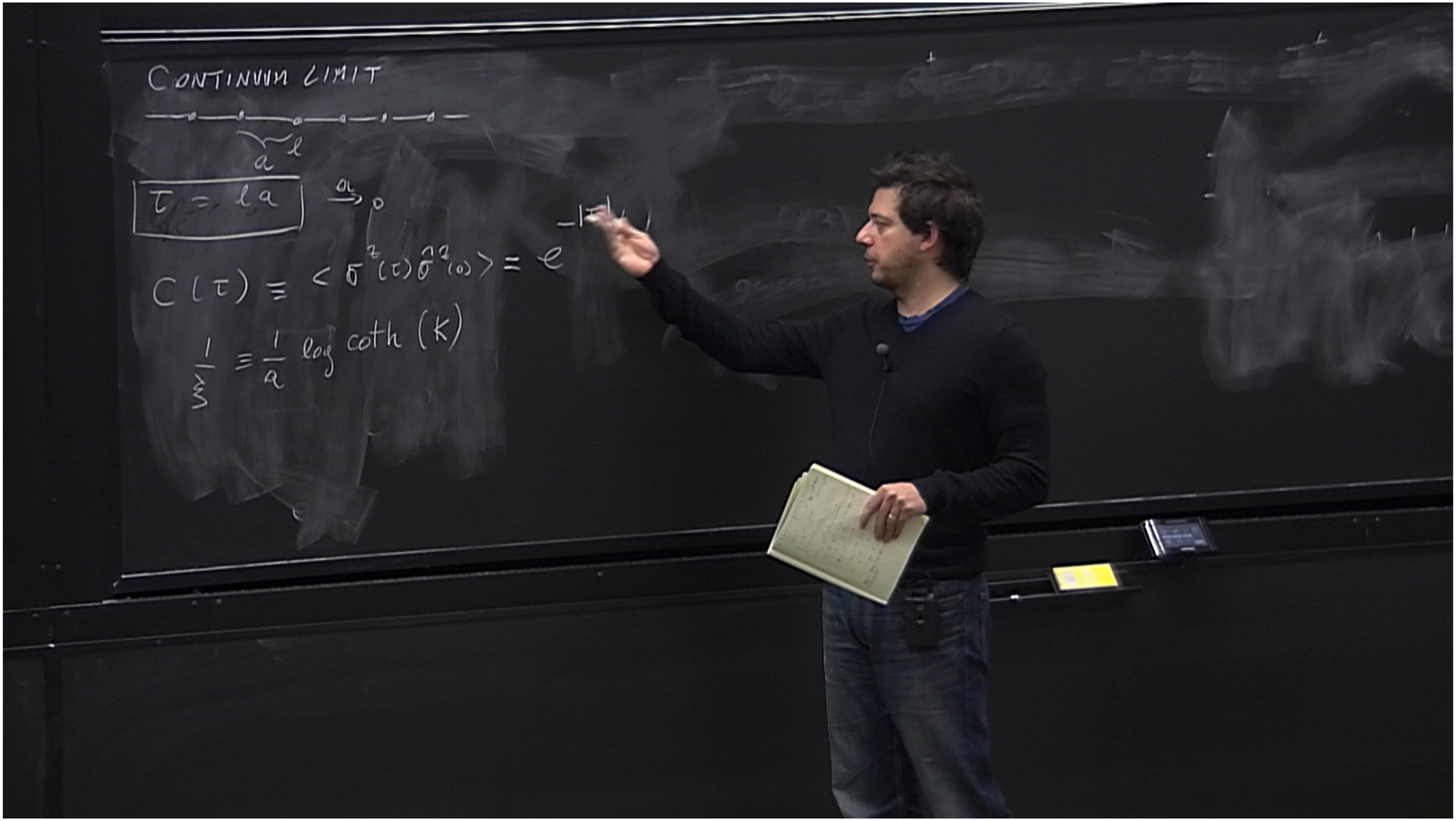
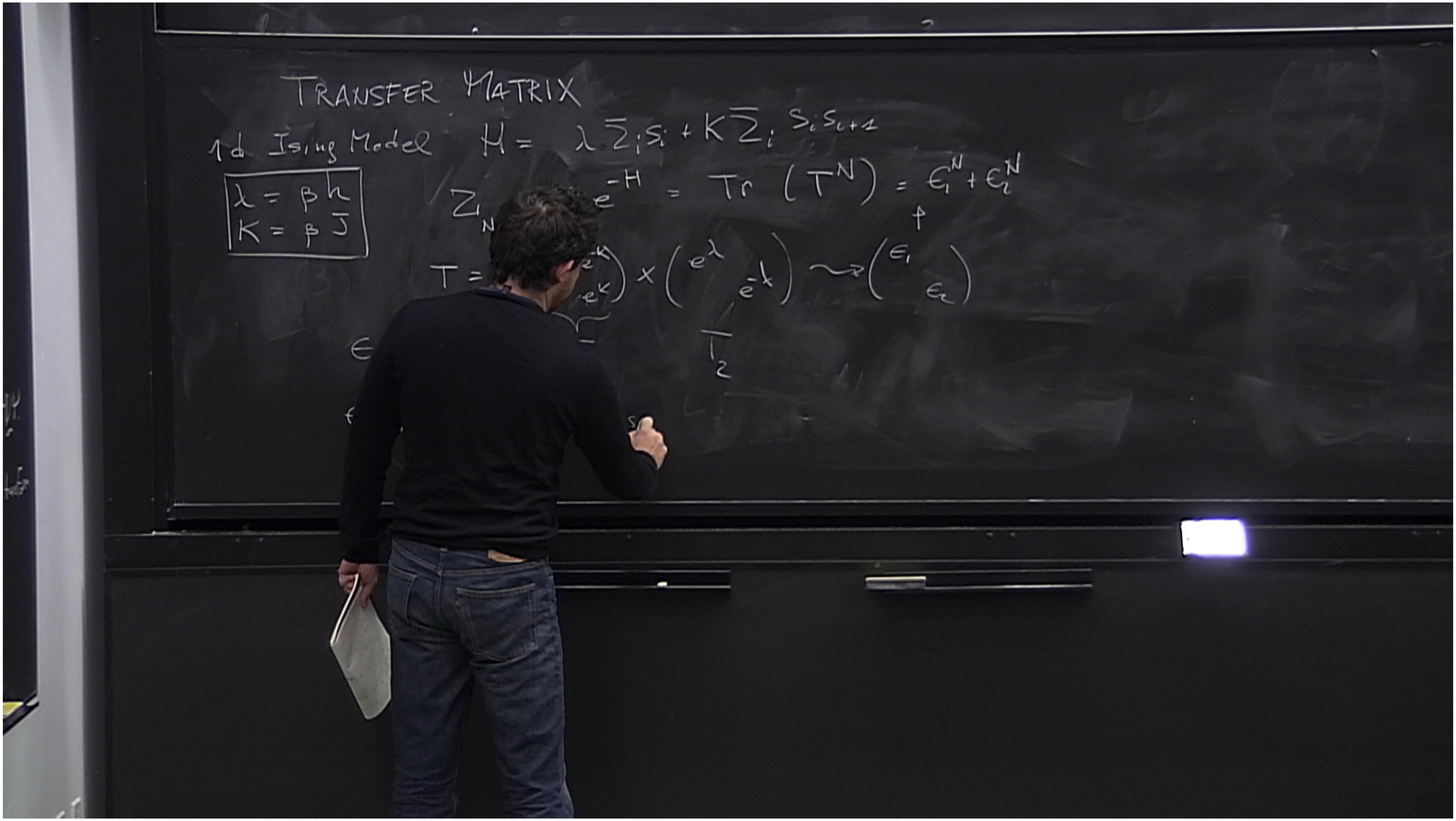
Title: Condensed Matter (Review) - Lecture 3

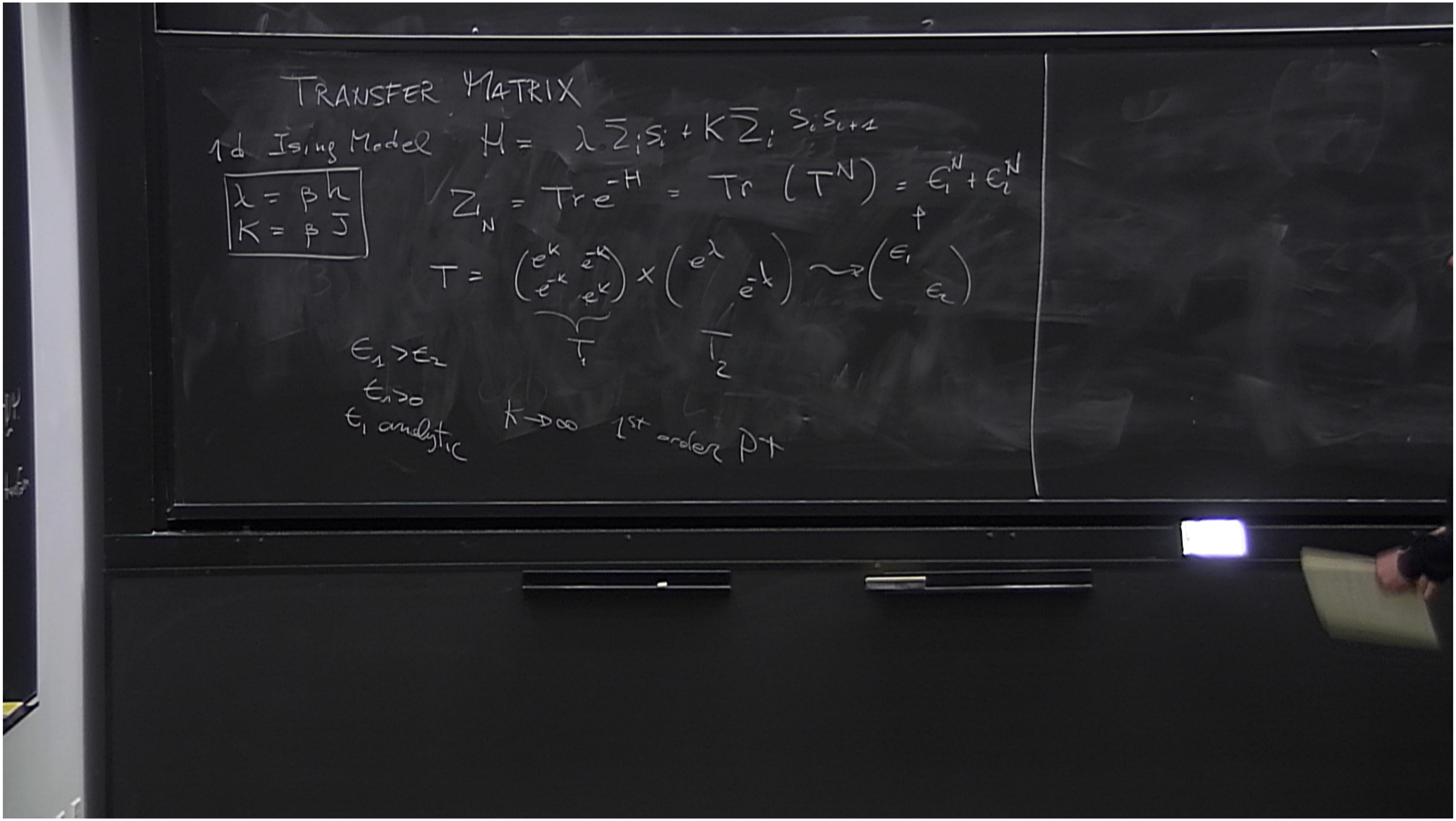
Date: Jan 04, 2012 10:15 AM

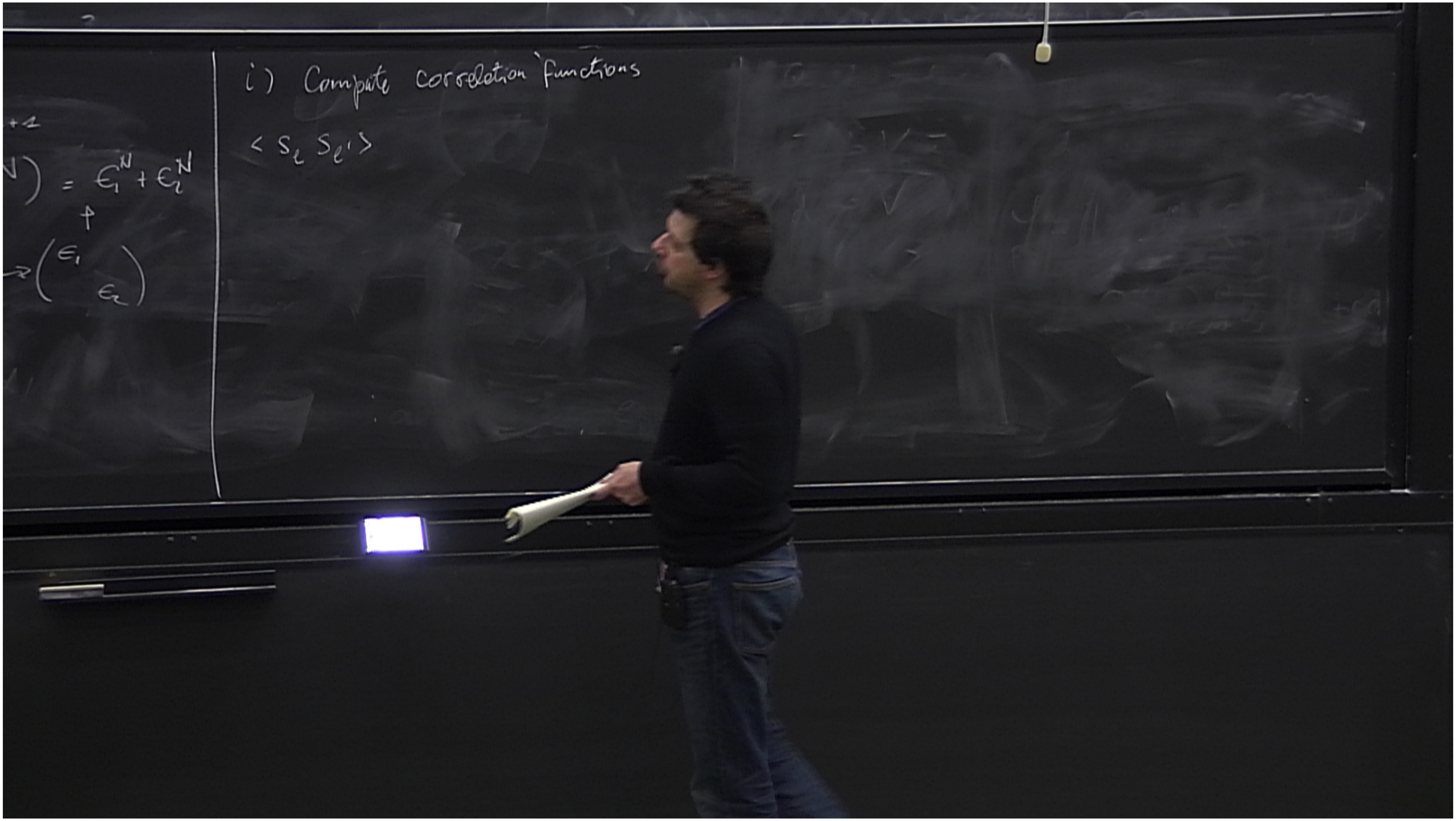
URL: http://pirsa.org/12010085

Abstract:

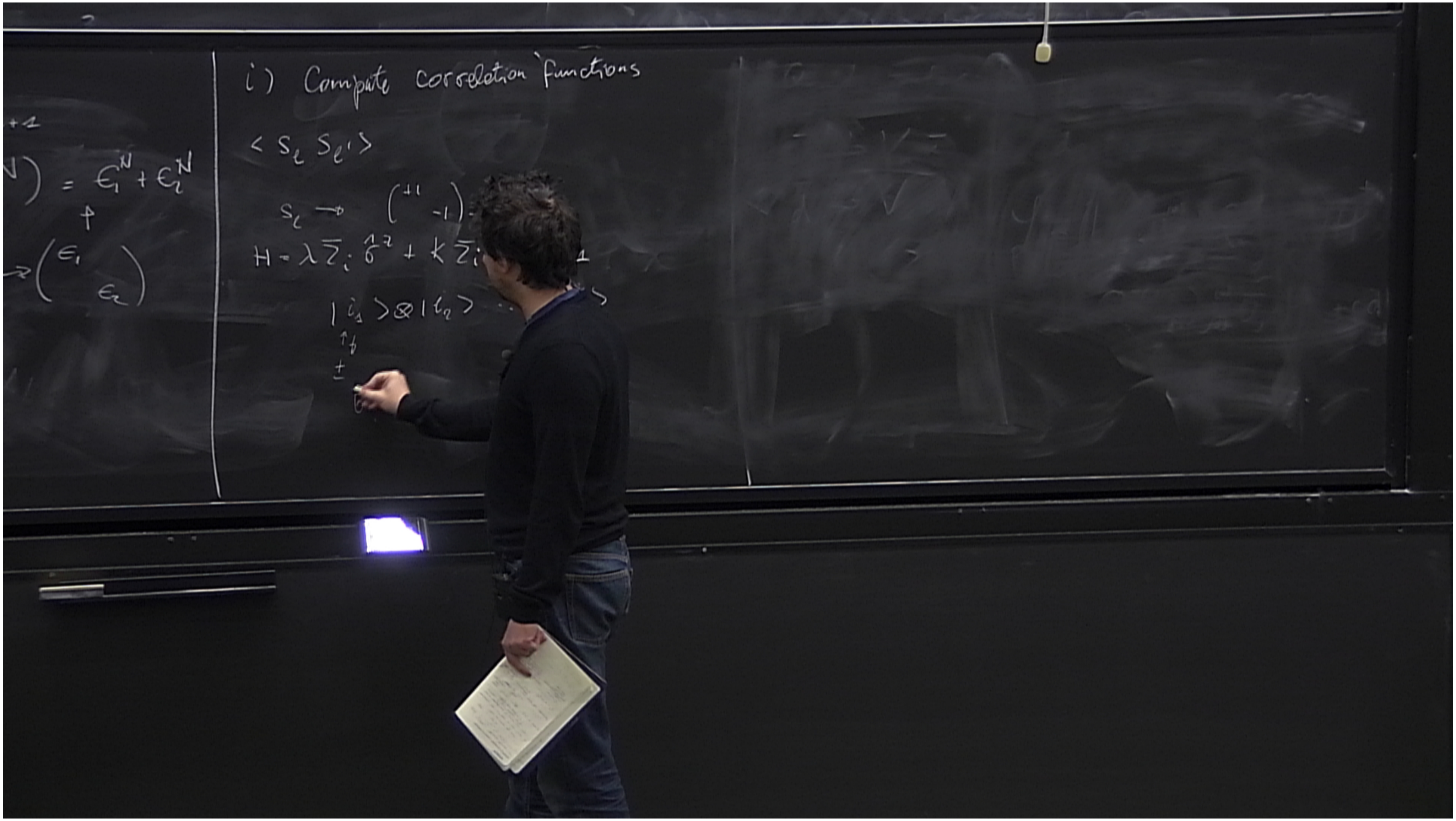

CONTINUUM LIMIT
τ = la
C(τ) ≡ < σ^z(τ) σ^z(0) > = e
1/ξ ≡ 1/a log coth (K)


TRANSFER MATRIX
1d Ising Model
H = λ Σ_i s_i + K Σ_i s_i s_{i+1}
λ = βh
K = βJ
= Tr (T^N) = ε_1^N + ε_2^N


TRANSFER MATRIX
1d Ising Model
$H = \lambda \sum_i s_i + K \sum_i s_i s_{i+1}$
$\lambda = \beta h$
$K = \beta J$
$Z_N = Tr\, e^{-H} = Tr\,(T^N) = \epsilon_1^N + \epsilon_2^N$
$T = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} \times \begin{pmatrix} e^\lambda & \\ & e^{-\lambda} \end{pmatrix} \rightsquigarrow \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$
T_1
T_2
$\epsilon_1 > \epsilon_2$
$\epsilon_1 > 0$
ϵ_1 analytic
$K \to \infty$
1st order PT


i) Compute correlation functions
< S_l S_l' >
= ε_1^N + ε_2^N


i) Compute correlation functions
< S_l S_l' >
S_l →
H = λ Σ_i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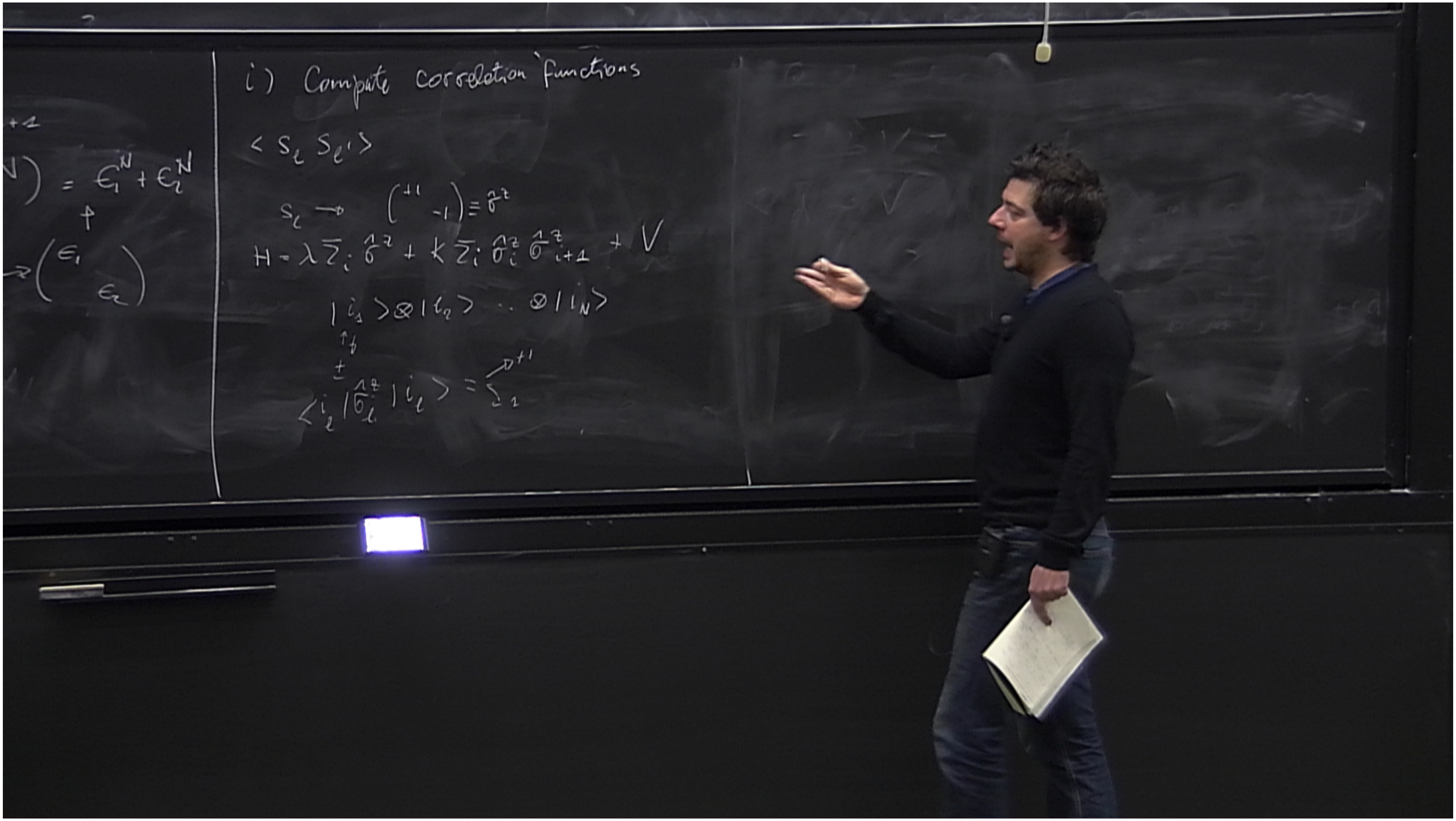
< S_l S_l' >


i) Compute correlation functions
< S_l S_l' >
S_l → (+1 -1) = σ^z
H = λ Σ_i σ^z + k Σ_i σ^z_i σ^z_i+1
| i_1 > ⊗ | i_2 > ... ⊗ | i_N >
< i_l | σ^z_l | i_l > = Σ_2 → +1
= ε_1^N + ε_2^N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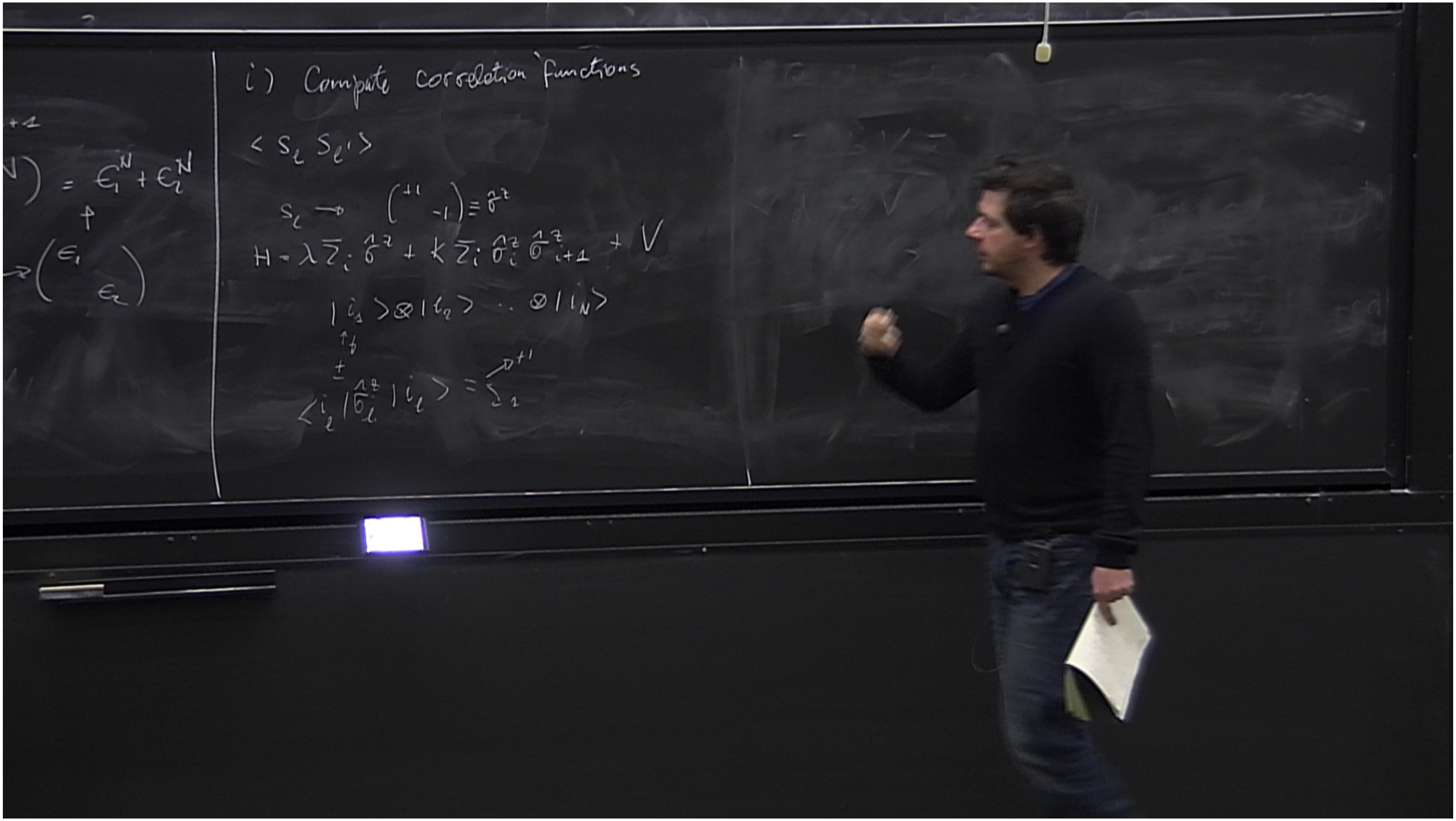
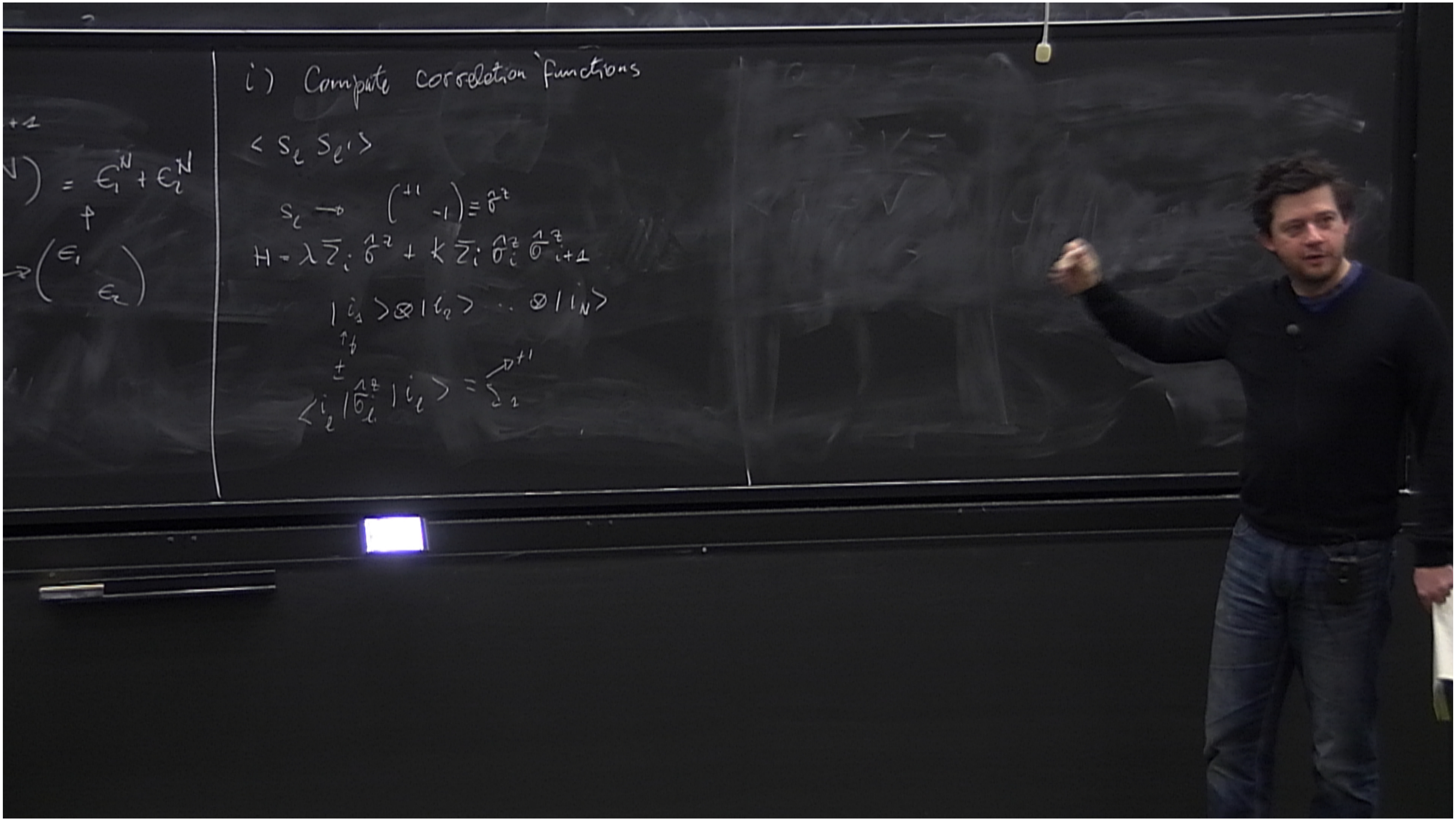
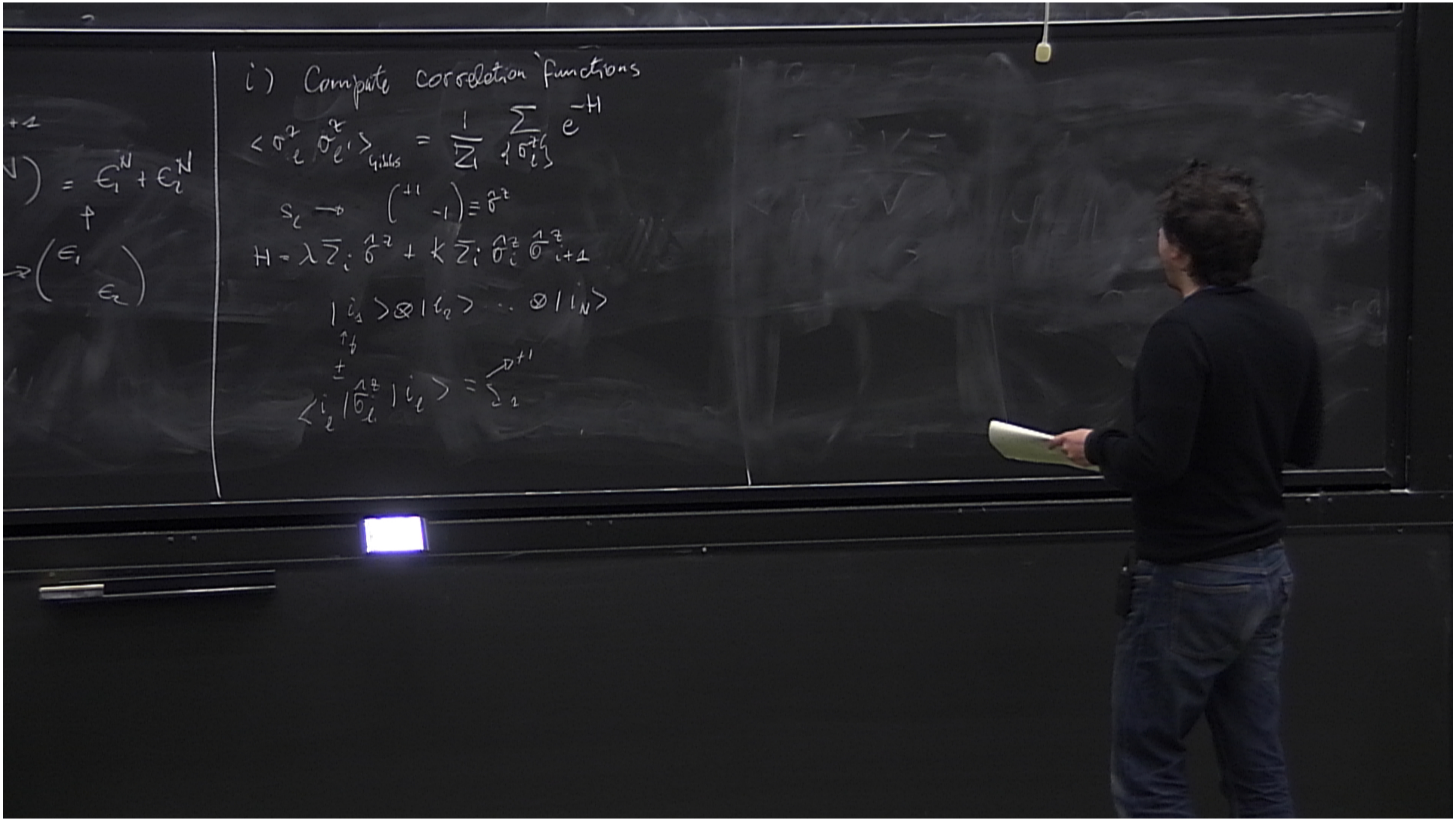
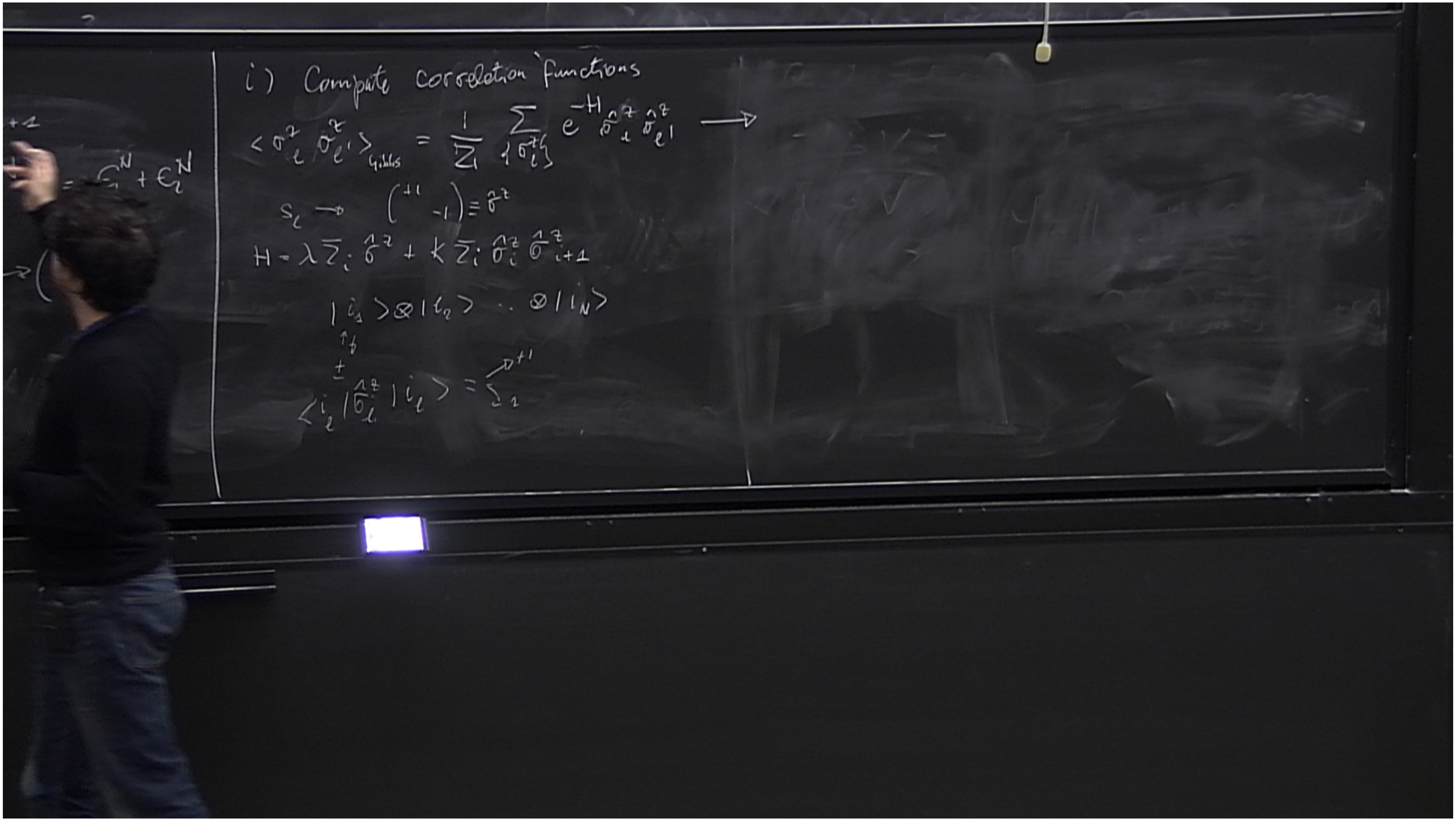

i) Compute correlation functions
< S_l S_l' >
S_l → (+1 -1) ≡ σ^z
H = λ Σ_i σ̂_i^z + k Σ_i σ̂_i^z σ̂_{i+1}^z + V
| i_1 > ⊗ | i_2 > ... ⊗ | i_N >
< i_l | σ̂_l^z | i_l > = i_l
= ε_1^N + ε_2^N


i) Compute correlation functions
< S_l S_l' >


i) Compute correlation functions
Gibbs
s → σ
H = λ Σ σ + k Σ σ σ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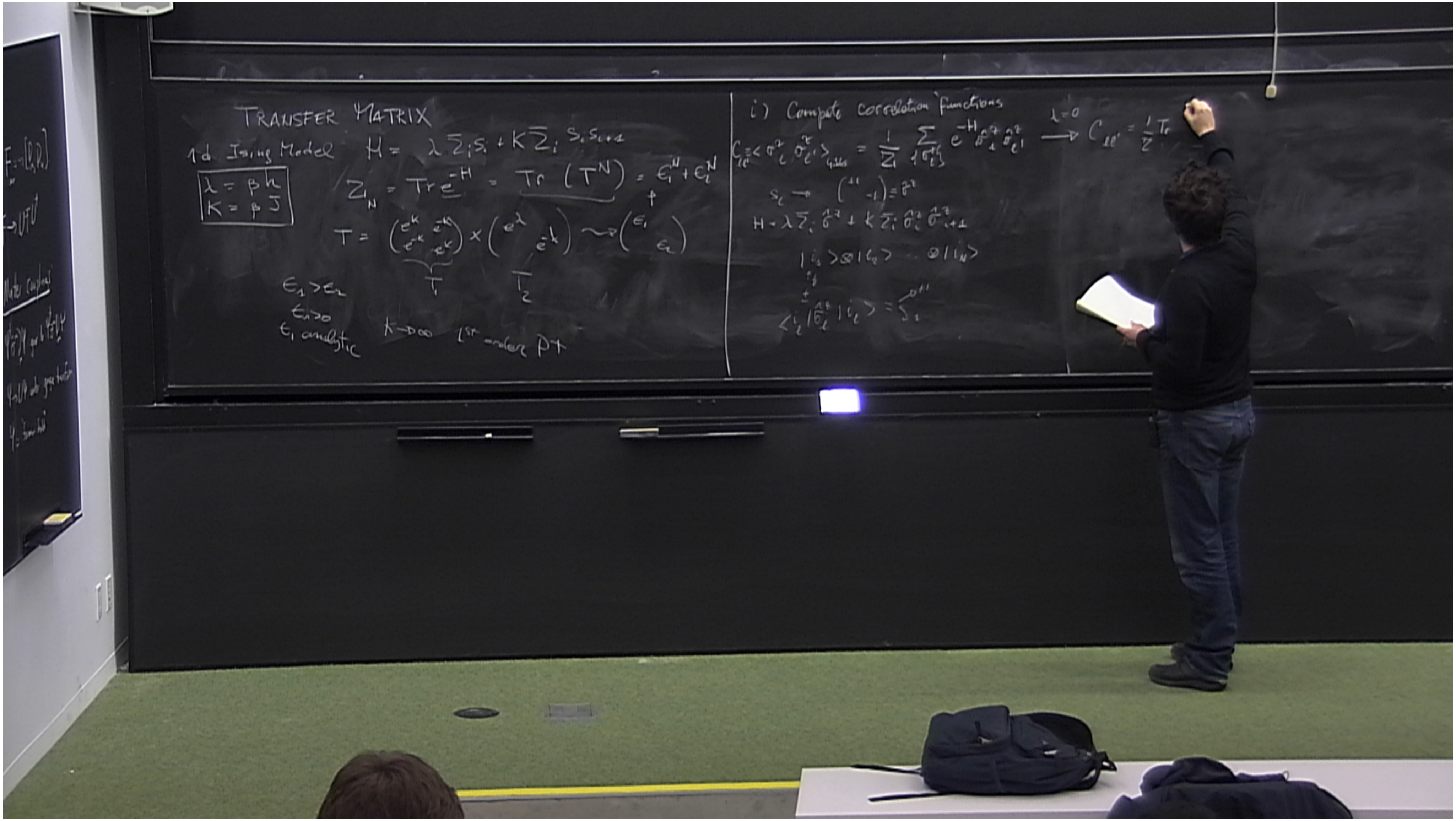
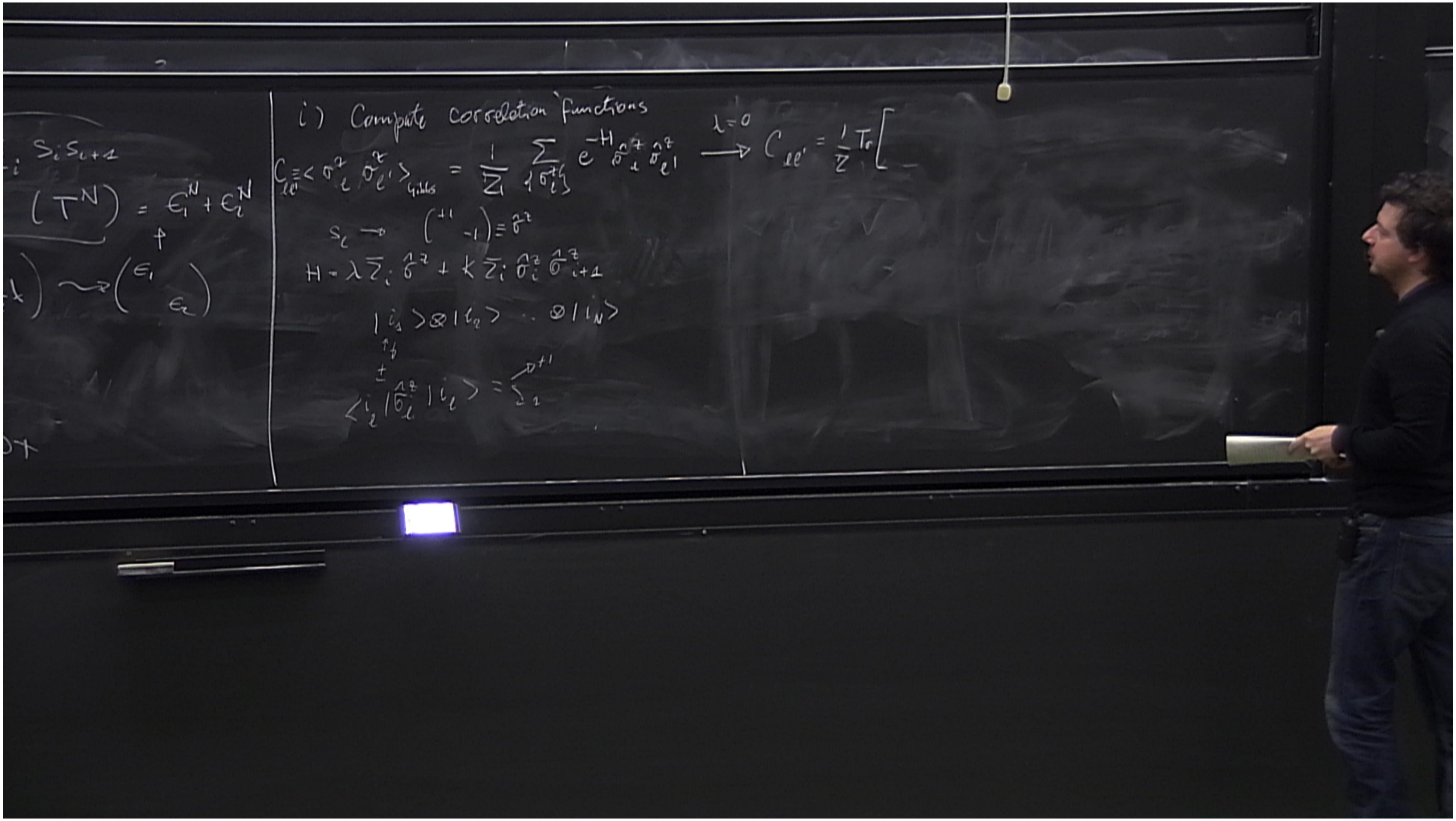
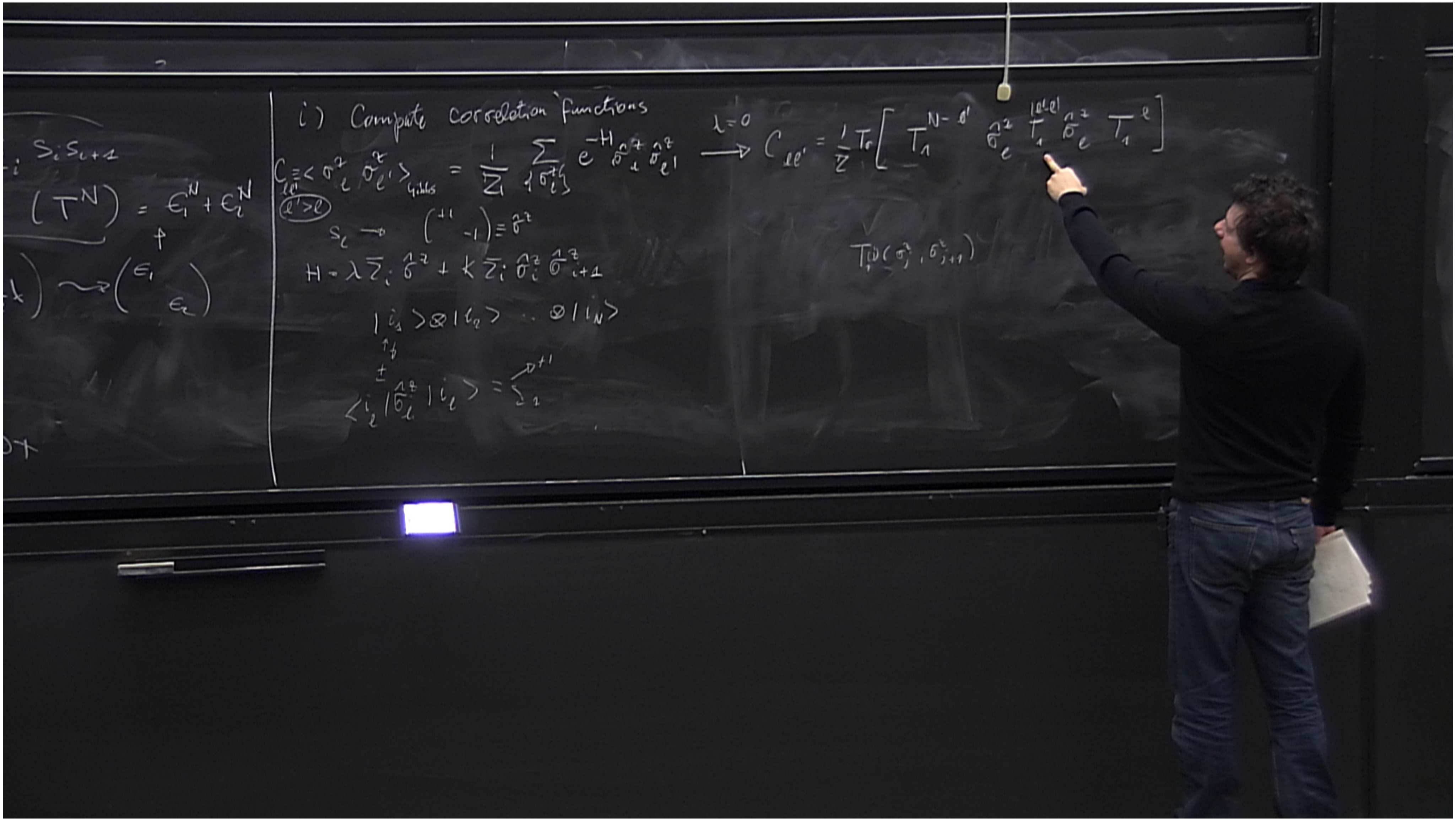



i) Compute correlation functions
λ=0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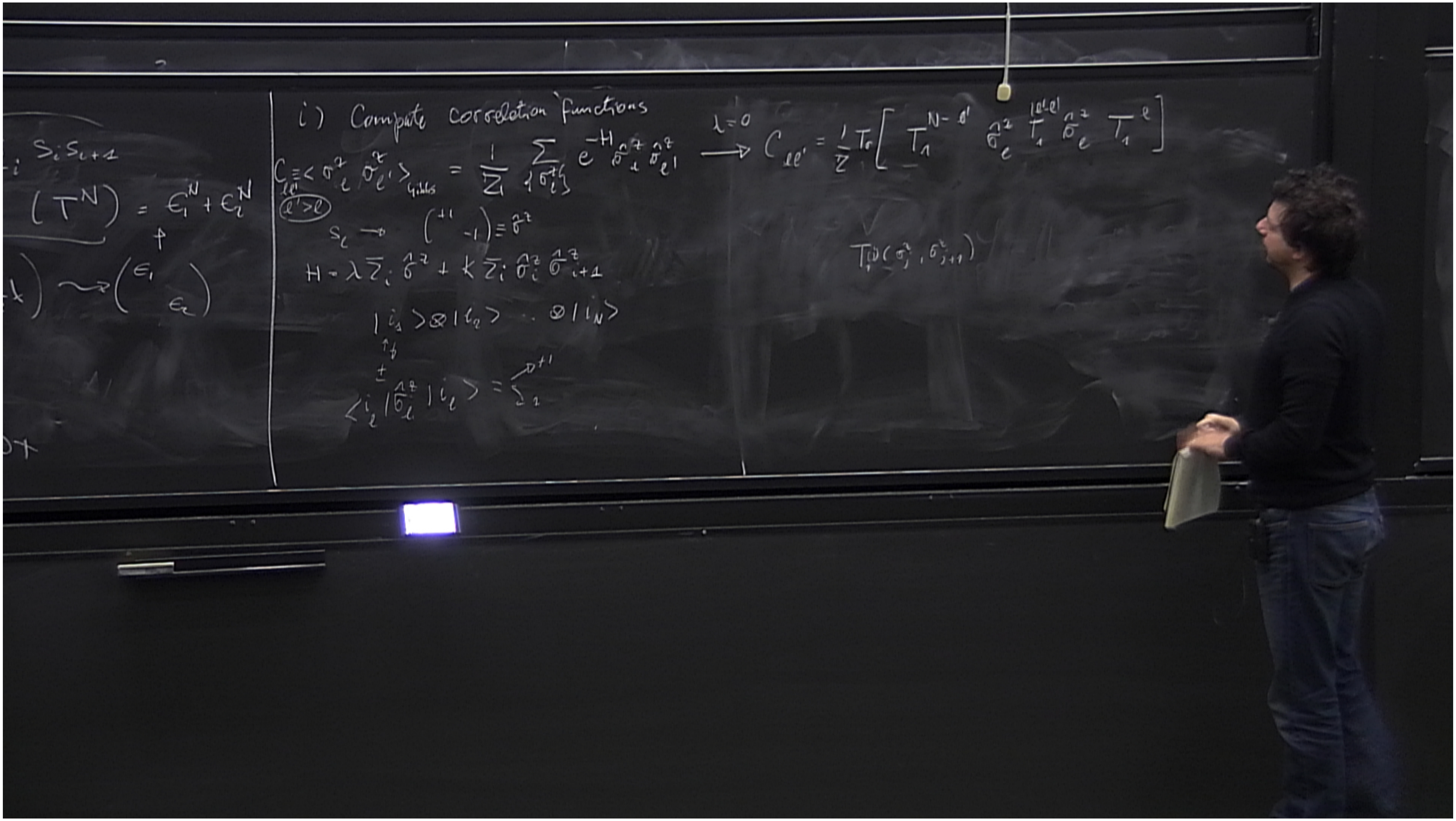
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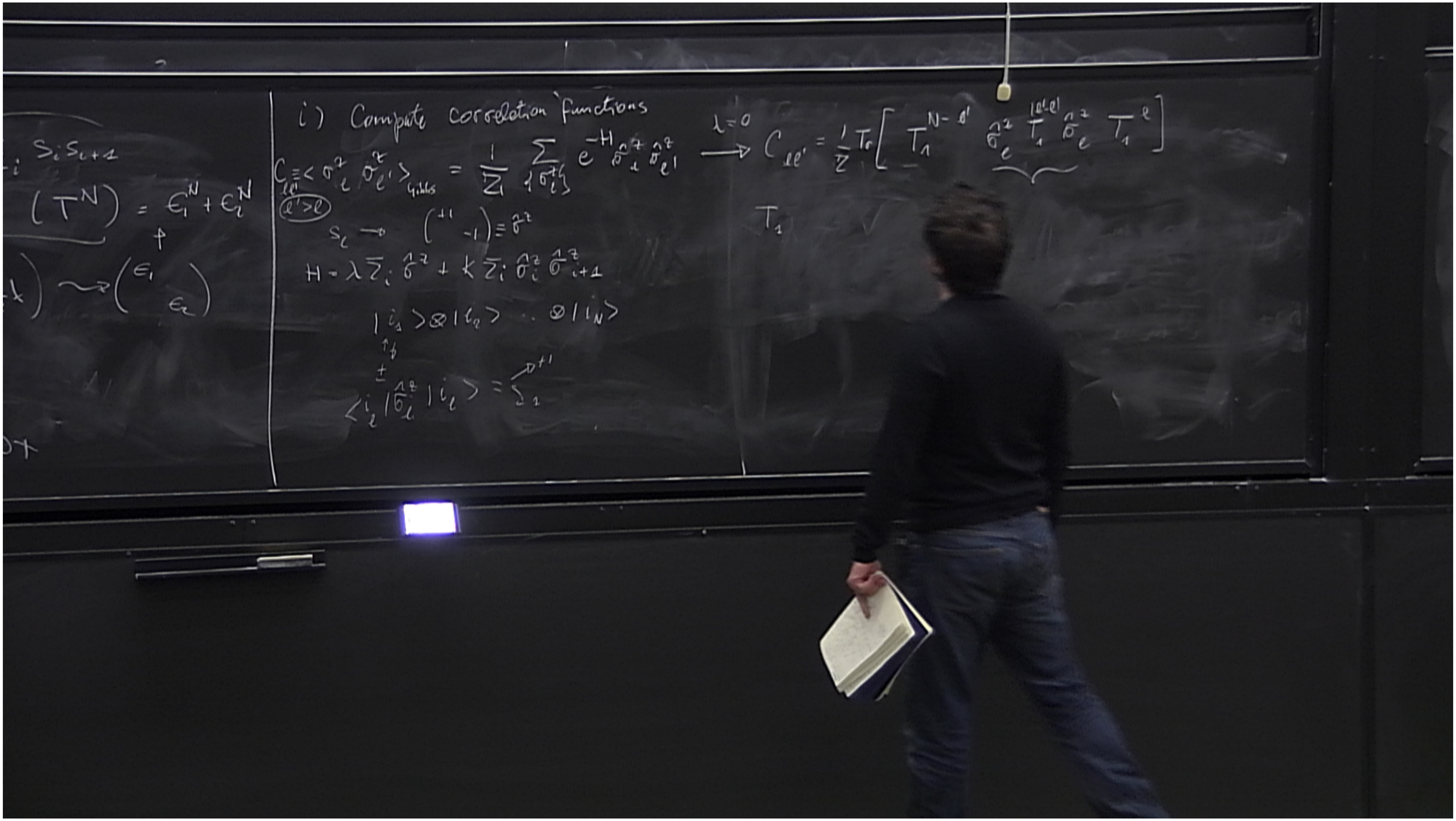
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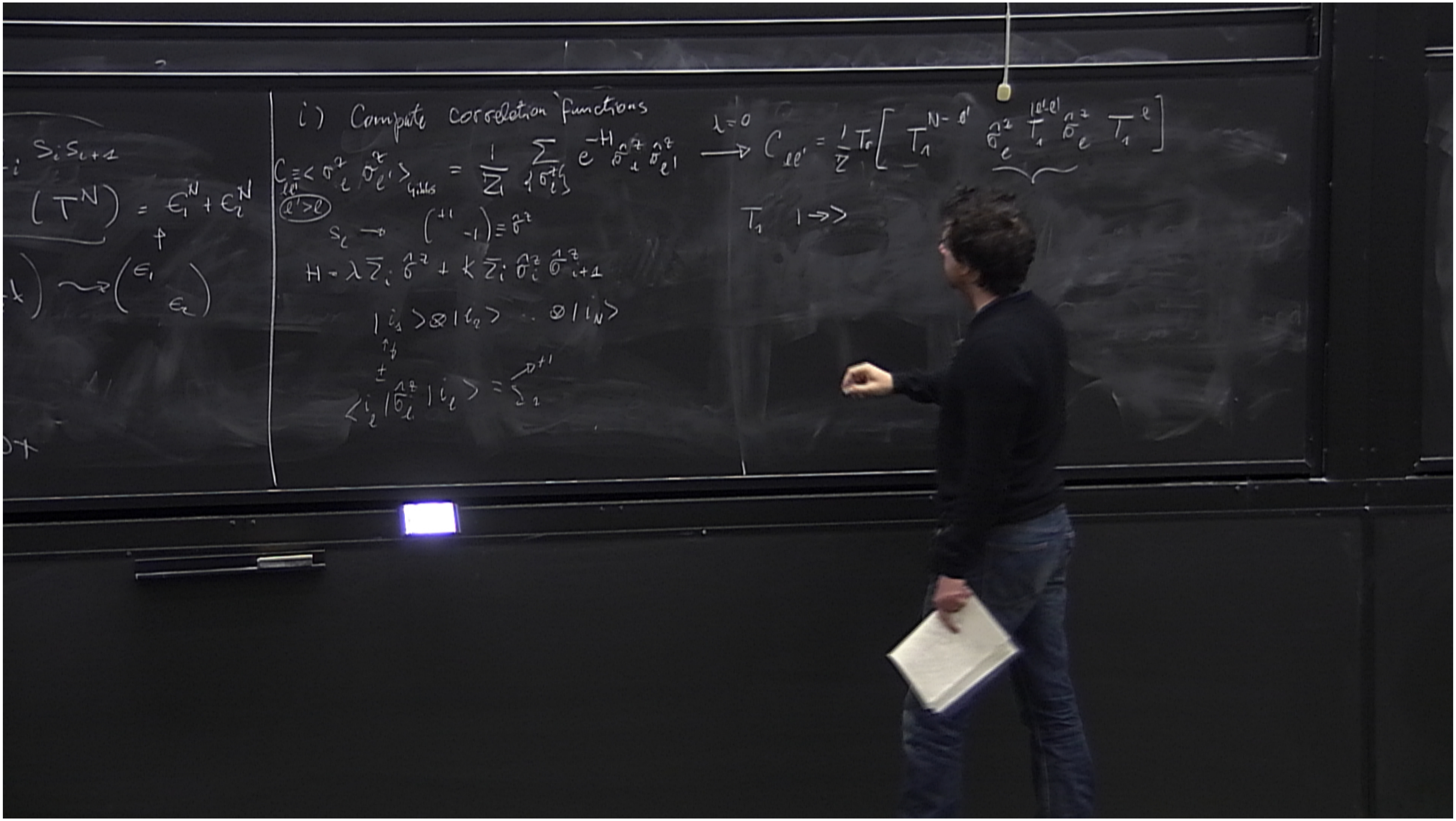
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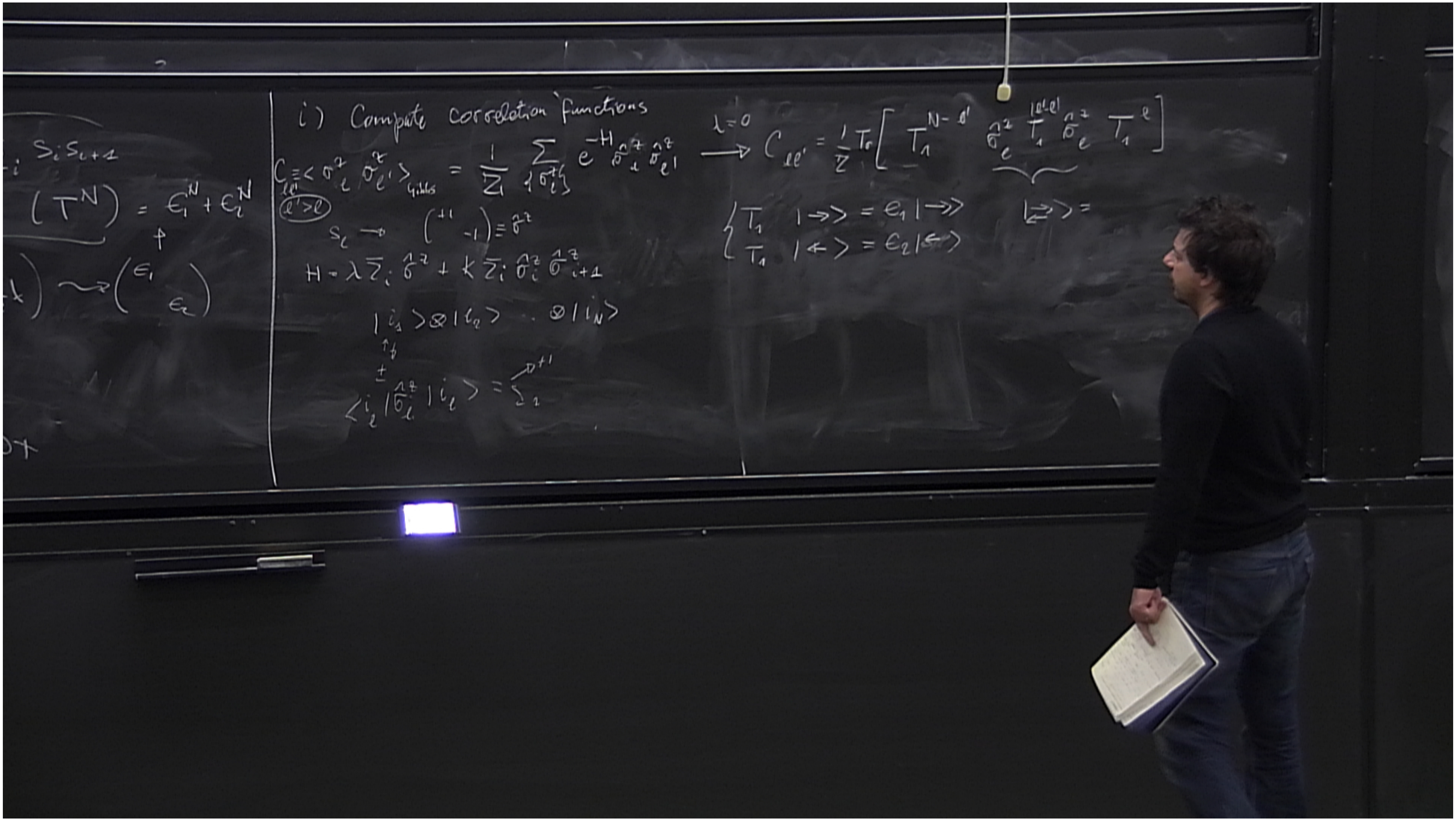
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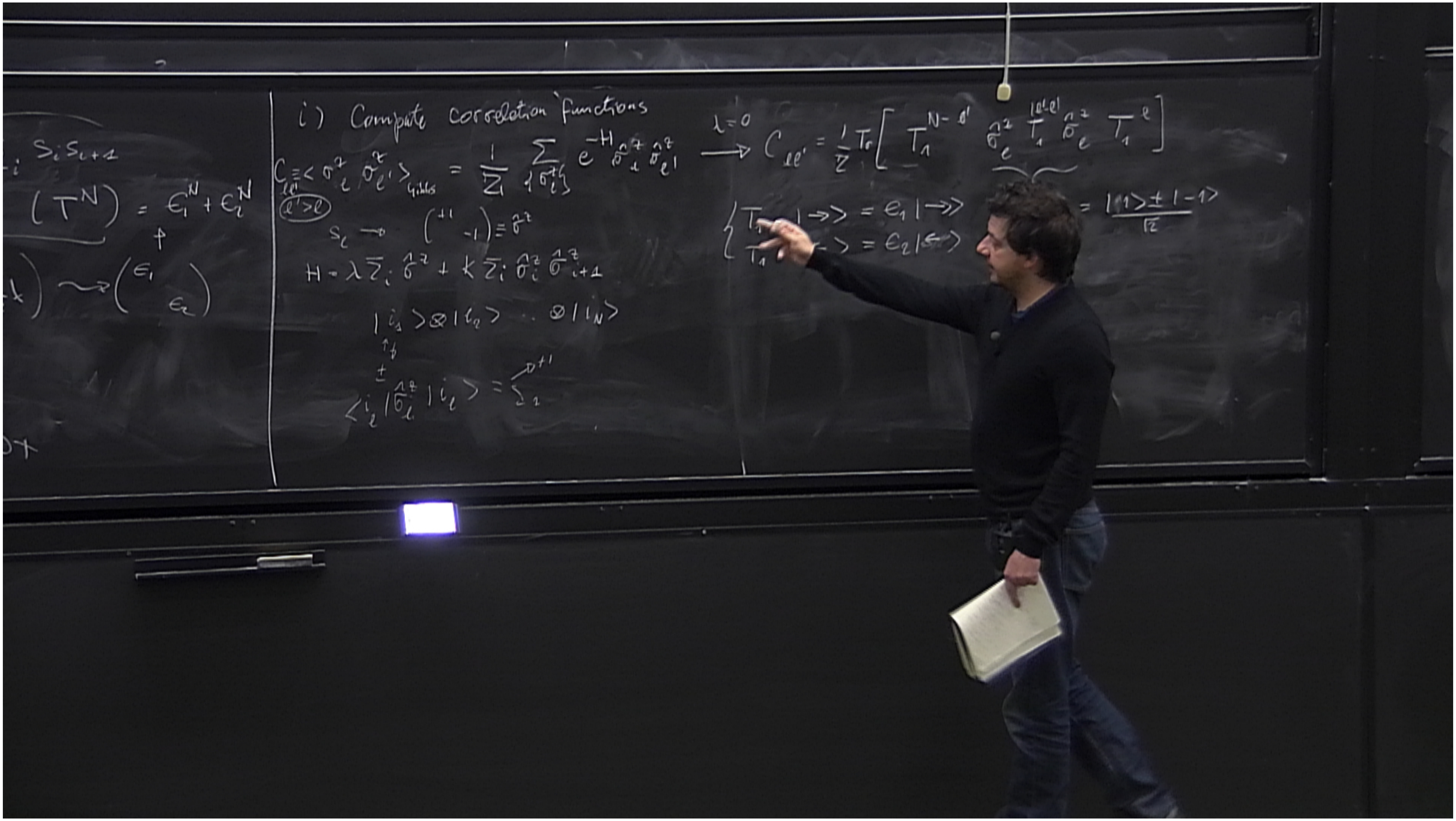
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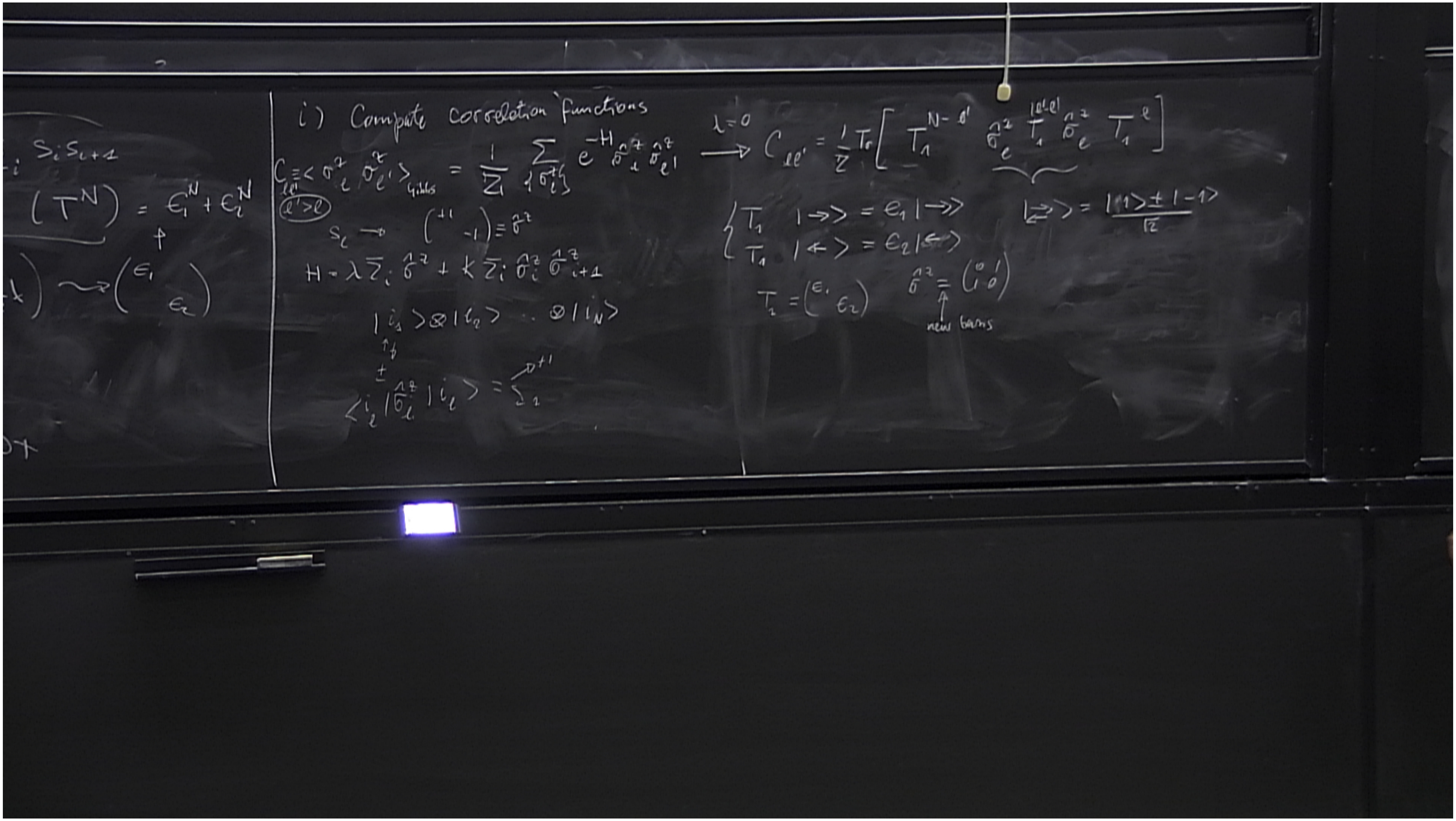
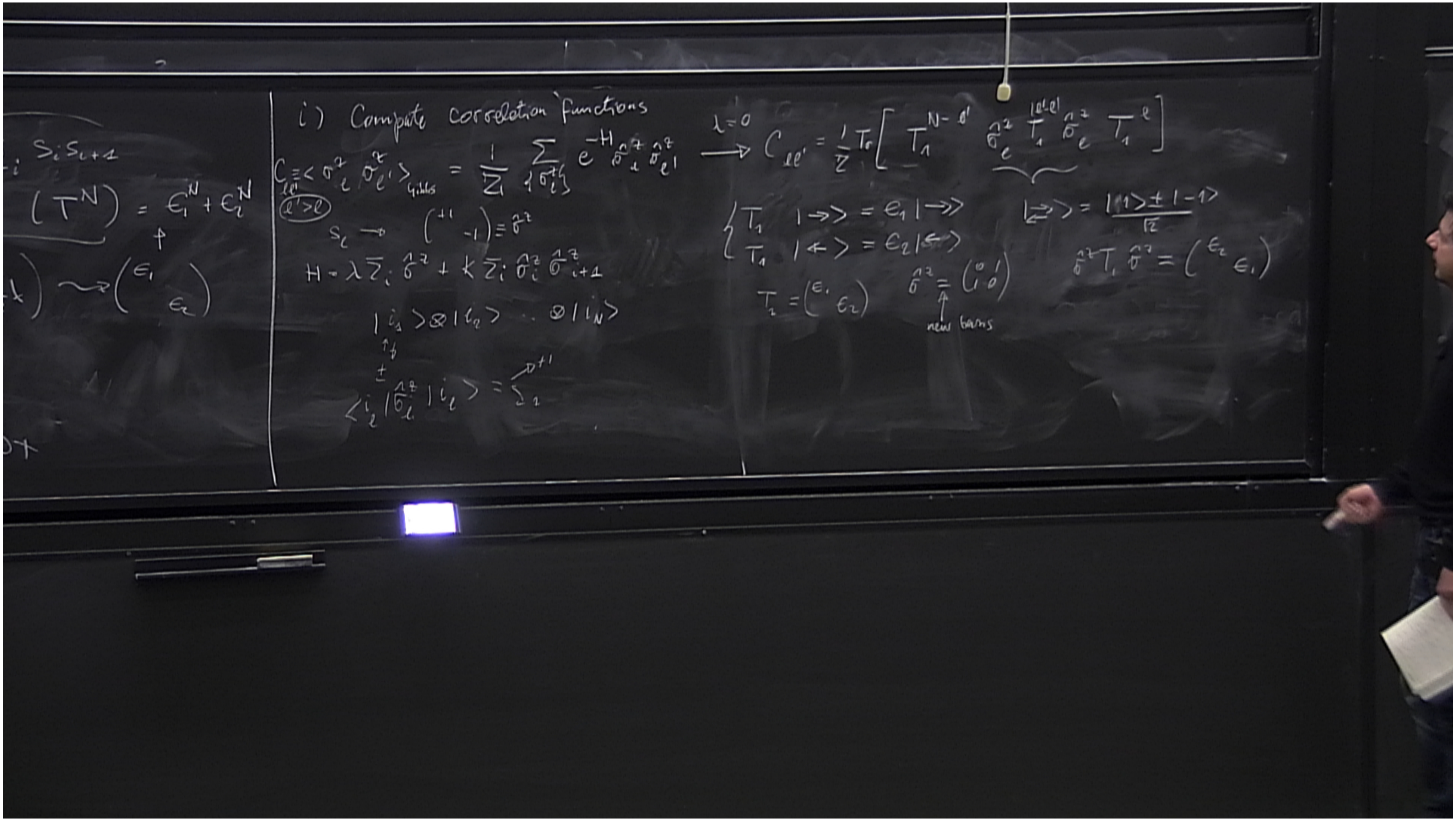

i) Compute correlation functions
new basis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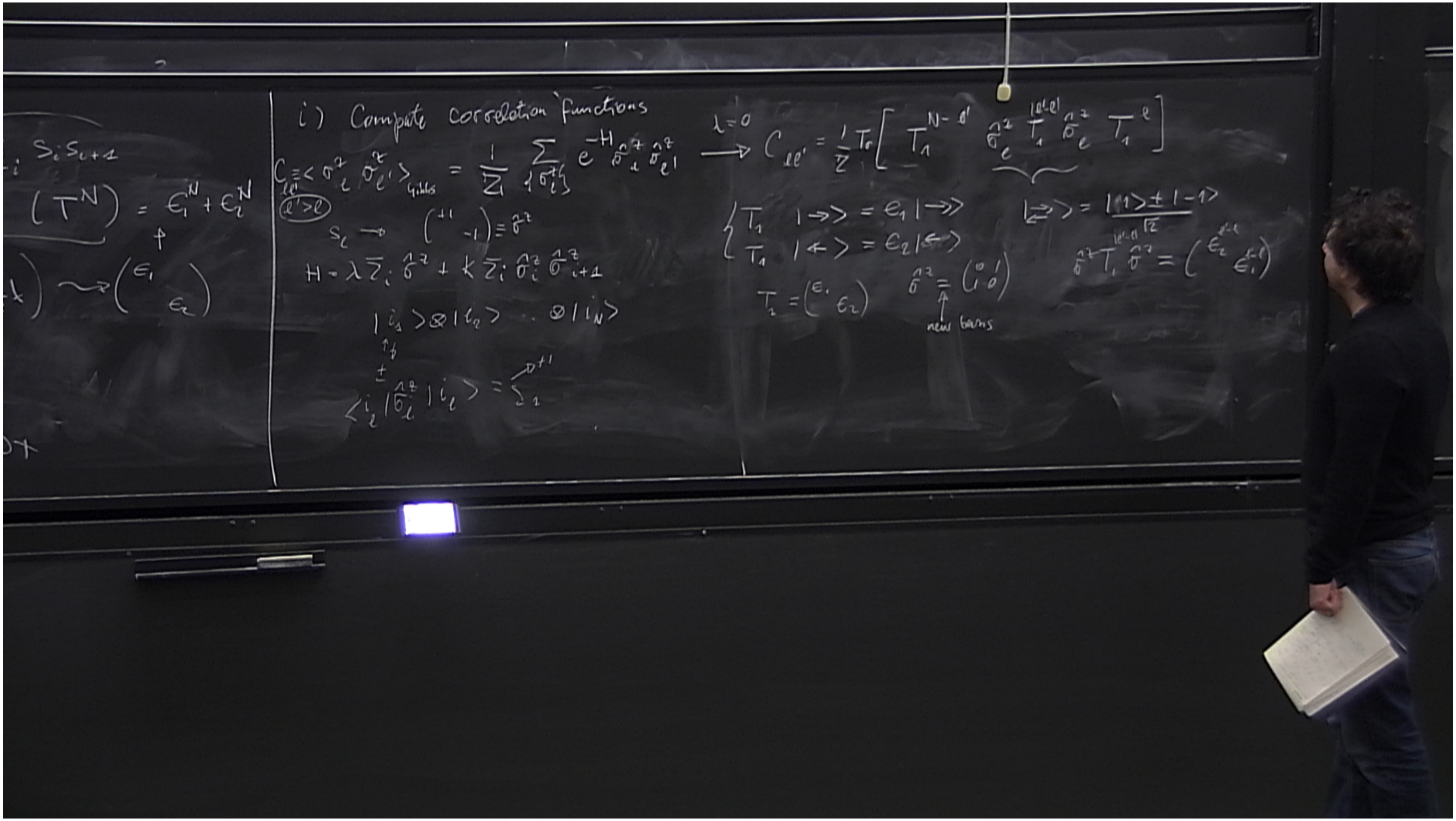
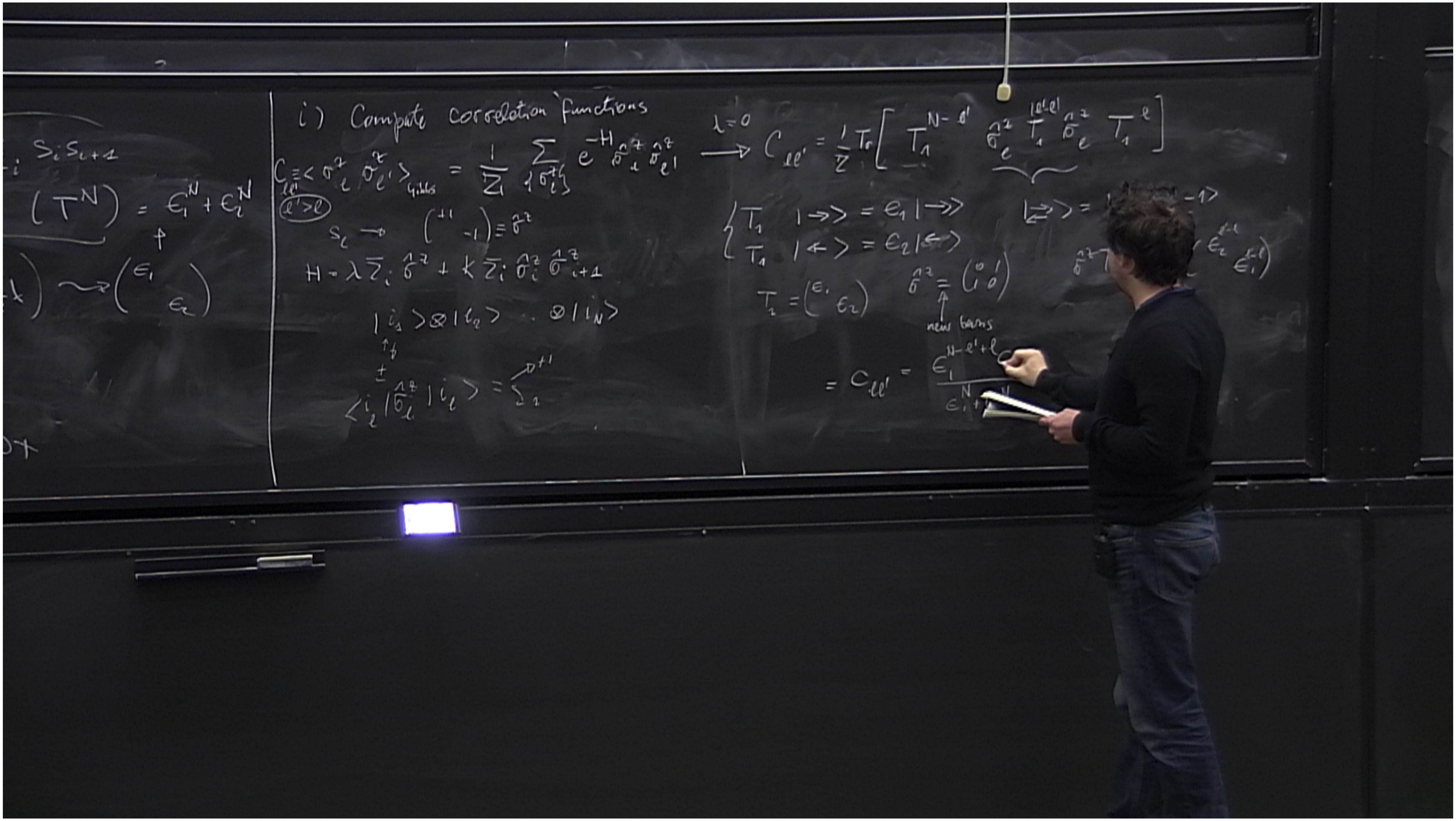

i) Compute correlation functions
new basis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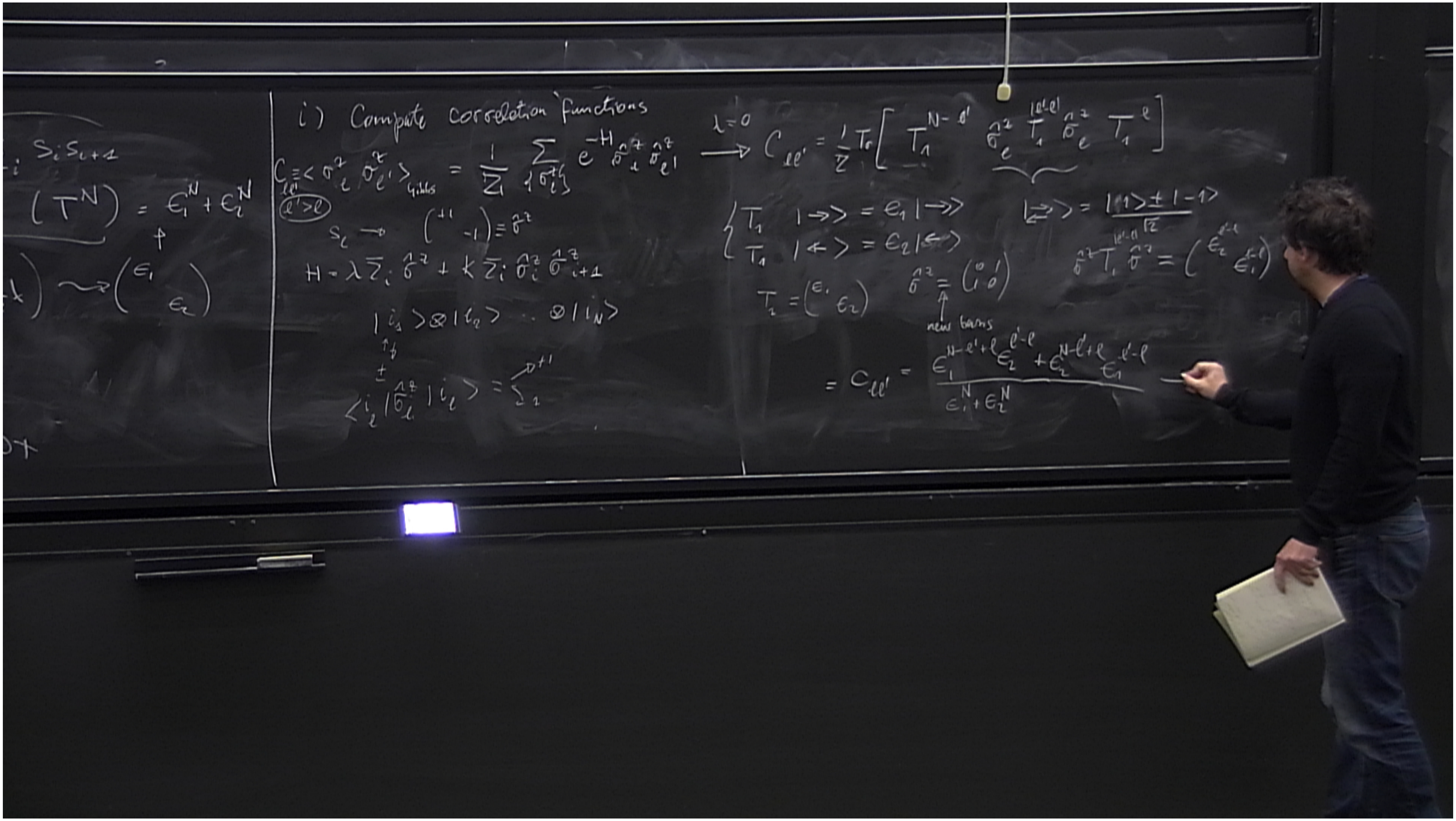

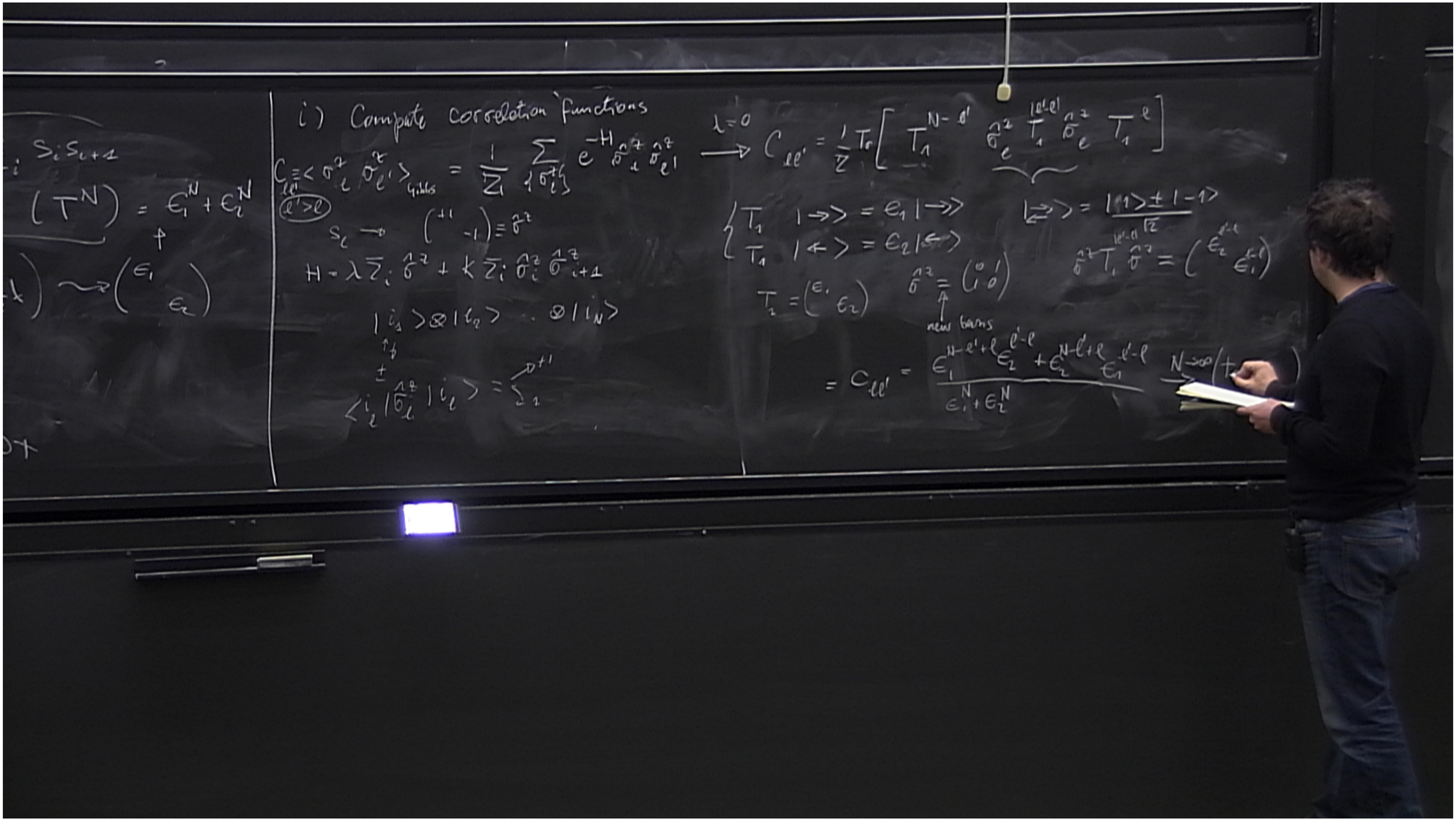
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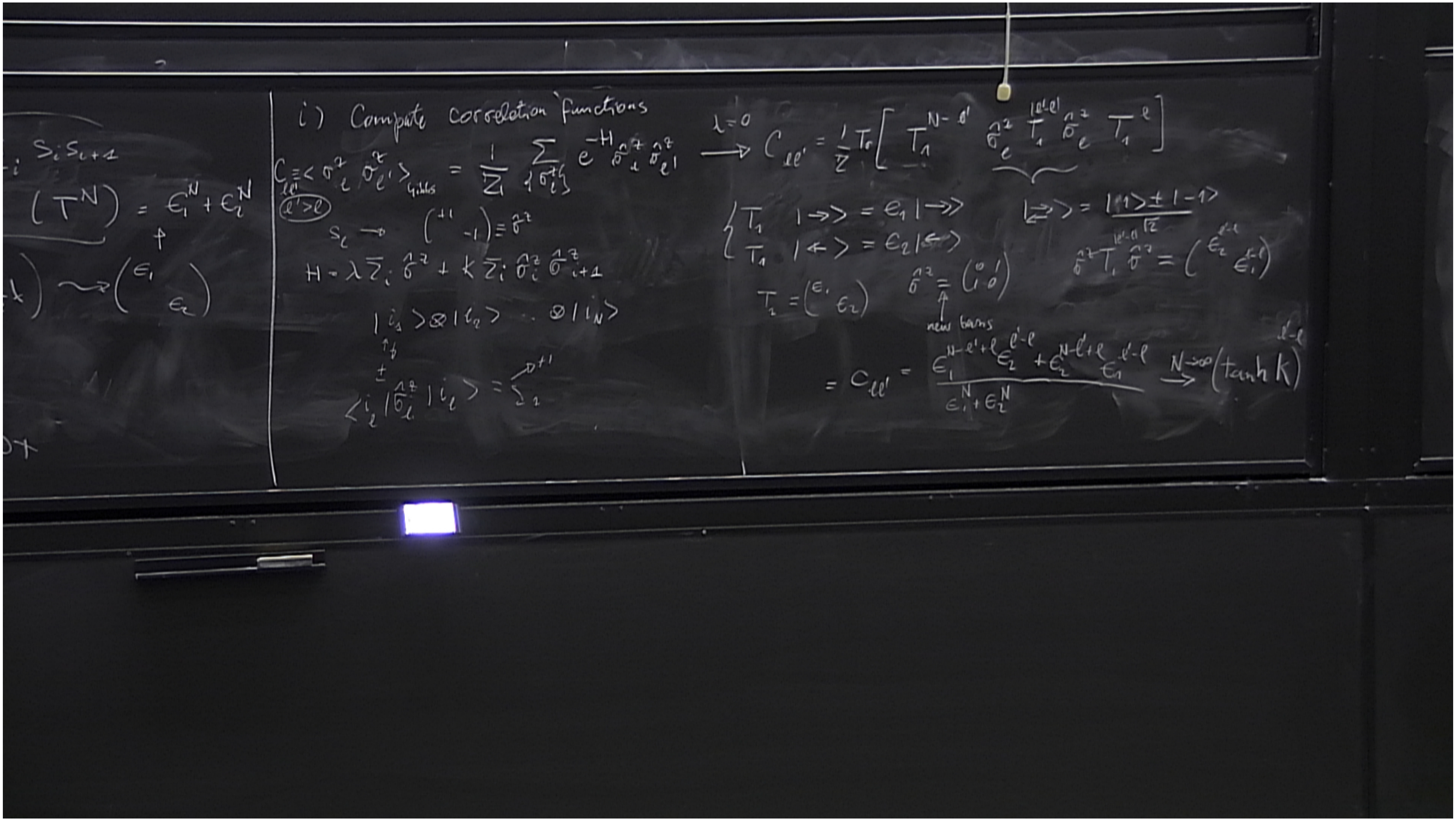
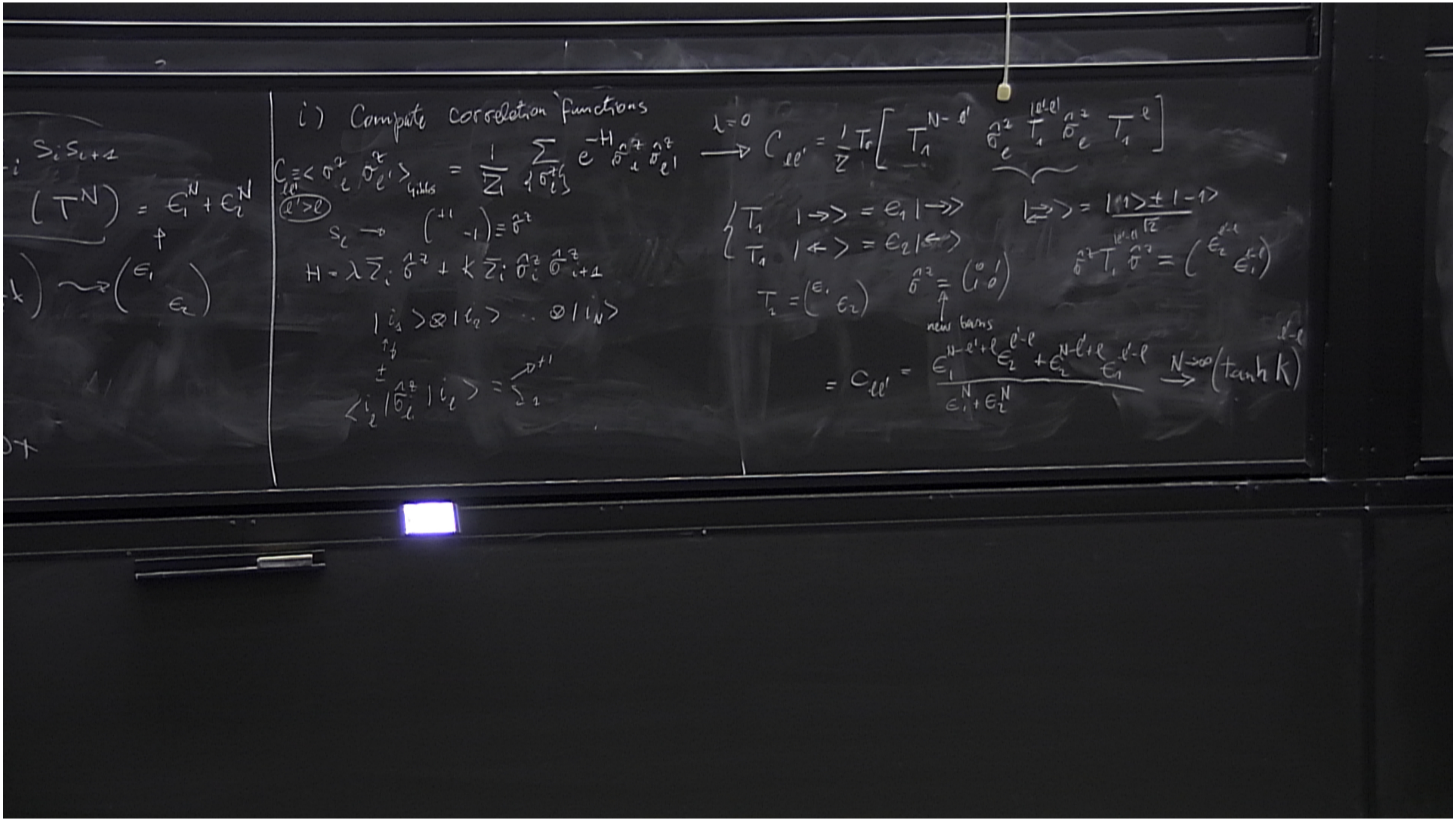
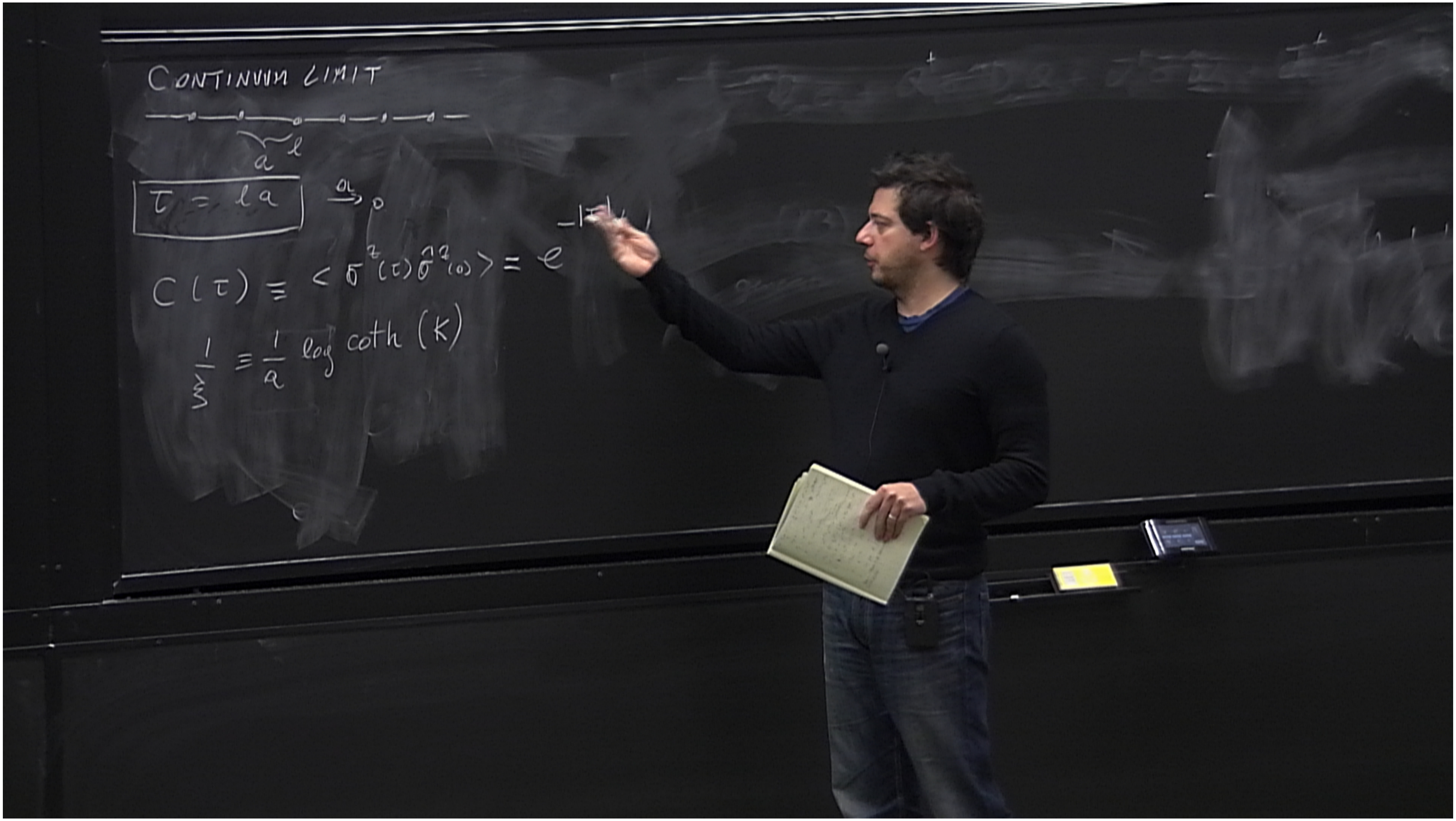
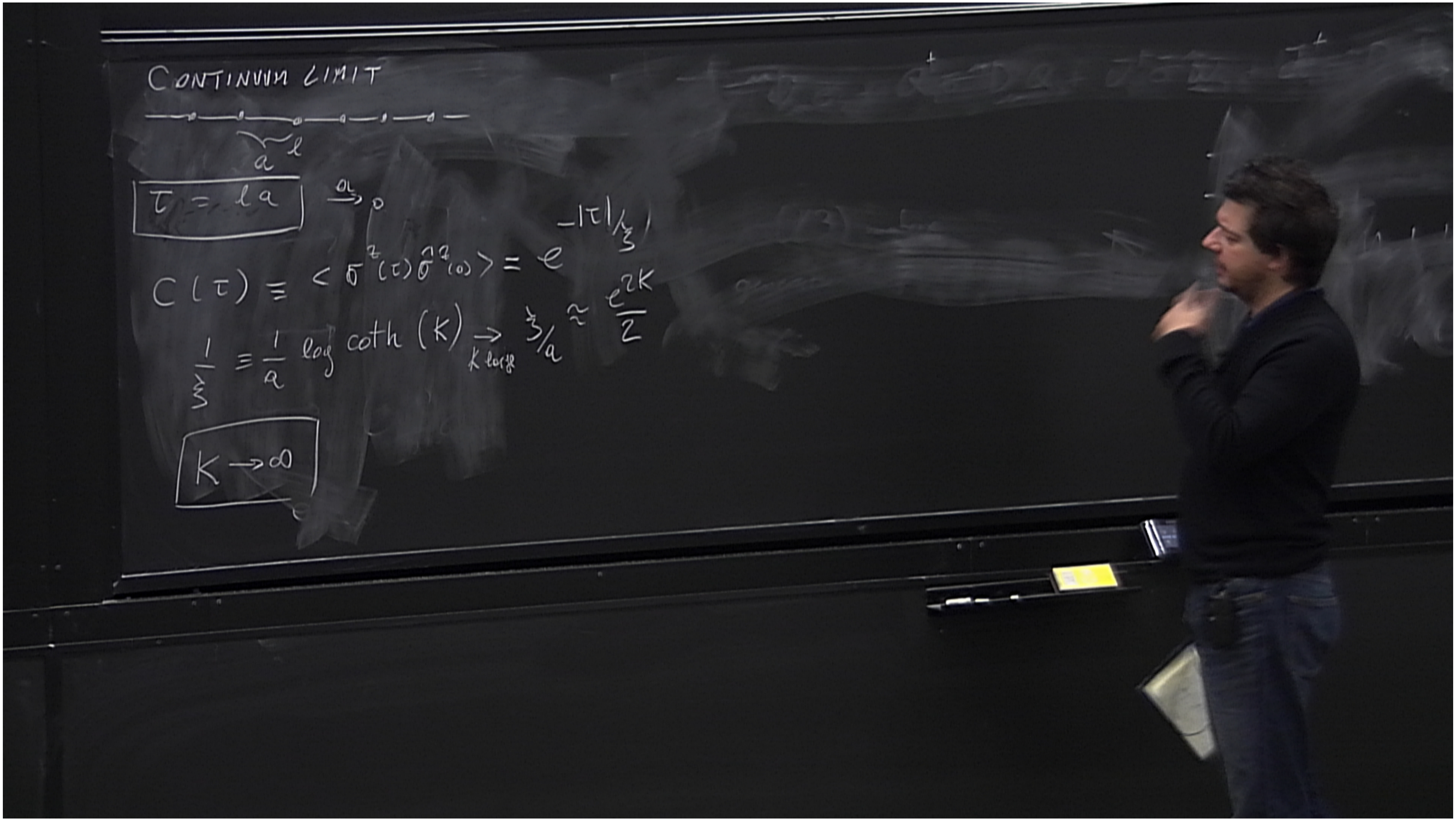
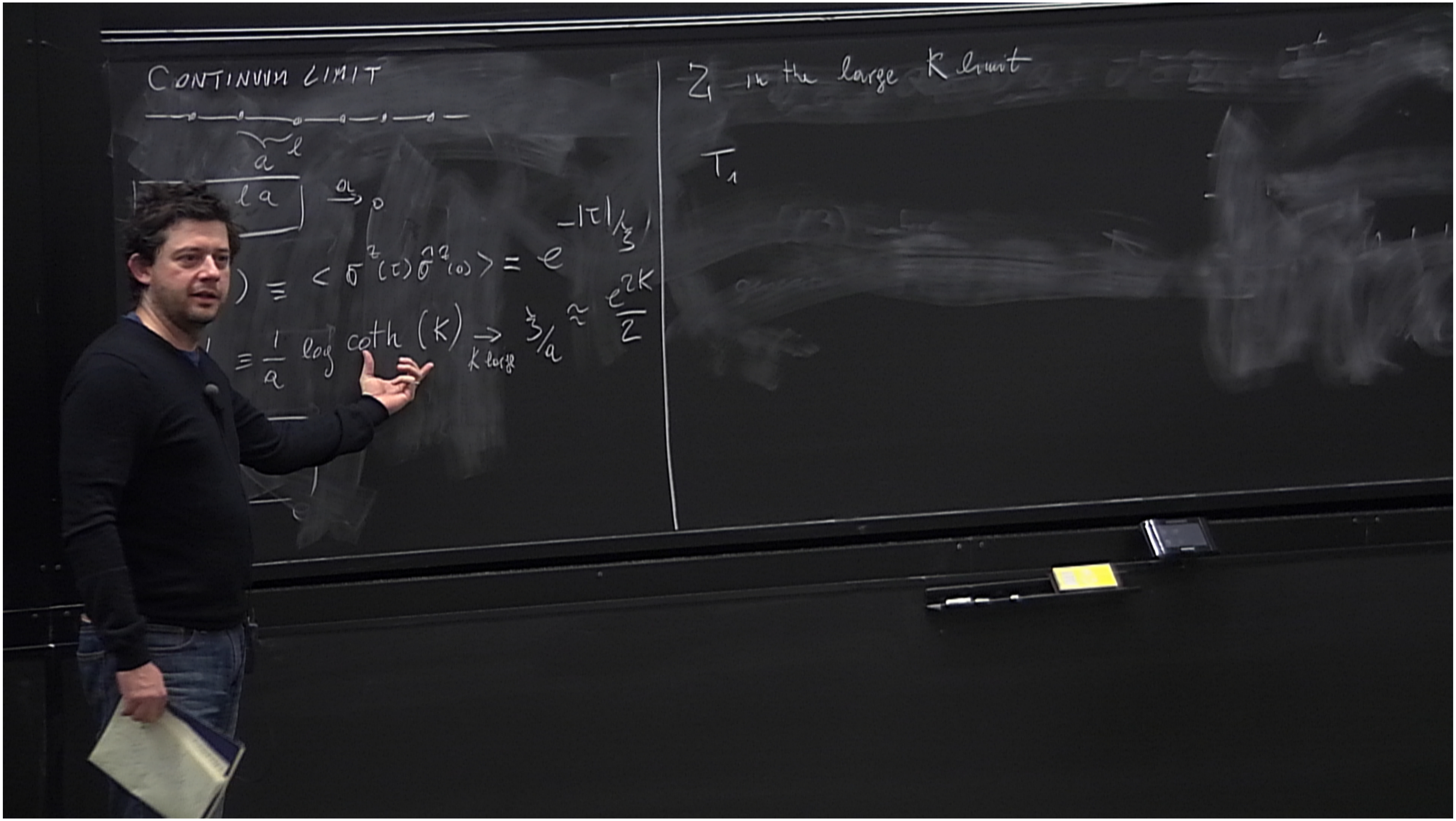

i) Compute correlation functions
new basis
N→∞ (tanh K)

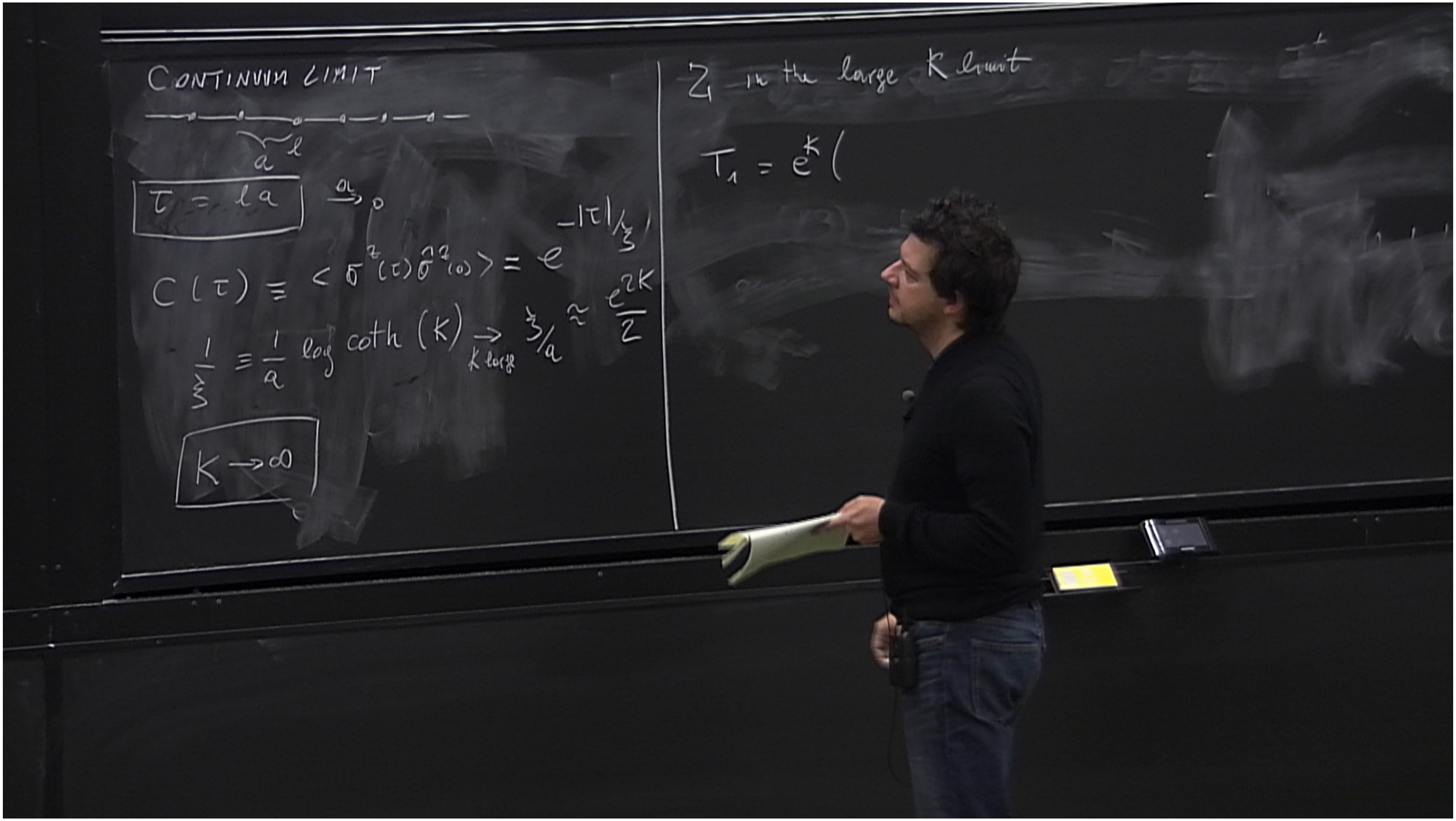


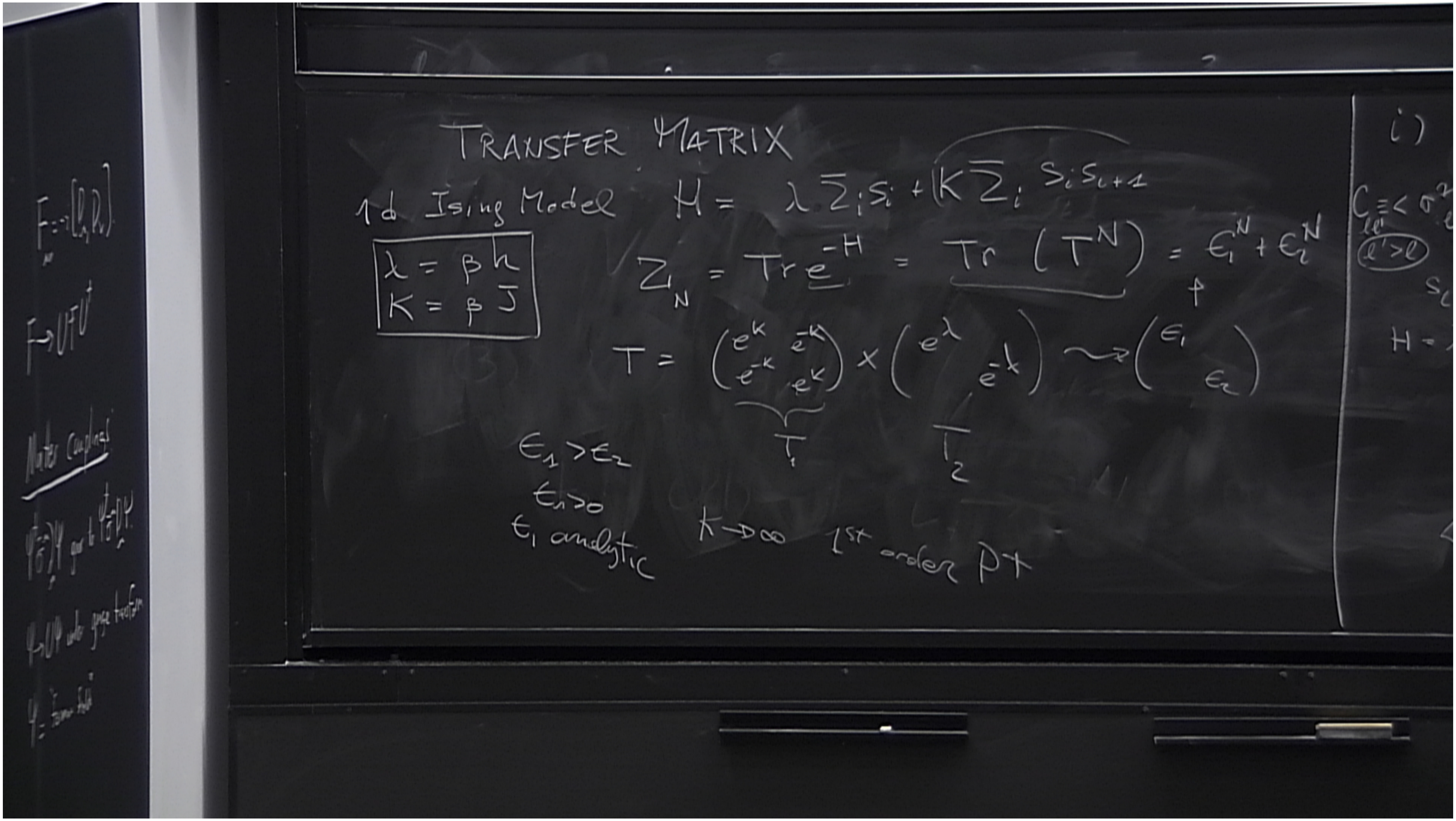
$S_i S_{i+1}$
$(T^N) = \epsilon_1^N + \epsilon_2^N$
$\sim\!\!> \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$
i) Compute correlation functions
$C_{\ell\ell'} = \langle \sigma^z_\ell \sigma^z_{\ell'} \rangle_{Gibbs} = \frac{1}{Z} \sum_{\{\sigma^z_i\}} e^{-H} \hat\sigma^z_\ell \hat\sigma^z_{\ell'}$
$\ell' > \ell$
$S_\ell \to \begin{pmatrix} +1 & \\ & -1 \end{pmatrix} = \hat\sigma^z$
$H = \lambda \sum_i \hat\sigma^z + k \sum_i \hat\sigma^z_i \hat\sigma^z_{i+1}$
$|i_1\rangle \otimes |i_2\rangle \ldots \otimes |i_N\rangle$
$\langle i_2 | \hat\sigma^z_\ell | i_\ell \rangle = \delta_2$
$\lambda = 0 \longrightarrow C_{\ell\ell'} = \frac{1}{Z} \mathrm{Tr} [T_1^{N-\ell'} \hat\sigma^z_{\ell'} T_1^{\ell'-\ell} \hat\sigma^z_\ell T_1^\ell]$
$T_1 |\rightarrow\rangle = \epsilon_1 |\rightarrow\rangle$
$T_1 |\leftarrow\rangle = \epsilon_2 |\leftarrow\rangle$
$|\rightleftarrows\rangle = \frac{|1\rangle \pm |-1\rangle}{\sqrt{2}}$
$T_1 = \begin{pmatrix} \epsilon_1 & \\ & \epsilon_2 \end{pmatrix}$
$\hat\sigma^z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
new basis
$\hat\sigma^z T_1^{\ell'-\ell} \hat\sigma^z = \begin{pmatrix} \epsilon_2^{\ell'-\ell} & \\ & \epsilon_1^{\ell'-\ell} \end{pmatrix}$
$= C_{\ell\ell'} = \frac{\epsilon_1^{N-\ell'+\ell} \epsilon_2^{\ell'-\ell} + \epsilon_2^{N-\ell'+\ell} \epsilon_1^{\ell'-\ell}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N\to\infty} (\tanh k)^{\ell'-\ell}$

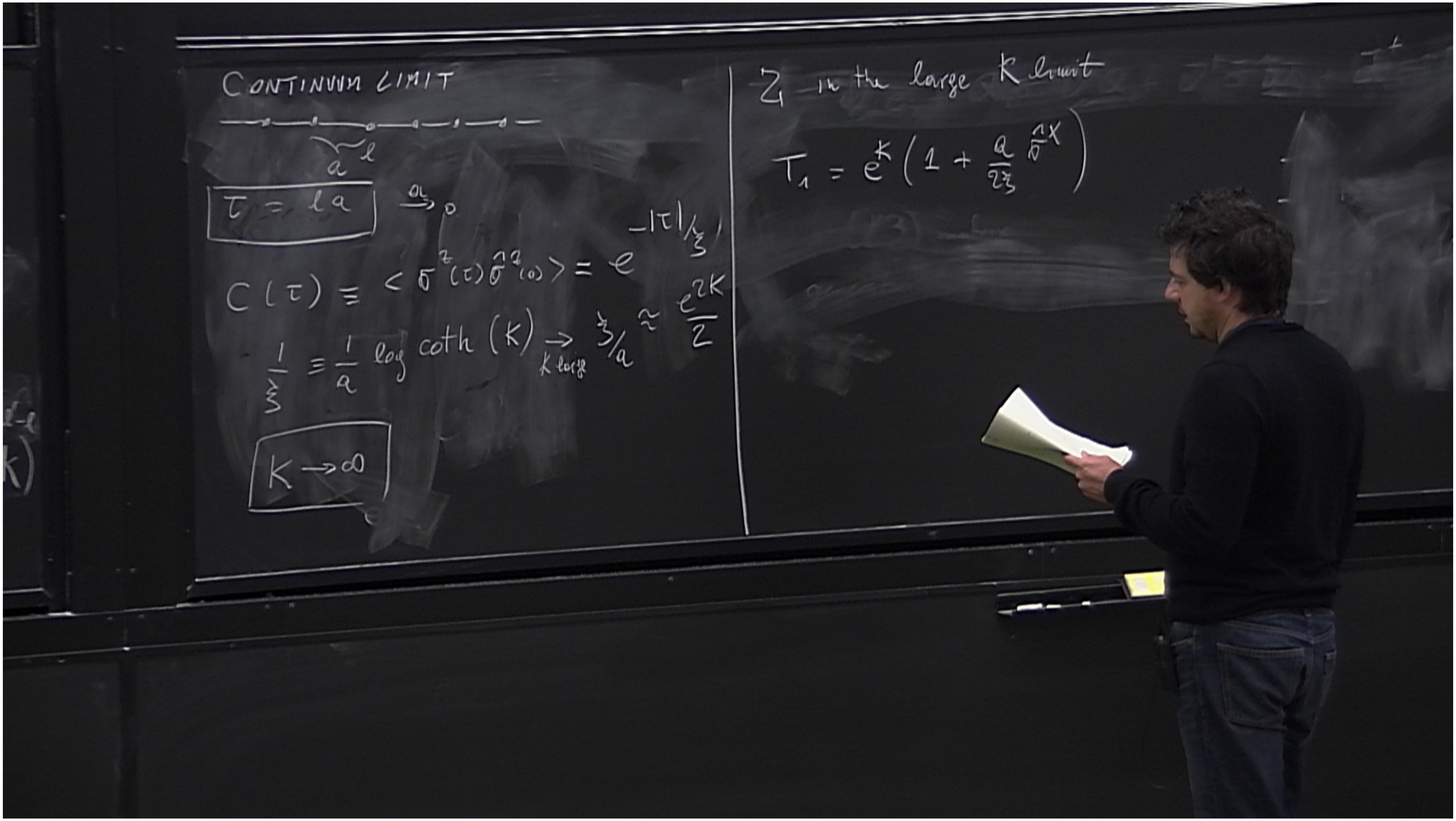

CONTINUUM LIMIT
$\tau = la$
$C(\tau) \equiv \langle \sigma^z(\tau)\sigma^z(0) \rangle = e$
$\frac{1}{\xi} \equiv \frac{1}{a} \log \coth (K)$

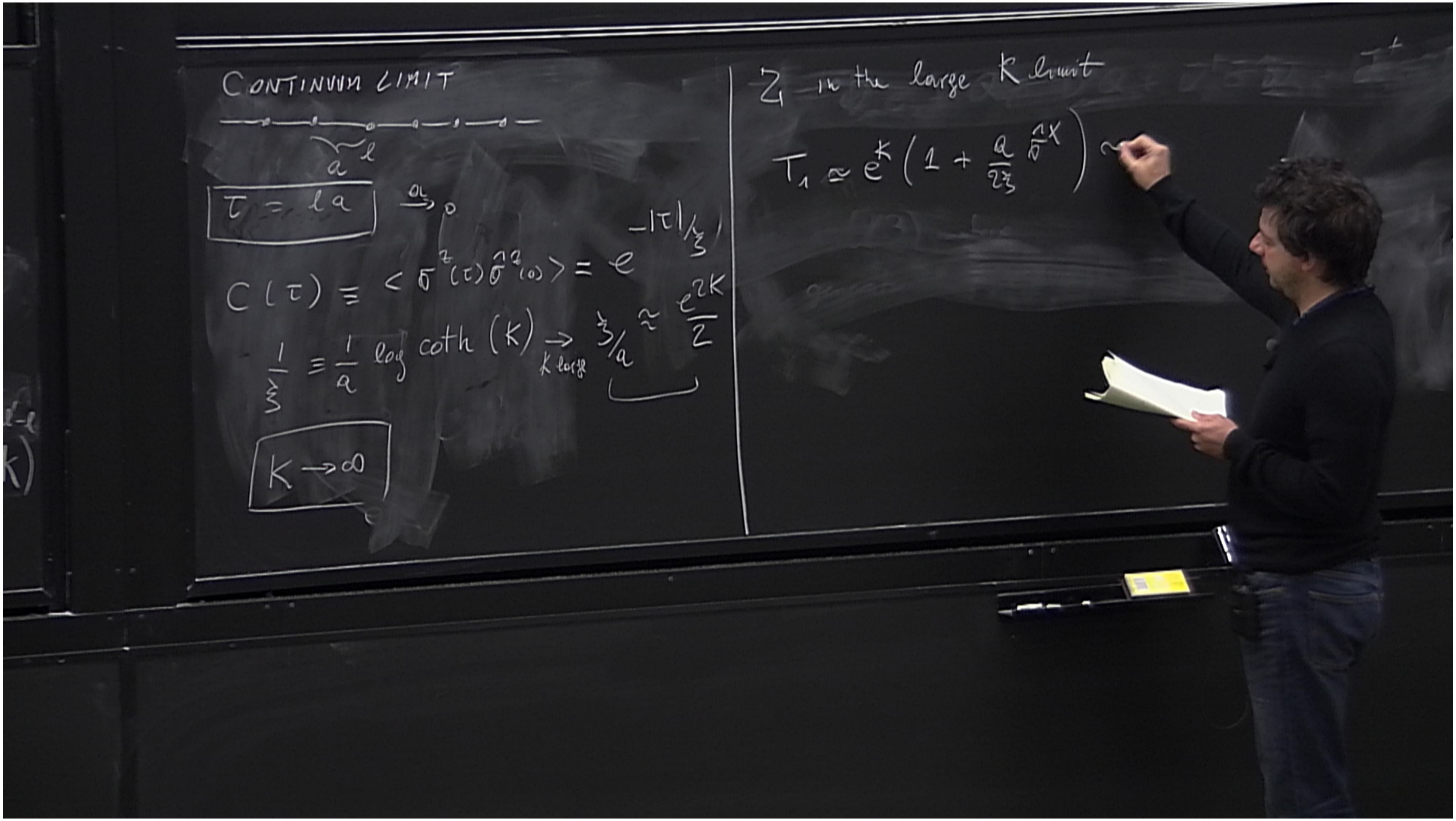

CONTINUUM LIMIT
τ = la
C(τ) ≡ <σ^z(τ)σ^z(0)> = e^{-|τ|/ξ}
1/ξ ≡ 1/a log coth(K)
K large
K → ∞

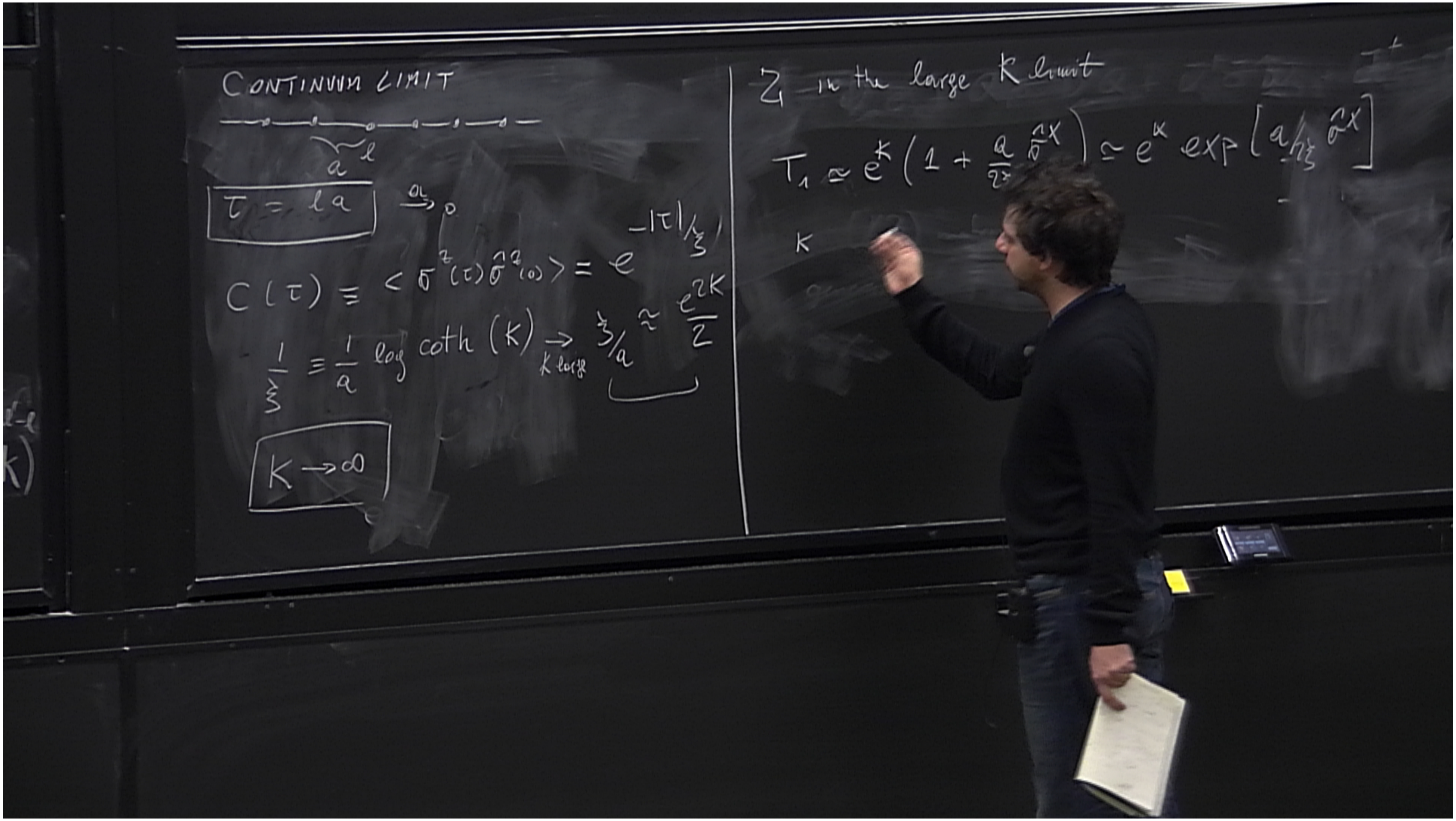

CONTINUUM LIMIT
ℓa
$a \to 0$
$= \langle \hat{\sigma}^z(\tau)\hat{\sigma}^z(0)\rangle = e^{-|\tau|/\xi}$
$= \frac{1}{a} \log \coth(K) \xrightarrow[K \text{ large}]{} \xi/a \approx \frac{e^{2K}}{2}$
Z_1 in the large K limit
T_1

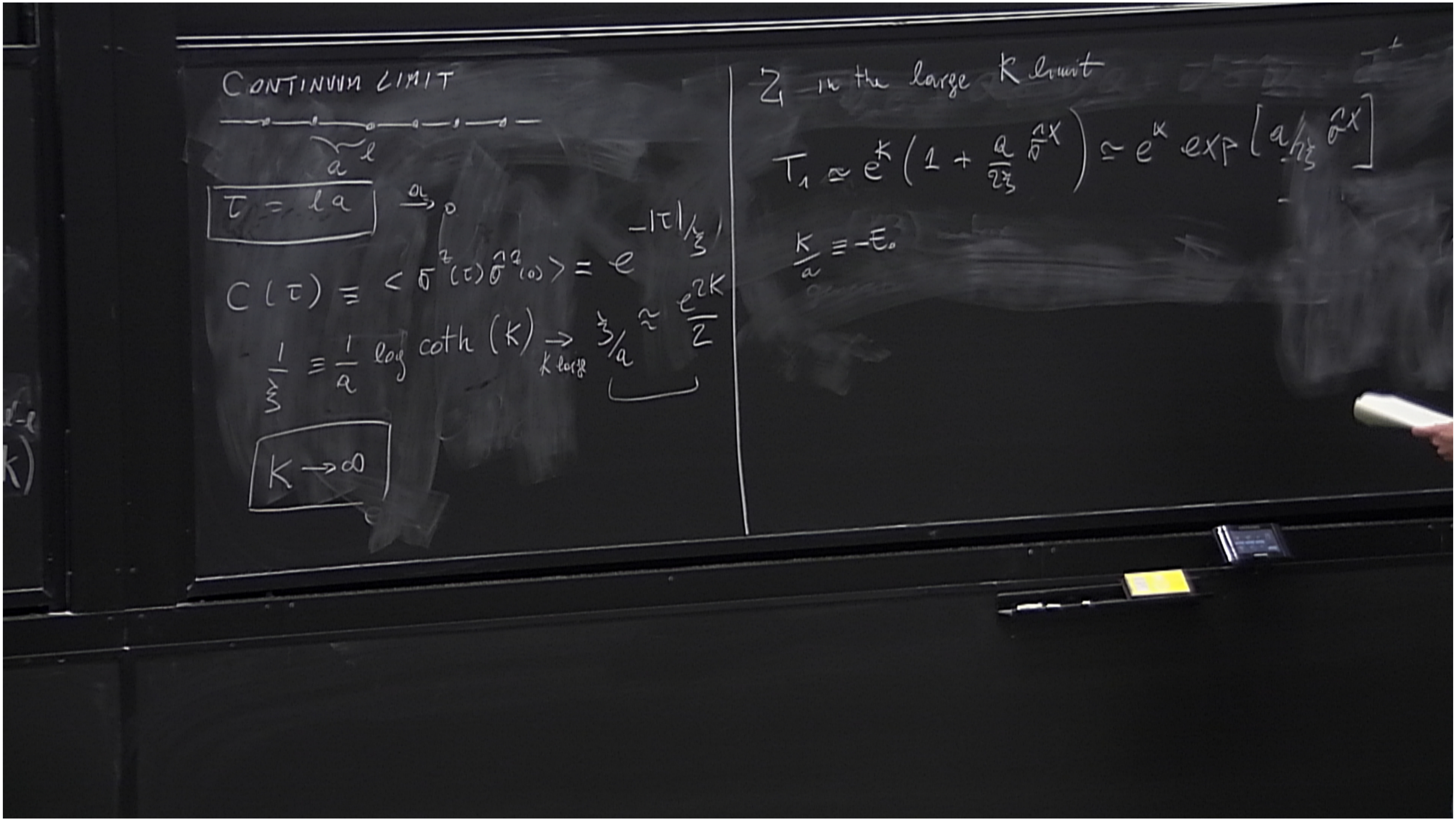

CONTINUUM LIMIT
$\tau = la$
$C(\tau) \equiv \langle \hat\sigma^z(\tau)\hat\sigma^z(0)\rangle = e^{-|\tau|/\xi}$
$\frac{1}{\xi} \equiv \frac{1}{a}\log\coth(K) \xrightarrow[K\,large]{} \xi/a \approx \frac{e^{2K}}{2}$
$K \to \infty$
Z_1 in the large K limit
$T_1 = e^K($

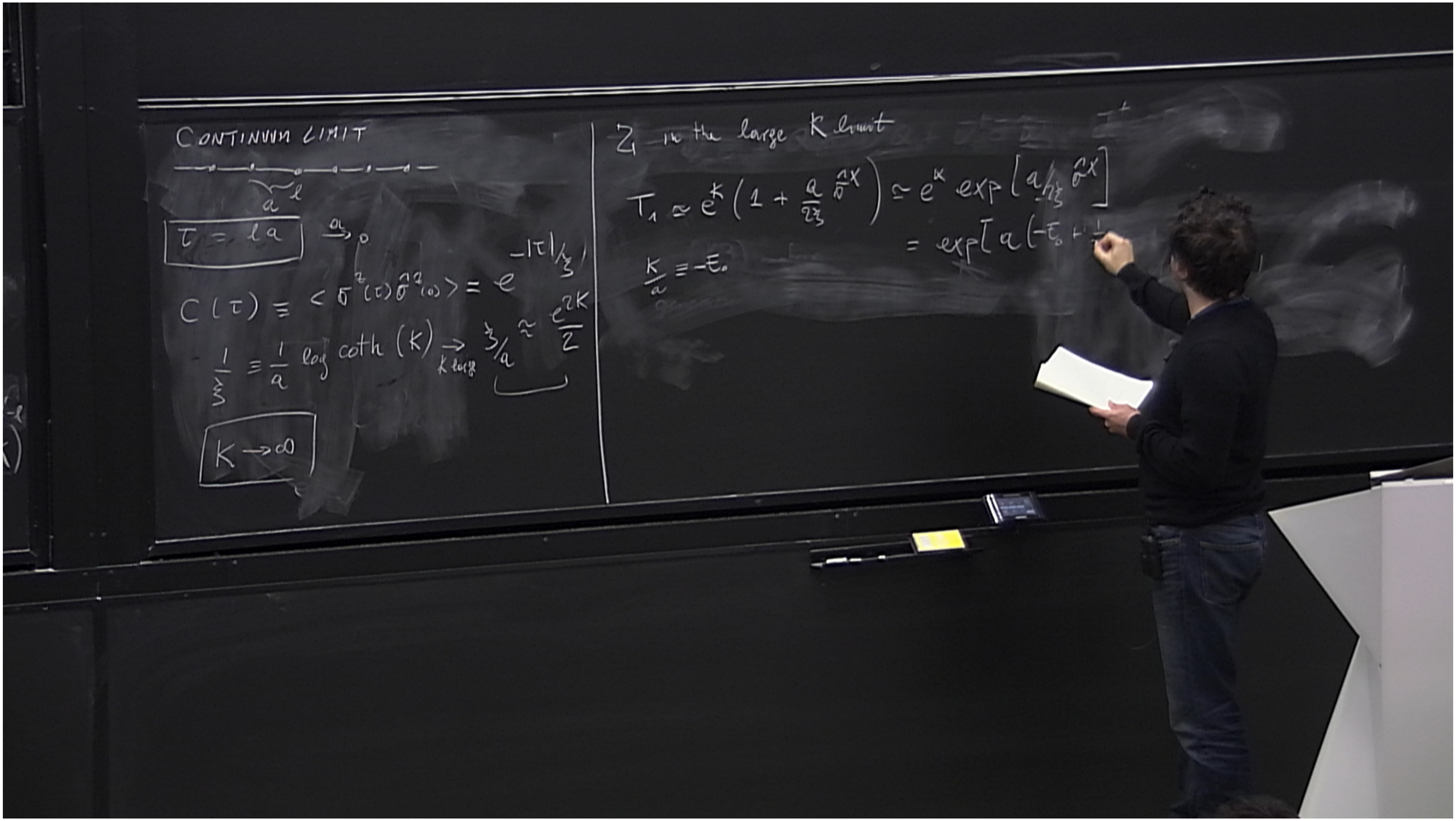

TRANSFER MATRIX
1d Ising Model
λ = βh
K = βJ
ε₁ > ε₂
ε₁ > 0
ε₁ analytic
K → ∞
1st order PT
Matter couplings
ψ → Uψ under gauge transform

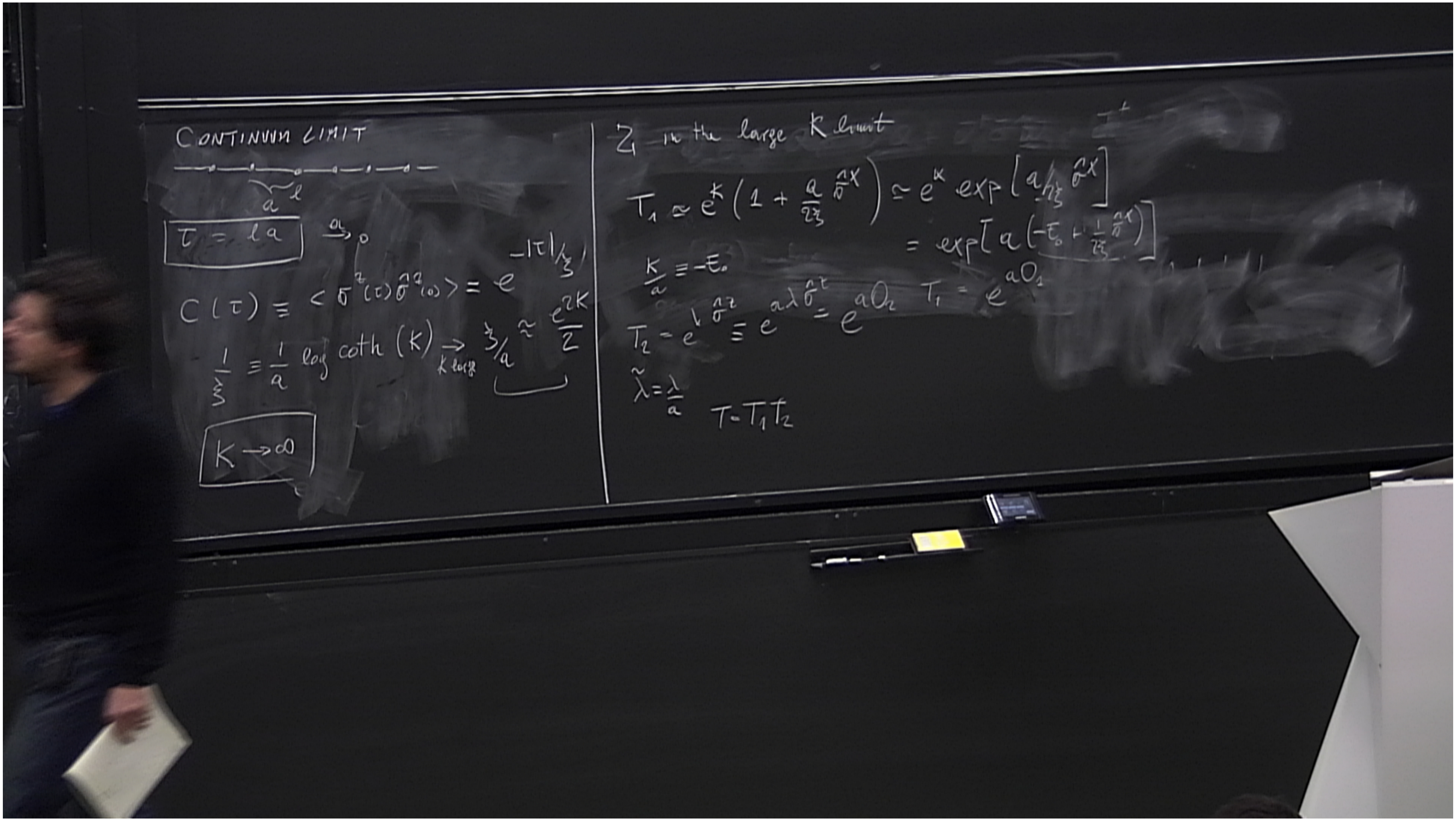

CONTINUUM LIMIT
$\tau = la$
$C(\tau) \equiv \langle \hat\sigma^z(\tau)\hat\sigma^z(0)\rangle = e^{-|\tau|/\xi}$
$\frac{1}{\xi} \equiv \frac{1}{a}\log\coth(K) \xrightarrow[K\ \text{large}]{} \xi/a \approx \frac{e^{2K}}{2}$
$K \to \infty$
Z_1 in the large K limit
$T_1 = e^K\left(1 + \frac{a}{2\xi}\hat\sigma^x\right)$

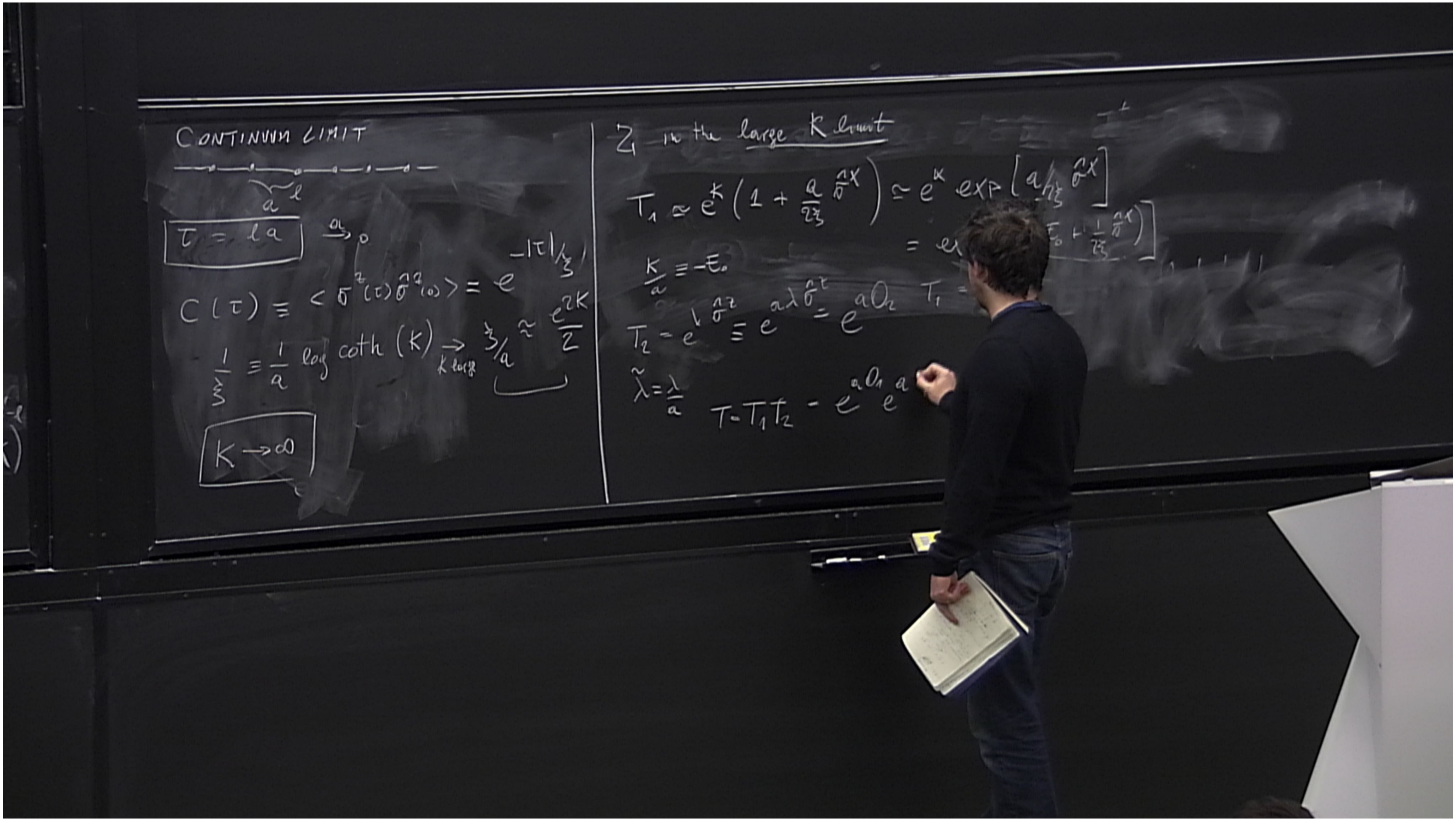

CONTINUUM LIMIT
τ = l a
C(τ) ≡ < σ^z(τ) σ^z(0) > = e^{-|τ|/ξ}
1/ξ ≡ 1/a log coth(K) → ξ/a ≈ e^{2K}/2
K large
K → ∞
Z_1 in the large K limit
T_1 ≈ e^K (1 + a/(2ξ) σ^x)

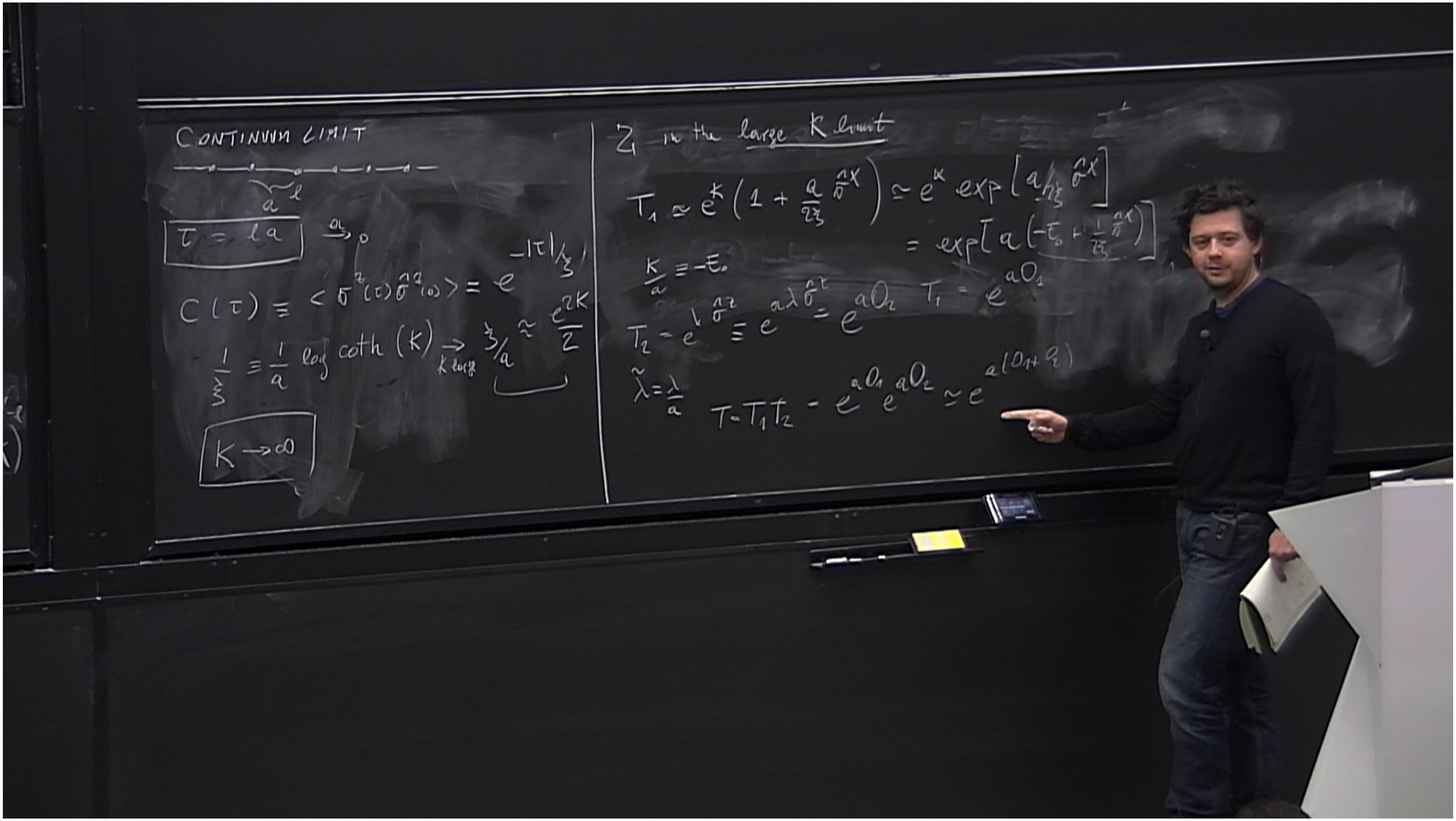

CONTINUUM LIMIT
in the large K limit

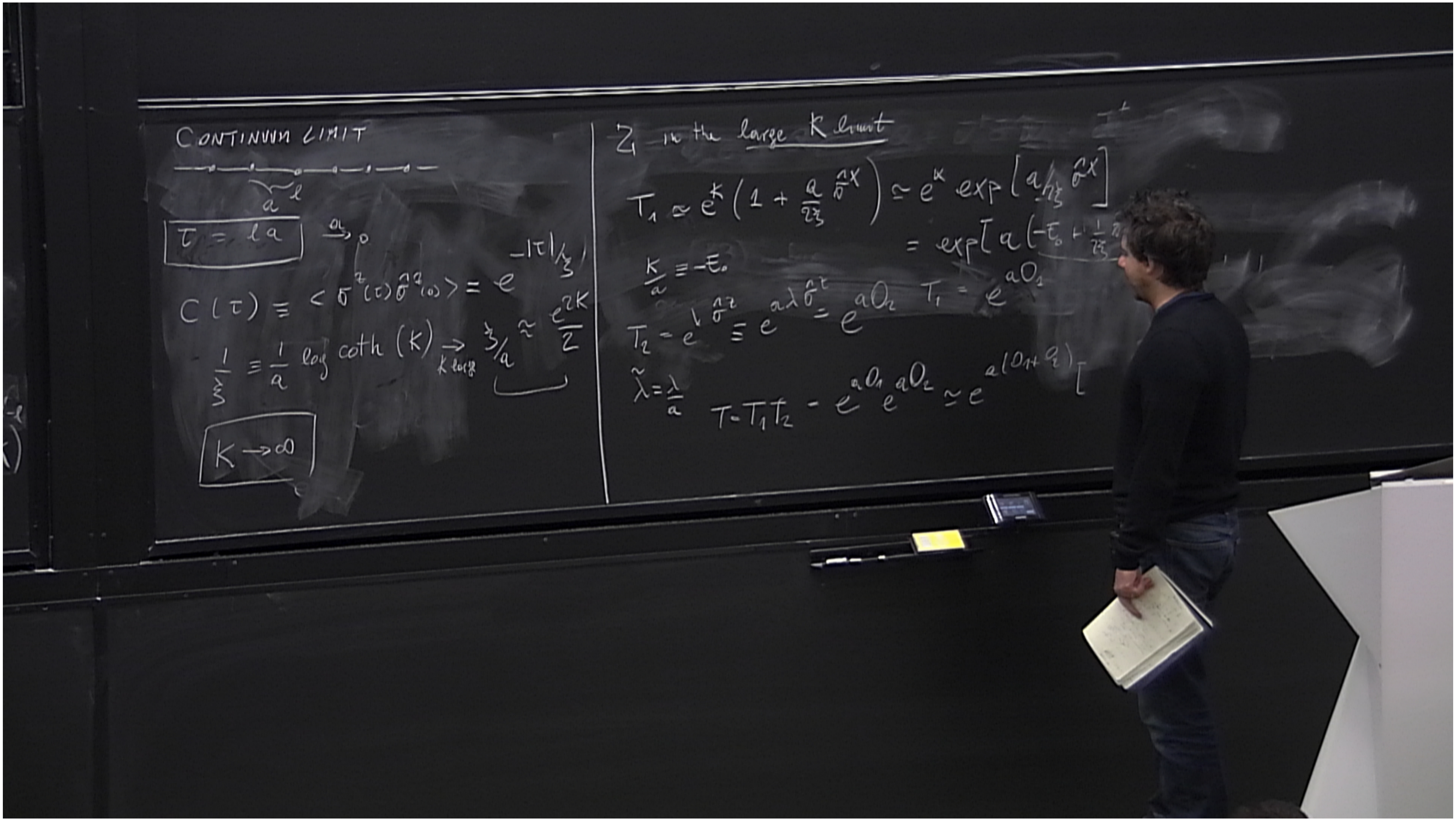

CONTINUUM LIMIT
τ = la
C(τ) ≡ <σ^z(τ) σ^z(0)> = e^{-|τ|/ξ}
1/ξ ≡ 1/a log coth(K) → ξ/a ≈ e^{2K}/2
K → ∞
Z_1 in the large K limit
T_1 ≃ e^K (1 + a/2ξ σ^x) ≃ e^K exp[a/2ξ σ^x]
K/a ≡ -E_0

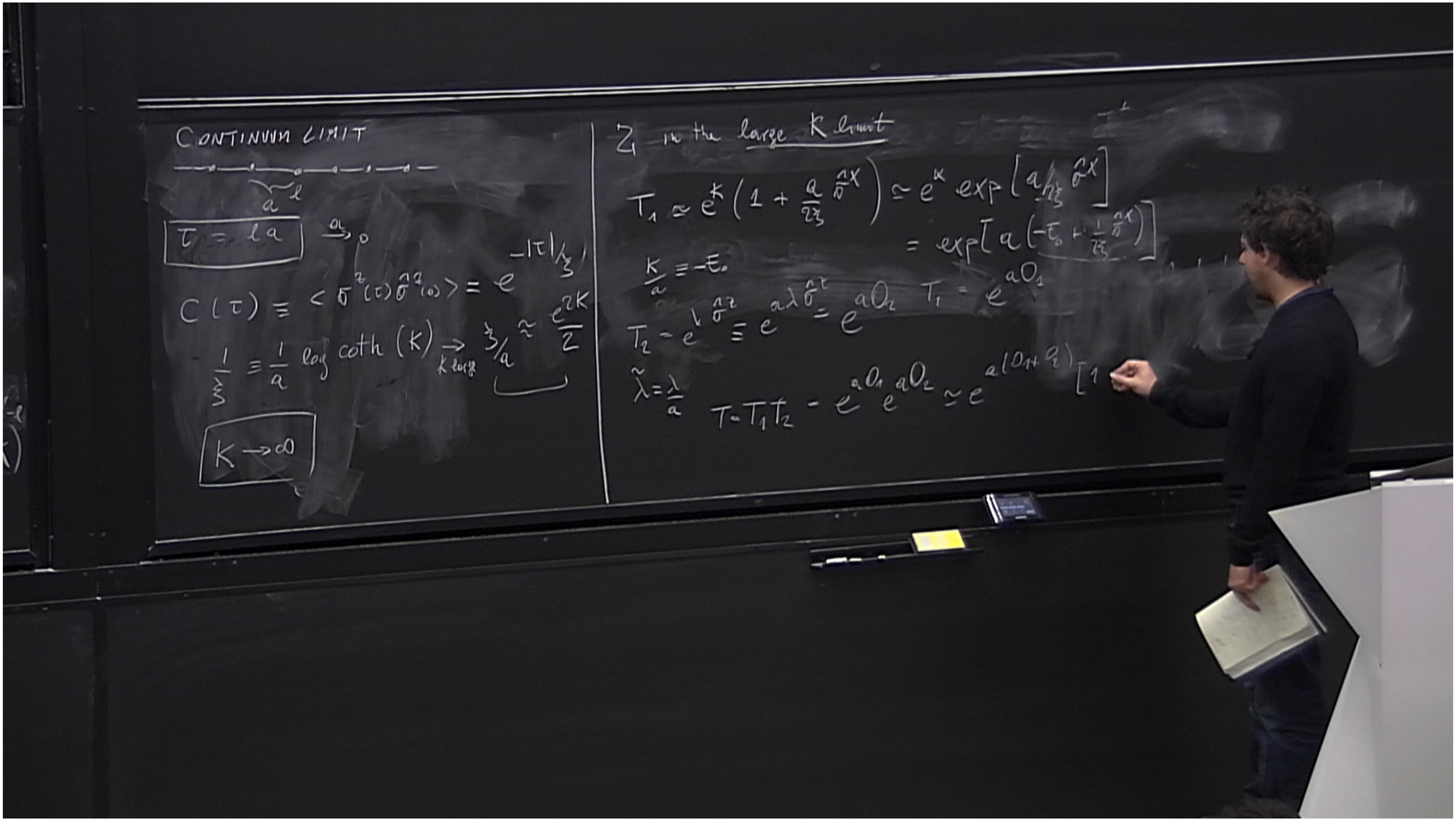

CONTINUUM LIMIT
τ = ℓa
C(τ) ≡ <σ^z(τ)σ^z(0)> = e^{-|τ|/ξ}
K → ∞
Z₁ in the large K limit
K/a ≡ -E₀

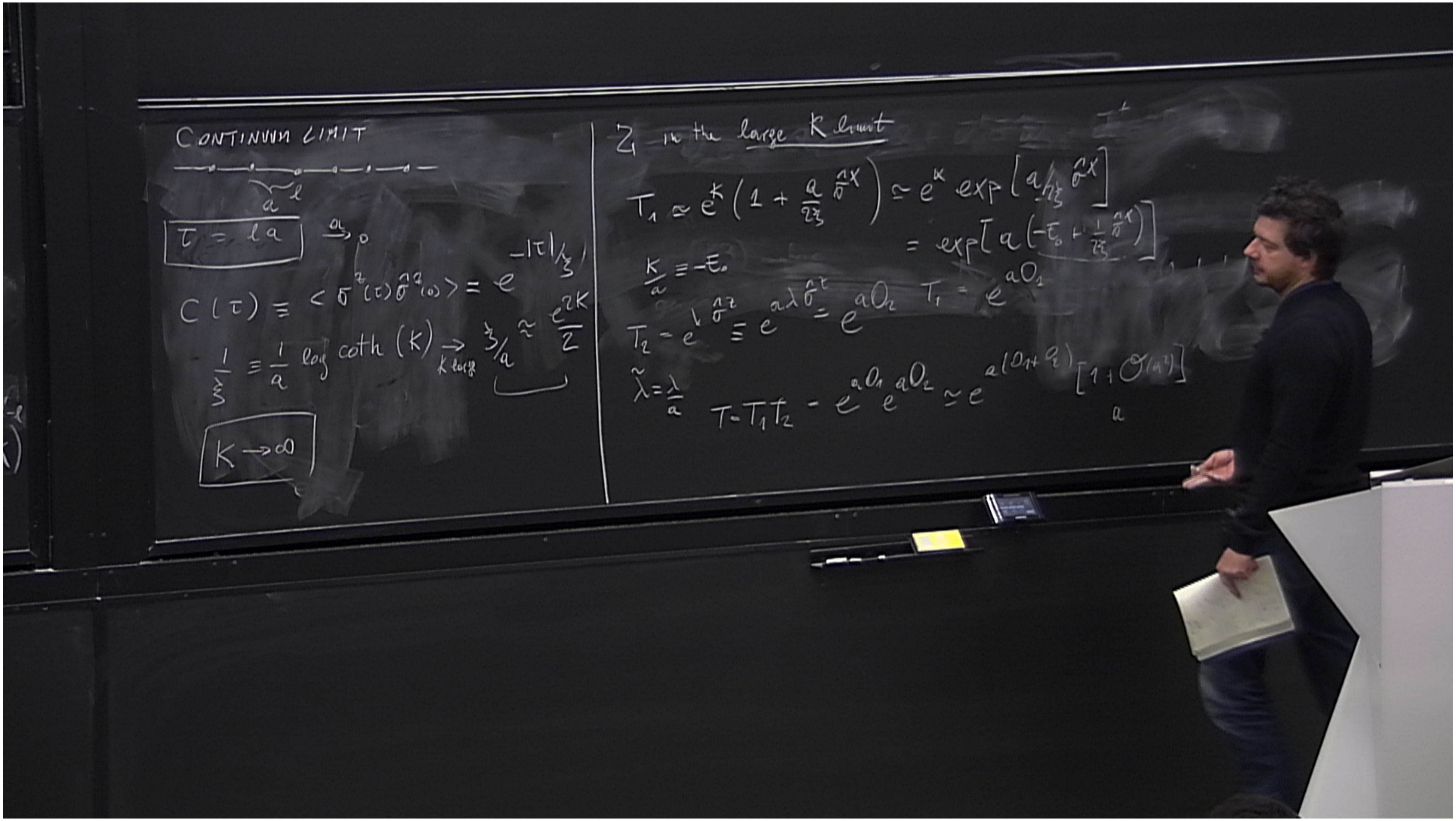

CONTINUUM LIMIT
τ = ℓa
K → ∞
Z1 in the large K limit
T = T1 T2

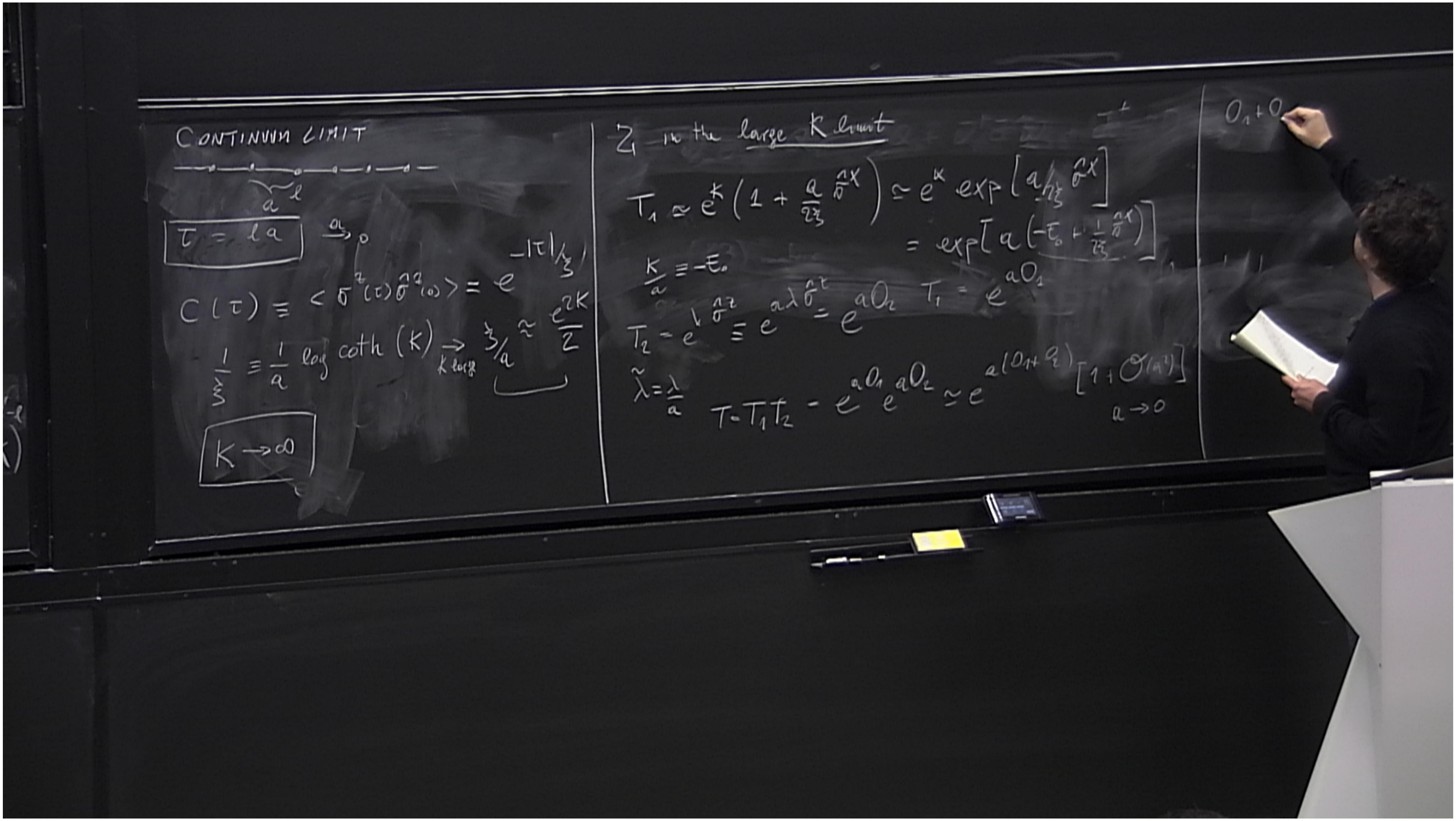


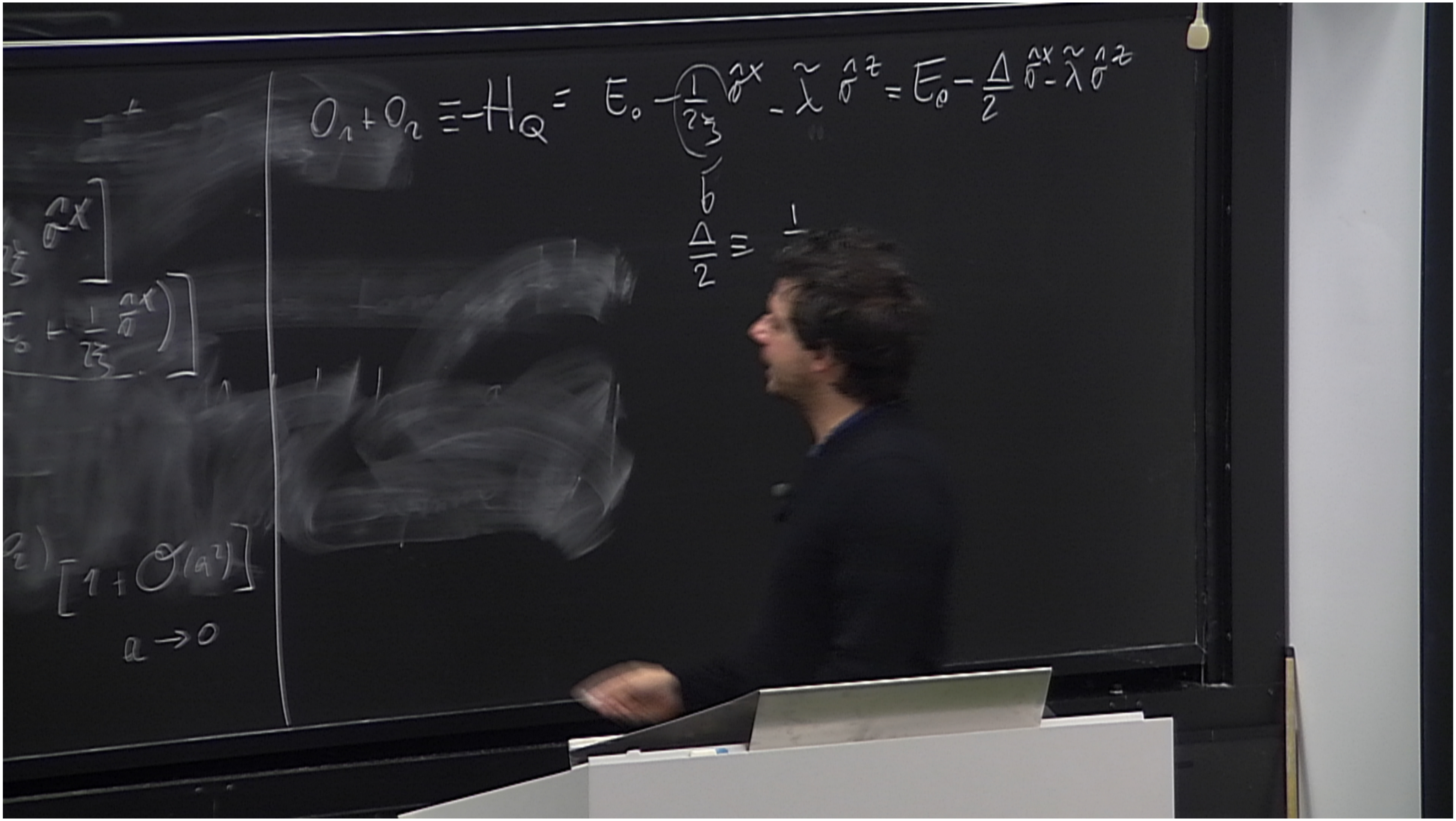

CONTINUUM LIMIT
τ = la
C(τ) ≡ < σ^z(τ) σ^z(0) > = e^{-|τ|/ξ}
1/ξ ≡ 1/a log coth(K) → ξ/a ≃ e^{2K}/2
K large
K → ∞
Z_1 in the large K limit
T_1 ≃ e^K (1 + a/2ξ σ^x) ≃ e^K exp[a/2ξ σ^x]
= exp[a(-E_0 + 1/2ξ σ^x)]
K/a ≡ -E_0
T_1 = e^{aO_1}
T_2 = e^{λ σ^z} ≡ e^{aλ̃ σ^z} = e^{aO_2}
λ̃ = λ/a
T = T_1 T_2 = e^{aO_1} e^{aO_2} ≃ e^{a(O_1+O_2)}

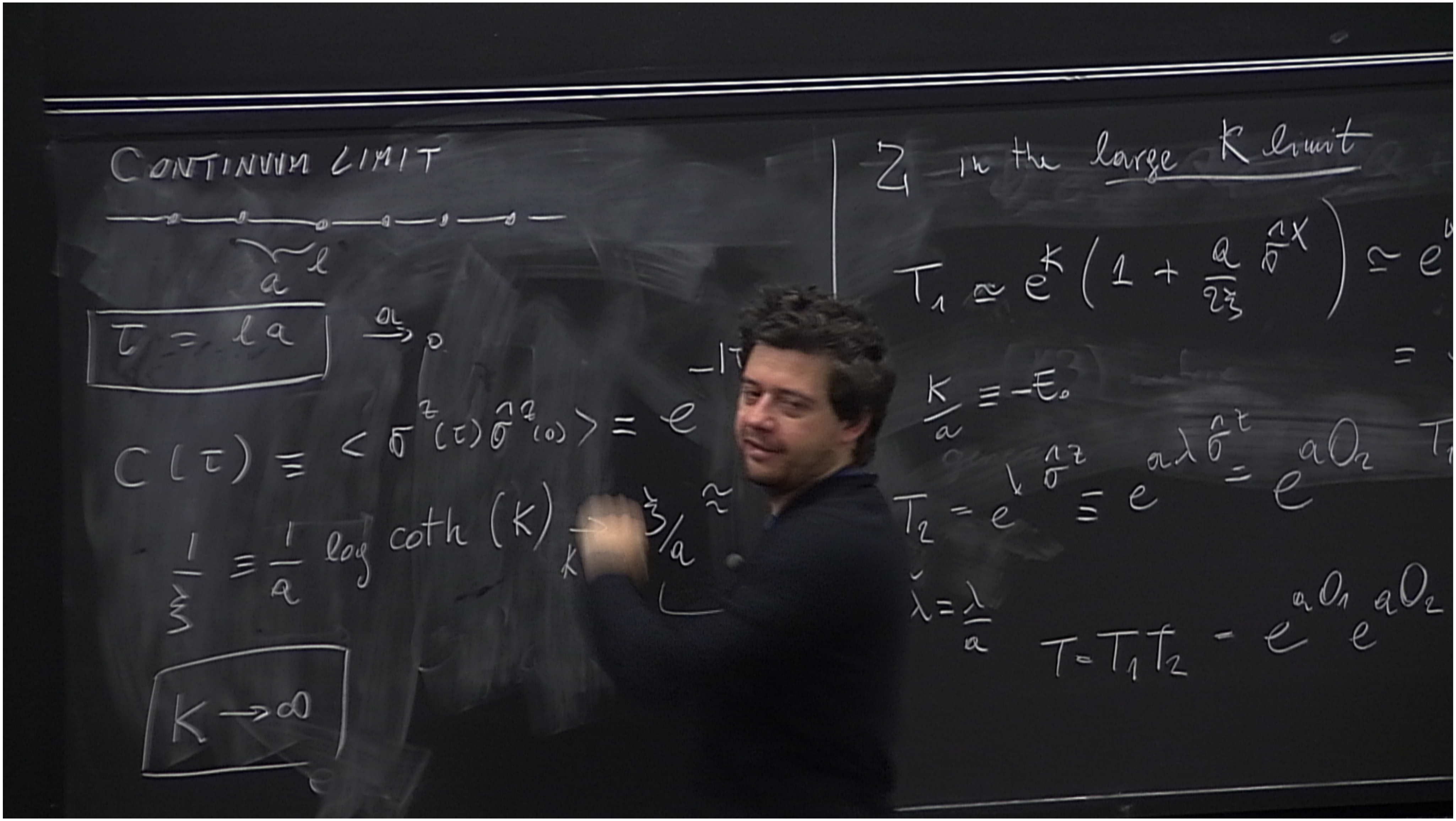

CONTINUUM LIMIT
τ = la
K → ∞
Z₁ in the large K limit
exp

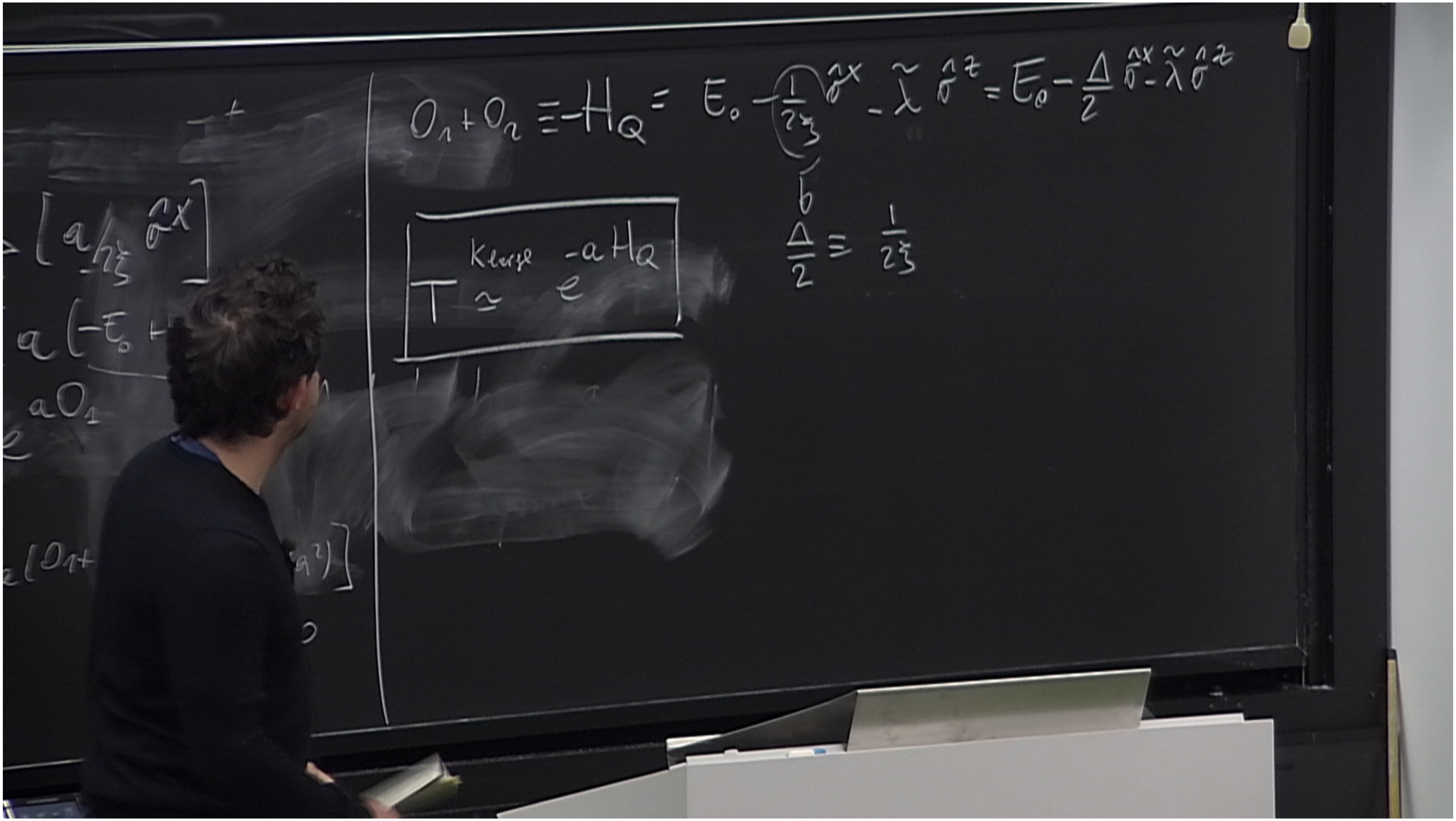

CONTINUUM LIMIT
in the large K limit

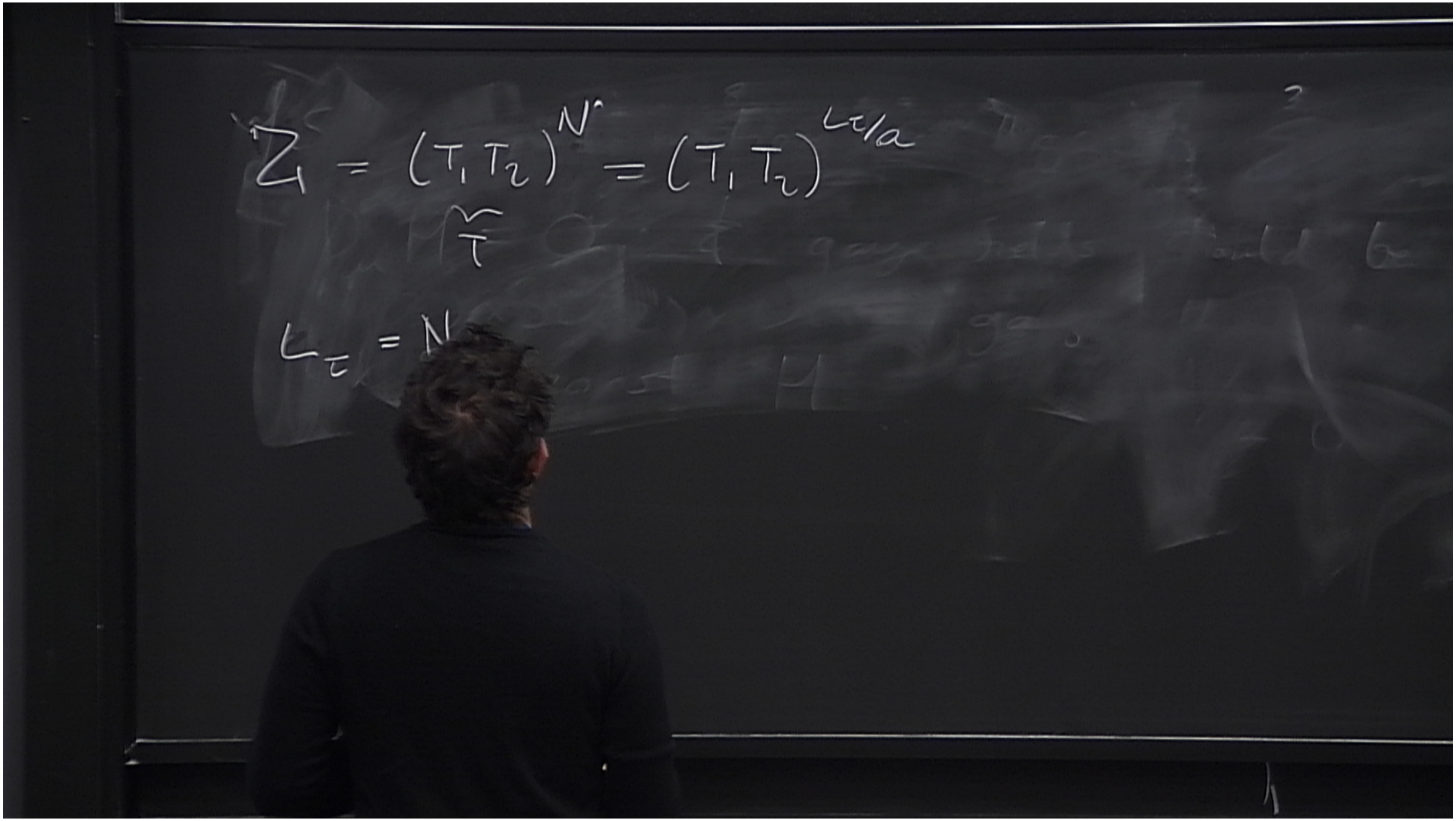

CONTINUUM LIMIT
τ = l a
K → ∞
Z1 in the large K limit
T = T1 T2

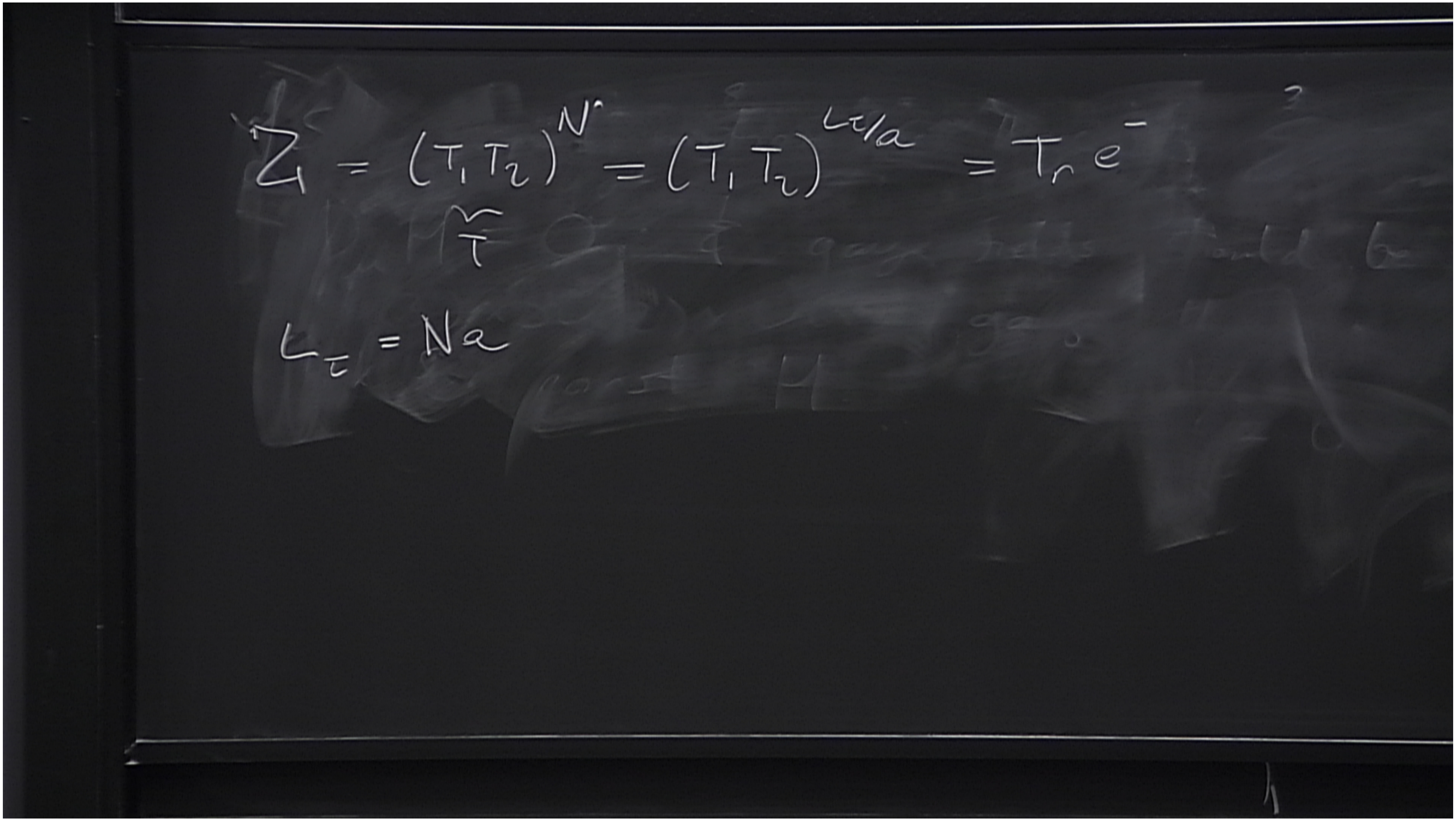

CONTINUUM LIMIT
τ = l a
C(τ) ≡ <σ^z(τ) σ^z(0)> = e^{-|τ|/ξ}
1/ξ ≡ 1/a log coth(K) → ξ/a ≃ e^{2K}/2
K large
K → ∞
Z_1 in the large K limit
T_1 ≃ e^K (1 + a/2ξ σ^x) ≃ e^K exp[a/2ξ σ^x]
= exp[a(−E_0 + 1/2ξ σ^x)]
K/a ≡ −E_0
T_1 = e^{aO_1}
T_2 = e^{l σ^z} ≡ e^{aλ σ^z} = e^{aO_2}
λ̃ = λ/a
T = T_1 T_2 = e^{aO_1} e^{aO_2} ≃ e^{a(O_1+O_2)} [1 + O(a^2)]
a → 0
O_1 + O

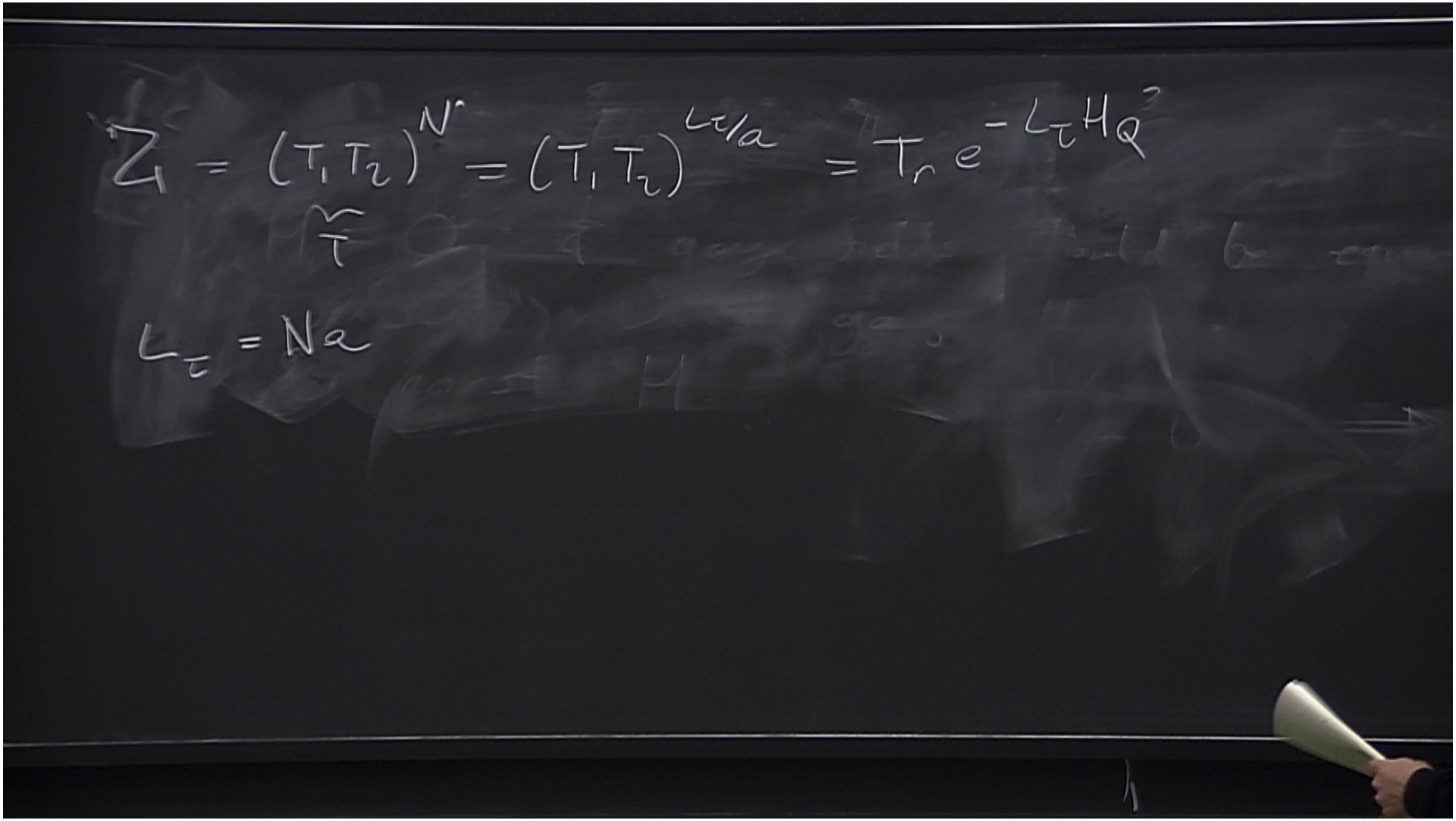

O1 + O2 ≡ −HQ = E0 − 1/2ξ σ̂x − λ̃ σ̂z = E0 − Δ/2 σ̂x − λ̃ σ̂z
[1 + O(a²)]
a → 0

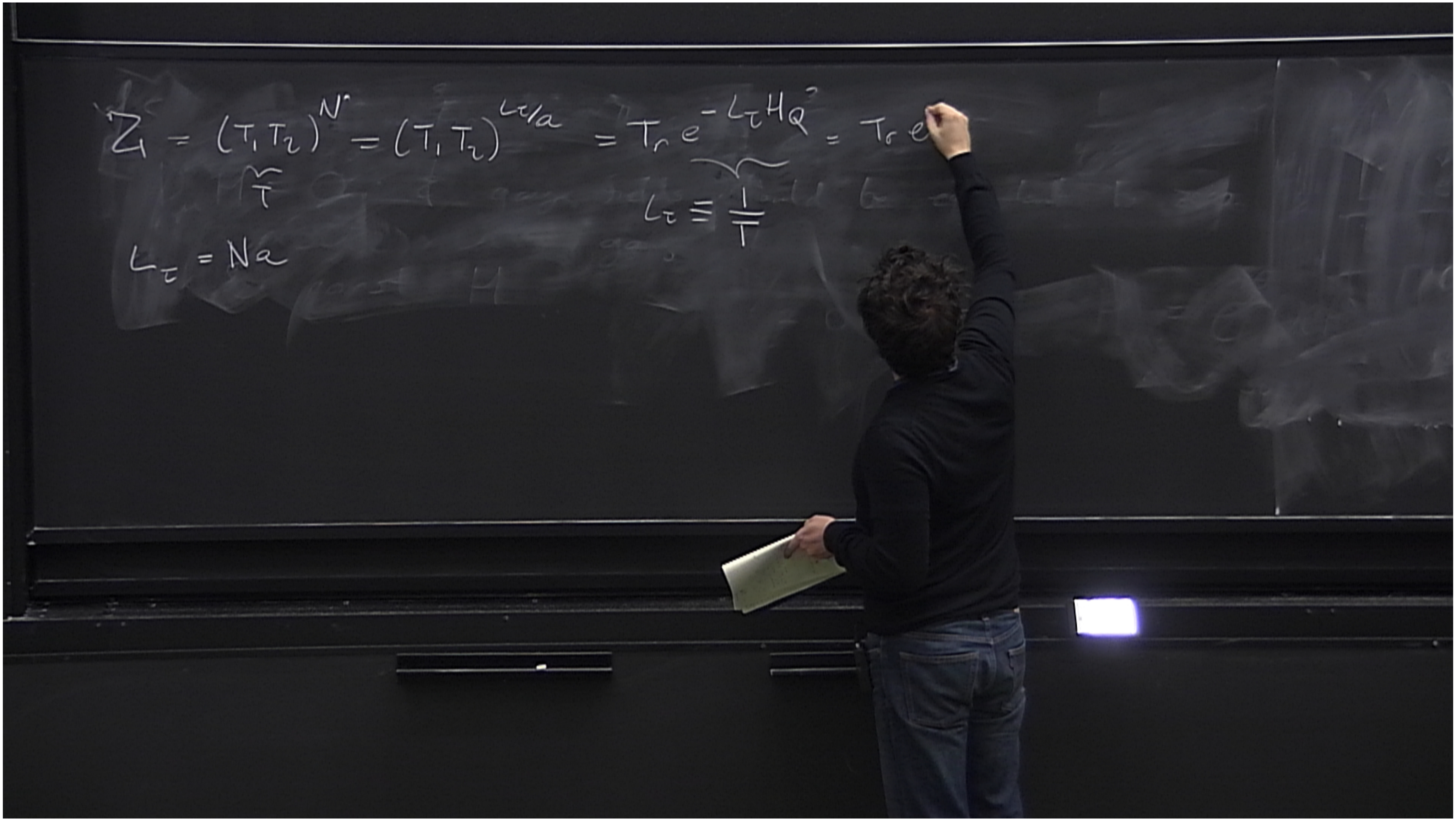

CONTINUUM LIMIT
τ = l a
C(τ) ≡ < σ^z(τ) σ^z(0) > = e
1/ξ ≡ 1/a log coth (K)
K → ∞
Z_1 in the large K limit
T_1 ≃ e^K (1 + a/2ξ σ^x) ≃ e
K/a ≡ −E_0
T_2 = e^{k σ^z} ≡ e^{aλ σ^z} = e^{aO_2}
λ = λ/a
T = T_1 T_2 = e^{aO_1} e^{aO_2}

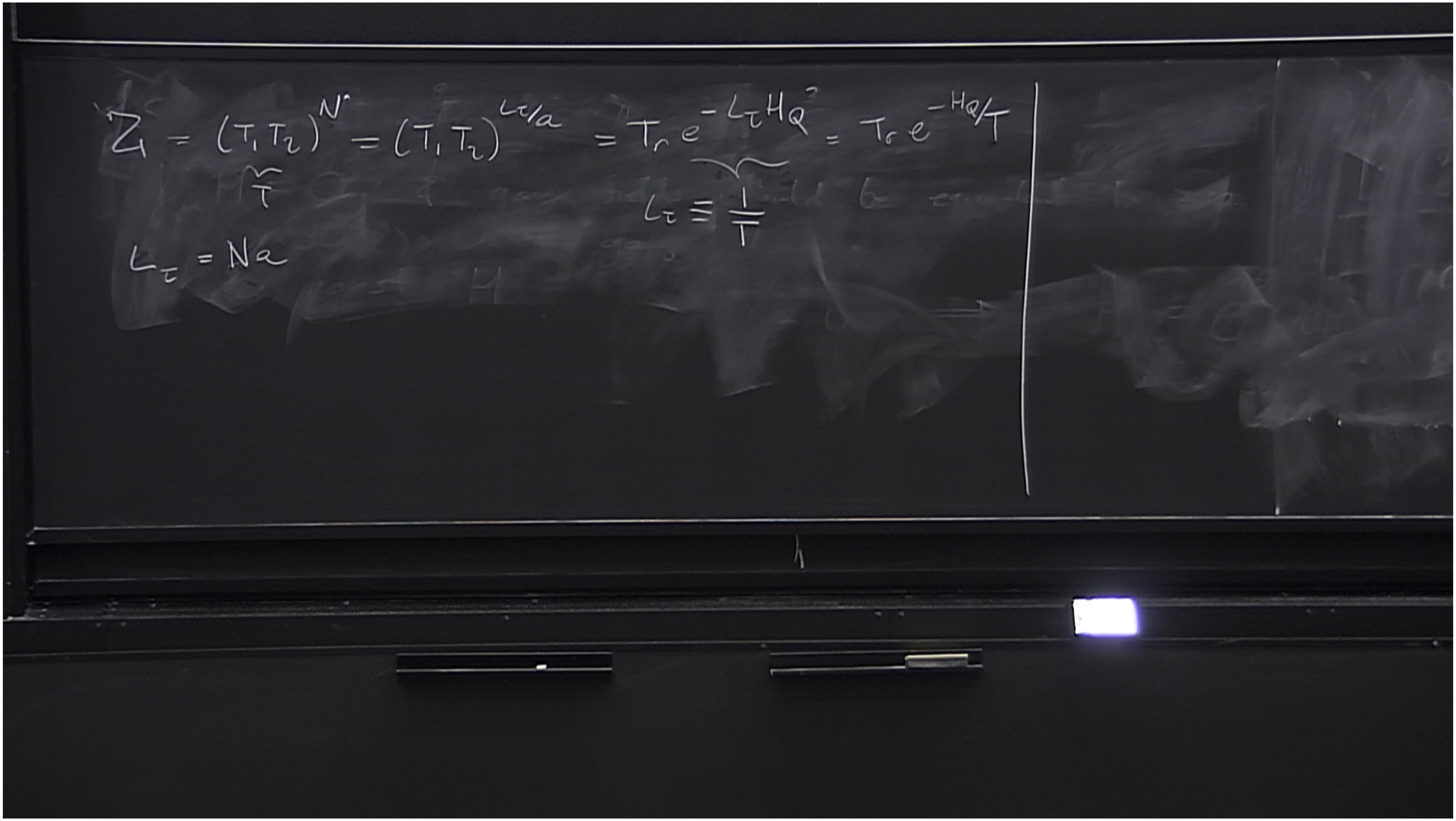

O1 + O2 ≡ −HQ = E0 − 1/2ξ σ̂x − λ̃ σ̂z = E0 − Δ/2 σ̂x − λ̃ σ̂z
Δ/2 ≡ 1/2ξ
T ≃ e^(−a HQ)
Klerge

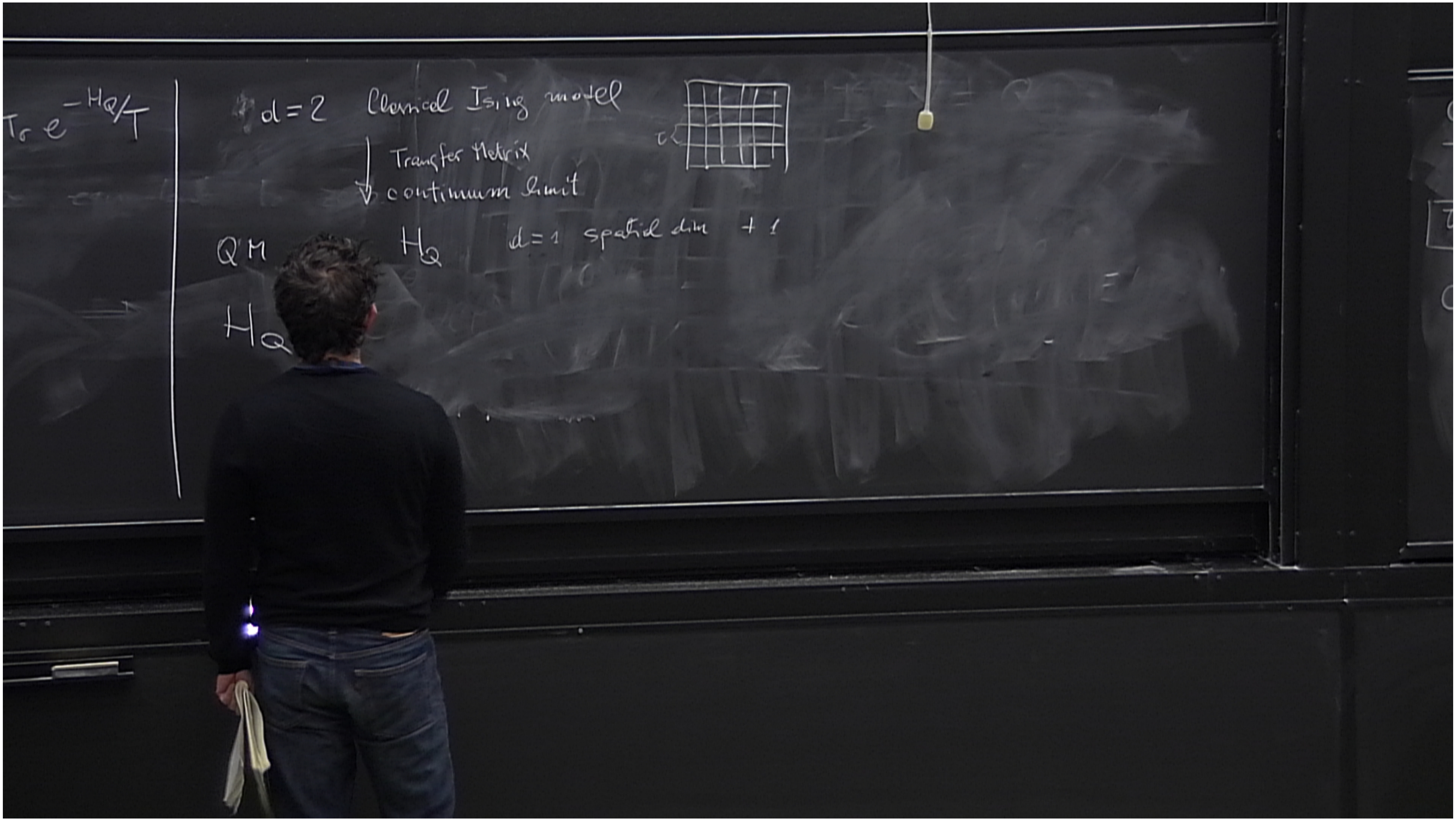

Z1 = (T1 T2)^N = (T1 T2)^Lτ/a
Lτ = N

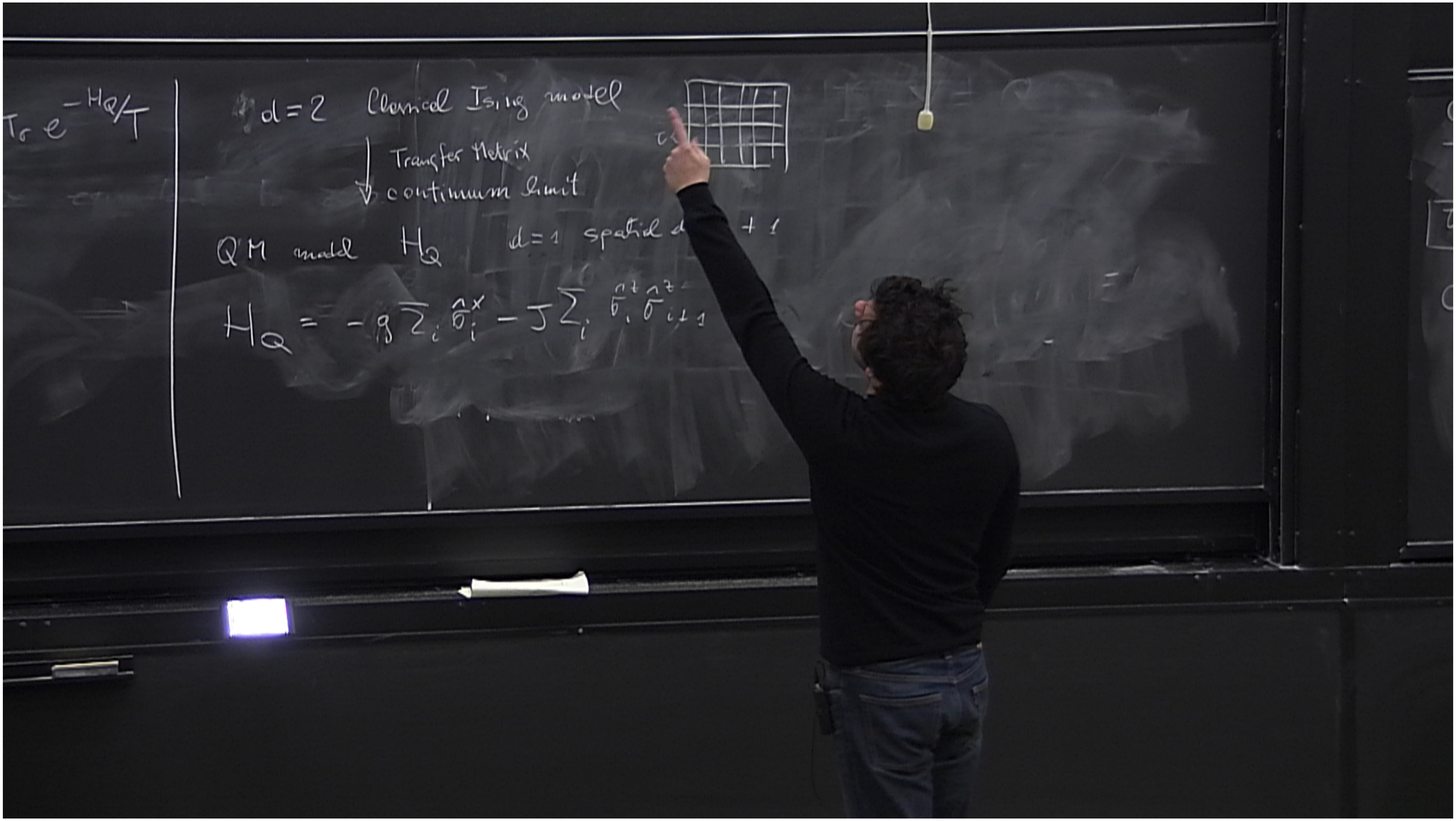

Z = (T1 T2)^N = (T1 T2)^Lτ/a = Tr e^-
Lτ = Na

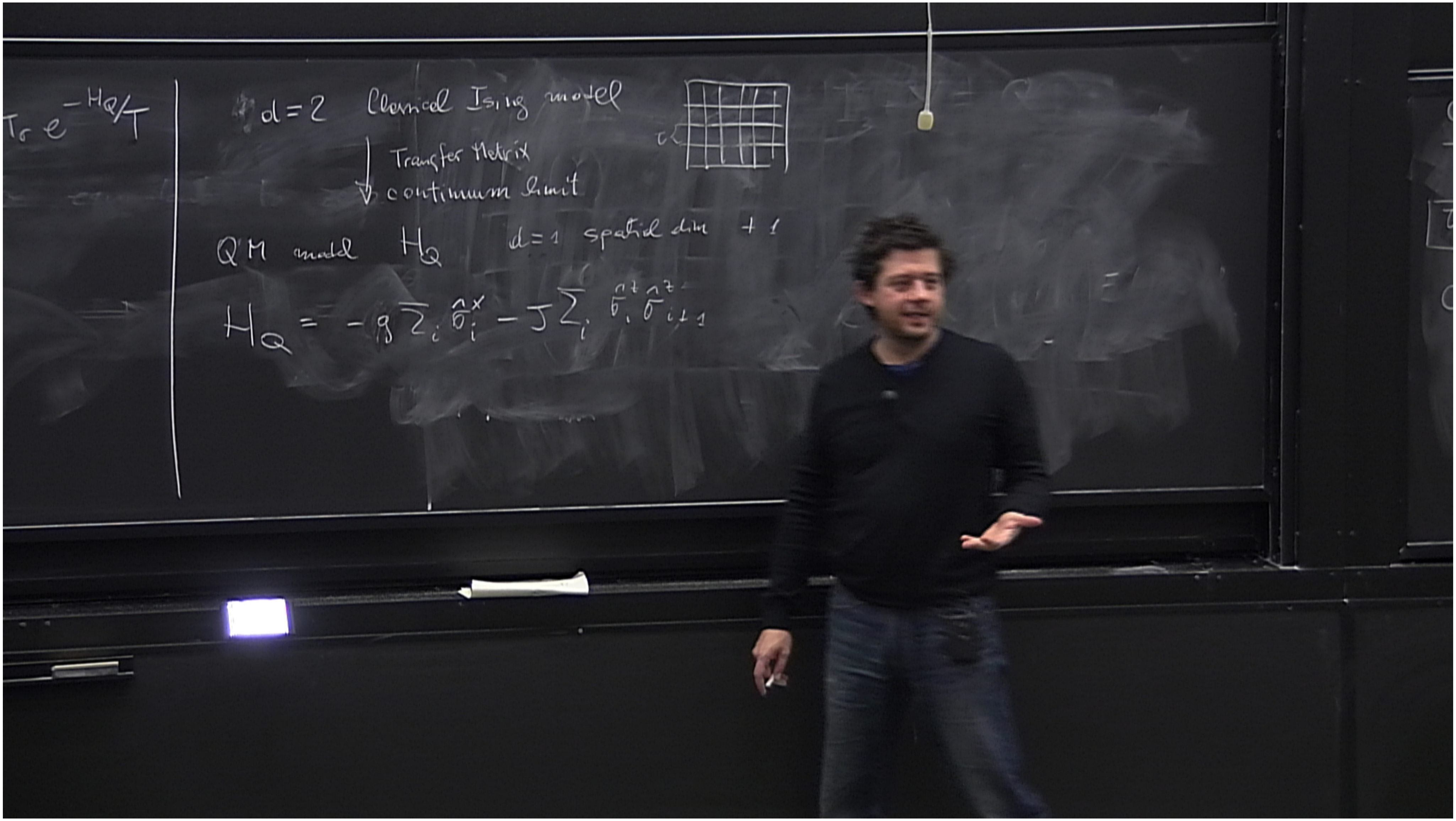

$Z = (T_1 T_2)^N = (T_1 T_2)^{L_\tau/a} = \mathrm{Tr}\, e^{-L_\tau H_Q}$
$L_\tau = Na$


$Z_1 = (T_1 T_2)^N = (T_1 T_2)^{L_\tau/a} = \mathrm{Tr}\, e^{-L_\tau H_Q} = \mathrm{Tr}\, e$
$L_\tau \equiv \frac{1}{T}$
$L_\tau = Na$

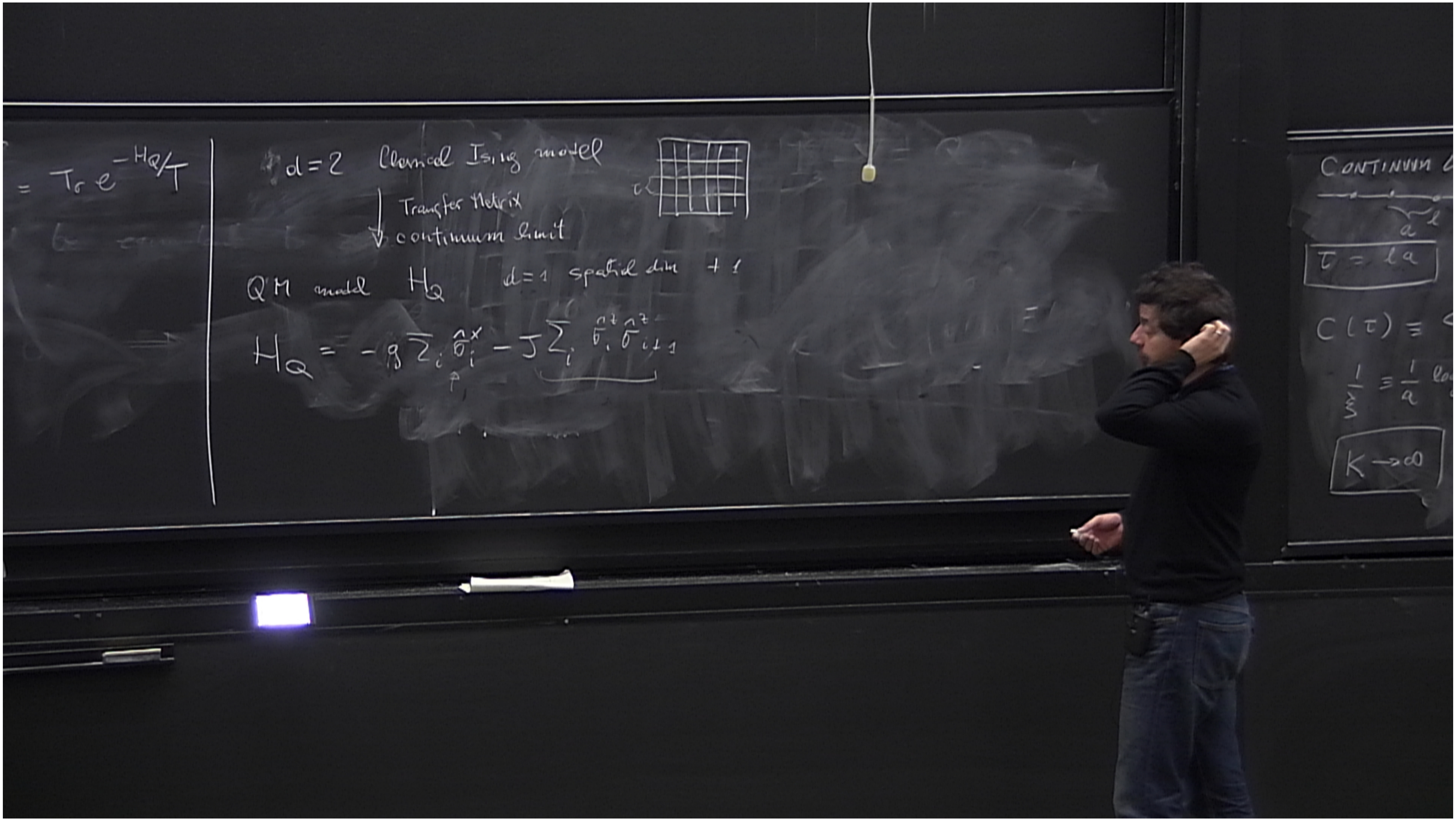

$Z_1 = (T_1 T_2)^N = (T_1 T_2)^{L_\tau/a} = \mathrm{Tr}\, e^{-L_\tau H_Q} = \mathrm{Tr}\, e^{-H_Q/T}$
$\tilde{T}$
$L_\tau \equiv \frac{1}{T}$
$L_\tau = Na$

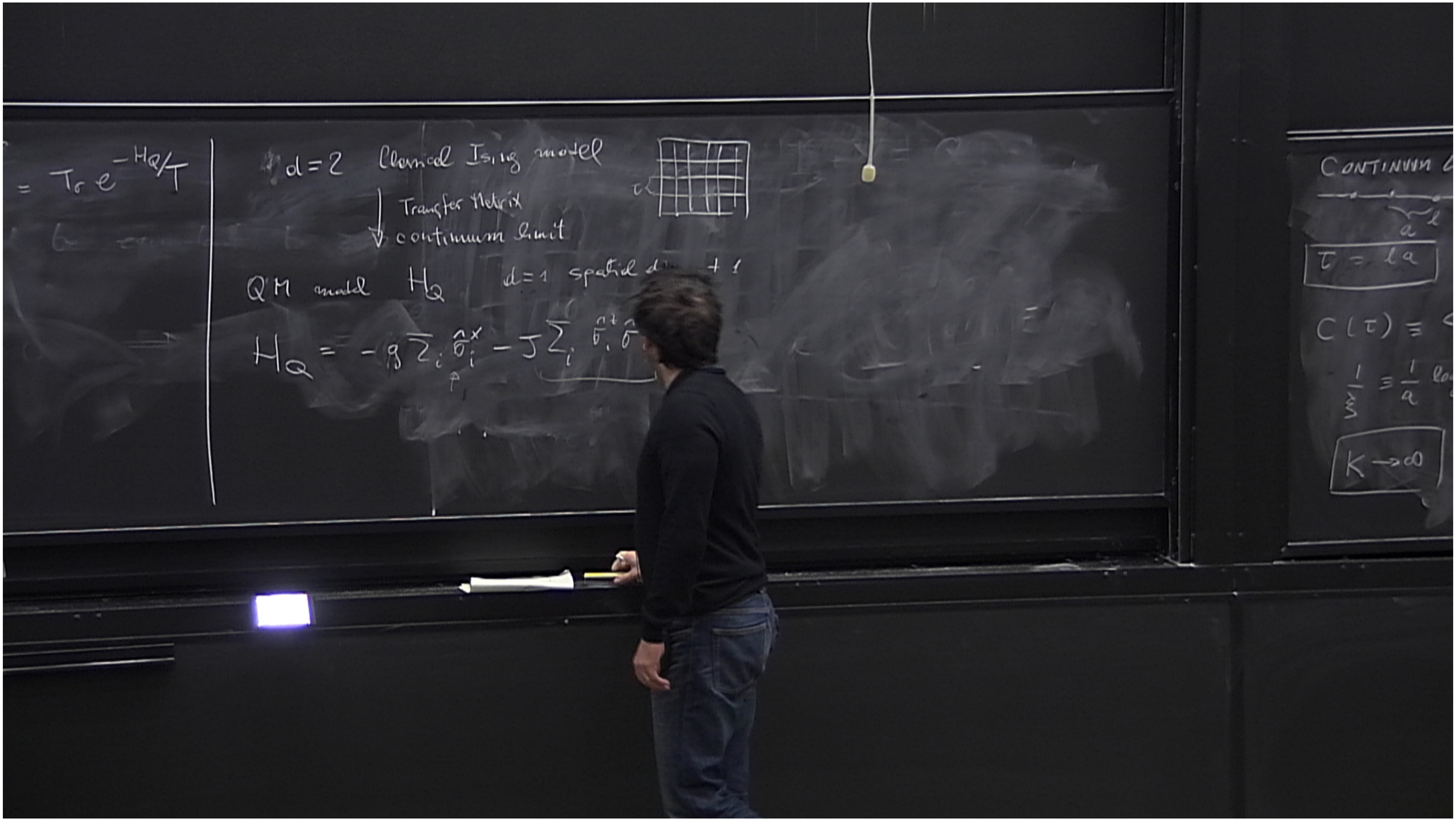

Tr e^{-HQ/T}
d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM
HQ
d = 1 spatial dim + 1
HQ

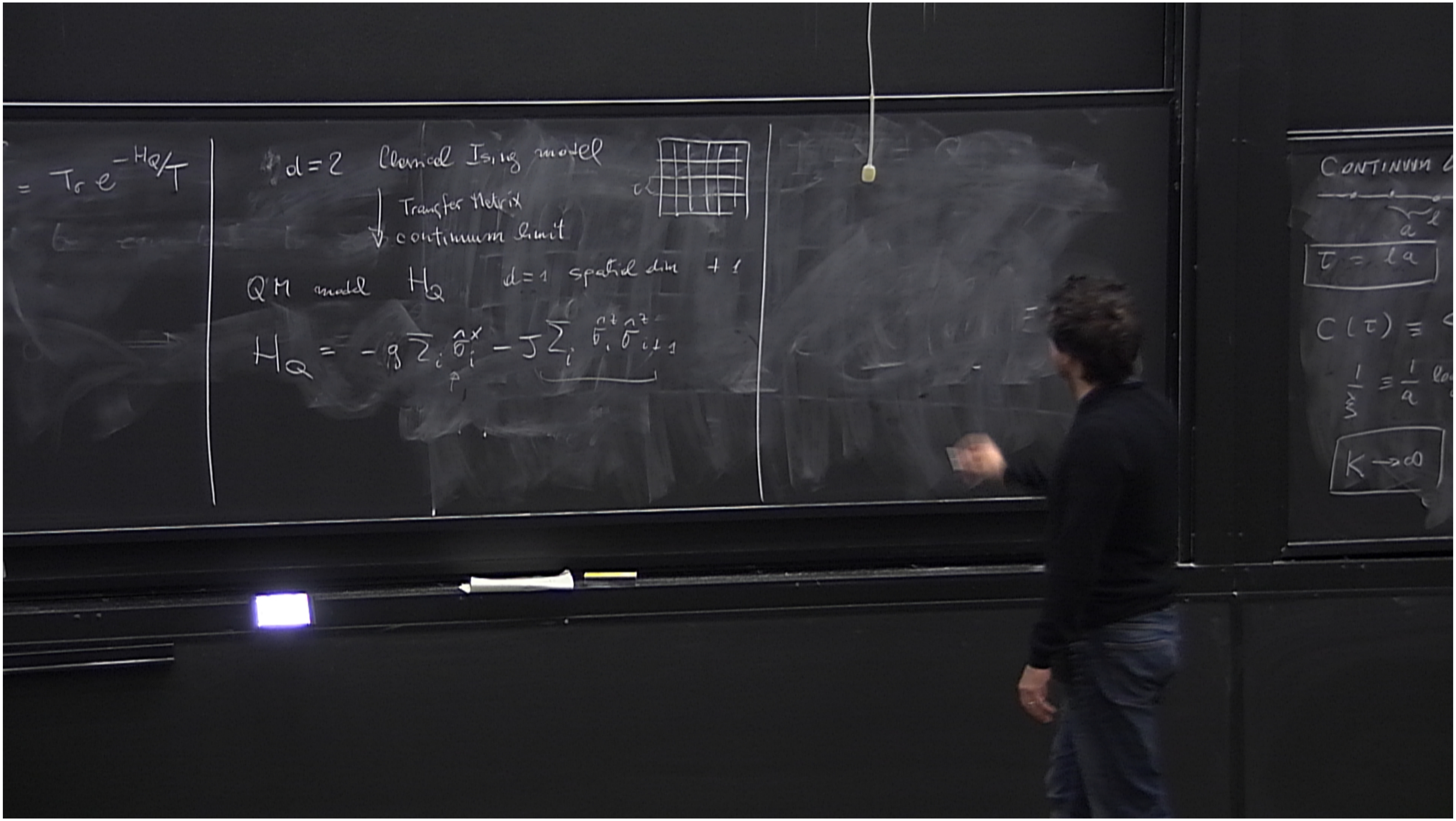

Tr e^{-HQ/T}
d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model HQ
d = 1 spatial
HQ = -g Σi σ̂i^x - J Σi σ̂i^z σ̂i+1^z



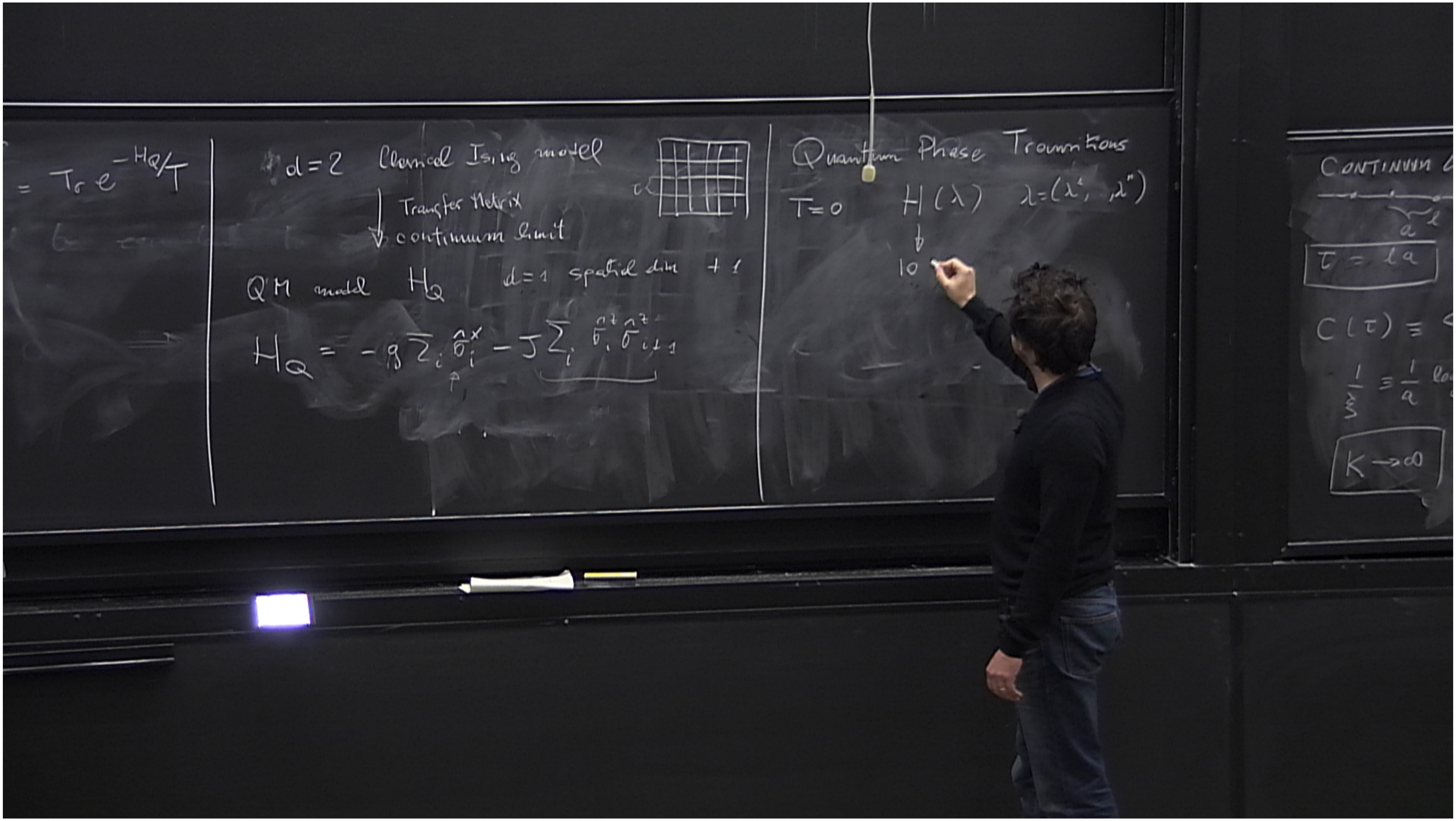
Tr e^{-HQ/T}
d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model HQ
d = 1 spatial dim + 1
HQ = - g Σi σ̂i^x - J Σi σ̂i^z σ̂i+1^z


d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model
d = 1 spatial dim + 1
CONTINUUM

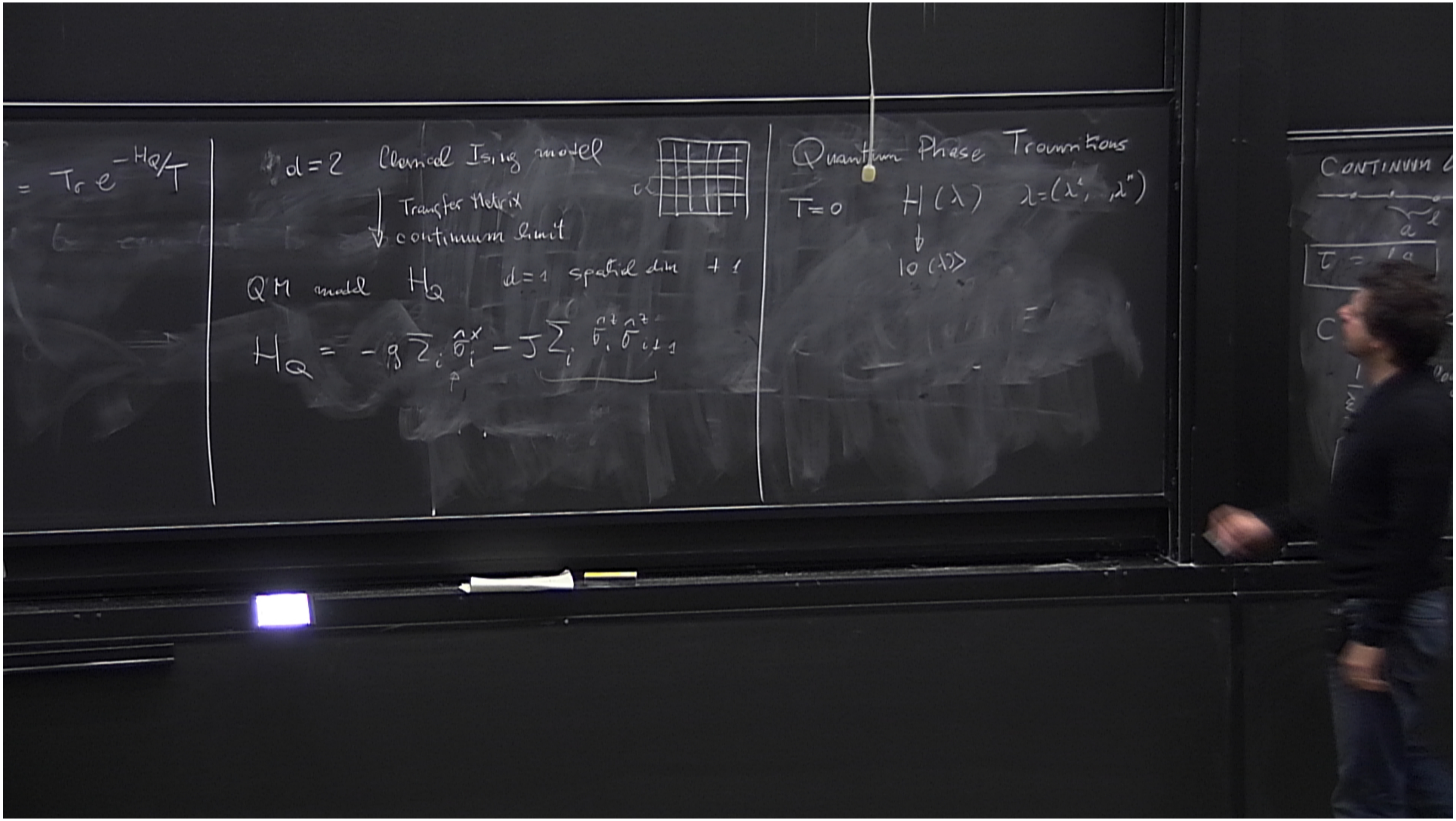

d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model H_Q
d = 1 spatial
CONTINUUM
τ = ℓa
C(τ) ≡
K → ∞

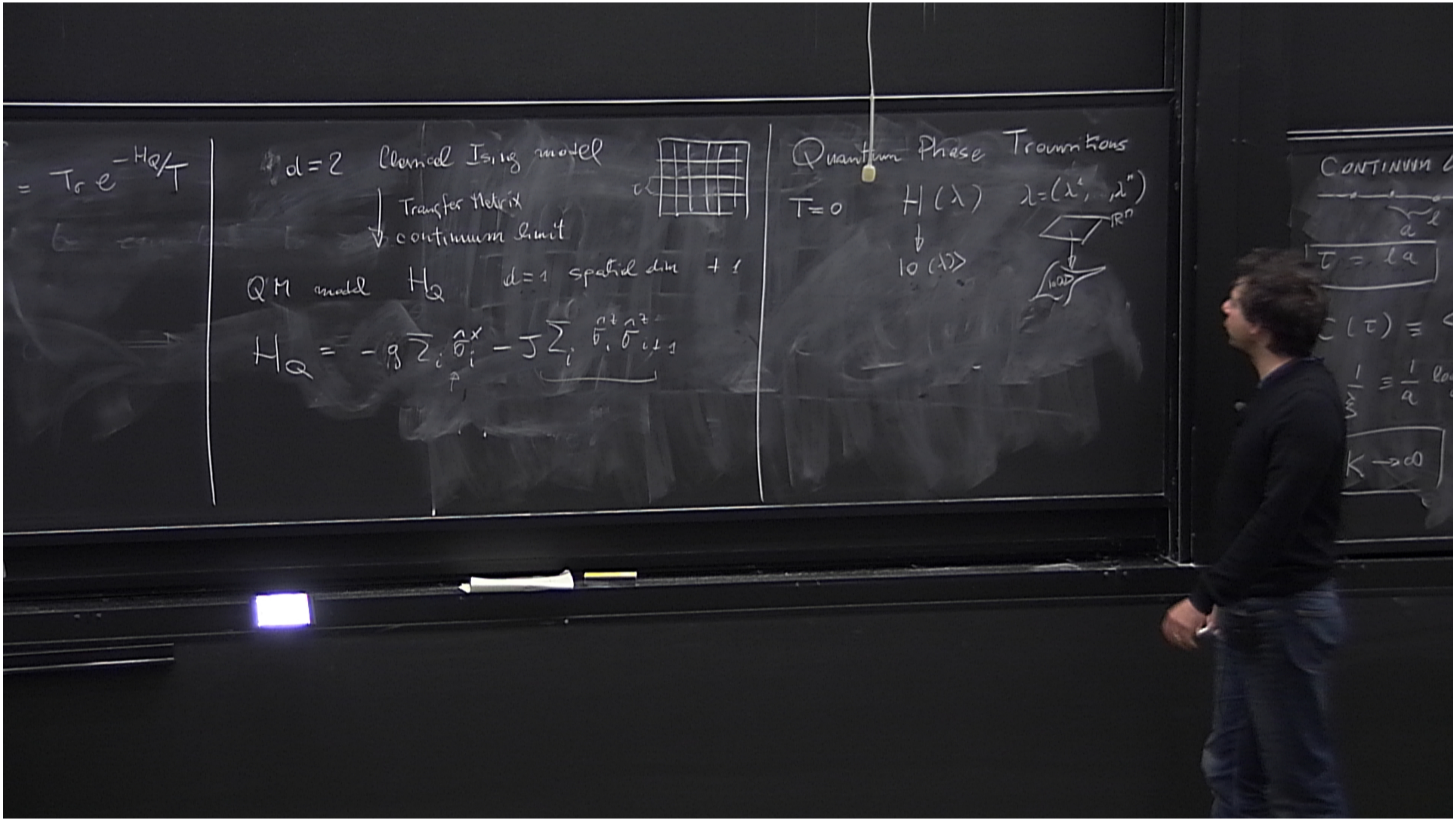


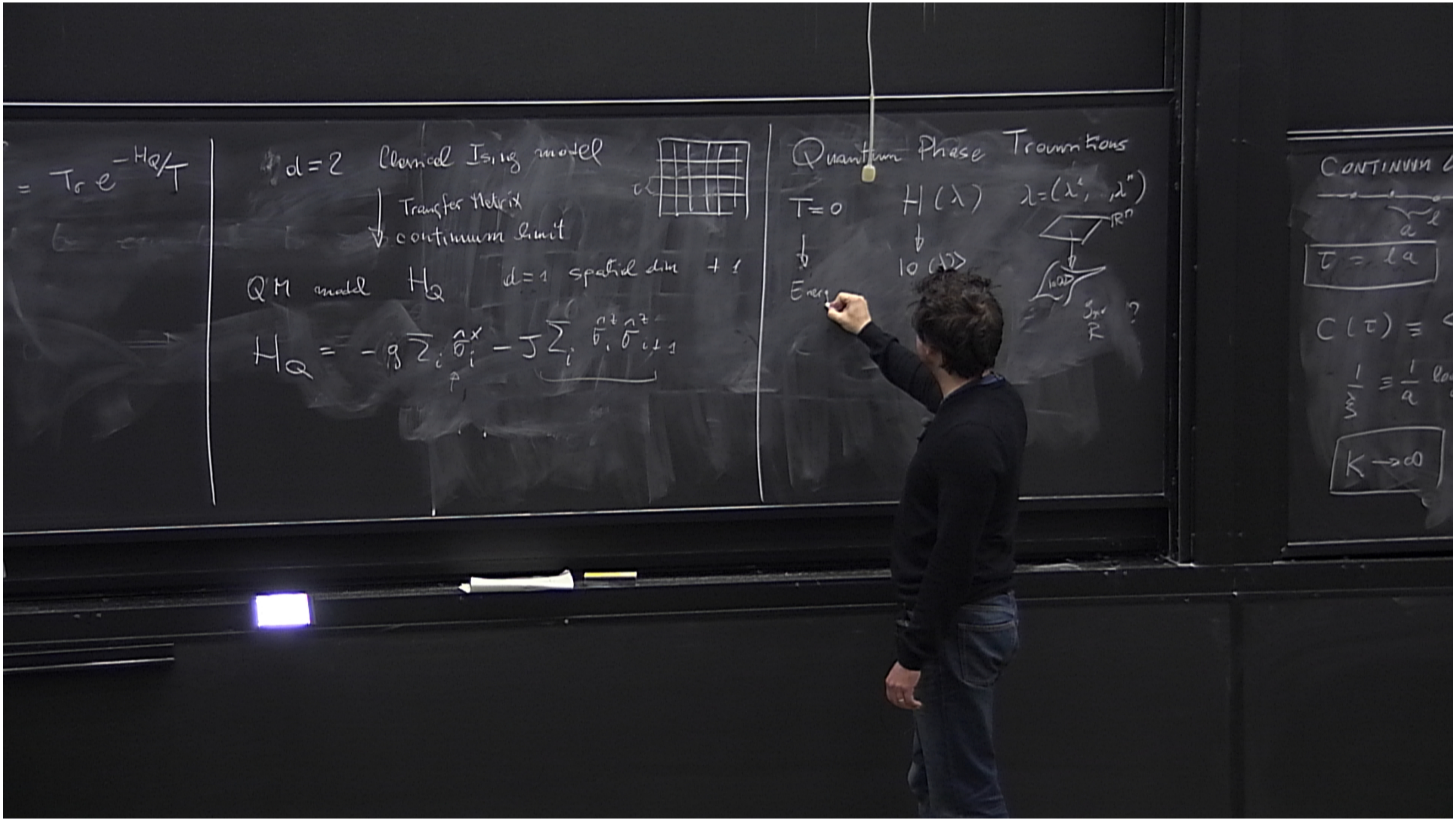
$= \mathrm{Tr}\, e^{-H_Q/T}$
d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model H_Q d = 1 spatial dim + 1
$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$
CONTINUUM
$\tau = \ell a$
$C(\tau) \equiv$
$K \to \infty$

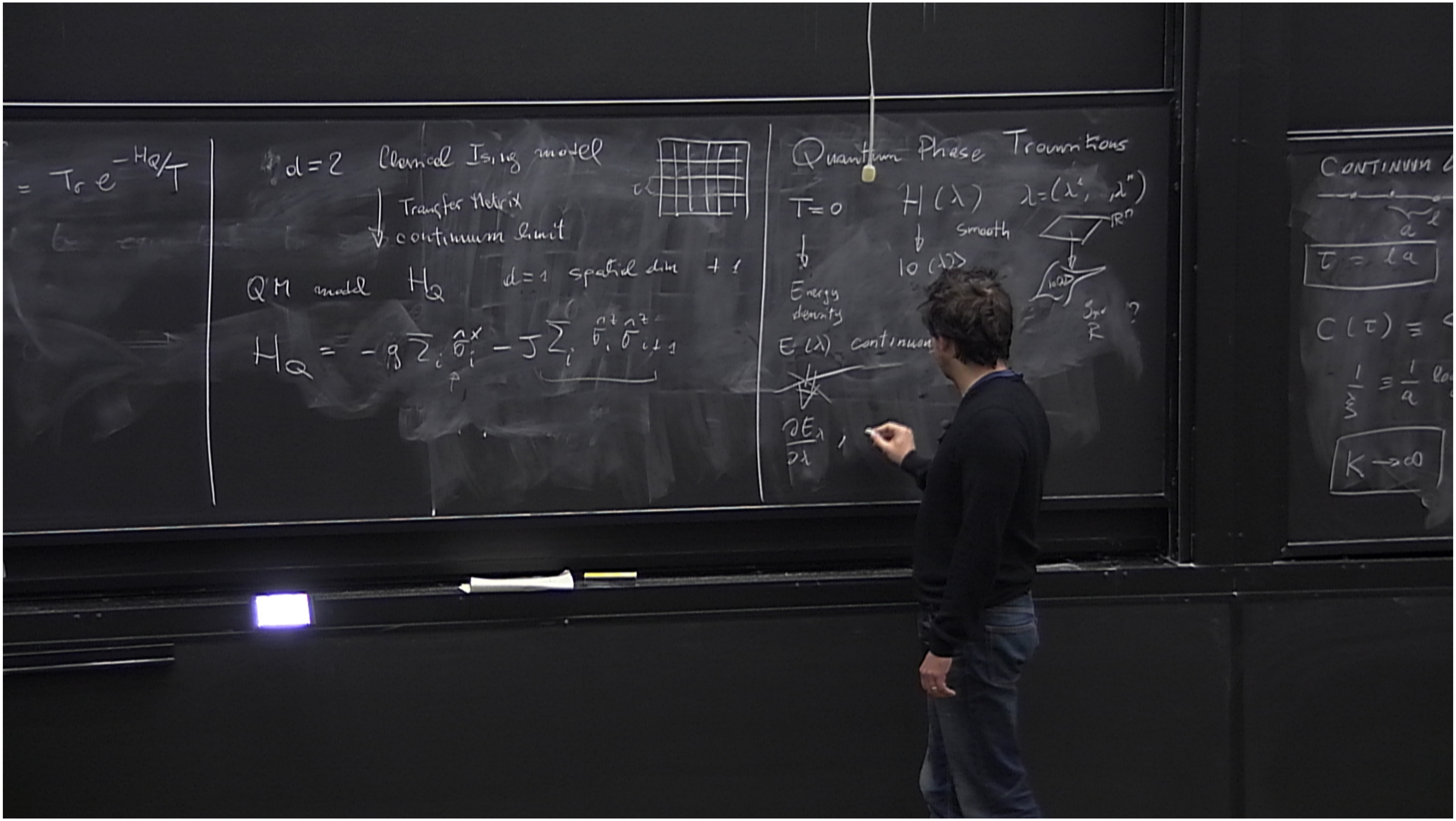


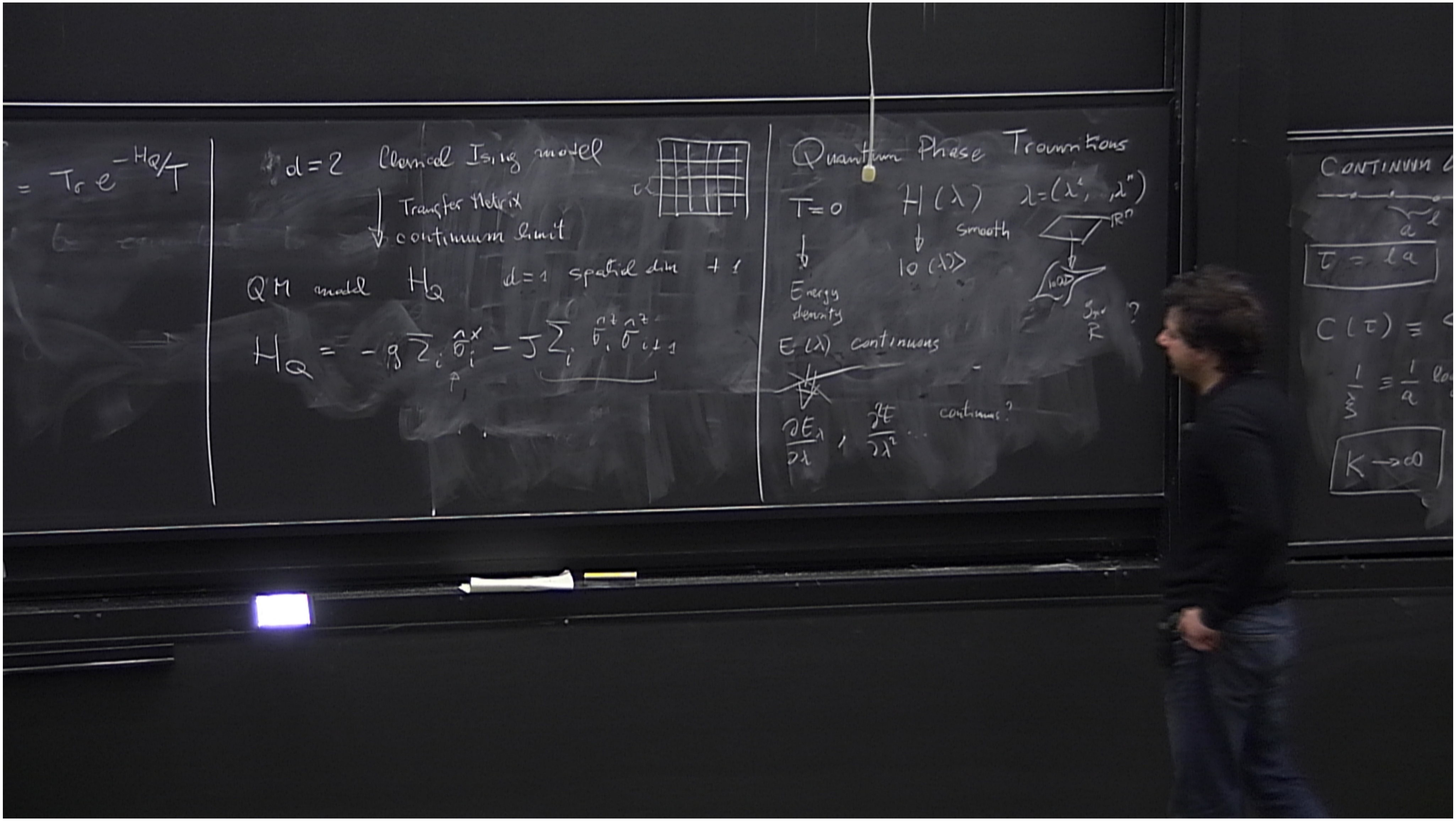
d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model
d = 1 spatial dim + 1
Quantum Phase Transitions
T = 0
CONTINUUM
K → ∞

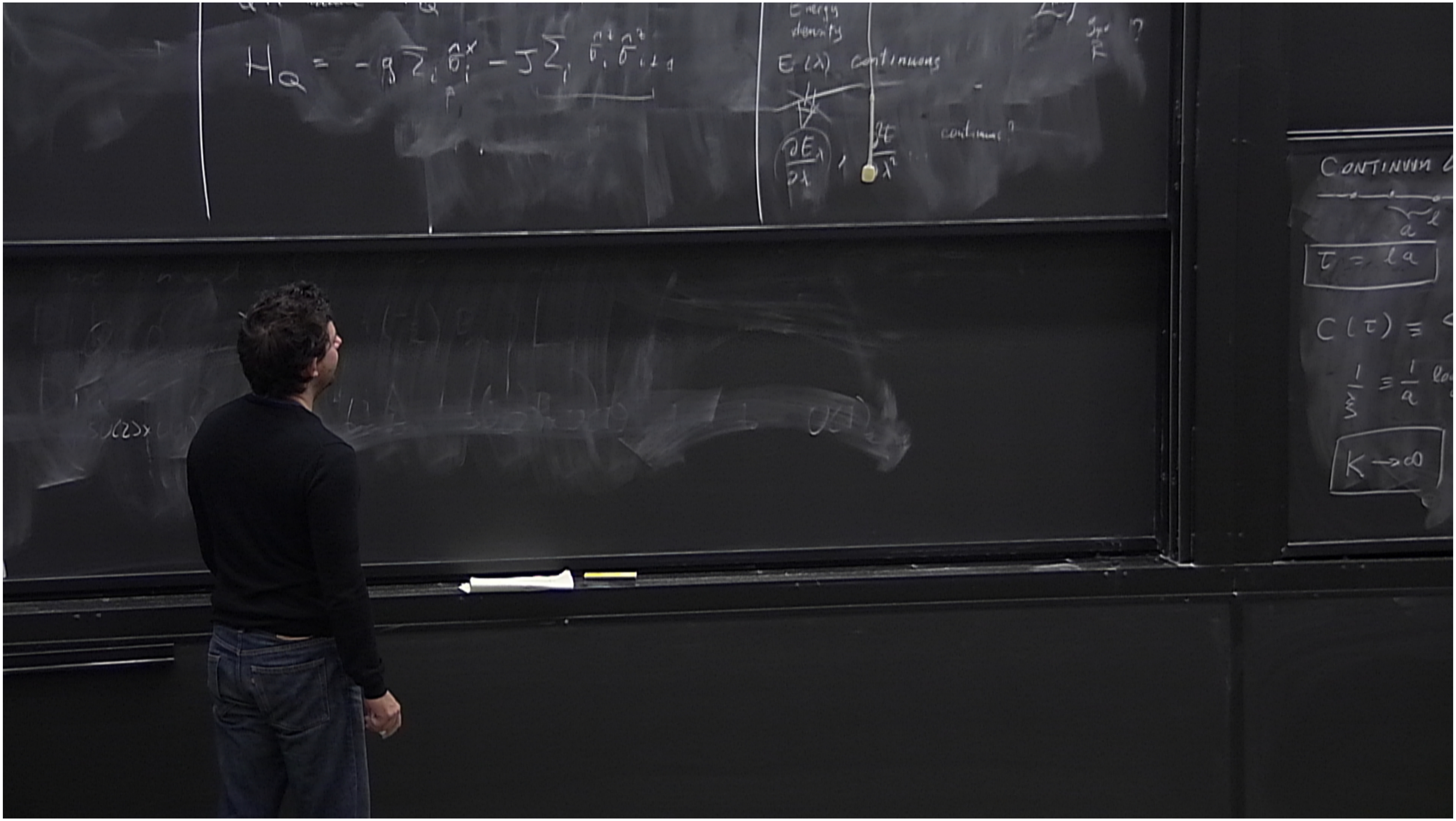

d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model H_Q
d = 1 spatial dim + 1
Quantum Phase Transitions
T = 0
H(λ)
CONTINUUM

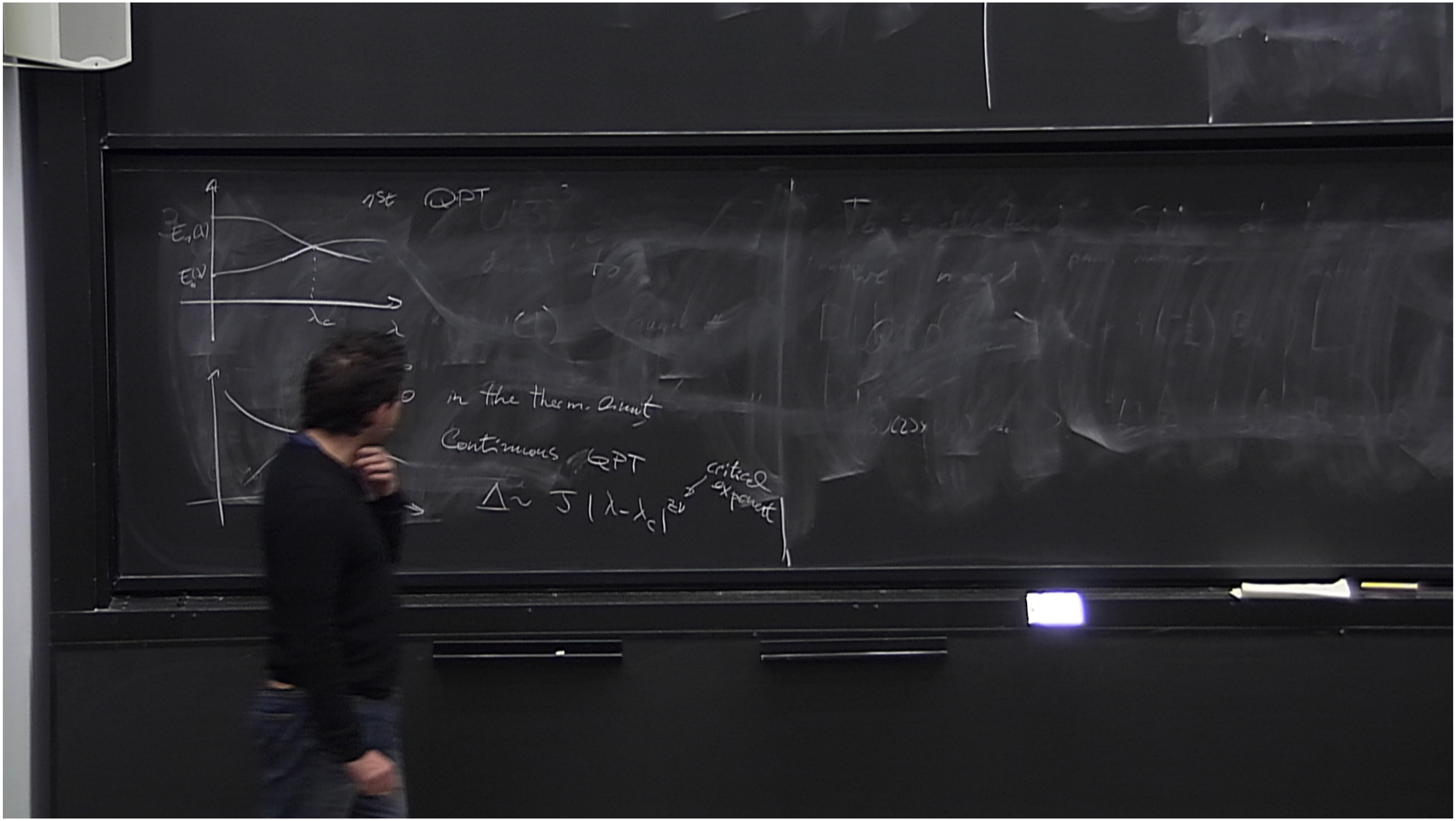

d=2 Classical Ising model
Transfer Matrix
continuum limit
QM model
d=1 spatial dim + 1
Quantum Phase Transitions
T=0
CONTINUUM

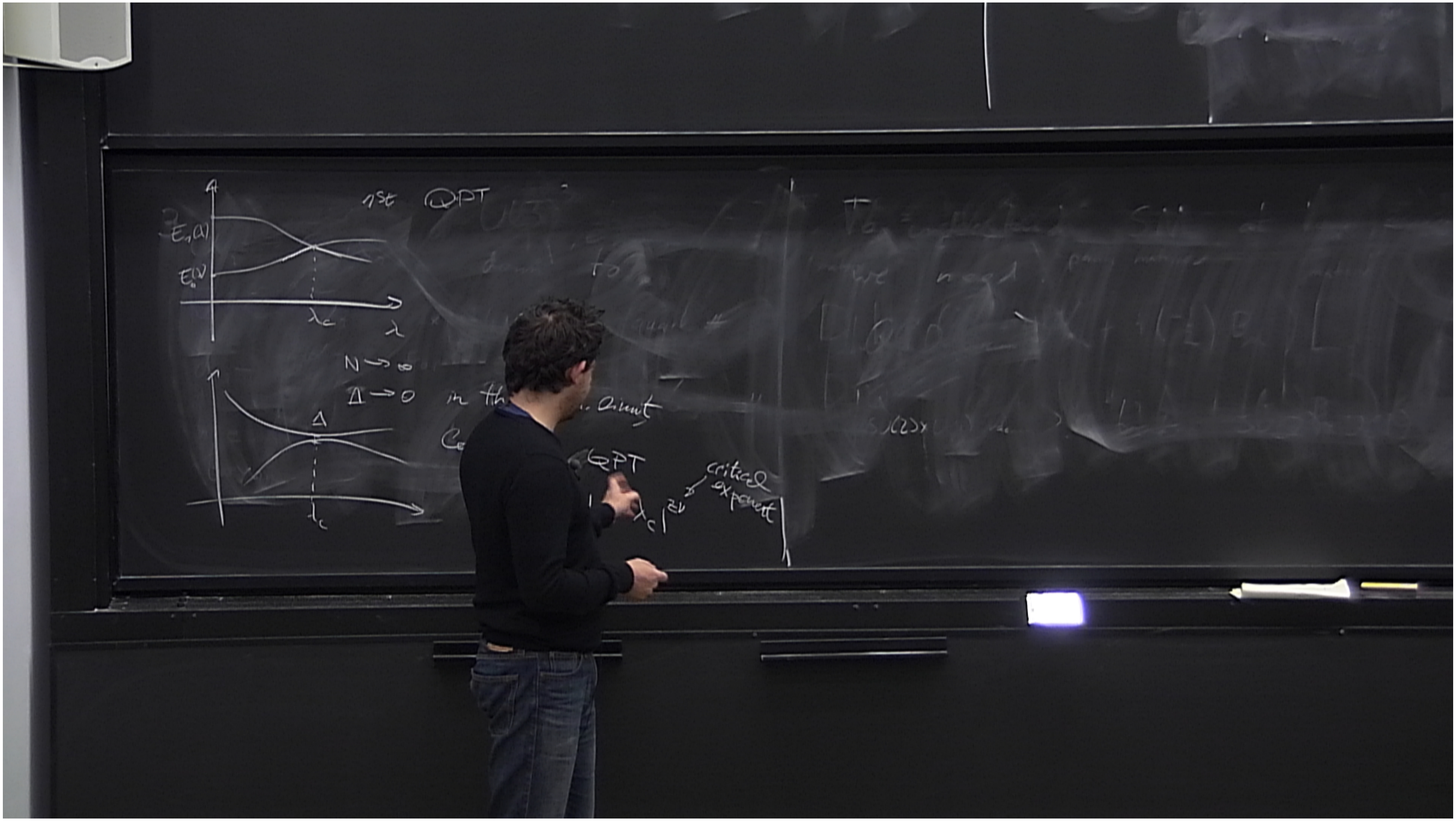

Classical Ising model
d = 2
Transfer Matrix
continuum limit
QM model
d = 1 spatial dim + 1
Quantum Phase Transitions
T = 0
Energy
CONTINUUM
K → ∞

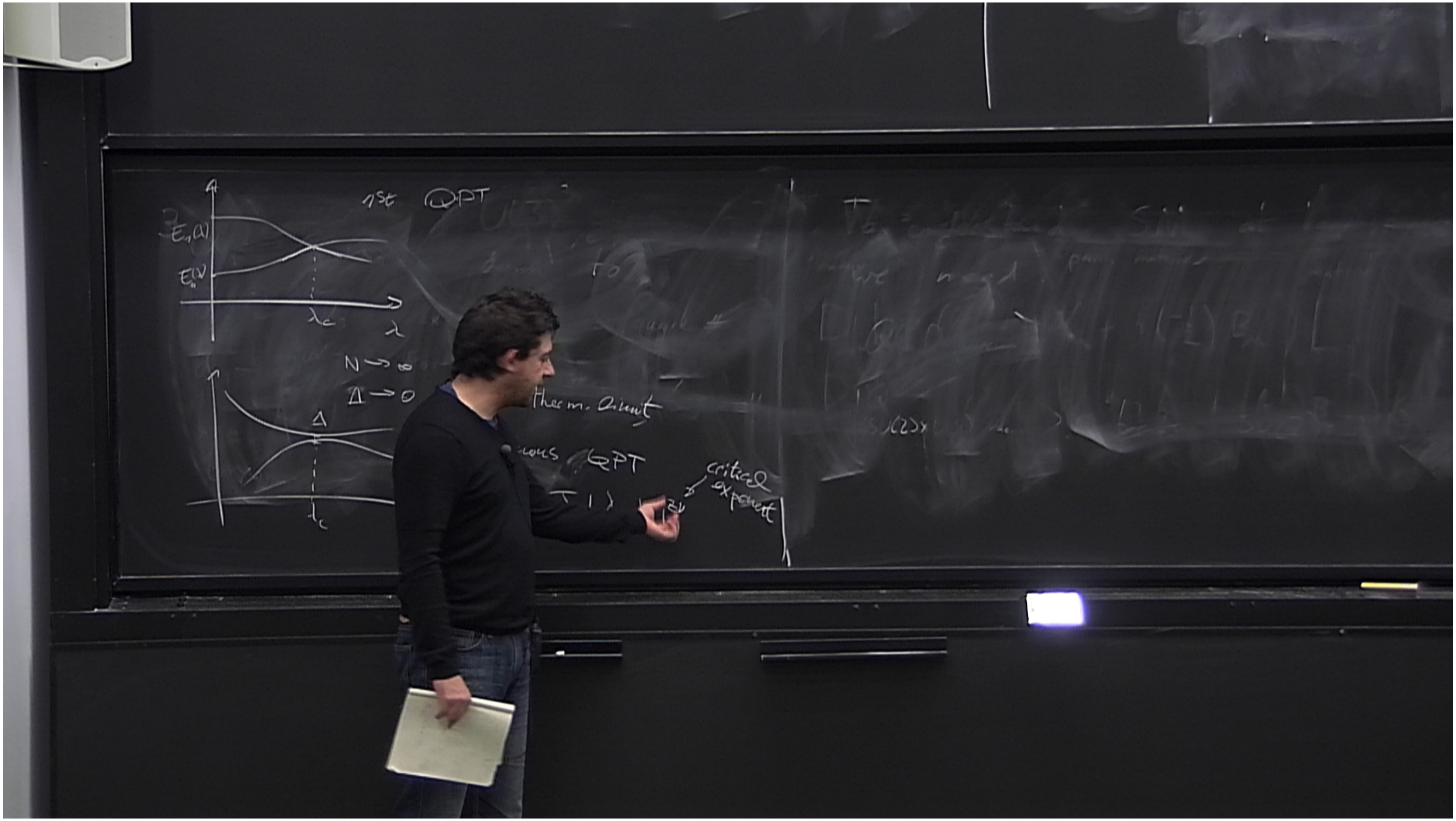

d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model
d = 1 spatial dim + 1
Quantum Phase Transitions
T = 0
smooth
Energy density
continuum
CONTINUUM
K → ∞


d = 2 Classical Ising model
Transfer Matrix
continuum limit
QM model H_Q d = 1 spatial dim + 1
$H_Q = -g \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$
Quantum Phase Transitions
T = 0
$H(\lambda)$ $\lambda = (\lambda^1, \ldots \lambda^n)$
smooth
$|0(\lambda)\rangle$
Energy density
$E(\lambda)$ continuous
$\frac{\partial E_\lambda}{\partial \lambda}$, $\frac{\partial^2 E}{\partial \lambda^2}$ … continuous?
CONTINUUM
$\tau = la$
$C(\tau) \equiv$
$K \to \infty$

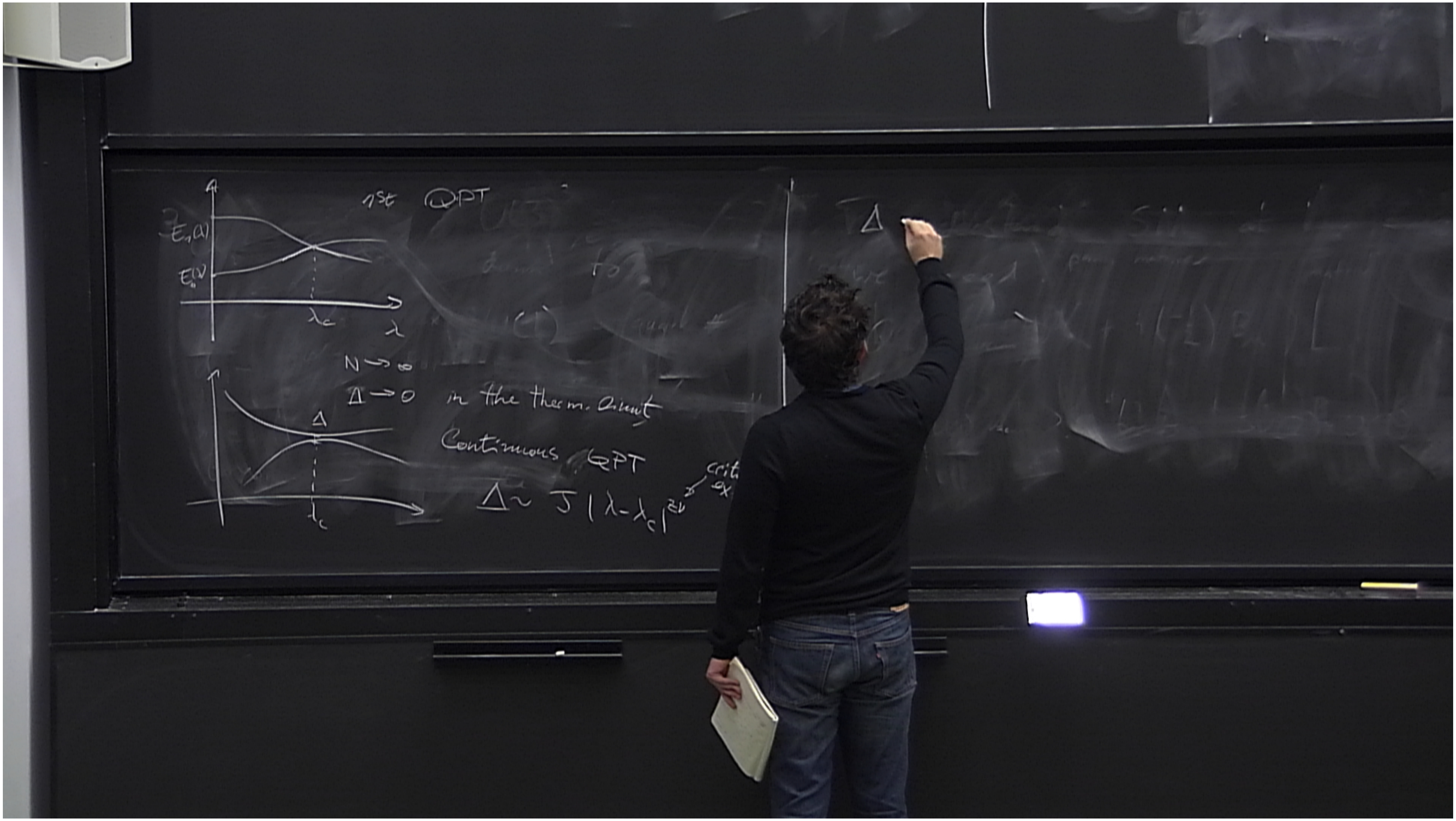

Energy
density
continuous
continuous?
CONTINUUM
τ = la
K → ∞

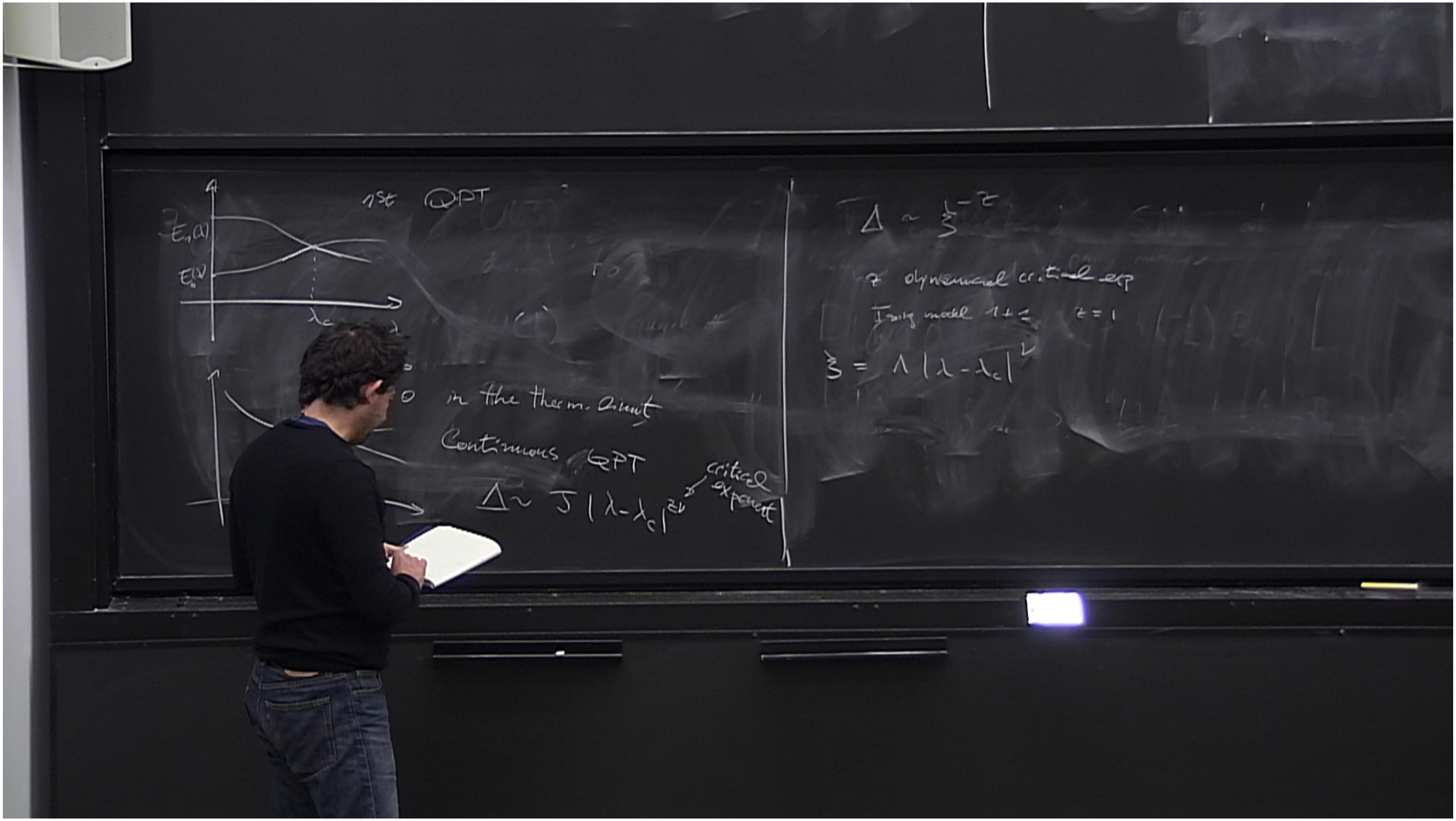

1st QPT
in the therm. limit
Continuous QPT
Δ ~ J | λ - λc |zν
critical exponent

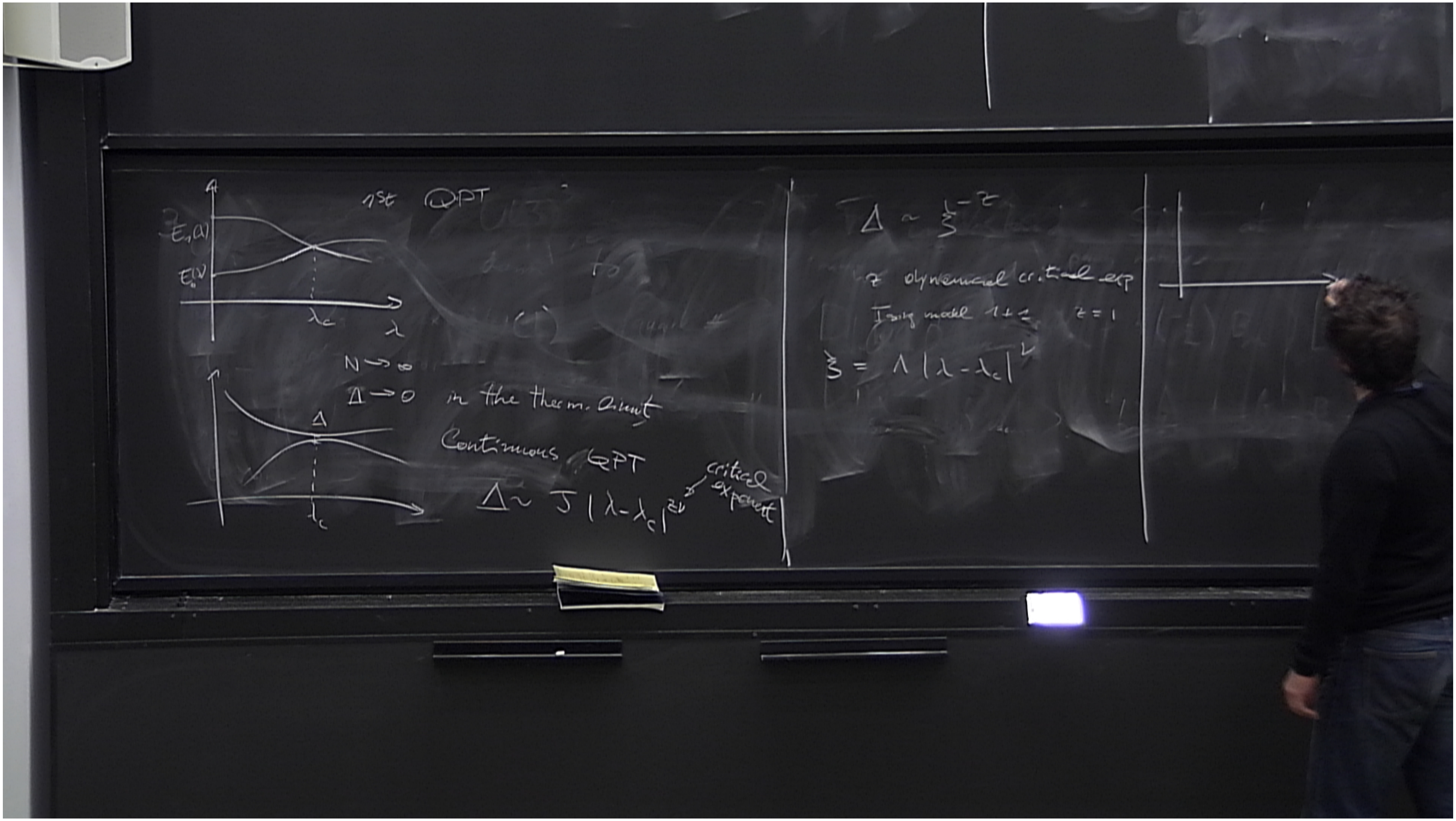

1st QPT
$E_1(\lambda)$
$E_0(\lambda)$
λ_c
λ
$N \to \infty$
$\Delta \to 0$ in the ... limit
Δ
QPT
critical exponent

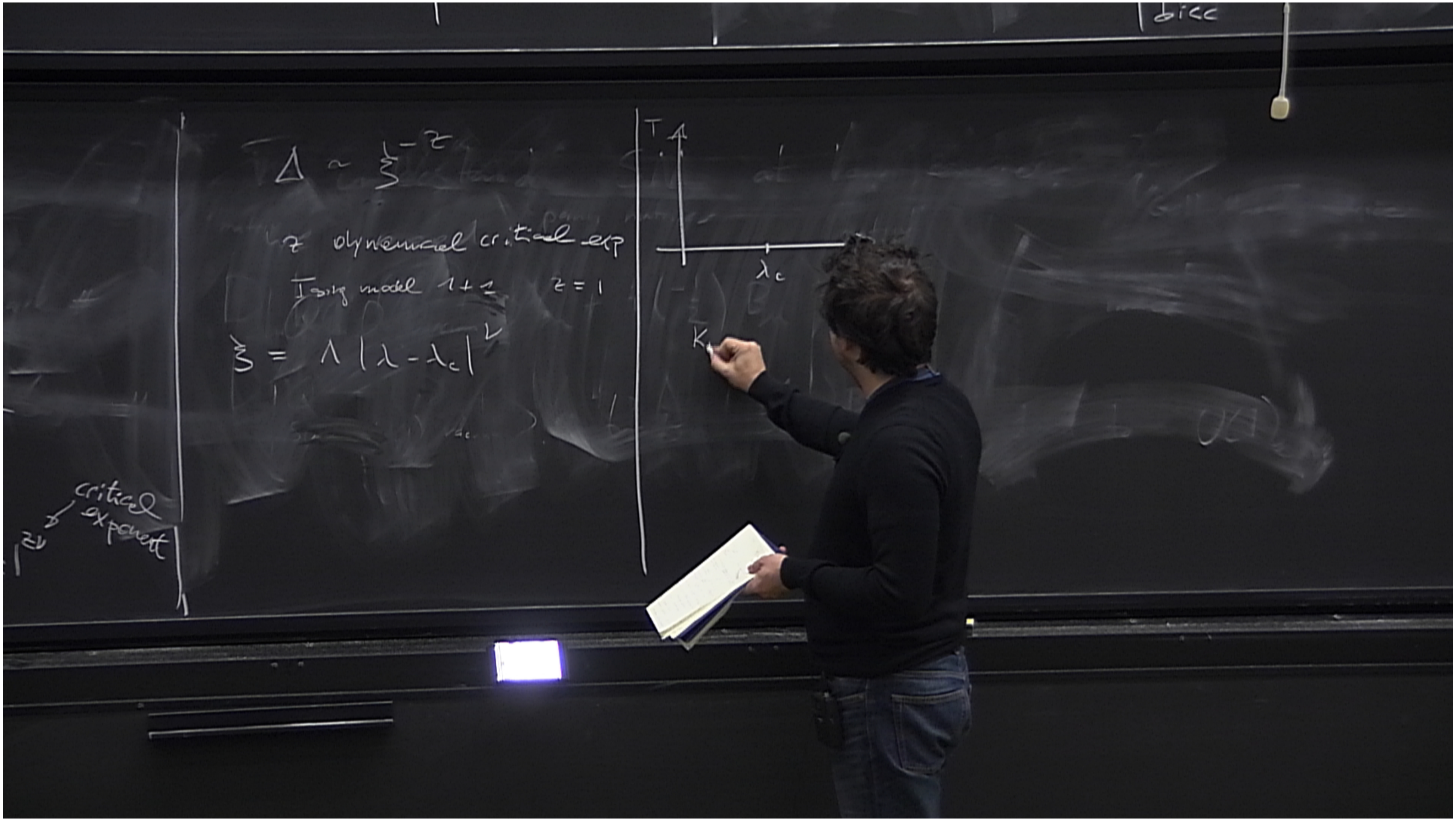


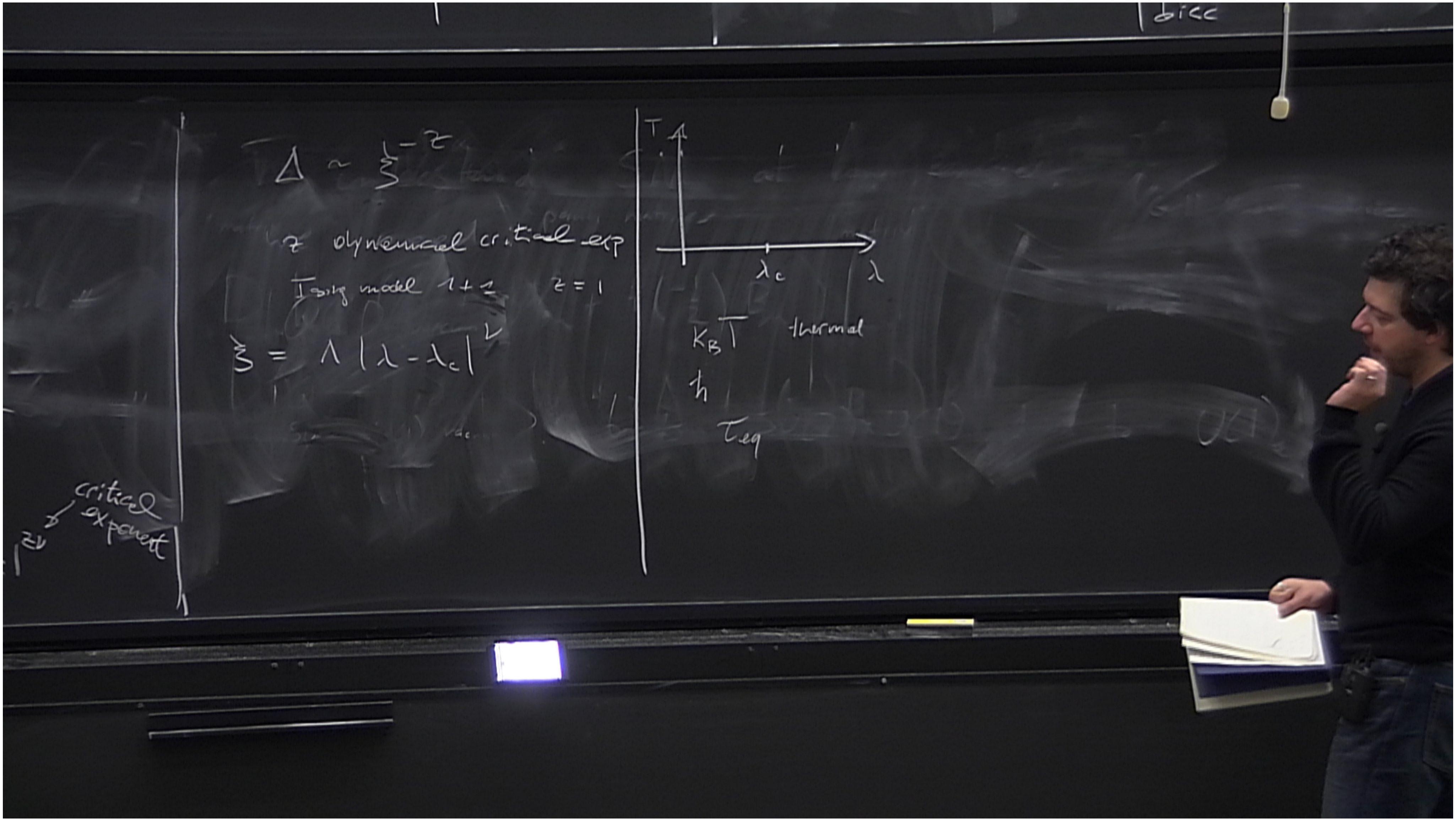
1st QPT
N → ∞
Δ → 0
therm. limit
QPT
critical exponent


1st QPT
N → ∞
Δ → 0 in the therm. limit
Continuous QPT
Δ ~ J | λ - λc |zν

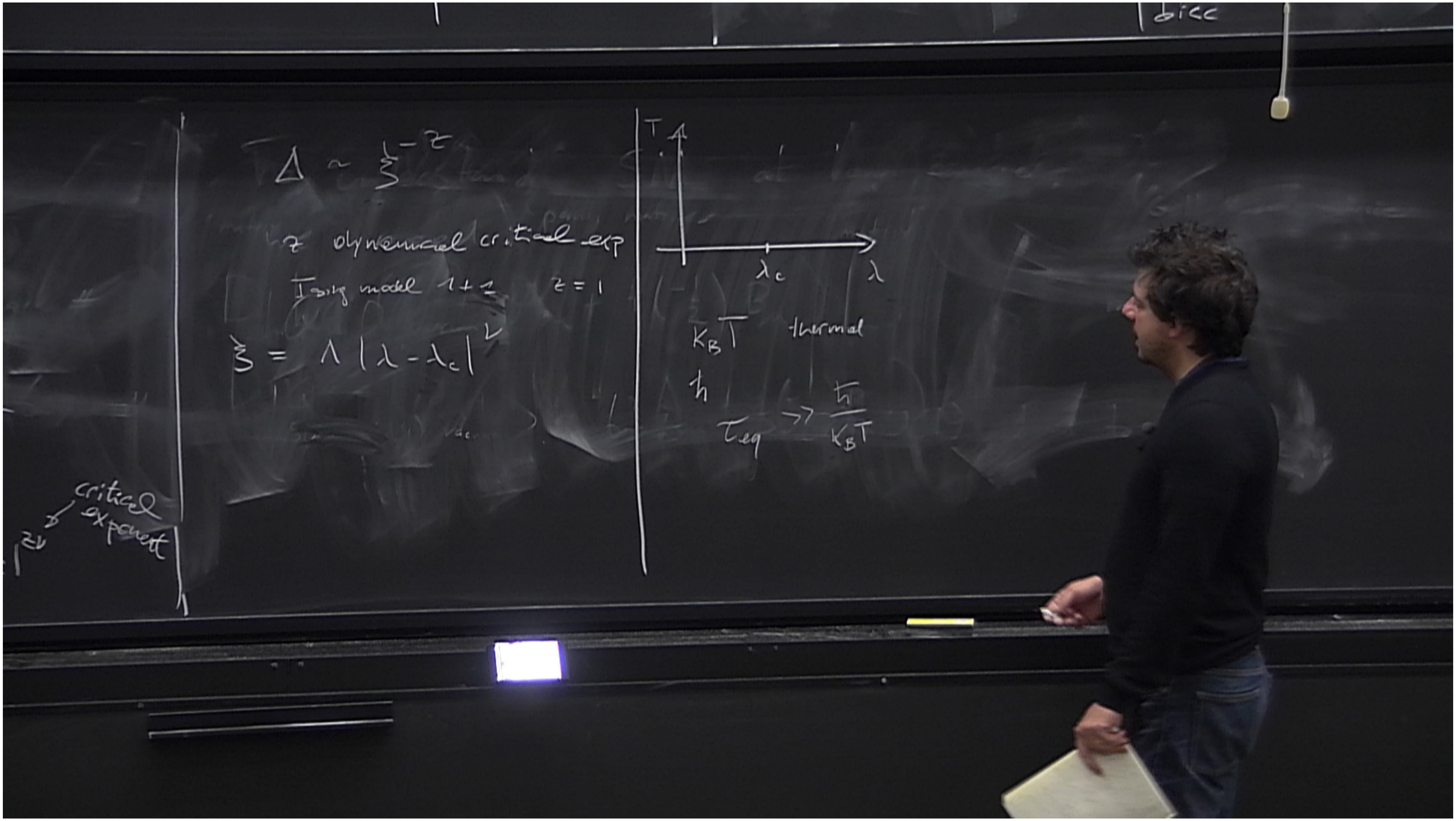

1st QPT
in the therm. limit
Continuous QPT
critical exponent
dynamical critical exp
Ising model 1+1
z = 1

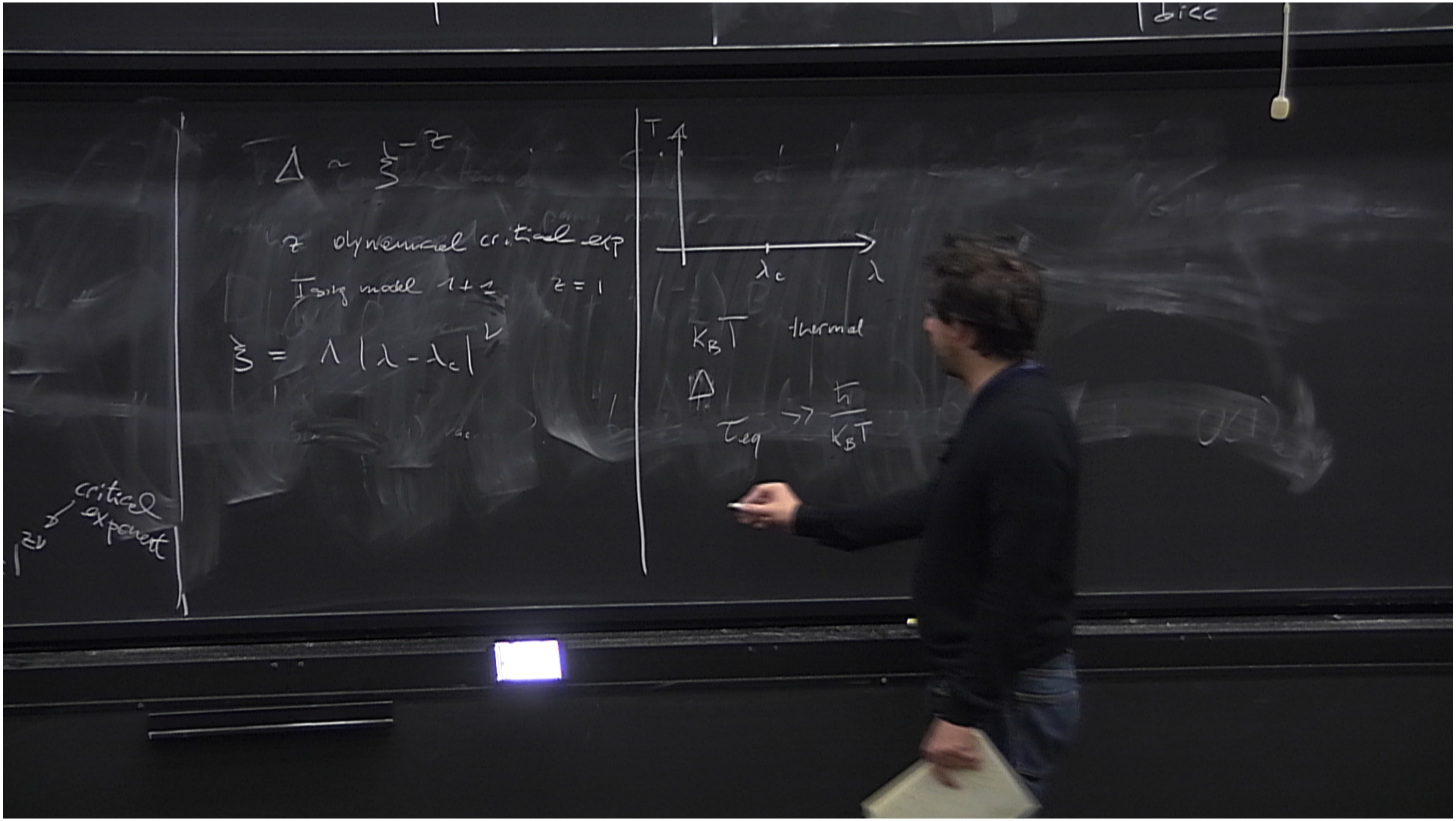

1st QPT
$E_1(\lambda)$
$E_0(\lambda)$
λ_c
λ
$N \to \infty$
$\Delta \to 0$ in the therm. limit
Δ
λ_c
Continuous QPT
$\Delta \sim J |\lambda - \lambda_c|^{z\nu}$
critical exponent
$\Delta \sim \xi^{-z}$
z dynamical critical exp
Ising model 1+1 z = 1
$\xi = \Lambda |\lambda - \lambda_c|^{\nu}$

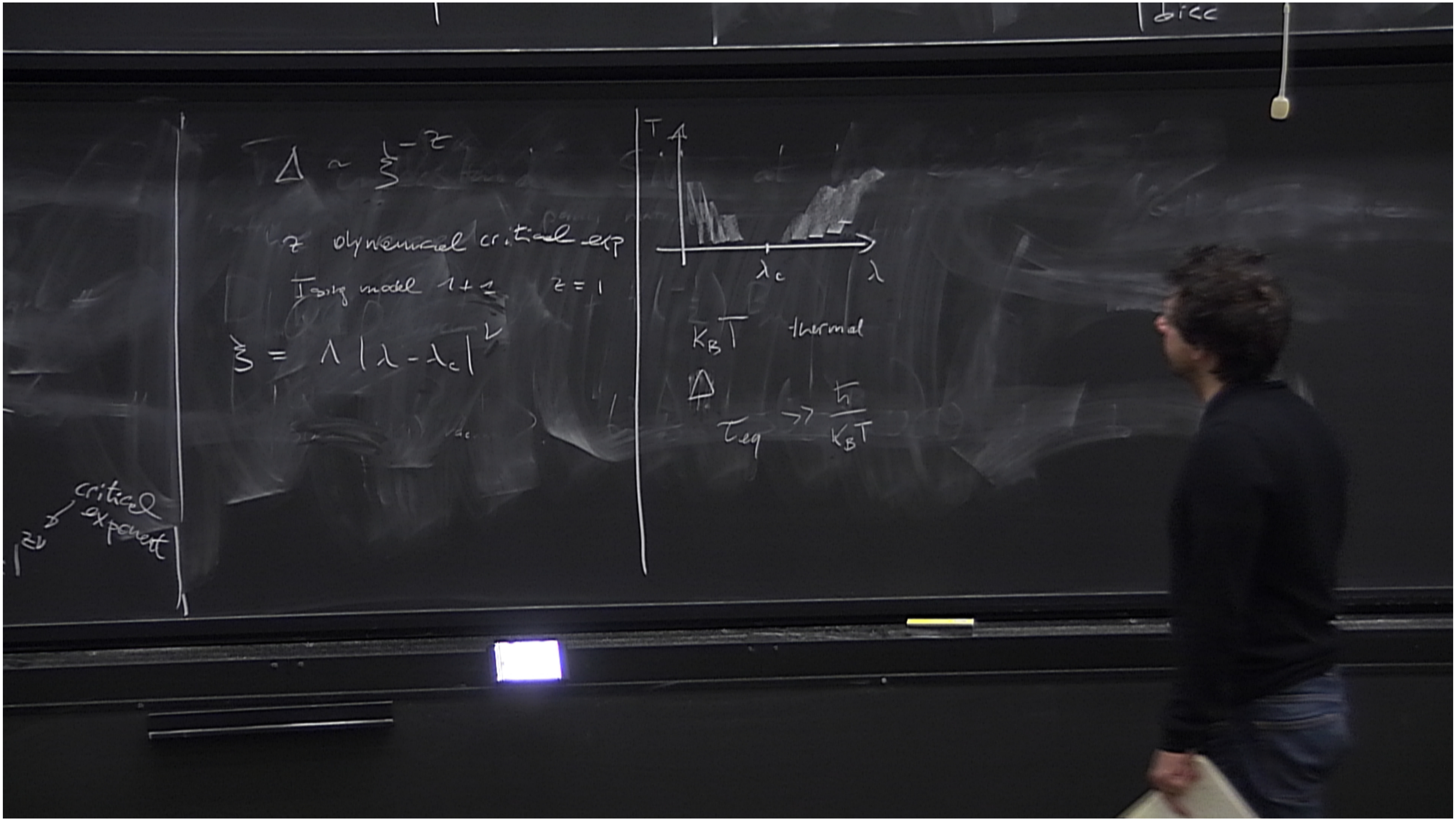

Δ ~ ξ^-z
z dynamical critical exp
Ising model 1+1   z = 1
ξ = Λ |λ - λc|^-ν
critical exponent
T
λc

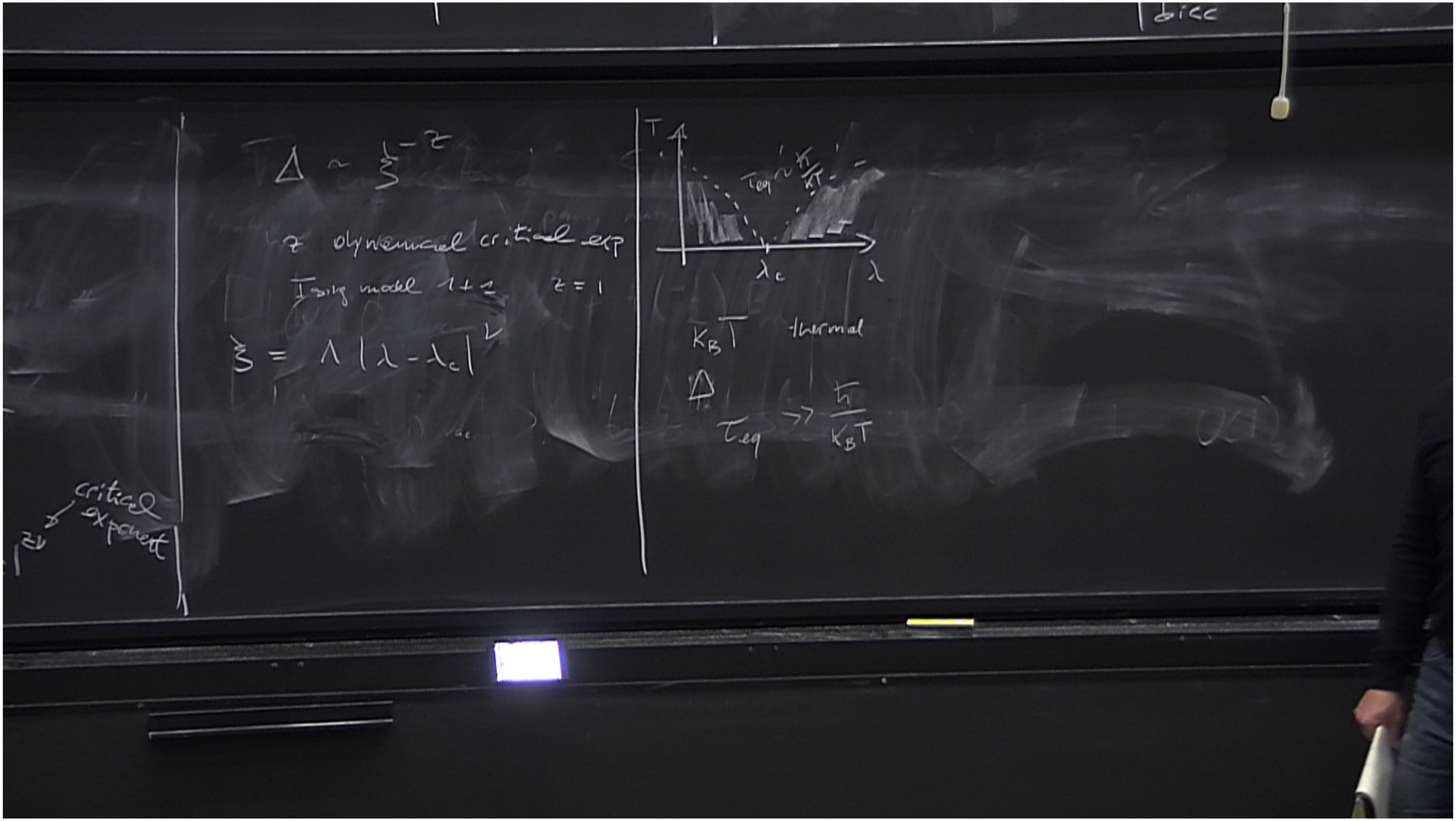


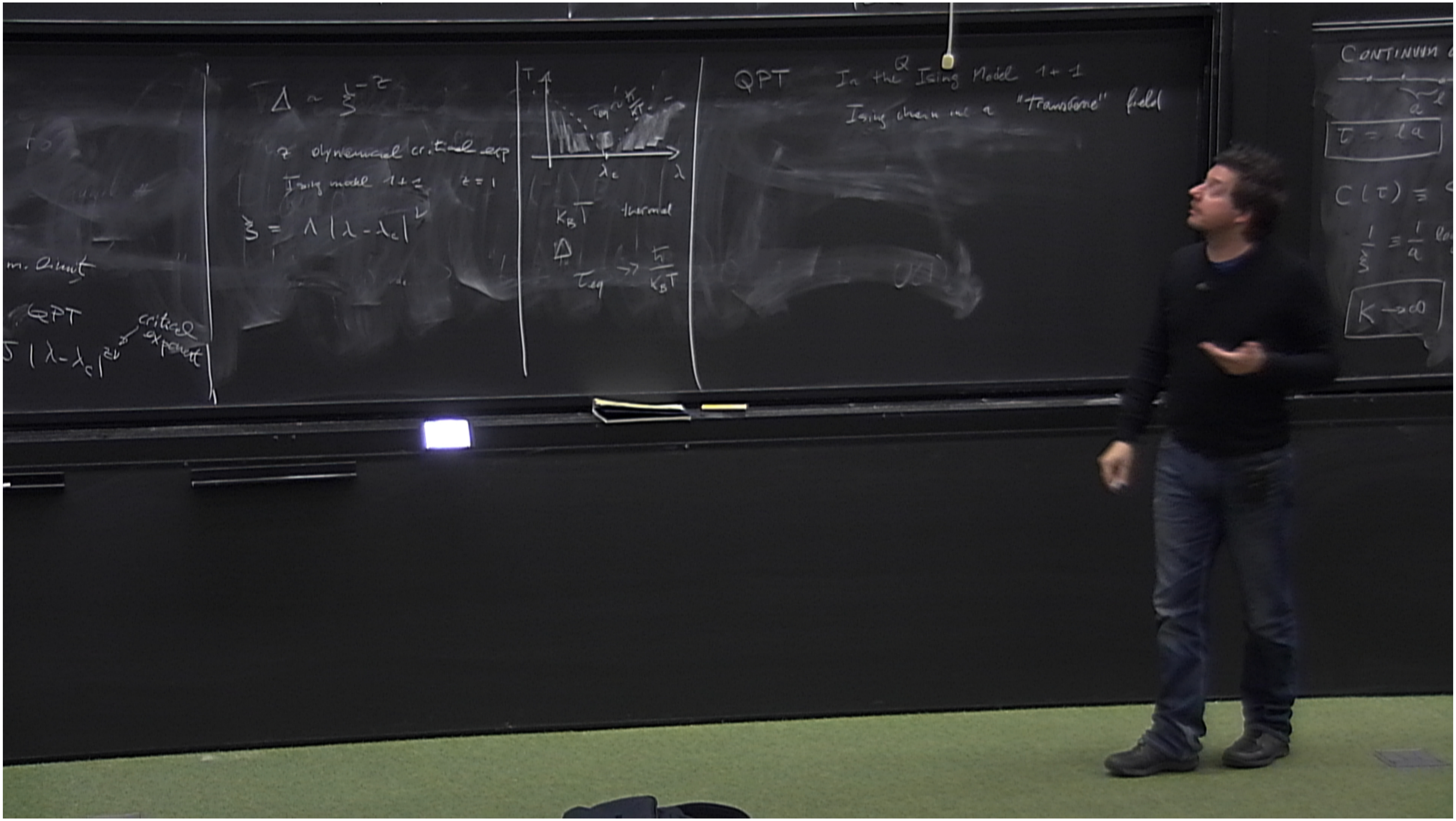
Δ ~ ξ^{-z}
z dynamical critical exp
Ising model 1+1   z = 1
ξ = Λ |λ - λ_c|^{-ν}
critical exponent
zν
T
λ_c
λ
k_B T   thermal
ħ
τ_eq

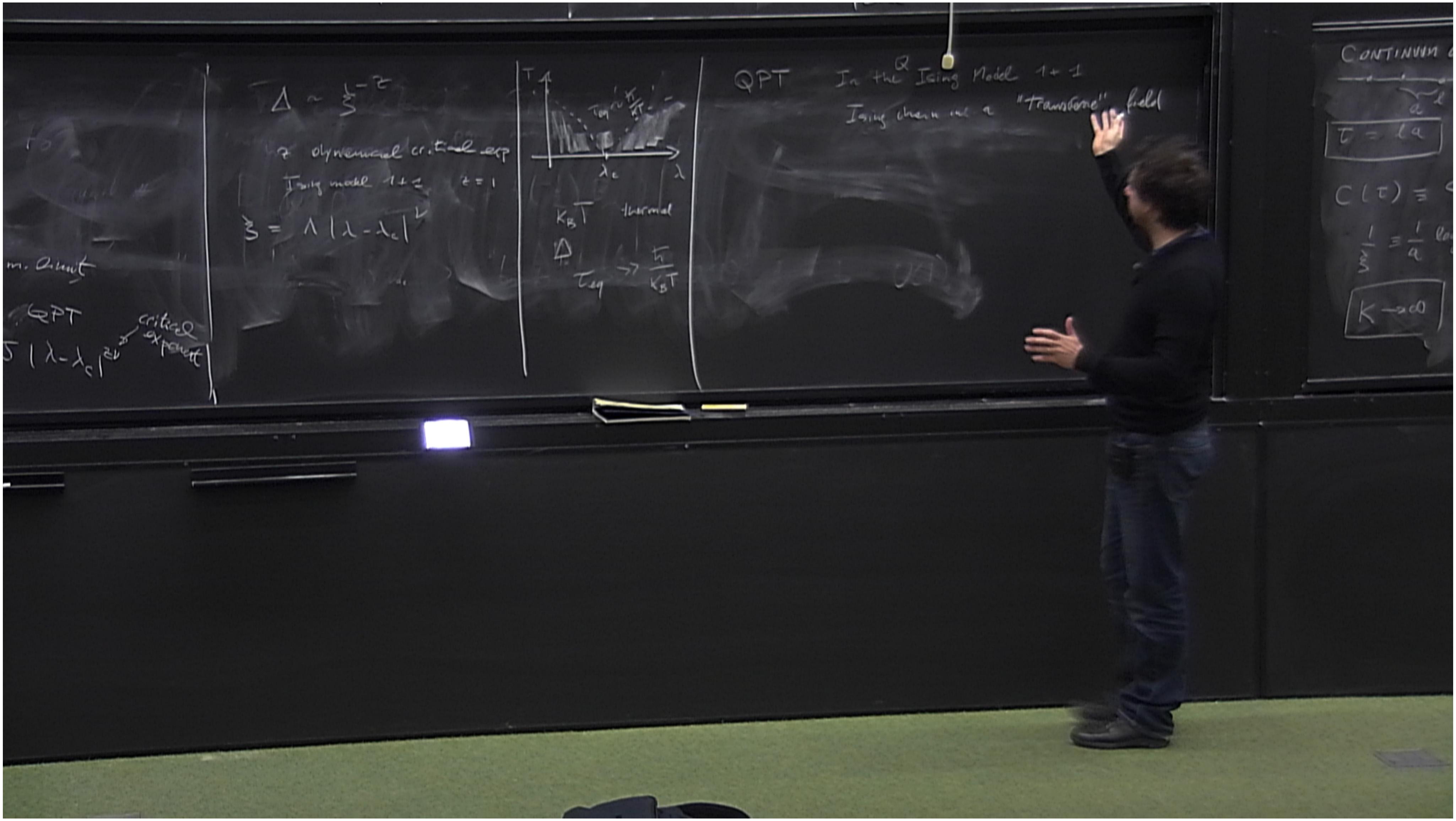

$\Delta \sim \xi^{-z}$
z dynamical critical exp
Ising model 1+1 z = 1
$\xi = \Lambda |\lambda - \lambda_c|^{\nu}$
critical exponent
$z\nu$
T
λ_c
λ
$k_B T$ thermal
$\hbar$
$\tau_{eq} \gg \frac{\hbar}{k_B T}$

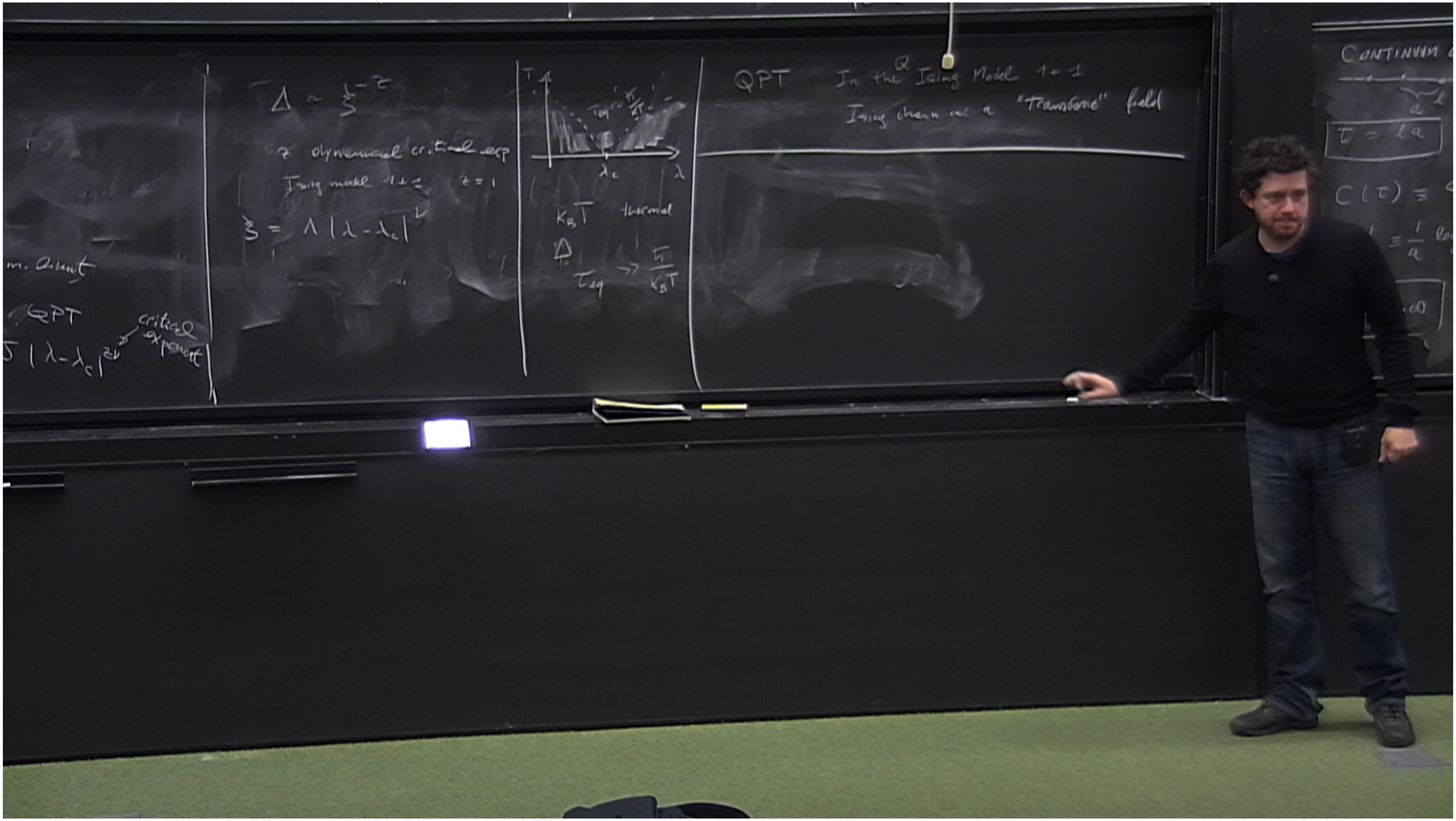

Δ ~ ξ^{-z}
z dynamical critical exp
Ising model 1+1 z = 1
ξ = Λ |λ - λ_c|^ν
critical exponent
zν
T
λ_c
λ
k_B T thermal
Δ
τ_eq >> ħ / k_B T

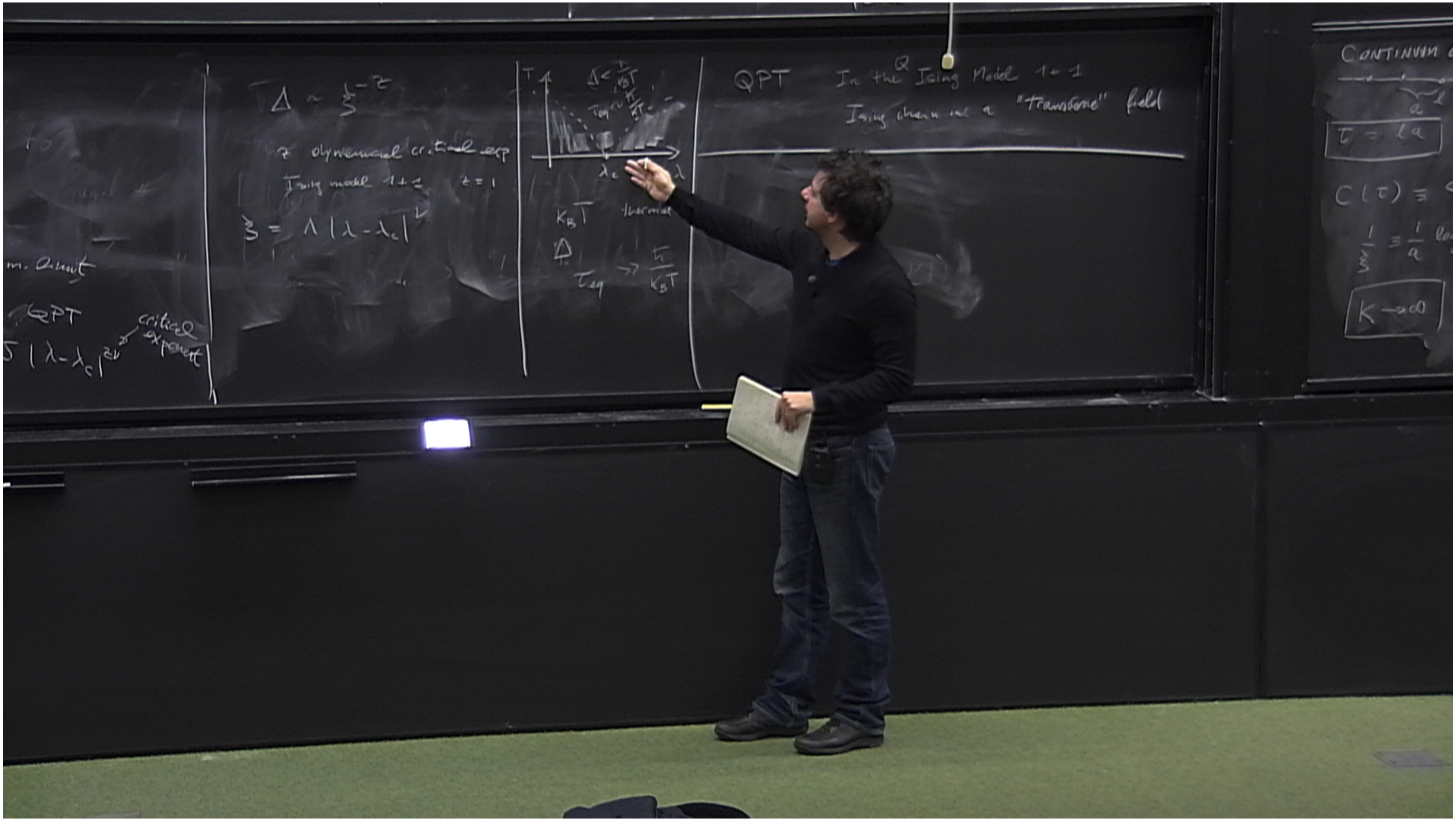

Δ ~ ξ^{-z}
z dynamical critical exp
Ising model 1+1 z = 1
ξ = Λ |λ - λ_c|^{-ν}
critical exponent
zν
T
λ_c
λ
k_B T thermal
Δ
τ_eq >> ħ / k_B T

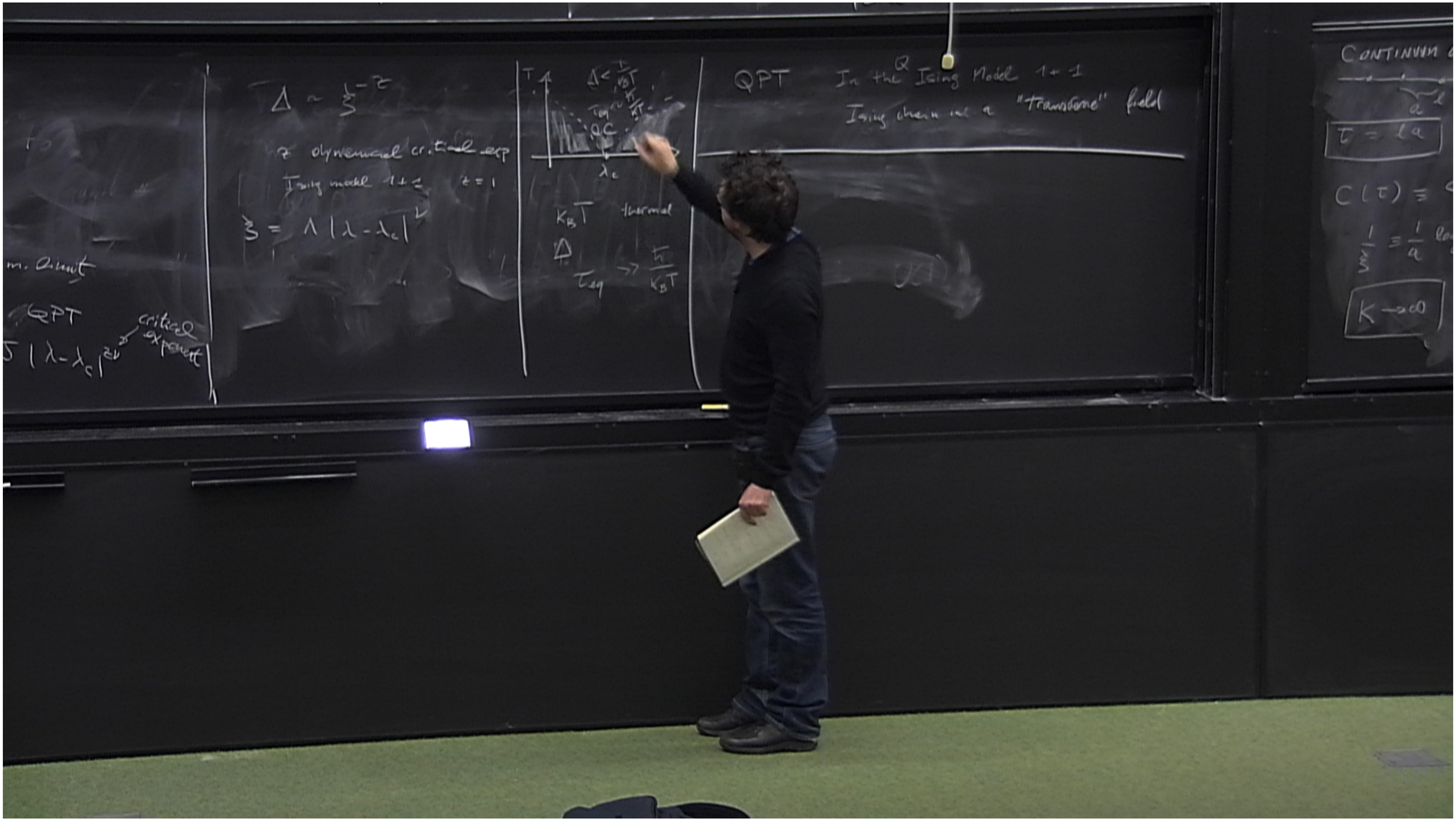

Δ ~ ξ^-z
z dynamical critical exp
Ising model 1+1 z = 1
ξ = Λ |λ - λc|^-ν
critical exponent
T
λc
λ
τeq ~ ħ/kT
KB T thermal
Δ
τeq >> ħ/KBT


QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
Ising model 1+1
thermal
QPT
critical exponent
CONTINUUM


QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
critical exponent
thermal
CONTINUUM

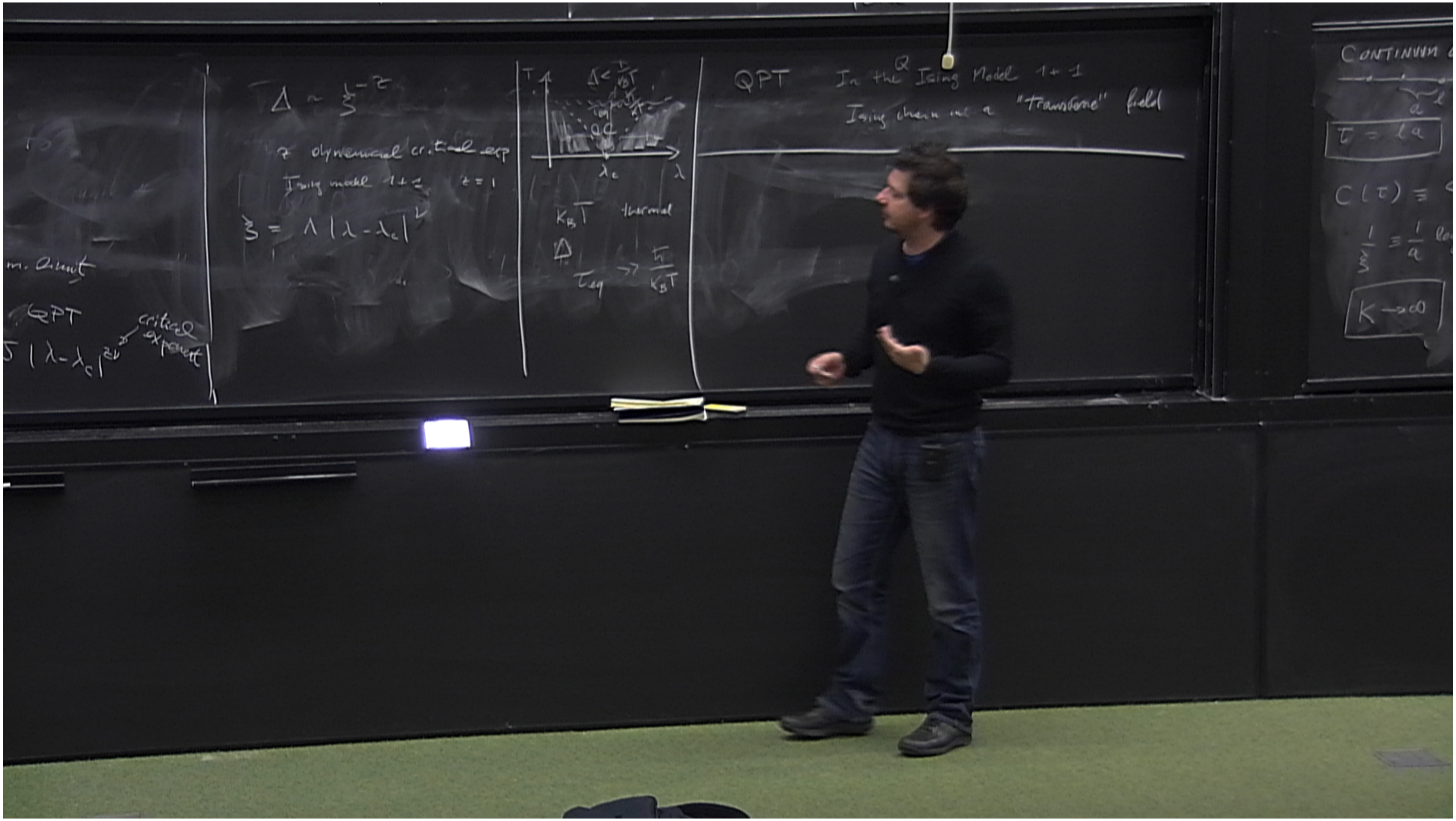

QPT In the Q Ising Model 1+1
Ising chain in a "Transverse" field
thermal
critical exponent
CONTINUUM

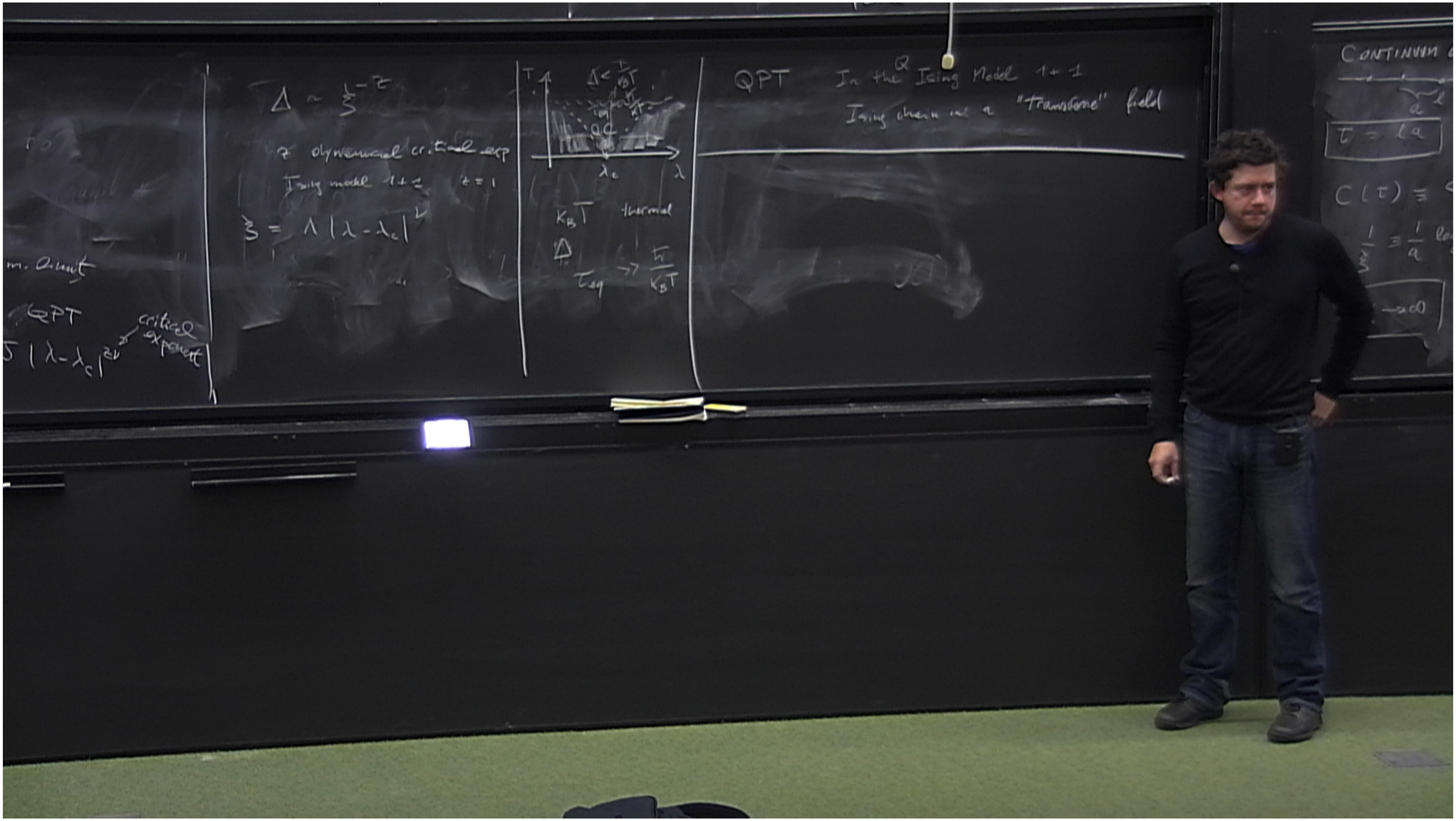

QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
Continuum




QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
thermal
critical exponent
CONTINUUM


QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
CONTINUUM
$\tau = \ell a$
$K \to \infty$
critical exponent
thermal




QPT
In the Q Ising Model 1+1
Ising chain in a "Transverse" field
thermal
critical exponent
CONTINUUM