow \rangle$$

$\uparrow$

$\pm$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle S_L S_{L'} \rangle$$

$$S_L \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$

$\pm$

$$\langle i_2 | \sigma_L^z | i_L \rangle = \sum_{\pm} \sigma_L^{\pm}$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$



i) Compute correlation functions

$$\langle S_L S_{L'} \rangle$$

$$S_L \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z + V$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$   
 $\downarrow$

$\pm$

$$\langle i_2 | \sigma_{i_1}^z | i_1 \rangle = \sum_{\pm} \sigma_{\pm}^z$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle S_L S_{L'} \rangle$$

$$S_L \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z + V$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$

$\pm$

$$\langle i_2 | \sigma_L^z | i_L \rangle = \sum_2^{\sigma^z}$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle S_i S_{i'} \rangle$$

$$S_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma_i^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$

$\pm$

$$\langle i_2 | \sigma_i^z | i_1 \rangle = \delta_{i_2, i_1 \pm 1}$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$N) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\rightarrow \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$\langle \sigma_i^z \sigma_{i'}^z \rangle_{\text{Gibbs}} = \frac{1}{Z} \sum_{\{\sigma_i^z\}} e^{-H}$$

$$s_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma_i^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$

$\pm$

$$\langle i_2 | \sigma_i^z | i_1 \rangle = \delta_{i_1 i_2}$$

i) Compute correlation functions

$$\langle \sigma_i^z \sigma_{i'}^z \rangle_{\text{Gibbs}} = \frac{1}{\sum_{\{\sigma_i^z\}} e^{-H_{\{\sigma_i^z\}}}} \rightarrow$$

$$s_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma_i^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$\uparrow$   
 $\downarrow$

$$\langle i_2 | \sigma_i^z | i_1 \rangle = \delta_{i_1 i_2}$$

$$= \sum_{i_1}^N + \sum_{i_2}^N$$

$\rightarrow$

$F = -\ln Z$   
 $\rightarrow UFD$   
 Matrix calculus  
 $\frac{\partial \ln Z}{\partial \lambda} = \langle S_i \rangle$   
 $\frac{\partial \ln Z}{\partial J} = \langle S_i S_{i+1} \rangle$   
 $\frac{\partial \ln Z}{\partial K} = \langle S_i S_{i+2} \rangle$

TRANSFER MATRIX

1d Ising Model  $H = \lambda \sum_i S_i + K \sum_i S_i S_{i+1}$

$\lambda = \beta h$   
 $K = \beta J$

$Z_N = \text{Tr} e^{-H} = \text{Tr} (T^N) = \epsilon_1^N + \epsilon_2^N$

$T = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix} \times \begin{pmatrix} e^{\lambda} & \\ & e^{-\lambda} \end{pmatrix} \sim \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$

$\epsilon_1 > \epsilon_2$   
 $\epsilon_1 > 0$   
 $\epsilon_1$  analytic  
 $K \rightarrow \infty$  1st order PT

i) Compute correlation functions

$C_{10} = \langle \sigma_i^z \sigma_{i+l}^z \rangle_{\text{quasi}} = \frac{1}{Z} \sum_{\{\sigma_i^z\}} e^{-H} \sigma_i^z \sigma_{i+l}^z \xrightarrow{\lambda=0} C_{10} = \frac{1}{2}$

$S_z \rightarrow \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = \sigma^z$

$H = \lambda \sum_i \sigma_i^z + K \sum_i \sigma_i^z \sigma_{i+1}^z$

$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$

$\langle i_1 | \sigma_i^z | i_1 \rangle = \delta_{i_1, i_1}$

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\left( \begin{array}{c} \epsilon_1 \\ \epsilon_2 \end{array} \right)$$

i) Compute correlation functions

$$C_{10} = \langle \sigma_i^z \sigma_{i+1}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i^z, \sigma_{i+1}^z}} \xrightarrow{\lambda=0} C_{10} = \frac{1}{Z} \text{Tr} [$$

$$S_i \rightarrow \begin{pmatrix} +1 & \\ & -1 \end{pmatrix} = \sigma^z$$

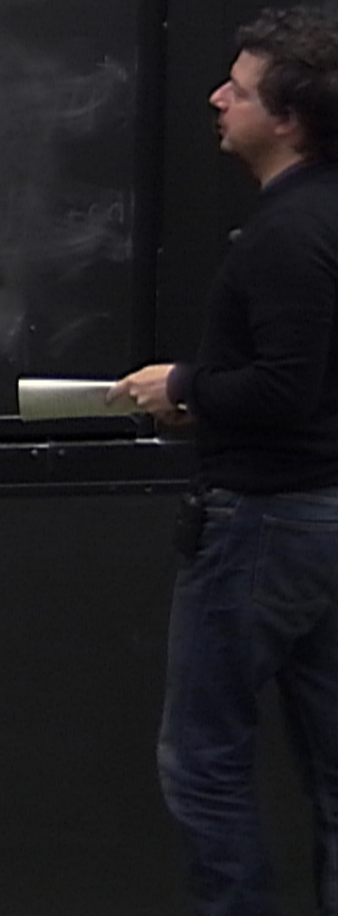
$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

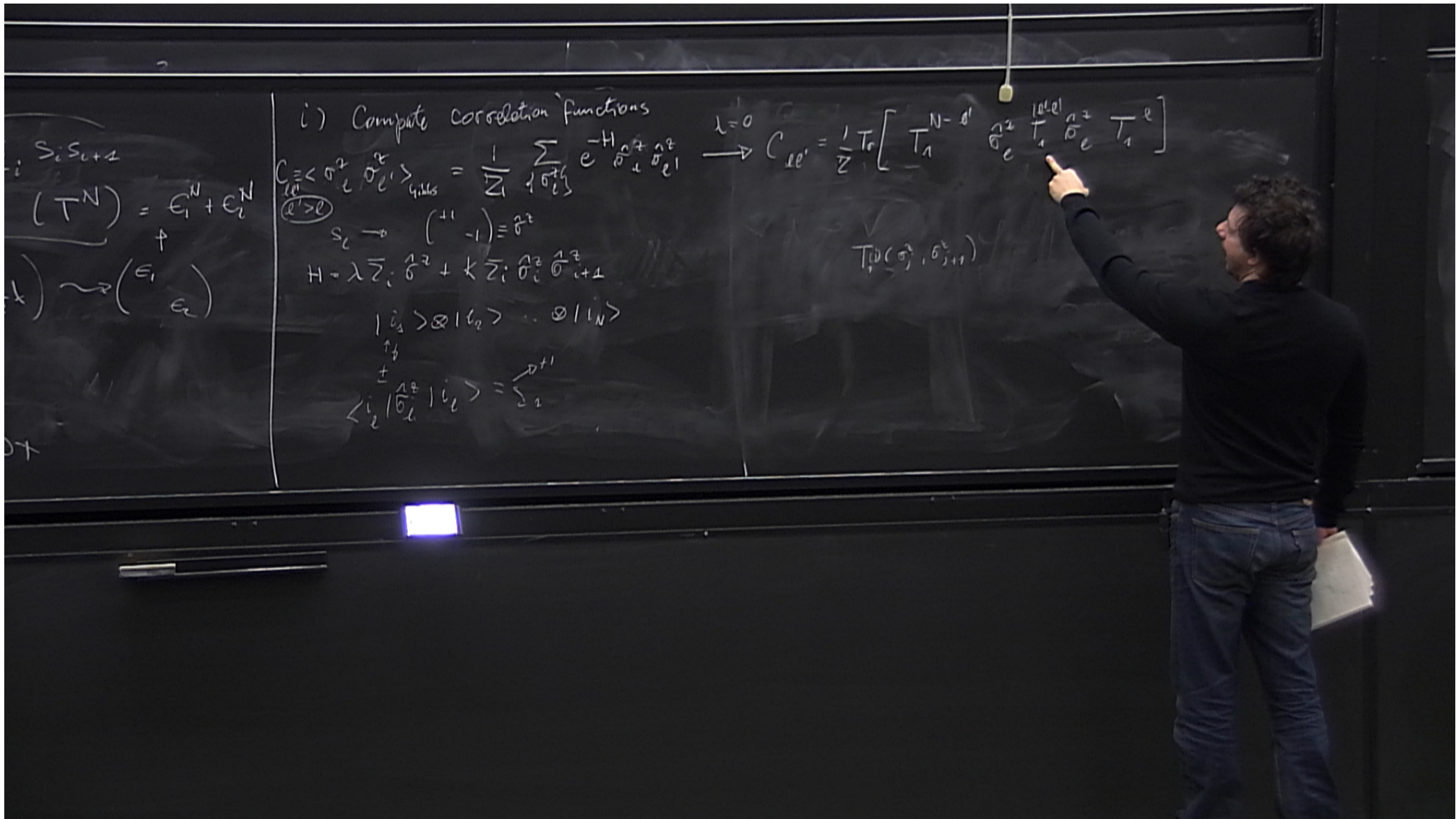
$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_i^z | i_2 \rangle = \sum_2^{\pm 1}$$





$S_i S_{i+1}$   
 $(TN) = \epsilon_1^N + \epsilon_2^N$   
 $\uparrow$   
 $\left( \begin{matrix} \epsilon_1 \\ \epsilon_2 \end{matrix} \right)$

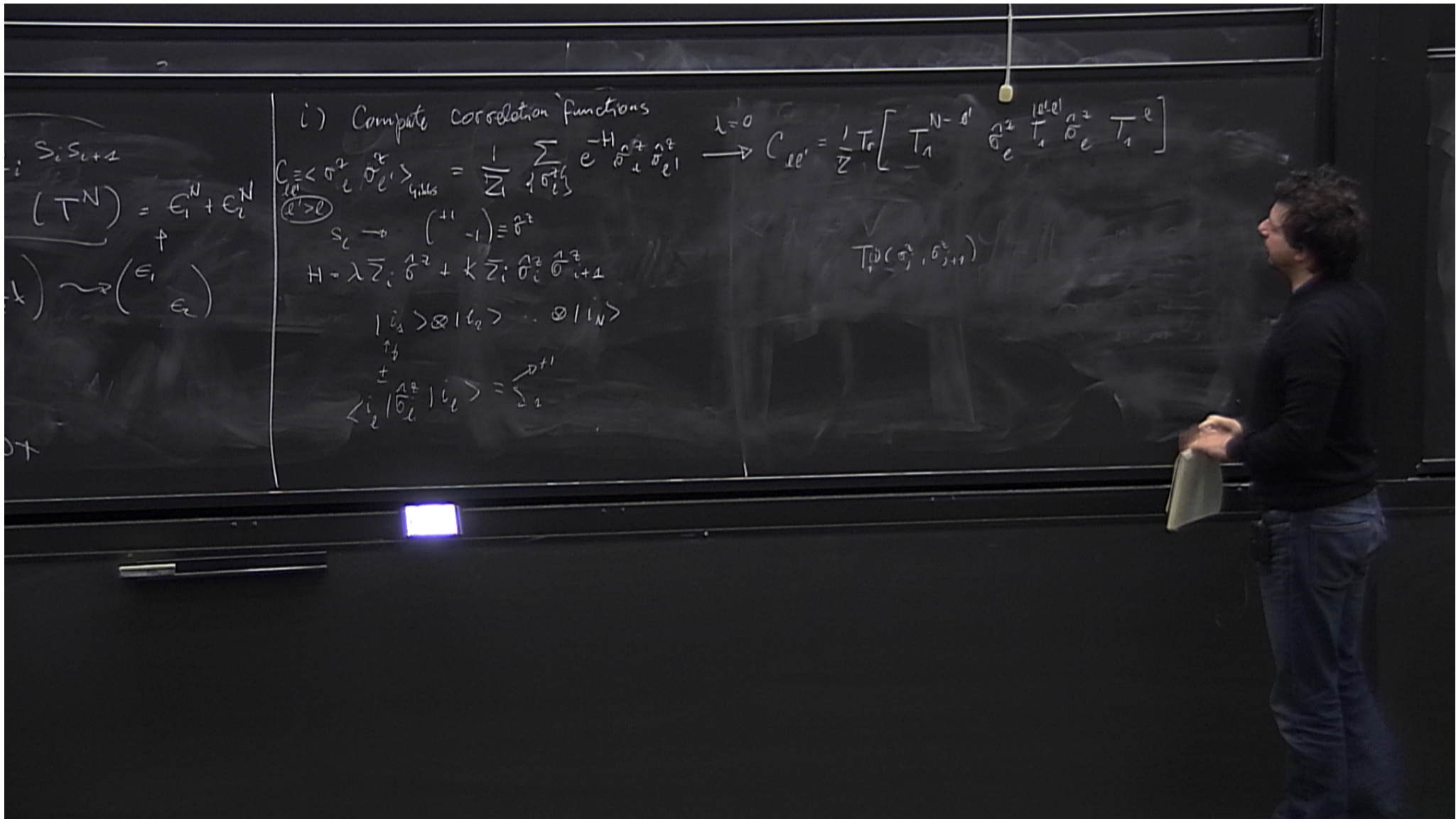
i) Compute correlation functions

$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_l^z, \sigma_{l'}^z}}$   
 $\epsilon_l > \epsilon_{l'}$   
 $s_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$   
 $H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$   
 $|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$   
 $\uparrow$   
 $\pm$   
 $\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_2$

$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \sigma_l^z T_1^{l-l'} \sigma_l^z T_1^l \right]$

$T_1(\sigma_i^z, \sigma_{i+1}^z)$





i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\left( \begin{matrix} \epsilon_1 \\ \epsilon_2 \end{matrix} \right)$$

$$C_{ll'} = \langle \sigma_l^x \sigma_{l'}^x \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_l^x, \sigma_{l'}^x}} \sigma_l^x \sigma_{l'}^x$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \begin{matrix} \sigma_l^x & \sigma_l^z \\ T_1 & \sigma_l^x \end{matrix} T_1^l \right]$$

$$s_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^x \sigma_{i+1}^x$$

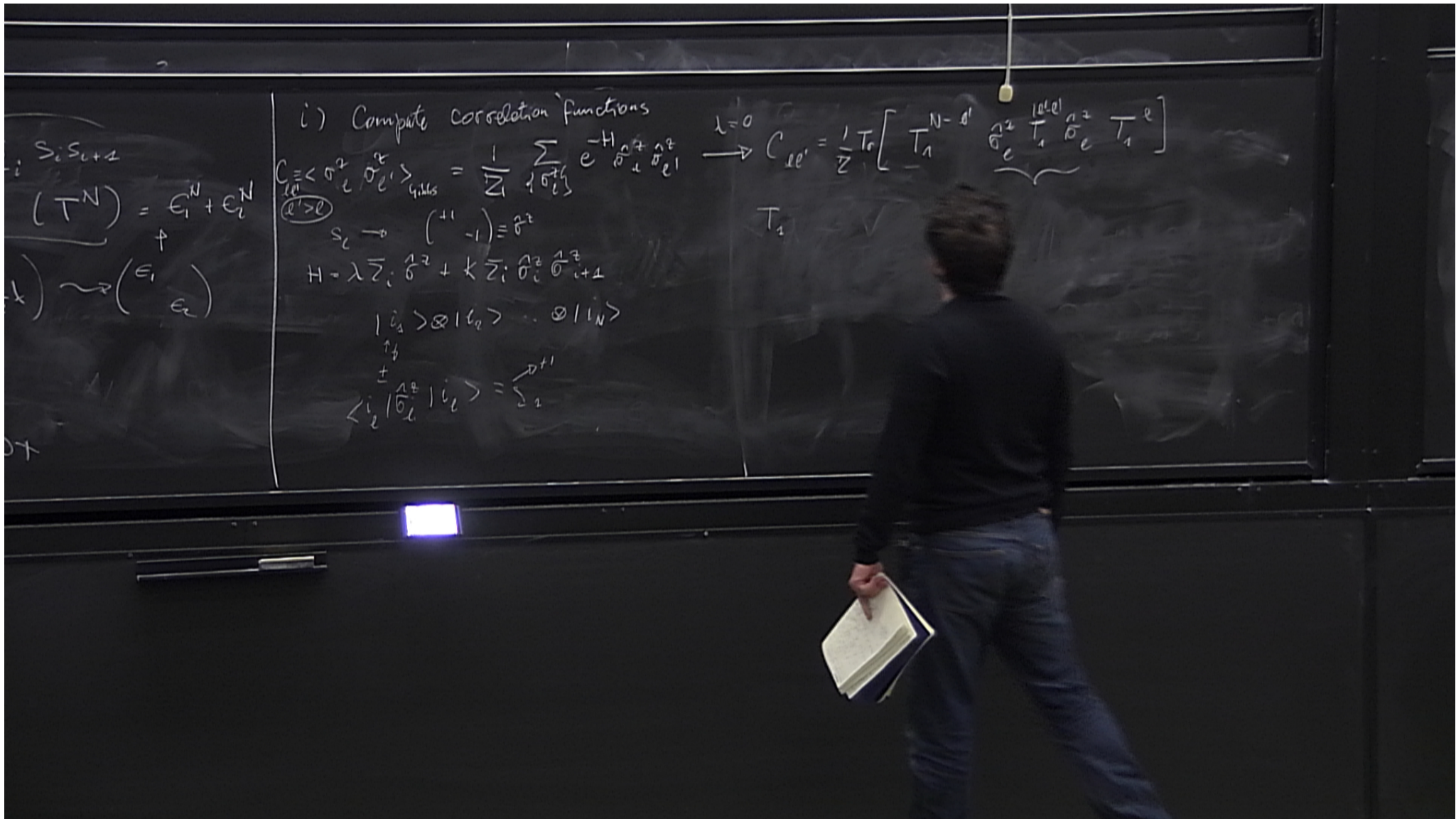
$$T_1(\sigma_i^z, \sigma_{i+1}^x)$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^x | i_2 \rangle = \sum_{\pm} \dots$$



i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\left( \begin{matrix} \epsilon_1 \\ \epsilon_2 \end{matrix} \right)$$

$$C_{ll'} = \langle \sigma_l^x \sigma_{l'}^x \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_l^x, \sigma_{l'}^x}} \sigma_l^x \sigma_{l'}^x$$

$$s_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

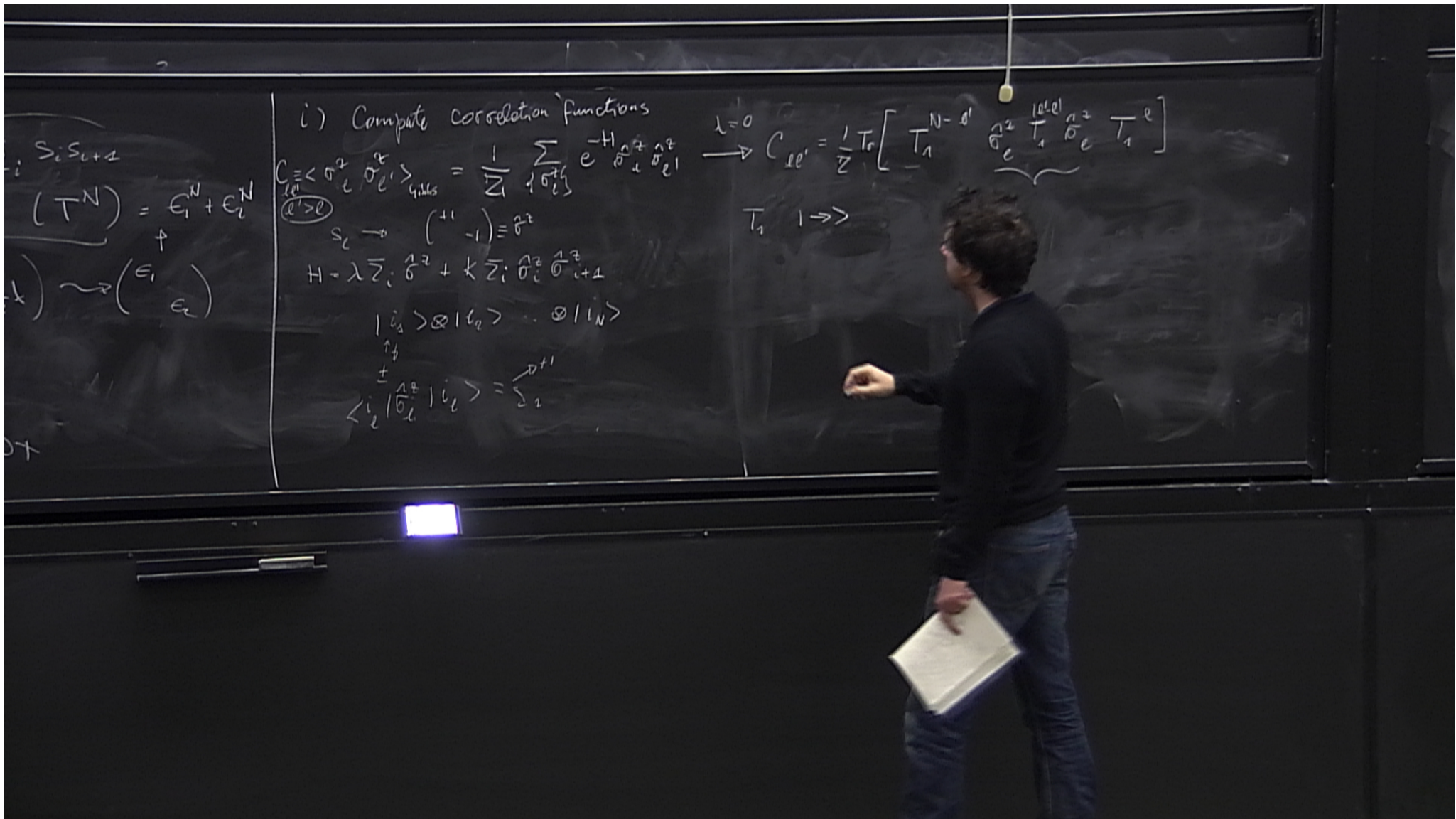
$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \pm 1$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{T_1} \sigma_{l'}^z T_1^l \right]$$

$T_1$



i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$C_{ll'} = \langle \sigma_l^x \sigma_{l'}^x \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_l^x, \sigma_{l'}^x}}$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^x T_1 \sigma_l^x}_{\text{tr}} T_1^l \right]$$

$$s_l \rightarrow \begin{pmatrix} +1 & -1 \end{pmatrix} = \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

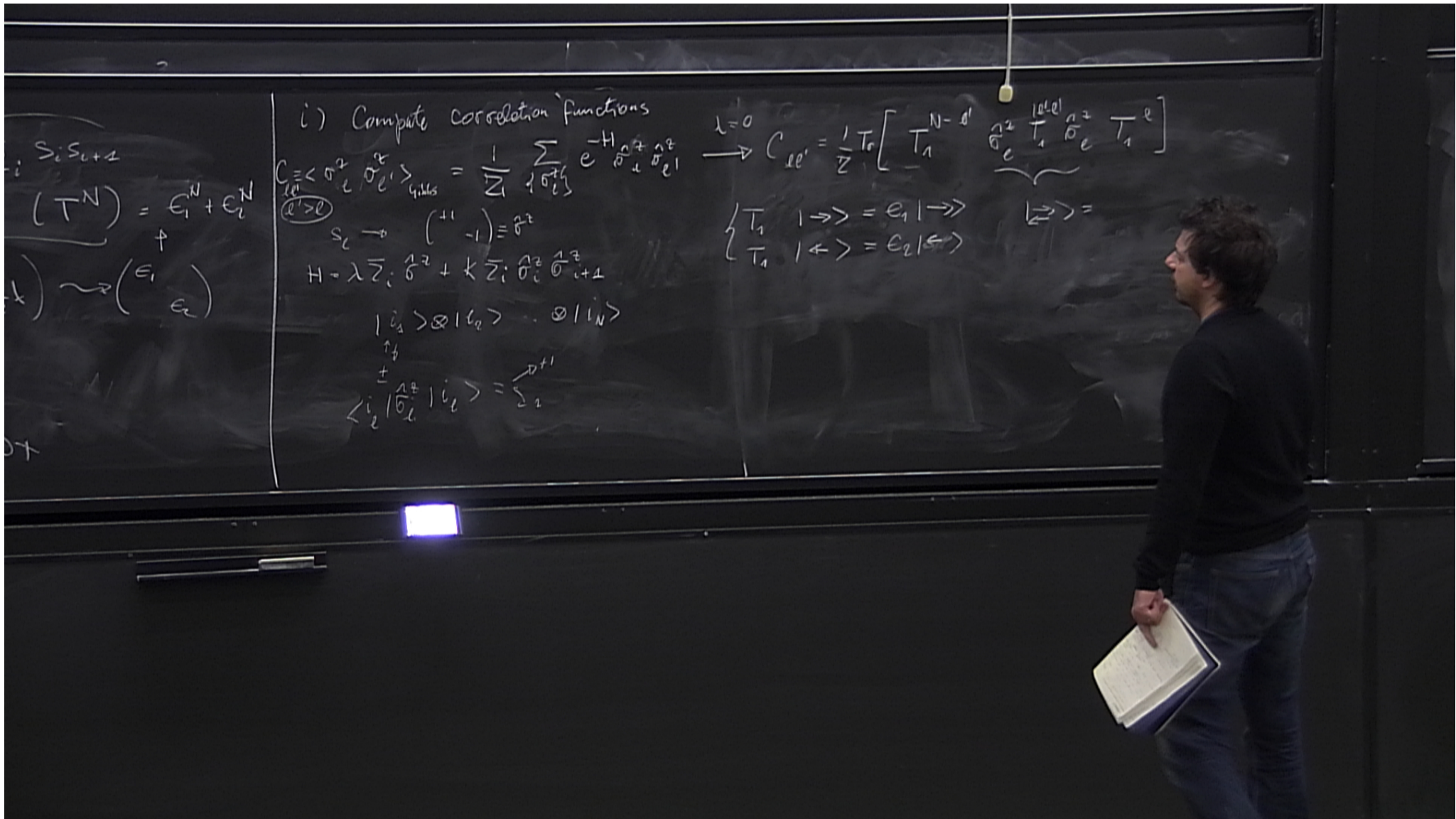
$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \frac{\lambda \pm}{\sigma_l} | i_1 \rangle = \sum_2^{\pm}$$

$T_1 \rightarrow$



i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$C_{ii'} = \langle \sigma_i^x \sigma_{i'}^x \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{i,i'}} \sigma_i^x \sigma_{i'}^x$$

$$S_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

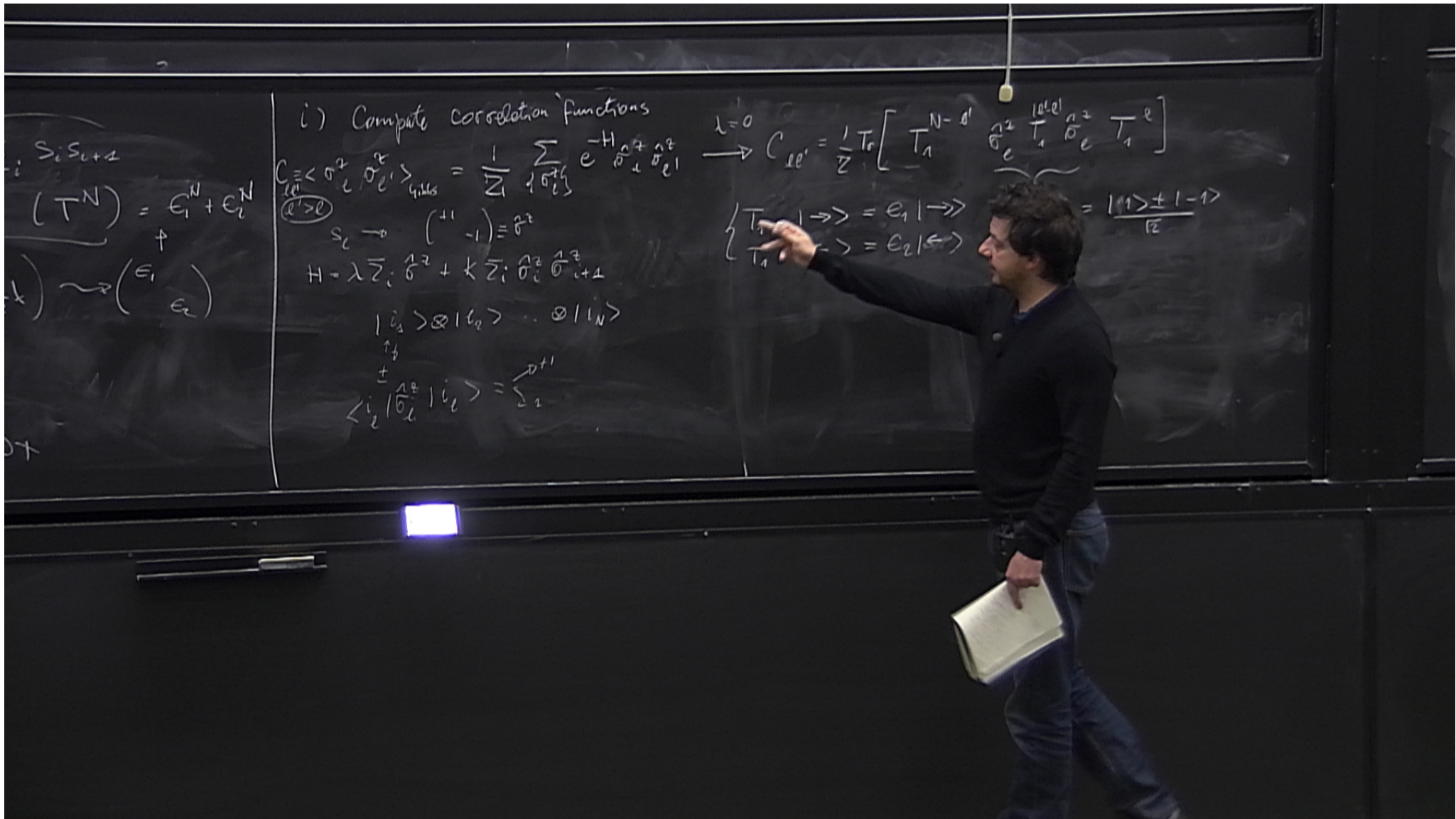
$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_i^z | i_2 \rangle = \sum_{\pm} \dots$$

$$\lambda=0 \rightarrow C_{ii'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-i'} \underbrace{\begin{pmatrix} \sigma_i^z & 1 \\ 1 & \sigma_i^z \end{pmatrix}}_{T_1^e} T_1^e \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases} \quad | \pm \rangle =$$



i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$C_{ll'} = \langle \sigma_l^x \sigma_{l'}^x \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma\}} e^{-H_{\sigma}^{\lambda=0}} \sigma_l^x \sigma_{l'}^x$$

$$s_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \pm 1$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{\begin{matrix} |0\rangle \\ T_1 \\ \sigma_l^z \\ T_1 \end{matrix}} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases} = \frac{| \uparrow \rangle \pm | \downarrow \rangle}{\sqrt{2}}$$

$S_i S_{i+1}$   
 $(TN) = \epsilon_1^N + \epsilon_2^N$   
 $\uparrow$   
 $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$

i) Compute correlation functions

$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i, \sigma_{i+1}}} \sigma_l^z \sigma_{l'}^z$   
 $\sigma_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$   
 $H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$   
 $|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$   
 $\uparrow$   
 $\pm$   
 $\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_2$

$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \sigma_l^z T_1^{l-1} \right]$

$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$   
 $| \pm \rangle = \frac{| +1 \rangle \pm | -1 \rangle}{\sqrt{2}}$   
 $T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$   
 $\sigma^{xz} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$   
 $\uparrow$   
 new basis

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i, \sigma_{i+1}}} \sigma_l^z \sigma_{l'}^z$$

$$S_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_2^{\pm 1}$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{\text{local}} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$$

$$| \pm \rangle = \frac{| \uparrow \uparrow \rangle \pm | \downarrow \downarrow \rangle}{\sqrt{2}}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

↑  
new basis

$$\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & \\ & \epsilon_1 \end{pmatrix}$$

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma\}} e^{-H_{\sigma}^{\lambda, k}} \sigma_l^z \sigma_{l'}^z$$

$$S_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^x$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm \frac{\lambda \pm k}{2} |i_1\rangle = \sum_2$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{\text{local}} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$$

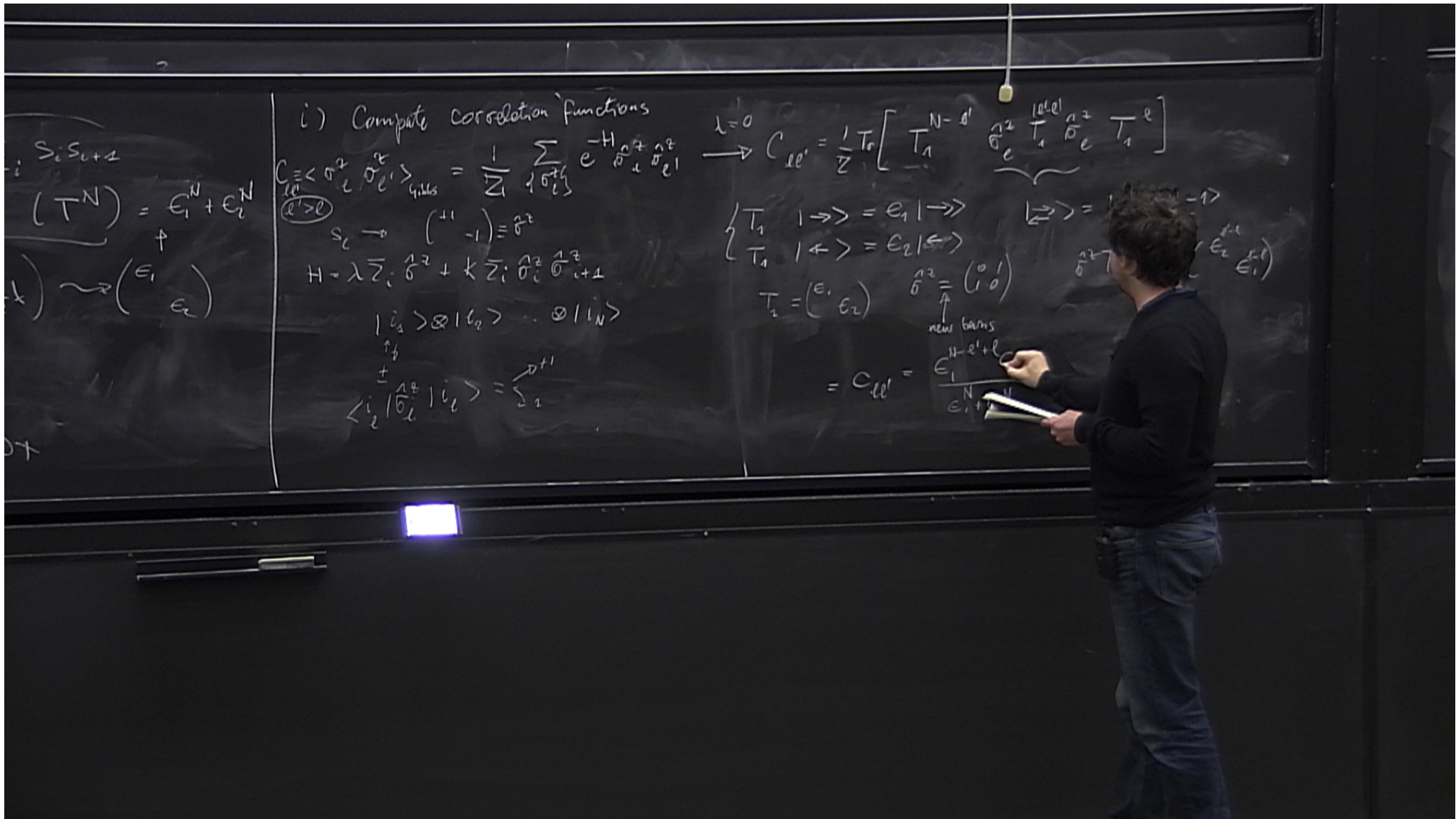
$$| \pm \rangle = \frac{| +1 \rangle \pm | -1 \rangle}{\sqrt{2}}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

↑  
new basis

$$\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & \\ & \epsilon_1 \end{pmatrix}$$





i) Compute correlation functions

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma\}} e^{-H_{\sigma}^{\lambda=0}} \sigma_l^z \sigma_{l'}^z$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{T_1} \sigma_{l'}^z T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle & | \rightarrow \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle & | \leftarrow \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

new basis

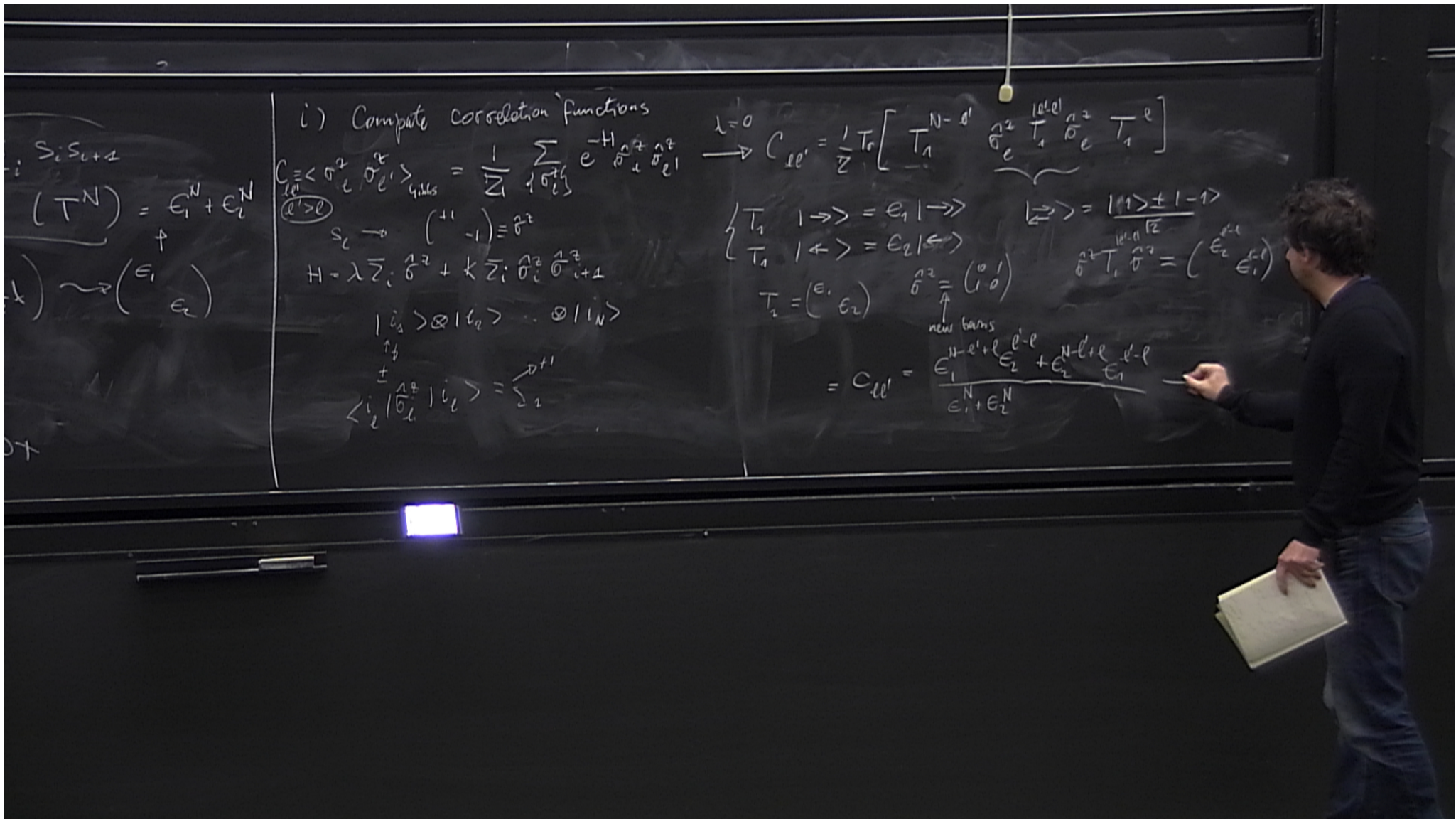
$$= C_{ll'} = \frac{\epsilon_1^{N-l'+l}}{\epsilon_1^N + \epsilon_2^N}$$

$$S_i S_{i+1} \rightarrow (TN) = \epsilon_1^N + \epsilon_2^N$$

$$H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_2$$



i) Compute correlation functions

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i, \sigma_{i+1}}} \sigma_l^z \sigma_{l'}^z$$

$$s_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} = \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_2$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{T_1} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$$

$$| \pm \rangle = \frac{| +1 \rangle \pm | -1 \rangle}{\sqrt{2}}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$$

$$\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & \\ & \epsilon_1 \end{pmatrix}$$

new basis

$$= C_{ll'} = \frac{\epsilon_1^{N-l'+l} \epsilon_2^{l-l} + \epsilon_2^{N-l'+l} \epsilon_1^{l-l}}{\epsilon_1^N + \epsilon_2^N}$$

$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i, \sigma_{i+1}}} \sigma_l^z \sigma_{l'}^z$$

$$s_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \sum_{\pm} \dots$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z}_{T_1} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$$

$$| \pm \rangle = \frac{| \uparrow \uparrow \rangle \pm | \downarrow \downarrow \rangle}{\sqrt{2}}$$

$$\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & \\ & \epsilon_1 \end{pmatrix}$$

$$\sigma^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

new basis

$$= C_{ll'} = \frac{\epsilon_1^{N-l'+l} \epsilon_2^{l-l} + \epsilon_2^{N-l'+l} \epsilon_1^{l-l}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N \rightarrow \infty} \dots$$



$$S_i S_{i+1}$$

$$(TN) = \epsilon_1^N + \epsilon_2^N$$

$$\uparrow$$

$$\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}$$

i) Compute correlation functions

$$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma_i, \sigma_{i+1}}} \sigma_l^z \sigma_{l'}^z$$

$$s_i \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$$

$$H = \lambda \sum_i \sigma_i^z \sigma_{i+1}^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$$

$$\uparrow$$

$$\pm$$

$$\langle i_2 | \sigma_l^z | i_2 \rangle = \dots$$

$$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z T_1 \sigma_l^z}_{\text{trick}} T_1^l \right]$$

$$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases} \quad | \pm \rangle = \frac{| +1 \rangle \pm | -1 \rangle}{\sqrt{2}}$$

$$T_2 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\uparrow$$

new basis

$$\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & \\ & \epsilon_1 \end{pmatrix}$$

$$= C_{ll'} = \frac{\epsilon_1^{N-l'+l} \epsilon_2^{l-l} + \epsilon_2^{N-l'+l} \epsilon_1^{l-l}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N \rightarrow \infty} \tanh k$$

$S_i S_{i+1}$   
 $(TN) = \epsilon_1^N + \epsilon_2^N$   
 $\uparrow$   
 $(\epsilon_1, \epsilon_2)$

i) Compute correlation functions

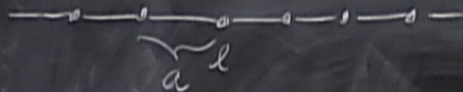
$C_{ll'} = \langle \sigma_l^z \sigma_{l'}^z \rangle_{\text{lim}} = \frac{1}{Z} \sum_{\{\sigma_i\}} e^{-H_{\sigma} + \lambda \sigma_l^z + \lambda' \sigma_{l'}^z}$   
 $\sigma_l \rightarrow \begin{pmatrix} +1 \\ -1 \end{pmatrix} \equiv \sigma^z$   
 $H = \lambda \sum_i \sigma_i^z + k \sum_i \sigma_i^z \sigma_{i+1}^z$   
 $|i_1\rangle \otimes |i_2\rangle \dots \otimes |i_N\rangle$   
 $\uparrow$   
 $\pm \frac{1}{\sqrt{2}} |i_1\rangle = \sum_2$

$\lambda=0 \rightarrow C_{ll'} = \frac{1}{Z} \text{Tr} \left[ T_1^{N-l'} \underbrace{\sigma_l^z T_1 \sigma_l^z}_{\text{local}} T_1^l \right]$

$\begin{cases} T_1 | \rightarrow \rangle = \epsilon_1 | \rightarrow \rangle \\ T_1 | \leftarrow \rangle = \epsilon_2 | \leftarrow \rangle \end{cases}$   
 $| \pm \rangle = \frac{| \uparrow \rangle \pm | \downarrow \rangle}{\sqrt{2}}$   
 $\sigma^z T_1 \sigma^z = \begin{pmatrix} \epsilon_2 & 0 \\ 0 & \epsilon_1 \end{pmatrix}$   
 $T_2 = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix}$   
 $\sigma^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$   
new basis

$= C_{ll'} = \frac{\epsilon_1^{N-l'+l} \epsilon_2^{l-l} + \epsilon_2^{N-l+l} \epsilon_1^{l-l}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N \rightarrow \infty} \tanh k$

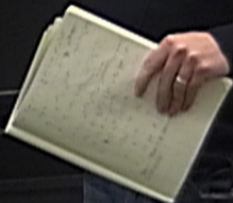
# CONTINUUM LIMIT



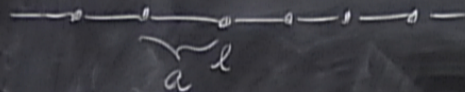
$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\xi^{-1} \equiv \frac{1}{a} \log \coth(K)$$



# CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

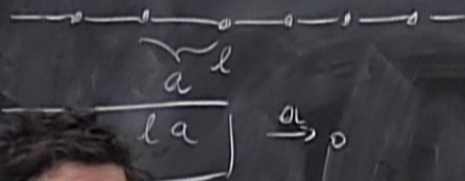
$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(k) \xrightarrow{k \text{ large}} \frac{3}{a} \approx \frac{e^2 k}{2}$$

$$K \rightarrow \infty$$



CONTINUUM LIMIT



$$\langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

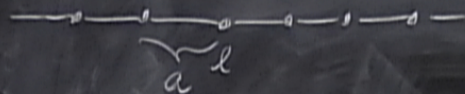
$$\xi \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

$Z_1$  in the large  $K$  limit

$T_1$



CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{4a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z_1$  in the large  $K$  limit

$$T_1 = e^K$$



# TRANSFER MATRIX

1d Ising Model  $H = \lambda \sum_i s_i + K \sum_i s_i s_{i+1}$

$$\begin{aligned} \lambda &= \beta h \\ K &= \beta J \end{aligned}$$

$$Z_N = \text{Tr} e^{-H} = \text{Tr} (T^N) = \epsilon_1^N + \epsilon_2^N$$

$$T = \begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix} \times \begin{pmatrix} e^{\lambda} & \\ & e^{-\lambda} \end{pmatrix} \rightsquigarrow \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$$

$\epsilon_1 > \epsilon_2$   
 $\epsilon_1 \rightarrow 0$   
 $\epsilon_1$  analytic

$K \rightarrow \infty$  1st order PT

i)  
 $C \equiv \langle \sigma_i^2 \rangle$   
 $\langle \sigma_i^2 \rangle > 0$   
 $H =$

$$F = -i \ln Z$$

$$F \rightarrow U F U^\dagger$$

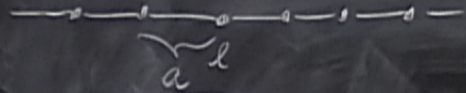
Matrix Computations

$$\psi \rightarrow U \psi \text{ and } \psi \rightarrow U^\dagger \psi$$

$$\psi \rightarrow U \psi \text{ under gauge transform}$$

$$\psi = \text{some field}$$

CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

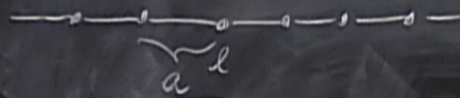
$$K \rightarrow \infty$$

$\xi$  in the large  $K$  limit

$$T_1 = e^K \left( 1 + \frac{a}{2\xi} \frac{\partial T_1}{\partial X} \right)$$



# CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

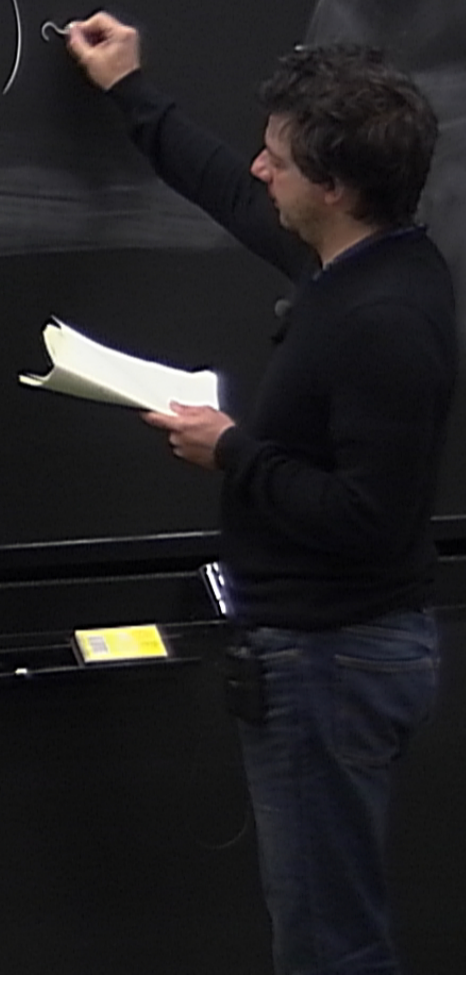
$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

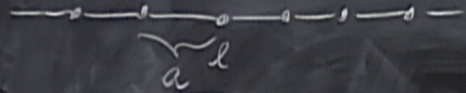
$$K \rightarrow \infty$$

$\xi$  in the large  $K$  limit

$$T_1 \approx e^K \left( 1 + \frac{a}{2\xi} \right)$$



# CONTINUUM LIMIT



$$\tau = la$$

$a \rightarrow 0$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

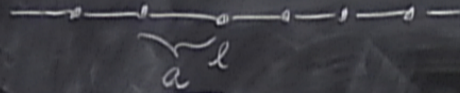
$$K \rightarrow \infty$$

$\sum$  in the large  $K$  limit

$$T_1 \approx e^K \left( 1 + \frac{a}{2} \frac{\partial^2 x}{\partial \tau^2} \right) \approx e^K \exp \left[ \frac{a}{2} \frac{\partial^2 x}{\partial \tau^2} \right]$$

$K$

# CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

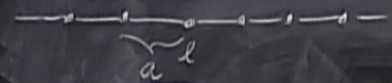
$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

$$T_1 \approx e^K \left( 1 + \frac{a}{2\xi} \right) \approx e^K \exp \left[ \frac{a}{2\xi} \right]$$

$$\frac{K}{a} \equiv -E_0$$

CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \psi^2(\tau) \psi^2(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

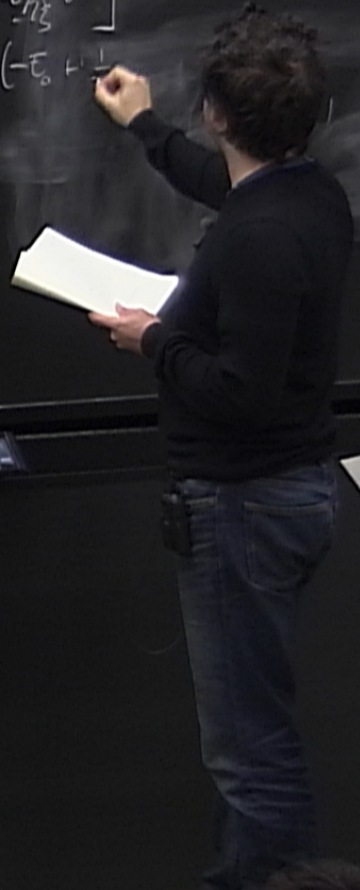
$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

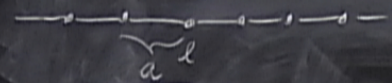
$$T_1 \approx e^K \left( 1 + \frac{a}{23} \frac{\partial^2 X}{\partial x^2} \right) \approx e^K \exp \left[ \frac{a}{23} \frac{\partial^2 X}{\partial x^2} \right]$$

$$= \exp \left[ a \left( -\epsilon_0 + \frac{1}{2} \right) \right]$$

$$\frac{K}{a} \equiv -\epsilon_0$$



CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

$$T_1 \approx e^K \left( 1 + \frac{a}{23} \frac{\partial^2 X}{\partial \tau^2} \right) \approx e^K \exp \left[ \frac{a}{23} \frac{\partial^2 X}{\partial \tau^2} \right]$$

$$= \exp \left[ a \left( -\epsilon_0 + \frac{1}{23} \frac{\partial^2 X}{\partial \tau^2} \right) \right]$$

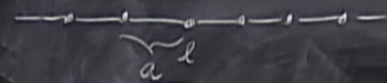
$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{a \frac{\partial^2 X}{\partial \tau^2}} \equiv e^{a \lambda \frac{\partial^2 X}{\partial \tau^2}} = e^{a O_2} \quad T_1 = e^{a O_1}$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2$$



CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{2a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

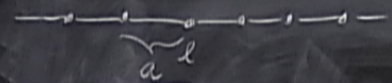
$$T_1 \approx e^K \left( 1 + \frac{a \frac{\partial^2 X}{\partial \sigma^2}}{23} \right) \approx e^K \exp \left[ \frac{a \frac{\partial^2 X}{\partial \sigma^2}}{23} \right]$$

$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{\frac{1}{2} \frac{\partial^2 X}{\partial \sigma^2}} \equiv e^{a \lambda \frac{\partial^2 X}{\partial \sigma^2}} = e^{a O_2} T_1$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2 = e^{a O_1} e^{a O_2}$$

CONTINUUM LIMIT



$$\tau = la \xrightarrow{a \rightarrow 0}$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{2a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

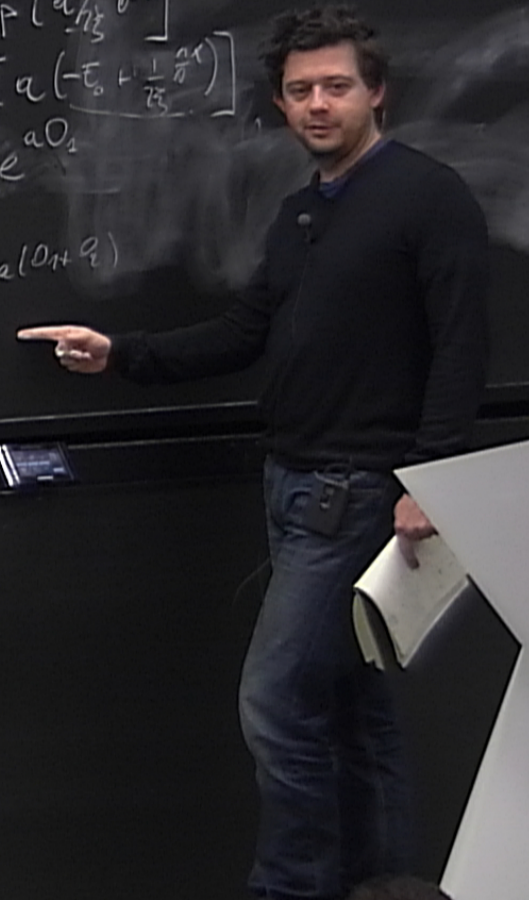
$$T_1 \approx e^K \left(1 + \frac{a}{23} \frac{\partial^2 X}{\partial \tau^2}\right) \approx e^K \exp\left[\frac{a}{23} \frac{\partial^2 X}{\partial \tau^2}\right]$$

$$= \exp\left[a\left(-\epsilon_0 + \frac{1}{23} \frac{\partial^2 X}{\partial \tau^2}\right)\right]$$

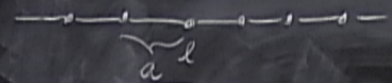
$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{a\lambda} \equiv e^{a\lambda} = e^{a\lambda} T_1 = e^{a\lambda}$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2 = e^{a\lambda} e^{a\lambda} \approx e^{2(a\lambda + \epsilon_0)}$$



CONTINUUM LIMIT



$$\tau = la$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{2a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

$$T_1 \approx e^K \left(1 + \frac{a}{2^3} \frac{\partial^2 X}{\partial \tau^2}\right) \approx e^K \exp\left[\frac{a}{2^3} \frac{\partial^2 X}{\partial \tau^2}\right]$$

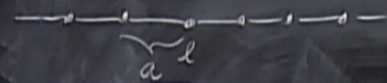
$$= \exp\left[a\left(-\frac{\epsilon_0}{2^3} + \frac{1}{2^3} \frac{\partial^2 X}{\partial \tau^2}\right)\right]$$

$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{a \frac{\partial^2 X}{\partial \tau^2}} \equiv e^{a \lambda \frac{\partial^2 X}{\partial \tau^2}} = e^{a O_2} \quad T_1 = e^{a O_1}$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2 = e^{a O_1} e^{a O_2} \approx e^{a(O_1 + O_2)}$$

CONTINUUM LIMIT



$$\tau = la$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{2a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

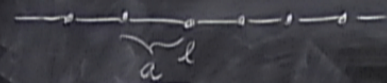
$$T_1 \approx e^K \left(1 + \frac{a}{2\xi}\right) \approx e^K \exp\left[\frac{a}{2\xi} K\right] = \exp\left[a\left(-\frac{\tau_0}{2} + \frac{1}{2\xi} K\right)\right]$$

$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{-\frac{a}{2\xi} K} \equiv e^{-\frac{a}{2\xi} K} = e^{-a\epsilon_0} T_1 = e^{-a\epsilon_0}$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2 = e^{-a\epsilon_0} e^{-a\epsilon_0} \approx e^{-2a\epsilon_0} [1]$$

CONTINUUM LIMIT



$$\tau = la$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{2a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

$$T_1 \approx e^K \left(1 + \frac{a}{23} \frac{\sigma^x}{\sigma^z}\right) \approx e^K \exp\left[\frac{a}{23} \frac{\sigma^x}{\sigma^z}\right]$$

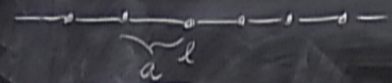
$$= \exp\left[a\left(-\epsilon_0 + \frac{1}{23} \frac{\sigma^x}{\sigma^z}\right)\right]$$

$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{a\sigma^z} \equiv e^{a\lambda \sigma^z} = e^{aO_2} \quad T_1 = e^{aO_1}$$

$$\tilde{\lambda} = \frac{1}{a} \quad T = T_1 T_2 = e^{aO_1} e^{aO_2} \approx e^{a(O_1 + O_2)} [1 + O(a^2)]$$

CONTINUUM LIMIT



$$\tau = la$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|/\xi}$$

$$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth(K) \xrightarrow{K \text{ large}} \frac{3}{a} \approx \frac{e^{2K}}{2}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

$$T_1 \approx e^K \left(1 + \frac{a}{23} \frac{\sigma^x}{\sigma^z}\right) \approx e^K \exp\left[\frac{a}{23} \frac{\sigma^x}{\sigma^z}\right]$$

$$= \exp\left[a\left(-\frac{\epsilon_0}{23} + \frac{1}{23} \frac{\sigma^x}{\sigma^z}\right)\right]$$

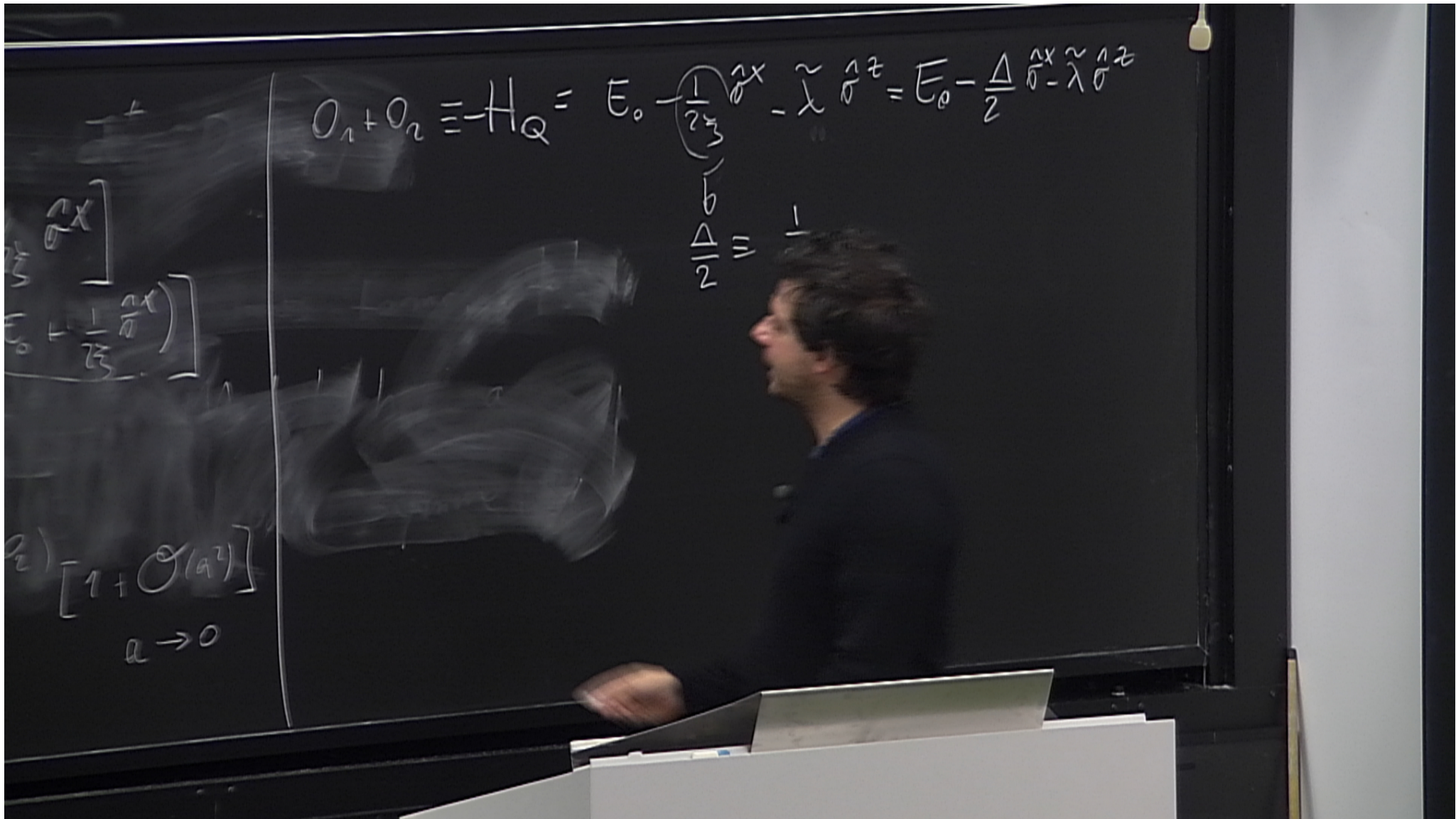
$$\frac{K}{a} \equiv -\epsilon_0$$

$$T_2 = e^{a\sigma^z} \equiv e^{a\lambda\sigma^z} = e^{aO_2} \quad T_1 = e^{aO_1}$$

$$\tilde{\lambda} = \frac{\lambda}{a} \quad T = T_1 T_2 = e^{aO_1} e^{aO_2} \approx e^{a(O_1 + O_2)} [1 + O(a^2)]$$

$$a \rightarrow 0$$

$O_1 + O_2$



$$O_1 + O_2 = -H_Q = E_0 - \left(\frac{1}{2\lambda}\right) \delta^x - \lambda \delta^z = E_0 - \frac{\Delta}{2} \frac{\delta^x}{\lambda} - \lambda \delta^z$$

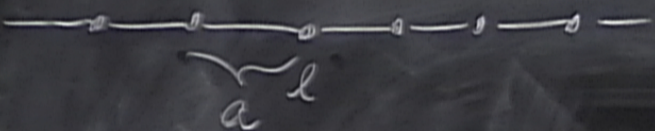
$$\frac{\Delta}{2} \frac{\delta^x}{\lambda} = \frac{1}{2}$$

$$E_0 + \left(\frac{1}{2} \frac{\delta^x}{\lambda}\right)$$

$$[1 + O(a^2)]$$

$a \rightarrow 0$

# CONTINUUM LIMIT



$$\tau = la$$

$$a \rightarrow 0$$

$$C(\tau) \equiv \langle \sigma^z(\tau) \sigma^z(0) \rangle = e^{-|\tau|}$$

$$\frac{1}{\lambda} \equiv \frac{1}{a} \log \coth(k) \xrightarrow{k \rightarrow \infty} \frac{3}{a}$$

$$K \rightarrow \infty$$

$Z$  in the large  $K$  limit

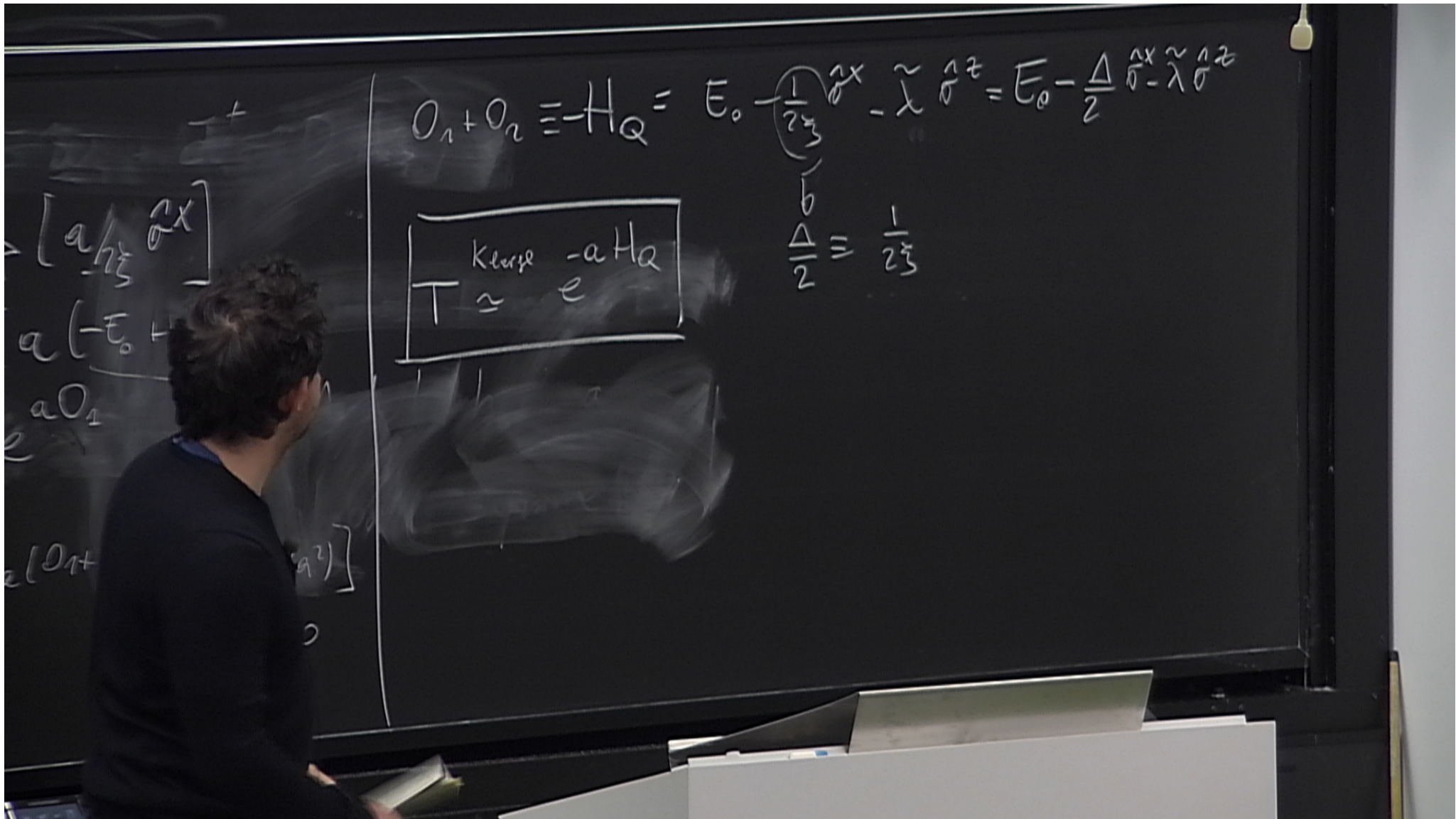
$$T_1 \approx e^K \left( 1 + \frac{a}{2\lambda} \sigma^x \right) \approx e^K$$

$$\frac{K}{a} \equiv -E_0$$

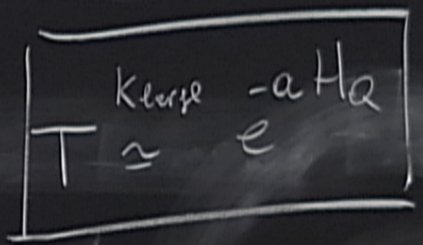
$$T_2 = e^{\lambda \sigma^z} \equiv e^{a \lambda \sigma^z} = e^{a D_2}$$

$$\lambda = \frac{1}{a} \quad T = T_1 T_2 = e^{a D_1} e^{a D_2}$$





$$O_1 + O_2 \equiv -H_Q = E_0 - \left(\frac{1}{23}\right) \sigma^x - \lambda \sigma^z = E_0 - \frac{\Delta}{2} \sigma^x - \lambda \sigma^z$$



$$\frac{\Delta}{2} \equiv \frac{1}{23}$$

$$\sum_1 = (T_1, T_2)^N = (T_1, T_2)^{L/a}$$

$$L = N$$

$$\sum_i = (T_1, T_2)^N = (T_1, T_2)^{L/a} = T_r e^{-}$$

$$L_z = Na$$

$$Z_1 = \underbrace{(T_1, T_2)}_{\tilde{T}}^{N'} = (T_1, T_2)^{L/a} = T_r e^{-L_T H Q^2}$$

$$L_T = N a$$

$$Z_1 = \underbrace{(T_1, T_2)^N}_{\tilde{T}} = (T_1, T_2)^{L_c/a} = \text{Tr} e^{-L_c H Q^2} = \text{Tr} e^{-L_c \frac{1}{T}}$$

$$L_c = Na$$

$$L_c \equiv \frac{1}{T}$$

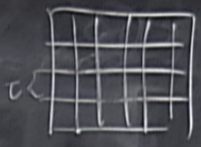
$$Z_1 = \underbrace{(T_1, T_2)}_{\tilde{T}}^{N'} = (T_1, T_2)^{L_c/a} = \text{Tr} e^{-L_c H Q^2} = \text{Tr} e^{-H Q/T}$$

$$L_c = N a$$

$$L_c \equiv \frac{1}{T}$$

$$T \propto e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM

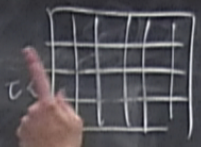
$H_Q$

$d=1$  spatial dim + 1

$H_Q$

$$T \sim e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

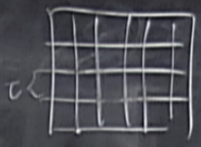
QM model  $H_Q$   $d=1$  spatial  $d=1$  + 1

$$H_Q = -g \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z$$



$$T \sim e^{-H_Q/T}$$

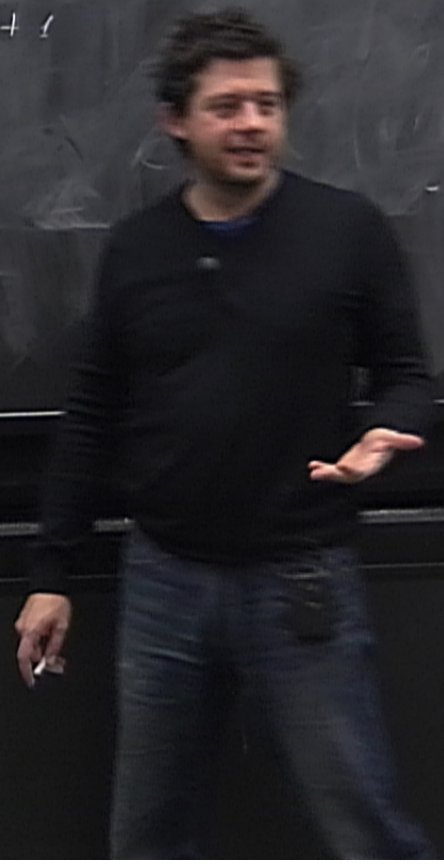
$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z$$



$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

CONTINUUM

$$\tau = la$$

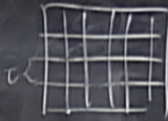
$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} la$$

$$K \rightarrow d$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial  $d+1$

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

CONTINUUM

$$\tau = la$$

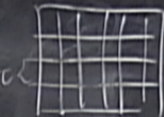
$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} la$$

$$K \rightarrow \infty$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
 $\downarrow$  continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

CONTINUUM

$$\tau = la$$

$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} la$$

$$K \rightarrow \infty$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Phase Transitions

$T=0$   $H(\lambda)$   $\lambda=(\lambda', \lambda'')$

↓  
10



CONTINUUM

$$\tau = la$$

$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} la$$

$$K \rightarrow \infty$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Phase Transitions

$$T=0 \quad H(\lambda) \quad \lambda = (\lambda', \lambda'')$$

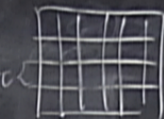
↓  
 $|\phi(\lambda)\rangle$

CONTINUUM

$$\tau = \tau_0$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



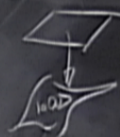
Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Phase Transitions

$T=0$   $H(\lambda)$   $\lambda = (\lambda^1, \lambda^2)$   
↓  
 $|0(\lambda)\rangle$



CONTINUUM

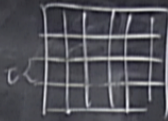
$$\tau = l a$$

$$\frac{1}{v} \equiv \frac{1}{a} l a$$

$$K \rightarrow cD$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



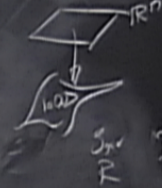
Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Quantum Phase Transitions

$T=0$   $H(\lambda)$   $\lambda = (\lambda', \lambda'')$   
↓  
Energy  $|\psi(\lambda)\rangle$



CONTINUUM

$$\tau = \lambda a$$

$$C(\tau) \equiv$$

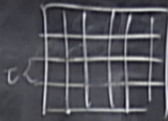
$$\frac{1}{\omega} \equiv \frac{1}{a} \text{ length}$$

$$K \rightarrow \infty$$



$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

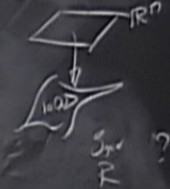
$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

### Quantum Phase Transitions

$T=0$   $H(\lambda)$   $\lambda = (\lambda', \lambda'')$   
↓ smooth  $\mathbb{R}^n$   
 $\rho(\lambda)$

Energy density  
 $E(\lambda)$  continuous

$$\frac{\partial E_1}{\partial \lambda}$$



### CONTINUUM

$$\tau = l a$$

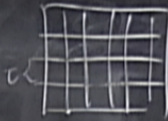
$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} l a$$

$$K \rightarrow \infty$$

$$= \text{Tr} e^{-H_Q/T}$$

$d=2$  classical Ising model



Transfer Matrix  
↓  
continuum limit

QM model  $H_Q$   $d=1$  spatial dim + 1

$$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

### Quantum Phase Transitions

$T=0$   $H(\lambda)$   $\lambda=(\lambda', \lambda'')$   
↓ smooth  $\mathbb{R}^n$   
 $|\phi(\lambda)\rangle$

Energy density

$E(\lambda)$  continuous

$$\frac{\partial E_1}{\partial \lambda}, \frac{\partial E}{\partial \lambda^2}$$

continuous?

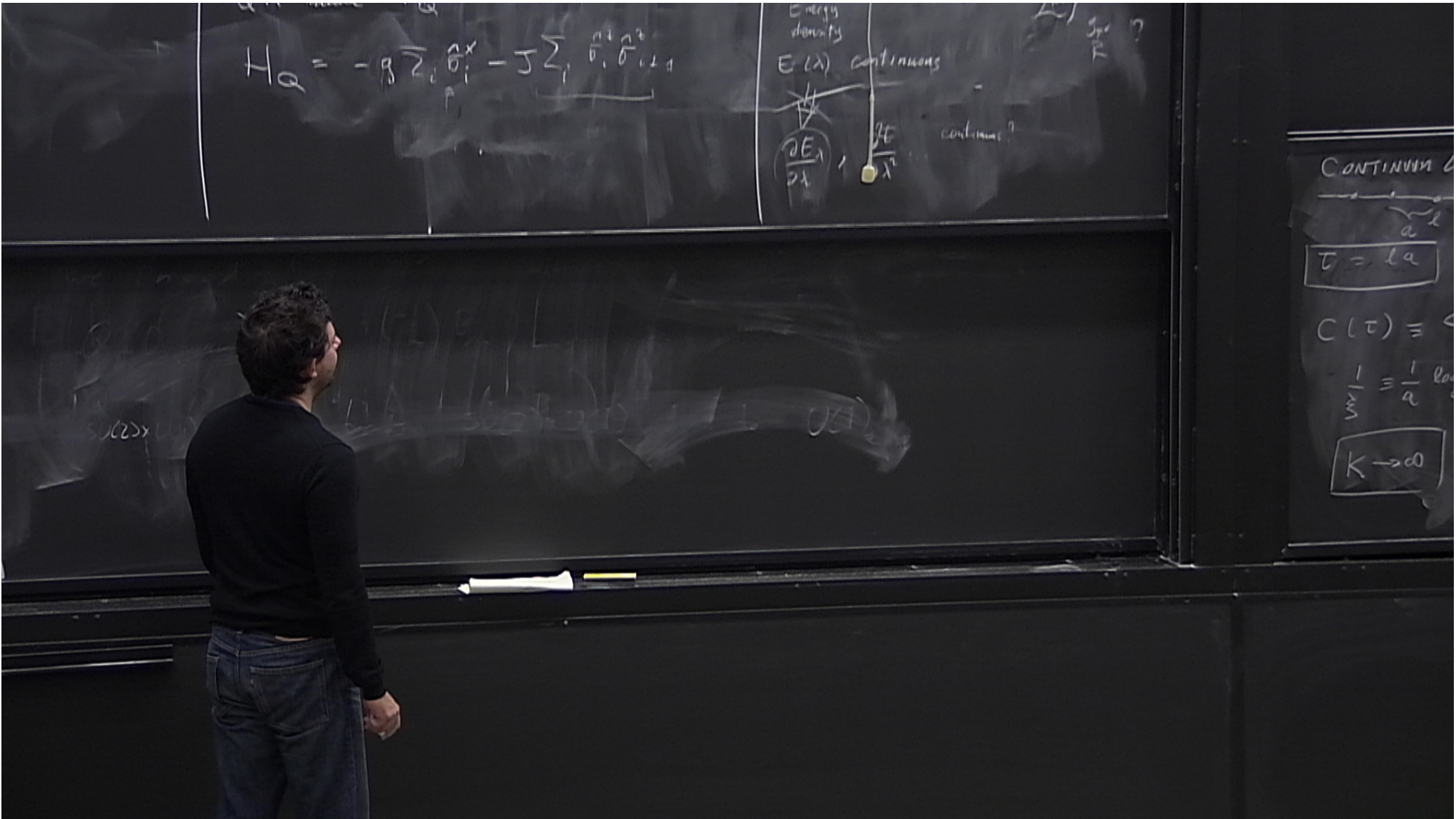
CONTINUUM

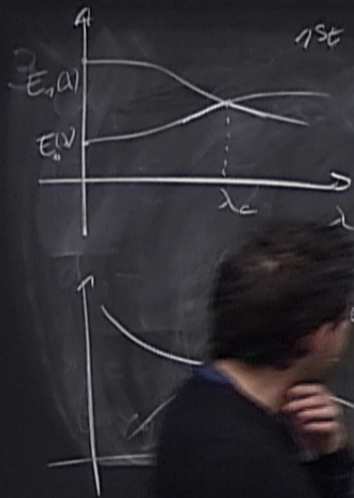
$$\tau = \lambda a$$

$$C(\tau) \equiv$$

$$\frac{1}{\omega} \equiv \frac{1}{a} \text{ length}$$

$$K \rightarrow \infty$$





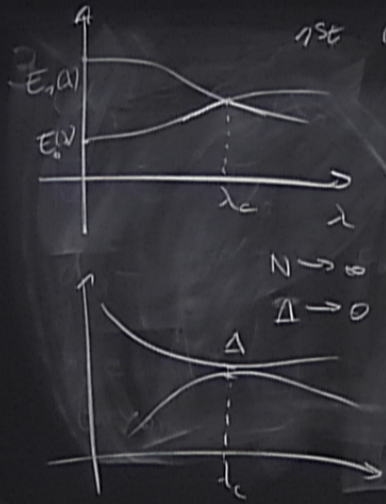
1st QPT

in the therm. limit

Continuous QPT

$$\Delta \sim J |\lambda - \lambda_c|^{2\nu}$$

critical exponent



1st QPT

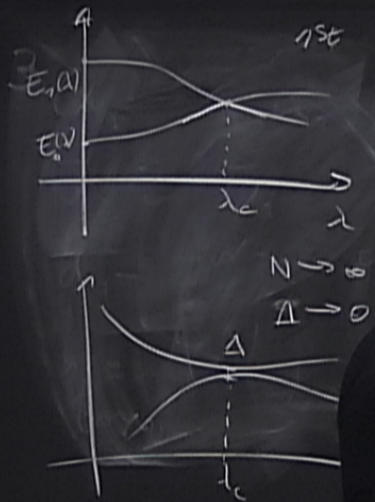
$N \rightarrow \infty$   
 $\Delta \rightarrow 0$

in the limit

$C_0$

QPT

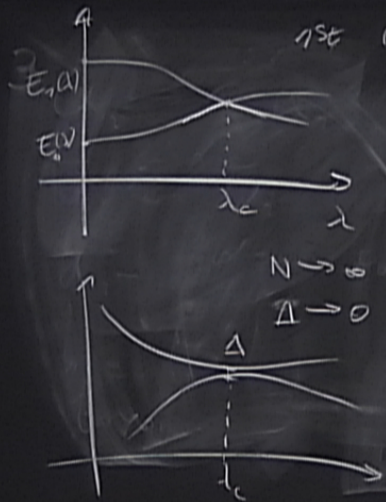
critical exponent



Therm. Orient

QPT

critical exponent



1st QPT

due to

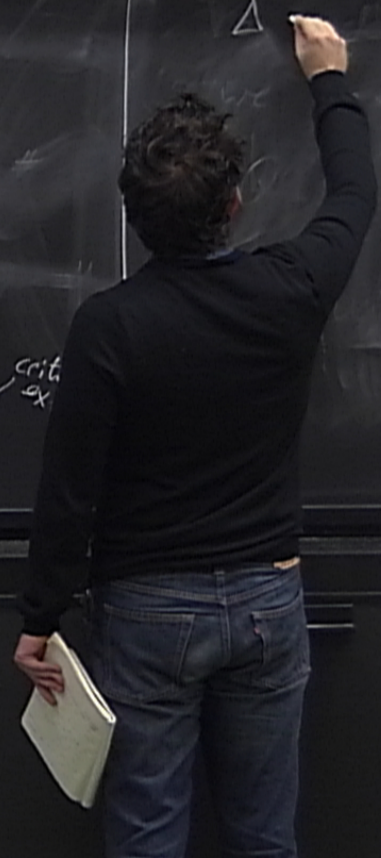
$N \rightarrow \infty$

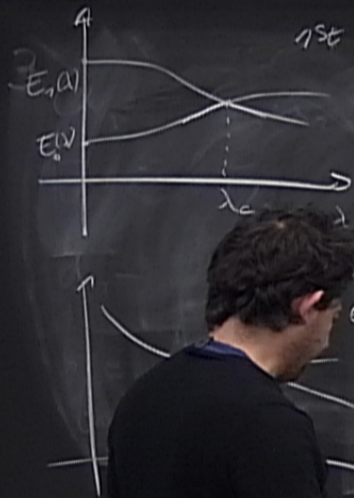
$\Delta \rightarrow 0$  in the therm. limit

Continuous QPT

$$\Delta \sim J |\lambda - \lambda_c|^{2\nu} \rightarrow \text{crit ex}$$

$\Delta$





1st QPT

in the therm. limit

Continuous QPT

$$\Delta \sim J |\lambda - \lambda_c|^{2\nu}$$

critical exponent

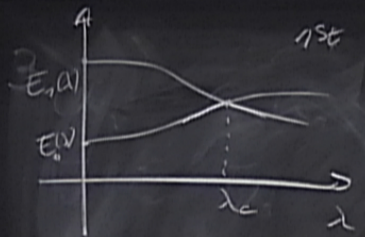
$$\Delta \sim |\lambda - \lambda_c|^{-z}$$

$z$  dynamical critical exp

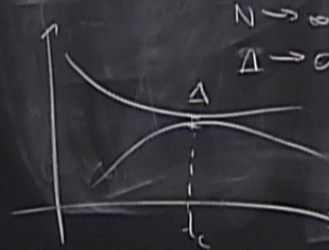
Ising model  $1+z = z = 1$

$$\xi = \lambda |\lambda - \lambda_c|^{-\nu}$$





1st QPT



$N \rightarrow \infty$

$\Delta \rightarrow 0$  in the therm. limit

Continuous QPT

$\Delta \sim J |\lambda - \lambda_c|^{2\nu}$  critical exponent

$\Delta \sim |\lambda - \lambda_c|^{-z}$

$z$  dynamical critical exp

Ising model  $1+z = 2 \Rightarrow z = 1$

$\xi = \lambda |\lambda - \lambda_c|^{-\nu}$

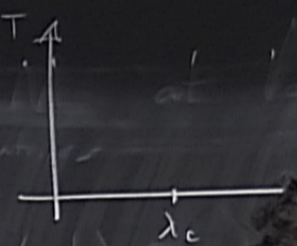


$$\Delta \sim \sum^{-z}$$

$z$  dynamical critical exp

Ising model  $1+z$   $z=1$

$$\xi = \lambda |\lambda - \lambda_c|^{-z}$$



$K$

critical exponent

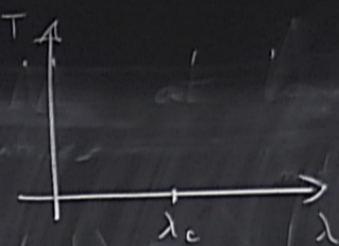
$$\Delta \sim \sum^{-z}$$

$z$  dynamical critical exp

Ising model  $1+z$   $z=1$

$$\xi = \Lambda |\lambda - \lambda_c|^{-\nu}$$

critical exponent



$k_B T$  thermal  
 $h$   
 $T_{eq}$



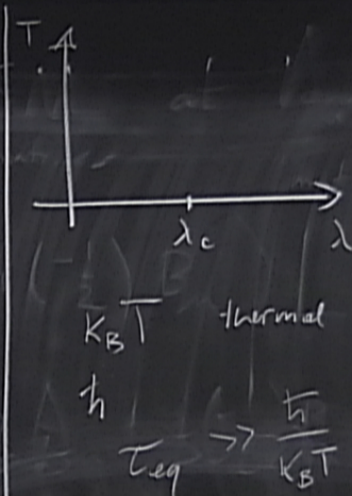
$$\Delta \sim \sum^{-z}$$

$z$  dynamical critical exp

Ising model  $1+z$   $z=1$

$$\xi = \Lambda |\lambda - \lambda_c|^{-\nu}$$

critical exponent



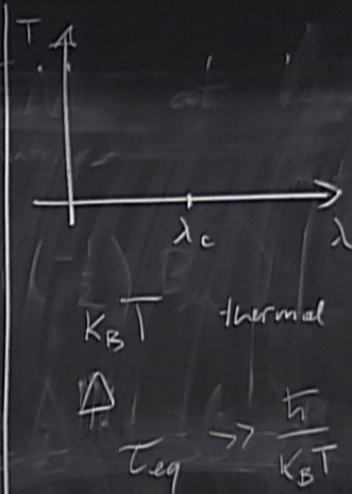
$$\Delta \sim |\lambda - \lambda_c|^{-z}$$

$z$  dynamical critical exp

Ising model  $1+z$   $z=1$

$$\xi = \Lambda |\lambda - \lambda_c|^{-\nu}$$

critical exponent

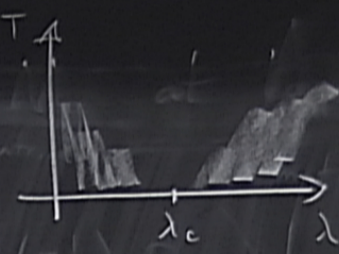


$$\Delta \sim \sum^{-z}$$

z dynamical critical exp

Ising model 1+1 z=1

$$\xi = \lambda |\lambda - \lambda_c|^{-\nu}$$



$k_B T$  thermal  
 $\Delta$   
 $\tau_{eq} \rightarrow \frac{t}{k_B T}$

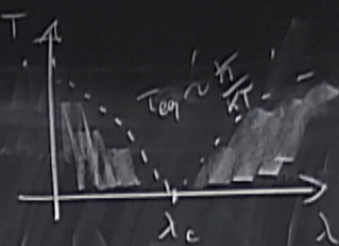
critical exponent  
 $\nu$

$$\Delta \sim |\lambda - \lambda_c|^{-z}$$

z dynamical critical exp

Ising model 1+1 z=1

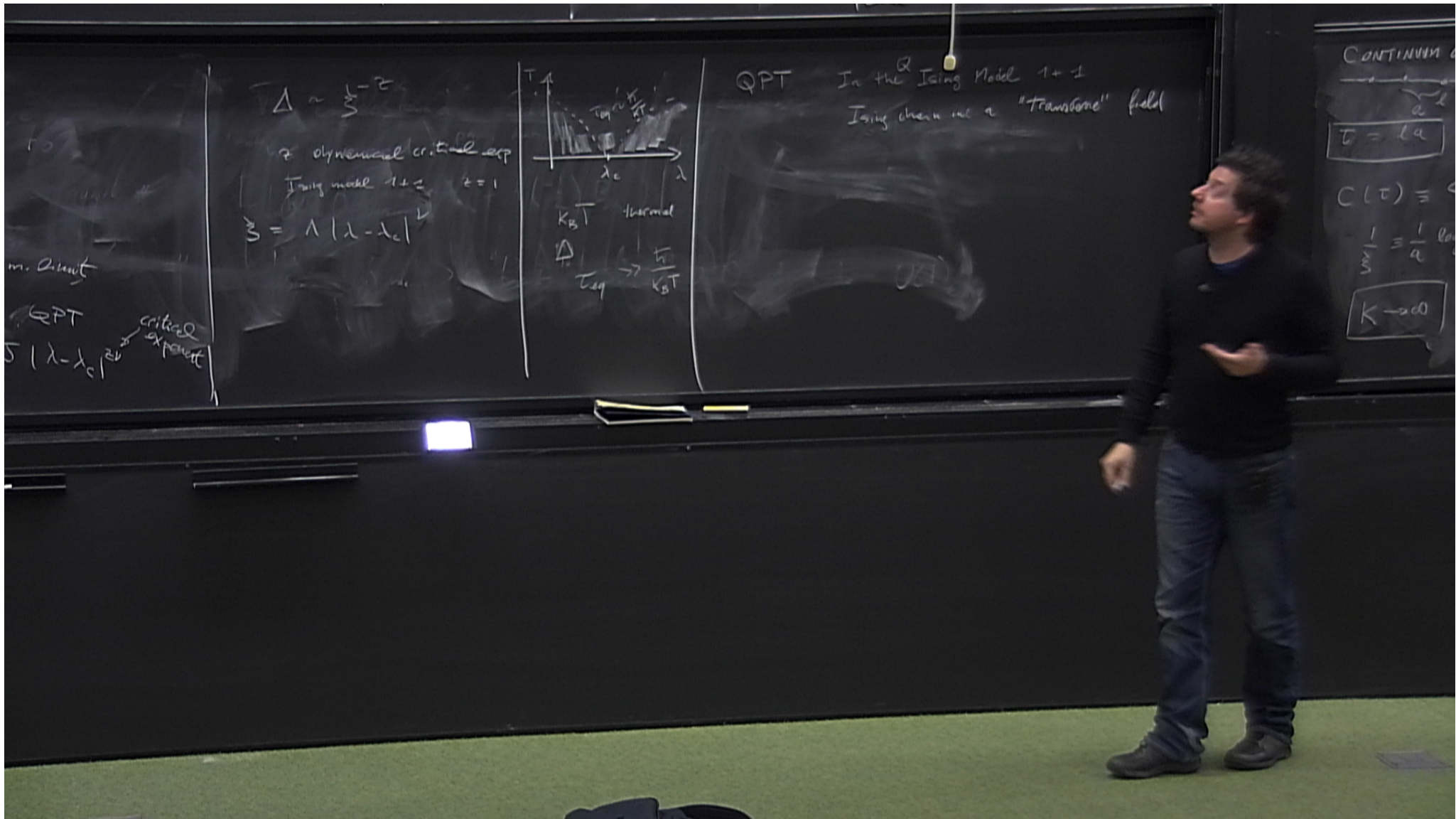
$$\xi = \lambda |\lambda - \lambda_c|^{-1}$$



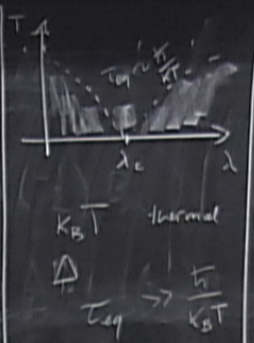
$K_B T$  thermal

$$\Delta \rightarrow \frac{h}{K_B T}$$

critical exponent  
z



$\Delta \sim \xi^{-z}$   
 dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$

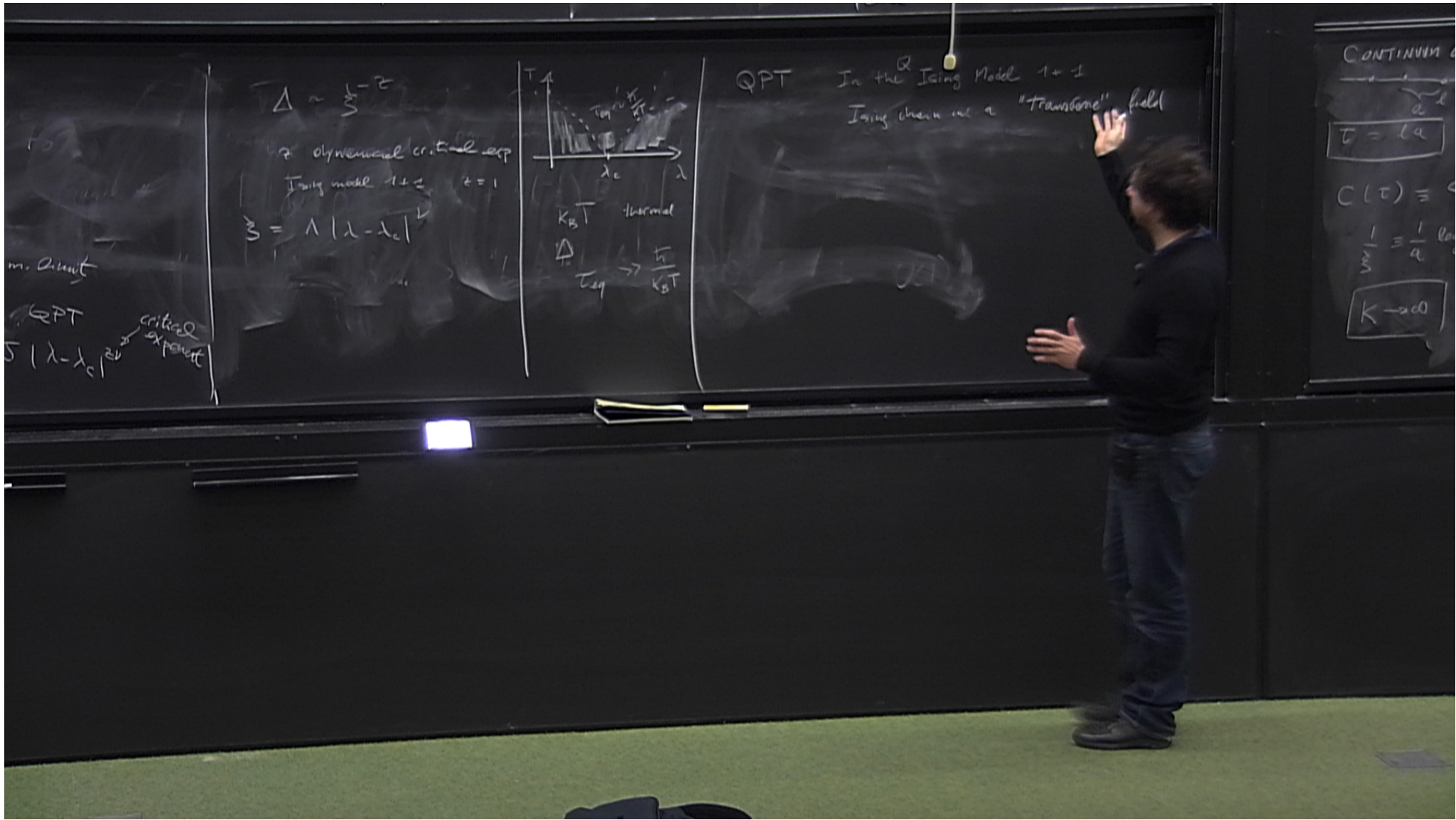


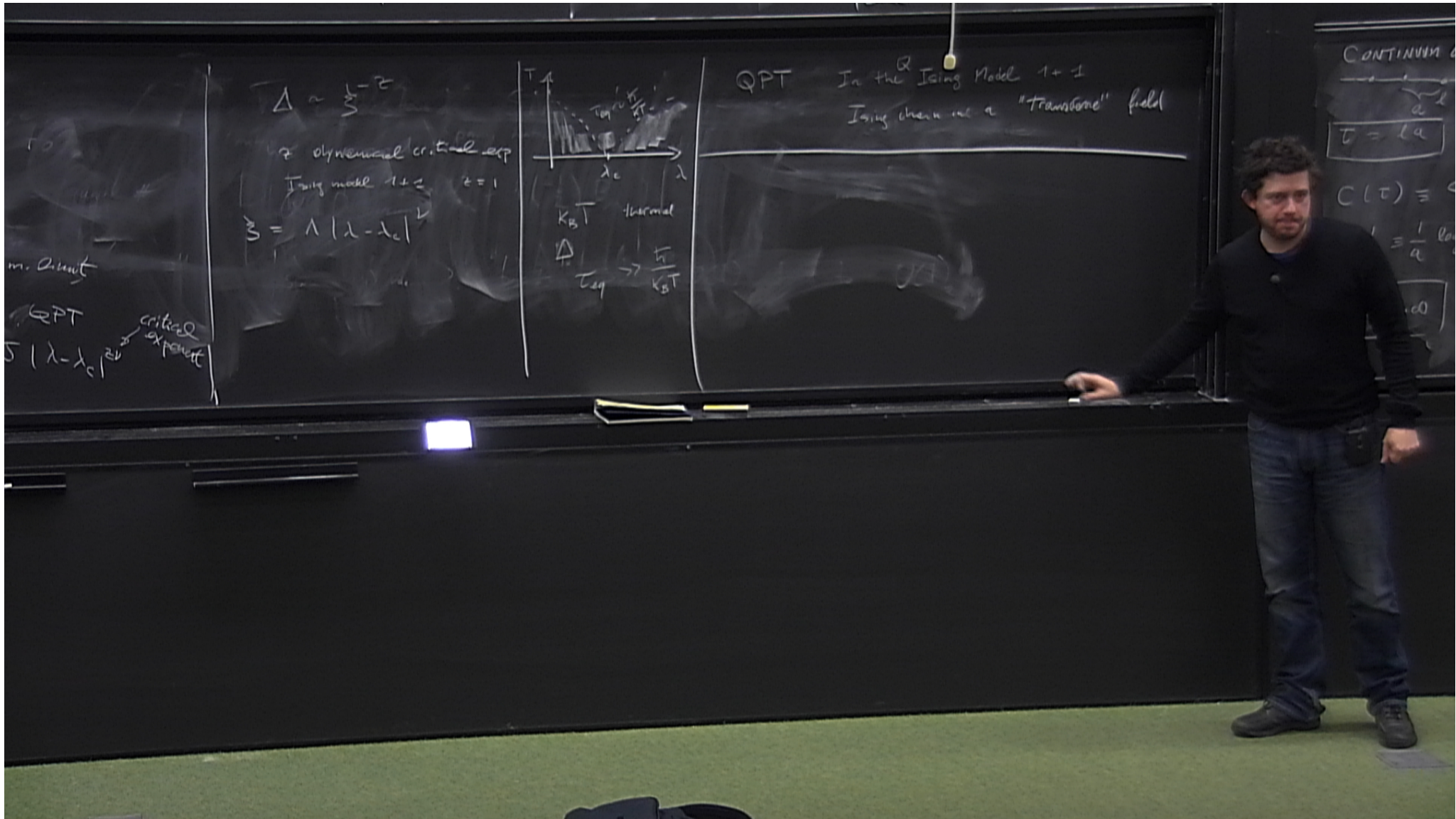
QPT In the  $1+1$  Ising Model  
 Ising chain in a "transverse" field

QPT  
 $\xi \sim |\lambda - \lambda_c|^{-\nu}$

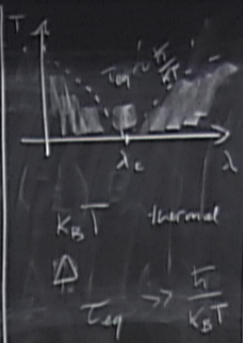
CONTINUUM  
 $\tau = \lambda a$   
 $C(\tau) \equiv$   
 $\frac{1}{\omega} \equiv \frac{1}{a} \ln$   
 $K \rightarrow \infty$







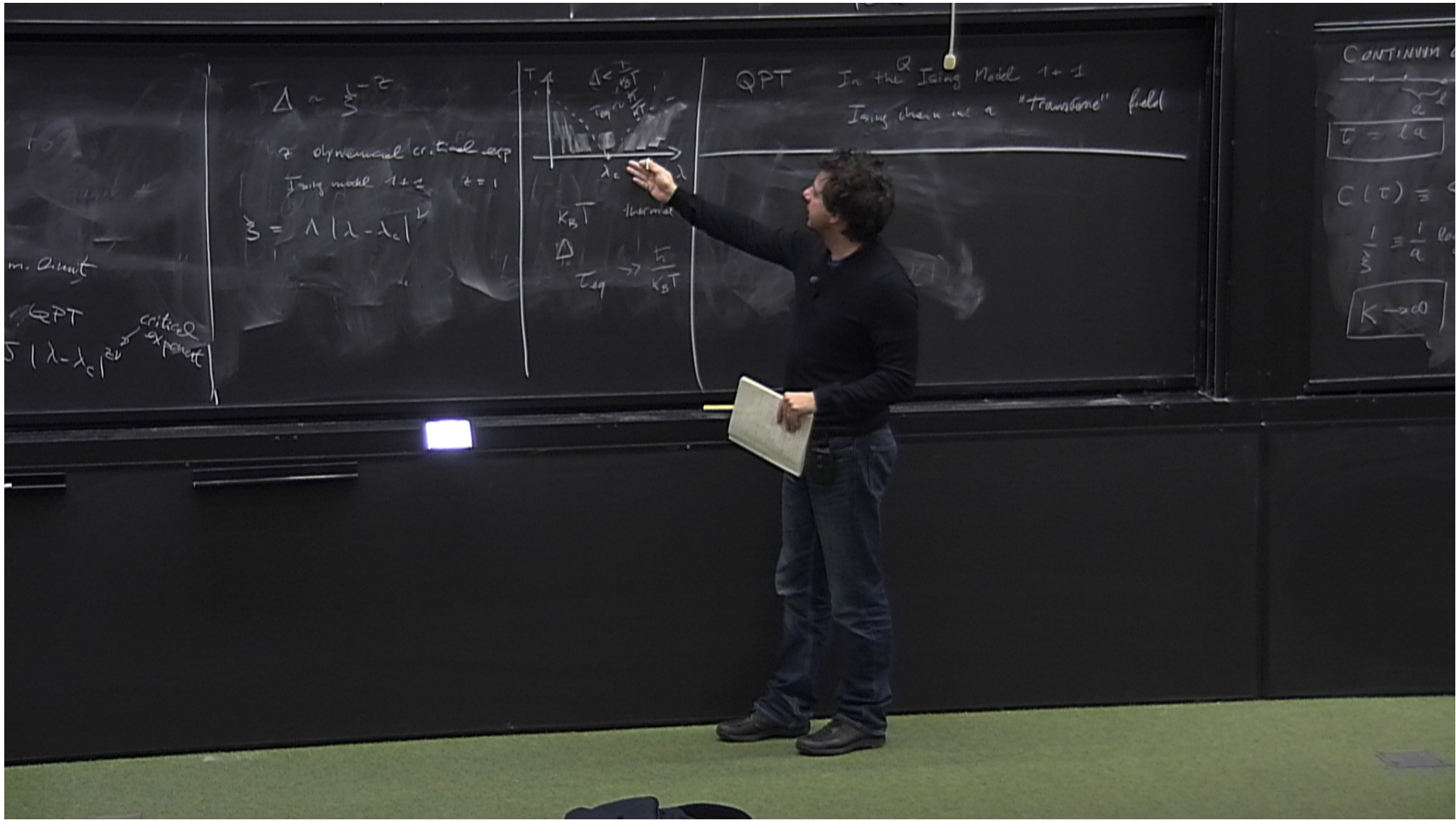
$\Delta \sim \xi^{-z}$   
 $z$  dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$

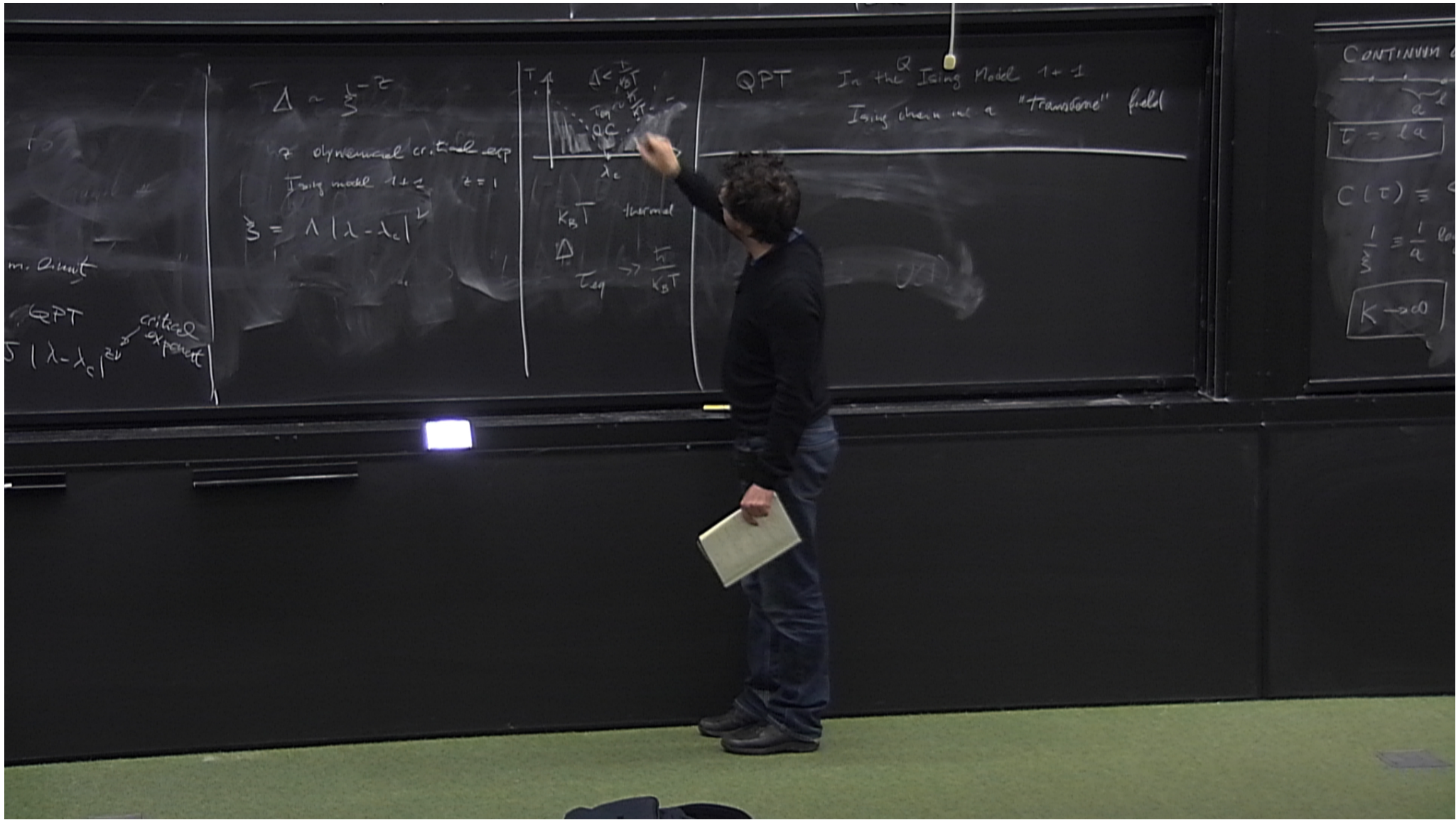


QPT In the Ising Model  $1+1$   
 Ising chain in a "transverse" field

QPT  
 $\xi \sim |\lambda - \lambda_c|^{-\nu}$  critical exponent

CONTINUUM  
 $\tau = \lambda a$   
 $C(\tau) \equiv$   
 $\equiv \frac{1}{a} \ln$

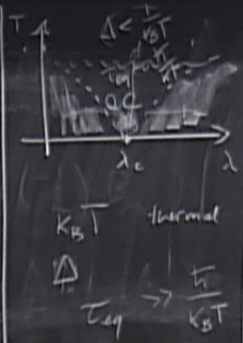








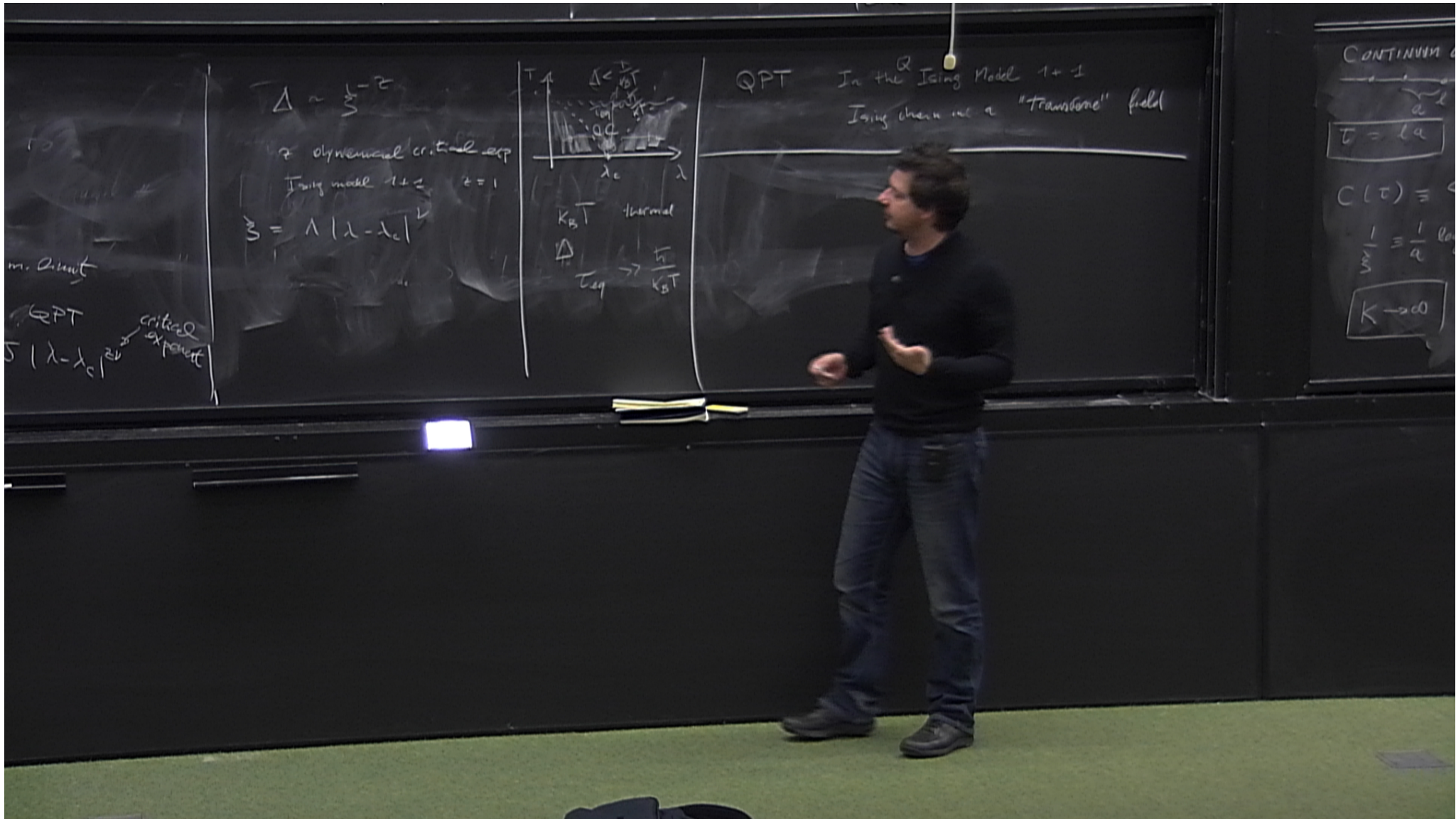
$\Delta \sim \xi^{-z}$   
 dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$



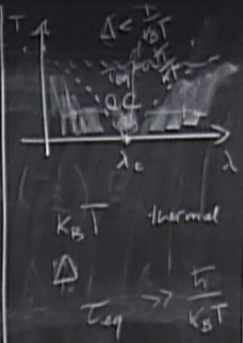
QPT In the Ising Model  $1+1$   
 Ising chain in a "transverse" field

QPT  
 $\xi \sim |\lambda - \lambda_c|^{-\nu}$  critical exponent

CONTINUUM  
 $\tau = \lambda a$   
 $C(\tau) \equiv$   
 $\frac{1}{\omega} \equiv \frac{1}{a} \ln$   
 $K \rightarrow \infty$



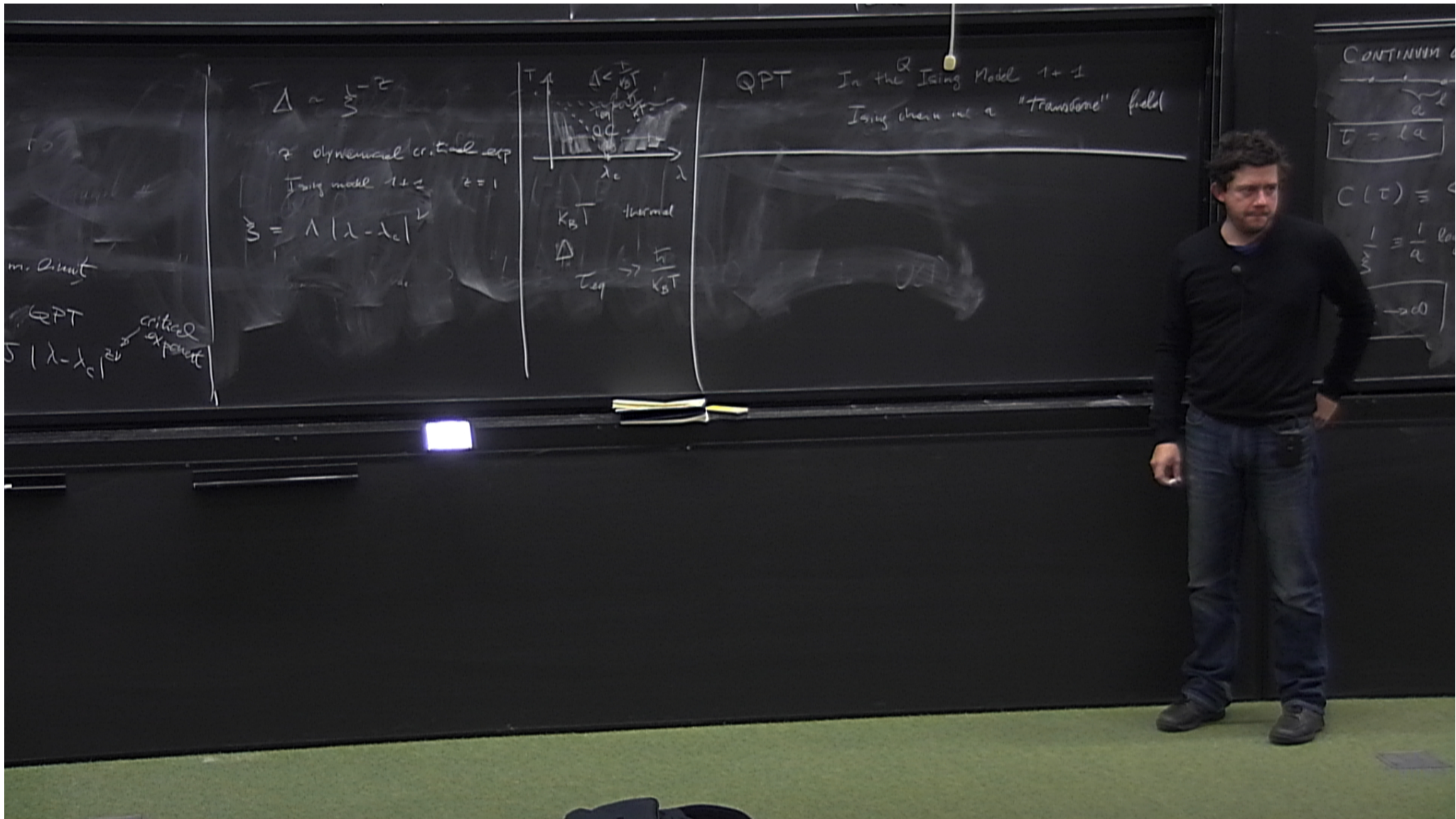
$\Delta \sim \xi^{-z}$   
 $z$  dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$



QPT In the  $1+1$  Ising Model  
 Ising chain in a "transverse" field

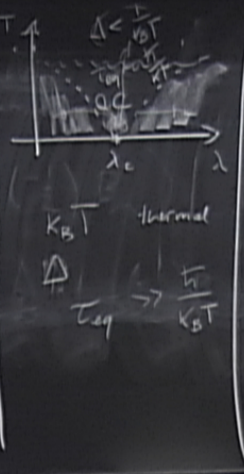
QPT  
 $\xi \sim |\lambda - \lambda_c|^{-\nu}$  critical exponent

CONTINUUM  
 $\tau = l a$   
 $C(\tau) \equiv$   
 $\frac{1}{\omega} \equiv \frac{1}{a}$   
 $K \rightarrow \infty$



$\Delta \sim \xi^{-z}$   
 dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$   
 QPT  
 $|\lambda - \lambda_c|^{-2\nu}$  critical exponent

$\Delta \sim \xi^{-z}$   
 dynamical critical exp  
 Ising model  $1+1$   $z=1$   
 $\xi = \lambda |\lambda - \lambda_c|^{-\nu}$



QPT In the Ising Model  $1+1$   
 Ising chain in a "transverse" field

CONTINUUM  
 $\tau = \lambda a$   
 $C(\tau) \equiv$   
 $\frac{1}{\omega} \equiv \frac{1}{a} \ln$   
 $\rightarrow \infty$